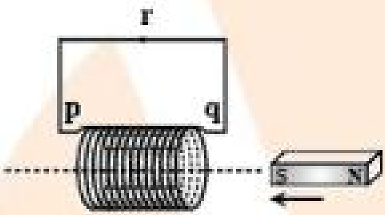
NCERT Solutions for Class 12

Physics

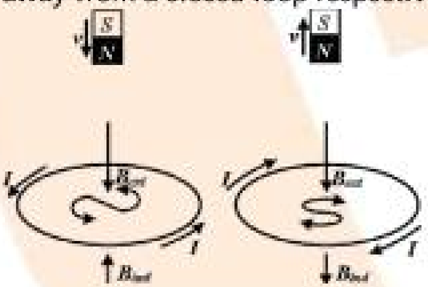
Chapter 6 – Electromagnetic Induction

1. Predict the direction of induced current in the situations described by the following figures (a) to (f).

a)

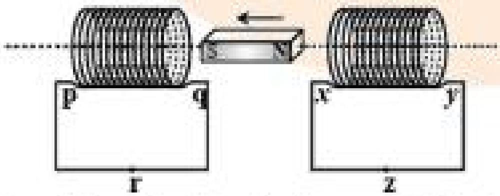


Ans: The direction of the induced current in a closed loop could be given by Lenz's law. The following pairs of figures show the direction of the induced current when the North pole of a bar magnet is moved towards and away from a closed loop respectively.



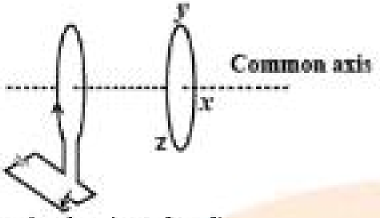
Now, by using Lenz's rule, the direction of the induced current in the given situation is found to be along qrpq.

b)



Ans: On using Lenz's law, we find the direction of the induced current here to be along prqp.

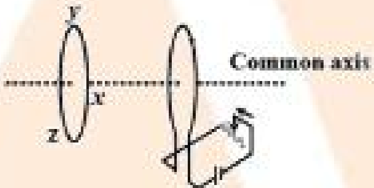
c)



(Tapping key just closed)

Ans: Using Lenz's law, we find the direction of the induced current to be along yzxy.

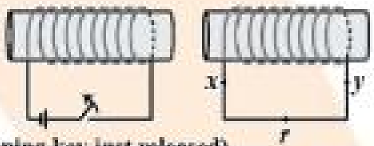
d)



Rheostat setting being changed

Ans: Using Lenz's law, we find the direction of the induced current to be along zyxz.

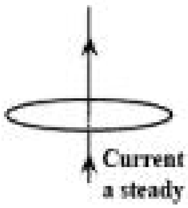
e)



(Tapping key just released)

Ans: Using Lenz's law, we found the direction of the induced current to be along xryx.

f)

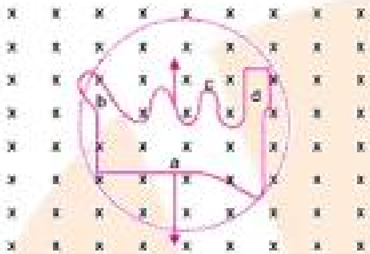


Current (I) decreasing at a steady rate

Ans: Here we find that, no current is induced since the field lines are lying in the same plane as that of the closed loop.

2. Use Lenz's law to determine the direction of induced current in the situations described by Figure:

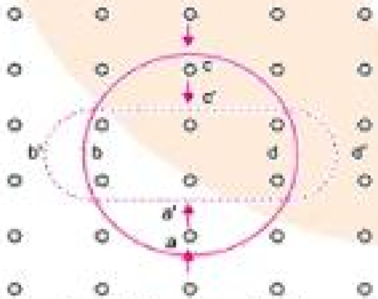
a) A wire of irregular shape turning into a circular shape;



Ans: According to the Lenz's law, the direction of the induced emf is such that it tends to produce a current that would oppose the change in the magnetic flux that produced it.

