Energy Prices and Household Heterogeneity: Monetary Policy in a Gas-TANK*

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First Version: October 18, 2022

This Version: April 14, 2023

Abstract

How does household heterogeneity affect the transmission of an energy price shock? What are the implications for monetary policy? We develop a small, open-economy TANK model that features labor and an energy import good as complementary production inputs (Gas-TANK). Given such complementarities, higher energy prices reduce the labor share of total income. Due to borrowing constraints, this translates into a drop in aggregate demand. Higher price flexibility insures firm profits from adverse energy price shocks, further depressing labor income and demand. We illustrate how the transmission of shocks in a RANK versus a TANK depends on the degree of complementarity between energy and labor in production and the degree of price rigidities. Optimal monetary policy is less contractionary in a TANK and can even be expansionary when credit constraints are severe. Finally, the contractionary effect of an energy price shock on demand cannot be generalized to alternate supply shocks, as the specific nature of the supply shock affects how resources are redistributed in the economy.

JEL Codes: F41, E52, E31, E21, E23

*The views expressed do not necessarily reflect the views of the Central Bank of Paraguay. We would like to thank Florin Bilbiie, Robin Braun, Cristiano Cantore, Ambrogio Cesa-Bianchi, Edouard Challe, Luca Fornaro, Andrea Gazzani, Jordi Gali, Diego Känzig, Tasos Karantounias, Michael Kumhof, Gergő Motyovszki, Omar Rachedi, Ricardo Reis, Frank Smets, and seminar/conference participants at the Bank of England, 25th Central Bank Macroeconomic Modeling Workshop, De Nederlandsche Bank 25th Annual Research Conference, and the Banque de France – Pierre Werner Chair (EUI) Conference for helpful comments and suggestions. We are especially grateful to Diego Känzig, Zaar Khan, and Jamie Lenney for sharing their data. All errors are our own.

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1 Introduction

In early 2022, energy prices rose to historically high levels as Russia's invasion of Ukraine increased the risk of disruptions to the energy trade (Figure 1). From the standpoint of an energy importer such as the UK or the EA, the developments in global energy prices represent a deterioration in the terms of trade. This implies a contraction in income flowing to domestic production inputs, including labor income. If households face limits in their access to financial markets, the contraction in income can translate into a drop in aggregate demand. That is, a supply shock can have demand side effects.

We highlight the demand side effects of this supply shock with a two-agent New Keynesian (TANK) model where agents differ in their sources of income and ability to smooth consumption. We use this setting to show that the implications for aggregate demand and inflation depend on how the cost of the energy price shock is distributed between the labor share and profit share of total income, and the degree of credit constraints. The model features two types of households: *constrained worker households*, who consume out of their labor income and have no access to financial markets, and *unconstrained households*, who earn firm profits and have free access to financial markets.[1] Our small, open-economy model also features labor and imported energy as complementary inputs in production. We assume a constant elasticity of substitution (CES) production technology with low elasticity of substitution between labor and energy, which allows the labor share of total income to fall as energy prices increase.

We show that the impact of energy prices on demand depends critically on the substitutability of production inputs and household heterogeneity. This is because the degree of substitutability among production inputs determines the response of workers' income to the shock. Due to borrowing constraints, this affects aggregate demand. Compared to the representative household in a RANK (representative agent New Keynesian) model, the constrained worker household will experience a stronger consumption response to the real income squeeze following an energy price shock because of its inability to smooth consumption by borrowing.[2] The channels we highlight are absent in the standard RANK model, which assumes all households are the same and that they can borrow to smooth consumption in the presence of adverse shocks. We illustrate this mechanism in a small, stylized model and embed it in medium scale model that is amenable for studying optimal policy.

The magnitude of these channels depends on the degree of price rigidity and the elasticity of substitution between energy and labor. Assuming production inputs are sufficiently difficult to substitute or that prices are sufficiently flexible, an energy price shock has a negative impact on aggregate demand.[3] This supply shock therefore has a self-correcting effect, as the consequent contraction in aggregate demand dampens inflationary pressures.

Is the demand contraction that follows a rise in energy prices a common feature of supply disturbances? We consider the dynamics following a productivity shock in our TANK model.[4] Both an in-

[1]Heterogeneous-agent New Keynesian (HANK) models are not analytically tractable, as they typically feature a wealth distribution that responds endogenously to aggregate shocks. Their complexity makes it difficult to identify the mechanisms at work. Debortoli and Galí (2017) show that a two-agent New Keynesian (TANK) model is able to capture fundamental properties of heterogeneous-agent models. Such models admit analytical solutions and can be extended to match the implications of HANK models, in terms of consistency with micro data and predictions for the macroeconomic effects of policy (Blanchard and Gali, 2007; Bilbiie, 2008; Cantore and Freund, 2021).

[2]In other words, while an energy price shock is a supply shock in a RANK model, it has elements of both a supply and demand shock in our TANK model.

[3]The aforementioned contraction in aggregate demand can be moderated by the behavior of markups. Given price rigidities, an increase in energy prices reduces firms' markups. This redistributes income in favor of constrained worker households, hence increasing aggregate demand. Instead, with higher price flexibility, firms are able to pass the cost of the more expensive energy to workers by raising prices.

[4]An energy price shock has also traditionally been modeled as a technology shock, or a shock that affects the productive capacity of the economy (Bruno and Sachs (1985), see Kilian (2008) for references).

crease in energy prices and an adverse productivity shock raise firms' marginal costs, leading to an increase in inflation. While the supply-side impact is the same, energy prices and productivity shocks yield opposing effects on the demand side. An adverse productivity shock leads to a fall in markups, as firms must hire more labor for the same amount of output. This increases constrained worker households' income, which boosts aggregate demand. However, an energy price shock in our model lowers constrained worker households' income and leads to a fall in economic activity. We conclude that no generalization can be made about the effects of supply shocks on aggregate demand, as the nature of the shock crucially affects the way resources are redistributed in the economy.

Next, we consider a normative question: what is the optimal response of monetary policy to an energy price shock in our model and how does it depend on the degree of household heterogeneity? In contrast to a RANK economy, energy price shocks in the TANK economy have both supply and demand side effects. On the one hand, higher energy prices place upward pressure on inflation, which calls for a monetary policy tightening. On the other hand, it restricts aggregate demand, which instead calls for a monetary loosening. In our baseline calibration, we find that in both the RANK and the TANK models, optimal monetary policy is contractionary in order to counteract the inflationary effect of the shock. However, in the TANK model, the negative impact of higher energy prices on aggregate demand mitigates inflationary pressures. An energy price shock therefore has a milder inflationary effect in the medium term, which requires a milder increase in the interest rate.[5] Finally, we explore conditions under which optimal policy may actually be expansionary in the presence of an adverse supply shock. We find that this is true when the share of financially constrained households increases.[6]

To sharpen our results, we first present the case where energy enters only as a production input as the baseline. We then consider the other extreme, where energy enters only as a component of households' consumption basket. Although energy consists of an equal proportion in both constrained and unconstrained households' consumption baskets, the energy price shock is still regressive.

With energy in the consumption basket, the effects of an energy price shock depend on the elasticity of substitution between energy and the domestically produced good. Given complementarities, costlier energy reduces the share of domestic goods in households' expenditure. As less resources are devoted to the purchase of domestically produced goods, households' income falls. While unconstrained households can maintain their consumption levels by borrowing from the foreign sector, constrained worker households must reduce their consumption, causing inequality to rise and aggregate demand to decline.

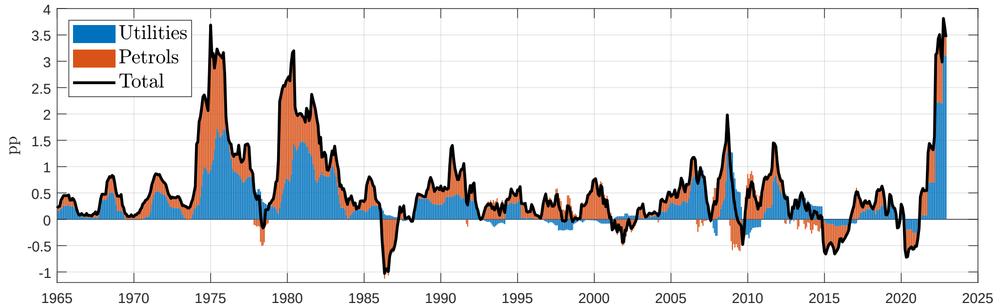
Moreover, when energy is in the consumption basket, inequality also rises due to the response of markups to the energy price shock. When energy is an input for firms, costlier energy transmits only gradually to the price of consumption goods, resulting in a decrease in markups. Profits partially absorb the effects of costlier energy, limiting the impact of the shock on the constrained worker households. However, when energy enters directly into the consumption basket, markups no longer absorb the shock, which exacerbates the impact of the shock on inequality.

[5] Recent work by Guerrieri et al. (2022a) and Caballero and Simsek (2022) also provides conditions under which optimal monetary policy is less contractionary in response to supply shocks.

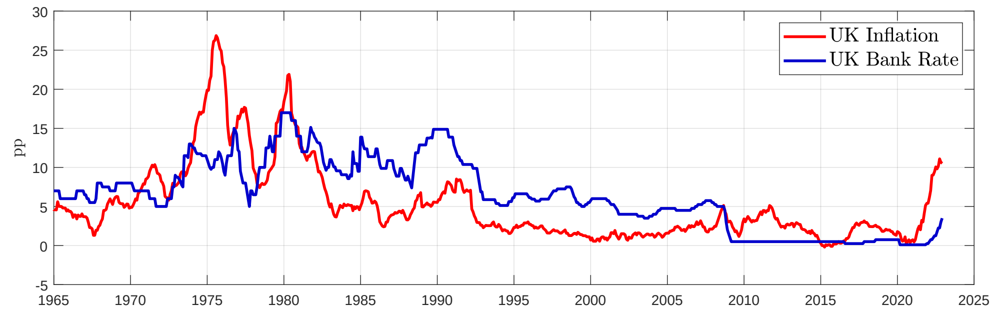
[6] Higher price flexibility also warrants more expansionary policy. As we emphasize throughout this paper, the demand effect of higher energy prices crucially depends on the evolution of firms' markups. If firms are able to increase prices to preserve markups, the costs of the energy price shock will be passed to workers, who will experience a more severe reduction in their income. Assuming a higher degree of price flexibility, constrained households experience a more pronounced drop in their income relative to unconstrained households, as reflected by the income gap. This leads to a deeper contraction in aggregate demand, which warrants looser monetary policy in the TANK model relative to its RANK counterpart.

FIGURE 1: UK Energy Prices and CPI Inflation

Notes: This panel shows the oil and gas spot prices for the UK, in £ per barrel and pence per therm, respectively. In mid-2022 the price of gas (blue line) had increased ten-fold, from an average of around 35 pence per therm before 2020 to a peak of around 350 pence per therm. Around the same time, the Sterling oil price (red line) reached an all-time high of 100£ per barrel.



Notes: A historical decomposition shows that these price increases have been a key driver of the high inflation rates that materialized in the UK in 2022. Almost 4 percentage points of the UK's 11% CPI inflation can directly be attributed to energy prices (blue bars). While the energy price shocks of the 1970s contributed to inflation mainly via increases in petrol prices, the shock of 2022 mainly contributed to inflation via an increase in utility prices.



Notes: We show the UK's CPI inflation and Bank Rate series. It is worth noting that inflation in the 1970s reached peaks above 20%, more than twice the peak of 2022/23, while the direct contribution of energy prices was broadly similar.

1.1 Related Literature

This paper contributes to a literature that emphasizes the demand side effects of an energy price shock.[7] While such shocks have traditionally been modeled as aggregate supply shocks or as technology shocks in domestic production, such approaches are unable to explain large fluctuations in real output (Kilian, 2008). More recent approaches place the main transmission channel on the demand side of the economy. That is, energy price shocks affect the economy primarily through their effect on consumer expenditures and firm investment expenditures. Hamilton (2008) provides evidence to show that energy price shocks mainly affect the economy through a disruption in consumers' and firms' spending on

[7] The mechanism in our model relies on complementarities between production inputs. Supply shocks with demand side effects can also be found in models with complementarities between consumption goods and distribution services (Corsetti et al., 2008) and complementarities among sectors (Guerrieri et al., 2022b; Cesa-Bianchi and Ferrero, 2021).

non-energy goods and services. Among firms, there is evidence that energy price shocks are perceived as shocks to product demand (Lee and Ni, 2002). Finally, among policymakers, an increase in energy prices is also thought to slow economic growth primarily through its effects on consumer spending (Natal, 2012).[8] This paper formalizes this intuition by allowing energy prices to affect aggregate demand through a heterogeneous impact on households depending on their sources of income and access to borrowing. To our knowledge, this is the first paper to explore this transmission channel and to study the optimal monetary policy response when accounting for such demand side effects of an energy price shock.

Recent studies have noted the distributional impact of the energy price shock due to its effect on the consumption baskets of heterogeneous households (Celasun et al., 2022; Bachmann et al., 2022; Battistini et al., 2022; Hobijn and Lagakos, 2005). An increase in energy prices can affect households' purchasing power through higher prices for energy products. Since poorer households spend a relatively large percentage of their income on energy, they receive a larger hit in terms of inflation when energy prices increase. We show that the shock can be regressive through an alternate channel, through a heterogeneous impact on households depending on their income sources and ability to smooth consumption.[9] Moreover, the shock also affects aggregate demand since financially constrained households will reduce purchases of other goods.[10] Känzig (2021) shows that carbon taxation imposes a larger burden on low-income households. The indirect, general equilibrium effects of carbon taxation via income and employment is estimated to be over 80%. In contemporaneous work, Pieroni (2022) also considers the transmission of an energy price shock in a heterogeneous agent model, but not the normative effects of monetary policy in an open economy setting.

Our paper also contributes to a literature that studies the transmission of shocks in a heterogeneous agent model. The interaction of household heterogeneity with nominal rigidities can amplify the contractionary effect of TFP shocks on employment (Furlanetto and Seneca, 2012) and fiscal policy shocks on output (Galí et al., 2007). However, we show that an interaction between household heterogeneity and production complementarity is crucial to generate the contractionary effect of an energy price shock on output. Our assumption of a CES production function with labor and energy allows for changes in energy prices to affect energy costs as a share of total income.[11]

More broadly, this paper builds on the vast literature that studies the implications of household heterogeneity for macroeconomic dynamics (Auclert et al., 2018; Bilbiie, 2008; Debortoli and Galí, 2017; Kaplan and Violante, 2018; Bilbiie, 2019; Acharya and Dogra, 2020; Bilbiie, 2020; Broer et al., 2020; Bilbiie and Ragot, 2021; Cantore and Freund, 2021; Bilbiie et al., 2022). Ravn and Sterk (2021) also show that a supply shock, namely productivity, can have effects on the demand side due to incomplete markets, sticky prices, and endogenous unemployment risk. The precautionary savings motive is central to their results, which is a different mechanism from ours.

Finally, we contribute to a literature that examines the implications of different monetary policy

[8]See Kilian (2008) for a discussion of this literature.

[9]In the UK, about X of all energy use takes the form of final consumption (the use of such products by consumers). The remainder involves energy being used in the production of non-energy goods and services (intermediate consumption).

[10]If these costs can be passed onto the final prices, then this affects households' purchasing power directly.

[11]The CES production function is a common feature of models that incorporate energy in the production function. Most recently, Bachmann et al. (2022) show that the losses to the German economy of an embargo on Russian energy imports depend crucially on the degree of substitutability between gas and other inputs. They show that the assumption of Leontief production is inconsistent with empirical evidence and leads to a number of implausible predictions with regard to the evolution of marginal products, prices, and expenditure shares. For example, production would drop one-for-one with energy supply in case of zero substitutability between production inputs. Note that in the case of a Cobb-Douglas production technology (elasticity of substitution equal to 1), energy prices have no impact on the labor share of total factor expenditure. Instead, they would only reduce firms' markups, redistributing income in favor of constrained worker households, which increases aggregate demand.

reactions to energy price shocks. The most closely related papers are Natal (2012) and Montoro (2012), which abstract from household heterogeneity. They show that when energy is a complementary input in production, an endogenous cost-push shock arises from the gap between the natural and efficient level of output. In Montoro (2012), a low elasticity of substitution between labor and energy leads to a trade-off between stabilizing output and inflation. This tradeoff is generated by the convexity of real marginal costs with respect to the real oil price, which produces a time-varying wedge between the marginal rate of substitution and the marginal productivity of labor. Eliminating the distortions in the steady state makes the wedge less sensitive to the energy price. Similarly, in Natal (2012), the impact of an energy price shock on the oil cost share (and therefore output) in the flexible prices and wages equilibrium is larger when the steady state distortion due to monopolistic competition is larger. Natural (distorted) output falls by more than efficient output, which increases the cost of strictly stabilizing inflation.[12] [13]

The rest of this paper is structured as follows. We discuss our model in Section 2, with an emphasis on the key features: household heterogeneity and product input complementarity. This leads us to Section 2.4, which shows how these features allow for the demand side effects of an energy price shock. Section 3 presents the baseline calibration and impulse response functions, which illustrates the transmission channels we discuss. We show how the magnitude of the various channels depend on the severity of credit constraints and the degree of substitutability between production inputs. In Section 4, we compare the dynamics of an energy price shock to alternate supply shocks. We consider optimal monetary policy in Section 5. Section 6 explores an extension with energy as a consumption good. Sensitivity checks are presented in Section 7. Finally, we discuss some empirical evidence in Section **??**. Section 8 concludes the paper.

2 Baseline model

We begin our discussion of the baseline model with a focus on two key model features: household heterogeneity and imported energy as a complementary input to production.

2.1 Household Heterogeneity

Unconstrained Households A fraction $(1-\omega)$ of households are financially *unconstrained* (denoted by u). They consume $C_{u,t}$, supply labor $N^h_{u,t}$ to unions, save in domestic (foreign) nominal riskless bonds $B_{u,t}$ $(B^*_{u,t})$, and receive profits from firm ownership $DIV^F_{u,t}$. Their lifetime utility is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{u,t}^{1-\sigma}-1}{1-\sigma} - \chi \frac{(N^h_{u,t})^{1+\varphi}}{1+\varphi} \right).$$

Unconstrained households maximize their lifetime utility subject to their budget constraint

$$W^h_t N^h_{u,t} + R_{t-1}B_{u,t-1} + \mathcal{E}_t R^*_{t-1} B^*_{u,t-1} + DIV^F_{u,t} + DIV^L_{u,t} = P_t C_{u,t} + B_{u,t} + \mathcal{E}_t B^*_{u,t} + T_{u,t}, \tag{2.1}$$

[12] Under Cobb-Douglas production, cost shares are constant regardless of the monopolistic competition distortion. This means that natural (distorted) output falls just as much as efficient output following an oil price shock, and perfectly stabIlizing prices is the optimal policy to follow.

[13] Several papers also study how monetary policy should respond depending on whether the energy price shock is demand or supply driven. Plante (2014) considers this question in a closed economy model, while Stevens (2015) considers this in an open economy model and finds that despite differences in the transmission of an energy demand and an energy supply shock, optimal monetary policy remains largely the same. However, Bodenstein et al. (2008) show that the source of an oil shock matters greatly for the optimal monetary policy response to fluctuations in energy prices.

where R_{t-1} (R^*_{t-1}) denotes the gross nominal rate of return on domestic (foreign) bonds, P_t is the price of the consumption good,[14] $\mathcal{E}_t$ is the nominal exchange rate, $DIV^F_{u,t}$ represents profits derived from firm ownership, $DIV^L_{u,t}$ are profits transferred to the household by labor unions and $T_{u,t}$ are government lump-sum transfers. The unconstrained household's consumption-savings Euler equation is given by

$$1 = \mathbb{E}_t \left[\Lambda_{u,t,t+1} \frac{R_t}{\Pi_{t+1}} \right], \tag{2.2}$$

where $\Pi_t \equiv P_t / P_{t-1}$, $\Lambda_{u,t,t+1} \equiv \beta \left(C_{u,t} / C_{u,t+1} \right)^{\sigma}$ and the UIP condition is given by

$$0 \quad = \quad \mathbb{E}_t \left[\Lambda_{u,t,t+1} \frac{1}{\Pi_{t+1}} \left(R_t - R^*_t \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) \right]. \tag{2.3}$$

Constrained Households The remaining fraction ω of households are *financially constrained* (denoted by c). They only receive labor income, hence their consumption is given by

$$P_t C_{c,t} = W^h_t N^h_{c,t} + DIV^L_{c,t} - T_{c,t}. \tag{2.4}$$

The wage received by households W^h_t is determined as a function of a weighted average of unconstrained and constrained households marginal rate of substitution. Aggregate consumption is

$$C_t = (1 - \omega) C_{u,t} + \omega C_{c,t}. \tag{2.5}$$

We define the consumption gap as the ratio between unconstrained and constrained consumption

$$\Gamma_t \equiv \frac{C_{u,t}}{C_{c,t}}. \tag{2.6}$$

2.2 Production Input Complementarity

Final good packers Final good packers operate in a competitive market. They produce the final good Z_t by combining a continuum of varieties $Z_t(i)$ with measure one so that $Z_t = \left(\int_0^1 (Z_t(i))^{\frac{\epsilon_z - 1}{\epsilon_z}} di \right)^{\frac{\epsilon_z}{\epsilon_z - 1}}$. Optimization implies the following demand function for variety i

$$Z_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_z} Z_t,$$

where $P_t \equiv \left(\int_0^1 (P_t(i))^{1-\epsilon_z} di \right)^{\frac{1}{1-\epsilon_z}}$ is the price of the final composite good. It can be shown that $P_t Z_t = \int_0^1 P_t(i) Z_t(i) di$.

Final good producers A continuum of final output producing firms, indexed by $i \in [0,1]$, operate in a monopolistically competitive environment. Hence, each firm produces a single-differentiated good and operates as a monopoly in its own market. A key element of our model is the production structure. Firm i produces the final output variety $Z_t(i)$ using the following CES production technology with imported

[14] We assume that households' consumption basket only consists of the domestically produced good.

energy ($E_t^z(i)$) and labor ($N_t(i)$) as inputs

$$Z_t(i) = \varepsilon_t^{TFP}\left((1-\alpha_{ez})^{\frac{1}{\psi_{ez}}}(N_t(i))^{\frac{\psi_{ez}-1}{\psi_{ez}}} + (\alpha_{ez})^{\frac{1}{\psi_{ez}}}(E_t^z(i))^{\frac{\psi_{ez}-1}{\psi_{ez}}}\right)^{\frac{\psi_{ez}}{\psi_{ez}-1}}, \tag{2.7}$$

where ε_t^{TFP} represents productivity and ψ_{ez} is the elasticity of substitution between energy and labor.

Cost Minimisation Cost minimization by final output goods producers yields the following demand functions for labor and energy, respectively[15]

$$\begin{aligned}
W_t &= (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}}\frac{MC_t^Z}{\tau_t^Z}\left(\frac{Z_t(i)}{N_t(i)}\right)^{\frac{1}{\psi_{ez}}}\left(\varepsilon_t^{TFP}\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \\
P_t^E &= (\alpha_{ez})^{\frac{1}{\psi_{ez}}}\frac{MC_t^Z}{\tau_t^Z}\left(\frac{Z_t(i)}{E_t^z(i)}\right)^{\frac{1}{\psi_{ez}}}\left(\varepsilon_t^{TFP}\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}},
\end{aligned}$$

where the Lagrange multiplier $MC_t^Z(i)$ is the (nominal) shadow cost of producing one more unit of final output, i.e. the nominal marginal cost, and $\tau_t^Z = \tau^Z\varepsilon_t^{\mathcal{M}_z}$ is a shock to final output marginal costs that is isomorphic to a markup shock.

Price Setting Firms face price stickiness à la Calvo, resetting prices in every period with probability $(1-\phi_z)$. A firm that is able to reset prices in period t chooses the price $P_t^\#$ that maximizes the sum of discounted profits subject to the demand faced in $t+s$

$$\mathbb{E}_t\sum_{s=0}^{\infty}(\phi_z)^s\{\Lambda_{u,t,t+s}(P_t^\# Z_{t+s|t} - MC_{t+s}^Z Z_{t+s|t})\} \quad s.t. \quad Z_{t+s|t} = \left(\frac{P_t^\#}{P_{t+s}}\right)^{-\epsilon_z} Z_{t+s}.$$

Profit maximization implies

$$\mathbb{E}_t\sum_{s=0}^{\infty}(\phi_z)^s\{\Lambda_{u,t,t+s}Z_{t+s|t}(P_t^\# - \mathcal{M}_z MC_{t+s|t}^Z)\} = 0,$$

where $\mathcal{M}_z \equiv \frac{\epsilon_z}{\epsilon_z - 1}$ is the desired final output price markup and $MC_{t+s|t}^Z$ the nominal marginal cost.

2.3 Remaining Features

2.3.1 Wage Stickiness

We incorporate wage stickiness following the standard in the literature (refer to Appendix (A.2)).

2.3.2 The World Block

The global demand schedule for the bundle of domestic exports X_t depends on the foreign currency price of domestic exports, $P_t^{EXP} = \frac{P_t}{\mathcal{E}_t}$, relative to the world non-energy export price, P_t^{X*}, and the world trade volume Y_t^*

$$X_t = \kappa^*\left(\frac{P_t^{EXP}}{P_t^{X*}}\right)^{-\varsigma^*} Y_t^*,$$

[15] See Appendix A.3 for details.

where parameter ς^* is the elasticity of substitution between differentiated export goods in the rest of the world.

2.3.3 Monetary policy

The central bank follows a Taylor rule that responds to deviations of (annual) inflation and employment from their targets,

$$R_t = R^{1-\theta_R} R_{t-1}^{\theta_R} \left(\frac{\Pi_t^{annual}}{\bar{\Pi}^{annual}} \right)^{\frac{(1-\theta_R)\theta_\Pi}{4}} \left(\hat{N}_t\right)^{(1-\theta_R)\theta_N}.$$

2.3.4 Shock Processes

The model includes a shock to the price of energy, which follows the exogenous process

$$log\left(\frac{P_t^{E*}}{P_t^*}\right) = \rho_E log\left(\frac{P_{t-1}^{E*}}{P_{t-1}^*}\right) + \varsigma_E \eta_t^E,$$

where $\frac{P_t^{E*}}{P_t^*}$ is the foreign currency price of energy relative to the price of the foreign final good, a shock to firms' productivity

$$log\left(\varepsilon_t^{TFP}\right) = \rho_{TFP} \log\left(\varepsilon_{t-1}^{TFP}\right) + \varsigma_{TFP} \eta_t^{TFP}$$

and a shock to firms' markup

$$log\left(\varepsilon_t^{\mathcal{M}_z}\right) = \rho_{\mathcal{M}_z} \log\left(\varepsilon_{t-1}^{\mathcal{M}_z}\right) + \varsigma_{\mathcal{M}_z} \eta_t^{\mathcal{M}_z}.$$

2.4 The demand side effects of an energy price shock

2.5 The IS equation

Taking employment as a measure of domestic value added (GDP), we can use the system of equations describing the economy to derive the following IS equation (see appendix)[16]

$$\hat{n}_t = \mathbb{E}_t \hat{n}_{t+1} - \frac{1}{\sigma}\frac{C_{ss}}{Z_{ss}}\left(\hat{r}_t - \mathbb{E}_t \hat{\pi}_{t+1}\right) + \omega \frac{C_{ss}}{Z_{ss}} \mathbb{E}_t \Delta \hat{\gamma}_{t+1} + \left(\frac{1-\alpha_{ez}+\psi_{ez}\alpha_{ez}}{1-\alpha_{ez}}\right) \mathbb{E}_t \Delta \hat{\varepsilon}_{t+1}^{tfp} - \psi_{ez}\frac{\alpha_{ez}}{1-\alpha_{ez}} \mathbb{E}_t \left(\Delta \hat{p}_{t+1}^E + \Delta \hat{\mu}_{t+1} + \Delta \hat{\tau}_{t+1}^Z\right) - \varsigma^* \mathbb{E}_t \Delta \hat{q}_{t+1} \quad (2.8)$$

Solving forward the IS equation, we obtain

[16] Capital letters without the time subscript represent steady state levels while lowercase letters denote variables in low deviation from steady state.

$$\hat{n}_t = \underbrace{-\frac{1}{\sigma}\frac{C_{ss}}{Z_{ss}}\mathbb{E}_t\sum_{k=0}^{\infty}(\hat{r}_{t+k}-\hat{\pi}_{t+k+1})}_{\text{Intertemporal substitution (-)}} - \underbrace{\omega\frac{C_{ss}}{Z_{ss}}\hat{\gamma}_t}_{\substack{\text{Demand effect from}\\ \text{credit constraints (+/-)}}} + \underbrace{\psi_{ez}\frac{\alpha_{ez}}{1-\alpha_{ez}}\hat{p}_t^E}_{\text{Intratemporal substitution (+)}} +$$

$$\varsigma^*\hat{q}_t - \left(\frac{1-\alpha_{ez}+\psi_{ez}\alpha_{ez}}{1-\alpha_{ez}}\right)\hat{\varepsilon}_t^{tfp} + \psi_{ez}\frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\hat{\mu}_t+\hat{\tau}_t^Z\right) \quad (2.9)$$

According to equation (2.9), GDP is dependent on the path of the real interest rate, the consumption gap ($\hat{\gamma}_t$) and the relative price of energy ($\hat{p}_t{}^E$). GDP also depends on foreign demand for the domestic good, which is determined by the real exchange rate ($\hat{q}_t$)[17]. An increase in the consumption gap reflects redistribution against the constrained workers. As constrained agents have a higher marginal propensity to consume, such redistribution causes a drop in aggregate demand that brings GDP down. The effect of the consumption gap on GDP is increasing in the share of constrained households.

The IS equation illustrates the channels through which an energy price shock affects economic activity. Since our model nests a RANK ($\omega = 0$), the channels present in a RANK are also present here. In a RANK, energy prices operate through two different channels. First, an increase in energy prices stimulates GDP through a higher relative price of energy ($\hat{p}_t{}^E$), which leads to substitution from imported energy towards the domestic labor input (intratemporal substitution effect). Second, given the inflationary pressures derived from the shock, the central bank responds by tightening monetary policy. The ensuing increase in the real rate contracts economic activity (intertemporal substitution effect). This interest rate channel captures the usual mechanism through which supply shocks depress economic activity in a RANK model. This means that in the RANK, the supply shock is not contractionary by itself. Instead, the economic downturn is a result of the monetary policy response to inflation. A new channel for supply shocks is present in the TANK economy, as indicated by the term involving the consumption gap. This term captures a demand side impact of the energy shock that operates through an income effect. The sign of this demand side effect depends on the response of the consumption gap to the shock. Under reasonable calibrations (where inputs are largely complements), an increase in the price of energy translates into a contraction in households' income, as more resources must be devoted to the purchase of the energy input. Given financial constraints, demand by worker households falls (as reflected by an increase in the consumption gap), leading to an economic recession.

Next, we discuss the aforementioned demand side effect that emerge from credit constraints by analyzing how the energy price shock affects the consumption gap.

2.6 The consumption gap

Letting $INC_{u,t}$ and $INC_{c,t}$ denote unconstrained and constrained households' current income, and using the budget constraints (2.4) and (2.1), we can express the consumption gap as follows[18]

$$\Gamma_t = \frac{INC_{u,t} + \mathcal{E}_t(R^*_{t-1}-1)B^*_{u,t-1} - \mathcal{E}_t\Delta B^*_{u,t}}{INC_{c,t}}.$$

[17] The real exchange rate is defined as $\mathcal{Q}_t \equiv \mathcal{E}_t\frac{P_t^*}{P_t}$.

[18] The expression for the consumption gap takes into account that domestic bonds must equal zero in equilibrium.

Defining the income gap between unconstrained and constrained households as $\Gamma_t^{inc} \equiv \frac{INC_{u,t}}{INC_{c,t}}$ we can rewrite the above equation as

$$\Gamma_t = \Gamma_t^{inc} + \frac{\mathcal{E}_t(R_{t-1}^* - 1)B_{u,t-1}^* - \mathcal{E}_t \Delta B_{u,t}^*}{INC_{c,t}}. \tag{2.10}$$

Equation (2.10) illustrates how an energy price shock affects the consumption gap through a differential impact on constrained and unconstrained households consumption. An unequal consumption response can have two sources. One source is through changes in the income gap, which reflects the different impact of the shock on current income (due to differences in income composition). The other source is access to borrowing (reflected in changes in foreign bond holdings $\Delta B_{u,t}^*$), which allows unconstrained households to insure their consumption from income fluctuations following an energy price shock.

Let's consider how these two components of the consumption gap are determined. From the economy's budget constraint, we know that the evolution of foreign bonds depends on the balance of trade. Therefore, the consumption gap can be rewritten as

$$\Gamma_t = \Gamma_t^{inc} - \frac{1}{1-\omega}\frac{TB_t}{INC_{c,t}}, \tag{2.11}$$

where $TB_t = P_t X_t - P_t^E E_t^z$ is the trade balance. Using the definitions of unconstrained and constrained total income and imposing $N_{u,t} = N_{c,t} = N_t$, the above expression can be written as follows

$$\Gamma_t = \underbrace{1 + \frac{1}{1-\omega}\frac{\mathcal{M}_t - 1}{\Xi_t^N}}_{\text{Income gap}} + \underbrace{\frac{1}{1-\omega}\left(\frac{1}{\Xi_t^N} - 1 - \frac{TB_t^{NM}}{INC_{c,t}}\right)}_{\text{Borrowing}}, \tag{2.12}$$

where $\mathcal{M}_t \equiv \frac{P_t}{MC_t^Z}$ is firms' average markup, $\Xi_t^N \equiv \frac{W_t N_t}{W_t N_t + P_t^E E_t^z}$ is the labor share in firms' total expenditure and $TB_t^{NM} \equiv P_t X_t$ is the balance of trade net of the energy imports. Equation (2.12) indicates that the effect of a change in energy prices on the consumption gap (and hence, on aggregate demand) is determined by the impact of the shock on two key variables, firms' markups ($\mathcal{M}_t$) and the labor share (Ξ_t^N).

The income gap (and hence, the consumption gap) depends positively on firms' markups, since an increase in the markup redistributes resources towards the unconstrained firm owners. The income gap also increases in response to a reduction of the labor share in total factor expenditure, since a reduction in the labor share redistributes resources against the constrained workers and towards the import of energy.

A reduction in the labor share also increases the consumption gap due to unconstrained households' ability to insure their consumption by borrowing. The reduction in the labor share reflects an increase in the resources devoted to import energy (an increase in the energy share), which must be financed via an increase in foreign debt. Borrowing from the foreign sector is used by unconstrained households to finance their consumption, hence increasing the consumption gap.

Finally, to understand how the energy price shock affects firms' average markup and the labor share, notice that these two objects are linked to the price of energy according to the following expressions[19]

$$\mathcal{M}_t = \frac{\varepsilon_t^{TFP} P_t}{\left((1-\alpha_{ez})W_t^{1-\psi_{ez}} + \alpha_{ez}\left(P_t^E\right)^{1-\psi_{ez}}\right)^{\frac{1}{1-\psi_{ez}}}} \tag{2.13}$$

$$\Xi_t^N = \left(1 + \frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{P_t^E}{W_t}\right)^{1-\psi_{ez}}\right)^{-1}. \tag{2.14}$$

Notice from (2.13) that, given price rigidities, an increase in energy prices (P_t^E) reduces firms' markups. This implies a redistribution of income in favor of workers, reflected in an reduction of the consumption gap (equation (2.12)). This boosts aggregate demand, and hence, activity (equation (2.9)).

Equation (2.14) shows that the impact of higher energy prices on the labor share crucially depends on the elasticity of substitution between energy and labor (ψ_{ez}). In the case of a Cobb-Douglas production technology ($\psi_{ez} = 1$) we have $\Xi_t^N = 1 - \alpha_{ez}$, implying that the price of energy has no impact on the labor share. If the elasticity of substitution is larger than one ($\psi_{ez} > 1$), higher energy prices increase the labor share. The reason is that costlier energy triggers a strong substitution from energy towards labor. The resulting redistribution of income in favor of workers is reflected in a reduction in the consumption gap, which boosts aggregate demand and activity. Alternatively, if energy cannot easily be substituted for by labor ($\psi_{ez} < 1$), an increase in energy prices reduces the labor share. The shock therefore redistributes against the constrained workers, increasing the consumption gap. The consequent drop in aggregate demand depresses economic activity.

As we will see later, the empirical evidence points to a low substitutability between labor and energy. In this scenario, we should expect that an increase in energy prices will reduce both the labor share and firms markups (i.e., the profit share). The relative impact of the shock on this two objects will determine whether the constrained workers of firm owners are mostly affected, and hence, the size of the demand side effect of the energy price shock.

3 Dynamic Responses under a Taylor Rule

3.1 Parameterization

We list the calibration for key model parameters in Table 1. To stay close to the literature, we calibrate our model using some common parameterizations. We assume a discount factor, β, of 0.9994. The elasticities of substitution across goods varieties (ϵ_z) and across worker types (ϵ_w) are both set to 11, which implies a markup of 10% in steady state. We assume goods prices and wages are adjusted with Calvo parameters $\phi_z = 0.66$ and $\phi_w = 0.75$. We set the response to inflation (θ_π) and slack (θ_n) in the Taylor rule to 1.5 and 0.25, respectively. The interest rate smoothing parameter (θ_R) is set to 0.9. The productivity process parameters are set to $\rho_{TFP} = 0.93$ and $\sigma^2_{TFP} = 0.07$. The energy price shock has persistence $\rho_E = 0.8$ and $\sigma^2_E = 1$ so that prices increase by 50% on impact. The mark up shock has persistence $\rho_{\mu_z} = 0.9$ and $\sigma^2_{\mu_z} = 1$. The population share of constrained worker households (ω) is set to 0.25.

We limit our discussion to the following key parameters: the elasticity of substitution between energy and labor in production ($\psi_{ez} = 0.15$) and the steady state share of energy in production ($\alpha_{ez} = 0.05$). There are a wide range of estimates for the elasticity of substitution between production inputs in the literature. Higher estimates, such as those provided by Bodenstein et al. (2012) (0.42) are motivated by estimates

[19] The expression for the labor share is obtained using firms' demand functions for energy and labor.

of the short-run price elasticity of oil demand from structural econometric models. Natal (2012) sets this parameter to 0.3, while Plante (2014) suggests a calibration of 0.25 so that the own price elasticity of oil is approximately -0.25. Montoro (2012) sets the value of the elasticity of substitution between oil and labor at 0.2, equal to the average value reported by Hamilton (2009). On the low end of estimates is Adjemian and Darracq Paries (2008) and Backus and Crucini (2000), at 0.09. However, their production function is Cobb-Douglas in labor and a capital services-energy mix, where the latter is combined via CES. Finally, Stevens (2015) suggests an elasticity of substitution between oil and value-added of 0.03, where value-added is a Cobb-Douglas function with labor and capital inputs. This parameter is equivalent to the short-run oil demand elasticity and is chosen to be consistent with reduced-form evidence on the slope of the oil demand curve that lie between 0 and 0.11.

Between the extreme cases of zero or infinite substitutability, the effects of an energy price shock on macroeconomic aggregates also depends on the share of energy in production. The share of energy in production ranges from 2% in Natal (2012) for the US, 4% in Bachmann et al. (2022) for Germany, and 5% in Stevens (2015).

