Review

- Types of Agents
- Problem formulation for goal based agents
    - States
    - Initial State
    - Actions
    - Goal
- Example of searching in the problem space

Search Implementation

- Open set
    - Group of states that haven’t been expanded yet
    - Group of states that are children of nodes that have been expanded
    - Search strategy chooses which open state to expand next
    - “set” as in collection of objects, not strictly mathematical set
    - Data structure depends on search strategy
- Closed set
    - Group of states that *have* been expanded
    - Often neglected (for space reasons), more later

Search Implementation

Pseudocode:

```
1. Initialize Open Set to contain initial state
2. Choose/remove one state from Open
   1. If chosen state is already closed jump to 2.4
   2. Check if chosen state is a goal state
         1. done if so
   3. Get child states of the chosen state using successor function
         1. Insert children into Open
         2. (Optional) Insert original state into Closed
   4. Repeat from (2)
```

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- Uninformed search
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  - Generate successors
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- Informed search
  - Given some idea of where to look for solutions

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  - Uniform-cost search
  - Depth-first search
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- Strategies differ by the order in which child nodes are expanded
  - i.e., removed from Open set

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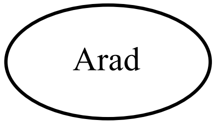
- Open set
  - Put new nodes at end, remove from front
  - Queue, first-in first-out (FIFO)

BFS Example



Open = { Arad }
Closed = { }

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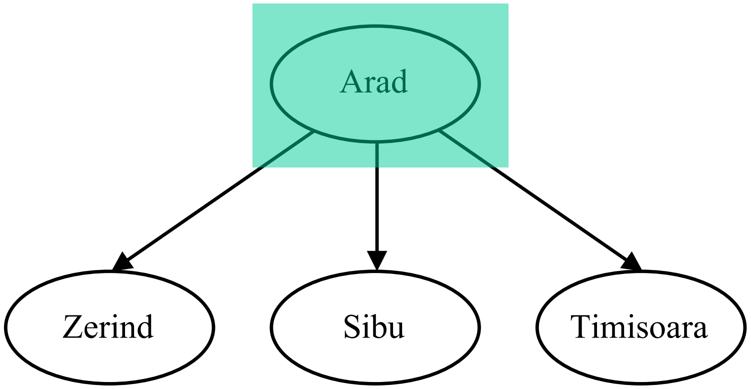

Open = { Zerind, Sibu, Timisoara }
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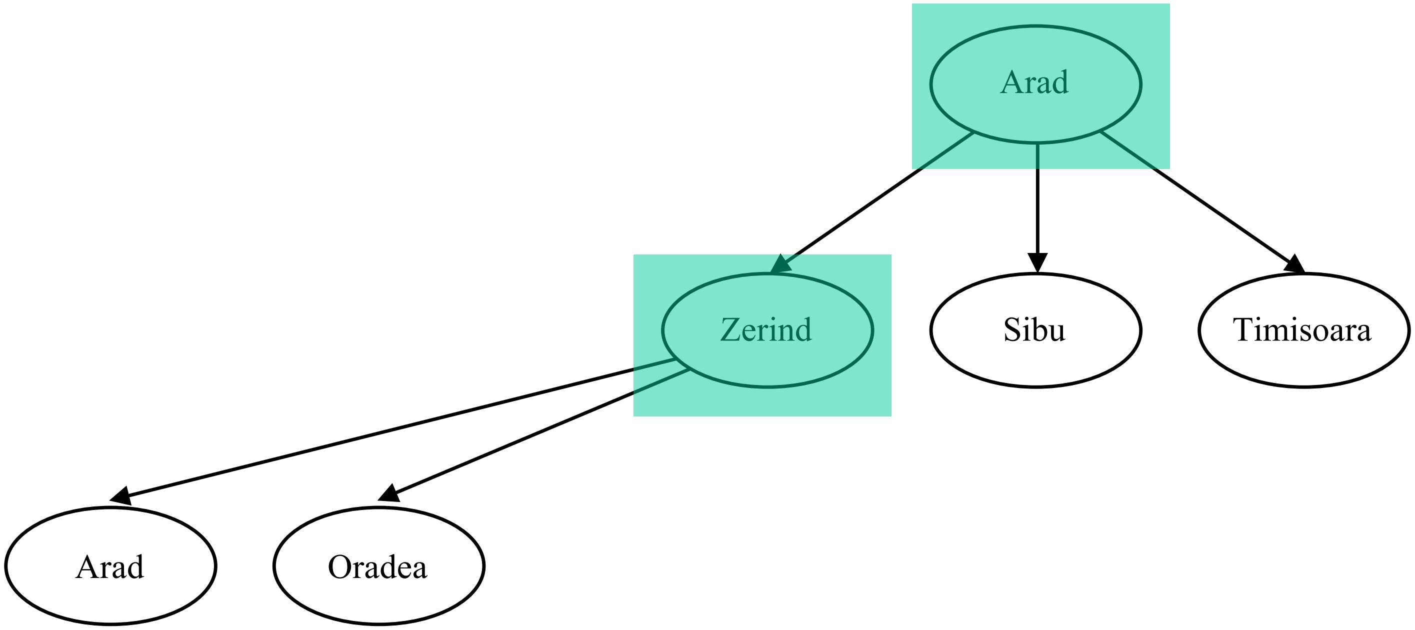
BFS Example



