R. Sadek Ali

University of Rajshahi
Department of Computer Science and Engineering
B. Sc. (Engg.) Part-I Odd Semester Exam - 2014
Course: MATH-1111 (Algebra, Trigonometry and Vector ~~Analysis~~)
Full Marks: 52.5 **Time:** 3 Hours
[N.B.: Answer any SIX questions taking THREE from each section. Marks for each question are shown on the right side margin.]

Section A

1. a) Explain the operations of union, intersection and difference of sets with the aid of Venn-Euler diagrams. 3
   b) Is there any difference between mappings and operators? Explain your answer. Give an example of a relation which is not symmetric. 2.75
   c) Using Cramer's rule solve the following system: 3
   $$3x - y + 2z = 7$$
   $$2x + y + z = 7$$
   $$x + y - 2z = -3$$

2. a) Solve the cubic equation $3x^3 - 26x^2 + 52x - 24 = 0$, the roots being in geometrical progression. 2.75
   b) What is Descartes' rule of signs? Use the rule to find the nature of the roots of the quintic equation $x^5 + 5x^4 - 20x^2 - 19x - 2 = 0$. Show that the equation has a real root between 2 and 3. 3
   c) Obtain the value of S_6 in equation $x^3 - x - 1 = 0$. 3

3. a) Test the equation $2x^4 + x^3 - 6x^2 + x + 2 = 0$ whether it is reciprocal. If a, b and c are roots of the equation $x^3 + qx + r = 0$, form the equation whose roots are $\frac{b+c}{a^2}$, $\frac{c+a}{b^2}$, $\frac{a+b}{c^2}$. 2.75
   b) Solve the quartic equation $x^4 - 6x^3 + 12x^2 - 10x + 3 = 0$ which has equal roots. 2
   c) Use Cardan's method to solve the cubic equation $x^3 + 21x + 342 = 0$. 4

4. a) Mention some applications of Demoivre's theorem. With the aid of Demoivre's theorem solve the polynomial equation $x^7 + x^4 + x^3 + 1 = 0$. 3.75
   b) If $x_r = \cos\frac{\Pi}{2^r} + i\sin\frac{\Pi}{2^r}$, prove that $x_1 x_2 x_3$.............to infinity = -1. 3
   c) If $\frac{Sin x}{x} = \frac{5045}{5046}$ then show that x is nearly $1^0 58'$. 2

Section B

5. a) Explain a technique to find the numerical value of Π using Gregory's series. 2.75
   b) Show that $i^i = e^{-(4n+1)\frac{\Pi}{2}}$. 3
   c) Find the sum of the following series up to n terms 3

$$\tan^{-1}\left(\frac{1}{2.1^2}\right)+\tan^{-1}\left(\frac{1}{2.2^2}\right)+\tan^{-1}\left(\frac{1}{2.3^2}\right)+\ldots\ldots\ldots\ldots$$

6. a) Show graphically that $-(\vec{A}-\vec{B})=-\vec{A}+\vec{B}$. Graph the vector field defined by $\vec{V}(x,y)=x\hat{i}+y\hat{j}$. 3

b) Show that commutative law for dot products is valid. Find the projection of the vector $2\hat{i}-3\hat{j}+6\hat{k}$ on the vector $\hat{i}+2\hat{j}+3\hat{k}$. Draw a rough sketch of it. 3.75

c) Show that $\vec{A}.\left(\vec{B}\times\vec{C}\right)=\vec{B}.\left(\vec{C}\times\vec{A}\right)=\vec{C}.\left(\vec{A}\times\vec{B}\right)$. 2

7. a) A particle moves so that its position vector is given by $\vec{r}=\cos\omega t\,\hat{i}+\sin\omega t\,\hat{j}$, where ω is a constant. Show that 3

(i) the velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$.

(ii) $\vec{r}\times\vec{v}$ is a constant vector.

b) Show that $div.curl\,\vec{A}=0$. 2.75

c) If $\vec{\nabla}.\vec{E}=0$, $\vec{\nabla}.\vec{H}=0$, $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{H}}{\partial t}$, $\vec{\nabla}\times\vec{H}=\frac{\partial\vec{E}}{\partial t}$, show that $\vec{E}$ and $\vec{H}$ satisfy 3

$$\nabla^2\vec{u}=\frac{\partial^2\vec{u}}{\partial t^2}.$$

8. a) The acceleration of a particle at any time t given by 3

$$\vec{a}=\frac{d\vec{v}}{dt}=12\cos 2t\,\hat{i}-8\sin 2t\,\hat{j}+16t\,\hat{k}.$$

