Presentation
Digital signal Processing

* Define LTI system?

A system satisfying both the linearity and the time invariance property is called LTI system.

Linearity:

Linear system is a system that possesses the property of superposition.

Time invariance:

A system is time invariant if the behaviour and characteristics of the system are fixed over time.

Discrete-time linear time invariant system:

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{elsewhere} \end{cases}$$

The impulse response completly characterizes the behaviours of any LTI system. The discrete-time unit impulse can be used to construct any discrete-time signal.

Representation of Discrete-time signals in Terms of Impulses:

$$x(n)\,\delta(n-k) = x(k)\,\delta(n-k)$$

$x(n)$ represents the input signal but $x(k)$ represents the magnitude of the input signal $x(n)$ at time k.

Then the eqn:

$$x(n) = \sum_{k=-\infty}^{+\infty} x(k)\,\delta(n-k)$$

here $\delta(n-k)$ time-shifted impulse sequence.

If the input signal $x(n) = u(n)$,

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

then any unit step signal can be represented as,

$$u(n) = \sum_{k=0}^{n} \delta(n-k)$$

Convolution Sum:

The 'convolution sum' and is denoted by the symbol $*$,

$$y(n) = \sum_{k=-\alpha}^{\alpha} x(k)\, h(n-k) = x(n) * h(n)$$

Problem 3.1:

Express the given signal sequence as a time shifted impluse –

$$x(n) = \{1, -2, 8, \underset{\uparrow}{4}, 5, -3, 7\}$$

Soln:

the arrow '↑' shows the value of data for $n = 0$.

n	-3	-2	-1	0	↑	2	3
$x(n)$	1	-2	8	4	5	-3	7

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$= \sum_{k=-3}^{+3} x(k)\,\delta(n-k)$$

$$= x(-3)\,\delta(n+3) + x(-2)\,\delta(n+2) + x(-1)\,\delta(n+1)$$

$$+ x(0)\,\delta(n) + x(1)\,\delta(n-1) + x(2)\,\delta(n-2) + x(3)\,\delta(n-3)$$

$$= \delta(n+3) - 2\,\delta(n+2) + 8\,\delta(n+1) + 4\,\delta(n) + 5\,\delta(n-1)$$

$$- 3\,\delta(n-2) + 7\,\delta(n-3)$$

Properties of Convolution:

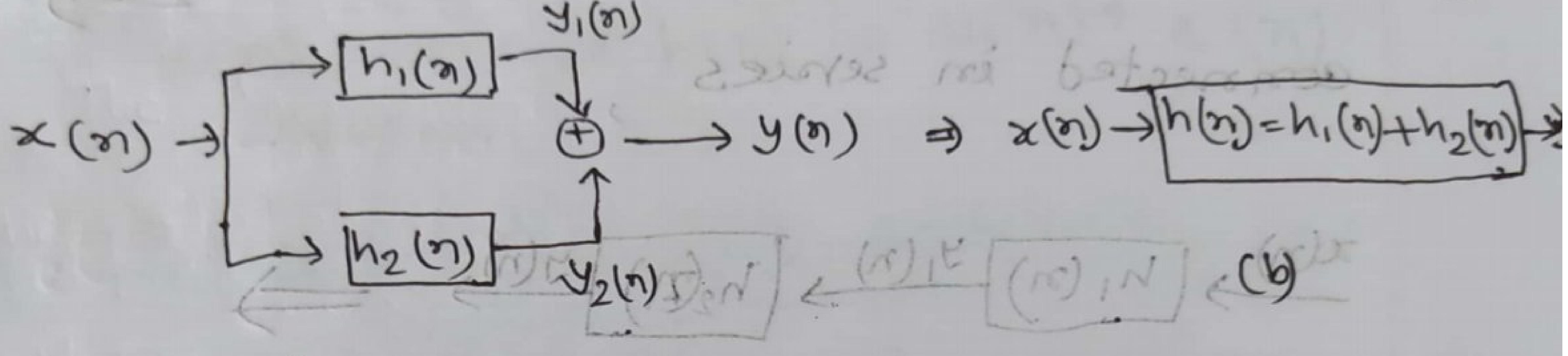
(i) Distributive property

(ii) Commutative Property

(iii) Associative Property

① Distributive Property:

Let us consider two LTI system with impulse responses $h_1(n)$ and $h_2(n)$ connected parallelly



(a)

Fig: (a) System Connected in parallel
(b) Equivalent representation

By defination of the distributive property,

$$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$$

$$\therefore y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k).$$

$$= x(n) * h(n)$$

Associative Property:

Let us, consider two LTI system with impulse responses $h_1(n)$ and $h_2(n)$ connected in series,

$$x(n) \rightarrow \boxed{h_1(n)} \xrightarrow{y_1(n)} \boxed{h_2(n)} \xrightarrow{y(n)} \quad \Longrightarrow$$

$$x(n) \rightarrow \boxed{h(n) = h_1(n) \cdot h_2(n)} \rightarrow y(n)$$

By the defination of associative property

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

Commutative Property:

let us consider two LTI system with impulse responses $h_1(n)$ and $h_2(n)$ connected in series,

$$x(n) \rightarrow \boxed{h_1(n)} \rightarrow \boxed{h_2(n)} \rightarrow y(n) \equiv x(n) \rightarrow \boxed{h_2(n)} \rightarrow \boxed{h_1(n)} \rightarrow y(n)$$

By defination of the commutative property,

$$y(n) = x(n) * h(n) = h(n) * x(n)$$

Properties of LTI system:

(I) LTI system with and without memory
(II) Invertibility of LTI system
(III) Causality of LTI system
(IV) Stability of LTI system
(V) The unit step response of LTI system

(I) LTI system with and without memory:

A system is memoryless if the output at any time depends only on the present input. This is true for the LTI system if and only if

$$h(n) = 0, \quad n \neq 0$$

Let us consider the impulse response of the form $h(n) = k\,\delta(n)$.

where $k = h(0)$, is a constant

The output of such a system is given by,

$$y(n) = \sum_{k=-\alpha}^{\alpha} x(k)\, h(n-k)$$

$$y(n) = \sum_{k=-\alpha}^{\alpha} x(k)\, k\,\delta(n-k)$$

$\therefore y(n) = k\, x(k)$, is a memoryless LTI system.

If $h(n) \neq 0$, $n \neq 0$, then the LTI system is called a memory system.

2. Invertibility of LTI system:

Let us consider the following figure —

$$x(n) \rightarrow \boxed{h(n)} \xrightarrow{y(n)} \boxed{h_1(n)} \rightarrow w(n) = x(n)$$

system 1   system 2

Fig: Invertibility of LTI system

The system response $h(n)$ results in an output $y(n)$ and the output of system 1 is given to system 2, whose response $h_1(n)$ results in a output $w(n)$, which is equal to the original input $x(n)$.

This is possible if

$$\boxed{h(n) * h_1(n) = \delta(n)}$$

Stability for LTI system :

A system is said to be stable if every bounded input produces a bounded output. The statement can be extended to LTI system also.

Let us consider a bounded input $x(n)$,

$$|x(n)| < M_x < \alpha \text{ for all } n$$

Suppose the bounded input is applied to an LTI system with unit impulse response $h(n)$, then using convolution sum, we obtain an expression for the output $y(n)$,

$$|y(n)| = \left| \sum_{k=-\alpha}^{\alpha} x(n)\, h(n-k) \right|$$

By the inequality relation, the magnitude of the sum of a set of numbers is no

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longer larger than the sum of the magnitudes of the numbers,

$$|y(n)| \le \sum_{-\alpha}^{\alpha} |x(k)| \, |h(n-k)|$$

or

$$|y(n)| \le \sum_{-\alpha}^{\alpha} |h(k)| \, |x(n-k)|$$

We know that,

$$|x(n)| \le m_x < \alpha$$

Therefore, $|x(n-k)| \le m_x < \alpha$ for all n and k

Substitute the equivalent relation of equation

$$|y(n)| \le m_x \sum_{k=-\alpha}^{\alpha} |h(k)| \text{ for all } k$$

The impulse response h(k) is absolute summable if, $\sum_{k=-\alpha}^{\alpha} |h(k)| < \alpha$

then the output of the LTI system y(n) is stable (bounded output). If the impulse response h(k) is not absolutely summable, then the system is a 'nonstable system'.

Causal system:

By the defination, for a discrete-time-causal LTI system, the impulse response $h(n)$ must be zero for $n<0$. The causality can be extended to convolution sum as ____

$$y(n) = \sum_{k=-\alpha}^{\alpha} x(k)\, h(n-k)$$

For a causal discrete-time-LTI system $h(n) = 0$ for $n<0$. Therefore, the output of a causal system must be expressed as,

$$y(n) = \sum_{k=0}^{\alpha} x(k)\, h(n-k)$$

a causal system cannot generate an output before an input is given to the system.