

# SML Homework

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CS 361

1. What are the types of the following expressions?

- `[(1,5), (2,3), (5,6)];`
  - `(int * int) list`
- `fun f(x:real) = true;`
  - `real -> bool`
- `map f;`
  - `'a list -> 'b list`

2. Provide expressions of the following types:

- `int * bool`
  - `(4, true)`
- `int list * bool`
  - `([5,8], true)`
- `int * real -> bool list`
  - `(1,3.4)-> [true, true]`

3. Write the following SML functions:

Write a recursive function that computes  $2^n$  for  $n \geq 0$ .

```
f(n) = 2^n
int -> int
fun exp(n) = if n=0 then
  1
  else exp(n-1) * 2;
```

---

```
fun fact n = if n=0 then 1
  else n * fact(n-1);
```

```
fun new_if (a,b,c) = if a then b else c;
```

Using `new_if`, write a function `new_fact` that is supposed to compute *fact*.

Explain why `new_fact` does not compute the factorial.

Note: How are recursive functions evaluated in SML?

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```
fun new_fact(a,b,c) = if a then b else c * fact(a, b,
c-1);

new_fact(1,1,5)
```

---

This `new_fact` doesn't run because our base case for factorial requires us to check if the number is equal to zero. In our ideal function, we are checking the value of `a` and then doing `b`, but this doesn't work if `a` itself relies on the value of `c`. We can't pass/check the value of `c` through `a`, so our base case in `new_fact` would never properly work. Furthermore, this would also rely on `b` always being equal to 1. If

that's the case, we should consider getting rid of the variable `b` and just requiring `1`, it makes no sense to have the base case in the function parameters like that.

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Define a function `circumference` that computes the circumference of a circle with respect to its radius. Use `pi` from the `Math` library.

---

```
fun circumference(r:real) =  
    2.0*Math.pi*r;  
  
circumference(14.0)
```

---

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How to use `map` to add 3 to each elements of a list

---

```
val L = [1, 1, 2, 3];  
fun addThree(x) = x + 3;  
map addThree L;
```

Write a function *move* that transforms a list  $[a_1, \dots, a_n]$  into a list  $[a_2, \dots, a_n, a_1]$ .

```
fun addOne(L) =  
    if L = [] then []  
    else  
        tl(L) @ [hd(L)];
```

4. Implement the datatype BinaryTree and all the functions that are provided in the lecture notes: lookup, inorder, preorder, postorder, left\_subtree, right\_subtree and label. Provide screenshots to show that your code is correct. Provide 2 tests for each function.

```

1 datatype 'a BinaryTree = bempty | bt of 'a * 'a BinaryTree * 'a BinaryTree ;
2
3 fun lookup(bempty, _) = false | lookup(bt(root:int, left, right), x:int) =
4   if (x = root) then true
5   else (
6     if (root > x) then lookup(left, x)
7     else lookup(right, x)
8   );
9
10 val test = bt(0, bempty, bt(1, bempty, bt(2, bt(3, bt(4, bempty, bempty)), bempty), bt(5, bempty, bempty)));
11
12 lookup(test, 1);
13 lookup(test, 9);

```

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```

Standard ML of New Jersey v110.78 [built: Thu Aug 31 03:45:42 2017]
- datatype 'a BinaryTree = bt of 'a * 'a BinaryTree * 'a BinaryTree | bempty
val lookup = fn : int BinaryTree * int -> bool
val test = bt (0,bempty,bt (1,bempty,bt #)) : int BinaryTree
val it = true : bool
val it = false : bool
-

```

```

1 datatype 'a BinaryTree = bempty | bt of 'a * 'a BinaryTree * 'a BinaryTree ;
2
3 fun inorder (bempty) = [] | inorder(bt(root:'a, left, right)) =
4   inorder(left) @ (root :: inorder(right));
5
6
7 val test = bt(0, bempty, bt(1, bempty, bt(2, bt(3, bt(4, bempty, bempty)), bempty), bt(5, bempty, bempty)));
8 val test2 = bt(4, bt(3, bempty, bt(2, bempty, bempty)), bt(1, bempty, bempty));
9
10 inorder(test);
11 inorder(test2);

```

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```

Standard ML of New Jersey v110.78 [built: Thu Aug 31 03:45:42 2017]
- datatype 'a BinaryTree = bt of 'a * 'a BinaryTree * 'a BinaryTree | bempty
val inorder = fn : 'a BinaryTree -> 'a list
val test = bt (0,bempty,bt (1,bempty,bt #)) : int BinaryTree
val test2 = bt (4,bt (3,bempty,bt #),bt (1,bempty,bempty)) : int BinaryTree
val it = [0,1,4,3,2,5] : int list
val it = [3,2,4,1] : int list
-

```

```

1 datatype 'a BinaryTree = bempty | bt of 'a * 'a BinaryTree * 'a BinaryTree ;
2
3 fun preorder (bempty) = [] | preorder(bt(root:'a, left, right)) =
4   root :: (preorder(left) @ preorder(right));
5
6
7 val test = bt(0, bempty, bt(1, bempty, bt(2, bt(3, bt(4, bempty, bempty), bempty), bt
8   (5, bempty, bempty))));
9 val test2 = bt(4, bt(3, bempty, bt(2, bempty, bempty)), bt(1, bempty, bempty));
10
11 preorder(test);
12 preorder(test2);

```

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Standard ML of New Jersey v110.78 [built: Thu Aug 31 03:45:42 2017]

