

Control Systems

CS2: Modelling and Identification of a DC Motor

2 Lab Task 5

5 Task description:

The aim of this task was to determine key model parameters of a DC motor using experimental data and analytical methods. These parameters are crucial for the accurate mathematical modeling of the motor's behavior, which is necessary for designing effective control systems of the motor. The parameters to be identified include (1) Armature resistance (R_A), (2) Armature inductance (L_A), (3) Motor constant (k_G), (4) friction model parameters (dry friction torque (τ_{F0}) and coefficient of viscous friction (α_F)), (5) and the moment of inertia (J) of the motor.

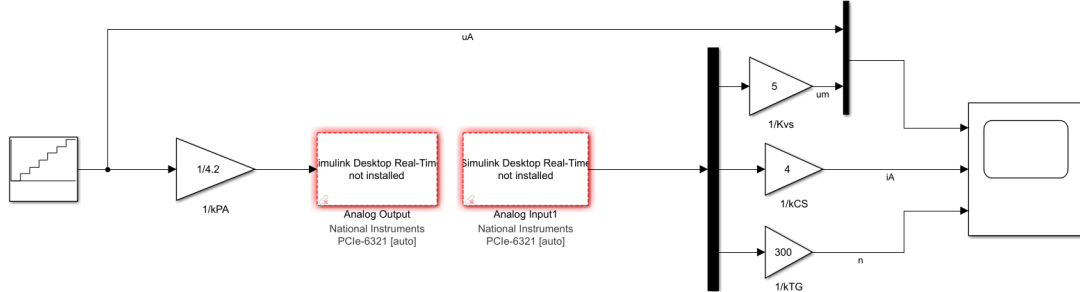


Figure 1: Simulink Model of the DC Motor

5.1 Armature resistance R_A and Armature inductance L_A :

To determine these parameters we used Kirchhoff's voltage law which is provided in 2 Task ([Equation(1) = $R_A \cdot i_A(t) + L_A \frac{di_A(t)}{dt} = u_A(t) - e_m(t)$] derived from figure(3)), In the blocked-rotor mode, the angular velocity and the emf ($e_m(t)$) of the motor are zero. By converting this first-order differential to a transfer function, we get this transfer function represents a model of a first-order system (PT1) (Equation(7)

$= \frac{I_A(s)}{U_A(s)} = \frac{\frac{1}{R_A}}{\frac{L_A}{R_A}s + 1}$). Now these parameters could be determined using the step response method used in the first experiment

