NCERT Solutions for Class 12

Physics

Chapter 13 – Nuclei

1.

a) Two stable isotopes of lithium ${}_3^6Li$ and ${}_3^7Li$ have respective abundances of 7.5% and 92.5% These isotopes have masses $6.01512u$ and $7.01600u$, respectively. Find the atomic mass of lithium.

Ans: We are given the following information:

Mass of ${}_3^6Li$ lithium isotope, $m_1 = 6.01512u$

Mass of ${}_3^7Li$ lithium isotope, $m_2 = 7.01600u$

Abundance of ${}_3^6Li$, $n_1 = 7.5\%$

Abundance of ${}_3^7Li$, $n_2 = 92.5\%$

The atomic mass of lithium atom is given by,

$$m = \frac{m_1 n_1 + m_2 n_2}{n_1 + n_2}$$

Substituting the given values, we get,

$$m = \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{92.5 + 7.5}$$

$\therefore m = 6.940934u$

Therefore, we found the atomic mass of lithium atom to be $6.940934u$.

b) Boron has two stable isotopes ${}_5^{10}B$ and ${}_5^{11}B$. Their respective masses are $10.01294u$ and $11.00931u$, and the atomic mass of boron is $10.811u$. Find the abundances of ${}_5^{10}B$ and ${}_5^{11}B$.

Ans: We are given:

Mass of ${}_5^{10}B$ Boron isotope, $m_1 = 10.01294u$

Mass of ${}_5^{11}B$ lithium isotope, $m_2 = 11.00931u$

Abundance of ${}_5^{10}B$, $n_1 = x\%$

Abundance of ${}_5^{11}B$, $n_2 = (100 - x)\%$

We know the atomic mass of boron to be, $m = 10.811u$

The atomic mass of lithium atom is given by,

$$m = \frac{m_1 n_1 + m_2 n_2}{n_1 + n_2}$$

Substituting the given values, we get,

$$10.811=\frac{10.01294\times x+11.00931\times(100-x)}{x+(100-x)}$$

$$\Rightarrow 1081.11=10.01294x+1100.931-11.00931x$$

$$\therefore x=\frac{19.821}{0.99637}=19.89\%$$

And, $100-x=80.11\%$

Therefore, we found the abundance of ${}^{10}_{5}B$ and ${}^{11}_{5}B$ to be 19.89% and 80.11% respectively.

2. The three stable isotopes of neon: ${}^{20}_{10}Ne$, ${}^{21}_{10}Ne$ and ${}^{22}_{10}Ne$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

Ans: We are given that:

Atomic mass of ${}^{20}_{10}Ne$, $m_1=19.99u$

Abundance of ${}^{20}_{10}Ne$, $\eta_1=90.51\%$

Atomic mass of ${}^{21}_{10}Ne$, $m_2=20.99u$

Abundance of ${}^{21}_{10}Ne$, $\eta_2=0.27\%$

Atomic mass of ${}^{22}_{10}Ne$, $m_3=21.99u$

Abundance of ${}^{22}_{10}Ne$, $\eta_3=9.22\%$

The average atomic mass of neon could be given as,

$$m=\frac{m_1\eta_1+m_2\eta_2+m_3\eta_3}{\eta_1+\eta_2+\eta_3}$$

Substituting the given values, we get,

$$m=\frac{19.99\times 90.51+20.99\times 0.27+21.99\times 9.22}{90.51+0.27+9.22}$$

$$\therefore m=20.1771u$$

The average atomic mass of neon is thus found to be $20.177u$.

3. Obtain the binding energy (in MeV) of a nitrogen nucleus $\left({}^{14}_{7}N\right)$, given $m\left({}^{14}_{7}N\right)=14.00307u$

Ans: We are given:

Atomic mass of nitrogen $\left({}_{7}N^{14}\right)$, $m=14.00307u$

A nucleus of ${}_{7}N^{14}$ nitrogen contains 7 protons and 7 neutrons.

Hence, the mass defect of this nucleus would be, $\Delta m = 7m_H + 7m_n - m$

Where,

Mass of a proton, $m_H = 1.007825u$

Mass of a neutron, $m_n = 1.008665u$

Substituting these values into the above equation, we get,

$\Delta m = 7 \times 1.007825 + 7 \times 1.008665 - 14.00307$

$\Rightarrow \Delta m = 7.054775 + 7.06055 - 14.00307$

$\therefore \Delta m = 0.11236u$

But we know that, $1u = 931.5 MeV / c^2$

$\Rightarrow \Delta m = 0.11236 \times 931.5 MeV / c^2$

Now, we could give the binding energy as,

$E_b = \Delta mc^2$

Where, c = speed of light $= 3 \times 10^8 ms^{-2}$

Now, $E_b = 0.11236 \times 931.5 \left(\frac{MeV}{c^2}\right) \times c^2$

$\therefore E_b = 104.66334 MeV$

Therefore, we found the binding energy of a Nitrogen nucleus to be $104.66334 MeV$.

4. Obtain the binding energy of the nuclei ${}^{56}_{26}Fe$ and ${}^{209}_{83}Bi$ in units of MeV from the following data: $m\left({}^{56}_{26}Fe\right) = 55.934939u$, $m\left({}^{209}_{83}Bi\right) = 208.980388u$

Ans: We are given the following:

Atomic mass of ${}^{56}_{26}Fe$, $m_1 = 55.934939u$

${}^{56}_{26}Fe$ nucleus has 26 protons and $56 - 26 = 30$ neutrons

Hence, the mass defect of the nucleus would be, $\Delta m = 26 \times m_H + 30 \times m_n - m_1$

Where, Mass of a proton, $m_H = 1.007825u$

Mass of a neutron, $m_n = 1.008665u$

Substituting these values into the above equation, we get,

$\Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$

$\Rightarrow \Delta m = 26.20345 + 30.25995 - 55.934939$

$\therefore \Delta m = 0.528461u$

But we have, $1u = 931.5 MeV / c^2$

$\Delta m = 0.528461 \times 931.5 MeV / c^2$

The binding energy of this nucleus could be given as,

$E_{b1} = \Delta mc^2$

Where, c = Speed of light

$\Rightarrow E_{b1} = 0.528461 \times 931.5\left(\frac{MeV}{c^2}\right) \times c^2$

$\therefore E_{b1} = 492.26MeV$

Now, we have the average binding energy per nucleon to be,

$B.E = \frac{492.26}{56} = 8.79MeV$

Also, atomic mass of ${}^{209}_{83}Bi$, $m_2 = 208.980388u$

We know that, ${}^{209}_{83}Bi$ nucleus has 83 protons and $209 - 83 = 126$ neutrons

Where,

Mass of a proton, $m_H = 1.007825u$

Mass of a neutron, $m_n = 1.008665u$

$\Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$

$\Rightarrow \Delta m' = 83.649475 + 127.091790 - 208.980388$

$\therefore \Delta m' = 1.760877u$

But we know, $1u = 931.5MeV / c^2$

Hence, the binding energy of this nucleus could be given as,

$E_{b2} = \Delta m' c^2 = 1.760877 \times 931.5\left(\frac{MeV}{c^2}\right) \times c^2$

$\therefore E_{b2} = 1640.26MeV$

Average binding energy per nucleon is found to be $= \frac{1640.26}{209} = 7.848MeV$

Hence, the average binding energy per nucleon is found to be $7.848MeV$.

