Chapter 4

Asset Volatility and Volatility Models

1. **Problem 1**. Daily SPY log returns.

   (a) The expected return is not significantly different from zero. The one-sample t-test cannot reject the null hypothesis of $\mu = 0$. The t-ratio is 0.265 with p-value 0.79. There are serial correlations in the log returns. The Ljung-Box statistics give $Q(10) = 40.38$ with p-value 1.45×10^{-5}. Using the residuals of an MA(2) model, the Ljung-Box statistics of the squared residuals give $Q(10) = 2141.1$, which is highly significant.

   (b) The fitted ARMA-GARCH model with Gaussian innovations is

   $$\begin{aligned} r_t &= 5.49 \times 10^{-4} + a_t - 0.069 a_{t-1}, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0,1) \\ \sigma_t^2 &= 2.31 \times 10^{-6} + 0.1 \times 10^{-7} a_{t-1}^2 + 0.113 a_{t-2}^2 + 0.871 \sigma_{t-1}^2, \end{aligned}$$

   where the coefficient of a_{t-1}^2 is not significant. Model checking statistics show that the fitted model is adequate except for the normality assumption. For instance, for the standardized residuals we have Q(10) = 7.26 with p-value 0.70 and for the squared standardized residuals we have Q(10) = 7.61 with p-value 0.67. The QQ-plot of the standardized residuals is in the top panel of Figure 4.1.

   (c) The ARMA-GARCH model with Student-t innovations is

   $$\begin{aligned} r_t &= 7.25 \times 10^{-4} + a_t - 0.058 a_{t-1}, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{7.63} \\ \sigma_t^2 &= 1.58 \times 10^{-6} + 0.0011 a_{t-1}^2 + 0.119 a_{t-2}^2 + 0.875 \sigma_{t-1}^2, \end{aligned}$$

   where, again, the coefficient of a_{t-1}^2 is not significant. Model checking statistics indicate that the model is adequate. For instance, for the standardized residual $\hat{\epsilon}_t$, we have $Q(10) = 7.84$ with p-value 0.64 and for $\hat{\epsilon}_t^2$ we have Q(10) = 5.73 with p-value 0.84. The QQ-plot of $\hat{\epsilon}_t$ is shown in the lower panel of Figure 4.1.

2. Again, the daily log returns of SPY index. Here we use *percentage* log returns. Also, we fixed $\delta = 2$ in the estimation.

   (a) The fitted MA-APARCH model is

$$\begin{aligned} r_t &= 0.017 + a_t - 0.065a_{t-1}, \quad a_t = \sigma_t\epsilon_t, \quad \epsilon_t \sim N(0,1) \\ \sigma_t^2 &= 0.02 + 1.0 \times 10^{-8}(|a_{t-1}| + 0.057a_{t-1})^2 + 0.045(|a_{t-2}| + 0.834a_{t-2})^2 + 0.904\sigma_{t-1}^2, \end{aligned}$$

   where the coefficient of a_{t-1}^2 is not significant. Model checking statistics show that, except for the normality assumption, the model is adequate. For the standardized residual $\hat{\epsilon}_t$, we have Q(10) = 7.00 with p-value 0.73 and for $\hat{\epsilon}_t^2$, we have Q(10) = 8.58 with p-value 0.57.

   (b) For Student-t innovations, the fitted model is

$$\begin{aligned} r_t &= 0.045 + a_t - 0.06a_{t-1}, \quad a_t = \sigma_t\epsilon_t, \quad \epsilon_t \sim t_{9.01} \\ \sigma_t^2 &= 0.014 + 1.0 \times 10^{-8}(|a_{t-1}| - 0.638a_{t-1})^2 + 0.049(|a_{t-2}| + 0.771a_{t-2})^2 + 0.907\sigma_{t-1}^2, \end{aligned}$$

   where, again, the coefficient of a_{t-1}^2 is not significant. Model checking statistics show that the model is adequate. For instance, for the standardized residuals $\hat{\epsilon}_t$, we have Q(10) = 8.33 with p-value 0.60, and for $\hat{\epsilon}_t^2$, we have Q(10) = 7.24 with p-value 0.70. The 1-step to 5-step ahead prediction of the model are

h	1	2	3	4	5
r_{t+h}	0.198	0.045	0.045	0.045	0.045
σ_{t+h}	2.011	2.166	2.142	2.132	2.121

3. Consider the monthly log returns of KO stock.

   (a) The expected log return is significantly different from zero; the one-sample t-test is 4.22 with p-value 2.82×10^{-5}. The log returns have no serial correlations as shown by the Ljung-Box statistic of Q(10) = 6.77 with p-value 0.75. However, the log returns have strong conditional heteroscedasticity because the Ljung-Box statistics of the squared returns (after mean adjustment) give Q(10) = 176.62 with p-value close to zero.