Open = { Sibu, Timisoara, Arad, Oradea }
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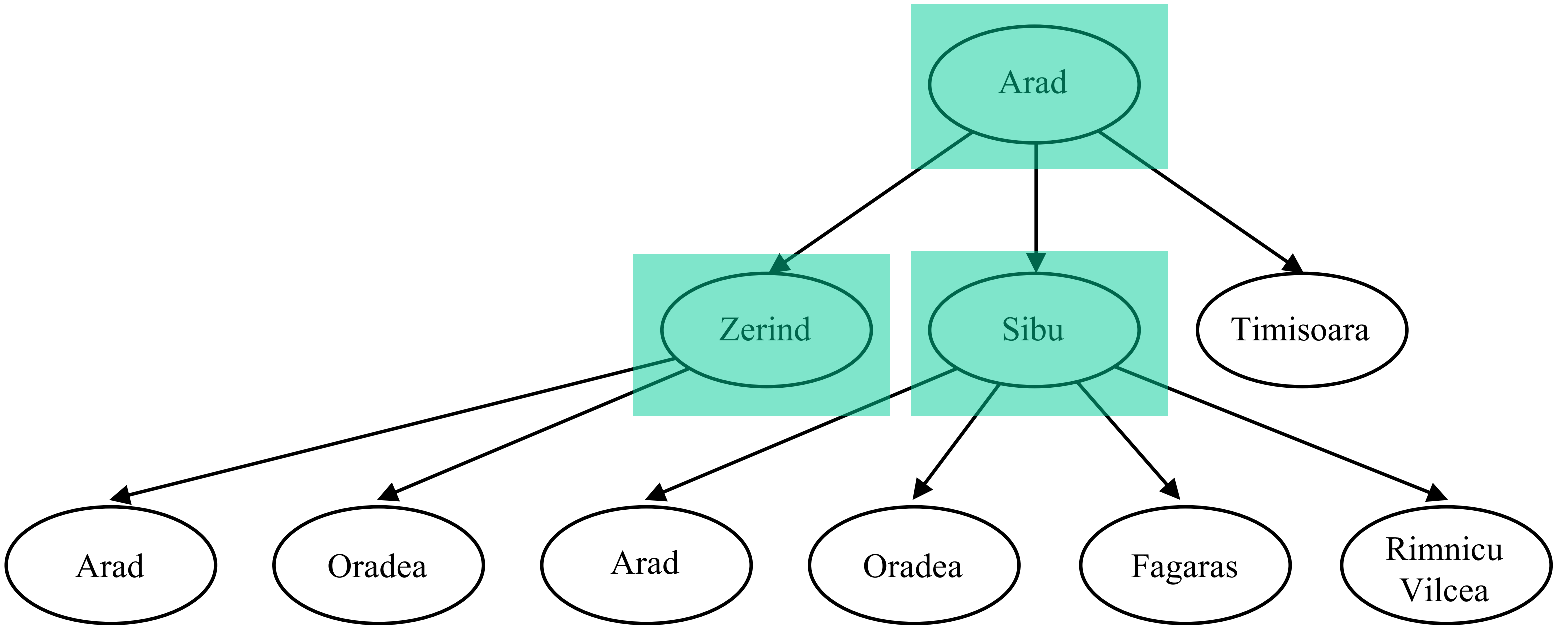

Open = { Timisoara, Arad, Oradea, Arad, Oradea, Fagaras, Rimnicu Vicea }

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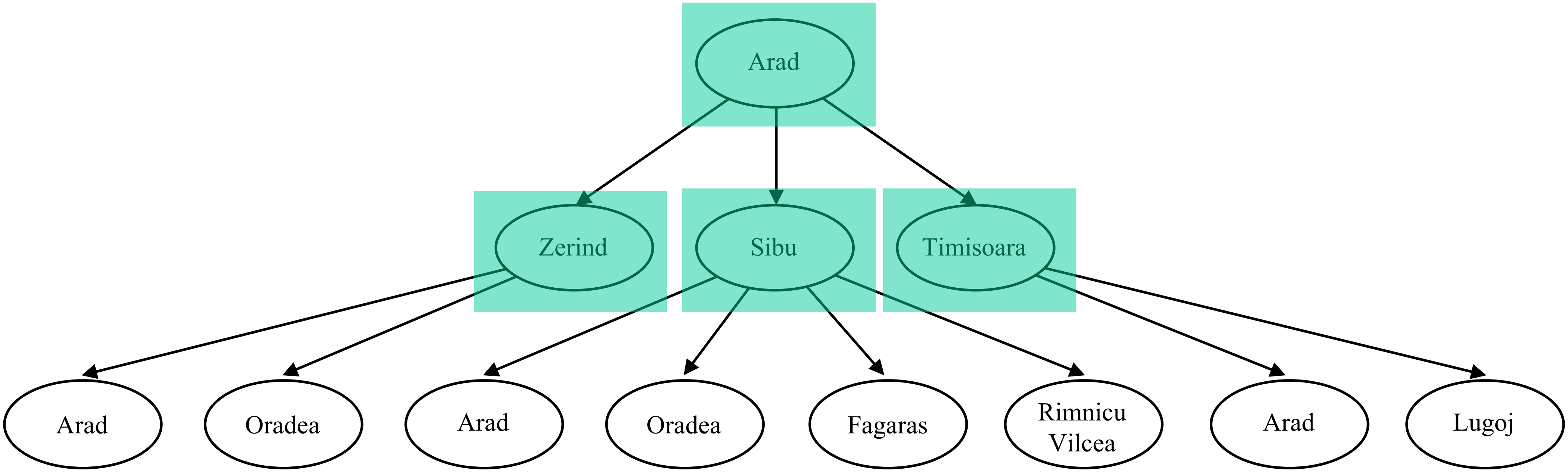

