signals: A signal is defined as any physical quantity that varies with time space or any other independent varibles.

signal types

i) continuous - time signal $x(t) = \sin(2\pi f_0 t)$

ii) Discrete - time signal $x(n) = \sin(2\pi f n t_s)$

* If signal takes on all possible values on a finite or an infinite range, then continuous signal.

* If the signals takes on values from a finite set of possible values then discrete signal.

Analog feature :

*) most of the real world signals are CT signal.

*) Often the only way we view this signals as through a transducer, a device that converts a CT signal to an electrical signal

*) common transducer are,

ear, eyes, nose but these are very complicated.

*) simpler transducer (human made)

voltmeters, microphones and pressure sensors

amplitude, frequency, phase

$$x(t) = \cancel{A}\,5\cos(3 \cdot 2\pi \cancel{f} t + 3.14)$$

stationary signal

let $x(t) = a\cos(2\pi f_1 t) + b\cos(3\pi f_2 t) + \cos(4\pi f_3 t)$

if f_1, f_2, f_3 does not change at any time then stationary signal.

otherwise non-stationary signal:

* Almost all signal are non stationary.

Scanned with CamScanner

Speech : is the message content or information conveyed

Noise : it wanted signal that interfere.

Voiced sound : Voiced sounds are those that make our vocal cords vibrate when they are produced.

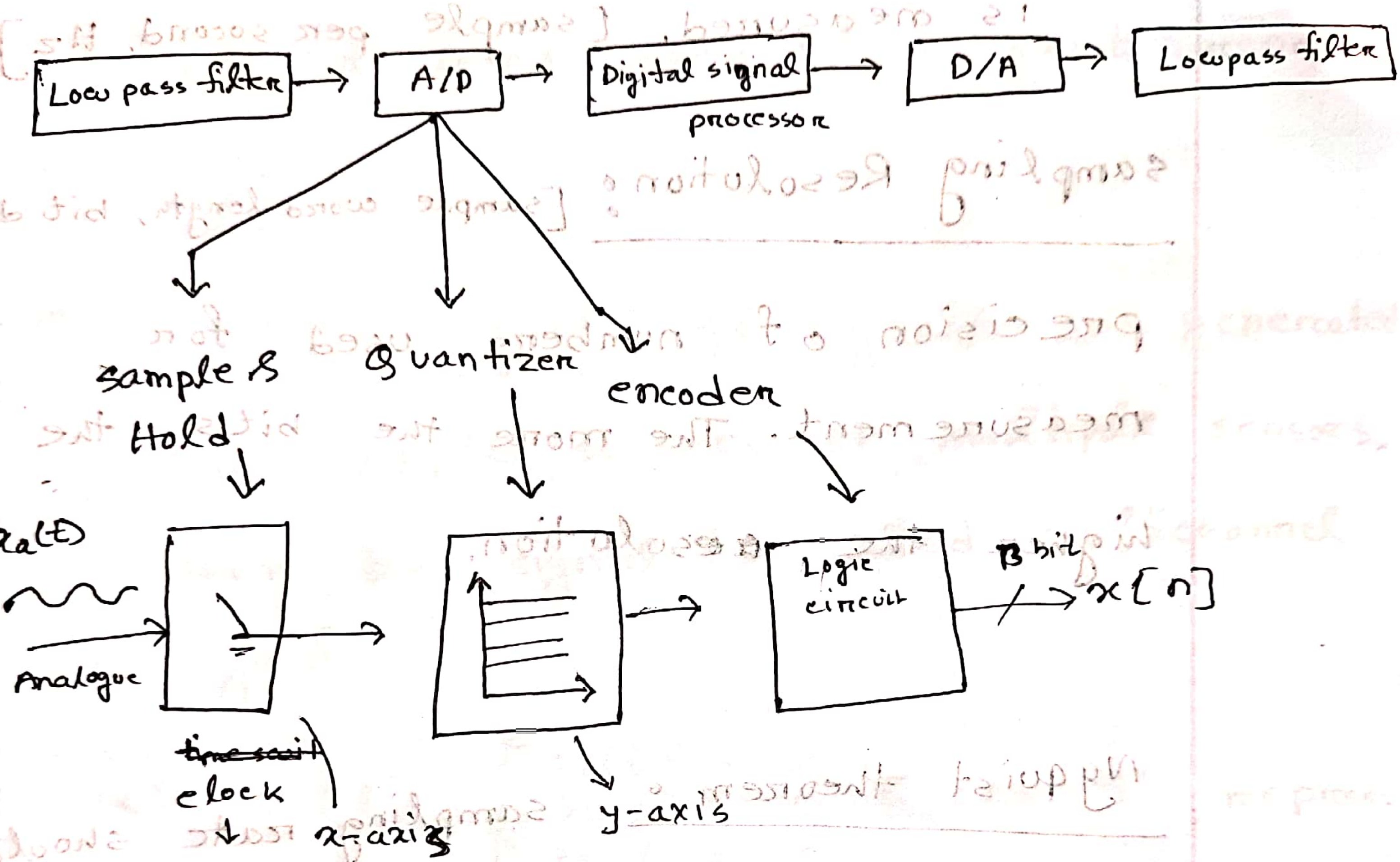
Voiceless/unvoiced : Voiceless sounds are produced from air passing through the mouth at different points.

Low pass filter : Let pass the frequency lower than a limit

High pass filter : Let pass high frequency than a limit

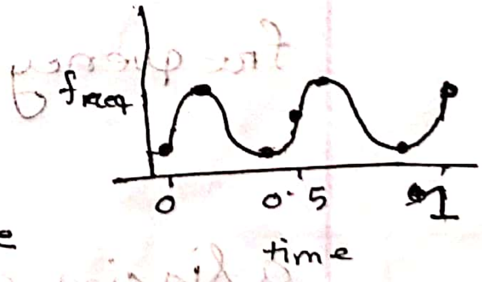
Band pass : Let only a range frequency pass the filter.

DSP system



~~So this etc~~

Let signal frequency 2 Hz.



so we need atleast 4 Hz sample rate

so, we can reconstruct the signal

when we need.

Step 1: sampling

Sampling Rate: How often analog signal is measured. [sample per second, Hz]

sampling Resolution: [sample word length, bit depth] precision of numbers used for measurement. The more the bits, the higher the resolution.

Nyquist theorem: sampling rate should be at least twice the max signal frequency.

aliasing ~~and~~ foldover: original signal cannot be re-construct, because of low sample rate.

Scanned with CamScanner

step 2 – Quantization

→ defining amplitude in y-axis

if 8 level then $\{2^3\}$ = 3 bit represent

→ aware 3

Multichannel signal: If signals are generated by multiple sources or multiple sensors, then the signal is called multichannel signal.

let $S_3(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{bmatrix}$ $S_3(t)$ is represented as vector of 3 signals so $S_3(t)$ is called multichannel signal.

Multi dimentional signal: If a signal is a function of more than one independent variable then it's called multidimentional signal.

continuous - time sinusoidal signal :

A simple harmonic oscillation is mathmetically described by the following continuous - time sinusoidal signal:

$$\underset{\text{analog}}{x_a(t)} = A\cos(\Omega t + \theta)$$

$$-\alpha < t < \alpha$$

$$\Omega = 2\pi F$$

F = continuous cycle per second

f = discrete cycle per sample

Analog sinusoidal signal properties :

1) For every fixed value of frequency F, $x_a(t)$ is periodic.

$$x_a(t + T_p) = x_a(t)$$

here $T_p = 1/F$ is fundamental period of the sinusoidal signal.

2) continuous - time sinusoidal signals with distinct frequencies are themselves distinct.

3) Increasing the frequency F results in an increase rate of oscillation of the signal.

<u>complex exponential signals :</u>

$$x_a(t) = A e^{j(\Omega t + \theta)}$$

which can be simplified by euler's identity, $e^{\pm j\theta} = \cos\theta \pm j \sin\theta$

$$\text{so, } x_a(t) = A e^{j(\Omega t + \theta)}$$

$$= A\{\cos(\Omega t + \theta) + j\sin(\Omega t + \theta)\}$$

P.t.o

some time for mathmetical convenience, we make a positive sinusoid to a complex sinusoid,

let, $\Omega t + \theta = \phi$

$$x_a(t) = A\cos(\Omega t + \theta)$$

$$= \cancel{\frac{A}{2}\{\cos(\Omega t + \theta)}$$

$$= \frac{A}{2}(\cos\phi + \sin\phi) + \frac{A}{2}(\cos\phi - \sin\phi)$$

$$= \frac{A}{2}e^{J\phi} + \frac{A}{2}e^{-J\phi}$$

$$= \frac{A}{2}(e^{J\phi} + e^{-J\phi})$$

(4) Discrete-time sinusoid:

$$x(n) = A\cos(\omega n + \theta) \quad \Big| \; \omega = 2\pi f$$

$$\omega = \frac{2\pi}{\text{sample per cycle}}$$

$$f = \frac{1}{\text{sample per cycle}}$$

$$\omega = 2\pi f$$

discrete time sinusoid propertyies

1. A discrete-time sinusoid is periodic only if its frequency 'f' is a rational number.

Means,

$x(n)$ is periodic w. period N $(N>0)$ if and only if,

$$x(n+N) = x(n)$$

so

$$\cos[2\pi f_0 (n+N) + \theta] = \cos(2\pi f_0 n + \theta)$$

$f_0 N = k$

k is an integer value.

