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Span of a vector:- The set of all possible vectors that you can reach with a linear combination of given pair of vectors is called span of vectors

vector space

V is a non-empty set

Operation based on addition & multiplication

Let u, v and $w \in V$ and constants $k, m \in R$

1. If $u, v \in V$ then $u+v \in V$ Add
2. If $k \in R$ and $u \in V$, then $k.v \in V$ Multiply
3. $u+v = v+u$ Commutative
4. $u+(v+w) = (u+v)+w$. Associative for addi
5. $\vec{0} \in \vec{V}$ then $0+u = u+0 = u$ Identity property for Addition
6. $-u \in V$ then $(u)+(-u) = 0 = (-u)+(u)$ Additive inverse
7. $k.(u+v) = ku+kv$ Distributive for Addition
8. $(k+m)u = ku+mu$ " " Multiplication
9. $(km)u = (ku)m = k(mu)$ Associative for Multiplicat
10. $1.u = u$ Identity for Multiplication.

If all the above condintion satisfied then given space is vector space

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Euclidean Inner Product [Scalar Product, Dot Product]

$u = (1, 1, 1)$ & $v = (0, 1, 2)$

$u \cdot v = (1, 1, 1) \cdot (0, 1, 2) = 0 + 1 + 2 = 3$

Euclidean Norm or length:

let, $u = (u_1, u_2, u_3)$ the euclidean length of u

$\|u\| = \sqrt{u \cdot u} = \sqrt{u_1^2 + u_2^2 + u_3^2}$ = Vector Norm

Unit vector: $v = \frac{u}{\|u\|}$

distance, $d(u, v) = \|u - v\| = \sqrt{(u_1 - v_1)^2 + (u_2 - v_2)^2 + (u_3 - v_3)^2}$

Inner product $= \frac{1}{4}\left[\|u+v\|^2 - \|u-v\|^2\right]$

Orthogonal $\Rightarrow u \cdot v = 0 \quad \theta = \frac{\pi}{2}$

Pythagorus: $\|u\|^2 + \|v\|^2 = \|u+v\|^2$

Parallelogram Identity: $\|u+v\|^2 + \|u-v\|^2 = 2\left[\|u\|^2 + \|v\|^2\right]$

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vector x vector = scalar

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Orthogonal Projection:- of vector u on v.

$$\underset{\text{of}}{\text{proj}_v} \underset{\text{on}}{u} = \frac{u \cdot v}{\|v\|^2} v$$

Vector component of u which is orthogonal to v.

$$u - \text{proj}_v u = u - \frac{u \cdot v}{\|v\|^2} v$$

Cross Product:-

$$u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \left((u_2 v_3 - u_3 v_2), -(u_1 v_3 - u_3 v_1), \right.$$

* Scalar triple Product. , u, v, w

$$u \cdot (v \times w) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$\underbrace{u \cdot v}_{\text{scalar}}$

Area of parallelogram

$$A = \|u \times v\|$$

Area of triangle

$$A = \frac{1}{2} \|u \times v\|$$

Inner Product
Scalar $u \cdot v = v \cdot u$

$(u + v) \cdot w = uw + v \cdot w$

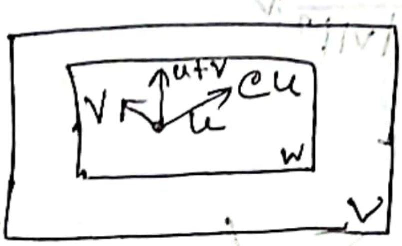
$u \cdot u = 0$

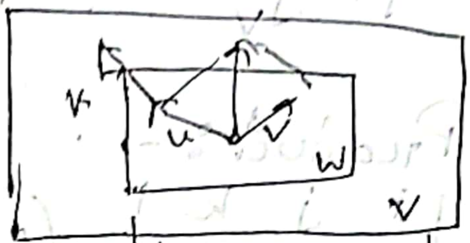
Cross Product
Vector $u \times v = -(v \times u)$

$(u+v) \times w = (u \times w) + (v + w)$

$u \times \vec{0} = \vec{0}$

Subspace:- a subset W of vector space V is called subspace of V if W is itself a vector space under the vector addition and scalar multiplication defined on the vector space V.





