## Instructions for the Applicant

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This template shall be used to fill in the project description by the *applicant under the call 09I03-03-V04 – Fellowships for excellent researchers R2-R4.* The structure of this form **must be maintained.** It is prepared to allow an efficient, transparent and fair evaluation of each application. Parts 1, 2 and 3 are identical to the evaluation criteria. Only complete applications, which address all the mandatory parts set out in this template and in the call, will be included in the evaluation process. If an application is approved, major changes in the content of the project will not be possible.

The description of the project shall not exceed 30 pages. All tables, numbers, references or other relevant information must be included in the project description and count towards the total page limit. The maximum limit must not be circumvented by inserting external hyperlinks. Content that exceeds the allowed page limit will not be taken into account and will not be subject to peer review, so it is recommended to comply with the maximum page limit.

Formal requirements for the description of the project:

The prescribed font to use in the description of the project is Times New Roman or Arial, minimum size 11. A font size of 10 is allowed in the tables; the applicant should not circumvent the page limit by artificially replacing the text with tables.

The size of the page is A4 with edges of at least 1.5 cm. Page numbering should be maintained.

Delete all instructions in this template highlighted in grey before the application is submitted.

Project description (template) for call 09I03-03-V04 Fellowships for excellent researchers R2-R4

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Title of the project: Chromatic problems and generalizations of Tutte polynomials

Short title of the project/Acronym: Chromatic problems

Category of researcher: R4

Researcher’s job type (full-time, part-time – in %): 100

Type of research[[2]](#footnote-3): basic

Identification of the entity involved in the implementation of the project:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Official name of the entity | Abbreviated name of the entity  Please indicate the short name of the entity that you will use throughout the application, max. 15 characters | Role in the project |
| 1 | Mathematical Institute of the Slovak Academy of Sciences | MI SAS | Applicant/host organisation |

## 1. Excellence

*Specific aspects that are relevant for this section*:

*The quality and adequacy of the proposed objectives of the project.*

Researcher, PNDr. Martin Kochol, PhD., DSc., has been dealing with various aspects of graph coloring for a long time. In this area, he achieved significant results and significantly contributed to the development of methodology in the given area. The project is aimed at continuing research in this area.

*Relevance of the problems/needs the project is focused on.*

The project is devoted to study pf generalizations of the Tutte, characteristic, chromatic and flow polynomials. We plan to study relations of this polynomials and methods of linear algebra for solutions of Tutte-Grothendieck invariants.

*How the project goes beyond the currently available solutions, procedures, etc. (“beyond the state of the art”).*

In the project we plan to introduce new classes of polynomials generalizing Tutte and characteristic polynomials and were not studied until now. We plan to extend methods of linear algebra for research of the Tutte-Grothendieck invariants resently introduced by the researcher.

*Appropriateness, timeliness and relevance of the proposed methodology to the objectives of the project.*

When studying the generalizations of Tutte polynomials, we propose to study and generalize both the principles applied so far, such as the principle of contraction and deletion, on the basis of which Tutte and characteristic polynomials are defined, but also Möbius functions, and in particular we plan to generalize the principle of cyclic bases that we introduced in the study of Tutte polynomials .

*The quality and adequacy of the researcher’s professional experience, expertise, competences and skills.*

The The researcher has been engaged in the study of graph coloring and related polynomials for a long time, achieving world-class results in this area (construction of new classes of snarks, study of Tutte conjectures on nowhere-zero 3- and 5-flows).

*The quality and adequacy of the host organisation in relation to the project and the researcher.*

The host organization, MÚ SAS, v.v.i., is a workplace focused on basic research in mathematics. Basic research in combinatorics has been significantly represented since the establishment of the host organization (Kotzig, Bosák, Tomasa). The host organization is the only workplace within the Slovak Academy of Sciences focused exclusively on basic research in mathematics and is among the top research instututions in Slovakia.

*The quality and adequacy of the conditions that the host organisation will ensure and provide for the researcher (e.g., additional training, supervision/mentoring, possibilities to build its own research team, etc.).*

The host researcher is a long-term employee of the host organization, thus has a built-in infrastructure (office, computer equipment, office equipment) and is established there for a long time (social infrastructure, etc.). The host organization has a long-established doctoral student education system. In the project, the researcher will build on the previous material and technical security.

*The quality of two-way knowledge transfer between the researcher and the host organisation.*

The project is oriented on the basic research in mathematics. In this type of research, the primary importance is given to the international cooperation with experts in the world. It is not a rule that there is an exchange of knowledge between the institution and the researcher.

### 

### 1.1 PROJECT OBJECTIVES

Describe objectives of the project – they should be clearly defined, realistic, measurable and achievable in the implementation of the project. For each objective, please also indicate how it will be verified and evaluated.

The aims of the project are

--introduction and study of generalizations of the Tutte and characteristic polynomials

--study of cyclic bases in connections with these invariants

--study of the Tutte polynomials in connections with methods of linear algebra applied for the Tutte-Grothendieck invariants.

1.2 RELEVANCE, QUALITY AND NOVELTY OF THE PROJECT

*Please briefly describe the current state in the area the project will focus on. How will the planned activities of the project address problems and challenges in this area? Why is necessary/appropriate to deal with such a project?*

Chromatic problems are at the centre of graph theory, a discipline at the interface of discrete mathematics, combinatorial optimization and computer science. These problems are important in order to understand the structure of graphs as well as the complexity of discrete algorithms. In their study, methods of cycles in graphs, linear algebra, groups, topologies, optimization methods and other techniques are used. In general, it can be said that chromatic problems are easy to formulate but very difficult to solve. They are directly related to complexity theory, as many of them belong to NP-complete problems.

