Project description for call 09I03-03-V04 Fellowships for excellent researchers R2-R4

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Title of the project: Fuzzy implication functions and their applications

Short title of the project/Acronym: FIFATA

Category of researcher: R2

Researcher’s job type: Full-time

Type of research: Independent

Identification of the entity involved in the implementation of the project:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Official name of the entity | Abbreviated name of the entity | Role in the project |
| 1 | Institute of Mathematics, Slovak Academy of Sciences | SAS | Applicant and host organisation |

## 1. Excellence

### PROJECT OBJECTIVES

* **Objective 1:** To compile, classify and relate the families of fuzzy implication functions introduced in the literature.

Due to the substantial volume of work devoted to the proposal of new classes of fuzzy implication functions and the study of novel additional properties published in the last ten years [1], the existing surveys [2], [3], [4] and monographs [5], [6] devoted to this topic do not reflect the current reality of the research field. For this reason, for the correct development of the rest of the proposed goals, it is necessary to first make a thorough review of the literature. Specifically, as the very first objective we propose to write an exhaustive survey on fuzzy implication functions in which the motivation behind the introduction of each family, their particular interest, their additional properties and their known intersections with other families will be highlighted. This document will not only be useful for the correct development of the project, but it is also intended to serve as a useful and unprecedented resource for any researcher interested in the subject. Furthermore, the completeness of this document will probably require the study of additional properties and/or intersections of some families that have not been previously investigated.

* **Objective 2:** To search for characterizations and intersections of families of fuzzy implication functions and to solve some related problems.

Although a large number of families of fuzzy implication functions have been introduced using various methods, these families can present intersection or even coincide [7]. For this reason, it is of great significance to study the additional properties that the operators of a certain family satisfy and to provide an axiomatic characterization of the new operators in the literature in order to find its possible relation with respect to those already known. As part of our project, we will focus on the characterization of families of fuzzy implication functions that have not been characterized yet and we will study the intersection between different families. Often the process to achieve these characterization results leads to the study of some related problems. In this way, we intend to provide new solutions to the completion of continuous pre-t-norms known in the regions related to the characterization of -implications.

* **Objective 3:** To find solutions or/and characterize fuzzy implication functions satisfying a certain additional property, focusing on those related to practical applications.

Since the definition of a fuzzy implication function is quite general, additional properties of these operators are usually considered. These properties come often in the form of functional equations which involve fuzzy implication functions and some of them, other operators as well. Although some of these additional properties have been studied only from a theoretical point of view, many of these additional properties play a major role for a particular application. The third objective is devoted to the study of some additional properties which are strictly related to practical applications. In particular, we will study the T-power invariance with respect to the powers of a non-Archimedean continuous t-norm [7], [8] , which may prove useful in approximate reasoning, and the monotonicity of the generalized modus ponens [9], which derives from the application of fuzzy implication functions in subgroup discovery.

* **Objective 4:** To propose new application areas for fuzzy implication functions within knowledge discovery.

Fuzzy implication functions have already been considered for various applications such as fuzzy control [10], [11], approximate reasoning [6] or image processing [12]. Nonetheless, most of the studies devoted to these operators are from a theoretical point of view. Therefore, it is of the utmost importance to search for new application domains in which fuzzy implication functions play a major role. In this objective we will delve on the potential of fuzzy implication functions in unexplored application areas. Specifically, we will design and implement algorithms of some data mining techniques within knowledge discovery like subgroup discovery, detection of exception rules or causal discovery which will be based on fuzzy implication functions. For each application, we will study the adequacy of the operators used and we will investigate which additional properties are required. Further, we will compare our approach with other perspectives to contextualize and point out the novelty and the strengths of our proposal.

* **Objective 5:** Applications in real-life problems.

In this last objective we will focus on the application of the techniques designed in Objective 4 to specific real-life problems. To obtain the required data and to validate the experiments we plan to collaborate with experts in the specific fields. Specifically, it is foreseen to maintain contact with Katarzyna Kaczmarek-Majer from the Systems Research Institute of the Polish Academy of Sciences and Magdalena Skorupska from the University of Warsaw for the analysis of data regarding Polish elections and with collaborators from the Health Research Institute of the Balearic Islands (IdISBa) for the analysis of medical data.

1.2 RELEVANCE, QUALITY AND NOVELTY OF THE PROJECT

Fuzzy logic was first introduced by L. A. Zadeh in the 1960s as a more adequate formalism to handle the imprecision of human reasoning [13]. In the narrow sense, fuzzy logic is a multi-valued logic in which truth degrees lie within the real interval , where 0 indicates “absolute falsity” and 1 indicates “absolute truth”. However, in the wider sense fuzzy logic is almost synonymous with the theory of fuzzy sets. Although fuzzy logic can be systematically studied as a multi-valued mathematical logic [14], the utmost motive of Zadeh’s ideas were to use fuzzy logic as a theory of approximate reasoning whereby truth degrees act as modifiers of the fuzzy statements they apply to. Nowadays, fuzzy logic has become a well-established discipline with several theoretical ramifications and a wide variety of contemporary application areas: computing with words [15], fuzzy control [16], decision making [17]–[19], image processing [20], data mining and machine learning [21], neural networks [22], genetic algorithms [23], knowledge discovery [24], [25], medicine [26], robotics [27]…

One of the most important branches of fuzzy logic corresponds to the study of fuzzy operators, which are used to operate between membership values or truth degrees. Traditionally, many fuzzy concepts were defined as a generalization of the corresponding one in classical logic. Following this reasoning, the main classical logic connectives have been generalized: the intersection or conjunction is defined as a fuzzy conjunction (usually a t-norm); the union or disjunction is defined as a fuzzy disjunction (usually a t-conorm); the negation or the complement is defined as a fuzzy negation; and the conditionals are represented by fuzzy implication functions. However, the study of fuzzy operators goes beyond logic connectives and it intersects with the study of aggregation functions. Aggregation functions (also called aggregation operators) are used for combining and merging values into a single one according to a certain objective. Since fuzzy operators play an important role in a wide variety of applications, many different types have been defined. To illustrate this fact, we refer the reader to some books exclusively devoted to this topic [6], [28]–[31]. Although other domains besides have been considered in the literature [32], [33], typically fuzzy operators are defined as functions that fulfil some set of conditions (monotonicity, continuity, associativity, commutativity, boundary conditions...). However, these conditions are usually general enough to allow the existence of many different operators of a certain kind. This results in the more specific study of different classes of operators that fulfil a certain set of conditions, in which desired additional properties apart from the ones in the definition of the operator can be included. Thus, from a more theoretical point of view, the study of fuzzy operators falls within the scope of functional equations [34], [35].

The submitted project is mainly devoted to the study of fuzzy implication functions and their applications. Fuzzy implication functions are defined as functions which are decreasing with respect to the first variable, increasing with respect to the second variable and they coincide with the classical implication in . It is well known that Boolean implications are employed in inference schemas like modus ponens, modus tollens, etc. In the fuzzy logic framework, fuzzy implication functions play an analogous role in the generalization of these schemas modelling the corresponding fuzzy conditionals which are called fuzzy IF-THEN rules. These rules are widely used in approximate reasoning, wherein from imprecise inputs and fuzzy premises or rules, imprecise conclusions are drawn. However, apart from inference systems based on fuzzy rules [36]–[38], fuzzy implication functions have been considered in other application areas like fuzzy mathematical morphology or data mining [39]. Partly motivated by their potential applications, the study of fuzzy implication functions has significantly grown in the last decades (see the bibliometric analysis in [1]). Indeed, some monographs [5], [6] and surveys [2], [3], [4] only devoted to the study of these operators have been published. From a theoretical perspective, the main research lines in this topic focus on the definition and study of different classes of fuzzy implication functions and the additional properties that they may satisfy.