2. A discrete time sinusoid is separated by an integer multiple of 2π are identical.

~~Dest~~

$$\cos[(\omega_0 + 2\pi)n + \theta] = \cos(2\pi n + \omega_0 n + \theta)$$

$$= \cos(\omega_0 n + \theta)$$

so, $x_k(n) = A \cos(\omega_k n + \theta)$

P.t.o

3 The highest rate of oscillation in a discrete-time sinusoid is attained when $\omega = \pi$ or $(-\pi)$ ~~or~~ equivalently

$f = \frac{1}{2}$ or $-\frac{1}{2}$

Example Problem: The implication of frequency relations, described by 2 anlog signals,

$$x_1(t) = \cos 2\pi 10 t$$
$$x_2(t) = \cos 2\pi 50 t$$

If we sample the 40 Hz, ($F_s = 40$) then the discrete signal is,

$$x_1(n) = \cos\left(2\pi \frac{10}{40} n\right) = \cos\left(\frac{\pi n}{2}\right)$$
$$x_2(n) = \cos\left(2\pi \frac{50}{40} n\right) = \cos\left(\frac{5\pi}{2} n\right)$$
$$= \cos\left\{2\pi n + \frac{\pi}{2} n\right\}$$
$$= \cos\left(\frac{\pi}{2} n\right)$$

Scanned with CamScanner

so, we see that after sampling, both discrete turned as one, so, we are sure that one of them will never be able to reconstruct (2nd one)

Folding Frequency: We can use $F_s/2$ or $\omega = \pi$ as the pivotal point and reflect or "fold" the alias frequency to the range $0 \le \omega \le \pi$. Since the point of reflection is $F_s/2$ $(\omega = \pi)$, the frequency $F_s/2$ $(\omega = \pi)$ is called the folding point.

Half of the sampling rate is folding frequency

Example 2

given,

$$x_a(t) = 3\cos(100\pi t)$$

a) minimum sample rate to avoid aliasing is 100.

b) If $F_s = 200$ then the discrete signal is,

$$x(n) = 3\cos\left(\frac{100\pi n}{200}\right)$$

$$= 3\cos\left(\frac{\pi}{2}n\right)$$

c) If $F_s = 75$ the the discrete signal is,

$$x(n) = 3\cos\left(\frac{100\pi n}{75}\right)$$

$$= 3\cos\left(\frac{4\pi}{3}n\right)$$

$$= 3\cos\left\{\left(2\pi - \frac{2\pi}{3}\right)n\right\}$$

$$= 3\cos\left\{2\pi n - \frac{2\pi}{3}n\right\}$$

$$= 3\cos\left(\frac{2\pi}{3}n\right)$$

d) What is the frequency $0 \leq F \leq F_s/2$ of a sinusoid that yields samples identical to those obtaind in part (c)

Answer: $F_s = 75$ Hz, (for c)

if we assume,

$$F = f F_s = 75 \times f$$

From eq-(3)

ωn

$2\pi f n = \frac{2\pi}{3} n$

$f = \frac{1}{3}$

$$F = 75 \times \frac{1}{3}$$

$$= 25$$

then the signal is,

$$y_a(t) = 3 \cos 2\pi F t$$

$$= 3 \cos 50\pi t$$

so, if now we sample $y_a(t)$ at 75 Hz

then $y(n) = 3 \cos\left(\frac{50\pi}{75} n\right)$

$$= 3 \cos\left(\frac{2\pi}{3} n\right)$$

Scanned with CamScanner

Example 3 :

given,

$$x_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12000\pi t$$

a) Nyquist rate of $x_a(t)$,

as $F_{max} = 6000$ Hz

so, $F_N = 2F_{max}$

$= 12000$.

Last sinusoidal Frequency

b) If F_N given $= 5000$,

discrete signal is,

$$x(n) = 3\cos\left(\frac{2\pi}{5}n\right) + 5\sin\left(\frac{6\pi}{5}n\right) + 10\cos\left(\frac{12\pi}{5}n\right)$$

$$= 3\cos\left(\frac{2\pi}{5}n\right) + 5\sin\left(2\pi n - \frac{4\pi}{5}n\right) + 10\cos\left(2\pi n + \frac{2\pi}{5}n\right)$$

$$= 3\cos\left(\frac{2\pi}{5}n\right) + 10\cos\left(\frac{2\pi}{5}n\right) - 5\sin\left(\frac{4\pi}{5}n\right)$$

$$= 13\cos\left(\frac{2\pi}{5}n\right) - 5\sin\left(\frac{4\pi}{5}n\right)$$

So using the ideal interpolation,

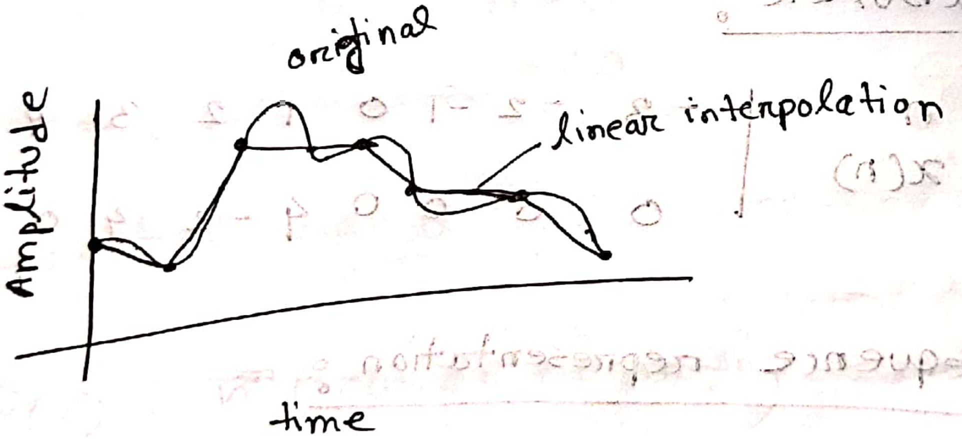
$$x(n) = 13\cos\left(\frac{2\pi}{5}n\right) - 5\sin\left(\frac{4\pi}{5}n\right)$$

$y_a(n)$ is,

$$= 13\cos\left(\frac{2\pi}{5} \times 5000\,t\right) - 5\sin\left(\frac{4\pi}{5} \times 5000\,t\right)$$

$$\Rightarrow 13\cos(2000\pi t) - 5\sin(4000\pi t)$$

Linear interpolation :



Discrete time signals and systems :

we can represent discrete time ~~fis~~ signal besides graphical 

1) Functional

$$x(n) = \begin{cases} 4 & \text{for } n = 1, 3 \\ -2 & \text{for } n = 2 \\ 0 & \text{else} \end{cases}$$

2) Tabular :

n	-3	-2	-1	0	1	2	3	4	5
$x(n)$	0	0	0	0	4	-2	4	0	0

3) sequence representation :

$$X(n) = \{ \underset{\substack{\uparrow \\ N=0}}{0}, 4, -2, 4, 0 \dots \}$$

~~Extras~~

unit sample signal

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

unit ~~spe~~ step signal

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

unit ramp signal :

$$u_r(n) = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

exponential signal

$$x(n) = r^n(\cos \phi n + j \sin \phi n) = r^n e^{j\phi n}$$

or $x(n) = \cancel{A} \; A\alpha^n$

sinusoidal wave

$$x(n) = A \sin(\omega n)$$

Delay & advance

let a sampling sequence is $\delta(n)$

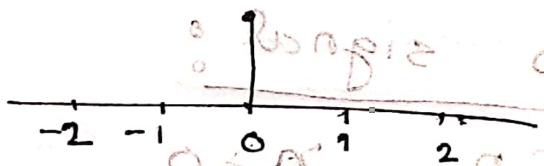
$\delta(n-n_0)$ is the shifted form of the sequence.

