NCERT Solutions for Class 11

Physics

Chapter 14 – Waves

1. A string of mass $2.50\ kg$ is under a tension of $200\ N$. The length of the stretched string is $20.0m$. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

Ans: It is provided that,
Mass of the string, $M = 2.50\ kg$
Tension in the string, $T = 200\ N$
String length, $l = 20.0\ m$
Mass per unit length, $\mu = \frac{M}{l} = \frac{2.50}{20} = 0.125 kgm^{-1}$
The transverse wave's velocity in the string is given by:
$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200}{0.125}} = \sqrt{1600} = 40 ms^{-1}$
Clearly, the time taken by disturbance to reach the other end is, $t = \frac{l}{v} = \frac{20}{40} = 0.50s$

2. A stone dropped from the top of a tower of height $300\ m$ high splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is $340\ ms^{-1}$? ($g = 9.8\ ms^{-2}$)
Ans: It is provided that,
Tower height, $s = 300\ m$
Stone's initial velocity, $u = 0$
Acceleration, $a = g = 9.8\ ms^{-2}$
Sound speed in air $= 340\ m/s$
The time that stone takes to strike the water in the pond can be estimated using the motion's second equation, as:
$s = ut_1 + \frac{1}{2}gt_1^2$
$\Rightarrow 300 = 0 + \frac{1}{2} \times 9.8 \times t_1^2$

We get,

$$t_1 = \sqrt{\frac{300 \times 2}{9.8}} = 7.82s$$

Time taken by the sound to reach the tower top, $t_2 = \frac{300}{340} = 0.88s$

Therefore, the time after which the sound of splash is heard, $t = t_1 + t_2$

$\Rightarrow t = 7.82 + 0.88 = 8.7s$

The time after which the sound of splash is heard is $8.7s$.

3. A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at $20^0C = 343ms^{-1}$.

Ans: It is provided that,

Steel wire's length, $l = 12$ m

Steel wire's mass, $m = 2.10$ kg

Velocity of the transverse wave, $v = 343\ ms^{-1}$

Mass per unit length, $\mu = \frac{m}{l} = \frac{2.10}{12} = 0.175kgm^{-1}$

For tension T, transverse wave's velocity can be calculated using the relation:

$$v = \sqrt{\frac{T}{\mu}}$$

$\Rightarrow T = v^2\mu$

$\Rightarrow T = (343)^2 \times 0.175 = 20588.575 \approx 2.06 \times 10^4 N$

Tension in the wire is $2.06 \times 10^4 N$.

4. Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air

a) is independent of pressure,

Ans: We have,

$$v = \sqrt{\frac{\gamma P}{\rho}} \qquad \text{......(i)}$$

Where,

Density, $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{V}$

M = Molecular weight of gas
V = Volume of gas
Hence, equation (i) becomes:

$$v=\sqrt{\frac{\gamma PV}{M}} \qquad ......(ii)$$

Now ideal gas equation for $n=1$ is:
$PV=RT$
For constant T, $PV=\text{Constant}$
Both M and γ are constants, $v=\text{Constant}$
Hence, the speed of sound is independent of the change in the pressure of the gas at a constant temperature.

b) increases with temperature,
Ans: We have,

$$v=\sqrt{\frac{\gamma P}{\rho}} \qquad ......(i)$$

Now ideal gas equation for $n=1$ is:
$PV=RT$

$$P=\frac{RT}{V} \qquad ......(ii)$$

Substituting (ii) in (i), we get:

$$v=\sqrt{\frac{\gamma RT}{V\rho}}=\sqrt{\frac{\gamma RT}{M}} \qquad ......(iii)$$

Where,
Mass, $M=\rho V$ is a constant
γ and R are also constants.
We get from equation (iii),
$v\propto\sqrt{T}$.
Hence, the sound speed in a gas is directly proportional to the square root of the gaseous medium's temperature, i.e., the sound speed increases with rise in the gaseous medium's temperature and vice versa.

c) increases with humidity.
Ans: Let v_m and v_d are the sound speed in moist air and dry air respectively and ρ_m and ρ_d are the densities of moist air and dry air respectively.

We have,

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

The speed of sound in moist air is:

$$v_m = \sqrt{\frac{\gamma P}{\rho_m}} \qquad \ldots\ldots(i)$$

The speed of sound in dry air is:

$$v_d = \sqrt{\frac{\gamma P}{\rho_d}} \qquad \ldots\ldots(ii)$$

On dividing equations (i) and (ii), we get:

$$\frac{v_m}{v_d} = \sqrt{\frac{\gamma P}{\rho_m} \times \frac{\rho_d}{\gamma P}} = \sqrt{\frac{\rho_d}{\rho_m}} \qquad \ldots\ldots(iii)$$

However, the presence of water vapour decreases the density of air, i.e.,

$\rho_d < \rho_m$

$\Rightarrow v_m > v_d$

Hence, the speed of sound in moist air is higher than it is in dry air. Thus, in a gaseous medium, the sound speed increases with humidity.

