Department of Computer Science and Engineering, RU Class Test-I (CO1)

Course: CSE4181 (Digital Image Processing) Time: 30min; Marks: 30

1. (a) What is image resolution? Differentiate between intensity level resolution and spatial resolution.

Answer:

Image resolution is the amount of detail an image contains. It can be measured in a number of ways, but in digital imaging, it's often measured as the number of pixels in the image. A pixel (short for picture element) is a single point or a tiny square in a graphic image stored in an ordered rectangular grid. It is the smallest element in a digital image. The more pixels used to represent an image, the closer the result can resemble the analogue original.

Aspect	Intensity Level Resolution	Spatial Resolution
Definition	Ability to distinguish various brightness levels	Ability to discern fine details or structures
Representation	Number of distinguishable intensity levels	Number of pixels or sensor elements
Measurement	Typically expressed in bits (e.g., 8-bit, 12-bit)	Expressed in pixels (e.g., megapixels)
Impact on Image Quality	Affects smoothness of gradients and transitions	Affects clarity, sharpness, and level of detail
Example	8-bit grayscale image with 256 intensity levels	12-megapixel camera capturing finer image details
Importance in Imaging	Determines precision of brightness representation	Determines level of detail and image clarity

https://medium.com/@gokcenazakyol/1-what-is-digital-image-processing-image-processing-2da13b5dfa9c

(b) Mention the main focuses of digital image processing. How much memory is required to store 512x521 image in (i) B/W (ii) grayscale image with 17 gray levels and (iii) color image with 6-bit quantization for each of fundamental color?

Answer:

Digital image processing manipulates and analyzes digital images with the help of some techniques and algorithms. DIP focuses on two major tasks: Improvement of pictorial information for human interpretation and processing of image data for storage, transmission and representation for autonomous machine perception.

(b) Given,

Image resolution ; $m \times n = 512 \times 521$

Image needs 1 bit to represent Color :-

i) B/W needs 1 bit to represent Color :-

$\therefore$ Total memory $= 512 \times 521 \times 1$

$= 266752$ bits

$= 33344$ bytes

$= 32.56$ kb (Ans).

ii) Grayscale image with 17 gray levels.

$2^n = L$ | n = no. of bits, L = no. of levels

$\log_a x = y$

$a^y = x$

$2^n = L$

$\log_2 L = n$

$n = \log_2 L$

$= \log_2 17$

$= 4.087$

≈ 5 bits

$\therefore$ Total memory $= 512 \times 521 \times 5 = 1333760$ bits

$= 166720$ bytes

$= 162.81$ KB. (Ans)

iii) Color image with 6-bit quantization fo each fundamental color.

There are 3 color channels – Red, Green & Blue

no. of bits needed total $= \cancel{4}3 \times 6 = 18$ bits.

