Circuits and Systems 2CJ4

Lab 5

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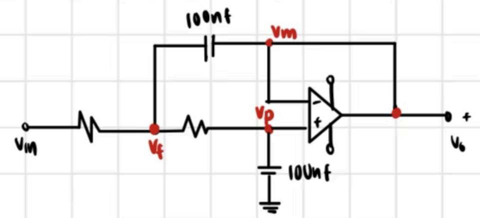
Set 5 Laboratory Experiment
Consider the shown Butterworth Low-Pass Filter. Take $R_1 = R_2 = 10\ K\Omega$ and $C_1 = C_2 = 100$ nF.

a. Derive an expression for the transfer function of the filter.

Lab 5 Analytical Calculations



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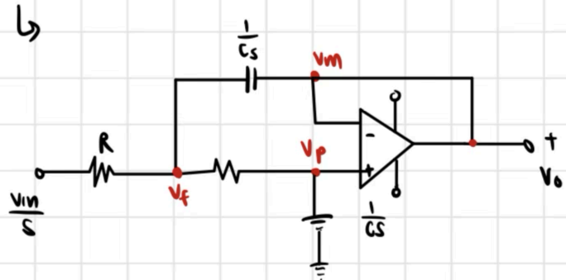


a) Transfer function

$$H(s) = \frac{V(s)}{X(s)} = \frac{V_{out}}{V_{in}}$$

$$V_m = V_p = V_f\left(\frac{Z_2}{Z_2+R}\right) = V_o$$

$$\hookrightarrow V_f = \frac{V_o(Z_2+R)}{Z_2}$$

KCL at Vf Node:

$$\frac{V_f - V_{in}}{R} + \frac{V_f - V_m}{Z_1} + \frac{V_f - V_p}{R} = 0$$

$$\frac{V_f}{R} - \frac{V_{in}}{R} + \frac{V_f}{Z_1} - \frac{V_o}{Z_1} + \frac{V_f}{R} - \frac{V_o}{R} = 0$$

$$\frac{2V_f}{R} - \frac{V_{in}}{R} + \frac{V_f}{Z_1} - \frac{V_o}{Z_1} - \frac{V_o}{R} = 0$$

$$V_f\left(\frac{2}{R} + \frac{1}{Z_1}\right) - V_o\left(\frac{1}{Z_1} + \frac{1}{R}\right) = \frac{V_{in}}{R}$$

$$\frac{V_o(Z_2+R)}{Z_2}\left(\frac{2}{R} + \frac{1}{Z_1}\right) - V_o\left(\frac{1}{Z_1} + \frac{1}{R}\right) = \frac{V_{in}}{R}$$

$$V_o\left[\frac{Z_2+R}{Z_2}\left(\frac{2}{R} + \frac{1}{Z_1}\right) - \frac{1}{Z_1} - \frac{1}{R}\right] = \frac{V_{in}}{R}$$

$$V_o\left[\left(1+\frac{R}{Z_2}\right)\left(\frac{Z}{R}+\frac{1}{Z_1}\right)-\frac{1}{Z_1}-\frac{1}{R}\right]=\frac{V_{in}}{R}$$

$$V_o\left[\frac{Z}{R}+\frac{1}{Z_1}+\frac{Z}{Z_2}+\frac{R}{Z_1 Z_2}-\frac{1}{Z_1}-\frac{1}{R}\right]=\frac{V_{in}}{R}$$

$$V_o\left[\frac{1}{R}+\frac{Z}{Z_2}+\frac{R}{Z_1 Z_2}\right]=\frac{V_{in}}{R}$$

Remember:

$z=\frac{1}{jwC}$

$j^2=-1$

$$\frac{V_o}{V_{in}}=\frac{1}{R}\left[\frac{1}{\frac{1}{R}+\frac{Z}{Z_2}+\frac{R}{Z_1 Z_2}}\right]$$

$$H(s)=\frac{1}{1+\frac{2R}{Z}+\frac{R^2}{Z^2}}$$

$$H(jw)=\frac{1}{1+2RjwC+R^2C^2j^2w^2}$$

$$\boxed{H(s)=\frac{1}{1+2RCs+R^2C^2s^2}}$$

$$\boxed{H(jw)=\frac{1}{1+2RjwC-R^2C^2w^2}}$$

$$\boxed{H(s)=\frac{V_o}{V_{in}}=\frac{1}{(1\times10^{-6})s^2+(2\times10^{-3})s+1}}$$

Transfer function

$$a+jb=\sqrt{a^2+b^2}$$

$$\boxed{|H(jw)|=\frac{1}{\sqrt{(1-R^2C^2w^2)^2+(2RwC)^2}}}$$

b. Evaluate the filter transfer function abs(V_o/V_i) using the transfer function derived in part (a) for the frequencies shown in the table.

Calculation:

when Subbing $s=j\omega$ in, then using that $\omega = 2\pi f$

$$|H(j\omega)| = \frac{1}{\sqrt{\left(-4\pi f\, C^2R^2 + 1\right)^2 + \left(4\pi f\, CR\right)^2}}$$

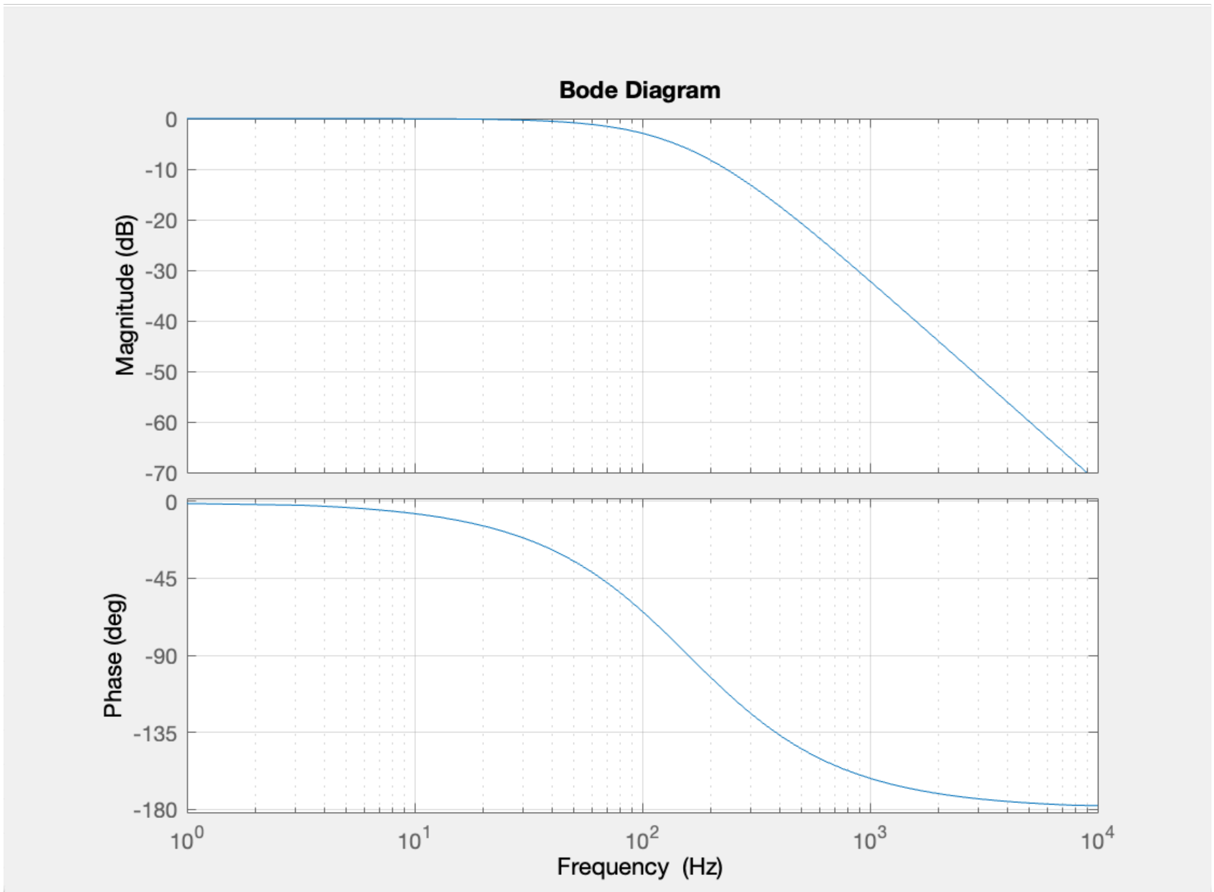
Sample Calculation for 50 Hz:

$$|H(j\omega)| = \frac{1}{\sqrt{\left(-4\pi(50)(1\times10^{-6}) + 1\right)^2 + \left(4\pi(50)(1\times10^{-3})\right)^2}}$$

Matlab Code:

```
clc; % Clear the command line
clear; % Remove all previous variables
% Component values
R1=10000;
R2=10000;
C1=100e-9;
C2=100e-9;
% Generic transfer function and parameters
numerator = 1;
denominator = [R1*R2*C1*C2 R1*C1+R2*C2 1];
% Plotting
H = tf(numerator,denominator);
p = bodeoptions('cstprefs');
p.FreqUnits = 'Hz';
bode(H,p)
grid on
% Cutoff frequency
Fc = 1/(2*pi*sqrt(R1*R2*C1*C2));
disp(['Fc = ' sprintf('%0.4f', Fc)]);
```

Figure 1: Bode Diagram



c. Measure the transfer function using the AD2 board and fill the corresponding components of the table below. Use a sine wave with an amplitude o f 2V and offset of 0V (Vcc = ±5V).

Figure 2: Circuit Set-Up For Butterworth Low-Pass Filter

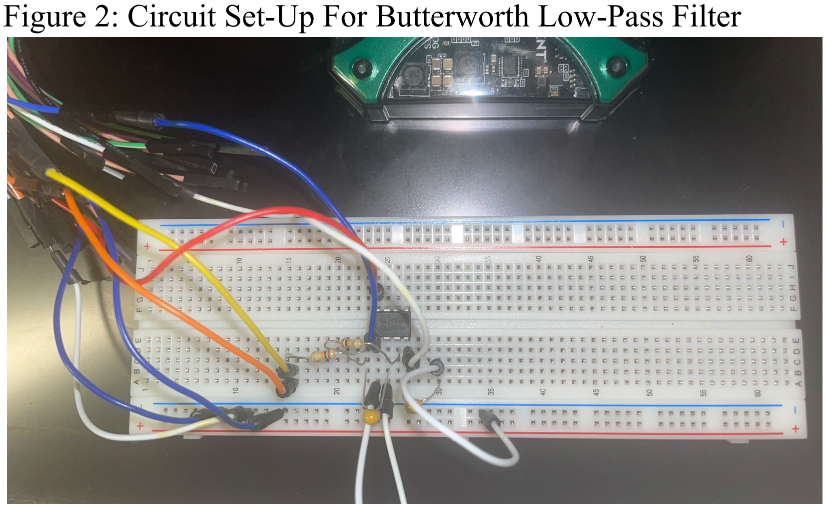


Table 1: Theoretical and Measures Gain Based on Different Frequencies

Frequency	abs (V_o/V_i) (analytical)	abs (V_o/V_i) (measured)
50 Hz	0.91	0.897
100 Hz	0.72	0.715
200 Hz	0.39	0.407
500 Hz	0.09	0.110
1 kHz	0.0247	0.028
1.1 kHz	0.021	0.023
1.2 kHz	0.0174	0.017
1.3 kHz	0.0147	0.0138
1.4 kHz	0.013	0.0155
1.5 kHz	0.011	0.0104
1.6 kHz	0.0098	0.0243
1.7 kHz	0.0089	0.0235
1.8 kHz	0.0078	0.0203
1.9 kHz	0.0070	0.0268
2 kHz	0.0062	0.0256
5 kHz	0.001	0.0303