5. You have learnt that a travelling wave in one dimension is represented by a function $y = f(x,t)$ where x and t must appear in the combination $x - vt$ or $x + vt$, i.e. $y = f(x \pm vt)$. Is the converse true? Examine if the following functions for y can possibly represent a travelling wave:

a) $(x - vt)^2$

Ans: No.

For $x = 0$ and $t = 0$, the function $(x + vt)^2$ becomes 0.

Hence, for $x = 0$ and $t = 0$, the function represents a point.

b) $\log\left[\frac{x + vt}{x_0}\right]$

Ans. Yes.

For $x = 0$ and $t = 0$, the function $\log\left(\frac{x + vt}{x_0}\right) = \log 0 = \infty$

Since the function does not converge to a finite value for $x = 0$ and $t = 0$, it does not represent a travelling wave.

c) $\frac{1}{(x+vt)}$

Ans: No.

For $x=0$ and $t=0$, the function

$$\frac{1}{x+vt}=\frac{1}{0}=\infty$$

Since the function does not converge to a finite value for $x=0$ and $t=0$, it does not represent a travelling wave.

The converse is not true. The requirement for a wave function of a travelling wave is that for all x and t values, wave function should have a finite value. Therefore, none can represent a travelling wave.

6. A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of

a) the reflected sound,

Ans: We have,

Frequency of the ultrasonic sound, $\nu=1000\text{kHz}=10^6\text{Hz}$

Speed of sound in air, $v_a=340\text{ms}^{-1}$

The wavelength (λ_r) of the reflected sound is given by:

$$\lambda_r=\frac{v_a}{\nu}=\frac{340}{10^6}=3.4\times10^{-4}\text{m}$$

b) the transmitted sound? Speed of sound in air is 340ms^{-1} and in water 1486ms^{-1}.

Ans: We have,

Ultrasonic sound's frequency, $\nu=1000\text{kHz}=10^6\text{Hz}$

Sound speed in water, $v_w=1486\text{m/s}$

The wavelength is given as: $\lambda_t=\frac{1486}{10^6}=1.49\times10^{-3}\text{m}$

7. A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7 kms^{-1}? The operating frequency of the scanner is 4.2 MHz.

Ans: It is provided that,

Sound speed in the tissue, $v=1.7\text{Kms}^{-1}=1.7\times10^3\text{ms}^{-1}$

Scanner's operating frequency, $\nu = 4.2\text{MHz} = 4.2 \times 10^6\text{Hz}$

The wavelength of sound wave in the tissue is given by:

$$\lambda = \frac{v}{\nu} = \frac{1.7 \times 10^3}{4.2 \times 10^6} = 4.1 \times 10^{-4}\text{m}$$

The wavelength of sound in the tissue is $4.1 \times 10^{-4}\text{m}$.

8. A transverse harmonic wave on a string is described by $y(x,t) = 3.0\sin\left(36t + 0.018x + \frac{\pi}{4}\right)$

Where x and y are in cm and t in s. The positive direction of x is from left to right.

a) Is this a travelling wave or a stationary wave?
If it is travelling, what are the speed and direction of its propagation?
Ans: The given equation is the equation of a travelling wave, moving from right to left because it is an equation of the type
$y(x,t) = A\sin(\omega t + kx + \phi)$

Here, $A = 3.0\text{cm}$, $\omega = 36\text{rad}^{-1}, k = 0.018\text{cm}$ and $\phi = \frac{\pi}{4}$

$\therefore$ Speed of wave propagation is given by,

$$v = \frac{\omega}{k} = \frac{36\text{rads}^{-1}}{0.018\text{cm}^{-1}} = \frac{36\text{rads}^{-1}}{0.018 \times 10^2\text{m}^{-1}} = 20\text{ms}^{-1}$$

The speed of wave propagation is 20ms^{-1}.

b) What are its amplitude and frequency?
Ans: Amplitude of wave, $A = 3.0\text{cm} = 0.03\text{m}$

Frequency of wave, $\nu = \frac{\omega}{2\pi} = \frac{36}{2\pi} = 5.7\text{Hz}$

c) What is the initial phase at the origin?

Ans: Initial phase at origin, $\phi = \frac{\pi}{4}\text{rad}$

d) What is the least distance between two successive crests in the wave?
Ans: Least distance between two successive crests in the wave,

$$\Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.018} = 349\text{cm} = 3.49\text{m}$$

9. For the wave described in Exercise 15.8, plot the displacement (y) versus (t) graphs for $x = 0$, 2 and 4 cm. What are the shapes of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another: amplitude, frequency or phase?

