CS5800: Algorithms — Virgil Pavlu

Homework 8

Name: Jenish Sunil Kothari
Collaborators:

Instructions:

- Make sure to put your name on the first page. If you are using the LaTeX template we provided, then you can make sure it appears by filling in the yourname command.
- Please review the grading policy outlined in the course information page.
- You must also write down with whom you worked on the assignment. If this changes from problem to problem, then you should write down this information separately with each problem.
- Problem numbers (like Exercise 3.1-1) are corresponding to CLRS 3^{rd} edition. While the 2^{nd} edition has similar problems with similar numbers, the actual exercises and their solutions are different, so make sure you are using the 3^{rd} edition.

1. Exercise 16.3-3

Amortized Analysis of Binomial Heaps

Solution:Suppose $f(n)$ represents the runtime of the EXTRACT-MIN operation for a heap containing n items. If we define the potential function $\phi = \sum_{k=1}^{n} f(k)$, the potential decreases by $f(n)$ whenever an EXTRACT-MIN operation is executed.Furthermore, it is permissible to increase the potential by $f(n)$ during each INSERT operation, since $f(n) = O(\log n)$, leading to an amortized cost of $O(\log n) + O(\log n) = O(\log n)$ for the INSERT operation.

Let $\phi = \sum_{k=1}^{n} f(k)$, where n is the number of items in the heap after the i-th operation. Initially, $\phi_0 = 0$ and $\phi_i \geq 0$ for all i. For an INSERT operation, we have $c_i' = c_i + \Delta\phi_i = \log n + f(n) = O(\log n)$. For an EXTRACT-MIN operation, the amortized cost is $c_i' = c_i + \Delta\phi_i = f(n) - f(n) = 0 = O(1)$.