DSP

Year Question Solve: 14, 15, 16, 18

2014

1.a) Define digital signal processing (DSP). [

- Digital Signal Processing (DSP): Digital signal processing (DSP) is the process of analyzing and modifying a signal to optimize or improve its efficiency or performance. It involves applying various mathematical and computational algorithms to analog and digital signals to produce a signal that's of higher quality than the original signal.

1.b) Write the advantages and limitation of DSP. [3.75]

- Advantages of DSP:

Digital signal processing processes several advantages over analog signal processing. They are -

- Greater accuracy: The tolerance of the circuit components used to design the analog filters affects the accuracy, whereas the DSP provides superior control of accuracy.

- **Cheaper:** In many applications, digital realization is comparatively cheaper than its analog counterpart.
- **Ease of Data Storage:** Digital signals can be easily stored on magnetic media without loss of fidelity and can be processed off-line in a remote laboratory.
- **Implementation of sophisticated algorithm**
The DSP allows to implement sophisticated algorithms when compared to its analog counterpart.
- **Flexibility in configuration:** A DSP system can be easily reconfigured by changing the program. Reconfiguration of an analog system involves redesign of system hardware.

• <u>Applicability of VLF signals:</u> The very Low frequency signals such as those occuring i[n] seismic application can be easily processed using a digital signal processor when compared to an analog processing syste[m] where inductors and capacitors needed would be physically very large in size.

• <u>Time sharing:</u> DSP allows the sharing of a given processor among a number of signals by time sharing thus reducing th[e] cost of processing a signal.

Limitations of DSP:

• <u>System Complexity:</u> System complexity increases in the digital processing of an analog signal because of devices such a[s] A/D and D/A converters and their associated filters.

• <u>Bandwidth limited by sampling rate:</u> Band limited signals can be sampled without information loss if the sampling rate i[s] more than twice the bandwidth. Therefore signals having extremely wide bandwidths require fast sampling rate A/D converter and fast digital signal processors. But there is practical limitations in the speed of operation of A/D converters and digital signal processors.

- **Power Consumption:** A variety of analog processing algorithms can be implemented using passive circuit elements like inductors, capacitors and resistors that do not need much power, whereas a DSP chip containing over 4 lakh transistors dissipates more power (1 watt).

1(c) Explain unit ramp sequence and exponen[tial] sequence of discrete-time signal. [4]

=> **Unit ramp sequence:** The unit ramp sequence is defined as:-

$$r(n) = n \quad \text{for } n \geq 0$$
$$= 0 \quad \text{for } n < 0$$

The graphical representation of $r(n)$ is shown in Fig-1.

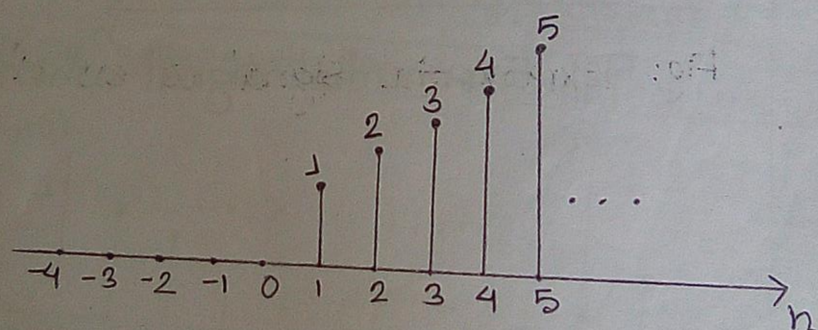


Fig-1: Unit ramp Sequence.

Exponential sequence: The exponential signa[l] is a sequence of the form -

$$x(n) = a^n \quad \text{for all } n .$$

When the value of $a > 1$, the sequenc[e] grows exponentially and when the value is $0 < a < 1$, the sequence decays exponentially. Note also that when $a < 0$, the discrete time exponential signal takes alternating signs.

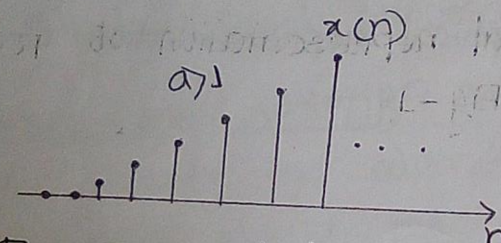
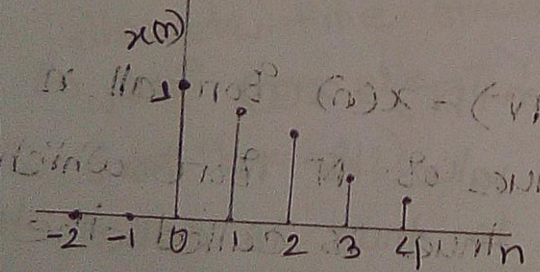
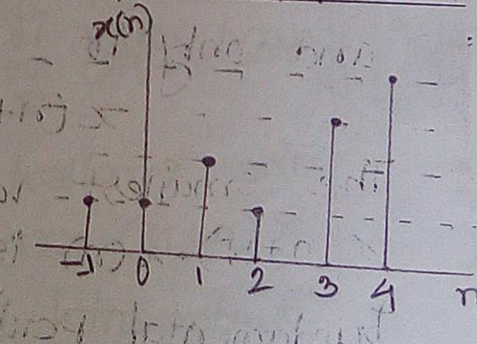


Fig: Exponential signal when $a > 1$.

2·a) Distinguish between discrete-time signals and discrete-valued signals. [I·75]

Discrete-time signals	Discrete-valued signals
Discrete-time signals are defined only at certain specific values of time.	If a signal takes on values from a finite set of possible valu It is said to be o discrete-valued sigr
These time instants need not be equidistant.	These values are equidistant.
Graphical representation:	Graphical representi
	

2.b) Discuss the properties by which the discrete-time sinusoids can be characterized. [3]

– The properties by which the discrete-time sinusoids can be characterized given below:–

1. A discrete-time sinusoid is periodic only if its frequency f is a rational number.

By definition, a discrete-time signal $x(n)$ is periodic with period N $(N>0)$ if and only if

$$x(n+N) = x(n) \text{ for all } n.$$

The smallest value of N for which $x(n+N) = x(n)$ is true is called the fundamental period.

