University of Rajshahi
Department of Computer Science and Engineering
B.Sc.in Engineering 1st Year 1st Semester Examination-2019
Course: **MATH 1211** [Differential and Integral Calculus]
Marks: 52.50
Time: 3 Hours
[Answer any six (06) questions taking three (03) from each section.]

Section-A

1.a) Define function and hence define domain codomain with an example. 3

b) Find the domain and range of $f(x) = |x+1| + |x|$ and also sketch the graph of $f(x)$. 3

c) Prove that every differential function is continuous but the converse is not true. 2.75

2.a) Define derivative of a function $f(x)$ at $x = c$. Show that the function 4

$$f(x) = \begin{cases} 3 + 2x \; for \; -\frac{3}{2} < x \le 0 \\ 3 - 2x \; for \; 0 < x \le \frac{3}{2} \end{cases}$$ is continuous at $x = 0$ but not differentiable at $x = 0$.

b) If $y = (\sin^{-1} x)^2$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$. 3

c) i) Evaluate $\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$ ii) Differentiate $x^{\sin^{-1} x}$ with respect to $\sin^{-1} x$. 1.75

3.a) For a given curved surface of a right circular cone when the volume is maximum, show that the semi-vertical angle is $\sin^{-1}\frac{1}{\sqrt{3}}$. 4

b) Give the geometrical interpretation of Mean value theorem. 3

c) Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 8$ in [1, 4]. 1.75

4.a) If $u = f(x^2 + 2xyz, y^2 + 2zx)$, prove that $(y^2 - zx)\frac{\partial u}{\partial x} + (z^2 - xy)\frac{\partial u}{\partial y} + (x^2 - yz)\frac{\partial u}{\partial z} = 0$. 4

b) Define pedal equation of a curve. Prove that the curve $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{a'} + \frac{y^2}{b'} = 1$ will cut orthogonally if $a - b = a' - b'$. 3

c) Find the asymptotes of the curve $x^2y^2 - a^2(x^2 + y^2) = 0$. 1.75

Section-B

5. Answer any three of the following: 8.75

i) $\int \frac{x+1}{3+2x-x^2} dx$ ii) $\int \frac{dx}{\sqrt{(2x^2+3x+4)}}$

iii) $\int \frac{dx}{5+4\cos x}$ iv) $\int \frac{dx}{\sqrt{(x^2-2x+3)}\ (x^2-2x+1)}$

6.a) Evaluate $\int_0^1 \frac{\ln(1+x)}{1+x^2} dx$. 4

b) Evaluate $\int_2^4 \frac{x-3}{\sqrt{(5-x)(x-1)}} dx$. 3

c) Write down the five general properties of definite integral. 1.75

7. a) Obtain a reduction formula for $\int sin^n x\, dx$ and hence deduce Walli's formula. 6

b) Evaluate $\int_0^{\frac{\pi}{2}} \sin x\, dx$ from the definition of definite integral as limit of a sum. 2.75

8. a) Find the volume generated by the revolution about x-axis of the area bounded by the loop of the curve $y^2 = x^2(2-x)$. 4

b) Find the area bounded by the curve $y^2 = x^3$ and the line $y = 2x$. 2

c) Find the length of the perimeter of the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. 2.75

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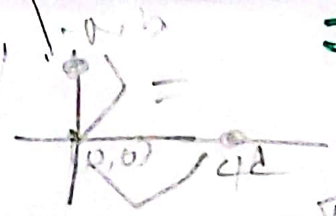
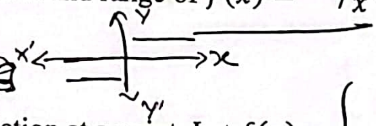


University of Rajshahi
Department of Computer Science and Engineering
B.Sc. Engineering Part I, **Even** Semester Examination **2018**
Course Code: MATH-**1211** Course Title: **Differential and Integral Calculus**
Time: **03** Hours Full Marks: **52.5**

Section A
Answer any **THREE** questions.

