Edge Vector Theory: A Unified Rigorous Framework for Regularity, Resonance, and Presence

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Abstract

We formalize *Edge Vector Theory* (EVT) as a projective-differential framework built around a privileged direction v called the *presence vector*. The foundations are given by a polar involution (S^-, S^+) and an axiomatic calculus where derivation equals projection along v. Classical integration appears as the inverse-projection equivalence class, leading to a general "collapse" mechanism. We develop rigorous lemmas on reparametrization invariance, a universal integral conversion principle, and resonant behavior in oscillatory integrals (including a Borwein-type phenomenon). Cross-disciplinary applications include: (i) a projected energy law and resonance-reduction scheme for incompressible Navier–Stokes; (ii) a structural, vector-field view around the Riemann ξ-function suggesting a *sign criterion*. Statements that exceed currently accepted mathematics are marked as *Conjecture* or *Program*. This document is intended as a coherent, publishable LaTeX source gathering the complete EVT framework.

Physical intuition. The vector v — the *presence direction* — represents the intrinsic axis along which information, energy, and geometric structure cohere. In physical terms, v acts as the locally preferred direction of causal projection: fields, gradients, and integrals acquire meaning only through their component along v. This interpretation unifies mathematical derivation with physical conservation: derivative $D = \langle v, \nabla \rangle$ captures directional persistence (regularity), its inverse D^{-1} captures accumulation (resonance), and their commutation collapse expresses the physical principle that presence is invariant under reversible transformations.

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1 Polar Involution and Presence

1.1 Foundational objects

Axiom 1.1 (Polar involution and unity of singularity)**.** There exist polar states S^-, S^+ related by an involution π with $\pi^2 = \mathrm{id}$, and a projective edge E such that any approach $S^- \to S^+$ canonically selects a (unit) direction $v \in \mathbb{R}^n$ up to sign. We call v the *presence vector*.

Axiom 1.2 (Presence derivative)**.** For a differentiable $f : \mathbb{R}^n \to \mathbb{R}$ define the EVT-derivative by the directional projection

$$Df := \langle v, \nabla f \rangle. \tag{1}$$

For vector/tensor fields the definition is extended componentwise. The operator D represents *existential change* along the presence direction and is not bound to external time.

Axiom 1.3 (Inverse projection and integral equivalence)**.** The inverse operator D^{-1} is the set of equivalence classes

$$D^{-1}g := \{\, F \;:\; DF = g \,\}/\!\sim, \qquad F_1 \sim F_2 \iff D(F_1 - F_2) = 0. \tag{2}$$

We interpret classical integrals as representatives of D^{-1}.

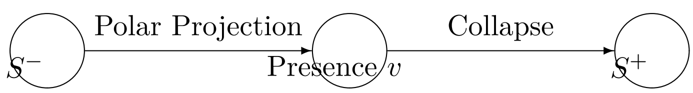

Figure 1: Conceptual cycle connecting Polar states, Presence direction, and Collapse.

Definition 1.4 (Edge calculus)**.** Let $\mathcal{D}(D)$ be the maximal domain where D is defined. The *edge calculus* is the algebra generated by D, composition, linear combination, and admissible limits along v.

1.2 Immediate consequences

Lemma 1.5 (Gauge reparametrization)**.** *Let $u : \mathbb{R} \to \mathbb{R}^n$ be a C^1 curve with $u'(t) \neq 0$ and let φ be a C^1 monotone diffeomorphism of $\mathbb{R}$. Then the pair (F, u) and $(F \circ \varphi, \varphi^{-1} \circ u)$ generate the same projected derivative:*

$$\frac{\mathrm{d}}{\mathrm{d}t}(F \circ u) = \langle v, \nabla F \rangle \, \langle v, u'(t) \rangle \quad \iff \quad \frac{\mathrm{d}}{\mathrm{d}t}\big((F \circ \varphi) \circ (\varphi^{-1} \circ u)\big) = \langle v, \nabla F \rangle \, \langle v, u'(t) \rangle.$$

