MATH2231: Numerical methods

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Date :

2018 1 a) What is numerical method? Write the reason to study it.

Numerical methods are those in which the mathematical problem is reformulated so it can be solved by arithmetic operation. Numerical methods involve large numbers of tedious arithmetic calculation. These methods have gained popularity due to the advacement in efficient computational tools such as digital computers and calculators.

Reasons:

1. Powerful problem solving technique and ~~tends~~ to can be used to handle large systems of equations.
2. It enable users to intelligently use commercial software package as well as design own algorithm.
3. ~~Atu~~ Numerical methods are efficient vehicles in learning to use computers.
4. It reinforces understanding of mathematics, where it reduces higher mathematics to basic arithmetic operations

1(b) Define 'accuracy', 'precision' and 'bias'.

Accuracy: Accuracy refers to the number of significant digits in a value. It refers to how closely a computed or measured value agrees with the true value. example: the number 57.396 is accurate to five siginificant digits.

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Date :

Precision: Refers to the number of decimal position i.e, the order of magnitude of the last digit in the value. The number 57.396 has a precision of 0.001 or 10^{-3}

→ ~~To~~ Refers to how closely individual computed or measured values agree with each other.

Bias: The systematic deviation of values from the true value.

1(c) what do you mean by significant figures? Write the rules for identifing significan figurs with examples.

Ans: The ~~significant~~ digits used to express a number ~~as as~~ are called significant figures. ~~These are~~ the numbers that can be used with ~~condi~~ confidence. They corresponds to the number ~~are~~ of certain digits plus one estimated digit.

The rules to identify:

1. ~~the digits used to~~ All nonzero digits are considered as significant, e.g. 9345, 129.9 have four significant figures.

2. All ~~nonzero digits are con~~ zeros between two nonzero digits are significant e.g 10011, 120.03 have 5 significant figures.

3. Leading zeros are not significant. e.g. 0·0012, 0·13 have two significant figures.

4. Only the trailing zeros or the zeroes to the right side of the decimal point are significant. Example: 0·7000 has 4, 7·00 has 3 and 0·70 has 2 significant figures.

5. When a number ends with zeros with no decimal point, then the trailing zeros are not significant. For example: 78000 has 2 significant figures, and 780 also has 2 significant figures.

1 (d) What is difference betn algebric and transcendental equations? (2)

Algebric Equation	Transcendental equation.
1. Any number that is solution to a polynomial with rational co-efficient	The number which are not the solutions to polynomial with rational co-efficient.
2. The equations of form $f(x)=0$ where $f(x)$ is purely a polynomial in x e.g $x^6-x^4-x^3-1=0$ is called algebric equation.	If $f(x)$ involves trigometrical, arithmetic or exponential term in it, then it is called transcendental equation. ex: $xe^x-2=0$ and $x\log_{10}x-1·2=0$.

SAT SUN MON TUE WED THU FRI

Date :

Algebric	Transcendental
3. An expression of the form $P_n(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_{n-1}x + a_n$ where $a_0, a_1, \dots, a_n$ are constants, n is positive integer, is called polynomial in x of degree n provided $a_0 \neq 0$.	If a function f(x) contains trigno metric, logarithmic, exponential etc, functions, then $f(x) = 0$ is a called transcendental equation.
ex. $f(x) = x^3 - 4x - 3 = 0$.	$f(x) = 3x - \cos x - 1 = 0$

Methods.

1) Bisection Method
2) Newton Raphson / Newton iterative Method
3) Regular Falsi Method / False position

2019

~~2 a) Define Bisec~~ 1 a) - 18 1 a

1 b) Explain inherent error, round off error and truncation error (3)

Inherent error: The error which are already present in the statement of problem before its solution is obtained.

Such are ~~are~~ either due to the given data being approximated or due to limitation of mathematical measurement.

This error is also known as input error. This contains two components, namely data errors and conversion errors.

types not necessary to write

Data error: Aka emperical error arises when data for a problem are obtained by some experimental means and are therefore, of limited accuracy and precision.

Conversion error: Aka representational error aries due to limitations of computer to store the data exactly.

Numerical Errors-

Numerical error are two types-

→ aka procedural error

1. Rounding off error: Rounding errors arise from the process of rounding off the numbers during the computation.

Occurs when a fixed number of digits are used to represent exact numbers.

They are numbers with large number of digits.

$\frac{22}{7} = 3.1428$. This process of droping unwanted digits is called rounding off. Similarly, 42.7893 will be rounded off upto 2 decimal digits as 42.79,

Trancation Error:- The error caused by using approximate results or on replacing are infinite process by a finite one.

If we are using a decimal computer having a fixed word length of 4 digits, rounding off of 13.658 gives 13.66 where as tranecation gives 13.65.

i.e. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots = x$ (say) is is replaced by $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} = \bar{x}$ (say) then the truncation error is $x - \bar{x}$

1.(c) How numbers are rounded-off? Give the ~~rou~~ rule. (2)

(a). Two general rule-

(a) If the number of rounding is followed by 5, 6, 7, 8 or 9 round the number up. Example 38 rounded to the nearest ten is 40.

(b) If the number of rounding is followed by 0, 1, 2, 3, or 4 round the number down.
Example: 33 rounded to nearest ten is 30

(9)

1(d) If $\frac{2}{3}$ is approximated to four significant (2) digits, find absolute, relative and percentage error.

$$x = \frac{2}{3},\ \bar{x} = 0.6667$$

The value of $\frac{2}{3} = 0.666666$ but it gets approximated to ~~for~~ four significant digits i.e 0.6667

Now, Absolute error $= |$exact value $-$ appox. value$|$

$$= \left|\frac{2}{3} - 0.6667\right|$$

$$= |0.000034|$$

$$= 6.6666 \times 10^{-5}$$

Relative error $= \dfrac{\text{Absolute error}}{\text{exact value}} = \dfrac{6.6666 \times 10^{-5}}{\frac{2}{3}}$

$$= 1 \times 10^{-4}$$

Percent of R.E = ~~Relative~~ error $\times 100\%$

$$= \left|\frac{x - \bar{x}}{x}\right| \times 100\%$$

$$= 0.01\ \%\ \text{Ans.}$$

.25
.3

2016

1.(c) How can you measure absolute, relative & percentage error? (2)

Absolute error: If x is the true value of a quantity and $\bar{x}$ is its approximate value, then absolute error is denoted by E_a.

$E_a = |\text{Exact value} - \text{Approximate value}|$

$E_a = |x - \bar{x}|$.

Relative error: The relative error is defined by

$$E_r = \left| \frac{\text{Exact value} - \text{Approximate value}}{\text{exact Value}} \right|$$

$$= \left| \frac{x - \bar{x}}{x} \right| = \frac{\text{Absolute error}}{\text{exact value}}$$

Percentage error: $E_p = \left| \frac{\text{exact value} - \text{Approx. value}}{\text{exact value}} \right| \times 100\%$

$$E_p = \left| \frac{x - \bar{x}}{x} \right| \times 100\%$$

$$= \frac{\text{absolute error}}{\text{exact value}} \times 100\%$$

15

1 (b) Why do occur numerical error? Explain different types of error?

Errors in solving an engineering or science problem can arise due to several factors. The error may be in the modeling technique. A mathematical model may be based on using assumption that are not acceptable. The error may arise from mistake in programs themselves or in the measurement of physical quantities. In application of numerical methods there are 5 types of error.

1. Absolute error: Difference between exact value & approximate value. $E_A = |x - \bar{x}|$

2. Relative error: Ratio of absolute error & exact value $= E_R = \frac{E_A}{x}$

3. Percentage error: The percentage error of a quantity is 100 times of its relative error.
$E_p = E_R \times 100\%$

4. Round off error: The difference between the calculated approximation of a number and its exact mathematical value due to rounding.
$\frac{1}{3}$ round off error $= \frac{1}{3} - 0.333333 = 0.00000033\bar{3}$.

5. Truncation error: The error made by truncating an infinite sum and approximating it by a finite sum. For instance, if we approximate the sine function by the first two non-zero term of its taylor series, as in $\sin(x) \approx x - \frac{1}{6}x^3$ for small x, the resulting error is a truncation error.

2014

1(a) Exact & Appoximate num? example.

Exact Number: A number which has a true value or real value correctly without rounding or without truncating any digits. For example, 23, 200, -56, 0, 0.73, 22/7, 3.143, 1/3, ~~sqr~~ $\sqrt{2}$ are all exact numbers.

If we say pi = 22/7 ; e = 2.7182818 . sq(2)=1.41, 1/7 = 0.143 we are considering that the right hand ~~hid~~ side number is the approximate value of its respective number on left hand side.

The ratio of circumference of circle to its diameter 3.142 is approximate number & pi is the exact value.

5.678881 is exact value It can be rounded to 4 decimals as 5.6789. Here 5.6789 is approximate value.

The approximate number is one that have uncertainty.

1(c) Round off following numbers to four significant figurs.

i) 38.46235 = 38.46

ii) 0.700029 = 0.7003

iii) 0.0022218 = 0.002222

iv) 19.235101 = 19.24

2019

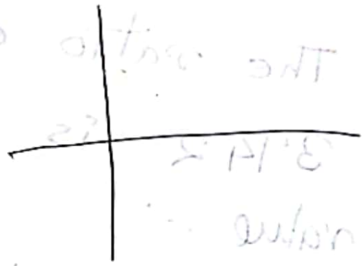
2 (a) Briefly discuss the method to obtain a root using False position method

Soln: Method for finding real root of a $\boxed{\text{nonlinear}}$ equation $f(x)=0$. ~~or~~ aka ~~falsi~~ regula-falsi or method of chords,

We choose two point's a and b such that f(a) and f(b) are ~~the~~ of opposite signs. Hence, the ~~p two~~ root must lie in between these points. Now, the equation of the chord joining the two points $[a, f(a)]$ and $[b, f(b)]$ given by

$$\frac{y - f(a)}{x - a} = \frac{f(b) - f(a)}{b - a} \quad \text{--- } (1)$$

On x-axis $y = 0$. then we get,



$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} \quad \text{--- } (2)$$

The steps:

Step 1: Find points x_0 and x_1 such that $x_0 < x_1$ and $f(x_0) \cdot f(x_1) < 0$.

Step 2: Take the interval $[x_0, x_1]$ and find next value, $x_2 = x_0 \cdot f(x_0) \frac{x_1 \cdot x_0}{f(x_1) \cdot f(x_0)}$

or,

$$x_2 = \frac{x_0 f(x_1) - x_2 f(x_0)}{f(x_1) - f(x_0)}$$

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Date :

Step 3: If $f(x_2)=0$ then x_2 is an exact root,

else if $f(x_0)\cdot f(x_2)<0$ then $x_1=x_2$

else if $f(x_2)\cdot f(x_1)<0$ then $x_0=x_2$

Step 4: Repeat step 2 and 3 untill $f(x_i)=0$ or $|f(x_i)|\le$ Accuracy.

1.6180

-0.6180

0

19 2(b) Use iterative method to find, correct to four significant figures, a real root of the equation $1+x^2=x^3$. 3.75

$\frac{d}{dx}(x^2+1)^{\frac{1}{3}}$

$\frac{1}{3}(x^2+1)^{\frac{1}{3}-1} 2x$

Soln: Given, $x^3-x^2-1=0$

$f(0)=-1$ $f(1)=-1$ (-ve) $f(2)=3$ (+ve)

~~f(1) & f(2) both roots lies close~~ Here $f(1)=-1<0$

and $f(2)=3>0$.

