NCERT Solutions for Class 12

Physics

Chapter 7 – Alternating Current

1. A 100Ω resistor is connected to a $220V$, $50Hz$ ac supply.

a) What is the rms value of current in the circuit?

Ans: It is given that,

Resistance, $R = 100\Omega$

Voltage, $V = 220V$

Frequency, $f = 50Hz$

It is known that,

$$I_{rms} = \frac{V_{rms}}{R}$$

$$\Rightarrow I_{rms} = \frac{220}{100} = 2.2A$$

Therefore, the rms value of current in the circuit is $I_{rms} = 2.2A$.

b) What is the net power consumed over a full cycle?

Ans: It is known that,

$$Power = V \times I$$

$$\Rightarrow Power = 220 \times 2.2$$

$$\Rightarrow Power = 484W$$

Therefore, the net power consumed over a full cycle is $484W$.

2.

a) The peak voltage of an ac supply is $300V$. What is the rms voltage?

Ans: It is given that,

Peak voltage of the ac supply, $V_0 = 300V$

It is known that,

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

$$\Rightarrow V_{rms} = \frac{300}{\sqrt{2}}$$

$\Rightarrow V_{rms} = 212.1V$

Therefore, the rms voltage is $212.1V$.

b) The rms value of current in an ac circuit is $10A$. What is the peak current?

Ans: It is given that,

Rms value of current in an ac circuit, $I_{rms} = 10A$

It is known that,

$I_0 = \sqrt{2} \times I_{rms}$

$\Rightarrow I_0 = 1.414 \times 10$

$\Rightarrow I_0 = 14.14A$

Therefore, the peak current is $14.14A$.

3. A $44mH$ inductor is connected to $220V$, $50Hz$ ac supply. Determine the rms value of the current in the circuit.

Ans: It is known that,

Inductance, $L = 44mH = 44 \times 10^{-3}H$

Voltage, $V = 220V$

Frequency, $f_L = 50Hz$

Angular frequency, $\omega_L = 2\pi f_L$

It is known that,

Inductive reactance, $X_L = \omega_L L = 2\pi f_L L$

$\Rightarrow X_L = 2 \times 3.14 \times 50 \times 44 \times 10^{-3}\Omega$

$\Rightarrow X_L = 13.8\Omega$

$I_{rms} = \frac{V}{X_L}$

$\Rightarrow I_{rms} = \frac{220}{13.82}$

$\Rightarrow I_{rms} = 15.92A$

Therefore, the rms value of the current in the circuit is $15.92A$.

4. A $60\mu F$ capacitor is connected to a $110V$, $60Hz$ ac supply. Determine the rms value of the current in the circuit.

Ans: It is given that,

Capacitance, $C = 60\mu F = 60 \times 10^{-6} F$

Voltage, $V = 110V$

Frequency, $f_c = 60Hz$

It is known that,

$$I_{rms} = \frac{V}{X_C}$$

$$X_C = \frac{1}{\omega_C C} = \frac{1}{2\pi f_C C}$$

$$\Rightarrow X_C = \frac{1}{2 \times 3.14 \times 60 \times 60 \times 10^{-6}}$$

$$\Rightarrow X_C = 44.248\Omega$$

$$\Rightarrow I_{rms} = \frac{110}{44.28}$$

$$\Rightarrow I_{rms} = 2.488A$$

Therefore, the rms value of the current in the circuit is $2.488A$.

5. In exercises 4 and 5 What is the net power absorbed by each circuit over a complete cycle? Explain your answer.

Ans: From the inductive circuit,

Rms value of current, $I_{rms} = 15.92A$

Rms value of voltage, $V_{rms} = 220V$

It is known that,

Net power absorbed, $P = V_{rms} \times I_{rms} \cos\phi$

Where,

ϕ is the phase difference between voltage and current

For a pure inductive circuit, the phase difference between alternating voltage and current is 90^0 i.e., $\phi = 90^0$

$$\Rightarrow P = 220 \times 15.92 \cos 90^0 = 0$$

Therefore, net power absorbed is zero in a pure inductive circuit.

In a capacitive circuit,

Rms value of current, $I_{rms} = 2.49A$

Rms value of voltage, $V_{rms} = 110V$

It is known that,

Net power absorbed, $P = V_{rms} \times I_{rms} \cos\phi$

Where,

ϕ is the phase difference between voltage and current

For a pure capacitive circuit, the phase difference between alternating voltage and current is 90^0 i.e., $\phi = 90^0$

$\Rightarrow P = 110 \times 2.49 \cos 90^0 = 0$

Therefore, net power absorbed is zero in a pure capacitive circuit.

6. Obtain the resonant frequency ω_r of a series LCR circuit with $L = 2.0H$, $C = 32\mu F$ and $R = 10\Omega$. What is the Q-value of this current?

Ans: It is given that,

Inductance, $L = 2H$

Capacitance, $C = 32\mu F = 32 \times 10^{-6} F$

$R = 10\Omega$

It is known that,

Resonant frequency, $\omega_r = \frac{1}{\sqrt{LC}}$

$\Rightarrow \omega_r = \frac{1}{\sqrt{2 \times 32 \times 10^{-6}}}$

$\Rightarrow \omega_r = \frac{1}{8 \times 10^{-3}}$

$\Rightarrow \omega_r = 125 \text{ rad/s}$

$Q - \text{value} = \frac{\omega_r L}{R}$

$\Rightarrow Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$\Rightarrow Q = \frac{1}{10}\sqrt{\frac{2}{32 \times 10^{-6}}}$

$\Rightarrow Q = \frac{1}{10 \times 4 \times 10^{-3}}$

$\Rightarrow Q = 25$

Therefore, the resonant frequency is 125rad / s and Q-value is 25.

