Topics are :

1. Properties of Fourier transform of discrete time signal
2. Frequency domain
3. Characteristics of linear time invariant system

Fourier transform:

A **Fourier transform** (**FT**) is a mathematical transform that decomposes functions depending on space or time into functions depending on spatial frequency or temporal frequency. That process is also called *analysis*.

Here are the properties of Fourier Transform:

Linearity Property

If $x(t) \xleftrightarrow{F.T} X(\omega)$ If $x(t) \xleftrightarrow{F.T} X(\omega)$
& $y(t) \xleftrightarrow{F.T} Y(\omega)$ & $y(t) \xleftrightarrow{F.T} Y(\omega)$

Then linearity property states that

$ax(t)+by(t) \xleftrightarrow{F.T} aX(\omega)+bY(\omega)$ $ax(t)+by(t) \xleftrightarrow{F.T} aX(\omega)+bY(\omega)$

Time Shifting Property

If $x(t) \xleftrightarrow{F.T} X(\omega)$ If $x(t) \xleftrightarrow{F.T} X(\omega)$

Then Time shifting property states that

$x(t-t_0) \xleftrightarrow{F.T} e^{-j\omega t_0} X(\omega)$ $x(t-t_0) \xleftrightarrow{F.T} e^{-j\omega t_0} X(\omega)$

Frequency Shifting Property

If $x(t) \xleftrightarrow{F.T} X(\omega)$ If $x(t) \xleftrightarrow{F.T} X(\omega)$

Then frequency shifting property states that

$e^{j\omega_0 t}.x(t) \xleftrightarrow{F.T} X(\omega-\omega_0)$ $e^{j\omega_0 t}.x(t) \xleftrightarrow{F.T} X(\omega-\omega_0)$

Time Reversal Property

If $x(t) \overset{F.T}{\longleftrightarrow} X(\omega)$

Then Time reversal property states that

$x(-t) \overset{F.T}{\longleftrightarrow} X(-\omega)$

Time Scaling Property

If $x(t) \overset{F.T}{\longleftrightarrow} X(\omega)$

Then Time scaling property states that

$x(at) \; \frac{1}{|a|} X\frac{\omega}{a}$

Differentiation and Integration Properties

If $x(t) \overset{F.T}{\longleftrightarrow} X(\omega)$

Then Differentiation property states that

$\frac{dx(t)}{dt} \overset{F.T}{\longleftrightarrow} j\omega . X(\omega)$

$\frac{d^n x(t)}{dt^n} \overset{F.T}{\longleftrightarrow} (j\omega)^n . X(\omega)$

and integration property states that

$\int x(t) dt \overset{F.T}{\longleftrightarrow} \frac{1}{j\omega} X(\omega)$

$\iiint ... \int x(t) dt \overset{F.T}{\longleftrightarrow} \frac{1}{(j\omega)^n} X(\omega)$

Multiplication and Convolution Properties

If $x(t) \overset{F.T}{\longleftrightarrow} X(\omega)$

$\& \; y(t) \overset{F.T}{\longleftrightarrow} Y(\omega)$

Then multiplication property states that

$x(t).y(t) \overset{F.T}{\longleftrightarrow} X(\omega) * Y(\omega)$

and convolution property states that

$x(t) * y(t) \overset{F.T}{\longleftrightarrow} \frac{1}{2\pi} X(\omega).Y(\omega)$

2020 - 5(C) :

Fourier series :

A **Fourier series** is a sum that represents a periodic function as a sum of sine and cosine waves.

Some application of Fourier transform :

There are many different applications of the Fourier Analysis in the field of science, and that is one of the main reasons why people need to know a lot more about it.

Apart from physics, this analysis can be used for the-

- Equalization of audio recordings
- Digital Radio Reception without any superheterodyne circuit
- Image Processing for removing periodic or anisotropic artefacts
- Cross-correlation of same types of images
- X-Ray Crystallography
- Passive Sonar which is used for classifying targets as per machinery noise
- Generation of sound spectrograms and so many other areas where people can benefit from this amazing analysis method.

2019 – 5 (a) :

Fourier series :

A **Fourier series** is a sum that represents a periodic function as a sum of sine and cosine waves.

- ## Some importance of FT in DSP:

(1)Fourier transform is a linear operator, if given the appropriate norm, it is still unitary operator.

(2) The inverse transformation of the Fourier transform is easy to find, and the form is very similar to the positive transformation.