$\therefore$ Roots lies between 1 & 2 $\quad x_0=\frac{1+2}{2}=1.5$

$x^3=x^2+1 \Rightarrow x_{i+1}=\sqrt[3]{x_i^2+1}=\phi(x)$

$\phi'(x)=\frac{1}{3}(x+1)^{-\frac{2}{3}}\cdot 2x$ $\quad \phi'(1.5)=\left|\frac{1}{3}(1.5^2+1)^{-\frac{2}{3}}\cdot 2x\right|=|0.1809|<1$

~~$x_1=\frac{1}{3}(x_0+1)^{-\frac{2}{3}}=0.0180$~~

$\frac{d}{dx}\left(\frac{1}{x}\right)$

$\frac{d}{dx}\left(\frac{x}{1}\right)$

$x_1=\sqrt[3]{x_0^2+1}=1.4812$ $\quad x_6=1.4659$

Ans

$x_2=\sqrt[3]{x_1^2+1}=1.4727$

$x_3=\sqrt[3]{x_2^2+1}=1.4688$

$x_4=\sqrt[3]{x_3^2+1}=1.4670$

$=1.4662$

$x_5=$

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2 (a) Define Bisection method. Find the real root of equation $x^3-2x-5=0$, using bisection method. 5·75

Bisection method: Let $f(x)$ be a continuous function and let a and b be real numbers such that $f(a)$ and $f(b)$ has opposite signs. Then there is a x^* in interval $[a,b]$ such that $F(x^*)=0$.

Then $c=\frac{(a+b)}{2}$ is an approximate solution with maximum possible error $\frac{b-a}{2}$.

If $f(c)$ and $f(a)$ are opposite sign then solution x^* is in the interval $[a,c]$. Then $d=\frac{c+a}{2}$ is approximate solⁿ but with max possible error $\frac{b-a}{4}$. Continuing this process n times we can reduce the max possible error to $\frac{b-a}{2^n}$.

Bisection method. Algorithm

1. Let a & b be ~~uppo~~ lower & upper limit respectively. such that $f(a) * f(b) < 0$.
2. Let $c=(a+b)/2$
3. If $f(a) * f(c) < 0$ the $b=c$ else $a=c$
4. If more accuracy is required go to step 2
5. Print the approximate solⁿ $(a+b)/2$

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2(b) Explain the advantages & disadvantage of bisection method

Given, $x^3 - 2x - 5 = 0$

x	0	1	2	3
$f(x)$	-5	-6	-1	16

$\therefore a = 2$
$b = 3$

2·09.

a	b	f(a)	f(b)	$c = \frac{a+b}{2}$	f(c)	decision
2	3	-1	3	2·5	5·625	b = c
2	2·5	-1	2·5	2·25	1·89	b = c
2	2·25	-1	1·89	2·125	0·346	b = c
2	2·125	-1	0·346	2·06	-0·378	a = c
2·06	2·125	-0·378	0·346	2·09	-0·05	a = c
2·09	2·125	-0·05	0·346	2·1075	0·145	b = c
2·09	2·107	-0·05	0·145	2·09	-0·05	b = c
2·09	2·09	-0·05	-0·05	2·09	-0·05	

$\therefore$ We approach the root at 2·09 Ans.

Advantages:

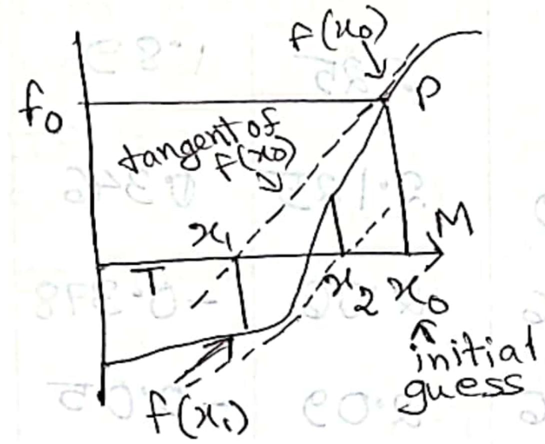
1. Simple and easy to implement.
2. One function evaluation per iteration.
3. The size of the interval containing the zero is reduced by 50% after each iteration
4. The number of iteration can be determined a priori
5. No knowledge of derivative needed
6. The function does not have to be differentiable

Disadvantage

1. Slow to coverage
2. Good intermediate approximation may be discarded.
3. We need two initial guesses a & b which bracket the root.
4. It is among the slowest method to find root.
5. When there are more than 1 root in an interval the bisection method can find only one of them.

2(a) Explain geometrical interpretation of Newton-Raphson method. (3)

Newton Raphson method: If x is the real root & x_0 is an initial approximation of the real root of an equation $f(x)=0$, $f'(x) \neq 0$. And $f(x)$ has the same sign between x_0 & x. Then, the tangent at $f(x_0)$ can lead to the real root x



Here,

The slope at x_1 is $\tan(PTM)$

$$\tan(PTM) = \frac{PM}{TM}$$

$$\tan(PTM) = \frac{f(x_0)}{h}$$

Again $\tan(PTM) = f'(x_0)$

$$\therefore f'(x_0) = \frac{f(x_0)}{h}$$

$$\Rightarrow h = \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = x_0 - h$$

$$= x_0 - \frac{f(x_0)}{f'(x_0)}$$

Similarly,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

~~[illegible]~~। এইটা ~~geo~~ geomatrical representation -
4 sheets এর accurate টা লিখেছি। এ টাও হয়।
এ টাও হয়

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)} \quad \begin{cases} f(a) * f(c) < 0 & b = c \\ f(b) * f(c) < 0 & a = c \end{cases}$$

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2 (c) Find a real root of equation $x^3 - x^2 - 1 = 0$ correct to 3 decimal places. by false position method. 2.75

1.465

⇒ The method consists in replacing the part of the curve betn the points $(x_a, f(x_a))$ and $(x_b, f(x_b))$.

The equation of chord joining the two points $(x_a, f(x_a))$ and $(x_b, f(x_b))$ is $x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

Given, $x^3 - x^2 - 1 = f(x)$

x	0	1	2
$f(x)$	-1	-1	3

Our root lies betn 1 & 2

1st iteration $f(1) = -1 < 0$ & $f(2) = 3 > 0$

$a = 1, b = 2$, $c = \frac{af(b) - bf(a)}{f(b) - f(a)}$

$= \frac{1.3 - 2(-1)}{3 - (-1)} = 1.25$

n	a	b	$f(a)$	$f(b)$	$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$	$f(c)$	Update
1	1	2	-1	3	1.250	-0.609	$a = c$
2	1.25	2	-0.609	3	0.340	-1.076	$a = c$
3	0.34	2	-1.076	3	0.778	-1.134	$a = c$
4	0.778	2	-1.134	3	1.113	-0.860	$a = c$
5	1.113	2	-0.860	3	1.311	-.466	$a = c$
6	1.311	2	-.466	3	1.404	-0.205	$a = c$
7	1.404	2	-0.205	3	1.442	-0.081	$a = c$
8	1.442	2	-0.081	3	1.457	-0.031	$a = c$
9	1.457	2	-0.031	3	1.463	-0.010	$a = c$
10	1.463	2	-0.010	3	1.465	-0.002	$a = c$
11	1.465	2	-0.002	3	1.465	-0.0007	

∴ The required Root = 1.465 Ans

2016

2(a) Describe the bisection method for finding root of equation $f(x)=0$ with merits & demerits.

For continuous equation of one variable $f(x)=0$

Step-1: Choose the lower guess x_l & upper limit x_u that bracket the root such that the function has opposite sign over interval $x_l \le x \le x_u$

Step-2: The estimation root, x_r is computed by using $x_r = \frac{x_l + x_u}{2}$, $c = \frac{a+b}{2}$

Step-3: Use the following evaluation to identify the subinterval that the root lies.

(a) If $f(x_l) \cdot f(x_r) < 0$ then root lies in lower subinterval (a, c)

Set $x_u = x_r$ $(b=c)$ & repeat step 2

(b) If $f(x_l) \cdot f(x_r) > 0$ then root lies in upper subinterval

Set $x_l = x_r$ $(a=c)$ & repeat step 2.

(c) If $f(x_l) \cdot f(x_r) = 0$ then root is equal to $x_r \cdot c$. Terminate the computation. (a, c)

Step-4: Calculate approximate ~~error~~ percent relative error.

$$\varepsilon_a = \left| \frac{x_r\,\text{present} - x_r\,\text{previous}}{x_r\,\text{present}} \right| \times 100\%$$

Step-5: Compare with if $\varepsilon_a < \varepsilon_s$ then stop computation. Otherwise go to step 2 & repeat the process by using the new interval.

16

2 (b) Describe Secant method for finding root of equation $f(x)=0$. 2.25

For a continuous function, $f(x)=0$.

Step-1: Choose initial value x_0 and x_1. Find $f(x_0)$ and $f(x_1)$.

Step-2: Compute the next estimate x_{i+1} by using secant method formula. $x_{i+1} = x_i - \left[\frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}\right]$

$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$

Step-3: Calculate the approximate percent relative error, ε_a. $\varepsilon_a = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \times 100\%$

Step-4: Compare ε_s with ε_a. If $\varepsilon_a < \varepsilon_s$ the computation is stopped. Otherwise repeat step 2.

No specific system to select x_0 & x_1

~~$3x^3 - x - 1 = 0$~~ $x_0 = -1$, $x_1 = 1$ $f(x) = e^{-x} - x$

i	$x_{i-1}(x_0)$	$x_i(x_1)$	$x_{i+1}(x_2)$	$f(x_0)$	$f(x_1)$	$f(x_2)$
1	0	1	0.6127	1	-0.6321	-0.0708
2	1	0.6127	0.5638	-0.632	-0.0708	0.00518
3	0.6127	0.5638	0.5671	-0.0708	0.00518	0.00024
4	0.5638	0.5671	0.5670	0.00518	0.00024	

$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$

$x_{i+1} = x_i - \left[\frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}\right]$

$x_2 = x_1 - \left[\frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}\right]$ 20

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CamScanner

2015

2(b) Find a real root of equation $x^3-2x-5=0$ correct to 4 decimal places by false position method.

2·09

Given, $x^3-2x-5=0=f(x)$

x	0	1	2	3
$f(x)$	−5	−6	−1	16

$\therefore f(2)=-1$ & $f(3)=16$

The root lies between 2 & 3.

$\therefore a=2,\ b=3$.

The formula for finding next values $c=\dfrac{af(b)-bf(a)}{f(b)-f(a)}$

if $f(a)*f(c)<0$ $b=c$ else $a=c$

nr	a	f(a)	b	f(b)	$c=\frac{af(b)-bf(a)}{f(b)-f(a)}$	f(c)	update
1	2	−1	3	16	2·0588	−·3984	a=c
2	2·0588	−·3984	3	16	2·0813	−·1472	a=c
3	2·0813	−·1472	3	16	2·0896	−0·0547	a=c
4	2·0896	−·0547	3	16	2·0927	−0·0202	a=c
5	2·0927	−0·0202	3	16	2·0939	−0·0075	a=c
6	2·939	−0·0075	3	16	2·0943	−0·0027	a=c
7	2·0943	−0·0027	3	16	2·0945	−0·0001	a=c
8	2·0945	−0·001	3	16	2·0945	−0·0004	

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2(a) Describe false position & Newton-Raphson method for finding root of an equation.

Newton-Raphson method

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1. Let x_0 be approximate root of $f(x)=0$ and
2. Let x_1 " correct root such that $x_1 = x_0 + h$ & $f(x_1)=0$
3. We have $h = -\frac{f(x_0)}{f'(x_0)}$ Negleting higher order derivatives of taylors series.
4. The successive approximation $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

example: $f(x)=x^3-2x-5$

$\frac{d}{dx}f(x) = 3x^2-2$

$f'(x) = 3x^2-2$

x	0	1	2	3
$f(x)$	-5	-6	-1	16

n	x_0	$f(x_0)$	$f'(x_0)$	x_1	update
1	2.5	5.625	16.75	2.1642	$x_0 = x_1$
2	2.1642	0.8079	12.051	2.0971	$x_0 = x_1$
3	2.0971	0.0289	11.1939	2.0946	$x_0 = x_1$
4	2.0946	0	11.1615	2.0946	$x_0 = x_1$

$f(2) = -1<0$ & $f(3)=16>0$

root lies betn 2 & 3

$x_0 = \frac{2+3}{2} = 2.5$

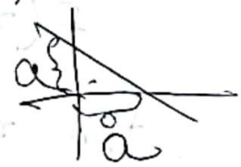
1st iteration

$f(x_0) = f(2.5) = 5.625$

$f'(x_0) = f'(2.5) = 16.75$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.5 - \frac{5.625}{16.75}$

$= 2.1642$



$\frac{x}{a} + \frac{y}{b} = 1$

CT.