Ans: All the waves have different phases.

The given transverse harmonic wave is:

$$y(x,t) = 3.0\sin\left(36t + 0.018x + \frac{\pi}{4}\right) \qquad \ldots\ldots(i)$$

For $x = 0$, the equation becomes:

$$y(0,t) = 3.0\sin\left(36t + \frac{\pi}{4}\right)$$

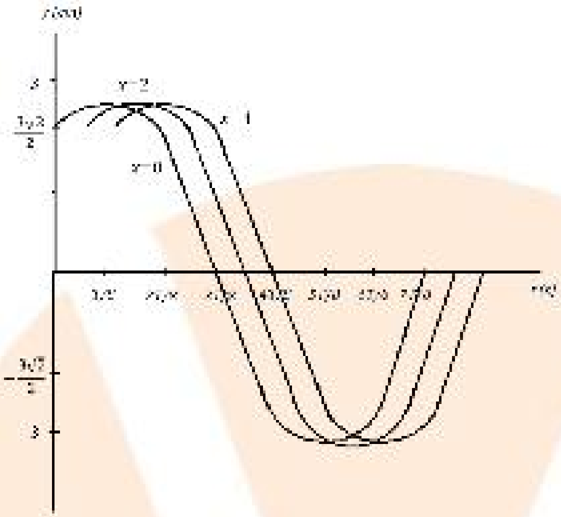
Also, $\omega = \frac{2\pi}{T} = 36\text{rad/s}$

$$\therefore T = \frac{2\pi}{36}\text{s}$$

Now, plotting y vs t graphs using the different values of t, as listed in the given table.

t(s)	0	$\frac{T}{8}$	$\frac{2T}{8}$	$\frac{3T}{8}$	$\frac{4T}{8}$	$\frac{5T}{8}$	$\frac{6T}{8}$	$\frac{7T}{8}$
y(cm)	$\frac{3\sqrt{2}}{2}$	3	$\frac{3\sqrt{2}}{2}$	0	$\frac{-3\sqrt{2}}{2}$	-3	$\frac{-3\sqrt{2}}{2}$	0

For $x = 0$, $x = 2$, and $x = 4$, the phases of the three waves will get altered. This is because amplitude and frequency are same for any change in x. The $y - t$ plots of the three waves are shown in the given figure.



10. For the travelling harmonic wave $y(x,t) = 2.0\cos 2\pi(10t - 0.0080x + 0.35)$

Where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of 4 m, 0.5 m, $\frac{\lambda}{2}$, $\frac{3\lambda}{4}$.

a) 4 m

Ans: Equation for a travelling harmonic wave is given by:

$$y(x,t) = 2.0\cos 2\pi(10t - 0.0080x + 0.35)$$

$$\Rightarrow y(x,t) = 2.0\cos(20\pi t - 0.016\pi x + 0.70\pi)$$

Where, Propagation constant, $k = 0.0160\pi$

Amplitude, $a = 2\text{cm}$

Angular frequency, $\omega = 20\pi\text{rad/s}$

Phase difference is given by:

$\phi = kx = \frac{2\pi}{\lambda}$

for $x = 4m = 400$ cm

$\Rightarrow \phi = 0.016\pi \times 400 = 6.4\pi$ rad

b) 0.5 m

Ans: Phase difference is given by:

$\phi = kr = \frac{2\pi}{\lambda}$

For $x = 0.5\text{ m} = 50\text{ cm}$

$\Rightarrow \phi = 0.016\pi \times 50 = 0.8\pi$ rad

c) $\frac{\lambda}{2}$

Ans: Phase difference is given by:

$\phi = kr = \frac{2\pi}{\lambda}$

For $x = \frac{\lambda}{2}$

$\Rightarrow \phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$ rad

d) $\frac{3\lambda}{4}$

Ans: Phase difference is given by:

$\phi = kr = \frac{2\pi}{\lambda}$

For $x = \frac{3\lambda}{4}$

$\Rightarrow \phi = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = 1.5\pi rad$

11. The transverse displacement of a string (clamped at its both ends) is given by $y(x,t) = 0.06\sin\frac{2\pi x}{3}\cos(120\pi t)$ Where x and y are in m and t in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg.
Answer the following:

a) Does the function represent a travelling wave or a stationary wave?
Ans: The general equation of stationary wave is given by:
$y(x,t) = A\sin(kx)\cos(\omega t)$
This given equation is similar to the equation of stationary wave:
$y(x,t) = 0.06\sin\frac{2\pi x}{3}\cos(120\pi t)$
Hence, the given function is a stationary wave.

b) Interpret the wave as a superposition of two waves travelling in opposite directions. What is the wavelength, frequency, and speed of each wave?
Ans: The transverse displacement of the string is given as:

$$y(x,t) = 0.06\sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t)$$

From above equation, $k = \frac{2\pi}{3}$

$\therefore$ Wavelength, $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\frac{2\pi}{3}} = 3m$

It is given that:

$120\pi = 2\pi\nu$

Frequency, $\nu = 60Hz$

Wave speed, $v = \nu\lambda = 60 \times 3 = 180ms^{-1}$

c) Determine the tension in the string.

