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Title of the project:

Some Aspects of Universal Algebra Constructions Applied to Classical Algebras

Short title of the project/Acronym: Some Aspects of Universal Algebra

Category of researcher: R2

Researcher’s job type: full-time

Type of research: independent

Identification of the entity involved in the implementation of the project:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Official name of the entity | Abbreviated name of the entity | Role in the project |
| 1 | Mathematical Institute Slovak Academy of Sciences | MI SAS | Applicant |

## 1. Excellence

### PROJECT OBJECTIVES

*The primary aim of this project is to achieve new results in universal algebra and in theory of classical algebraic structures as groups, rings, modules. The secondary aim is to capitalize on the results achieved at MI SAS for the development of European science.*

*We expect a progress in the following interconnected directions:*

1. *EKP property for algebras, in particular for rings and groups*. An algebra *A* has an endomorphism kernel property (EKP) if every congruence relation θ on *A* different from universal congruence *A × A* is the kernel of an endomorphism on *A*. This property has been intensively studied in recent years for various types of algebraic structures. Rings and groups are still open. EKP was investigated for distributive lattices (M.Ploščica) and unary algebras (E. Halušková) at MI SAS.
2. *Algebras with easy direct limits*. This topic is connected with endomorphisms of algebras again since it is about direct families in which all algebras are isomorphic. The direct limit construction is one of basic tools in Universal algebra to create new algebras from families of algebras. There are many interesting and open questions. We say that *A* is an algebra with easy direct limits (EDL) if every direct limit of *A* is isomorphic to a retract of *A*. It is known that every finite algebra is with EDL. A conjecture is that every simple algebra is with EDL. We recognized several examples of infinite groups and rings with EDL and we plan to generalize their properties.
3. *Classes of algebras closed with respect to direct limits*. Many classes of rings that are closed with respect to direct limits can be found in literature. We want to summarize them and study the lattice of classes of rings closed with respect to direct limits. It is possible that this work will contribute to the solution of problems of Universal Algebra mentioned in the next section.

*Our results will be presented at mathematical seminars, workshops, conferences* *in Slovakia, Poland and other European countries. We assume that it will be at least 8 presentations, 4 of them at international events. Further, we submit results of the project to publication in mathematical journals registred in databases WoS, SCOPUS. We suppose that we will prepare at least 3 papers.*

1.2 RELEVANCE, QUALITY AND NOVELTY OF THE PROJECT

The aim of the project is to use the universal algebra methods to study the properties of particular algebraic structures such as: commutative and non-commutative rings, groups, semigroups, modules, algebras over fields. The main focus of the research will be using definitions of direct limits to describe properties of such particular algebras. These limits occur naturally in universal algebra as well as in category theory.

One of the most important mathematical problems has long been the search for objects larger than the given ones that preserve selected properties, e.g. complex numbers were discovered as a generalization of real numbers. Over the centuries, algebraists have discovered many constructions that lead to finding new, larger objects using smaller ones. Among such constructions that have found more widespread use in ring theory, the following should be mentioned: commutative and noncommutative rings of polynomials in finitely (or infinitely) many variables over any ring, skew polynomial rings, localizations of commutative rings, rings of left or right fractions for any (noncommutative) ring, Martindale ring of quotients, direct and inverse limits of rings. Next, direct and inverse limits constructions have the advantage that they allow working with certain types of graphs.

Direct and inverse limits in ring theory, group theory, module theory are found in many abstract algebra textbooks such as:

1. M. Atiyah, I. Macdonald: *Introduction to Commutative Algebra*, Addison-Wesley, 1969.
2. M. Hazewinkel, N. Gubareni, V. V. Kirichenko: *Algebras, rings and modules. Vol. 1*, Mathematics and its applications 575, Springer, 2004.
3. T. Y. Lam: *Lectures on modules and rings*, Springer Graduate Texts in Math., Vol. 189, 1999.
4. J. J. Rotman: *An Introduction to Homological Algebra,* [Springer-Verlag New York Inc.](https://www.megaksiazki.pl/20000522_springer-verlag-new-york-inc) 2008.
5. R. Schmidt: *Subgroup lattices of groups*, Expositions in Math., Vol. 14, de Gruyter, 1994.

The recent results are in articles:

 M. Asgharzadeh, M. Dorreh, M.  Tousi: *Direct limits of Cohen–Macaulay rings, J Pure Appl Algebra,* Vol. 218, no. 9, 2014, 1730–1744.

 G. M. Bergman: Every module is an inverse limit of injectives, *Proceedings of the American Mathematical Society,* Vol 141, no. 4, 2013, 1177–83.

 G. R. Conner, C. Kent: *Inverse limits of finite rank free groups*, J. Group Theory, Vol. 15, no. 6, 2012,  823–829.

 D. Karim: *On Direct Limits of Finite Products of Fields as Subrings of a Commutative Ring*, Algebra Colloq., Vol 23, no. 2, 2016, 243—249.

 F. Leinen, O. Puglisi: *Free subgroups of inverse limits of iterated wreath products of non-abelian finite simple groups in primitive actions*,  J. Group Theory, Vol. 20, no. 4, 2017,  749–761.

 C.H. Silva de Souza, J.A. Novacoski , M. Spivakovsky: *Graded rings associated to valuations and direct limits*, J Pure Appl Algebra, Vol. 227, no. 5, 2023, 107296  
 D. C. Zhou, F. G.  Wang: The direct and inverse limits of *w*-modules, *Commun. Algebra,* Vol 44, no.6, 2016,  2495–2500.

In the mentioned sources, direct families are most often considered as sequences, i.e. indexed by a linearly ordered set of natural numbers. Universally, direct families are considered as indexed by an upward directed partial ordered set.

Algebras with EDL and direct limit closed classes of algebras from objectives of this project were not still considered for groups, rings, modules, algebras over fields.

