Propositional Logic

Propositional Logic: Syntax

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  - Logical connectives of symbols: $\wedge$, $\vee$, $\Leftrightarrow$, $\Rightarrow$, $\neg$
  - Also have parentheses to enclose each sentence: (…)

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  - Also have parentheses to enclose each sentence: (…)
- Sentences will be used for inference/problem-solving

Propositional Logic: Syntax

- If S is a sentence, $\neg S$ is a sentence
    - Not (negation)
- $S_1 \wedge S_2$ is a sentence, also $(S_1 \wedge S_2)$
    - And (conjunction)
- $S_1 \vee S_2$ is a sentence
    - Or (disjunction)
- $S_1 \Rightarrow S_2$ is a sentence
    - Implies (conditional)
- $S_1 \Leftrightarrow S_2$ is a sentence
    - Equivalence (biconditional)

Propositional Logic: Semantics

- Semantics defines the rules for determining the truth of a sentence (wrt a particular model)
- $\neg S$: is true iff S is false
- $S_1 \wedge S_2$: is true iff S_1 is true <u>and</u> S_2 is true
- $S_1 \vee S_2$: is true iff S_1 is true <u>or</u> S_2 is true
- $S_1 \Rightarrow S_2$: is true iff S_1 is false <u>or</u> S_2 is true

  (is false iff S_1 is true <u>and</u> S_2 is false)

  (if S_1 is true, then claiming that S_2 is true, otherwise make no claim)
- $S_1 \Leftrightarrow S_2$: is true iff $S_1 \Rightarrow S_2$ is true <u>and</u>

  $S_2 \Rightarrow S_1$ is true

Propositional Inference: Enumeration Method

- Truth tables can test for valid sentences
  - True under all possible interpretations in all possible worlds
- For a given sentence, make a truth table
  - Columns as the combinations of propositions in the sentence
  - Rows with all possible truth values for proposition symbols
- If sentence true in every row, then valid

Simple Wumpus Knowledge Base

- For simplicity, only deal with the pits
- Choose vocabulary
  - Let $P_{i,j}$ be True if there is a pit in $[i,j]$
  - Let $B_{i,j}$ be True if there is a breeze in $[i,j]$

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- KB sentences
  - FACT: "There is no pit in [1,1]"

$$R_1: \neg P_{1,1}$$

  - RULE: "There is breeze in adjacent neighbor of pit"

$$R_2: \; B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$$

$$R_3: \; B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$$

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Need rule for each square!

Wumpus Environment

- Given knowledge base
- Include percepts as move through environment (online)
- Need to “deduce what to do”
- Derive chains of conclusions that lead to the desired goal
  - Use inference rules

$$\frac{\alpha}{\beta}$$

Inference <u>rule</u>: “α derives β”

Knowing α is true, then β must also be true

Inference Rules for Prop. Logic

- Modus Ponens
  - From implication and premise of implication, can infer conclusion

$$\frac{\alpha \Rightarrow \beta, \;\; \alpha}{\beta}$$

Inference Rules for Prop. Logic

- And-Elimination
  - From conjunction, can infer any of the conjuncts

$$\frac{\alpha_1 \wedge \alpha_2 \wedge \cdots \wedge \alpha_n}{\alpha_i}$$

Inference Rules for Prop. Logic

- And-Introduction
  - From list of sentences, can infer their conjunction

$$\frac{\alpha_1, \alpha_2, \dots, \alpha_n}{\alpha_1 \wedge \alpha_2 \wedge \cdots \wedge \alpha_n}$$

Inference Rules for Prop. Logic

- Or-Introduction
  - From sentence, can infer its disjunction with anything else

$$\frac{\alpha_i}{\alpha_1 \vee \alpha_2 \vee \cdots \vee \alpha_n}$$

Inference Rules for Prop. Logic

- Double-Negation Elimination
  - From doubly negated sentence, can infer a positive sentence

$$\frac{\neg\neg\alpha}{\alpha}$$

Inference Rules for Prop. Logic

- Unit Resolution
  - From disjunction, if one of the disjuncts is false, can infer the other is true

$$\frac{\alpha \vee \beta, \quad \neg \beta}{\alpha}$$

Inference Rules for Prop. Logic

- Resolution
  - β cannot be both true and false
  - One of the other disjuncts must be true in one of the premises (implication is transitive)

$$\frac{\alpha \vee \beta,\quad \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