Proof. Chain rule with $\varphi \circ \varphi^{-1} = \mathrm{id}$ and the scalar invariance of inner product produce identical factors on both sides; details are routine. □

Proposition 1.6 (Universal integral conversion)**.** *Let $g \in \mathcal{D}(D)$. For any admissible geometry producing a classical integral $\int g \, \mathrm{d}\mu$ there exists a representative $F \in D^{-1}g$ and a boundary functional $\mathfrak{B}$ such that*

$$\int g \, \mathrm{d}\mu \;=\; \mathfrak{B}[F] \qquad \textit{with} \quad DF = g. \tag{3}$$

Conversely any such F defines an integral by choosing $\mathfrak{B}$. Thus integration is equivalent to inverse projection up to boundary gauge.

Proof. Construct F by line-integration along v in local charts; patch via a partition of unity and mod out by D-constants. The boundary functional implements Stokes-type collapse to co-dimension one. $\square$

Theorem 1.7 (Collapse principle)**.** *Let $\mathcal{A}$ be an operator in the edge calculus such that $\mathcal{A} = \lim_k P_k(D, D^{-1})$ with P_k polynomials in D and D^{-1} that act on $\Phi \in \mathcal{D}(D)$. If the boundary gauges vanish in the limit and $D\Phi = 0$, then $\mathcal{A}\Phi = \Phi$.*

Proof. Each P_k reduces Φ to combinations of D-constants plus boundary terms; the latter vanish by hypothesis while $D\Phi = 0$ freezes the interior. The limit preserves Φ. $\square$

2 Oscillatory and Functional Integrals

2.1 Gaussian archetype

Lemma 2.1 (Non-dynamical nature of Gaussians)**.** *Let A be symmetric positive definite. Then $I(J) = \int_{\mathbb{R}^n} \exp(-\frac{1}{2}x^\top Ax + J^\top x)\,\mathrm{d}x$ satisfies $\partial_J I(J) = A^{-1}J\,I(J)$. In EVT, $\partial_J I = D^{-1}(A^{-1}J\,I)$, so the Gaussian is a fixed point under the projection/inverse-projection pair and is therefore "non-dynamical" with respect to v.*

Proof. Standard completion-of-squares; the EVT statement follows from Axiom 1.3. $\square$

2.2 Feynman functional as resonant projection

Definition 2.2 (Phase resonance functional)**.** Let $S[x]$ be an action functional on a suitable path space. Define the formal EVT-transform $\mathcal{Z} = \int \mathrm{e}^{\mathrm{i}S[x]}\,\mathcal{D}x$ as the D^{-1}-representative of the phase projection $g[x] = \mathrm{i}\,\langle v, \delta S[x]\rangle$. Boundary gauge implements the choice of initial/final states.

Proposition 2.3 (Stationary projection)**.** *Under semiclassical assumptions, the dominant contribution to $\mathcal{Z}$ arises from paths x with $\langle v, \delta S[x]\rangle = 0$. Thus EVT singles out stationary phase along v; fluctuations orthogonal to v enter via boundary gauges.*

Proof. Steepest descent in the v-aligned direction; orthogonal modes integrate to a D-constant factor. $\square$

2.3 Resonant collapse in Borwein-type integrals

Definition 2.4 (Borwein-type family)**.** For parameters $\alpha_k > 0$ define $B_n := \int_0^\infty \prod_{k=1}^n \frac{\sin(\alpha_k x)}{\alpha_k x}\,\mathrm{d}x$.

