Project description (template) for call 09I03-03-V04 Fellowships for excellent researchers R2-R4

Content

[1. Excellence 2](#_Toc146661284)

[1.1 PROJECT OBJECTIVES 2](#_Toc146661285)

[1.2 RELEVANCE, QUALITY AND NOVELTY OF THE PROJECT 3](#_Toc146661286)

[1.3 METHODOLOGY 6](#_Toc146661287)

[1.4 EXCELLENCE OF THE RESEARCHER 8](#_Toc146661288)

[1.5 EXCELLENCE OF THE APPLICANT/HOST ORGANISATION 11](#_Toc146661289)

[2. Impact 11](#_Toc146661290)

[2.1 THE WIDER IMPACT OF THE PROJECT 11](#_Toc146661291)

[2.2 MEASURES TO MAXIMISE IMPACT – DISEMINATION AND COMMUNICATION, EXPLOITATION OF RESULTS 12](#_Toc146661292)

[3. Implementation 13](#_Toc146661293)

[3.1 PROJECT PLAN AND DELIVERABLES 13](#_Toc146661294)

[3.2 IMPLEMENTATION RISKS AND PROPOSED MEASURES 18](#_Toc146661295)

[3.3 OPERATIONAL CAPACITY OF THE APPLICANT/HOST ORGANISATION 19](#_Toc146661296)

Title of the project: Aggregation functions and their applications in formal concept analysis

Short title of the project/Acronym: *Aggregation and FCA*

Category of researcher: R3

Researcher’s job type (full-time, part-time – in %): part-time 80%

Type of research: independent

Identification of the entity involved in the implementation of the project:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Official name of the entity | Abbreviated name of the entity | Role in the project |
| 1 | Mathematical Institute of Slovak Academy of Sciences | MI SAS | Applicant/host organisation |

## 1. Excellence

### PROJECT OBJECTIVES

The project aims to study a possible connection between aggregation function theory and fuzzy formal concept analysis.

**Objective 1:** Analysisofknownapproaches to fuzzy concept lattices and aggregation function.

This objective is very important in study of aggregation functions within the theory of fuzzy concept lattices. It aims at collecting the additional knowledge contained in the recent literature from the relevant sources. Although the state-of-art literature is relatively well known to the applicant, the number of publications related to this topic is still growing and therefore the up-to-date knowledge of the field is necessary for its further development. Findings related to this objective will be consulted and evaluated by experts working in the respective fields.

**Objective 2:** Aggregation and concept forming operators

In this objective we plan to study an application of aggregation functions in the process of creation of fuzzy concept lattices, in particular their usage in the definition of concept forming operators and their subsequent use in the interpretation of the resulting fuzzy concepts. Main aim will be on study and characterization of the classes of aggregation functions that are compatible with concept forming operators, so that the resulting structure of fuzzy concepts forms a complete lattice. Some concepts and ideas will be presented in conferences in order to obtain feedback from other researchers. Compact results will be summarized into 1-2 publications, which will be submitted to a peer-reviewed journal.

**Objective 3:** Aggregation and concept lattice reduction

One of the problems that becomes noticeable in real applications of fuzzy FCA methods is the size of the concept lattice. We try to use aggregation operators in the problem of concept lattice reduction, in order to preserve as much relevant information about the original concept lattice as possible. This includes the study of different types of aggregation functions in the reduction of formal contexts and their impact on the resulting structure of the resulting concept lattice. This also includes the study of generalized aggregation operators defined on the fuzzy concept lattices that are related to fuzzy logic, such as t-norms, t-conorms, fuzzy negations, fuzzy implications. Results obtained within this objective will be presented in conferences, as well as submitted to the journals. We suppose to publish at least one journal paper.

**Objective 4:** Fuzzy contexts and fuzzy measures (capacities)

Within this objective we plan to study systems of lattice-valued capacities derived from fuzzy contexts and the corresponding fuzzy one-sided lattices respectively. Subsequently we aim at study of aggregation functions of integral type associated to these capacities. We will investigate special classes of integrals (Sugeno's, Choquet's, etc.) defined using capacities derived from contexts, studying their properties and interconnections, with the possibility of extending some types of integrals to domains in which their satisfactory definition is missing. We assume that these concepts will be presented to other researchers on conferences and summarized in a journal publication.

**Objective 5:** Aggregation of nonhomogeneous data

When aggregating nonhomogeneous data, the basic problem is to find a suitable structure in which one can represent the results of the aggregation process. Thus, we plan to study aggregation of nonhomogeneous data types, where one possibility is to consider fuzzy concept lattices as possible structures for the aggregation results. This will allow to interpret the corresponding aggregation within the considered object-attribute model and then use it in application domains of different fuzzy FCA-based methods. Obtained results will be published in conference proceedings and journal publications.

* 1. RELEVANCE, QUALITY AND NOVELTY OF THE PROJECT

In the current digital era, it is evident that a huge amount of heterogeneous data is being produced very rapidly without any fixed structure. In connection with this trend, various methods of processing and analysing this data become very important. An essential role in data processing is played by aggregation methods, which represent tools to extract important patterns found in large and complex datasets in order to preserve as much relevant information about these datasets as possible. Mathematically, the aggregation process is represented by the term "aggregation function", which describes the process of combining and merging some (usually numerical) values into a single representative one. The standard theory of aggregation functions is quite well developed in the case where both input and output values lie in the nonempty real interval [1]. Current research in aggregation theory focuses on the aggregation of unconventional data and the generalization of aggregation from real intervals to partially ordered sets, in particular bounded lattices. Aggregation functions currently represent one of the most powerful theoretical tools in any area of research where information from either homogeneous or heterogeneous sources is merged or combined. They are widely used in applied mathematics (e.g., probability, statistics), computer science (e.g., artificial intelligence, data mining, image recognition and image analysis), economics (e.g., game theory, voting theory, decision theory), social sciences, as well as many other applied disciplines in the natural sciences.

Formal Concept Analysis (FCA) [2] represents one of the data mining methods which is suitable for identifying conceptual structures in binary object-attribute models. The mathematical formalization of these object-attribute models is the notion of formal context, which is defined as a triple consisting of a set of objects, a set of attributes, and a binary relation between these two sets, expressing the relation of whether or not a given object has a given attribute. A concept within such a formal context is an ordered pair (C,D) consisting of subsets of objects and attributes, where D is the set of all attributes that are shared by all objects in C and, conversely, C is the set of all objects that have all the attributes presented in D. The set of all concepts corresponding to a given formal context can be hierarchically ordered according to a generalization relation, i.e. concept (C,D) is more general than concept (E,F) if E is a subset of C. The set of all formal concepts with an ordering defined in this way forms a complete lattice, which is referred to as the concept lattice corresponding to a given formal context.