if

$n_0 > 0$ ~~delayed~~ delayed shift

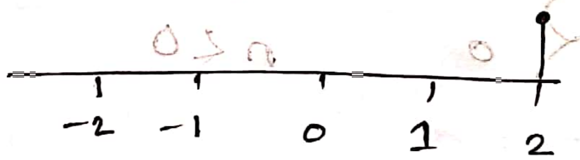
$n_0 < 0$ advanced ~~delayed~~ shift

$\delta(n) =$



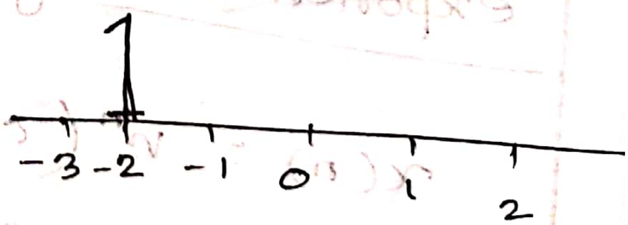
$\delta(n-2) =$, delayed

$n_0 > 0$



$\delta(n+2)$

$= \delta(n-(-2))$

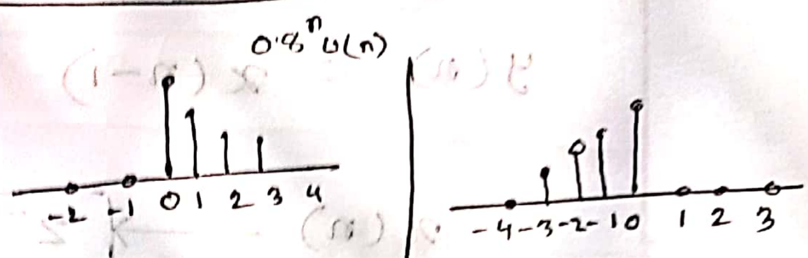


unit delay, $x(n) \xrightarrow{z^{-1}} y(n) = x(n-1)$

unit advance,

$x(n) \xrightarrow{z} y(n) = x(n+1)$

Time-reversal (folding) operation, $0.8^{-n}u(-n)$



folding

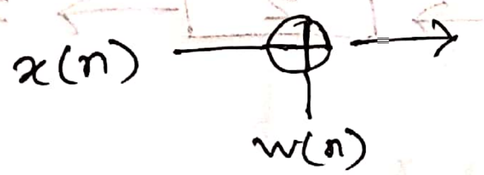
$$y(n) = x(-n)$$

Addition operation:

sample by sample addition

$$y(n) = x(n) + w(n)$$

adder,



$y(n) = x(n) + w(n)$

constant multiplier:

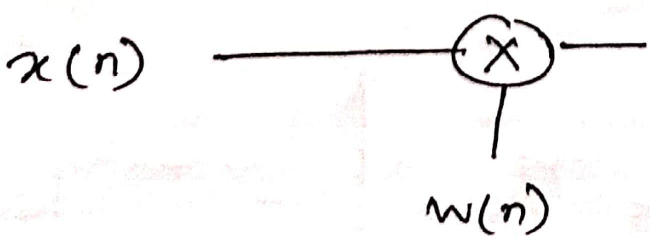
$y(n) = a\,x(n)$,

~~y=Ax~~

$a\,x(n) = y(n)$

signal multiplier/product/modulation:

$$y(n) = x(n) \cdot w(n)$$



$y(n) = x(n) \cdot w(n)$

V **unit delay**

$$y(n) = x(n-1)$$

$$x(n) \longrightarrow \boxed{z^{-1}} \longrightarrow x(n-1) = y(n)$$

VI ~~Ain~~ **unit advance**

$$y(n) = x(n+1)$$

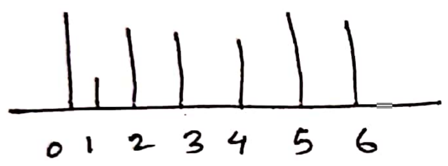
$$x(n) \longrightarrow \boxed{z} \longrightarrow x(n+1) = y(n)$$

wave – 6

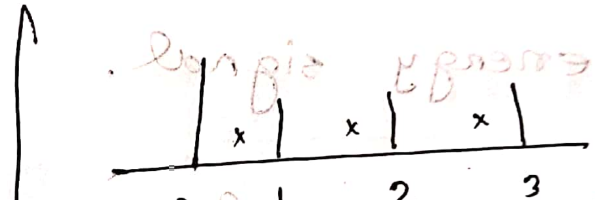
Decimation – down-sampling

$y(m) = x(mN)$

let $N = 2$ then,

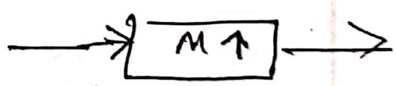


$x(m)$



x places values are discarded

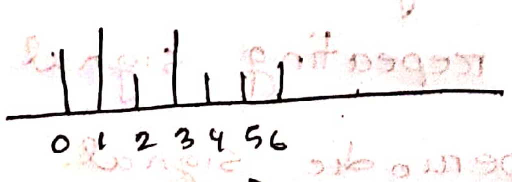
Interpolation – up-sampling



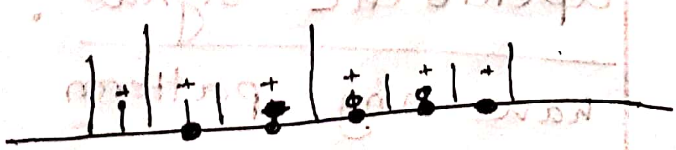
$y(m) = x(m/N)$

so in between 2 sample $x(m)$ we will insert $N-1$ '0' valued sample.

if $N=2$



$x(m)$



$y(m) = x(m/N)$

Classification of Discrete time signal

Energy signal: If a signal, whose energy is finite and power is zero is called energy signal.

power signal: If a signal's power is finite and energy is infinite is called power signal.

periodic signal: If a signal repeats the sequence of value exactly after a fixed length of time is called periodic signal.

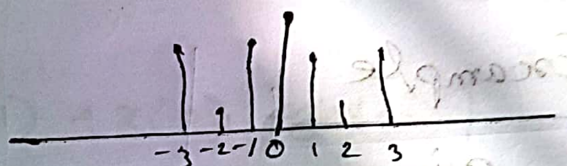
aperiodic signal: If signal does not have any pattern of repeating signal values is called aperiodic signal.

Scanned with CamScanner

Symmetric (even) signal:

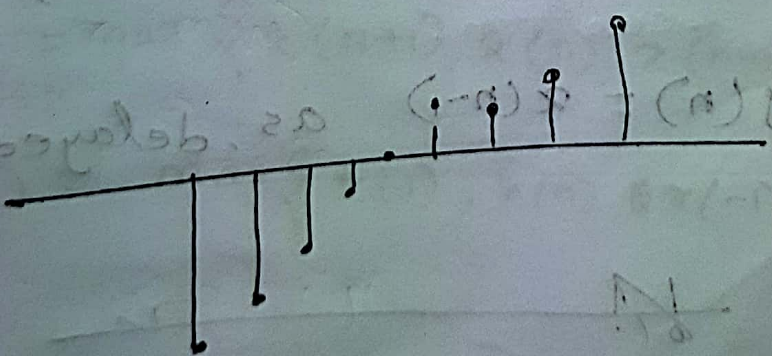
If a signal has same value for ~~even~~ positive and negative input is called symmetric signal.

$$x(n) = x(-n)$$



Antisymmetric (odd) signal:

if $x(-n) = -x(n)$ then odd signal



Discrete - time system : A system which takes discrete time signal as input and after processing outputs a discrete time signal.

Example

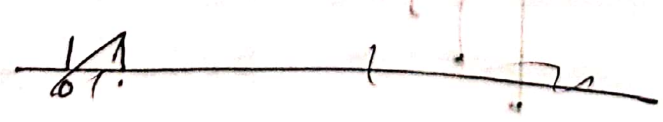
given,

$$x(n) = \begin{cases} |n| & -3 \le n \le 3 \\ 0 & \text{otherwise} \end{cases}$$

a) $y(n) = x(n)$ so, $x(n) = \{0, 3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3, 0\}$

so $y(n)$ is same as $x(n)$

b) $y(n) = x(n-1)$ as, delayed 1, so,



$$\{0, 3, 2, \underset{\uparrow}{1}, 0, 1, 2, 3, 0\}$$

Scanned with CamScanner

c) $y(n) = x(n+1)$ forward a unit,

$y(n) = \{ \ldots 0, 3, 2, 1, 0, 1, 2, 3, 0 \ldots \}$
$\uparrow$

d) $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$

so, $y(0) = \frac{1}{3}[x(1) + x(0) + x(-1)]$

$= \frac{1}{3}(1+1) = \frac{2}{3}$

$y(1) = \frac{1}{3}\{x(2) + x(1) + x(0)\} = \frac{1}{3}(2+1+0) =$

so, $y(n) = \{ \ldots 0, 1, \frac{5}{3}, 2, 1, \frac{2}{3}, 1, \frac{2}{3}, \frac{5}{3}, 1, 0 \ldots$
$\uparrow$