The definition of new families of fuzzy implication functions is motivated by the fact that, depending on the context and the proper rule and its behaviour, various fuzzy implication functions with different properties can be adequate [40]. Under this incentive, it is estimated that more than one hundred families of fuzzy implications functions have been proposed so far [41]. Nonetheless, the existing surveys and monographs focus on the classical families, and they do not consider the plenty of families that have been introduced in the last ten years.

For that reason, as part of this project (see Objective 1), we will make an up-to-date compilation of the families introduced in the literature so far. In this document we intend to collect the following information about each family: the motivation behind its definition, the additional properties that have been studied, and the known intersections with other families. This compilation will be publicly available so that it can serve as a helpful resource for researchers interested in fuzzy implication functions from both a theoretical and applied points of view. On the one hand, the survey will be useful for having a global view of the existing families and their additional properties to better contextualize the possible introduction of a new class or a new property. On the other hand, the document could be used as a source of reference for deciding which of the existing families fulfil the additional properties required for a practical problem. Also, the survey will point out open problems in the field. Further, it is intended to keep the document updated, taking into account the possible families of fuzzy implication functions to be introduced in the future.

Although it is clear that many families of fuzzy implication functions have been introduced in the literature, that does not mean that all of these families are significantly different. Indeed, since there exist several techniques for defining new families, two apparently different families may intersect or turn out to be exactly the same one [7]. For this reason, it is of the utmost importance to study the additional properties that the operators of a certain family satisfy and to provide an axiomatic characterization of the new operators in the literature in order to find its possible relation with respect to those already known. In this respect, the characterization of several families of fuzzy implication functions have already been achieved: (S,N)-implications with a continuous negation [42] or with a negation with one point of discontinuity [43], -implications obtained from left-continuous t-norms [44], [45], some -implications [46], Yager’s implications [47], -implications [48], probabilistic and survival -implications [7]; among others [8], [49]–[51]. Besides, the intersections between some of the families have also been studied [2], [6], [52]. However, there are still many families of fuzzy implication functions whose characterizations or intersections with others are unknown: some subfamilies of QL and D-implications, many of the generalizations of Yager’s implications, (S,N)-implications derived from a fuzzy negation with more than one point of discontinuity, R-implications derived from a t-norm which is not left-continuous... In the Objective 2 of the presented project, we will focus on the study of characterizations and intersections of these families. Further, since the problem of the study of characterization and intersections may be complex, it is well known that the characterization problem may lead to the study of other problems which fall outside the scope of the study of fuzzy implication functions. For instance, recently the characterization of -implications with a non-continuous negation has been related to the well-known problem of the completion of triangular norms [43]. As part of the Objective 2 we will also solve problems that are related to the study of characterizations and intersections of fuzzy implication functions.

Apart from the introduction of new families of fuzzy implication functions, the study and proposal of additional properties is also a hot topic right now[53]–[59]. These properties come often in the form of functional equations which involve fuzzy implication functions and some of them, other operators as well. The motivation behind the definition of these additional properties are diverse, but the most usual are: a large majority of them were introduced as the straightforward generalizations of classical logic tautologies to fuzzy logic; others point out some desirable or interesting analytical/algebraic properties of these functions; some were introduced since they appeared when solving a particular problem; finally, many of these properties aim to be useful in a particular problem or application. In Objective 3 we are particularly interested in the study of additional properties from this latter perspective. On the one hand, we will study and find solutions of additional properties related to practical applications like the invariance with respect to the powers of a non-Archimedean continuous t-norm [7], [8] or the monotonicity of the generalized modus ponens [9]. On the other hand, we will study which additional properties have to be considered or introduced for the practical applications corresponding to Objective 4.

Apart from their theoretical relevance, fuzzy implication functions also play an important role in many application areas like inference systems based on fuzzy rules [6], [10], [11], image processing [12] or data mining [60]. Recently, a novel research line for the applicability of fuzzy implication functions has been proposed in the context of knowledge discovery (see [61], [62] and [9]). In Objective 4 we propose to further explore this new research area by designing and implementing knowledge discovery algorithms based on fuzzy implication functions.

One of the most important aspects of the knowledge discovery algorithms is that the output should be easy to interpret by an expert, so he/she can derive his/her own conclusions without necessarily knowing all the specific details of its design. In accordance, fuzzy logic and particularly, the use of descriptive fuzzy rules, has been considered as a suitable description language to represent the knowledge provided by this kind of algorithms in a similar way to human reasoning. Indeed, heretofore several knowledge discovery algorithms based on fuzzy rules have been proposed [24], [25], [63]. However, even though IF-THEN rules are symbolically represented by a conditional, the generalization of conditionals in fuzzy logic - fuzzy implication functions - have not been considered for this kind of algorithms. Thus, differently from the existing fuzzy logic perspectives in the literature, we choose to interpret the conditional in an IF-THEN rule as a logical conditional rather than the co-occurrence of the antecedent and the consequent. Moreover, in contrast with many existing algorithms in the literature, the use of fuzzy implication functions makes our perspective adequate for numeric targets modelled as fuzzy linguistic variables. Some of the techniques we will consider are subgroup discovery, exception rules and causal discovery.

Finally, in Objective 5 we propose to test and use the techniques implemented in Objective 4 in real-life problems and to collaborate with experts in different areas for obtaining the data and validating our results. Up to now, we have maintained contact with experts in the analysis of Polish elections and the medical domain. However, it is planned to seek further collaborations in the development of the project. This part of the project highlights the researcher’s commitment to the dissemination and practical application of the proposed objectives.

The project aligns closely with the European Research Area’s objectives of promoting scientific research excellence, innovation and international collaboration. The project is of great novelty because, although it focuses on one of the well-established branches of fuzzy logic, it does so from a new and promising perspective. Indeed, the objectives of the project are in line with the latest concerns of the field [64], placing significant emphasis on practical applications. Our project also encourages international collaboration and engagement. On the one hand, contact with international collaborators for the data acquisition and the analysis of results is foreseen. On the other hand, the research outcomes will be disseminated through peer-reviewed publications, international conferences, and seminars, facilitating knowledge exchange and reinforcing Europe’s position as leader in mathematical research and its applications. In addition, this project promotes talent mobility because the applying researcher of the project is originally from an institution of a different country from the host organization. To promote transparency, all the implemented code will be available in open-access repositories and, as far as possible, the results of the project will be published in open-access journals.