steady-state gain $1/R_A = i_\infty / u_\infty$ and time constant $L_A/R_A = T$

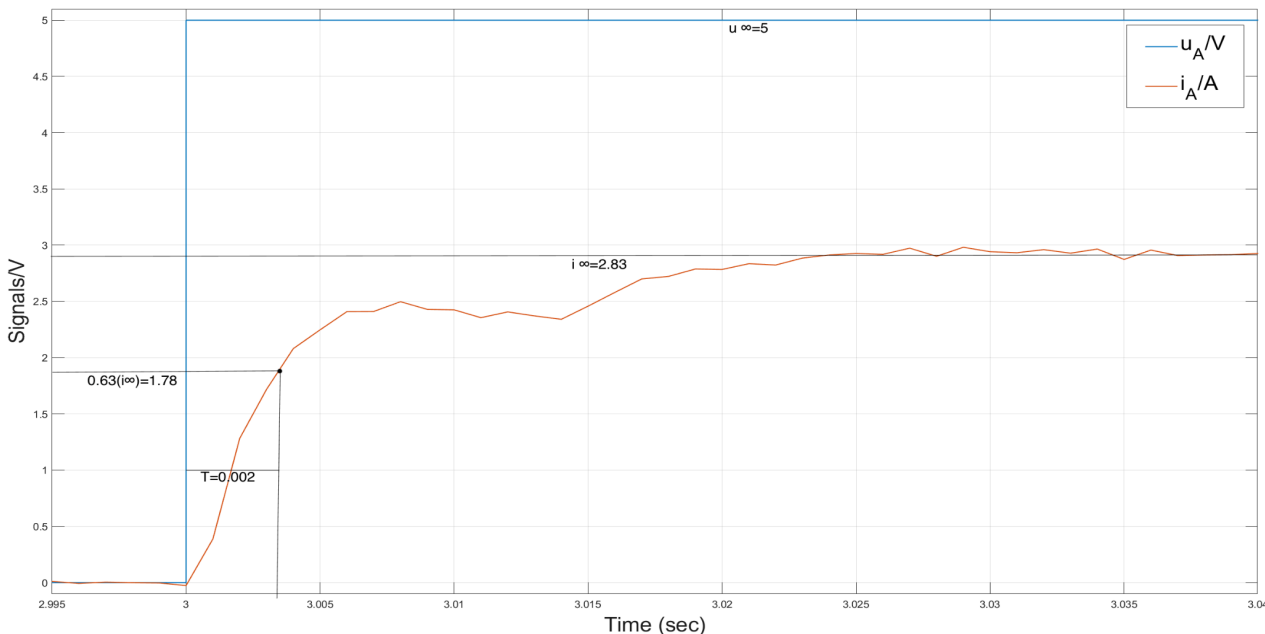


Figure 2: Armature step response

$$R_A = \frac{u_\infty}{i_\infty} = 1.766 \Omega$$

$$L_A = T(R_A) = 3.5 \text{ mH}$$

5.2 Motor/generator constant k_G :

The data measured during the coast-down phase of the motor is used to determine k_G . Determine the motor/generator constant k_G by plotting emf against angular velocity during the coast-down phase and calculating the slope of the graph.

- by using the basic fitting command in the 'Tools' menu of the Figure(3), the linear equation we got is $y = 0.127(x) + 0.00964$.
Slope

$$K_G = 0.127$$

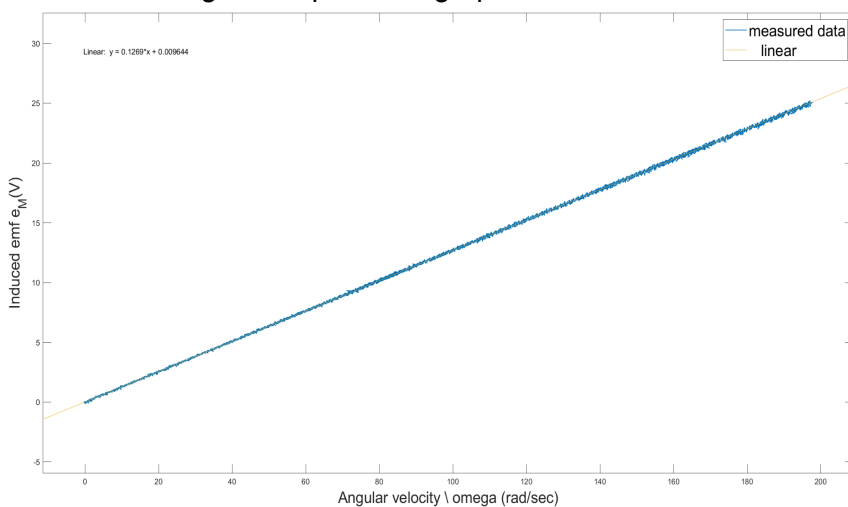


Figure 3: Induced emf (E_m) Vs Angular velocity (ω)

5.4 Parameters of the friction model:

To determine friction model parameters (dry friction torque (τ_{F0}) and coefficient of viscous friction (α_F)), we Consider an operating point, where the motor is running at constant speed without external load ($\tau_L = 0$) and ($\frac{d\omega(t)}{dt} = 0$), now from Newton's second law to rotor mechanics yields the relationship for the angular velocity ω and torque (τ_m) of the rotor (Equation(2)) we obtain [$\tau_F(t) = \tau_m(t) = k_G \cdot i_A(t)$] and then we determine the steady-state values for angular velocity and armature current at the end of each non-zero step. Multiply the current values with k_G , group the values in vectors and plot that data, then we use the command basic fitting to approximate a linear function and compare it with typical model for the friction torque ($\tau_F = \tau_{F0} + \alpha_F(\omega)$).

