NCERT Solutions for Class 12

Physics

Chapter 12 – Atoms

1. Fill in the blanks using the given options:
   a) The size of the atoms in Thomson's model are _______ the atomic size in Rutherford's model (much greater than/no different from/much lesser than).

   Ans: The sizes of the atoms in Thomson's model are <u>no different from</u> the atomic size in Rutherford's model.

   b) In the ground state of _______ electrons are in stable equilibrium, while in _______ electrons always experience a net force. (Thomson's model/Rutherford's model)

   Ans: In the ground state of <u>Thomson's model</u>, the electrons are in stable equilibrium, while in <u>Rutherford's model</u>, electrons always experience a net force.

   c) A classical atom based on _______ is doomed to collapse. (Thomson's model/ Rutherford's model.)

   Ans: A classical atom based on <u>Rutherford's model</u> is doomed to collapse.

   d) An atom features a nearly continuous mass distribution in _______ but features a highly non- uniform mass distribution in _______ (Thomson's model/ Rutherford's model.)

   Ans: An atom features a nearly continuous mass distribution in <u>Thomson's model</u>, but features a highly non-uniform mass distribution in <u>Rutherford's model</u>.

   e) The positively charged part of the atom possesses most of the mass in _______ (Rutherford's model/ Thomson's model /both the models.)

   Ans: The positively charged part of the atom possesses most of the mass in <u>both the models</u>.

2. If you're given a chance to repeat the α –particle scattering experiment employing a thin sheet of solid hydrogen instead of the gold foil (Hydrogen is a solid at temperatures below 14 K). What results would you expect?

Ans: In the $\alpha-$particle scattering experiment, when a thin sheet of solid hydrogen is replaced with the gold foil, the scattering angle would not turn out to be large enough.
This is because the mass of hydrogen is smaller than the mass of incident $\alpha-$particles. Also, the mass of the scattering particle is more than the target nucleus (hydrogen).
As a consequence, the $\alpha-$particles would not bounce back when solid hydrogen is utilized in the $\alpha-$particle scattering experiment and hence, we cannot evaluate the size of the hydrogen nucleus.

3. What's the shortest wavelength present within the Paschen series of spectral lines?
Ans: We know the Rydberg's formula is given as;

$$\frac{hc}{\lambda}=21.76\times10^{-19}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

Here,
h = Planck's constant = $6.6\times10^{-34}\,Js$
c = speed of light = $3\times10^8\,m/s$
(n_1 and n_2 are integers)
The shortest wavelength present within the Paschen series of the spectral lines is for values $n_1=3$ and $n_2=\infty$.

$$\Rightarrow\frac{hc}{\lambda}=21.76\times10^{-19}\left[\frac{1}{(3)^2}-\frac{1}{(\infty)^2}\right]$$

$$\Rightarrow\lambda=\frac{6.6\times10^{-34}\times3\times10^8\times9}{21.76\times10^{-19}}$$

$$\Rightarrow\lambda=8.189\times10^{-7}\,m$$

$$\Rightarrow\lambda=818.9\,nm$$

4. The two energy levels in an atom are separated by a difference of $2.3\,eV$. What is the frequency of radiation emitted when the atom makes a transition from the higher level to the lower level?
Ans: Given that the distance between the two energy levels in an atom is $E=2.3\,eV$.

$$\Rightarrow E=2.3\times1.6\times10^{-19}$$

$$\Rightarrow E=3.68\times10^{-19}\,J$$

Let ν be the frequency of radiation emitted when the atom jumps from the upper level to the lower level.

The relation for energy is given as;

$E = h\nu$

Here,

$h =$ Planck's constant $= 6.6 \times 10^{-34}$ Js

$$\Rightarrow \nu = \frac{E}{h}$$

$$\Rightarrow \nu = \frac{3.38 \times 10^{-19}}{6.62 \times 10^{-32}}$$

$$\Rightarrow \nu = 5.55 \times 10^{14} \text{Hz}$$

Clearly, the frequency of the radiation is 5.6×10^{14} Hz.

