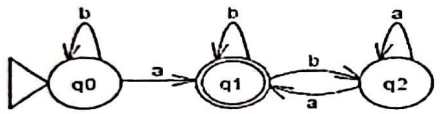
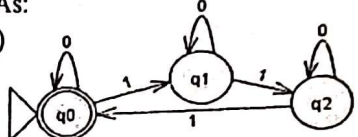


[N.B. Answer any Six questions taking Three from each section]

Section-A

1. a) Define alphabet, string and language. Discuss the basic operations of languages. 3
b) Define and classify Finite Automata (FA). 2.75
c) What are the ways by which a finite automata can be represented? Explain with example. 3
2. a) Construct minimal DFAs accepting the following languages: 3
i) $\{w \in \{a, b\}^* \mid w \text{ has even number of } a\text{'s}\}$
ii) $\{a^n b^m \mid m, n \geq 0\}$
b) Construct an NFA that accepts a binary language ending with 001. Convert it to a DFA using subset construction method. 4
c) Can an FA accept any formal language? If yes (or no), explain why? 1.75
3. a) Define regular expression. Write a regular expression for a language containing the strings starting and ending with same symbol over alphabet $\{a, b\}$. 2.5
b) Derive regular expressions from the following FAs: 3.5
i)  ii) 
- c) What is the regular language represented by the following left-linear grammar? 2.75
 $A \rightarrow Aa|Ab|Ba, B \rightarrow Cb, C \rightarrow \epsilon.$
4. a) Contrast between Moore machine and Mealy machine. 3
b) Construct a Mealy machine that takes binary numbers as input and produces 2's complement of that numbers as output. Assume that the string is read LSB to MSB and end carry is discarded. 3
c) Convert the above Mealy machine of 4(b) into Moore machine. 2.75

Section-B

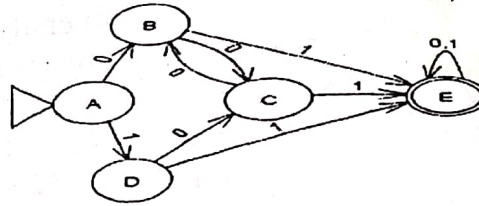
5. a) Write down the classification of problems. What are the tractable and intractable problems? Explain with examples. 2.75
b) What is formal language? Explain Chomsky's hierarchy of formal languages with relation to the corresponding grammar and computing machine. 4
c) What do you mean by derivation? Define sentence and sentential form. 2
6. a) Construct context-free grammars separately for each of the following languages: 4.75
i) $\{w \in \{a, b\}^* \mid w \text{ starts and end with different symbols}\}$
ii) $\{w \in \{a, b\}^* \mid w \text{ is palindrome}\}$
iii) $\{a^{2n} b^n \mid n \geq 1\}$.
b) Define Chomsky normal form. Convert the following CFG to Chomsky Normal Form (CNF): $S \rightarrow aX \mid Yb, X \rightarrow S \mid \epsilon, Y \rightarrow bY \mid b.$ 4
7. a) Define formally Pushdown Automata (PDA). There are two ways by which a context free language can be accepted. Explain, how? 2
b) Write down various operations of PDA. 3
c) Construct a PDA that accept the following language: $L = \{a^n b^m c^n \mid m, n \geq 1\}$. Write down the moves of the PDA for input string 'aabbcc' of the language L. 3.75
8. a) What is a Turing machine (TM)? What are the special features of TM? 3
b) What is the basic difference between 2-way FA and TM? 2
c) Briefly explain the different types of Turing machines. 3.75