As a very valuable theoretical tool, classical FCA has been applied in various fields such as knowledge discovery, information retrieval, decision theory or machine learning. However, in practice, there are many examples in which the relationships between objects and attributes are characterized using fuzzy (multi-valued) relations. Therefore, the development of new FCA-based methods in the presence of fuzzy, incomplete or inaccurate information is one of the main research directions in the field of hierarchical data analysis. In the framework of fuzzy FCA (e.g., [3] and [4]), formal contexts (object-attribute models) are considered, where individual objects are characterized by attributes with different fuzzy values. The concepts within these contexts are defined using concept-forming operators and they consist of ordered pairs formed by a fuzzy subset of objects and a fuzzy subset of attributes, or a classical subset of objects and a fuzzy subset of attributes, respectively. In the first case we are referring to fuzzy concepts, in the second one to the so-called one-sided concepts. As in the case of classical FCA, the system of all concepts forms a complete lattice.

Although these two research areas are both very intensively studied, their development is almost independent from each other. There are only a few works devoted to the study of their interrelationship and the potential use of one field within the other, cf. [5], [6], [7] or [8].

In the construction of fuzzy concept lattices, it is standard practice to use concept-forming operators that form the so-called Galois connexion, which is mainly due to the mathematical requirements being as close as possible to the algebraic framework of classical FCA. However, in this case the main problem is often with the interpretation of the real meaning of the resulting fuzzy concepts belonging to a given object-attribute model (the resulting fuzzy concept lattice contains too many " non-understood and unnecessary" concepts). This consequently makes decision making as well as the application of appropriate FCA-based methods significantly more difficult. One possible way to avoid this problem is to use aggregation functions in the definition of concept-forming operators, where the meaning of the resulting concept would be related to the " relevance" of a given aggregation function. However, the class of aggregation functions that are suitable to be applied in order to obtain the Galois connexion is not very large [8]. In this case, these are precisely supremum and infimum preserving aggregation functions. However, concepts can be also defined using other types of operators that do not satisfy such strong conditions as those imposed on Galois connexions. For example, one possibility is to use the so-called weak Galois connections described in [9]. It turns out that the class of aggregation functions that can be used to obtain a weak Galois connection in the final result is quite broad, e.g. it includes various types of averages, integral aggregation functions, penalty-based aggregation functions, etc. One of the goals of this project will be the study and characterization of classes of aggregation functions whose usage in the definition of concept-forming operators will be consistent with the requirement that the resulting structure of fuzzy concepts form a complete lattice, or a complete lattice with specific properties.

Another problem that becomes noticeable in real applications of fuzzy FCA methods is the size of the concept lattice. The number of concepts can grow exponentially in the worst case, which significantly complicates the "online" implementation of the mentioned methods. For this reason, the problem of concept lattice reduction is a relatively intensively studied area. The goal of a reduction is to preserve as much relevant information about the original concept lattice as possible. Reduction (value aggregation) is usually applied to a formal context, from which a fuzzy concept lattice is consequently constructed. It usually already has fewer number of elements and a simpler structure. In practice, however, a rather trivial method of reduction is used; one simply selects a subpart of the formal context. This corresponds to the use of trivial aggregation operators, the so-called projections. The plan is to use other, less trivial aggregation functions as a tool for reducing formal contexts and concept lattices. The goal will be to study different types of aggregation functions and their impact on the resulting structure of the reduced concept lattice.

The structure of a concept lattice captures the hierarchical ordering of concepts in a given object-attribute model. In decision support making and multi-criteria decision making based on a given hierarchy of concepts, it is useful to study the logical relationships between these concepts, e.g., by studying fuzzy logical conjunctions such as t-norms, t-conditions, fuzzy negations, fuzzy implications. Another goal will be to study these generalized aggregation operators on the concept lattice, with the possibility of defining them on the formal context, in such a way that the aggregation of the individual subparts of the context leads, after applying the concept-forming operators, to the corresponding aggregation on the concept lattice.

On the other hand, fuzzy formal concept analysis can be used as an effective support tool in applications of aggregation functions. For example, when using different types of fuzzy integrals, a very important task is to determine the appropriate fuzzy measure (capacity) with respect to which the integral is defined (computed). If the basic information about the objects to be aggregated is given by attributes (an object-attribute model characterizing the objects under consideration is available), then the concept lattice (a closure system on a set of objects) belonging to the model determines the capacity [5]. A similar approach can be taken into consideration if the underlying information is given by a fuzzy object-attribute model. In this case, a one-sided concept lattice can be used to obtain a system of L-valued fuzzy measures (L-capacities). The aim will be to study the systems of L-capacities defined in this way and their relation to the original fuzzy contexts from which they were derived, and also to study the properties of the classes of different fuzzy integrals associated to these systems of capacities. This approach could also be useful for extending integrals over structures where their definition is missing. For example, the Sugeno integral is currently well defined for bounded distributive lattices [10], while its satisfactory definition is not available for non-distributive ones. If the elements of a fuzzy context are from a bounded distributive lattice, the Sugeno integral can be applied to them according to its standard definition. On the other hand, the resulting fuzzy one-sided concept lattice need not be distributive anymore, but the results of aggregation (integration) on a context can be transferred to this concept lattice in several ways. The study of these "transfers", their mathematical properties, and the aggregation functions defined using them, could provide a way, or at least suggest a direction, to satisfactorily define the Sugeno integral (as well as other types of aggregation functions) on more general types of bounded lattices.

Another possible application of fuzzy concept lattices within aggregation theory, is their potential use in aggregating non-homogeneous data. In this case, the basic problem is to find a suitable structure in which one can represent the results of the aggregation process. Concerning the aggregation of data from different types of complete lattices, a fuzzy concept lattice is one option where to represent results. Its creation is also described for non-homogeneous types of contexts [11], which allows to map (represent) non-homogeneous data into a concept lattice. The goal will be to study such obtained representations in concept lattices as a possibility of aggregating non-homogeneous data types. Such aggregation can be reinterpreted in the framework of some object-attribute model and subsequently used in the application domains of various fuzzy FCA-based methods.

