Project 1 – SI1336 Simulation and modeling

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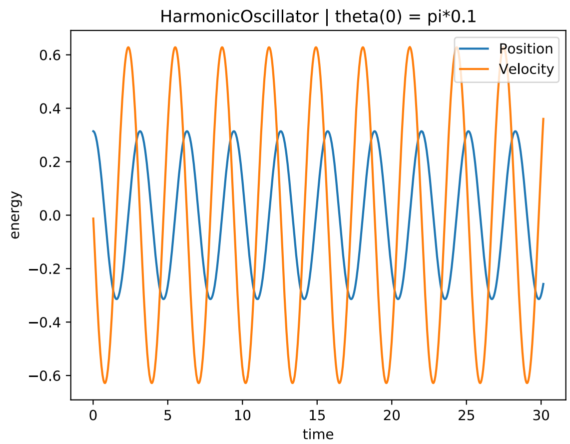
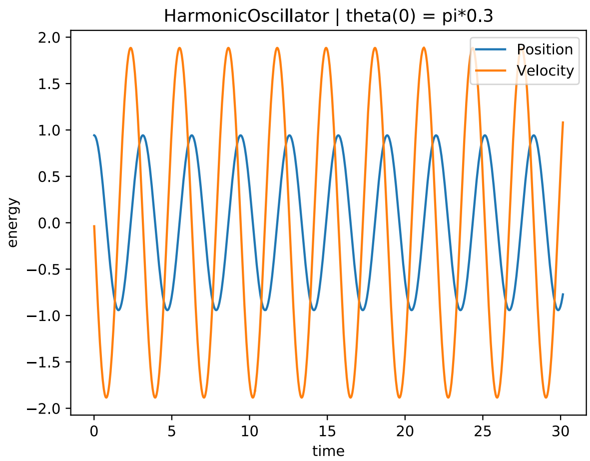
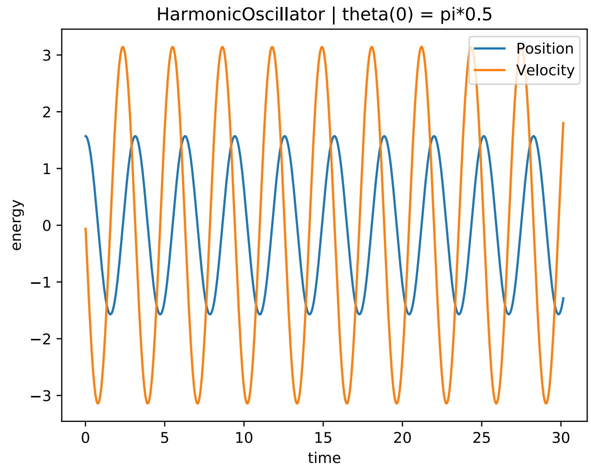
1.1. Consider initial conditions $\frac{\theta(0)}{\pi} = 0.1, 0.3, 0.5$. Study the dependence of the time step. Compare the different methods with each other. Although RK4 has a higher order accuracy than Verlet, the former is not good for many simulations. Why?