7. A charged $30\mu F$ capacitor is connected to a $27mH$ inductor. What is the angular frequency of free oscillations of the circuit?

Ans: It is given that,

Capacitance, $C = 30\mu F = 30\times 10^{-6} F$

Inductance, $L = 27mH = 27\times 10^{-3} H$

It is known that,

Angular frequency of free oscillations, $\omega_r = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \omega_r = \frac{1}{\sqrt{27\times 10^{-3}\times 30\times 10^{-6}}}$$

$$\Rightarrow \omega_r = \frac{1}{9\times 10^{-4}}$$

$$\Rightarrow \omega_r = 1.11\times 10^{3} rad/s$$

Therefore, the angular frequency of free oscillations of the circuit is $1.11\times 10^{3} rad/s$.

8. Suppose the initial charge on the capacitor in exercise 7 is $6mC$. What is the total energy stored in the circuit initially? What is the total energy at a later time?

Ans: It is known that,

Capacitance of the capacitor, $C = 30\mu F = 30\times 10^{-6} F$

Inductance of the capacitor, $L = 27mH = 27\times 10^{-3} H$

Charge on the capacitor, $Q = 6mC = 6\times 10^{-3} C$

It is known that,

Energy, $E = \frac{1}{2}\frac{Q^2}{C}$

$$\Rightarrow E = \frac{1}{2}\frac{(6\times 10^{-3})^2}{30\times 10^{-6}}$$

$$\Rightarrow E = \frac{6}{10} = 0.6J$$

Therefore, the energy stored in the circuit initially is $E = 0.6J$.

Total energy at later time will remain same as the initially stored i.e., $0.6J$ because energy is shared between the capacitor and the inductor.

9. A series LCR circuit with $R = 20\Omega$, $L = 1.5H$ and $C = 35\mu F$ is connected to a variable frequency $200V$ ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Ans: It is known that,

Resistance, $R = 20\Omega$

Inductance, $L = 1.5H$

Capacitance, $C = 35\mu F = 35\times10^{-6}F$

Voltage, $V = 200V$

It is known that,

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

At resonance, $X_L = X_C$

$\Rightarrow Z = R = 20\Omega$

$I = \frac{V}{Z} = \frac{200}{20}$

$\Rightarrow I = 10A$

Average power, $P = I^2R$

$\Rightarrow P = 10^2\times20$

$\Rightarrow P = 2000W$

Therefore, the average power transferred is $2000W$.

10. A radio can tune over the frequency range of a portion of MW broadcast band: ($800kHz$ to $1200kHz$). If its LC circuit has an effective inductance of $200\mu H$, what must be the range of its variable capacitor?

[Hint: For tuning, the natural frequency i.e., the frequency of free oscillations of the LC circuit should be equal to the frequency of the radio wave.]

Ans: It is given that,

The range of frequency (f) of a radio is $800kHz$ to $1200kHz$.

Effective inductance of the circuit, $L = 200\mu H = 200\times10^{-6}H$

It is known that,

Capacitance of variable capacitor for f_1 is $C_1 = \frac{1}{\omega_1^2 L}$

Where,

ω_1 is the angular frequency for capacitor for $f_1 = 2\pi f_1$

$\Rightarrow \omega_1 = 2 \times 3.14 \times 800 \times 10^3 \text{ rad / s}$

$\Rightarrow C_1 = \dfrac{1}{(2 \times 3.14 \times 800 \times 10^3)^2 \times 200 \times 10^{-6}}$

$\Rightarrow C_1 = 1.9809 \times 10^{-10} \text{F}$

$\Rightarrow C_1 = 198.1 \text{pF}$

$C_2 = \dfrac{1}{{\omega_2}^2 L}$

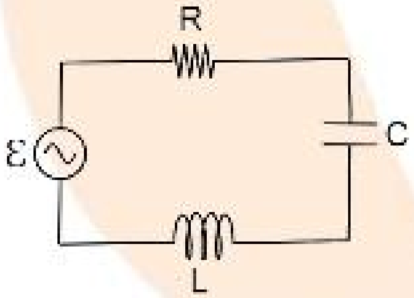
$\Rightarrow C_2 = \dfrac{1}{(2 \times 3.14 \times 1200 \times 10^3)^2 \times 200 \times 10^{-6}}$

$\Rightarrow C_2 = 0.8804 \times 10^{-10} \text{F}$

$\Rightarrow C_2 = 88.04 \text{pF}$

Therefore, the range of the variable capacitor is from 88.04pF to 198.1pF.

11. Figure shows a series LCR circuit connected to a variable frequency 230V source. $L = 5.0\text{H}$, $C = 80\mu\text{F}$, $R = 40\Omega$.



a) Determine the source frequency which drives the circuit in resonance.