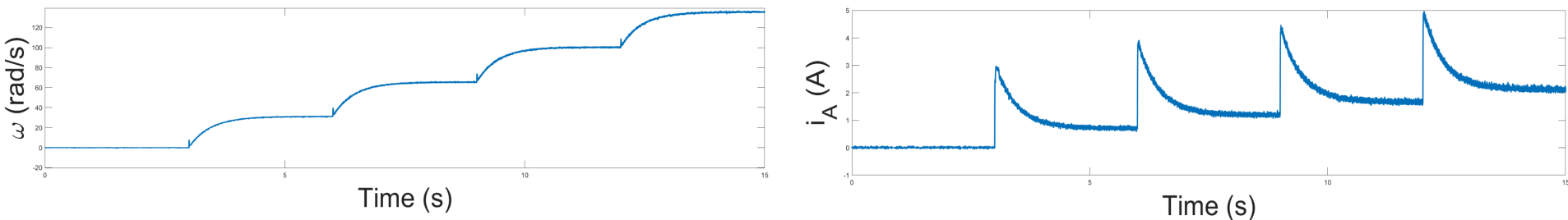


Figure 4: angular velocity and armature current (To determine steady-state value vectors)

These are the steady-state values we get from the above graph (Figure 4) :

- VecOmega = [30.086,64.805,99.357,135.77];
- VecTau=[0.688, 1.2, 1.65,2.10961]*kG;

by using the basic fitting command the linear equation we obtain [$y = 0.001702 (x) + 0.03885$] from figure(5).

So, by comparing it with $(\tau_F = \tau_{F0} + \alpha_F(\omega))$

$$\tau_{F0}= 0.03885 \text{ Nm}$$

$$\alpha_F=0.001702 \frac{\text{Nm}}{\text{rad/s}}$$

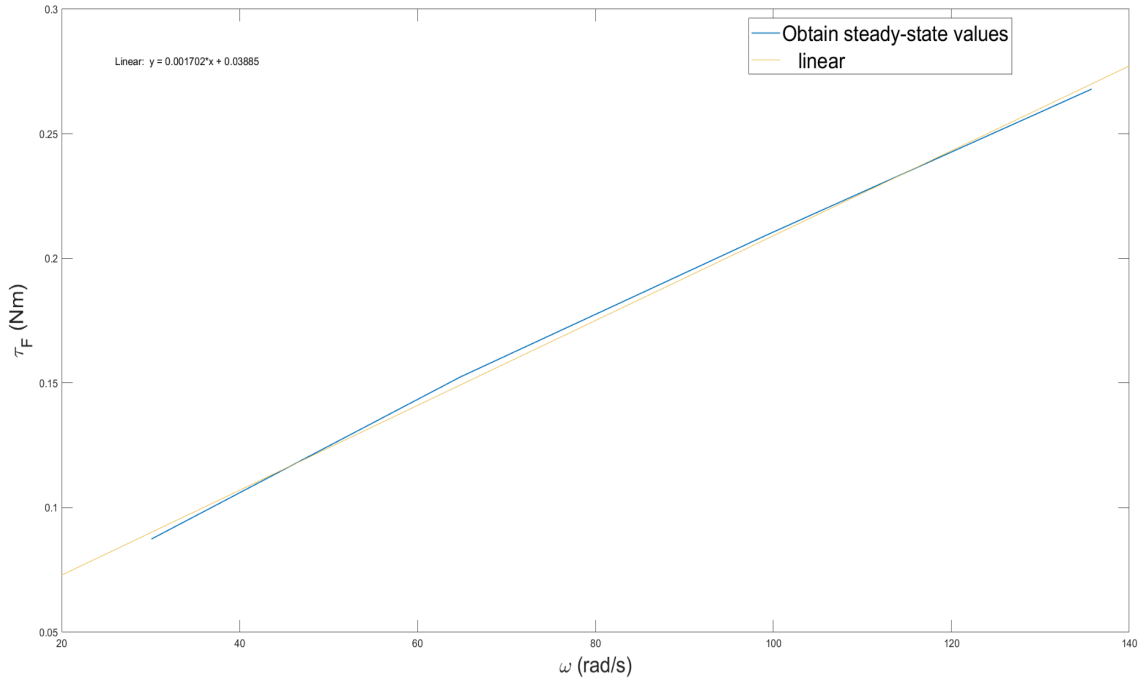


Figure 5:friction torque (τ_F) vs angular velocity (ω)

