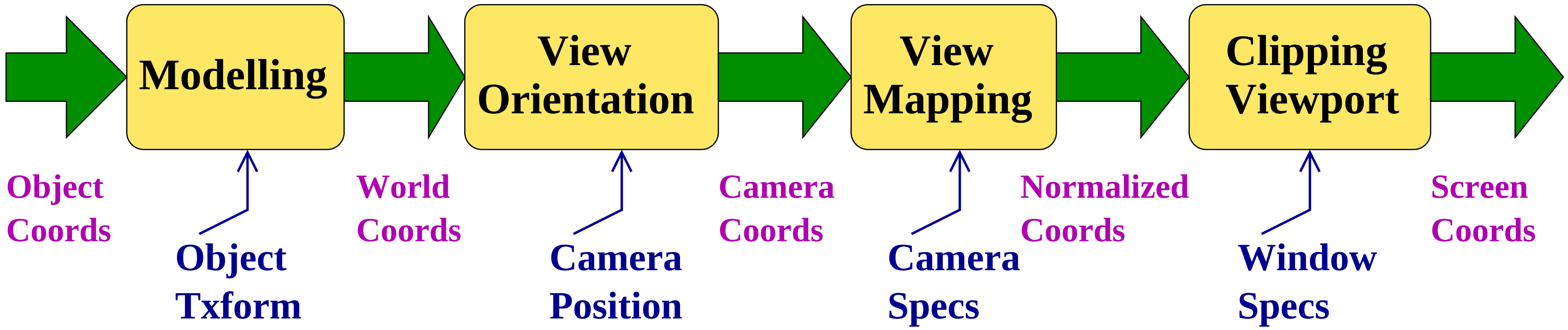
Computer Graphics Scan Conversion

Graphics in Practice: Summary

- Basic primitives: Points, Lines, Triangles/Polygons.
- Each constructed fundamentally from points.
- Points can be specified in different coordinate systems. The pipeline of operations on a point is:



Scan Conversion or **Rasterization**

- Primitives are defined using points, which have been mapped to the screen coordinates.

- In vector graphics, connect the points using a pen directly.

- In Raster Graphics, we create a discretized image of the whole screen onto the **frame buffer** first. The image is scanned automatically onto the display periodically.

- This step is called **Scan Conversion** or **Rasterization**.

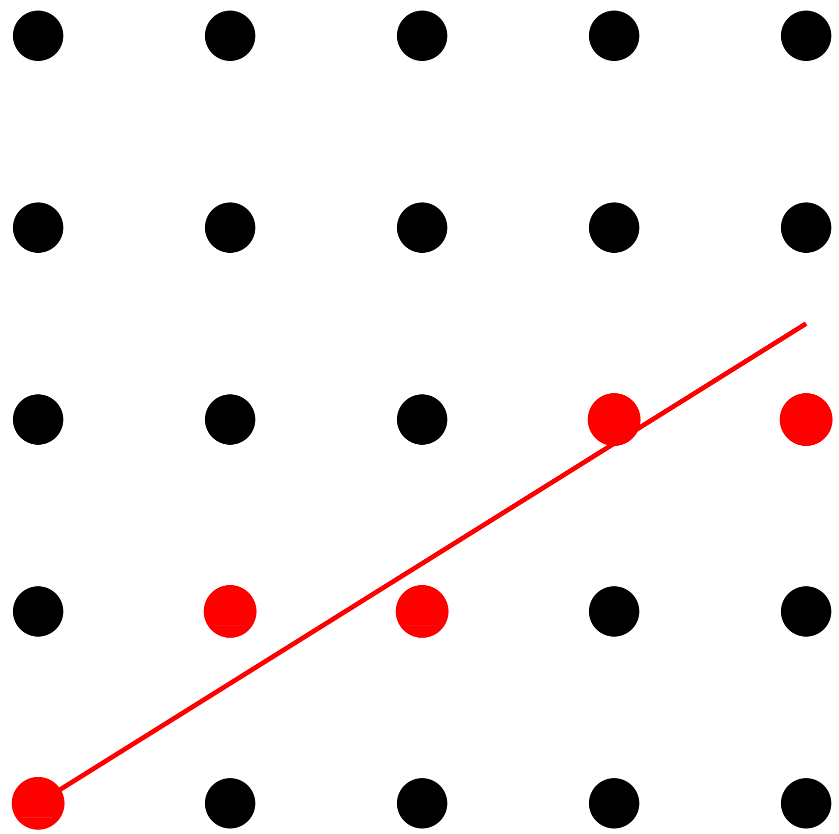
Scan Converting a Point

- The 3D point has been transformed to its screen coordinates (u, v).
- Round the coordinates to frame buffer array indices (i, j).
- Current colour is defined/known. Frame buffer array is initialized to the background colour.
- Perform: frameBuf[i, j] $\leftarrow$ currentColour
- The function WritePixel(x, y, colour) does the above.
- If *PointSize* > 1, assign the colour to a number of points in the neighbourhood!

Scan Converting a Line

- Identify the grid-points that lie on the line and colour them.
- Problem: Given two end-points on the grid, find the pixels on the line connecting them.
- Incremental algorithm or Digital Differential Analyzer (DDA) algorithm.
- Mid-Point Algorithm

Line on an Integer Grid



Incremental Algorithm

Function DrawLine(x_1, y_1, x_2, y_2, **colour**)

$\Delta x \leftarrow x_2 - x_1, \Delta y \leftarrow y_2 - y_1$, slope $\leftarrow \Delta y / \Delta x$

$x \leftarrow x_1, y \leftarrow y_1$

While ($x < x_2$)

WritePixel (x, round(y), **colour**)