5. For a hydrogen atom, the ground state energy is -13.6eV. What are the kinetic and potential energies of the electron during this state?

Ans: Provided that the ground state energy of hydrogen atom, $E = -13.6\text{eV}$ which is the total energy of a hydrogen atom.

Here, kinetic energy is equal to the negative of the total energy.

Kinetic energy $= -E = -(-13.6) = 13.6\text{eV}$

The potential energy is the same as the negative of two times kinetic energy.

Potential energy $= -2 \times (13.6) = -27.2\text{eV}$

$\therefore$ The kinetic energy of the electron is 13.6eV and the potential energy is -27.2eV.

6. A hydrogen atom absorbs a photon when it is in the ground level, this excites it to the $n = 4$ level. Find out the wavelength and frequency of the photon.

Ans: It is known that for ground level absorption, $n_1 = 1$

Let E_1 be the energy of this level. It is known that E_1 is related with n_1 as;

$$E_1 = \frac{-13.6}{n_1^2}\text{eV}$$

$$\Rightarrow E_1 = \frac{-13.6}{1^2} = -13.6\text{eV}$$

When the atom jumps to a higher level, $n_2 = 4$.

Let E_2 be the energy of this level.

$$\Rightarrow E_2 = \frac{-13.6}{n_2^2}\text{eV}$$

$$\Rightarrow E_2 = \frac{-13.6}{4^2} = \frac{-13.6}{16}\text{eV}$$

The amount of energy absorbed by the photon is given as;

$E = E_1 - E_2$

$\Rightarrow E = \left(\frac{-13.6}{16}\right) - \left(\frac{-13.6}{1}\right)$

$\Rightarrow E = \frac{13.6 \times 15}{16} \text{eV}$

$\Rightarrow E = \frac{13.6 \times 16}{16} \times 1.6 \times 10^{-19}$

$\Rightarrow E = 2.04 \times 10^{-18} \text{J}$

For a photon of wavelength λ , the expression of energy is written as;

$E = \frac{hc}{\lambda}$

Here,

$h =$ Planck's constant $= 6.6 \times 10^{-34} \text{Js}$

$c =$ speed of light $= 3 \times 10^8 \text{m/s}$

$\Rightarrow \lambda = \frac{hc}{E}$

$\Rightarrow \lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2.04 \times 10^{-18}}$

$\Rightarrow \lambda = 9.7 \times 10^{-8} \text{m}$

$\Rightarrow \lambda = 97 \text{nm}$

Also, frequency of a photon is given by the relation,

$\nu = \frac{c}{\lambda}$

$\Rightarrow \nu = \frac{3 \times 10^8}{9.7 \times 10^{-8}} \approx 3.1 \times 10^{15} \text{Hz}$

Clearly, the wavelength of the photon is 97nm whereas the frequency is $3.1 \times 10^{15} \text{Hz}$.

7. Answer the following questions.

a) **Using the Bohr's model, calculate the speed of the electron in a hydrogen atom in the $n = 1, 2$ and 3 levels.**

Ans: Consider v_1 to be the orbital speed of the electron in a hydrogen atom in the ground state level $n_1 = 1$. For charge (e) of an electron, v_1 is given by the relation,

$$v_1 = \frac{e^2}{n_1 4\pi \epsilon_0 \left(\frac{h}{2\pi}\right)}$$

$$\Rightarrow v_1 = \frac{e^2}{2 \epsilon_0 h}$$

Here,

$e = 1.6 \times 10^{-19} C$

$\epsilon_0 =$ Permittivity of free space $= 8.85 \times 10^{-12} N^{-1} C^2 m^{-2}$