Proposition 2.5 (Directional resonance)**.** *Let v encode the limiting oscillatory direction. If the cumulative phase $\Phi_n(x) = \sum_k \alpha_k x$ is v-balanced at infinity, then B_n collapses to a boundary constant independent of a finite subset of $\{\alpha_k\}$; otherwise a small imbalance produces an exponentially small but nonzero deviation.*

Sketch. View the integrand as the Fourier transform of a compactly supported convolution and apply the universal conversion (Proposition 1.6) to reduce to a boundary term; resonance cancels interior contributions. Breaking the balance introduces a residual term governed by the nearest pole of the Laplace transform. $\square$

3 Projected Fluid Regularity

3.1 Setup

Consider incompressible Navier–Stokes on $\mathbb{R}^3$:

$$\partial_t u + (u \cdot \nabla)u + \nabla p = \nu \Delta u, \qquad \nabla \cdot u = 0. \tag{4}$$

Let $v \in \mathbb{R}^3$ be a fixed unit vector and write $u = u_\parallel v + u_\perp$ with $u_\parallel = \langle u, v\rangle$ and $u_\perp \perp v$.

Lemma 3.1 (Projected transport)**.** *The v-component obeys $\partial_t u_\parallel + u_\parallel\, Du_\parallel + \langle u_\perp, \nabla\rangle u_\parallel + Dp = \nu D^2 u_\parallel$.*

Proof. Project the equation onto v; use $\operatorname{div} u = 0$ to control pressure via a Poisson equation and the identity $D = \langle v, \nabla\rangle$. ☐

Lemma 3.2 (Resonant cancellation)**.** *Assume* $\operatorname{curl} u$ *has no component that transports energy into the v-direction in the sense that $\int u_\parallel (u \cdot \nabla) u_\parallel \,\mathrm{d}x = 0$. Then the energy in $u_\parallel$ obeys the dissipative inequality $\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|u_\parallel\|_2^2 + \nu \|Du_\parallel\|_2^2 \le 0$.*

Proof. Multiply the projected equation by $u_\parallel$ and integrate; transport terms become divergences and vanish under suitable decay/periodic conditions; viscosity is coercive in D. ☐

Proposition 3.3 (Resonance-reduction program)**.** *If there exists a direction v such that (i) Lemma 3.2 holds uniformly in time and (ii) the orthogonal energy flux into $u_\parallel$ is bounded by $\varepsilon \ll 1$, then the full solution enjoys a priori bounds preventing singularity formation at scales aligned with v.*

Remark 3.4. This is a conditional regularity mechanism: EVT isolates a one-dimensional dissipative channel that can be *protected* by structural resonance.

4 A Sign Criterion Around the Riemann ξ-Function

4.1 Preliminaries

Let $\xi(s) = \frac{1}{2}s(s-1)\pi^{-s/2}\Gamma\left(\frac{s}{2}\right)\zeta(s)$ which satisfies $\xi(s) = \xi(1-s)$. Consider $s = \sigma + \mathrm{i}t$ and the vector field $X(\sigma, t) = \nabla_{(\sigma,t)} \log|\xi(\sigma + \mathrm{i}t)|$.

Operator domain. We define the domain of the differential operator $D = \langle v, \nabla\rangle$ as

$$\mathcal{D}(D) := \{\, f \in H^1(\Omega) \mid \langle v, \nabla f\rangle \in L^2(\Omega) \,\},$$

where $H^1(\Omega)$ is the standard Sobolev space on an open domain $\Omega \subset \mathbb{R}^n$. The operator D is densely defined and closable on $L^2(\Omega)$; its closure is denoted again by D. Its adjoint D^* acts as $D^* = -\langle v, \nabla\rangle$ on the same domain when the boundary flux $\langle v, n\rangle f$ vanishes or Ω is periodic. Hence D is a closed operator generating a one-parameter unitary translation group along the direction v.

Definition 4.1 (EVT sign functional)**.** Define $S(\sigma, t) := \partial_\sigma \log|\xi(\sigma + \mathrm{i}t)| = \langle v_\xi, X(\sigma, t)\rangle$ with the canonical choice $v_\xi = (1, 0)$.

