Radius/Momentum Calculation for S444 Experiment February 2020 - Overview

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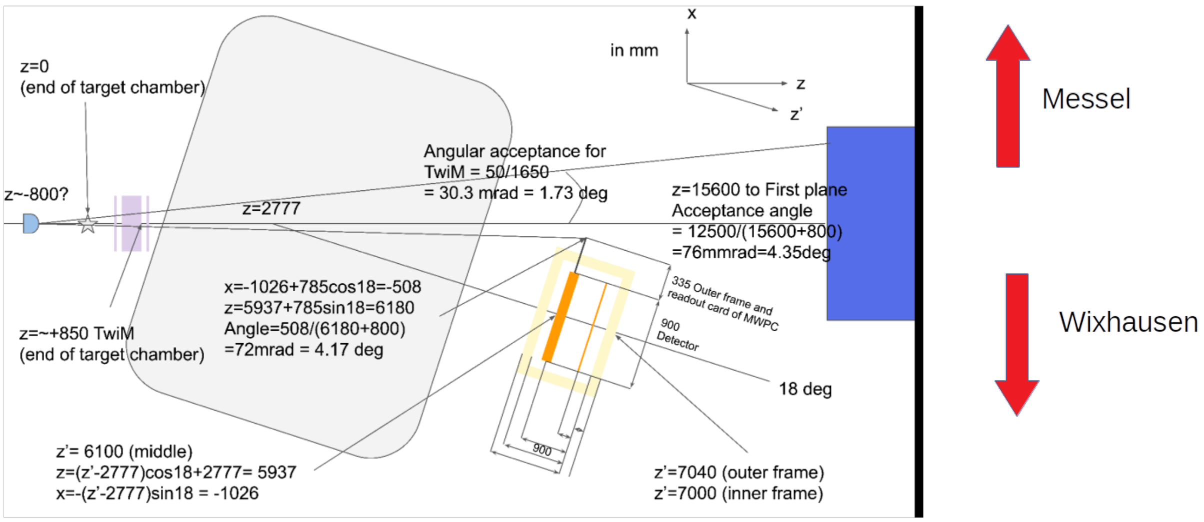
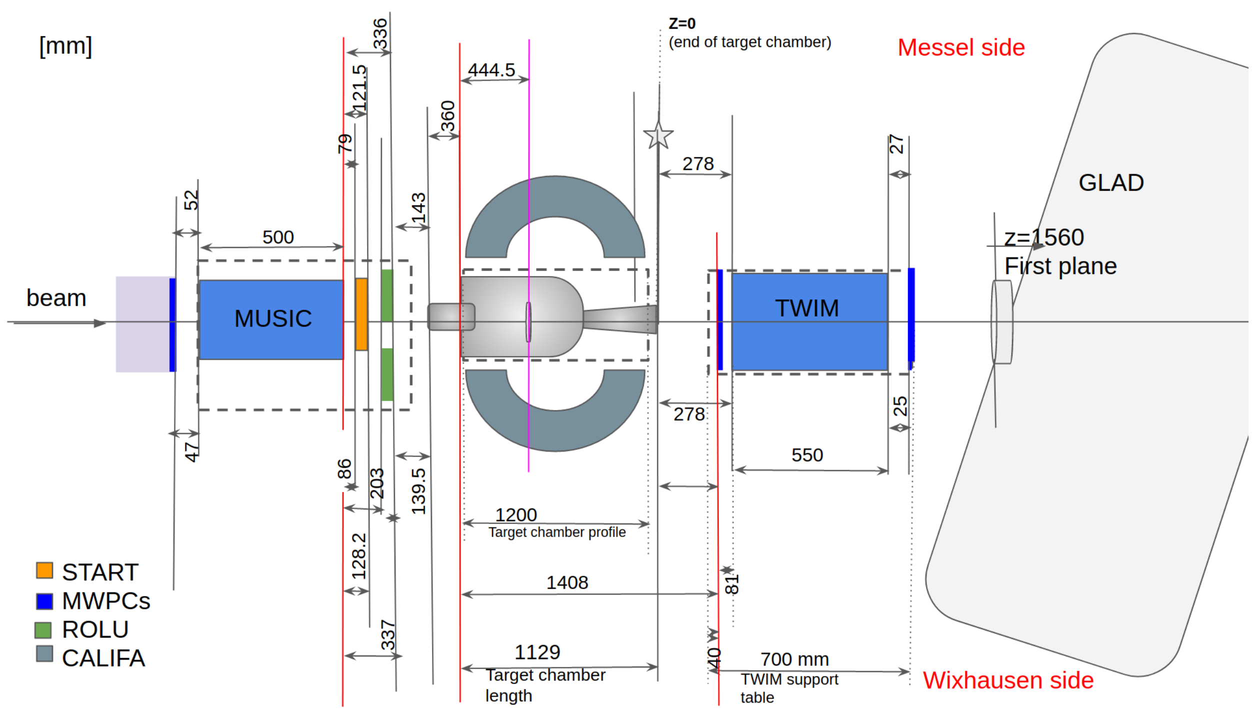
0.1 The Setup



z=0
(end of target chamber)
in mm
x
z
z'
z~-800?
z=2777
Angular acceptance for
TwiM = 50/1650
= 30.3 mrad = 1.73 deg
z=15600 to First plane
Acceptance angle
= 12500/(15600+800)
=76mmrad=4.35deg
x=-1026+785cos18=-508
z=5937+785sin18=6180
Angle=508/(6180+800)
=72mrad = 4.17 deg
335 Outer frame and readout card of MWPC
900 Detector
z=~+850 TwiM
(end of target chamber)
18 deg
900
z'= 6100 (middle)
z=(z'-2777)cos18+2777= 5937
x=-(z'-2777)sin18 = -1026
z'=7040 (outer frame)
z'=7000 (inner frame)
Messel
Wixhausen



[mm]
Z=0
(end of target chamber)
Messel side
336
121.5
444.5
360
79
278
27
GLAD
52
500
143
z=1560
First plane
beam
MUSIC
TWIM
278
25
47
86
203
139.5
550
1200
Target chamber profile
128.2
1408
81
START
MWPCs
ROLU
CALIFA
337
1129
Target chamber
length
40
700 mm
TWIM support
table
Wixhausen side

1 Geometry and relative position of the detectors in the beam direction

Here, the positions are given for the s444 and s467 experiments

z position of the MWPC0: zMW0 = -2520 mm
z position of the target: zT = -684.5 mm
z position of the MWPC1 in front of the Twin-MUSIC: zM1 = 279 mm
z position of the middle of the Twin-MUSIC: zTwin = 553 mm
z position of the MWPC2 after the Twin-MUSIC: zM2 = 854 mm
α tilted angle of GLAD (14 degrees): = 0.244 rad
effective length of GLAD: L_eff = 2067 mm
z middle of GLAD zGm = 2577mm
horizontal of the central path (18 degree) θ_out0 = pi/10 rad
z position of the MWPC3 after GLAD zM3 = 5937 mm
z position of the ToFwall zToFW = 6660.2 mm

Correspondence between the GLAD current and the magnetic field: I = 3584 A, B = 2.2 T

Positions of the TOFWPads:
1 $\Rightarrow$ Messel
27$\Rightarrow$ Wixhausen

2 RUNS used for calibration = SWEEP RUNS without target

RUN	Beam ion	Beam Energy [AmeV]	GLAD current [A]	Comments
36	12C primary	400	1444	before broken motor, here we see that tof is about 5ns faster. So they probably changed the position of the TOFW afterwards
37	12C primary	400	1444	it has be seen that motor drive not working
38	12C primary	400	1444	tof is back with new gates *magnet sweep 1444A
39	12C primary	400	1498	
40	12C primary	400	1501	
41	12C primary	400	1501	stopped with 1558 A
42	12C primary	400	1558	
43	12C primary	400	1558	stopped with 1653 A
44	12C primary	400	1653	
45	12C primary	400	1653	stopped with 1748 A
46	12C primary	400	1748	
47	12C primary	400	1748	stopped with 1843 A
48	12C primary	400	1843	
49	12C primary	400	1843	stopped with 1938 A
51	12C primary	400	1938	
52	12C primary	400	1938	stopped with 1444 A
53	12C primary	400	1444	
54	12C primary	400	1444	stopped with 1349 A
55	12C primary	400	1349	
56	12C primary	400	1349	stopped with 1254 A
57	12C primary	400	1254	
58	12C primary	400	1254	stopped with 1159
59	12C primary	400	1159	
60	12C primary	400	1159	stopped with 1064
61	12C primary	400	1064	
62	12C primary	400	1064	stopped with 1444 A
123	12C primary	650	1748	stopped with 1957
124	12C primary	650	1957	