5. A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of ${}^{63}_{29}Cu$ atoms (of mass $62.92960u$).

Ans: We are given:

Mass of a copper coin, $m' = 3g$

Atomic mass of ${}_{29}Cu^{63}$ atom, $m = 62.92960u$

The total number of ${}^{63}_{29}Cu$ atoms in the coin, $N = \frac{N_A \times m'}{\text{Mass number}}$

Where, $N_A =$ Avogadro's number $= 6.023 \times 10^{23}$ atoms / g

Mass number = 63g

$\Rightarrow N = \frac{6.023 \times 10^{23} \times 3}{63} = 2.868 \times 10^{22}$

${}_{29}Cu^{63}$ nucleus has 29 protons and $(63-29)=34$ neutrons
Mass defect of this nucleus would be, $\Delta m' = 29\times m_H + 34\times m_n - m$
Where,
Mass of a proton, $m_H = 1.007825u$
Mass of a neutron, $m_n = 1.008665u$
$\Delta m' = 29\times 1.007825 + 34\times 1.008665 - 62.9296 = 0.591935u$
Mass defect of all the atoms present in the coin would be,
$\Delta m = 0.591935\times 2.868\times 10^{22} = 1.69766958\times 10^{22}u$
But we have, $1u = 931.5MeV/c^2$
$\Rightarrow \Delta m = 1.69766958\times 10^{22}\times 931.5MeV/c^2$
Hence, the binding energy of the nuclei of the coin could be given as:
$E_b = \Delta mc^2 = 1.69766958\times 10^{22}\times 931.5\left(\frac{MeV}{c^2}\right)\times c^2$
$\therefore E_b = 1.581\times 10^{25}MeV$
But, $1MeV = 1.6\times 10^{-13}J$
$\Rightarrow E_b = 1.581\times 10^{25}\times 1.6\times 10^{-13}$
$\therefore E_b = 2.5296\times 10^{12}J$
This much energy is needed to separate all the neutrons and protons from the given coin.

6. Write the nuclear reactions for:
a) $\boldsymbol{\alpha}$ – decay of ${}^{226}_{88}Ra$
Ans: We know that, α is basically a nucleus of Helium $\left({}_2He^4\right)$ and β is an electron (e^- for β^- and e^+ for β^+). In every α -decay, there is a loss of 2 protons and 2 neutrons. In every β^+ -decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^- -decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.
For the given case, the nuclear reaction would be,
${}_{88}Ra^{226} \rightarrow {}_{86}Rn^{222} + {}_2He^4$

b) $\boldsymbol{\alpha}$ – decay of ${}^{242}_{94}Pu$
Ans: ${}^{242}_{94}Pu \rightarrow {}^{238}_{92}U + {}^4_2He$

c) $\boldsymbol{\beta}$ – decay of ${}^{32}_{15}P$
Ans: ${}^{32}_{15}P \rightarrow {}^{32}_{16}S + e^- + \bar{v}$

d) β – decay of $^{210}_{83}Bi$
Ans: $^{210}_{83}B \rightarrow ^{210}_{84}Po + e^{-} + \bar{v}$

e) β^{+} – decay of $^{11}_{6}C$
Ans: $^{11}_{6}C \rightarrow ^{11}_{5}B + e^{+} + v$

f) β^{+} – decay of $^{97}_{43}Tc$
Ans: $^{97}_{43}Tc \rightarrow ^{97}_{42}Mo + e^{+} + v$

g) Electron capture of $^{120}_{54}Xe$
Ans: $^{120}_{54}Xe + e^{+} \rightarrow ^{120}_{53}I + v$

7. A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to:
a) 3.125% of its original value?
Ans: We are said that, Half-life of the radioactive isotope $=$ T years
Original amount of the radioactive isotope $= N_0$
(a) After decay, let the amount of the radioactive isotope be N.
It is given that only 3.125% of N_0 remains after decay. Hence, we could write,

$$\frac{N}{N_0} = 3.125\% = \frac{3.125}{100} = \frac{1}{32}$$

But we know that, $\frac{N}{N_0} = e^{-\lambda t}$
Where, $\lambda =$ Decay constant and $t =$ Time

$$\Rightarrow -\lambda t = \frac{1}{32}$$
$$\Rightarrow -\lambda t = \ln 1 - \ln 32$$
$$\Rightarrow -\lambda t = 0 - 3.4657$$
$$\Rightarrow t = \frac{3.4657}{\lambda}$$

But, since $\lambda = \frac{0.693}{T}$

$$\Rightarrow t = \frac{3.466}{\left(\frac{0.693}{T}\right)}$$

$\therefore t \approx 5T$ years

Therefore, we found that the isotope will take about 5T years in order to reduce to 3.125% of its original value.

b) 1% of its original value?

Ans: After decay, let the amount of the radioactive isotope be N

It is given that only 1% of N_0 remains after decay. Hence, we could write:

$$\frac{N}{N_0} = 1\% = \frac{1}{100}$$

But we know, $\frac{N}{N_0} = e^{-\lambda t}$

$$\Rightarrow e^{-\lambda t} = \frac{1}{100}$$

$$\Rightarrow -\lambda t = \ln 1 - \ln 100$$

$$\Rightarrow -\lambda t = 0 - 4.602$$

$$\Rightarrow t = \frac{4.6052}{\lambda}$$

Since we have, $\lambda = \frac{0.639}{T}$

$$\Rightarrow t = \frac{4.6052}{\left(\frac{0.693}{T}\right)}$$

$$\therefore t = 6.645T \text{ years}$$

Therefore, we found that the given isotope would take about 6.645T years so as to reduce to 1% of its original value.

8. The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive ${}^{14}_{6}C$ present with the stable carbon isotope ${}^{12}_{6}C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of ${}^{14}_{6}C$, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of ${}^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Ans: We are given that:

Decay rate of living carbon-containing matter, $R = 15 \text{ decay / min}$

Let N be the number of radioactive atoms present in a normal carbon- containing matter.

Half life of ${}^{14}_{6}C$, $T_{\frac{1}{2}} = 5730$ years

The decay rate of the specimen obtained from the Mohenjodaro site:

$R' = 9$ decays / min

Let N' be the number of radioactive atoms present in the specimen during the Mohenjodaro period.

Therefore, we can relate the decay constant, λ and time, t as:

$$\frac{N'}{N} = \frac{R'}{R} = e^{-\lambda t}$$

$$\Rightarrow e^{-\lambda t} = \frac{9}{15} = \frac{3}{5}$$

$$\Rightarrow -\lambda t = \log_e \frac{3}{5} = -0.5108$$

$$\Rightarrow t = \frac{0.5108}{\lambda}$$

But we know,

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{5730}$$

$$\Rightarrow t = \frac{0.5108}{\left(\frac{0.693}{5730}\right)} = 4223.5 \text{ years}$$

Therefore, the approximate age of the Indus-Valley civilization is found to be 4223.5 years.

9. Obtain the amount of ${}^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of ${}^{60}_{27}Co$ is 5.3 years.

Ans: We know that,

The strength of the radioactive source could be given as,

$$\frac{dN}{dt} = 8.0\text{mCi}$$

$$\Rightarrow \frac{dN}{dt} = 8 \times 10^{-3} \times 3.7 \times 10^{10} = 29.6 \times 10^{7} \text{ decay / s}$$

Where, N is the required number of atoms.