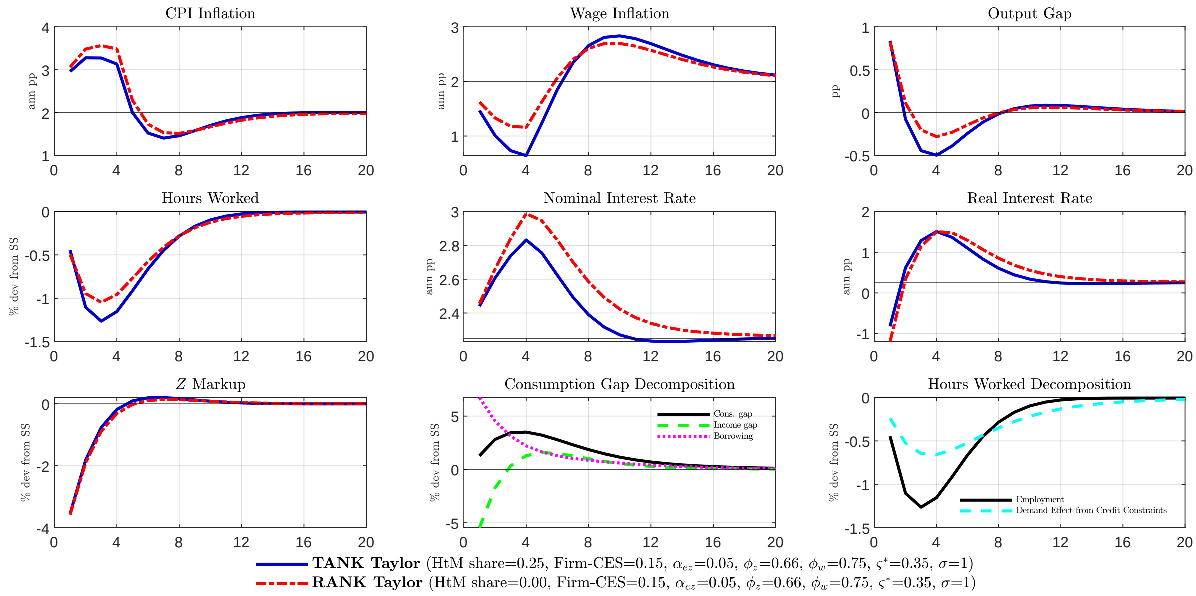
3.2 Impulse response functions

In Figure 2, we show the response to an increase in energy prices in the baseline model. In the RANK economy, an energy price shock places upward pressure on production costs, leading to a surge in inflation. The central bank responds by tightening monetary policy, which induces a contraction in activity. Relative to the RANK, the TANK economy experiences a deeper contraction. Moreover, while the recession in the RANK originates from the contractionary policy implemented by the central bank, in the TANK it is largely driven by the direct impact of higher energy prices on aggregate demand. Since production inputs are complementary in the TANK economy, higher energy prices reduce the labor share of total income, implying a drop in workers' earnings. Given borrowing constraints for worker households, this translates into a fall in aggregate demand. The employment decomposition in Figure 2 shows how much the fall in demand as a result of credit frictions contributes to the contraction in employment in the TANK economy. Due to the adverse effect of the energy price shock on demand, monetary policy in the TANK is looser.

The IRFs also illustrate how the energy price shock has a different effect on constrained workers' and unconstrained capitalists' consumption. Figure 2 contains a panel displaying the dynamics of the consumption gap and its drivers (see equation (2.12)). The consumption gap fluctuates due to households' unequal income composition (cyan dashed line) and unequal access to credit (pink dotted line). While workers' consumption largely falls due to the drop in their income, capitalists are able to insure their consumption by borrowing from the external sector. The unequal access to borrowing (pink dotted line) explains the increase in the consumption gap. Initially, a drop in firms' markups (due to costlier energy) results in a negative income gap, which limits the increase in the consumption gap. Over time, as firms pass the costs of the shock to worker households through an increase in prices, markups recover and the income gap goes up as well, further raising the gap in consumption.

In Section (7), we provide IRFs to illustrate how the strength of the channels discussed in this section depend on the degree of substitutability between energy and labor in production and the degree of credit frictions.

FIGURE 2: Dynamic Responses to a Global Energy Price Shock



4 The demand side effects of alternate supply shocks

Can the economic effects of an energy price shock be appropriately proxied with a TFP shock, since both shocks restrain supply? In this section, we explore whether the demand contraction that follows a rise in energy prices is a common feature of supply disturbances. Equations (2.12) to (2.14) are used to analyze the demand side effect of a disturbance to firms' TFP. For simplicity, assume a closed economy environment, where only labor is used in production. The consumption gap becomes

$$\Gamma_t = 1 + \frac{1}{1-\omega}\left(\mathcal{M}_t - 1\right), \tag{4.1}$$

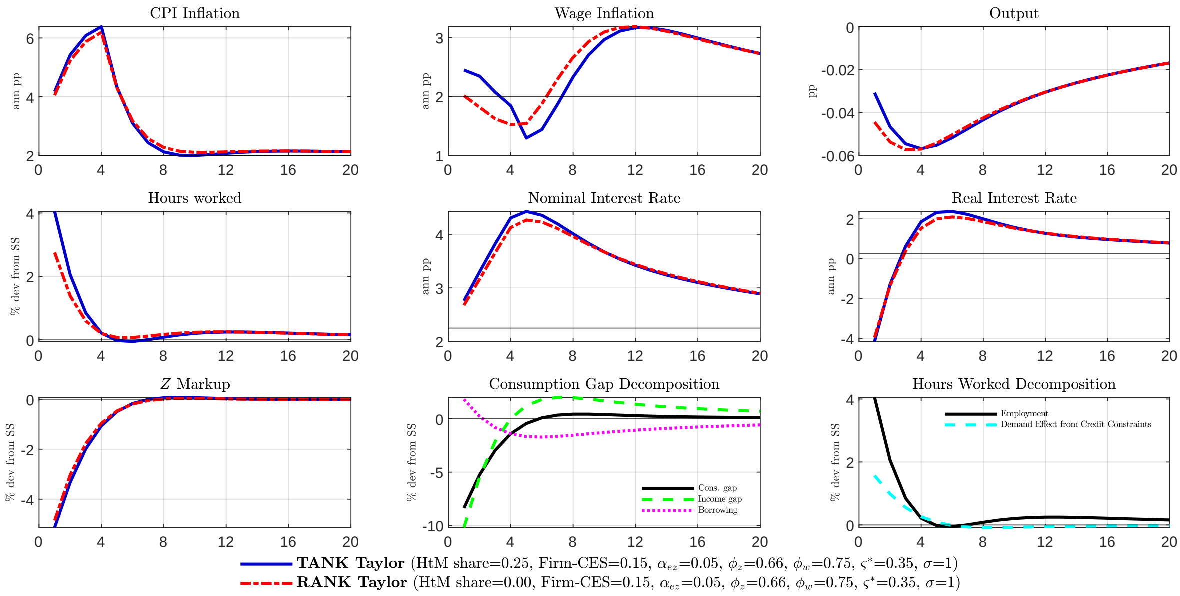
where

$$\mathcal{M}_t = \frac{\varepsilon^{TFP} P_t}{W_t}. \tag{4.2}$$

It is easy to check that an adverse TFP shock leads to a fall in markups. The reason is that with lower productivity firms must hire more labor to produce each unit of the good. This implies lower markups and an increase in workers' income. It follows that the consumption gap falls, leading to an increase in GDP (equation 2.9).

The IRFs to an adverse TFP shock in Figure 3 illustrate this intuition. Similar to the energy price shock, the TFP shock leads to higher marginal costs, which places upward pressure on inflation. The consequent response of the central bank to higher inflation leads to a drop in output. While both energy and TFP shocks generate similar supply side effects, this is not the case for the demand side effect. Lower TFP implies that more labor is required to produce each unit of the good, which explains the observed increase in employment. Workers' income thus increases, boosting aggregate demand. As a consequence, the TANK economy features a milder contraction in consumption and output relative to the RANK. Energy and TFP shocks therefore diverge in terms of their impact on demand. Whereas the former reduces workers' income, the latter increases it, leading to a different profile for aggregate demand. We conclude that no generalization can be made about the effects of supply shocks on aggregate demand, as the nature of the shock crucially affects the way resources are redistributed in the economy.

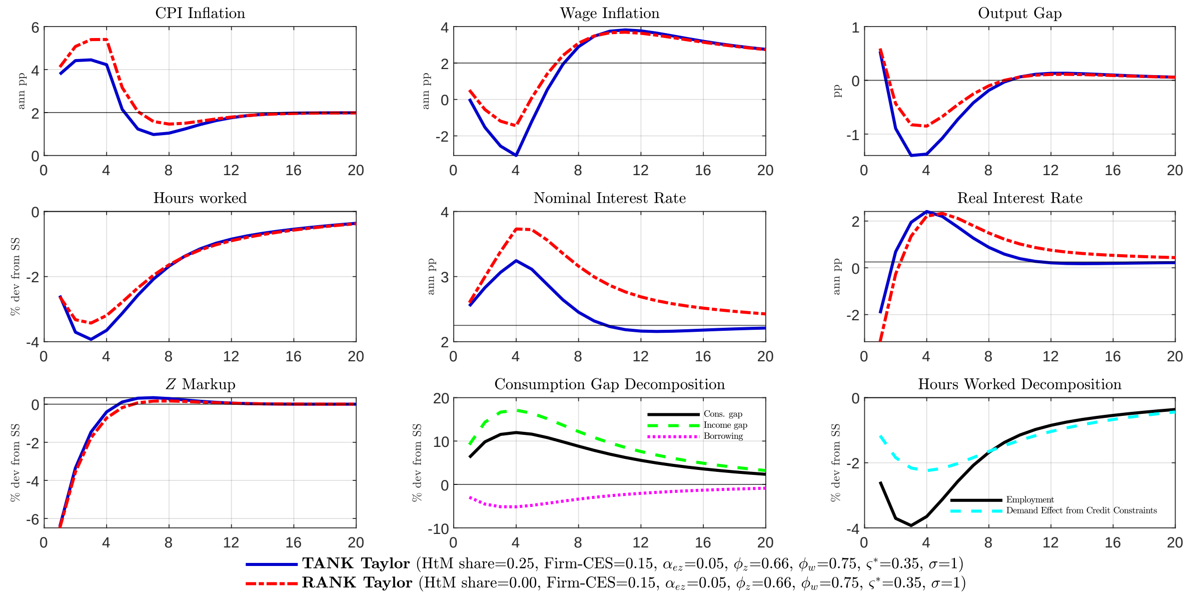
FIGURE 3: Dynamic Responses to a TFP Shock



Another supply disturbance frequently considered in the literature is a shock to firms' desired markup. In Figure 4 we show the IRFs after such a shock. A higher desired markup pushes inflation up, which the central bank responds to by raising the policy rate. Thus, on the supply side, the shock operates in a similar fashion to the energy and TFP disturbances. On the demand side, higher markups imply an increase in the profit share relative to the labor share of income (reflected in an increase in firms' markups in equation 4.1). The redistribution of resources against the constrained workers, as captured by a rise in the consumption gap, depresses aggregate demand. This explains the deeper fall in output experienced in the TANK.

Like the energy shock, a markup shock raises the consumption gap, hence depressing aggregate demand. However, in the case of a markup shock, the rise in the consumption gap is fully explained by the income gap, which goes up due to the unequal income composition between workers and firm owners. Instead, with the energy shock, the rise in the consumption gap is largely explained by an unequal access to international credit markets.

FIGURE 4: Dynamic Responses to a Markup Shock



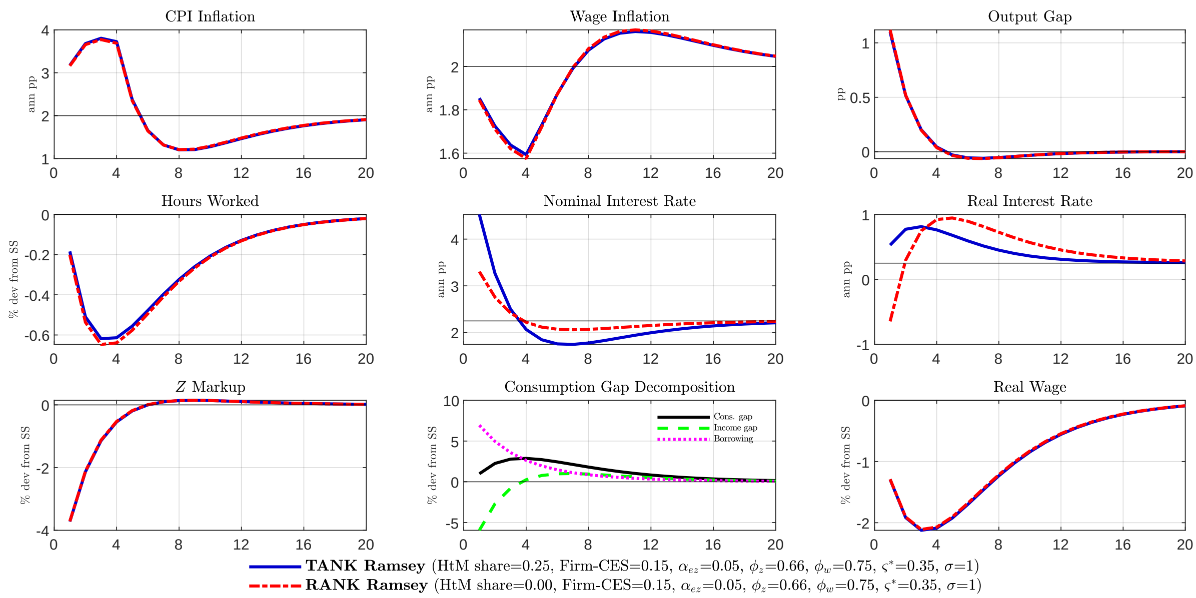
5 Optimal monetary policy

Next, we study the optimal monetary policy response to an energy price shock. To compute the optimal Ramsey policy, we assume an utilitarian central bank that attaches equal weights to the utility of all households.[20] Figure 5 presents the IRFs under the Ramsey policy. We compare the optimal policy in the TANK versus the RANK model. The figure shows that although optimal policy leads to very similar paths for inflation and employment in the two economies, the implementation is different. In both cases, the policymaker implements contractionary policy in order to counteract the inflationary effect of the shock. However, the required increase in the interest rate is milder in the TANK. This is explained by the direct contractionary effect of higher energy prices on households' income. In the TANK, the lower income translates into lower aggregate demand, which contains the inflationary pressures of the shock. Hence, a milder response of the central bank is needed.

Also, notice that the dynamics under the optimal policy closely resemble those with a Taylor rule. In fact, we can observe that under the Ramsey policy the central bank tolerates higher inflation compared to the policy regime with a Taylor rule.

[20] We compute the optimal Ramsey policy by maximizing households' lifetime utility subject to the non-linear system of equations that describe private agents' optimality conditions

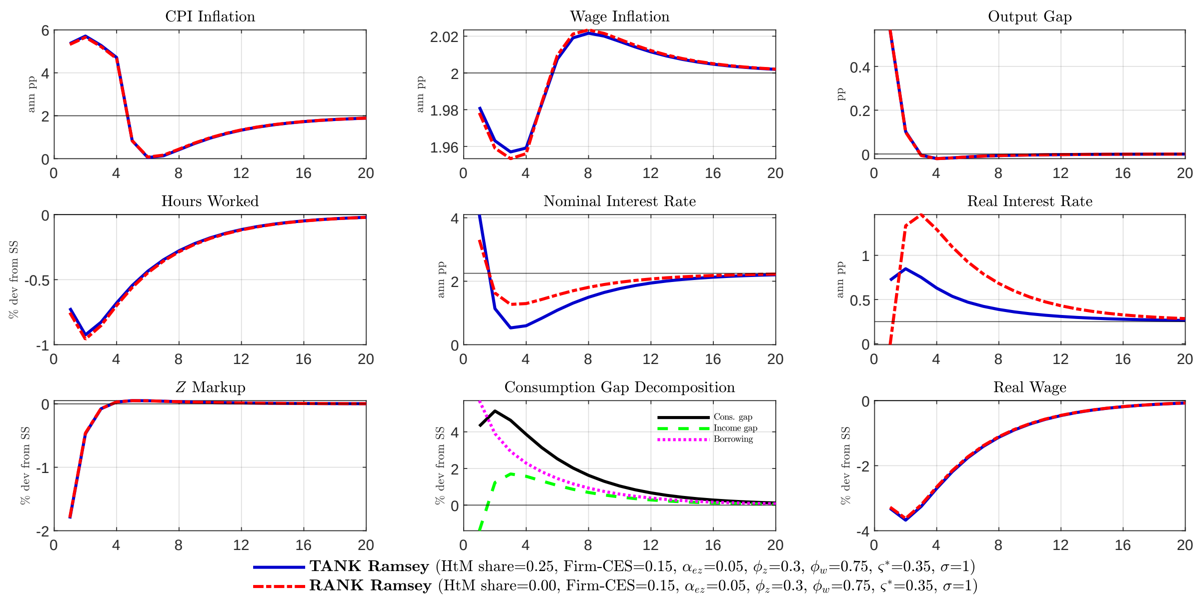
FIGURE 5: Dynamic Responses to a Global Energy Price Shock: Optimal Policy



The role of price rigidities As stressed earlier, the demand effect of higher energy prices depends on the evolution of firms' markups. If inflation remains contained in spite of the costlier energy input, firms largely absorb the costs of the shock. This would be reflected in a reduction in markups. Conversely, if prices go up strongly to preserve markups, firms can pass the costs of the shock to workers, who will experience a more severe reduction in their income. The degree to which prices react to the shock thus determines who takes the hit, and hence, its impact on aggregate demand. We then explore a scenario where firms raise prices more aggressively in response to the costlier energy in an attempt to preserve profits. To this end, we repeat the optimal policy exercise assuming a higher degree of price flexibility.[21] Results are presented in Figure 6. A comparison with Figure 5 illustrates that when firms react to an energy price shock by raising prices strongly, constrained households experience a more severe drop in their income relative to unconstrained households, as reflected by the income gap. Since the constrained households are more severely affected, there is a deeper contraction in aggregate demand. As a consequence, optimal monetary policy in the TANK is now much looser relative to its RANK counterpart.

[21] For this simulation we set the Calvo parameter to 0.3.

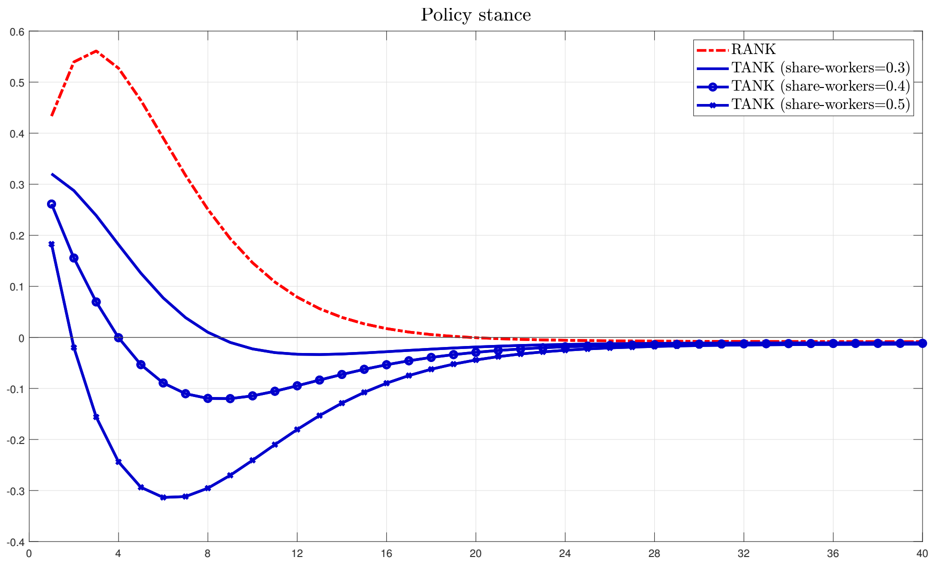
FIGURE 6: Dynamic Responses to a Global Energy Price Shock: Optimal Policy with Higher Price Flexibility



Next, we explore whether optimal policy may actually be expansionary in response to an adverse supply shock. We can expect that as the contractionary effect of the shock on demand strengthens, it should be optimal for the policymaker to loosen policy. For this exercise, we introduce a measure for the monetary policy stance, which indicates whether policy is contractionary or expansionary. From (2.2) we know that the demand of households whose consumption responds to interest rates is determined by the expected path of the real interest rate, rather than the current real rate. Therefore, we define the *policy stance* as $st_t \equiv \frac{1}{\sigma}\mathbb{E}_t \sum_{k=0}^{\infty}(r_t - \pi_{t+k+1})$.

Figure 7 presents the IRFs for the policy stance over an increasingly larger share of constrained agents, which allows the energy price shock to yield a correspondingly larger fall in households' consumption. In the RANK, monetary policy remains contractionary throughout the period of higher energy prices in order to counteract inflation. Meanwhile, in the TANK, the policy stance quickly turns expansionary as financial constraints become more severe. Optimal policy can be expansionary when the energy price shock has a larger adverse effect on the demand side.

FIGURE 7: Dynamic Responses to a Global Energy Price Shock: Optimal Policy with Stronger Credit Constraints



6 Extensions

6.1 Energy as a Consumption Good

So far we have considered the effects of an energy price shock when energy is used only as a production input. In this section, we extend our model to incorporate imported energy as a component of households' consumption basket. To be precise, assume now that unconstrained and constrained households' consumption is respectively given by the following CES aggregators

$$CES_{u,t} = \left((1-\alpha_{u,ec})^{\frac{1}{\psi_{ec}}} \left(C_{u,t}\right)^{\frac{\psi_{ec}-1}{\psi_{ec}}} + \alpha_{u,ec}^{\frac{1}{\psi_{ec}}} \left(E_{u,t}^{h}\right)^{\frac{\psi_{ec}-1}{\psi_{ec}}} \right)^{\frac{\psi_{ec}}{\psi_{ec}-1}}$$

and

$$CES_{c,t} = \left((1-\alpha_{c,ec})^{\frac{1}{\psi_{ec}}} \left(C_{c,t}\right)^{\frac{\psi_{ec}-1}{\psi_{ec}}} + \alpha_{c,ec}^{\frac{1}{\psi_{ec}}} \left(E_{c,t}^{h}\right)^{\frac{\psi_{ec}-1}{\psi_{ec}}} \right)^{\frac{\psi_{ec}}{\psi_{ec}-1}},$$

where $C_{u,t}$ is consumption of the domestically produced good for unconstrained households, $E_{u,t}^{h}$ is energy consumption for unconstrained households, and $\alpha_{u,ec}$ denotes the share of energy in unconstrained households' expenditure. The analagous variables with subscript c denote the counterparts for constrained households. Parameter ψ_{ec} is the elasticity of substitution between the domestic good and energy.

6.2 The consumption gap

With energy entering the consumption basket, the consumption gap is given by[22]

$$\Gamma_t = 1 + \underbrace{\frac{1}{1-\omega}\frac{\mathcal{M}_t - 1}{\Xi_t^N}}_{\text{Income gap}} + \underbrace{\frac{1}{1-\omega}\left(\frac{1}{\Xi_t^N} - 1 + \mathcal{M}_t\left(\frac{1}{\Xi_t^N}\right)\left(\frac{1}{\Xi_t^Z} - 1\right) - \frac{TB_t^{NM}}{INC_{c,t}}\right)}_{\text{Borrowing}},$$

where $\Xi_t^Z \equiv \frac{P_t C_t}{P_t C_t + P_t^E E_t^h}$[23] is households' expenditure share in the domestically produced good.

For simplicity, assume $\alpha_{ez} = 0$, so that energy is uniquely used as a final consumption good by households. In this case $\Xi_t^N = 1$, and the above expression reduces to

$$\Gamma_t = 1 + \underbrace{\frac{1}{1-\omega}(\mathcal{M}_t - 1)}_{\text{Income gap}} + \underbrace{\frac{1}{1-\omega}\left(\mathcal{M}_t\left(\frac{1}{\Xi_t^Z} - 1\right) - \frac{TB_t^{NM}}{INC_{c,t}}\right)}_{\text{Borrowing}}, \tag{6.1}$$

where

$$\mathcal{M}_t = \frac{\varepsilon_t^{TFP} P_t}{W_t}, \tag{6.2}$$

$$\Xi_t^Z = \left(1 + \frac{\alpha_{ec}}{1-\alpha_{ec}}\left(\frac{P_t^E}{P_t}\right)^{1-\psi_{ec}}\right)^{-1}. \tag{6.3}$$

With energy only in the consumption basket, the consumption gap depends negatively on the share of the domestic good in total households' expenditure (Ξ_t^Z) and positively on firms' average markup ($\mathcal{M}_t$).

The share of domestic goods in households' expenditure responds to changes in the price of energy, and the direction of this response depends on the elasticity of substitution between the domestic good and energy (ψ_{ec}) (equation 6.3). If the elasticity of substitution is lower than one ($\psi_{ec} < 1$), a higher price of energy reduces the expenditure share of the domestic good. As less resources are spent on the purchase of domestic products, households' income drops. While the unconstrained are able to maintain their consumption levels by borrowing from the foreign sector, the constrained workers must cut demand. The consumption gap therefore increases and aggregate demand goes down. If the elasticity of substitution is larger than one ($\psi_{ec} > 1$), the opposite is true, and higher energy prices reduce the consumption gap and boost demand. Finally, with an elasticity equal to one ($\psi_{ec} = 1$), the price of energy neither affects the share of the domestic good in households' expenditure nor the consumption gap. Notice that changes in the share of the domestic good in expenditure do not affect the income gap. They impact on the consumption gap only because households have unequal access to credit (i.e., through the borrowing term in equation (6.1)).

Equation (6.2) shows that the price of energy has no direct effect on markups, as energy does not enter

[22]For the following derivations we assume the share of energy in the consumption basket to be equal across unconstrained and constrained households ($\alpha_{u,ec} = \alpha_{c,ec} = \alpha_{ec}$).

[23]$C_t = (1-\omega)C_{u,t} + \omega C_{c,t}$ and $E_t^h = (1-\omega)E_{u,t}^h + \omega E_{c,t}^h$ denote the aggregate consumption of domestic and energy goods, respectively.

firms' production function. This is a key difference relative to the model with energy as a production input (equation 2.13). In that case, the higher price of energy gradually passed through to the price of the consumption good. Therefore, firms would partially absorb the impact of the costlier energy through a fall in markups. However, with energy in the consumption basket, energy prices instantaneously pass through to the price of the consumption good. Without a fall in markups to absorb the shock, worker households are more strongly affected by higher energy prices.

6.3 Impulse response functions

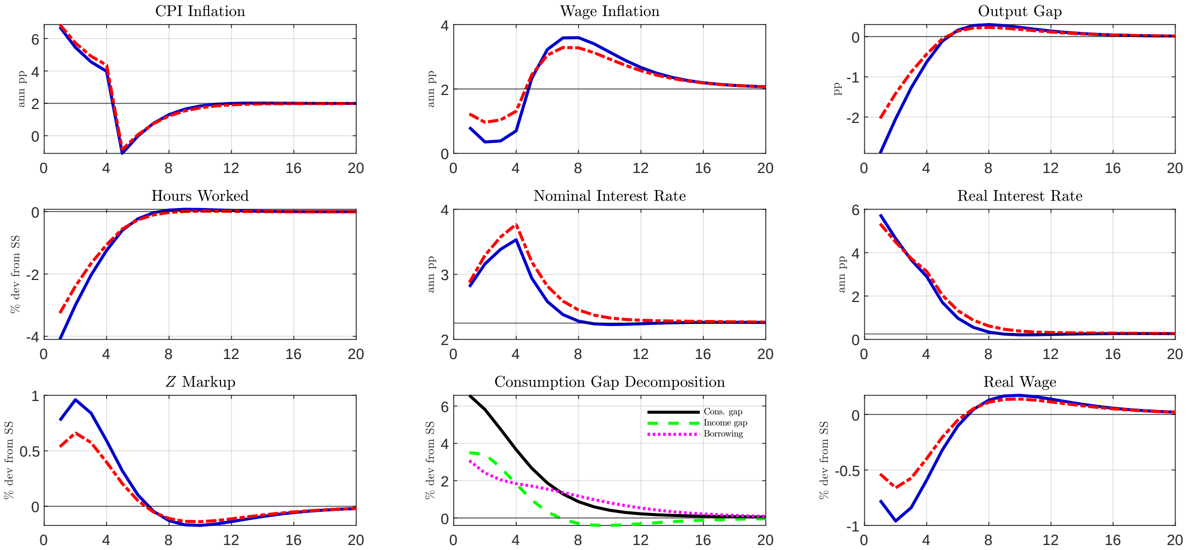
The IRFs in Figure 8 show the response to an energy price shock when energy is a component of households' consumption basket. Three additional parameters are needed relative to the baseline model calibration in Table 1: the proportion of energy in the consumption basket of constrained households ($\alpha_{c,ec} = 0.05$), the proportion of energy in the consumption basket of unconstrained households ($\alpha_{u,ec} = 0.05$), and $\psi_{ec} = 0.15$.[24] Households react to costlier energy by substituting it with the domestically produced good. However, given an elasticity of substitution lower than one, the share of the domestically produced good in total households' expenditure decreases. Consequently, domestic households' income falls. This leads to a decline in constrained agents' consumption, resulting in a larger consumption gap and lower aggregate demand. As a result, the TANK economy experiences a more severe contraction than the RANK economy.

Compared to the scenario where energy is only a production input, there is a larger impact on inequality, as we can see by the greater increase in the consumption gap in Figure 8. This is due to the income gap, which now goes up. The different evolution of the income gap relative to the case where energy is used as an input follows from the different response of firms' markups. With energy as an input for firms, production costs would go up due to the costlier energy. Due to price rigidities, markups would fall, partially absorbing the shock. With energy entering directly in households' consumption basket, markups do not attenuate the impact of the shock on inequality. Moreover, as observed in Figure 8, markups now increase. This is explained by the behavior of wages, which decrease as a consequence of a weaker economy.

In summary, this exercise demonstrates that the transmission of an energy price shock is similar when energy enters firms' production function and households' consumption basket. However, the key difference lies in the impact on inequality, as the income gap increases more when energy is a consumption good rather than a production input. For IRFs showing the the response to an energy price shock when energy is both a consumption good and a production input, see Figure H.1.

[24] We adopt this calibration to rule out effects that may result from a different proportion of energy in the consumption baskets of different households, or different elasticities of substitution between energy and non-energy goods/inputs for households versus firms.

FIGURE 8: Dynamic Responses to a Global Energy Price Shock: Energy only as a Consumption Good

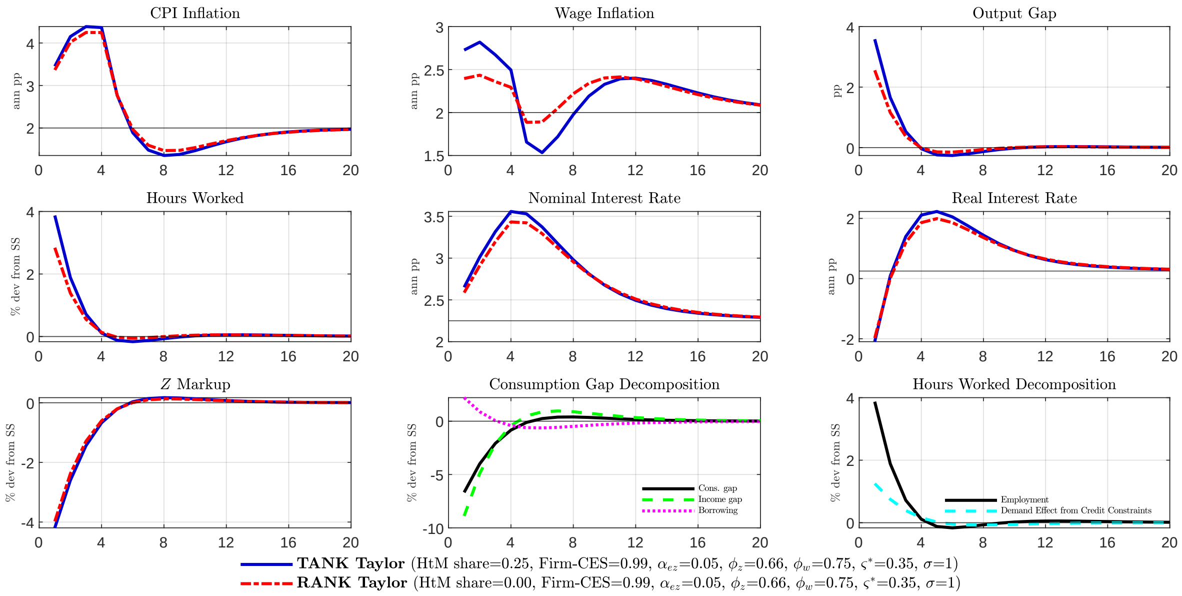


TANK Taylor (HtM share=0.25, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.05, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=1)
RANK Taylor (HtM share=0.00, HH-CES=0.15, α_{ec}=0.05, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=1)

7 Robustness checks

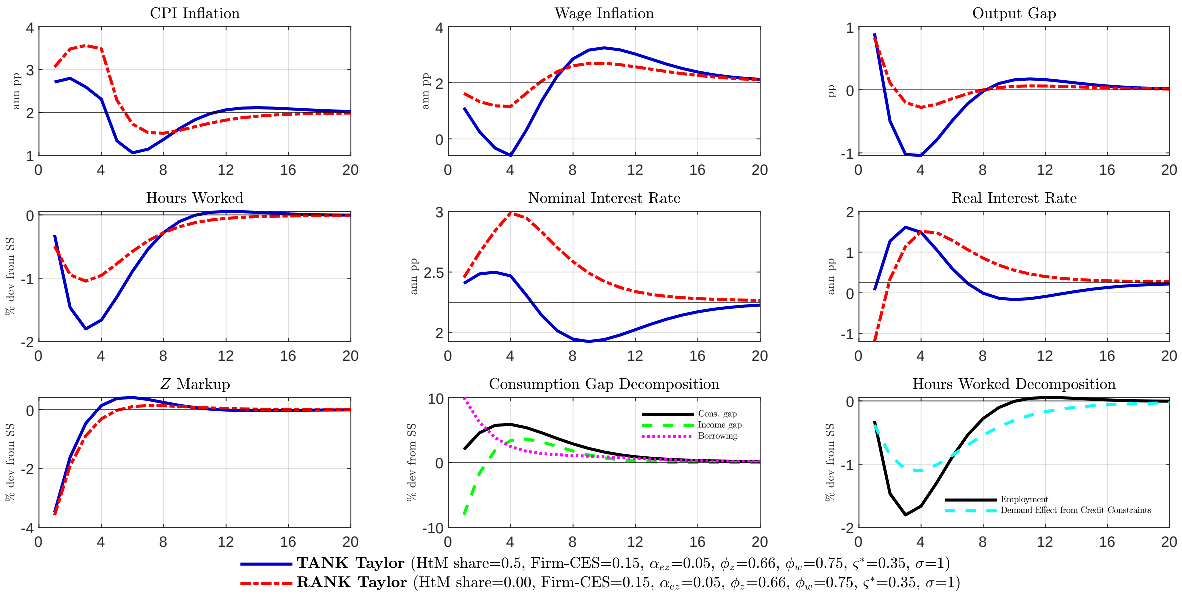
Role of complementarities For the case of energy as a production input, figure 9 illustrates the effects of the energy price shock under a higher degree of substitutability between labor and energy in production (Cobb-Douglas, $\psi_{ez} = 1$). The increase in the price of energy imports imply a fall in the relative price of labor, which now leads to greater degree of substitution towards labor. The labor demand schedule shifts upwards and employment increases. For $\psi_{ez} = 1$, higher employment fully compensates for the lower relative wage, leaving the labor share constant. While the labor share remains constant, firms' markups experience a reduction due to higher marginal costs. Redistribution in favor of constrained workers, reflected in a reduction of the consumption gap, boosts aggregate demand. Given the positive effect of the shock on demand, the TANK economy experiences a milder recession relative to its RANK counterpart.

FIGURE 9: Dynamic Responses to a Global Energy Price Shock: Cobb-Douglas instead of CES Production Function



Higher share of constrained households In the case of energy as a production input, figure 10 illustrates the effects of the energy shock when we assume a larger share of constrained households. Given more severe borrowing constraints, consumption becomes more responsive to the drop in households' income. It follows that the increase in energy prices induces a stronger fall in aggregate demand, leading to a deeper recession than in the baseline case (Figure 2).

FIGURE 10: Dynamic Responses to a Global Energy Price Shock: Higher Share of Constrained Households



8 Conclusion

We build an open economy model with household heterogeneity and low substitutability between energy and labor to highlight the demand side effects of an energy price shock. We show that an energy price shock has different effects on households, depending on their sources of income and borrowing constraints.

The transmission of an energy price shock to aggregate variables differs significantly from a RANK. An energy price shock reduces the labor share of total factor expenditures, thereby redistributing income against constrained worker households, which depresses aggregate demand. The increase in resources used for energy imports must be financed by an increase in debt. Borrowing from the foreign sector is used by unconstrained households to finance their consumption. This redistributes income in favor of unconstrained worker households, thereby depressing aggregate demand. The magnitude of these channels depend on the degree of price rigidity and the elasticity of substitution between energy and labor. An energy price shock therefore has a self-correcting effect, as the consequent contraction in economic activity dampens inflationary pressures.

In our model, an energy price shock has features of an adverse productivity shock, but there are important differences. Although the supply side effects of both shocks are the same in our model, the demand side effect is completely different. Both an adverse productivity shock and an energy price shock lead to an increase in inflation. However, while a negative productivity shock leads to an increase in aggregate demand, the opposite is true for an energy price shock.

The demand side effect of an energy price shock in our model implies that optimal monetary policy is less contractionary, relative to a RANK model. In some cases, it may even be expansionary (i.e., when credit constraints are severe).

We find similar results in an extension with energy in the consumption basket. In this scenario, inequality also increases due to the response of markups to the energy price shock. When energy is a production input, costlier energy transmits only gradually to the price of consumption goods, resulting in a decrease in markups. Profits partially absorb the effects of costlier energy, limiting the impact of the shock on the constrained worker households. However, when energy enters directly into the consumption basket, markups no longer absorb the shock, which exacerbates the impact of the shock on inequality.

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A Model Derivations

A.1 Households

A share $0 < \omega < 1$ of all households have access to domestic and international financial markets and are able to save and borrow in an unconstrained manner. The remaining share, $1-\omega$, are 'constrained' households. Those households directly consume their labor income. Unconstrained (constrained) household quantities are denoted with subscript u (c).

Unconstrained Households Members of unconstrained households consume, work, save, pay taxes and receive profits from firm ownership. Unconstrained household maximises their lifetime utility $\mathcal{U}_{u,s}$

$$\mathcal{U}_{u,s} = \mathbf{E}_s \left[\sum_{t=s}^{\infty} \beta^t \left\{ U_{u,t} \left(C_{u,t}, E^c_{c,t}, N^h_{u,t} \right) \right\} \right], \qquad \text{where} \quad U_{u,t} = \left[\frac{(CES_{u,t})^{1-\sigma} - 1}{1-\sigma} - \chi \frac{\left(N^h_{u,t} \right)^{1+\varphi}}{1+\varphi} \right]$$

$$\text{and} \quad CES_{u,t} = \left((1-\alpha_{u,ec})^{\frac{1}{\psi_{ec}}} (C_{u,t})^{\frac{\psi_{ec}-1}{\psi_{ec}}} + (\alpha_{u,ec})^{\frac{1}{\psi_{ec}}} \left(E^h_{u,t} \right)^{\frac{\psi_{ec}-1}{\psi_{ec}}} \right)^{\frac{\psi_{ec}}{\psi_{ec}-1}}.$$

$N^h_{u,t}$ is the labour supplied by the unconstrained household, φ is the inverse Frisch elasticity of labour supply and χ is the relative weight on the dis-utility of working. The total consumption bundle consumed by the unconstrained agent, $CES_{u,t}$, is a CES composite of a domestically produced non-energy consumption good $C_{u,t}$ and of an imported energy consumption good $E^h_{u,t}$ (where the superscript h indicates *h*ousehold rather than firm demand for energy). ψ_{ec} denotes the degree of the elasticity of substitution between non-energy consumption and energy consumption, $\alpha_{u,ec}$ denotes the share of energy in consumption. Utility is maximised subject to the budget constraint

$$W^h_t N^h_{u,t} - R_{t-1} B_{u,t-1} - \bar{R}^* B^*_{u,t-1} \mathcal{E}_t + DIV_{u,t} \quad = \quad P^C_t C_{u,t} + P^E_t E^h_{u,t} - B_{u,t} - B^*_{u,t} \mathcal{E}_t + T_{u,t} + P_t \mathcal{T}_u$$

where P^C_t is the price of the final consumption bundle, P_t is the price of final output and of the domestically produced non-energy consumption good, P^E_t is the price of energy in domestic currency, paid to the domestic firm that imports energy goods from abroad (i.e. a local gas station). W^h_t denotes the nominal wage received by households, $B_{u,t}$ and $B^*_{u,t}$ denote domestic and foreign nominal debt holdings, which provide a nominal gross returns of R_t and $\bar{R}^*$ to the household. $\mathcal{E}_t$ denotes the nominal exchange rate (domestic currency relative to foreign currency), $DIV_{u,t} = DIV^F_{u,t} + DIV^L_{u,t}$ are the profits made by monopolistic firms (F) and unions (L) that are re-distributed lump-sum to unconstrained households. Total firm profits consist of final output (Z) firm profits $DIV^F_{u,t} = DIV^Z_{u,t}$, $T_{u,t} = T^F_{u,t} + T^L_{u,t}$ are a lump-sum taxes imposed on unconstrained households (to subsidize firms costs in order to get a steady state in which the distortion from monopolistic competition is eliminated). $\mathcal{T}_u$ is a steady-state transfer from unconstrained to the constrained household in order to equate their steady state level of consumption.

