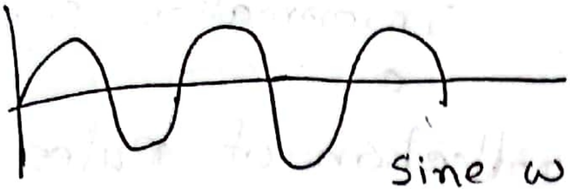
C.W

Shamim Reza Sir

07-06-22

Present
07-06-22
1. 13-Jency
2. 24-Janat
3. 42-Proma
4. 10-Natasha
5. 38-Nishi
6. 29-Fahim
7. " Niloy

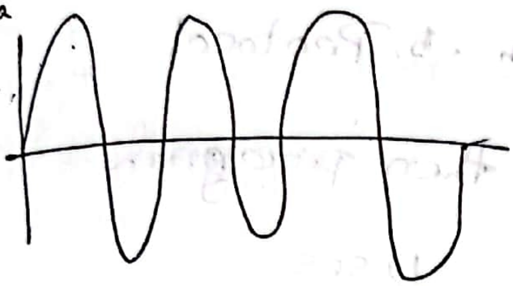
## DSP



sine wave.

Signal

mixed/
combined
Modulation



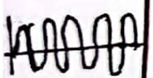
carrier signal.

Inosphere

Attenution → the reduction of power of a signal.

3 ways: of Modulation: the process.

1. AM: Amplitude : maximum distance from equilibrium



FM: Frequency; no.of complete wave in 1 second.

PM: Phase

Demodulator.

Demodulation: the process of deriving the main signal from ~~[illegible]~~.

Data Compression: cost effective.

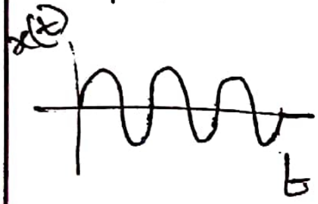
09-06-22

# Digital Signal Processing → 1

**Signal:** Physical quantity.

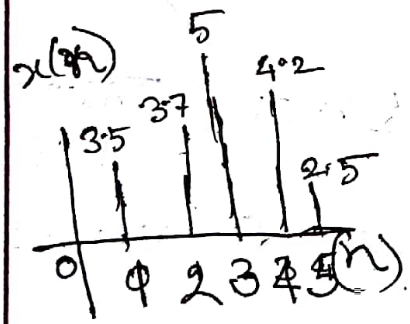
**Sampling:** Process in which a continuous time signal (continuous with respect to time) is

C.T ⟶ exists for all instance of time
প্রত্যেকটা time এর জন্য amplitude/value
থাকে



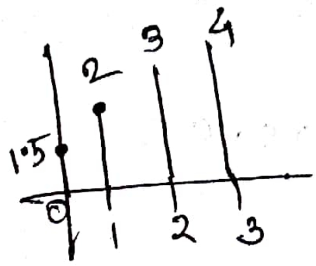

sampling

D.T → exists only for a certain/specific instance of time.




Nyquist theorem.

Continuous valued signal:
can take any value



## Discrete Valued Signal:

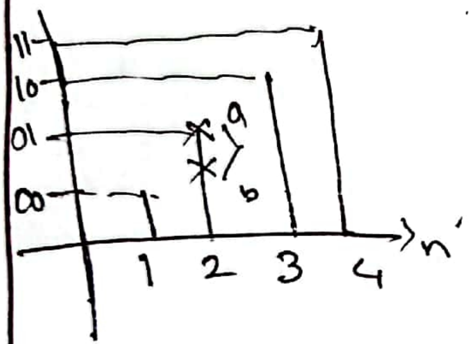
can take ~~ooo~~only limited set of value

Quantized Value

2 bit Quantization

$2^2 \Rightarrow 4$ combination.

quantization error.




For 2nd second the value we were supposed to get is at point a. but we got at b. this is caused by noise. This is called quantization error

1. .

DSP 4

# Classification of Signal

11-06-22
Saturday.

→ Continuous and discrete time signal
→ Deterministic and non-deterministic Signal
** → Even and Odd Signal.
→ Periodic and non-periodic Signal.
** → Energy and Power Signal
→ Real & Imaginary Signal.

1 26 - Fahim
2 12 - Riad
3 16 - Mithun
4 - Niloy
5 - Nahid
6 7 - Nadim
7 24 - Jannat
8 42 - Prema
9 33 - Nishi
10 07 - Bithi
11 13 - Jency

**CTS:** A signal is said to be continuous if it is defined for an instance of time.

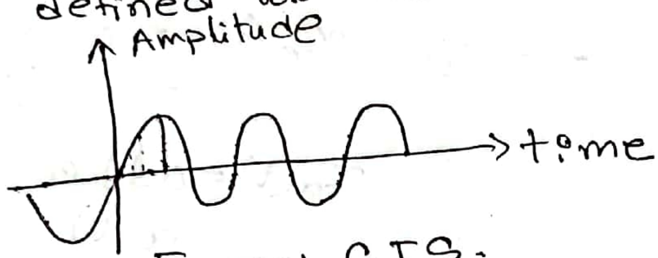
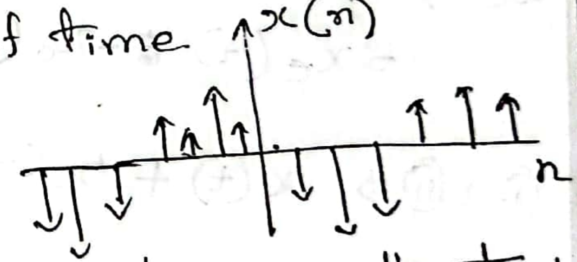


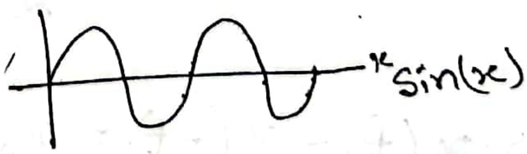

Figure: CTS.

**DTS:** A signal is said to be discrete when it is defined ~~for~~ at only discrete instance of time.

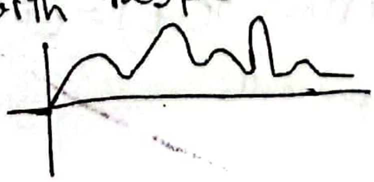



n = sample

Deterministic Signal.
Signals which can be defined exactly by a mathematical formula are known as D.S.
Follows a specific/certain pattern after certain time.




**NDS:** A signal is said to be non-deterministic if ~~there~~ it is uncertain with respect to it's value at some instance of time.
No pattern followed.

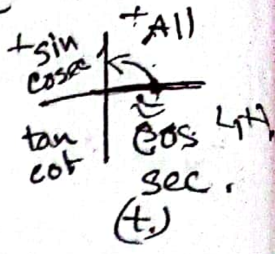


**Even Signal:** A continuous time signal $x(t)$ is said to be even. if $x(-t) = x(t)$ for all $t$.

$x(t) = t^2 \quad x(-t) = (-t)^2 = t^2 = x(t)$ even signal

$x(t) = \cos(t) \quad x(-t) = \cos(-t) = \cos t$ even

$x(t) = \sin(t) \quad x(-t) = \sin(-t) = -\sin t$ odd




**Even & Odd Components**

Any continuous time signal $x(t)$ can be written as sum of it's even and odd components. i.e. ~~$x(t) =$~~

$x(t) = x_e(t) + x_o(t)$ . —— ①

$x_e(t)$ = even components of $x(t)$

$x_o(t)$ = odd components of $x(t)$

$x(-t) = x_e(-t) + x_o(-t)$

$= x_e(t) - x_o(t)$ —— ⑪

$\because x_e(t) = x_e(-t)$

① + ⑪ ⇒ $x(t) + x(-t) = 2x_e(t)$ .

$\therefore x_e(t) = \dfrac{x(t) + x(-t)}{2}$ [Even Components]

① − ⑪ ⇒ $x(t) - x(-t) = x_o(t) + x_o(t)$ .