2. Discrete-time sinusoids whose frequenc are separated by an integer multiple of 2π are identical.

To prove this assertion, let us consider the sinusoid $\cos(\omega_0 n + \theta)$. It easily follows that

$$\cos[(\omega_0 + 2\pi)n + \theta] = \cos(\omega_0 n + 2\pi n + \theta) = \cos(\omega_0 n + \theta)$$

As a result, all sinusoidal sequences

$x_k(n) = A\cos(\omega_k n + \theta)$, $k = 0, 1, 2, \ldots$

where, $\omega_k = \omega_0 + 2k\pi$, $-\pi \le \omega_0 \le \pi$

are identical.

3. The highest rate of oscillation in a discrete-time sinusoid is attained when $\omega = \pi$ (or $\omega = -\pi$) or equivalently, $f = 1/2$ (or $f = -1/2$).

2. (a) Discuss the sampling process of analog signal [3]

— There are many ways to sample an analog signal, we limit our discussion to periodic or uniform sampling, whic is the type of sampling used most often in practice. This is described by the relation

$$x(n) = x_a(nT), \quad -\infty < n < \infty$$

Periodic sampling establishes a relationship between the time variables t and n of continuous-time and discrete-time signals, respectively. Indee these variables are linearly related through the sampling period T or, equivalently through the sampling rate $F_s = 1/T$ as

$$t = nT = \frac{n}{F_s}$$

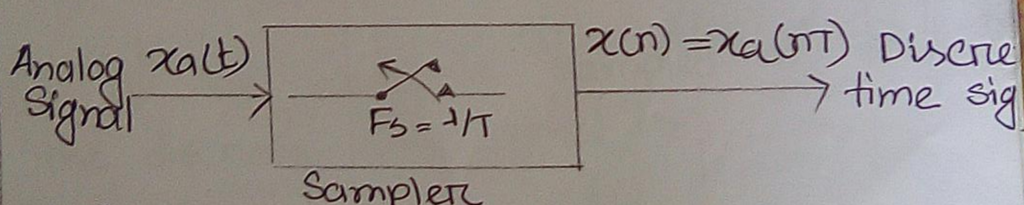
As a consequence of $t = nT = \frac{n}{F_s}$, there exists a relationship between the frequency variable F (or Ω) for analog signals and the frequency variable f (or ω) for discrete-time signals. To establish this relationship, consider an analog sinusoidal signal of the form

$$x_a(t) = A \cos(2\pi F t + \theta)$$

which, when sampled periodically at a rate $F_s = 1/T$ samples per second, yield

$$x_a(nT) \equiv x(n) = A \cos(2\pi F n T + \theta)$$

$$= A \cos\left(\frac{2\pi n F}{F_s} + \theta\right)$$



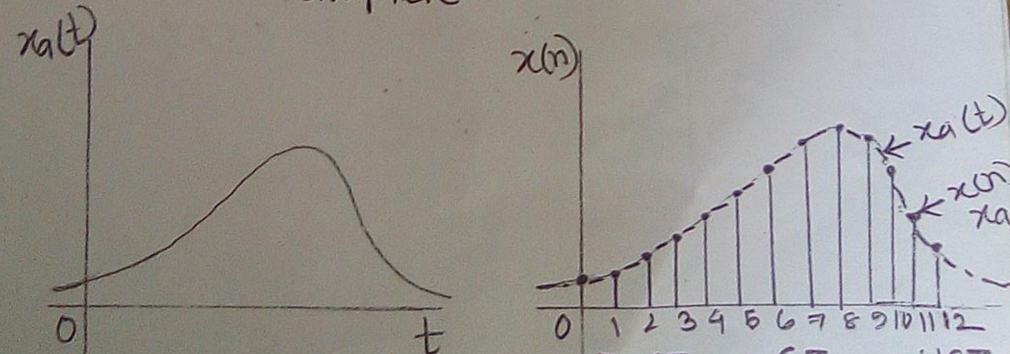


Fig: Periodic sampling of an analog signal.

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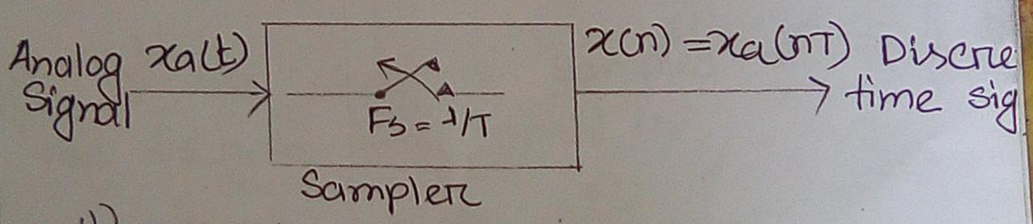

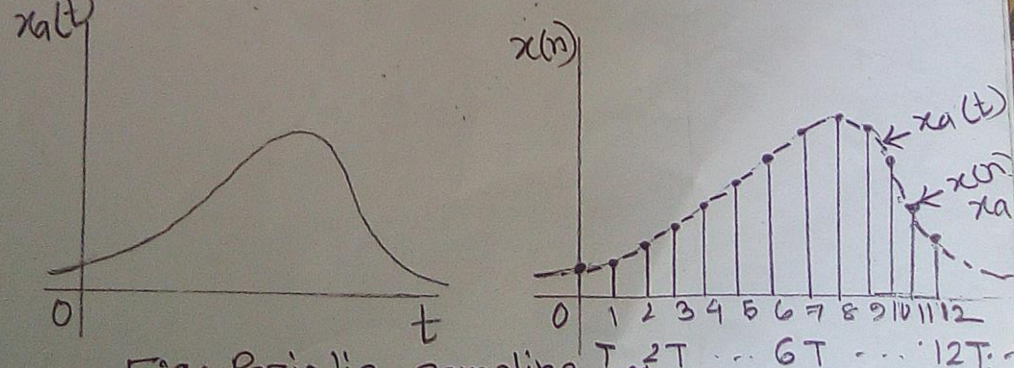

Fig: Periodic sampling of an analog signal.

2.d) Given the two analog sinusoidal signa[ls]
$x_1(t) = \cos 2\pi(10)t$ and $x_2(t) = \cos 2\pi(50$[)t]
are sampled at a rate of $F_s = 40$ Hz.
is there occur of any frequency
aliasing ? [1]

— Yes there is occur of frequency aliasing. The frequency $F_2 = 50$ Hz is an alias of the frequency $F_1 = 10$ Hz at the sampling rate of 40 samples per second.