Constructions of direct and inverse limits in the context of universal algebra belong to topics which are developed at MI SAS, mainly by J. Jakubík and E. Halušková. They published results related to general case of algebras as well to particular algebraic structures as mono-unary algebras, lattice ordered groups, cyclically ordered groups. Results for the universal case of algebras and the mentioned algebraic structures influence each other. It seems to be very hopeful to deal with other types of algebras in this context. Therefore, we see in our project an opportunity for productive interdisciplinary research with approach which is original and innovative in subjects of classical algebra.

Interdisciplinary means in areas Universal Algebra and some of branches of Classical Algebras as Group Theory, Ring Theory, Algebras over Fields, Commutative Algebras. We will investigate the questions formulated for general algebras in groups, rings, modules. In the case of unary algebras, such an investigation led to the discovery of a new class *K* of algebras with the property ***LL****(K) ≠* ***L****(K)*, where ***L****(K)* are direct limits of algebras of *K* and their isomorphic copies (Halušková, 1998). And conversely the universal formula for direct limit closed classes (Keisler, 1960) made possible to obtain many information about the lattice of direct limit closed classes of mono-unary algebras.

Let us mention two direct limit problems which are relevant in Universal Algebra and Model Theory (Halušková, 2016) and we will take them into account when solving the project.

1. Is every direct limit closed class axiomatic, i.e. determined by a set of formulas? It is known that there is a direct limit closed class of algebras which is not possible to express as an axiomatic one determined by some first-order sentences.
2. There is a class of first-order sentences ∆ such that every axiomatic class determined by some first-order sentences is closed with respect to direct limits if and only if it is an axiomatic class determined by some sentences from ∆. The question, whether an analogous statement about axiomatic classes generally is valid, is open. It is interesting even in the case of elementary classes.

Limit constructions in universal algebra are connected with endomorphisms of algebras, if we consider families of isomorphic algebras. It is very important to recognize what can produce such families for systematical study of these constructions. We hope to obtain some new results in this direction too. Endomorphisms of particular algebraic structures are currently being researched very intensively since so called endomorphism kernel property (EKP) and strong endomorphism kernel property (SEKP) have proven to be useful. The last published paper on this topic is

Jaroslav Guričan, Heghine Ghumashyan: [Strong endomorphism kernel property for finite Brouwerian semilattices and relative Stone algebras](https://articles.math.cas.cz/10.21136/MB.2023.0050-22), Math. Bohem., Published online on January 4, 2023. as doi: [10.21136/MB.2023.0050-22](https://articles.math.cas.cz/10.21136/MB.2023.0050-22)

In this paper you can find a list of 13 recent papers from this topic. We want to focus on EKP and SEKP for rings and linear spaces.

Our project is linked to European research space, since results or partial results will be presented and discussed at scientific events in various countries of Europe. Among them at

* regular algebra seminars in Warszaw,
* Workshops on General Algebra AAA, which are held twice a year at various universities in Europe, the next one will be the 104th in the series on February 8-11, 2024 in Blagoevgrad, Bulgaria

<https://aaa104.swu.bg/aaa104-2023/cfp>

* Summer Schools on General Algebra and Ordered Sets, the next one will be on September 2024 in Czechia.

**References:**

E. Halušková: On iterated direct limits of a monounary algebra, Contributions to General Algebra, 10 (Klagenfurt, 1997), Austria, Verlag Johannes Heyn, Klagenfurt, 1998, 189–195.

E. Halušková: On direct limit closed classes of algebras, Bulletin of the Section of Logic 45:3/4(2016), 155–169.

H. J. Keisler: Theory of Models with Generalized Atomic Formulas, The Journal of Symbolic Logic, Vol. 25, No. 1 (Mar.,1960), pp. 1–26.

1.3 METHODOLOGY

We will use methods and procedures typical for basic research in mathematics. In accordance with the goals of the project, we will focus on applying the results and methods of universal algebra to classical algebras. Further, we will investigate specially cases and then generalize them, do thought experiments, establish hypotheses and prove them (in some cases with the use of computing technology). We will follow the procedures of other mathematicians and creatively to develop them.

More specifically for individual directions of the project:

*1. EKP property for algebras, in particular for rings and groups*.

An algebra A has EKP if and only if every factor algebra of A is isomorphic to a subalgebra of A. So, we will focus on the question of how the property that A has an EKP is connected for the algebra A with properties of the congruence lattice of A. We are convinced that knowledge about the lattice of normal subgroups of a group and the lattice of ideals of a ring help to discover new algebras with EKP.

Challenge: Find new groups with EKP, since some classes of groups with EKP are described. All discovered groups with EKP are finite nilpotent groups, so we will start from infinite abelian ones.

*2. Algebras with easy direct limits*.

Firstly, we want to enlarge the list of groups and rings which are algebras with easy direct limits which we obtained on the research state 2022. Then we will try to generalize these examples to obtain statements which are useful to recognize if an algebra is with easy direct limits. Experiments will be directed both at properties specific to groups and rings, and at algebras in general.

Challenge: To answer the question if a simple algebra needs to be with easy direct limits. Here we need to choose appropriately simple algebras with endomorphisms that are not automorphisms. Then decide how to set up direct systems and analyze what happens when constructing limits.

*3. Classes of algebras closed with respect to direct limits*.

Finding out whether a class of rings is direct limit closed will mean first checking whether this class is axiomatic and, if so, whether the formulas that make it up are of a special form (Keisler, 1960 all formulas of finite length; Halušková, 2016 some formulas of infinite length).

Challenge: In many cases to check whether concrete class is axiomatic is complicated. And if so, the next non-trivial problem is to find out whether the sentences which determine it are equivalent to a set sentences of the form required for direct limits. We plan use the advantage of MI SAS that there are experts for theory of models (Repický, Eliáš) in this challenge.

