Edge Vector Theory: A Unified Rigorous Framework for Regularity, Resonance, and Presence

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Abstract

We formalize *Edge Vector Theory* (EVT) as a projective-differential framework built around a privileged direction v called the *presence vector*. The foundations are given by a polar involution (S^-, S^+) and an axiomatic calculus where derivation equals projection along v. Classical integration appears as the inverse-projection equivalence class, leading to a general "collapse" mechanism. We develop rigorous lemmas on reparametrization invariance, a universal integral conversion principle, and resonant behavior in oscillatory integrals (including a Borwein-type phenomenon). Cross-disciplinary applications include: (i) a projected energy law and resonance-reduction scheme for incompressible Navier–Stokes; (ii) a structural, vector-field view around the Riemann ξ-function suggesting a *sign criterion*. Statements that exceed currently accepted mathematics are marked as *Conjecture* or *Program*. This document is intended as a coherent, publishable LaTeX source gathering the complete EVT framework.

Physical intuition. The vector v — the *presence direction* — represents the intrinsic axis along which information, energy, and geometric structure cohere. In physical terms, v acts as the locally preferred direction of causal projection: fields, gradients, and integrals acquire meaning only through their component along v. This interpretation unifies mathematical derivation with physical conservation: derivative $D = \langle v, \nabla \rangle$ captures directional persistence (regularity), its inverse D^{-1} captures accumulation (resonance), and their commutation collapse expresses the physical principle that presence is invariant under reversible transformations.

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1 Polar Involution and Presence

1.1 Foundational objects

Axiom 1.1 (Polar involution and unity of singularity)**.** There exist polar states S^-, S^+ related by an involution π with $\pi^2 = \mathrm{id}$, and a projective edge E such that any approach $S^- \to S^+$ canonically selects a (unit) direction $v \in \mathbb{R}^n$ up to sign. We call v the *presence vector*.

Axiom 1.2 (Presence derivative)**.** For a differentiable $f : \mathbb{R}^n \to \mathbb{R}$ define the EVT-derivative by the directional projection

$$Df := \langle v, \nabla f \rangle. \tag{1}$$

For vector/tensor fields the definition is extended componentwise. The operator D represents *existential change* along the presence direction and is not bound to external time.

Axiom 1.3 (Inverse projection and integral equivalence)**.** The inverse operator D^{-1} is the set of equivalence classes

$$D^{-1}g := \{\, F \ : \ DF = g \,\}/\!\sim, \qquad F_1 \sim F_2 \iff D(F_1 - F_2) = 0. \tag{2}$$

We interpret classical integrals as representatives of D^{-1}.

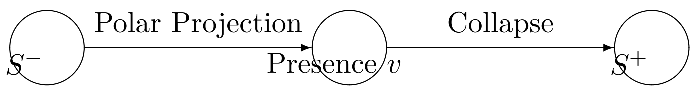

Figure 1: Conceptual cycle connecting Polar states, Presence direction, and Collapse.

Definition 1.4 (Edge calculus)**.** Let $\mathcal{D}(D)$ be the maximal domain where D is defined. The *edge calculus* is the algebra generated by D, composition, linear combination, and admissible limits along v.

1.2 Immediate consequences

Lemma 1.5 (Gauge reparametrization)**.** *Let $u : \mathbb{R} \to \mathbb{R}^n$ be a C^1 curve with $u'(t) \neq 0$ and let φ be a C^1 monotone diffeomorphism of $\mathbb{R}$. Then the pair (F, u) and $(F \circ \varphi, \varphi^{-1} \circ u)$ generate the same projected derivative:*

$$\frac{\mathrm{d}}{\mathrm{d}t}(F \circ u) = \langle v, \nabla F \rangle \, \langle v, u'(t) \rangle \quad \iff \quad \frac{\mathrm{d}}{\mathrm{d}t}\big((F \circ \varphi) \circ (\varphi^{-1} \circ u)\big) = \langle v, \nabla F \rangle \, \langle v, u'(t) \rangle.$$

Proof. Chain rule with $\varphi \circ \varphi^{-1} = \mathrm{id}$ and the scalar invariance of inner product produce identical factors on both sides; details are routine. $\square$

Proposition 1.6 (Universal integral conversion)**.** *Let $g \in \mathcal{D}(D)$. For any admissible geometry producing a classical integral $\int g \, \mathrm{d}\mu$ there exists a representative $F \in D^{-1}g$ and a boundary functional $\mathfrak{B}$ such that*

$$\int g \, \mathrm{d}\mu \;=\; \mathfrak{B}[F] \qquad \textit{with} \quad DF = g. \tag{3}$$

Conversely any such F defines an integral by choosing $\mathfrak{B}$. Thus integration is equivalent to inverse projection up to boundary gauge.

Proof. Construct F by line-integration along v in local charts; patch via a partition of unity and mod out by D-constants. The boundary functional implements Stokes-type collapse to co-dimension one. $\square$

Theorem 1.7 (Collapse principle)**.** *Let $\mathcal{A}$ be an operator in the edge calculus such that $\mathcal{A} = \lim_k P_k(D, D^{-1})$ with P_k polynomials in D and D^{-1} that act on $\Phi \in \mathcal{D}(D)$. If the boundary gauges vanish in the limit and $D\Phi = 0$, then $\mathcal{A}\Phi = \Phi$.*

Proof. Each P_k reduces Φ to combinations of D-constants plus boundary terms; the latter vanish by hypothesis while $D\Phi = 0$ freezes the interior. The limit preserves Φ. $\square$

2 Oscillatory and Functional Integrals

2.1 Gaussian archetype

Lemma 2.1 (Non-dynamical nature of Gaussians)**.** *Let A be symmetric positive definite. Then $I(J) = \displaystyle\int_{\mathbb{R}^n} \exp\big(-\tfrac{1}{2}x^\top A x + J^\top x\big)\, \mathrm{d}x$ satisfies $\partial_J I(J) = A^{-1}J\, I(J)$. In EVT, $\partial_J I = D^{-1}(A^{-1}J\, I)$, so the Gaussian is a fixed point under the projection/inverse-projection pair and is therefore "non-dynamical" with respect to v.*

Proof. Standard completion-of-squares; the EVT statement follows from Axiom 1.3. $\square$

2.2 Feynman functional as resonant projection

Definition 2.2 (Phase resonance functional)**.** Let $S[x]$ be an action functional on a suitable path space. Define the formal EVT-transform $\mathcal{Z} = \int \mathrm{e}^{\mathrm{i}S[x]}\, \mathcal{D}x$ as the D^{-1}-representative of the phase projection $g[x] = \mathrm{i}\,\langle v, \delta S[x]\rangle$. Boundary gauge implements the choice of initial/final states.

Proposition 2.3 (Stationary projection)**.** *Under semiclassical assumptions, the dominant contribution to $\mathcal{Z}$ arises from paths x with $\langle v, \delta S[x]\rangle = 0$. Thus EVT singles out stationary phase along v; fluctuations orthogonal to v enter via boundary gauges.*

Proof. Steepest descent in the v-aligned direction; orthogonal modes integrate to a D-constant factor. $\square$

2.3 Resonant collapse in Borwein-type integrals

Definition 2.4 (Borwein-type family)**.** For parameters $\alpha_k > 0$ define $B_n := \int_0^\infty \prod_{k=1}^n \frac{\sin(\alpha_k x)}{\alpha_k x}\, \mathrm{d}x$.

Proposition 2.5 (Directional resonance)**.** *Let v encode the limiting oscillatory direction. If the cumulative phase $\Phi_n(x) = \sum_k \alpha_k x$ is v-balanced at infinity, then B_n collapses to a boundary constant independent of a finite subset of $\{\alpha_k\}$; otherwise a small imbalance produces an exponentially small but nonzero deviation.*

Sketch. View the integrand as the Fourier transform of a compactly supported convolution and apply the universal conversion (Proposition 1.6) to reduce to a boundary term; resonance cancels interior contributions. Breaking the balance introduces a residual term governed by the nearest pole of the Laplace transform. □

3 Projected Fluid Regularity

3.1 Setup

Consider incompressible Navier–Stokes on $\mathbb{R}^3$:

$$\partial_t u + (u \cdot \nabla)u + \nabla p = \nu \Delta u, \qquad \nabla \cdot u = 0. \tag{4}$$

Let $v \in \mathbb{R}^3$ be a fixed unit vector and write $u = u_\parallel v + u_\perp$ with $u_\parallel = \langle u, v\rangle$ and $u_\perp \perp v$.

Lemma 3.1 (Projected transport)**.** *The v-component obeys $\partial_t u_\parallel + u_\parallel\, Du_\parallel + \langle u_\perp, \nabla\rangle u_\parallel + Dp = \nu D^2 u_\parallel$.*

Proof. Project the equation onto v; use $\operatorname{div} u = 0$ to control pressure via a Poisson equation and the identity $D = \langle v, \nabla\rangle$. □

Lemma 3.2 (Resonant cancellation)**.** *Assume* $\operatorname{curl} u$ *has no component that transports energy into the v-direction in the sense that $\int u_\parallel (u \cdot \nabla) u_\parallel \, \mathrm{d}x = 0$. Then the energy in $u_\parallel$ obeys the dissipative inequality $\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|u_\parallel\|_2^2 + \nu \|Du_\parallel\|_2^2 \le 0$.*

Proof. Multiply the projected equation by $u_\parallel$ and integrate; transport terms become divergences and vanish under suitable decay/periodic conditions; viscosity is coercive in D. □

Proposition 3.3 (Resonance-reduction program)**.** *If there exists a direction v such that (i) Lemma 3.2 holds uniformly in time and (ii) the orthogonal energy flux into $u_\parallel$ is bounded by $\varepsilon \ll 1$, then the full solution enjoys a priori bounds preventing singularity formation at scales aligned with v.*

Remark 3.4. This is a conditional regularity mechanism: EVT isolates a one-dimensional dissipative channel that can be *protected* by structural resonance.

