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Bachelor's thesis

# Probabilistic algorithms for computing the LTS estimate

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March 5, 2019

# Acknowledgements

THANKS to everybody

# Declaration

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# Abstrakt

V několika větách shrňte obsah a přínos této práce v českém jazyce.

**Klíčová slova** LTS odhad, lineÃąrnÃŋ regrese, optimalizace, nejmenÅąÃŋ usekanÃľ ÄŊtvrece, metoda nejmenÅąÃŋch ÄŊtvercÅŕ, outliers

# Abstract

The least trimmed squares (LTS) method is a robust version of the classical method of least squares used to find an estimate of coefficients in the linear regression model. Computing the LTS estimate is known to be NP-hard, and hence suboptimal probabilistic algorithms are used in practice.

**Keywords** LTS, linear regressin, robust estimator, least trimmed squares, ordinary least squares, outliers, outliers detection

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Chapter 2

# The Least trimmed squares

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#### 2.0.1.1 Problems

# Chapter 3

# Algorithms

## 3.1 FAST-LTS

In this section we'll introduce FAST-LTS algorithm[1]. It's, as well as in other cases, iterative algorithm. We'll discuss all main components of the algorithm starting with its core idea called concentration step which authors simply calls C-step.

### 3.1.1 C-step

We'll show that from existing LTS estimate $\hat{\boldsymbol{w}}_{\boldsymbol{old}}$ we can construct new LTS estimate $\hat{\boldsymbol{w}}_{\boldsymbol{new}}$ which objective function is less or equal to old one. Based on this property we'll be able to create sequence of LTS estimates which will lead to better results.

**Theorem 1:** Let's have dataset consisting of $x_1, x_2 ... x_n$ explanatory variables and its corresponding $y_1, y_2 ... y_n$ response variables where $x_i$

**Theorem 2:** BuÄŔ $f$ funkce, ktera ma[1] po ÄŊÃąstech spojitou derivaci na intervalu $\langle -T, T \rangle$. Potom Fourierova ÅŹada funkce $f$ na intervalu $(-T, T)$ konverguje na celÃľ mnoÅ"inÄŻ $\mathbb{R}$. OznaÄŊme $F$ jejÃŋ souÄŊtovou funkci, tzn.

$$F(x) := \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos \frac{k\pi x}{T} + b_k \sin \frac{k\pi x}{T} \,, \quad \forall x \in \mathbb{R} \,,$$

kde posloupnosti $(a_n)_{n=0}^{\infty}$ a $(b_n)_{n=1}^{\infty}$ jsou urÄŊeny vztahy . Potom platÃŋ:

(i) $F$ je periodickÃą funkce s periodou $2T$.

(ii) $F(x) = \dfrac{f(x+) + f(x-)}{2}$ pro kaÅ"dÃľ $x \in (-T, T)$.

(iii) $F(T) = F(-T) = \dfrac{f(-T+) + f(T-)}{2}$.

*Proof.* NeuvÃądÃŋme. □

```
Data: this text
Result: how to write algorithm with LaTeX2e
initialization;
while not at end of this document do
    read current;
    if understand then
        go to next section;
        current section becomes this one;
    else
        go back to the beginning of current section;
    end
end
```

In this section we'll describe FAST-LTS algorithm and it's main properties. The main idea of this algorthm is based on the fact that from one approximation of the algorithm we can compute another which can have lower objective function. TA DAAAAAAAAAAAAAAAA Thoerem 1: [2] Let w0 ... wp be the LTS estimate. for each data sample we can compute —y-wx—

Hlavni myslenka tohoto algoritmu spociva ve faktu,

V ÄDeskÃľ variantÄŻ naleznete Åąablony v souborech pojmenovanÃ · ch ve formÃątu prÃące_kÃşdovÃąnÃŋ.tex. Typ prÃące mÅŕÅ"e bÃ · t:

**BP** bakalÃąÅŹskÃą prÃące,

**DP** diplomovÃą (magisterskÃą) prÃące.

**UTF-8** kÃşdovÃąnÃŋ Unicode,

**ISO-8859-2** latin2,

**Windows-1250** znakovÃą sada 1250 Windows.

V pÅŹÃŋpadÄŻ nejistoty ohlednÄŻ kÃşdovÃąnÃŋ doporuÄDujeme nÃąsledujÃŋcÃŋ postup:

1. V opaÄDnÃľm pÅŹÃŋpadÄŻ postupujte dÃąle podle toho, jakÃ · operaÄDnÃŋ systÃľm pouÅ"ÃŋvÃąte:
    - v pÅŹÃŋpadÄŻ Windows pouÅ"ijte Åąablonu pro kÃşdovÃąnÃŋ Windows-1250,
    - jinak zkuste pouÅ"Ãŋt Åąablonu pro kÃşdovÃąnÃŋ ISO-8859-2.

V anglickÃľ variantÄŻ jsou Åąablony pojmenovanÃľ podle typu prÃące, moÅ"nosti jsou:

**bachelors** bakalÃąÅŹskÃą prÃące,

**masters** diplomovÃą (magisterskÃą) prÃące.

## 3.2 Exact algorithm

## 3.3 Feasible solution

## 3.4 MMEA

## 3.5 Branch and bound

## 3.6 Adding row

Chapter 4

# Experiments

## 4.1 Data

## 4.2 Results

## 4.3 Outlier detection

# Conclusion

# Bibliography

[1] Rousseeuw, P. J.; Driessen, K. V. An Algorithm for Positive-Breakdown Regression Based on Concentration Steps. In *Data Analysis: Scientific Modeling and Practical Application*, edited by M. S. W. Gaul, O. Opitz, Springer-Verlag Berlin Heidelberg, 2000, pp. 335–346.

[2] Rybicka, J. *LaTeX pro začátečníky*. Brno: Konvoj, third edition, ISBN 80-7302-049-1.

# Appendix A

# Datasets

**GUI** Graphical user interface

**XML** Extensible markup language

Appendix B

# Contents of enclosed CD

```
readme.txt ...................... the file with CD contents description
├── exe ............................. the directory with executables
├── src ............................. the directory of source codes
│   ├── wbdcm ............................. implementation sources
│   └── thesis ........ the directory of LaTeX source codes of the thesis
└── text ................................ the thesis text directory
    ├── thesis.pdf ..................... the thesis text in PDF format
    └── thesis.ps ....................... the thesis text in PS format
```