The **multidisciplinary** approach is in the methods we use to achieve results. **Interdisciplinarity** is that through the search for solutions to problems from universal algebra, we will enrich knowledge in the fields of classical algebra and conversly.

Principles of **open science** are respected through the active participation an international scientific events and the submission of contributions to the arXiv internet platform and to open access journals.

Data collection, gender equality are not relevant.

1.4 EXCELLENCE OF THE RESEARCHER

Małgorzata Jastrzębska is a specialist in classical algebras, i.e. the theory of rings, groups, modules. She has received PhD. degree under the supervision of a renowned expert prof. Jan Krempa from Warsaw in this field. She published as the author or coauthor 7 papers in journals registered in WOS and SCOPUS, 5 in other scientific press. Many of her results are about the use of lattices in ring theory. For example she described rings with Boolean lattices of one-sided annihilators (2021). She presented her work on many scientific mathematical seminars and conferences, across Europe and in Slovakia too. Last year, she worked for 3 months at MI SAS as part of a research stay through the agency SAIA and collaborated very intensively with several researchers from MI SAS. Inter alia she recognized several significant examples of infinite groups and rings with EDL during her stay. We are convinced that she has more than sufficient competence for this project, she is able to meet its objectives and it is very productive to support her research activities.

Curriculum Vitae

**Personal information**

First and last name: Małgorzata Jastrzębska

Identifier ORCID: 0000-0002-6318-5271

Date of birth: 27.07.1982

Nationality: Polish

**Education**

04/2016 – PhD

Faculty of Mathematics, Informatics and Mechanics, University of Warsaw, Poland

06/2006 – Master

Faculty of Natural Sciences, Academy of Podlasie in Siedlce (currently Siedlce University of Natural Sciences and Humanities), Poland

**Current position/positions**

10/2016 –  Assistant Professor,

Institute of Mathematics, Siedlce University of Natural Sciences and Humanities, Poland

**Previous positions**

10/2012 – 09/2016 – Assistant

Institute of Mathematics and Physics (current Institute of Mathematics), Siedlce University of Natural Sciences and Humanities, Poland

**Scholarships and awards**

07/2022 – 09/2022 – Scholarship 1 – Mathematical Institute, Slovak Academy of Sciences, Slovakia

**Teaching activities**

2007 – non-permanent staff member of Warsaw University – Linear algebra, Mathematical Analysis I Mathematical Analysis II, Warsaw University, Poland

2012 – assistant, assistant professor – Linear algebra, Abstract algebra, Discrete Mathematics, Siedlce University of Natural Sciences and Humanities, Poland

2020- Erasmus+ Teaching Mobility, Catholic University Ružomberok, Slovakia, Number of teaching hours: 8

2022- Erasmus+ Teaching Mobility, University of Žilina, Slovakia, Number of teaching hours: 8

**Organisation of scientific meetings**

2015 – Member of the organizing committee of the 8th International Workshop on Computer Algebra Systems in Teaching and Research (CASTR'2015), 50 participants, Poland

**Reviewing activities**

2022 — Member of the editorial board and reviewer of the monograph: Wybrane zagadnienia informatyki technicznej, Postawy matematyczne, Bialystok University of Technology, Poland

2019 — Member of the scientific editorial board of the monograph: *Nauka młodych. Przeszłość, teraźniejszość, przyszłość : XVIII Konferencja Studenckich i Doktoranckich Kół Naukowych, Siedlce University of Natural Sciences and Humanities, Poland*

2018 – Reviewer, Asian-European Journal of Mathematics

**Memberships of scientific societies**

2016 – Member, Research Network Polskie Towarzystwo Matematyczne, Poland

**Major collaborations**

* Jan Krempa, algebras over fields, Faculty of Mathematics, Informatics and Mechanics, University of Warsaw, Poland
* Andrzej Walendziak, RM-algebras, Institute of Mathematics, Siedlce University of Natural Sciences and Humanities, Poland
* Małgorzata Elżbieta Hryniewicka, non-unital rings, Institute of Mathematics, Bialystok University of Technology, Poland
* Ján Haluška, application of vector space to music, Mathematical Institute Slovak Academy of Sciences, Slovakia

**Overview of the researcher’s most important projects in the last 5 years** (max. 5)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Project name/identification** | **Source of funding** | **Budget (EUR)** | **Project period** | **The role of the researcher in the project** |
| UPH/WNSP/IM/zadaniebadawcze/60/20/B | state budget in the form of statutory funding and grants for University in Siedlce | 700 EUR | 2020-2021 | Team Member |
| UPH/WNSP/IM/zadaniebadawcze/60/20/B | state budget in the form of statutory funding and grants for University in Siedlce | 700 EUR | 2022 | Manager |
| UPH/WNSP/IM/zadaniebadawcze/145/23/B | state budget in the form of statutory funding and grants for University in Siedlce | 2000 EUR | 2023 | Team Member |