Ans: It is given that,

Voltage, $V = 230\text{V}$

Inductance, $L = 5.0\text{H}$

Capacitance, $C = 80\mu\text{F} = 80 \times 10^{-6}\text{F}$

Resistance, $R = 40\Omega$

It is known that,

Source frequency at resonance $= \frac{1}{\sqrt{LC}}$

$\Rightarrow \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50 \text{rad}/\text{s}$

Therefore, the source frequency of the circuit in resonance is $50 \text{rad}/\text{s}$.

b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

Ans: It is known that,

At resonance, Impedance, $Z =$ Resistance, R

$\Rightarrow Z = R = 40\Omega$

$I = \frac{V}{Z}$

$\Rightarrow I = \frac{230}{40} = 5.75\text{A}$

Amplitude, $I_0 = 1.414 \times I$

$\Rightarrow I_0 = 1.414 \times 5.75$

$\Rightarrow I_0 = 8.13\text{A}$

Therefore, the impedance of the circuit is 40Ω and the amplitude of current at resonating frequency is 8.13A.

c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

Ans: It is known that,

Potential drop, $V = IR$

Across resistor, $V_R = IR$

$\Rightarrow V_R = 5.75 \times 40 = 230\text{V}$

Across capacitor, $V_C = IX_C = \frac{I}{\omega C}$

$\Rightarrow V_C = 5.75 \times \frac{1}{50 \times 80 \times 10^{-6}}$

$\Rightarrow V_C = 1437.5\text{V}$

Across Inductor, $V_L = IX_L = I\omega L$

$\Rightarrow V_L = 5.75 \times 50 \times 5$

$\Rightarrow V_L = 1437.5V$

Across LC combination, $V_{LC} = I(X_L - X_C)$

At resonance, $X_L = X_C$

$\Rightarrow V_{LC} = 0$

Therefore, the rms potential drop across Resistor is $230V$, Capacitor is $1437.5V$, Inductor is $1437.5V$ and the potential drop across LC combination is zero at resonating frequency.

12. An LC circuit contains a $20mH$ inductor and a $50\mu F$ capacitor with an initial charge of $10mC$. The resistance of the circuit is negligible. Let the instant the circuit is closed be $t = 0$.

a) What is the total energy stored initially? Is it conserved during LC oscillations?

Ans: It is given that,

Inductance of the inductor, $L = 20mH = 20 \times 10^{-3} H$

Capacitance of the capacitor, $C = 50\mu F = 50 \times 10^{-6} F$

Initial charge on the capacitor, $Q = 10mC = 10 \times 10^{-3} C$

It is known that,

Total energy stored initially in the circuit, $E = \frac{1}{2}\frac{Q^2}{C}$

$$\Rightarrow E = \frac{(10 \times 10^{-3})^2}{2 \times 50 \times 10^{-6}} = 1J$$

Therefore, the total energy stored in the LC circuit will be conserved because there is no resistor $(R = 0)$ connected in the circuit.

b) What is the natural frequency of the circuit?

Ans: It is known that,

Natural frequency of the circuit, $\nu = \frac{1}{2\pi\sqrt{LC}}$

$$\Rightarrow \nu = \frac{1}{2\pi\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}}$$

$$\Rightarrow \nu = \frac{10^3}{2\pi} = 159.24\text{Hz}$$

Natural angular frequency, $\omega_r = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \omega_r = \frac{1}{\sqrt{20\times10^{-3}\times50\times10^{-6}}}$$

$$\Rightarrow \omega_r = \frac{1}{\sqrt{10^{-6}}} = 10^3\,\text{rad / s}$$

Therefore, the natural frequency is 159.24Hz and the natural angular frequency is $10^3\,\text{rad / s}$.

c) At what time is the energy stored (i) completely electrical (i.e., stored in the capacitor)? (ii) completely magnetic (i.e., stored in the inductor)?

Ans:

(i) Completely electrical

It is known that,

Time period for LC oscillations, $T = \frac{1}{\nu}$

$$\Rightarrow T = \frac{1}{159.24} = 6.28\text{ms}$$

Total charge on the capacitor at time t, $Q' = Q\cos\left(\frac{2\pi}{T}t\right)$

If energy stored is electrical, $Q' = \pm Q$

Therefore, it can be inferred that the energy stored in the capacitor is completely electrical at time, $t = 0, \frac{T}{2}, T, \frac{3T}{2}, \ldots\ldots\ldots\ldots$ where, $T = 6.3\text{ms}$.

(ii) Completely magnetic

Magnetic energy is maximum, when electrical energy Q' is equal to 0.

Therefore, it can be inferred that the energy stored is completely magnetic at time, $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \ldots\ldots\ldots$ where, $T = 6.3\text{ms}$.

d) At what times is the total energy shared equally between the inductor and the capacitor?

Ans: Consider, Q' be the charge on capacitor when total energy is equally shared between the capacitor and the inductor at time t.

When total energy is equally shared between the inductor and capacitor, the energy stored in the capacitor $= \frac{1}{2}$(maximum energy).

$$\Rightarrow \frac{1}{2}\frac{(Q')^2}{C} = \frac{1}{2}\left(\frac{1}{2}\frac{Q^2}{C}\right)$$

$$\Rightarrow \frac{1}{2}\frac{(Q')^2}{C} = \frac{1}{4}\frac{Q^2}{C}$$

$$\Rightarrow Q' = \frac{Q}{\sqrt{2}}$$

It is known that, $Q' = Q\cos\frac{2\pi}{T}t$

$$\Rightarrow \frac{Q}{\sqrt{2}} = Q\cos\frac{2\pi}{T}t$$

$$\Rightarrow \cos\frac{2\pi}{T}t = \frac{1}{\sqrt{2}} = \cos(2n+1)\frac{\pi}{4};\ n = 0,1,2,3.....$$

$$\Rightarrow t = (2n+1)\frac{T}{8}$$

Therefore, total energy is equally shared between the inductor and the capacitor at time, $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}$...........

e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

Ans: If a resistor is included in the circuit, then the total initial energy gets dissipated as heat energy in the circuit. The LC oscillation gets damped due to the resistance.