*(To see the corresponding bibliography, we refer the reader to the end of this document).*

1.3 METHODOLOGY

The methodology in our case is fundamentally based on study and constant dedication to achieve the proposed objectives, as evidenced by previously granted projects in which the applying researcher has been part of. It should be noted that the specific nature of the research implies that new questions may appear and, provided they are of interest, they may change the direction and methodology to a greater or lesser extent. Even so, we can outline a series of specific points to take into account, which will help guide the research in the best possible way and which will contribute to make our proposal viable:

* During the two years of the project, it is necessary to continue collecting and assimilating all the new bibliography that appears on the subject in order to be permanently up-to-date. We emphasize how prolific the study of fuzzy implication functions and their applications has been in the recent years. This point is especially relevant, since the novelty of the results presented will be guaranteed at all times and the techniques already used in the literature to solve similar problems will be used as much as possible.
* The researcher will assist to periodic meetings and seminars organized at the institute to keep abreast of other research lines, discuss problems and share the latest advances.
* It is necessary to preserve, and even strengthen, the contacts with the scientific community specialized in the field. Collaboration with other researchers can provide new data and contribute to the proper development of the project. Some of the specific objectives will be partly carried out with other foreign researchers with whom we maintain relationships. As we stated in Objective 5, for obtaining and analysing different type of data we have started a collaboration with Katarzyna Kaczmarek-Majer from the Systems Research Institute of the Polish Academy of Sciences, Magdalena Skorupska from the University of Warsaw and some collaborators from the Health Research Institute of the Balearic Islands (IdISBa). Besides, with respect to the study of fuzzy implication functions and aggregation functions, the researcher has already collaborated with Prof. Michal Baczynski (University of Silesia in Katowice), Balasubramaniam Jayaram (Indian Institute of Technology Hyderabad) and Andrea Mesiarová-Zemánková (Slovak Academy of Sciences Bratislava). In addition to these international relations, the researcher will also maintain the relationship with her current research team SCOPIA, especially with Prof. Sebastia Massanet, an expert in the area. Further, during the course of the project, new collaborations will be sought by attending international conferences and/or workshops.
* To ensure the transparency and reproducibility of the project, all the implementations will be done using an open-source programming language like Python. Further, all the corresponding code and simulations will be publicly available at the researcher’s personal repository.
* The research outcomes will be published in international peer-reviewed journals and proceedings of international conferences.

1.4 EXCELLENCE OF THE RESEARCHER

The candidate proposing the project is a young researcher who has recently obtained her Ph. D. degree from the University of Balearic Islands, Spain.

Although Raquel Fernández Peralta has still had a short research career given her youth, her soundness as a researcher in the field of fuzzy logic can be seen in her 7 journal papers published in Q1 journals by JCR as a first author. Apart from these journal papers, she also has 5 full papers and 6 abstracts published in international conferences proceedings which were personally presented by her, which shows her interest in the dissemination of her work and her presentation skills. All these publications are strictly related to the objectives of the presented project and show that the candidate is mature enough in the field for fulfilling the proposed goals.

The candidate has shown interest in being part of research projects since she was an undergraduate student. During her Bachelor’s degree she obtained a collaboration grant from the Spanish Ministry of Education, Culture and Sports with which she already initiated her research on the study of fuzzy implication functions. During her Master’s degree she obtained a collaboration fellowship at the Institute of Mathematical Sciences (ICMAT) in which she worked as a pre-doctoral student in a research project about Markov chains and epidemic modelling. Finally, for doing her Ph. D. she obtained one of the most renowned scholarships awarded by the Spanish government to finance her own project for her doctoral thesis, which was devoted to the study of fuzzy implication functions and their applications, the main topic of the presented project.

Although for the time being the researcher has carried out her activity in Spain, she has already collaborated with professors from other countries and she has done a three-months research stay at IIT Hyderabad, India, under the supervision of Prof. Balasubramian Jayaram.

Her interests have not been limited only to research activities, but she has also carried out teaching activities as a professor at the University of the Balearic Islands. Also, she has volunteered in other teaching activities like ESTALMAT (a program that seeks to guide and stimulate students aged 12-13 with exceptional mathematical talent), Mathematical Olympiad preparation classes and other dissemination activities organized by the University of the Balearic Islands. Further, she has been part of the organizing committee of several international conferences like BYMAT19, ICIAM19, CCIA19 and EUSFLAT23.

All these merits show that the candidate has great autonomy, initiative and courage to achieve high quality results.

Curriculum Vitae

(See the researcher's extended CV at [cv\_raquel\_fernandez\_peralta.pdf](https://1drv.ms/b/s!AiEY28bz83Ci4hk-TeJ10ollv554?e=sUYQMA))

**Personal information**

First and last name: Raquel Fernández Peralta

Identifier (ORCID): 0000-0003-1378-832X

Date of birth: August 27, 1996

Nationality: Spanish

**Education**

2023 – Ph. D. In Information and Communications Technology

Department of Mathematics and Computer Science, University of the Balearic Islands, Spain.

2019 – Master’s Degree In Statistical and Computational Information Processing

Faculty of Mathematics, Complutense University of Madrid, Spain.

2018 – Bachelor’s Degree In Mathematics

Department of Mathematics and Computer Science, University of the Balearic Islands, Spain.

**Previous positions**

09/2019 – 09/2023 – Pre-doctoral full-time researcher

Department of Mathematics and Computer Science, University of the Balearic Islands, Spain.

09/2018 – 07/2019 – Pre-doctoral full-time researcher

Institute of Mathematical Sciences (ICMAT), Spain.

**Scholarships and awards**

09/2019 – 09/2023 – Training programme for Academic Staff (FPU) Grant provided by the Spanish Ministry of Science, Innovation and Universities – Department of Mathematics and Computer Science, University of the Balearic Islands, Spain.

2022 – EDUIB-Santander Mobility Scholarship – Department of Mathematics and Computer Science,

University of the Balearic Islands, Spain.

09/2018 – 07/2019 – Collaboration fellowship at the ICMAT for master students, Institute of Mathematical

Sciences (ICMAT), Spain.

2018 – Best Final Degree Project – Social Council of the University of the Balearic Islands.

06/2018 – 08/2018 – Collaboration fellowship – Institute of Mathematical Sciences (ICMAT), Spain.

09/2017 – 07/2018 – Collaboration grant provided by the Spanish Ministry of Education, Culture and Sports – Department of Mathematics and Computer Science, University of the Balearic Islands, Spain.

**Teaching activities**

2020, 2021 and 2023 – Abstract Algebra I – Bachelor’s degree in Mathematics, Higher Polytechnic School,

University of the Balearic Islands, Spain. Total: 90 hours.

2021 and 2022 – Mathematics I – Bachelor’s degree in Biochemistry, Faculty of Science, University of the

Balearic Islands, Spain. Total: 90 hours.

**Organisation of scientific meetings**

2023– 13th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT 2023) –

Member of the organizing committee, Number of participants: 203, Spain.

2019 – 2nd BYMAT Conference. Bringing Young Mathematicians Together (BYMAT 2019) – Member of

the organizing committee, Number of participants: 200, Spain.

2019 – 9th International Congress on Industrial and Applied Mathematics (ICIAM 2019) – Volunteer,

Number of participants: 3000, Spain.

2019 – 22nd International Congress of the Catalan Association of Artificial Intelligence (CCIA 2019) –

Member of the organizing committee, Number of participants: 70, Spain.

**Reviewing activities**

Since 2022 – Reviewer, Fuzzy Sets and Systems (journal), 10 completed reviews.

Since 2022 – Reviewer, Kybernetika (journal), 1 completed review.

**Memberships of scientific societies**

Since 2019 – Member of EUSFLAT – European Society for Fuzzy Logic and Technology

**Major collaborations**

*Balasubramaniam Jayaram*, Connectives in Multivalued-Logic, Approximate Reasoning and Issues in High Dimensional Data Analysis, Institute of Technology Hyderabad, India.