**Overview of the researcher’s most important outputs** (max. 5)

|  |  |  |  |
| --- | --- | --- | --- |
| **Output name/identification** | **Type of output** | **Short description** | **The role of the researcher** |
| 2023,  Małgorzata Jastrzębska, Andrzej Walendziak, *Modal Operators on RM Algebras*,  Journal of Multiple-Valued Logic and Soft Computing, Vol. 40, 469–489. | Publication | Description of the properties of some algebras related to the logic. We consider some generalizations of BCK algebras such as for example  RM, tRM, aRM (= BH) algebras and others. We investigate modal operators on  such algebras. Furthermore, we establish connections between deductive systems and congruences for modal RM algebras. | co-author |
| 2022,  Ján Haluška, Małgorzata Jastrzębska,  *On a 4-Dimensional Subalgebra of the 12-Tone Equal Tempered Tuning*,  In : Computer Algebra Systems in Teaching and Research, Vol. 11, 20–37. | Publication | The article concerns algebraic description of some algebra related to the music.  An associative, commutative and distributive operation of multiplication on Euclidean vector space E\_4 is introduced by a skew circulant matrix. The resulting algebra W over R is isomorphic to CxC. The related algebraic, geometrical, and topological properties are given.There are subplanes of W isomorphic to the Gauss and Clifford complex number planes. A topology on W is given by a norm which is a sum of two norms. A hint how to apply this 4 dimensional algebra over R to the 12-tone Equally Tempered Tuning algebra is given. | co-author |
| 2022,  Talk „*On reduced rings*”, seminar „Ordered algebraic structures“ at Šafarik University, Faculty of Natural Sciences, Košice, Slovakia | Invited talk | The talk was about concerned an application of theory of lattices to describe reduced rings. The issues on the talk were closely related to the article: Małgorzata Jastrzębska, *Rings with Boolean Lattices of One-Sided Annihilators*, Symmetry, 2021, Vol. 13, no. 10:1909. | speaker |
| 2021,  Małgorzata Jastrzębska, *Zero-Divisor Graphs of Rings: Applications of Macaulay2 in Teaching and Research*, In: Computer Algebra Systems in Teaching and Research,  Vol. 10, 60–74. | Publication | The article shows how to use the Macaulay2 program to study the some properties of rings.  The aim of this paper is to present applications of Macaulay2 to determine and study graphs of zero-divisors, graphs of annihilating ideals and lattices of annihilators in associative, commutative rings | author |
| 2015,  Jan Krempa, Małgorzata Jastrzębska, *Lattices of annihilators in commutative algebras over fields*, Demonstratio Mathematica, , Vol. 48, no 4, p. 546-553. | Publication | The article concerns application of lattices to describe some classes of rings. Let K be any field and L any lattice. In this note we show that L is a sublattice of annihilators in an associative and commutative K-algebra. If L is finite, then our algebra is finite dimensional over K. | co-author |

1.5 EXCELLENCE OF THE APPLICANT/HOST ORGANISATION

Mathematical Institute of the Slovak Academy of Sciences is a scientific institute focused mainly on basic research in mathematics and theoretical informatics. The Institute has a long tradition in several important branches of pure and applied mathematics and participated in a number of successful projects in both basic and applied research, including projects of Frame Projects of EU, Structural projects of EU, and projects of domestic agencies APVV and VEGA. The researchers of the Institute belong to the top in their research, in a world-wide context, and are engaged in multiple collaborations with experts from internationally renowned institutions. In collaboration with the Commenius University, the Institute organises a PhD study program and many young scientists and students use the Slovak fellowship program SAIA for short term study stays at the institute.

Mathematical Institute SAS has been working for a long on topics from universal algebra, which are the subject of the project, currently E. Halušková and J. Pócs. These topics are also related to the issue of quantum structures, where we are a workplace of world importance. A comparative advantage is that experts in set theory, real and functional analysis also work at MI SAS. Workers regularly meet live at scientific seminars. This provides great potential for solving subtasks of the project.

The researcher will collaborate primarily with E. Halušková, who published 16 articles related to the project's issues in journals registered in databases WOS and Scopus. This cooperation successfully started in 2022, when M. Jastrzębska completed a 3-month research stay at MI SAS. The results of this stay were presented at the institute's seminars and at the international conference SSAOS which was from August 28 to September 2, 2022 in Tatranska Lomnica, Slovakia.

Mathematical Institute has a very good infrastructure with adequate hardware and software equipment. The library of Mathematical Institute SAS belongs to the best mathematical ones in Slovakia with access to many journal nets of the world publishing houses, scientific-information and technical databases, e.g. Mathematical Review, Zentralblatt Mathematik, Web of Knowledge, Scopus, etc. The regular seminars and consultations with researchers is expected to be an effective method of two-way knowledge transfer between the host organisation and the applicant.

## Impact

When solving the project, we will use standard mathematical procedures, they are **credible**. Expected results are at least 3 articles and 4 presentations at international scientific events. We estimated the number of articles according to the topics addressed in the project and taking into account the experience of M. Jastrzębska and th*e* possibilities of the employees of our institute with whom she will collaborate. We assume that the number of international mathematical events for which the topic of the project is relevant will be greater than 4. We have listed some of them in the section 1.2. Some we didn't list because we don't know if they will continue in post-covid era. Participation in workshops is of course limited by the physical and psychological capabilities of the researcher.

The **quality of the outputs** is guaranteed by the fact that the articles will be prepared for journals registered in the databases WoS and Scopus.

We consider active communication within our workplace on all matters related to the project to be the main **measure to maximize the results and impacts** of the project. Furthermore, regular information about the progress of the project at the institutional seminar as well as at the seminar Ordered Algebraic Structures, which is held at Faculty of Science Pavel Jozef Šafárik University in Košice.

### 2.1 THE WIDER IMPACT OF THE PROJECT

The expected impact of the project is to the following subjects according to the Mathematics Subject Classification (MSC) deployed from January 2020 by Mathematical Reviews and Zentralblatt für Mathematik:

08 General algebraic systems

12 Field theory and polynomials

13 Commutative algebra

15 Linear and multilinear algebra; matrix theory

16 Associative rings and algebras

17 Nonassociative rings and algebras

20 Group theory and generalizations

We expect impact beyond the direct scope of the project in some of the following subjects MSC:

01 General and overarching topics; collections

40 Sequences, series, summability

41 Approximations and expansions

92 Biology and other natural sciences

97 Mathematics education

**The direct and relevant scientific impact** is determined by expected results of the project. We expect at least 4 groups of scientific results

* New algebras with EKP or SEKP in rings and groups will be discovered.
* Some classes of rings and groups with EDL will be found and they will make it possible to obtain new results in systematic research of direct limit construction in Universal algebra.
* Overview of DL- classes of rings, groups, semigroups, modules, algebras over fields will be done together with a test if they are axiomatic.
* Some properties of the lattice of DL-classes of rings will be find out.