(red)	=	sweeping
(green)	=	stable GLAD current

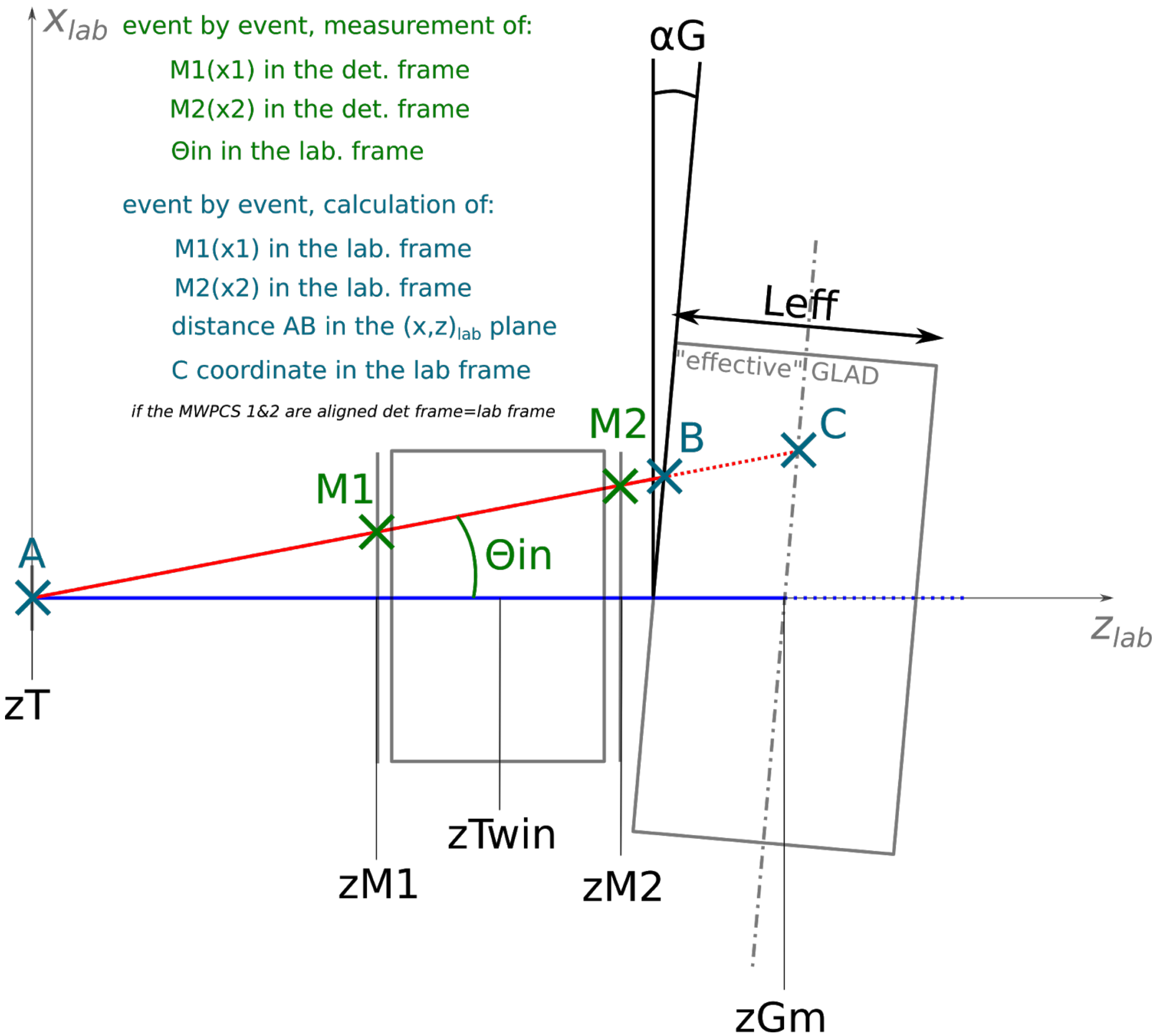
2.0.1 Other RUNS used for various checks:

RUN 70: 2 cm C target
RUN 80: 10.86 mm C target
RUN 81: 24.53 mm CH2 target
RUN 67: 24 mm CH2 target
RUN 68: 1 cm C target
RUN 79: 12.29 mm CH2 target
RUN 75: 21.98mm C target

3 Methods for flightpath reconstruction in the (x,z) plane

3.1 The "Kickplane" method

3.1.1 From MW0 to the entrance of GLAD, the ion is following a straight line

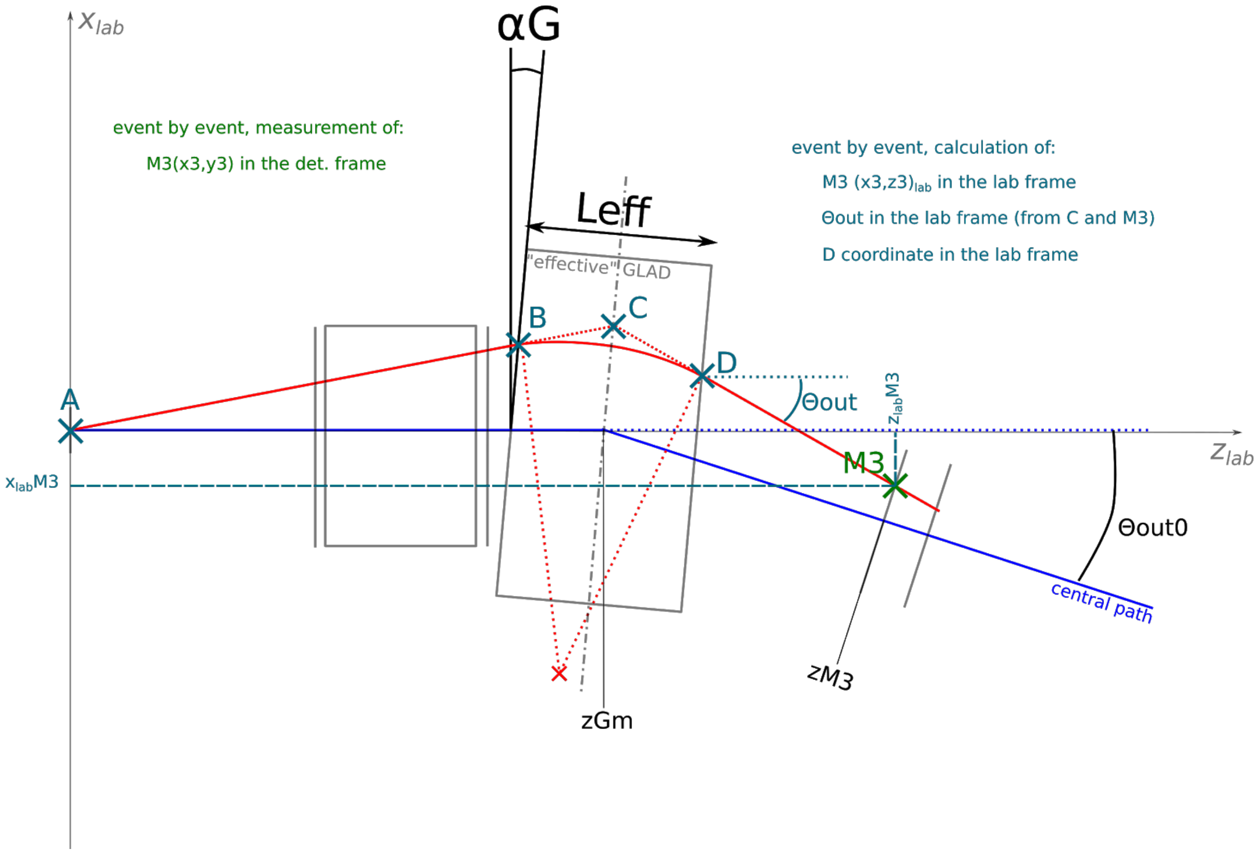


The straight line trajectory from MW0 to entrance before glad is defined by:
⇒ one absolute value before GLAD
absolute = calibrated position in mm in the laboratory frame
To get the position, use the position given by one MWPCs (1 or 2)

⇒ the theta angle (theta_in) before GLAD

Angle obtained from comining MWPC1 and MWPC2
(to get higher precision the drift time in TWIM MUSIC could be used)

3.1.2 From entrance to the exit of GLAD, the effective trajectory is circular



The circular trajectory is defined by:
⇒ one absolute position before GLAD B and angle theta_in (see: 3.1.1)
⇒ one absolute position at MWPC3

From this information the angle theta_out is constructed in follwing steps:

1. Extend the line of flight of the ion before the GLAD.
2. The point of intersection with the ”kickplane” (symmetry axis line of GLAD magnet) is the kickpoint C.
3. Draw a straight line between C and the absolute position at MWPC3 = M3.
4. theta_out is the positive angle between the z-beam direction and the line between C and M3.