$\therefore$ Memory required $= (18 \times 512 \times 521) = 4801536$ bits

$= 600192$ bytes

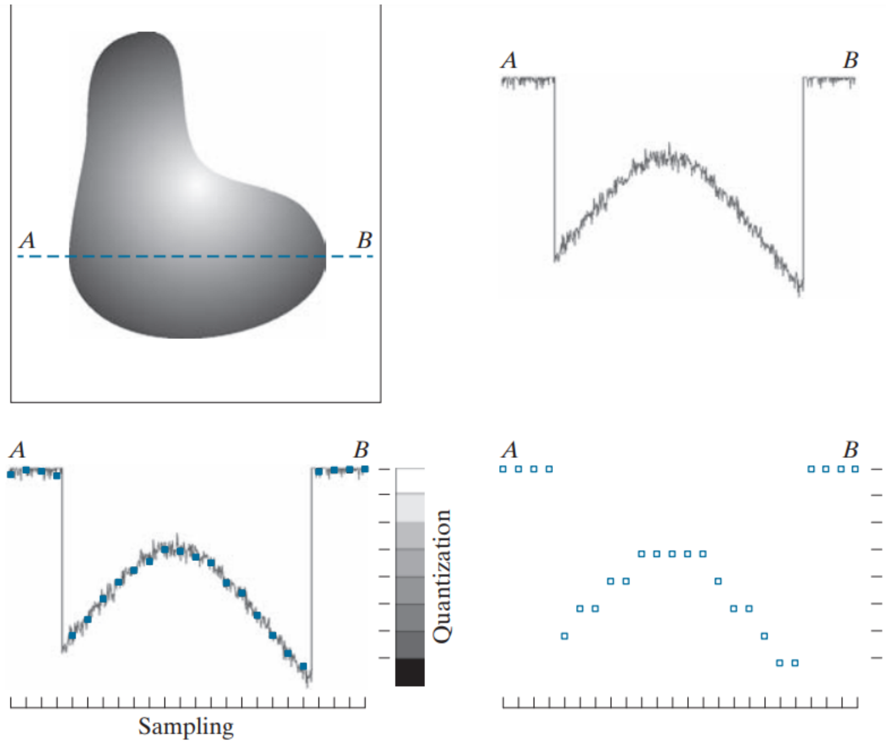
$= 586.126$ kb

(Ans)

(c) Define sampling and quantization with examples. What is the maximum possible sampling frequency?

Answer:

https://www.geeksforgeeks.org/difference-between-image-sampling-and-quantization/



c d

FIGURE 2.16 (a) Continuous image. (b) A scan line showing intensity variations along line AB in the continuous image. (c) Sampling and quantization. (d) Digital scan line. (The black border in (a) is included for clarity. It is not part of the image).

amplitude. To digitize it, we have to sample the function in both coordinates and also in amplitude. Digitizing the coordinate values is called *sampling*. Digitizing the amplitude values is called *quantization*.

The maximum possible sampling frequency is theoretically limited by the Nyquist-Shannon sampling theorem, which states that to accurately reconstruct a signal from its samples, the sampling frequency should be at least twice the highest frequency component present in the signal.

Mathematically, if f_{max} represents the maximum frequency in the signal, the Nyquist theorem suggests that the sampling frequency ($f_{sampling}$) should be at least $2 \times f_{max}$ to faithfully capture and reconstruct the original signal without aliasing (false or distorted representation of frequencies).

Department of Computer Science and Engineering, RU Class Test-**II (CO2**) on 17 Oct 2023

Course: CSE4181 (Digital Image Processing) Time: 30min; Marks: 30

1. (a) Define image histogram with example. Mention the information carried out by image histogram? Suppose you an image of size 200x200. It has four gray levels gl, g2, g3 and g4 consisting of 20%, 30%, 15% and 35% pixels respectively. Illustrate the its histogram.

Answer:

An image histogram is a gray-scale value distribution showing the frequency of occurrence of each gray-level value. For an image size of 1024 × 1024 × 8 bits, the abscissa ranges from 0 to 255; the total number of pixels is equal to 1024 × 1024. Modification of original histograms very often is used in image enhancement procedures.

3.3 HISTOGRAM PROCESSING

Let r_k, for $k = 0,1,2,\ldots,L-1$, denote the intensities of an L-level digital image, $f(x,y)$. The *unnormalized histogram* of f is defined as

$$h(r_k) = n_k \quad \text{for } k = 0,1,2,\ldots,L-1 \tag{3-6}$$

where n_k is the number of pixels in f with intensity r_k, and the subdivisions of the intensity scale are called *histogram bins*. Similarly, the *normalized histogram* of f is defined as

