NCERT Solutions for Class 11

Physics

Chapter 13 – Oscillations

1. Which of the following examples represent periodic motion?

(a) A swimmer completing one (return) trip from one bank of a river to the other and back.

Ans: As motion of the swimmer between the banks of the river is to and fro, it does not have a definite period. The time taken by the swimmer during his back-and-forth journey may not be the same. Hence, the swimmer's motion is not periodic.

(b) A freely suspended bar magnet displaced from its N-S direction and released.

Ans: If a magnet is displaced from its N-S direction and released, then the motion of the freely-suspended magnet is periodic. This is because the magnet oscillates about its position with a definite period of time.

(c) A hydrogen molecule rotating about its centre of mass.

Ans: If we consider a hydrogen molecule rotating about its centre of mass, it is observed that it comes to the same position after an equal interval of time. This type of motion is periodic motion.

(d) An arrow released from a bow.

Ans: When an arrow is released from a bow, it moves only in the forward direction. There is no motion repeated in equal intervals of time. Therefore, this motion is not periodic.

2. Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?

(a) the rotation of earth about its axis.

Ans: When the earth rotates about its axis, it comes to the same position in fixed intervals of time. Hence, it is a periodic motion. However, earth does not have a to and fro motion about its axis. Hence, it is not simple harmonic.

(b) motion of an oscillating mercury column in a U-tube.

Ans: In an oscillating mercury column in a U-tube, mercury moves to and from on the same path, about the fixed position, with a certain period of time. Hence, it is a simple harmonic motion.

(c) motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.

Ans: When a ball is released from a point slightly above the lower most point, it moves to and from about the lowermost point of the bowl. Also, the ball comes back to its initial position in the fixed interval of time, again and again. Thus, this motion is periodic as well as simple harmonic.

(d) general vibrations of a polyatomic molecule about its equilibrium position.

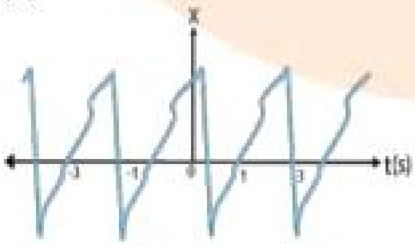
Ans: A polyatomic molecule possesses many natural frequencies of oscillation. Its vibration is the superposition of individual simple harmonic motions of a number of different molecules. Thus, it is not simple harmonic, but periodic.

3. Figure depicts four x-t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?

(a)

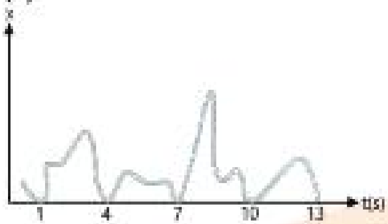
Ans: Graph in figure (a) is not a periodic motion. It is a unidirectional, linear uniform motion. Also, there is no repetition of motion in this case.

(b)



Ans: Graph in figure (b) is a periodic motion. The motion of the particle repeats itself after 2s. Thus, it is a periodic motion with a period of 2s.

(c)



Ans: Graph in figure (c) is not a periodic motion as the particle repeats the motion in one position only. For a periodic motion, the entire motion of the particle must be repeated in equal intervals of time.

(d)

Fig. 14.27

Ans: Graph in figure (d), the motion of the particle repeats itself after 2s. Thus, it is a periodic motion with a period of 2s.

4. Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):

(a) $\sin\omega t - \cos\omega t$

Ans: The given function is:

$\sin\omega t - \cos\omega t$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right]$$

$$= \sqrt{2}\left[\sin\omega t \times \cos\frac{\pi}{4} - \cos\omega t \sin\frac{\pi}{4}\right]$$

$$= \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

This function is SHM as it can be written in the form of $a\sin(\omega t + \phi)$. Its period is $2\pi/\omega$.

(b) $\sin^3\omega t$

Ans: The given function is:

$\sin^3 \omega t = \frac{1}{4}[3\sin \omega t - \sin 3\omega t]$

From the above equation it is clear that $\sin \omega t$ and $\sin 3\omega t$ represent simple harmonic motion individually and hence does their combination. Hence, given function is periodic but not simple harmonic.