1. (a) Define function. Find domain and range of $f(x) = {|x|}/{x}$ and draw the graph of f. **03**

$D_f = \mathbb{R} - \{0\}$
$R_f = [-1, 1]$



(b) Define continuity of a function at a point. Let $f(x) = \begin{cases} x : 0 \le x < \frac{1}{2} \\ 1 - x : \frac{1}{2} \le x < 1 \end{cases}$ **3.25**
Is this function continuous at x=1/2 ? Is it differentiable at x=1/2?

(c) Evaluate $\text{lt}_{x \to 0} \left(\frac{Sinx}{x}\right)^{1/x^2}$ **2.5**

2. (a) If $f(x) = \left(\frac{a+x}{b+x}\right)^{a+b+2x}$, show that $f'(0)=\left\{2Log\frac{a}{b} + \frac{b^2-a^2}{ab}\right\}\left(\frac{a}{b}\right)^{a+b}$ **03**
(b) If $Siny = xSin(a+y)$, prove that $\frac{dy}{dx} = \frac{Sin^2(a+y)}{Sina}$ **2.75**
(c) If $y = x^{n-1}logx$, then prove that $y_n = \frac{(n-1)!}{x}$ **03**

3. (a) State and prove Mean value theorem. **03**
(b) Show that the largest rectangle with a given perimeter is a square. **03**
(c) Expand $2x^3 + 7x^2 + x - 1$ in power of (x-2) **2.75**

4. (a) Show that $(3x^2y - 2y^2)dx + (x^3 - 4xy + 6y^2)dy$ can be written as an exact differential of a function $\varphi(x, y)$ and find this function. **03**
(b) Define homogeneous function. If $z = \sin^{-1}\frac{x^2+y^2}{x+y}$, show that $x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} =$ **2.75**
$tanz$
(c) Show that in the curve $by^2 = (x+a)^3$, the square of the subtangent varies as the subnormal. **03**

$\frac{\text{subtangent}^2}{\text{subnormal}} = \frac{\left(\frac{y}{\frac{dy}{dx}}\right)^2}{y\frac{dy}{dx}}$

Section B
Answer any **THREE** questions.

5. (a) Evaluate any three of the following **8.75**

(i) $\int \frac{dx}{\sqrt{x(1+x)^5}}$

(ii) $\int \frac{dx}{\sqrt{(x^2-7x+12)}}$

(iii) $\int \frac{dx}{(2x-3)\sqrt{(2x^2-3x+4)}}$

(iv) $\int (3x-2)\sqrt{(x^2+x+1)}dx$

6. (a) Evaluate $\int \frac{x^4+2x+6}{x^3+x^2-2x} dx$ 03

(b) Evaluate $\int_0^{\pi/2} \frac{Sinx}{Sinx+Cosx} dx$ 2.75

(c) If $I_n = \int_0^{\pi/4} tan^n x dx$ show that $I_n + I_{n-2} = \frac{1}{n-1}$ and deduce the value of I_5 03

7. (a) Evaluate $\int_0^a \frac{x^4}{\sqrt{a^2-x^2}} dx$ 03

(b) If $I_n = \int_0^{\pi/4} tan^n \theta d\theta$, show that $I_n = \frac{1}{n-1} - I_{n-2}$ 2.75

(c) Prove that $\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8} log2$ 03

8. (a) Find the length of the cardioide $r = a(1 + cos\theta)$ and show that arc of the upper half is bisected by $\theta = \pi/3$ 03

(b) Find the area of the segment cutoff from the parabola $y^2 = 2x$ by the straight line y=4x-1 03

(c) Evaluate $\int_{z=1}^{2} \int_{y=0}^{1} \int_{x=-1}^{1} (x^2 + y^2 + z^2) dxdydz$ 2.75

03

2.75

Odd fun: A fun with graph that is symmetric with respect to origin. A fun is odd ∀ $f(-x) = -f(x)$

University of Rajshahi

Department of Computer Science and Engineering

B.Sc.(Engg.), Part-1, Even Semester Examination-2017

Course: ~~CSE~~1211 (Differential and Integral Calculus) MATH

Total marks: 52.50 Time: 3 hours

[Answer three questions from each section]

