Binary Classification

This Lecture

- Two discriminative models:
  - logistic regression
  - perceptron

Classification

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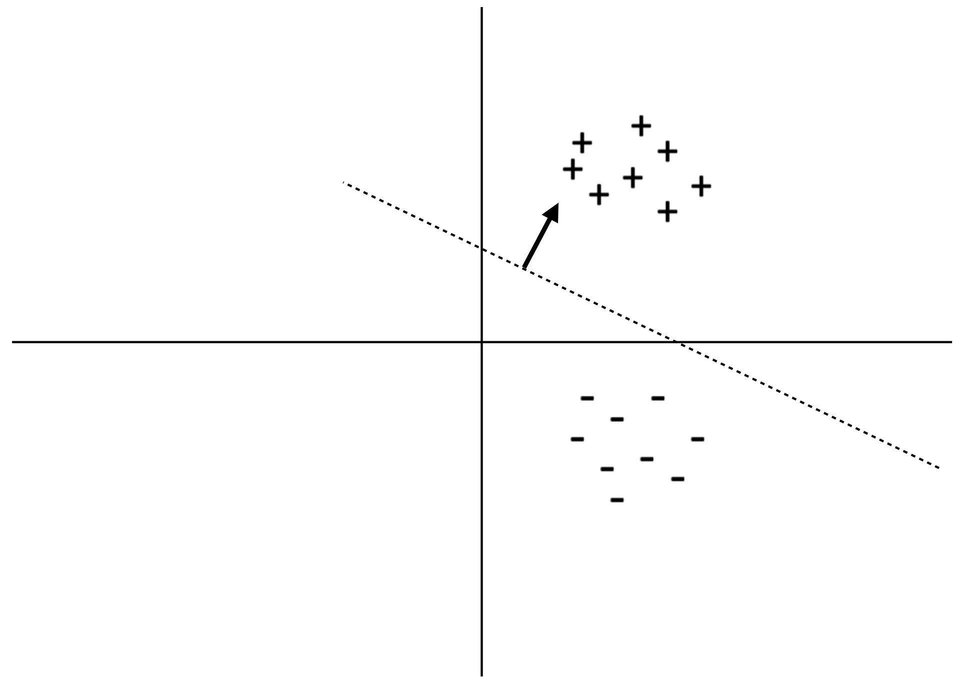

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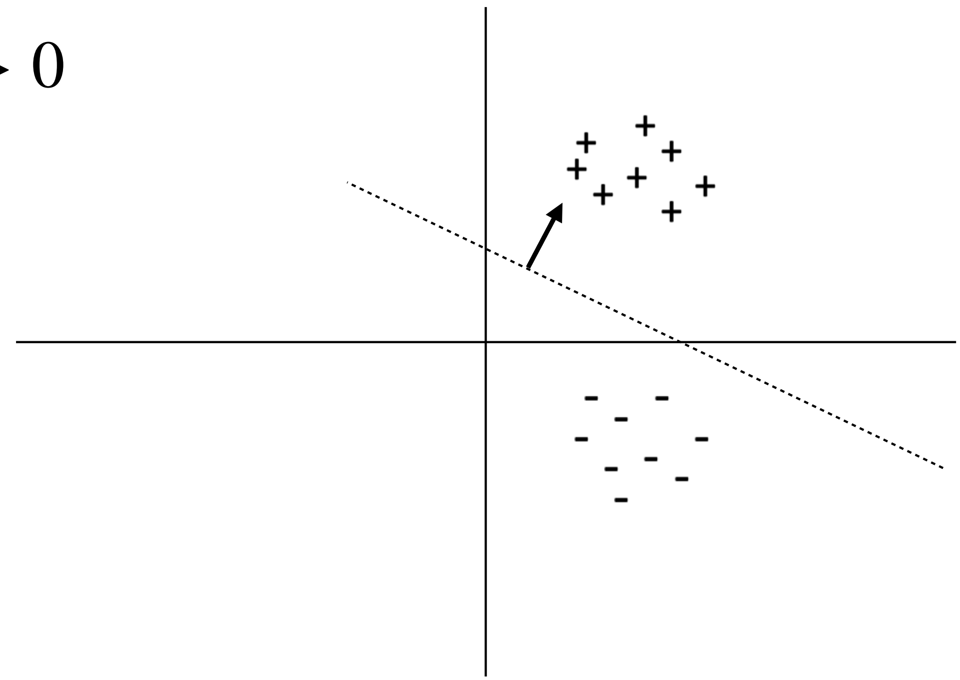
- Linear decision rule: $w^T f(x) + b > 0$



