2D TRANSFORMATIONS
COMPUTER GRAPHICS

2D Transformations

"Transformations are the operations applied to geometrical description of an object to change its position, orientation, or size are called geometric transformations".

Translation

- Translation is a process of changing the position of an object in a straight-line path from one co-ordinate location to another.
- We can translate a two dimensional point by adding translation distances, tx and ty.
- Suppose the original position is (x ,y) then new position is (x', y').
- Here x'=x + tx and y'=y + ty.

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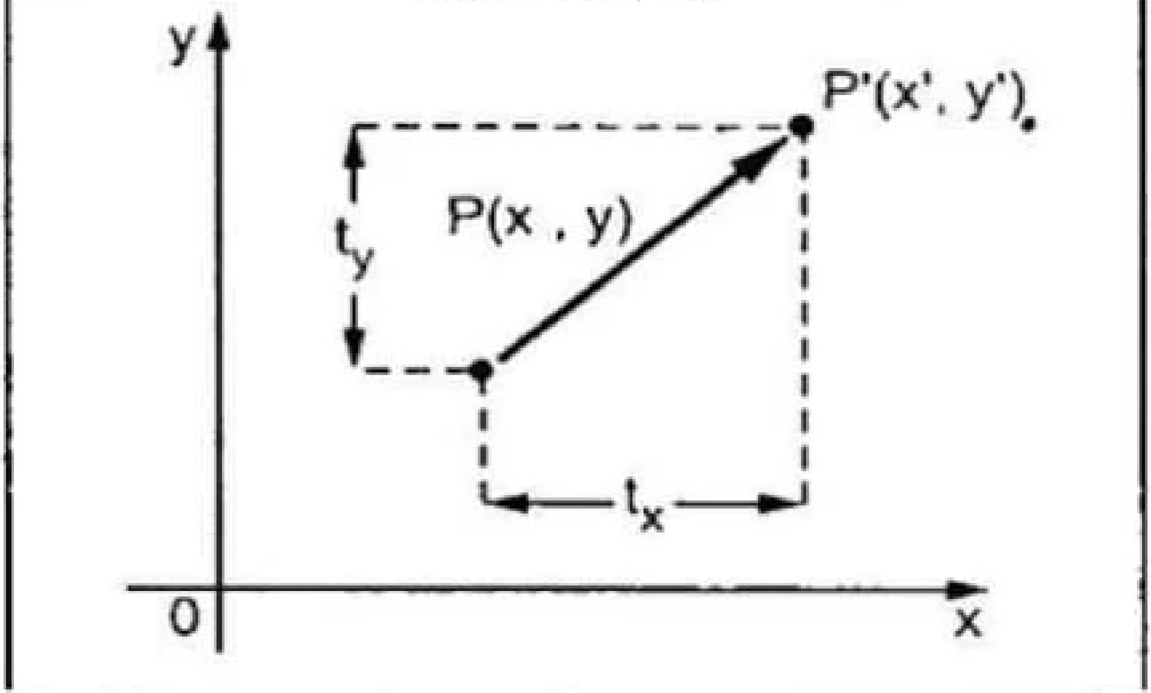
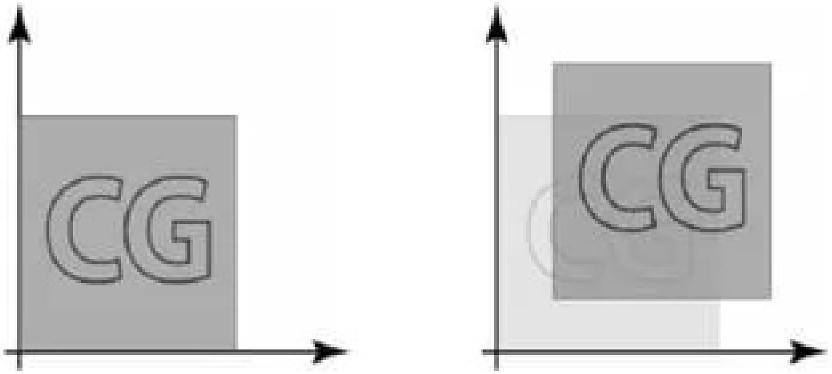


Translation



- Matrix form of the equations:
  X' = X + tx and Y' = Y + ty is

$$P = \begin{pmatrix} x \\ y \end{pmatrix} \qquad P' = \begin{pmatrix} x' \\ y' \end{pmatrix} \qquad T = \begin{pmatrix} tx \\ ty \end{pmatrix}$$

