Decision Tree

- A decision tree is a tree-like structure that is used as a model for classifying data.
- A decision tree decomposes the data into sub-trees made of other sub-trees and/or leaf nodes.
- A decision tree is made up of two types of nodes
 - Decision Nodes: These type of node have two or more branches
 - Leaf Nodes: The lowest nodes which represents decision

DataSet

Attributes

Classes

| Outlook | Temperature | Humidity | Windy | Play Golf |
|----------|-------------|----------|-------|-----------|
| Rainy | Hot | High | FALSE | No |
| Rainy | Hot | High | TRUE | No |
| Overcast | Hot | High | FALSE | Yes |
| Sunny | Mild | High | FALSE | Yes |
| Sunny | Cool | Normal | FALSE | Yes |
| Sunny | Cool | Normal | TRUE | No |
| Overcast | Cool | Normal | TRUE | Yes |
| Rainy | Mild | High | FALSE | No |
| Rainy | Cool | Normal | FALSE | Yes |
| Sunny | Mild | Normal | FALSE | Yes |
| Rainy | Mild | Normal | TRUE | Yes |
| Overcast | Mild | High | TRUE | Yes |
| Overcast | Hot | Normal | FALSE | Yes |
| Sunny | Mild | High | TRUE | No |

• Since decision trees are used for clarification, you need to determine the classes which are the basis for the decision.

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 is Play Golf column with
 classes Yes and No.

| | Classes | | | |
|----------|-------------|----------|-------|-----------|
| Outlook | Temperature | Humidity | Windy | Play Golf |
| Rainy | Hot | High | FALSE | No |
| Rainy | Hot | High | TRUE | No |
| Overcast | Hot | High | FALSE | Yes |
| Sunny | Mild | High | FALSE | Yes |
| Sunny | Cool | Normal | FALSE | Yes |
| Sunny | Cool | Normal | TRUE | No |
| Overcast | Cool | Normal | TRUE | Yes |
| Rainy | Mild | High | FALSE | No |
| Rainy | Cool | Normal | FALSE | Yes |
| Sunny | Mild | Normal | FALSE | Yes |
| Rainy | Mild | Normal | TRUE | Yes |
| Overcast | Mild | High | TRUE | Yes |
| Overcast | Hot | Normal | FALSE | Yes |
| Sunny | Mild | High | TRUE | No |

- Since decision trees are used for clarification, you need to determine the classes which are the basis for the decision.
- In this case, it it the last column, that
 is Play Golf column with
 classes Yes and No.
- Next determine the rootNode
 - we need to compute the entropy.
 - To compute the entropy, we create a frequency table for the classes

| | Classes | | | |
|----------|-------------|----------|-------|-----------|
| Outlook | Temperature | Humidity | Windy | Play Golf |
| Rainy | Hot | High | FALSE | No |
| Rainy | Hot | High | TRUE | No |
| Overcast | Hot | High | FALSE | Yes |
| Sunny | Mild | High | FALSE | Yes |
| Sunny | Cool | Normal | FALSE | Yes |
| Sunny | Cool | Normal | TRUE | No |
| Overcast | Cool | Normal | TRUE | Yes |
| Rainy | Mild | High | FALSE | No |
| Rainy | Cool | Normal | FALSE | Yes |
| Sunny | Mild | Normal | FALSE | Yes |
| Rainy | Mild | Normal | TRUE | Yes |
| Overcast | Mild | High | TRUE | Yes |
| Overcast | Hot | Normal | FALSE | Yes |
| Sunny | Mild | High | TRUE | No |

| | Classes | | | |
|----------|-------------|----------|-------|-----------|
| Outlook | Temperature | Humidity | Windy | Play Golf |
| Rainy | Hot | High | FALSE | No |
| Rainy | Hot | High | TRUE | No |
| Overcast | Hot | High | FALSE | Yes |
| Sunny | Mild | High | FALSE | Yes |
| Sunny | Cool | Normal | FALSE | Yes |
| Sunny | Cool | Normal | TRUE | No |
| Overcast | Cool | Normal | TRUE | Yes |
| Rainy | Mild | High | FALSE | No |
| Rainy | Cool | Normal | FALSE | Yes |
| Sunny | Mild | Normal | FALSE | Yes |
| Rainy | Mild | Normal | TRUE | Yes |
| Overcast | Mild | High | TRUE | Yes |
| Overcast | Hot | Normal | FALSE | Yes |
| Sunny | Mild | High | TRUE | No |