Total Consumption Expenditure Unconstrained households maximise their expenditure

$$\max_{C_{u,t}, E^h_{u,t}} \left\{ P^{CES_u}_t CES_{u,t} - P^C_t C_{u,t} - P^E_t E^h_{u,t} \right\} \quad s.t. \quad CES_{u,t} = \left((1-\alpha_{u,ec})^{\frac{1}{\psi_{ec}}} C_{u,t}^{\frac{\psi_{ec}-1}{\psi_{ec}}} + \alpha_{u,ec}^{\frac{1}{\psi_{ec}}} \left(E^h_{u,t} \right)^{\frac{\psi_{ec}-1}{\psi_{ec}}} \right)^{\frac{\psi_{ec}}{\psi_{ec}-1}}$$

which implies $\frac{\partial CES_{u,t}}{\partial C_{u,t}} = \frac{P^C_t}{P^{CES_u}_t}$ and $\frac{\partial CES_{u,t}}{\partial E^h_{u,t}} = \frac{P^E_t}{P^{CES_u}_t}$ so that the relative demand schedules are given by

$$C_{u,t} \quad = \quad \left(\frac{p^C_t}{p^{CES_u}_t} \right)^{-\psi_{ec}} (1-\alpha_{u,ec}) CES_{u,t} \tag{A.1}$$

$$E^h_{u,t} \quad = \quad \left(\frac{p^E_t}{p^{CES_u}_t} \right)^{-\psi_{ec}} (\alpha_{u,ec}) CES_{u,t}. \tag{A.2}$$

Optimality also implies

$$\begin{aligned}
p_t^{CES_u} CES_{u,t} &= p_t^C C_{u,t} + p_t^E E_{u,t}^h, \qquad P_t^{CES_u}/P_t = p_t^{CES_u}, P_t^C/P_t = p_t^C, P_t^E/P_t = p_t^E \\
\left(p_t^{CES_u}\right)^{1-\psi_{ec}} &= (1-\alpha_{u,ec})\left(p_t^C\right)^{1-\psi_{ec}} + \alpha_{u,ec}\left(p_t^E\right)^{1-\psi_{ec}} \\
p_t^{CES_u} &= \left[(1-\alpha_{u,ec})\left(p_t^C\right)^{1-\psi_{ec}} + \alpha_{u,ec}\left(p_t^E\right)^{1-\psi_{ec}}\right]^{\frac{1}{1-\psi_{ec}}} \qquad \text{(A.3)}
\end{aligned}$$

where P_t is the domestic final output price level.

Lagrangian for Lifetime Utility Maximisation Each unconstrained household solves the following Lagrangian in any arbitrary period t

$$\begin{aligned}
\mathcal{L}_{u,t} = \sum_{\mathcal{S}^t} \pi_{\mathcal{S}^t} \sum_{t=0}^{\infty} \beta^t \Bigg\{ U_{u,t} + \Lambda_{u,t} \Bigg[& W_t^h N_{u,t}^h - R_{t-1} B_{u,t-1} - \bar{R}^* B_{u,t-1}^* \mathcal{E}_t \\
& + DIV_{u,t}^F + DIV_{u,t}^L - P_t^{CES_u} CES_{u,t} + B_{u,t} + B_{u,t}^* \mathcal{E}_t - T_{u,t}^F - T_{u,t}^L - P_t \mathcal{T}_u \Bigg] \Bigg\}
\end{aligned}$$

where $\Lambda_{u,t}$ is the Lagrange multiplier for with the unconstrained households' resource constraint.

Optimal Choice of CES_u The first-order condition for unconstrained CES-composite consumption is

$$\Lambda_{u,t} = \frac{(CES_{u,t})^{-\sigma}}{P_t^{CES_u}}, \quad \lambda_{u,t} \equiv (CES_{u,t})^{-\sigma}. \qquad \text{(A.4)}$$

where we define the marginal utility of unconstrained CES-composite consumption as $\lambda_{u,t}$.

Optimal Choice of N_u^h The first order conditions for unconstrained household labor supply are

$$\Lambda_{u,t} = -\frac{U_{u,t}^N}{W_t^h} \quad \Leftrightarrow \quad W_t^h = MRS_{u,t} \equiv -\frac{U_{u,t}^N}{\Lambda_{u,t}}$$

We define $w_t^h \equiv W_t^h / P_t$ and denote the real wage and the marginal rate of substitution in real terms

$$\begin{aligned}
w_t^h &= mrs_{u,t} && \text{(A.5)} \\
mrs_{u,t} &= -\frac{U_{u,t}^N}{\lambda_{u,t}/p_t^{CES_u}} && \text{(A.6)} \\
U_{u,t}^N &= -\chi\left(N_{u,t}^h\right)^{\varphi} && \text{(A.7)}
\end{aligned}$$

Optimal Choice of B_u **and** B_u^* The domestic saving/CES-consumption Euler equation is then given by

$$(\Lambda_{u,t}) = \beta E_t(\Lambda_{u,t+1} R_t), \quad \frac{1}{P_t^{CES_u}} \lambda_{u,t} = \mathbf{E}_t\left[\beta \frac{1}{P_{t+1}^{CES_u}} \lambda_{u,t+1}\right] R_t, \quad 1 = \mathbf{E}_t\left[\beta \frac{\lambda_{u,t+1}}{\lambda_{u,t}} \frac{1}{\Pi_{t+1}^{CES_u}}\right] R_t. \qquad \text{(A.8)}$$

where

$$\Pi_t^{CES_u} \equiv \frac{P_t^{CES_u}}{P_{t-1}^{CES_u}} = \frac{p_t^{CES_u}}{p_{t-1}^{CES_u}} \frac{P_t}{P_{t-1}} = \frac{p_t^{CES_u}}{p_{t-1}^{CES_u}} \Pi_t \qquad \text{(A.9)}$$

and $\Pi_t = P_t / P_{t-1}$ refers to domestic final output price inflation. The foreign saving-consumption Euler equation is as follows

$$
\begin{aligned}
\frac{\partial \mathcal{L}_{u,t}}{\partial B^*_{u,t}} &= \pi_{\mathcal{S}^t} \beta^t \left(\Lambda_{u,t} \left[-\mathcal{E}_t \right] \right) + \beta^{t+1} \sum_{\mathcal{S}^{t+1} > \mathcal{S}^t} \pi_{\mathcal{S}^{t+1}} \left(\Lambda_{u,t+1} \bar{R}^* \mathcal{E}_{t+1} \right) = 0 \\
1 &= \beta \mathbf{E}_t \left(\frac{\Lambda_{u,t+1}}{\Lambda_{u,t}} \bar{R}^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) \Leftrightarrow 1 = \beta \mathbf{E}_t \left(\frac{\Lambda_{u,t+1}}{\Lambda_{u,t}} \bar{R}^* \frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t} \frac{\Pi_{t+1}}{\Pi^*_{ss}} \right)
\end{aligned}
$$

We use the definition of the real exchange rate

$$
\mathcal{Q}_t \equiv \mathcal{E}_t \frac{P^*_t}{P_t}
$$

P_t (P^*_t) denotes the domestic (foreign) final output price level. We can derive the uncovered interest rate parity (UIP) condition by combining the two Euler equations

$$
\mathbf{E}_t \left[\beta \frac{\lambda_{u,t+1}}{\lambda_{u,t}} \frac{1}{\Pi^{CES_u}_{t+1}} \left(R_t - \bar{R}^* \frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t} \frac{\Pi_{t+1}}{\Pi^*_{ss}} \right) \right] = 0. \tag{A.10}
$$

We define the unconstrained household's stochastic discount factor as

$$
\Lambda_{u,t,t+1} \equiv \mathbf{E}_t \left[\beta \frac{\lambda_{u,t+1}}{\lambda_{u,t}} \right] \tag{A.11}
$$

Unconstrained Household Budget in real terms

$$
\begin{aligned}
w^h_t N^h_{u,t} + \frac{R_{t-1} b_{u,t-1}}{\Pi_t} + \frac{\bar{R}^* b^*_{u,t-1} \mathcal{Q}_t}{\Pi^*_{ss}} + div^F_{u,t} + div^L_{u,t} &= p^C_t C_{u,t} + p^E_t E^h_{u,t} + b_{u,t} + b^*_{u,t} \mathcal{Q}_t \\
&\quad + t^F_{u,t} + t^L_{u,t} + \mathcal{T}_u
\end{aligned} \tag{A.12}
$$

Detrending Total Profits from Firm Ownership

$$
DIV^F_{u,t} = DIV^Z_{u,t}, \quad div^F_{u,t} \equiv \frac{1}{P_t} DIV^F_{u,t}, \quad div^F_{u,t} = div^Z_{u,t}. \tag{A.13}
$$

Constrained Households Members of constrained households consume, save and work to maximise their lifetime utility $\mathcal{U}_{c,s}$

$$
\mathcal{U}_{c,s} = \mathbf{E}_s \left[\sum_{t=s}^{\infty} \beta^t \left\{ U_{c,t} \left(C_{c,t}, E^h_{c,t}, N^h_{c,t} \right) \right\} \right], \quad U_{c,t} = \left[\frac{(CES_{c,t})^{1-\sigma} - 1}{1-\sigma} - \chi \frac{\left(N^h_{c,t} \right)^{1+\varphi}}{1+\varphi} \right]
$$

$$
\text{and} \quad CES_{c,t} = \left((1-\alpha_{c,ec})^{\frac{1}{\psi_{ec}}} \left(C_{c,t} \right)^{\frac{\psi_{ec}-1}{\psi_{ec}}} + (\alpha_{c,ec})^{\frac{1}{\psi_{ec}}} \left(E^h_{c,t} \right)^{\frac{\psi_{ec}-1}{\psi_{ec}}} \right)^{\frac{\psi_{ec}}{\psi_{ec}-1}}.
$$

where $CES_{c,t}$ is a CES composite of domestically produced non-energy goods $C_{u,t}$ and of imported energy goods $E^h_{c,t}$. The weight of energy in the consumption composite for the constrained household is $\alpha_{c,ec}$, potentially different from the unconstrained energy-weight $\alpha_{u,ec}$. $N^h_{c,t}$ is the constrained household's labor supply and φ is the elasticity of labor supply, χ is the relative weight on the disutility of working. Utility is maximised subject to the budget constraint

$$
W^h_t N^h_{c,t} - R_{t-1} B_{c,t-1} + DIV^L_{c,t} = P^C_t C_{c,t} + P^E_t E^h_{c,t} - B_{c,t} + P_t \vartheta_{c,t} + T^L_{c,t} - P_t \mathcal{T}_c, \quad \vartheta_{c,t} = \frac{\vartheta_c}{2} \left(\frac{B_{c,t}}{P_t} - \bar{b}_c \right)^2
$$

where $B_{c,t}$ denotes a domestic nominal risk-less debt, which provides a nominal gross returns of R_t to the constrained household and $DIV^L_{c,t}$ are the profits made by monopolistically competitive labor unions. $T^L_{c,t}$ is a transfer to the union in order to subsidize its cost.

Total Consumption Expenditure Constrained households maximise their expenditure

$$\max_{C_{c,t},E^h_{c,t}} \left\{ P^{CES_c}_t CES_{c,t} - P^C_t C_{c,t} - P^E_t E^h_{c,t} \right\} \quad s.t. \quad CES_{c,t} = \left((1-\alpha_{c,ec})^{\frac{1}{\psi_{ec}}} C_{c,t}^{\frac{\psi_{ec}-1}{\psi_{ec}}} + \alpha_{c,ec}^{\frac{1}{\psi_{ec}}} \left(E^h_{c,t} \right)^{\frac{\psi_{ec}-1}{\psi_{ec}}} \right)^{\frac{\psi_{ec}}{\psi_{ec}-1}}$$

so that the relative demand schedules are given by

$$\begin{aligned} C_{c,t} &= \left(p^C_t / p^{CES_c}_t \right)^{-\psi_{ec}} (1-\alpha_{c,ec}) CES_{c,t} && \text{(A.14)} \\ E^h_{c,t} &= \left(p^E_t / p^{CES_c}_t \right)^{-\psi_{ec}} (\alpha_{c,ec}) CES_{c,t}. && \text{(A.15)} \end{aligned}$$

Optimality also implies

$$P^{CES_c}_t CES_{c,t} = P^C_t C_{c,t} + P^E_t E^h_{c,t}, \qquad p^{CES_c}_t = \left[(1-\alpha_{c,ec}) \left(p^C_t \right)^{1-\psi_{ec}} + \alpha_{c,ec} \left(p^E_t \right)^{1-\psi_{ec}} \right]^{\frac{1}{1-\psi_{ec}}}. \quad \text{(A.16)}$$

Lagrangian Each constrained household solves the following Lagrangian in any arbitrary period t

$$\mathcal{L}_{c,t} = \sum_{\mathcal{S}^t} \pi_{\mathcal{S}^t} \sum_{t=0}^{\infty} \beta^t \left\{ U_{c,t} + \Lambda_{c,t} \left[W^h_t N^h_{c,t} - R_{t-1} B_{c,t-1} + DIV^L_{c,t} - P^{CES_c}_t CES_{c,t} + B_{c,t} - P_t \vartheta_{c,t} - T^L_{c,t} + P_t \mathcal{T}_c \right] \right\}$$

where $\Lambda_{c,t}$ is the constrained household Lagrange multiplier associated with the resource constraint.

Optimal Choice of CES_c The first-order condition for constrained CES-composite consumption is

$$\Lambda_{c,t} = \frac{U^{CES}_{c,t}}{P^{CES_c}_t}, \quad U^{CES}_{c,t} = (CES_{c,t})^{-\sigma}, \quad \lambda_{c,t} \equiv P_t \Lambda_{c,t} = \frac{(CES_{c,t})^{-\sigma}}{p^{CES_u}_t} \quad \text{(A.17)}$$

where we define the marginal utility of constrained household consumption as $\lambda_{c,t}$.

Optimal Choice of B_c The constrained household first order conditions for domestic bonds are

$$\begin{aligned} \frac{\partial \mathcal{L}_{c,t}}{\partial B_{c,t}} &= \pi_{\mathcal{S}^t} \beta^t \left\{ \Lambda_{c,t} \left[- \left(1 + P_t \frac{\partial \vartheta_{c,t}}{\partial B_{c,t}} \right) \right] \right\} + \pi_{\mathcal{S}^{t+1|t}} \beta^{t+1} \left\{ \Lambda_{c,t+1} \left[R_t \right] \right\} = 0 \\ 0 &= \left\{ \Lambda_{c,t} \left[- \left(1 + P_t \frac{\partial \vartheta_{c,t}}{\partial B_{c,t}} \right) \right] \right\} + \beta \mathbf{E}_t \left\{ \Lambda_{c,t+1} \left[R_t \right] \right\} \end{aligned}$$

The constrained household saving-consumption Euler equation is then given by

$$1 - \vartheta_c (b_{c,t} - \bar{b}_c) = \mathbf{E}_t \left[\beta \frac{U^{CES}_{c,t+1}}{U^{CES}_{c,t}} \frac{1}{\Pi^{CES_c}_{t+1}} \right] R_t, \text{ where } \Pi^{CES_c}_t = \frac{p^{CES_c}_t}{p^{CES_c}_{t-1}} \Pi_t, \text{ and } \Lambda_{c,t,t+1} \equiv \mathbf{E}_t \left[\beta \frac{\Lambda_{c,t+1}}{\Lambda_{c,t}} \right]$$

Optimal Choice of N^h_c The first order conditions for labor supply is

$$\Lambda_{c,t} = -\frac{U^N_{c,t}}{W^h_t} \quad \Leftrightarrow \quad W^h_t = MRS_{c,t} \equiv -\frac{U^N_{c,t}}{\Lambda_{c,t}}$$

We define $w_t^h \equiv W_t^h / P_t$ and denote the wage and the marginal rate of substitution in real terms.

$$\begin{aligned} mrs_{c,t} &= -\frac{U_{c,t}^N}{(CES_{c,t}^{-\sigma})/p_t^{CES_c}} && \text{(A.18)} \\ U_{c,t}^N &= -\chi\left(N_{c,t}^h\right)^{\varphi} && \text{(A.19)} \end{aligned}$$

We will consider two cases for the constrained household labour supply. In case 1, we follow the standard in the TANK literature and let the constrained household labour supply be determined by the unconstrained household's marginal rate of substitution. By doing so, we eliminate heterogeneity in hours worked among constrained and unconstrained households, preventing an implausible increase in labour supply whenever constrained households are hit by adverse shocks. In case 2, we relax this assumption and allow constrained households to supply labour according to their own marginal rate of substitution.

$$\begin{aligned} &\textbf{Case 1: Constrained Labour Supply is determined by } MRS_u \quad n_{c,t}^h = n_{u,t}^h \\ &\textbf{Case 2: Constrained Labour Supply is determined by } MRS_c \quad w_t^h = mrs_{c,t} \end{aligned} \tag{A.20}$$

Real-term constrained household budget ($\vartheta_c = \infty, \bar{b}_c = 0$)

$$p_t^C C_{c,t} + p_t^E E_{c,t}^h = w_t^h N_{c,t}^h + div_{c,t}^L - t_{c,t}^L + \mathcal{T}_c \tag{A.21}$$

We use

$$\begin{aligned} t_t^F &= t_t^Z && \text{(A.22)} \\ t_t^Z &= (1-\tau_t^Z)(w_t N_t^h + p_t^E E_t^z) && \text{(A.23)} \\ t_t^L &= (1-\tau_t^W) w_t^h N_t^h && \text{(A.24)} \end{aligned}$$

Aggregation and Market Clearing

$$\begin{aligned} C_t &= \omega C_{c,t} + (1-\omega) C_{u,t} && \text{(A.25)} \\ E_t^h &= \omega E_{c,t}^h + (1-\omega) E_{u,t}^h && \text{(A.26)} \\ N_t^h &= \omega N_{c,t}^h + (1-\omega) N_{u,t}^h && \text{(A.27)} \\ \Lambda_{t,t+1} &= (1-\omega)\Lambda_{u,t,t+1} && \text{(A.28)} \\ b_t &= \omega b_{c,t} + (1-\omega) b_{u,t} \\ div_t^F &= (1-\omega) div_{u,t}^F \\ div_t^L &= \omega div_{c,t}^L + (1-\omega) div_{u,t}^L \\ b_t^* &= (1-\omega) b_{u,t}^* \\ t_t^F &= (1-\omega) t_{u,t}^F \\ t_t^L &= \omega t_{c,t}^L + (1-\omega) t_{u,t}^L \end{aligned}$$

We define the 'consumer price index' as a population-weighted average price index of the constrained and unconstrained CES-consumption bundles such that

$$P_t^{CPI} = \omega P_t^{CES_c} + (1-\omega) P_t^{CES_u}, \quad \Leftrightarrow \quad p_t^{CPI} = \omega p_t^{CES_c} + (1-\omega) p_t^{CES_u}. \tag{A.29}$$

Domestic Bond Market Clearing We assume that domestic government bonds are in zero net supply

$$b_t = 0 \tag{A.30}$$

This implies that constrained and unconstrained households can lend to each other

$$0 = b_t = \omega b_{c,t} + (1-\omega) b_{u,t}, \qquad \Leftrightarrow \qquad \omega b_{c,t} = -(1-\omega) b_{u,t}$$

but recall the constrained household is subject to bond adjustment costs. We assume that $\vartheta_c = \infty$ and $\bar{b}_c = 0$ so that $b_{u,t} = 0 \; \forall t$.

Firm and Union Profits net of monopolistic competition correction subsidy

$$\begin{aligned} div_t^L - t_t^L &= w_t N_t - \tau_t^W w_t^H N_t^h - w_t^H N_t^h + \tau_t^W w_t^H N_t^h = w_t N_t - w_t^H N_t^h, \quad t_t^L = (1-\tau_t^L) w_t^h N_t^h \\ div_t^Z - t_t^Z &= Z_t - \tau_t^Z \left(w_t N_t + p_t^E E_t^z \right) - t_t^Z = Z_t - \left(w_t N_t + p_t^E E_t^z \right) \end{aligned}$$

Combine Firm Profits net of monopolistic competion correction subsidy

$$\begin{aligned} div_t^F - t_t^F &= Z_t - \left(w_t N_t + p_t^E E_t^z \right) + p_t^E E_t - p_t^E E_t + p_t^{EXP} \mathcal{Q}_t X_t - p_t^X X_t \\ div_t^F - t_t^F &= Z_t - w_t N_t - p_t^E E_t^z + p_t^{EXP} \mathcal{Q}_t X_t - p_t^X X_t \end{aligned}$$

Goods Market Clearing - Combine Household Budgets Recall the real-term household budgets

$$\begin{aligned} p_t^C C_{u,t} &= w_t^h N_{u,t}^h - \frac{R_{t-1}^* b_{u,t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} + div_{u,t}^F + div_{u,t}^L - t_{u,t}^F - t_{u,t}^L + b_{u,t}^* \mathcal{Q}_t - \mathcal{T}_u - p_t^E E_{u,t}^h \\ p_t^C C_{c,t} &= w_t^h N_{c,t}^h + div_{c,t}^L - t_{c,t}^L + \mathcal{T}_c - p_t^E E_{c,t}^h \end{aligned}$$

and re-arrange for consumption, pre-multiplied with their household-type share ω to get

$$\begin{aligned} p_t^C C_t &= (1-\omega) \left(w_t^h N_{u,t}^h - \frac{\bar{R}^* b_{u,t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} + div_{u,t}^F - t_{u,t}^F + div_{u,t}^L - t_{u,t}^L + b_{u,t}^* \mathcal{Q}_t - \mathcal{T}_u - p_t^E E_{u,t}^h \right) \\ &\quad + \omega \left(w_t^h N_{c,t}^h + div_{c,t}^L - t_{c,t}^L - \frac{\vartheta_c}{2} \left(b_{c,t} - \bar{b}_c \right)^2 + \mathcal{T}_c - p_t^E E_{c,t}^h \right), \quad (1-\omega)\mathcal{T}_u = \omega \mathcal{T}_c \end{aligned}$$

which can be simplified to

$$\begin{aligned} p_t^C C_t &= \underbrace{w_t^h N_t^h + div_t^L - t_t^L}_{= w_t N_t^h} - \frac{\bar{R}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} + \underbrace{div_t^F - t_t^F}_{Z_t - w_t N_t^h - p_t^E E_t^z + p_t^{EXP} \mathcal{Q}_t X_t - p_t^X X_t} \\ &\quad + b_t^* \mathcal{Q}_t - p_t^E E_t^h \end{aligned}$$

to get

$$\begin{aligned} p_t^C C_t + p_t^X X_t &= \underbrace{\left(b_t^* \mathcal{Q}_t - \frac{\bar{R}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} \right)}_{-nx_t} + Z_t + p_t^{EXP} \mathcal{Q}_t X_t - p_t^E (E_t^z + E_t^h) \\ p_t^C C_t + \underbrace{p_t^X X_t - \overbrace{p_t^{EXP} \mathcal{Q}_t}^{p_t^X} X_t}_{=0} + nx_t &= Z_t - p_t^E (E_t^z + E_t^h) \equiv \frac{INC_t}{P_t} \end{aligned}$$

and

$$p_t^C C_t + p_t^X X_t = \left(b_t^* \mathcal{Q}_t - \frac{\bar{R}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} \right) + Z_t + \underbrace{p_t^{EXP} \mathcal{Q}_t X_t - p_t^E (E_t^z + E_t^h)}_{nx_t}$$

and finally

$$p_t^C C_t + p_t^X X_t \quad = \quad Z_t \tag{A.31}$$

and the real trade balance nx_t is defined as

$$nx_t \quad = \quad p_t^{EXP} \mathcal{Q}_t X_t - p_t^E (E_t^z + E_t^h) = \frac{\bar{R}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} - b_t^* \mathcal{Q}_t \tag{A.32}$$

Consumption Gap Definition We define the consumption gap as the ratio between unconstrained and constrained consumption

$$\Gamma_t \equiv \frac{C_{u,t}}{C_{c,t}}. \tag{A.33}$$

Income Gap Definition We define the 'income' of the unconstrained and constrained households as

$$inc_{u,t} \quad \equiv \quad w_t N_{u,t}^h - \mathcal{T}_u + div_{u,t}^F - t_{u,t}^F = C_{u,t} + p_t^E E_{u,t}^h + \underbrace{\frac{R_{t-1}^* b_{u,t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} - b_{u,t}^* \mathcal{Q}_t}_{=-\frac{1}{1-\omega}\left(p_t^E (E_t^z + E_t^h) - p_t^X X_t\right)} \tag{A.34}$$

$$inc_{c,t} \quad \equiv \quad w_t N_{c,t}^h + \mathcal{T}_c = C_{c,t} + p_t^E E_{c,t}^h \tag{A.35}$$

The 'income gap' is the ratio between unconstrained and constrained income

$$\Gamma_t^{inc} = \frac{inc_{u,t}}{inc_{c,t}} = \frac{C_{u,t} + p_t^E E_{u,t}^h + nx_t/(1-\omega)}{C_{c,t} + p_t^E E_{c,t}^h} \tag{A.36}$$

It can be shown that

$$\begin{aligned} \Gamma_t^{inc} \quad &= \quad \Gamma_t \left(\frac{C_{c,t}}{inc_{c,t}} \right) + \frac{p_t^E E_{u,t}^h}{inc_{c,t}} + \frac{nx_t}{(1-\omega)\,(inc_{c,t})} \\ \Gamma_t \quad &= \quad \Gamma_t^{inc} \left(\frac{inc_{c,t}}{C_{c,t}} \right) - \frac{1}{(1-\omega) C_{c,t}} \left(p_t^X X_t - p_t^E (E_t^z + E_t^h) \right) - \frac{p_t^E E_{u,t}^h}{C_{c,t}} \end{aligned}$$

In the case in which households don't consume energy, $\alpha_{u,ec} = \alpha_{c,uc} = 0$ we would get

$$\Gamma_t \quad = \quad \Gamma_t^{inc} - \frac{1}{(1-\omega) C_{c,t}} \left(p_t^X X_t - p_t^E (E_t^z) \right)$$

A.2 Labor Packers and Unions

The introduction of wage stickiness into the model involves two types of agents: (i) perfectly competitive labor packers and (ii) monopolistically competitive unions. After households have chosen how much labor to supply in a given period, $N_{k,t}^h(j)$, $k \in \{u,c\}$, this labor is supplied to a union, in return for a nominal wage W_t^h. The union unpacks the homogenous labor supplied by households and differentiates it into different varieties $N_t(j)$, $j \in [0,1]$ and sells these units of labor varieties at wage $W_t(j)$. The union acts as monopolist since each labor variety is only imperfectly substitutable with each other.

Labor Packers Varieties $N_t(j)$ are assembled by labor packers according to a CES production function. $N_t(j)$ denotes the demand for a specific labor variety j and N_t denotes aggregate labor demand. ϵ_w is the elasticity of substitution between labor varieties and thus $\mathcal{M}_w = \epsilon_w/(\epsilon_w - 1)$ is the corresponding gross wage markup of monopolistically competitive unions. After the packers have assembled the labor bundle they sell it to firms at wage W_t who then use it in the production process. The packers' production function, and the implied demand schedule associated with the cost minimisation are

$$N_t = \left[\int_0^1 (N_t(j))^{\frac{\epsilon_w - 1}{\epsilon_w}} dj\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \quad N_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{\frac{\mathcal{M}_w}{1-\mathcal{M}_w}} N_t, \quad W_t \equiv \left(\int_0^1 (W_t(j))^{\frac{1}{1-\mathcal{M}_w}} dj\right)^{1-\mathcal{M}_w}$$

where W_t is the aggregate wage index. Optimal packer behaviour implies that $W_t N_t = \int_0^1 W_t(j) N_t(j) dj$.

Labor Unions Each individual labor union who sells its imperfectly substitutable labor variety $N_t^h(j)$ to the packer is subject to *nominal wage rigidities*. The probability that the union cannot reset its wage is ϕ_w. It is convenient to split the problem of a monopolistically competitive labor union into two steps: (i) the intra-temporal cost minimisation problem and (ii) the inter-temporal wage setting problem.

Cost Minimisation Problem A union will choose to minimise its costs $\tau^W W_t^h N_t^h(j)$ subject to meeting the packer's labor demand. The Lagrange multiplier $MC_t^W(j)$ is the union's (nominal) shadow cost of providing one more unit of labor, i.e. the nominal marginal cost and τ^W is a subsidy to marginal costs that eliminates the steady state distortion associated with monopolistic competition. Note that the Lagrange multiplier of an individual union j does not depend on its own quantities of inputs demanded, so that all unions have the same marginal costs $MC_t^W(j) = MC_t^W$. The wage paid to the household[25], W_t^h corresponds to the marginal rate of substitution so that $MC_t^W = \tau^W W_t^h = \tau^W MRS_t$. Recall that we use lower cases to denotes real (final output price level) terms $w_t^h \equiv W_t^h / P_t$ so that

$$mc_t^W = \tau^W w_t^h = \tau^W mrs_t \tag{A.37}$$

Wage Setting The objective of each union j is to maximise its nominal profits $DIV_t^L(j)$

$$DIV_t^L(j) = W_t(j) N_t^h(j) - \left\{\tau^W \left(W_t^h N_t^h(j)\right)\right\}, \quad div_t^L = \left(w_t - mc_t^W\right) N_t^h \tag{A.38}$$

With probability ϕ_w a union is stuck with its previous-period wage indexed to a composite

$$W_t(j) = \begin{cases} W_t^\#(j) & \textit{with probability: } 1 - \phi_w \\ W_{t-1}(j)\left(\left(\Pi_{ss}^W\right)^{1-\xi_w}\left(\Pi_{t-1}^W\right)^{\xi_w}\right) & \textit{with probability: } \phi_w \end{cases}$$

where $\xi_w = 0$ is the weight attached to the previous period wage inflation. Consider a union who can reset its wage in the current period $W_t(j) = W_t^\#(j)$ and who is then stuck with its wage until future period $t+s$. The wage in this case would be

$$W_{t+s}(j) = W_t^\#(j)\left(\Pi_{ss}^W\right)^{s(1-\xi_w)}\left(\prod_{g=0}^{s-1}\left(\left(\Pi_{t+g}^W\right)^{\xi_w}\right)\right) = W_t^\#(j)\left[\left(\Pi_{ss}^W\right)^{s(1-\xi_w)}\left(\frac{W_{t+s-1}}{W_{t-1}}\right)^{\xi_w}\right]$$

Subject to the above derived demand constraint and assuming that a union j always meets the demand for its labor at the current wage labor unions solve the following optimisation problem

$$\max_{W_t^\#(j)} E_t \sum_{s=0}^{\infty} (\phi_W)^s \Lambda_{u,t,t+s} P_{t+s}\left[\left(W_{t+s}(j)/P_{t+s} - mc_{t+s}^W\right) N_{t+s}^h(j)\right] \text{ s.t. } N_{t+s}^h(j) = \left(\frac{W_{t+s}(j)}{W_{t+s}}\right)^{-\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} N_{t+s}.$$

[25] We assume that both, unconstrained and constrained household receive the same wage.

Taking the derivative with respect to $W_t^{\#}(j)$ delivers the familiar wage inflation schedule

$$\frac{f_t^{W,1}}{f_t^{W,2}}\mathcal{M}_w = w_t^{\#} = \frac{W_t^{\#}}{W_t} = \left(\frac{1-\phi_W(\zeta_t^W)^{\frac{1}{\mathcal{M}_w-1}}}{1-\phi_W}\right)^{1-\mathcal{M}_w} \tag{A.39}$$

$$f_t^{W,1} = N_t\frac{mc_t^W}{w_t} + \phi_W\beta E_t\left[\left(\frac{U_{u,t+1}^{CES}}{U_{u,t}^{CES}}\right)\left(\frac{\Pi_{t+1}^W}{\Pi_{t+1}^{CES_u}}\right)\left(\frac{\Pi_{t+1}^W}{\Pi_{ss}^W}\right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w-1}} f_{t+1}^{W,1}\right] \tag{A.40}$$

$$f_t^{W,2} = N_t + \phi_W\beta E_t\left[\left(\frac{U_{u,t+1}^{CES}}{U_{u,t}^{CES}}\right)\left(\frac{\Pi_{t+1}^W}{\Pi_{t+1}^{CES_u}}\right)\left(\frac{\Pi_{t+1}^W}{\Pi_{ss}^W}\right)^{\frac{1}{\mathcal{M}_w-1}} f_{t+1}^{W,2}\right] \tag{A.41}$$

$$\zeta_t^W = \frac{\Pi_t^W}{\Pi_{ss}^W} \tag{A.42}$$

$$w_t = \frac{\Pi_t^W}{\Pi_t}w_{t-1} \tag{A.43}$$

$$\mathcal{D}_t^W = (1-\phi_W)\left(\frac{1-\phi_W\left(\zeta_t^W\right)^{\frac{1}{\mathcal{M}_w-1}}}{1-\phi_W}\right)^{\mathcal{M}_w} + \phi_W\left(\zeta_t^W\right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w-1}}\mathcal{D}_{t-1}^W \tag{A.44}$$

Wage dispersion is given by $\mathcal{D}^W$. Aggregate hours worked in the economy is given by $N_t^h = N_t\mathcal{D}_t^W$.

Calvo Wage Setting Derivation

$$\max_{W_t^{\#}(j)} E_t\sum_{s=0}^{\infty}(\phi_W)^s\Lambda_{u,t,t+s}\left[DIV_{t+s}^L(j)\right], \quad\Leftrightarrow\quad \max_{W_t^{\#}(j)} E_t\sum_{s=0}^{\infty}(\phi_W)^s\Lambda_{u,t,t+s}\left[DIV_{t+s}^L(j)\right]\frac{P_{t+s}}{P_{t+s}}$$

$$DIV_t^L(j) = W_t(j)N_t^h(j) - \tau^W W_t^h N_t^h(j),\ div_t^L(j) = \left(W_t(j)/P_t - mc_t^W\right)N_t^h(j),\ div_t^L(j) \equiv DIV_t^L(j)/P_t$$

$$\max_{W_t^{\#}(j)} E_t\sum_{s=0}^{\infty}(\phi_W)^s\Lambda_{u,t,t+s}P_{t+s}N_{t+s}\left[\frac{W_{t+s}(j)}{P_{t+s}}\left(\frac{W_{t+s}(j)}{W_{t+s}}\right)^{-\frac{\mathcal{M}_w}{\mathcal{M}_w-1}} - mc_{t+s}^W\left(\frac{W_{t+s}(j)}{W_{t+s}}\right)^{-\frac{\mathcal{M}_w}{\mathcal{M}_w-1}}\right]$$

Take the derivative and note that, if we assume $\xi_w = 0$, $W_{t+s}(j) = W_t^{\#}(j)(\Pi_{ss}^W)^s$

$$\begin{aligned}&E_t\sum_{s=0}^{\infty}(\phi_W)^s\Lambda_{u,t,t+s}P_{t+s}N_{t+s}\left[\frac{1}{P_{t+s}}\left(\frac{(\Pi_{ss}^W)^s}{W_{t+s}}\right)^{-\frac{\mathcal{M}_w}{\mathcal{M}_w-1}}\left(\frac{W_t}{W_t}W_t^{\#}\right)(\Pi_{ss}^W)^s\right]\\ &= E_t\sum_{s=0}^{\infty}(\phi_W)^s\Lambda_{u,t,t+s}P_{t+s}N_{t+s}\left[mc_{t+s}^W\left(\frac{(\Pi_{ss}^W)^s}{W_{t+s}}\right)^{-\frac{\mathcal{M}_w}{\mathcal{M}_w-1}}(\mathcal{M}_w)\right]\end{aligned}$$

$$\begin{aligned}&w_t^{\#}\frac{1}{\mathcal{M}_w}\left(E_t\sum_{s=0}^{\infty}(\phi_W)^s\Lambda_{u,t,t+s}P_{t+s}N_{t+s}\left[\frac{1}{P_{t+s}}\left(\frac{(\Pi_{ss}^W)^s}{W_{t+s}}\right)^{-\frac{\mathcal{M}_w}{\mathcal{M}_w-1}}\left(W_t(\Pi_{ss}^W)^s\right)\right]\right)\\ &= \left(E_t\sum_{s=0}^{\infty}(\phi_W)^s\Lambda_{u,t,t+s}P_{t+s}N_{t+s}\left[mc_{t+s}^W\left(\frac{1}{W_{t+s}}(\Pi_{ss}^W)^s\right)^{-\frac{\mathcal{M}_w}{\mathcal{M}_w-1}}\right]\right), \quad w_t^{\#} = \frac{W_t^{\#}}{W_t}\end{aligned}$$

$$w_t^{\#} = \mathcal{M}_w \frac{\left(E_t \sum_{s=0}^{\infty} (\phi_W)^s \Lambda_{u,t,t+s} P_{t+s} N_{t+s} \left[mc_{t+s}^W \left(\frac{1}{W_{t+s}} (\Pi_{ss}^W)^s \right)^{-\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} \right] \right)}{\left(E_t \sum_{s=0}^{\infty} (\phi_W)^s \Lambda_{u,t,t+s} P_{t+s} N_{t+s} \left[\frac{1}{P_{t+s}} \left(\frac{(\Pi_{ss}^W)^s}{W_{t+s}} \right)^{-\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} \left(\frac{P_t}{P_t} W_t (\Pi_{ss}^W)^s \right) \right] \right)} \frac{W_t^{-\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}}}{W_t^{-\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}}} \frac{P_t}{P_t}$$

$$w_t^{\#} = \mathcal{M}_w \frac{\left(E_t \sum_{s=0}^{\infty} (\phi_W)^s \Lambda_{u,t,t+s} \frac{P_{t+s}}{P_t} N_{t+s} \frac{mc_{t+s}^W}{w_t} \left[\left(\frac{W_{t+s}}{W_t} \frac{1}{(\Pi_{ss}^W)^s} \right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} \right] \right)}{\left(E_t \sum_{s=0}^{\infty} (\phi_W)^s \Lambda_{u,t,t+s} \frac{P_{t+s}}{P_t} N_{t+s} \left[\frac{P_t}{P_{t+s}} \left(\frac{W_{t+s}}{W_t} \frac{1}{(\Pi_{ss}^W)^s} \right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} (\Pi_{ss}^W)^s \right] \right)}$$

Introduce the definition $f_t^{W,1}$

$$\begin{aligned} f_t^{W,1} &\equiv E_t \sum_{s=0}^{\infty} (\phi_W)^s \Lambda_{u,t,t+s} \frac{P_{t+s}}{P_t} N_{t+s} \frac{mc_{t+s}^W}{w_t} \left(\frac{W_{t+s}}{W_t} \frac{1}{(\Pi_{ss}^W)^s} \right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} \\ f_{t+1}^{W,1} &= E_{t+1} \sum_{s=0}^{\infty} (\phi_W)^s \Lambda_{u,t+1,t+1+s} \frac{P_{t+1+s}}{P_{t+1}} N_{t+1+s} \frac{mc_{t+1+s}^W}{w_{t+1}} \left(\frac{W_{t+1+s}}{W_{t+1}} \frac{1}{(\Pi_{ss}^W)^s} \right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} \end{aligned}$$

Replace $s = k + 1$ so that

$$f_t^{W,1} = N_t \frac{mc_t^W}{w_t} + E_t \sum_{k=0}^{\infty} (\phi_W)^{k+1} \Lambda_{u,t,t+k+1} \frac{P_{t+k+1}}{P_t} N_{t+k+1} \frac{mc_{t+k+1}^W}{w_t} \left(\frac{W_{t+k+1}}{W_t} \frac{1}{(\Pi_{ss}^W)^{k+1}} \right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}}$$

$$f_t^{W,1} = N_t \frac{mc_t^W}{w_t}$$

$$+\phi_W E_t \left[E_{t+1} \left[\sum_{k=0}^{\infty} (\phi_W)^k \frac{\Lambda_{u,t+k+1}}{\Lambda_{u,t}} \frac{\Lambda_{u,t+1}}{\Lambda_{u,t+1}} \frac{P_{t+k+1}}{P_t} \frac{P_{t+1}}{P_{t+1}} N_{t+k+1} \frac{mc_{t+k+1}^W}{w_t} \frac{w_{t+1}}{w_{t+1}} \left(\frac{W_{t+k+1}}{W_t} \frac{1}{(\Pi_{ss}^W)^{k+1}} \frac{W_{t+1}}{W_{t+1}} \right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} \right] \right]$$

$$f_t^{W,1} = N_t \frac{mc_t^W}{w_t} + \phi_W E_t \left[E_{t+1} \left[\sum_{k=0}^{\infty} (\phi_W)^k \Lambda_{u,t+1,t+k+1} \frac{\Lambda_{u,t+1}}{\Lambda_{u,t}} \frac{P_{t+k+1}}{P_{t+1}} \frac{P_{t+1}}{P_t} N_{t+k+1} \cdot \right.\right.$$

$$\left.\left. \frac{mc_{t+k+1}^W}{w_{t+1}} \frac{w_{t+1}}{w_t} \left(\frac{W_{t+k+1}}{W_{t+1}} \frac{1}{(\Pi_{ss}^W)^k} \right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} \left(\frac{W_{t+1}}{W_t} \frac{1}{\Pi_{ss}^W} \right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} \right] \right]$$

$$f_t^{W,1} = N_t \frac{mc_t^W}{w_t} + \phi_W E_t \Lambda_{u,t,t+1} \frac{P_{t+1}}{P_t} \frac{w_{t+1}}{w_t} \left(\frac{W_{t+1}}{W_t} \frac{1}{\Pi_{ss}^W} \right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}}$$

$$\cdot \underbrace{\left[E_{t+1} \left[\sum_{k=0}^{\infty} (\phi_W)^k \Lambda_{u,t+1,t+k+1} \frac{P_{t+k+1}}{P_{t+1}} N_{t+k+1} \frac{mc_{t+k+1}^W}{w_{t+1}} \left(\frac{W_{t+k+1}}{W_{t+1}} \frac{1}{(\Pi_{ss}^W)^k} \right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} \right] \right]}_{f_{t+1}^{W,1}}$$

$$\begin{aligned} f_t^{W,1} &= N_t \frac{mc_t^W}{w_t} + \phi_W E_t \Lambda_{u,t,t+1} \Pi_{t+1}^W \left(\frac{\Pi_{t+1}^W}{\Pi_{ss}^W} \right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} f_{t+1}^{W,1} \\ f_t^{W,2} &= N_t + \phi_W E_t \Lambda_{u,t,t+1} \frac{\Pi_{t+1}^W}{\Pi_{t+1}^W} \Pi_{ss}^W \left(\frac{\Pi_{t+1}^W}{\Pi_{ss}^W} \right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} f_{t+1}^{W,2}, \quad \text{recall } \mathbf{E}_t \left[\Lambda_{u,t,t+1} \right] = \mathbf{E}_t \left[\beta \frac{U_{u,t+1}^{CES}}{U_{u,t}^{CES}} \frac{1}{\Pi_{t+1}^{CES_u}} \right] \end{aligned}$$

A.3 Firms

There are three domestic firm sectors in our model: (i) final output good producers, (ii) import good producers and (iii) export good producers. Final output firms are characterised by monopolistic competition and nominal rigidities.