Section-A

1 (a) Define even function. Show that $f(x) = 2\cos x + \sin^2 x - \frac{3}{x^2} + x^4$ is an even function and $f(x) = x\sin^2 x - x^3$ is an odd function. 3

$f(-x) = 2\cos(-x) + \sin^2(-x) - \frac{3}{(-x)^2} + (-x)^4 = 2\cos x + \sin^2 x - \frac{3}{x} + x^4$

$= x(\sin^2(-x)) - (-x)^3 = -x\sin^2 x + x^3 = -f(x) = -(x\sin^2 x - x^3) = -f(x).$

(b) If $f(x) = 3 + 2x \quad for \ -\frac{3}{2} < x \le 0$

$= 3 - 2x \quad for \ 0 < x < \frac{3}{2}$

Show that f(x) is continuous at $x = 0$ but $f'(0)$ does not exist. 3

(c) Evaluate $\lim_{x \to \frac{\pi}{2}} (sinx)^{logx}$. $1^{\log\frac{\pi}{2}} = 1$ 2.75

2 (a) Find the differential coefficient of $(sinx)^{cosx} + (cosx)^{sinx}$. [Pic] 2.75

(b) Differentiating $\cos^{-1}\frac{1-x^2}{1+x^2}$ w.r.to $tan^{-1}\frac{2x}{1-x^2}$. [Pic - Sir] 3

(c) If $y = \tan^{-1}x$ then 3

(i) $(1 + x^2)y_1 = 1$

and (ii) $(1 + x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$.

3 (a) Define partial derivatives. If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. 3

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(b) If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. 2.75

(c) Given $\frac{x}{2} + \frac{y}{3} = 1$, find the maximum value of xy and minimum value of $x^2 + y^2$. 3

445, 3-(ii)

4 (a) Show that the tangent at (a, b) to the curve $(\frac{x}{a})^3 + (\frac{y}{b})^3 = 2$ is $\frac{x}{a} + \frac{y}{b} = 2$. 3

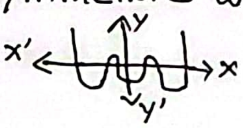
447, 10-(i)

(b) If $lx + my = 1$ touches the curve $(ax)^n + (by)^n = 1$ show that $(\frac{l}{a})^{\frac{n}{n-1}} + (\frac{m}{b})^{\frac{n}{n-1}} = 1$. 3

524 - 3

(c) Find the asymptotes of the curves $x^3 + 3x^2y - xy^2 - 3y^3 + x^2 - 2xy + 3y^2 + 4x + 5 = 0$. 2.75

Even fun: A fun with graph that is symmetric with respect to the y-axis. ∀ $f(-x) = f(x)$.



Section -B

8.75

5. Evaluate any three of the following
(i) $\int \frac{dx}{x(a+b\log x)}$ (ii) $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}} (\beta > \alpha)$ (iii) $\int \frac{dx}{5+4\cos x}$ (iv) $\int \left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right) dx$

6.(a) Show that $\int_0^a \frac{a^2-x^2}{(a^2+x^2)^2} dx = \frac{1}{2a}$. 2.75

(b) Evaluate $\lim_{n\to\infty} \left[\frac{1}{n+m} + \frac{1}{n+2m} + \dots\dots\dots + \frac{1}{n+nm}\right]$ 3

(c) Show that $\int_0^{\frac{\pi}{2}} \log \sin x \, dx = \int_0^{\frac{\pi}{2}} \log \cos x \, dx = \frac{\pi}{2} \log \frac{1}{2}$ 3

7.(a) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n \theta \, d\theta$, show that $I_n = \frac{1}{n-1} - I_{n-2}$. 2.75

(b) Prove that $u_n = \int_0^1 x^n \tan^{-1} x \, dx$ then $(n+1)u_n + (n-1)u_{n-2} = \frac{\pi}{2} - \frac{1}{n}$. 3