Half life of ${}^{60}_{27}Co$, $T_{\frac{1}{2}} = 5.3$years

$\Rightarrow T_{\frac{1}{2}} = 5.3 \times 365 \times 24 \times 60 \times 60 = 1.67 \times 10^8 s$

For decay constant λ, we could give the rate of decay as,

$\frac{dN}{dt} = \lambda N$

Where, $\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{1.67 \times 10^8} s^{-1}$

$\Rightarrow N = \frac{1}{\lambda}\frac{dN}{dt} = \frac{29.6 \times 10^7}{\left(\frac{0.693}{1.67 \times 10^8}\right)} = 7.133 \times 10^{16}$ atoms

Now for $_{27}Co^{60}$, Mass of Avogadro number of atoms $= 60g$

Then, mass of 7.133×10^{16} atoms $= \frac{60 \times 7.133 \times 10^{16}}{6.023 \times 10^{23}} = 7.106 \times 10^{-6} g$

Therefore, the amount of $_{27}Co^{60}$ that is required for the purpose is $7.106 \times 10^{-6} g$.

10. The half life of $^{90}_{38}Sr$ is 28years. What is the disintegration rate of 15mg of this isotope?

Ans: We know that,

Half life of $^{90}_{38}Sr$, $t_{\frac{1}{2}} = 28 years = 28 \times 365 \times 24 \times 3600 = 8.83 \times 10^8 s$

Mass of the isotope, $m = 15mg$

90g of $^{90}_{38}Sr$ atom contains Avogadro number of atoms. So, 15mg of $^{90}_{38}Sr$ contains,

$\frac{6.023 \times 10^{23} \times 15 \times 10^{-3}}{90} = 1.0038 \times 10^{20}$ number of atoms

Rate of disintegration would be, $\frac{dN}{dt} = \lambda N$

Where, λ is the decay constant given by, $\lambda = \frac{0.693}{8.83 \times 10^8} s^{-1}$

$\therefore \frac{dN}{dt} = \frac{0.693 \times 1.0038 \times 10^{20}}{8.83 \times 10^8} = 7.878 \times 10^{10}$ atoms / s

Therefore, we found the disintegration rate of 15mg of given isotope to be 7.878×10^{10} atoms / s.

11. Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}Au$ and the silver isotope $^{107}_{47}Ag$.

Ans: We know that,

Nuclear radius of the gold isotope $_{79}Au^{197} = R_{Au}$

Nuclear radius of the silver isotope $_{47}Ag^{107} = R_{Ag}$

Mass number of gold, $A_{Au} = 197$

Mass number of silver, $A_{Ag} = 107$

We also know that the ratio of the radii of the two nuclei is related with their mass numbers as:

$$\frac{R_{Au}}{R_{Ag}} = \left(\frac{A_{Au}}{A_{Ag}}\right)^{\frac{1}{3}} = 1.2256$$

Hence, the ratio of the nuclear radii of the gold and silver isotopes is found to be about 1.23.

12. Find the Q-value and the **kinetic energy of the emitted α**-particle in the α-decay of: [Given: $m\left(^{226}_{88}Ra\right) = 226.02540u$, $m\left(^{222}_{89}Rn\right) = 222.01750u$, $m\left(^{220}_{86}Rn\right) = 220.01137u$, $m\left(^{216}_{84}Po\right) = 216.00189u$]

a) $^{226}_{88}Ra$

Ans: We know that,

Alpha particle decay of $^{26}_{88}Ra$ emits a helium nucleus. As a result, its mass number reduces to $222 = (226-4)$ and its atomic number reduces to $86 = (88-2)$. This is shown in the following nuclear reaction:

$^{226}_{88}Ra \rightarrow ^{222}_{86}Ra + ^{4}_{2}He$

Q – value of emitted α – particle = (Sum of initial mass – Sum of final mass)c^2

Where, c = Speed of light

It is also given that:

$m\left(^{226}_{88}Ra\right) = 226.02540u$

$m\left(^{220}_{86}Rn\right) = 220.01137u$

$m\left(^{4}_{2}He\right) = 4.002603u$

On substituting these values into the above equation,

Q value $= \left[226.02540 - (222.01750 + 4.002603)\right]uc^2$

Q value $= 0.005297uc^2$

But we know, $1u = 931.5 MeV / c^2$
$\Rightarrow Q = 0.005297 \times 931.5 \approx 4.94 MeV$

Kinetic energy of the α particle $= \left(\frac{\text{Mass number after decay}}{\text{Mass number before decay}} \right) \times Q$

$\therefore K.E_\alpha = \frac{222}{226} \times 4.94 = 4.85 MeV$

Hence, the Kinetic energy of the alpha particle is found to be $4.85 MeV$.

b) ${}^{220}_{86}Rn$

Ans: We know that, Alpha particle decay of ${}^{220}_{86}Rn$ could be given as,

${}^{220}_{86}Rn \rightarrow {}^{216}_{84}Po + {}^{4}_{2}He$

We are also given,

Mass of ${}^{220}_{86}Rn = 220.01137u$

Mass of ${}^{216}_{84}Po = 216.00189u$

Now, Q value could be given as,

$Q - \text{value} = \left[220.01137 - (216.00189 + 4.00260)\right] \times 931.5 \approx 641 MeV$

Now, we have the kinetic energy as,

$K.E_\alpha = \left(\frac{220-4}{220} \right) \times 6.41 = 6.29 MeV$

The kinetic energy of the alpha particle is found to be $6.29 MeV$.

13. The radionuclide ${}^{11}_{6}C$ decays according to,

${}^{11}_{6}C \rightarrow {}^{11}_{5}B + e^+ + \nu$; $T_{\frac{1}{2}} = 20.3 min$

The maximum energy of the emitted positron is $0.960 MeV$.

Given the mass values: $m\left({}^{11}_{6}C\right) = 11.011434u$ and $m\left({}^{11}_{6}B\right) = 11.009305u$

Calculate Q and compare it with the maximum energy of the positron emitted.

Ans: The given nuclear reaction is,

${}^{11}_{6}C \rightarrow {}^{11}_{5}B + e^+ + \nu$

Half life of ${}^{11}_{6}C$ nuclei, $T_{\frac{1}{2}} = 20.3 min$

Atomic masses are given to be:

$m\left({}^{11}_{6}C\right) = 11.011434u$

$m\left({}^{11}_{6}B\right) = 11.009305u$

Maximum energy that is possessed by the emitted positron would be 0.960MeV. The change in the Q - value (ΔQ) of the nuclear masses of the ${}^{11}_{6}C$

$$\Delta Q=\left[m'\left({}_{6}C^{11}\right)-\left[m'\left({}^{11}_{5}B\right)+m_e\right]\right]c^2 \ldots\ldots (1)$$

Where, $m_e=$ Mass of an electron or positron $= 0.000548u$

$c=$ Speed of light

$m'=$ Respective nuclear masses

If atomic masses are used instead of nuclear masses, then we will have to add $6m_e$ in the case of ${}^{11}C$ and $5m_e$ in case of ${}^{11}B$.

Hence, equation (1) would now reduce to,

$$\Delta Q=\left[m\left({}_{6}C^{11}\right)-m\left({}^{11}_{5}B\right)-2m_e\right]c^2$$

Where, $m\left({}_{6}C^{11}\right)$ and $m\left({}^{11}_{5}B\right)$ are the atomic masses.

