

6. (a) Distinguish between parametric and non-parametric statistical tests. Discuss the advantages and disadvantages of non-parametric test. 2.5
- (b) Derive sign test, stating clearly the assumptions made for small sample case. 3.5
- (c) Use the sign test to see whether there is a difference between the number of days required to collect an account receivable before and after a new collection policy. Use the 0.05 significance level. 2.75

Before: 33 36 41 32 39 47 34 29 32 34 40 42

After: 35 29 38 34 37 47 36 32 30 34 41 38

6. (c) Use sign test to see whether there is a difference between number of days required to collect an account receivable before and after a new collection policy. Use the 0.05 significance level. Before: 33 36 41 32 39 47 34 29 32 34 40 42 35 29 38 34 37 47 36 32 30 34 41 38

Solⁿ:

No. 1 to 12	1	2	3	4	5	6	7	8	9	10	11	12
Before	33	36	41	32	39	47	34	29	32	34	40	42
After	35	29	38	34	37	47	36	32	30	34	41	38
Sign	+	-	-	+	-	0	+	+	-	0	+	-

There are 5 + signs and 5 - signs.

$$\therefore n = 5 + 5 = 10.$$

H_0 : ~~Diff~~ No significant difference betⁿ no. of days

H_a : Significant difference betⁿ no. of days.

At $\alpha = 0.05$ (one-tailed) and $n = 10$, the critical value is 0.

The test statistic x is the smaller number of + sign or - sign, so, $x = 4$.

4 is greater than critical value, so we fail to reject H_0 .

\therefore There is not enough evidence at 5% level to ~~claim~~ support the claim of having significant difference.

4. a) What do you mean by statistical hypothesis? Distinguish between simple and composite hypothesis. Let a random sample of size n is drawn from a normal population with mean μ and known variance σ^2 . How would you test the hypothesis that mean is equal to μ_0 ? 1+1.75
+3
- b) The average IQ of university female students in Bangladesh is suspected to be more than the average 110 for all students. A random sample of 64 female students yielded a sample average IQ of 115.5 and standard deviation of 20. Can you conclude that the average score of the female students is really more than 110? [$Z_{0.05}=1.64$] 3

2016 4b) The average IQ of university female students in Bangladesh is suspected to be more than the average 110 for all students. A random sample of 64 female students yielded a sample of average IQ of 115.5 and standard deviation of 20. Can you conclude that the average score of female students is really more than 110? [$Z_{0.05}=1.64$]

Solution: Consider, $H_0: \mu = 110$ vs $H_a: \mu > 110$

$$H_a: \mu > 110$$

$$|z| = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{115.5 - 110}{20 / \sqrt{64}}$$

$$= 2.2$$

Here

$$n = 64$$

$$\bar{x} = 115.5$$

$$\sigma = 20$$

$$\mu = 110$$

\therefore Calculated value, $z_{cal} = 2.2$

\therefore Tabulated value, $z_{tab} = 1.64$ at 5% level of significance

$\therefore z_{cal} > z_{tab}$ i.e. Reject H_0 .

So, it can be concluded that the average IQ of university female students in Bangladesh is not more than the average value 110 at 5% level of significance.

5. a) Define $r \times c$ contingency table. Show that in case of 2×2 contingency table, the test statistics becomes $\chi^2 = \frac{N(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$, also mention Yate's correction for continuity. 1+3
+1.75
3
- b) In a psychological test, 70 out of 100 boys came out successful while 60 out of 100 girls of the same age group as the boys passed the test. Do the data provide any evidence of difference in respect of abilities between the genders?

2016 5 (b) In a psychological test, 70 out of 100 boys came out successful while 60 out of 100 girls of the same age group as the boys passed the test. Do the data provide any evidence of difference in respect of abilities between the gender.

Solution: The Observed data,

	Male	Female	
Pass	70	60	130
Fail	30	40	70
	100	100	200

We consider,

~~H_0~~ Relation

H_0 : Difference in respect to gender

H_1 : No difference in respect to gender.

Estimated data.