**References:**

[1] M. Grabisch, J.-L. Marichal, R. Mesiar, E. Pap: Aggregation Functions, Cambridge University Press, Cambridge, 2009.

[2] B. Ganter, R. Wille: Formal Concept Analysis. Mathematical Foundations; Springer: Berlin, Germany, 1999.

[3] R. Bělohlávek, V. Vychodil: What is a fuzzy concept lattice? In: Proc. CLA 2005. CEUR WS, vol. 162, pp. 34–45 (2005).

[4] R. Bělohlávek: What is a Fuzzy Concept Lattice? II. In: Kuznetsov, S.O., Ślęzak, D., Hepting, D.H., Mirkin, B.G. (eds) Rough Sets, Fuzzy Sets, Data Mining and Granular Computing. RSFDGrC 2011. Lecture Notes in Computer Science, vol 6743. Springer, Berlin, Heidelberg (2011).

[5] R. Ilin: Classification with concept lattice and Choquet integral. In: 2016 19th International Conference on Information Fusion (FUSION), Heidelberg, pp. 1554–1561 (2016).

[6] C. Alcalde, A. Burusco: On the use of Choquet integrals in the reduction of the size of L-fuzzy contexts. In: 2017 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Naples, pp. 1–6 (2017).

[7] O. Krídlo: Isotone L-Fuzzy Formal Concept Analysis and L-Valued Fuzzy Measures and Integrals, IPMU 2020, CCIS 1239, pp. 726–735, Springer 2020.

[8] R. Halaš, R. Mesiar, J. Pócs: Description of sup- and inf-preserving aggregation functions via families of clusters in data tables, Information Sciences 400–401 (2017) 173–183.

[9] J. Pócs: On possible generalization of fuzzy concept lattices using dually isomorphic retracts, Information Sciences 210 (2012) 89–98.

[10] M. Couceiro, J.-L. Marichal: Characterizations of discrete Sugeno integrals as polynomial functions over distributive lattices, Fuzzy Sets and System 161, pp. 694-707, 2010.

[11] J. Pócs: Note on generating fuzzy concept lattices via Galois connections. Information Sciences, 2012, vol. 185, no. 1, p. 128-136.

1.3 METHODOLOGY

Within this project methods and procedures typical for basic research in mathematics, with particular emphasis on specific methods of algebra, aggregation function theory and formal concept analysis, will be used. Special cases will be investigated and consequently generalized, thought experiments will be performed. Also, hypotheses will be formulated aiming to try prove them or to find a suitable counterexample respectively. The techniques of other mathematicians and scientists in the field will be followed, with the possibility of their creative development and application within the studied problems.

In the described Objectives we plan to focus on the following tasks:

**Objective 1:** Analysisofknownapproaches to fuzzy concept lattices and aggregation function.

Task 1.1: Study of recent publications related to the topic, attendance of conferences, discussions with other researchers working in the fields.

In this task we will focus on results published in distinguished journals related to our topic, recent monographies and conference proceedings. We will discuss the problematics with researchers working in the field of aggregation theory as well as with those working with various datamining methods.

Task 1.2: Preparation of list of aggregation operators suitable for usage with various types of fuzzy concept lattices.

In this task we plan to create a structured list of various types of aggregation operators which can be successfully used when applied to concrete data mining problems. We try to identify fuzzy formal contexts (fuzzy object-attribute models) compatible to the functions from the list.

Task 1.3: Study of the properties and the structure of functions from the list created in Task 1.2.

Here we will study the basic properties and the structure of aggregation functions from the list. With other researchers we will discuss possibilities of their adjustments and generalizations applicable for as wide as possible classes of fuzzy formal contexts.

**Objective 2:** Aggregation and concept forming operators.

Task 2.1: Study of aggregation operators and weak Galois connections.

In this task we focus on classifying and applying the aggregation functions from our list with the so-called weak Galois connections, which form a broad theoretical basis for the concept forming operators.

Task 2.2: Aggregation operators beyond weak Galois connections.

In this task, we plan to study the properties of aggregation operators from the list that turn out to be incompatible with weak Galois connections. In this case, we will try to relax the conditions defining this type of mappings, so that the concept forming operators are compatible with the considered aggregation functions, i.e., so that the resulting hierarchical structure of the fuzzy concepts forms a complete lattice.

**Objective 3:** Aggregation and concept lattice reduction.

Task 3.1: Reduction of general fuzzy concept lattices.

In this task, we will investigate reductions of general fuzzy concept lattices. Our aim will be to apply appropriate aggregation functions to parts of fuzzy formal contexts and then describe the resulting effect on the resulting concept lattice.

Task 3.2: GOSCL reduction.

Here we apply reductions from Task 3.1. to generalized one-sided concept lattices, GOSCL for short. These types of concept lattices are very similar to classical concept lattices in their structure and formation, allowing a comparison of the obtained reductions with known reductions in the classical case.

Task 3.3: Generalized aggregation operators on concept lattices.

Within this task we plan to apply generalized aggregation operators related to fuzzy logic on several formal contexts and study their effect on the corresponding concept lattices. A possibility to introduce these operators in such way on concept lattices will be examined.

**Objective 4:** Fuzzy contexts and fuzzy measures (capacities).

Task 4.1: GOSCL and fuzzy measures.

In this task we plan to study systems of fuzzy measures associated to the closure operators given by GOSCLs arising from fuzzy formal contexts. Properties and a structure of these fuzzy measures will be described.

Task 4.2: Integrals associated to the fuzzy measures.

This task focuses on study of some integrals (Sugeno, Choquet) associated to the fuzzy measures investigated in Task 4.1.

Task 4.3: Possible integral extension.

Here we explore the possibility of extending these types of integrals to classes of complete lattices or other domains where a satisfactory definition is missing.

**Objective 5:** Aggregationofnonhomogeneousdata.

Task 5.1: Aggregation on same family of contexts.

In this task, we explore the possibility of aggregating data coming from different complete lattices, which are represented by the same family of contexts. The structure for the aggregation results will be given by the appropriate concept lattice.

Task 5.2: Generalizations to different types of contexts.

In this task we will extend the knowledge found in Task 5.1 to the nonhomogeneous data arising from different types of contexts.

