Decision Tree

- A decision tree is a tree-like structure that is used as a model for classifying data.
- A decision tree decomposes the data into sub-trees made of other sub-trees and/or leaf nodes.
- A decision tree is made up of two types of nodes
    - *Decision Nodes*: These type of node have two or more branches
    - *Leaf Nodes*: The lowest nodes which represents decision

DataSet

Attributes | Classes

Outlook	Temperature	Humidity	Windy	Play Golf
Rainy	Hot	High	FALSE	No
Rainy	Hot	High	TRUE	No
Overcast	Hot	High	FALSE	Yes
Sunny	Mild	High	FALSE	Yes
Sunny	Cool	Normal	FALSE	Yes
Sunny	Cool	Normal	TRUE	No
Overcast	Cool	Normal	TRUE	Yes
Rainy	Mild	High	FALSE	No
Rainy	Cool	Normal	FALSE	Yes
Sunny	Mild	Normal	FALSE	Yes
Rainy	Mild	Normal	TRUE	Yes
Overcast	Mild	High	TRUE	Yes
Overcast	Hot	Normal	FALSE	Yes
Sunny	Mild	High	TRUE	No

Step 1: Determine the Decision Column

- Since decision trees are used for clarification, you need to determine the classes which are the basis for the decision.

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Overcast	Cool	Normal	TRUE	Yes
Rainy	Mild	High	FALSE	No
Rainy	Cool	Normal	FALSE	Yes
Sunny	Mild	Normal	FALSE	Yes
Rainy	Mild	Normal	TRUE	Yes
Overcast	Mild	High	TRUE	Yes
Overcast	Hot	Normal	FALSE	Yes
Sunny	Mild	High	TRUE	No

Step 1: Determine the Decision Column

- Since decision trees are used for clarification, you need to determine the classes which are the basis for the decision.

- In this case, it it the last column, that is *Play Golf* column with classes ***Yes*** and ***No***.

- Next determine the rootNode
  - we need to compute the entropy.
  - To compute the entropy, we create a frequency table for the classes

Attributes				Classes
Outlook	**Temperature**	**Humidity**	**Windy**	**Play Golf**
Rainy	Hot	High	FALSE	No
Rainy	Hot	High	TRUE	No
Overcast	Hot	High	FALSE	Yes
Sunny	Mild	High	FALSE	Yes
Sunny	Cool	Normal	FALSE	Yes
Sunny	Cool	Normal	TRUE	No
Overcast	Cool	Normal	TRUE	Yes
Rainy	Mild	High	FALSE	No
Rainy	Cool	Normal	FALSE	Yes
Sunny	Mild	Normal	FALSE	Yes
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Overcast	Mild	High	TRUE	Yes
Overcast	Hot	Normal	FALSE	Yes
Sunny	Mild	High	TRUE	No

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$\underbrace{\hspace{10em}}_{}$

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Overcast	Hot	High	FALSE	Yes
Sunny	Mild	High	FALSE	Yes
Sunny	Cool	Normal	FALSE	Yes
Sunny	Cool	Normal	TRUE	No
Overcast	Cool	Normal	TRUE	Yes
Rainy	Mild	High	FALSE	No
Rainy	Cool	Normal	FALSE	Yes
Sunny	Mild	Normal	FALSE	Yes
Rainy	Mild	Normal	TRUE	Yes
Overcast	Mild	High	TRUE	Yes
Overcast	Hot	Normal	FALSE	Yes
Sunny	Mild	High	TRUE	No

Play Golf(14)	
Yes	No
9	5

Step 2: Calculating Entropy for the classes (Play Golf)

- In this step, you need to calculate the entropy for the Decision Column (Play Golf)
- *Entropy(PlayGolf) =* ***E****(5-,9+)*

Play Golf(14)	
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$$Entropy(S) = \sum_{i=1}^{c} -p_i log_2 p_i$$

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Yes	No
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$$Entropy(S) = \sum_{i=1}^{c} -p_i log_2 p_i$$

$$Entropy(PlayGolf) = -p_{yes} log_2(p_{yes}) - p_{no} log_2(p_{no})$$

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$$Entropy(PlayGolf) = -p_{yes} log_2(p_{yes}) - p_{no} log_2(p_{no})$$

Play Golf(14)	
Yes	No
9	5

$$\boldsymbol{E(PlayGolf)} = E(5,9)$$

$$= -\left(\frac{9}{14}\log_2\frac{9}{14}\right) - \left(\frac{5}{14}\log_2\frac{5}{14}\right)$$

$$= -(0.357\ \log_2 0.357) - (0.643\ \log_2 0.643)$$

$$= \mathbf{0.94}$$

Step 3: Calculate Entropy for Other Attributes After Split

For the other four attributes, we need to calculate the entropy after each of the split.

