

# Theory of Computation

Subject : 2018

Date : \_\_\_\_\_

- 1(a) Define Alphabet, string, language. Discuss basic operations<sup>3</sup> of language.

Alphabet: An alphabet is a finite set of symbols, such as  $\{a, b\}$  or  $\{0, 1\}$  or  $\{A, B, C, \dots, Z\}$ .

We will usually use the Greek letter  $\Sigma$  to denote the alphabet. Example,  $\Sigma = \{0, 1\}$  is an alphabet of binary digits.

String: A String is a finite sequence of symbols selected from some alphabet. It is generally denoted as  $w$ . For example for alphabet  $\Sigma = \{0, 1\}$ ,  $w = 010101$  is a string.

A string over  $\Sigma$  is a finite sequence of symbols in  $\Sigma$ . For a string  $w$ ,  $|w|$  stands for the length (the no. of symbols) of  $w$ , and the length of a string denoted as  $|w|$ .

## Note Language

$\Sigma^*$  is the set of all possible strings (often power set (need not be unique have or we can say multiset) of strings) so this implies that language is a subset of  $\Sigma^*$ .

→ Empty string is the string with zero occurrence of symbol represented as  $\epsilon$ .

Subject : .....

Date : 

--	--	--

Number of strings (of length 2) generated over alphabets  $\{a, b\}$  - aa, ab, ba, bb.

length of string  $|w|=2$ , Number of strings = 4.

Language: A language over  $\Sigma$  (alphabet) is a subset of  $\Sigma^*$  (the set of all strings over  $\Sigma$ ).

This means that language  $L$  is subset of  $\Sigma^*$ .

An example is English language, where the collection of legal English words is a set of strings over the alphabet that consists of all the letters.

The language formed over ' $\Sigma$ ' can be finite or infinite.

Example of Finite language.

$$\Sigma = \{a, e, y\}$$

$$L_1 = \{\text{set of strings of } 2\}$$

$$L_1 = \{xy, xx, ye, yy\}$$

Example of Infinite language:

$$L_1 = \{\text{set of all strings starting with 'b'}\}$$

$$L_1 = \{ba, baa, babb, bbb, \dots\}$$

2018

## 1(b) Define &amp; Classify finite automata

Finite automata is an abstract computing device. It is a mathematical model of a system with discrete inputs, outputs, states and set of transitions, from state to state that occurs on input symbols from alphabet  $\Sigma$ .

Its representation:

- Graphical (Transition Diagrams or Transition table)
- Tabular (Transition table)
- Mathematical (Transition function or Mapping function)

Formal definition of FA.

A finite automata is 5-tuples; they are -

$$M(Q, \Sigma, \delta, q_0, F)$$

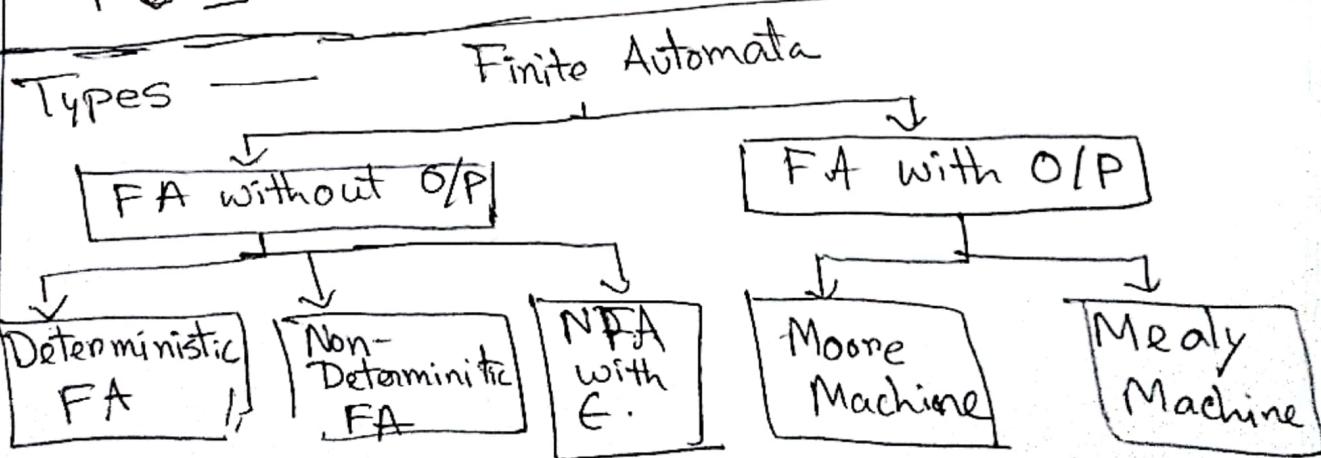
$Q$  : finite set of states.

$\Sigma$  : finite set of alphabets

$\delta$  :  $Q \times \Sigma \rightarrow Q$  transition function

$q_0 \in Q$  : start state or initial state

$F \subseteq Q$  : Set of accept states / final states.



## 1(b) Define and Classify Finite Automata

Ans: FA is a simple idealized machine used to recognize patterns within input taken from some character set (or alphabet). The job of an Finite Automaton is to accept or reject an input depending on whether the pattern defined by FA occurs in input.

A. D-Finite Automaton consists of: (Deterministic)

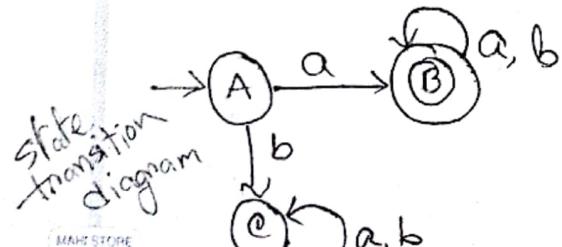
- 1) A finite set  $S$  of  $N$  states:  $Q$
- 2) a special start state /input alpha.
- 3) a set of final (or accepting) state.
- 2) A finite input alphabet:  $\Sigma$
- 3) The initial state:  $q_0 \in Q$
- 4) The set of accepting states:  $A \subseteq Q$
- 5) The transition function:  $\delta: Q \times \Sigma \rightarrow Q$

DFA

A Finite automaton (FA) is a 5-tuple  $(Q, \Sigma, q_0, A, \delta)$  where

for any element  $q$  of  $Q$  and any symbol  $\alpha \in \Sigma$ , we interpret  $\delta(q, \alpha)$  as the state of which the FA moves, if  $q_0$  is in state  $q$  and receives the input  $\alpha$ .