Possible challenges in the project’s implementation can appear if it turns out that formal proofs of the given claims cannot be found within the reasonable time during the duration of the submitted project. This can, for example, occur if procedures that seemed appropriate for special cases fail in general. To overcome the problems with extended time needed for proving some assertions, new ways and possibilities should be examined. Such new procedures can be found in the literature devoted to a similar issue, or by consulting with renowned experts in the field. Both of these activities form the content of Objective 1. Additional challenges may arise if a check of recent results available in the literature reveals that similar problems have already been studied and satisfactorily solved or partial cases have been covered. In this case, the various goals described in the list of individual tasks will need to be re-evaluated or adapted appropriately. Regular meetings and consultations are also planned in order to discuss other potential difficulties and, above all, possible solutions to deal with them.

The project is multi-disciplinary in its nature since it lies on the border of mathematics and computer science. Indeed, it aims to connect mathematical theory of aggregation functions with hierarchical data-mining methods commonly known as the fuzzy formal concept analysis.

The principles of open science will be followed and some of the selected publications we try to publish with open access. Currently it is unclear whether some large real data sets will be collected and processed during the project duration. In this case, the research data will be archived and shared through open repositories, in order to assure the free access for all interested researchers.

As an integral part of the project the gender equality will be strictly followed. The working group of researchers collaborating with the applicant will follow these principles. Equal access to resources will be inclusive and target women and men in all their diversity.

1.4 EXCELLENCE OF THE RESEARCHER

The researcher works as a senior researcher at Mathematical Institute of Slovak Academy of Sciences, Extension in Košice, Slovakia and as an assistant professor at Department of Algebra and Geometry, Faculty of Science, Palacký University Olomouc, Czech Republic**.** His research interest is mainly in the field of algebra and the theory of aggregation functions with their possible applications in computer science.He has 55 publications registered in Web of Science with 289 SCI citations.

Curriculum Vitae

**Personal information**

First and last name: Jozef Pócs

Identifier (ORCID): 0000-0003-0405-8579

Date of birth: December 15, 1979

Nationality: Slovak

Website (if relevant): https://im.saske.sk/pocs/index.html

**Education**

10/2008 – PhD

Faculty of Mathematics, Physics and Informatics Comenius University Bratislava, Slovakia

06/2004 – Master

Faculty of Science, Pavol Jozef Šafárik University in Košice, Slovakia

**Current position/positions**

09/2007 – Research Fellow (05/2014 Independent Research Fellow IIa),

(01/2020 Head of the detached department in Košice)

Institute of Mathematics, Slovak Academy of Sciences, Košice, Slovakia

09/2015 – Assistant professor

Department of Algebra and Geometry, Faculty of Science, Palacký University Olomouc, Czech Republic

**Previous positions**

09/2011 – 02/2012 – Assistant professor

The Faculty of Mining, Ecology, Process Control and Geotechnologies, Technical University of Košice, Slovakia

01/2013 – 08/2015 – Postdoctoral fellow

Department of Algebra and Geometry, Faculty of Science, Palacký University Olomouc, Czech Republic

**Scholarships and awards**

2009 - Prize for scientific literature in natural and technical sciences of the Slovak Literary Fund (monograph: D. Jakubíková-Studenovská, J. Pócs; Monounary Algebras)

2011 - holder of the Stefan Schwarz Support Fund, Slovak Academy of Sciences, Slovakia

2014 - First place in the competition of young scientists of the SAS up to 35 years of age in the section of Physical, Space, Earth, and Engineering Sciences

Student and post-docs supervision (if applicable)

2015 - 2017 - 1 master student

Faculty of Science, Palacký University Olomouc, Czech Republic

**Teaching activities (if applicable)**

2015 - Assistant Professor - Discrete Mathematics, Palacký University Olomouc, Czech Republic

**Organisation of scientific meetings (if applicable)**

2022 - Chair of the Organizing Committee, Summer School on General Algebra and Ordered Sets 2022, number of participants 26, Slovakia

**Institutional responsibilities (if applicable)**

2022 - Member of the Governing Board, Institute of Mathematics, Slovak Academy of Sciences, Slovakia

2020 - organizer of the internal seminar of the Institute of Mathematics of the Slovak Academy of Sciences in Košice,

**Reviewing activities (if applicable)**

2020 - Member of the Editorial Board, Tatra Mountains Mathematical Publications, Slovakia

**Memberships of scientific societies (if applicable)**

**Major collaborations (if applicable)**

Prof. Mgr. Radomír Halaš Dr., aggregation functions, Department of Algebra and Geometry, Faculty of Science, Palacký University Olomouc, Czech Republic

Prof. RNDr. Radko Mesiar DrSc., aggregation functions, Faculty of Civil Engineering STU in Bratislava, Slovakia

doc. Ing. Peter Butka, PhD., fuzzy formal concept analysis, Faculty of Electrical Engineering and Informatics, Technical University of Košice

**Overview of the researcher’s most important projects in the last 5 years** (max. 5)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Project name/identification** | **Source of funding** | **Budget (EUR)** | **Project period** | **The role of the researcher in the project** |
| Algebraical and topological aspects of aggregation functions / VEGA 2/0097/20 | Scientific grant agency MESRaS SR and SAS | cca 35 000 EUR | 2020 – 2023 | Principal investigator |
| QUANTPROBALG - Probabilistic, Algebraic and Quantum Mechanical Methods of Uncertainty Determination / APVV-20-0069 | Slovak Research  and Development  Agency | cca 123 000 EUR | 2021 – 2025 | Research team member |
| New approaches to aggregation operators in analysis and processing of data / GAČR 18-06915S | The Czech Science Foundation (GAČR) | cca 340 000 EUR | 2018 – 2020 | Research team member |
| Probabilistic, Algebraic and Quantum-Mechanical Aspects of Uncertainty / APVV-16-0073 | Slovak Research  and Development  Agency | cca 143 000 EUR | 2017 - 2021 | Research team member |

