Understanding Randomess Through Win and Loss in Tiến Lên Card Game

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What is Tiến Lên?

Introduction:

Tiến Lên, which translates to "forward" in Vietnamese, is a captivating card game known for its simple yet strategic gameplay. Players take turns discarding individual or combinations of cards, attempting to be the first to get rid of all the cards. The deck is shuffled at random before each game.
This simulation employs the Monte Carlo method to analyze the financial outcomes of a competitive card game scenario involving four players across multiple repetitions. In this simulated scenario, each player participates in a winner-takes-all card game, where the winner receives a reward of $3 and the losers each contribute $1 to the pot. By conducting multiple repetitions of the game and tracking the net profit or loss for each player, this simulation aims to provide insights into the potential financial implications and risk exposure associated with such a gambling scenario, shedding light on the variability and convergence of outcomes over repeated plays.

Methodology

Preliminary rules:

Standard 52-card deck
Number of players: 4
Number of cards dealt to each player per game: 13
Cards are ranked from highest to lowest as follows: 2 A K Q J 10 9 8 7 6 5 4 3
Suits are ranked from highest to lowest: Hearts ♥ – Diamonds ♦ – Clubs ♣ – Spades ♠

Shuffling and Dealing Techniques:

The act of shuffling the deck, in this case is done by an algorithm, introduces a level of unpredictability, ensuring that no player knows the exact sequence of cards they will receive. This inherent randomness creates an environment where chance influences each player's starting hand and subsequent decisions.

Basic Gameplay:

Players take turns discarding single cards or card combinations to a central face-up pile. The goal is to avoid becoming the last person to hold any cards. The starting player is the one hold the 3♠ card. As long as the 3♠ completes a sequence or is a part of an X-of-a-kind play, it can be played using any strategy. Each player then continues in a clockwise direction. On each turn, if a player have no moves, they can pass and forfeit their hand until the next round. When three out of the four players fail to take their turn, the round is over. The person who didn't pass win the round and can start a new round.

After each round of the game, the losing players are required to contribute $1 each to the pot, while the winner collects $3 from the accumulated funds.

Winning Statistics

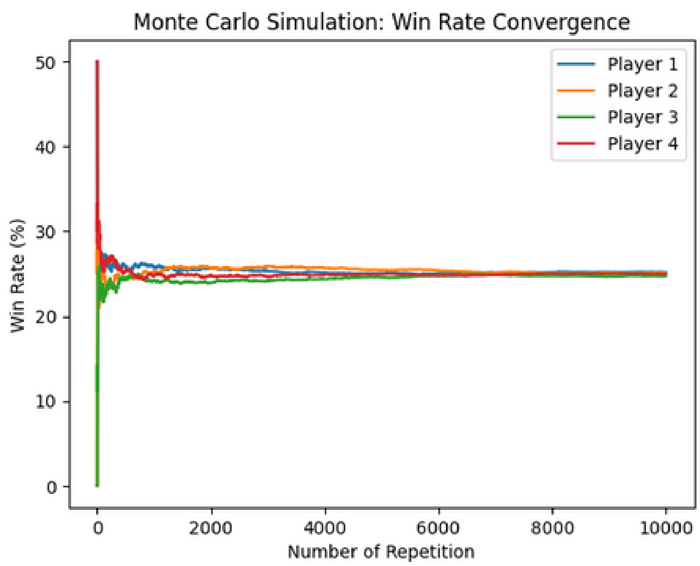
Players' hand example:
Player 1's hand: 2♥, 2♣, A♥, Q♥, 10♥, 9♥, 9♠, 8♦, 8♣, 7♦, 7♠, 5♦, 4♥
Player 2's hand: 2♦, A♠, J♠, 10♠, 9♣, 7♣, 6♥, 6♦, 6♣, 6♠, 5♣, 4♠, 3♦
Player 3's hand: 2♠, A♦, A♣, K♦, K♣, Q♦, J♥, 10♣, 8♥, 8♠, 7♥, 5♠, 4♦
Player 4's hand: K♥, K♠, Q♠, Q♣, J♦, J♣, 10♦, 9♦, 5♥, 4♣, 3♥, 3♣, 3♠

Win Rates:

We generate 10000 simulations first to ensure the statistical significance of the experiment:

- Player 1: 2531 wins (25.31% win rate)
- Player 2: 2558 wins (25.58% win rate)
- Player 3: 2457 wins (24.57% win rate)
- Player 4: 2454 wins (24.54% win rate)

Each player's win rate converged close to the mean of 25%, as expected by the normal distribution. Despite potential fluctuations in individual game outcomes, the aggregated results demonstrate a clear trend towards equilibrium.



