Administrative Details

- HW 3 is posted
    - Due on March 4
    - Available until March 6
- MidTerm on March 6
    - Syllabus:
        - Everything covered in class up to Feb 28
    - Cheat Sheet:
        - One page written on both sides

Decision Tree

Supervised Learning: find f

- Given: Training set $\{(x_i, y_i) \mid i = 1 \ldots n\}$
- Find: A good approximation to $f : X \rightarrow Y$

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Examples: what are X and Y?

- Spam Detection
  - Map email to {Spam,Ham}
- Digit recognition
  - Map pixels to {0,1,2,3,4,5,6,7,8,9}
- Stock Prediction
  - Map new, historic prices, etc. to $\mathbb{R}$ (the real numbers)

A Supervised Learning Problem

- Consider a simple, Boolean dataset:
  - $f: X \rightarrow Y$
  - $X = \{0,1\}^4$
  - $Y = \{0,1\}$
- Question 1: How should we pick the *hypothesis space*, the set of possible functions f?
- Question 2: How do we find the best f in the hypothesis space?

Dataset:

Example	x_1	x_2	x_3	x_4	y
1	0	0	1	0	0
2	0	1	0	0	0
3	0	0	1	1	1
4	1	0	0	1	1
5	0	1	1	0	0
6	1	1	0	0	0
7	0	1	0	1	0

Most General Hypothesis Space

Consider all possible boolean functions over four input features!

Dataset:

x_1	x_2	x_3	x_4	y
0	0	0	0	?
0	0	0	1	?
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	?
1	0	0	0	?
1	0	0	1	1
1	0	1	0	?
1	0	1	1	?
1	1	0	0	0
1	1	0	1	?
1	1	1	0	?
1	1	1	1	?

Most General Hypothesis Space

Consider all possible boolean functions over four input features!

- 2^{16} possible hypotheses
- 2^{9} are consistent with our dataset
- How do we choose the best one?

Dataset:

x_1	x_2	x_3	x_4	y
0	0	0	0	?
0	0	0	1	?
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	?
1	0	0	0	?
1	0	0	1	1
1	0	1	0	?
1	0	1	1	?
1	1	0	0	0
1	1	0	1	?
1	1	1	0	?
1	1	1	1	?

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Consider all possible boolean functions over four input features!

- 2^{16} possible hypotheses
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0	0	0	0	?
0	0	0	1	?
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	?
1	0	0	0	?
1	0	0	1	1
1	0	1	0	?
1	0	1	1	?
1	1	0	0	0
1	1	0	1	?
1	1	1	0	?
1	1	1	1	?

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1	0	0	1	0	0
2	0	1	0	0	0
3	0	0	1	1	1
4	1	0	0	1	1
5	0	1	1	0	0
6	1	1	0	0	0
7	0	1	0	1	0

A Restricted Hypothesis Space

Consider all conjunctive boolean functions.

- 16 possible hypotheses
- None are consistent with our dataset
- How do we choose the best one?

Dataset:

Rule	Counterexample
$\Rightarrow y$	1
$x_1 \Rightarrow y$	3
$x_2 \Rightarrow y$	2
$x_3 \Rightarrow y$	1
$x_4 \Rightarrow y$	7
$x_1 \wedge x_2 \Rightarrow y$	3
$x_1 \wedge x_3 \Rightarrow y$	3
$x_1 \wedge x_4 \Rightarrow y$	3
$x_2 \wedge x_3 \Rightarrow y$	3
$x_2 \wedge x_4 \Rightarrow y$	3
$x_3 \wedge x_4 \Rightarrow y$	4
$x_1 \wedge x_2 \wedge x_3 \Rightarrow y$	3
$x_1 \wedge x_2 \wedge x_4 \Rightarrow y$	3
$x_1 \wedge x_3 \wedge x_4 \Rightarrow y$	3
$x_2 \wedge x_3 \wedge x_4 \Rightarrow y$	3
$x_1 \wedge x_2 \wedge x_3 \wedge x_4 \Rightarrow y$	3