Answer any three questions from each part

Part-A

1. Suppose that $G_1 = (V_1, \{a, b\}, S_1, P_1)$ and $G_2 = (V_2, \{a, b\}, S_2, P_2)$ are CFGs, and $V_1 \cap V_2 = \emptyset$.
 - a) It is easy to see that no matter what G_1 and G_2 are, the CFG $G_u = (V_u, \{a, b\}, S_u, P_u)$ defined by $V_u = V_1 \cup V_2$, $S_u = S_1$, and $P_u = P_1 \cup P_2 \cup \{S_1 \rightarrow S_2\}$ generates every string in $L(G_1) \cup L(G_2)$. Find grammars G_1 and G_2 (you can use $V_1 = \{S_1\}$ and $V_2 = \{S_2\}$) and a string $x \in L(G_u)$ such that $x \notin L(G_1) \cup L(G_2)$. 3
 - b) As in part (a), the CFG $G_c = (V_c, \{a, b\}, S_c, P_c)$ defined by $V_c = V_1 \cup V_2$, $S_c = S_1$, and $P_c = P_1 \cup P_2 \cup \{S_1 \rightarrow S_1 S_2\}$ generates every string in $L(G_1)L(G_2)$. Find grammars G_1 and G_2 (again with $V_1 = \{S_1\}$ and $V_2 = \{S_2\}$) and a string $x \in L(G_c)$ such that $x \notin L(G_1)L(G_2)$. 3
 - c) The CFG, $G = (V, \{a, b\}, S, P)$ defined by $V = V_1$, $S = S_1$, and $P = P_1 \cup \{S_1 \rightarrow S_1 S_1\}$ generates every string in $L(G_1)^*$. Find a grammar G_1 with $V_1 = \{S_1\}$ and a string $x \in L(G^*)$ such that $x \notin L(G)$. 2.75
2. a) Write down formal definition of a DFA. 1
 - b) Construct m-DFAs accepting the following languages: 6
 - i) $\{w \in \{a, b\}^* \mid w \text{ starts and end with 'a'}\}$
 - ii) $\{w \in \{0, 1\}^* \mid w \text{ is a binary number divisible by 3}\}$
 - iii) $\{a^n b^m c^l \mid l, m, n \geq 1\}$
 - c) Why the language $L = \{a^n b^n \mid n \geq 1\}$ is not regular? Explain. 1.75
3. a) Define regular expression. 1
 - b) Derive regular expressions from the following languages: 3
 - i) $\{w \in \{a, b\}^* \mid w \text{ has exactly two a's}\}$
 - ii) $\{w \in \{a, b\}^* \mid w \text{ starts and ends with same symbol}\}$.
 - c) Given a regular expression $x(x | y)^* z$. Convert it to a DFA via ϵ -NFA. 4.75

4. a) Minimize the following NFA:



- b) Construct a Moore Machine that takes set of all strings over $\{a, b\}$ as input and prints '1' as output for every occurrence of 'baa' as substring, otherwise print '0'. Using the machine produce an output string for the input string 'abaaababaa'.

4

- c) Let N be the set of natural numbers, T the set of nonempty subsets of N , and P the set of partitions of N into two nonempty subsets.

1.75

Suppose, $f : T \rightarrow P$ is defined by the formula $f(A) = \{A, N - A\}$ (in other words, for a nonempty subset A of N , $f(A)$ is the partition of N consisting of the two subsets A and $(N - A)$). Is f a bisection from T to P ? Justify your answer.

Part-B

5. a) The TM shown in Figure 5.1, computes a function from $\{a, b\}^*$ to $\{a, b\}^*$. For any string $x \in \{a, b\}^*$, describe the string $f(x)$.

5

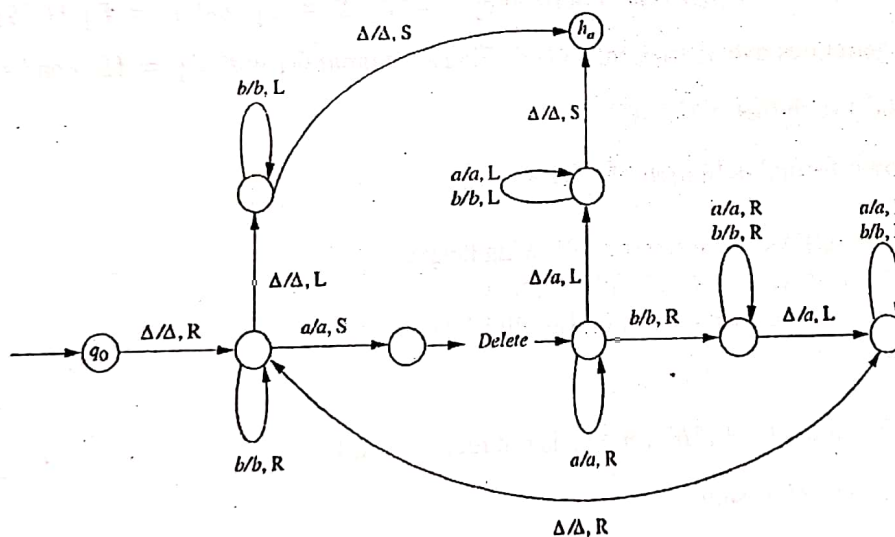


Figure 5.1: TM

- b) Draw a transition diagram for a TM with input alphabet $\{0, 1\}$ that interprets the input string as the binary representation of a nonnegative integer and adds 1 to it.