- E(PlayGolf, Outlook)
- E(PlayGolf, Temperature)
- E(PlayGolf, Humidity)
- E(PlayGolf,Windy)

The entropy for two variables is calculated using the formula.

$$Entropy(\mathrm{S}, \mathrm{T}) = \sum_{c \in T} P(c)E(c)$$

The easiest way to approach this calculation is to create a frequency table for the two variables

Step 3: Calculate Entropy for Other Attributes After Split

E(PlayGolf, Outlook) Calculation:

To calculate **E**(PlayGolf, Outlook), we would use the formula below:

$$\boldsymbol{E}(PlayGolf, Outlook) = \boldsymbol{P}(Sunny)\boldsymbol{E}(Sunny) + \boldsymbol{P}(Overcast)\boldsymbol{E}(Overcast) + \boldsymbol{P}(Rainy)\boldsymbol{E}(Rainy)$$

		PlayGolf(14)		
		Yes	No	
Outlook	Sunny	3	2	5
	Overcast	4	0	4
	Rainy	2	3	5

E(PlayGolf, Outlook) = ***P****(Sunny)* ***E****(3,2) + P(Overcast)* ***E****(4,0) +* ***P****(rainy) E(2,30)*

$$\boldsymbol{E}(PlayGolf, Outlook) = \frac{5}{14}\boldsymbol{E}(3{,}2) + \frac{4}{14}\boldsymbol{E}(4{,}0) + \frac{5}{14}\boldsymbol{E}(2{,}3)$$

Step 3: Calculate Entropy for Other Attributes After Split

E(PlayGolf, Outlook) Calculation:

		PlayGolf(14)		
		Yes	No	
Outlook	Sunny	3	2	5
	Overcast	4	0	4
	Rainy	2	3	5

$$\boldsymbol{E}(PlayGolf, Outlook) = \frac{5}{14}\boldsymbol{E}(3,2) + \frac{4}{14}\boldsymbol{E}(4,0) + \frac{5}{14}\boldsymbol{E}(2,3)$$

$$\boldsymbol{E}(\boldsymbol{Sunny}) = E(3,2)$$

$$= -\left(\frac{3}{5}\log_2\frac{3}{5}\right) - \left(\frac{2}{5}\log_2\frac{2}{5}\right)$$

$$= -(0.60\ \log_2 0.60) - (0.40 \log_2 0.40)$$

$$= -(0.60 * 0.737) - (0.40 * 0.529)$$

$$= \mathbf{0.971}$$

Step 3: Calculate Entropy for Other Attributes After Split

E(PlayGolf, Outlook) Calculation:

		PlayGolf(14)		
		Yes	No	
Outlook	Sunny	3	2	5
	Overcast	4	0	4
	Rainy	2	3	5

$$E(PlayGolf, Outlook) = \frac{5}{14}E(3,2) + \frac{4}{14}E(4,0) + \frac{5}{14}E(2,3)$$

$$\boldsymbol{E(Sunny)} = E(3,2)$$

$$= -\left(\frac{3}{5}\log_2\frac{3}{5}\right) - \left(\frac{2}{5}\log_2\frac{2}{5}\right)$$

$$= -(0.60\ \log_2 0.60) - (0.40 \log_2 0.40)$$

$$= -(0.60 * 0.737) - (0.40 * 0.529)$$

$$= \mathbf{0.971}$$

$$\boldsymbol{E(Overcast)} = E(4,0)$$

$$= -\left(\frac{4}{4}\log_2\frac{4}{4}\right) - \left(\frac{0}{4}\log_2\frac{0}{4}\right)$$

$$= -(0) - (0)$$

$$= \mathbf{0}$$

Step 3: Calculate Entropy for Other Attributes After Split

E(PlayGolf, Outlook) Calculation:

		PlayGolf(14)		
		Yes	No	
Outlook	Sunny	3	2	5
	Overcast	4	0	4
	Rainy	2	3	5

$$E(PlayGolf, Outlook) = \frac{5}{14}\boldsymbol{E}(3{,}2) + \frac{4}{14}\boldsymbol{E}(4{,}0) + \frac{5}{14}\boldsymbol{E}(2{,}3)$$

$$\boldsymbol{E(Sunny)} = E(3{,}2)$$

$$= -\left(\frac{3}{5}\log_2\frac{3}{5}\right) - \left(\frac{2}{5}\log_2\frac{2}{5}\right)$$

$$= -(0.60\ \log_2 0.60) - (0.40 \log_2 0.40)$$

$$= -(0.60 * 0.737) - (0.40 * 0.529)$$

$$= \mathbf{0.971}$$

$$\boldsymbol{E(Overcast)} = E(4{,}0)$$

$$= -\left(\frac{4}{4}\log_2\frac{4}{4}\right) - \left(\frac{0}{4}\log_2\frac{0}{4}\right)$$

$$= -(0) - (0)$$

$$= \mathbf{0}$$

$$\boldsymbol{E(Rainy)} = E(2{,}3)$$

$$= -\left(\frac{2}{5}\log_2\frac{2}{5}\right) - \left(\frac{3}{5}\log_2\frac{3}{5}\right)$$

$$= -(0.40\ \log_2 0.40) - (0.6 \log_2 0.60)$$

$$= \mathbf{0.971}$$

Step 3: Calculate Entropy for Other Attributes After Split

E(PlayGolf, Outlook) Calculation:

		PlayGolf(14)		
		Yes	No	
Outlook	Sunny	3	2	5
	Overcast	4	0	4
	Rainy	2	3	5

$$E(PlayGolf, Outlook) = \frac{5}{14}E(3,2) + \frac{4}{14}E(4,0) + \frac{5}{14}E(2,3)$$

$$\begin{aligned}
\boldsymbol{E(Sunny)} &= E(3,2) \\
&= -\left(\frac{3}{5}\log_2\frac{3}{5}\right) - \left(\frac{2}{5}\log_2\frac{2}{5}\right) \\
&= -(0.60\ \log_2 0.60) - (0.40 \log_2 0.40) \\
&= -(0.60 * 0.737) - (0.40 * 0.529) \\
&= \mathbf{0.971}
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{E(Overcast)} &= E(4,0) \\
&= -\left(\frac{4}{4}\log_2\frac{4}{4}\right) - \left(\frac{0}{4}\log_2\frac{0}{4}\right) \\
&= -(0) - (0) \\
&= \mathbf{0}
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{E(Rainy)} &= E(2,3) \\
&= -\left(\frac{2}{5}\log_2\frac{2}{5}\right) - \left(\frac{3}{5}\log_2\frac{3}{5}\right) \\
&= -(0.40\ \log_2 0.40) - (0.6 \log_2 0.60) \\
&= \mathbf{0.971}
\end{aligned}$$

E(4,0) = 0;
E(2,3) = E(3,2)

Step 3: Calculate Entropy for Other Attributes After Split

E(PlayGolf, Outlook) Calculation:

		PlayGolf(14)		
		Yes	No	
Outlook	Sunny	3	2	5
	Overcast	4	0	4
	Rainy	2	3	5

E(PlayGolf, Outlook) = P(Sunny) E(3,2) + P(Overcast) E(4,0) + P(rainy) E(2,3)

$$\boldsymbol{E}(PlayGolf\,, Outlook) = \frac{5}{14}\boldsymbol{E}(3{,}2) + \frac{4}{14}\boldsymbol{E}(4{,}0) + \frac{5}{14}\boldsymbol{E}(2{,}3)$$

$$= \frac{5}{14}0.971 + \frac{4}{14}0.0 + \frac{5}{14}0.971$$

$$= 0.357 * 0.971 + 0.0 + 0.357 * 0.971$$

$$= 0.693$$

Step 3: Calculate Entropy for Other Attributes After Split

E(PlayGolf, Temperature) Calculation

		PlayGolf(14)		
		Yes	No	
Temperature	Hot	2	2	4
	Cold	3	1	4
	Mild	4	2	6

E(PlayGolf, Temperature) = P(Hot) E(2,2) + P(Cold) E(3,1) + P(Mild) E(4,2)

E (PlayGolf, Temperature) = **4/14 * E**(Hot) + **4/14 * E**(Cold) + **6/14 * E**(Mild)

E (PlayGolf, Temperature) = **4/14 * E**(2, 2) + **4/14 * E**(3, 1) + **6/14 * E**(4, 2)

E (PlayGolf, Temperature) = 4/14 * -(2/4 log 2/4) – (2/4 log 2/4)

+ 4/14 * -(3/4 log 3/4) – (1/4 log 1/4)

+ 6/14 * -(4/6 log 4/6) – (2/6 log 2/6)