(c) Show that $\int_0^1 x^{n-1}(1-x)^{m-1} dx = \int_0^1 x^{m-1}(1-x)^{n-1} dx = \frac{1.2.3 \dots\dots (m-1)}{n(n+1) \dots\dots (n+m-1)}$. 3

8.(a) Find the area of quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ between the major and minor axes. 4

(b) Find the length of the arc of the parabola $y^2 = 4ax$ measured from the vertex to one extremity of the latus rectum. 4.75

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① Maximum & minimum
② partial derivation
③ tangent and normal

University of Rajshahi
Department of Computer Science and Engineering
B.Sc. Engg. Part-1 Even Semester, Examination-2016
Course: MATH-1211 (Differential and Integral Calculus)

Time: 3 Hours | Full Marks: 52.5

Answer any six questions taking three from each group

PART: A

1 (a) Define domain and range of a function. Find the domain and range of the function f defined by $f(x) = \frac{x-3}{2x+1}$. 2.75

(b) State L' Hospital's rule. Evaluate $\lim_{x\to 0}(\cos x)^{\cot^2 x}$. 355-6 3

(c) Define continuity of functions. Show that f(x) =|x| is continuous at x=0 but f '(x) does not exist. 3

2 (a) If siny=xsin(a+y), prove that $dy/dx = \frac{\sin^2(a+y)}{\sin a}$. 3

(b) If $y = e^{a\sin^{-1}x}$, show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$ [259 (4) 2.75

(c) State and prove Mean Value theorem. 3

3 (a) Show that the maximum value of x+1/x is less than its minimum value. 2.75

(b) If $u=F(x^2+y^2+z^2)f(xy+yz+zx)$, then show that $(y-z)\frac{\partial u}{\partial x} + (z-x)\frac{\partial u}{\partial y} + (x-y)\frac{\partial u}{\partial z} = 0$. (2017 3a) 3

(c) Define homogeneous functions. If $u = \cos^{-1}(\frac{x+y}{\sqrt{x}+\sqrt{y}})$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$. 416 3 (ii) 3

444 (4)

4 (a) If xcosα+ysinα=p touches the curve $\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1$, show that $(a\cos\alpha)^{\frac{m}{m-1}} + (b\sin\alpha)^{\frac{m}{m-1}} = p^{\frac{m}{m-1}}$. 3

(b) If lx+my=1 is normal to the parabola $y^2=4ax$, then prove that $al^3+2alm^2=m^2$. 441 10(ii) 2.75

(c) Define asymptotes. Find the asymptotes of $x^2y^2-4(x-y)^2+2y-3=0$. 3

524 (11)

PART: B

5 (a) Evaluate any three of the followings: 8.75

(i) $\int\sqrt{\frac{a+x}{x}}dx$ (ii) $\int\frac{xe^x}{(1+x)^2}dx$

(iii) $\int\frac{x+\sin x}{1+\cos x}dx$ (iv) $\int\sqrt{2ax-x^2}dx$

6 (a) Evaluate $\int_0^{\pi/2}\frac{dx}{3+5\cos x}$. 3

(b) Show that $\int_0^{\pi} x\log\sin x\,dx = \frac{\pi^2}{2}\log\frac{1}{2}$. 3

(c) Evaluate $\lim_{n\to\infty}[\frac{1^2}{n^3+1^3} + \frac{2^2}{n^3+2^3} + \cdots + \frac{n^2}{n^3+n^3}]$ 2.75

7 (a) Show that $\int_0^{\pi/2}\sin^n x\,dx = \frac{(n-1)}{n}\frac{(n-3)}{n-2}\cdots\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}$, where n is a positive integer. 3

(b) Evaluate $\int_0^{\pi/2}\frac{\sin^2 x}{\sin x+\cos x}dx$. 2.75

(c) If $I_n = \int_0^{\pi/4}\tan^n\theta\, d\theta$, show that $I_n = \frac{1}{n-1} - I_{n-2}$. 3