⇒ $x_o(t) = \dfrac{x(t) - x(-t)}{2}$ [odd Components]

Q.
* Find the even & odd components. $x(t) = e^{j2t}$
* Describe even & odd components

Given, $x(t) = e^{J2t}$

$$x_e(t) = \frac{x(t)+x(-t)}{2} = \frac{e^{J2t} + e^{-J2t}}{2}$$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

$$= \frac{\cos 2t + J\cancel{\sin 2t} + \cos 2t - J\cancel{\sin 2t}}{2}$$

$$= \frac{2\cos 2t}{2}$$

$$= \cos 2t \text{ . Ans}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$= \frac{e^{J2t} - e^{-J2t}}{2}$$

$$= \frac{\cancel{\cos 2t} + J\sin 2t - \cancel{\cos 2t} + J\sin 2t}{2} = \frac{2J\sin 2t}{2}$$

$$= J\sin 2t \text{ . Ans}$$

$$x(t) = \cos\left(\omega_0 t + \frac{\pi}{3}\right).$$

$$x_e(t) = \frac{x(t)+x(-t)}{2}$$

$$= \frac{\cos\left(\omega_0 t + \frac{\pi}{3}\right) + \cos\left(\omega_0(-t) + \frac{\pi}{3}\right)}{2}$$

$$= \frac{\cos\left(\omega_0 t + \frac{\pi}{3}\right) + \cos\left(-\omega_0 t + \frac{\pi}{3}\right)}{2}$$

$$= \frac{\cos\left(\omega_0 t + \frac{\pi}{3}\right) + \cos\left(-\left(\omega_0 t - \frac{\pi}{3}\right)\right)}{2}$$

$$= \frac{\cos\left(\omega_0 t + \frac{\pi}{3}\right) + \cos\left(\omega_0 t - \frac{\pi}{3}\right)}{2}$$

$$= \frac{\cancel{2}\cos\omega_0 t \cdot \cos\frac{\pi}{3}}{\cancel{2}} = \frac{1}{2}\cos\omega_0 t$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$= \frac{\cos\left(\omega_0 t + \frac{\pi}{3}\right) - \cos\left(\omega_0 t - \frac{\pi}{3}\right)}{2}$$

$$= \frac{-\left(\cos\left(\omega_0 t - \frac{\pi}{3}\right) - \cos\left(\omega_0 t + \frac{\pi}{3}\right)\right)}{2}$$

$$= \frac{-\cancel{2}\sin\omega_0 t \sin\frac{\pi}{3}}{\cancel{2}}$$

$$= -\frac{\sqrt{3}}{2}\sin\omega_0 t$$

Ans

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B).$$

Cisco Packet Tracer

Math lab. ~~DSS~~ DSP 15-06-22 ~~Thu~~ Wednesday.

24 Jannat
13 Jency
7 Bithi
33 Nishi.
15 Mashrafi
16 Mithun
14 Sadia
19 Niloy
34 Shorno
26 Fahim
10 Nahid
08 Nadim

## ** Energy & Power Signal.

A continuous or discrete signal is said to be energy signal if its energy is finite & power is zero.

$$0 < E < \infty \; ; \; P = 0$$

2020 -> 1 no question upload

For continuous signal,

(i) $E = \int_{-\infty}^{\infty} x^2(t)\, dt$ → Real valued signal

$e^{-t} / e^{t}$ →

(ii) $E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$ → Complex values signal $e^{j(2t+\frac{\pi}{4})}$

For discrete signal,

(i) $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

Power: A continuous or discrete time signal is said to be power signal. If it's power is finite. energy is infinite. $0 < P < \infty \; ; \; E = \infty$

Fourier transform, Z-transform

Continuous

$$P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt \rightarrow \text{Real}$$

$\to$ sample
$n \Rightarrow$ discret
$t \to$ continuous

$$P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt \rightarrow \text{complex.}$$

Discrete. $P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

* $x(n) = \left(\frac{1}{4}\right)^n u(n)$ is it energy or power signal

We know

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{4}\right)^n \cdot u(n)\right|^2$$

unit step signal,
For 0 to $\infty$ = value = 1

$$= \sum_{n=0}^{\infty} \left[\left(\frac{1}{4}\right)^n\right]^2$$

$$\boxed{\sum_{-\infty}^{\infty} a^n = \frac{1}{1-a}}$$

$$= \sum_{n=0}^{\infty} \left|\left(\frac{1}{4}\right)^2\right|^n$$

$$= \sum_{n=0}^{\infty} \left|\frac{1}{16}\right|^n$$

$$= \sum_{n=0}^{\infty} [0.0625]^n$$

$$= \frac{1}{1-0.0625}$$

$$= \frac{1}{0.9375} = 1.067$$

$$P = \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

For discrete in Power

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|\left(\frac{1}{4}\right)^n u(n)\right|^2$$

$$\sum_{0}^{n} a^n = \frac{a^{N+1}-1}{a-1}$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{0}^{N} \left|\left(\frac{1}{4}\right)^n\right|^2$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{0}^{N} \left|\frac{1}{16}\right|^n$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{0}^{n} (0.0625)^n$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \cdot \frac{0.0625^{N+1} - 1}{0.625 - 1}$$

$$= \frac{1}{2\alpha+1} \cdot \frac{0.0625^{\alpha+1} - 1}{0.0625 - 1}$$

$$= \frac{1}{\alpha} \cdot \frac{0.625^{\alpha} - 1}{0.625 - 1}$$

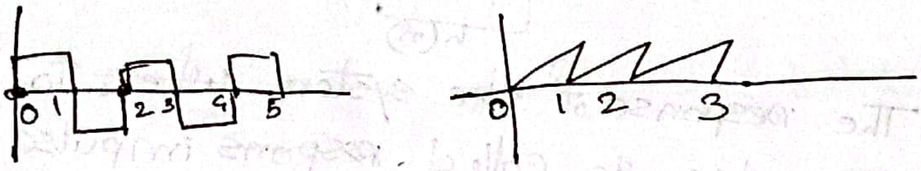
$$= 0$$

$\therefore$ $E = 1.067$ & $P = 0$ i.e the given signal is energy signal.

# Periodic & Aperiodic Signal.

PS: A continuous time signal $x(t)$ is said to be periodic if it satisfies $x(t+T)=x(t)$ for all 't'. ~~সর্বনিম্ন যে value এর জন্যে~~.

The smallest value of T which satisfies the above equation is fundamental period.




* Find the fundamental period of $x(t) = A\sin(\omega_0 t + \theta)$ (is it periodic / aperiodic).

$$x(t+T) = A\sin(\omega_0(t+T)+\theta)$$
$$= A\sin(\omega_0 t + \omega_0 T + \theta).$$

Let, $\omega_0 T = 2\pi$ then,

$$x(t+T) = A\sin(2\pi + \omega_0 t + \theta).$$
$$= A\sin(\omega_0 t + \theta)$$
$$= x(t) \text{ . i.e periodic}$$

$$T = \frac{2\pi}{\omega_0}$$

Periodic : $x(t) = x(t+T)$, $T > 0$ where $T$ = period of a cycle, which is an integer value

C.W.

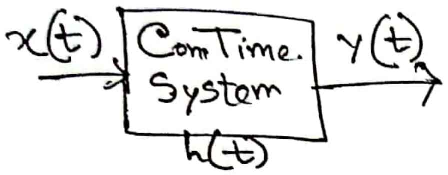
# DSP

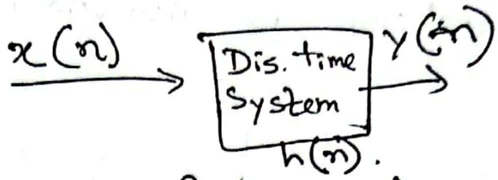
16-06-22
Thursday.

07-Bithi
13-Inc
16-Mithn
24-Jant
26-Fahim

System ~~Signal~~: Any physical device which performs certain operation on the input signal and produces an output. Ex:-, Amplifier, Filter.

Weak signal —▷— Strong Signal.




Impulse Response: The response of the system ~~when~~ to an applied ~~re~~ impulse is called ~~respons~~ impulse response of the system. It is represented as h(t) or h(n).

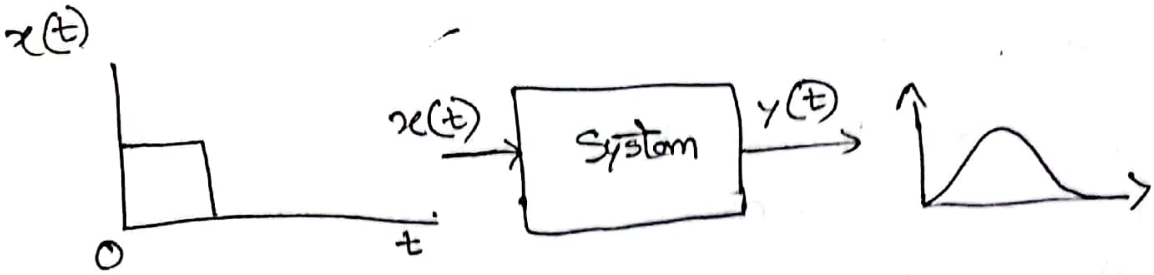
*Difference between digital signal & analog signal

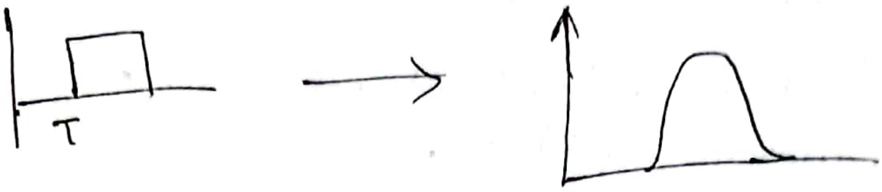
Both Dis & Con. time system are classified ~~invariant or varient system~~ in

1. Time invarient and varient system.
ii. Static ~~&~~ or dynamic system.
iii. Causal or Non-causal "
iv. Linear or Non-linear "
v. Stable or un-stable "

not done

1. ~~Thee~~ time shift in the input results the same corresponding time shift in the output. Then it is called time invarient system.




Check whether the following system is time invariant or varient system. (i) $y(t) = t \cdot x(t)$

$$y(t, t_1) = t \; x(t - t_1) \quad \text{—— (i)}$$

1. Shift
2. Replace.

$$y(t - t_1) = (t - t_1) \; x(t - t_1) \quad \text{— (ii)}$$

(time varient)

$$y(t) = t^2 x(t).$$
$$y(t, t_1) = t^2 x(t - t_1)$$
$$y(t - t_1) = (t - t_1)^2 x(t - t_1).$$

time varient

$$y(t) = x(t^2)$$
$$y(t, t_1) = x(t^2 - t_1)$$
$$y(t - t_1) = x(t - t_1)^2.$$

time varient

$$y(t) = e^{2x(t)}.$$
$$y(t, t_1) = e^{2x(t - t_1)}$$
$$y(t - t_1) = e^{2x(t - t_1)}$$

invarient.

