ICE 3261
Assignment-2

1. Find the nyquist interval and nyquist rate for the continuous time signal given below-

$$x(t) = \frac{1}{2\pi}\cos(4000\pi t)\cos(1000\pi t).$$

Ans: Given, $m_1(t) = \frac{1}{2\pi}\cos(4000\pi t)$

$m_2(t) = \cos(1000\pi t)$.

$\omega_{m_1} = 4000\pi = 2\times 2000\pi$ $\therefore f_{m_1} = 2000\,Hz$

$\omega_{m_2} = 1000\pi = 2\times 500\pi$ $\therefore f_{m_2} = 500\,Hz$

~~$\omega_{s_1} = 2\omega_{m_1}$~~

$f_{s_1} = 2f_{m_1}$
$= 2\times 2000$
$= 4000\,Hz$

$f_{s_2} = 2f_{m_2}$
$= 2\times 500$
$= 1000\,Hz$

$\therefore$ Nyquist rate $= f_{s_1} + f_{s_2}$
$= 4000 + 1000$
$= \boxed{5000\ Hz}$.

$\therefore$ Nyquist interval, $T_s = \frac{1}{5000} = 2\times 10^{-4}\,s$
$= 0.2\,ms$

2. Find the Nyquist rate for the continuous-time signal given below- $x(t) = \frac{\sin(4\times 10^3\pi t)}{\pi t}$

Ans: Given, $\omega_m = 4\times 10^3\pi$

$\omega_s = 2\omega_m = 4\times 10^3\pi \times 2 = 8\times 10^3\pi$

$\therefore f_s = \frac{\omega_s}{2\pi} = \frac{8\times 10^3\pi}{2\pi} = 4\times 10^3\,Hz$

3. Given $x(t)$ with Nyquist rate ω_0. Determine nyquist rate for continuous-time signal
$y(t) = x(t) \cos\omega_0 t$

Nyquist rate of $x(t)$, ω_0 is f_0

~~Nyquist~~ For $\cos\omega_0 t$, ω_0 is message angular phase.

$\therefore \omega_{s_2} = 2\omega_0$

$\therefore f_{s_2} = \frac{2\omega_{s_2}}{2\pi} = \frac{2 f_{s_2} 2\pi}{2\pi} = f_{s_2}$

$\therefore$ Nyquist rate $= f_0 + f_{s_2}$ Ans

4. Determine nyquist sampling rate & nyquist sampling interval for the following signals.
(a) $\text{sinc}^2(100\pi t)$ (b) $0.01\,\text{sinc}^2(100\pi t)$
(c) $\text{sinc}(100\pi t) + \text{sinc}^2(60\pi t)$ (d) $\text{sinc}(100\pi t)\,\text{sinc}(50\pi t)$

(a) Ans. $x(t) = \text{sinc}(100\pi t)\ \&\ \text{sinc}(100\pi t) = \frac{\sin(100\pi t)}{100\pi t} \cdot \frac{\sin(100\pi t)}{100\pi t}$

$\omega_{m_1} = 100\pi$ $\qquad \omega_{m_2} = 100\pi$

$\omega_{s_1} = 2\omega_{m_1} = 200\pi$ $\qquad \omega_{s_2} = 2 \times \omega_{m_2} = 200\pi$

$\therefore f_{s_1} = \frac{\omega_{s_1}}{2\pi} = 100\,\text{Hz}$ $\qquad f_{s_2} = \frac{\omega_{s_2}}{2\pi} = 100\,\text{Hz}$

$\therefore$ Nyquist rate $= f_{s_1} + f_{s_2} = 200\,\text{Hz}$ Nyquist interval, $= \frac{1}{200} = 5\ \text{ms}$

(b) Given, $x(t) = 0.01\,\text{sinc}^2(100\pi t)$

$$= \frac{0.01}{(100\pi t)^2}\sin(100\pi t)\sin(100\pi t)$$

$\omega_{m_1} = 2\pi f_1 = 2\pi\, 50$ $\quad\therefore f_{s_1} = 2\times f_1 = 100$

$\omega_{m_2} = 2\pi f_2 = 2\pi\, 50$ $\quad f_{s_2} = 2\times f_2 = 100$

$\therefore$ Nyquist rate $= f_{s_1} + f_{s_2} = 200$ Hz

$\therefore$ Nyquist interval $= \frac{1}{200} = 5\times 10^{-3}$ s $= 5$ ms

(c) Given, $x(t) = \text{sinc}(100\pi t) + \text{sinc}^2(60\pi t)$

$$= \frac{1}{100\pi t}\sin(100\pi t) + \frac{1}{(60\pi t)^2}\text{sinc}(60\pi t)\,\text{sinc}(60\pi t)$$

$m_1(t) = \sin(100\pi t)$ $\therefore \omega_{m_1} = 2\pi\, 50$ $\quad\therefore f_{m_1} = 50$ Hz

