Exponential Distribution compare to Central limit Theorem. A simulation exercise

I D

```
# set seed for reproducability, set lambda to 0.2, 40 samples, 1000 simulations
set.seed(31)
lambda <- 0.2
n <- 40
simulations <- 1000
```

```
# simulate
simulated_exponentials <- replicate(simulations, rexp(n, lambda))
simExp = function(n, lambda){
    mean(rexp(n,lambda))    }
simul = data.frame(ncol=2,nrow=simulations)
names(simul) = c("Sample","Mean")
for (i in 1:simulations)
{    simul[i,1] = i
    simul[i,2] = simExp(n,lambda)}
```

```
# calculate mean of exponentials
means_exponentials <- apply(simulated_exponentials, 2, mean)
```

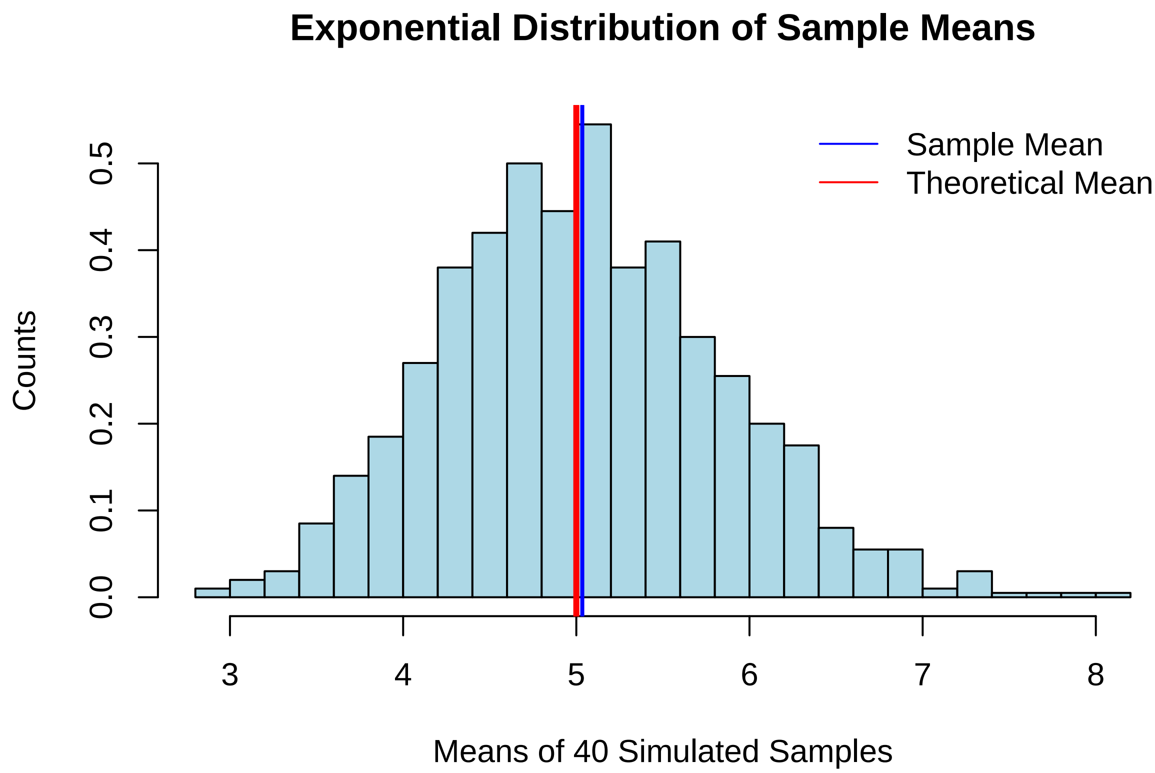
```
#Question 1 Show where the distribution is centered at and compare it to the theoretical center of the
analytical_mean <- mean(means_exponentials)
analytical_mean
```

```
## [1] 4.993867
```

```
# analytical mean
theory_mean <- 1/lambda
theory_mean
```

```
## [1] 5
```

```
# visualization
meanSample = mean(simul$Mean)
meanTheory = 1/lambda
hist(simul$Mean, breaks = 30, prob = TRUE,col = "lightblue",
     main="Exponential Distribution of Sample Means",
     xlab="Means of 40 Simulated Samples", ylab = "Counts")
abline(v = meanTheory, col= "red", lwd = 3)
abline(v = meanSample, col = "blue",lwd = 2)
legend('topright', c("Sample Mean", "Theoretical Mean"),
       bty = "n",    lty = c(1,1), col = c(col = "blue", col = "red"))
```