Now, we have the change in Q value as,

$$\Delta Q=[11.011434-11.009305-2\times 0.000548]c^2=\left(0.001033c^2\right)u$$

But we know, $1u=931.5\text{MeV}/c^2$

$\therefore \Delta Q=0.001033\times 931.5\approx 0.962\text{MeV}$

We see that the Q value is almost comparable to the maximum energy of the emitted positron.

14. The nucleus ${}^{23}_{10}Ne$ decays by β^- emission. Write down the β decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$m\left({}^{23}_{10}Ne\right)=22.994466u$

$m\left({}^{23}_{11}Na\right)=22.989770u$

Ans: We know that: In β^- emission, the number of protons increases by 1, and one electron and an antineutrino are emitted from the parent nucleus. β^- emission of the nucleus could be given by,

$${}^{23}_{10}Ne \rightarrow {}^{23}_{11}Na+e^-+\bar{\nu}+Q$$

It is also given that:

Atomic mass of ${}^{23}_{10}Ne=22.994466u$

Atomic mass of ${}^{23}_{11}Na=22.989770u$

Mass of an electron, $m_e=0.000548u$

Q value of the given reaction could be given as:

$$Q=\left[m\left({}^{23}_{10}Ne\right)-\left[m\left({}^{23}_{11}Na\right)+m_e\right]\right]c^2$$

There are 10 electrons in ${}_{10}Ne^{23}$ and 11 electrons in ${}^{23}_{11}Na$. Hence, the mass of the electron is cancelled in the Q-value equation.

$Q = [22.994466 - 22.9897770]c^2 = (0.004696c^2)u$

But we have, $1u = 931.5 MeV / c^2$

$\Rightarrow Q = 0.004696 \times 931.5 = 4.374 MeV$

The daughter nucleus is too heavy as compared to that of e^- and $\bar{v}$. Hence, it carries negligible energy. The kinetic energy of the antineutrino is found to be nearly zero.

Hence, the maximum kinetic energy of the emitted electrons is almost equal to the Q-value, i.e., $4.374 MeV$.

15. The Q-value of a nuclear reaction $A + b \rightarrow C + d$ is defined by $Q = [m_A + m_b - m_C - m_d]c^2$ where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

Atomic masses are given to be: $m({}^2_1H) = 2.014102u$, $m({}^3_1H) = 3.016049u$,

$m({}^{12}_6C) = 12.000000u$, $m({}^{20}_{10}Ne) = 19.992439u$

a) ${}^1_1H + {}^3_1H \rightarrow {}^2_1H + {}^2_1H$

The given nuclear reaction is:

${}^1_1H + {}^3_1H \rightarrow {}^2_1H + {}^2_1H$

Atomic mass of ${}^1_1H = 1.007825u$

Atomic mass of ${}^3_1H = 3.0164049u$

Atomic mass of ${}^2_1H = 2.014102u$

According to the question, the Q-value of the reaction could be written as:

$Q = [m({}^1_1H) + m({}^3_1H) - 2m({}^2_1H)]c^2$

$\Rightarrow Q = [1.007825 + 3.016049 - 2 \times 2.014102]c^2 = (-0.00433c^2)u$

But we know, $1u = 931.5 MeV / c^2$

$\therefore Q = -0.00433 \times 931.5 = -4.0334 MeV$

The negative Q-value of this reaction shows that the given reaction is endothermic.

b) ${}^{12}_6C + {}^{12}_6C \rightarrow {}^{20}_{10}Ne + {}^4_2He$

We are given that,

Atomic mass of ${}^{12}_6C = 12.0u$

Atomic mass of ${}^{12}_{10}Ne = 19.992439u$
Atomic mass of ${}^{4}_{2}He = 4.002603u$
The Q-value here could be given as,
$$Q = \left[2m\left({}^{12}_{6}C\right) - m\left({}^{20}_{10}Ne\right) - m\left({}^{4}_{2}He\right)\right]c^2$$
$$\Rightarrow Q = [2 \times 12.0 - 19.992439 - 4.002603]c^2 = \left(0.004958c^2\right)u = 0.004958 \times 931.5$$
$$\therefore Q = 4.618377MeV$$
Since the Q-value is found to be positive, the reaction could be considered exothermic.

16. Suppose, we think of fission of a ${}^{56}_{26}Fe$ nucleus into two equal fragments of ${}_{13}Al^{28}$. Is the fission energetically possible? Argue by working out Q of the process. Given: $m\left({}^{56}_{26}Fe\right) = 55.93494u$ and $m\left({}^{28}_{13}Al\right) = 27.98191u$

Ans: We know that the fission of ${}^{56}_{26}Fe$ could be given as,
$${}^{56}_{26}Fe \rightarrow 2\,{}^{28}_{13}Al$$
We are also given, atomic masses of ${}^{56}_{26}Fe$ and ${}^{28}_{13}Al$ as $55.93494u$ and $27.98191u$ respectively.
The Q-value here would be given as,
$$Q = \left[m\left({}^{56}_{26}Fe\right) - 2m\left({}^{28}_{13}Al\right)\right]c^2$$
$$\Rightarrow Q = [55.93494 - 2 \times 27.98191]c^2 = \left(-0.02888c^2\right)u$$
But, $1u = 931.5MeV / c^2$
$$\therefore Q = -0.02888 \times 931.5 = -26.902MeV$$
The Q value is found to be negative and hence we could say that the fission is not possible energetically. In order for a reaction to be energetically possible, the Q-value must be positive.

17. The fission properties of ${}^{239}_{94}Pu$ are very similar to those of ${}^{235}_{92}U$. The average energy released per fission is $180MeV$. How much energy, in MeV, is released if all the atoms in 1kg of pure ${}^{239}_{94}Pu$ undergo fission?

Ans: We are given that the average energy released per fission of ${}^{239}_{94}Pu$, $E_{av} = 180MeV$

The amount of pure ${}_{94}Pu^{239}$, $m = 1kg = 1000g$

Avogadro number, $N_A = 6.023 \times 10^{23}$

Mass number of ${}^{239}_{94}Pu = 239g$

1 mole of $_{94}Pu^{239}$ contains Avogadro number of atoms.

1g of $_{94}Pu^{239}$ contains $\left(\frac{N_A}{\text{mass number}} \times m\right)$ atoms

$$\Rightarrow \left(\frac{6.023\times10^{23}}{239}\times1000\right) = 2.52\times10^{24}\text{atoms}$$

Total energy released during the fission of 1kg of $^{239}_{94}Pu$ could be calculated as:

$$E = E_{av} \times 2.52\times10^{24} = 180\times2.52\times10^{24} = 4.536\times10^{26}\,\text{MeV}$$

Therefore, 4.536×10^{26} MeV is released if all the atoms in 1kg of pure $_{94}Pu^{239}$ undergo fission.

18. A 1000MW fission reactor consumes half of its fuel in 5.00 y. How much $^{235}_{92}U$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of $^{235}_{92}U$ and that this nuclide is consumed only by the fission process.

Ans: We are said that the half life of the fuel of the fission reactor, $t_{\frac{1}{2}} = 5\text{years}$

$$\Rightarrow t_{\frac{1}{2}} = 5 \times 365 \times 24 \times 60 \times 60\ \text{s}$$

We know that in the fission of 1g of $^{235}_{92}U$ nucleus, the energy released is equal to 200MeV.

