Control Systems

CS3: Design and performance evaluation of PID control

1 Task description :-

The primary aim of this task is to design P, PI, and PIDT1 controllers for different plants using MATLAB. Calculate and tune controller parameters to achieve desired performance metrics from the Bode Plot. Implement the designed controllers in Simulink and evaluate their performance in real-time simulations. Gain hands-on experience with controller design and performance evaluation, understanding the impact of various parameters on system behavior.

2 Introduction :-

1. **P - Controller**(Proportional controller), K_R is known as the controller gain.

   Transfer Function : $G_R(s) = K_R$

2. **PI - Controller**(Proportional-Integral controller),K_R is the P-controller gain and T_I is the integrator time constant.

   Transfer Function : $G_R(s) = K_R\left(1 + \frac{1}{T_I.(s)}\right)$

3. **PIDT1 Controller** (Proportional-Integral-Derivative controller) with a first order low pass filter , K_R is the P-controller gain, T_I is the integrator time constant, T_D is the derivative time constant and T_V is the filter.

   Transfer Function : $G_R(s) = K_R\left(1 + \frac{1}{T_I.(s)} + \frac{T_D.(s)}{T_V.(s)+1}\right)$

3 The plants used in this task :-

Plant A : A second order plant with one pole at origin.

Transfer Function : $G_{SA}(s) = \frac{1.5}{(0.1(s)+1).(s)}$

Plant B : A Plant with 3 real poles.

Transfer Function : $G_{SB}(s) = \frac{1.5}{(1+0.1(s)).(1+0.5(s)).(1+2(s))}$

Plant C : A Plant with a Complex pole and a real pole.

Transfer Function : $G_{SC}(s) = \frac{1.5}{(1+0.1(s)).((\frac{s}{5})^2 + 2\,.\,0.4\,(\frac{s}{5}) + 1)}$

4 Parameter Calculation :-

(4.1) Plant A :-

(4.1.1) P controller

Calculate the controller gain K_R for a phase margin of $\phi_M = 60°$.

As, $\phi_M = \phi(\omega_{gc}) + 180, \ \therefore \ \phi_M(\omega_{gc}) = -120$

$\boxed{\omega_{gc} = 5.8 \text{ rad/s}}$

Now, to compute controller gain K_R , A_dB(ω_{gc}) = (-13) must be shifted to zero. Then that value must be compensated by the K_R

$$K_R = 10^{\left(\frac{-A\_dB(\omega_{gc})}{20}\right)} = 10^{\left(\frac{13}{20}\right)} = \boxed{4.466}$$

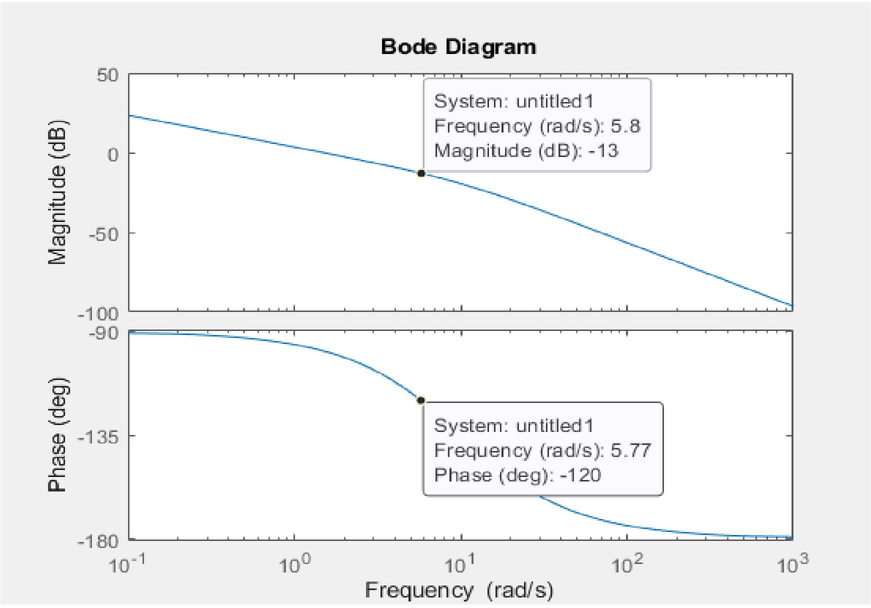


Figure 1 (plant A): Bode plot to determine controller gain K_R (P).

(4.1.2) PI controller

Determine the value for T_I so that the phase curve has a maximum value of -135°.

∴ At $T_I = 0.58$ phase curve has the max value of -135°.

Now,Calculate the controller gain K_R for a phase margin of φ_M = 45°, ∴ $\varphi(\omega_{gc})$ = -135 => ω_{gc} = 4.25 rad/s Around

$$K_R = 10^{\left(\frac{-A\_dB(\omega_{gc})}{20}\right)} = 10^{\left(\frac{9.13}{20}\right)} = \boxed{2.85}$$

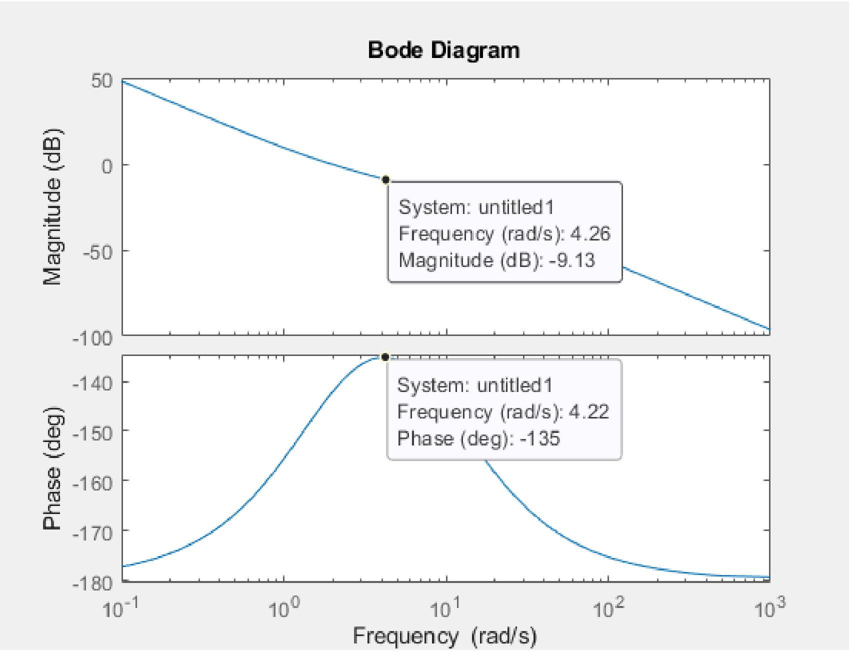


Figure 2 (plant A): Bode plot to determine controller gain K_R(PI).

(4.2) Plant B :-

(4.2.1) P controller

Same as plant A (4.1.1) at ϕ_M = 60°.

