Subband Adaptive Filters

ECE 251C Final Project

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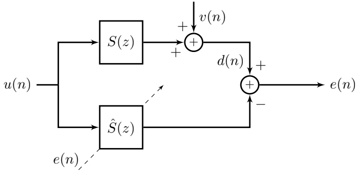
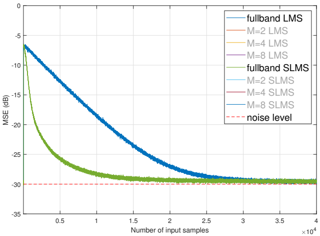
Outline

Background & Motivation



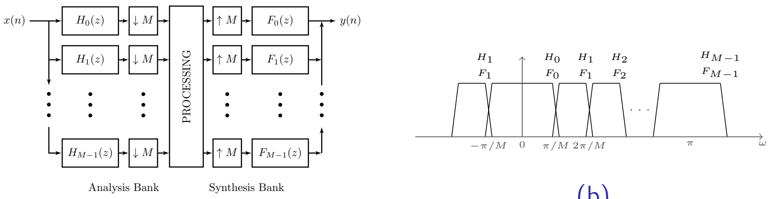
(a) System identification using a fullband adaptive filter.



(b) Different Convergence behaviors.

Figure: Can we improve the convergence behavior by using subband decomposition in adaptive filtering? Why and how?

M-channel Maximally Decimated CMFB



(a) (b)

Figure: M-channel maximally-decimated CMFB.

Types	Properties		Design
Pseudo QMF	real coefficients	near PR (NPR)	PF
PR QMF		PR	PF+QCLS

Table: Two types of cosine modulated filter banks (PF means prototype filter, QCLS means quadratic-constrained least-squares).

M-channel Maximally Decimated Cosine Modulated Pseudo QMF Banks

Given a linear-phase, low-pass PF $P_0(z) = \sum_{n=0}^{N-1} p_0(n) z^{-n}$, the analysis filters $H_k(z)$ and the synthesis filters $F_k(z)$ are obtained by

$$h_k(n) = 2p_0(n) cos\left((2k+1)\frac{\pi}{2M}(n - \frac{N-1}{2}) + (-1)^k \frac{\pi}{4}\right), \tag{1}$$

$$f_k(n) = h_k(N-n) \tag{2}$$

for $0 \leq n \leq N-1$ and $0 \leq k \leq M-1$ where the $h_k(n)$ and $f_k(n)$ do not have linear phase in general. However, the distortion function $T(z)$ has linear phase.

Cosine Modulated PR Systems (1/3)

Theorem for forcing losslessness in causal FIR systems [1]

Let the PF $P_0(z)$ be a linear phase filter with length $N+1=2mM$ for some integer m. Let $G_k(z), 0 \leq k \leq 2M-1$ be the $2M$ polyphase components of $P_0(z)$. Then the $M \times M$ polyphase component matrix $\mathbf{E}(z)$ is paraunitary if and only if $G_k(z)$ satisfy the pairwise power complementary conditions

$$\tilde{G}_k(z)G_k(z)+\tilde{G}_{M+k}(z)G_{M+k}(z)=\alpha, 0 \leq k \leq M-1. \tag{3}$$

for some $\alpha > 0$.

Example [2]:

$$P_0(n)=\begin{cases}\frac{1}{\sqrt{4M}}, & (mM-M \leq k \leq (mM+M-1) \\ 0, & \text{otherwise}\end{cases}. \tag{4}$$

Cosine Modulated PR Systems (2/3)

We aim to minimize the stopband energy and maintain the PR property.

