University of Rajshahi
Department of Computer Science and Engineering
B.Sc. Engg. Part – II, Semester – Even, Examination 2019
Course: MATH 2241 (Linear Algebra)

Marks: 52.50 Time: 3 Hours

[Answer any Six (06) questions taking at least three (03) from each section.]

## Section-A

1.a) Define a vector space. Let $X$ be any non-empty set and $K$ be an arbitrary field. Define addition and scalar multiplication on $V$, the set of all functions $f: X \rightarrow K$, so that $V$ is a vector space. 3

b) Define subspaces of a vector space with example. Show that the intersection of any two subspaces is also a subspace. 3

c) Determine whether or not $W$ is a subspace of $\mathbb{R}^3$ if (i) $W = \{(a,b,0): a,b \in \mathbb{R}\}$. (ii) $\{(a,b,c): a^2+b^2+c^2 \leq 1\} = W$. 2.75

2.a) Define pivot position and pivot column in a matrix. 2

b) Let $u = (1,-2,-5)$, $v = (2,5,6)$, and $b = (7,4,-3)$. Determine whether $b$ can be generated as a linear combination of $u$ and $v$. 3.75

c) Let $u = (1,-2,3)$, $v = (5,-13,-3)$, and $b = (-3,8,1)$. Span$\{u,v\}$ is a plane through the origin in $R^3$. Is $b$ in that plane? 3

3.a) Given the system 2

$$x_1 + 2x_2 - x_3 = 4$$
$$-5x_2 + 3x_3 = 1.$$

Write the system as a matrix times a vector form, i.e., $Ax = b$ form.

b) Determine if the following homogeneous system has a nontrivial solution. Then describe the solution set (if any). If there is no nontrivial solution set then explain why? 4.75

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$-3x_1 - 2x_2 + 4x_3 = 0$$
$$6x_1 + x_2 - 4x_3 = 0.$$

c) Let $u_1 = (1,2,3)$, $u_2 = (4,5,6)$, and $u_3 = (2,1,0)$. Determine if the set $\{u_1, u_2, u_3\}$ is linearly independent. 2

4.a) Define a linear transformation $T: R^2 \rightarrow R^2$ by 2.75

$$T(x) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix}$$

Find the images under $T$ of $u = (4,1)$, $v = (2,3)$, and $u + v = (6,4)$.

b) Define kernel and range of a linear transformation $T$ from a vector space $V$ into a vector space $W$. 2

c) Define inner product of vector space, Cauchy-Schwarz inequality, Triangle inequality, and Hilbert space. 4

## Section-B

5.a) Define vector space. Given $u$ and $v$ in a vertor space $W$, let $H = \text{Span}\{u, v\}$. Show that $H$ is a subspace of $W$. 3.75

b) Define null space. Let $A = \begin{pmatrix} 1 & -3 & -2 \\ -5 & 9 & 1 \end{pmatrix}$ be a matrix, and let $u = (5,3,-2)$. Determine if 2

$u$ belongs to the null space of $A$.

c) Let $A = \begin{pmatrix} 2 & 4 & -21 \\ -2 & -5 & 7\ 3 \\ 3 & 7 & -86 \end{pmatrix}$ 3

i) If the column space of $A$ is a subspace of $R^k$, what is $k$?

ii) If the null space of A is a subspace of $R^k$, what is $k$?

6.a) Define basis and dimension with an example. Find a basis of the subspace $U = \{(a, b, c, d): b + c + d = 0\}$ of $\mathbb{R}^4$. 2.75

b) Define matrix representation of a linear operator. Verify that $[T]_f[V]_f = [T(V)]_f$ , where $T(x, y) = (5x + y, 3x - 2y)$ and the basis $f$ is given by $\{f_1 = (1,2), f_2 = (2,3)\}$. 3

c) Define row space of a $m \times n$ matrix. Find bases for the row space, the column space, and the null space of the matrix A. 3

$$A = \begin{pmatrix} -2 & -5 & 8\ 0-17 \\ 1 & 3 & -5\ 1\ 5 \\ 3 & 11 & -197\ 1 \\ 1 & 7-13 & 5\ -3 \end{pmatrix}$$

7.a) Consider two bases $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$ for a vector space $V$, such that $b_1 = 4c_1 + c_2$ and $b_2 = -6c_1 + c_2$. 3

Suppose $x = 3b_1 + b_2$, so we have $[x]_B = (3,1)$. Find $[x]_C$.

b) What do you mean by orthogonal complement of subspace $W$ of $R^n$? 2.75

Let $y = (7,6)$ and $u = (4,2)$. Find the orthogonal projection of $y$ onto $u$. Then write $y$ as the sum of two orthogonal vectors, one in Span $\{u\}$ and one orthogonal to $u$.

c) The set $S = \{u_1, u_2, u_3\}$ is an orthogonal basis for $R^3$. 3

where $u_1 = (3,1,1)$, $u_2 = (-1,2,1)$, $u_3 = \left(\frac{-1}{2}, -2, \frac{7}{2}\right)$. book-358

Express the vector $y = (6,1,-8)$ as a linear combination of the vectors in $S$.

8.a) Let $A = \begin{pmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{pmatrix}$. An eigenvalue of A is 2. Find a basis for the corresponding eigenspace. 287 3

b) For any subset $W$ of a vector space $V$, define $W^\perp$. Let W be the subspace of $\mathbb{R}^5$ spanned by $u = (1, 2, 3, -1, 2)$ and $v = (2, 4, 7, 2, -1)$. Find basis of the orthogonal complement $W^\perp$, of $W$. 3

c) Define orthonormal set. Find orthonormal basis $\{u_1, u_2, u_3\}$ from the basis $\{v_1 = (1, 1, 1), v_2 = (0, 1, 1), v_3 = (0, 0, 1)\}$. 2.75