8 (a) Find the area bounded by the curves $y^2=4ax$ and $x^2=4ay$. 2.75

(b) If s is the arc length of $3ay^2=x(x-a)$ measured from the origin to the point (x, y), show that $3s^2=4x^2+3y^2$. 3

(c) Show that the volume of the solid produced by the revolution of the loop of the curve $y^2(a+x)=x^2(a-x)$ about x-axis is $2\Pi a^3(\log 2-2/3)$. 3

University of Rajshahi
Department of Computer Science and Engineering
B. Sc. (Engg.) Part-I. Even Semester. Examination 2015
Course: MATH1211 (Differential and Integral Calculus)
Full Marks: 52.5 Time: 3 Hours

[Answer any Six questions taking three from each part]

Part A

domain

1. a) Find the ~~domain~~ domain and range of the function $\frac{x-3}{2x+1}$ and also find its inverse function, if exists. 3

b) $f(x)$ is defined as follows: 3

CSE

$$f(x) = 0, \; x=0$$
$$= x, \; x>0$$
$$= -x, \; x<0$$

Draw the graph of the function. Does $f'(x)$ exist at $x=0$? Justify your answer.

c) Examine the continuity of the function $f(x)$ at $x=3/2$ where 2.75

$$f(x) = \begin{cases} 3 - 2x, & 0 \leq x < 3/2 \\ -3 - 2x, & x \geq 3/2 \end{cases}$$

2. a) Define differentiability of a function. Prove that every finitely derivable function is continuous. 3

b) If $y = e^{ax} \sin bx$, prove that $y_2 - 2ay_1 + (a^2 + b^2)y = 0$. 2.75

c) If $y = \sin(a\sin^{-1}x)$, with the help of Leibnitz's theorem prove that 3

$$y_{n+2}(1 - x^2) - (2n + 1)xy_{n+1} - (n^2 - a^2)y_n = 0$$

$f(b)-f(a)=(b-a)f'(\xi)$.

3. a) State and prove the Rolle's theorem. 3

b) Verify the mean value theorem for $f(x) = 2x^2 - 7x + 10$, $a=2$, $b=5$. 2.75

c) Expand $e^x \cos x$ in a finite series in power of x with Lagrange's remainder using Maclaurin series. 3

310-2(x)

4. a) Define maxima and minima of a function. Find the maximum and minimum values of $2x^3 - 9x^2 + 12x - 3$. 3

b) State Euler's theorem. Verify Euler's theorem for the function $u = \sin\frac{x^2+y^2}{xy}$. 3

c) Find the asymptotes of $x^3 + 2x^2y - xy^2 - 2y^3 + xy + y^2 - 1 = 0$. 2.75

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523 2(ai)

Part B

5. a) Integrate the following with respect to x (any two) 3

i). $\int \frac{dx}{(1+x)\sqrt{(1+2x+x^2)}}$ ii). $\int \frac{dx}{a+b\sin x}$ iii). $\int \sqrt{\{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}\}}dx$

b) Integrate $\int \frac{\log(\log x)}{x} dx$

c) Evaluate $\int \frac{dx}{\cos x\,(5+3\cos x)}$. 3

2.75

6. a) Evaluate $\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x} dx$. 2.75

b) Prove that $\int_0^1 \frac{dx}{(1+x^2)\sqrt{(1-x^2)}} = \frac{\pi}{2\sqrt{2}}$. 3

c) Evaluate $\lim_{n\to\infty} \sum_{r=0}^{n} \frac{n}{n^2+r^2}$. 3

7. a) Prove that $\int_0^{\pi/4} \log(1+\tan\theta)\,d\theta = \frac{\pi}{8}\log 2$. 3

b) Prove that $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x} dx = \frac{1}{2}\pi(\pi-2)$. 2.75

c) Obtain the reduction formula for $\int \frac{dx}{(x^2+a^2)^{n/2}}$. 3

Hence, find the value of $\int \frac{dx}{(x^2+a^2)^{7/2}}$.

