Introduction to Machine Learning

NYU K12 STEM Education: Machine Learning

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Course Details

- Course Website
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Tell the class about yourself

- Name
- Grade
- In which city/town are you currently living?
- What is your favourite movie?
- What is the IMDB score of this movie!
- What is the category of this movie? *(thriller/drama/action, etc.)*
- Rate your coding experience from 1 *(no experience)* to 5 *(plenty of experience)*!
- We will visualize this dataset using Python tomorrow! (Link to sheet)

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Outline

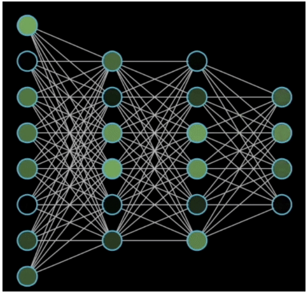
Why is Machine Learning important?

- Most recent exciting technology
- We use these algorithms dozens of times a day:
  - Search Engines
  - Recommendations
- Machine learning is an important component to achieve artificial general intelligence (AGI)

What is Machine Learning

Definition

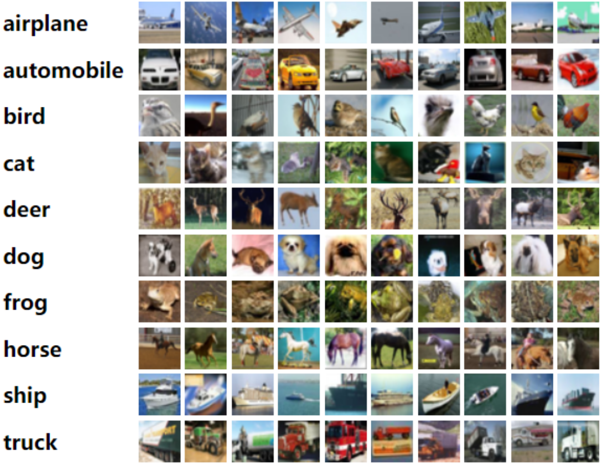
Machine Learning is a field of study that gives computers the ability to learn without being explicitly programmed.



Example: Digit Recognition

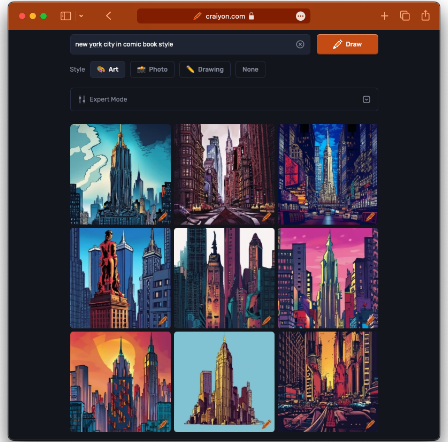
- Challenges with expert approach:
  - Simple expert rule breaks down in practice
  - Difficult to translate our knowledge into code
- Machine Learning approach:
  - Learned systems do very well on image recognition problems