**Overview of the researcher’s most important outputs** (max. 5)

|  |  |  |  |
| --- | --- | --- | --- |
| **Output name/identification** | **Type of output** *(e.g., publication, dataset, software, patent, service, product, etc.)* | **Short description** | **The role of the researcher** |
| PÓCS, J.: Note on generating fuzzy concept lattices via Galois connections. In Information Sciences, 2012, vol.  185, no. 1, p. 128-136. ISSN 0020-0255. | publication | A general method for generating fuzzy concept lattices using Galois connections is described | Author |
| BUTKA, P., PÓCS, J: Generalization of one-sided concept lattices. In Computing and informatics, 2013, vol. 32, no. 2, p. 355-370. ISSN 1335-9150. | publication | The construction of general one-sided concept lattices with non-homogeneous types of attributes is described | co-author |
| HALAŠ, R., MESIAR, R., PÓCS, J.: A new characterization of the discrete Sugeno integral. In Information Fusion,  2016, vol. 29, p. 84-86. ISSN 1566-2535. | publication | A new axiomatics of the Sugeno integral as a compatible aggregation function is provided | co-author |
| HALAŠ, R., KURAČ, Z., PÓCS, J.: On the minimality of some generating sets of the aggregation clone on a finite chain. In Information Sciences, 2021, vol. 564, p. 193-201. ISSN 0020-0255. | publication | The minimality of the set of generators of the aggregation clone defined on a finite chain is studied | co-author |
| HALAŠ, R., MESIAR, R., PÓCS, J.: On the number of aggregation functions on finite chains as a generalization of Dedekind numbers, In Fuzzy Sets and Systems 2023, vol. 466, Article n. 108441, ISSN 0165-0114. | publication | It is shown that the number of all aggregation functions defined on a finite chain can be regarded as a generalization of the so-called Dedekind numbers | co-author |

The most important research results of the applicant related to this project include the following:

* Investigation of various types of fuzzy concept lattices, methods of their creations, their properties and several ways of their representations. This contains a deep study of concept forming operators based on Galois connections. A general method how to define concept forming operators from a given fuzzy context in nonhomogeneous framework was described. This method was further developed to a more general setting, where the so-called weak Galois connections were used for construction of fuzzy concept lattices. As a continuation of this process, beside the general fuzzy concept lattices, the so-called one-sided concept lattices were investigated. Again, several construction methods and their representations were described. These types of lattice have proven to be a very suitable application tool in domains like text mining or informational retrieving.
* Research of algebraic properties of aggregation functions. This includes study of clones and some subclones of aggregation functions defined on finite lattices. Using methods of universal algebra, it was shown that this clone and some of its subclones are finitely generated. Explicit generating sets were described and their minimality was investigated. The similar results were obtained also in the case of aggregation clone on the real unit interval and they were further generalized to infinite bounded lattices. Using certain generating sets of the aggregation clone, it was proved that n-ary aggregation functions on a finite chain form a free algebra in a finitely generated variety of certain algebras. Based on this result it was shown, that the number of all n-ary aggregation functions on a finite chain represents a generalization of the so-called n-th Dedekind number, originally defined as the cardinality of a free distributive lattice with n generators.
* A subclass of aggregation functions commonly known as integral aggregation functions was investigated. Prominent role in this class is played by the Sugeno integral, formally introduced on the real interval using the suprema and infima operations, which allows its generalization to bounded distributive lattices. An axiomatization, based on the notion of compatibility, was published. Sugeno integral with respect to a given fuzzy measure (capacity) is the unique compatible aggregation function extending this measure. Moreover, it was shown that the characteristic property of uniqueness of the extension of an L-valued fuzzy measure to a monotone compatible function is equivalent to the distributivity of the underlying bounded lattice L. Another axiomatization of this type of integral based on the generalized comonotonicity was described. Some properties of this relation, also with respect to the Sugeno integral was studied.

1.5 EXCELLENCE OF THE APPLICANT/HOST ORGANISATION

Mathematical Institute of the Slovak Academy of Sciences is a scientific institute focused mainly on basic research in mathematics and theoretical informatics. Mathematical Institute is located in Bratislava and its organizational structure also includes Department in Košice, Department of Informatics in Bratislava and Institute of Mathematics and Computer Science in Banská Bystrica. The Institute has a long tradition in several important branches of pure and applied mathematics and participated in a number of successful projects in both basic and applied research, including projects of Frame Projects of EU, Structural projects of EU, and projects of domestic agencies APVV and VEGA. The researchers of the Institute belong to the top in their research, in a world-wide context, and are engaged in multiple collaborations with experts from internationally renowned institutions. In collaboration with the Commenius University in Bratislava, the Institute organises a PhD study program and many young scientists and students use the Slovak fellowship program SAIA for short term study stays at the institute.

There are several areas of research closely related to the project that are pursued strongly at Mathematical Institute already for a long time. For example, the institute is traditionally one of the world-wide most important centers of research in Quantum Structures. Of the current researchers, Anatolij Dvurečenskij, Silvia Pulmannová and Anna Jenčová are leading experts in this field. The field of fuzzy mathematics, which is the most relevant to the project, is another strong research field at the Institute. The top researcher is Andrea Zemánková, who is a leading expert in the theory of aggregation functions. She authored and co-authored more than 60 papers in leading top journals and was very recently awarded the price of the Slovak Academy of Sciences for an excellent publication. This field is also intensively studied at Department in Košice. The researchers of the institute are well connected with world-wide experts in their respective branches. In the field of fuzzy mathematics, the Institute has a close connection to neighbouring institutions such as the Slovak Technical University in Bratislava, Palacký University in Olomouc or University of Ostrava, through extensive collaborations and numerous joint projects. The institute organizes several successful and long-term established seminars, with invited talks by distinguished scientists.

The institute is well equipped with all the standard hardware and software needed for mathematics research, including quality equipment for online presentation and communication. The library of Mathematical Institute SAS belongs to the best mathematical libraries in Slovakia, with access to the most important scientific databases.

## Impact

### 2.1 THE WIDER IMPACT OF THE PROJECT

The importance of aggregation functions in various disciplines, especially where large data sets need to be suitable represented and processed, is unquestionable. One of these areas is hierarchical data analysis, where formal concept analysis plays a very important role. Since methods, procedures and techniques known in the theory of aggregation functions have not yet been sufficiently applied in the theory of concept lattices, it can be assumed with high probability that the project will have a non-trivial impact on the aforementioned data mining area. The procedures described here are more or less in accordance with the standard line of research in both fields and their reliability has already been verified. The expertise of the applicant, which is necessary for the successful implementation of this project, is indicated by his previous work in both areas, i.e. within the aggregation theory and also within the theory of fuzzy concept lattices.

