

### Lesson Summary

When you're trying to visualize how a pattern will grow, it helps to have a toolbox of different strategies.

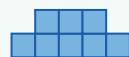
Here are the first three figures in a pattern and a table showing the number of tiles at each stage. You could determine how many tiles there will be in Figure 7 by:

- Drawing the next four figures and adding a row of 3 each time.
- Continuing the table, increasing the number of tiles by 3 in each new row.
- Noticing that the number of tiles is 3 times the figure number, plus 2, and then calculating the tiles for Figure 7.

**Figure 1**



**Figure 2**



**Figure 3**

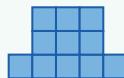


Figure	Number of Tiles
1	5
2	8
3	11

### Things to Remember:

# Lesson Practice

A1.1.01

Name: ..... Date: ..... Period: .....

Problems 1–3: Here is a visual pattern.

Figure 1

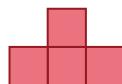


Figure 2

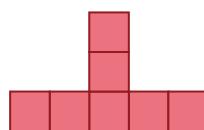
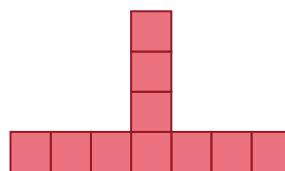
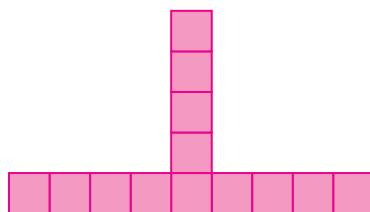


Figure 3



1. Draw Figure 4.



2. Complete the table with the number of tiles in each figure.

Figure	1	2	3	4	...	10
Number of Tiles	4	7	10	13	...	31

3. Which expression represents the number of tiles in Figure 11?

- A.  $1 + 3(11)$   
B.  $1 + 4(11)$   
C.  $4 + 3(11)$   
D.  $4 + 1(11)$

For Problems 4–5: Create your own pattern.

4. Draw Figures 1 and 3. Then complete the table. *Patterns vary.*

Figure 1



Figure 2

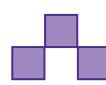


Figure 3

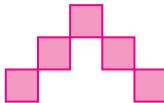


Figure	Number of Tiles
1	1
2	3
3	5

5. How many tiles will Figure 6 of your pattern have?

*Responses vary depending on the patterns students create.*

# Lesson Practice

## A1.1.01

Name: ..... Date: ..... Period: .....

### Spiral Review

6. Create two different patterns that begin with the numbers 5 and then 10.

*Patterns vary.*

**Pattern A:** 5, 10, **15**, **20**, **25**

**Pattern B:** 5, 10, **20**, **40**, **80**

Describe how each pattern is changing.

*Responses vary depending on the patterns students create.*

**Pattern A:** Add 5 each time.

**Pattern B:** Double the previous number.

7. Create two different patterns that each include the numbers 2 and 12.

*Patterns vary.*

**Pattern C:** **2, 4, 6, 8, 10, 12**

**Pattern D:** **2, 12, 22, 32, 42**

8. Determine the value of each expression.

Expression	Value
$4 + 4 + 4 + 4$	<b>16</b>
$4 \cdot 4 \cdot 4$	<b>64</b>
$4(3)$	<b>12</b>
$3^4$	<b>81</b>

9. Select *all* the expressions that are equal to 16.

A.  $8^2$

B.  $2^4$

C.  $2^8$

D.  $4^2$

E.  $16^1$

### Reflection

- Put a smiley face next to a problem you were stuck on and then figured out.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

A **sequence** is a list of numbers in a particular order.

Sequences can change in predictable ways. Two examples of predictable change are constant differences and constant ratios.

A sequence has a **constant difference** when the difference between any two consecutive terms is the same.

$$\begin{array}{cccc} +6 & +6 & +6 \\ \curvearrowright & \curvearrowright & \curvearrowright \\ 5, & 11, & 17, & 23 \end{array}$$

A sequence has a **constant ratio** when the ratio between any two consecutive terms is the same.

$$\begin{array}{cccc} \times 4 & \times 4 & \times 4 \\ \curvearrowright & \curvearrowright & \curvearrowright \\ 2, & 8, & 32, & 128 \end{array}$$

Sequences can also change in predictable ways that are neither a constant difference or ratio.

$$\begin{array}{cccc} -1 & -2 & -3 \\ \curvearrowright & \curvearrowright & \curvearrowright \\ 13, & 12, & 10, & 7 \end{array}$$

Once you know how a sequence changes, you can use that change to determine unknown terms.

Here is an example:

- The known terms in this sequence have a constant ratio of 0.5.
- The value of the unknown term should be 0.5 times the term before, 40, or 2 times the term after, 10.
- That means the unknown term is 20.

$$\begin{array}{cccc} \times 0.5 & & \times 0.5 \\ \curvearrowright & & \curvearrowright \\ 80, & 40, & ?, & 10, & 5 \end{array}$$

## Things to Remember:

# Lesson Practice

## A1.1.02

Name: ..... Date: ..... Period: .....

- 1.** Determine the *constant ratio* for each sequence.

Sequence	Constant Ratio
256, 128, 64	0.5 or $\frac{1}{2}$
18, 54, 162	3
0.8, 0.08, 0.008	0.1 or $\frac{1}{10}$

- 2.** Determine the *constant difference* for each sequence.

Sequence	Constant Difference
12, 17, 22	5
102, 85, 68	-17
$\frac{1}{4}, 1, \frac{7}{4}$	0.75 or $\frac{3}{4}$

- 3.** Here's the start of a sequence: 1, -1, ...

- (a) Write two different rules the sequence could follow.

Rule 1: Constant ratio of -1

Rule 2: Constant difference of -2

- (b) Determine the next three terms for each rule.

Rule 1: 1, -1, **-1**, **-1**, **-1**

Rule 2: 1, -1, **-3**, **-5**, **-7**

**Problems 4–7:** Fill in the blanks to complete each sequence. Each sequence has a constant difference.

**4.** -3, -2, **-1**, **0**, 1

**5.** **1**, 13, 25, **37**, **49**

**6.** 1, 0.25, **-0.5**, -1.25, **-2**

**7.** 92, **89**, **86**, **83**, 80

**Problems 8–9:** Here are Sequence A and Sequence B. One sequence has a constant difference and one has a constant ratio.

- 8.** Complete the table.

Sequence A	70, 90, 110, ...	Constant ratio or Constant difference	Fourth term: <b>130</b>
Sequence B	10, 20, 40, ...	Constant ratio or Constant difference	Fourth term: <b>80</b>

- 9.** Which sequence will have the greater 10th term? Show or explain your thinking.

**Sequence B.** Explanations vary. Sequence B will have a greater 10th term. Even though the beginning terms in Sequence B are less than the terms in Sequence A, they increase at a faster rate.

## Lesson Practice

### A1.1.02

Name: ..... Date: ..... Period: .....

10. A sequence has a first term of 6 and a constant ratio of -2. What are the first four terms of the sequence?

A. 6, 4, 2, 0

B. -2, 4, 10, 16

C. 6, 8, 10, 12

D. 6, -12, 24, -48

## Spiral Review

11. Square  $A$  has an area of 64 square feet. Select *all* the expressions that are equal to the side length of this square in feet.

A.  $\sqrt{8}$

B.  $\sqrt{64}$

C. 4

D. 8

E.  $\frac{64}{2}$

12. The points (7, 21) and (-5, 17) lie on a line. What is the slope of the line?

$\frac{4}{12}$  (or equivalent)

13. Determine the value of each expression when  $n = 4$ .

Expression	$n^2 - 5$	$n(n + 6)$	$3n^2$
Value When $n = 4$	11	40	48

## Reflection

- Put a heart next to the problem you found most interesting.
- Use this space to ask a question or share something you're proud of.

### Lesson Summary

There are several ways to define, or describe, a sequence. When you define a sequence recursively, you're determining each term using the previous term.

A **recursive definition** of a sequence includes a first term and a rule for finding every term that follows.

Here are some examples of recursive definitions for this sequence: 32, 16, 8, 4, 2, 1, 0.5.

**First term:** 32

**Rule:** Half of the previous term

**First term:** 32

**Rule:** Constant ratio of  $\frac{1}{2}$

**First term:** 32

**Rule:** Multiply the previous term by 0.5

### Things to Remember:

# Lesson Practice

## A1.1.03

Name: ..... Date: ..... Period: .....

**Problems 1–2:** Use the sequence  $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$

1. Write a recursive definition for this sequence.

First term:  $\frac{1}{2}$

Rule: Constant ratio of  $\frac{1}{2}$   
(or equivalent)

2. Write the next three terms of the sequence.

$\frac{1}{16}, \frac{1}{32}, \frac{1}{64}$

3. Complete the recursive definition for each sequence.

	Sequence A	Sequence B	Sequence C	Sequence D
Sequence	2, 4, 6, 8, ...	5, 7, 9, 11, ...	50, 25, 0, -25, ...	$\frac{1}{3}, 1, 3, 9, \dots$
First Term	2	5	50	$\frac{1}{3}$
Constant Difference or Constant Ratio?	Constant difference	Constant difference	Constant difference	Constant ratio
Rule	2	2	-25	3

4. This sequence has a constant difference of 5. Fill in the missing terms.

-3, 2, 7, 12, 17

**Problems 5–6:** Here is the start of a sequence: 1, 5, ...

5. Write a rule and the next three terms the sequence could follow.

Rule: Constant difference of 4

Terms: 9, 13, 17

6. Write a *different* rule and the next three terms the sequence could follow.

Rule: Constant ratio of 5

Terms: 25, 125, 625

7. Remy is studying this sequence: 20, ?, 80. Remy thinks the missing term could be less than the first and third terms. Is Remy correct? Explain your thinking.

Yes. Explanations vary. The sequence could have a constant ratio of -2, which would make the second term -40.

# Lesson Practice

A1.1.03

Name: ..... Date: ..... Period: .....

8. A swimming pool is filling with water at a rate of 17 gallons per minute. The pool started with 100 gallons of water. Write a recursive definition to model the situation.

First term: **100**

Rule: **Constant difference of 17**

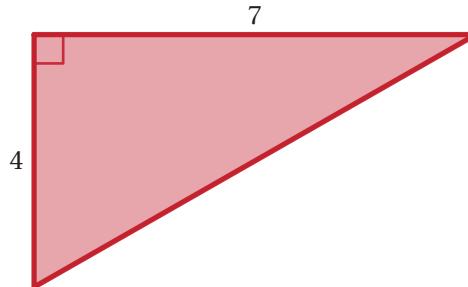
## Spiral Review

9. Explain how you know that  $\sqrt{42}$  is between 6 and 7.

**Responses vary.**  $\sqrt{36} = 6$  and  $\sqrt{49} = 7$ .  $\sqrt{42}$  is between 6 and 7 because 42 is between 36 and 49.

10. Determine the exact length of the unlabeled side.

**$\sqrt{65}$**



11. Write each expression as a single power of 10.

Expression	$\frac{10^7}{10^3}$	$10^2 \cdot 10^5$	$10^0 \cdot 10^9$	$\frac{10^5}{10^0}$
Single Power of 10	<b><math>10^4</math></b>	<b><math>10^7</math></b>	<b><math>10^9</math></b>	<b><math>10^5</math></b>

## Reflection

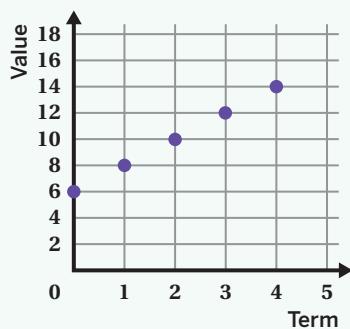
- Put a star next to a problem you could explain to a classmate.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

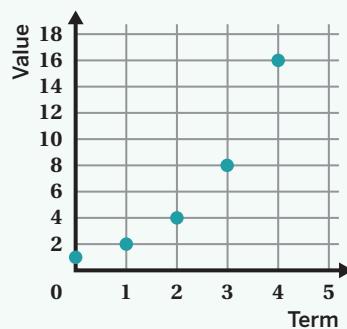
Sequences that change by a constant difference are called **arithmetic sequences**, while sequences that change by a constant ratio are called **geometric sequences**.

Sequences can be represented in multiple ways: as a list of numbers, in a table, on a graph, or with a recursive definition. In each representation, there are ways to identify if the sequence is arithmetic, geometric, or neither. Here are some examples of graphs of different kinds of sequences.

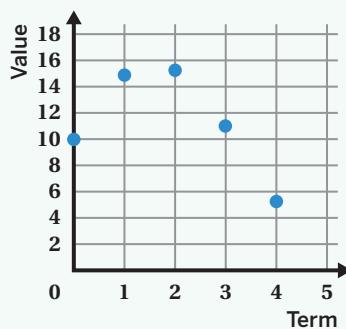
Arithmetic



Geometric



Neither



## Things to Remember:

# Lesson Practice

## A1.1.04

Name: ..... Date: ..... Period: .....

1. Determine whether each sequence is arithmetic, geometric, or neither.

Sequence	1000, 200, 40, 8	2, 4, 16, 256	10, 20, 30, 40	500, 100, 20, 4
Arithmetic, Geometric, or Neither	Geometric	Neither	Arithmetic	Geometric

2. Complete each arithmetic sequence with its missing terms.

$$-2, 4, \underline{10}, 16, \underline{22}$$

$$11, 111, \underline{211}, \underline{311}, 411$$

$$\underline{5}, 7.5, 10, \underline{12.5}, \underline{15}$$

$$5, \underline{-4}, -13, -22, \underline{-31}$$

3. Complete each geometric sequence with its missing terms.

$$\underline{1}, 5, 25, \underline{125}, 625$$

$$-1, \underline{6}, -36, 216, \underline{-1296}$$

$$10, 5, \underline{2.5}, \underline{1.25}, 0.625$$

$$\underline{-4}, \underline{12}, -36, 108, \underline{-324}$$

**Problems 4–6:** Here are the graphs of two sequences.

4. For Sequence A, describe a way to produce a new term from the previous term.

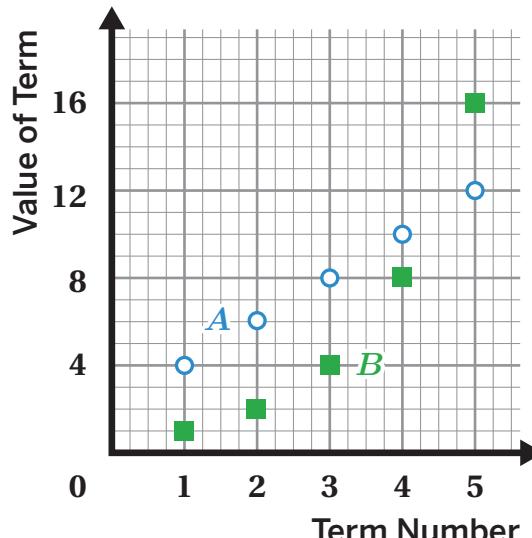
*Responses vary. Add 2 to the previous term.*

5. For Sequence B, describe a way to produce a new term from the previous term.

*Responses vary. Multiply the previous term by 2.*

6. Which of these is a geometric sequence? Explain your thinking.

*Sequence B. Explanations vary. Sequence B is a geometric sequence because it has the same multiplier from one term to the next: 2.*



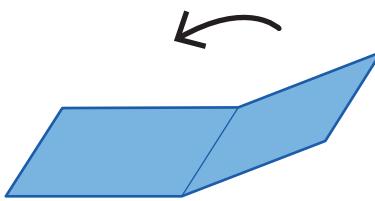
# Lesson Practice

## A1.1.04

Name: ..... Date: ..... Period: .....

7. Ahmed is trying to see how many times he can fold a piece of paper in half. The thickness of the piece of paper he starts with is 0.001 centimeters. How thick will the paper be after 10 folds?

**1.024 centimeters**



## Spiral Review

8. Select *all* the sets of side lengths that form a right triangle.

A. 2, 3, 5

B.  $\sqrt{7}, 9, \sqrt{88}$

C.  $\sqrt{12}, 6, \sqrt{48}$

D. 4, 5,  $\sqrt{41}$

E. 4, 5, 9

9. If a line has a negative slope and contains the point (4, 6), which of these points could it also contain?

A. (7, 6)

B. (3, 1)

C. (6, 5)

D. (5, 8)

10. Evaluate each expression.

Expression	$\left(\frac{1}{3}\right)^2$	$\left(\frac{1}{3}\right)^{-2}$
Value	$\frac{1}{9}$	9

## Reflection

1. Circle a problem you're still curious about.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

An **explicit definition** is a formula that determines the value of any term in a sequence using the term number. An explicit definition of a sequence can be written as an expression or in words.

Here are two situations and the explicit expressions that define them.

**Situation:** Julian's family is 250 miles away from home. They drive 50 miles toward their home for every  $n$  hours.

**Expression:**  $250 - 50n$

The expression represents the number of miles Julian's family is from home after  $n$  hours.

- 250 represents the starting distance.
- -50 represents the distance traveled each hour.
- $-50n$  represents the total distance traveled.
- $n$  represents the number of hours driven.

**Situation:** A scientist tracks a group of 250 bugs. Each month, the number of bugs increases by 1.5 times, for  $n$  months.

**Expression:**  $250 \cdot 1.5^n$

The expression represents the number of bugs after  $n$  months.

- 250 represents the initial number of bugs.
- 1.5 represents the ratio that the bugs grow by each month.
- $n$  represents the number of months.

**Things to Remember:**

# Lesson Practice

## A1.1.05

Name: ..... Date: ..... Period: .....

1. Jamar had 80 followers on social media. His number of followers *tripled* every month for 4 months. Select *all* the expressions that represent Jamar's followers after 4 months.

- A.  $80 \cdot 3 \cdot 3 \cdot 3 \cdot 3$   
 B.  $80 + 4^3$   
 C.  $80 \cdot 4 \cdot 4 \cdot 4$   
 D.  $80 + 3 + 3 + 3 + 3$   
 E.  $80 \cdot 3^4$

2. The weather forecast predicts a snowfall rate of 0.75 inches of snow per hour overnight. There are already 9 inches of snow on the ground in Anushka's neighborhood.

Complete the table to show the amount of snow on the ground, in inches, after  $n$  hours.

Hours, $n$	0	1	2	3	...	8
Amount of Snow (in.)	9	9.75	10.5	11.25	...	15

3. The population of a city was 100,000 in 1970 and has doubled 3 times since then. Select *all* the expressions that represent the population today.

- A. 300000  
 B. 800000  
 C.  $100000 \cdot 2 \cdot 2 \cdot 2$   
 D.  $100000 \cdot 3^2$   
 E.  $100000 \cdot 2^3$

**Problems 4–5:** Here is a table representing a pattern.

4. Circle the equation that represents the table.

- A.  $y = 60 + \frac{1}{2}x$   
 B.  $y = 60 \cdot \left(\frac{1}{2}\right)^x$   
C.  $y = 60 \cdot 2^x$   
D.  $y = 60 - \frac{1}{2}x$

x	y
0	60
1	30
2	15
3	7.5
4	3.75

5. Explain your thinking.

*Explanations vary. The starting value is 60 and the common ratio is  $\frac{1}{2}$ .*

## Lesson Practice

A1.1.05

Name: ..... Date: ..... Period: .....

**Problems 6–7:** Maia earns money by shoveling snow in her neighborhood. She starts the winter with \$52 in her bank account and will deposit \$10 for every driveway she shovels.

6. Complete the table with Maia's account balance, in dollars, after she shovels  $n$  driveways.

Number of Driveways, $n$	0	1	2	3
Account Balance (\$)	52	62	72	82

7. Write an explicit expression for this situation.

$52 + 10n$

**Problems 8–9:** A group of students tracked the number of likes on a social media post. They represented this number using the equation  $n = 40 \cdot 1.5^t$ , where  $n$  is the total number of likes and  $t$  is the number of days since the students started counting.

8. Explain what the 40 and 1.5 mean in this situation.

40 means that there were 40 likes on the social media post when the students started counting. 1.5 means that the number of likes on the post grew by 1.5 times every day.

9. How many likes do the students predict there will be 2 days after they started counting?

90 likes

## Spiral Review

**Problems 10–11:** Write each expression using an exponent.

10.  $\left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right)$   
 $\left(\frac{4}{5}\right)^3$

11.  $9.3 \cdot 9.3 \cdot 9.3 \cdot 9.3 \cdot 9.3$   
 $9.3^5$

12. An arithmetic sequence starts with 10, 5, ... .

Explain how you would calculate the value of the 100th term.

Responses vary. Starting with the first term, I can add the constant difference multiplied by 99 to get the 100th term. In this example, the first term is 10 and the constant difference is -5, so the 100th term would be  $10 + (-5)(99)$  or -485.

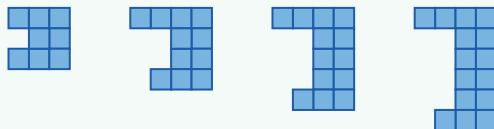
## Reflection

- Put a star next to a problem you spent the most time on.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can write an explicit expression for a pattern or sequence in multiple equivalent ways, just by referencing different term numbers.

Here's an example of a pattern and its matching sequence.

**Figure 1    Figure 2    Figure 3    Figure 4**

$$7 + 2(1) \quad 7 + 2(2) \quad 7 + 2(3) \quad 7 + 2(4)$$

**Sequence**

9, 11, 13, 15  
Term 1

$$9 + 2(n - 1)$$

One expression that represents this sequence is  $9 + 2(n - 1)$ ; this expression uses Term 1 as the starting value and  $n - 1$  to calculate the change.

The expression  $7 + 2n$  also represents this sequence, using Term 0 as the starting value, and  $n$  to calculate the change.

**Things to Remember:**

# Lesson Practice

## A1.1.06

Name: ..... Date: ..... Period: .....

1. Match each sequence to its explicit expression.

### Sequence

a. 4, 10, 16, 22

b. 4, 12, 36, 108

c. 160, 80, 40, 20

d. 320, 320.5, 321, 321.5

### Explicit Expression

**c.**  $320 \cdot \left(\frac{1}{2}\right)^n$

**a.**  $4 + 6(n - 1)$

**d.**  $320 + \frac{1}{2}(n - 1)$

**b.**  $4 \cdot 3^{(n - 1)}$

2. Select *all* the expressions that could represent the number of tiles in Figure  $n$  of this pattern.

A.  $5 + n$

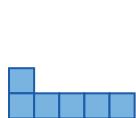


Figure 1



Figure 2

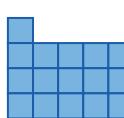


Figure 3

B.  $5n + 1$

C.  $6n - 1$

D.  $6 + 5(n - 1)$

E.  $10n - 4$

Problems 3–4: Here is a visual pattern.

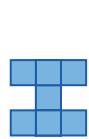


Figure 1

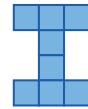


Figure 2

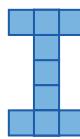


Figure 3

3. Complete the table with the number of tiles in each figure.

4. Write an explicit expression for the number of tiles in Figure  $n$ .

- 6 +  $n$  (or equivalent)
- 7 +  $(n - 1)$  (or equivalent)

Figure	# of Tiles
1	7
2	8
3	9
4	10
...	...
15	21

5. The first four terms of a geometric sequence are: 6, 18, 54, 162. Which explicit expression can be used to describe this sequence?

A.  $2 + 3n$

B.  $3^n$

**C.**  $2 \cdot 3^n$

D.  $6 \cdot 3^n$

## Lesson Practice

### A1.1.06

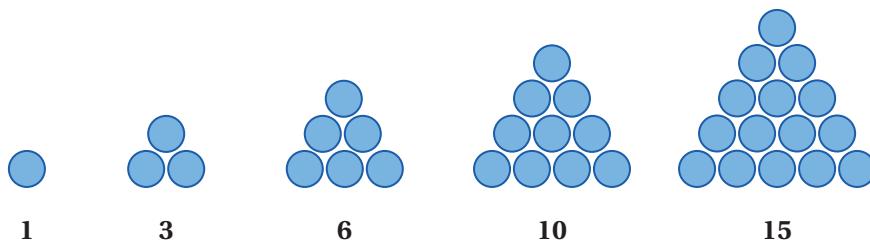
Name: ..... Date: ..... Period: .....

6. The first four terms in a sequence are 15, 22, 29, 36. Write two explicit expressions for Term  $n$  of this sequence.

Expression 1: **15 + 7( $n - 1$ ) (or equivalent)**

Expression 2: **8 + 7 $n$  (or equivalent)**

7. A *triangular number* can be represented as the number of dots in an equilateral triangle.



What is the 8th triangular number?

**36**

## Spiral Review

**Problems 8–10:** Determine the value of each expression when  $x = 3$ .

8.  $7x - 8$

**13**

9.  $5(x + 9)$

**60**

10.  $\frac{x - 6}{4x}$

**$\frac{1}{4}$  (or equivalent)**

**Problems 11–13:** Decide whether each sequence is arithmetic, geometric, or neither.

Circle one.

11. 25, 5, 1, ...

Arithmetic

Geometric

Neither

12. 25, 19, 13, ...

Arithmetic

Geometric

Neither

13. 25, 52, 25, 52, ...

Arithmetic

Geometric

Neither

## Reflection

- Put a heart next to the problem you feel most confident about.
- Use this space to ask a question or share something you're proud of.

### Lesson Summary

You can solve one-variable equations by creating *equivalent equations*. To create equivalent equations, use moves that keep the equation balanced, such as combining like terms or using inverse operations to move a variable from one side of the equation to the other.

Here is an example of a set of moves that keep an equation balanced:

$$-3m + 5 + m = 2(6m + 3)$$

This is the original equation.

$$-2m + 5 = 12m + 6$$

Combine like terms on the left and distribute on the right.

$$5 = 14m + 6$$

Add  $2m$  to each side of the equation.

$$-1 = 14m$$

Subtract 6 from each side of the equation.

$$\frac{-1}{14} = m$$

Divide each side of the equation by 14.

All of the equations created at each step of this solution process are *equivalent equations* because they have the same solution:  $m = -\frac{1}{14}$ .

### Things to Remember:

# Lesson Practice

A1.2.02

Name: ..... Date: ..... Period: .....