13. A coil of inductance 0.5H and resistance 100Ω is connected to a $240\text{V}, 50\text{Hz}$ ac supply.

a) What is the maximum current in the coil?

Ans: It is given that,

Inductance of the inductor, $L = 0.5H$

Resistance of the resistor, $R = 100\Omega$

Potential of the supply voltage, $V = 240V$

Frequency of the supply, $\nu = 50Hz$

It is known that,

Peak voltage, $V_0 = \sqrt{2}V$

$\Rightarrow V_0 = \sqrt{2} \times 240$

$\Rightarrow V_0 = 339.41V$

Angular frequency of the supply, $\omega = 2\pi\nu$

$\Rightarrow \omega = 2\pi \times 50 = 100\pi rad/s$

Maximum current in the circuit, $I_0 = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}}$

$\Rightarrow I_0 = \frac{339.41}{\sqrt{(100)^2 + (100\pi)^2 (0.50)^2}} = 1.82A$

Therefore, the maximum current in the coil is $1.82A$.

b) What is the time lag between the voltage maximum and the current maximum?

Ans: It is known that,

Equation for voltage, $V = V_0 \cos\omega t$

Equation for current, $I = I_0 \cos(\omega t - \phi)$

Where,

ϕ is the phase difference between voltage and current.

At time $t = 0$, $V = V_0$ (voltage is maximum)

If $\omega t - \phi = 0$ i.e., at $t = \frac{\phi}{\omega}$, $I = I_0$ (current is maximum)

Therefore, the time lag between maximum voltage and maximum current is $\frac{\phi}{\omega}$.

$\Rightarrow \tan\phi = \frac{\omega L}{R}$

$\Rightarrow \tan\phi = \frac{2\pi \times 50 \times 0.5}{100} = 1.57$

$\Rightarrow \phi = \tan^{-1}(1.57)$

$\Rightarrow \phi = 57.5 = \frac{57.5\pi}{180}\text{rad}$

Time lag, $t = \frac{\phi}{\omega}$

$\Rightarrow t = \frac{57.5\pi}{180 \times 2\pi \times 50}$

$\Rightarrow t = 3.19 \times 10^{-3}\text{s}$

$\Rightarrow t = 3.2\text{ms}$

Therefore, the time lag between the maximum voltage and maximum current is 3.2ms.

14. Obtain the answers (a) to (b) in Exercise 13 if the circuit is connected to a high frequency supply $(240\text{V}, 10\text{kHz})$. Hence, explain the statement that at very high frequency, an inductor in a circuit nearly amounts to an open circuit. How does an inductor behave in a dc circuit after the steady state?

Ans: It is given that,

Inductance of the inductor, $L = 0.5\text{H}$

Resistance of the resistor, $R = 100\Omega$

Potential of the supply voltage, $V = 240\text{V}$

Frequency of the supply, $\nu = 10\text{kHz} = 10^4\text{Hz}$

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 10^4 \text{rad/s}$

Peak Voltage, $V_0 = V\sqrt{2} = 110\sqrt{2}\text{V}$

a) Maximum current, $I_0 = \frac{V_0}{\sqrt{R^2 + \omega^2 C^2}}$

$\Rightarrow I_0 = \frac{240\sqrt{2}}{\sqrt{(100)^2 + (2\pi \times 10^4)^2 \times (0.5)^2}} = 1.1 \times 10^{-2}\text{A}$

Therefore, the maximum current in the coil is $1.1 \times 10^{-2}\text{A}$.

b) The time lag between maximum voltage and maximum current is $\frac{\phi}{\omega}$.

For phase difference ϕ : $\tan\phi = \frac{\omega L}{R}$

$\Rightarrow \tan\phi = \frac{2\pi \times 10^4 \times 0.5}{100} = 100\pi$

$\Rightarrow \phi = \tan^{-1}(100\pi)$

$\Rightarrow \phi = 89.82^\circ = \frac{89.82\pi}{180}$ rad

Time lag, $t = \frac{\phi}{\omega}$

$\Rightarrow t = \frac{89.82\pi}{180 \times 2\pi \times 10^4}$

$\Rightarrow t = 25 \times 10^{-6}$s

$\Rightarrow t = 25\mu s$

Therefore, the time lag between the maximum voltage and maximum current is $25\mu s$.

It can be observed that I_0 is very small in this case.

Thus, at high frequencies, the inductor amounts to an open circuit.

In a dc circuit, after a steady state is achieved, $\omega = 0$. Thus, inductor L behaves like a pure conducting object.

15. A $100\mu F$ capacitor in series with a 40Ω resistance is connected to a $110V, 60Hz$ supply.

a) What is the maximum current in the circuit?

