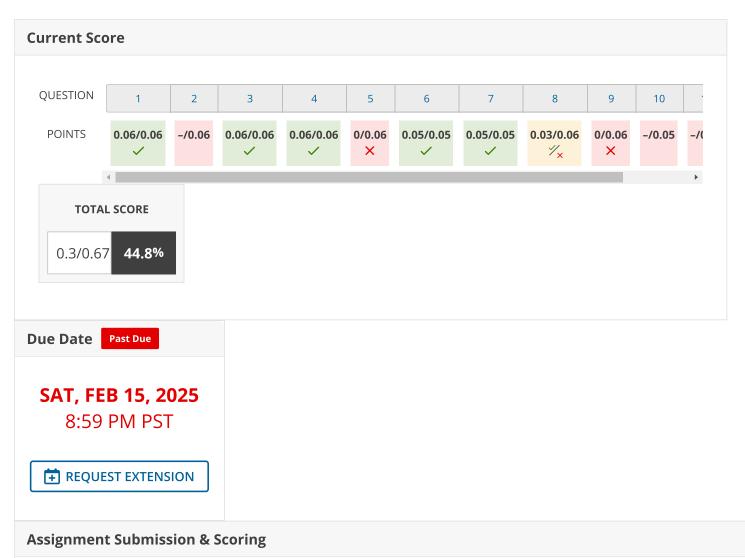


← CIS-021-10673-202510, Spring 2025

# 4.3: Direct Proof & Counterexp III: Rational Numb (Homework)

INSTRUCTOR
Brian
Weathersby
Solano College, CA



## **Assignment Submission**

For this assignment, you submit answers by question parts. The number of submissions remaining for each question part only changes if you submit or change the answer.

# **Assignment Scoring**

Your last submission is used for your score.

# The due date for this assignment has passed.

Your work can be viewed below, but no changes can be made.

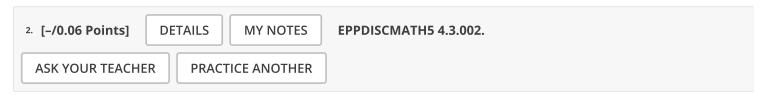
**Important!** Before you view the answer key, decide whether or not you plan to request an extension. Your Instructor may not grant you an extension if you have viewed the answer key. Automatic extensions are not granted if you have viewed the answer key.



1. [0.06/0.06 Points] DETAILS MY NOTES EPPDISCMATH5 4.3.001.

PREVIOUS ANSWERS ASK YOUR TEACHER PRACTICE ANOTHER

Show that the following number is rational by writing it as a ratio of two integers.



Show that the following number is rational by writing it as a ratio of two integers.



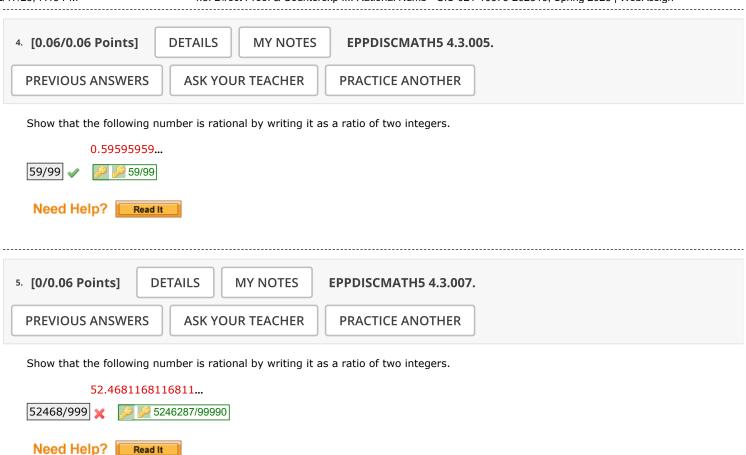


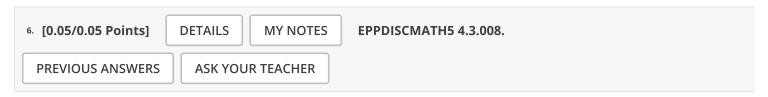
Show that the following number is rational by writing it as a ratio of two integers.

$$\frac{4}{7} + \frac{2}{5}$$
34/35  $\checkmark$  34/35

Need Help? Read It

Read It



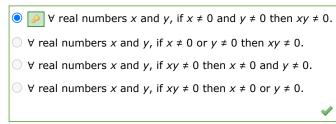


The zero product property says that if a product of two real numbers is 0, then one of the numbers must be 0.

