

6. (a) Distinguish between parametric and non-parametric statistical tests. Discuss the advantages and disadvantages of non-parametric test. 2.5
- (b) Derive sign test, stating clearly the assumptions made for small sample case. 3.5
- (c) Use the sign test to see whether there is a difference between the number of days required to collect an account receivable before and after a new collection policy. Use the 0.05 significance level. 2.75

Before: 33 36 41 32 39 47 34 29 32 34 40 42

After: 35 29 38 34 37 47 36 32 30 34 41 38

6. (c) Use sign test to see whether there is a difference between number of days required to collect an account receivable before and after a new collection policy. Use the 0.05 significance level. Before: 33 36 41 32 39 47 34 29 32 34 40 42 35 29 38 34 37 47 36 32 30 34 41 38

Sol<sup>n</sup>:

No. 1 to 12	1	2	3	4	5	6	7	8	9	10	11	12
Before	33	36	41	32	39	47	34	29	32	34	40	42
After	35	29	38	34	37	47	36	32	30	34	41	38
Sign	+	-	-	+	-	0	+	+	-	0	+	-

There are 5 + signs and 5 - signs.

$$\therefore n = 5 + 5 = 10.$$

$H_0$ : ~~Diff~~ No significant difference bet<sup>n</sup> no. of days

$H_1$ : Significant difference bet<sup>n</sup> no. of days.

At  $\alpha = 0.05$  (one-tailed) and  $n = 10$ , the critical value is 0.

The test statistic  $x$  is the smaller number of + sign or - sign, so,  $x = 4$ .

4 is greater than critical value, so we fail to reject  $H_0$ .

$\therefore$  There is not enough evidence at 5% level to ~~claim~~ support the claim of having significant difference.



4. a) What do you mean by statistical hypothesis? Distinguish between simple and composite hypothesis. Let a random sample of size  $n$  is drawn from a normal population with mean  $\mu$  and known variance  $\sigma^2$ . How would you test the hypothesis that mean is equal to  $\mu_0$ ? 1+1.75  
+3
- b) The average IQ of university female students in Bangladesh is suspected to be more than the average 110 for all students. A random sample of 64 female students yielded a sample average IQ of 115.5 and standard deviation of 20. Can you conclude that the average score of the female students is really more than 110? [ $Z_{0.05}=1.64$ ] 3

2016 4b) The average IQ of university female students in Bangladesh is suspected to be more than the average 110 for all students. A random sample of 64 female students yielded a sample of average IQ of 115.5 and standard deviation of 20. Can you conclude that the average score of female students is really more than 110? [ $Z_{0.05}=1.64$ ]

Solution: Consider,  $H_0: \mu = 110$  vs  $H_a: \mu > 110$

$$H_a: \mu > 110$$

$$|Z| = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{115.5 - 110}{20/\sqrt{64}}$$

$$= 2.2$$

Here

$$n = 64$$

$$\bar{X} = 115.5$$

$$\sigma = 20$$

$$\mu = 110$$

$\therefore$  Calculated value,  $z_{cal} = 2.2$

$\therefore$  Tabulated value,  $z_{tab} = 1.64$  at 5% level of significance

$\therefore z_{cal} > z_{tab}$  i.e. Reject  $H_0$ .

So, it can be concluded that the average IQ of university female students in Bangladesh is not more than the average value 110 at 5% level of significance.



5. a) Define  $r \times c$  contingency table. Show that in case of  $2 \times 2$  contingency table, the test statistics becomes  $\chi^2 = \frac{N(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$ , also mention Yate's correction for continuity. 1+3  
+1.75  
3
- b) In a psychological test, 70 out of 100 boys came out successful while 60 out of 100 girls of the same age group as the boys passed the test. Do the data provide any evidence of difference in respect of abilities between the genders?

2016 5 (b) In a psychological test, 70 out of 100 boys came out successful while 60 out of 100 girls of the same age group as the boys passed the test. Do the data provide any evidence of difference in respect of abilities between the gender.

Solution: The Observed data,

We consider,

~~$H_0$~~  Relation

$H_0$ : Difference in respect to gender

$H_1$ : No difference in respect to gender.

Estimated data.