*Michał Baczyński*, Fuzzy logic, Fuzzy implications, Artificial Intelligence, Computational intelligence, Functional equations, Faculty of Science and Technology, University of Silesia in Katowice, Poland.

*Andrea Mesiarová-Zemánková*, Aggregation theory, Associative functions on bounded lattices, Multi-polar aggregation, and Non-additive measures and integrals, Mathematical Institute, Slovak Academy of Sciences, Slovakia.

*Sebastia Massanet*, fuzzy sets theory and some related fields, such as fuzzy connectives, specially fuzzy implication functions and aggregation functions, functional equations, fuzzy mathematical morphology, and its applications to image processing and decision making, University of the Balearic Islands, Spain.

*Katarzyna Kaczmarek-Majer*, data mining, signal processing and forecasting, fuzzy systems and computational statistics, Polish Academy of Sciences, Warsaw, Poland.

*Magdalena Skorupsk*, Elections, Voter Turnout (Electoral Behaviour), University of Warsaw, Poland.

**Overview of the researcher’s most important projects in the last 5 years** (max. 5)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Project name/identification** | **Source of funding** | **Budget (EUR)** | **Project period** | **The role of the researcher in the project** |
| FPU18/05664: New characterizations of some families of fuzzy implication functions and their intersections. Applications to subgroup discovery | Spanish Ministry of Education, Culture and Sports | 65.688,00€ | 24/09/2019  -  14/09/2023 | Main recipient |
| TIN2016-75404-P:  Soft Computing techniques for the treatment of uncertainty in image processing | Spanish Ministry of Economy, Industry and Competitiveness | 125.235,00€ | 30/12/2016  -  29/06/2021 | Research team member |
| PID2020-113870GB-I00: Development of Soft Computing tools for Clinical Diagnostic Assistance and Emergency Management (HESOCODICE) | Spanish State Research Agency/ Ministry of Science and Innovation | 99.099,00€ | 01/09/2021  -  31/08/2025 | Research team member |

**Overview of the researcher’s most important outputs** (max. 5)

|  |  |  |  |
| --- | --- | --- | --- |
| **Output name/identification** | **Type of output** | **Short description** | **The role of the researcher** |
| R. Fernández-Peralta, S. Massanet, A. Mesiarová-Zemánková, A. Mir:  *A general framework for the characterization of (S, N)-implications with a non-continuous negation based on completions of t-conorms.*  Published in Fuzzy Sets and Systems 441, 1-32 (2022) | publication | The equivalence between the problem of the characterization of (S,N)-implications with a non-continuous negation and the problem of the completion of t-norms is proved. The completion problem is then solved for the eight regions of interest when the negation N has only 1 point of discontinuity and the corresponding t-norm is the minimum or it is cancellative. | Concept  Formal analysis  Writing |
| R. Fernández-Peralta, S. Massanet, A. Mir:  *On strict T-power invariant implications: Properties and intersections.*  Published in Fuzzy Sets and Systems 423, 1-28 (2021) | publication | The so-called strict T-power invariant implications, those implications which are invariant with respect to powers of a strict t-norm, are analysed. From this study, it is proved that there are members of this family satisfying important properties. This analysis leads to the characterization of the intersection of this family with the most usual classes of fuzzy implication functions. | Concept  Formal analysis  Writing |
| R. Fernández-Peralta, S. Massanet, A. Mir:  *Study of Two Families of Generalized Yager’s Implications for Describing the Structure of Generalized (h,e)-Implications.*  Published in Mathematics 9(13), 1490 (2021) | publication | The additional properties of generalized (h,e)-implications are studied and a representation theorem that describes the structure of a generalized (h,e)-implication in terms of two families of fuzzy implication functions is obtained. These two families can be interpreted as particular cases of the (f,g) and (g,f)-implications, which are two families of fuzzy implication functions that generalize the well-known Yager’s implications. The behavior and additional properties of these two families are also studied in detail. | Concept  Formal analysis  Writing |
| R. Fernández-Peralta, S. Massanet, A. Mir:  *Characterization of generalized (h, e)-implications based on the characterization of (f, e) and (g, e)-implications.*  Published in Information Sciences 612, 1145-1170 (2022) | publication | The open problem of the characterization of generalized (h,e)-implications is studied and totally solved. The characterization is based on the characterization of two families which are generalizations of the well-known Yager’s implications, called (f,e) and (g,e)-implications and two new additional properties of fuzzy implication functions which are modifications of the law of importation. | Concept  Formal analysis  Writing |
| R. Fernández-Peralta, S. Massanet, A. Mesiarová-Zemánková, A. Mir:  *Determination of the continuous completions of conditionally cancellative pre-t-norms associated with the characterization of (S,N)-implications: Part I.*  Published in Fuzzy Sets and Systems 468, 108614 (2023) | publication | The continuous completions of conditionally cancellative pre-t-norms defined in the regions linked to the characterization of (S,N)-implications when S is a continuous t-conorm and N is a fuzzy negation with one point of discontinuity are determined. | Concept  Formal analysis  Writing |

The candidate’s most important outputs consist of 7 journal papers published in Q1 journals by JCR as a first author and 5 full papers and 6 abstracts published in international conference proceedings. The 5 highlighted papers are devoted to the characterization of fuzzy implication functions and the study of their additional properties. These publications contain high-quality results related to well-known and complex open problems within the field fuzzy implication functions. For instance, the first paper corresponds to new advances in the renowned problem of the characterization of -implications with a non-continuous negation, which was highlighted as an open problem for several years [6], [65]. This publication contains a detailed discussion of the problem and provides new significant advances for its resolution. In addition, it relates this open problem to a classic problem in the study of fuzzy operators: the completion of t-norms. Therefore, the results obtained are not only relevant to the study of fuzzy implication functions but also extend beyond that scope. The complexity of these two problems is underscored by the fact that two more papers related to the subject have been published. One of them corresponds to Publication 5, in which all the continuous completions of some conditionally cancellative pre-t-norms are explicitly constructed. Publication 2 corresponds to the characterization and investigation of all fuzzy implication functions satisfying the invariance property with respect to a strict t-norm, which involved the study and resolution of several functional equations. From the results of this publication, we obtained parametric fuzzy implication functions satisfying valuable additional properties. Finally, the main results of Papers 3 and 4 correspond to the characterization of three families of fuzzy implication functions. To solve the corresponding problems, we used different techniques which involved the study of the interrelation between families of fuzzy implication functions to describe its structure and the proposal of new additional properties.

1.5 EXCELLENCE OF THE APPLICANT/HOST ORGANISATION

The Slovak Academy of Sciences was instituted in 1942 by Parliament of the Slovak Republic, under the name Slovak Academy of Sciences and Arts (Slovenská akadémia vied a umení-SAVU). During the post‐World War II period the SAVU was transformed into Slovenská akadémia vied - Slovak Academy of Sciences (SAS). Nowadays, the Slovak Academy of Sciences successfully presents itself nationally and worldwide through the results in basic and applied research. It consists of 48 organisations, of which 22 are budgetary and 26 are contributory. The Academy publishes 57 scientific and professional journals and 8 yearbooks. There are 51 scientific and scholarly associations, societies and unions affiliated with the SAS, which operate in accordance with the Law on Civic Associations. The Assembly, the Scientific Council, and the Presidium are the Academy’s self-governing bodies.