ω_{gc} = 1.59 rad/s

$$K_R = 10^{\left(\frac{-A\_dB(\omega_{gc})}{20}\right)} = 10^{\left(\frac{9.18}{20}\right)} = \boxed{2.88}$$

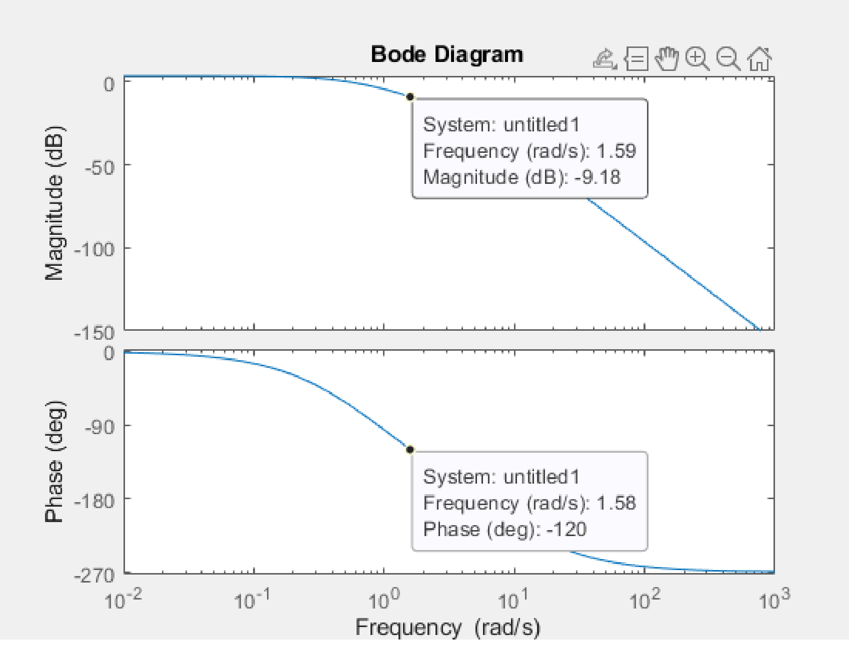


Figure 3 (plant B): Bode plot to determine controller gain K_R (P).

(4.2.2) PI controller

Determine the value for T_I so that the slowest pole of the plant is canceled by the zero of the controller.

In plant B slowest pole of the plant = -0.5

and zero of the controller = - $\frac{1}{T_I}$

∴ $T_I = 2$

From the Bode plot ω_{gc} = 0.915 rad/s

$$K_R = 10^{\left(\frac{-A\_dB(\omega_{gc})}{20}\right)} = 10^{\left(\frac{2.59}{20}\right)} = \boxed{1.34}$$

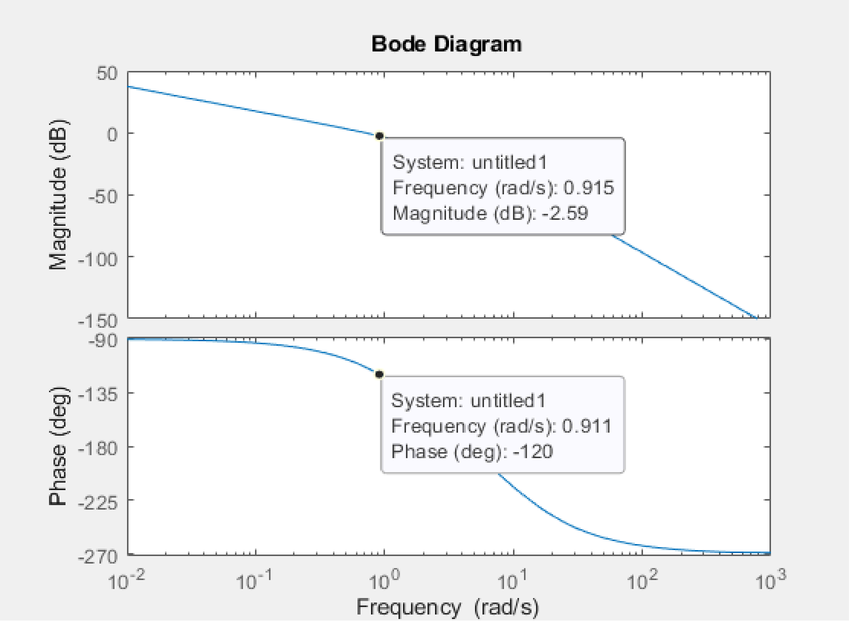


Figure 4 (plant B): Bode plot to determine controller gain K_R (PI).

(4.2.3) PIDT1 controller

Cancel the two slow poles of the plant with controller zeros.
Set T_V equal to 20% of the second largest time constant of the plant:-

∴ $T_V = \frac{20(0.5)}{100} = 0.1$

Numerator of controller :-

$$[(T_V.(s) + 1).(T_I.(s)) + (T_V.(s) + 1) + T_I.(s).(T_D.(s))]$$

two slowest poles denominator of plant :-

(1+0.5(s)).(1+2(s))

∴ $[T_I.(T_V) + T_I.(T_D)](s)^2 + [T_I + T_V](s) + 1 = s^2 + 2.5(s) + 1$

From that :- $[T_I.(T_V) + T_I.(T_D)] = 1$, And $[T_I + T_V] = 2.5$

From both equations $T_I = 2.4$ and $T_D = 0.31$

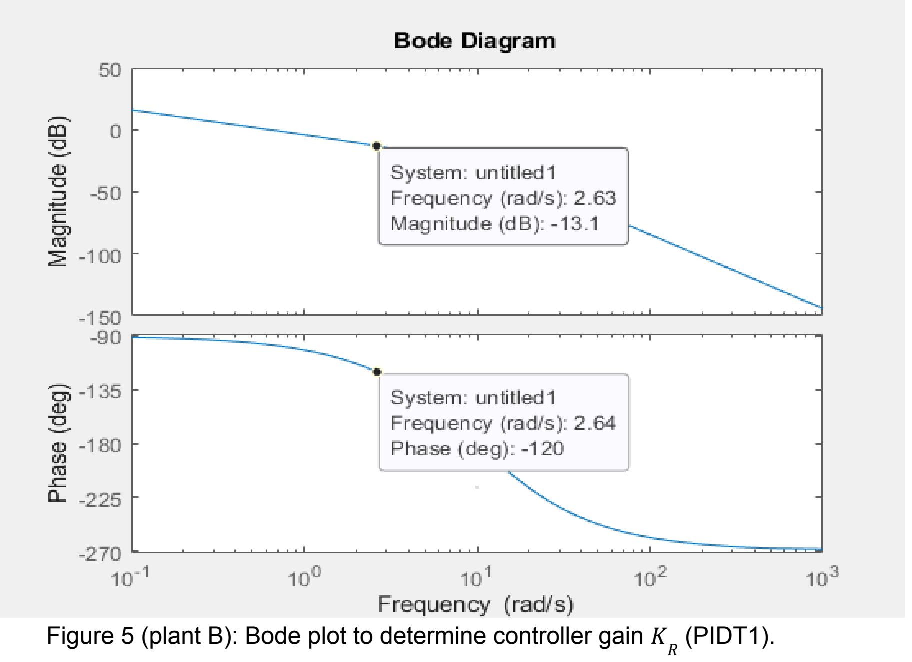


Figure 5 (plant B): Bode plot to determine controller gain K_R (PIDT1).

