Z-Transform

* What is Z-transform?

→ In mathematics & signal processing, the Z-transform converts a discrete-time signal, which is a sequence of real or complex numbers, into complex freequency domain representation. It can be considered as a discrete time equivalent of the laplace transform.

Note : Z-transform use in Discrete time signal.

Types of Z-transform

* There are two types of Z-transform

① One-sided Z-transform.

② Two sided Z-transform.

One sided Z-transform

The One sided Z-transform of Discrete time signal (DTS) is defined as

$$Z\{x(n)\} = \sum_{n=0}^{\alpha} x(n)\, z^{-n}.$$

Two sided z-transform

The two sided z-transform of discrete time signal (DTS) is defined as

$$z\{x(n)\} = \sum_{n=-\alpha}^{\alpha} x(n) z^{-n}$$

Note : In one & two sided z-transform the z is always defined as a complex variable.

problems of One-sided z-transform.

* Find the z-transform of $x(n) = \{\underset{\uparrow}{1}, 2, 3, 2\}$

soln :

→ we know,

$$z\{x(n)\} = \sum_{n=-\alpha}^{\alpha} x(n) z^{-n}$$

$$= \sum_{0}^{3} x(n) z^{-n}$$

$$= x(0) z^{-0} + x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3}$$

$$= x(0) \cdot 1 + x(1) \frac{1}{z} + x(2) \frac{1}{z^2} + x(3) \frac{1}{z^3}$$

$$= x_0 + \frac{x(1)}{z} + \frac{x(2)}{z^2} + \frac{x(3)}{z^3}$$

$$= 1 + \frac{2}{z} + \frac{3}{z^2} + \frac{2}{z^3}$$

that is the required z transformation of the formula

*please solve ↓

→ Find the z-transform of $x(n) = \{3, 2, -1, -4, \underset{\uparrow}{1}\}$

→ Find the z-transform of $x(n) = \{2, -1, 3, 2, 0, 1\}$

→ Find the z-transform of $x(n) = \{1, \underset{\uparrow}{2}, 3, 4, 5\}$

→ Find the z-transform of $x(n) = \{0, 0, 1, 2, 3, 4, 5, 1\}$

An another problem

* Find the z-transform of $x(n) = u(n)$

Sol^n: we know,

$$Z\{x(n) = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

condition for $u(n)$

$u(n) = 1 \; ; \; n \geq 0$

$\quad\quad 0 \; ; \; n < 0$

$$= \sum_{0}^{\infty} u(n) z^{-n}$$

$$= \sum_{0}^{\infty} z^{-n}$$

$$= \sum_{0}^{\infty} (z^{-1})^{n}$$

$$= \frac{1}{1-z^{-1}}$$

$$= \frac{1}{1-\frac{1}{z}}$$

$$= \frac{z}{z-1}$$

that is the required sol^n of z-transform.

* Find the z-transform of $x(n) = 0.3^n u(n)$

here 0.3^n ; $n \geq 0$

0 ; $n < 0$

Now,

$$x(z) = \sum_{-\alpha}^{\alpha} x(n) z^{-n}$$

$$= \sum_{0}^{\alpha} 0.3^n z^{-n}$$

$$= \sum_{0}^{\alpha} (0.3z^{-1})^n$$

$$= \frac{1}{1-0.3z^{-1}}$$

$$= \frac{1}{1-\frac{0.3}{z}}$$

$$= \frac{z}{z-0.3}$$

that is the required sol^n of z-transform.

* Find the z-transform of $x(n) = 0.8^n u(-n-1)$

here 0.8^n ; $n \le -1$

0 ; $n \ge 0$

Now, $X(z) = Z\{x(n)\} = \sum_{n=-\alpha}^{\alpha} x(n) z^{-n}$

$$= \sum_{n=-\alpha}^{\alpha} 0.8^n z^{-n}$$

$$= \sum_{-\alpha}^{-1} 0.8^n z^{-n}$$

$$= \sum_{1}^{\alpha} 0.8^{-n} z^n$$

$$= \sum_{1}^{\alpha} (0.8^{-1} z)^n$$

$$= \sum_{0}^{\alpha} (0.8^{-1} z)^n - 1$$

$$= \frac{1}{0.8^{-1} z} - 1$$

$$= \frac{1}{1 - \frac{z}{0.8}} - 1$$

$$= \frac{0.8 - 0.8 + z}{0.8 - z}$$

$$= \frac{z}{0.8 - z} = \frac{-z}{z - 0.8}$$ Ans

Freequency Analysis of continuos Time - Signal

* what is continuous time signal ?

→ A signal is said to be continuos if it is defined for on instance of time.