$L(m)$  = set of all strings start with  $a$        $\Sigma = \{a, b\}$   
 $= \{a, ab, aab, abb, aaab, \dots\}$



Finite automaton

String: aabb  
 $A \rightarrow B \rightarrow B \rightarrow B \rightarrow B \Rightarrow$  Yes in language

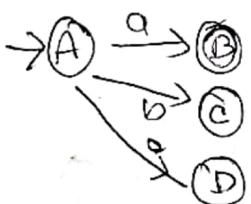
String: baba  
 $A \rightarrow C \rightarrow C \rightarrow C \not\rightarrow$  Not in language

# Introduction to NFA

SAT SUN MON TUE WED THU FRI

Date: \_\_\_\_\_

## Non-deterministic Finite Automata:



From one to 0, 1 or multiple no. of states.



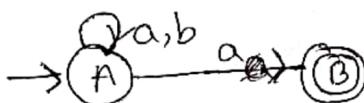
Deterministic

5-Tuple  $(Q, \Sigma, q_0, A, \delta)$  where

$\{q\} = \text{finite set of states}$   
 $\{\Sigma\} = \text{finite input alphabet}$   
 $\{q_0 \in Q\} = \text{initial state}$   
 $\{A \subseteq Q\} = \text{set of accepting states.}$

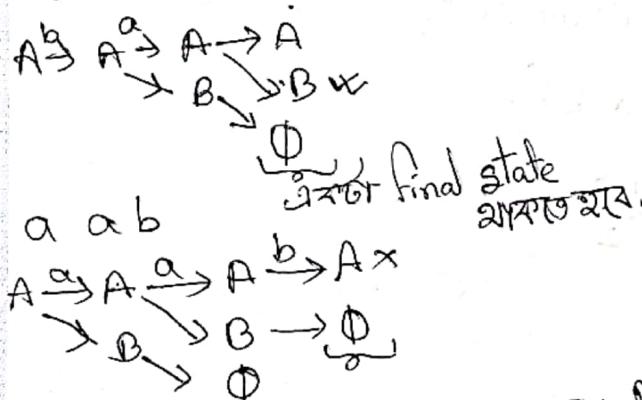
$\delta: Q \times (\Sigma \cup \{A\}) \rightarrow 2^Q$  (subset of  $Q$ )  $S: Q \times \Sigma \rightarrow 2^Q$

$\delta \neq S$    
 different from DFA  
 Example:  $L = \{ \text{ends with } a \} . \Sigma = \{a, b\}$   
 $= \{a, aa, aaa, ba, \dots\}$



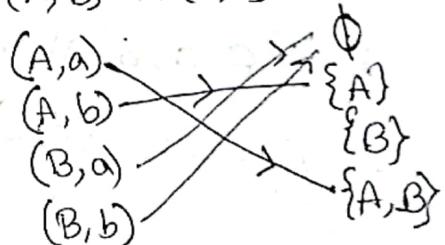
same alphabet 'a' द्वारा एक state ने  
travel possible & last state/final  
state की transition द्वारा अनेक रूप  
में होती है।

baa.



$Q \times \Sigma \rightarrow 2^Q$

$\{A, B\} \times \{a, b\}$



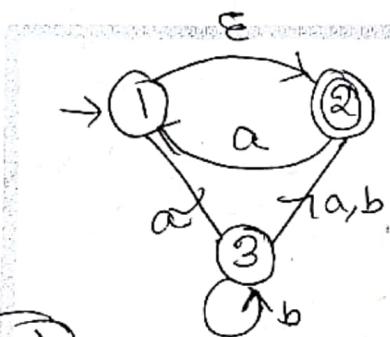
All DFA are NFA.

# Notes DFA :  $Q \times \Sigma \rightarrow Q$   
NFA :  $Q \times \Sigma \rightarrow 2^Q$

# $\epsilon$ -NFA to DFA Conversion

SAT SUN MON TUE WED THU FRI

Date: \_\_\_\_\_



Step-1

	a	b
$\rightarrow 1$	{3}	$\emptyset$
*2	{1}	$\emptyset$
3	*{2}	{2,3}

STT of  $\epsilon$ -NFA

Step-2

$$\epsilon\text{-closure}(1) = \{1, 2\}$$

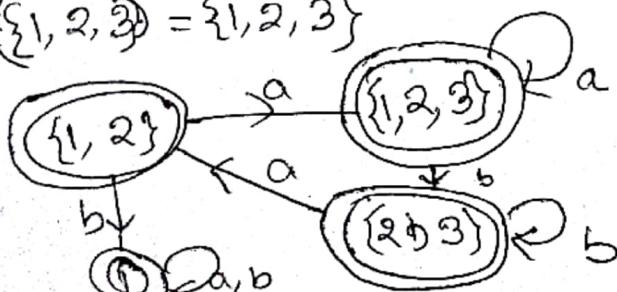
$$\epsilon\text{-closure}(2) = \{2\}$$

$$\epsilon\text{-closure}(3) = \{3\}$$

Step-3

	a	b
$\rightarrow \{1, 2\}$	*{1, 2, 3}	$\emptyset$
*{1, 2, 3}	*{1, 2, 3}	*{2, 3}
*{2, 3}	*{1, 2}	*{2, 3}

$$E(\{1, 2, 3\}) = \{1, 2, 3\}$$



Rules

- 1) STT for  $\epsilon$ -NFA
- 2) Find  $\epsilon$ -closure for all states
- 3) Make STT for DFA  
 $E(\bigcup_{n \in N} E(\{n\}))$

$$4) q'_0 = E(\{q_0\})$$

$\epsilon$ -NFA মানে একটা

$\epsilon$ -edge আছে

$\epsilon$ -closure মানে এলো যে state টাকে আমরা select করছি ত্রুটি এ পেলে মানে কোন value না পেলে কোন কোন state এ যেতে পাবে।