1. Which equation is equivalent to  $6x + 9 = 12$ ?

- A.  $x + 9 = 6$   
**B.**  $2x + 3 = 4$   
C.  $3x + 9 = 6$   
D.  $6x + 12 = 9$

2. Write another equation that is equivalent to  $6x + 9 = 12$ .

*Responses vary.*  $6x = 3$

3. Select all the equations that are equivalent to  $\frac{-8x - 6}{2} = 15$ .

- A.  $4x + 3 = 15$        B.  $\frac{1}{2}(-4x - 3) = 15$   
 C.  $-4x - 3 = 15$        D.  $-8x - 6 = 30$   
 E.  $8x + 6 = 30$

**Problems 4–6:** Solve each equation.

4.  $26 - 2x = 3(x + 2)$

*x = 4*

5.  $\frac{4x - 6}{2} = x - 8$

*x = -5*

6.  $\frac{1}{4}x - 5 = x - 14$

*x = 12*

**Problems 7–9:** Paz made a mistake when solving  $-3(x + 7) = 24$  for  $x$ .

7. What is one thing that Paz did well?

*Responses vary.*

- She subtracted 7 from both sides to make  $7 - 7 = 0$ .
- She tried to get rid of -3 as a first step.
- She organized her work clearly.

8. What is one thing that Paz did incorrectly?

*Responses vary.* She added 3 to both sides instead of dividing by -3.

Polina

Step 1:  $-3(x + 7) = 24$

Step 2:  $x + 7 = 27$

Step 3:  $x = 20$

## Lesson Practice

A1.2.02

Name: ..... Date: ..... Period: .....

9. Which equation is equivalent to  $0.05n + 0.1d = 3.65$ ?

- A.  $5n + d = 365$
- B.  $0.5n + d = 365$
- C.  $5n + 10d = 365$
- D.  $0.05d + 0.1n = 365$

10. Select *all* the moves that could be the first step to solving the equation

$$4(x + 3) = 8x - 4 + 12x.$$

- A. Divide each side by 4.
  - B. Take away  $12x$  on the right side.
  - C. Distribute 4 on the left side.
  - D. Combine like terms on the right side.
  - E. Add 4 to the right side.
11. Determine a value for  $x$  that makes this equation true:  $9x - 4(x - 3) = 27 + 2x$ . Show or explain your thinking.

*x = 5. Work varies.*

$$9x - 4x + 12 = 27 + 2x$$

$$5x + 12 = 27 + 2x$$

$$3x + 12 = 27$$

$$3x = 15$$

$$x = 5$$

## Spiral Review

12. Rio scored 409 points in a video game. This was 223 more points than Sadia scored,  $s$ . Which equation does *not* represent this situation?

- A.  $223 = 409 - s$
- B.  $s = 409 - 223$
- C.  $s = 409 + 223$
- D.  $223 + s = 409$

## Reflection

1. Put a question mark next to a response you'd like to compare with a classmate's.
2. Use this space to ask a question or share something you're proud of.

### Lesson Summary

Not all one-variable linear equations have a single solution. Some linear equations have *infinitely many solutions*, and some have *no solution*.

In the process of solving, there is a difference between equations with one solution, no solution, or infinitely many solutions:

- In an equation with one solution, a *single value* of  $x$  will make the equation true.
- In an equation with no solution, *no value* of  $x$  will make the equation true.
- In an equation with infinitely many solutions, *any value* of  $x$  will make the equation true.

Here are examples of equations with one solution, no solution, and infinitely many solutions.

#### One Solution

$$\begin{aligned}3x + 4 &= 2x + 10 \\3x &= 2x + 6 \\x &= 6\end{aligned}$$

#### No Solution

$$\begin{aligned}2x + 4 &= 2x + 10 \\4 &= 10\end{aligned}$$

This is *never true!*

#### Infinitely Many Solutions

$$\begin{aligned}2(x + 5) &= 2x + 10 \\2x + 10 &= 2x + 10 \\10 &= 10\end{aligned}$$

This is *always true!*

If the variable in an equation is eliminated during the solving process, the equation has either no solution or infinitely many solutions. If the statement remaining is false, the equation has no solution. If the statement remaining is true, the equation has infinitely many solutions.

### Things to Remember:

# Lesson Practice

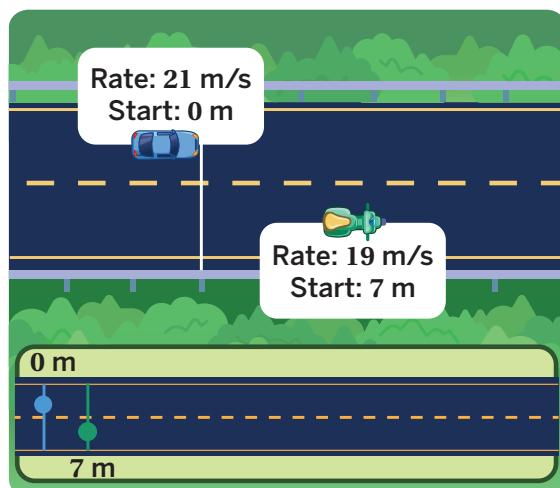
A1.2.03

Name: ..... Date: ..... Period: .....

- The car and scooter are moving at constant speeds. The equation  $21t = 19t + 7$  represents the time,  $t$ , when they will be in the same position.

When will the car and scooter be in the same position?

**3.5 seconds**



**Problems 2–3:** The equation  $10t = 2.5t$  represents the time,  $t$ , when two vehicles will be in the same position.

- When will these two vehicles be in the same position? Circle one.

Once

Never

Always

- Explain how you know.

*Explanations vary.  $t = 0$  makes the equation true because  $10(0) = 2.5(0)$ . No other values make the equation true, so the vehicles will be in the same position once.*

- Here is Kiandra's work to solve  $16x = 10x$ . She says there is no solution.

Is this correct? Show or explain your thinking.

*No. Explanations vary. If Kiandra subtracted  $10x$  from each side of the equation, she would get  $6x = 0$ . If she then divided by 6, she would get a solution:  $x = 0$ .*

**Kiandra**

$$\begin{array}{rcl} 16x &=& 10x \\ \hline x & & x \\ 16 &=& 10 \end{array}$$

*There is no solution.*

- Group the equations based on their number of solutions.

$$5t = 3t$$

$$2t = 10 - 2t$$

$$15 - 3(t + 5) = -3t$$

$$4t + 7 = 4(t + 2)$$

$$6t + 2 = -3 + 6t$$

One Solution	No Solution	Infinitely Many Solutions
$5t = 3t$ $2t = 10 - 2t$	$4t + 7 = 4(t + 2)$ $6t + 2 = -3 + 6t$	$15 - 3(t + 5) = -3t$

## Lesson Practice

A1.2.03

Name: ..... Date: ..... Period: .....

**Problems 6–7:** Create two different equations that each have a solution of  $x = 1$ .

6. Fill in each blank using the whole numbers 0 to 9 only once each.

$$\boxed{\phantom{0}}x + \boxed{\phantom{0}} = \boxed{\phantom{0}}x + \boxed{\phantom{0}}$$

**Responses vary.**

- $1x + 8 = 2x + 7$  and  $3x + 6 = 4x + 5$
- $7x + 1 = 6x + 2$  and  $5x + 3 = 8x + 0$

$$\boxed{\phantom{0}}x + \boxed{\phantom{0}} = \boxed{\phantom{0}}x + \boxed{\phantom{0}}$$

7. Explain what you notice about your equations.

**Explanations vary.** The digits used on each side of each equation sum to the same total.

## Spiral Review

8. Select all the equations where  $x = 2$  is a solution.

A.  $\frac{x}{4} = 8$

B.  $19 = 2(x + 6) + 3$

C.  $2x + 10 = 2x + 8$

D.  $5 - 3x = -1$

E.  $4 - x = x$

9. Select all the expressions that are equivalent to  $2(x + 3)$ .

A.  $(x + 3) \cdot 2$

B.  $2x + 5$

C.  $2x + 3 \cdot 2$

D.  $2x + 3$

E.  $2x + 6$

10. Select the expression that is equivalent to  $6 - 2(x + 1)$ .

A.  $4(x + 1)$

B.  $7 - 2x$

C.  $4 - 2x$

D.  $4x + 4$

## Reflection

1. Put a heart next to the problem you're most proud of.
2. Use this space to ask a question or share something you're proud of.



### Lesson Summary

Two-variable linear equations can be written in different forms. Sometimes the different forms of an equation can reveal information that is useful for solving problems. Depending on what information you are looking for, you might choose to use one form or the other.

Here is an example of two equivalent equations that represent the number of seats and handholds in possible in a subway car that has 300 square feet of floor space. They are each written in different forms, and they each reveal different information about the situation. In each equation,  $t$  is the seating capacity and  $d$  is the standing capacity.

$$4t + 2d = 300$$

- Each seat takes up 4 square feet of floor space.
- Each standing spot requires 2 square feet of floor space.
- The total amount of floor space is 300 square feet.

$$d = 150 - 2t$$

- When there are no seats ( $t = 0$ ), 150 passengers can stand in the car.
- For every seat that is added, the number of spots for standing decreases by 2.



### Things to Remember:

# Lesson Practice

A1.2.04

Name: ..... Date: ..... Period: .....

**Problems 1–4:** Adriana spent \$24 on fruit punch and apple juice. Fruit punch costs \$3 per bottle. Apple juice costs \$2 per bottle.

1. How many bottles of *fruit punch* could Adriana buy if she didn't get any apple juice?

**8 bottles**

2. How many bottles of *apple juice* could Adriana buy if she did not get any fruit punch?

**12 bottles**

Adriana wrote the equation  $3f + 2a = 24$  to represent the situation.

3. Use the equation to help you complete the table.

4. Which equation represents the same relationship?

A.  $a = 8 - \frac{2}{3}f$

B.  $a = 8 - \frac{3}{2}f$

C.  $a = 12 - \frac{2}{3}f$

D.  $a = 12 - \frac{3}{2}f$

<i>f</i>	<i>a</i>
2	<b>9</b>
6	<b>3</b>

**Problems 5–6:** Here is an equation:  $2x + 4y = 80$ .

5. Use the equation to help you complete the table.

6. Which equation represents the same relationship?

A.  $y = 20 - 2x$

B.  $y = 40 - 2x$

C.  $y = 20 - \frac{1}{2}x$

D.  $y = 40 - \frac{1}{2}x$

<i>x</i>	<i>y</i>
6	<b>17</b>
12	<b>14</b>

**Problems 7–8:** Nia is buying bananas and apples at the farmer's market.

Bananas cost \$0.50 each. Apples cost \$1.00 each.

7. Select *all* the combinations of bananas and apples that Nia could buy for exactly \$3.50.

A. 1 banana and 3 apples

B. 5 bananas and 1 apple

C. 1 banana and 2 apples

D. 3 bananas and 2 apples

E. 5 bananas and 2 apples

8. The equation  $0.5b + 1a = 3.50$  represents the number of bananas and apples that Nia can buy for \$3.50. Solve this equation for *a*.

**$a = 3.50 - 0.5b$**

## Lesson Practice

A1.2.04

Name: ..... Date: ..... Period: .....

### Spiral Review

9. Select *all* the expressions that are equivalent to  $8 - 12 - (6 + 4)$ .

- A.  $(6 + 4) - 8 - 12$   
 B.  $8 - 6 - 12 + 4$   
 C.  $8 - 12 - 6 - 4$   
 D.  $8 - (6 + 4) - 12$   
 E.  $(8 - 12) - 6 + 4$

10. Which equation is equivalent to  $\frac{1}{3}m + \frac{1}{2}n = 9$ ?

- A.  $3m + 2n = 9$        B.  $2m + 3n = 54$   
C.  $m + 3n = 27$       D.  $2m + n = 18$

11. Explain how you know that Equation A and Equation B are equivalent.

**Equation A**

$$48 - 5x = 13$$

**Equation B**

$$5x = 35$$

*Responses vary.*

- The solution to each equation is  $x = 7$ .
- If you subtract 48 from both sides of Equation A, the equation would be  $-5x = -35$ . Then if you divide or multiply both sides by -1, Equation A would be equivalent to Equation B.

12. Create two equivalent equations by filling in the blanks using the whole numbers 0 to 9 only once.

$$\boxed{\phantom{0}}x + \boxed{\phantom{0}}y = \boxed{\phantom{0}}$$

$$y = \boxed{\phantom{0}} - \boxed{\phantom{0}}x$$

*Responses vary.*

- $6x + 2y = 8$  and  $y = 4 - 3x$
- $8x + 2y = 6$  and  $y = 3 - 4x$

### Reflection

- Circle the problem you feel most confident about.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

Equations, tables, and graphs are all different ways to model a situation. The graph of a linear equation represents all the pairs of values that are solutions to the equation (make the equation true).

Linear equations can be written in different but equivalent forms. Rearranging equations helps reveal new information, such as the *x*-intercept and *y*-intercept, which we can see in a graph, table, or description of a situation.

Let's say a lemonade stand sold lemonade for \$3 per cup and cookies for \$2 each. The stand made \$12.  $\ell$  represents the number of cups of lemonade sold and  $c$  represents the number of cookies sold. This situation can be represented in many different ways:

## Equation

$$3\ell + 2c = 12$$

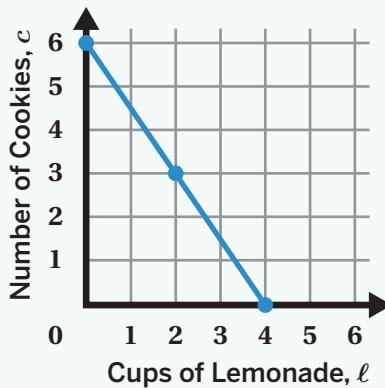
Equation Solved for  $c$ 

$$c = 6 - \frac{3}{2}\ell$$

## Table

$\ell$	0	2	4
$c$	6	3	0

## Graph



## Things to Remember:

# Lesson Practice

A1.2.06

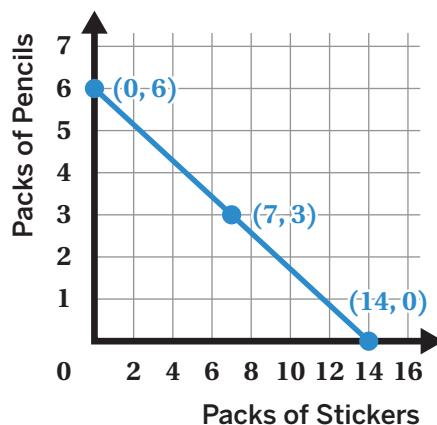
Name: ..... Date: ..... Period: .....

**Problems 1–2:** Here is a graph about school supplies.

- A teacher spent \$21 on packs of stickers and packs of pencils for her class.
  - Stickers cost \$1.50 per pack.
  - Pencils cost \$3.50 per pack.

Show or explain how you know that this graph represents this situation.

*Responses vary.* I know this graph represents this situation because each coordinate pair corresponds to a total of \$21. When I multiply the  $x$ -value by 1.50 and the  $y$ -value by 3.50, the total is always 21.



- Circle a coordinate pair and explain what it means in this situation.

(0, 6)

(7, 3)

(14, 0)

*Responses vary.*

- (0, 6) means that the teacher bought 0 packs of stickers and 6 packs of pencils for \$21.
- (7, 3) means that the teacher bought 7 packs of stickers and 3 packs of pencils for \$21.
- (14, 0) means that the teacher bought 14 packs of stickers and 0 packs of pencils for \$21.

- Circle the line that represents  $12 = 3x + 4y$ .

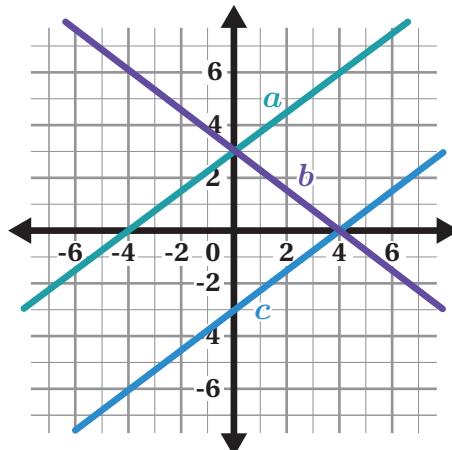
Line a

Line b

Line c

Show or explain how you know.

*Explanations vary.* When I solve  $12 = 3x + 4y$  for  $y$ , I get  $y = 3 - \frac{3}{4}x$ . Line b is the only choice that starts at 3 on the  $y$ -axis and has a negative slope.



- Which equation is equivalent to  $15x + 3y = 2$ ?

A.  $y = \frac{2}{3} + 5x$

B.  $y = \frac{2}{3} - 5x$

C.  $y = 2 - 15x$

D.  $y = 2 - 5x$

- Match each equation with its equivalent equation.

a.  $4x + 6y = 20$

b.  $3x - 6y = 16$

c.  $2x - 3y = 10$

d.  $-3x + 6y = 16$

d.  $y = \frac{8}{3} + \frac{1}{2}x$

c.  $y = -\frac{10}{3} + \frac{2}{3}x$

a.  $y = \frac{10}{3} - \frac{2}{3}x$

b.  $y = -\frac{8}{3} + \frac{1}{2}x$

# Lesson Practice

A1.2.06

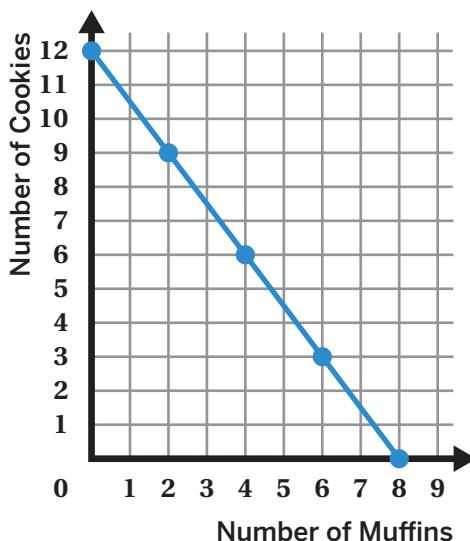
Name: ..... Date: ..... Period: .....

6. Jordan had a bake sale. Muffins were \$3 each and cookies were \$2 each. Jordan earned \$24.

Here is a graph of Jordan's situation.

Select *all* the combinations of muffins and cookies Jordan could have sold.

- A. 0 muffins and 8 cookies
- B. 9 muffins and 2 cookies
- C. 2 muffins and 9 cookies
- D. 6 muffins and 4 cookies
- E. 4 muffins and 6 cookies



## Spiral Review

7. A tourist is trying to predict the price of their next taxi ride.

The tourist recorded their last three rides in the table.

How much can they expect to pay to travel 10 miles?  
**\$38**

Distance (mi)	Price (\$)
2	10
4	17
7	27.50

8. Select *all* the equations where  $x = -2$  is a solution.

- A.  $4x = 4 + 2x$
- B.  $2(x + 5) = x + 8$
- C.  $3x - 5 = 1$
- D.  $19 = 2(x - 6) + 3$
- E.  $5 + 3x = -1$

9. Solve  $-3x + 4y = 28$  for  $y$ .

$$y = \frac{3}{4}x + 7 \text{ (or equivalent)}$$

10. Solve  $6x - 3y = 36$  for  $y$ .

$$y = -12 + 2x \text{ (or equivalent)}$$

## Reflection

1. Put a star next to a problem you could explain to a classmate.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

We can use *inequalities* to model situations with constraints. A **constraint** is a limitation on what values are possible in a model or situation.

Here is an example of a situation, the constraint, and the inequality that models them.

Situation	Constraint	Inequality
Tasia is ordering pizza for a party. Each plain pizza costs \$12 and there is a delivery fee of \$8.	Tasia can spend <i>up to</i> \$140.	$12p + 8 \leq 140$ Where $p$ represents the number of pizzas.

When writing inequalities to model situations, you can use the symbols  $>$ ,  $\geq$ ,  $<$ , and  $\leq$  to represent the nature of the constraint. Terms like *greater than*, *less than*, *at most*, *at least*, or *up to* can help you determine which inequality symbol to use.

Constraints modeled by inequalities produce a set of solutions. When using inequalities to model constraints, it is important to note that not all solutions to inequalities are viable solutions or make sense for the situation.

**Things to Remember:**

# Lesson Practice

## A1.2.08

Name: ..... Date: ..... Period: .....

1. For each constraint, write the letter of the matching inequality.

- a.  $x \leq 10$       A.  $x$  is less than or equal to 10.  
b.  $x > 10$       A.  $x$  is at most 10.  
c.  $x \geq 10$       C.  $x$  is greater than or equal to 10.  
B.  $x$  is greater than 10.  
C.  $x$  is at least 10.
2. Marquis wants to work at least 20 hours a week to earn enough money to go to a concert. Which inequality represents  $x$ , the number of hours Marquis wants to work?  
A.  $x > 20$       B.  $x < 20$       C.  $x \leq 20$       D.  $x \geq 20$
3. Demetrius can spend as much as \$50 on shirts. Shirts,  $s$ , cost \$16 each at a nearby store. Which inequality represents this situation?  
A.  $16s \geq 50$       B.  $16s \leq 50$       C.  $50s \geq 16$       D.  $50s \leq 16$

Explain your thinking.

*Explanations vary. 16s represents \$16 for each shirt and 50 represents \$50, which is the most that Demetrius can spend. He can spend \$50 or anything less than that, so I used the symbol  $\leq$ .*

4. List three values for  $x$  that would make  $8 + 2x \leq 20$  true.

**Responses less than or equal to 6 are considered correct.**

$$x = \underline{\quad 0 \quad}, \quad x = \underline{\quad 1 \quad}, \quad x = \underline{\quad \frac{5}{3} \quad}$$

**Problems 5–7:** Write an inequality for each constraint. Use  $t$  for time (in hours).

5. Trevon practices the clarinet at least 1 hour each day.

$$\underline{t \geq 1}$$

6. At some colleges, students must work 20 hours or less per week.

$$\underline{t \leq 20}$$

7. The American Academy of Pediatrics recommends teenagers play video games for no more than 2 hours each day.<sup>1</sup>

$$\underline{t \leq 2}$$

<sup>1</sup> Source: American Academy of Pediatrics

## Lesson Practice

A1.2.08

Name: ..... Date: ..... Period: .....

**Problems 8–9:** Tell a story about each inequality. Specify the constraint and what the variable represents.

8.  $x \geq 3$

**Responses vary.**

- Sahana needs to practice playing the clarinet at least 3 hours per week.
- $x$  represents the number of hours that she practices each week.

9.  $5 > y$

**Responses vary.**

- Your height must be less than 5 feet to ride the kids' roller-coaster at the fair.
- $y$  represents your height in feet.

10. Fatima makes \$9.25 per hour,  $h$ , plus \$150 in commissions. She created an inequality that could be used to find the number of hours she needs to work to make at least \$510 on her next paycheck.

Her inequality is  $9.25 + 150h \leq 510$ . Find the mistakes in Fatima's inequality and correct it.

**Responses vary. There are two mistakes in Fatima's inequality:**

- She misidentified what number to multiply variable  $h$  by. She multiplied it by 150 when it should be multiplied by 9.25.
- She used the wrong inequality symbol for the phrase "at least". It should be  $\geq$  instead of  $\leq$ .
- The correct inequality is  $9.25h + 150 \geq 510$ .

## Spiral Review

**Problems 11–13:** Solve each equation.

11.  $4x - 6 = 12 - 2x$

$x = 3$

12.  $\frac{1}{3}x - 8 = 12 - 3x$

$x = 6$

13.  $2x + 7 - 3x = \frac{5}{2}$

$x = \frac{9}{2}$  (or equivalent)

## Reflection

1. Circle a problem you want to talk to a classmate about.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

The *solution set* of an inequality contains all the values that make the inequality true. You can represent a solution set on a number line by marking the *boundary point* and then shading the region of values that make the inequality true. To identify the boundary point, you can solve the equation that corresponds to the inequality. Then you can test one or more values to determine whether the *solution set* is greater than or less than the boundary point.

Here is an example of how you can determine and represent the solution set for  $2x - 4 \geq 8$ :

Determine the boundary point:

$$2x - 4 = 8$$

$$2x = 12$$

$$x = 6$$

Determine all the solutions:

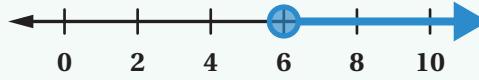
$$2x - 4 \geq 8$$

$$2(0) - 4 \geq 8$$

$4 \geq 8$  **False!**

- 6 is the boundary point and the  $\geq$  symbol means it is included in the solution set.
- Since this statement is false when we substitute 0, we know 0 is not in the solution set.

When a boundary point is not included in the solution set, this is represented with an open circle on the number line. Here is an example of the solution  $-0.5 > x$  graphed on a number line.



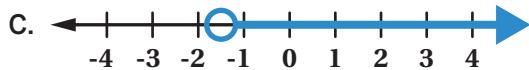
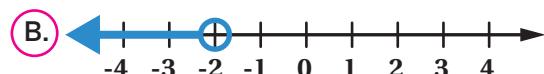
## Things to Remember:

## Lesson Practice

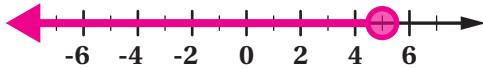
A1.2.09

Name: ..... Date: ..... Period: .....

1. Which graph represents the solutions to  $2x < -4$ ?



2. Create a graph of the solutions to  $2x \leq 10$ .



3. Leo is trying to solve  $-15 + x < -14$ . He knows the boundary point is at  $x = 1$ . How can he determine whether the solutions are  $x < 1$  or  $x > 1$ ?

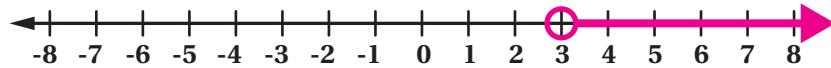
*Explanations vary. Leo could test a point that is less than 1, like 0. If that point is a solution to  $-15 + x < -14$ , then  $x < 1$  are all the solutions. If it's not, then  $x > 1$  must be all the solutions.*

4. Diego says that  $x = 5$  is a solution to  $-3x > 9$  because when you divide both sides by  $-3$ , you get  $x > -3$ . Is this correct? Explain your thinking.

*No. Explanations vary. I know  $x = 5$  is not a solution because when you substitute 5 for  $x$  in  $-3x > 9$ , you get a false statement.*

5. Solve and graph the inequality  $5x + 7 > 22$ .

$x > 3$



6. What are the solutions to  $-2(2 + c) + 4 < 6$ ?

- A.  $c > -3$   
B.  $c < -3$   
C.  $c > -2$   
D.  $c < -2$

## Lesson Practice

A1.2.09

Name: ..... Date: ..... Period: .....