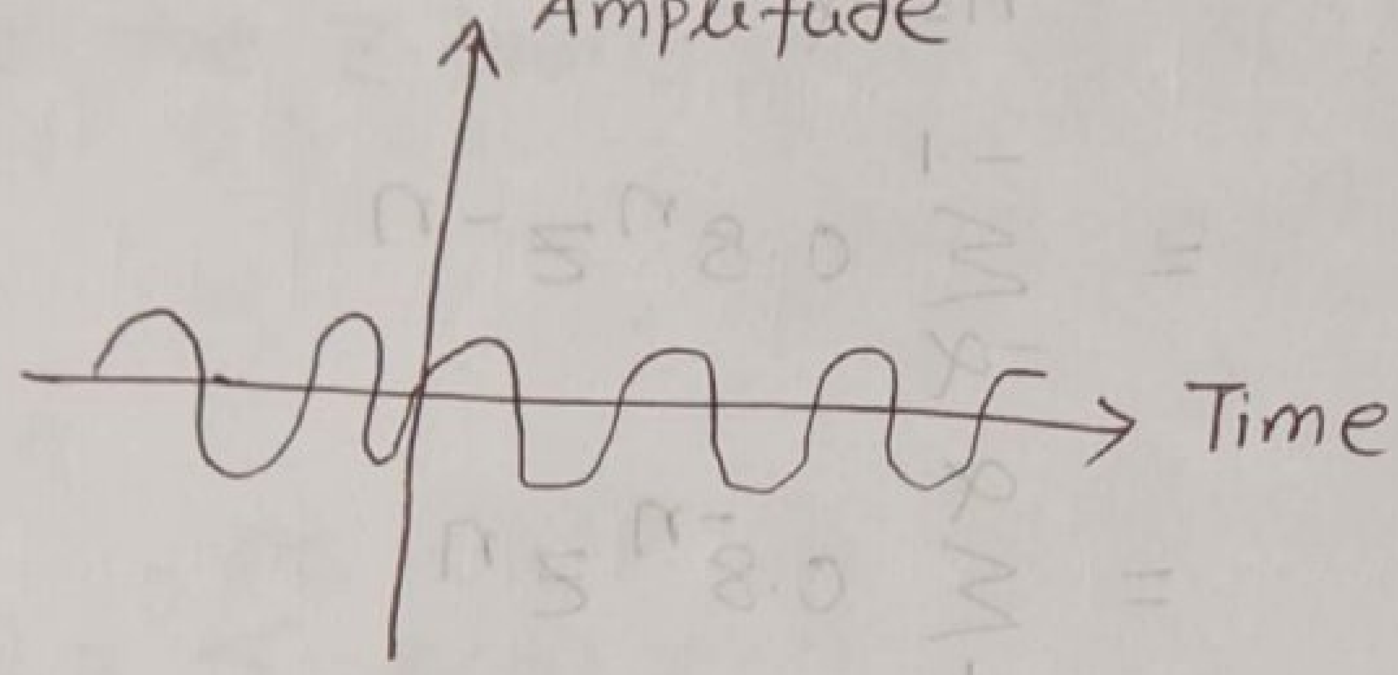

figure : continuous time signal.

* what is Continuous time furier series ?

→ A continuous time signal $x(t)$ with period T can be represented by a continuous furier series. i.e

$$x(t) = \sum_{K=-\alpha}^{\alpha} x(K) \exp\left(j \frac{2\pi}{T} Kt\right)$$

where $x(K)$ is given by $x(K) = \frac{1}{T} \int_T x(t) \exp\left(-j \frac{2\pi}{T} Kt\right) dt$

here $x(K)$ is called the specturm of $x(t)$

Freequency Analysis of Discrete time signal.

* what is Discrete Time signal (DTS)?

→ A signal is said to be discrete when it is defined on discrete instance of time.