$$p(r_k) = \frac{h(r_k)}{MN} = \frac{n_k}{MN} \tag{3-7}$$

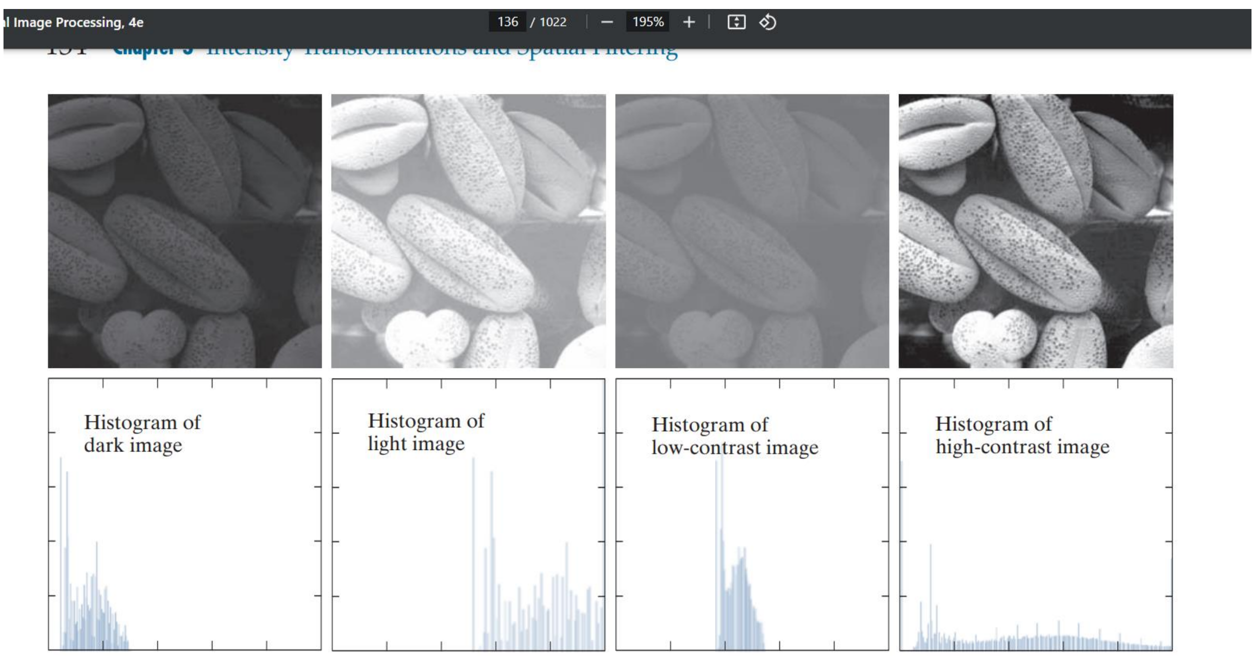
where, as usual, M and N are the number of image rows and columns, respectively. Mostly, we work with normalized histograms, which we refer to simply as *histograms* or *image histograms*. The sum of $p(r_k)$ for all values of k is always 1. The components of $p(r_k)$ are estimates of the probabilities of intensity levels occurring in an image. As you will learn in this section, histogram manipulation is a fundamental tool in image processing. Histograms are simple to compute and are also suitable for fast hardware implementations, thus making histogram-based techniques a popular tool for real-time image processing.

Information carried by an image histogram:

1. **Intensity Distribution**: It displays how many pixels in the image have specific intensity values, providing insights into the distribution of brightness levels throughout the image. For grayscale

images, the histogram typically shows the distribution of intensity levels from darkest (0 or black) to brightest (255 or white).

2. **Brightness and Contrast**: Histograms reveal the overall brightness and contrast of an image. Peaks and valleys in the histogram indicate areas of higher or lower frequency of pixel intensities, highlighting regions of different brightness levels.
3. **Dynamic Range:** The width of the histogram spread indicates the dynamic range of the image. A wide spread implies a larger range of tonal values, potentially indicating a higher level of detail in the image.
4. **Underexposed or Overexposed Areas**: Histograms help identify underexposed (too dark) or overexposed (too bright) areas in an image. If intensity values are primarily clustered at the extremes (left for underexposed, right for overexposed), it indicates potential loss of detail in those areas.
5. **Color Channel Distribution (for Color Images):** In color images, separate histograms for each color channel (Red, Green, Blue) provide insights into the distribution of color intensities, allowing adjustments for color balance and tone mapping.
6. **Image Quality and Processing**: Histogram analysis aids in assessing image quality, understanding the effects of image processing operations, and guiding adjustments such as brightness/contrast modifications, equalization, or normalization.