| Play Golf(14) | | |
|---------------|---|--|
| Yes No | | |
| 9 | 5 | |

- In this step, you need to calculate the entropy for the Decision Column (Play Golf)
- Entropy(PlayGolf) = E(5-,9+)

| Play Golf(14) | | |
|---------------|----|--|
| Yes | No | |
| 9 | 5 | |

- In this step, you need to calculate the entropy for the Decision Column (Play Golf)
- Entropy(PlayGolf) = E(5-,9+)

$$Entropy(S) = \sum_{i=1}^{c} -p_i log_2 p_i$$

| Play Golf(14) | | |
|---------------|----|--|
| Yes | No | |
| 9 | 5 | |

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$$Entropy(S) = \sum_{i=1}^{c} -p_i log_2 p_i$$

 $Entropy(PlayGolf) = -p_{yes}log_2(p_{yes}) - p_{no}log_2(p_{no})$

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| Play Golf(14) | | |
|---------------|----|--|
| Yes | No | |
| 9 | 5 | |

$$E(PlayGolf) = E(5,9)$$

$$= -\left(\frac{9}{14}\log_2\frac{9}{14}\right) - \left(\frac{5}{14}\log_2\frac{5}{14}\right)$$

$$= -(0.357\log_2 0.357) - (0.643\log_2 0.643)$$

$$= 0.94$$

For the other four attributes, we need to calculate the entropy after each of the split.

- E(PlayGolf, Outlook)
- E(PlayGolf, Temperature)
- E(PlayGolf, Humidity)
- E(PlayGolf,Windy)

The entropy for two variables is calculated using the formula.

$$Entropy(S,T) = \sum_{c \in T} P(c)E(c)$$

The easiest way to approach this calculation is to create a frequency table for the two variables

E(PlayGolf, Outlook) Calculation:

To calculate **E**(PlayGolf, Outlook), we would use the formula below:

$$E(PlayGolf, Outlook) = P(Sunny)E(Sunny) + P(Overcast)E(Overcast) + P(Rainy)E(Rainy)$$

| | | PlayGolf(14) | | |
|---------|----------|--------------|----|---|
| | | Yes | No | |
| Outlook | Sunny | 3 | 2 | 5 |
| | Overcast | 4 | 0 | 4 |
| | Rainy | 2 | 3 | 5 |

$$E(PlayGolf, Outlook) = P(Sunny) E(3,2) + P(Overcast) E(4,0) + P(rainy) E(2,3)$$

$$E(PlayGolf, Outlook) = \frac{5}{14}E(3,2) + \frac{4}{14}E(4,0) + \frac{5}{14}E(2,3)$$

| | | PlayGolf(14) | | |
|---------|----------|--------------|----|---|
| | | Yes | No | |
| Outlook | Sunny | 3 | 2 | 5 |
| | Overcast | 4 | 0 | 4 |
| | Rainy | 2 | 3 | 5 |

$$E(PlayGolf,Outlook) = \frac{5}{14}E(3,2) + \frac{4}{14}E(4,0) + \frac{5}{14}E(2,3)$$

```
E(Sunny) = E(3,2)
= -\left(\frac{3}{5}\log_2\frac{3}{5}\right) - \left(\frac{2}{5}\log_2\frac{2}{5}\right)
= -(0.60 \log_2 0.60) - (0.40 \log_2 0.40)
= -(0.60 * 0.737) - (0.40 * 0.529)
= 0.971
```

| | | PlayGolf(14) | | |
|---------|----------|--------------|----|---|
| | | Yes | No | |
| Outlook | Sunny | 3 | 2 | 5 |
| | Overcast | 4 | 0 | 4 |
| | Rainy | 2 | 3 | 5 |