E (PlayGolf, Temperature) = 5/14 * 1.0

+ 4/14 * 1.811

+ 5/14 * 0.918

= 0.911

Step 3: Calculate Entropy for Other Attributes After Split

E(PlayGolf, Humidity) Calculation

		PlayGolf(14)		
		Yes	No	
Humidity	High	3	4	7
	Normal	6	1	7

E (PlayGolf, Humidity) = **7/14 * E**(High) + **7/14 * E**(Normal)

E (PlayGolf, Humidity) = **7/14 * E**(3, 2) + **7/14 * E**(4, 0)

E (PlayGolf, Humidity) = 7/14 * -(3/7 log 3/7) – (4/7 log 4/7)

+ 7/14 * -(6/7 log 6/7) – (1/7 log 1/7)

E (PlayGolf, Humidity) = 7/14 * 0.985

+ 7/14 * 0.592

= 0.788

Step 3: Calculate Entropy for Other Attributes After Split

E(PlayGolf, Windy) Calculation

		PlayGolf(14)		
		Yes	No	
Windy	TRUE	3	3	6
	FALSE	6	2	8

$$E\,(\text{PlayGolf, Windy}) = \mathbf{6/14 * E(\text{True})} + \mathbf{8/14 * E(\text{False})}$$

$$E\,(\text{PlayGolf, Windy}) = \mathbf{6/14 * E(3,3)} + \mathbf{8/14 * E(6,2)}$$

$$E\,(\text{PlayGolf, Windy}) = 6/14 * -(3/6 \log 3/6) - (3/6 \log 3/6)$$

$$+\ 8/14 * -(6/8 \log 6/8) - (2/8 \log 2/8)$$

$$E\,(\text{PlayGolf, Windy}) = 6/14 * 1.0$$

$$+\ 8/14 * 0.811$$

$$= \mathbf{0.892}$$

Step 3: Calculate Entropy for Other Attributes After Split

1. E(PlayGolf, Outlook) = **0.693**
2. E(PlayGolf, Temperature) = **0.911**
3. E(PlayGolf, Humidity) = **0.788**
4. E(PlayGolf,Windy) = **0.892**

Step 4: Calculating Information Gain for Each Split

- The next step is to calculate the information gain for each of the attributes.
- The information gain is calculated from the split using each of the attributes.
- Then the attribute with the largest information gain is used for the split.
- The information gain is calculated using the formula:

$$\text{Gain}(S,T) = \text{Entropy}(S) - \text{Entropy}(S,T)$$

Step 4: Calculating Information Gain for Each Split

Gain(PlayGolf, Outlook) = Entropy(PlayGolf) – Entropy(PlayGolf, Outlook)
= 0.94 – 0.693 **= 0.247**

Gain(PlayGolf, Temperature) = Entropy(PlayGolf) – Entropy(PlayGolf, Temperature)
= 0.94 – 0.911 ***= 0.029***

Gain(PlayGolf, Humidity) = Entropy(PlayGolf) – Entropy(PlayGolf, Humidity)
= 0.94 – 0.788 ***= 0.152***

Gain(PlayGolf, Windy) = Entropy(PlayGolf) – Entropy(PlayGolf, Windy)
= 0.94 – 0.892 ***= 0.048***

Step 4: Calculating Information Gain for Each Split

Gain(PlayGolf, Outlook) = Entropy(PlayGolf) – Entropy(PlayGolf, Outlook)
= 0.94 – 0.693 **= 0.247**