	Male	Female	
Pass	65	65	130
Fail	35	35	70
	100	100	200

	Male	Female	
Pass	70	60	130
Fail	30	40	70
	100	100	200

$$E_{ij} = \frac{R_i C_j}{N}$$

$$E_{11} = \frac{100 \times 130}{200} = 65$$

$$E_{12} = \frac{100 \times 70}{200} = 35$$

$$E_{21} = \frac{100 \times 65}{200} = 32.5$$

$$E_{22} = \frac{100 \times 35}{200} = 17.5$$

$$\begin{aligned} \chi^2 &= \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \\ &= \frac{(70-65)^2}{65} + \frac{(60-65)^2}{65} + \frac{(30-35)^2}{35} + \frac{(40-35)^2}{35} \\ &= 2.197 \end{aligned}$$

$$\text{So, } \chi^2_{\text{cal}} = 2.197 < \chi^2_{\text{tab}}(0.05) = 3.84$$

$\therefore$  We fail to reject the Null hypothesis,  $H_0$ .

So, there is Evidence of difference in respect of abilities between the gender.



6. a) What do you mean by non-parametric test? Discuss its importance. Describe the testing procedure of the run test. 1+1.75  
+3  
3
- b) The following sequence is purported to be a set of random integers from 0 to 99. Use the run's test to test the hypothesis of the randomness at  $\alpha=0.05$  significance level. The sequence is

28, 4, 23, 98, 44, 10, 6, 25, 54, 81, 12, 6, 4, 33, 67, 55, 71, 66, 22, 18, 49, 85

2016: 6 ⑥ The following sequence is purported to be a set of random integers from 0 to 99. Use the run's test to test the hypothesis of randomness at  $\alpha=0.05$  significance level. The sequence is,

28, 4, 23, 98, 44, 10, 6, 25, 54, 81, 12, 6, 4, 33, 67, 55, 71, 66, 22, 18, 49, 85.

Sol<sup>n</sup>:  $H_0$ : Sequence is Random,  $H_1$ : Sequence is not Random

The data sequenced in ascending order:

4, 6, 6, 10, 12, 18, 22, 23, 25, 28, 33, 44, 49, 54, 55, 66, 67, 71, 81, 85, 98.

$$\text{Median, } m = \frac{\left( \left( \frac{n}{2} \right) + 1 \right)^{\text{th}} \text{ term} + \frac{n}{2}^{\text{th}} \text{ term}}{2} = \frac{\left( \frac{22}{2} + 1 \right)^{\text{th}} + \left( \frac{22}{2} \right)^{\text{th}}}{2}$$

$$= \frac{12^{\text{th}} + 11^{\text{th}}}{2} = \frac{28 + 33}{2} = 30.5$$

4, 6, 6, 10, 12, 18, 22, 23, 25, 28, 33, 44, 49, 54, 55, 66, 67, 71, 81, 85, 98  
 - - - + + - - - + + - - - + + + + + - - + +  
 28, 4, 23, 98, 44, 10, 6, 25, 54, 81, 12, 6, 4, 33, 67, 55, 71, 66, 22, 18, 49, 85  
 - - - + + - - - + + - - - + + + + + - - + +

number of run,  $G = 8$

number of (-ve) sign,  $n_1 = 11$

number of (+ve) sign,  $n_2 = 11$

At  $\alpha = 0.05$ ,  $n_1 = 11$  &  $n_2 = 11$  the tabulated value, lower critical

lower critical value = 7, Higher critical value = 17

Number of Runs, = 8.  $H_0$  fail to reject

The set is random.

|         |                    |          |
|---------|--------------------|----------|
| $G < 7$ | $7 \leq G \leq 17$ | $G > 17$ |
| Reject  | Do not Reject      | Reject   |



4. (a) What do you mean by a statistical hypothesis? Describe different steps for testing statistical hypothesis. Write down the procedure to test the significance of regression coefficient. 1+2.75+1
- (b) A random sample of 10 persons is selected as follows: 5, 2, 0, 4, 16, 14, 10, 11, 6, 8. Do you think that the average schooling year of the persons in population is 5? (Tabulated value at 5% with 9 d.f. is 2.26) 4

2015. 4(b) A random sample of 10 person is selected as follows: 5, 2, 0, 4, 16, 14, 10, 11, 6, 8. Do you think that the average schooling year of the persons in population is 5? (Tabulated value at 5% with 9 d.f. is 2.26)

Solution: The standard deviation is not given. For sample value,  $n = 10$ ,

$$|t| = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{7.6 - 5}{5.16/\sqrt{10}}$$

$$= 1.59$$

Calculated value,

$$\therefore t_{cal} = 1.59$$

Tabulated value,  $t_{tab} = 2.26$  at

5% level of significance.

