Binary Search Trees

Instructor: Jeeho Ryoo

Announcements

- Quiz today
    - V1 vs V2
- Quiz next week (last quiz)
- Assignment 3 comments
- Quiz 2 and 3 grades will be released this weekend

Designing a Set

How would we implement Set?

- Add
- Contains
- Remove

First Try

Store all the elements in an **unsorted** array or linked list

- What is the Big-Oh of contains?
- What is the Big-Oh of adding an element?
- What is the Big-Oh of removing an element?

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
3	8	9	7	5	12	4	8	1	6	75

Another attempt

What if we **sorted** the array?

What is the Big-Oh of contains?

What is the Big-Oh of adding an element?

What is the Big-Oh of removing an element?

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
2	5	6	8	11	13	17	22	23	29	31

Binary Search

Fast way to search for elements in a **sorted array**
Looping through elements one by one is slow [O(N)]
Idea:
Jump to the middle element:

- if the middle is what we're looking for, we're done. Hooray!
- if the middle is too small – we rule out the entire **left side** of elements smaller than the middle element
- if the middle is too big – we rule out the entire **right side** of elements bigger than the middle element

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
2	5	6	8	11	13	17	22	23	29	31

Binary Search in Action

Search for 8:

0	1	2	3	4	5	6	7	8	9	10
2	5	6	8	11	13	17	22	23	29	31

Binary Search in Action

Search for 8:

middle

0	1	2	3	4	5	6	7	8	9	10
2	5	6	8	11	13	17	22	23	29	31

Binary Search in Action

Search for 8:

Look at 13

it's too big, so we rule out indices 5-10

middle

0	1	2	3	4	5	6	7	8	9	10
2	5	6	8	11	13	17	22	23	29	31

Binary Search in Action

Search for 8:

Look at 13

it's too big, so we rule out indices 5-10

Pick the new middle of the remaining elements

Look at 6:

middle

0	1	2	3	4	5	6	7	8	9	10
2	5	6	8	11	13	17	22	23	29	31

Binary Search in Action

Search for 8:

Look at 13

it's too big, so we rule out indices 5-10

Pick the new middle of the remaining elements

Look at 6:

it's too small, so we rule out indices 0-3

middle

0	1	2	3	4	5	6	7	8	9	10
2	5	6	8	11	13	17	22	23	29	31

Binary Search in Action

Search for 8:

Look at 13

it's too big, so we rule out indices 5-10

Pick the new middle of the remaining elements

Look at 6:

it's too small, so we rule out indices 0-3

Look at 8:

it's just right! We return true

middle

0	1	2	3	4	5	6	7	8	9	10
2	5	6	8	11	13	17	22	23	29	31

Binary Search in Action

Search for 7:

middle

0	1	2	3	4	5	6	7	8	9	10
2	5	6	8	11	13	17	22	23	29	31

Binary Search in Action

Search for 7:

Look at 13

it's too big, so we rule out indices 5-10

middle

0	1	2	3	4	5	6	7	8	9	10
2	5	6	8	11	13	17	22	23	29	31

Binary Search in Action

Search for 7:

Look at 13

it's too big, so we rule out indices 5-10

Pick the new middle of the remaining elements

Look at 6:

middle

0	1	2	3	4	5	6	7	8	9	10
2	5	6	8	11	13	17	22	23	29	31

Binary Search in Action

Search for 7:

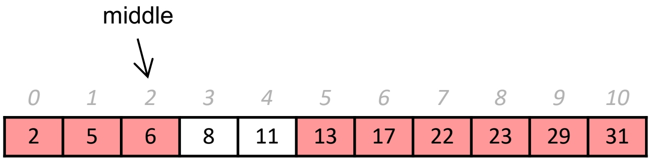
Look at 13

it's too big, so we rule out indices 5-10

Pick the new middle of the remaining elements

Look at 6:

it's too small, so we rule out indices 0-3



Binary Search in Action

Search for 8:

Look at 13

it's too big, so we rule out indices 5-10

Look at 6:

it's too small, so we rule out indices 0-3

Look at 8:

it's too big! We rule out elements 3-4

middle

0	1	2	3	4	5	6	7	8	9	10
2	5	6	8	11	13	17	22	23	29	31

Binary Search in Action

Search for 8:
Look at 13
 it's too big, so we rule out indices 5-10
Look at 6:
 it's too small, so we rule out indices 0-3
Look at 8:
 it's too big! We rule out elements 3-4
No elements left to search – we return false

middle

0	1	2	3	4	5	6	7	8	9	10
2	5	6	8	11	13	17	22	23	29	31

Sorted Array

What if we **sorted** the array?

