Gradual C0: Symbolic Execution for Efficient Gradual Verification

JENNA DIVINCENZO, Carnegie Mellon University, USA IAN MCCORMACK, Carnegie Mellon University, USA HEMANT GOUNI, University of Minnesota, USA JACOB GORENBURG, Haverford College, USA MONA ZHANG, Columbia University, USA CONRAD ZIMMERMAN, Brown University, USA JOSHUA SUNSHINE, Carnegie Mellon University, USA ÉRIC TANTER, University of Chile, Chile JONATHAN ALDRICH, Carnegie Mellon University, USA

Current static verification techniques such as separation logic support a wide range of programs. However, such techniques only support complete and detailed specifications, which places an undue burden on users. To solve this problem, prior work proposed gradual verification, which handles complete, partial, or missing specifications by soundly combining static and dynamic checking. Gradual verification has also been extended to programs that manipulate recursive, mutable data structures on the heap. Unfortunately, this extension does not reward users with decreased dynamic checking as specifications are refined. In fact, all properties are checked dynamically regardless of any static guarantees. Additionally, no full-fledged implementation of gradual verification exists so far, which prevents studying its performance and applicability in practice.

We present Gradual C0, the first practical gradual verifier for recursive heap data structures, which targets C0, a safe subset of C designed for education. To minimize insertion of dynamic checks and support efficiency, Gradual C0 adopts symbolic execution at its core, instead of the backwards reasoning of weakest liberal preconditions used in prior work. Our approach addresses technical challenges related to symbolic execution with imprecise specifications, heap ownership, and branching in both program statements and specification formulas. Finally, we provide the first empirical performance evaluation of a gradual verifier, and found that on average, Gradual C0 decreases run-time overhead between 50-90% compared to the fully-dynamic approach used in prior work. Further, the worst-case scenarios for performance are predictable and avoidable. This work paves the way towards evaluating gradual verification at scale.

CCS Concepts: • Theory of computation \rightarrow Logic and verification; Separation logic.

1 INTRODUCTION

Separation logic [Reynolds 2002] supports the modular static verification of heap-manipulating programs. The \mapsto operator asserts both ownership of a heap location and its value, *e.g.* $x.f \mapsto 2$ states that the location x.f is uniquely owned and contains the value 2. The *separating conjunction* ensures memory disjointness: $x.f \mapsto 2 * y.f \mapsto 2$ states that the heap locations x.f and y.f are distinct (*i.e.* $x \neq y$), are each owned, and each contain the value 2. *Implicit dynamic frames*

*This material is based upon work supported by a Google PhD Fellowship award and the National Science Foundation under Grant Nos. CCF-1901033, DGE1745016, and DGE2140739. É. Tanter is partially funded by the ANID FONDECYT Regular Project 1190058 and the Millennium Science Initiative Program: code ICN17_002. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation, Google, ANID, or the Millennium Science Initiative.

Authors' addresses: Jenna DiVincenzo, Carnegie Mellon University, USA, jlwise@andrew.cmu.edu; Ian McCormack, Carnegie Mellon University, USA, icmccorm@cs.cmu.edu; Hemant Gouni, University of Minnesota, USA, hemant@hemantgouni. com; Jacob Gorenburg, Haverford College, USA, jgorenburg@haverford.edu; Mona Zhang, Columbia University, USA, mz2781@columbia.edu; Conrad Zimmerman, Brown University, USA, me@conradz.com; Joshua Sunshine, Carnegie Mellon University, USA, sunshine@cs.cmu.edu; Éric Tanter, University of Chile, Chile, etanter@dcc.uchile.cl; Jonathan Aldrich, Carnegie Mellon University, USA, jonathan.aldrich@cs.cmu.edu.

(IDF) [Smans et al. 2009] is an alternative to separation logic, which asserts ownership of a heap location and its value separately. Ownership is ensured through *accessibility predicates* such as acc(x.f). In IDF, acc(x.f) * x.f == 2 states that x.f is uniquely owned and contains the value 2. *Recursive abstract predicates* [Parkinson and Bierman 2005; Smans et al. 2009] further support verifying recursive heap data structures, such as trees, lists, and graphs. Abstract predicates can be thought of as pure boolean functions. For example, the following predicate specifies that a list is acyclic, *i.e.* acyclic(1) denotes that all heap locations in list 1 are distinct by recursively generating accessibility predicates for each node in 1, joined with the separating conjunction:

```
predicate acyclic(Node n) = n == null ? true : acc(n.val) * acc(n.next) * acyclic(n.next)
```

While these techniques allow users to specify and verify more code than ever before, formal verification is still largely unused due to the burden it places on its users. CompCert [Leroy 2009], an optimizing compiler, took 6 person-years to verify [Kästner et al. 2017]; verifying seL4 required 20 person-years compared to the only 2.2 person-years required to write the kernel itself [Klein et al. 2009]. In practice, engineering organizations may not be willing to pay the cost of fully specifying and proving code; can they instead derive significant benefits from *partial* specifications?

Inspired by gradual typing [Siek and Taha 2006], Bader et al. [2018] proposed gradual verification to allow users to write imprecise (i.e. partial) specifications, complemented by run-time checks where necessary. An imprecise formula can be fully unknown, written ?, or combine a static part with the unknown, as in ? * x.f == 2. Wise et al. [2020] extend Bader et al. [2018]'s minimal system by designing and formalizing the first gradual verifier for recursive heap data structures. It supports the strengthening of imprecise specifications with accessibility predicates and abstract predicates; and thus, also (in theory) the run-time verification of these constructs. During static verification, an imprecise specification can be optimistically strengthened (in non-contradictory ways) by the verifier to support proof goals. Wherever such strengthenings occur, dynamic checks are inserted to preserve soundness. Gradual verification smoothly supports the spectrum between static and dynamic verification. This is captured by properties adapted from gradual typing [Siek et al. 2015], namely the gradual guarantee, stating that the verifier will not flag static or dynamic errors for specifications that are correct but imprecise, and the fact that gradual verification conservatively extends static verification, i.e. they coincide on fully-precise programs.

Regarding performance, the seminal work of Siek and Taha [2006] describes a translation that *selectively* inserts run-time casts to compensate for optimistic static checking in the presence of imprecision. Follow-up work on gradual typing performance has shown that minimizing the insertion of dynamic checks does not trivially correlate with overall execution performance; the nature of the inserted checks (such as higher-order function wrappers) as well as their location in the overall execution flow of a program can have drastic and hard-to-predict consequences [Campora et al. 2018; Feltey et al. 2018; Takikawa et al. 2016]. In this work, we are first and foremost concerned about efficiency in terms of minimizing the insertion of dynamic checks using statically available information. Indeed, while Wise et al. [2020] prove soundness, conservative extension, and the gradual guarantee, their gradual verifier is not efficient in that sense, because all memory safety and functional properties are checked dynamically regardless of the precision of specifications. Furthermore, their approach has neither been implemented nor validated in practice. We aim at a practical implementation to empirically evaluate the relation between minimizing check insertion and observed runtime performance in gradual verification.

A key insight of this work is that the reliance on *weakest liberal preconditions* [Dijkstra 1975] for static reasoning in a gradual verifier, as manifest in prior work [Wise et al. 2020], compromises the efficiency of the gradual verifier by enforcing too many dynamic checks to be inserted. To address this, we propose a switch of perspective, from backward reasoning to forward reasoning, and

propose to use *symbolic execution* [King 1976] as a foundational technique for gradual verification. Specifically, this paper presents the design, implementation and validation of Gradual C0—the first gradual verifier for imperative programs manipulating recursive heap data structures¹. Gradual C0 targets C0, a safe subset of C designed for education, with appropriate support (and pedagogical material) for dynamic verification. Technically, Gradual C0 is built on top of the Viper static verifier [Müller et al. 2016], which uses symbolic execution and supports both IDF and recursive abstract predicates.

We address new technical challenges in gradual verification related to these differences:

- Gradual Co's symbolic execution algorithm is responsible for statically verifying programs with imprecise specifications and producing minimized run-time checks. In particular, Gradual Co tracks the branch conditions created by program statements and specifications to produce run-time checks for corresponding execution paths. At run time, branch conditions are assigned to variables at the branch point that introduced them, which are then used to coordinate the successive checks as required. Further, Gradual Co creates run-time checks by translating symbolic expressions into specifications—reversing the symbolic execution process.
- The run-time checks produced by Gradual C0 contain branch conditions, simple logical expressions, accessibility predicates, separating conjunctions, and predicates. Each of these constructs is specially translated into source code that can be executed at run time for dynamic verification. Logical expressions are turned into assertions. Accessibility predicates and separating conjunctions are checked by tracking and updating a set of owned heap locations. Finally, predicates are translated into recursive boolean functions.

We evaluate the performance of Gradual C0 by emulating Takikawa et al. [2016]'s performance lattice method, exploring the performance characteristics for partial specifications of four common data structures. Statically, we observe that as more specifications are added, more verification conditions can be statically discharged. Though imprecision introduces unavoidable run-time checks, gradual verification decreases run-time overhead by an average of 50-90% compared to dynamic verification (and thus Wise et al. [2020]'s approach). Sources of run-time overhead correspond to the predictions made in prior work, and our study shows that the gradual guarantee holds empirically for our tool across thousands of sampled imprecise specifications.

2 SYMBOLIC EXECUTION VS. WEAKEST LIBERAL PRECONDITIONS

A key insight of this work is to recognize that the approach to gradual verification of Wise et al. [2020], which relies on weakest liberal preconditions (called WLP), is at odds with the objective of inserting as little runtime checks as possible to handle imprecision. This section informally explains this observation via a simple list insertion program, showing that the backwards reasoning inherent to WLP requires the verifier to choose between inserting dynamic checks too early (jeopardizing the gradual guarantee) or inserting too many dynamic checks (incurring unnecessary overhead). We illustrate how relying on symbolic execution instead addresses this tension.

Fig. 1 shows the implementation of a linked list and method for inserting a new node at the end of a non-empty list, called insertLast. This program is inspired by an introductory example of Wise et al. [2020], written here in C0 [Arnold 2010], a safe subset of C designed for education. Note, insertLast traverses the list using a while loop, and thus diverges if the list is cyclic. Therefore, insertLast is gradually specified for both memory safety and acyclicity.

The gradual specification of insertLast uses both precise and imprecise specifications highlighted in gray and yellow respectively. The definition of the acyclic predicate (line 4) is left imprecise, as ?. Acyclicity of list insertion is specified in insertLast's pre- and postconditions (lines 7-8).

¹Gradual C0 is hosted on Github: https://github.com/gradual-verification/gvc0.

Further, insertLast's precondition ensures insertLast only operates on non-empty lists with the list != NULL clause (line 7), and introduces optimism into the verification with ? (&& is separating conjunction). The loop invariant, ? && y!= NULL on line 13, is also imprecise. So only non-nullness of the current node (y!= NULL) is enforced at every iteration of the loop, either statically or dynamically. Accessibility predicates for memory safety are implicitly enforced: the imprecise formulas in Fig. 1 (lines 4, 7, and 13) avoid specifying accessibility predicates. Thus, the only interesting properties that can be verified by this gradual specification is whether y!= NULL is preserved by the loop, and whether heap accesses are justified. More details about the verification are given in §2.1 and §2.2.

```
struct Node { int val; struct Node *next; };
2
    typedef struct Node Node;
3
    //@ predicate acyclic(Node* root) = ?;
4
5
    Node* insertLast(Node* list, int val)
6
7
       //@ requires ? && acyclic(list) && list != NULL;
8
      //@ ensures acyclic(\result);
9
10
         //@ unfold acyclic(list);
11
        Node* y = list;
        while (y->next != NULL)
12
13
         //@ loop_invariant ? && y != NULL;
           { y = y->next; }
14
15
        y->next = alloc(struct Node);
        y->next->val = val;
16
17
        y->next->next = NULL;
18
         //@ fold acyclic(list);
19
         return list;
20
    }
     Imprecise specification
                                Precise specification
```

Fig. 1. Partially specified non-empty linked list insertion

As an aside, insertLast's specification also includes (un)folding the acyclic predicate. Unfolding acyclic (line 10) consumes the *predicate instance* acyclic(list) and introduces its imprecise body (?), hence optimism, into the analysis. In contrast, folding acyclic(list) (line 18) consumes its body in favor of its instance, hence satisfying the postcondition. Using folds and unfolds to control the availability of predicate information treats predicates *iso-recursively*; while an *equi-recursive* interpretation treats predicates as their complete unfolding [Summers and Drossopoulou 2013]. Like WLP, Gradual C0 uses iso-recursion for static checking and equi-recursion for dynamic checking.

2.1 Gradually Verifying List Insertion with Weakest Liberal Preconditions

Fig. 2 shows how Wise et al. [2020]'s weakest liberal preconditions approach (*gWLP*) gradually verifies insertLast from Fig. 1. WLP begins with insertLast's postcondition acyclic(\result)

on line 39 and is applied to each program statement step-by-step until the start of insertLast is reached on line 6. The specifications highlighted in purple are intermediate conditions produced by WLP as it is applied in this backward fashion. Each intermediate condition is minimally sufficient to verify the following program statement and intermediate condition. For example, ? on line 35 satisfies the body of acyclic(list) on line 36, which in turn satisfies acyclic(list) on line 37 (i.e. acyclic(\result), line 39). Similarly, the assignments to y->next->next and y->next->val on lines 34 and 32 require access to those locations in addition to y->next. Therefore, the intermediate condition on lines 30-31 contains accessibility predicates for those locations joined with ? (? satisfies the following intermediate condition on line 35). Alloc on line 29 provides access to y->next->next and y->next->val, so only ? && acc(y->next) is required by line 28.

When WLP cannot soundly propagate a condition backwards, a *consistent implication* \Longrightarrow check is injected. These checks are required at the beginning of a method (lines 6-7) or loop body (lines 18-20), at the end of a loop with an imprecise invariant (lines 26-27), after unfolding an abstract predicate with an imprecise body (lines 10-11), and after a method call with an imprecise postcondition. So a consistent implication is created at lines 26-27. The loop invariant and negated loop guard are joined for the premise (line 26), and the conclusion is the current intermediate

Gradual C0 1:5

```
? && acc(y->next->next) &&
1
    Node* insertLast(Node* list, int val)
                                                            21
2
       /*@ requires ? && acyclic(list)
                                                            22
                                                                            acc(y->next) && y->next != NULL
3
                         && list != NULL: @*/
                                                            23
                                                                         y = y -> next;
4
       //@ ensures acyclic(\result);
                                                                         ? && y != NULL && acc(y->next)
                                                            24
5
                                                            25
        ? && acyclic(list) && list != NULL
6
                                                            26
                                                                    ? && y != NULL && y->next == NULL \Longrightarrow
7

? && acyclic(list)

                                                                       ? && acc(y->next)
                                                            27
        ? && acyclic(list)
8
                                                                    ? && acc(y->next)
                                                            28
9
        //@ unfold acyclic(list);
                                                            29
                                                                    y->next = alloc(struct Node);
        ? ⇒ ? && list != NULL &&
10
                                                            30
                                                                    ? && acc(y->next) && acc(y->next->next) &&
11
           acc(list->next)
                                                            31
                                                                     acc(y->next->val)
        ? && list != NULL && acc(list->next)
12
                                                                    y->next->val = val;
                                                            32
        Node* y = list;
13
                                                            33
                                                                    ? && acc(y->next) && acc(y->next->next)
        ? && y != NULL && acc(y->next)
14
                                                            34
                                                                    y->next->next = NULL;
15
        while (y->next != NULL)
                                                            35
                                                                    ?
          //@ loop_invariant ? && y != NULL;
16
                                                            36
                                                                    //@ fold acyclic(list);
17
                                                            37
                                                                    acyclic(list)
        ? && y != NULL && acc(y->next) && y->next != NULL
18
                                                           38
                                                                    return list;
19

⇒ ? && acc(y->next->next) &&

                                                            39
                                                                    acyclic(\result)
                acc(y->next) && y->next != NULL
20
                                                            40
                                                                }
                 Intermediate condition produced by WLP
                                                                Dynamically checked conclusion of \widetilde{\Rightarrow}
                 Premise of consistent implication \Longrightarrow
                                                               Statically checked conclusion of \Longrightarrow
```

Fig. 2. The gradual verification of insertLast from Fig. 1 using WLP

condition (line 27). The premise does not statically imply the conclusion, but can do so optimistically with imprecision. Therefore, the conclusion is dynamically checked as highlighted in red. Note that if the premise of a consistent implication cannot imply the conclusion (e.g. ? && x == 3 \Longrightarrow x != 3), then the program is statically rejected.