মানে এলো কোন string না দেখে অহি state-কে গো কোম্বক্ষু যেতে পাবে।

$$\begin{aligned} q'_0 &= E(\{q_0\}) \\ &= E(1) \\ &= \{1, 2\} \end{aligned}$$

$$\{1, 2\} \xrightarrow{a} \{1, 3\} \quad \text{১৩}$$

এবাব দুই কোনো  $\epsilon$ -closure কোর করতে হবে।

$$E(\{1, 3\}) = \{1, 2, 3\}$$

$$\epsilon\text{-closure}(1) \cup \epsilon\text{-closure}(3)$$

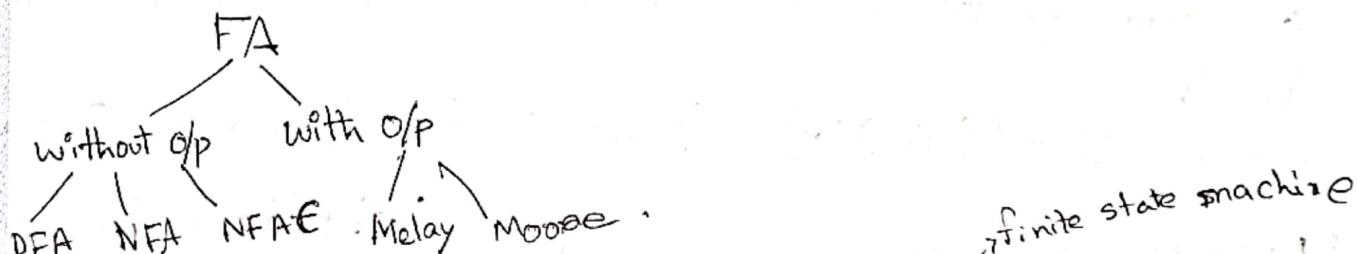
$$\{1, 2\} \cup \{3\} = \{1, 2, 3\}$$

3)  $G_3 = (\{S, A, B\}, \{a, b\}, S, \{S \rightarrow AB, A \rightarrow aA|a, B \rightarrow bB|b\})$

$$\begin{array}{llll} S \rightarrow AB & S \rightarrow AB & S \rightarrow AB & S \rightarrow AB \\ \rightarrow ab & \rightarrow aAbB & \rightarrow aAb & \rightarrow abB \\ & \rightarrow aabb & \rightarrow aab & \rightarrow abb \end{array}$$

$$L(G_3) = \{ab, \tilde{a}b^r, \tilde{a}^b, ab^r, \dots\}$$

$$= \{a^m b^n \mid m \geq 1 \text{ and } n \geq 1\} \quad m, n \geq 1$$



Melay Machine: A melay machine is a FSM whose output depends on the present state as well as the present input. [O/P depends on  $\rightarrow$  Present state + I/P]

6 tuples:  $(Q, \Sigma, O, \delta, X, q_0)$

$Q = Q$  is finite set of states

$\Sigma = \Sigma$  is finite set of symbols called I/P alphabet

$O = O$  is set of symbols called O/P alphabet

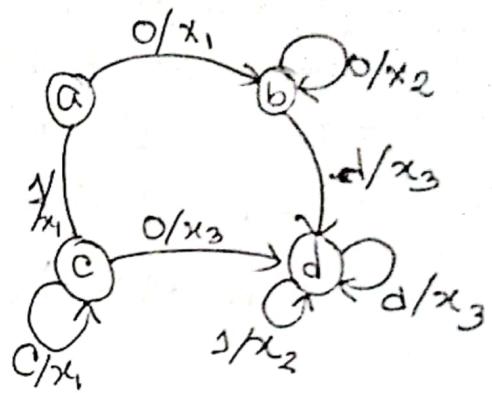
$\delta = \delta$  is I/P transition func  $\delta: Q \times \Sigma \rightarrow Q$

$X = X$  is O/P transition function where  $X: Q \times \Sigma \rightarrow O$

$q_0 = q_0$  is initial state from where any input is processed ( $q_0 \in Q$ )

Example: State table of Mealy Machine

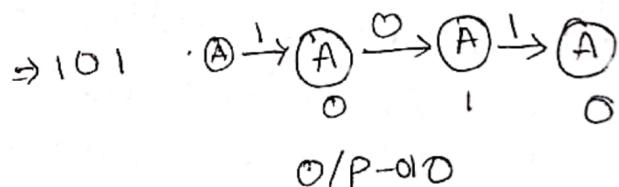
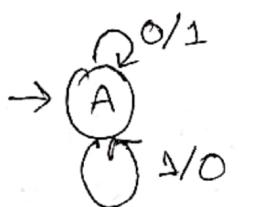
Present State	Next State			
	Input=0		Input=1	
	State	O/P	State	O/P
$\rightarrow a$	b	$x_1$	c	$x_1$
b	b	$x_2$	d	$x_3$
c	d	$x_3$	c	$x_1$
d	d	$x_3$	d	$x_2$



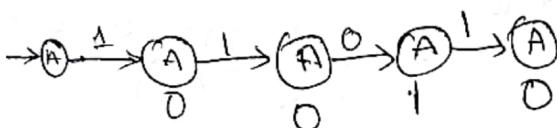
Example: Construct Mealy M/c that produce is complement of any binary input string.

$$\text{Sol: } \Sigma = \{0, 1\}$$

$$\Rightarrow 01001 - \text{I/P} \\ \text{Is Compl} \\ 10110 \xrightarrow{\text{O/P}} 0/\text{P}$$



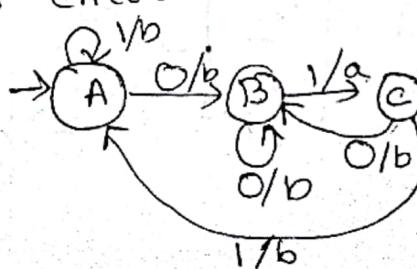
$$1101 \Rightarrow \text{i/P}$$



$$0010 \Rightarrow \text{O/P}$$

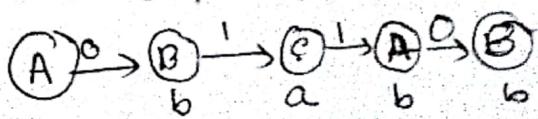
example: Construct a mealy m/c that points 'a' whenever the sequence '01' is encountered in any i/p binary string.

$$\text{Sol: } \Sigma = \{0, 1\} \text{ I/P} \\ \text{O/P } O = \{a, b\}$$



$$\frac{01}{a} \frac{01}{a} \frac{001}{a}$$

$$0110 \dots$$

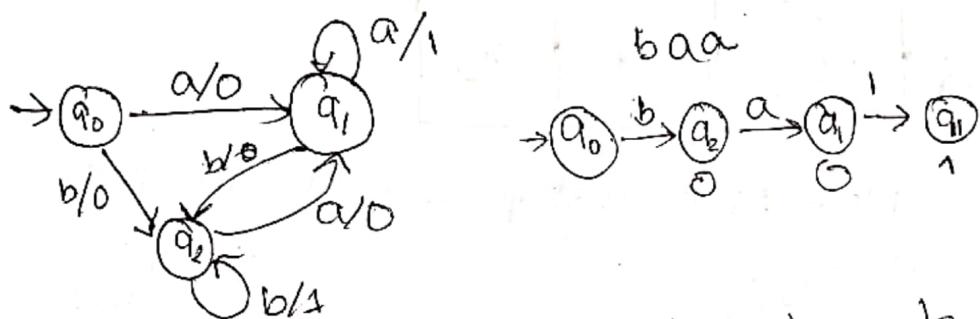


Mealy m/c accepting lang. consisting of strings from  $\Sigma^*$ , where  $\Sigma = \{a, b\}$  & the strings should end with either aa & bb.

$$\text{Soln: } \Sigma = \{a, b\} \quad O/P \cdot O = \{0, 1\}$$

String should end with either aa or bb.