5.5 Moment of inertia J:

The aim of this task is to experimentally determine the moment of inertia of the motor by analyzing its coast-down phase. Applying Newton’s second law to rotor mechanics yields the following relationship: $J \frac{d\omega(t)}{dt} = \tau_m(t) - \tau_F(t) - \tau_L(t)$. During the deceleration phase $i_A(t) = 0$, and thus $\tau_m(t) = 0$ because $\tau_m(t) = k_G \cdot i_A(t)$. Since there is no external load, $\tau_L(t) = 0$. $\therefore J \frac{d\omega(t)}{dt} = -\tau_F(t) \Rightarrow J = \frac{-\tau_F(t)}{\frac{d\omega(t)}{dt}}$. After plotting the $\omega(t)$ against time, we found the slope at $\omega(t) = 100 \text{ rad/s}$.

$$\left. \frac{d\omega(t)}{dt} \right|_{\omega p=100 \text{ rad/s}} = -12.72 \frac{\text{rad}}{\text{s}^2}$$

Then we used the friction curve to find the friction torque $\tau_F(t)$:

$$\tau_F \Big|_{\omega p=100 \text{ rad/s}} = 0.0749 \text{ Nm}$$

Lastly we calculated the moment of inertia:

$$J = \frac{\tau_F}{\frac{d\omega(t)}{dt}} \Big|_{\omega p=100 \text{ rad/s}} = 0.00558 \text{ Kg.m}^2$$

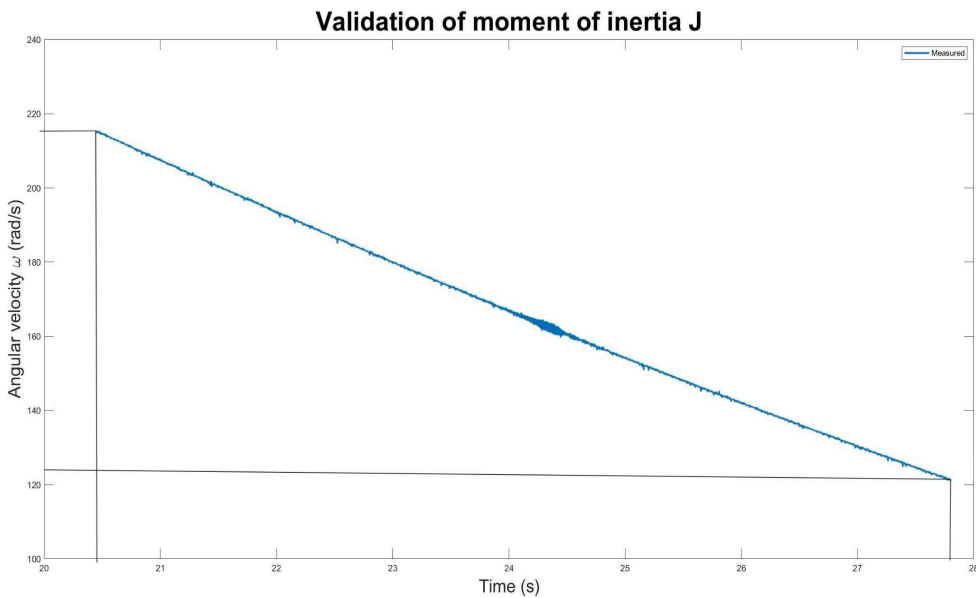


Figure 6: Angular Velocity vs Time Curve

5.5.5 Validating the Model Lastly, after modeling and identification we need to validate the calculated parameters. We did this by actuating the real system and the simulated system with the same input and comparing the outputs. Some of the parameters of the model had to be slightly tuned to achieve a better fit to the real curve. After tuning estimated value of moment of inertia $J = 0.0066$ and in the end, we got satisfactory results with the model simulation curve almost fitting the measured curve.