1. (a) image size: $200 \times 200 = 40000$

4 gray level: $g_1 \to 20\%$ $g_2 \to 30\%$, $g_3 \to 15\%$, $g_4 \to 35\%$

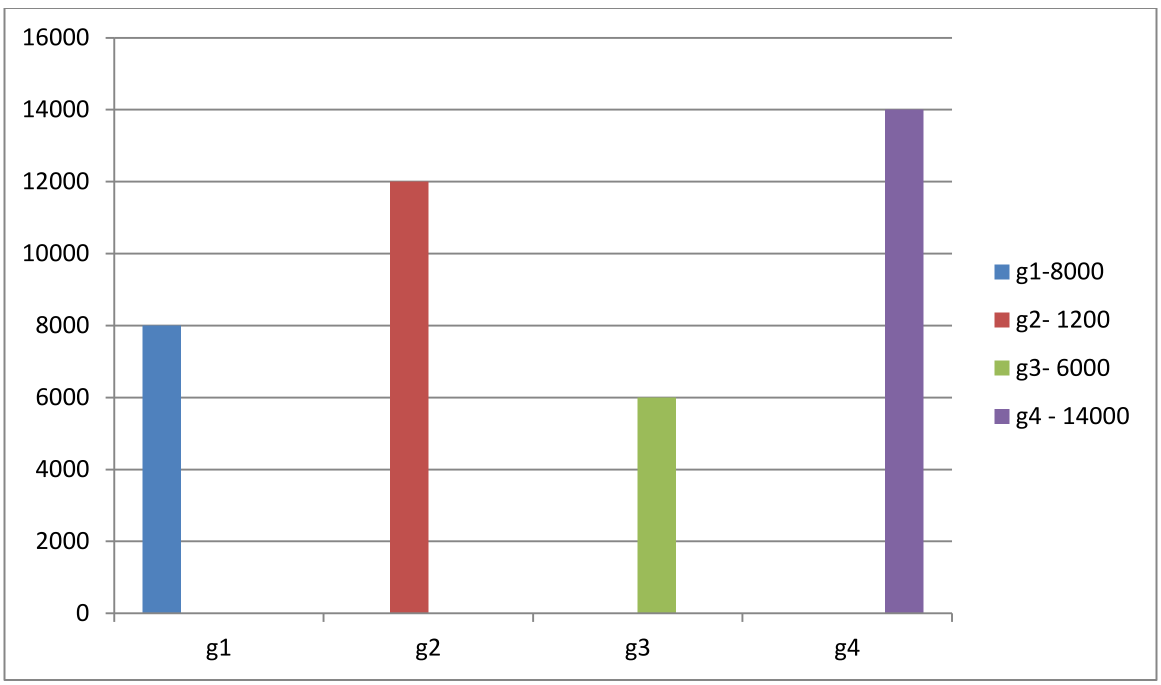
$\therefore r_k = g_1$, $P(r_k) = 0.2$ $\quad n_k = P(r_k) \times MN = 0.2 \times 40000$
$= 8000$

$\therefore r_k = g_2$, $P(r_k) = .3$ $\quad n_k = P(r_k) \times MN = 0.3 \times 40000$
$= 12000$

$\therefore r_k = g_3$, $P(r_k) = .15$ $\quad n_k = P(r_k) \times MN = .15 \times 4000$
$= 6000$

$\therefore r_k = g_4$, $P(r_k) = .35$ $\quad n_k = P(r_k) \times MN = .35 \times 4000$
$= 14000$