Example	x_1	x_2	x_3	x_4	y
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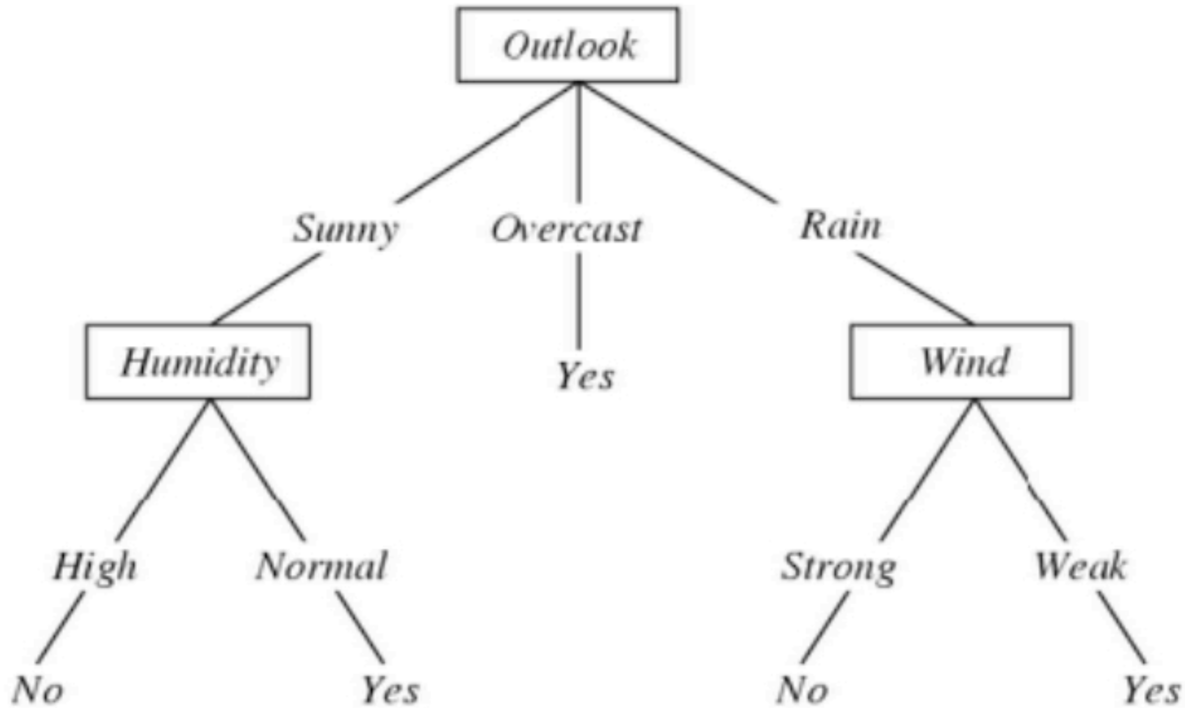
5	0	1	1	0	0
6	1	1	0	0	0
7	0	1	0	1	0

Simple Training Data Set

Day	Outlook	Temperature	Humidity	Wind	PlayTennis?
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

A Decision tree for

f: <Outlook, Temperature, Humidity, Wind> → PlayTennis?