Ans: It is given that,

Capacitance of the capacitor, $C = 100\mu F = 100 \times 10^{-6} F$

Resistance of the resistor, $R = 40\Omega$

Supply voltage, $V = 110V$

Frequency oscillations, $\nu = 60Hz$

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 60 rad/s$

It is known that,

For a RC circuit, Impedance: $Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

Peak Voltage, $V_0 = V\sqrt{2} = 110\sqrt{2}V$

Maximum current; $I_0 = \frac{V_0}{Z}$

$$\Rightarrow I_0 = \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$\Rightarrow I_0 = \frac{110\sqrt{2}}{\sqrt{(40)^2 + \frac{1}{(120\pi)^2 (10^{-4})^2}}}$$

$$\Rightarrow I_0 = \frac{110\sqrt{2}}{\sqrt{1600 + \frac{1}{(120\pi)^2 (10^{-4})^2}}} = 3.24A$$

Therefore, the maximum current in the circuit is $3.24A$.

b) What is the time lag between the current maximum and the voltage maximum?

Ans: It is known that,

In a capacitor circuit, the voltage lags behind the current by a phase angle of ϕ.

$$\tan\phi = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega CR}$$

$$\Rightarrow \tan\phi = \frac{1}{120\pi \times 10^{-4} \times 40} = 0.6635$$

$$\Rightarrow \phi = \tan^{-1}(0.6635)$$

$$\Rightarrow \phi = 33.56^{\circ} = \frac{33.56\pi}{180}\text{rad}$$

It is known that,

Time lag, $t = \frac{\phi}{\omega}$

$$\Rightarrow t = \frac{33.56\pi}{180 \times 120\pi}$$

$$\Rightarrow t = 1.55 \times 10^{-3}\text{s}$$

$$\Rightarrow t = 1.55\text{ms}$$

Therefore, the time lag between maximum current and maximum voltage is 1.55ms.

16. Obtain the answers to (a) and (b) in Exercise 15 if the circuit is connected to a $110\text{V}, 12\text{kHz}$ supply? Hence, explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in a dc circuit after the steady state.

Ans: It is given that,

Capacitance of the capacitor, $C = 100\mu F = 100 \times 10^{-6} F$

Resistance of the resistor, $R = 40\Omega$

Supply voltage, $V = 110V$

Frequency oscillations, $\nu = 12kHz = 12 \times 10^3 Hz$

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 12 \times 10^3 rad/s = 24\pi \times 10^3 rad/s$

Peak Voltage, $V_0 = V\sqrt{2} = 110\sqrt{2}V$

a) It is known that,

For a RC circuit, Impedance: $Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

Maximum current; $I_0 = \frac{V_0}{Z}$

$$\Rightarrow I_0 = \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$\Rightarrow I_0 = \frac{110\sqrt{2}}{\sqrt{(40)^2 + \frac{1}{(24\pi \times 10^3)^2 (10^{-4})^2}}}$$

$$\Rightarrow I_0 = \frac{110\sqrt{2}}{\sqrt{1600 + \left(\frac{10}{24\pi}\right)^2}} = 3.9A$$

Therefore, the maximum current in the circuit is $3.9A$.

b) It is known that,

In a capacitor circuit, the voltage lags behind the current by a phase angle of ϕ.

$$\tan\phi = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega CR}$$

$$\Rightarrow \tan\phi = \frac{1}{24\pi \times 10^{3} \times 10^{-4} \times 40} = \frac{1}{96\pi}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{1}{96\pi}\right)$$

$$\Rightarrow \phi = 0.2^{\circ} = \frac{0.2\pi}{180}\text{rad}$$

It is known that,

Time lag, $t = \frac{\phi}{\omega}$

$$\Rightarrow t = \frac{0.2\pi}{180 \times 24\pi \times 10^{3}}$$

$$\Rightarrow t = 0.04 \times 10^{-6}\text{s}$$

$$\Rightarrow t = 0.04\mu\text{s}$$

Therefore, the time lag between maximum current and maximum voltage is $0.04\mu s$.

It can be concluded that ϕ tends to become zero at high frequencies. At a high frequency, capacitor C acts as a conductor.

In a dc circuit, after the steady state is achieved, $\omega = 0$. Therefore, capacitor C amounts to an open circuit.

17. Keeping the source frequency equal to the resonating frequency of the series LCR circuit, if three elements, L,C and R are arranged in parallel, show that the total current in the parallel LCR circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements and source specified in Exercise 11 for this frequency.

Ans: It is given that,

An inductor (L), a capacitor (C) and a resistor (R) is connected in parallel with each other in a circuit where,

Inductance, $L = 5.0\text{H}$

Capacitance, $C = 80\mu\text{F} = 80 \times 10^{-6}\text{F}$

Resistance, $R = 40\Omega$

Potential of the voltage source, $V = 230V$

It is known that,

Impedance (Z) of the given LCR circuit is given as:

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

Where,

ω is the angular frequency

At resonance: $\frac{1}{\omega L} - \omega C = 0$

$$\Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \omega = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50 rad/s$$

Therefore, the magnitude of Z is maximum at $50 rad/s$ and the total current is minimum.

Rms current flowing through inductor L: $I_L = \frac{V}{\omega L}$

$$\Rightarrow I_L = \frac{230}{50 \times 5} = 0.92A$$

Rms current flowing through capacitor C: $I_C = \frac{V}{\frac{1}{\omega C}} = \omega CV$

$$\Rightarrow I_C = 50 \times 80 \times 10^{-6} \times 230 = 0.92A$$

Rms current flowing through resistor R: $I_R = \frac{V}{R}$

$$\Rightarrow I_R = \frac{230}{40} = 5.75A$$

Current rms value in inductor is $0.92A$, in capacitor is $0.92A$ and in resistor is $5.75A$.

