University of Rajshahi

Department of Computer Science and Engineering

B. Sc. (Engg.) Part-2 (Odd Semester) Examination-2020

Course: MATH2111 (Matrices and Differential Equations)

Full Marks: 52.5 **Time: 3 Hours**

[Answer six questions taking any three from each section]

Section A

1. a) Define the inverse of a matrix. Find the inverse of 3.00

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

b) Define an involutory matrix. If a matrix A is involutory, then what is the inverse of A? 2.00

c) Define the transpose of a matrix. Prove that $(AB)^t = B^t A^t$. When a matrix is called symmetric? 3.75

2. a) What is the adjoint of a matrix? Find the adjoint of 3.00

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

b) If A is non-singular, then show that $adj\ (adj\ A) = |A|^{n-2} .\ A$ 3.00

c) Prove that the adjoint of a symmetric matrix is itself symmetric. 2.75

3. a) Define the rank of a matrix. Find the rank of the matrix: 3.00

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$$

b) Reduce the matrix: 3.00

$$A = \begin{bmatrix} 1 & 2 & -2 & 3 & 1 \\ 1 & 3 & -2 & 3 & 0 \\ 2 & 4 & -3 & 6 & 4 \\ 1 & 1 & -1 & 4 & 6 \end{bmatrix}$$

to its canonical form, find its rank and then reduce to its normal form.

c) Solve the following equations by Crammer's rule 2.75

$$x + y + z = 6$$
$$x - y + z = 2$$
$$2x + y - z = 1$$

4. a) Define eigenvalues and eigenvectors of an n −square matrix. What is meant by characteristic polynomial of an n −square matrix? Find the characteristic roots and characteristic vectors of the following matrix: 4.75

$$\begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$$

b) Define null space and nullity of a matrix. State Cayley-Hamilton theorem. Use the mentioned theorem to find the inverse of the matrix given below: 4.00

$$\begin{bmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{bmatrix}$$

Section B

5. a) Define ordinary differential equations. Solve the homogeneous differential equation $\frac{dy}{dx} = \frac{3y^2-x^2}{2xy}$. 2.75

b) Define Bernoulli differential equations. Identify the differential equation $\frac{dy}{dx} + \frac{3y}{x} = 6x^2$ and solve it. 3.00

c) The population x of a certain city satisfies the logistic law $\frac{dx}{dt} - \frac{1}{100}x = -\frac{1}{10^8}x^2$, where time t is measured in years. Find the population of the city at any time. 3.00

6. a) Define initial value problems. A circuit has in series a constant electromotive force of 40 V, a resistor of 10Ω, and an inductor of 0.2 H. Find the current at time $t > 0$ if the initial current is zero. 2.75

b) Define Clairaut's equation. Find a one-parameter family of solutions of the equation $y = px + p^2$, where $p \equiv \frac{dy}{dx}$. Find singular solution, if exists, of the given equation which is not a member of the one-parameter family of solutions. 3.00

c) Use the method of undetermined coefficients to solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\, sin\, x$. 3.00

7. a) Define Reccati's equation. State under what conditions, Reccati's equation reduces to a linear and Bernoulli's equation? Solve: $\frac{dy}{dx} + y = xy^3$ 3.00

b) Solve $(D^2 + 4)y = x^2e^{2x}$ by operator method. 2.75

c) Define regular singular point and examine the regular singular point of $x^2y'' - xy' + 8(x^2 - 1)y = 0$ 3.00

8. a) Convert the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ with the initial conditions $y(0) = 1,\ y'(0) = 0$ into an integral equation. Finally, identify your obtained integral equation. 2.75

b) An LTIC system is in zero state. Its response $y(t)$ is described by the differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = \frac{df}{dt} + f(t)$. Find the response of the system if the input is given by $f(t) = 3e^{-5t}u(t)$ 2.00

c) Solve the following one-dimensional heat equation by the method of separation of variables 4.00

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < 1, t \geq 0$

subject to the initial profile

i) $u(x, 0) = 1,\ 0 < x < 1$ and the Dirichlet boundary conditions

ii) $u(0, t) = 0, t \geq 0$ and

iii) $u(1, t) = 0, t \geq 0$.