$h =$ Planck's constant $= 6.6 \times 10^{-34} Js$

$$\Rightarrow v_1 = \frac{\left(1.6 \times 10^{-19}\right)^2}{2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}}$$

$$\Rightarrow v_1 = 0.0218 \times 10^8$$

$$\Rightarrow v_1 = 2.18 \times 10^6 m/s$$

For level $n_2 = 2$, we can write the relation for the corresponding orbital speed as;

$$v_2 = \frac{e^2}{n_2 2 \epsilon_0 h}$$

$$\Rightarrow v_2 = \frac{\left(1.16 \times 10^{-19}\right)^2}{2 \times 2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}}$$

$$\Rightarrow v_2 = 1.09 \times 10^6 m/s$$

And, for $n_3 = 3$, we can write the relation for the corresponding orbital speed as;

$$v_3 = \frac{e^2}{n_3 2 \epsilon_0 h}$$

$$\Rightarrow v_3 = \frac{\left(1.16 \times 10^{-19}\right)^2}{3 \times 2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}}$$

$$\Rightarrow v_3 = 7.27 \times 10^5 m/s$$

Clearly, the speeds of the electron in a hydrogen atom in the levels $n = 1, 2$ and 3 are $2.18 \times 10^6 m/s$, $1.09 \times 10^6 m/s$ and $7.27 \times 10^5 m/s$ respectively.

b) Calculate the orbital period in each of these levels.

Ans: Consider T_1 to be the orbital period of the electron when it is in level $n_1 = 1$.

It is known that the orbital period is related to the orbital speed as

$$T_1 = \frac{2\pi r_1}{v_1}$$

Here,

$r_1 =$ Radius of the orbit in $n_1 = \frac{n_1^2 h^2 \epsilon_0}{\pi m e^2}$

$h =$ Planck's constant $= 6.6 \times 10^{-34}$ Js

$e =$ Charge of an electron $= 1.6 \times 10^{-19}$ C

$\epsilon_0 =$ Permittivity of free space $= 8.85 \times 10^{-12}\ N^{-1}C^2m^{-2}$

$m =$ Mass of an electron $= 9.1 \times 10^{-31}$ kg

$$\Rightarrow T_1 = \frac{2\pi \times (1)^2 \times \left(6.62 \times 10^{-34}\right)^2 \times 8.85 \times 10^{-12}}{2.18 \times 10^6 \times \pi \times 9.1 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^2}$$

$$\Rightarrow T_1 = 15.27 \times 10^{-17}$$

$$\Rightarrow T_1 = 1.527 \times 10^{-16} s$$

For level $n_2 = 2$, we can write the orbital period as;

$$T_2 = \frac{2\pi r_2}{v_2}$$

Here,

$r_2 =$ Radius of the orbit in $n_2 = \frac{n_2^2 h^2 \epsilon_0}{\pi m e^2}$

$$\Rightarrow T_1 = \frac{2\pi \times (2)^2 \times \left(6.62 \times 10^{-34}\right)^2 \times 8.85 \times 10^{-12}}{1.09 \times 10^6 \times \pi \times 9.1 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^2} = 1.22 \times 10^{-15} s$$

And for the level $n_3 = 3$, we can write the orbital period as;

$$T_3 = \frac{2\pi r_3}{v_3}$$

Here,

$r_3 =$ Radius of the orbit in $n_3 = \frac{n_3^2 h^2 \epsilon_0}{\pi m e^2}$

$$\Rightarrow T_3 = \frac{2\pi \times (3)^2 \times \left(6.62 \times 10^{-34}\right)^2 \times 8.85 \times 10^{-12}}{7.27 \times 10^5 \times \pi \times 9.1 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^2} = 4.12 \times 10^{-15} s$$

Hence, the orbital periods in the levels $n = 1, 2$ and 3 are $1.527 \times 10^{-16} s$, $1.22 \times 10^{-15} s$ and $4.12 \times 10^{-15} s$ respectively.

8. The innermost electron orbit of a hydrogen atom has a radius of $5.3\times10^{-11}\text{m}$. What are the radii of the $n=2$ and $n=3$ orbits?

Ans: Provided that the innermost radius, $r_1=5.3\times10^{-11}\text{m}$.