3.a) Consider the analog signal $x_a(t) = 3\cos 10($

(i) Determine the minimum sampling rate required to avoid aliasing.

(ii) Suppose that the signal is sampled at the rate $F_s = 200\,Hz$. What is the discre time signal obtained after sampling? [1].

(i) The frequency of the analog signal is $F = 50\,Hz$. Hence the minimum sampling rate required to avoid aliasin is $F_s = 100\,Hz$.

(ii) If the signal is sampled at $F_s = 200$ the discrete-time signal is,

$$x(n) = 3\cos\frac{100\pi}{200}n$$

$$= 3\cos\frac{\pi}{2}n$$

3.b) Suppose two discrete signals are represented by the following two finite duration sequences.

$$x_1 = \{3, -1, \underset{\uparrow}{-2}, 5, 0, 4, -1\}$$
$$x_2 = \{\underset{\uparrow}{3}, -1, -2, 5, 0, 4, -1\}$$

Are these two signals different or not? Explain. [1.5]

– Here,

$$x_1 = \{3, -1, \underset{\uparrow}{-2}, 5, 0, 4, -1\}$$
$$x_2 = \{\underset{\uparrow}{3}, -1, -2, 5, 0, 4, -1\}$$

These two signals are different. Because sequence with the time origin $(n=0)$ indicated by the symbol $\uparrow$. The x_1 an x_2 signals have different time origin $(n=0)$ indicated by the symbol $\uparrow$.

The graphical representation of x_1 and x_2 are,

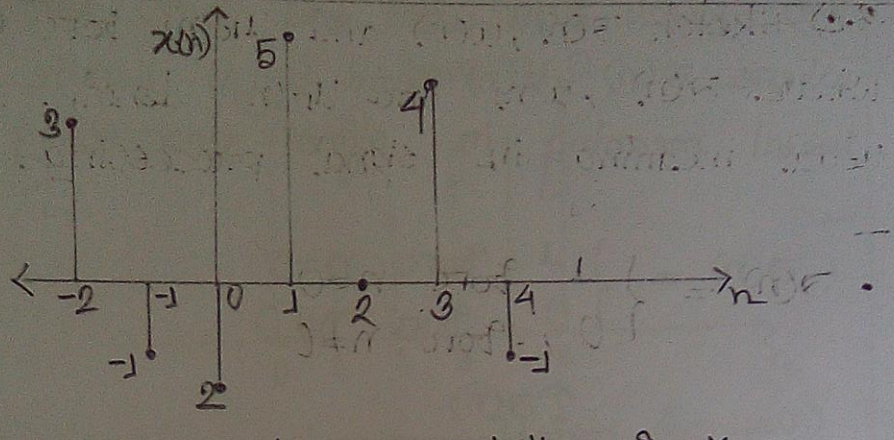

Fig-1: Graphical representation of x_1.

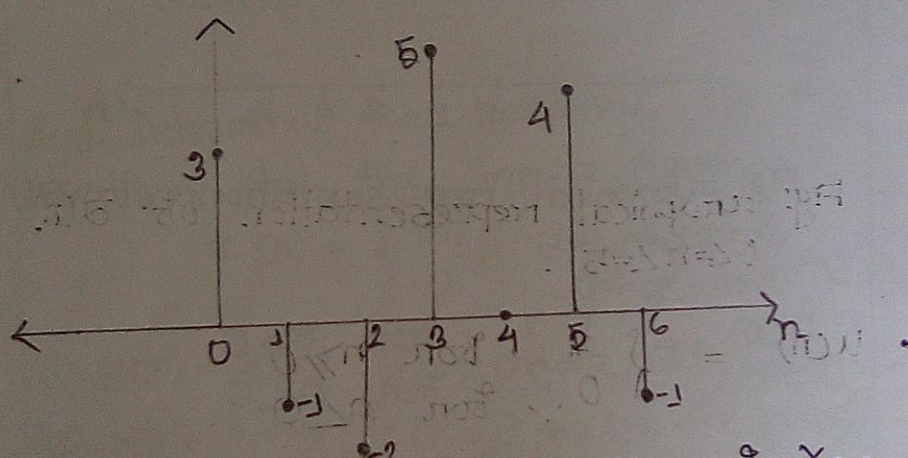

Fig-2: Graphical representation of x_2.

From Fig-1 and Fig-2 we can clearly say that the x_1 and x_2 signals are different.

3.(c) Sketch $\delta(n)$, $u(n)$ and $u_r(n)$ for $0<=n<$
where $\delta(n)$, $u(n)$ and $u_r(n)$ denote their usual meaning in signal processing. [1.5]

- $$\delta(n) = \begin{cases} 1 & \text{for } n=0 \\ 0, & \text{for } n\neq 0 \end{cases}$$

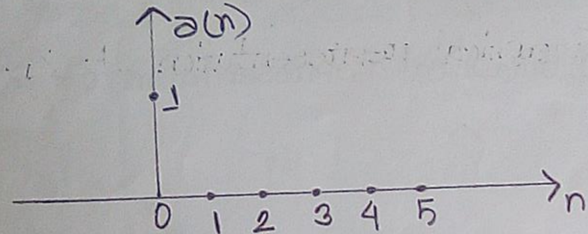

Fig: Graphical representation of $\delta(n)$ for $0<=n<=5$.

- $$u(n) = \begin{cases} 1, & \text{for } n\geq 0 \\ 0, & \text{for } n<0 \end{cases}$$

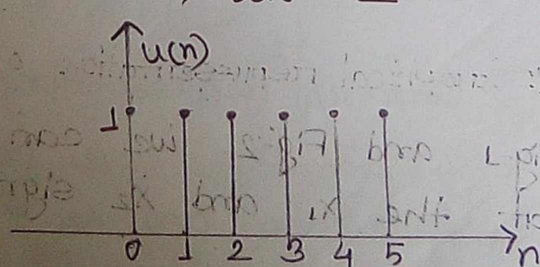

Fig: Graphical representation of $u(n)$ for $0<=n<=5$.