(c) $3\cos(\pi / 4 - 2\omega t)$

Ans: The given function is:

$3\cos(\pi / 4 - 2\omega t) = 3\cos(2\omega t - \pi / 4)$

This function represents simple harmonic motion because it can be written in the form $a\cos(\omega t + \phi)$

Period $= \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

(d) $\cos \omega t + \cos 3\omega t + \cos 5\omega t$

Ans: The given function is:

$\cos \omega t + \cos 3\omega t + \cos 5\omega t$

From the above equation it is clear that $\cos \omega t$, $\cos 2\omega t$ and $\cos 3\omega t$ represent simple harmonic motion individually and hence does their combination. Hence, given function is periodic but not simple harmonic

(e) $\exp(-\omega^2 t^2)$

Ans: The given function is:

$\exp(-\omega^2 t^2)$ which is an exponential function. As exponential functions do not repeat themselves, hence it is a non-periodic motion.

(f) $1 + \omega t + \omega^2 t^2$

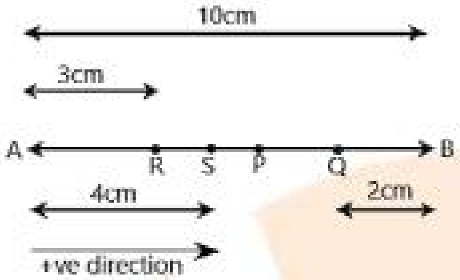
Ans: The given function is:

$1 + \omega t + \omega^2 t^2$

It is non-periodic.

5. A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

Ans: Consider the figure given in the question. The two extreme positions, A and B are of a SHM. The positive direction of velocity is considered from A to B. The acceleration and the force, along AP are positive and BP are negative.



(a) at the end A,

$\Rightarrow$ The particle that is executing SHM is momentarily at rest being its extreme position of motion at the end A. Hence, its velocity is zero. Acceleration is positive as it is directed along AP, force is also positive as the force is directed along AP.

(b) at the end B,

$\Rightarrow$ Velocity is zero at the end B. As acceleration and force are directed along BP, hence they are negative.

(c) at the mid-point of AB going towards A,

$\Rightarrow$ Along the direction towards A, at the midpoint of AB, the particle is at its mean position P and has a tendency to move along PA. Thus, velocity is positive. Both acceleration and force are zero.

(d) at 2 cm away from B going towards A,

$\Rightarrow$ The position of particle at 2 cm away from B going towards A is at Q. At this position it has the tendency to move along QP, which is negative direction. Therefore, velocity, acceleration and force all are positive.

(e) at 3 cm away from A going towards B, and

$\Rightarrow$ The position of particle at 3 cm away from A going towards B is at R. It has a tendency to move along RP, which is positive direction. Here, velocity, acceleration all are positive.

(f) at 4 cm away from B going towards A.

$\Rightarrow$ The position of particle at 4 cm away from A going towards A, is at S. It has a tendency to move along SA, which is negative direction. Thus, velocity is negative but acceleration is directed towards mean position, along SP. Hence it is positive and also force is positive similarly.

6. Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?

(a) $a = 0.7x$
(b) $a = -200x^2$
(c) $a = -10x$
(d) $a = 100x^3$

Ans: For SHM, acceleration and displacement are related by the relation (c) $a = -10x$ (of the form $a = -kx$).

7. The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t) = A\cos(\omega t + \varphi)$. If the initial $(t = 0)$ position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is πs^{-1}. If instead of the cosine function, we choose the sine function to describe the SHM: $x(t) = B\sin(\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions?

Ans: In the above question it is given that:
Displacement at $t = 0$, is $x = 1\text{cm}$.
Initial velocity, $v = \omega$ cm/sec.
We have,

$x(t) = A\cos(\omega t + \varphi)$

$1 = A\cos(\omega \times 0 + \varphi) = A\cos\varphi$

$A\cos\varphi = 1$ ..... (i)

Velocity, $v = \frac{dx}{dt}$

$\omega = -A\omega\sin(\omega t + \varphi)$

$1 = A\sin(\omega \times 0 + \varphi) = -A\sin\varphi$

$A\sin\varphi = -1$ ...... (ii)

Squaring and adding equations (i) and (ii), we get:

$A^2(\sin^2\varphi + \cos^2\varphi) = 1 + 1 \Rightarrow A^2 = 2 \Rightarrow A = \sqrt{2}\text{cm}$

Dividing equation (ii) by equation (i), we get:

$\tan\varphi = -1$

$\Rightarrow \varphi = 3\pi/4, 7\pi/4.....$

SHM is given by:

$x(t) = B\sin(\omega t + \alpha)$

Substituting the given values in this equation, we get:

$1 = B\sin[\omega \times 0 + \alpha] = 1 + 1$

$B\sin\alpha = 1$ ...... (iv)

Squaring and adding equations (iii) and (iv), we get:

$B^2(\sin^2\alpha + \cos^2\alpha) = 1 + 1 \Rightarrow B^2 = 2 \Rightarrow B = \sqrt{2}\text{cm}$

Dividing equation (iii) by equation (iv), we get:

$\tan\alpha = 1 \Rightarrow \alpha = \pi/4, 5\pi/4.....$

8. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

Ans: In the above question it is given that:

Maximum mass that the scale can read is $M = 50$ kg.

Maximum displacement of the spring

$=$ Length of the scale, $l = 20$ cm $= 0.2$ m

Time period, $T = 0.6$ s

Maximum force exerted on the spring is $F = Mg$.

where,

g = acceleration due to gravity $= 9.8\ \text{m/s}^2$

$F = 50 \times 9.8 = 490$ N

$\therefore$ Spring constant, $k = \frac{F}{l} = \frac{490}{0.2} = 2450\text{Nm}^{-1}$.

Mass m, is suspended from the balance.

Time period,

$$T = t = 2\pi\sqrt{\frac{m}{k}}$$

$$\Rightarrow m = \left(\frac{T}{2\pi}\right) \times k = \left(\frac{0.6}{2 \times 3.14}\right)^2 \times 2450 = 22.36\text{kg}$$

Weight of the body $= mg = 22.36 \times 9.8 = 219.167$ N

Therefore, the weight of the body is 219 N.

9. A spring having with a spring constant $1200\ \text{N m}^{-1}$ is mounted on a horizontal table as shown in figure. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released.

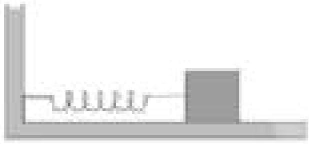


Fig 14.28

Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass.

Ans: In the above question it is given that:

Spring constant is $k = 1200 Nm^{-1}$

Mass, $m = 3$ kg

Displacement, $A = 2.0$ cm $= 0.02$ cm

(i)Frequency of oscillation v, is given by:

$$\upsilon = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Here, T is time period.

$$\Rightarrow \upsilon = \frac{1}{2\pi}\sqrt{\frac{1200}{3}} = 3.18 m/s$$

Thus, the frequency of oscillations is 3.18 cycles per second.

(ii) Maximum acceleration (a) is given by:

$a = \omega^2 A$

Where,

A= maximum displacement

$$\alpha = \frac{k}{m}A = \frac{1200 \times 0.02}{3} = 8 ms^{-2}$$

Thus, the maximum acceleration of the mass is $8 ms^{-2}$.

(iii)Maximum velocity is

$$v_{max} = A\omega = A\sqrt{\frac{k}{m}} = 0.02 \times \sqrt{\frac{1200}{3}} = 0.4 m/s$$

Hence, the maximum velocity of the mass is 0.4 m/s.

10. In Question 9, let us take the position of mass when the spring is unstarched as x = 0, and the direction from left to right as the positive direction of x-axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch (t = 0), the mass is

(a) at the mean position,

(b) at the maximum stretched position, and

(c) at the maximum compressed position.

In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

Ans: In the above question it is given that:

Distance travelled by the mass sideways is $a = 2.0\text{ cm}$.

Angular frequency of oscillation is given by:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3}} = 20\text{rad / s}$$

(a) As time is noted from the mean position, initial phase is 0. Hence, Displacement, $x = 2\sin 20t$.

(b) When the body is at maximum stretched position, it is at the extreme right position, with an initial phase of $\pi / 2$ rad.

Then, displacement $x = \alpha \sin\left(\omega t + \frac{\pi}{2}\right) = \alpha \cos \omega t = 2\cos 20t$.

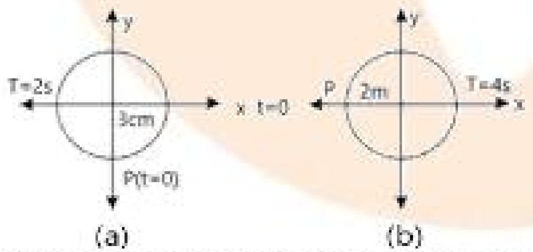
(c) The body is at left position at maximum compressed position, with an initial phase of $3\pi / 2$ rad.