- What is the Big-Oh of contains?
  - O(log N)
- What is the Big-Oh of adding an element?
  - O(N)
- What is the Big-Oh of removing an element?
  - O(N)

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
2	5	6	8	11	13	17	22	23	29	31

A Modification

Problem: an array is slow to insert into or remove from

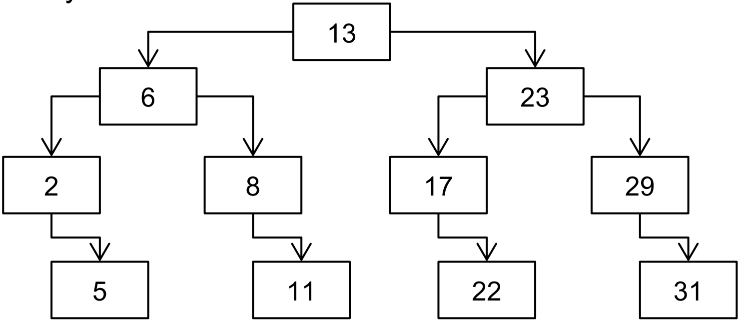
Our solution was a **linked list** – have each element connected to one other element

- Easy to add/remove elements
- Can't skip elements – need to go in order

Maybe we can find some way to implement the jumps necessary for binary search...

A Modification

What are all the possible paths binary search could take on this array?



0	1	2	3	4	5	6	7	8	9	10
2	5	6	8	11	13	17	22	23	29	31

A Modification

We always jump to one of two elements in binary search (depending on if the element we're looking at is too big or too small)

What if we had a Linked List where we stored two pointers, allowing us to make those jumps quickly?

Binary Search Tree

A **tree** is a data structure where each element (**parent**) stores two or more pointers to other elements (its **children**)

- A doubly-linked list doesn't count because, just like outside of computer science, a child can not be its own ancestor

Each node in a **binary tree** has two pointers

- Some of these pointers may be `NULL` (just like in a linked list)
- We'll see examples of non-binary trees in future lectures

A **binary search tree** is a binary tree with special ordering properties that make it easy to do binary search

Similar to a Linked List:

- Each element in its own block of memory
- Have to travel through pointers (can't skip "generations")

(Binary) TreeNode

```
struct TreeNode {
    int data; // assume that the tree stores ints
    TreeNode *left;
    TreeNode *right;
};
```

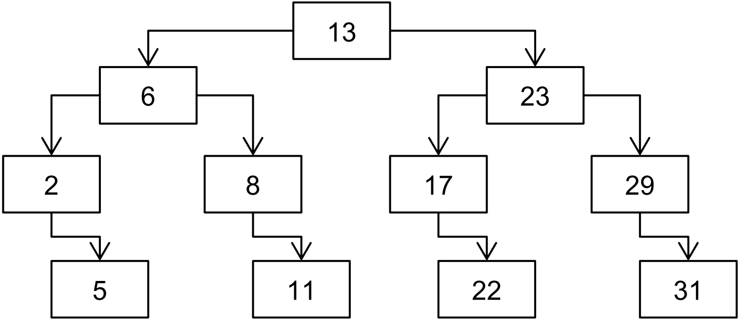
Binary Search Trees

A binary search tree has the following property:

All elements to the left of an element are smaller than that element

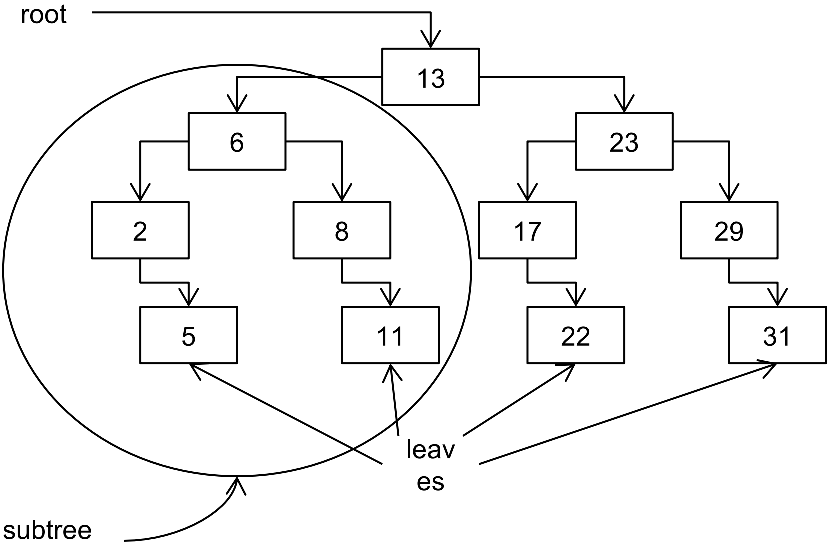
All elements to the right of an element are bigger than that element

Just like our sorted array!



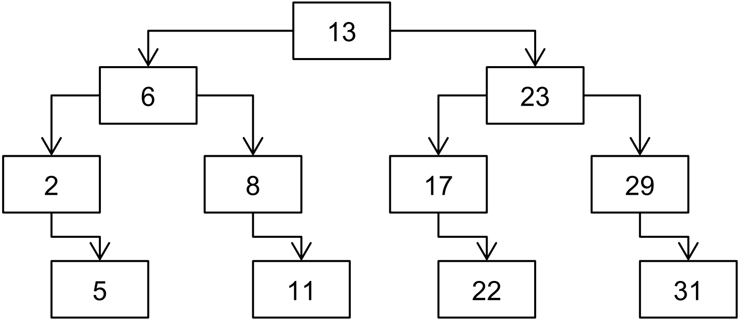
0	1	2	3	4	5	6	7	8	9	10
2	5	6	8	11	13	17	22	23	29	31

Tree anatomy



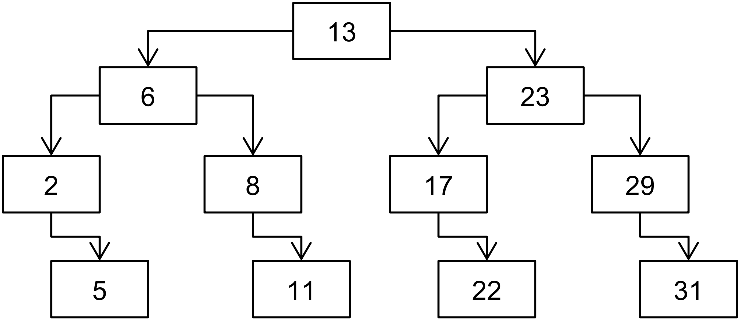
BST Contains

- How would you search a BST for an element?



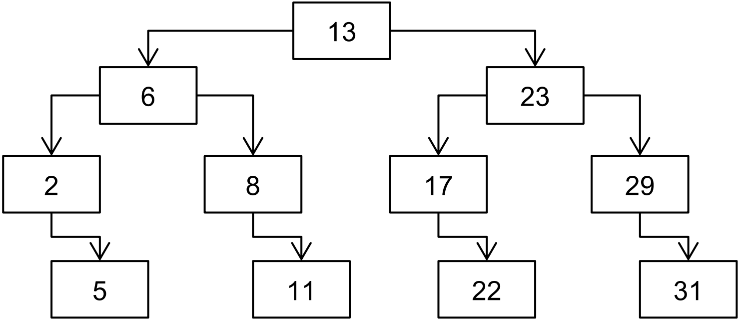
BST Contains

- How would you search a BST for an element?
- Start at root:
  - If root is too big, go left (entire right subtree is too big)
  - If root is too small, go right (entire left subtree is too small)



