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Title of the project: Structure of non-commutative associative functions on real intervals

Short title of the project/Acronym: SONCAFORI

Category of researcher: R3

Researcher’s job type: full-time researcher

Type of research: independent

Identification of the entity involved in the implementation of the project:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Official name of the entity | Abbreviated name of the entity | Role in the project |
| 1 | Mathematical Institute, Slovak Academy of Sciences | MI SAS | Host organisation |

## 1. Excellence

### 1.1 Project objectives

**Objective 1**: Analysis of known non-commutative associative functions (NCAFs)

This objective is crucial for the further study of NCAFs and it aims at collecting the additional knowledge contained in the recent literature from the relevant fields. The NCAFs used both in theory and applications will be identified and their structure will be studied. Although the state-of-art literature is well known to the researcher, the amount of publications related to this topic is still growing and therefore the up-to-date knowledge of the field is necessary for its further development. Findings related to this objective will be summarized in a state-of art report and evaluated by an expert.

**Objective 2**: Construction methods and characterizations of non-commutative associative aggregation functions (NCAAFs) with (local) neutral element.

In the commutative case, functions with a neutral element (like t-norms, t-conorms and uninorms) and with local neutral elements (like nullnorms and *n*-uninorms) are the most studied and applied classes of aggregation functions. Therefore, the study of their counterparts in the non-commutative case is an important task. In this objective we would like to extend our work on idempotent pseudo-uninorms and pseudo-uninorms with continuous Archimedean underlying functions to all pseudo-uninorms with continuous underlying functions and then to pseudo-*n*-uninorms with continuous underlying functions. Further generalizations will be studied as well. Results will be summarized into 1-2 publications, which will be submitted to a high-rank journal and peer-reviewed.

**Objective 3**: Construction methods for NCAFs

In this task we plan to explore construction methods for NCAFs. Since in the case of Archimedean NCAAFs several properties, like for example the continuity, imply commutativity, we will focus mainly on non-Archimedean functions and on generalizations of commutative summand-based methods (like for example ordinal or *z*-ordinal sum) to the non-commutative case. Generalizations of other commutative construction methods, related to ordinal sum, to the non-commutative case will be considered as well. Our concepts will be presented to other researchers on a conference and summarized in a journal publication, which will be peer-reviewed.

**Objective 4**: Representation of NCAAFs continuous around the diagonal

In this task we will use construction methods developed in Objective 3 for decomposition and representation of NCAAFs. We expect that nice characterizations can be achieved for functions defined on real intervals, which are continuous around the main diagonal – similarly as it was achieved in the commutative case. However, due to the lack of commutativity, new approaches to this decomposition have to be adopted. Obtained results will be published in conference proceedings and journal publications, which will be peer-reviewed.

**Objective 5**: Generalizations and applications

In this task we will extend the construction methods developed in Objective 3 to more general non-associative and non-commutative functions, e.g., pseudo-overlap functions and pseudo-grouping functions, and we will justify usefulness of our construction methods by applications of constructed functions (both associative and non-associative) in problems from multi-criteria decision making and in other domains like for example, natural language processing, depending on the suitability of the constructed functions to corresponding problems from these domains. Adequacy of our concepts will be verified on applications and compared to other existing methods.

Depending on time necessary for solving the above described tasks, we will investigate also a complementary task of representation of NCAAFs defined on more general bounded lattices. We will start with simpler structures, such as horizontal sums of chains, and then we will move to more general structures.

**Objective 6**: Monitoring the progress of the project, summarization and dissemination of results

Results obtained in Objectives 1-5 will be summarized into a comprehensive summary and corresponding reports will be produced. The researcher’s web page containing description of the project, information about researcher, data, methods and code related to applications, and a link to publications will be launched. The progress of the project will be closely monitored for timely solution of any problems and, if necessary, preventive measures will be taken to mitigate the negative impact on the project. In this task we will also focus on dissemination of results via journal publications, conference lectures, contributions in proceedings, seminar talks and discussions with experts as well as with young researchers. Finally, a new research proposal will be drafted. In addition to the interim and the final report, we will produce a brochure that will summarize the main achievements of the project. Our conclusions will be evaluated by experts.

1.2 RELEVANCE, QUALITY AND NOVELTY OF THE PROJECT

The class of associative (aggregation) functions is very important both in theory and in applications, which is confirmed by an intensive study of this class of functions in the last century, while associative aggregation functions (such as t-norms, t-conorms, uninorms, nullnorms, etc.) were successfully applied in many scientific fields like probability, statistic, many-valued logic, decision theory, artificial intelligence, neural networks, image processing, data fusion, however, also in economics and social sciences.

Note that although various particular fusion methods were already known and applied in ancient Egypt and Greece, the aggregation theory itself is a relatively new field of research. Since the 1980‘s, aggregation theory has become an established branch of mathematics and computer science, focusing primarily on design methods and analysis of real functions at intervals, as documented in rich literature, such as the monographs [BBC 2016], [BPC 2007], [G 2015], [GMMP 2009], [NT 2007], [D 2018], etc. The recent growing demand for aggregation has initiated intensive development of new aggregation methods that allow processing of data from different sources or modalities, e.g. collected on social media [M-ZKA 2018]. Modern aggregation techniques propose strategies how to handle data with different credibility [JXChMY 2022], coming from various contexts, or even data where part of the information is corrupted or missing [DeTCP 2008], [D 2012], [ŠCBBD 2019]. Based on the bipolar and multi-polar tasks coming, e.g., from expert systems like MYCIN or PROSPECTOR, or from the field of game theory, classification and even international relations - aggregation on bipolar and multi-polar scales were also considered [YR 1996], [DeBF 1999], [GL 2005], [M-ZH 2016]. Motivated by practical applications of aggregation in computer science, social sciences, biology and other fields, research in aggregation theory focuses on development of aggregation functions in various spaces as subspaces of Riesz spaces, bounded lattices and posets, sets equipped with ternary relations (in particular, betweenness relations), etc. (see [DH 2020],[P-FDeB 2019],[S 2006])

The associativity of an aggregation function ensures that the *n*-ary form of a binary aggregation function is uniquely determined, while a new input is easily added to the previous output without any expensive computations. The results related to associative functions originate in two main fields, the semigroup theory (see e.g., [Abe 26], [Acz 49]) and the domain of functional equations (see e.g., [Fau 55], [MS 57]). However, up to recently, a greater emphasis was given to commutative associative functions (see e.g., [AFS 06], [KMP 00]). In last twenty years, the demand for non-commutative aggregation functions is constantly growing, since in some kinds of applications commutative functions simply cannot capture a non-commutative nature of the fusion process and therefore are unsuitable to model these processes. Examples of such applications comprise aggregation of time-dependent data, natural language processing, multi-criteria decision making, modelling of non-commutative logical connectives and many others. In fact, all aggregation processes where the value of the input and the source of the input are dependent require a non-commutative aggregation function (for example in the case when input sources have a different reliability).