$\neg\beta$ is true

$$\frac{\alpha \vee \beta,\ \neg\beta}{\alpha}$$

β is true

$$\frac{\neg\beta \vee \gamma,\ \beta}{\gamma}$$

Monotonicity

- A logic is monotonic if when add new sentences to KB,
  - All sentences entailed by original KB are still entailed by the new larger KB

TASK: Find the Wumpus

Can we infer that the Wumpus is in cell (1,3), given our percepts and environment rules?

A = agent

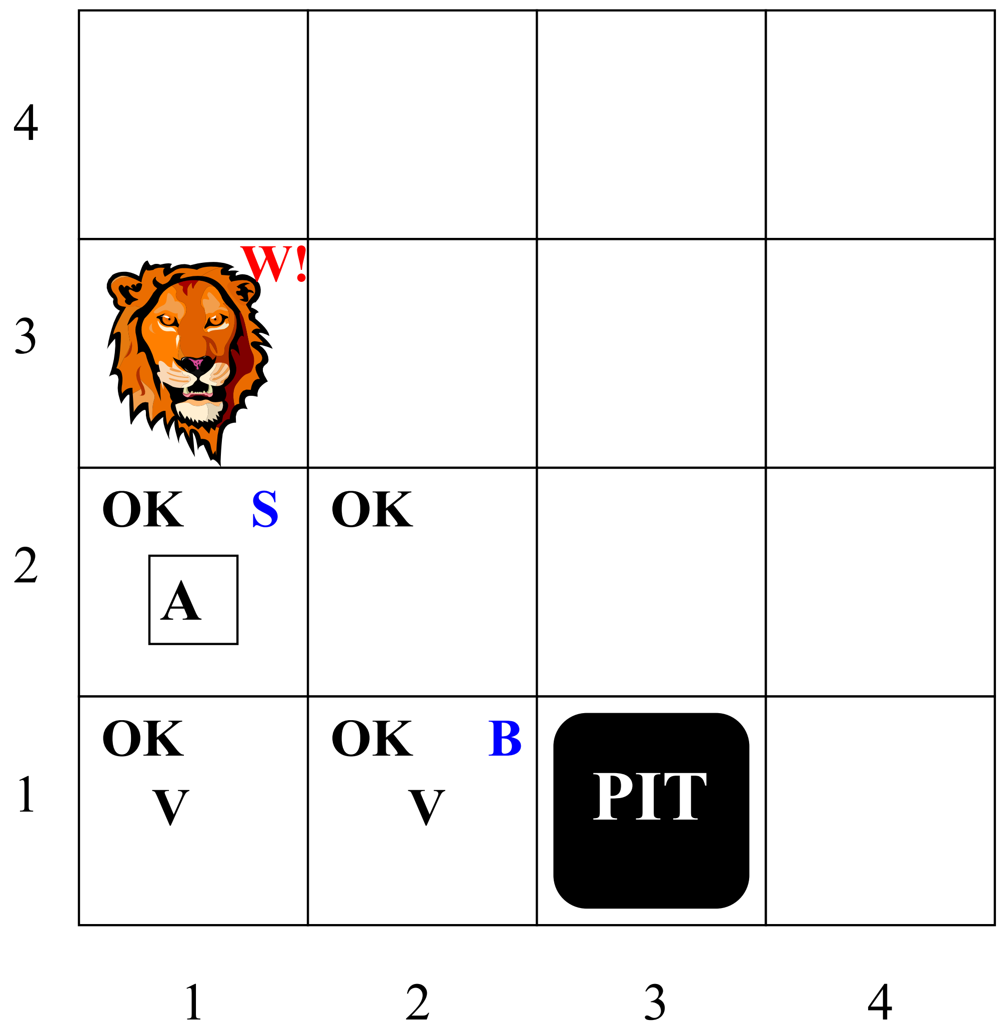