Final Output Goods Sector Final output goods production involves two types of agents: (i) perfectly competitive final output packers and (ii) monopolistically competitive final output producers.

Final Output Good packers Final output packers demand and aggregate infinitely many varieties of final output goods $Z_t(i)$, $i \in [0,1]$ into a final output good Z_t. $Z_t(i)$ denotes the demand for a specific variety i of the final output good and Z_t denotes the aggregate demand of the final output good. ϵ_z is the elasticity of substitution and $\mathcal{M}_z = \epsilon_z/(\epsilon_z - 1)$ is the corresponding gross markup of monopolistically competitive final output good producers. Final output packers purchase a single variety at given prices $P_t(i)$ and sell the final output good Z_t at price P_t to a sectoral retailer who transforms the final output good into consumption and export goods. The packers' CES production function, and the implied demand schedule associated with the cost minimisation are

$$Z_t = \left[\int_0^1 (Z_t(i))^{1-\frac{1}{\epsilon_z}} di\right]^{\frac{\epsilon_z}{\epsilon_z - 1}}, \quad Z_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{\frac{\mathcal{M}_z}{1-\mathcal{M}_z}} Z_t, \quad P_t \equiv \left(\int_0^1 (P_t(i))^{\frac{1}{1-\mathcal{M}_z}} di\right)^{1-\mathcal{M}_z}$$

where P_t is the price index and optimal behaviour implies $P_t Z_t = \int_0^1 P_t(i) Z_t(i) di$.

Final Output Good Producers Each variety $Z_t(i)$ that the final output good packer demands and assembles is produced and supplied by a single *monopolistically competitive* final output producer $i \in [0,1]$ according to the final output CES production function

$$Z_t(i) \quad = \quad \varepsilon_t^{TFP} \left((1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} (N_t(i))^{\frac{\psi_{ez}-1}{\psi_{ez}}} + (\alpha_{ez})^{\frac{1}{\psi_{ez}}} (E_t^z(i))^{\frac{\psi_{ez}-1}{\psi_{ez}}} \right)^{\frac{\psi_{ez}}{\psi_{ez}-1}} \tag{A.45}$$

The production inputs demanded by a specific firm i are labor $N_t(i)$ and imported energy goods $E_t^z(i)$. α_{ez} denotes the share of energy in production and ψ_{ez} denotes the elasticity of substitution between labor and the import good. Both, labour is provided by monopolistically competitive unions. Moreover, firm i purchases energy imports E_t^z from the importer.

Each individual final output producer is subject to *nominal rigidities*. The probability that they cannot reset their price is ϕ_z. We split the firms problem into two steps: (i) the intra-temporal cost minimisation problem and (ii) the inter-temporal price setting problem.

Cost Minimisation Problem A final output firm chooses its inputs to minimise its costs

$$\min_{N_t(i), E_t^z(i)} \left\{ \tau_t^Z \left(W_t N_t(i) + P_t^E E_t^z(i) \right) \right\}, \qquad s.t. \quad Z_t(i) \geq \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} Z_t.$$

The Lagrangian is given by

$$\mathcal{L}_t^Z = -\tau_t^Z \left(W_t N_t(i) + P_t^E E_t^z(i) \right) + MC_t^Z(i) \left(Z_t(i) - \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} Z_t \right)$$

and the Lagrange multiplier $MC_t^Z(i)$ is the (nominal) shadow cost of producing one more unit of final output, e.g. the nominal marginal cost and $\tau_t^Z = \tau^Z \varepsilon_t^{\mathcal{M}_z}$ is a shock to final output marginal costs that is isomorphic to a price markup shock process. The optimality conditions are given by

$$w_t = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} \frac{mc_t^Z}{\tau_t^Z} \left(\frac{Z_t(i)}{N_t(i)}\right)^{\frac{1}{\psi_{ez}}} \left(\varepsilon_t^{TFP}\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \quad \text{(A.46)}$$

$$p_t^E = (\alpha_{ez})^{\frac{1}{\psi_{ez}}} \frac{mc_t^Z}{\tau_t^Z} \left(\frac{Z_t(i)}{E_t^z(i)}\right)^{\frac{1}{\psi_{ez}}} \left(\varepsilon_t^{TFP}\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \quad \text{(A.47)}$$

Combine the first order conditions to obtain

$$\frac{W_t}{P_t^E} = \left(\frac{1-\alpha_{ez}}{\alpha_{ez}}\right)^{\frac{1}{\psi_{ez}}} \left(\frac{N_t(i)}{E_t^z(i)}\right)^{-\frac{1}{\psi_{ez}}}$$

Rearrange to obtain the optimal trade-off between production factors as a function of their relative price,

$$\frac{N_t(i)}{E_t^z(i)} = \frac{1-\alpha_{ez}}{\alpha_{ez}} \left(\frac{W_t}{P_t^E}\right)^{-\psi_{ez}}.$$

Factor Demand Schedules Combine the optimality condition with the production function

$$Z_t(i) = \left((1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} \left(\frac{1-\alpha_{ez}}{\alpha_{ez}} \left(\frac{W_t}{P_t^E}\right)^{-\psi_{ez}} E_t^z(i)\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} + \alpha_{ez}^{\frac{1}{\psi_{ez}}} \left(E_t^z(i)\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}}\right)^{\frac{\psi_{ez}}{\psi_{ez}-1}}$$

$$Z_t(i) = \alpha_{ez}^{-1} \left(\frac{(1-\alpha_{ez})W_t^{1-\psi_{ez}} + \alpha_{ez}(P_t^E)^{1-\psi_{ez}}}{(P_t^E)^{1-\psi_{ez}}}\right)^{\frac{\psi_{ez}}{\psi_{ez}-1}} E_t^z(i)$$

Rearrange to obtain the demand function for $E_t^z(i)$,

$$E_t^z(i) = \alpha_{ez} \left(\frac{P_t^E}{\left((1-\alpha_{ez})W_t^{1-\psi_{ez}} + \alpha_{ez}(P_t^E)^{1-\psi_{ez}}\right)^{\frac{1}{1-\psi_{ez}}}}\right)^{-\psi_{ez}} Z_t(i).$$

Equivalently, for $N_t(i)$ we obtain the following demand function

$$N_t(i) = (1-\alpha_{ez}) \left(\frac{W_t}{\left((1-\alpha_{ez})W_t^{1-\psi_{ez}} + \alpha_{ez}(P_t^E)^{1-\psi_{ez}}\right)^{\frac{1}{1-\psi_{ez}}}}\right)^{-\psi_{ez}} Z_t(i).$$

Final Output Marginal Cost To obtain the marginal cost, raise the first order condition with respect to $N_t(i)$ to the power $1-\psi_{ez}$ and multiply by $1-\alpha_{ez}$,

$$\begin{aligned}
(1-\alpha_{ez})W_t^{1-\psi_{ez}} &= (1-\alpha_{ez})^{\frac{1-\psi_{ez}}{\psi_{ez}}+1} (MC_t^Z(i))^{1-\psi_{ez}} (Z_t(i))^{\frac{1-\psi_{ez}}{\psi_{ez}}} N_t(i)^{-\frac{1-\psi_{ez}}{\psi_{ez}}} \\
\left((1-\alpha_{ez})W_t^{1-\psi_{ez}} + \alpha_{ez}(P_t^E)^{1-\psi_{ez}}\right)^{\frac{1}{1-\psi_{ez}}} &= \Bigg((1-\alpha_{ez})^{\frac{1-\psi_{ez}}{\psi_{ez}}+1} (MC_t^Z(i))^{1-\psi_{ez}} Z_t(i)^{\frac{1-\psi_{ez}}{\psi_{ez}}} (N_t(i))^{-\frac{1-\psi_{ez}}{\psi_{ez}}} \\
&\quad + \alpha_{ez}^{\frac{1-\psi_{ez}}{\psi_{ez}}+1} (MC_t^Z(i))^{1-\psi_{ez}} (Z_t(i))^{\frac{1-\psi_{ez}}{\psi_{ez}}} (E_t^z(i))^{-\frac{1-\psi_{ez}}{\psi_{ez}}}\Bigg)^{\frac{1}{1-\psi_{ez}}}.
\end{aligned}$$

Rearrange to obtain the marginal cost,

$$\left((1-\alpha_{ez})W_t^{1-\psi_{ez}}+\alpha_{ez}(P_t^E)^{1-\psi_{ez}}\right)^{\frac{1}{1-\psi_{ez}}} = \left((1-\alpha_{ez})^{\frac{1}{\psi_{ez}}}N_t(j)^{\frac{\psi_{ez}-1}{\psi_{ez}}}+\alpha_{ez}^{\frac{1}{\psi_{ez}}}M_{j,t}^{\frac{\psi_{ez}-1}{\psi_{ez}}}\right)^{\frac{1}{1-\psi_{ez}}} MC_t^Z(i)(Z_t(i))^{\frac{1}{\psi_{ez}}}$$

$$MC_t^Z(i) = MC_t^Z = \left((1-\alpha_{ez})W_t^{1-\psi_{ez}}+\alpha_{ez}(P_t^E)^{1-\psi_{ez}}\right)^{\frac{1}{1-\psi_{ez}}}$$

Note that the Lagrange multiplier of an individual final output producing firm i does not depend on its own quantities of labor demanded, so that all final output firms have the same multiplier $MC_t^Z(i) = MC_t^Z$.

Price Setting The objective of each final output producing firm is to maximise its nominal profits

$$DIV_t^Z(i) = P_t(i)Z_t(i) - \left\{\tau_t^Z\left(W_tN_t(i)+P_t^EE_t^z(i)\right)\right\} \quad \Leftrightarrow \quad div_t^Z = \left(1-mc_t^Z\right)Z_t. \tag{A.48}$$

With probability ϕ_z a firm is stuck with its previous-period price indexed to a composite of previous-period inflation and steady state inflation so that

$$P_t(i) = \begin{cases} P_t^\#(i) & \textit{with probability: } 1-\phi_z \\ P_{t-1}(i)\left((\Pi_{ss})^{1-\xi_z}(\Pi_{t-1})^{\xi_z}\right) & \textit{with probability: } \phi_z \end{cases}$$

where $\xi_z \in [0,1]$ is the weight attached to previous period inflation. Consider a firm who can reset its price in the current period $P_t(i) = P_t^\#(i)$ and who is then stuck with its price until future period $t+s$. The price in this case would be

$$P_{t+s}(i) = P_t^\#(i)\left[(\Pi_{ss})^{s(1-\xi_z)}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\xi_z}\right].$$

Final output good producing firms solve the following optimisation problem

$$\max_{P_t^\#(i)} E_t\sum_{s=0}^{\infty}(\phi_Z)^s\beta^{t+s}\frac{\lambda_{u,t+s}}{\lambda_{u,t}}\left[P_{t+s}(i)Z_{t+s|t}(i) - MC_{t+s}^ZZ_{t+s|t}(i)\right]$$

subject to the above derived demand constraint and assuming that a firm z always meets the demand for its good at the current price. $Z_{t+s|t}(i)$ denotes the final output supplied in period $t+s$ by a firm i that last reset its price in period t. If one substitutes the demand schedule and $P_{t+s}(i)$ into the objective function one obtains

$$\begin{aligned}\max_{P_t^\#(i)} E_t\sum_{s=0}^{\infty}(\phi_Z)^s\beta^{t+s}\frac{\lambda_{u,t+s}}{\lambda_{u,t}}&\left[\left(P_t^\#(i)\right)^{1-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}}\left(\left[(\Pi_{ss})^{s(1-\xi_z)}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\xi_z}\right]\right)^{1-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}}\left[\left(\frac{1}{P_{t+s}}\right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}}Z_{t+s}\right]\right.\\ &\left.-MC_{t+s}^Z\left[\left(P_t^\#(i)\right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}}\left(\frac{\left[(\Pi_{ss})^{s(1-\xi_z)}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\xi_z}\right]}{P_{t+s}}\right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}}Z_{t+s}\right]\right].\end{aligned}$$

Taking the derivative with respect to $P_t^\#(i)$ delivers the familiar price inflation schedule (B.46)

$$\frac{f_t^{Z,1}}{f_t^{Z,2}}\mathcal{M}_z = \left[\frac{1-(\phi_Z)\left(\zeta_t^Z\right)^{\frac{-1}{1-\mathcal{M}_z}}}{1-\phi_Z}\right]^{1-\mathcal{M}_z} \tag{A.49}$$

$$f_t^{Z,1} = Z_t mc_t^Z + \phi_Z \beta E_t \left[\left(\frac{U_{u,t+1}^{CES}}{U_{u,t}^{CES}} \right) \left(\frac{\Pi_{t+1}}{\Pi_{t+1}^{CES_u}} \right) \left(\frac{\Pi_{t+1}}{\Pi_{ss}} \right)^{\frac{\mathcal{M}_Z}{\mathcal{M}_Z - 1}} f_{t+1}^{Z,1} \right] \tag{A.50}$$

$$f_t^{Z,2} = N_t + \phi_Z \beta E_t \left[\left(\frac{U_{u,t+1}^{CES}}{U_{u,t}^{CES}} \right) \left(\frac{\Pi_{t+1}}{\Pi_{t+1}^{CES_u}} \right) \left(\frac{\Pi_{t+1}}{\Pi_{ss}^Z} \right)^{\frac{1}{\mathcal{M}_Z - 1}} f_{t+1}^{Z,2} \right] \tag{A.51}$$

$$\zeta_t^Z = \frac{\Pi_t}{(\Pi_{ss})^{1-\xi_z} (\Pi_{t-1})^{\xi_z}} \tag{A.52}$$

$$\mathcal{D}_t^Z = (1 - \phi_Z) \left(\frac{1 - \phi_Z \left(\zeta_t^Z \right)^{\frac{1}{\mathcal{M}_z - 1}}}{1 - \phi_Z} \right)^{\mathcal{M}_z} + \phi_Z \left(\zeta_t^Z \right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} \mathcal{D}_{t-1}^Z. \tag{A.53}$$

Aggregation implies $\int_0^1 Z_t(i) di = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} Z_t di = Z_t \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} di$ where we define price dispersion as $\mathcal{D}_t^Z \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} di$ which can be written recursively

Calvo Price Setting Derivation

$$\max_{P_t^\#(j)} E_t \sum_{s=0}^{\infty} (\phi_Z)^s \Lambda_{u,t,t+s} \left[DIV_{t+s}^Z(j) \right], \quad \Leftrightarrow \quad \max_{P_t^\#(j)} E_t \sum_{s=0}^{\infty} (\phi_Z)^s \Lambda_{u,t,t+s} \left[DIV_{t+s}^Z(j) \right] \frac{P_{t+s}}{P_{t+s}}$$

$$DIV_t^Z(j) = P_t(j) Z_t(j) - MC_t^Z(j) Z_t(j),\ div_t^Z(j) = \left(P_t(j)/P_t - mc_t^Z \right) Z_t(j),\ div_t^Z(j) \equiv DIV_t^Z(j)/P_t$$

$$\max_{P_t^\#(j)} E_t \sum_{s=0}^{\infty} (\phi_Z)^s \Lambda_{u,t,t+s} P_{t+s} Z_{t+s} \left[\frac{P_{t+s}(j)}{P_{t+s}} \left(\frac{P_{t+s}(j)}{P_{t+s}} \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} - mc_{t+s}^Z \left(\frac{P_{t+s}(j)}{P_{t+s}} \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_Z - 1}} \right]$$

Take the derivative and note that, if we assume $\xi_z = 0$, $P_{t+s}(j) = P_t^\#(j)(\Pi_{ss})^s$

$$E_t \sum_{s=0}^{\infty} (\phi_z)^s \Lambda_{u,t,t+s} P_{t+s} Z_{t+s} \left[\frac{1}{P_{t+s}} \left(\frac{(\Pi_{ss})^s}{P_{t+s}} \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} \left(\frac{P_t}{P_t} P_t^\# \right) (\Pi_{ss})^s \right]$$
$$= E_t \sum_{s=0}^{\infty} (\phi_z)^s \Lambda_{u,t,t+s} P_{t+s} Z_{t+s} \left[mc_{t+s}^Z \left(\frac{(\Pi_{ss})^s}{P_{t+s}} \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} (\mathcal{M}_z) \right]$$

$$p_t^\# \frac{1}{\mathcal{M}_z} \left(E_t \sum_{s=0}^{\infty} (\phi_z)^s \Lambda_{u,t,t+s} P_{t+s} Z_{t+s} \left[\frac{1}{P_{t+s}} \left(\frac{(\Pi_{ss})^s}{P_{t+s}} \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} \left(P_t (\Pi_{ss})^s \right) \right] \right)$$
$$= \left(E_t \sum_{s=0}^{\infty} (\phi_z)^s \Lambda_{u,t,t+s} P_{t+s} Z_{t+s} \left[mc_{t+s}^Z \left(\frac{1}{P_{t+s}} (\Pi_{ss})^s \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} \right] \right), \quad p_t^\# = \frac{P_t^\#}{P_t}$$
$$p_t^\# = \mathcal{M}_z \frac{\left(E_t \sum_{s=0}^{\infty} (\phi_z)^s \Lambda_{u,t,t+s} P_{t+s} Z_{t+s} \left[mc_{t+s}^Z \left(\frac{1}{P_{t+s}} (\Pi_{ss})^s \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} \right] \right)}{\left(E_t \sum_{s=0}^{\infty} (\phi_z)^s \Lambda_{u,t,t+s} P_{t+s} Z_{t+s} \left[\frac{1}{P_{t+s}} \left(\frac{(\Pi_{ss})^s}{P_{t+s}} \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} \left(\frac{P_t}{P_t} P_t (\Pi_{ss})^s \right) \right] \right)} \frac{P_t^{-\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}}}{P_t^{-\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}}} \frac{P_t}{P_t}$$

$$p_t^{\#} = \mathcal{M}_z \frac{\left(E_t \sum_{s=0}^{\infty} (\phi_z)^s \Lambda_{u,t,t+s} \frac{P_{t+s}}{P_t} Z_{t+s} \frac{mc_{t+s}^Z}{1} \left[\left(\frac{P_{t+s}}{P_t} \frac{1}{(\Pi_{ss})^s} \right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} \right] \right)}{\left(E_t \sum_{s=0}^{\infty} (\phi_z)^s \Lambda_{u,t,t+s} \frac{P_{t+s}}{P_t} Z_{t+s} \left[\frac{P_t}{P_{t+s}} \left(\frac{P_{t+s}}{P_t} \frac{1}{(\Pi_{ss})^s} \right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} (\Pi_{ss})^s \right] \right)}$$

Introduce the definition $f_t^{Z,1}$

$$\begin{aligned} f_t^{Z,1} &\equiv E_t \sum_{s=0}^{\infty} (\phi_z)^s \Lambda_{u,t,t+s} \frac{P_{t+s}}{P_t} Z_{t+s} \frac{mc_{t+s}^Z}{1} \left(\frac{P_{t+s}}{P_t} \frac{1}{(\Pi_{ss})^s} \right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} \\ f_{t+1}^{Z,1} &= E_{t+1} \sum_{s=0}^{\infty} (\phi_Z)^s \Lambda_{u,t+1,t+1+s} \frac{P_{t+1+s}}{P_{t+1}} Z_{t+1+s} \frac{mc_{t+1+s}^Z}{1} \left(\frac{P_{t+1+s}}{P_{t+1}} \frac{1}{(\Pi_{ss})^s} \right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} \end{aligned}$$

Replace $s = k + 1$ so that

$$f_t^{Z,1} = Z_t \frac{mc_t^Z}{1} + E_t \sum_{k=0}^{\infty} (\phi_Z)^{k+1} \Lambda_{u,t,t+k+1} \frac{P_{t+k+1}}{P_t} Z_{t+k+1} \frac{mc_{t+k+1}^Z}{1} \left(\frac{P_{t+k+1}}{P_t} \frac{1}{(\Pi_{ss})^{k+1}} \right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}}$$

$$f_t^{Z,1} = Z_t \frac{mc_t^Z}{1}$$

$$+\phi_Z E_t \left[E_{t+1} \left[\sum_{k=0}^{\infty} (\phi_Z)^k \frac{\Lambda_{u,t+k+1}}{\Lambda_{u,t}} \frac{\Lambda_{u,t+1}}{\Lambda_{u,t+1}} \frac{P_{t+k+1}}{P_t} \frac{P_{t+1}}{P_{t+1}} Z_{t+k+1} \frac{mc_{t+k+1}^Z}{1} \left(\frac{P_{t+k+1}}{P_t} \frac{1}{(\Pi_{ss})^{k+1}} \frac{P_{t+1}}{P_{t+1}} \right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} \right] \right]$$

$$f_t^{Z,1} = Z_t \frac{mc_t^Z}{1} + \phi_z E_t \left[E_{t+1} \left[\sum_{k=0}^{\infty} (\phi_z)^k \Lambda_{u,t+1,t+k+1} \frac{\Lambda_{u,t+1}}{\Lambda_{u,t}} \frac{P_{t+k+1}}{P_{t+1}} \frac{P_{t+1}}{P_t} Z_{t+k+1} \cdot \right. \right.$$

$$\left. \left. \frac{mc_{t+k+1}^Z}{1} \left(\frac{P_{t+k+1}}{P_{t+1}} \frac{1}{(\Pi_{ss})^k} \right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} \left(\frac{P_{t+1}}{P_t} \frac{1}{\Pi_{ss}} \right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} \right] \right]$$

$$f_t^{Z,1} = Z_t \frac{mc_t^Z}{1} + \phi_Z E_t \Lambda_{u,t,t+1} \frac{P_{t+1}}{P_t} \left(\frac{P_{t+1}}{P_t} \frac{1}{\Pi_{ss}} \right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}}$$

$$\cdot \underbrace{\left[E_{t+1} \left[\sum_{k=0}^{\infty} (\phi_z)^k \Lambda_{u,t+1,t+k+1} \frac{P_{t+k+1}}{P_{t+1}} Z_{t+k+1} \frac{mc_{t+k+1}^Z}{1} \left(\frac{W_{t+k+1}}{W_{t+1}} \frac{1}{(\Pi_{ss})^k} \right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} \right] \right]}_{f_{t+1}^{Z,1}}$$

$$\begin{aligned} f_t^{Z,1} &= Z_t mc_t^Z + \phi_Z E_t \Lambda_{u,t,t+1} \Pi_{t+1} \left(\frac{\Pi_{t+1}}{\Pi_{ss}} \right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} f_{t+1}^{Z,1} \\ f_t^{Z,2} &= Z_t + \phi_Z E_t \Lambda_{u,t,t+1} \frac{\Pi_{t+1}}{\Pi_{t+1}} \Pi_{ss} \left(\frac{\Pi_{t+1}}{\Pi_{ss}} \right)^{\frac{\mathcal{M}_Z}{\mathcal{M}_Z - 1}} f_{t+1}^{Z,2}, \quad \text{recall } \mathbf{E}_t \left[\Lambda_{u,t,t+1} \right] = \mathbf{E}_t \left[\beta \frac{U_{u,t+1}^{CES}}{U_{u,t}^{CES}} \frac{1}{\Pi_{t+1}^{CES_u}} \right] \end{aligned}$$

Energy Import Sector Each energy import good $E_t^j, j \in \{z,h\}$ that the final output good producer or the household demands is supplied by a perfectly competitive energy importer. Energy import firms buy a homogenous tradeable energy good on the world market from foreign energy exporters at foreign currency energy price $P_t^{E,*}$. One can transform this into domestic currency by multiplying by the *nominal* exchange rate so that $P_t^E \equiv P_t^{E,*}\mathcal{E}_t$. If for example (from the UK's perspective as the domestic economy) the nominal exchange rate was $\mathcal{E}_t = 0.5$ £/\$ and the importer purchases oil on the world market for $P_t^{E,*} = 100\$$ this would correspond to $P_t^E = (100\$) * (0.5£/\$) = 50£$. The importers then transform the homogenous good they purchased $E_t^e = X_t^{e,*}$. After the importers have transformed the energy import good they sell it to domestic final output producers. The cost minimisation problem of importers takes the simple form $\min_{E_t^*} \left\{P_t^{E,*}E_t^*\right\}$ $s.t.$ $E_t^* \geq E_t = E_t^z + E_t^c$. The optimality conditions are given by

$$\mathcal{L} = -\left(P_t^{E,*}\mathcal{E}_t E_t^*\right) + P_t^E E_t^*, \quad \frac{\partial \mathcal{L}}{\partial E_t^*} = 0 \Leftrightarrow P_t^{E,*}\mathcal{E}_t = P_t^E \Leftrightarrow P_t^{E,*}\mathcal{E}_t \frac{1}{P_t}\frac{P_t^*}{P_t^*} = \frac{P_t^E}{P_t} \Leftrightarrow p_t^{E,*}\mathcal{Q}_t = p_t^E \quad \text{(A.54)}$$

We assume that the global energy export price level follows the exogenous process

$$p_t^{E,*} = \left(p_{ss}^{E,*}\right)^{1-\rho_E}\left(p_{t-1}^{E,*}\right)^{\rho_E}\varepsilon_t^E. \quad \text{(A.55)}$$

Non-Energy Export Sector Exports X_t are produced by a perfectly competitive export good firm. They buy a homogenous non-energy export good on the domestic market from final output retailers at domestic-currency price P_t^X. The 'production' of non-energy export goods works via a simple transformation of final output goods into the expenditure components C, X, so that the supply of a specific export good is given by $X_t = Z_t^X$. The objective of each export good firm is to maximise its nominal profits DIV_t^X

$$DIV_t^X = P_t^{EXP}\mathcal{E}_t X_t - P_t^X Z_t^X \Leftrightarrow div_t^X = \left(p_t^{EXP}\mathcal{Q}_t - p_t^X\right)X_t$$

which implies

$$\frac{\partial DIV_t^X}{\partial X_t} = \left(p_t^{EXP}\mathcal{Q}_t - p_t^X\right) = 0 \quad \Leftrightarrow p_t^X = p_t^{EXP}\mathcal{Q}_t \quad \text{(A.56)}$$

Retailers There is a continuum of perfectly competitive retailers defined on the unit interval, who buy final output goods from the final output good packers at price P_t and convert them into differentiated goods representing each expenditure component: non-energy consumption and non-energy export goods. Retailer r in sector N converts goods using the following linear technology:

$$N_t(r) = Z_t^N(r), \text{ for } N \in \{C, X\}$$

where the input $Z_t^N(r)$ is the amount of the final output good bundle Z_t demanded by retail firm r in expenditure sector N and where the final good bundle, Z_t, is defined by its above stated CES aggregator. Each retailer r in sector N chooses its input $Z_t^N(r)$ to maximise profits, taking the price of its output, $P_t^N, N \in \{C, X\}$ and the price of the final output good, P_t as given. They solve

$$\max_{Z_t^N(r)} P_t^N Z_t^N(r) - P_t Z_t^N(r)$$

with first-order condition given by

$$P_t^N = P_t, \quad N \in \{C, X\} \quad \Leftrightarrow p_t^X = P_t^X / P_t = 1, \quad \text{(A.57)}$$
$$p_t^C = P_t^C / P_t = 1 \quad \text{(A.58)}$$

A.4 Monetary Policy

The monetary policy maker follows a simple rule for the nominal interest rate in which it responds to persistent deviations of annual CPI inflation, $\Pi_t^{CPI,a}$, from its target, $\bar{\Pi}^{CPI,a} = 2\%$, and a measure of the output gap, $\hat{Y}_t$. This gives the following rule:

$$R_t = \bar{R}^{1-\theta_R} R_{t-1}^{\theta_R} \left(\frac{\Pi_t^{CPI,a}}{\bar{\Pi}^{CPI,a}} \right)^{\frac{(1-\theta_R)\theta_\Pi}{4}} \left(\hat{Y}_t \right)^{(1-\theta_R)\theta_Y} \tag{A.59}$$

with

$$\Pi_t^{CPI} = \frac{P_t^{CPI}}{P_{t-1}^{CPI}} = \frac{p_t^{CPI}}{p_{t-1}^{CPI}} \Pi_t \tag{A.60}$$

$$\Pi_t^{CPI,a} = \frac{P_t^{CPI}}{P_{t-4}^{CPI}} = \frac{P_t^{CPI}}{P_{t-1}^{CPI}} \frac{P_{t-1}^{CPI}}{P_{t-2}^{CPI}} \frac{P_{t-2}^{CPI}}{P_{t-3}^{CPI}} \frac{P_{t-3}^{CPI}}{P_{t-4}^{CPI}} = \Pi_t^{CPI} \Pi_t^{CPI,lag1} \Pi_t^{CPI,lag2} \Pi_{t-1}^{CPI,lag2} \tag{A.61}$$

$$\Pi_t^{CPI,lag1} = \Pi_{t-1}^{CPI} \tag{A.62}$$

$$\Pi_t^{CPI,lag2} = \Pi_{t-1}^{CPI,lag1}, \quad \text{where} \bar{\Pi}^{CPI,a} = \left(\bar{\Pi}^{CPI} \right)^4 \tag{A.63}$$

$$\hat{Y}_t \equiv \frac{N_t}{N_t^{flex}} \tag{A.64}$$

where N_t^{flex} is the level of employment under flexible prices and wages, $\bar{R}$ is the steady state nominal interest rate consistent with steady-state inflation being at target.

A.5 The World Block

The global demand schedule for the bundle of domestic non-energy exports X_t depends on the foreign currency price of domestic non-energy exports, P_t^{EXP}, relative to the world non-energy export price, P_t^{X*}, and on the world trade volume Z_t^*:

$$X_t = \kappa^* \left(\frac{P_t^{EXP}}{P_t^{X*}} \right)^{-\varsigma^*} Y_{ss}^* \quad \Leftrightarrow \quad X_t = \kappa^* \left(\frac{p_t^{EXP}}{p_{ss}^{X*}} \right)^{-\varsigma^*} Y_{ss}^* \tag{A.65}$$

where the parameter ς^* is the elasticity of substitution between differentiated export goods in the rest of the world. κ^* can be interpreted as a shifter of the world's preference for domestic exports.

B Summary of Model Equations

Unconstrained Households

$$C_{u,t} = \left(\frac{p_t^C}{p_t^{CESu}} \right)^{-\psi_{ec}} (1-\alpha_{u,ec}) CES_{u,t} \tag{B.1}$$

$$E_{u,t}^h = \left(\frac{p_t^E}{p_t^{CESu}} \right)^{-\psi_{ec}} (\alpha_{u,ec}) CES_{u,t} \tag{B.2}$$

$$p_t^{CESu} = \left[(1-\alpha_{u,ec}) \left(p_t^C \right)^{1-\psi_{ec}} + \alpha_{u,ec} \left(p_t^E \right)^{1-\psi_{ec}} \right]^{\frac{1}{1-\psi_{ec}}} \tag{B.3}$$

$$\lambda_{u,t} = (CES_{u,t})^{-\sigma} \tag{B.4}$$

$$w_t^h = mrs_{u,t} \tag{B.5}$$

$$mrs_{u,t} = -\frac{U_{u,t}^N}{\lambda_{u,t} / p_t^{CESu}} \tag{B.6}$$

$$U_{u,t}^N = -\chi \left(N_{u,t}^h \right)^{\varphi} \tag{B.7}$$

$$1 = E_t \left[\Lambda_{u,t,t+1} \left(\Pi_{t+1}^{CESu} \right)^{-1} \right] R_t \tag{B.8}$$

$$\Pi_t^{CESu} = \frac{p_t^{CESu}}{p_{t-1}^{CESu}} \Pi_t \tag{B.9}$$

$$0 = E_t \left[\beta \frac{\lambda_{u,t+1}}{\lambda_{u,t}} \frac{1}{\Pi_{t+1}^{CESu}} \left(R_t - \bar{R}^* \frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t} \frac{\Pi_{t+1}}{\Pi_{ss}^*} \right) \right] \tag{B.10}$$

$$\Lambda_{u,t,t+1} \equiv E_t \left[\beta \frac{\lambda_{u,t+1}}{\lambda_{u,t}} \right] \tag{B.11}$$

$$p_t^C C_{u,t} = w_t N_{u,t}^h + \frac{\left(b_t^* \mathcal{Q}_t - \frac{\bar{R}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} + div_t^F - t_t^F \right)}{1-\omega} - \mathcal{T}_u - p_t^E E_{u,t}^h \tag{B.12}$$

$$div_t^F = div_t^Z \tag{B.13}$$

Constrained Households

$$C_{c,t} = \left(\frac{p_t^C}{p_t^{CESc}} \right)^{-\psi_{ec}} (1-\alpha_{c,ec}) CES_{c,t} \tag{B.14}$$

$$E_{c,t}^h = \left(\frac{p_t^E}{p_t^{CESc}} \right)^{-\psi_{ec}} (\alpha_{c,ec}) CES_{c,t} \tag{B.15}$$

$$p_t^{CESc} = \left[(1-\alpha_{c,ec}) \left(p_t^C \right)^{1-\psi_{ec}} + \alpha_{c,ec} \left(p_t^E \right)^{1-\psi_{ec}} \right]^{\frac{1}{1-\psi_{ec}}} \tag{B.16}$$

$$\lambda_{c,t} = (CES_{c,t})^{-\sigma} \tag{B.17}$$

$$w_t^h = mrs_{c,t} \tag{B.18}$$

$$mrs_{c,t} = -\frac{U_{c,t}^N}{\lambda_{c,t} / p_t^{CESc}} \tag{B.19}$$

$$U_{c,t}^N = -\chi \left(N_{c,t}^h \right)^{\varphi} \tag{B.20}$$

$$p_t^C C_{c,t} = w_t N_{c,t}^h + \mathcal{T}_c - p_t^E E_{c,t}^h \tag{B.21}$$

Aggregation, Market Clearing, Definitions

$$\Gamma_t = \frac{C_{u,t}}{C_{c,t}} \tag{B.22}$$

$$t_t^F = t_t^Z \tag{B.23}$$

$$t_t^Z = (1-\tau_t^Z)(w_t N_t^h + p_t^E E_t^z) \tag{B.24}$$

$$t_t^L = (1-\tau_t^W) w_t^h N_t^h \tag{B.25}$$

$$C_t = \omega C_{c,t} + (1-\omega) C_{u,t} \tag{B.26}$$

$$E_t^h = \omega E_{c,t}^h + (1-\omega) E_{u,t}^h \tag{B.27}$$

$$N_t^h = \omega N_{c,t}^h + (1-\omega) N_{u,t}^h \tag{B.28}$$

$$\Lambda_{t,t+1} = (1-\omega) \Lambda_{u,t,t+1} \tag{B.29}$$

$$p_t^{CPI} = \omega p_t^{CESc} + (1-\omega) p_t^{CESu} \tag{B.30}$$

$$b_t = 0 \tag{B.31}$$

$$p_t^C C_t + p_t^X X_t = Z_t \tag{B.32}$$

$$NFA_t = p_t^{EXP} \mathcal{Q}_t X_t - p_t^{E,*} \mathcal{Q}_t (E_t^z + E_t^h) \tag{B.33}$$

Labour Unions

$$mc_t^W = \tau^W w_t^h = \tau^W mrs_t \tag{B.34}$$

$$div_t^L = \left(w_t - mc_t^W\right) N_t^h \tag{B.35}$$

$$\frac{f_t^{W,1}}{f_t^{W,2}} \mathcal{M}_w = w_t^\# = \left(\frac{1-\phi_W (\zeta_t^W)^{\frac{1}{\mathcal{M}_w - 1}}}{1-\phi_W}\right)^{1-\mathcal{M}_w} \tag{B.36}$$

$$f_t^{W,1} = N_t mc_t^W / w_t + \phi_W \beta E_t \left[\left(U_{u,t+1}^{CES}/U_{u,t}^{CES}\right)\left(\Pi_{t+1}^W/\Pi_{t+1}^{CESu}\right)\left(\Pi_{t+1}^W/\Pi_{ss}^W\right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w-1}} f_{t+1}^{W,1}\right] \tag{B.37}$$

$$f_t^{W,2} = N_t + \phi_W \beta E_t \left[\left(U_{u,t+1}^{CES}/U_{u,t}^{CES}\right)\left(\Pi_{t+1}^W/\Pi_{t+1}^{CESu}\right)\left(\Pi_{t+1}^W/\Pi_{ss}^W\right)^{\frac{1}{\mathcal{M}_w-1}} f_{t+1}^{W,2}\right] \tag{B.38}$$

$$\zeta_t^W = \frac{\Pi_t^W}{(\Pi_{ss}^W)^{1-\xi_W}(\Pi_{t-1}^W)^{\xi_W}} \tag{B.39}$$

$$w_t = \frac{\Pi_t^W}{\Pi_t} w_{t-1} \tag{B.40}$$

$$\mathcal{D}_t^W = (1-\phi_W)\left(\frac{1-\phi_W\left(\zeta_t^W\right)^{\frac{1}{\mathcal{M}_w-1}}}{1-\phi_W}\right)^{\mathcal{M}_w} + \phi_W\left(\zeta_t^W\right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w-1}} \mathcal{D}_{t-1}^W. \tag{B.41}$$