- $u_r(n) = \begin{cases} n, & \text{for } n \geq 0 \\ 0, & \text{for } n < 0 \end{cases}$

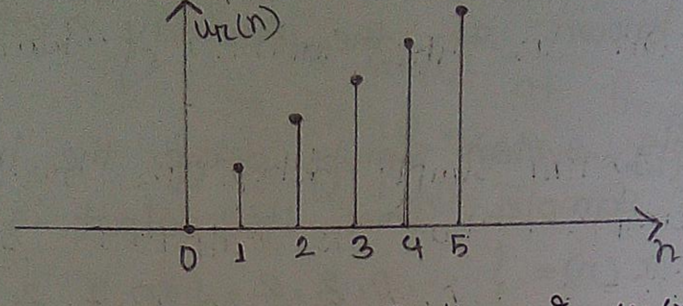

Fig: Graphical representation of $u_r(n)$ for $0 <= n <= 5$.

3.d) Determine the response of the followin
systems to the input signal

$$x(n) = \begin{cases} |n|, & -3 \leq n \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

(i) $y(n) = x(n)$ (ii) $y(n) = x(n-1)$ (iii) $y(n) = x(n$
(iv) $y(n) = 1/3\,[x(n+1) + x(n) + x(n-1)]$ [4]

– First, we determine explicitly the sample values of the input signal

$$x(n) = \{\ldots, 0, 3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3, 0, \ldots\}$$

Next we determine the output of each system using its input-output relationship.

(i) In this case the output is exactly the same as the input signal. Such a system is known as the identity system.

(ii) This system simply delays the input by one sample. Thus its output is given by

$$x(n) = \{\ldots, 0, 3, 2, \underset{\uparrow}{1}, 0, 1, 2, 3, 0, \ldots\}$$

(iii) In this case the system "advances" the input one sample into the future. For example, the values of the output at time $n=0$ is $y(0) = x(1)$. The response of this system to the giver input is

$$x(n) = \{\ldots, 0, 3, 2, 1, 0, \underset{\uparrow}{1}, 2, 3, 0, \ldots$$

(iv) The output of this system at any time is the mean value of the present, the immediate past and the immediate future samples.

For example, the output at time $n = 0$ is

$$y(0) = \frac{1}{3}\,[x(-1) + x(0) + x(1)]$$
$$= \frac{1}{3}\,[1 + 0 + 1]$$
$$= \frac{2}{3}$$

Repeating this computation for every value n, we obtain the output signal

$$y(n) = \{\ldots, 0, 1, 5/3, 2, 1, \underset{\uparrow}{2/3}, 1, 2, 5/3$$

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1.(a) Give a brief introduction about the different parts of a digital signal processing. [2.75]

- Block diagram of a digital signal processing system:

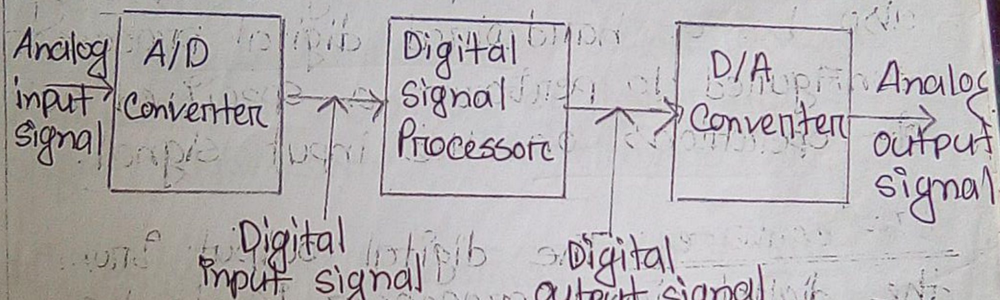


Fig: Digital signal processing system.

A/D Converter: To perform the processing digitally, there is a need for an interface between the analog signal and the digital signal processor. This interface is called an analog-to-digital (A/D) converter. The output of the A/D converter is a digital signal that is appropriate as an input of the digital processor.

Digital Signal Processor: The digital signal processor may be a large programmable digital computer or a small microprocess programmed to perform the desired operations on the input signal. It may also be a hardwired digital processor configured to perform a specified set of operations on the input signal.

D/A Converter: The digital output from the digital signal processor is to be given to the user in analog form, suc as in speech communications, we mu provide another interface from the digital domain to the analog domain such an interface is called a digit to analog (D/A) converter.

1.c) Consider the following input-output equatio[n] of signal :

(i) $y(n) = x(n) + \frac{1}{x(n-1)}$ (ii) $y(n) = nx(n)$

Determine whether the system is linear or non-linear. [3]

(i) Law of Additivity :

$x_1(n) \rightarrow \text{sys} \rightarrow y_1(n) = x_1(n) + \frac{1}{x_1(n-1)}$

$x_2(n) \rightarrow \text{sys} \rightarrow y_2(n) = x_2(n) + \frac{1}{x_2(n-1)}$

$y_1(n) + y_2(n) = x_1(n) + x_2(n) + \frac{1}{x_1(n-1) + x_2(n-1)}$

$x_1(n) + x_2(n) \rightarrow \text{sys.} \rightarrow y'(n) = x_1(n) + \frac{1}{x_1(n-1)} + \frac{1}{x_2(n-1)}$ --- (ii)

Eqn (i) and eqn (ii) are not equal and the superposition principle is not satisfied.

So, the system is non-linear.

(ii) Law of Additivity:

$x_1(n) \rightarrow sys. \rightarrow y_1(n) = n.x_1(n)$

$x_2(n) \rightarrow sys. \rightarrow y_2(n) = n.x_2(n)$

$y_1(n) + y_2(n) = n[x_1(n) + x_2(n)]$ —— (i)

$x_1(n) + x_2(n) \rightarrow sys. \rightarrow y'(n) = n[x_1(n) + x_2(n)]$ —— (ii)

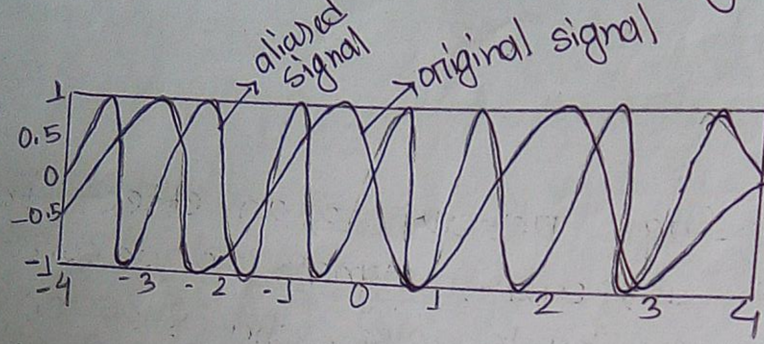
Eqn (i) and eqn (ii) are equal and the superposition principle is satisfied. Hence the system is linear.