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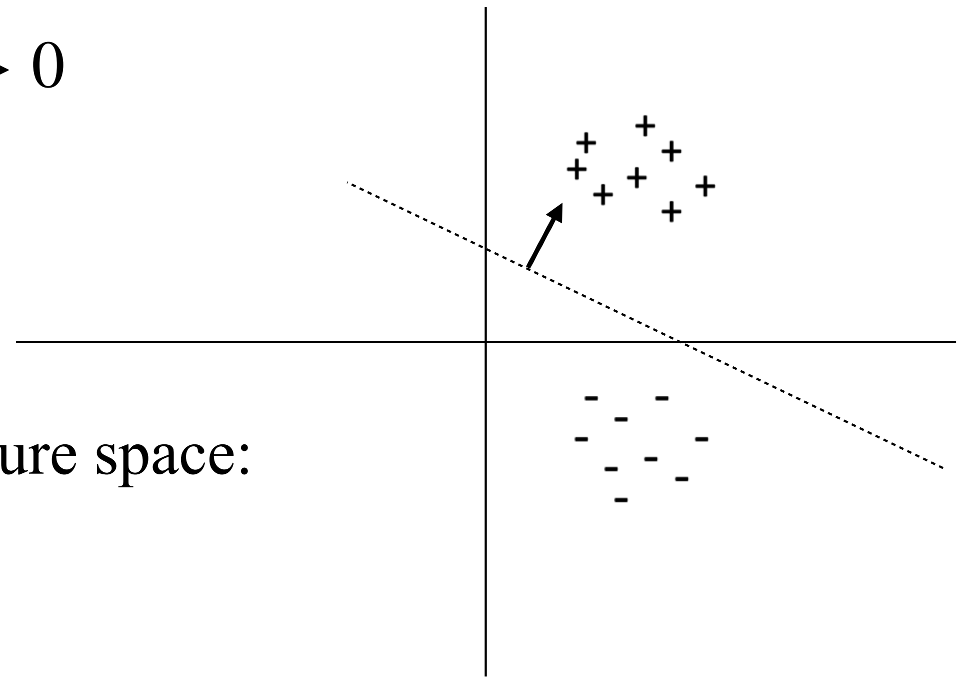

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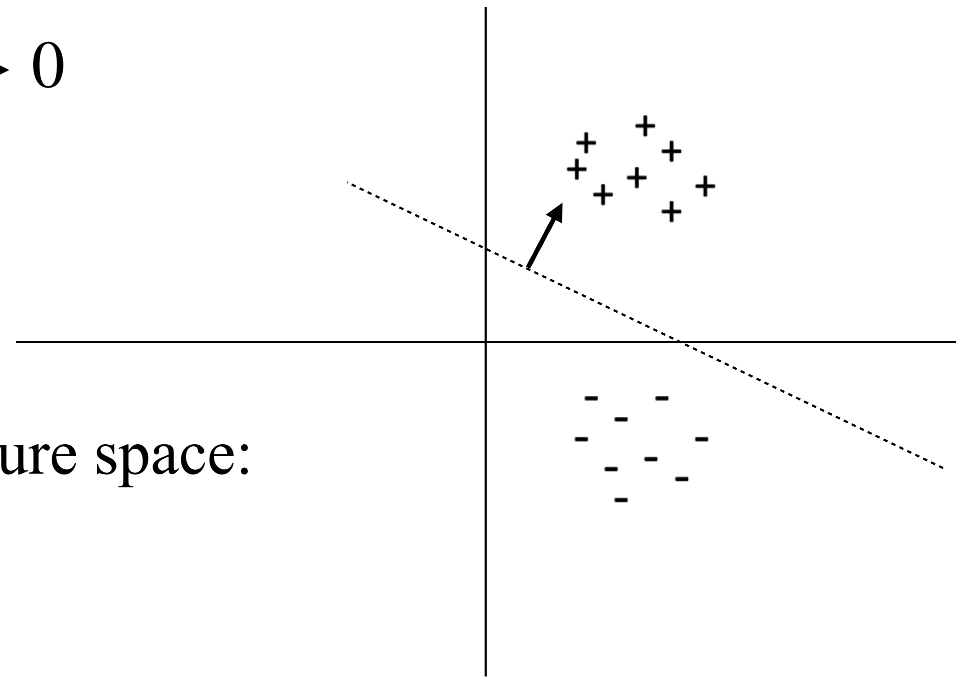

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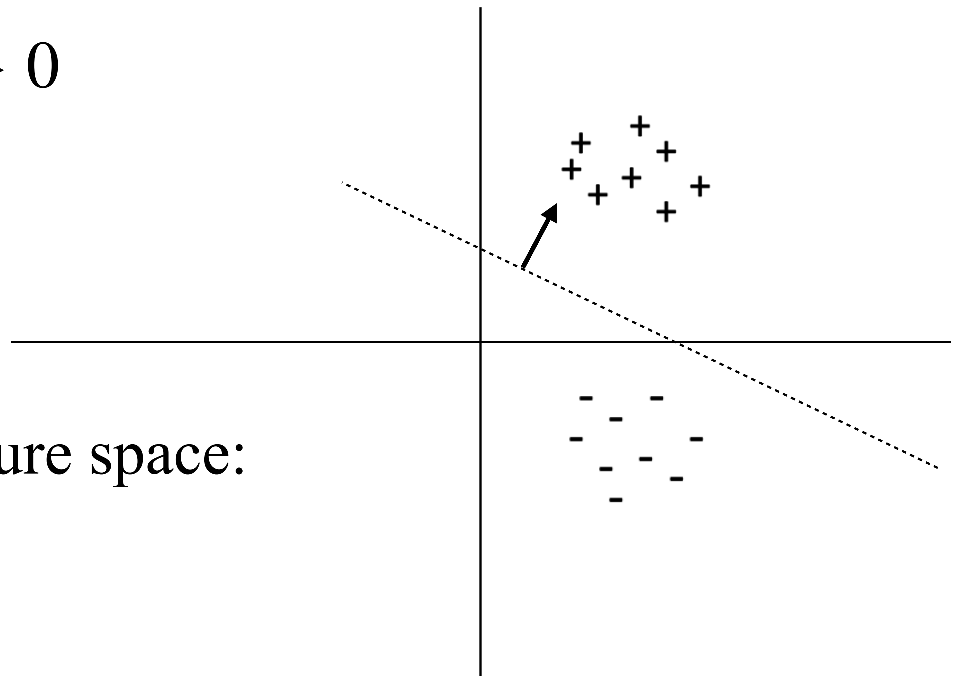
$$w^T f(x) > 0$$