Open = { Arad, Oradea, Arad, Oradea, Fagaras, Rimnicu Vicea, Arad, Lugoj }

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BFS Properties

- Complete (good)
  - If branching factor *b* is finite

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- Optimality (good)
  - Not in general, shallowest may not be optimal path cost
  - Optimal if path cost non-decreasing function of node depth

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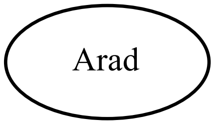
- Open set
  - Remove nodes in order of increasing path cost
  - Priority queue

UCS Example



Open = { Arad(0) }

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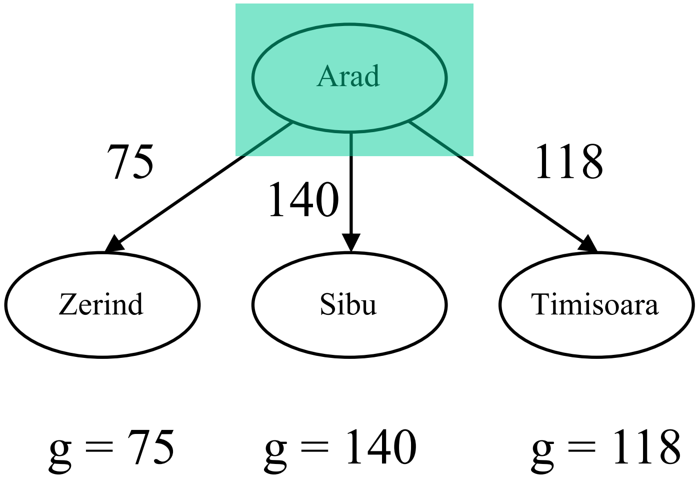

Open = { Zerind(75), Timisoara(118), Sibu(140) }
Closed = { Arad }

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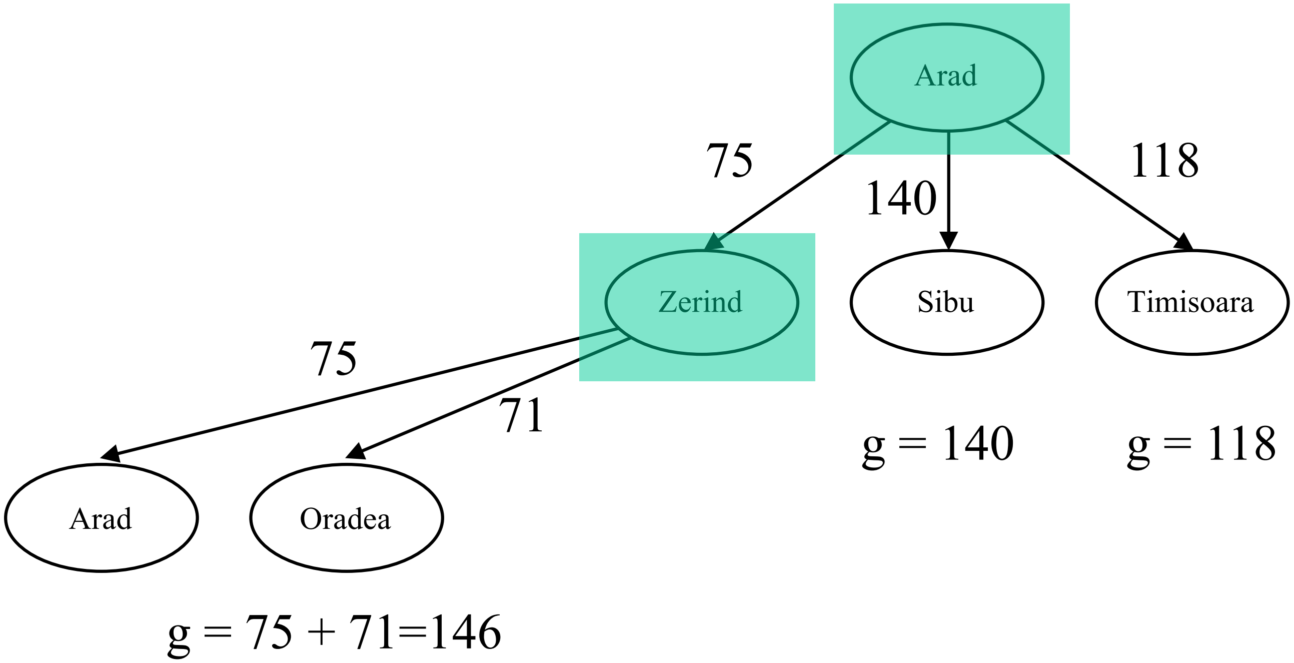

Open = { Timisoara(118), Sibu(140), Oradea(146), Arad(150) }
Closed = { Arad, Zerind }