2.a) What is ... with an ex... antialiasi...

2.a) What is aliasing effect? Discuss it with an example. Why do you need antialiasing filter in signal processing?

- Aliasing refers to an effect that causes different signals to become indistinguishable (or aliases of one another) when sampled. Aliasing occu when a system is measured at an insufficient sampling rate.

It also refers to the distortion or artifact those results when the sign is reconstructed from samples is differen from the original continuous signal. Aliasing can occur in signals sampled in time, for instance digital audio is referred to as temporal aliasing. Aliasing can also occur in spatially sampled signals for instance digital images. Aliasing in spatially sam signals is called spatial aliasing.



To avoid aliasing we can choose very high sampling frequency. But sampling at very h frequencies introduces numerical errors. Therefore, to avoid aliasing errors caus by the undesired high frequency signals, analog lowpass filter, called anti-aliasin filter is used prior to sampler to filter high frequency components befor the signal is sampled.

Q.b) Define up sampling and down samplin
with example. Why do you need to chan
the sampling rate of a signal? [3]

2.C) Prove that the highest rate of oscillation in a discrete sinusoidal is attained when $\omega = \pi$ or $\omega = -\pi$. [2.75]

- To illustrate this property, let us investigate the characteristics of the sinusoidal signal sequence, $x(n) = \cos\omega_0 n$

When the frequency varies from 0 to π
To simplify the argument, we take values of $\omega_0 = 0, \pi/8, \pi/4, \pi/2, \pi$ corresponding t $f = 0, 1/16, 1/8, 1/4, 1/2$ which result in periodic sequences having periods $N = \infty$, 16, 8, 4, 2. We note that the period of the sinusoid decreases as the frequenc increases. In fact, we can see that the rate of oscillation increases as the frequency increases.

To see what happens for $\pi \le \omega_0 \le 2\pi$, we consider the sinusoids with frequenc $\omega_1 = \omega_0$ and $\omega_2 = 2\pi - \omega_0$. Note that as u

varies from π to 2π, ω_2 varies from π
0. It can be easily seen that

$$x_1(n) = A\cos\omega_1 n = A\cos\omega_0 n$$

$$x_2(n) = A\cos\omega_2 n = A\cos(2\pi - \omega_0)n$$

$$= A\cos(-\omega_0 n) = x_1 n$$

Hence ω_2 is an alias of ω_1. If we had used a sine function instead of cosine function, the result would basically be the same, except for a 180° phase difference between sinusoids $x_1(n)$ and $x_2(n)$. In a case, as we increase the relative frequency ω_0 of a discrete time sin from π to 2π, its rate of oscillation decreases. For $\omega_0 = 2\pi$ the result is a constant signal, as in the case for $\omega_0 = 0$. Obviously, for $\omega_0 = \pi$ (or $f = 1/2$) or $\omega_0 = -\pi$ (or $f = -1/2$) we have the highest rate of oscillation

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3. a) Describe stable and unstable syste[m] with example. [3]

- Stable system: The system is said to be stable only when the output is bounded for bounded input at each or every instant of time.

For example –

$$y(t) = x^2(t)$$

Let the input is u(t) (unit step boun[ded] input) then the output $y(t) = u^2(t) =$
= bounded output

Hence, the system is stable

Unstable system: The system is said t[o] be unstable when the output is unbounded for bounded input at each and every instant of time.

For example –

$$y(t) = \int x(t)\,dt$$

Let the input is u(t) (unit step bound input) then the output $y(t) = \int u(t)dt$ = ramp signal (unbounded because amplitude of ramp is not finite it goes to infinite when $t \to$ infinite)

Hence, the system is unstable.

3.b) Explain convolution sum of signals.

- Convolution is a mathematical operation used to express the relation between input and output of an LTI system. It relates input, output and impulse response of LTI system as

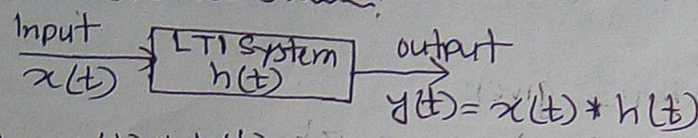
$$y(n) = x(n) * h(n)$$

Where, $y(n)$ = output of LTI
$x(n)$ = input of LTI
$h(n)$ = impulse response of LTI
$*$ = convolution operation

There are two types of convolutions:

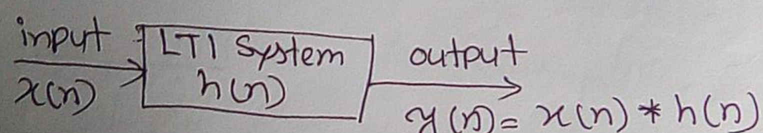
- Continuous convolution.
- Discrete convolution.

- **Continuous Convolution** –



$$y(t) = x(t) * h(t)$$
$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$
(or)
$$= \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$$

- Discrete Convolution –



$$y(n) = x(n) * h(n)$$
$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$
(or)
$$= \sum_{k=-\infty}^{\infty} x(n-k)\, h(k)$$

3.c) Consider the input $x[n] = \{1, 0, 2, 3\}$ of a LTI system and impulse response $h[n] = \{1, 2, 1, 3\}$. Find out the convoluti sum of output $y[n]$ of the LTI syste

— Here given,

$x[n] = \{1, 0, 2, 3\}$

$h[n] = \{1, 2, 1, 3\}$

h(n) \ x(n)	1	0	2	3
1	1	0	2	3
2	2	0	4	6
1	1	0	2	3
3	3	0	6	9

The convolution sum,

$$y(n) = \{1, 2, 3, 10, 8, 9, 9\}$$

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1.a) Same as 2014. 1 (a, b).

1.b) Same as 2014 1 (c).

1.c) Draw the block diagram of the following discrete-time system.