1 mole, i.e., 235g of $^{235}_{92}U$ contains 6.023×10^{23} atoms.

1g of $^{235}_{92}U$ contains $\frac{6.023\times10^{23}}{234}$ atoms

The total energy generated per gram of $^{235}_{92}U$ is calculated as:

$$E = \frac{6.023\times10^{23}}{235}\times200\text{MeV}/\text{g} = \frac{200\times6.023\times10^{23}\times1.6\times10^{-19}\times10^{6}}{235} = 8.20\times10^{10}\,\text{J}/\text{g}$$

The reactor operator operates only 80% of the time. Therefore, the amount of $^{235}_{92}U$ consumed in 5years by the 1000MW fission reactor could be calculated as,

$$\frac{5\times80\times60\times60\times365\times24\times1000\times10^{6}}{100\times8.20\times10^{10}}\text{g} \approx 1538\text{kg}$$

So, the initial amount of $^{235}_{92}U = 2\times1538 = 3076\text{kg}$

Hence, we found the initial amount of uranium to be 3076kg.

19. How long can an electric lamp of 100W be kept glowing by fusion of 2.0kg of deuterium? Take the fusion reaction as

$${}_1^2H + {}_1^2H \rightarrow {}_2^3He + n + 3.27MeV$$

Ans: The fusion reaction is given to be:

$${}_1^2H + {}_1^2H \rightarrow {}_2^3He + n + 3.27MeV$$

Amount of deuterium, $m = 2$ kg

1 mole, i.e., 2 g of deuterium contains 6.023×10^{23} atoms.

2.0 kg of deuterium contains $\frac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{26}$ atoms atoms

It could be inferred from the given reaction that when two atoms of deuterium fuse, $3.27MeV$ energy is released.

Therefore, the total energy per nucleus released in the fusion reaction would be:

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26}\,MeV = \frac{3.27}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{6}$$

$$\therefore E = 1.576 \times 10^{14}\,J$$

Power of the electric lamp is given to be, $P = 100W = 100J/s$, that is, the energy consumed by the lamp per second is 100J.

Now, the total time for which the electric lamp glows could be calculated as,

$$t = \frac{1.576 \times 10^{14}}{100} = \frac{1.576 \times 10^{14}}{100 \times 60 \times 60 \times 24 \times 365}$$

$$\therefore t \approx 4.9 \times 10^{4}\,\text{years}$$

Hence, the total time for which the electric lamp glows is found to be 4.9×10^{4} years.

20. Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0fm.)

Ans: When two deuterons collide head-on, the distance between their centres, d could be given as:

Radius of 1st deuteron + Radius of 2nd deuteron

Radius of a deuteron nucleus $= 2fm = 2 \times 10^{-15}m$

$$\Rightarrow d = 2 \times 10^{-15} + 2 \times 10^{-15} = 4 \times 10^{-15}m$$

Also, charge on a deuteron = Charge on an electron $= e = 1.6 \times 10^{-19}C$

Potential energy of the two-deuteron system could be given by,

$$V = \frac{e^2}{4\pi\varepsilon_0 d}$$

Where, ε_0 is the permittivity of free space.

Also, $\frac{1}{4\pi\varepsilon_0}=9\times10^9\,Nm^2C^{-2}$

$$\Rightarrow V=\frac{9\times10^9\times\left(1.6\times10^{19}\right)^2}{4\times10^{15}}J=\frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{4\times10^{-15}\times\left(1.6\times10^{-19}\right)}eV$$

$\therefore V=360keV$

Therefore, we found the height of the potential barrier of the two-deuteron system to be 360keV.

21. From the relation $R=R_0A^{\frac{1}{3}}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e., independent of A).

Ans: We know the expression for nuclear radius to be:

$$R=R_0A^{\frac{1}{3}}$$

Where, R_0 is a Constant and A is the mass number of the nucleus

Nuclear matter density would be,

$$\rho=\frac{\text{Mass of the nucleus}}{\text{Volume of the nucleus}}$$

Now, let m be the average mass of the nucleus, then, mass of the nucleus $=mA$

Nuclear density,

$$\rho=\frac{mA}{\frac{4}{3}\pi R^3}=\frac{3mA}{4\pi\left(R_0A^{\frac{1}{3}}\right)^3}=\frac{3mA}{4\pi R_0^{\,3}A}$$

$$\therefore\rho=\frac{3m}{4\pi R_0^{\,3}}$$

Therefore, we found the nuclear matter density to be independent of A and it is found to be nearly constant.

22. For the β^+(positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, **say, the K− shell, is captured by the nucleus and a neutrino is emitted).**

$$e^+ + {}^A_ZX \rightarrow {}^{A}_{Z-1}Y+\nu$$

Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed **but not vice−versa.**

Ans: Let the amount of energy released during the electron capture process be Q_1. The nuclear reaction could be written as:

$e^{+} + {}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + \nu + Q_1 \ldots\ldots (1)$

Let the amount of energy released during the positron capture process be Q_2. The nuclear reaction could be written as:

${}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + e^{+} + \nu + Q_2 \ldots\ldots (2)$

Let, $m_N\left({}_{Z}^{A}X\right)$ be the nuclear mass of ${}_{Z}^{A}X$,

$m_N\left({}_{Z-1}^{A}Y\right)$ be the nuclear mass of ${}_{Z-1}^{A}Y$

$m\left({}_{Z}^{A}X\right)$ be the atomic mass of ${}_{Z}^{A}X$

$m\left({}_{Z-1}^{A}Y\right)$ be the nuclear mass of ${}_{Z-1}^{A}Y$

m_e be the mass of an electron, c be the speed of light, then, the Q-value of the electron capture reaction could be given as,

$$Q_1 = \left[m_N\left({}_{Z}^{A}X\right) + m_e - m_N\left({}_{Z-1}^{A}Y\right)\right]c^2$$

$$\Rightarrow Q_1 = \left[m\left({}_{Z}^{A}X\right) - Zm_e + m_e - m\left({}_{Z-1}^{A}Y\right) + (Z-1)m_e\right]c^2$$

$$\Rightarrow Q_1 = \left[m\left({}_{Z}^{A}X\right) - m\left({}_{Z-1}^{A}Y\right)\right]c^2 \ldots\ldots (3)$$

The Q-value of the positron capture reaction could be given as,

$$Q_2 = \left[m_N\left({}_{Z}^{A}X\right) - m_N\left({}_{Z-1}^{A}Y\right) - m_e\right]c^2$$

$$\Rightarrow Q_1 = \left[m\left({}_{Z}^{A}X\right) - Zm_e - m\left({}_{Z-1}^{A}Y\right) + (Z-1)m_e - m_e\right]c^2$$

$$\Rightarrow Q_1 = \left[m\left({}_{Z}^{A}X\right) - m\left({}_{Z-1}^{A}Y\right) - 2m_e\right]c^2 \ldots\ldots (4)$$

It can be inferred that if $Q_2 > 0$, then; Also, if $Q1 > 0$, it does not necessarily mean that $Q2 > 0$. In other words, we could say that if β^{+} emission is energetically allowed, then the electron capture process is necessarily allowed, but not vice-versa. This is so because the Q-value must be positive for an energetically-allowed nuclear reaction.