Describe false position method & find the root of $f(x)=0$.

The false ~~b~~ position method requires two initial guesses x_a and x_b, such that $f(x)=0$ and $f_a(x_a)$ & $f(x_b)$ has opposite signs.

a = Lower limit

b = Upper limit

Let, $f(x)=0$ be given eq^n

Suppose $f(0) = -ve$

$f(1) = -ve$

$f(2) = -ve$

$f(3) = +ve$

$\because f(2) = -ve$ and $f(3) = +ve$, the root lies between 2 and 3.

Let $x_0 = 2$ and $x_1 = 3$

$\therefore f(x_0) = -ve$ and $f(x_1) = +ve$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

Suppose, $x_2 = +ve$ then,

$f(x_0) = -ve$ & $f(x_2) = +ve$

the root lies bet^n 2 & x_2

$$x_3 = \frac{2 f(x_0) - x_0 f(2)}{f(x_0) - f(2)}$$

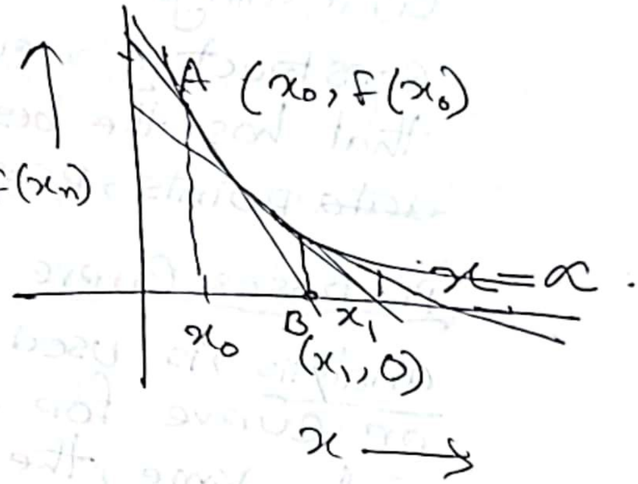
~~or~~ $x_3 = -ve$ then

$f(x_3) = -ve$ & $f(x_2) = +ve$

$$x_4 = \frac{x_3 f(x_2) - x_2 f(x_3)}{f(x_3) - f(x_2)}$$

and repeat till ~~reach~~ desired accuracy is achieved

Geomatrical interpetation



$f(x) = 0$

$y = f(x)$

let $y = f(\alpha) = 0$

$y_0 = f(x_0)$

$y - y_1 = \left(\frac{dy}{dx}\right)_{x_0} (x - x_0)$

$y - y_0 = f'(x_0)(x - x_0)$

$0 - y_0 = f'(x_0)(x_1 - x_0)$

$\frac{-f(x_0)}{f'(x_0)} = x_1 - x_0$

$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$\vdots$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Section - B

19. 4.a) Define Curve fitting. Explain the purpose of it.

Curve fitting: Curve fitting is the process of constructing a curve or mathematical function, that has the best fit to a series of data ~~data~~ points, possibly subject to constraints.

Purpose: Curve fitting also known as regression analysis, is used to find the "best fit" line or curve for a series of data points. Most of the time, the curve fit will produce an equation that can be used to find points anywhere along the curve.

19 4(b) Describe the least square curve fitting procedure for a straight line.

Ans Let, $y = a_0 + a_1 x$ be a straight line to be fitted to the given data (x_i, y_i). Then we have

$$S = [y_1 - (a_0 + a_1 x_1)]^2 + [y_2 - (a_0 + a_1 x_2)]^2 + \cdots + [y_n - (a_0 + a_1 x_n)]^2$$

For S to be minimum,

$$\frac{dS}{da_0} = 0 = -2[y_1 - (a_0 + a_1 x_1)] - 2[y_2 - (a_0 + a_1 x_2)] - \cdots - 2[y_n - (a_0 + a_1 x_n)]$$

and $$\frac{dS}{da_1} = 0 = -2x_1 [y_1 - (a_0 + a_1 x_1)] - 2x_2 [y_1 - (a_0 + a_1 x_2)] - \cdots - 2x_m [y_1 - (a_0 + a_1 x_m)]$$

$y = mx + c$

$y = a_1 x + a_0$

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The above equations simplify to

$$ma_0 + a_1(x_1 + x_2 + \dots + x_m) = y_1 + y_2 + \dots + y_m$$

and $a_0(x_1 + x_2 + \dots + x_m) + a_1(x_1^2 + x_2^2 + \dots + x_m^2) = x_1y_1 + x_2y_2 + \dots + x_my_m$

or more compactly to,

$$ma_0 + a_1\sum_{i=1}^{m} x_i = \sum_{i=1}^{m} y_i \quad \text{--- (1)}$$

$$a_0\sum_{i=1}^{m} x_i + a_1\sum_{i=1}^{m} x_i^2 = \sum_{i=1}^{m} x_iy_i \quad \text{--- (2)}$$

We can now easily solve a_0 and a_1,

$$A_0 = \bar{Y} - A_1\bar{X}$$

$$A_1 = \frac{m\sum_{i=1}^{m} x_iy_i - \sum_{i=1}^{m} x_i \sum_{i=1}^{m} y_i}{m\sum_{i=1}^{m} x_i^2 - \left(\sum_{i=1}^{m} x_i\right)^2}$$

19 4(c) Find value of a_0 and a_1 so that $y = a_0 + a_1x$ fits the data given in table:

$n = 5$

x	0	1	2	3	4
y	1.0	2.9	4.8	6.7	8.6

x	y	x^2	xy
0	1.0	0	0
1	2.9	1	2.9
2	4.8	4	9.6
3	6.7	9	20.1
4	8.6	16	34.4
10	24	30	67

$5a_0 + 10a_1 = 24$

$10a_0 + 30a_1 = 67$

$a_0 = 1, \; a_1 = 1.9$

$$\begin{cases} ma_0 + \sum x_i a_1 = \sum y \\ \sum x_i a_0 + \sum x_i^2 a_1 = \sum xy \end{cases}$$

2018

4a- 19 4a,

4 (b) Describe the least square curve fitting procedure for a polynomial of degree n.

$y = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ be fitted to the dat points (x_i, y_i), $i = 1, 2, \dots m$. We then have,

$$S = [y_1 - (a_0 + a_1x_1 + \dots + a_nx_1^n)]^2 + [y_2 - (a_0 + a_1x_2 + \dots + a_nx_2^n)]^2 + \dots + [y_m - (a_0 + a_1x_m + \dots + a_nx_m^n)]^2$$

Differentiating S w.r.t a_0

$$\frac{dS}{da_0} = -2[y_1 - (a_0 + a_1x_1 + \dots + a_nx_1^n)] - 2[y_2 - (a_0 + a_1x_2 + \dots + a_nx_2^n)] - \dots$$

$$\frac{dS}{da_1} = -2x_1[y_1 - (a_0 + a_1x_1 + \dots + a_nx_1^n)] - 2x_2[y_2 - (a_0 + a_1x_2 + \dots + a_nx_2^n)] - 2x_1[y_m - (a_0 + a_1x_1 + \dots + a_nx_m^n)]$$

$\vdots$

$$\frac{dS}{da_n} = -2x_1^n[y_1 - (a_0 + a_1x_1 + \dots + a_nx_1^n)] - 2x_2^n[y_2 - (a_0 + a_1x_2 + \dots + a_nx_2^n)] - \dots - 2x_m^n[y_m - (a_0 + a_1x_1 + \dots + a_nx_m^n)]$$

$$ma_0 + a_1\sum_{i=1}^{m} x_i + \dots + a_n\sum_{i=1}^{m} x_i^n = \sum_{i=1}^{m} y_i$$

$$a_0\sum_{i=1}^{m} x_i + a_1\sum_{i=1}^{m} x_i^2 + \dots + a_n\sum_{i=1}^{m} x_i^{n+1} = \sum_{i=1}^{m} x_iy_i$$

$\vdots$

$$a_0\sum_{i=1}^{m} x_i^n + a_1\sum_{i=1}^{m} x_i^{n+1} + \dots + a_n\sum_{i=1}^{m} x_i^{2n} = \sum_{i=1}^{m} x_i^n y_i$$

Date :

1(c) Derive a polynomial of degree 2 to the data points in the table:

x	0	1	2
y	1.0	6.0	17.0

Soln:

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
1	6	1	1	1	6	6
2	17	4	8	16	34	68
3	24	5	9	17	40	74

$m = \text{size} = 3$

We know that,

$m a_0 + \Sigma x_i a_1 + \Sigma x_i^2 a_2 = \Sigma y_i$ $\quad 3a_0 + 3a_1 + 5a_2 = 24$

$\Sigma x_i a_0 + \Sigma x_i^2 a_1 + \Sigma x_i^3 a_2 = \Sigma x_i y_i$ $\quad 3a_0 + 5a_1 + 9a_2 = 40$

$\Sigma x_i^2 a_0 + \Sigma x_i^3 a_1 + \Sigma x_i^4 a_2 = \Sigma x_i^2 y_i$ $\quad 5a_0 + 9a_1 + 17a_2 = 74$

$a_0 = 1,\ a_1 = 2\ \&\ a_2 = 3$

The polynomial is : $1a_0 + 2a_1 + 3a_2 = 0$

$m a_0 + a_1 \Sigma x_1 + a_2 \Sigma x_1^2 = \Sigma y$

$a_0 \Sigma x + a_1 \Sigma x^2 + a_2 \Sigma x^3 = \Sigma xy$

$a_0 \Sigma x^2 + a_1 \Sigma x^3 + a_2 \Sigma x^4 = \Sigma x^2 y$

(17) 4 a,b − 19

4(c) The exponential func $y = ae^{bx}$ is fitted to data:

x	0	0.5	1	1.5	2	2.5
y	0.1	0.45	2.15	9.15	40.35	180.75

Given,

$y = ae^{bx}$

$\ln y = \ln ae^{bx}$

$\ln y = \ln a + \ln e^{bx}$

$\ln y = \ln a + bx \ln e$

$\ln y = \ln a + bx$

$a = 0.102$
$b = 2.996$

$m a_0 + \sum x\, a_1 = \sum y$

$\sum x a_0 + \sum x^2 a_1 = \sum xy$

$a_1 = \dfrac{m\sum xy - \sum x \sum y}{m\sum x^2 - (\sum x)^2}$

$a_0 = \bar{y} - a_1 \bar{x}$

Setting $\ln y = Y$, $\ln a = a_0$ and $b = a_1$ we get,

$Y = a_0 + a_1 x$

$m = 6$

−1.429
2.478

$X = x$	y	$\ln y = Y$	x^2	xy
0	0.1	−2.3	0	0
0.5	0.45	−0.799	0.25	−.3995
1	2.15	0.765	1	0.765
1.5	9.15	2.214	2.25	3.321
2	40.35	3.698	4	7.396
2.5	180.75	5.197	6.25	12.9925
7.5	~~232.95~~	8.775	13.75	24.075

$a = e^{a_0}$

$b = a_1$

$a = e^{-2.282}$
$= .102$

$\therefore b = 2.996$

$6a_0 + 7.5a_1 = 8.775$ —— (i)

$7.5a_0 + 13.75a_1 = 24.075$ —— (ii)

$a_0 = -2.282$

$a_1 = 2.996$

2016

Find a and b. $y = ae^{bx}$ $m=4$

$\ln y = \ln a + bx$

$Y = a_0 + a_1 x$

x	y	$\ln y = Y$	x^2	xY
1.0	40.17	3.693	1	3.693
1.2	73.196	4.293	1.44	5.1516
1.4	133.372	4.893	1.96	6.8502
1.6	243.02	5.493	2.56	8.7888
5.2		18.372	6.96	24.4836

$a = 1.999$
$b = 3$

$$m a_0 + \Sigma x a_1 = \Sigma Y$$

$$\Sigma x a_0 + \Sigma x^2 a_1 = \Sigma xY$$

$$4a_0 + 5.2a_1 = 18.372 \quad (1)$$

$$5.2a_0 + 6.96a_1 = 24.4836 \quad (11)$$

$a_0 = 0.693$ $a_1 = 3$

$\therefore a = e^{a_0} = e^{0.693} = 1.99$

$b = 3$ Ans.