Then,

Displacement, $x = \alpha \sin\left(\omega t + \frac{3\pi}{2}\right) = -\alpha \cos \omega t = -2\cos 20t$

The functions differ in initial phase. They neither differ in amplitude nor in frequency.

11. Figures correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e. clockwise or anti-clockwise) are indicated on each figure.



(a) (b)

Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.

Ans:

(a) In the figure (a) it is given that:

Time period is $t = 2\text{ s}$.

Amplitude is $A = 3\text{ cm}$.

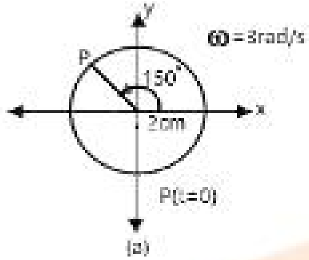
When, $t = 0$, the radius vector OP makes an angle $\pi/2$ with the positive x-axis.
Phase angle $= \pi/2$.
Hence, the equation of simple harmonic motion for the x-projection of OP, at the time t, and is given by the displacement equation:
$$x = A\cos\left[\frac{2\pi t}{T}+\phi\right] = 3\cos\left[\frac{2\pi t}{2}+\frac{\pi}{2}\right] = -3\sin\left(\frac{2\pi t}{2}\right)\text{cm}$$
$\Rightarrow x = -3\sin\pi t\,\text{cm}$, which represents the SHM equation here.

(b) In the figure (b) it is given that:
Time Period, $t = 4$ s.
Amplitude, $a = 2$ m.
At time $t = 0$, OP makes an angle π with the x-axis, in the anticlockwise direction,
Phase angle $= \pi$
Thus, the equation of simple harmonic motion for the x-projection of OP, at the time t, is
$$x = A\cos\left[\frac{2\pi t}{T}+\phi\right] = 2\cos\left[\frac{2\pi t}{4}+\pi\right]$$
$\Rightarrow x = -2\cos\left(\frac{\pi}{2}t\right)$m, which represents the SHM equation here.

12. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial ($t = 0$) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

(a) $x = -2\sin(3t + \pi/3)$

Ans: Given equation is
$$x = -2\sin(3t+\pi/3) = -2\cos(3t+\pi/3+\pi/2) = -2\cos(3t+5\pi/6)$$
If this equation is compared with the standard SHM equation
$$x = A\cos\left[\frac{2\pi t}{T}+\phi\right]$$
Therefore, $A = 2\text{cm}$,
$\phi = 5\pi/6$,
$\omega = 2\pi/t = 3\text{rad/s}$
The motion of the particle is shown in fig (a).



(b) $x = \cos(\pi/6 - t)$

Ans: Given equation is

$x = \cos(\pi/6 - t) = \cos(t - \pi/6)$

If this equation is compared with the standard SHM equation

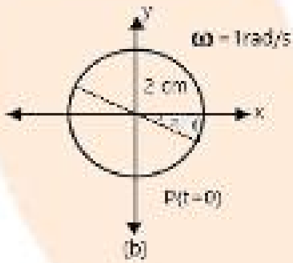
$x = A\cos\left[\frac{2\pi t}{T} + \phi\right]$

Therefore, $A = 1cm$,

$\phi = -\pi/6$,

$\omega = 2\pi/t = 1rad/s$

The motion of the particle is shown in fig (b).



(c) $x = 3\sin(2\pi t + \pi/4)$

Ans: Given equation is

$x = 3\sin(2\pi t + \pi/4) = -3\cos(2\pi t + \pi/4 + \pi/2) = -3\cos(2\pi t + 3\pi/4)$

If this equation is compared with the standard SHM equation

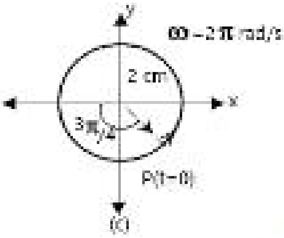
$x = A\cos\left[\frac{2\pi t}{T} + \phi\right]$

Therefore, $A = 3cm$,

$\phi = 3\pi/4$,

$\omega = 2\pi/t = 2rad/s$

The motion of the particle is shown in fig (c).