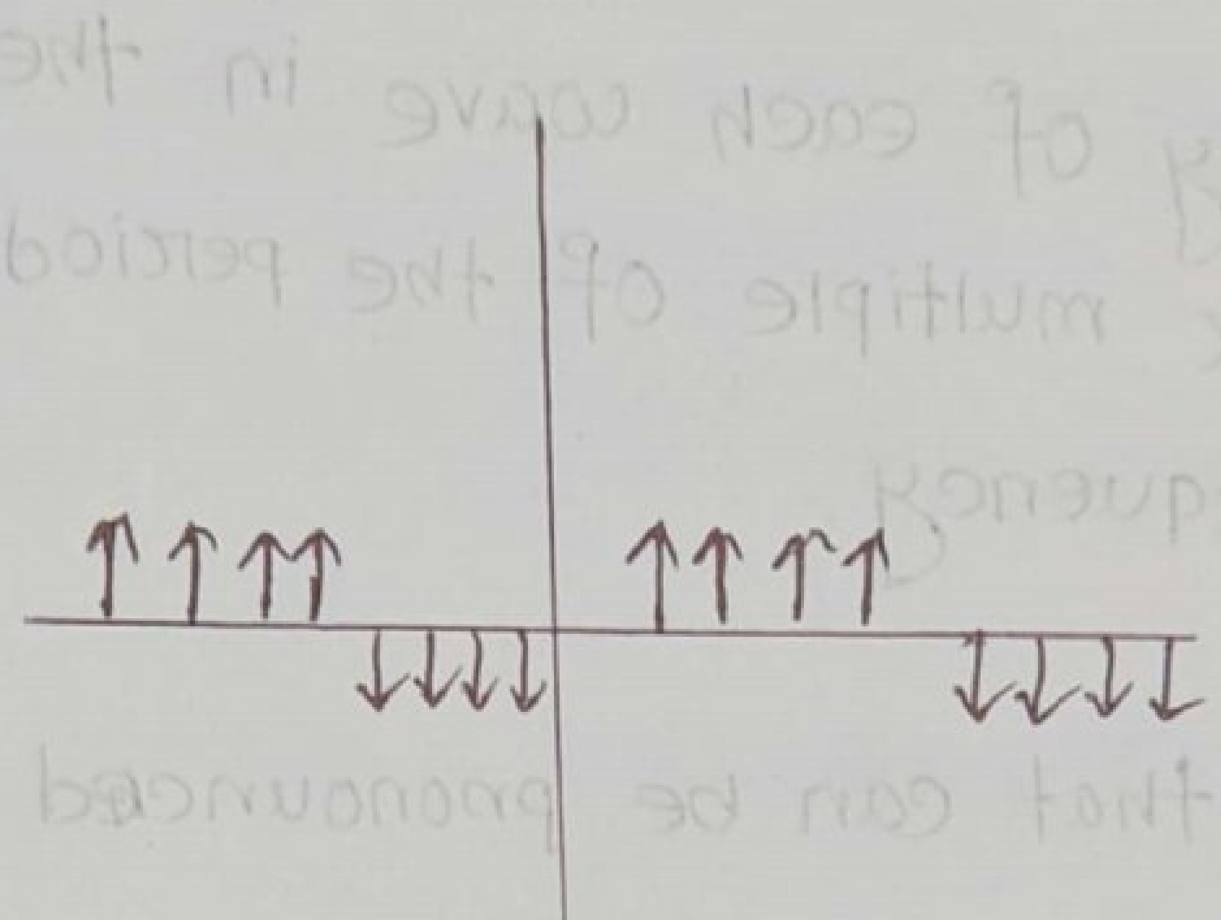

figure : Discrete Time signal.

* what is Discrete time furier series ?

→ A discrete time signal $x(N)$ which period N can be represented by a ~~continuous~~ Discrete furier series . i.e

$$\tilde{x}[N] = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{x}[k] \, e^{j \frac{2\pi}{N} \cdot kn}$$

there are only N different frequencies of period N.

Question solve

2020 - 5(c)

* Define fourier series. Write down some application of fourier transform.

Soln: A Fourier series is a sum that reppents a periodic function as a sum of sin and cosine waves. The freequency of each wave in the sum, or harmonic is an integer multiple of the periodic functions fundamental freequency.

→ A Fourier (that can be pronounced for- YAY) series is a specific type of infinite mathematical series that involves trigonometric functions. Fourier series are the ones that are used in applied mathematics, and especially in the field of physics and electronics, to express periodic functions such as those that comprise communication signal waveforms.

It is analogous to a taylor series, that represents functions as possibility infinite sums of the monomial terms.

07-a

* Define DFT and IDFT equation.

→ Freequency domain representation is not converient representation for a DTS, x(n), hence the fourier transform is sampled to obtain a freequency domain sequence x(K), this is called Discrete Fourier Transform.

$$x(K) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi}{N} \cdot Kn} \; ; \quad 0 \leq K \leq N-1$$

→ The Inverse DFT transforms N discrete freequency samples to the same number of discrete time samples. The IDFT has a form very similar to the DFT.

the formula for IDFT is

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} x\left(\frac{2\pi}{N} \cdot K\right) \cdot e^{\frac{j2\pi}{N} \cdot Kn}$$

07-b

* Define symmetry property of DFT equation.

→ The DFT of a real valued Discrete-time signal has a special symmetry, in which the real part of the transform values are DFT even symmetric and the imaginary part is DFT odd symmetric, as illustrated in the equation.

$$x(e^{j\omega}) = \sum_{n=-\alpha}^{\alpha} x(n)e^{j\omega} = x(e^{j(-\omega)}).$$

07-c

* Define DFT leakage.

→ spectral leakage occurs when a non-integer number of periods of a signal is sent to the DFT (Discrete Fourier Transform). spectral leackage lets a single tone signal be spread among several freequencies after the DFT operation. This makes it hard to find the actual freeequency of the signal.

2019-05 (a)

* Define fourier series. point out some importance of fourier transform in DSP.

→ Importance of Fourier transform

① Fourier transform decomposes a signal into its frequency components.

② Used in telecommunications, data compression, digital signal processing. fast multipilication of polynomials.

05-b

* Explain the DFT leakage problem with example.

→

06-a

* what is window function?

→ A window function is a mathematical function that applies a weighting (often between 0 and 1) to each discrete time series in a finite set.