Time varient system: results different.

main কাজ হবে 1st এ $-t_1$ করব 2nd এ t এর $t - t_1$ দিয়ে replace করব

## Static or dynamic.

A continuous or discrete time system is said to be static or memoryless system. If the output at any instance of ~~doesn~~ depends on input at that instant time only. Otherwise it is dynamic system.

① $y(t) = x(t-3)$

$y(0) = x(-3)$ — $x$ এর value past value এ
$y(1) = x(-2)$
∴ dynamic

② $x(n) = x^2(n)$

$x(0) = x^2(0)$ — Present value এর উপর only
$x(1) = x^2(1)$
$x(2) = x^2(2)$
↳ Static

Static = memoryless
dynamic = requires memory

③ $y(t) = x(2t)$

$y(0) = x(0)$
$y(1) = x(2)$ — future value এর উপর depend
↳ dynamic

C.W

|Absent| me
Shovno
natasha
nishi

DSP

18-06-22
Saturday

7. Bithi
8 Nadim
16 Mithun
24 Jannat
26 Fahim
27 Mou
42 Prema
19 Niloy vai

*** Casual and non-casual (anticasual) system

A continous or discrete time system is said to be casual if the output at any instant of time depends on present and past values of input. But not on future values of input.

If the output depends on future values of input also then the system is called non-casual or anti-casual.

① $y(t) = x^2(t) + x(-4)$

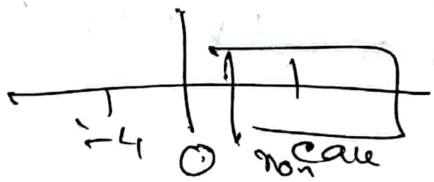
$y(0) = x^2(0) + x(-4)$

↑ Present value ↑ Past value

(ii) $y(t) = x(2-t) + x(t-4)$

$y(0) = x(2) + x(-4)$

↑ future ↑ past




## DSP block diagram:


Figure: Block diagram of DSP system.

## Elementary signal:

The standard elementary discrete time signals as follows:

1. Unit step sequence
2. Unit ramp sequence.
3. Unit impulse sequence
4. Sinosoidal

5. Unit parabolic sequence

6. Real exponential

7. Complex

**1. Unit Step Function:** The discrete time unit step sequence $u(n)$ is defined as $u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$.

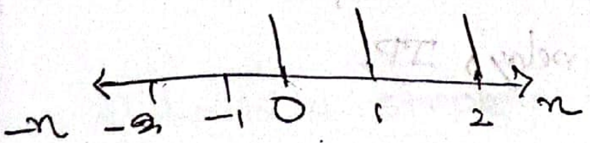
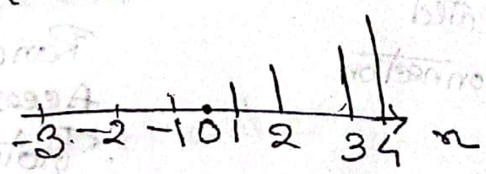



Figure: Discrete time unit step seq.

**2. Unit ramp sequence:** The discrete time unit ramp sequence $r(n)$ is that sequence which start at $n=0$ and increases linearly with time and is defined as

$$r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$



CuWa DSP 17-07-22

## Z-transform ***

Discrete time signal analysis of ↑

Laplace
Continuous time signal analysis of ↑

**two sided Z-transform (bilateral)**

The two sided Z-transform of discrete time signal $x(n)$ is defined as $Z\{x(n)\} = \sum_{n=-\infty}^{+\infty} x(n) z^{-n}$ where z is complex variable

$$\boxed{x(n) \xrightarrow{ZT} X(z)}$$

One sided: $x(n)$ defined as $Z\{x(n)\} = \sum_{n=0}^{\infty} x(n) z^{-n}$
(unilateral)

Inverse Z-transform: The inverse Z-transform of $X(z)$ is defined as $x(n) = \frac{1}{2\pi j} \oint_C z^{n-1} X(z) dz$

***Properties of Z.T***

Scalability, Convulation, linearity

***ROC: Region of Convergence.**

the Range of values of z for which the summation of Z-transform i.e. $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$ converges. (for getting finite value) is called ROC.

Find the z-transform of $x(n) = \{\underset{\uparrow}{1}, 2, 3, 2\}$ (indices 0, 1, 2, 3)

$$Z\{x(n)\} = \sum_{0}^{3} x(n) z^{-n} = x(0) z^{0} + x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3}$$

$$= 1 z^{0} + 2 z^{-1} + 3 z^{-2} + 2 z^{-3}$$

$$= 1 + \frac{2}{z} + \frac{3}{z^2} + \frac{2}{z^3}$$

For $z = 0$ value of $Z(x(n)) = \infty$

$\therefore$ ROC for all values of $z$ except $z=0$.
Ans.

ROC for entire z-plane except $z=0$.

* $x(n) = \{3, 2, -1, -4, \underset{\uparrow}{1}\}$

$-4 \quad -3 \quad -2 \quad -1 \quad 0$

$$Z\{x(n)\} = \sum_{n=-4}^{0} x(n) z^{-n}$$

$$= x(-4) z^{-(-4)} + x(-3) z^{-(-3)} + x(-2) z^{-(-2)} + x(-1) z^{-(-1)} + x(0) z^{-0}$$

$$= 3z^4 + 2z^3 + (-1)z^2 + (-4) z + 1.$$

$$= 3z^4 + 2z^3 - z^2 - z + 1.$$

$x(n)$ of IT

For $z = \infty$ the value of $z\{x(n)\} = \alpha$.

$\therefore$ ROC for entire z-plane except $z = \alpha$
Ans

* $x(n) = \{\underset{\uparrow}{\overset{0}{2}}, \overset{1}{-1}, \overset{2}{3}, \overset{3}{2}, \overset{4}{0}, \overset{5}{1}\}$

$$z\{x(n)\} = \sum_{0}^{5} x(n) z^{-n} = 2z^{-0} + (-1)z^{-1} + 3z^{-2} + 2z^{-3} + 0z^{-4} + 1z^{-5}$$

$$= 2\cdot1 - z^{-1} + \frac{3}{z^2} + \frac{2}{z^3} + \frac{1}{z^5}$$

$$= 2 - \frac{1}{z} + \frac{3}{z^2} + \frac{2}{z^3} + \frac{1}{z^5}$$

ROC for entire z-plane except $z=0$ An

# CNL

1. Switch and ~~Ed~~ End Device Connection

192.168.10.1

2. Router and End-divice.

3. Static Routing. with connecting 4 routers.

4. Dynamic " " " "

7
8
10
13
16
~~20~~ 26.
27
34
19

24

## z-transform

18-07-22

DSP

7
8
10
13
16
26
27
33
34
42
19

Find the z-transform of the following signal

$$x(n) = u(n) = 1 \;;\; n \geq 0$$
$$= 0 \;;\; n < 0$$

$$z\{x(n)\} = \sum_{-\infty}^{+\infty} x(n)\, z^{-n}$$

$$= \sum_{n=0}^{\infty} u(n)\, z^{-n}$$

$$\sum_{n=0}^{\infty} c^n = \frac{1}{1-c} \;;\; 0 < |c| < 1$$

$$= \sum_{n=0}^{\infty} z^{-n}$$

What is condition to converge? (ROC).

$$= \sum_{n=0}^{\infty} (z^{-1})^n$$

$$= \frac{1}{1-z^{-1}}$$

$$= \frac{z}{z-1}$$

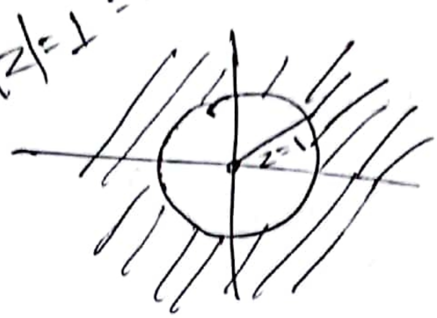
Condition for Convergence:-

$0 < |e| < 1$

$0 < |z^{-1}| < 1.$

$0 < \left|\frac{1}{z}\right| < 1.$

$|z| = 1$ = unit radius circle



$\frac{1}{|z|} < 1$

$|z| > 1$. for infinte. $\therefore$ ROC for ~~all values~~ entire z-plane except $|z| = 1$.

$\frac{1}{1-1^{-1}} = \frac{1}{1-\frac{1}{1}} = \frac{1}{1-1} = \frac{1}{0} = \infty$

H.W. Properties of ROC***

Z-trans / ROC diff la

$\boxed{\text{Linear} \\ \text{Non-linear}}$ → [superposition]

# DSP (In DSP Lab).

Find the z-transform

$x(n) = 0.3^n u(n)$.

$u(n) = 1 ; n \geq 0$
$\quad\quad 0 ; n < 0$

$x(n) = 0.3^n ; n \geq 0$
$\quad\quad 0 ; n < 0$.

