Project description (template) for call 09I03-03-V04 Fellowships for excellent researchers R2-R4

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Title of the project: Chromatic problems and generalizations of Tutte polynomials

Short title of the project/Acronym: Chromatic problems

Category of researcher: R4

Researcher’s job type (full-time, part-time – in %): 100

Type of research[[2]](#footnote-3): basic

Identification of the entity involved in the implementation of the project:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Official name of the entity | Abbreviated name of the entity | Role in the project |
| 1 | Mathematical Institute of the Slovak Academy of Sciences | MI SAS | Applicant/host organisation |

## 1. Excellence

Researcher, RNDr. Martin Kochol, PhD., DSc., has been dealing with various aspects of graph coloring for a long time. In this area, he achieved significant results and significantly contributed to the development of methodology in the given area. The project is aimed at continuing research in this area.

The project is devoted to study of generalizations of the Tutte, characteristic, chromatic and flow polynomials. We plan to study relations of this polynomials and methods of linear algebra for solutions of Tutte-Grothendieck invariants.

In the project we plan to introduce new classes of polynomials generalizing Tutte and characteristic polynomials and were not studied until now. We plan to extend methods of linear algebra for research of the Tutte-Grothendieck invariants resently introduced by the researcher.

When studying the generalizations of Tutte polynomials, we propose to study and generalize both the principles applied so far, such as the principle of contraction and deletion, on the basis of which Tutte and characteristic polynomials are defined, but also Möbius functions, and in particular we plan to generalize the principle of cyclic bases that we introduced in the study of Tutte polynomials .

The The researcher has been engaged in the study of graph coloring and related polynomials for a long time, achieving world-class results in this area (construction of new classes of snarks, study of Tutte conjectures on nowhere-zero 3- and 5-flows).

The host organization, MÚ SAS, v.v.i., is a workplace focused on basic research in mathematics. Basic research in combinatorics has been significantly represented since the establishment of the host organization (Kotzig, Bosák, Tomasa). The host organization is the only workplace within the Slovak Academy of Sciences focused exclusively on basic research in mathematics and is among the top research instututions in Slovakia.

The host researcher is a long-term employee of the host organization, thus has a built-in infrastructure (office, computer equipment, office equipment) and is established there for a long time (social infrastructure, etc.). The host organization has a long-established doctoral student education system. In the project, the researcher will build on the previous material and technical security.

The project is oriented on the basic research in mathematics. In this type of research, the primary importance is given to the international cooperation with experts in the world. It is not a rule that there is an exchange of knowledge between the institution and the researcher.

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### 1.1 PROJECT OBJECTIVES

The aims of the project are

--introduction and study of generalizations of the Tutte and characteristic polynomials

We study generalizations of Tutte and characteristic polynomials in connections with homomorphisms of some classes of matroids. In this way we generalize polynomials charcterizing nowhere-zero chains in graphs, that we have introduced recently. We plan prepare two scientific paper one devoted to generalization of characteristic polynomilas and one devoted to generalizations of the Tutte polynomials.

--study of cyclic bases in connections with these invariants

Cyclic bases we have introduced by study of the Tutte and chromatic polynomials. We plan to apply this approach also for generalizations of the Tutte and characteristic polynomials. This approach we plan to apply in the papers devoted to the Tutte and characteristic polynomials.

--study of the Tutte polynomials in connections with methods of linear algebra applied for the Tutte-Grothendieck invariants.

We plan to study basic and advanced properties of the Tutte and characteristic polynomials. We plan to study connection of the coefficients arising by study some Tutte-Grothendieck invariants with generalizations of the Tutte and characteristic polynomials. We plan to prepare at least one scientific paper devoted to study of properties of generalizations of the Tutte and chromatic polynomials and at least one paper devoted to study of coefficients arising by study of some Tutte-Grothendieck invariants.

1.2 RELEVANCE, QUALITY AND NOVELTY OF THE PROJECT

Chromatic problems are at the centre of graph theory, a discipline at the interface of discrete mathematics, combinatorial optimization and computer science. These problems are important in order to understand the structure of graphs as well as the complexity of discrete algorithms. In their study, methods of cycles in graphs, linear algebra, groups, topologies, optimization methods and other techniques are used. In general, it can be said that chromatic problems are easy to formulate but very difficult to solve. They are directly related to complexity theory, as many of them belong to NP-complete problems.

A dual approach to chromatic problems is represented by nowhere-zero flows. A graph has a nowhere-zero k-flow if its edges can be oriented and evaluated with numbers from 1 to k-1 so that each vertex of the graph satisfies Kirchhoff's law (the sum of the values of the edges entering a vertex is equal to the sum of the values of the edges leaving it). Tutte's classic result says that a planar graph can be k-colored if and only if its dual graph has a nowhe-zero k-flow. Among the most famous problems in graph theory are 3- and 5-flow conjectures of Tutte. The 3-flow conjecture is that every graph without edge cuts of cardinality 1 and 3 has a nowhere-zero 3-flow. The 5-flow conjecture is that every bridgeless graph admits a nowhere-zero 5-flow.