The wire is here is expanding to form a circle, which means that force would be acting outwards on each part of wire because of the magnetic field (acting in the downwards direction). Now, the direction of induced current should be such that it will produce magnetic field in the upward direction (towards the reader). Therefore, the force on wire will be towards inward direction, i.e., induced current would be flowing in anticlockwise direction in the loop from cbad.

b) A circular loop being deformed into a narrow straight wire.



Ans: On deforming the shape of a circular loop into a narrow straight wire, the flux piercing the surface decreases. Therefore, the induced current flows along abcd according to Lenz's law.

3. A long solenoid with 15 turns per cm has a small loop of area $2.0cm^2$

placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0A to 4.0A in 0.1s, what is the induced emf in the loop while the current is changing?

Ans: We are given the following information:

Number of turns on the solenoid $= 15\text{turns/cm} = 1500\text{turns/m}$

Number of turns per unit length, $n = 1500$ turns

The solenoid has a small loop of area, $A = 2.0\text{cm}^2 = 2 \times 10^{-4}\text{m}^2$

Current carried by the solenoid changes from 2A to 4A.

Now, the change in current in the solenoid, $di = 4 - 2 = 2\text{A}$

Change in time, $dt = 0.1\text{ s}$

Induced emf in the solenoid could be given by Faraday's law as:

$$\varepsilon = \frac{d\phi}{dt} \quad \text{..............................} (1)$$

Where, induced flux through the small loop, $\phi = BA$ .... (2)

Equation (1) would now reduce to:

$$\varepsilon = \frac{d}{dt}(BA) = A\mu_0 n \times \left(\frac{di}{dt}\right)$$

Substituting the given values into this equation, we get,

$$\varepsilon = 2 \times 10^{-4} \times 4\pi \times 10^{-7} \times 1500 \times \frac{2}{0.1}$$

$$\therefore \varepsilon = 7.54 \times 10^{-6}\text{V}$$

Therefore, the induced voltage in the loop is found to be, $\varepsilon = 7.54 \times 10^{-6}\text{V}$.

4. A rectangular wire loop of sides 8cm and 2cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is in a direction normal to the:

a) longer side? For how long does the induced voltage last in this case?

Ans: We are given the following,

Length of the rectangular wire, $l = 8\text{cm} = 0.08\text{ m}$

Width of the rectangular wire, $b = 2\text{cm} = 0.02\text{m}$

Now, the area of the rectangular loop,

$A = lb = 0.08 \times 0.02 = 16 \times 10^{-4}\text{m}^2$

Magnetic field strength, $B = 0.3\text{T}$

Velocity of the loop, $v = 1\text{ cm/s} = 0.01\text{ m/s}$

Emf developed in the loop could be given as:

$\varepsilon = Blv$

Substituting the given values,

$\varepsilon = 0.3 \times 0.08 \times 0.01 = 2.4 \times 10^{-4}\text{V}$

Time taken to travel along the width, $t = \frac{\text{Distance travelled}}{\text{velocity}} = \frac{b}{v}$

$\Rightarrow t = \frac{0.02}{0.01} = 2s$

Therefore, the induced voltage is found to be $2.4 \times 10^{-4} V$ which lasts for 2s.

b) shorter side of the loop? For how long does the induced voltage last in this case?

Ans: We know that, Emf developed in the loop could be given as:

$\varepsilon = Blv$

Substituting the given values,

$\varepsilon = 0.3 \times 0.02 \times 0.01 = 0.6 \times 10^{-4} V$

Time taken to travel along the width, $t = \frac{\text{Distance travelled}}{\text{velocity}} = \frac{l}{v}$

$\Rightarrow t = \frac{0.08}{0.01} = 8s$

Therefore, the induced voltage is found to be $0.6 \times 10^{-4} V$ which lasts for 8s.

