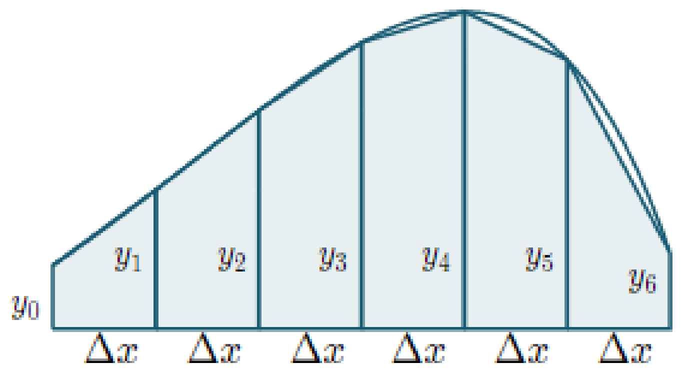
Simpson's Rule

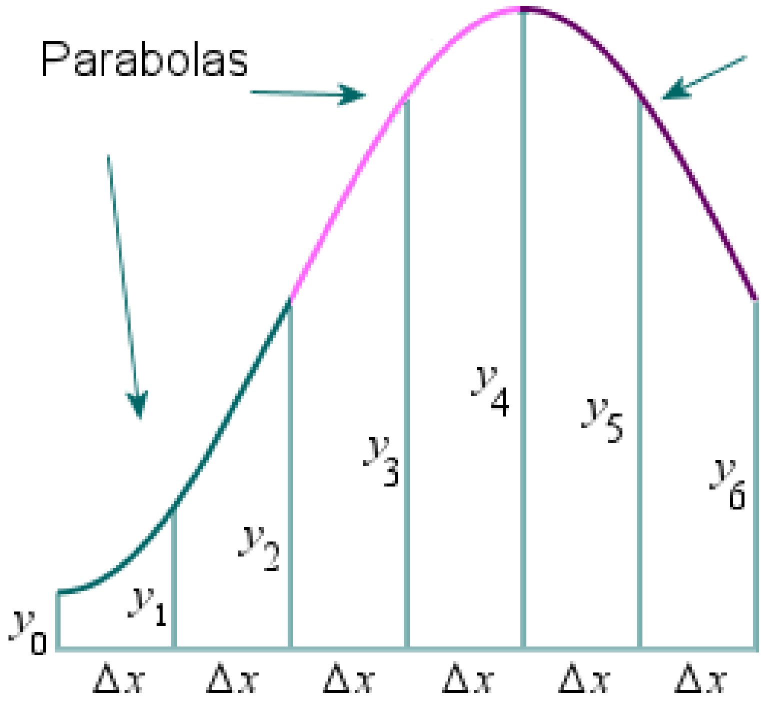
Simpson's Rule

- In Trapezoidal Rule, we used **straight lines** to model a curve and learned that it was an improvement over using rectangles for finding areas under curves because we had much less "missing" from each segment.



Area under curve using trapezoidal Rule

- We seek an even better approximation for the area under a curve.
- In **Simpson's Rule**, we will use **parabolas** to approximate each part of the curve. This proves to be very efficient since it's generally more accurate than the other numerical methods we've seen.



Simpson's 1/3 Rule

$$\int_a^b f(x)\,dx = h/3[(y_0+y_n) + 4(y_1+y_3+y_5+....+y_{n-1})+2(y_2+y_4+y_6+.....+y_{n-2})]$$

which is known as *Simpson's 1/3-rule*, or simply Simpson's rule. It should be noted that this rule requires the division of the whole range into an even number of subintervals of width *h*.

h is the interval size given by h = (b – a) / n
n is number of subintervals or interval limit

Simpson’s 3/8 Rule

$$\int_a^b f(x)\,dx = 3h/8[(y_0+y_n)+3(y_1+y_2+y_4+y_5+....+y_{n-1})+2(y_3+y_6+y_9+.....+y_{n-3})]$$

The Simpsons’s 3/8 rule was developed by Thomas Simpson. This method is used for performing numerical integrations. This method is generally used for numerical approximation of definite integrals. Here, parabolas are used to approximate each part of curve.

h is the interval size given by h = (b – a) / n
n is number of subintervals or interval limit

Example-1

Use the Simpson’s 1/3 Rule to estimate: $\int_0^1 \mathrm{e}^{\,\mathrm{x}}\, dx$

Solution

Let us divide the range (0,1) into **six equal parts** by taking h = 1/6.

When, $x_0 = 0$ then $y_0 = e^0 = 1$

Now, when;

- $x_1 = x_0 + h = 1/6$, then $\mathbf{y_1} = e^{1/6} = 1.1813$
- $x_2 = x_1 + h = 2/6 = 1/3$ then, $\mathbf{y_2} = e^{1/3} = 1.3956$
- $x_3 = x_2 + h = 3/6 = 1/2$ then $\mathbf{y_3} = e^{1/2} = 1.6487$
- $x_4 = x_3 + h = 4/6\ 2/3$ then $\mathbf{y_4} = e^{2/3} = 1.9477$
- $x_5 = x_4 + h = 5/6$ then $\mathbf{y_5} = e^{5/6} = 2.3009$
- $x_6 = x_5 + h = 6/6 = 1$ then $\mathbf{y_6} = e^1 = 2.7182$

Solution

We know by Simpson's ⅓ rule which is;

$$\int_a^b f(x)\,dx = h/3[(y_0+y_n) + 4(y_1+y_3+y_5+....+y_{n-1})+2(y_2+y_4+y_6+.....+y_{n-2})]$$

Therefore,

$\int_0^1 e^x dx = h/3\ [(y_0 + y_6\ ) +4\ (y_1 + y_3 + y_5\ ) + 2(y_2 + y_4 + y_6\ )]$

$=1/18[(1+2.718)+4(1.1813+1.6487+2.3009)+2(1.39561+1.9477)]$

$= 0.055[3.7182 + 20.52422 + 6.6866]$

$= 1.71828$

Example-2

Use Simpson’s 1/3 Rule to Evaluate log x dx within limit 4 to 5.2

Solution:

Hence the approximation of above integral 1.827 is using Simpson's 1/3 rule.

Example-3

Find Solution using Simpson's 3/8 rule

x	0	0.1	0.2	0.3	0.4
y	1	0.9975	0.99	0.9776	0.8604

Solution:

The value of table for *x* and *y*

x	0	0.1	0.2	0.3	0.4
y	1	0.9975	0.99	0.9776	0.8604

Solution:

Using Simpson's 3/8 Rule

$$= \frac{3h}{8}\left[\left(y_0 + y_4\right) + 2\left(y_3\right) + 3\left(y_1 + y_2\right)\right]$$

$$= \frac{3 \times 0.1}{8}[(1 + 0.8604) + 2 \times (0.9776) + 3 \times (0.9975 + 0.99)]$$

$$= \frac{3 \times 0.1}{8}[(1 + 0.8604) + 2 \times (0.9776) + 3 \times (1.9875)]$$

$$= 0.36668$$