While in commutative case we have several established construction methods (such as, e.g., ordinal sum and *z*-ordinal sum, see [Cli 54], [M-Z 22a]) and a number of classes of commutative, associative aggregation functions were already completely characterized, construction methods and characterization results for NCAAFs are still missing. Moreover, it was shown that using (*z*-)ordinal sum we can decompose (partially-)continuous commutative, associative aggregation functions into simple semigroups, which either posses an additive generator, or they are trivial (see [M-Z 22b]). A similar decomposition for NCAAFs is not known, even for the class of non-commutative semi-t-operators, which is the class that covers all continuous associative aggregation functions. The aim of this research is to fill this gap and develop construction methods for NCAFs, which will also allow us to decompose special classes of NCAAFs (such as non-commutative semi-t-operators) into simple semigroups, similarly as it can be done in the commutative case. Of course, ordinal and *z*-ordinal sum can be also used for construction of non-commutative functions, however, in such a case this non-commutativity is a property of the corresponding summands and not of the construction itself. If, for example, summands are defined on intervals then the non-commutativity of a function constructed by (*z*-)ordinal sum can appear only around the main diagonal, which need not be sufficient in real world applications. Note that decomposition of an associative function into simple semigroups is useful both in theory as well as in applications and it plays a role in construction as well as in fitting of a function to real observed data, where one can focus on separate areas and design the function differently for distinct reference levels of inputs, and see interaction between these areas, while fitting of a function on distinct areas separately considerably reduces the computational complexity. From the theoretical point of view, the decomposition via ordinal sum facilitates proofs of several properties, for example of the associativity - which is needed to validate various construction methods, or properties like modularity, distributivity, cross-migrativity and others, since it is then enough to verify these properties on separate areas and to show under which conditions the selected construction method preserves the discussed property.

Good knowledge of the structure of the fusion function and its possible decomposability is crucial for any application and therefore this project is of key importance in the development of all applications based on non-commutative fusion processes. The applicability and relevancy of the project is ensured by the persisting demand for non-commutative aggregation functions from several fields of science, while we will show its basic applications also in Objective 5. In this task we plan to cooperate with the distinguished group of researchers around Professor Humberto Bustince from Public University of Navarre, Spain. The quality of planned theoretical results are ensured by previous results of the researcher and her collaboration with distinguished researchers, like group around Dr. Sebastia Massanet from University of the Balearic Islands, Spain, group around Dr. Michal Holčapek from Institute for Research and Applications of Fuzzy Modelling, University of Ostrava, Czech Republic, or group around Prof. Yong Su from Suzhou University of Science and Technology, China, which will lead to a synergic effect.

As confirmed by the above mentioned present and intended cooperation, this project will promote international collaboration and by its innovative results it will promote the scientific excellence. This will contribute to Europe’s scientific competitiveness in the field of applied mathematics and secure a leading position of European researchers in aggregation theory.

The research outcomes will be disseminated through peer-reviewed publications, lectures on conferences and seminars, and via knowledge exchange with European research institutions (both personally and online).

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1.3 METHODOLOGY

From a theoretical point of view, the implementation of the project requires expert knowledge of mathematical theories and methods of their applications, including universal algebra, mathematical analysis and aggregation theory. From an applicational point of view, the implementation needs collection of data, algorithmizing and programming. In the described Objectives we plan to focus on the following tasks:

**Objective 1**: Analysis of known non-commutative associative functions

Task 1.1: Study of recent publications related to the topic and attendance on conferences related to the topic

Task 1.2: Create a list of non-commutative associative functions used both in applications and in theoretical studies

Task 1.3: Study of the properties and the structure of functions from the list created in Task 1.2

In Task 1.1 we will focus on results published in distinguished journals related to our topic, such as journals from Elsevier, IEEE, Taylor & Francis, Springer Verlag, etc., recent monographies, edited volumes and respected conferences such as IPMU, EUSFLAT, AGOP, etc. In Task 1.2 we plan to create a concise structured list of non-commutative associative functions that appear in publications and conferences from Task 1.1 and in Task 1.3 we will study the basic properties and the structure of functions from this list.

**Objective 2**: Construction methods and characterizations of non-commutative associative aggregation functions with (local) neutral element

Task 2.1: Construction and characterization of pseudo-uninorms with continuous underlying functions

Task 2.2: Construction and characterization of pseudo-*n*-uninorms with continuous underlying functions.

Task 2.3: Further generalizations

Recently we were able to characterize idempotent pseudo-uninorms and pseudo-uninorms with continuous Archimedean underlying functions. In Task 2.1, we would like to extend these characterizations and describe the construction and representation of all pseudo-uninorms with continuous underlying functions. In Task 2.2, we will replace the global neutral element by *n* local neutral elements and we will describe the construction and characterization of pseudo-*n*-uninorms with continuous underlying functions. Further generalizations will be studied in Task 2.3.

**Objective 3**: Construction methods for NCAFs

Task 3.1: Development of construction methods for non-commutative associative functions

Task 3.2: Development of construction methods for non-commutative associative aggregation functions

In Task 3.1 we will study and develop construction methods for NCAFs, which will have a true non-commutative nature, i.e., these methods will construct non-commutative functions even from commutative summands. In Task 3.2 we will focus on construction of NCAAFs defined on chains, which means that also the monotonicity should be ensured by the corresponding construction method. We will identify which methods, or which additional conditions ensure that the constructed non-commutative associative function is monotone.

**Objective 4**: Representation of non-commutative associative aggregation functions

continuous around the diagonal

Task 4.1: Decomposition of semi-t-operators and NCAAFs continuous around the diagonal

Task 4.2: Representation of related non-commutative functions.

In Task 4.1 we will first describe the decomposition of semi-t-operators into simple semigroups via methods obtained in Objective 3 and then we will try to extend these results to all NCAAFs continuous around the diagonal. In Task 4.2 we will describe which functions from Objective 1 and Objective 2 can be represented by methods obtained in Objective 3.

**Objective 5**: Generalizations and applications

Task 5.1: Generalizations of construction methods to related non-commutative functions

Task 5.2: Applications to MCDM and related domains

Task 5.3 (complementary): Generalizations to bounded lattices

In Task 5.1 we will extend construction methods obtained in Objective 3 to some related non-associative functions, such as pseudo-overlap functions and pseudo-grouping functions. In Task 5.2 we plan to apply these non-commutative functions obtained by our methods (both associative and non-associative) in problems from multi-criteria decision making and in other domains like, for example, natural language processing - depending on the suitability of the constructed functions to corresponding problems from these fields. In this task we plan to closely cooperate with our international partners to obtain useful real-life applications. After identifying a suitable problem and collecting/obtaining relevant data, our work will be focused on algorithmizing and programming. In Task 5.3 (which will be covered if there are no time delays) we will extend our results achieved for non-commutative functions defined on real intervals to functions defined on general bounded lattices. Since the monotonicity of the function is dependent on the individual structure of its domain we will focus first on easier structures, like horizontal sums of chains, and then to more complicated structures.

**Objective 6**: Surveillance of the progress of the project, summarization and dissemination of results

Task 6.1: To closely monitor the progress of the project

Task 6.2: Summarize results obtained in the project, launch researcher’s website

Task 6.3: Dissemination of results

Task 6.4: Research proposal

In Task 6.1 we will record the progress of the project, especially focusing on an early detection of any time delays, or other problems in project implementation, and report which measures were taken for mitigation of the negative impact on the project. In Task 6.2 we will summarize results obtained in the project and their significance for the theoretical study and applications. We will also introduce the researcher’s web page. In Task 6.3 we will disseminate the results of the project in the form or journal publications, conference contributions, seminar lectures, etc. Finally, Task 6.4 is dedicated to writing a proposal for a new project with the involvement of the applicant.

The proposed methodology aligns closely with the project’s objectives. We will follow the same line of research as it was done in the case of commutative associative functions, however, without commutativity we expect that several proofs that rely on commutativity should be modified or completely rewritten. While in the commutative case the proofs were based mainly on the synergy of the associativity and the commutativity, in the case of non-commutative aggregation functions we plan to build the proofs on the monotonicity and we expect that these proofs will be much more complicated as in the commutative case.

The planned objectives and the proposed methodology follow standard lines of research, while results achieved in the commutative case can serve as a guideline for reaching the results also in the non-commutative case. Therefore, the proposed methods and procedures ensure the achievement of the project’s objectives.