Design a PR CMFB using QCLS [3]

$$\text{minimize } \Phi(\mathbf{p}) \quad \text{subject to} \quad \mathbf{p}^T \mathbf{Q}_{l,n} \mathbf{p} = c_n \tag{5}$$

where

$$\begin{aligned} 0 \leq & n \leq 2m - 2, \\ 0 \leq & l \leq \frac{M}{2} - 1, \end{aligned} \tag{6}$$

and

$$\Phi(\mathbf{p}) = \int_{\omega_s}^{\pi} |P(e^{j\omega})|^2 d\omega. \tag{7}$$

Cosine Modulated PR Systems (3/3)

To derive the matrices $\mathbf{Q}_{l,n}$, firstly note that the polyphase components $G_l(z)$ can be written as

$$G_l(z) = \mathbf{p}^T \mathbf{V}_l \mathbf{e}, \tag{8}$$

where

$$\begin{aligned} \mathbf{p} &= \begin{bmatrix} p(0) & p(1) & \cdots & p(2mM-1) \end{bmatrix}^T \\ \mathbf{e} &= \begin{bmatrix} 1 & z^{-1} & \cdots & z^{-(m-1)} \end{bmatrix}^T \\ [\mathbf{V}_l]_{i,j} &= \begin{cases} 1, & i = l + 2jM \\ 0, & \text{otherwise} \end{cases}, \mathbf{V}_l \in \mathbb{R}^{2mM \times m}. \end{aligned} \tag{9}$$

Then, Eq. 3 can be expressed as

$$\mathbf{p}^T \left[\mathbf{V}_l \mathbf{e}\mathbf{e}^T \mathbf{V}_l^T + \mathbf{V}_{M+l} \mathbf{e}\mathbf{e}^T \mathbf{V}_{M+l}^T \right] \mathbf{p} = \alpha, \tag{10}$$

where we let

$$\mathbf{Q}_{l,n} = \mathbf{V}_l \mathbf{e}\mathbf{e}^T \mathbf{V}_l^T + \mathbf{V}_{M+l} \mathbf{e}\mathbf{e}^T \mathbf{V}_{M+l}^T. \tag{11}$$

Least Mean Square (LMS) Algorithm

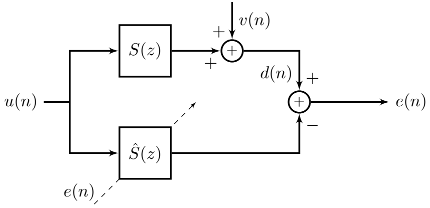


Figure: A system identification problem.

LMS algorithm

$$\hat{\mathbf{s}}(n+1) = \hat{\mathbf{s}}(n) + \mu \mathbf{u}(n) e(n). \tag{12}$$

By exploiting the properties of the input signal $u(n)$ and the target system $S(z)$, the convergence behavior can be improved.

Proportionate Adaptation

Proportionate adaptation for LMS [4]

$$\hat{\mathbf{s}}(n+1) = \hat{\mathbf{s}}(n) + \mu \mathbf{G}(n)\mathbf{u}(n)e(n), \tag{13}$$

where

$$\mathbf{G}(n) = \operatorname{diag}\left(g_0(n), g_1(n), \ldots, g_{L-1}(n)\right) \tag{14}$$

is called the "step-size control matrix" and $g_i(n)$ is a function of the current $\hat{\mathbf{s}}_i(n)$.

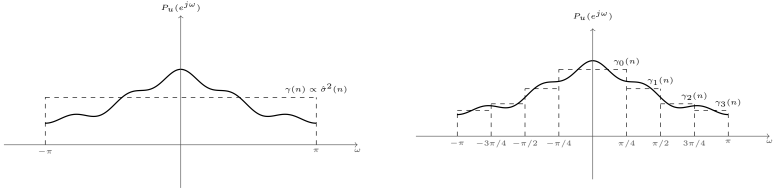
Different algorithms have been proposed to update $\mathbf{G}(n)$ which utilizes the properties of the target system $S(z)$. It does not exploit the properties of the input signal $\mathbf{u}(n)$.