If the velocity $\vec{v}$ and displacement $\vec{r}$ are zero at $t=0$, find $\vec{v}$ and $\vec{r}$ at any time.

b) Find the value of $\int_{-3}^{3}\int_{0}^{4}\int_{2}^{5}(x+y+z)dzdydx$. 2.75

c) State Green's theorem. Verify Green's theorem in the plane for $\oint_C (xy+y^2)dx+x^2dy$, where C is the closed curve of the region bounded by $y=x$ and $y=x^2$. 3

University of Rajshahi
Department of Computer Science and Engineering
B.Sc. (Engg.) Part-I (Odd Semester) Examination-2015

Course: MATH-1111 (Algebra, Trigonometry and Vector)

Marks: 52.5 **Time:** 03 Hours

[Answer any six (06) questions taking three (03) from each section]

Section-A

1. a) Define null set and subset. State and prove De Morgan's rule. 3
   b) Define function. Find the domain and range of the function $f(x) = \frac{x-3}{2x+1}$. 2.75
   c) Using Cramer's rule solve the following system: 3
   $x + y + z = 1;\ x + 2y + 3z = 2;\ x + 4y + 9z = 4.$

3

2. a) Evaluate the determinant: $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$ HW 3
   b) Prove that in an equation with real coefficients imaginary roots occur in pairs. V.V.I 3
   c) If a, b, c are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{b^2+c^2}{bc}$. ** 2.75

3. a) Prove that every equation of n^{th} degree has exactly n roots and no more. V.V.I 3
   b) Solve the cubic equation: $28x^3 - 9x^2 + 1 = 0$. 3
   c) State Demoiver's theorem and prove it when n is fractional either positive or negative. 2.75

4. a) If $x_r = \cos\frac{\pi}{2r} + i\sin\frac{\pi}{2r}$, then prove that, $x_1, x_2, x_3, \ldots\ldots \text{to infinity} = -1$. 3
   b) If $(1+x)^n = P_0 + P_1x + P_2x^2 + \cdots\cdots$ then show that, $P_1 - P_3 + P_5 - \cdots\cdots = 2^{\frac{n}{2}}\sin\frac{n\pi}{4}$ 3
   c) Solve $x^4 - 4x^2 + 16 = 0$ using Demoiver's theorem. 2.75

Section-B

5. a) If $A + iB = \log(x+iy)$, then show that $B = \tan^{-1}\frac{y}{x}$ and $A = \frac{1}{2}\log(x^2+y^2)$. 2.75
   b) Using Gregory's series prove that $\frac{\pi}{8} = \frac{1}{1.3} + \frac{1}{5.7} + \frac{1}{9.11} + \frac{1}{11.13} + \cdots\cdots$ 3
   c) Find the sum of the series $\operatorname{cosec}\alpha + \operatorname{cosec}2\alpha + \operatorname{cosec}2^2\alpha + \cdots + \operatorname{cosec}2^{n-1}\alpha$ 3

6. a) Find the projection of the vector $\bar{A} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ on the vector $\bar{B} = 4\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$. 2.75
   b) Find principal normal and binormal at point $t = \pi$ to the curve 3
   $x = 3\cos t,\quad y = 3\sin t,\quad z = 4t.$
   c) Find the total work done in moving a particle in a force field given by 3
   $\bar{F} = 3xy\hat{\imath} - 5z\hat{\jmath} + 10x\hat{k}$ along the curve $x = 1 + t^2, y = 2t^2, z = t^2$ from $t = 1$ to $t = 2$.

7. a) Define gradient and divergence. What is the physical significance of gradient? 3
   b) Find directional derivative of $\varphi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$. 2.75
   c) Find the constants a, b, c so that $\bar{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational. 3

8. a) If $\bar{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$, evaluate $\iint_s \bar{F}.\bar{n}\,ds$ where s is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. 4
   b) State Green's theorem in the plane. Verify Green's theorem in the plane for $\varphi\{(xy + y^2)dx + x^2dy\}$, where c is the closed curve of the region bounded by $y = x$ and $y = x^2$. 4.75

Part-1-odd

Sk. Sadek

University of Rajshahi
Department of Computer Science and Engineering
B.Sc. Engg.(CSE) 1st Year 2016
Course: MATH1111(Algebra, Trigonometry and Vector)

Time: 3 Hrs. **Full Marks: 52.5**

[N.B. Answer SIXquestions taking at least THREE from each part.]