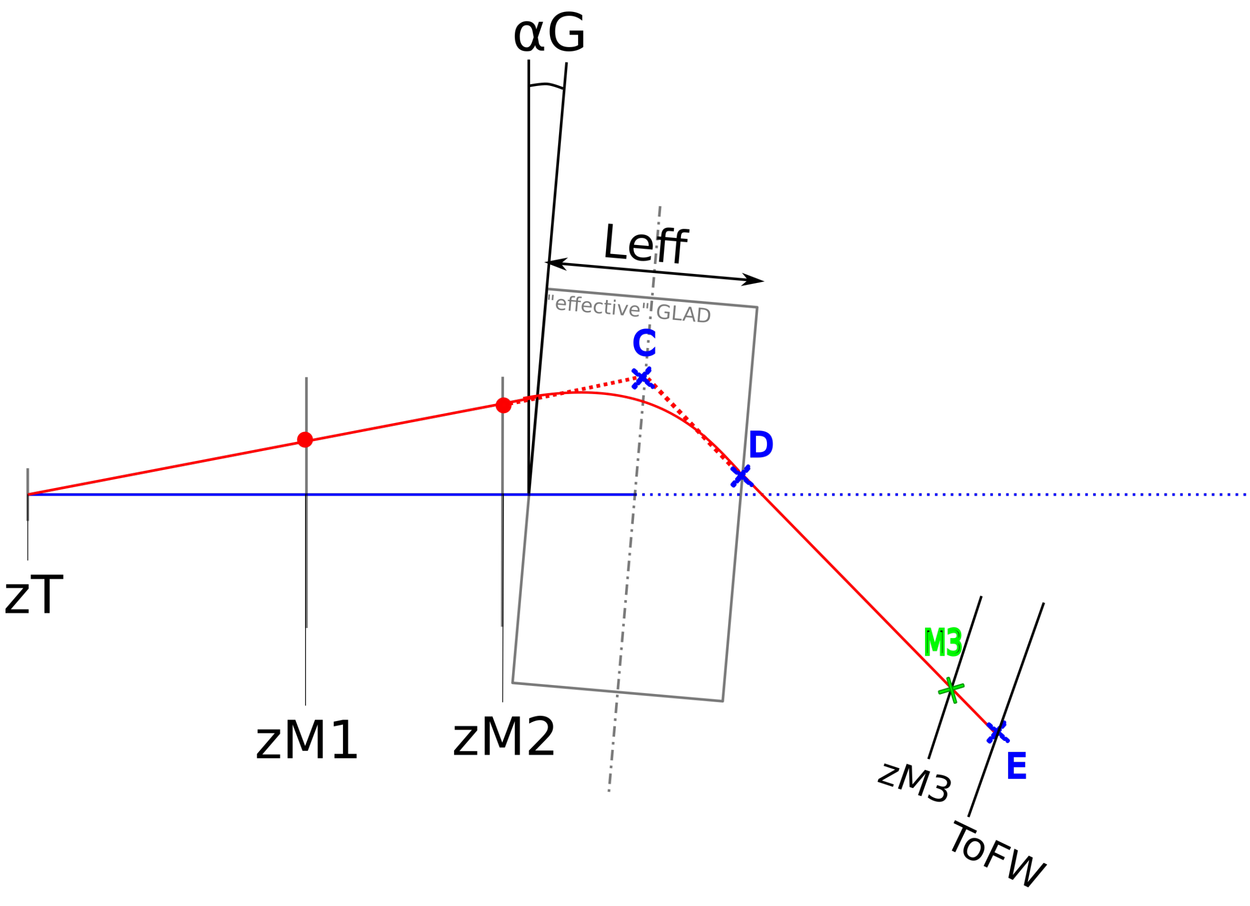
The curvature radius ρ is given by[1]:
$\rho = \frac{\text{L\_eff}}{2\cdot\sin\left(\frac{theta\_in}{2}+\frac{theta\_out}{2}\right)\cdot\cos(\delta)}$

With δ:
$\delta = \arctan\left(\left|\frac{\frac{cos(theta\_out)-\cos(theta\_in)}{\sin(theta\_out)+\sin(theta\_in)}+\tan(\alpha)}{1-\frac{cos(theta\_out)-\cos(theta\_in)}{\sin(theta\_out)+\sin(theta\_in)}\cdot\tan(\alpha)}\right|\right)$
The full derivation can be found in the appendix.

The circular trajectory is then given by:
$\omega = 2 * \left|\arcsin\left[\frac{BD}{2\cdot\rho}\right]\right|$

with BD = length of the BD segment

3.1.3 After GLAD up to the TOFW, the trajectory is a straight line



The straight line trajectory from D to E is definded by:

[1]for consistency checks the $\cos(\delta)$ term can be omitted, as it plays a minor role

⇒ **the output angle from GLAD theta_out**
⇒ **one absolute position after GLAD in the laboratory frame M3**

With this information the straight line trajectory lenght after GLAD can be measured. It starts at the exit point of GLAD D and follows the straigh line (characterized by the angle theta_out and the absolute position at MWPC3) until the intersection with the TofW (middle position of the ToFWall zToFW= $6660.2mm$, tilted angle= $18°$).

Finally the pathlength in the (x,z) plane from the target position to the ToFW is given by:
P = AB + $\rho \cdot \omega$ +DE

where:
A = (x,z) position at the target point
B = (x,z) position at the GLAD entry point
D = (x,z) position at the GLAD exit point
E = (x,z) position where the constructed trajectory line hits the ToFW

The assumption for the "Kickplane" method is that the kickpoint for each event lies on the predefinded Kickplane, the symmetry axis line of the GLAD magnet.

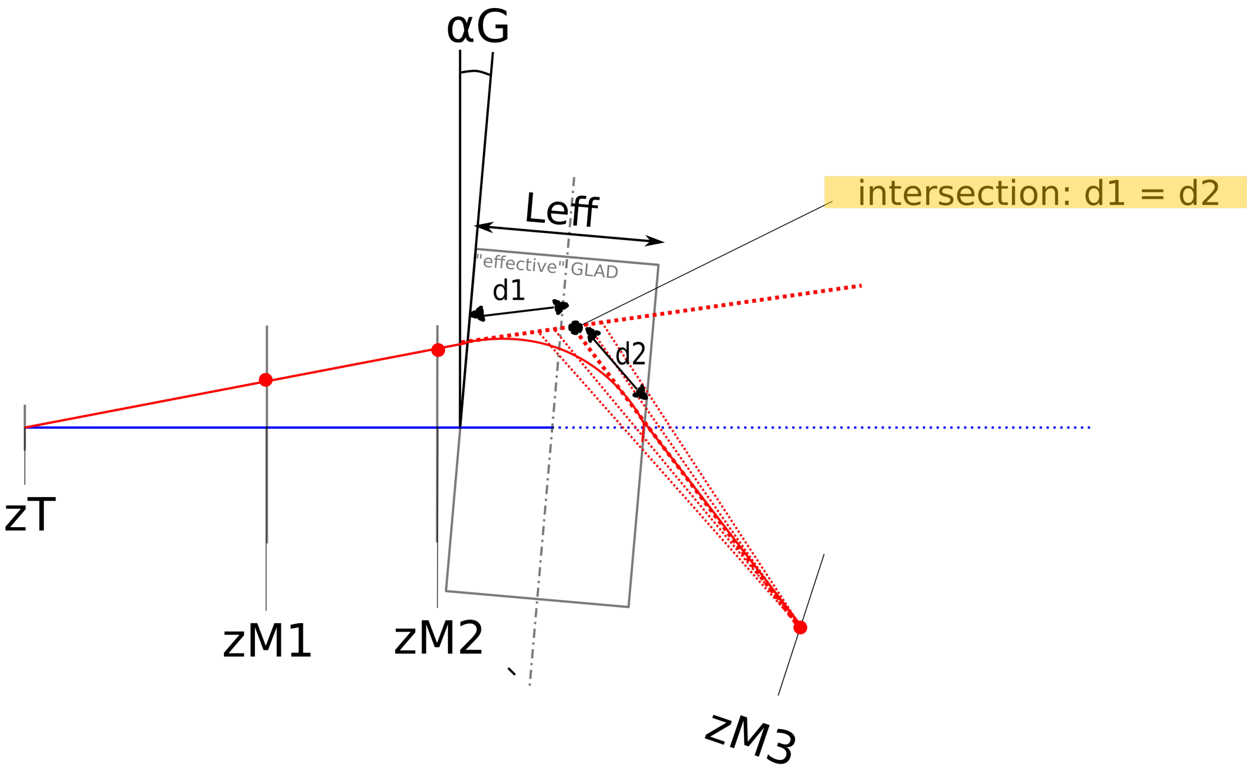
3.2 The "Fit-Track" method

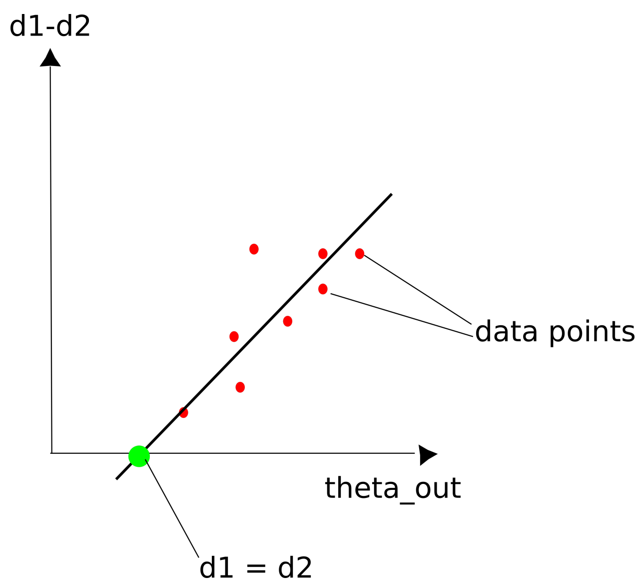
For the "Fit-Track" method the assumption that the kickpoint C lies on the symmetry axis line of the GLAD magnet is rejected. Instead following algorithm is applied: [2]

1. Extend the line of flight of the ion before the GLAD.
2. Draw a line from the point MW3 to C (as constructed with the "Kickplane" method).
3. Now sweep the straight line after the kickpoint, leaving the position MW3 unchanged but sweeping the intersection point along the inline beam.
4. For each sweeping step plot theta_out versus (d1-d2) where d1 is the distance betweeen B and the point of intersection and d2 the distance between D and the intersection point accordingly.
5. 50 sweeping steps are performed.
6. Fit the final theta_out versus (d1-d2) plot with linear least square fit.

[2] This algorithm is motivated from https://www.blogs.uni-mainz.de/fb08-kernphysik/files/2018/09/PHDThesis_OlgaBertini.pdf, section 3.4

7. Find the intersection of the abscissa. The corresponding theta_out value is now the corrected one which should be used for the calculation of the radius.





