Numerical Methods

By

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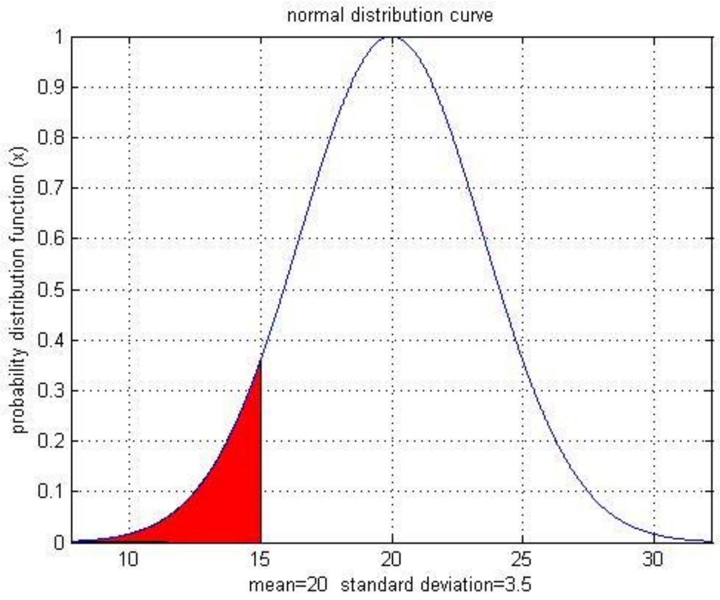
Numerical Methods

- Numerical methods are techniques by which mathematical problems are formulated so that they can be solved with arithmetic operations.
- A numerical method is a complete and unambiguous set of procedures for the solution of a problem, together with computable error estimates. The study and implementation of such methods is the province of numerical analysis.
- Although there are many kinds of numerical methods, they have one common characteristic: **they invariably involve large numbers of tedious arithmetic calculations**.

Why Numerical Methods?

- To solve problems that cannot be solved exactly

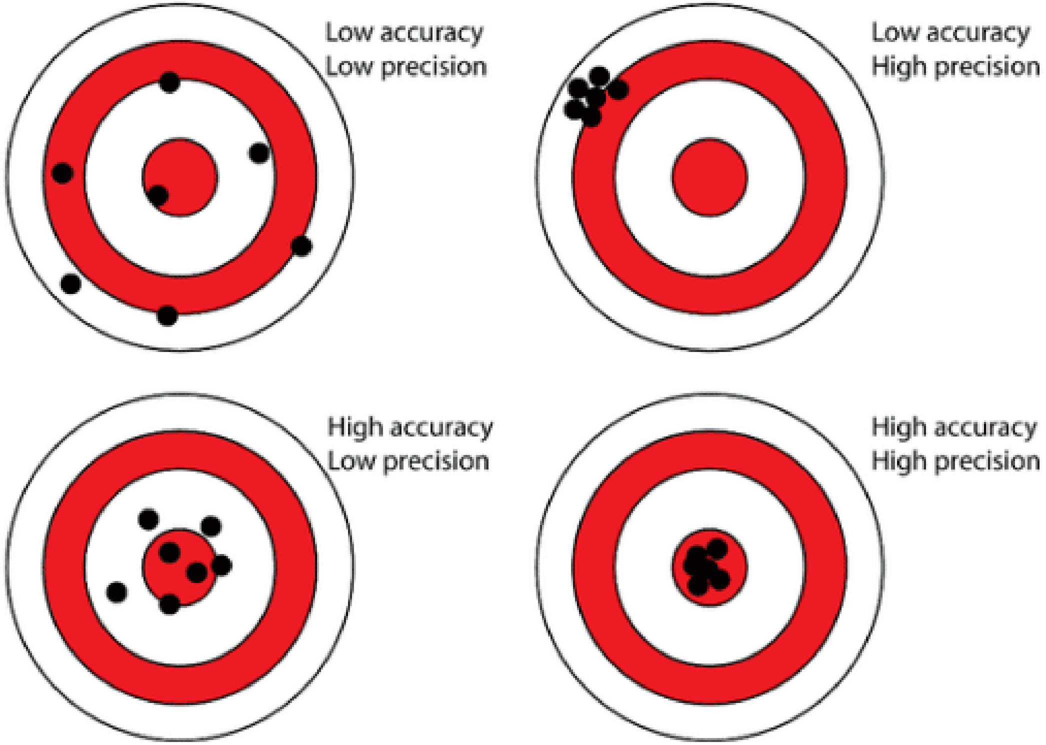
$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{u^2}{2}}\,du$$



- “Solve” problems with no analytic solution
    - Non-linear equations
    - Complex behaviors

ACCURACY vs PRECISION

- Accuracy means how close a **measured value** is to the **actual value**.
- Precision means how close the **measured values** are to each other.



ERROR

- Numerical errors arise from the use of approximations to represent exact mathematical operations and quantities.

$$\textbf{Error} = \textbf{True value} - \textbf{Approximation} \quad \textbf{(1)}$$

Why measure errors?

- To determine the accuracy of numerical results.
- To develop **stopping criteria** for iterative algorithms.

Round-off Errors

- **Round-off errors**, which result when numbers having limited significant figures are used to represent exact numbers.

- **Ex**. a = 0.43424632

the variable a can be rounded off to a significant figure 0.434. This process is called rounding off the number. Then **Round-off errors will be :**

0.43424632 - 0.434

Truncation Errors

- These include ***truncation errors***, which result when approximations are used to represent exact mathematical procedures.
- In computing applications, truncation error is the discrepancy that arises from executing a finite number of steps to approximate an infinite process.

Truncation Errors Example

- For example, the infinite series (2) adds up to exactly 1.

$$1/2 + 1/4 + 1/8 + 1/16 + 1/32 \ldots \qquad (2)$$

- However, if we truncate the series to only the first four terms, we get 1/2 + 1/4 + 1/8 + 1/16 = 15/16, producing a truncation error of 1 - 15/16, or 1/16.