Harmonic Oscillator comparison

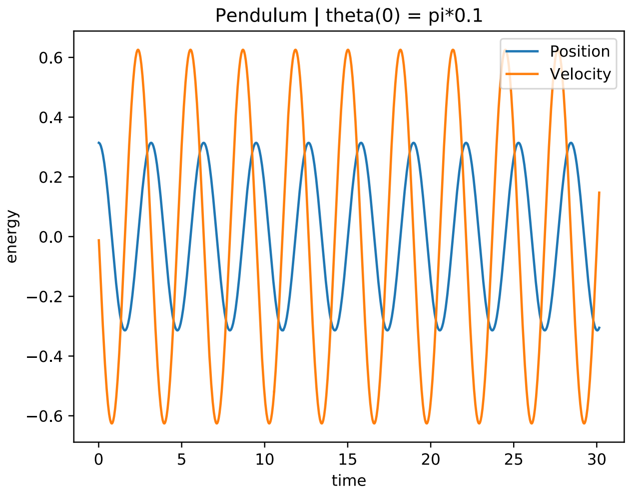


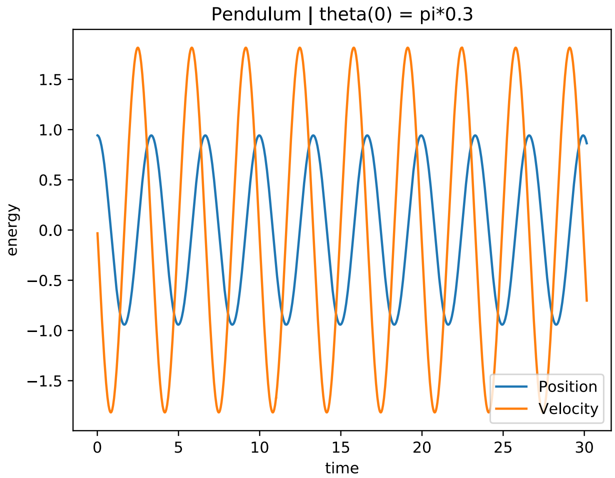


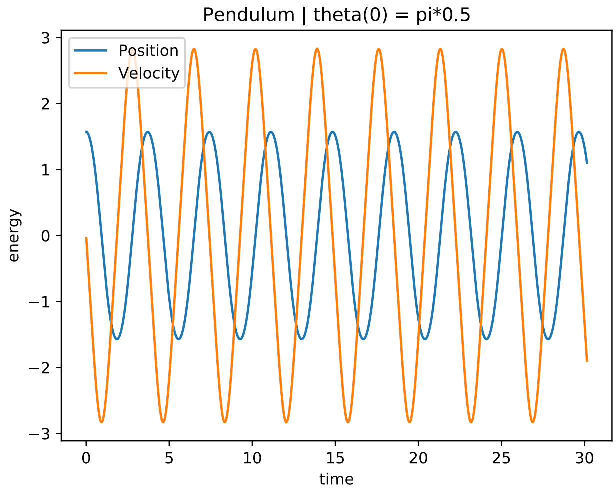


As the initial angle increases, the amplitude of both the position and velocity curve increases.

Pendulum comparison

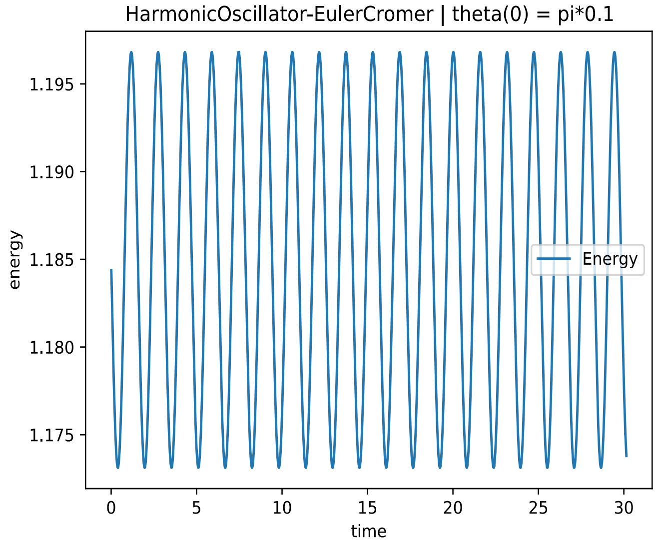


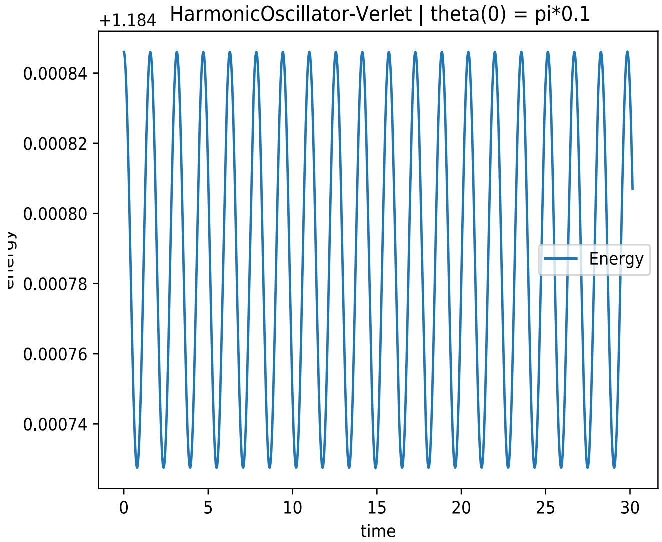


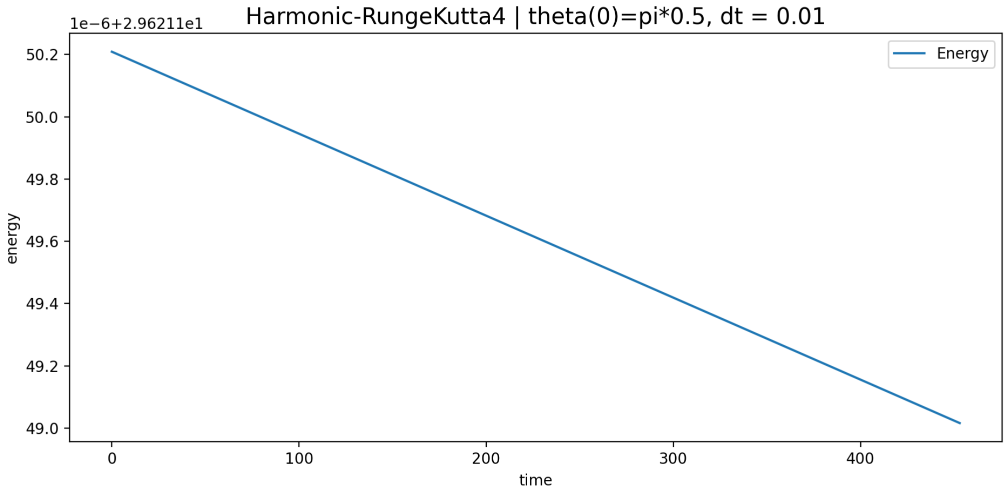


As the initial angle increases, the amplitude of both the position and velocity curve increases as well as the period increases.

