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**University of Rajshahi**

**B.Sc. (Engg.) Part-2 (Odd Semester) Examination-2018**

**Course: MATH2111 (Matrix and Differential Equation)**

**Marks: 52.5 Time: 3:00 Hours**

[N.B. Answer any **Six** questions taking **Three** from each section.]

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## Section-A

1.(a) Define matrix multiplication. Prove that matrix multiplication is associative. 2.75

(b) Define a symmetric matrix with an example. For any matrix $A$, prove that $AA'$ and $A'A$ are symmetric. 3

(c) Define inverse of a matrix. If $A$ and $B$ are invertible matrices then prove that $(AB)^{-1} = B^{-1}A^{-1}$. 3

2.(a) For any $n$-square matrix $A$, prove that $\operatorname{adj}(\operatorname{adj}(A)) = |A|^{n-2}A$. Hence find $|\operatorname{adj}(\operatorname{adj}(A))|$. 3

(b) Reduce the matrix $\begin{pmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{pmatrix}$ into echelon form and determine its rank. 2.75

(c) Find the inverse of $A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$, using elementary row operations. 3

3.(a) Solve the system of equations: 2.75

$$\begin{aligned} 2x + 3y &= 3 \\ x - 2y &= 5 \\ 3x + 2y &= 7. \end{aligned}$$

(b) Determine the values of $k$ such that the following system in unknowns $x, y, z$ has (i) a unique solution, (ii) no solution, (iii) more than one solution: 3

$$\begin{aligned} kx + y + z &= 1 \\ x + ky + z &= 1 \\ x + y + kz &= 1. \end{aligned}$$

(c) Prove that all eigenvalues of a Hermitian matrix are real. 3

4.(a) Define similar matrices. Prove that if two matrices $A$ and $B$ are similar then they have the same eigenvalues. 2.75

(b) Let $A = \begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix}$. Find a nonsingular matrix $P$ such that $P^{-1}AP$ is diagonal. 3

(c) If $X_1$ and $X_2$ are eigenvectors of a matrix belonging to distinct eigenvalues $\lambda_1$ and $\lambda_2$ then prove that $X_1$ and $X_2$ are linearly independent. 3

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## Section-B

5.(a) Find the differential equations of all circles which have their centres on $x$ axis and have a given radius. 3

(b) Solve the differential equation: $\frac{dy}{dx} = x^3y^3 - xy$. 3

(c) Prove that the differential equation $(\sin x \cos y + e^{2x})dx + (\cos x \sin y + \tan y)\, dy = 0$ is exact and solve it. 2.75

6. Solve the following differential equations:

$D = \frac{dy}{dx}$

(a) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = \sin 2x$. 3

(b) $(D^3 - 7D - 6)y = e^{2x}\, x^2$. 3

(c) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$. 2.75

7.(a) Solve the differential equation by the method of variation of parameters: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^x$. 4

(b) Solve the differential equation by the method of operator factorization: $[xD^2 + (1-x)D - 2(1+x)]y = e^{-x}\,(1-6x)$. 4.75

8.(a) Solve the partial differential equations by Charpit's method $z^2 = pqxy$. 4

(b) Solve the partial differential equation $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$ by Lagrange's method and hence find its integral surface containing the straight line $x + y = 0,\ z = 1$. 4.75

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# University of Rajshahi
## Department of Computer Science and Engineering
## B.Sc.(Engg.) Part-2 (Odd Semester) Examination-2017
**Course: MATH 2111 (Matrix and Differential Equation)**

Time: 3 Hours Marks: 52.5

(Answer any **Six** of the following questions taking **three** from each section.)

**Section-A**

1.(a) Define Horizontal matrix, Sub matrix, unit matrix and diagonal matrix with example. 2.75

(b) Define periodic matrix and idempotent. If A is an idempotent matrix, show that the matrix B = I − A is also idempotent and AB = 0 = BA. 3

(c) Define nilpotent matrix and orthogonal matrix with example. If A and B be two orthogonal matrices of same order, show that AB is also orthogonal. 3

2.(a) Define transposed matrix, symmetric matrix and skew-symmetric matrix. If A is any square matrix, show that $A + A'$ is symmetric matrix. 3

(b) Define rank of a matrix. Reduce the matrix 3

$$\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

to normal form and find its rank.

(c) If $A$ and $B$ are two $n \times n$ matrices, show that $Adj(AB) = Adj\ B\ .Adj\ A$. 2.75

3.(a) Define inverse of a matrix. Find the inverse of 3

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}.$$

(b) Find the solution of the system of equations 2.75

$$x + 2y - 3z = 1$$
$$2x + 5y - 8z = 4$$
$$3x + 8y + 3z = 7$$

using matrix method.