To verify the loop body, WLP starts with the loop invariant joined with accessibility predicates required for the loop guard (line 24). For variable assignment, like the one at line 23, WLP substitutes the right-hand side of the assignment (y->next) for the left-hand side (y) in the current intermediate condition. This produces the intermediate condition before the assignment on lines 21-22. Additionally, an accessibility predicate is added for y->next. Upon reaching the start of the loop body, a consistent implication (lines 18-20) is inserted to check the current intermediate condition (lines 19-20). Here, the premise (line 18) is the loop invariant, guard, and accessibility predicates required by the guard. The loop guard and its accessibility predicate imply part of the conclusion (the current intermediate condition), so that part is statically discharged as highlighted in green (line 20). The rest is dynamically checked, as before (line 19). The condition prior to the loop (line 14) consists of the loop invariant and acc(y->next) for the loop guard. As before, WLP performs substitution on the condition on line 14 for the assignment on line 13 to produce the condition on line 12. This condition becomes the conclusion of the consistent implication (lines 10-11) introduced for the unfold on line 9. The premise is the body of acyclic(list), i.e.?. The conclusion is entirely dynamically checked, because? provides no static information. The condition prior to the unfold (line 8) contains the predicate that is unfolded (acyclic(list)) and ?. The ? represents information not contained within acyclic(list). Finally, insertLast's precondition (the premise on line 6) is used to check the condition prior to the unfold (the conclusion on line 7). Since acyclic(list) is implied by the precondition, the conclusion is discharged statically.

2.2 Gradually Verifying List Insertion with Symbolic Execution

```
1
    Node* insertLast(Node* list, int val)
                                                              29
                                                                      ? && y != NULL && acc(y->next) &&
       /*@ requires ? && acyclic(list) &&
2
                                                              30
                                                                         y->next == NULL
3
                              list != NULL; @*/
                                                              31
                                                                      ? && y != NULL && acc(y->next) &&
4
       //@ ensures acyclic(\result);
                                                              32
                                                                         y->next == NULL \Longrightarrow acc(y->next)
5
                                                              33
                                                                      y->next = alloc(struct Node);
        ? && acyclic(list) && list != NULL
6
                                                                      ? && y != NULL && y->next != NULL &&
                                                              34
        ? && acyclic(list) && list != NULL
7
                                                                       acc(y->next) && acc(y->next->next) &&
                                                              35
           ⇒ acyclic(list)
8
                                                                       acc(y->next->val)
        //@ unfold acyclic(list);
9
                                                              37
                                                                       ? && ... && acc(y->next->next) &&
        ? && list != NULL
10
        Node* v = list;
                                                                         acc(y->next->val) \cong acc(y->next->val)
11
                                                              38
        ? && list != NULL && y == list
12
                                                              39
                                                                      v -> next -> val = val:
                                                                       ? && ... && acc(y->next->next) &&
13
         ? && list != NULL && y == list
                                                              40
                                                                       acc(y->next->val) && y->next->val == val
                                                              41
14
            ? && ... && acc(y->next->next) \widetilde{\Rightarrow}
                y != NULL
                                                              42
15
        while (y->next != NULL)
                                                              43
                                                                          acc(y->next->next)
16
17
           //@ loop_invariant ? && y != NULL;
                                                                      y->next->next = NULL;
                                                              44
18
                                                              45
                                                                       ? && ... && acc(y->next->next) &&
        ? && y != NULL && acc(y-next) && y->next != NULL _{46}
19
                                                                       y->next->next == NULL
20
         ? && y != NULL && acc(y->next) &&
                                                              47
                                                                      ? && ... && acc(y->next->next) &&
           y->next != NULL \Longrightarrow acc(y->next)
21
                                                                         y->next->next == NULL \widetilde{\Rightarrow} ?
                                                              48
22
             y = y->next;
                                                              49
                                                                      //@ fold acyclic(list);
23
             ? && y' != NULL && acc(y'->next) &&
                                                              50
                                                                       ? && acyclic(list)
                y'->next != NULL && y == y'->next
24
                                                                       return list;
                                                              51
             ? && y' != NULL && acc(y'->next) &&
25
                                                              52
                                                                       ? && acyclic(list) && list == \result
              y'->next != NULL && y == y'->next \widetilde{\Rightarrow}
                                                                       ? && acyclic(list) && list == \result \widetilde{\Rightarrow}
26
                                                              53
27
                ? && acc(y->next) && y != NULL
                                                              54
                                                                         acyclic(\result)
28
           }
                                                                  }
                                                              55
              Intermediate condition produced by Gradual C0
                                                                Dynamically checked conclusion of \Longrightarrow
                                                                     Statically checked conclusion of \Longrightarrow
          Premise of consistent implication \Longrightarrow
```

Fig. 3. The gradual verification of insertLast from Fig. 1 using Gradual C0

Fig. 3 demonstrates how Gradual C0 gradually verifies insertLast from Fig. 1 using symbolic execution instead of weakest liberal preconditions. We abstract details of symbolic execution to facilitate the comparison with $\widehat{\text{WLP}}$. For example, intermediate conditions (highlighted in purple) are represented as formulas rather than symbolic states. Also, we use consistent implication $\widehat{\Rightarrow}$ instead of the algorithm implemented in Gradual C0 (§3.2).

Gradual C0 works forward: it starts with insertLast's precondition (line 6) and analyzes each program statement until it reaches insertLast's end (line 54). Intermediate conditions created during this process are highlighted in purple. Here, an intermediate condition contains maximal information propagated forward from the previous program statement and intermediate condition. If Gradual C0 cannot soundly propagate a condition forward, a consistent implication check is injected. This time, checks are required for preconditions at method calls, loop invariants at loop entry (lines 13-15) and loop body end (lines 25-27), predicates at unfolds (lines 7-8), predicate bodies at folds (lines 47-48), asserted formulas, and postconditions at the end of method bodies (lines 53-54). Additionally, checks are also produced for accessibility predicates required by program statements (lines 14, 20-21, 27, 31-32, 37-38, and 42-43). Therefore, a consistent implication is generated for the unfold at line 9: the premise (line 7) is the current condition, *i.e.* insertLast's precondition; the conclusion is acyclic(list) (line 8) from the unfold, which is statically discharged (green).

After the check, acyclic(list) is consumed by the unfold leaving ? && list != NULL, then joined with ? from acyclic(list)'s body (line 10). The assignment on line 11 adds y == 1 ist to

? && list != NULL on line 12. At the loop on line 16, Gradual C0 checks (lines 14-15) that the loop invariant (line 17) holds and accesses to heap locations in the guard are justified with accessibility predicates (e.g. acc(y-next)). The premise statically proves y != NULL from the loop invariant in the conclusion (line 15). In contrast, acc(y-next) is dynamically checked thanks to ? (line 14).

To verify the loop body, Gradual C0 starts with the loop invariant, loop guard, and accessibility predicates required by the loop guard (line 19). The accessibility predicate acc(y-next) required by the loop guard statically verifies (in the consistent implication on lines 20-21) access to y-next in the assignment on line 22. Then, the previous value for y, which is reassigned on line 22, is versioned as y' in the current condition, and the resulting condition is joined with y = y'-next on lines 23-24. This becomes the premise of the consistent implication on lines 25-27 at the end of the loop, which preserves the loop invariant and access to heap locations in the loop guard (the conclusion). The y = NULL clause from the invariant is statically proven (green), and acc(y-next) from the loop guard is checked dynamically (red).

Following the loop, Gradual C0 begins with the loop invariant, negated loop guard, and accessibility predicates required by the loop guard (lines 29-30). Then, this after loop condition contains acc(y-next), which statically verifies access to y-next (lines 31-32) in the alloc statement on line 33. The alloc adds acc(y-next-val) and acc(y-next-next) to the after loop condition (lines 34-36). These predicates statically prove access to y-next-val and y-next-next in the following two assignment statements (lines 39 and 44). Thus, y-next-val == val followed by y-next-next == NULL are added to the current intermediate condition resulting in the condition on lines 45-46.

When Gradual C0 reaches the fold on line 49, the body of the predicate must be true, hence the consistent implication on lines 47-48. Since acyclic(list)'s body is ?, the implication succeeds trivially. Then, the body of acyclic(list) is replaced in the current condition with acyclic(list) itself to produce the next condition ? && acyclic(list) on line 50. Since the body is ?, all of the current condition is conservatively replaced, and ? in the next condition represents any residual information from the replacement. Finally, insertLast's postcondition acyclic(\result) is discharged statically on lines 53-54 by ? && acyclic(list) && list == \result, which was created from ? && acyclic(list) and returning list on line 51.

2.3 Weakest Liberal Preconditions vs. Symbolic Execution

We now reflect on the two gradual verification approaches and the reasons that pushed us to adopt symbolic execution for implementing Gradual C0. First of all, observe that $\overline{\text{WLP}}$ produces additional run-time checks compared to Gradual C0, which are unnecessary: in Fig. 2, list != NULL on line 10 and acc(y-next) after the loop (line 27) could safely be dropped. For example, acc(list-next) implies list != NULL, so dynamically checking acc(list-next) at the same program point as list != NULL is sufficient for soundness. Further, the run-time check for acc(y-next-next) in the loop body (line 19) ensures access to heap locations in the loop guard are justified for every loop iteration, including the last (when execution breaks out of the loop). Thus, acc(y-next) holds after the loop (line 26) and the run-time check on line 27 could be avoided.

One could be tempted to modify WLP to eagerly move dynamic checks as early as possible. However, doing so would break the dynamic gradual guarantee as formulated by Wise et al. [2020], which states that if a verified program takes a step at run time, then a less precise version of the program is guaranteed to take the same step. To see why, consider that if the unfold on line 9 in insertLast were removed (making the specification less precise), then the condition ? && list != NULL && acc(list->next) on line 12 would become the premise of the consistent implication at the start of the method (line 7) and a dynamic check would be inserted there. If, at run time, the executing function does not have permission to list->next, it will fail at the beginning of the

method, instead of taking several steps and failing before testing the loop condition, which is where permission to list->next is actually needed. This problem is inherent to WLP: moving checks earlier is the only way that checks can be eliminated via static reasoning, so any implementation based on WLP must choose between breaking the gradual guarantee and inserting run-time checks that are in fact unnecessary.

Adopting forward reasoning, as in symbolic execution, allows for a minimized insertion of dynamic checks without jeopardizing the dynamic gradual guarantee. Indeed, by propagating information forward, Gradual C0 can statically prove list != NULL and acc(y->next). After the assignment on line 11, list (which equals y) is clearly non-null. Also, Gradual C0 ensures access to heap locations in the loop guard are justified for every loop iteration at the end of the loop body on line 27. Thus, acc(y->next) clearly holds statically directly after the loop. Likewise, Gradual C0 does not attempt to verify access to y->next (list->next) until right before it is needed, i.e. right before the loop on line 14. As a result, Gradual C0 verifies acc(y->next) and y != NULL on lines 14-15 when the unfold is present, and will still try to verify both clauses at the same program point when the unfold is missing. Further, removing unfold does not change how insertLast executes at run time. Therefore, since acc(y--next) and y != NULL are true at lines 14-15 when the unfold is present, they are guaranteed to be true at the same program point when the unfold is missing. In general for Gradual C0, imprecision does not affect how a program executes and only weakens what is checked with consistent implications—in contrast, WLP may strengthen the conclusion of an implication. Thus, any correctly-verified program is guaranteed to take the same execution steps when made less precise, so Gradual C0 easily adheres to the dynamic gradual guarantee.

Additionally, in our subjective implementation experience, we found that it is more difficult to come up with algorithms for gradual verification working backwards compared to forwards. Another pragmatic reason to prefer symbolic execution over weakest precondition is that forward reasoning allows Gradual C0 to produce more digestible error messages than WLP when dynamic checks fail. For example, an error message for **acc**(y->next) prior to the loop (line 14), which accesses y->next, is easier to understand than one for **acc**(list->next) further up in the program.

3 GRADUAL CO

Gradual C0 is a working gradual verifier for the C0 programming language [Arnold 2010] that is built on top of the Viper static verifier [Müller et al. 2016]. Gradual C0 is structured in two major sub-systems: 1) the gradual verification pipeline and 2) the C0 pipeline. The gradual verification pipeline is responsible for statically verifying C0 programs as in §2.2 and producing run-time checks for soundness. First, a C0 program is translated into a Gradual Viper program by Gradual C0's frontend module, GVC0. Next, the Gradual Viper module uses a symbolic execution approach that handles imprecise formulas to statically verify the Gradual Viper program. Wherever imprecise formulas are optimistically strengthened in support of proofs, Gradual Viper creates a description of needed run-time checks. Finally, GVC0 takes these run-time checks and combines them with the original C0 program to produce a sound, gradually-verified program. The C0 pipeline takes this C0 program and feeds it to the C0 compiler, CC0, which is used to execute the program. Note that because Gradual Viper operates on its own language—namely the Viper language plus imprecise formulas—Gradual Viper can support multiple frontend languages, not just C0. We chose to build a C0 frontend first because C0 is a pedagogical version of C designed with dynamic verification in mind, and plan to use it in the classroom.

This section describes the implementation of Gradual Viper and GVC0's design and illustrate the concepts via example. §3.1 discusses how C0 programs are translated to Gradual Viper programs, along with modifications made to both C0 and Viper. Then, §3.2 details Gradual Viper's symbolic

```
x \in VAR
                                                        (variables) S \in STRUCTNAME
                                                                                                                             (struct names)
v \in VAL
                                                             (values) f \in FIELDNAME
                                                                                                                              (field names)
e \in EXPR
                                                      (expressions) p \in PREDNAME
                                                                                                                         (predicate names)
s \in STMT
                                                       (statements) m \in METHODNAME
                                                                                                                             (method names)
op ∈ +, -, /, *, ==, !=, <=, >=, <, >
                                                        (operators)
                                                       (a) Shared abstract syntax definitions
                     ::= struct predicate method
                                                                                               ::= \overline{field} \overline{predicate} \overline{method}
                                                                                    field ::= field f:T
         struct ::= struct S \{ \overline{T f} \}
                                                                                 predicate ::= predicate p(\overline{x:T}) \{ \widetilde{\phi} \}
       predicate ::= //@predicate p(\overline{T x}) = \widetilde{\phi}
                                                                                  method ::= method m(\overline{x:T}) returns (\overline{y:T})
         method ::= \tilde{T} m(\overline{T x}) contract \{ s \}
                                                                                                     contract { s }
        contract ::= //@requires \widetilde{\phi}; //@ensures \widetilde{\phi};
                                                                                 contract ::= requires \widetilde{\phi} ensures \widetilde{\phi}
       invariant ::= //@loop_invariant \widetilde{\phi}
                                                                                      T
                                                                                              ::= Int | Bool | Ref
                    := \frac{\mathsf{struct} \ S}{\mathsf{struct} \ S} \mid \mathsf{int} \mid \mathsf{bool} \mid \frac{\mathsf{char}}{\mathsf{char}} \mid T *
                                                                                              ::= s; s \mid var x : T \mid x := e
             \tilde{T}
                     ::= void \mid T
                                                                                                 x.f := e \mid x := \text{new}(\overline{f})
                    ::= s; s | T x | T x = e | x = e
             s
                       | l = e | e | assert(e) | //@assert \phi
                                                                                                 | \overline{x} := m(\overline{e}) | \text{assert } \phi
                        | //@fold p(\overline{\tilde{e}}) | //@unfold p(\overline{\tilde{e}})
                                                                                                 | fold acc(p(\overline{e})) | unfold acc(p(\overline{e}))
                        \mid if (e) { s } else { s }
                                                                                                 | if (e) { s } else { s }
                        | while (e) invariant \{s\}
                                                                                                | while (e) invariant \phi \{s\}
                                                                                              := v \mid x \mid op(\overline{e}) \mid e.f
                        for (s; e; s) invariant \{s\}
                                                                                              ::= result | id
                                                                                      x
                      ::= v \mid x \mid op(\overline{e}) \mid e \rightarrow f \mid *e \mid m(\overline{e})
                                                                                              ::= n \mid \text{null} \mid \text{true} \mid \text{false}
                      \mid alloc(T) \mid e ? e : e
                                                                                              ::=? \&\& \phi \mid \theta
             \tilde{e}
                     := v \mid x \mid op(\overline{\tilde{e}}) \mid \tilde{e} \rightarrow f \mid \star \tilde{e}
                                                                                      \theta
                                                                                               ::= self-framed \phi
             l
                     := x \rightarrow f \mid \star x \mid l \rightarrow f \mid \star l
                                                                                               := e \mid \mathsf{acc}(p(\overline{e})) \mid \mathsf{acc}(e.f) \mid \phi \&\& \phi
                     ::= \result | id
                                                                                                 |e?\phi:\phi
             x
                     := n \mid c \mid NULL \mid true \mid false
                                                                                              (c) Gradual Viper abstract syntax
             \widetilde{\phi}
                     ::= ? \&\& \phi \mid \theta
             \theta
                      ::= self-framed \phi
             φ
                     := \tilde{e} \mid p(\overline{\tilde{e}}) \mid acc(l) \mid \phi \&\& \phi
                        |\tilde{e}?\phi:\phi
                          (b) GVC0 abstract syntax
                   Representation differs slightly in GVC0 vs.
                                                                                        Functionality in GVC0 that requires non-
                   Gradual Viper
                                                                                        trivial translation to Gradual Viper
```

Fig. 4. Abstract syntax comparison for GVC0 and Gradual Viper

execution approach and how it produces run-time checks. Finally, §3.3 focuses on how GVC0 turns run-time checks from Gradual Viper into C0 code for dynamic verification.

3.1 Translating C0 Source Code to Gradual Viper Source Code

The C0 language, with its minimal set of language features and its existing support for specifications, serves well as the source language for our implementation. As its name suggests, C0 borrows heavily from C, but its feature set is reduced to better suit its intended purpose as a tool in computer science education [Arnold 2010]. It is a memory-safe subset of C that forbids casts, pointer arithmetic, and pointers to stack-allocated memory. All pointers are created with heap allocation, and de-allocation is handled by a garbage collector.