$\boxed{\omega_{gc} = 2.64 \text{ rad/s}}$ and $K_R = 10^{\left(\frac{-A\_dB(\omega_{gc})}{20}\right)} = 10^{\left(\frac{13.1}{20}\right)} = \boxed{4.51}$

(4.3) Plant C :-

(4.3.1) P controller

Same as plant A (4.1.1) at ϕ_M = 60°.

$$\boxed{\omega_{gc} = 5.11 \text{ rad/s}}$$

$$K_R = 10^{\left(\frac{-A\_dB(\omega_{gc})}{20}\right)} = 10^{\left(\frac{-4.21}{20}\right)} = \boxed{0.613}$$

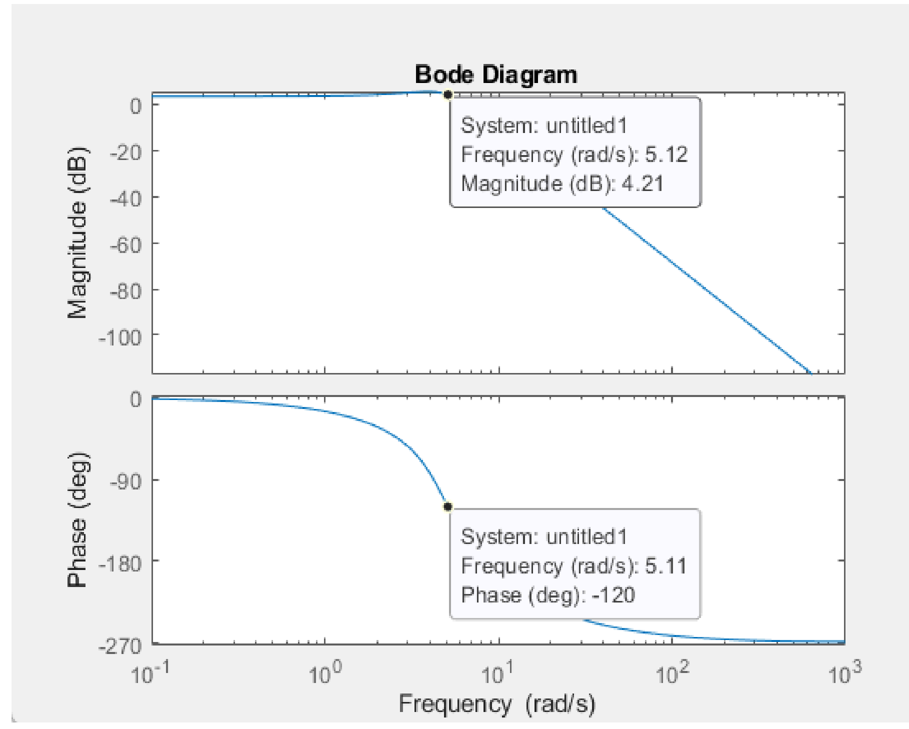


Figure 6 (plant C): Bode plot to determine controller gain K_R (P).

(4.3.2) PI controller

Set T_I = 1/ω_0, where ω_0 is the natural frequency of the conjugate complex pole-pair of Plant C.

Roots of complex poles of plant C are :-

$$s = (-2 \pm \sqrt{21})$$

$\therefore \omega_0 = |s| = \sqrt{(2)^2 + (21)} = 5$ so, $\boxed{T_I = 0.2}$

Same as plant A (4.1.1) at ϕ_M = 60°.

$$\boxed{\omega_{gc} = 3.39 \text{ rad/s}}$$

$$K_R = 10^{\left(\frac{-A\_dB(\omega_{gc})}{20}\right)} = 10^{\left(\frac{-10.4}{20}\right)} = \boxed{0.30}$$

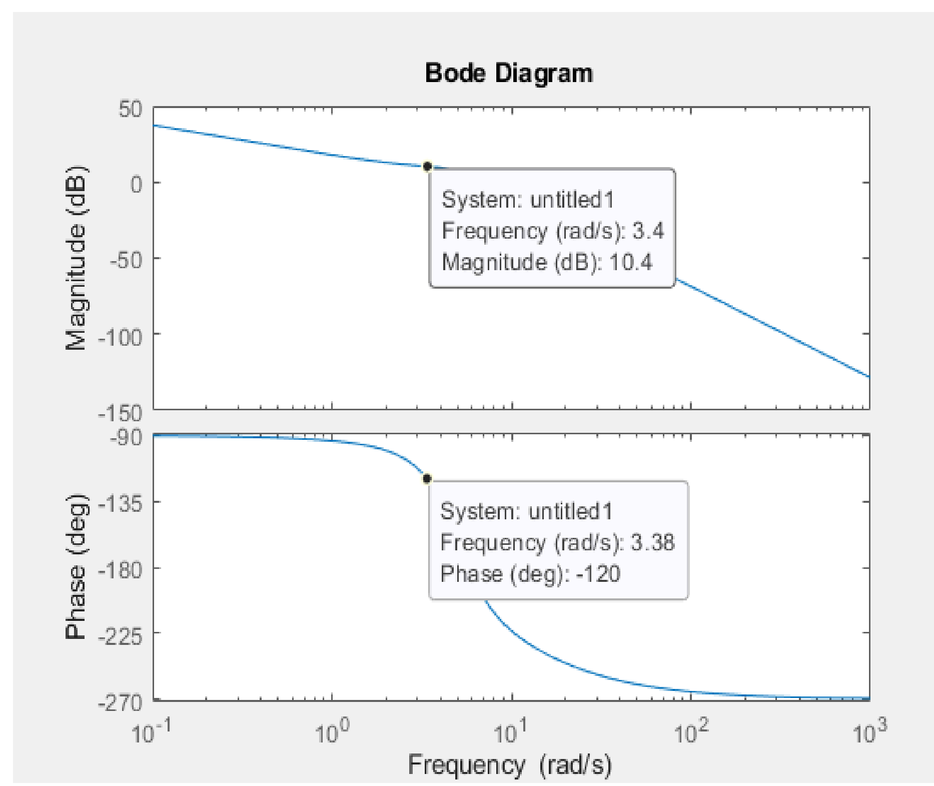


Figure 7 (plant C): Bode plot to determine controller gain K_R (PI).

(4.3.3) PIDT1 controller

Same as plant B (4.2.3):-

Set T_V = 0.2/ω_0, where ω_0= 5 from (4.3.2) is the natural frequency of the conjugate complex pole-pair of Plant C.

