4.25

$$\alpha = \begin{pmatrix} 1 & 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 3 & 2 & 1 & 4 & 2 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$\rightarrow$ $-R_1$
$\quad\;\; -3R_1$
$$\begin{pmatrix} 1 & 0 & 1 & 2 & 0 \\ 0 & 0 & -1 & -2 & 1 \\ 0 & 2 & -2 & -2 & 2 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$\rightarrow$
$\quad\;\; -2R_4$
$$\begin{pmatrix} 1 & 0 & 1 & 2 & 0 \\ 0 & 0 & -1 & -2 & 1 \\ 0 & 0 & -2 & -4 & 2 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$\rightarrow$
$\quad \div 2$
$$\begin{pmatrix} 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -2 & 1 \\ 0 & 0 & -1 & -2 & 1 \end{pmatrix}$$

$\rightarrow$
$\quad\;\; x-1$
$\quad\;\; -R_3$
$$\begin{pmatrix} 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\quad\;\; -R_3$
$\rightarrow$
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{pmatrix}$$

$\underbrace{\qquad\qquad\qquad}_{\text{rank } 3}$

$\therefore$ Max # rxns $= 5 - 3 = 2$

$$\begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{pmatrix} = - \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 2 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} s_4 \\ s_5 \end{pmatrix}$$

$$\Rightarrow \quad s_1 = -s_5$$
$$s_2 = -s_4$$
$$s_3 = -2s_4 + s_5$$
$$s_4 = s_4 \qquad \Rightarrow r_1$$
$$s_5 = s_5 \qquad \Rightarrow r_2$$

$$\Rightarrow \quad s_2 + 2s_3 \longrightarrow s_4$$

$$s_1 \longrightarrow s_3 + s_5$$