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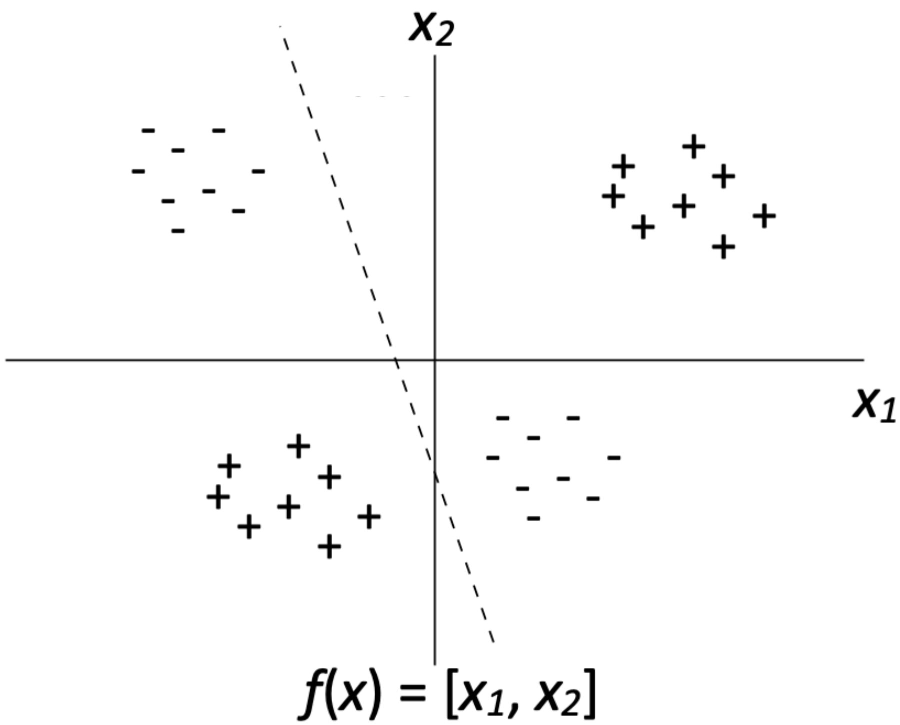
↓

[0.5, 1.6, 0.3, **1**]

Linear functions are powerful!



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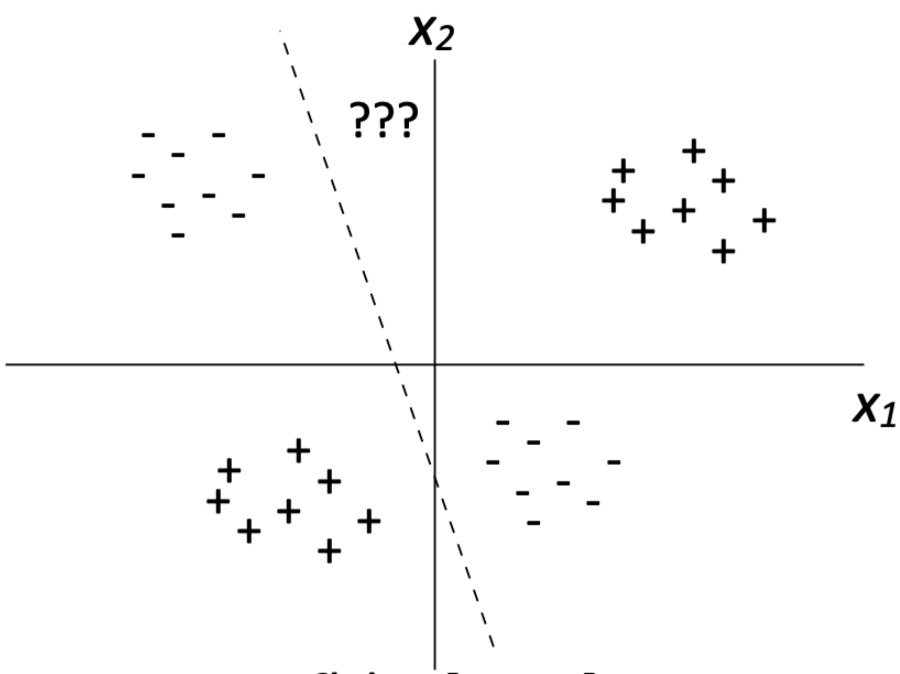