Assume that the string ends with aa or bb.  
generates  $O/P = 1$   
else  $O/P = 0$ .



Moore: Is a FSM whose output depends on only the present state. A moore Machine can be described by

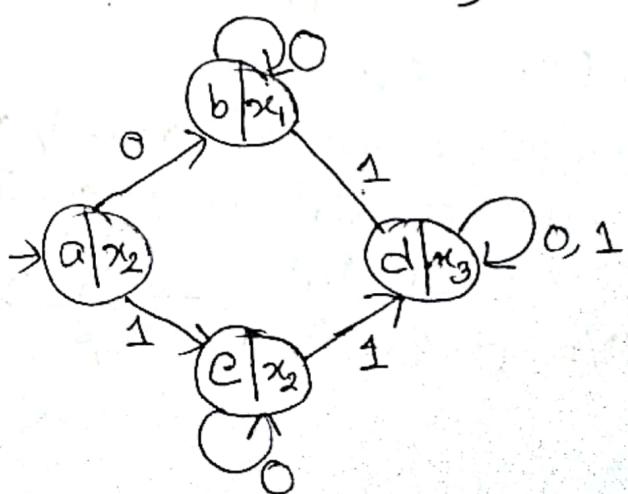
a 6 tuple  $(Q, \Sigma, O, \delta, x, q_0)$   
 $Q$ : finite set of states,  $\Sigma$ : finite set of symbols i/p alphabet

$O$ : finite set of o/p alphabet,  $\delta$ : i/p transition func  $\delta: Q \times \Sigma \rightarrow Q$

$x$ : o/p transition func  $x: Q \rightarrow O$ ,  $q_0$ : initial state from where any i/p is processed

$(q_0 \in Q)$

Present state	Next state		Output
	Input=0	Input=1	
$\rightarrow a$	b	c	$x_2$
b	b	d	$x_1$
c	c	d	$x_2$
d	d	d	$x_3$



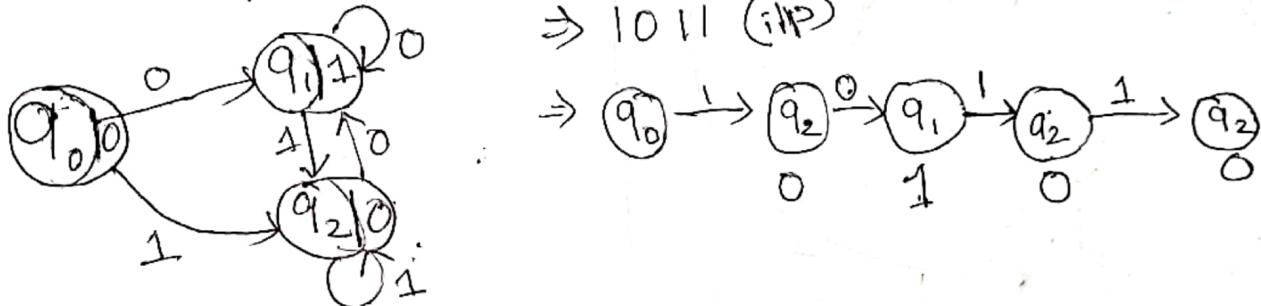
SAT SUN MON TUE WED THU FRI

Date \_\_\_\_\_

Example: Design Moore M/c to generate 1's complement of a given binary no.

Sol<sup>n</sup> If i/p 0 → O/P 1  
1 → 0

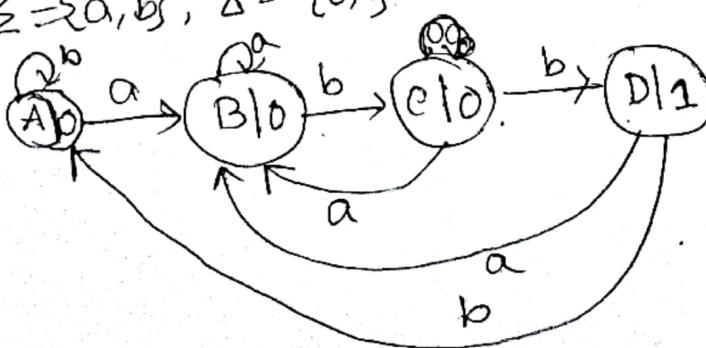
3 states are required → Start state, Second for tracing 0's as i/p & produce 1 as O/P, third is tracing 1's as i/p & produce 0 as O/P.



Current State	next state		O/P
	0	1	
$\rightarrow q_0$	$q_1$	$q_2$	0
$q_1$	$q_1$	$q_2$	1
$q_2$	$q_1$	$q_2$	0

\*Construct a Moore Machine that counts the occurrence of sequence 'abb' in any input strings over  $\{a, b\}$

$$\Sigma = \{a, b\}, \Delta = \{0, 1\}$$



$$A \xrightarrow{a} B \xrightarrow{b} C \xrightarrow{b} D$$

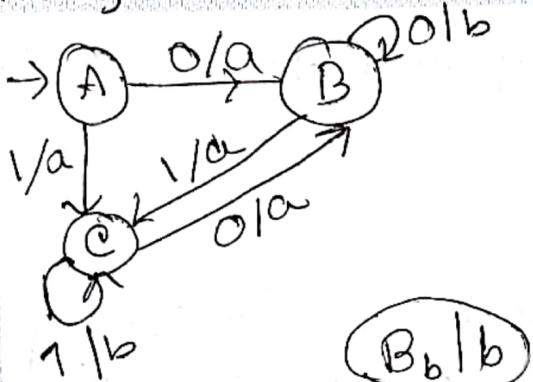
0 0 0 1

$$A \xrightarrow{a} B \xrightarrow{b} C \xrightarrow{b} A \xrightarrow{b} D \xrightarrow{a} B \xrightarrow{b} C \xrightarrow{b} D$$