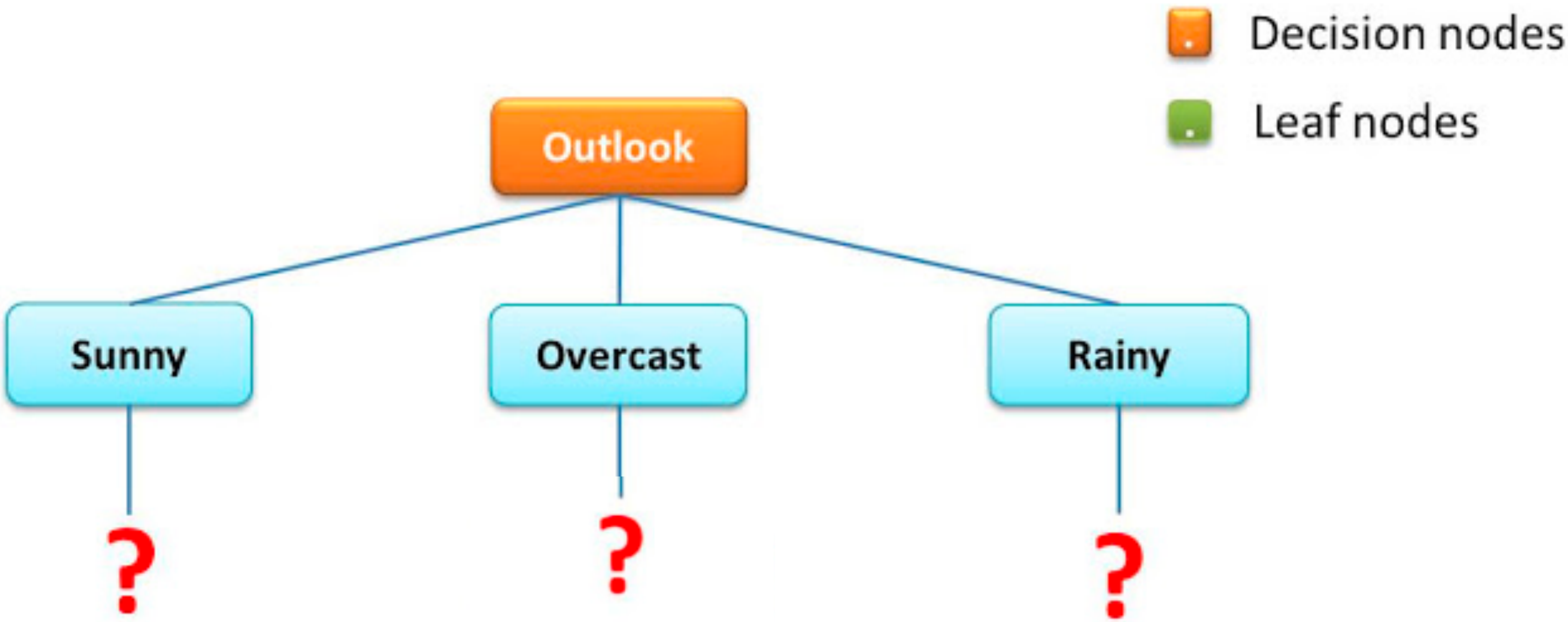
Gain(PlayGolf, Temperature) = Entropy(PlayGolf) – Entropy(PlayGolf, Temperature)
= 0.94 – 0.911 ***= 0.029***

Gain(PlayGolf, Humidity) = Entropy(PlayGolf) – Entropy(PlayGolf, Humidity)
= 0.94 – 0.788 ***= 0.152***

Gain(PlayGolf, Windy) = Entropy(PlayGolf) – Entropy(PlayGolf, Windy)
= 0.94 – 0.892 ***= 0.048***

Step 5: Perform the First Split

From our calculation, the highest information gain comes from Outlook. Therefore the split will look like this:



Step 5: Perform the First Split

Outlook	Temperature	Humidity	Windy	Play Golf
Sunny	Mild	Normal	FALSE	Yes
Sunny	Mild	High	FALSE	Yes
Sunny	Cool	Normal	FALSE	Yes
Sunny	Cool	Normal	TRUE	No
Sunny	Mild	High	TRUE	No
Overcast	Hot	High	FALSE	Yes
Overcast	Mild	High	TRUE	Yes
Overcast	Hot	Normal	FALSE	Yes
Overcast	Cool	Normal	TRUE	Yes
Rainy	Hot	High	FALSE	No
Rainy	Hot	High	TRUE	No
Rainy	Mild	High	FALSE	No
Rainy	Cool	Normal	FALSE	Yes
Rainy	Mild	Normal	TRUE	Yes

Overcast outlook requires no further split because it is just one homogeneous group.
So we have a leaf node.