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2015

4. a) Derive ~~Langrange's interpolation formular for~~ unequal distance. Find value of tan(0.05) for following table.

4 (b)

	x	y	Δ	Δ^2	Δ^3	Δ^4
x_1	.10	y_1 0.1003				
			y_2-y_1 0.0508			
x_2	.15	y_2 0.1511		y_2-y_1 0.0008		
			y_3-y_2 0.0516		0.0002	
x_3	.20	y_3 0.2027		0.0010		0.0002
			y_4-y_3 0.0526		0.0004	
x_4	.25	y_4 0.2553		0.0014		
			y_5-y_4 0.054			
x_5	.30	y_5 0.3093				

To find tan(0.05) we have

$h = 0.05$

$x_0 = 0.1$

$x_n = 0.05$

$$0.05 = 0.10 + p(0.05)$$

$h = \frac{x_n - x_0}{r}$

$$\Rightarrow p = \frac{0.05-0.10}{0.05} = -1$$

$r = \frac{0.05-0.1}{0.05} = -1$

$r = \frac{x-x_0}{h}$

Hence, according to Newton's forward difference interpolation formula,

$$\tan(0.05) = 0.1003 + (-1)0.0508 + \frac{(-1)(-1-1)}{2}(0.0008) + \frac{(-1)(-1-1)(-1-2)}{6}(0.0002) + \frac{(-1)(-1-1)(-1-2)(-1-3)}{24}(0.0002)$$

$$= 0.0503$$

$$y = y_1 + r\Delta y_1 + \frac{r(r-1)}{2!}\Delta^2 + \frac{r(r-1)(r-2)}{3!}\Delta^3 + \frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 + \cdots$$

h = difference

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4.(a) Derive Langrange's interpolation formula for unequal distance.

Ans: Let, $y(x)$ be continuous and differentiable $(n+1)$ times in the interval (a,b). Given $(n+1)$ unequally distanced points $(x_0, y_0), (x_1, y_1), \ldots (x_n, y_n)$, We wish to find a polynomial of degree n, say $L_n(x)$. Such that

$$L_n(x_i) = y(x_i) = y_i \;;\quad i = 0, 1, 2, \ldots n.$$

Then polynomial is $L_n(x) = \sum_{i=0}^{n} l_i(x) y_i$

where, $$l_i(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_n)}{(x_i-x_0)(x_i-x_1)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)}$$

which obviously satisfies the condition,

$$l_i(x_j) = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$$

If we set,

$$\pi_{n+1}(x) = (x-x_0)(x-x_1)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_n)$$

$$\pi'_{n+1}(x_i) = \frac{d}{dx}\left[\pi_{n+1}(x)\right]_{x=x_i}$$

$$= (x_i-x_0)(x_i-x_1)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)$$

So, $$l_i(x) = \frac{\pi_{n+1}(x)}{(x-x_i)\,\pi'_{n+1}(x_i)}$$

Hence, $$L_n(x) = \sum_{i=0}^{n} \frac{\pi_{n+1}(x)}{(x-x_i)\,\pi'_{n+1}(x_i)} \cdot y_i .$$

This is lagrange interpolation formula.

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Date :

2014

4 (b) Find a_0 & a_1 so that $y = a_0 + a_1 x$ fits the data given in table.

x	y	x^2	xy
1	2.4	1	2.4
2	3.1	4	6.2
3	3.5	9	10.5
4	4.2	16	16.8
6	5.0	36	30
8	6.0	64	48
$\Sigma x = 24$	$\Sigma y = 24.2$	$\Sigma x^2 = 130$	$\Sigma xy = 113.9$

$m = 6$

$a = 2.02$
$b = 0.5$

$$m a_0 + \Sigma x . a_1 = \Sigma y$$

$$\Sigma x a_0 + \Sigma x^2 a_1 = \Sigma xy$$

$$6 a_0 + 24 a_1 = 24.2$$

$$24.2 a_0 + 130 a_1 = 113.9$$

$$a_0 = 2.07 .$$

$$a_1 = 0.5$$

2013

4(b) Develop an interpolation polynomial for following data using the finite difference approach. Estimate the $f(x)$ for $x = 2.7$.

x	1	2	3
$f(x)$	3	5	8

x	$f(x)$	Δ	Δ^2
1.	3.		
		2	
			1
2	5.		
		3	
3	8		

$h = 2 - 1 = 1.$

$r = \frac{x - x_0}{h} = \frac{x-1}{1} = (x-1)$

We know, N.F.I.F :

$$y_r = y_0 + r\Delta + \frac{r(r-1)}{2!}\Delta^2 + \frac{r(r-1)(r-2)}{3!}\Delta^3$$

$$= 3 + (x-1)\cdot 2 + \frac{(x-1)(x-1-1)}{2!}\cdot 1$$

$$= 3 + 2x - 2 + \frac{(x-1)(x-2)}{2}$$

$$= 3 + 2x - 2 + \frac{x^2 - 2x - x + 2}{2}$$

$$= 3 + 2x - 2 + \frac{x^2}{2} - \frac{3x}{2} + 1 = \frac{x^2}{2} + \frac{x}{2} + 2$$

$= 6.9$

N.B.I.F:

$$y_r = y_n + r\nabla + \frac{r(r+1)}{2!}\nabla^2 + \frac{r(r-1)(r-2)}{3!}\nabla^3$$

$r = \frac{x - x_n}{h}$

$= \frac{x-3}{1}$

$= \frac{x-3}{1}$

$$= 8 + (x-3)3 + \frac{(x-3)(x-3+1)}{2!}\times 1$$

$$= 8 + 3x - 9 + \frac{(x-3)(x-2)}{2}$$

$$= 8 + 3x - 9 + \frac{x^2 - 2x - 3x + 6}{2}$$

$$= 3x - 1 + \frac{x^2}{2} - \frac{5x}{2} + 3 = \frac{x^2}{2} + \frac{x}{2} + 2$$

x-এর irregular difference এর জন্য Newtons divided difference

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Date :

(b) Find form of fun^c $y(x)$ from following table:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	-12			
		12		
1	0		0	
		12		0
3	12		0	
		12		
4	24			

$r = \frac{x - x_0}{h}$

$= \frac{x - 0}{}$

Now, using newton's ~~forward~~ divided ~~interpolation~~ diference formula

$$\cancel{y_r = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y} + \dots$$

$$f(x) = f(x_1) + (x-x_1)\underset{x_2}{\Delta} f(x_1) + (x-x_1)(x-x_2)\underset{x_2 x_3}{\Delta^2} f(x_1)$$
$$+ \dots + (x-x_1)(x-x_2) \dots (x-x_n)\underset{x_2 x_{n+1}}{\Delta^n} f(x) + \dots$$

Arguments 0, 1, 3, 4

$$f(x) = f(0) + (x-0)\underset{1}{\Delta} f(0) + (x-0)(x-1)\underset{1,3}{\Delta^2} f(0) +$$
$$(x-0)(x-1)(x-3)\underset{1,3,4}{\Delta^3} f(0) \cancel{+ (x-0)(x-1)(x-3)(x-4)}$$

Δ^4 এ $(x-4)$ লেখা যাবে না, last এর element এর জন্য এই $(x-4)$ এর set নেয়া লাগবেনা

2012

4(b) Find func $y(x)$.

x	0	1	3	4
y	-12	0	12	24

এখানে ~~হয়~~ দেখে x এর value গুলোর difference irregular তাই Newton's divided difference interpolation use করতে হবে।

আগের page এ কিছু info দেওয়া আছে

~~Formula~~ Arguments: 0, 1, 3, 4

$$f(x) = f(0) + (x-0)\underset{1}{\Delta} f(0) + (x-0)(x-1)\underset{1,3}{\Delta^2} f(0) + (x-0)(x-1)(x-3)\underset{1,3,4}{\Delta^3} f(0)$$

$$\frac{y_1 - y_0}{x_1 - x_0}$$

x	y	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	-12 $f(0)$			
		$\frac{0--12}{1-0} = 12$ $\Delta f(0)$		
1	0		$\frac{6-12}{3-0} = -2$ $\Delta^2 f(0)$	
		$\frac{12-0}{3-1} = 6$		$\frac{2-(-2)}{4-0} = 1$ $\Delta^3 f(0)$
3	12		$\frac{12-6}{4-1} = 2$	
		$\frac{24-12}{4-3} = 12$		
4	24			

$$f(x) = -12 + (x-0)\,12 + (x-0)(x-1)(-2) + (x-0)(x-1)(x-3)\,1$$

$$= -12 + 12x - 2x(x-1) + x(x-1)(x-3$$

(19 - 1d) sa

19

5 (a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ for $h = 0.5$ and 0.125 using Trapezoidal rule (correct to three decimal places) (3)

$\int_0^1 \frac{dx}{1+x^2}$ $h = 0.5$ & 0.125

Here, $a = 0$, $b = 1$, $f(x) = \frac{1}{1+x^2}$ $h = 0.5$

$h = \frac{b-a}{n}$

$\Rightarrow 0.5 = \frac{1-0}{n}$

$\Rightarrow n = \frac{1}{.5}$

$\Rightarrow n = 2$

	a	$a+h$	$a+2h$
x	0	$0+0.5$ 0.5	$0+2\times0.5$ $=1$
$f(x)$	$f(0) = \frac{1}{1+0}$ $=1$	$f(0.5) = \frac{1}{1+.5}$ $= \frac{4}{5}$	$f(1) = \frac{1}{1+1^2}$ $= \frac{1}{2}$
	y_0	y_1	y_2

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[(F+L) + 2(y_1 + y_2 + \cdots + y_{n-1})\right]$$

$$\int_0^1 \frac{1}{1+x^2}\,dx = \frac{h}{2}\left[(y_0 + y_2) + 2y_1\right]$$

$$\left[\tan^{-1} x\right]_0^1 = \frac{0.5}{2}\left[\left(1 + \frac{1}{2}\right) + 2\times\frac{4}{5}\right]$$

$$= \frac{1}{4}\left[\frac{3}{2} + \frac{8}{5}\right]$$

$\tan^{-1} 1 - \tan^{-1} 0 = 0.775$

$\cancel{\tan^{-1}\tan}\frac{\pi}{4} = 0.775$

$\frac{\pi}{4} = 0.775$

$\pi = 3.1$ (Approx.)

