# Measuring Stellar Abundance of Sodium

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March 26, 2025

## 1 Motivation

The study of stellar atmospheres is key to understanding the elemental composition of stars, which plays a crucial role in shaping the formation and composition of its planetary system. Since planets form from the same protoplanetary disks as their host star, studying stellar abundances provides insight into the elemental makeup of planets. A fundamental technique used to study the stellar atmosphere is known as spectroscopic analysis, which can be used to measure the presence and abundance of different elements using absorption and emission lines from a star's spectrum.

Sodium in particular serves as an important indicator of both stellar and planetary atmospheric properties. Commonly observed in exoplanet atmospheres, sodium's absorption lines can reveal temperature, pressure, and atmospheric escape processes.

This project aims to analyze the solar spectrum in the vicinity of the Na D lines to estimate parameters related to sodium abundance in the Sun's atmosphere. By measuring the equivalent width, computing the excitation and ionization states using the Boltzmann and Saha equations, and determining the relative abundance of sodium to hydrogen, we can infer the total sodium column density. These calculations help to both characterize the Sun and provide a reference point for understanding the abundance of sodium in planetary atmospheres.

The broader implication of this study is to explore the relationship between the atmosphere of exoplanets and their host stars. The results will further theories of planetary formation and aid in assessing the habitability of exoplanets. Although this study focuses on the Sun, the same processes can be applied to other stellar systems to further explore the connection between planets and their stars.

## 2 Methods

To analyze the sodium content in the Sun's atmosphere, we examined the Na D absorption lines in the solar spectrum using observational data. We plotted the region around 5887.5 Åto 5892.5 Å, indicated by two vertical red lines in Figure 1. This region contains the Na D absorption lines.

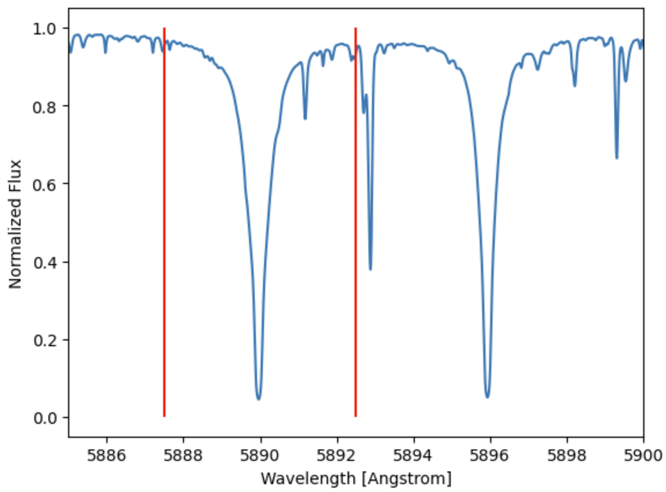


Figure 1: A plot of normalized flux against wavelength in Å, where the red vertical lines indicate the region the Na D absorption line can be found.

### 2.1 Equivalent Width

The equivalent width (EW) quantifies the total absorption strength of the sodium feature, representing the number of atoms that absorb along the line of sight in Å. We compute the equivalent width by summing the difference between the continuum level and the observed flux within the sodium absorption region, then multiplying by the wavelength interval, and finally normalizing.

### 2.2 Excitation State

The ratio of sodium atoms in the excited state (3p) to the ground state (3s) is determined using the Boltzmann equation:

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp(-\frac{E_2 - E_1}{k_B T})$$

where $g_1$ and $g_2$ are the degeneracies of the ground and excited states, $E_1$ and $E_2$ are their respective energies, $T$ is the temperature of the photosphere, and $k_B$ is the Boltzmann constant.

## 2.3 Ionization State

The fraction of ionized sodium (Na II) to neutral sodium (Na I) is computed using the Saha equation:

$$\frac{Na_{II}}{Na_{I}} = \frac{2k_BT}{P_e}\frac{Z_{II}}{Z_I}(\frac{2\pi m_e k_B T}{h^2})^{3/2}\exp(-\frac{\chi}{k_BT})$$

where $P_e$ is the electron pressure, $Z_I$ and $Z_{II}$ are partition functions, $\chi$ is the ionization energy, and $h$ is Planck's constant.