(d) $x = 2\cos \pi t$

Ans: Given equation is

$x = 2\cos \pi t$

If this equation is compared with the standard SHM equation

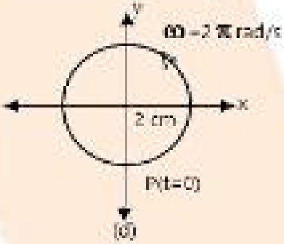
$$x = A\cos\left[\frac{2\pi t}{T} + \phi\right]$$

Therefore, $A = 2cm$,

$\phi = 0$,

$\omega = \pi rad / s$

The motion of the particle is shown in fig (d).



13. Figure (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. (b) is stretched by the same force F.

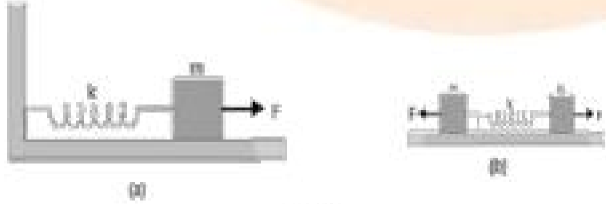


Fig 14.30

(a) What is the maximum extension of the spring in the two cases?
Ans: Consider one block system.
If a force F, is applied to the free end of the spring, then extension l, is produced. For the maximum extension,
i.e., $F = kl$
Where, k is the spring constant.
Thus, the maximum extension produced in the spring is $l = \frac{F}{k}$.
Now consider the two-block system:
in this case the displacement (x) produced is $x = \frac{l}{2}$.
Therefore, net force $= 2kx = 2k\left(\frac{l}{2}\right)$
$\Rightarrow l = \frac{F}{k}$
Thus, the maximum extension of the spring in both cases will be $\frac{F}{k}$.

(b) If the mass in Fig. (a) and the two masses in Fig. (b) are released, what is the period of oscillation in each case?
Ans: In figure (a) if x is the extension in the spring, when mass m is returning to its mean position after being released free, then restoring force on the mass is $F = -kx$, i.e., $F \propto x$. As, this F is directed towards mean position of the mass, hence the mass attached to the spring will execute SHM.
Spring factor = spring constant = k
inertia factor = mass of the given mass = m
As time period,
$T = 2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}} = 2\pi\sqrt{\frac{m}{k}}$
If we consider figure (b), we have a two body system of spring constant k and reduced mass,
$\mu = \frac{m \times m}{m + m} = \frac{m}{2}$
Inertia factor $= m/2$
Spring factor $= k$
Hence, time period, $T = 2\pi\sqrt{\frac{m/2}{k}} = 2\pi\sqrt{\frac{m}{2k}}$.

14. The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rad/min, what is its maximum speed?
Ans: In the above question it is given that:
Angular frequency of the piston is $200\ \text{rad / min}$.
Stroke $= 1.0\ \text{m}$
Amplitude, is $A = 1.0/2 = 0.5\ \text{m}$.
The maximum speed (v_{max}) of piston is given by:
$v_{max} = A\omega = 200 \times 0.5 = 100\text{m / min}$

15. The acceleration due to gravity on the surface of moon is 1.7m / s^2. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? (g on the surface of earth is 9.8m / s^2)
Ans: In the above question it is given that:
Acceleration due to gravity on the surface of moon is $g' = 1.7\text{m / s}^2$.
Acceleration due to gravity on the surface of earth is $g = 9.8\text{m / s}^2$.
Time period of a simple pendulum on earth is $T = 3.5\ \text{s}$.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$l = \frac{T^2}{4\pi^2}g = \frac{(3.5)^2}{4(3.14)^2}(9.8)$$

Where,
l is the length of the pendulum.
The length of pendulum remains constant

On moon's surface, time period, $T' = 2\pi\sqrt{\frac{l}{g'}}$

$$\Rightarrow T' = 2\pi\sqrt{\frac{\frac{(3.5)^2}{4(3.14)^2}(9.8)}{1.7}} = 8.4\text{s}$$

Therefore, Hence, the time period of the simple pendulum on the surface of moon is 8.4s.

16. Answer the following questions:
(a) Time period of a particle in SHM depends on the force constant k and mass m of the particle:

$T = 2\pi\sqrt{\frac{m}{k}}$. A simple pendulum executes SHM approximately. Why then is the time period of a pendulum independent of the mass of the pendulum?

Ans: We know that:

The time period of the simple pendulum is $T = 2\pi\sqrt{\frac{m}{k}}$

For a simple pendulum, k is expressed in terms of mass m, as: $k \propto m$,

Where, $\frac{m}{k}$ is a constant.