18. A circuit containing an $80mH$ inductor and a $60\mu F$ capacitor in series is connected to a $230V, 50Hz$ supply. The resistance of the circuit is negligible.

a) Obtain the current amplitude and rms values.

Ans: It is given that,

Inductance, $L = 80mH = 80 \times 10^{-3} H$

Capacitance, $C = 60\mu F = 60 \times 10^{-6} F$

Supply voltage, $V = 230V$

Frequency, $\nu = 50Hz$

Angular frequency, $\omega = 2\pi\nu = 100\pi rad/s$

Peak voltage, $V_0 = V\sqrt{2} = 230\sqrt{2}V$

It is known that,

Maximum current: $I_0 = \dfrac{V_0}{(\omega L - \dfrac{1}{\omega C})}$

$$\Rightarrow I_0 = \frac{230\sqrt{3}}{(100\pi \times 80 \times 10^{-3} - \dfrac{1}{100\pi \times 60 \times 10^{-6}})}$$

$$\Rightarrow I_0 = \frac{230\sqrt{3}}{(8\pi - \dfrac{1000}{6\pi})} = -11.63A$$

The negative sign is because $\omega L < \dfrac{1}{\omega C}$

Amplitude of maximum current, $|I_0| = 11.63A$

$$\Rightarrow I = \frac{I_0}{\sqrt{2}} = \frac{-11.63}{\sqrt{2}}$$

$\Rightarrow I = -8.22A$, which is the rms value of current.

b) Obtain the rms values of potential drops across each element.

Ans: It is known that,

Potential difference across the inductor, $V_L = I \times \omega L$

$\Rightarrow V_L = 8.22 \times 100\pi \times 80 \times 10^{-3}$

$\Rightarrow V_L = 206.61V$

Potential difference across the capacitor, $V_C = I \times \dfrac{1}{\omega C}$

$\Rightarrow V_C = 8.22 \times \frac{1}{100\pi \times 60 \times 10^{-6}}$

$\Rightarrow V_C = 436.3V$, which is the rms value of potential drop.

c) What is the average power transferred to the inductor?

Ans: Average power transferred to the inductor is zero as actual voltage leads the current by $\frac{\pi}{2}$.

d) What is the average power transferred to the capacitor?

Ans: Average power transferred to the capacitor is zero as actual voltage lags the current by $\frac{\pi}{2}$.

e) **What is the total average power absorbed by the circuit? ['Average' implies 'averaged over one cycle'.]**

Ans: The total average power absorbed (averaged over one cycle) is zero.

19. Suppose the circuit in Exercise 18 has a resistance of 15Ω . Obtain the average power transferred to each element of the circuit, and the total power absorbed.

Ans: It is given that,

Average power transferred to the resistor $= 788.44W$

Average power transferred to the capacitor $= 0W$

Total power absorbed by the circuit $= 788.44W$

Inductance of inductor, $L = 80mH = 80 \times 10^{-3}H$

Capacitance of capacitor, $C = 60\mu F = 60 \times 10^{-6}F$

Resistance of resistor, $R = 15\Omega$

Potential of voltage supply, $V = 230V$

Frequency of signal, $\nu = 50Hz$

Angular frequency of signal, $\omega = 2\pi\nu = 2\pi \times (50) = 100\pi rad/s$

It is known that,

Impedance, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$$\Rightarrow Z=\sqrt{(15)^2+\left(100\pi(80\times10^{-3})-\frac{1}{\left(100\pi\times60\times10^{-6}\right)}\right)^2}$$

$$\Rightarrow Z=\sqrt{(15)^2+(25.12-53.08)^2}=31.728\Omega$$

Now,

$$I=\frac{V}{Z}$$

$$\Rightarrow I=\frac{230}{31.728}=7.25A$$

The elements are connected in series to each other. Therefore, impedance of the circuit is given as current flowing in the circuit,

Average power transferred to resistance is given as: $P_R=I^2R$

$$\Rightarrow P_R=(7.25)^2\times15=788.44W$$

Average power transferred to capacitor, $P_C=$ Average power transferred to inductor, $P_L=0$

Total power absorbed by the circuit: $P_T=P_R+P_C+P_L$

$P_T=788.44+0+0=788.44W$

Therefore, the total power absorbed by the circuit is $788.44W$.

20. A series LCR circuit with $L=0.12H, C=480nF, R=23\,\Omega$ is connected to a $230V$ variable frequency supply.

a) What is the source frequency for which current amplitude is maximum? Obtain this maximum value.

Ans: It is given that,

Inductance, $L=0.12H$

Capacitance, $C=480nF=480\times10^{-9}F$

Resistance, $R=23\Omega$

Supply voltage, $V=230V$

Peak voltage, $V_0=230\times\sqrt{2}=325.22V$

It is known that,

Current flowing in the circuit, $I_0 = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

Where,
I_0 is maximum at resonance.