Modified LMS

Modified LMS [5]

$$\hat{\mathbf{s}}(n+1) = \hat{\mathbf{s}}(n) + \frac{\mu}{\hat{\sigma}^2(n) + \delta}\mathbf{u}(n)e(n). \tag{15}$$

This exploits the power of the input signal $u(n)$, the convergence behavior can be slightly enhanced.



(a) The input spectrum is estimated by a scalar.

(b) Is this a better estimate? What does it lead to? Subbands?

Figure: An explanation of the term $\hat{\sigma}^2(n)$ in modified LMS.

Transform-Domain LMS

The basic idea behind transform-domain LMS

$$\mathbf{u}'(n) = \mathbf{A}\mathbf{u}(n) \text{ where } \mathbf{A}\mathbf{A}^T = \mathbf{A}^T\mathbf{A} = \mathbf{I}. \tag{16}$$

Modify the input signal $\mathbf{u}(n)$ to be applied to the adaptive filter such that the conditioning number $\kappa(\mathbf{\Phi}_{\mathbf{u}'})$ of the corresponding correlation matrix $\mathbf{\Phi}_{\mathbf{u}'}$ is smaller than $\kappa(\mathbf{\Phi}_{\mathbf{u}})$.

The best scenario

The elements of $\mathbf{u}'(n)$ are uncorrelated, namely, the matrix $\mathbf{\Phi}_{\mathbf{u}'}$ is diagonal.

Question

Can we find a transformation $\mathbf{A}$ diagonalize $\mathbf{\Phi}_{\mathbf{u}}$?

Notice that analysis filter banks are actually transformations.

Structure of the SAF (1/6)

$$\hat{S}(z) = \hat{S}_0(z^M) + z^{-1}\hat{S}_1(z^M) + \dots + z^{-M+1}\hat{S}_{M-1}(z^M) \qquad (17)$$

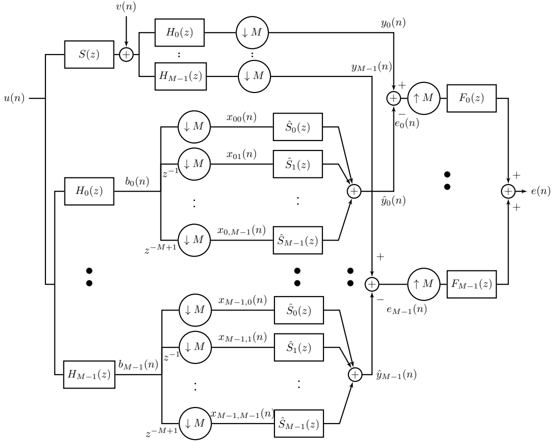


Figure: SAF for M-band case.

Structure of the SAF (2/6)

Define a cost function as

$$J(n) = E[\alpha_0 e_0^2(n) + \alpha_1 e_1^2(n) + \ldots + \alpha_{M-1} e_{M-1}^2(n)] \tag{18}$$

where we use $e_i(n)$ to adapt the coef. of these filters, and α_i are proportional to the inverse of the powers of $b_i, \forall i$.

The gradient-based algorithm for adaptation is given by

$$\hat{s}_{ki}(n+1) = \hat{s}_{ki}(n) - \mu \frac{\partial J}{\partial \hat{s}_{ki}} \tag{19}$$

$$\frac{\partial J}{\partial \hat{s}_{ki}} = \sum_{l=0}^{M-1} 2\alpha_l E(e_l(n) \frac{\partial e_l(n)}{\partial \hat{s}_{ki}}) \tag{20}$$

where $\hat{s}_{ki}$ is the ith coefficients of $\hat{S}_k(z)$, and μ is the step size.