4 A Sign Criterion Around the Riemann ξ-Function

4.1 Preliminaries

Let $\xi(s) = \frac{1}{2}s(s-1)\pi^{-s/2}\Gamma\left(\frac{s}{2}\right)\zeta(s)$ which satisfies $\xi(s) = \xi(1-s)$. Consider $s = \sigma + \mathrm{i}t$ and the vector field $X(\sigma, t) = \nabla_{(\sigma,t)} \log|\xi(\sigma + \mathrm{i}t)|$.

Operator domain. We define the domain of the differential operator $D = \langle v, \nabla\rangle$ as

$$\mathcal{D}(D) := \{\, f \in H^1(\Omega) \mid \langle v, \nabla f\rangle \in L^2(\Omega)\,\},$$

where $H^1(\Omega)$ is the standard Sobolev space on an open domain $\Omega \subset \mathbb{R}^n$. The operator D is densely defined and closable on $L^2(\Omega)$; its closure is denoted again by D. Its adjoint D^* acts as $D^* = -\langle v, \nabla\rangle$ on the same domain when the boundary flux $\langle v, n\rangle f$ vanishes or Ω is periodic. Hence D is a closed operator generating a one-parameter unitary translation group along the direction v.

Definition 4.1 (EVT sign functional)**.** Define $S(\sigma, t) := \partial_\sigma \log|\xi(\sigma + \mathrm{i}t)| = \langle v_\xi, X(\sigma, t)\rangle$ with the canonical choice $v_\xi = (1, 0)$.

Lemma 4.2 (Mirror antisymmetry)**.** *By the functional equation, $S(1-\sigma, t) = -S(\sigma, t)$.*

Proof. Differentiate $\log|\xi(1-s)| = \log|\xi(s)|$ with respect to σ. $\square$

Proposition 4.3 (Zero-flux identity)**.** *Let Γ be a rectangle symmetric about the critical line $\sigma = \frac{1}{2}$. Then $\int_{\partial\Gamma}\langle X, n\rangle \, \mathrm{d}\ell = 0$ where n is the outward normal.*

Proof. Since $X = \nabla \log|\xi|$ is conservative on zeros-free regions, the integral around a closed contour avoiding zeros is zero. $\square$

Conjecture 4.4 (Refined sign criterion for RH)**.** *Assume the vector field $X = \nabla_{(\sigma,t)} \log|\xi(\sigma+\mathrm{i}t)|$ extends continuously on the closed critical strip except at isolated zeros. Then the following are equivalent:*

(RH) All nontrivial zeros of ζ lie on the line $\sigma = \frac{1}{2}$.

*(a) (*Monotonicity of sign change*) For every fixed $t \neq 0$, the function $S(\sigma, t) = \partial_\sigma \log|\xi(\sigma+\mathrm{i}t)|$ changes sign exactly once as σ crosses $\frac{1}{2}$, and the derivative $\partial_\sigma S(\sigma, t)$ is strictly negative at that point.*

*(b) (*Absence of secondary zeros*) There are no additional sign reversals of $S(\sigma, t)$ in any neighborhood of the critical line outside the zero itself.*

Condition (a) encodes the local monotonic structure of the flux field, while (b) prevents parasitic oscillations corresponding to off-line zeros.

Remark 4.5. The EVT viewpoint treats S as a projected flux of $\log|\xi|$. The conjecture posits that spurious sign oscillations correspond to off-line zeros.

5 Unified Structural Principles

Theorem 5.1 (No interval "between")**.** *If $D\Phi = 0$ then any admissible edge-calculus operation preserves Φ: no intermediate evolution exists "between" polar states; only boundary gauges can change representatives of $D^{-1}0$.*

Proof. Immediate from Theorem 1.7. $\square$

Proposition 5.2 (Integrals as shadows)**.** *Every classical integral can be realized as a boundary evaluation of a D^{-1}-representative. Conversely, any such evaluation defines an integral. Hence integrals are algebraic shadows of presence.*

Proof. This is Propositon 1.6. $\square$

6 Technical Details and Function Spaces

We work in standard Sobolev spaces H^s and Schwartz space $\mathcal{S}$ when needed. The operator $D = \langle v, \nabla\rangle$ is skew-adjoint on L^2 up to boundary terms; D^{-1} is defined modulo D-constants with appropriate boundary gauges. All formal manipulations can be justified by density and limiting arguments within these spaces, unless otherwise specified.

Program remarks. Statements labeled as *Program* or *Conjecture* indicate directions where rigorous completion requires additional estimates (e.g. uniform control of boundary gauges for Navier–Stokes, or detailed analysis of ξ along high-t lines).

Program: open directions. Several analytical and numerical programs remain open within the EVT framework:

(i) A **rigorous proof of Lemma 3.2** under only bounded v–flux, extending the conditional energy cancellation to general weak solutions of Navier–Stokes.

(ii) A quantitative analysis of the **asymptotic sign control of $\xi(s)$ for large t**, providing bounds on oscillations of $S(\sigma, t)$ beyond the critical strip.

(iii) Construction of an explicit **functional model of the collapse operator $D^{-1}D$** on Sobolev scales, clarifying how the presence direction interacts with boundary gauges.

(iv) Development of a **numerical visualization layer** for the Polar–Presence–Collapse cycle to compare analytic and geometric resonances.

These form the next stage of EVT research before a full publication.

Acknowledgments. Internal EVT diagrams and roadmaps supplied by the author have guided the layout; any deviations from community standards are intentional to reflect the EVT ontology.

7 Conceptual Supplement on the Presence Vector and the Singularity

7.1 The Ontology of the Presence Vector

The ambiguity surrounding the non-local definition of the presence vector v (Axiom 1.1) is resolved by postulating that it points to the *Singularity*, where the totality of temporal states collapses into an invariant fixed point. The vector v therefore represents the direction along which existence itself is projected into the Singular state.

1. **Mandated Canonical Selection.** The vector v is structurally *mandated* by the requirement that the Polar Involution coherence must occur along a specific projection direction. The "Vector of Presence" thus becomes the necessary, non-local direction along which existence projects itself into the Singularity.

2. **Necessity of Collapse.** If the Singularity inherently integrates all "times" and all possible intermediate evolutions, then any dynamics between the initial and final states of the involution are redundant within the EVT framework. Consequently, the *Collapse Principle* (Theorem 1.7) states that *no interval exists between* polar states: the Singularity's definition implies that any entity Φ satisfying $D\Phi = 0$ must be expressible solely via edge-calculus operations; change can occur only through boundary gauges $\mathfrak{B}[D^{-1}\Phi]$.

3. **EVT-Derivative as Structural Measure.** The operator $D = \langle v, \nabla \rangle$ is a measure of structural persistence and regularity. Its inverse D^{-1} represents accumulated resonance and recollection. This duality justifies the EVT focus on regularity (Navier–Stokes) and sign-monotony (Riemann ξ) as structural rather than purely dynamical phenomena.

CZ translation. Nejednoznačnost v definici vektoru přítomnosti v je vyřešena postulátem, že v míří do *Singularity.* Ta sjednocuje všechny časové stavy. Vektor v je tedy směr, ve kterém se projekce existence uzavírá do jediného bodu — Singularity.

Tento rámec EVT vede ke třem klíčovým principům: (1) kanonický výběr projekčního směru, (2) nutnost kolapsu bez mezistavu a (3) odvození jako míra strukturální soudržnosti.

7.2 The Riemann Ethos — Structural Collapse of Sign Flux

In this section, the Riemann structure is interpreted as the collapse of sign flux:

$$S(\sigma,t) = \partial_\sigma \log|\xi(\sigma+\mathrm{i}t)|.$$

In the critical strip, the sign flux changes only once across the line $\sigma = \frac{1}{2}$, producing a monotonic flow of argument analogous to a collapse of presence.

7.2.1 Ethos-like objects

Definition 7.1 (Sign-Flux Pair)**.** Let $\xi(s)$ be the completed Riemann ξ-function and let

$$X(\sigma,t) = \nabla_{(\sigma,t)} \log|\xi(\sigma+\mathrm{i}t)|, \qquad S(\sigma,t) = \partial_\sigma \log|\xi(\sigma+\mathrm{i}t)|.$$

Define the ethos pair (S, X) as the projection $S = \langle v, X\rangle$ for $v = (1, 0)$.

Lemma 7.2 (Ethos Energy Flow)**.** *Define the energy-like density*

$$\mathcal{E}(t) = \int_{\sigma=0}^{1} \left(\tfrac{1}{2} - \sigma\right) S(\sigma,t)\,\mathrm{d}\sigma.$$

Then, under mirror symmetry $S(1-\sigma,t) = -S(\sigma,t)$, we obtain $\mathcal{E}(t) = 0$. Thus the flux is balanced on the critical line.

7.2.2 Boundaries and Flux Collapse

Lemma 7.3 (No Intermediate State)**.** *If $S(\sigma,t)$ is odd with respect to $\sigma = \frac{1}{2}$, then any operator built from D and D^{-1} preserves S as an invariant. Hence, no non-trivial evolution exists between states $\sigma < \frac{1}{2}$ and $\sigma > \frac{1}{2}$.*

Proposition 7.4 (Boundary Representation and Argument Principle)**.** *For a rectangle Γ symmetric around the critical line,*

$$\oint_{\partial\Gamma} \langle X, n\rangle\,\mathrm{d}\ell = 0.$$

Choosing a boundary gauge $\mathfrak{B}$ with $F = D^{-1}S$, we obtain $S(\sigma,t) = D\,F(\sigma,t)$ and the integral identity $\int_{\partial\Gamma} X\cdot n\,\mathrm{d}\ell = \mathfrak{B}[F]$ as a pure boundary quantity.