$X(z)$

$$= Z\{x(n)\} = \sum_{-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{-\infty}^{\infty} 0.3^n u(n) z^{-n}$$

$$= \sum_{0}^{\infty} 0.3^n z^{-n}$$

$$= \sum_{0}^{\infty} \left(0.3 z^{-1}\right)^n$$

$$= \sum_{0}^{\infty} \left(\frac{0.3}{z}\right)^n$$

$$= \frac{1}{1 - \frac{0.3}{z}}$$ [value of z-transform)

$$= \frac{z}{z - 0.3}$$

ROC for entire z-plane for $z > 0.3$

or

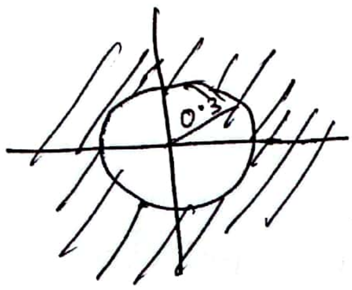
Exterior of circle with radius 0.3 in z-plane

$0 \leq \left|\frac{0.3}{z}\right| \leq 1$

$\frac{0.3}{|z|} < 1$

$0.3 < |z|$

$|z| > 0.3$



$x(n) = 0.8^n u(-n-1)$

$u(-n-1) = 1 \;;\; n \le -1$

$= 0 \;;\; n \ge 0$

$x(n) = 0.8^n \;;\; n \le -1$

$0 \;;\; n \ge 0$

$Z\{x(n)\} = \sum_{-\infty}^{\infty} 0.8^n u(-n-1) z^{-n}$

$= \sum_{-\infty}^{-1} 0.8^n z^{-n}$

$= \sum_{1}^{\infty} 0.8^{-n} z^{n}$

$\frac{1}{1-c}$

$= \sum_{1}^{\infty} (0.8^{-1} z)^n$

$= \sum_{0}^{\infty} (0.8^{-1} z)^n - 1$

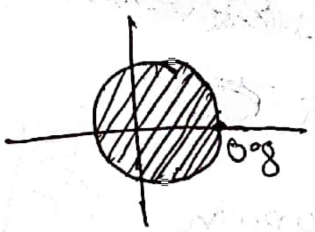
$= \frac{1}{1 - \frac{z}{0.8}} - 1$

$= \frac{0.8}{0.8 - z} - 1$

$= \frac{0.8 - 0.8 + z}{0.8 - z}$

$= \frac{z}{0.8 - z}$

$= -\frac{z}{z - 0.8}$




$|0.8^{-1} z| < 1$

$\frac{|z|}{0.8} < 1$

$|z| < 0.8$

$u(-n-1) = 1 \;;\; n \le -1$

$-\boxed{n} - 1 = 0$

$-(-1) - 1$

$+1 - 1$

$0$

$1$

$$x(n) = 0.3^n u(n) + 0.8^n u(-n-1)$$

$$= \frac{z}{z-0.3} + \frac{-z}{z-0.8}$$

$$= \frac{z}{z-0.3} - \frac{z}{z-0.8}$$

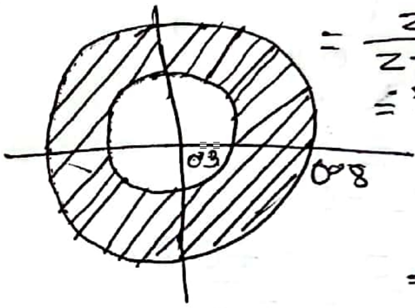
$$= \frac{z(z-0.8) - z(z-0.3)}{(z-0.3)(z-0.8)}$$

$$= \frac{\cancel{z^2} - 0.8z - \cancel{z^2} + 0.3z}{z^2 - 0.8z - 0.3z + 0.24}$$

$$= \frac{-0.5z}{z^2 - 1.1z + 0.24}$$ Ans

$$\frac{0.3}{|z|} < 1 \qquad 0.3 < |z|$$

$$\frac{|z|}{0.8} < 1 \qquad |z| < 0.8$$




$(n-1)$

$1-1=0$

$2-1=1$

$$u(n) = \begin{cases} 1 ; & n \geq 1 \\ 0 ; & n \leq 0 \end{cases}$$

$$\sum_{-\infty}^{\infty} u(n-1) z^{-n}$$

$$z^{-n} = \sum_{1}^{\infty} (z^{-1})^n$$

$$= \sum_{0}^{\infty} (z^{-1})^n - 1$$

$$= \frac{1}{1-\frac{1}{z}} - 1$$

$$= \frac{z}{z-1} - 1$$

$$= \frac{z-z+1}{z-1}$$

$$= \frac{1}{z-1}$$

$$= -\frac{1}{1-z}$$

Ans

C.W. DSP

21-07-22
Thursday.

8
10
13
16
26
27
33
34
19

Determine inverse z-transform.

$$x(z) = \frac{3+2z^{-1}+z^{-2}}{1-3z^{-1}+2z^{-2}}$$

$$1-3z^{-1}+2z^{-2}\ \big)\ 3+2z^{-1}+z^{-2}\ \big(\ 3+11z^{-1}+28z^{-2}+62z^{-3}$$

$$\begin{array}{l} 3-9z^{-1}+6z^{-2} \\ (-)\ (+)\ (-) \\ \hline 11z^{-1}-5z^{-2} \\ 11z^{-1}-33z^{-2}+22z^{-3} \\ (-)\ (+)\ (-) \\ \hline 28z^{-2}-22z^{-3} \\ 28z^{-2}-84z^{-3}+56z^{-4} \\ (-)\ (+)\ (-) \\ \hline 62z^{-3}-56z^{-4} \\ 62z^{-3}-186z^{-4}+124z^{-5} \\ \hline 130z^{-4}-124z^{-5} \end{array}$$

$$x(z) = 3+11z^{-1}+28z^{-2}+62z^{-3}$$
$$= 3z^{0}+11z^{-1}+28z^{-2}+62z^{-3}$$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=0}^{\infty} x(n)z^{-n}$$

$$= x(0)+x(1)z^{-1}+x(2)z^{-2}+x(3)z^{-3}+\dots$$

$$= 3z^{0}+11z^{-1}+28z^{-2}+62z^{-3}+\dots$$

$$x(z) = \{\underset{\uparrow}{3}, 11, 28, 62, \dots\}$$

Determine inverse z-transform. $x(z) = \dfrac{z^2+2z}{z^3-3z^2+4z+1}$

ROC : $|z|>1$.

$|z|>1$.

$|z|>r$ condition

For $|z|>1$ means it's causal system or right sided sequences

For such case $z^{-n}$ i.e. the power of z must be negative. $\therefore$ Do in decending order.

$$z^3-3z^2+4z+1 \,\big)\, z^2+2z \,\big(\, z^{-1}+5z^{-2}+11z^{-3}+12z^{-4} \ldots$$

$$\underset{(-)}{z^2} \underset{(+)}{-3z} \underset{(-)}{+4} \underset{(-)}{+z^{-1}}$$

$$5z-4-z^{-1}$$

$$\underset{(-)}{5z} \underset{(+)}{-15} \underset{(-)}{+20z^{-1}} \underset{(-)}{+5z^{-2}}$$

$$11 \quad -21z^{-1}-5z^{-2}$$

$$\underset{(-)}{11} \underset{(+)}{-33z^{-1}} \underset{(-)}{+44z^{-2}} \underset{(-)}{+11z^{-3}}$$

$$12z^{-1}-49z^{-2}-11z^{-3}$$

$$\underset{(-)}{12z^{-1}} \underset{(+)}{-36z^{-2}} \underset{(-)}{+48z^{-3}} \underset{(-)}{+12z^{-4}}$$

$$-13z^{-2}-59z^{-3}-19z^{-4}$$

$$x(z) = z^{-1}+5z^{-2}+11z^{-3}+12z^{-4}+\ldots$$

$$x(n) = \sum_{n=-\alpha}^{\alpha} x(n)z^{-n} = x(0)+x(1)z^{-1}+x(2)z^{-2}+x(3)z^{-3}+x(4)z^{-4}+\ldots$$

$$= 0+1z^{-1}+5z^{-2}+11z^{-3}+12z^{-4}\ldots$$

$x(0)=0$ $\quad x(2)=5$

$x(1)=1$ $\quad x(3)=11$

$x(4)=12$

$$x(n) = \{\underset{\uparrow}{0}, 1, 5, 11, 12, \ldots\}$$

non-causal/anti causal for NOT right sided. $|z| > \infty$

↳ causal for > sign

∴ z at power inverse.

$$x(z) = \frac{z^2+z+2}{z^3-2z^2+3z+4} \quad \text{ROC } |z|<1$$

arrange in ascending order.

$$\frac{2+z+z^2}{4+3z-2z^2+z^3}$$

$4 \times x = 0.5$

$x = \frac{1}{8}$

$4 \times x = 2.375$

$$4+3z-2z^2+z^3 \overline{\Big) \; 2+z+z^2 \;} \Big| \; \frac{1}{2} - \frac{1}{8}z + \frac{19}{32}z^2$$

$2 + \frac{3}{2}z - z^2 + \frac{z^3}{2}$

(-) (-) (+) (-)

$-0.5z + 2z^2 - 0.5z^3$

$-0.5z - 0.375z^2 + 0.25z^3 - 0.125z^4$

(+) (+) (-) (+)

$2.375z^2 - 0.75z^3 + 0.125z^4$

M-1 Q3 28

$$x(z) = \frac{1}{2} - \frac{1}{8}z + \frac{19}{32}z^2$$

$$x(z) = \ldots\ldots + \frac{19}{32}z^{-(-2)} - \frac{1}{8}z^{-(-1)} + \frac{1}{2}z^{-(0)}$$

$$x(n) = \{\ldots, \frac{19}{32}, -\frac{1}{8}, \underset{\uparrow}{\frac{1}{2}}\}$$

24-07-22

# DSP

## DFT

What is DFT

What is DTS

Frequency domain Sampling & reconstruction of

DTS (Discrete time Signal). sequence.