Ans: The transverse wave's velocity travelling in a string is given by the relation:

$$v=\sqrt{\frac{T}{\mu}} \quad ......(i)$$

Where, Velocity of the transverse wave, $v=180ms^{-1}$

Mass of the string, $m=3.0\times10^{-2}kg$

Length of the string, $l = 1.5$ m

Mass per unit length of the string, $\mu=\frac{m}{l}=\frac{3.0}{1.5}\times10^{-2}=2\times10^{-2}kgm^{-1}$

Tension in the string $=$ T

From equation (i), tension can be obtained as:

$T=v^2\mu$

$\Rightarrow T=(180)^2\times2\times10^{-2}$

$\Rightarrow T=648N$

The tension in string is $648N$.

12.

i) For the wave on a string described in Question 11, do all the points on the string oscillate with the same

a) frequency,

Ans: Yes, all the points on the string vibrate with the same frequency, except at the nodes which are having zero frequency.

b) phase,

Ans: Yes, all the points in any oscillating loop have the same phase, except at the nodes.

c) amplitude? Explain your answers.

Ans: No, all the points in any oscillating loop have different vibration amplitudes.

ii) What is the amplitude of a point 0.375 m away from one end?

Ans: The given equation is:

$$y(x,t)=0.06\sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t)$$

For x = 0.375 m and t = 0

$$\text{Amplitude} = 0.06\sin\left(\frac{2\pi}{3}x\right)\cos 0$$

$$\Rightarrow a = 0.06\sin\left(\frac{2\pi}{3}\times 0.375\right)\times 1$$

$$\Rightarrow a = 0.06\sin(0.25\pi) = 0.06\sin\left(\frac{\pi}{4}\right)$$

$$\Rightarrow a = 0.06\times\frac{1}{\sqrt{2}} = 0.042\text{m}$$

The value of amplitude is 0.042m.

13. Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent, (i) a travelling wave, (ii) a stationary wave or (iii) none at all:

a) $y = 2\cos(3x)\sin(10t)$

Ans: This equation demonstrates a stationary wave because the harmonic terms kx and ωt seem separately in the equation.

b) $y = 2\sqrt{x - vt}$

Ans: This equation is not having any harmonic term. Therefore, it is not representing either a stationary wave or travelling wave.

c) $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$

Ans: This equation demonstrates a travelling wave as it is having harmonic terms kx and ωt are in $kx - \omega t$ combination.

d) $y = \cos x \sin t + \cos 2x \sin 2t$

Ans: This equation demonstrates a stationary wave because it is having harmonic terms kx and ωt separately in the equation. It is the superposition of two stationary waves.

14. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is $3.5 \times 10^{-2}\text{kg}$ and its linear mass density is $4.0 \times 10^{-2}\text{kg m}^{-1}$. What is

a) the speed of a transverse wave on the string, and
Ans: Provided that,
Mass of the wire, $m = 3.5\times10^{-2}\,kg$

Linear mass density, $\mu = \frac{m}{l} = 4.0\times10^{-2}\,kgm^{-1}$

Frequency of vibration, $\nu = 45Hz$

Length of the wire, $l = \frac{m}{\mu} = \frac{3.5\times10^{-2}}{4.0\times10^{-2}} = 0.875m$

The wavelength of the stationary wave (λ) is given by:

$\lambda = \frac{2l}{n}$ where, $n =$ number of nodes

For fundamental node, $n = 1$:

$\lambda = 2l$

$\lambda = 2\times0.875 = 1.75m$

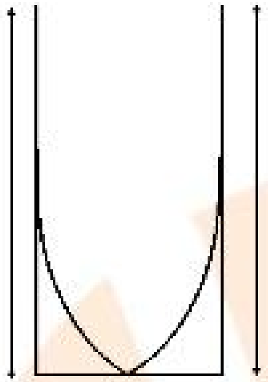
The speed of the transverse wave in the string is given as:

$v = \nu\lambda = 45\times1.75 = 78.75ms^{-1}$

The speed of transverse wave is $78.75ms^{-1}$.

b) the tension in the string?
Ans: The tension produced in the string is given by the relation:
$T = v^2\mu$
$\Rightarrow T = (78.75)^2\times4.0\times10^{-2} = 248.06N$
The tension in the string is $248.06N$.

