MATLAB 5:

Analytical Calculations:

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2 regions to consider:

1) $R < r \rightarrow Q_{enclosed} = 0, \therefore D = 0$

2) $\oint D_r \cdot ds = 4\pi r^2 \rho_s$

$D_r 4\pi R^2 = 4\pi r^2 \rho_s$

$D_r = \frac{\rho_s r^2}{R^2} a_r$

$D_r = \frac{\rho_s}{R^2} a_r$

$W_E = \frac{1}{2} \iiint_V \frac{|D|^2}{\varepsilon_0} dv$

$= \frac{1}{2\varepsilon_0} \int_0^{2\pi} \int_0^{\pi} \int_2^3 \left(\frac{\rho_s^2}{R^4}\right) \cdot R^2 r \sin\theta \, dr \, d\theta \, d\phi$

$= \frac{2\pi \rho_s^2}{\varepsilon_0} \int_2^3 \frac{1}{R^2} \cdot dv$

$= \frac{2\pi \rho_s^2}{\varepsilon_0} - \frac{1}{R} \Big]_2^3$

$= \frac{2\pi (2\times10^{-6})^2}{\varepsilon_0} \left(\frac{1}{6}\right)$

$W_E = 0.473 \, J/m^2$

MatLab Code:

```
%Jenisha Thevarajah
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e = 1e-9/(36*pi);
Ps = 2e-6;

r_upper = 3.0;
theta_upper = pi;
phi_upper = (2*pi);

r_lower = 2.0;
theta_lower = 0;
phi_lower = 0;

num_of_r_steps = 50;
dr = (r_upper-r_lower)/num_of_r_steps; %r increment

num_of_phi_steps = 50;
dphi = (phi_upper-phi_lower)/num_of_phi_steps; %phi increment

num_of_theta_steps = 50;
dtheta = (theta_upper-theta_lower)/num_of_theta_steps; %theta increment

WE = 0;

for k=1:num_of_phi_steps
    for j=1:num_of_theta_steps
        for i=1:num_of_r_steps
            r = r_lower+0.5*dr+(i-1)*dr;
            theta = theta_lower+0.5*dtheta+(j-1)*dtheta;
            phi = phi_lower+0.5*dphi+(k-1)*dphi;

            eMag = (Ps)/(e*r^2);
            dV = r^2*sin(theta)*dr*dtheta*dphi;
            dWE = 0.5*e*eMag*eMag*dV;

            WE = WE+dWE;
        end
    end
end

disp("Electric Energy: ");
disp(WE);
```

Output:

```
>> assignment5
Electric Energy:
    0.4738
```