CS302
Assignment 3

1. (15 points) Give the runtime for each of the following

   (a)
```
for (int i = 0; i < n; i++)
    for (int j = 0; j < n * n; j++)
      std::cout << "Insert generic message here\n";
```

   (b)
```
for (int i = 1; i < n; i = 2 * i)
    for (int j = n; j > 0; j = j / 3)
      std::cout << "Is this the krusty krab?\n";
```

   (c)
```
for (int i = 0; i < sqrt(n); i++)
    for (j = 0; j < i; j++)
      std::cout << "NO THIS IS PATRICK!!!\n";
```

2. Prove or disprove each of the following

(a) (5 points) $2^{\sqrt{n}} = O(2^n)$

(b) (5 points) $\Theta(\log^k n) = O(n)$ where k is any constant larger than 1, hint: what is the derivative of $\ln^k n$?

(c) (5 points) If $f(n) = O(g(n))$ and $g(n) = O(h(n))$ then $f(n) = O(h(n))$

3. (10 points) If we are given an unsorted list of size n, and we wish to perform a series of searches. Suppose algorithm $\mathcal{A}$ is to sort the array first using merge sort and then performing several binary searches, and suppose algorithm $\mathcal{B}$ is to use linear search several times, at least how many searches would we need to have in order for algorithm $\mathcal{A}$ to have a better runtime over algorithm $\mathcal{B}$? Please show your explanation/work.

4. (15 points) Rearrange the following functions from smallest asymptotic growth rate to the largest asymptotic growth, if any functions have the same asymptotic growth rate, please indicate that (use $<$ to denote the function on the right has a larger growth rate and use $=$ to denote same growth rate, for example $n < n^2$ and $3n = 4n$)

1.00001^n $\quad n^{0.00001}$ $\quad \log_5 2^n$ $\quad \log_2 n!$ $\quad n^2 + 300n - n \ln n$ $\quad \log_2 n \log_2 2^n$ $\quad 64!$ $\quad n^{100}$ $\quad \pi^n$ $\quad e^\pi$

5. (15 points) Perform insertion sort on the following list, show the steps of the inner loop and show when an iteration of the outer loop finishes.

$$5, 7, 1, 3, 10, 2, 0$$

6. (15 points) Perform mergesort on the following list, draw the recursive tree to shows how the problem is broken up and how the merge is done.

$$10, 11, 5, 16, 33, 21, 12, 9, 5, 40, 18, 22, 13, 4, 13, 1$$

7. (15 points) Show the execution of quicksort on the following link, please choose the leftmost element in each list as the pivot.

$$5, 1, 8, 2, 11, 7, 4, 12, 9$$