5. A $1.0m$ long metallic rod is rotated with an angular frequency of $400 rads^{-1}$ about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of $0.5T$ parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Ans: We are given the following:

Length of the rod, $l = 1m$

Angular frequency, $\omega = 400 rad/s$

Magnetic field strength, $B = 0.5T$

One end of the rod has zero linear velocity, while the other end has a linear velocity of $l\omega$.

Average linear velocity of the rod, $v = \frac{l\omega + 0}{2} = \frac{l\omega}{2}$

Emf developed between the centre and the ring,

$\varepsilon = Blv = Bl\left(\frac{l\omega}{2}\right) = \left(\frac{Bl^2\omega}{2}\right)$

On substituting the given values,

$\therefore \varepsilon = \frac{0.5 \times (1)^2 \times 400}{2} = 100V$

Therefore, the emf developed between the centre and the ring is $100V$.

6. A circular coil of radius $8.0cm$ and 20 turns is rotated about its vertical diameter with an angular speed of $50 rad s^{-1}$ in a uniform horizontal magnetic field of magnitude $3.0 \times 10^{-2} T$. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?

Ans: We are given:

Max induced emf $= 0.603V$

Average induced emf $= 0V$

Max current in the coil $= 0.0603A$

Average power loss $= 0.018W$ (Power comes from the external rotor)

Radius of the circular coil, $r = 8cm = 0.08m$

Area of the coil, $A = \pi r^2 = \pi \times (0.08)^2 m^2$

Number of turns on the coil, $N = 20$

Angular speed, $\omega = 50 rad/s$

Magnetic field strength, $B = 3 \times 10^{-2} T$

Resistance of the loop, $R = 10 \Omega$

Maximum induced emf could be given as:

$\varepsilon = N\omega AB = 20 \times 50 \times \pi \times (0.08)^2 \times 3 \times 10^{-2}$

$\therefore \varepsilon = 0.603V$

The maximum emf induced in the coil is found to be $0.603V$.

Over a full cycle, the average emf induced in the coil is found to be zero.

Maximum current is given as:

$I = \frac{\varepsilon}{R}$

$\Rightarrow I = \frac{0.603}{10}$

$\Rightarrow I = 0.0603A$

Average power loss due to joule heating:

$\therefore P = \frac{eI}{2} = \frac{0.603 \times 0.0603}{2} = 0.018W$

We know that the current induced in the coil would produce a torque opposing the rotation of the coil. Since the rotor is an external agent, it must supply a torque to counter this torque in order to keep the coil rotating uniformly. Hence, dissipated power comes from the external rotor.

7. A horizontal straight wire 10m long extending from east to west is falling with a speed of 5.0ms^{-1}, at right angles to the horizontal **component of the earth's magnetic field,** $0.30\times10^{-4}\text{Wbm}^{-2}$.

a) What is the instantaneous value of the emf induced in the wire?

Ans: We are given the following:

Length of the wire, $l = 10\text{m}$

Falling speed of the wire, $v = 5.0\text{m/s}$

Magnetic field strength, $B = 0.3\times10^{-4}\text{Wbm}^{-2}$

Emf induced in the wire is thus found to be,

$\varepsilon = Blv$

$\Rightarrow \varepsilon = 0.3\times10^{-4}\times5\times10$

$\therefore \varepsilon = 1.5\times10^{-3}\text{V}$

Hence, the emf induced in the wire is $\varepsilon = 1.5\times10^{-3}\text{V}$.

b) What is the direction of the emf?

Ans: Using Fleming's rule, we find that the direction of the induced emf is from West to East.

c) Which end of the wire is at the higher electrical potential?

Ans: The eastern end of the wire is the end that is at higher potential.

8. Current in a circuit falls from 5.0A to 0.0A in 0.1s. If an average emf of 200V induced, give an estimate of the self-inductance of the circuit.