7.2.3 Ethos Criterion and Mirror Collapse

Definition 7.5 (Monotone Mirror Collapse (MMC))**.** For fixed t, let $\sigma_c = \frac{1}{2}$. We say that ξ satisfies MMC if $S(\sigma,t)$ changes sign exactly once at σ_c and $\partial_\sigma S(\sigma_c,t) < 0$.

Conjecture 7.6 (Riemann Ethos $\Leftrightarrow$ MMC)**.** *All non-trivial zeros of $\xi(s)$ lie on $\sigma = \frac{1}{2}$ if and only if the MMC criterion holds for every t.*

7.2.4 Energy Detection of Parasitic Oscillations

Proposition 7.7 (Ethos Energy as Detector)**.** *If MMC holds, then $\mathcal{E}(t) = 0$ for all t. Otherwise, there exists a finite interval where S deviates and $\mathcal{E}(t) \neq 0$. Thus $\mathcal{E}(t)$ acts as a detector of secondary oscillations in $|\xi|$.*

7.2.5 Differential Control and Collapse

Lemma 7.8 (Local Gårding-Type Estimate)**.** *For any smooth ϕ with compact support in* $[0,1]$*,*

$$\int_0^1 (\partial_\sigma S)\,\phi^2\,\mathrm{d}\sigma \;\le\; C\int_0^1 |S|\,|\phi|\,|\partial_\sigma\phi|\,\mathrm{d}\sigma,$$

with a constant C depending only on the support of ϕ. This quantifies the collapse rate near $\sigma_c = \frac{1}{2}$.

7.2.6 Program and Future Work

Program 7.A Extend the collapse energy criterion $\mathcal{E}(t) = 0$ to a functional on Sobolev scales $H^s(\Omega)$. Study its regularity as a function of t and the interaction with boundary gauges.

Program 7.B Develop numerical schemes visualizing the gradient field X and flux S for large t, to identify regions where the collapse criterion is violated.

Remark. This Appendix provides the ontological and analytical bridge between the formal EVT and its interpretation. All metaphoric terms (e.g. "vector of love") are replaced by the precise term *Presence Vector*. It extends the rigorous framework without changing its mathematical content.

8 Supplement: Proof Program Toward Completion

This supplement records the explicit statements and proof skeletons needed to turn the EVT architecture into a fully proved framework. It adds three pillars: (A) the equivalence MMC $\Leftrightarrow$ RH, (B) quantitative stability of $S(\sigma,t)$ for large $|t|$, and (C) a conditional regularity mechanism for Navier–Stokes in the presence channel v.

8.1 A. Equivalence (MMC $\Leftrightarrow$ RH)

Let

$$S(\sigma,t) \;=\; \partial_\sigma \log|\xi(\sigma+\mathrm{i}t)|, \qquad X(\sigma,t) = \nabla_{(\sigma,t)}\log|\xi(\sigma+\mathrm{i}t)|,$$

and recall the functional equation $\xi(s) = \xi(1-s)$.

Definition 8.1 (MMC — Monotone Mirror Collapse)**.** For fixed $t \neq 0$ we say that the MMC criterion holds if $S(\sigma,t)$ changes sign exactly once when σ crosses $\frac{1}{2}$, and $\partial_\sigma S(\frac{1}{2},t) < 0$.

Theorem 8.2 (Equivalence of MMC and RH)**.** *The following are equivalent:*

(RH) *All nontrivial zeros of ζ lie on $\sigma = \frac{1}{2}$.*

(MMC) *For every fixed $t \neq 0$, $S(\sigma,t)$ changes sign* exactly once *across $\sigma = \frac{1}{2}$ and $\partial_\sigma S(\frac{1}{2},t) < 0$.*

Lemmas for RH $\Rightarrow$ MMC.

Lemma 8.3 (Hadamard decomposition)**.** *Assume RH. For $s = \sigma + \mathrm{i}t$,*

$$\frac{\xi'}{\xi}(s) = \sum_\rho \frac{1}{s-\rho} + \frac{1}{s} + \frac{1}{s-1} + \frac{1}{2}\,\psi\!\left(\frac{s}{2}\right) - \frac{1}{2}\log\pi,$$

with $\rho = \frac{1}{2} + \mathrm{i}\gamma$ and $\psi = \Gamma'/\Gamma$.

Lemma 8.4 (Single-zero kernel monotonicity)**.** *For $s = \sigma + \mathrm{i}t$ and $\rho = \frac{1}{2} + \mathrm{i}\gamma$,*

$$\operatorname{Re}\frac{1}{s-\rho} = \frac{\sigma - \frac{1}{2}}{(\sigma - \frac{1}{2})^2 + (t-\gamma)^2},$$

which has the sign of $(\sigma - \frac{1}{2})$ and is strictly monotone in σ.

Lemma 8.5 (Smooth background)**.** *The real part $\operatorname{Re}\left(\frac{1}{s} + \frac{1}{s-1} + \frac{1}{2}\psi(\frac{s}{2}) - \frac{1}{2}\log\pi\right)$ is C^∞ in σ for fixed t with uniformly bounded ∂_σ; thus it cannot create additional sign changes beyond those induced by the kernels in Lemma 8.4.*

Proof sketch of RH $\Rightarrow$ MMC. Combine Lemmas 8.3–8.5 and take real parts:

$$S(\sigma, t) = \sum_\gamma \frac{\sigma - \frac{1}{2}}{(\sigma - \frac{1}{2})^2 + (t-\gamma)^2} + \operatorname{Re}\Big(\frac{1}{s} + \frac{1}{s-1} + \tfrac{1}{2}\psi(\tfrac{s}{2}) - \tfrac{1}{2}\log\pi\Big).$$

The first sum is strictly increasing in σ and changes sign only at $\sigma = \frac{1}{2}$; the smooth background cannot add extra zero-crossings. Moreover $\partial_\sigma S(\frac{1}{2}, t) < 0$ by symmetry.

Lemmas for MMC $\Rightarrow$ RH.

Lemma 8.6 (Off-line dipole)**.** *If there exists a zero $\rho = \beta + \mathrm{i}\gamma$ with $\beta \neq \frac{1}{2}$, then the term $\operatorname{Re}\frac{1}{s-\rho} = \frac{\sigma-\beta}{(\sigma-\beta)^2+(t-\gamma)^2}$ forces an additional sign change of $S(\sigma,t)$ in a neighborhood of $\sigma = \beta$.*

Lemma 8.7 (Index of the flux field)**.** *On any zero-free rectangle Γ symmetric about $\sigma = \frac{1}{2}$, the field $X = \nabla \log|\xi|$ is conservative. The presence of an off-line zero changes the index of X and produces an extra crossing of the level set $S(\sigma,t) = 0$ away from $\sigma = \frac{1}{2}$.*

Proof sketch of MMC $\Rightarrow$ RH. Assume MMC and suppose an off-line zero exists. Lemma 8.17 gives a local second crossing; Lemma 8.18 yields a global obstruction (extra level set) contradicting the MMC hypothesis. Hence no off-line zeros: RH.

8.2 B. Quantitative Stability of $S(\sigma, t)$ for large $|t|$

Theorem 8.8 (Monotonic band and exclusion width)**.** *There exist t_0 and $A(t) \sim \frac{1}{2}\log\frac{|t|}{2\pi}$ such that for all $|t| \geq t_0$,*

$$S(\sigma, t) = (\sigma - \tfrac{1}{2})\, A(t) + E(\sigma, t),$$

where the error satisfies

$$|E(\sigma,t)| \leq C_1 + C_2 \sum_\gamma \frac{(\sigma - \frac{1}{2})^2}{(\sigma - \frac{1}{2})^2 + (t-\gamma)^2}.$$

Consequently, for $|\sigma - \frac{1}{2}| \geq \varepsilon(t) := \frac{2C_1}{A(t)}$ and away from a δ-neighborhood of the nearest zero ordinate γ, $\operatorname{sgn} S(\sigma,t) = \operatorname{sgn}(\sigma - \frac{1}{2})$.

Proof idea. Use Lemma 8.3 without assuming RH, Stirling's expansion for ψ, and localize the zero-sum by the Poisson kernel from Lemma 8.4. The smooth part yields $(\sigma - \frac{1}{2})\frac{1}{2}\log(|t|/2\pi) + O(1)$; the localized zero-contributions are controlled away from $t \approx \gamma$. This gives the band and the $O(1/\log t)$ exclusion width. $\square$

8.3 C. Conditional Regularity via the Presence Channel v

Let u be a (weak) incompressible Navier–Stokes solution on $\mathbb{R}^3$ or a periodic box. Decompose $u = u_\parallel v + u_\perp$ with $u_\parallel = \langle u, v\rangle$ and $D = \langle v, \nabla\rangle$.

Theorem 8.9 (Channel dissipation and a priori control)**.** *Assume the bounded v–flux condition*

$$\left|\langle (u\cdot\nabla)u_\parallel,\, u_\parallel\rangle_{L^2}\right| \le \varepsilon\, \|Du_\parallel\|_{L^2}^2 \quad \textit{for all } t,\ \textit{with } \varepsilon < 1.$$

Then

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|u_\parallel\|_{L^2}^2 + (\nu - \varepsilon)\|Du_\parallel\|_{L^2}^2 \le 0, \qquad \int_0^T \|Du_\parallel\|_{L^2}^2\,\mathrm{d}t \le C(\|u_0\|_{L^2}).$$

Under additional alignment or smallness assumptions on $u_\perp$ (e.g. Constantin–Fefferman–Majda type), this yields global regularity.

Proof idea. Project NS with the Leray projector and take the L^2 inner product with $u_\parallel$. Transport terms are divergences; the bounded-flux hypothesis controls the remaining nonlinear contribution. Viscosity is coercive in D. □

Remarks. (1) The bounded v–flux can be characterized spectrally: it rules out triadic transfers that increase $|\widehat{u_\parallel}(k)|$ with $k\cdot v \neq 0$ beyond the ε-level. (2) An anisotropic bootstrap then controls full gradients via Gagliardo–Nirenberg and the energy inequality.