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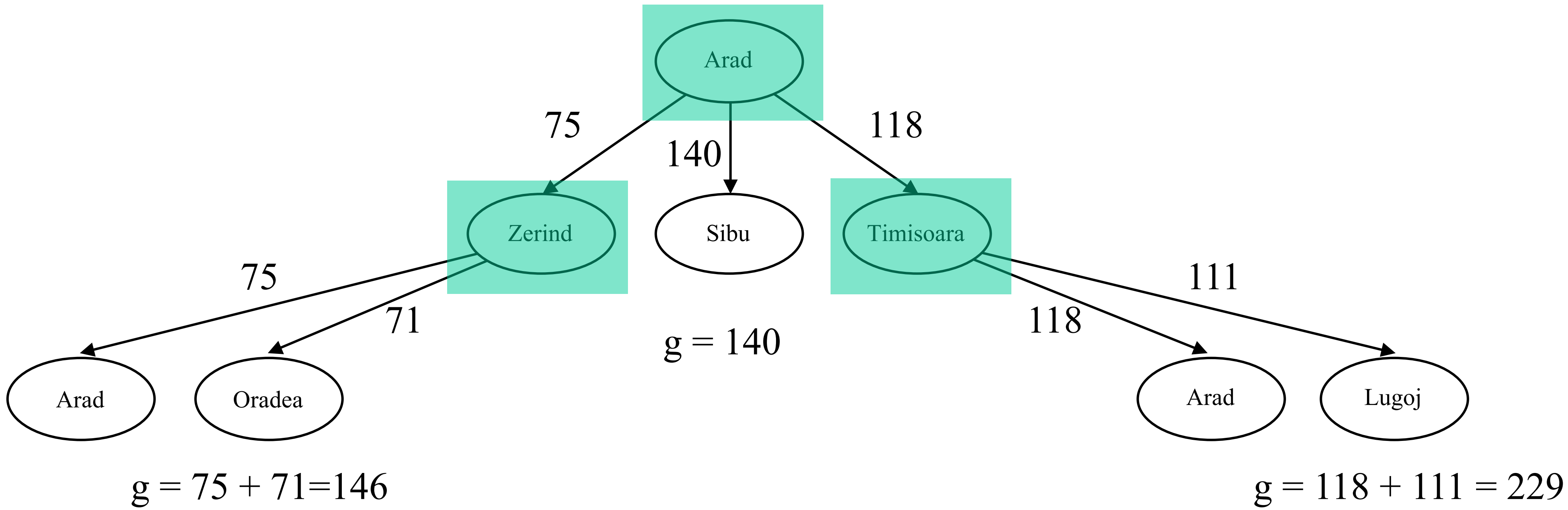

Open = { Sibu(140), Oradea(146), Arad(150), Lugoj(229), Arad(236) }

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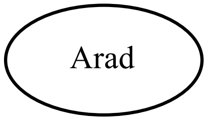
DFS Example



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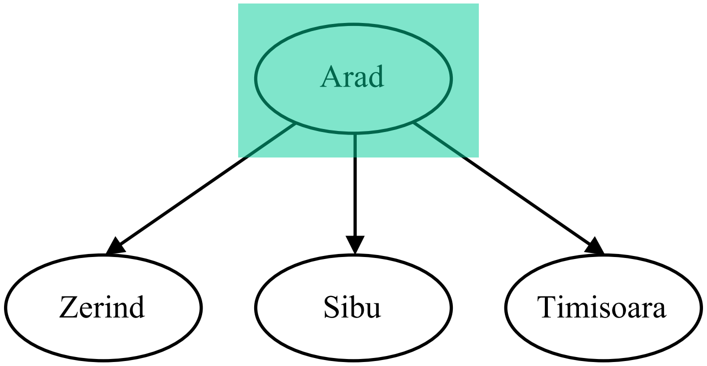

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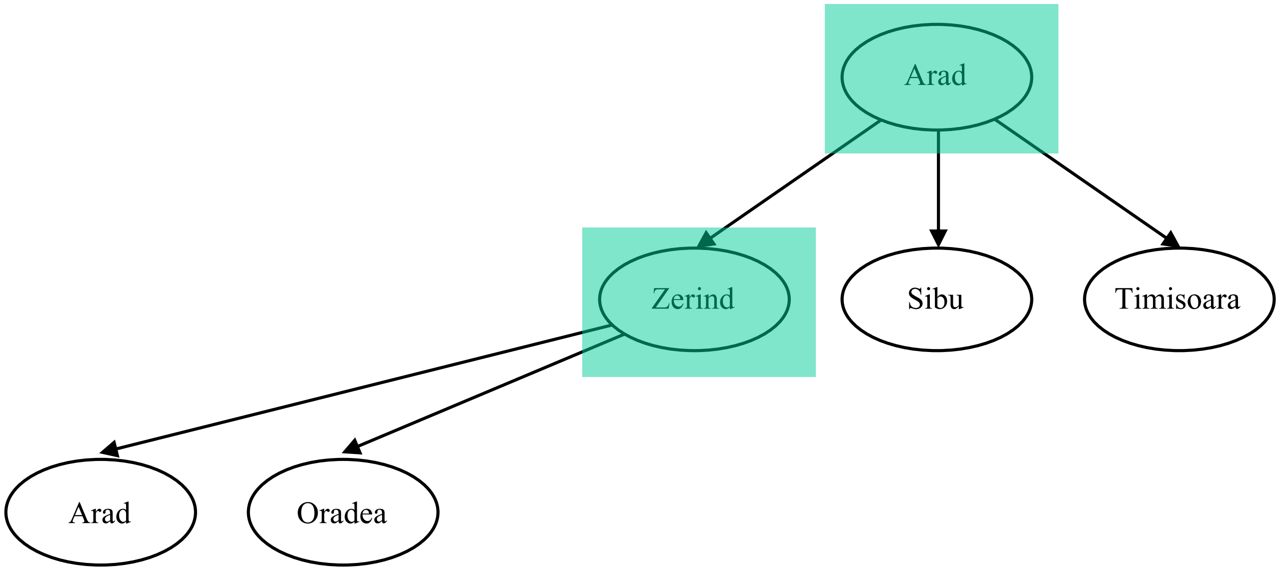