	Male	Female	
Pass	65	65	130
Fail	35	35	70
	100	100	200

$$E_{ij} = \frac{R_i C_j}{N}$$

$$E_{11} = \frac{100 \times 130}{200} = 65$$

$$E_{12} = \frac{100 \times 130}{200} = 65$$

$$E_{21} = \frac{100 \times 70}{200} = 35$$

$$E_{22} = \frac{100 \times 70}{200} = 35$$

$$\begin{aligned} \chi^2 &= \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \\ &= \frac{(70-65)^2}{65} + \frac{(60-65)^2}{65} + \frac{(30-35)^2}{35} + \frac{(40-35)^2}{35} \\ &= 2.197 \end{aligned}$$

$$\text{So, } \chi^2_{\text{cal}} = 2.197 < \chi^2_{\text{tab}}(0.05) = 3.84$$

\therefore We fail to reject the Null hypothesis, H_0 .

So, there is Evidence of difference in respect of abilities between the gender.

6. a) What do you mean by non-parametric test? Discuss its importance. Describe the testing procedure of the run test. 1+1.75
+3
3
- b) The following sequence is purported to be a set of random integers from 0 to 99. Use the run's test to test the hypothesis of the randomness at $\alpha=0.05$ significance level. The sequence is

28, 4, 23, 98, 44, 10, 6, 25, 54, 81, 12, 6, 4, 33, 67, 55, 71, 66, 22, 18, 49, 85

2016: 6 ⑥ The following sequence is purported to be a set of random integers from 0 to 99. Use the run's test to test the hypothesis of randomness at $\alpha=0.05$ significance level. The sequence is,

28, 4, 23, 98, 44, 10, 6, 25, 54, 81, 12, 6, 4, 33, 67, 55, 71, 66, 22, 18, 49, 85.

Solⁿ: H_0 : Sequence is Random, H_1 : Sequence is not Random

The data sequenced in ascending order:

4, 6, 6, 10, 12, 18, 22, 23, 25, 28, 33, 44, 49, 54, 55, 66, 67, 71, 81, 85, 98.

$$\text{Median, } m = \frac{\left(\left(\frac{n}{2} \right) + 1 \right)^{\text{th}} \text{ term} + \frac{n}{2}^{\text{th}} \text{ term}}{2} = \frac{\left(\frac{22}{2} + 1 \right)^{\text{th}} + \left(\frac{22}{2} \right)^{\text{th}}}{2}$$

$$= \frac{12^{\text{th}} + 11^{\text{th}}}{2} = \frac{28 + 33}{2} = 30.5$$

4, 6, 6, 10, 12, 18, 22, 23, 25, 28, 33, 44, 49, 54, 55, 66, 67, 71, 81, 85, 98

28, 4, 23, 98, 44, 10, 6, 25, 54, 81, 12, 6, 4, 33, 67, 55, 71, 66, 22, 18, 49, 85

number of run, $G = 8$
 number of (-ve) sign, $n_1 = 11$
 number of (+ve) sign, $n_2 = 11$

At $\alpha = 0.05$, $n_1 = 11$ & $n_2 = 11$ the tabulated value, lower critical value = 7, higher critical value = 17

Number of Runs, = 8. H_0 fail to reject

The set is random.

$G < 7$	$7 \leq G \leq 17$	$G > 17$
Reject	Do not Reject	Reject

4. (a) What do you mean by a statistical hypothesis? Describe different steps for testing statistical hypothesis. Write down the procedure to test the significance of regression coefficient. 1+2.75+1
- (b) A random sample of 10 persons is selected as follows: 5, 2, 0, 4, 16, 14, 10, 11, 6, 8. Do you think that the average schooling year of the persons in population is 5? (Tabulated value at 5% with 9 d.f. is 2.26) 4

2015. 4(b) A random sample of 10 person is selected as follows: 5, 2, 0, 4, 16, 14, 10, 11, 6, 8. Do you think that the average schooling year of the persons in population is 5? (Tabulated value at 5% with 9 d.f. is 2.26)

Solution: The standard deviation is not given. For sample value, $n = 10$,

$$|t| = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{7.6 - 5}{5.16/\sqrt{10}}$$

$$= 1.59$$

Calculated value,

$$\therefore t_{cal} = 1.59$$

Tabulated value, $t_{tab} = 2.26$ at

5% level of significance.

\therefore Calculated value < tabulated value,

\therefore ~~Do not~~ Accept H_0 (Null hypothesis)

\therefore The average schooling year of person is 5 years.