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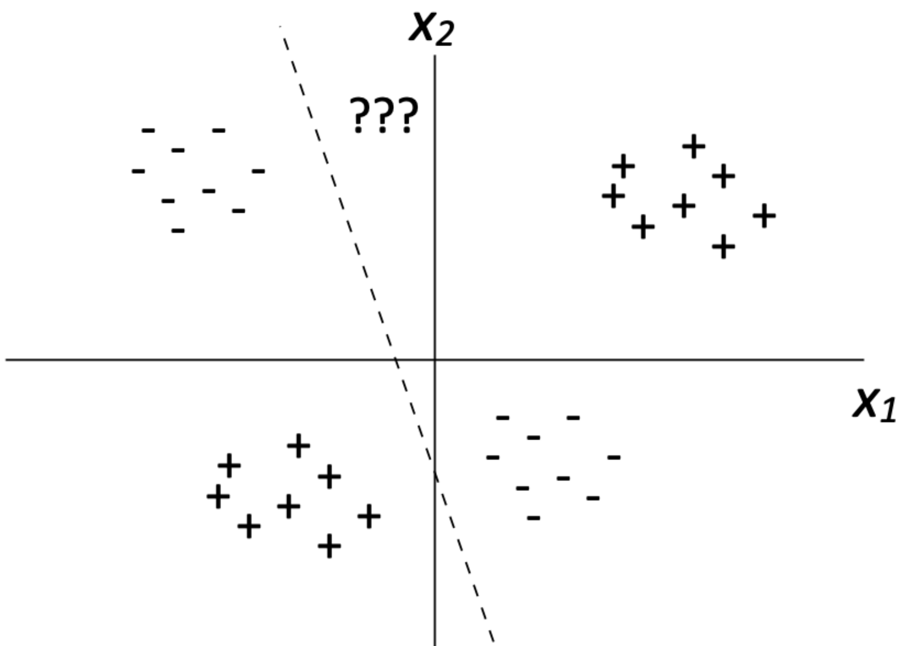

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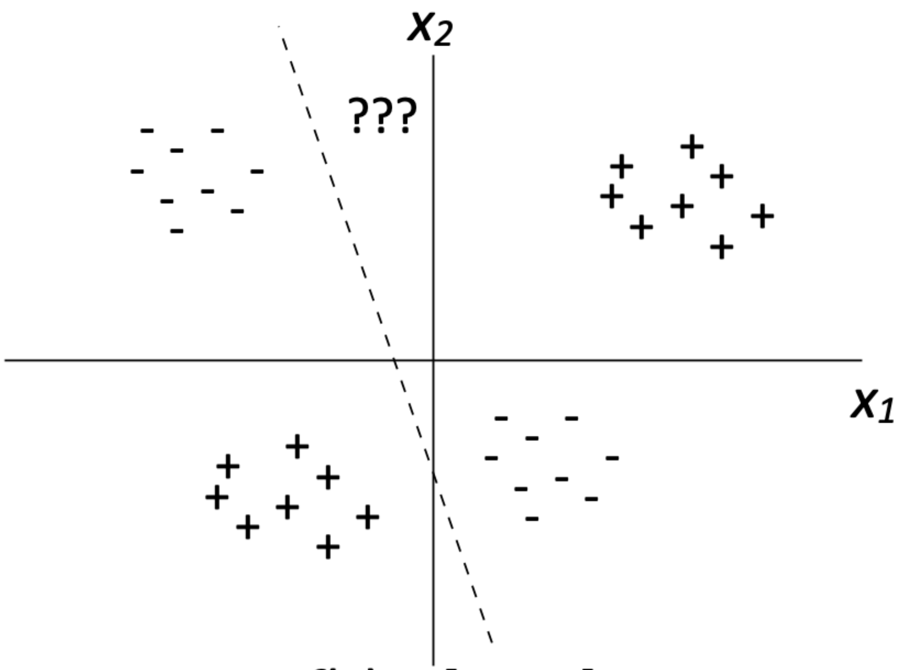

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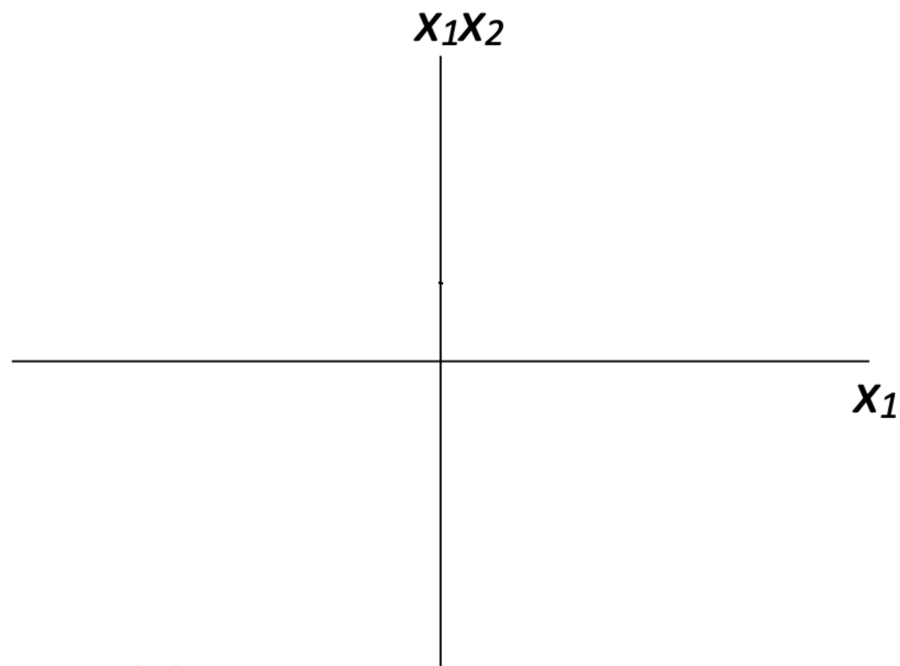