23. In a periodic table the average atomic mass of magnesium is given as $24.312u$. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are:

${}_{12}^{24}Mg(23.98504u)$, ${}_{12}^{25}Mg(24.98584u)$ and ${}_{12}^{26}Mg(25.98259u)$

The natural abundance of ${}_{12}Mg^{24}$ is 78.99% by mass. Calculate the abundances of other two isotopes.

Ans: We are given:

Average atomic mass of magnesium, $m = 24.312$ u

Mass of magnesium ${}_{12}^{24}Mg$ isotope, $m_1 = 23.98504$ u

Mass of magnesium $^{25}_{12}Mg$ isotope, $m_2 = 24.98584$ u

Mass of magnesium $^{26}_{12}Mg$ isotope, $m_3 = 25.98259$ u

Abundance of $^{24}_{12}Mg$, $\eta_1 = 78.99\%$

Abundance of $^{25}_{12}Mg$, $\eta_2 = x\%$

Now, the abundance of $^{26}_{12}Mg$, $\eta_3 = 100 - x - 78.99\% = (21.01 - x)\%$

Also, we have the relation for the average atomic mass as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$\Rightarrow 24.312 = \frac{23.98504 \times 78.99 + 24.98584 \times x + 25.98259 \times (21.01 - x)}{100}$$

$$\Rightarrow 0.99675x = 9.2725255$$

$$\therefore x \approx 9.3\%$$

And, $21.01 - x = 11.71\%$

Therefore, we found the abundance of $^{25}_{12}Mg$ to be 9.3% and that of $^{26}_{12}Mg$ to be 11.71%.

24. The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei $^{41}_{20}Ca$ and $^{27}_{13}Al$ from the following data:

$m\left(^{40}_{20}Ca\right) = 39.962591u$, $m\left(^{41}_{20}Ca\right) = 40.962278u$, $m\left(^{26}_{13}Al\right) = 25.986995u$,

$m\left(^{27}_{13}Al\right) = 26.981541u$

Ans: For a neutron removal from $_{20}Ca^{41}$ nucleus, the corresponding nuclear reaction could be written as,

$$^{41}_{20}Ca \rightarrow {}^{40}_{20}Ca + {}^{1}_{0}n$$

We are given:

$$m\left(^{40}_{20}Ca\right) = 39.962591u$$

$$m\left(^{41}_{20}Ca\right) = 40.962278u$$

$$m\left(_0n^1\right) = 1.008665u$$

Now, the mass defect for this reaction could be given by,

$$\Delta m = m\left(^{40}_{20}Ca\right) + \left(^{1}_{0}n\right) - m\left(^{41}_{20}Ca\right)$$

$$\Rightarrow \Delta m = 39.962591 + 1.008665 - 40.962278 = 0.008978u$$

But we know, $1u = 931.5MeV / c^2$

$$\Rightarrow \Delta m = 0.008978 \times 931.5MeV / c^2$$

Now, we could calculate the energy required for the neutron removal by,

$E = \Delta mc^2$

$\Rightarrow E = 0.008978 \times 931.5 = 8.363007 MeV$

For the case of ${}^{27}_{13}Al$, the neutron removal reaction could be written as,

${}^{27}_{13}Al \rightarrow {}^{26}_{13}Al + {}^{1}_{0}n$

We are given,

$m\left({}^{26}_{13}Al\right) = 25.986995u$

$m\left({}^{27}_{13}Al\right) = 26.981541u$

Now, the mass defect here could be given by,

$\Delta m = m\left({}^{26}_{13}Al\right) + m\left({}^{1}_{0}n\right) - m\left({}^{27}_{13}Al\right)$

$\Rightarrow \Delta m = 25.986895 + 1.008665 - 26.981541 = 0.014019u$

$\Rightarrow \Delta m = 0.014019 \times 931.5 MeV / c^2$

Therefore, the energy that is required for the removal of neutron would be,

$E = \Delta mc^2 = 0.014019 \times 931.5$

$\therefore E = 13.059 MeV$

25. A source contains two phosphorous radio nuclides ${}^{32}_{15}P\left(T_{\frac{1}{2}} = 14.3d\right)$ and ${}^{33}_{15}P\left(T_{\frac{1}{2}} = 25.3d\right)$ Initially, 10% of the decays come from ${}^{33}_{15}P$. How long must one wait until 90% do so?

Ans: We are given:

Half life of ${}^{32}_{15}P\left(T_{\frac{1}{2}} = 14.3d\right)$

Half life of ${}^{33}_{15}P\left(T_{\frac{1}{2}} = 25.3d\right)$

Now, we know that nucleus decay is 10% of the total amount of decay.

Also, the source has initially 10% of ${}^{32}_{15}P$ nucleus and 90% of ${}^{32}_{15}P$ nucleus.

Suppose after t days, the source has 10% of ${}^{32}_{15}P$ nucleus and 90% of ${}^{33}_{15}P$ nucleus.

Initially we have:

Number of ${}^{33}_{15}P$ nucleus $= N$

Number of ${}^{32}_{15}P$ nucleus $= 9N$

Finally:

Number of ${}^{33}_{15}P$ nucleus $= 9N'$

Number of ${}^{32}_{15}P$ nucleus $= N'$

For ${}^{32}_{15}P$ nucleus, we could write the number ratio as:

$$\frac{N'}{9N} = \left(\frac{1}{2}\right)^{\frac{t}{T_{\frac{1}{2}}}}$$

$$\Rightarrow N' = 9N(2)^{\frac{-t}{14.3}} \ldots\ldots. (1)$$

Now, for ${}^{33}_{15}P$, we could write the number ratio as,

$$\frac{9N'}{N} = \left(\frac{1}{2}\right)^{\frac{t}{T'_{\frac{1}{2}}}}$$

$$\Rightarrow 9N' = N(2)^{\frac{-t}{25.3}} \ldots\ldots. (2)$$

We could now divide equation (1) by equation (2) to get,

$$\frac{1}{9} = 9 \times 2^{\left(\frac{t}{25.3} - \frac{t}{14.3}\right)}$$

$$\Rightarrow \frac{1}{81} = 2^{\left(-\frac{11t}{25.3 \times 14.3}\right)}$$

$$\Rightarrow \log 1 - \log 81 = \frac{-11t}{25.3 \times 14.3} \log 1$$

$$\Rightarrow \frac{-11t}{25.3 \times 14.3} = \frac{0 - 1.908}{0.301}$$

$$\therefore t = \frac{25.3 \times 14.3 \times 1.908}{11 \times 0.301} \approx 208.5 \text{days}$$

Therefore, we found that it would take about 208.5days for 90% decay of ${}^{33}_{15}P$.

26. Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α – particle. Consider the following decay processes:

$${}^{223}_{88}Ra \rightarrow {}^{209}_{82}Pb + {}^{14}_{6}C$$

$${}^{223}_{88}Ra \rightarrow {}^{219}_{86}Rn + {}^{4}_{2}He$$

Calculate the Q-values for these decays and determine that both are energetically allowed.