Ans: We are given the following:

Initial current, $I_1 = 5.0\text{A}$

Final current, $I_2 = 0.0\text{A}$

Change in current, $dI = I_1 - I_2 = 5\text{A}$

Time taken for the change, $t = 0.1\text{s}$

Average emf, $\varepsilon = 200\text{V}$

For self-inductance (L) of the coil, we have the relation for average emf that could be given as:

$$\varepsilon = L\frac{di}{dt}$$

$$\Rightarrow L = \frac{\varepsilon}{\left(\frac{di}{dt}\right)}$$

Substituting the given values we get,

$$\therefore L = \frac{200}{\left(\frac{5}{0.1}\right)} = 4H$$

Therefore, we found the self induction in the coil to be 4H.

9. A pair of adjacent coils has a mutual inductance of $1.5H$. If the current in one coil changes from 0 to $20A$ in $0.5s$, what is the change of flux linkage with the other coil?

Ans: We are given the following,

Mutual inductance of a pair of coils, $\mu = 1.5H$

Initial current, $I_1 = 0A$

Final current, $I_2 = 20A$

Change in current, $dI = I_2 - I_1 = 20 - 0 = 20A$

Time taken for the change, $t = 0.5s$

Induced emf, $\varepsilon = \frac{d\phi}{dt}$ …………………. (1)

Where, $d\phi$ is the change in the flux linkage with the coil.

Emf is related with mutual inductance could be given as:

$\varepsilon = \mu \frac{dI}{dt}$…………………….. (2)

Equating equations (1) and (2), we get,

$$\frac{d\phi}{dt} = \mu \frac{dI}{dt}$$

$$\Rightarrow d\phi = 1.5 \times (20)$$

$$\therefore d\phi = 30Wb$$

Hence, we found the change in the flux linkage to be 30Wb.

10. A jet plane is travelling towards west at a speed of $1800km/h$. What is the voltage difference developed between the ends of the wing having a span of $25m$, **if the Earth's magnetic field at the location has a magnitude** of $5\times10^{-4}T$ and the dip angle is 30^o.

Ans: Speed of the jet plane, $v = 1800\ km/h = 500\ m/s$

Wingspan of jet plane, $l = 25m$

Earth's magnetic field strength, $B = 5.0\times10^{-4}T$
Angle of dip, $\delta = 30^\circ$
Vertical component of Earth's magnetic field could be given by,
$B_V = B\sin\delta$
$\Rightarrow B_V = 5\times10^{-4}\sin30^\circ = 2.5\times10^{-4}T$
Voltage difference between the ends of the wing can be calculated as,
$\varepsilon = B_V \times l \times v$
Substituting the given values,
$\Rightarrow \varepsilon = 2.5\times10^{-4}\times25\times500$
$\therefore \varepsilon = 3.125V$
Hence, the voltage difference developed between the ends of the wings is $3.125V$.

11. Suppose the loop in Exercise 6.4 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of $0.3T$ at the rate of $0.02Ts^{-1}$. If the cut is joined and the loop has a resistance of 1.6Ω, how much power is dissipated by the loop as heat? What is the source of this power?
Ans: We are given,
Sides of the rectangular loop are 8cm and 2cm. Hence, area of the rectangular wire loop would be,
$A = \text{length} \times \text{width}$
Initial value of the magnetic field, $B' = 0.3T$
Rate of decrease of the magnetic field, $\frac{dB}{dt} = 0.02T/s$
Emf developed in the loop is given as:
$\varepsilon = \frac{d\phi}{dt}$
Where,
$\varepsilon = \frac{d(AB)}{dt} = \frac{AdB}{dt}$
$\Rightarrow \varepsilon = 16\times10^{-4}\times0.02 = 0.32\times10^{-4}V$
Resistance of the loop, $R = 1.6\Omega$
The current induced in the loop could be given as:
$i = \frac{\varepsilon}{R}$
Substituting the given values,

$\Rightarrow i = \dfrac{0.32\times10^{-4}}{1.6} = 2\times10^{-5}A$

Power dissipated in the loop in the form of heat could be given as:

$P = i^2R$

$\Rightarrow P = \left(2\times10^{-5}\right)^2\times1.6$

$\therefore P = 6.4\times10^{-10}W$

The source of this heat loss is an external agent, which is responsible for changing the magnetic field with time.