Structure of the SAF (3/6)

Combining the results of Eq.19 and Eq.20, we obtain

$$\hat{s}_{ki}(n+1) = \hat{s}_{ki}(n) + 2\mu \sum_{l=0}^{M-1} \alpha_l E(e_l(n) x_{lk}(n-i)) \tag{21}$$

The LMS adaptation equation are represent by replacing the true gradient by the instantaneous gradient

$$\hat{s}_{ki}(n+1) = \hat{s}_{ki}(n) + 2\mu \sum_{l=0}^{M-1} \alpha_l e_l(n) x_{lk}(n-i) \tag{22}$$

Structure of the SAF (4/6)

From the above figure, we have

$$E_l(z) = Y_l(z) - \sum_{k=0}^{M-1} X_{lk}(z)\hat{S}_k(z) \tag{23}$$

$$Y_l(z) = \sum_{l=0}^{M-1} S_k(z)X_{lk}(z) \tag{24}$$

$$E_l(z) = \sum_{k=0}^{M-1} X_{lk}(z)[S_k(z) - \hat{S}_k(z)] \tag{25}$$

Taking the inverse z transform

$$e_l(n) = \sum_{k=0}^{M-1} \mathbf{x}_{lk}^T(n)\mathbf{v}_k(n) \tag{26}$$

where $\mathbf{x}_{lk}^T(n) = [x_{lk}(n), x_{lk}(n-1), ..., x_{lk}(n-(L/2)+1)]$
, and $\mathbf{v}_k(n) = \mathbf{s}_k - \hat{\mathbf{s}}_k(n)$ is the coefficient error vector.

Combining Eq.22 and Eq.26, the recursive relations for the coefficient error vector can be obtained in vector form as

$$\begin{bmatrix} \mathbf{v}_0(n+1) \\ \mathbf{v}_1(n+1) \\ \dots \\ \mathbf{v}_{M-1}(n+1) \end{bmatrix} = \begin{bmatrix} \mathbf{v}_0(n) \\ \mathbf{v}_1(n) \\ \dots \\ \mathbf{v}_{M-1}(n) \end{bmatrix} - 2\mu \begin{bmatrix} \alpha_0 \mathbf{A}_0(n) & \alpha_1 \mathbf{A}_1(n) & \dots & \alpha_{M-1} \mathbf{A}_{M-1}(n) \end{bmatrix} \begin{bmatrix} \mathbf{v}_0(n) \\ \mathbf{v}_1(n) \\ \dots \\ \mathbf{v}_{M-1}(n) \end{bmatrix} \tag{27}$$

where $\mathbf{A}_k(n) = \begin{bmatrix} \mathbf{x}_{k0}(n)\mathbf{x}_{k0}^T(n) & \mathbf{x}_{k0}(n)\mathbf{x}_{k1}^T(n) & \dots & \mathbf{x}_{k0}(n)\mathbf{x}_{k,M-1}^T(n) \\ \mathbf{x}_{k1}(n)\mathbf{x}_{k0}^T(n) & \mathbf{x}_{k1}(n)\mathbf{x}_{k1}^T(n) & \dots & \mathbf{x}_{k1}(n)\mathbf{x}_{k,M-1}^T(n) \\ \dots & \dots & \dots & \dots \\ \mathbf{x}_{k,M-1}(n)\mathbf{x}_{k0}^T(n) & \mathbf{x}_{k,M-1}(n)\mathbf{x}_{k1}^T(n) & \dots & \mathbf{x}_{k,M-1}(n)\mathbf{x}_{k,M-1}^T(n) \end{bmatrix}$

Structure of the SAF (6/6)

Rewrite the expectation form of Eq.27 as

$$E\begin{bmatrix} \mathbf{v}_0(n+1) \\ \mathbf{v}_1(n+1) \\ \dots \\ \mathbf{v}_{M-1}(n+1) \end{bmatrix} = [\mathbf{I}_L - 2\mu\mathbf{\Phi}]E\begin{bmatrix} \mathbf{v}_0(n) \\ \mathbf{v}_1(n) \\ \dots \\ \mathbf{v}_{M-1}(n) \end{bmatrix} \tag{28}$$

where $\mathbf{\Phi} = \sum_{k=0}^{M-1} \alpha_k E[\mathbf{A}_k(n)]$, and $\mathbf{I}_L$ is the identity matrix of order L.

