Prep Course
Module 2
Lecture 1

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Monte Carlo Estimate

Given that n random variables $Y_1, \dots, Y_n$ are IID (independent and identically distributed) and $\mathbb{E}[Y_i] = \mu$ where $1 \leq i \leq n$. Let's define

$$\hat{C}_n = \frac{Y_1 + \cdots + Y_n}{n} \implies \mathbb{E}[\hat{C}_n] = \mu$$

As per the law of large numbers, with probability 1,

$$\hat{C}_n \to \mu \ \text{ as } \ n \to \infty$$

$\hat{C}_n$ is called a Monte Carlo estimate for μ.

Interview Question

What is the expectation of the number of times you need to roll a fair die until you get two 6's in a row?

Answer:

- Let x denote the unconditional expectation of the number of rolls needed to see two 6's in a row and x_1 denote the expected number of how many more rolls are needed given that the current roll is a 6.

$$x = 1 + \frac{5}{6}x + \frac{1}{6}x_1$$

$$x_1 = 1 + \frac{5}{6}x + \frac{1}{6} \cdot 0$$

$$\Rightarrow x = 42$$

See code_M2L01.ipynb