3.75

6. a) Construct context-free grammars separately for each of the following languages: 4.75
 i) $\{w \in \{a, b\}^* \mid w \text{ starts and end with different symbols}\}$
 ii) $\{w \in \{a, b\}^* \mid w \text{ is even length string}\}$
 iii) $\{a^n b^m c^n \mid m, n \geq 1\}$.
- b) Explain the necessity of normal form. Convert the following CFG to Chomsky Normal Form (CNF): $S \rightarrow aX \mid bS \mid ab \mid X, X \rightarrow aX \mid a \mid \epsilon$. 4
7. a) Construct pushdown automata that accept the following languages: 3.75
 i) $L = \{a^n c b^n \mid n \geq 1\}$, ii) $M = \{a^{m+n} b^m c^n \mid m, n \geq 1\}$
- b) Write down the moves of PDA for input string 'aacb' of the above language L. 2.5
- c) Suppose m_1 and m_2 are integers representing months ($1 \leq m_i \leq 12$), and d_1 and d_2 are integers representing days (d_i is at least 1 and no larger than the number of days in month m_i). For each i , the pair (m_i, d_i) can be thought of as representing a date; for example, (9, 18) represents September 18. We wish to write a logical proposition involving the four integers that says (m_1, d_1) comes before (m_2, d_2) in the calendar. Find such a proposition that is a disjunction of two propositions (i.e., combines them using \vee). 2.5
8. a) Show that if L can be accepted by a multi-tape TM with time complexity f , then L can be accepted by a one-tape machine with time complexity $O(f^2)$. 2.75
- b) Construct Turing machines to recognize each of the languages: 4
 $L = \{0^n 1^n 2 \mid n \geq 1\}$, $M = \{\text{all the strings containing substring '010' over } \{0, 1\}\}$
- c) Write down instantaneous descriptions of the TM for input string '00112' of the above language, L . 2

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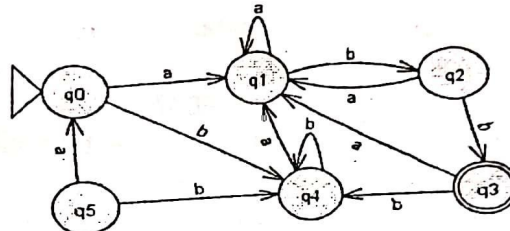
Course: CSE-4121 (Theory of Computation)

Full Marks-52.5 Time: 3 hours

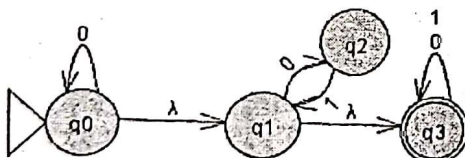
[N.B. Answer any six questions taking THREE from each of the groups]

Part-A

1. a) Define: i) Finite Automata (FA) and ii) Transition diagram. 3
 b) What are the applications of automata theory? 1.75
 c) Prove by Mathematical Induction $1+2+3+\dots+n=n(n+1)/2$. 4
2. a) Define regular expression. Convert the following regular expression in DFAs 4.75
 i) $(a + b)^*$, ii) $ab(a + b)^*$.
 b) Minimize the DFA. 4



3. a) Convert an NFA to DFA for the language containing "all strings in $\{0, 1\}^*$ in which the 2nd (or 3rd) symbol from the right hand side is 0". 5
 b) Given a left-linear grammar G with the set of production rules: $\{A \rightarrow Ba | Ab | b, B \rightarrow Ca | Bb, \text{ and } C \rightarrow Aa | Cb | a\}$. Draw a finite automata that accepts the language $L(G)$. 3.75
4. a) Convert the following ϵ -NFA to NFA (where, lambda stands for ϵ). 3.75



- b) What is Moore machine and Mealy machine? 2
 c) Construct a Mealy machine that takes binary no's as input and produces 2's complement of that numbers as output. Assume the string is read LSB to MSB and end carry is discarded. 3

Part-B

5. a) What is Computation? Briefly discuss the evolution of Theory of Computation. 3.75
 b) Define ambiguous grammar. 1
 c) What are the three ways to simplify a context free grammar? 2
 d) Differentiate sentences vs. sentential forms. 2
6. a) What is the function of grammar? How to ^{get} a string from a grammar? 2
 b) Construct context-free grammars that generate the following languages over $\{0, 1\}$: 4
 i). $\{w: w \text{ is any string of the symbols}\}$, ii) $\{0^n 1^{2n} \mid n \geq 1\}$, iii) $\{w: w \text{ is palindrome}\}$.
 c) What are languages generated by the CFG grammar with the following indicated productions: 2.75
 i) $S \rightarrow SaS \mid b \mid \epsilon$
 ii) $S \rightarrow aAa \mid bAb \mid a \mid b \mid \epsilon, A \rightarrow aA \mid bA \mid \epsilon$.
7. a) Define Pushdown Automata. 1.75
 b) Compare NFA and PDA. 2
 c) What is the significance of PDA? 1
 d) Define Chomsky normal form. Convert the following CFG to Chomsky normal form: 4
 $S \rightarrow XaX \mid bX \mid Y, X \rightarrow XaX \mid XbX \mid \epsilon, Y \rightarrow ab$.
8. a) Let $G = (V, \Sigma, R, S)$ be the context-sensitive grammar consists of the production rules: 3
 i) $S \rightarrow aSBC \mid aBC$ ii) $CB \rightarrow BC$ iii) $aB \rightarrow ab$ iv) $bB \rightarrow bb$ v) $bC \rightarrow bc$ vi) $cC \rightarrow cc$.
 Prove that $aabbcc \in L(G)$. 5.75
 b) Define Turing machine. Construct a Turing machine accepting the language $\{0^n 1^n 2^n \mid n \geq 1\}$.