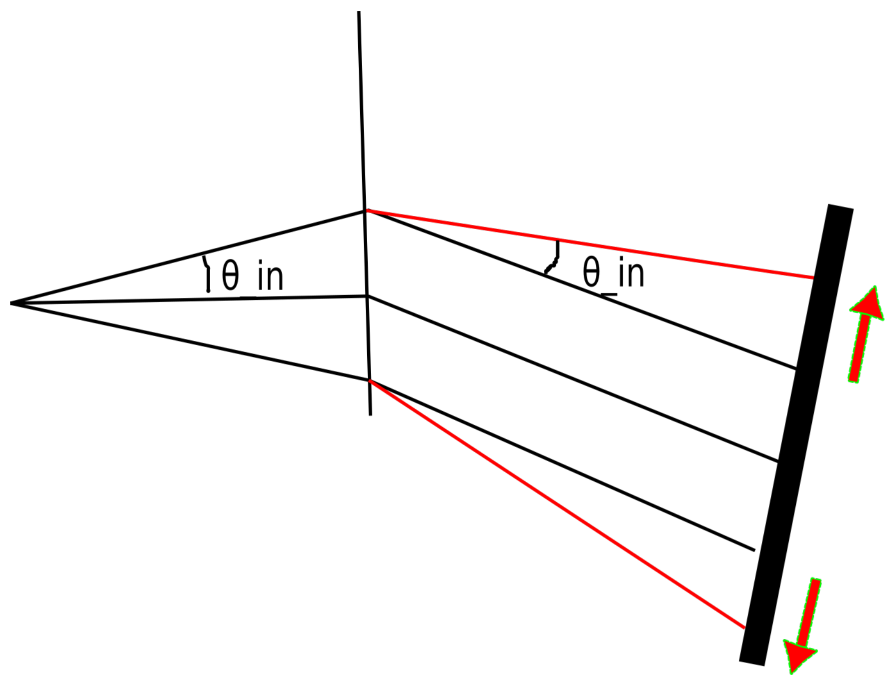
3.3 The "Theta_in correction" method

Here the "Kickplane" method is used and subsequently the theta_out is corrected by theta_in. That means:

theta_out_corr = theta_out - theta_in.

Consequentely the theta_in dependence of ρ vanishes(neglecting the $\cos(\delta)$ term):

$$\rho = \frac{\text{L\_eff}}{2{\cdot}\sin\left(\frac{theta\_in}{2}+\frac{theta\_out\_corr}{2}\right)} = \frac{\text{L\_eff}}{2{\cdot}\sin\left(\frac{theta\_out}{2}\right)}$$



3.4 Final method: "Advanced Fit-Track" method

The same track finding algorithm as for the "Fit-Track" method is used with the only difference that the value for *theta_in* is calculated from the fit of *theta_out* vs. xMW3:

The parameters of the linear fit are used for the calculation of *theta_in* :

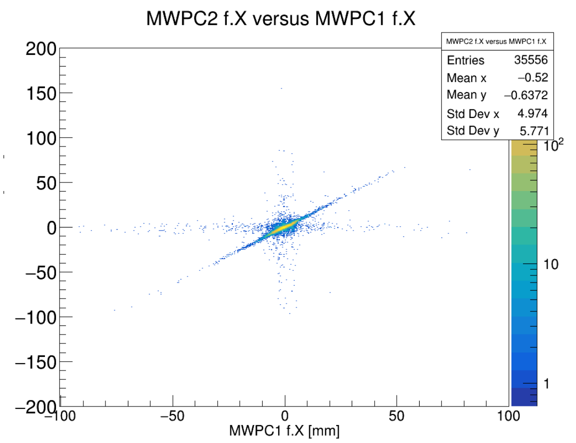
$$theta\_in = \alpha - a \cdot x\_MW3 - b$$

With **a** being the slope and **b** the offset of the fit. This method prevents from adding up the errors from theta_in measurement.
α corresponds to the mean value of the theta _out from the fit of *theta_out* vs. xMW3.

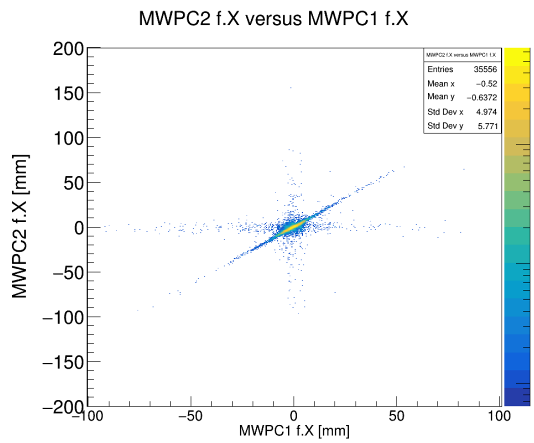
4 Plots

In this section all the plots for the various track finding algorithms are presented. For the calculation of the theta_in angle MWPC1 and MWPC2 are used. Alternatively MWPC0 and MWPC2 could be used, to get a longer lever arm (work in progress ...).

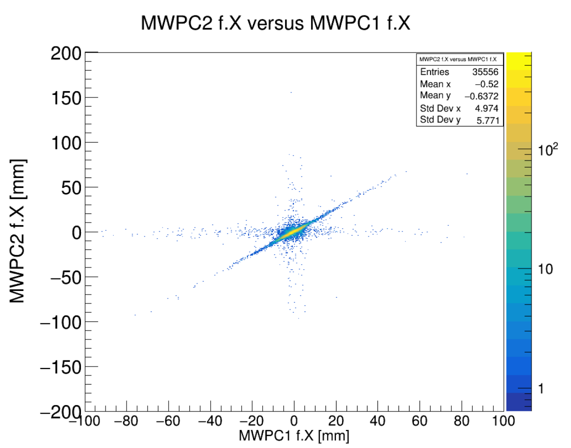
4.1 MWPC1 vs MWPC2 - x position



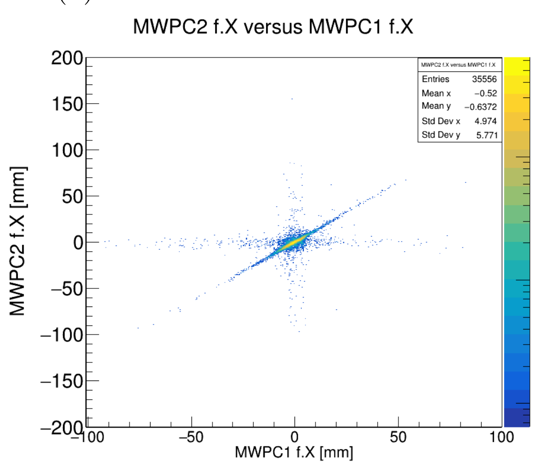
(a) "Kickplane-Method"



(b) "Theta _in correction-Method"



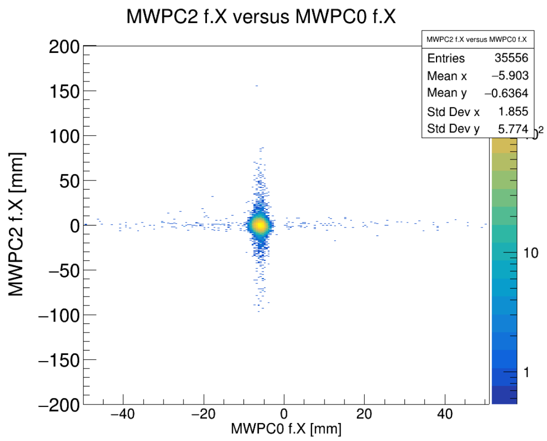
(c) "Fit-Track-Method"



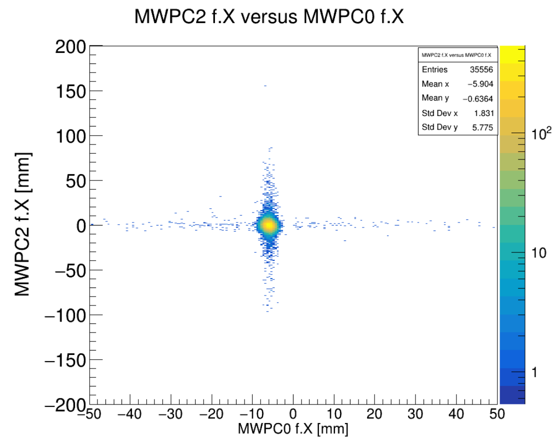
(d) "Advanced Fit-Track-Method"

Figure 1: MWPC1 vs MPWPC2 - x position for sweep runs 39-61.