1.0
1.3
1.6
3.3
3.4
4.2
1.9

DTS $\xrightarrow{\text{transform}}$ $\underbrace{\text{Equivalent F.D (frequency domain)}}$

to analyze করতে হবে then → Fourier transform. $X(e^{j\omega})$

$x(n)$

not so convenient to analyze DTS. (করা যায়)

So, we sample ~~DF~~

Frequency Domain ~~repr.~~ is not convenient representation for a DTS $x(n)$.

Hence, the Fourier transform is sampled to obtain a frequency domain sequence, $X(k)$.

this is called Discrete Fourier transform

Fourier transform $\longrightarrow$ Sample $\rightarrow$ a Fourier Domain Sequence

$X(e^{j\omega})$ $\qquad$ $X(k)$

$\downarrow$

called Discrete Fourier transform

DFT is a powerful tool for frequency analysis of DTS.

$F.T = X(e^{j\omega})$

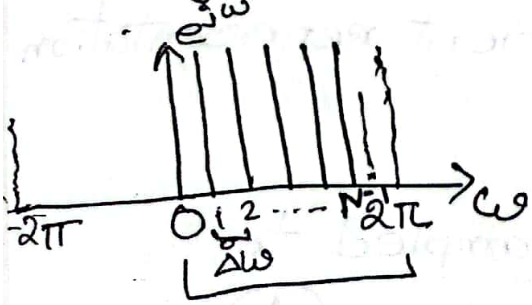
$DFT = X(k)$

Lets Consider a non-periodic discrete time signal $x(n)$. Now, the F.T. of $x(n)$ can be written as- F.T of $x(n) \longrightarrow X(e^{j\omega})$

F.T of $x(n)$

$$X(e^{j\omega}) = X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$X(e^{j\omega})$ is periodic with fundamental frequency ranging from $0 < \omega < 2\pi$




lets take N equidistance Sample in the interval $0 \leq \omega \leq 2\pi$ 0 to N-1 th sample

$$\Delta\omega = \frac{2\pi}{N}$$

$$\Delta\omega = \frac{2\pi}{N}$$

$$\Delta\omega = \frac{2\pi}{N}\cdot k \quad [k = \# \text{of sample}] \quad ; 0 \le k \le N-1$$

$k=0$, $\Delta\omega = 0$

$k=1$, $\Delta\omega = \frac{2\pi}{N}$

$k=2$, $\Delta\omega = \frac{4\pi}{N}$ [Dis from 0 to 2]

$$x(e^{j\omega}) = x(\omega) = \sum_{n=-\alpha}^{\alpha} x(n) e^{-j\omega n} \quad \text{--- (I)}$$

$$x\left(\frac{2\pi}{N}\cdot k\right) = \sum_{n=-\alpha}^{\alpha} x(n) e^{-j\left(\frac{2\pi k}{N}\right) n} \longleftarrow \text{(II)}$$

$$x\left(\frac{2\pi}{N} k\right) = \sum_{n=-\alpha}^{\alpha} x(n) e^{-j\left(\frac{2\pi}{N}\right) kn} \quad ; 0 \le k \le n-1$$

Here, $x(n)$ has infinite no. of summation with N sample ranging from ~~0 to~~ $0-(N-1)$

$$X\left(\frac{2\pi}{N}\cdot k\right) = \sum_{n=-N}^{-1} x(n) e^{-j\frac{2\pi}{N}\cdot kn} + \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}\cdot kn} + \sum_{n=N}^{2N-1} x(n) e^{-j\frac{2\pi}{N}kn} + \cdots$$

$+N$

$---- +$

$\text{(III)}$

$$X\left(\frac{2\pi}{N}\cdot k\right) = \sum_{l=-\alpha}^{\alpha} \sum_{n=lN}^{lN+N-1} x(n) e^{-j\frac{2\pi}{N}kn} \quad \text{--- (4)}$$

compact version of (III).

$$n = n - lN$$

$$lN = n - n$$

$$lN = 0$$

$$X\left(\frac{2\pi}{N} \cdot k\right) = \sum_{l=-\alpha}^{\alpha} \sum_{n=0}^{N-1} x(n-lN)\, e^{-j\frac{2\pi}{N}kn}$$

$$X\left(\frac{2\pi}{N} \cdot k\right) = \sum_{n=0}^{N-1} \underbrace{\sum_{l=-\alpha}^{\alpha} x(n-lN)\, e^{-j\frac{2\pi}{N} \cdot kn}}_{\text{periodic signal of } N}$$

$$x(n-lN) = x_p(n)$$

$$\boxed{X\left(\frac{2\pi}{N} k\right) = \sum_{n=0}^{N-1} x_p(n)\, e^{-j\frac{2\pi}{N} \cdot kn}} \quad (5)$$

D.F.T equation.

Q What is DFT?

* Properties of DFT?

Using Fourier Series ~~any~~ periodic signal can be written as-

$$x_p(n) = \sum_{k=0}^{N-1} a_k \cdot e^{j\frac{2\pi}{N} \cdot kn} \; ; 0 < n < N-1 \quad (6)$$

Here,

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x_p(n) \cdot e^{-j\frac{2\pi}{N} kn} \; ; 0 \le K \le N-1 \quad (7)$$

Fourier coefficient

Compare 5 & 7: 5/7 ⇒

$$\frac{x\left(\frac{2\pi}{N}\cdot k\right)}{a_k} = \frac{1}{\frac{1}{N}} = \frac{1}{\frac{1}{N}}$$

$$\frac{x\left(\frac{2\pi}{N}\cdot k\right)}{a_k} = N$$

$$a_k = \frac{1}{N}\cdot x\left(\frac{2\pi}{N}\cdot k\right).$$

$$x_p(n) = \sum_{k=0}^{N-1} \frac{1}{N}\, x\left(\frac{2\pi}{N}\cdot k\right) \cdot e^{j\frac{2\pi}{N}\cdot kn}$$

$$\boxed{x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} x\left(\frac{2\pi}{N}\cdot k\right) e^{j\frac{2\pi}{N}\cdot kn}} \quad (8)$$

Inverse Discrete fourier transform

IDFT

* Define IDFT

C.W.

7
10
13
16
26
27
33
34
42
19
5 ema apu
16 Sumon vai

DSP

$0 \leq k \leq N-1$

DFT → $x(n)$

* DFT $\{x(n)\} = x(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}k.n}$;

↳ main DFT equation.

$$DFT\{x(n)\} = X(k) = \sum_{n=0}^{N-1} x(n)\, \omega_N^{kn}$$

$$\omega_K = e^{-j\frac{2\pi}{N}}$$

$\omega_n$ = Phase factor / twiddle factor.

Q: What is phase factor?

~~IDFT~~ IDFT $\{x(k)\} = x(n) = \frac{1}{N}\sum_{k=0}^{N-1} x(k)\, e^{j\frac{2\pi}{N}.kn}$

* Compute the N-point (4, 8) DFT of the signal –

(a) $x(n) = \delta(n)$

We know, from DFT equ$^n$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}.kn}$$

$$= \sum_{n=0}^{N-1} \delta(n)\, e^{-j\frac{2\pi}{N}.kn}$$

$$= \delta(0) . e^{-\frac{2\pi}{N}.k.0}$$

$$= \delta(0) . e^{0}$$

$$= 1$$

Ans.

$\delta(n) = 1 ; n = 0$
$= 0 ; n \neq 0$

(b) $x(n) = \delta(n - n_0)$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}.kn}$$

$$= \sum_{n=0}^{N-1} \delta(n - n_0)\, e^{-j\frac{2\pi}{N}.kn}$$

$$= \sum_{n=0}^{N-1} \delta(n_0 - n_0)\, e^{-j\frac{2\pi}{N}.kn_0}$$

$n - n_0 = 0$
$n = n_0$

$= 1 e^{-\frac{2\pi}{N} k.n}$

$= e^{-\frac{2\pi}{N} \cdot kn}$.