The numbers of nowhere-zero flows and colourings in graphs are determined by flow and chromatic polynomials, whose dichromatic generalization is the Tutte polynomial. Tutte polynomials have applications far beyond the horizon of combinatorics, including areas such as knot theory, coding theory, statistical physics, etc., [BO], [EMM]. The researcher introduced new interpretations of Tutt's polynomials on matroids and matroid perspectives based on the so-called cyclic bases [K21], [K22b]. The researcher also generalized chromatic polynomials on graphs by introducing the so-called assigning polynomials that characterize numbers of nowhere-zero chains in graphs (non-homogenous variants of nowhere-zero flows) [K22a]. The researcher studied the properties of nowhere-zero flows in the context of the number of these flows on a selected set of edges, and showed that they satisfy certain quantitative relations that can be characterized by the language of linear algebra [K15]. Later he introduced these rules for other Tutte-Grothedieck invariants [K23a].

The researcher in the project plans to generalize assigning polynomials to different classes of matroids, to introduce and investigate their dichromatic variants, to investigate their structure especially from the point of view of cyclic bases. We also plan to study these new polynomials in terms of classical combinatorial functions such as the Möbius function and the zeta function. The connection with these functions and the Tutte polynomials was already known in [Cr].

In the project, we also plan to study the connection between generalizations of the Tutte polynomials and the application of linear algebra in solving problems related to Tutte-Grothendieck invariants. We plan to show that the coefficients arising by applications of methods of linear algebra can be evaluated by these polynomials.

[BO] T. Brylawski, J. Oxley, The Tutte polynomial and its applications, in: Matroid Applications, (N. White, Editor),Cambridge University Press, Cambridge (1992), 123-225.

[Cr] H.H. Crapo, The Tutte polynomial, Aequationes Math. 3 (1969) 211–229.

[EMM] J. Ellis-Monaghan, I. Moffat (Editors), Handbook of the Tutte Polynomial and Related Topics, CRC Press, Boca Raton, FL, 2022.

[K02] M. Kochol, Superposition and constructions of graphs without nowhere-zero k-flows, European Journal of Combinatorics 23 (2002) 281–306.

[K15] M. Kochol, Quantitative methods for nowhere-zero flows and edge-colorings, in: M. Dehmer, F. Emmert-Streib, Quantitative Graph Theory: Theoretical Foundations and Applications, pp. 141–180, Chapman and Hall/CRC Press, Boca Raton, FL, 2015.

[K21] M. Kochol, Interpretations of the Tutte and characteristic polynomials of matroids, Journal of Algebraic Combinatorics 53 (2021) 1–9.

[K22a] M. Kochol, Polynomials counting nowhere-zero chains in graphs, The Electronic Journal of Combinatorics 29(1) (2022) #P1.19.

[K22b] M. Kochol, Interpretations for the Tutte polynomials of morphisms of matroids, Discrete Applied Mathematics 322 (2022) 210–216.

[K23a] M. Kochol, Linear algebraic relations among cardinalities of sets of matroid functions, Mathematics 11(11) (2023) 2570.

Generalizations of the Tutte polynomials as dichromatic assigning polynomials have not been studied so far. Coefficients arising from the linear algebra methods introduced by us for the study of Tutte-Grothendieck invariants in connection with generalizations of the Tutte polynomials were also not studied. We also plan to generalize the interpretations of these polynomials by studying cyclic bases.

These are new perspectives on the given issue, which were only recently introduced by the researcher. The innovativeness and originality of the issue is also proven by the fact that the researcher's previous results in this area have been published in top mathematical journals focused on combinatorics.

The study of the Tutte polynomials and chromatic problems is in the interest of the worldwide scientific community, which is also strongly represented in Europe. We note that in a recent publication dedicated to the overview of results and applications of the Tutte polynomials [EMM], there are a lot of researchers from Europe (the editors work in the Netherlands and the United Kingdom of Great Britain and Northern Ireland).

1.3 METHODOLOGY

The project will be implemented through basic research in mathematics. We will apply standard methods of mathematics and combinatorics. The basic method used in the study of Tutt polynomials is the principle of contraction and omission of elements. It is an approach applicable to several invariants of the theory of graphs and matroids, consisting in the fact that the value of a given invariant on a matroid can be expressed by the values of invariants on matroids arising after the omission and contraction of (any) element of the matroid. Alternatively, properties of classical combinatorial functions such as Möbius and zeta functions can be used. We plan to apply variants of this principle in the project as well. We also plan to apply linear algebra methods.

We plan to use the proposed methods in mathematical proofs of the planned goals.

We assume to use mathematical proofs by implementing the mentioned methods. In the case of applying linear algebra methods for solving problems on specific invariants, it is also possible to use calculations (these had to be used when calculating the ranks of huge matrices when determining circuit restrictions for the smallest counterexamples of the 5-flow conjecture).

We do not envisage the use of research data or other research outputs within the project (as a rule, this is not used in basic research in mathematics).

Since the researcher plans to work independently, it is not necessary to take into account the interpersonal relations between the project workers. Basic research in mathematics essentially applied principles of open science and FAIR access to research data in a natural way.