7. Create an inequality and the graph of its solutions by filling in each blank using the numbers 0 to 9 only once.

$$\boxed{\phantom{0}}x + \boxed{\phantom{0}} > \boxed{\phantom{0}}x$$



$$x < \boxed{\phantom{0}}$$

### Spiral Review

**Problems 8–10:** A community pool offers two different membership plans.

Moon wants to spend no more than \$48 at the community pool this month.

Plan A	Plan B
\$4 per visit	An initial \$12 fee, then \$2 per visit

8. What is the most number of visits Moon can make with Plan A?

**12 visits**

9. What is the most number of visits Moon can make with Plan B?

**18 visits**

10. After how many visits will the cost of both plans be the same?

**6 visits**

### Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can solve any one-variable inequality by solving its corresponding equation to determine the boundary point, then testing values to determine the rest of the solutions.

Here's an example of how you could solve the inequality  $10 > -3x - 2$ .

Step 1: Solve the corresponding equation:

$$10 = -3x - 2$$

$$10 = -3x - 2$$

$$12 = -3x$$

The solution to this equation is  $-4$ , so the boundary point is  $x = -4$ .

$$-4 = x$$

Step 2: Test values to determine the rest of the solutions.

In this example, we tested  $x = -5$  and  $x = 0$ .

$$x = -5$$

$$x = 0$$

$$10 > -3(-5) - 2$$

$$10 > -3(0) - 2$$

$$10 > 13$$

$$10 > -2$$

**False!**

**True!**

When  $x = -5$ , the inequality is false, so  $x = -5$  is not a solution. This means that the solutions are greater than  $-4$ .

The solutions to an inequality do not always have the same inequality symbol as the original inequality. Since the solutions are *greater than*  $-4$ , the solution set can be written as  $-4 < x$  or  $x > -4$ .

**Things to Remember:**

# Lesson Practice

A1.2.10

Name: ..... Date: ..... Period: .....

1. Write three values of  $n$  that make this inequality true:  $-5(n + 1) > 10$ .

**Responses less than -3 are considered correct.**

2. Here is an inequality:  $7x + 6 < 3x + 2$ . Select *all* values that are solutions.

A.  $x = 1$

B.  $x = 0$

C.  $x = -1$

D.  $x = -2$

E.  $x = -8$

**Problems 3–6:** Solve each inequality.

3.  $4x + 5 \geq 37$

$x \geq 8$

4.  $-3x + 4 \geq 12$

$x \leq -\frac{8}{3}$  (or equivalent)

5.  $-6 + \frac{x}{2} < 7$

$x < 26$

6.  $-8x - 6 > 2x - 26$

$x < 2$

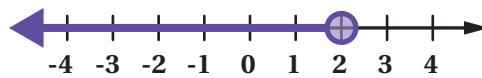
7. Which inequality is represented by the graph?

A.  $2x + 6 < 10$

B.  $2x + 6 \geq 10$

C.  $-2x + 6 \geq 10$

D.  $-2x - 6 \geq -10$



## Lesson Practice

A1.2.10

Name: ..... Date: ..... Period: .....

8. Make the two inequalities equivalent by filling in each blank using the numbers 0 to 9 only once.

**Responses vary.**

- $2x + 7 < 4x + 1$  and  $x > 3$
- $5x + 6 < 8x + 0$  and  $x > 2$

$$\boxed{\phantom{0}} x + \boxed{\phantom{0}} < \boxed{\phantom{0}} x + \boxed{\phantom{0}}$$

$$x > \boxed{\phantom{0}}$$

## Spiral Review

**Problems 9–11:** Rewrite each expression as a single power of 10.

9.  $\frac{10^2 \cdot 10^5}{10^4}$   
 $10^3$

10.  $(10^4)^2 \cdot (10^2)^2$   
 $10^{12}$

11. Imani is going shopping with a budget of \$125. Which inequality represents the amount of dollars,  $x$ , that Imani can spend while shopping?

- A.  $x \leq 125$       B.  $x \geq 125$       C.  $x > 125$       D.  $x < 125$

## Reflection

1. Put a star next to the problem you think is the most important.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

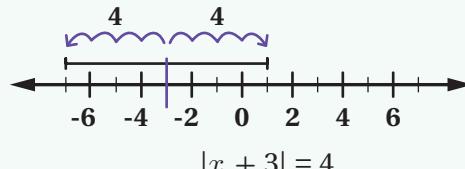
The *absolute value* of a number is its distance from 0 on a number line. Equations with absolute values often have two solutions because there are often two numbers that are the same distance from a number.

For example, the equation  $|x + 3| = 4$  has two solutions,  $x = 1$  and  $x = -7$ , because 1 and -7 are both 4 units away from -3.

You can determine the solutions using a number line or by solving two equations.

To determine the solutions to an inequality with an absolute value, you can:

- Step 1: Solve for the boundary points.
- Step 2: Determine whether the boundary points are included in the solution set. 1 and -7 are *not* included.
- Step 3: Test a value between the two boundary points to decide which values make the inequality true.
- Step 4: Graph all the solutions.



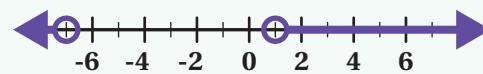
$$\begin{aligned} |x + 3| &= 4 \\ x + 3 &= 4 & -(x + 3) &= 4 \\ x &= 1 & x &= -7 \end{aligned}$$

$$|x + 3| > 4$$

Boundary points:  
 $x = 1$        $x = -7$

Test  $x = 0$ :  
 $|(0) + 3| > 4$

**False!**



## Things to Remember:

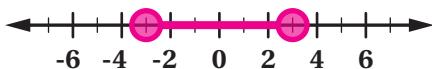
## Lesson Practice

A1.2.11

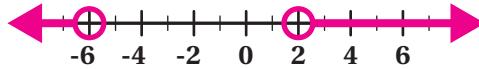
Name: ..... Date: ..... Period: .....

**Problems 1–4:** Graph all the solutions to each inequality or equation.

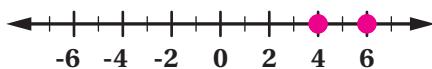
1.  $|x| \leq 3$



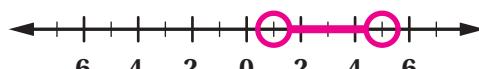
2.  $|x + 2| > 4$



3.  $|x - 5| = 1$

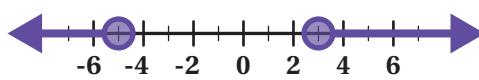


4.  $2 > |x - 3|$

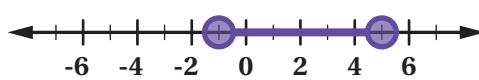


5. What value of  $p$  would make  $|x - p| \geq 4$  match the graph?

-1



6. Which inequality's solutions are shown by the graph?



A.  $|x + 5| < -1$

B.  $|x - 2| \leq 3$

C.  $|x - 2| \geq 3$

D.  $|x + 3| \leq 2$

7. List three numbers that are not solutions to  $|x + 1| \geq 2$ .

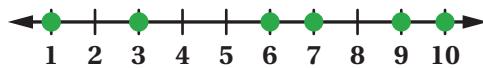
Responses between -3 and 1 are considered correct.

# Lesson Practice

A1.2.11

Name: ..... Date: ..... Period: .....

8. Omar is thinking of a number somewhere on this number line. Here are some guesses from Omar's friends.



No guess was more than 5 away. Only one guess was correct. What is Omar's number?

Explain your thinking.

6. *Explanations vary.* The number has to be 6 because every other number has at least one guess that is more than 5 away.

## Spiral Review

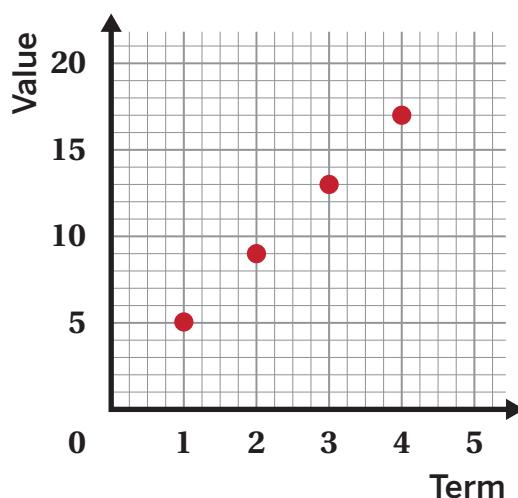
Problems 9–11: Here is the graph of a sequence.

9. Is this sequence arithmetic, geometric, or something else? Explain your thinking.

**Arithmetic.** *Explanations vary.* The value of each consecutive term increases by 4.

10. What is the value of the 5th term?

21



11. What is the value of the  $n$ th term?

$4n + 1$  (or equivalent)

## Reflection

1. Circle the problem you enjoyed doing the most.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

The solutions to a two-variable inequality are all of the ordered pairs that make the inequality true.

Here's a situation that can be modeled by an inequality: Marco is making bracelets. Each bracelet needs to cost no more than \$10. Planet beads cost \$1 and oval beads cost \$2. Marco wants to know if he can make a bracelet with 3 planet beads and 4 oval beads.

Marco is wondering if  $(4, 3)$  is a solution to the inequality  $2x + y \leq 10$ , where  $x$  represents the number of \$2 beads and  $y$  represents the number of \$1 beads.

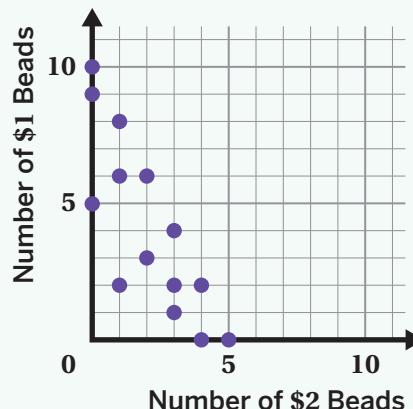
To check, Marco substitutes  $x = 4$  and  $y = 3$  into the inequality:

$$2(4) + (3) \leq 10$$

$$8 + (3) \leq 10$$

$$11 \leq 10 \quad \text{False!}$$

That means that  $(4, 3)$  is *not* a solution and Marco cannot make a bracelet with 3 planet beads and 4 oval beads while staying within his budget.



## Things to Remember:

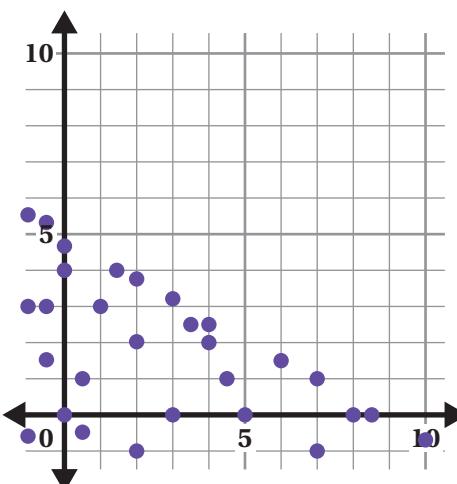
# Lesson Practice

A1.2.13

Name: ..... Date: ..... Period: .....

1. This graph shows some solutions to  $5x + 9y < 45$ . Select all of the points that are also solutions.

- A. (1, 1)
- B. (4, 0)
- C. (10, 4)
- D. (0, 10)
- E. (6, -1)



2. Write at least three coordinate pairs that are solutions to the inequality  $x \leq y$ .

*Responses vary. (0, 1), (-2, 2), (3, 5)*

**Problems 3–4:** Tyler can spend up to \$45 on hats and socks. A hat costs \$10 and a pair of socks costs \$2.50.

- $h$  is the number of hats.
- $s$  is the number of pairs of socks.

3. Which inequality represents this situation?

- A.  $10h + 2.50s > 45$
- B.  $10h + 2.50s < 45$
- C.  $10h + 2.50s \geq 45$
- D.  $10h + 2.50s \leq 45$

4. Explain how you know that  $h = 2$  and  $s = 1$  are solutions to this situation.

*Explanations vary. If you substitute 2 in place of  $h$  and 1 in place of  $s$ , you get  $10(2) + 2.50(1) \leq 45$ , which is true because  $22.50 \leq 45$ .*

5. Marc is making a bracelet for his sister using beads that cost \$0.50 and \$0.75. He cannot spend more than \$8.00 on the bracelet.

- $x$  is the number of \$0.50 beads.
- $y$  is the number of \$0.75 beads.

Marc says that the inequality  $0.5x + 0.75y \geq 8$  represents all the bracelets he can make. Do you agree with him? Explain your thinking.

*No. Explanations vary. Marc's inequality says that he can spend \$8.00 or more on the bracelet. He can spend \$8.00 or less.*

# Lesson Practice

A1.2.13

Name: ..... Date: ..... Period: .....

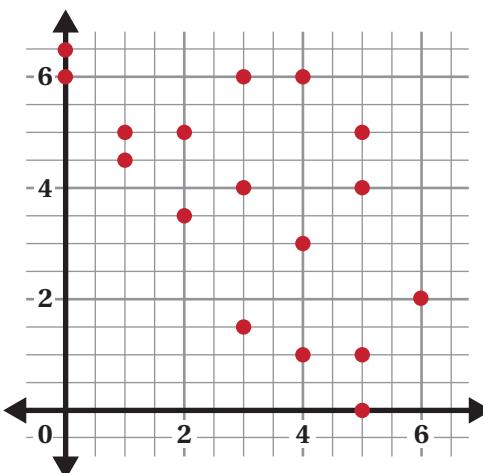
6. This graph shows some solutions to  $3x + 2y \geq 12$ .

Write an ordered pair for another point that is a solution to this inequality.

**Responses vary.** (6, 4)

7. Write an ordered pair for a point that is *not* a solution to  $3x + 2y \geq 12$ .

**Responses vary.** (2, 1)



## Spiral Review

**Problems 8–9:** Write an inequality for each situation.

8. Duri will stay warm in a sleeping bag when the temperature is at least  $30^{\circ}\text{F}$ .

Use  $t$  to represent temperatures at which Duri will stay warm.

$$t \geq 30$$

9. Duri's backpack needs to weigh less than 45 pounds. Use  $w$  to represent the weight of the backpack.

$$w < 45$$

**Problems 10–11:** Here is an equation:  $6x + 2y = 36$ .

10. For each value of  $x$ , determine the value of  $y$ .

$x$	$y$
2	12
4	6

11. Which equation represents the same relationship?

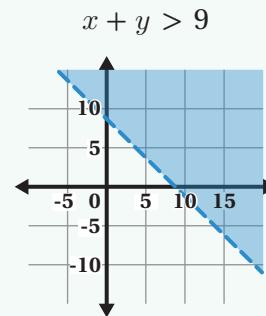
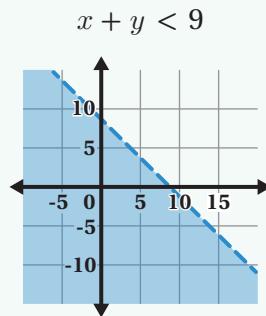
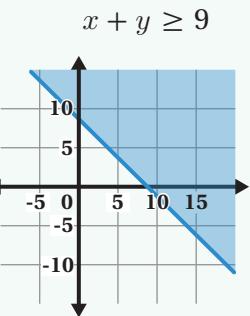
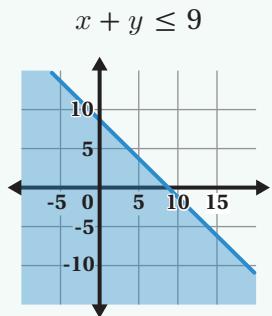
- A.  $y = 6 - 3x$       B.  $y = 18 - 3x$       C.  $y = 18 - \frac{1}{3}x$       D.  $y = 6 - \frac{1}{3}x$

## Reflection

- Circle the problem you enjoyed doing the most.
- Use the space to ask a question or share something you're proud of.

## Lesson Summary

The solutions to a two-variable linear inequality can be represented on a graph as a *half-plane*. A **boundary line** separates the plane into the region that contains solutions and the region that does not. The shaded area represents all of the solutions, which are the values of  $(x, y)$  that make the inequality true.



A solid line means that the points on the boundary line are included in the solutions. This is represented by the  $\leq$  and  $\geq$  symbols.

A dashed line means the points on the boundary line are *not* included in the solutions. This is represented by the  $<$  and  $>$  symbols.

To determine which of the half-planes is the **solution region**, you can test points on either side of the *boundary line* to see whether they make the inequality true or false.

## Things to Remember:

# Lesson Practice

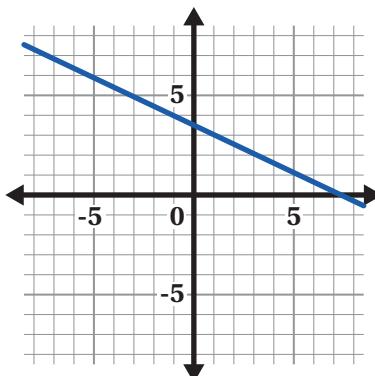
A1.2.14

Name: ..... Date: ..... Period: .....

1. Here is a graph of the equation  $x + 2y = 7$ .

Which of these points is a solution to the inequality  $x + 2y < 7$ ?

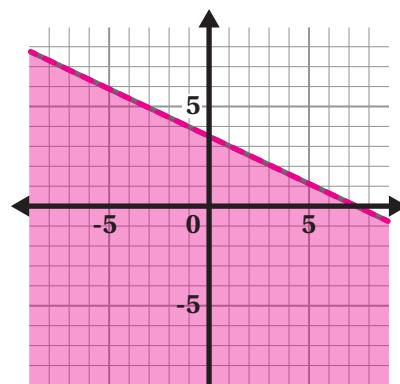
- A. (0, 0)
- B. (10, 0)
- C. (7, 0)
- D. (0, 7)



2. Graph all the solutions to  $x + 2y < 7$ .

Explain your thinking.

*Explanations vary.* The line is dashed because the inequality sign is  $<$ . When I plugged (0, 0) into the inequality, I got a true statement, so I shaded that side of the line.

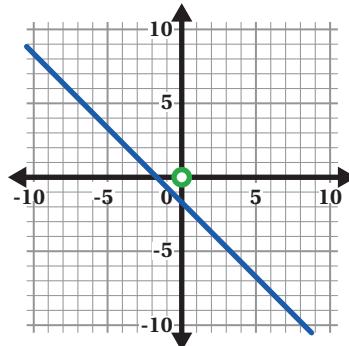


3. Here is an inequality:  $x + y \leq -2$ .

- Ada graphed the equation  $x + y = -2$ .
- Ada noticed that (0, 0) is not a solution to  $x + y \leq -2$ .

How can Ada use this information to graph the solutions to this inequality?

*Explanations vary.* Since the inequality symbol in this problem is  $\leq$ , the line should be solid. Ada knows that (0, 0) is not a solution, so Ada should shade the side of the line that does not include that point.

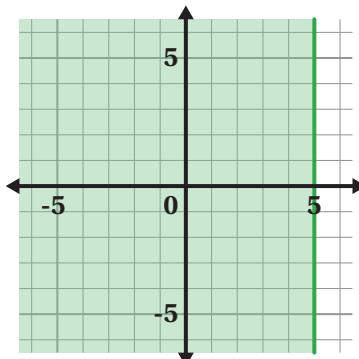


4. Brianna is creating a graph for the inequality  $2x - 5y > 10$ . She says that since the inequality has a greater-than symbol, she should shade the region above the line  $2x - 5y = 10$ . Is Brianna correct? Explain your thinking.

*No. Explanations vary.* If I plug in a point that's above the line  $2x - 5y = 10$ , such as (2, 0), it makes the inequality false.

5. Which inequality is shown on the graph?

- A.  $y \leq 5$
- B.  $y \geq 5$
- C.  $x \geq 5$
- D.  $x \leq 5$



# Lesson Practice

A1.2.14

Name: ..... Date: ..... Period: .....

**Problems 6–7:** A food truck only sells hot dogs and hamburgers. They want to sell 50 items or more each day.

- $x$  represents the number of hot dogs sold.
- $y$  represents the number of hamburgers sold.

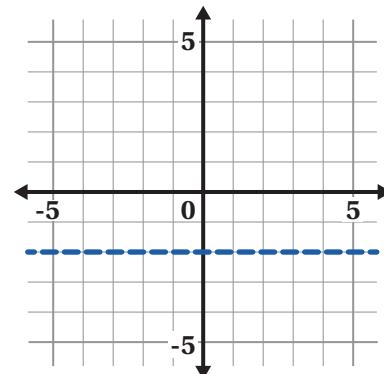
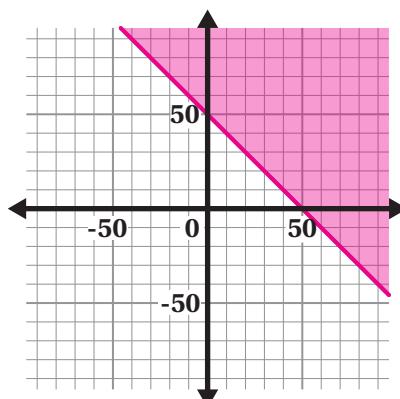
**6.** Which inequality represents this situation?

- A.  $x + y > 50$       B.  $x + y < 50$   
**C.**  $x + y \geq 50$       D.  $x + y \leq 50$

**7.** Complete the graph so that it represents all the solutions to the inequality for this situation.

**8.** Leah-James started to graph the inequality  $y > -2$  by graphing a dashed line at  $y = -2$ . How might Leah-James decide where to shade?

**Responses vary.** Pick a point above or below the line. Plug that point into the inequality and see if it makes the inequality true. If it does, shade the region that point is in. If it doesn't, shade the other region.



## Spiral Review

**9.** The equation of line  $a$  is  $y = -2x - 1$ . Select *all* the points that are on line  $a$ .

- A.  $(-2, 3)$        B.  $(0, 4)$        C.  $(1, -3)$   
 D.  $(-1, -3)$        E.  $(2, 5)$

**10.** Lola can spend up to \$15 on pens and notebooks. A pen costs \$2 and a notebook costs \$1.50. Using  $p$  for the number of pens and  $n$  for the number of notebooks, write an inequality that represents this situation.

$$2p + 1.5n \leq 15 \text{ (or equivalent)}$$

**11.** A golf ball weighs 1.6 ounces and an empty metal bucket weighs 12 ounces. Neel adds golf balls one at a time to the empty metal bucket. How many golf balls will be in the bucket when the total weight is 20 ounces?

**5 golf balls**

## Reflection

1. Put a smiley face next to the problem you learned from most.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

Different types of questions lead to different types of data:

- **Quantitative data** has values that are numbers, measurements, or quantities instead of words. It's sometimes called *numerical data*.  
*How many pets do you have?* is a question that produces quantitative data.
- **Categorical data** has values that are categories, such as colors, words, or zip codes.  
*What's your favorite animal?* is a question that produces categorical data.

It's important to be specific when writing survey questions, so you can gather the exact type of data you need.

## Things to Remember:

# Lesson Practice

## A1.3.01

Name: ..... Date: ..... Period: .....

1. Which would *not* be a good survey question?
  - A. What grade are you in?
  - B. How many books did you read last year?
  - C. How many inches are in 1 foot?
  - D. How many pets do you have?
2. Determine which type of data these questions produce.

Question	Categorical or Quantitative?
How old are you?	Quantitative
Do you have any pets?	Categorical
How many siblings do you have?	Quantitative

3. Select *all* the questions that would produce quantitative data.
  - A. How many people live in your home?
  - B. What is your favorite breakfast food?
  - C. How did you travel to school this morning?
  - D. How many minutes did it take you to get ready this morning?
  - E. What is the last thing you ate or drank?
4. Callen claims that players on the school basketball teams are taller than players on the soccer teams. Write two survey questions that Callen could ask to investigate the claim.  
Question 1: **Responses vary. How tall are you, in feet and inches?**  
Question 2: **Responses vary. What sports team(s) do you play on?**
5. Deven and Erendirani want to know about the types of food their classmates prefer. Write a survey question that would give them *categorical* data about their classmates' food preferences.  
**Responses vary.**
  - What kind of food do you like best?
  - Do you prefer sweet or savory food?

**Problems 6–7:** Here are some responses to the question: *What is your birthday?*

January 7

March 18

December 23

6. Jaylin is not sure whether the data is categorical or quantitative. Explain why this type of data is unclear.  
**Explanations vary. Birthdays can have numbers in them, but they don't work like regular numbers. Two people can also have the same birthday in different years.**
7. What is another question that might generate data that is unclear?  
**Responses vary. What is your favorite number? What is your area code?**

# Lesson Practice

A1.3.01

Name: ..... Date: ..... Period: .....

8. Think about your community. What information would you like to know about the people in it? Write an example of one question that will produce *quantitative* data and one question that will produce *categorical* data about your community. **Responses vary.**

Quantitative	Categorical
How many years have you lived in our community?	What is your favorite place in our community?

## Spiral Review

9. Determine the value of  $x$  that makes the equation  $5(3x - 2) = -55$  true.

$$x = -3$$

**Problems 10–11:** A scientist is studying how two different types of bacteria grow. This table represents the number of bacteria cells in the days since her experiment began.

	Day 0	Day 1	Day 2	Day 3	Day 4
Bacteria A (number of cells)	1	3	9	27	?
Bacteria B (number of cells)	80	100	120	140	?

10. If the patterns continue, which type of bacteria will there be more of after 4 days? **Bacteria B**

11. If the patterns continue, which type of bacteria will there be more of after 10 days? **Bacteria A**

**Problems 12–13:** Tamiya is organizing two sets of marbles.

12. How many marbles are in each set?

Set A: **40**

Set B: **25**

	Green Marbles	Marbles That Are Not Green
Set A	17	23
Set B	13	12

13. Across both sets, how many marbles does Tamiya have in total that are *not* green?  
**35 marbles**

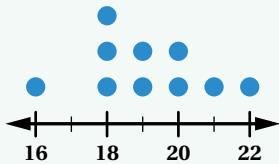
## Reflection

- Circle the problem you feel least confident about.
- Use this space to ask a question or share something you're proud of.

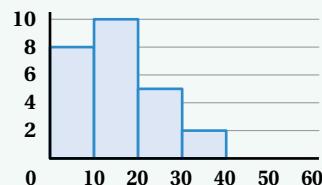
## Lesson Summary

It can be helpful to visualize quantitative data using dot plots or histograms.

*Dot plots* present each data point as a dot at its value on a number line. Dots with the same value are stacked on top of one another.