A dual approach to chromatic problems is represented by nowhere-zero flows. A graph has a nowhere-zero k-flow if its edges can be oriented and evaluated with numbers from 1 to k-1 so that each vertex of the graph satisfies Kirchhoff's law (the sum of the values of the edges entering a vertex is equal to the sum of the values of the edges leaving it). Tutte's classic result says that a planar graph can be k-colored if and only if its dual graph has a nowhe-zero k-flow. Among the most famous problems in graph theory are 3- and 5-flow conjectures of Tutte. The 3-flow conjecture is that every graph without edge cuts of cardinality 1 and 3 has a nowhere-zero 3-flow. The 5-flow conjecture is that every bridgeless graph admits a nowhere-zero 5-flow.

The numbers of nowhere-zero flows and colourings in graphs are determined by flow and chromatic polynomials, whose dichromatic generalization is the Tutte polynomial. Tutte polynomials have applications far beyond the horizon of combinatorics, including areas such as knot theory, coding theory, statistical physics, etc., [BO], [EMM]. The researcher introduced new interpretations of Tutt's polynomials on matroids and matroid perspectives based on the so-called cyclic bases [K21], [K22b]. The researcher also generalized chromatic polynomials on graphs by introducing the so-called assigning polynomials that characterize numbers of nowhere-zero chains in graphs (non-homogenous variants of nowhere-zero flows) [K22a]. The researcher studied the properties of nowhere-zero flows in the context of the number of these flows on a selected set of edges, and showed that they satisfy certain quantitative relations that can be characterized by the language of linear algebra [K15]. Later he introduced these rules for other Tutte-Grothedieck invariants [K23a].

The researcher in the project plans to generalize assigning polynomials to different classes of matroids, to introduce and investigate their dichromatic variants, to investigate their structure especially from the point of view of cyclic bases. We also plan to study these new polynomials in terms of classical combinatorial functions such as the Möbius function and the zeta function. The connection with these functions and the Tutte polynomials was already known in [Cr].

In the project, we also plan to study the connection between generalizations of the Tutte polynomials and the application of linear algebra in solving problems related to Tutte-Grothendieck invariants. We plan to show that the coefficients arising by applications of methods of linear algebra can be evaluated by these polynomials.

[BO] T. Brylawski, J. Oxley, The Tutte polynomial and its applications, in: Matroid Applications, (N. White, Editor),Cambridge University Press, Cambridge (1992), 123-225.

[Cr] H.H. Crapo, The Tutte polynomial, Aequationes Math. 3 (1969) 211–229.

[EMM] J. Ellis-Monaghan, I. Moffat (Editors), Handbook of the Tutte Polynomial and Related Topics, CRC Press, Boca Raton, FL, 2022.

[K02] M. Kochol, Superposition and constructions of graphs without nowhere-zero *k*-flows, European Journal of Combinatorics 23 (2002) 281–306.

[K15] M. Kochol, Quantitative methods for nowhere-zero flows and edge-colorings, in: M. Dehmer, F. Emmert-Streib, Quantitative Graph Theory: Theoretical Foundations and Applications, pp. 141–180, Chapman and Hall/CRC Press, Boca Raton, FL, 2015.

[K21] M. Kochol, Interpretations of the Tutte and characteristic polynomials of matroids, Journal of Algebraic Combinatorics 53 (2021) 1–9.

[K22a] M. Kochol, Polynomials counting nowhere-zero chains in graphs, The Electronic Journal of Combinatorics 29(1) (2022) #P1.19.

[K22b] M. Kochol, Interpretations for the Tutte polynomials of morphisms of matroids, Discrete Applied Mathematics 322 (2022) 210–216.

[K23a] M. Kochol, Linear algebraic relations among cardinalities of sets of matroid functions, Mathematics 11(11) (2023) 2570.

*How does the project go beyond the currently available solutions, procedures, etc.? What makes it original and innovative?*

Generalizations of the Tutte polynomials as dichromatic assigning polynomials have not been studied so far. Coefficients arising from the linear algebra methods introduced by us for the study of Tutte-Grothendieck invariants in connection with generalizations of the Tutte polynomials were also not studied. We also plan to generalize the interpretations of these polynomials by studying cyclic bases.

These are new perspectives on the given issue, which were only recently introduced by the researcher. The innovativeness and originality of the issue is also proven by the fact that the researcher's previous results in this area have been published in top mathematical journals focused on combinatorics.

*Describe the link between the project and its activities with the European Research Area.*

The study of the Tutte polynomials and chromatic problems is in the interest of the worldwide scientific community, which is also strongly represented in Europe. We note that in a recent publication dedicated to the overview of results and applications of the Tutte polynomials [EMM], there are a lot of researchers from Europe (the editors work in the Netherlands and the United Kingdom of Great Britain and Northern Ireland).

1.3 METHODOLOGY

*Describe how the project will be implemented. Describe the methods and procedures used in each activity and their appropriateness and interconnection. Describe the concepts, models, assumptions underlying the proposed project methodology.*

The project will be implemented through basic research in mathematics. We will apply standard methods of mathematics and combinatorics. The basic method used in the study of Tutt polynomials is the principle of contraction and omission of elements. It is an approach applicable to several invariants of the theory of graphs and matroids, consisting in the fact that the value of a given invariant on a matroid can be expressed by the values of invariants on matroids arising after the omission and contraction of (any) element of the matroid. Alternatively, properties of classical combinatorial functions such as Möbius and zeta functions can be used. We plan to apply variants of this principle in the project as well. We also plan to apply linear algebra methods.

*Describe how the proposed methods and procedures will ensure the achievement of the project’s objectives.*

We plan to use the proposed methods in mathematical proofs of the planned goals.

*Describe possible challenges in the implementation of the project in relation to the methodology and the proposed way to overcome them.*

We assume to use mathematical proofs by implementing the mentioned methods. In the case of applying linear algebra methods for solving problems on specific invariants, it is also possible to use calculations (these had to be used when calculating the ranks of huge matrices when determining circuit restrictions for the smallest counterexamples of the 5-flow conjecture).

*Describe the use and management of research data and other research outputs within the project. If the project collects data and/or other research outputs (except publications), how the data/outputs of the research will be managed. Describe how open science principles are integrated in the project.*

We do not envisage the use of research data or other research outputs within the project (as a rule, this is not used in basic research in mathematics).