```

- datatype 'a BinaryTree = bt of 'a * 'a BinaryTree * 'a BinaryTree | bempty
val preorder = fn : 'a BinaryTree -> 'a list
val test = bt (0,bempty,bt (1,bempty,bt #)) : int BinaryTree
val test2 = bt (4,bt (3,bempty,bt #),bt (1,bempty,bempty)) : int BinaryTree
val it = [0,1,2,3,4,5] : int list
val it = [4,3,2,1] : int list
-

```

```

1 datatype 'a BinaryTree = bempty | bt of 'a * 'a BinaryTree * 'a BinaryTree ;
2
3 fun postorder (bempty) = [] | postorder(bt(root:'a, left, right)) =
4   (postorder(left) @ postorder(right)) @ (root :: []);
5
6
7 val test = bt(0, bempty, bt(1, bempty, bt(2, bt(3, bt(4, bempty, bempty), bempty), bt
8   (5, bempty, bempty))));
9 val test2 = bt(4, bt(3, bempty, bt(2, bempty, bempty)), bt(1, bempty, bempty));
10
11 postorder(test);
12 postorder(test2);

```

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```

- datatype 'a BinaryTree = bt of 'a * 'a BinaryTree * 'a BinaryTree | bempty
val postorder = fn : 'a BinaryTree -> 'a list
val test = bt (0,bempty,bt (1,bempty,bt #)) : int BinaryTree
val test2 = bt (4,bt (3,bempty,bt #),bt (1,bempty,bempty)) : int BinaryTree
val it = [4,3,5,2,1,0] : int list
val it = [2,3,1,4] : int list
-

```

```

1 datatype 'a BinaryTree = bempty | bt of 'a * 'a BinaryTree * 'a BinaryTree ;
2
3 exception label_has_nil_argument;
4
5 fun left_subtree bempty = bempty | left_subtree(bt(_, left, _)) = left;
6
7
8 val test = bt(0, bempty, bt(1, bempty, bt(2, bt(3, bt(4, bempty, bempty), bempty), bt
  (5, bempty, bempty))));
9 val test2 = bt(4, bt(3, bempty, bt(2, bempty, bempty)), bt(1, bempty, bempty));
10
11 left_subtree(test);
12 left_subtree(test2);

```

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```

Standard ML of New Jersey v110.78 [built: Thu Aug 31 03:45:42 2017]
- datatype 'a BinaryTree = bt of 'a * 'a BinaryTree * 'a BinaryTree | bempty
exception label_has_nil_argument
val left_subtree = fn : 'a BinaryTree -> 'a BinaryTree
val test = bt (0,bempty,bt (1,bempty,bt #)) : int BinaryTree
val test2 = bt (4,bt (3,bempty,bt #),bt (1,bempty,bempty)) : int BinaryTree
val it = bempty : int BinaryTree
val it = bt (3,bempty,bt (2,bempty,bempty)) : int BinaryTree
-

```

```

datatype 'a BinaryTree = bempty | bt of 'a * 'a BinaryTree * 'a BinaryTree ;
exception label_has_nil_argument;

fun right_subtree bempty = bempty | right_subtree(bt(_, _, right)) = right;

val test = bt(0, bempty, bt(1, bempty, bt(2, bt(3, bt(4, bempty, bempty), bempty), bt
  (5, bempty, bempty))));
val test2 = bt(4, bt(3, bempty, bt(2, bempty, bempty)), bt(1, bempty, bempty));

right_subtree(test);
right_subtree(test2);

```

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```

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- datatype 'a BinaryTree = bt of 'a * 'a BinaryTree * 'a BinaryTree | bempty
exception label_has_nil_argument
val right_subtree = fn : 'a BinaryTree -> 'a BinaryTree
val test = bt (0,bempty,bt (1,bempty,bt #)) : int BinaryTree
val test2 = bt (4,bt (3,bempty,bt #),bt (1,bempty,bempty)) : int BinaryTree
val it = bt (1,bempty,bt (2,bt #,bt #)) : int BinaryTree
val it = bt (1,bempty,bempty) : int BinaryTree
-

```



```

1 datatype 'a BinaryTree = bempty | bt of 'a * 'a BinaryTree * 'a BinaryTree ;
2
3 exception label_has_nil_argument;
4
5 fun label bempty = raise label_has_nil_argument | label(bt(value, _, _)) = value;
6
7
8 val test = bt(0, bempty, bt(1, bempty, bt(2, bt(3, bt(4, bempty, bempty), bempty
9   ), bt(5, bempty, bempty))));
10 val test2 = bt(4, bt(3, bempty, bt(2, bempty, bempty)), bt(1, bempty, bempty));
11 label(test);
12 label(test2);

```

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```

Standard ML of New Jersey v110.78 [built: Thu Aug 31 03:45:42 2017]
- datatype 'a BinaryTree = bt of 'a * 'a BinaryTree * 'a BinaryTree | bempty
exception label_has_nil_argument
val label = fn : 'a BinaryTree -> 'a
val test = bt (0,bempty,bt (1,bempty,bt #)) : int BinaryTree
val test2 = bt (4,bt (3,bempty,bt #),bt (1,bempty,bempty)) : int BinaryTree
val it = 0 : int
val it = 4 : int
-

```