The abstract syntax for C0 programs supported by Gradual C0 is given in Fig. 4b, *i.e.* GVC0's abstract syntax. GVC0 programs are made of struct and method declarations that largely follow C syntax. What differs from C is GVC0's specification language. Methods may specify constraints on their input and output values as side-effect-free gradual formulas $\widetilde{\phi}$, usually in //@requires or //@ensures clauses in the method header. Loops and abstract predicates contain invariants and

bodies respectively that are made of gradual formulas. Such formulas $\widetilde{\phi}$ are imprecise formulas ? && ϕ or complete boolean formulas ϕ (Note, in this case, ϕ must be self-framed as defined in IDF). A formula ϕ joins boolean values, boolean operators, predicate instances, accessibility predicates, and conditionals via the separating conjunction && . GVC0 programs also contain //@fold p($\overline{\widetilde{e}}$) and //@unfold p($\overline{\widetilde{e}}$) statements for predicates and //@assert ϕ statements for convenience.

Now, GVC0 programs are converted to Gradual Viper programs before verification. Thus, we also give the abstract syntax for Gradual Viper in Fig. 4c. The two languages are roughly 1-to-1, but there are some differences as highlighted in yellow (trivial) and blue (non-trivial) in Fig. 4. For example, for loops in GVC0 are rewritten as while loops in Gradual Viper, and alloc(struct T) expressions are translated to new statements containing struct T's fields. Additionally, GVC0 allows method calls, allocs, and ternaries in arbitrary expressions, while Gradual Viper only allows such constructs in corresponding program statements². Therefore, GVC0 uses fresh temporary variables to version expressions containing the aformentioned constructs into program statements in Gradual Viper. The temporary variables are then used in the original expression in place of the corresponding method call, alloc, or ternary. Nested field assignments, such as x->y->z = a, are similarly expanded into multiple program statements using temporary variables. Value type pointers in GVC0 are rewritten as pointers to single-value structs that can be easily translated into Gradual Viper syntax. Finally, assert(e) statements are essentially ignored; e is translated into Gradual Viper syntax to verify its heap accesses, but e is not asserted (i.e. e is discarded).

Note that GVC0 does not support array and string values since gradually verifying any interesting properties about such constructs requires non-trivial extensions to current gradual verification theory. Similarly, the Gradual Viper language, in contrast to the Viper language, does not support the aforementioned constructs and fractional permissions.

3.2 Gradual Viper: Symbolic Execution for Gradual Verification

In this section, we describe Gradual Viper's symbolic execution based algorithm that supports the static verification of imprecise formulas. Our algorithm extends Viper's symbolic execution engine, and so Gradual Viper's design is heavily influenced by Müller et al. [2016]'s work. Like Viper, Gradual Viper's symbolic execution algorithm consists of 4 major functions: eval, produce, consume, and exec. The functions evaluate expressions, produce (inhale) and consume (exhale) formulas, and execute program statements respectively. Following Viper's lead, our 4 functions are defined in continuation-passing style, where the last argument of each of the aforementioned functions is a continuation Q. The continuation is a function that represents the remaining symbolic execution that still needs to be performed. Note that the last continuation returns a boolean (λ _ . success() or λ _ . failure()), indicating whether or not symbolic execution was successful.

A Gradual Viper program is checked by examining each of its method and predicate definitions to ensure they are well-formed. The formal definitions are given in the supplementary material, §A.5. Intuitively, for each method, we define symbolic values for the method arguments, and then create an initial symbolic state by calling the produce function on the method precondition. We then call the exec function on the method body, which symbolically executes the body and ensures that all operations are valid based on that precondition. Finally, we invoke the consume function on the final symbolic state and the postcondition, verifying that the former implies the latter. Throughout these operations a set of run-time checks is built up, which (along with success or failure) is the ultimate result of gradual verification.

The rest of this section is outlined as follows. Run-time checks and the collections that hold them are described in §3.2.1. We define symbolic states in §3.2.2 and preliminaries in §3.2.3. Finally,

²Note, ternaries correspond to if statements

the 4 major functions of our algorithm are given in their own sections: eval §3.2.4, produce §3.2.5, consume §3.2.6, and exec §3.2.7. Note, throughout this section, we make clear where Viper has been extended to support imprecise formulas with yellow highlighting in figures. We also use blue highlighting to indicate extensions for run-time check generation and collection.

3.2.1 **Run-time checks**. Run-time checks produced by Gradual Viper are collected in the \Re set. A run-time check is a 4-tuple (bcs_c, origin_c, location_c, ϕ_c), where bcs_c is a set of branch conditions, origin_c and location_c denote where the run-time check is required in the program, and ϕ_c is what must be checked. A branch condition in bcs_c is also a tuple of (origin_e, location_e, e), where origin_e and location_e define the program location at which Gradual Viper's execution branches on the condition e. A location is the AST element in the program where the branch or check occurs, denoted as a formula ϕ_l . Sometimes, the condition being checked is defined elsewhere in the program (e.g. in the precondition of a method) but we need to relate it to the method being verified. The origin is used to do this. It is none when the condition is in the method being verified; otherwise, it contains a method call, fold, unfold, or special loop statement from the method being verified that referenced the check specified in the location. An example run-time check is: ({(none, x > 2, $\neg(x > 2)$)}, z := m(y), acc(y.f), acc(y.f)). The check is for accessing y.f, and it is required for m's precondition element acc(y.f) at the method call statement z := m(y). The check is only required when $\neg(x > 2)$, which is evaluated at the program point where the AST element x > 2 exists. Since $\neg(x > 2)$'s origin is none, it comes from an if or assert statement.

Further, \mathcal{R} is used to collect run-time checks down a particular execution path in Gradual Viper. \mathcal{R} is a 3-tuple (bcs $_p$, origin $_p$, rcs $_p$) where bcs $_p$ is the set of branch conditions collected down the execution path p, origin $_p$ is the current origin that is set and reset during execution, and rcs is the set of run-time checks collected down p. Two auxiliary functions are used to modify \mathcal{R} : addcheck and addbc. The addcheck function takes an \mathcal{R} collection \mathcal{R}_{arg} , a location ϕ_l for a check, and the check itself, and returns a copy of \mathcal{R}_{arg} with the run-time check added to \mathcal{R}_{arg} .rcs. If necessary, addcheck uses \mathcal{R}_{arg} .origin and substitution to ensure ϕ_l and the check refer to the correct context. For example, let ϕ_l and check ϕ_c come from asserting a precondition for z:=m(y). Then, addcheck performs the substitutions: $\phi_l[t\mapsto m_{arg}]$ (precondition declaration context) and $\phi_c[t\mapsto y]$ (method call context) where t is the symbolic value for y. The addbc function operates similarly to addcheck but for branch conditions.

3.2.2 **Symbolic State**. We use $\sigma \in \Sigma$ to denote a symbolic state, which represents an intermediate condition from §2. A symbolic state is a 6-tuple (isImprecise, h_7 , h, γ , π , \Re) consisting of a boolean isImprecise, a symbolic heap h_7 , another symbolic heap h, a symbolic store γ , a path condition π , and a collection \Re (defined previously, §3.2.1). Since symbolic states represent intermediate conditions from §2, symbolic states may be imprecise. We therefore use the boolean isImprecise to record whether or not the state is imprecise. A symbolic store maps local variables to their symbolic values, and a path condition (defined in §3.2.3) contains constraints on symbolic values that have been collected on the current verification path.

A symbolic heap is a multiset of heap chunks for fields or predicates that are currently accessible. A field chunk $id(r; \delta)$ (representing expression r.id) consists of the field name id, the receiver's symbolic value r, and the field's symbolic value δ —also referred to as the snapshot of a heap chunk. For a predicate chunk $id(args; \delta)$, id is the predicate name, args is a list of symbolic values that are arguments to the predicate, and δ is the snapshot of the predicate. A predicate's snapshot represents the values of the heap locations abstracted over by the predicate. The symbolic heap $h_{?}$ contains heap chunks that are accessible due to optimism in the symbolic execution, while h contains heap chunks that are statically accessible. Further, unlike $h_{?}$, h maintains the invariant that its heap chunks are separated in memory, i.e. h's heap chunks can be joined successfully by the separating

conjunction. The *empty symbolic state* is $\sigma_0 = (\text{isImprecise} := \text{false}, h_? := \emptyset, h := \emptyset, \gamma := \emptyset, \pi := \emptyset, \mathcal{R} := (\emptyset, \text{none}, \emptyset)).$

3.2.3 **Preliminaries**. We introduce a few preliminary definitions here that will be helpful later. A path condition π is a stack of tuples (id, bc, pcs). An id is a unique identifier that determines the constraints on symbolic values that have been collected between two branch points in execution. The bc entry is the symbolic value for the branch condition from the first of two branch points, and pcs is the set of constraints that have been collected. Branch points can be from if statements and logical conditionals in formulas. Functions pc-all, pc-add, and pc-push manipulate path conditions and are formally defined in the supplement Fig. 13. The pc-all function collects and returns all the constraints in π , pc-add adds a new constraint to π , and pc-push adds a new stack entry to π . Similarly, snapshots for heap chunks have their own related functions: unit, pair, first, and second. The constant unit is the empty snapshot, pair constructs pairs of snapshots, and first and second deconstruct pairs of snapshots into their sub-parts. Further, fresh is used to create fresh snapshots, symbolic values, and other identifiers depending on the context. The havoc function similarly updates a symbolic store by assigning a fresh symbolic value to each variable in a given collection of variables. Finally, $check(\pi, t) = pc-all(\pi) \Rightarrow t$ queries the underlying SAT solver to see if the given constraint t is valid in a given path condition π (i.e. π proves or implies t).

3.2.4 **Symbolic execution of expressions**. The symbolic execution of expressions by the eval function is defined in Fig. 5. Using the current symbolic state, eval evaluates an expression to a symbolic value t and returns t and the current state to the continuation Q. Variable values are looked up in the symbolic store and returned. For $op(\bar{e})$, its arguments \bar{e} are each evaluated to their symbolic values \bar{t} . A symbolic value $op'(\bar{t})$ is then created and returned with the state after evaluation. Each op has a corresponding symbolic value op' of the same arity. For example, $e_1 + e_2$ results in the symbolic value $add(t_1, t_2)$ where e_1 and e_2 evaluate to t_1 and t_2 respectively.

Finally, the most interesting rule is for fields e.f. The receiver e is first evaluated to t resulting in a new state σ_2 . Then, eval looks for a heap chunk for t.f first in the current heap $h.^3$ If a chunk exists, then the heap read succeeds and σ_2 and the chunk's snapshot δ is returned to the continuation. If a chunk does not exist in h, then eval looks for a chunk in the optimistic heap $h_?$, and if found the chunk's snapshot is returned with σ_2 . If a heap chunk for t.f is not found in either heap, then the heap read can still succeed when σ_2 is imprecise. As long as $t \neq \text{null}$ does not contradict the current path condition $\sigma_2.\pi$ (the call to assert, supplementary material Fig. 21), σ_2 's imprecision optimistically provides access to t.f. Therefore, a run-time check for $\operatorname{acc}(e_t.f)$ is created and added to σ_2 's set of run-time checks (highlighted in blue). Note that $e_t.f$ is used in the check rather than t.f, because—unlike t which is a symbolic value—the expression e_t can be evaluated at run time. Specifically, translate (described in supplementary material Fig. 17) is called on t with the current state σ_2 to compute e_t . Additionally, the AST element e.f is used to denote the check's location.

Afterwards, a fresh snapshot δ is created for t.f's value, and a heap chunk $f(t; \delta)$ for t.f and δ is created and added to σ_2 's optimistic heap passed to the continuation. Similarly, the constraint $t \neq \text{null}$ is added to σ_2 's path condition. By adding $f(t; \delta)$ to the optimistic heap, the following accesses of t.f are statically verified by the optimistic heap, which reduces the number of run-time checks produced. Finally, verification of the heap read for t.f fails when none of the aforementioned cases are true. Fig. 14 and Fig. 15 in the supplementary material define variants of eval, called eval-p and eval-c, that are used in produce and consume respectively. The eval-p variant does not introduce run-time checks and eval-c does not extend the optimistic heap and path condition.

³Heap lookup in eval also looks for heap chunks that are aliases (according to the path condition) to the chunk in question.

Gradual C0 1:13

```
eval(\sigma, t, Q)
                               = Q(\sigma, t)
                               =Q(\sigma, \sigma.\gamma(x))
eval(\sigma, x, Q)
eval(\sigma_1, op(\overline{e}), Q) = eval(\sigma_1, \overline{e}, (\lambda \sigma_2, \overline{t} \cdot Q(\sigma_2, op'(\overline{t}))))
eval(\sigma_1, e.f, Q) = eval(\sigma_1, e, (\lambda \sigma_2, t.
                                  if (\exists f(r; \delta) \in \sigma_2.h . \mathsf{check}(\sigma_2.\pi, r = t)) then
                                     Q(\sigma_2, \delta)
                                  else if (\exists \ f(r; \ \delta) \in \sigma_2.h_? . \mathsf{check}(\sigma_2.\pi, \ r = t)) then
                                      Q(\sigma_2, \delta)
                                  else if (\sigma_2.isImprecise) then
                                      res, \_ := assert(\sigma_2.isImprecise, \sigma_2.\pi, t \neq \text{null})
                                      e_t := translate(\sigma_2, t)
                                      \mathcal{R}' := \operatorname{addcheck}(\sigma_2.\mathcal{R}, e.f, \operatorname{acc}(e_t.f))
                                      \delta := fresh
                                      res \wedge Q(\sigma_2\{h_2 := \sigma_2.h_2 \cup f(t; \delta), \pi := pc\text{-add}(\sigma_2.\pi, \{t \neq null\}), \mathcal{R} := \mathcal{R}'\}, \delta)
                                  else failure())
                            Handles imprecision
                                                                         Handles run-time check generation and collection
```

Fig. 5. Rules for symbolically executing expressions

Fig. 6. Rules for symbolically producing formulas

3.2.5 **Symbolic production of formulas**. Produce (Fig. 6) is responsible for adding information to the symbolic state, in particular, the path condition and the heap h. Producing an imprecise formula makes the symbolic state imprecise. The produce rule for an expression e evaluates e to its symbolic value and produces it into the path condition. The produce rules for accessibility predicates containing fields and predicates are similar, so we focus on the rule for fields only. The field e.f in acc(e.f) first has its receiver e evaluated to a symbolic value t. Then, using the parameter δ a fresh heap chunk $f(t; \delta)$ is created and added to the heap before invoking the continuation. Note, the disjoint union θ ensures $f(t; \delta)$ is not already in the heap before $f(t; \delta)$ is added; otherwise, verification fails. Further, acc(e.f) implies $e \neq nul1$ and so that fact is recorded in the path condition as $t \neq nul1$. When the separating conjunction ϕ_1 && ϕ_2 is produced, ϕ_1 is first produced into the symbolic state, followed by ϕ_2 . Finally, to produce a conditional, Gradual Viper branches on the symbolic value t for the condition e splitting execution along two different paths.

```
\begin{split} \operatorname{exec}(\sigma_1,\ x.f := e,\ Q) &= \operatorname{eval}\left(\sigma_1,\ e,\ (\lambda\ \sigma_2,\ t\ .\ \operatorname{consume}\left(\sigma_2,\ \operatorname{acc}(x.f),\ (\lambda\ \sigma_3,\ \_\ .\ \right. \right. \\ &\qquad \qquad \operatorname{produce}\left(\sigma_3,\ \operatorname{acc}(x.f)\ \&\&\ x.f = t,\ pair(\operatorname{fresh},\ unit),\ Q))))) \\ \operatorname{exec}(\sigma_1,\ \overline{z} := m(\overline{e}),\ Q) &= \operatorname{eval}\left(\sigma_1,\ \overline{e},\ (\lambda\ \sigma_2,\ \overline{t}\ .\ \right. \\ &\qquad \qquad \mathcal{R}' := \sigma_2.\mathcal{R}\{\operatorname{origin}:=(\sigma_2,\ \overline{z}:=m(\overline{e}),\ \overline{t})\} \\ &\qquad \qquad \operatorname{consume}\left(\sigma_2\{\ \mathcal{R}:=\mathcal{R}'\ \},\ meth_{pre}[\overline{meth_{args}\mapsto t}],\ (\lambda\ \sigma_3,\ \delta\ .\ \sigma_4:=\sigma_3\{\gamma:=\operatorname{havoc}(\sigma_3.\gamma,\ \overline{z})\} \\ &\qquad \qquad \operatorname{produce}\left(\sigma_4,\ meth_{post}[\overline{meth_{args}\mapsto t}][\overline{meth_{ret}\mapsto z}],\ \operatorname{fresh}, \\ &\qquad \qquad (\lambda\ \sigma_5\ .\ Q(\sigma_5\{\ \mathcal{R}:=\sigma_5.\mathcal{R}\{\operatorname{origin}:=\operatorname{none}\}\ \})))))) \end{split}
```

Fig. 7. Select rules for symbolically executing program statements

Along one path ϕ_1 is produced into the state under the assumption that t is true, and along the other path ϕ_2 is produced under the $\neg t$ assumption. Both paths follow the continuation to the end of its execution, and a branch condition corresponding to the t assumption made is added to the symbolic state. Paths are pruned when they are infeasible (the assumption about t would contradict the current path conditions). The definition of branch and other details are in the supplementary material §A.2. Note, produce does not add any run-time checks to the symbolic state.