0 0 0 1 0 0 1

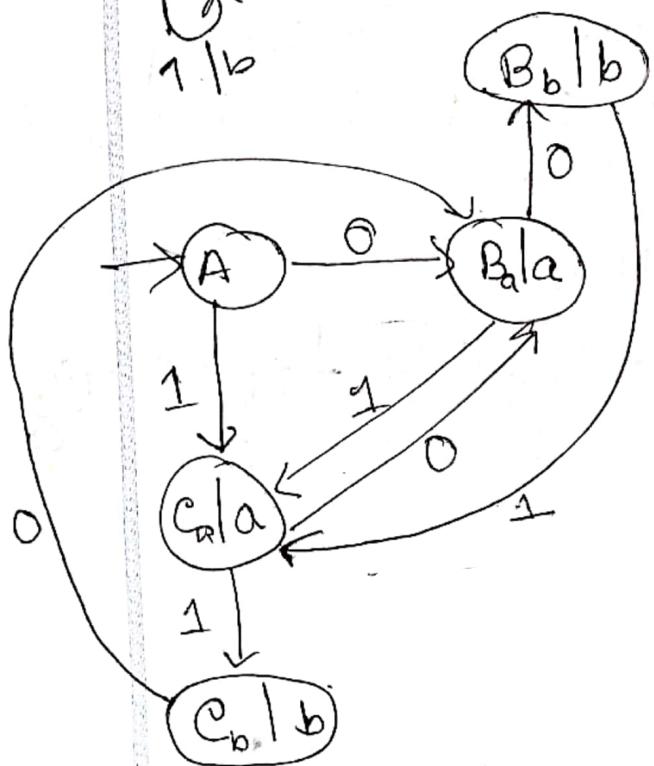
## Conversion

Mealy to Moore



Mealy to Moore  $\Rightarrow$   
no. of states increased

Moore  $\rightarrow$  Mealy  $\Rightarrow$  no. of  
states were one same.



2019

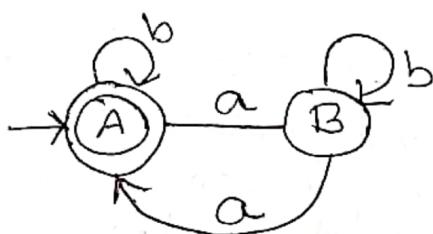
Subject : .....

Date : \_\_\_\_\_

2(a) Construct m DFA accepting the following languages:

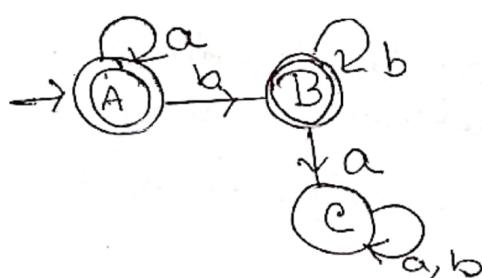
i)  $\{w \in \{a,b\}^* \mid w \text{ has even number of } a's\}$

$$L(M) = \{ \epsilon, baab, aba, aabaa, abab, \dots \}$$



ii)  $\{a^m b^n \mid m, n \geq 0\}$  [এটায় sequence matters কৰা, a এর পরে must b আসবে]

$$L(M) = \{ \epsilon, a, b, ab, aab, abb, aabb, \dots \}$$



[b এর পরে a  
accepted হা]

bax  
(abv  
caa .. -br  
(b .. -ax  
(babax  
~~bax~~

3(a) Define regular expression. Write a regular for a language containing the strings starting & ending with same symbol over alphabet {a,b}

Regular expression: If  $\Sigma$  is an alphabet, the set  $R$  of regular languages over  $\Sigma$  is defined as follows.

Regular languages over  $\Sigma$  are elements of  $R$ , and for every  $a \in \Sigma$ , the language  $\{a\}$  is in  $R$ .

①. The language  $\emptyset$  is an element of  $R$ , and for every

2018

- 2(b) Construct an NFA that accepts a binary language ending with 001. Convert it to a DFA using subset construction method.

Solution: Given,  $\Sigma = \{0, 1\}$

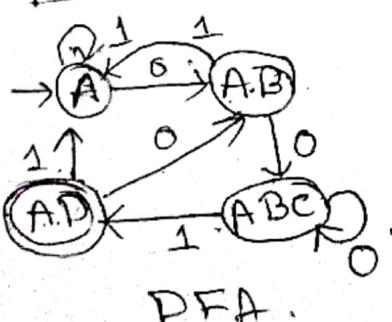
The  $L(M) = \{\text{Binary w/w ends with 001}\}$



The state transition table of the NFA.

Digital Products

	0	1
A	{A, B}	{A}
B	{C}	{}
C	{}	*{D}
D	{}	{}



DFA.

The STT for DFA using STT of NFA.

	0	1
[A]	[AB]	[A]
[AB]	[ABC]	[A]
[ABC]	[ABC]	*[AD]
*[AD]	[AB]	[A]

② For any two language  $L_1$  and  $L_2$  in  $R$ , the three languages  $L_1 \cup L_2$ ,  $L_1 L_2$  and  $L_1^*$  are elements of  $R$ .

~~The lang-the~~

Regular expression: The language accepted by finite automata can be easily described by simple expression called Regular expression. It is the most efficient way to represent any language.

The languages accepted by some regular expression are referred to as Regular string.

A regular expression can also be described as a sequence of pattern that defines a string.

Regular expression are used to match character combinations in string.

Example:

In a regular expression,  $\infty$  means Zero or more occurrence of  $x$ . It can generate

$$\{ \epsilon, x, xx, xxx, xxxx, \dots \}$$

In a regular expression  $\infty$  means One or more occurrence of  $x$ . It can generate

$$\{ x, xx, xxx, \dots \}$$

The various operations on regular language are -

Union: If  $L$  &  $M$  are two regular language then their union  $\rightarrow L \cup M$  is also a union.

$$1. L \cup M = \{ s | s \text{ is in } L \text{ or } s \text{ is in } M \}$$

Intersection: If  $L$  and  $M$  are two regular language then their intersection is also an intersection.

$$1. L \cap M = \{ st | s \text{ is in } L \text{ and } t \text{ is in } M \}$$

Kleen Closure: If  $L$  is a regular language then its Kleen closure  $L^*$  will also be a regular language.

1.  $L^* = \text{zero or more occurrence of language } L$ .

Example 1: The regular expression of language accepting all combination of 'a's over the set  $\Sigma = \{a\}$ .

Soln: All combination of "a" means 'a' may be zero, single double, and so on. If 'a' is appearing zero times, that is null string.

$$R. E \Rightarrow R = a^*$$

Example-2: ... all ...  
of 'a's except null string  
over the set  $\Sigma = \{a\}$

Soln: Regular expression for language

$$L = \{a, aa, aaa, \dots\}$$

This set indicates that there is no null string.

$$R = a^*$$

3. All string containing any number of 'a's and 'b's.

$$n. e = (a+b)^*$$

$$L = \{\epsilon, a, aa, bb, b, ab, ba, \dots\}$$

Subject : .....