1.4 EXCELLENCE OF THE RESEARCHER

Curriculum Vitae

**Personal information**

First and last name: Martin Kochol

Identifier: www.mat.savba.sk/~kochol

Date of birth: May 11, 1961

Nationality: Slovak Republic

Website (if relevant): www.mat.savba.sk/~kochol

**Education**

11/1990 – PhD. Mathematical Institute of the Slovak Academy of Sciences, Slovakia

06/1985 – RNDr. Faculty of Mathematics and Physics, Comenius University, Slovakia

**Current position/positions**

05/2007 – Principar Research Fellow, I.

Mathematical Institute of the Slovak Academy of Sciences, Bratislava, Slovakia

05/2006 – DSc. Akademy of Sciences of Czech Republic, Czech Republic.

**Previous positions**

06/1993 – 05/2007 – Senior Research Fellow, IIa

Mathematical Institute of the Slovak Academy of Sciences, Bratislava, Slovakia

11/1990 – 06/1993 - Research Fellow, IIb,

Mathematical Institute of the Slovak Academy of Sciences, Bratislava, Slovakia

08/1987 – 11/1990 – Research Emploee

Mathematical Institute of the Slovak Academy of Sciences, Bratislava, Slovakia

09/1985 – 07/1987 - Assistant

Institute of Applied mathematics and Computing Technique, Comenius University, Bratislava, Slovakia

**Scholarships and awards**

03/1997 – 02/1998 – JSPS Postdoctoral Fellowship – Keio University, Yokohama, Japan, Japan Society for the Promotion of Science (JSPS), Japan

03/1998 – 08/1999 – Alexander von Humboldt Fellowship – Freie Universität Berlin, Germany, Alexander von Humboldt Foundation, Germany

1985 – Rector Prize for the Study and Research Results – Comenius University, Slovakia

1995 – Vinner of Competion of Young Mathematicians – Union of Slovak Mathematics and Physics, Slovakia

Student and post-docs supervision (if applicable)

2007 – 2010 - three finished PhD Students

Faculty of Natural Sciences, University of Žilina, Slovakia

**Teaching activities (if applicable)**

2020 – external pedagogical position – course: Analyzis and Complesity of Algorithms, Faculty of Informatics and Information Technologies, Slovak Technical Univerzity, Bratislava, Slovakia

2008 – external pedagogical position – course: Discrete Mathematics and Logic, Faculty of Electrical Engineering, Slovak Technical Univerzity, Bratislava, Slovakia

2007 – external pedagogical and research position – course for PhD students: Combinatorics and Grpah Theory, Faculty Natural Sciences, University of Žilina, Slovakia

2003 – visiting position – two courses: Second Year Accelerated Calculus, Vanderbilt University, Nashville, TN, USA

2002 – visiting position – two courses: Combinatorics and Linear and Discrete Mathematics, Georgia Institute of Technology, Atlanta, GA, USA

1990 – 1995 – external pedagogical position – courses: Linear Algebra, Discrete Mathematics, matematika, Faculty of Electrical Engineering, Slovak Technical Univerzity, Bratislava, Slovakia

1988 – 1989 – external pedagogical position – courses: Algebra and Algebra and Theoretical Arithmetics, Faculty of Mathematics and Physics, Comenius University, Bratislava, Slovakia

**Organisation of scientific meetings (if applicable)**

**Institutional responsibilities (if applicable)**

2003 – 2004 – nenber of the Scientific Council of the Faculty, Faculty of Natural Sciences, University of

Žilina, Slovakia

1998 – 2002 – member of the Commitee for Defence of PhD Thesis in Branch 11-11-9 Discrete

Mathematics, Ministry of Education, Slovakia

2006 – 2010 – organization of internal seminar for PhD students, Graph Theory, Faculty of Natural

Sciences, University of Žilina, Slovakia

2008 – member of a Commitee for Defence of a PhD Thesis, Faculty of Natural Sciences, University of

Žilina, Slovakia.

**Reviewing activities (if applicable)**

2008 – 2017 – member of Editorial Board, International Journal of Combinatorics, Hindawi, India

Reviewer for Mathematical Reviews, Zentralblatt, Journal of Combinatorial Theory Seires A and Series B, Journal of Graph Theory, Discrete Mathematics, Discrete Applied Mathematics, European Journal of Combinatorics, The Electronic Journal of Combinatorics, Annals of Combinatorics, Journal of Combinatorics, Annals of Mathematics, etc.