$\therefore \boxed{T_V = \frac{0.2}{5} = 0.04}$

$\therefore [T_I.(T_V) + T_I.(T_D)](s)^2 + [T_I + T_V](s) + 1 = (\frac{s}{5})^2 + 2(0.4)(\frac{s}{5}) + 1$

From that :- $[T_I.(T_V) + T_I.(T_D)] = \frac{1}{25}$, And $[T_I + T_V] = \frac{2(0.4)}{5}$

From both equations $\boxed{T_I = 0.12 \text{ and } T_D = 0.29}$

Same as plant A (4.1.1) at ϕ_M = 60°.

$$\boxed{\omega_{gc} = 3.8 \text{ rad/s}}$$

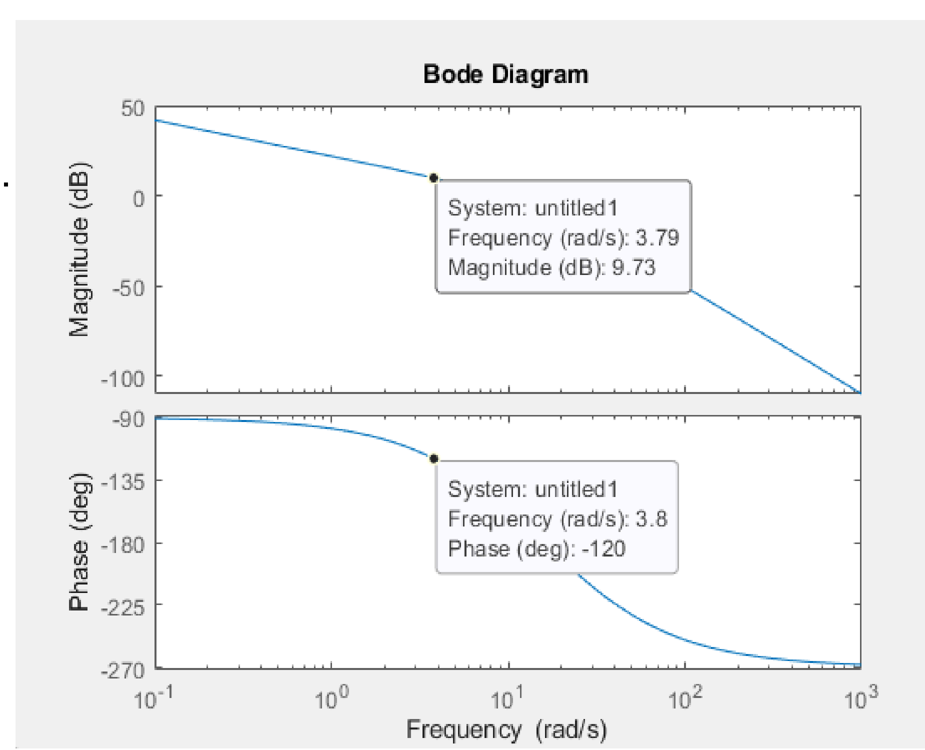


Figure 8 (plant C): Bode plot to determine controller gain K_R (PIDT1).

$$K_R = 10^{\left(\frac{-A\_dB(\omega_{gc})}{20}\right)} = 10^{\left(\frac{-9.79}{20}\right)} = \boxed{0.323}$$

Plant	Controller	$\phi_M[°]$	ω_{gc}	K_R	T_I	T_D	T_V
A	P	60	5.8	4.46	-	-	-
	PI	45	4.25	2.85	0.57	-	-
B	P	60	1.59	2.88	-	-	-
	PI	60	0.915	1.34	2	-	-
	PIDT1	60	2.64	4.51	2.4	0.31	0.1
C	P	60	5.11	0.613	-	-	-
	PI	60	3.39	0.3	0.2	-	-
	PIDT1	60	3.8	0.323	0.12	0.293	0.04

Table 1: Controller parameters

5 Implementation of PID Controller :-

1. Plant A :

Set point Tracking and Disturbance Rejection results for P and PI controllers.

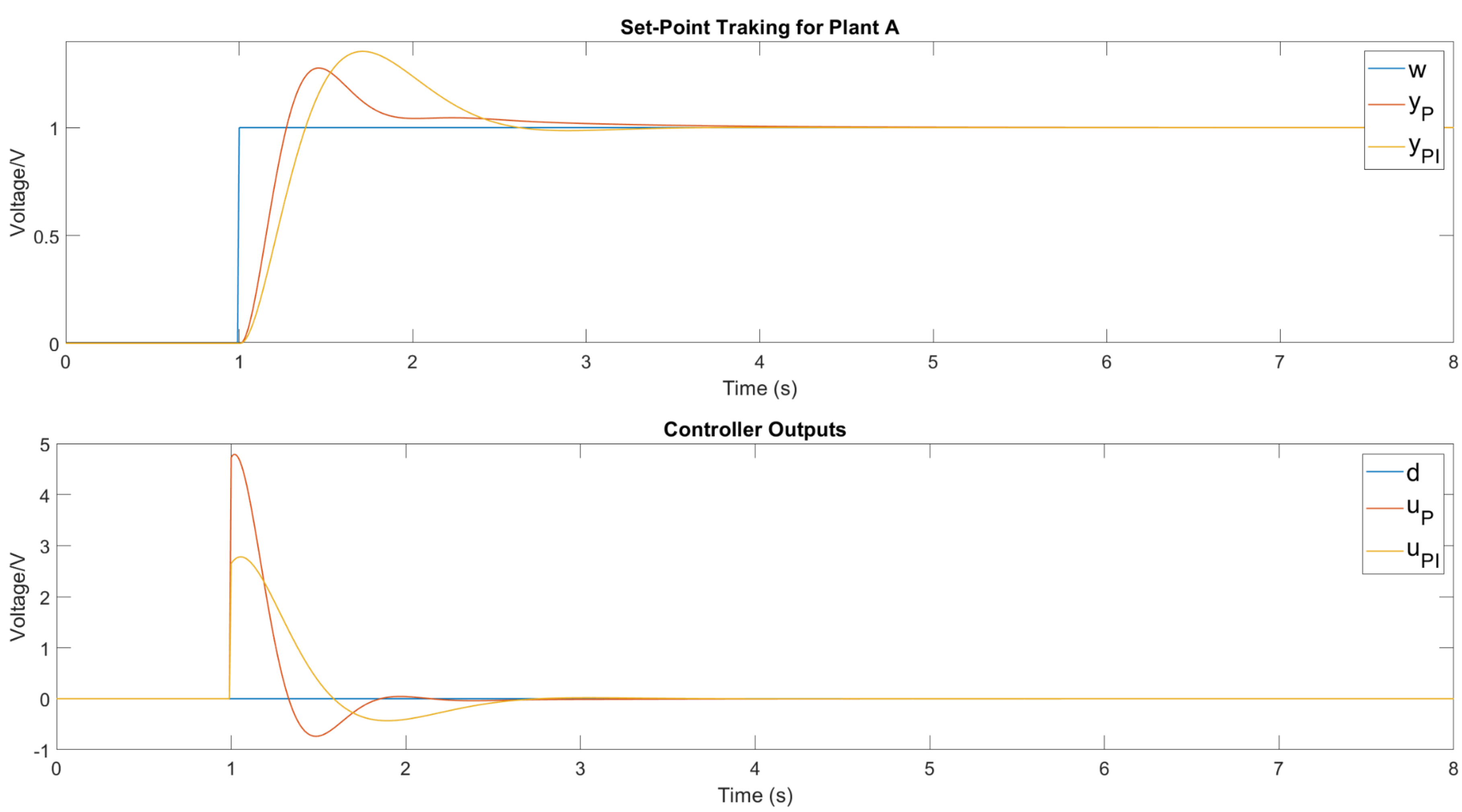
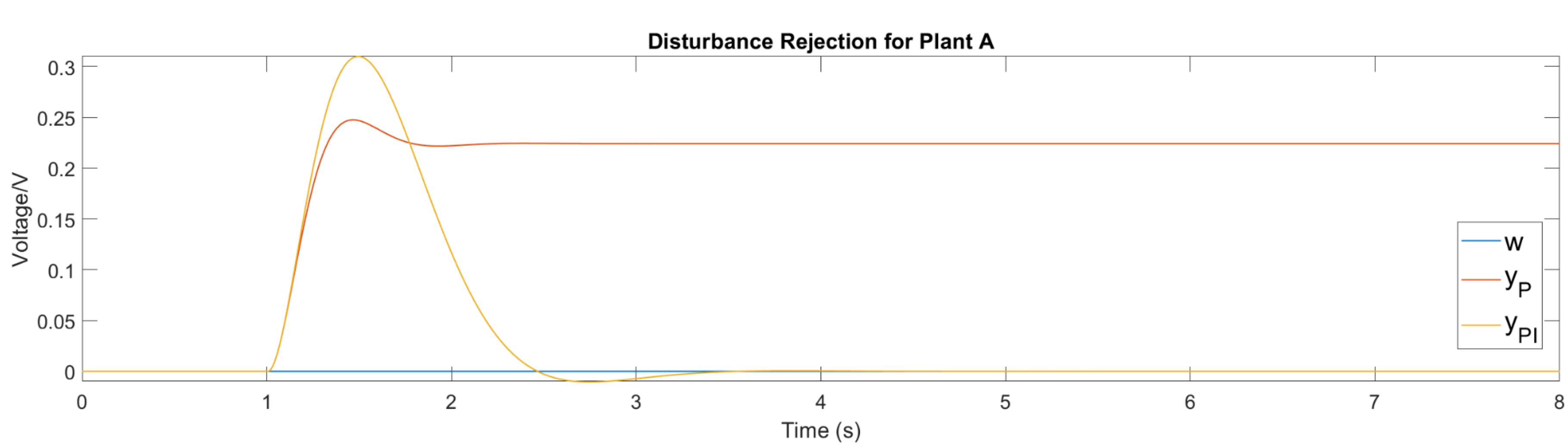