$x(n) = a^n \quad 0 \le n \le N-1$

$x(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N} \cdot kn}$

$= \sum_{n=0}^{N-1} a^n e^{-j\frac{2\pi}{N} \cdot n}$

$= \sum_{n=0}^{N-1} \left(a e^{-j\frac{2\pi}{N}k}\right)^n$

$= \frac{\left(a e^{-j\frac{2\pi}{N}k}\right)^0 - \left(a \cdot e^{-j\frac{2\pi}{N}k}\right)^{N-\cancel{1}+\cancel{1}}}{1 - a e^{-j\frac{2\pi}{N}k}}$

$= \frac{1 - \left(a e^{-j\frac{2\pi}{\cancel{N}}k}\right)^{\cancel{N}}}{1 - a e^{-j\frac{2\pi}{N}k}}$

$= \frac{1 - a^N e^{-j2\pi k}}{1 - a e^{-j\frac{2\pi}{N}k}}$

$= \frac{1 - a^N}{1 - a e^{-j\frac{2\pi}{N}k}}$

$$\boxed{\text{Formula} \quad \sum_{N_1}^{N_2} a^n = \frac{a^{N_1} - a^{N_2+1}}{1-a}}$$

$x(n) = a^n = 1 \; ; n = 0$

$$\boxed{\begin{array}{l} e^{-j2\pi k} \\ = \cos 2\pi k - j \sin 2\pi k \\ k = 0, 1, 2, 3, \ldots \\ \cos(2\pi 0) - \sin(2\pi \cdot 0) \\ \text{then } 1 - 0 = 1 \end{array}}$$

27-07-22 - Wednesday.

C.W

13
16

33
34
18 → 19
— 16

CN & DSP → Assignment Mid Question.

## DSP

* Derivation v.v.I. For exam.

Find the 4 point DFT of the sequence $x(n) = 1 ; 0 \le n \le 2$
$0$ ; otherwise.

Soln: $N=4$. Sketch the magnitude and phase spectrum.

$$x(n) = \{ \begin{matrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 0 \end{matrix} \}$$ → position, → DFT points.

$$* \; X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn} \; ; 0 \le k \le N-1$$

$$= \sum_{n=0}^{3} x(n) e^{-j\frac{2\pi}{N}kn} \; ; 0 \le k \le 3$$

$$x(k) = x(0) + x(1).e^{-j\frac{2\pi}{N}.k.1} + x(2).e^{-j\frac{2\pi}{N}.k.2}$$

$$x(3).e^{-j\frac{2\pi}{k}.k\,3}$$

$e^{-j\theta}$

$= \cos\theta - j\sin\theta$

If $k = 1$

$$x(1) = x(0)1 + 1.e^{-j\frac{2\pi}{4}.1} + 1\,e^{-j\frac{2\pi}{4}.1.2} + 0..$$

Calculator
complex
radian

$$= 1 + e^{-j\frac{\pi}{2}} + e^{-j\frac{\pi}{1}} + 0$$

$$= 1 + \left(\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}\right) + \cos 2\pi - j\sin 2\pi$$

$$= 1 + 0 - j + 1 . - 0$$

$$= 2 - j$$

If $k=0$, $x(0) = 1 + 1\times 1 + 1\times 1 + 0 = 3$

$$k=1,\quad x(1) = 1 + 1.e^{j\frac{\cancel{2}\pi}{\cancel{4}_2}.1.1} + 1.e^{-j\frac{2\pi}{4}.1.2} + 0$$

$$= 1 + e^{-j\frac{\pi}{2}} + e^{-j\pi}$$

$$= 1 + \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} + \cos\pi - j\sin\pi$$

$$= 1 + 0 - j + (-1) + 0$$

$$= -j$$

$$x(2) = x(0) + x(1)\, e^{-j\frac{2\pi}{4}\cdot 2\cdot 1} + x(2)\, e^{-j\frac{2\pi}{4}\cdot 2\cdot 2} + 0$$

$$= 1 + 1\times e^{-j\pi} + 1\times e^{-j2\pi} + 0$$

$$= 1 + \cos\pi - j\sin\pi + \cos 2\pi - j\sin 2\pi = \cancel{1} - \cancel{1}\, 0 + 1 - 0 = 1$$

$$x(3) = x(0) + x(1)\, e^{-j\frac{2\pi}{4}\cdot 3} + x(2)\, e^{-j\frac{2\pi}{4}\cdot 3\cdot 2} + 0$$

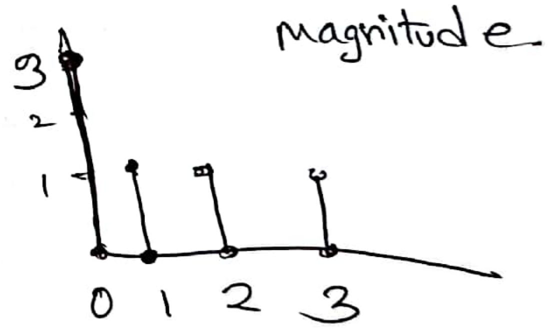
$$= 1 + e^{-j\frac{3\pi}{2}} + e^{-j3\pi}$$

$$= 1 + \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2} + \cos 3\pi - j\sin 3\pi = \cancel{1} + 0 - j - \cancel{1} + 0 = j$$

$$X(k) = \{3, -j, 1, j\}$$

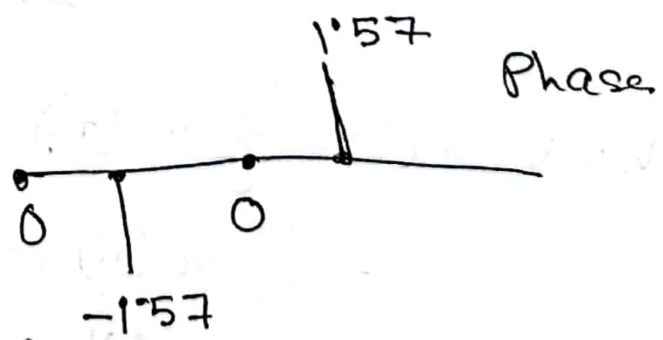
Magnitude: $|x(k)| = \{3, 1, 1, 1\}$

Phase: $\angle x(k) = \{0, -1.57, 0, 1.57\}$




3·(shift +) + spectrum = 3

shift = 0



C.W DSP 28-07-22 Thursday.

8
13
16
26
33
42
19
5
16

* Compute the DFT of the following sequence:- For 4 point

$x(n) = 1 ; \quad 0 \le n \le 1$

$\quad\quad 0 ; \quad$ otherwise

$N = 4$

$N-1 = 3$

$x(n) = \{\underset{0}{1}, \underset{1}{1}, \underset{2}{0}, 0\}$

According to formula, DFT equation-

$$X(k) = \sum_{0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N} \cdot kn}$$

$$= \sum_{0}^{3} x(n) e^{-j\frac{2\pi}{N} k \cdot n}$$ generalized formula

$0 \le k \le N-1$

$$= \sum_{0}^{3} x(n) e^{-j\frac{2\pi}{4} k \cdot n} \quad \text{——— (1)}$$

$k=0, \quad x(0) = \sum_{0}^{3} x(n) e^{-j\frac{2\pi}{4} \cdot 0 \cdot n}$

$$= \sum_{0}^{3} x(n) e^{0}$$

$$= \sum_{0}^{3} x(n)$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 1 + 0 + 0$$

$$= 2$$

$k=1, \quad x(1) = \sum_{0}^{3} x(n) e^{-j\frac{\pi}{2} \cdot 1 \cdot n}$

$$= \sum_{0}^{3} x(n) e^{-j\frac{\pi}{2} n}$$

$$= x(0) e^{-j\frac{\pi}{2} \cdot 0} + x(1) e^{-j\frac{\pi}{2} \cdot 1}$$

$$= 1 \times 1 + 1 \times e^{-j\frac{\pi}{2}}$$

$$= 1 + e^{-j\frac{\pi}{2}}$$

$= 1 + \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = 1 + 0 - j = 1 - j$

$k = 2 \quad x(2) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi}{2}\cdot 2\cdot n}$

$= \sum_{n=0}^{3} x(n)\, e^{-j\pi n}$

$= x(0)\, e^{-j\pi\cdot 0} + x(1)\, e^{-j\pi\cdot 1} + 0 + 0$

$= 1 + e^{-j\pi}$

$= 1 + \cos\pi - j\sin\pi$

$= \cancel{1} - \cancel{1} - j\times 0$

$= 0$

$e^{-j3\pi} = -1 \qquad e^{j\pi} = -1$

$e^{-j\frac{3\pi}{2}} = j \qquad e^{-j\pi} = -1$

$e^{-j\frac{9\pi}{2}} = -j$

$e^{j\frac{9\pi}{2}} = j \qquad e^{j\frac{\pi}{2}} = j$

$e^{j2\pi} = 1$

$e^{j3\pi} = -1 \qquad e^{-j\frac{\pi}{2}} = -j$

$k = 3 \quad x(3) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi}{2}\cdot 3n} \qquad e^{j\frac{3\pi}{2}} = -j$

$= \sum_{n=0}^{3} x(n)\, e^{-j\frac{3}{2}\pi n}$

$= x(0)\, e^{-j\frac{3}{2}\pi\cdot 0} + x(1)\cdot e^{-j\frac{3}{2}\pi\cdot 1} + 0 + 0$

$= 1 + e^{-j\frac{3}{2}\pi}$

$= 1 + \cos\frac{3}{2}\pi - j\sin\frac{3}{2}\pi$

$= 1 + 0 \cancel{+} (-j)$

$= 1 + j$

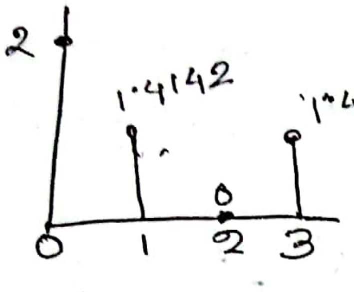
$x(k) = \{2 \quad 1-j \quad 0 \quad 1+j\}$