The matrix $\mathbf{\Phi}$ is positive definite. The mean coefficient error vector converges to zero asymptotically if the step size is chosen according to $0 < \mu < \frac{2}{\lambda_{max}}$, where λ_{max} is the maximum eigenvalue of $\mathbf{\Phi}$.

Convergence Analysis (1/4)

Recall

$$\mathbf{\Phi} = \sum_{k=0}^{M-1} \alpha_k \mathbf{\Phi}_k \tag{29}$$

where $\mathbf{\Phi}_k = E(\begin{bmatrix} \mathbf{x}_{k0}(n) \\ \mathbf{x}_{k1}(n) \\ \dots \\ \mathbf{x}_{k,M-1}(n) \end{bmatrix} \begin{bmatrix} \mathbf{x}_{k0}^T(n) & \mathbf{x}_{k1}^T(n) & \dots & \mathbf{x}_{k,M-1}^T(n) \end{bmatrix})$

is a correlation matrix with elements $r_k(.)$.

$$r_k(m) = E[b_k(n) b_k(n+m)] \tag{30}$$

Convergence Analysis (2/4)

The power spectrum of $u(n)$ is shown in the following figure

$$P_u(e^{j\omega}) = \sum_{k=0}^{M-1} \gamma_k (P_0(e^{j(\omega-\omega_k)}) + P_0(e^{j(\omega+\omega_k)})) \tag{31}$$

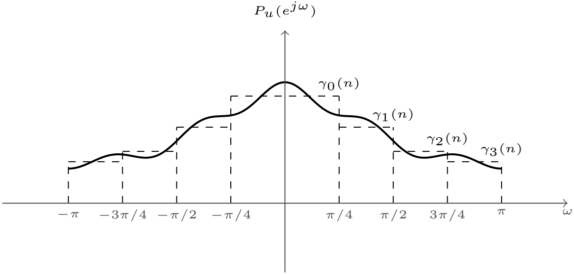


Figure: Assume that the input spectrum is piece-wise flat

Convergence Analysis (3/4)

The power spectrum of $b_k(n)$ is given by

$$\begin{aligned} P_{b_k}(e^{j\omega}) &= P_u(e^{j\omega})|H_k(e^{j\omega})|^2 \\ &= P_u(e^{j\omega})|P_0(e^{j(\omega-\omega_k})|^2 + P_u(e^{j\omega})|P_0(e^{j(\omega+\omega_k})|^2 \\ &= \gamma_k[P_0(e^{j(\omega-\omega_k)}) + P_0(e^{j(\omega+\omega_k)})] \end{aligned} \tag{32}$$

The inverse FT is

$$r_k(n) = 2\gamma_k p_0(n) cos(\omega_k n) \tag{33}$$

By designing $\alpha_k = c/\gamma_k$, where c is a constant, then

$$r(n) = \sum_{k=0}^{M-1} \alpha_k r_k(n) = 2cp_0(n) \sum_{k=0}^{M-1} cos(\omega_k n). \tag{34}$$

We can express $\mathbf{\Phi}$ as a matrix with diagonal elements $r(0)$.

Convergence Analysis (4/4)

Because of the nature of $P_0(e^{j\omega})$, its inverse FT is a sinc function, $r(n)$ is nonzero only for $n = 0$. That is, $\mathbf{\Phi} = r(0)\mathbf{I}$. The eigenvalue spread of $\mathbf{\Phi}$ is unity.

- The convergence rate of LMS algorithm is fastest under this condition.
- As we increasing the number of bands M, it tends to a scalar multiple of $\mathbf{I}$, provided the analysis filters are of better quality.