$$E(PlayGolf,Outlook) = \frac{5}{14}E(3,2) + \frac{4}{14}E(4,0) + \frac{5}{14}E(2,3)$$

$$E(Sunny) = E(3,2)$$

$$= -\left(\frac{3}{5}\log_2\frac{3}{5}\right) - \left(\frac{2}{5}\log_2\frac{2}{5}\right)$$

$$= -(0.60\log_2 0.60) - (0.40\log_2 0.40)$$

$$= -(0.60 * 0.737) - (0.40 * 0.529)$$

$$= 0.971$$

$$E(Overcast) = E(4,0)$$

$$= -\left(\frac{4}{4}\log_2\frac{4}{4}\right) - \left(\frac{0}{4}\log_2\frac{0}{4}\right)$$

$$= -(0) - (0)$$

$$= 0$$

| | | PlayGolf(14) | | |
|---------|----------|--------------|----|---|
| | | Yes | No | |
| | Sunny | 3 | 2 | 5 |
| Outlook | Overcast | 4 | 0 | 4 |
| | Rainy | 2 | 3 | 5 |

$$E(PlayGolf, Outlook) = \frac{5}{14}E(3,2) + \frac{4}{14}E(4,0) + \frac{5}{14}E(2,3)$$

$$E(Sunny) = E(3,2)$$

$$= -\left(\frac{3}{5}\log_2\frac{3}{5}\right) - \left(\frac{2}{5}\log_2\frac{2}{5}\right)$$

$$= -(0.60\log_2 0.60) - (0.40\log_2 0.40)$$

$$= -(0.60 * 0.737) - (0.40 * 0.529)$$

$$= 0.971$$

$$E(Overcast) = E(4,0)$$

$$= -\left(\frac{4}{4}\log_2\frac{4}{4}\right) - \left(\frac{0}{4}\log_2\frac{0}{4}\right)$$

$$= -(0) - (0)$$

$$= 0$$

$$E(Rainy) = E(2,3)$$

$$= -\left(\frac{2}{5}\log_2\frac{2}{5}\right) - \left(\frac{3}{5}\log_2\frac{3}{5}\right)$$

$$= -(0.40\log_2 0.40) - (0.6\log_2 0.60)$$

$$= 0.971$$

| | | PlayGolf(14) | | |
|---------|----------|--------------|----|---|
| | | Yes | No | |
| | Sunny | 3 | 2 | 5 |
| Outlook | Overcast | 4 | 0 | 4 |
| | Rainy | 2 | 3 | 5 |

$$E(PlayGolf, Outlook) = \frac{5}{14}E(3,2) + \frac{4}{14}E(4,0) + \frac{5}{14}E(2,3)$$

$$E(Sunny) = E(3,2)$$

$$= -\left(\frac{3}{5}\log_2\frac{3}{5}\right) - \left(\frac{2}{5}\log_2\frac{2}{5}\right)$$

$$= -(0.60\log_2 0.60) - (0.40\log_2 0.40)$$

$$= -(0.60 * 0.737) - (0.40 * 0.529)$$

$$= 0.971$$

$$E(Overcast) = E(4,0)$$

$$= -\left(\frac{4}{4}\log_2\frac{4}{4}\right) - \left(\frac{0}{4}\log_2\frac{0}{4}\right)$$

$$= -(0) - (0)$$

$$= 0$$

$$E(Rainy) = E(2,3)$$

$$= -\left(\frac{2}{5}\log_2\frac{2}{5}\right) - \left(\frac{3}{5}\log_2\frac{3}{5}\right)$$

$$= -(0.40\log_2 0.40) - (0.6\log_2 0.60)$$

$$= 0.971$$

$$E(4,0) = 0;$$

 $E(2,3) = E(3,2)$

| | | PlayG | olf(14) | |
|---------|----------|-------|---------|---|
| | | Yes | No | |
| | Sunny | 3 | 2 | 5 |
| Outlook | Overcast | 4 | 0 | 4 |
| | Rainy | 2 | 3 | 5 |