The Mathematical Institute of Slovak Academy of Sciences (MI SAS) is a scientific institute concentrated mainly on basic research in mathematics and theoretical informatics. On January 1, 2022, it was transformed into a public research institution. Its collaborators have participated in many important and successful projects in both basic and applied research, including EU Framework and Structural projects and Slovak national projects APVV and VEGA. Many of its collaborators belong to the top in their research in a world-wide context. The Mathematical Institute publishes the major journal Mathematica Slovaca (called Matematicko-fyzikálny sborník during 1951–1952, Matematicko-fyzikálny časopis during 1953–1966, and Matematický časopis during 1967–1975) and the Tatra Mountains Mathematical Publications. In collaboration with the Commenius University, the institute organizes a PhD study program and other tasks of the institute include in particular: education of young researchers, cooperation with national and international scientific and research institutions, publication of scientific journals and books, collecting and sorting information about scientific results, preparation of various expertises for the government and government organizations, provision of scientific information for institutions and citizens of Slovakia, cooperation and assistance to universities in teaching and its organization, transferring the results of scientific research into practice and popularization of science.

The field of fuzzy mathematics is a prominent research area within the institute. The top researcher is Andrea Zemánková, who is a leading expert in fuzzy mathematics and aggregation functions. She authored and co-authored more than 60 papers in leading journals and was very recently awarded the price of the Slovak Academy of Sciences for an excellent publication. Further, Andrea Zemánková has recently presented relevant results in the concrete field of the study of fuzzy implication functions, which were achieved in collaboration with the researcher of the project [43], [66], [67]. This fact demonstrates that the expertise of Andrea Zemánková in the field of aggregation functions is of great value for the fulfilment of the concrete objectives of the presented project. We intend to continue the collaboration with Andrea Zemánková during the course of the project. Fuzzy mathematics is also strong in the Košice branch of the institute, with Jozef Pócs as the leading scientist.

Further, the researchers of the institute are well connected with world-wide experts in their respective branches. Specifically, in the field of aggregation, the institute has a close connection to neighboring institutions such as the Slovak University of Technology in Bratislava and Institute for Research and Applications of Fuzzy Modelling, University of Ostrava, Czech Republic through extensive collaborations and numerous joint projects. Also, the institute is visited by many distinguished mathematical experts from all over the world and many young scientists use the fellowship program SAIA for study stays at the institute. The regular seminars visited by international and national experts, as well as by young scientists, and consultations with local and visiting researchers are expected to be an effective method of two-way knowledge transfer between the host organization and the researcher.

## Impact

### 2.1 THE WIDER IMPACT OF THE PROJECT

The objectives presented in this project are designed for having a relevant and positive impact at different scales.

The fulfilment of Objective 1 aims to have a great impact on the fuzzy logic community, since the philosophy of the compendium of fuzzy implication functions is to make available an exhaustive and comprehensive reference document for any researcher interested in the study of these operators. Furthermore, it is hoped that the document could be updated over time in order to keep track of fuzzy implication functions as they are introduced into the literature. Thus, the publication of this document will have a long-term impact as it can serve as the basis for the future development of fuzzy implication functions.

In addition, by studying characterizations and intersections of families of fuzzy implication functions as part of Objective 2 we aim to help eliminate the redundancy in the field and to understand better the relation between the classes introduced in the literature. This will have a short-term impact in the field, since it will point out which families are more significantly different from others, and this will facilitate the optimal progression of the research on fuzzy implication functions. Also, from Objectives 3 and 4 we will stress the importance of fuzzy implication functions for practical applications which will also justify and contextualize the huge number of existing families and additional properties. These new results will have a short-term impact for the data mining techniques proposed in this project, but also a medium-term impact for other future application areas.

Furthermore, Objectives 4 and 5 are intended to have an impact beyond the field of mathematics. Since the techniques used will be based on fuzzy rules, the output will be represented in natural language that can be easily interpreted by humans. Therefore, all the implemented techniques aim to have a high interpretability, which will make them accessible to a much wider public than other techniques with a more complex output. Further, since the implementation of these techniques will be publicly available, we believe that our techniques will not only be useful for the concrete problems of interest for our international collaborators, but also for anyone that may be interested in using them. Further, it is clear that the fulfilment of these objectives will have a short-term impact, because we are going to design and implement new knowledge discovery techniques that will be used by experts for analysing real datasets that refer to current research topics. However, we believe that our rigorous methodology will also have a long-term impact for future application areas based on fuzzy implication functions.

Additionally, it is estimated that the results derived from the proposed objectives will be well received in the academic community because they align with the current priorities within the field [64].

The possibility of carrying out this project would have a great impact on the candidate's research career for various reasons. First, it would give her the opportunity to continue her research career in a prestigious research centre. Secondly, it would give her the chance to work in a different institution than the one where she did her doctoral thesis, so she will be able to nurture from different points of view and she also will be able to contribute with her own. Finally, she would be able to show her independence and maturity to conduct her own research project.

One potential negative short-term impact could be that our techniques could be misinterpreted and/or confused with other similar techniques that are not based on fuzzy logic or fuzzy implication functions. To mitigate this possible inconvenience our perspective will be compared with others based on other methodologies and we will strive to clearly explain and contextualize the advantages and differences of our new perspective.

The expected results of the project include 4 to 7 high-quality publications, both in prestigious peer-reviewed journals and international conference proceedings. This estimate is based on the candidate's excellence and publications on similar topics.

Potential obstacles to the planned impact of the project may be the lack of data, the impossibility of making the collected data available or the computational complexity of the designed algorithms. To address these potential obstacles, it is essential to conduct a thorough risk assessment, plan accordingly, and remain adaptable throughout the research project. We will address data scarcity by exploring data augmentation techniques and seeking partnerships for additional data sources. Data accessibility challenges will be tackled through secure data-sharing protocols and compliance measures. To mitigate algorithmic complexity, we will optimize the code, and we will consider alternative modelling approaches.

### 2.2 MEASURES TO MAXIMISE IMPACT – DISEMINATION AND COMMUNICATION, EXPLOITATION OF RESULTS

The dissemination of the results of this project contemplates different approaches:

1. *Publication of the most important results in high-impact scientific journals.*

To disseminate and validate externally the results as much as possible, the main results will be presented in scientific peer-reviewed journals. Among the journals that we will take into account we can highlight IEEE Transactions on Fuzzy Systems, Fuzzy Sets and Systems, Information Sciences, Iranian Journal of Fuzzy Systems, Mathematica Slovaca...

1. *Talks in scientific conferences related to the field of the project.*

The attendance and participation at specialized conferences is necessary for an adequate transfer and public dissemination of the results. Moreover, these conferences empower joint collaborations with other experts in the fields related to the project, and also facilitate the researcher to keep on track with novel techniques. Among the conferences that we will consider we can highlight EUSFLAT, IPMU, FUZZ-IEEE, FSTA, IEEE WCCI, IFSA…

1. *Meetings with our international collaborators.*

During the project periodic meetings with the collaborators of the project will be organized. In these meetings we will discuss the most recent advances and the next steps for a coordinated evolution of the project.

1. *Attendance to seminars.*

The researcher will attend the different seminars organized at the institute to keep abreast of other lines of research, disseminate the project and discuss possible common problems and collaborations.