Not in subspace

$\vec{u} + \vec{v} \in W$

$2.\vec{u} \in W$

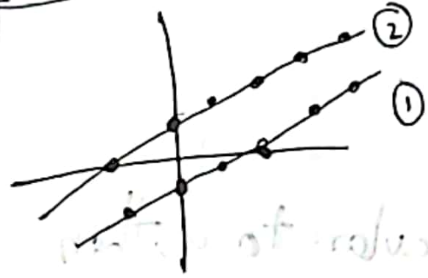
$0 \in \vec{W}$

Vector Space:- A non-empty set V of objects called vectors, on which are defined two operations, called vector addition & scalar multiplication.

$$\text{(i)}\quad x - 2y = 2 \quad \text{— (i)}$$

$$2x - 4y = -2 \quad \text{— (ii)}$$

Row Picture



For $x - 2y = 2 \Rightarrow y = \frac{x-2}{2}$ — (i)

y	-1	0	1	2	3	4	5
x	-1.5	-1	-0.5	0	0.5	1	1.5

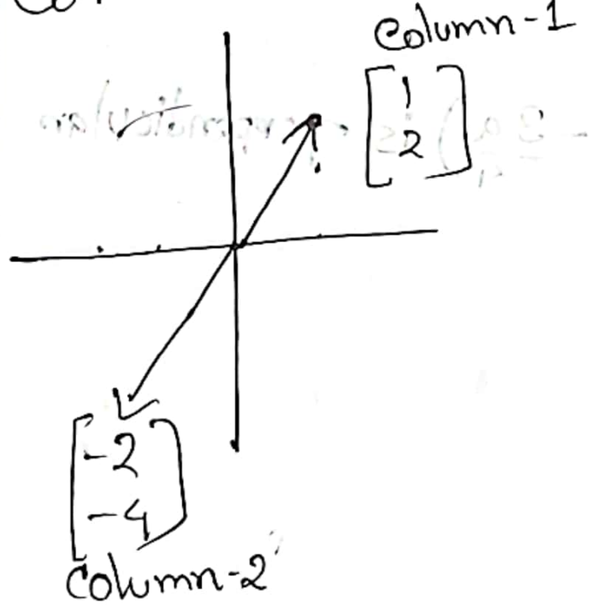
For $2x - 4y = -2$, $y = \frac{x+1}{2}$ — (ii)

y	-1	0	1	2	3	4	5
x	0	.5	1	1.5	2	2.5	3

As we can see the two lines are parallel there is no solution.

Column picture

$$x - 2y = 2$$
$$2x - 4y = -2$$



$$\Rightarrow \begin{bmatrix}1 & -2\\2 & -4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$$

$$\therefore \left\{ \begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}-2\\-4\end{bmatrix} \right\}$$

The Column's dont combine to give $b = \begin{bmatrix}2\\-2\end{bmatrix}$

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1(c) Unit vector u in direction v is $u = \dfrac{v}{\|v\|}$

$v = (3,4)$ $\quad \|v\| = \sqrt{3^2+4^2} = 5$

$u = \dfrac{(3,4)}{5} = \left(\dfrac{3}{5}, \dfrac{4}{5}\right)$

Let W is unit vector perpendicular to u, then

$w = (a,b)$

$(a,b) \cdot \left(\dfrac{3}{5}, \dfrac{4}{5}\right) = 0$

$\dfrac{3a}{5} + \dfrac{4b}{5} = 0$

$3a + 4b = 0$

$b = -\dfrac{3a}{4}$

$\therefore$ Any vector in form $\left(a, -\dfrac{3a}{4}\right)$ is perpendicular to u.

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2 (a) Given, $H = \{(a-3b, b-a, a, b)\}$

$$\begin{bmatrix} a-3b \\ b-a \\ a \\ b \end{bmatrix} = a\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$H = \text{Span}\{v_1, v_2\}$ where v_1 & v_2 are vectors of $\mathbb{R}^4$

Row-echelon Form

(1) All non-zero rows above any rows of all zero

(2) each leading entry of a row, is in column to right

(3) All entries in column below a leading entry are zeros.

Reduced Row echelon form (Hermite form).

(4) The leading entry in each non-zero row is 1.

(5) Each leading 1 is only non-zero entry in its column

Every matrix is equivalent to one & only one Reduced R.E.M.

Gaussian Elimination –

1. left most Column (non zero Column pivot Column).

2.