Method comparison







Both the Euler-Cromer and Velocity Verlet method are sinusoidal and thereby energy-preserving. RK4 is a non-symplectic method, which means that it is a non-energy-preserving method. This is why the energy curve is decreasing over time and why RK4 is an inferior integrator compared to Euler-Cromer and Velocity Verlet.

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1.2. Which system (HO or pendulum) has a larger period?

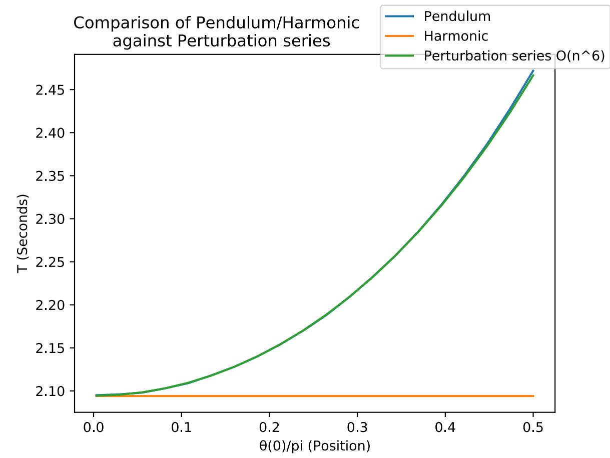


Figure 1

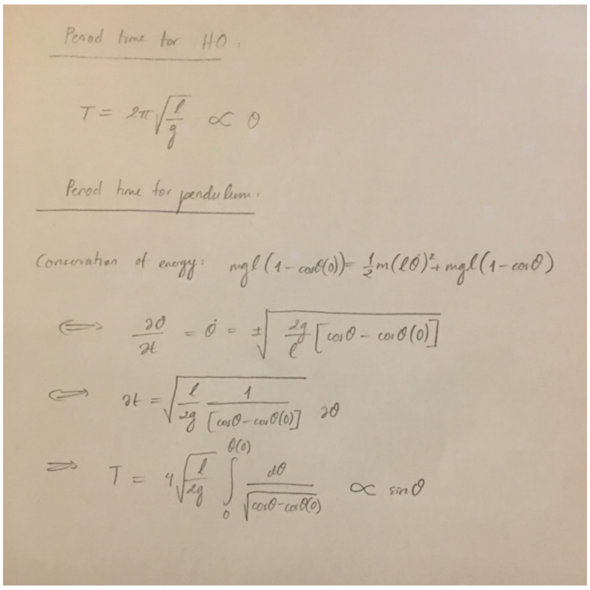


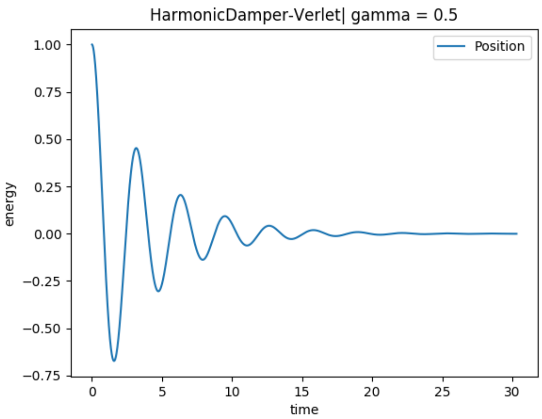
Figure 2

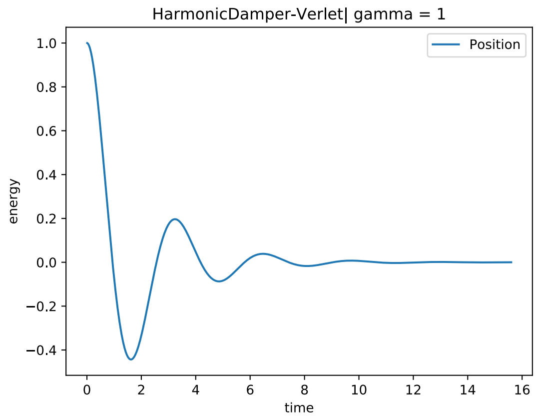
As observed from figure 1, the harmonic oscillator has a constant period whereas the pendulum's period increases as the initial boundary increases. The method to calculate these curves was to define the acceleration and the energy for both oscillators. Additionally, a piece of code from the internet, calculating the period as a function of acceleration, energy and initial boundary θ, was then used (file 1.2.py).

