Numerical Methods

Nonlinear Equations: Roots

- Objective is to find a solution of

  $F(x) = 0$

  Where F is a polynomial or a transcendental function, given explicitly.
- Exact solutions are not possible for most equations.
- A number $x \pm e$, ($e > 0$) is an approximate solution of the equation if there is a solution in the interval [x-e,x+e]. e is the maximum possible error in the approximate solution.
- With unlimited resources, it is possible to find an approximate solution with arbitrarily small e.

Nonlinear Equations: Roots

- If the function F is calculated using floating point numbers then the rounding-off errors spread the solution to a finite interval.
- Example
- Let $F(x) = (1 – x)^6$. This function has a zero at x = 1. Range of the function is nonnegative real numbers.
- It is not possible to obtain an answer that is more accurate than second decimal place.

Show graph

Bisection Method

Let *F(x)* be a continuous function and let *a* and *b* be real numbers such that *f(a)* and *f(b)* have opposite signs. Then there is a x* in interval [a,b] such that *F(x*) = 0*.

Then c = (a + b)/2 is an approximate solution with maximum possible error (b – a)/2.

If f(c) and f(a) have opposite signs then the solution x* is in the interval [a,c]. Then, again, d = (c + a)/2 is an approximate solution but with max possible error (b – a)/4.

Else the solution is in the interval [c,b]. The approximate solution now is (c+b)/2 with max possible error (b-a)/4.

Continuing this process n times we can reduce the max possible error to $(b-a)/2^n$.

Bisection Method

Algorithm

1. Let a and b be such that f(a)*f(b) < 0
2. Let c = (a + b) / 2
3. If f(a) * f(c) < 0 then b = c
   else a = c
4. If more accuracy is required go to step 2
5. Print the approximate solution (a + b)/2

Example

Regula Falsi Method

Algorithm

1. Let a and b be such that f(a)*f(b) < 0
2. Let c = (f(b)*a – f(a) * b) / (f(b) – f(a))
3. If f(a) * f(c) < 0 then b = c
   else a = c
4. If more accuracy is required go to step 2
5. Print the approximate solution (a + b)/2

Example

Terminating Iterative Method

- Let eps > 0 be a small number.
- |b – a| < eps
- |b – a| / |a| < eps
- | f(c) | < eps
- Number of iterations

Newton Raphson Method

Algorithm

1. Input xOld
2. xNew = xOld – f(xOld) / f’(xOld)
3. If not satisfied go to step 2
4. Print xNew

Fixed Point Iterations

Convert the given equation in the form $x = g(x)$

Examples

$x^2 - 1 = 0$ can be written as

$x = 1 / x$ or

$x = x^2 + x - 1$

$x^2 + x - 2 = 0$ can be written as

$x = 2 - x^2$

$x = \text{sqrt}(2 - x)$