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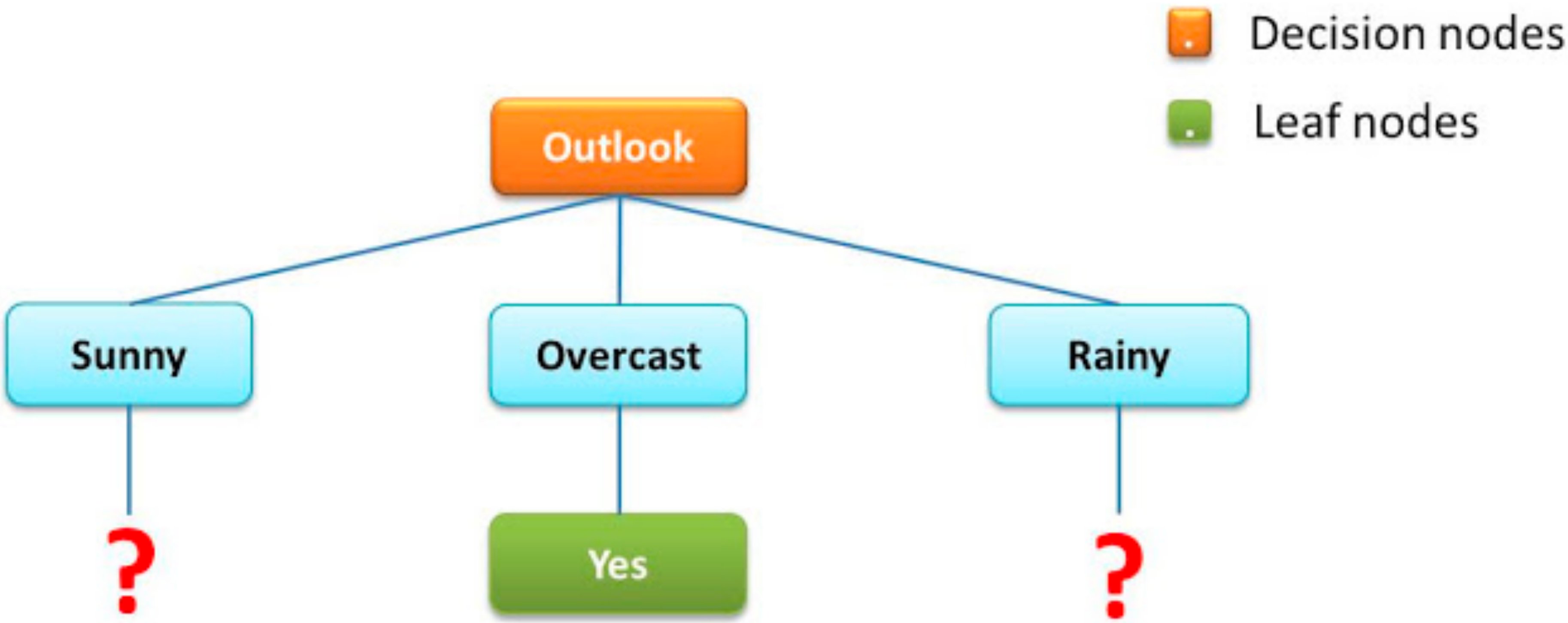


Overcast outlook requires no further split because it is just one homogeneous group. So we have a leaf node.

Step 6: Perform Further Splits

The Sunny and the Rainy attributes needs to be split

The Rainy outlook can be split using either Temperature, Humidity or Windy.

Question: What attribute would best be used for this split?

- *Gain(PlayGolf, Outlook=Rainy, Temperature) =*
  Entropy(PlayGolf, Outlook=Rainy) – Entropy(PlayGolf, Outlook=Rainy, Temperature)
- *Gain(PlayGolf, Outlook=Rainy, Humidity) =*
  Entropy(PlayGolf, Outlook=Rainy) – Entropy(PlayGolf, Outlook=Rainy, Humidity)
- *Gain(PlayGolf, Outlook=Rainy, Windy) =*
  Entropy(PlayGolf, Outlook=Rainy) – Entropy(PlayGolf, Outlook=Rainy, Windy)

Step 6: Perform Further Splits

The Sunny and the Rainy attributes needs to be split

The Rainy outlook can be split using either Temperature, Humidity or Windy.

Question: What attribute would best be used for this split?