At resonance: $\omega_R L - \frac{1}{\omega_R C} = 0$

Where,
ω_R is the resonance angular frequency

$\omega_R = \frac{1}{\sqrt{LC}}$

$\Rightarrow \omega_R = \frac{1}{\sqrt{0.12 \times 480 \times 10^{-9}}}$

$\Rightarrow \omega_R = 4166.67 rad/s$

Resonant frequency, $v_R = \frac{\omega_R}{2\pi}$

$\Rightarrow v_R = \frac{4166.67}{2 \times 3.14} = 663.48Hz$

Maximum current, $(I_0)_{Max} = \frac{V_0}{R}$

$\Rightarrow (I_0)_{Max} = \frac{325.22}{23} = 14.14A$

b) What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of this maximum power.

Ans: It is known that,

Maximum average power absorbed by the circuit; $(P_V)_{Max} = \frac{1}{2}(I_0)^2_{Max} R$

$\Rightarrow (P_V)_{Max} = \frac{1}{2} \times (14.14)^2 \times 23$

$\Rightarrow (P_V)_{Max} = 2299.3W$

Therefore, the resonant frequency, $v_R = 663.48Hz$

c) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?

Ans: It is known that,

The power transferred to the circuit is half the power at resonant frequency.

Frequencies at which power transferred is half, $= \omega_R \pm \Delta\omega = 2\pi(\nu_R \pm \Delta\nu)$

Where,

$$\Delta\omega = \frac{R}{2L}$$

$$\Rightarrow \Delta\omega = \frac{23}{2 \times 0.12} = 95.83 \text{rad / s}$$

Therefore, the change in frequency, $\Delta\nu = \frac{1}{2\pi}\Delta\omega$

$$\Delta\nu = \frac{95.83}{2\pi} = 15.26\text{Hz}$$

$$\nu_R + \Delta\nu = 663.48 + 15.26 = 678.74\text{Hz}$$

$$\nu_R - \Delta\nu = 663.48 - 15.26 = 648.22\text{Hz}$$

Therefore, at 648.22Hz and 678.74Hz frequencies, the power transferred is half.

At these frequencies, current amplitude: $I' = \frac{1}{\sqrt{2}} \times (I_0)_{Max}$

$$\Rightarrow I' = \frac{14.14}{\sqrt{2}} = 10\text{A}$$

Therefore, the current amplitude is 10A.

d) What is the Q-factor of the given circuit?

Ans: It is known that,

Q-factor of the given circuit, $Q = \frac{\omega_r L}{R}$

$$\Rightarrow Q = \frac{4166.67 \times 0.12}{23} = 21.74$$

Therefore, the Q-factor of the given circuit is 21.74.

21. Obtain the resonant frequency and Q-factor of a series LCR circuit with $L = 3.0\text{H}$, $C = 27\mu\text{F}$ and $R = 7.4\Omega$. It is desired to improve the sharpness

of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.

Ans: It is given that,

Inductance, $L = 3.0H$

Capacitance, $C = 27\mu F = 27 \times 10^{-6} F$

Resistance, $R = 7.4\Omega$

It is known that,

At resonance, angular frequency of the source for the given LCR series circuit is

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \omega_r = \frac{1}{\sqrt{3 \times 27 \times 10^{-6}}}$$

$$\Rightarrow \omega_r = \frac{10^3}{9} = 111.11 rad/s$$

Therefore, the resonant frequency is $111.11 rad/s$.

Q-factor of the series, $Q = \frac{\omega_r L}{R}$

$$\Rightarrow Q = \frac{111.11 \times 3}{7.4} = 45.0446$$

Therefore, the Q-factor is 45.0446.

To improve the sharpness of the resonance by reducing 'full width at half maximum' by a factor of 2 without changing ω_r, reduce the resistance to half.

$$\Rightarrow R = \frac{7.4}{2} = 3.7\Omega$$

Therefore, required resistance is 3.7Ω.

22. Answer the following questions:

a) In any ac circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for rms voltage?

Ans: Yes, in any ac circuit, the applied instantaneous voltage is equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit.

The same is not true for rms voltage because voltages across different elements may not be in phase.

b) A capacitor is used in the primary circuit of an induction coil.

Ans: Yes, a capacitor is used in the primary circuit of an induction coil.

This is because, when the circuit is broken, a high induced voltage is used to charge the capacitor to avoid sparks.

c) An applied voltage signal consists of a superposition of a dc voltage and an ac voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the dc signal will appear across C and the ac signal across L.

Ans: The dc signal will appear across capacitor C because for dc signals, the impedance of an inductor L is negligible while the impedance of a capacitor C is very high (almost infinite).

Therefore, a dc signal appears across C.

For an ac signal of high frequency, the impedance of L is high and that of C is very low.

Thus, an ac signal of high frequency appears across L.

d) A choke coil in series with a lamp is connected to a dc line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no **change in the lamp's brightness. Predict the corresponding observations** if the connection is to an ac line.

Ans: When an iron core is inserted in the choke coil (which is in series with a lamp connected to an ac line), the lamp will glow dimly.

This is because the choke coil and the iron core increase the impedance of the circuit.

e) Why is choke coil needed in the use of fluorescent tubes with ac mains? Why can we not use an ordinary resistor instead of the choke coil?

Ans: As the choke coil reduces the voltage across the tube without wasting much power, it is used in the fluorescent tubes with ac mains. An ordinary resistor cannot be used instead of choke coil because it wastes power in the form of heat.

23. A power transmission line feeds input power at 2300V to a step-down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230V ?