*Histograms* group data into rectangular bins. The height of each rectangle shows how many values are in that bin.



A dot plot is useful for observing the frequency of individual data, and the number of points in a data set. Histograms are useful for representing large data sets and observing the shape of a distribution.

## Things to Remember:

# Lesson Practice

## A1.3.04

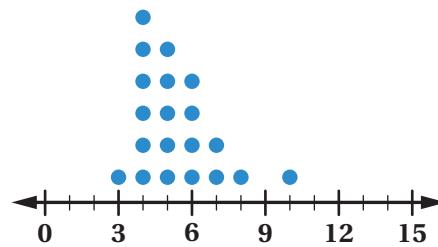
Name: ..... Date: ..... Period: .....

**Problems 1–2:** A class made this dot plot to explore how many letters are in their first names.

- How many letters are in the shortest first name?  
**3**
- Dyani says that most students have 10 letters in their first name.

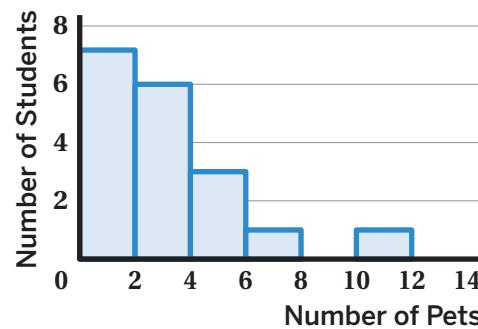
Is Dyani correct? Explain your thinking.

**No. Explanations vary. Most of the dots on the dot plot are located above 4, 5, and 6, not above 10.**

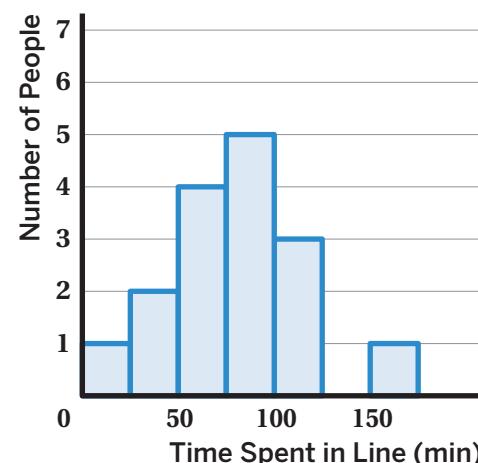


**Problems 3–5:** Jayden created a histogram about the number of pets students have.

- What is the bin width of the histogram?  
**2**
- How many students have less than 4 pets?  
**13 students**
- What is a number of pets that no students have?  
**Responses vary. 9 pets**



- A group of 15 friends went to a theme park for the day. Each person kept track of how many minutes they spent waiting in line. This histogram shows their data. Select all the statements that must be true.
  - A. 1 person was in line for at least 150 minutes.
  - B. 3 people were in line for less than 50 minutes.
  - C. Most people spent over 100 minutes in line.
  - D. 1 person spent 0 minutes in line.
  - E. No one spent exactly 130 minutes in line.



- Imani collected this data about the outdoor temperature, in degrees Fahrenheit, at noon over the last two weeks:

50.0	55.4	57.2	60.8	64.4	59.0	60.9
68.9	60.8	60.8	64.4	55.9	55.9	57.9

Would this data be best displayed using a dot plot or histogram? Explain your thinking.

**Histogram. Explanations vary. The data values involve a wide variety of numbers, including many decimals.**

# Lesson Practice

A1.3.04

Name: ..... Date: ..... Period: .....

## Spiral Review

**Problems 8–9:** Vihaan is investigating whether there is an association between wearing sneakers and participating on an athletic team. This table summarizes the data Vihaan collected from a survey of students.

	On Athletic Team	Not on Athletic Team	Total
Wear Sneakers	16%	17%	33%
Don't Wear Sneakers	32%	35%	67%
Total	48%	52%	100%

8. Interpret what 32% means in this situation.

**32% of students surveyed are on an athletic team and don't wear sneakers.**

9. Is there evidence of an association between wearing sneakers and participating on an athletic team?

**No. Explanations vary. The percent of students who are on an athletic team and wear sneakers, 16%, is very similar to the percent of students who are not on an athletic team and wear sneakers, 17%.**

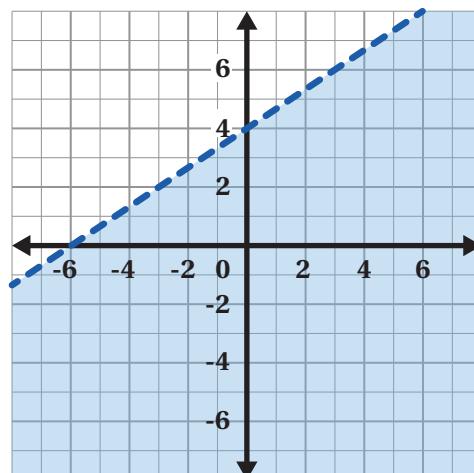
10. Which inequality is shown on the graph?

A.  $y > \frac{2}{3}x - 6$

B.  $y \leq -\frac{2}{3}x + 4$

C.  $y > \frac{2}{3}x + 4$

D.  $y < \frac{2}{3}x + 4$



11. What is the median of this data set? 2, 4, 5, 5, 7, 9, 10, 11, 11, 11, 21

**9**

## Reflection

- Put a smiley face next to the problem you learned from most.
- Use this space to ask a question or share something you're proud of.

### Lesson Summary

A box plot is one way to visualize quantitative data.

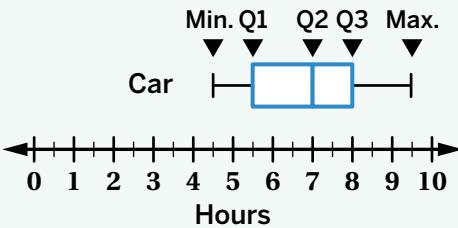
The data is divided into four sections using five values: the *minimum*, the *maximum*, and three *quartiles*. Quartiles divide a data set into four sections, or quarters. Each quarter represents 25% of the data.

Quartile 1 (Q1) is the median of the lower half of the data. Quartile 2 (Q2) is the median of the entire data set, which divides the data into two halves. Quartile 3 (Q3) is the median of the upper half of the data.

The minimum, maximum, median, and quartiles are all examples of statistics. A statistic is a single number that measures something about a data set.

Box plots are useful for representing large data sets, especially those with extreme values. They are less helpful for seeing individual data points.

Box plots can be used to make statements about percentages of data. This box plot represents data on road trip travel times. According to the box plot, 50% of travelers had a travel time of under 7 hours. However, 75% of the recorded travel times were less than 8 hours. The longest road trip recorded was 9.5 hours.



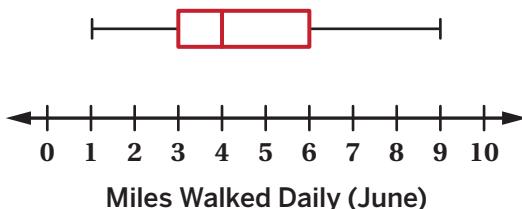
### Things to Remember:

# Lesson Practice

A1.3.05

Name: ..... Date: ..... Period: .....

**Problems 1–2:** Ricardo used a fitness app to track how many miles he walked each day in June. Here is a box plot of Ricardo's data.



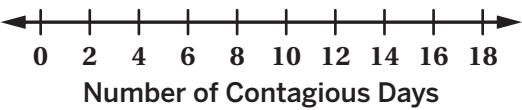
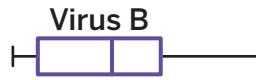
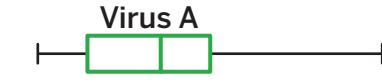
1. Determine each of these statistics:

Min	Q1	Median	Q3	Max
1	3	4	6	9

2. Select *all* of the questions that the box plot can answer.

- A. Did Ricardo walk more than 8 miles any day in June?
- B. How many times did Ricardo walk more than 8 miles in a day?
- C. Did Ricardo walk more on weekends or weekdays?
- D. About how often did Ricardo walk 4 miles or more in a day?
- E. Did Ricardo walk exactly 5 miles on any day in June?

**Problems 3–5:** A team of scientists wanted to know how long people sick with certain viruses were contagious. They studied 2 groups of 500 people and measured how many days each person was contagious.



3. Approximately what percentage of people with Virus B were contagious for more than 8 days?

- (A) 25%    (B) 50%    (C) 75%    (D) 33%

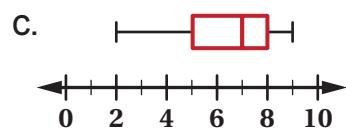
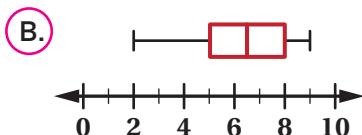
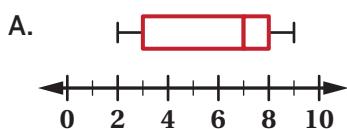
4. How do the two viruses compare?

*Responses vary. Generally, Virus A is contagious for a longer time than Virus B. There is also a wider range in the data for Virus A.*

5. Citlali says that the box plot for Virus A tells us that at least one person was contagious for exactly 14 days. Is Citlali correct? Explain your thinking.

*No. Explanations vary. The right section outside of the box tells us that about 25% of people were contagious for 10–17 days. It does not show us if any people were contagious for exactly 14 days.*

6. Determine which box plot represents the data set: 2, 3, 5, 5, 6, 6, 6, 7, 7, 7, 8, 8, 8, 9



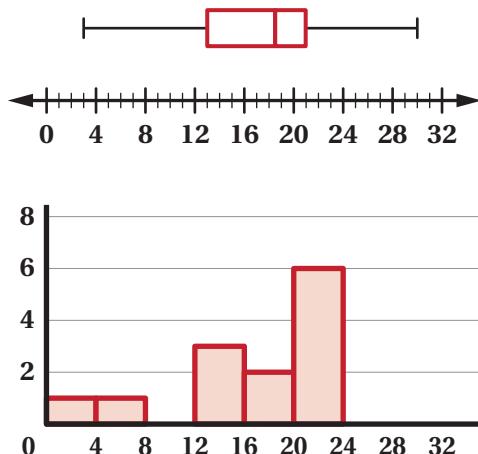
## Lesson Practice

A1.3.05

Name: ..... Date: ..... Period: .....

7. Here is a box plot and a histogram. Could these two representations have come from the same set of data? Explain your thinking.

No. Explanations vary. The box plot shows a maximum of about 30, but there is no data on the histogram in the 28–32 interval.

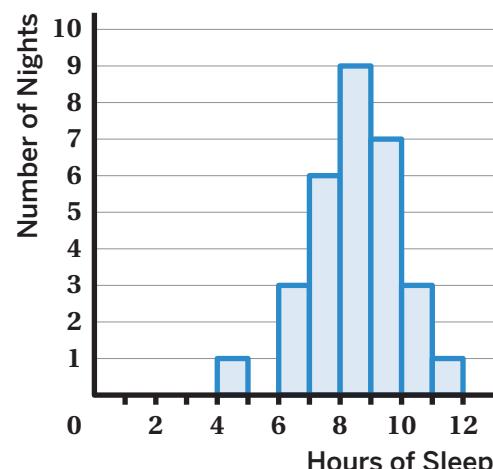


## Spiral Review

8. This histogram shows how many hours of sleep Dakota got each night in September.

How many nights did Dakota sleep less than 7 hours?

4 nights



9. Select all the ordered pairs that are solutions to the inequality,  $2x - 3y > 18$ .

- A. (6, -4)  
 B. (-4, 6)  
 C. (6, -2)  
 D. (0, 0)  
 E. (0, -8)

10. Calculate the mean of the data set: 1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15

8

## Reflection

- Put a star next to a problem where you revised your thinking.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

Here are some terms that can help us describe the *shape* of a data set.

- A data set is **bell-shaped** when most of the data is at the center and there are fewer points farther from the center. When presented in a dot plot or histogram, the data looks like a bell.
- A data set is **bimodal** when it has two very common data values. These appear in a dot plot or histogram as two peaks.
- A data set is **skewed** when more values are concentrated on one end of the data than the other.
- A data set is **symmetric** when you can draw a vertical line of symmetry through it.
- A data set is **uniform** when the data values are evenly distributed.

## Things to Remember:

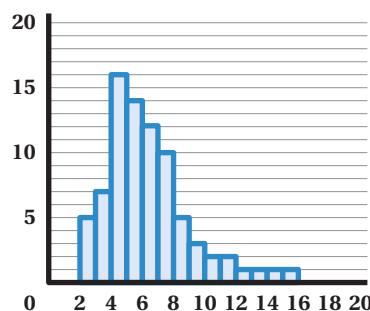
# Lesson Practice

A1.3.06

Name: ..... Date: ..... Period: .....

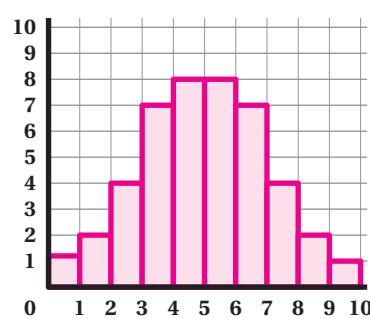
1. Which term best describes this histogram?

- A. Symmetric
- B. Skewed
- C. Uniform
- D. Bimodal



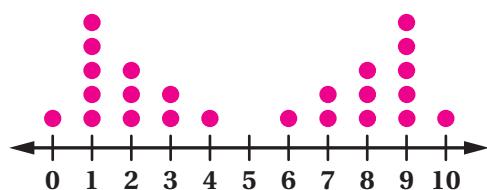
2. Create a histogram that is bell-shaped.

*Responses vary. Sample shown on graph.*



3. Create a dot plot that is *symmetric but not uniform*. Plot at least eight points.

*Responses vary. Sample shown on graph.*



## Spiral Review

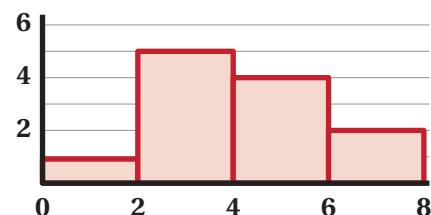
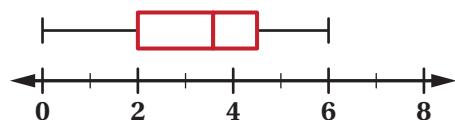
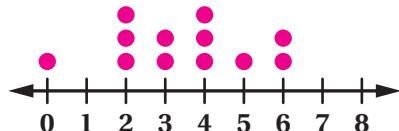
**Problems 4–5:** Here is a box plot and a histogram of the same data set.

4. What is the value of Q1?

**2**

5. Create a dot plot that could also represent this data set.

*Responses vary. Sample shown on graph*



# Lesson Practice

A1.3.06

Name: ..... Date: ..... Period: .....

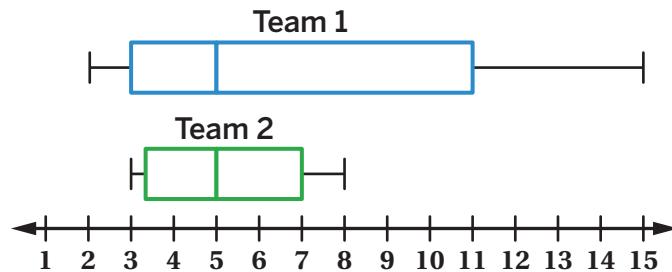
**Problems 6–7:** The players on two different football teams were surveyed about the number of hours they spend in the weight room each week. Here are the box plots for each team's data.

6. What is one *similarity* between the two data sets?

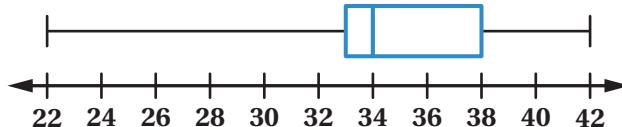
**Responses vary.** The median for both data sets is 5 hours.

7. What is one *difference* between the two data sets?

**Responses vary.** The Team 1 data is skewed, while the Team 2 data is more symmetric.



8. This box plot shows the number of points scored by a cross country team at 12 different meets.



What percentage of meets did the team score between 33 and 38 points?

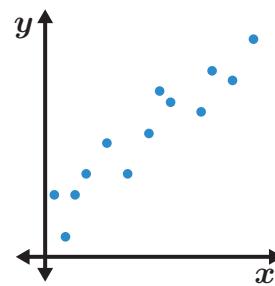
50%

9. Which equation is equivalent to  $5x + 3y = 30$ ?

- A.  $y = 10 - 5x$       B.  $y = 10 - \frac{5}{3}x$       C.  $y = 30 - 5x$       D.  $y = 10 + \frac{5}{3}x$

10. Select *all* the terms that describe the association in this scatter plot.

- A. Linear association  
 B. Non-linear association  
 C. Positive association  
 D. Negative association  
 E. No association



## Reflection

- Put a question mark next to a response you'd like to compare with a classmate's.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

A **measure of center** is a single number that summarizes all of the values in a data set.

*Mean* and *median* are measures of center that are used to describe a typical value of a data set.

- The mean is also called the average of a data set. To calculate the mean, you can add up all the data values, and divide by the number of data points.
- The median is the middle value of a data set when the values are in numerical order. If there are two values in the middle of the data set, then the median is the middle of those two values.

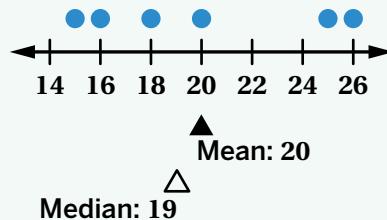
Here is one way to determine the mean of the data in this dot plot.

$$15 + 16 + 18 + 20 + 25 + 26 = 120$$

$$120 \div 6 = 20$$

The median is the value halfway between 18 and 20, so the median is 19.

When a data set includes extreme values that are much larger or smaller than most of the data, the value of the mean and median can be very different. Extreme values impact the mean more than they impact the median.

**Things to Remember:**

# Lesson Practice

A1.3.07

Name: ..... Date: ..... Period: .....

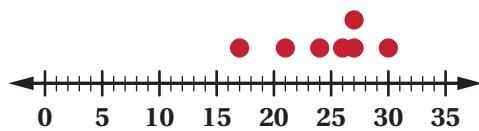
1. Determine the mean and median of each data set.

Data Set	Mean	Median
27, 30, 33	30	30
0, 100, 100, 100, 100	80	100
3, 5, 7, 15	7.5	6

2. Seven people estimated how many marbles there were in a jar. Determine the mean and median of the estimates.

Mean: 24.57

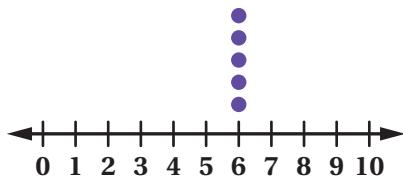
Median: 26



3. Here is a dot plot. If you added 4 and 9 to the data set, which statistic would change? Circle one.

Mean    Median    Both    Neither

Explain your thinking.

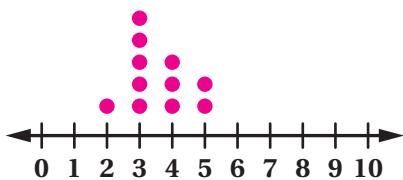


*Explanations vary. Since the total value and the number of points would change, the mean would change, too. The median wouldn't change because we'd be adding a value that's higher and a value that's lower than the group of points, so the middle point would still be in the middle.*

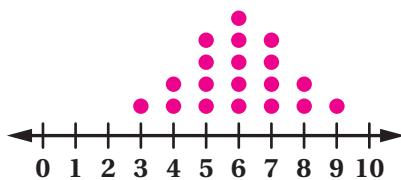
**Problems 4–5:** Create a dot plot that matches each description.

**Responses vary. Samples shown on graphs.**

4. A median of 3 and a mean that is greater than the median.



5. Bell-shaped, with a median of 6.



# Lesson Practice

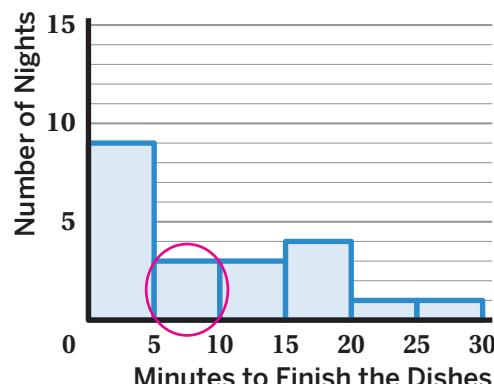
A1.3.07

Name: ..... Date: ..... Period: .....

**Problems 6–7:** For the past few weeks, Anand kept track of how long it took him to do the dishes each night. This histogram shows the results organized in 5-minute bins.

6. Circle the bin that contains the median.

*Response shown on histogram.*

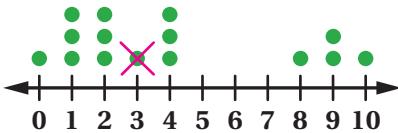


7. Explain how you know.

*Explanations vary.* The median is the middle night, so I counted the total number of nights and found that the median would be the 11<sup>th</sup> night. The first interval includes 9 nights, so the 11<sup>th</sup> night must be in the 5-to-10-minute interval.

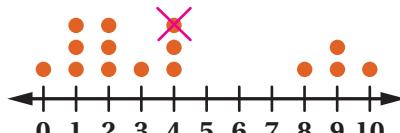
8. Remove one value to change the mean but not the median.

*Responses vary.*



9. Remove one value to change the median but not the mean.

*Responses vary.*



## Spiral Review

10. Select all the expressions that are equivalent to  $2(x + 3)$ .

- A.  $2x + 6$        B.  $2 \cdot x + 3$        C.  $x + 6$   
 D.  $(x + 3) \cdot 2$        E.  $2 \cdot x + 3 \cdot 2$

**Problems 11–12:** Evaluate each expression. Write your solution in scientific notation.

11.  $(2 \cdot 10^4)(6 \cdot 10^5)$

$1.2 \cdot 10^{10}$

12.  $\frac{3 \cdot 10^{-8}}{2 \cdot 10^{-3}}$

$1.5 \cdot 10^{-5}$

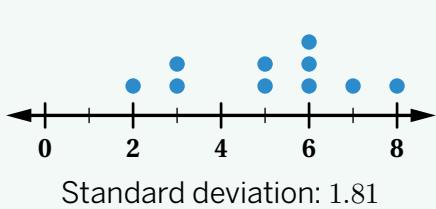
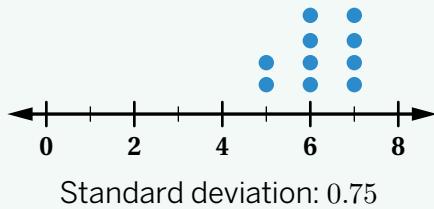
## Reflection

- Put a star next to the problem you understood best.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

The **standard deviation** of a data set is a **measure of spread**, or a way to measure how spread out the values in a data set are from its mean. Measures of spread can help you describe how consistent a data set is. If a data set is very consistent and much of the data clustered together, the standard deviation will be small. If the data is spread out, the standard deviation will be larger.

For example, compare these two dot plots.

**Dot Plot A****Dot Plot B**

The data in Dot Plot A has a greater standard deviation than the data in Dot Plot B because the data in A is more spread out, or variable. Since the data values in Dot Plot B are more consistent, B has a lower standard deviation.

One way to calculate the standard deviation of a data set is to use the Desmos Graphing Calculator. In the calculator, use the functions menu or type `stdevp( )` and then insert a list with the data values. For example, to calculate the standard deviation of Data Set A, you can type `stdevp([ 2, 3, 3, 5, 5, 6, 6, 6, 7, 8])`.

**Things to Remember:**

# Lesson Practice

## A1.3.08

Name: ..... Date: ..... Period: .....

**Problems 1–2:** Use a graphing calculator to determine the mean and standard deviation of each data set. Round your answer to the nearest hundredth.

1. 14, 14, 15, 16, 16, 18, 18, 19

Mean: ..... **16.25**.....

Standard Deviation: ..... **1.79**.....

2. 2, 3, 6, 6, 8, 8, 10, 11

Mean: ..... **6.75**.....

Standard Deviation: ..... **2.95**.....

**Problems 3–4:** Moon and Alejandro are both on the track team. Each day in May, they recorded the average distance they threw the javelin.

3. Based on the data, who is the more consistent thrower?

**Alejandro**

	Mean (ft)	Standard Deviation (ft)
Alejandro	128	1.2
Moon	130	2.6

4. If a throw of 160 feet was added to each data set, whose standard deviation would change more?

**Alejandro**

5. Order the data sets from smallest to largest standard deviation.

Use a graphing calculator to help with your thinking.

Set A	Set B	Set C
7, 10, 12, 14, 17, 19	2, 2, 5, 22, 25, 25	4, 8, 9, 10, 12, 13
Set C	Set A	Set B

**Smallest Standard Deviation**

**Largest Standard Deviation**

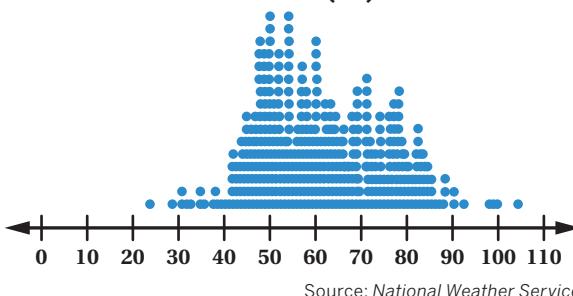
6. Here are the high temperatures in Seattle for each day of 2021. If the data points for the hottest and coldest temperatures were removed, would the standard deviation increase, decrease, or stay the same? Circle one.

Increase

**Decrease**

Stay the same

**High Temperatures in Seattle in 2021 (°F)**



Source: National Weather Service

# Lesson Practice

## A1.3.08

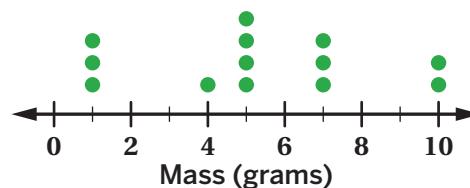
Name: ..... Date: ..... Period: .....

7. Here is a data set: 4, 5, 5, 6, 7, 9.

Select *all* of the changes that will make the standard deviation *smaller*.

- A. Add a 2 to the data set.  
 B. Add a 6 to the data set.  
 C. Remove the 4 from the data set.  
 D. Remove the 9 from the data set.  
 E. Increase each value by 10.