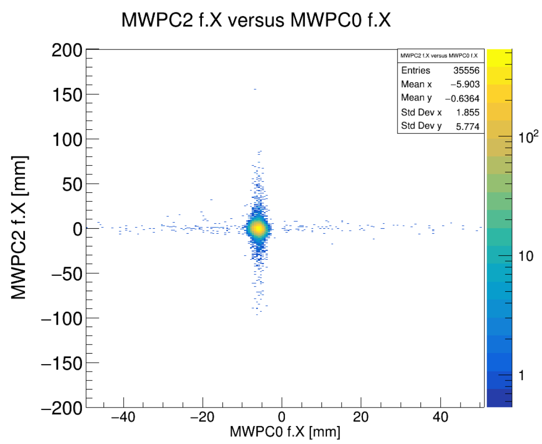
4.2 MW0 vs MW2- x position



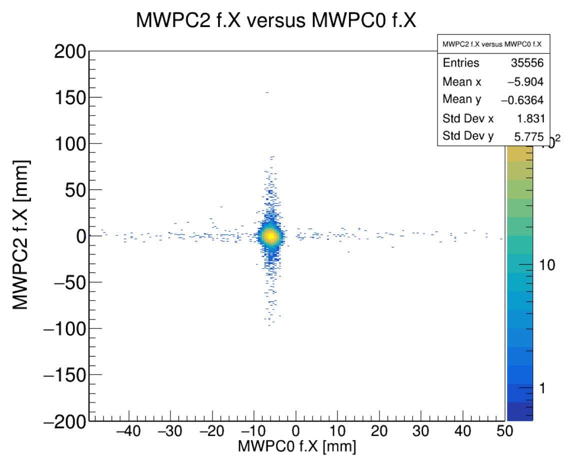
(a) "Kickplane-Method"



(b) "Theta _in correction-Method"



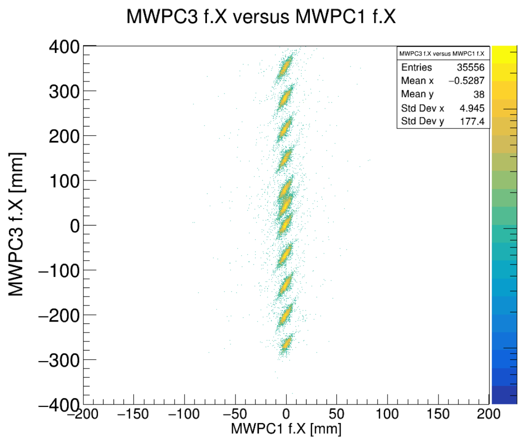
(c) "Fit-Track-Method"



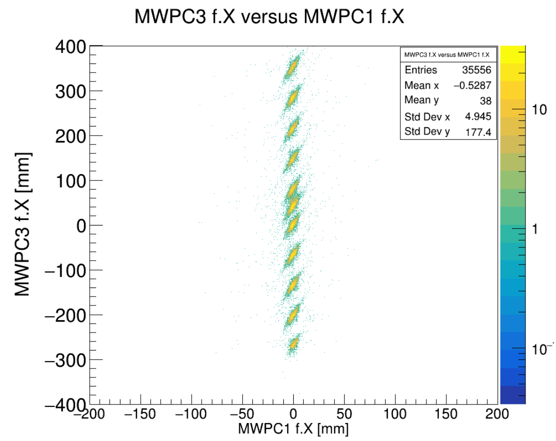
(d) "Advanced Fit-Track-Method"

Figure 2: MWPC2 vs MWPC0 - x position for sweep runs 39-61.

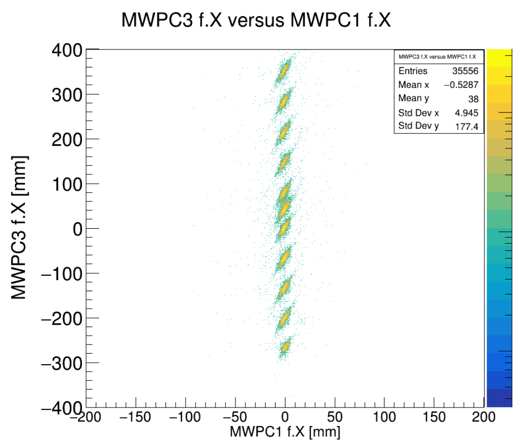
4.3 MW1 vs MW3 - x position



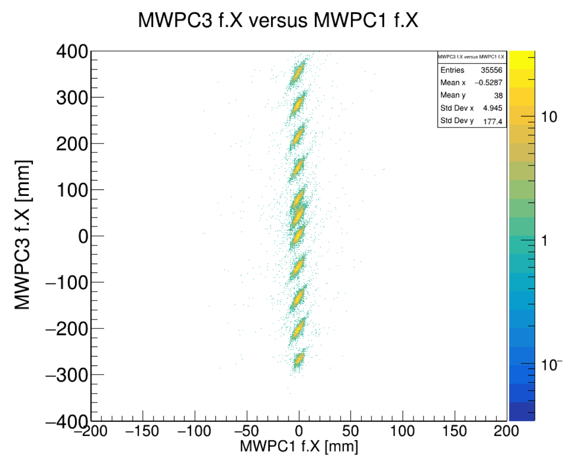
(a) "Kickplane-Method"



(b) "Theta _in correction-Method"



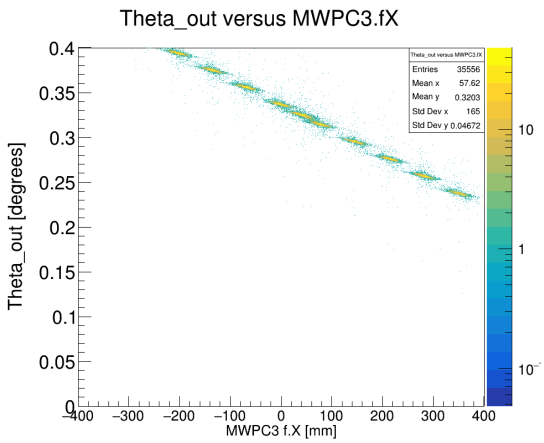
(c) "Fit-Track-Method"



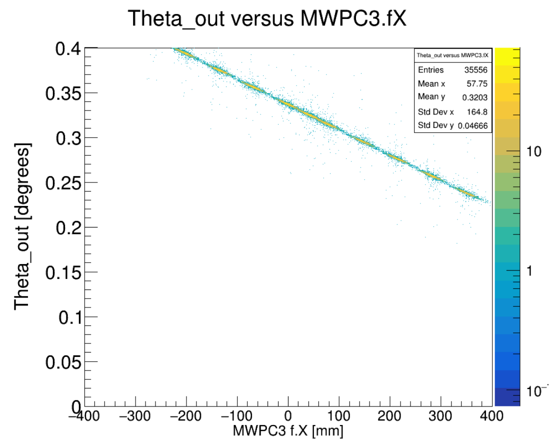
(d) "Advanced Fit-Track-Method"

Figure 3: MWPC3 vs MWPC1 - x position for sweep runs 39-61.

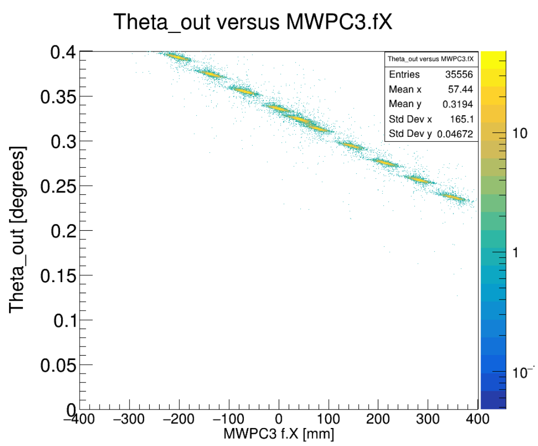
4.4 theta_out vs MW3- x position



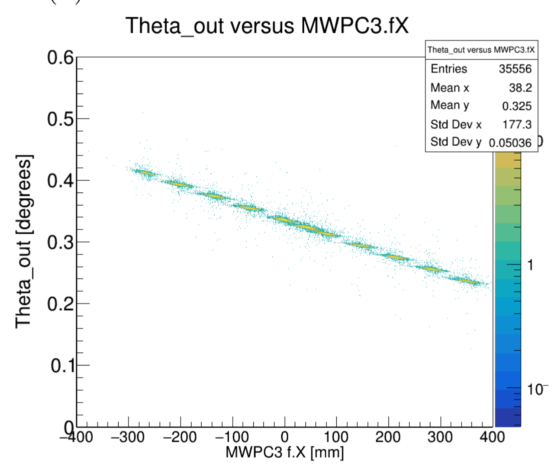
(a) "Kickplane-Method"



(b) "Theta_in correction-Method"



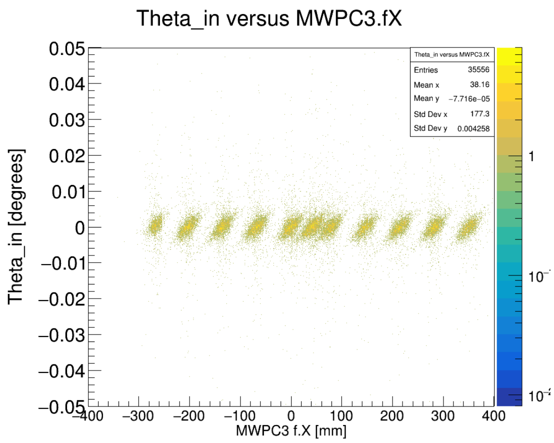
(c) "Fit-Track-Method"



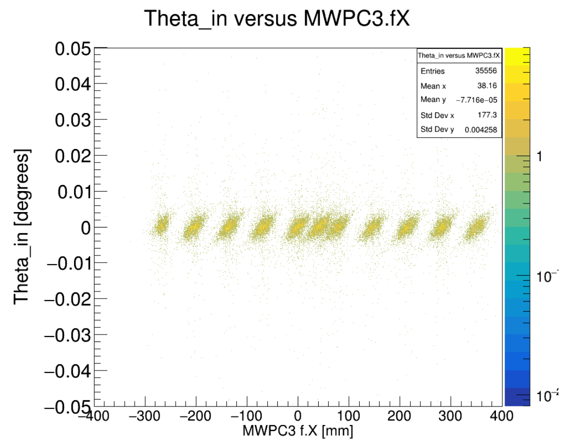
(d) "Advanced Fit-Track-Method"

Figure 4: Theta_out vs MWPC3 x position for sweep runs 39-61.