Ans: Consider a ${}^{14}_{6}C$ emission nuclear reaction,

$${}^{223}_{88}Ra \rightarrow {}^{209}_{82}Pb + {}^{14}_{6}C$$

We know that:

Mass of ${}^{223}_{88}Ra$, $m_1 = 223.01850u$

Mass of ${}^{14}_{6}C$, $m_3 = 14.00324u$

Now, the Q-value of the reaction could be given as:

$$Q=(m_1-m_2-m_3)c^2$$

$$\Rightarrow Q=(223.01850-208.98107-14.00324)c^2=\left(0.03419c^2\right)u$$

But we have, $1u=931.5\text{MeV}/c^2$

$$\Rightarrow Q=0.03419\times 931.5$$

$$\therefore Q=31.848\text{MeV}$$

Hence, the Q-value of the nuclear reaction is found to be 31.848 MeV. Since the value is positive, the reaction is energetically allowed.

Now consider a ${}_2^4He$ emission nuclear reaction:

$${}_{88}^{223}Ra\rightarrow {}_{86}^{229}Rn+{}_2^4He$$

We know that:

Mass of ${}_{88}^{223}Ra$, $m_1=223.01850$

Mass of ${}_{82}^{219}Rn$, $m_2=219.00948$

Mass of ${}_2^4He$, $m_3=4.00260$

Q-value of this nuclear reaction could be given as:

$$Q=(m_1-m_2-m_3)c^2$$

$$\Rightarrow Q=(223.01850-219.00948-4.00260)c^2$$

$$\Rightarrow Q=\left(0.00642c^2\right)u$$

$$\therefore Q=0.00642\times 931.5=5.98\text{MeV}$$

Therefore, the Q-value of the second nuclear reaction is found to be 5.98MeV. Since the value is positive, we could say that the reaction is energetically allowed.

27. Consider the fission of ${}_{92}^{238}U$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are ${}_{58}^{140}Ce$ and ${}_{44}^{99}Ru$. Calculate Q for this fission process. The relevant atomic and particle masses are:

$$m\left({}_{92}^{238}U\right)=238.05079u$$

$$m\left({}_{58}^{140}Ce\right)=139.90543u$$

$$m\left({}_{44}^{99}Ru\right)=98.90594u$$

Ans: We are given:

In the fission of ${}_{92}^{238}U$, 10 $\beta-$particles decay from the parent nucleus. The nuclear reaction can be written as:

$${}_{92}^{238}U+{}_0^1n\rightarrow {}_{58}^{140}Ce+{}_{44}^{99}Ru+10\,{}_{-1}^{0}e$$

It is also given that:

Mass of a nucleus of $^{238}_{92}U$, $m_1 = 238.05079u$

Mass of a nucleus of $^{140}_{58}Ce$, $m_2 = 139.90543u$

Mass of nucleus of $^{99}_{44}Ru$, $m_3 = 98.90594u$,

Mass of a neutron $^{1}_{0}n$, $m_4 = 1.008665u$

Q-value of the above equation would be,

$$Q = \left[m'\left(^{238}_{92}U\right) + m\left(^{1}_{0}n\right) - m'\left(^{140}_{58}Ce\right) - m'\left(^{99}_{44}Ru\right) - 10m_e\right]c^2$$

Where, m' = Represents the corresponding atomic masses of the nuclei

$$m'\left(^{238}_{92}U\right) = m_1 - 92m_e$$

$$m'\left(^{140}_{58}Ce\right) = m_2 - 58m_e$$

$$m'\left(^{99}_{44}Ru\right) = m_3 - 44m_e$$

$$m\left(^{1}_{0}n\right) = m_4$$

$$Q = \left[m_1 - 92m_e + m_4 - m_2 + 58m_e - m_3 + 44m_e - 10m_e\right]c^2$$

$$\Rightarrow Q = \left[m_1 + m_4 - m_2 - m_3\right]c^2 = \left[238.0507 + 1.008665 - 139.90543 - 98.90594\right]c^2$$

$$\Rightarrow Q = \left[0.247995c^2\right]u$$

But $1u = 931.5MeV/c^2$

$\therefore Q = 0.247995 \times 931.5 = 231.007MeV$

Therefore, the Q-value of the fission process is found to be 231.007MeV.

28. Consider the D−T reaction (deuterium−tritium fusion)

$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + n$

a) Calculate the energy released in MeV in this reaction from the data:

$m\left(^{2}_{1}H\right) = 2.014102u$, $m\left(^{3}_{1}H\right) = 3.016049u$

Ans: Consider the D-T nuclear reaction,

$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + n$

We are also given that:

Mass of $^{2}_{1}H$, $m_1 = 2.014102u$

Mass of $^{3}_{1}H$, $m_2 = 3.016049u$

Mass of $^{4}_{2}He$, $m_3 = 4.002603u$

Mass of $^{1}_{0}n$, $m_4 = 1.008665u$

Now, the Q-value of the given D-T reaction would be:

$$Q = \left[m_1 + m_2 - m_3 - m_4\right]c^2$$

$\Rightarrow Q = [2.014102 + 3.016049 - 4.002603 - 1.008665]c^2$

$\Rightarrow Q = [0.018883c^2]u$

But $1u = 931.5MeV/c^2$

$\therefore Q = 0.018883 \times 931.5 = 17.59MeV$

b) Consider the radius of both deuterium and tritium to be approximately 2.0fm. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction? (Hint: Kinetic energy required for one fusion event = average thermal kinetic energy available with the interacting particles $2\left(\frac{3kT}{2}\right)$; **k = Boltzmann's constant, T = absolute** temperature.)

Ans: We are given:

Radius of deuterium and tritium, $r \approx 2.0fm = 2 \times 10^{-15}m$

Distance between the two nuclei at the moment when they touch each other,

$d = r + r = 4 \times 10^{-15}m$

Charge on the deuterium nucleus $= e$

Charge on the tritium nucleus $= e$

Hence, the repulsive potential energy between the two nuclei could be given as:

$$V = \frac{e^2}{4\pi\varepsilon_0 d}$$

Where, $\varepsilon_0 =$ Permittivity of free space

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm^2c^{-2}$$

$$\Rightarrow V = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15}} = 5.76 \times 10^{-14} J = \frac{5.76 \times 10^{-14}}{1.6 \times 10^{-19}}$$

$\therefore V = 3.6 \times 10^5 eV = 360keV$

Therefore, $5.76 \times 10^{-14}J$ or $360keV$ of kinetic energy (KE) is needed to overcome the coulomb repulsion between the two nuclei.

However, we are also given that:

$$KE = 2 \times \frac{3kT}{2}$$

Where, $k =$ Boltzmann constant

$T =$ Temperature required for triggering the reaction

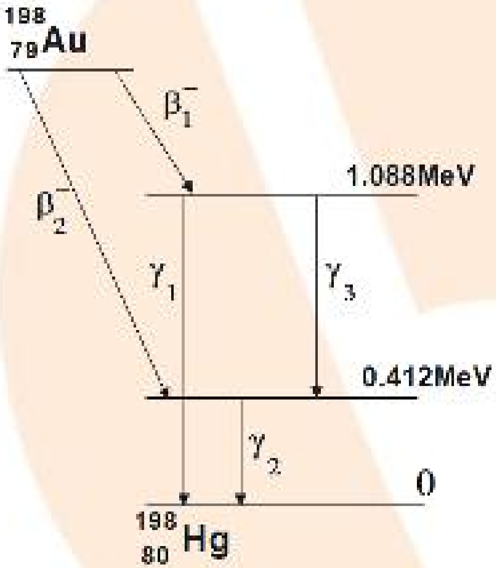
$\therefore T = \frac{KE}{3k} = \frac{5.76\times10^{-14}}{3\times1.38\times10^{-23}} = 1.39\times10^{9}K$

Therefore, we found that the gas must be heated to a temperature of $1.39\times10^{9}K$ to initiate the reaction.