$x \leftarrow x + 1,\ \ y \leftarrow y +$ slope

EndWhile

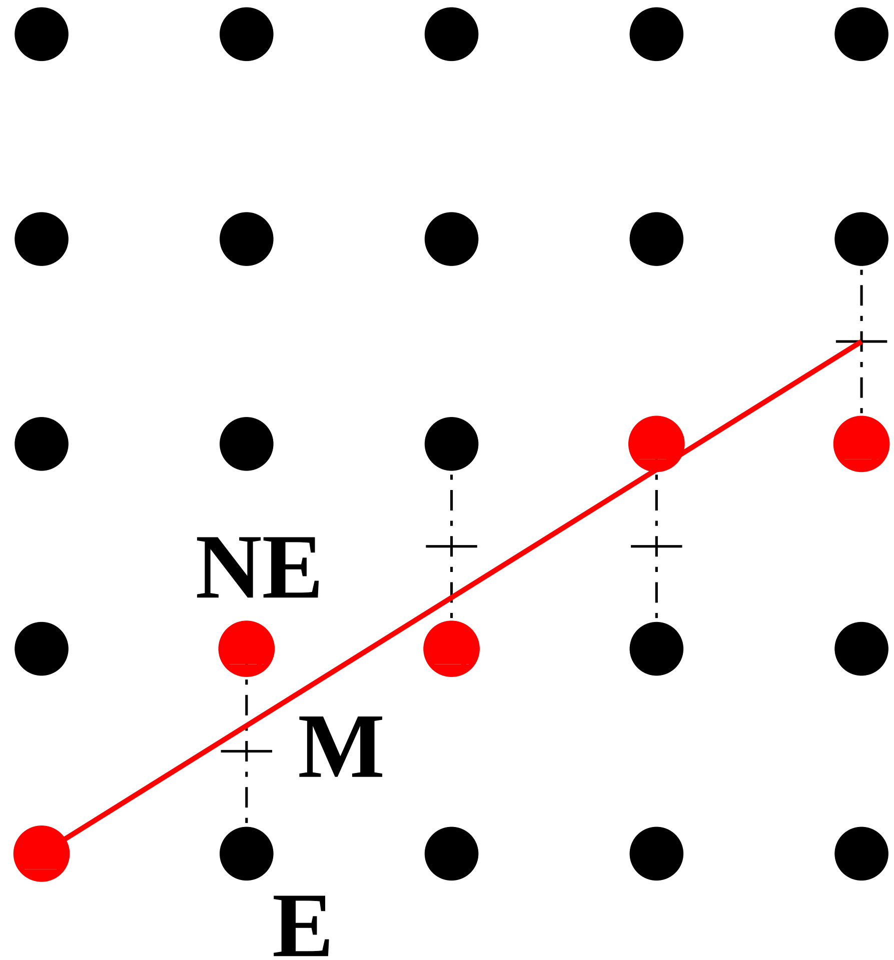
WritePixel (x_2, y_2, **colour**)

EndFunction

Points to Consider

- If abs(slope) > 1, step through y values, adding inverse slopes to x at each step.
- Simple algorithm, easy to implement.
- Need floating point calculations (add, round), which are expensive.
- Can we do with integer arithmetic only?
  Yes: **Bresenham's Algorithm**
- We will study a simplified version of it called the
  Mid-Point Line Algorithm.

Two Options at Each Step!



Mid-Point Line Algorithm

- Line equation: $ax + by + c = 0, \;\; a > 0$.
  Let $0 <$ slope $= \Delta y / \Delta x = -a/b < 1.0$
- $F(x, y) = ax + by + c > 0$ for below the line, < 0 for above.
- **NE** if $d = F(\mathbf{M}) > 0$; **E** if $d < 0$; else any!
- $d_{\mathbf{E}} = F(M_{\mathbf{E}}) = d + a, \quad d_{\mathbf{NE}} = d + a + b$.
- Therefore, $\Delta_{\mathbf{E}} = a, \quad \Delta_{\mathbf{NE}} = a + b$.
- Initial value: $d_0 = F(x_1 + 1, y_1 + \frac{1}{2}) = a + b\ /2$
- Similar analysis for other slopes. Eight cases in total.

Pseudocode

Function DrawLine (l, m, i, j, **colour**)

$a \leftarrow j - m,\ b \leftarrow (l - i),\ x \leftarrow l,\ y \leftarrow m$

$d \leftarrow 2a + b,\ \Delta_E \leftarrow 2a,\ \Delta_{NE} \leftarrow 2(a + b)$

While ($x < i$)

WritePixel(x, y, **colour**)

if ($d < 0$)   // East

$d \leftarrow d + \Delta_E,\ x \leftarrow x + 1$

else   // North-East

$d \leftarrow d + \Delta_{NE},\ x \leftarrow x + 1,\ y \leftarrow y + 1$

EndWhile

WritePixel(i, j, **colour**)

EndFunction

Example: $(10, 10)$ **to** $(20, 17)$

$$F(x, y) = 7x - 10y + 30,\ a = 7,\ b = -10$$
$$d_0 = 2 * 7 - 10 = 4,\ \Delta_{\mathbf{E}} = 2 * 7 = 14,\ \Delta\mathbf{NE} = -6$$

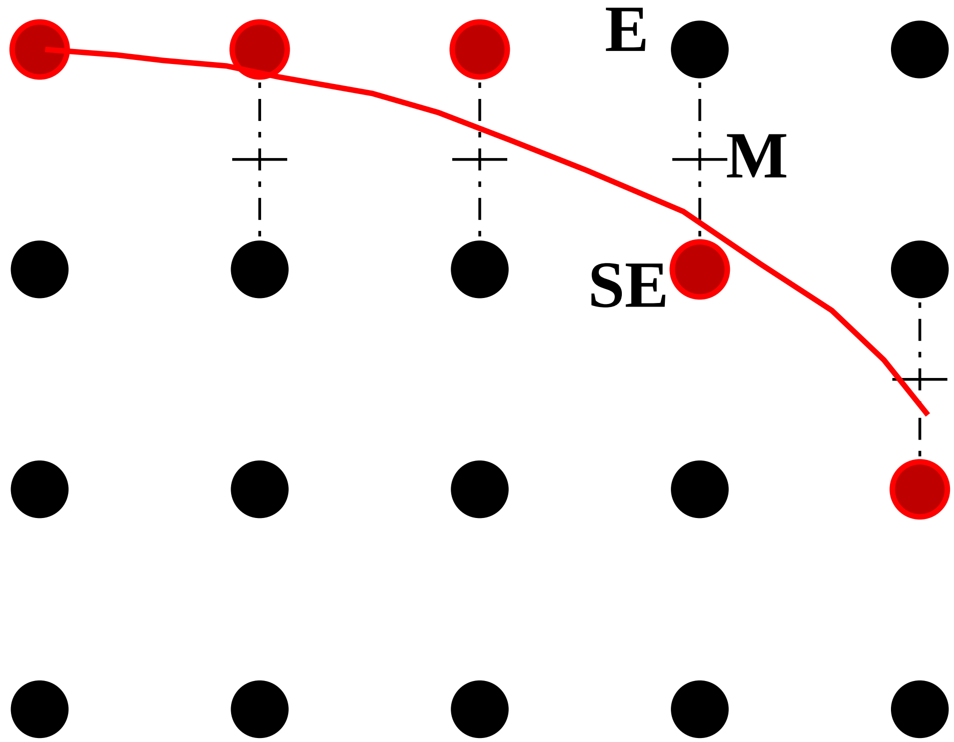
$$d > 0 : \mathbf{NE}\ (11, 11),\ d = 4 + -6 = -2$$
$$d < 0 : \mathbf{E}\ (12, 11),\ d = -2 + 14 = 12$$
$$d > 0 : \mathbf{NE}\ (13, 12),\ d = 12 + -6 = 6$$
$$d > 0 : \mathbf{NE}\ (14, 13),\ d = 6 + -6 = 0$$
$$d = 0 : \mathbf{E}\ (15, 13),\ d = 0 + 14 = 14$$
$$d > 0 : \mathbf{NE}\ (16, 14),\ d = 14 + -6 = 8$$

Later, **NE** (17, 15), **NE** (18, 16), **E** (19, 16), **NE** (20, 17).