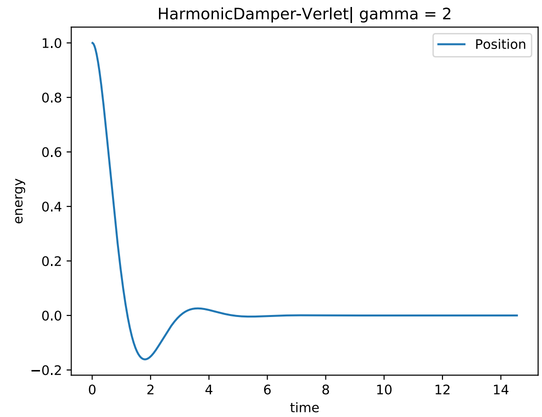
The harmonic oscillator's acceleration is proportional to θ, while the pendulum's acceleration is proportional to $sin(\theta)$. Consequently, it is reasonable that the period time for the pendulum is greater because $\theta \geq sin(\theta)$ for large values of θ. This results in a smaller force acting on the pendulum, which according to Newton's second law makes it accelerate at a slower pace.

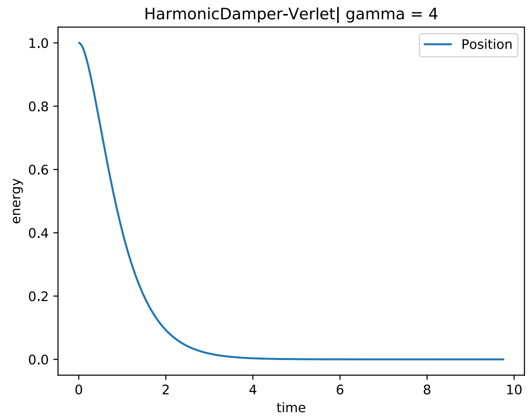
From figure 2, which are the calculations for the period time, the latter T is what the perturbation series converges to with infinite terms. That is also why the two curves in figure 1 collapse into each other.

1.3. Estimate the relaxation time τ. Critical damping γ_c? Relation between τ and γ?









The plots above illustrates the harmonic damper for $\gamma = 0.5, 1, 2$ and 4

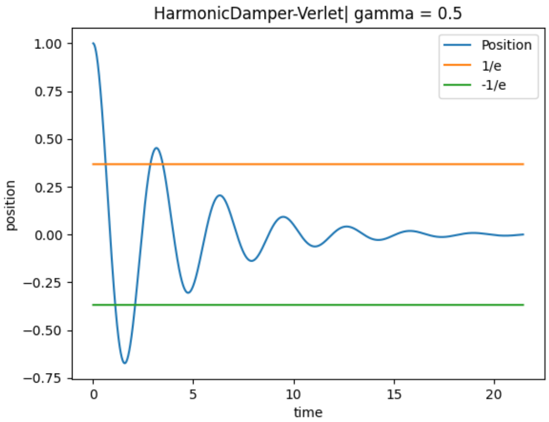


Figure 1

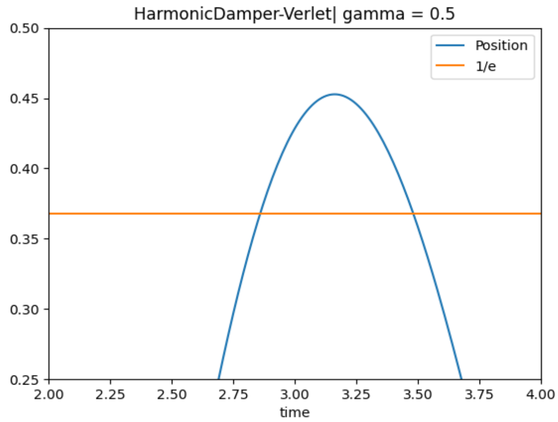


Figure 2

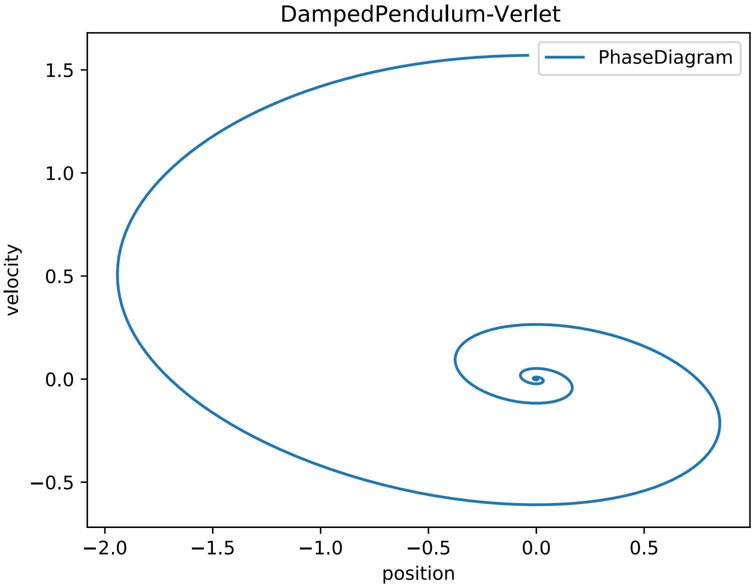
In order to determine the relaxation τ time for $\gamma = 0.5$, the position dependence on time was plotted. The relaxation time, which is the time when the absolute value of the amplitude has reduced to $1/e \approx 0.37$ of its value, was then estimated by simply looking at the plot (figure 2) and trying to approximate the crossover point between the curves. For $\gamma = 0.5 \rightarrow \ \tau \approx 3.5$ (I approximated it to roughly three last time).

