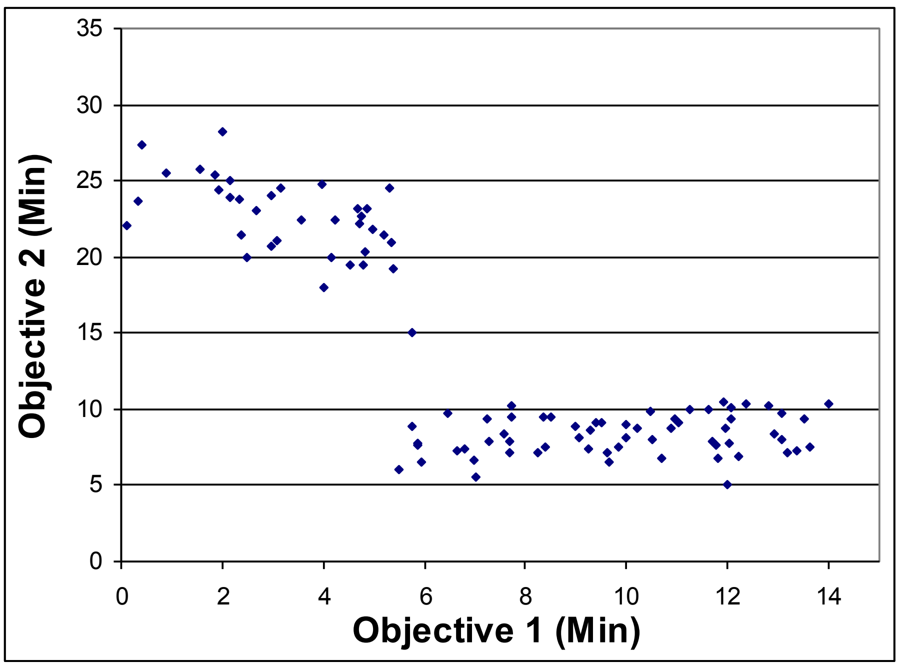
CEE 5290/COM S 5722/ORIE 5340 Heuristic Methods for Optimization
Homework 12: Multi-Objective Optimization
Assigned: Friday, November 14, 2014
Due: Friday, November 21, 2014 @ noon
Prof. Shoemaker Office hour: MWF 2:30-3:30

1. **Pareto Front**

(a) Consider the figure below that plots all feasible solutions in 2-D objective space where both objectives are to be minimized.

- Circle the non-dominated solutions on the figure (i.e. hand in this page with answers written on it). How many non-dominated solutions are there? __________
- Is the tradeoff curve convex? YES or NO



(b) A student is considering four schools for graduate studies. A national magazine has ranked 50 universities under four different criteria that are of interest to the student. These are criterion A: Educational Rank; criterion B: Engineering Job placement; criterion C: Student Selectivity; criterion D: Research Activity. The table below shows the rankings for these criteria for four schools to which student is considering applying. A rank of 1 is the best possible.

School	Criteria			
	A	B	C	D
University of Disney Land	1	1	11	1
Bedlam College	1	8	9	7
Hard Knocks U	8	12	4	6
Space Cadet Academy	8	2	20	14

The student wishes to narrow down the choice of schools for final consideration. In the event of a tie in ranks both schools are equally important, i.e. neither school dominates the other. Identify the Pareto-optimal set of schools that the student should consider for graduate studies. Also for each school specify if it is dominated or not and if yes by whom?

2. **Using NSGA-II**

This problem is to be done using NSGA toolbox,
MATLAB version can be downloaded from
http://www.mathworks.com/matlabcentral/fileexchange/10429-nsga-ii-a-multi-objective-optimization-algorithm
Python version can be downloaded from
http://code.google.com/p/nsga-ii-python/downloads/list

The problem we will try to solve is the cantilever beam design problem (CBD), which Professor Shoemaker mentioned in class. The design problem has two decision variables, length (l) and diameter (d) (Deb et al. 2001). The two conflicting objectives of this design problem are

1) Weight minimization.
2) Deflection minimization.

There are some constraints in the problem as well (which can be handled using penalty function approach).
The algebraic formulation of the problem is as follows:

$$Minimize\ f_1(d,l) = \rho\frac{\pi d^2}{4}l$$

$$Minimize\ f_2(d,l) = \delta = \frac{64Pl^3}{3E\pi d^4}$$

Subject to:

$$\frac{32Pl}{\pi d^3} \leq S_y$$

$$\delta \leq \delta_{max}$$

$$10 \leq d \leq 50mm,\ 200 \leq l \leq 1000mm$$

Where $\rho = 7.8 * 10^{-6} kg/mm^3$, $P = 10^6 kg.mm.s^{-2}$,
$E = 2.07 * 10^8\ kg.mm^{-1}s^{-2}$, $S_y = 3 * 10^5 kg.mm^{-1}s^{-2}$, $\delta_{max} = 5mm$