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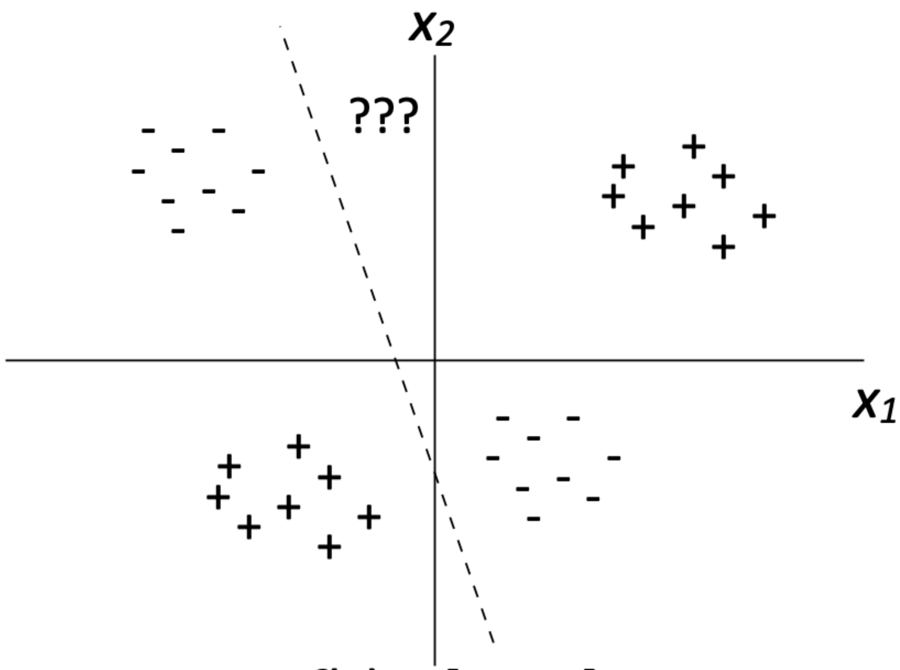

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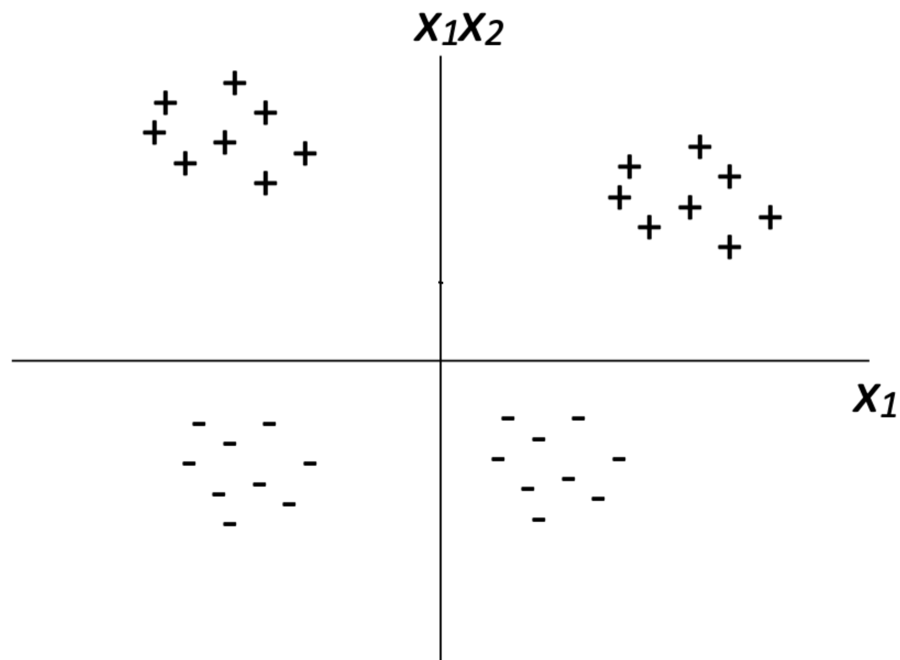