**Problems 8–9:** Here is a dot plot showing the weights of fish.



8. Circle the weight of fish that would change the standard deviation the *most* if added to the data set.

1 gram      4 grams      5 grams      7 grams      10 grams

9. Circle the weight that would change the standard deviation the *least* if added to the data set.

1 gram      4 grams      5 grams      7 grams      10 grams

## Spiral Review

**Problems 10–12:** Rewrite each expression as a single power.

10.  $(12^3)^5$

11.  $\frac{6^4}{6^5}$

$12^{15}$

$6^{-1}$

12.  $5^7 \cdot 5^2 \cdot 5$

$5^{10}$

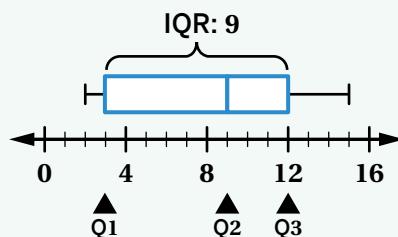
## Reflection

- Put a star next to a problem you could explain to a classmate.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

The *interquartile range* (IQR) is a statistic that measures the spread of a data set. In a box blot, the IQR is the width of the box. Measures of spread, like IQR, help us determine how consistent the data within a set is. The IQR represents the middle half of the data set, and we calculate it by determining the distance from Q1 to Q3. This makes IQR a more resistant measure of spread for skewed data than standard deviation.

Here is an example of a data set where  $Q_1 = 3$  and  $Q_3 = 12$ . The IQR of this data set is 9, because  $12 - 3 = 9$ .

**Things to Remember:**

# Lesson Practice

## A1.3.10

Name: ..... Date: ..... Period: .....

1. Create two different data sets, each with 5 numbers or less and a median of 13.

*Responses vary.*

Data Set 1: **2, 7, 13, 15, 30**

Data Set 2: **0, 13, 13, 20**

**Problems 2–4:** These box plots represent the points scored per game for three different football teams over the course of a season.

2. Which team generally scored the most points?

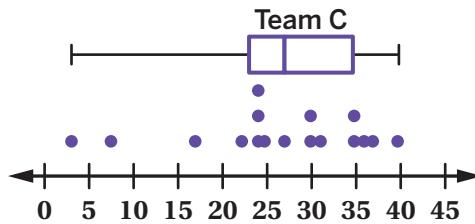
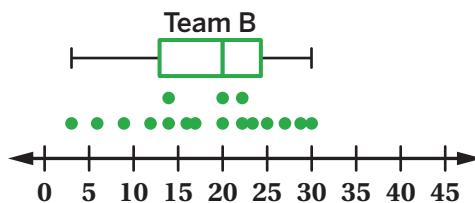
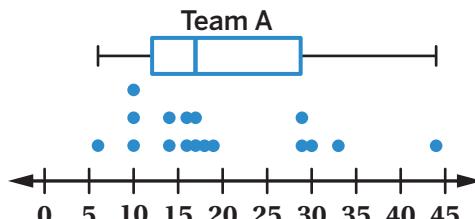
**Team C**

3. Which team scored the most points in a single game?

**Team A**

4. Which team was the most consistent?  
Explain your thinking.

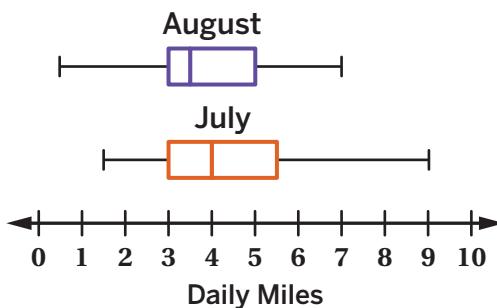
**Team B.** *Explanations vary.* The spread is smallest for Team B, which means the scores are the most consistent.



**Problems 5–6:** Mariana used a fitness app to track how many miles she walked each day in July and August. Here is a box plot of her data.

5. Which month has a smaller IQR?

**August**



6. What does the IQR tell you about Mariana's walking habits in July and August?

*Responses vary.*

- The distance that Mariana walked each day in August was more consistent.
- There is more variability in the walk data for July.

## Lesson Practice

A1.3.10

Name: ..... Date: ..... Period: .....

7. Here is a data set: [2, 2, 4, 4, 5, 5, 6, 7, 9, 15].

If you added 24 as a new data point, how would the IQR change?  
Explain your thinking.

The IQR would increase. Explanations vary. Adding 24 would increase the spread of the whole data set, which would also increase the IQR.

8. Here is a different data set: [3, 6, 7, 23].

Determine a number you could add to make an IQR of 15.

16 or -3

## Spiral Review

**Problems 9–11:** For each equation, write the positive solution as a whole number or by using square root or cube root notation.

9.  $t^3 = 729$

$t = 9$

10.  $a^2 = 27$

$a = \sqrt{27}$

11.  $m^3 = 12$

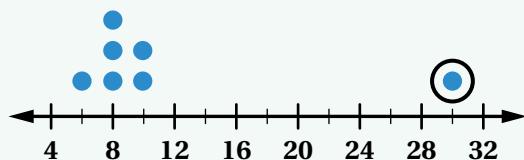
$m = \sqrt[3]{12}$

## Reflection

- Put a star next to the problem you understood best.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

Outliers are data values that are far from the other values in the data set. In this data set, the circled data point is an outlier.



You can identify outliers using dot plots, box plots, and technology tools such as graphing calculators. You can also identify outliers using the *IQR*. Outliers are values further than 1.5 times the IQR below Q1 or above Q3.

When deciding which measure of center is appropriate to represent a data set, it's important to identify any outliers. Outliers have a big impact on the *mean*, but they don't impact the *median* very much.

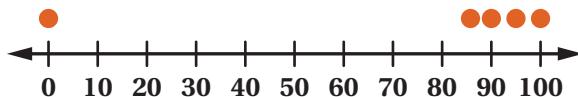
## Things to Remember:

# Lesson Practice

## A1.3.11

Name: ..... Date: ..... Period: .....

**Problems 1–2:** Ricardo got the following scores on five class assignments: 87, 90, 0, 95, and 100. Ricardo's teacher lets students decide whether their final score will be the mean or the median of those five scores.



1. Would using the mean or the median give Ricardo the higher final score?

Median

2. Explain your thinking.

*Explanations vary. Ricardo did really well on 4 of his 5 assignments, so reporting the median would give him the higher score.*

**Problems 3–4:** This histogram represents the number of text messages that 20 people received in one day.

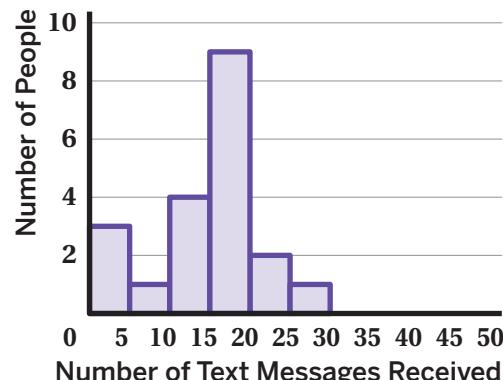
3. How many people received 15 or more text messages in one day?

12 people

4. If you added a person who received 52 text messages to the data set, would the mean or median change more? Circle one.

Mean

Median



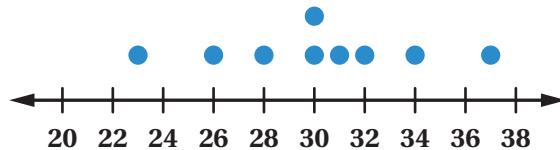
Explain your thinking. *Explanations vary. The mean will change more because 52 is much bigger than the rest of the numbers, so it will increase the total number of text messages by a lot but only increase the number of data points by 1.*

5. Here is a data set:

26, 30, 31, 32, 28, 30, 34, 37, 23.

Julian says that 37 is an outlier because it is the maximum value. What could you say to Julian to help him understand his mistake?

*Responses vary. I would tell Julian that the maximum is not always an outlier and that he can use a graphing calculator to determine whether or not any point is an outlier.*



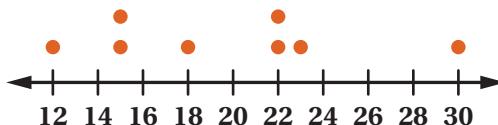
## Lesson Practice

A1.3.11

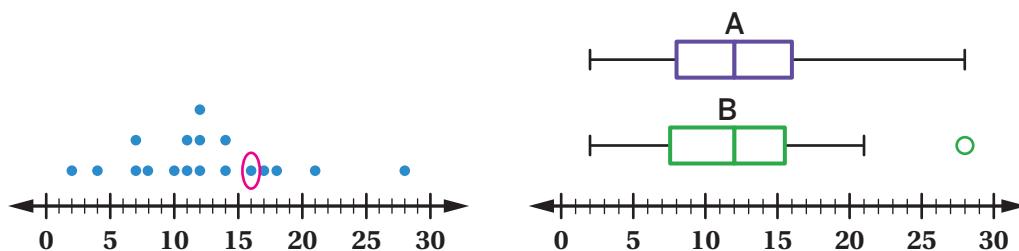
Name: ..... Date: ..... Period: .....

6. This dot plot represents the ages of different people in a bike shop. Which data point would be an outlier if it were added to the data set?

A. 44      B. 8      C. Both      D. Neither



7. The dot plot represents the data in box plot A. Circle one point to remove from the dot plot so that it represents box plot B instead. *Responses vary.*



## Spiral Review

**Problems 8–10:** Evaluate each expression. Write your answer as a fraction.

8.  $\frac{2}{3} + \frac{1}{5} = \frac{13}{15}$

9.  $\frac{2}{5} - \frac{3}{8} = \frac{1}{40}$

10.  $\frac{6}{15} + \frac{8}{20} = \frac{4}{5}$

## Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

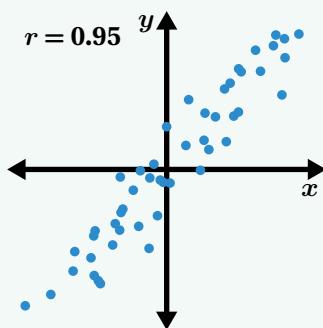
**Lesson Summary**

A *scatter plot* is a graph of plotted points that shows the relationship, or association, between two variables. When there is a linear association between the variables, you can describe the *strength* and *direction* of the association using the **correlation coefficient** (also called the **r-value**). The correlation coefficient is a number between -1 and 1 that describes the strength and direction of a linear association between two variables in a scatter plot.

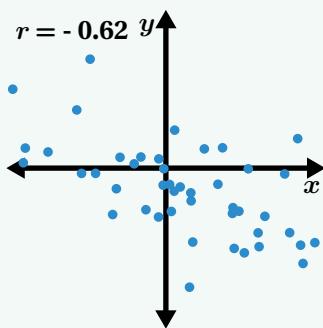
The closer the *r*-value is to 0, the weaker the linear association. The closer the *r* value is to -1 or 1, the stronger the linear association.

These graphs show examples of different associations and their correlation coefficients.

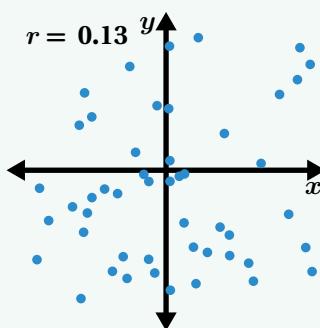
**Positive Association  
Strong Association**



**Negative Association**



**Positive Association  
Weak Association**

**Things to Remember:**

# Lesson Practice

A1.3.13

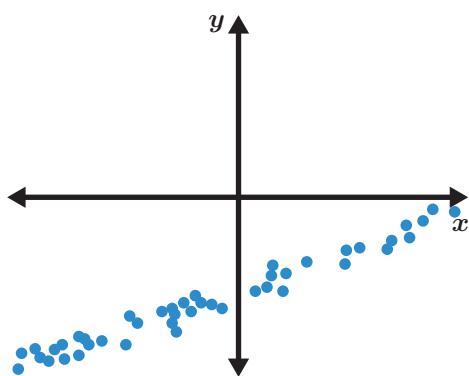
Name: ..... Date: ..... Period: .....

**Problems 1–4:** Determine whether each scatter plot has a strong linear relationship, weak linear relationship, or no linear relationship.

1. **Strong**

Weak

None

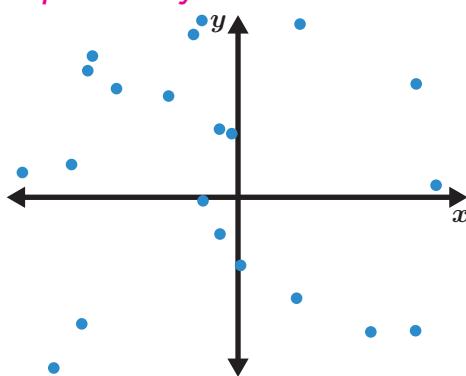


2. Strong

*Responses vary.*

Weak

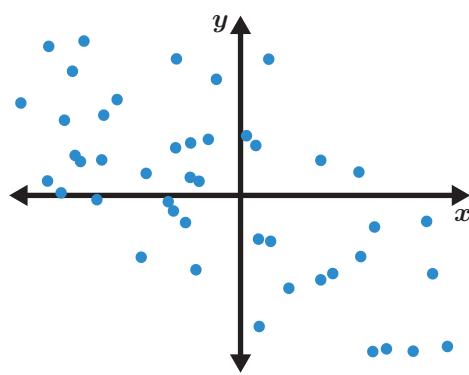
None



3. Strong

**Weak**

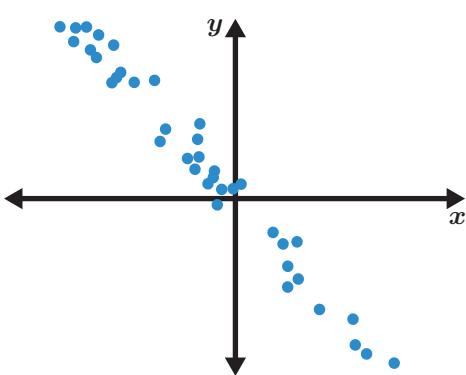
None



4. **Strong**

Weak

None

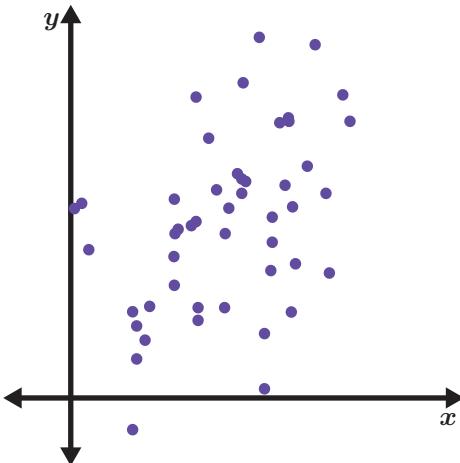


5. Which number could be the correlation coefficient for this scatter plot?

- A. 0.4      B. -0.4  
C. 0.9      D. -0.9

Explain your thinking.

*Explanations vary. The correlation is not that strong, so it has to be 0.4 or -0.4. I picked 0.4 because the trend is going in the direction of a positive slope.*



6. A scatter plot has a correlation coefficient of  $r = 0.85$ . What does this tell you about the data? *Responses vary. This tells me that the data has a strong positive correlation.*

# Lesson Practice

A1.3.13

Name: ..... Date: ..... Period: .....

7. Fill in each blank using the numbers 0 to 9 only once to create a set of four points that have a correlation coefficient between -0.1 and 0.1. Use the a graphing calculator to help with your thinking.

*Responses vary.*

$$\left( \boxed{1}, \boxed{9} \right), \left( \boxed{2}, \boxed{3} \right)$$

$$\left( \boxed{5}, \boxed{8} \right), \left( \boxed{4}, \boxed{6} \right)$$

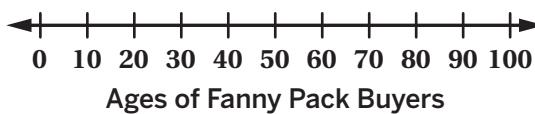
## Spiral Review

8. An online store is curious about who buys the fanny packs they sell.

Use the box plots to make a claim about the ages of fanny pack buyers this year compared to ten years ago.

*Responses vary.*

- The median age is lower this year than it was ten years ago, so there are more young people buying fanny packs.
- The ages are more consistent this year than ten years ago.
- This year there is an outlier, which means there is one person who is significantly older than the rest of the people who bought fanny packs.



**Problems 9–10:** Evaluate each expression. Write your answer as a fraction.

9.  $\frac{3}{16} \cdot 8 = \frac{3}{2}$  (or equivalent)

10.  $\frac{5}{6} \cdot \frac{3}{4} \cdot \frac{14}{10} = \frac{7}{8}$  (or equivalent)

## Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

### Lesson Summary

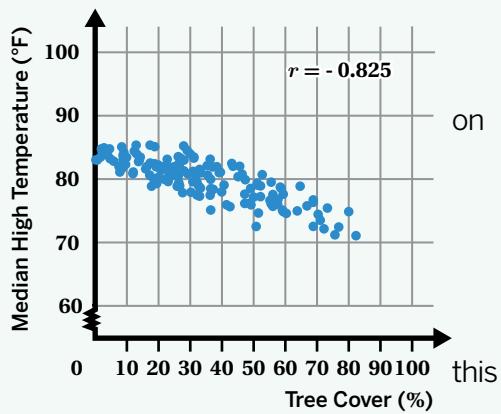
You can use a correlation coefficient to analyze the relationship between two variables and determine whether there is an association between them. The correlation coefficient, or *r*-value, describes the strength and direction of the relationship that may exist between two variables.

- A positive *r*-value means that as one variable increases, the other variable also increases.
- A negative *r*-value means that as one variable increases, the other variable decreases.
- The closer the *r*-value is to 1 or -1, the stronger the correlation.

People may use correlations in data to understand and address issues in their community.

For example, this scatter plot shows data tree cover and temperature for 150 blocks in Detroit, Michigan.

The *r*-value is -0.825. This means there is a negative and strong relationship between the amount of tree cover and median high temperature in Detroit neighborhoods. Community members may use correlation to advocate for more trees to be planted in different neighborhoods across the city.



### Things to Remember:

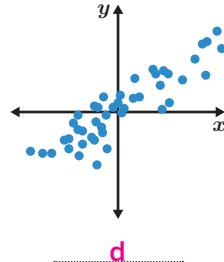
# Lesson Practice

## A1.3.14

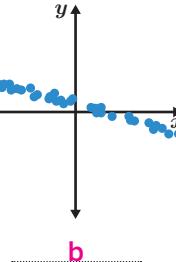
Name: ..... Date: ..... Period: .....

1. Match each scatter plot to its  $r$ -value.

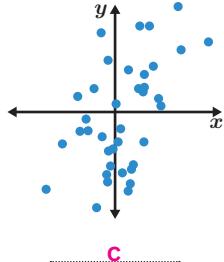
a.  $r = -0.7$



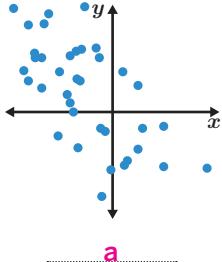
b.  $r = -0.99$



c.  $r = 0.7$



d.  $r = 0.9$



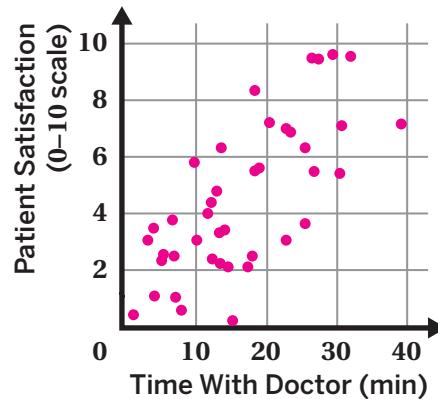
2. A medical clinic wanted to know about the experiences of their patients. They looked at the following variables:

- The number of minutes spent with a doctor.
- Patient satisfaction (on a 0-10 scale).

They found the variables have a *weak, positive relationship*.

Make a scatter plot that could represent this data.

*Responses vary. Sample shown on graph.*



3. Noah creates a scatter plot showing the relationship between the number of free throws taken in a basketball game and the final score. The correlation coefficient for the line of best fit is 0.76.

Are the number of free throws and the final score correlated? Explain your thinking.

*Explanations vary. A correlation coefficient of 0.76 shows a correlation. Also, the variables should be correlated because more free throws create more chances to score points.*

**Problems 4–6:** Martina is interested in learning about different education systems around the world. Martina found data about two variables:

- The average class size.
- The scores on a standardized test.

Here is a scatter plot for the data.

4. The point  $(18.369, 555)$  represents Slovakia. What do the coordinates tell you about Slovakia?

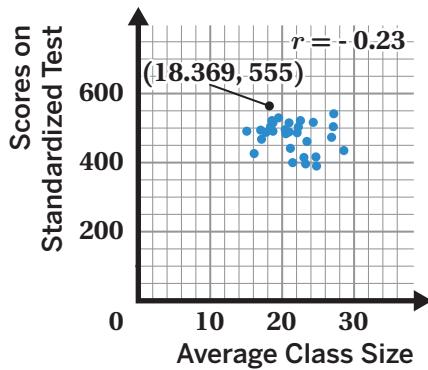
*Responses vary. The coordinates tell me that the average class size in Slovakia is about 18 students and the average score on a standardized test is 555.*

5. Based on the  $r$ -value, what relationship is there between the variables? Circle one.

Positive    Negative    None

6. What is the strength of the relationship? Circle one.

Weak    Strong



Source: OECD

# Lesson Practice

## A1.3.14

Name: ..... Date: ..... Period: .....

7. The correlation coefficient for a given set of data is  $r = 0.962$ . Select all of the conclusions you can make about the data.

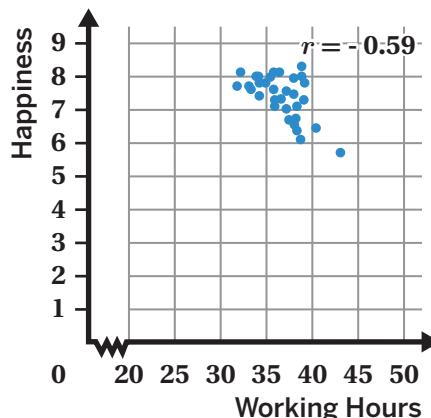
- A. There is a strong association between the two variables.
- B. There is a weak association between the two variables.
- C. There is no association between the two variables.
- D. As one variable *increases*, the other variable *increases*.
- E. As one variable *increases*, the other variable *decreases*.

8. Saanvi is interested in learning more about life in different countries. She found data about two variables:

- Average happiness (on a 1–10 scale).
- The average number of hours people worked in a week.

What does the  $r$ -value tell you about the relationship?

**Responses vary. The  $r$ -value is negative, so it tells me that the more hours someone works, the lower their average happiness tends to be.**



Sources: ILOSTAT and OECD

## Spiral Review

**Problems 9–10:** This table shows the statistics for Rio's quiz scores in math class and science class.

	Median (%)	Mean (%)	Standard Deviation (%)	IQR (%)
Math Class	85	85	7.5	6
Science Class	80	85.5	13.4	30

9. Were Rio's quiz scores more consistent in math or science class? Explain your thinking.

**Math class. Explanations vary. Both the standard deviation and the IQR are smaller for math class.**

10. If Rio scored 65% on a math quiz, how would the standard deviation change? Circle your choice and explain your thinking.

Increase

Decrease

No change

**Explanations vary. The standard deviation would increase because 65% is far from the median and the mean.**

## Reflection

1. Put a smiley face next to a problem you were stuck on and then figured out.
2. Use this space to ask a question or share something you're proud of.

### Lesson Summary

While the correlation coefficient can help you understand the general relationship between two variables, a *line of fit* can help you make predictions about specific values in a data set. Points along the line of fit represent the likely value of unknown data in the data set.

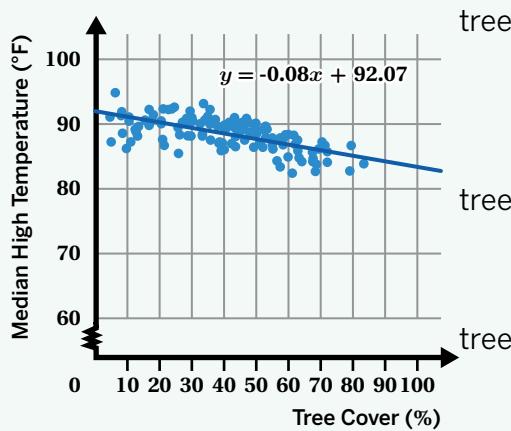
You can use the equation of the line of fit to better understand the data. The  $y$ -intercept represents a potential initial value and the slope of the line describes the rate that the variables change in relationship to one another.

For example, this scatter plot shows data on cover and temperature for 150 blocks in Austin, Texas. The equation of the line of fit is  $y = -0.08x + 92.07$ .

The slope is  $-0.08$ . This means that when the cover increases by 1% in Austin, the predicted temperature decreases by  $0.08^{\circ}\text{F}$ .

The  $y$ -intercept is  $92.07$ . This means that if the cover in Austin is 0%, the predicted temperature is  $92.07^{\circ}\text{F}$ .

You can use the line to predict that if a block in Austin has 80% tree cover, the temperature will be about  $85^{\circ}\text{F}$ .



### Things to Remember:

# Lesson Practice

A1.3.15

Name: ..... Date: ..... Period: .....

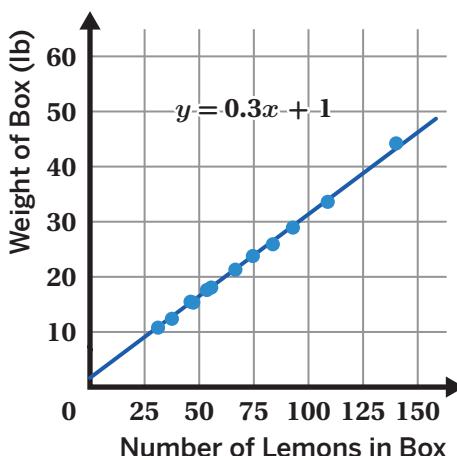
**Problems 1–2:** A store receives 12 boxes that contain many different numbers of lemons. They weigh each box and then count the lemons. Here is a scatter plot of that data.