Scan Converting Circles

- Need to consider only with centre at origin: $x^2 + y^2 = r^2$.
- For arbitrary centre, add (x_c, y_c) to each point.
- 8-way symmetry: Only an eighth of the circle need to be scan converted!
- If (x, y) on circle, $(\pm x, \pm y)$, $(\pm y, \pm x)$ are also on the circle!
- Easy way: $y = \sqrt{r^2 - x^2}$, but floating point calculations!

Back to Two Points??



- Choice between E and SE neighbours between the vertical and the 45 degree lines.

Mid-Point Circle Algorithm

- Circle equation: $x^2 + y^2 - r^2 = 0$
- $F(x, y) = x^2 + y^2 - r^2 < 0$ for inside circle, > 0 for outside.
- **SE** if $d = F(\mathbf{M}) > 0$; **E** if $d < 0$; else any!
- $d_{\mathbf{E}} = F(M_{\mathbf{E}}) = d + 2x + 3, \quad d_{\mathbf{SE}} = d + 2(x - y) + 5$.
- Therefore, $\Delta_{\mathbf{E}} = 2x + 3, \quad \Delta_{\mathbf{SE}} = 2(x - y) + 5$.
- Initial value: $d_0 = F(1, r - \frac{1}{2}) = \frac{5}{4} - r$

Pseudocode

Function DrawCircle (r, **colour**)

$x \leftarrow 0,\ \ y \leftarrow r,\ \ d \leftarrow 1 - r$

CirclePoints (x, y, **colour**)

While ($x < y$)

if ($d < 0$) // East

$d \leftarrow d + 2 * x + 3,\ x \leftarrow x + 1$

else // South-East

$d \leftarrow d + 2 * (x - y) + 5,\ x \leftarrow x + 1,\ y \leftarrow y - 1$

CirclePoints (x, y, **colour**)

EndWhile

EndFunction

Eliminate Multiplication?

- **Current selection is E:** What are the new Δ's?
  $\Delta'_{\mathbf{E}} = 2(x+1)+3 = \Delta_{\mathbf{E}} + 2$
  $\Delta'_{\mathbf{SE}} = 2(x+1-y)+5 = \Delta_{\mathbf{SE}} + 2$
- **Current selection is SE:** What are the new Δ's?
  $\Delta'_{\mathbf{E}} = 2(x+1)+3 = \Delta_{\mathbf{E}} + 2$
  $\Delta'_{\mathbf{SE}} = 2(x+1-(y-1))+5 = \Delta_{\mathbf{SE}} + 4$
- if ($d < 0$)   // East
  $d \leftarrow d + \Delta_E,\ \Delta_E$ += $2,\ \Delta_{\mathbf{SE}}$ += $2,\ x$++
  else   // South-East
  $d \leftarrow d + \Delta_{\mathbf{SE}},\ \Delta_{\mathbf{E}}$ += $2,\ \Delta_{\mathbf{SE}}$ += $4,\ x$++, $y = y - 1$

Patterned Line

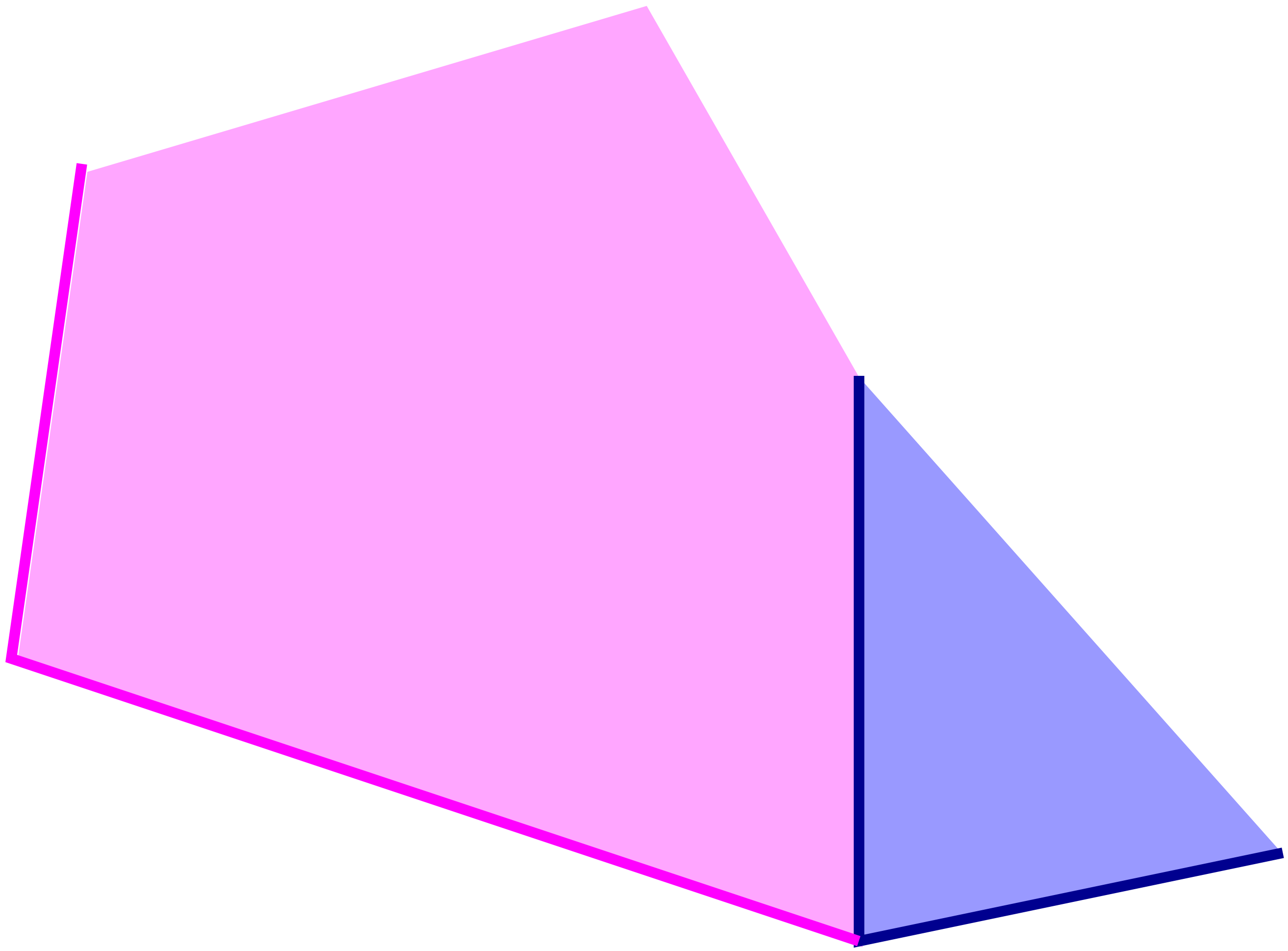
- Represent the pattern as an array of booleans/bits, say, 16 pixels long.
- Fill first half with 1 and rest with 0 for dashed lines.
- Perform WritePixel(x, y) only if pattern bit is a 1.

```
if (pattern[i])   WritePixel(x, y)
```

where **i** is an index variable starting with 0 giving the ordinal number (modulo 16) of the pixel from starting point.