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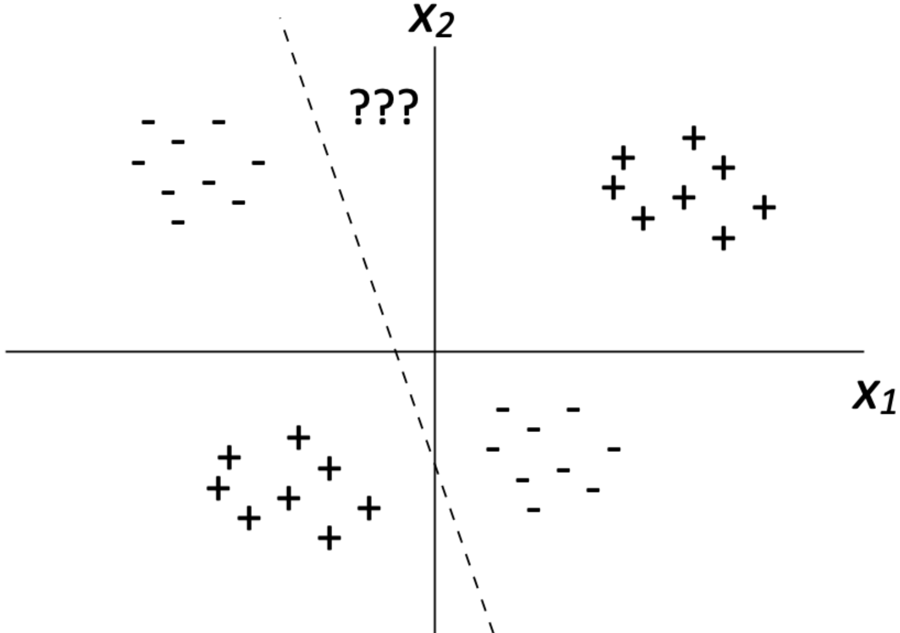

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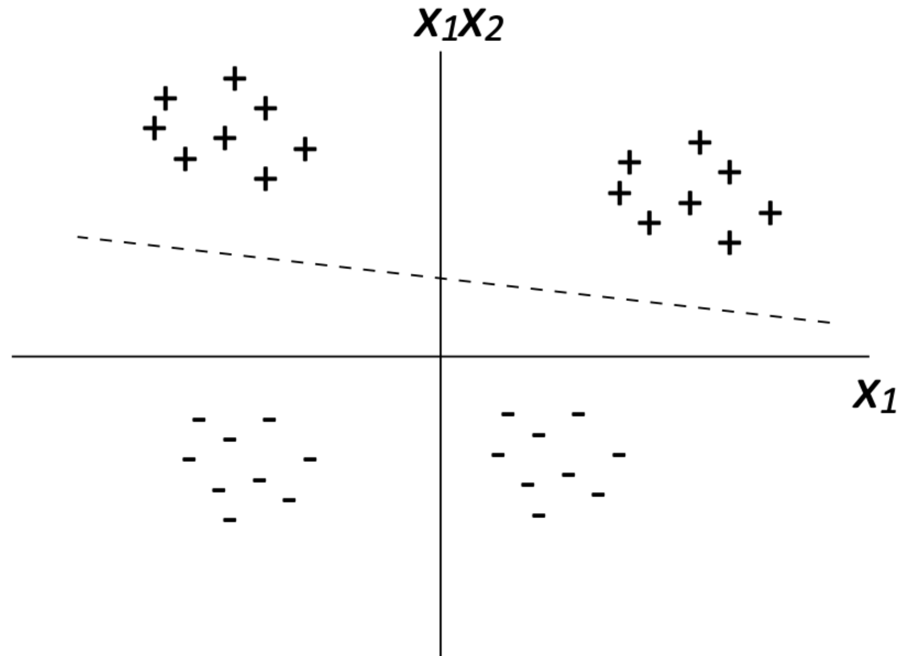

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Classification: Sentiment Analysis

this movie was great! would watch again Positive

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- Requires *indexing* the features (mapping them to axes)
- More sophisticated feature mappings possible:
    - character n-grams, parts of speech, lemmas, …

Maximum Likelihood Estimation

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features (i)

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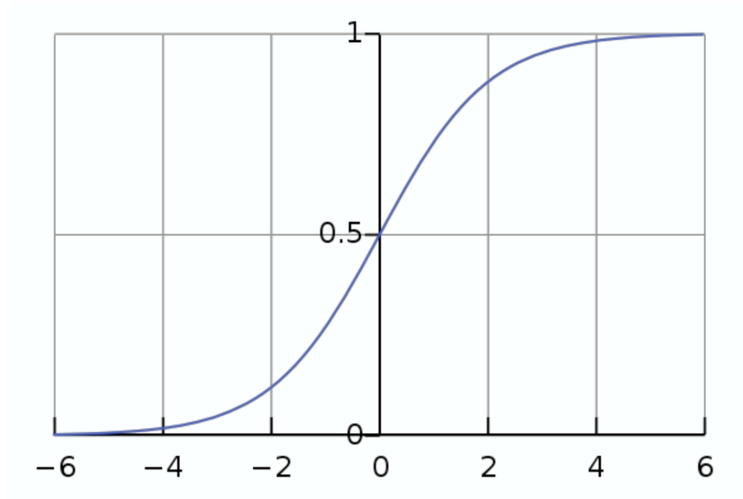
- Imagine a coin flip which is heads with probability *p*
- Observe (H, H, H, T) and maximize likelihood: $\prod_{j=1}^{m} P(y_j) = p^3(1-p)$
- Easier: maximize *log* likelihood