Possible challenges in the project’s implementation can appear in the complexity of the proofs characterizing individual classes of non-commutative associative functions. To overcome the problems with extended time needed for characterization of the most important classes of such functions, we have made Task 5.3 complementary, i.e., we will perform the research needed in Task 5.3 only in the case when all goals described in other tasks from Objectives 1-5 are achieved. Additional challenges may arise in Objective 5, which follows from the complex nature of real-world applications. To overcome possible difficulties, we will collaborate in this task with distinguished experts with a long-year experiences in work with applications described in the corresponding objective. Regular meetings (online or personal) will be planned to discuss possible difficulties and further approach. Since some researchers from the group of Prof. Bustince visit Slovakia regularly, the travel expenses in this case will be reduced. Early solution of any difficulties encountered during the research will be secured by continuous monitoring of the progress and appropriate adjustments of methodology.

The project will follow the principles of open science and some publications (depending on the fee in the selected journals) will be publish with open access. Research data will be archived and shared through open repositories, while all data protection principles will be met. The collection of data will follow transparent data processing procedures. All data, code and output of our analysis will be accessible via researcher’s website linked to the web page of the institute. This will make our findings available to all researchers for study and verification of our methods, which will hopefully lead to new collaborations in the subject and will contribute to the further development of the studied area.

The multi- and interdisciplinary approach is crucial for this project since on the one hand, it uses knowledge of several domains, such as semigroup theory, functional equations, algebra, mathematical analysis, etc., and on the other hand, non-commutative aggregation functions are needed in many areas, such as time dependent data, natural language processing, fusion of source-dependent data, multi-criteria decision making, but also in social sciences, etc. The project will contribute to a better understanding of non-commutative functions across several domains. The working group of researchers collaborating with the applying researcher will follow the principles of gender balance and strictly follow the gender equity.

1.4 EXCELLENCE OF THE RESEARCHER

The researcher works as a senior researcher at Mathematical Institute of Slovak Academy of Sciences, Slovakia and partially as a researcher of grade IV at the Institute for Research and Applications of Fuzzy Modelling, University of Ostrava, Czech Republic. She also worked at Trinity College Dublin, Ireland as a research assistant and later as an independent researcher. In 2022 she obtained DrSc. (doctor scientiarum) in mathematics, which is the highest academic degree in Slovakia. Her main research interests are the aggregation theory, associative functions on bounded lattices, multi-polar aggregation, and non-additive measures and integrals. The researcher has also experience in preparing and contributing to European projects: CAS-AIR - Computer-assisted system for analysis and information for business recovery (Horizon 2020, Call: H2020-DRS-2014, Role: Partner/Work package leader, Evaluation score 10.5/15, the project was not shortlisted), SLANDAIL (European Community’s Seventh Framework Programme under Grant 607691, Role: Team member). She has 64 publications registered in Web of Science (which are mainly research papers published in journals in Q1) with 536 SCI citations.

Curriculum Vitae

**Personal information**

First and last name: Andrea Zemánková

Identifier(ORCID):  <https://orcid.org/0000-0003-4022-0086>

Date of birth: July 2, 1979

Nationality: Slovak

Website (if relevant): https://www.mat.savba.sk/~zemankova

**Education**

06/2005 – PhD

Mathematical Institute, Slovak Academy of Sciences, Slovakia

06/2002 – Master/Doctor

Faculty of Mathematics Physics and Informatics, Comenius University, Slovakia

**Current position/positions**

04/2009 – Senior Researcher

Mathematical Institute, Slovak Academy of Sciences, Slovakia

08/2021 – Researcher of Grade IV

Institute for Research and Applications of Fuzzy Modeling, University of Ostrava, Czech Republic

**Previous positions**

2011 – 2012 – Research Fellow

Trinity College Dublin, Ireland

2007 – 2009 – Research Assistant

Trinity College Dublin, Ireland

**Scholarships and awards**

2014 – **Visegrad Group Academies Young Researcher Award 2014**  – Academies of V4

2013 – 2019 – **Program Fellowship of Slovak Academy of Sciences** – Mathematical Institute, Slovak Academy of Sciences, Slovakia

2011 – **Price of the President of Slovak Republic** **for young scientists**

2010 – 2011 – **AXA Research Fund Post-doctoral fellowship** – Trinity College Dublin, Ireland

2010 – **Price of the Ministry of Education, Science, Research and Sport of Slovak Republic**.

2008 – *Competition of young researchers SAS (under 35 years)*: **2. price for I. science department** – Slovak Academy of Sciences, Slovakia

2006 – *Scientist of SR*: **honorary appreciation.** Ministry of Education, Science, Research and Sport of Slovak Republic.

Student and post-docs supervision (if applicable)

2023 – Number of PhD Students: 1

Department of Mathematics and Descriptive Geometry, Faculty of Civil Engineering, STU

**Teaching activities (if applicable)**

**2007-2009** **Demonstrator for course of Fuzzy logic (**Department of Computer Science, Trinity College Dublin, Dublin 2, Ireland)

**2009** **Demonstrator for course of Research methods** (Department of Computer Science, Trinity College Dublin, Dublin 2, Ireland)

**2004-2006** **Demonstrator for course of Mathematics I and II** (Faculty of Management, Comenius University (FMUK), Odbojárov 10, P.O.BOX 95, 820 05 Bratislava, Slovakia)

**2004-2006** **Lecturer for course of Mathematics I, II and III** (Európska vzdelávacia akadémia, n.o., Nobelova 16, 836 14 Bratislava, Slovakia)

**2003 Demonstrator for course of Mathematics** (Integrals and differential equations) (Faculty of Civil Engineering, Slovak University of Technology (STU), Radlinského 11, 810 05 Bratislava, Slovakia)

**Organisation of scientific meetings (if applicable)**

* **Member of the program committee:** MDAI 2015, MDAI 2016, MDAI 2017, MDAI 2018, MDAI 2019, MDAI 2020, MDAI 2021, MDAI 2022, MDAI 2023, IFSA-EUSFLAT 2021

**Reviewing activities (if applicable)**

* **Regular reviewer for** Fuzzy Sets and Systems**,** Information Sciences, Iranian Journal of Fuzzy Systems, IEEE Transactions on Fuzzy Systems, International Journal of Approximate Reasoning, Soft Computing, Mathematica Slovaca.
* **Occasional reviewer for** Kybernetika, Computers and Mathematics with Applications, Journal of Nonlinear Analysis, Journal of Aerospace Information Systems.
* **Referee for VEGA projects (Slovak grant agency)**.

**Major collaborations (if applicable)**

* Dr. Sebastia Massanet and Dr. Arnau Mir from Dept. of Mathematics and Computer Science, University of the Balearic Islands, Palma, Spain – fuzzy implications
* Doc. RNDr. Michal Holčapek, PhD. and Doc. Ing. Antonín Dvořák, PhD. from Institute for Research and Applications of Fuzzy Modelling, University of Ostrava – aggregation functions on bounded lattices
* Prof. RNDr. Radko Mesiar, DrSc. from Department of Mathematics and Descriptive Geometry, Faculty of Civil Engineering, Slovak University of Technology in Bratislava, Slovakia – aggregation operators and non-additive measures and integrals
* Prof. Khurshid Ahmad from Trinity College Dublin, Ireland – multi-polar aggregation and non-additive measures and integrals
* Prof. Yong Su from Suzhou University of Science and Technology, China – idempotent uninorms on bounded lattices
* Dr. Zhudeng Wang from School of Mathematics and Statistics, Yancheng Teachers University, China - idempotent uninorms on bounded lattices