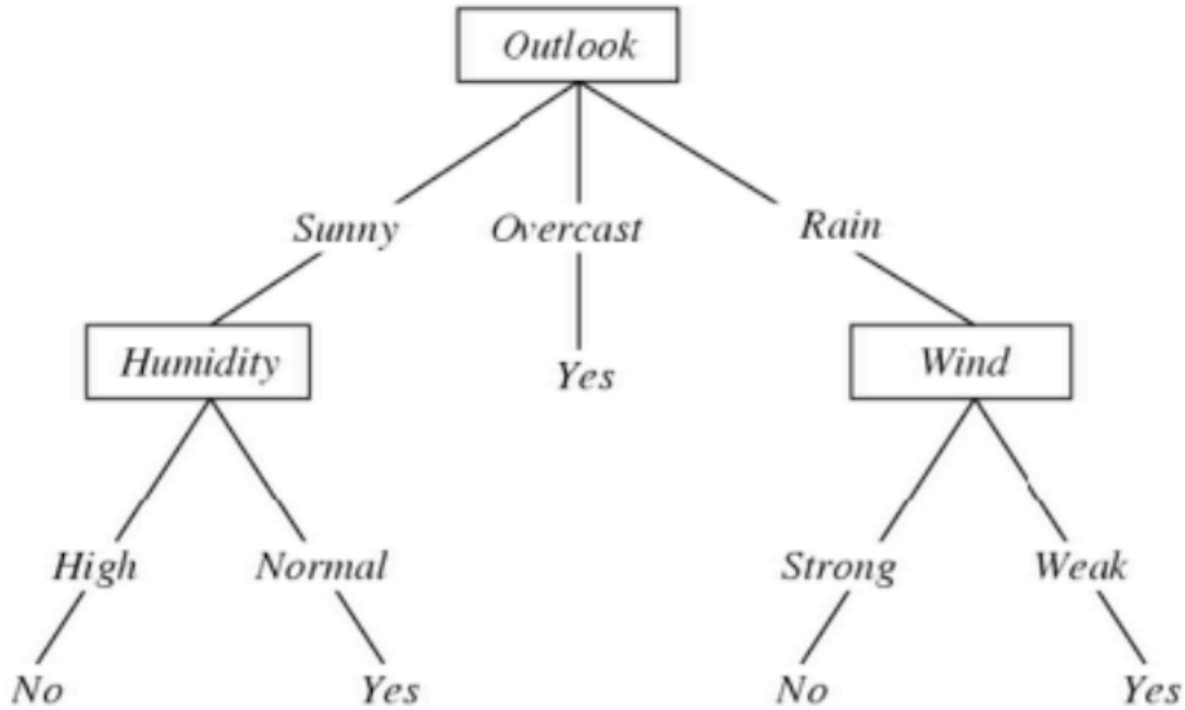
Each internal node: test one discrete-valued attribute X_i

Each branch from a node: selects one value for X_i

Each leaf node: predict Y (or $P(Y|X \in \text{leaf})$)

A Decision tree for

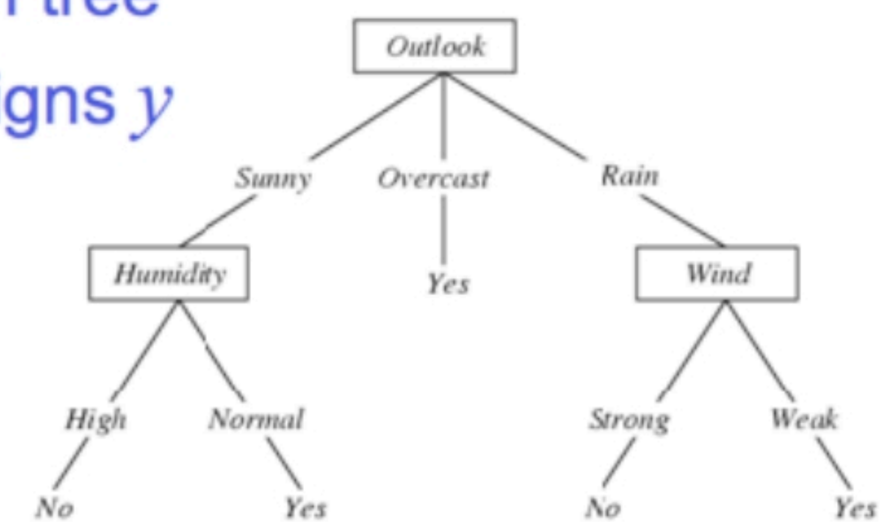
f: <Outlook, Temperature, Humidity, Wind> → PlayTennis?



