Part 3 (The Leslie Matrix) Discussion:

1. In the given Leslie matrix, there are several different meanings to the data. In the first row, the numbers [0, 1.2, 1.1, 0.9, 0.1, 0, 0, 0, 0] represent fecundity, or the per capita average number of female offspring reaching *n1* born from a mother of age class *x*. From this, we can conclude that of age class 1, which includes “newborns” from age 0 to 9, the average number of female offspring that they produce is 0, which makes sense because they are merely children. Of age class 2, which includes 10 to 19 year olds, the average number of female offspring that they produce is 1.2, which also makes sense with the increasing number of teenage pregnancies in modern times. As we move onto each successive class, the fecundity decreases, until it becomes 0 at the age class that includes 50 to 59 year olds, which makes sense because women above the age of 50 (or even 40, since the fecundity is 0.1 for the age class of 40-49 year olds) hardly produce any offspring. The main years that an average woman produces offspring are from their late teens to their mid to late 30s.

Furthermore, the numbers in the matrix that form a diagonal represent the fraction of individuals who survive from age class *x* to age class *x+1*. In this scenario, the population of the city we are examining has been divided into nine age groups: 0-9, 10-19, 20-29, 30-39, 40-49, 50-59, 60-69, 70-79, and 80+. From this, we learn that in this population, 0.7 (70%) of all “newborns” ages 0-9 survive to age 10, 0.85 (85%) of all 10-19 year olds survive to age 20, and so on. By age class 60-69, the rate at which they survive to the next class is 0.77 (77%), and after that, the survival rate of 70-79 year olds to ages 80+ is 0.4 (40%). All of these data were influenced and can be explained by various social factors, such as health behaviors, clinical care, socioeconomic status, and the physical environment that they live in.

For the social factors that affect these numbers, health behaviors include alcohol and tobacco use, and diet and exercise. As we progress through the age groups, older people tend to develop more sicknesses and let go of taking care of their health, which we can see with the decline from a 0.8 survival rate to a 0.77 rate between the age groups of 50-59 and 60-69. As for clinical care, it correlates with the socioeconomic status of that person, since the education, employment, income, and safety within their community all determines the quality of care that they receive. People living in poorer areas have less access to care than do people with more income, which can affect the health of those people. Finally, another social factor that could influence these numbers is the quality of the environment. If the city in question is one that produces smog and has a lot of manufacturing buildings, the population’s health may be affected by constant exposure to dangerous chemicals. Overall, the general population, as they get older, their health begins to deteriorate, which makes the average expectancy 78.7 years.

1. For this problem, I wrote a code (see population.py) using the equation: . By plugging in the Leslie matrix, A, and the given initial x(0), or the population distribution of the city in question in 2000, I was able to find the population distribution by raising A to the k power, which is how many decades ahead you would like to see the population for. Then, you simply multiply the two together and the resulting vector will have the populations for each age group in that decade. To find the total population, you can add every entry in the vector together.

From running the code, I found the following:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Age Group | 2010 | 2020 | 2030 | 2040 | 2050 |
| 0-9 | 635000 | 518750 | 816240 | 965648.5 | 1341322.675 |
| 10-19 | 147000 | 444500 | 363125 | 571368 | 675953.95 |
| 20-29 | 161500 | 124950 | 377825 | 308656.25 | 485662.8 |
| 30-39 | 162000 | 145350 | 112455 | 340042.5 | 277790.625 |
| 40-49 | 189000 | 145800 | 130815 | 101209.5 | 306038.25 |
| 50-59 | 176000 | 166320 | 128304 | 115117.2 | 89064.36 |
| 60-69 | 136000 | 140800 | 133056 | 102643.2 | 92093.76 |
| 70-79 | 92400 | 104720 | 108416 | 102453.12 | 79035.264 |
| 80+ | 36000 | 36960 | 41888 | 43366.4 | 40981.248 |
| Total: | 1734900 | 1828150 | 2212124 | 2650504.67 | 3387942.932 |

Between the years 2010 and 2020, the total population changed by 1.054. Between the years 2020 and 2030, the total population changed by 1.21. Between the years 2030 and 2040, the total population changed by 1.198. Finally, between the years 2040 and 2050, the total population changed by 1.278.

1. By using the program, I calculated the largest eigenvalue of the Leslie matrix A to be approximately 1.2886562339. This tells me that as we progress through the years, the population will continuous increase at a rate of or around 1.2887; thus, the population will become stable in the sense that it will increase at approximately the same rate each year, but unstable in that the population will never reach a limit and stay at a constant number. The power method goes through as many iterations as it takes to get the eigenvalue to eight digits of accuracy, and as many iterations as it takes until , where is the error tolerance that is an input for the program.

When running this program, with an error tolerance of 1e-10, it takes about 50 iterations determine the approximate eigenvalue. Since the power method is essentially the equation that the Leslie matrix is iterated through, this number of iterations can be interpreted to mean that it takes 50 decades in this population to approximate the average change in population of the city.

1. To get the population distributions and total population of 2030, 2040, and 2050 after decreasing the birth rate of the second age group by half in 2020, I took the first two iterations (first two decades) of the original Leslie matrix and used those numbers as the first two decades’ populations. Then, I found the first iteration of the altered matrix and multiplied the result by the population distribution of 2020 of the original matrix. Since the first iteration of the altered matrix will only occur in 2030, we can use that first as the “third” iteration, the second as the “fourth,” and the third as the “fifth” and so on.

If we are able to decrease the birth rate of the second age group (1.2) by half in 2020, the predictions for 2030, 2040, and 2050 are as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| Age Group | 2030 | 2040 | 2050 |
| 0-9 | 720480 | 816947.5 | 1148215.075 |
| 10-19 | 301385 | 504336 | 571863.25 |
| 20-29 | 309995 | 256177.25 | 428685.6 |
| 30-39 | 112455 | 278995.5 | 230559.525 |
| 40-49 | 130815 | 101209.5 | 251095.95 |
| 50-59 | 128304 | 115117.2 | 89064.36 |
| 60-69 | 133056 | 102643.2 | 92093.76 |
| 70-79 | 108416 | 102453.12 | 79035.264 |
| 80+ | 41888 | 43366.4 | 40981.248 |
| Total: | 1986794 | 2321245.67 | 2931594.032 |

Between the years 2030 and 2040, the total population changed by 1.168. Between the years 2040 and 2050, the total population changed by 1.263. Oddly, the approximate largest eigenvalue of A is again, 1.2886562339. Regarding the population in the long run, this means that in both cases, the change between populations of two consecutive decades will reach its largest value, which is the largest eigenvalue calculated. This ultimately means that between every decade, the maximum amount the population can grow in a decade is 1.2886562339.