Figure 9 : Set-Point Tracking for plant A.



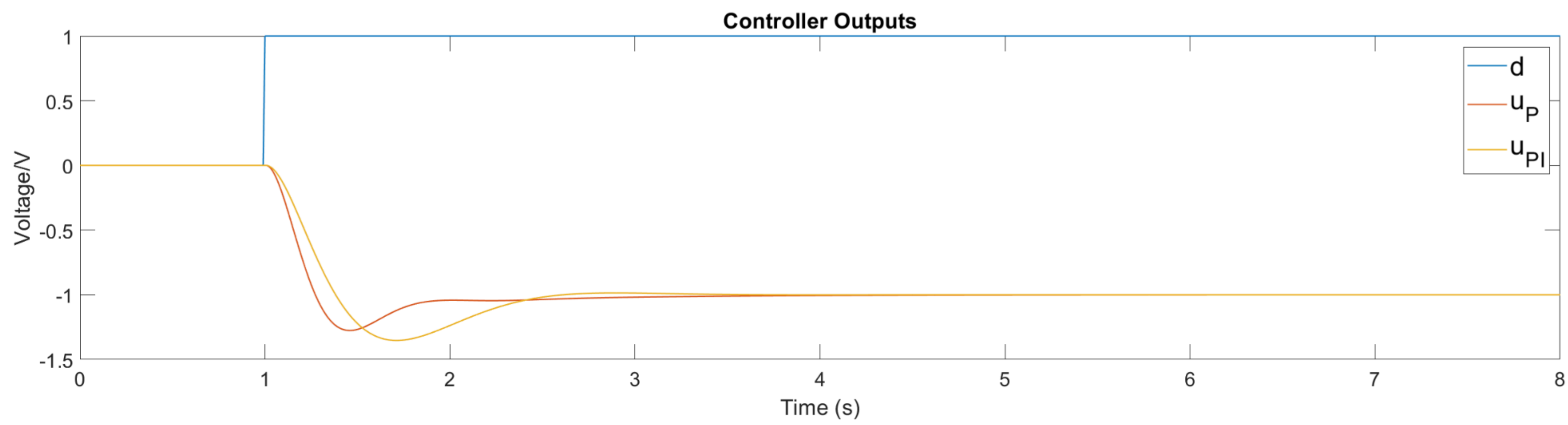


Figure 10 : Disturbance Rejection for plant A.

2. Plant B :

Set point Tracking and Disturbance Rejection results for P, PI, PIDT1 controllers.