Basis. → the variable on which the whole system depends

on. $\begin{bmatrix} 1 & 0 & 9 & 8 \\ 0 & 1 & 3 & 4 \end{bmatrix}$ $\quad x_1 + 9x_3 = 8 \quad x_1 = 8 - 9x_3$

$x_2 + 3x_3 = 4 \quad x_2 = 4 - 3x_3$

$x_3 = x_3 \quad x_3 = x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 - 9x_3 \\ 4 - 3x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} -9 \\ -3 \\ 1 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 2 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & -3 & -4 & 3 \\ 0 & 1 & 8 & 5 & -7 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

x_3 free

$0 = 1$ False

No Solution

Inconsistent

vector – quantity that has both a length (also called magnitude)

$v = \vec{v} = \begin{bmatrix} 60 \\ 0 \end{bmatrix}$ vector is Column vector

$\vec{v} = \begin{bmatrix} -12 \\ 9 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \in \mathbb{R}^n$

$\vec{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}, \vec{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$ are equal if

① same direction & same length

$v_1 = w_1$ for all $1 \leq i \leq n$

$$\begin{bmatrix} -4 \\ -5 \end{bmatrix} = x_1 \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{e_1} + x_2 \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{e_2}$$

Standard Column vector

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Linear Combination:- Given vectors $v_1, v_2 \dots v_p$ in $\mathbb{R}^n$ and scalar $c_1, c_2, \dots c_p$ the vector y defined by

$$y = c_1v_1 + c_2v_2 + \dots + c_pv_p = \sum_{i=1}^{p} c_i v_i$$

is called a linear combination of vector $v_1, v_2, \dots v_p$ with weights $c_1, c_2 \dots c_p$.

Span of a set of vector:- Let $v_1, v_2, \dots v_p$ be vectors in $\mathbb{R}^n$. The set of all linear combination of vectors $v_1, v_2, \dots v_p$ is called subset of $\mathbb{R}^n$ spanned by $v_1, v_2 \dots v_p$. The set $\text{Span}\{v_1, v_2 \dots v_p\}$ is the collection of all vectors in $\mathbb{R}^n$ then can be written in form $c_1v_1 + c_2v_2 + \dots + c_pv_p = \sum_{i=1}^{p} c_i v_i$

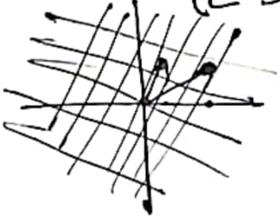
Q whether y is in $\text{Span}\{v_1, v_2, \dots v_p\}$;

$\Rightarrow$ solve $c_1v_1 + c_2v_2 + \dots + c_pv_p = y$

augmented matrix $\begin{bmatrix} v_1 & v_2 & \dots & v_p & y \end{bmatrix}$

column vectors

$\text{Span}\left\{\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}\right\}$



$\mathbb{R}^2$

$\text{Span}\left\{\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}$

already using the first 2 we can get all values in $\mathbb{R}^2$, So the last ~~value is ereq~~ column is redundant

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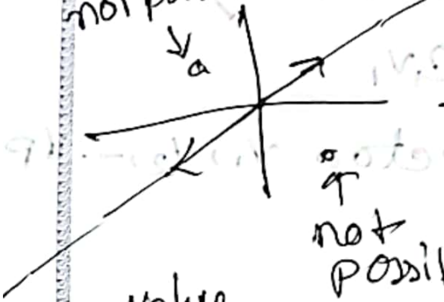
3. $\text{Span}\left\{\begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}-4\\-2\end{bmatrix}\right\}$ here $\begin{bmatrix}-4\\-2\end{bmatrix} = -2\begin{bmatrix}2\\1\end{bmatrix}$

not possible



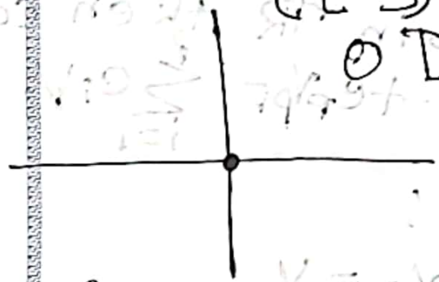
1 line
1D

not possible

all value along the line possible, So $\mathbb{R}$

4. $\text{span}\left\{\begin{bmatrix}0\\0\end{bmatrix}\right\}$

0 Dimension.



$\mathbb{R}^3$

Span $\left\{\begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$

$\left\{c\begin{bmatrix}0\\1\\1\end{bmatrix}\right\} = \left\{\begin{bmatrix}0\\c\\c\end{bmatrix}\right\}$

All vectors parallel to the line

$\text{span}\left\{\begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$

$a\begin{bmatrix}0\\1\\0\end{bmatrix} + b\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\a\\b\end{bmatrix}$

Plane
y z plane.