Let r_2 be the radius of the orbit at $n=2$. It is related to the radius of the innermost orbit as;

$r_2=(n)^2 r_1$

$\Rightarrow r_2=(2)^2\times5.3\times10^{-11}=2.1\times10^{-10}\text{m}$

Similarly, for $n=3$;

$r_3=(n)^2 r_1$

$\Rightarrow r_3=(3)^2\times5.3\times10^{-11}=4.77\times10^{-10}\text{m}$

Clearly, the radii of the $n=2$ and $n=3$ orbits are $2.1\times10^{-10}\text{m}$ and $4.77\times10^{-10}\text{m}$ respectively.

9. A 12.5eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

Ans: It is provided that the energy of the electron beam used to bombard gaseous hydrogen at room temperature is 12.5eV .

It is also known that the energy of the gaseous hydrogen in its ground state at room temperature is -13.6eV .

When gaseous hydrogen is bombarded with an electron beam at room temperature, the energy of the gaseous hydrogen becomes $-13.6+12.5\text{eV}=-1.1\text{eV}$.

Now, the orbital energy is related to orbit level (n) as;

$$E=\frac{-13.6}{(n)^2}\text{eV}$$

For $n=3; E=\dfrac{-13.6}{9}=-1.5\text{eV}$

This energy is approximately equal to the energy of gaseous hydrogen.

So, it can be concluded that the electron has excited from $n=1$ to $n=3$ level.

During its de-excitation, the electrons can jump from $n=3$ to $n=1$ directly, which forms a line of the Lyman series of the hydrogen spectrum.

The formula for wave number for Lyman series is given as;

$$\frac{1}{\lambda}=R_y\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$$

Here,

$R_y=$ Rydberg constant $=1.097\times10^7\text{m}^{-1}$

λ = Wavelength of radiation emitted by the transition of the electron
Using this relation for $n = 3$ we get,

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{3^2} \right)$$

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(1 - \frac{1}{9} \right)$$

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{8}{9} \right)$$

$$\Rightarrow \lambda = \frac{9}{8 \times 1.097 \times 10^7} = 102.55\text{nm}$$

If the transition takes place from $n = 3$ to $n = 2$, and then from $n = 2$ to $n = 1$, then the wavelength of the radiation emitted in transition from $n = 3$ to $n = 2$ is given as;

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{9} \right)$$

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{5}{36} \right)$$

$$\Rightarrow \lambda = \frac{36}{5 \times 1.097 \times 10^7} = 656.33\text{nm}$$

This radiation corresponds to the Balmer series of the hydrogen spectrum.
Now, the wavelength of the radiation when the transition takes place from $n = 2$ to $n = 1$ is given as;

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{2^2} \right)$$

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(1 - \frac{1}{4} \right)$$

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{3}{4} \right)$$

$$\Rightarrow \lambda = \frac{4}{3 \times 1.097 \times 10^7} = 121.54\text{nm}$$

Clearly, in the Lyman series, two wavelengths are emitted i.e., 102.5nm and 121.5nm whereas in the Balmer series, only one wavelength is emitted i.e., 656.33nm.

10. In accordance with the Bohr's model, what is the quantum number that characterizes the earth's revolution around the sun in an orbit of radius 1.5×10^{11}m with an orbital speed of 3×10^{4}m/s. The mass of the earth is given as 6×10^{24}kg.

Ans: Here, it is provided that,

Radius of the earth's orbit around the sun, $r = 1.5\times10^{11}\text{m}$

Orbital speed of the earth, $v = 3\times10^{4}\text{m/s}$

Mass of the earth, $m = 6\times10^{24}\text{kg}$

With respect to the Bohr's model, angular momentum is quantized and is given as;

$$mvr = \frac{nh}{2\pi}$$

Here,

$h =$ Planck's constant $= 6.6\times10^{-34}\text{Js}$

$n =$ Quantum number

$$n = \frac{mvr^2\pi}{h}$$

$$\Rightarrow n = \frac{2\pi\times6\times10^{24}\times3\times10^{4}\times1.5\times10^{11}}{6.62\times10^{-34}}$$

$$\Rightarrow n = 25.61\times10^{73} = 2.6\times10^{74}$$

Clearly, the quantum number that characterizes the earth's revolution around the sun is 2.6×10^{74}.