Symbolic consumption of formulas. The goals of consume are 3-fold: 1) given a symbolic state σ and formula $\widetilde{\phi}$, check whether $\widetilde{\phi}$ is established by σ , i.e. $\widetilde{\phi}_{\sigma} \cong \widetilde{\phi}$ where $\widetilde{\phi}_{\sigma}$ is the formula which represents the state σ , 2) produce and collect run-time checks that are minimally sufficient for σ to establish ϕ soundly, *i.e.* the red and green highlighting in Fig. 3, and 3) remove accessibility predicates and predicates that are asserted in $\overline{\phi}$ from σ . The rules for consume are described in great detail in supplement §A.3. We give an abstract description here. For the first and second goals, heap chunks representing accessibility predicates and predicates in ϕ are looked up in the heap h and optimistic heap h_7 from σ . When σ is precise, the heap chunks must be in h or verification fails. If σ is imprecise, then the heap chunks are always justified either by the heaps or imprecision. Run-time checks for heap chunks that are verified by imprecision are collected in σ . \mathcal{R} . Clauses in ϕ containing logical expressions are first evaluated to a symbolic value t, which is then checked against σ 's path condition π . If σ is precise, then pc-all $(\pi) \Rightarrow t$ must hold (i.e. the constraints in π prove t) or verification fails. In contrast, when σ is imprecise, \wedge pc-all $(\pi) \wedge t$ must hold (i.e. t does not contradict constraints in π) otherwise verification fails. In this case, a run-time check is added to $\sigma.R$ for the set of residual symbolic values in t that cannot be proved statically by π . Finally, fields used in ϕ must have corresponding heap chunks in h when σ and ϕ are precise; otherwise when σ or ϕ are imprecise, field access can be justified by either the heaps or imprecision. A run-time check containing an accessibility predicate for the field is added to σ .R when imprecision is relied on.

The third goal of consume is to remove heap chunks $\overline{hc_i}$ representing accessibility predicates and predicates in $\widetilde{\phi}$ from σ , and in particular, from heaps h and $h_{\widehat{i}}$. When σ and $\widetilde{\phi}$ are both precise, the heap chunks in $\overline{hc_i}$ are each removed from h ($h_{\widehat{i}}$ is empty here). If $\widetilde{\phi}$ is imprecise, then all heap chunks in both heaps are removed as they may be in $\overline{hc_i}$ or $\widetilde{\phi}$ may represent them with imprecision. Finally, when σ is imprecise and $\widetilde{\phi}$ is precise, any heap chunks in h or $h_{\widehat{i}}$ that overlap with or may potentially overlap with (thanks to σ 's imprecision) heap chunks in $\overline{hc_i}$ are removed.

Gradual C0 1:15

3.2.7 **Symbolic execution of statements**. Select rules for exec, which symbolically executes program statements, are given in Fig. 7. Here we provide an intuition; the full set of rules are listed in the supplementary material $\S A.4$. The exec function takes a symbolic state σ , program statement stmt, and continuation Q. Then, exec symbolically executes stmt using σ to produce a potentially modified state σ' , which is passed to the continuation. That is, exec is the main verification function: it produces the intermediate conditions in Fig. 3 ($\S 2.2$) and calls eval and consume to produce consistent implications \Longrightarrow (Fig. 3 $\S 2.2$) where necessary.

Symbolic execution of field assignments first evaluates the right-hand side expression e to the symbolic value t. Any field reads in e are either directly or optimistically verified using σ_1 . Then, the resulting state σ_2 must establish write access to x.f in consume, i.e. $\sigma_2 \cong \operatorname{acc}(x.f)$. Calling consume also removes the field chunk for $\operatorname{acc}(x.f)$ from σ_2 (if it is in there) resulting in σ_3 . Therefore, the call to produce can safely add a fresh field chunk for $\operatorname{acc}(x.f)$ alongside x.f = t to σ_3 before it is passed to the continuation Q. Under the hood, run-time checks are collected and passed to Q.

The method call rule evaluates the arguments \bar{e} to symbolic values \bar{t} , consumes the method precondition (substituting arguments with \bar{t}) while making sure the origin is set properly for check and branch condition insertion, havoes existing assumptions about the variables being assigned to, produces knowledge from the postcondition, and finally continues after resetting the origin to none. An in-depth explanation is in the supplementary material, along with the other exec rules.

3.3 Dynamic Verification: Translating Run-time Checks into C0 Source Code

After static verification, Gradual Viper returns a collection of run-time checks **R** that are required for soundness to GVC0. Then, GVC0 creates a C0 program from both the run-time checks in \Re and the original C0 program, and passes the resulting program to the C0 compiler to be executed at run time. For example, consider the C0 program in Fig. 8 that implements a method for inserting a new node at the end of a list, called insertLastWrapper. Note, when passed a non-empty list, insertLastWrapper calls insertLast from Fig. 1 to perform insertion (line 17). Further, insertLastWrapper is gradually specified: its precondition is ? (line 7)—requiring unknown information—and its postcondition is acyclic(\result) (line 8)—ensuring the list after insertion is acyclic. Note, the acyclic predicate is fully specified (is precise) (lines 1-4). Fig. 8 also contains run-time checks generated by Gradual Viper for insertLastWrapper, as highlighted in blue. The first check (lines 15-16) ensures the list 1 sent to insertLast (line 17) is acyclic, and is only required when 1 is non-empty (non-null). The second check (lines 20-21) ensures the list returned from insertLastWrapper is acyclic, and is only required when insertLastWrapper's parameter 1 is empty (null). These checks are not executable by the C0 compiler; therefore, GVC0 takes the program and checks in Fig. 8 and returns the executable program in Fig. 9. That is, GVC0 translates branch conditions (lines 15 and 20), predicates (lines 16 and 21), accessibility predicates (acc(1->val) and acc(1->next) in acyclic's body, lines 1-4), and separating conjunctions (also in acyclic's body) from Gradual Viper into C0 source code. We discuss the aforementioned translations via example in the rest of this section: §3.3.1, §3.3.2, and §3.3.3 respectively. While not in the insertLastWrapper example, GVC0 translates checks of simple logical expressions into C0 assertions: e.g. assert($y \ge 0$);.

3.3.1 **Translating branch conditions**. Run-time checks contain branch conditions that denote the execution path a check is required on. For example, in Fig. 8 acyclic(\result) should only be checked at lines 20-21 when 1 == NULL, as indicated by the branch condition (none,1 == NULL,1 == NULL). Therefore, GVC0 first translates the condition 1 == NULL into C0 code. The origin and location pair (none,1 == NULL) tells GVC0 that 1 == NULL must be evaluated at the program point in insertLastWrapper containing the 1 == NULL AST element. As a result, in Fig. 9 a boolean variable

```
1:16
```

```
/*@ predicate acyclic(Node* 1) =
                                                           Node* insertLastWrapper(Node* 1, int val,
1
                                                       1
2
           1 == NULL ? true :
                                                             OwnedFields* _ownedFields)
                                                       2
             acc(1->val) && acc(1->next) &&
3
                                                       3
                                                          {
4
             acyclic(1->next) ;@*/
                                                             bool _cond_1 = 1 == NULL;
                                                       4
5
                                                       5
                                                             if (1 == NULL) {
6
    Node* insertLastWrapper(Node* 1, int val)
                                                               1 = alloc(struct Node);
                                                       6
7
      //@ requires ?;
                                                       7
                                                                1->_id = addStructAcc(_ownedFields, 2);
8
      //@ ensures acyclic(\result);
                                                       8
                                                                1->val = val; 1->next = NULL;
9
                                                       9
                                                             } else {
10
      if (1 == NULL) {
                                                                if (!_cond_1) { acyclic(l, _ownedFields); }
                                                      10
11
        1 = alloc(struct Node);
                                                      11
                                                                OwnedFields* _tempFields =
12
        1->val = val;
        1->next = NULL;
                                                                     initOwnedFields(_ownedFields->instCntr);
13
                                                      12
      } else {
14
                                                      13
                                                                sep_acyclic(l, _tempFields);
       (\{(none, 1 == NULL, \neg(1 == NULL))\},
15
                                                      14
                                                                1 = insertLast(1,val, _ownedFields);
         (l=insertLast(l,val), acyclic(l), acyclic(l))
16
                                                      15
17
         1 = insertLast(1, val);
                                                             if (_cond_1) { acyclic(l, _ownedFields); }
                                                      16
18
      }
                                                      17
                                                              OwnedFields* _tempFields1 =
19
       return 1:
                                                                  initOwnedFields(_ownedFields->instCntr);
                                                      18
       (\{(none, 1 == NULL, 1 == NULL)\},
20
                                                      19
                                                             sep_acyclic(l, _tempFields1);
         (none, acyclic(\result), acyclic(\result))
21
                                                      20
                                                             return 1;
22
                                                           }
                                                      21
        Run-time checks from Gradual Viper
                                                                        Run-time checks from GVC0
```

Fig. 8. Original insertLastWrapper program with run-time checks from Gradual Viper

Fig. 9. GVC0 generated insertLastWrapper program

_cond_1 is introduced on line 4 to hold the value of 1 == NULL. The condition variable _cond_1 is then used in the C0 run-time check for acyclic(\result) later in the program (line 16). To reduce run-time overhead, _cond_1 is also used in the check for acyclic(1) on line 10, which relies on the same branch point (none, 1 == NULL). While not demonstrated here, GVC0 supports short-circuit evaluation of conditions on the same execution path.

- **Translating predicates.** Now that GVC0 has translated the branch conditions in Fig. 8 into condition variables, GVC0 can use the variables to develop C0 run-time checks. The Gradual Viper check ($\{(none, 1 == NULL, \neg(1 == NULL))\}$, (1=insertLast(1, val), acyclic(1), acyclic(1))) is translated into if (!_cond_1) {acyclic(1,_ownedFields);} on line 10 in Fig. 9. GVC0 places this CO check according to the origin, location pair ((l=insertLast(1, val),acyclic(1)), which points to the program point just before the call to insertLast on line 14. The branch condition becomes the if statement with condition !_cond_1 (§3.3.1), and acyclic(1) is turned into the method call acyclic(1,_ownedFields). The acyclic method implements acyclic's predicate body as C0 code: it asserts true for empty lists and recursively verifies accessibility predicates (using _ownedFields) for nodes in non-empty lists. For efficiency, separation of list nodes is encoded separately on lines 11-13. We discuss the dynamic verification of accessibility predicates and the separating conjunction in C0 code next (§3.3.3). Finally, a similar C0 check is created for acyclic(\result) on lines 16-19.
- Translating accessibility predicates and separating conjunctions. GVC0 implements run-time tracking of owned heap locations in C0 programs to verify accessibility predicates and uses of the separating conjunction. An owned field is a tuple (id, field) where id is an integer identifying a struct instance and field is an integer indexing a field in the struct. Then, the OwnedFields struct instance called _ownedFields contains currently owned fields, and all struct definitions in a program are modified to contain an _id field. When a new struct instance is created—such as allocating a new node on line 6 in Fig. 9—the _id field is initialized with the value of a global integer instCntr

that uniquely identifies the instance. The call to library method addStructAcc on line 7 performs this functionality and then increments instCntr. It also adds all fields in the struct instance (e.g. 1-val:(1--id,0), 1--next:(1--id,1), and 1--id:(1--id,2)) to _ownedFields.

For methods with an imprecise⁴ pre- or postcondition, like insertLastWrapper, a parameter that initializes _ownedFields is added to the method's declaration (line 2, Fig. 9). When a method's precondition is imprecise, then any caller will pass all of its owned fields to the method, as on line 14 for the call to insertLast. After execution, the callee method returns all of its owned fields to the caller. When a method's precondition is precise, then any caller only passes its owned fields specified by the precondition to the method. If the method's postcondition is imprecise, then after execution the callee method returns all of its owned fields as before; otherwise, only the owned fields specified by the postcondition are returned. Finally, in precisely specified methods (no external—pre- and postconditions—or internal—loop invariants, unfolds, folds, etc.—specifications contain imprecision), GVC0 does not implement any _ownedFields tracking.

Now, _ownedFields tracking is used to verify accessibility predicates and uses of the separating conjunction. Run-time checks for accessibility predicates are turned into assertions that ensure the presence of their heap location in _ownedFields. For example, acc(1->val) looks like assertAcc(_ownedFields,1->_id,0); in C0 code, where 0 is the index for val in the Node struct. Wherever GVC0 must check separation of heap locations,—such as for the nodes in list 1 at lines 10-13—it creates (with the library method initOwnedFields) an auxiliary data structure _tempFields of type OwnedFields. We check that heap cells are disjoint by adding them one at a time to _tempFields and failing if the cell is already there. GVC0 generates a sep_X method for each predicate X to actually perform the separation check; and when done, discards _tempFields, as its purpose was only to check separation. Similar checks are created for the acyclic(\result) check on lines 16-19.

4 EMPIRICAL EVALUATION

The seminal work on gradual typing [Siek and Taha 2006] selectively inserts run-time casts in support of optimistic static checking: for instance, whenever a function application is deemed well-typed only because of imprecision—such as passing an argument of the unknown type to a function that expects an integer—the type-directed cast insertion procedure inserts a runtime check. But if the application is definitely well-typed, no cast is inserted. This approach ensures that a fully-precise program does not incur any overhead related to runtime type checking. While it is tempting to assume that more precision necessarily results in better performance, the reality has been shown to be more subtle: both the nature of the inserted checks (such as higher-order function wrappers) as well as when/how often they are executed is of utmost importance [Muehlboeck and Tate 2017; Takikawa et al. 2016], and anticipating the performance impact of precision is challenging [Campora et al. 2018].

The performance of gradual verification has never been studied until now, due to the lack of a working gradual verifier. Here, we explore the relation between minimizing dynamic check insertion with statically available information and observed run-time performance in gradual verification with Gradual C0. Specifically, we explore the performance characteristics of Gradual C0 for thousands of partial specifications generated from four data structures, as inspired by Takikawa et al. [2016]'s work in gradual typing. In particular, we observe how adding or removing individual atomic formulas and? within a specification impacts the degree of static and dynamic verification and, as a result, the run-time overhead of the program. Additionally, we compare the run-time performance of Gradual C0 to a fully dynamic approach, as readily available in C0. The aforementioned ideas are captured in the following research questions:

⁴Here, a formula is also imprecise if it contains predicates that expose? when fully unrolled—an equi-recursive treatment.

Example	Unverified Complexity	# Specs	Contents of Complete Spec						
Example	Chvermed Complexity		Fold	Unfold	Pre.	Post.	Pred. Body	Loop Inv.	
Binary Search Tree	$O(n \log(n))$	3344	43	23	0/20/20/23	0/22/3/23	6/6/7/4	0/0/4/2	
Linked List	O(n)	1728	17	10	8/6/15/5	4/5/6/5	4/3/4/3	4/2/5/2	
Composite	$O(n \log(n))$	2577	28	15	0/10/2/12	0/11/1/12	32/9/17/3	0/3/2/3	
AVI.	$O(n \log(n))$	3056	25	14	3/4/5/9	3/6/9/9	25/8/21/3	1/1/2/1	

Table 1. Description of benchmark examples. For each example, the table shows the complexity of the test program without verification, the number of sampled partial specifications, and the distribution of specification elements for the complete specification. Element counts are formatted as "Accessibility Predicate/Predicate Instance/Boolean Expression/Imprecision"

- **RQ1**: As specifications are made more precise, can more verification conditions be eliminated statically?
- **RQ2**: Does gradual verification result in less run-time overhead than a fully dynamic approach?
- **RQ3**: Are there particular types of specification elements that have significant impact in run-time overhead, and can high overhead be avoided?

4.1 Creating Performance Lattices

We define a *complete* specification as being statically verifiable when all ?s are removed, and then a *partial* specification as a subset of formulas from a complete specification that are joined with ?. Like Takikawa et al. [2016], we model the gradual verification process as a series of steps from an unspecified program to a statically verifiable specification where, at each step, an *element* is added to the current, partial specification. An element is an atomic conjunct (excluding boolean primitives) in any type of method contract, assertion, or loop invariant. We form a lattice of partial specifications by varying which elements of the complete specification are included. We also similarly vary the presence of ? in formulas that are complete—contain the same elements as their counterparts in the statically verifiable specification—and have related fold and unfold statements in the partial specification. Otherwise, ? is always added to incomplete formulas. This strategy creates lattices where the bottom entry is an empty specification containing only ?s and the top entry is a statically verifiable specification. A *path* through a lattice is the set of specifications created by appending *n* elements or removing ?s one at a time from the bottom to the top of the lattice. The large array of partial specifications created in each lattice closely approximates the ones supported by the gradual guarantee.

To illustrate the aforementioned approach, consider the following loop invariant:

```
//@ loop_invariant sortedSeg(list, curr, curr->val) && curr->val <= val;</pre>
```

The invariant is made of two elements: the sortedSeg predicate instance and the boolean expression curr->val <= val;. The lattice generated for a program with this invariant has five unique specifications: four contain a combination of the two elements joined with ?, and the fifth is the complete invariant above.