Figure 3: AD2 Measurement of V_{in} and V_{out} Channels for 50Hz

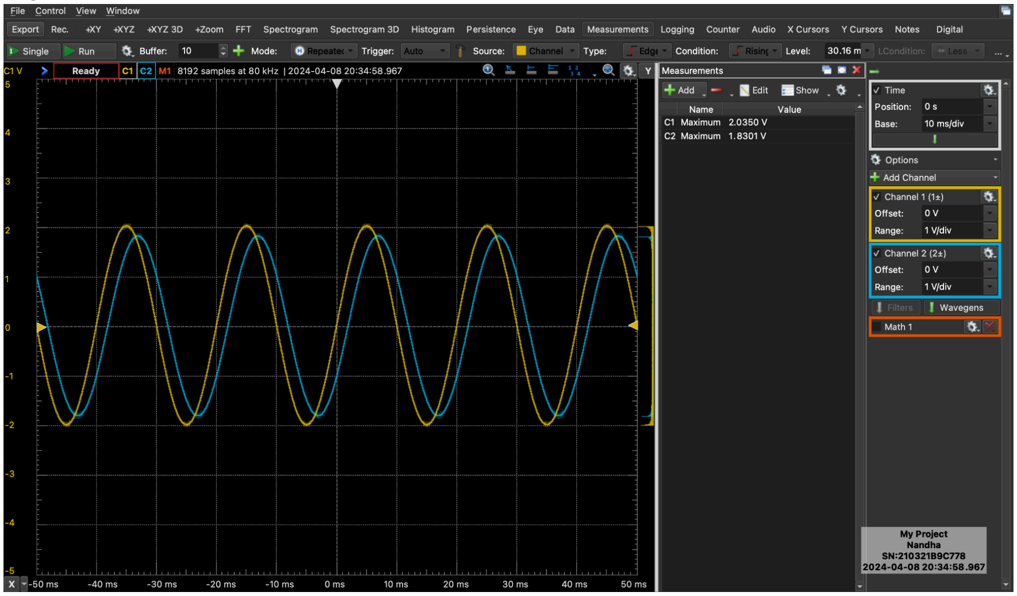
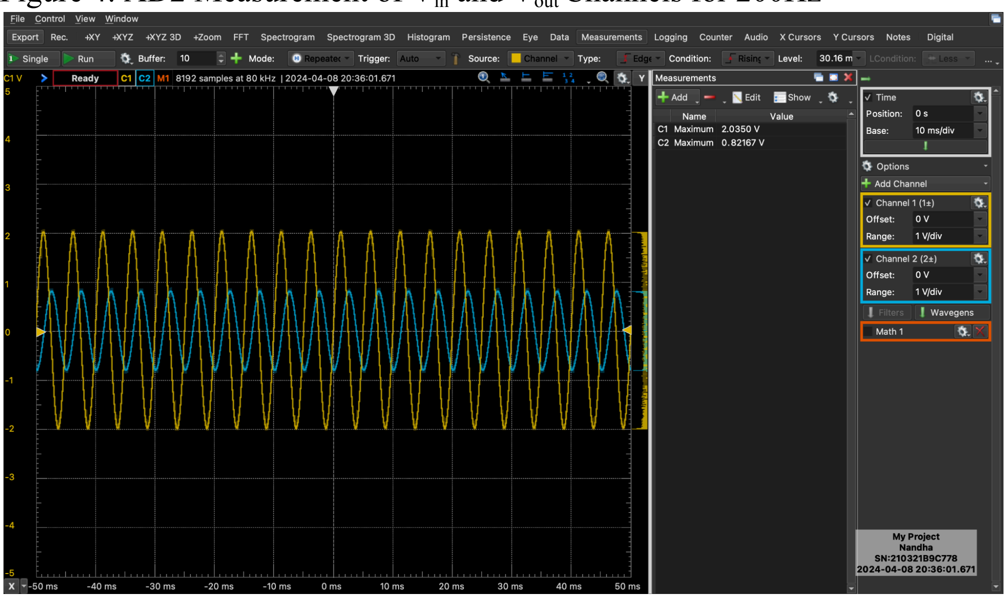


Figure 4: AD2 Measurement of V_{in} and V_{out} Channels for 200Hz



d. What is the cut-off frequency of the filter?