Thus, the time period T, of a simple pendulum is independent of the mass of the bob.

(b) The motion of a simple pendulum is approximately simple harmonic for small angle oscillations. For larger angles of oscillation, a more involved analysis shows that T is greater than $2\pi\sqrt{\frac{l}{g}}$

Think of a qualitative argument to appreciate this result.

Ans: If we consider a simple pendulum, the restoring force acting on the bob of the pendulum is given by: $F = -mg\sin\theta$

Where,

F = Restoring force

m = Mass of the bob

g = Acceleration due to gravity

θ=Angle of displacement

For small θ, $\sin\theta \approx \theta$

For large θ, $\sin\theta > \theta$

This decreases the effective value of g.

Thus, the time period increases as:

$T = 2\pi\sqrt{\frac{l}{g}}$ where, l is the length of the simple pendulum.

(c) A man with a wristwatch on his hand falls from the top of a tower. Does the watch give correct time during the free fall?

Ans: The time shown by the wrist watch of a man falling from the top of a tower is not affected by the fall. As a wristwatch does not work on the principle of a simple pendulum, it is not affected by the acceleration due to gravity during free fall. The working of wrist watch depends on spring action.

(d) What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity?
Ans: If a simple pendulum mounted in a cabin falls freely under gravity, its acceleration is zero. Thus, the frequency of the oscillation of this pendulum is zero.

17. A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?
Ans: From the above question it is clear that the bob of the simple pendulum will experience the acceleration due to gravity and the centripetal acceleration provided by the circular motion of the car.
Acceleration due to gravity = g
Centripetal acceleration $a_c = v^2 / r$
where,
v is the uniform speed of the car
R is the radius of the track
Hence, effective acceleration is given as:

$$g' = \sqrt{g^2 + {a_c}^2}$$

$$g' = \sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}$$

Time period,

$$T = 2\pi\sqrt{\frac{l}{g'}}\frac{dy}{dx}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{v^4}{r^2}}}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}}}$$, which represents the time period here.

18. Cylindrical piece of cork of density of base area A and height h floats in a liquid of density ρ_1. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period $T = 2\pi\frac{h\rho}{\rho_1 g}$, where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).

Ans: In the above question it given that:

Base area of the cork $= A$

Height of the cork $= h$

Density of the liquid $= \rho_1$

Density of the cork $= \rho$

In equilibrium:

Weight of the cork $=$ Weight of the liquid displaced by the floating cork

If the cork is depressed slightly by x, some extra water of a certain volume is displaced. Hence, an extra up-thrust acts upward and provides the restoring force to the cork.

Upthrust $=$ Restoring force, F $=$ Weight of the extra water displaced

$F = -(\text{Volume} \times \text{Density} \times g)$

Volume $=$ Area $\times$ Distance through which the cork is depressed

$\Rightarrow F = -Ax\rho_1 g \quad \ldots\ldots \text{(i)}$

According to the force law:

$F = kx \Rightarrow k = F/x$

where, k is constant.

$k = F/x = -A\rho_1 g \ldots\ldots \text{(ii)}$

The time period of the oscillations of the cork: $T = 2\pi\sqrt{\frac{m}{k}} \ldots.. \text{(iii)}$

where,

$m =$ Mass of the cork

$m =$ Volume of the cork $\times$ Density

$m =$ Base area of the cork $\times$ Height of the cork $\times$ Density of the cork

$m = Ah\rho$

Hence, the expression for the time period becomes:

$$T = 2\pi\sqrt{\frac{Ah\rho}{A\rho_1 g}} = 2\pi\sqrt{\frac{h\rho}{\rho_1 g}}$$

19. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is

maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.

Ans: Consider,

Area of cross – section of the U – tube $= A$

Density of the mercury column $= \rho$

Acceleration due to gravity = g

Restoring force, F = Weight of the mercury column of a certain height

$$F = -(\text{Volume} \times \text{Density} \times g)$$

$$F = -(2A \times h \times \rho \times g) = -2A\rho gh$$

$$= k \times \text{Displacement in one of the arms (h)}$$

Where,

2h is the height of the mercury column in the two arms

k is a constant, given by

$k = -F/h = 2A\rho g$

Time period, $T = 2\pi\sqrt{\frac{m}{A\rho g}}$

where,

m is the mass of the mercury column

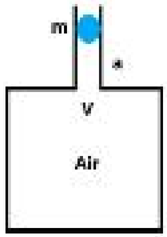
Let l be the length of the total mercury in the U-tube

Mass of mercury, m = Volume of mercury $\times$ Density of mercury $= Al\rho$

$$T = 2\pi\sqrt{\frac{Al\rho}{2A\rho g}} = 2\pi\sqrt{\frac{l}{2g}}$$

Thus, the mercury column executes simple harmonic motion with time period $2\pi\sqrt{\frac{l}{2g}}$.