We see the **societal impact** during the durationof the project.It isin the enrichment of the collectives in which the researcher will work and also the events in which he will participate. Scientific employees of MI SAS and participants in mathematical workshops can benefit from activities of the project. Benefits are in new ideas that can develop and enrich their own work or in contributing to the output of this project, which will be realized by possible co-authorship of the article.We expect new collaborations.

The project has no **economic impact** at present. The research direction we propose here might lead to a better mathematical models of the world we live in and processes that take place in it. That means that this impact may increase in the future.

This project will give the researcher an opportunity to broaden her horizon, and to integrate her research with that of other research programs. Furthermore, the project would give the researcher more opportunities to visit conferences, which would also contribute to the growth of her network, to the possibilities for new collaborations, and to her visibility within the science community. All these aspects will have the **impact on researcher´s career**.The **researcher's skills** will develop in the ability to use the concepts, methods and procedures of universal algebra in her specialization. This has a short-term significance for the fulfilment of the project's goals, as well as a long-term one for interdisciplinary communication and cooperation. Let's illustrate this with the concept of retraction, which MJ adopted for her field last year during a research stay at MI SAS.

The notion of retract is important in universal algebra for many reasons, and as mentioned in section 1.1 of this project description, it is one of the key ones in this project. However, it is not used in classical algebra, so it was necessary to find out what it actually is. MJ found out that in the case of groups, retracts are those subgroups from which it is possible to obtain the original group by a direct sum with some normal subgroup of the same group, and in the case of rings, such subrings from which it is possible to obtain the original ring by a direct sum with some ideal of the same ring.

As we already mentioned impact on the applicant/host organization will be societal. However, we expect much more that the project will enrich the research of Emília Halušková. Further, results for groups can impact the research on fuzzy mathematics at MI SAS, since MV-algebras correspond with lattice ordered groups with a strong unit.

There is no potential negative impact of this research..

Potential obstacles to the planned impact of the project: This project has no potential legislative barrier. Competition might be a potential factor that impacts this project.   
This applies especially to EKP and SEKP for groups. Two papers was published for the finite case:

*J. Fang, Z.-J. Sun*: Finite abelian groups with the strong endomorphism kernel property. Acta Math. Sin., Engl. Ser. 36 (2020), 1076-1082.

H. Ghumashyan, J. Guričan: Endomorphism kernel property for finite groups, Math. Bohemica, vol. 147 (2022), issue 3, 347-358.

Finite groups are not solved completely. We plan start our project with this topic, concentrate on infinite groups to avoid an obstacle. Further, the cooperation with J. Guričan from Faculty of Mathematics, Physics an Informatics Comenius University in Bratislava is possible.

**Monitored data**The least numberof papers submitted to WoS/Scopus registered journals: 3  
The least number of presentations on international scientific conferences: 4  
The least number of other scientific presentations: 4

### 2.2 MEASURES TO MAXIMISE IMPACT – DISEMINATION AND COMMUNICATION, EXPLOITATION OF RESULTS

In order to maximize impact of the project results, the outcomes of the project will be published in distinguished journals with a good accessibility. Beside this, our achievements will be presented on conferences and workshops related to the scientific field of the project such as AAA series, SSAOS.

Any significant result of the project will be published immediately as preprint on arXiv, [https://arxiv.org](https://arxiv.org/). Then followed up, and validated by publications in a peer-reviewed journal. Furthermore, we will use the institute website to inform about the project.

Regular seminars organized by the MI SAS, or by institutes which the researcher will visit in the time of the duration of the project will help to spread results to a wider audience. The seminar of workplace MI SAS in Košice attend all scientific employees of this workplace. Therefore there is a great variability of opinions, observations, approaches and there is no interest in explaining all details of subject. The seminar Ordered algebraic structures at Faculty of Science Pavol Jozef Šafárik University in Košice is the seminar for experts in the area of universal algebra. There are detailed discussions and presentations expected. The seminar Algebra at Faculty of Mathematics, Informatics and Mechanics University of Warszaw is devoted to classical algebra and structures as rings, groups, semigroups, algebras over fields.

In addition to the above, ensure training for the researcher in preparing presentations and in presenting results both to the public and to the scientific community.

Technology transfer, commercialisation of project outputs and managing intellectual property rights are irrelevant in this project.

## Implementation

We divided the implementation of the project into four work packages WP1-WP4. The activities in them are directly determined by the tasks listed below in the tables for individual work packages. Their **feasibility** results from the nature of the mathematical work, the processes we use at MI SAS, experiences of the researcher and negligible implementation risks.

The **quality** of the project is guaranteed by the following facts:

* the fulfilment of the project's goals is a contribution to the development of European science,
* we emphasize the public presentation of the achieved results,
* MI SAS outputs.

Highly focused mathematical work is required for WP1-WP3. Its **efficiency** is guaranteed by the fact that each of these packages represents the researcher's concentration on exactly 1 topic from the project's objectives. In both the first and second packages, the key concept is the notion of endomorphism of the algebraic structure we are dealing with. In the second and third ones, we deal with the construction of the direct limit. Therefore, we assume that the transition between the packages will be smooth and in both WP2 and WP3 it will be possible to use the experience and some of the results of the work on the previous package. Another efficiency factor is that WP4 does not contain mathematical research and allows the researcher to use the time when concentrated mathematical work is not possible for the benefit of the project. Finally, the operational capacity of MI SAS is the important factor of efficiency.

3.1 PROJECT PLAN AND DELIVERABLES

The project is divided into these 4 work packages:

WP1 ................ EKP in rings and groups,

WP2 ................ Algebras with EDL,

WP3 ................ DL-classes,  
WP4 ................ Dissemination.