e) $y(n) = \max\{x(n+1), x(n), x(n-1)\}$

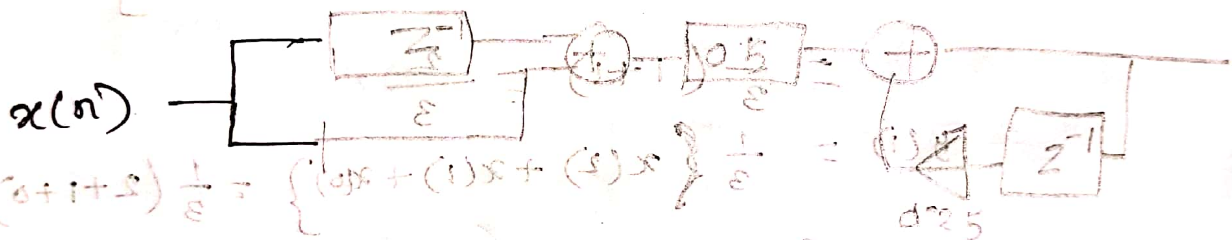
$y(0) = \max\{x(1), x(0), x(-1)\} = 1$

$\{ \ldots 0, 3, 3, 3, 2, 1, 2, 3, 3, 3, 0 \ldots \}$
$\uparrow$

Scanned with CamScanner

Example : using basic building blocks, sketch the block diagram representation of the discrete time system described by the input output relation.

$$y(n) = \frac{1}{4} y(n-1) + \frac{1}{2} x(n) + \frac{1}{2} x(n-1)$$



Block diagram representation of an equation.

classification of

Discrete time system :

1) static v/s dynamic system : memoryless and

output depends on instant input. ~~on~~ not

not past of future input. $y(n) = n x(n) + b x^3(n)$

If system has memory and output

depends on past/& future input is called

dynamic system.

$$y(n) = x(n) + 3x(x-1) + \sum x(n-k)$$

discrete time

Digital system

classification:

time invariant: (shift invariant) if input output characteristics of a system do not change with time is called time invariant system.

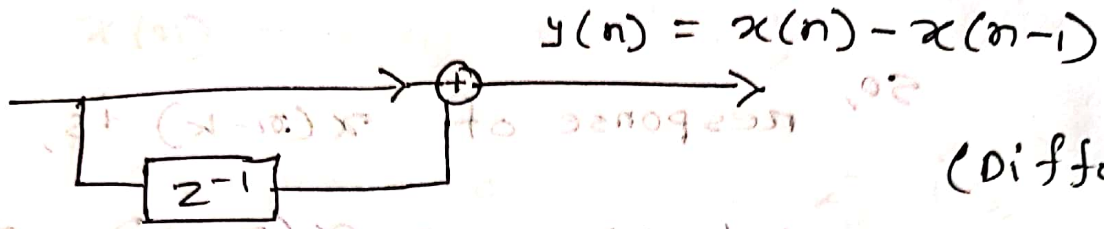
$x(n) \xrightarrow{\tau} y(n)$ is time/shift invariant if and only if,

$$x(n-k) \xrightarrow{\tau} y(n-k)$$

time variant: otherwise time variant.

Examples

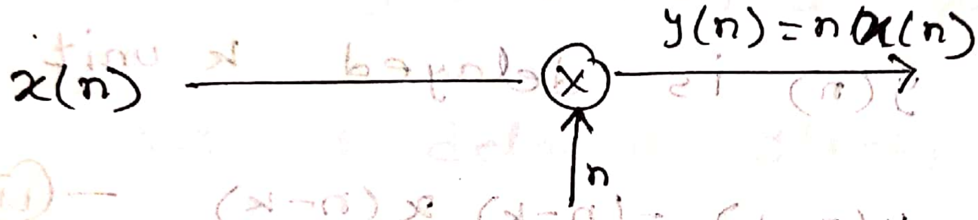
1)



(Differentiator)

a

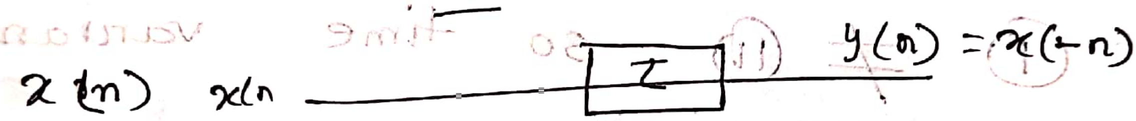
ii)



"time multiplier"

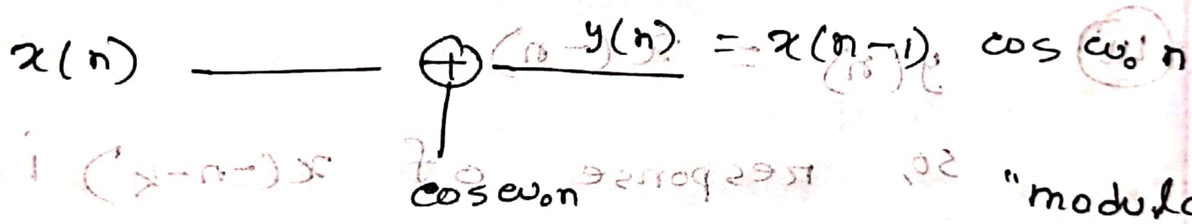
b

iii)



"Folder"

iv)



"modulator"

check time variant or not,

1) $y(n) = x(n) - x(n-1)$,

so, $y(n,k) = x(n-k) - x(n-k-1)$ — (i)

now delay k unit,

$$y(n-k) = x(n-k) - x(n-k-1) \quad \text{— (ii)}$$

(i) and (ii) are equivalent

P.T.O

(ii) $y(n) = n\, x(n)$

so, response of $x(n-k)$ is,

$y(n,k) = n\, x(n-k)$ — (i)

if $y(n)$ is delayed k unit,

$y(n-k) = (n-k)\, x(n-k)$ — (ii)

(i) $\neq$ (ii) so time variant

(iii) $y(n) = x(-n)$

so, response of $x(-n-k)$ is,

$y(n,k) = x(-n-k)$ — (i)

but for $y(n-k)$, it's

$y(n-k) = x\{-(n-k)\}$

$= x(-n+k)$ — (ii)

$1 \neq 11$ so variant,

(iv) given,

$$x(n) = x(n) \cdot \cos(\omega_0 n)$$

so, response of $x(n-k)$ is,

$$y(n,k) = x(n-k) \cdot \cos \omega_0 n \quad \text{--- (i)}$$

but is delayed $y(n-k)$,

$$y(n-k) = x(n-k) \cos(\omega_0 \{n-k\}) \quad \text{--- (ii)}$$

(i) $\neq$ (ii) so, time variant.

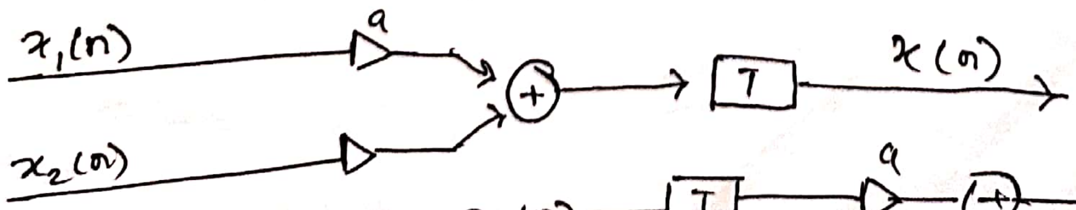
Linear system v/s non-linear system:

A system is linear if it satisfied the superposition theorem means if and only if,

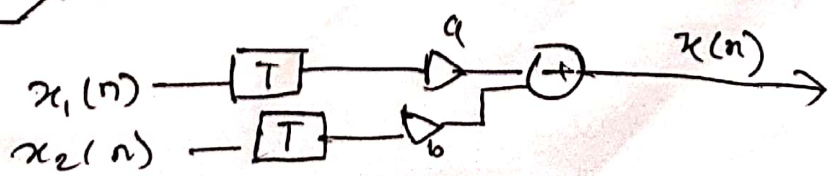
function is not powered or exponential

$$T(a_1 x_1(n) + a_2 x_2(n))$$

$$= a_1 T[x_1(n)] + a_2 T[x_2(n)]$$



it is same as,



So

a) $y(n) = n\,x(n)$ linear

b) $y(n) = x(n^2)$ linear

c) $y(n) = x^2(n)$ non-linear

d) $y(n) = A\,x(n) + B$ linear

e) $y(n) = e^{x(n)}$ non-linear

causal vs noncausal system

If a output of a system at any time depends only on present and past input but not on future input is called then the system is called causal system.

Otherwise non-causal.

stable v/s unstable

An arbitrary ~~relaxed~~ system is said to be bounded input- bounded output (BIBO) stable if and only if every bounded input produces a bounded output.

otherwise unstable.