*Describe how the following aspects are taken into account in the project: multi- and interdisciplinary approach, principles of open science, FAIR access to research data, gender equality in research.*

Basic research in mathematics essentially applied similar principles in a natural way without the need for specific emphasis. Since the researcher plans to work independently, it is not necessary to take into account the interpersonal relations between the project workers.

1.4 EXCELLENCE OF THE RESEARCHER

Describe the quality and adequacy of the researcher’s professional experience and expertise in relation to the proposed project. Please support the description by the CV below.

Curriculum Vitae

**Personal information**

First and last name: Martin Kochol

Identifier (*ORCID or Researcher ID or other – specify)*: www.mat.savba.sk/~kochol

Date of birth: May 11, 1961

Nationality: Slovak Republic

Website (if relevant): www.mat.savba.sk/~kochol

**Education**

05/2006 – DSc. Akademy of Sciences of Czech Republic, Czech Republic.

11/1990 – PhD Mathematical Institute of the Slovak Academy of Sciences, Slovakia

06/1985 – RNDr. Faculty of Mathematics and Physics, Comenius University, Slovakia

**Current position/positions**

05/2007 – Principar Research Fellow, I.

Mathematical Institute of the Slovak Academy of Sciences, Bratislava, Slovakia

**Previous positions**

06/1993 – 05/2007 – Senior Research Fellow, IIa

Mathematical Institute of the Slovak Academy of Sciences, Bratislava, Slovakia

11/1990 – 06/1993 - Research Fellow, IIb,

Mathematical Institute of the Slovak Academy of Sciences, Bratislava, Slovakia

08/1987 – 11/1990 – Research Emploee

Mathematical Institute of the Slovak Academy of Sciences, Bratislava, Slovakia

09/1985 – 07/1987 - Assistant

Institute of Applied mathematics and Computing Technique, Comenius University, Bratislava, Slovakia

**Scholarships and awards**

03/1997 – 02/1998 – JSPS Postdoctoral Fellowship – Keio University, Yokohama, Japan, Japan Society for the Promotion of Science (JSPS), Japan

03/1998 – 08/1999 – Alexander von Humboldt Fellowship – Freie Universität Berlin, Germany, Alexander von Humboldt Foundation, Germany

1985 – Rector Prize for the Study and Research Results – Comenius University, Slovakia

1995 – Vinner of Competion of Young Mathematicians – Union of Slovak Mathematics and Physics, Slovakia

Student and post-docs supervision (if applicable)

2007 – 2010 - three finished PhD Students

Faculty of Natural Sciences, University of Žilina, Slovakia

**Teaching activities (if applicable)**

2020 – external pedagogical position – course: Analyzis and Complesity of Algorithms, Faculty of Informatics and Information Technologies, Slovak Technical Univerzity, Bratislava, Slovakia

2008 – external pedagogical position – course: Discrete Mathematics and Logic, Faculty of Electrical Engineering, Slovak Technical Univerzity, Bratislava, Slovakia

2007 – external pedagogical and research position – course for PhD students: Combinatorics and Grpah Theory, Faculty Natural Sciences, University of Žilina, Slovakia

2003 – visiting position – dve courses: Second Year Accelerated Calculus, Vanderbilt University, Nashville, TN, USA

2002 – visiting position – dve courses: Combinatorics and Linear and Discrete Mathematics, Georgia Institute of Technology, Atlanta, GA, USA

1990 – 1995 – external pedagogical position – courses: Linear Algebra, Discrete Mathematics, matematika, Faculty of Electrical Engineering, Slovak Technical Univerzity, Bratislava, Slovakia

1988 – 1989 – external pedagogical position – courses: Algebra and Algebra and Theoretical Arithmetics, Faculty of Mathematics and Physics, Comenius University, Bratislava, Slovakia

**Organisation of scientific meetings (if applicable)**

**Institutional responsibilities (if applicable)**

2003 – 2004 – nenber of the Scientific Council of the Faculty, Faculty of Natural Sciences, University of

Žilina, Slovakia

1998 – 2002 – member of the Commitee for Defence of PhD Thesis in Branch 11-11-9 Discrete

Mathematics, Ministry of Education, Slovakia

2006 – 2010 – organization of internal seminar for PhD students, Graph Theory, Faculty of Natural

Sciences, University of Žilina, Slovakia

2008 – member of a Commitee for Defence of a PhD Thesis, Faculty of Natural Sciences, University of

Žilina, Slovakia.

**Reviewing activities (if applicable)**

2008 – 2017 – member of Editorial Board, International Journal of Combinatorics, Hindawi, India

Reviewer for Mathematical Reviews, Zentralblatt, Journal of Combinatorial Theory Seires A and Series B, Journal of Graph Theory, Discrete Mathematics, Discrete Applied Mathematics, European Journal of Combinatorics, The Electronic Journal of Combinatorics, Annals of Combinatorics, Journal of Combinatorics, Annals of Mathematics, etc.

**Memberships of scientific societies (if applicable)**

1985 – member, Union of Slovak Mathematicians and Physicists

1999 – member, Humboldt club in Slovensku

2007 – member, Mathematical Programming Society

2003 – 2016, member, American Mathematical Society

**Major collaborations (if applicable)**

The researcher cooperates or cooperated with many scientists in the world:

Hikoe Enomoto, Keio University, Yokohama, Japan

Martin Aigner, Freie Universität Berlin, Germany

Jaroslav Nešetřil, Charles University, Praha, Czech Republic

Herbert Flieschner, Austrian Academy of Scientists, Austria

Andreas Huck, Universität Hannover, Germany

Andre Raspaud, Bordeaux University, France

Vadim Lozin, University of Warwick, Coventry, UK

Iain Moffatt, Royal Holloway University of London, Egham, UK

Mark Ellingham, Vanderbilt university, Nashvill, TN, USA

József Balogh, University of Illinoi at Urbana-Champaign, Illinoi, USA

László Lovász, Eötvös Loránd University, Budapest, Hungary

Gyula O.H. Katona, Alfréd Rényi Institute of Mathematics, Budapest, Hungary

**Overview of the researcher’s most important projects in the last 5 years** (max. 5)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Project name/identification** | **Source of funding** | **Budget (EUR)** | **Project period** | **The role of the researcher in the project** |
| Chromatic Problems  and Polynomials  2/0042/22 | VEGA, Scientific Grant Agency of Ministry of education and SAS | 3444 € | 2022 – 2025 | Head of the project |
| Chromatic Problems  in Combinatorics  2/0024/18 | VEGA, Scientific  Grant Agency of  Ministry of education  and SAS | 5204 € | 2018 – 2021 | Head of the project |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

**Overview of the researcher’s most important outputs** (max. 5)

|  |  |  |  |
| --- | --- | --- | --- |
| **Output name/identification** | **Type of output** *(e.g., publication, dataset, software, patent, service, product, etc.)* | **Short description** | **The role of the researcher** |
| M. Kochol, Snarks without  small cycles, Journal of  Combinatorial Theory Series  B, 67 (1996) 34–47 | publication | Construction of snarks with  arbitray large girth, a  counterexample to a conjecture of  Jaeger and Swart | author |
| M. Kochol, An equivalent  version of the 3-flow  conjecture, Journal of  Combinatorial Theory Series B, 83 (2001) 258–261 | publication | Reduction of the 3-flow conjecture to edge-5-connected graphs | author |
| M. Kochol, Polyhedral  embeddings of snarks in  orientable surfaces,  Proceedings of the  American Mathematical  Society, 137 (2009) 1613–  1619 | publication | Construction of snarks with  polyhedral embedding in an  orientable surface of genus at least  5, a counterexample to a  conjecture of Grünbaum |  |
| M. Kochol, Reduction of the  5-flow conjecture to  cyclically 6-edge-connected  snarks, Journal of  Combinatorial Theory Series B, 90 (2004) 139–145 | publication | Reduction of the 5-flow conjecture  on 3-regular graphs with cyclic  connectivity at least 6 | author |
| M. Kochol, Smallest  counterexample to the 5-  flow conjecture has girth at  least eleven, Journal of  Combinatorial Theory Series B, 100 (2010) 381–389 | publication | Reduction of the 5-flow conjecture  on 3-regular graphs with girth at  least 11 | author |

Briefly describe your most important research and/or innovation achievements.

The most important results of the researcher are the introduction of superposition as a construction of graphs without nowhere-zero flows and the introduction of linear algebra techniques to solve problems of some Tutte-Grothendieck invariants. These are the outputs of basic research in mathematics consisting of the publication of results in scientific journals.

The general principle of superposition is studied in detail in [K02]. Using this method, we constructed new classes of snarks (non-trivial 3-regular graphs without edge-3-coloring). This is an important class of graphs, the study of which has much to do with the solution of the most important open problems in combinatorics (the 5-flow conjecture and the cycle double cover conjecture) as with complexity theory. We constructed snarks with arbitrarily large girth [K96], disproving the Jeager and Swart conjecture of 1980, and polyhedrally embeddable snarks in any orientable surface of genus at least 5 [K09], disproving Grünbaum's conjecture of 1968. The superposition method was also used in the proof that the smallest counterexample to the 3-flow conjecture of Tutte (every graph without edge cuts of order 1 and 3 has a nowhere-zero 3-flow) is edge 5-connected [K01]. It is known that every edge 6-connected graph has a nowhere-zero 3-flow [LTWZ13].

The general approach of introducing linear algebra methods for solving some Tutte-Grothendieck invariants is introduced in [K23a]. We previously used these methods to investigate the smallest counterexamples to Tutt's 5-flow conjecture (every bridgeless graph has a nowhere-zero 5-flow). We proved that the smallest counterexample to the 5-flow hypothesis is a cyclic edge-6-continuous snark [K04] and has girth at least 11 [K10]. We note that it is not known whether such snarks even exist.

We have also introduced a tiling principle on Möbius strip, and these ideas we have used by constructions of infinite families of crossing-critical graphs with crossing number at least 2 [K87]. We have also constructed asymtotically optimal approximation of n-dimensional balls by polytopes [K94] and prove the cycle double cover conjecture for a class of 3-regular graphs with low degree of non edge-3-colorability [HK95].

The reseacher has 580 ciation on his papers, majority of them in databases of Web of Science and SCOPUS but also in monographs and Googe Scholar.

[K87] M. Kochol: Construction of crossing-critical graphs, Discrete Mathematics 66 (1987) 311–313.

[K94] M. Kochol: Constructive approximation of a ball by polytopes, Mathematica Slovaca 44 (1994)

99–105.

[HK95] A. Huck, M. Kochol: Five cycle double covers of some cubic graphs, Journal of Combinatorial

Theory Series B 64 (1995) 119–125.

[K96] M. Kochol: Snarks without small cycles, Journal of Combinatorial Theory Series B 67 (1996) 34–47

[K01] M. Kochol: An equivalent version of the 3-flow conjecture, Journal of Combinatorial Theory Series

B 83 (2001) 258–261.

[K02] M. Kochol: Superposition and constructions of graphs without nowhere-zero k-flows, European

Journal of Combinatorics 23 (2002) 281–306.

[K04] M. Kochol: Reduction of the 5-flow conjecture to cyclically 6-edge-connected snarks, Journal of

Combinatorial Theory Series B 90 (2004) 139-145.

[K09] M. Kochol: Polyhedral embeddings of snarks in orientable surfaces, Proceedings of the American

Mathematical Society, 137 (2009) 1613–1619.

[K10] M. Kochol, Smallest counterexample to the 5-flow conjecture has girth at least eleven, Journal of

Combinatorial Theory, Series B 100 (2010) 381-389.