- we can write it,
  P'= P + T

- Translate a polygon with co-ordinates A(2,5) B(7,10) and C(10,2) by 3 units in X direction and 4 units in Y direction.
- $A' = A + T$

$$= \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \end{pmatrix}$$

- $B' = B + T$

$$= \begin{pmatrix} 7 \\ 10 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 10 \\ 14 \end{pmatrix}$$

- $C' = C + T$

$$= \begin{pmatrix} 10 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ 7 \end{pmatrix}$$

Rotation

- A two dimensional rotation is applied to an object by repositioning it along a circular path in the xy plane.
- Using standard trigonometric equations , we can express the transformed co-ordinates in terms of θ *and* $\emptyset$ *as*

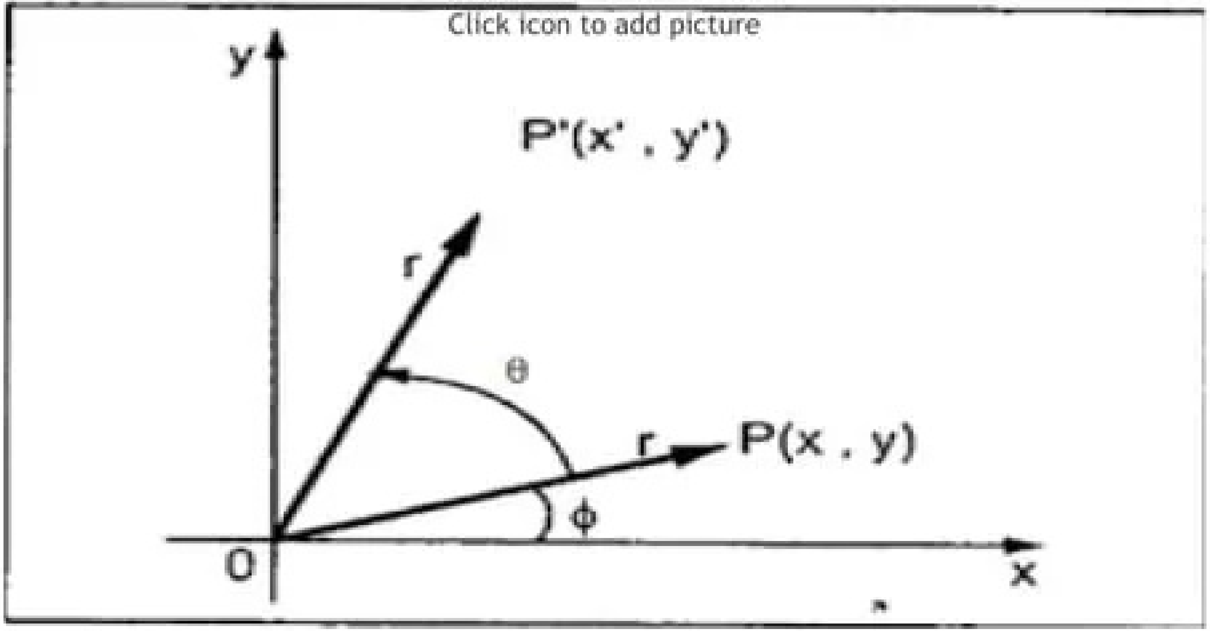
$$x' = r\cos(\phi + \theta) = r\cos\phi\cos\theta - r\sin\phi\sin\theta$$

$$y' = r\sin(\phi + \theta) = r\cos\phi\sin\theta + r\sin\phi\cos\theta$$

- The original co-ordinates of the point is

$$x = r\cos\phi$$

$$y = r\sin\phi$$

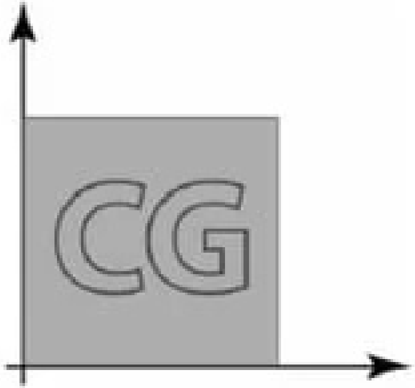


After substituting equation 2 in equation 1 we get

x'=x cosθ $- y \sin \theta$

Y'=x sinθ + y cosθ

Rotation



- That equation can be represented in matrix form

$$\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- we can write this equation as,