**Memberships of scientific societies (if applicable)**

1985 – member, Union of Slovak Mathematicians and Physicists

1999 – member, Humboldt club in Slovakia

2007 – member, Mathematical Programming Society

2003 – 2016, member, American Mathematical Society

**Major collaborations (if applicable)**

The researcher cooperates or cooperated with many scientists in the world:

Hikoe Enomoto, Keio University, Yokohama, Japan

Martin Aigner, Freie Universität Berlin, Germany

Jaroslav Nešetřil, Charles University, Praha, Czech Republic

Herbert Flieschner, Austrian Academy of Scientists, Austria

Andreas Huck, Universität Hannover, Germany

Andre Raspaud, Bordeaux University, France

Vadim Lozin, University of Warwick, Coventry, UK

Iain Moffatt, Royal Holloway University of London, Egham, UK

Mark Ellingham, Vanderbilt university, Nashvill, TN, USA

József Balogh, University of Illinoi at Urbana-Champaign, Illinoi, USA

László Lovász, Eötvös Loránd University, Budapest, Hungary

Gyula O.H. Katona, Alfréd Rényi Institute of Mathematics, Budapest, Hungary

**Overview of the researcher’s most important projects in the last 5 years** (max. 5)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Project name/identification** | **Source of funding** | **Budget (EUR)** | **Project period** | **The role of the researcher in the project** |
| Chromatic Problems  and Polynomials/  2/0042/22 | VEGA, Scientific Grant Agency of Ministry of Education and SAS | 3444 € | 2022 – 2025 | Head of the project |
| Chromatic Problems  in Combinatorics/  2/0024/18 | VEGA, Scientific Grant Agency of Ministry of  Education and SAS | 5204 € | 2018 – 2021 | Head of the project |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

**Overview of the researcher’s most important outputs** (max. 5)

|  |  |  |  |
| --- | --- | --- | --- |
| **Output name/identification** | **Type of output** | **Short description** | **The role of the researcher** |
| M. Kochol, Snarks without  small cycles, Journal of  Combinatorial Theory Series  B, 67 (1996) 34–47 | publication | Construction of snarks with  arbitrary large girth, a  counterexample to a conjecture of  Jaeger and Swart | author |
| M. Kochol, An equivalent  version of the 3-flow  conjecture, Journal of  Combinatorial Theory Series B, 83 (2001) 258–261 | publication | Reduction of the 3-flow conjecture to edge-5-connected graphs | author |
| M. Kochol, Polyhedral  embeddings of snarks in  orientable surfaces,  Proceedings of the  American Mathematical  Society, 137 (2009) 1613–  1619 | publication | Construction of snarks with  polyhedral embedding in an  orientable surface of genus at least  5, a counterexample to a  conjecture of Grünbaum |  |
| M. Kochol, Reduction of the  5-flow conjecture to  cyclically 6-edge-connected  snarks, Journal of  Combinatorial Theory Series B, 90 (2004) 139–145 | publication | Reduction of the 5-flow conjecture  on 3-regular graphs with cyclic  connectivity at least 6 | author |
| M. Kochol, Smallest  counterexample to the 5-  flow conjecture has girth at  least eleven, Journal of  Combinatorial Theory Series B, 100 (2010) 381–389 | publication | Reduction of the 5-flow conjecture  on 3-regular graphs with girth at  least 11 | author |

The most important results of the researcher are the introduction of superposition as a construction of graphs without nowhere-zero flows and the introduction of linear algebra techniques to solve problems of some Tutte-Grothendieck invariants. These are the outputs of basic research in mathematics consisting of the publication of results in scientific journals.

The general principle of superposition is studied in detail in [K02]. Using this method, we constructed new classes of snarks (non-trivial 3-regular graphs without edge-3-coloring). This is an important class of graphs, the study of which has much to do with the solution of the most important open problems in combinatorics (the 5-flow conjecture and the cycle double cover conjecture) as with complexity theory. We constructed snarks with arbitrarily large girth [K96], disproving the Jeager and Swart conjecture of 1980, and polyhedrally embeddable snarks in any orientable surface of genus at least 5 [K09], disproving Grünbaum's conjecture of 1968. The superposition method was also used in the proof that the smallest counterexample to the 3-flow conjecture of Tutte (every graph without edge cuts of order 1 and 3 has a nowhere-zero 3-flow) is edge 5-connected [K01]. It is known that every edge 6-connected graph has a nowhere-zero 3-flow [LTWZ13].

The general approach of introducing linear algebra methods for solving some Tutte-Grothendieck invariants is introduced in [K23a]. We previously used these methods to investigate the smallest counterexamples to Tutt's 5-flow conjecture (every bridgeless graph has a nowhere-zero 5-flow). We proved that the smallest counterexample to the 5-flow hypothesis is a cyclic edge-6-continuous snark [K04] and has girth at least 11 [K10]. We note that it is not known whether such snarks even exist.

We have also introduced a tiling principle on Möbius strip, and these ideas we have used by constructions of infinite families of crossing-critical graphs with crossing number at least 2 [K87]. We have also constructed asymtotically optimal approximation of n-dimensional balls by polytopes [K94] and prove the cycle double cover conjecture for a class of 3-regular graphs with low degree of non edge-3-colorability [HK95].

[K87] M. Kochol: Construction of crossing-critical graphs, Discrete Mathematics 66 (1987) 311–313.

[K94] M. Kochol: Constructive approximation of a ball by polytopes, Mathematica Slovaca 44 (1994)

99–105.

[HK95] A. Huck, M. Kochol: Five cycle double covers of some cubic graphs, Journal of Combinatorial

Theory Series B 64 (1995) 119–125.