~~Derive Romberg Integration formula~~ (4)

$$\int_0^1 \frac{1}{1+x^2}\,dx \quad h = 0.125 = \frac{1}{8}$$

$$= \int_a^b \cancel{f(x)}\,dx. \quad b = 1,\ a = 0,\ f(x) = \frac{1}{1+x^2}.$$

$$h = \frac{b-a}{n}$$

$$\Rightarrow 0.125 = \frac{1-0}{n}$$

$$\Rightarrow n = \frac{1}{0.125} = 8,$$

	a	a+h	a+2h	a+3h	a+4h	a+5h	a+6h	a+7h	a+8h
x	0	$0+\frac{1}{8} = \frac{1}{8}$	$0+\frac{2}{8} = \frac{1}{4}$	$0+\frac{3}{8} = \frac{3}{8}$	$0+\frac{4}{8} = \frac{1}{2}$	$0+\frac{5}{8} = \frac{5}{8}$	$0+\frac{6}{8} = \frac{3}{4}$	$0+\frac{7}{8} = \frac{7}{8}$	$0+\frac{8}{8} = 1$
$f(x) = \frac{1}{1+x^2}$	$f(0) = \frac{1}{1+0^2} = 1$	$f(\frac{1}{8}) = \frac{1}{1+(\frac{1}{8})^2} = \frac{64}{65}$	$f(\frac{1}{4}) = \frac{1}{1+(\frac{1}{4})^2} = \frac{16}{17}$	$f(\frac{3}{8}) = \frac{1}{1+(\frac{3}{8})^2} = \frac{64}{73}$	$f(\frac{1}{2}) = \frac{1}{1+\frac{1}{2^2}} = \frac{4}{5}$	$f(\frac{5}{8}) = \frac{1}{1+(\frac{5}{8})^2} = \frac{64}{89}$	$f(\frac{3}{4}) = \frac{1}{1+(\frac{3}{4})^2} = \frac{16}{25}$	$f(\frac{7}{8}) = \frac{1}{1+(\frac{7}{8})^2} = \frac{64}{113}$	$f(1) = \frac{1}{1+1^2} = \frac{1}{2}$
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

$$\int_0^1 \frac{1}{1+x^2}\,dx = \frac{h}{2}\left[(y_0+y_8) + 2(y_1+y_2+y_3+y_4+y_5+y_6+y_7)\right]$$

$$[\tan x]_0^1 = \frac{0.125}{2}\left[\left(1+\frac{1}{2}\right) + 2\left(\frac{64}{65}+\frac{16}{17}+\frac{64}{73}+\frac{4}{5}+\frac{64}{89}+\frac{16}{25}+\frac{65}{113}\right)\right]$$

$$= \frac{1}{16}\left[\frac{3}{2} + 11.07\right]$$

$$\cancel{\tan^{-1}\tan}\frac{\pi}{4} - 0 = 0.78585331 83$$

$$\frac{\pi}{4} = 0.7858533183$$

$$\pi = 3.1434132 73 \text{ Approx.}$$

5(b) Derive Romberg Integration formula (4)

Ans: Composite Trapezoidal Rule $\xrightarrow[\text{Extrapolation}]{\text{Richardson}}$ Romberg.

The Composite Trapezoidal rule,

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a)+f(b) + 2\left(\sum_{i=1}^{n-1} f(x_i)\right)\right] + O(h^2)$$

where $h = \frac{b-a}{n}$ and ~~$x_i = x_0$~~

Sub interval

Let, $n = 2^{k-1}$, $h = \frac{b-a}{2^{k-1}}$

$k=2$
$n=2^{2-1}=2$

$k=1$
$n=2^{1-1}=1$

$k=1 \Rightarrow h = \frac{b-a}{2^0} = b-a$

$k=2 \rightarrow h = \frac{b-a}{2^{2-1}} = \frac{b-a}{2}$

$k=3 \rightarrow h = \frac{b-a}{2^{3-1}} = \frac{b-a}{2^2} = \frac{b-a}{4}$

$\vdots$

$R_{k,1} \longrightarrow$ denotes $\rightarrow$ CTR.

$$R_{1,1} = \frac{h_1}{2}\left[f(a)+f(b)\right]$$

$$= \frac{b-a}{2}\left[f(a)+f(b)\right]$$

$$R_{2,1} = \frac{h_2}{2}\left[f(a)+f(b) + 2f(a+h_2)\right]$$

$$= \frac{b-a}{4}\left[f(a)+f(b)+2f(a+h_2)\right]$$

$$= \frac{1}{2}\times\frac{b-a}{2}\left[f(a)+f(b)+2f(a+h_2)\right]$$

$$= \frac{1}{2}\left[\frac{b-a}{2}\{f(a)+f(b)\} + (b-a) f(a+h_2)\right]$$

$$R_{21} = \frac{1}{2}\left[R_{1,1} + h_1 f(a+h_2)\right]$$

$$R_{k,1} = \frac{1}{2}\left[R_{k-1,1} + h_{k-1} \sum_{i=1}^{2^{k-2}} f(a+(2i-1)h_k)\right]$$

CTR expressed differently.

$$R_{k,j} = R_{k,j-1} + \frac{R_{k,j-1} - R_{k-1,j-1}}{4^{j-1} - 1}$$

$$R_{2,2} = R_{2,1} + \frac{R_{2,1} - R_{1,1}}{4^{2-1} - 1}$$

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a)+f(b)+2\sum_{i=1}^{n-1}(x_i)\right]$$

$h = \frac{b-a}{n}$, let, if $n = 2^{k-1}$ then, $h = \frac{b-a}{2^{k-1}}$

then $k=1 \Rightarrow h_1 = \frac{b-a}{2^0} = b-a$

$k=2 \Rightarrow h_2 = \frac{b-a}{2}$

$K=3 \rightarrow h_3 = \frac{b-a}{2^2} = \frac{b-a}{4}$

$k=4 \rightarrow h_4 = \frac{b-a}{2^3} = \frac{b-a}{8}$

$k=2$
$n = 2^{k-1} = 2^{2-1} = 2$

Let, $R_{1,1} = \frac{h_1}{2}\left[f(a)+f(b)\right]$ $\quad h = b-a$

$R_{2,1} = \frac{h_2}{2}\left[f(a)+f(b)+2f(a+h_2)\right]$ $\quad h_2 = \frac{b-a}{2}$, $h_2 = \frac{h}{2}$

$= \frac{b-a}{4}\left[f(a)+f(b)+2f(a+h_2)\right]$

$= \frac{h_1}{4}\left[f(a)+f(b)+2f(a+\frac{h}{2})\right]$

$R_{3,1} = \frac{h_1}{8}\left[f(a)+f(b)+2\{f(a+\frac{h}{4})+f(a+\frac{h}{2})\}\right]$

$R_{k,2} = R_{k,1} + \frac{1}{3}(R_{k,1} - R_{k-1,1})$ $\quad k = 2, 3, \ldots$

$R_{2,2} = R_{2,1} + \frac{1}{3}(R_{2,1} - R_{1,1})$

$R_{3,2} = R_{3,1} + \frac{1}{3}(R_{3,1} - R_{3,1})$

SAT SUN MON TUE WED THU FRI

Date :

$$R_{k,3} = R_{k,2} + \frac{1}{15}\left(R_{k,2} - R_{k-1,2}\right)$$

$$R_{k,j} = R_{k,j} + \frac{1}{4^{j-1}-1}\left(R_{k,j-1} - R_{k-1,j-1}\right) \quad k = j, j+1, \ldots$$

$O(h^2)$ $\quad O(h^4)$ $\quad O(h^6)$ $\ldots\ldots$ $O(h^{2n})$

$R_{1,1}$

$R_{2,1}$ $\quad R_{2,2}$

$R_{3,1}$ $\quad R_{3,2}$ $\quad R_{3,3}$

$\vdots$ $\quad \vdots$ $\quad \vdots$ $\qquad R_{n,n}$

$R_{n,1}$ $\quad R_{n,2}$ $\quad R_{n,3}$

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Date :

5ⓒ State any two difference between direct & iterative method for solving system of equation. 1.75

Topic	Direct Method	Iterative Method
1. Number of Operation	Finite	Finite
2. Accuracy of solution	Exact to machine precision	An approximation that coverges to an exact solution
Remark	Require large memory storage	• Require a very good initial set of displacement to be known. • Numerical condition heavily affect convergence
Popular Methods	1. Gaussian elimination (standard) 2. LU decomposition 3. Thomas algorithm	1. Gauss-Seidel Method (standard) 2. Jacobi Method 3. Newton-Raphson method

2018 5ⓐ Solve following system using Jacobi iterative technique.

$3x_1 + x_2 - 2x_3 = 9$ $\qquad$ $3x + y - 2z = 9$

$-x_1 + 4x_2 - 3x_3 = -8$ $\qquad$ $-x + 4y - 3z = -8$

$x_1 - x_2 + 4x_3 = 1$ $\qquad$ $x - y + 4z = 1$

Sol^n: We write the given equation in the form of, $x_1 = \frac{1}{3}(9 - x_2 + 2x_3)$

$$x_2 = \frac{1}{4}(-8 + x_1 + 3x_3)$$

$$x_3 = \frac{1}{4}(1 - x_1 + x_2)$$

First approx, $x_{1_0} = 0$, $x_{2_0} = 0$, $x_{3_0} = 0$

$$\cancel{x_1 = \frac{1}{20}(-8 +}$$

$$x_{1_1} = \frac{1}{3}(9 - x_{2_0} + 2x_{3_0}) = 3$$

$$x_{2_1} = \frac{1}{4}(-8 + x_{1_0} + 3x_{3_0}) = -2$$

$$x_{3_1} = \frac{1}{4}(1 - x_{1_0} + x_{2_0}) = 0.25$$

2nd approx,

$$x_{1_2} = \frac{1}{3}(9 - x_{2_1} + 2x_{3_1}) = \frac{1}{3}(9 - (-2) + 2(.25)) = \frac{23}{6} = 3.833$$

$$x_{2_2} = \frac{1}{4}(-8 + x_{1_1} + 3x_{3_1}) = \frac{1}{4}(-8 + 3 + 3.(0.25)) = -\frac{17}{16} \quad 1.062$$

$$x_{3_2} = \frac{1}{4}(1 - x_{1_1} + x_{2_1}) = \frac{1}{4}(1 - 3 + (-2)) = -1$$

3rd approx,

$$x_{1_3} = \frac{1}{3}(9 - x_{2_2} + 2x_{3_2}) = \frac{1}{3}\left(9 - \left(-\frac{17}{16}\right) + 2(-1)\right) = 2.68$$

$$x_{2_3} = \frac{1}{4}(-8 + x_{1_2} + 3x_{3_2}) = \frac{1}{4}\left(-8 + \frac{23}{6} + 3(-1)\right) = -1.79$$

$$x_{3_3} = \frac{1}{4}(1 - x_{1_2} + x_{2_2}) = \frac{1}{4}\left(1 - \frac{23}{6} + \left(-\frac{17}{16}\right)\right) = -0.97$$

4th approximation.

$$x_{1_4} = \frac{1}{3}(9 - x_{2_3} + 2x_{3_3}) = \frac{1}{3}(9-(-1.79)+2(-0.97)) = 2.95$$

$$x_{2_4} = \frac{1}{4}(-8 + x_{1_3} + 3x_{3_3}) = \frac{1}{4}(-8+2.68+3(-0.97)) = -2.05$$

$$x_{3_4} = \frac{1}{4}(1 - x_{1_3} + x_{2_3}) = \frac{1}{4}(1-2.68+(-1.79)) = -0.86$$

5th approximation;

$$x_{1_5} = \frac{1}{3}(9 - x_{2_4} + 2x_{3_4}) = \frac{1}{3}(9-(-2.05)+2(-0.86)) = 3.11$$

$$x_{2_5} = \frac{1}{4}(-8 + x_{1_4} + 3x_{3_4}) = \frac{1}{4}(-8+(2.95)+3(-0.86)) = -1.9$$

$$x_{3_5} = \frac{1}{4}(1 - x_{1_4} + x_{2_4}) = \frac{1}{4}(1-2.95+(-2.05)) = -1$$

6th approximation,

$$x_{1_6} = \frac{1}{3}(9 - x_{2_5} + 2x_{3_5}) = \frac{1}{3}(9-(-1.9)+2(-1)) = 2.966$$

$$x_{2_6} = \frac{1}{4}(-8 + x_{1_5} + 3x_{3_5}) = \frac{1}{4}(-8+3.11+3(-1)) = -1.97$$

$$x_{3_6} = \frac{1}{4}(1 - x_{1_5} + x_{2_5}) = \frac{1}{4}(1-3.11+(-1.9)) = -1.00$$