  P' = P . R

- Where R is a rotation matrix and it is given as

$$R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- A point (4,3) is rotated counterclockwise by angle of 45.
  find the rotation matrix and the resultant point.
- $$R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \cos 45 & \sin 45 \\ -\sin 45 & \cos 45 \end{pmatrix}$$

$$= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$P' = \begin{bmatrix} 4 & 3 \end{bmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$= \begin{bmatrix} 4/\sqrt{2} - 3/\sqrt{2} & 4/\sqrt{2} + 3/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 7/\sqrt{2} \end{bmatrix}$$

Scaling

- A scaling transformation changes the size of an object.
- This operation can be carried out for polygons by multiplying the co-ordinates values (x , y) of each vertex by scaling factors Sx and Sy to produce the transformed co-ordinates (x' , y').

$$x' = x \cdot S_x$$

$$y' = y \cdot S_y$$

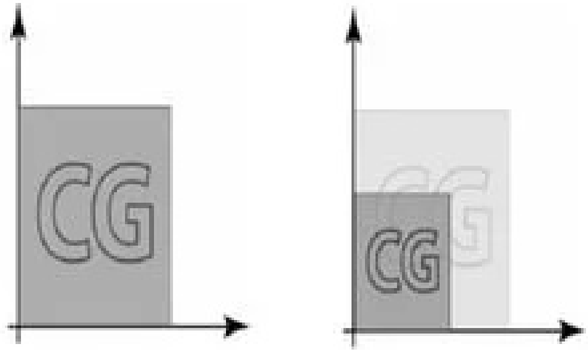
- In the matrix form

$$\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{pmatrix} S_x & 0 \\ 0 & S_y \end{pmatrix}$$

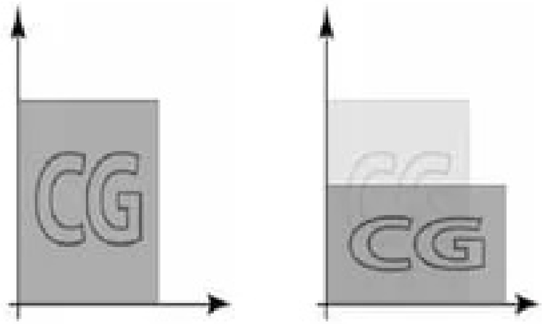
$$= P \cdot S$$

Scaling

- Uniform Scaling



Un-uniform Scaling



Homogeneous co-ordinates for Translation

- The homogeneous co-ordinates for translation are given as

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ tx & ty & 1 \end{pmatrix}$$

- Therefore , we have

$$\begin{bmatrix} x' & y' & 1 \end{bmatrix} = \begin{bmatrix} x & y & 1 \end{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ tx & ty & 1 \end{pmatrix}$$

$$= \begin{bmatrix} x + tx & y + ty & 1 \end{bmatrix}$$

Homogeneous co-ordinates for rotation

- The homogeneous co-ordinates for rotation are given as

$$R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Therefore , we have

$$\begin{pmatrix} x' & y' & 1 \end{pmatrix} = \begin{pmatrix} x & y & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} x\cos\theta - y\sin\theta & x\sin\theta + y\cos\theta & 1 \end{pmatrix}$$

Homogeneous co-ordinates for scaling

- The homogeneous co-ordinate for scaling are given as

$$S = \begin{pmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Therefore , we have

$$\begin{bmatrix} x' & y' & 1 \end{bmatrix} = \begin{bmatrix} x & y & 1 \end{bmatrix} \begin{pmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} x \, . \, Sx & y \, . \, Sy & 1 \end{bmatrix}$$

Composite Transformations

(A) Translations

If two successive translation vectors (t_{x1},t_{y1}) and (t_{x2},t_{y2}) are applied to a coordinate position P, the final transformed location P' is calculated as: -

$$P' = T(t_{x2},t_{y2}) \cdot \{T(t_{x1},t_{y1}) \cdot P\}$$
$$= \{T(t_{x2},t_{y2}) \cdot T(t_{x1},t_{y1})\} \cdot P$$

Where P and P' are represented as homogeneous-coordinate column vectors. We can verify this result by calculating the matrix product for the two associative groupings. Also, the composite transformation matrix for this sequence of transformations is: -

$$\begin{vmatrix} 1 & 0 & t_{x2} \\ 0 & 1 & t_{y2} \\ 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} \bullet & 0 & t_{x1} \\ 0 & 1 & t_{y1} \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} \bullet & 0 & t_{x1}+t_{x2} \\ 0 & 1 & t_{y1}+t_{y2} \\ 0 & 0 & 1 \end{vmatrix}$$

Or, $\quad T(t_{x2},t_{y2}) \cdot T(t_{x1},t_{y1}) = T(t_{x1}+t_{x2},\ t_{y1}+t_{y2})$

Which demonstrate that two successive translations are additive.