Z Firms

$$Z_t \mathcal{D}_t^Z = \varepsilon_t^{TFP}\left((1-\alpha_{ez})^{\frac{1}{\psi_{ez}}}(N_t)^{\frac{\psi_{ez}-1}{\psi_{ez}}} + (\alpha_{ez})^{\frac{1}{\psi_{ez}}}(E_t^z)^{\frac{\psi_{ez}-1}{\psi_{ez}}}\right)^{\frac{\psi_{ez}}{\psi_{ez}-1}} \tag{B.42}$$

$$w_t = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} \frac{mc_t^Z}{\tau_t^Z}\left(\frac{Z_t\mathcal{D}_t^Z}{N_t}\right)^{\frac{1}{\psi_{ez}}}\left(\varepsilon_t^{TFP}\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \tag{B.43}$$

$$p_t^E = (\alpha_{ez})^{\frac{1}{\psi_{ez}}} \frac{mc_t^Z}{\tau_t^Z}\left(\frac{Z_t\mathcal{D}_t^Z}{E_t^z}\right)^{\frac{1}{\psi_{ez}}}\left(\varepsilon_t^{TFP}\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \tag{B.44}$$

$$div_t^Z = \left(1-mc_t^Z\right) Z_t \tag{B.45}$$

$$\frac{f_t^{Z,1}}{f_t^{Z,2}}\mathcal{M}_z = \left[\frac{1-(\phi_Z)\left(\zeta_t^Z\right)^{\frac{-1}{1-\mathcal{M}_z}}}{1-\phi_Z}\right]^{1-\mathcal{M}_z} \tag{B.46}$$

$$f_t^{Z,1} = Z_t mc_t^Z + \phi_Z\beta E_t\left[\left(U_{u,t+1}^{CES}/U_{u,t}^{CES}\right)\left(\Pi_{t+1}/\Pi_{t+1}^{CESu}\right)\left(\Pi_{t+1}/\Pi_{ss}\right)^{\frac{\mathcal{M}_Z}{\mathcal{M}_Z-1}} f_{t+1}^{Z,1}\right] \tag{B.47}$$

$$f_t^{Z,2} = N_t + \phi_Z\beta E_t\left[\left(U_{u,t+1}^{CES}/U_{u,t}^{CES}\right)\left(\Pi_{t+1}/\Pi_{t+1}^{CESu}\right)\left(\Pi_{t+1}/\Pi_{ss}^Z\right)^{\frac{1}{\mathcal{M}_Z-1}} f_{t+1}^{Z,2}\right] \tag{B.48}$$

$$\zeta_t^Z = \frac{\Pi_t}{(\Pi_{ss})^{1-\xi_Z}(\Pi_{t-1})^{\xi_Z}} \tag{B.49}$$

$$\mathcal{D}_t^Z = (1-\phi_Z)\left(\frac{1-\phi_Z\left(\zeta_t^Z\right)^{\frac{1}{\mathcal{M}_z-1}}}{1-\phi_Z}\right)^{\mathcal{M}_z} + \phi_Z\left(\zeta_t^Z\right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z-1}}\mathcal{D}_{t-1}^Z \tag{B.50}$$

$$p_t^E = p_t^{E,*}\mathcal{Q}_t \tag{B.51}$$

$$p_t^{E,*} = \left(p_{ss}^{E,*}\right)^{1-\rho_E}\left(p_{t-1}^{E,*}\right)^{\rho_E}\varepsilon_t^E \tag{B.52}$$

$$p_t^X = p_t^{EXP}\mathcal{Q}_t \tag{B.53}$$

$$p_t^X = 1 \tag{B.54}$$

$$p_t^C = 1 \tag{B.55}$$

Monetary Policy

$$R_t = R^{1-\theta_R} R_{t-1}^{\theta_R}\left(\frac{\Pi_t^{CPI,a}}{\bar{\Pi}^{CPI,a}}\right)^{\frac{(1-\theta_R)\theta_\Pi}{4}}\left(\hat{Y}_t\right)^{(1-\theta_R)\theta_Y} \tag{B.56}$$

$$\Pi_t^{CPI} = \frac{P_t^{CPI}}{P_{t-1}^{CPI}} = \frac{p_t^{CPI}}{p_{t-1}^{CPI}}\Pi_t \tag{B.57}$$

$$\Pi_t^{CPI,a} = \frac{P_t^{CPI}}{P_{t-4}^{CPI}} = \frac{P_t^{CPI}}{P_{t-1}^{CPI}}\frac{P_{t-1}^{CPI}}{P_{t-2}^{CPI}}\frac{P_{t-2}^{CPI}}{P_{t-3}^{CPI}}\frac{P_{t-3}^{CPI}}{P_{t-4}^{CPI}} = \Pi_t^{CPI}\Pi_t^{CPI,lag1}\Pi_t^{CPI,lag2}\Pi_{t-1}^{CPI,lag2} \tag{B.58}$$

$$\Pi_t^{CPI,lag1} = \Pi_{t-1}^{CPI} \tag{B.59}$$

$$\Pi_t^{CPI,lag2} = \Pi_{t-1}^{CPI,lag1} \tag{B.60}$$

$$\hat{Y}_t = \frac{L_t}{L_t^{flex}} \tag{B.61}$$

World

$$X_t = \kappa^*\left(\frac{p_t^{EXP}}{p_{ss}^{X*}}\right)^{-\varsigma^*} Y_{ss}^* \tag{B.62}$$

Shocks

$$\log\varepsilon_t^{TFP} = \rho_{TFP}\log\varepsilon_{t-1}^{TFP} - \varsigma_{TFP}\eta_t^{TFP}, \qquad \eta_t^{TFP}\sim N(0,1) \tag{B.63}$$

$$\log\varepsilon_t^{\mathcal{M}_z} = \rho_{\mathcal{M}_z}\log\varepsilon_{t-1}^{\mathcal{M}_z} - \varsigma_{\mathcal{M}_z}\eta_t^{\mathcal{M}_z}, \qquad \eta_t^{\mathcal{M}_z}\sim N(0,1) \tag{B.64}$$

$$\log\varepsilon_t^E = \varsigma_E\eta_t^E, \qquad \eta_t^E\sim N(0,1). \tag{B.65}$$

C Log-linearisation

C.1 Unconstrained Households

$$
\begin{aligned}
C_{u,t} &= \left(\frac{p^C_t}{p^{CES_u}_t}\right)^{-\psi_{ec}} (1-\alpha_{u,ec}) CES_{u,t} \\
C_{u,ss}\left(1+\hat{c}_{u,t}\right) &= \left(\frac{p^C_{ss}}{p^{CES_u}_{ss}}\right)^{-\psi_{ec}} (1-\alpha_{u,ec}) CES_{u,ss} \left(1-\psi_{ec}(\hat{p}^C_t-\hat{p}^{CES_u}_t)+\widehat{ces}_{u,t}\right) \\
\hat{c}_{u,t} &= \widehat{ces}_{u,t} - \psi_{ec}(\hat{p}^C_t-\hat{p}^{CES_u}_t) && \text{(C.1)} \\
E^h_{u,t} &= \left(\frac{p^E_t}{p^{CES_u}_t}\right)^{-\psi_{ec}} (\alpha_{u,ec}) CES_{u,t} \\
\hat{e}^h_{u,t} &= \widehat{ces}_{u,t} - \psi_{ec}(\hat{p}^E_t-\hat{p}^{CES_u}_t) && \text{(C.2)} \\
p^{CES_u}_t &= \left[(1-\alpha_{u,ec})\left(p^C_t\right)^{1-\psi_{ec}} + \alpha_{u,ec}\left(p^E_t\right)^{1-\psi_{ec}}\right]^{\frac{1}{1-\psi_{ec}}} \\
(p^{CES_u}_{ss})^{1-\psi_{ec}}\hat{p}^{CES_u}_t &= (1-\alpha_{u,ec})\left(p^C_{ss}\right)^{1-\psi_{ec}}\hat{p}^C_t + \alpha_{u,ec}\left(p^E_{ss}\right)^{1-\psi_{ec}}\hat{p}^E_t \\
\hat{p}^{CES_u}_t &= (1-\alpha_{u,ec})\hat{p}^C_t + \alpha_{u,ec}\hat{p}^E_t && \text{(C.3)} \\
\lambda_{u,t} &= (CES_{u,t})^{-\sigma} \\
\hat{\lambda}_{u,t} &= -\sigma\widehat{ces}_{u,t} && \text{(C.4)} \\
w^h_t &= mrs_{u,t} \\
\hat{w}^h_t &= \widehat{mrs}_{u,t} && \text{(C.5)} \\
mrs_{u,t} &= \frac{\chi\left(N^h_{u,t}\right)^{\varphi} p^{CES_u}_t}{\lambda_{u,t}} \\
\widehat{mrs}_{u,t} &= \left(\varphi\hat{n}^h_{u,t} + \hat{p}^{CES_u}_t - \hat{\lambda}_{u,t}\right) && \text{(C.6)} \\
U^N_{u,t} &= -\chi\left(N^h_{u,t}\right)^{\varphi} \\
U^N_{u,ss}\hat{u}^N_{u,t} &= -\chi\left(N^h_{u,ss}\right)^{\varphi}\varphi\hat{n}^h_{u,t} \\
\hat{u}^N_{u,t} &= \varphi\hat{n}^h_{u,t} && \text{(C.7)} \\
1 &= E_t\left[\Lambda_{u,t,t+1}\left(\Pi^{CES_u}_{t+1}\right)^{-1}\right]R_t \\
1 &= \left[\Lambda_{u,ss}\left(\Pi^{CES_u}_{ss}\right)^{-1}\right]R_{ss} \\
1 &= \left[\Lambda_{u,ss}\left(\Pi^{CES_u}_{ss}\right)^{-1}\right]R_{ss}\left(1+E_t\hat{\Lambda}_{u,t,t+1} - E_t\hat{\pi}^{CES_u}_{t+1} + \hat{r}_t\right) \\
0 &= E_t\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} - E_t\hat{\pi}^{CES_u}_{t+1} + \hat{r}_t && \text{(C.8)} \\
\Pi^{CES_u}_t &= \frac{p^{CES_u}_t}{p^{CES_u}_{t-1}}\Pi_t \\
\hat{\pi}^{CES_u}_t &= \hat{p}^{CES_u}_t - \hat{p}^{CES_u}_{t-1} + \hat{\pi}_t && \text{(C.9)}
\end{aligned}
$$

$$1 = E_t\left[\beta \frac{\lambda_{u,t+1}}{\lambda_{u,t}} \frac{1}{\Pi_{t+1}^{CES_u}} \left(\bar{R}^* \frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t} \frac{\Pi_{t+1}}{\Pi_{ss}^*}\right)\right]$$

$$0 = E_t\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} - E_t\hat{\pi}_{t+1}^{CES_u} + E_t\hat{\pi}_{t+1} + \hat{q}_{t+1} - \hat{q}_t \tag{C.10}$$

$$\hat{\Lambda}_{u,t,t+1} = E_t\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} \tag{C.11}$$

$$p_t^C C_{u,t} = w_t N_{u,t}^h + \frac{\left(b_t^* \mathcal{Q}_t - \frac{\bar{R}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} + div_t^F - t_t^F\right)}{1-\omega} - p_t^E E_{u,t}^h - \mathcal{T}_u$$

$$p_{ss}^C C_{u,ss}\left(\hat{p}_t^C + \hat{c}_{u,t}\right) = \frac{\left(b_{ss}^* \mathcal{Q}_{ss}(\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^* \mathcal{Q}_{ss}}{\Pi_{ss}^*}(\hat{b}_{t-1}^* + \hat{q}_t) + div_{ss}^F \widehat{div}_t^F - t_{ss}^F \hat{t}_t^F\right)}{1-\omega}$$
$$+ w_{ss} N_{u,ss}^h\left(\hat{w}_t + \hat{n}_{u,t}^h\right) - p_{ss}^E E_{u,ss}^h\left(\hat{p}_t^E + \hat{e}_{u,t}^h\right), \mathcal{Q}_{ss} = 1, p_{ss}^C = 1, p_{ss}^E = 1, \hat{p}_t^C = 0$$

$$C_{u,ss}\hat{c}_{u,t} = \frac{\left(b_{ss}^*(\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^*}{\Pi_{ss}^*}(\hat{b}_{t-1}^* + \hat{q}_t) + div_{ss}^F \widehat{div}_t^F - t_{ss}^F \hat{t}_t^F\right)}{1-\omega}$$
$$+ w_{ss} N_{u,ss}^h\left(\hat{w}_t + \hat{n}_{u,t}^h\right) - E_{u,ss}^h\left(\hat{p}_t^E + \hat{e}_{u,t}^h\right) \tag{C.12}$$

$$\widehat{div}_t^F = \widehat{div}_t^Z \tag{C.13}$$

C.2 Constrained Households

$$C_{c,t} = \left(\frac{p_t^C}{p_t^{CES_c}}\right)^{-\psi_{ec}} (1-\alpha_{c,ec}) CES_{c,t}$$

$$\hat{c}_{c,t} = \widehat{ces}_{c,t} - \psi_{ec}(\hat{p}_t^C - \hat{p}_t^{CES_c}) \tag{C.14}$$

$$E_{c,t}^h = \left(\frac{p_t^E}{p_t^{CES_c}}\right)^{-\psi_{ec}} (\alpha_{c,ec}) CES_{c,t}$$

$$\hat{e}_{c,t}^h = \widehat{ces}_{c,t} - \psi_{ec}(\hat{p}_t^E - \hat{p}_t^{CES_c}) \tag{C.15}$$

$$p_t^{CES_c} = \left[(1-\alpha_{c,ec})\left(p_t^C\right)^{1-\psi_{ec}} + \alpha_{c,ec}\left(p_t^E\right)^{1-\psi_{ec}}\right]^{\frac{1}{1-\psi_{ec}}}$$

$$\hat{p}_t^{CES_c} = (1-\alpha_{c,ec})\hat{p}_t^C + \alpha_{c,ec}\hat{p}_t^E \tag{C.16}$$

$$\lambda_{c,t} = (CES_{c,t})^{-\sigma}$$

$$\hat{\lambda}_{c,t} = -\sigma\widehat{ces}_{c,t} \tag{C.17}$$

$$\hat{w}_t^h = \widehat{mrs}_{c,t} \tag{C.18}$$

$$mrs_{c,t} = -\frac{U_{c,t}^N}{\lambda_{c,t}/p_t^{CES_c}}$$

$$\widehat{mrs}_{c,t} = \left(\varphi\hat{n}_{c,t}^h + \hat{p}_t^{CES_c} - \hat{\lambda}_{c,t}\right) \tag{C.19}$$

$$\hat{u}_{u,t}^N = \varphi\hat{n}_{u,t}^h \tag{C.20}$$

$$p_t^C C_{c,t} = w_t N_{c,t}^h + \mathcal{T}_c - p_t^E E_{c,t}^h$$

$$C_{c,ss}\hat{c}_{c,t} = w_{ss} N_{c,ss}^h\left(\hat{w}_t + \hat{n}_{c,t}^h\right) - E_{c,ss}^h\left(\hat{p}_t^E + \hat{e}_{c,t}^h\right) \tag{C.21}$$

C.3 Aggregation, Market Clearing, Definitions

$$\Gamma_t = \frac{C_{u,t}}{C_{c,t}}$$
$$\hat{\gamma}_t = \hat{c}_{u,t} - \hat{c}_{c,t} \tag{C.22}$$
$$t_t^F = t_t^Z$$
$$\hat{t}_t^F = \hat{t}_t^Z \tag{C.23}$$
$$t_t^Z = (1-\tau_t^Z)(w_t N_t^h + p_t^E E_t^z)$$
$$t_{ss}^Z \hat{t}_t^Z = w_{ss} N_{ss}^h(\hat{w}_t + \hat{n}_t^h) - \tau_{ss}^Z w_{ss} N_{ss}^h(\hat{\tau}_t^Z + \hat{w}_t + \hat{n}_t^h) + p_{ss}^E E_{ss}^z(\hat{p}_t^E + \hat{e}_t^z) - \tau_{ss}^Z p_{ss}^E E_{ss}^z(\hat{\tau}_t^Z + \hat{p}_t^E + \hat{e}_t^z) \tag{C.24}$$
$$t_t^L = (1-\tau_t^W) w_t^h N_t^h$$
$$t_{ss}^L \hat{t}_t^L = w_{ss}^h N_{ss}^h(\hat{w}_t^h + \hat{n}_t^h) - \tau_{ss}^W w_{ss}^h N_{ss}^h(\hat{\tau}_t^W + \hat{w}_t^h + \hat{n}_t^h) \tag{C.25}$$
$$C_t = \omega C_{c,t} + (1-\omega) C_{u,t}$$
$$C_{ss}\hat{c}_t = \omega C_{c,ss}\hat{c}_{c,t} + (1-\omega) C_{u,ss}\hat{c}_{u,t}, \quad C_{ss} = C_{u,ss} = C_{c,ss}$$
$$\hat{c}_t = \omega \hat{c}_{c,t} + (1-\omega)\hat{c}_{u,t} \tag{C.26}$$
$$E_t^h = \omega E_{c,t}^h + (1-\omega) E_{u,t}^h$$
$$E_{ss}^h \hat{e}_t^h = \omega E_{c,ss}^h \hat{e}_{c,t}^h + (1-\omega) E_{u,ss}^h \hat{e}_{u,t}^h \tag{C.27}$$
$$N_t^h = \omega N_{c,t}^h + (1-\omega) N_{u,t}^h$$
$$N_{ss}^h \hat{n}_t^h = \omega N_{c,ss}^h \hat{n}_{c,t}^h + (1-\omega) N_{u,ss}^h \hat{n}_{u,t}^h \tag{C.28}$$
$$\Lambda_{t,t+1} = (1-\omega)\Lambda_{u,t,t+1}$$
$$\hat{\Lambda}_{t,t+1} = \hat{\Lambda}_{u,t,t+1} \tag{C.29}$$
$$p_t^{CPI} = \omega p_t^{CES_c} + (1-\omega) p_t^{CES_u}$$
$$p_{ss}^{CPI}\hat{p}_t^{CPI} = \omega p_{ss}^{CES_c}\hat{p}_t^{CES_c} + (1-\omega) p_{ss}^{CES_u}\hat{p}_t^{CES_u} \tag{C.30}$$
$$b_t = 0 \tag{C.31}$$
$$p_t^C C_t + p_t^X X_t = Z_t$$
$$C_{ss}\hat{c}_t + X_{ss}\hat{x}_t = Z_{ss}\hat{z}_t \tag{C.32}$$
$$NFA_t = p_t^{EXP}\mathcal{Q}_t X_t - p_t^{E,*}\mathcal{Q}_t(E_t^z + E_t^h)$$
$$NFA_{ss}\widehat{nfa}_t = p_{ss}^{EXP} X_{ss}(\hat{p}_t^{EXP} + \hat{q}_t + \hat{x}_t) - p_{ss}^{E,*} E_{ss}^z(\hat{p}_t^{E,*} + \hat{q}_t + \hat{e}_t^z) - p_{ss}^{E,*} E_{ss}^h(\hat{p}_t^{E,*} + \hat{q}_t + \hat{e}_t^h) \tag{C.33}$$

C.4 Labour Unions

$$mc_t^W = \tau^W w_t^h = \tau^W mrs_t$$
$$\hat{mc}_t^W = \hat{w}_t^h = \widehat{mrs}_t \tag{C.34}$$
$$div_t^L = \left(w_t - mc_t^W\right) N_t^h$$
$$div_{ss}^L \widehat{div}_t^L = w_{ss} N_{ss}^h(\hat{w}_t + \hat{n}_t^h) - mc_{ss}^W N_{ss}^h(\hat{mc}_t^W + \hat{n}_t^h) \tag{C.35}$$

$$\frac{f_t^{W,1}}{f_t^{W,2}}\mathcal{M}_w = w_t^{\#} = \left(\frac{1-\phi_W(\zeta_t^W)^{\frac{1}{\mathcal{M}_w-1}}}{1-\phi_W}\right)^{1-\mathcal{M}_w}$$

$$\left(\frac{f_t^{W,1}}{f_t^{W,2}}\mathcal{M}_w\right)^{\frac{1}{1-\mathcal{M}_w}} = \left(\frac{1}{1-\phi_W}\right)\left(1-\phi_W(\zeta_t^W)^{\frac{1}{\mathcal{M}_w-1}}\right)$$

$$\left(\frac{f_{ss}^{W,1}}{f_{ss}^{W,2}}\mathcal{M}_w\right)^{\frac{1}{1-\mathcal{M}_w}} = \left(\frac{1}{1-\phi_W}\right) - \left(\frac{1}{1-\phi_W}\right)\left(\phi_W(\zeta_{ss}^W)^{\frac{1}{\mathcal{M}_w-1}}\right)$$

$$\left(\frac{f_{ss}^{W,1}}{f_{ss}^{W,2}}\mathcal{M}_w\right)^{\frac{1}{1-\mathcal{M}_w}}\left(\left(\frac{1}{1-\mathcal{M}_w}\right)(\hat{f}_t^{w,1}-\hat{f}_t^{w,2})\right) = -\left(\frac{1}{1-\phi_W}\right)\left(\phi_W(\zeta_{ss}^W)^{\frac{1}{\mathcal{M}_w-1}}\right)\left(\frac{1}{\mathcal{M}_w-1}\hat{\zeta}_t^W\right)$$

$$\zeta_{ss}^W = 1$$

$$\left(\frac{1}{1-\phi_W}\right)(1-\phi_W)\left(\left(\frac{1}{1-\mathcal{M}_w}\right)(\hat{f}_t^{w,1}-\hat{f}_t^{w,2})\right) = -\left(\frac{1}{1-\phi_W}\right)\phi_W\left(\frac{1}{\mathcal{M}_w-1}\hat{\zeta}_t^W\right)$$

$$(1-\phi_W)\left(\hat{f}_t^{w,1}-\hat{f}_t^{w,2}\right) = \phi_W\left(\hat{\zeta}_t^W\right) \tag{C.36}$$

$$f_t^{W,1} = \frac{1}{w_t}mc_t^W N_t + \phi_W E_t\left[\beta\frac{\lambda_{u,t+1}}{\lambda_{u,t}}\frac{\Pi_{t+1}^W}{\Pi_{t+1}^{CES_u}}(\zeta_{t+1}^W)^{\frac{\mathcal{M}_w}{\mathcal{M}_w-1}}f_{t+1}^{W,1}\right]$$

$$f_{ss}^{W,1} = \frac{1}{w_{ss}}mc_{ss}^W N_{ss} + \phi_W\left[\beta\frac{\Pi_{ss}^W}{\Pi_{ss}^{CES_u}}(\zeta_{ss}^W)^{\frac{\mathcal{M}_w}{\mathcal{M}_w-1}}f_{ss}^{W,1}\right], \quad N_{ss}=1, \Pi_{ss}^W=\Pi_{ss}, \zeta_{ss}=1$$

$$f_{ss}^{W,1} = \frac{mc_{ss}^W}{w_{ss}}\frac{1}{(1-\phi_W\beta)}$$

$$f_{ss}^{W,1}\hat{f}_t^{W,1} = \frac{mc_{ss}^W}{w_{ss}}\left(-\hat{w}_t+\hat{mc}_t^W+\hat{n}_t\right) + \phi_W\beta f_{ss}^{W,1}E_t\left[\hat{\lambda}_{u,t+1}-\hat{\lambda}_{u,t}+\hat{\pi}_{t+1}^W-\hat{\pi}_{t+1}^{CES_u}+\left(\frac{\mathcal{M}_w}{\mathcal{M}_w-1}\right)\hat{\zeta}_{t+1}^W+\hat{f}_{t+1}^{W,1}\right]$$

$$\hat{f}_t^{W,1} = (1-\phi_w\beta)\left(\hat{n}_t+\hat{mc}_t^W-\hat{w}_t\right)+\phi_W\beta E_t\left[\hat{\lambda}_{u,t+1}-\hat{\lambda}_{u,t}+\hat{\pi}_{t+1}^W-\hat{\pi}_{t+1}^{CES_u}+\left(\frac{\mathcal{M}_w}{\mathcal{M}_w-1}\right)\hat{\zeta}_{t+1}^W+\hat{f}_{t+1}^{W,1}\right] \tag{C.37}$$

$$f_t^{W,2} = N_t + \phi_W E_t\left[\frac{\lambda_{u,t+1}}{\lambda_{u,t}}\frac{\Pi_{t+1}^W}{\Pi_{t+1}^{CES_u}}(\zeta_{t+1}^W)^{\frac{1}{\mathcal{M}_w-1}}f_{t+1}^{W,2}\right]$$

$$f_{ss}^{W,2} = \frac{1}{(1-\phi_W\beta)}$$

$$f_{ss}^{W,2}\hat{f}_t^{W,2} = N_{ss}\hat{n}_t + \phi_W\beta f_{ss}^{W,2}\left(E_t\left[\hat{\lambda}_{t+1}-\hat{\lambda}_t+\hat{\pi}_{t+1}^W-\hat{\pi}_{t+1}^{CES_u}+\left(\frac{1}{\mathcal{M}_w-1}\right)\hat{\zeta}_{t+1}^W+\hat{f}_{t+1}^{W,2}\right]\right)$$

$$\hat{f}_t^{W,2} = (1-\phi_W\beta)\hat{n}_t + \phi_W\beta\left(E_t\left[\hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{\pi}_{t+1}^W - \hat{\pi}_{t+1}^{CES_u} + \left(\frac{1}{\mathcal{M}_w - 1}\right)\hat{\zeta}_{t+1}^W + \hat{f}_{t+1}^{W,2}\right]\right) \tag{C.38}$$

$$\zeta_t^W = \frac{\Pi_t^W}{(\Pi_{ss}^W)^{1-\xi_w}(\Pi_{t-1}^W)^{\xi_w}}$$

$$\hat{\zeta}_t^W = \left(\hat{\pi}_t^W - \xi_w\hat{\pi}_{t-1}^W\right) \tag{C.39}$$

$$\hat{w}_t = \hat{\pi}_t^W - \hat{\pi}_t + \hat{w}_{t-1} \tag{C.40}$$

$$\hat{d}_t^W = 0 \tag{C.41}$$

C.5 Z Firms

$$Z_t\mathcal{D}_t^Z = \varepsilon_t^{TFP}\left((1-\alpha_{ez})^{\frac{1}{\psi_{ez}}}(N_t)^{\frac{\psi_{ez}-1}{\psi_{ez}}} + (\alpha_{ez})^{\frac{1}{\psi_{ez}}}(E_t^z)^{\frac{\psi_{ez}-1}{\psi_{ez}}}\right)^{\frac{\psi_{ez}}{\psi_{ez}-1}}$$

$$\left(Z_t\mathcal{D}_t^Z\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}}\left(\varepsilon_t^{TFP}N_t\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} + (\alpha_{ez})^{\frac{1}{\psi_{ez}}}\left(\varepsilon_t^{TFP}E_t^z\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}}$$

$$(Z_{ss})^{\frac{\psi_{ez}-1}{\psi_{ez}}} = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} + (\alpha_{ez})^{\frac{1}{\psi_{ez}}}(E_{ss}^z)^{\frac{\psi_{ez}-1}{\psi_{ez}}}, \quad \mathcal{D}_{ss}^Z = 1, \varepsilon_{ss}^{TFP} = 1, N_{ss} = 1$$

$$(Z_{ss})^{\frac{\psi_{ez}-1}{\psi_{ez}}}(\hat{z}_t)\left(\frac{\psi_{ez}-1}{\psi_{ez}}\right) = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}}\left(\hat{\varepsilon}_t^{tfp} + \hat{n}_t\right)\left(\frac{\psi_{ez}-1}{\psi_{ez}}\right) + (\alpha_{ez})^{\frac{1}{\psi_{ez}}}(E_{ss}^z)^{\frac{\psi_{ez}-1}{\psi_{ez}}}\left(\hat{\varepsilon}_t^{tfp} + \hat{e}_t^z\right)\left(\frac{\psi_{ez}-1}{\psi_{ez}}\right)$$

$$(Z_{ss})^{\frac{\psi_{ez}-1}{\psi_{ez}}}(\hat{z}_t) = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}}\left(\hat{\varepsilon}_t^{tfp} + \hat{n}_t\right) + (\alpha_{ez})^{\frac{1}{\psi_{ez}}}(E_{ss}^z)^{\frac{\psi_{ez}-1}{\psi_{ez}}}\left(\hat{\varepsilon}_t^{tfp} + \hat{e}_t^z\right)$$

$$\hat{z}_t = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}}\left(\frac{1}{Z_{ss}}\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}}\left(\hat{\varepsilon}_t^{tfp} + \hat{n}_t\right) + (\alpha_{ez})^{\frac{1}{\psi_{ez}}}\left(\frac{E_{ss}^z}{Z_{ss}}\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}}\left(\hat{\varepsilon}_t^{tfp} + \hat{e}_t^z\right)$$

$$\hat{z}_t = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}}(1-\alpha_{ez})^{\frac{\psi_{ez}-1}{\psi_{ez}}}\left(\hat{\varepsilon}_t^{tfp} + \hat{n}_t\right) + (\alpha_{ez})^{\frac{1}{\psi_{ez}}}(\alpha_{ez})^{\frac{\psi_{ez}-1}{\psi_{ez}}}\left(\hat{\varepsilon}_t^{tfp} + \hat{e}_t^z\right)$$

$$\hat{z}_t = \hat{\varepsilon}_t^{tfp} + (1-\alpha_{ez})\hat{n}_t + \alpha_{ez}\hat{e}_t^z \tag{C.42}$$

$$w_t = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}}\frac{mc_t^Z}{\tau_t^Z}\left(\frac{Z_t\mathcal{D}_t^Z}{N_t}\right)^{\frac{1}{\psi_{ez}}}\left(\varepsilon_t^{TFP}\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}}$$

$$w_{ss} = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}}\frac{mc_{ss}^Z}{\tau_{ss}^Z}\left(\frac{Z_{ss}}{N_{ss}}\right)^{\frac{1}{\psi_{ez}}}$$

$$w_{ss}\hat{w}_t = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}}\frac{mc_{ss}^Z}{\tau_{ss}^Z}\left(\frac{Z_{ss}}{N_{ss}}\right)^{\frac{1}{\psi_{ez}}}\left(\hat{mc}_t^Z - \hat{\tau}_t^Z + \frac{1}{\psi_{ez}}(\hat{z}_t - \hat{n}_t) + \left(\frac{\psi_{ez}-1}{\psi_{ez}}\hat{\varepsilon}_t^{tfp}\right)\right)$$

$$\hat{w}_t = \hat{mc}_t^Z - \hat{\tau}_t^Z + \frac{1}{\psi_{ez}}(\hat{z}_t - \hat{n}_t) + \left(\frac{\psi_{ez}-1}{\psi_{ez}}\right)\hat{\varepsilon}_t^{tfp} \tag{C.43}$$

$$p_t^E = (\alpha_{ez})^{\frac{1}{\psi_{ez}}}\frac{mc_t^Z}{\tau_t^Z}\left(\frac{Z_t\mathcal{D}_t^Z}{E_t^z}\right)^{\frac{1}{\psi_{ez}}}\left(\varepsilon_t^{TFP}\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}}$$

$$\hat{p}_t^E = \hat{mc}_t^Z - \hat{\tau}_t^Z + \frac{1}{\psi_{ez}}(\hat{z}_t - \hat{e}_t^z) + \left(\frac{\psi_{ez}-1}{\psi_{ez}}\right)\hat{\varepsilon}_t^{tfp} \tag{C.44}$$

$$div_t^Z = \left(1 - mc_t^Z\right) Z_t$$

$$div_{ss}^Z \widehat{div}_t^Z = Z_{ss}\hat{z}_t - mc_{ss}^Z Z_{ss}(\hat{mc}_t^Z + \hat{z}_t) \tag{C.45}$$

$$\frac{f_t^{Z,1}}{f_t^{Z,2}}\mathcal{M}_z = \left[\frac{1-(\phi_Z)\left(\zeta_t^Z\right)^{\frac{-1}{1-\mathcal{M}_Z}}}{1-\phi_Z}\right]^{1-\mathcal{M}_z}$$

$$\hat{\zeta}_t^Z = (1-\phi_Z)/\phi_Z\left(\hat{f}_t^{Z,1} - \hat{f}_t^{Z,2}\right) \tag{C.46}$$

$$f_t^{Z,1} = mc_t^Z Z_t + \phi_Z E_t\left[\frac{\lambda_{u,t+1}}{\lambda_{u,t}}\frac{\Pi_{t+1}}{\Pi_{t+1}^{CES_u}}(\zeta_{t+1}^Z)^{\frac{\mathcal{M}_z}{\mathcal{M}_z-1}} f_{t+1}^{Z,1}\right]$$

$$\hat{f}_t^{Z,1} = (1-\phi_z\beta)\left(\hat{z}_t + \hat{mc}_t^Z\right) + \phi_Z\beta E_t\left[\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} + \hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{CES_u} + \left(\frac{\mathcal{M}_z}{\mathcal{M}_z-1}\right)\hat{\zeta}_{t+1}^Z + \hat{f}_{t+1}^{Z,1}\right] \tag{C.47}$$

$$f_t^{Z,2} = Z_t + \phi_Z E_t\left[\frac{U_{u,t+1}^{CES}}{U_{u,t}^{CES}}\frac{\Pi_{t+1}}{\Pi_{t+1}^{CES_u}}(\zeta_{t+1}^Z)^{\frac{1}{\mathcal{M}_z-1}} f_{t+1}^{Z,2}\right]$$

$$\hat{f}_t^{Z,2} = (1-\phi_Z\beta)\hat{z}_t + \phi_Z\beta\left(E_t\left[\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} + \hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{CES_u} + \left(\frac{1}{\mathcal{M}_Z-1}\right)\hat{\zeta}_{t+1}^Z + \hat{f}_{t+1}^{Z,2}\right]\right) \tag{C.48}$$

$$\zeta_t^Z = \Pi_t/(\Pi_{ss})^{1-\xi_z}(\Pi_{t-1})^{\xi_z}, \quad \hat{\zeta}_t^Z = \hat{\pi}_t, \xi_z = 0 \tag{C.49}$$

$$\hat{d}_t^Z = 0 \tag{C.50}$$

$$p_t^E = p_t^{E,*}\mathcal{Q}_t$$

$$\hat{p}_t^E = \hat{p}_t^{E,*} + \hat{q}_t \tag{C.51}$$

$$p_t^{E,*} = \left(p_{ss}^{E,*}\right)^{1-\rho_E}\left(p_{t-1}^{E,*}\right)^{\rho_E}\varepsilon_t^E$$

$$\hat{p}_t^{E,*} = \rho_E\hat{p}_{t-1}^{E,*} + \hat{\varepsilon}_t^E \tag{C.52}$$

$$p_t^X = p_t^{EXP}\mathcal{Q}_t$$

$$0 = \hat{p}_t^{EXP} + \hat{q}_t \tag{C.53}$$

$$p_t^X = 1, p_t^C = 1$$

$$\hat{p}_t^X = 0 \tag{C.54}$$

$$\hat{p}_t^C = 0 \tag{C.55}$$

C.6 Monetary Policy and World

$$\hat{r}_t = \theta_R\hat{r}_{t-1} + (1-\theta_R)\left(\theta_\Pi/4\hat{\pi}_t^{CPI,a} + \theta_Y\hat{y}_t\right) \tag{C.56}$$

$$\hat{\pi}_t^{CPI} = \hat{p}_t^{CPI} - \hat{p}_{t-1}^{CPI} + \hat{\pi}_t \tag{C.57}$$

$$\hat{\pi}_t^{CPI,a} = \hat{\pi}_t^{CPI} + \hat{\pi}_t^{CPI,lag1} + \hat{\pi}_t^{CPI,lag2} + \hat{\pi}_{t-1}^{CPI,lag2} \tag{C.58}$$

$$\hat{\pi}_t^{CPI,lag1} = \hat{\pi}_{t-1}^{CPI} \tag{C.59}$$

$$\hat{\pi}_t^{CPI,lag2} = \hat{\pi}_{t-1}^{CPI,lag1} \tag{C.60}$$

$$\hat{y}_t = \hat{n}_t - \hat{n}_t^{flex} \tag{C.61}$$

$$X_t = \kappa^*\left(\frac{p_t^{EXP}}{p_{ss}^{X*}}\right)^{-\varsigma^*} Y_{ss}^*, \quad \hat{x}_t = -\varsigma^*\hat{p}_t^{EXP} \tag{C.62}$$

D Summary of loglinear system

Unconstrained Households

$$\hat{c}_{u,t} = \widehat{ces}_{u,t} - \psi_{ec}(\hat{p}^C_t - \hat{p}^{CESu}_t) \tag{D.1}$$

$$\hat{e}^h_{u,t} = \widehat{ces}_{u,t} - \psi_{ec}(\hat{p}^E_t - \hat{p}^{CESu}_t) \tag{D.2}$$

$$\hat{p}^{CESu}_t = (1-\alpha_{u,ec})\hat{p}^C_t + \alpha_{u,ec}\hat{p}^E_t \tag{D.3}$$

$$\hat{\lambda}_{u,t} = -\sigma\widehat{ces}_{u,t} \tag{D.4}$$

$$\hat{w}^h_t = \widehat{mrs}_{u,t} \tag{D.5}$$

$$\widehat{mrs}_{u,t} = \left(\varphi\hat{n}^h_{u,t} + \hat{p}^{CESu}_t - \hat{\lambda}_{u,t}\right) \tag{D.6}$$

$$\hat{u}^N_{u,t} = \varphi\hat{n}^h_{u,t} \tag{D.7}$$

$$0 = \mathbf{E}_t\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} - \mathbf{E}_t\hat{\pi}^{CESu}_{t+1} + \hat{r}_t \tag{D.8}$$

$$\hat{\pi}^{CESu}_t = \hat{p}^{CESu}_t - \hat{p}^{CESu}_{t-1} + \hat{\pi}_t \tag{D.9}$$

$$0 = \mathbf{E}_t\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} - \mathbf{E}_t\hat{\pi}^{CESu}_{t+1} + \mathbf{E}_t\hat{\pi}_{t+1} + \hat{q}_{t+1} - \hat{q}_t \tag{D.10}$$

$$\hat{\Lambda}_{u,t,t+1} = \mathbf{E}_t\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} \tag{D.11}$$

$$\begin{aligned} C_{u,ss}\hat{c}_{u,t} = {} & \frac{\left(b^*_{ss}(\hat{b}^*_t + \hat{q}_t) - \frac{R^* b^*_{ss}}{\Pi^*_{ss}}(\hat{b}^*_{t-1} + \hat{q}_t) + div^F_{ss}\widehat{div}^F_t - t^F_{ss}\hat{t}^F_t\right)}{1-\omega} \\ & + w_{ss}N^h_{u,ss}\left(\hat{w}_t + \hat{n}^h_{u,t}\right) - E^h_{u,ss}\left(\hat{p}^E_t + \hat{e}^h_{u,t}\right) \end{aligned} \tag{D.12}$$

$$\widehat{div}^F_t = \widehat{div}^Z_t \tag{D.13}$$

Constrained Households

$$\hat{c}_{c,t} = \widehat{ces}_{c,t} - \psi_{ec}(\hat{p}^C_t - \hat{p}^{CESc}_t) \tag{D.14}$$

$$\hat{e}^h_{c,t} = \widehat{ces}_{c,t} - \psi_{ec}(\hat{p}^E_t - \hat{p}^{CESc}_t) \tag{D.15}$$

$$\hat{p}^{CESc}_t = (1-\alpha_{c,ec})\hat{p}^C_t + \alpha_{c,ec}\hat{p}^E_t \tag{D.16}$$

$$\hat{\lambda}_{c,t} = -\sigma\widehat{ces}_{c,t} \tag{D.17}$$

$$\hat{w}^h_t = \widehat{mrs}_{c,t} \tag{D.18}$$

$$\widehat{mrs}_{c,t} = \left(\varphi\hat{n}^h_{c,t} + \hat{p}^{CESc}_t - \hat{\lambda}_{c,t}\right) \tag{D.19}$$

$$\hat{u}^N_{u,t} = \varphi\hat{n}^h_{u,t} \tag{D.20}$$

$$C_{c,ss}\hat{c}_{c,t} = w_{ss}N^h_{c,ss}\left(\hat{w}_t + \hat{n}^h_{c,t}\right) - E^h_{c,ss}\left(\hat{p}^E_t + \hat{e}^h_{c,t}\right) \tag{D.21}$$

$$\hat{\gamma}_t = \hat{c}_{u,t} - \hat{c}_{c,t} \tag{D.22}$$

$$\hat{t}^F_t = \hat{t}^Z_t \tag{D.23}$$

$$\begin{aligned} t^Z_{ss}\hat{t}^Z_t = {} & w_{ss}N^h_{ss}(\hat{w}_t + \hat{n}^h_t) - \tau^Z_{ss}w_{ss}N^h_{ss}(\hat{\tau}^Z_t + \hat{w}_t + \hat{n}^h_t) + p^E_{ss}E^z_{ss}(\hat{p}^E_t + \hat{e}^z_t) \\ & - \tau^Z_{ss}p^E_{ss}E^z_{ss}(\hat{\tau}^Z_t + \hat{p}^E_t + \hat{e}^z_t) \end{aligned} \tag{D.24}$$

$$t^L_{ss}\hat{t}^L_t = w^h_{ss}N^h_{ss}(\hat{w}^h_t + \hat{n}^h_t) - \tau^W_{ss}w^h_{ss}N^h_{ss}(\hat{\tau}^W_t + \hat{w}^h_t + \hat{n}^h_t) \tag{D.25}$$

$$\hat{c}_t = \omega\hat{c}_{c,t} + (1-\omega)\hat{c}_{u,t} \tag{D.26}$$

$$E^h_{ss}\hat{e}^h_t = \omega E^h_{c,ss}\hat{e}^h_{c,t} + (1-\omega)E^h_{u,ss}\hat{e}^h_{u,t} \tag{D.27}$$

$$N^h_{ss}\hat{n}^h_t = \omega N^h_{c,ss}\hat{n}^h_{c,t} + (1-\omega)N^h_{u,ss}\hat{n}^h_{u,t} \tag{D.28}$$

$$\hat{\Lambda}_{t,t+1} = \hat{\Lambda}_{u,t,t+1} \tag{D.29}$$

$$p^{CPI}_{ss}\hat{p}^{CPI}_t = \omega p^{CESc}_{ss}\hat{p}^{CESc}_t + (1-\omega)p^{CESu}_{ss}\hat{p}^{CESu}_t \tag{D.30}$$

$$b_t = 0 \tag{D.31}$$

$$C_{ss}\hat{c}_t + X_{ss}\hat{x}_t = Z_{ss}\hat{z}_t \tag{D.32}$$

$$NFA_{ss}\widehat{nfa}_t = p^{EXP}_{ss}X_{ss}(\hat{p}^{EXP}_t + \hat{q}_t + \hat{x}_t) - p^{E,*}_{ss}E^z_{ss}(\hat{p}^{E,*}_t + \hat{q}_t + \hat{e}^z_t) - p^{E,*}_{ss}E^h_{ss}(\hat{p}^{E,*}_t + \hat{q}_t + \hat{e}^h_t) \tag{D.33}$$