8. a) Find the area included between the ellipses 3

$x^2+2y^2=a^2$ and $2x^2+y^2=a^2$

b) Show that the length of the area of the evolute $27ay^2 = 4(x-2a)^3$ of the parabola $y^2=4ax$, from the cusp to one of the points where the evolute meets the parabola is $2a(3\sqrt{3}-1)$. 3

c) Find the volume and the surface area of the solid generated by revolving the cardioids $r=a(1-\cos\theta)$ about the initial line. 2.75

Department of Computer Science and Engineering
B. Sc. (Engg) Part-I Even Semester Examination 2014
Course: MATH-1211 (Differential and Integral Calculus)
Full Marks: 52.5 Duration: 3(Three) Hours
Answer 06(Six) questions taking any 03(Three) questions from each section in separate answer script

Section - A

1. a) Draw the graph of $y = x - [x]$, where $[x]$ denotes the greatest integer not greater than x. 2.75
b) Define the differentiability of a function at $x = a$. Let $f(x)$ be defined by 3

$$f(x) = \begin{cases} x\sin\frac{1}{x}, & when\ x \neq 0 \\ 0, & when\ x = 0 \end{cases}$$

Examine the differentiability of $f(x)$ at $x = 0$. CSE
c) Evaluate $\lim_{x\to 0}(\sin x)^{2\tan x}$ 370 5(ii)/(iv) 3

2. a) If $y = \sin(10\sin^{-1}x)$, then use Leibnitz's theorem to show that $(1-x^2)y_{12} - 21xy_{11} = 0$. 3
b) Differentiate $\tan^{-1}\frac{\sqrt{(1+x^2)}-1}{x}$ with respect to $\tan^{-1}x$. [Pic] 2.75
c) If $y = x^{2n}$, where n is a positive integer, show that $y_n = 2^n\{1.3.5.\dots\dots\dots(2n-1)\}x^n$. 3

Problem
3. a) State Mean Value theorem. In the Mean Value theorem $f(h) = f(0) + hf'(\theta h), 0 < \theta < 1$, show that the limiting value of θ as $h \to 0$ is $1/2$, according as $f(x) = \cos x$.
V.V.I.
b) If $f(x)$ be a maximum at $x = c$ and if $f'(c)$ exists, then show that $f'(c) = 0$. 2.75
2017-3(c)
c) Given $x/2 + y/3 = 1$, find the maximum value of xy and minimum value of $x^2 + y^2$. 3

4. a) Define homogeneous function for n variables. Verify Euler's theorem for $u = \tan^{-1}\frac{x^3+y^3}{x-y}$. 3
393 .124
b) Define subtangent and subnormal. Show that for the curve $by^2 = (x+a)^3$, the square of the subtangent varies as the subnormal. 4Y 3
c) Prove that the asymptotes of the cubic $(x^2 - y^2)y - 2ay^2 + 5x - 7 = 0$ form a triangle of area a^2. 2.75

Section- B

5. a) Integrate the following: 3
i. $\int \frac{e^x - 1}{e^x + 1}dx$.
ii. $\int \sqrt{\frac{x}{a-x}}dx$.
b) Evaluate the integral $\int \frac{e^x}{x}(1 + x\log x)\,dx$.
c) Integrate $\int \frac{dx}{5+4\sin x}$ 2.75

6. a) State the Fundamental Theorem of Integral Calculus. Evaluate $\int_0^a \sqrt{a^2 - x^2}\,dx$. 3
b) Prove that $\int_2^e \left\{\frac{1}{\log x} - \frac{1}{(\log x)^2}\right\}dx = e - \frac{2}{\log 2}$. 3
c) Evaluate $\lim_{n\to\infty}\left[\frac{n!}{n^n}\right]^{1/n}$. 3

7. a) If $I_n = \int_0^{\pi/4}\tan^n\theta\,d\theta$, show that $I_n = \frac{1}{n-1} - I_{n-2}$. Hence find the value of $\int_0^{\pi/4}\tan^6 x\,dx$. 2.75
b) Obtain a reduction formula for $\int \frac{dx}{(a+b\sin x)^n}$. 3
c) If $u_n = \int_0^{\pi/2} x^n \sin x\,dx\ (n > 0)$. Prove that $u_n + n(n-1)u_{n-2} = n(\pi/2)^{n-1}$. 3