[K96] M. Kochol: Snarks without small cycles, Journal of Combinatorial Theory Series B 67 (1996) 34–47

[K01] M. Kochol: An equivalent version of the 3-flow conjecture, Journal of Combinatorial Theory Series

B 83 (2001) 258–261.

[K02] M. Kochol: Superposition and constructions of graphs without nowhere-zero *k*-flows, European

Journal of Combinatorics 23 (2002) 281–306.

[K04] M. Kochol: Reduction of the 5-flow conjecture to cyclically 6-edge-connected snarks, Journal of

Combinatorial Theory Series B 90 (2004) 139-145.

[K09] M. Kochol: Polyhedral embeddings of snarks in orientable surfaces, Proceedings of the American

Mathematical Society, 137 (2009) 1613–1619.

[K10] M. Kochol, Smallest counterexample to the 5-flow conjecture has girth at least eleven, Journal of

Combinatorial Theory, Series B 100 (2010) 381-389.

[LTWZ13] L.M. Lovász, C. Thomassen, Y.Wu, C.-Q. Zhang, Nowhere-zero 3-fows and modulo

-orientations, Journal of Combinatorial Theory Series B 103 (2013) 587-598.

[K23a] M. Kochol, Linear algebraic relations among cardinalities of sets of matroid functions,

1.5 EXCELLENCE OF THE APPLICANT/HOST ORGANISATION

The host organization, MI SAS, is an organization devoted to basic research in mathematics. The host institution is the only institute of the Slovak Academy of Sciences devoted exlusively to basic research in mathematics and belongs to top research organization in Slovakia. Basic research devoted to combinatorics was introduced in the host organization from its beginning.

The researcher is a long-term employee of the host organization. That's why it can seamlessly build on the previous work. Any change of the host organization within Slovakia for a given project has no meaning either from an organizational point of view or from the point of view of the quality of the research workplace. Any change would only cause difficulties from an organizational as well as a personal point of view.

The host organization will provide the researcher with technical support (office, office supplies, computer equipment, access to literature through the library and access to various databases). The host organization does not expect special monitoring of the researcher, as he is a senior researcher who works independently for a long time and achieves excellent scientific results that are well received by the professional public.

The host organization has a long-standing system of doctoral studies in cooperation with the Faculty of Mathematics, Physics and Informatics of the Comenius University in Bratislava. The researcher also has experience in supervising doctoral students. Until now, he supervised three doctoral students who successfully completed their doctoral studies under his supervision.

The scientific and professional staff of the host organization regularly (annually) announce the achieved results, thereby also participating in the preparation of annual reports of the MI SAS as well as Accreditation reports. All these reports are publicly accessible. Information on ongoing and completed projects within the MI SAS is also published. This practice will also be applied in the case of this project, which will ensure a transparent transfer of knowledge between the researcher and the workplace.

## Impact

The researcher has been working in basic research for a long time and mainly focuses on combinatorics and theoretical informatics. In the proposed project, he plans to continue researching the issue in which he has been working recently and achieved interesting results that were published in top journals focused on combinatorics, discrete mathematics and theoretical informatics. It is natural to assume output in the form of publications in journals of similar quality. As a rule, basic research in mathematics has the most important outputs in the form of publications in scientific journals.

The researcher realizes his scientific career by achieving the results of basic research and publishing them in the scientific literature. Also, the host organization, focused on basic research in mathematics, realizes outputs by publishing the results of scientific and professional workers. Since the goal of the project is basic research and achieving results, it is a natural realization of the work of the researcher as well as the host organization.

As a rule, basic research in mathematics has a direct impact on the field of knowledge for the scientific community in the world. We anticipate this impact in the project as well.

We expect results from the very current and intensively studied issue of discrete mathematics, chromatic problems in combinatorics. Therefore, it is possible to expect quality results important for scientific knowledge. As part of the project, we expect the preparation of at least four scientific articles by the researcher, which represents a sufficient output of the project even from a quantitative point of view.

The researcher's previous scientific activity represents a sufficient qualitative guarantee for achieving the planned goals of the

The project is focused on basic research in mathematics, it will be carried out by a researcher, therefore specific management of intellectual property rights is not foreseen. project.

### 2.1 THE WIDER IMPACT OF THE PROJECT

The output of the project will be new knowledge of basic research. The form of output will be the publication of articles in professional journals and their presentation at scientific conferences. This is the impact of the project. Since the applications of Tutte polynomials extend beyond the boundaries of combinatorics, it is possible to expect similar applications to the results of the project in the long term.

By working on the project, the researcher will gain new knowledge regarding Tutt's polynomials and their applications. High-quality results are expected, which automatically have a positive impact on the researcher's career growth.

The project envisages the output of several articles published in high-quality scientific journals, which is the goal of the host organization as a scientific workplace.

The target group that will benefit from the project is the scientific community engaged in basic research in mathematics. This is a target group that generally benefits from the results of basic research in mathematics.

The direct impact is assumed to be new scientific knowledge in mathematics.

As it is basic research in mathematics, we do not expect any negative impact

We assume the participation of one researcher (project leader).

Patent applications for basic research in mathematics are not applicable, so these are not expected.