7th Approximation,

$$x_{1_7} = \frac{1}{3}(9 - x_{2_6} + 2x_{3_6}) = \frac{1}{3}(9-(-1.97)+2(-1.00)) = 2.99$$

$$x_{2_7} = \frac{1}{4}(-8 + x_{1_6} + 3x_{3_6}) = \frac{1}{4}(-8+(2.966)+3(-1.00)) = -2.00$$

$$x_{3_7} = \frac{1}{4}(1 - x_{1_6} + x_{2_6}) = \frac{1}{4}(1-2.966+(-1.97)) = -0.98$$

$\therefore$ values of 6th & 7th iteration are same, we can stop

$$x_1 = 2.99, \quad x_2 = -2, \quad x_3 = -0.98$$

Jacobi তে আগের set এর x, y, z এর Am value use করে, আর gauss-seidel এ ~~সাথে~~ সাথে সাথে যখন x/ y,/ z পাওয়া যাই সেই immediate value গুলো use করে।

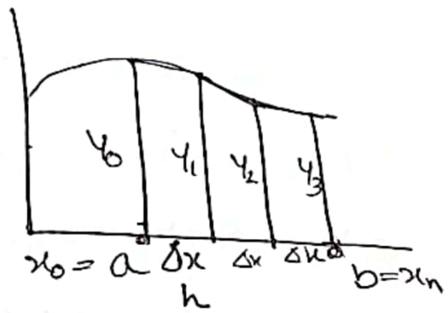
SAT SUN MON TUE WED THU FRI

Date :

17

5(a) Describe the geometrical meaning of Trapezoidal rule. (2)

Ans: The geometrical significance of trapezoidal rule is that the curve $y=f(x)$ is replace by n straight lines joining the points (x_0, y_0) and (x_1, y_1), (x_1, y_1) and (x_2, y_2), ---- (x_{n-1}, y_{n-1}) and (x_n, y_n). The area bounded by the curve $y=f(x)$, the ordinates $x=x_0$ and $x=x_n$ and the x-axis is then approximately equivalent to the sum of the area of the n trapeziums obtained.



17

5(b) Explain Gaussian Elimination method to solve linear system of equation. (4)

Ans. Let the linear system of equations in n unknowns be given by,

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

There are two steps in gaussian elimination.

SAT SUN MON TUE WED THU FRI

Date :

Step-1: Eliminate the unknowns to obtain upper triangular system. To eliminate x_1 from 2nd equation multiply it by $(-a_{11}/a_{21})$ and add to 1st equation to obtain,

$$-\frac{a_{11}}{a_{21}} a_{22}x_2 + \cdots + \left(-\frac{a_{11}}{21}\right) a_{2n} x_n = \left(-\frac{a_{11}}{a_{21}}\right) b_2$$

Let write it

$$a'_{22}x_2 + \cdots + a'_{2n}x_n = b'_2$$

Similarly eliminate x_1 from all equation except 1st equation.

And by this way eliminate other variable from below equations and get upper triangle

Now the upper triangle form

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a'_{22}x_2 + \cdots + a'_{2n}x_n = b'_2$$

$$\vdots$$

$$a'_{nn}x_n = b'_n$$

Step 2: Now solve the equation to get required solution.

From the last equation of the system we obtain,

$$x_n = \frac{b_n^{(n-1)}}{a_n^{(n-1)}}$$

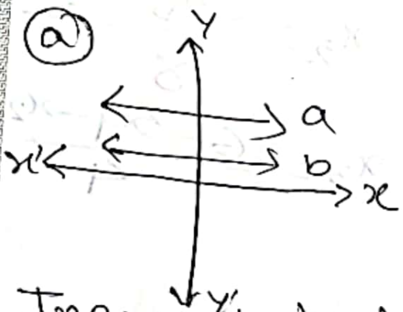
Similarly we can solve for all unkown.

2016

5 (a) Classify System of Linear Equation and explain them based on graphical representation. 8.75

Ans: Linear equations can be represented in various ways. Let us consider two straight lines and make different graph & ~~make~~ representation.

(a)

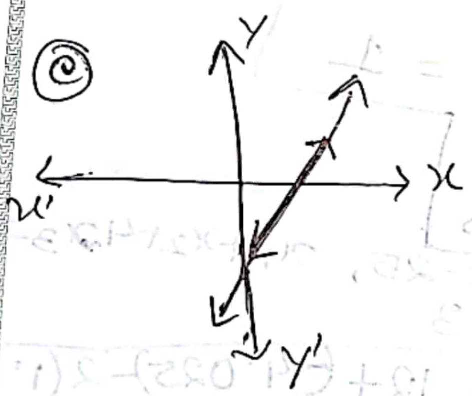


Inconsistant : A system of equation ~~that~~ that has no solution

(b)

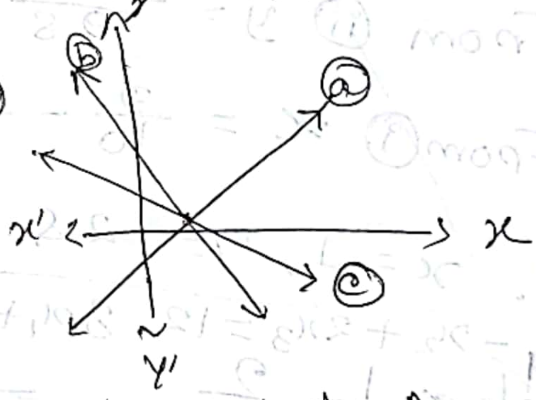
Consistant : A system of equations that has one or more solutions.

(c)



Dependent : A system of equation that has ~~infinity~~ infinite no of solution.

(d)



Independent : A system of equation that has only one solution.

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Date :

5 (b) Solve the following set of simultaneous equations using the Gauss-elimination method (5)

$2x - 4y + 6z = 5$ —— (i)

$x + 3y - 7z = 2$ —— (ii)

$7x + 5y + 9z = 4$ —— (iii)

Ans: First multiply 2^{nd} & 3^{rd} equation by (-2) and $\left(-\frac{2}{7}\right)$ respectively and add to 1^{st} equation to get 4^{th} and 5^{th} equation.

$$\cancel{2x} - 4y + 6z = 5$$
$$-\cancel{2x} - 6y + 14z = -4$$
$$-10y + 20z = 1 \text{ —— (iv)}$$

$$\cancel{2x} - 4y + 6z = 5$$
$$-\cancel{2x} - \frac{10}{7}y - \frac{18}{7}z = -\frac{8}{7}$$
$$-\frac{38}{7}y + \frac{24}{7}z = \frac{27}{7}$$
$$-38y + 24z = 27 \text{ —— (v)}$$

We multiply (v) with $\left(-\frac{5}{19}\right)$ and add to 4^{th} equation to get,

$-38a = -10$

$38a = 10$

$a = \frac{10}{38} = \frac{5}{19}$

$$-\cancel{10y} + 20z = 1$$
$$\frac{5}{19} \times \cancel{38y} - \frac{5}{19} \times 24z = -\frac{5}{19} \times 27$$
$$\frac{260}{19}z = -\frac{116}{19}$$
$$260z = -116$$
$$z = -\frac{29}{65}$$

The upper-triangular form,

$$2x - 4y + 6z = 5$$
$$-10y + 20z = 1$$
$$z = -\frac{29}{65}$$

Solving these equations,

$$x = \frac{241}{130}, \quad y = -\frac{129}{130}$$
$$z = -\frac{29}{65}$$

(2014

Reduced Row echelon form ↑

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Date :

5 (b) Solve following system by Gauss-Jordan Method.

$$2x+y+z=10$$
$$3x+2y+3z=18$$
$$x+4y+9z=16.$$

$x=7$
$y=-9$
$z=5$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{array}\right]$$

$3-\frac{3}{2}\times 2$
$2-\frac{3}{2}\times 1$
$3-\frac{3}{2}\times 1$
$18-\frac{3}{2}\times 10$

$$\sim\left[\begin{array}{ccc|c} 2 & 1 & 1 & 10 \\ 0 & \frac{1}{2} & \frac{3}{2} & 3 \\ 0 & \frac{7}{2} & \frac{17}{2} & 11 \end{array}\right] \quad R_2=R_2-\frac{3}{2}R_1, \quad R_3=R_3-\frac{1}{2}R_1$$

$1-\frac{1}{2}\times 2$
$4-\frac{1}{2}\times 1$
$9-\frac{1}{2}\times 1$
$16-\frac{1}{2}\times 10$

$$\sim\left[\begin{array}{ccc|c} 2 & 1 & 1 & 10 \\ 0 & \frac{1}{2} & \frac{3}{2} & 3 \\ 0 & 0 & -2 & -10 \end{array}\right] \quad R_3=R_3-(7)R_2$$

$\frac{7}{2}-\frac{1}{2}x=0$
$\frac{7}{2}=\frac{x}{2}$
$x=7$

$$\sim\left[\begin{array}{ccc|c} 2 & 1 & 0 & 5 \\ 0 & \frac{1}{2} & 0 & -\frac{9}{2} \\ 0 & 0 & -2 & -10 \end{array}\right] \quad R_1=R_1+\frac{1}{2}(R_3), \quad R_2=R_2+\frac{3}{4}R_3$$

$\frac{17}{2}-7\frac{3}{2}$
$\frac{17}{2}-\frac{21}{2}$
$5-7\times 3$

$$\sim\left[\begin{array}{ccc|c} 2 & 0 & 0 & 14 \\ 0 & \frac{1}{2} & 0 & -\frac{9}{2} \\ 0 & 0 & -2 & -10 \end{array}\right] \quad R_1=R_1-2R_2$$

$\frac{3}{2}-2x=0$
$\frac{3}{4}=x$

$$\sim\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$x,y,z=(7,-9,5)$

Ans

13

Reduced Row-echelon form

5 (b) Solve Gauss-Jordan method:

$$4x - 2y + 3z = 15.7$$
$$-2x + 4y - z = -14.1$$
$$3x + y - 3z = -4.2$$

$$\left[\begin{array}{ccc|c} 4 & -2 & 3 & \frac{157}{10} \\ -2 & 4 & -1 & -\frac{141}{10} \\ 3 & 1 & -3 & -\frac{21}{5} \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{3}{4} & \frac{157}{40} \\ -2 & 4 & -1 & -\frac{141}{10} \\ 3 & 1 & -3 & -\frac{21}{5} \end{array}\right] \quad R_1 = \frac{1}{4}R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{3}{4} & \frac{157}{40} \\ 0 & 3 & \frac{1}{2} & -\frac{25}{4} \\ 0 & \frac{5}{2} & -\frac{21}{4} & -\frac{639}{40} \end{array}\right] \quad \begin{array}{l} R_2 = R_2 + 2R_1 \\ R_3 = R_3 + (-3)R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{3}{4} & \frac{157}{40} \\ 0 & 1 & \frac{1}{6} & -\frac{25}{12} \\ 0 & \frac{5}{2} & -\frac{21}{4} & -\frac{639}{40} \end{array}\right] \quad R_2 = \frac{1}{3}R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{3}{4} & \frac{157}{40} \\ 0 & 1 & \frac{1}{6} & -\frac{25}{12} \\ 0 & 0 & -\frac{17}{3} & -\frac{323}{30} \end{array}\right] \quad R_3 = R_3 + \left(-\frac{5}{2}\right)R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{3}{4} & \frac{157}{40} \\ 0 & 1 & \frac{1}{6} & -\frac{25}{12} \\ 0 & 0 & 1 & \frac{19}{10} \end{array}\right] \quad R_3 = -\frac{3}{17} \times R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{3}{4} & \frac{157}{40} \\ 0 & 1 & 0 & -\frac{12}{5} \\ 0 & 0 & 1 & \frac{19}{10} \end{array}\right] \quad R_2 = R_2 + \left(\frac{-1}{6}\right) \times R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & 0 & \frac{5}{2} \\ 0 & 1 & 0 & -\frac{12}{5} \\ 0 & 0 & 1 & \frac{19}{10} \end{array}\right] \quad R_1 = R_1 + \left(-\frac{3}{4}\right) R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{13}{10} \\ 0 & 1 & 0 & -\frac{12}{5} \\ 0 & 0 & 1 & \frac{19}{10} \end{array}\right] \quad R_1 = R_1 + \frac{1}{2} R_2$$

An

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Date :

Main idea of Gauss-Seidel.