Experiment	I	II
Property of target system $S(z)$	Quasi-Sparse	
Length of $S(z)$, L	80	
Input signal $y(n)$	White	Color (first-order AR process)
System noise $N(n)$	White Gaussian noise (WGN)	
System noise level	-30 dB	
Adaptation	LMS and SLMS ($p = 1.5$) [6]	
Monte Carlo Runs	2000	

Table: Experimental setup for the SAF using fullband, $M = 2$, $M = 4$ and $M = 8$.

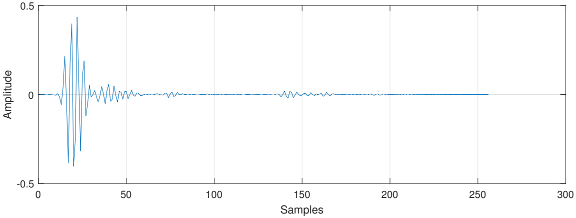


Figure: The target system $\mathbf{s}$.

Simulation Results & Comparison (2/5)

Experiment I

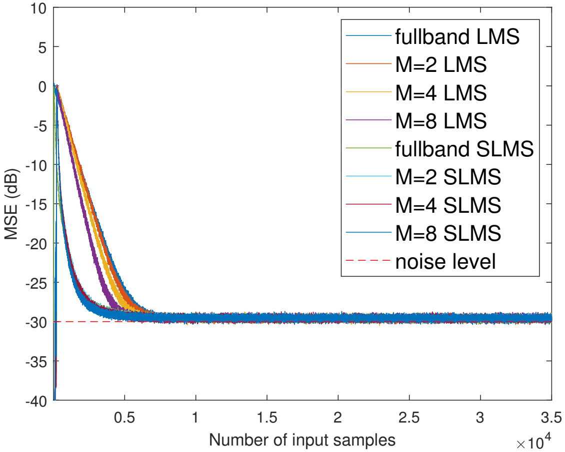


Figure: Convergence behavior for experiment I.

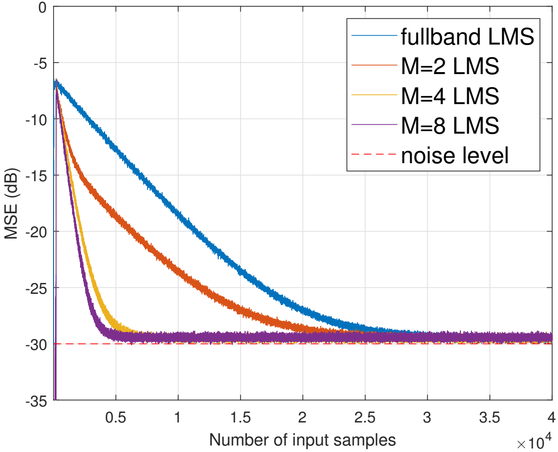


Figure: Convergence behavior for LMS adaptation with fullband, $M = 2$, $M = 4$ and $M = 8$ (the first part of experiment II).

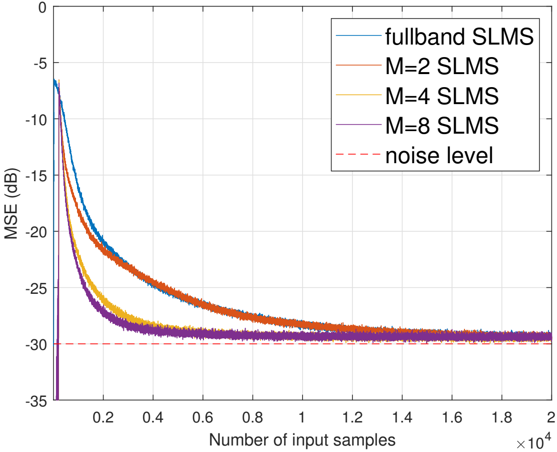


Figure: Convergence behavior for SLMS adaptation with fullband, $M = 2$, $M = 4$ and $M = 8$ (the second part of experiment II).

