LINEAR TIME INVARIANT SYSTEM AS FREQUENCY SELECTIVE FILTERS, INVERSE SYSTEM AND DE CONVOLUTION, ADAPTIVE FILTER

Fahim Ahammed Firoz
ID: 1938520126
Odd semester, Part-3
Dept. of CSE
Imperial College of Engineering

TABLE OF CONTENTS

Points for discussion

WHAT IS LTI SYSTEM? [2020-4(a), 2018-3(a)]

A system that possesses two basic properties namely linearity and time invariant is know as Linear Time-Invariant system or LTI System.

Linear scaling: $x \longrightarrow \boxed{\text{L}} \longrightarrow y$ $\qquad \alpha x \longrightarrow \boxed{\text{L}} \longrightarrow \alpha y$

Additively : $x_1 \longrightarrow \boxed{\text{L}} \longrightarrow y_1$ $\qquad x_2 \longrightarrow \boxed{\text{L}} \longrightarrow y_2$ $\qquad x_1 + x_2 \longrightarrow \boxed{\text{L}} \longrightarrow y_1 + y_2$

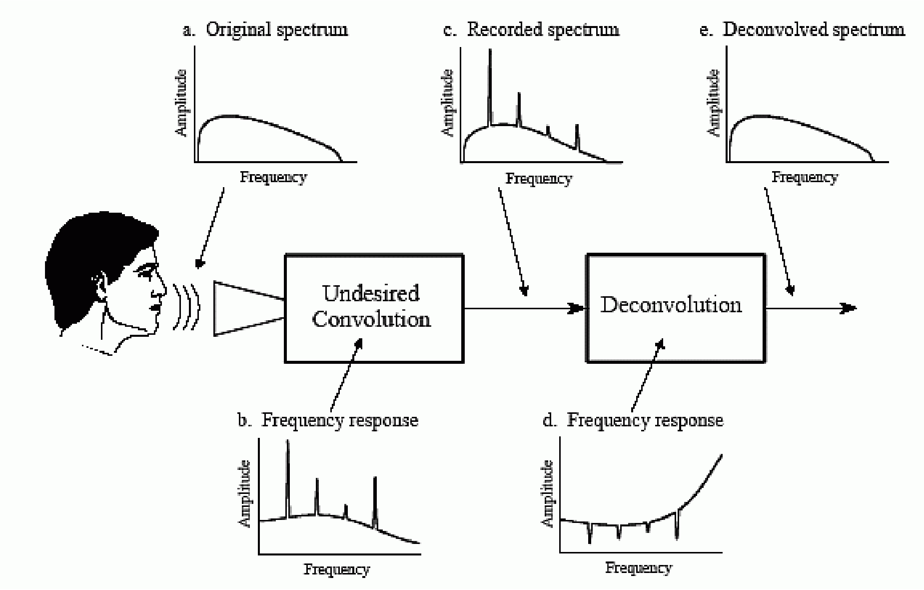
Time Invariant: $x(t) \longrightarrow \boxed{\text{TI}} \longrightarrow y(t)$ $\qquad x(t - t_0) \longrightarrow \boxed{\text{TI}} \longrightarrow y(t - t_0)$

WHAT IS INVERSE SYSTEM? [2020-6(c)]

A system is called inverse system if it produces distinct output signals for distinct input signals.

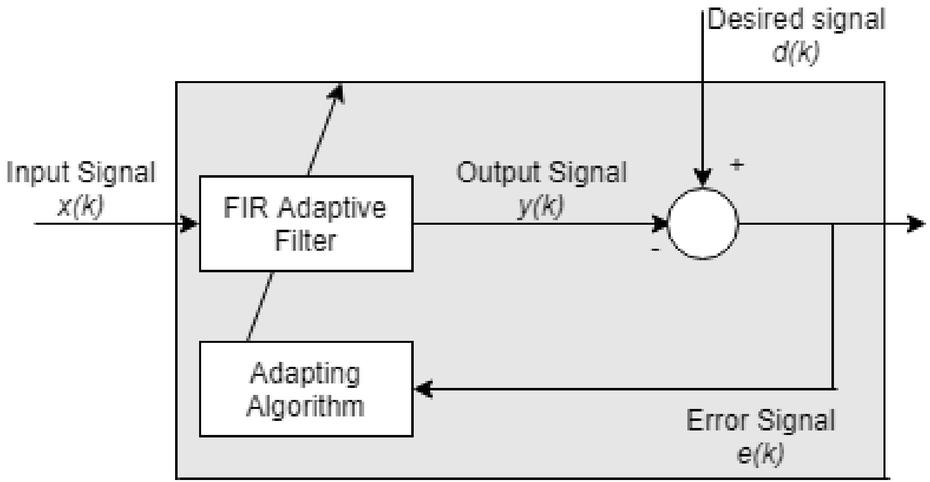
WHAT IS DECONVOLUTION?

Deconvolution is the process of filtering a signal to compensate for an undesired convolution. The goal of deconvolution is to recreate the signal as it existed before the convolution took place.



WHAT IS ADAPTIVE FILTER?

An adaptive filter is a digital filter that has self-adjusting characteristics. It also a digital filter whose coefficients change with an objective to make the filter converge to an optimal state.