δ ~~Impulse system~~

PA-0

Impulse response:

response of a system to a unit impulse is called impulse response. ie $x(n) = \delta(n)$ then output $h(n)$ is the impulse response.

$$\delta(n) \longrightarrow \boxed{\text{system}} \longrightarrow y(n) = T[\delta(n)] = h(n)$$

given

$$x(n) = [2, \underset{\uparrow}{4}, 0, 3]$$

resolve the sequence $x(n)$ into a sum of weighted impulse sequence,

sol:

$$x(n) = 2\,\underset{\substack{\uparrow \\ \text{foreward}}}{\delta(n+1)} + 4\,\delta(n) + 3 \times \underset{\substack{\uparrow \\ \text{delay}}}{\delta(n+2)}$$

convolution : is a mathmetical operation which express input, output relation of an LTI system.

*. **convolution:** of two signal gives a 3rd signal.

*. **convolution:** can give zerostate response of the system.

For discrete (continuous) input, output relation we need discrete (continuous) convolution.

continuous, $y(t) = x(t) * h(t)$ — input $x(t)$; convolution; impulse response $h(t)$

$$= \int_{-\alpha}^{\alpha} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\alpha}^{\alpha} h(\tau)\, x(t-\tau)\, d\tau$$

P.t.o

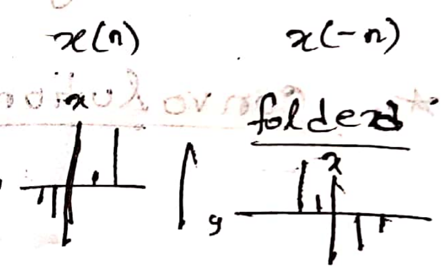
discrete convolution

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$\sum_{k=-\infty}^{\infty} x(k) * h(n-k)$$



This has 4 step

1) Folding : (mirror value on x=0)

fold $h(k)$ ~~at~~ on $k=0$, to get $h(-k)$

2) Shifting : shift $h(-k)$ by n_0

3) multiplication : multiply $x(k)$ by $h(n_0-k)$

4) summation : sum of all the product value.

Example :

let
fold→ $h(n) = \{1, \underset{\uparrow}{5}, 4, 2\}$, $h(-n) = \{2, 4, 5, 1\}$

and $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

Now multiply

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & & \\ & & 2 & 4 & 5 & 1 \\ \hline & & 8 & & & \end{array}$$

,

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & & \\ & 2 & 4 & 5 & 1 & \\ \hline & & 6+16 & & & \end{array}$$

= 22

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 2 & 4 & 5 & 1 \\ \hline \end{array}$$

4+12+20

= 36

do not use it

P.t.o

Given:

$$x(n) = \{ \underset{\uparrow}{1}, 2, 3 \} \rightarrow m_1 = 3$$

$$h(n) = \{ \underset{\uparrow}{1}, 1 \} \rightarrow m_2 = 2$$

find $y(n) = x(n) \overset{\text{conv}}{*} h(n)$

$$y(n) = \sum_{m=-\infty}^{\infty} x(m)\, h(n-m) \rightarrow m_1 + m_2 - 1$$

$n = 4$

now length of $y(n), n = m_1 + m_2 - 1$

here $x(n)$ range $\quad x_L\ 0 - x_h\ 2$

$h(n) \quad h_L\ 0 - h_h\ 1$

$\therefore n_L = x_L + h_L = 0 + 0 = 0$

$n_h = x_h + h_h = 2 + 1 = 3$

$$y(n) = \sum_{m=0}^{2} x(m)\, h(n-m)$$

$0 \leq n \leq 3$

P.T.O

$$y(0) = \sum_{m=0}^{2} x(m)\, h(-m)$$

h(k) does not exist, then h(k) = 0

$$= x(0)\,h(0) + x(1)\,h(-1) + x(2)\,h(-2)$$

$$= 1\times 1 + 2\times 0 + 3\times 0 = 1$$

n = 1,

$$y(1) = \sum_{m=0}^{2} x(m)\, h(1-m)$$

$$= x(0)\,h(1) + x(1)\,h(0) + x(2)\,h(-1)$$

$$= 1\times 1 + 2\times 1 + 0 = 3$$

n = 2,

$$y(2) = \sum_{m=0}^{2} x(m)\, h(2-m)$$

$$= x(0)\,h(2) + x(1)\,h(1) + x(2)\,h(0)$$

$$= 0 + 2\times 1 + 3\times 1 = 5$$

n = 3,

$$y(3) = \sum_{m=0}^{2} x(m)\, h(3-m)$$

$$= x(0)\,h(3) + x(1)\,h(2) + x(2)\,h(1)$$

$$= 1\times 0 + 2\times 0 + 3\times 1 = 3$$

$$y(n) = \{\underset{\uparrow}{1}, 3, 5, 3\}$$

convolution v/s ~~corellacorrelation~~ correlation

correlation : it is the degree of similarity between two different signals.

cross-correlation : Correlating two different sequence is called cross-correlation.

Auto-correlation : correlating between two same sequence.

wave 8

z-transform: This transforms to complex dom to analysis the discrete time signal of LTI system.

Laplace-transform: This transform analysis the continuous time signal of any LTI system.

Direct z-transform

$$X(z) = \sum_{n=-\alpha}^{\alpha} x(n) z^{-n}$$

means

$z[x(n)] = X(z)$

$= \sum_{n=} x(n) z^{-n}$

Inverse z

$x(n) \xleftrightarrow{z} X(z)$

ROC: is the set of all values of z for which X(z) attains a finite value.

PTO

Example:

Given ~~ROC~~ finite duration signal & determine z-transform.

a) $x_1(n) = \{1, 2, 5, 7, 0, 1\}$

$$X_1(z) = 1 + 2z^{-1} + 5z^{-2} + 7z^{-3} + z^{-5}$$

ROC: entire z-plane except $z = 0$

Note: i) If Right sided z-transform, then ROC all but $z \neq 0$

ii) If both sided z-transform, then, ROC all but $z \neq 0$ & $z \neq \infty$

iii) If left sided z-transform, then ROC all but ~~except~~ $z \neq \infty$

b) $x_2(n) = \{1, 2, \underset{\uparrow}{5}, 7, 0, 1\}$

$X(z) = z^2 + 2z + 5 + 7z^{-1} + z^{-3}$

As both sided, ROC, all but $z = 0$ a

c) $x_3(n) = \{0, 0, 1, 2, 5, 7, 0, 1\}$

$X(z) = z^{-2} + 2z^{-3} + 5 \times z^{-4} + 7 \times z^{-5} + z^{-}$

as Right sided, ROC is all but

d) $x_4(n) = \{2, 4, \underset{\uparrow}{5}, 7, 0, 1\}$

$X(z) = 2z^2 + 4z + 5 + 7z^{-1} + z^{-3}$

~~Exam~~

Example

given $x(n) = \left(\frac{1}{2}\right)^n u(n)$, determine z transform.

$$x(n) = \left\{1, \frac{1}{2}, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3, \cdots \left(\frac{1}{2}\right)^n \cdots\right\}$$

so the z transform of $x(n)$ is infinite power series,

$$x(z) = 1 + \frac{1}{2} z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \cdots \left(\frac{1}{2}\right)^n z^{-n}$$

$$= \sum_{n=0}^{\alpha} \left(\frac{1}{2}\right)^n z^{-n} = \sum_{n=0}^{\alpha} \left(\frac{1}{2} \times z^{-1}\right)^n$$

we know that,

$$1 + A + A^2 + \cdots = \frac{1}{1-A} \quad \text{if } |A| < 1$$

so, $x(z) = \dfrac{1}{1 - \left(\frac{1}{2} \times z^{-1}\right)}$

so here,

$$\left|\frac{1}{2} z^{-1}\right| < 1$$

$$\frac{1}{2} < |z|$$

$$\text{ROC} : |z| > \frac{1}{2}$$

P.T.O

Furier trans form : tran forms the time do main signal to a frequency domain signal representation.

French mathmetician found that, any periodic wave form can be expressed as a series harmonically related sinusoids.

$$f(t) = \frac{1}{2}a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \ldots$$
$$b_1 \sin \omega t + b_2 \sin 2\omega t + \ldots$$

so,

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega t) + b_n \sin(n\omega t)\right)$$

here, $n=1$ the fundamental wave
$n=2$ 2nd wave
$n=3$ 3rd wave

Even signal: if $x(t)$ is even then,

$b_k = 0$, then,

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos k\omega_0 t$$

$$\omega_0 = \frac{2\pi}{T_0}$$

Odd signal: If $x(t)$ is odd, then, $a_k = 0$

and it's fourier series contains,

$$x(t) = \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t)$$

$$\omega_0 = \frac{2\pi}{T_0}$$

Even and odd signal came from

Trigonometric Fourier series.