   (b) The fitted GARCH(1,1) model is

$$\begin{aligned} r_t &= 0.0124 + a_t, \quad a_t = \sigma_t\epsilon_t, \quad \epsilon_t \sim N(0,1) \\ \sigma_t^2 &= 2.59 \times 10^{-4} + 0.099a_{t-1}^2 + 0.83\sigma_{t-1}^2. \end{aligned}$$

   Model checking statistics show that, except for the normality assumption, the model adequately describes the mean and volatility of the log returns. For instance, consider the standardized residuals $\hat{\epsilon}_t$. We have Q(10) = 9.88 with p-value 0.45, indicating no serial correlations in $\hat{\epsilon}_t$. Similarly, for the $\hat{\epsilon}_t^2$ series, we have Q(10) = 12.54 with p-value 0.25.

(c) The fitetd GARCH(1,1) model with Student t innovations is

$$\begin{aligned} r_t &= 0.0127 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{7.10} \\ \sigma_t^2 &= 0.00022 + 0.104 a_{t-1}^2 + 0.836 \sigma_{t-1}^2. \end{aligned}$$

This model is adequate based on the available model checking statistics. For instance, for the standardized residuals $\hat{\epsilon}_t$, we have Q(10) = 10.29 with p-value 0.42. For the $\hat{\epsilon}_t^2$ series, we have Q(10) = 11.67 with p-value 0.31.

h	1	2	3	4	5
r_{t+h}	0.0127	0.0127	0.0127	0.0127	0.0127
σ_{t+h}	0.0448	0.0460	0.0470	0.0479	0.0488

4. Consider the monthly log returns, in percentages, of KO stock.

(a) The fitted TAGRCH model is

$$\begin{aligned} r_t &= 1.157 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0,1) \\ \sigma_t^2 &= 3.035 + (0.048 + 0.080 N_{t-1}) a_{t-1}^2 + 0.823 \sigma_{t-1}^2 \end{aligned}$$

where the estimate 0.048 and 0.080 have t-ratio 1.61 and 1.79, respectively, but other estimates are significant at the 5% level. Let $\hat{\epsilon}_t$ be the standardized residuals. The one-sample t-test shows that the expected value of ϵ_t is not significantly different from zero, the Ljung-Box statistics of $\hat{\epsilon}_t$ show Q(10) = 10.08 with p-value 0.43, and those of $\hat{\epsilon}_t^2$ give Q(10) = 10.40 with p-value 0.41. Consequently, the fitted TGARCH model is adequate. Based on the model and 5% significance level, the leverage effect is not statistically significant.

(b) The fitted NGARCH model is

$$\begin{aligned} r_t &= 1.464 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0,1) \\ \sigma_t^2 &= 1.155 + 0.868 \sigma_{t-1}^2 + 0.098 (a_{t-1} - 0.111 \sigma_{t-1})^2, \end{aligned}$$

where all estimates, but 0.111, are significant at the 5% level. Let $\hat{\epsilon}_t$ denotes the standardized residuals. The one-sample t ratio of $\hat{\epsilon}_t$ is -1.50 with p-value 0.133. For $\hat{\epsilon}_t$ series, the Ljung-Box statistics give Q(10) = 11.10 with p-value 0.35, and for the $\hat{\epsilon}_t^2$ series, the Ljung-Box statistics show Q(10) = 11.05 with p-value 0.35. Therefore, the model is adequate for the log return series. The leverage effect is, again, not significant at the 5% level.

The QQ-plots of the standardized residuals for the TGARCH(1,1) and NGARCH(1,1) models are given in Figure 4.3.

5. Daily log return of Procter and Gamble stock from September 1, 2001 to September 30, 2011.

(a) Yes, there are serial correlations in the daily log returns; see sample ACF or the Ljung-Box statistics that given Q(10) = 52.02 with p-value 1.13×10^{-7}.

(b) An MA(2) model is suggested by the sample ACF and the fitted model is

$$r_t = 3.0 \times 10^4 + (1 - 0.118B - 0.066B^2)a_t,$$

where the constant term is not significantly different from zero and other parameters are significant at the 5% level.

(c) Let $x_t = 100a_t$, where a_t is the residual of the MA(2) model. The Ljung-Box statistics of x_t^2 show that the residuals have ARCH effects. In particular, we have Q(10) with p-value close to zero.