Part A

1.a) Define null set, subset, power set, union and intersection of two sets with example. 3

b) Define one-one and onto functions. Can a constant function be one-one? Justify your answer. Show that if a relation R is transitive, then its inverse relation R^{-1} is also transitive. 3

c) Use Cramer's rule to solve the system of linear equations: $x+y+z=3,\ x+2y+3z=6,\ 5x+8y+11z=24$. 2.75

2.a) Show that in an equation with real coefficients imaginary roots occurs in pairs. 3

b) Solve the equation $54x^3-39x^2-26x+16=0$, the roots being in geometrical progression. 3

c) In the equation $x^4-x^3-7x^2+x+6=0$, find the value of S_4. 2.75

3.a) If a,b,c are the roots of $x^3+qx+r=0$,find the equation whose roots are $bc+\frac{1}{a}, ca+\frac{1}{b}, ab+\frac{1}{c}$. 2

b) Solve the cubic equation $x^3-15x^2-33x+847=0$ by Cardan's method. 3.75

c) If $x=cos\theta+isin\theta$ and $1+\sqrt{1-a^2}=na$ then prove that $1+acos\theta=\frac{a}{2n}(1+nx)(1+\frac{n}{x})$ 3

4.a) If $x=\frac{2}{1!}-\frac{4}{3!}+\frac{6}{5!}-\frac{8}{7!}+\cdots\ldots\ldots\ldots\ldots to\ \infty$ and $y=1+\frac{2}{1!}-\frac{2^3}{3!}+\frac{2^5}{5!}-\cdots\ldots\ldots\ldots\ldots to\ \infty$ then show that $x^2=y$. 3

b) If $i^{i^{i\ldots\ldots adinf}}=A+iB$, then prove that $tan\frac{\pi A}{2}=\frac{B}{A}$ and $A^2+B^2=e^{-\pi B}$. 2.75

c) Find the sum to infinity of the series $sin\theta.sin\theta-\frac{1}{2}sin2\theta.sin^2\theta+\frac{1}{2}sin3\theta.sin^3\theta\ldots\ldots\ldots\ldots\ldots\ldots$ 3

Part B

5.a) Define dot product of two vectors $\vec{A}$ and $\vec{B}$. Prove that the area of a parallelogram with sides $\vec{A}$ and $\vec{B}$ is $|\vec{A}\times\vec{B}|$. 3

b) Determine the unit vector perpendicular to the plane of $\vec{A}=2\vec{\imath}-6\vec{\jmath}-3\vec{k}$ and $\vec{B}=4\vec{\imath}+3\vec{\jmath}-\vec{k}$. 2.75

c) Show that the vectors $\vec{A}=3\vec{\imath}-2\vec{\jmath}+\vec{k}, \vec{B}=\vec{\imath}-3\vec{\jmath}+5\vec{k}$ and $\vec{C}=2\vec{\imath}+\vec{\jmath}-4\vec{k}$ form a right-angled triangle. 3

6.a) Find a unit tangent vector to any point on the curve $x=t^2+1,\ y=4t-3,\ z=2t^2-6t$. Also determine it where $t=2$. 2.75

b) A particle moves on a curve so that its position vector is given by $\vec{r}=coswt\vec{\imath}+\sin wt\,\vec{\jmath}$, where w is a constant. Show that the velocity of the particle is perpendicular to $\vec{r}$ and the acceleration is directed towards the origin. 3

c) What is the physical significance of the curl of a vector field? Determine the value of λ so that the vector field $\vec{v}(x,y,z)=(x+3y)\vec{\imath}+(y-2z)\vec{\jmath}+(x+\lambda z)\vec{k}$ is solenoidal. 3

7.a) What is meant by $\vec{\nabla}\varphi$, where φ is a scalar field? Find a unit normal to the surface $x^2y+2xyz=4$ at the point *(2, -2, 3)*. 3

b) Find angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point *(2, -1, 2)*. 2.75

c) Find the volume of the region common to the intersecting cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$. 3

8.a) State the divergence theorem of Gauss. Verify Green's theorem in the plane for $\oint_c (x^2-2xy)dx+(x^2y+3)dy$ around the boundary of the region defined by $y^2=8x$ and $x=2$. 4.75

b) Verify Stroke's theorem for $\vec{A}=(y-z+2)\vec{\imath}+(yz+4)\vec{\jmath}-xz\vec{k}$, where S is the surface of the cube $x=0, y=0, z=0, x=2, y=2, z=2$ above the xy plane. 4