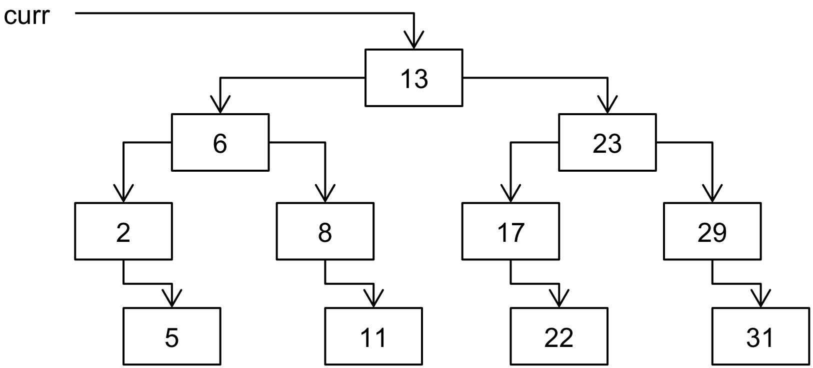
Trees and Recursion

- Trees are fundamentally **recursive (**subtrees are smaller trees)
- Start at root:
  - If root is too big, go left (entire right subtree is too big)
  - If root is too small, go right (entire left subtree is too small)



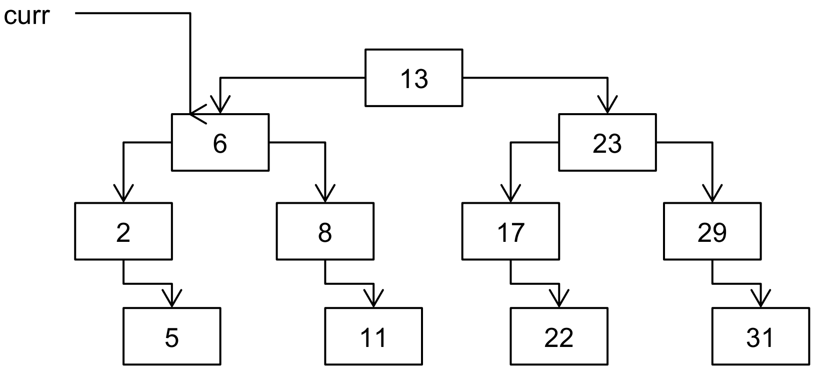
Trees and Contains

- Search for 5
- Start at root:
  - If root is too big, go left (entire right subtree is too big)
  - If root is too small, go right (entire left subtree is too small)



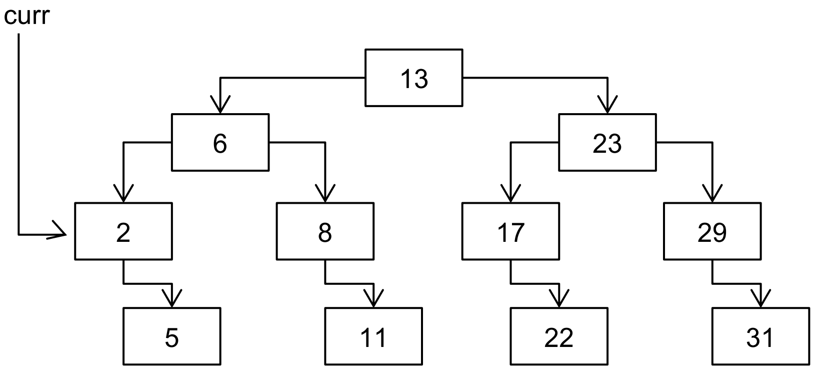
Trees and Contains

- Search for 5
- Start at root:
  - If root is too big, go left (entire right subtree is too big)
  - If root is too small, go right (entire left subtree is too small)