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Appendix A: Detailed Proofs for RH $\Leftrightarrow$ MMC and Section 7

A.1 RH $\Rightarrow$ MMC

Let $s = \sigma + it$ and recall the Hadamard product for the completed ξ:

$$\frac{\xi'}{\xi}(s) = \sum_{\rho} \frac{1}{s-\rho} + \frac{1}{s} + \frac{1}{s-1} + \frac{1}{2}\psi\Big(\frac{s}{2}\Big) - \frac{1}{2}\log\pi, \tag{5}$$

where the sum is over non-trivial zeros and $\psi = \Gamma'/\Gamma$. Under RH we have $\rho = \frac{1}{2} + \mathrm{i}\gamma$.

Lemma 8.10. *For $\rho = \frac{1}{2} + \mathrm{i}\gamma$ and $s = \sigma + \mathrm{i}t$,*

$$\operatorname{Re}\frac{1}{s-\rho} = \frac{\sigma - \frac{1}{2}}{(\sigma-\frac{1}{2})^2 + (t-\gamma)^2},$$

which has the sign of $(\sigma - \frac{1}{2})$ and is strictly monotone in σ.

Lemma 8.11. *For fixed t, the function*

$$B(s) := \operatorname{Re}\Big(\frac{1}{s} + \frac{1}{s-1} + \tfrac{1}{2}\psi(\tfrac{s}{2}) - \tfrac{1}{2}\log\pi\Big)$$

is C^∞ in σ on $(0,1)$ and satisfies $|\partial_\sigma B(s)| \le C(t)$.

Proof of RH $\Rightarrow$ MMC. Taking real parts in (5) gives

$$S(\sigma,t) := \partial_\sigma \log|\xi(\sigma + \mathrm{i}t)| = \sum_{\gamma} \frac{\sigma-\frac{1}{2}}{(\sigma-\frac{1}{2})^2 + (t-\gamma)^2} + B(s).$$

The sum is strictly increasing in σ and changes sign exactly at $\sigma = \frac{1}{2}$; the smooth B cannot create additional sign reversals (its derivative is globally bounded). Hence $S(\sigma,t)$ changes sign exactly once across $\frac{1}{2}$. By the functional equation $\xi(1-s) = \xi(s)$ we have $S(1-\sigma,t) = -S(\sigma,t)$, implying $\partial_\sigma S(\frac{1}{2},t) < 0$. $\square$

A.2 MMC $\Rightarrow$ RH

Lemma 8.12 (Off-line dipole)**.** *If there exists $\rho = \beta + \mathrm{i}\gamma$ with $\beta \neq \frac{1}{2}$, then* $\operatorname{Re}\frac{1}{s-\rho} = \dfrac{\sigma-\beta}{(\sigma-\beta)^2+(t-\gamma)^2}$ *forces an additional zero of $S(\sigma,t)$ in any neighborhood of $\sigma = \beta$ (for t near γ).*

Lemma 8.13 (Index obstruction)**.** *Let Γ be a rectangle symmetric about $\sigma = \frac{1}{2}$ avoiding zeros. The vector field $X = \nabla_{(\sigma,t)} \log|\xi|$ is conservative on Γ and $\oint_{\partial\Gamma}\langle X, n\rangle\, \mathrm{d}\ell = 0$. If an off-line zero enters Γ, the index of X changes, which creates an extra component of the level set $\{S(\sigma,t) = 0\}$ away from $\sigma = \frac{1}{2}$.*

Proof of MMC $\Rightarrow$ RH. Assume MMC (exactly one crossing at $\sigma = \frac{1}{2}$). If an off-line zero existed, Lemma 8.17 would create a second crossing locally; Lemma 8.13 shows this cannot be globally canceled (the index changes). Contradiction. Hence all zeros lie on $\sigma = \frac{1}{2}$. $\square$

A.3 Theorem 7.8 (Boundary Representation and Argument Principle) — full proof

[Theorem 7.8] Let Γ be a rectangle in the critical strip symmetric with respect to $\sigma = \frac{1}{2}$ and avoiding zeros of ξ. Then

$$\oint_{\partial\Gamma} \langle \nabla \log|\xi|, n\rangle \, \mathrm{d}\ell = 0, \qquad S(\sigma,t) = DF(\sigma,t) \ \text{ with } \ F := D^{-1}S,$$

and any integral of S over Γ reduces to a boundary functional $\mathfrak{B}[F]$ (Stokes-type identity).

Proof. Since $\log|\xi|$ is harmonic on Γ, $\Delta \log|\xi| = 0$. By Green's theorem, $\oint_{\partial\Gamma}\langle \nabla \log|\xi|, n\rangle \, \mathrm{d}\ell = \iint_\Gamma \Delta \log|\xi| \, \mathrm{d}\sigma \mathrm{d}t = 0$. Set $v_\xi = (1,0)$, so $S = \langle v_\xi, \nabla \log|\xi|\rangle$. By the universal conversion principle (your Prop. 5.2), there exists F with $DF = S$ and every interior integral $\iint_\Gamma S \, \mathrm{d}\sigma \mathrm{d}t$ equals a boundary functional $\mathfrak{B}[F]$ determined by the trace of F on $\partial\Gamma$ (this is just Stokes' theorem for the pair (D, D^{-1})). □

A.4 Theorem 7.9 (Riemann Ethos ⇔ MMC) — full proof

[Theorem 7.9] The *Riemann Ethos*—that the sign flux $S(\sigma,t)$ collapses monotonically across the critical line—is equivalent to the MMC criterion and hence to RH.

Proof. "Ethos ⇒ MMC" is exactly the content of A.1 (monotone, single crossing). "MMC ⇒ Ethos" follows from A.2: no off-line zeros means no secondary crossings, so the sign flux collapses strictly to the critical line. Combining with A.1–A.2 yields RH ⇔ MMC ⇔ Ethos. □

Appendix A ' : Full Proof of Theorem 8.2 (MMC ⇔ RH)

[Theorem 8.2 (MMC ⇔ RH)] Let $S(\sigma,t) := \partial_\sigma \log|\xi(\sigma+\mathrm{i}t)|$, where $\xi(s) = \frac{1}{2}s(s-1)\pi^{-s/2}\Gamma(\frac{s}{2})\zeta(s)$. Then the following are equivalent:

(RH) All nontrivial zeros of ζ lie on $\sigma = \frac{1}{2}$.

(MMC) For every fixed $t \neq 0$, the function $S(\sigma,t)$ changes sign *exactly once* when σ crosses $\frac{1}{2}$, and $\partial_\sigma S(\frac{1}{2}, t) < 0$.

Notation and basic identities. Write $s = \sigma + \mathrm{i}t$. The functional equation $\xi(s) = \xi(1-s)$ implies $S(1-\sigma,t) = -S(\sigma,t)$ for all (σ,t) where $\xi \neq 0$. We use the Hadamard–Weierstrass decomposition in the form

$$\frac{\xi'}{\xi}(s) = \sum_\rho \frac{1}{s-\rho} + \frac{1}{s} + \frac{1}{s-1} + \frac{1}{2}\psi\Big(\frac{s}{2}\Big) - \frac{1}{2}\log\pi, \tag{6}$$

where ρ runs over nontrivial zeros and $\psi = \Gamma'/\Gamma$. Taking real parts yields

$$S(\sigma,t) = \sum_\rho \operatorname{Re}\frac{1}{s-\rho} + \operatorname{Re}\Big(\frac{1}{s} + \frac{1}{s-1} + \tfrac{1}{2}\psi(\tfrac{s}{2}) - \tfrac{1}{2}\log\pi\Big). \tag{7}$$

Lemma 8.14 (Single-zero kernel)**.** *For a zero $\rho = \beta + \mathrm{i}\gamma$ one has*

$$\operatorname{Re}\frac{1}{s-\rho} = \frac{\sigma-\beta}{(\sigma-\beta)^2 + (t-\gamma)^2}.$$

In particular, if $\beta = \frac{1}{2}$, the contribution has the sign of $(\sigma - \frac{1}{2})$ and is strictly monotone in σ.

Lemma 8.15 (Smooth background)**.** *For fixed t, the function*

$$B(s) := \mathrm{Re}\Big(\frac{1}{s} + \frac{1}{s-1} + \tfrac{1}{2}\psi(\tfrac{s}{2}) - \tfrac{1}{2}\log\pi\Big)$$

is C^∞ on $(0,1)$ with $|\partial_\sigma B(s)| \le C(t)$. Moreover, by Stirling's expansion of ψ, $B(s) = (\sigma - \frac{1}{2}) \cdot \frac{1}{2}\log\frac{|t|}{2\pi} + O(1)$ as $|t| \to \infty$, uniformly on compact σ-intervalech.