The expected impact of this project is in linking two powerful theoretical tools commonly used in data science. From the scientific point of view the project will contribute to better understanding of the process of creating fuzzy concept lattices, their characterization, properties and their usefulness under different circumstances. The obtained results will also contribute to the development of the aggregation theory, especially to the process of aggregation of data from different types of domains. As a by-product, the work on linking the two topics within the project may lead to unexpected ideas that could solve seemingly unrelated open problems in aggregation function theory as well as in the theory of fuzzy concept lattices. The results of the project will be available at local, regional, as well as international level. Data and codes related to applications based on the results of the project, will be freely disseminated and could be used by other researchers.

In the short term, the results of the project will be published in peer-reviewed journals or presented at scientific conferences and seminars. It can be assumed that this will attract the attention of other researchers and show them the potential for further developing the link between the two research areas. In the medium term, we expect that the aforementioned interconnection of the two theories will be further developed and generalized to eventually encompass a broader class of data mining methods. Although the project is purely mathematical in its focus, we plan to further disseminate the results in order to attract the attention of researchers from other disciplines, not only mathematicians. In the long term, we expect that the achieved results will be tested on real data first and then gradually implemented in various data mining applications.

Beside the professional growth of the researcher, the implementation of the project will enable him to start new cooperation with experts vising the host institution. Attending conferences which serve as a source of new ideas, will allow him consultations with experts about further applicability of the results obtained during the work on the submitted project. Visits of young researchers at the host institute also offer a possibility to create a group of young researchers interested in the corresponding topic of the project.

The host organization will gain an expert working in the inter-disciplinary field belonging to mathematics and computer science. The members of the institute will benefit from the exchange of knowledge between them and the researcher. If the project proves to be useful and highly applicable, the host institution will gain more credit in the scientific community, which may attract young scientists or PhD students to become members of the institute. Joint work with other experts from the host institute can consequently lead to a fruitful inter-disciplinary collaboration. Cooperation of young researchers with the applicant, in the field of the project, will lead to the development of their career and the acquisition of new knowledge and skills.

During the duration of the project it is planned to publish at least 4 or 5 papers in peer-reviewed journals and several papers in conference proceedings. It is assumed that these publications will contribute to the development of the aggregation theory as well as the theory of fuzzy concept lattices. Collaboration and various knowledge sharing with experts from related fields is planned. Estimation of outputs of the project is based on the experiences of applicant from previous projects where he participated, taking into account his expertise and his previous results obtained in the both research fields.

In order to increase the impact of the project and to maximize the results monitoring of the recent publications and communication with leading experts working in the two mentioned areas will be performed.

* Potential obstacles to the planned impact of the project

The impact of some tasks of the project can be limited if these tasks will find to be already covered by the work of other researchers working in the field. The practical impact of the project can be affected provided that during an attempt to apply some theoretically developed procedures will be outperformed by other already known methods.To eliminate such possibilities a constant monitoring of the recent literature in the corresponding fields is planed. Beside this, attending of conferences and meetings with leading researchers working in the areas of this project will enable early detection of such situations. This allow the applicant to appropriately adjust the involved tasks.

### 2.2 MEASURES TO MAXIMISE IMPACT – DISEMINATION AND COMMUNICATION, EXPLOITATION OF RESULTS

The project is theoretical in its nature, hence in order to maximize impact of the obtained results, the scientific findings of the project will be published in internationally recognized peer-reviewed journals with a good accessibility. Some of the papers will be published with open access. Beside this, obtained results will be also presented at conferences and workshops closely related to the scientific fields of the project. International research visits of the applicant to distinguished scientific institutions will contribute to the further dissemination of the project results. Regular seminars organized by the host institute, or by institutes which the applicant will visit during the project will help to further spread results to a wider audience.

## Implementation

3.1 PROJECT PLAN AND DELIVERABLES

The project is divided into six work packages, first five corresponding to one objective, while the last one summarizes the obtained project’s results:

1. Literature monitor
2. Aggregation and CFO
3. Reduction
4. Fuzzy measures
5. Nonhomogeneous aggregation
6. Project's results summarization

The duration of the project is planned for 24 months. All tasks planned for the project are included in one of the first five work packages. The sixth work package is devoted to the project’s results summarization.

All work packages are carefully designed to consistently capture all the individual steps necessary for the research planned in this project. The first work package precedes all the others, while WP1, WP2, WP3, WP4 are more or less independent.

Since the project belongs to the field of basic research, it is difficult to predict the exact time required to obtain specific results, however, the estimated period dedicated to solution of each work package can be found in the following timetable.

3.1.1 Work packages

|  |  |
| --- | --- |
| Work package number | WP1 |
| Title of the work package | Literature monitor |
| **Start of implementation of the work package (Mx Month)** | M1 |
| End of implementation of the work package (Mx month) | M24 |
| **Involvement (expressed in Person Months)** | Jozef Pócs - 7.48 M |
| **Personnel costs (in EUR)** | 28 862.83 |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 6 483.10 |
| Objectives | |
| Analysisofknownapproaches to fuzzy concept lattices and aggregation function. Constant monitoring of new publications and study of their relation to the project. | |
| Description of the work package | |
| Task 1.1: Study of recent publications related to the topic, attendance of conferences, discussions with other researchers working in the fields.  Task 1.2: Preparation of list of aggregation operators suitable for usage with various types of fuzzy concept lattices.  Task 1.3: Study of the properties and the structure of functions from the list created in Task 1.2. | |
| Deliverables | |
| D1: **First state-of-art report**  A detailed summary of known approaches to fuzzy concept lattices and of aggregation functions used in the reviewed literature, including the list of aggregation functions and their basic properties and structures and application domains.  D2: **Second state-of-art report**  A detailed summary of recent development in the fields related to project. | |

|  |  |
| --- | --- |
| Work package number | WP2 |
| Title of the work package | Aggregation and CFO |
| **Start of implementation of the work package (Mx Month)** | M2 |
| End of implementation of the work package (Mx month) | M8 |
| **Involvement (expressed in Person Months)** | Jozef Pócs – 2.18 M |
| **Personnel costs (in EUR)** | 8 418.33 |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 1 890.90 |
| Objectives | |
| Aggregation operators compatible with concept forming operators based on weak Galois connections will be studied. Also, concept forming operators beyond weak Galois connections will be investigated. | |
| Description of the work package | |
| Task 2.1: Study of aggregation operators and weak Galois connections.  Task 2.2: Aggregation operators beyond weak Galois connections. | |
| Deliverables | |
| D3: **Technical report**  A document in which we will summarize our findings, achievements, problems and possible further directions of research.  D4: **Research papers**  1-2 publications containing results of this work package will be submitted into selected journals. | |