(c) Determine the value of $a$ so that the following system in unknown $x, y$ and $z$ has *(i)* no solution, *(ii)* more than one solution *(iii)* a unique solution: 3

$$x + y - z = 1, \qquad 2x + 3y + az = 3 \quad and \quad x + ay + 3z = 2$$

4.(a) Show that every matrix is zero of its characteristics polynomial. 4

(b) Define eigenvalue and eigenvector. Find all eigenvalues and the corresponding eigenvectors of the matrix 4.75

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

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## Section-B

5.(a) Define order and degree of differential equation. Find the order and degree of the differential equation 2.75

$$2\frac{d^3y}{dx^3} + 3\left(\frac{d^2y}{dx^2}\right)^4 + \frac{dy}{dx} + y = \sin 4x$$

(b) Define homogeneous ODE. Solve the ODE $(2xy + 3y^2)dx - (2xy + x^2)dy = 0$ 3

(c) Define exact differential equation. Solve $3x(xy - 2) + (x^3 + 2y)dy = 0$ 3

6.(a) Define Bernoulli's equation. Solve $\frac{dy}{dx} + \frac{y}{x}logy = \frac{y}{x^2}(logy)^2$ 2.75

(b) Solve *(i)* $y + px = p^2x^4$ *(ii)* $y = 2px + y^2p^3$ where $p = \frac{dy}{dx}$ 3+3

7.(a) Solve *(i)* $(D^3 - D^2 - 6D)y = 1 + x^2$ *(ii)* $(D^2 + 4)y = cosx$ 3+3

(b) Using variation of parameters to find the general solution $4y'' - 4y' - 8y = 8e^{-t}$ 2.75

8.(a) Define regular singular point. Find the general solution of $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 + 2)y = 0$ in power of x about $x_0 = 0$ 5.75

(b) Define Laplace transformation. Fine the solution of $f''(t) + 3f'(t) + 2f(t) = 4t$ where $f(0) = f'(0) = 0$ using Laplace transformation. 3

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## University of Rajshahi

**Department of Computer Science and Engineering**

B. Sc. (Engg.) Part-2 Odd Semester Examination-2016

Course: MATH2111 (Matrix and Differential Equation)

Full Marks: 52.5 Duration: 3(Three) Hours

**Answer 06(Six) questions taking any 03(Three) questions from each part.**

### Part-A

1. (a) Let A and B be matrices of order $m \times n$ and $n \times p$ respectively. Prove that $(AB)' = B'A'$. 3
   (b) Define symmetric matrix and skew symmetric matrix. Give an example of each kind. Prove that the diagonal elements of a skew symmetric matrix are all zero. 3
   (c) Let A and B be $n$-square non-singular matrices. Prove that AB is non-singular and $(AB)^{-1} = B^{-1}A^{-1}$. 2.75

2. (a) For what value of $\lambda$, the system of equations fail to have a solution: 3
$$3x - y + \lambda z = 1$$
$$2x + y + z = 2$$
$$x + 2y - \lambda z = -1$$
   (b) State Cayley-Hamilton theorem and verify it for the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$. 3
   (c) Define linear dependence and linear independence of a set of vectors. Determine whether or not the vectors [1,2,3], [2,3,4], [3,5,7] are linearly dependent. 2.75

3. (a) Apply Cramer's rule to solve the equations: 4
$$x + y + z = 1,\ x + 2y + z = 2,\ x + y + 2z = 0$$
   (b) Determine the values of $a$ and $b$ so that the system of equations 4.75
$x + 2y + z = 1,\ 3x + y + 2z = b,\ ax - y + 4z = b^2$ has
(i) a unique solution, (ii) no solution and (iii) many solutions.

4. (a) For any square matrix A, define $adj(\overset{A}{\cancel{B}})$. Prove that $A\,adj(A) = |A|I$. 3
   (b) Find the adjoint and inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$. 3
   (c) Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ to echelon form and find its rank. 2.75

### Part-B

5. (a) Define *degree* and *order*. Find the differential equation of the family of circles 3
$$x^2 + y^2 + 2gx + 2fy + c = 0.$$
   (b) Identify and solve $(x^2 + y^2)dx - 2xydy = 0$. 3
   (c) Solve by the variation of parameters: $y'' + y = cos^2(x)$ 2.75

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6. Solve the following differential equations: 2.75

(a) $$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = xe^{-x}.$$

3

(b) $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos(x).$$

3

(c) $$\frac{d^2y}{dx^2} + a^2y = x\cos(ax).$$

8.75

7. Define regular singular points. Find the general solution of $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ by series method. Test the convergency of the series.