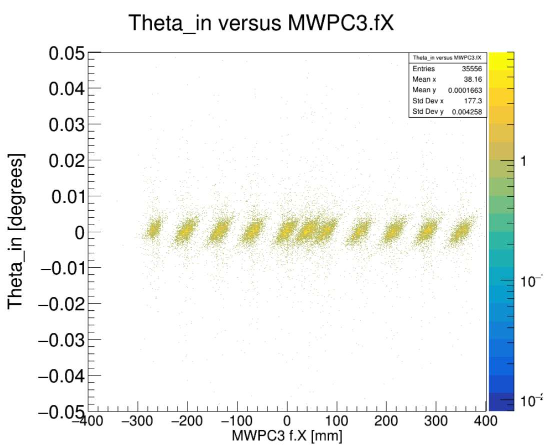
4.5 theta_in vs MW3 - x position



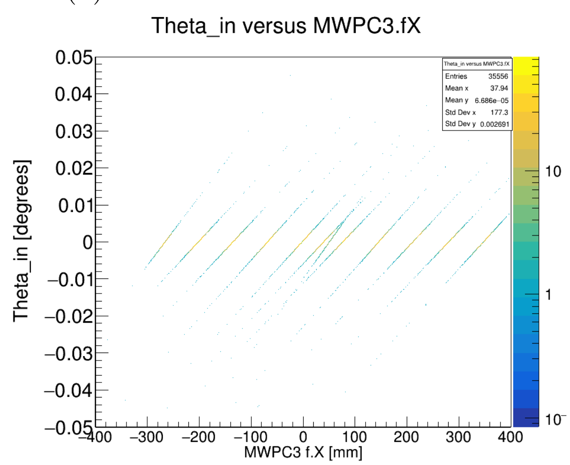
(a) "Kickplane-Method"



(b) "Theta _in correction-Method"



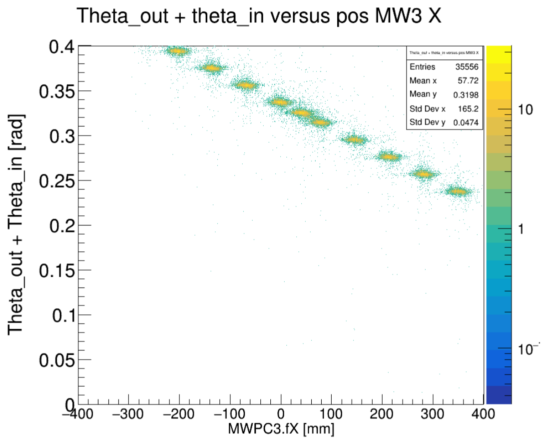
(c) "Fit-Track-Method"



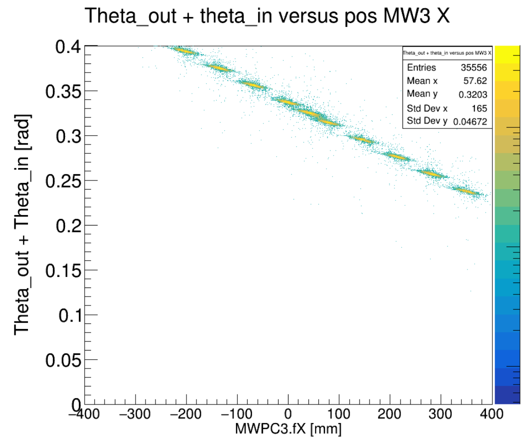
(d) "Advanced Fit-Track-Method"

Figure 5: Theta _in vs MWPC3 x position for sweep runs 39-61.

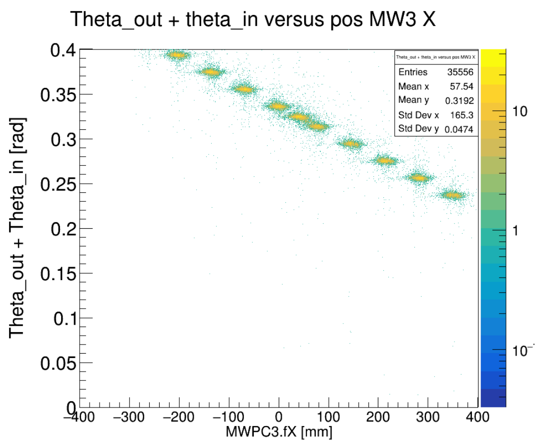
4.6 theta_in+theta_out vs MW3- x position



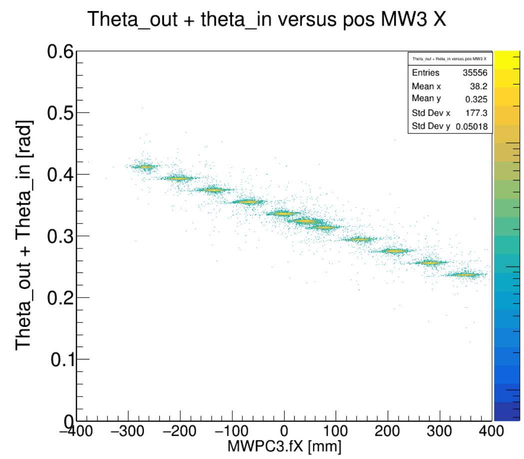
(a) "Kickplane-Method"



(b) "Theta _in correction-Method"



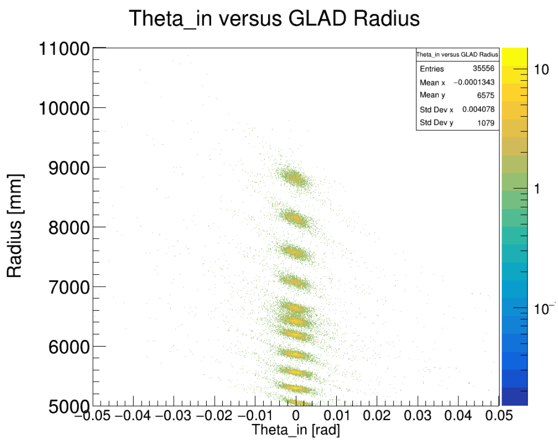
(c) "Fit-Track-Method"



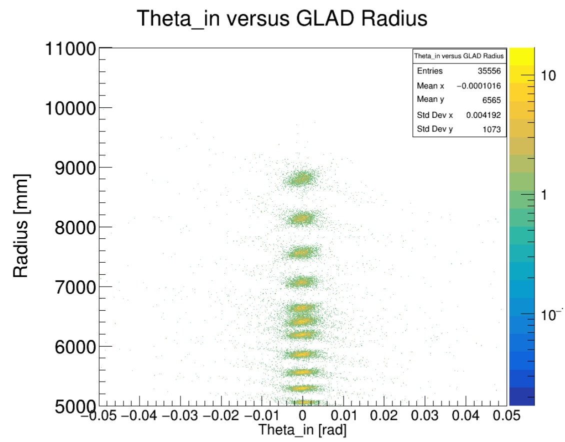
(d) "Advanced Fit-Track-Method"

Figure 6: Theta _out + theta _in vs MWPC3 x position for sweep runs 39-61.

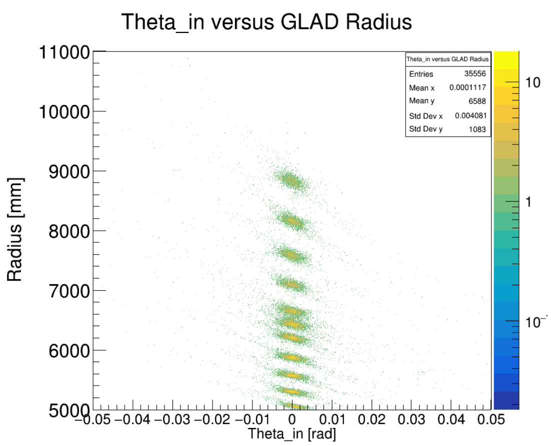
4.7 theta_in vs Radius



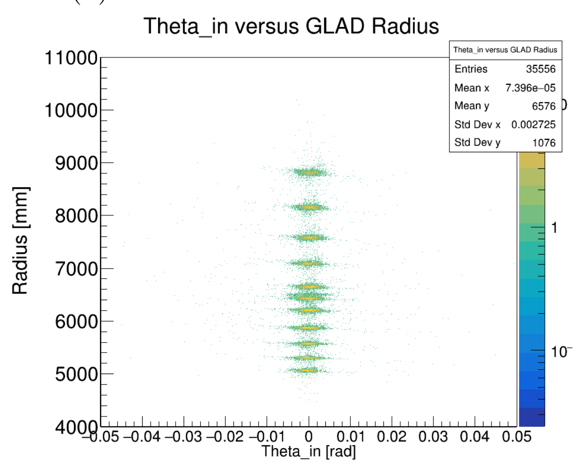
(a) "Kickplane-Method"



(b) "Theta _in correction-Method"



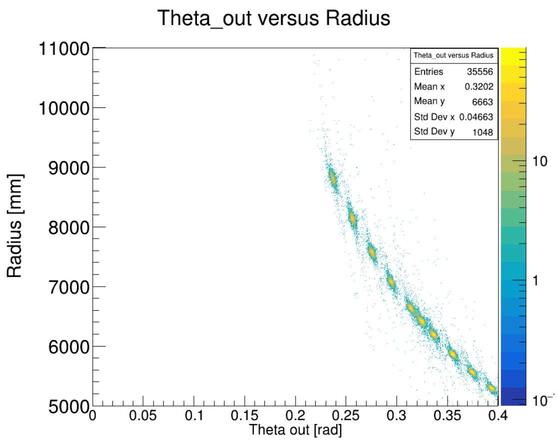
(c) "Fit-Track-Method"



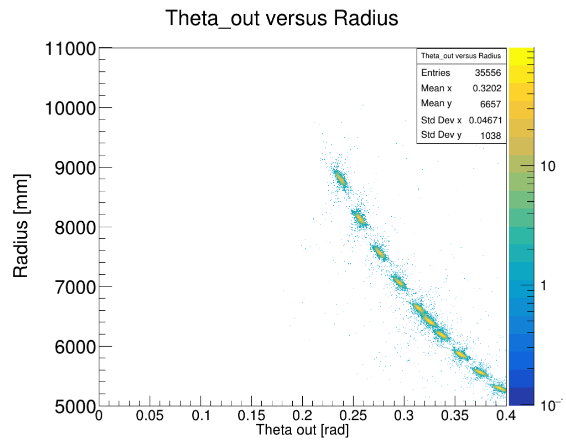
(d) "Advanced Fit-Track-Method"

Figure 7: Theta _in vs GLAD Radius for sweep runs 39-61.