Net Profit and Loss

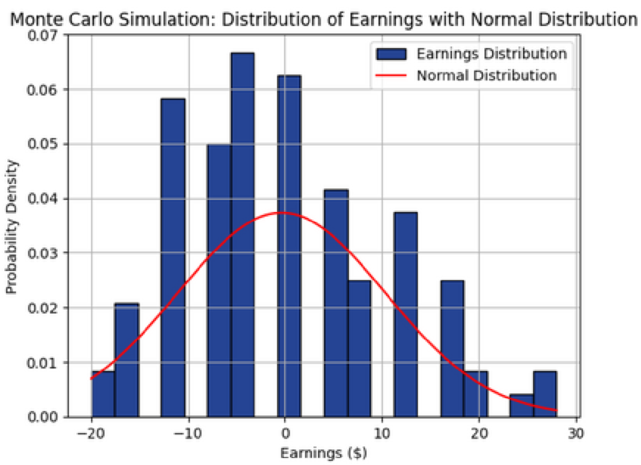
At the equilibrium of 25% win rate, player will break even. Any rate above 25% will result in a profit and vice versa.
Net profits or losses:

- Player 1: $124.00 (25.31% win rate)
- Player 2: $232.00 (25.58% win rate)
- Player 3: $-172.00 (24.47% win rate)
- Player 4: $-184.00 (24.54% win rate)

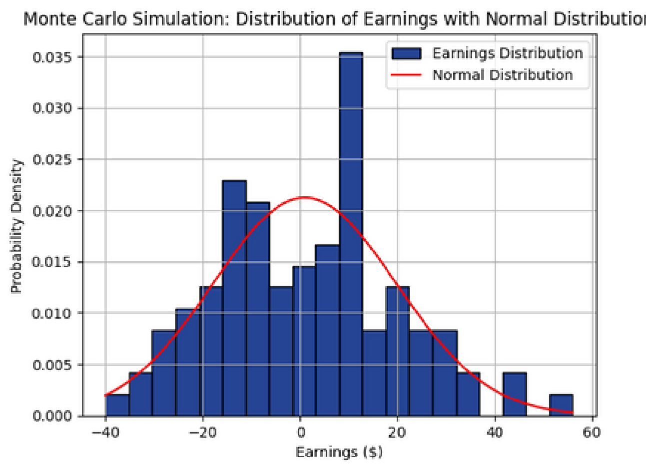
Despite the small margin of difference in win rates among the players, the resulting net profits or losses exhibit substantial disparities.

The expected return is $3.72, suggesting that, on average, a player can anticipate earning approximately $3.72 per game.
The standard deviation is approximately $30.04. This indicates that the earnings distribution has a considerable amount of variability or volatility. A higher standard deviation implies greater variability in earnings, reflecting a higher level of risk associated with the game.

However, if we assume a more realistic gambling timeline of 40 simulations for example (roughly 2 hours of gameplay, assuming 3 minutes per game), the disparities in win rates among players can be quite noticeable.
Upon conducting 40 simulations, we observed a fatter tail in the winning distribution in each player



40 iterations (2 hours gameplay)



100 iterations (5 hours gameplay)

The expected return drops down to only $0.01 per game. The standard deviation is approximately $27.28.
Value at Risk (VaR) at 95.0% confidence level is -40.0. This means that there is a 5% chance that the maximum potential loss that the player might experience is $40.0. In other words, there is a 5% chance that a player could lose all the 40 gameplays.

Other Factors In Influencing the Game

Cards Counting:

In Tiến Lên, card counting is a simple example of when there is a finite number of outcomes, each of which is equally likely, we can just count up the possibilities. We already know the probability of each suit is 1/4 and each card is 1/52. As the game progresses, by observing the cards that have been played a player can narrow down the potential cards that opponents might hold, a strategy particularly effective in the later stages . With more information about the cards played and those still in play, players can make more precise calculations about the probability of their opponents holding specific cards. This enables them to anticipate opponents' moves and adjust their strategies to optimize their chances of winning.

However, it's important to note that despite this strategy, victory remains heavily influenced by initial card distribution. If a player receives low-ranking cards or lacks counterplay options, their options can be limited, making it challenging to secure a victory as they might be forced to continually forfeit turns.

Impact of Shuffling Technique on Randomness and Strategy:

In our simulation, we employed Python's random shuffle, but in reality, various shuffling techniques can introduce bias into the distribution of cards. A suboptimal shuffling technique can introduce bias into the distribution, leading to deviations from true randomness. Players may capitalize on the observed patterns or tendencies within the deck, leveraging their insights to make more informed decisions. This shifts the balance away from pure randomness and encourages players to adopt adaptive strategies that exploit the deck's perceived irregularities.