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos k\omega_0 t + b_k \sin k\omega_0 t)$$

$$\Rightarrow a_k = \frac{2}{T_0} \int_{T_0} x(t) \cos k\omega_0 t \, dt$$

$$\Rightarrow b_k = \frac{2}{T_0} \int_{T_0} x(t) \sin k\omega_0 t \, dt$$

$$\omega_0 = \frac{2\pi}{T}$$

Harmonic fourier series

$$x(t) = C_0 \sum_{k=1}^{\alpha} C_k \cos(k\omega_0 t - \theta_k)$$

$$\omega_0 = \frac{2\pi}{T_0}$$

here,

$$C_0 = \frac{a_0}{2}, \quad C_k = \sqrt{a_k^2 + b_k^2}, \quad \theta_k = \tan^{-1}\frac{b_k}{a_k}$$

Continuous fourier transform

continuous fourier transc:

$$X(\omega) = \int_{-\alpha}^{\alpha} x(t)\, e^{-j\omega t}\, dt$$

Inverse, fourier transform,

$$x(t) = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} X(\omega)\, e^{j\omega t}\, d\omega$$

P.t.o

Discrete furier transform

has similarity with continuous signal

$$X(m) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi mn/N}$$

$$= \sum_{n=0}^{N-1} x(n) \left[\cos(2\pi mn/N) - j\sin(2\pi mn/N)\right.$$

here,

$X(m)$ = m'th DFt output component

m = index of dft output

$x(n)$ = sequence of input sample

n = time domain index of input sample

N = number of sample in the input sequence.

Inverse furier transform

$$x(n) = \frac{1}{N} \sum_{m=0}^{N-1} X(m)\, e^{j2\pi nm/N}$$

$$= \frac{1}{n} \sum_{m=0}^{N-1} X(m) \left[\cos(2\pi mn/N) + j\sin(2\pi mn/N)\right]$$

Here, in DFT,

Amplitude of $X(m)$:

$$X(m) = X_{real}(m) + j X_{imag}(m)$$

magnitude of $X(m)$,

$$X_{mag}(m) = |X(m)| = \sqrt{X_{real}(m)^2 + X_{imag}(m)^2}$$

phase of $X(m)$,

$$X_\phi(m) = \tan^{-1}\left(\frac{X_{imag}(m)}{X_{real}(m)}\right)$$

power spectrum of $X(m)$

$$X_{PS}(m) = \left(X_{mag}(m)\right)^2 = X_{real}(m)^2 + X_{imag}(m)^2$$

P.t.o

Features of DFT

i) there is one to one correspondence between $x(n)$ and $X(k)$

ii) There is an extremely fast algorithm called FFT.

iii) The DFT is closely related to discrete Furier series and the Fouries transform

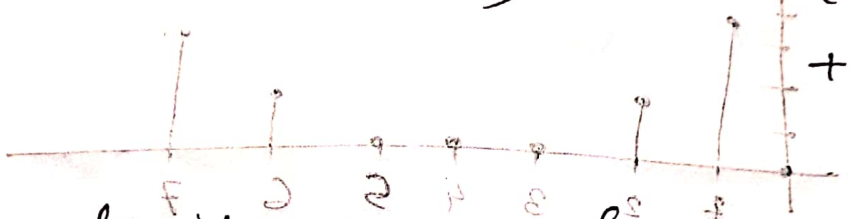
iv) DFT is the appropriate fourier representation for digital computer realization because it is discrete and finite length in both time and frequency domain.

Example: Do 8 point DFT on a the below signal,

$$x(t) = \sin(2\pi \cdot 1000t) + 0.5\sin(2\pi \cdot 2000t + 3\pi/4)$$

$$x(n) = x(nt_s)$$

$$= \sin(2\pi \cdot 1000\, nt_s) + 0.5\sin(2\pi \cdot 2000\, nt_s + 3\pi/4)$$



Let's sample it in 8000 sample/s

and we get (rough of a page)

$$\sum_{n=0}^{7} x(n) = [\,0 - j0,\ 0 - j4,\ 1.414 + j1.414,$$

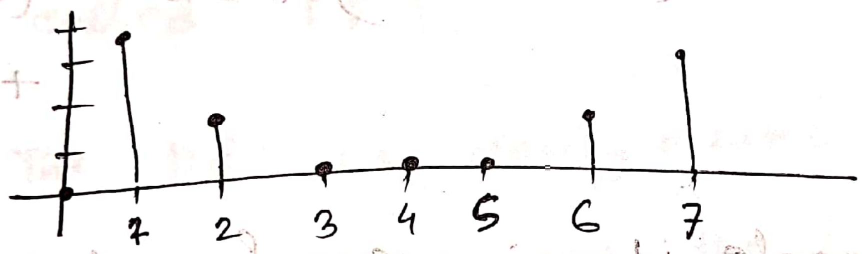
$$0 - j0,\ 0 - j0,\ 0 - j0,\ 1.414 + j1.414,\ 0.0 + j4$$

	mag	phase
$X(0) =$	0	0
$X(1)$	4	$-90°$
$X(2)$	2	$45°$
$X(3)$	0	$0°$
$X(4)$	0	0
$X(5)$	2	$45°$

Scanned with CamScanner

X(6)	2	45°
X(7)	9	90

Let's plot the magnitude,



Plotted X(m) magnitude

Above property is called DFT symmetry.

$$X(m) = X(N-m)$$

magnitude folds, $[0 - \frac{n}{4})$ to $[\frac{n}{2}, n-1)$

and that's why X[0] is always 0 cause $\Sigma x(n) = 0$

$X[m] = X[n-m]$

স্মৃতি কাইয়ুম

DFT magnitude: When a real input signal contains a sine wave component of peak amplitude A_0 with an integral number of cycles over N input samples, the output magnitude of the DFT for that particular sine wave is,

$$M_r = A_0 N/2$$

if DFT input is a complex sinusoid,

$$M_c = A_0 N$$

so as magnitude can be N/2 times of input magnitude, so our register must be able to hold the N/2 times magnitude.

DFT symmetry:

DFT Frequency Axis:

$$f_{analysis}(m) = \frac{m f_s}{N}$$

if
$f_s = 8000$
$N = 8$

$$f_{analysis}(1) = \frac{1 \times 8000}{8} = 1000 \text{ Hz}$$

DFT Linearity: DFT of the sum of two signals is equal to the sum of the transform of each signal.

if, DFT of $x_1(n) = X_1(m)$
& " $x_2(n) = X_2(m)$

then,

$$X_{sum}(m) = X_1(m) + X_2(m),$$

proof,

$$X_{sum}(m) = \sum_{n=0}^{N-1} [x_1(n) + x_2(n)] e^{-j2\pi nm/N}$$

$$= \sum_{n=0}^{N-1} x_1(n) e^{-j2\pi nm/N} + \sum_{n=0}^{N-1} x_2(n) e^{-j2\pi nm/N}$$

$$= X_1(m) + X_2(m)$$

Inverse DFT

we know,

$$x(n) = \frac{1}{N}\sum_{m=0}^{N-1} X(m)\left[\cos(2\pi nm/N) + j\sin(2\pi nm/N)\right]$$

But we know that $x(n)$ does not contain imagnary part, so let's remove that,

$X(m) = a_m + j b_m$ so,

$$x(n) = \frac{1}{N}\sum_{m=0}^{N-1}(a_m + jb_m)\left[\cos(2\pi nm/N) + j\sin(2\pi nm/N)\right]$$

$$= \frac{1}{N}\sum_{m=0}^{N-1}\left\{a_m\cos(2\pi nm/N) - b_m\sin(2\pi nm/N)\right\}$$

$$+ j\,\frac{1}{N}\sum_{m=0}^{N-1}\left\{a_m\sin(2\pi nm/N) + b_m\cos(2\pi nm/N)\right\}$$

as imagnary part will turn to 0,

$$j\,\frac{1}{N}\sum_{m=0}^{N-1}\left\{a_m\sin(2\pi nm/N) + b_m\cos(2\pi nm/N)\right\}$$

$= 0$ so,

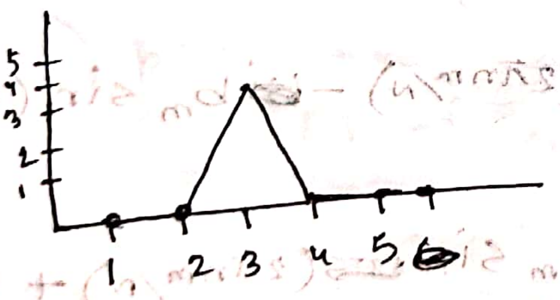
IDFT of, $x(m)$ is,

$$x(n) = \frac{1}{N} \sum_{m=0}^{N-1} \left\{ a_m \cos(2\pi nm/N) - b_m \sin(2\pi nm/N) \right\}$$

DFT Leakage: When the spectral content of your signal does not correspond to an available spectral line.