Labour Unions

$$\hat{mc}_t^W = \hat{w}_t^h = \widehat{mrs}_t \tag{D.34}$$

$$div_{ss}^L \widehat{div}_t^L = w_{ss} N_{ss}^h (\hat{w}_t + \hat{n}_t^h) - mc_{ss}^W N_{ss}^h (\hat{mc}_t^W + \hat{n}_t^h) \tag{D.35}$$

$$\hat{\zeta}_t^W = \frac{1-\phi_W}{\phi_W} \left(\hat{f}_t^{w,1} - \hat{f}_t^{w,2} \right) \tag{D.36}$$

$$\begin{aligned} \hat{f}_t^{W,1} &= (1-\phi_w \beta) \left(\hat{n}_t + \hat{mc}_t^W - \hat{w}_t \right) \\ &\quad + \phi_W \beta E_t \left[\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} + \hat{\pi}_{t+1}^W - \hat{\pi}_{t+1}^{CESu} + \left(\frac{\mathcal{M}_w}{\mathcal{M}_w - 1} \right) \hat{\zeta}_{t+1}^W + \hat{f}_{t+1}^{W,1} \right] \end{aligned} \tag{D.37}$$

$$\begin{aligned} \hat{f}_t^{W,2} &= (1-\phi_W \beta) \hat{n}_t \\ &\quad + \phi_W \beta \left(E_t \left[\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} + \hat{\pi}_{t+1}^W - \hat{\pi}_{t+1}^{CESu} + \left(\frac{1}{\mathcal{M}_w - 1} \right) \hat{\zeta}_{t+1}^W + \hat{f}_{t+1}^{W,2} \right] \right) \end{aligned} \tag{D.38}$$

$$\hat{\zeta}_t^W = \hat{\pi}_t^W, \xi_w = 0 \tag{D.39}$$

$$\hat{w}_t = \hat{\pi}_t^W - \hat{\pi}_t + \hat{w}_{t-1} \tag{D.40}$$

$$\hat{d}_t^W = 0 \tag{D.41}$$

Z Firms

$$\hat{z}_t = \hat{\varepsilon}_t^{tfp} + (1-\alpha_{ez}) \hat{n}_t + \alpha_{ez} \hat{e}_t^z \tag{D.42}$$

$$\hat{w}_t = \hat{mc}_t^Z - \hat{\tau}_t^Z + \frac{1}{\psi_{ez}} (\hat{z}_t - \hat{n}_t) + \left(\frac{\psi_{ez} - 1}{\psi_{ez}} \right) \hat{\varepsilon}_t^{tfp} \tag{D.43}$$

$$\hat{p}_t^E = \hat{mc}_t^Z - \hat{\tau}_t^Z + \frac{1}{\psi_{ez}} (\hat{z}_t - \hat{e}_t^z) + \left(\frac{\psi_{ez} - 1}{\psi_{ez}} \right) \hat{\varepsilon}_t^{tfp} \tag{D.44}$$

$$div_{ss}^Z \widehat{div}_t^Z = Z_{ss} \hat{z}_t - mc_{ss}^Z Z_{ss} (\hat{mc}_t^Z + \hat{z}_t) \tag{D.45}$$

$$\hat{\zeta}_t^Z = \frac{1-\phi_Z}{\phi_Z} \left(\hat{f}_t^{Z,1} - \hat{f}_t^{Z,2} \right) \tag{D.46}$$

$$f_t^{Z,1} = mc_t^Z Z_t + \phi_Z E_t \left[\Lambda_{u,t,t+1} (\zeta_{t+1}^Z)^{\frac{\mathcal{M}_Z}{\mathcal{M}_Z - 1}} f_{t+1}^{Z,1} \right]$$

$$\hat{f}_t^{Z,1} = (1-\phi_z \beta) \left(\hat{z}_t + \hat{mc}_t^Z \right) + \phi_Z \beta E_t \left[\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} + \hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{CESu} + \left(\frac{\mathcal{M}_z}{\mathcal{M}_z - 1} \right) \hat{\zeta}_{t+1}^Z + \hat{f}_{t+1}^{Z,1} \right] \tag{D.47}$$

$$\hat{f}_t^{Z,2} = (1-\phi_Z \beta) \hat{z}_t + \phi_Z \beta \left(E_t \left[\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} + \hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{CESu} + \left(\frac{1}{\mathcal{M}_Z - 1} \right) \hat{\zeta}_{t+1}^Z + \hat{f}_{t+1}^{Z,2} \right] \right) \tag{D.48}$$

$$\hat{\zeta}_t^Z = \hat{\pi}_t, \xi_z = 0 \tag{D.49}$$

$$\hat{d}_t^Z = 1 \tag{D.50}$$

$$\hat{p}_t^E = \hat{p}_t^{E,*} + \hat{q}_t \tag{D.51}$$

$$\hat{p}_t^{E,*} = \rho_E \hat{p}_{t-1}^{E,*} + \hat{\varepsilon}_t^E \tag{D.52}$$

$$0 = \hat{p}_t^{EXP} + \hat{q}_t \tag{D.53}$$

$$\hat{p}_t^X = 0 \tag{D.54}$$

$$\hat{p}_t^C = 0 \tag{D.55}$$

Monetary Policy and World

$$\hat{r}_t = \theta_R \hat{r}_{t-1} + (1-\theta_R) \left(\theta_\Pi / 4 \hat{\pi}_t^{CPI,a} + \theta_Y \hat{y}_t \right) \tag{D.56}$$

$$\hat{\pi}_t^{CPI} = \hat{p}_t^{CPI} - \hat{p}_{t-1}^{CPI} + \hat{\pi}_t \tag{D.57}$$

$$\hat{\pi}_t^{CPI,a} = \hat{\pi}_t^{CPI} + \hat{\pi}_t^{CPI,lag1} + \hat{\pi}_t^{CPI,lag2} + \hat{\pi}_{t-1}^{CPI,lag2} \tag{D.58}$$

$$\hat{\pi}_t^{CPI,lag1} = \hat{\pi}_{t-1}^{CPI} \tag{D.59}$$

$$\hat{\pi}_t^{CPI,lag2} = \hat{\pi}_{t-1}^{CPI,lag1} \tag{D.60}$$

$$\hat{y}_t = \hat{n}_t - \hat{n}_t^{flex} \tag{D.61}$$

$$\hat{x}_t = -\varsigma^* \hat{p}_t^{EXP} \tag{D.62}$$

E Reduce the loglinear system

E.1 Unconstrained HH loglinear system

Step 1: $\hat{p}_t^C = 0$, take out $\hat{\Lambda}$, $\hat{\lambda}_{u,t}$, $\hat{u}^N$

$$
\begin{aligned}
\hat{c}_{u,t} &= \widehat{ces}_{u,t} - \psi_{ec}(\hat{p}_t^C - \hat{p}_t^{CES_u}), \qquad \hat{c}_{u,t} = \widehat{ces}_{u,t} + \psi_{ec}\hat{p}_t^{CES_u} \\
\hat{e}_{u,t}^h &= \widehat{ces}_{u,t} - \psi_{ec}(\hat{p}_t^E - \hat{p}_t^{CES_u}) \\
\hat{p}_t^{CES_u} &= (1-\alpha_{u,ec})\hat{p}_t^C + \alpha_{u,ec}\hat{p}_t^E, \hat{p}_t^C = 0, \qquad \hat{p}_t^{CES_u} = \alpha_{u,ec}\hat{p}_t^E \\
\hat{w}_t^h &= \widehat{mrs}_{u,t} \\
\widehat{mrs}_{u,t} &= \left(\varphi\hat{n}_{u,t}^h + \hat{p}_t^{CES_u} - \hat{\lambda}_{u,t}\right) \\
-\sigma\widehat{ces}_{u,t} &= -\mathbf{E}_t[\sigma\widehat{ces}_{u,t+1}] - \mathbf{E}_t\hat{\pi}_{t+1}^{CES_u} + \hat{r}_t \\
\hat{\pi}_t^{CES_u} &= \hat{p}_t^{CES_u} - \hat{p}_{t-1}^{CES_u} + \hat{\pi}_t \\
-\sigma\widehat{ces}_{u,t} &= -\mathbf{E}_t[\sigma\widehat{ces}_{u,t+1}] - \mathbf{E}_t\hat{\pi}_{t+1}^{CES_u} + \mathbf{E}_t\hat{\pi}_{t+1} + \hat{q}_{t+1} - \hat{q}_t \\
C_{u,ss}\hat{c}_{u,t} &= \frac{\left(b_{ss}^*(\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^*}{\Pi_{ss}^*}(\hat{b}_{t-1}^* + \hat{q}_t) + div_{ss}^F\widehat{div}_t^F - t_{ss}^F\hat{t}_t^F\right)}{1-\omega} + w_{ss}N_{u,ss}^h\left(\hat{w}_t + \hat{n}_{u,t}^h\right) - E_{u,ss}^h\left(\hat{p}_t^E + \hat{e}_{u,t}^h\right)
\end{aligned}
$$

Step 2

$$
\begin{aligned}
\hat{c}_{u,t} &= \widehat{ces}_{u,t} + \psi_{ec}\hat{p}_t^{CES_u} \\
\hat{e}_{u,t}^h &= \widehat{ces}_{u,t} - \psi_{ec}(\hat{p}_t^E - \hat{p}_t^{CES_u}) \\
\hat{p}_t^{CES_u} &= \alpha_{u,ec}\hat{p}_t^E \\
\hat{w}_t^h &= \left(\varphi\hat{n}_{u,t}^h + \hat{p}_t^{CES_u} - \hat{\lambda}_{u,t}\right) \\
\widehat{ces}_{u,t} &= \mathbf{E}_t[\widehat{ces}_{u,t+1}] - \frac{1}{\sigma}\left(\hat{r}_t - \mathbf{E}_t\hat{\pi}_{t+1}^{CES_u}\right) \\
\hat{\pi}_t^{CES_u} &= \hat{p}_t^{CES_u} - \hat{p}_{t-1}^{CES_u} + \hat{\pi}_t = \alpha_{u,ec}(\hat{p}_t^E - \hat{p}_{t-1}^E) + \hat{\pi}_t \\
\widehat{ces}_{u,t} &= \mathbf{E}_t[\widehat{ces}_{u,t+1}] - \frac{1}{\sigma}\left(\hat{q}_{t+1} - \hat{q}_t - \mathbf{E}_t\hat{\pi}_{t+1}^{CES_u} + \mathbf{E}_t\hat{\pi}_{t+1}\right) \\
C_{u,ss}\hat{c}_{u,t} &= \frac{\left(b_{ss}^*(\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^*}{\Pi_{ss}^*}(\hat{b}_{t-1}^* + \hat{q}_t) + div_{ss}^F\widehat{div}_t^F - t_{ss}^F\hat{t}_t^F\right)}{1-\omega} + w_{ss}N_{u,ss}^h\left(\hat{w}_t + \hat{n}_{u,t}^h\right) - E_{u,ss}^h\left(\hat{p}_t^E + \hat{e}_{u,t}^h\right)
\end{aligned}
$$

Step 3

$$
\begin{aligned}
\widehat{ces}_{u,t} &= \hat{c}_{u,t} - \psi_{ec}\alpha_{u,ec}\hat{p}_t^E \\
\hat{e}_{u,t}^h &= \hat{c}_{u,t} - \psi_{ec}\alpha_{u,ec}\hat{p}_t^E - \psi_{ec}(1-\alpha_{u,ec})\hat{p}_t^E \\
\hat{w}_t^h &= \left(\varphi\hat{n}_{u,t}^h + \alpha_{u,ec}\hat{p}_t^E + \sigma\widehat{ces}_{u,t}\right) \\
\hat{c}_{u,t} - \psi_{ec}\alpha_{u,ec}\hat{p}_t^E &= \mathbf{E}_t[\hat{c}_{u,t+1} - \psi_{ec}\alpha_{u,ec}\hat{p}_{t+1}^E] - \frac{1}{\sigma}\left(\hat{r}_t - \mathbf{E}_t\left[\alpha_{u,ec}(\hat{p}_{t+1}^E - \hat{p}_t^E) + \hat{\pi}_{t+1}\right]\right) \\
\hat{c}_{u,t} - \psi_{ec}\alpha_{u,ec}\hat{p}_t^E &= \mathbf{E}_t[\hat{c}_{u,t+1} - \psi_{ec}\alpha_{u,ec}\hat{p}_{t+1}^E] - \frac{1}{\sigma}\left(\hat{q}_{t+1} - \hat{q}_t - \mathbf{E}_t\left[\alpha_{u,ec}(\hat{p}_{t+1}^E - \hat{p}_t^E) + \hat{\pi}_{t+1}\right] + \mathbf{E}_t\hat{\pi}_{t+1}\right) \\
C_{u,ss}\hat{c}_{u,t} &= \frac{\left(b_{ss}^*(\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^*}{\Pi_{ss}^*}(\hat{b}_{t-1}^* + \hat{q}_t) + div_{ss}^F\widehat{div}_t^F - t_{ss}^F\hat{t}_t^F\right)}{1-\omega} \\
&\quad + w_{ss}N_{u,ss}^h\left(\hat{w}_t + \hat{n}_{u,t}^h\right) - E_{u,ss}^h\left(\hat{p}_t^E + \hat{e}_{u,t}^h\right)
\end{aligned}
$$

Step 4: boil down further

$$
\begin{aligned}
\hat{w}_t^h &= \varphi \hat{n}_{u,t}^h + \sigma \hat{c}_{u,t} + \hat{p}_t^E \alpha_{u,ec}(1-\sigma\psi_{ec}) \\
\hat{c}_{u,t} &= \mathbf{E}_t[\hat{c}_{u,t+1}] - \frac{1}{\sigma}\left(\hat{r}_t - \mathbf{E}_t\left[\alpha_{u,ec}(\hat{p}_{t+1}^E - \hat{p}_t^E) + \hat{\pi}_{t+1}\right]\right) + \psi_{ec}\alpha_{u,ec}\left(\hat{p}_t^E - \mathbf{E}_t\hat{p}_{t+1}^E\right)
\end{aligned}
$$

$$
\hat{r}_t - \mathbf{E}_t\hat{\pi}_{t+1} = \mathbf{E}_t\hat{q}_{t+1} - \hat{q}
$$

$$
\begin{aligned}
\hat{c}_{u,t} &= \frac{1}{C_{u,ss}} \frac{\left(b_{ss}^*(\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^*}{\Pi_{ss}^*}(\hat{b}_{t-1}^* + \hat{q}_t) + div_{ss}^F \widehat{div}_t^Z - t_{ss}^Z \hat{t}_t^Z\right)}{1-\omega} \\
&\quad + w_{ss}\frac{N_{u,ss}^h}{C_{u,ss}}\left(\hat{w}_t + \hat{n}_{u,t}^h\right) - \frac{E_{u,ss}^h}{C_{u,ss}}\left(\hat{c}_{u,t} + \hat{p}_t^E(1-\psi_{ec})\right)
\end{aligned}
$$

- note that $E_{u,ss}^h = 0$ if $\alpha_{u,ec} = 0$
- use the the Z profit conditions and the tax equations

$$
\begin{aligned}
div_t^Z - t_t^Z &= Z_t - w_t N_t - p_t^E E_t^z + p_t^{EXP} \mathcal{Q}_t X_t - p_t^X X_t \\
div_{ss}^Z \widehat{div}_t^Z - t_{ss}^Z \hat{t}_t^Z &= Z_{ss}\hat{z}_t - w_{ss}N_{ss}(\hat{w}_t + \hat{n}_t) - E_{ss}^z(\hat{p}_t^E + \hat{e}_t^z) + X_{ss}(\hat{x}_t) - X_{ss}(\hat{x}_t)
\end{aligned}
$$

E.2 Constrained HH loglinear system

Step 1: take out $\hat{\lambda}_{c,t}$, $\hat{p}_t^C = 0$

$$
\begin{aligned}
\hat{c}_{c,t} &= \widehat{ces}_{c,t} - \psi_{ec}(\hat{p}_t^C - \hat{p}_t^{CES_c}) \\
\hat{e}_{c,t}^h &= \widehat{ces}_{c,t} - \psi_{ec}(\hat{p}_t^E - \hat{p}_t^{CES_c}) \\
\hat{p}_t^{CES_c} &= (1-\alpha_{c,ec})\hat{p}_t^C + \alpha_{c,ec}\hat{p}_t^E \\
\hat{\lambda}_{c,t} &= -\sigma\widehat{ces}_{c,t} \\
\hat{w}_t^h &= \widehat{mrs}_{c,t} \\
\widehat{mrs}_{c,t} &= \left(\varphi\hat{n}_{c,t}^h + \hat{p}_t^{CES_c} - \hat{\lambda}_{c,t}\right) \\
\hat{u}_{u,t}^N &= \varphi\hat{n}_{u,t}^h \\
C_{c,ss}\hat{c}_{c,t} &= w_{ss}N_{c,ss}^h\left(\hat{w}_t + \hat{n}_{c,t}^h\right) - E_{c,ss}^h\left(\hat{p}_t^E + \hat{e}_{c,t}^h\right)
\end{aligned}
$$

which implies

$$
\begin{aligned}
\widehat{ces}_{c,t} &= \hat{c}_{c,t} - \psi_{ec}\alpha_{c,ec}\hat{p}_t^E \\
\hat{e}_{c,t}^h &= \widehat{ces}_{c,t} - \psi_{ec}\hat{p}_t^E(1-\alpha_{c,ec}) \\
\hat{w}_t^h &= \left(\varphi\hat{n}_{c,t}^h + \alpha_{c,ec}\hat{p}_t^E + \sigma\widehat{ces}_{c,t}\right) \\
\hat{c}_{c,t} &= w_{ss}\frac{N_{c,ss}^h}{C_{c,ss}}\left(\hat{w}_t + \hat{n}_{c,t}^h\right) - \frac{E_{c,ss}^h}{C_{c,ss}}\left(\hat{p}_t^E + \hat{e}_{c,t}^h\right)
\end{aligned}
$$

and

$$
\begin{aligned}
\hat{w}_t^h &= \varphi\hat{n}_{c,t}^h + \sigma\hat{c}_{c,t} + \alpha_{c,ec}\hat{p}_t^E(1-\sigma\psi_{ec}) \\
\hat{c}_{c,t} &= w_{ss}\frac{N_{c,ss}^h}{C_{c,ss}}\left(\hat{w}_t + \hat{n}_{c,t}^h\right) - \frac{E_{c,ss}^h}{C_{c,ss}}\left(\hat{c}_{c,t} + \hat{p}_t^E(1-\psi_{ec})\right)
\end{aligned}
$$

- note that $E_{c,ss}^h = 0$ if $\alpha_{c,ec} = 0$

E.3 Remaining HH loglinear system

Step 1

$$
\begin{aligned}
\hat{\gamma}_t &= \hat{c}_{u,t} - \hat{c}_{c,t} \\
t^Z_{ss}\hat{t}^Z_t &= w_{ss}N^h_{ss}(\hat{w}_t + \hat{n}^h_t) - \tau^Z_{ss}w_{ss}N^h_{ss}(\hat{\tau}^Z_t + \hat{w}_t + \hat{n}^h_t) + p^E_{ss}E^z_{ss}(\hat{p}^E_t + \hat{e}^z_t) \\
&\quad - \tau^Z_{ss}p^E_{ss}E^z_{ss}(\hat{\tau}^Z_t + \hat{p}^E_t + \hat{e}^z_t) \\
\hat{c}_t &= \omega\hat{c}_{c,t} + (1-\omega)\hat{c}_{u,t} \\
\hat{e}^h_t &= \omega\frac{E^h_{c,ss}}{E^h_{ss}}\hat{e}^h_{c,t} + (1-\omega)\frac{E^h_{u,ss}}{E^h_{ss}}\hat{e}^h_{u,t} \\
\hat{n}^h_t &= \omega\frac{N^h_{c,ss}}{N^h_{ss}}\hat{n}^h_{c,t} + (1-\omega)\frac{N^h_{u,ss}}{N^h_{ss}}\hat{n}^h_{u,t} \\
\hat{p}^{CPI}_t &= \hat{p}^E_t\left(\omega\alpha_{c,ec} + (1-\omega)\alpha_{u,ec}\right) \\
\hat{z}_t &= \frac{C_{ss}}{Z_{ss}}\hat{c}_t + \frac{X_{ss}}{Z_{ss}}\hat{x}_t \\
NFA_{ss}\widehat{nfa}_t &= X_{ss}(\underbrace{\hat{p}^{EXP}_t + \hat{q}_t}_{=0} + \hat{x}_t) - E^z_{ss}(\hat{p}^{E,*}_t + \hat{q}_t + \hat{e}^z_t) - E^h_{ss}(\hat{p}^{E,*}_t + \hat{q}_t + \hat{e}^h_t)
\end{aligned}
$$

Step 2

$$
\begin{aligned}
\hat{\gamma}_t &= \hat{c}_{u,t} - \hat{c}_{c,t} \\
\hat{c}_t &= \omega\hat{c}_{c,t} + (1-\omega)\hat{c}_{u,t} \\
\hat{e}^h_t &= \omega\frac{E^h_{c,ss}}{E^h_{ss}}\hat{e}^h_{c,t} + (1-\omega)\frac{E^h_{u,ss}}{E^h_{ss}}\hat{e}^h_{u,t} \\
\hat{n}^h_t &= \omega\frac{N^h_{c,ss}}{N^h_{ss}}\hat{n}^h_{c,t} + (1-\omega)\frac{N^h_{u,ss}}{N^h_{ss}}\hat{n}^h_{u,t} \\
\hat{p}^{CPI}_t &= \hat{p}^E_t\left(\omega\alpha_{c,ec} + (1-\omega)\alpha_{u,ec}\right) \\
\hat{z}_t &= \frac{C_{ss}}{Z_{ss}}\hat{c}_t + \frac{X_{ss}}{Z_{ss}}\hat{x}_t \\
\widehat{nfa}_t &= \frac{X_{ss}}{NFA_{ss}}\hat{x}_t - \hat{p}^E_t\left(\frac{E^z_{ss} + E^h_{ss}}{NFA_{ss}}\right) - \frac{E^z_{ss}}{NFA_{ss}}\hat{e}^z_t - \frac{E^h_{ss}}{NFA_{ss}}\hat{e}^h_t
\end{aligned}
$$

Thinking about C gap Combine

$$
\begin{aligned}
\hat{\gamma}_t &= \hat{c}_{u,t} - \hat{c}_{c,t} \\
\frac{\hat{c}_t - (1-\omega)\hat{c}_{u,t}}{\omega} &= \hat{c}_{c,t} \\
\hat{\gamma}_t &= \hat{c}_{u,t} - \frac{\hat{c}_t - (1-\omega)\hat{c}_{u,t}}{\omega} \\
\hat{c}_t &= \hat{c}_{u,t} - \omega\hat{\gamma}_t \quad \Leftrightarrow \quad \hat{c}_{u,t} = \hat{c}_t + \omega\hat{\gamma}_t
\end{aligned}
$$

E.4 Aggregate demand (AD) non-policy block

The household block can be combined into the non-policy aggregate demand block

$$\hat{w}_t^h = \varphi \hat{n}_{u,t}^h + \sigma \hat{c}_{u,t} + \hat{p}_t^E \alpha_{u,ec}(1-\sigma\psi_{ec}) \tag{E.1}$$

$$\begin{aligned}\hat{c}_t &= \mathbf{E}_t[\hat{c}_{t+1}] + \mathbf{E}_t[\omega \Delta \hat{\gamma}_{t+1}] - \frac{1}{\sigma}\left(\hat{r}_t - \mathbf{E}_t\left[\alpha_{u,ec}(\hat{p}_{t+1}^E - \hat{p}_t^E) + \hat{\pi}_{t+1}\right]\right) \\ &\quad + \psi_{ec}\alpha_{u,ec}\left(\hat{p}_t^E - \mathbf{E}_t \hat{p}_{t+1}^E\right)\end{aligned} \tag{E.2}$$

$$\hat{r}_t = \mathbf{E}_t \hat{q}_{t+1} - \hat{q} + \mathbf{E}_t \hat{\pi}_{t+1} \tag{E.3}$$

$$\begin{aligned}\hat{c}_{u,t} &= \frac{1}{C_{u,ss}} \frac{\left(b_{ss}^*(\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^*}{\Pi_{ss}^*}(\hat{b}_{t-1}^* + \hat{q}_t)\right)}{1-\omega} \\ &\quad + \frac{1}{C_{u,ss}} \frac{\left(Z_{ss}\hat{z}_t - w_{ss}N_{ss}(\hat{w}_t + \hat{n}_t) - E_{ss}^z(\hat{p}_t^E + \hat{e}_t^z) + X_{ss}(\hat{x}_t) - X_{ss}(\hat{x}_t)\right)}{1-\omega} \\ &\quad + w_{ss}\frac{N_{u,ss}^h}{C_{u,ss}}\left(\hat{w}_t + \hat{n}_{u,t}^h\right) - \frac{E_{u,ss}^h}{C_{u,ss}}\left(\hat{c}_{u,t} + \hat{p}_t^E(1-\psi_{ec})\right)\end{aligned} \tag{E.4}$$

$$\hat{w}_t^h = \varphi \hat{n}_{c,t}^h + \sigma \hat{c}_{c,t} + \alpha_{c,ec}\hat{p}_t^E(1-\sigma\psi_{ec}) \tag{E.5}$$

$$\hat{c}_{c,t} = w_{ss}\frac{N_{c,ss}^h}{C_{c,ss}}\left(\hat{w}_t + \hat{n}_{c,t}^h\right) - \frac{E_{c,ss}^h}{C_{c,ss}}\left(\hat{c}_{c,t} + \hat{p}_t^E(1-\psi_{ec})\right) \tag{E.6}$$

$$\hat{\gamma}_t = \hat{c}_{u,t} - \hat{c}_{c,t} \tag{E.7}$$

$$\hat{c}_t = \omega \hat{c}_{c,t} + (1-\omega)\hat{c}_{u,t}, \quad \hat{c}_{u,t} = \hat{c}_t + \omega\hat{\gamma}_t \tag{E.8}$$

$$\hat{e}_t^h = \omega \frac{E_{c,ss}^h}{E_{ss}^h}\hat{e}_{c,t}^h + (1-\omega)\frac{E_{u,ss}^h}{E_{ss}^h}\hat{e}_{u,t}^h \tag{E.9}$$

$$\hat{n}_t^h = \omega \frac{N_{c,ss}^h}{N_{ss}^h}\hat{n}_{c,t}^h + (1-\omega)\frac{N_{u,ss}^h}{N_{ss}^h}\hat{n}_{u,t}^h \tag{E.10}$$

$$\hat{p}_t^{CPI} = \hat{p}_t^E\left(\omega\alpha_{c,ec} + (1-\omega)\alpha_{u,ec}\right) \tag{E.11}$$

$$\hat{z}_t = \frac{C_{ss}}{Z_{ss}}\hat{c}_t + \frac{X_{ss}}{Z_{ss}}\hat{x}_t \tag{E.12}$$

$$\widehat{nfa}_t = \frac{X_{ss}}{NFA_{ss}}\hat{x}_t - \hat{p}_t^E\left(\frac{E_{ss}^z + E_{ss}^h}{NFA_{ss}}\right) - \frac{E_{ss}^z}{NFA_{ss}}\hat{e}_t^z - \frac{E_{ss}^h}{NFA_{ss}}\hat{e}_t^h \tag{E.13}$$

IS equations features the consumption gap

$$\hat{c}_t = \mathbf{E}_t[\hat{c}_{t+1}] + \mathbf{E}_t[\omega\Delta\hat{\gamma}_{t+1}] - \frac{1}{\sigma}\left(\hat{r}_t - \mathbf{E}_t[\hat{\pi}_{t+1}] - \alpha_{u,ec}(1-\sigma\psi_{ec})\mathbf{E}_t\left[\Delta\hat{p}_{t+1}^E\right]\right)$$

- if energy enters the unconstrained consumption basket, then $\alpha_{u,ec} > 0$
- if energy is close to a Leontief, hard to substitute good, ψ_{ec} is very low, $(1-\sigma\psi_{ec})$ is large, the effect of energy in the C basket is then aggravated

Show in a seperate step that the C gap is affected by energy, even if $\alpha_{u,ec} = 0$

$$\hat{\gamma}_t = \hat{c}_{u,t} - \hat{c}_{c,t}$$

$$\begin{aligned}
\hat{\gamma}_t = &\left(\frac{1}{C_{u,ss}} \frac{\left(b^*_{ss}(\hat{b}^*_t + \hat{q}_t) - \frac{\bar{R}^* b^*_{ss}}{\Pi^*_{ss}} (\hat{b}^*_{t-1} + \hat{q}_t) \right)}{1-\omega} \right. \\
&+ \frac{1}{C_{u,ss}} \frac{\left(Z_{ss}\hat{z}_t - w_{ss}N_{ss}(\hat{w}_t + \hat{n}_t) - E^z_{ss}(\hat{p}^E_t + \hat{e}^z_t) + X_{ss}(\hat{x}_t) - X_{ss}(\hat{x}_t) \right)}{1-\omega} \\
&\left. + w_{ss}\frac{N^h_{u,ss}}{C_{u,ss}} \left(\hat{w}_t + \hat{n}^h_{u,t} \right) - \frac{E^h_{u,ss}}{C_{u,ss}} \left(\hat{c}_{u,t} + \hat{p}^E_t(1-\psi_{ec}) \right) \right) \\
&- \left(w_{ss}\frac{N^h_{c,ss}}{C_{c,ss}} \left(\hat{w}_t + \hat{n}^h_{c,t} \right) - \frac{E^h_{c,ss}}{C_{c,ss}} \left(\hat{c}_{c,t} + \hat{p}^E_t(1-\psi_{ec}) \right) \right)
\end{aligned}$$

If $\alpha_{u,ec} = \alpha_{x,ec} = 0$, then $E^h_{c,ss} = E^h_{u,ss} = 0$

$$\hat{\gamma}_t = \hat{c}_{u,t} - \hat{c}_{c,t}$$

$$\begin{aligned}
\hat{\gamma}_t = &\left(\frac{1}{C_{u,ss}} \frac{\left(b^*_{ss}(\hat{b}^*_t + \hat{q}_t) - \frac{\bar{R}^* b^*_{ss}}{\Pi^*_{ss}} (\hat{b}^*_{t-1} + \hat{q}_t) \right)}{1-\omega} \right. \\
&+ \frac{1}{C_{u,ss}} \frac{\left(Z_{ss}\hat{z}_t - w_{ss}N_{ss}(\hat{w}_t + \hat{n}_t) - E^z_{ss}(\hat{p}^E_t + \hat{e}^z_t) + X_{ss}(\hat{x}_t) - X_{ss}(\hat{x}_t) \right)}{1-\omega} \\
&\left. + w_{ss}\frac{N^h_{u,ss}}{C_{u,ss}} \left(\hat{w}_t + \hat{n}^h_{u,t} \right) \right) - \left(w_{ss}\frac{N^h_{c,ss}}{C_{c,ss}} \left(\hat{w}_t + \hat{n}^h_{c,t} \right) \right)
\end{aligned}$$

Energy shocks matter even if $\alpha_{u,ec} = \alpha_{c,ec} = 0$ since they directly affect firm profits, and indirectly affect labour demand, and hence household labour income, which affects unconstrained and constrained households differently.

E.5 Reduce the Union and Firm loglinear system to get the AS block

Combine Union Equations to Wage Philips Curve

$$\begin{aligned}
&\hat{w}^h_t = \widehat{mrs}_t \quad \hat{\zeta}^W_t = \frac{1-\phi_W}{\phi_W}\left(\hat{f}^{w,1}_t - \hat{f}^{w,2}_t \right) \\
&\hat{f}^{W,1}_t = (1-\phi_w\beta)\left(\hat{n}_t + \widehat{mrs}_t - \hat{w}_t\right) + \phi_W\beta E_t\left[\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} + \hat{\pi}^W_{t+1} - \hat{\pi}^{CES_u}_{t+1} + \left(\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}\right)\hat{\zeta}^W_{t+1} + \hat{f}^{W,1}_{t+1} \right] \\
&\hat{f}^{W,2}_t = (1-\phi_W\beta)\hat{n}_t + \phi_W\beta\left(E_t\left[\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} + \hat{\pi}^W_{t+1} - \hat{\pi}^{CES_u}_{t+1} + \left(\frac{1}{\mathcal{M}_w - 1}\right)\hat{\zeta}^W_{t+1} + \hat{f}^{W,2}_{t+1} \right] \right) \\
&\hat{\zeta}^W_t = \hat{\pi}^W_t, \xi_w = 0, \ \hat{w}_t = \hat{\pi}^W_t - \hat{\pi}_t + \hat{w}_{t-1}
\end{aligned}$$

Next

$$\begin{aligned}
&\hat{f}^{W,1}_t - \hat{f}^{W,2}_t = (1-\phi_w\beta)\left(\widehat{mrs}_t - \hat{w}_t\right) + \phi_W\beta E_t\left[\hat{\zeta}^W_{t+1} + \hat{f}^{W,1}_{t+1} - \hat{f}^{W,2}_{t+1} \right] \\
&\frac{\phi_W}{1-\phi_W}\hat{\zeta}^W_t = \hat{f}^{w,1}_t - \hat{f}^{w,2}_t, \ \hat{\zeta}^W_{t+1} + \frac{\phi_W}{1-\phi_W}\hat{\zeta}^W_{t+1} = \hat{\zeta}^W_{t+1} + \hat{f}^{w,1}_{t+1} - \hat{f}^{w,2}_{t+1}, \ \frac{1}{1-\phi_W}\hat{\zeta}^W_{t+1} = \hat{\zeta}^W_{t+1} + \hat{f}^{w,1}_{t+1} - \hat{f}^{w,2}_{t+1} \\
&\frac{\phi_W}{1-\phi_W}\hat{\zeta}^W_t = (1-\phi_w\beta)\left(\widehat{mrs}_t - \hat{w}_t\right) + \phi_W\beta E_t\left[\frac{1}{1-\phi_W}\hat{\zeta}^W_{t+1} \right]
\end{aligned}$$

So we have the wage inflation system as follows

$$\hat{\pi}_t^W = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t \tag{E.14}$$

$$\hat{\pi}_t^W = \frac{(1-\phi_w\beta)(1-\phi_W)}{\phi_W}\left(\hat{w}_t^h - \hat{w}_t\right) + \beta E_t\left[\hat{\pi}_{t+1}^W\right] \tag{E.15}$$

Energy shocks affect the wage PC via $\hat{w}_t^h - \hat{w}_t$

Combine Z Firm Equations to Domestic Z Price Philips Curve

$$\hat{z}_t = \hat{\varepsilon}_t^{tfp} + (1-\alpha_{ez})\hat{n}_t + \alpha_{ez}\hat{e}_t^z \tag{E.16}$$

$$\hat{mc}_t^Z = \hat{w}_t + \hat{\tau}_t^Z - \frac{1}{\psi_{ez}}(\hat{z}_t - \hat{n}_t) - \left(\frac{\psi_{ez}-1}{\psi_{ez}}\right)\hat{\varepsilon}_t^{tfp} \tag{E.17}$$

$$\hat{mc}_t^Z = \hat{p}_t^E + \hat{\tau}_t^Z - \frac{1}{\psi_{ez}}(\hat{z}_t - \hat{e}_t^z) - \left(\frac{\psi_{ez}-1}{\psi_{ez}}\right)\hat{\varepsilon}_t^{tfp} \tag{E.18}$$

$$\hat{\pi}_t = \frac{(1-\phi_z\beta)(1-\phi_z)}{\phi_z}\left(\hat{mc}_t^Z\right) + \beta E_t\left[\hat{\pi}_{t+1}\right] \tag{E.19}$$

$$\hat{p}_t^E = \hat{p}_t^{E,*} + \hat{q}_t \tag{E.20}$$

$$\hat{p}_t^{E,*} = \rho_E\hat{p}_{t-1}^{E,*} + \hat{\varepsilon}_t^E \tag{E.21}$$

$$\hat{q}_t = -\hat{p}_t^{EXP}$$

Monetary Policy and World

$$\hat{r}_t = \theta_R\hat{r}_{t-1} + (1-\theta_R)\left(\theta_\Pi/4\hat{\pi}_t^{CPI,a} + \theta_Y(\hat{n}_t - \hat{n}_t^{flex})\right) \tag{E.22}$$

$$\hat{\pi}_t^{CPI,a} = \hat{\pi}_t^{CPI} + \hat{\pi}_{t-1}^{CPI} + \hat{\pi}_{t-2}^{CPI} + \hat{\pi}_{t-3}^{CPI} \tag{E.23}$$

$$\hat{x}_t = \varsigma^*\hat{q}_t \tag{E.24}$$

F Combine AD, AS, World and Policy Block

Household Demand

$$\hat{c}_t = \mathbf{E}_t[\hat{c}_{t+1}] + \mathbf{E}_t[\omega\Delta\hat{\gamma}_{t+1}] - \frac{1}{\sigma}\left(\hat{r}_t - \mathbf{E}_t\left[\hat{\pi}_{t+1}\right] - \alpha_{u,ec}(1-\sigma\psi_{ec})\mathbf{E}_t\left[\Delta\hat{p}^E_{t+1}\right]\right) \tag{F.1}$$

$$\begin{aligned}\hat{c}_{u,t} = {} & \frac{N^h_{u,ss}}{E^h_{u,ss}+C_{u,ss}}\left(\hat{w}_t + \hat{n}^h_{u,t}\right) - \alpha_{u,ec}\left(1-\psi_{ec}\right)\hat{p}^E_t + \frac{\left(b^*_{ss}(\hat{b}^*_t + \hat{q}_t) - \frac{\bar{R}^* b^*_{ss}}{\Pi^*_{ss}}(\hat{b}^*_{t-1} + \hat{q}_t)\right)}{\left(E^h_{u,ss}+C_{u,ss}\right)(1-\omega)} \\ & + \frac{\left(Z_{ss}\left(\hat{\varepsilon}^{tfp}_t + (1-\alpha_{ez})\hat{n}_t + \alpha_{ez}\hat{e}^z_t\right) - (\hat{w}_t + \hat{n}_t) - E^z_{ss}(\hat{p}^E_t + \hat{e}^z_t) + X_{ss}(\hat{x}_t) - X_{ss}(\hat{x}_t)\right)}{\left(E^h_{u,ss}+C_{u,ss}\right)(1-\omega)}\end{aligned} \tag{F.2}$$

$$\hat{c}_{c,t} = \frac{N^h_{c,ss}}{C_{c,ss}+E^h_{c,ss}}\left(\hat{w}_t + \hat{n}_{c,t}\right) - \alpha_{c,ec}\left(1-\psi_{ec}\right)\hat{p}^E_t, \quad \frac{E^h_{c,ss}}{E^h_{c,ss}+C_{c,ss}} = \alpha_{c,ec} \tag{F.3}$$

$$\hat{\gamma}_t = \hat{c}_{u,t} - \hat{c}_{c,t} \tag{F.4}$$

$$\hat{c}_t = \omega\hat{c}_{c,t} + (1-\omega)\hat{c}_{u,t}, \quad \hat{c}_{u,t} = \hat{c}_t + \omega\hat{\gamma}_t \tag{F.5}$$