15. A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency $340\,Hz$) when the tube length is $25.5\,cm$ or $79.3\,cm$. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected.
Ans: It is provided that,
Frequency of the turning fork, $\nu = 340\,Hz$
Since the given pipe is attached with a movable piston at one end, it will behave as a pipe with one end closed and the other end open, as shown in the given figure.



Such a system gives odd harmonics. The relation of fundamental note in a closed pipe is given by:

$l_1 = \frac{\lambda}{4}$

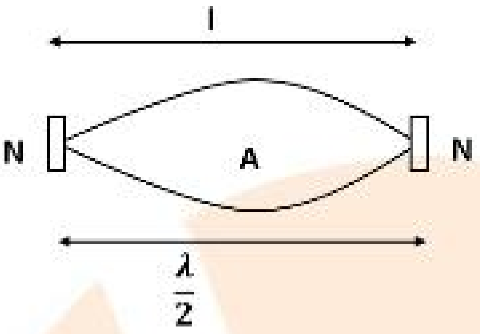
Where, length of the pipe $l_1 = 25.5\text{cm} = 0.255\text{m}$

$\Rightarrow \lambda = 4l_1 = 4 \times 0.255 = 1.02\text{m}$

The relation of sound speed is given by:

$v = \nu\lambda = 340 \times 1.02 = 346.8\text{ms}^{-1}$

16. A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 kHz. What is the speed of sound in steel?
Ans: We have,
Length of the steel rod, $l = 100\text{ cm} = 1\text{ m}$
Fundamental frequency of vibration, $\nu = 2.53\text{kHz} = 2.53 \times 10^3\text{Hz}$
An antinode (A) is formed at its centre, and nodes (N) are formed at its two ends when the rod is plucked at its middle, as shown in the given figure.



The distance between two successive nodes is $\frac{\lambda}{2}$

$\therefore l = \frac{\lambda}{2}$

$\lambda = 2l = 2 \times 1 = 2m$

The sound speed in steel is given by:

$v = \nu\lambda$

$\Rightarrow v = 2.53 \times 10^3 \times 2$

$\Rightarrow v = 5.06 \times 10^3 ms^{-1}$

$\Rightarrow v = 5.06 kms^{-1}$

The speed of sound in steel is $5.06 kms^{-1}$.

17. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open? (Speed of sound in air is 340 ms^{-1}).

Ans: Provided that,

Length of the pipe, $l = 20\text{ cm} = 0.2\text{ m}$

Source frequency $=$ n^{th} normal mode of frequency, $\nu_n =$ 430 Hz

Speed of sound, $v = 340\text{ m/s}$

In a closed pipe, the n^{th} normal mode of frequency is given by the relation: $v_n = (2n-1)\frac{v}{4l}$

Where, n is an integer $= 0,1,2,3.....$

$$\Rightarrow 430 = \frac{(2n-1)340}{4 \times 0.2}$$

$$\Rightarrow 2n-1 = \frac{430 \times 4 \times 0.2}{340}$$

$$\Rightarrow 2n-1 = 1.01$$

$$\Rightarrow n \approx 1$$

Clearly, the vibration frequency's first mode is resonantly excited by the given source. In a pipe open at both ends, the n^{th} mode of vibration frequency is given by:

$$v_n = \frac{nv}{2l}$$

$$\Rightarrow n = \frac{2lv_n}{v}$$

$$\Rightarrow n = \frac{2 \times 0.2 \times 430}{340} = 0.5$$

Since the number of the vibration mode has to be an integer, the given source does not generate a resonant vibration in an open pipe.

18. Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B?

Ans: It is provided that,

Frequency of string A, $f_A = 324\text{Hz}$

Frequency of string B $= f_B$

Beat's frequency, $n = 6\text{ Hz}$

Beat's frequency is given as:

$n = |f_A \pm f_B|$

$\Rightarrow 6 = 324 \pm f_B$

$\Rightarrow f_n =$ 330 Hz or 318 Hz

Frequency decreases with a reduction in the tension in a string because frequency is directly proportional to the tension's square root. It is given as:

$v \propto \sqrt{T}$

Hence, the beat frequency cannot be 330 Hz

$\therefore f_B = 318\text{Hz}$

19. Explain why (or how):

a) In a sound wave, a displacement node is a pressure antinode and vice versa,
Ans: A node is a point where the vibration amplitude is the minimum and pressure is the greatest. An antinode is a point where the maximum vibration amplitude and pressure are the lowest. Therefore, a displacement node is a pressure antinode and vice versa.