8. a) Show that the area bounded by the parabola $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. 2.75
b) Find the whole length of the loop of the curve $3ay^2 = x(x-a)^2$. 3
c) Find the volume of the solid generated by revolving the cardioide $r = a(1 - \cos\theta)$ about the initial line. 2.75 3

University of Rajshahi
Department of Computer Science and Engineering
B.Sc. Engg. Part-I Even Semester, Examination-2013
Course: MATH-1211 (Differential and Integral Calculus)
Full Marks: 52.5
Time: 3 Hours

Answer six questions taking any three from each Section

Section: A

1 (a) Find the domain and range of the function $\frac{x^2-1}{x-1}$. Also sketch the graph. 3

(b) Define continuity at a point. If the function $f(x) = \begin{cases} -\frac{x^2-16}{x-4} & if\ x \neq 4 \\ a & if\ x = 4 \end{cases}$ is continuous at point 4, what is the value of a? 2.75

2d5 -1(b)

(c) Evaluate $\boxed{\lim_{n\to 0}\left(\frac{\tan x}{x}\right)}$. 3

CSE

2 (a) Show that $f(x)=|x|$ is not differentiable at $x=0$. 2.75
(b) State Leibnitz's theorem. If $y = (\sin^{-1} x)^2$, then prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$. 3
(c) On a dark night, a thin man 6 feet tall walks away from a lamp post 24 feet high at the rate of 5 mph. How fast is the end of his shadow moving? How fast is the shadow lengthening? 3

3 (a) State Rolle's Theorem. Verify Roll's theorem for $f(x) = x^2-2x-3$ on $[-1, 3]$ and give geometrical interpretation. 4
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(b) Suppose that $f(0) = -3$ and $f'(x) \leq 5$ for all values of x. How large can f(2) be possible? 3
(c) Find two positive numbers whose sum is 100 and the sum of whose square is minimum. 1.75

4 (a) Find the shortest distance from the point P (1, 0) to the parabola $x=y^2$. 3
(b) State Euler's Theorem. If $u = x\phi(y/x) + \phi(y/x)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x\phi(y/x)$. 3
(c) Find the condition that the conics $ax^2+by^2=1$ and $a_1x^2+b_1y^2=1$ shall cut orthogonally. 2.75

Section: B

5 (a) Find the antidevivative of the following: 6

(i) $f(x) = \cos(2\cot^{-1}(\sqrt{\frac{1-x}{1+x}}))$.

(ii) $f(x) = \log(x + \sqrt{x^2 + a^2})$.

(b) A rocket shot straight up from the ground hits the ground 8 seconds later. Find its maximum height using integration. 2.75

6 (a) Using the definition of definite integral Evalute. 4

(i) $\int_a^b e^x\, dx$.

(ii) $\lim_{n\to\infty}[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}]$.

(b) Evaluate $\int_a^b \frac{\log x}{x} dx$. 3

(c) Prove that $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx = \frac{\pi}{4}$. 1.75

7 a) Evaluate $\int \cos^m x\, dx$ and hence $\int_0^{\pi/2} \cos^6 x\, dx$. 3
b) Obtain the reduction formula for $\int x^m(\log x)^n dx$. 2.75
c) If $u_n = \int_0^{\pi/2} \theta \sin^n\theta\, d\theta$ and $n > 1$, then prove that $u_n = \frac{n-1}{n} u_{n-2} + \frac{1}{n^2}$. 3

8 a) Find the area of the region bounded by the x-axis and one of the arc of y=sinx. 2.75
b) If s be the length of an arc of $3ay^2=x(x-a)^2$ measured from the origin to the point (x, y), then show that $3s^2=4x^2+3y^2$. 3
c) Find the volume of a paraboloid of revolution formed by revolving the parabola $y^2=4ax$ about x-axis and bounded by the section $x=x_1$ 3