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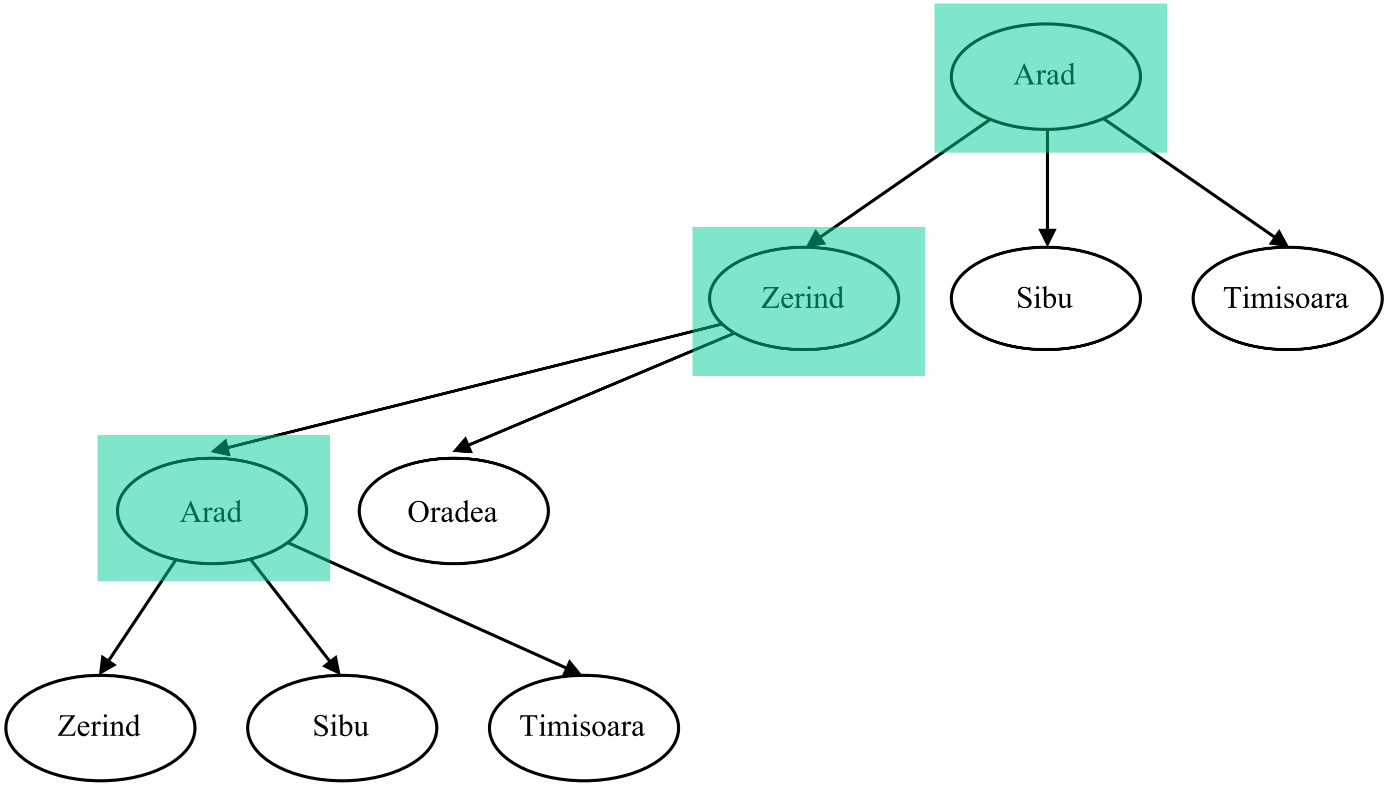