(d) The fitted EGARCH model is

$$\begin{aligned} x_t &= -0.0089 + a_t, \quad a_t = \sigma_t\epsilon_t, \quad \epsilon_t \sim N(0,1) \\ \ln(\sigma_t^2) &= -0.0622 + 0.0857(|\epsilon_{t-1}| - 0.749\epsilon_{t-1}) + 0.983\ln(\sigma_{t-1}^2), \end{aligned}$$

where all estimates, but -0.0089, are significant at the 5% level. Let $\hat{\epsilon}_t$ be the standardized residuals. Model checking statistics of $\hat{\epsilon}_t$ and $\hat{\epsilon}_t^2$ show that the fitted EGARCH(1,1) model is adequate. Specifically, the one-sample test statistic of $\hat{\epsilon}_t = 0.28$ with p-value 0.78. The Ljung-Box statistics of $\hat{\epsilon}_t$ give Q(10) = 4.04 with p-value 0.95 and that of $\hat{\epsilon}_t^2$ show Q(10) = 8.15 with p-value 0.61.

6. Daily prices and volatilities of Apple stock from January 2007 to November 2011.

(a) The three volatility series are shown in Figure 4.4.

(b) The fitted GARCH model with Student-t innovations is

$$\begin{aligned} r_t &= 0.00215 + a_t, \quad a_t = \sigma_t\epsilon_t, \quad \epsilon_t \sim t_{7.18} \\ \sigma_t^2 &= 6.88 \times 10^{-6} + 0.082a_{t-1}^2 + 0.906\sigma_{t-1}^2 \end{aligned}$$

where all estimates are significant at the 5% level. Based on the standardized residuals $\hat{\epsilon}_t$, the model checking statistics support the fitted model. For instance, Q(10) = 8.84 with p-value 0.55 for $\hat{\epsilon}_t$, and Q(10) = 9.40 with p-value 0.49.

To compare the three volatility estimates, we consider the summary statistics of the three volatility estimates:

Type	Min	Max	Mean	Stdev	Skewness	Kurtosis
YZ63	0	0.0129	0.0046	0.00255	1.054	1.600
YZ32	0	0.0156	0.0046	0.00262	1.66	3.820
GARCH	0.0106	0.0701	0.0228	0.0095	2.095	5.724

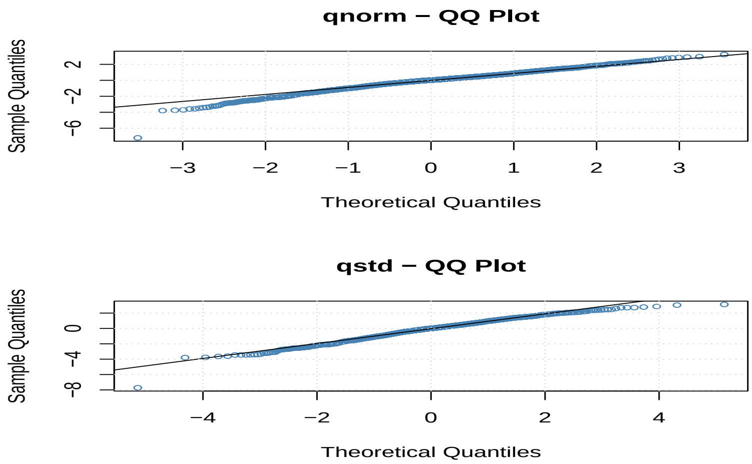

Figure 4.1: QQ plots for the fitted ARMA-GARCH Models for the SPY index log returns. The upper panel is for a fitted Gaussian MA(1)-GARCH(2,1) model and the lower panel for a Student-t MA(1)-GARCH(2,1) model.

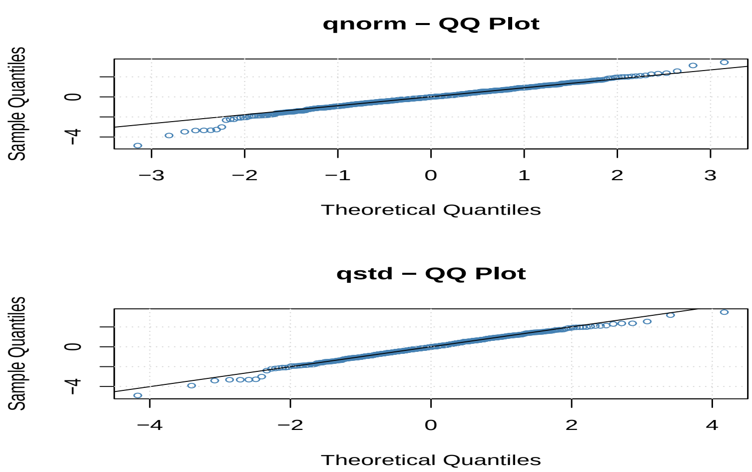

Figure 4.2: QQ plots for the fitted ARMA-PARCH Models for the SPY index log returns. The upper panel is for a fitted Gaussian MA(1)-PARCH(2,1) model and the lower panel for a Student-t MA(1)-PARCH(2,1) model.