[LTWZ13] L.M. Lovász, C. Thomassen, Y.Wu, C.-Q. Zhang, Nowhere-zero 3-fows and modulo

-orientations, Journal of Combinatorial Theory Series B 103 (2013) 587-598.

[K23a] M. Kochol, Linear algebraic relations among cardinalities of sets of matroid functions,

1.5 EXCELLENCE OF THE APPLICANT/HOST ORGANISATION

Describe the applicant/host organisation and its excellence.

The host organization, MI SAS, is an organization devoted to basic research in mathematics. The host institution is the only institute of the Slovak Academy of Sciences devoted exlusively to basic research in mathematics and belongs to top research organization in Slovakia. Basic research devoted to combinatorics was introduced in the host organization from its beginning.

*What is the comparative advantage of implementing this project within the given host organisation, why is the applicant the ideal hosting organisation to implement the project?*

The researcher is a long-term employee of the host organization. That's why it can seamlessly build on the previous work. Any change of the host organization within Slovakia for a given project has no meaning either from an organizational point of view or from the point of view of the quality of the research workplace. Any change would only cause difficulties from an organizational as well as a personal point of view

*Describe the conditions that the applicant/host organisation will create and provide for the researcher for the implementation of the project. In the case of supervision/mentoring, describe the person of the supervisor/mentor, his/her quality and adequacy in relation to the researcher and the project.*

The host organization will provide the researcher with technical support (office, office supplies, computer equipment, access to literature through the library and access to various databases). The host organization does not expect special monitoring of the researcher, as he is a senior researcher who works independently for a long time and achieves excellent scientific results that are well received by the professional public.

*Describe the conditions the researcher will be provided with to build his/her own team (particularly relevant for R3 and R4 researchers’ categories).*

The host organization has a long-standing system of doctoral studies in cooperation with the Faculty of Mathematics, Physics and Informatics of the Comenius University in Bratislava. The researcher also has experience in supervising doctoral students. Until now, he supervised three doctoral students who successfully completed their doctoral studies under his supervision.

*Describe the quality and how two-way knowledge transfer between the applicant/host organisation and the researcher will be ensured.*

The scientific and professional staff of the host organization regularly (annually) announce the achieved results, thereby also participating in the preparation of annual reports of the MI SAS as well as Accreditation reports. All these reports are publicly accessible. Information on ongoing and completed projects within the MI SAS is also published. This practice will also be applied in the case of this project, which will ensure a transparent transfer of knowledge between the researcher and the workplace.

## Impact

*Specific aspects that shall be taken into account in this section:*

*The credibility of the proposed procedures, the likelihood that the project will achieve the expected results and will have the expected impact.*

The researcher has been working in basic research for a long time and mainly focuses on combinatorics and theoretical informatics. In the proposed project, he plans to continue researching the issue in which he has been working recently and achieved interesting results that were published in top journals focused on combinatorics, discrete mathematics and theoretical informatics. It is natural to assume output in the form of publications in journals of similar quality. As a rule, basic research in mathematics has the most important outputs in the form of publications in scientific journals.

*The assumption of a positive impact on the further career of the researcher, the assumption of a positive impact on the applicant/host organisation.*

The researcher realizes his scientific career by achieving the results of basic research and publishing them in the scientific literature. Also, the host organization, focused on basic research in mathematics, realizes outputs by publishing the results of scientific and professional workers. Since the goal of the project is basic research and achieving results, it is a natural realization of the work of the researcher as well as the host organization.

*The significance of the expected impact – on the given area of knowledge and the scientific community, on the economy, on society, on the environment.*

As a rule, basic research in mathematics has a direct impact on the field of knowledge for the scientific community in the world. We anticipate this impact in the project as well.

*Adequacy of expected results and impact of the project – qualitative and quantitative.*

We expect results from the very current and intensively studied issue of discrete mathematics, chromatic problems in combinatorics. Therefore, it is possible to expect quality results important for scientific knowledge. As part of the project, we expect the preparation of at least four scientific articles by the researcher, which represents a sufficient output of the project even from a quantitative point of view.

*The appropriateness and quality of the proposed measures to maximise the results and impact of the project.*

The researcher's previous scientific activity represents a sufficient qualitative guarantee for achieving the planned goals of the project.

*The quality of the proposed intellectual property rights management strategy for project results (if applicable).*

The project is focused on basic research in mathematics, it will be carried out by a researcher, therefore specific management of intellectual property rights is not foreseen.

### 2.1 THE WIDER IMPACT OF THE PROJECT

Describe the expected impact of the project in the short, medium and long term. What impact will the implementation of the project have beyond its direct scope and after completion of its implementation?

The output of the project will be new knowledge of basic research. The form of output will be the publication of articles in professional journals and their presentation at scientific conferences. This is the impact of the project. Since the applications of Tutte polynomials extend beyond the boundaries of combinatorics, it is possible to expect similar applications to the results of the project in the long term.

Describe the impact of project implementation on the researcher’s further career and the development of his/her skills.

By working on the project, the researcher will gain new knowledge regarding Tutt's polynomials and their applications. High-quality results are expected, which automatically have a positive impact on the researcher's career growth.

Describe the impact of the project implementation on the applicant/host organisation.

The project envisages the output of several articles published in high-quality scientific journals, which is the goal of the host organization as a scientific workplace.

Identify the individual target groups that will benefit from the activities and achievement of the project objectives, describe the impact of the project on these groups.

The target group that will benefit from the project is the scientific community engaged in basic research in mathematics. This is a target group that generally benefits from the results of basic research in mathematics.

Describe the direct and relevant scientific, economic, environmental, societal impact of the project (or other, if relevant).

The direct impact is assumed to be new scientific knowledge in mathematics.

Identify and describe the potential negative impact of the project and what the proposed measures to eliminate/mitigate it are.

As it is basic research in mathematics, we do not expect any negative impact Describe the specific expected results and impacts of the project activities (qualitative and quantitative), which will bring significant and direct benefits measurable within the monitored data.

