2020. 7(a)

Define DFT and IDFT equation.

⇒ DFT : Freequency domain representation is not Convenient representation for a DTS, $x(n)$, hence the fourier transform is sampled to obtain a freequency domain sequence $x(k)$, this is Called Discrete Fourier Transform.

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}\cdot kn} \quad ; \quad 0 \leq k \leq N-1$$

IDFT : The Inverse DFT transforms N discrete freequency samples to the same number of discrete time sample. The IDFT has a form very Similar to the DFT.

The formula for IDFT is,

$$x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} x\left(\frac{2\pi}{N}\cdot k\right)\cdot e^{j\frac{2\pi}{N}\cdot kn}$$

2020. 7(b)

Define Symmetry property of DFT equation.

⇒ The DFT of a real valued Discrete-time Signal has a special Symmetry, in which the real part of the transform Values are DFT even Symmetric and the imaginary part is DFT odd symmetric, as illustrated in the equation.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{j\omega} = x(e^{j(-\omega)})$$

2020. 7(c)

Define DFT leakage.

⇒ Spectral leakage occurs when a non-Integer number of periods of a signal is sent to the DFT (Discrete Fourier Transform). Spectral leackage lets a single tone Signal be spread among Several frequencies after the DFT operation. This makes it hard to find the actual freeequency of the Signal.

2020 . 7(d)

Define Hamming Window in both TD and FD.

⇒ The Hamming window is a taper formed by using a raised cosine with Non-zero end-points, Optimized to minimize the nearest side lode. Number of point in the Output window. If zero or less, an empty array is returned. When True (default), generates a Symmetric window, for use in filter design.

2020. 8(b)

Determine the 4 point DFT of the Sequence $x(n)=\{1,0,1,1\}$

$\Rightarrow x(n)=\{1,0,1,1\}$

$$ATF : x(k) = \sum_{0}^{N-1} x(n).e^{-J\frac{2\pi}{N}k(n)}$$

$$x(k) = \sum_{0}^{3} x(n).e^{-J\frac{2\pi}{4}k(n)}$$

$$= \sum_{0}^{3} x(n)e^{-J\frac{\pi}{2}k(n)}.$$

$K=\{0,1,2,3\}$

If $k=0$

$$= \sum_{0}^{3} x(n)e^{-0}$$

$$= \sum_{0}^{3} x(n)$$

$$= x(0)+x(1)+x(2)+x(3)$$

$$= 1+0+1+1$$

$$= 3$$

If $k=1$

$$= \sum_{0}^{3} x(n).e^{-J\frac{\pi}{2}n}$$

$$= \sum_{0}^{3} x(n)e^{-J\frac{\pi}{2}n}$$

$$= x(0).e^{-0}+x(1).e^{-j\frac{\pi}{2}.1}+x(2).e^{-j\pi}+x(3).e^{-j\frac{\pi}{2}3}$$

$$= 1.1+0+1.e^{-J\pi}+1.e^{-J\frac{\pi}{2}.3}$$

$$= 1+(-1)+J$$

$$= J$$

If $K=2$

$$= \sum_{0}^{3} x(n).e^{-J\frac{\pi}{2}.2n}$$

$$= \sum_{0}^{3} x(n)\, e^{-J\pi n}$$

$$= x(0).e^{-0}+0+x(2)e^{-J\pi}+x(3).e^{-J3\pi}$$

$$= 1.1+0+e^{-J2\pi}+e^{-J3\pi}$$

$$= 1+0+J-1$$

$$= J$$

If $K=3$

$$= \sum_{0}^{3} x(n)e^{-J\frac{\pi}{2}.3n}$$

$$= \sum_{0}^{3} x(n)\, e^{-J\frac{3\pi}{2}n}$$

$$= x(0).e^{-0}+0+x(2).e^{-J\frac{3\pi}{2}.2}+x(3).e^{-J\frac{3\pi}{2}.3}$$

$$= 1.1+0+e^{-J3\pi}+e^{-J\frac{9\pi}{2}}$$

$$= 1+0-1-J$$

$$= -J$$

$$\therefore x(K) = \{3, J, J, -J\}$$

2019. 5(a)

Define fourier series. Point out some important of Fourier transform in DSP.

⇒ Fourier Series : A Fourier series is an expansion of a periodic function f(x) in terms of an infinite sum of sines and Cosines.

Some importance of Fourier transform in DSP:

(i) Fourier transform is a linear operator, if given the appropriate norm, it is still unitary operator.

(ii) The inverse transformation of the Fourier transform is easy to find, and the form is very similar to the positive transformation.

(iii) The sinusoidal basis function is an Eigen function of the differential operation, so that the solution of the linear differential equation can be transformed into the algebraic equation of the Constant ~~coff~~ Coefficient.