$$\sum_{j=1}^{m} \log P(y_j) = 3\log p + \log(1-p)$$

Logistic Regression

Logistic Regression

$$P(y = +|x) = \text{logistic}(w^\top x)$$



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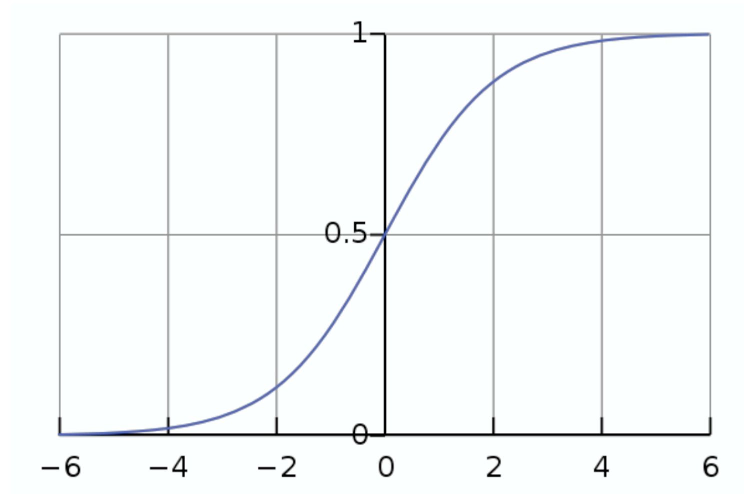

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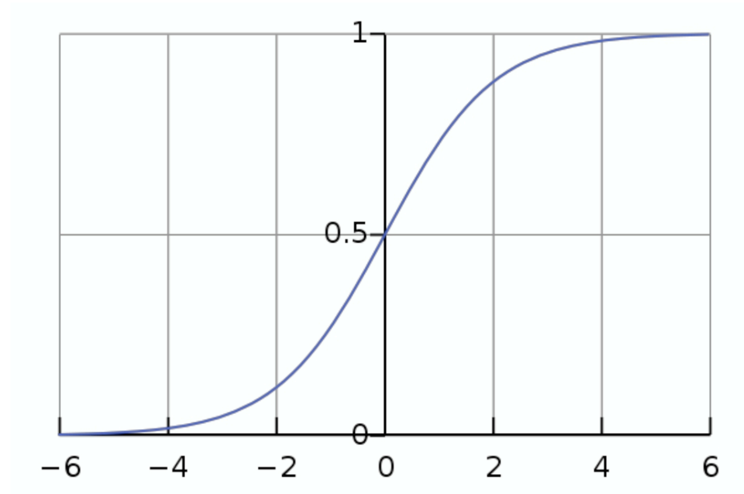

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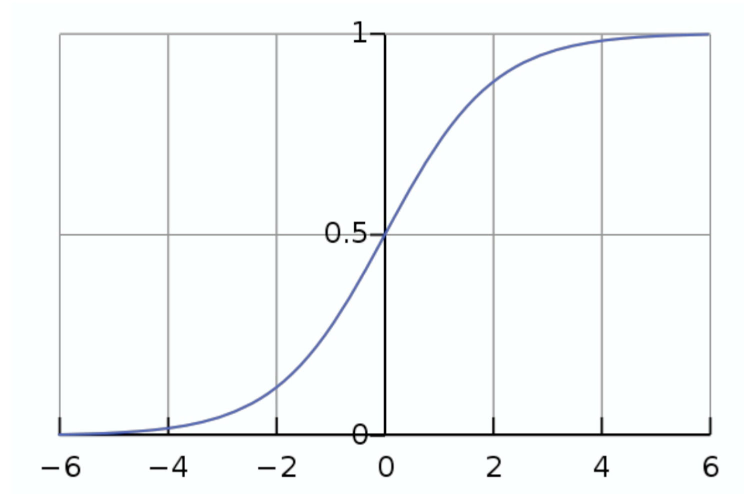

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$$\mathcal{L}(x_j, y_j=+) = \log P(y_j=+|x_j)$$

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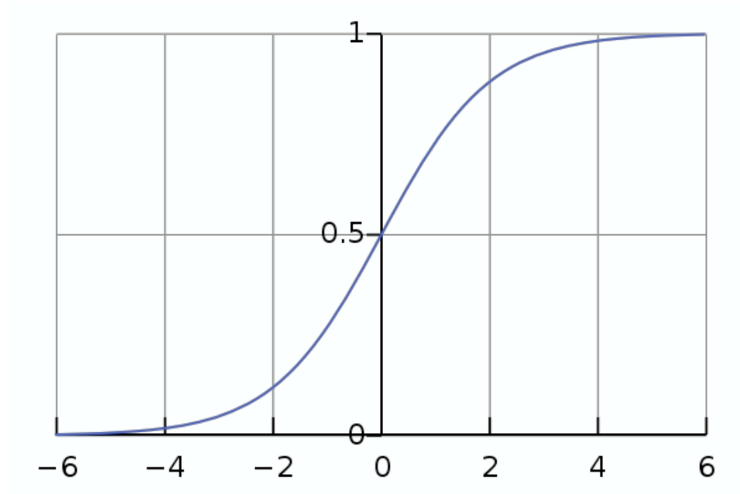