Ans: It is given that,

Input voltage, $V_1 = 2300\text{V}$

Number of turns in primary coil, $n_1 = 4000$

Output voltage, $V_2 = 230\text{V}$

Number of turns in secondary coil , $n_2 = ?$

It is known that,

Voltage is related to number of terms: $\frac{V_1}{V_2} = \frac{n_1}{n_2}$

$$\Rightarrow \frac{2300}{230} = \frac{4000}{n_2}$$

$$\Rightarrow n_2 = \frac{4000 \times 230}{2300} = 400$$

Therefore, the number of turns in the second winding is 400.

24. At a hydroelectric power plant, the water pressure head is at a height of 300m and the water flow available is $100\text{m}^3/\text{s}$. If the turbine generator efficiency is 60% , estimate the electric power available from the plant ($g = 9.8\text{m}/\text{s}^2$).

Ans: It is known that,

Height of water pressure head, $h = 300\text{m}$

Volume of water flow per second, $V = 100\text{m}^3/\text{s}$

Efficiency of turbine generator, $n = 60\% = 0.6$

Acceleration due to gravity, $g = 9.8\text{m}/\text{s}^2$

Density of water, $\rho = 10^3 \text{kg}/\text{m}^3$

It is known that,

Electric power available from the plant $= \eta \times h\rho gV$

$$\Rightarrow P = 0.6 \times 300 \times 10^3 \times 9.8 \times 100$$

$$\Rightarrow P = 176.4 \times 10^6 \text{W}$$

$\Rightarrow P = 176.4MW$

Therefore, the estimated electric power available from the plant is $176.4MW$.

25. A small town with a demand of $800kW$ of electric power at $220V$ is situated $15km$ away from an electric plant generating power at $440V$. The resistance of the two-wire line carrying power is 0.5Ω perkm. The town gets power from the line through a $400-220V$ step-down transformer at a sub-station in the town.

a) Estimate the line power loss in the form of heat.

Ans: It is given that,

Total electric power required, $P = 800kW = 800 \times 10^3 W$

Supply voltage, $V = 220V$

Voltage at which electric plant is generating power, $V' = 440V$

Distance between the town and power generating station, $d = 15km$

Resistance of the two wire lines carrying power $= 0.5\Omega / km$

Total resistance of the wires, $R = (15 + 15)0.5 = 15\ \Omega$

A step-down transformer of rating $4000 - 220V$ is used in the sub-station.

Input voltage, $V_1 = 4000V$

Output voltage, $V_2 = 220V$

It is known that,

Rms current in the wire lines: $I = \dfrac{P}{V_1}$

$\Rightarrow I = \dfrac{800 \times 10^3}{4000} = 200A$

Line power loss $= I^2 R$

$\Rightarrow (200)^2 \times 15$

$\Rightarrow 600 \times 10^3 W = 600kW$

Therefore, the line power loss is $600kW$.

b) How much power must the plant supply, assuming there is negligible power loss due to leakage?

Ans: Assuming that there is negligible power loss due to leakage of the current:

Total power supplied by the plant $= 800kW + 600kW = 1400kW$

Therefore, the plant must supply $1400kW$ of power.

c) Characterise the step up transformer at the plant.

Ans: It is known that,

Voltage drop in the power line $=$ IR

$\Rightarrow V = 200 \times 15 = 3000V$

Total voltage transmitted from the plant $= 3000 + 4000 = 7000V$

The power generated is $440V$.

Therefore, the rating of the step-up transformer situated at the power plant is $440V - 7000V$.

26. Do the same exercise as above with the replacement of the earlier transformer by a $40,000 - 220V$ step-down transformer (Neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred?

Ans: It is given that,

Total electric power required, $P = 800kW = 800 \times 10^3 W$

Supply voltage, $V = 220V$

Voltage at which electric plant is generating power, $V' = 440V$

Distance between the town and power generating station, $d = 15km$

Resistance of the two wire lines carrying power $= 0.5\Omega / km$

Total resistance of the wires, $R = (15 + 15)0.5 = 15\ \Omega$

The rating of a step-down transformer is $40000V - 220V$.

Input voltage, $V_1 = 40000V$

Output voltage, $V_2 = 220V$

a) It is known that,

Rms current in the wire lines: $I = \frac{P}{V_1}$

$$\Rightarrow I = \frac{800 \times 10^3}{40000} = 20A$$

Line power loss $= I^2R$

$\Rightarrow (20)^2 \times 15$

$\Rightarrow 6 \times 10^3 W = 6kW$

Therefore, the line power loss is $6kW$.

b) Assume that there is negligible power loss due to leakage of the current:
Total power supplied by the plant $= 800\text{kW} + 6\text{kW} = 806\text{kW}$
Therefore, the plant must supply 806kW of power.

c) It is known that,
Voltage drop in the power line $= \text{IR}$
$\Rightarrow \text{V} = 20 \times 15 = 300\text{V}$
Total voltage transmitted from the plant $= 300 + 40000 = 40300\text{V}$
The power generated in the plant is generated at 440V.
Therefore, the rating of the step-up transformer situated at the power plant is $440\text{V} - 40300\text{V}$.

Power loss during transmission $= \frac{600}{1400} \times 100 = 42.8\%$
In previous exercise the power loss due to the same reason is
$= \frac{6}{806} \times 100 = 0.744\%$
As the power loss is less for a high voltage transmission, High voltage transmissions are preferred for this purpose.