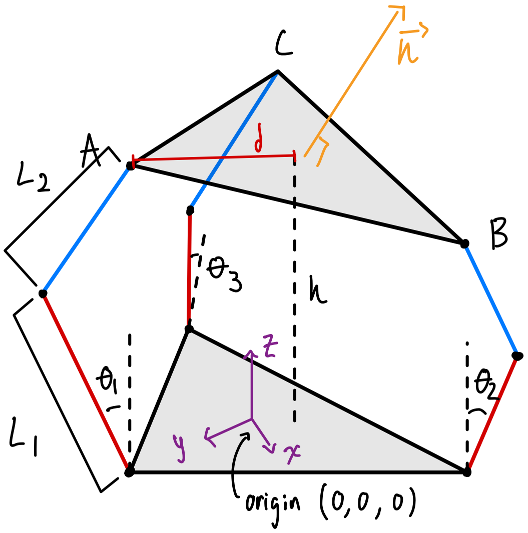
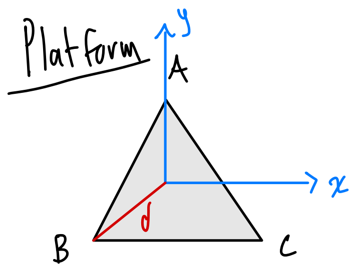


Input : θ , ϕ , h → height of platform

x-axis rotation

y-axis rotation

Output : θ_1 , θ_2 , θ_3



Initial coords of triangle

$$A_i = \begin{bmatrix} 0 \\ d \\ h \end{bmatrix} \quad B_i = \begin{bmatrix} -\frac{d\sqrt{3}}{2} \\ -\frac{d}{2} \\ h \end{bmatrix} \quad C_i = \begin{bmatrix} \frac{d\sqrt{3}}{2} \\ -\frac{d}{2} \\ h \end{bmatrix}$$

Centroid :
$$\begin{bmatrix} \left(0 - \frac{d\sqrt{3}}{2} + \frac{d\sqrt{3}}{2}\right)/3 \\ \left(d - \frac{d}{2} - \frac{d}{2}\right)/3 \\ (h + h + h)/3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ h \end{bmatrix}$$

Rotation Matrix

- Only rotating about x & y

$$R = R_y R_x = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

- only change z-values because each arm will move in a straight line

$$R = [-\sin\phi, \ \cos\phi\sin\theta, \ \cos\phi\cos\theta]$$

Applying R to each position vector

$$R \cdot A_i = A_R = \begin{bmatrix} 0 \\ d \\ \cos\phi \sin\theta \cdot d + \cos\phi\cos\theta \cdot h \end{bmatrix} \qquad R \cdot B_i = B_R = \begin{bmatrix} -\frac{d\sqrt{3}}{2} \\ -\frac{d}{2} \\ \sin\phi \frac{d\sqrt{3}}{2} - \cos\phi \sin\theta \frac{d}{2} + \cos\phi \cos\theta h \end{bmatrix}$$

$$R \cdot C_i = C_R = \begin{bmatrix} \frac{d\sqrt{3}}{2} \\ -\frac{d}{2} \\ -\sin\phi \frac{d\sqrt{3}}{2} - \cos\phi \sin\theta \frac{d}{2} + \cos\phi\cos\theta h \end{bmatrix}$$

Centroid after rotation

$$\begin{bmatrix} \left(0 - \frac{d\sqrt{3}}{2} + \frac{d\sqrt{3}}{2}\right)/3 \\ \left(d - \frac{d}{2} - \frac{d}{2}\right)/3 \\ \left(\cancel{\sin\phi \frac{d\sqrt{3}}{2} - \sin\phi \frac{d\sqrt{3}}{2}} + \cancel{\cos\phi \sin\theta \left(d - \frac{d}{2} - \frac{d}{2}\right)} + \cos\phi\cos\theta \cdot 3h\right)/3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cos\phi\cos\theta h \end{bmatrix}$$

Translation Vector (T)

- So that the platform remains at the same height

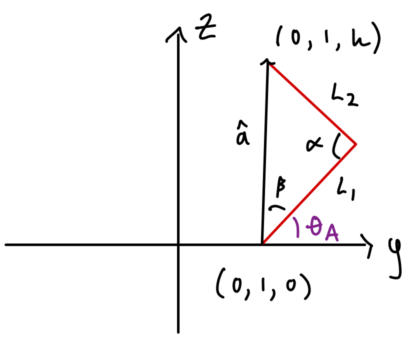
$$T = C_i - C_R = \begin{bmatrix} 0 \\ 0 \\ h - \cos\phi\cos\theta h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ h(1 - \cos\phi\cos\theta) \end{bmatrix}$$