$$E(PlayGolf, Outlook) = P(Sunny) E(3,2) + P(Overcast) E(4,0) + P(rainy) E(2,3)$$

$$E(PlayGolf, Outlook) = \frac{5}{14}E(3,2) + \frac{4}{14}E(4,0) + \frac{5}{14}E(2,3)$$

$$= \frac{5}{14}0.971 + \frac{4}{14}0.0 + \frac{5}{14}0.971$$

$$= 0.357 * 0.971 + 0.0 + 0.357 * 0.971$$

$$= 0.693$$

E(PlayGolf, Temperature) Calculation

| | | PlayGo | olf(14) | |
|-------------|------|--------|---------|---|
| | | Yes | No | |
| Temperature | Hot | 2 | 2 | 4 |
| | Cold | 3 | 1 | 4 |
| | Mild | 4 | 2 | 6 |

```
E(PlayGolf, Temperature) = P(Hot) E(2,2) + P(Cold) E(3,1) + P(Mild) E(4,2)
```

```
E (PlayGolf, Temperature) = 4/14 * E(Hot) + 4/14 * E(Cold) + 6/14 * E(Mild)

E (PlayGolf, Temperature) = 4/14 * E(2, 2) + 4/14 * E(3, 1) + 6/14 * E(4, 2)

E (PlayGolf, Temperature) = 4/14 * -(2/4 log 2/4) - (2/4 log 2/4) + 4/14 * -(3/4 log 3/4) - (1/4 log 1/4) + 6/14 * -(4/6 log 4/6) - (2/6 log 2/6)

E (PlayGolf, Temperature) = 5/14 * 1.0 + 4/14 * 1.811 + 5/14 * 0.918
```

= 0.911

E(PlayGolf, Humidity) Calculation

| | | PlayGolf(14) | | |
|----------|--------|--------------|----|---|
| | | Yes | No | |
| Humidity | High | 3 | 4 | 7 |
| | Normal | 6 | 1 | 7 |

```
E (PlayGolf, Humidity) = 7/14 * E(High) + 7/14 * E(Normal)

E (PlayGolf, Humidity) = 7/14 * E(3,4) + 7/14 * E(6,1)

E (PlayGolf, Humidity) = 7/14 * -(3/7 \log 3/7) - (4/7 \log 4/7)

+ 7/14 * -(6/7 \log 6/7) - (1/7 \log 1/7)

E (PlayGolf, Humidity) = 7/14 * 0.985

+ 7/14 * 0.592
```

= 0.788

E(PlayGolf, Windy) Calculation

| | | PlayGolf(14) | | |
|-------|-------|--------------|----|---|
| | | Yes | No | |
| Windy | TRUE | 3 | 3 | 6 |
| | FALSE | 6 | 2 | 8 |

```
E (PlayGolf, Windy) = 6/14 * E(True) + 8/14 * E(False)
```

$$E (PlayGolf, Windy) = 6/14 * E(3, 3) + 8/14 * E(6, 2)$$

- 1. E(PlayGolf, Outlook) = **0.693**
- 2. E(PlayGolf, Temperature) = **0.911**
- 3. E(PlayGolf, Humidity) = **0.788**
- 4. E(PlayGolf, Windy) = **0.892**

Step 4: Calculating Information Gain for Each Split

- The next step is to calculate the information gain for each of the attributes.
- The information gain is calculated from the split using each of the attributes.
- Then the attribute with the largest information gain is used for the split.
- The information gain is calculated using the formula:

Gain(S,T) = Entropy(S) - Entropy(S,T)

Step 4: Calculating Information Gain for Each Split

```
Gain(PlayGolf, Outlook) = Entropy(PlayGolf) - Entropy(PlayGolf, Outlook)

= 0.94 - 0.693 = 0.247

Gain(PlayGolf, Temperature) = Entropy(PlayGolf) - Entropy(PlayGolf, Temperature)

= 0.94 - 0.911 = 0.029

Gain(PlayGolf, Humidity) = Entropy(PlayGolf) - Entropy(PlayGolf, Humidity)

= 0.94 - 0.788 = 0.152

Gain(PlayGolf, Windy) = Entropy(PlayGolf) - Entropy(PlayGolf, Windy)

= 0.94 - 0.892 = 0.048
```

