University of Rajshahi

Department of Computer Science and Engineering

B. Sc. (Engg) Part-II Even Semester Examination 2020

Course: MATH 2241 (Linear Algebra)

Full Marks: 52.5 Duration: 3 (Three) Hours

Answer 06 (Six) questions taking any 03 (Three) from each section

Section-A

1. a) Define a vector in $\mathbb{R}^n$. What do you mean by linear combination of vectors in $\mathbb{R}^n$? 3

   b) Consider the system of linear equations 3

$$x - 2y = 2$$
$$2x - 4y = -2$$

   Draw row picture and column picture, and explain the solution of the system of the linear equations based on the pictures.

   c) Find a unit vector u in the direction of $v = (3, 4)$. Find a unit vector w that is perpendicular to u. How many possibilities for w? 2.75

2 - 2 2-7

2. a) Define a *vector space*. Let H be the set of all vectors of the form $(a - 3b, b - a, a, b)$, where a and b are arbitrary scalars. Show that H is a subspace of $\mathbb{R}^4$. 3.75

   b) Define *Null space*. Find a spanning set for the null space of the matrix 3

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}.$$

   c) Let $A = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$ and $u = (3, -2, -1, 0)$. 2

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   Determine if u is in $Nul\ A$. Could u be in $Col\ A$?

3. a) What do you mean by linearly dependent and linearly independent set of vectors? Define basis for a vector subspace H. 2

   b) Consider figure 3(a) and let $H = \text{Span}\{u, v, w\}$ and $w = u + v$. 3.75

   Show that $\text{Span}\{u, v, w\} = \text{Span}\{u, v\}$. Then find a basis for the subspace H.

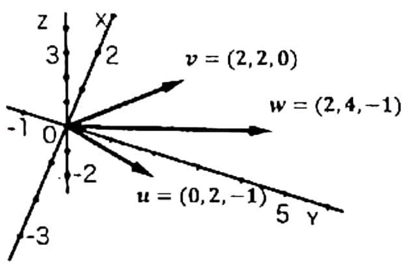

Figure - 3(a)

   c) Let $u = (2, 1)$, $v = (-1, 1)$, $x = (4, 5)$, and $\mathcal{B} = \{u, v\}$. Find the coordinate vector $[x]_{\mathcal{B}}$ of x relative to $\mathcal{B}$. 2

   d) Let $v_1 = (1, -2, 2)$ and $v_2 = (-3, 7, -8)$. Is $\{v_1, v_2\}$ a basis for $\mathbb{R}^2$? 1

4. a) Consider the bases $\{e_1 = (1,0),\ e_2 = (0,1)\}$ and $\{f_1 = (1,3),\ f_2 = (2,5)\}$ of $\mathbb{R}^2$. 2.25+3
(i) Find the transition matrix P from $\{e_i\}$ to $\{f_i\}$.
(ii) Show that $[T]_f = P^{-1}[T]_e P$ for the linear operator T on $\mathbb{R}^2$ defined by
$T(x,y) = (2y, 3x - y)$.

b) Let T be the linear operator on $\mathbb{R}^2$ defined by $T(x,y) = (4x - 2y, 2x + y)$. 3.5
Verify that $[T]_f[v]_f = [T(v)]_f$ for any vector $v \in \mathbb{R}^2$.

Section-B

5. a) Consider figure 5(a). Is v an eigenvector of the matrix A ? 1

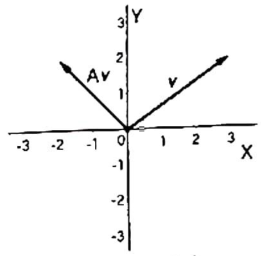

Figure- 5(a)

b) Show that 7 is an eigenvalue of matrix $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$, and find the corresponding eigenvectors. 3.75

c) Prove that the set $\{v_1, v_2, \dots\dots, v_r\}$ is linearly independent where $v_1, v_2, \dots\dots, v_r$ are eigenvectors that correspond to distinct eigenvalues $\lambda_1, \lambda_2, \dots\dots, \lambda_r$ of an $n \times n$ matrix A. 4

6. a) Define ***similarity*** of two matrices. Prove that two $n \times n$ ***similar*** matrices A and B have the same characteristic polynomial and hence the same eigenvalues. 3

b) Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a formula for A^k, given that $A = PDP^{-1}$, 3.75
where $P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$.

c) Suppose A is a 2×2 matrix. One eigenvalue of A is $\lambda_1 = 0.8 - 0.6i$ and its corresponding eigenvector $v_1 = (-2 - 4i, 5)$. Find another eigenvalue and its corresponding eigenvector. 2

7. a) Define inner product of two vectors and length of a vector. 1

b) Let W be the subspace of $\mathbb{R}^2$ spanned by $x = \left(\frac{2}{3}, 1\right)$. Find a unit vector z that is a basis for W. 2

c) The set $S = \{u_1, u_2, u_3\}$ is an orthogonal basis for $\mathbb{R}^3$, where $u_1 = (3,1,1)$, $u_2 = (-1,2,1)$ and $u_3 = \left(-\frac{1}{2}, -2, \frac{7}{2}\right)$. Express the vector $y = (6,1,-8)$ as a linear combination of the vectors in S. 3
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d) Let $u_1 = (2,5,-1)$, $u_2 = (-2,1,1)$, and $y = (1,2,3)$. Observe that $\{u_1, u_2\}$ is an orthogonal basis for $W = \text{Span}\,\{u_1, u_2\}$. Write y as the sum of a vector in W and a vector orthogonal to W. 2.75

8. a) Define an inner product space. State and prove Cauchy Schwartz inequality in an inner product space. 4.5

b) Use Gram-Schmidt orthogonalization process to transform the basis $\{v_1 = (1,1,1), v_2 = (0,1,1), v_3 = (0,0,1)\}$ of $\mathbb{R}^3$ into an orthonormal basis $\{u_i\}$. 4.25