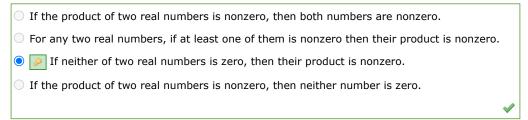
(a) Which of the following expresses the zero product property formally using quantifiers and variables?

$\bigcirc$ $\forall$ real numbers $x$ and $y$ , if $x = 0$ or $y = 0$ then $xy = 0$ .
$\bigcirc$ $\forall$ real numbers $x$ and $y$ , if $x = 0$ and $y = 0$ then $xy = 0$ .
$\bigcirc$ $\forall$ real numbers $x$ and $y$ , if $xy = 0$ then $x = 0$ and $y = 0$ .
$\bullet$ $\forall$ real numbers $x$ and $y$ , if $xy = 0$ then $x = 0$ or $y = 0$ .
<b>✓</b>

(b) Which of the following is the contrapositive of the zero product property?



(c) Which of the following is an informal version (without quantifier symbols or variables) for the contrapositive of the zero product property?



Need Help? Read It



Consider the following statement.

The cube of any rational number is a rational number.

(a) Write the statement formally using a quantifier and a variable.

(b) Construct a proof for the statement by selecting sentences from the following scrambled list and putting them in the correct order.

Therefore,  $r^3$  is a quotient of integers with a nonzero denominator, and so  $r^3$  is rational.

Suppose r is the cube of any rational number.

By substitution, 
$$r^3 = \left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$
.

Suppose r is any rational number.

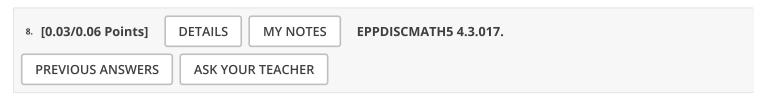
By definition of rational,  $r = \frac{a}{b}$  where a and b are integers and  $b \neq 0$ .

Now  $a^3$  and  $b^3$  are integers because products of integers are integers, and  $b^3 \neq 0$  by the zero product property.

#### Proof:

- 1. Suppose r is any rational number.  $\checkmark$  Suppose r is any rational number.
- 2. By definition of rational, r = a/b where a and b are integers and  $b \neq 0$ .  $\checkmark$  By definition of rational, r = a/b where a and b are integers and  $b \neq 0$ .
- 3. By substitution,  $r^3 = (a/b)^3 = (a^3)/(b^3)$ . By substitution,  $r^3 = (a/b)^3 = (a^3)/(b^3)$ .
- 4. Now  $a^3$  and  $b^3$  are integers because products of integers are integers, and  $b^3 \neq 0$  by the zero product property. Now  $a^3$  and  $b^3$  are integers because products of integers are integers, and  $b^3 \neq 0$  by the zero product property.
- Therefore,  $r^3$  is a quotient of integers with a nonzero denominator, and so  $r^3$  is rational. Therefore,  $r^3$  is a quotient of integers with a nonzero denominator, and so  $r^3$  is rational.

Need Help? Read It

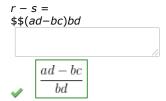


Prove the following statement directly from the definition of rational number.

The difference of any two rational numbers is a rational number.

**Proof**: Suppose r and s are any two rational numbers. By definition of rational,  $r = \frac{a}{b}$  and  $s = \frac{c}{d}$  for some real numbers  $x = \frac{c}{d}$  integers a, b, c, and d with  $a + c \neq 0$   $x = \frac{c}{d}$  for some a and a integers a, b, a, a, and a with a integers a, a, a, a, a integers a, a, a, a, a integers a, a, a, a, a integers a, a, a integers a, a, a integers a, a integers a integers a, a, a integers a integers

Write r - s in terms of a, b, c, and d as a quotient of two integers whose numerator and denominator are simplified as much as possible. The result is the following.



Both the numerator and the denominator are integers because

products and differences of integers are rational numbers 🗶 🔑 products and differences of integers are integers

In addition,  $bd \neq 0$  by the zero product property  $\checkmark$   $\nearrow$  zero product property .

Hence r-s is a quotient  $\checkmark$  Quotient of two integers with a nonzero denominator, and so by definition of rational, r-s is rational.

Need Help? Read It Watch It



Consider the following statement.