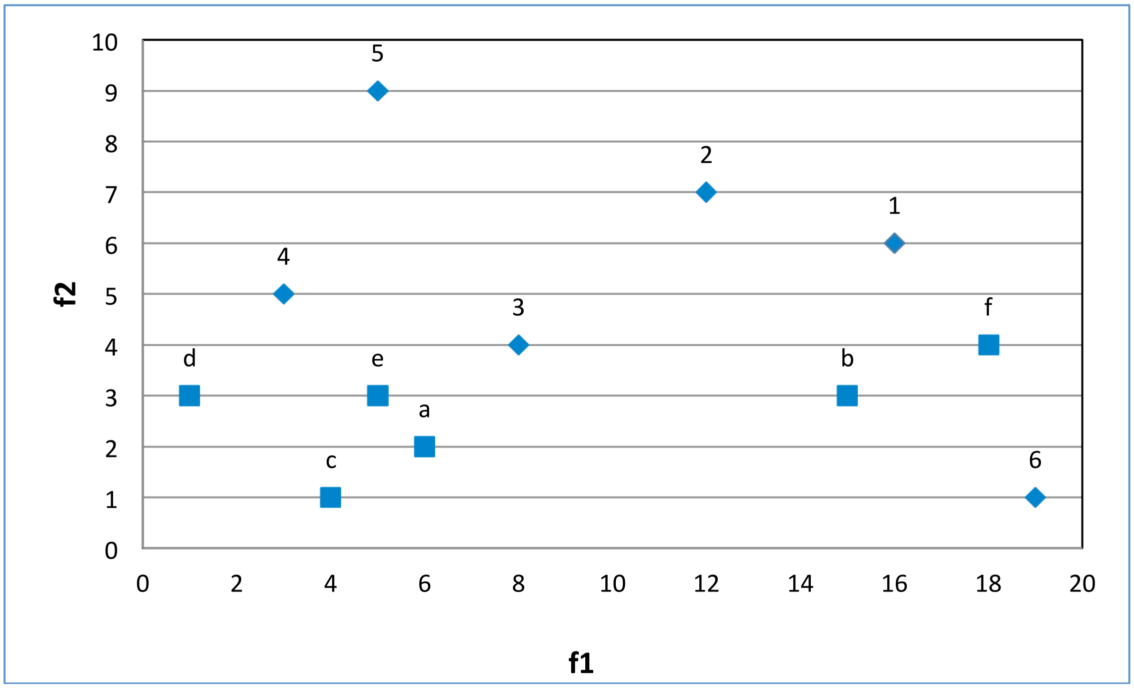
(a) Show a plot of the pareto optimal solutions
(b) What is the mean and standard deviation of the weight of the beam for the solutions on the pareto front?
(c) What is the mean and standard deviation of the deflection of the beam for the solutions on the pareto front?
(d) Can you say which solution is the best for this problem? Why or why not?

3. **NSGA-II**

(a) You are using NSGA-II to solve a multi-objective optimization problem where you want to minimize f_1 and maximize f_2. Assume for this problem that f_1 has a minimum of 0 and a maximum of 20, and that f_2 has a minimum of 0 and a maximum of 10. Be sure to explain your reasoning as you give your answers

Given the points from the parent and offspring populations in generation t, which are located in the table below, **find the parent population for generation t+1 using the NSGA-II approach.**

Parents			**Offspring**		
Solution	**f1**	**f2**	**Solution**	**f1**	**f2**
1	16	6	a	6	2
2	12	7	b	15	3
3	8	4	c	4	1
4	3	5	d	1	3
5	5	9	e	5	3
6	19	1	f	18	4



Part (b) - For a different problem, you have generated the new parent population and you are going to perform crowded tournament selection to determine the mating pool. Given the information in the table below, if you use the following pairs, **what is the mating pool**?

Pairs: (f,c), (2,e), (3,5), (5,f), (c,3), (e,2)

Front 1					Sorting in		Distance
Solution	**x1**	**x2**	**f1**	**f2**	**f1**	**f2**	
f	2.32	6.48	2.32	18.08	second	second	2.15
2	2.64	5.64	2.64	14.60	third	first	Inf
c	0.84	0.96	0.84	23.60	first	third	Inf

Front 2					Sorting in		Distance
Solution	**x1**	**x2**	**f1**	**f2**	**f1**	**f2**	
e	0.88	2.24	0.88	28.36	first	fourth	Inf
5	1.24	3.56	1.24	24.40	third	second	2.51
3	3.16	8.56	3.16	15.88	fourth	first	Inf
a	1.08	3.48	1.08	27.72	second	third	0.47