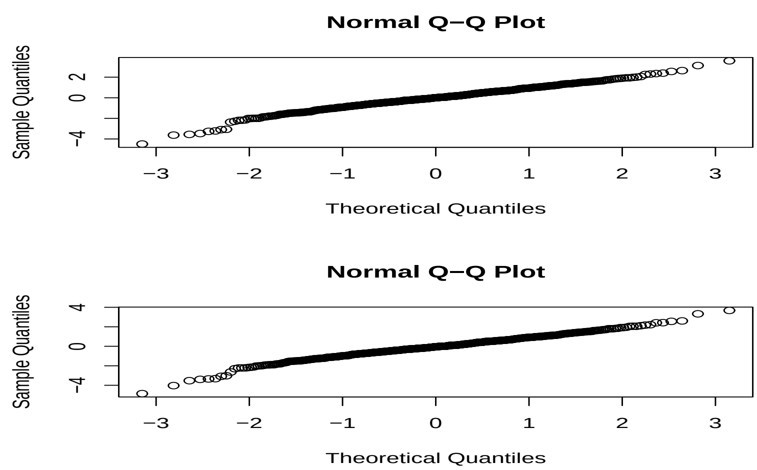

Figure 4.3: QQ plots for the fitted TGARCH(1,1) and NGARCH(1,1) models for the monthly log returns, in percentages, of KO stock.

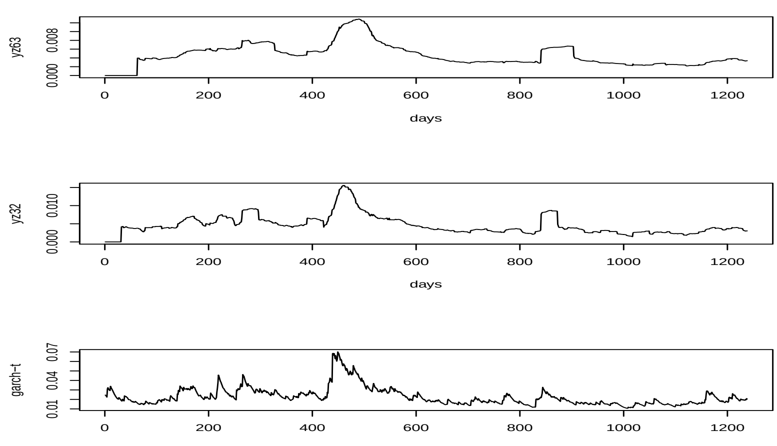

Figure 4.4: Volatility series of daily log returns of Apple stock. The volatility series are based on the methods YZ64, YZ32, and GARCH(1,1), respectively.

