Imperial College of Engineering

Boikali, Khulna

Affiliated by Rajshahi University
Code : 385

Assignment-1.

Submitted by,
Name: Teresa Jency Bala
Department: Computer Science & Engineering
ID: 1938520113
Part : 3
Semester: Odd

Course Code: CSE 3131
Course Title: Digital Signal Processing.
Topic: Mid term question Solution.

Submitted to,
Shamim
Lecturer, ICE

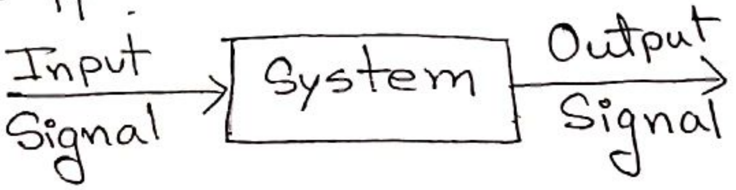
1.

(a) Define system and signal processing. Mention the application of DSP.

Answer:

<u>System:</u> Any physical device which performs certain operation on the input signal and produces an output. Example: Amplifier, filter.

<u>Signal Processing:</u> Signal processing involves a systematic analysis, modification and synthesize signals to extract a meaning out of it.



Some applications of DSP includes:

1. Speech and audio processing. Speech recognition and analysis, noice filtering, echo cancellation.

2. Process image and video, involves - compression, enhancement, reconstruction of videos and images.

3. Military and Telecommunication. Eg:- Sonar, and navigation, radar tracking, modulation and demodulation. etc.

4. Healtcare and Biomedical sector: Analysis of ECG, EEGs, and X-ray signals.

5. Consumer electronics: Smartphone, tv, digital camera etc has dsp embedded in it, to accelerate it's performance.

So, these are some applications of dsp.

(b) Describe various elementary signals. (3)

Answer:

The standard elementary signals are -

1. Exponential signal
   a) Real exponential signal
   b) Complex exponential signal
2. Sinusoidal signal.
3. Step signal
4. Impulse signal
5. Ramp signal.

Real exponential signal: Signal as $x(t) = B e^{\alpha t}$

Where $\alpha > 0$ (positive) & $\alpha < 0$ (negative)

Complex exponential signal: $x(t) = e^{j\omega_0 t}$

where $\omega_0 = \frac{2\pi}{T}$

Sinusoidal Signal: The continuous time version of a sinusoidal signal $x(t)$ in it's general form may be written as $X(t) = A\cos(\omega t + \phi)$
A = Amplitude, ω = frequency (radian/s) &
ϕ = Phase angle (radians).

Step Function: Continuous-time step function is denoted as $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

Impulse Function:- A derivative of the step function $u(t)$ with respect to time denoted as $\delta(t) = 0,\ t \neq 0$ & $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

In discrete-time impulse function denoted as
$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$

Ramp Function: Integral of Step function $u(t)$ is ramp function of unit slope. The continous-time ramp function is $r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$

(c) Differentiate between energy and power signal. Determine whether $x(n)$ is an energy or a power signal $x(n) = 0.25^n u(n)$.

Answer:

Energy Signal	No.	Power Signal
A signal is reffered to as an energy signal if and only if total energy of the signal satisfies condition $0 < E < \infty$	1.	Signal is reffered to as power signal if & only if total power of the signal satisfies the condition. $0 < P < \infty$
For energy signal $P = 0$	2.	For power signal $E = \infty$
For DT S, $E = \sum_{n=-\infty}^{\infty} \lvert x(n) \rvert^2$	3.	For discrete time Signal, $P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \lvert x(n) \rvert^2$
Generally deterministic and aperiodic signals are energy signals.	4	Generally random and periodic signals are power signals.

Given, $x(n) = 0.25^n u(n)$

We know $E = \sum_{n=-\alpha}^{\alpha} |x(n)|^2$

$$= \sum_{n=-\alpha}^{\alpha} \left|(0.25)^n u(n)\right|^2$$

$$= \sum_{n=0}^{\alpha} \left|(0.25)^n\right|^2$$

$$= \sum_{n=0}^{\alpha} \left|(0.25)^2\right|^n$$

$$= \sum_{n=0}^{\alpha} [0.0625]^n \qquad \left[\because \sum_{n=0}^{\alpha} z^n = \frac{1}{1-z}\right]$$

$$= \frac{1}{\cancel{0}1-0.0625}$$

$$= 1.0667 \qquad \therefore 0 < E = 1.0667 < \alpha$$

$$P = \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|(0.25)^n u(n)\right|^2$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{n=0}^{N} \left|(0.25)^n\right|^2$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{n=0}^{N} (0.0625)^n$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \cdot \frac{0.0625^{N+1} - 1}{0.0625 - 1}$$

$$= \frac{1}{2\alpha+1} \cdot \frac{0.0625^{\alpha+1} - 1}{0.0625 - 1}$$

$$= 0$$

$\therefore E = 1.0667$ & $P = 0$ i.e the given signal energy signal.

2 (a) What is Z-transform. Write down the properties of ROC.

Answer: The method to analysis and represent a discrete time signal of LTI (Linear time invariant) in the frequency domain is z-transform. It can be considered as a discrete time equivalent of the Laplace transformation,

Z-transform of $x(n)$ will convert the time domain signal of $x(n)$ to z-domain signal $x(z)$ where the signal becomes the function of a complex variable z.