1. A new box weighs 30 pounds. Approximately how many lemons are in the box?

**95 lemons. Responses between 90 and 97 are considered correct.**

2. The equation for the line that best fits this data is  $y = 0.3x + 1$ . What do the numbers 0.3 and 1 mean in this context? **Responses vary.**

**The number 0.3 means that as you add each new lemon, the weight of the box tends to increase by 0.3 pounds. The number 1 means that the weight of the box is 1 pound before any lemons are added.**



**Problems 3–4:** The store collects data about the weight of other types of fruit boxes. The table shows equations for the line of best fit for each type of fruit box, where  $y$  represents the weight of a box and  $x$  represents the number of fruits in a box.

3. Which fruit is heavier? Circle your choice.

Lemons **Pomegranates** They are the same

Fruit	Line of Best Fit
Orange	$y = 0.45x + 0.95$
Lemon	$y = 0.3x + 1$
Pomegranate	$y = 1.05x + 1.15$
Mango	$y = 0.85x + 1.25$

Explain your thinking.

**Explanations vary. I can tell that pomegranates are heavier because the slope of the line of fit for pomegranates is higher, so the weight of a box of pomegranates increases more each time one is added.**

4. Order these fruits by their weight.

Lemon
Orange
Mango
Pomegranate

**Lightest**

**Heaviest**

5. Sora collects data about the number of bananas she buys at the store,  $x$ , and the total weight of the bananas in pounds,  $y$ . If Sora graphed her data, which value would be closest to the slope of the line of best fit?

- A. -4  
**C. 0.4**  
D. 4

# Lesson Practice

A1.3.15

Name: ..... Date: ..... Period: .....

**Problems 6–8:** A restaurant gathered data about how long customers had to wait and how many staff members were working. The slope of the line of fit is  $-1.62$ . The  $r$ -value is  $-0.9$ .

6. What does the slope of  $-1.62$  mean in this situation?

*Responses vary.*  $-1.62$  means that for each additional staff member working, the customer wait time tends to decrease by 1.62 minutes.

7. What does the  $r$ -value of  $-0.9$  mean in this situation?

*Responses vary.* The number  $-0.9$  means that there is a strong negative correlation between the number of staff and wait time.

8. Madison says that for any scatter plot, if the  $r$ -value is negative, then the slope must also be negative. Is this true or false? Circle one.

True

False

Explain your thinking.

*Explanations vary.* The  $r$ -value and the slope of the line of fit both describe the trend of the data, so if the data shows a negative trend, they must both be negative.

## Spiral Review

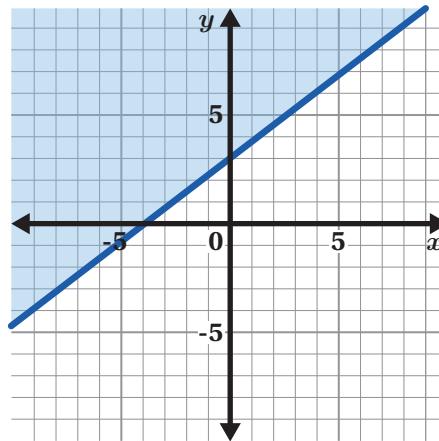
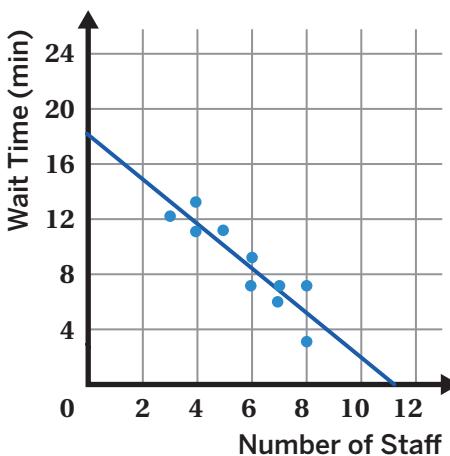
9. Select all the coordinate pairs that are solutions to  $y = -\frac{1}{2}x + 10$ .

- A.  $\left(0, \frac{2}{3}\right)$     B.  $(-10, 15)$     C.  $(4, 12)$     D.  $(2, 9)$     E.  $(5, 8)$

10. Here is the graph of the inequality  $y \geq 0.75x + 3$ .

Select all the coordinate pairs that are solutions to this inequality.

- A.  $(-9, -5)$   
 B.  $(-3, 2)$   
 C.  $(0, 3)$   
 D.  $(2, -3)$   
 E.  $(6, 8)$



## Reflection

- Put a heart next to the problem you feel most confident about.
- Use this space to ask a question or share something you're proud of.

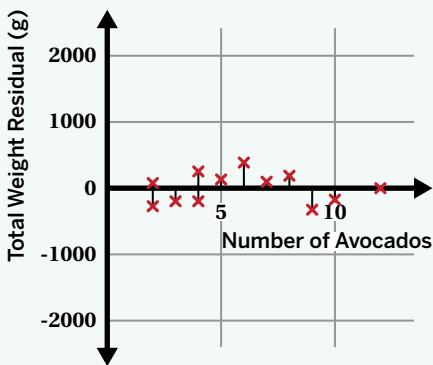
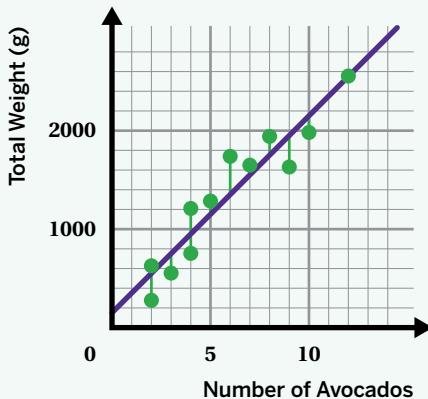
### Lesson Summary

You can use residuals to determine how well a line fits a data set. A **residual** is the difference between the actual y-value of a data point and the value predicted by the line of best fit.

Here is a scatter plot with data on the number of avocados and their weights. The residuals are represented with lines connecting each point to the line of best fit.

You can also create a residual plot to analyze how well a line fits a data set. A **residual plot** is a scatter plot of residual values for a data set.

Here is the residual plot of the graph of avocado weights. The closer a point is to the x-axis, the closer that point is to the line of best fit. A line is a good fit for the data if the points on the residual plot are close to the x-axis and are randomly dispersed above and below the axis.



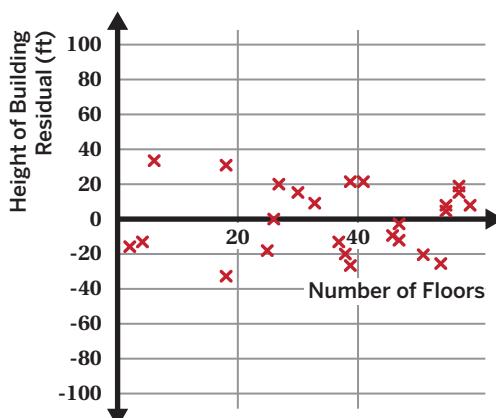
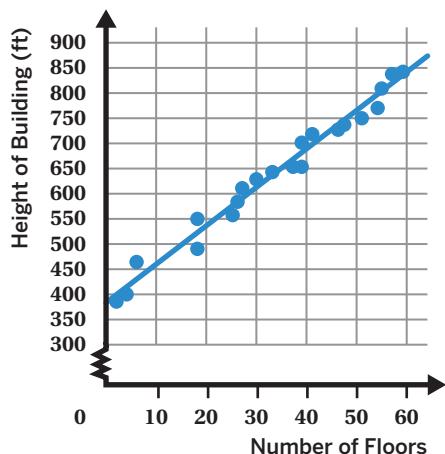
### Things to Remember:

# Lesson Practice

A1.3.16

Name: ..... Date: ..... Period: .....

**Problems 1–2:** The scatter plot shows the heights of a group of buildings, the number of floors in each building, and a line that best fits the data. The residual plot is also shown.



- Predict the height of a building that has 50 floors.

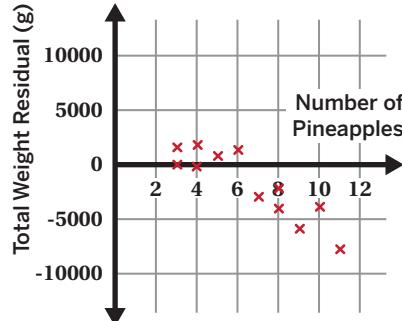
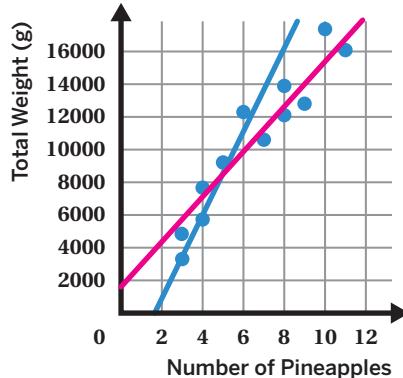
**Responses between 750 and 770 feet are considered correct.**

- How can you tell that the graphed line is a good fit for the data? Use the residual plot if it helps with your thinking.

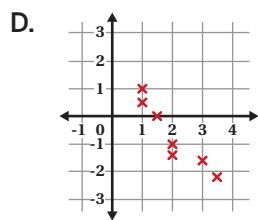
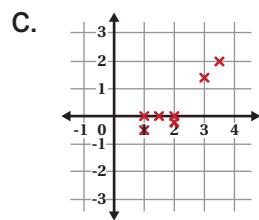
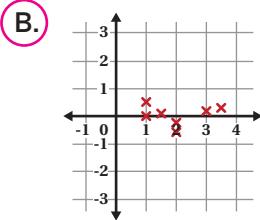
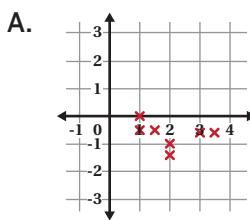
**Explanations vary. The line is a good fit because the points on the residual plot are close to the  $x$ -axis, and they are both above and below the  $x$ -axis.**

- Here is a scatter plot and its corresponding residual plot.

Draw a better line of fit on the scatter plot.



- These residual plots are from the same set of data, but each one represents a different line of fit. Which residual plot shows the best line of fit?



Explain your thinking.

**Explanations vary. All of the values in the graph I chose are close to the  $x$ -axis. The other three graphs have some points that are much higher or much lower.**

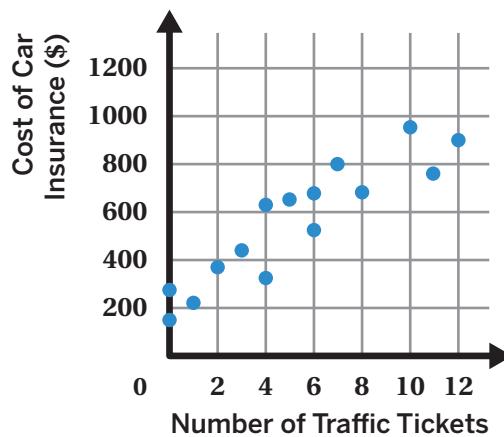
# Lesson Practice

A1.3.16

Name: ..... Date: ..... Period: .....

**Problems 5–6:** This scatter plot shows the number of traffic tickets and the cost of car insurance for 16 people.

5. Which  $r$ -value could represent the correlation coefficient for this data?
- A. 0.25      B. -0.25  
**C.** 0.9      D. -0.9



6. Which equation could represent the line of best fit?
- A.  $y = -62x + 220$       **B.**  $y = 62x + 220$       C.  $y = -220x + 62$       D.  $y = 220x + 62$

Explain your thinking.

*Explanations vary.*

- I drew a line to fit the data. My line started near 250 and had a positive slope, so I picked  $y = 62x + 220$ .
- I can see that the trend has a positive slope but is not steep enough to be increasing by \$220 for each ticket, so I picked  $y = 62x + 220$ .

## Spiral Review

**Problems 7–9:** Solve each equation for  $x$ .

7.  $y = 3x + 5$

$$\frac{y-5}{3} = x$$

8.  $4y - 2x = 10$

$$x = -5 + 2y$$

9.  $y = \frac{1}{6}x$

$$6y = x$$

10. Are there any outliers in this data set? Use a calculator if it helps with your thinking.

[1, 1, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 20]

Circle one. **Yes** No

Explain your thinking.

*Explanations vary.*

- I used the Desmos Graphing Calculator to create a box plot. When I clicked “Exclude outliers,” I saw that 20 was an outlier.
- 20 is an outlier because it’s 14 units away from the next highest number, and 14 units is bigger than the spread of the rest of the data.

## Reflection

- Put a star next to the problem you think is the most important.
- Use this space to ask a question or share something you’re proud of.

### Lesson Summary

The **line of best** fit is the line on a scatter plot that best represents the trend created by the points in a data set.

Instead of sketching a line of fit, you can use a graphing calculator to precisely generate the equation of the line of best fit from a scatter plot. The equation of the line of best fit can help you interpret information about a situation, or allow you to substitute values into the equation to make predictions. You can also use a graphing calculator to calculate the correlation coefficient for a given data set.

Here is an example of a line of best fit generated by a graphing calculator, and the information about the line that a graphing calculator will show you.

$$y = 0.351312x + 1.31984$$

STATISTICS

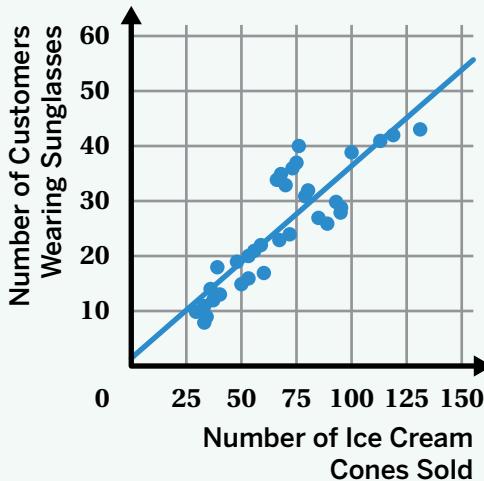
$$r_2 = 0.7642$$

$$r = 0.8742$$

RESIDUALS

plot residuals

plot residuals



### Things to Remember:

# Lesson Practice

A1.3.17

Name: ..... Date: ..... Period: .....

**Problems 1–3:** This scatter plot shows the distances of some one-way flights and their ticket prices.

- The equation for the line of best fit is  $y = 0.13x + 224$ .

What does 0.13 mean in this situation?

**0.13 means that for every mile that the distance increases, the price of a ticket tends to increase by \$0.13.**

- The distance from Phoenix, Arizona, to Jacksonville, Florida, is 1,795 miles.

Use the equation of line of best fit to predict the cost of a plane ticket from Phoenix to Jacksonville.

**\$457.35**

- Do you think the prediction is accurate? Use the  $r$ -value to explain your thinking.

**Responses and explanations vary. I don't think that the prediction is accurate because the  $r$ -value is only 0.43, which is not a strong correlation.**

**Problems 4–6:** This scatter plot shows the distances of some non-stop flights and their lengths in minutes.

- The equation for the line of best fit is  $y = 0.12x + 37$ .

What does 0.12 mean in this situation?

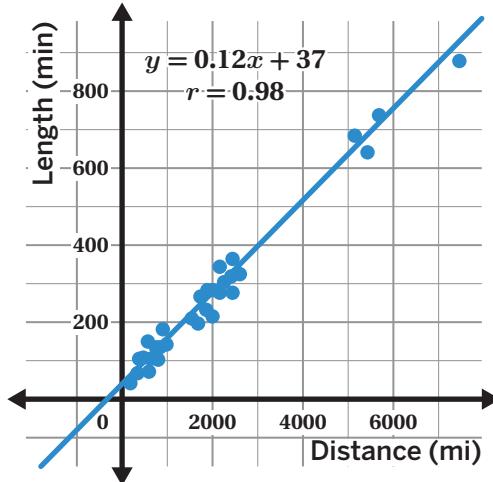
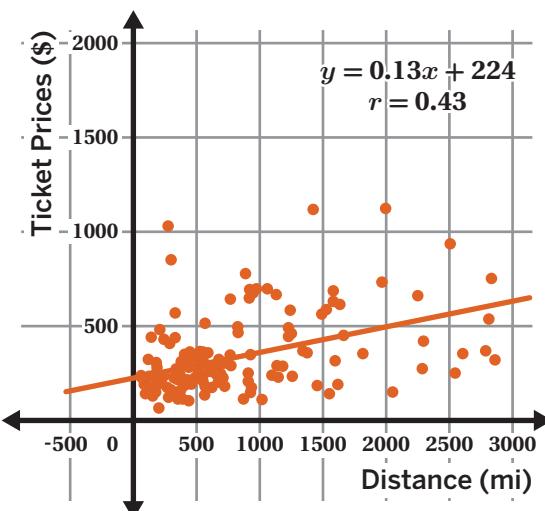
**0.12 means that for every mile that the distance increases, the length of a flight tends to increase by 0.12 minutes.**

- Use the equation of line of best fit to predict the length, in minutes, of a direct flight from Phoenix to Jacksonville (1,795 miles).

**252.4 minutes (or 4.2 hours)**

- Do you think the prediction is accurate? Use the  $r$ -value to explain your thinking.

**Responses and explanations vary. Yes, I think that the prediction is accurate because the  $r$ -value is 0.98, which shows a very strong linear association.**



# Lesson Practice

A1.3.17

Name: ..... Date: ..... Period: .....

**Problems 7–9:** Kwasi wanted to know the relationship between the ages of cars and their values. He found some data on the ages of several cars (in years) and their sale prices (in dollars).

7. Describe the relationship between the age of a car and its sale price.

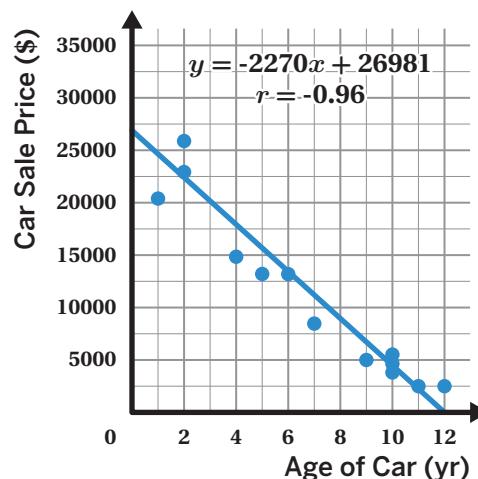
*Responses vary. As the age of the car increases the sales price decreases.*

8. Do you think one of the variables causes the other? Explain your thinking.

*Responses and explanations vary. Yes. As cars get older it makes sense that they would sell for a lower price.*

9. What else might affect this relationship? Explain your thinking.

*Explanations vary. The mileage on the car and its maintenance would also affect the sale price.*



## Spiral Review

**Problems 10–11:** Here is a data set: [1, 1, 2, 2, 3, 3, 7, 8, 9, 10, 11, 35]

10. Complete the table. Use a graphing calculator if it helps with your thinking.

Min.	Q1	Median	Q3	Max.
1	2	5	9.5	35

11. Are there any outliers in this data set? Circle one.  Yes  No

**Problems 12–14:** Solve each equation for  $y$ .

12.  $7 = 6x - y$

$y = 6x - 7$

13.  $3y + 15x = 24$

$y = 8 - 5x$

14.  $4y - x = 44$

$y = \frac{1}{4}x + 11$

## Reflection

- Circle the question that you are least confident about.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

We can represent rules with verbal descriptions or as a table of inputs and outputs.

All sets of inputs and outputs are called relations. A *function* is a special kind of relation that assigns exactly one output to each possible input.

You can determine whether a rule is a function by organizing the inputs and outputs into a table. If one input has multiple possible outputs, then the rule is not a function.

Here are two examples.

**Rule A** takes an integer and outputs an integer that is one less.

Input	Output
1	0
2	1
2	1
4	3

In this relationship, Rule A is a function because each input has exactly one output.

**Rule B** takes a number and outputs a random number that is greater.

Input	Output
0	2
0	10
-2	0
-1.6	-1.2

In this relationship, Rule B is *not* a function because each input has multiple outputs.

**Things to Remember:**

# Lesson Practice

A1.4.01

Name: ..... Date: ..... Period: .....

1. Rule A takes any word as an input and writes the word backwards as an output.

Is Rule A a function?  
Explain your thinking.

**Yes. Explanations vary.** Rule A is a function because any word written backwards results in only one sequence of letters.

Input	hat	sock	racecar
Output	tah	kcos	racecar

Problems 2–3: Here is Rule B.

2. Is Rule B a function?  
Explain your thinking.

**No. Explanations vary.** Rule B is not a function because there are two different outputs for the input 6.

Input	4	6	6	5
Output	blue	purple	yellow	white

3. Predict what the output could be when the input is 3.

**Responses vary.** It can be any three-letter color, such as red.

Problems 4–5: Here is Rule C.

4. Is Rule C a function?  
Explain your thinking.

**Yes. Explanations vary.** Rule C is a function because every input only has one output.

Input	6	8	12	14
Output	4	6	10	?

5. Predict the missing output for Rule C. **12**

6. This table shows the total number of days in each month of a given year.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
28		X										
29		X										
30				X		X			X		X	
31	X		X		X		X	X		X		X

Imagine a rule where the input is a month and the output is the number of days in that month. Does this rule represent a function? Explain your thinking.

**No. Explanations vary.** This rule does not represent a function because the input February has two possible outputs of either 28 or 29 days.

# Lesson Practice

A1.4.01

Name: ..... Date: ..... Period: .....

7. A machine uses Rule D to turn inputs into outputs. The table shows two inputs and their outputs.

Hoang tried the input 4 again and the output was not 27. He claims that this is enough information to determine whether or not Rule D is a function.

Is Hoang correct? Explain how you know.

**Yes. Explanations vary.** If the input 4 results in any output value other than 27, then Rule D is not a function. This is because a function only has one output for every input.

**Rule D**

Input	Output
3	21
4	27

## Spiral Review

8. Complete the table using this rule:  
Add 2 to the input, then multiply by 3 to get the output.

Input	-5	0	4
Output	-9	6	18

**Problems 9–10:** A school sells two types of tickets for a play: adult tickets and student tickets. Adult tickets are \$5 each and student tickets are \$2 each. The school collects \$400 total.

9. Write an equation where  $x$  represents the number of adult tickets sold and  $y$  represents the number of student tickets sold.

$$5x + 2y = 400$$

10. How many of each ticket type could the school sell to collect \$400?

**Responses vary.**

Adult tickets: ..... 40 ..... Student tickets: ..... 100 .....

## Reflection

- Put a star next to the problem you spent the most time on.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

**Function notation** is a way to write the inputs and outputs of a function. For example,  $f(4) = 9$  is a statement written in function notation. It says that when the input of the function  $f$  is 4, the output is 9. In other words, when the value of the *independent variable* is 4, the value of the *dependent variable* is 9.

This table shows some input-output pairs for the function  $s(t)$ , that determine the price of a slice of pizza based on the number of toppings

$s(2) = 2.75$  is a statement written in function notation.

- $s(2)$  can be read as “ $s$  of two.”
- For this situation, the number of toppings is the *independent variable*,  $t$ , and the price of a slice of pizza is the *dependent variable*,  $s(t)$ .
- $s(2) = 2.75$  means the price of a slice of pizza with 2 toppings is \$2.75.

**Menu**

Slice of Pizza
\$1.75 plus \$0.50 per topping

Number of Toppings	Price (\$)
0	1.75
1	2.25
2	2.75

**Things to Remember:**

# Lesson Practice

A1.4.02

Name: ..... Date: ..... Period: .....

**Problems 1–2:** The function  $f(t)$  models the temperature, in degrees Celsius,  $t$  hours after midnight.

1. Select the equation that represents the statement: *At 1 AM, the temperature was 20°C.*

A.  $f(100) = 20$       B.  $f(20) = 100$       C.  $f(1) = 20$       D.  $f(20) = 1$

2. Use function notation to represent each statement.

The temperature at 2 AM. ....

$f(2)$

The temperature was the same at 9 AM and at 11 AM. ....

$f(9) = f(11)$

The temperature was higher at 9 AM than at 2 AM. ....

$f(9) > f(2)$

$t$  hours after midnight, the temperature was 24°C. ....

$f(t) = 24$

3. A restaurant sells three different salads. They use the functions  $c(x)$ ,  $g(x)$ , and  $p(x)$  to represent the cost of their caesar, garden, and pasta salads in dollars, with  $x$  additional ingredients added. Explain the meaning of each statement. *Explanations vary.*

$g(0) = 10$

A garden salad with no additional ingredients costs \$10.

$g(3) > c(1)$

A garden salad with 3 additional ingredients costs more than a caesar salad with 1 additional ingredient.

$p(2) < g(3)$

A pasta salad with 2 additional ingredients costs less than a garden salad with 3 additional ingredients.

**Problems 4–6:** Use the table to determine the missing values in the function statements.

4.  $f(\underline{\hspace{1cm} 5 \hspace{1cm}}) = 23$

5.  $f(-5) = \underline{\hspace{1cm} 17 \hspace{1cm}}$

6.  $f(\underline{\hspace{1cm} -2 \hspace{1cm}}) = -5$

$x$	$f(x)$
-5	17
-2	-5
5	23

# Lesson Practice

A1.4.02

Name: ..... Date: ..... Period: .....

**Problems 7–9:** Desmos Pizza's online menu offers small, medium, and large pizzas. Fill in each blank to make each equation true.

**MENU**  
Small: \$12 plus \$1 per topping  
Medium: \$15 plus \$2 per topping  
Large: \$18 plus \$3 per topping

7.  $s(7) = m(\underline{\hspace{1cm}2\underline{\hspace{1cm}}})$     8.  $m(\underline{\hspace{1cm}9\underline{\hspace{1cm}}}) = l(5)$     9.  $l(\underline{\hspace{1cm}0\underline{\hspace{1cm}}}) = s(\underline{\hspace{1cm}6\underline{\hspace{1cm}}})$

Responses vary.

## Spiral Review

10. Here are Rules A and B.

Which rule is a function? Circle one.