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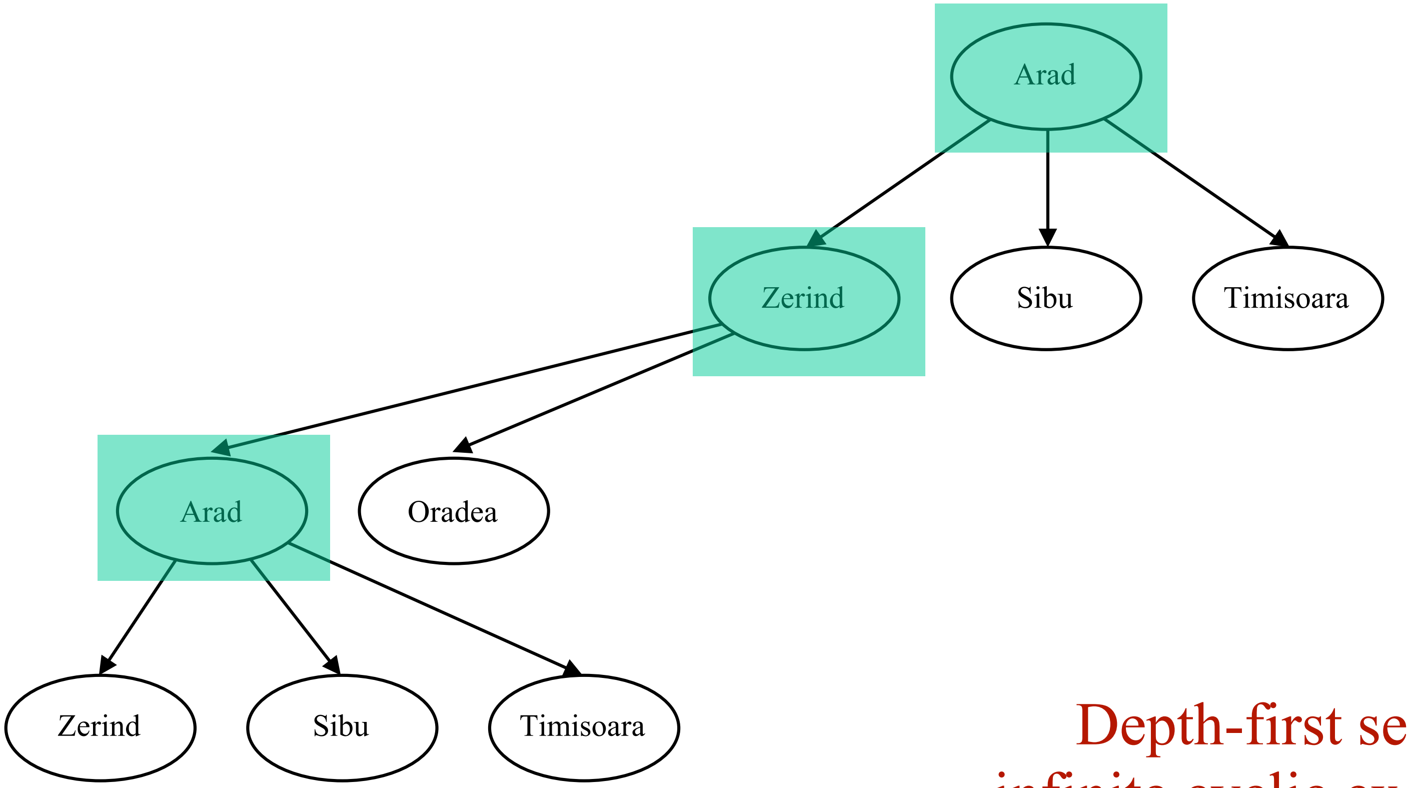

Depth-first search can perform infinite cyclic excursions if not check for repeated-states!

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  - *O(mb),* linear space (keep only leaf nodes "as you go")
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- Optimality (bad)
  - No, it returns the first deepest solution, so it could miss a shallower solution it has not yet seen (even at low depth)

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- Time and space complexity of depth-first search

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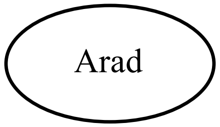
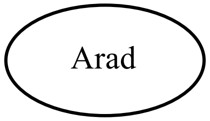
- Preferred method with large search space and depth of solution not known
  -

IDS Example [**with l=0]**



[STOP]

IDS Example [**with l=1**]



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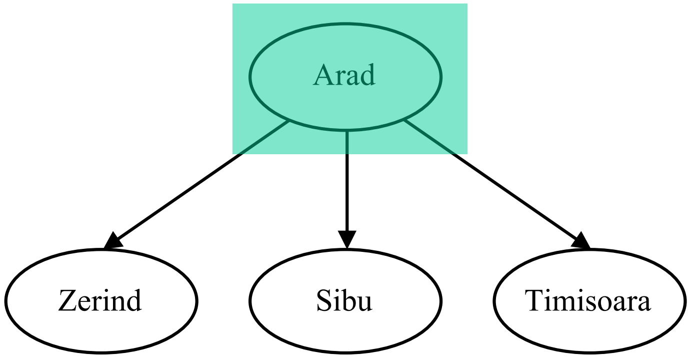
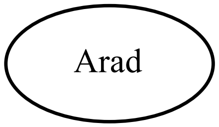


[STOP]

IDS Example [**with l=2**]



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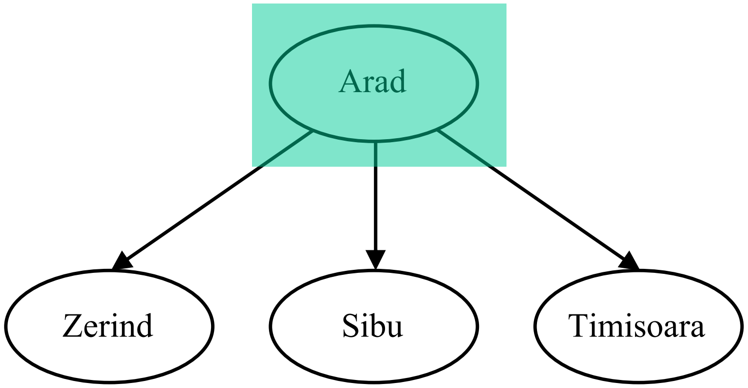