$y[n] = 0.5x[n-3] + 0.25x[n-2] + 0.5x[n-4] + 0.5y$

where $x(n)$ is the input and $y(n)$ is the output of the system. [3.75]

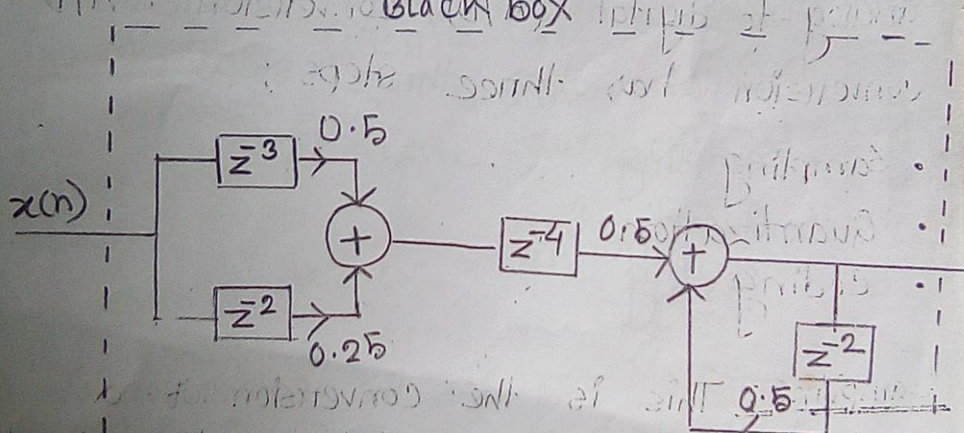

Fig: Block diagram realizations of the system $y(n) = 0.5x[n-3] + 0.25x[n-2] + 0.5x[n-4] + 0.5y[n-2]$.

2· a) Describe analog signal to digital signal (A/D) conversion process in brief with figure. [5]

– To process analog signals by digital means it first necessary to convert them into digital form; that is to convert them to a sequence numbers having finite precision. This process is called analog-to digital (A/D) conversion. A/D conversion has three steps:

- Sampling
- Quantization
- Coding.

Sampling: This is the conversion of a continuous-time signal into a discrete time signal. It is called samples. Thus if $x_a(t)$ is the input to the sampler, the output is $x_a(nT) = x(n)$, where T is the sampling interval.

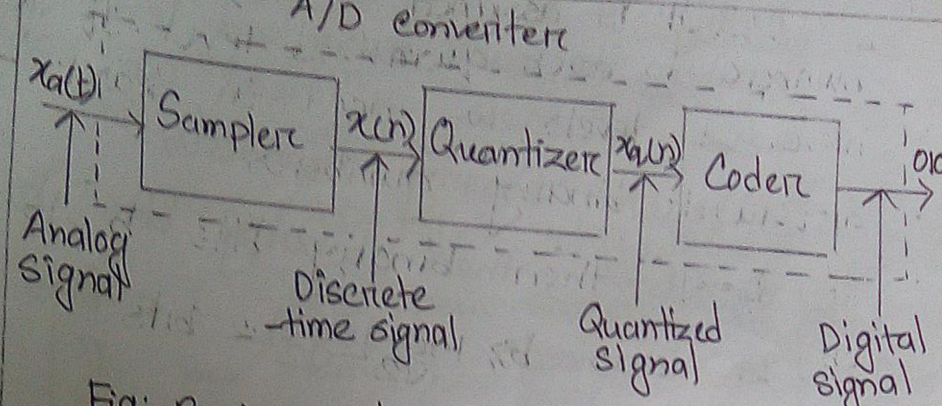

Fig: Basic parts of an analog-to digital (A/D) converter.

<u>Quantization:</u> Quantization is the process of converting a continuous range of values into a finite range of discrete values.

We denote the quantization operation on the samples $x(n)$ as $Q[x(n)]$ and let $x_q(n)$ denote the sequence of quantized samples at the output of the quantizer. Hence

$$x_q(n) = Q[x(n)]$$

Quantization Error, $e_q(n) = x_q(n) - x(n)$

<u>Coding:</u> It is assigned a unique binary number to each quantized level. If we have L levels we need at least L different binary numbers.

If $L=4$, then binary bits b, is

$2^b \geq L$ needs $b \geq \log_2 L = 2$ bits.

2.b) Consider the analog signal $x_a(t) = 10 \sin 400\pi t + 25 \cos 550\pi t - 15 \cos 450\pi t$

Determine the sampling rate to avoid aliasing and maximum magnitude of the signal. [2]

—

2.c) If the
$L = 128$, th
will be re
— Here, b

2.c) If the number of quantization levels $L = 128$, then how many bits per sample will be required? [1.75]

— Here, $b \geq \log_2 L = \log_2 128 = 7$

So 7 'bits per sample will be requi[red] if the number of quantization level $L = 128$.

3.a) Describe cross-correlation function. Write the application of cross-correlati[on] [3.75]

— The cross-correlation between a pair of signals $x(n)$ and $y(n)$ is given by.

$$\gamma_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)\, y(n-l) \quad l = 0, \pm 1, \pm 2, \pm$$

The index l is the shift (lag) paramet[er]. The order of subscripts xy indicate[s] that $x(n)$ is the reference sequenc[e] that remains unshifted in time whereas the sequence $y(n)$ is shifted l units in time with respect to $x(n)$.

If we wish to fix $y(n)$ and to shift $x(n)$, then correlation of two sequences can be written as

$$\gamma_{yx}(l) = \sum_{n=-\infty}^{\infty} y(n)\, x(n-l)$$

$$= \sum_{n=-\infty}^{\infty} y(n+l)\, x(n) \quad \text{--- (ii)}$$

If the time shift $l=0$, then we get.

$$\gamma_{xy}(0) = \gamma_{yx}(0) = \sum_{n=-\infty}^{\infty} x(n)\, y(n) -$$

Comparing eq^n (i) and with eq^n (ii) we get,

$$\gamma_{xy}(l) = \gamma_{yx}(-l)$$

where $\gamma_{yx}(-l)$ is the folded version of $\gamma_{yx}(l)$ about $l=0$

We can rewrite eq^n (i) as,

$$\gamma_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)\, y[-(l-n)]$$

$$= x(l) * y(-l)$$

7

Applications of cross-correlation:

- Measuring fast signal decay
- Laser microscopy
- Measuring pulse broadening and distort
- Fluorescence spectroscopy
- Measuring musical beats (estimating pit
- Scanning pulsar frequencies.
- Analyzing spatial patterns.
- Used in X-ray diffraction data analysis
- Security system design (pattern recognition utilized)
- Water traffic monitoring.