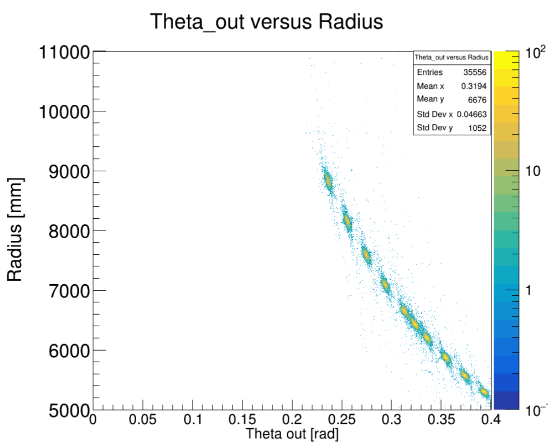
4.8 theta_out vs Radius



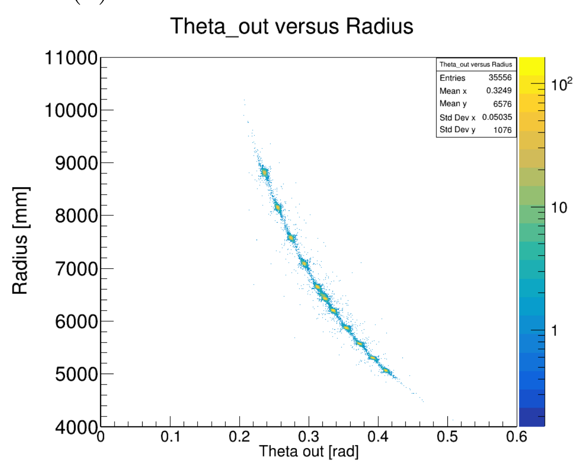
(a) "Kickplane-Method"



(b) "Theta _in correction-Method"



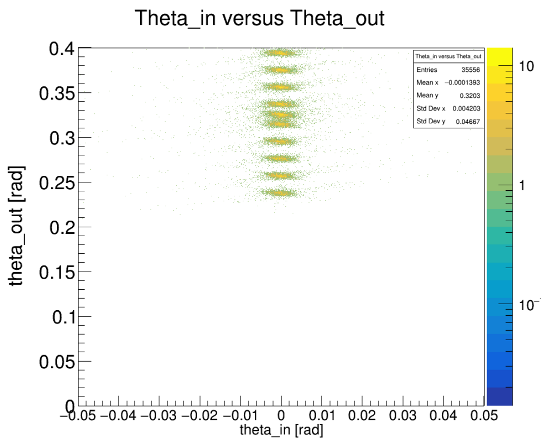
(c) "Fit-Track-Method"



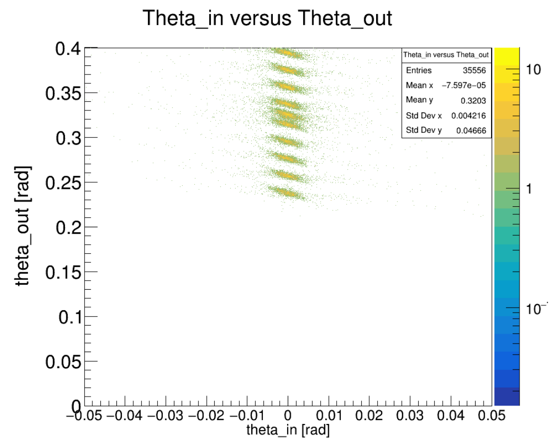
(d) "Advanced Fit-Track-Method"

Figure 8: Theta _out vs GLAD Radius for sweep runs 39-61.

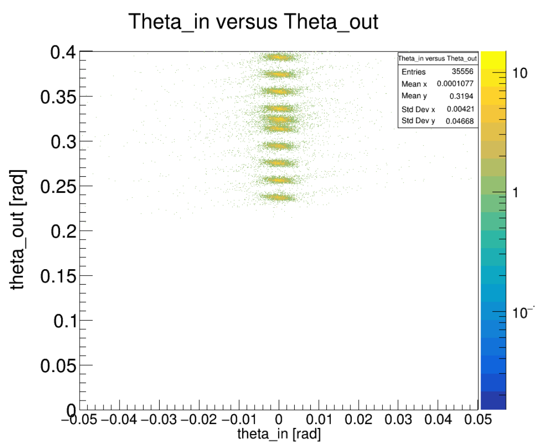
4.9 theta_in vs theta_out



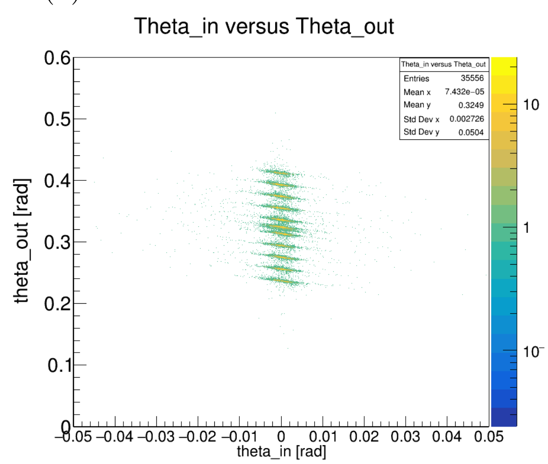
(a) "Kickplane-Method"



(b) "Theta _in correction-Method"



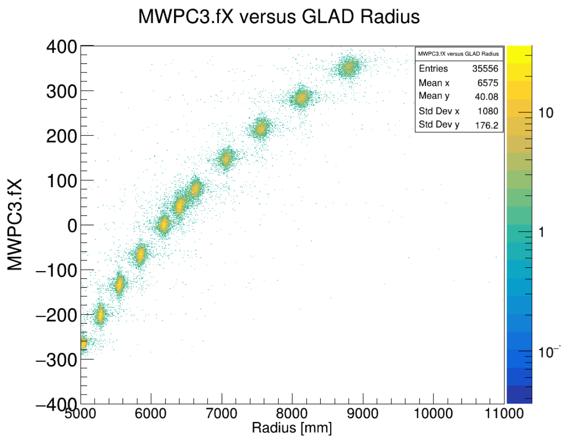
(c) "Fit-Track-Method"



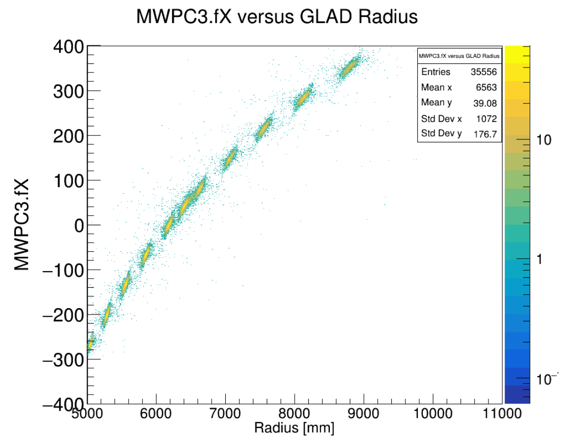
(d) "Advanced Fit-Track-Method"

Figure 9: Theta _in vs theta _out for sweep runs 39-61.