(B) Rotations

Two successive rotations applied to point P produce the transformed position: -

$$P' = R(\Theta_2) \cdot \{R(\Theta_1) \cdot P\}$$
$$= \{R(\Theta_2) \cdot R(\Theta_1)\} \cdot P$$

By multiplication the two rotation matrices, we can verify that two successive rotations are additive:

$$R(\Theta_2) \cdot R(\Theta_1) = R(\Theta_1 + \Theta_2)$$

So that the final rotated coordinates can be calculated with the composite rotation matrix as: -

$$P' = R(\Theta_1 + \Theta_2) \cdot P$$

(C) Scaling

Concatenating transformation matrices for two successive scaling operations produces the following composite scaling matrix: -

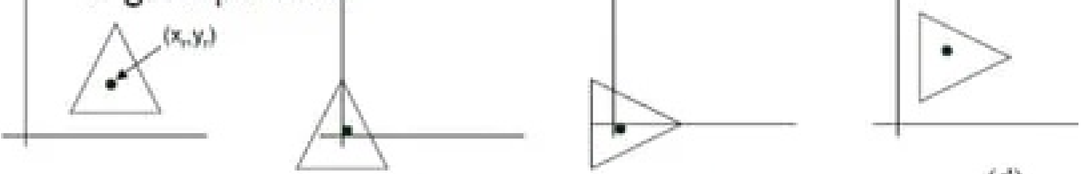
$$\begin{vmatrix} S_{x2} & 0 & 0 \\ 0 & S_{y2} & 0 \\ 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} S_{x1} & 0 & 0 \\ 0 & S_{y1} & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} S_{x1} . S_{x2} & 0 & 0 \\ 0 & S_{y1} . S_{y2} & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Or, $$S(S_{x2}, S_{y2}) . S(S_{x1}, S_{y1}) = S(S_{x1} . S_{x2}, S_{y1} . S_{y2})$$

The resulting matrix in this case indicates that successive scaling operations are multiplicative.

General pivot point rotation

- Translate the object so that pivot-position is moved to the coordinate origin
- Rotate the object about the coordinate origin
- Translate the object so that the pivot point is returned to its original position



(a) Original Position of Object and pivot point

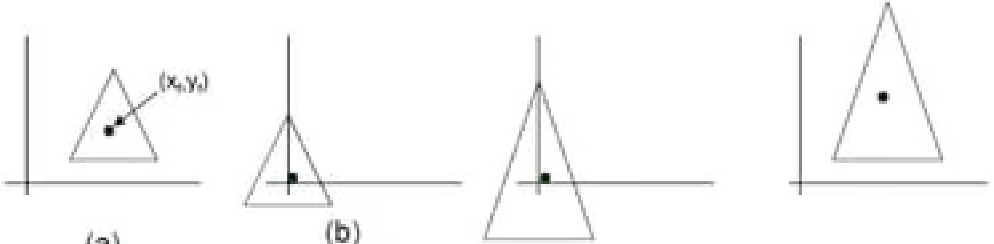
(b) Translation of object so that pivot point (x_r,y_r) is at origin

(c) Rotation was about origin

(d) Translation of the object so that the pivot point is returned to position (x_r,y_r)

General fixed point scaling

- Translate object so that the fixed point coincides with the coordinate origin
- Scale the object with respect to the coordinate origin
- Use the inverse translation of step 1 to return the object to its original position



(a) Original Position of Object and Fixed point

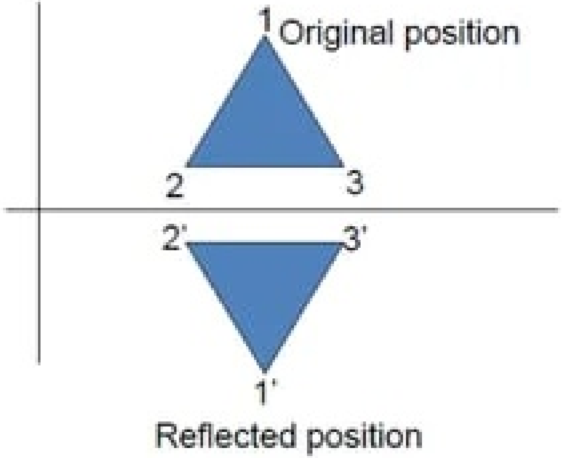
(b) Translation of object so that fixed point (x_f, y_f) is at origin

(c) scaling was about origin

(d) Translation of the object so that the Fixed point is returned to position (x_f, y_f)