20. An air chamber of volume V has a neck area of cross section a into which a ball of mass m just fits and can move up and down without any friction (Fig.14.33). Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal [see figure].



Ans: In the above question it given that:

Volume of the air chamber $= V$

Area of cross $-$ section of the neck $= a$

Mass of the ball $= m$

The pressure inside the chamber $=$ atmospheric pressure.

Let the ball be depressed by x units. As a result of this depression, there would be a decrease in
the volume and an increase in the pressure inside the chamber.

Decrease in the volume of the air chamber, $\Delta V = ax$

$$\text{Volumetric Strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{ax}{V}$$

$$\text{Bulk modulus of air, } B = \frac{\text{Stress}}{\text{Strain}} = \frac{-p}{\frac{ax}{V}}$$

Here, stress is the increase in pressure. The negative sign indicates that pressure increases with decrease in volume.

$p = -Bax / V$

The restoring force acting on the ball,

$F = p \times a$

$$= -\frac{Ba^2x}{V} \qquad \ldots\ldots \text{(i)}$$

In simple harmonic motion, the equation for restoring force is:

$F = -kx \quad \ldots\ldots \text{(ii)}$

where, k is the spring constant

Comparing equations (i) and (ii), we get:

$$k = -\frac{Ba^2}{V}$$

Time Period,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{Vm}{B\alpha^2}}$$

21. You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of

(a) the spring constant k

Ans: In the above question it is given that:

Mass of the automobile, $m = 3000\text{ kg}$

Displacement in the suspension system, $x = 15\text{ cm} = 0.15\text{ m}$

There are 4 springs in parallel to the support of the mass of the automobile.

The equation for the restoring force for the system is $F = -4kx = mg$.

Where, k is the spring constant of the suspension system.

Time period, $T = 2\pi\sqrt{\frac{m}{4k}}$

And

$k = mg/4x = 3000 \times 10/4 \times 0.15 = 5000 = 5 \times 10^4\text{ Nm}$

Spring Constant, $k = 5 \times 10^4\text{ Nm}$.

(b) the damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.

Ans: We know that each wheel supports a mass, $M = 3000/4 = 750\text{ kg}$.

For damping factor b, the equation for displacement is written as:

$x = x_0 e^{-bt/2M}$

The amplitude of oscillation decreases by $50\ \%$.

$x = x_0/2$

$\Rightarrow x_0/2 = x_0 e^{-bt/2M}$

$\log_e 2 = bt/2M$

$b = 2M \log_e 2/t$

Where,

$$T = 2\pi\sqrt{\frac{m}{4k}} = 2\pi\sqrt{\frac{3000}{4 \times 5 \times 10^4}} = 0.7691\text{s}$$

$$\Rightarrow b = \frac{2 \times 750 \times 0.693}{0.7691} = 1351.53\text{kg/s}$$

Hence, the damping constant of the spring is 1351.53kg/s.

22. Show that for a particle in linear SHM the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

Ans: Consider,

The equation of displacement of a particle executing SHM at an instant t is given as:

$x = A\sin\omega t$

where,

A is the amplitude of oscillation,

Angular frequency=$\omega=\sqrt{\frac{k}{M}}$

The velocity of the particle is given by $v = dx/dt = A\omega\cos\omega t$.