**Overview of the researcher’s most important projects in the last 5 years** (max. 5)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Project name/identification** | **Source of funding** | **Budget (EUR)** | **Project period** | **The role of the researcher in the project** |
| VEGA 1/0036/23: Advanced approaches to data aggregation and applications | Scientific grant agency MESRaS SR and SAS | Proposed budget: 116 306 EUR (SAS - 12 000 EUR) | 2023-2026 | Principal investigator for SAS |
| APVV-20-0069: Probabilistic, Algebraic and Quantum Mechanical Methods of Uncertainty Determination | Slovak Research  and Development  Agency | 123 000 EUR | 2021-2025 | Research team member |
| VEGA 1/0006/19:  New trends in theory of aggregation and their applications | Scientific grant agency MESRaS SR and SAS | 76 034 EUR (SAS - 6 916 EUR) | 2019-2022 | Principal investigator for SAS |
| APVV-16-0073: Probabilistic, Algebraic and Quantum-Mechanical Aspects of Uncertainty | Slovak Research  and Development  Agency | 143 674 EUR | 2017-2021 | Research team member |
| Program Fellowship of Slovak Academy of Sciences | Slovak Academy of Sciences | 148 776 EUR | 2013-2015,  2019-2021 | Principal investigator |

**Overview of the researcher’s most important outputs** (max. 5)

|  |  |  |  |
| --- | --- | --- | --- |
| **Output name/identification** | **Type of output** | **Short description** | **The role of the researcher** |
| A. Mesiarová-Zemánková, Commutative, associative and non-decreasing functions continuous around diagonal, Iranian Journal of Fuzzy Systems 19(2), (2022), 31-48 | publication | The class of all commutative, associative aggregation functions continuous around the main diagonal is shown to coincide with the class of all functions that can be obtained as a *z*-ordinal sum of semigroups related to continuous t-norms, t-conorms, representable uninorms (i.e., semigroups with an additive generator) and trivial semigroups (defined on a single point). | All parts |
| A. Mesiarová-Zemánková, Structure of Uninorms with Continuous Diagonal Functions. In: On Logical, Algebraic, and Probabilistic Aspects of Fuzzy Set Theory (S. Saminger, R. Mesiar, eds.), Book series: Studies in Fuzziness and Soft Computing 336, Springer, 2016, pp. 109-135 | book chapter | In this chapter, the structure of uninorms which are continuous on some special parts of the unit square is discussed. Representation theorems for uninorms with continuous underlying functions, based on the set of discontinuity points of such a uninorm and the ordinal sum construction for semigroups, are presented. Further generalizations are also discussed. | All parts |
| A. Mesiarová-Zemánková, Multi-polar t-conorms and uninorms. Information Sciences 301 (2015), 227–240 | publication | The correspondence between uninorms/nullnorms and bipolar t-conorms/t-norms was shown. The structure and construction of bipolar and *n*-polar t-norms, t-conorms and uninorms was described. This paper has 42 SCI citations. | All parts |
| R. Mesiar, A. Mesiarová-Zemánková, The Ordered Modular Averages, IEEE Transactions on Fuzzy Systems 19(1) (2011), 42-50 | publication | Ordered modular averages (OMAs) generalize ordered weighted averages operators, with the replacement of the additivity property by the modularity. OMAs can be seen as a symmetrized idempotent modular aggregation functions, characterized by comonotone modularity. Each OMA can be seen also as a copula-based integral with respect to a symmetric capacity. Special OMAs are characterized, such as associative operators, operators with neutral element, or operators with an annihilator. Some construction methods for OMAs are also discussed. This paper has 43 SCI citations. | Concept  Formal analysis  Writing |
| A. Mesiarová, Continuous triangular subnorms, Fuzzy Sets and Systems 142 (1) (2004), 75-83 | publication | Continuous triangular subnorms are shown to be ordinal sums of Archimedean continuous t-subnorms with at most one proper t-subnorm summand. Special attention is paid to generated continuous t-subnorms. An application of continuous t-subnorms to the construction of left-continuous t-norms is shown. Several illustrative examples are included. This paper has 34 SCI citations. | All parts |

The most important research of the applying researcher includes the following.

• Deep study of triangular norms, related t-subnorms, their residual implications and their additive generators. This research covers results on t-norms continuous around the diagonal, completions of t-norms known on some subarea of the unit interval (including the completion problem related to the characterization of (S,N)-fuzzy implications), Lipschitz stability of t-norms, their approximations, boundaries and comparisons. Results on cancellative t-norms, semi-divisible t-norms, *n*-contractive t-norms, convex combinations of t-norms, uninorms and t-subnorms are also included.

• The results on non-additive measures and integrals cover overview of fuzzy integrals, their geometric interpretation, the study of the linearity and generalized linearity of fuzzy integrals, definition of new concepts for bipolar, multi-polar and level-dependent fuzzy integrals, proposal of generated fuzzy measures yielding aggregation functions with varying number of inputs and characterization of several special classes of fuzzy measures.  
• Introduction of multi-polarity into aggregation, where unipolar and bipolar aggregation was only assumed, study of (extended) multi-polar aggregation functions (e.g., t-norms, t-conorms, averaging functions) including their applications in classification systems, study of multi-polar measures and integrals. Applications (citations) of these results can be found in various fields, beside the aggregation theory for example in theory of fuzzy graphs, fuzzy set theory, multi-criteria decision-making, fuzzy integrals, supervised learning, and others.  
• Ordinal sums of uninorms and characterization of uninorms with continuous underlying functions: the problem of a complete characterization of uninorms with continuous underlying functions was in the centre of attention for a long time, however, despite numerous results this problem was solved only for special cases. In the collection of works of applying researcher these uninorms were completely characterized using the ordinal sum construction and characterizing functions. Similarly, several classes of uninorms, such that their underlying functions are continuous inside the unit square, were characterized.  
• Introduction of *z*-ordinal sum, which extends ordinal sum for partially ordered families of semigroups and thus enables to construct non-decreasing functions on the unit interval which have an annihilator inside the unit interval. By using the *z*-ordinal sum the class of *n*-uninorms with continuous underlying functions was completely characterized. This characterization was later extended to all commutative, associative aggregation functions continuous around the diagonal. The structure of all monotone functions defined on a real interval, constructed via (*z*-)ordinal sum was also shown.

• Characterization and construction of special classes of aggregation functions defined on general bounded lattices, such as idempotent uninorms, and t-norms with values in a sublattice.

1.5 EXCELLENCE OF THE APPLICANT/HOST ORGANISATION

The Slovak Academy of Sciences was instituted in 1942 by Parliament of the Slovak Republic, under the name Slovak Academy of Sciences and Arts (Slovenská akadémia vied a umení). During the post‐World War II period the SAVU was transformed into Slovenská akadémia vied - Slovak Academy of Sciences (SAS). Nowadays, the Slovak Academy of Sciences successfully presents itself nationally and worldwide through the results in basic and applied research. It consists of 48 organisations, of which 22 are budgetary and 26 are contributory. The Academy publishes 57 scientific and professional journals and 8 yearbooks. There are 51 scientific and scholarly associations, societies and unions affiliated with the SAS, which operate in accordance with the Law on Civic Associations. The Assembly, the Scientific Council, and the Presidium are the Academy’s self-governing bodies.

The Mathematical Institute of Slovak Academy of Sciences (MI SAS) is a scientific institute concentrated mainly on basic research in mathematics and theoretical informatics. On January 1, 2022, it was transformed into a public research institution. Its collaborators have participated in many important and successful projects in both basic and applied research, including EU Framework and Structural projects and Slovak national projects APVV and VEGA. Many of its collaborators belong to the top in their research in a world-wide context. The Mathematical Institute publishes the major journal Mathematica Slovaca (called Matematicko-fyzikálny sborník during 1951–1952, Matematicko-fyzikálny časopis during 1953–1966, and Matematický časopis during 1967–1975) and the Tatra Mountains Mathematical Publications. In collaboration with the Comenius University, the Institute organises a PhD study program and other tasks of the institute include in particular: education of young researchers, cooperation with national and international scientific and research institutions, publication of scientific journals and books, collecting and sorting information about scientific results, preparation of various expertises for the government and government organizations, provision of scientific information for institutions and citizens of Slovakia, cooperation and assistance to universities in teaching and its organization, transferring the results of scientific research into practice and popularization of science.