1. *The implementation of the designed techniques will be publicly available.*

The source code of the implemented techniques will be publicly available in a GitHub repository to allow their use and improvement.

1. *Dissemination events and outreach activities.*

The researcher will take part on dissemination events for sharing the research findings with a wider audience.

1. *The personal website of the researcher will be used to publish useful resources and disseminate our research.*

Any other resources originated from the project that could be useful for the community will be published in the researcher personal website (https://github.com/rferper).

## Implementation

3.1 PROJECT PLAN AND DELIVERABLES

The project consists of five working packages:

1. Literature review
2. Characterizations and Intersections
3. Study of additional Properties
4. Knowledge Discovery algorithms
5. Real-life applications

The working packages we have specified are in line with the proposed objectives and they define the concrete structure of the work plan to be followed to successfully fulfil the corresponding goals. These working packages are described in more detail with objectives and deliverables in the following Subsection 3.1.1. The project is planned to take 24 months and the implementation of the individual working packages is illustrated in the following Gantt diagram.

In all the work packages we have estimated involvement needed for the fulfilment of the corresponding deliverables. Further, we have also indicated an approximate of other eligible costs, which will be spend on intrinsic project costs like: books, attendance to international conferences and/or workshops, open-access publications, computational resources, and so on.

3.1.1 Work packages

|  |  |
| --- | --- |
| Work package number | WP1 |
| Title of the work package | Literature Review |
| **Start of implementation of the work package (Mx Month)** | M01 |
| End of implementation of the work package (Mx month) | M04 |
| **Involvement (expressed in Person Months)** | Raquel Fernández Peralta – 3,5 person months |
| **Personnel costs (in EUR)** | 14.850,50 eur |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 5.646,60 eur |
| Objectives | |
| * To do an exhaustive review of research papers and publications on the study of families of fuzzy implication functions and their additional properties. * To compile and classify the families of fuzzy implication functions introduced in the literature in terms of their construction method. * To gather the additional properties and intersections studied for each family. * To study some additional properties and/or intersections for the completeness of the corresponding survey. * To write a report about the current state of the field and to highlight open problems. | |
| Description of the work package | |
| In this work package, we focus on the study of the literature about fuzzy implication functions and, specifically, in the compilation of the families of fuzzy implication functions introduced in the literature jointly with their additional properties and their intersections with other families. | |
| Deliverables | |
| D01. Literature review report.  To write an exhaustive survey of the families of fuzzy implication functions introduced in the literature in which the motivation behind the introduction of each family, their particular interest, their additional properties and their known intersections with other families will be specified. The report will be published in the arXiv repository.  D02. State-of-the-art analysis.  To write a report in which we analyse the current situation of the field and we highlight open-problems. | |

|  |  |
| --- | --- |
| Work package number | WP2 |
| Title of the work package | Characterizations and Intersections |
| **Start of implementation of the work package (Mx Month)** | M03 |
| End of implementation of the work package (Mx month) | M12 |
| **Involvement (expressed in Person Months)** | Raquel Fernández Peralta – 6,5 person months |
| **Personnel costs (in EUR)** | 27.579,50 eur |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 10.486,60 eur |
| Objectives | |
| * To study the problem of the characterization of different families of fuzzy implication functions. * To study the intersections between various families of fuzzy implication functions. * To solve problems related to the characterizations considered. | |
| Description of the work package | |
| In this work package, we will study open problems regarding the characterization of some families of fuzzy implication functions and their intersections with other families. Further, we will also study problems that are related to the study of characterizations, like the problem of the completion of continuous t-norms. | |
| Deliverables | |
| D03. New characterizations and intersections of some families of fuzzy implication functions.  We will provide new results regarding the problem of the characterization and study of intersection of families of fuzzy implication functions. Part of the results will be written as a full paper which will be submitted to an international peer-review journal.  D04. New advances on related problems.  We will provide new results regarding the problems related to the characterization of families of fuzzy implication functions. Part of the results will be written as a full paper which will be submitted to an international peer-review journal. | |

|  |  |
| --- | --- |
| Work package number | WP3 |
| Title of the work package | Study of additional properties |
| **Start of implementation of the work package (Mx Month)** | M08 |
| End of implementation of the work package (Mx month) | M18 |
| **Involvement (expressed in Person Months)** | Raquel Fernández Peralta – 5 months |
| **Personnel costs (in EUR)** | 21.215,00 eur |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 8.066,60 eur |
| Objectives | |
| * To provide new solutions of some additional properties. * To highlight the importance of some additional properties for practical applications. * To thoroughly investigate the additional properties pointed out in WP4. If necessary, we will define and find solutions of potential new additional properties. | |
| Description of the work package | |
| In this work package, we will study some additional properties which are strictly related to practical applications. We will characterize the fuzzy implication functions satisfying the corresponding additional property and/or we will provide new solutions. Also, we will study the additional properties required for the algorithms designed in WP4. | |
| Deliverables | |
| D05. *Interim report.*  Interim report on the results achieved in the middle of the project realization and discussion about the fulfilment of the results yet to be obtained.  D06. *New solutions of additional properties.*  We will provide new results regarding the problem of finding new solutions of additional properties. Part of the results will be written as a full paper which will be submitted to an international peer-review journal.  D07. Attendance to international conference.  The applying researcher will assist to at least one international conference to disseminate the results of this working package by submitting a conference abstract/full paper. | |

|  |  |
| --- | --- |
| Work package number | WP4 |
| Title of the work package | Knowledge discovery algorithms |
| **Start of implementation of the work package (Mx Month)** | M12 |
| End of implementation of the work package (Mx month) | M22 |
| **Involvement (expressed in Person Months)** | Raquel Fernández Peralta – 6 months |
| **Personnel costs (in EUR)** | 25.458,00 eur |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 9.680,00 eur |
| Objectives | |
| * To investigate the literature on knowledge discovery with the aim of properly comparing our new perspectives with others. * To design knowledge discovery algorithms based on fuzzy implication functions. * To study which additional properties should be imposed to the operators used in the designed algorithms. * To implement the corresponding algorithms. | |
| Description of the work package | |
| In this work package, we will deeply study the literature on knowledge discovery techniques, and we will design new algorithms based on fuzzy implication functions. All the new techniques will be implemented in an open-source programming language and they will be published in a public repository. Moreover, we will thoroughly discuss which additional properties should be satisfied by the fuzzy operators used in our algorithms in order to have the desired behaviour. These properties will be studied in WP3. | |
| Deliverables | |
| D08. *Literature review report.*  We will write a report in which we gather information about other knowledge discovery algorithms in the literature. This document will be crucial for contextualizing and validating our new techniques with respect to the existing ones.  D09. *Design and propose different knowledge discovery algorithms.*  We will propose new techniques for knowledge discovery based on fuzzy implication functions. Some of the new proposals will be included in a full paper which will be submitted to an international peer-review journal or/and the proceedings of an international conference.  D10. *Implementation of the designed algorithms.*  We will implement the algorithms designed and we will publish the code in a public repository.  D11. Attendance to international conferences.  The applying researcher will attend to at least one international conference to disseminate the results of this working package by submitting a conference abstract/full paper. | |