We expect at least four publications in professional journals. We also plan to present the results at scientific conferences.

Now we are not planning direct international cooperation within the planned project. It is also a consequence of the researcher's limited funds for travel expenses. In the case of a successful application with planned financial support, it is possible to expect a significant increase in the researcher's contacts with scientists working on related issues who are exclusively abroad.

The results and impact of the project were carried out by the researcher on the basis of the previous results in the given issue, his previous experience and also the previous results and methods used in the given issue.

As the project is focused on basic research, we do not foresee any potential obstacles until the cancellation of the hosting organization or deterioration of the overall social climate due to force majeure (pandemic, war, natural disaster, etc.)

### 2.2 MEASURES TO MAXIMISE IMPACT – DISEMINATION AND COMMUNICATION, EXPLOITATION OF RESULTS

To maximize the impact, we plan to publish the results in top scientific journals focused on combinatorics and also present the results at scientific conferences.

The project focused on basic research in mathematics does not envisage specific technology transfer and commercialization of outputs.

The results of the project will be publicly accessible to the scientific community, as is usual for basic research in mathematics.

Intellectual property rights are secured by relevant periodicals. For basic research in mathematics, we do not assume the protection of commercial use.

## Implementation

Dividing the project into stages and packages is difficult to implement in this type of project, because the individual stages overlap with each other. Nevertheless, the project is formally divided into 4 basic packages. These are results that follow the researcher's recent work, which guarantees their feasibility.

Individual work packages are related to each other and interconnected. The basis is the research of generalizations of Tutte polynomials, which the other packages follow.

As a result of each of the packages, we plan at least one publication in a scientific journal. The project is planned for 2 years, while the time to publish a scientific article is several years for some journals. It can be expected that at the end of the project solution period, the results will be prepared in the form of manuscripts and sent to the editorial offices of the journals. The specific date of the final publication cannot be guaranteed in advance, as it may take several years.

We plan to present the results of the project at scientific conferences. Most scientific conferences focused on combinatorics do not have an established system of publications in proceedings, as is customary in computer science. Therefore, we plan to present the results mainly at high-quality international conferences focused on combinatorics, while we direct our publication activity towards publication in scientific journals.

We do not assume implementation risks. We consider the proposed measures to be sufficient, as they are based on the researcher's previous experience.

There is a real risk of quarantine measures that could limit participation in scientific conferences. Participation in scientific conferences is only an additional form of presentation of results in the given project, while we consider publication in scientific journals to be the most important, where there are no restrictions related to quarantine measures.

The project is focused on basic research in mathematics. The researcher plans to work independently, which includes preparing and writing articles. We plan to provide technical and infrastructural support for the project within the host organization.

As part of the project, we are also planning a contribution to the research team in the amount of 2500 € per month, which we plan to use as expenses for collaborators in the case of massive calculations as well as potential doctoral students.

3.1 PROJECT PLAN AND DELIVERABLES

The project is focused on basic research in mathematics. The researcher plans to work independently, which includes preparing and writing articles. We plan to provide technical and infrastructural support for the project within the host organization.

We plan 6 months for each of the packages. We plan to deal with individual packages according to the following plan: generalizations of characteristic polynomials in months M1-M6, generalizations of the Tutte polynomials in months M7-M12, basic properties of generalized Tutte polynomials in months M13-M18, the Tutte polynomials and coefficients in the application of linear algebra in combinatorics in months M19-M24.

For the entire project, lasting 24 months, we are planning Personnel costs of 189 504 € (consisting of salary costs for the researcher of 129 504 € and an allowance for the research team of 60 000 €) and Other eligible costs excluding personnel costs without VAT of 55 680 € (consisting of a research allowance excluding VAT in the amount of 26 000 € and a contribution to indirect costs excluding VAT for the institution of 29 680 € as indicated in Appendix No. 1, where these items are automatically calculated from the monthly contribution to the research team in the amount of 2500 €). We indicate ¼ of the respective amount for each of the packages.

3.1.1 Work packages

|  |  |
| --- | --- |
| Work package number | 1 |
| Title of the work package | Generalizations of characteristic polynomials |
| **Start of implementation of the work package (Mx Month[[3]](#footnote-4))** | M1 |
| End of implementation of the work package (Mx month) | M6 |
| **Involvement (expressed in Person Months)[[4]](#footnote-5)** | 6 Person Months |
| **Personnel costs (in EUR)[[5]](#footnote-6)** | 47 376 € |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 13 920 € |
| Objectives | |
| Get generalizations of characteristic polynomials, study their relationship with classical characteristic polynomials and various implementations of these polynomials | |
| Description of the work package | |
| We will focus on generalizations of characteristic polynomials and their relationship with classical characteristic polynomials. We also plan to study different implementations of these polynomials. | |
| Deliverables | |
| Output number: 1. It will be a scientific article devoted to generalizations of characteristic polynomials, their connection with classical characteristic polynomials and various implementations of these polynomials. | |