12. A square loop of side 12cm with its sides parallel to X and Y axes is moved with a velocity of 18cm in the positive x-direction in an environment containing a magnetic field in the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient of $10^{-3}Tcm^{-1}$ along the negative x-direction (that is it increases by $10^{-3}Tcm^{-1}$ as one moves in the negative x-direction), and it is decreasing in time at the rate of $10^{-3}Ts^{-1}$. Determine the direction and magnitude of the induced current in the loop if its resistance is $4.50m\Omega$.

Ans: We are given,

Side of the square loop, $s = 12cm = 0.12m$

Area of the square loop, $A = 0.12\times0.12 = 0.0144m^2$

Velocity of the loop, $v = 8cm/s = 0.08m/s$

Gradient of the magnetic field along negative x-direction,

$\dfrac{dB}{dx} = 10^{-3}Tcm^{-1} = 10^{-1}Tm^{-1}$

And, rate of decrease of the magnetic field,

$\dfrac{dB}{dt} = 10^{-3}Ts^{-1}$

Resistance of the loop,

$R = 4.5m\Omega = 4.5\times10^{-3}\Omega$

Rate of change of the magnetic flux due to the motion of the loop in a non-uniform magnetic field is given as:

$\dfrac{d\phi}{dt} = A\times\dfrac{dB}{dx}\times v$

$\Rightarrow \dfrac{d\phi}{dt} = 144\times10^{-4}m^2\times10^{-1}\times0.08$

$\Rightarrow \dfrac{d\phi}{dt} = 11.52\times10^{-5}Tm^2s^{-1}$

Rate of change of the flux due to explicit time variation in field B is given as:

$$\frac{d\phi'}{dt} = A \times \frac{dB}{dx}$$

$$\Rightarrow \frac{d\phi'}{dt} = 144 \times 10^{-4} \times 10^{-3} = 1.44 \times 10^{-5}\,Tm^2s^{-1}$$

Since the rate of change of the flux is the induced emf, the total induced emf in the loop can be calculated as:

$$e = 1.44 \times 10^{-5} + 11.52 \times 10^{-5}$$

$$= 12.96 \times 10^{-5}\,V$$

$\therefore$ Induced current, $i = \frac{e}{R}$

$$\Rightarrow i = \frac{12.96 \times 10^{-5}}{4.5 \times 10^{-3}}$$

$$\therefore i = 2.88 \times 10^{-2}\,A$$

Therefore, the direction of the induced current is such that there is an increase in the flux through the loop along the positive z-direction.

13. It is desired to measure the magnitude of field between the poles of a powerful loudspeaker magnet. A small flat search coil of area $2cm^2$ with 25 closely wound turns, is positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction). The total charge flown in the coil (measured by a ballistic galvanometer connected to coil) is $7.5mC$. The combined resistance of the coil and the galvanometer is 0.50Ω. Estimate the field strength of the magnet.