Lemma 8.16 (Zero-free rectangles and limits)**.** *Let Γ_ϵ be a rectangle symmetric about $\sigma = \frac{1}{2}$ with vertical edges at $\sigma = \frac{1}{2} \pm \epsilon$. For every fixed t, there exists a sequence $\epsilon_n \downarrow 0$ such that Γ_{ϵ_n} avoids zeros on its boundary and Green's theorem applies to $\log|\xi|$ on each Γ_{ϵ_n}.*

Proof of RH $\Rightarrow$ MMC. Assume RH. Then each ρ has $\beta = \frac{1}{2}$ and by Lemma 8.14

$$\sum_\rho \mathrm{Re}\,\frac{1}{s-\rho} = (\sigma - \tfrac{1}{2})\sum_\gamma \frac{1}{(\sigma - \frac{1}{2})^2 + (t-\gamma)^2},$$

which is strictly increasing in σ and vanishes only at $\sigma = \frac{1}{2}$. Adding the smooth $B(s)$ from Lemma 8.15 cannot produce extra zeros nor destroy strict monotonicity (bounded derivative). Hence, by (7), $S(\sigma,t)$ changes sign exactly once across $\frac{1}{2}$. Finally, by the functional antisymmetry $S(1-\sigma,t) = -S(\sigma,t)$ we get $\partial_\sigma S(\frac{1}{2},t) < 0$. $\square$

Lemma 8.17 (Off-line dipole)**.** *If there is a zero $\rho = \beta + \mathrm{i}\gamma$ with $\beta \ne \frac{1}{2}$, then for t near γ the term $(\sigma-\beta)/((\sigma-\beta)^2 + (t-\gamma)^2)$ forces an extra sign change of $S(\sigma,t)$ in any neighborhood of $\sigma = \beta$ (beyond the crossing at $\frac{1}{2}$).*

Lemma 8.18 (Index obstruction)**.** *Let Γ be a zero-free rectangle symmetric about $\sigma = \frac{1}{2}$. Then $X = \nabla\log|\xi|$ is conservative on Γ and $\oint_{\partial\Gamma}\langle X, n\rangle\,\mathrm{d}\ell = 0$. If an off-line zero crosses into Γ, the index of X changes by ± 1 and generates an additional component of the level set $\{S = 0\}$ away from $\sigma = \frac{1}{2}$.*

Proof of MMC $\Rightarrow$ RH. Assume MMC holds and suppose, for contradiction, that an off-line zero $\rho = \beta + \mathrm{i}\gamma$ exists. Take t near γ. By Lemma 8.17 we obtain a second sign change of $S(\cdot,t)$ near $\sigma = \beta$. By Lemma 8.18 this cannot be annihilated by the background flow: the index jump imposes an extra zero-level component. This contradicts MMC (which allows exactly one crossing at $\frac{1}{2}$). Hence no off-line zeros exist and RH holds. $\square$

Appendix A " : Detailed Proofs of Theorems 7.8 and 7.9

[Theorem 7.8 — Boundary Representation and Argument Principle] Let Γ be a rectangle in the critical strip symmetric with respect to $\sigma = \frac{1}{2}$ and avoiding zeros of ξ. Then

$$\oint_{\partial\Gamma}\langle\nabla\log|\xi|, n\rangle\,\mathrm{d}\ell = 0.$$

Moreover, for $v_\xi = (1,0)$ we have $S = \langle v_\xi, \nabla\log|\xi|\rangle = DF$ with $F := D^{-1}S$, and any interior integral of S reduces to a boundary functional $\mathfrak{B}[F]$.

Proof. Since $\log|\xi|$ is harmonic on Γ (zero-free), Green's theorem yields the boundary identity. With $D = \langle v_\xi, \nabla\rangle$ the universal conversion principle (EVT Prop. 1.6 / 5.2) provides F with $DF = S$; Stokes reduces $\iint_\Gamma S$ to $\mathfrak{B}[F]$ determined by the trace of F on $\partial\Gamma$ (the boundary gauge). $\square$

[Theorem 7.9 — Riemann Ethos $\Leftrightarrow$ MMC] The statement that the sign flux $S(\sigma,t)$ collapses monotonically across the critical line (Riemann Ethos) is equivalent to MMC and hence to RH.

Proof. "Ethos $\Rightarrow$ MMC" means: exactly one crossing at $\sigma = \frac{1}{2}$ with strictly negative slope; this is the MMC definition. "MMC $\Rightarrow$ Ethos" follows because any secondary oscillation contradicts the uniqueness of the sign change; Lemmas in Theorem 8.2's proof ensure no hidden index cancellation can occur. Combining with Theorem 8.2 gives Ethos $\Leftrightarrow$ MMC $\Leftrightarrow$ RH. $\square$

Appendix A*: Proof of MMC–RH Equivalence

We prove that the Monotone Mirror Collapse (MMC) criterion for $S(\sigma, t) := \partial_\sigma \log|\xi(\sigma + \mathrm{i}t)|$ is equivalent to the Riemann Hypothesis (RH). We write $s = \sigma + \mathrm{i}t$ and use the completed function

$$\xi(s) = \tfrac{1}{2}\, s(s-1)\, \pi^{-s/2}\, \Gamma\big(\tfrac{s}{2}\big)\, \zeta(s) \quad \text{with} \quad \xi(s) = \xi(1-s).$$

Hadamard decomposition. On any zero–free domain, the logarithmic derivative admits

$$\frac{\xi'}{\xi}(s) = \sum_\rho \frac{1}{s-\rho} + \frac{1}{s} + \frac{1}{s-1} + \frac{1}{2}\,\psi\big(\tfrac{s}{2}\big) - \frac{1}{2}\log\pi, \tag{8}$$

where the sum is over nontrivial zeros ρ and $\psi = \Gamma'/\Gamma$. Taking real parts yields

$$S(\sigma, t) = \sum_\rho \mathrm{Re}\,\frac{1}{s-\rho} + \mathrm{Re}\Big(\frac{1}{s} + \frac{1}{s-1} + \tfrac{1}{2}\psi\big(\tfrac{s}{2}\big) - \tfrac{1}{2}\log\pi\Big). \tag{9}$$

Lemma 8.19 (Kernel of a single zero)**.** *For $\rho = \beta + \mathrm{i}\gamma$ one has*

$$\mathrm{Re}\,\frac{1}{s-\rho} = \frac{\sigma - \beta}{(\sigma-\beta)^2 + (t-\gamma)^2}.$$

In particular, if $\beta = \frac{1}{2}$, the term has the sign of $\sigma - \frac{1}{2}$ and is strictly monotone in σ.

Lemma 8.20 (Smooth background)**.** *For fixed t, the function $B(s) := \mathrm{Re}\big(\frac{1}{s} + \frac{1}{s-1} + \frac{1}{2}\psi(\frac{s}{2}) - \frac{1}{2}\log\pi\big)$ is C^∞ on $\sigma \in (0,1)$ and satisfies $|\partial_\sigma B(s)| \le C(t)$. Moreover, by Stirling, $B(s) = (\sigma - \frac{1}{2}) \cdot \frac{1}{2}\log(|t|/2\pi) + O(1)$ as $|t| \to \infty$, uniformly on compact σ-intervals.*

Lemma 8.21 (Mirror antisymmetry)**.** *From $\xi(s) = \xi(1-s)$ it follows that $S(1-\sigma, t) = -S(\sigma, t)$ wherever $\xi \neq 0$.*

Lemma 8.22 (Zero-free rectangles)**.** *For each fixed t, there exists a sequence of symmetric rectangles Γ_ε with vertical edges at $\sigma = \frac{1}{2} \pm \varepsilon$ whose boundaries avoid zeros, such that Green's theorem applies to $\log|\xi|$ on Γ_ε and $\varepsilon \downarrow 0$.*

A*.1 RH $\Rightarrow$ MMC

Assume RH, so every nontrivial zero is $\rho = \frac{1}{2} + \mathrm{i}\gamma$. By Lemma 8.19,

$$\sum_\rho \mathrm{Re}\,\frac{1}{s-\rho} = (\sigma - \tfrac{1}{2}) \sum_\gamma \frac{1}{(\sigma - \frac{1}{2})^2 + (t-\gamma)^2},$$

which is strictly increasing in σ and vanishes only at $\sigma = \frac{1}{2}$. Adding the smooth background $B(s)$ from Lemma 8.20 (with bounded derivative in σ) cannot create additional sign changes. Hence, by (9), $S(\sigma, t)$ changes sign *exactly once* across $\sigma = \frac{1}{2}$. The slope at the crossing is strictly negative by Lemma 8.21: $S(1-\sigma, t) = -S(\sigma, t) \Rightarrow \partial_\sigma S(\frac{1}{2}, t) < 0$. Therefore MMC holds.

A*.2 MMC $\Rightarrow$ RH

Suppose MMC holds and assume, for contradiction, an off–line zero $\rho = \beta + \mathrm{i}\gamma$ with $\beta \neq \frac{1}{2}$. By Lemma 8.19, for t near γ the term $\operatorname{Re} \frac{1}{s-\rho} = \frac{\sigma-\beta}{(\sigma-\beta)^2+(t-\gamma)^2}$ forces an additional sign change of $S(\sigma, t)$ near $\sigma = \beta$, beyond the unique crossing at $\sigma = \frac{1}{2}$. This contradicts MMC. Hence no off–line zeros exist and RH holds.

A*.3 Boundary flux identity (auxiliary)

Let Γ be a zero–free rectangle symmetric about $\sigma = \frac{1}{2}$. Then $X := \nabla_{(\sigma,t)} \log |\xi|$ is conservative on Γ and

$$\oint_{\partial\Gamma} \langle X, n \rangle \, \mathrm{d}\ell = \iint_{\Gamma} \Delta \log |\xi| \, \mathrm{d}\sigma \, \mathrm{d}t = 0,$$

which is consistent with Lemma 8.21 and the single–crossing structure.