|  |  |
| --- | --- |
| Work package number | WP5 |
| Title of the work package | Real-life Applications |
| **Start of implementation of the work package (Mx Month)** | M20 |
| End of implementation of the work package (Mx month) | M24 |
| **Involvement (expressed in Person Months)** | Raquel Fernández Peralta – 3 months |
| **Personnel costs (in EUR)** | 12.729,00 eur |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 4.840,00 eur |
| Objectives | |
| * To study different case studies using the datasets obtained from our international collaborators using the techniques designed and implemented in WP4. * To discuss the obtained results with the corresponding experts. * To compare our results with other perspectives. | |
| Description of the work package | |
| The techniques designed and implemented in WP4 will be used in different case studies and the corresponding results will be analysed by experts in the corresponding field. We will validate the results obtained by our algorithms and we will discuss why our new techniques outperform others in the literature. | |
| Deliverables | |
| D12. *Case study reports.*  We will write reports in which we study and analyse in collaboration with experts of the corresponding field the information provided by our algorithms.  D13. *Comparative analyses and effectiveness.*  We will compare the performance of our algorithms with respect to other proposals in the literature.  D14. Attendance to international conferences.  The applying researcher will attend to at least one international conference to disseminate the results of this working package by submitting a conference abstract/full paper.  D15. *Final report.*  Final report summarizing achievements accomplished while implementing the project. | |

3.1.2 List of work packages:

|  |  |  |  |
| --- | --- | --- | --- |
| Work package number | Title of the work package | **Start of activities** | **End of activities** |
| WP1 | Literature review | M01 | M04 |
| WP2 | Characterizations and Intersections | M03 | M12 |
| WP3 | Study of Additional Properties | M08 | M18 |
| WP4 | Knowledge Discovery algorithms | M12 | M22 |
| WP5 | Real-life applications | M20 | M24 |

3.1.3 List of deliverables:

Hereunder, it can be found the list of the concrete deliverables specified in each work package. The specific implementation time of the deliverables is only indicative and can occur also in other project months. The number of publications in each package is an estimate and it will depend on the progress made in each objective and the relevance of the results obtained. Nonetheless, the new results will be rigorously written and shared with our collaborators on an ongoing basis to facilitate their fast publication.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Deliverable number | Deliverable | Work package number | Type | Access and dissemination | Method of verification | Delivery (project implementation month) |
| D01 | Literature review report | WP1 | report | public | expert review | M04 |
| D02 | State-of-the art analysis | WP1 | report | non-public | expert review | M04 |
| D03 | New characterizations and intersections of some families of fuzzy implication functions | WP2 | publication | public | peer review | M12 |
| D04 | New advances on related problems | WP2 | publication | public | peer review | M12 |
| D05 | Interim Report | WP3 | report | non-public | project evaluation | M12 |
| D06 | New solutions of additional properties | WP3 | publication | public | peer review | M16 |
| D07 | Attendance to international conferences | WP3 | publication | public | participant feedback | M16 |
| D08 | Literature review report | WP4 | report | public | expert review | M17 |
| D09 | Design and propose different knowledge discovery algorithms | WP4 | publication | public | peer review | M20 |
| D10 | Implementation of the designed algorithms | WP4 | software | public | expert review | M22 |
| D11 | Attendance to international conferences | WP4 | publication | public | participant feedback | M22 |
| D12 | Case study reports | WP5 | report | non-public | expert review | M24 |
| D13 | Comparative analyses and effectiveness | WP5 | report | public | performance metrics | M24 |
| D14 | Attendance to international conferences | WP5 | publication | public | participant feedback | M24 |
| D15 | Final Report | WP5 | report | non-public | project evaluation | M24 |

3.1.4 List of milestones:

The work packages we have specified correspond to the concrete objectives of the presented project, nonetheless, they contribute to more general milestones. First, by providing an exhaustive survey of the families of fuzzy implication functions introduced in the literature and by studying characterizations and intersections we will make easier for the researchers in the community to have a global view of all the existing families of fuzzy implication functions and their relationships. Second, the new advances on the study of additional properties of fuzzy implication functions will disclose important knowledge about the structure of these operators. Third, by proposing new knowledge discovery techniques based on fuzzy implication functions we will show the potential and significance of these operators for practical applications. Also, the study of additional properties related to practical applications will highlight valuable families. Finally, we expect that our results will be of enough significance and novelty to be published in high-quality peer-reviewed international journals and to be presented at international conferences.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Milestone number | Milestone | Work package number | Method of verification | Expected time to reach the milestone (project month) |
| MS1 | To disclose the interrelation between families of fuzzy implication functions introduced in the literature. | WP1, WP2 | Literature review | 10 project months |
| MS2 | To understand better the structure of families of fuzzy implication functions in the literature and to find solutions of additional properties. | WP2, WP3 | Peer review | 11,5 project months |
| MS3 | Show the potential of fuzzy implication functions in applications | WP3, WP4, WP5 | Peer review | 14 project months |
| MS4 | High-quality publications | WP2, WP3, WP4 | Peer review | 17,5 project months |

3.2 IMPLEMENTATION RISKS AND PROPOSED MEASURES

3.2.1 Risks of implementation:

The project has different risks associated with it that could make it difficult to meet the specified schedule. Specifically, the most significant risks correspond to:

* *Lack of data and data quality (low risk).* Although we have already contacted some experts in the field of the analysis of Polish elections and the medical domain, due to the novelty of the datasets to be considered not all the corresponding data have been collected yet. This could result in an insufficient amount of data for the analysis of the WP5.
* *Computational complexity (medium risk).* Since the techniques to be designed and implemented in WP4 are totally novel, we have not yet estimated their computational complexity. Thus, a potential risk is that the demands of the techniques to be implemented may lead to a high computational complexity.
* *Publication delays (low risk).* It is well known that the rigorous peer-review process of prestigious journals may take a long time. This inconvenience could affect the estimated deadlines for the deliverables corresponding to journal publications.

However, for each risk an action plan is available so that possible inconvenience will not conflict with the quality or quantity of the results.

|  |  |  |
| --- | --- | --- |
| **Description of the risk of implementation** | **Work package** | Proposed measures for risk mitigation or elimination |
| Lack of data and data quality (low risk) | WP4, WP5 | Exploring data augmentation techniques and seeking partnerships for additional data sources. |
| Computational complexity (medium risk) | WP4 | Depending on the computational demands of the techniques considered and the datasets obtained, the computational demands of the experiments could be high. In order to mitigate this adverse situation different measures will be considered: code optimization, parallel computing, the use of a supercomputer, the consideration of different heuristics… |
| Publication delays (low risk) | WP2, WP3, WP4 | New results will be rigorously written and shared with our collaborators on an ongoing basis to facilitate their fast publication. We will also consider the possibility of publishing our results as arXiv preprints. |

3.3 OPERATIONAL CAPACITY OF THE APPLICANT/HOST ORGANISATION

The Mathematical Institute at SAS is well equipped and beside standard hardware and software equipment necessary for the project, it has one of the best mathematical libraries in Slovakia with access to journal nets of the main world publishing houses. Also, it has access to the Slovak Centre of Scientific and Technical Information (http://www.cvtisr.sk/) – which provides additional mathematical processing software suite capabilities. In addition, the institute has access to a supercomputer which can be used for complex calculations and simulations.