R Output. Edited.

```
> da=read.table("d-spy-0111.txt",header=T)
> rtn=log(da$rtn+1)
> t.test(rtn)
        One Sample t-test
data:  rtn
t = 0.2651, df = 2534, p-value = 0.7909
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -0.0004633792  0.0006082874

> Box.test(rtn,lag=10,type="Ljung")
        Box-Ljung test
data:  rtn
X-squared = 40.3801, df = 10, p-value = 1.452e-05

> acf(rtn)
> m1=arima(rtn,order=c(0,0,2))
> m1
Call: arima(x = rtn, order = c(0, 0, 2))

Coefficients:
          ma1      ma2  intercept
      -0.0838  -0.0626      1e-04
s.e.   0.0199   0.0202      2e-04

sigma^2 estimated as 0.0001871:  log likelihood = 7283.07,  aic = -14558.13
> tsdiag(m1)
> Box.test(m1$residuals,lag=10,type="Ljung")
        Box-Ljung test

data:  m1$residuals
X-squared = 12.3197, df = 10, p-value = 0.2642

> Box.test(m1$residuals^2,lag=10,type="Ljung")
        Box-Ljung test

data:  m1$residuals^2
X-squared = 2141.095, df = 10, p-value < 2.2e-16

> require(fGarch)
> m2=garchFit(~arma(0,2)+garch(2,1),data=rtn,trace=F)
> summary(m2)
Title: GARCH Modelling
Call:
 garchFit(formula = ~arma(0, 2) + garch(2, 1), data = rtn, trace = F)

Mean and Variance Equation:
 data ~ arma(0, 2) + garch(2, 1)
```

```
Conditional Distribution: norm

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      5.492e-04   1.535e-04    3.577 0.000347 ***
ma1    -6.739e-02   1.893e-02   -3.560 0.000371 ***
ma2    -3.049e-02   2.139e-02   -1.426 0.153998
omega   2.341e-06   4.948e-07    4.732 2.22e-06 ***
alpha1  1.000e-08   1.207e-02    0.000 0.999999
alpha2  1.136e-01   1.791e-02    6.346 2.21e-10 ***
beta1   8.700e-01   1.454e-02   59.819  < 2e-16 ***
---
Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  350.232   0
 Shapiro-Wilk Test  R    W      0.9834774 0
 Ljung-Box Test     R    Q(10)  6.344909  0.7855011
 Ljung-Box Test     R    Q(15)  13.21293  0.5858551
 Ljung-Box Test     R    Q(20)  16.54518  0.6822788
 Ljung-Box Test     R^2  Q(10)  7.430835  0.6842409
 Ljung-Box Test     R^2  Q(15)  9.294003  0.86165
 Ljung-Box Test     R^2  Q(20)  10.41108  0.9600909
 LM Arch Test       R    TR^2   8.237079  0.7663398

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.293135 -6.277015 -6.293150 -6.287287

> m2=garchFit(~arma(0,1)+garch(2,1),data=rtn,trace=F)
> summary(m2)
Title: GARCH Modelling
Call:
 garchFit(formula = ~arma(0, 1) + garch(2, 1), data = rtn, trace = F)

Mean and Variance Equation:
 data ~ arma(0, 1) + garch(2, 1)

Conditional Distribution: norm
Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      5.489e-04   1.580e-04    3.473 0.000514 ***
ma1    -6.896e-02   1.940e-02   -3.555 0.000378 ***
omega   2.312e-06   4.917e-07    4.702 2.57e-06 ***
alpha1  1.000e-08   1.219e-02    0.000 0.999999
alpha2  1.132e-01   1.797e-02    6.297 3.03e-10 ***
beta1   8.707e-01   1.452e-02   59.977  < 2e-16 ***
---
Log Likelihood:
 7982.539    normalized:  3.148931
```

```
Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  333.3606  0
 Shapiro-Wilk Test  R    W      0.9841873 3.436301e-16
 Ljung-Box Test     R    Q(10)  7.262873  0.7004168
 Ljung-Box Test     R    Q(15)  14.26091  0.5058402
 Ljung-Box Test     R    Q(20)  17.64679  0.6106636
 Ljung-Box Test     R^2  Q(10)  7.614365  0.6664474
 Ljung-Box Test     R^2  Q(15)  9.43197   0.8538705
 Ljung-Box Test     R^2  Q(20)  10.39754  0.9603776
 LM Arch Test       R    TR^2   8.401265  0.7530396

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.293127 -6.279310 -6.293139 -6.288115
> plot(m2)

> m3=garchFit(~arma(0,1)+garch(2,1),data=rtn,trace=F,cond.dist="std")
> summary(m3)
Title: GARCH Modelling
Call:
 garchFit(formula = ~arma(0, 1) + garch(2, 1), data = rtn, cond.dist = "std",
    trace = F)

Mean and Variance Equation:
 data ~ arma(0, 1) + garch(2, 1)

Conditional Distribution: std

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      7.245e-04   1.518e-04    4.773 1.81e-06 ***
ma1    -5.821e-02   1.908e-02   -3.051  0.00228 **
omega   1.582e-06   4.945e-07    3.200  0.00137 **
alpha1  1.116e-03   1.179e-02    0.095  0.92457
alpha2  1.185e-01   1.999e-02    5.928 3.07e-09 ***
beta1   8.746e-01   1.610e-02   54.308  < 2e-16 ***
shape   7.625e+00   1.173e+00    6.500 8.05e-11 ***
---
Log Likelihood:
 8020.441    normalized:  3.163882

Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  465.3614  0
 Shapiro-Wilk Test  R    W      0.9823552 0
 Ljung-Box Test     R    Q(10)  7.839162  0.6445439
 Ljung-Box Test     R    Q(15)  14.20829  0.5097941
 Ljung-Box Test     R    Q(20)  17.70566  0.6067911
```

```
 Ljung-Box Test     R^2  Q(10)  5.727105  0.8376459
 Ljung-Box Test     R^2  Q(15)  8.292801  0.911508
 Ljung-Box Test     R^2  Q(20)  10.0217   0.9677767
 LM Arch Test       R    TR^2   6.328886  0.898604

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.322241 -6.306121 -6.322256 -6.316393

> plot(m3)
>
#### percentage log returns
> rtn=rtn*100
> m4=garchFit(~arma(0,1)+aparch(2,1),data=rtn,delta=2,include.delta=F,trace=F)
Warning message:
In sqrt(diag(fit$cvar)) : NaNs produced
> summary(m4)
Title:  GARCH Modelling
Call:
 garchFit(formula = ~arma(0, 1) + aparch(2, 1), data = rtn, delta = 2,
    include.delta = F, trace = F)

Mean and Variance Equation:
 data ~ arma(0, 1) + aparch(2, 1)

Conditional Distribution: norm

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      1.702e-02   1.606e-02    1.060 0.289274
ma1    -6.517e-02   1.973e-02   -3.302 0.000959 ***
omega   2.008e-02   3.683e-03    5.452 4.98e-08 ***
alpha1  1.000e-08          NA       NA       NA
alpha2  4.532e-02   2.414e-02    1.877 0.060453 .
gamma1  5.653e-02          NA       NA       NA
gamma2  8.341e-01   4.638e-01    1.798 0.072098 .
beta1   9.043e-01   1.132e-02   79.862  < 2e-16 ***
---
Log Likelihood:
 -3652.02    normalized:  -1.440639

Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  270.0283  0
 Shapiro-Wilk Test  R    W      0.9854461 1.921587e-15
 Ljung-Box Test     R    Q(10)  6.995874  0.7258344
 Ljung-Box Test     R    Q(15)  13.40647  0.5709336
 Ljung-Box Test     R    Q(20)  16.05418  0.7132576
 Ljung-Box Test     R^2  Q(10)  8.578766  0.572491
 Ljung-Box Test     R^2  Q(15)  11.13476  0.7429845
```

```
 Ljung-Box Test     R^2  Q(20)  11.96281  0.9173502
 LM Arch Test       R    TR^2   10.24446  0.594524

Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
2.887590 2.906013 2.887570 2.894274
> m5=garchFit(~arma(0,1)+aparch(2,1),data=rtn,delta=2,include.delta=F,trace=F,cond.dist="std")
Warning message:
In sqrt(diag(fit$cvar)) : NaNs produced
> summary(m5)
Title: GARCH Modelling
Call:
 garchFit(formula = ~arma(0, 1) + aparch(2, 1), data = rtn, delta = 2,
    cond.dist = "std", include.delta = F, trace = F)

Mean and Variance Equation:
 data ~ arma(0, 1) + aparch(2, 1)

Conditional Distribution: std

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      4.498e-02   1.566e-02    2.873 0.004068 **
ma1    -6.008e-02   1.921e-02   -3.127 0.001765 **
omega   1.449e-02   3.794e-03    3.819 0.000134 ***
alpha1  1.000e-08   6.030e-07    0.017 0.986769
alpha2  4.907e-02   2.560e-02    1.916 0.055303 .
gamma1 -6.378e-01          NA       NA       NA
gamma2  7.708e-01   4.243e-01    1.817 0.069242 .
beta1   9.072e-01   1.346e-02   67.376  < 2e-16 ***
shape   9.008e+00   1.588e+00    5.672 1.41e-08 ***
---
Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  358.2849  0
 Shapiro-Wilk Test  R    W      0.9840144 2.73303e-16
 Ljung-Box Test     R    Q(10)  8.327021  0.5969272
 Ljung-Box Test     R    Q(15)  14.11435  0.5168749
 Ljung-Box Test     R    Q(20)  16.73132  0.670345
 Ljung-Box Test     R^2  Q(10)  7.244771  0.7021526
 Ljung-Box Test     R^2  Q(15)  10.42895  0.7919261
 Ljung-Box Test     R^2  Q(20)  11.75531  0.9242404
 LM Arch Test       R    TR^2   8.523088  0.7430344

Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
2.866581 2.887308 2.866556 2.874101

> predict(m5,5)
  meanForecast meanError standardDeviation
```

```
1   0.19750004  2.011487         2.011487
2   0.04497752  2.165896         2.162522
3   0.04497752  2.142467         2.138524
4   0.04497752  2.132012         2.128136
5   0.04497752  2.120621         2.116763
>

#### Problem 3 ###
> da=read.table("m-ko-6111.txt",header=T)
> rtn=log(da$ko+1)
> t.test(rtn)
        One Sample t-test
data:  rtn
t = 4.2198, df = 608, p-value = 2.819e-05
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.005655242 0.015501584

> Box.test(rtn,lag=10,type="Ljung")
        Box-Ljung test

data:  rtn
X-squared = 6.7711, df = 10, p-value = 0.7469

> Box.test((rtn-mean(rtn))^2,lag=10,type="Ljung")