Monitored data may include, for example:

* + number of excellent students, PhD candidates and researchers implementing the project;
  + number of patent applications;
  + number of publications;
  + number of collaborations (international, with private sector, application sphere),
  + Others.

We assume the participation of one researcher (project leader).

Patent applications for basic research in mathematics are not applicable, so these are not expected.

We expect at least four publications in professional journals. We also plan to present the results at scientific conferences.

Now we are not planning direct international cooperation within the planned project. It is also a consequence of the researcher's limited funds for travel expenses. In the case of a successful application with planned financial support, it is possible to expect a significant increase in the researcher's contacts with scientists working on related issues who are exclusively abroad.

When designing project results and impact as part of the monitored data, describe the basis on which the estimate, benchmarks, statistical data, etc. were made.

The results and impact of the project were carried out by the researcher on the basis of the previous results in the given issue, his previous experience and also the previous results and methods used in the given issue.

* Potential obstacles to the planned impact of the project

Describe potential barriers, conditions (e.g., legislative, competition or others that go beyond the scope and duration of the project) that may affect the desired results and impact. Identify whether these factors can evolve over time and the ways you address them.

(This does not include the implementation risks of the project, which will be described below)

As the project is focused on basic research, we do not foresee any potential obstacles until the cancellation of the hosting organization or deterioration of the overall social climate due to force majeure (pandemic, war, natural disaster, etc.)

### 2.2 MEASURES TO MAXIMISE IMPACT – DISEMINATION AND COMMUNICATION, EXPLOITATION OF RESULTS

*What tools and measures do you choose to maximise the impact of project results and deliverables? Describe what communication and sharing tools you will use, list the planned communication activities and target audience that will be targeted both during and after the project.*

*(In the planned communication activities, take into consideration the target groups and how to reach them through different tools)*

To maximize the impact, we plan to publish the results in top scientific journals focused on combinatorics and also present the results at scientific conferences.

*How will possible technology transfer, commercialisation of project outputs, etc. be ensured?*

The project focused on basic research in mathematics does not envisage specific technology transfer and commercialization of outputs.

Describe the measures to exploit the results of the project even after its completion. Describe the measures for the use of research data and other research outputs after the completion of the project implementation.

The results of the project will be publicly accessible to the scientific community, as is usual for basic research in mathematics.

*If relevant, describe the strategy for managing intellectual property rights in relation to the results of the project. How will their protection and the possibility of commercial use be ensured? Please briefly describe what requirements will need to be met in order for the results of the project to be exploited by intellectual property and how you intend to meet these conditions.*

Intellectual property rights are secured by relevant periodicals. For basic research in mathematics, we do not assume the protection of commercial use.

## Implementation

*Specific aspects that shall be taken into account in this section*:

Quality and efficiency of the project plan, feasibility of planned activities.

Dividing the project into stages and packages is difficult to implement in this type of project, because the individual stages overlap with each other. Nevertheless, the project is formally divided into 4 basic packages. These are results that follow the researcher's recent work, which guarantees their feasibility.

The coherence and logical framework of the work packages and the adequacy of the resources allocated to them, the adequacy of the proposed milestones and deliverables.

Individual work packages are related to each other and interconnected. The basis is the research of generalizations of Tutt's polynomials, which the other packages follow.

As a result of each of the packages, we plan at least one publication in a scientific journal. The project is planned for 2 years, while the time to publish a scientific article is several years for some journals. It can be expected that at the end of the project solution period, the results will be prepared in the form of manuscripts and sent to the editorial offices of the journals. The specific date of the final publication cannot be guaranteed in advance, as it may take several years.

We plan to present the results of the project at scientific conferences. Most scientific conferences focused on combinatorics do not have an established system of publications in proceedings, as is customary in computer science. Therefore, we plan to present the results mainly at high-quality international conferences focused on combinatorics, while we direct our publication activity towards publication in scientific journals.

Estimation of implementation risks, quality of proposed measures.

We do not assume implementation risks. We consider the proposed measures to be sufficient, as they are based on the researcher's previous experience.

There is a real risk of quarantine measures that could limit participation in scientific conferences. Participation in scientific conferences is only an additional form of presentation of results in the given project, while we consider publication in scientific journals to be the most important, where there are no restrictions related to quarantine measures.

Capacities (personnel, professional, technical, infrastructure, other) of the applicant/host organisation.

The project is focused on basic research in mathematics. The researcher plans to work independently, which includes preparing and writing articles. We plan to provide technical and infrastructural support for the project within the host organization.

3.1 PROJECT PLAN AND DELIVERABLES

Describe the overall structure of the project plan, which consists of individual work packages, their interconnection, logical and chronological relation.

The project is focused on basic research in mathematics. The researcher plans to work independently, which includes preparing and writing articles. We plan to provide technical and infrastructural support for the project within the host organization.

Describe the timeline of the project – specify project duration and the time frame for the implementation of each work package (e.g., Gantt diagram).

We plan 6 months for each of the packages. We plan to deal with individual packages according to the following plan: generalizations of characteristic polynomials in months M1-M6, generalizations of the Tutte polynomials in months M7-M12, basic properties of generalized Tutte polynomials in months M13-M18, the Tutte polynomials and coefficients in the application of linear algebra in combinatorics in months M19-M24.

Describe the planned work packages (template of the table below) and add a separate table for each work package. The number of work packages should reflect the scale and complexity of the project.