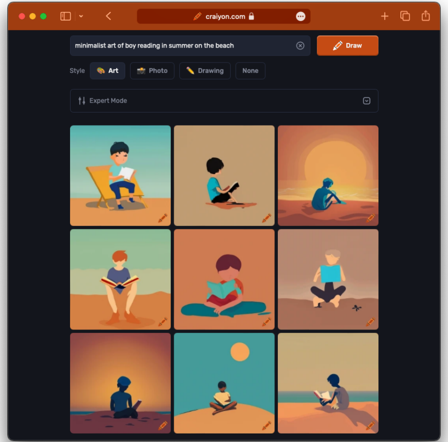
Example: Image Classification



A break to look at cats

Example: Dall.E





Outline

Week 1

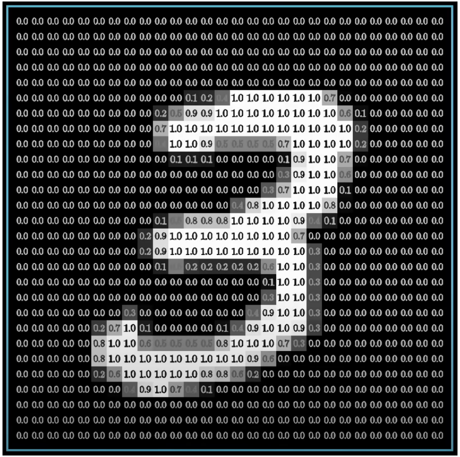
- Day 1 : Introduction to Machine Learning
- Day 2 : Supervised Learning - Linear Regression
- Day 3 : Feature Engineering and Model Evaluation
- Day 4 : Supervised Learning - Classification
- Day 5 : Mini Project

Week 2

- Day 6 : Introduction to Deep Learning and Neural Networks
- Day 7 : Convolutional Neural Networks
- Day 8 : Advanced Deep Learning Topics
- Day 9 : Ethics and Future of AI
- Day 10 : Final Project

Outline

Why do we learn Vectors and Matrices?



Matrices

A matrix is a rectangular array of numbers (or other mathematical objects).

Example:

$$A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \quad M = \begin{bmatrix} 1 & 3 \\ 2 & -1 \\ -4 & 5 \end{bmatrix}$$

Size of a Matrix

The size of a matrix is defined by the number of rows and columns it contains.

Example:

Matrix A has 2 rows and 2 columns, hence the size of the matrix is 2×2

$$A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

Size of a Matrix

Similarly, matrix M is of size $\cdots$?

$$M = \begin{bmatrix} 1 & 3 \\ 2 & -1 \\ -4 & 5 \end{bmatrix}$$

Size of a Matrix

In general, a matrix A of size $m \times n$ is given as,

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

where a_{ij} represents the i^{th} row and j^{th} column element.

Size of a Matrix

$$M = \begin{bmatrix} 1 & 3 \\ 2 & -1 \\ -4 & 5 \end{bmatrix}$$

- $m_{31} = \cdots ?$
- $m_{22} = \cdots ?$

Matrix Addition

Matrices of the same size may be added together, element-wise.

Example:

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 8 \\ 7 & 1 \end{bmatrix}$$

$$\therefore C = A + B = \begin{bmatrix} 1+0 & 1+8 \\ 2+7 & 1+1 \end{bmatrix} = \begin{bmatrix} 1 & 9 \\ 9 & 2 \end{bmatrix}$$

Matrix Subtraction

Similarly, matrices of the same size may be subtracted together, element-wise.

Example:

$$\therefore D = A - B = \begin{bmatrix} 1-0 & 1-8 \\ 2-7 & 1-1 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ -5 & 0 \end{bmatrix}$$

Matrix Scalar Multiplication

Matrices can be scaled by a number. The resulting matrix is computed element-wise. This operation is called scalar multiplication.

Example:

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \quad c = 3$$

$$\therefore c \cdot A = \begin{bmatrix} 1 \times 3 & 1 \times 3 \\ 2 \times 3 & 1 \times 3 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 6 & 3 \end{bmatrix}$$

Matrices Exercise

$$\begin{bmatrix} 1 & 3 \\ 2 & -1 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 9 & 2 \\ -7 & 6 \\ 3 & 1 \end{bmatrix} = \cdots ?$$

$$\begin{bmatrix} 1 & 3 \\ 2 & -1 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 9 & 2 \\ -7 & 6 \\ 3 & 1 \end{bmatrix} = \cdots ?$$

$$2 \cdot \begin{bmatrix} 1 & 9 \\ 3 & -2 \end{bmatrix} = \cdots ?$$

Transpose of a Matrix

The transpose of a matrix is formed by swapping rows and columns.

Example:

$$M = \begin{bmatrix} 1 & 3 \\ 2 & -1 \\ -4 & 5 \end{bmatrix}$$

$$\therefore M^T = \begin{bmatrix} 1 & 2 & -4 \\ 3 & -1 & 5 \end{bmatrix}$$

A transposed matrix is denoted as M^T

Outline

Vectors

Matrices with a single row are called **row vectors**. A row vector is a $1 \times n$ matrix, consisting of a single row of n elements.