$\therefore$  Calculated value < tabulated value,

$\therefore$  ~~Do not~~ Accept  $H_0$  (Null hypothesis)

$\therefore$  The average schooling year of person is 5 years.

Here,

$$\bar{x} = \frac{\sum x}{n} = \frac{5+2+0+4+16+14+10+11+6+8}{10}$$

$$= 7.6$$

$$s^2 = \frac{1}{n-1} \sum (x - \bar{x})^2$$

$$= \frac{(5-7.6)^2 + (2-7.6)^2 + (0-7.6)^2 + (4-7.6)^2 + (16-7.6)^2 + (14-7.6)^2 + (10-7.6)^2 + (11-7.6)^2 + (6-7.6)^2 + (8-7.6)^2}{10-1}$$

$$= 26.71$$

$$s = 5$$

$$\therefore s = \sqrt{26.71} = 5.16$$

$H_0$ : Average schooling year of person is 5.

$H_a$ : Average schooling year of person not 5



6. (a) What is contingency table? What is form of  $\chi^2$  test statistic in case of a  $2 \times 2$  contingency table?

(b) For given information in the following table, test at level of significance 0.05 that whether level of education affects the job performance. [ $\chi^2_{0.05,4} = 13.3$ ]

4.75

| Job Performance | Level of Education |         |            |     |
|-----------------|--------------------|---------|------------|-----|
|                 | Below primary      | College | University |     |
| Excellent       | 10                 | 40      | 10         | 60  |
| Good            | 30                 | 30      | 20         | 80  |
| Fair            | 10                 | 30      | 20         | 60  |
|                 | 50                 | 100     | 50         | 200 |

2015] 6(b) For given information in following table, test at level of significance 0.05 that whether level of education affects the job performance [ $\chi^2_{0.05,4} = 13.3$ ]

Soln

$H_0$ : No affect

$H_a$ : Affect

| Job Performance | Level of education |         |            | Total |
|-----------------|--------------------|---------|------------|-------|
|                 | Below Primary      | College | University |       |
| Excellent       | 10                 | 40      | 10         | 60    |
| Good            | 30                 | 30      | 20         | 80    |
| Fair            | 10                 | 30      | 20         | 60    |
| Total           | 50                 | 100     | 50         | 200   |

Expected data :

$$E_{11} = \frac{60 \times 50}{200} = 15$$

$$E_{12} = \frac{100 \times 60}{200} = 30$$

$$E_{21} = \frac{50 \times 80}{200} = 20$$

$$E_{22} = \frac{100 \times 80}{200} = 40$$

$$E_{31} = \frac{50 \times 60}{200} = 15$$

$$E_{32} = \frac{100 \times 60}{200} = 30$$

$$\therefore \chi^2_{cal} = \sum \frac{(O-E)^2}{E} = 17.47$$

$$\text{Tabulated } \chi^2_{0.05,4} = 13.3$$

$$\chi^2_{cal} > \chi^2_{tab}$$

$\therefore$  Reject  $H_0$ .

i.e. The level of education affects job performance

| Job Performance | Level of education |         |                     | Total |
|-----------------|--------------------|---------|---------------------|-------|
|                 | Below Primary      | College | University          |       |
| Excellent       | 15                 | 30      | $\frac{60-45}{=15}$ | 60    |
| Good            | 20                 | 40      | $\frac{80-60}{=20}$ | 80    |
| Fair            | 15                 | 30      | $\frac{60-45}{=15}$ | 60    |
| Total           | 50                 | 100     | 50                  | 200   |

Calculation for CHI-Square

| O         | E   | (O-E) | (O-E) <sup>2</sup> | (O-E) <sup>2</sup> /E |
|-----------|-----|-------|--------------------|-----------------------|
| 10        | 15  | -5    | 25                 | 1.66                  |
| 40        | 30  | 10    | 100                | 3.33                  |
| 10        | 15  | -5    | 25                 | 1.66                  |
| 30        | 20  | 10    | 100                | 5                     |
| 30        | 40  | -10   | 100                | 2.5                   |
| 20        | 20  | 0     | 0                  | 0                     |
| 10        | 15  | -5    | 25                 | 1.66                  |
| 30        | 30  | 0     | 0                  | 0                     |
| 20        | 15  | 5     | 25                 | 1.66                  |
| Total 200 | 300 | 0     |                    | 17.47                 |