b) Bats can ascertain distances, directions, nature, and sizes of the obstacles **without any "eyes",**
Ans: Bats emit powerful high-frequency ultrasonic sound waves. These waves get reflected toward them by obstructions. A bat takes a reflected wave and measures the distance, size, direction, and nature of an obstacle with the aid of its brain senses.

c) A violin note and sitar note may have the same frequency, yet we can distinguish between the two notes,
Ans: The overtones generated by a sitar and a violin, and the powers of these overtones, are different. Hence, one can differentiate between the notes produced by a sitar and a violin even if they have a similar vibration frequency.

d) Solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and
Ans: Solids have a shear modulus. They can provide shearing stress. Since fluids

do not hold any definite shape, they produce shearing stress. The transverse wave propagation is such that it provides shearing stress in a medium. The propagation of such a wave is probable only in solids and not in gases. Both solids and fluids have their respective bulk moduli. They can provide compressive stress. Hence, longitudinal waves can pass through solids and fluids.

e) the shape of a pulse gets distorted during propagation in a dispersive medium.
Ans: A pulse is a combination of waves having various wavelengths. These waves move in a dispersive medium with varying velocities, depending on the medium's nature. This results in the shape distortion of a wave pulse.

20. A train, standing at the outer signal of a railway station blows a whistle of frequency $400\ \text{Hz}$ in still air.

i) What is the frequency of the whistle for a platform observer when the train

a) approaches the platform with a speed of $10\ \text{ms}^{-1}$,
Ans: Provided that,
Frequency of the whistle, $\nu = 400\ \text{Hz}$
Speed of the train, $v_T = 10\ \text{m/s}$
Speed of sound, $v = 340\ \text{m/s}$
The whistle's apparent frequency (ν') as the train approaches the platform is given by:

$$\nu' = \left(\frac{v}{v - v_T}\right)\nu$$

$$\Rightarrow \nu' = \left(\frac{340}{340 - 10}\right) \times 400 = 412.12\text{Hz}$$

b) recedes from the platform with a speed of $10\ \text{ms}^{-1}$?
Ans: The apparent frequency (ν'') of the whistle as the train recedes from the platform is given by the relation:

$$\nu'' = \left(\frac{v}{v + v_T}\right)\nu$$

$$\Rightarrow \nu'' = \left(\frac{340}{340 + 10}\right) \times 400 = 388.57\text{Hz}$$

The apparent frequency of the whistle is 388.57Hz.

ii) What is the speed of sound in each case? The speed of sound in still air can be taken as $340\ ms^{-1}$.

Ans: The apparent change in sound frequency is caused by the relative motions of the source and the observer. These relative motions generate no effect on the sound speed. Therefore, the sound speed in the air in both cases remains the same, i.e., $340\ m/s$.

21. A train, standing in a station-yard, blows a whistle of frequency $400\ Hz$ in still air. The wind starts blowing in the direction from the yard to the station with at a speed of $10\ ms^{-1}$. What is the frequency, wavelength, and speed of sound for an observer standing on the station's platform? Is the situation exactly identical to the case when the air is still and the observer runs towards the yard at a speed of $10\ ms^{-1}$? The speed of sound in still air can be taken as $340\ ms^{-1}$.

Ans: For the stationary observer:

Frequency of the sound generated by the whistle, $\nu = 400\ Hz$

Speed of sound $=\ 340\ m/s$

Velocity of the wind, $v = 10\ m/s$

As there is no relative motion between the observer and the source, the sound's frequency heard by the observer will be the similar as that produced by the source, i.e., $400\ Hz$.

The wind is blowing towards the observer. Hence, the sound's effective speed increases by 10 units, i.e., $v_e = 340 + 10 = 350 m/s$

The wavelength (λ) of the sound heard by the observer is given by:

$$\lambda = \frac{v_e}{\nu} = \frac{350}{400} = 0.875m$$

For the running observer:

Velocity of the observer, $v_0 = 10\ m/s$

The observer is moving towards the source. As a result, the relative motions of the source and the observer, there is a frequency change (ν')

This is given by the relation:

$$\nu' = \left(\frac{v + v_0}{v}\right)\nu$$

$$\Rightarrow \nu' = \left(\frac{340 + 10}{340}\right) \times 400 = 411.76Hz$$

Since the air is still so, the effective speed of sound $= 340 + 0 = 340\ m/s$

The source is at rest. Hence, the sound's wavelength will not change, i.e., λ remains $0.875\ m$.

Hence, the two given situations are not exactly identical.