Household Labour Supply

$$\hat{n}_{u,t} = \varphi^{-1}\hat{w}^h_t - \varphi^{-1}\sigma\hat{c}_{u,t} - \varphi^{-1}\alpha_{u,ec}(1-\sigma\psi_{ec})\hat{p}^E_t \tag{F.6}$$

$$\hat{n}_{c,t} = \varphi^{-1}\hat{w}^h_t - \varphi^{-1}\sigma\hat{c}_{c,t} - \varphi^{-1}\alpha_{c,ec}(1-\sigma\psi_{ec})\hat{p}^E_t \tag{F.7}$$

$$\hat{n}_t = \omega N^h_{c,ss}\hat{n}_{c,t} + (1-\omega)N^h_{u,ss}\hat{n}_{u,t} \tag{F.8}$$

Market Clearing

$$\hat{c}_t = \frac{Z_{ss}}{C_{ss}}\left(\hat{\varepsilon}^{tfp}_t + (1-\alpha_{ez})\hat{n}_t + \alpha_{ez}\hat{e}^z_t\right) - \frac{Z_{ss}}{X_{ss}}\hat{x}_t \tag{F.9}$$

Wage and Price Setting

$$\hat{\pi}^W_t = \frac{(1-\phi_W\beta)(1-\phi_W)}{\phi_W}\left(\hat{w}^h_t - \hat{w}_t\right) + \beta E_t\left[\hat{\pi}^W_{t+1}\right], \quad \hat{\pi}^W_t = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t \tag{F.10}$$

$$\hat{\pi}_t = \frac{(1-\phi_z\beta)(1-\phi_z)}{\phi_z}\left(\hat{mc}^Z_t\right) + \beta E_t\left[\hat{\pi}_{t+1}\right] \tag{F.11}$$

$$\hat{mc}^Z_t = \hat{w}_t + \hat{\tau}^Z_t - \frac{1}{\psi_{ez}}(\alpha_{ez}\hat{e}^z_t - \alpha_{ez}\hat{n}_t) - \hat{\varepsilon}^{tfp}_t \tag{F.12}$$

$$\hat{mc}^Z_t = \hat{p}^E_t + \hat{\tau}^Z_t - \frac{1}{\psi_{ez}}((1-\alpha_{ez})\hat{n}_t - (1-\alpha_{ez})\hat{e}^z_t) - \hat{\varepsilon}^{tfp}_t \tag{F.13}$$

World, Exports, Exchange Rate, NFA

$$\hat{x}_t = \varsigma^*\hat{q}_t \tag{F.14}$$

$$\hat{r}_t = \mathbf{E}_t\hat{q}_{t+1} - \hat{q} + \mathbf{E}_t\hat{\pi}_{t+1} \tag{F.15}$$

$$\widehat{nfa}_t = \frac{X_{ss}}{NFA_{ss}}\hat{x}_t - \hat{p}^E_t\left(\frac{E^z_{ss}+E^h_{ss}}{NFA_{ss}}\right) - \frac{E^z_{ss}}{NFA_{ss}}\hat{e}^z_t - \left(\omega\frac{E^h_{c,ss}}{NFA_{ss}}\hat{e}^h_{c,t} + (1-\omega)\frac{E^h_{u,ss}}{NFA_{ss}}\hat{e}^h_{u,t}\right) \tag{F.16}$$

$$\hat{p}^E_t = \hat{p}^{E,*}_t + \hat{q}_t \tag{F.17}$$

$$\hat{p}^{E,*}_t = \rho_E\hat{p}^{E,*}_{t-1} + \hat{\varepsilon}^E_t \tag{F.18}$$

Monetary Policy

$$\hat{r}_t = \theta_R \hat{r}_{t-1} + (1-\theta_R)\left(\frac{\theta_\Pi \hat{\pi}_t^{CPI,a}}{4} + \theta_Y(\hat{n}_t - \hat{n}_t^{flex})\right), \qquad \hat{\pi}_t^{CPI,a} \equiv \sum_{j=0}^{3} \hat{\pi}_{t-j}^{CPI} \tag{F.19}$$

$$\hat{\pi}_t^{CPI} = \hat{\pi}_t + \Delta \hat{p}_t^E \left(\omega \alpha_{c,ec} + (1-\omega)\alpha_{u,ec}\right) \tag{F.20}$$

Shocks

$$\hat{\varepsilon}_t^{TFP} = \rho_{TFP} \hat{\varepsilon}_{t-1}^{TFP} - \varsigma_{TFP} \eta_t^{TFP} \tag{F.21}$$

$$\hat{\varepsilon}_t^{\mathcal{M}_z} = \rho_{\mathcal{M}_z} \hat{\varepsilon}_{t-1}^{\mathcal{M}_z} - \varsigma_{\mathcal{M}_z} \eta_t^{\mathcal{M}_z} \tag{F.22}$$

$$\hat{\varepsilon}_t^{E} = \varsigma_E \eta_t^E \tag{F.23}$$

G Calibration

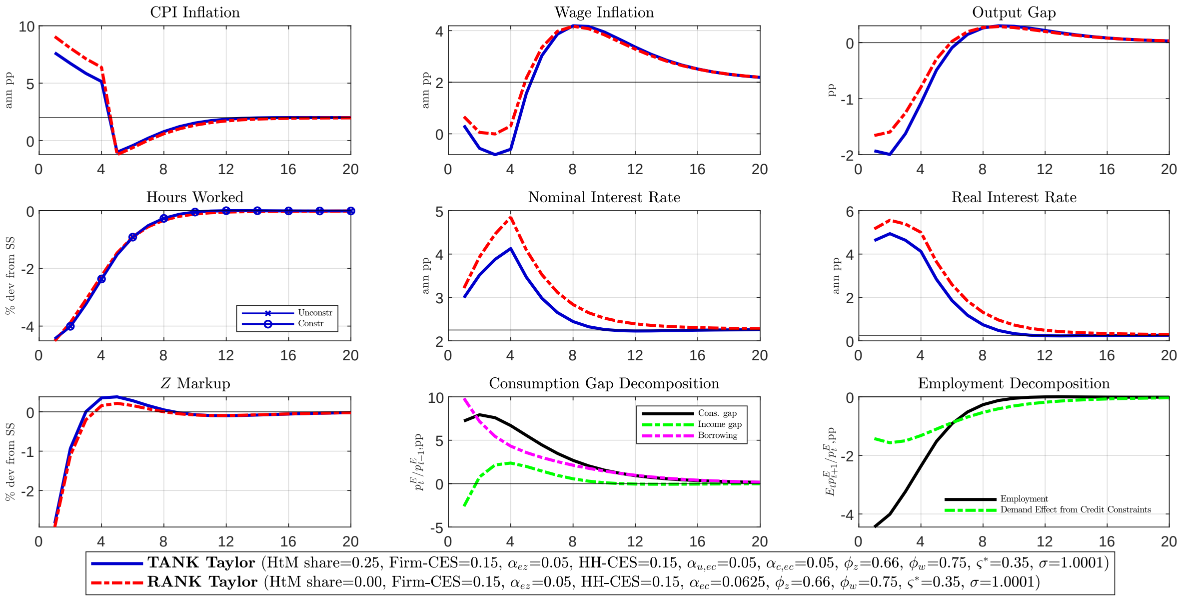
Table 1 presents the calibration of our baseline model.

TABLE 1: PARAMETER VALUES

Parameter	Definition	Value	Source/Target
Households			
β	Household Discount Factor	0.9994	Annual net nominal rate $r_{ss} \approx 2.25\%$
σ	Household Risk Aversion	2.0000	Literature
χ	Utility Weight of Labour	1.4102	$L_{ss} = 1$
φ	Inverse Frisch Elasticity	2	Literature
ω	Share of constrained Households	0.2500	Literature
ϑ_c	Domestic Debt adjustment cost for constrained HH	∞	Hand-to-mouth
$\alpha_{u,ec}$	Energy share in consumption, unconstrained	0.05	5 % energy share in production
$\alpha_{c,ec}$	Energy share in consumption, constrained	0.10	10 % energy share in production
ψ_{ec}	CES Degree between energy and non-energy in consumption	0.15	UK estimates
Labour Unions			
ϵ_w	Elasticity of substitution for labour	11.0000	10 % gross wage markup
ϕ_w	Calvo Wage Adjustment	0.7500	Avg lifetime of wages 4Q
Firms			
α_{ez}	Energy share in production	0.05	5 % energy share in production
ψ_{ez}	CES Degree between energy and labour in production	0.3	UK estimates
ϵ_z	Elasticity of substitution for goods	11.0000	10 % gross final goods markup
ϕ_z	Calvo Price Adjustment	0.6600	Avg lifetime of prices 3Q
Monetary Policy			
θ_Π	Interest Rate Sensitivity to Inflation	1.5000	Literature
θ_Y	Interest Rate Sensitivity to Output	0.1250	Literature
θ_R	Interest Rate Smoothing	0.9000	Literature
$\bar{\Pi}$	Inflation Target	1.0050	2% Target
World Trade			
κ^*	Foreign preference for domestic exports	0.2632	export share X_{ss}/Z_{ss}=0.25
ς^*	Price elasticity of world demand for domestic exports	0.35	COMPASS
Shock Processes			
ρ_{TFP}	Persistence of TFP Shock	0.93	Fernald 2014
$\rho_{\mathcal{M}_z}$	Persistence of Price Markup Shock	0.9	
ρ_E	Persistence of Global Energy Price Shock	0.8	Fall of energy price by 50% after 4Q
ς_{TFP}	Stdev of TFP Shock	0.007	Fernald 2014
$\varsigma_{\mathcal{M}_z}$	Stdev of Price Markup Shock	0.01	
ς_E	Stdev of Global Energy Price Shock	0.1	10 stdev shock leads to 100% increase on impact

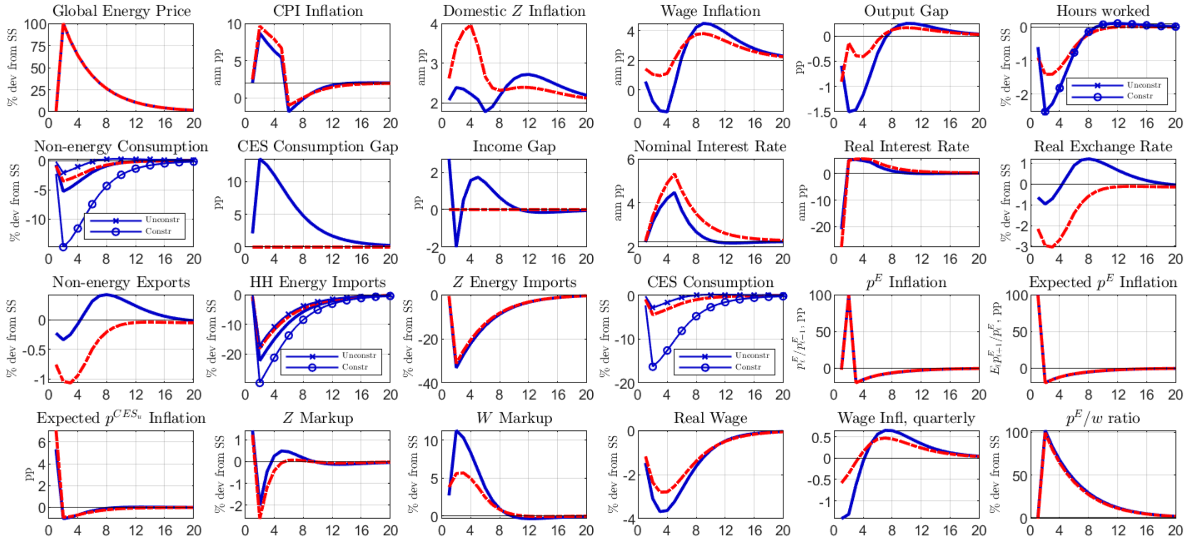
H Impulse Response Functions under Baseline Calibration

FIGURE H.1: Dynamic Responses to a Global Energy Price Shock: Taylor and Ramsey policy for TANK vs RANK

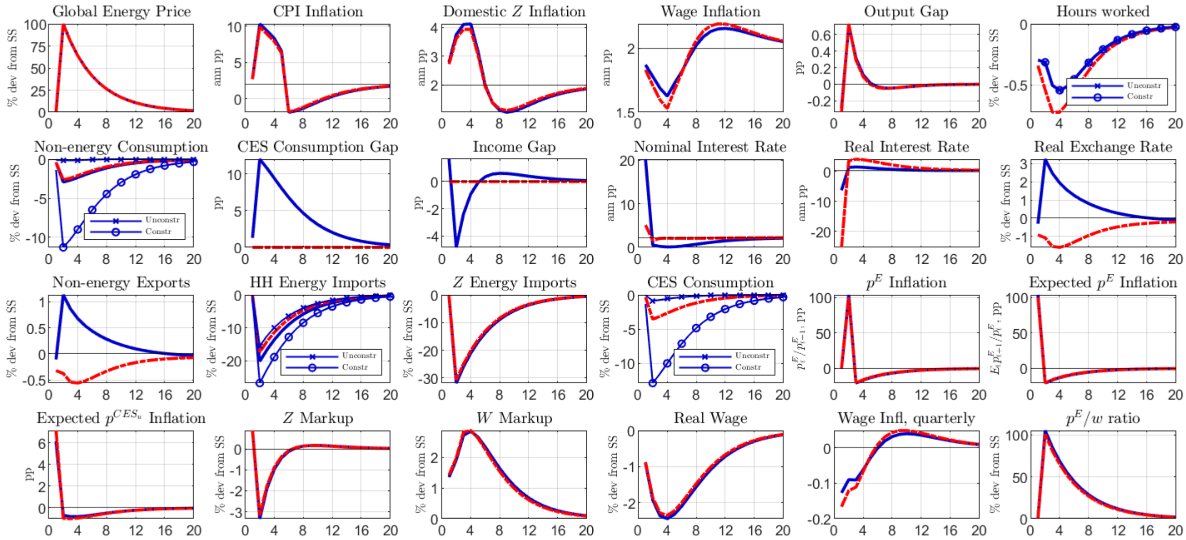


H.1 Energy News Shock

FIGURE H.2: Dynamic Responses to a Global Energy Price News Shock: Taylor and Ramsey policy for TANK vs RANK



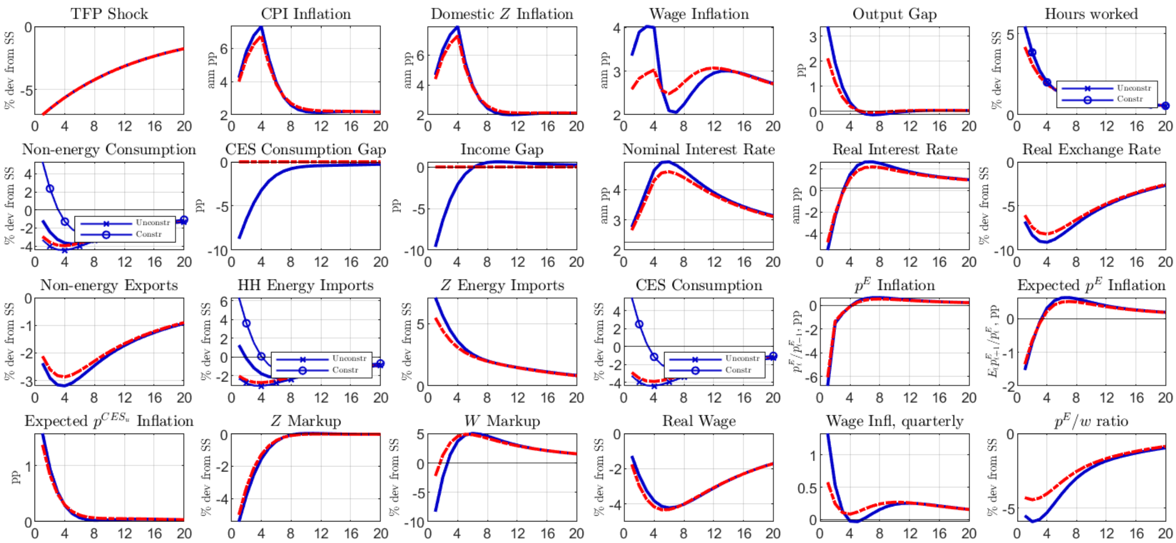
TANK Taylor (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)
RANK Taylor (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)



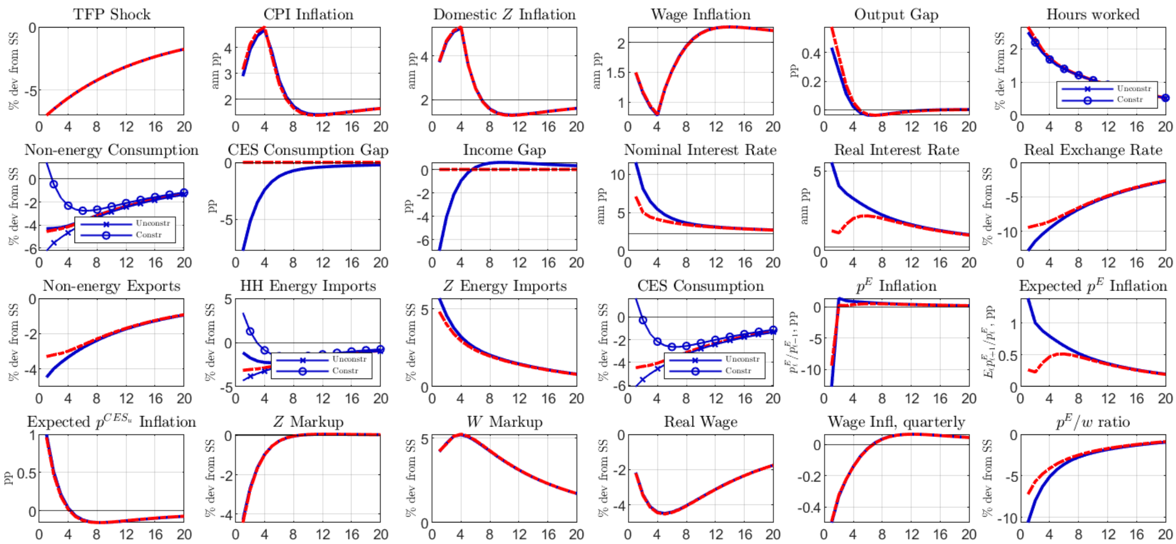
TANK Ramsey (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)
RANK Ramsey (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)

H.2 TFP Shock

FIGURE H.3: Dynamic Responses to a TFP Shock: Taylor and Ramsey policy for TANK vs RANK



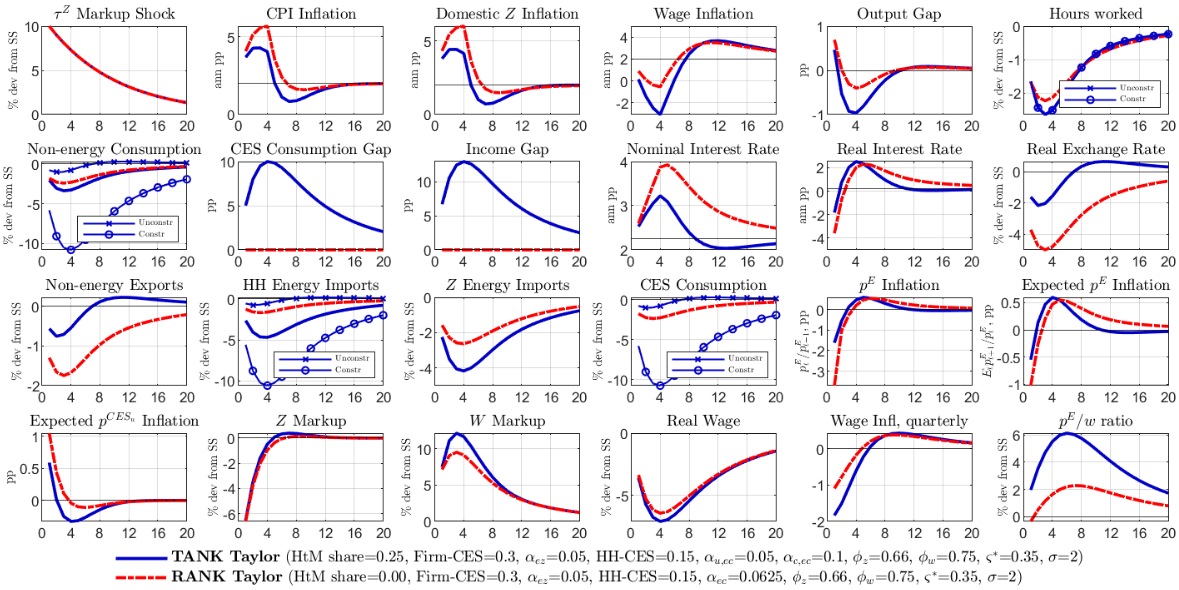
TANK Taylor (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)
RANK Taylor (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)

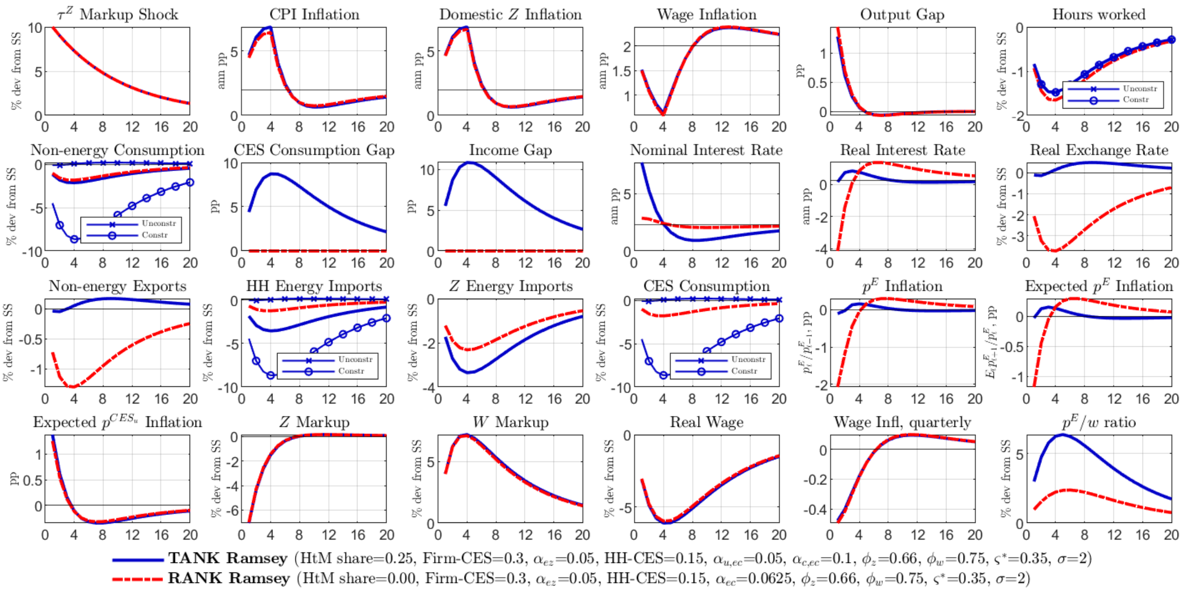


TANK Ramsey (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)
RANK Ramsey (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)

H.3 Markup Shock

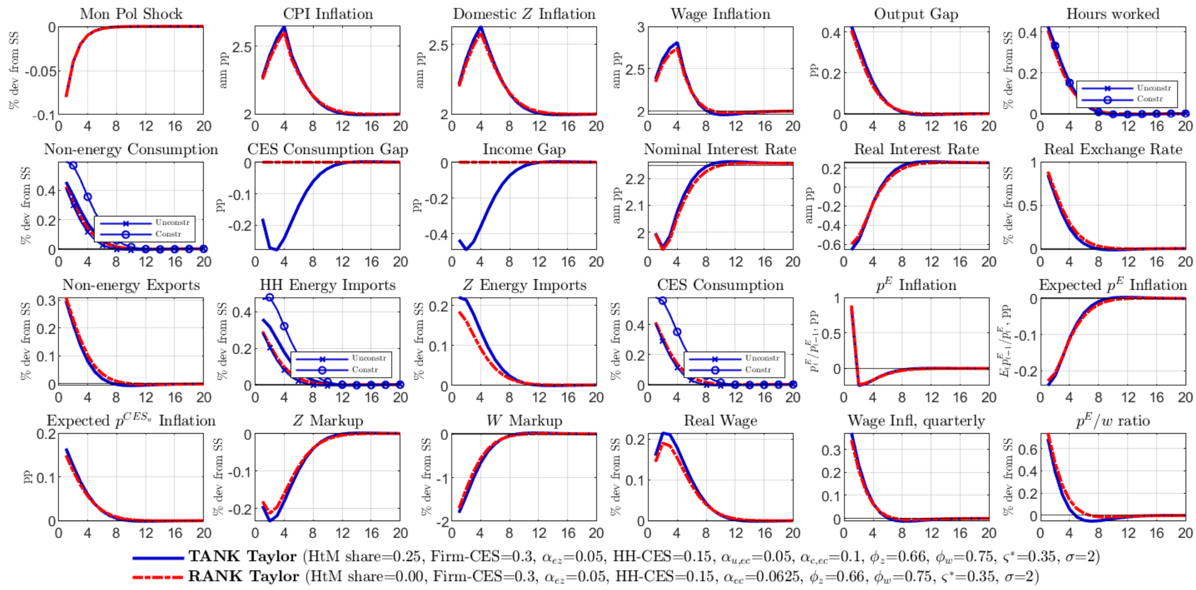
FIGURE H.4: Dynamic Responses to a Markup Shock: Taylor and Ramsey policy for TANK vs RANK





H.4 Monetary Policy Shock

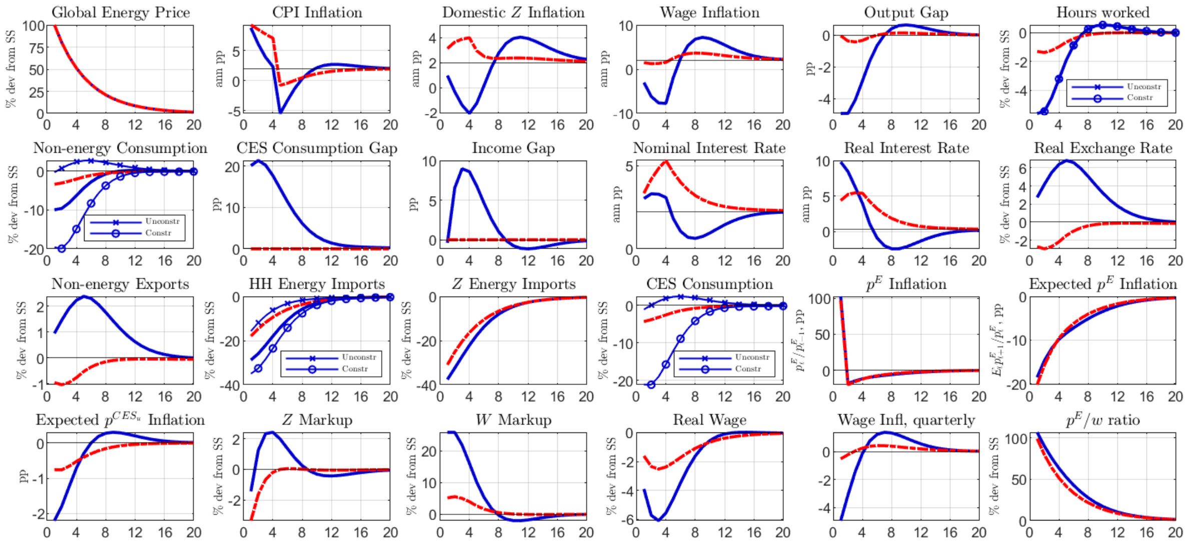
FIGURE H.5: Dynamic Responses to a Monetary Policy Shock: Taylor and Ramsey policy for TANK vs RANK



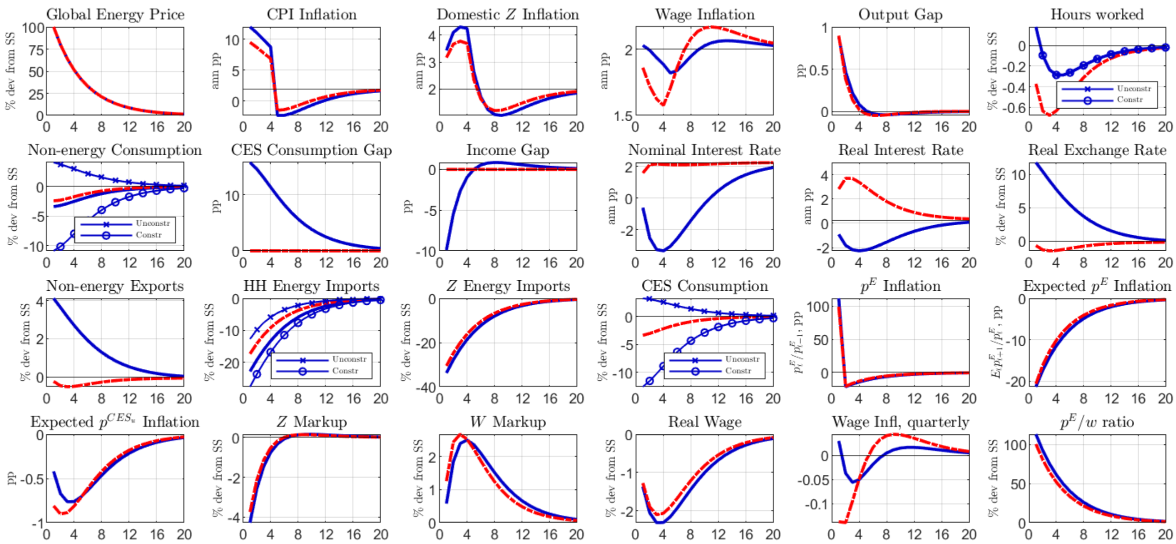
I Impulse Response Functions under Alternative Calibrations

I.1 Energy Shock - Higher Share of Constrained Agents

FIGURE I.6: Dynamic Responses to a Global Energy Price Shock: Higher Share of Constrained Agents



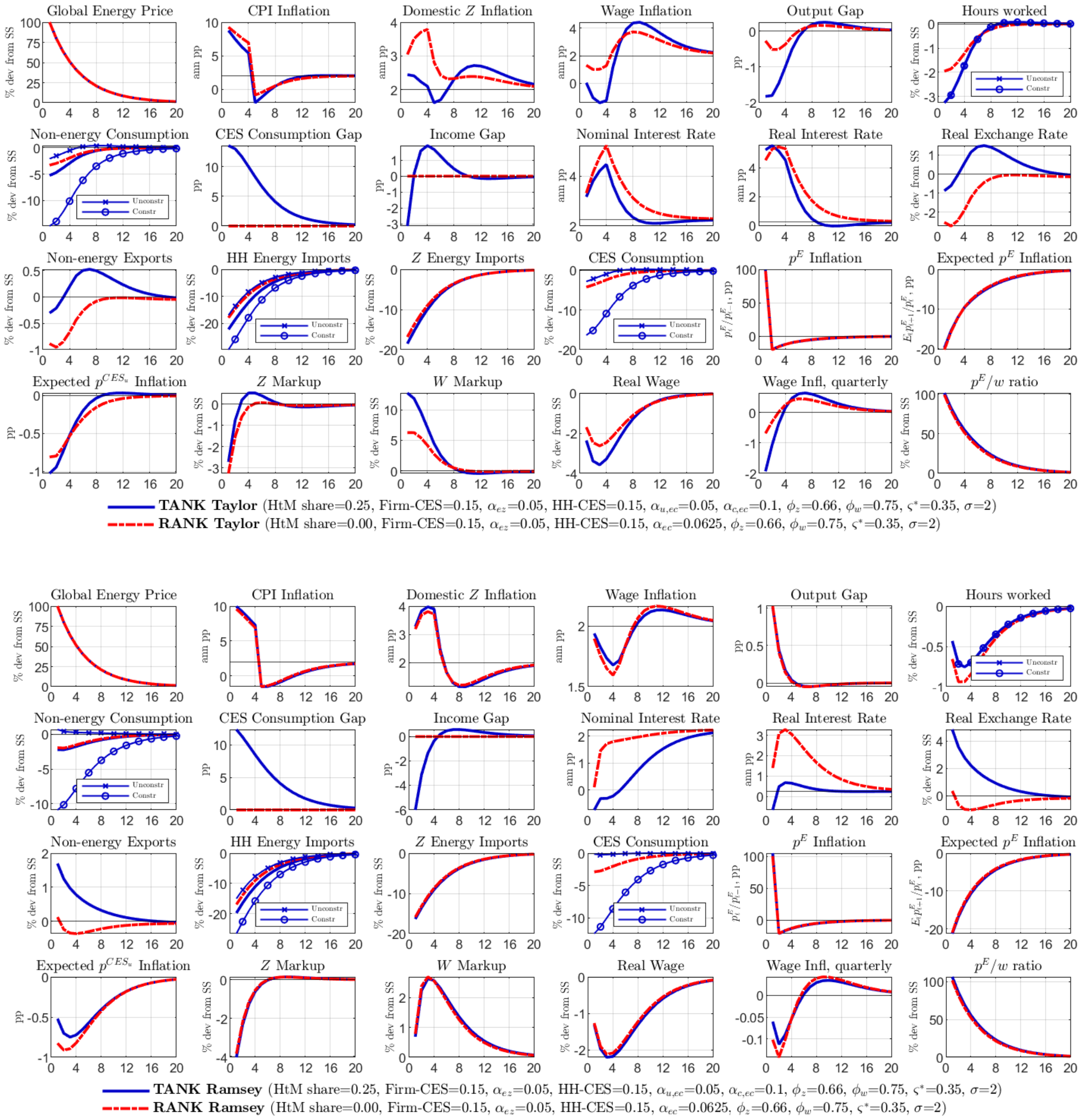
TANK Taylor (HtM share=0.5, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)
RANK Taylor (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)



TANK Ramsey (HtM share=0.5, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)
RANK Ramsey (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)

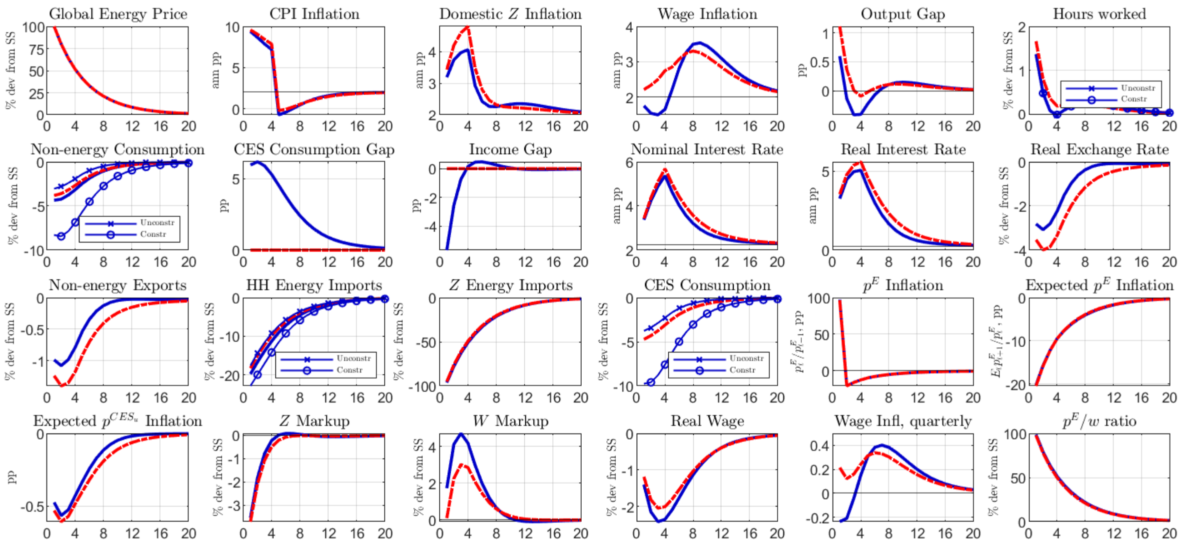
I.2 Energy Shock - Similar Substitutability of Energy in Production and Consumption

FIGURE I.7: Dynamic Responses to a Global Energy Price Shock: Similar Substitutability of Energy in Production and Consumption

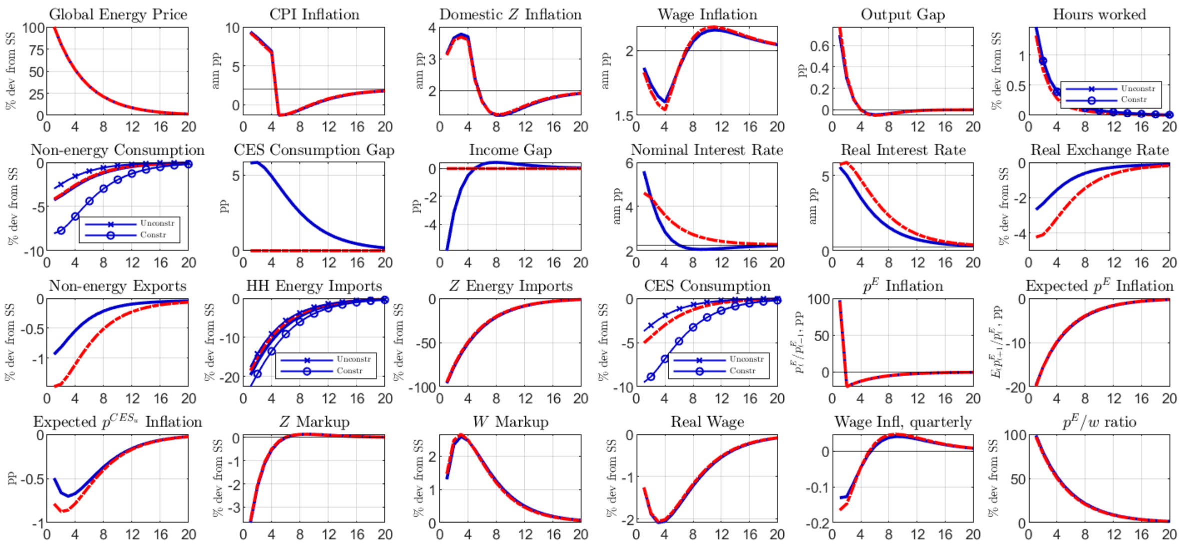


I.3 Energy Shock - Higher Substitutability of Energy in Production

FIGURE I.8: Dynamic Responses to a Global Energy Price Shock: Higher Substitutability of Energy in Production