       Box-Ljung test

data:  (rtn - mean(rtn))^2
X-squared = 176.6227, df = 10, p-value < 2.2e-16

> Box.test(rtn^2,lag=10,type="Ljung")
        Box-Ljung test

data:  rtn^2
X-squared = 165.1094, df = 10, p-value < 2.2e-16

> m1=garchFit(~garch(1,1),data=rtn,trace=F)
> summary(m1)
Title: GARCH Modelling
Call:
 garchFit(formula = ~garch(1, 1), data = rtn, trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)

Conditional Distribution: norm
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     1.237e-02   2.267e-03    5.455 4.90e-08 ***
omega  2.592e-04   8.641e-05    3.000   0.0027 **
```

```
alpha1 9.878e-02   2.261e-02    4.368 1.25e-05 ***
beta1  8.298e-01   3.393e-02   24.458  < 2e-16 ***
---

Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  83.09438  0
 Shapiro-Wilk Test  R    W      0.9828977 1.470435e-06
 Ljung-Box Test     R    Q(10)  9.877629  0.4512942
 Ljung-Box Test     R    Q(15)  18.69547  0.2278667
 Ljung-Box Test     R    Q(20)  21.54345  0.3657889
 Ljung-Box Test     R^2  Q(10)  12.54335  0.2503354
 Ljung-Box Test     R^2  Q(15)  13.04873  0.5985339
 Ljung-Box Test     R^2  Q(20)  14.33025  0.8133643
 LM Arch Test       R    TR^2   10.79746  0.5463519

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-2.841816 -2.812838 -2.841901 -2.830543

> m2=garchFit(~garch(1,1),data=rtn,trace=F,cond.dist="std")
> summary(m2)
Title: GARCH Modelling
Call:
 garchFit(formula = ~garch(1, 1), data = rtn, cond.dist = "std",   trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)
Conditional Distribution: std

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     0.0127078   0.0021240    5.983 2.19e-09 ***
omega  0.0002228   0.0000909    2.451 0.014256 *
alpha1 0.1042104   0.0283506    3.676 0.000237 ***
beta1  0.8359406   0.0380904   21.946  < 2e-16 ***
shape  7.0967684   1.8562635    3.823 0.000132 ***
---
Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  84.74462  0
 Shapiro-Wilk Test  R    W      0.9829203 1.493566e-06
 Ljung-Box Test     R    Q(10)  10.28596  0.4157731
 Ljung-Box Test     R    Q(15)  18.9476   0.2161183
 Ljung-Box Test     R    Q(20)  21.6197   0.3614981
 Ljung-Box Test     R^2  Q(10)  11.67235  0.3075833
 Ljung-Box Test     R^2  Q(15)  11.94762  0.6829891
 Ljung-Box Test     R^2  Q(20)  13.25639  0.8661143
 LM Arch Test       R    TR^2   9.814017  0.6322729
```

```
Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-2.879667 -2.843445 -2.879800 -2.865576

> plot(m1)
> plot(m2)

> predict(m2,5)
  meanForecast  meanError standardDeviation
1   0.01270783 0.04482957        0.04482957
2   0.01270783 0.04595843        0.04595843
3   0.01270783 0.04699500        0.04699500
4   0.01270783 0.04794910        0.04794910
5   0.01270783 0.04882910        0.04882910
>
### Problem 4 ####
> rtn=rtn*100
> source("Tgarch11.R")
> m3=Tgarch11(rtn)
[1] 1941.82
  0:     1941.8202:  1.05784  3.82719 0.100000 0.100000 0.800000

Coefficient(s):
       Estimate  Std. Error  t value   Pr(>|t|)
mu    1.1574837   0.2283502  5.06890 4.0013e-07 ***
omega 3.0348847   1.0524401  2.88366  0.0039308 **
alpha 0.0488281   0.0301259  1.62080  0.1050609
gam1  0.0804687   0.0448357  1.79475  0.0726943 .
beta  0.8233350   0.0379449 21.69819 < 2.22e-16 ***
---
[1] "residuals"  "volatility" "par"
> sresi=m3$residuals/m3$volatility
> t.test(sresi)
        One Sample t-test

data:  sresi
t = -0.2222, df = 608, p-value = 0.8243
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -0.08865464  0.07063479

> Box.test(sresi,lag=10,type='Ljung')
        Box-Ljung test

data:  sresi
X-squared = 10.0802, df = 10, p-value = 0.4335

> Box.test(sresi^2,lag=10,type='Ljung')
        Box-Ljung test
```

```
data:  sresi^2
X-squared = 10.399, df = 10, p-value = 0.4062

> qqnorm(sresi)
> source("Ngarch.R")
> m4=Ngarch(rtn)

Estimation results of NGARCH(1,1) model:
estimates:  1.464186 1.154998 0.86844 0.09782551 0.1113541
std.errors:  0.2236933 0.4166718 0.02274459 0.02149025 0.1596848
t-ratio:  6.545508 2.771961 38.18226 4.552088 0.6973369
> names(m4)
[1] "residuals"  "volatility"
> sresi=m4$residuals/m4$volatility
> t.test(sresi)
        One Sample t-test
data:  sresi
t = -1.5042, df = 608, p-value = 0.133
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -0.1432922  0.0189921