As for the critical damping γ_c, the harmonic damper was plotted with different values of γ until the curve did not pass $x = 0$ anymore. It could be concluded that the position curve was sufficiently damped and fulfilled the requirement for $\gamma = \gamma_c \approx 4$.

The relation between the relaxation time τ and gamma γ is that when γ increases, the relaxation time τ decreases.

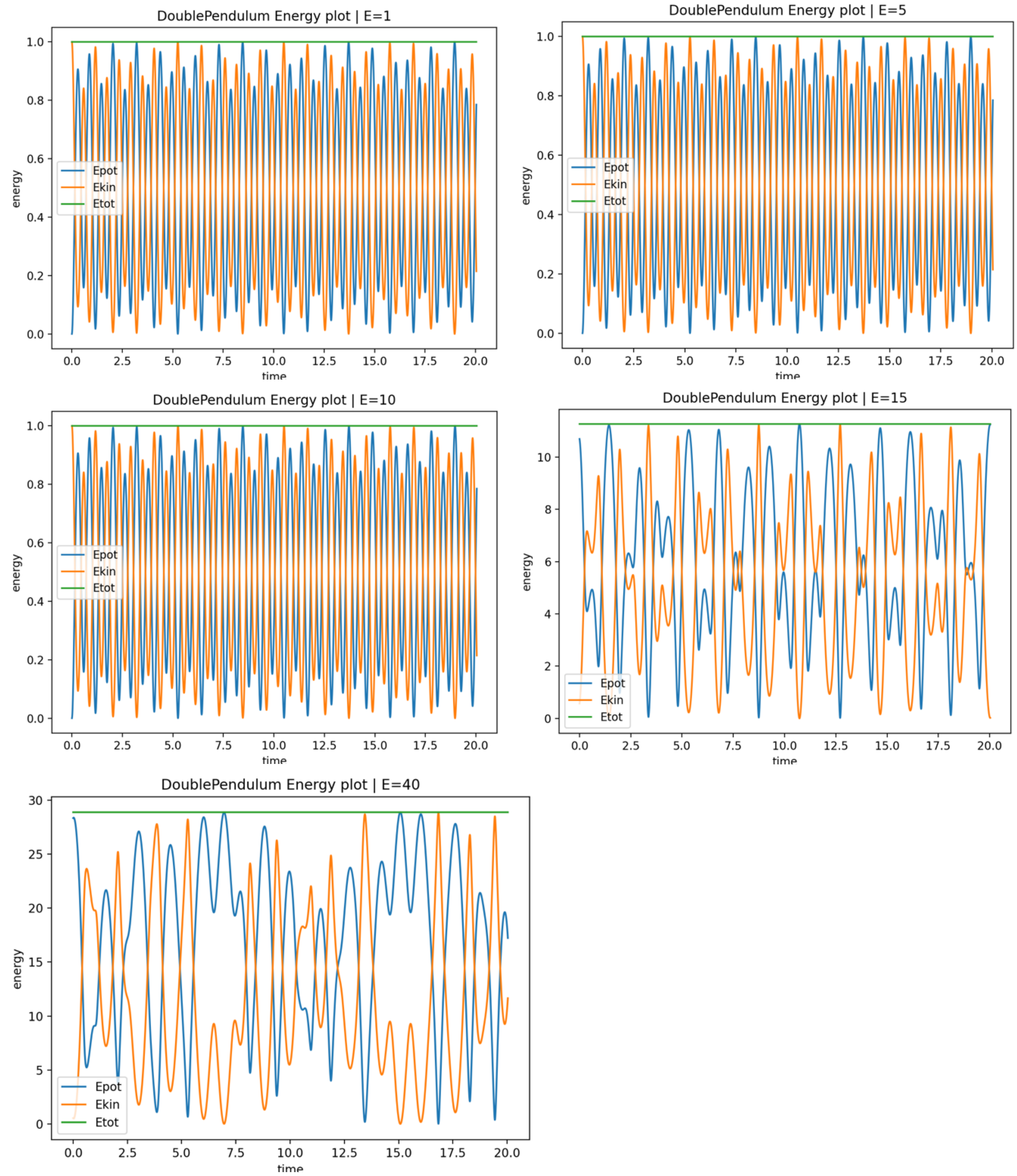
Sidenote: Apologies from my side that this did not get explained in the edited version 1.0. I corrected the report the first time under stress and thought you referred to task 1.2 for some reason.