Shared Points/Edges

- It is common to have points common between two lines and edges between two polygons.
- They will be scan converted **twice**. Not efficient. Sometimes harmful.
- Solution: Treat the intervals closed on the left and open on the right. **[**x_m, x_M**) & [**y_m, y_M**)**
- Thus, edges of polygons on the top and right boundaries are not drawn.



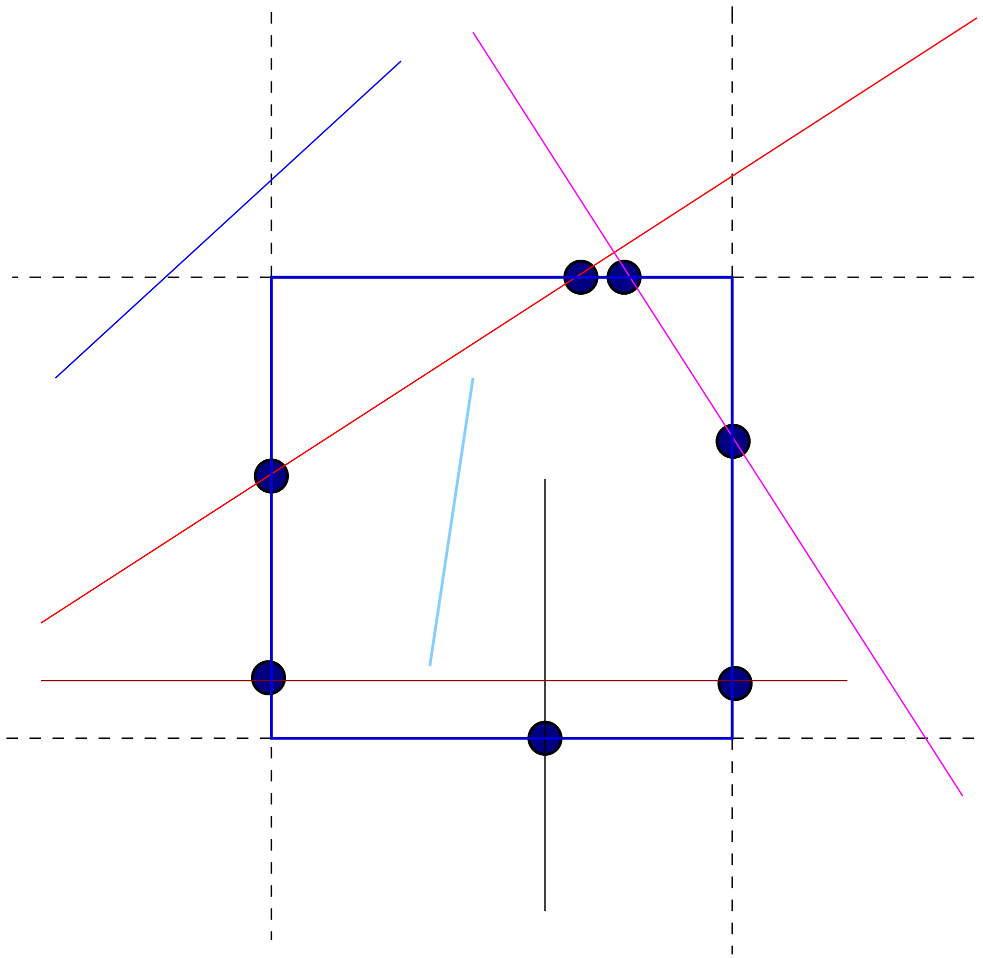
Clipping

- Often, many points map to outside the range in the normalized 2D space.
- Think of the FB as an infinite canvas, of which a small rectangular portion is sent to the screen.
- Let's get greedy: draw only the portion that is visible. That is, **clip** the primitives to a *clip-rectangle*.
- **Scissoring:** Doing scan-conversion and clipping together.

Clipping Points

- Clip rectangle: (x_m, y_m) to (x_M, y_M).
- For (x, y): $\quad x_m \leq x \leq x_M, \quad y_m \leq y \leq y_M$
- Can use this to clip any primitives: Scan convert normally. Check above condition before writing the pixel.
- Simple, but perhaps we do more work than necessary.
- Analytically clip to the rectangle, then scan convert.

Clipping Lines



Intersecting Line Segments

- Infinite line equation: $ax + by + c = 0$. Not good for line segments!

- $P = P_1 + \mathbf{t}\ (P_2 - P_1),\ 0 \leq t \leq 1$.

- Represent sides of clip-rectangles and lines for clipping this way, with two parameters **t** and **s**. Solve for s, t. Both should be within $[0, 1]$.

Cohen-Sutherland Algorithm

- Identify line segments that can be accepted trivially.
- Identify line segments that can be rejected trivially.
- For the rest, identify the segment that falls within the clip-rectangle.
- For ease of this, assign **outcodes** to each of the 9 regions.

