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Matlab 4

Analytical Calculations:

Set 4: Analytical Calculations

Surface charge density: $\rho_s = 2.0\ \mu C/m^2$, $\rho < 1.0m$, $z = 0$ and zero else where

↳ Find E at $P(\rho = 0, z = 1.0)$

$\rho_s = 2.0\ \mu C/m^2$

$dq = \rho_s \cdot da$

$da = \rho d\phi \cdot d\rho$

$E = z \int dE \cdot \cos\theta$

$$dE = \frac{dq}{4\pi\varepsilon_0 r^2}$$

$$dE = \frac{\rho_s \cdot \rho d\phi d\rho}{4\pi\varepsilon_0 (z^2+\rho^2)}$$

$$E = z \int \frac{\rho_s \cdot \rho d\phi d\rho}{4\pi\varepsilon_0 (z^2+\rho^2)} \cos\theta$$

$$= \frac{\rho_s}{4\pi\varepsilon_0} z \int_{\rho=0}^{1} \int_0^{2\pi} \frac{\rho}{(z^2+\rho^2)} \cdot \frac{z}{\sqrt{z^2+\rho^2}} d\phi d\rho$$

$$E = \frac{z \rho_s z}{2\varepsilon_0} \int_0^1 \frac{\rho d\rho}{(z^2+\rho^2)^{3/2}}$$

$z^2+y^2 = v^2$
$\rho d\rho = v dv$

↳ use 'v' substitution

$$\int \frac{\rho d\rho}{(z^2+\rho^2)^{3/2}} = \int \frac{v dv}{(v^2)^{3/2}} = \int \frac{1}{v^2} dv$$

$$= v^{-2} dv$$

$$= -\frac{1}{v}$$

$$\int_0^1 \frac{\rho d\rho}{(z^2+\rho^2)^{3/2}} = \left(-\frac{1}{v}\right)_z^{\sqrt{z^2+1}}$$

$$= \frac{1}{z} - \frac{1}{\sqrt{z^2+1}}$$

$$E = \frac{z \rho_s}{2\varepsilon_0} z \left[\frac{1}{z} - \frac{1}{\sqrt{z^2+1}}\right]$$

$$E = \frac{z \rho_s}{2 \varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+1}}\right]$$

$$\underline{E}\,(\rho = 0,\ z = 1) = \frac{\underline{z}\ 2\times10^{-6}}{2.885\times10^{-12}}\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$E = 3.308 \times 10^4$$

MatLab Code:

```
%initalize
Epsilono=8.854e-12;
D=2e-6;
P=[0 0 1];
E=zeros(1,3);


%initialize discretization
Number_of_rho_Steps=100;
Number_of_phi_Steps=100;


rho_lower=0; %the lower boundary of rho
rho_upper=1; %the upper boundary of rho
phi_lower=0; %the lower boundary of phi
phi_upper=2*pi; %the upper boundary of phi


drho=(rho_upper- rho_lower)/Number_of_rho_Steps;
dphi=(phi_upper- phi_lower)/Number_of_phi_Steps;

ds=drho*dphi;
dQ=D*ds;


for j=1: Number_of_phi_Steps
for i=1: Number_of_rho_Steps
rho = rho_lower + drho/2 + (i-1) * drho; % Rho component of the center of a grid
phi = phi_lower + dphi/2 + (j-1) * dphi; % Phi component of the center of a grid
R = P - [rho * cos(phi), rho * sin(phi), 0]; % Vector from the center of the grid to the observation point
RMag = norm(R); % Magnitude of vector R
ds = drho * rho * dphi; % Area of a single grid
dQ = D * ds; % Charge on a single grid
E = E + (dQ / (4 * pi * Epsilono * RMag^3)) * R; % Contribution to the electric field
end
end

disp(E);
```

Output:

```
1.0e+04 *

 0.0000   -0.0000    3.3081
```