Decision Tree Learning

Problem Setting:

- Set of possible instances X
  - each instance x in X is a feature vector
  - e.g., <*Humidity=low, Wind=weak, Outlook=rain, Temp=hot*>
- Unknown target function $f : X \rightarrow Y$
  - Y=1 if we play tennis on this day, else 0
- Set of function hypotheses $H=\{ h \mid h : X \rightarrow Y \}$
  - each hypothesis h is a decision tree
  - trees sorts x to leaf, which assigns y



Decision Tree Learning

Problem Setting:

- Set of possible instances X
  - each instance x in X is a feature vector
    $x = < x_1, x_2 \dots x_n>$
- Unknown target function $f : X \rightarrow Y$
  - Y is discrete-valued
- Set of function hypotheses $H=\{ h \mid h : X \rightarrow Y \}$
  - each hypothesis h is a decision tree

Input:

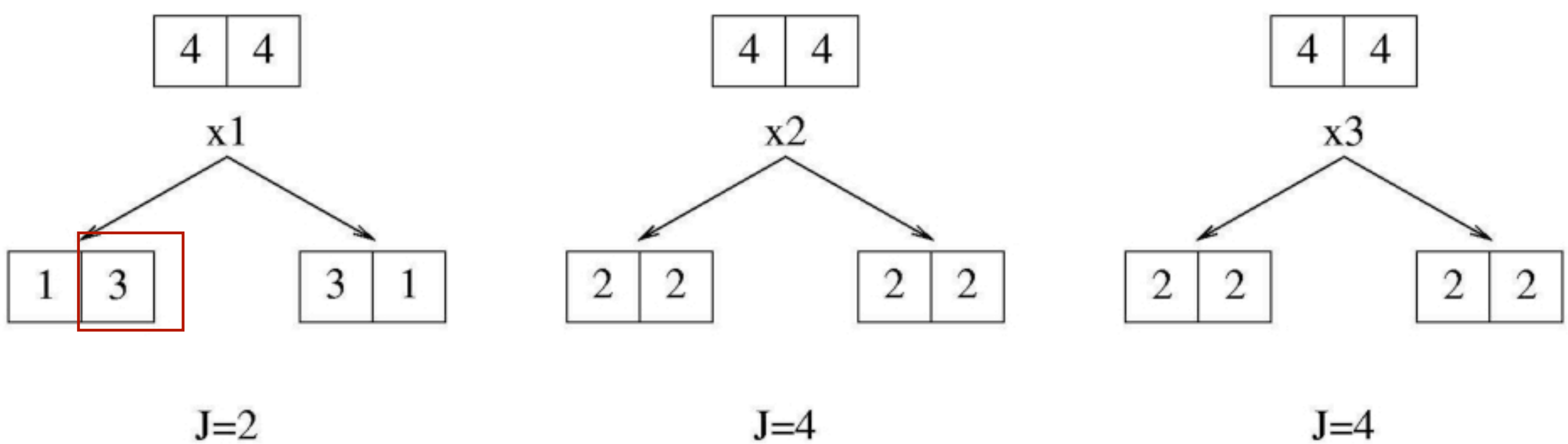
- Training examples $\{<x^{(i)},y^{(i)}>\}$ of unknown target function f

Output:

- Hypothesis $h \in H$ that best approximates target function f

Choosing the Best Attribute—An Example

x_1	x_2	x_3	y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



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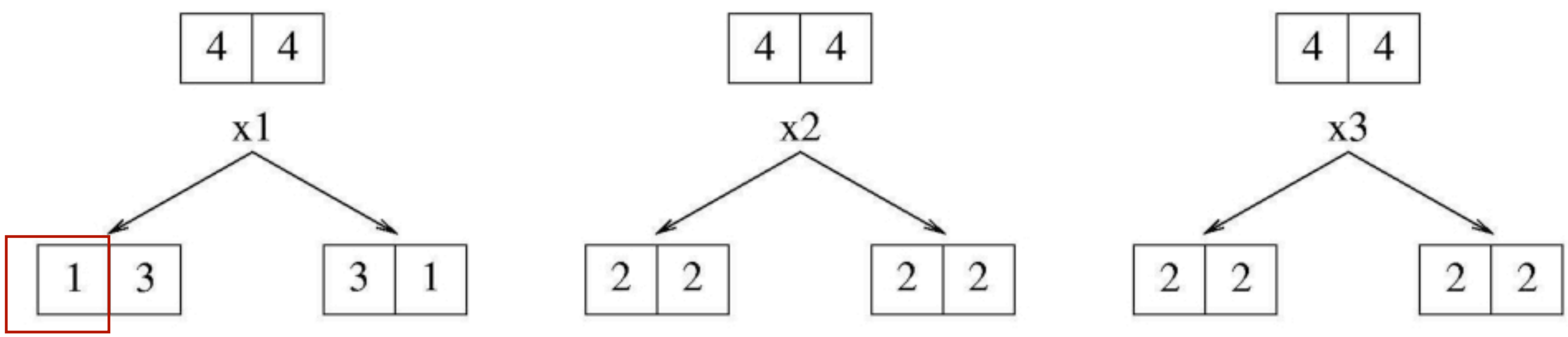