2020. 5(c)

Find 4-point DFT of the following sequence

$$x(n) = \{1, 0, -1, 0\}$$

$\Rightarrow$ AFF: $x(k) = \sum_{0}^{N-1} x(n) \cdot e^{-J\frac{2\pi}{N}kn}$

$$= \sum_{0}^{3} x(n) \cdot e^{-J\frac{2\pi}{4}kn} \quad \text{--- (1)} \quad 0 \leq k \leq N-1$$

$K = 0, 1, 2, 3$

If $K = 0$

$$x(0) = \sum_{0}^{3} x(n) e^{-0}$$

$$x(0) = \sum_{0}^{3} x(n) \cdot 1 \Rightarrow x(0) = \sum_{0}^{3} x(n)$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 0 + (-1) + 0$$

$$= 1 - 1$$

$$= 0$$

If $k = 1$

$$x(1) = \sum_{0}^{3} x(n) e^{-J\frac{2\pi}{4} \cdot 1n}$$

$$x(1) = \sum_{0}^{3} x(n) e^{-J\frac{2\pi}{4} \cdot n}$$

$$= x(0) \cdot e^{-0} + x(1) \cdot e^{-J\frac{2\pi}{4} \cdot 1} + x(2) \cdot e^{-J\frac{2\pi}{4} 2} + x(3) \cdot e^{-J\frac{2\pi}{4} \cdot 3}$$

P.T.O

$= 1.1 + 0 + (-1).(-1) + 0$

$= 1+1$

$= 2$

If $k=2$

$$x(2) = \sum_{0}^{3} x(n) e^{-J\frac{2\pi}{4}kn}$$

$$= \sum_{0}^{3} x(n) e^{-J\frac{2\pi}{4}2.n}$$

$$= x(0).e^{-0} + x(1).e^{-J\frac{2\pi}{4}2.1} + x(2).e^{-J\frac{2\pi}{4}2.2} + x(3).e^{-J\frac{2\pi}{4}.3.1}$$

$= 1.1 + 0 + (-1).e^{-J2\pi} + 0$

$= 1-1$

$= 0$.

If $k=3$

$$x(3) = \sum_{0}^{3} x(n) e^{-J\frac{2\pi}{4}3n}$$

$$= x(0).e^{-0} + 0 + x(2) e^{-J\frac{2\pi}{4}3.3} + 0$$

$$= 1.1 + 0 + (-1) e^{-J\frac{18\pi}{4}} + 0$$

$= 1 + (-1)(-j)$

$= 1+j$

$\therefore x(k) = \{0, 2, 0, 1+j\}$

2018. 5(a)

Define Fourier series of a Continuous time signal.

⇒ The CT Fourier ~~series~~ Series Considers periodic signals. A signal $x(t)$ is periodic with period T

in Case: $\forall t : x(t+T) = x(t)$.

Note that in case x is periodic with period T it is also periodic with period 2T. This leads us to the definite of the fundamental period T_0 being the smallest value such that $x(t+T_0) = x(t)$. The fundamental frequency then is $f_0 = 1/T_0$.

2018. 5(b)

What is fourier integral? Write down some properties of Continuous time fourier transform.

⇒ Fourier integral: A formula for the decomposition of a nonperiodic function into harmonic Component whose frequencies rang over a Continuous set of values.

Some properties of Continuous time fourier transform:

Property of CTFT	Time Domain $x(t)$	Frequency Domain $x(\omega)$
Linearity Property	$ax_1(t)+bx_2(t)$	$aX_1(\omega)+bX_2(\omega)$
Time shifting property	$x(t \pm t_0)$	$e^{\pm j\omega t_0} X(\omega)$
Frequency shifting Property	$e^{\pm j\omega_0 t} x(t)$	$X(\omega \pm \omega_0)$
Time reversal property	$x(-t)$	$x(-\omega)$

Property of CTFT	Time Domain $x(t)$	Frequency Domain $X(\omega)$
Time Scaling property	$x(at)$	$\frac{1}{\lvert a \rvert} X\left(\frac{\omega}{a}\right)$
Time Differentiation Property	$\frac{d}{dt} x(t)$	$j\omega X(\omega)$
Frequency Derivative property	$t.x(t)$	$J \frac{d}{d\omega} X(\omega)$
Time integration Property	$\int_{-\alpha}^{\alpha} x(t)\, dT$	$\frac{X(\omega)}{j\omega}$
Modulation property	$x(t) \cos \omega_0 t$ $x(t) \sin \omega_0 t$	$\frac{1}{2}[X(\omega-\omega_0)+X(\omega+\omega_0)]$ $\frac{1}{2j}[X(\omega-\omega_0) \cancel{+} - X(\omega+\omega_0)]$