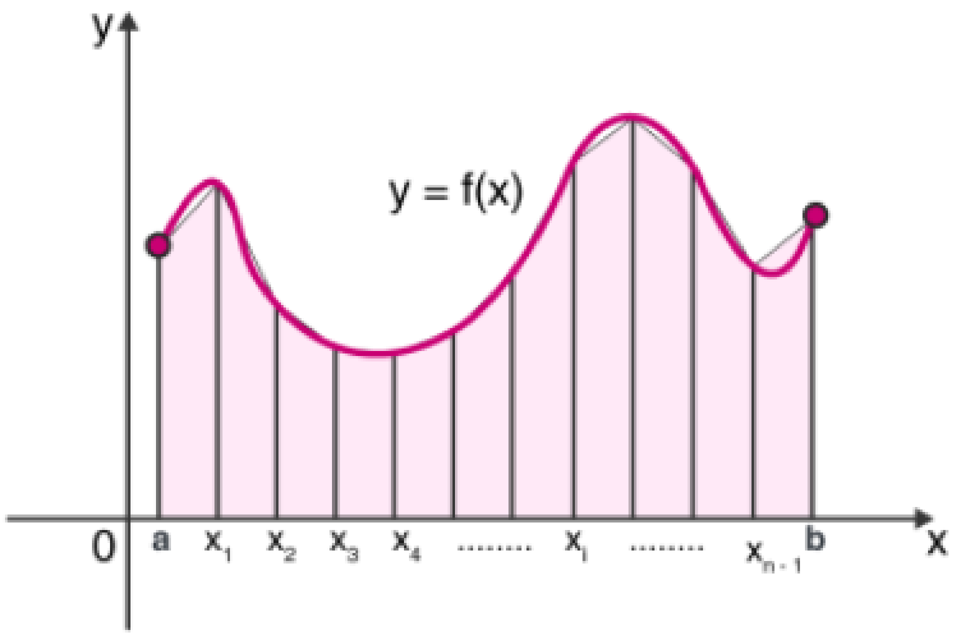
Trapezoidal Rule

Trapezoidal Rule

- The trapezoidal rule, also known as the trapezoid rule or trapezium rule, is a technique for approximating the **definite integral**.
- The trapezoidal rule works by approximating **the region under the graph** of the function *f(x)* as a trapezoid and calculating its area.
- The trapezoidal rule is to find the exact value of a definite integral using a numerical method.
- This rule is mainly based on the **Newton-Cotes formula** which states that one can find the exact value of the integral as an nth order polynomial.

Trapezoidal Rule

- **Trapezoidal Rule** is a rule that evaluates the area under the curves by dividing the total area into smaller trapezoids rather than using rectangles.



Trapezoidal Rule

Assume that f(x) be a continuous function on the given interval [a, b]. Now divide the intervals [a, b] into n equal subintervals with each of width,

$\Delta x = (b-a)/n$, Such that $a = x_0 < x_1 < x_2 < x_3 < \ldots\ldots < x_n = b$

Then the Trapezoidal Rule formula for area approximating the definite integral $\int_a^b f(x)\,\mathrm{d}x$ is given by:

$$\int_a^b f(x)\,\mathrm{d}x = \frac{\Delta x}{2}\left(f(x_0) + 2\,f(x_1) + 2\,f(x_2) + \cdots + 2\,f(x_{n-1}) + f(x_n)\right).$$

Where, $x_i = a + \Delta x$

If $n \rightarrow \infty$, R.H.S of the expression approaches the definite integral.

Example

Use the trapezoidal rule with *n = 8* to estimate: $\int_1^5 \sqrt{1+x^2}\,\mathrm{d}x$

Solution

Given, function : $\int_{1}^{5} \sqrt{1+x^2}\,\mathrm{d}x$

we know that, a=1, b=5 and n=8.

Now, substitute the values in the formula, we get

$\Delta x = (b-a)/n$

$= (5-1)/8$

$= 1/2$

Solution

Now, divide the interval into 8 subintervals with the length of $\Delta x = 1/2$, with the following endpoints,

$$a=1, 3/2, 2, 5/2, 3, 7/2, 4, 9/2, 5 = b$$

$\sqrt{1 + x^2}$

Solution

Now, compute the functions with these endpoints,

$f(x_0) = f(1) = \sqrt{2} = 1.4142135623731$

$2f(x_1) = 2f(3/2) = \sqrt{13} = 3.60555127546399$

$2f(x_2) = 2f(2) = 2\sqrt{5} = 4.47213595499958$

$2f(x_3) = 2f(5/2) = \sqrt{29} = 5.3851648071345$

$2f(x_4) = 2f(3) = 2\sqrt{10} = 6.32455532033676$

$2f(x_5) = 2f(7/2) = \sqrt{53} = 7.28010988928052$

$2f(x_6) = 2f(4) = 2\sqrt{17} = 8.24621125123532$

$2f(x_7) = 2f(9/2) = \sqrt{85} = 9.21954445729289$

$2f(x_8) = 2f(5) = \sqrt{26} = 5.09901951359278$

Now, substitute the values in the trapezoidal rule formula,

$$\int_a^b f(x)\,\mathrm{d}x = \frac{\Delta x}{2}\left(f(x_0) + 2\,f(x_1) + 2\,f(x_2) + \cdots + 2\,f(x_{n-1}) + f(x_n)\right).$$

= 1/4 (1.4142135623731 + 3.60555127546399 + 4.47213595499958 + 5.3851648071345 + 6.32455532033676 +7.28010988928052 + 8.24621125123532 + 9.21954445729289 + 5.09901951359278)

= 1/4(51.0465060317)

= 12.7616265079

Which can be approximately written as 12.76

Hence, $\int_1^5 \sqrt{1+x^2}\,dx \approx 12.76$

Example-2

Approximate the area under the curve y = f(x) between x =0 and x=8 using Trapezoidal Rule with n = 4 subintervals. A function f(x) is given in the table of values.

x	0	2	4	6	8
f(x)	3	7	11	9	3

Solution

The Trapezoidal Rule formula for n= 4 subintervals is given as:

T_4 =(Δx/2)[**$f(x_0)$+ $f(x_4)$**+ $2f(x_1)$+ $2f(x_2)$+$2f(x_3)$]

Here the subinterval width Δx = (8-0)/4=2.

Now, substitute the values from the table, to find the approximate value of the area under the curve.

$\int_0^8 f(x)\,\mathrm{d}x$ =(2/2)[3 + 3 + 2(7)+ 2(11)+2(9)]

= 3 + 14 + 22+ 18+3 = 60

Therefore, the approximate value of area under the curve using Trapezoidal Rule is 60.

Find Solution using Trapezoidal rule

x	0	0.1	0.2	0.3	0.4
y	1	0.9975	0.99	0.9776	0.8604

Solution by Trapezoidal Rule is 0.38953