B = breeze

OK = safe square

S = stench

V = visited

W = wumpus



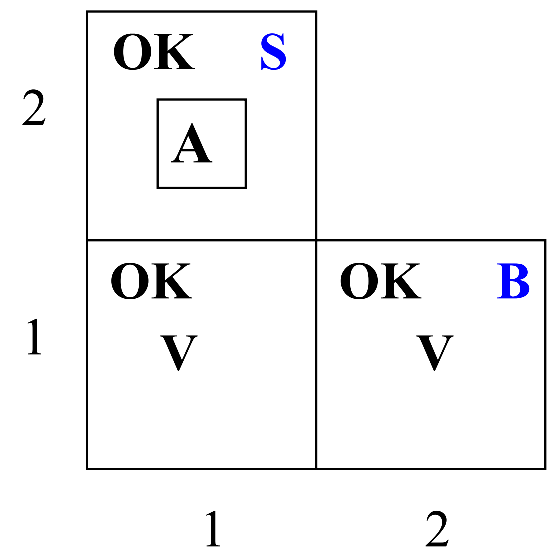
Wumpus Knowledge Base

- Percept sentences (facts) “at this point”

$$\neg S_{1,1} \quad \neg B_{1,1}$$

$$\neg S_{2,1} \quad B_{2,1}$$

$$S_{1,2} \quad \neg B_{1,2}$$



A = agent

B = breeze

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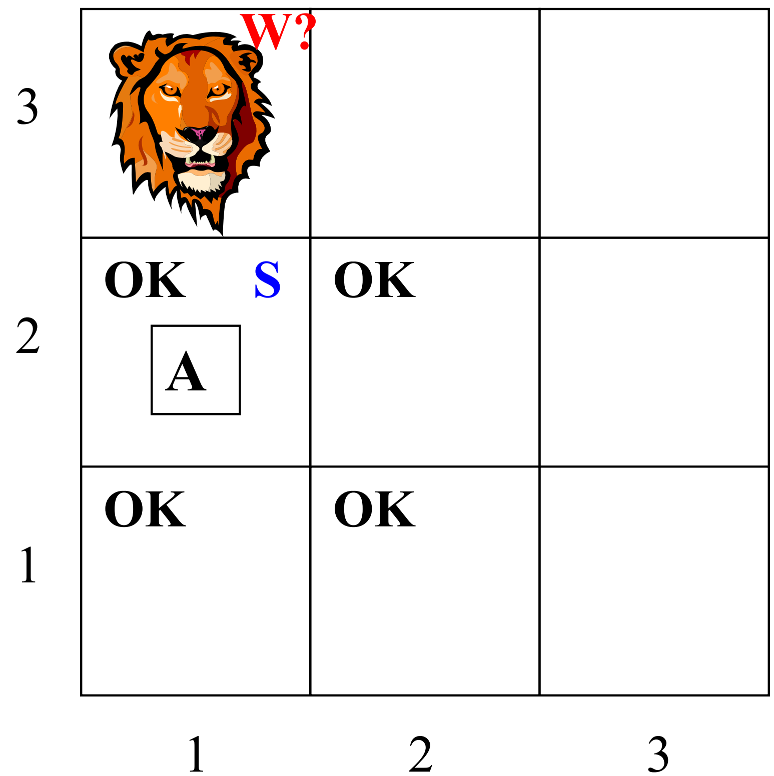
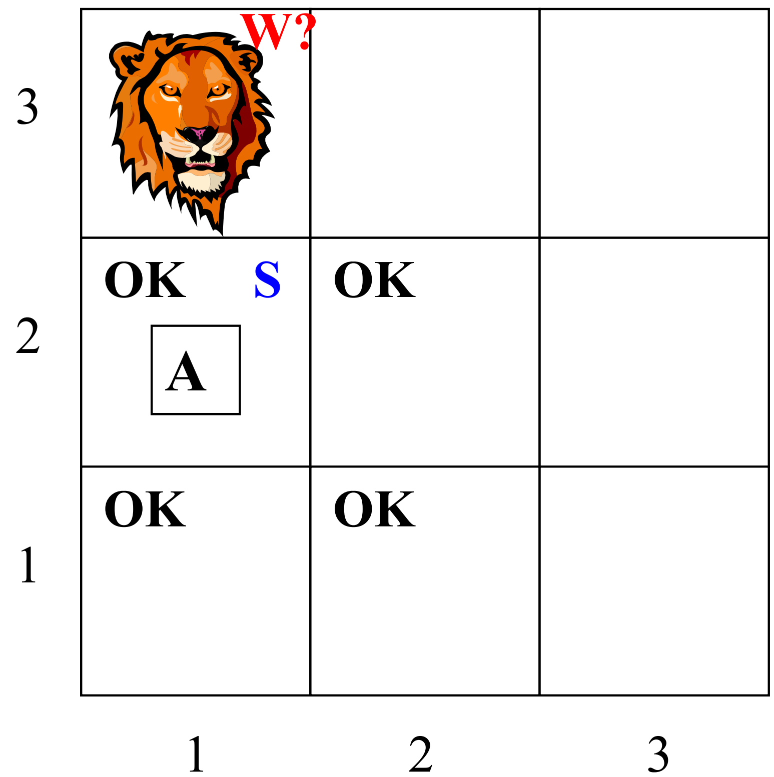
Environment Rules