Here,

$$\bar{x} = \frac{\sum x}{n} = \frac{5+2+0+4+16+14+10+11+6+8}{10}$$

$$= 7.6$$

$$s^2 = \frac{1}{n-1} \sum (x - \bar{x})^2$$

$$= \frac{(5-7.6)^2 + (2-7.6)^2 + (0-7.6)^2 + (4-7.6)^2 + (16-7.6)^2 + (14-7.6)^2 + (10-7.6)^2 + (11-7.6)^2 + (6-7.6)^2 + (8-7.6)^2}{10-1}$$

$$= 26.71$$

$$s = 5$$

$$\therefore s = \sqrt{26.71} = 5.16$$

H_0 : Average schooling year of person is 5.

H_a : Average schooling year of person not 5

6. (a) What is contingency table? What is form of χ^2 test statistic in case of a 2×2 contingency table?

(b) For given information in the following table, test at level of significance 0.05 that whether level of education affects the job performance. [$\chi^2_{0.05,4} = 13.3$]

4.75

Job Performance	Level of Education			
	Below primary	College	University	
Excellent	10	40	10	60
Good	30	30	20	80
Fair	10	30	20	60
	50	100	50	200

2015] 6(b) For given information in following table, test at level of significance 0.05 that whether level of education affects the job performance [$\chi^2_{0.05,4} = 13.3$]

Soln

H_0 : No affect

H_a : Affect

Job Performance	Level of education			Total
	Below Primary	College	University	
Excellent	10	40	10	60
Good	30	30	20	80
Fair	10	30	20	60
Total	50	100	50	200

Expected data :

$$E_{11} = \frac{60 \times 50}{200} = 15$$

$$E_{12} = \frac{100 \times 60}{200} = 30$$

$$E_{21} = \frac{50 \times 80}{200} = 20$$

$$E_{22} = \frac{100 \times 80}{200} = 40$$

$$E_{31} = \frac{50 \times 60}{200} = 15$$

$$E_{32} = \frac{100 \times 60}{200} = 30$$

Job Performance	Level of education			Total
	Below Primary	College	University	
Excellent	15	30	$\frac{60-45}{=15}$	60
Good	20	40	$\frac{80-60}{=20}$	80
Fair	15	30	$\frac{60-45}{=15}$	60
Total	50	100	50	200

Calculation for CHI-Square

O	E	(O-E)	(O-E) ²	(O-E) ² /E
10	15	-5	25	1.66
40	30	10	100	3.33
10	15	-5	25	1.66
30	20	10	100	5
30	40	-10	100	2.5
20	20	0	0	0
10	15	-5	25	1.66
30	30	0	0	0
20	15	5	25	1.66
Total 200	300	0		17.44

$$\therefore \chi^2_{cal} = \sum \frac{(O-E)^2}{E} = 17.47$$

$$\text{Tabulated } \chi^2_{0.05,4} = 13.3$$

$$\chi^2_{cal} > \chi^2_{tab}$$

\therefore Reject H_0 .

i.e. The level of education affects job performance.

Section - B

4. a) Define simple hypothesis and critical region. Let x_1, x_2, \dots, x_n and y_1, y_2, \dots, y_m be two random samples drawn from two normal populations $N(\mu_i, \sigma^2)$, $i=1,2$ respectively. If σ^2 is unknown, how would you test null hypothesis that $H_0: \mu_1 = \mu_2$ 8.75
- b) Sample mean weight of 20 CSE students is 50kg and 10 ICE students is 45kg. if the sample variances of weights are 25 and 16, test whether the mean weights of CSE students is greater than the mean weights of ICE students. (Use $t_{0.05,28}=1.64$)

2014/4/2014 Sample mean weight of 20 CSE students is 50kg and 10 ICE students is 45kg. if the sample variance of weights are 25 & 16, test whether the mean weight of CSE students is greater than the mean weights of ICE students. (USE $t_{0.05,28}=1.64$)

Solution: Consider, $H_0: \mu_1 = \mu_2$ i.e. no significant difference between mean $H_a: \mu_1 > \mu_2$ Re. level α .

Let, $H_0: \mu_1 = \mu_2$ i.e. no significant difference between mean weight of CSE & ICE students.