J=2 J=4 J=4

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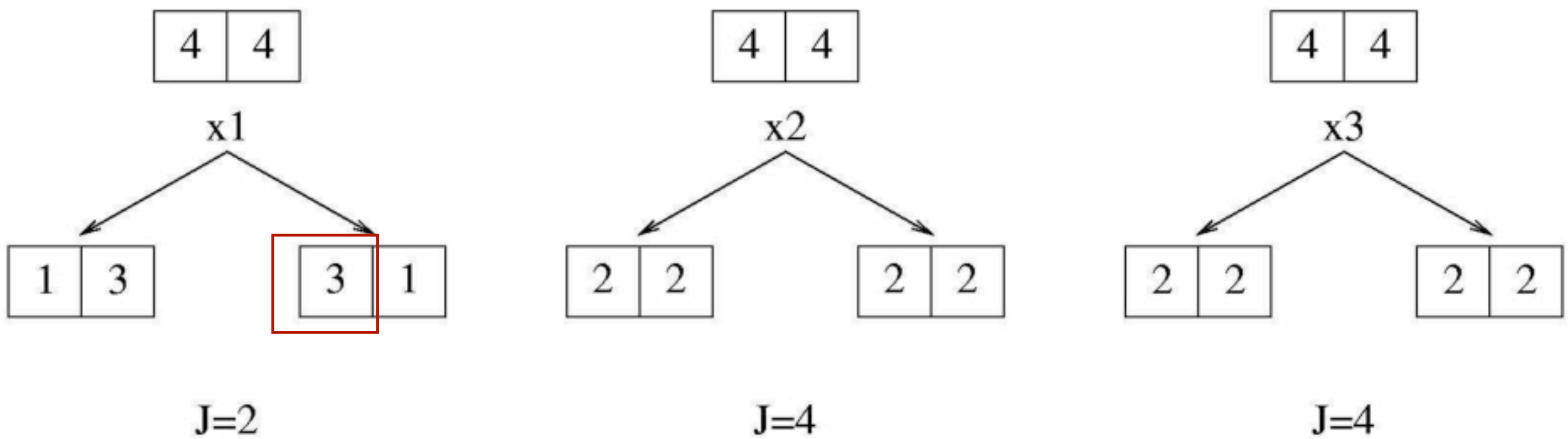

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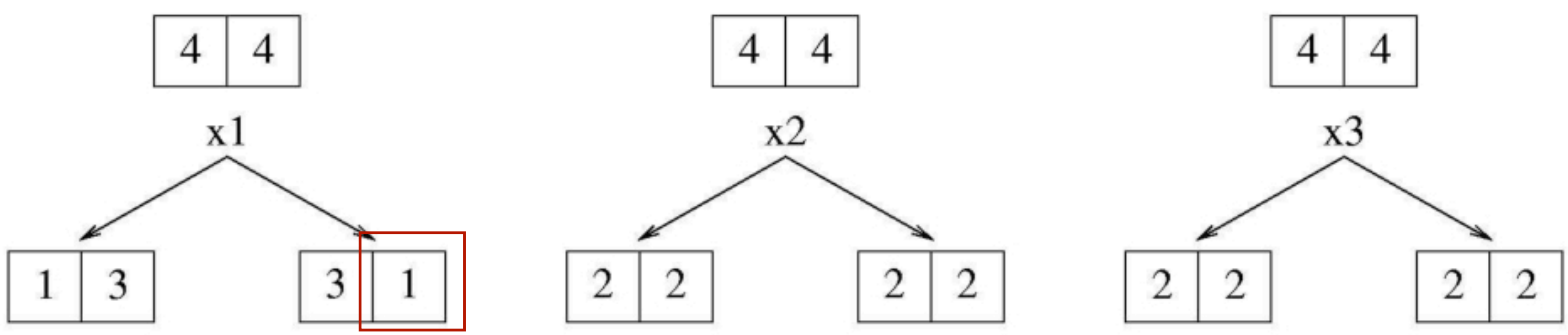