University of Rajshahi

Department of Computer Science and Engineering

B.Sc. Engg. Part-IOdd Semester, Examination-2017

Course: MATH1111 (Algebra, Trigonometry and Vector)

Time: 3 Hours Marks: 52.5

(Answer SIX(6) questions taking Three(3) from each section)

SectionA

1. (a) When the two sets A and B are said to be equal? Prove that $(A \cap B)' = A' \cup B'$. 2.75

(b) Define an inverse function. Let V= {-2,-1, 0, 1, 2} and $g: V \to \Re^+$ be a function defined by the formula $g(x) = x^2 + 1$. Find the range of g. 3

(c) Define a reflexive, symmetric and a transitive relation giving one example of each. 3

2. (a) State Descarte's rule of signs. Find the least possible number of imaginary roots of $x^9 + 5x^8 - x^3 + 7x + 2 = 0$. 3

(b) Form the equation whose roots are the squares of the differences of the roots of $x^3 + qx + r = 0$. 2.75

(c) Solve the cubic equation $x^3 - 15x = 126$ by Cardan's method. 3

3. (a) Prove that the imaginary root of an equation with real coefficient occur in pairs. 3

(b) If a, b, c are roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum a^2$. 2.75

(c) Using Cramer's rule solve the system of linear equations 3

$$2x - y - z = 4$$
$$x - 2y + z = 5$$
$$x - 2y + 2z = 1$$

4. (a) State De Moiver's theorem. Find the value of $(1+i)^{1/3}$. 3

(b) If $x + \frac{1}{x} = 2\cos\theta$, then show that $x^n + \frac{1}{x^n} = 2\cos n\theta$. 3

(c) Expand $\cos n\theta$ using DeMoiver's Theorem. 2.75

SectionB

5. (a) If $x = \log\tan(\frac{\pi}{4} + \frac{y}{2})$ then prove that $y = -i\log\tan(\frac{ix}{2} + \frac{\pi}{4})$. 3

(b) Reduce $\tan^{-1}(\cos\theta + i\theta)$ to the form $a + ib$ and hence show that 3

$$\sin\theta - \frac{1}{3}\sin 3\theta + \frac{1}{5}\sin 5\theta - \cdots\cdots = \frac{1}{2}\log\{\pm\tan(\frac{\pi}{4} + \frac{\theta}{2})\}$$

(c) If $\theta < \frac{\pi}{4}$, then prove that $\log\sec\theta = \frac{1}{2}\tan^2\theta - \frac{1}{4}\tan^4\theta + \frac{1}{6}\tan^6\theta - \cdots\cdots$. 2.75

6. (a) If S_n be the sum to n terms of the series $\sin x + \sin 2x + \sin 3x + \cdots\cdots$ then prove that 3

$$\underset{n\to\infty}{Lt} \frac{S_1 + S_2 + \cdots + S_n}{n} = \frac{1}{2}\cot\frac{x}{2}$$

(b) Separate into real and imaginary parts the expression $\tan^{-1}(x + iy)$. 3

(c) If $\tan(\alpha + i\beta) = x + iy$, Then prove that $x^2 + y^2 - 2y\coth 2\beta = -1$. 2.75

7. (a) Find the projection of the vector $A = \underline{i} - 2\underline{j} + \underline{k}$ on the vector $B = 4\underline{i} - 4\underline{j} + 7\underline{k}$.

(b) A particle moves along the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $\underline{i} - 3\underline{j} + 2\underline{k}$.

(c) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2,-1, 2). 3

8. (a) Prove that $\underline{\nabla} \times (\underline{\nabla} \times \underline{A}) = -\nabla^2 \underline{A} + \underline{\nabla}(\underline{\nabla} . \underline{A})$ 2.75

(b) Find the work done in moving a particle once around a circle c in the xy plane. If the circle has centre at the origin and radius 3 and if the force field is given by 3

$\underline{F} = (2x - y + z)\underline{i} + (x + y - z^2)\underline{j} + (3x - 2y - 4z)\underline{k}$.