Simulation Results & Comparison (5/5)

Experiment II

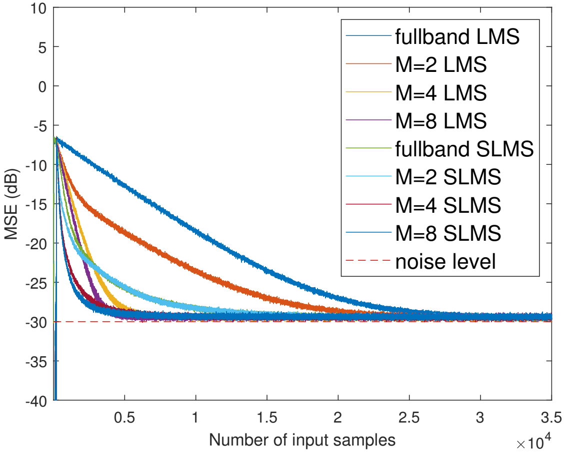


Figure: Convergence behavior for experiment II (overall).

A Real-World Problem (1/2)

Adaptive Feedback Cancellation (AFC)

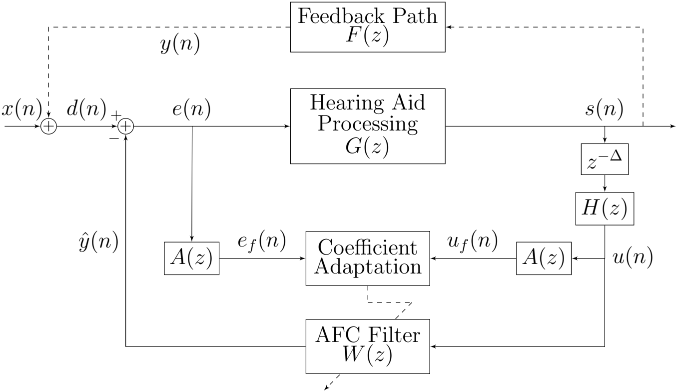


Figure: Block diagram of the AFC framework [7]. The pre-filter $A(z)$ is utilized to whiten (flatten) the spectra of the input signals in order to reduce the correlation. Can we get rid of the $A(z)$ by using SAF and achieve higher maximum stable gain (MSG)?

A Real-World Problem (2/2)

Adaptive Feedback Cancellation (AFC)

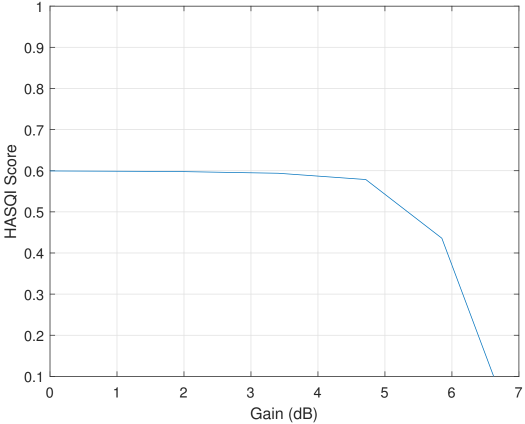


Figure: Finding the MSG by plotting the HASQI score v.s. gain [7]. We used a $M = 8$ SAF (LMS adaptation) with $L = 256$.

Conclusion & Problems & Future Work

Conclusion: SAF

- decorrelates the input signal; the conditioning number is smaller; faster convergence.
- is just a transform-domain LMS.
- can be incorporated with sparsity-promoting LMS algorithm even though the length of SAF is shorter than the original one.

Problems

- aliasing happens if the transformed coefficients are modified
- the predefined basis (analysis bank) may not diagonalize the correlation matrix
- how to find the eigenvectors of the correlation matrix?

Future Work

- using non-maximally decimated FB to deal with the aliasing
- apply Gram Schmidt to find the basis ?

References

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