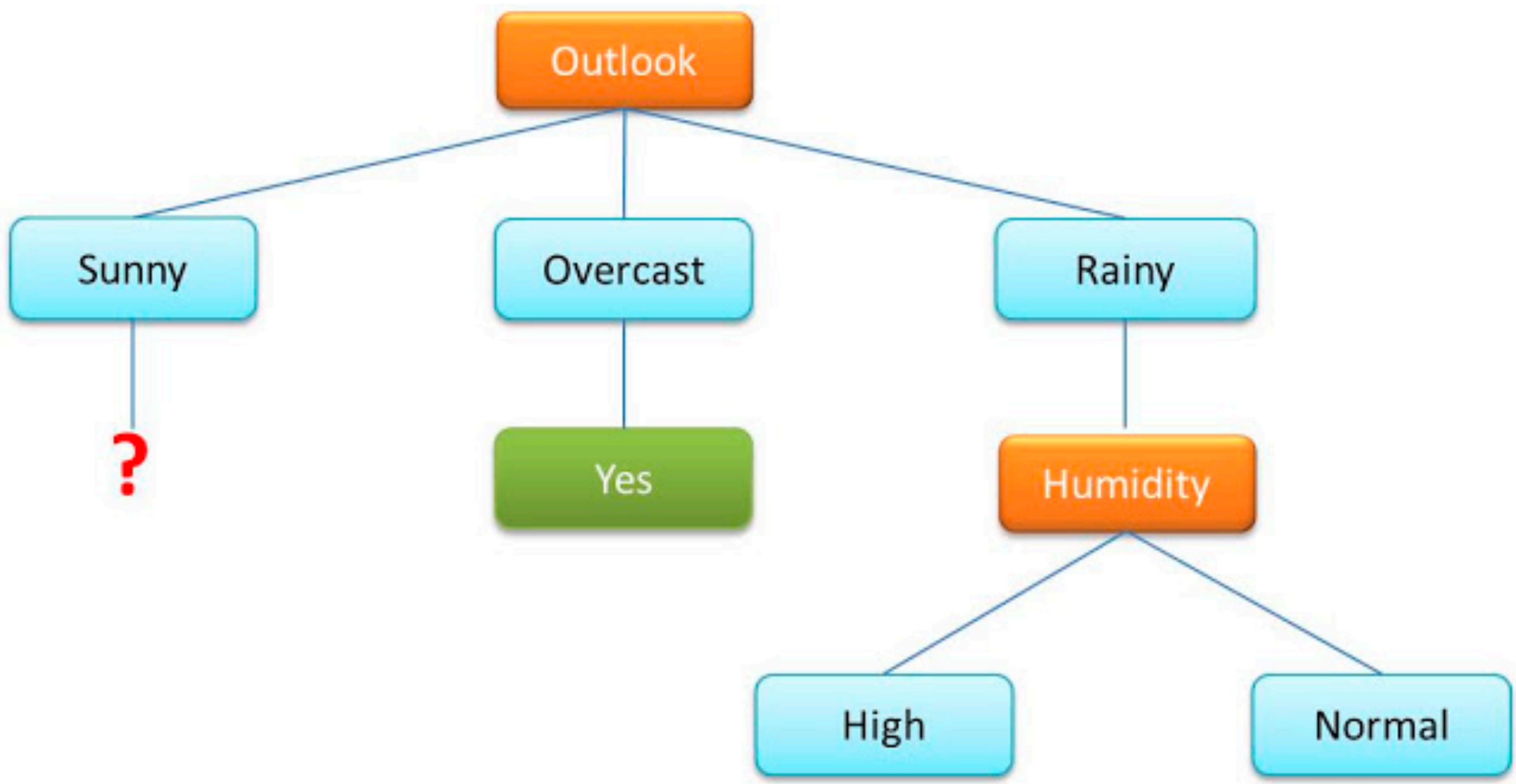
Humidity, produces homogenous groups.

Outlook	Temperature	Humidity	Windy	Play Golf
Rainy	Hot	High	FALSE	No
Rainy	Hot	High	TRUE	No
Rainy	Mild	High	FALSE	No

Rainy	Cool	Normal	FALSE	Yes
Rainy	Mild	Normal	TRUE	Yes

- *Gain(PlayGolf, Outlook=Rainy, Humidity) =*
  Entropy(PlayGolf, Outlook=Rainy) – Entropy(PlayGolf, Outlook=Rainy, Humidity)=
  Entropy(PlayGolf, Outlook=Rainy) – 0

