Physics Experiments II
Signal Processing and Noise

Exercise Class 2: Signal Processing

January 2025

Problem 1: Birdwatcher - Aliasing and frequency resolution

Bob the birdwatcher has biked to the Veluwe on a beautiful Saturday morning to record the sounds of a range of different animals. His microphone has a range from DC up to 20 kHz, you can assume for a) that this microphone does not create (electronic) noise.

(a) From his watch tower Bob has spotted a bellowing deer and wants to record its call so that he can look up what species it is at home. What is the minimum sampling frequency necessary to characterise the sound when the highest frequency is 500 Hz and there is no other noise?

(b) What is the lowest sampling frequency needed when the microphone does produce (electronic) noise within its recording range and you want to ensure that no high frequency noise is present on the recording of the deer's call?

Bob records the signal for 10 seconds, with a sampling frequency of 60 kSamples/second.

(c) What is the frequency resolution when we use a computer to Fourier transform this signal?

(d) Bob finds an unexpected peak in his spectrum at 25 kHz. Bob suspects the amplifier of his microphone to be creating high frequency noise. What are the possible frequencies for this noise?

Problem 2: Spectral Leakage

In 1978 Fredric Harris published an overview of the spectral leakage properties of more than 20 different windows. Included below is a table from that article with the properties of two different windows:

TABLE I
WINDOWS AND FIGURES OF MERIT

WINDOW	HIGHEST SIDE-LOBE LEVEL (dB)	SIDE LOBE FALL-OFF (dB/OCT)	COHERENT GAIN	EQUIV. NOISE BW (BINS)	3.0-dB BW (BINS)	SCALLOP LOSS (dB)	WORST CASE PROCESS LOSS (dB)	6.0-dB BW (BINS)	OVERLAP CORRELATION (PCNT) 75% OL	50% OL
RECTANGLE	-13	-6	1.00	1.00	0.89	3.92	3.92	1.21	75.0	50.0
TRIANGLE	-27	-12	0.50	1.33	1.28	1.82	3.07	1.78	71.9	25.0

In this assignment we want to understand the side lobe fall off in dB per octave (frequency doubling), which are given in the table: -6 for a rectangular window and -12 for the triangular window. For a definition of dB per octave refer to page 30 in the reader.

(a) Look up the Fourier Transform $U(\omega)$ of the square window $u(t)$ with length $T = 1$:

$$u(t) = \begin{cases} 1, & |t| < 1/2 \\ 0, & |t| > 1/2 \end{cases}$$

Sketch the result for $|\omega| < 10\pi$ rad/s.

The side lobes are the maxima of the absolute value of the Fourier Transform of the window (not including the global maximum at $\omega = 0$). This is roughly when the sine is maximum, you can also find a more exact expression by differentiating for instance.

(b) Give the **approximate** expressions for the side lobes' values ω_i and show that the side lobes fall of with $|U(\omega_i)| \propto \frac{1}{\omega_i}$.

(c) Use your answer from b) to show that the side lobe fall off for the rectangular window is indeed about -6 dB/octave (see also the footnote on page 30 in the reader).

The triangular window v(t) is given by the following function:

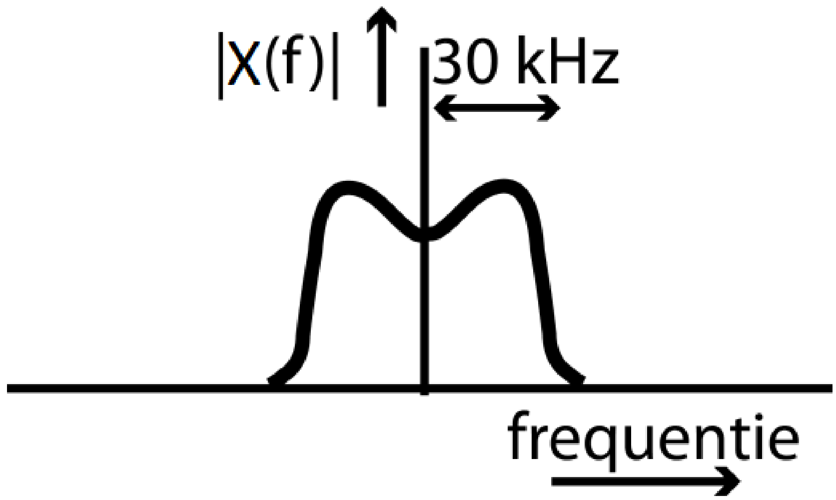
$$v(t) = \begin{cases} 1 - |2t|, & |t| < 1/2 \\ 0, & |t| > 1/2 \end{cases}$$

(d) Show that $v(t) = 2 \cdot \tilde{u}(t) \otimes \tilde{u}(t)$ where $\tilde{u}(t)$ is the rectangular window with $T = 1/2$. *Hint: sketch the functions.* Now use the convolution theorem to determine the Fourier Transform of the triangular window. Do the side lobes fall off at a rate of -12dB/octave?

(e) From d) we see that the triangular window has smaller side lobes than the rectangular window. Even so, the rectangular window is used for a range of measurements, what do you think is the main advantage of using the rectangular window?

Problem 3: Radio: Fourier Transforms and Convolution

Parts from the exam 'Signaalverwerking en Ruis' 2013



The figure above shows the frequency spectrum of an audio signal we wish to send over the radio. In the lecture, as well as in example 2 on page 44 of the reader, we have seen an example of using amplitude modulation (AM) to do so. From the convolution theorem it followed that for the radio signal $u(t)$ (eq 2.7 in the reader):

$$u(t) = x(t)A_1 \cos(\omega_d t)\,, \tag{1}$$

$$U(\omega) = \frac{1}{2\pi} X(\omega) \otimes A_1 \left(\pi(\delta(\omega - \omega_d) + \delta(\omega + \omega_d))\right)\,, \tag{2}$$

where $x(t)$, $X(\omega)$ are the original audio signal and its Fourier transform, and A_1, ω_d are the amplitude and angular frequency of the carrier signal, and of course $\omega = 2\pi f$.

(a) Reproduce/make a quick sketch of $|U(f)|$.

(b) We now wish to listen to the radio, i.e. retrieve the original audio signal from the modulated one. To do so our radio receiver again multiplies the radio signal with a carrier wave of the same frequency (we tune in to the correct radio station):

$$y(t) = u(t)A_2 \cos(\omega_d t)\,.$$

Again using the convolution theorem, give an expression for $Y(\omega)$ and/or make a sketch of the resulting $|Y(f)|$ by using the translation properties of a convolution with the delta function.

(c) What further operation must we perform to retrieve our original audio signal $x(t)$? Show/argue that it will have an amplitude factor $\frac{1}{2}A_1A_2$.

We now wish to save our recovered audio from the radio on a CD *(a 'boomer' storage medium, introduced in 1988 as replacement of gramophone records.)* on which we can save $740 \cdot 10^6$ bytes (1 byte = 8 bits). We sample the audio signal at 44 kHz with 16 bits per sample. To achieve a stereo sound, we record two separate channels. We further filter our signal before we sample.

(d) Give the maximum frequency of the audio that is possible in the recording. Explain why it doesn't matter that we don't record signals above that frequency. (1 point)

(e) Explain why all signals above this maximum frequency have to be filtered before we sample. (1 point)

(f) Calculate how many minutes of radio we can save on the CD. (1 point)