$x_i \sim N(0, \sigma^2)$

$E[x_i] = 0$

$\sigma_{x_i}^2 = \sigma^2$

$w_i \sim Uniform(-a, a)$

$E[w_i] = 0$

$\sigma_{w_i}^2 = (a - (-a))^2/12 = (2a)^2/12 = a^2/3$

$$\xi = \sum_i^n w_i x_i$$

Goal

$E[\xi] = 0$

$\sigma_\xi^2 = 1$

Solution for zero mean

$$E[\xi] = \sum E[x_i w_i] = \sum E[x_i] E[w_i] = 0$$

First equality from lienarity of E

Second equality from independence of x_i and w_i

So $E[\xi] = 0$ and does not depend on a

Solution for unary variance

$$\sigma_\xi^2 = E[\xi^2] - E^2[\xi] = E[\xi^2] = E[(\sum_i^n w_i x_i)(\sum_i^n w_i x_i)] = E[\sum_{i,j}^n w_i w_j x_i x_j] = \sum_{i,j}^n E[w_i w_j x_i x_j] = \sum_{i,j}^n E[w_i w_j] E[x_i x_j] = \sum_i^n E[w_i^2] E[x_i^2]$$

$E[x_i^2] = \sigma^2$

$E[w_i^2] = a^2/3$

$$\sigma_\xi^2 = \sum_i^n E[w_i^2] E[x_i^2] = n\sigma^2 a^2/3$$

We want $\sigma_\xi^2 = 1$ thus

$n\sigma^2 a^2/3 = 1$

$a = (3/(n\sigma))^{1/2} = (3/n)^{1/2} 1/\sigma$