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Matlab 2: Set 2

Exercise: $r=0$, and $r=2$

$\phi=45°$, $\phi=90°$, $\theta=45°$, $\theta=90°$

- Find enclosed volume and area of closed surface

Volume:

$$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$V = \iiint dv$$

$$= \iiint r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \int_{r=0}^{r=2} \int_{\theta=45}^{\theta=90°} \int_{\phi=45}^{\phi=90°} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \int_0^2 r^2 dr \times \int_{45}^{90} \sin\theta \, d\theta \times \int_{45}^{90} d\phi$$

$\sin 90° = \pi/2$

$\sin 45° = \pi/4$

$$= \frac{r^3}{3}\Big|_0^2 \times \left(-\cos\theta\right)\Big|_{\pi/4}^{\pi/2} \times \phi\Big|_{\pi/4}^{\pi/2}$$

$$= \frac{1}{3}(2^3-0^3) \times \left(-\cos\left(\tfrac{\pi}{2}\right) + \cos\left(\tfrac{\pi}{4}\right)\right) \times \left(\tfrac{\pi}{2} - \tfrac{\pi}{4}\right)$$

$\frac{8}{3}$ $\qquad$ $0 + \frac{\sqrt{2}}{2}$ $\qquad$ $\frac{\pi}{4}$

$$= \frac{8}{3} \times \left(\frac{\sqrt{2}}{2}\right) \times \left(\frac{\pi}{4}\right)$$

$$V = \frac{\sqrt{2}\,\pi}{3} \, m^2$$

$$V = 1.481 \, m^3$$

Surface Area:

$$S_1 = \int_{\pi/4}^{\pi/2}\int_{\pi/4}^{\pi/2} r^2 \sin\theta \, d\theta \, d\phi \quad (r=2)$$

$$= 2^2 \int_{\pi/4}^{\pi/2} \sin\theta \, d\theta \int_{\pi/4}^{\pi/2} d\phi$$

$$= 4 \times \left(-\cos\theta \Big|_{\pi/4}^{\pi/2}\right) \times \left(\theta \Big|_{\pi/4}^{\pi/2}\right)$$

$$= 4 \times \underbrace{\left(-\cos\pi/2 + \cos\pi/4\right)}_{0 + \frac{\sqrt{2}}{2}} \times \underbrace{\left(\frac{\pi}{2} - \frac{\pi}{4}\right)}_{\pi/4}$$

$$= 4 \times \left(\frac{\sqrt{2}}{2}\right) \times \left(\frac{\pi}{4}\right)$$

$$S_1 = \frac{\sqrt{2}\pi}{2} m^2$$

$$S_2 = \int_{\pi/4}^{\pi/2}\int_{0}^{2} r \, d\theta \, dr$$

$$= \int_0^2 r \, dr \times \int_{\pi/4}^{\pi/2} d\theta$$

$$= \frac{r^2}{2}\Big|_0^2 \times \theta \Big|_{\pi/4}^{\pi/2}$$

$$= \frac{1}{2}(2^2 - 0^2) \times \left(\frac{\pi}{2} - \frac{\pi}{4}\right)$$

$$= 2 \times \frac{\pi}{4}$$

$$S_2 = \frac{\pi}{2} m^2$$

$$S_2 = S_3$$

$$S_4 = \int_{\pi/4}^{\pi/2}\int_{0}^{2} r \sin\theta \, dr \, d\theta$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{r^2}{2}\Big|_0^2 + \phi \Big|_{\pi/4}^{\pi/2} \qquad \left(\sin\left(\tfrac{\pi}{4}\right) \rightarrow \frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}} \times \frac{2^2}{2} \times \left(\frac{\pi}{2} - \frac{\pi}{4}\right)$$

$$S_4 = \frac{\pi}{2\sqrt{2}}$$

$$S_5 = \int_0^2 \int_{\pi/4}^{\pi/2} dr \cdot r \sin\theta \, d\phi \;\Big|\; \theta = \frac{\pi}{2}$$

$$= \sin\theta \int_0^2 r \cdot dr \int_{\pi/4}^{\pi/2} d\phi$$

$$= \left(\sin(\pi/2)\right)\left(\frac{(2)^2}{2}\right)\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$$