$H_a: \mu_1 > \mu_2$ i.e. Mean weight of CSE students is greater than that of ICE students.

$$\begin{aligned}
 t_{cal} &= \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}} \\
 &= \frac{50 - 45}{\sqrt{\frac{25}{20} + \frac{16}{10}}} \\
 &= \frac{5}{\sqrt{1.25 + 1.6}} \\
 &= 1.75
 \end{aligned}$$

Here,

$$\begin{aligned}
 n_1 &= 20 \\
 n_2 &= 10 \\
 \bar{x}_1 &= 50 \\
 \bar{x}_2 &= 45 \\
 \sigma_1^2 &= 25 \\
 \sigma_2^2 &= 16
 \end{aligned}$$

$\therefore t_{cal} = 1.75 > t_{tab} = 1.64 \therefore$ Reject H_0 at 5% level of significance.

\therefore The mean weight of CSE students is greater than mean weight of ICE students.

6. a) Describe the procedure of Fisher's exact test for testing the independence of two binary variables. 4

b) For given information in the following table, test at level of significance 0.05 that whether class attendance affects the examination score. 4.75

Class attendance	Examination score	
	A+	Less than A+
Less than 80%	3	6
80% and above	7	4

2014/ G6) For given information in the following table, test at level of significance 0.05 that whether class attendance affect examination score.

Solⁿ

H_0 : No affect

H_a : Affect

Class attendance	Exam score		Total
	A+	less than A+	
less than 80%	3	6	9
80% and above	7	4	11
Total	10	10	20

Expected values

$$E_{11} = \frac{10 \times 9}{20} = 4.5$$

$$E_{21} = \frac{10 \times 11}{20} = 5.5$$

Class attendance	Exam Score		Total
	A+	less than A+	
less than 80%	4.5	$9 - 4.5 = 4.5$	9
80% & above	5.5	5.5	11
Total	10	10	20

$$\therefore \chi^2_{cal} = \sum \left[\frac{(O - E)^2}{E} \right]$$

$$= 1.818$$

$$\chi^2_{0.05, 1} = 3.84$$

$$\therefore \chi^2_{cal} < \chi^2_{tab}$$

O	E	(O - E)	(O - E) ²	(O - E) ² /E
3	4.5	-1.5	2.25	0.5
6	4.5	1.5	2.25	0.5
7	5.5	1.5	2.25	0.409
4	5.5	-1.5	2.25	0.409
20		0		1.818

\therefore Fail to reject.

H_0 accepted at 5% level of significance and we may conclude that there is no significant affect of class attendance over the examination score.

Questions that I couldn't solve, please check, Thank you

2014

5. a) Define type I error and type II error. Describe the procedure of testing the hypothesis that population proportion is equal to a specified value p_0 . 5.75
- (b) A nutritionist claims that 80 percent of the pre-school children in a certain country have protein-deficient diet. A sample survey reveals that it is true for 244 children out of 300. Is the nutritionist justified in his claim? Use a significant level of 0.01 [$Z_{0.01} = 2.33$] 3

2017

4. (a) Distinguish between Type 1 and Type 2 errors. Define: (i) Power of a test, (ii) Level of significance and (iii) Degree of freedom. Describe the procedure for Testing of Hypothesis. 1.75 +2 +2
- (b) The coefficient of correlation obtained from a random sample of 20 pairs is 0.50. Test the population correlation coefficient ($\rho=0$) at 5% level of significance. [$t_{0.05,18} = 2.10$]. 03

5. (a) When do you use independent samples t-test? $n =$ 2017 1+4
- Researchers are interested in the mean level of some enzyme in a certain population. They take a sample of 10 individuals, determine the level of enzyme in each and compute a sample mean 22. It is known that the variable of interest is approximately normally distributed with a variance of 45. Can you conclude that the mean enzyme level in this population is different from 25 at the 5% level of significance? [$Z_{0.05} = 1.96$].
- (b) For a simple random sample of adults, IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. A simple random sample of 13 statistics professors yields a standard deviation of $s=7.2$. Assume that IQ scores of statistics professors are normally distributed and use a 0.05 significance level to test the claim that $\sigma = 15$. [The tabulated value χ^2 with d.f. 12 at 5% level of significance are 4.404 and 23.337]. 3.75