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3. Span $= \left\{ \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} \right\}$

$$a\begin{bmatrix}1\\0\\0\end{bmatrix} + b\begin{bmatrix}0\\1\\0\end{bmatrix} + c\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}a\\b\\c\end{bmatrix}$$

All of $\mathbb{R}^3$ 3D

Span $\left\{ \begin{bmatrix}1\\0\\-2\end{bmatrix}, \begin{bmatrix}0\\1\\-1\end{bmatrix} \right\}$

$$\begin{bmatrix}4\\5\\-1\end{bmatrix} = x_1\begin{bmatrix}1\\0\\-2\end{bmatrix} + x_2\begin{bmatrix}0\\1\\-1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 4\\0 & 1 & 5\\-2 & -1 & -1\end{bmatrix}$$

$$\approx \begin{bmatrix}1 & 0 & 4\\0 & 1 & 5\\0 & -1 & 7\end{bmatrix}$$

$$\approx \begin{bmatrix}1 & 0 & 4\\0 & 1 & 5\\0 & 0 & 12\end{bmatrix}$$

$\therefore\ 0 \neq 12$

$\therefore$ no solution

4. Span $= \left\{ \begin{bmatrix}1\\0\\-2\end{bmatrix}, \begin{bmatrix}0\\1\\-1\end{bmatrix} \right\}$

$$a\begin{bmatrix}1\\0\\-2\end{bmatrix} + b\begin{bmatrix}0\\1\\-1\end{bmatrix}$$

$$\begin{bmatrix}6\\-2\\-10\end{bmatrix} = x_1\begin{bmatrix}1\\0\\-2\end{bmatrix} + x_2\begin{bmatrix}0\\1\\-1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 6\\0 & 1 & -2\\-2 & -1 & -10\end{bmatrix}$$

$$\approx \begin{bmatrix}1 & 0 & 6\\0 & 1 & -2\\0 & -1 & 2\end{bmatrix}$$

$$\approx \begin{bmatrix}1 & 0 & 6\\0 & 1 & -2\\0 & 0 & 0\end{bmatrix}$$

$x_1 = 6$
$x_2 = -2$
$x_3 = x_3$

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Let v_1, v_2, v_3 and w be vectors in $\mathbb{R}^n$. Prove that if w is in span$\{v_1, v_2, v_3\}$ then span$\{v_1, v_2, v_3, w\}$ = Span$\{v_1, v_2, v_3\}$.

Soln: $\vec{w} = a\vec{v_1} + b\vec{v_2} + c\vec{v_3}$

Consider a ~~vector~~ vector space $u_1 = a\vec{v_1} + b\vec{v_2} + c\vec{v_3}$ which is ~~of~~ span ~~of~~ $\{v_1, v_2, v_3\}$

We can write it as $u_1 = 0.\vec{w} + a\vec{v_1} + b\vec{v_1} + c\vec{v_3}$

Consider another vector space, $u_2 = x_1 w + x_2\vec{v_1} + x_3\vec{v_2} + x_4\vec{v_3}$ which is span$\{v_1, v_2, v_3, w\}$

$u_2 = x_1(a\vec{v_1} + b\vec{v_2} + c\vec{v_3}) + x_2\vec{v_1} + x_3\vec{v_2} + x_4\vec{v_3}$

$= (x_1 a + x_2)v_1 + (x_1 b + x_3)\vec{v_2} + (x_1 c + x_4)\vec{v_3}$

which canbe spanned in space $\{v_1, v_2, v_3\}$

So, for w ~~is~~ in span $\{v_1, v_2, v_3\}$ then span$\{v_1, v_2, v_3, w\}$ = span $\{v_1, v_2, v_3\}$.