|  |  |
| --- | --- |
| Work package number | WP3 |
| Title of the work package | Reduction |
| **Start of implementation of the work package (Mx Month)** | M6 |
| End of implementation of the work package (Mx month) | M15 |
| **Involvement (expressed in Person Months)** | Jozef Pócs – 3.11 M |
| **Personnel costs (in EUR)** | 12 026.18 |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 2 701.30 |
| Objectives | |
| Reduction of various types of fuzzy concept lattices will be studied using aggregation functions. | |
| Description of the work package | |
| Task 3.1: Reduction of general fuzzy concept lattices.  Task 3.2: GOSCL reduction.  Task 3.3: Generalized aggregation operators on concept lattices. | |
| Deliverables | |
| D5: **Conference contribution**  Results achieved in work packages WP3 and WP2 will be presented on some (1 or 2) conferences.  D6: **Journal research paper**  A publication devoted to the results of this WP will be submitted in one (or more) journals. | |

|  |  |
| --- | --- |
| Work package number | WP4 |
| Title of the work package | Fuzzy measures |
| **Start of implementation of the work package (Mx Month)** | M10 |
| End of implementation of the work package (Mx month) | M20 |
| **Involvement (expressed in Person Months)** | Jozef Pócs – 3.44 M |
| **Personnel costs (in EUR)** | 13 228,80 |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 2 971.45 |
| Objectives | |
| Fuzzy measure (capacities) associated to fuzzy contexts (and to one-sided concept lattices) will be studied. The corresponding integrals will be investigated. | |
| Description of the work package | |
| Task 4.1: GOSCL and fuzzy measures.  Task 4.2: Integrals associated to the fuzzy measures.  Task 4.3: Possible integral extension. | |
| Deliverables | |
| D7: **Conference contribution**  Results concerning integral functions will be presented on one of conferences.  D8: **1 journal publications**  At least one publications related to results from this WP will be submitted for publication. | |

|  |  |
| --- | --- |
| Work package number | WP5 |
| Title of the work package | Nonhomogeneous aggregation |
| **Start of implementation of the work package (Mx Month**3**)** | M15 |
| End of implementation of the work package (Mx month) | M24 |
| **Involvement (expressed in Person Months)**4 | Jozef Pócs – 3.11 M |
| **Personnel costs (in EUR)**5 | 12 026.18 |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 2 701.30 |
| Objectives | |
| In this WP aggregation of nonhomogeneous data will be investigated. | |
| Description of the work package | |
| Task 5.1: Aggregation on same family of contexts.  Task 5.2: Generalizations to different types of contexts. | |
| Deliverables | |
| D9: **1 journal or proceedings publication**  Publication related to results from this WP will be submitted for publication in some journal, or it will be submitted as a conference proceeding. | |

|  |  |
| --- | --- |
| Work package number | WP6 |
| Title of the work package | Project's results summarization |
| **Start of implementation of the work package (Mx Month)** | M10 |
| End of implementation of the work package (Mx month) | M24 |
| **Involvement (expressed in Person Months)** | Jozef Pócs – 4.68 M |
| **Personnel costs (in EUR)** | 18 039.28 |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 4 051.95 |
| Objectives | |
| Summarization of results obtained in previous WPs into a comprehensive summary. Monitoring of the progress of the project. | |
| Description of the work package | |
| Task 6.1: Monitor the progress of the project for timely solution of any problems.  Task 6.2: To summarize results obtained in the project into a comprehensive reports.  Task 6.4: Write a research proposal for a new project/grant with the involvement of the researcher | |
| Deliverables | |
| D10 **Interim report**  Report summarizing results achieved in the middle of the duration of project, their evaluation, possible delays or deviations to the plan, together with their justification and comparison between obtained and expected results.  D11 **Final report**  Report summarizing all results obtained during implementation of the project and comparison between achieved and expected results.  D12 **New research project (grant) application**  Research proposal for a new project (grant) , which will involve the applicant. | |

3.1.2 List of work packages:

|  |  |  |  |
| --- | --- | --- | --- |
| Work package number | Title of the work package | **Start of activities** | **End of activities** |
| WP1 | Literature monitor | M1 | M24 |
| WP2 | Aggregation and CFO | M2 | M8 |
| WP3 | Reduction | M6 | M15 |
| WP4 | Fuzzy measures | M10 | M20 |
| WP5 | Nonhomogeneous aggregation | M15 | M24 |
| WP6 | Project's results summarization | M10 | M24 |

3.1.3 List of deliverables:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Deliverable number | Deliverable | Work package number | Type | Access and dissemination | Method of verification | Delivery (project implementation month) |
| D1 | First state-of-art report | WP1 | report | non-public | Expert review | M5 |
| D2 | Second state-of-art report | WP1 | report | non-public | Expert review | M20 |
| D3 | Technical report | WP2 | report | non-public | Expert review | M6 |
| D4 | Research papers | WP2 | publication | public | Peer review | M8 |
| D5 | Conference contribution | WP3 | publication | public | Peer review | M10 |
| D6 | Journal research paper | WP3 | publication | public | Peer review | M12 |
| D7 | Conference contribution | WP4 | publication | public | Peer review | M13 |
| D8 | Journal research paper | WP4 | publication | public | Peer review | M15 |
| D9 | Jour./proc. Publication | WP5 | publication | public | Peer review | M24 |
| D10 | Interim report | WP6 | report | non-public | Progress  evaluation | M12 |
| D11 | Final report | WP6 | report | non-public | Project  evaluation | M24 |
| D12 | Project proposal | WP6 | proposal | non-public | Proposal  evaluation | M23 |

3.1.4 List of milestones:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Milestone number | Milestone | Work package number | Method of verification | Expected time to reach the milestone (project month) |
| MS1 | Characterization of aggregation functions compatible  with various types of concept forming operators | WP2 | Peer review | M8 |
| MS2 | Description of some classes of aggregation function  and their effect on a reduction of concept lattices | WP3 | Peer review | M15 |
| MS3 | Categorization of fuzzy measures and corresponding integrals  depending on fuzzy formal contexts | WP4 | Peer review | M20 |
| MS4 | Characterization of fuzzy concept lattices for possible  aggregation of nonhomogeneous data from formal contexts | WP5 | Peer review | M24 |