Step 4: Calculating Information Gain for Each Split

```
Gain(PlayGolf, Outlook) = Entropy(PlayGolf) - Entropy(PlayGolf, Outlook)

= 0.94 - 0.693 = 0.247

Gain(PlayGolf, Temperature) = Entropy(PlayGolf) - Entropy(PlayGolf, Temperature)

= 0.94 - 0.911 = 0.029

Gain(PlayGolf, Humidity) = Entropy(PlayGolf) - Entropy(PlayGolf, Humidity)

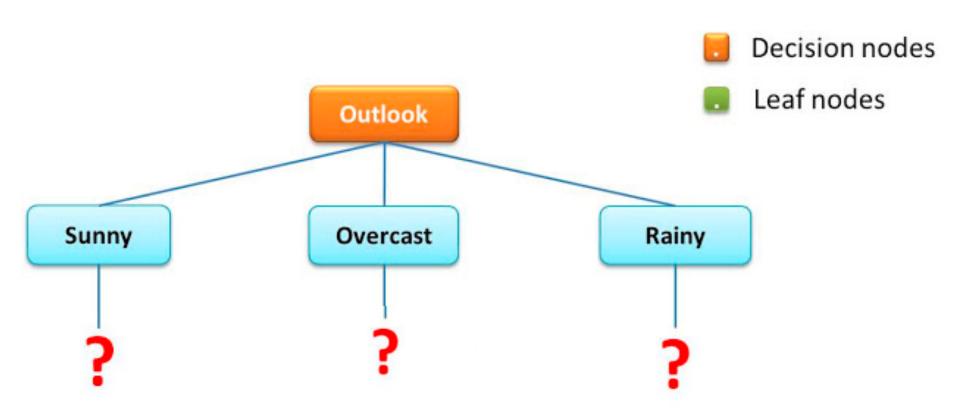
= 0.94 - 0.788 = 0.152

Gain(PlayGolf, Windy) = Entropy(PlayGolf) - Entropy(PlayGolf, Windy)

= 0.94 - 0.892 = 0.048
```

Step 5: Perform the First Split

From our calculation, the highest information gain comes from Outlook. Therefore the split will look like this:



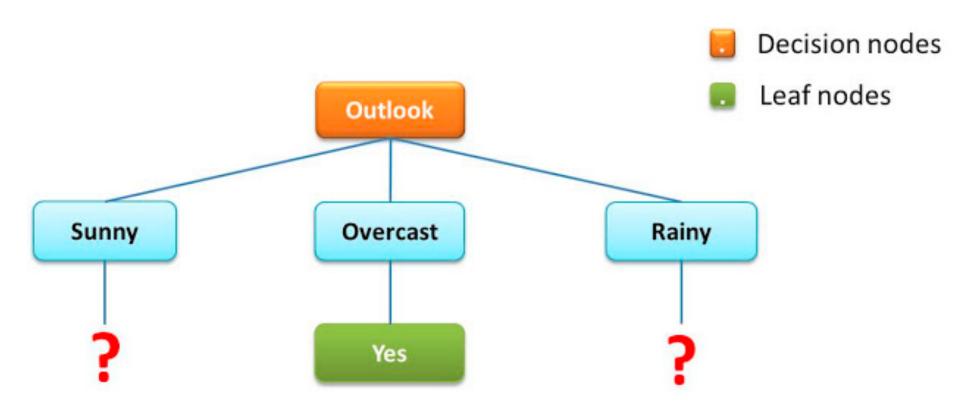
Step 5: Perform the First Split

| Outlook | Temperature | Humidity | Windy | Play Golf |
|----------|-------------|----------|-------|-----------|
| Sunny | Mild | Normal | FALSE | Yes |
| Sunny | Mild | High | FALSE | Yes |
| Sunny | Cool | Normal | FALSE | Yes |
| Sunny | Cool | Normal | TRUE | No |
| Sunny | Mild | High | TRUE | No |
| Overcast | Hot | High | FALSE | Yes |
| Overcast | Mild | High | TRUE | Yes |
| Overcast | Hot | Normal | FALSE | Yes |
| Overcast | Cool | Normal | TRUE | Yes |
| Rainy | Hot | High | FALSE | No |
| Rainy | Hot | High | TRUE | No |
| Rainy | Mild | High | FALSE | No |
| Rainy | Cool | Normal | FALSE | Yes |
| Rainy | Mild | Normal | TRUE | Yes |

Overcast outlook requires no further split because it is just one homogeneous group. So we have a leaf node.