Ans: We are given the following:

Area of the small flat search coil, $A = 2cm^2 = 2 \times 10^{-4}m^2$

Number of turns on the coil, $N = 25$

Total charge flowing in the coil, $Q = 7.5mC = 7.5 \times 10^{-3}C$

Total resistance of the coil and galvanometer, $R = 0.50\Omega$

Induced current in the coil,

$$I = \frac{\text{Induced emf}\,(\varepsilon)}{R} \quad \ldots\ldots\ldots (1)$$

Induced emf is given as:

$$\varepsilon = -N\frac{d\phi}{dt} \quad \ldots\ldots\ldots (2)$$

Where, $d\phi$ = Induced flux

Combining equations (1) and (2), we get

$$I = -\frac{N\frac{d\phi}{dt}}{R}$$

$$Idt = -\frac{N}{R}d\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Initial flux through the coil, $\phi_i = BA$

Where,

$B =$ Magnetic field strength

Final flux through the coil, $\phi_f = 0$

Integrating equation (3) on both sides, we have

$$\int Idt = -\frac{N}{R}\int_{\phi_i}^{\phi_f} d\phi$$

But total charge could be given as, $Q = \int Idt$

$$\Rightarrow Q = \frac{-N}{R}(\phi_f - \phi_i) = \frac{-N}{R}(-\phi_i) = \frac{+N\phi_i}{R}$$

$$Q = \frac{NBA}{R}$$

$$\Rightarrow B = \frac{QR}{NA}$$

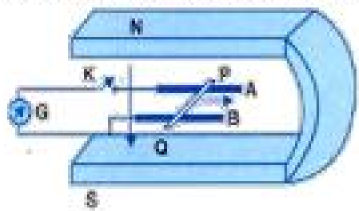
Substituting the given values, we get,

$$\Rightarrow B = \frac{7.5\times10^{-3}\times0.5}{25\times2\times10^{-4}}$$

$\therefore B = 0.75T$

Therefore, the field strength of the magnet is found to be $0.75T$.

14. Figure shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod $= 15cm$, $B = 0.50T$, resistance of the closed loop containing the rod $= 9.0m\Omega$. Assume the field to be uniform.



Suppose K is open and the rod is moved with a speed of $12\text{cm}\cdot\text{s}^{-1}$ in the direction shown. Give the polarity and magnitude of the induced emf .

Ans: We are given:

Length of the rod, $l = 15\text{cm} = 0.15\text{m}$

Magnetic field strength, $B = 0.50\text{T}$

Resistance of the closed loop, $R = 9\text{m}\Omega = 9\times10^{-3}\Omega$

Induced emf $= 9\text{mV}$

Here, polarity of the induced emf is such that end P shows positive while end Q shows negative ends.

Speed of the rod, $v = 12\text{cm/s} = 0.12\text{m/s}$

We know that the induced emf could be given as: $\varepsilon = Bvl$

Substituting the given values, we get,

$\varepsilon = 0.5\times0.12\times0.15$

$\Rightarrow \varepsilon = 9\times10^{-3}\text{v}$

$\therefore \varepsilon = 9\text{mV}$

Therefore, the magnitude of the induced emf is found to be $\varepsilon = 9\text{mV}$ an =d the polarity of the induced emf is such that end P shows positive while end Q shows negative.

a) Is there an excess charge built up at the ends of the rods when K is open? What if K is closed?

Ans: Yes; when key K is closed, excess charge could be maintained by the continuous flow of current. When key K is open, there is excess charge built up at both rod ends but when key K is closed, excess charge is maintained by the continuous flow of current.

b) With K open and the rod moving uniformly, there is no net force on the electrons in the rod PQ even though they do experience magnetic force due to the motion of the rod. Explain.

Ans: Magnetic force is cancelled by the electric force that is set-up due to the excess charge of opposite nature at both rod ends. There is no net force on the electrons in rod PQ when key K is open and the rod would move uniformly. This is because magnetic force is cancelled by the electric force set-up due to the excess charge of opposite nature at both ends of the rods.

c) What is the retarding force on the rod when K is closed?