Region Outcodes

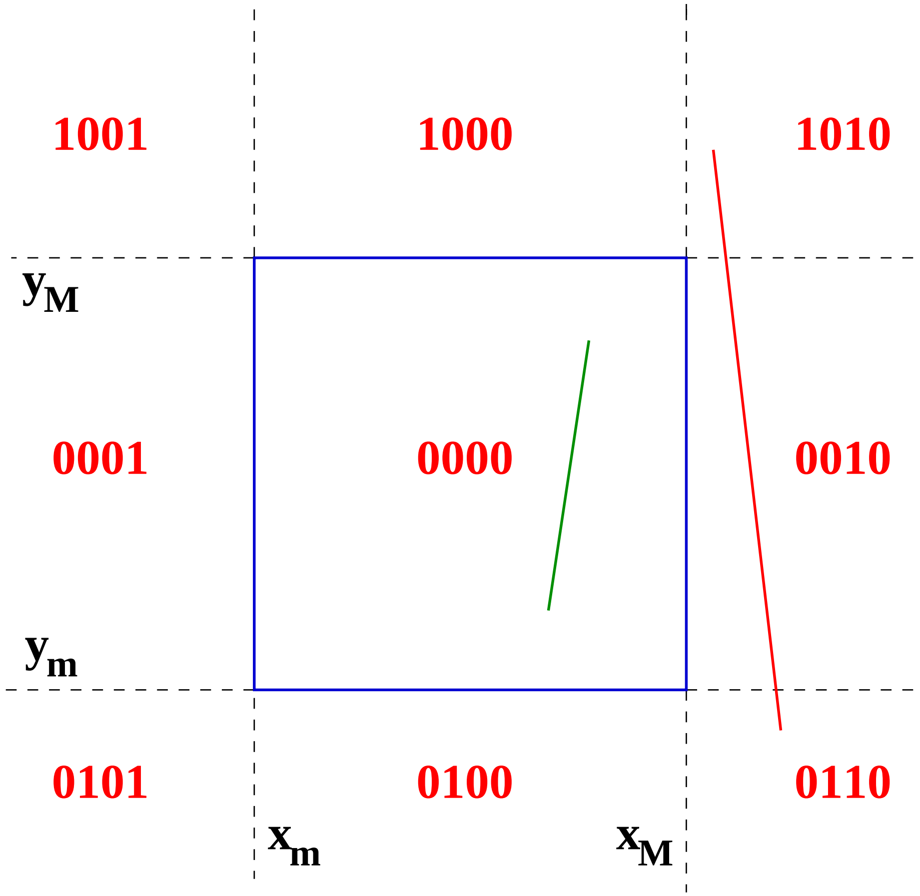
Bits from left to right:

$y > y_M$

$y < y_m$

$x > x_M$

$x < x_m$



Overall Algorithm

- Accept: code1 | code0 == 0
- Reject: code1 & code0 != 0
- Else, identify one of the boundaries crossed by the line segment and clip it to the inside.
- Do it in some order, say, TOP, RIGHT, BOTTOM, LEFT.
- We also have: TOP = 1000, BOTTOM = 0100, LEFT = 0001, RIGHT = 0010

Intersecting with Right/Top

if (code & RIGHT) // Intersects right boundary

// Adjust right boundary to the intersection with x_M

$y \leftarrow y0 + (y1 - y0) * (x_M - x0) \ / \ (x1 - x0)$

$x \leftarrow x_M$

ComputeCode(x, y)

if (code & TOP) // Intersects top boundary

// Adjust top boundary to the intersection with y_M

$x \leftarrow x0 + (x1 - x0) * (y_M - y0) \ / \ (y1 - y0)$

$y \leftarrow y_M$

ComputeCode(x, y)

Whole Algorithm

0 code0 $\leftarrow$ ComputeCode(x0, y0), code1 $\leftarrow \cdots$

1 if (! (code1 $|$ code0)) Accept and Return

2 if (code1 & code0) Reject and Return

3 code $\leftarrow$ code1 ? code1 : code0

4 if (code & TOP) Intersect with y_M line.

5 elsif (code & RIGHT) Intersect with x_M line.

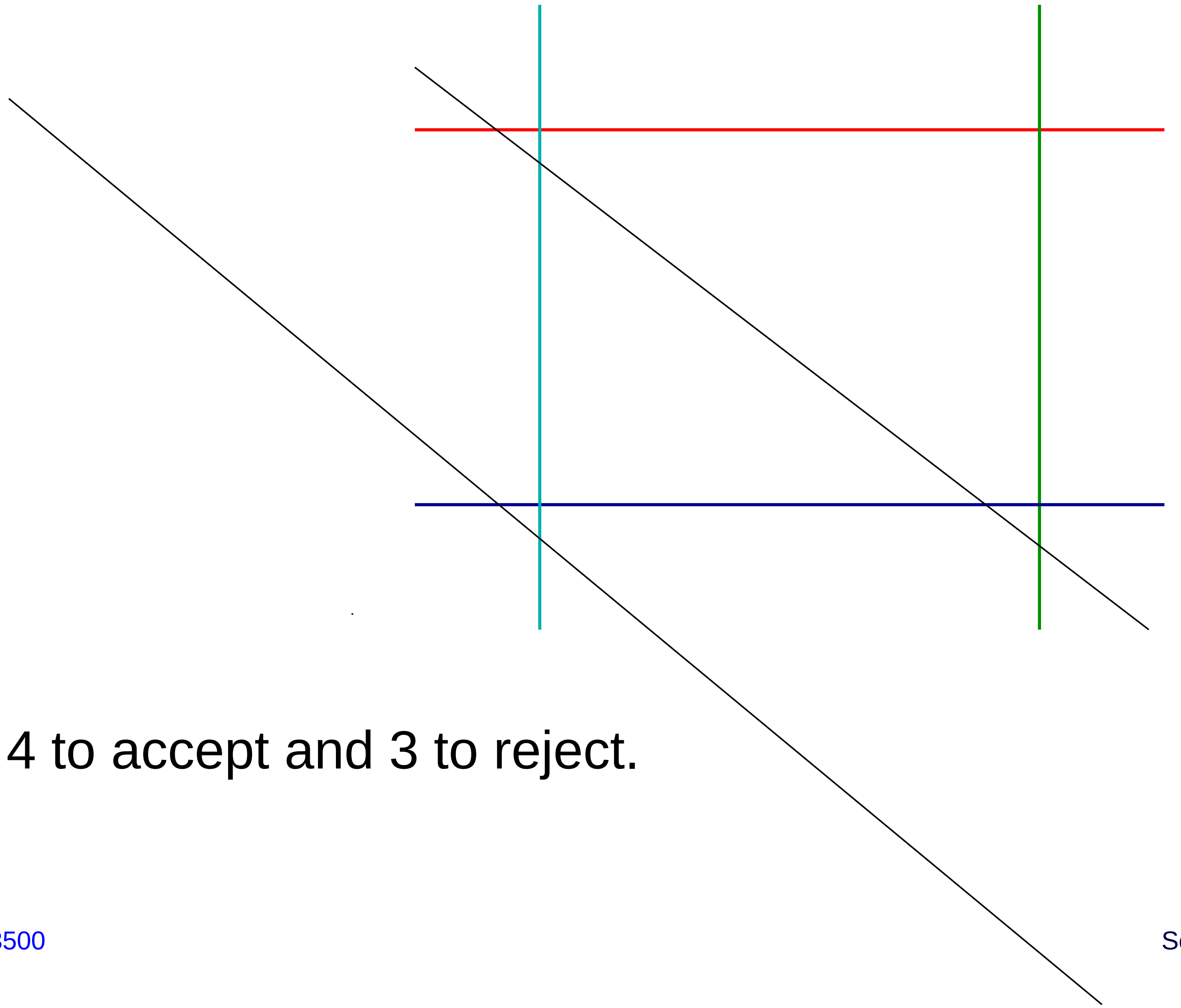
6 elsif (code & BOTTOM) Intersect with y_m line.

7 elsif (code & LEFT) Intersect with x_m line.

8 if (code == code1) Replace EndPoint1.