8. (a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $u(x,0) = 3\sin(2\pi x)$, $u(0,t) = 0$, $u(1,t) = 0$ where $0 < x < 1,\ t > 0$ by Laplace transformation. 4.75

(b) Write down Helmholtz's equation and solve it. 4

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**University of Rajshahi**

**Department of Computer Science & Engineering**

B.Sc. (Engg.) Part-II Odd Semester Examination 2015

Course: MATH-2111 (Matrix and Differential Equation)

Full Marks: 52.5 Duration: 3(Three) Hours

**Answer 6 (Six) questions taking any 3(Three) from each part**

## Part-A

1. a) Define matrix multiplication. Prove that matrix multiplication is associative. 3
   b) Prove that every square matrix can be expressed uniquely as a sum of a symmetric and a skew symmetric matrix. 3
   c) If A and B are $n$-square matrices then prove that A and B commute if and only if A-kI and B-kI commute for scalar k. 2.75

2. a) For any square matrix $A$ and $B$, prove that $adj(AB)=adj(B)adj(A)$. 3
   b) Find the inverse of $A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$, using elementary row operations. 3
   c) Define linear dependence and independence of a set of vectors. Determine whether or not the following vectors are linearly dependent $x_1=[1, 2, -3, 4]$, $x_2=[3, -1, 2, 1]$, $x_3=[1, -5, 8, -7]$ 2.75

3. a) Determine the value of $k$ such that the system in unknowns $x, y, z$ has (i) a unique solution (ii) no solution, (iii) more than one solution 3
$$x + y + kz = 2$$
$$3x + 4y + 2z = k$$
$$2x + 3y - z = 1$$
   b) Reduce the matrix $A=\begin{pmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{pmatrix}$ into echelon form and determine its rank. 2.75
   c) Define an eigenvalue and associated vector of a square matrix. Find eigenvalues and associated eigenvectors of the matrix $A=\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$. 3

4. a) If $x_1$ and $x_2$ are eigenvectors of a matrix $A$ belonging to $m$ eigenvalue $\lambda$ then prove that any linear combination $c_1x_1+c_2x_2$ of $x_1$ and $x_2$ is also an eigenvector of $A$ belonging to the eigenvalue $\lambda$ provided $c_1x_1+c_2x_2\neq0$. 3
   b) If $A$ is an $n$-square matrix. Prove that A and A′ have the same eigenvalues. 2.75
   c) Let $A=\begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix}$. Find a nonsingular matrix P such that $P^{-1}AP$ is diagonal. 3

## Part-B

5. a) Define variable separable equation. Find an explicit solution of the initial value problem $x^2\frac{dy}{dx} = y - xy, y(-1) = -1$ by separating variabless. 3
   b) Find the general solution of the differential equation $y' + 3x^2y = x^2$. Give the largest interval over which the solution is defined. Is there any transient term in the general solution? 3
   c) Solve the equation by using an appropriate substitution $x\frac{dy}{dx} + y = \frac{1}{y^2}$. 2.75

6. Solve the following differential equations:
   a) $y'' + 4y' + 4y = 2x + 6$ 3
   b) $y'' - y = x^2e^x + 5$ 3
   c) $y'' - y' - 12y = e^{4x}$ 2.75

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## PART-B

5.(a) Define an idempotent matrix. Prove that the matrix 3

$$A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$ is idempotent.

(b) Define a Skew-Hermitian matrix. Prove that every complex square matrix can be uniquely expressed as a sum of a Hermitian and a Skew-Hermitian matrices. 3

(c) Prove that (AB)'=B'A' 2.75

6.(a) Prove that $A^{-1} = \frac{1}{|A|}(adjA)$ 3

(b) Define rank of matrix. Find the rank of $A = \begin{bmatrix} 1 & 5 & 9 \\ 4 & 8 & 12 \\ 7 & 11 & 15 \end{bmatrix}$ 2.75

(c) Find the inverse of $A = \begin{bmatrix} 2 & -4 & -2 \\ 4 & 6 & 2 \\ 0 & 10 & -4 \end{bmatrix}$ 3

7.(a) Reduce $A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$ to the normal form. 3

(b) For what values of λ the equations 3

$x+y+z=1,\ x+2y+4z=\lambda\ and\ x+4y+10z=\lambda^2$ has a solution and solve them completely.

(c) Solve the systems of equations 2.75

$x+y+z=6,\ x-y+z=2\ and\ 2x+y-z=1$ by Krammer's rule/matrix method.

8.(a) Determine the eigen values and corresponding eigen vectors of the matrix 4.75

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

(b) State and Prove Cayley-Hamilton theorem. 4

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