~~$\therefore f_s = 2f_{m_1} = 50\times 2 = 100$ Hz~~

$m_2(t) = \frac{1}{(60\pi t)^2}\sin(60\pi t)\sin(60\pi t)$

$\omega_{m_{21}} = 60\pi$ $\quad\therefore \omega_{m_{21}} = 2\pi f_{m_{21}} = 2\pi\, 30$

$f_{m_{21}} = 30$ Hz $\quad\therefore f_{s_{21}} = 2 f_{m_{21}} = 60$ Hz

$\omega_{m_{22}} = 60\pi$ $\quad\therefore \omega_{m_{22}} = 2\pi f_{m_{22}} = 2\pi\, 30$

$\therefore f_{m_{22}} = 30$ Hz. $\quad\therefore f_{s_{22}} = 2 f_{m_{22}} = 60$

$\therefore f_s = f_{s_{21}} + f_{s_{22}} = 120$ Hz

$\therefore$ Nyquist rate $= 120$ Hz.

$\therefore$ Nyquist interval $= \frac{1}{120} = 8.3\times 10^{-3}$

$= 8.3$ ms

(d) Given, $x(t) = \frac{1}{100\pi t}\sin(100\pi t)\sin(50\pi t) \times \frac{1}{50\pi t}$

$\omega_{m_1} = 2\pi f_{m_1} = 100\pi$ $\therefore f_{m_1} = 50$ Hz.

$\omega_{m_2} = 2\pi f_{m_2} = 50\pi$ $\therefore f_{m_2} = 25$ Hz

$\therefore f_{s_1} = 2 f_{m_1} = 100$ Hz, $f_{s_2} = 2 \times f_{m_2} = 50$ Hz.

$\therefore$ Nyquist rate, $f_{s_1} + f_{s_2} = 150$ Hz.

$\therefore$ Nyquist interval, $\frac{1}{150} = 6.66 \times 10^{-3}$ s $= 6.67$ ms

5. The Nyquist sampling rate for the signal,

$x(t) = \frac{\sin(500\pi t)}{\pi t} \times \frac{\sin(700\pi t)}{\pi t}$ is -

$= \frac{1}{\pi^2 t^2}\sin(500\pi t)\sin(700\pi t)$

$\therefore m_1(t) = \frac{1}{\pi^2 t^2}\sin(500\pi t)$ $\therefore \omega_{m_1} = 500\pi$

$\therefore \omega_{s_1} = 2\omega_{m_1} = 1000\pi$

$\therefore f_{s_1} = \frac{\omega_{s_1}}{2\pi} = \frac{1000\pi}{2\pi} = 500$ Hz

$m_2(t) = \sin(700\pi t)$ $\therefore \omega_{m_1} = 700\pi$

$\therefore \omega_{s_1} = 2\omega_{m_1} = 2 \times 700\pi = 1400\pi$ $\therefore f_{s_2} = \frac{\omega_{s_2}}{2\pi} = \frac{1400\pi}{2\pi} = 700$

$\therefore$ Nyquist Rate, $f_s = f_{s_1} + f_{s_2} = 500 + 700 = 1200$ Hz

$\therefore$ Nyquist interval $= \frac{1}{1200} = 8.3 \times 10^{-4}$ s $= 0.83$ ms

(6) Determine Nyquist Rate of following signals:

(a) $10\cos(40t)$ (b) $10\cos(40t) + 10\sin(40t)$.

(c) $20\sin(40t)\sin(60t)$ (d) $40\,\text{sinc}(20t)$.

(a) $x(t) = 10\cos(40t)$.

$\therefore \omega_m = 2\pi f_m = 2\pi 20$

$\therefore f_m = 20$

$\therefore f_s = 2 \times f_m = 40\text{Hz}$

Ans

(b) Given, $x(t) = 10\cos(40t) + 10\sin(40t)$

$\omega_{m_1} = 2\pi f_{m_1} = 2\pi 20$

$\omega_{m_2} = 2\pi f_{m_2} = 2\pi 20$

$\therefore f_s = 2 f_{m_1} = 2 \times 20 = 40\text{Hz}$

Ans

(c) Given,

$x(t) = 20\sin(40t)\sin(60t)$.

$\omega_{m_1} = 2\pi f_{m_1} = 2\pi 20$

$f_{m_1} = 20$.

$\therefore f_{s_1} = 2 \times 20 = 40\text{Hz}$.

$m_2(t) = \sin(60t)$

$\omega_{m_2} = 2\pi f_{m_2} = 2\pi 30$

$f_{m_2} = 30$

$\therefore f_{s_2} = 2 \times 30 = 60\text{Hz}$

$\therefore$ Nyquist Rate, $f_s = 40 + 60$
$= 100\text{ Hz}$.

Ans

(d) $x(t) = 40\,\text{sinc}(20t)$

$= \frac{40}{20t}\sin(20t)$

$= \frac{2}{t}\sin(20t)$

$\omega = 2\pi f = 2\pi 10$

$\therefore f_m = 10\text{ Hz}$

$\therefore f_s = 2 \times f_m$
$= 20\text{ Hz}$.

Ans