3.b) Determine the cross-correlation sequence $r_{xy}(l)$ of the sequences

$$x(n) = \{\ldots, 0, 0, 2, -1, 3, 4, \underset{\uparrow}{1}, 2, -3, 0, 0, \ldots\}$$

$$y(n) = \{\ldots, 0, 0, 1, -1, 2, -2, \underset{\uparrow}{4}, 1, -2, 1, 0, 0, \ldots\}$$

$$x(l) = \{\ldots, 0, 0, 2, -1, 3, 4, \underset{\uparrow}{1}, 2, -3, 0, 0, \ldots\}$$

$$y(l) = \{\ldots, 0, 0, 1, -1, 2, -2, \underset{\uparrow}{4}, 1, -2, 1, 0, 0, \ldots\}$$

$$y(-l) = \{\ldots, 0, 0, 1, -2, 1, \underset{\uparrow}{4}, -2, 2, -1, 1, 0, 0, \ldots\}$$

$$r_{xy}(l) = x(l) * y(-l)$$

$y(-l)$ \ $x(l)$	2	-1	3	4	1	2	-3
1	2	-1	3	4	1	2	-3
-2	-4	2	-6	-8	-2	-4	6
1	2	-1	3	4	1	2	-3
4	8	-4	12	16	4	8	-12
-2	-4	2	-6	-8	-2	-4	6
2	4	-2	6	8	2	4	-6
-1	-2	1	-3	-4	-1	-2	3
1	2	-1	3	4	1	2	-3

$\therefore r_{xy}(l) = \{2, -5, 7, 5, -12, 20, 0, \underset{\uparrow}{13}, 7, -15, 13, -7, 5, -$

ID-1637820107

2018

1.a) Why signal processing is needed? Draw the block diagram of DSP system and briefly introduce the different componen of the system. [3]

– Signal processing needed: Signals need to be processed so that the information that they contain can be displayed, analyzed, or converted to another type of signal that may be of use. In real-world, analog products detect sign such as sound, light, temperature or pressure and manipulate them. Converte such as an Analog-to-Digital converter then take the real-world signal and turn it into the digital format of 1's and 0's.

[Same as 2016 1(a)]

1.b) Write the mathematical expression of a discrete time sinusoidal signal and draw the signal with $\omega = \pi/6$ and $\phi = \pi/3$. $\boxed{3}$

— A discrete-time sinusoidal signal may be expressed as,

$$x(n) = A \cos(\omega n + \theta), \quad -\infty < n < \infty$$

where n is an integer variable, called th sample number, A is the amplitude of the sinusoid, ω is the frequency in radians per sample and θ is the phase in radians. If instead of ω we use the frequency variable f defined by

$$\omega \equiv 2\pi f$$

$$x(n) = A \cos(2\pi f n + \theta), \quad -\infty < n < \infty$$

3

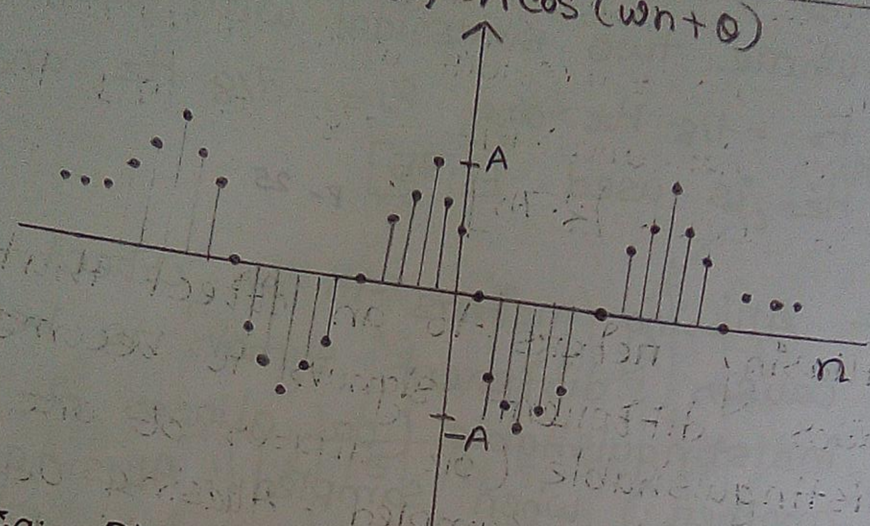

Fig. Discrete-time sinusoidal signal with $\omega =$ and $\theta = \pi/3$)

1.(c) Show that the highest rate of oscillat in discrete-time sinusoidal is attained when $f = 1/2$ or $f = -1/$ [2.5]

— [Same as 2016 - 2(c)]

4

2. a) What is aliasing effect. Show the where two sinusoidal with frequencies $F_0 = 1/8$ Hz and $F_1 = -7/8$ Hz and $F_s =$ Hz is used. $\boxed{2.75}$ P- 25

- Aliasing refers to an effect that causes different signals to become indistinguishable (or aliases of one another) when sampled. Aliasing occur when a system is measured at an insufficient sampling rate.

The effect where two sinusoidal with frequencies $F_0 = 1/8$ Hz and $F_1 = -7/8$ Hz and $F_s = 1$ Hz given below:

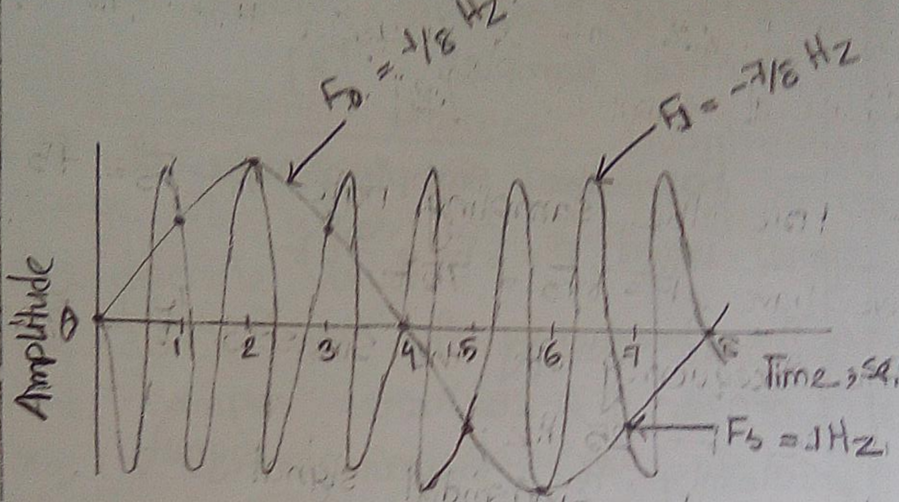

Fig:1 The effect where two sinusoidal with frequencies $F_0 = 1/8$ Hz and $F_1 = -$ and $F_s = 1$ Hz.