|  |  |
| --- | --- |
| Work package number | 2 |
| Title of the work package | Generalizations of the Tutte polynomials |
| **Start of implementation of the work package (Mx Month[[6]](#footnote-7))** | M7 |
| End of implementation of the work package (Mx month) | M12 |
| **Involvement (expressed in Person Months)[[7]](#footnote-8)** | 6 Person Months |
| **Personnel costs (in EUR)[[8]](#footnote-9)** | 47 376 € |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 13 920 € |
| Objectives | |
| Get generalizations of the Tutte polynomials, study their relationship with the classical Tutte polynomials and various implementations of these polynomials | |
| Description of the work package | |
| We will focus on generalizations of the Tutte polynomials and their relationship with the classical Tutte polynomials. We also plan to study different implementations of these polynomials. | |
| Deliverables | |
| Output number: 2. It will be a scientific article devoted to generalizations of the Tutte polynomials, their connection with the classical Tutte polynomials and various implementations of these polynomials. | |

|  |  |
| --- | --- |
| Work package number | 3 |
| Title of the work package | Properties of the generalized Tutte and chromatic polynomials |
| **Start of implementation of the work package (Mx Month[[9]](#footnote-10))** | M13 |
| End of implementation of the work package (Mx month) | M18 |
| **Involvement (expressed in Person Months)[[10]](#footnote-11)** | 6 Person Months |
| **Personnel costs (in EUR)[[11]](#footnote-12)** | 47 376 € |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 13 920 € |
| Objectives | |
| To gain knowledge about the properties of generalized Tutte and characteristic polynomials. | |
| Description of the work package | |
| We will study the basic properties of generalized Tutte and characteristic polynomials. | |
| Deliverables | |
| Output number: 3. It will be a scientific article devoted to the properties of generalized Tutte and characteristic polynomials. | |

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| --- | --- |
| Work package number | 4 |
| Title of the work package | The Tutte polynomials and coefficients in the application of linear algebra in combinatorics |
| **Start of implementation of the work package (Mx Month[[12]](#footnote-13))** | M19 |
| End of implementation of the work package (Mx month) | M24 |
| **Involvement (expressed in Person Months)[[13]](#footnote-14)** | 6 Person Months |
| **Personnel costs (in EUR)[[14]](#footnote-15)** | 47 376 € |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 13 920 € |
| Objectives | |
| To gain knowledge about the coefficients arising from the application of linear algebra in combinatorics and their connection with generalized Tutte polynomials. | |
| Description of the work package | |
| We will focus on coefficients arising from the application of linear algebra in combinatorics and study their connection with generalized Tutte polynomials. | |
| Deliverables | |
| Output number: 4. It will be a scientific article devoted to the coefficients arising from the application of linear algebra in combinatorics and to study their connection with generalized Tutte polynomials. | |

3.1.2 List of work packages:

|  |  |  |  |
| --- | --- | --- | --- |
| Work package number | Title of the work package | **Start of activities** | **End of activities** |
| 1 | Generalizations of characteristic polynomials | M1 | M6 |
| 2 | Generalizations of the Tutte polynomials | M7 | M12 |
| 3 | Properties of the generalized Tutte and chromatic  polynomials | M13 | M18 |
| 4 | The Tutte polynomials and coefficients in the application  of linear algebra in combinatorics | M19 | M24 |
|  |  |  |  |

3.1.3 List of deliverables:

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| --- | --- | --- | --- | --- | --- | --- |
| Deliverable number | Deliverable | Work package number | Type | Access and dissemination | Method of verification | Delivery (project implementation month) |
|  |  |  |  | P= public  N= non-public, |  |  |
| 1 | Interim  Report | 1 and 2 | Report | N | report | M13 |
| 2 | Final  Report | 1, 2, 3, 4 | Report | N | report | M24 |
| 3 | Scientific  papers | 1, 2, 3, 4 | Published  papers in  scientific  journals or  Technical  Reports | P | Publication  or report | continuously |
| 4 | Outputs or  presentations  in scientific  conferences | 1, 2, 3, 4 | Talks or  outputs in  conferences | P | Publication  or report | continuously |
|  |  |  |  |  |  |  |

3.1.4 List of milestones:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Milestone number | Milestone | Work package number | Method of verification | Expected time to reach the milestone (project month) |
| 1 | Characteristic and the  Tutte polynomials | 1, 2 | Interim Report | M13 |
| 2 | End of the project | 3, 4 | Final Report | M24 |
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3.2 IMPLEMENTATION RISKS AND PROPOSED MEASURES

3.2.1 Risks of implementation:

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| --- | --- | --- |
| **Description of the risk of implementation[[15]](#footnote-16)** | **Work package** *(one or more)* | Proposed measures for risk mitigation or elimination |
| low | 1, 2, 3, 4 | Work on project |
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3.3 OPERATIONAL CAPACITY OF THE APPLICANT/HOST ORGANISATION

The researcher plans to work independently in basic research in mathematics. Office space, computer technology and access to literature will be provided within the host organization and will be supported and partially funded within the project as well. No special infrastructure in the form of laboratories necessary for experimental sciences is required. For the time being, the necessity of access to special computing technology for the purpose of robust calculations. In the event that the need for these calculations is proven during the course of the project, we expect to cover it with the planned expenses from the project.