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$$\text{span}\left\{ \overset{\vec{a}_1}{\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}}, \overset{\vec{a}_2}{\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}} \right\} \quad b = \begin{bmatrix} 6 \\ -2 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = Ax \,?$$

$$x_1 \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = b = \begin{bmatrix} 6 \\ -2 \\ -10 \end{bmatrix}$$

$$Ax = x_1 \vec{a}_1 + x_2 \vec{a}_2 + x_3 \vec{a}_3$$

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -2 \\ -2 & -1 & -10 \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -2 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore x_1 = 6, \; x_2 = -2, \; x_3 = x_3$

$$A = \begin{array}{cccc} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 & \overset{x_4 \vec{a}_4}{\vec{a}_4} \end{array}$$

$$A = \begin{bmatrix} 2 & -4 & 3 & 1 \\ 6 & 2 & 1 & 9 \\ 1 & 0 & 2 & -1 \end{bmatrix} \quad x = \begin{bmatrix} -1 \\ -1 \\ 5 \\ 2 \end{bmatrix}$$

$$x_1 \vec{a}_1 + x_2 \vec{a}_2 + x_3 \vec{a}_3 + x_4 \vec{a}_4$$

$$= -1 \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix} + (-1) \begin{bmatrix} -4 \\ 2 \\ 0 \end{bmatrix} + 5 \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 1 \\ 9 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ -6 \\ -1 \end{bmatrix} + \begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix} + \begin{bmatrix} 15 \\ 5 \\ 10 \end{bmatrix} + \begin{bmatrix} 2 \\ 18 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 19 \\ 15 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ -10 \end{bmatrix}$$

$x_1 = 6$

$x_2 = -2$

$-2x_1 - x_2 = -10$

Ax is only defined when the number of rows in x

The equation $Ax = b$ has a sol^n (is consistent) if & only if b is a linear Combination of the column of matrix A.

augmented matrix
$$\begin{bmatrix} a_1 & a_2 & \cdots & a_n & b \end{bmatrix}$$

$$x_1 a_1 + x_2 a_2 + \cdots + x_n a_n = b$$

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Q: General all vectors $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ in $\mathbb{R}^3$ in span$\left\{ \left\{ \underset{\vec{a}_1}{\begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}}, \underset{\vec{a}_2}{\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}}, \underset{\vec{a}_3}{\begin{bmatrix} 6 \\ 8 \\ 6 \end{bmatrix}} \right\} \right\}$

$$\begin{bmatrix} 1 & 1 & 6 & b_1 \\ 1 & 0 & 8 & b_2 \\ -3 & 2 & 6 & b_3 \end{bmatrix}$$

$-3+$

$$\sim \begin{bmatrix} 1 & 1 & 6 & b_1 \\ 0 & -1 & 2 & b_2 - b_1 \\ 0 & 5 & 24 & 3b_1 + b_3 \end{bmatrix}$$

$5b_2 - 5b_1 + 3b_1 + b_3$

$$\sim \begin{bmatrix} 1 & 1 & 6 & b_1 \\ 0 & -1 & 2 & b_2 - b_1 \\ 0 & 0 & 34 & -2b_1 + 5b_2 + b_3 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 3 & 1 & 4 & 2 \\ 6 & 2 & 9 & 2 \\ -2 & 0 & -4 & 0 \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & -4 \\ 0 & 2 & 3 & -10 \\ 0 & 0 & -2 & 4 \end{bmatrix} \begin{matrix} R_2' \to R_2 - 3R_1 \\ R_3' \to R_3 - 6R_1 \\ R_4' \to R_4 + 2R_1 \end{matrix}$$

$$\approx \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & -2 & 4 \end{bmatrix} R_3' \to R_3 - 2R_2'$$

$$\approx \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_4 \to R_4 + 2R_3$$

there 4th Row has no pivot & so not spall all of $\mathbb{R}^4$

Properties of Matrix Vector product

$A(u+v) = Au + Av$

$A(cv) = c(Av)$

n = n.o. of unknow.
r = rank

Homogeneous Soln:- $Ax = 0$, trivial solution

$n = r$, linearly independent

$(x_1, x_2, x_3) = 0$,

consistent

Non-trivial soln: $Ax = b$ A has one or more free variable

Find Sol^n set of homogeneous system.