Rule A    **Rule B**    Both    Neither

Rule A

Rule B

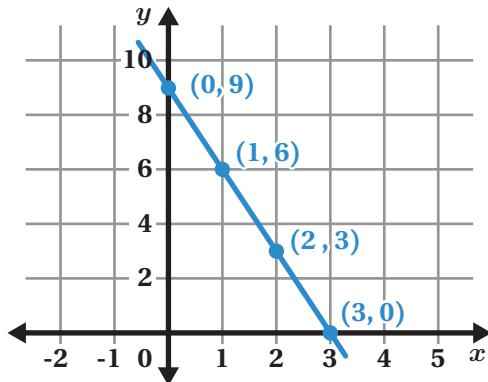
Input	Output	Input	Output
4	2	1	5
9	-3	2	9
9	3	2	9

11. Here are two data sets. Which one has the larger standard deviation? Circle one.

**Data Set A:** 6, 7, 8, 8, 8, 8.5, 9

**Data Set B:** 4, 7, 8, 8, 9, 12, 12

12. Here's a graph of a relationship and its table of values.



x	y
0	9
1	6
2	3
3	0

Write an equation to represent this relationship.  **$y = -3x + 9$  (or equivalent)**

## Reflection

- Put a heart next to a problem you understand well.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can represent function rules with equations, verbal descriptions, and tables.

For example, the function  $s(t)$  describes the relationship between the cost of a slice of pizza and the number of toppings,  $t$ .

**Description**

**Menu**  
Slice of Pizza  
\$1.75 plus \$0.50  
per topping

**Table**

Number of Toppings	Price (\$)
0	1.75
1	2.25
2	2.75

**Equation**

$$s(t) = 1.75 + 0.50t$$

You can use the equation to determine different values of the function.

Let's determine the value of  $s(4)$ :

$$s(4) = 1.75 + 0.50(4)$$

$$s(4) = 3.75$$

This means the price of a slice of pizza with 4 toppings is \$3.75.

**Things to Remember:**

# Lesson Practice

A1.4.03

Name: ..... Date: ..... Period: .....

1. Let  $f(x) = 2x + 5$ .

Calculate the value of each function notation expression.

The first value is already completed.

Expression	Value
$f(0)$	5
$f(4)$	13
$f(6)$	17
$f(-3)$	-1

**Problems 2–3:** A toy factory makes toy bunnies. Each toy bunny holds a carrot. A bunny's height,  $h(x)$ , is three times the length of the carrot,  $x$ .

2. Complete the table.

$x$	1	2	3	4	5	6
$h(x)$	3	6	9	12	15	18

3. Write an equation for the function  $h(x)$ .

$$h(x) = 3x$$

4. Here are two functions:  $f(x) = -15x + 80$  and  $g(x) = 10x + 25$ . Which is greater? Circle one.

$f(2)$     or     $g(2)$

Explain your thinking.

*Explanations vary.  $f(2) = 50$  and  $g(2) = 45$ , so  $f(2)$  is greater.*

**Problems 5–6:** The function  $p(s)$  models the perimeter of a square of side length  $s$ .

The perimeter is represented by the equation  $p(s) = 4s$ .

5. What is the value of  $p(20)$ ?

$$p(20) = 80$$

6. What does your answer mean in this situation?

*When the side length of a square is 20, the perimeter of the square is 80.*

## Lesson Practice

A1.4.03

Name: ..... Date: ..... Period: .....

7. Model rockets are created in various sizes. The height of a rocket in inches,  $h(x)$ , depends on the radius of the base of the rocket in inches,  $x$ .

Use the table to write an equation for  $h(x)$  that outputs the height of the rocket with a base radius of  $x$ .

$$h(x) = 4x + 1$$

Radius (in.), $x$	Height (in.), $h(x)$
1	5
3	13
5	21
10	41

**Problems 8–9:** The function  $w(t)$  models the weight of a pumpkin, in pounds, as a function of how many months,  $t$ , it has been growing. Explain the meaning of each statement.

8.  $w(2) = 5$  At 2 months, the pumpkin weighed 5 pounds.

9.  $w(6) > w(4)$  The pumpkin weighed more at 6 months than it did at 4 months.

## Spiral Review

**Problems 10–11:** Here are the first four figures in a pattern.

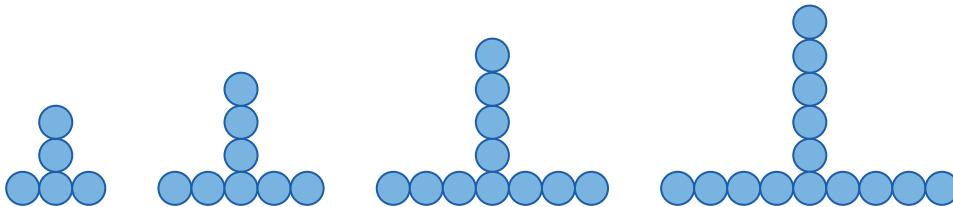


Figure 1

Figure 2

Figure 3

Figure 4

10. How many dots will be in Figure 5?

17 dots

11. Write an equation for the number of dots,  $d(n)$ , in Figure  $n$ .

$$d(n) = 3n + 2$$

## Reflection

1. Circle a problem you want to talk to a classmate about.
2. Use this space to ask a question or share something you're proud of.



**Lesson Summary**

A graph can reveal in more detail what is happening during a situation. Here is an example.

The function  $h(t)$  represents the height of the cart on the Ferris wheel at time  $t$ .

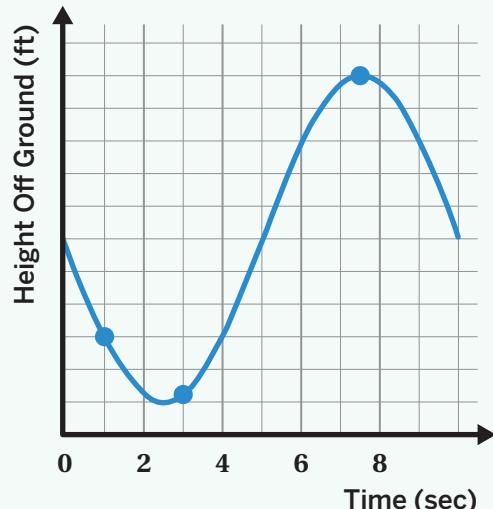
We can use the graph to describe many parts of the situation. For example:

- At around 7.5 seconds, the Ferris wheel cart is at its maximum height.
- $h(1)$  is greater than  $h(3)$ . This means the Ferris wheel cart was higher off the ground at 1 second than at 3 seconds.

While we can use the graph to describe many things, there are lots of things the graph cannot describe.

For example:

- How much fun the people are having
- How many people are riding the Ferris wheel

**Things to Remember:**

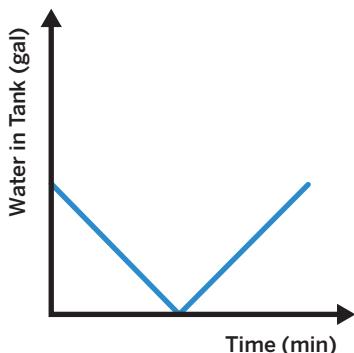
# Lesson Practice

A1.4.04

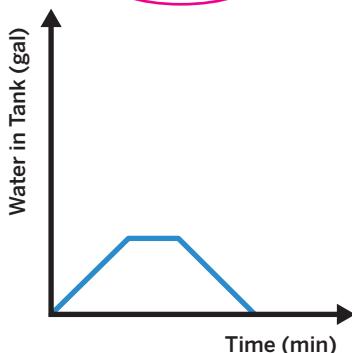
Name: ..... Date: ..... Period: .....

1. An empty water tank is filled until it is half full. Two minutes later, it drains until it is empty again. Which graph could represent this situation? Circle your choice.

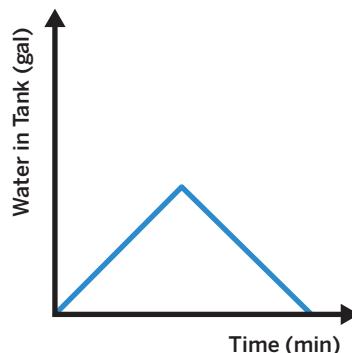
Graph A



Graph B



Graph C



**Problems 2–4:** Prisha rode her bike around town. Her fitness tracker made a graph to represent the distance she was from her home at any given time during her ride.

2. How many minutes was Prisha's bike ride?

15 minutes

3. At what time was Prisha the farthest distance from her home?

7 minutes

4. How long did Prisha rest during her ride?

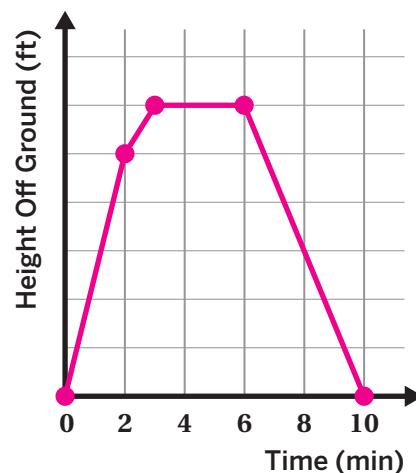
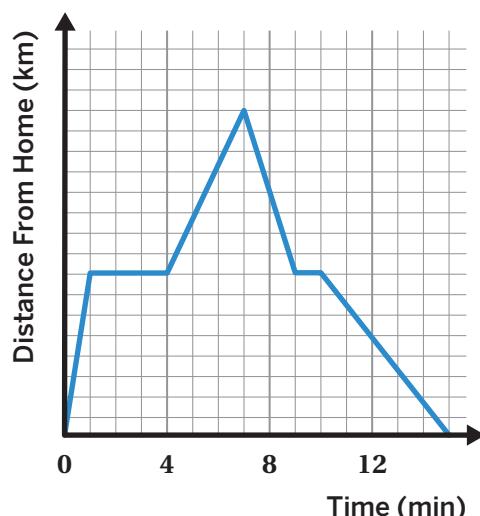
She rested for 4 minutes in total.

5. Here is some information about a hot air balloon ride.

- Ascends (goes up) quickly for 2 minutes.
- Ascends slowly for another minute until it reaches its maximum height.
- Maintains its maximum height for 3 minutes.
- Descends (goes down) for the next 4 minutes until it lands on the ground.

Make a graph that could represent the height of a hot air balloon over time.

Graphs vary.



# Lesson Practice

A1.4.04

Name: ..... Date: ..... Period: .....

**Problems 6–7:** Aba described yesterday morning at school like this:

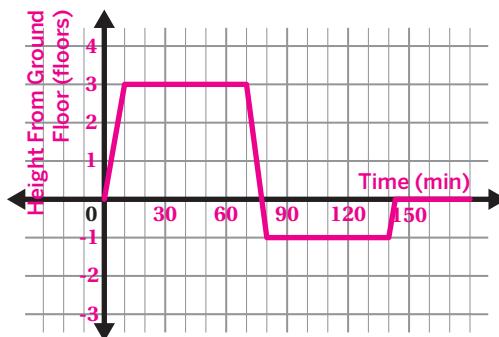
I entered the school on the ground floor, then walked up the stairs to the third floor to attend an hour-long class. Afterward, I had an hour-long class in the basement, then I went up to the ground floor and sat outside to eat my lunch.

6. Label both axes.

**Responses vary.**

7. Sketch a possible graph of Aba's height from the ground floor as a function of time.

**Responses vary.**



**Problems 8–9:** Here is a graph of speed and time.

8. Which sport could this graph represent?

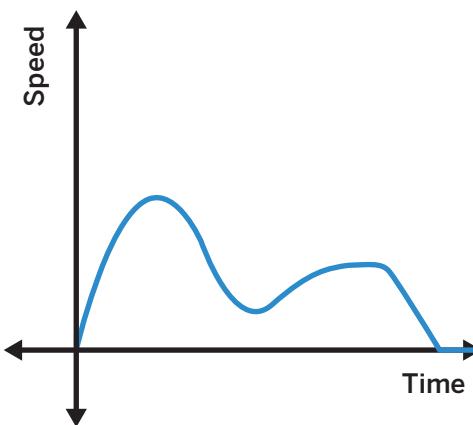
**Responses vary.**

- A. Fishing      B. Skydiving  
C. 100-yard sprint      D. Golf  
E. Soccer

9. Describe how you think that sport fits the graph.

**Explanations vary.**

- Soccer. The player runs the ball down the field, then slows down to change direction, then speeds up again before they stop to kick the ball.
- 100-yard sprint. The runner gets faster until reaching their top speed and then slow down before speeding up again to finish the race.



## Spiral Review

10. The function  $p(t)$  represents the height of water in a bathtub, in inches, after  $t$  minutes.

Match each sentence to its equation.

- a. After 20 minutes, the bathtub is empty.  
b. The bathtub starts out with no water.  
c. After 10 minutes, the height of the water is 4 inches.  
d. The height of the water is 10 inches after 4 minutes.  
e. The height of the water is  $w$  inches after  $t$  minutes.

- c.  $p(10) = 4$   
e.  $p(t) = w$   
a.  $p(20) = 0$   
b.  $p(0) = 0$   
d.  $p(4) = 10$

## Reflection

- Put a question mark next to a problem you're feeling unsure of.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

We can use the key features of a graph to help us describe a function or sketch a possible graph of a function. Here is an example. Let's analyze the graph of this function.

**Minimum:** The lowest point on a graph.

(-1, -3)

**Maximum:** The highest point on a graph.

(3, 1)

**Positive:** The  $x$ -values where the function has positive outputs; the graph is *above* the  $x$ -axis.

$x > 2$

**Negative:** The  $x$ -values where the function has negative outputs; the graph is *below* the  $x$ -axis.

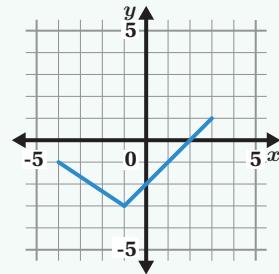
$x < 2$

**Increasing:** The  $x$ -values where the graph is sloping upward as you read the graph from left to right. As the inputs increase, the outputs also increase.

$x > -1$

**Decreasing:** The  $x$ -values where the graph is sloping downward as you read the graph from left to right. As the inputs increase, the outputs decrease.

$x < -1$

**Things to Remember:**

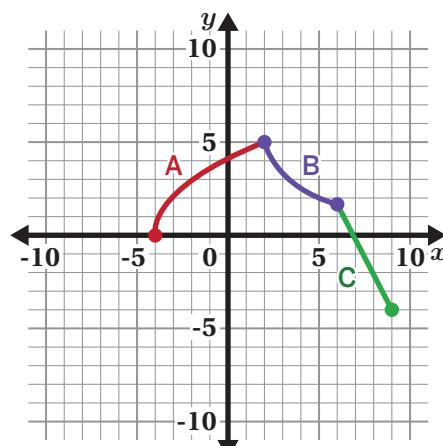
# Lesson Practice

A1.4.05

Name: ..... Date: ..... Period: .....

1. Select *all* the true statements about this graph.

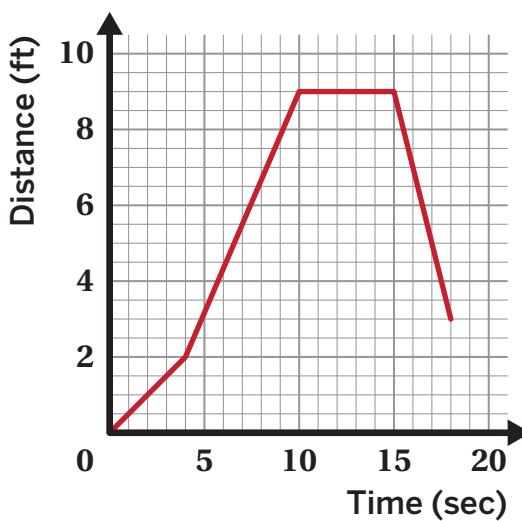
- A. This graph is a function.
- B. Part A is decreasing.
- C. Part B is decreasing.
- D. The maximum is at  $(2, 5)$ .
- E. The minimum is at  $(-4, 0)$ .



2. Manuel is watching his younger sibling at the park. The graph represents the distance Manuel is from his sibling as a function of time.

Describe Manuel's distance from his sibling over time. Use terms that you learned in this lesson.

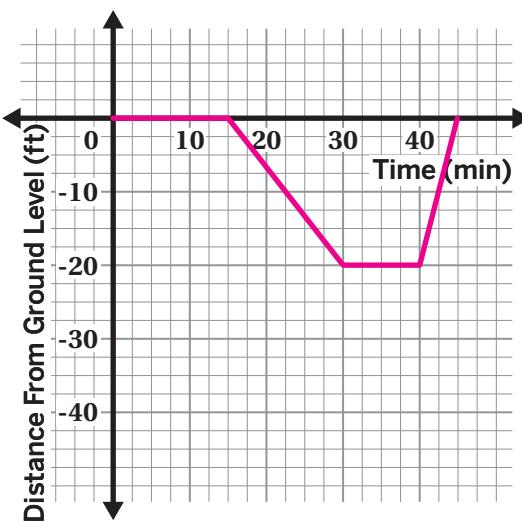
**Responses vary.** Manuel starts out right next to his sibling. Manuel's distance from his sibling increases for about 4 seconds to about 2 feet. His distance from his sibling continues to increase to a maximum distance of 9 feet, where he stays for 5 seconds. Then Manuel's distance from his sibling decreases to 3 feet in about 3 seconds.



3. Ivory goes on a tour of a cave. The tour starts at ground level.

- The tour stays at ground level for 15 minutes.
- Then the tour descends (goes down) for 15 minutes to a depth of 20 feet below ground level.
- The tour stays at this level for 10 minutes.
- The tour spends the last 5 minutes ascending (going up) to ground level.

Sketch a graph describing Ivory's elevation as a function of time.



# Lesson Practice

A1.4.05

Name: ..... Date: ..... Period: .....

**Problems 4–6:** Here is a table that lists Seattle's temperatures for one day.

4. How many hours after midnight is the minimum temperature?

1 hour

5. How many hours after midnight is the maximum temperature?

4 hours

6. Between what hours is the temperature decreasing?

Between 4 and 5 hours

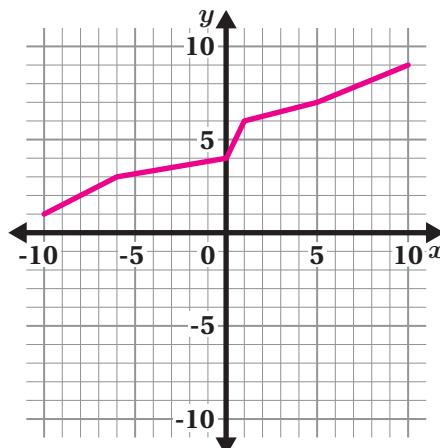
7. Here are three statements:

- The function is always positive.
- The function is always increasing.
- The function is always decreasing.

Sketch a graph of a function so that two of the statements are true and one is false.

Graphs vary.

Hours After Midnight	Temperature (°F)
1	32
2	33
3	35
4	36
5	35



## Spiral Review

**Problems 8–10:** Determine each quotient.

8.  $\frac{5}{6} \div \frac{2}{3} = \frac{5}{4}$

(or equivalent)

9.  $\frac{8}{6} \div \frac{4}{3} = 1$

(or equivalent)

10.  $\frac{12}{20} \div \frac{18}{16} = \frac{8}{15}$

(or equivalent)

## Reflection

- Put a star next to your favorite problem.
- Use this space to ask a question or share something you're proud of.

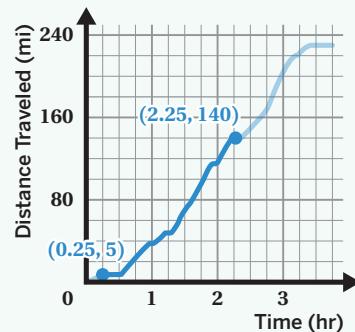
## Lesson Summary

Functions can have different rates of change over different intervals. The **average rate of change** is equivalent to the slope of the line between two points.

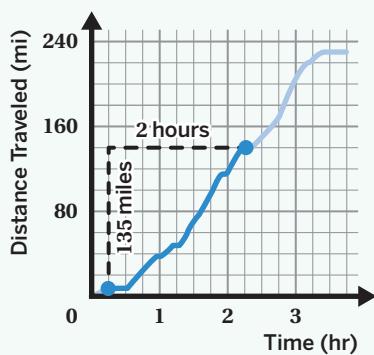
This graph represents Troy's car trip.

We can calculate the average rate of change over an **interval**, a specific length between two points, like the interval from 0.25 to 2.25 hours.

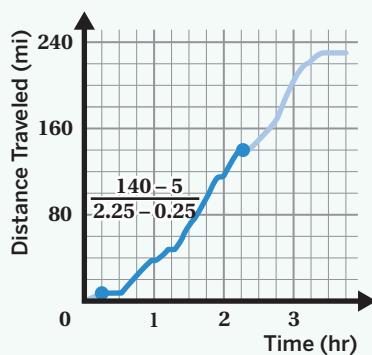
Here are two different strategies.



Strategy 1



Strategy 2



The average rate of change for the interval 0.25 to 2.25 hours is  $\frac{135}{2} = 67.5$ .

That means that Troy's average speed was 67.5 miles per hour in that interval.

## Things to Remember:

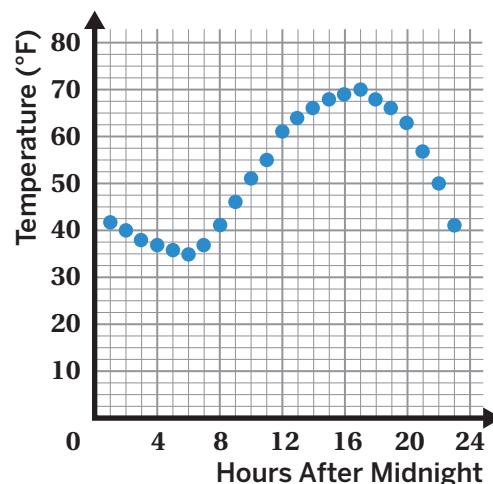
# Lesson Practice

A1.4.06

Name: ..... Date: ..... Period: .....

- The temperature was recorded at several times in a 24-hour period. Function  $t(n)$  gives the temperature in degrees Fahrenheit  $n$  hours after midnight. Use the graph to determine if the average rate of change for each interval is *positive* (+), *negative* (-), or *zero*. Place a check in the appropriate column.

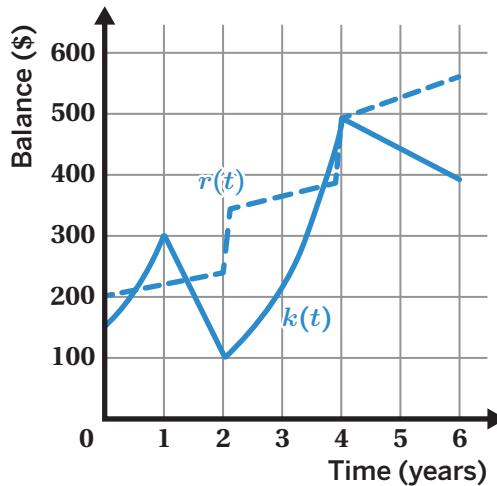
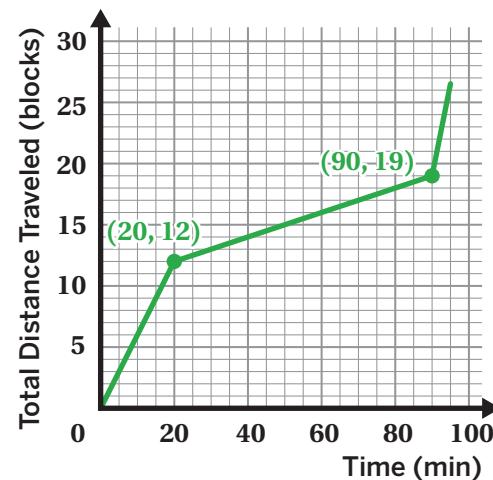
	+	-	0
$n = 1$ to $n = 5$			
$n = 5$ to $n = 7$		<input checked="" type="checkbox"/>	
$n = 10$ to $n = 20$	<input checked="" type="checkbox"/>		



**Problems 2–3:** This graph shows the total distance in city blocks,  $d(t)$ , that Pilar walked as a function of time in minutes,  $t$ .

- Determine the average rate of change between  $t = 20$  and  $t = 90$ .  
**0.1**
- What do you think the average rate of change you calculated means in this situation?  
**Responses vary. The average rate of change means that Pilar walked 0.1 blocks per minute or  $\frac{1}{10}$  of a block per minute, from 20 minutes to 90 minutes into her walk.**
- $r(t)$  and  $k(t)$  model the savings account balances of Rafael and Katie after  $t$  years. Select all the statements that are true.

- A. Katie has a lower average rate of change in the last two years.
- B. Katie's balance is always less than Rafael's.
- C.  $r(2) = 100$
- D. Rafael's balance is increasing from year 0 to year 6.
- E. Rafael has a higher average rate of change in the first four years.



# Lesson Practice

A1.4.06

Name: ..... Date: ..... Period: .....

**Problems 5–7:** This table shows the population of a city from 1988 to 2016.

5. Determine the average rate of change for  $p(t)$  between 1992 and 2000.

**-1,125 people per year**

6. State two values of  $t$  that create an interval with a negative rate of change.

**Responses vary. 2008 and 2012.**

7. State two values of  $t$  that create an interval with a positive rate of change.

**Responses vary. 1988 and 2016.**

8. Match each interval to its average rate of change.

**Interval**

**Average Rate of Change**

a.  $X$  to  $Y$

c.  $\frac{1}{5}$

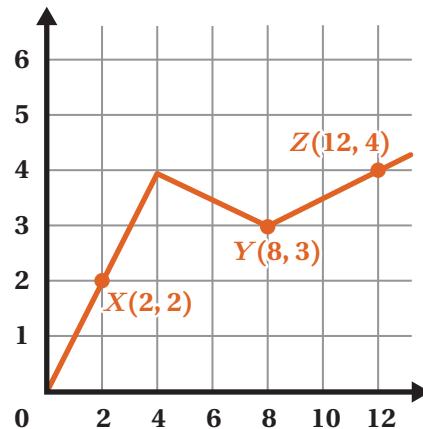
b.  $Y$  to  $Z$

b.  $\frac{1}{4}$

c.  $X$  to  $Z$

a.  $\frac{1}{6}$

Year, $t$	Population, $p(t)$
1988	35,700
1992	42,700
1996	33,100
2000	33,700
2004	45,000
2008	48,400
2012	40,900
2016	43,000



## Spiral Review

9. Jada is walking to school. The function  $d(t)$  gives the distance from school, in meters,  $t$  minutes since Jada left home. Which equation represents this statement? *Jada is 600 meters from school after 5 minutes.*

- (A)  $d(5) = 600$       (B)  $d(600) = 5$       (C)  $t(5) = 600$       (D)  $t(600) = 5$
10. Complete the arithmetic sequence with the missing terms:  $\underline{\hspace{1cm}}$ , 6,  $\underline{\hspace{1cm}}$ , 22, 30

## Reflection

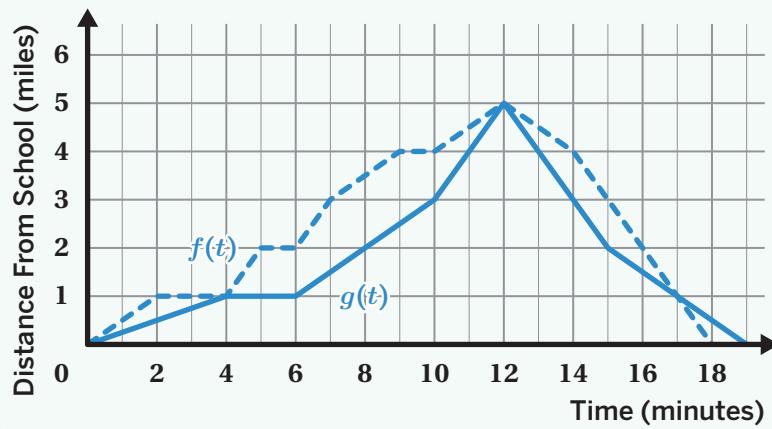
- Put a heart next to the problem you're most proud of.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

We can analyze functions by comparing their key features at different intervals and then use function notation to describe them.