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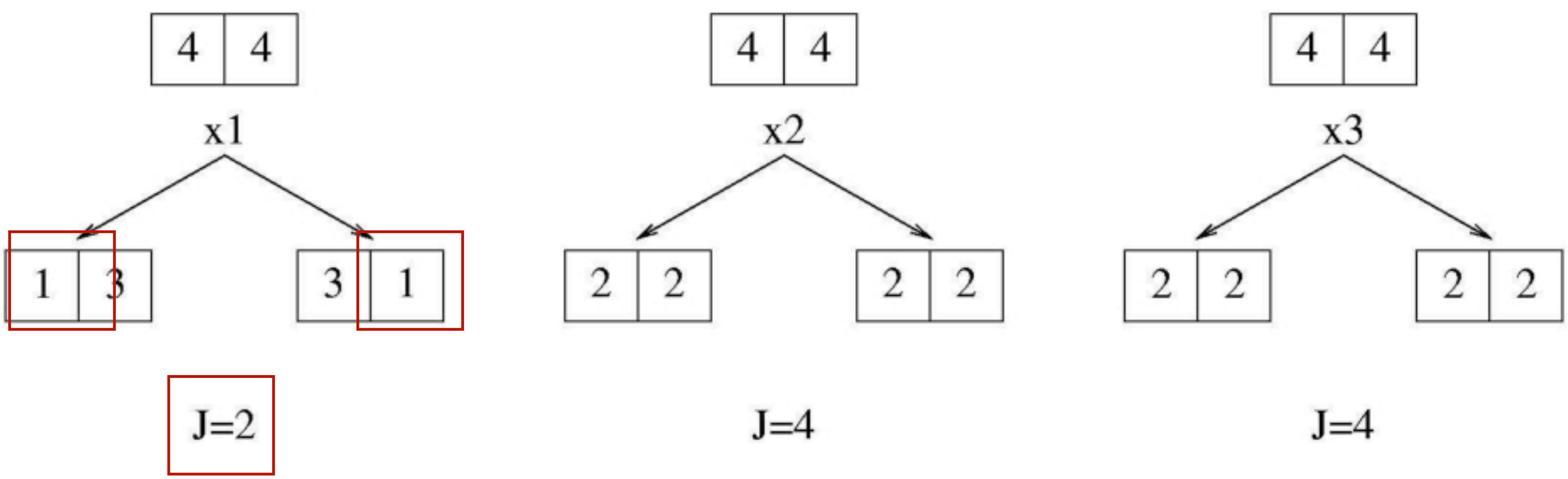
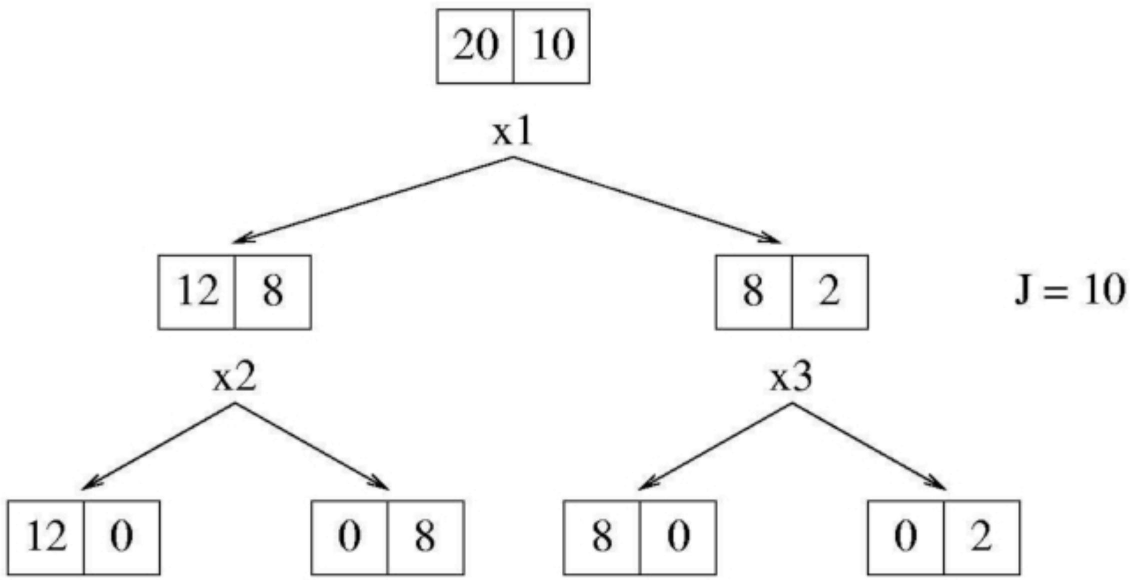