Step 6: Perform Further Splits



Step 6: Perform Further Splits

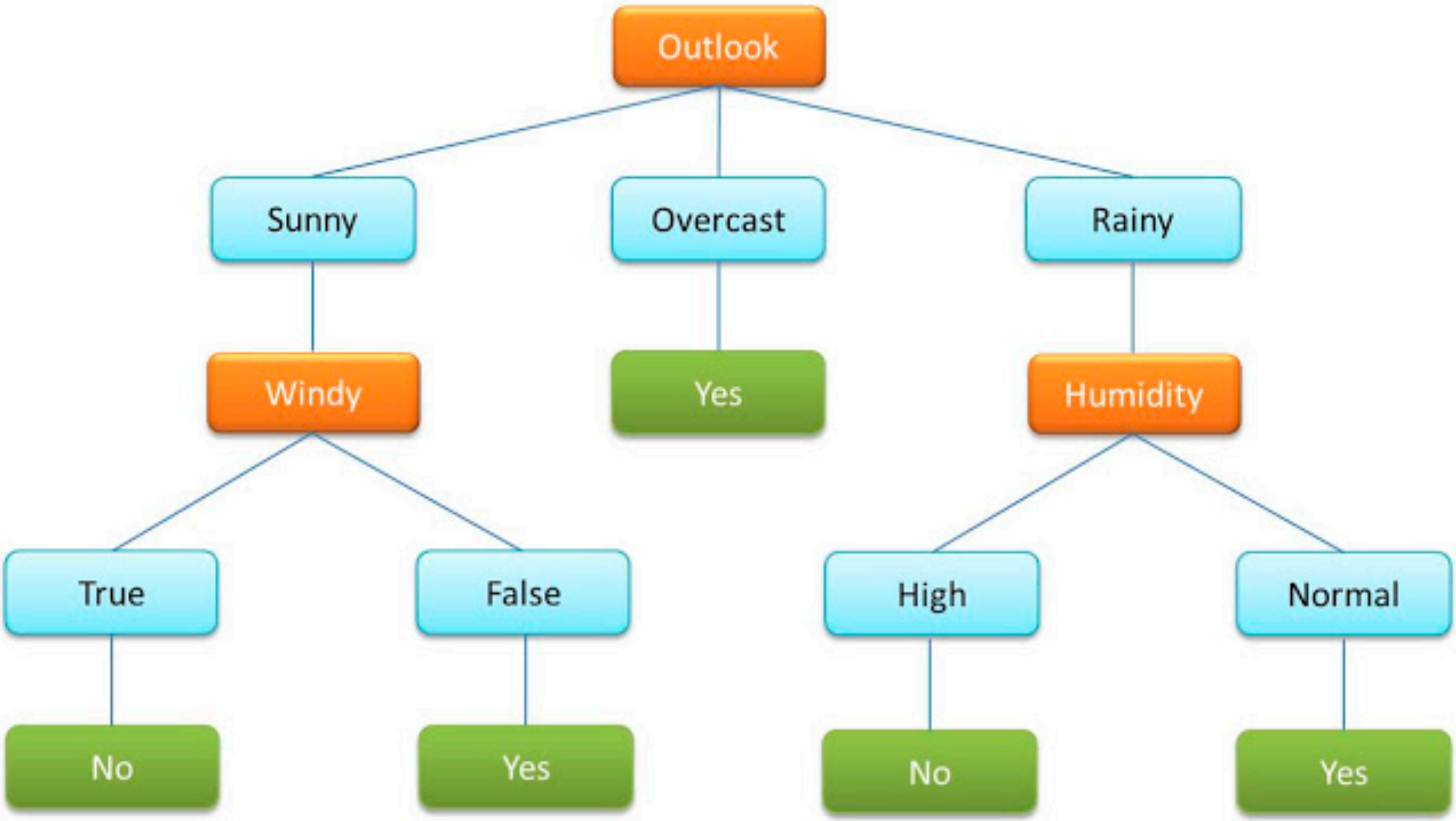
The Rainy outlook can be split using either Temperature, Humidity or Windy.

Question: What attribute would best be used for this split? Why?
Answer: **Windy** . Because it produces homogeneous groups.

Outlook	Temperature	Humidity	Windy	Play Golf
Sunny	Mild	Normal	FALSE	Yes
Sunny	Mild	High	FALSE	Yes
Sunny	Cool	Normal	FALSE	Yes

Sunny	Cool	Normal	TRUE	No
Sunny	Mild	High	TRUE	No

Step 6: Perform Further Splits



ID3 Algorithm

ID3 (S, A, V)

Let:
S = Learning Set
A = Attibute Set
V = Attribute Values

```
Begin
    Load learning sets  and create decision tree root node(rootNode),
    Add learning set S into root not as its subset

    For rootNode, compute Entropy(rootNode.subset)

    If Entropy(rootNode.subset) == 0 (subset is homogeneous)
        return a leaf node

    If Entropy(rootNode.subset)!= 0 (subset is not homogeneous)
        compute Information Gain for each attribute left (not been used for spliting)
        Find attibute A with Maximum(Gain(S,A))
        Create child nodes for this root node and add to rootNode in the decision tree

    For each child of the rootNode
        Apply ID3(S,A,V)
        Continue until a node with Entropy of 0 or a leaf node is reached
End
```