The researchers of the institute are highly qualified professionals, many of whom are world leading experts in their scientific fields and the institute has a rich network of collaborating national and international experts in aggregation and multi-criteria decision‐making. Creative environment visited by many international experts and seminars which are organized by the institute provide an excellent opportunity for discussions, exchange of ideas and interdisciplinary cooperation.

The applying researcher will be provided with full institutional support and will have access to all necessary resources and infrastructure. Access to the supercomputer will be granted upon request. Research and technical staff of the institute is prepared to help the researcher with any technical issues that can appear during the implementation of the project. Discussions with experts from the institute working in similar scientific fields will contribute to fruitful interdisciplinary cooperation that will enrich the obtained results and maximize the impact of this project.

3.3.1 Description of the research/innovation infrastructure of the applicant/host organisation that is necessary for the implementation of the project:

|  |  |
| --- | --- |
| Name of infrastructure or equipment | Short description |
| Standard hardware and software equipment | All hardware and software which is needed in this type of research is available to researcher (including laptops, desktops, internet connection, computational software, etc.) |
| Communication tools | Special video conference system for communication with foreign collaborators |
| Library | One of the best mathematical libraries in Slovakia |
| Access to main scientific databases | Access to journal nets of the main world publishing houses, scientific-information and technical databases, like e.g., Science Direct, Scopus, WoS, SpringerLink, SpringerNATURE, JSTOR, PLOS, PNAS, Taylor & Francis, Wiley Online Library, etc. |
| Supercomputer Devana | Supercomputer Devana with an estimated output of 800 TFlop/s, which can be used for complex calculations and simulations |

3.3.2 List of the five most important projects of the applicant/host organisation and their relevance to the proposed project (in the last 5 years):

|  |  |  |
| --- | --- | --- |
| Project name/identification | Programme/scheme/grant provider | Short description |
| VEGA 1/0036/23: Advanced approaches to data aggregation and applications | Scientific grant agency MESRaS SR and SAS | The project is devoted to basic research in the field of aggregation theory and applications of aggregation functions in various industries. Construction methods and properties of aggregation on structures that generalize real intervals are studied. Based on the requirements from applied fields, several new modifications of standard monotonicity, as well as other properties, were introduced and studied in this project. The corresponding types of aggregation functions are currently analysed and applied in the field of image processing, in decision-making problems, in statistical modelling and in other areas. |
| VEGA 1/0006/19:  New trends in theory of aggregation and their applications | Scientific grant agency MESRaS SR and SAS | The project contributed to the basic research in the domain of the aggregation theory and its applications, according to the latest trends and their development. Achieved theoretical goals include deep study of directional and ordered directional monotonicity in the domain of (pre-)aggregation functions and related functions, of k-additive and related aggregation functions on different real scales, of sub- and the super-decomposition integrals. Further results were focused on aggregation of data from bipolar and multipolar scales, including extensions of aggregation functions to more complex domains, such as the lattices or posets. These results were applied in the problems from the domain of image processing. |
| NFP313011T683 Mathematical support of quantum technologies | ITMS-2014+/ European Regional Development Fund | The main goal of the project was to stabilize a high-quality research team(s) of the institute in the domain of mathematical structures and functions, to implement independent research and development activity with the relevant project outputs, to obtain new knowledge about mathematical structures and functions. |
| APVV-20-0069: Probabilistic, Algebraic and Quantum Mechanical Methods of Uncertainty Determination | Slovak Research  and Development  Agency | The project studies the mathematical foundations of quantum mechanics and uncertainty using the state of art methods of quantum structures. Its goals are focused on deepening the knowledge of partial and total algebras such as effect algebras, MV-algebras, synaptic algebras, orthomodular unions, BL-algebras, EMV-algebras, wEMV-algebras, residue unions and their non-commutative generalizations and states on them in connection with partially ordered groups. Aggregation methods are used to combine selected measurement values ​​into one aggregation function. The uncertainty inherent in quantum measurements is analyzed from the perspective of states and quantum channels with a focus on applications in quantum mechanics, quantum information theory and the description of incompatibility measures. |
| APVV-16-0073: Probabilistic, Algebraic and Quantum-Mechanical Aspects of Uncertainty | Slovak Research  and Development  Agency | The project was focused on obtaining original results for the description of the uncertainty related to quantum structures and on the uncertainty contained in the convex structure of quantum-mechanical measurements. The research was especially concentrated on study of partial and total algebras such as effect algebras, synaptic algebras, MV-algebras and their non-commutative generalizations, as well as on the description of quantum channels, states, estimation and testing tasks of quantum-mechanical processes, categorical properties and aggregation functions. |

3.3.3 List of maximum five most important outputs of the applicant/host organisation relevant to the submitted project:

|  |  |  |
| --- | --- | --- |
| Output name/identification | **Type of output** | Short description |
| A. Mesiarová:  *Continuous triangular subnorms*  Published in Fuzzy Sets and Systems 142 (1), 75-83 (2004) | publication | Triangular subnorms are associative commutative non-decreasing operations on the unit interval, upper bounded by the minimum. Continuous triangular subnorms are shown to be ordinal sum of Archimedean continuous t-subnorms with at most one proper t-subnorm summand. Special attention is paid to generated  continuous t-subnorms. An application of continuous t-subnorms to the construction of left-continuous t-norms is shown. Several illustrative examples are included. This paper has 54 SCI citations. |
| A. Mesiarová-Zemánková:  *Multi-polar t-conorms and uninorms.*  Published in Information Sciences 301 (2015), pp. 227-240 | publication | The correspondence between uninorms/nullnorms and bipolar t-conorms/t-norms was shown. The structure and construction of bipolar and n-polar t-norms, t-conorms and uninorms was described. This paper has 42 SCI citations. |
| A. Mesiarová-Zemánková:  *Characterization of uninorms with continuous underlying t-norm and t-conorm by their set of discontinuity points*  Published in IEEE Transactions on Fuzzy Systems 26 (2), 705-714 (2017) | publication | Uninorms with continuous underlying t-norm and t-conorm are discussed and properties of the set of discontinuity points of such a uninorm are shown. This set is proved to be a subset of the graph of a special symmetric, u-surjective, nonincreasing set-valued function, which gives us a necessary condition for a uninorm to have continuous underlying functions. A sufficient condition for a uninorm to have continuous underlying operations is also given. Several examples are included. This paper has 38 SCI citations. |
| A. Mesiarová-Zemánková:  *Structure of Uninorms with Continuous Diagonal Functions.*  In: On Logical, Algebraic, and Probabilistic Aspects of Fuzzy Set Theory (S. Saminger, R. Mesiar, eds.), Book series: Studies in Fuzziness and Soft Computing 336, Springer, 2016, pp. 109-135 | book chapter | In this chapter, the structure of uninorms which are continuous on some special parts of the unit square is discussed. Representation theorems for uninorms with continuous underlying functions, based on the set of discontinuity points of such a uninorm and the ordinal sum construction for semigroups, are presented. Further generalizations are also discussed. |
| HALAŠ, R. and PÓCS, J. –  *On the clone of aggregation functions on bounded lattices.*  Published in Information Sciences 329 (2016), pp. 381-389 | publication | The paper studies aggregation functions on lattices via clone theory approach. It is shown that the set of generating aggregation functions consists only of at most *n* unary functions, at most *n* binary functions, and lattice operations ∧, ∨, and all aggregation functions of *L* are composed of them by usual term composition. This approach is shown to work also for infinite lattices. |

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