With the Jacobi Method, the values of $x_i^{(k)}$ obtained in the k^{th} iteration remain unchanged untill the entire $(k+1)^{th}$ iteration. has been calculated. With the Gauss-Seidel method, we use the new values $x_i^{(k+1)}$ as soon as they are known. For example, once we have compted $x_1^{(k+1)}$ for first equation, its value is then used in the second equation to obtain the new x_2^{k+1}, and so on.

Method:-

For each $k \geq 1$ generate components x_i^k of $x^{(k)}$ from x^{k-1} by,

$$x_i^{(k)} = \frac{1}{a_{ii}}\left[-\sum_{j=1}^{i-1}\left(a_{ij}x_j^{(k)}\right) - \sum_{j=i+1}^{n} a_{ij}x_j^{(k-1)} + b_i\right], i=1,2,\dots n$$

Namely

$$a_{11}x_1^k = -a_{12}x_2^{k-1} - \dots - a_{1n}x_n^{(k-1)} + b_1$$

$$a_{21}x_1^k + a_{22}x_2^k = -a_{23}x_3^{k-1} - \dots - a_{2n}x_n^{k-1} + b_2$$

$$\vdots$$

$$a_{n1}x_1^k + a_{n2}x_2^{(k)} + a_{nn}x_n^{(k)} = b_n.$$

Matrix for Gauss-seidel method

$$(D-L)x^k = Ux^{k-1} + b$$

$$x^{(k)} = (D-L)^{-1}Ux^{k-1} + (D-L)^{-1}b$$

Define $T_g = (D-L)^{-1}U$ and $C_g = (D-L)^{-1}b$,

$$x^k = T_g x^{k-1} + C_g \quad k=1,2,3.$$

19

6 (a) From Taylor series $y(x)$, find $y(0.1)$ correct to four decimal places if $y(x)$ satisfies $y' = x - y^2$ and $y(0) = 1$ (4)

Given, $y' = x - y^2$, $x_0 = 0$, $y_0 = 1$.

Now, $y(0) = 1$

$$y' = x - y^2 \quad ; y'(0) = 0 - 1^2 = -1$$

$$y'' = 1 - 2yy' \quad ; y''(0) = 1 - 2.1(-1) = 3$$

$$y''' = \cancel{1} - 2yy'' - 2y'y'$$
$$= \cancel{1} - 2yy'' - 2(y')^2 \quad ; y'''(0) = \cancel{1} - 2.1(3) - 2(-1)^2 = -8$$

$$y^{iv} = -2yy''' - 2y''y' - 4y'y''$$
$$= -2yy''' - 6y'y'' \quad ; y^{iv}(0) = -2.1(-8) - 6.(-1)\,3 = 34$$

Putting in taylor series.

$$y = y_0 + (x-x_0)y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0''' + \frac{(x-x_0)^4}{4!}y_0^{iv} + \cdots$$

$$= 1 + x(-1) + \frac{(x-0)^2}{2!}.3 + \frac{(x-0)^3}{3!}(-8) + \frac{(x-0)^4}{4!}.34 + \cdots$$

$$= 1 - x + \frac{3x^2}{2} - \frac{4x^3}{3} + \frac{17x^4}{12} - \cdots$$

~~= 0.9138~~

$\therefore y(0.1) = 0.9138$

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$y(x_n)$

(b) Determine value of y using Euler's formula when $x = 0.1$ given that $y(0) = 1$ and $y' = x^2 + y$ and $h = 0.05$ $y(x_n = y_n)$

$x_0 = y_0$ $\frac{dy}{dx}$

Solⁿ: Given, $f(x,y) = \frac{dy}{dx} = y' = x^2 + y$; $x_0 = 0$, $y_0 = 1$, $x_n = 0.1$

$h = 0.05$. $h = \frac{x_n - x_0}{n}$

$\Rightarrow 0.05 = \frac{0.1 - 0}{n}$

$\Rightarrow n = \frac{0.1}{0.05}$

$= 2$ $y_0 + hf(x,y) = Y_1$

x	y	$\frac{dy}{dx} = x^2 + y$	Old $y + (0.05)\frac{dy}{dx}$ = new y
0.0	1.00	1.00	$1.00 + 0.05(1.00) = 1.05$
0.05	1.05	1.0525	$1.05 + (0.05)(1.0525) = 1.10262$
0.1	1.102625	1.112625	$1.102625 + (0.05)(1.112625) = 1.1582$
0.15	1.158256	1.180756	$1.158256 + (0.05)(1.180756) = 1.2173$

3 steps at 4

2018 - 5 (b) - s same as above

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2018

6 ⓐ For the following set of data: Construct a finite-difference table and numerically evaluate the first, second, third derivative at $x=1$ using forward differences.

x	$f(x)$	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
$0 = x_0$	$0 = y_0$				
		0.5			
1	0.5		-0.25		
		0.25		0.04	
2	0.75		-0.21		0.33
		0.04		0.37	
3	0.79		0.16		
		0.2			
4	0.99				

$r = \frac{x - x_0}{h}$

$r = \frac{1-0}{1}$

$= 1$

$$y(x_0) = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0$$

$$= 0 + 1 \times 0.5 + \frac{1(1-1)}{2!}(-0.25) + \frac{1(1-1)(1-2)}{3!}(0.04) + \dots$$

$$= 0.5 + 0 + 0$$

$$= 0.5$$

$r = 1$, $r = 1/4$

$i = 2, 3, 4, 5$

$2, 3, 4, 5$

$n - j + 1$

$= 5 - 2 + 1$

$= 3 + 1 = 4$

$y'(x_0) = \frac{1}{h}$

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① Trapozoidal Rule ($n=1$) :-

$$\int_{a=x_0}^{b=x_n} y\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\right]$$

used for any value of n.

② Simpson's 1/3 Rule ($n=2$) [multiple of 2]

$$\int_{a=x_0}^{b=x_n} y\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+y_5+\cdots+y_{n-1})+2(y_2+y_4+y_6+\cdots+y_{n-2})\right]$$

n should be multiple of 2.

③ Simpson's 3/8 Rule ($n=3$)

$$\int_{a=x_0}^{b=x_n} y\,dx = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+y_5+y_7+\cdots+y_{n-1})+2(y_3+y_6+y_9+\cdots+y_{n-3})\right]$$

[n should be muliple of 3]

④ Wedd's Rule [$n=6$] - n should be multiple of 6.

$$\int_{a=x_0}^{b=x_n} y\,dx = \frac{3h}{10}\left[(y_0+y_n)+5(y_1+y_7+y_{13}+\cdots)+(y_2+y_8+y_{14}+\cdots)+6(y_3+y_9+y_{15}+\cdots)+(y_4+y_{10}+y_{16}+\cdots)+5(y_5+y_{11}+y_{17}+\cdots)\right]$$

2012

6 (a) Modified Euler's Method.

In order to get better approximate value of y, we first obtain y_1 by Euler's method i.e. initial approximation $y_1(0)$ by using formula

$$y_1(0) = y_1 + h f(x_0, y_0).$$

Then we modify it by using modified Euler's method using formula $y_1^{(n+1)} = y_0 +$

$$\frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(n)})\right].$$

Put $n = 0, 1, 2$

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right] \rightarrow \text{1st approximation}$$

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right] \rightarrow \text{2nd approximation}$$

and so on.

This procedure will be terminated depending upon the accuracy required. If two consecutive values of $y_1(k)$ and $y_1^{(k+1)}$ are equal, then $y_1 = y_1^{(k)}$

Proceeding in the same manner, $y_2, y_3, \ldots$ are calculated.

False $\frac{af(b)-bf(a)}{f(b)-f(a)}$ $b-\frac{f(b)\,(b-a)}{f(b)-f(a)}$ $Ax=B$
$x=A^{-1}B$

Forward elimination
back sub

Pitfalls of Naive
① Division by 0.
→ 0 pivot
→ diagonal becomes 0

② → Large Round-off error.
→ $\begin{bmatrix} 1 \\ 1.5 \\ .99 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

③ Ill condition ?
Gauss solution by partial pivoting
। আগে diagonal এ highest absolute value টা বসাতে হবে

Newton's forward

$$y_n(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots$$

Newton's Backward

$$y_n(x) = y_n + p\Delta y_n + \frac{p(p+1)}{2!}\Delta^2 y_n + \frac{p(p+1)(p+2)}{3!}\Delta^3 y_n + \ldots$$

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Date :

2012

5 (a) Derive Simpson's 1/3 rule for numerical integration.

Soln: The general formula for Newtons forward difference in integral form

$$\int_{x_0}^{x_n} y\,dx = nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{n(n-2)^2}{24}\Delta^3 y_0 + \cdots\right]$$

Now putting $n=2$ i.e replacing the curve by $\frac{n}{2}$ arcs of second-degree polynomials. We have

then $$\int_{x_0}^{x_2} y\,dx = 2h\left(y_0 + \Delta y_0 + \frac{1}{6}\Delta^2 y_0\right)$$

$$= \frac{h}{3}\left(y_0 + 4y_1 + y_2\right)$$

Similarly, $$\int_{x_2}^{x_4} y\,dx = \frac{h}{3}\left(y_2 + 4y_3 + y_4\right)$$

and finally, $$\int_{x_{n-2}}^{x_n} y\,dx = \frac{h}{3}\left(y_{n-2} + 4y_{n-1} + y_n\right)$$

Summing up we obtain.

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\Big[y_0 + 4(y_1 + y_3 + y_5 + \cdots + y_{n-1}) + 2(y_2 + y_4 + y_6 + y_8 + \cdots + y_{n-2}) + y_n\Big]$$

which is known as simpson's 1/3 rule. This rule requires the division of whole range into an even number of subintervals of width h.

n^3-4n^2+4n

$n(n^2-4n+4)$

$(n-2)^2$

$p(p-1)(p-2)$

$(p^2-p)(p-2)$

$p^3-2p^2-p^2+2p$

p^3-3p^2+2p

direct integral form used.

Newton's Forward diff.

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

Differenciating w.r.t to p.