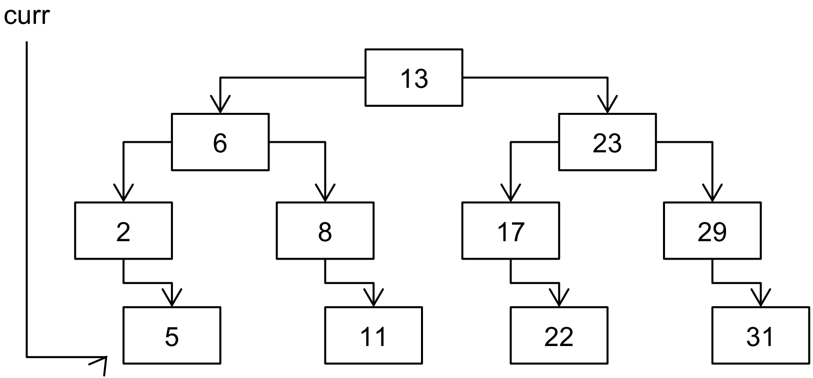
Trees and Contains

- Search for 5
- Start at root:
  - If root is too big, go left (entire right subtree is too big)
  - If root is too small, go right (entire left subtree is too small)



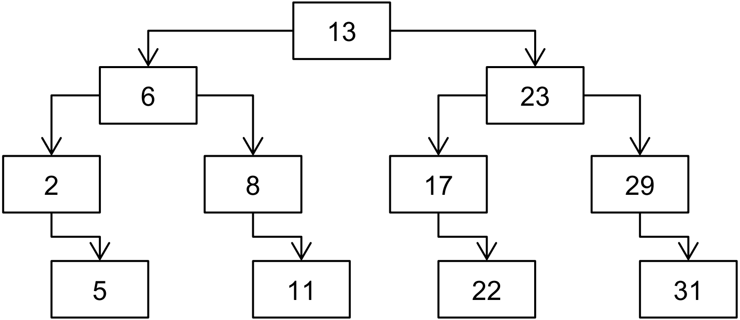
Trees and Contains

- Search for 5
- Start at root:
  - If root is too big, go left (entire right subtree is too big)
  - If root is too small, go right (entire left subtree is too small)



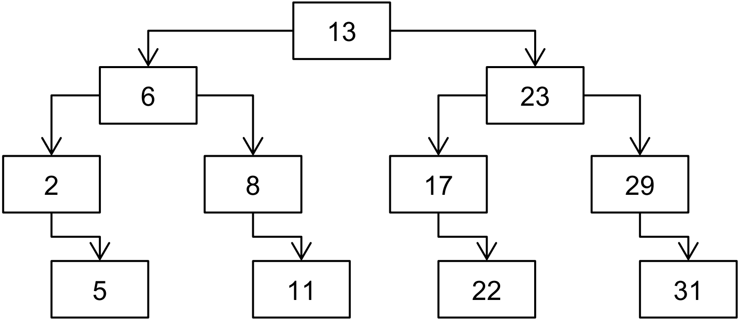
Printing Trees

- We need to be able to print our Set
- How would we print a tree?



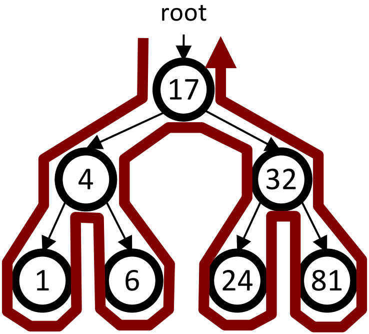
Printing Trees

- How would we print a tree?
  - Need to recurse both left and right
  - **Traverse the tree!**
    - Most tree problems involve traversing the tree



Traversal trick

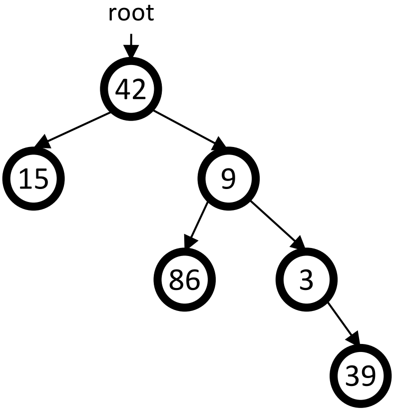
- To quickly generate a traversal:
  - Trace a path counterclockwise.
  - As you pass a node on the proper side, process it.
    - pre-order: left side
    - in-order: bottom
    - post-order: right side
- What kind of traversal does a for-each loop in a Set do?