$$R_1: \; \neg S_{1,1} \Rightarrow \; \neg W_{1,1} \wedge \neg W_{1,2} \wedge \neg W_{2,1}$$



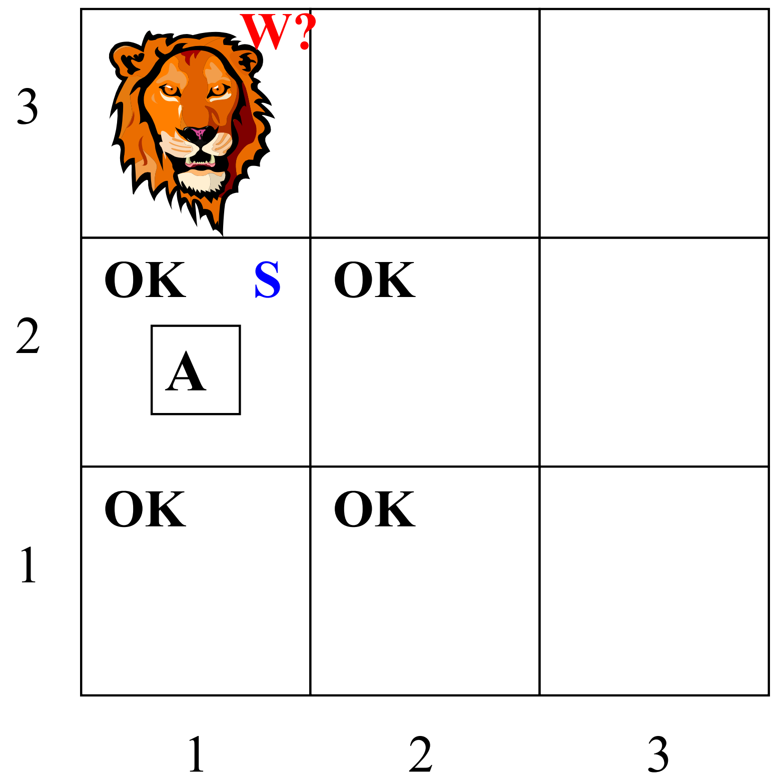
Environment Rules

$$R_2\colon\ \neg S_{2,1} \Rightarrow \neg W_{1,1} \wedge \neg W_{2,1} \wedge \neg W_{2,2} \wedge \neg W_{3,1}$$