$x_1 + 3x_2 + 7x_3 = 0$

$x_2 + 3x_3 = 0$

$-2x_1 - 4x_2 - 8x_3 = 0$

$$\begin{bmatrix} 1 & 3 & 7 & 0 \\ 0 & 1 & 3 & 0 \\ -2 & -4 & -8 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & 7 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 2 & 6 & 0 \end{bmatrix} R_3' \to R_3 + 2R_1$$

$$\sim \begin{bmatrix} 1 & 3 & 7 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_3' \to R_3 - 2R_2$$

x_3 is free

$$\sim \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ infinite no. of sol^n

$x_1 - 2x_3 = 0$ $\qquad x_1 = 2x_3$

$x_2 + 3x_3 = 0$ $\qquad x_2 = -3x_3$

$x_3 = x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_3 \\ 3x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \quad \therefore v = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

For $x_3 = 0$ the gives trivial sol^h

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Find homogeneous System

$3x_1 - 2x_2 + x_3 = 0$
$-6x_1 + 4x_2 - 2x_3 = 0$

$$\begin{bmatrix} 3 & -2 & 1 & 0 \\ 6 & 4 & -2 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2/3 & 1/3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Same plane

x_2 & x_3 free variable.

$$x_1 - \frac{2x_2}{3} + \frac{x_3}{3} = 0$$

$$x_1 = \frac{2x_2}{3} - \frac{x_3}{3}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{3}x_2 - \frac{x_3}{3} \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} \frac{2}{3} \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -\frac{1}{2} \\ 0 \\ 1 \end{bmatrix}$$ vector form

$$= x_2 u + x_3 v.$$

implicit Sol^n : $3x_1 - 2x_2 + 3x_3 = 0$

explicit Sol^n : $x_1 = \frac{2}{3}x_2 - \frac{1}{3}x_3$

vector form: x.

Parametric V.F. : $s\begin{bmatrix} \frac{2}{3} \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -\frac{1}{3} \\ 0 \\ 1 \end{bmatrix} = su + tv$ $s \& t \in \mathbb{R}$

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Nonhomogeneous

$$x_1 + 3x_2 + 7x_3 = 4$$
$$x_2 + 3x_3 = 5$$
$$-2x_1 - 4x_2 - 8x_3 = 2$$

$$\begin{bmatrix} 1 & 3 & 7 & 4 \\ 0 & 1 & 3 & 5 \\ -2 & -4 & -8 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & 7 & 4 \\ 0 & 1 & 3 & 5 \\ 0 & 2 & 6 & 10 \end{bmatrix}$$

$R_3 = R_3 - 2R_2$

$$\sim \begin{bmatrix} 1 & 3 & 7 & 4 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & -11 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 - 2x_3 = -11$$
$$x_2 + 3x_3 = 5$$

$$x_1 = -11 + 2x_3$$
$$x_2 = 5 - 3x_3$$
$$x_3 = x_3$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -11 + 2x_3 \\ 5 - 3x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 5 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = p + t\vec{v}$$

$$t \in \mathbb{R}$$

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trivial $x = 0 = \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$n = n$

linearly independent only 1 Solution

Determine whether $x_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 6 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 6 \\ -10 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ has a Non-Trivial sol

$\boxed{-2} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \boxed{2} \begin{bmatrix} -1 \\ 6 \\ 3 \end{bmatrix} + \boxed{1} \begin{bmatrix} 6 \\ -10 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow -2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 6 \\ 3 \end{bmatrix} = - \begin{bmatrix} 6 \\ -10 \\ -6 \end{bmatrix}$

$\Rightarrow 2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + (-2) \begin{bmatrix} -1 \\ 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -10 \\ -6 \end{bmatrix}$

we can express one vector as linear Combination of 2 other vector

$\therefore$ the sol^n is Non-Trivial & is linearly dependent

$$\left[\begin{array}{ccc|c} 2 & -1 & 6 & 0 \\ 1 & 6 & -10 & 0 \\ 0 & 3 & -6 & 0 \end{array}\right]$$

$6-(2)\times(-10)$

$6+20$

$-1-2\times 6$

$=1$

$$= \left[\begin{array}{ccc|c} 1 & 6 & -10 & 0 \\ 2 & -1 & 6 & 0 \\ 0 & 3 & -6 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 6 & -10 & 0 \\ 0 & -13 & 26 & 0 \\ 0 & 3 & -6 & 0 \end{array}\right] \quad R_2 = R_2 - 2R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 6 & -10 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 3 & -6 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} R_1' \to R_1 - 6R_2 \\ R_3' \to R_3 - 3R_2 \end{array}$$

$x_1 + 2x_3 = 0$

$x_2 - 2x_3 = 0$

$x_3 = x_3$

$x_1 = -2x_3$

$x_2 = 2x_3$

$x_3 = x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$$

infinit no. of sol^n $\therefore$ non-trivial

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Linear Independence

Let $\{x_1, v_2, v_3 \dots v_p\}$ denote a set of p vectors each in $\mathbb{R}^n$. We say the set of vectors is Linearly independent if vector equation $x_1 v_1 + x_2 v_2 + \dots + x_p v_p = 0$ has only a trivial sol^n, $x_i = 0$, $1 \le i \le p$.