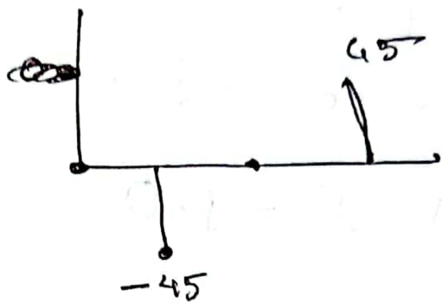
Magnitude : $|x(k)| = \{2 \quad \sqrt{2} \quad 0 \quad \sqrt{2}\}$

$\qquad\qquad\qquad\qquad 1.4142 \qquad\quad 1.4142$

Phase : $\angle x(k) = \{0 . \ -0.785 \quad 0 \ . \ 0.78\}$

$\qquad\qquad\qquad -45^\circ \qquad\qquad 45^\circ$




* Compute 4 point DFT of following sequence $x(n) = \{1, 0, 0, 1\}$ sketch magnitude & phase
$\qquad\qquad\qquad\qquad 0 \ \ 1 \ \ 2 \ \ 3$

$$x(k) = \sum_{0}^{N-1} x(n) \, e^{-j\frac{2\pi}{N} \cdot kn}$$

$$= \sum_{0}^{3} x(n) \, e^{-j\frac{2\pi}{4} k \cdot n}$$

$$= \sum_{0}^{3} x(n) \, e^{-j\frac{\pi}{2} k \cdot n}$$

For $k=0$, $x(0) = \sum_{0}^{3} x(n) \, e^{-j\frac{\pi}{2} 0 \cdot n}$

$$= \sum_{0}^{3} x(n) \, 1$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 0 + 0 + 1 = 2$$

For $k=1$ $x(1) = \sum_{0}^{3} x(n) e^{-j\frac{2\pi}{2}\cdot 1\cdot n}$

$$= \sum_{0}^{3} x(n) e^{-j\frac{\pi}{2}n}.$$

$$= x(0) e^{-j\frac{\pi}{2}\cdot 0} + 0 + 0 + x(3) e^{-j\frac{\pi}{2}\cdot 3}$$

$$= 1 + e^{-j\frac{3}{2}\pi}$$

$$= 1 + \cos\frac{3}{2}\pi - j\sin\frac{3}{2}\pi$$

$$= 1 + 0 - j(-1)$$

$$= 1 + j$$

$k=2$ $x(2) = \sum_{0}^{3} x(n) e^{-j\frac{\pi}{2}\cdot 2\cdot n} = \sum_{0}^{3} x(n) e^{-j\pi n}$

$$= x(0) e^{-j\pi\cdot 0} + 0 + 0 + x(3) e^{-j\pi 3}$$

$$= 1 + \cos 3\pi - j\sin 3\pi = 1 + (-1) - j(0) = 0$$

$k=3$, $x(3) = \sum_{0}^{3} x(n) e^{-j\frac{\pi}{2}\cdot 3n} = \sum_{0}^{3} x(n) \cdot e^{-j\frac{3}{2}\pi\cdot n}$

$$= x(0) e^{-j\frac{3}{2}\pi\cdot 0} + 0 + 0 + x(3) e^{-j\frac{3}{2}\pi\cdot 3}$$

$$= 1 + e^{-j\frac{9}{2}\pi}$$

$$= 1 + \cos\frac{9}{2}\pi - j\sin\frac{9}{2}\pi$$

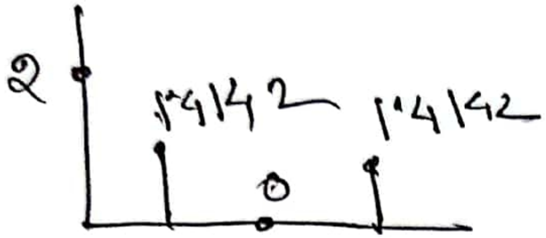
$$= 1 + 0 - j(1)$$

$$= 1 - j$$

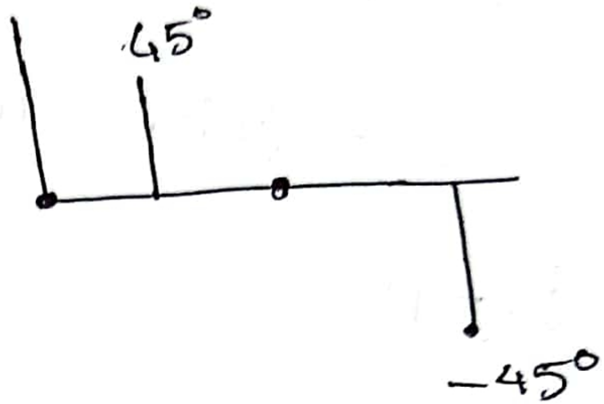
$\therefore x(k) = \{2 \quad 1+j \quad 0 \quad 1-j\}$

$|x(k)| = \{2 \quad 1.414 \quad 0 \quad 1.4142\}$

$\angle x(k) = \{0 \quad 45 \quad 0 \quad -45\}$




$|x(k)|$




DFT → Math

↳ Matrix method* H.W

↳ sequence, point, values etc

## DSP

Find the IDFT of $X(k) = \{6, -2+2j, -2, -2-2j\}$

We know,

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) \cdot e^{j \frac{2\pi}{N} kn} \qquad N=4$$

$$= \frac{1}{4} \sum_{k=0}^{3} x(k) \cdot e^{j \frac{2\pi}{4} k \cdot n}$$

$$= \frac{1}{4} \cancel{\sum_{k=0}^{3}} \left[ x(0) + x(1) e^{j \frac{2\pi}{4} \cdot 1 \cdot n} + X(2) e^{j \frac{2\pi}{4} \cdot 2n} + X(3) e^{j \frac{2\pi}{4} \cdot 3n} \right]$$

$n=0$, $x(0) = \frac{1}{4} [X(0) + x(1) + X(2) + X(3)]$

$$= \frac{1}{4} [6 - 2 + \cancel{2j} - 2 - 2 - \cancel{2j}]$$

$$= \frac{1}{4} \times 0 = 0.$$

$n=1$, $x(1) = \frac{1}{4} \left[ X(0) \cancel{e^{j \frac{2\pi}{4}}} + x(1) e^{j \frac{2\pi}{4} \cdot 1} + x(2) e^{j \frac{2\pi}{4} \cdot 2 \cdot 1} + X(3) e^{j \frac{2\pi}{4} \cdot 3 \cdot 1} \right]$

$$= \frac{1}{4} [6 + (-2+2j) j + (-2)(-1) + (-2-2j)(-j)]$$

$$= \frac{1}{4} [6 - \cancel{2j} - 2 + \cancel{2} + \cancel{2j} \cancel{-2}]$$

$$= \frac{1}{4} \times 4$$

$$= 1$$

$n=2,\ x(2) = \frac{1}{4}\left[x(0) + x(1)\, e^{j\frac{2\pi}{4}\cdot 1\cdot 2} + x(2)\, e^{j\frac{2\pi}{4}\cdot 2\cdot 2}\right.$
$\left. + x(3)\, e^{j\frac{2\pi}{4}\cdot 3\cdot 2}\right]$

$= \frac{1}{4}\left[6 + (-2+2j)\, e^{j\pi} + (-2)\, e^{j2\pi} + (-2-2j)\, e^{j3\pi}\right]$

$= \frac{1}{4}\left[6 + (-2+2j)(-1) + (-2)\,1 + (-2-2j)(-1)\right]$

$= \frac{1}{4}\left[6 + \cancel{2} - \cancel{2j} - \cancel{2} + 2 + \cancel{2j}\right]$

$= \frac{1}{4} \times 8$

$= 2$

$n=3,\ x(3) = \frac{1}{4}\left[x(0) + x(1)\, e^{j\frac{2\pi}{4}\cdot 3} + x(2)\, e^{j\frac{2\pi}{4}\cdot 2\cdot 3} + \right.$
$\left. x(3)\, e^{j\frac{2\pi}{4}\cdot 3\cdot 3}\right]$

$= \frac{1}{4}\left[6 + (-2+2j)\, e^{j\frac{3\pi}{2}} + (-2)\, e^{3j} + (-2-2j)\, e^{j\frac{9\pi}{2}}\right]$

$= \frac{1}{4}\left[6 + (-2+2j)(-j) + (-2)(-1) + (-2-2j)(j)\right]$

$= \frac{1}{4}\left[6 + \cancel{2j} + 2 + 2 - \cancel{2j} + 2\right]$

$= \frac{1}{4} \times 12$

$= 3$

$x(n) = \{0, 1, 2, 3\}$

H.W

IDFT
Math
previous classes

DSP - Presentation

Bithi → anti-aliasing ? aliasing.

Nadim → convulation signal.