## Section - B

4. a) Define simple hypothesis and critical region. Let  $x_1, x_2, \dots, x_n$  and  $y_1, y_2, \dots, y_m$  be two random samples drawn from two normal populations  $N(\mu_i, \sigma^2)$ ,  $i=1,2$  respectively. If  $\sigma^2$  is unknown, how would you test null hypothesis that  $H_0: \mu_1 = \mu_2$  8.75
- b) Sample mean weight of 20 CSE students is 50kg and 10 ICE students is 45kg. if the sample variances of weights are 25 and 16, test whether the mean weights of CSE students is greater than the mean weights of ICE students. (Use  $t_{0.05,28}=1.64$ )

2014/4/2014 Sample mean weight of 20 CSE students is 50kg and 10 ICE students is 45kg. if the sample variance of weights are 25 & 16, test whether the mean weight of CSE students is greater than the mean weights of ICE students. (USE  $t_{0.05,28}=1.64$ )

Solution: Consider,  $H_0: \mu_1 = \mu_2$  i.e. no significant difference between mean  $H_a: \mu_1 > \mu_2$  Re. level  $\alpha$ .

Let,  $H_0: \mu_1 = \mu_2$  i.e. no significant difference between mean weight of CSE & ICE students.

$H_a: \mu_1 > \mu_2$  i.e. Mean weight of CSE students is greater than that of ICE students.

$$\begin{aligned}
 t_{cal} &= \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}} \\
 &= \frac{50 - 45}{\sqrt{\frac{25}{20} + \frac{16}{10}}} \\
 &= \frac{5}{\sqrt{1.25 + 1.6}} \\
 &= 1.75
 \end{aligned}$$

Here,

$$\begin{aligned}
 n_1 &= 20 \\
 n_2 &= 10 \\
 \bar{x}_1 &= 50 \\
 \bar{x}_2 &= 45 \\
 \sigma_1^2 &= 25 \\
 \sigma_2^2 &= 16
 \end{aligned}$$

$\therefore t_{cal} = 1.75 > t_{tab} = 1.64 \therefore$  Reject  $H_0$  at 5% level of significance.

$\therefore$  The mean weight of CSE students is greater than mean weight of ICE students.



6. a) Describe the procedure of Fisher's exact test for testing the independence of two binary variables. 4

b) For given information in the following table, test at level of significance 0.05 that whether class attendance affects the examination score. 4.75

| Class attendance | Examination score |              |
|------------------|-------------------|--------------|
|                  | A+                | Less than A+ |
| Less than 80%    | 3                 | 6            |
| 80% and above    | 7                 | 4            |

2014/ G6) For given information in the following table, test at level of significance 0.05 that whether class attendance affect examination score.

Sol<sup>n</sup>

$H_0$ : No affect

$H_a$ : Affect

| Class attendance | Exam score |              | Total |
|------------------|------------|--------------|-------|
|                  | A+         | less than A+ |       |
| less than 80%    | 3          | 6            | 9     |
| 80% and above    | 7          | 4            | 11    |
| Total            | 10         | 10           | 20    |

Expected values

$$E_{11} = \frac{10 \times 9}{20} = 4.5$$

$$E_{21} = \frac{10 \times 11}{20} = 5.5$$

| Class attendance | Exam Score |                 | Total |
|------------------|------------|-----------------|-------|
|                  | A+         | less than A+    |       |
| less than 80%    | 4.5        | $9 - 4.5 = 4.5$ | 9     |
| 80% & above      | 5.5        | 5.5             | 11    |
| Total            | 10         | 10              | 20    |

$$\therefore \chi^2_{cal} = \sum \left[ \frac{(O - E)^2}{E} \right]$$

$$= 1.818$$

$$\chi^2_{0.05, 1} = 3.84$$

$$\therefore \chi^2_{cal} < \chi^2_{tab}$$

| O  | E   | (O - E) | (O - E) <sup>2</sup> | (O - E) <sup>2</sup> /E |
|----|-----|---------|----------------------|-------------------------|
| 3  | 4.5 | -1.5    | 2.25                 | 0.5                     |
| 6  | 4.5 | 1.5     | 2.25                 | 0.5                     |
| 7  | 5.5 | 1.5     | 2.25                 | 0.409                   |
| 4  | 5.5 | -1.5    | 2.25                 | 0.409                   |
| 20 |     | 0       |                      | 1.818                   |

$\therefore$  Fail to reject.

$H_0$  accepted at 5% level of significance and we may conclude that there is no significant affect of class attendance over the examination score.