1.4. Discuss the phase space portrait.



The damping part results in a decrease in value in both velocity and position. The spiral is a result of the pendulum oscillating back and forth from equilibrium.

1.5a. Visually determine whether the steady state behavior is regular or appears to be chaotic. Are there some values of E for which all the trajectories appear regular? Are there values of E for which all trajectories appear chaotic? Are there values of E for which both types of trajectories occur?



With randomized initial conditions q1 and q2 we get the energy plots above. As observed from the plots, the trajectories tend to be regular for E=1, 5, 10 while they become more chaotic as E increases.

1.5b. Plot the phase space diagrams with p1 vs. q1 and p2 vs. q2. Are these plots more useful to determine the nature of the trajectories?

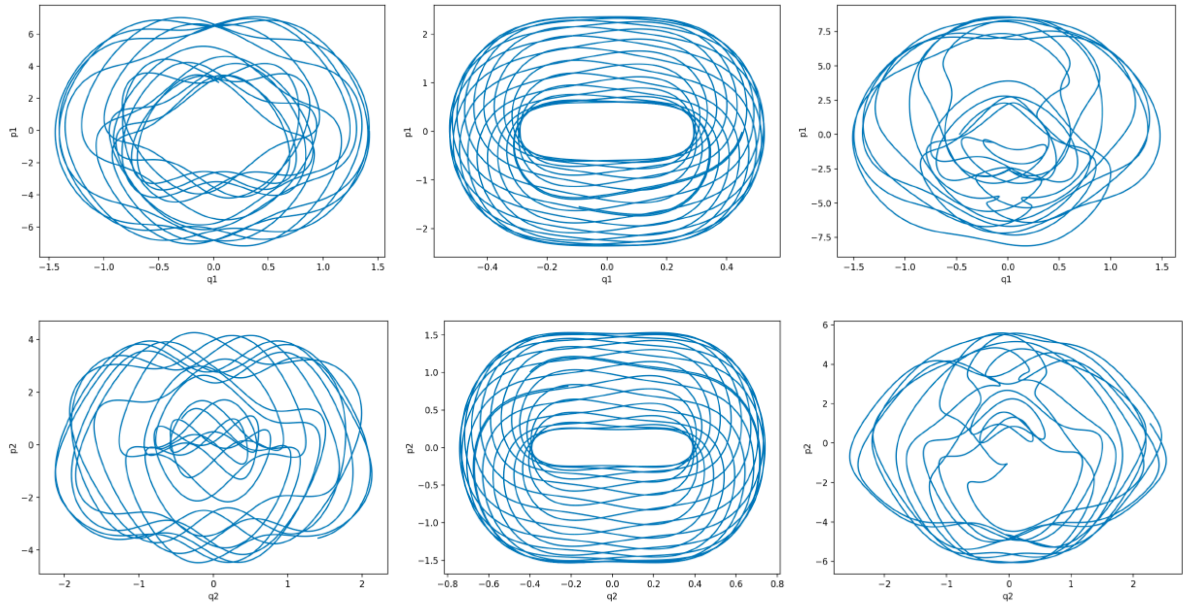


Figure 2: The six above plots display the phase space diagrams for E=1, 5, 10 respectively

