CENGAGE | WEBASSIGN

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← CIS-021-10673-202510, Spring 2025

4.3: Direct Proof & Counterexp III: Rational Numb (Homework)

INSTRUCTOR
Brian Weathersby
Solano College, CA

Current Score

QUESTION	1	2	3	4	5	6	7	8	9	10
POINTS	0.06/0.06	–/0.06	0.06/0.06	0.06/0.06	0/0.06	0.05/0.05	0.05/0.05	0.03/0.06	0/0.06	–/0.05

TOTAL SCORE

0.3/0.67 **44.8%**

Due Date Past Due

SAT, FEB 15, 2025
8:59 PM PST

REQUEST EXTENSION

Assignment Submission & Scoring

Assignment Submission

For this assignment, you submit answers by question parts. The number of submissions remaining for each question part only changes if you submit or change the answer.

Assignment Scoring

Your last submission is used for your score.

The due date for this assignment has passed.

Your work can be viewed below, but no changes can be made.

Important! Before you view the answer key, decide whether or not you plan to request an extension. Your Instructor may not grant you an extension if you have viewed the answer key. Automatic extensions are not granted if you have viewed the answer key.

REQUEST EXTENSION

1. **[0.06/0.06 Points]** DETAILS MY NOTES **EPPDISCMATH5 4.3.001.**

PREVIOUS ANSWERS ASK YOUR TEACHER PRACTICE ANOTHER

Show that the following number is rational by writing it as a ratio of two integers.

$$-\frac{41}{7}$$

-41/7 ✓ -41/7

Need Help? Read It

2. **[–/0.06 Points]** DETAILS MY NOTES **EPPDISCMATH5 4.3.002.**

ASK YOUR TEACHER PRACTICE ANOTHER

Show that the following number is rational by writing it as a ratio of two integers.

3.8033

(No Response) 38033/10000

Need Help? Read It

3. **[0.06/0.06 Points]** DETAILS MY NOTES **EPPDISCMATH5 4.3.003.**

PREVIOUS ANSWERS ASK YOUR TEACHER PRACTICE ANOTHER

Show that the following number is rational by writing it as a ratio of two integers.

$$\frac{4}{7} + \frac{2}{5}$$

34/35 ✓ 34/35

Need Help? Read It

4. **[0.06/0.06 Points]** DETAILS MY NOTES **EPPDISCMATH5 4.3.005.**

PREVIOUS ANSWERS ASK YOUR TEACHER PRACTICE ANOTHER

Show that the following number is rational by writing it as a ratio of two integers.

0.59595959...

59/99 ✓ 59/99

Need Help? Read It

5. **[0/0.06 Points]** DETAILS MY NOTES **EPPDISCMATH5 4.3.007.**

PREVIOUS ANSWERS ASK YOUR TEACHER PRACTICE ANOTHER

Show that the following number is rational by writing it as a ratio of two integers.

52.4681168116811...

52468/999 ✗ 5246287/99990

Need Help? Read It

6. **[0.05/0.05 Points]** DETAILS MY NOTES **EPPDISCMATH5 4.3.008.**

PREVIOUS ANSWERS ASK YOUR TEACHER

The zero product property says that if a product of two real numbers is 0, then one of the numbers must be 0.

(a) Which of the following expresses the zero product property formally using quantifiers and variables?

- ∀ real numbers x and y, if $x = 0$ or $y = 0$ then $xy = 0$.
- ∀ real numbers x and y, if $x = 0$ and $y = 0$ then $xy = 0$.
- ∀ real numbers x and y, if $xy = 0$ then $x = 0$ and $y = 0$.
- (selected) ∀ real numbers x and y, if $xy = 0$ then $x = 0$ or $y = 0$.

(b) Which of the following is the contrapositive of the zero product property?

- (selected) ∀ real numbers x and y, if $x \neq 0$ and $y \neq 0$ then $xy \neq 0$.
- ∀ real numbers x and y, if $x \neq 0$ or $y \neq 0$ then $xy \neq 0$.
- ∀ real numbers x and y, if $xy \neq 0$ then $x \neq 0$ and $y \neq 0$.
- ∀ real numbers x and y, if $xy \neq 0$ then $x \neq 0$ or $y \neq 0$.