## 2.4 Total Sodium Column Density and Relative Abundance

The column density represents the number of absorbing atoms per unit area along the line of sight, in atoms/cm$^2$. To determine it from spectral absorption features, we use the curve of growth, which describes how the EW of an absorption line changes as a function of the column density. The curve of growth is displayed in Figure 2, and the column density comes from solving for N.

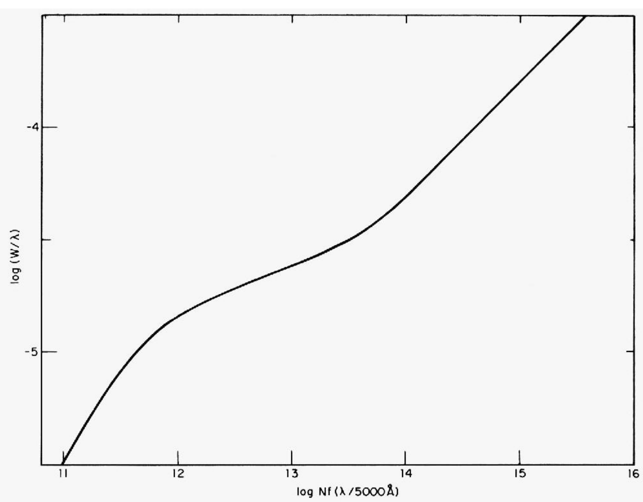


**Figure 9.22** A general curve of growth for the Sun. (Figure from Aller, *Atoms, Stars, and Nebulae*, Revised Edition, Harvard University Press, Cambridge, MA, 1971.)

Figure 2: Curve of growth

The relative abundance of sodium to hydrogen is then computed using the total column density of hydrogen, expressed in both linear and logarithmic form.

## 3 Results

In this study, we analyzed the absorption features of sodium in the solar spectrum to determine its equivalent width, column density, and relative abundance. The key findings from our calculations can be found in the following table.

| **Parameter** | **Value** |
|---|---|
| Ratios | |
| $N_2/N_1$ | 0.044 |
| $Na_{II}/Na_I$ | 2510.75 |
| | |
| Number density (atoms/cm$^2$) | |
| Na in ground state | $2.07 \times 10^{18}$ |
| Neutral Na | $2.16 \times 10^{18}$ |
| Ionized Na | $7.61 \times 10^{16}$ |
| Total Na in all states | $2.16 \times 10^{18}$ |
| | |
| Relative Abundance (atoms/cm$^2$) | |
| $N_{Na}/N_H$ | $3.27 \times 10^{-6}$ |
| $[Na/H]$ | 7.88 |

The ratio of ionized sodium to neutral sodium indicates a high degree of ionization in the sodium population. The total number density of sodium in all states is consistent with the neutral sodium value due to the dominance of neutral sodium. The relative abundance indicates a high abundance of sodium compared to hydrogen.

## 4 Conclusion

The study of sodium in the Sun's atmosphere has provided valuable insights into its composition and the processes governing stellar and planetary formation. By analyzing the sodium D absorption lines, we were able to estimate the equivalent width, column density, and the relative abundance of sodium to hydrogen. Our results show a significant portion of sodium remains in the neutral state, with a smaller fraction in the excited and ionized states. The number densities of neutral sodium and total sodium were found to be $2.16 \times 10^{18}$ atoms/cm$^2$, with a relatively small ionized sodium fraction at $7.61 \times 10^{16}$ atoms/cm$^2$.

These findings contribute to a deeper understanding of the sodium abundance in stellar atmospheres, and they have broader implications for planetary science. Since planets form from the same protoplanetary disks as their host stars, the study of stellar composition can provide valuable insights about the elemental makeup of planets. Future work could extend these methods to other stars and exoplanetary systems to explore the relationship between stellar and planetary compositions. A better understanding of these processes will be crucial for advancing theories of planetary formation and assessing the potential habitability of exoplanets. Ultimately, this research serves a stepping stone toward the use of stellar spectroscopic analysis as a tool for characterizing distant planetary systems.