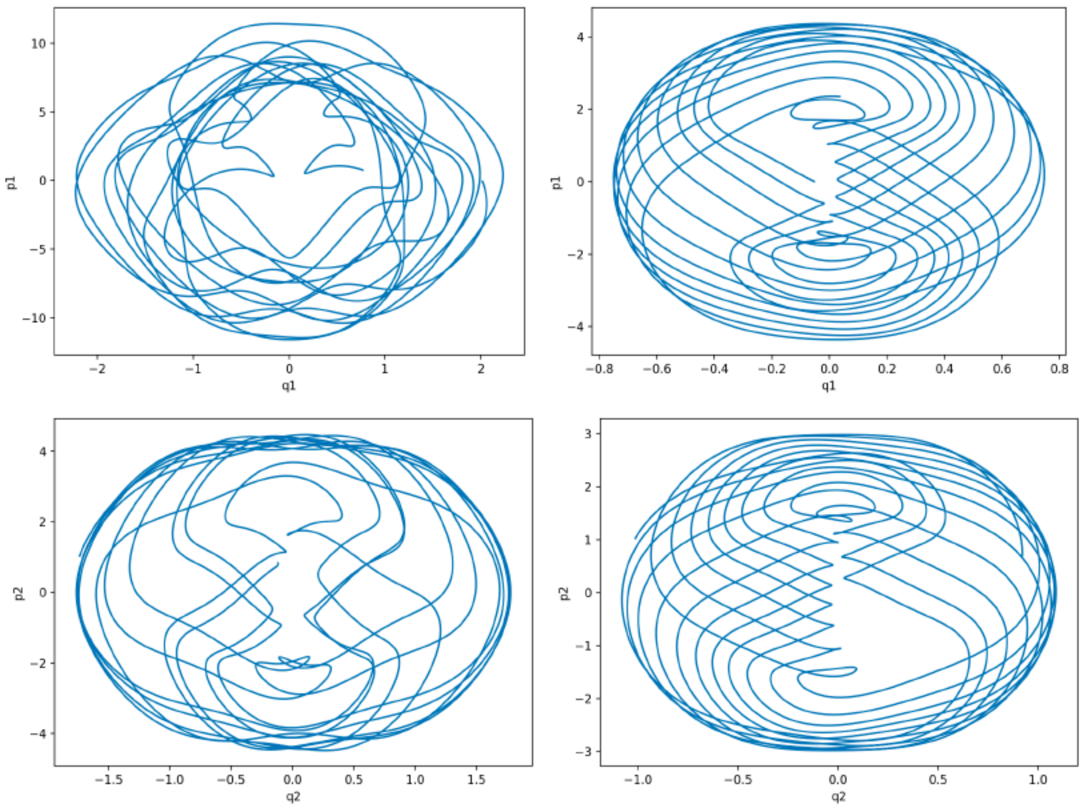
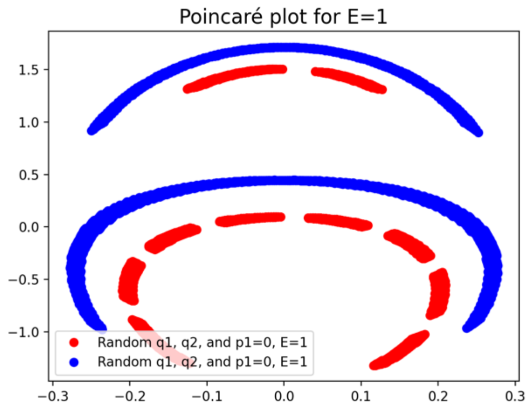
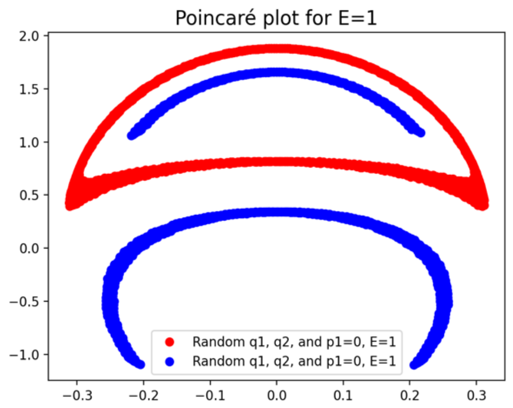


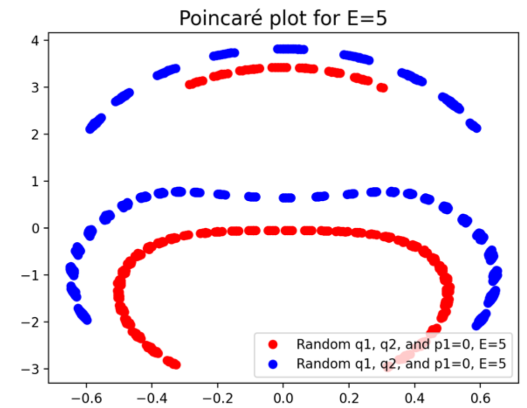
Figure 3: The four above plots display the phase space diagrams for E=15, 40 respectively

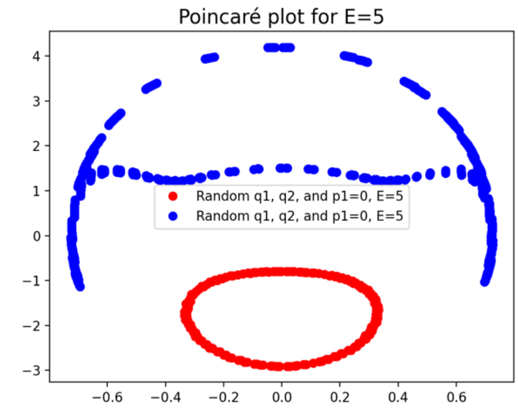
The initial conditions are as before randomized. With the help of phase space diagrams, it is easier to spot patterns and symmetries and thereby determine the nature of the trajectories. Based on these plots, it seems like the trajectories are regular for E=1, 5, 10, while they become more chaotic for E=15, 40 as one requirement for a dynamic system to be chaotic is to have dense orbits.