❑ Drive convolution equation from the response of LTI system to arbitrary inputs. [2020-4(b), 2018-3(a), 2016-3(b)]

Characterization of LTI system by means of its impulse response: Let us give $\delta(n)$ unit impulse as the input to a LTI system. The output of this system is called impulse response $h(n)$. Refer figure-2. By the time invariant property of DT-LTI system the response to the applied impulse at any time n_0 is $h(n-n_0) = T[\delta(n-n_0)]$.
(2)

Also by linearity property of superposition, for $x(n_1)\delta(n-n_1) + x(n_2)\delta(n-n_2)$ inputs the corresponding output must be equal to $x(n_1)h(n-n_1) + x(n_2)h(n-n_2)$,

where $x(n_1)$ and $x(n_2)$are amplitudes of $x(n)$ at $n = n_1$ and $n = n_2$ respectively. The discrete input sequence $x(n)$ is represented as $x(n) = --x(0)\delta(n) + x(1)\delta(n-1) + x(2)\delta(n-2) + \cdots \ldots \ldots \ldots \ldots . +x(n_0)\delta(n-n_0) + ---$

OR

$x(n) = T[x(n)] = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$
(may be finite or infinite) (3)

By observing the relation between $\delta(n)$ and $h(n)$ as given in equation (2), the output of DT-LTI system is

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \qquad (4)$$

This equation is called convolution sum. It is characterization of DT-LTI system by unit impulse response[4].

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \text{ OR } y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) \qquad (5)$$

- If h(n) = {1, 2, 3, 4} and x(n) = {2, 5, 7, 8}. Find out y(n) = h(n) * x(n).
- [2020-4(c)]

Given input signal is x(n) = {2, 5, 7, 8}
LTI system is h(n) = {1, 2, 3, 4}

According to traditional method-

h(n) \ x(n)	2	5	7	8
1	2	5	7	8
2	4	10	14	16
3	6	15	21	24
4	8	20	28	32

y(n) = { 2, 9, 23, 45, 57, 52, 32 }

- [ ] Find **y(n)** if **x(n) = n+2 for $\boldsymbol{0 \le n \le 3}$** and $\boldsymbol{h(n) = a^n u(n)}$ for all n. [2018-3(b)]

We have-

$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$

Given x(n) = n+2 for $0 \le n \le 3$ and $h(n) = a^n u(n)$ for all n.

h(n) = 0 for n<0; so the system is causal x(n) is causal finite sequence whose value is zero for n>3.

Therefore,

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$
$$=\sum_{k=0}^{3} x(k+2)\, a^{n-k}\, u(n-k)$$
$$= 2a^n u(n) + 3a^{n-1}u(n-1)$$
$$= 4a^{n-2}u(n-2) + 5a^{n-3}u(n-3)$$

❑ Determine if the following system are **time-invariant or time variant**, y(n) =x (-n). [2018-3(c)]

Here given,

y(n) = x(-n)

Step-1: $y(n) \overset{n}{\rightarrow} y(n-n_0) = x(-n-n_0)$ ---------------- (1)

Step-2: $\mathrm{x}(n) \overset{n_0}{\rightarrow} x(n-n_0) \rightarrow System \rightarrow x(-n-n_0) = x(-n+n_0)$

The output of equation (1) and equation(2) are different. So it is time variant system.

❑ Consider the following input-output equations of signal:

$(\boldsymbol{i})\ \boldsymbol{y(n) = x(n) + \frac{1}{x(n-1)}}$ $\qquad$ $(\boldsymbol{ii})\boldsymbol{y(n) = nx(n)}$

Determine whether the system is linear or non-linear. [2016-1(c), 2019-1(c)]

(i) Law of additivity:

$$x_1(n) \rightarrow sys \rightarrow y_1 = x_1(n) + \frac{1}{x_1(n-1)} \qquad \text{-------------------(1)}$$

$$x_2(n) \rightarrow sys \rightarrow y_2 = x_2(n) + \frac{1}{x_2(n-1)} \qquad \text{-------------------(2)}$$

$$y_1(n) + y_2(n) = x_1(n) + x_2(n) + \frac{1}{x_1(n-1)+x_2(n-1)} \qquad \text{-------- --- ----(3)}$$

$$x_1(n) + x_2(n) \rightarrow sys \rightarrow y'(n) = x_1(n) + \frac{1}{x_1(n-1)} + \frac{1}{x_2(n-1)} \qquad \text{--------------(4)}$$

Equation (3) and (4) are not equal and the superposition principle is not satisfied. So the system is non-linear.

(ii) Law of additivity:

$x_1(n) \rightarrow sys \rightarrow y_1 = nx_1(n)$ - (1)

$x_2(n) \rightarrow sys \rightarrow y_2 = x_2(n)$ - (2)

$y_1(n) + y_2(n) = n[x_1(n) + x_2(n)]$ -- - - - - - - - - - - - - - (3)

$x_1(n) + x_2(n) \rightarrow sys \rightarrow y'(n) = n[x_1(n) + x_2(n)]$ ---------------(4)

Equation (3) and (4) are equal and the superposition principle is satisfied. So the system is linear.

- Consider the input x(n) = {1, 0, 2, 3 } of a LTI system and impulse response h(n)={ 1, 2, 3, 4 } . Find out the convolution sum of output y(n) of the LTI system.

[2016-3(c)]

Given input signal is x(n) = {1, 0, 2, 3}

LTI system is h(n) = {1, 2, 1, 3}

According to traditional method-

h(n) \ x(n)	1	0	2	3
1	1	0	2	3
2	2	0	4	6
1	1	0	2	3
3	3	0	6	9

y(n) = { 1, 2, 3, 10, 8, 9, 9 }

Thank you