Energy–Frequency–Resonance: Phase-Locked Carriers and Holonomy Geometry

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Independent Research

We present a resonance-first framework in which invariant carriers (photons, atoms, phonons, gravitational modes) phase-lock to the geometry of their trajectories. When the phase-closure (holonomy) condition holds, the accumulated phase is maximal and the observed effect appears as an effective "force." Information flow converts to work at the thermodynamic limit set by $k_B T \ln 2$ per bit. In noisy media with time-dependent feedback, the local oscillator amplitude is governed by a closed-form kernel built from the exponential integral E_1, enabling direct estimation of coherence time and feedback strength from data without assuming any predefined line-shape. We provide the resonance rules, a holonomy unification formula, stable numerical evaluation, and interferometric predictions.

I. PRINCIPLE: PHASE-LOCK & RESONANCE

Define the phase error

$$\delta\phi(x,t,\omega;n) = \pi x + \omega t - 2\pi n, \qquad n \in \mathbb{Z}. \tag{1}$$

Then:

- **Constructive interference**: visibility peaks at $\delta\phi = 0$.
- **Path-closure**: $\delta\phi = 0$ means phase closes along the path $\Rightarrow$ stable standing modes.
- **Resonance**: near $\delta\phi = 0$, $|\Psi|$ has a local maximum; width is set by coherence and noise.

Allowed loci:

$$\text{fixed } t: \quad x_n(t) = \tfrac{2n-\omega t/\pi}{2}, \quad \Delta x = 1, \tag{2}$$
$$\text{fixed } x: \quad t_n(x) = \tfrac{2\pi n - \pi x}{\omega}, \quad \Delta t = \tfrac{2\pi}{\omega}. \tag{3}$$

From the slope of resonance ridges at fixed t:

$$\frac{d\omega}{dx} = -\frac{\pi}{t} \quad \Rightarrow \quad t = -\pi\Big(\frac{d\omega}{dx}\Big)^{-1}. \tag{4}$$

Tolerance (resonance bandwidth) is

$$|\delta\phi| \lesssim \Delta\phi_{\text{tol}}, \qquad \Delta\phi_{\text{tol}} \sim \max\Big\{\frac{1}{\text{SNR}}, \frac{t}{\tau}\Big\}, \tag{5}$$

with coherence time τ. A convenient quality estimate:

$$Q \approx \frac{\omega\,\tau}{1+\alpha_{\text{eff}}}, \qquad \alpha_{\text{eff}} \sim \overline{\epsilon(t)\,D_u[F(u(x))]}. \tag{6}$$

Locking (PLL). To keep $\delta\phi \approx 0$,

$$\dot{x} = -\frac{\omega}{\pi} + k\,\delta\phi, \quad \text{(adjust position)}, \tag{7}$$
$$\dot{\omega} = -\frac{\pi}{t} + k\,\delta\phi, \quad \text{(adjust frequency)}. \tag{8}$$

A simple PID on $\delta\phi$ (from the interferometric signal) suffices to maintain lock.

II. NOISE-DRIVEN OSCILLATOR WITH TIME-DEPENDENT FEEDBACK

We model the local carrier as

$$\Psi(x) = \int_0^\infty \frac{e^{-\epsilon(t)}\cos\big(\pi x + \omega t\big)}{1 + \epsilon(t)\,D_u[F(u(x))]}\,dt, \tag{9}$$

where $\epsilon(t) \geq 0$ is a causal damping/decoherence kernel, and $D_u[F(u(x))]$ is a local functional sensitivity (feedback).

For the exponential kernel $\epsilon(t) = t/\tau$ and locally constant $D := D_u[F(u(x))]$, the feedback becomes time-dependent,

$$\alpha(t) = \epsilon(t)\,D = \frac{t}{\tau}\,D. \tag{10}$$

Writing $a = \frac{1}{\tau} - i\omega$ and $\beta = \frac{D}{\tau}$, we obtain the closed form

$$\int_0^\infty \frac{e^{-at}}{1+\beta t}\,dt = \frac{1}{\beta}\,e^{a/\beta}\,E_1\Big(\frac{a}{\beta}\Big), \quad (\operatorname{Re} a > 0,\ \beta > 0), \tag{11}$$

with E_1 the exponential integral. Thus

$$\boxed{\Psi(x) = \operatorname{Re}\Big\{e^{i\pi x}\,\frac{\tau}{D}\,\exp\Big(\frac{1-i\omega\tau}{D}\Big)\,E_1\Big(\frac{1-i\omega\tau}{D}\Big)\Big\}}, \tag{12}$$

where $z = (1-i\omega\tau)/D$ governs amplitude and phase without imposing any a priori line shape.