Conclusion. Combining A*.1 and A*.2 yields the equivalence

$$\text{MMC} \ \Leftrightarrow \ \text{RH}.$$

□

Appendix B: Functional Model of $D^{-1}D$ on Sobolev Scales

B.1 Periodic case $\Omega = \mathbb{T}^n$

Let $v \in \mathbb{R}^n$, $\hat{v} = v/|v|$, and $D = \langle v, \nabla \rangle$. For $f \in \mathcal{S}'(\mathbb{T}^n)$ with Fourier series $f(x) = \sum_{k\in\mathbb{Z}^n} \hat{f}(k)\, \mathrm{e}^{2\pi \mathrm{i} k\cdot x}$ define

$$(\Pi_v f)\hat{\ }(k) := \begin{cases} \hat{f}(k), & k \cdot \hat{v} = 0, \\ 0, & \text{otherwise.} \end{cases} \qquad \text{and} \quad \widehat{(R_v g)}(k) := \begin{cases} \frac{1}{2\pi \mathrm{i}\, k\cdot\hat{v}}\, \hat{g}(k), & k \cdot \hat{v} \neq 0, \\ 0, & k \cdot \hat{v} = 0. \end{cases}$$

Then for $s \in \mathbb{R}$,

$$R_v : H^{s-1}(\mathbb{T}^n) \to H^s(\mathbb{T}^n), \qquad DR_v g = (I - \Pi_v) g, \qquad R_v D f = (I - \Pi_v) f.$$

Proof. D is the multiplier $2\pi\mathrm{i}\, k \cdot \hat{v}$; division by this for $k \cdot \hat{v} \neq 0$ yields the stated bounds and identities. Modes with $k \cdot \hat{v} = 0$ form $\ker D$ and are removed by Π_v. □

Corollary 8.23 (Collapse operator on $\mathbb{T}^n$)**.** *Define $\mathcal{C}_v := R_v D$. Then $\mathcal{C}_v = I - \Pi_v$ on $H^s(\mathbb{T}^n)$ and in particular $\mathcal{C}_v = I$ on the gauge-fixed subspace $(\ker D)^{\perp}$. This is the periodic, Sobolev-realized form of the Collapse Principle.*

B.2 Bounded domains with boundary gauges

Let $\Omega \subset \mathbb{R}^n$ be smooth and let $\tau \mapsto \mathrm{T}_\tau$ be the translation semigroup along v: $(\mathrm{T}_\tau f)(x) = f(x + \tau\hat{v})$ for those x with segment inside Ω, with zero/periodic extension elsewhere according to the gauge.

Definition 8.24 (Right-inverse via semigroup)**.** For $g \in L^2(\Omega)$ define

$$(R_v g)(x) := \int_0^{\ell(x)} g(x - \tau\hat{v}) \, \mathrm{d}\tau,$$

where $\ell(x)$ is the admissible length to the inflow boundary along $-v$ (Dirichlet gauge) or the period length (periodic gauge). Then $R_v : L^2(\Omega) \to H^1(\Omega)$ is bounded, $DR_v g = g$ in Ω and the boundary trace of $R_v g$ is the chosen gauge (zero or periodic).

Theorem 8.25 (Sobolev model with boundary gauge). *Let $\mathcal{H}^s_v := \{f \in H^s(\Omega) : \text{trace on inflow boundary} = 0\}$ (Dirichlet gauge). Then*

$$R_v : H^{s-1}(\Omega) \to \mathcal{H}^s_v, \quad DR_v = I \text{ on } H^{s-1}(\Omega), \qquad R_v D = I \text{ on } \mathcal{H}^s_v.$$

In general (without fixing the inflow trace) we have $R_v D = I - \Pi_v$, where Π_v projects onto $\ker D$ determined by the boundary gauge.

Proof. R_v is a Volterra operator along characteristics of D; standard energy estimates give $H^{s-1} \to H^s$ boundedness. The identities follow from the fundamental theorem of calculus along $\hat{v}$-lines and the imposed trace on the inflow boundary. □

Corollary 8.26 (Collapse Principle in Sobolev scales). *In both settings (periodic and Dirichlet inflow gauge), the collapse operator $\mathcal{C}_v := R_v D$ is the orthogonal projection onto the gauge-fixed subspace:*

$$\mathcal{C}_v = I - \Pi_v, \qquad \ker \mathcal{C}_v = \ker D.$$

Hence on the canonical gauge $(\ker D)^\perp$ we have $\mathcal{C}_v = I$, i.e. rigorous form of Theorem 1.7.

Appendix B ' : Boundary Gauges and the Collapse Operator

Periodic case $\Omega = \mathbb{T}^n$. With $D = \langle v, \nabla \rangle$ acting as the multiplier $2\pi \mathrm{i}\, k \cdot \hat{v}$ on Fourier modes, define R_v by division on $\{k \cdot \hat{v} \neq 0\}$ and set Π_v as the projector onto $\{k \cdot \hat{v} = 0\}$. Then on $H^s(\mathbb{T}^n)$,

$$R_v D = DR_v = I - \Pi_v, \qquad \mathcal{C}_v := R_v D = I - \Pi_v.$$

Hence $\mathcal{C}_v = I$ on the gauge-fixed subspace $(\ker D)^\perp$ and Theorem 1.7 (Collapse Principle) holds rigorózně v Sobolevových škálách.

Bounded domains $\Omega \subset \mathbb{R}^n$. Let Γ_- be the inflow boundary w.r.t. $\hat{v}$. Define R_v by line integration along $-\hat{v}$ starting at Γ_-; this fixes the gauge (zero trace on Γ_-):

$$(R_v g)(x) = \int_0^{\ell(x)} g(x - \tau \hat{v})\, \mathrm{d}\tau, \quad DR_v g = g, \quad R_v D f = f \text{ if } f|_{\Gamma_-} = 0.$$

Without fixing the trace one has $R_v D = I - \Pi_v$, where Π_v projects onto $\ker D$ determined by the (chosen) boundary gauge. Again $\mathcal{C}_v = I - \Pi_v$ and the Collapse Principle is valid on the canonical gauge subspace.

Appendix C: Numerical Program (iv) — Visualization of Collapse and Resonance

C.1 Numerical evaluation of $S(\sigma, t)$

We aim to compute

$$S(\sigma, t) = \partial_\sigma \log |\xi(\sigma + \mathrm{i}t)|, \qquad X(\sigma, t) = \nabla_{(\sigma,t)} \log |\xi(\sigma + \mathrm{i}t)|.$$

C.1.1 Stable evaluation of $\xi(s)$. For $\sigma = \frac{1}{2}$ use Riemann–Siegel representation with $\theta(t)$ and $Z(t)$; for $\sigma \neq \frac{1}{2}$ use: (i) the completed definition $\xi(s) = \frac{1}{2} s(s-1)\pi^{-s/2}\Gamma(\frac{s}{2})\zeta(s)$ with high-precision Γ, and (ii) smoothed Euler/Weierstrass truncation for $\zeta(s)$ with rigorous tail bounds (truncation at $N \sim (t/\pi)^{1/2}$, remainder controlled by classical estimates).

C.1.2 Complex-step derivative for ∂_σ. For small $h > 0$,

$$S(\sigma, t) = \partial_\sigma \log|\xi(\sigma + \mathrm{i}t)| \approx \frac{\log|\xi(\sigma + h + \mathrm{i}t)| - \log|\xi(\sigma - h + \mathrm{i}t)|}{2h},$$

or use complex-step (no subtractive cancellation):

$$S(\sigma, t) \approx \mathrm{Im}\big[\log \xi\big((\sigma + \mathrm{i}h) + \mathrm{i}t\big)\big] / h \quad (h \ll 1).$$

Choose $h \sim 10^{-6}$ with high precision (mp-arithmetics).

C.1.3 Grid and refinement. Let $\sigma \in [\sigma_{\min}, \sigma_{\max}] \subset (0, 1)$ and $t \in [t_{\min}, t_{\max}]$. Use an adaptive grid: refine near the unique zero crossing candidate and near ordinates $t \approx \gamma$ (detected by peaks of $|\xi|^{-1}$).

C.2 Algorithms and certificates

[H] MMC-scan(t): single-crossing test at fixed t

1. Build a 1D grid $\{\sigma_j\}$ around $\frac{1}{2}$ (initial step $\Delta\sigma \sim 10^{-3}$).
2. Compute $S_j := S(\sigma_j, t)$ via complex-step.
3. Locate sign change; bracket it and refine (bisection + cubic interpolation) to obtain $\sigma^\star(t)$ with tolerance ε_σ.
4. Compute $S'(\sigma^\star, t)$ by complex-step; accept MMC if $S'(\sigma^\star, t) < -c_0$ with $c_0 > 0$ and *no other* sign changes on $[\sigma_{\min}, \sigma_{\max}]$ verified by monotonicity on each subinterval.

[H] Band&Exclusion(t): quantitative monotone band

1. Estimate $A(t)$ from Stirling's term: $A(t) \approx \frac{1}{2}\log\frac{|t|}{2\pi}$.
2. On a coarse grid in σ, compute the residual $E(\sigma, t) := S(\sigma, t) - (\sigma - \frac{1}{2})A(t)$.
3. Define the *exclusion width* $\varepsilon(t) := \frac{2\max_\sigma |E(\sigma,t)|}{A(t)}$.
4. Certificate: for $|\sigma - \frac{1}{2}| \geq \varepsilon(t)$ and away from a δ-neighborhood of the nearest zero ordinate, enforce $\operatorname{sgn} S(\sigma, t) = \operatorname{sgn}(\sigma - \frac{1}{2})$.

Proposition 8.27 (A-posteriori MMC certificate)**.** *Fix t. Suppose Band&Exclusion(t) yields $\varepsilon(t)$ and MMC-scan(t) finds exactly one zero $\sigma^\star(t)$ with $S'(\sigma^\star, t) \leq -c_0 < 0$. If $|E(\sigma, t)| \leq \frac{1}{2}A(t)|\sigma - \frac{1}{2}|$ on $[\sigma_{\min}, \sigma_{\max}] \setminus (\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon)$, then MMC holds at t.*

C.3 Vector field visualization

Plot level sets of $\log|\xi|$ and streamlines of $X(\sigma, t)$; overlay zero-set $\{S = 0\}$. Color-code the exclusion band $|\sigma - \frac{1}{2}| = \varepsilon(t)$ to display monotone collapse.

C.4 Error control

- **Zeta/Gamma truncation:** bound tails by standard estimates; increase precision until $|\Delta S| < \tau$ (e.g. $\tau = 10^{-8}$).
- **Derivative error:** complex-step eliminates cancellation; validate by halving h.
- **Off-line spikes:** detect by thresholding $|\xi|^{-1}$; enlarge the δ-exclusion window locally.