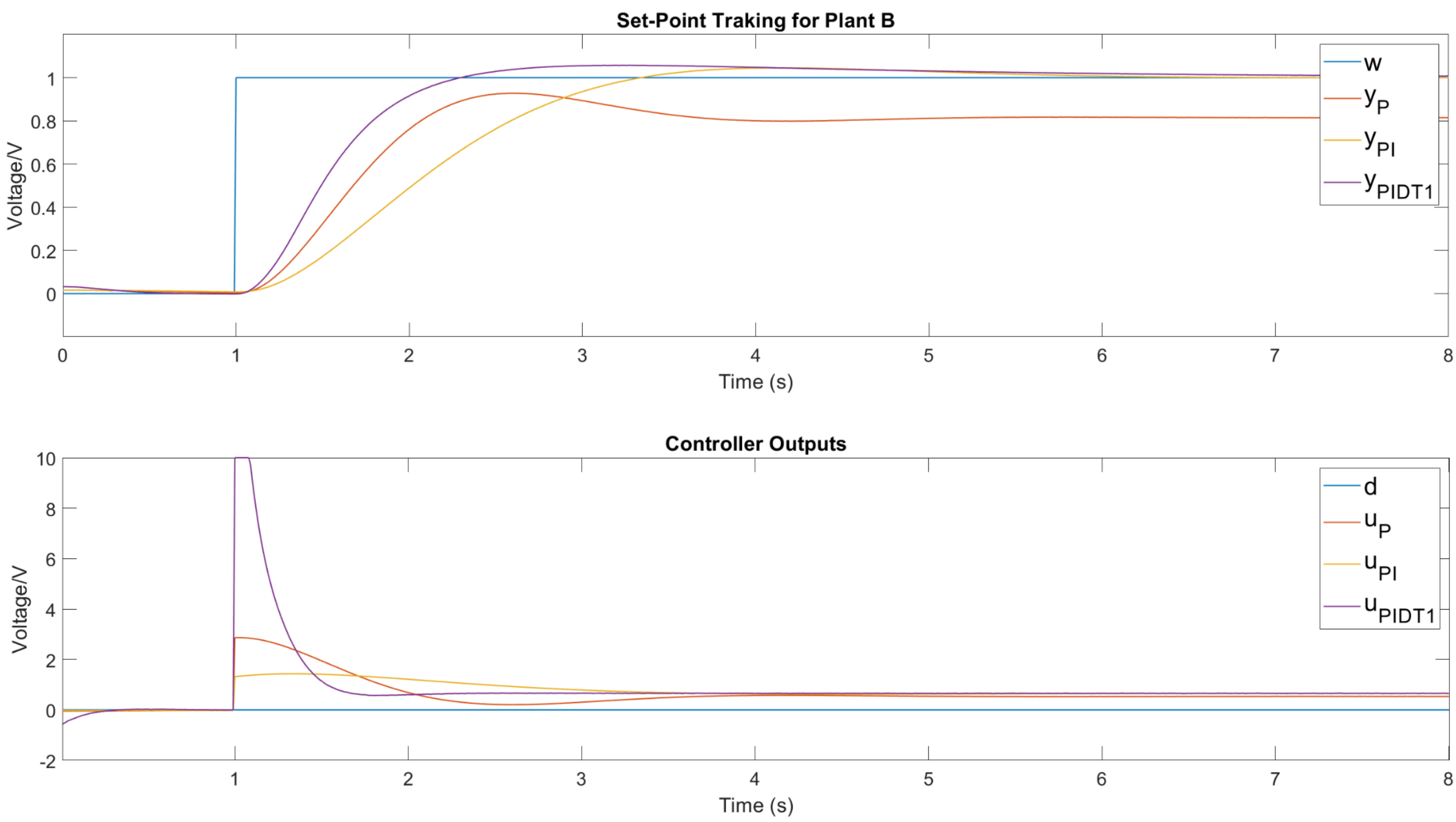


Figure 11 : Set-Point Tracking for plant B.

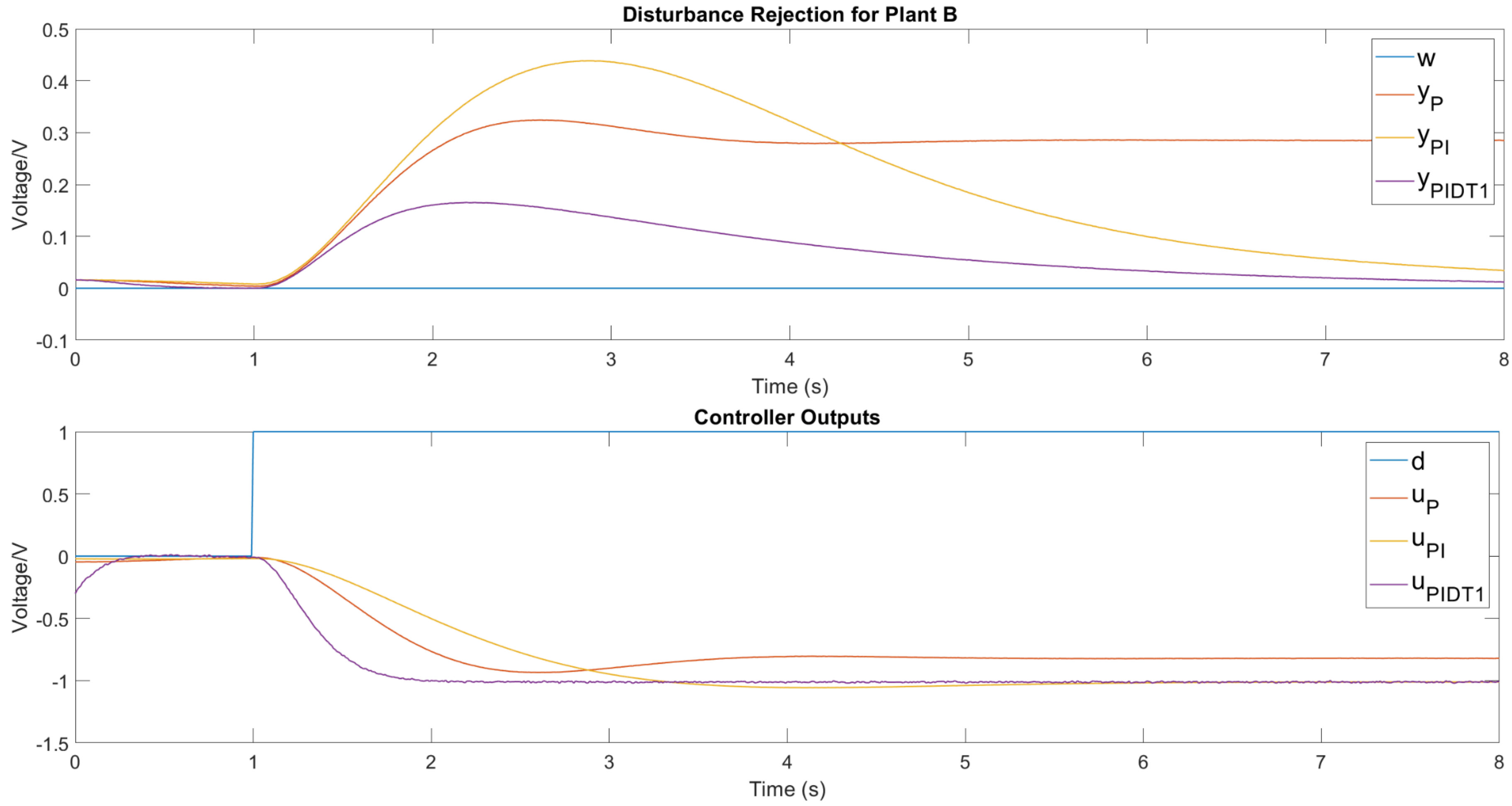


Figure 12 : Disturbance Rejection for plant B.

3. Plant C :

Set point Tracking and Disturbance Rejection results for P, PI, PIDT1 controllers.