Additional Exercise:

22. A travelling harmonic wave on a string is described by

$$y(x,t) = 7.5\sin\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

a) What are the displacement and velocity of oscillation of a point at $x = 1$ cm, and $t = 1$ s? Is this velocity equal to the velocity of wave propagation?

Ans: The given harmonic wave is:

$$y(x,t) = 7.5\sin\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

For $x = 1$ cm and $t = 1$ s,

$$y(1,1) = 7.5\sin\left(0.0050 + 12 + \frac{\pi}{4}\right)$$

$$\Rightarrow y(1,1) = 7.5\sin\left(12.0050 + \frac{\pi}{4}\right)$$

$$\Rightarrow y(1,1) = 7.5\sin\theta$$

$$\text{Where, } \theta = 12.0050 + \frac{\pi}{4} = 12.0050 + \frac{3.14}{4} = 12.78\text{rad}$$

$$\Rightarrow \theta = \frac{180}{3.14} \times 12.78 = 732.81^\circ$$

$$\therefore y(1,1) = 7.5\sin(732.81^\circ)$$

$$\Rightarrow y(1,1) = 7.5\sin(90 \times 80 + 12.81^\circ) = 7.5\sin 12.81^\circ$$

$$\Rightarrow y(1,1) = 7.5 \times 0.2217$$

$$\Rightarrow y(1,1) = 1.6629 \approx 1.663\text{cm}$$

The velocity of the oscillation at a given point and time is given as:

$$v = \frac{d}{dt}y(x,t) = \frac{d}{dt}\left[7.5\sin\left(0.0050x + 12t + \frac{\pi}{4}\right)\right]$$

$$\Rightarrow v = 7.5 \times 12\cos\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

At $x = 1$ cm and $t = 1$s,

$v = y(1,1) = 90\cos\left(12.005 + \frac{\pi}{4}\right)$

$\Rightarrow v = 90\cos(732.81^\circ) = 90\cos\left(90\times 8 + 12.81^\circ\right)$

$\Rightarrow v = 90\cos(12.81^\circ)$

$\Rightarrow v = 90\times 0.975 = 87.75 cm/s$

Now, the equation of a propagating wave is given by:

$y(x,t) = a\sin(kx + \omega t + \phi)$

Where, $k = \frac{2\pi}{\lambda}$

$\lambda = \frac{2\pi}{k}$ and $\omega = 2\pi\nu$

$\Rightarrow \nu = \frac{\omega}{2\pi}$

Speed, $v = \nu\lambda = \frac{\omega}{k}$

Where, $\omega = 12 rad/s$

$k = 0.0050 m^{-1}$

$\Rightarrow v = \frac{12}{0.0050} = 2400 cm/s$

Hence, the velocity of the wave oscillation at $x = 1$ cm and $t = 1$ s is not equal to the wave propagation's velocity.

b) Locate the points of the string which have the same transverse displacements and velocity as the $x = 1$ cm point at $t = 2$ s, 5 s and 11 s.

Ans: The relation of propagation constant with wavelength is given by:

$k = \frac{2\pi}{\lambda}$

$\Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\times 3.14}{0.0050} = 1256 cm = 12.56 m$

Therefore, all the points at distances $n\lambda(n=\pm1,\pm2,...\text{and so on})$, i.e., $\pm12.56\text{m}, \pm25.12\text{m},.....$ and so on for $x=1\text{ cm}$, will have the same displacement as the $x=1\text{ cm}$ points at $t=2\text{ s}$, 5 s, and 11 s.

23. A narrow sound pulse (for example, a short pip by a whistle) is sent across a medium.

a) Does the pulse have a definite (i) frequency, (ii) wavelength, (iii) speed of propagation?
Ans: (a) The sound pulse does not have a constant wavelength or frequency. However, the sound speed vibration remains the same, equal to the sound speed in that medium.

b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to $\frac{1}{20}$ or 0.05 Hz?
Ans: The short pip produced after every 20 s does not implies that the frequency of the whistle is $\frac{1}{20}$ or 0.05 Hz. It means that 0.05 Hz is the frequency of the repetition of the pip of the whistle.