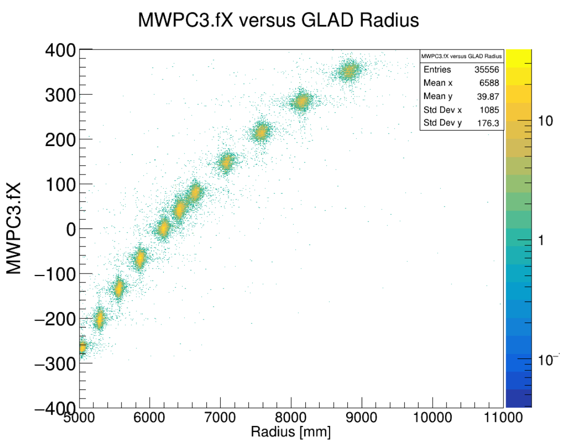
4.10 MW3 vs Radius - x position



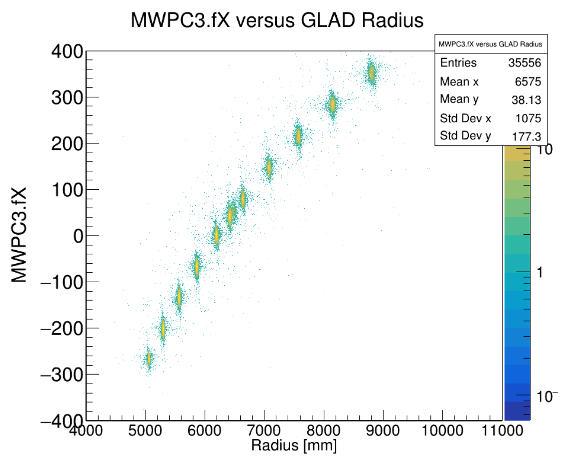
(a) "Kickplane-Method"



(b) "Theta _in correction-Method"



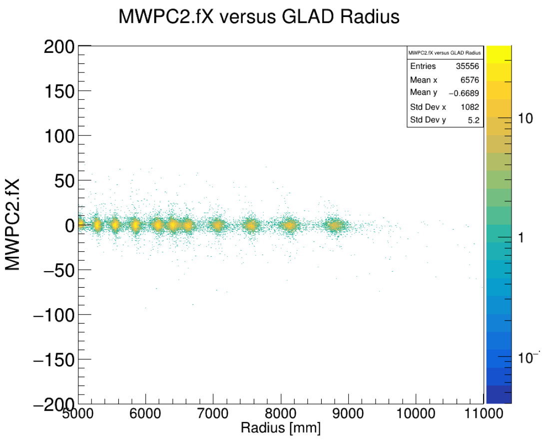
(c) "Fit-Track-Method"



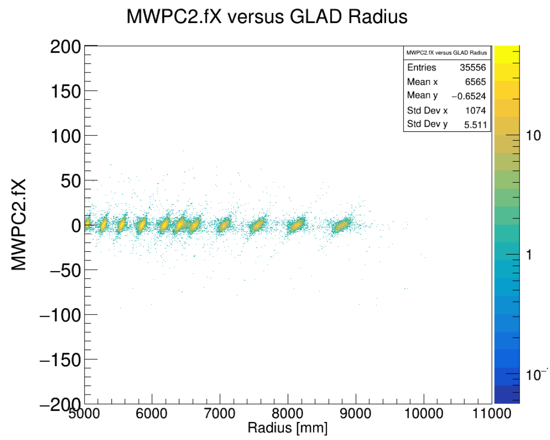
(d) "Advanced Fit-Track-Method"

Figure 10: MWPC3 x position vs GLAD Radius for sweep runs 39-61.

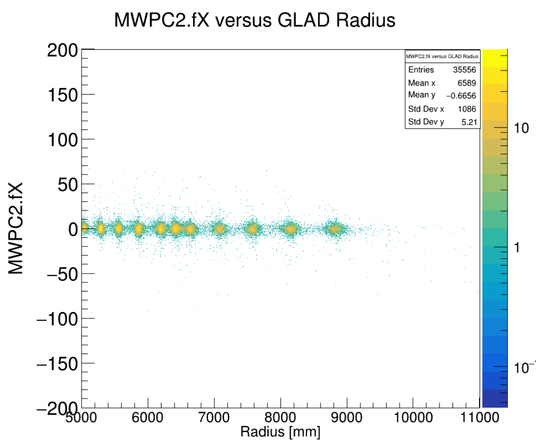
4.11 MW2 vs Radius - x position



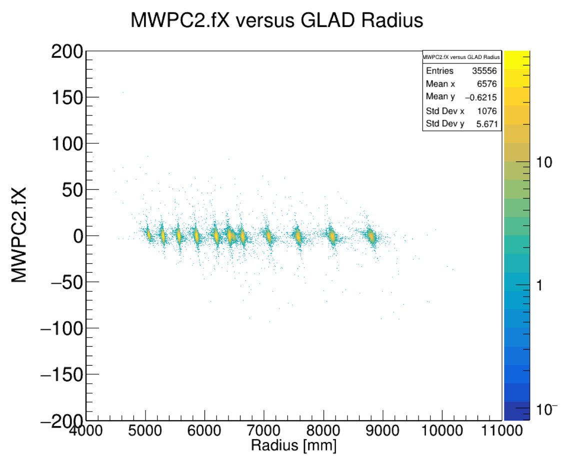
(a) "Kickplane-Method"



(b) "Theta_in correction-Method"



(c) "Fit-Track-Method"



(d) "Advanced Fit-Track-Method"

Figure 11: MWPC2 x position vs GLAD Radius for sweep runs 39-61.

5 Relative momentum resolution "Advanced Fit-Track-Method"

The momentum resolution is calculated from the radius-calculation as $\rho \sim p$. From that follows:

$\frac{\Delta p}{p} = \frac{\Delta \rho}{\rho}$

For the evaluation of the resolutions for the various sweep runs the two dimensional plot "MWPC3.fX versus GLAD Radius" is projected on the abscissa. The resulting 1D plot is fitted with a gaussian. The mean value from the fit corresponds to ρ and the σ to $\Delta\rho$ respectively.

Runnr.	$\overline{\rho}$	σ	rel. resolution
39	6.48911e+03	2.13410e+01	3.289e-03
40	6.42396e+03	1.56313e+01	2.433e-03
42	6.20033e+03	1.42716e+01	2.301e-03
44	5.86854e+03	1.29449e+01	2.206e-03
46	5.57125e+03	1.31369e+01	2.358e-03
48	5.30203e+03	1.07044e+01	2.018e-03
51	5.06232e+03	1.07182e+01	2.117e-03
53	6.64556e+03	1.66141e+01	2.500e-03
55	7.08710e+03	1.91895e+01	2.708e-03
57	7.57474e+03	2.16135e+01	2.853e-03
59	8.15089e+03	2.22151e+01	2.725e-03
61	8.81055e+03	2.72038e+01	3.088e-03

With mean relative resolution $\frac{\Delta\rho}{\rho}$ = 2.55e-03.

6 Limiting Radius/Momentum resolution factors

The radius/momentum resolution is limited by:

- Energy straggling
- Angular straggling
- Angular resolution of MWPC1/2
- position resolution of MWPC3 (higher order??)

6.1 Engergy straggling

For the error calculation the mean energy inside the GLAD was used and the corresponding standard deviation. Generally for a charged particle drifting through a magnetic field the radius of the curved path the particle is following is defined as:

$\rho = \frac{\gamma \cdot \beta \cdot m}{q \cdot B}$
(considering c = 1)
Energy straggling has an effect on β and γ respectively. The measurement uncertainty with respect to β can be calculated as follows:
$\frac{d\rho}{d\beta} = \frac{1}{(\sqrt{1-\beta^2})^3} \cdot \frac{m}{qB}$

$\Delta\rho_\beta = \frac{d\rho}{d\beta} \cdot \Delta\beta = \frac{\Delta\beta}{(\sqrt{1-\beta^2})^3} \cdot \frac{m}{qB}$

with relative measurement uncertainty $\frac{\Delta\rho_\beta}{\rho} = 5.434e - 04$

6.2 Angular straggling

For the error calculation originating from angular straggling it is considered only angular straggling starting from the backend of the MWPC1. That means for this error calculation it is assumed to have a perfect focussed beam undergoes no broadening until it hits the MWPC1 (always at the same x-y-z position). Angular straggling broadens the beam focus and affects therefore both Θ_in and Θ_out.
(angular straggling for Θ_in can be neglected, higher order...)
For the calculation of measurement uncertainty with respect to Θ_out we use the simplified geometrical formula of the radius:
$\rho = \frac{L_{eff}}{2 \cdot \sin(\frac{\Theta in + \Theta out}{2})}$[3]
From that follows:
$\Delta\rho_{\Theta out} = -\rho \cdot \frac{1}{2 \cdot \tan(\frac{\Theta in + \Theta out}{2})} \cdot \Delta\Theta out$
with $\Delta\Theta out = 5.431e - 04$
Hence:
$\left|\frac{\Delta\rho\Theta out}{\rho}\right| = \frac{1}{2 \cdot \tan(\frac{\Theta in + \Theta out}{2})} \cdot \Delta\Theta out$ =1.724e-03

6.3 Angular resolution of MWPC1/2

$\Delta\rho_{\Theta in} = -\rho \cdot \frac{1}{2 \cdot \tan(\frac{\Theta in + \Theta out}{2})} \cdot \Delta\Theta in$
with $\Delta\Theta_{in} = \frac{2 \cdot \Delta x}{L}$[4] and L the (z-)distance between MWPC1 and MWPC2 (= 575mm).

[3]on the following pages I set $\Theta in = 0$ and Θout to the value given by the GLAD field calculator(Mass = 12, Charge = 6, Energy = 400 A/MeV, Current = 1444), see http://web-docs.gsi.de/~land/glad/.
[4]$\Theta_{in} = \frac{x_2 - x1}{L}$, $\Delta\Theta_{in} = \left|\frac{d\Theta_{in}}{dx_2}\right| \cdot \Delta x_2 + \left|\frac{d\Theta_{in}}{dx_1}\right| \cdot \Delta x_1 = \frac{2 \cdot \Delta x}{L}$

With $\Delta x = 0.1mm$ it follows:

$\left|\frac{\Delta\rho_{\Theta_{in}}}{\rho}\right| = $ 1.104e-03.

Adding up the above errors quadratically we get:
$\left|\frac{\Delta\rho}{\rho}\right| = \sqrt{(5.434 \cdot 10^{-4})^2 + (1.724 \cdot 10^{-3})^2 + (1.104 \cdot 10^{-3})^2} = 2.12 \cdot 10^{-3}$