<Part 1 Plots>

<Part 1 Discussion>

**Q) Why is it justified to use the LU or QR-factorizations as opposed of calculating an inverse matrix?**

It is justified to use the LU or QR-factorizations as opposed to calculating an inverse matrix because when compared to directly calculating the inverse of a matrix, the solutions given by using these factorizations are more numerically stable, as we can see with their condition numbers. A condition number is essentially a measure of how “conditioned” a matrix is: if the number is low, the matrix is well-conditioned, if the number is high, the matrix is ill-conditioned. Therefore, if a matrix has an ill-conditioned condition number, the solutions are unstable, or more sensitive, with respect to small changes in data. LU and QR-factorizations have low condition numbers, while simply calculating an inverse matrix can result in a high condition number.

Also, with trying to find the inverse of a matrix A, you must solve *n* sets of equations with the *n* columns of the identity matrix, which can take a long time and be quite tedious. However, with LU and QR-factorizations, for calculations of each column of the inverse of the A matrix, the coefficient matrix A does not change. So, if these methods are used, the factorization needs to be done only once, with the forward substitution *n* times, and the back substitution *n* times.

**Q) What is the benefit of using LU or QR-factorizations in this way? (Your answer should consider the benefit in terms of conditioning error.)**

There are a few benefits to using LU or QR-factorizations in this way; one of them comes from using forward and backward substitution. For example, if you have a lower-triangular matrix, you can use forward substitution; one the other hand, if you have an upper-triangular matrix, you can use backward substitution. This essentially means that, through a program on the computer, to solve a matrix, all you have to do is to run the LU or QR decompositions for matrix A, and then use forward or backward substitution when necessary. After substitution, you will have solved the equation Ax = b.

Back to the condition number, however, the benefits of using these decompositions clearly outweigh the benefits of calculating an inverse matrix, which has hardly any. The condition number, or conditional error, of a nonsingular matrix A is found with the equation κ. A is a well-conditioned matrix if κ(A) is close to 1, meaning the relative error in *x* is not much larger than the relative error in *b*. A is an ill-conditioned matrix if κ(A) is large, meaning the relative error in *x* can be much larger than the relative error in *b*. The matrices the LU and QR-factorizations produce are generally well-conditioned matrices, with low conditional error.