In addition to the main part of the institute located in Bratislava, the institute has departments also in Košice and Banská Bystrica and a separate department of theoretical informatics located in Bratislava. The theory of aggregation functions is studied both in Bratislava, as well as in department located in Košice, with J. Pócs as the leading scientist. Since the project goals are focused mainly on associative functions, it also contributes to the solution of special algebraic problems. The algebraic research of the former members of the institute had a great worldwide impact. Recall, for example, such famous institute members, as academicians Š. Schwarz and J. Jakubík. The institute is traditionally one of the world's most important research centres of Quantum Structures, with A. Dvurečenskij and S. Pulmannová who are leading experts in this field. They are both former presidents of the International Quantum Structures Association (IQSA), they co-authored a fundamental monograph in this field (New Trends in Quantum Structures) and contributed hundreds of papers in top journals with thousands of citations. Furthermore, A. Jenčová is an internationally renowned expert in quantum information theory, both in finite dimensional setting and in von Neumann algebras, and in quantum foundations. Note that she was awarded the Birkhoff-von Neumann prize of the IQSA in 2014. Integrals form a very important special class of aggregation functions. The theory of measure and integral was deeply studied on the institute, especially by world-renowned experts B. Riečan, I. Dobrakov and L. Mišík (as an example of their work we recall the monograph B. Riečan, T. Neubrunn: Integral, Measure and Ordering).

The researchers of the institute are well connected with world-wide experts in their respective branches. In the field of aggregation, the Institute has a close connection to neighbouring institutions such as the Slovak University of Technology in Bratislava and Institute for Research and Applications of Fuzzy Modelling, University of Ostrava, Czech Republic through extensive collaborations and numerous joint projects. The institute organizes several successful and long-standing established seminars, such as the seminar on Quantum structures, seminar BBTS on topology, and recently also the seminar on Category theory and applications, with invited talks given by distinguished scientists. The seminar on Uncertainty Modelling is traditionally organized by the nearby Faculty of Civil Engineering, Slovak University of Technology. These seminars offer an excellent opportunity for exchanging ideas and discussions.

Mathematical Institute has a very good infrastructure with adequate hardware and software equipment. The library of Mathematical Institute SAS belongs to the best mathematical ones in Slovakia with access to many journal nets of the world publishing houses, scientific-information and technical databases, e.g. Mathematical Review, Zentralblatt Mathematik, Web of Knowledge, Scopus, etc. The institute is visited by many distinguished mathematical experts from all over the world and many young scientists use the fellowship program SAIA for study stays at the institute. The regular seminars visited by international and national experts, as well as by young scientists, and consultations with local and visiting researchers is expected to be an effective method of two-way knowledge transfer between the host organisation and the researcher. We also plan to involve young scientists in the solution of this project, including current and possible new PhD students.

## Impact

### 2.1 THE WIDER IMPACT OF THE PROJECT

The importance of aggregation functions in daily life is indisputable, commencing with basic summation and multiplication that everybody uses on a regular basis up to complex functions used in control systems, management, engineering and in fact nearly in any system where a collection of data should be represented by, or compressed to, a smaller group of representatives used in further processes. While aggregation of source independent data can be modelled by commutative functions, in the case when the source is important (like for example for sources with different reliability, time dependent data, etc.) aggregation by a non-commutative function is more suitable. Since construction methods and descriptions of the main classes of non-commutative (associative) functions are still missing, a high likelihood that the project will have the expected impact is anticipated. The proposed procedures follow the standard line of research and since similar procedures were successfully applied in the case of commutative associative functions, their reliability is already verified. Flexibility, creativity and expertise needed for a successful completion of this project are ensured by the professional approach of the applying researcher, as confirmed by her previous work.

The expected impact of the project is twofold. From the scientific point of view the project will contribute to a better understanding of non-commutative functions, their characterization, construction, properties and their usefulness in different scenarios. The obtained results will contribute to the development of the aggregation theory, theory of semigroups, functional equations, mathematical analysis and other domains. In the case of applications, we expect the impact of the project especially in multi-criteria decision making, neural networks, image processing, but also in social sciences and related domains. The use of non-commutative functions will help decision makers to better describe the relation between the value and the source of data (e.g., incorporate different modalities, unequal credibility, data imbalance, etc.) and obtain more precise recommendations, which is crucial especially in emergency management, but also in economy, environment management, etc. Deeper understanding of non-commutative functions also contributes to better imbalance detection, which can be used especially in management and suppression of social and gender inequality.

In the short term, results of the project published in distinguished journals, or presented on main conferences and seminars will attract attention of other researchers, showing them the structure, construction methods and properties of the described non-commutative functions. In the medium term the theory of non-commutative aggregation functions will be developed further to encompass all necessary generalizations, the methods will be fine-tuned and more basic applications will appear. In the long term the complex theory on non-commutative fusion functions will be available to researchers as well as to practitioners, the corresponding results will be used in various fields of science and applied in industrial applications. The results of the project will be disseminated at local, regional, European and international level. Data, methodology and code related to applications, as well as the list of publications, based on the results of the project, will be freely disseminated and could be used by European researchers and companies.

Beside the professional growth, the implementation of the project will enable applying researcher to start new cooperation with experts vising the host institution, as well as to attend conferences which serve as a source of new ideas, methods and consultations with experts about further applicability of the results obtained during the work on the current project. Visits of young researchers at the host institute also offer a possibility to create a group of young researchers interested in the corresponding topic of the project. Participation on seminars and meetings with experts working for the host institute, as well as experts from nearby universities and visiting experts, guarantee to improve the researcher’s comprehension of new areas of science and thus acquire better cross-disciplinary understanding useful in further research. New skills obtained during the solution of Objective 5, focused on the applied part of the project will help the applying researcher in future work in applied research.

The host organization will gain an expert working in the field of aggregation theory and non-additive measures and integrals, while the members of the institute will benefit from the exchange of knowledge between them and the researcher. The usefulness and applicability of this project will enhance the reputation of the institute that will host it, which can attract new PhD students or other researchers to work within the group of collaborators on the institute. Joint work with other experts from the host institute will lead to flourishing inter-disciplinary collaboration. The cooperation of young researchers with the applying researcher, in the field of the project, will lead to the development of their career and the acquisition of new knowledge and skills. In the case of interest, a small working team contributing to the project will be created.

The project's anticipated outcomes involve at least 7 publications in prominent international scientific journals and several contributions in conference proceedings. These publications will contribute to the development of several scientific domains and will serve as a basic guide for selection of non-commutative functions in applications. Cooperation and sharing of knowledge with experts from related as well as from more distinct fields is planned, and involvement of PhD student(s) is expected. Estimations of outputs of the project are based on the experiences of the applying researcher from previous projects where she participated, considering her expertise and her intensive cooperation with leading foreign and national scientific groups.

Constant monitoring of the recent publications, communication with leading experts working in the area of the project and collaboration with main national and international research groups will allow to maximize the results. Carefully selected journals with a high impact factor and participation in major conferences will further increase the impact of the project

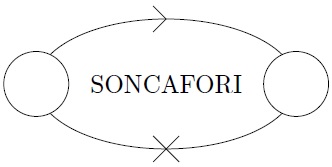
* Potential obstacles to the planned impact of the project

The impact of some tasks of the project can be limited if these tasks are already covered by the work of other researchers working in the field. To eliminate such possibilities, Objective 1 is focused on constant monitoring of the recent literature in the corresponding domain. Beside this, attendance on conferences and meetings (online or personal) with leading groups of researchers working in the area of this project will enable early detection of such situation which will allow the researcher to appropriately adjust the involved tasks.