•pre-order:	17	4	1	6	32	24	81
•in-order:	1	4	6	17	24	32	81
•post-order:	1	6	4	24	81	32	17

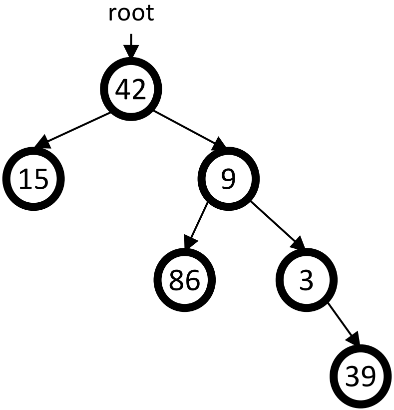
Traversal exercise

- Give pre-, in-, and post-order traversals for the following tree:



Traversal exercise

- Give pre-, in-, and post-order traversals for the following tree:

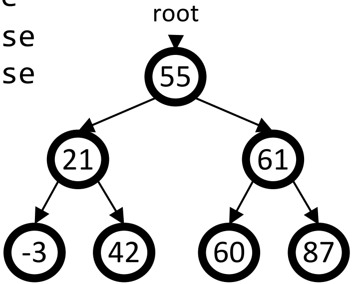


  - pre: 42 15 9 86 3 39
  - in: 15 42 86 9 3 39
  - post: 15 86 39 3 9 42

Exercise: contains

Write a function **`contains`** that accepts a tree node pointer as its parameter and searches the tree for a given integer, returning `true` if found and `false` if not.

```
contains(root, 87) → true
contains(root, 60) → true
contains(root, 63) → false
contains(root, 44) → false
```

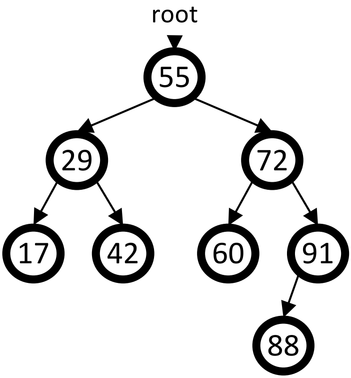


contains solution

```
// Returns whether this BST contains the given
integer.
// Assumes that the given tree is in valid BST
order.
int contains(TreeNode* node, int value) {
    if (node == NULL) {
        return false;   // base case: not found here
    } else if (node->data == value) {
        return true;    // base case: found here
    } else if (node->data > value) {
        return contains(node->left, value);
    } else {            // root->data < value
        return contains(node->right, value);
    }
}
```

getMin/getMax

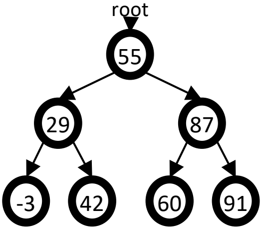
Sorted arrays can find the smallest or largest element in O(1) time (how?)
How could we get the same values in a binary search tree?



getMin/Max solution

```
// Returns the minimum/maximum value from this BST.
// Assumes that the tree is a nonempty valid BST.
int getMin(TreeNode* root) {
    if (root->left == NULL) {
        return root->data;
    } else {
        return getMin(root->left);
    }
}

int getMax(TreeNode* root) {
    if (root->right == NULL) {
        return root->data;
    } else {
        return getMax(root->right);
    }
}
```



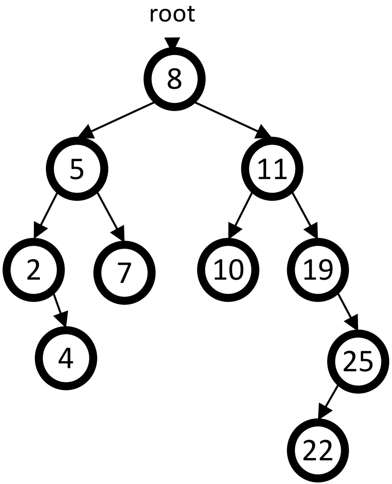
Adding to a BST

Suppose we want to add new values to the BST below.