The kinetic energy of the particle is:

$E_k = 1/2Mv^2 = 1/2MA^2\omega^2\cos^2\omega t$

The potential energy of the particle is:

$E_p = 1/2kx^2 = 1/2M^2\omega^2A^2\sin^2\omega t$

For time period T, the average kinetic energy over a single cycle is given as:

$$(E_k)_{Avg} = \frac{1}{T}\int_0^T E_k dt$$

$$= \frac{1}{T}\int_0^T \frac{1}{2}MA^2\omega^2\cos^2\omega t dt$$

$$= \frac{1}{2T}MA^2\omega^2\left[1 + \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{1}{2T}MA^2\omega^2(T)$$

$$= \frac{1}{4}MA^2\omega^2 \qquad \ldots\ldots (i)$$

And, average potential energy over one cycle is given as:

$$(E_p)_{Avg} = \frac{1}{T}\int_0^T E_p dt$$

$$= \frac{1}{T}\int_0^T \frac{1}{2}MA^2\omega^2\sin^2\omega t dt$$

$$= \frac{1}{2T}M\omega^2A^2\int_0^T \frac{(1+\cos 2\omega t)}{2}dt$$

$$=\frac{1}{4T}M\omega^2A^2(T)$$

$$=\frac{M\omega^2A^2}{4} \qquad \ldots\ldots (ii)$$

It can be interpreted from equations (i) and (ii) that the average kinetic energy for a given time period is equal to the average potential energy for the same time period.

23. A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillations is found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire. (Torsional **spring constant α is defined by the relation** $J = -\alpha\theta$. where J is the restoring couple and θ the angle of twist)

Ans: In the above question it given that:

Mass of the circular disc, $m = 10$ kg

Radius of the disc, $r = 15\text{ cm} = 0.15\text{ m}$

The torsional oscillations of the disc has a time period, $T = 1.5$ s

The moment of inertia of the disc is:

$$I = 1/2mr^2$$

$$= 1/2(10)(0.15)^2$$

$$= 0.1125\text{kg}/\text{m}^2$$

Time period,

$$T = 2\pi\sqrt{\frac{I}{\alpha}}$$

And

$$\alpha = 4\pi^2 I / T^2$$

$$= 4\pi^2\left(\frac{0.1125}{1.5^2}\right)$$

$$= 1.972\text{N}/\text{rad}$$

Therefore, the torsional spring constant of the wire is $1.972\text{N}/\text{rad}$.

24. A body describes simple harmonic motion with amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the body when the displacement is (a) 5 cm, (b) 3 cm, (c) 0cm.

Ans: In the above question it is given that:

Amplitude, r= 5 cm = 0.05 m

$$r = 5\text{cm} = 0.05\text{m}$$

Time period, T = 0.2 s.

$\omega = 2\pi / T = 2\pi / 0.2 = 10\pi$ rad/s

When displacement is y, then acceleration, $A = -\omega^2 y$.

Velocity is $V = \omega\sqrt{r^2 - y^2}$

Now,

Case (a) When y = 5cm = 0.05 m

$A = -(10\pi)^2 \times 0.05 = -5\pi^2 m/s^2$

$V = 10\pi\sqrt{(0.05)^2 - (0.05)^2} = 0$

Case (b) When y = 3cm = 0.03 m

$A = -(10\pi)^2 \times 0.03 = -3\pi^2 m/s^2$

$V = 10\pi\sqrt{(0.05)^2 - (0.03)^2} = 0.4\pi m/s$

Case (c) When y = 0cm = 0 m

$A = 0 m/s^2$

$V = 10\pi\sqrt{(0.05)^2 - (0)^2} = 0.5\pi m/s$

25. A mass attached to a spring is free to oscillate, with angular velocity ω, in a horizontal plane without friction or damping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time $t = 0$. Determine the amplitude of the resulting oscillations in terms of the parameters ω, x_0 and v_0. [Hint: Start with the equation $x = a\cos(\omega t + \theta)$ and note that the initial velocity is negative.]

Ans: In the above question it is given that:

$x = a\cos(\omega t + \theta)$

Where,

A is the amplitude,

x is the displacement,

θ is the phase constant.

Then,

$v = dx/dt = -A\omega\sin(\omega t + \theta)$

At $t = 0$, $x = x_0$

$x_0 = A\cos\theta$ ...... (i)

And $dx/dt = -v_0 = A\omega\sin\theta$

$A\sin\theta = v_0/\omega$ ...... (ii)

Squaring and adding (i) an (ii),

$$A^2(\cos^2\theta + \sin^2\theta) = x_0^{\ 2} + \left(\frac{v_0^{\ 2}}{\omega^2}\right)$$

$$\Rightarrow A = \sqrt{x_0^{\ 2} + \left(\frac{v_0}{\omega}\right)^2}$$

Thus, the resulting amplitude is $\sqrt{x_0^{\ 2} + \left(\frac{v_0}{\omega}\right)^2}$.