> Box.test(sresi,lag=10,type='Ljung')
        Box-Ljung test
data:  sresi
X-squared = 11.1025, df = 10, p-value = 0.3496

> Box.test(sresi^2,lag=10,type='Ljung')
        Box-Ljung test
data:  sresi^2
X-squared = 11.0523, df = 10, p-value = 0.3535

> qqnorm(sresi)

### Problem 5 #####
> da=read.table("d-pg-0111.txt",header=T)
> pg=log(da$rtn+1)
> acf(pg)
> Box.test(pg,lag=10,type='Ljung')
        Box-Ljung test
data:  pg
X-squared = 52.0181, df = 10, p-value = 1.132e-07

> m1=arima(pg,order=c(0,0,2))
> m1
Call:arima(x = pg, order = c(0, 0, 2))

Coefficients:
          ma1      ma2  intercept
      -0.1179  -0.0658      3e-04
```

```
s.e.   0.0200   0.0206      2e-04

sigma^2 estimated as 0.0001415:  log likelihood = 7637.19,  aic = -15266.38
> tsdiag(m1)
> xt=m1$residuals*100
> Box.test(xt^2,lag=10,type='Ljung')
        Box-Ljung test
data:  xt^2
X-squared = 745.5259, df = 10, p-value < 2.2e-16

> source("Egarch.R")
> m2=Egarch(xt)
  Estimation results of EGARCH(1,1) model:
estimates:  -0.008872955 -0.06219673 0.08574628 -0.7485869 0.9832538
std.errors:  0.01953147 0.009196044 0.01258558 0.1406611 0.003837151
t-ratio:  -0.4542902 -6.763422 6.813059 -5.321918 256.2458
> names(m2)
[1] "residuals"  "volatility"
> sresi=m2$residuals/m2$volatility
> t.test(sresi)
        One Sample t-test
data:  sresi
t = 0.2819, df = 2534, p-value = 0.778
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -0.03340254  0.04462039

> Box.test(sresi,lag=10,type='Ljung')
        Box-Ljung test
data:  sresi
X-squared = 4.0433, df = 10, p-value = 0.9454

> Box.test(sresi^2,lag=10,type='Ljung')
        Box-Ljung test
data:  sresi^2
X-squared = 8.154, df = 10, p-value = 0.6138

> qqnorm(sresi)
>
#### Problem 6 ####
> getSymbols("AAPL",from="2007-01-02",to="2011-11-30")
[1] "AAPL"
> price=log(as.numeric(AAPL$AAPL.Adjusted))
> Open=log(as.numeric(AAPL$AAPL.Open))
> High=log(as.numeric(AAPL$AAPL.High))
> Low=log(as.numeric(AAPL$AAPL.Low))
> Close=log(as.numeric(AAPL$AAPL.Close))
> source("yz.R")
> m1=yz(Open,High,Low,Close,window=63)
> m2=yz(Open,High,Low,Close,window=32)
```

```
> names(m1)
[1] "yzsq"
> rtn=diff(price)
> acf(rtn)

> m3=garchFit(~garch(1,1),data=rtn,trace=F,cond.dist="std")
> summary(m3)
Title: GARCH Modelling
Call:
 garchFit(formula = ~garch(1, 1), data = rtn, cond.dist = "std",  trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)

Conditional Distribution: std

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     2.151e-03   5.134e-04    4.189 2.80e-05 ***
omega  6.883e-06   3.443e-06    1.999   0.0456 *
alpha1 8.414e-02   2.055e-02    4.094 4.25e-05 ***
beta1  9.058e-01   2.215e-02   40.893  < 2e-16 ***
shape  7.176e+00   1.375e+00    5.218 1.81e-07 ***
---
Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  131.7623  0
 Shapiro-Wilk Test  R    W      0.9877552 1.123946e-08
 Ljung-Box Test     R    Q(10)  8.841581  0.5472014
 Ljung-Box Test     R    Q(15)  13.81759  0.5394059
 Ljung-Box Test     R    Q(20)  20.99753  0.3972781
 Ljung-Box Test     R^2  Q(10)  9.403493  0.4942858
 Ljung-Box Test     R^2  Q(15)  12.46673  0.643413
 Ljung-Box Test     R^2  Q(20)  18.04057  0.5847354
 LM Arch Test       R    TR^2   12.51185  0.4054959

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-4.884436 -4.863753 -4.884469 -4.876657

> v3=m3@sigma.t
> length(v3)
[1] 1238
> names(m1)
[1] "yzsq"
> v1=sqrt(m1$yzsq)
> v2=sqrt(m2$yzsq)
> basicStats(v1)
                    v1
nobs        1238.000000
```

```
Minimum       0.000000
Maximum       0.012859
Mean          0.004599
Median        0.003861
Sum           5.693403
SE Mean       0.000072
Variance      0.000007
Stdev         0.002550
Skewness      1.053753
Kurtosis      1.599435
> basicStats(v2)
                    v2
nobs       1238.000000
Minimum       0.000000
Maximum       0.015566
Mean          0.004603
Median        0.003856
Sum           5.698959
SE Mean       0.000074
Variance      0.000007
Stdev         0.002620
Skewness      1.661996
Kurtosis      3.819679
> basicStats(v3)
                    v3
nobs       1238.000000
Minimum       0.010629
Maximum       0.070121
Mean          0.022829
Median        0.020147
Sum          28.261865
SE Mean       0.000271
Variance      0.000091
Stdev         0.009523
Skewness      2.095204
Kurtosis      5.724303
```