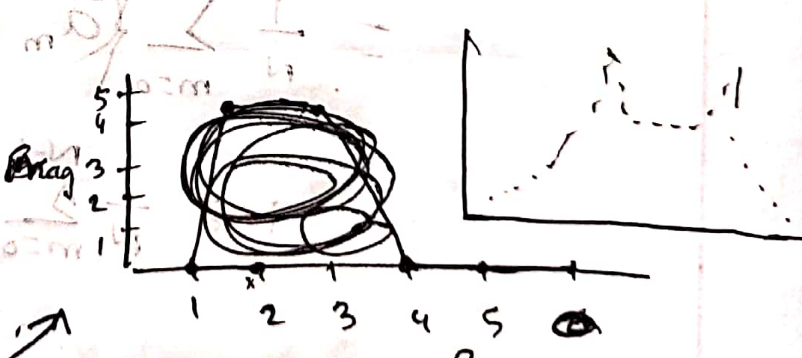
But if

5 V sine wave 3 Hz



5 point DFT

5 V sine wave at 2.5 Hz



as 2.5 Hz is no any spectral line so it's been distributed to 2 & 3 and thats introduces leakage.

DFT sink function

$$X(m) = \frac{N}{2} \cdot \frac{\sin[\pi(k-m)]}{\pi(k-m)}$$

sink function $\frac{\sin(x)}{x}$

Windowing

A machanism to minimize leakage.

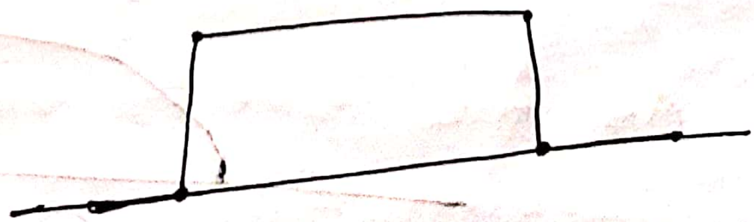
now in DFT

$$X_w(m) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nm/N}$$

we multiply a window function to reduce leakage. for every N-point DFT.

$$X_w(m) = \sum_{n=0}^{N-1} W(n)\, x(n)\, e^{-j2\pi nm/N}$$

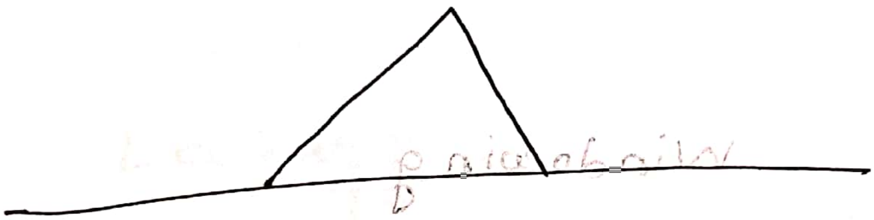
Rectangular window : $w(n) = 1$ w $n \geq 0$



rectangular window

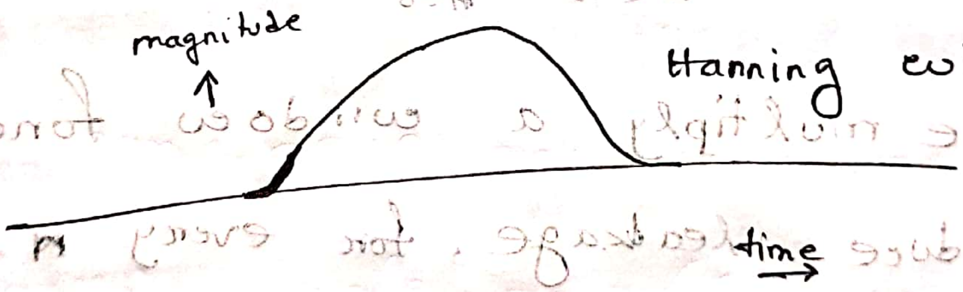
Triangular window:

$$w(n) = \begin{cases} \dfrac{n}{N/2} & ; \text{ if } 0 \le n \le N/2 \\ 2 - \dfrac{n}{N/2} & ; \; N/2 \le n \le N \end{cases}$$



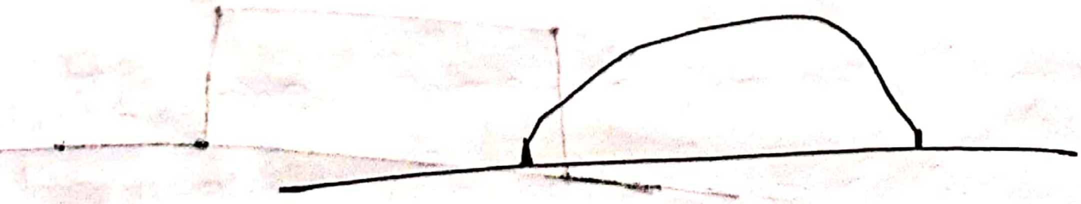
Hanning window:

$$w(n) = 0.5 - 0.5 \cos\left(\frac{2\pi n}{N-1}\right)$$

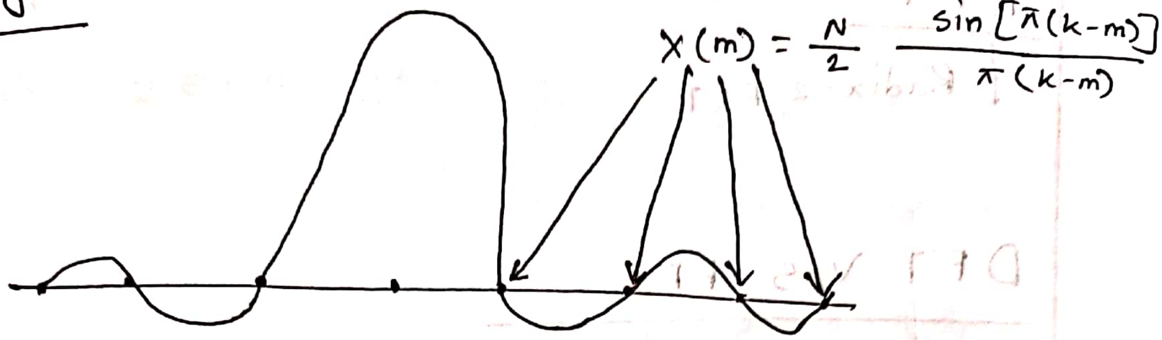


Hamming window:

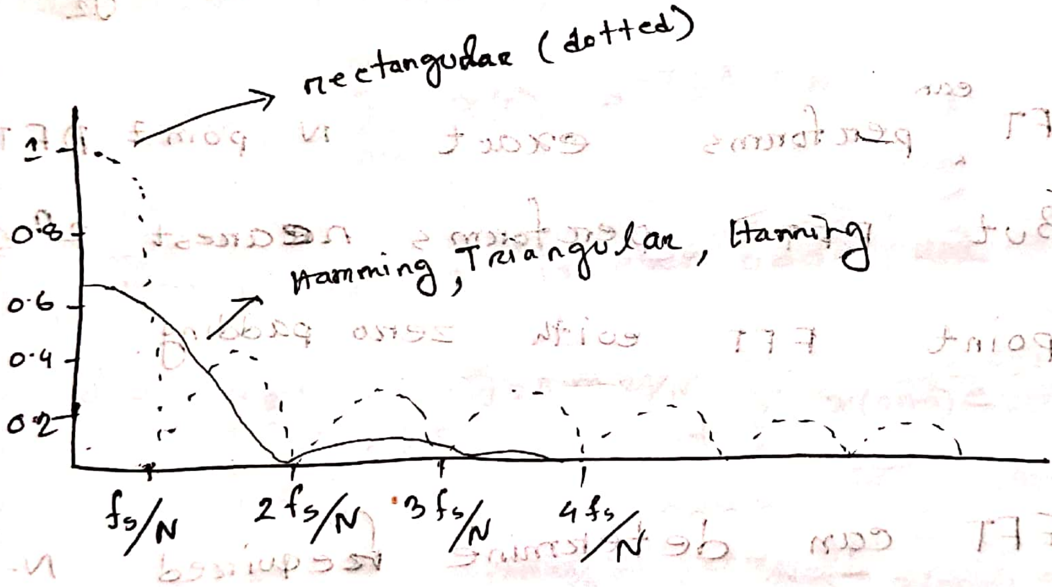
$$w(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right)$$



Leakage



Now using window



FFT

Radix-2 FFT

DFT V/S FFT

i) DFT calculation for N point is N^2

ii) FFT " " N point $\frac{N}{2} \times \log_2 N$

iii) DFT can performs exact N point DFT

- But FFT performs nearest $2^n \cong N$ point FFT with zero padding.

iv) FFT can determine required N-point for a sample rate (F_s) which helps to detect aliasing.

$$N = \frac{f_s}{\text{desired resolution}}$$

v) As we come to know, larger N give better frequency resolution, so FFT manipulates data befor transformation with zero padding.