The analytics mean is 5.006 the theoretical mean 5. The center of distribution of averages of 40 exponentials is very close to the theoretical center of the distribution. # Question 2 Show how variable it is and compare it to the theoretical variance of the distribution..

```
# standard deviation of distribution
standard_deviation_dist <- sd(means_exponentials)
standard_deviation_dist
```

```
## [1] 0.7931608
```

```
# standard deviation from analytical expression
standard_deviation_theory <- (1/lambda)/sqrt(n)
standard_deviation_theory
```

```
## [1] 0.7905694
```

```
# variance of distribution
variance_dist <- standard_deviation_dist^2
variance_dist
```

```
## [1] 0.6291041
```

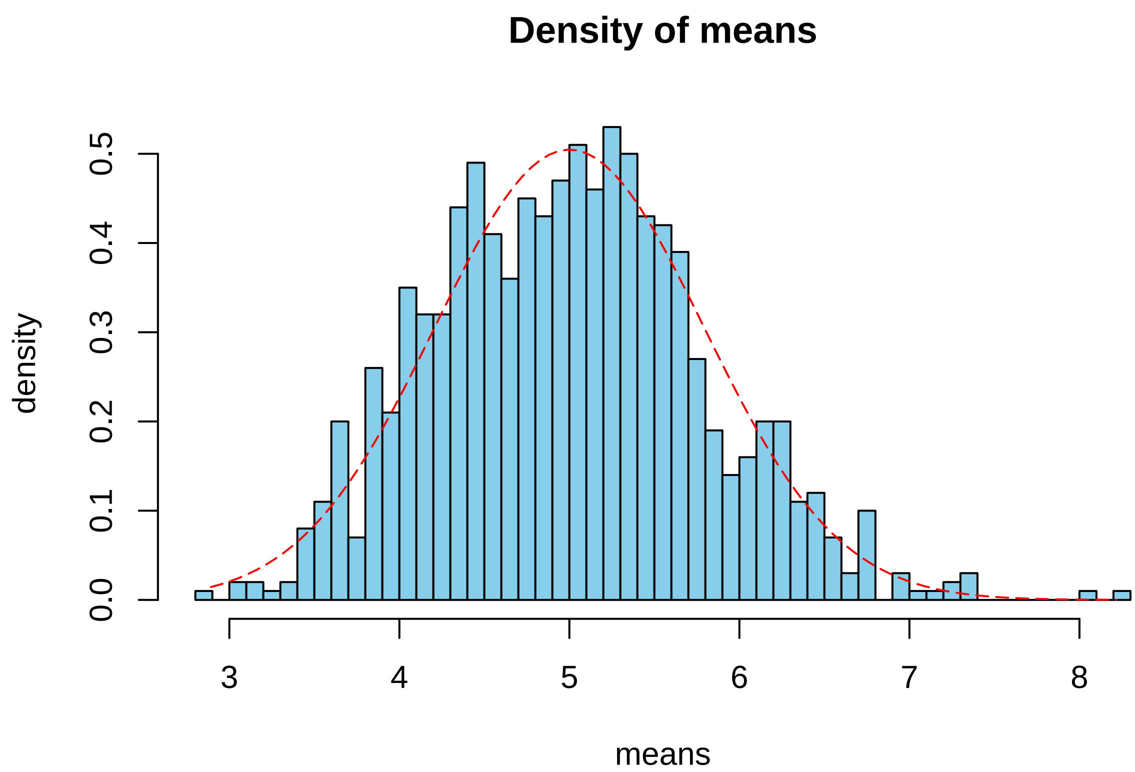
```
# variance from analytical expression
variance_theory <- ((1/lambda)*(1/sqrt(n)))^2
variance_theory
```

```
## [1] 0.625
```

Standard Deviation of the distribution is 0.793 with the theoretical SD calculated as 0.7905. The Theoretical variance is calculated as ((1 / ??) * (1/???n))2 = 0.625. The actual variance of the distribution is 0.629

Question 3

```
# Show that the distribution is approximately normal.
xfit <- seq(min(means_exponentials), max(means_exponentials), length=100)
yfit <- dnorm(xfit, mean=1/lambda, sd=(1/lambda/sqrt(n)))
hist(means_exponentials,breaks=n,prob=T,col="skyblue",xlab = "means",main="Density of means",ylab="dens
lines(xfit, yfit, pch=22, col="red", lty=5)
```



Due to Due to the central limit theorem (CLT), the distribution of averages of 40 exponentials is very close to a normal distribution.