9 else Replace EndPoint0.

10 Goto step 1.



4 to accept and 3 to reject.

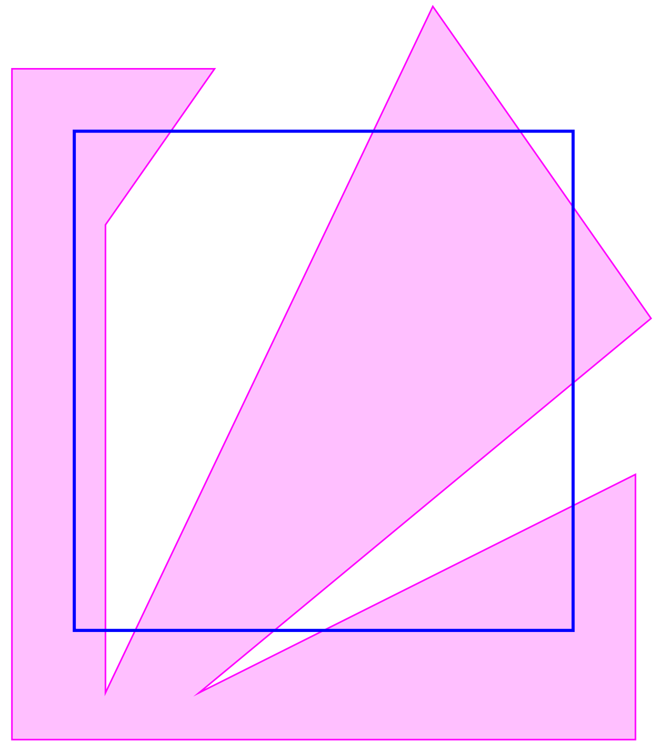
Discussion

- Simple logical operations to check intersections etc.
- Not efficient, as external intersections are not eliminated.
- In the worst case, 3 intersections may be computed and then the line segment could be rejectd.
- 4 intersections may be computed before accepting a line segment.

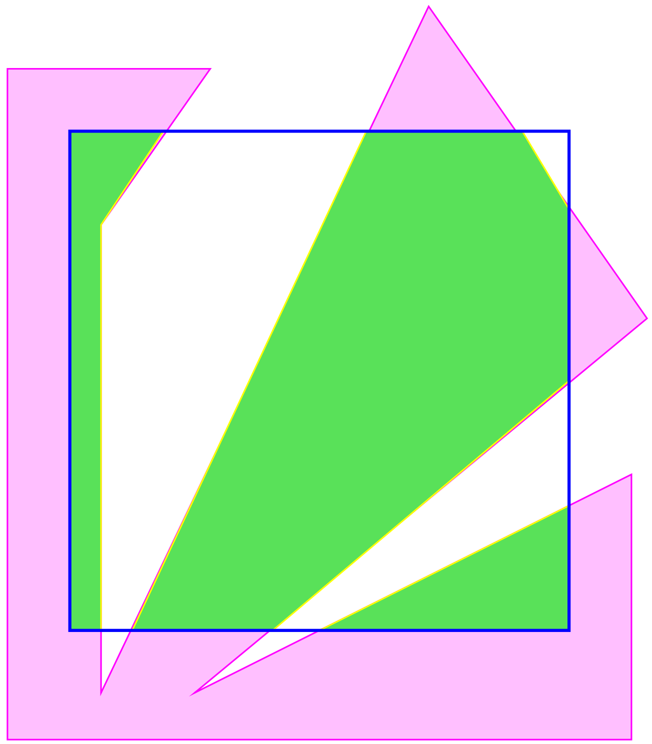
Clipping Polygons

- Restrict drawing/filling of a polygon to the inside of the clip rectangle.
- A convex polygon remains convex after clipping.
- A concave polygon can be clipped to multiple polygons.
- Can perform by intersecting to the four clip edges in turn.

An Example



An Example



Sutherland-Hodgman Algorithm

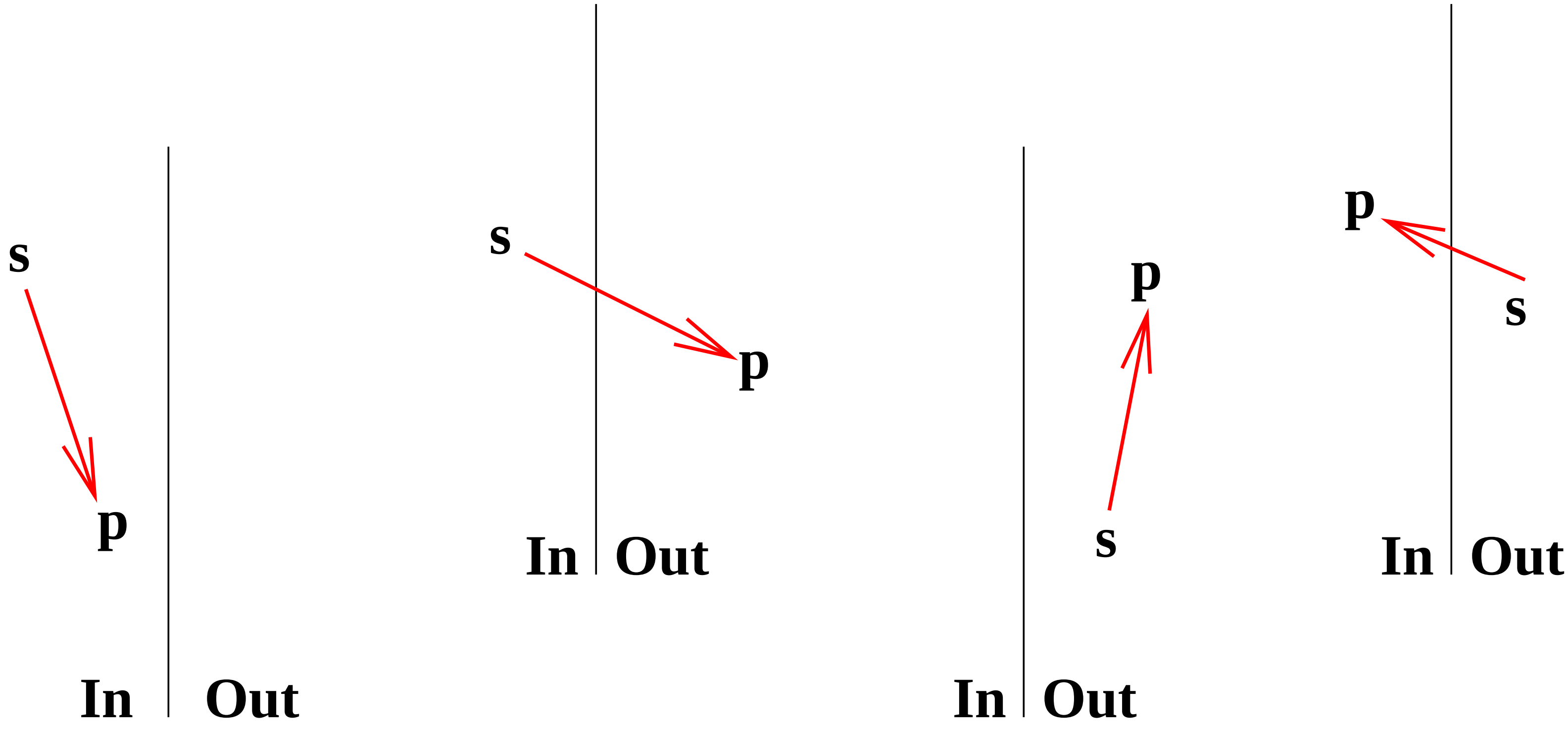
- *Input:* A list of vertices $v_1, v_2, \cdots, v_n$. Implied edges from v_i to v_{i+1} and from v_n to v_1.
- *Output:* Another list of vertices giving the clipped polygon.
- *Method:* Clip the entire polygon to the infinite line for each clip edge in turn.
- Four passes, the output of each is a partially clipped polygon used as input to the next.
- Post-processing to eliminate degenerate edges.