(3) The sinusoidal basis function is an Eigen function of the differential operation, so that the solution of the linear differential equation can be transformed into the algebraic equation of the constant coefficient. In a linear time-invariant physical system, the frequency is invariant in nature, so that the response of the system to complex stimuli can be obtained by combining its response to different frequency sinusoidal signals

2018 – 5(a) :

The CT Fourier Series considers periodic signals. A signal $x(t)$ is periodic with period T

in case:

$$\forall t : x(t+T) = x(t)$$

Note that in case x

is periodic with period T it is also periodic with period $2T$ (or nT). This leads us to the definition of the fundamental period T_0 being the smallest value such that $x(t+T_0)=x(t)$. The fundamental frequency then is $f_0=1/T_0$.

5 (b)

A formula for the decomposition of a nonperiodic function into harmonic components whose frequencies range over a continuous set of values.

Some properties of Continuous time Fourier transform:

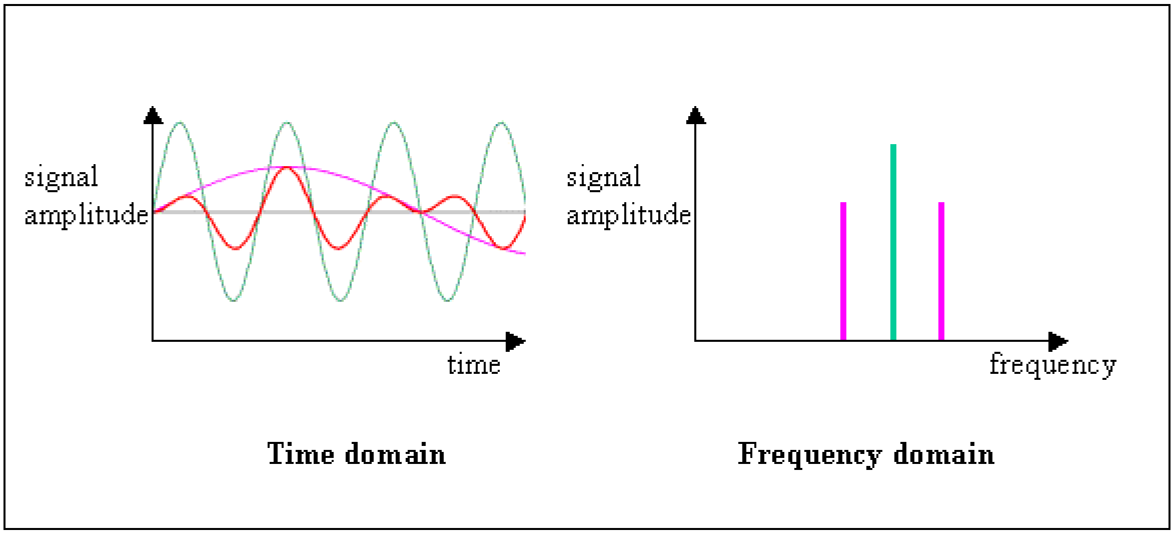
Property of CTFT	Time Domain x(t)	Frequency Domain X(ω)
Linearity Property	$ax_1(t) + bx_2(t)$	$aX_1(\omega) + bX_2(\omega)$
Time Shifting Property	$x(t \pm t_0)$	$e^{\pm j\omega t_0} X(\omega)$
Frequency Shifting Property	$e^{\pm j\omega_0 t} x(t)$	$X(\omega \mp \omega_0)$
Time Reversal Property	x(-t)	$x(-\omega)$
Time Scaling Property	x(at)	$\frac{1}{\lvert a\rvert} X(\frac{\omega}{a})$
Time Differentiation Property	$\frac{d}{dt} x(t)$	$j\omega X(\omega)$
Frequency Derivative Property	$t.\, x(t)$	$j\frac{d}{d\omega} X(\omega)$
Time Integration Property	$\int_{-\infty}^{\infty} x(t) d\tau$	$\frac{X(\omega)}{j\omega}$
Convolution Property	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Multiplication Property	$x_1(t)x_2(t)$	$\frac{1}{2\pi}[X_1(\omega) * X_2(\omega)]$
Duality or Symmetry Property	X(t)	$2\pi x(-\omega)$
Modulation Property	$x(t)\ cos\ \omega_0 t$	$\frac{1}{2}[X(\omega - \omega_0) + X(\omega + \omega_0)]$
	$x(t)\ sin\ \omega_0 t$	$\frac{1}{2j}[X(\omega - \omega_0) - X(\omega + \omega_0)]$
Conjugation Property	x*(t)	$x * (-\omega)$
Autocorrelation Property	R(τ)	$\lvert X(-\omega)\rvert^2$