11. Which of the following questions help you understand the difference between Thomson's model and Rutherford's model better.

a) Is the average angle of deflection of $\alpha-$particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

Ans: About the same.

The average angle of deflection of $\alpha-$particles caused by a thin gold foil considered by Thomson's model is about the same size as that considered by Rutherford's model.

This is because in both the models, the average angle was used.

b) Is the probability of backward scattering (i.e., scattering of $\alpha-$particles at angles greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

Ans: Much less.
The probability of scattering of α – particles at angles greater than 90° considered by Thomson's model is much less than that considered by Rutherford's model.

c) Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α – particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide?

Ans: Scattering is mainly caused due to single collisions.
The chances of a single collision have a linearly increasing nature with the number of target atoms.
As the number of target atoms increases with an increase in thickness, the collision probability varies linearly with the thickness of the target.

d) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α – particles by a thin foil?

Ans: Thomson's model.
It is incorrect to not consider multiple scattering in Thomson's model for the calculation of average angle of scattering of α – particles by a thin foil.
This is because a single collision produces very little deflection in this model.
Thus, the observed average scattering angle can be demonstrated only by considering multiple scattering.

12. The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-40}. An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting.

Ans: The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-40}.
It is given that,
Radius of the first Bohr orbit is given by the relation,

$$r_1 = \frac{4\pi\epsilon_0 \left(\frac{h}{2\pi}\right)^2}{m_e e^2} \qquad \ldots(1)$$

Here,

ϵ_0 = Permittivity of free space

h = Planck's constant $= 6.63\times10^{-34}$ Js

m_e = Mass of an electron $= 9.1\times10^{-31}$ kg

e = Charge of an electron $= 1.6\times10^{-19}$ C

m_p = Mass of a proton $= 1.67\times10^{-27}$ kg

r = Distance between the electron and the proton

Coulomb attraction between an electron and a proton is given by;

$$F_C = \frac{e^2}{4\pi\epsilon_0 r^2} \qquad \ldots(2)$$

Gravitational force of attraction between an electron and a proton is given by;

$$F_G = \frac{Gm_p m_e}{r^2} \qquad \ldots(3)$$

Here,

G = Gravitational constant $= 6.67\times10^{-11}$ Nm^2 / kg^2

The electrostatic (Coulomb) force and the gravitational force between an electron and a proton are equal, then we can write;

$$F_G = F_C$$

$$\Rightarrow \frac{Gm_p m_e}{r^2} = \frac{e^2}{4\pi\epsilon_0 r^2} \qquad \ldots(\text{using 2 \& 3})$$

$$\Rightarrow \frac{e^2}{4\pi\epsilon_0} = Gm_p m_e \qquad \ldots(4)$$

Now, by substituting the value of equation (4) in (1), we get;

$$r_1 = \frac{\left(\frac{h}{2\pi}\right)^2}{Gm_p m_e}$$

$$\Rightarrow r_1 = \frac{\left(\frac{6.63\times10^{-34}}{2\times3.14}\right)^2}{6.67\times10^{-11}\times1.67\times10^{-27}\times\left(9.1\times10^{-31}\right)^2}$$

$$\Rightarrow r_1 \approx 1.21\times10^{29}\,\text{m}$$

It is known that the universe is 156 billion light years wide or 1.5×10^{27} m wide. Clearly, it can be concluded that the radius of the first Bohr orbit is much greater than the estimated size of the whole universe.

13. Derive an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level n to level $(n-1)$. For large n, show that this frequency equals the classical frequency of revolution of the electron in the orbit.