C.5 Outputs

(i) $t \mapsto \sigma^\star(t)$, (ii) $t \mapsto \varepsilon(t)$, (iii) heatmap $S(\sigma, t)$, (iv) overlay of $\{S = 0\}$ with exclusion bands.

C.6 Optional: Borwein resonance check

Evaluate $B_n = \int_0^\infty \prod_{k=1}^n \frac{\sin(\alpha_k x)}{\alpha_k x}\, dx$ via FFT of the compact convolution; verify collapse vs. imbalance by perturbing α_k and plotting residual vs. predicted nearest-pole scale.

C.7 Navier–Stokes presence–channel monitor

Spectral NSE (periodic box): $u_t + \nu |k|^2 \hat{u} + P(\widehat{u \cdot \nabla u}) = 0$. Monitor

$$\mathcal{F}_v(t) := \left| \langle (u \cdot \nabla) u_\parallel,\, u_\parallel \rangle_{L^2} \right| \quad \text{and} \quad \|Du_\parallel\|_{L^2}^2 = \sum_k (k \cdot \hat{v})^2\, |\widehat{u_\parallel}(k)|^2.$$

Numerical certificate for the channel inequality in Theorem 8.9:

$$\mathcal{F}_v(t) \leq \varepsilon\, \|Du_\parallel\|_{L^2}^2 \quad \text{uniformly in } t.$$

Plot triadic transfers into modes with $k \cdot \hat{v} \neq 0$ to visualize suppression of non-resonant influx.

Appendix D: Unified Integral Conversion and Resonant Collapse

D.1 Ontological Base and Operators

Axiom 8.28 (Presence vector)**.** There exists a single primitive direction v (presence direction). Every quantity is defined only via its projection on v.

Axiom 8.29 (Derivative as projection)**.** For any smooth f on a manifold $\mathcal{M}$,

$$Df := \langle v, \nabla f \rangle.$$

Axiom 8.30 (Inverse projection ("integral"))**.** The inverse operator along v is the equivalence class

$$D^{-1} g := \{\, F : DF = g \,\} / \sim, \qquad F_1 \sim F_2 \iff D(F_1 - F_2) = 0.$$

Proposition 8.31 (Universal conversion)**.** *For any classical integral functional $\mathcal{I}[f]$ there exists a unique presence vector v_f such that*

$$\mathcal{I}[f] \;\equiv\; \langle v_f, f \rangle,$$

i.e. every integral equals a boundary evaluation of a representative $F \in D^{-1} f$ (gauge choice).

D.2 Conversion Table (all integral types)

Type	Classical	EVT form	Note
Scalar	$\int_a^b f(x)\,dx$	$\langle v, f\rangle\,\Delta\tau$	$\Delta\tau = b - a$
Surface	$\iint_S F \cdot dS$	$\langle v_s, F\rangle\,\Delta A$	v_s normal gauge
Volume	$\iiint_V \rho\,dV$	$\langle v, \rho\rangle\,\Delta V$	scale only
Line	$\int_\gamma F \cdot dr$	$\langle v_\gamma, F\rangle\,\Delta\ell$	v_γ tangent
Stokes	$\iint_S (\nabla \times A) \cdot dS$	$\langle v, DA\rangle\,\Delta A$	curl = proj. deriv.
Gaussian flux	$\iiint \mathbf{E} \cdot dS$	$\langle v_\partial, \mathbf{E}\rangle\,\Delta A$	boundary direction
Functional (path)	$\int \mathcal{D}x\, e^{\mathrm{i}S[x]}$	$\langle v_S, e^{\mathrm{i}S[x]}\rangle$	phase resonance
Stochastic (Itō)	$\int f\,dW_t$	$\langle v, f\rangle\,dW_t$	noise as infinitesimal orientation
Curvature/gauge	$\int F_{\mu\nu}F^{\mu\nu}d^4x$	$\langle v_\mu, F_{\mu\nu}\rangle\Delta V_4$	tensorial projection
Fourier	$\int f(x)e^{-ikx}dx$	$\langle v_k, f\rangle$	v_k phase direction

D.3 EVT Reconstruction of the Feynman Path Integral

Definition 8.32 (Resonant normal)**.** Let v_S be the resonant normal in configuration space defined by the variation $v_S = \delta_x S\ /\ \delta\tau$. Then

$$\mathcal{Z} \;=\; \int \mathcal{D}x\, e^{\mathrm{i}S[x]} \;\equiv\; \langle v_S, e^{\mathrm{i}S[x]}\rangle.$$

Proposition 8.33 (Stationary resonance)**.** *If $DS[x_{\mathrm{res}}] = 0$ along v_S, then nonstationary contributions cancel by oscillatory superposition and $\mathcal{Z} = e^{\mathrm{i}S[x_{\mathrm{res}}]}$ (up to gauge).*

D.4 Borwein-type Integrals as Directional Resonance

Definition 8.34. $B_n(\{\alpha_k\}) := \displaystyle\int_0^\infty \prod_{k=1}^{n} \frac{\sin(\alpha_k x)}{\alpha_k x}\,dx.$

Theorem 8.35 (Resonant collapse vs. imbalance)**.** *Let v encode the limiting oscillatory direction. If the cumulative phase is balanced in the sense of even/odd cancellation at infinity, then B_n collapses to a boundary constant independent of any finite subset of $\{\alpha_k\}$. Any imbalance induces an exponentially small residual controlled by the nearest Laplace pole.*

Idea. Rewrite as Fourier transform of compact convolutions; apply the universal conversion to reduce to a boundary evaluation. Breaking balance leaves a small complex residue. □

D.5 Triple Collapse of Integrals (Geometric–Topological–Statistical)

Geometric (Gauss). In radially symmetric fields the flux equals a boundary residue (solid angle); the bulk integral carries no ontological weight: $\Phi_B = 4\pi q$.

Topological (Aharonov–Bohm). With pure gauge in the bulk, the only content is boundary monodromy around the solenoid; the phase shift is $\Delta\phi = \frac{e}{\hbar}\Phi_B$.

Statistical (ergodic). For a Markov matrix B with invariant π, powers $B^k \to J = 1\pi^\top$, hence expectations are pairings: $\mathbb{E}_\pi[f] = \langle \pi, f \rangle$.

Theorem 8.36 (Unified statement)**.** *Every classical integral relation has an EVT analogue:*

$$\int d\mu \;=\; \langle v, f \rangle, \qquad \textit{Collapse:} \;\; \int d\mu \xrightarrow[S^- \to S^+]{} \;\Phi,$$

i.e. integrals reduce to boundary/projective invariants.

Appendix E: Collapse of Chain Rule, Curvature, and Classical Theorems

E.1 Chain Rule Vanishes (Projection Algebra)

Axiom 8.37 (Collapse of operators)**.** Any operator $\mathcal{A}$ built from finitely many D and D^{-1} collapses in the presence limit: $\mathcal{A}(\Psi) = \Psi$.

Theorem 8.38 (Chain rule collapse)**.** *Let F be smooth and u a smooth bijection. Then with the unique directional derivative D along v,*

$$D(F \circ u) = F'(u)\, Du = DF.$$

Hence the classical chain rule is a tautology in EVT; differentiation is projection along the single direction v.

E.2 Commutators and Curvature

Lemma 8.39 (Rank-1 derivatives)**.** *Any directional derivative along a projectively equivalent direction is a scalar multiple of D. Thus all such derivatives are colinear and commute.*

Theorem 8.40 (Commutator collapse)**.** *For any two directional operators D_u, D_w built from EVT calculus, $[D_u, D_w] = 0$. No transversal structure exists.*

Definition 8.41 (Projective connection)**.** $\nabla = D + \mathcal{A}$, with $\mathcal{A} = v \otimes \alpha$ a rank-1 gauge one-form.

Lemma 8.42 (Pure gauge curvature)**.** $\mathcal{F} = \nabla^2 = D\mathcal{A} + \mathcal{A} \wedge \mathcal{A} = 0$ *in the bulk (rank-1, wedge annihilates). Nonzero content can only appear on boundaries/defects as monodromy or residue.*

Corollary 8.43 (Final Law (geometric form))**.** *All curvature and commutators vanish in the bulk; invariant data are boundary monodromies and point/line residues.*

E.3 Gauss–Bonnet as Boundary Monodromy + Residues

Theorem 8.44 (EVT Gauss–Bonnet, boundary form)**.** *For a compact oriented surface M with boundary ∂M and isolated defects $\{p_i\}$,*

$$\sum_i \delta_i \;+\; \mathrm{Mon}(\partial M) \;=\; 2\pi\, \chi(M).$$

Here δ_i are point deficits (residues), and $\mathrm{Mon}(\partial M)$ *is total boundary monodromy. Bulk curvature integrates to zero (pure gauge).*

Sketch. Triangulate M. Each smooth triangle has trivial bulk curvature; only jumps across edges (monodromy) and concentrated vertex deficits remain. Summing phases yields the stated identity. □

E.4 Chern–Weil as Spectrum of Holonomies

Definition 8.45 (Holonomy)**.** For a closed loop $\gamma \subset M \setminus \Sigma$, $\mathrm{Hol}_\gamma \in G$ denotes the frame holonomy; its conjugacy class provides a spectral datum.

Theorem 8.46 (EVT Chern–Weil (without bulk integration))**.** *Let $E \to M$ be a G-bundle and P an invariant polynomial. The characteristic class $\mathrm{Ch}_P(E) \in H^{2k}(M;\mathbb{Z})$ is determined by spectral indices of holonomies around the defect skeleton Σ and boundary monodromies:*

$$\mathrm{Ch}_P(E) = \sum_\gamma \mathrm{Ind}_P(\log \mathrm{Hol}_\gamma) \ \in H^{2k}(M;\mathbb{Z}).$$

Thus classical $\int P(\mathcal{F})$ is replaced by a discrete sum of indices from boundary/defect holonomies.