### 2.2 MEASURES TO MAXIMISE IMPACT – DISEMINATION AND COMMUNICATION, EXPLOITATION OF RESULTS

In order to maximize impact of the project results, the outcomes of the project will be published in distinguished journals with a good accessibility, and some of the papers will be published with open access. Beside this, our achievements will be presented on main conferences, workshops, webinars related to the scientific field of the project. International research visits of applying researcher to distinguished scientific institutions will contribute to the further dissemination of the project results.

Data, code and outputs related to applications will be accessible through researcher’s web site. For easy recognition of the project we will use the following logo:



The project results, publications, data and software developed in Objective 5 will be available even after completion of the project which will allow verification, comparison of other methods and the basis for further research.

Regular seminars organized by the host institute, or by institutes which the applying researcher will visit in the time of the duration of the project will help to spread results to a wider audience.

In addition to the interim and the final report, we will produce a brochure that will summarize the main achievements of the project. This brochure will be available to all researchers visiting the institute, as well as to all interested researchers during study stays and conferences attended by the researcher.

## Implementation

3.1 PROJECT PLAN AND DELIVERABLES

The project is divided into six work packages, each corresponding to one objective:

1. Literature monitor
2. Special NCAAFs
3. Construction methods
4. NCAAFs and continuity
5. Applications
6. Dissemination

The duration of the project is planned for 24 months. All tasks planned for the project are accordingly included in one of the work packages, which are naturally interdependent. The first work package WP1 serves as a basis for other work packages as it focuses on creation of a list of non-commutative functions that are used in theory and applications, which will serve as a starting point for the study of such functions, determination of the main properties required from functions applied in various domains and verification of our approaches. Possible interactions with the project will be identified as well. WP2 focuses on a simpler case of NCAAFs with (local) neutral element, which is a specific case that will be used in more general cases studied in WP3 and WP4. WP3 is dedicated to the development of construction methods for NCAFs, while these methods will be also used in WP4 for decomposition and characterization of NCAAFs continuous around the diagonal. All results obtained in WP1-4 will be used in WP5 where further generalizations will be studied and the results will be applied to a problem from multi-criteria decision making (or a related domain - if it turns out to be more appropriate). In WP6 we will summarize all results of the project and disseminate its results. All work packages are carefully designed to consistently capture all the individual steps necessary for the research planned in this project.

Due to the nature of the research proposed in the project, it is difficult to predict the exact time required to obtain specific results, however, the estimated period dedicated to solution of each work package can be found in the following timetable.

## 

More details on project work packages can be found below. Research team will include the applying researcher and one or more PhD students. In addition to personnel costs, the project budget will be used to cover participation in conferences, research stays and access to publications.

3.1.1 Work packages

|  |  |
| --- | --- |
| Work package number | WP1 |
| Title of the work package | Literature monitor |
| **Start of implementation of the work package (Mx Month)** | M1 |
| End of implementation of the work package (Mx month) | M21 |
| **Involvement (expressed in Person Months)** | Andrea Zemánková - 3 person months  Researcher R1 – 1.5 person months |
| **Personnel costs (in EUR)** | 17 469 EUR |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 5 807 EUR |
| Objectives | |
| Identification of NCAFs used both in theory and applications and summarization of basic observations about their structure. Constant monitoring of new publications and study of their relation to the project. | |
| Description of the work package | |
| Task 1.1: Study of recent publications related to the topic and attendance on conferences related to the topic  Task 1.2: Create a list of non-commutative associative functions used both in applications and in theoretical studies  Task 1.3: Study of the properties and the structure of functions from the list created in Task 1.2 | |
| Deliverables | |
| D1: **First state-of-art report**  A detailed summary of non-commutative approaches and functions used in the reviewed literature, including the list of non-commutative (associative) functions and their basic properties and structures.  D2: **Second state-of-art report**  A detailed summary of recent development in the fields related to the project, including the updated list of functions and properties. | |

|  |  |
| --- | --- |
| Work package number | WP2 |
| Title of the work package | Special NCAAFs |
| **Start of implementation of the work package (Mx Month)** | M2 |
| End of implementation of the work package (Mx month) | M9 |
| **Involvement (expressed in Person Months)** | Andrea Zemánková - 3 person months  Researcher R1 – 1.5 person months |
| **Personnel costs (in EUR)** | 17 469 EUR |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 5 807 EUR |
| Objectives | |
| To obtain construction methods and characterizations for non-commutative associative aggregation functions with (local) neutral element, i.e., for pseudo-uninorms, pseudo-*n*-uninorms and related functions. Compare properties of commutative counterparts of these functions and see which are preserved also in the non-commutative case. | |
| Description of the work package | |
| Task 2.1: Construction and characterization of pseudo-uninorms with continuous underlying functions  Task 2.2: Construction and characterization of pseudo-*n*-uninorms with continuous underlying functions.  Task 2.3: Further generalizations to functions related to pseudo-*n*-uninorms | |
| Deliverables | |
| D3: **Technical report**  A document in which we will summarize our findings, achievements, problems and possible further directions of research.  D4: **Research papers**  1-2 publications containing results of this work package, i.e., describing the structure of NCAAFs with (local) neutral element will be submitted into carefully selected journal from a distinguished publishing house, such as Elsevier, IEEE, Taylor & Francis, etc. | |

|  |  |
| --- | --- |
| Work package number | WP3 |
| Title of the work package | Construction methods |
| **Start of implementation of the work package (Mx Month)** | M4 |
| End of implementation of the work package (Mx month) | M11 |
| **Involvement (expressed in Person Months)** | Andrea Zemánková - 3 person months  Researcher R1 – 1.5 person months |
| **Personnel costs (in EUR)** | 17 469 EUR |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 5 807 EUR |
| Objectives | |
| Proposal of construction methods for non-commutative associative functions. Description of the necessary and sufficient conditions which ensure that such a construction method generates a monotone function. Comparison of results of this work package with results from WP2. | |
| Description of the work package | |
| Task 3.1: Development of construction methods for non-commutative associative functions  Task 3.2: Development of construction methods for non-commutative associative aggregation functions | |
| Deliverables | |
| D5: **Conference contribution**  Results achieved in this work package and in WP2 will be presented on one of the main conferences, like for example IPMU, EUSFLAT, AGOP, or similar.  D6: **Journal research paper**  A publication related to results from this WP will be submitted for publication in one (or more) distinguished journals. | |

|  |  |
| --- | --- |
| Work package number | WP4 |
| Title of the work package | NCAAFs and continuity |
| **Start of implementation of the work package (Mx Month)** | M6 |
| End of implementation of the work package (Mx month) | M18 |
| **Involvement (expressed in Person Months)** | Andrea Zemánková - 5 person months  Researcher R1 – 2.5 person months |
| **Personnel costs (in EUR)** | 29 115 EUR |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 9 679 EUR |
| Objectives | |
| Representation of non-commutative associative aggregation functions continuous around the diagonal. Identification of functions from deliverable D1 which allow a similar representation/decomposition. | |
| Description of the work package | |
| Task 4.1: Decomposition of semi-t-operators and NCAAFs continuous around the diagonal  Task 4.2: Representation of related non-commutative functions | |
| Deliverables | |
| D7: **Conference contribution**  Representation results for partially continuous non-commutative associative functions will be presented on one of the main conferences  D8: **2 journal publications**  At least two publications related to results from this WP will be submitted for publication in distinguished scientific journals | |