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$$\frac{dy}{dp} = \Delta y_0 + \frac{2p-1}{2!}\Delta^2 y_0 + \frac{3p^2-3p+2}{6}\Delta^3 y_0 + \cdots$$

For max

$$I = \int_{x_0}^{x}\left[y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots\right]dx$$

$\because x = x_0 + ph$, $dx = h\,dp$ and hence the above integral becomes.

$$I = h\int_0^n\left[y_0 + p\Delta y_0 + \frac{p^2-p}{2!}\Delta^2 y + \frac{p^3-3p^2+2p}{3!}\Delta^3 y_0 + \cdots\right]$$

$$= h\left[py_0 + \frac{p^2}{2}\Delta y_0 + \frac{\frac{p^3}{3}-\frac{p^2}{2}}{2!}\Delta^2 y + \frac{\frac{p^4}{4}-\frac{3p^3}{3}+\frac{2p^2}{2}}{3!}\right.$$

$$= h\left[ny_0 + \frac{n^2}{2}\Delta y_0 + \frac{\frac{2n^3-3n^2}{6}}{2!}\Delta^2 y + \frac{\frac{n^4-4n^3+4n^2}{4}}{3!}\Delta^3\right.$$

$$= nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y + \frac{n(n^2-4n-4)}{4\times 3!}\Delta^3 y_0 + \cdots\right.$$

$$= nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y + \frac{n(n-2)^2}{24}\Delta^3 y_0 + \cdots\right]$$

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$\frac{4,3,2}{2,3,1}$

$4x$

$\frac{2n^3-3p^2}{6}$

$C = A + B \iff c_{ij} = a_{ij} + b_{ij} \quad \forall i,j$

$C = AB \iff c_{ij} = \sum_{k=1}^{m} a_{ik} b_{kj} \quad \forall i,j$

Naive Gaussian Elemination.
↳ 2 steps →

1. Forward elimination - Row echelon form
2. Backward Substitution

$-3x_4 = -3 \Rightarrow x_4 = 1$

$2x_3 - 5x_4 = -9$

$2x_3 - 5.1 = -9$

$2x_3 = -9 + 5$

$x_3 = -2$

$-4x_2 + 2\times(-2) + 2\times 1 = -6$

$-4x_2 - 4 + 2 = -6$

$-4x_2 - 2 = -6$

$-4x_2 = -4$

$x_2 = 1$

Unique	No Solution	Infinite
$\lvert A\rvert \neq 0$	$\det(A) = 0$	$\det(A) = 0$
reduced matrix has no zero rows	1 or more zero rows - corresponding B elements $\neq 0$	1 or more zero rows corresponding B elements $= 0$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} X = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$x_2 = \alpha$

$x_1 + 2x_2 = 2$

$x_1 + 2\alpha = 2$

$x = 2 - 2\alpha$

$x = 1 - \alpha$

Gaussian Elemination with Scaled Partial Pivoting.

Trapezoidal. $n=1$

$$\int_{a=x_0}^{b=x_n} y\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+y_3+\cdots y_{n-1})\right]$$

Simpson's 1/3 $n=2$ [multiple of 2]

$$\int_{a=x_0}^{b=x_n} y\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+y_5+\cdots+y_{n-1}) + 2(y_2+y_4+y_6+\cdots y_{n-2})\right]$$

Simpson's 3/8 $n=3$ [multiple of 3]

$$\int_{a=x_0}^{b=x_n} y\,dx = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+y_5+\cdots+y_{n-1}) + 2(y_3+y_6+y_9+\cdots y_{n-3})\right]$$

n দেওয়া থাকলে ঐটাই নিব না হলে multiple একটা ধরে নিব।

$$h=\frac{b-a}{n}$$

x	a	$a+h$	$a+2h$	--- ...	$a+nh$
$f(x)$	y_0	y_1	y_2	y_3	y_4

Newton's Divided difference. ~~x 300~~

$\boxed{x = 301}$? y ~~y 2.4~~

x	y	Δ	Δ^2	Δ^3
300	2.4771			
		0.0058		
			-0.0044	
304	2.4829			0.0058
		0.0014		
305	2.4843		0.0014	
		0.0028		
307	2.4871			

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Newton's divided difference

$x = 301 \quad y = ?$

$\Delta f(x_1) = \frac{y_2 - y_1}{x_2 - x_1}$

x	y	Δ	Δ^2	Δ^3
300	2.4771 $f(300)$			
		$\frac{2.4829 - 2.4771}{304 - 300} = 0.00145$ $\Delta f(x_1)$		
304	2.4829		$\frac{0.0014 - 0.00145}{305 - 300} = -0.00001$ $\Delta^2 f(x_2)$	
		$\frac{2.4843 - 2.4829}{305 - 304} = 0.0014$		$\frac{0 - (-0.00001)}{307 - 300} = 0.0000014$ $\Delta^3 f(x_1)$
305	2.4843		$\frac{0.0014 - 0.0014}{307 - 304} = 0$	
		$\frac{2.4871 - 2.4843}{307 - 305} = 0.0014$		
307	2.4871			

Main formula: $f(x) = f(x_1) + (x - x_1)\Delta_{x_2} f(x_1) + (x - x_1)(x - x_2)\Delta^2_{x_2, x_3} f(x_1)$

$+ \_\_ + (x - x_1)(x - x_2) \ldots (x - x_n) \Delta^n_{x_2 \ldots x_{n+1}} f(x)$

$f(x) = f(300) + (x - 300)(0.00145) + (x - 300)(x - 304)(-0.00001)$
$+ (x - 300)(x - 304)(x - 305)(0.0000014)$

$f(301) = f(300) + (301 - 300)(0.00145) + (301 - 300)(301 - 304)(-0.00001)$
$+ (301 - 300)(301 - 304)(301 - 305)(0.0000014)$

$= 2.4785$

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x	y	x^2	xy
0	1	0	0
1	2.9	1	2.9
2	4.8	4	9.6
3	6.7	9	20.1
4	8.6	16	34.4
10	24	30	67

$a=1$
$b=1.9$

$m=5$

$y=ae^{bx}$

$\ln y = \ln a + bx$

$Y = a_0 + a_1 x$

$ma_0 + \Sigma x a_1 = \Sigma y$

$\Sigma x a_0 + \Sigma x^2 a_1 = \Sigma xy$

$5a_0 + 10a_1 = 24$

$10a_0 + 30a_1 = 67$

$a_0 = 1 \quad a_1 = 1.9$

An

x	y	$\ln y = Y$	x^2	ΣxY
0	1	0	0	0
1	2.9	1.06	1	1.06
2	4.8	1.56	4	3.12
3	6.7	1.90	9	5.7
4	8.6	2.15	16	8.6
10		6.67	30	18.48

$a = 1.36$
$b = 0.51$

$5a_0 + 10a_1 = 6.67$

$10a_0 + 30a_1 = 18.48$

$a_0 = 0.306$

$a_1 = 0.514$

$\ln a = a_0$

$a = e^{a_0} = 1.36$

An

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$$I = \int_0^1 \frac{dx}{1+x} \quad h = 0.25 .$$

Trapezoidal
simpson 1/3
n 3/8

$$h = \frac{b-a}{n}$$

$$n = \frac{b-a}{h} = \frac{1-0}{0.25} = 4.$$

	$a=0$	$a+h$	$a+2h$	$a+3h$	$a+4h$
	0	$0+0.25$	0.5	0.75	1
$f(x) = \frac{1}{1+x}$	$\frac{1}{1+0}$	$\frac{1}{1.25}$	$\frac{1}{1.5}$	$\frac{1}{1.75}$	$\frac{1}{2}$
	y_0	y_1	y_2	y_3	y_4

0.8 .666 0.571

$$\text{trapezoidal} = \frac{h}{2}\left[(y_0+y_4) + 2(y_1+y_2+y_3)\right]$$

$$= \frac{0.25}{2}\left[\left(1+\frac{1}{2}\right) + 2\left(\frac{1}{1.25} + \frac{1}{1.5} + \frac{1}{1.75}\right)\right]$$

$$= 0.697$$

$$\text{Simpson } 1/3 = \frac{h}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2(y_2)\right]$$

$$= \frac{0.25}{3}\left[\left(1+\frac{1}{2}\right) + 4\left(\frac{1}{1.25} + \frac{1}{1.75}\right) + \left(2 \times \frac{1}{1.5}\right)\right]$$

$$\cancel{= 1.039} = 0.693$$

$$\text{Simpson } 3/8 = \frac{3h}{8}\left[(y_0+y_4) + 3(y_1+y_2) + 2(y_3)\right]$$

$$= \frac{3h}{8}\left[\left(1+\frac{1}{2}\right) + 3 \times \left(\frac{1}{1.25} + \frac{1}{1.5}\right) + 2 \times \frac{1}{1.75}\right]$$

$$= 0.66$$

Jacobi-

$3x_1 + x_2 - 2x_3 = 9$

$-x_1 + 4x_2 - 3x_3 = -8$

$x_1 - x_2 + 4x_3 = 1$

$$x_1 = \frac{9 - x_2 + 2x_3}{3}$$

$$x_2 = \frac{-8 + x_1 + 3x_3}{4}$$

$$x_3 = \frac{1 - x_1 + x_2}{4}$$

~~1st~~ 1st itiration. $x_1 = 0,\ x_2 = 0,\ x_3 = 0$.

$$x_1 = \frac{9 - 0 + 0}{3} = 3$$

$$x_2 = \frac{-8}{4} = -2$$

$$x_3 = \frac{1}{4} = 0.25$$

2nd $x_1 = 3,\ x_2 = -2,\ x_3 = 0.25$

$$x_1 = \frac{9 + 2 + 2 \times 0.25}{3} = 3.83$$

$$x_2 = \frac{-8 + 3 + 3 \times 0.25}{4} = -1.07$$

$$x_3 = \frac{1 - 3 - 2}{4} = -1$$

x_1 বের করার

~~আর~~ Gauss-seidel method-এর ক্ষেত্রে

But x_2 & x_3 এর value বের করার সময় x_1 এর $/ x_2$ latest value use হবে।

somoy x_2 & x_3

x_2 & x_3 আগেরটা নাহয়

2nd iteration

$$x_1 = \frac{9 - (-2) + 2 \times 0.5}{3} = 3.83$$

$$x_2 = \frac{-8 + 3.83 + 3 \times .25}{4} = -0.855$$

$$x_3 = \frac{1 - 3.83 + (-0.855)}{4} = -0.921$$

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Lagranges Interpolation

$$y(x) = \frac{y_1(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} + \frac{y_2(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} +$$

$$\frac{y_3(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} + \frac{y_4(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)}$$

evaluate $\sqrt{155}$ by using Lagrange's Interpolation formula from following data

x	150 (x_1)	152 (x_2)	154 (x_3)	156 (x_4)
$y=\sqrt{x}$	12.247 (y_1)	12.329 (y_2)	12.410 (y_3)	12.490 (y_4)

3.15 saturday

0,1,2 $$L_2 = \frac{y_0(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + \frac{y_1(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + \frac{y_2(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$y(0.1)$, $y(x)$? $y' = x-y^2$

Given, $\frac{dy}{dx} = x - y^2$ $x_0=0, y_0=1$

$y' = x - y^2$; $y'(0) = 0-1^2 = -1$

$y'' = 1 - 2yy'$; $y''(0) = 1-2.1.(-1) = 3$

$y''' = -2yy'' - 2y'y'$; $y'''(0) = -2.1.3 - 2(-1)^2 = -8$

$= -2yy'' - 2(y')^2$

T.S. $y = y_0 + (x-x_0)y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0'''$

euler

$h=0.05$, $x_0=0$, $y_0=1$, $y'=x^2+y$. $h = \frac{x_n - x_0}{n} \Rightarrow n = \frac{x_n - x_0}{h} = \frac{.1-0}{0.05}$

$x_n = 0.1$ $= 2$

	x	y	$\frac{dy}{dx}=x^2+y$	$y + h\frac{dy}{dx}$
0	0.0	1	0^2+1	$1+0.05(1)$
1	0.05			
2	0.1			

Newton's forward difference interpolation for equal interval.

Let, $y=f(x)$ and $y_0, y_1, y_2, \cdots y_n$ are values corresponding to points $x_0, x_0+h, x_0+2h, \cdots$ x_0+nh. Suppose we want to find $f(x)=y$ at point $x=x_0+ph$. $\left(\text{or, } p=\frac{x-x_0}{h}\right)$

We kno by the definition of E (shifting operator)

$$E^p f(x) = f(x+ph)$$

$$\Rightarrow E^p f(x_0) = f(x_0+ph)$$

$$\Rightarrow f(x_0+ph) = \cancel{y(x_0+ph)}\ E^p f(x_0)$$

$$\therefore y(x) = f(x) = \cancel{f(x_0)} = E^p y_0$$

$$= (1+\Delta)^p y_0 \quad \left[\because E = 1+\Delta\right]$$

We know, by binomial expansion of

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}+\frac{n(n-1)(n-2)}{3!}+\cdots$$

$$\therefore (1+\Delta)^p y_0$$

$$= \left(1+p\Delta+\frac{p(p-1)\Delta^2}{2!}+\frac{p(p-1)(p-2)\Delta^3}{3!}+\cdots\right)y_0$$

$$= y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$