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$$\begin{aligned}\mathcal{L}(x_j, y_j=+) &= \log P(y_j=+|x_j) \\ &= \sum_{i=1}^{n} w_i x_{ji} - \log\left(1+\exp\left(\sum_{i=1}^{n} w_i x_{ji}\right)\right)\end{aligned}$$

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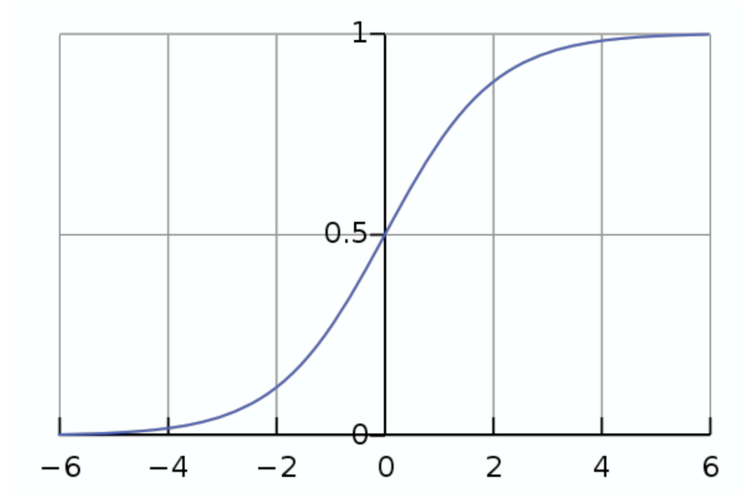

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sum over features

Logistic Regression

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$$\frac{\partial \mathcal{L}(x_j, y_j)}{\partial w_i} =$$

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deriv
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deriv of exp

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$$= x_{ji} - x_{ji} \frac{\exp\left(\sum_{i=1}^{n} w_i x_{ji}\right)}{1 + \exp\left(\sum_{i=1}^{n} w_i x_{ji}\right)}$$

Logistic Regression

- Recall that , for positive instances, $y_j = 1$ and for negative instances, $y_j = 0$
- Gradient of w_i on positive example = $x_{ji}(y_j - P(y_j = + \mid x_j))$
    - If P(+) is close to 1, make very little update
    - Otherwise make w_i look more like x_{ji} , which will increase P(+)

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- Can combine these gradient as : $x_j(y_j - P(y_j = 1 \mid x_j))$

Logistic Regression: Summary

- Model

$$P(y=+|x) = \frac{\exp(\sum_{i=1}^{n} w_i x_i)}{1+\exp(\sum_{i=1}^{n} w_i x_i)}$$

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$$\text{argmax}_y P(y|x)$$

$$P(y=1|x)\geq 0.5 \Leftrightarrow w^\top x \geq 0$$

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- Learning: gradient ascent on the (regularized) discriminative log-likelihood

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Logistic Regression

$$w \leftarrow w + x(1 = P(y = 1 \mid x))$$

$$w \leftarrow w - x(1 = P(y = 1 \mid x))$$

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Logistic Regression

$$w \leftarrow w + x(1 = P(y = 1 \mid x))$$

$$w \leftarrow w - x(1 = P(y = 1 \mid x))$$

- Guaranteed to eventually separate the data if the data are separable

Comparing Gradient Updates (Reference)

Logistic regression

$x(y - P(y = 1|x))$ = $x(y - \text{logistic}(w^\top x))$

Perceptron

$(2y - 1)x$ if classified incorrectly

0 else

y = 1 for pos,
0 for neg

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Sentiment Analysis

this movie was great! would watch again +

the movie was gross and overwrought, but I liked it +

this movie was not really very enjoyable —

- Bag-of-words doesn’t seem sufficient (discourse structure, negation)

- There are some ways around this:
    - extract bigram feature for “not X” for all X following the not
    - character n-grams, parts of speech, lemmas, …

Sentiment Analysis

	Movie Reviews	Product Reviews
Unigram only	64.1	42.91
Bigram only	76.15	69.62
Trigram only	76.1	71.37
(Uni + Bi) gram	77.15	72.94
(Uni + Bi + Tri) gram	**80.15**	**78.67**

Table 1: Results of Simple NGram

	Movie Reviews	Product Reviews
POS-(U + B + T)-JJ	75.00	50.425
POS-(U + B + T)-RB	65.50	36.76
POS-(U + B + T)-(JJ + RB)	**76.50**	**62.06**

Table 2: Results of POS-Tagged NGram. U = Unigram, B = Bigram, T = Trigram

Bakliwal et al. (2011)

Recap

- Logistic regression, and perceptron are closely related

- Perceptron inference require taking maxes, logistic regression has a similar update but is “softer” due to its probabilistic nature

- All gradient updates: “make it look more like the right thing and less like the wrong thing”