Where should the value 14 be added?

Where should 3 be added? 7?

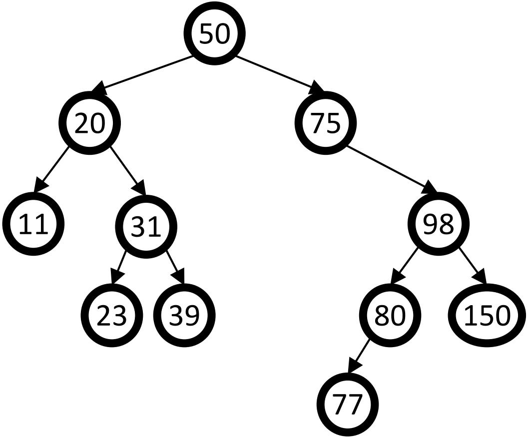
If the tree is empty, where should a new value be added?



Adding exercise

Draw what a binary search tree would look like if the following values were added to an initially empty tree in this order:

50
20
75
98
80
31
150
39
23
11
77

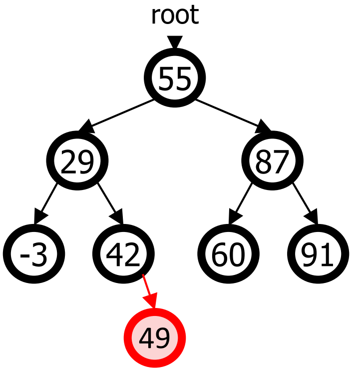


Exercise: add

Write a function **add** that adds a given integer value to the BST.

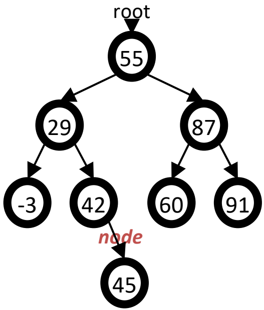
Add the new value in the proper place to maintain BST ordering.

```
add(root, 49);
```



Add Solution

```
void add(TreeNode* node, int value) {
    if (node == NULL) {
        node = New_TreeNode(value);
    } else if (node->data > value) {
        add(node->left, value);
    } else if (node->data < value) {
        add(node->right, value);
    }
}
```

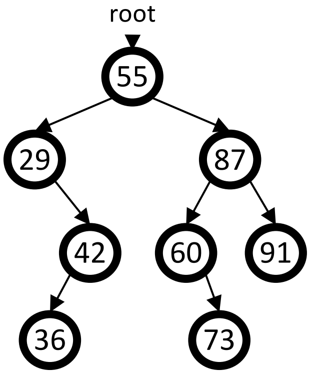


Free Tree

To avoid leaking memory when discarding a tree, we must free the memory for every node.

- Like most tree problems, often written *recursively*
- must free the node itself, and its left/right subtrees

- this is another *traversal* of the tree
- should it be pre-, in-, or post-order?



Free tree solution

```
void freeTree(TreeNode* node) {
    if (node == NULL) {
        return;
    }
    freeTree(node->left);
    freeTree(node->right);
    free(node);
}
```

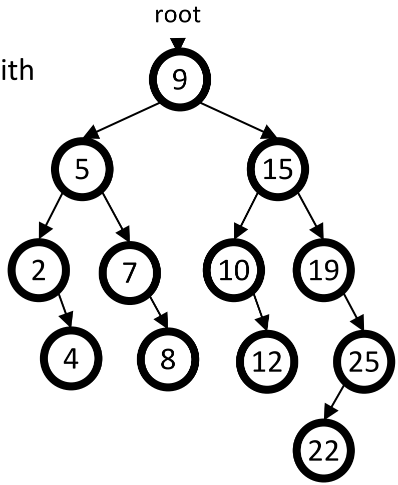
Removing from a BST

Suppose we want to **remove** values from the BST below.