Date :

\* Write regular expression for a language containing the string starting and end ending with same symbol over alphabet  $\{a, b\}$ .

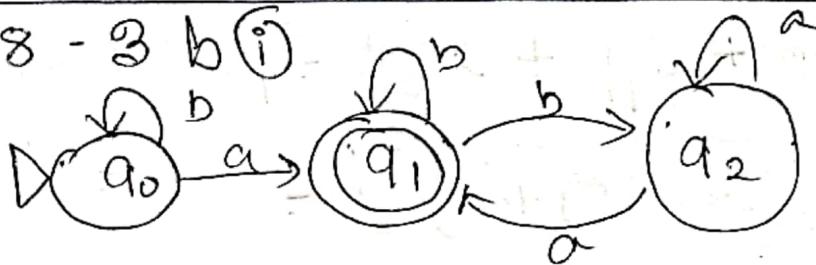
$$\underline{a \cup b} \cup \underline{b \in^* b} \cup \underline{a \in^* a}$$
$$a + b + b(a+b)^*b + a(a+b)^*a$$

$L = \{w \mid w \text{ contains at least two } 1's\}$   $\Sigma = \{0, 1\}$

$$\begin{aligned} & \in^* 1 \in^* 1 \in^* \\ & (0+)^* + 1 + (0+)^* + 1 + (0+)^* \end{aligned}$$

$L = \{11, -$

2018 - 3 b i



Hence,

$$q_0 = \epsilon + q_0 b \quad \textcircled{1}$$

$$q_1 = q_0 a + q_1 b + q_2 a \quad \textcircled{11}$$

$$q_2 = q_1 b + q_2 a \quad \textcircled{111}$$

From  $\textcircled{1} \Rightarrow q_0 = \epsilon + q_0 b$  [Arden's Theorem]

$$\begin{aligned} &= \epsilon b^* \\ q_0 &= b^* \rightarrow \textcircled{11} \end{aligned}$$

$$\begin{aligned} R &= Q + RP \\ &= QP^* \end{aligned}$$

From  $\textcircled{111} \Rightarrow q_2 = q_1 b + a q_2$ 

$$= q_1 b a^* \quad [\text{Arden's Theorem}]$$

$$\therefore q_2 = q_1 b a^* \quad \textcircled{111}$$

Putting values from  $\textcircled{11}$  &  $\textcircled{111}$  in  $\textcircled{111}$  we get,

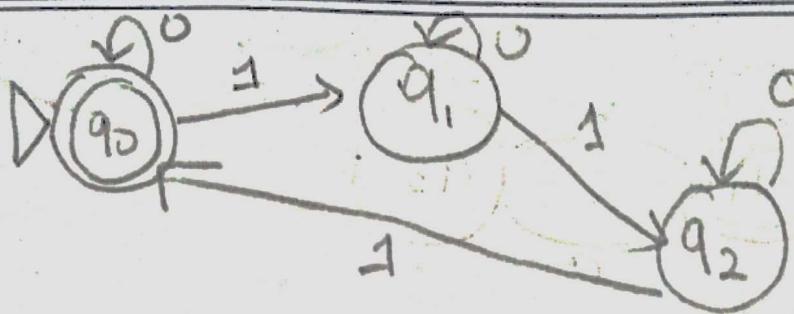
$$q_1 = b^* a + q_1 b + q_1 b a^* a$$

$$= b^* a + (b + b a^* a) q_1$$

$$q_1 = b^* a (b + b a^* a)^*$$

Ans.

3(b) ii)



$$q_0 = \epsilon + q_0'0 + q_0''1 \rightarrow \textcircled{1}$$

$$q_1 = q_1'0 + q_1''1 \rightarrow \textcircled{II}$$

$$q_2 = q_2'0 + q_2''1 \rightarrow \textcircled{III}$$

$$\textcircled{IV} \Rightarrow q_2 = q_1'' + q_2'0$$

$$= q_1''0^* \rightarrow \textcircled{IV}$$

$$\textcircled{V} \Rightarrow q_1 = q_0''1 + q_1'0$$

$$= q_0''10^* \rightarrow \textcircled{VI}$$

$$\textcircled{I} \Rightarrow q_0 = \epsilon + q_2''1 + q_0'0$$

$$= \epsilon + q_1''10^*1 + q_0'0 \quad [\text{from } \textcircled{IV}]$$

$$= \epsilon + q_0''10^*10^*1 + q_0'0 \quad [\text{from } \textcircled{VI}]$$

$$= \epsilon + (10^*10^*1 + 0)'q_0$$

$$q_0 = \epsilon (10^*10^*1 + 0)^*$$

$$= (10^*10^*1 + 0)^*$$

left is non-terminal

Subject: .....

Date: [ ]

3(c) What is the regular language represented by the following left-linear grammar? 2.75

$$A \rightarrow Aa \mid Ab \mid Ba, B \rightarrow Cb, C \rightarrow \epsilon$$

$$A \rightarrow Aa$$

$$\rightarrow Aba \quad [A \rightarrow Ab]$$

$$\rightarrow Baba \quad [A \rightarrow Ba]$$

$$\rightarrow Cbaba \quad [B \rightarrow Cb]$$

$$\rightarrow \epsilon baba \quad [C \rightarrow \epsilon]$$

$$\rightarrow baba$$

$$A \rightarrow Ab$$

$$\rightarrow Abb \quad [A \rightarrow Ab]$$

$$\rightarrow Abb b \quad [A \rightarrow Ab]$$

$$\rightarrow Babbb \quad [A \rightarrow Ba]$$

$$\rightarrow Cbabbb \quad [B \rightarrow Cb]$$

$$\rightarrow \epsilon babbb \quad [C \rightarrow \epsilon]$$

$$\rightarrow babbb$$

$$A \rightarrow Ba$$

$$\Rightarrow Cba \quad [B \rightarrow Cb]$$

$$\Rightarrow \epsilon ba \quad [Cb \rightarrow \epsilon]$$

$$\Rightarrow ba$$

$$A \rightarrow Aa$$

$$\rightarrow Aaa \quad [A \rightarrow Aa]$$

$$\rightarrow Aaaa \quad [A \rightarrow Aa]$$

$$\rightarrow Baaaa \quad [A \rightarrow Ba]$$

$$\rightarrow Cbaaaa \quad [B \rightarrow Cb]$$

$$\rightarrow \epsilon baaaa \quad [C \rightarrow \epsilon]$$

$$\rightarrow baaaa$$

$$A \neq Aa$$

$$\rightarrow Aba$$

$$\rightarrow Abba$$

$$\rightarrow Ba bba$$

$$\rightarrow Cbabba$$

$$\rightarrow babba$$

$$L(G) = \{ba, baba, babbb, baaaa, \dots\}$$

$$= \{bab^m a^n \mid m, n \geq 1\}$$

$$\underline{ba} \underline{b^*} \underline{a^*}$$

4(i)

## Contrast between Moore machine & Mealy machine

### Moore Machine

Output depends only on current state.