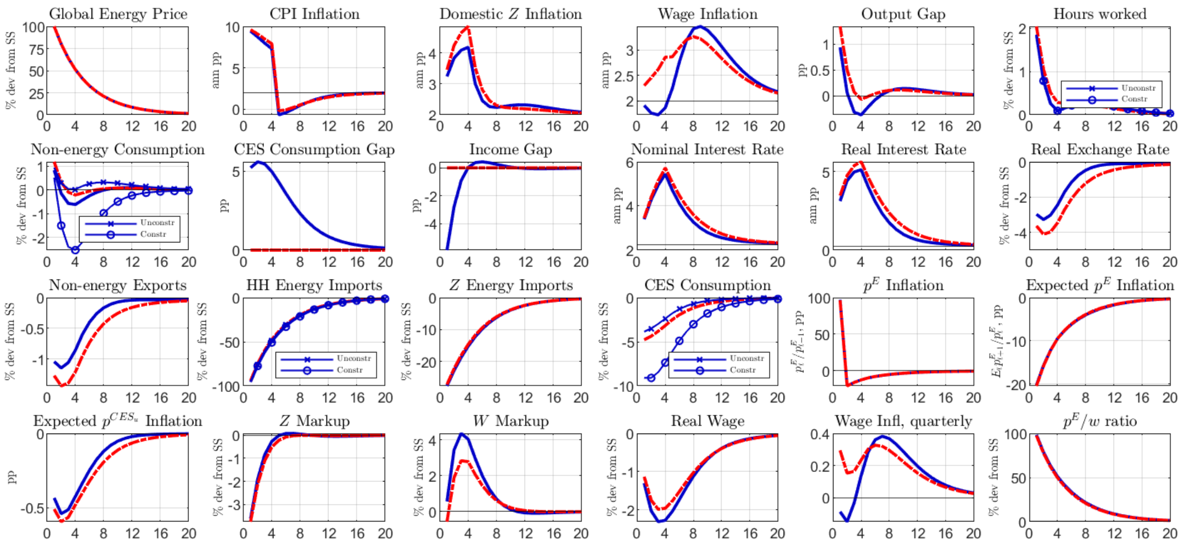
TANK Taylor (HtM share=0.25, Firm-CES=0.99, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)
RANK Taylor (HtM share=0.00, Firm-CES=0.99, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)



TANK Ramsey (HtM share=0.25, Firm-CES=0.99, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)
RANK Ramsey (HtM share=0.00, Firm-CES=0.99, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)

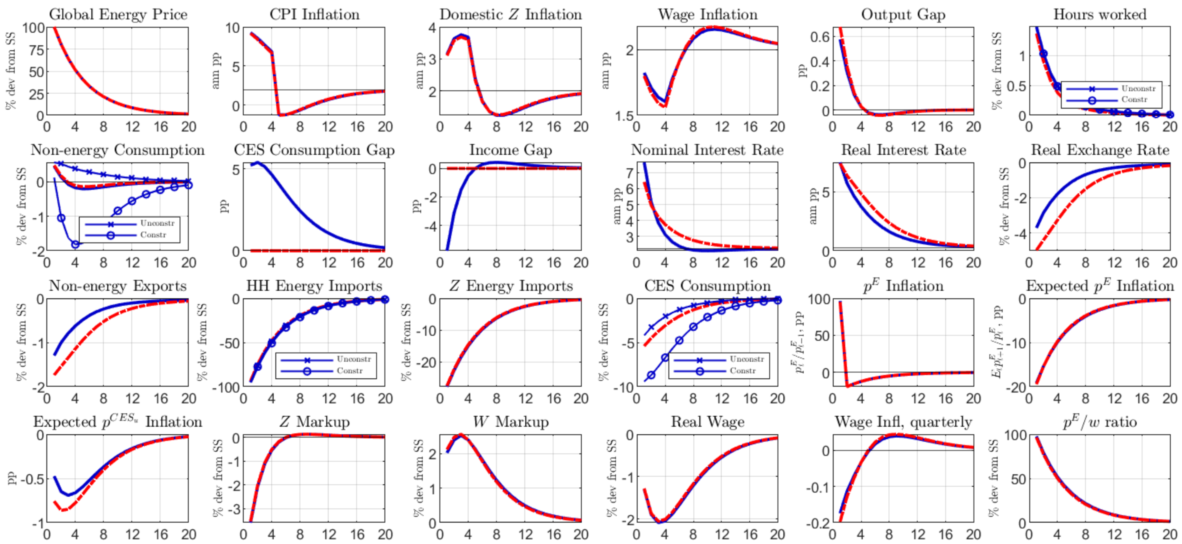
I.4 Energy Shock - Higher Substitutability of Energy in Consumption

FIGURE I.9: Dynamic Responses to a Global Energy Price Shock: Higher Substitutability of Energy in Consumption



TANK Taylor (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.99, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)

RANK Taylor (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.99, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)

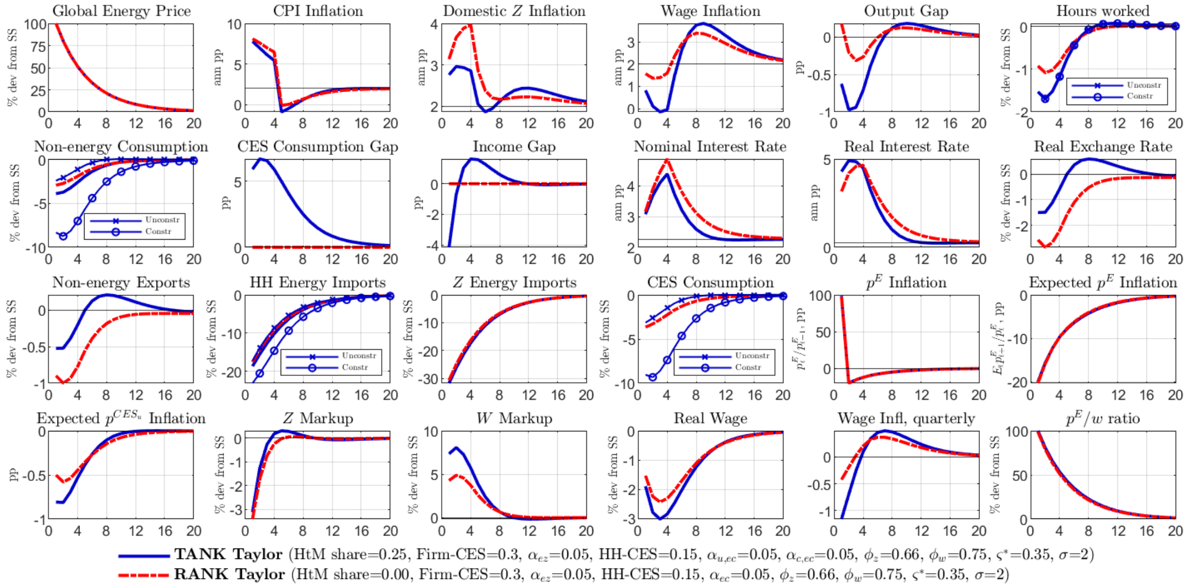


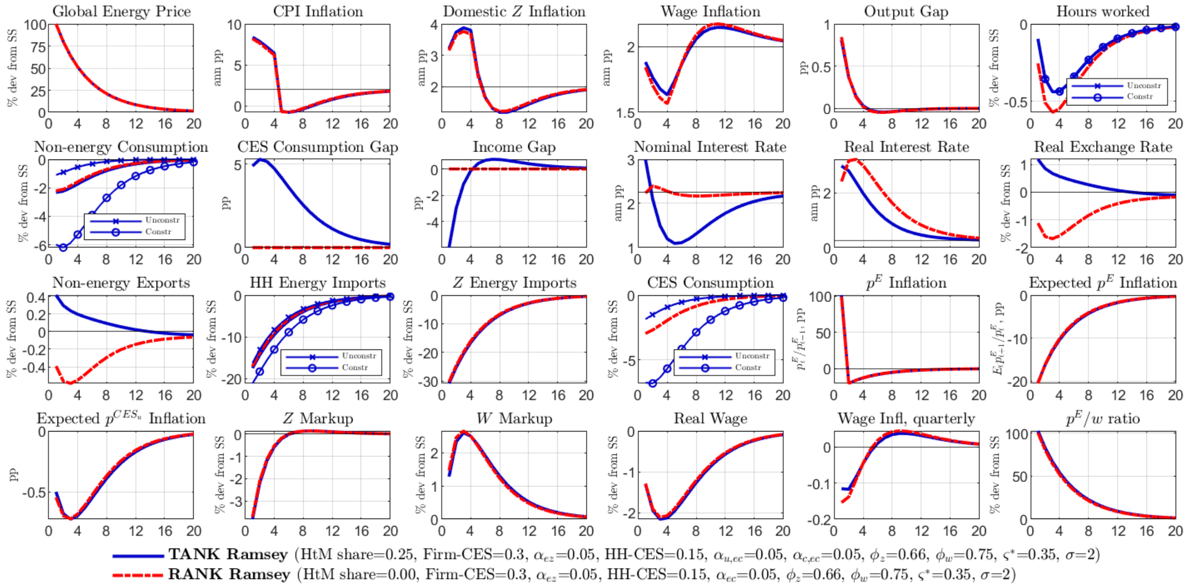
TANK Ramsey (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.99, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)

RANK Ramsey (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.99, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)

I.5 Energy Shock - Similar Share of Energy in Production and Consumption

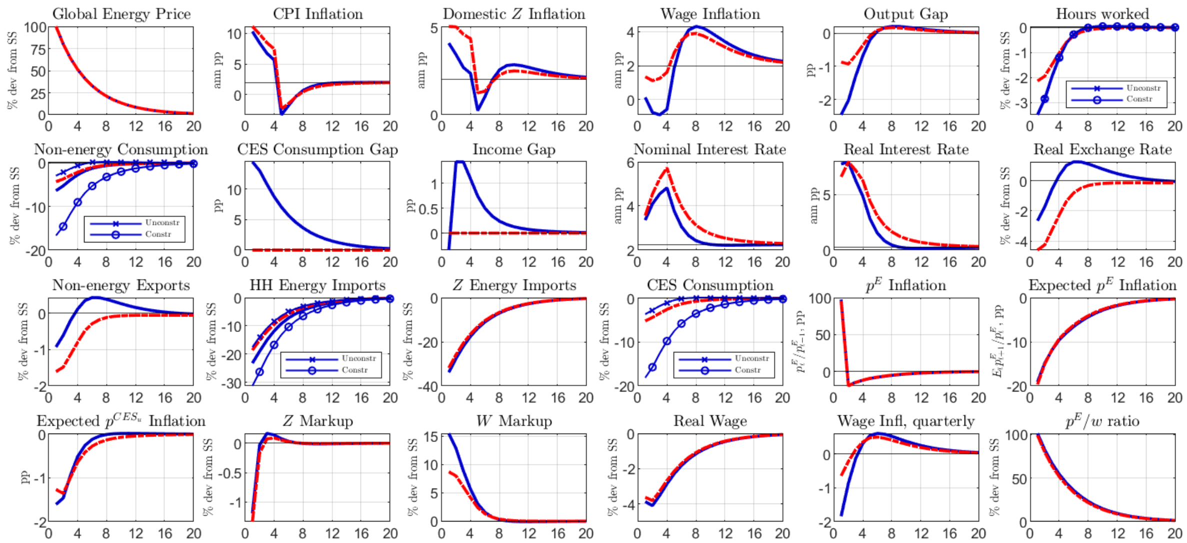
FIGURE I.10: Dynamic Responses to a Global Energy Price Shock: Similar Share of Energy in Production and Consumption



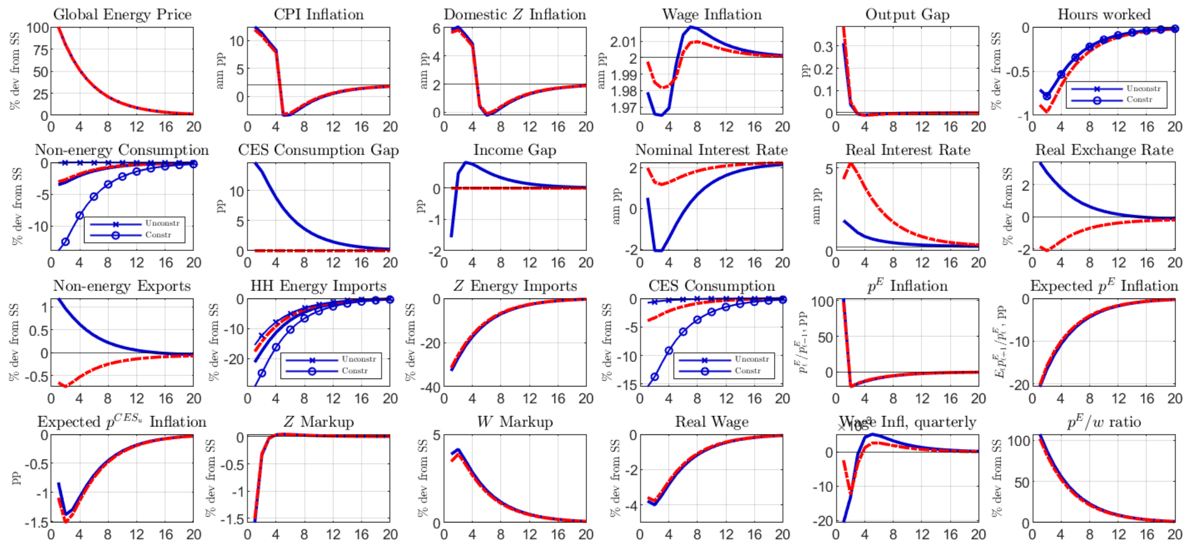


I.6 Energy Shock - Prices More Flexible

FIGURE I.11: Dynamic Responses to a Global Energy Price Shock: Prices More Flexible



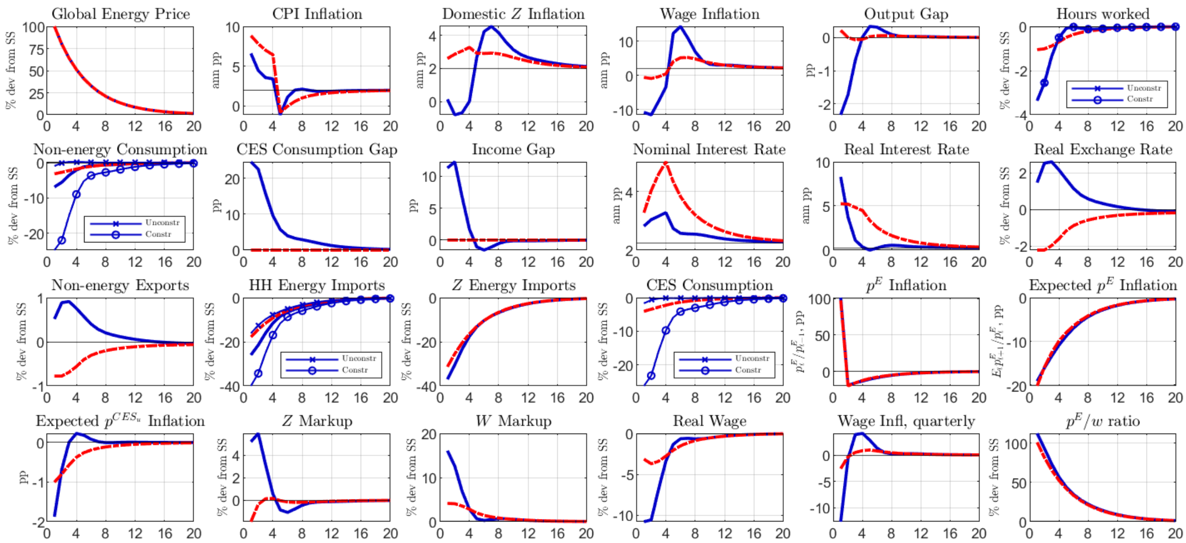
TANK Taylor (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.25, ϕ_w=0.75, ς^*=0.35, σ=2)
RANK Taylor (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.25, ϕ_w=0.75, ς^*=0.35, σ=2)



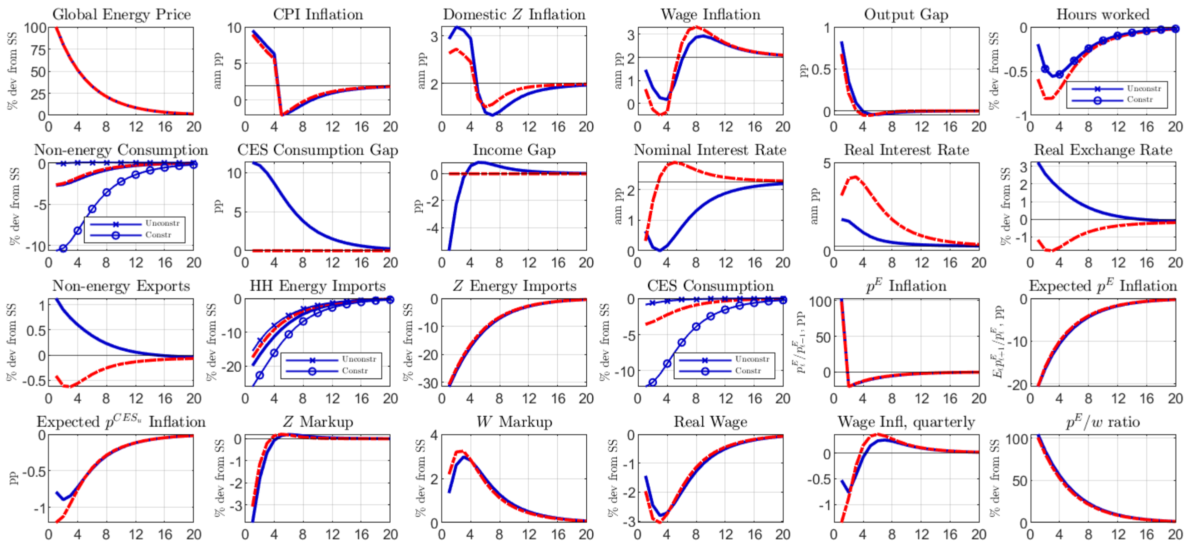
TANK Ramsey (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.25, ϕ_w=0.75, ς^*=0.35, σ=2)
RANK Ramsey (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.25, ϕ_w=0.75, ς^*=0.35, σ=2)

I.7 Energy Shock - Wages More Flexible

FIGURE I.12: Dynamic Responses to a Global Energy Price Shock: Wages More Flexible



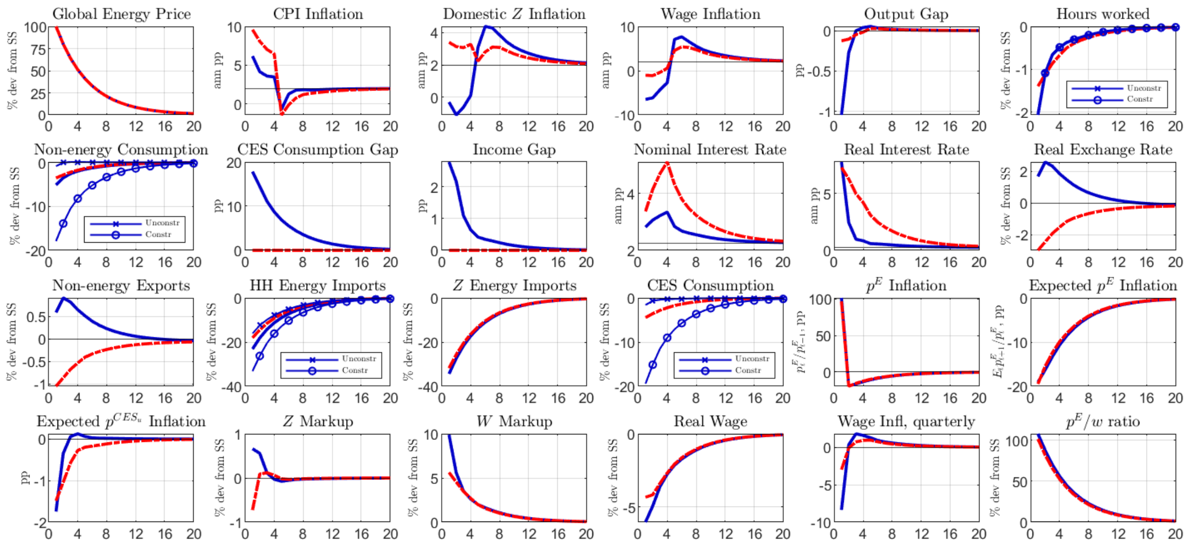
TANK Taylor (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.25, ς^*=0.35, σ=2)
RANK Taylor (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.25, ς^*=0.35, σ=2)



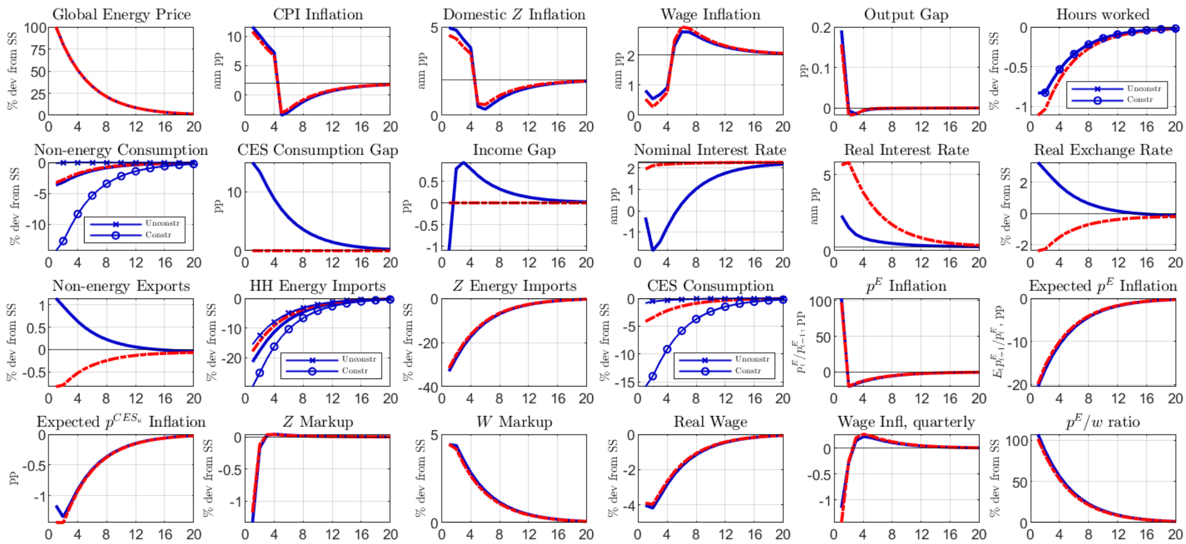
TANK Ramsey (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.25, ς^*=0.35, σ=2)
RANK Ramsey (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.25, ς^*=0.35, σ=2)

I.8 Energy Shock - Prices and Wages More Flexible

FIGURE I.13: Dynamic Responses to a Global Energy Price Shock: Prices and Wages More Flexible



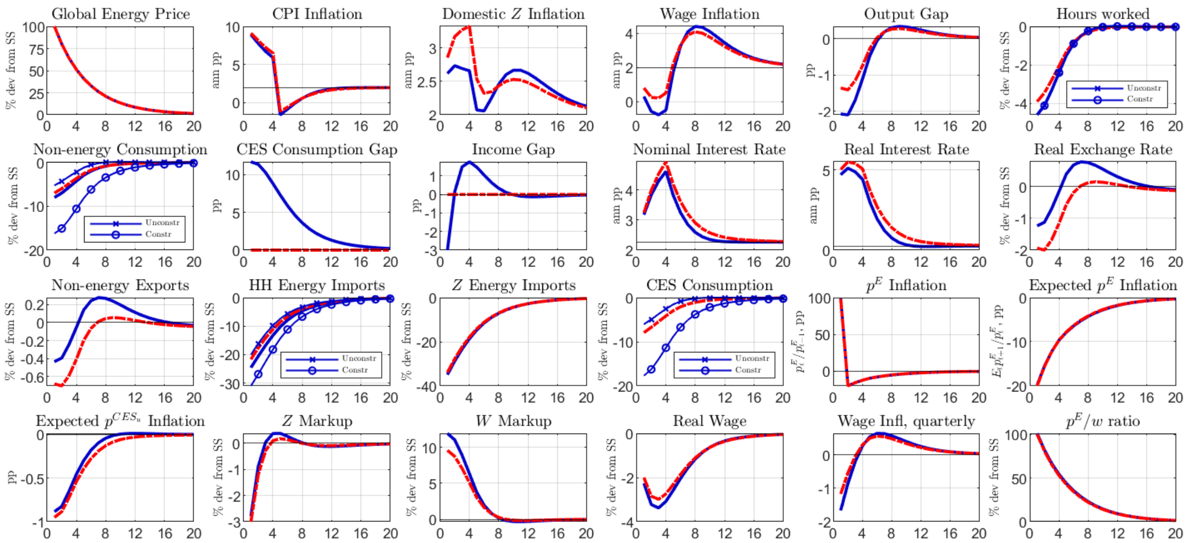
TANK Taylor (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.25, ϕ_w=0.25, ς^*=0.35, σ=2)
RANK Taylor (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.25, ϕ_w=0.25, ς^*=0.35, σ=2)



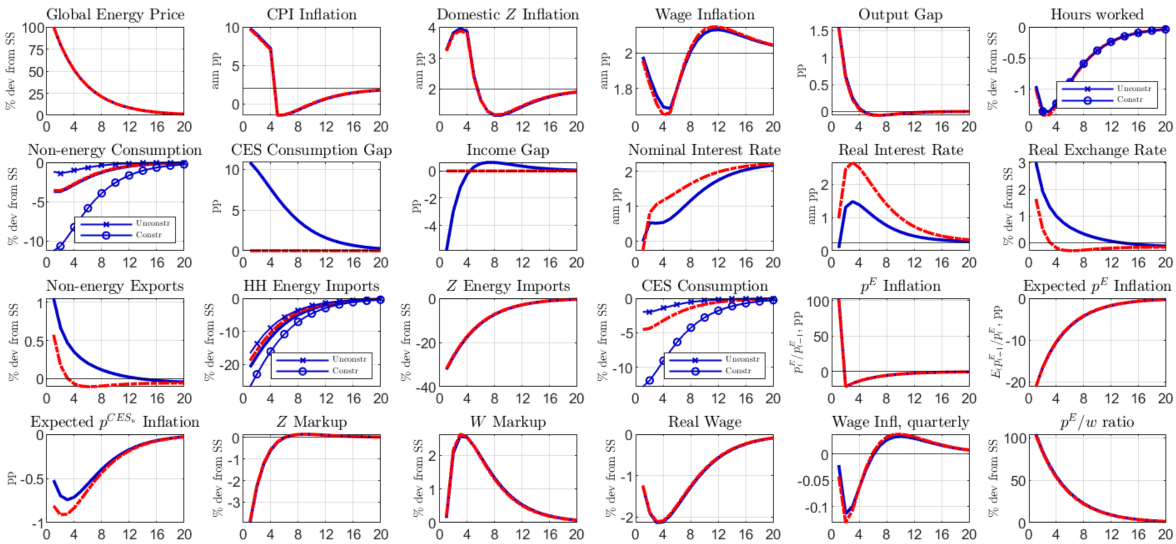
TANK Ramsey (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.25, ϕ_w=0.25, ς^*=0.35, σ=2)
RANK Ramsey (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.25, ϕ_w=0.25, ς^*=0.35, σ=2)

I.9 Energy Shock - Risk Aversion Parameter equals 1

FIGURE I.14: Dynamic Responses to a Global Energy Price Shock: Risk Aversion Parameter equals 1



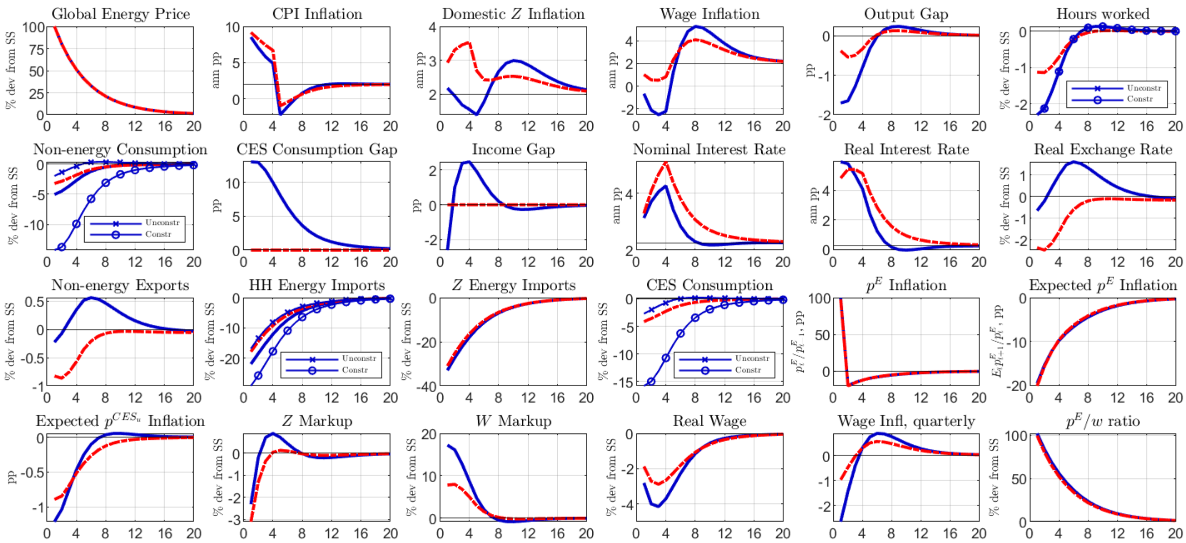
TANK Taylor (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=1.0001)
RANK Taylor (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=1.0001)



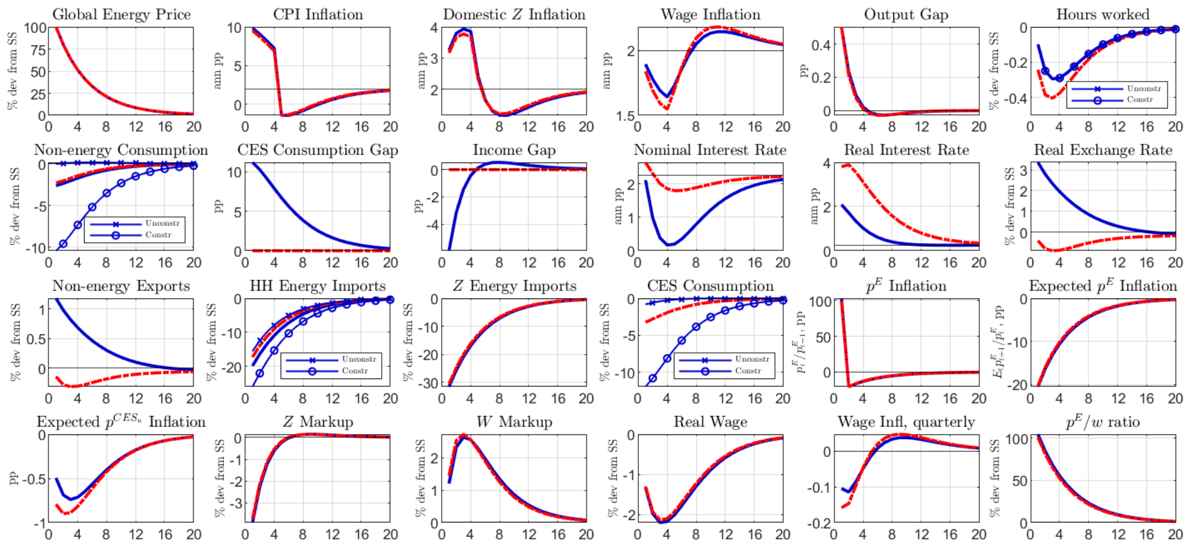
TANK Ramsey (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=1.0001)
RANK Ramsey (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=1.0001)

I.10 Energy Shock - Inverse Frisch Parameter equals 5

FIGURE I.15: Dynamic Responses to a Global Energy Price Shock: Inverse Frisch Parameter equals 5



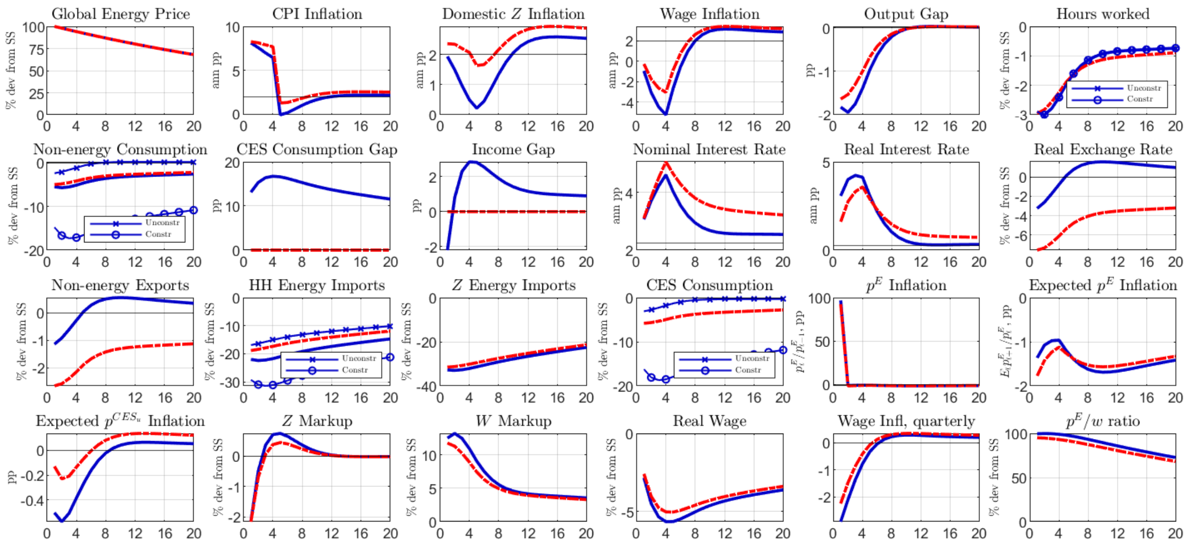
TANK Taylor (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)
RANK Taylor (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)



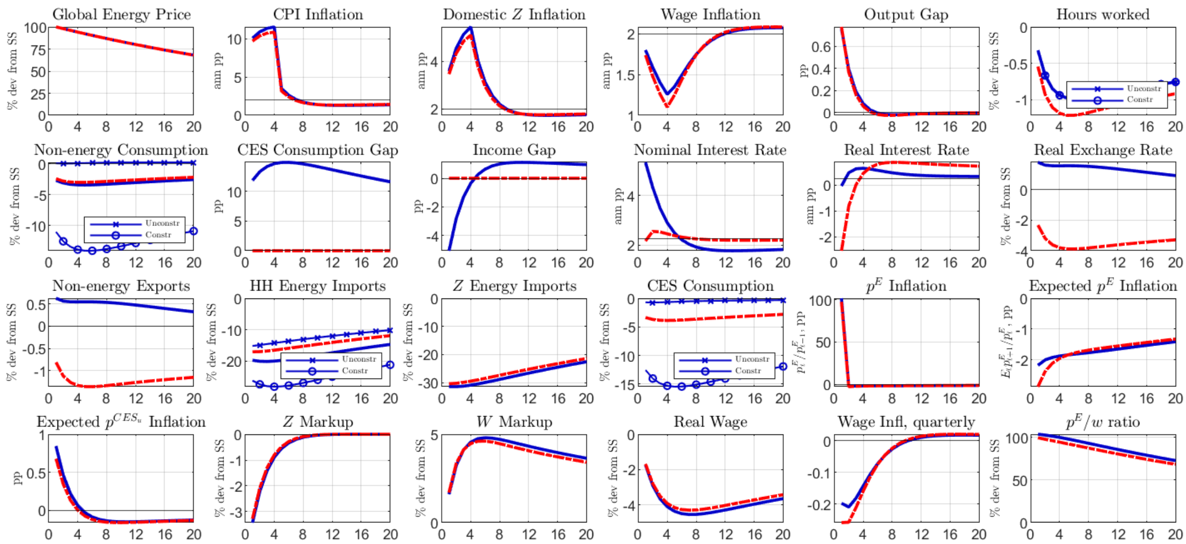
TANK Ramsey (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)
RANK Ramsey (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)

I.11 Energy Shock - Higher Persistence

FIGURE I.16: Dynamic Responses to a Global Energy Price Shock: Higher Persistence



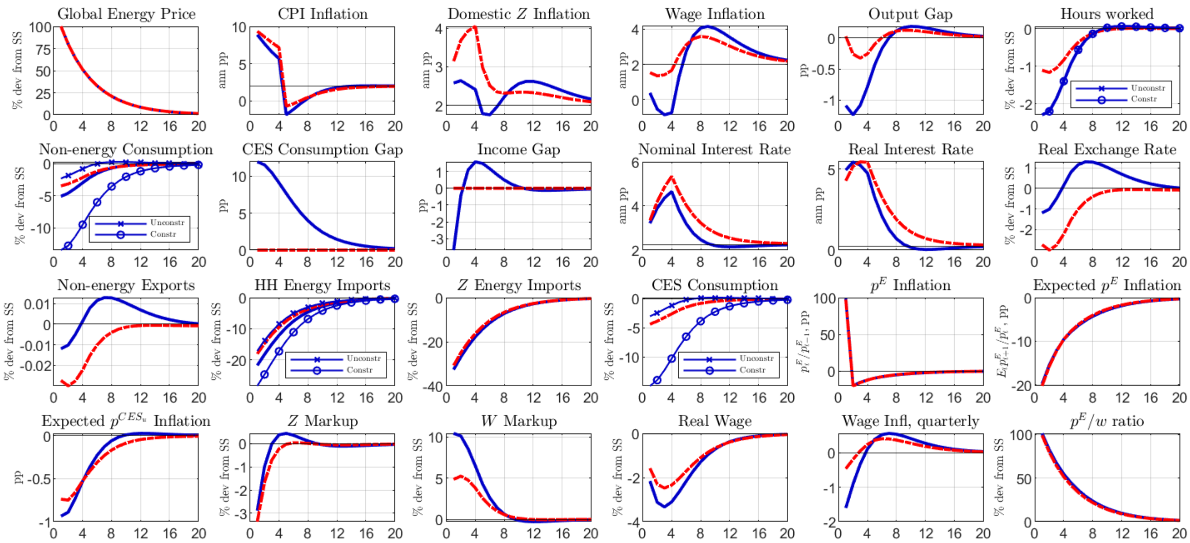
TANK Taylor (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)
RANK Taylor (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)



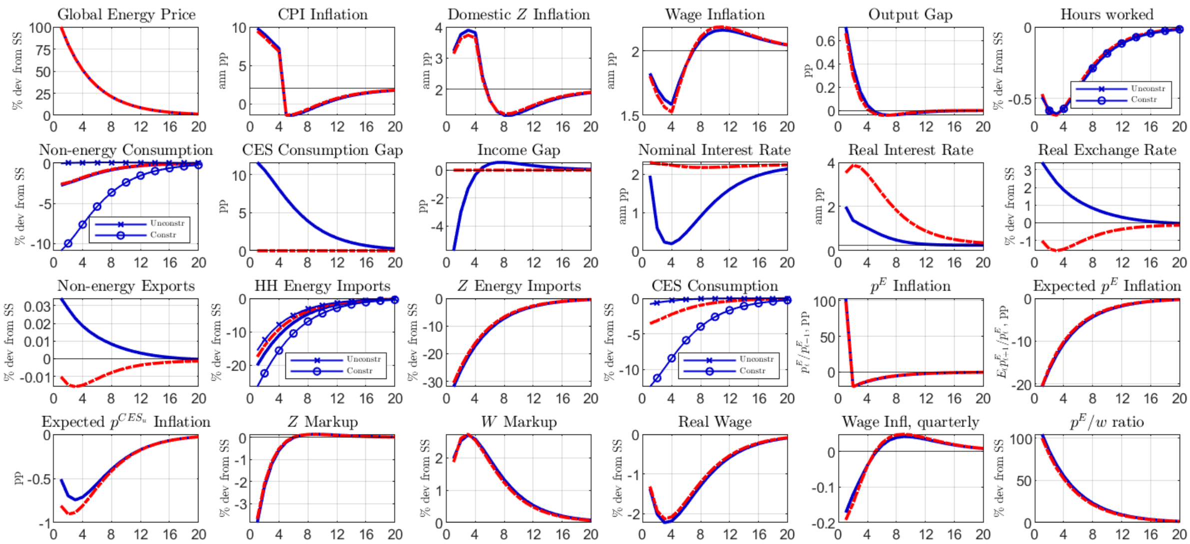
TANK Ramsey (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)
RANK Ramsey (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.35, σ=2)

I.12 Energy Shock - Lower Export Sensitivity

FIGURE I.17: Dynamic Responses to a Global Energy Price Shock: Lower Export Sensitivity



TANK Taylor (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.01, σ=2)
RANK Taylor (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.01, σ=2)



TANK Ramsey (HtM share=0.25, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, $\alpha_{u,ec}$=0.05, $\alpha_{c,ec}$=0.1, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.01, σ=2)
RANK Ramsey (HtM share=0.00, Firm-CES=0.3, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.0625, ϕ_z=0.66, ϕ_w=0.75, ς^*=0.01, σ=2)