Conclusion

Tiến Lên, cherished as a source of entertainment in Vietnamese culure, offers a captivating exploration of the interplay between chance and outcome. Despite its seemingly modest nature, the game's slender margin of win rates holds the potential for substantial profit or loss, casting a revealing light on the volatility inherent in its gameplay. In just a short span of two hours, a mere $1 wager per round can yield remarkable windfalls or profound setbacks, echoing the swift currents of a dynamic market landscape.

While Tien Len serves as a captivating microcosm of probability dynamics, it remains a contained example with distinct limitations. The finite nature of its gameplay, governed by a fixed deck of cards with a finite number of shuffling permutations, offers a stark contrast to the boundless complexities of financial markets. Market risk, unlike the predictable confines of a card game, unfolds within a vast and ever-evolving ecosystem shaped by myriad variables and unforeseeable events. In this broader context, the parallels drawn from Tien Len offer valuable insights into risk management principles, yet they must be tempered with an appreciation for the unique nuances and challenges inherent in navigating the dynamic terrain of financial markets.

Appendix

Law of Large Numbers:

The Law of Large Numbers states that as the number of independent, random events increases, the average of these events will converge to the true probability or expected value.

Let $\overline{X}n$ be the average win rate for each player, and X1,...,Xn are independent outcome of each gameplay (Orloff and Bloom, 2017) :

$$\overline{X}_n = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{1}{n}\sum_{i=1}^{n} X_i.$$

An intriguing observation is the convergence of win rates toward the 25% mark as the number of simulations increases. This convergence aligns with the principle of the Law of Large Numbers, where repeated trials tend to stabilize outcomes around their theoretical probabilities.

What exactly is randomness in this case?

A deck of 52 cards can be randomized by shuffling 52! ways. To achieving full randomness implies that the resulting distribution of cards is independent of the previous order of the deck. Markov chain mixing time refers to the number of shuffles required for the deck to become sufficiently randomized. It quantifies the speed at which a Markov chain reaches a state where further shuffling doesn't significantly alter the distribution of the cards (Diaconis 1996).

Some of the common shuffling techniques are:

- **Riffle:** Split the cards into two halves and interlace them (Aldous and Diaconis 1986). In a paper by Diaconis and Bayer in 1992 stated that "seven shuffles are enough to randomize a deck of cards".
- **Overhand:** repeatedly taking a small portion of cards from the top of the deck and placing them in the opposite hand's pile. This is a popular technique among casual players but has been proven to be the least efficient since it's prone to clumps of cards. Robin Pemantle in 1989 showed that the mixing time of the overhand shuffle with respect to variation distance is between order n^2 and order n^2logn. That is well over 4000 shuffles (Cousins 2019).
- **Faro:** the deck is split into two equal halves and then the cards from each half are interwoven perfectly, with one card from each half being placed alternately. The faro shuffle doesn't have a mixing time due to its controlled nature. An "out-shuffle" maintains the original top and bottom cards, while an "in-shuffle" places the original top card as the second card and the original bottom card second to last. In 1989, Diaconis, Graham, and Kantor showed that the order of a perfect in-shuffle and out-shuffle is 2 mod (2n ± 1).

Valid cards or combinations:

- A single card
- A pair of the same rank: 5♥ 5♠
- A triplet of the same rank: 8♦ 8♣ 8♠
- A quartet of the same rank: J♥ J♦ J♣ J♠
- A sequence of 3 or more cards, regardless of suit: 7♣ 8♦ 9♣
- A double sequence of 3 or more pairs, regardless of suit, as 3♣ 3♠ 4♥ 4♦ 5♣ 5♦
- Impossible sequence: K A 2 of any suit.

The ranking of the highest card in each combination determines the winning combination. If 7♥ 8♦ 9♣ is led, it can be beaten by 7♠ 8♠ 9♦, because the highest card of the second sequence (9♦) outranks the highest card of the first sequence (9♣).

Winning against the 2's are presented as follows:

- A single 2 may be beaten by any quartet:
  3♥ 3♦ 3♣ 3♠
- A single 2 may also be beaten by any double sequence of 3+ pairs:
  9♣ 9♠ 10♥ 10♦ J♣ J♦
- A pair of 2s may be beaten by any double sequence of 4+ pairs:
  9♣ 9♠ 10♥ 10♦ J♣ J♦ Q♠ Q♥
- A triplet of 2s may be beaten by any double sequence of 5+ pairs:
  9♣ 9♠ 10♥ 10♦ J♣ J♦ Q♠ Q♥ K♦ K♣

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