More states than Mealy machine

Synchronous output and state generation

The design of Moore model is easy

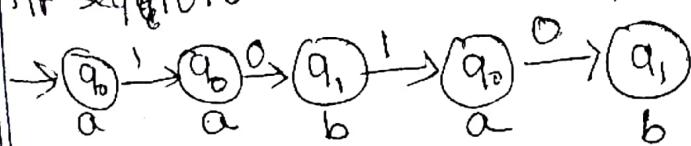
Output is a set of state

To design, more hardware is necessary

Can refer a counter as a Moore machine



Input seq: 1010 - 01 P seq: aabab



### Difference

1. States

2. Output generation

3. Design

4. Output type

5. Hard-wire

6. Counter

7. Example

### Mealy Machine

Output depends on the present state as well as on the present input.

Fewer states than Moore machine.

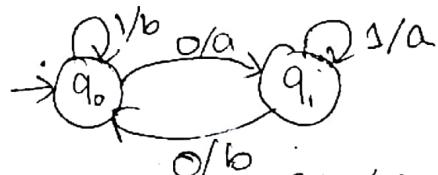
Asynchronous output generation.

The design of Mealy model is complex

Output is a set of transition

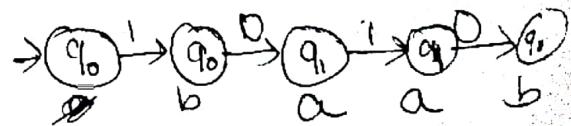
It's easier to design with less hardware.

Can not refer a counter as Mealy machine



If sequence #1010 =

O/P → baab



4(b) Construct a Mealy machine that takes binary numbers as input and produces 2's complement of that number as output. Assume that the string is read ~~LSP~~ ESB to MSB and end carry is discarded.

Ans: 2's Complement logic:- First calculate 1's complement of binary number, Convert 1 to 0 and 0 to 1 and then add 1 to it. For example: if binary number is 1011 then 1's complement is 0100 and its 2's complement 0101

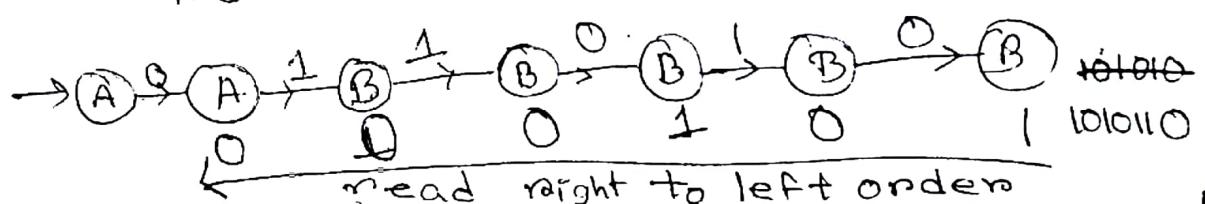
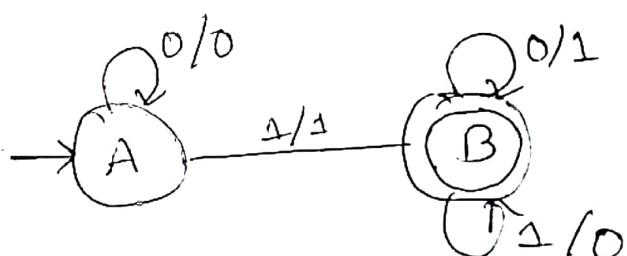
Design Mealy machine:

1. Take initial state A.
2. If there are  $n$  number of zeros at initial state, it will remain at initial state.
3. Whenever 1st input ~~is~~ 1 is found then it gives output 1 and goes to state B.
4. In state B, if input is zero, output will be 1. And if input is 1 then output will be 0.
5. And then set state B as final state.

The approach:

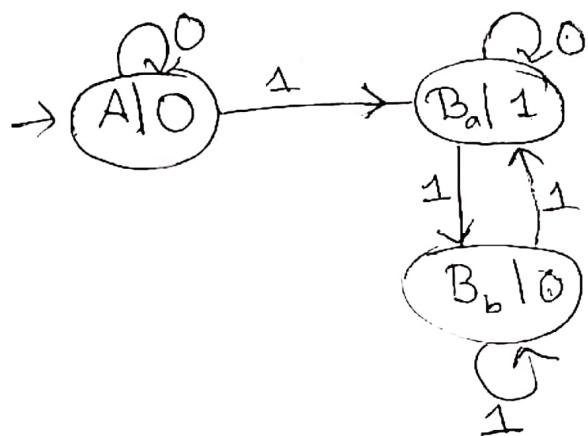
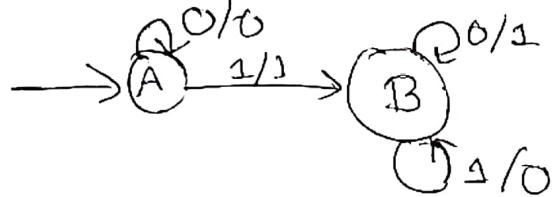
1. Start from right to left
2. Ignore all 0's
3. When 1 comes ignore it and then take 1's complement of every digit

example: 010110. Read right to left.



4(c) Convert above Mealy machine of 4(b) into Moore machine.

Sol: The Mealy machine for 2's Complement,



## ~~Chomsky Hierarchy of Grammar with respect to formal language.~~

A grammar  $G$  can be defined as  $G = (V, T, P, S)$

$V$  = Finite set of variables,  $T$  = Finite set of terminals

$P$  = Production,  $S$  = Start Production

The production rule must be in the form  $x \rightarrow B$

Type-0: Unrestricted Grammar: There is

no restriction on the grammar rules of these types of language. These languages can be efficiently modeled by Turing machine. Example:  $bAa \rightarrow aa, S \rightarrow s$

Type-1: Context Sensitive Grammar: Represents

context sensitive language. The rules:

① The context  $s.G$  has more than one symbol on left hand side of their production rules.

② No. of symbols on L.H.S must not exceed no. of symbols on R.H.S.

③ The rule  $A \rightarrow E$  is not allowed unless  $A$  is start symbol. It doesn't occur in R.H.S of any rule.