Ans: We know that the retarding force exerted on the rod could be given by, $F = IBl$

Where,

$I =$ Current flowing through the rod

Substituting the given values, we get,

$I = \frac{e}{R} = \frac{9\times10^{-3}}{9\times10^{-3}} = 1A$

$\Rightarrow F = 1\times0.5\times0.15$

$\therefore F = 75\times10^{-3}N$

Therefore, we found the retarding force on the rod when the key K is closed to be,

$F = 75\times10^{-3}N$

d) How much power is required (by an external agent) to keep the rod moving at the same speed $(=12cm\cdot s^{-1})$ when K is closed? How much power is required when K is open?

Ans: We are given:

Speed of the rod, $v = 12cm/s = 0.12m/s$

Now, power could be given as:

$P = Fv$

Substituting the given values, we get,

$\Rightarrow P = 75\times10^{-3}\times0.12$

$\Rightarrow P = 9\times10^{-3}W$

$\therefore P = 9mW$

Therefore, we found the power that is required (by an external agent) to keep the rod moving at the same speed $(=12cm\cdot s^{-1})$ when K is closed to be $P = 9mW$ and when key K is open, no power is expended.

e) How much power is dissipated as heat in the closed circuit? What is the source of this power?

Ans: We know that,

Power dissipated as heat , $P=I^2R$

$\Rightarrow P = (1)^2\times9\times10^{-3}$

$\therefore P = 9mW$

The power dissipated as heat in the closed circuit is found to be $P = 9mW$ and the source of this power is found to be an external agent.

f) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular?

Ans: In this case, no emf would be induced in the coil because the motion of the rod does not cut across the field lines.

15. An air-cored solenoid with length $30cm$, area of cross-section $25cm^2$

and number of turns 500, carries a current of 2.5A. The current is suddenly switched off in a brief time of 10^{-3}s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.

Ans: We are given the following:

Length of the solenoid, $l = 30\text{cm} = 0.3\text{m}$

Area of cross-section, $A = 25\text{cm}^2 = 25\times10^{-4}\text{m}^2$

Number of turns on the solenoid, $N = 500$

Current in the solenoid, $I = 2.5\text{A}$

Current flows for time, $t = 10^{-3}\text{s}$

Average back emf , $e = \frac{d\phi}{dt}$ .... (1)

Where,

$d\phi$ = Change in flux=NAB .... (2)

Where, B = Magnetic field strength $= \mu_0 \frac{NI}{l}$ .... (3)

Where, μ_0 = Permeability of free space $= 4\pi\times10^{-7}\,\text{TmA}^{-1}$

Substituting equations (2) and (3) in equation (1), we get,

$$\varepsilon = \frac{\mu_0 N^2 IA}{lt}$$

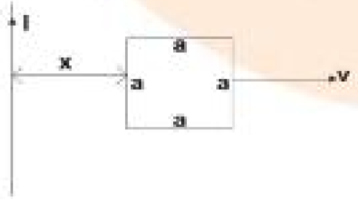
$$\Rightarrow \varepsilon = \frac{4\pi\times10^{-7}\times(500)^2\times2.5\times25\times10^{-4}}{0.3\times10^{-3}}$$

$\therefore \varepsilon = 6.5\text{V}$

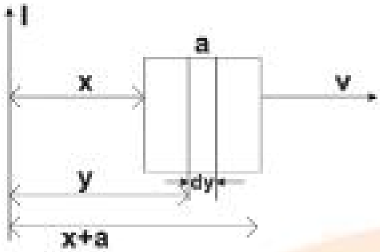
Hence, the average back emf induced in the solenoid is found to be 6.5V.

16.

a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown in below figure.



Ans: Consider a small element dy in the loop at a distance y from the long straight wire (as shown in the given figure).