Example:

$$U = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

Vectors

Matrices with a single column are called **column vectors**. A column vector is a $n \times 1$ matrix, consisting of a single column of n elements.

(We consider column vectors by default)

Example:

$$V = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Vector Addition

Vectors of the same dimensions may be added together, element-wise.

Example:

$$V = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad W = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$\therefore X = V + W = \begin{bmatrix} 1+4 \\ 2+5 \\ 3+6 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$$

Vector Subtraction

Similarly, vectors of the same dimensions may be subtracted together, element-wise.

Example:

$$\therefore Y = W - V = \begin{bmatrix} 4-1 \\ 5-2 \\ 6-3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

Vector Scalar Multiplication

Vectors can be scaled by a number. The resulting vector is computed element-wise.

Example:

$$V = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad c = 5$$

$$\therefore c \cdot V = \begin{bmatrix} 1 \times 5 \\ 2 \times 5 \\ 3 \times 5 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix}$$

Vector Dot Product

Vector Dot Product (a.k.a. **Inner Product**) is the sum of the products of the corresponding entries of the two vectors.

Example:

$$\therefore V \cdot W = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 1 \times 4 + 2 \times 5 + 3 \times 6 = 4 + 10 + 18 = 32$$

Norm of a Vector

The **norm of a vector** (l^2-norm) for a vector $Z = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ is given as,

$$||Z||_2 = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

The l^2-norm is denoted as $||Z||_2$ or $||Z||$

The **squared norm** is the square of the norm of a vector.

$$||Z||_2^2 = \left(\sqrt{3^2 + 4^2}\right)^2 = 25$$

The squared norm is denoted as $||Z||_2^2$ or $||Z||^2$

Vector Exercise

$$P = \begin{bmatrix} 3 \\ 2 \\ 9 \\ 4 \end{bmatrix} \quad Q = \begin{bmatrix} 1 \\ 9 \\ 0 \\ 3 \end{bmatrix}$$

- $3Q + 2P = \cdots ?$
- $Q \cdot Q = \cdots ?$
- $||Q||^2 = \cdots ?$
- $P \cdot Q = \cdots ?$
- $||P|| \cdot ||Q|| = \cdots ?$

Outline

1. Introduction
2. Course Outline
3. Matrices
4. Vectors
5. Matrix Multiplication
6. Matrix Inverse
7. Python

Matrix Multiplication

Two matrices, A and B, can be multiplied together provided their shapes meet the criteria:

- Number of columns of $A =$ Number of rows of B
- Result is a matrix of shape (Number of rows of $A \times$ Number of columns of B)

Matrix Multiplication

Example:

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 1 \\ 0 & -1 \\ 3 & 1 \end{bmatrix}$$

Size of A is 2×3, size of B is 3×2. Therefore, the resulting matrix C is of size 2×2

$$A \times B = C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

The entries of C are given by the dot product of the corresponding row of A and the corresponding column of B.

Matrix Multiplication

$$c_{11} = \begin{bmatrix} 2 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix} = 2 \times -3 + -1 \times 0 + 0 \times 3 = -6$$

$$c_{12} = \begin{bmatrix} 2 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 2 \times 1 + -1 \times -1 + 0 \times 1 = 3$$

$$c_{21} = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix} = -1 \times -3 + 0 \times 0 + 1 \times 3 = 6$$

$$c_{22} = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = -1 \times 1 + 0 \times -1 + 1 \times 1 = 0$$

Matrix Multiplication

$$\therefore C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} -6 & 3 \\ 6 & 0 \end{bmatrix}$$

To sum up: Multiplication of two matrices is defined if and only if the number of columns of the left matrix is the same as the number of rows of the right matrix. If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then their matrix product $A \times B$ is the $m \times p$ matrix whose entries are given by dot product of the corresponding row of A and the corresponding column of B.