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Otherwise a set of vectors $\{v_1, v_2, \dots v_p\}$ is said to be linearly dependent if there exists weight $c_1, c_2, \dots c_p$ not all zero such that

$$c_1 v_1 + c_2 v_2 + \dots + c_p v_p = 0.$$

Q: Determine $S = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$ is Linear independent or not.

Ans: Consider, $x_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 4 & 0 \end{bmatrix}$$

$A.\vec{x} = 0$

$$\sim \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad R_2' = R_2 - 2R_1$$

$\therefore$ linearly independent

~~xQ=0~~

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad R_1' \Rightarrow R_1 - 2.R_2$$

$x_1 = 0$, $x_2 = 0$ $\therefore x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

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1. If possible ~~ex~~ give example of 1 vector in $\mathbb{R}^3$ so that it's linearly independent set.

$S = \{\vec{v_1}\}$ $\quad x_1\vec{v_1} = 0 \quad S = \left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\right\} \quad x_1\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

2. If possible, give example of set of 2 vector in $\mathbb{R}^3$ that is linearly independent

$S = \{\vec{v_1}, \vec{v_2}\}$, $\quad x_1\vec{v_1} + x_2\vec{v_2} = 0 \quad x_1\begin{bmatrix}1\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

here if we have a span where 1 vector is scalar multiple of other then not linearly independent.

$\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}2\\0\\0\end{bmatrix} \quad x_1\begin{bmatrix}1\\0\\0\end{bmatrix} + x_2\begin{bmatrix}2\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ Linearly dependent

3. If possible set of 3 vectors in $\mathbb{R}^3$ i.e linearly independent

$S = \left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$

4. If possible set of 4 vectors in $\mathbb{R}^3$. $\mathbb{R}^3$ already has set of 3 vectors, any 4th vector will be just combination of current vectors. So, not possible

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Interpretation of Matrix $Ax=b$.

Mapping: $f: x \to x^2$ $\mathbb{R} \to \mathbb{R}$

$f: x \to 7x$ $\mathbb{R} \to \mathbb{R}$

$f: \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \to \begin{bmatrix} x_2 \\ x_1 \\ x_1+x_2 \end{bmatrix} : \mathbb{R}^2 \to \mathbb{R}^3$

$f: \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \to \begin{bmatrix} 7x_1 \\ 7x_2 \end{bmatrix}$ $\mathbb{R}^2 \to \mathbb{R}^2$

Map: $T: x \mapsto Ax$

A transformation (also called function or mapping) T from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a rule that assigns each vector x in $\mathbb{R}^n$ a vector $T(x)$ in $\mathbb{R}^m$.

→ The set of inputs $\mathbb{R}^n$ is domain of T.

→ Space where the input are mapped to, $\mathbb{R}^m$ is called co-domain of T.

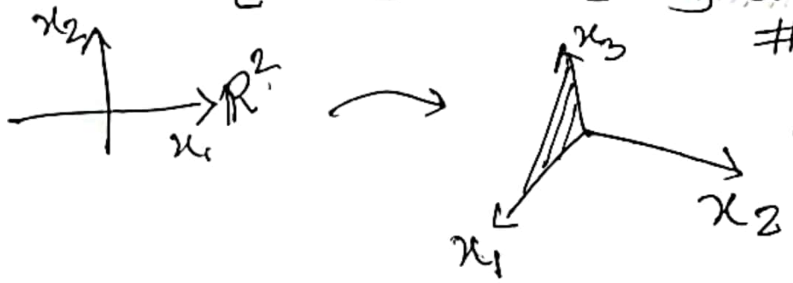
→ For each x in domain, vector $T(x)$ in $\mathbb{R}^m$ is called Image of x

→ Set of all images (outputs) is called range of T

$T(x) = Ax = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ 0 \\ x_2 \end{bmatrix}$

Domain $= \mathbb{R}^2 =$ # of column in A

Codomain $= \mathbb{R}^3 =$ # of row in A



Range = $x_1 x_3$ plane

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