Deriving radix-2 FFT from DFT

Our DFT equation is,

$$X(m) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi mn/N}$$

Let's divide it in even odd part

$$X(m) = \sum_{n=0}^{N/2-1} x(2n) e^{-j2\pi m 2n/N} + \sum_{n=0}^{N/2-1} x(2n+1) e^{-j2\pi m(2n+1)/N}$$

$$= \sum_{n=0}^{N/2-1} x(2n) e^{-j2\pi m 2n/N} + e^{-j2\pi m/N} \sum_{n=0}^{N/2-1} x(2n+1) e^{-j2\pi m(2n)/N} \quad \text{--- (i)}$$

We'll define,

$$W_N = e^{-j2\pi/N}$$

So $$X(m) = \sum_{n=0}^{N/2-1} x(2n) W_N^{2mn} + W_N^{m} \sum_{n=0}^{N/2-1} x(2n+1) W_N^{2nm} \quad \text{--- (ii)}$$

P.t.o

we clearly see there is 2 part in summation form in the both n part, there is $W_N^{\bullet}$ factor mathmatician calls it twiddle factor.

Now,

$$W_N^2 = e^{-j2\pi 2/N} = e^{-j2\pi/(N/2)} = W_{N/2}$$

so we put this in equation (II)

$$X(m) = \sum_{n=0}^{N/2-1} x(2n)\, W_{N/2}^{nm} + W_N^m \sum_{n=0}^{N/2-1} x(2n+1)\, W_{N/2}^{nm} \quad \text{— (III)}$$

we now can relate $X(m+N/2)$ from $X(m)$

before that we calculate the effect on twiddle factor.

$$W_{N/2}^{n(m+N/2)} = W_{N/2}^{nm} \cdot e^{-j2\pi n \times \frac{2N}{2N}}$$

$$= W_{N/2}^{nm} \cdot 1 \quad \text{— (IV)} \qquad \Big| \; e^{-j2\pi n} = 1$$

$$W_N^{m+N/2} = W_N^{m}\, e^{-j2\pi n \frac{N}{2N}}$$

$$= W_N^m\, e^{-j\pi n} \quad \text{— } \cancel{(V)} \qquad \Big| \; e^{-j\pi n} = -1$$

$$= -W_N^m \quad (12)$$

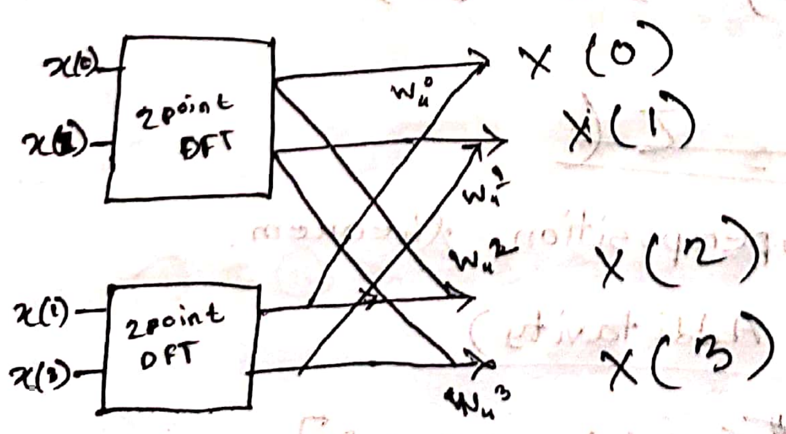
Apply them to equation (11) to find

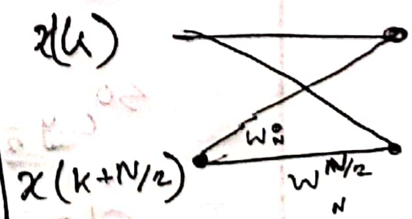
$$X(m+N/2) = \sum_{n=0}^{N/2-1} x(2n) W_{N/2}^{nm} - W_N^m \sum_{n=0}^{N/2-1} x(2n+1) W_{N/2}^{nm}$$

so we need not to perform any sine ; cosine multiplication to get $X(m+N/2)$

we just change the sign of twiddle factor W_N^m and that will do it.

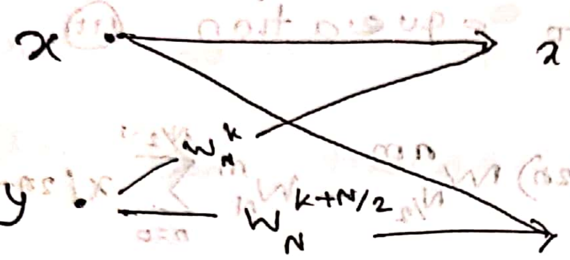
also we will get N-point FFT from 2, N/2-point FFT





Scanned with CamScanner

Radix-2 FFT Butterfly structure



Linear system v/s non-linear system

a) $y(n) = n\,x(n)$

Answer

so for two sequence,

$y_1(n) = n\,x_1(n)$

$y_2(n) = n\,x_2(n)$

so ~~$y_3(n) = T($~~

Check superposition theorem.

LOA (Law of Additavity)

$y_3 = T[a_1 x_1(n) + a_2 x_2(n)]$

$= n\{a_1 x_1(n) + a_2 x_2(n)\} = n a_1 x_1(n) + n a_2 x_2(n)$

again,

$$a_1 y_1(n) + a_2 y_2(n)$$

$$= a_1 n\, x_1(n) + a_2 n\, x_2(n) \quad — (ii)$$

equation (i) & (ii) are identical so linear

b) $y(n) = x(n^2)$

$$y_1(n) = x_1(n^2)$$
$$y_2(n) = x_2(n^2)$$

$$y_3(n) = T[a_1 x_1(n) + a_2 x_2(n)]$$

$$= a_1 x_1(n^2) + a_2 x_2(n^2) \quad — (i)$$

Again,

$$a_1 y_1(n) + a_2 y_2(n)$$

$$= a_1 x_1(n^2) + a_2 x_2(n^2) \quad — (ii)$$

(i) & (ii) are equal so linear

P.t.o

c) $y(n) = x^2(n)$

then,

$$y_1(n) = x_1^2(n)$$
$$y_2(n) = x_2^2(n)$$

$$y_3(n) = T[a_1 x_1(n) + a_2 x_2(n)]$$
$$= [a_1 x_1(n) + a_2 x_2(n)]^2$$
$$= a_1^2 x_1^2(n) + a_2^2 + x_2^2(n) + 2a_1 a_2 x_1(n) x_2(n) \quad \text{--- (I)}$$

Now,

$$a_1 y_1(n) + a_2 y_2(n)$$
$$= a_1 x_1^2(n) + a_2 x_2^2(n) \quad \text{--- (II)}$$

(I) $\neq$ (II) so non-linear

d) $y(n) = A\, x(n) + B$

$$y_1(n) = A\, x_1(n) + B$$
$$y_2(n) = A\, x_2(n) + B$$

then,

$$y_3(n) = T[a_1 x_1(n) + a_2 x_2(n)]$$
$$= A[a_1 x_1(n) + a_2 x_2(n)] + B \quad \text{--- (1)}$$

P.T.O

on the other hand,

$$a_1 y_1(n) + a_2 y_2(n)$$

$$= a_1(A x_1(n) + B) + a_2(A x_2(n) + B)$$

$$= A a_1 x_1(n) + A a_2 x_2(n) + a_1 B + a_2 B \quad \text{— (II)}$$

equation (I) & (II) will be equal if

$B = 0$, linear

$B \neq 0$, non linear

e) $y(n) = e^{x(n)}$

$$y_1(n) = e^{x_1(n)}$$
$$y_2(n) = e^{x_2(n)}$$

then,

$$y_3(n) = T[a_1 x_1(n) + a_2 x_2(n)]$$

$$= e^{a_1 x_1(n) + a_2 x_2(n)} \quad \text{— (I)}$$

But,

$$a_1 y_1(n) + a_2 y_2(n)$$

$$= a_1 e^{x_1(n)} + a_2 e^{x_2(n)} \quad \text{— (II)}$$

(I) $\neq$ (II) so non-linear.

Scanned with CamScanner

FIR V/S IIR

FIR: Finite impulse response

1) Impulse response has finite length
2) All zero filter (only numerator in the transfer function)
3) Inherently stable
4) Implemented on convolution, non-recursive

IIR: Infinite impulse response

i) Impulse response has infinite length
ii) All-pole or pole-zero mix filter
iii) Can be stable or unstable depending on pole location.
v) Implemented using differene equation
vi) recursive.