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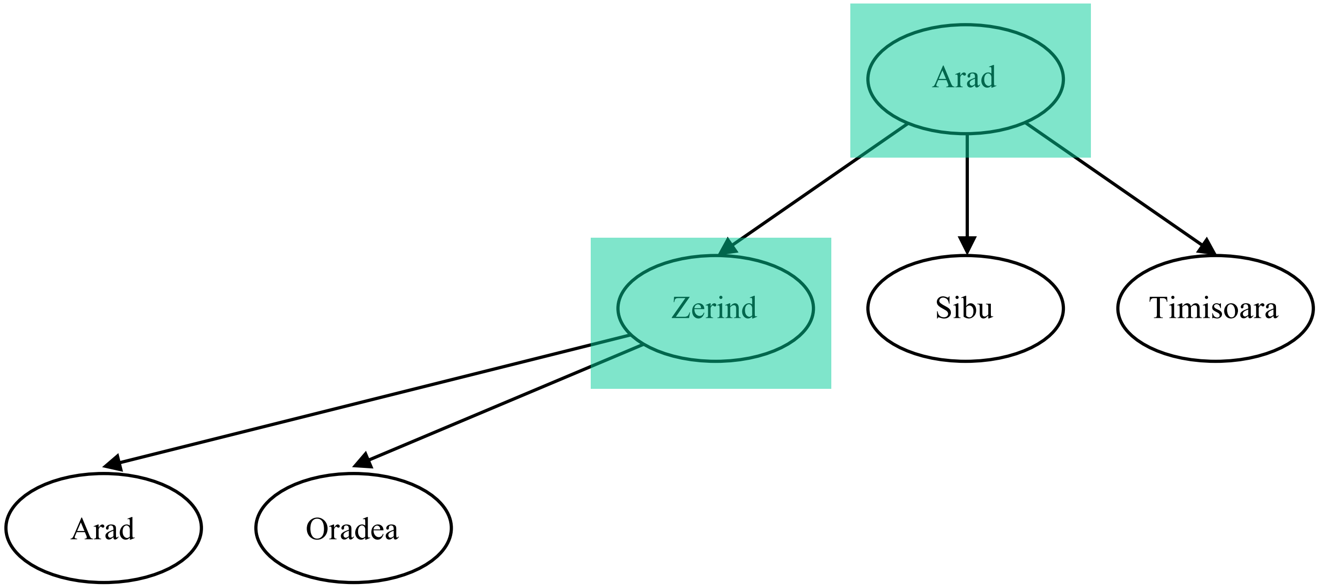

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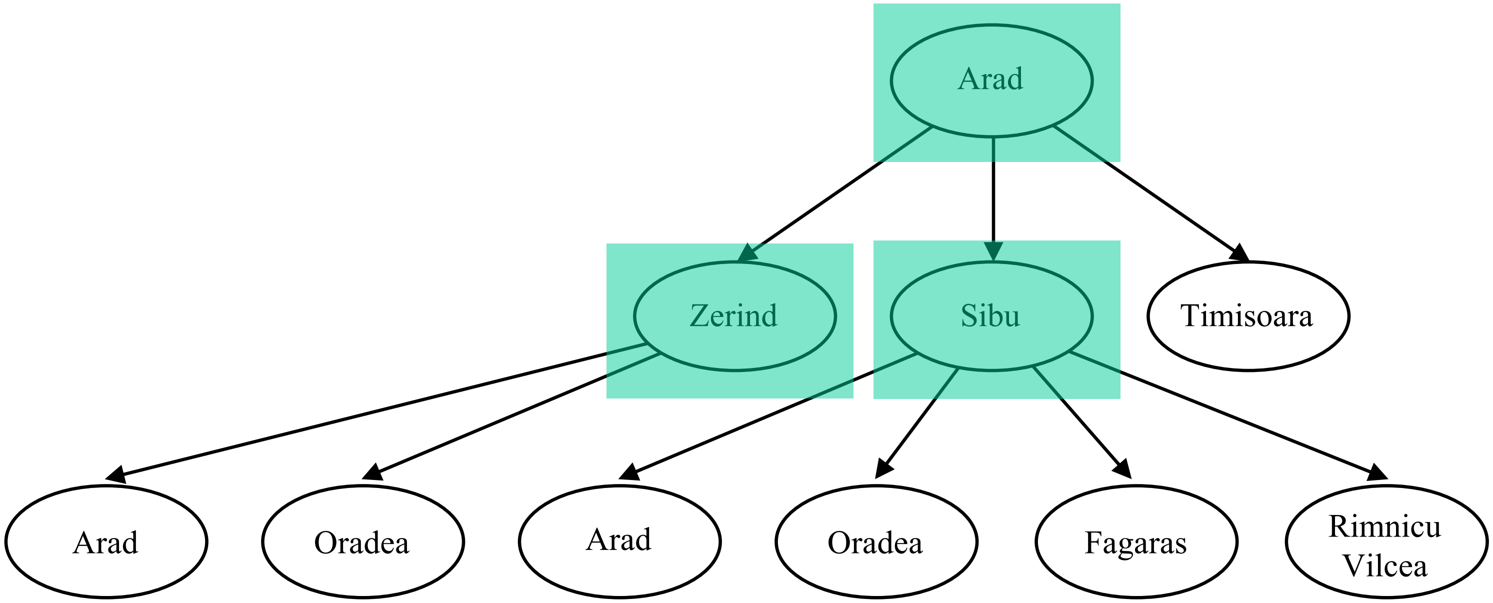