Other transformations

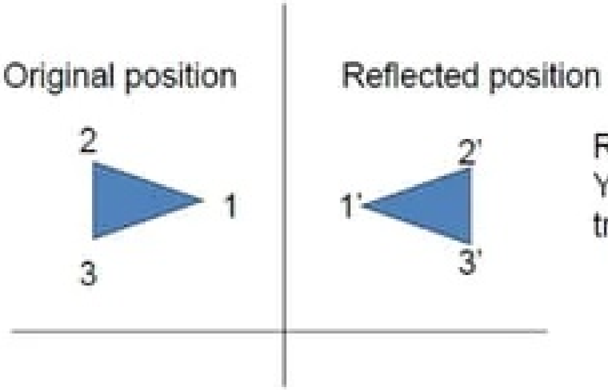
- **Reflection** is a transformation that produces a mirror image of an object. It is obtained by rotating the object by 180 deg about the reflection axis



Reflection about the line y=0, the X- axis , is accomplished with the transformation matrix

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

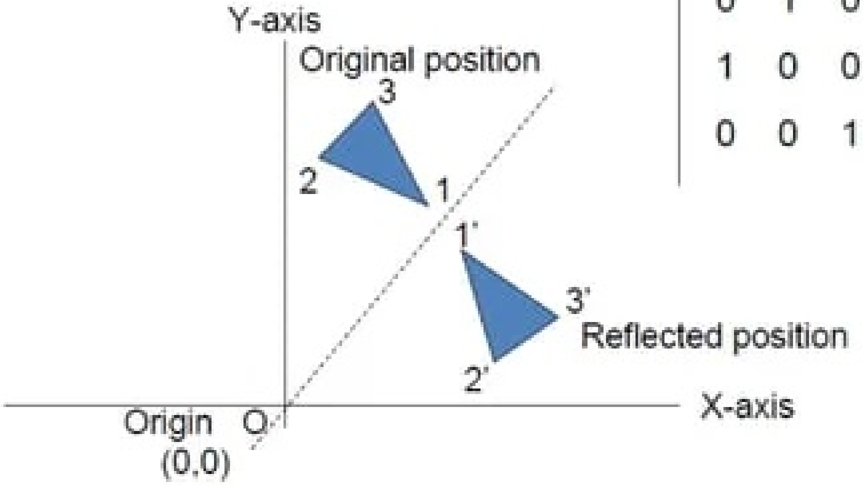
Reflection



Reflection about the line x=0, the Y- axis , is accomplished with the transformation matrix

$$\begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Reflection of an object w.r.t the straight line y=x

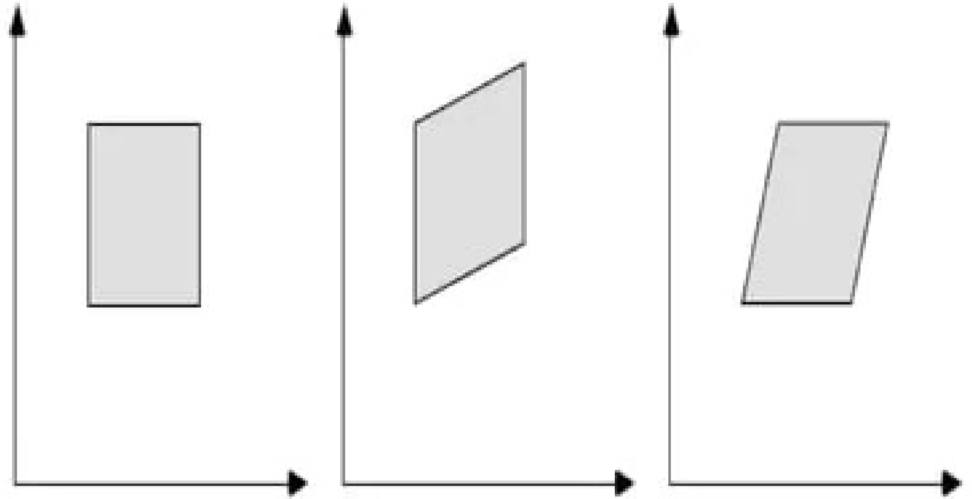


$$\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Shear Transformations

- Shear is a transformation that distorts the shape of an object such that the transformed shape appears as if the object were composed of internal layers that had been caused to slide over each other
- Two common shearing transformations are those that shift coordinate x values and those that shift y values

Shears



Original Data y Shear x Shear

CONCLUSION

- To manipulate the initially created object and to display the modified object without having to redraw it, we use Transformations.

THANK YOU