4.2 Data Structures

To apply this methodology, we implemented and fully specified four recursive heap data structures with Gradual C0: binary search tree, sorted linked list, composite tree, and AVL tree. Each data structure has a test program that contains its implementation and a main function that adds elements to the structure based on a workload parameter ω . We design the test programs to incur as little run-time overhead as possible outside of structure size and run-time checks. For each example and corresponding test program, Table 1 displays the distribution of elements in the complete specification, as well as the run-time complexity of the test program and the number of unique partial specifications generated by our benchmarking tool.

Binary Search Tree (BST). The implementation of the binary search tree is typical; each node contains a value and pointers to left and right nodes. We statically specify memory safety and preservation of the binary search tree property—that is, any node's value is greater than any value in its left subtree and less than any value in its right subtree. The test program creates a root node with value ω and sequentially adds and removes a set of ω values in the range $[0, 2\omega]$. Note that values are removed in the same order they were added.

Linked List. We implement a linked list with insertion similar to the one given in Fig. 1 and described in §2. Insertion is statically specified for memory safety as well as preservation of list sortedness. Its test program creates a new list and inserts ω arbitrary elements.

Composite. The composite data structure is a binary tree where each node tracks the size of its subtree—this is verified by its specification along with memory safety. Its test program starts with a root node and builds a tree of size ω by randomly descending from the root until a node without a left or right subtree is reached. A new node is added in the empty position, and then traversal backtracks to the root.

AVL Tree. The implementation of AVL tree with insertion is standard except that the height of the left and right subtrees is stored in each node (instead of the overall height of the tree). This allows us to easily state the AVL balanced property—for every node in the tree the height difference between its left and right children is at most 1—without using functions or ghost variables, which Gradual C0 does not currently support. In addition to specifying the AVL balanced property for insertion, we also specify memory safety. The AVL test program starts with a root node and builds a tree of size ω by inserting randomly valued nodes into the tree using balanced insertion.

4.3 Experimental Setup

With upwards of 100 elements in the specifications for each data structure, it is combinatorially infeasible to fully explore every partial specification. Therefore, unlike Takikawa et al. [2016], we proceed by *sampling* a subset of partial specifications in a lattice, rather than executing them all. Specifically, we sample 16 unique paths through the lattice from randomized orderings of specification elements. Every step is executed with three workloads chosen arbitrarily to ensure observable differences in timing. Each timing measurement is the median of 10 iterations. Programs were executed on four physical Intel Core i5-4250U 1.3GHz Cores with 16 GB of RAM.

We introduce two baseline verifiers to compare Gradual C0 against. The *dynamic verifier* transforms every specification into a run-time check and inserts accessibility predicate checks for field dereferences—thereby emulating a fully dynamic verifier. The *framing verifier* only performs the accessibility predicate checks, and therefore represents the minimal dynamic checks that must be performed in a language that checks ownership.⁵

4.4 Evaluation

Fig. 10 shows how the total number of verification conditions (proof obligations) changes as more of each benchmark is specified (green curve). The figure also similarly shows the number of verification conditions that are statically verified as each benchmark is specified (purple curve). From the green curve, we see that even when there are no specifications, there are verification conditions, *e.g.* before a field is accessed, the object reference must be non-null and the field must be owned. Some of these verification conditions can be verified statically as illustrated by the purple curve. As more of a benchmark is specified, there are more verification conditions (green curve); but

⁵These framing checks could fail, for example, if some function lower in the call stack owns data that is accessed by the currently-executing function.

Mean Verification Conditions

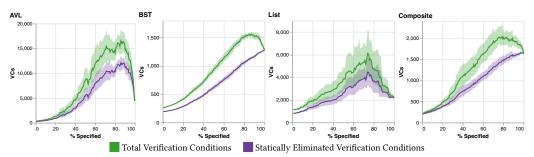


Fig. 10. For each example, the average quantity of verification conditions and the subset that were eliminated statically at each level of specification completeness across all paths sampled. Shading indicates the standard deviation.

Mean Execution Time Over All Paths

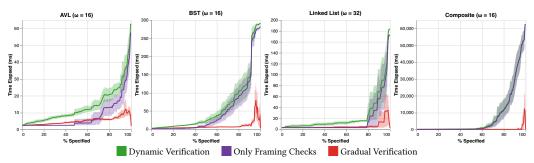


Fig. 11. The mean time elapsed at each step over the 16 paths sampled. Shading indicates the confidence interval of the mean for each verification type.

99th Percentile Changes in Run-time Overhead

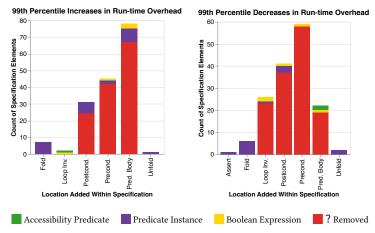


Fig. 12. The quantity of specification elements, grouped by type and location, that caused the highest $(P_{99\%})$ increases and decreases in time elapsed out of every path sampled

also, more of these verification conditions are discharged statically and do not have to be checked dynamically (purple curve). Towards the right end of the plots, the two curves converge until they meet when all the verification conditions are discharged statically. As a result, the answer to **RQ1** is *yes*. Note, the number of verification conditions does decrease when enough of the benchmark is specified. This is due to being able to prune execution paths with new static information.

The plots in Fig. 11 display the run-time performance (in red) of dynamically checking the verification conditions from Fig. 10. The plots also show how the run-time performance of the dynamic verifier (in green) and framing verifier (in purple) change as more of each benchmark is specified. The green lines show that as more properties are specified, the cost of run-time verification increases. With Gradual C0, some of these properties can be checked statically; therefore, the runtime cost of gradual verification, shown in red, is always lower than the cost of pure run-time verification.

Notably, the green and purple lines in all plots exhibit dramatic increases in run-time performance starting at around 60-80% specified all the way to 100%, where performance ends orders of magnitude higher than where it started to incline. This is caused by the owned fields passing strategy employed at method boundaries (described in §3.3.3) to verify memory safety at run time. To respect precondition abstractions, only owned fields specified by a callee's precondition are passed by the caller to the callee when the precondition is precise. Similarly, when a callee's postcondition is precise, then only the owned fields specified by the postcondition are passed back to the caller. Computing owned fields from precise contracts is costly; and even more-so for contracts containing recursive predicates like in our benchmarks. Further, our benchmarks call such methods frequently during execution. As a result, execution time increases significantly at each path step where one of the aforementioned methods gets a precise pre or postcondition from ? removal. This, of course, happens more frequently as more of a benchmark is specified. At 100% specified every method contract is precise, and so the owned fields passing strategy is used at every method call and return leading to the highest run-time costs for the dynamic and framing verifiers. Unlike these verifiers, Gradual C0 checks fully-specified methods completely statically and does not use the owned field passing strategy for calls to these methods. As a result, looking at the red lines, Gradual C0 is not as heavily affected by this phenomena as the other verifiers—the dramatic increases start at 90% specified or higher and the peaks are significantly less costly. Additionally, once a critical mass of specifications have been written, Gradual Co's cost decreases until the run-time verification cost is zero—which is the same as running the raw C0 version of the benchmark. If the spikes around 90% specified are too costly, production gradual verifiers can reduce them by employing more optimal permission passing strategies.

Table 2 displays summary statistics for Gradual C0's performance on every sampled partial specification compared to the dynamic verification baseline. Depending on the workload and example, on average Gradual C0 reduces run-time overhead by 2.4-91.9% (Table 2, Column 3) compared to the dynamic verifier. In fact, the speed-ups are more substantial as ω increases: -49.7%, -75.2%, -46.4%, and -91.9% at the largest ω values compared to -2.4%, -34.5%, -10.2%, and -61.3% at the lowest. While Gradual C0 generally improves performance, there are some outliers in the data (Table 2, Column 5) where Gradual C0 is slower than dynamic verification by 11.2-91.9%. Fortunately, for lattice paths that produce these poor-performing specifications, gradual verification still outperforms dynamic verification (on average) for 68.5-99.0% (Table 2, Column 7) of all steps. Further, these outliers are partially due to the bookkeeping we insert to track conditionals, which is unoptimized and could be improved.

Fig. 11 displays the average run-time cost across all paths under each of our benchmarks and verifiers. In all the plots, Gradual C0 significantly outperforms the dynamic verifier—the red lines are always well below the green ones. Therefore, the answer to **RQ2** is *yes*. For some early parts of

Example	ω	$\% \Delta t$, GV vs. DV			% Steps	GV < DV f	% Paths GV < DV			
		Mean	St. Dev.	Max	Min	Mean	St. Dev.	Max	Min.	% Tauls GV < DV
AVL	4	-2.4	11.9	91.9	-43.9	68.5	14.4	96.9	42.9	0
	8	-21.4	17.1	64.7	-79.7	94.0	4.9	100.0	83.9	6.3
	16	-49.7	22.5	28.6	-97.0	97.9	2.0	100.0	92,7	12.5
BST	4	-34.5	20.8	21.5	-76.3	97.8	1.6	99.0	93.3	0.0
	8	-59.8	24.8	11.2	-96.4	99.0	0.8	100.0	97.1	12.5
	16	-75.2	22	17.5	-99.4	99.0	0.9	100.0	96.7	12.5
Linked List	8	-10.2	14.8	66.2	-55.3	72.0	18.9	98.1	36.1	0
	16	-28.2	28.2	35.3	-90.1	72.6	20.0	99.1	41.7	0
	32	-46.4	41.2	26.8	-99.0	74.6	18.8	99.1	38.0	0
Composite	4	-61.3	26.2	45.7	-98.8	98.1	2.1	100.0	93.8	37.5
	8	-83.7	21.2	15.3	-99.9	98.5	1.7	100.0	94.4	37.5
	16	-91.9	19.3	15.5	-100.0	98.6	1.5	100.0	94.4	18.8

Table 2. Summary statistics for the performance of each example over 16 paths at selected workloads (ω), comparing gradual verification (GV) against dynamic verification (DV). The grouped column "% in Δt , GV. vs. DV." displays summary statistics for the percent decrease in time elapsed for each step when using GV versus DV. The column "% Steps GV-DV for Paths DV-GV" shows the distribution of steps that performed best under GV that were part of paths containing steps that performed better under DV. The final column shows the percentage of paths in which every step performed better under GV.

the path the cost of Gradual C0 is comparable to or exceeds the cost of the framing checks, but towards the end of the path, static optimization kicks in and Gradual C0 begins to significantly outperform it. Overall, Gradual C0 and thus gradual verification outperforms dynamic verification and framing checks for each of our examples to varying degrees, and is consistently better than both on average past the point of $2/3^{\rm rds}$ static completion.

Fig. 12 captures the impact that different types of specification elements (accessibility predicates, predicates, and boolean expressions) have on Gradual Co's run-time performance when specified in different locations. It also captures the impact removing? from a formula has on performance. Elements that when added or? that when removed from one step in a lattice path to another increase run-time overhead significantly (in the top 1%) are counted in the left sub-figure, and ones that decrease run-time overhead significantly (top 1%) are counted in the right sub-figure. The count for accessibility predicates is colored in green, predicates in purple, boolean expressions in yellow, and ? removal in red. Clearly, removing? from one step to another has the most significant impact on performance either way, whether it be increasing or decreasing run-time cost. When? is removed from preconditions, postconditions, and predicate bodies, run-time cost significantly increases. This corresponds with the dramatic increases in Fig. 11 caused by the owned fields passing strategy: removal of? in the aforementioned locations leads to precise pre- and postconditions that trigger the use of this costly strategy. Eventually, a critical mass of specifications are written so that when ?s are removed further this costly strategy is no longer necessary (i.e. when callee methods are full statically verified) and so run-time performance improves dramatically—the downward trends seen prior to full static specification in Fig. 11. This suggests a strategy for avoiding high checking costs: specify frequently-executed code in critical-mass chunks that are fully statically verifiable, leaving boundaries between statically and dynamically verified code in places that are executed less frequently. Overall, the answer to RQ3 is yes; we have identified some key contributors to run-time overhead, whose optimization is a promising direction for future work, and we have also identified strategies for minimizing run time overhead in practice.

Threats to Validity. Our test programs were executed on multiple devices, each with the same CPU and memory configuration. However, we did not otherwise control for differences in performance between devices. While the test programs we used are of sufficient complexity to demonstrate interesting empirical trends, they are not representative of all software. Further, the baseline we used for dynamic verification is entirely unoptimized as we naively insert a check for each written element of a specification. Due to computational constraints, only a small subset of over 2^{100}

possible imprecise specifications were sampled, and we did not use a formal criteria to choose our workload values. Finally, our partial specification generation strategy (§4.1) could be improved to further approximate those supported by the gradual guarantee. As such, while our results reveal interesting trends, including significant performance improvements by Gradual C0 over dynamic verification, more work is needed to validate the robustness of those trends.

4.5 Qualitative Experience with AVL Tree

Notably, it was our experience that the incrementality of gradual verification was very helpful for developing a complete specification of the AVL tree example. In particular, a run-time verification error from a partial specification helped us realize the contract for the rotateRight helper function was not general enough. We fully specified rotateRight and proved it correct. However, insert's preand postconditions were left as?, and so static verification could not show us that the contract proved for rotateRight was insufficiently general. Nevertheless, we ran the program; gradual verification inserted run-time checks, and the precondition for rotateRight failed. This early notification allowed us to identify the problem with the specification and fix it immediately. Otherwise, we would have had to get deep into the static verification of insert—a complicated function, 50 lines long, with lots of tricky logic and invariants—before discovering the error, and a lot of verification work built on the faulty specification would have had to be redone. Interestingly, it is conventional wisdom that one of the benefits of static checking is that you get feedback early, when it is easier to correct mistakes. Here, we encountered a scenario where gradual verification had a similar benefit over static verification! We found an error (in a specification) earlier than we would have otherwise, presumably saving time.

5 RELATED WORK

Much of the closely related work, particularly previous work on gradual verification [Bader et al. 2018; Wise et al. 2020], gradual typing [Herman et al. 2010; Siek and Taha 2006; Siek et al. 2015; Takikawa et al. 2016], and verification [Müller et al. 2016; Parkinson and Bierman 2005; Reynolds 2002; Smans et al. 2009], has been discussed throughout the paper. We detail the differences between our work and Wise et al. [2020]'s work (the most closely related work to ours) in §1 and §2.

Another closely related work is soft contract verification [Nguyen et al. 2014], which verifies dynamic contracts statically where possible and dynamically where necessary by utilizing symbolic execution. This hybrid technique does not rely on a notion of *precision*, which is central to gradual approaches and their metatheory [Siek et al. 2015]. Nguyen et al. [2014] use symbolic execution results directly to discharge proof obligations where possible, while Gradual C0 strengthens symbolic execution results to discharge proof obligations adhering to the theory of imprecise formulas from Wise et al. [2020]. Further, Nguyen et al. [2014]'s work is targeted at dynamic functional languages, while our work focuses on imperative languages. We also build in memory safety as a default, while Nguyen et al. [2014] do not.

Additional related work in gradual typing includes richer type systems such as gradual refinement types [Lehmann and Tanter 2017] and gradual dependent types [Eremondi et al. 2019; Lennon-Bertrand et al. 2022]. These systems focus on pure functional programming, while Gradual C0 targets imperative programs. There is an extensive body of work on optimizing run-time checks in gradual type systems. Muehlboeck and Tate [2017] show that in languages with nominal type systems, such as Java, gradual typing does not exhibit the usual slowdowns induced by structural types. Feltey et al. [2018] reduce run-time overhead from redundant contract checking by contract wrappers. They eliminate unnecessary contract checking by determining—across multiple contract checking boundaries for some datatype or function call—whether some of the contracts being

checked imply others. While the results in §4 are promising, we may be able to draw from the extensive body of work in gradual typing to achieve further performance gains.

Finally, work in formal verification contains approaches that try to reduce the specification burden of users—a goal of Gradual C0. Furia and Meyer [2010] infer loop invariants with heuristics that weaken postconditions into invariants. When that approach fails, verification also fails because invariants are missing. Similarly, several tools (Smallfoot [Berdine et al. 2005], jStar [Distefano and Parkinson J 2008], and Chalice [Leino et al. 2009]) use heuristics to infer fold and unfold statements for verification. In contrast, Gradual C0 does not fail solely because invariants, folds, or unfolds are missing; imprecision begets optimism. However, Gradual C0 may benefit from similar heuristic approaches by leveraging additional static information to further reduce run-time overhead.

6 CONCLUSION

Gradual verification is a promising approach to supporting incrementality and enhance adoptability of program verification. Users can focus on specifying and verifying the most important properties and components of their systems and get immediate feedback about the consistency of their specifications and the correctness of their code. By relying on symbolic execution, Gradual C0 overcomes several limitations of prior work on gradual verification of heap-manipulating programs. The experimental results show that our approach can reduce overhead dramatically compared to purely dynamic checking and confirms performance trends speculated in prior work. While more work remains to extend gradual verification (and Gradual C0) to the expressiveness of state-of-theart static program verifiers, we believe the symbolic-execution approach to gradual verification shows promise to make verification more adoptable in software development practice.