Frequency domain:

The Frequency Domain refers to the analytic space in which mathematical functions or signals are conveyed in terms of frequency, rather than time. For example, where a time-domain graph may display changes over time, a frequency-domain graph displays how much of the signal is present among each given frequency band. It is possible, however, to convert the information from a time-domain to a frequency-domain. An example of such transformation is a Fourier transform. The Fourier transform converts the time function into a set of sine waves that represent different frequencies. The frequency-domain representation of a signal is known as the "spectrum" of frequency components.

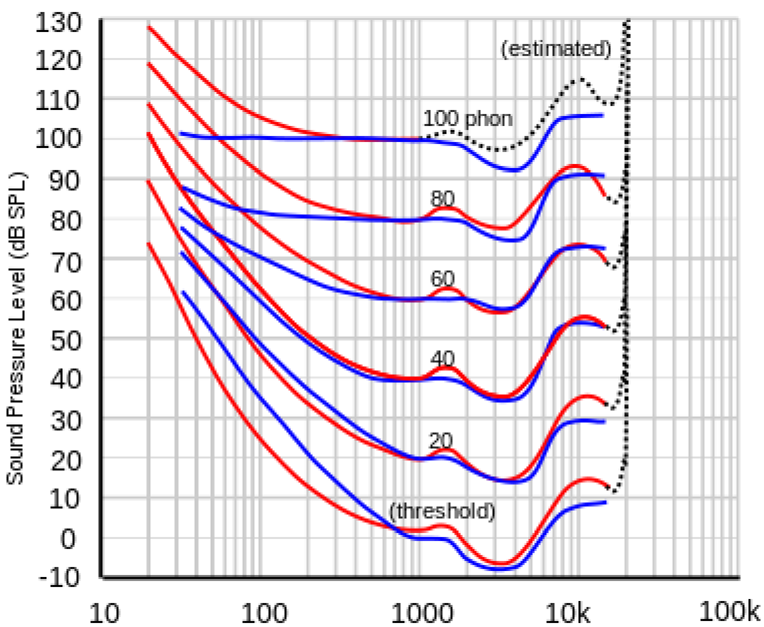
How does the Frequency Domain work?

The Frequency domain works by allowing a representation of the qualitative behavior of a system, as well as characteristics of the way the system response to changes in bandwidth, gain, phase shift, harmonics, etc. A discipline in which the frequency domain is used for graphical representation is in music. Often audio producers and engineers display an audio signal within a frequency domain in order to better understand the shape and character of an audio signal.


signal
amplitude
time
Time domain
signal
amplitude
frequency
Frequency domain

Applications of Frequency Domain

For example, auditory sounds exist between a range of 20-20,000Hz, and some frequencies are harder for the human ear to withstand. The frequency 3,400Hz is a harsh frequency (the sound of babies crying), and the human ear is specifically tuned to respond viscerally to that sound. An audio engineer may reduce the strength of that frequency in the frequency domain using an audio equalizer. By displaying the audio signal in the frequency domain, an engineer can boost and reduce signals to make the sounds more pleasant for the human ear. The Fletcher-Munson curve is a widely used function that lays atop the frequency domain that audio engineers often reference when mixing various frequencies. The function's curve selectively boosts and reduces frequencies to allow the audio engineer to raise the gain of the signal while mitigating the unpleasant sounds



Equal-loudness contours (red) (from ISO 226:2003 revision)
Fletcher–Munson curves shown (blue) for comparison

Characteristics of linear time invariant system:

Linear time-invariant systems (LTI systems) are a class of systems used in signals and systems that are both linear and time-invariant. Linear systems are systems whose outputs for a linear combination of inputs are the same as a linear combination of individual responses to those inputs. Time-invariant systems are systems where the output does not depend on *when* an input was applied. These properties make LTI systems easy to represent and understand graphically.

LTI systems are superior to simple state machines for representation because they have more memory. LTI systems, unlike state machines, have a memory of past states and have the ability to predict the future. LTI systems are used to predict long-term behavior in a system. So, they are often used to model systems like power plants. Another important application of LTI systems is electrical circuits. These circuits, made up of inductors, transistors, and resistors, are the basis upon which modern technology is built.