(c) Which of the following is an informal version (without quantifier symbols or variables) for the contrapositive of the zero product property?

- If the product of two real numbers is nonzero, then both numbers are nonzero.
- For any two real numbers, if at least one of them is nonzero then their product is nonzero.
- (selected) If neither of two real numbers is zero, then their product is nonzero.
- If the product of two real numbers is nonzero, then neither number is zero.

Need Help? Read It

7. **[0.05/0.05 Points]** DETAILS MY NOTES **EPPDISCMATH5 4.3.014.**

PREVIOUS ANSWERS ASK YOUR TEACHER PRACTICE ANOTHER

Consider the following statement.

The cube of any rational number is a rational number.

(a) Write the statement formally using a quantifier and a variable.

$\forall r$ ✔ $\forall r$ if r is a rational number ✔ r is a rational number , then r^3 is a rational number ✔ r^3 is a rational number .

(b) Construct a proof for the statement by selecting sentences from the following scrambled list and putting them in the correct order.

Therefore, r^3 is a quotient of integers with a nonzero denominator, and so r^3 is rational.

Suppose r is the cube of any rational number.

By substitution, $r^3 = \left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$.

Suppose r is any rational number.

By definition of rational, $r = \frac{a}{b}$ where a and b are integers and $b \neq 0$.

Now a^3 and b^3 are integers because products of integers are integers, and $b^3 \neq 0$ by the zero product property.

Proof:

1. Suppose r is any rational number. ✔ Suppose r is any rational number.
2. By definition of rational, $r = a/b$ where a and b are integers and $b \neq 0$. ✔ By definition of rational, $r = a/b$ where a and b are integers and $b \neq 0$.
3. By substitution, $r^3 = (a/b)^3 = (a^3)/(b^3)$. ✔ By substitution, $r^3 = (a/b)^3 = (a^3)/(b^3)$.
4. Now a^3 and b^3 are integers because products of integers are integers, and $b^3 \neq 0$ by the zero product property. ✔ Now a^3 and b^3 are integers because products of integers are integers, and $b^3 \neq 0$ by the zero product property.
5. Therefore, r^3 is a quotient of integers with a nonzero denominator, and so r^3 is rational. ✔ Therefore, r^3 is a quotient of integers with a nonzero denominator, and so r^3 is rational.

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8. **[0.03/0.06 Points]** DETAILS MY NOTES **EPPDISCMATH5 4.3.017.**

PREVIOUS ANSWERS ASK YOUR TEACHER

Prove the following statement directly from the definition of rational number.

The difference of any two rational numbers is a rational number.

Proof: Suppose r and s are any two rational numbers. By definition of rational, $r = \frac{a}{b}$ and $s = \frac{c}{d}$ for some real numbers ✗ integers a, b, c, and d with $a + c \neq 0$ ✗ $b \neq 0$ and $d \neq 0$.

Write $r - s$ in terms of a, b, c, and d as a quotient of two integers whose numerator and denominator are simplified as much as possible. The result is the following.

$r - s =$
$$(ad-bc)bd

✓ $\frac{ad - bc}{bd}$

Both the numerator and the denominator are integers because products and differences of integers are rational numbers ✗ products and differences of integers are integers .

In addition, $bd \neq 0$ by the zero product property ✓ zero product property .

Hence $r - s$ is a quotient ✓ quotient of two integers with a nonzero denominator, and so by definition of rational, $r - s$ is rational.

Need Help? Read It Watch It

9. **[0/0.06 Points]** DETAILS MY NOTES **EPPDISCMATH5 4.3.022.**

PREVIOUS ANSWERS ASK YOUR TEACHER PRACTICE ANOTHER

Consider the following statement.

If a is any odd integer, then $a^2 + a$ is even.

Use the properties of even and odd integers listed in Example 4.3.3 and repeated below to evaluate whether the statement is true or false. Indicate which properties you use to justify your reasoning.

1. The sum, product, and difference of any two even integers are even.
2. The sum and difference of any two odd integers are even.
3. The product of any two odd integers is odd.
4. The product of any even integer and any odd integer is even.
5. The sum of any odd integer and any even integer is odd.
6. The difference of any odd integer minus any even integer is odd.
7. The difference of any even integer minus any odd integer is odd.