REFERENCES

Rob Arnold. 2010. *C0, an imperative programming language for novice computer scientists.* Master's thesis. Department of Computer Science, Carnegie Mellon University.

Johannes Bader, Jonathan Aldrich, and Éric Tanter. 2018. Gradual Program Verification. In *International Conference on Verification, Model Checking, and Abstract Interpretation*. Springer, 25–46.

Josh Berdine, Cristiano Calcagno, and Peter W O'hearn. 2005. Smallfoot: Modular automatic assertion checking with separation logic. In *International Symposium on Formal Methods for Components and Objects*. Springer, 115–137.

John Peter Campora, Sheng Chen, and Eric Walkingshaw. 2018. Casts and costs: harmonizing safety and performance in gradual typing. *PACM on Programming Languages* 2, ICFP (Sept. 2018), 98:1–98:30.

Edsger W. Dijkstra. 1975. Guarded Commands, Nondeterminacy and Formal Derivation of Programs. *Commun. ACM* 18, 8 (August 1975), 453–457.

Dino Distefano and Matthew J Parkinson J. 2008. jStar: Towards practical verification for Java. ACM Sigplan Notices 43, 10 (2008), 213–226.

Joseph Eremondi, Éric Tanter, and Ronald Garcia. 2019. Approximate Normalization for Gradual Dependent Types. *Proc. ACM Program. Lang.* 3, ICFP, Article 88 (jul 2019), 30 pages. https://doi.org/10.1145/3341692

Daniel Feltey, Ben Greenman, Christophe Scholliers, Robert Bruce Findler, and Vincent St-Amour. 2018. Collapsible Contracts: Fixing a Pathology of Gradual Typing. *Proc. ACM Program. Lang.* 2, OOPSLA, Article 133 (oct 2018), 27 pages. https://doi.org/10.1145/3276503

Carlo Alberto Furia and Bertrand Meyer. 2010. Inferring loop invariants using postconditions. In Fields of logic and computation. Springer, 277–300.

David Herman, Aaron Tomb, and Cormac Flanagan. 2010. Space-efficient gradual typing. 23, 2 (June 2010), 167–189.

Daniel Kästner, Xavier Leroy, Sandrine Blazy, Bernhard Schommer, Michael Schmidt, and Christian Ferdinand. 2017. Closing the gap—the formally verified optimizing compiler CompCert. In SSS'17: Safety-critical Systems Symposium 2017. CreateSpace, 163–180.

James C. King. 1976. Symbolic Execution and Program Testing. Commun. ACM 19, 7 (July 1976), 385-394.

Gerwin Klein, Kevin Elphinstone, Gernot Heiser, June Andronick, David Cock, Philip Derrin, Dhammika Elkaduwe, Kai Engelhardt, Rafal Kolanski, Michael Norrish, et al. 2009. seL4: Formal verification of an OS kernel. In *Proceedings of the ACM SIGOPS 22nd symposium on Operating systems principles*. ACM, 207–220.

Nico Lehmann and Éric Tanter. 2017. Gradual Refinement Types. SIGPLAN Not. 52, 1 (jan 2017), 775–788. https://doi.org/10.1145/3093333.3009856

- K Rustan M Leino, Peter Müller, and Jan Smans. 2009. Verification of concurrent programs with Chalice. In Foundations of Security Analysis and Design V. Springer, 195–222.
- Meven Lennon-Bertrand, Kenji Maillard, Nicolas Tabareau, and Éric Tanter. 2022. Gradualizing the Calculus of Inductive Constructions. *ACM Transactions on Programming Languages and Systems* 44, 2 (June 2022). https://doi.org/10.1145/3495528
- Xavier Leroy. 2009. Formal verification of a realistic compiler. *Commun. ACM* 52, 7 (2009), 107–115. http://xavierleroy.org/publi/compcert-CACM.pdf
- Fabian Muehlboeck and Ross Tate. 2017. Sound Gradual Typing is Nominally Alive and Well. *Proc. ACM Program. Lang.* 1, OOPSLA, Article 56 (oct 2017), 30 pages. https://doi.org/10.1145/3133880
- P. Müller, M. Schwerhoff, and A. J. Summers. 2016. Viper: A Verification Infrastructure for Permission-Based Reasoning. In *Verification, Model Checking, and Abstract Interpretation (VMCAI) (LNCS, Vol. 9583)*, B. Jobstmann and K. R. M. Leino (Eds.). Springer-Verlag, 41–62.
- Phúc C. Nguyen, Sam Tobin-Hochstadt, and David Van Horn. 2014. Soft Contract Verification. In Proceedings of the 19th ACM SIGPLAN International Conference on Functional Programming (Gothenburg, Sweden) (ICFP '14). Association for Computing Machinery, New York, NY, USA, 139–152. https://doi.org/10.1145/2628136.2628156
- Matthew Parkinson and Gavin Bierman. 2005. Separation logic and abstraction. In *ACM SIGPLAN Notices*, Vol. 40. ACM, 247–258.
- John C Reynolds. 2002. Separation logic: A logic for shared mutable data structures. In Logic in Computer Science, 2002. Proceedings. 17th Annual IEEE Symposium on. IEEE, 55–74.
- Jeremy G Siek and Walid Taha. 2006. Gradual typing for functional languages. In *Scheme and Functional Programming Workshop*, Vol. 6. 81–92.
- Jeremy G Siek, Michael M Vitousek, Matteo Cimini, and John Tang Boyland. 2015. Refined criteria for gradual typing. In LIPIcs-Leibniz International Proceedings in Informatics, Vol. 32. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik.
- Jan Smans, Bart Jacobs, and Frank Piessens. 2009. Implicit dynamic frames: Combining dynamic frames and separation logic. In European Conference on Object-Oriented Programming. Springer, 148–172.
- Alexander J Summers and Sophia Drossopoulou. 2013. A formal semantics for isorecursive and equirecursive state abstractions. In European Conference on Object-Oriented Programming. Springer, 129–153.
- Asumu Takikawa, Daniel Feltey, Ben Greenman, Max S. New, Jan Vitek, and Matthias Felleisen. 2016. Is Sound Gradual Typing Dead? SIGPLAN Not. 51, 1 (jan 2016), 456–468. https://doi.org/10.1145/2914770.2837630
- Jenna Wise, Johannes Bader, Cameron Wong, Jonathan Aldrich, Éric Tanter, and Joshua Sunshine. 2020. Gradual verification of recursive heap data structures. *Proceedings of the ACM on Programming Languages* 4, OOPSLA (2020), 1–28.

A SUPPLEMENTARY MATERIAL FOR GRADUAL CO: SYMBOLIC EXECUTION FOR EFFICIENT GRADUAL VERIFICATION

```
\begin{split} & \text{pc-add}(\pi,\ t) = \text{Let}\ (id,\ bc,\ pcs) :: \text{suffix match}\ \pi \\ & (id,\ bc,\ pcs \cup \{t\}) :: \text{suffix} \\ & \text{pc-push}(\pi,\ id,\ bc) = (id,\ bc,\ \emptyset) :: \pi \\ & \text{pc-all}(\pi) = \text{foldl}(\pi,\ \emptyset,\ (\lambda\ (id_i,\ bc_i,\ pcs_i),\ all_i\ .\ all_i \cup \{bc_i\} \cup pcs_i)) \end{split}
```

Fig. 13. Path condition helper functions

```
eval-p(\sigma, t, Q)
                                      = Q(\sigma, t)
eval-p(\sigma, x, Q)
                                      = Q(\sigma, \sigma.\gamma(x))
\operatorname{eval-p}(\sigma_1,\ op(\overline{e}),\ Q) = \operatorname{eval-p}(\sigma_1,\ \overline{e},\ (\lambda\ \sigma_2,\ \overline{t}\ .\ Q(\sigma_2,\ op'(\overline{t}))))
eval-p(\sigma_1, e.f, Q) = eval-p(\sigma_1, e, (\lambda \sigma_2, t.
                                         if (\exists \ f(r; \ \delta) \in \sigma_2.h . \mathrm{check}(\sigma_2.\pi, \ r=t)) then
                                             Q(\sigma_2, \delta)
                                         else if (\exists \ f(r; \ \delta) \in \sigma_2.h_? . \mathsf{check}(\sigma_2.\pi, \ r = t)) then
                                             Q(\sigma_2, \delta)
                                         else if (\sigma_2.\mathrm{isImprecise}) then
                                              if (\sigma_2.\mathcal{R}.\text{origin} = (\_, \text{ unfold } \mathbf{acc}(\_), \_)) then
                                                  e_t := translate(\sigma_2, t)
                                                  \mathcal{R}' := addcheck(\sigma_2.\mathcal{R}, e.f, acc(e_t.f))
                                             else
                                                  \mathcal{R}' := \sigma_2.\mathcal{R}
                                             \delta := fresh
                                             Q(\sigma_2\{\ h_?:=\sigma_2.h_?\cup f(t;\ \delta),\ \pi:=\operatorname{pc-add}(\sigma_2.\pi,\ \{t\neq\operatorname{null}\}),\ \mathcal{R}:=\mathcal{R}'\ \},\ \delta)
                                         else failure())
```

Fig. 14. Rules for symbolically executing expressions without introducing run-time checks (except for a special case for unfold)

In eval-p (Fig. 14), a special case (highlighted in blue) for unfold statements is added that creates run-time checks for field accesses in the unfolded predicate's body. This case ensures soundness when introducing branch condition variables in C0 programs during run-time verification. In our implementation of Gradual C0, these checks are optimized further as they are only produced for branch conditions in the predicate body rather than for the whole body.

```
=Q(\sigma, t)
eval-c(\sigma, t, Q)
                                    =Q(\sigma, \sigma.\gamma(x))
eval-c(\sigma, x, Q)
eval-c(\sigma_1, op(\overline{e}), Q) = eval-c(\sigma_1, \overline{e}, (\lambda \ \sigma_2, \overline{t} . Q(\sigma_2, \ op'(\overline{t})))
eval-c(\sigma_1, e.f, Q) = eval-c(\sigma_1, e, (\lambda \sigma_2, t.
                                      if (\exists f(r; \delta) \in \sigma_2.h . \mathsf{check}(\sigma_2.\pi, r = t)) then
                                          Q(\sigma_2, \delta)
                                       else if (\exists f(r; \delta) \in \sigma_2.h_? . \mathsf{check}(\sigma_2.\pi, r = t)) then
                                          Q(\sigma_2, \delta)
                                       else if (\sigma_2.isImprecise) then
                                          res, \_ := assert(\sigma_2.isImprecise, \sigma_2.\pi, t \neq \text{null})
                                          e_t := translate(\sigma_2, t)
                                          \mathcal{R}' := \operatorname{addcheck}(\sigma_2.\mathcal{R}, e.f, \operatorname{acc}(e_t.f))
                                          res \wedge Q(\sigma_2 \{ \mathcal{R} := \mathcal{R}' \}, fresh)
                                       else failure())
```

Fig. 15. Rules for symbolically executing expressions without modifying the optimistic heap and path condition

A.1 Diff and Translate

Algorithm 1 Generating minimal checks

```
1: function Diff(\phi)
         conjuncts \leftarrow CNF(\phi)
 2:
          \phi' \leftarrow \emptyset
 3:
 4:
          for c \leftarrow conjuncts do
              if !check(c) then
 5:
                   \phi' \leftarrow \phi' + c
 6:
              end if
 7:
         end for
 8:
         return \phi'
10: end function
```

Fig. 16. Algorithm for computing the diff between two symbolic values

The DIFF (Fig. 16) function finds a minimal run-time check from an optimistically asserted formula containing statically known information. It accomplishes this by first performing a standard transformation to conjunctive normal form (CNF) on the optimistically asserted formula, to extract the maximal number of top level conjuncts. It then attempts to call *check()* on each conjunct; it accumulates each conjunct for which the call does not succeed. The set of conjuncts which could not be statically discharged are returned as the final check.

The translate (Fig. 17) function lifts symbolic values to concrete values. Most symbolic values are directly translated to their concrete counterparts via recursive descent; the exception is variables, whose concrete values must be reconstructed by searching the program state known by the verifier. This is done by retrieving the states of the symbolic store, which contains mappings from concrete variables to symbolic variables, and the heap, which contains field and predicate permissions. When translate encounters a symbolic variable, it first retrieves all possible aliasing information from

Algorithm 2 Variable resolution procedure

```
1: function Translate-var(s, v)
         store \leftarrow \emptyset
 2:
         if s.oldStore then
 3:
             store \leftarrow s.oldStore
 4:
         else
 5:
 6:
             store \leftarrow s.store
         end if
 7:
         aliasList \leftarrow aliases(v, s.pathConditions + +s.heap + +s.optimisticHeap)
 8:
         heap \leftarrow s.heap + +s.optimisticHeap
 9:
         outputs \leftarrow \emptyset
10:
         for v \leftarrow aliases do
11:
             if c \leftarrow store.lookup(v) then
12.
                  outputs \leftarrow outputs + c
13.
             else
14.
                  if h \leftarrow heap.lookup(v) \&\&c \leftarrow store.lookup(h) then
15.
                      outputs \leftarrow outputs + c
16:
                  end if
17.
             end if
18.
         end for
19.
         return selectLongest(outputs)
20.
21: end function
```

Fig. 17. Translate's procedure for resolving variables

Gradual Viper's state. This includes all variables known to be equivalent to the translation target according to the path condition and the heap. If the translation target or one of its aliases exists as a value in the symbolic store, then the translator finds a key corresponding to it in the store and returns it. Note that multiple valid keys may exist for a particular symbolic variable, because Gradual Viper may have determined that multiple concrete values are equivalent at a particular program point. If the translation target is a field, then only the top level receiver (the variable on which fields are being accessed) or one of its aliases will exist in the store. The fields being accessed are resolved by mapping their corresponding heap entries, or any aliased heap entries, to a value in the symbolic store, and resolving the store entry as described. In particular contexts, TRANSLATE may be asked to translate a precondition for a method call, or a predicate body for an (un)fold statement. In these cases, an old store attached to the current symbolic state as described in 3.2.7 is retrieved, and its symbolic store and heap are used for translation. This causes variables in a precondition or predicate to be resolved to their concrete values at the call site, or site of unfolding. This enables run-time checks produced via translate to be straightforwardly emitted to the frontend. The portion of translate related to translating variables is shown in Fig. 17.

A.2 Symbolic production of formulas

The rules for produce are given in Fig. 6. Essentially, produce takes a formula and snapshot δ (mirroring the structure of the formula) and adds the information in the formula to the symbolic state, which is then returned to the continuation Q. An imprecise formula ? & $\& \phi$ has its static part ϕ produced into the current state σ alongside second(δ). Note the snapshot δ for an imprecise formula

looks like (unit, second(δ)) where unit is the snapshot for ? and second(δ) is the snapshot for ϕ . An imprecise formula also turns σ imprecise to produce the unknown information represented by ? into σ . For example, if the state is represented by the formula θ , then this rule results in ? && θ && ϕ . A symbolic value t is produced into the path condition of the current state σ . Also, the snapshot δ for t must be unit, so this fact is also stored in σ 's path condition. Then, σ is passed to Q.

The produce rule for expression e, first evaluates e to its symbolic value t using eval-p. Then, t is produced into the path condition of the current state σ_2 using the aforementioned symbolic value rule. Imprecision in the symbolic state can always provide accessibility predicates for fields also in the state. Therefore, when fields in e are added to an imprecise state, heap chunks for those fields do not have to already be in the state, e.g. the state ? && true becomes ? && true && e. This functionality is permitted by eval-p. Similarly, an imprecise formula always provides accessibility predicates for fields in its static part, e.g. the state true and produced formula ? && e results in the state ? && true && e. The goal of produce is not to assert information in the state, but rather add information to the state. So we reduce run-time overhead by ensuring no run-time checks are produced by produce even for verifying field accesses.

The rules for producing field and predicate accessibility predicates into the state σ_1 operate in a very similar manner. Thus, we will focus on the rule for fields only. The field e.f in $\mathbf{acc}(e.f)$ first has its receiver e evaluated to t by eval-pc, resulting in σ_2 . Then, using the parameter δ a fresh heap chunk $f(t; \delta)$ is created and added to σ_2 's heap h, which represents $\mathbf{acc}(e.f)$ in the state. Note, the disjoint union θ ensures $f(t; \delta)$ is not already in the heap before adding $f(t; \delta)$ in there. If the chunk is in the heap, then verification will fail. Further, $\mathbf{acc}(e.f)$ implies $e \neq \text{null}$ and so that fact is recorded in σ_2 's path condition as $t \neq \text{null}$.

When the separating conjunction ϕ_1 && ϕ_2 is produced, ϕ_1 is first produced and then afterwards ϕ_2 is produced into the resulting symbolic state. Note that the snapshot δ is split between the two formulas using first(δ) and second(δ). Finally, to produce a conditional, Gradual Viper branches on the symbolic value t for the condition e splitting execution along two different paths. Along one path only the true branch ϕ_1 is produced into the state, and along the other path only the false branch ϕ_2 is produced. Both paths follow the continuation to the end of its execution. More details about branching are provided next, as we describe Gradual Viper's branch function.