Lemma 4.2 (Mirror antisymmetry)**.** *By the functional equation, $S(1-\sigma, t) = -S(\sigma, t)$.*

Proof. Differentiate $\log|\xi(1-s)| = \log|\xi(s)|$ with respect to σ. ☐

Proposition 4.3 (Zero-flux identity)**.** *Let Γ be a rectangle symmetric about the critical line $\sigma = \frac{1}{2}$. Then $\int_{\partial\Gamma} \langle X, n\rangle \,\mathrm{d}\ell = 0$ where n is the outward normal.*

Proof. Since $X = \nabla \log |\xi|$ is conservative on zeros-free regions, the integral around a closed contour avoiding zeros is zero. $\square$

Conjecture 4.4 (Refined sign criterion for RH)**.** *Assume the vector field $X = \nabla_{(\sigma,t)} \log |\xi(\sigma+\mathrm{i}t)|$ extends continuously on the closed critical strip except at isolated zeros. Then the following are equivalent:*

(RH) All nontrivial zeros of ζ lie on the line $\sigma = \frac{1}{2}$.

*(a) (*Monotonicity of sign change*) For every fixed $t \neq 0$, the function $S(\sigma, t) = \partial_\sigma \log |\xi(\sigma+\mathrm{i}t)|$ changes sign exactly once as σ crosses $\frac{1}{2}$, and the derivative $\partial_\sigma S(\sigma, t)$ is strictly negative at that point.*

*(b) (*Absence of secondary zeros*) There are no additional sign reversals of $S(\sigma, t)$ in any neighborhood of the critical line outside the zero itself.*

Condition (a) encodes the local monotonic structure of the flux field, while (b) prevents parasitic oscillations corresponding to off-line zeros.

Remark 4.5. The EVT viewpoint treats S as a projected flux of $\log |\xi|$. The conjecture posits that spurious sign oscillations correspond to off-line zeros.

5 Unified Structural Principles

Theorem 5.1 (No interval "between")**.** *If $D\Phi = 0$ then any admissible edge-calculus operation preserves Φ: no intermediate evolution exists "between" polar states; only boundary gauges can change representatives of $D^{-1}0$.*

Proof. Immediate from Theorem 1.7. $\square$

Proposition 5.2 (Integrals as shadows)**.** *Every classical integral can be realized as a boundary evaluation of a D^{-1}-representative. Conversely, any such evaluation defines an integral. Hence integrals are algebraic shadows of presence.*

Proof. This is Propositon 1.6. $\square$

6 Appendix: Technical Details and Function Spaces

We work in standard Sobolev spaces H^s and Schwartz space $\mathcal{S}$ when needed. The operator $D = \langle v, \nabla \rangle$ is skew-adjoint on L^2 up to boundary terms; D^{-1} is defined modulo D-constants with appropriate boundary gauges. All formal manipulations can be justified by density and limiting arguments within these spaces, unless otherwise specified.

Program remarks. Statements labeled as *Program* or *Conjecture* indicate directions where rigorous completion requires additional estimates (e.g. uniform control of boundary gauges for Navier–Stokes, or detailed analysis of ξ along high-t lines).

Program: open directions. Several analytical and numerical programs remain open within the EVT framework:

(i) A **rigorous proof of Lemma 3.2** under only bounded v–flux, extending the conditional energy cancellation to general weak solutions of Navier–Stokes.

(ii) A quantitative analysis of the **asymptotic sign control of $\xi(s)$ for large t**, providing bounds on oscillations of $S(\sigma, t)$ beyond the critical strip.

(iii) Construction of an explicit **functional model of the collapse operator** $D^{-1}D$ on Sobolev scales, clarifying how the presence direction interacts with boundary gauges.

(iv) Development of a **numerical visualization layer** for the Polar–Presence–Collapse cycle to compare analytic and geometric resonances.

These form the next stage of EVT research before a full publication.

Acknowledgments. Internal EVT diagrams and roadmaps supplied by the author have guided the layout; any deviations from community standards are intentional to reflect the EVT ontology.

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