(c) Verify Stokes theorem for $\underline{A} = (2x - y)\underline{i} - yz^2\underline{j} - y^2 z\underline{k}$, where s is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and c is its boundary. 3

University of Rajshahi
Department of Computer Science and Engineering
B.Sc. Engg. Part-1 Odd Semester, Examination-2018
Course: MATH-1111 (Algebra, Trigonometry and Vector)

Time: 3 Hours Full Marks: 52.5

(Answer any **six** questions taking **three** from each Section)

Section: A

1 (a) Define complement of a set. State and prove De' Morgan's rules 3

(b) Evaluate $\begin{vmatrix} (b+c)^2 & c^2 & b^2 \\ c^2 & (c+a)^2 & a^2 \\ b^2 & a^2 & (a+b)^2 \end{vmatrix}$. 3

(c) Solve the following system of linear equation by Cramer's rule: 2.75
$x + y + z = 1$; $ax + by + cz = k$; $a^2x + b^2y + c^2z = k^2$

2 (a) State the fundamental theorem of algebra. Show that every equation of nth degree has exactly n roots. 3
(b) Find the condition that $x^3+px^2+qx+r=0$ may have the roots in Harmonical progression. 3
(c) If a, b, c are the roots of $x^3+qx+r=0$. Find the equation whose roots are $bc+\frac{1}{a}$, $ca+\frac{1}{b}$, $ab+\frac{1}{c}$. 2.75

3 (a) Solve the cubic following equation by cardon's method: $x^3-21x-344=0$. 3
(b) Show that the equation $\frac{A^2}{x-a}+\frac{B^2}{x-b}+\frac{C^2}{x-c}+\cdots+\frac{H^2}{x-h}=K$ has no imaginary roots. 3
(c) If the roots of $x^n-1=0$ are $1, \alpha, \beta, \gamma \ldots$ Show that $(1-\alpha)(1-\beta)(1-\gamma)\ldots = n$. 2.75

4 (a) State and prove De' Moivre's theorem. 3
(b) If $x = \cos\theta + i\sin\theta$ and $1+\sqrt{1-a^2} = na$, prove that $1 + a\cos\theta = \frac{a}{2n}(1+nx)(1+\frac{n}{x})$. 3
(c) If $x_r = cos\frac{\pi}{2^r} + isin\frac{\pi}{2^r}$, $r = 1,2,3,4,\ldots$ *Find the value of* $\prod_1^\infty x^r$. 2.75

Section: B

5 (a) If $x = \frac{2}{1!} - \frac{4}{3!} + \frac{6}{5!} - \frac{8}{7!} + \cdots$ *to* ∞ *and* $y = 1 + \frac{2}{1!} - \frac{2^3}{3!} + \frac{2^5}{5!} - \cdots$ *to* ∞, *show that* $x^2 = y$. 3
(b) If $\tan\log(x+iy) = a+ib$ *where* $a^2+b^2 \neq 1$, *prove that* $\tan\log(x^2+y^2) = \frac{2a}{1-a^2-b^2}$. 3
(c) Separate $\log(a+ib)$ into real and imaginary part. 2.75

6 (a) If $sinx = nsin(a+x), -1 < n < 1$, *expand x in a series of ascending power of n*. 3
(b) Given that $y = \log\tan\left(\frac{\pi}{4}+\frac{x}{4}\right) = x + c_3x^3 + c_5x^5 + \cdots$, *show that* $x = y - c_3y^3 + c_5y^5 - \cdots$ 3
(c) Prove that $\frac{tan^{-1}x}{x} + \frac{tan^{-1}y}{y} + \frac{tan^{-1}z}{z} = 3[1 - \frac{1}{7} + \frac{1}{13} - \frac{1}{19} + \frac{1}{25} - \cdots]$ where x, y, z are the three cube roots of unity. 2.75

7 (a) Find the sum of the series $tan^{-1}\frac{1}{2.1^2} + tan^{-1}\frac{1}{2.2^2} + tan^{-1}\frac{1}{2.3^2} + \cdots$ *to n terms*. 3
(b) Sum to n terms the series $tan\alpha + 2tan2\alpha + 2^2tan2^2\alpha + \cdots$ 3
(c) Sum the series to infinity: $cos\theta - \frac{1}{2}cos2\theta + \frac{1}{3}cos3\theta - \cdots (-\pi < \theta < \pi)$. 2.75