3.1.1 Work packages

Template of the table for the work package (1 work package = 1 table):

|  |  |
| --- | --- |
| Work package number | 1 |
| Title of the work package | Generalizations of characteristic polynomials |
| **Start of implementation of the work package (Mx Month[[3]](#footnote-4))** | M1 |
| End of implementation of the work package (Mx month) | M6 |
| **Involvement (expressed in Person Months)[[4]](#footnote-5)** | 6 Person Months |
| **Personnel costs (in EUR)[[5]](#footnote-6)** | 32 376 € |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 12 656,4 € |
| Objectives | |
| Get generalizations of characteristic polynomials, study their relationship with classical characteristic polynomials and various implementations of these polynomials | |
| Description of the work package | |
| Where appropriate, please also provide a breakdown per task level.  We will focus on generalizations of characteristic polynomials and their relationship with classical characteristic polynomials. We also plan to study different implementations of these polynomials. | |
| Deliverables | |
| Identify deliverables in numbered list and describe each deliverable in more detail  Output number: 1. It will be a scientific article devoted to generalizations of characteristic polynomials, their connection with classical characteristic polynomials and various implementations of these polynomials. | |

|  |  |
| --- | --- |
| Work package number | 2 |
| Title of the work package | Generalizations of the Tutte polynomials |
| **Start of implementation of the work package (Mx Month[[6]](#footnote-7))** | M7 |
| End of implementation of the work package (Mx month) | M12 |
| **Involvement (expressed in Person Months)[[7]](#footnote-8)** | 6 Person Months |
| **Personnel costs (in EUR)[[8]](#footnote-9)** | 32 376 € |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 12 656,4 € |
| Objectives | |
| Get generalizations of the Tutte polynomials, study their relationship with the classical Tutte polynomials and various implementations of these polynomials | |
| Description of the work package | |
| Where appropriate, please also provide a breakdown per task level.  We will focus on generalizations of the Tutte polynomials and their relationship with the classical Tutte polynomials. We also plan to study different implementations of these polynomials. | |
| Deliverables | |
| Identify deliverables in numbered list and describe each deliverable in more detail  Output number: 2. It will be a scientific article devoted to generalizations of the Tutte polynomials, their connection with the classical Tutte polynomials and various implementations of these polynomials. | |

|  |  |
| --- | --- |
| Work package number | 3 |
| Title of the work package | Properties of the generalized Tutte and chromatic polynomials |
| **Start of implementation of the work package (Mx Month[[9]](#footnote-10))** | M13 |
| End of implementation of the work package (Mx month) | M18 |
| **Involvement (expressed in Person Months)[[10]](#footnote-11)** | 6 Person Months |
| **Personnel costs (in EUR)[[11]](#footnote-12)** | 32 376 € |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 12 656,4 € |
| Objectives | |
| To gain knowledge about the properties of generalized Tutte and characteristic polynomials. | |
| Description of the work package | |
| Where appropriate, please also provide a breakdown per task level.  We will study the basic properties of generalized Tutte and characteristic polynomials. | |
| Deliverables | |
| Identify deliverables in numbered list and describe each deliverable in more detail  Output number: 3. It will be a scientific article devoted to the properties of generalized Tutte and characteristic polynomials. | |

|  |  |
| --- | --- |
| Work package number | 4 |
| Title of the work package | The Tutte polynomials and coefficients in the application of linear algebra in combinatorics |
| **Start of implementation of the work package (Mx Month[[12]](#footnote-13))** | M19 |
| End of implementation of the work package (Mx month) | M24 |
| **Involvement (expressed in Person Months)[[13]](#footnote-14)** | 6 Person Months |
| **Personnel costs (in EUR)[[14]](#footnote-15)** | 32 376 € |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 12 656,4 € |
| Objectives | |
| To gain knowledge about the coefficients arising from the application of linear algebra in combinatorics and their connection with generalized Tutt polynomials. | |
| Description of the work package | |
| Where appropriate, please also provide a breakdown per task level.  We will focus on coefficients arising from the application of linear algebra in combinatorics and study their connection with generalized Tutt polynomials. | |
| Deliverables | |
| Identify deliverables in numbered list and describe each deliverable in more detail  Output number: 1. It will be a scientific article devoted to the coefficients arising from the application of linear algebra in combinatorics and to study their connection with generalized Tutt polynomials. | |

3.1.2 List of work packages (template):

|  |  |  |  |
| --- | --- | --- | --- |
| Work package number | Title of the work package | **Start of activities** *(specify month of project implementation)* | **End of activities** *(specify month of project implementation)* |
| 1 | Generalizations of characteristic polynomials | M1 | M6 |
| 2 | Generalizations of the Tutte polynomials | M7 | M12 |
| 3 | Properties of the generalized Tutte and chromatic  polynomials | M13 | M18 |
| 4 | The Tutte polynomials and coefficients in the application  of linear algebra in combinatorics | M19 | M24 |
|  |  |  |  |

3.1.3 List of deliverables (template):

Mandatory deliverables shall be at least:

* Interim report on the implementation and achievements of the project submitted at mid-term of project implementation
* Final report on the achievements of the project presented at the end of the project implementation
* Researcher’s publications in scientific and/or professional journals
* Outputs in the conference proceedings with the active participation of the researcher
* Submission/award of a research project(s)/grant(s) with the involvement of the researcher

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Deliverable number | Deliverable | Work package number | Type | Access and dissemination | Method of verification | Delivery (project implementation month) |
|  |  |  | Report, publication, prototype, software, patent, other (please specify) ... | P= public  N= non-public, limited only to team/host organisation, grant provider for reporting purpose |  |  |
| 1 | Interim  Report | 1 and 2 | Report | N | report | M13 |
| 2 | Final  Report | 1, 2, 3, 4 | Report | N | report | M24 |
| 3 | Scientific  papers | 1, 2, 3, 4 | Published  papers in  scientific  journals or  Technical  Reports | P | Publication  or report | continuously |
| 4 | Outputs or  presentations  in scientific  conferences | 1, 2, 3, 4 | Talks or  outputs in  conferences | P | Publication  or report | continuously |
|  |  |  |  |  |  |  |

The applicant shall define the number of planned mandatory deliverables and define other deliverables relevant to its project.

3.1.4 List of milestones (template):

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Milestone number | Milestone | Work package number | Method of verification | Expected time to reach the milestone (project month) |
| 1 | Characteristic and the  Tutte polynomials | 1, 2 | Interim Report | M13 |
| 2 | End of the project | 3, 4 | Final Report | M24 |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

3.2 IMPLEMENTATION RISKS AND PROPOSED MEASURES

Describe the approach to risk management in the implementation of the project.

