HW 3 生命科學數學 B04B01036 林妤真

Homework logistic growth

Show analytically: $\dfrac{dN}{dt} = rN(1-\dfrac{N}{K}) \longrightarrow N_t = \dfrac{K}{1+Ce^{-rt}}$

Hint: partial fraction.

Due date:
No late homework acceptable without consulting me in advance.

Solution:

$$\frac{dN}{dt} = rN\left(1-\frac{N}{k}\right)$$

$$\frac{1}{N\left(1-\frac{N}{k}\right)}\,dN = r\,dt$$

$$\frac{k}{N(k-N)}\,dN = r\,dt$$

< partial fraction >

$$\int\left(\frac{1}{N}+\frac{1}{k-N}\right)dN = \int r\,dt$$

$$\ln N - \ln(k-N) = rt + \text{Const.}$$

$$\ln\left(\frac{N_t}{k-N_t}\right) = rt + \text{Const.}$$

$$\frac{N_t}{k-N_t} = \frac{1}{c}e^{rt} \xrightarrow{t=0} C = \frac{k-N_0}{N_0}$$

$$k-N_t = N_t \cdot c \cdot e^{-rt}$$

$$k = N_t\left(1+Ce^{-rt}\right)$$

$$N_t = \frac{k}{1+ce^{-rt}}, \quad C = \frac{k-N_0}{N_0}$$

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