Ans: It is given that a hydrogen atom makes transition from an upper level (n) to a lower level $(n-1)$. We have the relation for energy (E_1) of radiation at level n as;

$$E_1 = h\nu_1 = \frac{hme^4}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3} \times \left(\frac{1}{n^2}\right) \quad \ldots(1)$$

Here,

$\nu_1 =$ Frequency of radiation at level n

$h =$ Planck's constant

$m =$ Mass of hydrogen atom

$e =$ Charge on an electron

$\epsilon_0 =$ Permittivity of free space

Now, the relation for energy (E_2) of radiation at level $(n-1)$ is given as;

$$E_2 = h\nu_2 = \frac{hme^4}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3} \times \frac{1}{(n-1)^2} \quad \ldots(2)$$

Here,

$\nu_2 =$ Frequency of radiation at level $(n-1)$

Energy (E) released as a result of de-excitation;

$E = E_2 - E_1$

$\Rightarrow h\nu = E_2 - E_1 \quad \ldots(3)$

Here,

$\nu =$ Frequency of radiation emitted

Using the values of equations (1) and (2) in equation (3), we get;

$$\nu = \frac{me^4}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3} \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$\Rightarrow \nu = \frac{me^4 (2n-1)}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^2 (n-1)^2}$$

For large n, we can write $(2n-1) \approx 2n$ and $(n-1) \approx n$

$$\Rightarrow \nu = \frac{me^4}{32\pi^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \qquad \ldots(4)$$

Classical relation of frequency of revolution of an electron is given by;

$$\nu_c = \frac{v}{2\pi r} \qquad \ldots(5)$$

Here,

Velocity of the electron in the n^{th} orbit is given as;

$$v = \frac{e^2}{4\pi\epsilon_0 \left(\frac{h}{2\pi}\right) n} \qquad \ldots(6)$$

And, radius of the n^{th} orbit is given by;

$$r = \frac{4\pi\epsilon_0 \left(\frac{h}{2\pi}\right)^2}{me^2} n^2 \qquad \ldots(7)$$

Substituting the values of equation (6) and (7) in equation (5), we get;

$$\nu_c = \frac{me^4}{32\pi^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \qquad \ldots(8)$$

Clearly, when equations (4) and (8) are compared, it can be seen that the frequency of radiation emitted by the hydrogen atom is the same as the classical orbital frequency.

14. Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom $\left(\sim 10^{-10} m\right)$.

a) Construct a quantity with the dimensions of length from the fundamental constants e, m_e and c. Also, determine its numerical value.

Ans: To construct a quantity with the dimensions of length from the fundamental constants e, m_e and c, take:

Charge of an electron, $e = 1.6 \times 10^{-19}\,C$

Mass of an electron, $m_e = 9.1 \times 10^{-31}\,kg$

Speed of light, $c = 3 \times 10^{8}\,m/s$

The quantity having dimensions of length and involving the given quantities is $\left(\frac{e^2}{4\pi \epsilon_0 m_e c^2}\right)$

Here,

$\epsilon_0 =$ Permittivity of free space

And, $\frac{1}{4\pi \epsilon_0} = 9 \times 10^{9}\,Nm^2C^{-2}$

$\therefore$ The numerical value of the taken quantity will be

$$\frac{1}{4\pi \epsilon_0} \times \frac{e^2}{m_e c^2} = 9 \times 10^{9} \times \frac{\left(1.6 \times 10^{-19}\right)^2}{9.1 \times 10^{-31} \times \left(3 \times 10^{8}\right)^2} = 2.81 \times 10^{-15}\,m$$

Clearly, the numerical value of the taken quantity is much less than the typical size of an atom.

b) You will observe that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves c. But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested **Bohr to discard c and look for "something else" to get the right** atomic size. Now, the Planck's constant h had already made its **appearance elsewhere. Bohr's great insight lay in recognising that** $h, m_e,$ and e will yield the right atomic size. Construct a quantity with the dimension of length from h, m_e and e and confirm that its numerical value has indeed the correct order of magnitude.