Algorithm Detail

- Process edges one by one and clip it to a line.
- Start with the edge $E(v_n, v_1)$.
- Compare the current edge $E(v_{i-1}, v_i)$ with the current clip line. Clip it to lie within the clip rectangle.
- Repeat for the next edge $E(v_i, v_{i+1})$. Till all edges are processed.
- When processing $E(v_{i-1}, v_i)$, treat v_{i-1} as the **in** vertex and v_i as the **out** vertex.

At Each Step ...

- **in** vertex: $\mathbf{s}$ (already handled). **out** vertex: $\mathbf{p}$. Four cases:



Function SuthHodg()

```
p ← last(inVertexList)    // Copy, not remove
while (notEmpty(inVertexList))
    s ← p,  p ← removeNext(inVertexList)
    if (inside(p, clipBoundary))
        if (inside(s, clipBoundary))
            addToList(p, outVertexList)    // Case 1
        else  i ← intersect(s, p, clipBoundary)    // Case 4
            addToList(i, outVertexList),  addToList(p, outVertexList)
    elsif (inside(s, clipBoundary))    // Case 2
        addToList(intersect(s, p, clipBoundary), outVertexList)
```

Complete Algorithm

- Invoke SuthHodg() 4 times for each clip edge as clipBoundary.
- The outVertexList after one run becomes the inVertexList for the next.
- Uses list data structures to implement polygons.
- Function inside() determines if a point is in the **inside** of the clip-boundary. We can define it as “being on the left when looking from first vertex to the second”.
- Can be extended to clip to any convex polygonal region!

Filled Rectangles

- Write to all pixels within the rectangle.

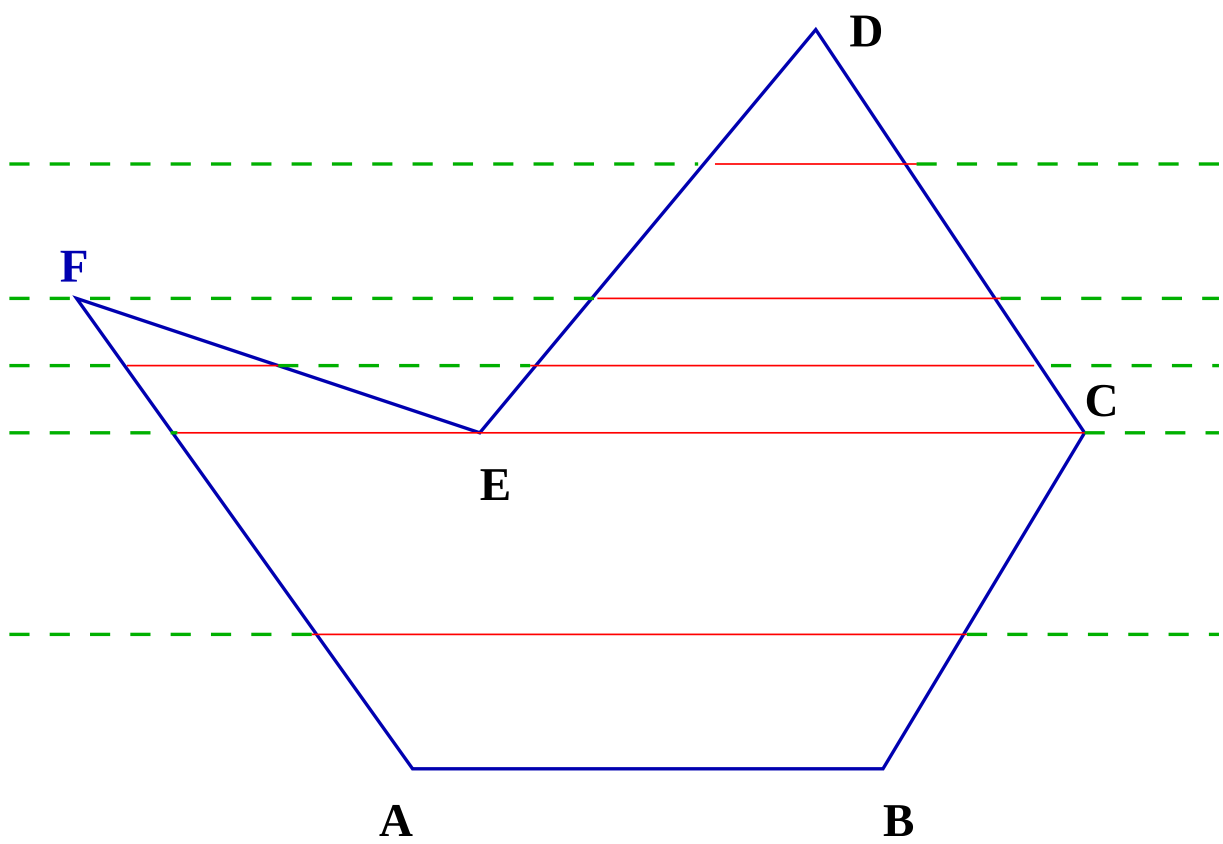
Function FilledRectangle (x_m, x_M, y_m, y_M, **colour**)
    for $x_m \leq x \leq x_M$ do
        for $y_m \leq y \leq y_M$ do
            WritePixel (x, y. **colour**)
EndFunction

- How about non-upright rectangles? General polygons?

Filled Polygons

- For each scan line, identify **spans** of the polygon interior. Strictly interior points only.
- For each scan line, the **parity** determines if we are inside or ouside the polygon. Odd is inside, Even is outside.
- Trick: End-points count towards parity enumeration only if it is a y_{min} point.
- Span extrema points and other information can be computed during scan conversion. This information is stored in a suitable data structure for the polygon.