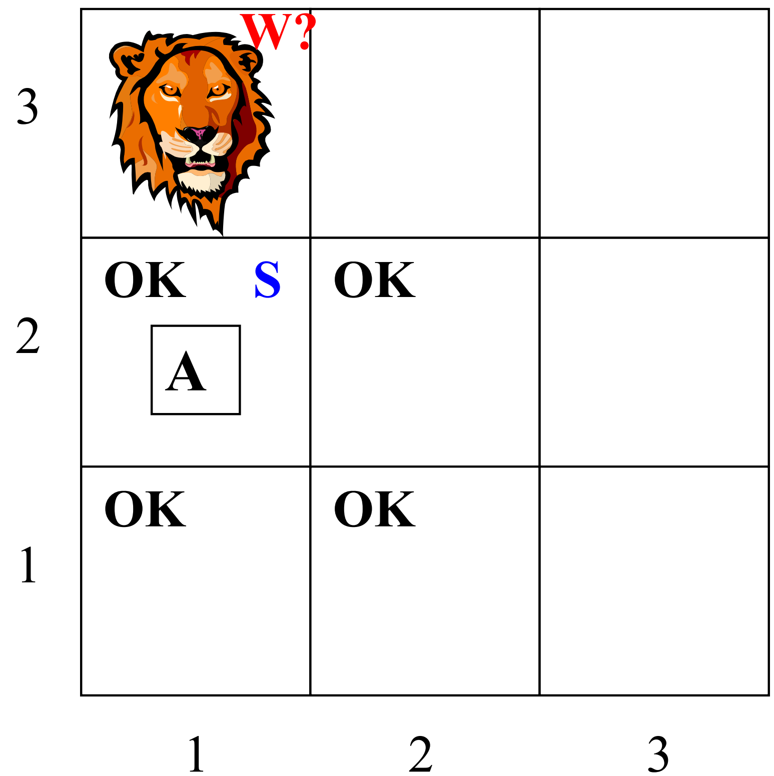
Environment Rules

$$R_3: \neg S_{1,2} \Rightarrow \neg W_{1,1} \wedge \neg W_{1,2} \wedge \neg W_{2,2} \wedge \neg W_{1,3}$$



Environment Rules

$$R_4\text{:}\;\; S_{1,2} \Rightarrow \; W_{1,3} \vee W_{1,2} \vee W_{2,2} \vee W_{1,1}$$



Conclude $W_{1,3}$?

- Does the Wumpus reside in square (1,3) ?
- In other words, can we <u>infer</u> $W_{1,3}$ from our knowledge base?

$$KB \vdash_i W_{1,3}$$

Conclude $W_{1,3}$ (Step #1)

- *Modus Ponens* $\dfrac{\alpha \Rightarrow \beta,\ \alpha}{\beta}$

$$R_1:\ \neg S_{1,1} \Rightarrow \neg W_{1,1} \wedge \neg W_{1,2} \wedge \neg W_{2,1}$$

Percept: $\neg S_{1,1}$

Infer

$$\neg W_{1,1} \wedge \neg W_{1,2} \wedge \neg W_{2,1}$$

Conclude $W_{1,3}$ (Step #2)

- *And-Elimination*

$$\frac{\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n}{\alpha_i}$$

$$\neg W_{1,1} \wedge \neg W_{1,2} \wedge \neg W_{2,1}$$

Infer

$$\neg W_{1,1} \quad \neg W_{1,2} \quad \neg W_{2,1}$$

Conclude $W_{1,3}$ (Step #3)

- *Modus Ponens* $$\frac{\alpha \Rightarrow \beta,\ \alpha}{\beta}$$

R_2: $\neg S_{2,1} \Rightarrow \neg W_{1,1} \wedge \neg W_{2,1} \wedge \neg W_{2,2} \wedge \neg W_{3,1}$

Percept: $\neg S_{2,1}$

Infer

$\neg W_{1,1} \wedge \neg W_{2,1} \wedge \neg W_{2,2} \wedge \neg W_{3,1}$

And-Elimination $$\frac{\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n}{\alpha_i}$$

$\neg W_{1,1} \quad \neg W_{2,1} \quad \neg W_{2,2} \quad \neg W_{3,1}$

Conclude $W_{1,3}$ (Step #4)