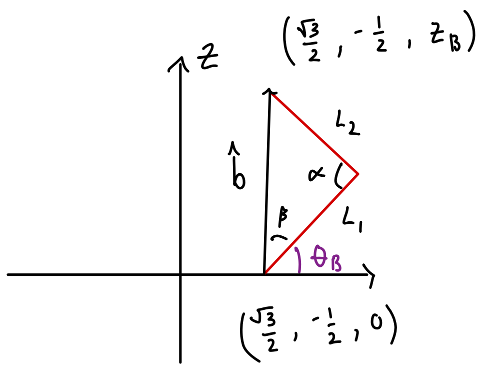
$$T_f A_R = A_f = \begin{bmatrix} 0 \\ d \\ \cos\phi\sin\theta\cdot d + \cos\phi\cos\theta\cdot h + h(\cos\phi\cos\theta - 1) \end{bmatrix} = \begin{bmatrix} 0 \\ d \\ \underbrace{\cos\phi\sin\theta\cdot d + h}_{z_A} \end{bmatrix}$$

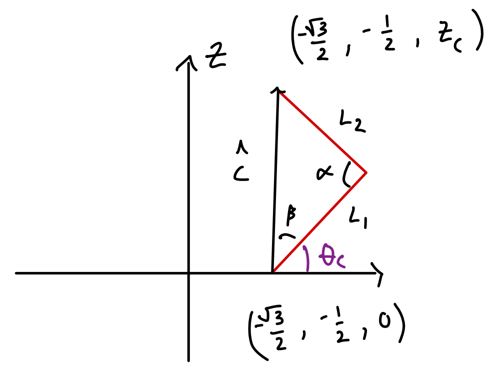
$$T_f B_R = B_f = \begin{bmatrix} -\frac{d\sqrt{3}}{2} \\ -\frac{d}{2} \\ \sin\phi\frac{d\sqrt{3}}{2} - \cos\phi\sin\theta\frac{d}{2} + \cos\phi\cos\theta h \end{bmatrix} = \begin{bmatrix} -\frac{d\sqrt{3}}{2} \\ -\frac{d}{2} \\ \underbrace{\sin\phi\frac{d\sqrt{3}}{2} - \cos\phi\sin\theta\frac{d}{2} + h}_{z_B} \end{bmatrix}$$

$$T_f C_R = C_f = \begin{bmatrix} \frac{d\sqrt{3}}{2} \\ -\frac{d}{2} \\ -\sin\phi\frac{d\sqrt{3}}{2} - \cos\phi\sin\theta\frac{d}{2} + \cos\phi\cos\theta h \end{bmatrix} = \begin{bmatrix} \frac{d\sqrt{3}}{2} \\ -\frac{d}{2} \\ \underbrace{-\sin\phi\frac{d\sqrt{3}}{2} - \cos\phi\sin\theta\frac{d}{2} + h}_{z_c} \end{bmatrix}$$

Inverse Kinematics







$|\hat{a}| = z_A$

$|\hat{b}| = z_B$

$|\hat{c}| = z_c$

$$\alpha = \cos^{-1}\left(\frac{L_1^2 + L_2^2 - z_A^2}{2L_1L_2}\right)$$

$$\beta = \cos^{-1}\left(\frac{L_1^2 + z_A^2 - L_2^2}{2L_1 z_A}\right)$$

$$\theta_A = 90 - \beta$$

$$\alpha = \cos^{-1}\left(\frac{L_1^2 + L_2^2 - z_B^2}{2L_1L_2}\right)$$

$$\beta = \cos^{-1}\left(\frac{L_1^2 + z_B^2 - L_2^2}{2L_1 z_B}\right)$$

$$\theta_B = 90 - \beta$$

$$\alpha = \cos^{-1}\left(\frac{L_1^2 + L_2^2 - z_c^2}{2L_1L_2}\right)$$

$$\beta = \cos^{-1}\left(\frac{L_1^2 + z_c^2 - L_2^2}{2L_1 z_c}\right)$$

$$\theta_c = 90 - \beta$$