Magnetic flux associated with element $dy, d\phi = BdA$

Where, dA = Area of element $dy = a\,dy$

B = magnetic field at distance $y = \dfrac{\mu_0 I}{2\pi y}$

I = Current in the wire

μ_0 = Permeability of free space $= 4\pi \times 10^{-7}\ H/m$

Carrying out the substitutions accordingly, we get,

$$\Rightarrow d\phi = \frac{\mu_0 Ia}{2\pi}\frac{dy}{y}$$

$$\Rightarrow \phi = \frac{\mu_0 Ia}{2\pi}\int \frac{dy}{y}$$

Now, the limit of y will be from x to $a + x$, on applying the limits we get,

$$\Rightarrow \phi = \frac{\mu_0 Ia}{2x}\int_x^{a+x} \frac{dy}{y}$$

$$\Rightarrow \phi = \frac{\mu_0 Ia}{2\pi}\left[\log_e y\right]_x^{a+x}$$

$$\Rightarrow \phi = \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{a+x}{x}\right)$$

For mutual inductance M, the flux could be given as:

$\phi = MI$

$$\Rightarrow MI = \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{a}{x}+1\right)$$

$$\therefore M = \frac{\mu_0 a}{2\pi}\log_e\left(\frac{a}{x}+1\right)$$

Therefore, the expression for the mutual inductance between the given long straight wire and the square loop of side a is found to be,

$$M = \frac{\mu_0 a}{2\pi}\log_e\left(\frac{a}{x}+1\right)$$

b) Now assume that the straight wire carries a current of 50A and the loop is moved to the right with a constant velocity, $v = 10 m/s$. Calculate the induced emf in the loop at the instant when $x = 0.2m$. Take $a = 0.1m$ and assume that the loop has a large resistance.

Ans: We know that, the Emf induced in the loop, $\varepsilon = B'av = \left(\frac{\mu_0 I}{2\pi x}\right)av$

We are given the following,

$I = 50A$

$x = 0.2m$

$a = 0.1m$

$v = 10m/s$

On substituting the given values into the equation, we get,

$$\varepsilon = \frac{4\pi \times 10^{-7} \times 50 \times 0.1 \times 10}{2\pi \times 0.2}$$

$\therefore \varepsilon = 5 \times 10^{-5} V$

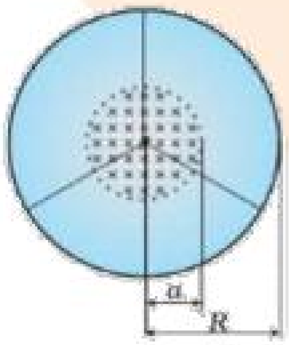
Therefore, induced emf in the loop at the given instant is found to be, $\varepsilon = 5 \times 10^{-5} V$.

17. A line charge λ per unit length is lodged uniformly onto the rim of a wheel of mass M and radius R. The wheel has light non-conducting spokes and is free to rotate without friction about its axis (figure). A uniform magnetic field extends over a circular region within the rim. It is given by,

$A = -B_0 k (r \le a; a < R)$

$A = 0$ (otherwise)

What is the angular velocity of the wheel after the field is suddenly switched off?



Ans: We know that, the Line charge per unit length

$$= \lambda = \frac{\text{Total charge}}{\text{Length}} = \frac{Q}{2\pi r}$$

Where,

r = Distance of the point within the wheel

Mass of the wheel = M

Radius of the wheel = R

Magnetic field, $\bar{B} = -B_0 k$

At distance r, the magnetic force would be balanced by the centripetal force i.e.,

$$BQv = \frac{Mv^2}{r}$$

Where,

v = linear velocity of the wheel

$$\Rightarrow B2\pi r\lambda = \frac{Mv}{r}$$

$$\Rightarrow v = \frac{B2\pi\lambda r^2}{M}$$

$$\therefore \text{Angular velocity}, \omega = \frac{v}{R} = \frac{B2\pi\lambda r^2}{M}$$

For $r \leq$ and $a < R$, we would get:

$$\omega = \frac{2\pi B_0 a^2 \lambda}{MR} k$$

Therefore, we found the angular velocity of the wheel after the field is suddenly switched off to be given as,

$$\omega = \frac{2\pi B_0 a^2 \lambda}{MR} k$$