$\therefore$ histogram:-

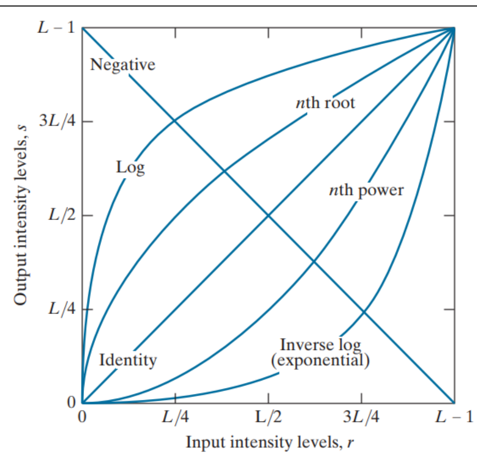


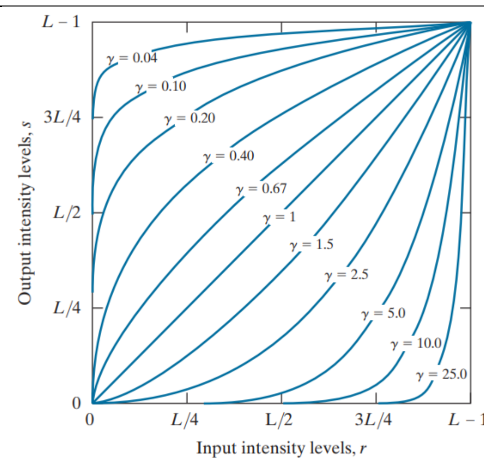
(b) Differentiate between log transform and power lay transform with necessary figure. Quantitatively explain the situation when log transform is effective.

differences between Logarithmic Transformation and Power Law Transformation:

Aspect	Logarithmic Transformation	Power Law Transformation
Formula	$s = c\log(1+r)$ where c is a constant and it is assumed that r ≥ 0	$S = cr^{\gamma}$ where c and γ are positive constants
Purpose	Reduces the range of values, reduces skewness in data, useful for handling data spanning multiple orders of magnitude	Modifies the distribution of data, useful for data with heavy tails or following a power-law distribution
Effect on Data	Compresses the scale, brings large values closer, transforms skewed data towards a more linear relationship	Can stretch or compress the data, alters the shape of the distribution, particularly tail behavior
Interpretation	Makes data easier to interpret, especially for visualizations and analysis	Useful for fitting data to certain models, revealing underlying relationships, or handling extreme values
Example	$\log_{10}(1000)=3$, $\ln(e)=1$	$2^3=8$, $3^2=9$
Applicability	Suitable for handling exponential growth or decay, wide-ranging data	Useful for power-law relationships, heavy-tailed distributions, or data with varying behavior across scales

Image





A log transform is effective in certain situations, particularly when dealing with data that exhibits a wide range of values or a highly skewed distribution. Quantitatively, the effectiveness of a log transform can be understood based on the characteristics of the data and the objectives of the transformation:

1. **Handling Skewed Distributions:**
    - **Skewed Data:** Data that is heavily skewed toward larger values or has a long tail in its distribution.
    - **Effect:** A log transform compresses the scale, bringing larger values closer together. It reduces the magnitude of extreme values, thereby reducing the skewness in the data distribution.
2. **Equalizing Variances:**
    - **Heteroscedasticity:** When the variance of the data differs across the range of values.
    - **Effect:** A log transform can stabilize variance by reducing the impact of large values, making the variance more consistent across the dataset.
3. **Linearization of Relationships:**
    - **Non-linear Relationships:** Data exhibiting non-linear relationships between variables.
    - **Effect:** Applying a log transform to one or more variables in such relationships can help linearize the overall relationship, making it easier to interpret or analyze using linear models.
4. **Handling Multiplicative Relationships:**
    - **Multiplicative Effects:** When relationships between variables involve multiplicative effects rather than additive effects.
    - **Effect:** A log transform converts multiplicative relationships into additive relationships, making it easier to model and interpret the data using linear methods.

Quantitatively, the effectiveness of the log transform can be assessed by examining statistical measures or properties of the data before and after transformation. Some quantitative indicators include:

- **Skewness:** A decrease in skewness after the log transform indicates a reduction in the asymmetry of the distribution.
- **Variance Stabilization:** Log transformation can stabilize variance, making it more homogeneous across the dataset.
- **Linear Relationships:** Assessing the linearity of relationships between variables, such as through correlation coefficients or regression analyses, before and after log transformation.