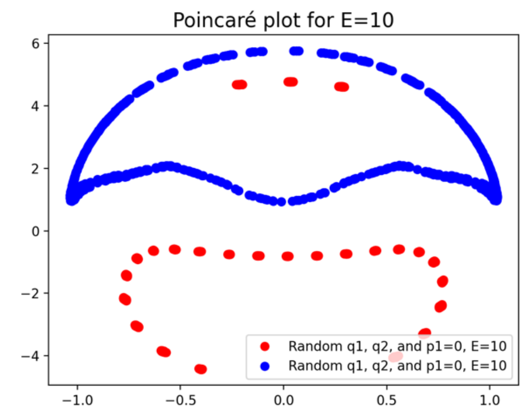
1.5c. Describe the different type of behavior for the Poincaré maps.

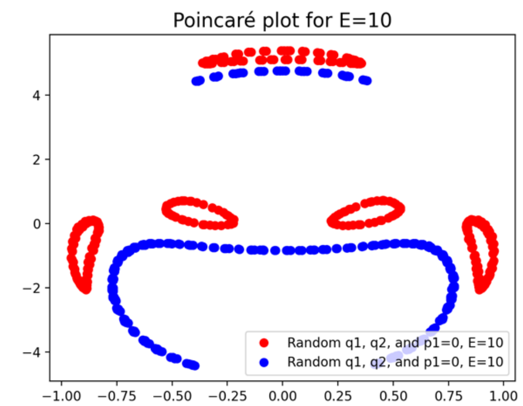


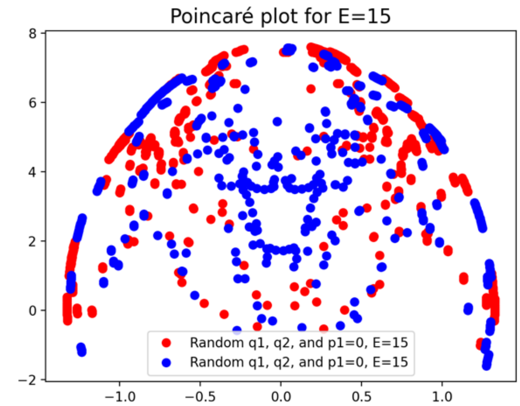


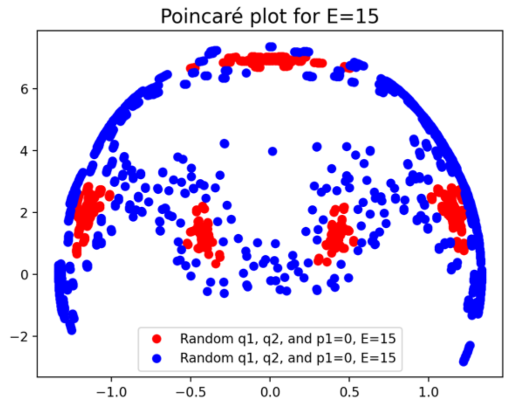


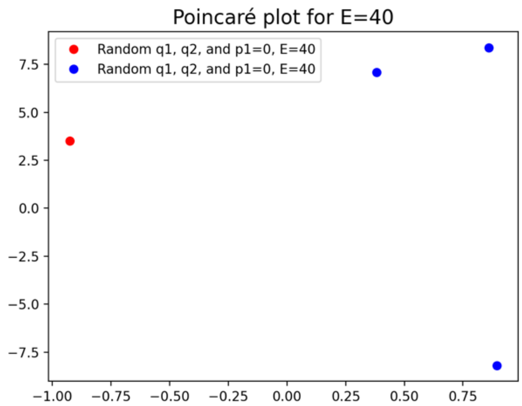


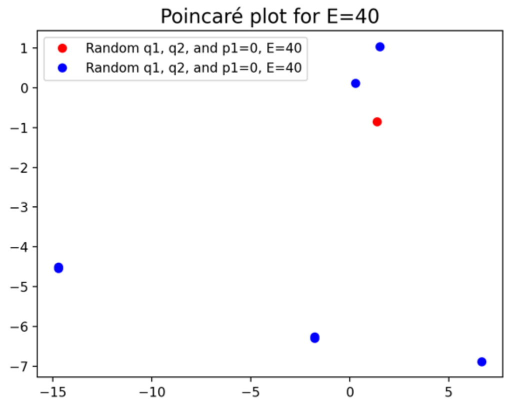












The plots above illustrates the Poincaré map for a double pendulum with p1 plotted versus q1 for p1=0 and randomized values on q1, q2 and p2 for different values on E. Unfortunately, as I had trouble implementing the code for many different initial value I could only overlap two Poincaré maps per plot, each with different initial values (represented by blue and red dots).

The conclusions that can be drawn from the Poincaré maps is that the trajectories are more regular for small values on the energy, whereas they become more chaotic as the energy increases. This can be displayed as the dots getting more and more scattered as the energy increases. This align with the results derived in earlier tasks.