Parity Checking



Edge Coherence

- If scan line y intersects with an edge E, it is likely that $y+1$ also does. (Unless intersection is the y_{max} vertex.)
- When moving from y to $y+1$, the X-coordinate goes from x to $x+1/m$. $1/m = (x_2 - x_1)/(y_2 - y_1) = \Delta x \ / \ \Delta y$
- Store the integer part of x, the numerator (Δx) and the denominator (Δy) of the fraction separately.
- For next scan line, add Δx to numerator. If sum goes $> \Delta y$, increment integer portion, subtract Δy from numerator.

Scan Converting Filled Polygons

- Find intersections of each scan line with polygon edges.
- Sort them in increasing X-coordinates.
- Use parity to find interior spans and fill them.
- Most information can be computed during scan conversion. A list of intersecting polygons stored for each scan line.
- Use edge coherence for the computation otherwise.

Special Concerns

- Fill only strictly interior pixels: Fractions rounded up when even parity, rounded down when odd.
- Intersections at integer pixels: Treat interval closed on left, open on right.
- Intersections at vertices: Count only y_m vertex for parity.
- Horizontal edges: Do not count as y_m!

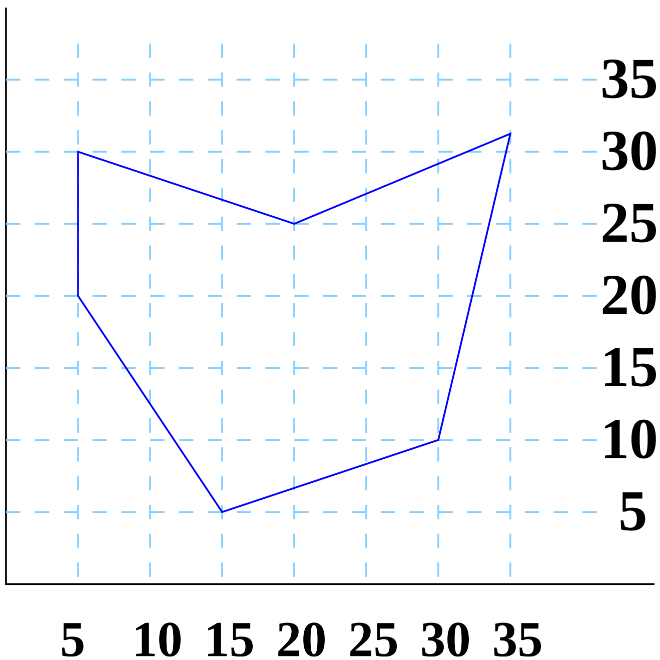
Filled Polygon Scan Conversion

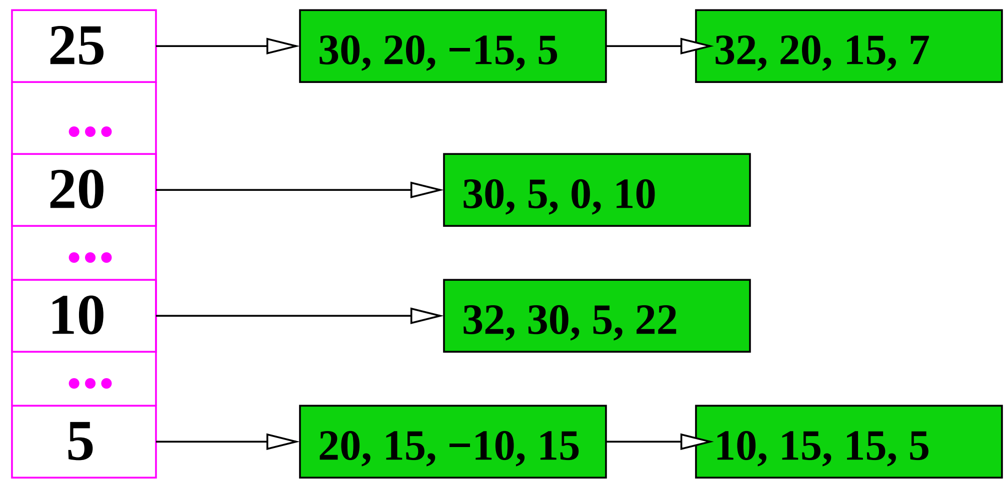
- Perform all of it together. Each scan line should not be intersected with each polygon edge!
- Edges are known when polygon vertices are mapped to screen coordinates.
- Build up an edge table while that is done.
- Scan conversion is performed in the order of scan lines. Edge coherence can be used; an active edge table can keep track of which edges matter for the current scan line.

Edge Table for a Polygon

- Construct a bucket-sorted table of edges, sorted into buckets of y_m for the edge.
- Each bucket y contains a list of edges with $y = y_m$, in the increasing order of x coordinate of the lower end point.
- Each edge is represented by its y_M for the edge, x of the lower (that is y_m) point, and the slope as a rational number.
- This is the basis for constructing the Active Edge Table to compute the spans.

Polygon and Edge Table





Active Edge Tables

- Start with the lowest y value and an empty AET.
- Insert edges from bucket y of ET to AET.
  (They have $y = y_m$ and are sorted on x.)
- Remove edges from AET where y is the y_M point.
- Between pairs of AET entries lie spans. Fill them.
- Compute next point on edge using coherence. (Increment y by 1 and numerator by Δx, etc. Or vice versa)
- Continue above 4 steps till ET and AET are empty.

Active Edge Table: Snapshots

y = 15 AET → 20, 9, −10, 15 → 32, 31, 5, 22

y = 22 AET → 30, 5, 0, 10 → 32, 32, 5, 22

y = 27 AET → 30, 5, 0, 10 → 30, 14, −15, 5 → 32, 24, 15, 7 → 32, 34, 5, 22

Pattern Filling

- A rectangular bit-map with the desired pattern can be used to fill the interior with a pattern.
- If *pattern*$(i \mod M, j \mod N)$, draw pixel, else ignore.
- i, j are row, col indices. Lower left corner at 0 and 0.
- M, N are the pattern height and width.

Scan Conversion: Summary

- Filling the frame buffer given 2D primitives.
- Convert an analytical description of the basic primitives into pixels on an integer grid in the frame buffer.
- Lines, Polygons, Circles, etc. Filled and unfilled primitives.
- Efficient algorithms required since scan conversion is done repeatedly.
- 2D Scan Conversion is all, even for 3D graphics.

Scan Conversion: Summary

- High level primitives (point, line, polygon) map to window coordinates using transformations.
- Creating the display image on the Frame Buffer is important. Needs to be done efficiently.
- Clipping before filling FB to eliminate futile effort.
- After clipping, line remains line, polygons can become polygons of greater number of sides, etc.
- General polygon algorithm for clipping and scan conversion are necessary.