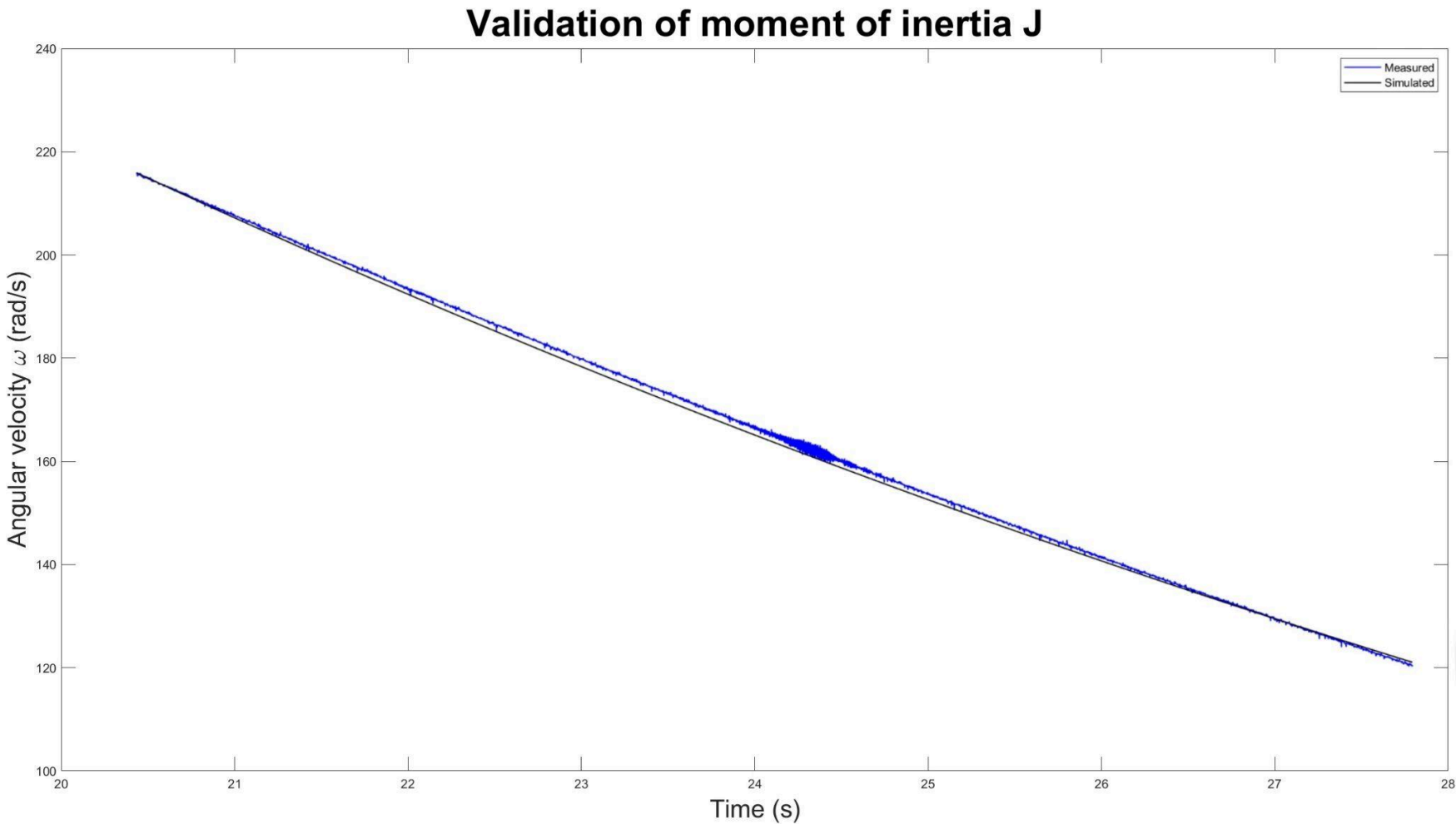


Figure 7: Validation of the Model