$$= 1 \times 2 \times \pi/4$$

$$S_5 = \pi/2$$

$$S_{closed} = S_1 + S_2 + S_3 + S_4 + S_5$$

$$= \frac{\sqrt{2}\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2\sqrt{2}} + \frac{\pi}{2}$$

$$= 2.221 + 1.57 + 1.57 + 1.11 + 1.57$$

$$\boxed{S_{closed} = 8.041 \, m^2}$$

MatLab Code:

```
%intialize
V = 0;

S1 = 0;
S2 = 0;
S3 = 0;
S4 = 0;
S5 = 0;

radius = 0;
theta = pi/4;
phi = pi/4;

radius_discretization = 100;
theta_discretization = 100;
phi_discretization = 100;

dradius = (2 - 0)/radius_discretization; %the radial increment
dtheta = (pi/2 - pi/4)/theta_discretization; %the theta increment
dphi = (pi/2 - pi/4)/phi_discretization; %the phi increment

% Calculates the volume of the enclosed surface
for k = 1:radius_discretization
   for j = 1:theta_discretization
       for i = 1:phi_discretization
           dV = (radius^2) * sin(theta) * dradius * dtheta * dphi; % add contribution to the volume
           V = V + dV;
       end
       theta = theta + dtheta; % increment
   end
   radius = radius + dradius; % increment
   theta = pi/4; % reset theta
end


radius1 = 2; %radius of S1
for k = 1:theta_discretization
   for i = 1:phi_discretization
       S1 = S1 + (radius1^2) * sin(theta) * dphi * dtheta;
   end
   theta = theta + dtheta;
end
%%the following routine calculates the area of S3 and S4
radius = 0; %reset radius to it's lower boundary
theta1 = pi/4;
theta2 = pi/2;

```

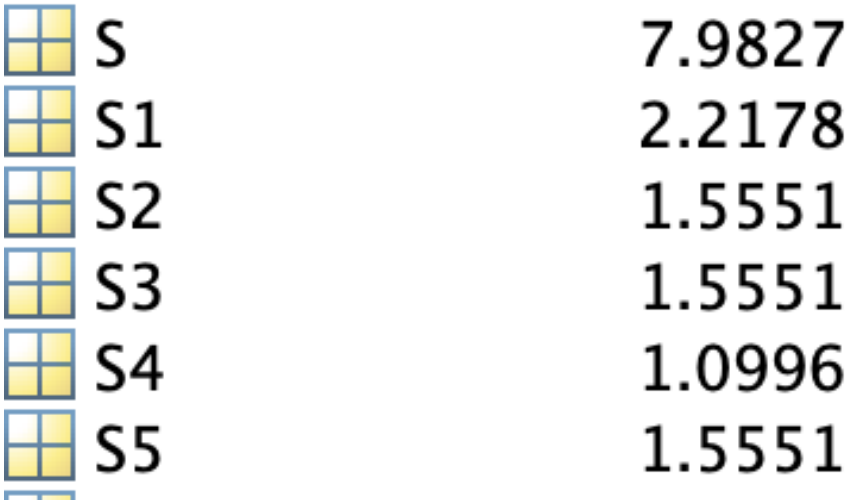
```
for j=1: radius_discretization
   for i=1: phi_discretization
       S4 = S4 + radius * sin(theta1) * dphi * dradius; %get contribution to the area of S4
       S3 = S3 + radius * sin(theta2) * dphi * dradius; %get contribution to the area of S3
   end
   radius = radius + dradius;
end

%%the following routing calculate the area of S2
radius = 0; %reset radial to it's lower boundary
for k=1:theta_discretization
   for j=1:radius_discretization
       S2 = S2 + radius * dtheta * dradius;%S2 area
   end
   radius = radius + dradius;
end

S5=S3;%area of S5 is equal to S3

%Total surface
S=S1+S2+S3+S4+S5;%the area of the closed surface and volume of closed surface
disp(['Volume of Surface: ' num2str(V)]);
disp(['Area of Closed Surface: ' num2str(S)]);
```

Output:




```
>> assignment2
Volume of Surface: 1.4564
Area of Closed Surface: 7.9827
>>
```

There is a very small discrepancy between my theoretical calculations and MatLab calculated answers; however, this may have occurred due to rounding calculations within my theoretical calculations. Matlab answers are more precise compared to my written calculations as matlab accounts of all decimal points.