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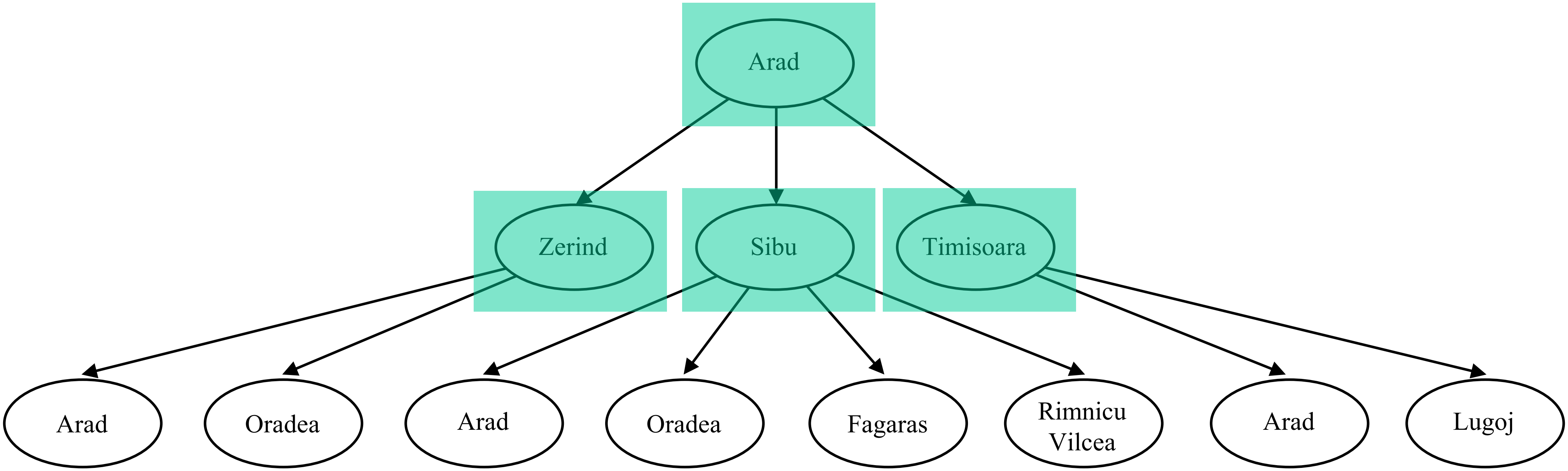

[STOP]

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IDS combines benefits of depth-first and breadth-first search

IDS Properties

- Nodes generated when goal at depth *d*
  - (bottom level; root is depth 0)
  - with branching factor *b*
  - $(d)b + (d\text{-}1)b^2 + \ldots + (1)b^d$ => $O(b^d)$

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  - $\underbrace{(d)b}_{\text{Children of root generated } d \text{ times}} + (d\text{-}1)b^2 + \ldots + \underbrace{(1)b^d}_{\text{Nodes at bottom generated once}}$ => $O(b^d)$

Note: root node always generated/available

Cost: IDS vs. DLS

b	Worst (Asymp)
2	2x
3	1.5x
4	1.33x

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- Compare to Breadth-first search
  - $b + b^2 + \ldots + b^d + (b^{d+1} - b\ )$ => $O(b^{d+1})$

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Assume goal is last node (e.g., rightmost) at depth *d*

These are nodes already put in the queue/stack from the non-goal nodes at depth *d* (though not examined later)

Depth-first search does <u>not</u> have these extra nodes

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Depth-first search does not have these extra nodes

- Comparison for $b = 10$ and $d = 5$
  - #nodes(IDS) = 123,450
  - #nodes(BFS) = 1,111,100

When to use IDS

- Preferred uninformed search method when
  - there is a <u>large search space</u> and
  - the <u>depth of the solution is not known</u>.

Notes: Looping

- Loop: Returning to a node previously visited on path
- How to avoid?
  - Keep a list of previously visited nodes → Closed set
- Cost?
  - BFS/UCS: Minimal, Closed set is always smaller than Open
  - DFS: Worsens space cost to similar to BFS

Notes: Exponential Complexity

- So far, describe time/space complexity as exponential in terms of branching factor and depth. E.g., $O(b^d)$
- But if we avoid looping, will only visit each state once!
- In terms of number of states *n*, complexity can be $O(bn)$ or even $O(n)$!!
- For problems with limited number of states, these algorithms are very fast
  - Consider Romania problem, only 20 states

Notes: Recursive DFS

- Possible to implement DFS (and related) as recursive functions
    - Think of child states as “new” initial states
- Reduces space cost to *O(d)*
- Loops
    - Only need to store/check current path (previous recursion levels)
    - No additional space cost
- Recursive implementation is typical

QuestionS