|  |  |
| --- | --- |
| Work package number | WP5 |
| Title of the work package | Applications |
| **Start of implementation of the work package (Mx Month)** | M6 |
| End of implementation of the work package (Mx month) | M22 |
| **Involvement (expressed in Person Months)** | Andrea Zemánková - 6 person months  Researcher R1 - 3 person months |
| **Personnel costs (in EUR)** | 34 938 EUR |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 11 615 EUR |
| Objectives | |
| Extension of construction methods to more general non-associative and non-commutative functions, e.g., pseudo-overlap functions and pseudo-grouping functions. Applications of non-commutative functions in problems from the field of multi-criteria decision making, or a similar field.  Complementary task - representation of non-commutative associative aggregation functions defined on more general bounded lattices. | |
| Description of the work package | |
| Task 5.1: Generalizations of construction methods to related non-commutative functions  Task 5.2: Applications to MCDM and/or related domains  Task 5.3 (complementary): Generalizations to bounded lattices | |
| Deliverables | |
| D9: **Data and methods**  Database with data, list of methods and code used in applications  D10: **3 journal publications**  Publications related to results from this WP will be submitted for publication in some of the distinguished journals | |

|  |  |
| --- | --- |
| Work package number | WP6 |
| Title of the work package | Dissemination |
| **Start of implementation of the work package (Mx Month)** | M8 |
| End of implementation of the work package (Mx month) | M24 |
| **Involvement (expressed in Person Months)** | Andrea Zemánková - 4 person months  Researcher R1 - 2 person months |
| **Personnel costs (in EUR)** | 23 292 EUR |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) | 7 743 EUR |
| Objectives | |
| Summarization of results obtained in previous WPs into a comprehensive summary. Monitoring of the progress of the project. Dissemination of results. | |
| Description of the work package | |
| Task 6.1: To closely monitor the progress of the project for timely solution of any problems.  Task 6.2: To summarize results obtained in the project into a comprehensive summary and produce corresponding reports and researcher’s web page.  Task 6.3: Dissemination of results via journal publications, conference lectures, contributions in proceedings, seminar talks and discussions with experts as well as with young researchers. Preparation of a promotional brochure.  Task 6.4: Write a research proposal for a new project/grant with the involvement of the applicant | |
| Deliverables | |
| D11 **Interim report**  Report summarizing results achieved in the middle of the duration of the project, their evaluation, possible delays or deviations to the plan, together with their justification and comparison between obtained and expected results.  D12 **Website**  Researcher’s web page containing project description, information about researcher, data, methods and code related to applications, a link to publications and additional information about the project  D13 **Final report**  Report summarizing all results obtained during implementation of the project and comparison between achieved and expected results.  D14 **New research project (grant) application**  Research proposal for a new project (grant), which will involve the applicant.  D15 **Brochure**  Promotional booklet that will contain basic information about the project, the applicant organization, the researcher and the main achievements of the project. | |

3.1.2 List of work packages:

|  |  |  |  |
| --- | --- | --- | --- |
| Work package number | Title of the work package | **Start of activities** | **End of activities** |
| WP1 | Literature monitor | M1 | M21 |
| WP2 | Special NCAAFs | M2 | M9 |
| WP3 | Construction methods | M4 | M11 |
| WP4 | NCAAFs and continuity | M6 | M18 |
| WP5 | Applications | M6 | M22 |
| WP6 | Dissemination | M8 | M24 |

3.1.3 List of deliverables:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Deliv. num. | Deliverable | WP# | Type | Access and dissemination | Method of verification | Delivery |
| D1 | First state-of-art report | WP1 | report | non-public | Expert review | M5 |
| D2 | Second state-of-art report | WP1 | report | non-public | Expert review | M20 |
| D3 | Technical report | WP2 | report | non-public | Booklet | M6 |
| D4 | Research papers | WP2 | publication | public | Peer review | M7 |
| D5 | Conference contribution | WP3 | publication | public | Peer review | M8 |
| D6 | Journal research paper | WP3 | publication | public | Peer review | M10 |
| D7 | Conference contribution | WP4 | publication | public | Peer review | M15 |
| D8 | Research papers | WP4 | publication | public | Peer review | M18 |
| D9 | Data and methods | WP5 | database | public | Expert review | M20 |
| D10 | Research papers | WP5 | publication | public | Peer review | M22 |
| D11 | Interim report | WP6 | report | non-public | Progress  evaluation | M12 |
| D12 | Researcher’s web page | WP6 | website | public | Accessibility  check | M20 |
| D13 | Final report | WP6 | report | non-public | Project evaluation | M24 |
| D14 | Project proposal | WP6 | proposal | non-public | Proposal  evaluation | M24 |
| D15 | Promotional brochure | WP6 | booklet | public | Expert feedback | M24 |

3.1.4 List of milestones:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Milestone number | Milestone | Work package number | Method of verification | Expected time to reach the milestone (project month) |
| MS1 | Complete characterization of NCAAFs with (local) neutral element | WP2 | peer review | M9 |
| MS2 | Definition and verification of construction methods for NCAFs | WP3 | peer review | M11 |
| MS3 | Complete characterization of partially continuous NCAAFs | WP4 | peer review | M18 |
| MS4 | Verification of obtained results on applications | WP5 | real-life applications | M22 |

3.2 IMPLEMENTATION RISKS AND PROPOSED MEASURES

A list of risks and problems that may hinder the smooth implementation of the project consists mainly of problems with publication delays, either caused by the publisher and an unexpectedly long review process, or if the studied problem turns out to be more complicated than expected. It can also happen that some minor tasks planned in this project will be solved by other researchers in the meantime, or the task will be easier than expected (i.e., it will take less time than we have estimated). Further, we can encounter issues with data accessibility for applications and based on experience from previous years we must also consider the possibility of a pandemic, which can cause a lockdown or similar obstacles. Due to the nature of the research proposed in this project, we do not expect any legislative issues. In order to avoid these problems, each risk will be assessed in terms of its severity and likelihood of occurrence. We will identify its triggers and indicators and develop the corresponding mitigation strategies and contingency plans.

In Task 6.1 from WP6 we plan to closely monitor the progress of the project, which includes also updating the list of potential risks and proposing new strategies how to avoid them. We will focus mainly on proactive measures that will diminish the likelihood of risk occurrence. For the case of some unforeseen difficulties we will also develop contingency plans, which will be triggered if a new risk is identified. The proposed methods will efficiently eliminate, or diminish the impact of identified risks on the project and ensure a successful completion of all planned tasks. This will help to maximize the impact of the project and guarantee its smooth progress.

3.2.1 Risks of implementation:

|  |  |  |
| --- | --- | --- |
| **Description of the risk of implementation (severity)** | **Work package** | Proposed measures for risk mitigation or elimination |
| Publication delays (medium) | W2,W3,W4,W5 | Check journal’s average publication time. Monitor the progress. Contact Editor after the internal deadline is crossed. |
| Task is too complicated (low) | W4,W5 | Contact experts working in the field for cooperation on the solution of the task. In the worst-case scenario – if none of the experts would be able to solve the task, skip it and focus on subsequent tasks and solve complementary Task 5.3 from WP5. |
| Task is already solved or it is easier than expected (low) | W2,W3 | Focus on subsequent tasks and solve complementary Task 5.3 from WP5. The generality of this task offers sufficient amount of problems for further study. |
| Data not available (low) | W5 | Use a different dataset. Select another problem. Search for problems with available data. Contact colleagues working with applications. |
| Pandemic (medium) | All WPs | Focus on online conferences, collaboration via online tools and use home office. |

3.3 OPERATIONAL CAPACITY OF THE APPLICANT/HOST ORGANISATION

The Mathematical Institute at SAS is well equipped and beside standard hardware and software equipment necessary for the project it has one of the best mathematical libraries in Slovakia with access to journal nets of the main world publishing houses, and access to the Slovak Centre of Scientific and Technical Information (http://www.cvtisr.sk/) – which provides additional mathematical processing software suite capabilities. In addition, the institute has access to a supercomputer which can be used for complex calculations and simulations.