MI SAS has basic office equipment as well as basic computing equipment, which we plan to restore through the project.

3.3.1 Description of the research/innovation infrastructure of the applicant/host organisation that is necessary for the implementation of the project:

|  |  |
| --- | --- |
| Name of infrastructure or equipment | Short description |
| Supercomputer DEVANA | with an available total performance of around 800 TFlop/s |
| Standard computer equipment and  internet access | is available and will be continuously updated |
| office | is provided in standard equipment |
| Access to databases | Science Direct, Scopus, WoS, SpringerLink, SpringerNATURE,  JSTOR, PLOS, PNAS, Taylor & Francis, Wiley Online Library, etc. |
|  |  |

3.3.2 List of the five most important projects of the applicant/host organisation and their relevance to the proposed project (in the last 5 years):

|  |  |  |
| --- | --- | --- |
| Project name/identification | Programme/scheme/grant provider | Short description |
| Chromatic Problems  and Polynomials 2/0042/22 | VEGA, Scientific Grant Agency of Ministry of Education and SAS | Head of the project |
| Chromatic Problems in  Combinatorics 2/0024/18 | VEGA, Scientific Grant Agency of Ministry of Education and SAS | Head of the project |
|  |  |  |
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3.3.3 List of maximum five most important outputs of the applicant/host organisation relevant to the submitted project:

|  |  |  |
| --- | --- | --- |
| Output name/identification | **Type of output** | Short description |
| M. Kochol, Edge cut splitting  formulas for Tutte-  Grothendieck invariants,  Journal of Combinatorial  Theory Series B125 (2017)  114-131 | publication | Characterization of the Tutte-Grothendieck  graph invariants using components arising  from the edge cuts of a graph |
| M. Kochol, Interpretations of  The Tutte and characteristic  polynomials of matroids,  Journal of Algebraic  Combinatorics 53 (2021) 1–9 | publication | Interpretations of the Tutte and  characteristic polynomials of matroids  using the newly introduced cyclic bases |
| M. Kochol, Polynomials  counting nowhere-zero chains  in graphs, The Electronic  Journal of Combinatorics  29(1) (2022) #P1.19 | publication | Polynomials are introduced using which  it is possible to express numbers of nowhere-  zero chains of graphs |
| M. Kochol, One-to-one  correspondence between  interpretations of the Tutte  polynomials, Journal of  Combinatorial Theory  Series B 162 (2023) 134–143 | publication | It is introduced A bijection between  Interpretations of the Tutte polynomials  on matroid perspectives using cyclic bases  and a classical interpretation based on  internal and external activities |
| M. Kochol, Linear algebraic  relations among cardinalities  of sets of matroid functions,  Mathematics 11(11) (2023)  2570 | publication | It is introduced a Quantitative expression of  numbers of Tutte-Grothendieck invariants  with respect to the selected element sets  of a matroid |

1. After completion of the document, update the content. [↑](#footnote-ref-2)
2. Choose one type of research. [↑](#footnote-ref-3)
3. The months are indicated ascending from the start of the project, i.e., the month in which the project started is M1. [↑](#footnote-ref-4)
4. In case of involvement of the research team in the project implementation, it is necessary to identify the individual positions, identify the category of researcher R1-R4 for researchers and determine the level of involvement for all members of the research team. [↑](#footnote-ref-5)
5. Where the research team is involved in the project implementation, the personnel costs will include personnel costs of the researcher and the members of the research team. [↑](#footnote-ref-6)
6. The months are indicated ascending from the start of the project, i.e., the month in which the project started is M1. [↑](#footnote-ref-7)
7. In case of involvement of the research team in the project implementation, it is necessary to identify the individual positions, identify the category of researcher R1-R4 for researchers and determine the level of involvement for all members of the research team. [↑](#footnote-ref-8)
8. Where the research team is involved in the project implementation, the personnel costs will include personnel costs of the researcher and the members of the research team. [↑](#footnote-ref-9)
9. The months are indicated ascending from the start of the project, i.e., the month in which the project started is M1. [↑](#footnote-ref-10)
10. In case of involvement of the research team in the project implementation, it is necessary to identify the individual positions, identify the category of researcher R1-R4 for researchers and determine the level of involvement for all members of the research team. [↑](#footnote-ref-11)
11. Where the research team is involved in the project implementation, the personnel costs will include personnel costs of the researcher and the members of the research team. [↑](#footnote-ref-12)
12. The months are indicated ascending from the start of the project, i.e., the month in which the project started is M1. [↑](#footnote-ref-13)
13. In case of involvement of the research team in the project implementation, it is necessary to identify the individual positions, identify the category of researcher R1-R4 for researchers and determine the level of involvement for all members of the research team. [↑](#footnote-ref-14)
14. Where the research team is involved in the project implementation, the personnel costs will include personnel costs of the researcher and the members of the research team. [↑](#footnote-ref-15)
15. Indicate the probability of risk occurrence (low, medium, high) and the severity of the risk (low, medium, high) [↑](#footnote-ref-16)