$x(n)$ = Discrete time signal

$X(z)$ = z-transform of $x(n)$

Properties of ROC :-

1. The ROC of Z-transform is a ring or disc in the z-plane centred at the origin

2. ROC of z-transform cannot contain any poles.

3. ROC of Z-transform of an LTI stable system contains the unit circle.

4. For $x(n) = \delta(n)$ i.e impulse sequence is the only sequence whose ROC of Z-transform is the entires Z-plane

5. If $x(n)$ is a finite duration causal sequence (i.e right sided sequence) then ROC is entire z-plane except at $z=0$.

6. If $x(n)$ is a finite duration anti-causal sequence (i.e left sided) then ROC is entire z-plane except $z = \alpha$.

7. If $x(n)$ is a finite duration Two-sided sequence, then its ROC is entire z-plane except $z=0$ and $z = \alpha$.

(b) Find the Z-transform and ROC of the following finite duration signal. (4)

i) $x(n) = \{1, 2, \underset{\uparrow}{-1}, 2, 3\}$

ii) $x(n) = 0.8^n u(-n-1)$.

i) Given, $x(n) = \{1, 2, \underset{\uparrow}{-1}, 2, 3\}$

We know, $X(z) = \sum_{n=-\alpha}^{\alpha} x(n) z^{-n}$

$$= \sum_{n=-2}^{2} x(n) z^{-n}$$

$$= x(-2)\, z^{-(-2)} + x(-1)\, z^{-(-1)} + x(0)\, z^{-0} + x(1)\, z^{-1}$$

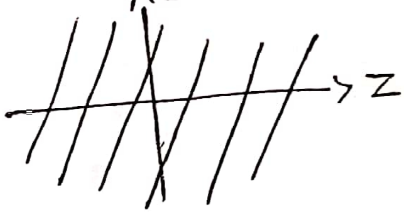
$$x(2)\, z^{-2}$$

$$= 1\, z^{2} + 2z + (-1)\, z^{0} + 2 z^{-1} + 3 z^{-2}$$

$$= z^{2} + 2z - \cancel{+} 1 + \frac{2}{z} + \frac{3}{z^{2}}$$

For $z = 0$, $X(z) = \infty$, $z = \infty$. then $X(z) = \infty$

$\therefore$ ROC for entire z-plane except $z = 0$ & $z = \infty$.



(ii) Given, $x(n) = 0.8^{n} u(-n-1)$

$$X(z) = \sum_{-\infty}^{\infty} x(n)\, z^{-n}$$

$$= \sum_{-\infty}^{\infty} 0.8^{n} u(-n-1)\, z^{-n}$$

$$= \sum_{-\infty}^{-1} 0.8^{n} z^{-n}$$

$$= \sum_{\infty}^{1} 0.8^{-n} z^{n}$$

$$= \sum_{1}^{\infty} (0.8^{-1} z)^{n}$$

$$= \sum_{0}^{\infty} (0.8^{-1} z)^{n} - 1$$

$$= \frac{1}{1 - 0.8^{-1} z} - 1$$

$$= \frac{1}{1 - \frac{z}{0.8}} - 1$$

$$= \frac{0.8}{0.8 - z} - 1$$

$$= \frac{0.8 - 0.8 + z}{0.8 - z}$$

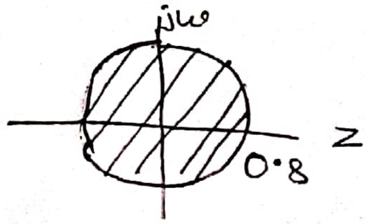
$$= \frac{z}{0.8 - z}$$

$$|0.8^{-1} z| < 1$$

$$\frac{|z|}{0.8} < 1$$

$$|z| < 0.8$$

$\therefore$ ROC entire z-plane for $|z| < 0.8$



(8)

© Check whether the equations are, i. causal or anti-causal, static or dynamic system and ii. is linear or non-linear system.

i. $y(t) = x^2(t) + x(t-6)$ ii. $y(n) = n x(n)$

Answer: Given,

① $y(t) = x^2(t) + x(t-6)$.

$y(0) = x^2(0) + x(0-6) = x^2(0) + x(-6)$

$y(1) = x^2(1) + x(1-6) = x^2(1) + x(-5)$

$y(7) = x^2(7) + x(7-6) = x^2(7) + x(1)$.

∴ The system is causal system. And as depends on both present & past also dynamic system.

$y_1(t) = x_1^2(t) + x_1(t-6)$

$y_2(t) = x_2^2(t) + x_2(t-6)$

$y_3(t) = a y_1(t) + b y_2(t)$
$= a x_1^2(t) + a x_1(t-6) + b x_2^2(t) + b x_2(t-6)$

$$\xrightarrow[+bx_2(t)]{ax_1(t)} \boxed{sys} \longrightarrow y_3'(t)$$

$y_3'(t) =$

$[a x_1(t) + b x_2(t)]^2 + a x_1(t-6) + b x_2(t-6)$

$= a^2 x_1^2(t) + 2ab\, x_1(t) x_2(t) + b^2 x_2^2(t) + a x_1(t-6) + b x_2(t-6)$

∵ $y_3(t) \neq y_3'(t)$

∴ Non-linear.

(ii) Given, $y(n) = n \cancel{\gamma} x(n)$.

$y(0) = 0 . x(0)$
$y(1) = 1 . x(1)$
$y(2) = 2\, x(2)$

At it depends on present input only the system is causal.
Also static system.

For two input sequences $x_1(n)$ and $x_2(n)$, the corresponding output are

$$y_1(n) = n x_1(n)$$
$$y_2(n) = n\, x_2(n)$$

$$y_3(n) = a y_1(n) + b y_2(n) = a n x_1(n) + b n x_2(n)$$
$$= n\left(a x_1(n) + b x_2(n)\right)$$

$$a x_1(n) + b x_2(n) \longrightarrow \boxed{\text{system}} \longrightarrow y_3'(n)$$

$$y_3'(n) = n\left[a x_1(n) + b x_2(n)\right]$$

$\therefore y_3 = y_3'$ $\therefore$ The system is linear