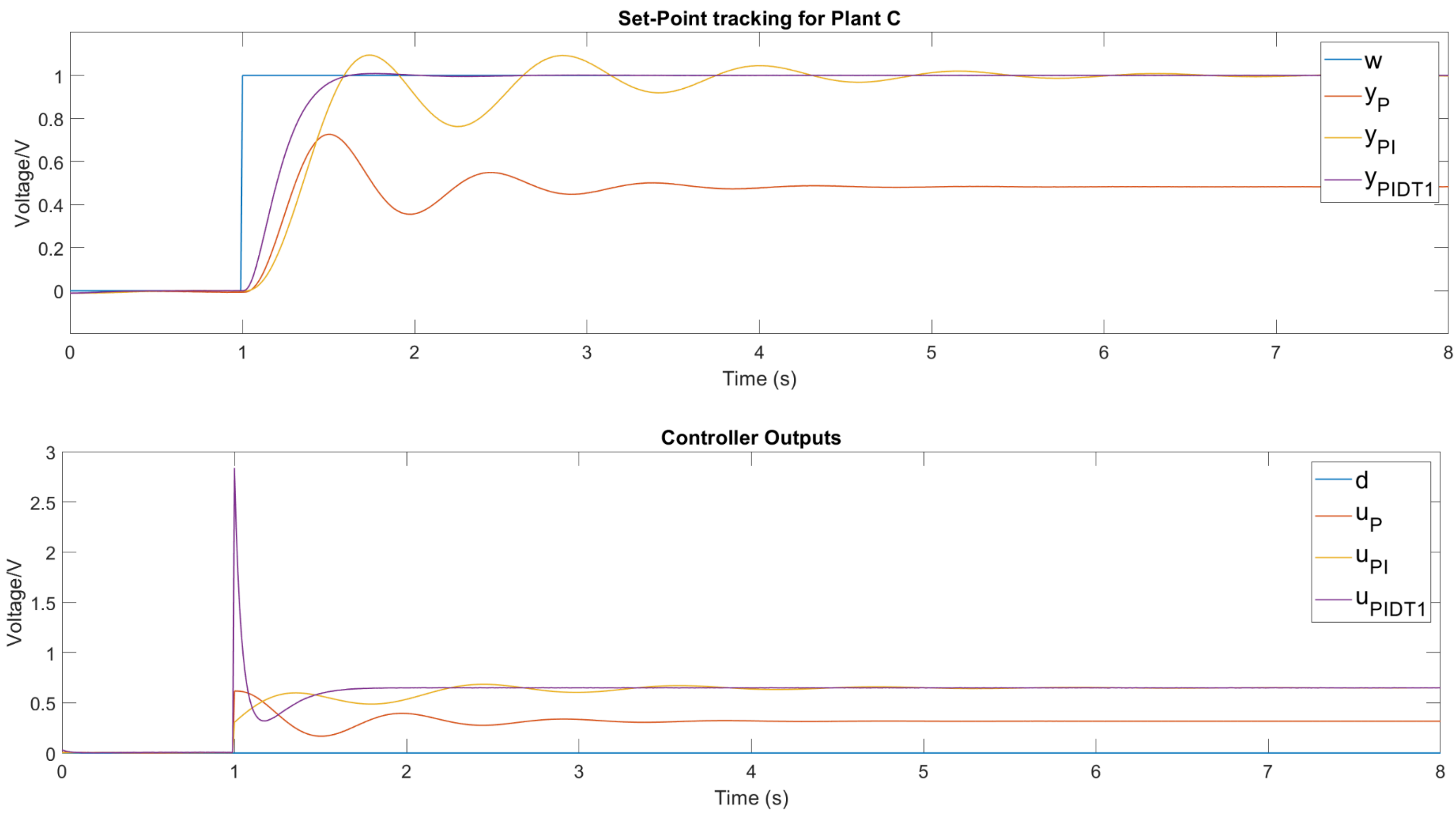


Figure 13 : Set-Point Tracking for plant C.

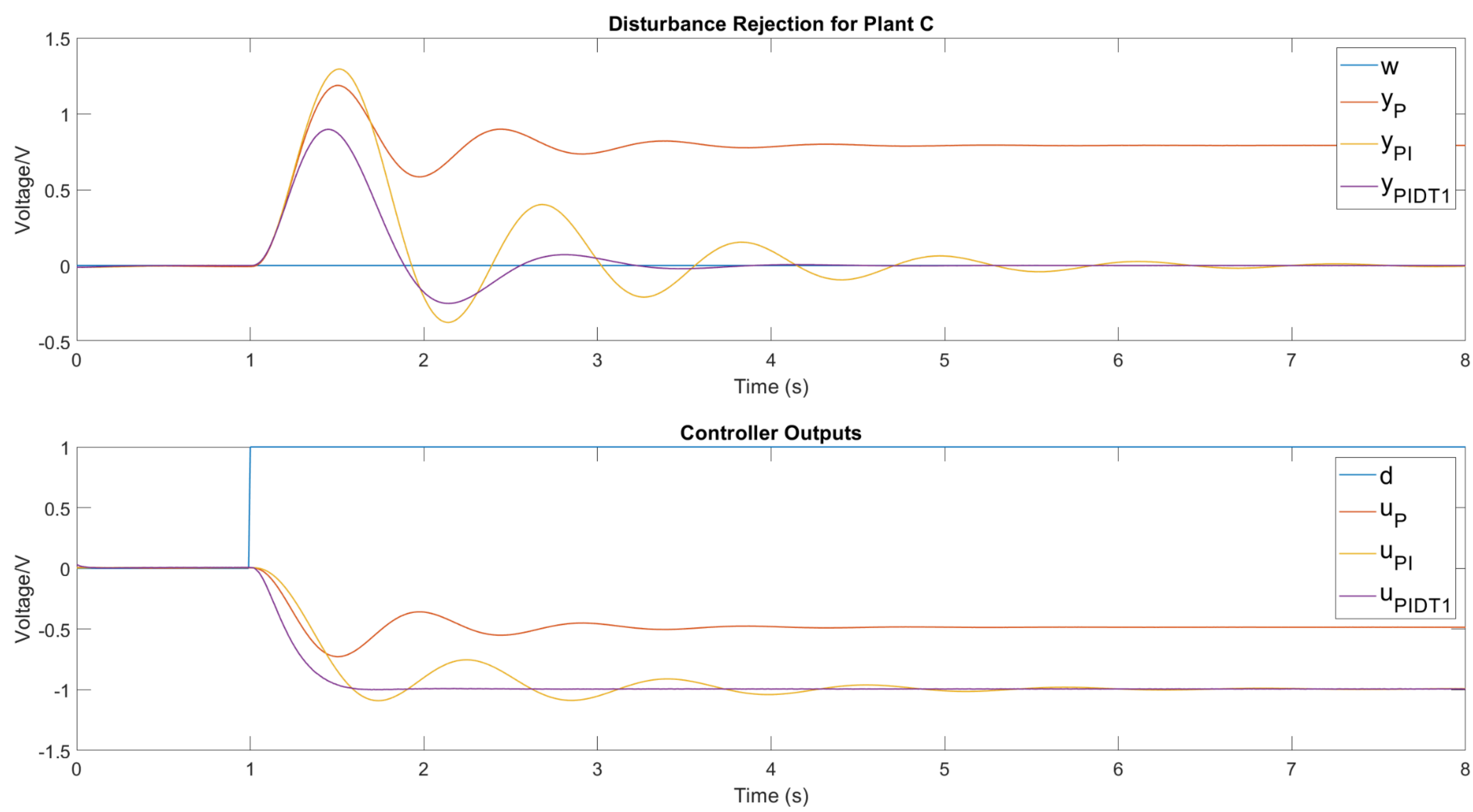
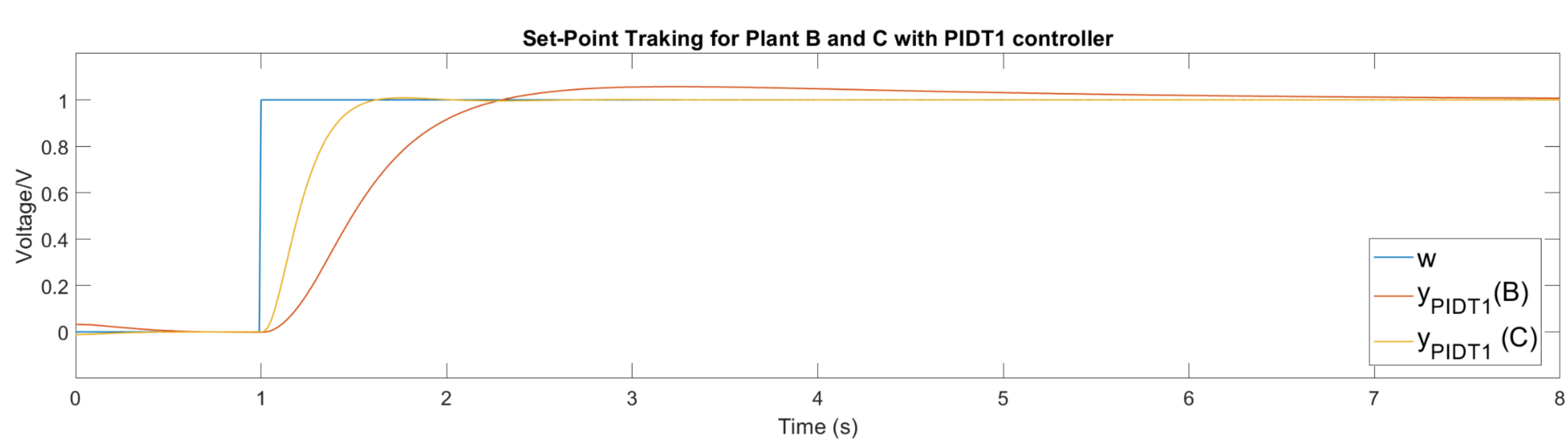