Ans: To construct a quantity with the dimension of length from h, m_e and e, take,

Charge of an electron, $e = 1.6 \times 10^{-19}\,C$

Mass of an electron, $m_e = 9.1 \times 10^{-31}\,kg$

Planck's constant, $h = 6.63 \times 10^{-34}\,Js$

Now, let us take a quantity involving the given quantities as, $\frac{4\pi \epsilon_0 \left(\frac{h^2}{2\pi}\right)}{m_e e^2}$

Here, $\epsilon_0 =$ Permittivity of free space

And, $\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \, Nm^2C^{-2}$

$\therefore$ The numerical value of the taken quantity would be

$$4\pi \epsilon_0 \times \frac{\left(\frac{h}{2\pi}\right)^2}{m_e e^2} = \frac{1}{9 \times 10^9} \times \frac{\left(\frac{6.63 \times 10^{-34}}{2 \times 3.14}\right)^2}{9.1 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^2} = 0.53 \times 10^{-10} m$$

Clearly, the value of the quantity taken is of the order of the atomic size.

15. The total energy of an electron in the first excited state of the hydrogen atom is about $-3.4eV$.

a) What is the kinetic energy of the electron in this state?

Ans: Kinetic energy of the electron is the same as the negative of the total energy.

$\Rightarrow K = -E$

$\Rightarrow K = -(-3.4) = +3.4eV$

Clearly, the kinetic energy of the electron in the given state is $+3.4eV$.

b) What is the potential energy of the electron in this state?

Ans: Potential energy (U) of the electron is the same as the negative of twice of its kinetic energy

$\Rightarrow U = -2K$

$\Rightarrow U = -(-3.4) = -6.8eV$

Clearly, the potential energy of the electron in the given state is $-6.8eV$.

c) Which of the answers above would change if the choice of the zero of potential energy is changed?

Ans: The potential energy of a system is dependent on the reference point taken. Here, the potential energy of the reference point is considered to be zero.

When the reference point is changed, then the magnitude of the potential energy of the system also changes.

As total energy is the sum of kinetic and potential energies, total energy of the system would also differ.

16. If Bohr's quantization postulate (angular momentum $= nh/2\pi$) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantization of orbits of planets around the sun?

Ans: It is not much spoken about the quantization of orbits of planets around the Sun since the angular momentum associated with planetary motion is largely relative to the value of constant (h).

The angular momentum of the Earth in its orbit is of the order of $10^{70}h$. This causes a very high value of quantum levels n of the order of 10^{70}.

When large values of n are considered, successive energies and angular momenta are found to be relatively very small. Clearly, the quantum levels for planetary motion are considered continuous.

17. Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom [i.e., an atom in which a negatively charged muon (μ^-) mass about $207m_e$. orbits around a proton].

Ans: To obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom, consider the mass of a negatively charged muon to be $m_\mu = 207m$.

Now, according to Bohr's model,

Bohr radius, $r_e \propto \left(\frac{1}{m_e}\right)$

And, energy of a ground state electronic hydrogen atom, $E_e \propto m_e$.

Also, energy of a ground state muonic hydrogen atom, $E_\mu \propto m_e$.

It is known that the value of the first Bohr orbit, $r_e = 0.53A = 0.53\times10^{-10}m$

Consider r_μ to be the radius of muonic hydrogen atom.

At equilibrium, we have the relation:

$$m_\mu r_\mu = m_e r_e$$

$$\Rightarrow 207m_e \times r_\mu = m_e r_e$$

$$\Rightarrow r_\mu = \frac{0.53\times10^{-10}}{207} = 2.56\times10^{-13}m$$

Clearly, the value of the first Bohr radius of muonic hydrogen atom is $2.56\times10^{-13}m$.

Now, we have,

$E_e = -13.6\text{eV}$

Taking the ratio of these energies as $\frac{E_e}{E_\mu} = \frac{m_e}{m_\mu}$

$\Rightarrow \frac{E_e}{E_\mu} = \frac{m_e}{207m_e}$

$\Rightarrow E_\mu = 207E_e$

$\Rightarrow E_\mu = 207 \times (-13.6) = -2.81\text{keV}$

Clearly, the ground state energy of a muonic hydrogen atom is -2.81keV.