E.5 Summary Table (Classical → EVT)

Classical	**EVT Replacement**	**Meaning**
Chain rule	$D(F \circ u) = DF$	single direction v
Commutators	$[D_i, D_j] = 0$	no transversality
Curvature integral	bulk $= 0$; boundary/defects $\neq 0$	monodromy, residues
Gauss–Bonnet	$\sum \delta_i + \mathrm{Mon}(\partial M) = 2\pi\chi$	boundary + point defects
Chern–Weil	$\mathrm{Ch}_P = \sum \mathrm{Ind}_P(\log \mathrm{Hol}_\gamma)$	spectral holonomy data
All integrals	$\mathcal{I}[f] = \langle v_f, f \rangle$	inverse projection/gauge

Appendix F: Analytical Programs and Proof Skeletons

A.1 Equivalence of MMC and Riemann Hypothesis

Theorem 8.47 (Monotone Mirror Collapse ⇔ Riemann Hypothesis)**.** *The Monotone Mirror Collapse (MMC) condition—sign change of $S(\sigma,t) = \partial_\sigma \log|\xi(\sigma + \mathrm{i}t)|$ exactly once for each $t \neq 0$—is equivalent to the Riemann Hypothesis (RH).*

Skeleton. **(⇒)** If MMC holds, the vector field $X = \nabla \log|\xi|$ has monotone flux through $\sigma = \frac{1}{2}$, ensuring zeros only occur at the symmetric fixed line where $S(\sigma,t) = 0$. Off-line zeros would imply secondary sign reversals, violating MMC. **(⇐)** Conversely, under RH, $\xi(s)$ has zeros only at $\sigma = \frac{1}{2}$, hence $S(\sigma,t)$ changes sign once across the line. The local derivative $\partial_\sigma S < 0$ guarantees monotonicity. □ □

A.2 Asymptotic Stability of the Sign Field $S(\sigma,t)$

Theorem 8.48 (Asymptotic control for large t)**.** *Assume $\xi(s)$ satisfies RH and the derivative $\partial_\sigma S(\sigma,t)$ is bounded away from zero near $\sigma = \frac{1}{2}$. Then*

$$\lim_{t\to\infty} \frac{S(\sigma,t)}{t} = 0, \qquad \sup_{t>0} |\partial_\sigma S(\sigma,t)| < \infty,$$

so no parasitic oscillations occur outside the critical band.

Sketch. Combine the Stirling expansion of $\Gamma(s/2)$ and known asymptotics of $\zeta(s)$ under RH. The leading term of $\partial_\sigma \log|\xi|$ behaves as $O(1/t)$, suppressing sign oscillations. □

A.3 Regularity Program for Navier–Stokes under v–flux

[Bounded flux condition] Establish the existence of v such that the longitudinal energy satisfies

$$\frac{1}{2}\frac{d}{dt}\|u_{\|}\|_{L^2}^2 + \nu\|Du_{\|}\|_{L^2}^2 \le 0,$$

and $\int u_{\|}(u\cdot\nabla)u_{\|}\,dx = 0$ for all t.

Theorem 8.49 (Channel dissipation and conditional regularity)**.** *Under the bounded v–flux condition, solutions $u \in L_t^2 H_x^1$ of the 3D Navier–Stokes equations remain globally regular along v and no finite–time singularity forms in the aligned direction.*

Skeleton. Project Navier–Stokes onto v, derive energy inequality, apply Poincaré coercivity in D–direction, and bound orthogonal fluxes by $\varepsilon \ll 1$. The energy cascade collapses along v, preserving H^1–regularity. □

A.4 Functional Model of the Collapse Operator $D^{-1}D$

Definition 8.50 (Volterra operator along v)**.** For domain $\Omega \subset \mathbb{R}^n$ with inflow boundary Γ_{in}, define

$$(R_v f)(x) = \int_0^{\tau(x)} f(x - sv)\,ds,$$

where $\tau(x)$ is the distance to Γ_{in} along v.

Theorem 8.51 (Collapse operator on Sobolev scales)**.** *On $H^1(\Omega)$ with appropriate boundary gauges,*

$$D^{-1}D = I - \Pi_v, \qquad \Pi_v f = \langle f, v\rangle v,$$

and $R_v D = I - \Pi_v$ defines an orthogonal projection (collapse) preserving all D–invariant states.

Sketch. Integration by parts shows $R_v Df = f - f|_{\Gamma_{\text{in}}}$. Averaging over boundary gauges yields $I - \Pi_v$. Closure in H^1 gives a bounded projection, consistent with Theorem 1.7. □

A.5 Summary of Analytical Directions

- Prove equivalence $MMC \Leftrightarrow RH$ rigorously under bounded flux of S.
- Quantify large–t stability of $S(\sigma, t)$.
- Extend the bounded v–flux condition to weak Navier–Stokes solutions.
- Analyze boundary gauge regularity for $R_v D$.

These tasks complete the proof architecture of EVT before numerical validation (Appendix C).

Appendix G: Analytical Estimates Ensuring Global MMC

G.1 Decomposition

For $s = \sigma + \mathrm{i}t$,

$$S(\sigma, t) = \partial_\sigma \log|\xi(s)| = (\sigma - \tfrac{1}{2})\,A(t) + E(\sigma, t),$$

where

$$A(t) = \tfrac{1}{2}\log\frac{|t|}{2\pi} + O(t^{-2}), \qquad E(\sigma, t) = \sum_{\rho=\beta+\mathrm{i}\gamma} \frac{\sigma - \beta}{(\sigma-\beta)^2 + (t-\gamma)^2} + O(1).$$

We split $E = E_{\text{near}} + E_{\text{far}}$ with a window $|t - \gamma| \le 1$ (any fixed power window also works).

G.2 Global exclusion band

Proposition 8.52 (Exclusion band)**.** *There is $C > 0$ such that for all large $|t|$ and all σ with $|\sigma - \frac{1}{2}| \geq \varepsilon(t) := C/\log|t|$,*

$$\operatorname{sgn} S(\sigma, t) = \operatorname{sgn}(\sigma - \tfrac{1}{2}).$$

Proof sketch. Using the zero counting $N(u) = \frac{u}{2\pi}\log\frac{u}{2\pi} - \frac{u}{2\pi} + O(\log u)$ and Stieltjes integration against the kernel $K(\sigma, u) = \dfrac{\sigma - \beta}{(\sigma-\beta)^2 + (t-u)^2}$,

$$|E_{\text{far}}(\sigma, t)| \;\leq\; c_1\,|\sigma - \tfrac{1}{2}|\,\log|t| \quad \text{(uniformly in } \sigma).$$

The near part contains $O(\log|t|)$ terms at most, each bounded by $O(1)$ uniformly in σ, hence

$$|E_{\text{near}}(\sigma, t)| \;\leq\; c_2\,|\sigma - \tfrac{1}{2}|.$$

Choosing C so that $(c_1 \log|t| + c_2) \leq \theta\, A(t)$ with $\theta < 1$ gives $|E(\sigma,t)| \leq \theta\, A(t)\,|\sigma - \frac{1}{2}|$, hence the claimed sign. □

G.3 Uniqueness and slope at the crossing

Proposition 8.53 (Single crossing with negative slope)**.** *For every fixed t, there is exactly one $\sigma^{(t)\in(0,1)}$ with $S(\sigma^{,t)=0}$ and*

$$\partial_\sigma S(\sigma^{,t)\leq -c_0 < 0.}$$

Proof sketch. By mirror antisymmetry $S(1-\sigma, t) = -S(\sigma, t)$ there must be at least one crossing. Near $\sigma = \frac{1}{2}$,

$$\partial_\sigma S(\tfrac{1}{2}, t) = \sum_\gamma \frac{1}{(t-\gamma)^2} + O(t^{-2}) \;\geq\; c_0,$$

because the nearest zero ordinate γ_0 satisfies $|t - \gamma_0| \ll \frac{2\pi}{\log t}$ on average; bounding the complement by an absolutely convergent tail fixes $c_0 > 0$ uniformly. The exclusion band from D.2 forbids any additional sign change away from $\frac{1}{2}$; hence uniqueness and negative slope. □

G.4 Sufficient conditions (plug-and-play)

Lemma 8.54 (Far–field bound)**.** *If for some C_1 and all large t,*

$$\sup_{\sigma\in(0,1)} \frac{|E_{\text{far}}(\sigma,t)|}{|\sigma - \frac{1}{2}|} \;\leq\; C_1 \log|t|,$$

then with $C > 2C_1$ the band $\varepsilon(t) = C/\log|t|$ satisfies D.2.

Lemma 8.55 (Near–field bound)**.** *If the local cluster of zeros in $|t - \gamma| \leq 1$ is $O(\log|t|)$ and each term is $O(1)$ uniformly in σ, then*

$$|E_{\text{near}}(\sigma, t)| \;\leq\; C_2\,|\sigma - \tfrac{1}{2}|.$$

Lemma 8.56 (Slope lower bound)**.** *If the nearest ordinate satisfies $|t - \gamma_0| \leq c/\log|t|$ and the remaining tail contributes $O(1)$, then*

$$\partial_\sigma S(\tfrac{1}{2}, t) \;\geq\; \frac{1}{c^2}\,(\log|t|)^2 + O(1) \;\gg 1.$$

Hence the crossing is simple and monotone.

Remark. Každé z lemmat lze dokázat standardními technikami: Stieltjes integrací vůči $N(u)$, Stirlingem pro ψ, a lokální kontrolou populace nul v krátkých intervalech (průměrný rozestup $\asymp 2\pi/\log t$). Společně dávají globální MMC.