Step 5: Perform the First Split

From our calculation, the highest information gain comes from Outlook. Therefore the split will look like this:



Overcast outlook requires no further split because it is just one homogeneous group. So we have a leaf node.

The Sunny and the Rainy attributes needs to be split

The Rainy outlook can be split using either Temperature, Humidity or Windy.

Question: What attribute would best be used for this split?

- Gain(PlayGolf, Outlook=Rainy, Temperature) = Entropy(PlayGolf, Outlook=Rainy) - Entropy(PlayGolf, Outlook=Rainy, Temperature)
- Gain(PlayGolf, Outlook=Rainy, Humidity) =
 Entropy(PlayGolf, Outlook=Rainy) Entropy(PlayGolf, Outlook=Rainy, Humidity)
- Gain(PlayGolf, Outlook=Rainy, Windy) =
 Entropy(PlayGolf, Outlook=Rainy) Entropy(PlayGolf, Outlook=Rainy, Windy)

The Sunny and the Rainy attributes needs to be split

The Rainy outlook can be split using either Temperature, Humidity or Windy.

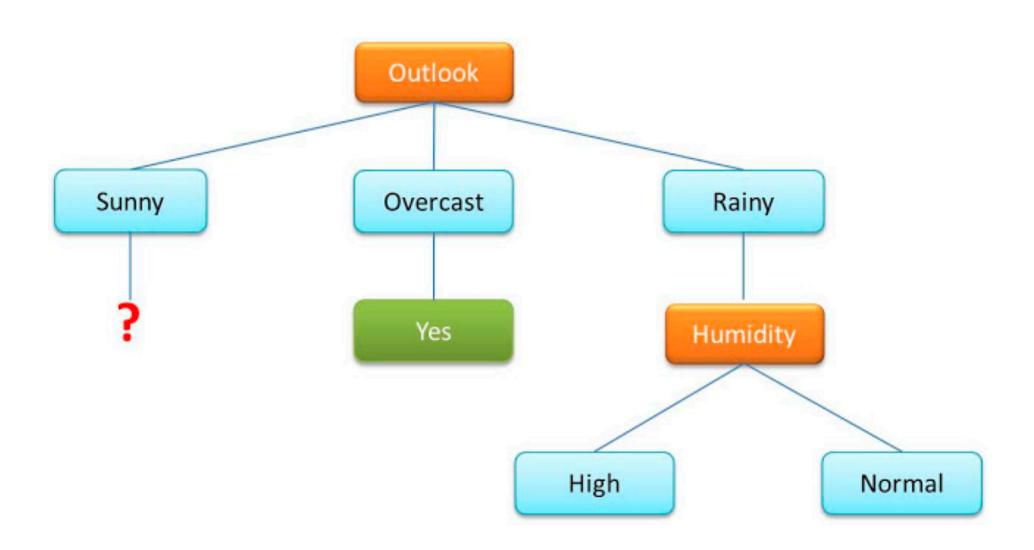
Question: What attribute would best be used for this split?