The researchers of the institute are highly qualified professionals, many of whom are world leading experts in their scientific fields and the institute has a rich network of collaborating national and international experts in aggregation and multi-criteria decision‐making. Creative environment visited by many international experts and seminars which are organized by the institute provide an excellent opportunity for discussions, exchange of ideas and interdisciplinary cooperation.

The applying researcher will be provided with full institutional support and will have access to all necessary resources and infrastructure. Access to the supercomputer will be granted upon request. Research and technical staff of the institute is prepared to help researcher with any technical issues that can appear during the implementation of the project. Discussions with experts from the institute working in similar scientific fields will contribute to fruitful interdisciplinary cooperation that will enrich the obtained results and maximize the impact of this project.

3.3.1 Description of the research/innovation infrastructure of the applicant/host organisation that is necessary for the implementation of the project:

|  |  |
| --- | --- |
| Name of infrastructure or equipment | Short description |
| Standard hardware and software equipment | All hardware and software which is needed in this type of research is available to researcher (including laptops, desktops, internet connection, computational software, etc.) |
| Communication tools | Special video conference system for communication with foreign collaborators |
| Library | One of the best mathematical libraries in Slovakia |
| Access to main scientific databases | Access to journal nets of the main world publishing houses, scientific-information and technical databases, like e.g., Science Direct, Scopus, WoS, SpringerLink, Springer NATURE, JSTOR, PLOS, PNAS, Taylor & Francis, Wiley Online Library, etc. |
| Supercomputer Devana | Supercomputer Devana with an estimated output of 800 TFlop/s, which can be used for complex calculations and simulations |

3.3.2 List of the five most important projects of the applicant/host organisation and their relevance to the proposed project (in the last 5 years):

|  |  |  |
| --- | --- | --- |
| Project name/identification | Programme/scheme/grant provider | Short description |
| NFP313011T683 Mathematical support of quantum technologies | ITMS-2014+/ European Regional Development Fund | The main goal of the project was to stabilize a high-quality research team(s) of the institute in the domain of mathematical structures and functions, to implementation independent research and development activity with the relevant project outputs, to obtain new knowledge about mathematical structures and functions. |
| VEGA 1/0036/23: Advanced approaches to data aggregation and applications | Scientific grant agency MESRaS SR and SAS | The project is devoted to basic research in the field of aggregation theory and applications of aggregation functions in various industries. Construction methods and properties of aggregation on structures that generalize real intervals are studied. Based on the requirements from applied fields, several new modifications of standard monotonicity, as well as other properties, were introduced and studied in this project. The corresponding types of aggregation functions are currently analysed and applied in the field of image processing, in decision-making problems, in statistical modelling and in other areas. |
| APVV-20-0069: Probabilistic, Algebraic and Quantum Mechanical Methods of Uncertainty Determination | Slovak Research  and Development  Agency | The project studies the mathematical foundations of quantum mechanics and uncertainty using the state of art methods of quantum structures. Its goals are focused on deepening the knowledge of partial and total algebras such as effect algebras, MV-algebras, synaptic algebras, orthomodular unions, BL-algebras, EMV-algebras, wEMV-algebras, residue unions and their non-commutative generalizations and states on them in connection with partially ordered groups. Aggregation methods are used to combine selected measurement values ​​into one aggregation function. The uncertainty inherent in quantum measurements is analyzed from the perspective of states and quantum channels with a focus on applications in quantum mechanics, quantum information theory and the description of incompatibility measures. |
| VEGA 1/0006/19:  New trends in theory of aggregation and their applications | Scientific grant agency MESRaS SR and SAS | The project contributed to the basic research in the domain of the aggregation theory and its applications, according to the latest trends and their development. Achieved theoretical goals include deep study of directional and ordered directional monotonicity in the domain of (pre-)aggregation functions and related functions, of k-additive and related aggregation functions on different real scales, of sub- and the super-decomposition integrals. Further results were focused on aggregation of data from bipolar and multipolar scales, including extensions of aggregation functions to more complex domains, such as the lattices or posets. These results were applied in problems from the domain of image processing. |
| APVV-16-0073: Probabilistic, Algebraic and Quantum-Mechanical Aspects of Uncertainty | Slovak Research  and Development  Agency | The project was focused on obtaining original results for the description of the uncertainty related to quantum structures and on the uncertainty contained in the convex structure of quantum-mechanical measurements. The research was especially concentrated on study of partial and total algebras such as effect algebras, synaptic algebras, MV-algebras and their non-commutative generalizations, as well as on the description of quantum channels, states, estimation and testing tasks of quantum-mechanical processes, categorical properties and aggregation functions. |

3.3.3 List of maximum five most important outputs of the applicant/host organisation relevant to the submitted project:

|  |  |  |
| --- | --- | --- |
| Output name/identification | **Type of output** | Short description |
| HOLÁ, Ľ. - HOLÝ, D. and MOORS, W. - USCO and Quasicontinuous  Mappings. Berlin: Walter de Gruyter, 2021. 295 p. Studies in Mathematics, 81. Accessible on https://doi.org/10.1515/9783110750188-201. ISBN 978-3-11-075015-7 | monography | This book generalizes the notion of a single-valued continuous function to usco and quasicontinuous mapings  and it presents the most fundamental properties of these mappings.  Topological properties of the space of all usco (quasicontinuous)  mappings acting between two given topological spaces, including the countability, metrisability, and completeness of these spaces are explored. It also highlights their utility across many areas of mathematics. |
| DVUREČENSKIJ, A. and PULMANNOVÁ, S. New Trends in Quantum Structures. Dordrecht : Kluwer Academic ; Bratislava : Ister Science,  2000. 541+xvi pp. Accessible on https://doi.org/10.1007/978-94-017-2422-7. ISBN 0-7923-6471-6. | monography | This book presents some of the newest trends in the mathematical theory of algebraic structures arising in quantum mechanics. Properties of D-posets and effect algebras are studied, they are shown to be equivalent and closely related to partially ordered Abelian groups. Their tensor products and generalizations are also studied. MV-algebras and their quantum counter-  part QMV-algebras, BCK-algebras and σ-complete MV-algebras are studied as well. The representation theorem enables to characterize MV-observables and to construct a calculus of MV-algebras. |
| MESIAROVÁ-ZEMÁNKOVÁ, A -  Multi-polar t-conorms and uninorms. INFORMATION SCIENCES  301 (2015), pp. 227-240 | publication | The correspondence between uninorms/nullnorms and bipolar t-conorms/t-norms was shown. The structure and construction of bipolar and *n*-polar t-norms, t-conorms and uninorms was described. This paper has 42 SCI citations. |
| MESIAROVÁ-ZEMÁNKOVÁ, A. -Characterization of idempotent *n*-uninorms. FUZZY SETS AND SYSTEMS 427 (2022), pp. 1-22 | publication | The *z*-ordinal sum construction which allows to construct aggregation functions defined on a real interval, with annihilator in the interior of this interval (which is not possible via standard ordinal sum) was introduced. Idempotent *n*-uninorms were shown to be in one-to-one correspondence with special lower semi-lattices defined on the unit interval. |
| HALAŠ, R. and PÓCS, J. - On the clone of aggregation functions on bounded lattices. INFORMATION SCIENCES 329 (2016), pp. 381-389 | publication | The paper studies aggregation functions on lattices via clone theory approach. It is shown that the set of generating aggregation functions consists only of at most *n* unary functions, at most *n* binary functions, and lattice operations ∧, ∨, and all aggregation functions of *L* are composed of them by usual term composition. This approach is shown to work also for infinite lattices. |