The branch function in Fig. 18 is used to split the symbolic execution into two paths in a number of places in our algorithm: during the production or consumption of logical conditionals and during the execution of if statements. One path (Q_t) is taken under the assumption that the parameter t is true, and the other $(Q_{\neg t})$ is taken under the assumption that t is false. For each path, a branch condition corresponding to the assumption made is added to σ . \mathcal{R} , as highlighted in blue. Additionally, paths may be pruned using check when Gradual Viper knows for certain a path is infeasible (the assumption about t would contradict the current path conditions). Now, normally, if either of the two paths fail verification, then branch marks verification as failed (\(\lambda \) the results). This is still true when σ (the current state) is precise. However, when σ is imprecise, branch can be more permissive as highlighted in yellow. If verification fails on one of two paths only (one success, one failure), then branch returns success (\vee the results). In this case, a run-time check (highlighted in blue) is added to \Re to force run-time execution down the success path only. Of course, two failures result in failure and two successes result in success (V the results). No run-time checks are produced in these cases, as neither path can be soundly taken or both paths can be soundly taken at run time respectively. Note that Gradual Viper being flexible in the aforementioned way is critical to adhering to the gradual guarantee at branch points.

```
\begin{aligned} & \operatorname{branch}(\sigma,\ e,\ t,\ Q_{t},\ Q_{\neg t}) = \\ & (\pi_{T},\ \mathcal{R}_{T}) := (\operatorname{pc-push}(\sigma.\pi,\ \operatorname{fresh},\ t),\ \operatorname{addbc}(\sigma.\mathcal{R},\ e,\ e)) \\ & (\pi_{F},\ \mathcal{R}_{F}) := (\operatorname{pc-push}(\sigma.\pi,\ \operatorname{fresh},\ \neg t),\ \operatorname{addbc}(\sigma.\mathcal{R},\ e,\ \neg e)) \\ & \operatorname{if}\ (\sigma.\operatorname{isImprecise})\ \operatorname{then} \\ & res_{T} := (\operatorname{if}\ \neg \operatorname{check}(\sigma.\pi,\ \neg t)\ \operatorname{then}\ Q_{t}(\sigma\{\pi\ :=\ \pi_{T},\ \mathcal{R}\ :=\ \mathcal{R}_{T}\ \})\ \operatorname{else}\ \operatorname{failure}()) \\ & res_{F} := (\operatorname{if}\ \neg \operatorname{check}(\sigma.\pi,\ t)\ \operatorname{then}\ Q_{\neg t}(\sigma\{\pi\ :=\ \pi_{F},\ \mathcal{R}\ :=\ \mathcal{R}_{F}\ \})\ \operatorname{else}\ \operatorname{failure}()) \\ & \operatorname{if}\ ((res_{T}\land \neg res_{F})\lor (\neg res_{T}\land res_{F}))\ \operatorname{then} \\ & \mathcal{R}' := \operatorname{addcheck}(\sigma.\mathcal{R},\ e,\ (\operatorname{if}\ (res_{T})\ \operatorname{then}\ e\ \operatorname{else}\ \neg e)) \\ & \mathcal{R} := \mathcal{R}\ \cup \mathcal{R}'.\operatorname{rcs.last} \\ & res_{T}\lor res_{F} \\ & \operatorname{else} \\ & (\operatorname{if}\ \neg \operatorname{check}(\sigma.\pi,\ \tau)\ \operatorname{then}\ Q_{\tau}(\sigma\{\pi\ :=\ \pi_{T},\ \mathcal{R}\ :=\ \mathcal{R}_{F}\ \})\ \operatorname{else}\ \operatorname{success}()) \\ & \wedge \\ & (\operatorname{if}\ \neg \operatorname{check}(\sigma.\pi,\ t)\ \operatorname{then}\ Q_{\neg t}(\sigma\{\pi\ :=\ \pi_{F},\ \mathcal{R}\ :=\ \mathcal{R}_{F}\ \})\ \operatorname{else}\ \operatorname{success}()) \\ & & \wedge \\ & \text{Handles imprecision} \qquad \qquad \operatorname{Handles}\ \operatorname{run-time}\ \operatorname{check}\ \operatorname{generation}\ \operatorname{and}\ \operatorname{collection} \end{aligned}
```

Fig. 18. branch function definition

A.3 Symbolic consumption of formulas

```
 \begin{aligned} \operatorname{consume}(\sigma_1,\ \theta,\ Q) &= \sigma_2 \ := \sigma_1\{\ h,\ \pi \ := \operatorname{consolidate}(\sigma_1.h,\ \sigma_1.\pi)\ \} \\ & \operatorname{consume}'(\sigma_2,\ \sigma_2.\operatorname{isImprecise},\ \sigma_2.h_7,\ \sigma_2.h,\ \theta,\ (\lambda\ \sigma_3,\ h_7',\ h_1,\ \delta_1\ . \\ & Q(\sigma_3\{\ h_7 \ := h_7',\ h\ := h_1\},\ \delta_1))) \end{aligned}   \begin{aligned} \operatorname{consume}(\sigma_1,\ ?\&\&\ \phi,\ Q) &= \sigma_2 \ := \sigma_1\{\ h,\ \pi \ := \operatorname{consolidate}(\sigma_1.h,\ \sigma_1.\pi)\ \} \\ & \operatorname{consume}'(\sigma_2,\ \operatorname{true},\ \sigma_2.h_7,\ \sigma_2.h,\ \phi,\ (\lambda\ \sigma_3,\ h_7',\ h_1,\ \delta_1\ . \\ & Q(\sigma_3\{\operatorname{isImprecise} := \operatorname{true},\ h_7 \ := \ \emptyset,\ h\ := \ \emptyset\ \},\ pair(unit,\ \delta_1)))) \end{aligned}   \end{aligned}  Handles imprecision  \end{aligned}  Handles run-time check generation and collection  \end{aligned}
```

Fig. 19. Rules for symbolically consuming formulas (1/3)

The goals of consume are 3-fold: 1) given a symbolic state σ and formula $\widetilde{\phi}$ check whether $\widetilde{\phi}$ is established by σ , *i.e.* $\widetilde{\phi}_{\sigma} \cong \widetilde{\phi}$ where $\widetilde{\phi}_{\sigma}$ is the formula which represents the state σ , 2) produce and collect run-time checks that are minimally sufficient for σ to establish $\widetilde{\phi}$ soundly, *i.e.* the red and green highlighting in Fig. 3, and 3) remove accessibility predicates and predicates that are asserted in $\widetilde{\phi}$ from σ . Note that \cong is the consistent implication described in §2 and formally defined by Wise et al. [2020]. The rules for consume are given in Fig. 19.

The consume function always begins by consolidating information across the given heap $\sigma_1.h$ and path condition $\sigma_1.\pi$. The invariant on the heap $\sigma_1.h$ ensures all heap chunks in $\sigma_1.h$ are separated in memory, e.g. $f(x; \delta_1) \in \sigma_1.h$ and $f(y; \delta_2) \in \sigma_1.h$ implies $x \neq y$. Similarly, $f(x; \delta_1) \in \sigma_1.h$ implies $x \neq null$. Therefore, such information is added to the path condition $\sigma_1.\pi$ during consolidation. Further, consolidate ensures $\sigma_1.h$ and $\sigma_1.\pi$ are consistent, i.e. do not contain contradictory information. We use the definition of consolidate from [Müller et al. 2016], without repeating it here.

```
consume'(\sigma, f_?, h_?, h, (e,t), Q)
                                                                                                           = res, \bar{t} := assert(\sigma.isImprecise, \sigma.\pi, t)
                                                                                                                \mathcal{R}' := \operatorname{addcheck}(\sigma.\mathcal{R}, e, \operatorname{translate}(\sigma, \overline{t}))
                                                                                                                res \wedge Q(\sigma \{ \mathcal{R} := \mathcal{R}' \}, \frac{h_2}{h_2}, h, unit)
consume'(\sigma_1, f_?, h_?, h, e, Q)
                                                                                                            = eval-c (\sigma_1 { isImprecise := f_? }, e, (\lambda \sigma_2, t .
                                                                                                       consume'(\sigma_2{ isImprecise := \sigma_1.isImprecise }, f_?, h_?, h, (e, t), Q)))
\operatorname{consume}'(\sigma_1, \ \ \frac{f_?}{f_?}, \ \ h, \ \operatorname{acc}(p(\overline{e})), \ Q) = \frac{\operatorname{eval-c}}{\operatorname{eval-c}}(\sigma_1\{\text{ isImprecise } := f_?\}, \ \overline{e}, \ (\lambda \ \sigma_2, \ \overline{t} \ ...)
                                                                                                                 \sigma_3 := \sigma_2 \{ \text{isImprecise} := \sigma_1. \text{isImprecise} \}
                                                                                                                  (h_1, \delta_1, b_1) := \text{heap-rem-pred}(\sigma_3.\text{isImprecise}, h, \sigma_3.\pi, p(\bar{t}))
                                                                                                                 if (\sigma_3.isImprecise) then
                                                                                                                      (h'_2, \delta_2, b_2) := \text{heap-rem-pred}(\sigma_3.\text{isImprecise}, h_2, \sigma_3.\pi, p(\bar{t}))
                                                                                                                      if (b_1 = b_2 = false) then
                                                                                                                            \mathcal{R}' := \operatorname{addcheck}(\sigma_3.\mathcal{R}, \operatorname{acc}(p(\overline{e})), \operatorname{acc}(p(\overline{e})))
                                                                                                                      else \mathcal{R}' := \sigma_3.\mathcal{R}
                                                                                                                       Q(\sigma_3\{\mathcal{R} := \mathcal{R}'\}, h'_2, h_1, (if (b_1) then \delta_1 else \delta_2))
                                                                                                                 else if (b_1) then Q(\sigma_3, \sigma_3.h_2, h_1, \delta_1)
                                                                                                                 else failure() ))
consume'(\sigma_1, f_1, h_2, h_3, h_4, acc(e.f), g_1 = eval-c (g_1 (isImprecise := g_2), g_2, g_3, g_4, g_4, g_5, g_7, g_8, g_8, g_9, g_9
                                                                                                                 \sigma_3 := \sigma_2 \{ \text{isImprecise} := \sigma_1. \text{isImprecise} \}
                                                                                                                res, \bar{t} := assert(\sigma_3.isImprecise, \sigma_3.\pi, t \neq \text{null})
                                                                                                                 \mathcal{R}' := \operatorname{addcheck}(\sigma_3.\mathcal{R}, \operatorname{acc}(e.f), \operatorname{translate}(\sigma_3, \overline{t}))
                                                                                                                 (h_1, \delta_1, b_1) := heap-rem-acc(\sigma_3.isImprecise, h, \sigma_3.\pi, f(t))
                                                                                                                 if (\sigma_3.isImprecise) then
                                                                                                                       (h_2', \delta_2, b_2) := heap-rem-acc(\sigma_3.isImprecise, h_2, \sigma_3.\pi, f(t))
                                                                                                                      if (b_1 = b_2 = false) then
                                                                                                                            \mathcal{R}'' := addcheck(\mathcal{R}', acc(e.f), acc(translate(\sigma_3, t).f))
                                                                                                                      else \mathcal{R}'' := \mathcal{R}'
                                                                                                                       Q(\sigma_3 \{ \mathcal{R} := \mathcal{R}'' \}, h'_2, h_1, (if (b_1) \text{ then } \delta_1 \text{ else } \delta_2))
                                                                                                                  else if (b_1) then Q(\sigma_3\{\mathcal{R} := \mathcal{R}'\}, \sigma_3.h_?, h_1, \delta_1)
                                                                                                                  else failure() )))
                                                                                                                          Handles run-time check generation and collection
```

Fig. 19. Rules for symbolically consuming formulas (2/3)

After consolidation, consume calls a helper function consume', which performs the major functionality of consume. Along with the state σ_2 from consolidation, consume' accepts a boolean flag, optimistic heap $\sigma_2.h_7$, regular heap $\sigma_2.h$, the formula to be consumed $\widetilde{\phi}$, and a continuation. The boolean flag sent to consume' controls how σ_2 provides access to fields in $\widetilde{\phi}$. When $\widetilde{\phi}$ is precise (is θ), then σ_2 provides access to fields in θ through heap chunks or imprecision where applicable. Therefore, in this case, the boolean flag is set to $\sigma_2.isImprecise$. However, when $\widetilde{\phi}$ is imprecise

Fig. 19. Rules for symbolically consuming formulas (3/3)

(i.e. ? && ϕ), then the boolean flag is set to true so access to fields in $\widetilde{\phi}$ is always justified: first by σ_2 if applicable and second by imprecision in $\widetilde{\phi}$. Copies of the optimistic heap $\sigma_2.h_?$ and regular heap $\sigma_2.h$ are sent to consume' where heap chunks from $\widetilde{\phi}$ are removed from them. If consume' succeeds, then when $\widetilde{\phi}$ is precise execution continues with the residual heap chunks. When $\widetilde{\phi}$ is imprecise execution continues with empty heaps, because $\widetilde{\phi}$ may require and assert any heap chunk in σ_2 . Residual heap chunks are instead represented by imprecision, i.e. execution continues with an imprecise state. Finally, consume' also sends snapshots collected for removed heap chunks to the continuation.

Rules for consume' can also be found in Fig. 19. Cases for expressions e, the separating conjunction ϕ_1 && ϕ_2 , and logical conditionals e? ϕ_1 : ϕ_2 are straightforward. Expressions are evaluated to symbolic values that are then consumed with the corresponding rule. In a separating conjunction, ϕ_1 is consumed first, then afterward ϕ_2 is consumed. The rule for logical conditionals evaluates the condition e to a symbolic value, and then uses the branch function to consume ϕ_1 and ϕ_2 along different execution paths. The case for $\mathbf{acc}(p(\overline{e}))$ is also very similar to the case for $\mathbf{acc}(e.f)$ that we discuss later in this section.

When a symbolic value t is consumed, the current state σ must establish t, i.e. $\sigma \cong t$, or verification fails. The assert function (defined in Fig. 21) implements this functionality. In particular, assert returns success() when π can statically prove t or when σ is imprecise and t does not contradict constraints in π —here, t is optimistically assumed to be true. Otherwise, assert returns failure(). When assert succeeds, it also returns a set of symbolic values \bar{t} that are residuals of t that cannot be proved statically by π . If t is proven entirely statically, then assert returns true. A run-time check is created for the residuals \bar{t} and is added to σ to be passed to the continuation Q. Note that translate is used to create an expression from \bar{t} that can be evaluated at run time. Further, the location e is the expression that evaluates to t and is passed to consume' alongside t. The heaps h? and t0 are passed unmodified to t2 alongside the snapshot t3 unit.

The consume' rule for accessibility predicates $\mathbf{acc}(e.f)$, first evaluates the receiver e to t using eval-c, the current state σ_1 , and the parameter $f_?$. The parameter $f_?$ is the boolean flag mentioned previously. Assigning $f_?$ to σ_1 .isImprecise during evaluation allows $f_?$ to control whether or not imprecision verifies field accesses. This occurs in all of the consume' rules where expressions and thus fields are evaluated. After evaluation, the isImprecise field is reset resulting in σ_3 , and assert is used to ensure the receiver t is non-null. If $t \neq \text{null}$ is optimistically true, a run-time check for $t \neq \text{null}$ at location $\mathbf{acc}(e.f)$ is created and added to σ_3 . \mathcal{R} . Next, heap-rem-acc is used to remove the heap chunks from heap h that overlap with or may potentially overlap with $\mathbf{acc}(e.f)$ in

Gradual C0 1:33

memory. The heap-rem-acc function is formally defined alongside a similar function for predicates (heap-rem-pred) in the Fig. 20. If a field chunk is not statically proven to be disjoint from $\mathbf{acc}(e.f)$, then it is removed. Further, since predicates are opaque, Gradual Viper cannot tell whether or not their predicate bodies overlap with $\mathbf{acc}(e.f)$. Therefore, predicate chunks are almost always considered to potentially overlap with $\mathbf{acc}(e.f)$. The only time this is not the case is if they both exist in the heap h, which ensures its heap chunks do not overlap in memory. The heap-rem-acc function also checks that $\mathbf{acc}(e.f)$ has a corresponding heap chunk in h. If so, its snapshot δ_1 is returned and b_1 is assigned true. Otherwise, a fresh snapshot is returned with false. If the current state σ_3 is imprecise, then heap chunks are similarly removed from h_7 and $\mathbf{acc}(e.f)$ is checked for existence in h_7 . If a field chunk for $\mathbf{acc}(e.f)$ is not found in either heap, then a run-time check is generated for it and passed to the continuation Q alongside the two heaps after removal and $\mathbf{acc}(e.f)$'s snapshot. Without imprecision, consume' will fail when a field chunk for $\mathbf{acc}(e.f)$ is not found in h.

```
heap-rem-pred(isImprecise, h, \ \pi, \ p(\bar{t}))= if \exists \ (p(\bar{r}; \ \delta) \in h \ . \ \operatorname{check}(\pi, \ \bigwedge \overline{t=r})) then  (h \setminus \{p(\bar{r}; \ \delta)\}, \ \delta, \ \operatorname{true})  else (\emptyset, \ \operatorname{fresh}, \ \operatorname{false}) heap-rem-acc(isImprecise, h, \ \pi, \ f(t))=h':=\operatorname{foldl}(h, \ \emptyset, \ (\lambda \ f_{src}(\bar{r}; \ \delta), \ h_{dst} \ .  if (\neg(|\bar{r}|=1) \ \| \ \neg(f=f_{src}) \ \| \ \neg \operatorname{check}(\operatorname{isImprecise}, \ \pi, \ t=r)) then  h_{dst} \cup f_{src}(\bar{r}; \ \delta)  else  h_{dst})) if \exists \ f(r; \ \delta) \in h \ . \ \operatorname{check}(\pi, \ t=r) \ \text{then}   (h', \ \delta, \ \operatorname{true})  else  h':=\operatorname{foldl}(h', \ \emptyset, \ (\lambda \ f_{src}(\bar{r}; \ \delta), \ h_{dst} \ .  if  (f_{src}(\bar{r}; \ \delta) \ \text{is a field chunk}) \ \text{then}   h_{dst} \cup f_{src}(\bar{r}; \ \delta)  else  h_{dst})) else  h_{dst}))  (h', \ \operatorname{fresh}, \ \operatorname{false})
```