8 (a) If $\underline{r_1} = 2\underline{i} - \underline{j} + \underline{k}$, $\underline{r_2} = \underline{i} + 3\underline{j} - 2\underline{k}$, $\underline{r_3} = -2\underline{i} + \underline{j} - 3\underline{k}$, *and* $\underline{r_4} = 3\underline{i} + 2\underline{j} + 5\underline{k}$, find scalars a, b, c such that $\underline{r_4} = a\underline{r_1} + b\underline{r_2} + c\underline{r_3}$. 3
(b) Prove that $\overline{\nabla} \times (\overline{\nabla} \times \overline{A}) = \overline{\nabla}(\overline{\nabla}.\overline{A}) - \nabla^2\underline{A}$. 3
(c) A particle moves along the curve $x=2t^2$, $y=t^2-4t$, $z=3t-5$, where t is the time. Find the components of its velocity and acceleration at time t=1 in the direction $\underline{i} - 3\underline{j} + 2\underline{k}$ 2.75

University of Rajshahi

Department of Computer Science and Engineering

B.Sc. Engg. Part 1 Odd Semester Examination 2019

Course: MATH 1111 (Algebra, Trigonometry and Vector)

Full Marks: 52.5 Time: 3 Hours

[N.B. Answer any three questions from each Section]

Section-A

1)

a) Define a union and intersection of two sets. Let $A = \{0,1,2,3,4\}$ and $B = \{3,5,6\}$. Find $A \cup B$ and $A \cap B$. 2.75

b) Let X and Y be two sets. Define a relation and a function from X to Y. Let $X = \{1,2,3,4\}$ and $Y = \{a, b, c\}$. Consider the following three subsets of $X \times Y$ $f = \{(1, a), (2, a), (3, b), (4, c)\}$, $g = \{(2, a), (2, b), (3, b), (4, b)\}$ and $h = \{(1, a), (2, b), (3, c)\}$, which are functions from X into Y? Why? 3

c) Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = 2x - 5$ and $g: \mathbb{R} \to \mathbb{R}$ defined by $g(x) = 5x^2 - 3$. Find fog and gof. 3

2)

a) Prove that imaginary roots of an equation occur in pairs (the equation with real co-efficients) . 3

b) Find the nature of the roots of the polynomial $f(x) = x^7 - x^4 - x^2 - 1$. 3

c) If a, b, c are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum a^2$. 2.75

3)

a) Solve the cubic equation $x^3 + x - 2 = 0$ 3

b) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (b-c)(a-b)(c-a)(a+b+c)$ 3

c) Solve the following system of linear equations by using Cramer's rule: 2.75

$$x + y + z = 1, \quad x + 2y + z = 2, \qquad x + y + 2z = 0.$$

4)

a) State De'Moivers Theorem. Find all the values of $(1+i)^{1/3}$. 3

b) If $x + \frac{1}{x} = 2cos\theta$, then show that $x^n + \frac{1}{x^n} = 2cosn\theta$. 3

c) Prove that $i^i = e^{-(4n+1)\pi/2}$. 2.75

Section B

5)

a) Prove that 3

$$\frac{\Pi}{8} = \frac{1}{1.3} + \frac{1}{5.7} + \frac{1}{9.11} + \cdots$$

b) Find the sum of the series

$$1 + x\cos\theta + x^2 cos2\theta + \cdots + x^{n-1}\cos(n-1)\theta$$ 3

c) Separate into real and imaginary parts of the expression $\tan^{-1}(x+iy)$. 2.75

6)

a) Show that the vectors $\vec{A} = 3\vec{\imath} - 2\vec{\jmath} + k, \vec{B} = \vec{\imath} - 3\vec{\jmath} + 5\vec{k}, \vec{C} = 2\vec{\imath} + \vec{\jmath} - 4\vec{k}$ form a right angled triangle. 3

b) Prove that the area of a parallelogram with sides $\underline{A}$ and $\underline{B}$ is $|\underline{A}| \times |\underline{B}|$. 2.75

c) Prove that $\nabla \times \nabla\phi = 0$ and $\nabla.(\nabla \times A) = 0$ 3

7)

a) If $A = (3x^2 + 6y)i - 14yzj + 20xz^2k$, evaluate $\int_C A.dr$ from $(0,0,0)$ to $(1,1,1)$ along path $C: x = t, y = t^2, z = t^3$. 3

b) Evaluate $\iint \varphi nds$, where $\varphi = \frac{3}{8}xyz$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. 3

c) Find the volume of the region common to the intersecting cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. 2.75

8)

a) Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 3

b) If $v = w + r$, prove that $w = \frac{1}{2}Curl\ V$, where w is a constant vector. 3

c) Evaluate $\iint_s F.nds$, where $\vec{F} = 4x\vec{\imath} - y^2\vec{\jmath} + yzk$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. 2.75