Matrix Multiplication Exercise

$$\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \cdots ?$$

$$\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \times \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \cdots ?$$

$$\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} = \cdots ?$$

Matrix Multiplication

- In general, $A \times B \neq B \times A$
- $(A \times B)^T = B^T \times A^T$

Matrix Multiplication Exercise

$$X = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & -1 \end{bmatrix} \quad Y = \begin{bmatrix} 3 & 1 \\ 0 & -1 \\ -2 & 3 \end{bmatrix} \quad Z = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$$

- $XY = \cdots ?$
- $YX = \cdots ?$
- $Z^T Y = \cdots ?$

Identity Matrix

- An identity matrix of size n is a $n \times n$ square matrix with ones on the main diagonal and zeros elsewhere.
- When an identity matrix is multiplied with another matrix A, the result is equal to A. It is analogous to multiplying a number by 1.

Example: Identity matrix of size 2 is given as,

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Identity Matrix

So, if we have matrix A,

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

$$\therefore I \times A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = A$$

Outline

Inverse of a Matrix

A inverse of a matrix, denoted as A^{-1}, is a matrix such that it satifies the following condition:

$$A \times A^{-1} = A^{-1} \times A = I$$

where A is a $n \times n$ invertible matrix, and I is the identity matrix of size n.

Inverse of a Matrix

Think of it like a reciprocal of a number.

A number x and it's reciprocal is given as $x^{-1} = \frac{1}{x}$ are multiplied together, the result is 1.

For example, $x = 2$

$$x \times x^{-1} = x \times \frac{1}{x} = 2 \times \frac{1}{2} = 1$$

Inverse of a Matrix

Inverse of a matrix is hard to compute by hand. But for a 2×2 size matrix, the formula is given as,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{a \cdot d - b \cdot c} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

The matrix inverse does not always exist. Can you tell when that is the case for 2×2 matrices based on the formula given above?

Inverse of Matrix Application

When is matrix inverse useful? We can use it to solve systems of linear equations!

Consider the following equations:

$$x + 2y = 5$$
$$3x + 5y = 13$$

This can be written in a matrix form as,

$$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \end{bmatrix}$$

Inverse of Matrix Application

$$\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}^{-1}\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}^{-1}\begin{bmatrix}5\\13\end{bmatrix}$$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}^{-1}\begin{bmatrix}5\\13\end{bmatrix}$$

$$\therefore \begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}^{-1}\begin{bmatrix}5\\13\end{bmatrix}$$

Now, we can easily solve the system of equations and get the solution for x and y

Outline

Setting up Python

- Google Colab:
  - Interactive programming online
  - No installation
  - Free GPU for 12 hours
- Your task:
  - Register a Google account and set up Google Colab
  - Run `print('hello world!')`
  - Open Python Basics Demo from Course Website

Python Basics

- Program
  - We write operations to be executed on variables
- Variables
  - Referencing and interacting with items in the program
- If-Statements
  - Conditionally execute lines of code
- Functions
  - Reuse lines of code at any time

Python Basics

- Lists
  - Store an ordered collection of data
- Loops
  - Conditionally re-execute code
- Strings
  - Words and sentences are treated as lists of characters
- Classes (advanced)
  - Making your own data-type. Functions and variables made to be associated with it too.

Python Basics

- Write a function to find the second largest number in a list (Hint: use `sort()`)
- Define a class `Student`
- Use the `__init()__` function to assign the values of two attributes of the class: `name` and `grade`
- Define a function `study()` with an argument time in minutes. When calling this function, it should be printed "(the student's name) has studied for (time) minutes"

NumPy Basics

A Python package is a collection of code people wrote for other users to run directly. Today, we learn how to use the package NumPy for linear algebra.

- Open NumPy Basics Demo from Course Website
- Your task: use NumPy functions to compute the exercises we did earlier this morning. Compare the results.