Choosing the Best Attribute (3)

Unfortunately, this measure does not always work well, because it does not detect cases where we are making "progress" toward a good tree.



A Better Heuristic From Information Theory

Let V be a random variable with the following probability distribution:

$P(V=0)$	$P(V=1)$
0.2	0.8

The *surprise*, $S(V=v)$ of each value of V is defined to be

$$S(V=v) = -\lg P(V=v).$$

An event with probability 1 gives us zero surprise.

An event with probability 0 gives us infinite surprise!

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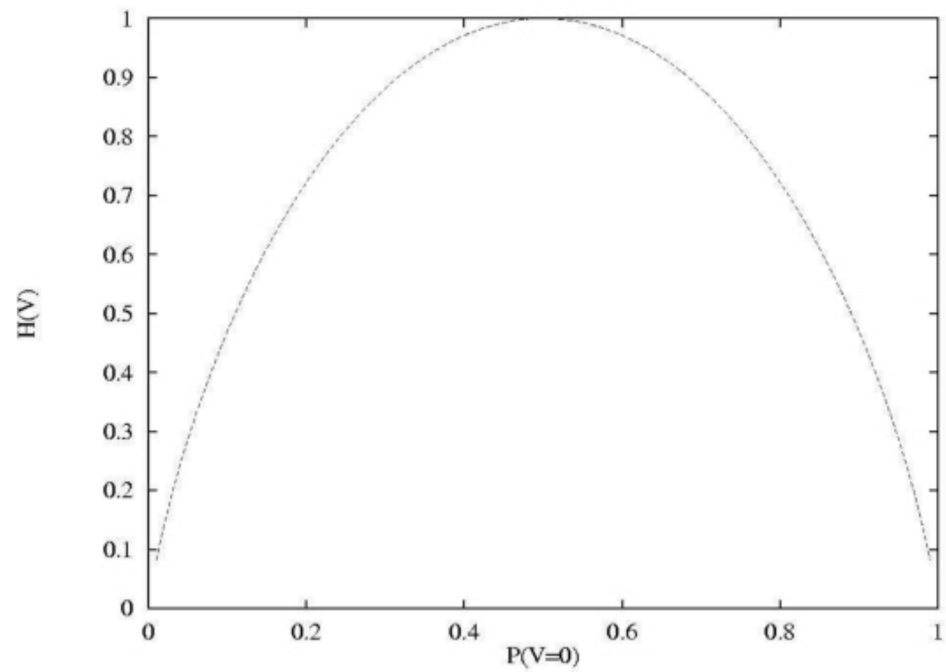
This is also called the *description length* of $V = v$.
Fractional bits only make sense if they are part of a longer message (e.g., describe a whole sequence of coin tosses).

Entropy

The *entropy* of V, denoted $H(V)$ is defined as follows:

$$H(V) = \sum_{v=0}^{1} -P(H = v) \lg P(H = v).$$

This is the average surprise of describing the result of one "trial" of V (one coin toss).



Entropy can be viewed as a measure of uncertainty.