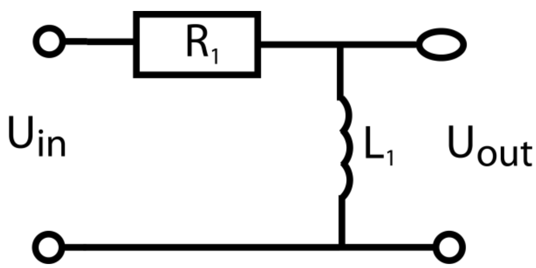
# Physics Experiments II
## Signal Processing and Noise

Exercise Class 1: Electronic Circuits

January 2025

### Problem 1: RL Filter

*Herhaling PE1*




(a) Show that the differential equation for the output signal $U_{out}(t)$ with parameters $R_1$, $L_1$ and an input voltage $U_{in}(t)$ is equal to

$$U_{out}(t) = \frac{L_1}{R_1}\left(\frac{dU_{in}(t)}{dt} - \frac{dU_{out}(t)}{dt}\right) .$$

(b) Solve the differential equation for $U_{out}(t) = u_{out}e^{i\omega t}$ and $u_{in}(t) = u_{in}e^{i\omega t}$ where $u_{out}$ and $u_{in}$ are scalars. With solve we mean that you gain an expression for $u_{out}$ as a function of $u_{in}$.

When the function $U_{in}(t) = u_{in}e^{st}$ (with $s = i\omega$) is used to solve a differential equation it is commonly called a test function in physics. You have seen in your mathematics courses that this function is a basis of eigenfunctions of a homogeneous time-invariant differential equation with constant coefficients.

(c) Check your solution by using the complex impedances of the resistor and inductor in the frequency domain (see Table 1.1 in the reader) to directly calculate the solution without using the differential equation.

(d) Write your solution in the form $H(\omega) = u_{out}/u_{in}$. What is the order and cut-off frequency of the transfer function?

(e) **Sketch** both the Bode magnitude plot and Bode phase plot for $R_1 = 10\,\Omega$ and $L_1 = 0.1\,\text{mH}$. Is this a high- or a low-pass filter?

## Problem 2: Coupling of filters

We use the filter of the first assignment and connect it to another, identical filter but now with values $R_2$ and $L_2$. Both filters have the same cut-off frequency and we have $R_1 = 10\,\Omega$ and $L_1 = 0.1\,\text{mH}$.

(a) Write down from the first exercise the transfer function of each filter separately with their cut-off frequencies.

(b) Give the requirements for $R_2$ so that the second filter does not load the first filter. Also give an example of appropriate values for $R_2$ and $L_2$.

(c) Sketch the Bode magnitude plot and the Bode phase plot for the case where the second filter **does not** load the first filter, what is the difference with the Bode plot of Problem 1? (Sketch both plots on the same graph).

(d) **Extra**: Determine $H_{tot}(\omega)$ for the general case, when the second filter **does** load the first filter.

## Problem 3: Moving Average

*From the exam Signal Processing and Noise 2016*
Consider the following moving average with $T$ the averaging time:

$$y(t) = \frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x(\tau)d\tau \tag{1}$$

(a) Show that this operation is linear and time invariant. (2 points)

(b) Show/argue that the impulse response function $h(t)$ is given by a rectangular function of width $T$ and height $\frac{1}{T}$. (2 points)

(c) Derive the transfer function $H(\omega)$ and argue why the moving average is considered a low pass filter. (2 points)

**The next question is about chapter 2 of the reader.**
To prevent aliasing, researcher Alice filters the signal with this moving average and samples with a sampling rate $\frac{2}{T}$.

(d) Argue from the answer given at c) if Alice's approach is correct or needs improvement. (2 points)

## Problem 4: Series LCR circuit

*Additional practice chapter 1 and repetition PE1*
Below is the figure of an series LCR circuit.

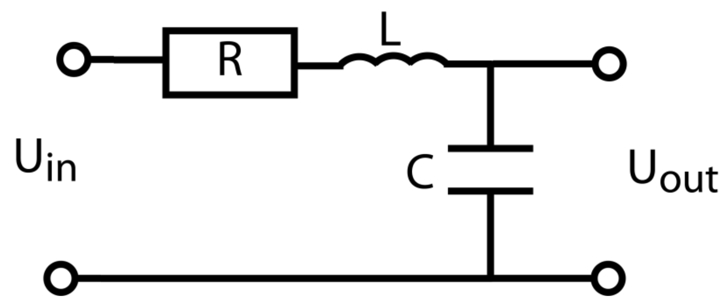



Figure 1: LCR circuit.

(a) Give the transfer function of this circuit.

(b) Show that your answer in a) can be written as:

$$H(\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_{LC}}\right)^2 + i\frac{\omega}{\omega_{LC} Q}}$$

Express $Q$ and $\omega_{LC}$ in terms of $L$, $R$ en $C$.

From now on assume that $Q \gg 1$.

(c) What is the amplitude and phase of the transfer function for $\omega = 0$, $\omega = \omega_{LC}$, and $\omega \gg \omega_{LC}$?

(d) Use your answer from c) to sketch the Bode plots. Indicate $\omega_{LC}$ and give the ratio of the amplitude between $\omega = 0$, and $\omega = \omega_{LC}$. Also specify the gradient of the curve at high frequencies in dB/decade.

(e) **Extra:** What is the full width of the peak at $1/\sqrt{2}$ of the maximum in the amplitude? Express this in terms of $Q$ and $\omega_{LC}$. *Hint: Use* $1/\sqrt{2} = |1/(1+i)| = |1/(1-i)|$ *and if necessary a first order approximation in* $\omega/\omega_{LC}$.

## Problem 5: Extra: prove general solution LDE with constant coefficients

In the lecture we used the fact that $x(t) = e^{i\omega t}$ are eigenfunctions of LTI systems to show that the LDE

$$a_0 x + a_1 \dot{x} + a_2 \ddot{x} + ... + a_n x^{(n)} = b_0 y + b_1 \dot{y} + b_2 \ddot{y} + ... + b_m y^{(m)} . \qquad (2)$$

is solved for general inputs $x(t)$ by

$$Y(\omega) = \frac{a_0 + a_1(i\omega) + a_2(i\omega)^2 + ... + a_n(i\omega)^n}{b_0 + b_1(i\omega) + b_2(i\omega)^2 + ... + b_m(i\omega)^m} X(\omega), \qquad (3)$$

where $X(\omega)$, $Y(\omega)$ are the Fourier transforms of $x(t)$, $y(t)$.

Prove eq. (3) without relying on $e^{i\omega t}$ being eigenfunctions, by taking the FT of both sides of eq. (2), and using the properties of the FT $\mathcal{F}[ax(t) + by(t)] = aX(\omega) + bY(\omega)$ and $\mathcal{F}\left[\frac{dx(t)}{dt}\right] = i\omega X(\omega)$.