- *Modus Ponens* $$\frac{\alpha \Rightarrow \beta,\ \alpha}{\beta}$$

R_4: $S_{1,2} \Rightarrow W_{1,3} \lor W_{1,2} \lor W_{2,2} \lor W_{1,1}$

Percept: $S_{1,2}$

Infer

$$W_{1,3} \lor W_{1,2} \lor W_{2,2} \lor W_{1,1}$$

Conclude $W_{1,3}$ (Step #5)

- *Unit Resolution* $\dfrac{\alpha \vee \beta, \quad \neg\beta}{\alpha}$

$W_{1,3} \vee W_{1,2} \vee W_{2,2} \vee W_{1,1}$ from Step #4

$\neg W_{1,1}$ *from Step #2*

Infer

$W_{1,3} \vee W_{1,2} \vee W_{2,2}$

Conclude $W_{1,3}$ (Step #6)

- *Unit Resolution* $\dfrac{\alpha \vee \beta, \quad \neg\beta}{\alpha}$

$W_{1,3} \vee W_{1,2} \vee W_{2,2}$ from Step #5

$\neg W_{2,2}$ *from Step #3*

Infer

$W_{1,3} \vee W_{1,2}$

Conclude $W_{1,3}$ (Step #7)

- *Unit Resolution* $\dfrac{\alpha \vee \beta, \quad \neg\beta}{\alpha}$

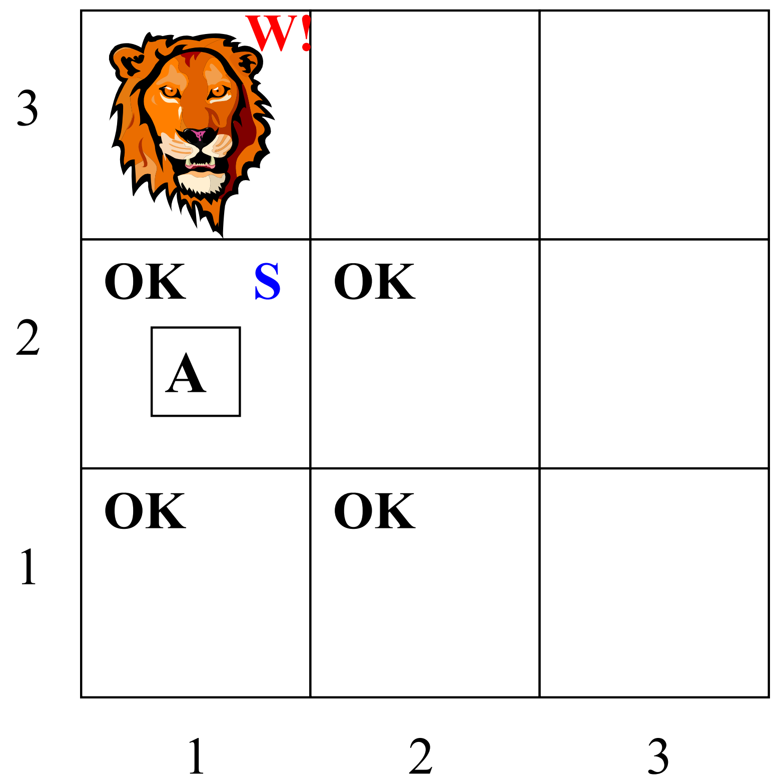
$W_{1,3} \vee W_{1,2}$ from Step #6

$\neg W_{1,2}$ *from Step #2*

Infer

$W_{1,3} \rightarrow$ *The wumpus is in cell 1,3!!!*

Wumpus in $W_{1,3}$



Summary

- Propositional logic commits to existence of facts about the world being represented
  - Simple syntax and semantics
- Proof methods
  - Truth table
  - Inference rules
    - Modus Ponens
    - And-Elimination
    - And/Or-Introduction
    - Double-Negation Elimination
    - Unit Resolution
    - Resolution
- Propositional logic quickly becomes impractical