Removing a leaf like 4 or 22 is easy.

What about removing 2? 19?

How can you remove a node with two large subtrees under it, such as 15 or 9?



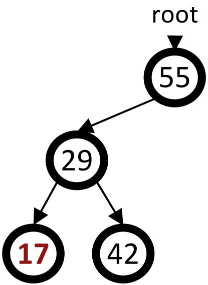
Cases for removal

1. a **leaf**:
2. a node with a **left child only**:
3. a node with a **right child only**:

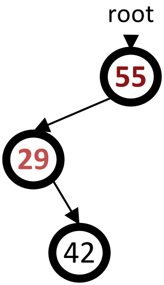
Replace with `null`

Replace with left child

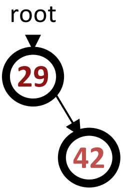
Replace with right child

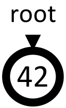


`remove(root, 17);`



`remove(root, 55);`





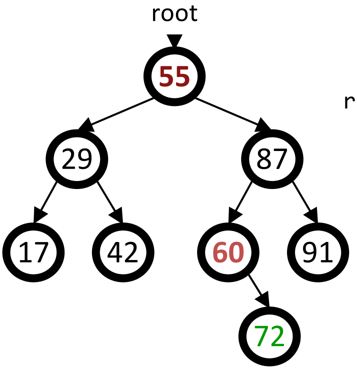
`remove(root, 29);`

Cases for removal

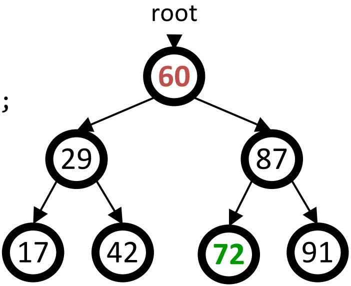
4. a node with **both** children:

   replace with **min from right**

   (replacing with **max from left** would also work)



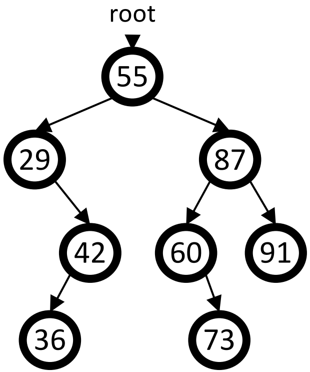
`remove(root, 55);`



Exercise: remove

Add a function **`remove`** that accepts a root pointer and removes a given integer value from the tree, if present. Remove the value in such a way as to maintain BST ordering.

```
remove(root, 73);
remove(root, 29);
remove(root, 87);
remove(root, 55);
```



remove solution

```
// Removes the given value from this BST, if it
exists.
// Assumes that the given tree is in valid BST
order.
void remove(TreeNode* node, int value) {
    if (node == NULL) {
        return;
    } else if (value < node->data) {
        remove(node->left, value);    // too small;
go left
    } else if (value > node->data) {
        remove(node->right, value);   // too big;
go right
    } else {
        // value == node->data; remove this node!
        // (continued on next slide)
        ...
```

remove solution

```
        // value == node->data; remove this node!
        if (node->right == NULL) {
            // case 1 or 2: no R child; replace w/
left
            TreeNode* trash = node;
            node = node->left;
            free(trash);
        } else if (node->left == NULL) {
            // case 3: no L child; replace w/ right
            TreeNode* trash = node;
            node = node->right;
            free(trash);
        } else {
            // case 4: L+R both; replace w/ min
from right
            int min = getMin(node->right);
            remove(node->right, min);
            node->data = min;
        }
    }
}
```

Order or Procedures

We saw how to add to a binary search tree. Does it matter what order we add in?

- Try adding: 50, 20, 75, 98, 80, 31, 150
- Now add the same numbers but in sorted order: 20, 31, 50, 75, 80, 98, 150