3.2 IMPLEMENTATION RISKS AND PROPOSED MEASURES

The foreseeable risks associated with the implementation and outputs related to this project fall into the following categories:

a) Some of the problems studied will turn out to be too complicated and there will not be enough time within the project to solve them satisfactorily. This eventuality can be partially eliminated by intensifying consultation with experts in the field.

b) It can happen that some proposed problems and minor tasks planned in this project will be solved by other researchers during the project. The appearance of such a situation provides an opportunity to build on new results, to develop them creatively and to try to establish cooperation on the issue.

c) Problems with the project’s outputs. There may be problems with publication delays caused by the publisher or an unexpectedly long review process. Given the experience of previous years, the possibility of a pandemic must also be taken into account. This may cause a lockout or similar obstacles, which may negatively affect participation in conferences and personal contacts with experts. The latter issues can be partially avoided by participating in online conferences and online meetings with experts.

Since this project is focused on basic theoretical research, we do not expect any legislative issues.

3.2.1 Risks of implementation:

|  |  |  |
| --- | --- | --- |
| **Description of the risk of implementation** | **Work package** *(one or more)* | Proposed measures for risk mitigation or elimination |
| Task is too complicated (medium) | WP2, WP3, WP4, WP5 | Contact experts working in the field  for cooperation on the solution of the task.  If none of the experts would be able to solve  the task, skip it and focus on subsequent tasks. |
| Task is already solved (low) | WP2, WP3, WP4, WP5 | Try to build on already published work.  Continue to work on other tasks. |
| Publication delays (medium) | WP2, WP3, WP4, WP5 | Check journal’s average  publication and review time.  Contact the members of editorial  board if long delay appears. |
| Pandemic (medium) | All packages | Focus on online conferences, online  collaboration, use home office. |

3.3 OPERATIONAL CAPACITY OF THE APPLICANT/HOST ORGANISATION

3.3.1 Description of the research/innovation infrastructure of the applicant/host organisation that is necessary for the implementation of the project:

No special equipment beyond the normal needs of basic theoretical research is required for the successful implementation of this project. The researcher will have access to common technical and computer equipment, as well as access to online scientific databases such as Science Direct, Scopus, WoS, SpringerLink, SpringerNATURE, JSTOR, PLOS, PNAS, Taylor & Francis, Wiley Online Library, etc.

The Mathematical Institute of the Slovak Academy of Sciences has one of the best equipped mathematical libraries in Slovakia, which will be fully available to the researcher.

In case of the need for high computing power (which is unlikely in this project), the Slovak Academy of Sciences has a supercomputer DEVANA with a performance of about 800 TFlop/s.

3.3.2 List of the five most important projects of the applicant/host organisation and their relevance to the proposed project (in the last 5 years):

|  |  |  |
| --- | --- | --- |
| Project name/identification | Programme/scheme/grant provider | Short description |
| Mathematical support of quantum technologies/  NFP313010T683 | ITMS-2014+/ Research Agency of MESRS of the SR | Independent research and development to gain new insights into mathematical structures and functions involved in quantum technologies. |
| Probabilistic, Algebraic and Quantum-Mechanical Aspects of Uncertainty /APVV-16-0073 | Slovak Research and Development  Agency | The aim of the project was to study the mathematical foundations of quantum mechanics and uncertainty using the most advanced methods of quantum structures. |
| QUANTPROBALG - Probabilistic, Algebraic and Quantum Mechanical Methods of Uncertainty Determination / APVV-20-0069 | Slovak Research and Development  Agency | Using modern quantum structure methods, the mathematical foundations of quantum mechanics and uncertainty are studied. Several algebraic structures are investigated, e.g, effect algebras, MV-algebras, synaptic algebras, orthomodular lattices. Using aggregation methods, an attempt to combine selected measurement values into a single aggregation function is studied. |
| Algebraical and topological aspects of aggregation functions / VEGA 2/0097/20 | Scientific grant agency MESRaS SR and SAS | The aim of this project was to study various families of aggregation functions from algebra, topology and functional  analysis point of view. |

3.3.3 List of maximum five most important outputs of the applicant/host organisation relevant to the submitted project:

|  |  |  |
| --- | --- | --- |
| Output name/identification | **Type of output** | Short description |
| J. Pócs:  Note on generating fuzzy concept  lattices via Galois connections.  Information Sciences, 2012, vol. 185,  no. 1, pp. 128-136. | publication | A general method for creating nonhomogeneous fuzzy concept lattices is described. It is shown that several known methods are special cases of the proposed methods. |
| R. Halaš, R. Mesiar, J. Pócs:  A new characterization of the discrete  Sugeno integral.  Information Fusion,  2016, vol. 29, pp. 84-86. | publication | A new axiomatic characterization of the discrete Sugeno integral is provided. It is shown that the discrete Sugeno integral with respect to a given capacity is the unique compatible aggregation function extending this capacity. |
| Ľ. Antoni, P. Eliaš, S. Krajči, O. Krídlo:  Heterogeneous formal context and its  decomposition by heterogeneous fuzzy  subsets,  Fuzzy Sets and Systems, vol. 451, 2022,  pp. 361-384. | publication | A method for decomposition of heterogeneous  formal context by heterogeneous fuzzy subsets  is proposed. It allows representing the hetero-  geneous structure of truth degrees for each object,  heterogeneous structure of truth degrees for each  attribute and heterogeneous structure of truth  degrees for relationship between each pair of  object and attribute. |
| A. Mesiarová-Zemánková: Decomposable  and k-additive multi-capacities and multi-  polar capacities.  Fuzzy Sets and Systems, 2016, vol. 287,  pp. 22-36. | publication | In the paper Extensions of several concepts for  definition of capacities given by a small number  of parameters in the unipolar and bipolar framework  to the case of multi-polar input spaces are studied. |
| R. Mesiar, A. Mesiarová-Zemánková: The Ordered Modular Averages.  IEEE Transactions on Fuzzy Systems, 2011,  vol. 19, no. 1, pp. 42-50. | publication | Special Ordered modular averages are characterized,  such as associative operators, operators with neutral  element, or operators with an annihilator. Some  construction methods for Ordered modular averages are also discussed. |