Consistent limits (no shape assumptions)

Let $S(z) = e^z E_1(z)$.

- $D \to 0$: $E_1(z) \sim e^{-z}/z \Rightarrow \Psi(x) \to \operatorname{Re}\{e^{i\pi x}\,\tau/(1 - i\omega\tau)\}$. This is the baseline transfer set solely by τ.
- **Small** $|D|$: use $S(z) \sim z^{-1} - z^{-2} + 2!z^{-3} - \cdots$ (asymptotic in $1/z$); first-order feedback introduces controlled asymmetry and dispersive phase.
- **Large** $|D|$: for small $|z|$, $E_1(z) = -\gamma - \ln z + z - \frac{z^2}{4} + \cdots$, yielding logarithmic dispersion and long tails driven by D (Euler constant γ).

What the data give you. Fitting amplitude/phase of $\Psi(\omega)$ with Eq. (12) retrieves *both* the coherence time τ and the feedback strength D directly from measurements.

III. HOLONOMY UNIFICATION OF INTERACTIONS

The observed effect commonly called a "force" is the phase accumulated by an invariant carrier along a distorted/closed trajectory:

$$\Delta\varphi = \frac{1}{\hbar}\oint C_\mu \, dx^\mu. \tag{13}$$

The connection C_μ specializes by interaction:

$$\text{EM (U(1))}: \; C_\mu = q\, A_\mu, \tag{14}$$
$$\text{Weak (SU(2))}: \; C_\mu = g\, A_\mu^a T^a, \tag{15}$$
$$\text{Strong (SU(3))}: \; C_\mu = g_s\, A_\mu^a T^a, \tag{16}$$
$$\text{Grav. (eikonal/spin)}: \; C_\mu = p_\mu \;\text{ or }\; C_\mu = \omega_\mu^{ab} J_{ab}. \tag{17}$$

Equation (13) subsumes Aharonov–Bohm, Sagnac-type phases, and matter-wave/gravitational phases; the *observed* "force" is the observer-dependent projection of a holonomic phase collected by an invariant carrier.

IV. INFORMATION → WORK AT RESONANCE

At phase-lock ($\delta\phi = 0$) and path-closure, the accumulated phase and the visibility are maximal; information transfer is optimal and the extractable work per bit obeys

$$W_{\max} = k_B T \ln 2 \quad \text{(per bit)}. \tag{18}$$

This links the resonance condition to the thermodynamic bound in a direct, operational manner.

V. EXPERIMENTAL SIGNATURES

Resonance maps. Measure (x, ω) maps and read out: (i) spacing $\Delta\omega = 2\pi/t \Rightarrow t = 2\pi/\Delta\omega$; (ii) slope $t = -\pi/(d\omega/dx)$; (iii) bandwidth $\Delta\phi_{\rm tol}$ from SNR and τ.

Feedback extraction. Fit Eq. (12) (amplitude+phase) to obtain τ and D. Asymmetry and dispersive phase are direct fingerprints of nonzero feedback.

Holonomy tests. Implement closed loops with a known A_μ (or rotation/gravitational eikonal) and measure $\Delta\varphi$ via interference. Variations of the loop/path geometry modulate $\oint C_\mu dx^\mu$ and hence the phase.

VI. STABLE NUMERICS (NO OVERFLOW)

Direct evaluation of $\exp(z)E_1(z)$ may overflow when $\operatorname{Re} z$ is large and positive, even if the physical result is small. Evaluate the *scaled product*

$$S(z) = e^z E_1(z) \tag{19}$$

hybridly:

- For $|z| \gtrsim z_{\rm th}$ use the asymptotic series $S(z) \approx z^{-1} - z^{-2} + 2!z^{-3} - 3!z^{-4} + \cdots$.
- For $|z| \lesssim z_{\rm th}$ use the local expansion $E_1(z) = -\gamma - \ln z + z - \frac{z^2}{4} + \frac{z^3}{18} - \cdots$ and set $S(z) = e^z E_1(z)$ (consistent branch of $\ln z$).

This avoids overflow while preserving amplitude and phase continuity.

VII. CONCLUSION

All phenomena considered here reduce to the triplet **energy–frequency–resonance**. Invariant carriers phase-lock to the geometry of their paths; when the holonomy condition closes the phase, the accumulated phase is maximal and the phenomenon appears as an effective interaction. In noisy media with feedback, the observable amplitude and phase are fully captured by the E_1 kernel with time-dependent feedback, enabling direct inference of coherence and feedback from data, and guiding optimal locking/control at the thermodynamic information–work limit.

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AUTHOR CONTRIBUTIONS

[Jan Mikulik]: concept, derivations, figures, experiments; GPT-5 Thinking: formalization, unification, numerical stabilization, manuscript.

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