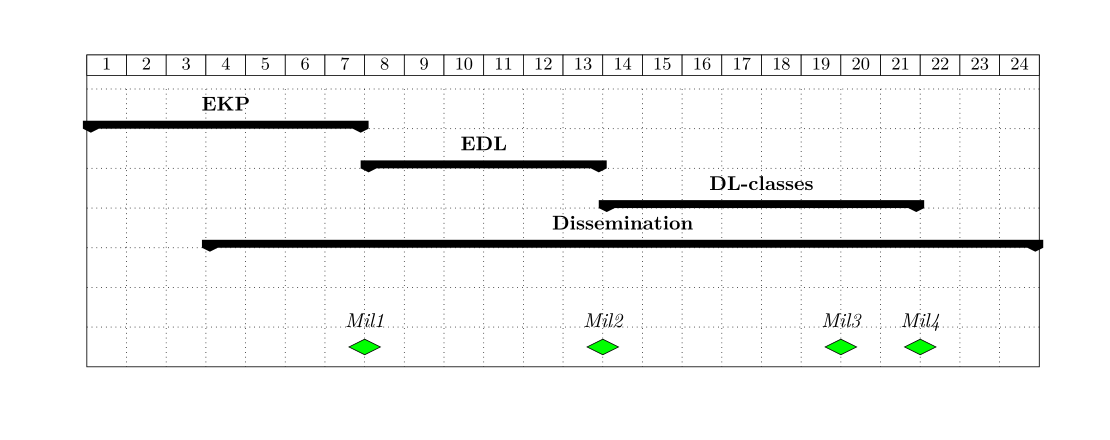
The first tree WPs correspond to the main directions and objectives of the project. Their content is intensive scientific work. The first one starts at the beginning of the project. They will follow immediately after each other. „DL-classes“ is the abbreviation of „direct limit closed classes“ or „classes which are closed with respect to direct limits”. We plan to stop the third package three months before the end of project. The reason is that we want to be concentrated to the tasks of the fourth work package in this time.

The last work package WP4 processes the results WP1-WP3. It serves for communication of project results and ongoing evaluation. We expect that first more relevant results of the project will appear in the fourth month and thus it will be possible to start fulfilling tasks of the work package 4. It will last until the end of the project.

As milestones, we determined the 4 most important expected scientific results of the project:

*Mil1............* New algebras with EKP or SEKP in rings and groups,  
*Mil2............* Some rings and groups with EDL and application of them in Universal algebra,  
*Mil3............* Overview of DL- classes of rings, groups, semigroups, modules, algebras over fields,  
*Mil4* ........... Some properties of the lattice of DL-classes of rings.

The duration of the project is planned for 24 months.   
All work packages are carefully designed to consistently capture all the individual steps necessary for the research planned in this project. Since the project belongs to the field of basic mathematical research, it is difficult to predict the exact time required to obtain specific results. However, the estimated period dedicated to solution of each work package and time to achieve milestones *Mil1*-*Mil4* can be found in the following timetable:



We plan to use the financial resources for publishing in Open Access, traveling to conferences and meetings with experts, buying a laptop and to pay for a lecturer to train the researcher's presentation skills.

3.1.1 Work packages

|  |  |
| --- | --- |
| Work package number | WP1 |
| Title of the work package | **EKP** |
| **Start of implementation of the work package** | M1 |
| End of implementation of the work package | M7 |
| **Involvement (expressed in Person Months)** | 6,2 Person Months |
| **Personnel costs (in EUR)** | 26 306,60 |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 3 286 + 3000 = 6286 |
| Objectives | |
| O1.1: Describe some classes of rings with EKP or SEKP.  O1.2: Describe some new classes of groups with EKP or SEKP | |
| Description of the work package | |
| Task 1.1: Study of recent publications related to the topic.  Task 1.2: Experiment with specific algebraic structures. Create a list of rings and groups, which came out with EKP or SEKP.  Task 1.3: Analyze properties of structures from the list created in Task 1.2. Task 1.4: Establishing hypotheses and proving them. | |
| Deliverables | |
| D1: **EKP seminar contributions** At least 2 seminar contributions about obtained and expected results of this work package will be given at scientific seminars held in Košice.  D2: **EKP paper** At least one article related to the topics of EKP or SEKP of algebraic structures investigated in this work package will be submitted for publication in a journal indexed in Elsevier/Scopus. | |

|  |  |
| --- | --- |
| Work package number | WP2 |
| Title of the work package | **EDL** |
| **Start of implementation of the work package** | M8 |
| End of implementation of the work package | M13 |
| **Involvement** | 4,4 Person Months |
| **Personnel costs (in EUR)** | 18 662,2 |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 2 332 + 4000 = 6332 |
| Objectives | |
| O2.1: Describe some classes of rings, groups with EDL.  O2.2: To solve whether the statement “If an algebra is simple, then it is with EDL” is true. | |
| Description of the work package | |
| Task 2.1: Do an overview of results about algebraic structures with EDL.  Task 2.2: Generalize obtained examples of groups and rings.  Task 2.3: Investigate EDL in structures defined on the set of complex numbers.  Task 2.4: Establish conjectures about simple algebras and prove them. | |
| Deliverables | |
| D3: **EDL seminar contributions** At least one seminar contributions about obtained and expected results of this work package will be given at scientific seminars held in Košice.  D4: **EDL paper** At least one article related to the topic of algebras with easy direct will be submitted for publication in a journal indexed in Elsevier/Scopus. | |