Fig-1, shows an example of aliasing, where two sinusoids with frequencies $F_0 = \frac{1}{8}$ Hz and $F_1 = -\frac{7}{8}$ Hz yield identical samples when a sampling rate of $F_s = 1$ Hz is used. When $K = -1$, $F_0 = F_1 + F_s = (-\frac{7}{8} + 1)$ Hz

$$= \frac{1}{8} \text{ Hz.}$$

6

analog

2.b) Consider the signal: $x_a(t) = 3\cos 100\pi t$

Suppose $F_s = 75$ Hz. What is the frequency $0 < F < F_s/2$ of a sinusoid that yields identical samples. [3]

- For the sampling rate of $F_s = 75$ Hz

We have, $F = f F_s = 75 f$

The frequency of the sinusoid $f = 1/3$.

Hence $F = 25$ Hz.

Clearly, the sinusoidal signal

$$y_a(t) = 3\cos 2\pi F t$$
$$= 3\cos 50\pi t$$

Sampled at $F_s = 25$ samples/s yields indentical samples. Hence $F = 50$ Hz is an alias of $F = 25$ for the sampling rate $F_s = 75$ Hz.

6

analog

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2.(c) What is quantization process. Define quantization step size and quantization error with examples. [3]

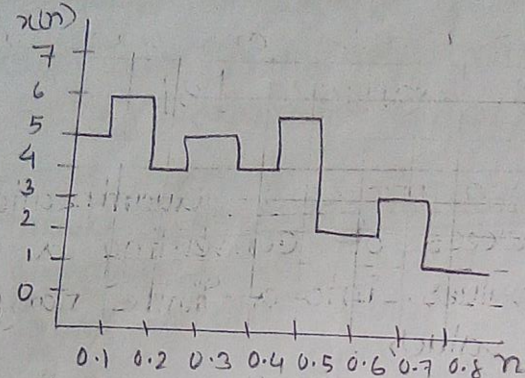
→ <u>Quantization process:</u> Quantization is the process of converting a continuous range of values into a finite range of discrete values.

We denote the quantization operation on the samples $x(n)$ as $Q[x(n)]$ and let $x_q(n)$ denote the sequence of quantized samples at the output of the quantizer. Hence,

$$x_q(n) = Q[x(n)]$$

<u>Quantization step size:</u> The spacing between the two adjacent representation levels is called a quantum or quantization step-size.

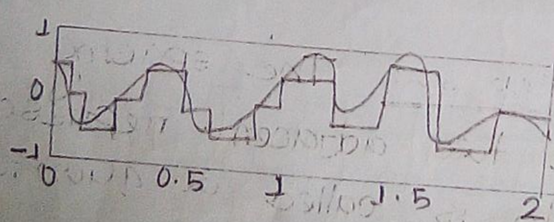
For example -



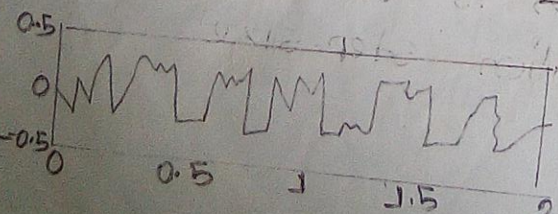
<u>Quantization error</u>: The difference between an input value and its quantized value is called a quantization error.

Quantization error, $e_q(n) = x_q(n) - x(n)$

For example -



Original and quantized signal



Quantization Error

9

3. a) Define LTI system. Discuss the respon of LTI systems to aribitrary inputs. [2-

– LTI system: An LTI system is one which possesses two of the basic properties using linearity and time-invariance.

Linearity: An LTI system obeys superpositio principle which states that the output of the system to a weight sum of inputs is equal to the corresponding weighted sum of the outputs to each of the individual inputs.

Time-invariance: If the input-output relation of a system does not vary with time, th system is said to be time-invariant system.

[Same as 2016-1(b)]

3.b) Find $y(n)$ if $x(n) = n+2$ for $0 \le n \le 3$ and $h(n) = a^n u(n)$ for all n. - [3]

- We have,

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Given,

$$x(n) = n+2 \text{ for } 0 \le n \le 3$$
$$h(n) = a^n u(n) \text{ for all } n$$

$h(n) = 0$ for $n < 0$ so the system is causal. $x(n)$ is causal finite sequence whose valu is zero for $n > 3$.

Therefore,

$$y(n) = \sum_{k=0}^{3} x(k)\, h(n-k)$$
$$= \sum_{k=0}^{3} (k+2)\, a^{n-k}\, u(n-k)$$
$$= 2a^n u(n) + 3a^{n-1} u(n-1)$$
$$= 4a^{n-2} u(n-2) + 5a^{n-3} u(n-3)$$

3.c) Define causal and non-causal system with examples. Determine if the following system are time-invariant or time-variant $y(n) = x(-n)$. [3]

— Causal system: A system is said to be causal if the output of the system at any time n depends only on present and past inputs, but do not depend on future inputs.

Example: (i) $y(t) = x(t)$
(ii) $y(t) = x(t) + x(t-1)$

Non-causal system: A system is said to be non-causal if the output of the system at any time n depends on past, present, future inputs.

Example: (i) $y(t) = x(t+2)$
(ii) $y(t) = x(t) + x(t-1) + x(t+1)$

Here given,

$y(n) = x(-n)$

Step-1: $y(n) \xrightarrow{n_0} y(n-n_0) = x(-n-n_0)$ —— (I)

Step-2: $x(n) \xrightarrow{n_0} x(n-n_0) \longrightarrow sys \longrightarrow x[-(n-n_0)]$

$= x(-n+n_0)$

The output of eqn (I) and eqn (II) are different so it is time-variant system. Page - 1.57

Thank you