Humidity, produces homogenous groups.

| Outlook | Temperature | Humidity | Windy | Play Golf |
|---------|-------------|----------|-------|-----------|
| Rainy | Hot | High | FALSE | No |
| Rainy | Hot | High | TRUE | No |
| Rainy | Mild | High | FALSE | No |

| Rainy Rainy | Cool | Normal | FALSE | Yes | |
|----------------|------|--------|-------|-----|--|
| Rainy | Mild | Normal | TRUE | Yes | |

Gain(PlayGolf, Outlook=Rainy, Humidity) =
 Entropy(PlayGolf, Outlook=Rainy) - Entropy(PlayGolf, Outlook=Rainy, Humidity)=
 Entropy(PlayGolf, Outlook=Rainy) - 0

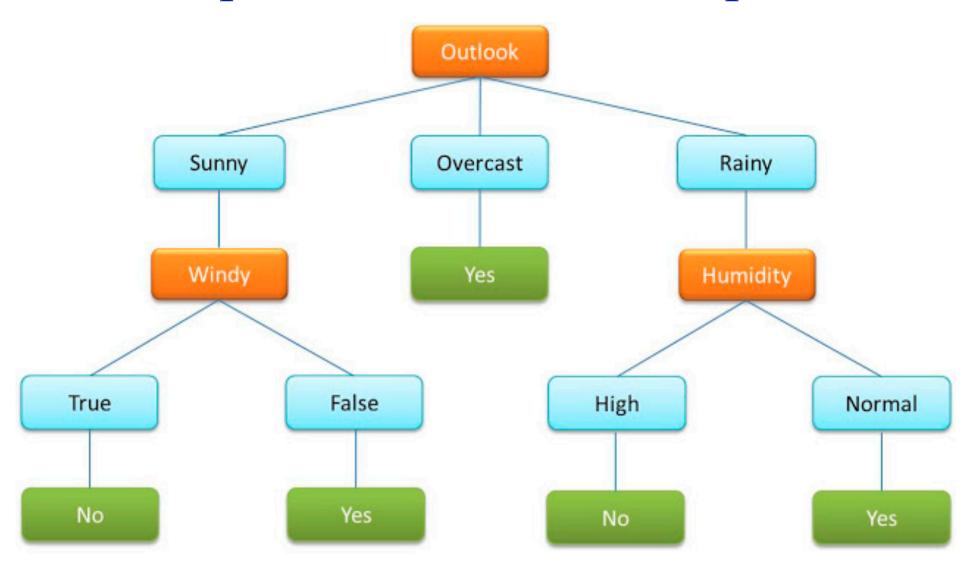


The Rainy outlook can be split using either Temperature, Humidity or Windy.

Question: What attribute would best be used for this split? Why? Answer: **Windy** . Because it produces homogeneous groups.

| Outlook | Temperature | Humidity | Windy | Play Golf |
|---------|-------------|----------|-------|-----------|
| Sunny | Mild | Normal | FALSE | Yes |
| Sunny | Mild | High | FALSE | Yes |
| Sunny | Cool | Normal | FALSE | Yes |

| Sunny | Cool | Normal | TRUE | No | |
|-------|------|--------|------|----|--|
| Sunny | Mild | High | TRUE | No | |



ID3 Algorithm

ID3 (S, A, V) Let: S = Learning Set A = Attibute Set V = Attribute Values Begin Load learning sets and create decision tree root node(rootNode), Add learning set S into root not as its subset https://github.com/jeniyat/cse_5521/blob/master/diabetes.csv For rootNode, compute Entropy(rootNode.subset) If Entropy(rootNode.subset) == 0 (subset is homogeneous) return a leaf node If Entropy(rootNode.subset)!= 0 (subset is not homogeneous) compute Information Gain for each attribute left (not been used for spliting) Find attibute A with Maximum(Gain(S,A)) Create child nodes for this root node and add to rootNode in the decision tree For each child of the rootNode Apply ID3(S,A,V) Continue until a node with Entropy of 0 or a leaf node is reached

End

Continuous Valued Attributes

Create a discrete attribute to test continuous

- \bullet Temperature = 82.5
- (Temperature > 72.3) = t, f

Temperature: 40 48 60 72 80 90 Play Golf: No No Yes Yes Yes No

Unknown Attribute Values

What if some examples are missing values of A? Use training example anyway, sort through tree

- If node n tests A, assign most common value of A among other examples sorted to node n
- Assign most common value of A among other examples with same target value
- Assign probability p_i to each possible value v_i of A Assign fraction p_i of example to each descendant in tree

Classify new examples in same fashion