|  |  |
| --- | --- |
| Work package number | WP3 |
| Title of the work package | **DL-classes** |
| **Start of implementation of the work package** | M14 |
| End of implementation of the work package | M21 |
| **Involvement** | 5,9 Person Months |
| **Personnel costs (in EUR)** | 25 033,70 |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 3127 + 4000 = 7127 |
| Objectives | |
| O3.1: An overview of known classes of rings, groups, semigroups, modules, algebras over fields closed with respect to direct limits.  O3.2: Study the lattice of classes of rings closed with respect to direct limits. | |
| Description of the work package | |
| Task 3.1: Study of publications related to the topic.  Task 3.2: Prepare a list of classes closed with respect to direct limits from relevant publications studied in Task 3.1.  Task 3.3: Detect which of classes fom Task 3.2 are axiomatic.  Task 3.4: Detect if axiomatic classes from Task 3.3 are equivalent to the known formula for classes closed with respect to direct limits  Task 3.4: Find some properties of the lattice of classes of rings closed with respect to direct limits. | |
| Deliverables | |
| D5: **Presentations about DL-classes** At least 2 seminar contributions about obtained and expected results of this work package will be given at scientific seminars held in Košice.  D6: **Paper on DL-classes** At least one article related to the direct limit closed classes of will be submitted for publication in a journal indexed in Elsevier/Scopus. | |

|  |  |
| --- | --- |
| Work package number | WP4 |
| Title of the work package | **Dissemination** |
| **Start of implementation of the work package** | M4 |
| End of implementation of the work package | M24 |
| **Involvement** | 7,5 Person Months |
| **Personnel costs (in EUR)** | 31 822,50 |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 3 975 +15 000 = 18 975 |
| Objectives | |
| O4.1: Summarize results obtained in the project and their significance for the theoretical study and applications.  O4.2: Presentations of results on international scientific events | |
| Description of the work package | |
| Task 4.1:Summarize EKP results obtained in WP1 and their significance for the theoretical study and applications.  Task 4.2: Presentation of EKP results on an international scientific event.  Task 4.3:Summarize EDL results obtained in WP2 and their significance for the theoretical study and applications.  Task 4.4: Presentation of EDL results on an international scientific event.  Task 4.5:Summarize results obtained in WP3 and their significance for the theoretical study and applications.  Task 4.6: Presentation of results about direct limit closed classes of algebras on an international scientific event. | |
| Deliverables | |
| D7: **Presentations on conferences**  We will present results of this project at least three high-level international scientific conferences, while at least two of them will not be in Slovakia.  D8: **Interim report**  Report on the implementation and achievements of the project at mid-term of project implementation will be given.  D9: **New research plan**  We will develop a list of open questions and a research proposal for the next period with the active participation of the researcher.  D10: **Final report**  Report summarizing all results obtained during implementation of the project and comparison between achieved and expected results will be prepared. It will be presented at the end of the project implementation. | |

3.1.2 List of work packages:

|  |  |  |  |
| --- | --- | --- | --- |
| Work package number | Title of the work package | **Start of activities** | **End of activities** |
| 1 | EKP | M1 | M7 |
| 2 | EDL | M8 | M15 |
| 3 | DL-classes of algebras | M16 | M21 |
| 4 | Dissemination | M4 | M24 |

3.1.3 List of deliverables:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Deliverable number | Deliverable | Work package number | Type | Access and dissemination | Method of verification | Delivery |
| D1 | EKP seminar contributions | 1 | talk | P | Expert feedback | M4, M7 |
| D2 | EKP paper | 1 | publication | P | Peer review | M7 |
| D3 | EDL seminar contribution | 2 | talk | P | Expert feedback | M13 |
| D4 | EDL paper | 2 | publication | P | Peer review | M13 |
| D5 | Presentations about DL-classes | 3 | talk | P | Expert feedback | M17,M21 |
| D6 | Paper on DL-classes | 3 | publication | P | Peer review | M21 |
| D7 | Presentations on conferences | 1-4 | talk | P | Expert feedback | M10, M18,M23 |
| D8 | Interim report | 1,2,4 | report | N | Progress evaluation | M13 |
| D9 | New research plan | 1-4 | proposal | N | Proposal evaluation | M23 |
| D10 | Final report | 1-4 | report | N | Project evaluation | M24 |

3.1.4 List of milestones

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Milestone number | Milestone | Work package number | Method of  verification | Expected time to reach the milestone |
| 1 | New algebras with EKP or SEKP  in rings and groups | 1 | Peer review | M7 |
| 2 | Some rings and groups with EDL  and application of them in Universal algebra | 2 | Peer review | M13 |
| 3 | Overview of DL- classes of rings, groups, semigroups, modules, algebras over fields | 3 | Expert review | M19 |
| 4 | Some properties of the lattice  of DL-classes of rings | 3 | Peer review | M21 |

3.2 IMPLEMENTATION RISKS AND PROPOSED MEASURES

A list of risks and problems that may hinder the smooth implementation of the project consists mainly of problems with publication delays, either caused by publisher and an unexpectedly long review process, or if the studied problem turns out to be more complicated than expected. Further, due to the experiences from past years we must also take into account the possibility of a pandemic, which can cause a lockdown or similar obstacles. Since this project is focused on basic research, we do not expect any legislative issues. In order to avoid these problems, each risk will be assessed in terms of its severity and likelihood of occurrence. We will identify its triggers and indicators and develop the corresponding mitigation strategies and contingency plans.