Additionally, for specific applications such as signal processing or image analysis, metrics related to signal-to-noise ratio (SNR) improvements or dynamic range expansion can quantitatively demonstrate the effectiveness of log transformations

(c) Explain the effects of mask size in spatial domain image filtering. Derive Laplacian Mask.

Answer:

In spatial domain image filtering, mask size (also known as kernel size or filter size) refers to the dimensions of the filter applied to process the image. This filter, often represented as a matrix or a window, moves over the entire image and performs operations such as blurring, sharpening, edge detection, or noise reduction.

The effects of mask size in spatial domain image filtering can be understood in terms of the following aspects:

1. Spatial Resolution:

Smaller Mask Size: A smaller mask size typically leads to less smoothing or blurring effect on the image. It can preserve finer details and edges but might be less effective in reducing noise or smaller imperfections.

Larger Mask Size: A larger mask size results in more smoothing or blurring. It tends to reduce finer details and edges, but it can effectively remove noise or smaller variations in the image.

2. Computation and Processing Time:

Smaller Mask Size: Smaller masks require less computation and processing time since they involve fewer calculations per pixel.

Larger Mask Size: Larger masks demand more computational resources as they involve a greater number of calculations for each pixel in the image.

3. Filtering Effects:

Smaller Mask Size: When the mask is small, the influence of each pixel's neighboring pixels on the output is limited. This can lead to less noticeable changes in the image and may not effectively capture broader patterns.

Larger Mask Size: Larger masks have a wider reach and consider more neighboring pixels. They can capture broader patterns and structures in the image, resulting in more pronounced changes in the appearance of the image.

4. Edge Preservation and Artifacts:

Smaller Mask Size: Smaller masks are better at preserving edges and finer details in the image. However, they might not effectively remove larger noise artifacts or imperfections.

Larger Mask Size: Larger masks can cause more blurring around edges and boundaries, potentially leading to loss of detail or oversmoothing. However, they can effectively suppress larger noise artifacts or imperfections.

5. Trade-offs:

There's often a trade-off between noise reduction and detail preservation. Smaller masks tend to preserve details but might not reduce noise effectively, while larger masks can reduce noise but might blur important details.

Choosing the appropriate mask size depends on the specific characteristics of the image, the intended filtering operation, and the balance needed between noise reduction and detail preservation.

The Laplacian

The Laplacian is defined as follows:

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

where the partial 1st order derivative in the x direction is defined as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

and in the y direction as follows:

$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

The Laplacian (cont…)

So, the Laplacian can be given as follows:

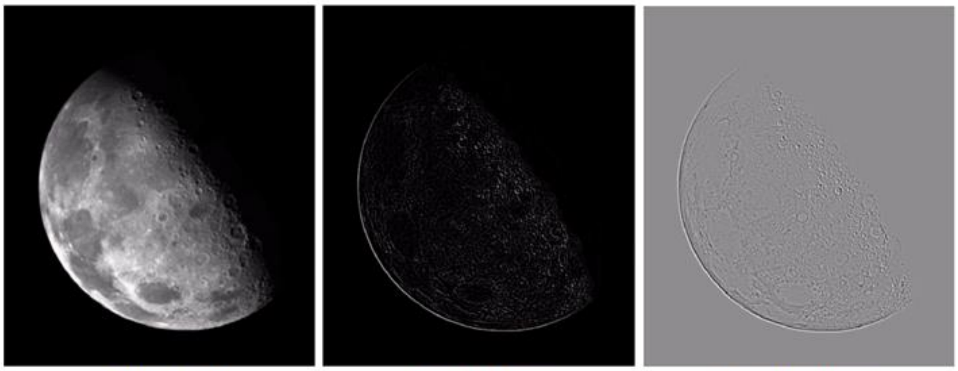
$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

We can easily build a filter based on this

0	1	0
1	-4	1
0	1	0

The Laplacian (cont…)

Applying the Laplacian to an image we get a new image that highlights edges and other discontinuities



Original Image | Laplacian Filtered Image | Laplacian Filtered Image Scaled for Display