Figure 14 : Disturbance Rejection for plant C.

4. PIDT1:

Controller for Plant B and C Set point Tracking results for PIDT1 controller of plant B and C.



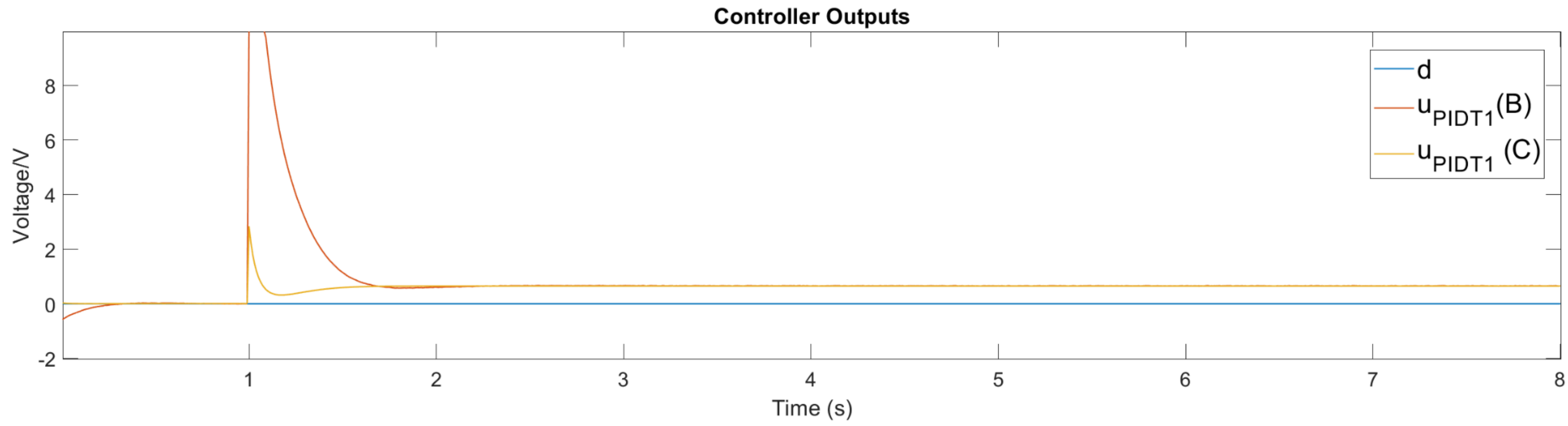


Figure 13 : Set-Point Tracking for plant B and C .

6 Results :-

Plant	Controller	Setpoint Tracking				Disturbance Rejection	
		e_{∞} [%]	M_p[%]	t_r [s]	$t_{s,5\%}$[s]	y_{∞}	$t_{s,5\%}$[s]
A	P	0	13	0.22	0.72	0.225	0.72
	PI	0	34	0.25	1.52	0	1.31
B	P	19.1	15	0.69	3.51	0.28	2.72
	PI	1.01	5.9	1.38	4.2	0.031	6.39
	PIDT1	0.62	5.3	0.69	1.29	0.011	6.86
C	P	52.8	53	0.2	3.21	0.77	3.22
	PI	0	13.4	0.37	3.82	0.01	5.19
	PIDT1	0	6.5	0.323	0.96	0	2.30

Table 2: : Control loop performance indicators

7 Conclusion :-

In this lab, we designed and evaluated P, PI, and PIDT1 controllers for three different plants using MATLAB and Simulink. The performance of each controller was assessed based on rise time, overshoot, steady-state error, settling time, and disturbance rejection capabilities. The key findings from this exercise are as follows:

- P Controller: The P controller demonstrated a good rise time but exhibited a noticeable overshoot and steady-state error. Additionally, its disturbance rejection characteristics were poor, as the system struggled to reach the setpoint in the presence of disturbances.

- PI Controller: The PI controller effectively reduced the steady-state error, providing better accuracy over time. However, it introduced longer settling times due to oscillations, and the overshoot was higher than that observed with the PIDT1 controller but lower than with the P controller. The disturbance rejection was good in case of plant A and moderate in case of plant B and C, improving upon the P controller but not reaching the performance of the PIDT1 controller.

- PIDT1 Controller: The PIDT1 controller outperformed both the P and PI controllers across all performance metrics. It achieved almost no steady-state error, a reasonable rise time, acceptable settling time, and low overshoot. Moreover, the PIDT1 controller exhibited excellent disturbance rejection capabilities, maintaining a very low steady-state output even in the presence of disturbances.