3.2.1 Risks of implementation (template):

|  |  |  |
| --- | --- | --- |
| **Description of the risk of implementation[[15]](#footnote-16)** | **Work package** *(one or more)* | Proposed measures for risk mitigation or elimination |
| low | 1, 2, 3, 4 | Work on project |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

3.3 OPERATIONAL CAPACITY OF THE APPLICANT/HOST ORGANISATION

Describe the capacities (staff, professional, technical, infrastructure and others) of the applicant/host organisation that are necessary for the successful implementation of the project. Describe their relevance to the project and how they will be made available to the researcher or how the access to them will be ensured during the implementation of the project.

The researcher plans to work independently in basic research in mathematics. Office space, computer technology and access to literature will be provided within the host organization and will be supported and partially funded within the project as well. No special infrastructure in the form of laboratories necessary for experimental sciences is required. For the time being, the necessity of access to special computing technology for the purpose of robust calculations. In the event that the need for these calculations is proven during the course of the project, we expect to cover it with the planned expenses from the project.

MI SAS has basic office equipment as well as basic computing equipment, which we plan to restore through the project.

3.3.1 Description of the research/innovation infrastructure of the applicant/host organisation that is necessary for the implementation of the project (template):

|  |  |
| --- | --- |
| Name of infrastructure or equipment | Short description |
| computer | It is available and will be continuously updated |
| printers | It is available and will be continuously updated |
| office | It is available |
| library | Part of MI SAS It is available and will be continuously updated |
|  |  |

3.3.2 List of the five most important projects of the applicant/host organisation and their relevance to the proposed project (in the last 5 years) (template):

|  |  |  |
| --- | --- | --- |
| Project name/identification | Programme/scheme/grant provider | Short description |
| Chromatic Problems  and Polynomials 2/0042/22 | VEGA, Scientific Grant Agency of Ministry of education and SAS | Head of the project |
| Chromatic Problems in  Combinatorics 2/0024/18 | VEGA, Scientific Grant Agency of Ministry of education and SAS | Head of the project |
|  |  |  |
|  |  |  |
|  |  |  |

3.3.3 List of maximum five most important outputs of the applicant/host organisation relevant to the submitted project (Template):

|  |  |  |
| --- | --- | --- |
| Output name/identification | **Type of output** *(e.g., publication, dataset, software, patent, service, product, etc.)* | Short description |
| M. Kochol, Edge cut splitting  formulas for Tutte-  Grothendieck invariants,  Journal of Combinatorial  Theory Series B125 (2017)  114-131 | publication | Characterization of the Tutte-Grothendieck  graph invariants using components arising  from the edge cuts of a graph |
| M. Kochol, Interpretations of  The Tutte and characteristic  polynomials of matroids,  Journal of Algebraic  Combinatorics 53 (2021) 1–9 | publication | Interpretations of the Tutte and  characteristic polynomials of matroids  using the newly introduced cyclic bases |
| M. Kochol, Polynomials  counting nowhere-zero chains  in graphs, The Electronic  Journal of Combinatorics  29(1) (2022) #P1.19 | publication | Polynomials are introduced using which  it is possible to express numbers of nowhere-  zero chains of graphs |
| M. Kochol, One-to-one  correspondence between  interpretations of the Tutte  polynomials, Journal of  Combinatorial Theory  Series B 162 (2023) 134–143 | publication | It is introduced A bijection between  Interpretations of the Tutte polynomials  on matroid perspectives using cyclic bases  and a classical interpretation based on  internal and external activities |
| M. Kochol, Linear algebraic  relations among cardinalities  of sets of matroid functions,  Mathematics 11(11) (2023)  2570 | publication | It is introduced aQuantitative expression of  numbers of Tutte-Grothendieck invariants  with respect to the selected element sets  of a matroid |

1. After completion of the document, update the content. [↑](#footnote-ref-2)
2. Choose one type of research. [↑](#footnote-ref-3)
3. The months are indicated ascending from the start of the project, i.e., the month in which the project started is M1. [↑](#footnote-ref-4)
4. In case of involvement of the research team in the project implementation, it is necessary to identify the individual positions, identify the category of researcher R1-R4 for researchers and determine the level of involvement for all members of the research team. [↑](#footnote-ref-5)
5. Where the research team is involved in the project implementation, the personnel costs will include personnel costs of the researcher and the members of the research team. [↑](#footnote-ref-6)
6. The months are indicated ascending from the start of the project, i.e., the month in which the project started is M1. [↑](#footnote-ref-7)
7. In case of involvement of the research team in the project implementation, it is necessary to identify the individual positions, identify the category of researcher R1-R4 for researchers and determine the level of involvement for all members of the research team. [↑](#footnote-ref-8)
8. Where the research team is involved in the project implementation, the personnel costs will include personnel costs of the researcher and the members of the research team. [↑](#footnote-ref-9)
9. The months are indicated ascending from the start of the project, i.e., the month in which the project started is M1. [↑](#footnote-ref-10)
10. In case of involvement of the research team in the project implementation, it is necessary to identify the individual positions, identify the category of researcher R1-R4 for researchers and determine the level of involvement for all members of the research team. [↑](#footnote-ref-11)
11. Where the research team is involved in the project implementation, the personnel costs will include personnel costs of the researcher and the members of the research team. [↑](#footnote-ref-12)
12. The months are indicated ascending from the start of the project, i.e., the month in which the project started is M1. [↑](#footnote-ref-13)
13. In case of involvement of the research team in the project implementation, it is necessary to identify the individual positions, identify the category of researcher R1-R4 for researchers and determine the level of involvement for all members of the research team. [↑](#footnote-ref-14)
14. Where the research team is involved in the project implementation, the personnel costs will include personnel costs of the researcher and the members of the research team. [↑](#footnote-ref-15)
15. Indicate the probability of risk occurrence (low, medium, high) and the severity of the risk (low, medium, high) [↑](#footnote-ref-16)