29. Obtain the maximum kinetic energy of β – particles, and the radiation frequencies of γ decays in the decay scheme shown in figure. You are given that:

$m\left({}^{198}Au\right) = 197.968233u$

$m\left({}^{198}Hg\right) = 197.966760u$



Ans: It can be observed from the given γ-decay diagram that γ_1 decays from the 1.088MeV energy level to the 0MeV energy level. Hence, the energy corresponding to γ_1-decay is given as:

$E_1 = 1.088 - 8 = 1.088MeV$

$\Rightarrow h\nu_1 = 1.088\times1.6\times10^{-19}\times10^{6}J$

Where, Planck's constant $h = 6.6\times10^{-34}Js$

ν_1 = Frequency of radiation radiated by γ_1 – decay

$\nu_1 = \frac{E_1}{h}$

$\Rightarrow \nu_1 = \frac{1.088\times1.6\times10^{-19}\times10^{6}}{6.6\times10^{-34}} = 2.637\times10^{20}Hz$

It can be observed from the given $\gamma-$decay diagram that γ_2 decays from the 0.412MeV energy level to the 0MeV energy level.
Now, the energy corresponding to γ_2 -decay could be given as:

$E_2 = 0.412 - 0 = 0.412\text{MeV}$

$\Rightarrow h\nu_2 = 0.412 \times 1.6 \times 10^{-19} \times 10^6\,\text{J}$

Where, $\nu_2 =$ Frequency of radiation radiated by $\gamma_2 -$decay

$$\nu_2 = \frac{E_2}{h} = \frac{0.412 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 9.988 \times 10^{19}\,\text{Hz}$$

It can be observed from the given γ-decay diagram that γ_3-decays from the 1.088MeV energy level to the 0.412MeV energy level.
Now, the energy corresponding to γ_3 -decay is given as:

$E_3 = 1.088 - 0.412 = 0.676\text{MeV}$

$\Rightarrow h\nu_3 = 0.676 \times 10^{-19} \times 10^6$

Where, $\nu_3 =$ Frequency of radiation radiated by $\gamma_3 -$decay

$$\nu_3 = \frac{E_3}{h} = \frac{0.676 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 1.639 \times 10^{20}\,\text{Hz}$$

Mass of $m\left({}^{198}_{78}\text{Au}\right) = 197.968233\text{u}$

Mass of $m\left({}^{198}_{80}\text{Hg}\right) = 197.966760\text{u}$

$1\text{u} = 931.5\text{MeV}/c^2$

Energy of the highest level could be given as:

$E = \left[m\left({}^{198}_{78}\text{Au}\right) - m\left({}^{198}_{80}\text{Hg}\right)\right] = 197.968233 - 197.966760 = 0.001473\text{u}$

$\Rightarrow E = 0.001473 \times 931.5 = 1.3720995\text{MeV}$

β_1 decays from the 1.3720995MeV level to the 1.088MeV level

Maximum kinetic energy of the β_1 particle $= 1.3720995 - 1.088$

$\Rightarrow K.E = 0.2840995\text{MeV}$

β_2 decays from the 1.3720995MeV level to that of the 0.412MeV level. Now, we find the maximum kinetic energy of the β_2 particle to be,

$K.E_{max} = 1.3720995 - 0.412 = 0.9600995\text{MeV}$

Therefore, we found the maximum kinetic energy of the β_2 particle to be 0.9600995MeV.

30. Calculate and compare the energy released by
a) fusion of 1.0kg of hydrogen deep within Sun and
Ans: We are given:
Amount of hydrogen, $m = 1\text{ kg} = 1000\text{g}$

1 mole, i.e., 1g of hydrogen $\left({}_1^1H\right)$ contains 6.023×10^{23} atoms.

That is, 1000g of ${}_1^1H$ contains 6.023×10^{23} atoms.

Within the sun, four ${}_1^1H$ nuclei combine and form one ${}_2^4He$ nucleus. In this process $26MeV$ of energy is released.

Hence, the energy released from the fusion of 1 kg ${}_1^1H$ is:

$$E_1=\frac{6.023\times10^{23}\times26\times10^3}{4}=39.1495\times10^{26}\,MeV$$

Therefore, we found the energy released during the fusion of 1kg ${}_1^1H$ is:

$$E_1=\frac{6.023\times10^{23}\times26\times10^3}{4}=39.1495\times10^{26}\,MeV$$

Hence, the energy released during the fusion of 1kg of ${}_1^1H$ to be $39.1495\times10^{26}\,MeV$.

b) the fission of 1.0kg of ${}^{235}U$ in a fission reactor.

Ans: We are given:

Amount of ${}_{92}U^{235}=1000gm$

1 mole, i.e., 235g of ${}_{92}^{235}U$ contains 6.023×10^{23} atoms.

1000g of ${}_{92}^{235}U$ contains $\frac{6.023\times10^{23}\times1000}{235}$ atoms

We know that the amount of energy released in the fission of one atom of ${}_{92}^{235}U$ is $200MeV$. Therefore, energy released from the fission of 1kg of ${}_{92}^{235}U$ is:

$$E_2=\frac{6\times10^{23}\times1000\times200}{235}=5.106\times10^{26}\,MeV$$

$$\frac{E_1}{E_2}=\frac{39.1495\times10^{26}}{5.106\times10^{26}}=7.67\approx8$$

Hence, we found the energy released during the fusion of 1kg of hydrogen is nearly 8 times the energy released during the fusion of 1kg of uranium.

31. Suppose India had a target of producing, by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (i.e., conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of ${}^{235}U$ to be about $200MeV$.

Ans: We are given the following:

Amount of electric power to be generated, $P = 2\times10^5\,MW$, 10% of this amount has to be obtained from nuclear power plants.

Amount of nuclear power, $P_1 = \frac{10}{100}\times 2\times10^5 = 2\times10^4\,MW$

$\Rightarrow P_1 = 2\times10^4\times10^6\,J/s = 2\times10^{10}\times3600\times24\times365\,J/y$

Heat energy released per fission of a ^{235}U nucleus, $E = 200$ MeV

Efficiency of a reactor $= 25\%$

Hence, the amount of energy converted into the electrical energy per fission is calculated as:

$$\frac{25}{100}\times200 = 50MeV = 50\times1.6\times10^{-19}\times10^6 = 8\times10^{-12}\,J$$

The number of atoms required for fission per year would be:

$$\frac{2\times10^{10}\times60\times60\times24\times365}{8\times10^{-12}} = 78840\times10^{24}\,\text{atoms}$$

1 mole, i.e., 235g of U^{235} contains 6.023×10^{23} atoms

That is, the mass of 6.023×10^{23} atoms of $U^{235} = 235g = 235\times10^{-3}\,kg$

Also, the mass of:

$$78840\times10^{24}\,\text{atoms of }U^{235} = \frac{235\times10^{-3}}{6.023\times10^{23}}\times78840\times10^{24} = 3.076\times10^4\,kg$$

Hence, the mass of uranium needed per year is found to be, $3.076\times10^4\,kg$.