- ( ) The statement is true. $a^2 = a \cdot a$ is a product of odd integers and thus is even by property 4. Therefore, $a^2 + a$ is a sum of an even and an odd integer and thus is even by property 2.
- ( ) The statement is true. $a^2 = a \cdot a$ is a product of odd integers and thus is odd by property 3. Therefore, $a^2 + a$ is a sum of odd integers and thus is even by property 2.
- ( ) The statement is false. $a^2 = a \cdot a$ is a product of odd integers and thus is odd by property 3. Therefore, $a^2 + a$ is a sum of odd integers and thus is odd by property 4.
- (•) The statement is false. $a^2 = a \cdot a$ is a product of odd integers and thus is even by property 4. Therefore, $a^2 + a$ is a sum of an even and an odd integer and thus is odd by property 5.

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10. **[–/0.05 Points]** DETAILS MY NOTES **EPPDISCMATH5 4.3.025.** ASK YOUR TEACHER

Prove that if r is any rational number, then $3r^2 - 2r + 4$ is rational.

The following properties may be used in your proof.

Property 1:

Every integer is a rational number.

Property 2:

The sum of any two rational numbers is rational.

Property 3:

The product of any two rational numbers is rational.

Note: Property 1 is Theorem 4.3.1, Property 2 is Theorem 4.3.2, and Property 3 is Exercise 15 in Section 4.3.

Using these properties, choose explanations for each step in the given proof.

Statement	Explanation
Suppose r is a rational number.	Starting point.
3, −2, 4 are rational numbers.	*(No Response)* Property 1
r^2 is a rational number.	*(No Response)* Property 3
$3r^2$ and $-2r$ are rational numbers.	*(No Response)* Property 3
$3r^2 - 2r = 3r^2 + (-2)r$ is a rational number.	*(No Response)* Property 2
Therefore, $3r^2 - 2r + 4$ is a rational number.	*(No Response)* Property 2

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11. **[–/0.06 Points]** DETAILS MY NOTES **EPPDISCMATH5 4.3.030.** ASK YOUR TEACHER

Consider a quadratic equation of the form $x^2 + bx + c = 0$, where b and c are rational numbers. Fill in the blanks in the following proof that if one solution is rational, then the other solution is also rational.

Proof: Suppose $x^2 + bx + c = 0$ is any quadratic equation where b and c are rational numbers, and suppose one solution r is rational. Call the other solution s. Then $x^2 + bx + c = (x - r)(x - s)$. Multiply out $(x - r)(x - s)$ and set it equal to $x^2 + bx + c$ to obtain $x^2 + bx + c = x^2 +$ ((No Response) $-r - s$)x + (No Response) rs . Equate coefficients and solve for s in terms of b and r to obtain s = (No Response) $-b - r$. Since (No Response) b and r are (No Response) rational and since (No Response) differences of rational numbers are rational, we conclude that (No Response) $-b - r$ is rational, and so s is rational.

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12. **[0/0.06 Points]** DETAILS MY NOTES **EPPDISCMATH5 4.3.038.**

PREVIOUS ANSWERS ASK YOUR TEACHER

Consider the following statement.

The sum of any two rational numbers is a rational number.

The statement is true, but the following proposed proof is incorrect.

Proposed proof:

1. Suppose r and s are any rational numbers.
2. By definition of rational, $r = \frac{a}{b}$ and $s = \frac{c}{d}$ for some integers a, b, c, and d with $b \neq 0$ and $d \neq 0$.
3. By substitution, $r + s = \frac{a}{b} + \frac{c}{d}$.
4. Thus $r + s$ is a sum of two fractions, which is a fraction.
5. So $r + s$ is a rational number since a rational number is a fraction.

Identify the error(s) in the proposed proof. (Select all that apply.)

- [ ] The second sentence should say $b = 0$ and $d = 0$ instead of $b \neq 0$ and $d \neq 0$.
- [ ] The fourth sentence assumes that a sum of two fractions is a fraction, which is equivalent to assuming what is to be proved.
- [ ] The first sentence claims that r and s are rational numbers, which is equivalent to assuming what is to be proved.
- [ ] The second sentence should say $a \neq 0$ and $c \neq 0$ instead of $b \neq 0$ and $d \neq 0$.
- [x] To prove the statement, r and s must have the same denominator.

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