④ Type-1 should be in type-0.  $V \rightarrow T$ .

Count symbol of  $V$  is less than or equal to  $T$ .

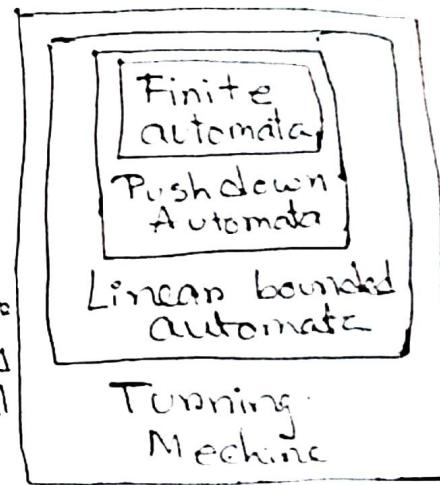
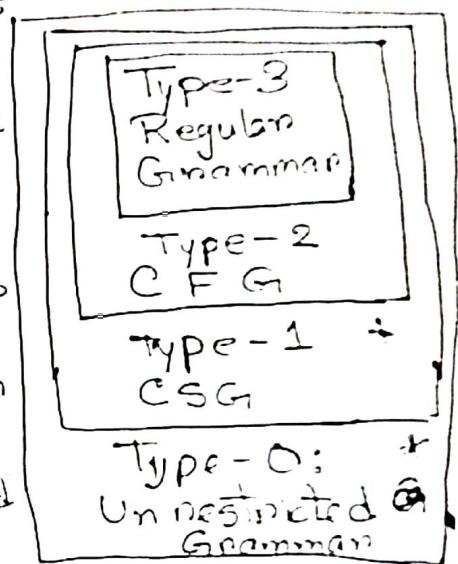
$S \rightarrow AT, T \rightarrow xy, A \rightarrow a$ .

Type-2: Context free grammar: Regular languages

represented by CFG. Type-2 should be in type-1 represented by any single non-terminal. Allows  $A \rightarrow \alpha$  where  $A$  is any combination of terminals and non-terminals.  $A \rightarrow aBb, A \rightarrow b, B \rightarrow a$

Type-3: Regular Grammar: Contains Regular language of those languages which can be described using regular expression. Can be modeled by NFA or DFA. Type-3 is most restricted form of grammar. Type-3 should be Type-2 and Type-1. Type-3 should be in form  $V \rightarrow T^* V / T^*$

example:  $A \rightarrow XY$



Ques No.

6 (a) What do you mean by derivation? what is the function of it? Define sentence & sentential form. (1.75)

18/18/60

Derivation is a sequence of production rules. It is used to get input string through these production rules. During parsing, we have to take two decisions.

These are:

- ① We have to decide the non-terminal which is to be replaced.
- ② We have to decide the production rules by the non-terminal will be replaced.

Sentential form: Is the start symbol  $S$  of a grammar or any string in  $(V \cup T)^*$  that can be derived from  $S$ .

Consider linear grammar  $\{S, B\}, \{a, b\}, S, \{S \rightarrow aS, S \rightarrow B, B \rightarrow bB, B \rightarrow \lambda\}$ . A derivation using this grammar might look like  $S \rightarrow aS \rightarrow aB \rightarrow abB \rightarrow abbB \rightarrow abb$ . Each of  $\{S, aS, abB, abb\}$  is a sentential form. Because this grammar is linear, each sentential form has at most one variable. Hence there is never any choice about which variable to expand next.

b) Construct context-free grammar that generates the

(P)

following languages.

L = { ... }

i)  $\{w \in \{a, b\}^*$   
with same symbols

$L = \{\epsilon, 0, 1, 00, 11, 010, 000, 1001, 0101010, \dots\}$

$0(0+1)^*0 \mid 01(0+1)^*1 \mid \epsilon \mid 1 \mid 0$

Let, Start variable,  $S \rightarrow 0S_10 \mid 1S_11 \mid 10 \mid \epsilon$

$S \rightarrow 0x0 \mid 1x1 \mid 1 \mid 0 \quad S_1 \rightarrow S_10 \mid S_11 \mid \epsilon$

$x \rightarrow 0X \mid 1X \mid \epsilon$

$CFG_1, G_1 = \{V, T, P, S\}$

Subject : .....

Date : [ ] [ ] [ ]

6(a) Construct Context-free grammar separately for  
 $\{w \in \{a, b\}^* \mid w \text{ starts } \& \text{ end with different symbols}\}$

$$\begin{array}{l} \text{Soln: } \\ P: \left\{ \begin{array}{l} S \rightarrow aAb \mid bAa \\ A \rightarrow aA \mid bA \mid \epsilon \end{array} \right. \end{array}$$

$$L = \{ab, ba, abab, aaab, baaa, baba, \dots\}$$

$CFG_1, G_1 = (\{S, A\}, \{a, b\}, P, S)$

(i)  $w \in \{a, b\}^* \mid w \text{ is palindrome} \}$  [even length & odd length both]

$$\text{Soln: } L = \{aba, baab, bab, \dots\}$$

$$S \rightarrow aSa$$

$$V = \{S\}$$

odd length

even length

$$S \rightarrow bSb$$

$$T = \{a, b\}$$

$$S \rightarrow aSa$$

$$S \rightarrow bSb$$

$$S \rightarrow a$$

$$\Rightarrow abSba$$

$$S \rightarrow b$$

$$\Rightarrow abbba$$

$$S \rightarrow \epsilon$$

অথবা

$$S \rightarrow aSa \mid bSb \mid a\epsilon b$$

$CFG_1, G_1 = (\{S\}, \{a, b\}, \{\epsilon\}, P, S)$

Subject : .....

Date : \_\_\_\_\_

(iii)  $\{a^n b^n \mid n \geq 1\}$

$L = \{aab, aaaabb; aaaaaaabb, \dots\}$

$S \rightarrow aaSb$

$S \rightarrow aab$

Po:  $S \rightarrow aaSb | aab$

CFG,  $G_1 = (\{S\}, \{a, b\}, P, S)$

- 6) b) Define Chomsky normal form. Convert the following CFG to Chomsky normal form (CNF):
- $S \rightarrow aXb \mid Yb, X \rightarrow S \mid \epsilon, Y \rightarrow bY \mid b$
- ~~CNF is~~ A context free grammar is in CNF if all the production rules satisfy one of the following conditions:
1. Start symbol generating  $\epsilon$ . For example
  2. A non-terminal generating two non-terminal  
example,  $S \rightarrow AB$
  3. A non-terminal generating a terminal.  
example,  $S \rightarrow a$