24. One end of a long string of linear mass density $8.0\times10^{-3}\text{kg m}^{-1}$ is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At $t=0$, the left end (fork end) of the string $x=0$ has zero transverse displacement $(y=0)$ and is moving along positive y-direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement y as function of x and t that describes the wave on the string.
Ans: The equation of a travelling wave propagating along the positive y-direction is given by the displacement equation:

$$y(x,t)=a\sin(\omega t-kx) \quad ......(i)$$

Linear mass density, $\mu=8.0\times10^{-3}\text{kgm}^{-1}$

Frequency of the tuning fork, $\nu=256\text{ Hz}$

Amplitude of the wave, $a = 5.0\text{ cm} = 0.05\text{ m}$ ......(ii)

Mass of the pan, $m = 90\text{ kg}$

Tension in the string, $T = mg = 90x9.8 = 882\text{ N}$

The transverse wave's velocity is given by:

$$v = \sqrt{\frac{T}{\mu}}$$

$$\Rightarrow v = \sqrt{\frac{882}{8.0\times10^{-3}}} = 332\text{m/s}$$

Angular frequency, $\omega = 2\pi\nu$

$$\Rightarrow w = 2\times3.14\times256$$

$$\Rightarrow w = 1608.5 = 16\times10^{3}\text{rad/s} \text{......(iii)}$$

Wavelength, $\lambda = \frac{v}{\nu} = \frac{332}{256}\text{m}$

Propagation constant, $k = \frac{2\pi}{\lambda}$

$$\Rightarrow k = \frac{2\times3.14}{332/256} = 4.84\text{m}^{-1} \text{......(iv)}$$

Substituting the values from equations (ii), (iii), and (iv) in equation (i), we get,

$$y(x,t) = 0.05\sin(1.6\times10^{3}t - 4.84x)\text{m}$$

This describes the wave of the string.

25. A SONAR system fixed in a submarine operates at a frequency 40 kHz. An enemy submarine moves towards the SONAR with a speed of 360 kmh^{-1}. What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450 ms^{-1}.

Ans: It is provided that,

SONAR system's operating frequency, $\nu = 40\text{ kHz}$

Speed of the enemy submarine, $v_s = 360\text{km}/\text{h} = 100\ \text{m}/\text{s}$
Sound speed in water, $v = 1450\ \text{m}/\text{s}$
The source is at rest and the observer is moving towards it. Hence, the apparent frequency (ν') received and reflected by the submarine is given by:

$$\nu' = \left(\frac{v + v_s}{v}\right)\nu$$

$$\Rightarrow \nu' = \left(\frac{1450 + 100}{1450}\right) \times 40 = 42.76\text{kHz}$$

The frequency (ν'') received by the enemy submarine is given by:

$$\nu'' = \left(\frac{v}{v - v_s}\right)\nu'$$

Where, $v_s = 100\ \text{m}/\text{s}$

$$\Rightarrow \nu'' = \left(\frac{1450}{1450 - 100}\right) 42.72 = 45.93\text{kHz}$$

26. Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically, the speed of S wave is about $4.0\ \text{kms}^{-1}$, and that of P wave is $8.0\ \text{kms}^{-1}$. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur?

Ans: Let v_s and v_p be the velocities of S and P waves respectively.

Let L be the distance between the seismograph and the epicentre.

We have, $L = v_s t_s \quad \ldots\ldots(i)$

$L = v_p t_p \quad \ldots\ldots(ii)$

Where, t_s and t_p are the respective times taken by the S and P waves to reach the seismograph from the epicentre.

It is given that, $v_p = 8\text{km}/\text{s}$

$v_s = 4\text{km}/\text{s}$

From equations (i) and (ii), we have:

$v_s t_s = v_p t_p$

$4t_s = 8t_p$

$t_s = 2t_p \quad ......(iii)$

It is also given that:

$t_s - t_p = 4\,\text{min} = 240\text{s}$

$\Rightarrow 2t_p - t_p = 240$

$\Rightarrow t_p = 240$

And $t_s = 2 \times 240 = 480\text{s}$

From equation (ii), we get:

$L = 8 \times 240 = 1920\text{ km}$

Clearly, the earthquake occurs at a distance of 1920 km from the seismograph.

27. A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall?

Ans: Bat's ultrasonic beep frequency, $\nu = 40\text{ kHz}$

Velocity of the bat, $v_b = 0.03v$

Where, v is the velocity of sound in air

The apparent frequency of the sound striking the wall is given by:

$$\nu' = \left(\frac{v}{v - v_b}\right)\nu$$

$$\Rightarrow \nu' = \left(\frac{v}{v - 0.03v}\right) \times 40$$

$$\Rightarrow \nu' = \frac{40}{0.97}\text{kHz}$$

This frequency is reflected by the stationary wall ($v_s = 0$) toward the bat.

The frequency (ν'') of the received sound is given by the relation:

$$\nu'' = \left(\frac{v + v_b}{v}\right)\nu'$$

$$\Rightarrow \nu'' = \left(\frac{v + 0.03v}{v}\right) \times \frac{40}{0.97}$$

$$\Rightarrow \nu'' = \frac{1.03 \times 40}{0.97} = 42.47\text{kHz}$$

The frequency of the received sound is 42.47kHz.