Mithun → Window

Fahim → Inverse, deconvulation

$h(n)$ = linear time invarient system $\{1, 2, 3, 4\}$

$x(n)$ = input. $= \{2, 5, 7, 8\}$

Find $y(n) = h(n) * x(n)$ $= \{2, 9, 23,$

| | 2 | 5 | 7 | 8 |
|---|---|---|---|---|
| 1 | 2 | 5 | 7 | 8 |
| 2 | 4 | 10 | 14 | 16 |
| 3 | 6 | 15 | 21 | 24 |
| 4 | 8 | 20 | 28 | 32. |

$y(n) = x(-n)$

$y(n, n_0) = x(-n-n_0)$

$y(n-n_0) = x(-(n-n_0))$
$= x(-n+n_0)$

∴ time varient

# DSP → 31·08·22

7

Model test → 8th September

10

13

24

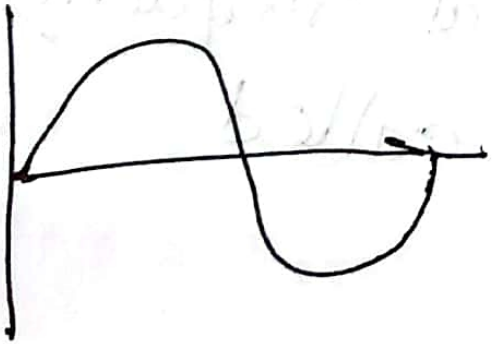
## Sampling theorem

26

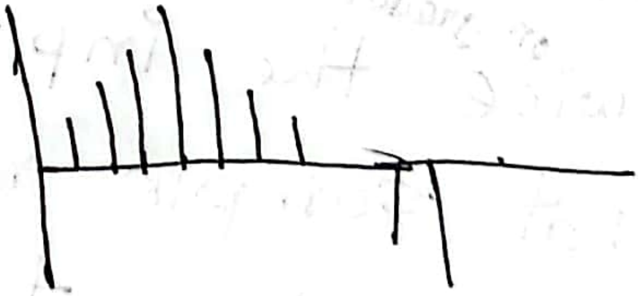
33

34

16



D.t →



Sampling theorem :-

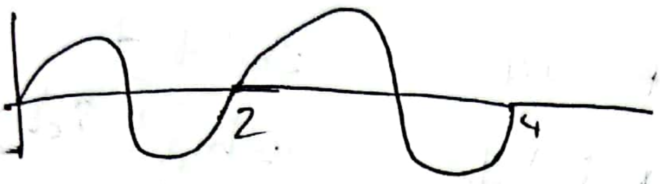
Continuous form of a time varient signal can be represented in the discrete form of a signal with the help of samples and the sampled signal can be recovered to original form when the sampling frequency ($f_s$) having greater frequency value then or equal to the highest input signal frequency ($f_{max}$). $F_s \geq 2 F_{max}$.

⇒ Any sampling frequency ($F_s$) less then twice the input signal frequency will cause a effect. this effect is know as aiasing effect.

When sampling Frequency equals or greater twice the input signal frequency that sample rate is called nyquist rate.

aliasing effects:

1. Frequencreny: complete wave in 1 second

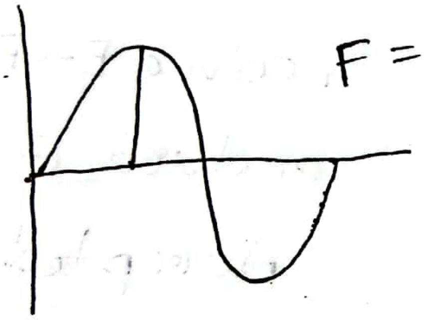



$F = 0{\cdot}5\,Hz$

$-\propto < F \propto \propto$

Normalized, $f = \dfrac{F}{F_s}$ = frequency cont. time signal

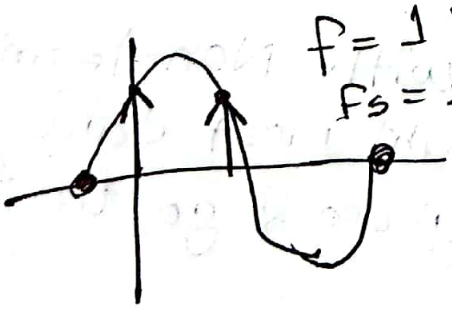
$F_s$ = Sampling



$F = 1\,Hz$

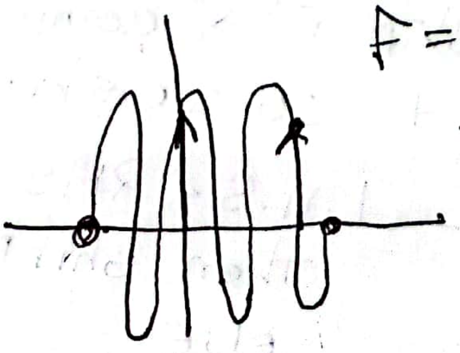
$F_s = 2\,Hz$

unability to detect which ~~frequency~~ a sample is derived from frequency is aliasing effect.



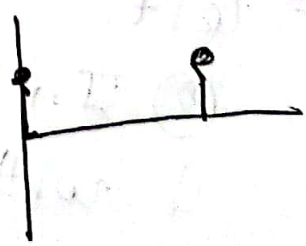
$f = 1\,Hz$

$f_s = 2\,Hz$



$F = 3\,Hz$

$f_s = 2\,Hz$



# DSP - 04-09-22

Consider an Analog Signal

$x_a = 2\cos(50\pi t)$

10

13 Q1: Determine the minimum sampling rate

16 required to avoid aliasing?

24

26 $F_s \geq 2F$ (input frequency)

27

32 *** analog signal $x_a(t) = A\cos(2\pi F t)$

42 important

analog signal input frequency: $F$

19

5 Here,

16 $x_a = 2\cos(2 \cdot \pi \cdot 25 t)$

$\therefore F_s \geq 2F$

$\geq 2 \times 25$

$\geq 50$

$\therefore$ the minimum sampling rate required

$\therefore f_s \geq 50$

* Important equations–

$$x_a(t) = A\cos(2\pi F t)$$
$$x(n) = x_a(nt)$$

Q2: Suppose the signal is sampled at the rate $F_s = 200$ Hz. What is the discrete time signal obtained after sampling.

VVI Model test + Final

$$x(n) = x_a(nT) \qquad T = \frac{1}{F_s}$$
$$= A\cos(2\pi F(nT))$$
$$= A\cos(2\pi n FT)$$
$$= A\cos(2\pi n F \cdot \frac{1}{F_s})$$
$$= 2\cos(\cancel{2}\cdot\pi\cdot n\cdot \cancel{25}\cdot\frac{1}{\cancel{200}\ \cancel{100}\ 4})$$
$$= 2\cos(\pi \cdot n \cdot \frac{1}{4})$$

$x(n) = 2\cos(\frac{\pi}{4} n)$ i.e discrete time signal with $F_s = 200$ Hz.

Q: ~~b~~ $x_a(t) = 2\cos(50\pi t)$

$0 < F < \frac{F_s}{2}$ ∴ f

Q3: What is the frequency that generate samples a sinusoid identical to obtained in $Q_2$.

$x(n) = A\cos(2\pi f n)$ ← discrete time signal.

We know, $x(n) = A\cos(2\pi f n)$ -① &

$x(n) = 2\cos(\frac{\pi}{4} \cdot n)$ ⑪ from $Q_2$

Comparing ① & ⑪ ⇒

$x(n) = 2\cos(2\pi \cdot \frac{1}{8} \cdot n)$

$\therefore f = \frac{1}{8}$

We know, $f = \frac{F}{F_s}$

$F = f \times f_s$

$= \frac{1}{\cancel{8}_{2}} \times \cancel{200}^{\cancel{50}\,25}$

$= 25 Hz.$

$\therefore x(t) = 2\cos(2\pi F t)$

$= 2\cos(2\pi\, 25\, t)$

Q4: Consider, analog signal;

$x_a(t) = 3\cos(100\pi t) + 10\sin(400\pi t)$

$- \cos(600\pi t)$

* what is the Nyquist rate of the signal

$$x_a(t) = 3\cos(2\pi \underbrace{50}_{f_1} t) + 10\sin(2\pi \underbrace{200}_{f_2} t) - \cos(2\pi \underbrace{300}_{f_3} t)$$

Here, $f_1 = 50\,Hz$, $f_2 = 200\,Hz$, $f_3 = 300\,Hz$

$\therefore F_{max} = 300\,Hz$

input equation এর maximum frequency এর twice or greater frequency rate is nyquist rate

$\therefore f_N = 2 \times F_{max}$
$= 2 \times 300$
$= 600\,Hz$.

Q4:- $x(t) = 3\cos(400\pi t) + 5\sin(800\pi t) + \cancel{10\cos(1400}$
$10\cos(14000\pi t)$

Find → Nyquist rate?
→ If we sample this signal using sampling rate $F_s = 600$ sample/s. What is discrete time signal after sampling.

a) Here,
$x(t) = 3\cos(2\pi 200t) + 5\sin(2\pi 400t) + 10\cos(2\pi 7000t)$

$F_1 = 200\,Hz$
$F_2 = 400\,Hz$
$f_3 = 7000\,Hz$.

$\therefore f_{max} = 7000\,Hz$

$\therefore F_N = 2 \times 7000\,Hz$
$= 14000\,Hz$

Nyquist rate
$F_N \geq 14000\,Hz$.

(b) We know,

$$x_n = x_a(nT)$$

$$= x_a\left(n \cdot \frac{1}{F_s}\right)$$

$$x_n = x_a\left(\frac{n}{6000}\right)$$