Cutoff frequency:

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

$$f_c = \frac{1}{2\pi\sqrt{(100\times10^{-9})(10\times10^{-3})^2}}$$

$$f_c = 159.154\ Hz$$

Figure 5: Cut-off Frequency Calculated by Matlab

Editor – /Users/bluesidetan/Documents/MATLAB/Lab5_2CJ4.m

visualization.m | Lab5_2CJ4.m

```
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% Component values
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R2=10000;
C1=100e-9;
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% Generic transfer function and parameters
numerator = 1;
denominator = [R1*R2*C1*C2 R1*C1+R2*C2 1];

% Plotting
H = tf(numerator,denominator);
p = bodeoptions('cstprefs');
p.FreqUnits = 'Hz';
bode(H,p)
grid on

% Cutoff frequency
Fc = 1/(2*pi*sqrt(R1*R2*C1*C2));
disp(['Fc = ' sprintf('%0.4f', Fc)]);
```

Command Window

```
Fc = 159.1549
>>
```

e. How do the theoretical and measured results compare? Comment on your results.

Table 2: Theoretical and Measures Gain with Percent Error

Frequency	abs (V_o/V_i) (analytical)	abs (V_o/V_i) (measured)	Percent Error
50 Hz	0.91	0.897	1.42%
100 Hz	0.72	0.715	0.69%
200 Hz	0.39	0.407	4.35%
500 Hz	0.09	0.110	22.2%
1 kHz	0.0247	0.028	13.3%
1.1 kHz	0.021	0.023	9.52%
1.2 kHz	0.0174	0.017	2.298%
1.3 kHz	0.0147	0.0138	6.12%
1.4 kHz	0.013	0.0155	19.2%
1.5 kHz	0.011	0.0104	5.45%
1.6 kHz	0.0098	0.0243	147.2%
1.7 kHz	0.0089	0.0235	164.1%
1.8 kHz	0.0078	0.0203	160.3%
1.9 kHz	0.0070	0.0268	282.8%
2 kHz	0.0062	0.0256	312.90%
5 kHz	0.001	0.0303	2930%

Percent Error Calculations:

Percent Error at 50Hz

$$\left|\frac{0.897-(0.91)}{0.91}\right| \times 100\% \;=\; 1.42\%$$

Percent Error at 100Hz

$$\left|\frac{0.715-(0.72)}{0.72}\right| \times 100\% \;=\; 0.69\%$$

Percent Error at 200Hz

$$\left|\frac{0.407-(0.39)}{0.39}\right| \times 100\% = 4.35\%$$

Percent Error at 2kHz

$$\left|\frac{0.0256-(0.0062)}{0.0062}\right| \times 100\% = 312.90\%$$

Percent Error at 5kHz

$$\left|\frac{0.303-(0.001)}{0.001}\right| \times 100\% = 2930\%$$

Explanation:

Initially, the measured values align closely with the analytical values, as observed at the start of Table 1 which made the percent error very low. However, as the frequency rises, the percent error seems to grow at a very quick rate. The cut-off frequency which is 159.15Hz seems to impact the percent error, as it starts to grow from this point. As the frequency increases, the AD2 does not have enough precision to measure the V_{out} values as the filter's response has begun to attenuate, and essentially ends up skewing the values. However, it is noteworthy that beyond the frequency cutoff, 159.15Hz, the measured results are very close to each other and have a trend towards a near-zero slope. This suggests that while the theoretical measurements do not fully account for frequency cutoff effects and other non-idealities, the behavior of the filter beyond the cutoff frequency remains consistent and aligns closely with expectations.