Fig. 20. Heap remove function definitions

```
\mathsf{check}(\pi,\ t) = \mathsf{pc-all}(\pi) \Rightarrow t \mathsf{check}(\mathsf{isImprecise},\ \pi,\ t) = \begin{cases} \mathsf{true}, & \mathsf{true} & \mathsf{if}\ \mathsf{check}(\pi,\ t) \\ \mathsf{true}, & \mathsf{diff}(\mathsf{pc-all}(\pi),t) & \mathsf{if}\ (\mathsf{isImprecise} \land (\bigwedge \mathsf{pc-all}(\pi) \land t)_\mathsf{SAT}) \\ \mathsf{false}, & \mathsf{0} & \mathsf{otherwise} \end{cases} \mathsf{assert}(\mathsf{isImprecise},\ \pi,\ t) = \begin{cases} \mathsf{success}(), & \mathsf{\overline{t}} & \mathsf{if}\ (b = \mathsf{true}) \ \mathsf{where}\ b, & \mathsf{\overline{t}} & \mathsf{:=}\ \mathsf{check}(\mathsf{isImprecise},\ \pi,\ t) \\ \mathsf{failure}(), & \mathsf{0} & \mathsf{otherwise} \end{cases} Handles imprecision \mathsf{m} = \mathsf{Handles} \ \mathsf{run-time}\ \mathsf{check}\ \mathsf{generation}\ \mathsf{and}\ \mathsf{collection}
```

Fig. 21. Check and assert function definitions

A.4 Symbolic execution of statements

The exec rules for sequence statements, variable declarations and assignments, allocations, and if statements are pretty much unchanged from Viper. The only difference is that Gradual Viper's

```
= exec(\sigma_1, s_1, (\lambda \sigma_2 . exec(\sigma_2, s_2, Q)))
exec(\sigma_1, s_1; s_2, Q)
exec(\sigma, var x : T, Q)
                                                                                                 =Q(\sigma\{\gamma := \text{havoc}(\sigma.\gamma, x)\})
                                                                                                  = eval (\sigma_1, e, (\lambda \sigma_2, t \cdot Q(\sigma_2\{\gamma := \sigma_2.\gamma[x \mapsto t]\})))
\operatorname{exec}(\sigma_1, x := e, Q)
                                                                                                  = eval (\sigma_1, e, (\lambda \sigma_2, t \cdot consume (\sigma_2, acc(x.f), (\lambda \sigma_3, ...))
\operatorname{exec}(\sigma_1, x.f := e, Q)
                                                                                                          produce (\sigma_3, \mathbf{acc}(x.f) \&\& x.f = t, pair(fresh, unit), Q))))
\operatorname{exec}(\sigma, x := \operatorname{new}(\overline{f}), Q)
                                                                                                  = produce (\sigma\{\gamma := \text{havoc}(\sigma.\gamma, x)\}, \overline{\text{acc}(x.f)}, \text{ fresh}, Q)
\operatorname{exec}(\sigma_1, \ \overline{z} := m(\overline{e}), \ Q)
                                                                                                  = eval (\sigma_1, \overline{e}, (\lambda \sigma_2, \overline{t})).
                                                                                                         \mathcal{R}' := \sigma_2 . \mathcal{R} \{ \text{origin} := (\sigma_2, \overline{z} := m(\overline{e}), \overline{t}) \}
                                                                                                         consume (\sigma_2 \{ \mathcal{R} := \mathcal{R}' \}, meth_{pre} [\overline{meth_{args} \mapsto t}], (\lambda \sigma_3, \delta).
                                                                                                             \sigma_4 := \sigma_3 \{ \gamma := \text{havoc}(\sigma_3.\gamma, \overline{z}) \}
                                                                                                              produce (\sigma_4, meth_{post}[\overline{meth_{args} \mapsto t}][\overline{meth_{ret} \mapsto z}], fresh,
                                                                                                                  (\lambda \ \sigma_5 \ . \ Q(\sigma_5 \{ \mathcal{R} := \sigma_5.\mathcal{R}\{\text{origin} := \text{none}\} \})))))))
                                                                                                  = consume (\sigma_1, \phi, (\lambda \sigma_2, \delta)).
exec(\sigma_1, assert \phi, Q)
                                                                                                         well-formed (\sigma_2, ? \&\& \phi, \delta, (\lambda \sigma_3).
                                                                                                             Q(\sigma_1\{\pi := \sigma_3.\pi, \mathcal{R} := \sigma_3.\mathcal{R}\}))))
                                                                                                  = eval (\sigma_1, \overline{e}, (\lambda \sigma_2, \overline{t}).
\operatorname{exec}(\sigma_1, \text{ fold } \operatorname{acc}(p(\overline{e})), \ Q)
                                                                                                         \mathcal{R}' := \sigma_2.\mathcal{R}\{\text{origin} := (\sigma_2, \text{ fold } \mathsf{acc}(p(\overline{e})), \overline{t})\}
                                                                                                         consume (\sigma_2 \{ \mathcal{R} := \mathcal{R}' \}, pred_{body}[\overline{pred_{args} \mapsto t}], (\lambda \sigma_3, \delta).
                                                                                                              produce (\sigma_3 \{ \mathcal{R} := \sigma_3.\mathcal{R} \{ \text{origin} := \text{none} \} \}, \text{ acc}(p(\overline{t})), \delta,
                                                                                                                 Q)))))))
\operatorname{exec}(\sigma_1, \text{ unfold } \operatorname{acc}(p(\overline{e})), \ Q)
                                                                                                 = eval (\sigma_1, \overline{e}, (\lambda \sigma_2, \overline{t})).
                                                                                                         \mathcal{R}' := \sigma_2.\mathcal{R}\{\text{origin} := (\sigma_2, \text{ unfold } \mathsf{acc}(p(\overline{e})), \overline{t})\}
                                                                                                         consume (\sigma_2 \{ \mathcal{R} := \mathcal{R}' \}, \text{ acc}(p(\overline{t})), (\lambda \sigma_3, \delta).
                                                                                                              produce (\sigma_3, pred_{body}[\overline{pred_{args} \mapsto t}], \delta, (\lambda \sigma_4).
                                                                                                                 Q(\sigma_4 \{ \mathcal{R} := \sigma_4.\mathcal{R}\{\text{origin} := \text{none}\} \})))))))
\operatorname{exec}(\sigma_1, \text{ if } (e) \{ stmt_1 \} \text{ else } \{ stmt_2 \}, Q) = \frac{\operatorname{eval}}{\operatorname{eval}}(\sigma_1, e, (\lambda \sigma_2, t))
                                                         branch (\sigma_2, e, t, (\lambda \sigma_3 \cdot \text{exec}(\sigma_3, stmt_1, Q)), (\lambda \sigma_3 \cdot \text{exec}(\sigma_3, stmt_2, Q)))))
                                          Handles imprecision
                                                                                                 Handles run-time check generation and collection
```

Fig. 22. Rules for symbolically executing program statements (1/2)

versions of eval, produce, branch, and consume (defined previously) are used instead of Viper's. Statements in a sequence are executed one after another, and variable declarations introduce a fresh symbolic value for the variable into the state. Variable assignments evaluate the right-hand side to a symbolic value and update the variable in the symbolic store with the result. Allocations produce fresh heap chunks for fields into the state. Finally, if statements have their condition evaluated and then branch is used to split execution along two paths to symbolically execute the true and false branches.

Symbolic execution of field assignments first evaluates the right-hand side expression e to the symbolic value t with the current state σ_1 and eval. Any field reads in e are either directly or optimistically verified using σ_1 . Then, the resulting state σ_2 must establish write access to x.f in consume, i.e. $\sigma_2 \cong acc(x,f)$. The call to consume also removes the field chunk for acc(x,f) from

```
\operatorname{exec}(\sigma_1, \text{ while } (e) \text{ invariant } \overline{\phi} \text{ } \{ \text{ } stmt \text{ } \}, \text{ } Q) = \gamma_2 := \operatorname{havoc}(\sigma_1.\gamma, \overline{x})
                                         res_{body} := well-formed (
                                             \sigma_1\{\text{isImprecise}:=\text{false},\ h_?:=\emptyset,\ h:=\emptyset,\ \gamma:=\gamma_2,
                                                  \mathcal{R} := \sigma_1.\mathcal{R}\{\text{origin} := (\sigma_1, \text{ while } (e) \text{ invariant } \widetilde{\phi} \in \mathcal{M} \}, \text{ beginning}\}\}
                                             \frac{\delta}{\delta} && e, fresh, (\lambda \ \sigma_3 \{ \mathcal{R} := \sigma_3 . \mathcal{R} \{ \text{origin} := \text{none} \} \}.
                                                 exec(\sigma_3, stmt, (\lambda \sigma_4).
                                                     consume (\sigma_4 \{ \mathcal{R} := \sigma_4. \mathcal{R} \{ \text{origin} := (\sigma_4, \text{ while } (e) \text{ invariant } \widetilde{\phi} \{ \text{ stmt } \}, \text{ end} ) \} \}
                                                         \widetilde{\phi}, (\lambda \ \sigma_5, \ \_ . \Re := \Re \cup \sigma_5.\Re.rcs ; success()))))))
                                          res_{after} := consume (
                                             \sigma_1\{\mathcal{R}:=\sigma_1.\mathcal{R}\{\text{origin}:=(\sigma_4, \text{ while } (e) \text{ invariant } \widetilde{\phi} \text{ } \{ \text{ } \textit{stmt } \}, \text{ before})\}\} ,
                                             \widetilde{\phi}, (\lambda \ \sigma_2, \ \_.
                                                 produce (\sigma_2 \{ \gamma := \gamma_2, \sigma_2. \mathcal{R} \{ \text{origin} := (\sigma_2, \text{ while } (e) \text{ invariant } \widetilde{\phi} \{ \text{ } stmt \}, \text{ after} ) \} \}
                                                      \tilde{\phi} &&!e, fresh, Q)))
                                         if (\sigma_1.isImprecise) then
                                             if (\neg res_{body} \land res_{after}) then
                                                 \mathcal{R}' := addcheck(\sigma_1.\mathcal{R}, e, \neg e)
                                                 \Re := \Re \cup \mathcal{R}'.rcs.last
                                              (\neg res_{body} \lor res_{after}) \land (res_{body} \lor res_{after})
                                             res_{bodu} \land res_{after}
                         where \overline{x} are variables modified by the loop body
                                       Handles imprecision
                                                                                          Handles run-time check generation and collection
```

Fig. 22. Rules for symbolically executing program statements (2/2)

 σ_2 (if it is in there) resulting in σ_3 . Therefore, the call to produce can safely add a fresh field chunk for $\mathbf{acc}(x.f)$ alongside x.f = t to σ_3 before it is passed to the continuation Q. Under the hood, run-time checks are collected where required for soundness and passed to Q.

The exec rule for method calls similarly uses eval to evaluate the given args \overline{e} to symbolic values \overline{t} , asserts the method's precondition $meth_{pre}$ holds in the current state, consumes the heap chunks in the precondition, and produces the method's postcondition $meth_{post}$ into the continuation. Runtime checks are also collected where necessary (under the hood) and passed to the continuation. Note that the origin field of \mathcal{R} is set to $\overline{z}:=m(\overline{e})$ before consuming $meth_{pre}$ and reset to none after producing $meth_{post}$. Setting the origin indicates that run-time checks or branch conditions for $meth_{pre}$ or $meth_{post}$ should be attached to the method call statement rather than where they are declared. The origin arguments σ_2 and \overline{t} are used to reverse the substitution $[meth_{args} \mapsto t]$ in run-time checks and branch conditions for $meth_{pre}$ and $meth_{post}$. The rule for (un)folding predicates operates the same as for method calls where $meth_{pre}$ is the predicate body (predicate instance) and $meth_{post}$ is the predicate instance (predicate body). The origin is set to fold $acc(p(\overline{e}))$ and unfold $acc(p(\overline{e}))$ respectively.

In contrast, ϕ in assert ϕ maintains a none origin field, because ϕ 's use and declaration align at the same program location assert ϕ . The assert rule relies on consume to assert ϕ holds in

the current state σ_1 . If the consume succeeds, the state σ_1 is passed to the continuation nearly unmodified. Path condition constraints from ϕ hold in σ_1 either directly or optimistically. Therefore, these constraints are added to σ_1 to avoid producing run-time checks for them in later program statements. Run-time checks from the consume are also passed to the continuation. Note that ϕ is checked for well-formedness here (Fig. 23). A formula is well-formed if it contains? or accessibility predicates that verify access to the formula's fields (self-framing). Additionally, the formula cannot contain duplicate accessibility predicates or predicate instances. Finally, well-formed adds the formula's information to the given symbolic state. Here, ϕ does not need to be self-framed, and so it is joined with? in the call to well-formed.? verifies access to all of ϕ 's fields.

```
\label{eq:well-formed} \begin{split} \mathsf{well-formed}(\sigma_1,\ \widetilde{\phi},\ \delta,\ Q) &= \mathsf{produce}(\sigma_1,\ \widetilde{\phi},\ \delta,\ (\lambda\ \sigma_2\ . \\ &\quad \mathsf{produce}(\sigma_1\{\ \pi\ :=\ \sigma_2.\pi\ \},\ \widetilde{\phi},\ \delta,\ Q))) \end{split}
```

Fig. 23. Well-formed formula function definition

Finally, while the while loop rule is the largest rule and looks fairly complex, it just combines ideas from other rules that are discussed in great detail in this section and from the branch rule described in §A.2.

A.5 Valid program

A Gradual Viper program is valid if all of its method and predicate declarations are verified successfully as defined in Fig. 24. In particular, a method m's declaration is verified first by checking well-formedness of m's precondition $meth_{pre}$ and postcondition $meth_{post}$ using the empty state σ_0 (well-formedness is described in §A.4). Note, fresh symbolic values are created and added to σ_0 for m's argument variables \overline{x} and return variables \overline{y} . If $meth_{pre}$ and $meth_{post}$ are well-formed, then the body of m ($meth_{body}$) is symbolically executed (§A.4) starting with the symbolic state σ_1 containing $meth_{pre}$. Recall, well-formed additionally produces the formula that is being checked into the symbolic state. The symbolic state σ_2 is produced after the symbolic execution of $meth_{body}$. Then, $meth_{post}$ is checked for validity against σ_2 , i.e. σ_2 must establish $meth_{post}$ (§A.3). If $meth_{post}$ is established, then verification succeeds; and as a result, the run-time checks collected during verification are added to \mathfrak{R} (highlighted in blue). A valid predicate p is simply valid if p's body $pred_{body}$ is well-formed. As before, fresh symbolic values are created for p's argument variables \overline{x} . Note, no run-time checks are added to \mathfrak{R} here, because well-formedness checks do not produce any run-time checks.

```
 \text{verify(method} \quad m(\overline{x:T}) \text{ returns } (\overline{y:T})) = \underset{\text{well-formed}}{\text{well-formed}} (\sigma_0\{\gamma := \sigma_0.\gamma[\overline{x} \mapsto \text{fresh}][\overline{y} \mapsto \text{fresh}]\}, \quad meth_{pre}, \quad \text{fresh,} \quad (\lambda \ \sigma_1 \ . \\ \text{well-formed} \left(\sigma_1\{\underset{\text{isImprecise}}{\text{isImprecise}} := \text{false,} \ h_? := \emptyset, \quad h := \emptyset\}, \quad meth_{post}, \\ \text{fresh,} \quad (\lambda \ \_. \text{ success())}) \\ \wedge \\ \text{exec} \left(\sigma_1, \quad meth_{body}, \quad (\lambda \ \sigma_2 \ . \\ \text{consume} \left(\sigma_2, \quad meth_{post}, \quad (\lambda \ \sigma_3, \ \_. \\ \\ \Re := \Re \cup \sigma_3.\Re.\text{rcs} \ ; \quad \text{success())}))))) \\ \text{verify(predicate} \quad p(\overline{x:T})) \\ \text{= well-formed} \quad (\sigma_0\{\gamma := \sigma_0.\gamma[\overline{x} \mapsto \text{fresh}]\}, \quad pred_{body}, \quad \text{fresh,} \quad (\lambda \ \_. \text{ success())}) \\ \text{Handles imprecision} \quad \text{= Handles run-time check generation and collection}
```

Fig. 24. Rules defining a valid Gradual Viper program