If a is any odd integer, then  $a^2 + a$  is even.

Use the properties of even and odd integers listed in Example 4.3.3 and repeated below to evaluate whether the statement is true or false. Indicate which properties you use to justify your reasoning.

- 1. The sum, product, and difference of any two even integers are even.
- 2. The sum and difference of any two odd integers are even.
- 3. The product of any two odd integers is odd.
- 4. The product of any even integer and any odd integer is even.
- 5. The sum of any odd integer and any even integer is odd.
- 6. The difference of any odd integer minus any even integer is odd.
- 7. The difference of any even integer minus any odd integer is odd.
- The statement is true. a² = a · a is a product of odd integers and thus is even by property 4. Therefore, a² + a is a sum of an even and an odd integer and thus is even by property 2.
  The statement is true. a² = a · a is a product of odd integers and thus is odd by property 3. Therefore, a² + a is a sum of odd integers and thus is even by property 2.
  The statement is false. a² = a · a is a product of odd integers and thus is odd by property 3. Therefore, a² + a is a sum of odd integers and thus is odd by property 4.
  The statement is false. a² = a · a is a product of odd integers and thus is even by property 4. Therefore, a² + a is a sum of an even and an odd integer and thus is odd by property 5.

Need Help? Read It

10. [-/0.05 Points]

**DETAILS** 

**MY NOTES** 

EPPDISCMATH5 4.3.025.

**ASK YOUR TEACHER** 

Prove that if r is any rational number, then  $3r^2 - 2r + 4$  is rational.

The following properties may be used in your proof.

# Property 1:

Every integer is a rational number.

# Property 2:

The sum of any two rational numbers is rational.

# Property 3:

The product of any two rational numbers is rational.

Note: Property 1 is Theorem 4.3.1, Property 2 is Theorem 4.3.2, and Property 3 is Exercise 15 in Section 4.3.

Using these properties, choose explanations for each step in the given proof.

Statement	Explanation
Suppose $r$ is a rational number.	Starting point.
3, -2, 4 are rational numbers.	(No Response) Property 1
$r^2$ is a rational number.	(No Response) Property 3
$3r^2$ and $-2r$ are rational numbers.	(No Response) Property 3
$3r^2 - 2r = 3r^2 + (-2)r$ is a rational number.	(No Response) Property 2
Therefore, $3r^2 - 2r + 4$ is a rational number.	(No Response) Property 2

Need Help?

Read It

11. [-/0.06 Points]

**DETAILS** 

**MY NOTES** 

EPPDISCMATH5 4.3.030.

**ASK YOUR TEACHER** 

Consider a quadratic equation of the form  $x^2 + bx + c = 0$ , where b and c are rational numbers. Fill in the blanks in the following proof that if one solution is rational, then the other solution is also rational.

Need Help?

Read It

12. [0/0.06 Points] DETAILS MY NOTES EPPDISCMATH5 4.3.038.

PREVIOUS ANSWERS

**ASK YOUR TEACHER** 

Consider the following statement.

The sum of any two rational numbers is a rational number.

The statement is true, but the following proposed proof is incorrect.

### Proposed proof:

- 1. Suppose r and s are any rational numbers.
- 2. By definition of rational,  $r=\frac{a}{b}$  and  $s=\frac{c}{d}$  for some integers a,b,c, and d with  $b\neq 0$  and  $d\neq 0$ .
- 3. By substitution,  $r + s = \frac{a}{h} + \frac{c}{d}$ .
- 4. Thus r + s is a sum of two fractions, which is a fraction.
- 5. So r + s is a rational number since a rational number is a fraction.

Identify the error(s) in the proposed proof. (Select all that apply.)

 $\Box$  The second sentence should say b=0 and d=0 instead of  $b\neq 0$  and  $d\neq 0$ .

The fourth sentence assumes that a sum of two fractions is a fraction, which is equivalent to assuming what is to be proved.

 $\square$  The first sentence claims that r and s are rational numbers, which is equivalent to assuming what is to be proved.

The second sentence should say  $a \neq 0$  and  $c \neq 0$  instead of  $b \neq 0$  and  $d \neq 0$ .

 $\checkmark$  To prove the statement, r and s must have the same denominator.

Need Help?

Read It

Home My Assignments

Request Extension

Copyright © 1998 - 2025 Cengage Learning, Inc. All Rights Reserved TERMS OF USE PRIVACY