3.2.1 Risks of implementation:

|  |  |  |
| --- | --- | --- |
| **Description of the risk of implementation (severity)** | **Work package** | Proposed measures for risk mitigation or elimination |
| Publication delays (medium) | WP1, WP2, WP3 | Check journal’s average publication time. Monitor the progress. Contact Editor after the internal deadline is crossed. |
| Task is too complicated (low) | WP1, WP2, WP3 | Contact experts working in the field for cooperation on the solution of the task. In the worst-case scenario – if none of the experts would be able to solve the task or give actionable recommendations, skip it and focus on subsequent tasks. |
| Pandemic (medium) | All WPs | Focus on online conferences, collaboration via online tools and use home office |

3.3 OPERATIONAL CAPACITY OF THE APPLICANT/HOST ORGANISATION

Research in mathematics doesn’t require much **staff, technical, infrastructure capacities***.* This is also the case with the implementation of this project. The most essential is an office with a desk, a working computer with good securityand access to internet*.* MI SAS has all this capacitiesavailable for every of its employeewithout restrictions and thereforeduring the whole project. Further, access to databasesisimportantfor a researcher*.* MI SAS provides access to scientific databases as Science Direct, Scopus, WoS, SpringerLink, SpringerNATURE, JSTOR, PLOS, PNAS, Taylor & Francis, Wiley Online Library, etc. Moreover, it provides access to many scientific journals and databases and high-level library service.

The researcher will work at the MI SAS workplace in Košice and cooperate especially with E. Halušková. There is a creative and friendly atmosphere at these workplace. In addition to universal algebra, **professionals** in the field of logic, complex analysis, and set theory (P. Eliáš, J. Haluška, J. Pócs, M. Repický) will be available here. It is likely that questions from these areas will arise when solving the project, so it is a great advantage to have the opportunity to contact and consult with them immediately. Renowned experts in mathematical structures work at MI SAS outside of Košice (Bratislava: A. Dvurečenskij, A. Jenčová, S. Pulmannová, Banská Bystrica: R. Nedela). The researcher will have the opportunity to consult with them online or visit them in person.

3.3.1 Description of the research/innovation infrastructure of the applicant/host organisation that is necessary for the implementation of the project:

|  |  |
| --- | --- |
| Name of infrastructure or equipment | Short description |
| Office | Desk, computer, internet |
| Supercomputer | MI SAS has access to the supercomputer DEVANA with performance 800 TFlop/s |
| Databases | MI SAS provides access to scientific databases as Science Direct, Scopus, WoS, SpringerLink, SpringerNATURE, JSTOR, PLOS, PNAS, Taylor & Francis, Wiley Online Library  PNAS, Taylor & Francis, Wiley Online Library, etc.  the Web of Science, Mathematical Reviews etc. |

3.3.2 List of the five most important projects of the applicant/host organisation and their relevance to the proposed project (in the last 5 years):

|  |  |  |
| --- | --- | --- |
| Project name/ identification | Programme/scheme/ grant provider | Short description |
| Mathematical support of quantum technologies/  NFP313010T683 | ITMS-2014+ | The aim of this project was to enable the main project activity: Independent research and development in the field of obtaining new knowledge about mathematical structures and functions. Among others algebraic structures, effect algebras, synapsis algebras, MV-algebras have been studied. |
| Probabilistic, algebraic and quantum mechanical methods of uncertainty determination/  APVV-20-0069 | APVV | Project led by prof. A. Dvurečeskij focuses on the mathematical foundations of quantum mechanics and uncertainty. Solvers deepen their knowledge of partial and total algebras such as effect algebras, MV-algebras, synaptic algebras, orthomodular unions, BL-algebras, EMV-algebras, wEMV-algebras, residue unions and their non-commutative generalizations and states on them in connection with partially organized groups. |
| Algebraic and topological aspects of aggregation functions/ VEGA 2/0097/20 | VEGA | J. Pocs et al. are working on the rapidly developing topic of aggregation functions. Methods and results from universal algebra, e.g. from clone theory, they have multiple uses here. |
| Selected Problems in Universal Algebra and Lattice Theory/ VEGA 2/0044/16 | VEGA | As part of this project, DL-classes of general algebras and properties EKP, SEKP for distributive lattices and mono-unary algebras were investigated. Responsible solver: M. Ploščica |
| Exceptional structures in discrete mathematics*/* APVV-19-0308 | APVV | Responsible solver is R.Nedela. Inter alia a representation of the automorphism group of a graph in a unimodular group and various properties of snarks has been investigated. |

3.3.3 List of maximum five most important outputs of the applicant/host organisation relevant to the submitted project:

|  |  |  |
| --- | --- | --- |
| Output name/identification | **Type of output** | Short description |
| A. Dvurečenskij, S. Pulmannová: *New Trends in Quantum Structures*. Dordrecht : Kluwer Academic ; Bratislava : Ister Science,  2000. 541+xvi pp. Accessible on https://doi.org/10.1007/978-94-017-2422-7. ISBN 0-7923-6471-6. | monography | This book presents some of the newest trends in the mathematical theory of algebraic structures arising in quantum mechanics. Properties of D-posets and effect algebras are studied, they are shown to be equivalent and closely related to partially ordered Abelian groups. |
| E. Halušková: *Some monounary algebras with EKP* Math. Bohem. 145 (2020), 401-414. | publication | All mono-unary algebras with an injective operation and EKP are described. |
| E. Halušková – M. Ploščica: *On direct limits of finite algebras*, Contributions to General Algebra, 11 (Olomouc, 1998, Velke Karlovice, 1998), Verlag Johannes Heyn, Klagenfurt, 1999, 101–104. | publication | The main result of this paper is that every finite algebra is the algebra with EDL. |
| E. Halušková: *On direct limit closed classes of algebras*, Bulletin of the Section of Logic 45:3/4(2016), 155–169. | publication | Algebras of type F are considered as structures of the model theory. Itis shown that the Keisler sentence from 1960 determines all first-order defined DL-classes. Further, DL-classes which are models of infinitely long sentence with finitely many quantifiers are obtained. |
| E. Halušková: *Monounary algebras with easy direct limits*, Miskolc Math. Notes 19(2018), No.1, 291–302. | publication | If A is a monounary with EDL, then A is countable and the number of retracts of A is not equal to the cardinality of integers. Further, we will see that the number of non-isomorphic monounary algebras with EDL is uncountable. |