For instance, here are some true statements about these two graphs:

- When  $t = 4$ ,  $f(t) = g(t)$ .
- $f(8) > g(8)$
- $f(12) = g(12)$
- $f(15) > g(15)$
- $f(t)$  and  $g(t)$  have the same maximum.
- $f(t)$  and  $g(t)$  are both decreasing from 12 to 15 minutes.
- $f(t)$  and  $g(t)$  have the same average rate of change from 5 to 6 minutes.

**Things to Remember:**

# Lesson Practice

A1.4.07

Name: ..... Date: ..... Period: .....

**Problems 1–2:** Mai built a model race car for a school competition.

$m(t)$  represents the distance of Mai's car, in meters, after  $t$  seconds.

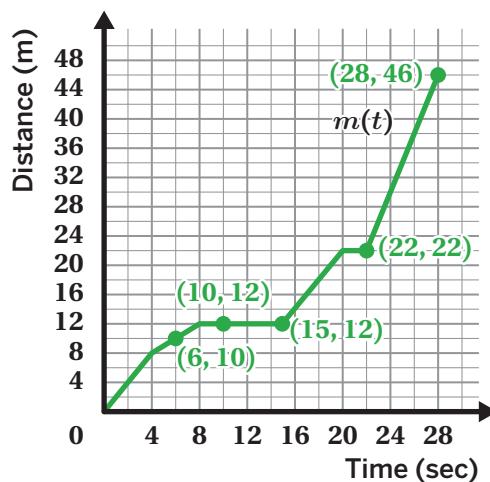
- Use the graph to determine the missing value in each function statement.

$$m(\underline{\quad 6 \quad}) = 10$$

$$m(10) = \underline{\quad 12 \quad}$$

$$m(22) = \underline{\quad 22 \quad}$$

$$m(\underline{\quad 28 \quad}) = 46$$



- Over what interval did Mai's car travel the slowest?

- A. 0 to 4 seconds    B. 4 to 8 seconds    C. 8 to 15 seconds    D. 15 to 20 seconds

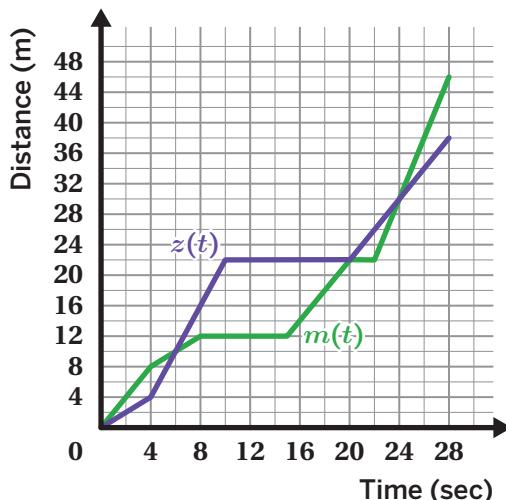
**Problems 3–6:** Zion also built a model race car for the school competition.

$z(t)$  represents the distance of Zion's car, in meters, after  $t$  seconds.

Did Zion or Mai have the greater average rate of change over the following intervals? Explain your thinking.

- $t = 4$  to  $t = 8$

**Zion.** Explanations vary. The average rate of change is 3 meters per second for Zion's car and 1 meter per second for Mai's car.



- $t = 10$  to  $t = 20$

**Mai.** Explanations vary. The average rate of change is 1 meter per second for Mai's car and 0 meters per second for Zion's car.

- Name a time when Zion's and Mai's cars had traveled the same distance.

0 seconds, 6 seconds, 20 seconds, or 24 seconds

- Select all the true statements.

A.  $m(t)$  has a greater maximum than  $z(t)$ .

B.  $z(t)$  and  $m(t)$  have the same minimum at  $(6, 10)$ .

C.  $z(20) = m(20)$

D.  $m(15) > z(15)$

E.  $m(t)$  and  $z(t)$  both increase from 22 to 28 seconds.

## Lesson Practice

A1.4.07

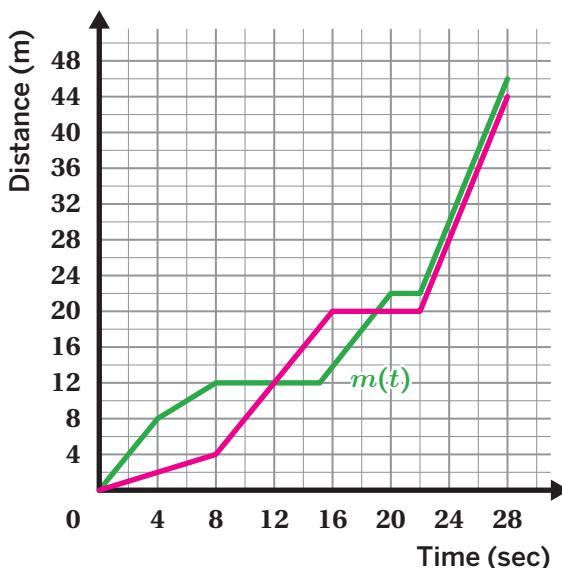
Name: ..... Date: ..... Period: .....

7. Parv built a race car to race against Mai.

Use this information to make a graph that could represent the distance of Parv's race car,  $p(t)$ , after  $t$  seconds:

- $p(8) < m(8)$
- $p(12) = m(12)$
- The average rate of change of  $p(t)$  and  $m(t)$  is the same from  $t = 22$  to  $t = 28$ .
- $m(t)$  has a greater maximum than  $p(t)$ .

**Graphs vary. Sample shown.**



## Spiral Review

**Problems 8–10:** Nekeisha goes for a bike ride.  $d(t)$  represents Nekeisha's distance from home, in miles,  $t$  minutes after she leaves.

Explain the meaning of each statement in context:

$$d(0) = 0$$

Nekeisha starts her bike ride at home.

$$d(30) = d(60)$$

Nekeisha is the same distance from home 30 minutes and 60 minutes into her bike ride.

$$d(90) = 0$$

Nekeisha is back home 90 minutes into her bike ride.

## Reflection

1. Put a star next to the problem you understood best.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

The *domain* and *range* of a function can each be described using a **compound inequality**, which is two or more inequalities joined together. You can write a compound inequality using symbols or using the words “and” or “or.”

A graph can help you visualize the domain and range of a function, making it easier to describe them using compound inequalities.

**Domain:** The width of the function, or how far left and right the function goes. The domain is also the set of all inputs to the function, or all values of the independent variable. The domain of this function is all the values of  $t$  from 0 to 15.

$$t \geq 0 \text{ and } t \leq 15$$

$$0 \leq t \text{ and } t \leq 15$$

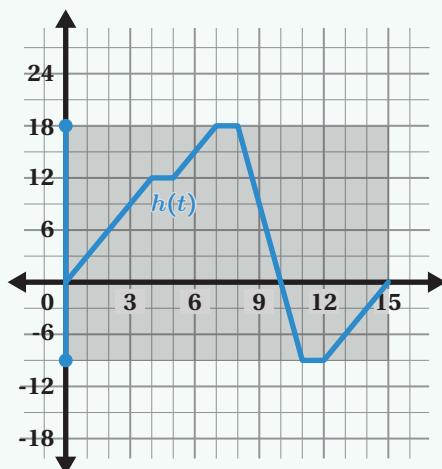
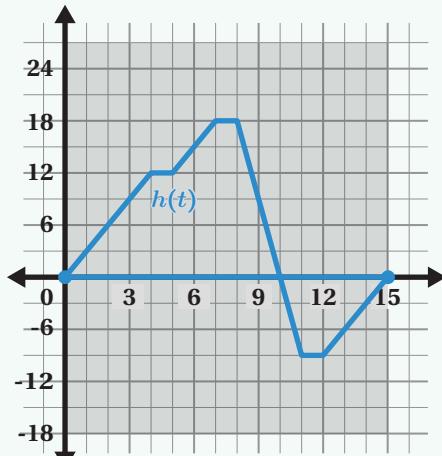
$$0 \leq t \leq 15$$

**Range:** The height of the function, or how far up and down the function goes. The range is also the set of all outputs of the function, or all values of the dependent variable. The range of this function is all the values of  $h(t)$  from -9 to 18.

$$h(t) \geq -9 \text{ and } h(t) \leq 18$$

$$-9 \leq h(t) \text{ and } h(t) \leq 18$$

$$-9 \leq h(t) \leq 18$$



## Things to Remember:

# Lesson Practice

A1.4.09

Name: ..... Date: ..... Period: .....

**Problems 1–2:** Valeria and Thiago disagree about the domain of  $f(x)$ .

- Valeria says the domain is  $-1.5 \leq x \leq 2$
- Thiago says the domain is  $-4.375 \leq x \leq 3.5$

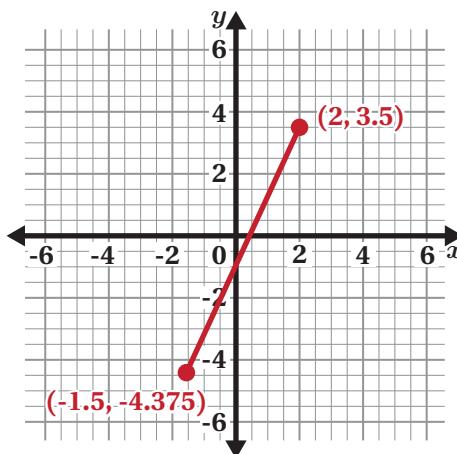
1. Whose answer is correct? Circle one.

Valeria's

Thiago's

2. Explain why the other person's answer is incorrect.

*Explanations vary. Thiago gave the output values for the function. That's the range.*

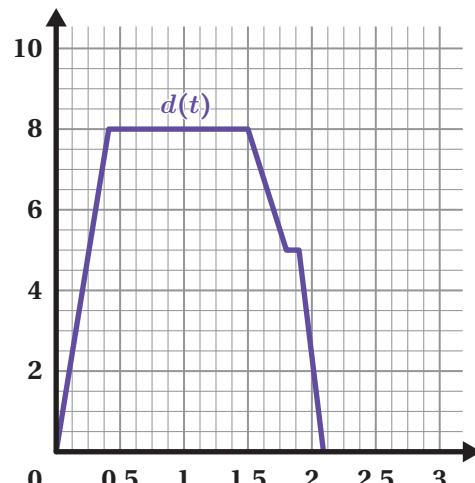


**Problems 3–5:** Haru bikes to his friend's house.

After a while, he heads home. On the way, he stops at the store to buy a bottle of water.  $d(t)$  represents Haru's distance from his house, in kilometers, after  $t$  hours. This graph shows Haru's distance over time.

3. Which inequality describes the domain of  $d(t)$ ?

- A.  $0 \leq d(t) \leq 2.1$       B.  $0 \leq d(t) \leq 8$   
 C.  $0 \leq t \leq 2.1$       D.  $0 \leq t \leq 8$



4. Which inequality describes the range of  $d(t)$ ?

- A.  $0 \leq d(t) \leq 2.1$       B.  $0 \leq d(t) \leq 8$   
 C.  $0 \leq t \leq 2.1$       D.  $0 \leq t \leq 8$

5. If Haru had not stopped at the store, would that change the domain or the range?

Circle one.

Domain      Range      Both      Neither

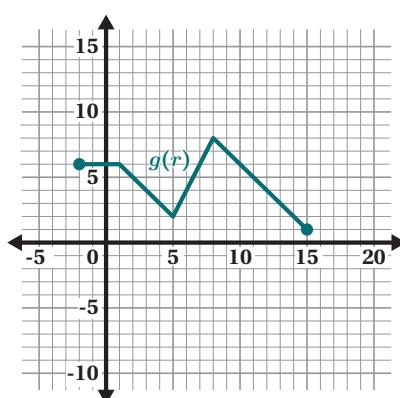
**Problems 6–7** Here is the graph of  $g(r)$ .

6. Write a compound inequality to describe the domain.

$-2 \leq r \leq 15$

7. Write a compound inequality to describe the range.

$1 \leq g(r) \leq 8$



# Lesson Practice

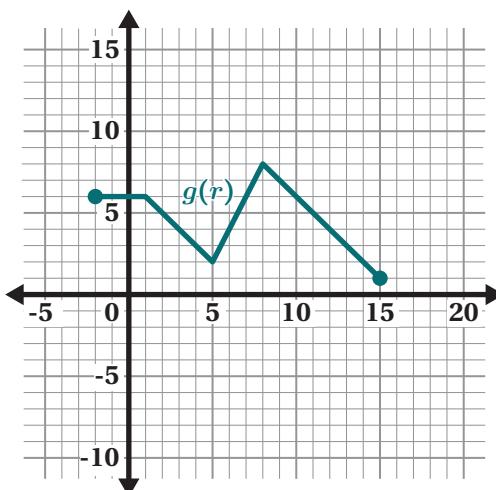
A1.4.09

Name: ..... Date: ..... Period: .....

## Spiral Review

8. Determine the average rate of change for each interval on the graph of  $g(r)$ .

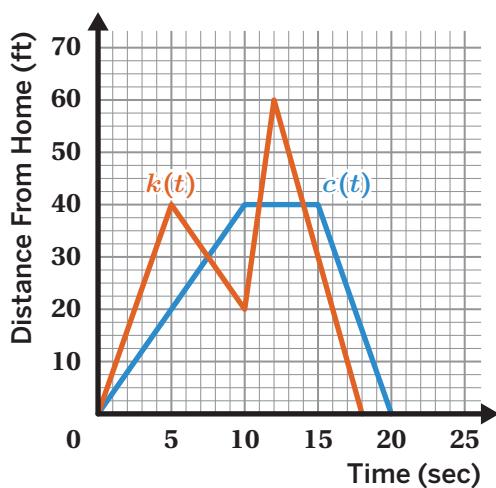
Interval	Average Rate of Change
$r = -2$ to $r = 1$	0 (or equivalent)
$r = -2$ to $r = 5$	$-\frac{4}{7}$ (or equivalent)
$r = -2$ to $r = 8$	$\frac{1}{5}$ (or equivalent)



9. Functions  $c(t)$  and  $k(t)$  represent the distance of two cats from home after  $t$  seconds.

Select all the true statements.

- A.  $k(5) > c(5)$
- B.  $k(t)$  and  $c(t)$  have the same domain and range.
- C.  $k(t)$  keeps increasing from 0 to 13 seconds.
- D.  $k(11) = c(11)$
- E. Both cats return home.



10. Tickets to the state fair cost \$10 each. The function  $c(t) = 10t$  gives the total cost in dollars,  $c(t)$ , for the number of tickets purchased,  $t$ . Select all the values that are possible outputs for  $c(t)$ .

- A. 0
- B. 70
- C. 105
- D. 880
- E. 963

## Reflection

- Put a star next to a problem where you revised your thinking.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can restrict a function's domain or range to highlight specific portions of a graph. Inequalities are one way to represent these restrictions symbolically.

Here is an example. Let's restrict the domain and range of  $h(x)$  to highlight the interval from  $(-3, 7)$  to  $(6, 1)$ .

To restrict the domain, use the  $x$ -values of each ordered pair:

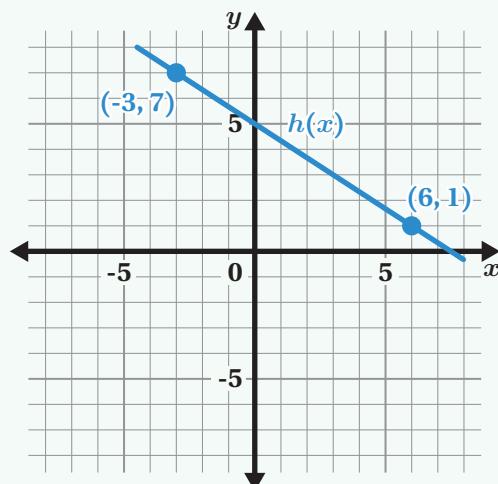
**Domain:**  $-3 \leq x \leq 6$

When you restrict the domain, you are restricting the  $x$ -values, which should be included when you write the inequality.

To restrict the range, use the  $y$ -values of each ordered pair:

**Range:**  $1 \leq h(x) \leq 7$

When you restrict the range, you are restricting the  $y$ -values, or the output values of  $h(x)$ .

**Things to Remember:**

# Lesson Practice

A1.4.10

Name: ..... Date: ..... Period: .....

**Problems 1–2:** Here is the graph of  $f(x)$ .

1. What is the domain of  $f(x)$ ?

A.  $x \geq 0$

B.  $x \geq -2$

C.  $x \leq -2$

D.  $x \geq 1$

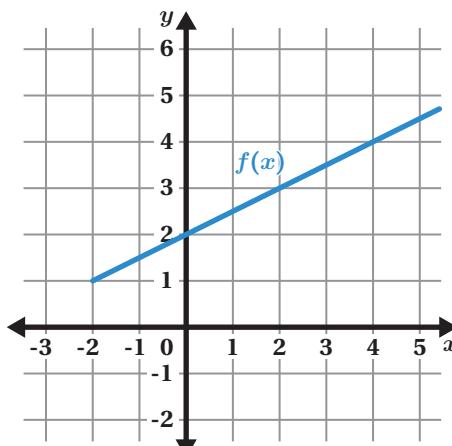
2. What is the range of  $f(x)$ ?

A.  $f(x) \geq 0$

B.  $f(x) \geq -2$

C.  $f(x) \leq 1$

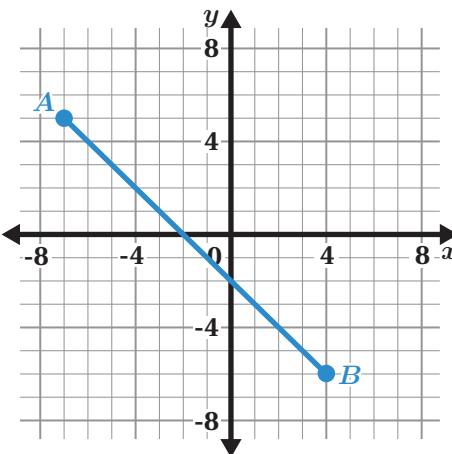
D.  $f(x) \geq 1$



3. Fill in the blanks for the domain and range of  $y = -x - 2$  from point A to point B.

..... ≤  $x$  ≤ .....

..... ≤  $y$  ≤ .....



**Problems 4–5:** Precious leaves her home to go to the grocery store. This is her path:

- She walks to the store, which is 5 blocks away, at a speed of half a block per minute.
- She is in the store for 10 minutes.
- She runs back home at a speed of 1 block per minute.

The graph shows part of her path.

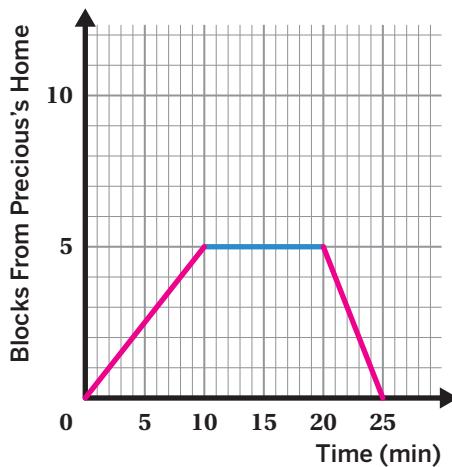
4. Sketch the missing pieces of the graph of Precious's path.
5. Which equation represents the beginning piece of Precious's path? Circle your choice.

A.  $y = 0.5x$   
 $\{0 < x < 5\}$

B.  $y = 0.5x$   
 $\{0 < x < 10\}$

C.  $y = -x$   
 $\{20 < x < 25\}$

D.  $y = -x + 15$   
 $\{10 < x < 20\}$



Explain your thinking.

*Explanations vary. The slope of the line is the speed and the domain is the number of minutes. Since Precious travels from her home to the grocery store at a speed of half a block per minute from time 0 to 10 seconds, I picked choice B.*

## Lesson Practice

A1.4.10

Name: ..... Date: ..... Period: .....

**Problems 6–8:** Sneha tries to connect points  $A$  to  $B$  to  $C$  to  $D$  with three segments. She graphs three lines but needs help restricting each line's domain. Determine the domain restriction for each of the lines.

6. Domain restriction for  $A$  to  $B$ :

$$-4 < x < -2$$

7. Domain restriction for  $B$  to  $C$ :

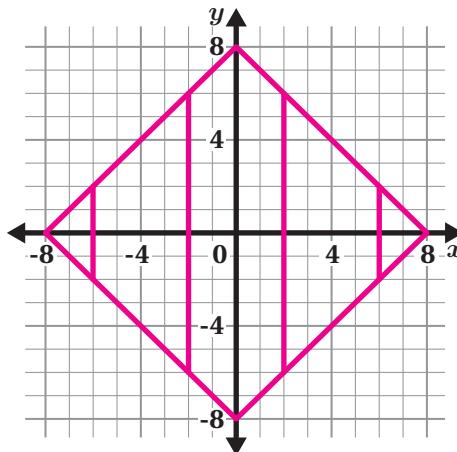
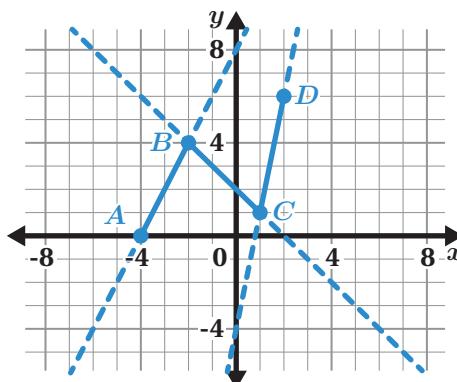
$$-2 < x < 1$$

8. Domain restriction for  $C$  to  $D$ :

$$1 < x < 2$$

9. Create a design on the graph using at least six lines. Then write the equations with their domain or range restriction to represent each line segment in your design. *Responses vary.*

$y = -x + 8$	$\{0 < x < 8\}$
$y = x - 8$	$\{0 < x < 8\}$
$y = x + 8$	$\{-8 < x < 0\}$
$y = -x - 8$	$\{-8 < x < 0\}$
$x = 6$	$\{-2 < y < 2\}$
$x = -6$	$\{-2 < y < 2\}$
$x = -2$	$\{-6 < y < 6\}$
$x = 2$	$\{-6 < y < 6\}$



## Spiral Review

10. Elena and Alexis are deciding between two cafeteria meal plans. They estimate that they will buy 15 meals from the cafeteria each month.

Which meal plan would cost them less? Circle one.

Plan A

Plan B

Explain your choice.

*Explanations vary.* They should choose Plan B because 15 meals will cost \$37.50 for Plan A, but only \$30 for Plan B.

### Meal Plans

**Plan A:** \$2.50 per meal

**Plan B:** \$30 per month

## Reflection

- Circle the problem you enjoyed doing the most.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

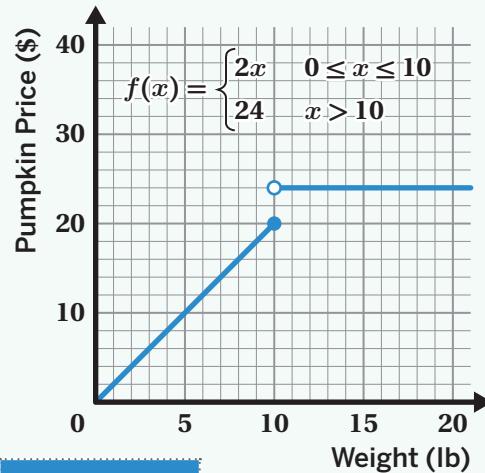
A **piecewise-defined function** is a function in which different rules apply to different intervals in its domain. You can use a graph or an equation to evaluate a piecewise-defined function.

Let's look at an example. The function  $f(x)$  represents the price of a pumpkin with a weight of  $x$  pounds.

You can use the graph to evaluate  $f(4)$  and  $f(15)$ .

- The point  $(4, 8)$  is on the graph, so  $f(4) = 8$ .
- The point  $(15, 24)$  is on the graph, so  $f(15) = 24$ .

You can also use the equation to evaluate  $f(4)$  and  $f(15)$ .



Value	Domain Interval	Equation	Evaluate
$f(4)$	$x = 4$ is in $0 \leq x \leq 10$	$f(x) = 2x$	$f(4) = 2(4) = 8$
$f(15)$	$x = 15$ is in $x > 10$	$f(x) = 24$	$f(15) = 24$

## Things to Remember:

# Lesson Practice

A1.4.11

Name: ..... Date: ..... Period: .....

**Problems 1–3:** Here is the piecewise-defined function,  $g(x)$ . Determine each value.

$$g(x) = \begin{cases} -3x & 0 \leq x < 2 \\ 12 & x \geq 2 \end{cases}$$

1.  $g(0) = \underline{\hspace{2cm}} \textcolor{red}{0}$

2.  $g(2) = \underline{\hspace{2cm}} \textcolor{red}{12}$

3.  $g(4) = \underline{\hspace{2cm}} \textcolor{red}{12}$

**Problems 4–5:** A parking garage charges \$5 to park for less than 1 hour, \$10 to park for 1 to 3 hours, and \$12 to park for more than 3 hours. Let  $c(t)$  represent the price of parking, in dollars, for  $t$  hours.

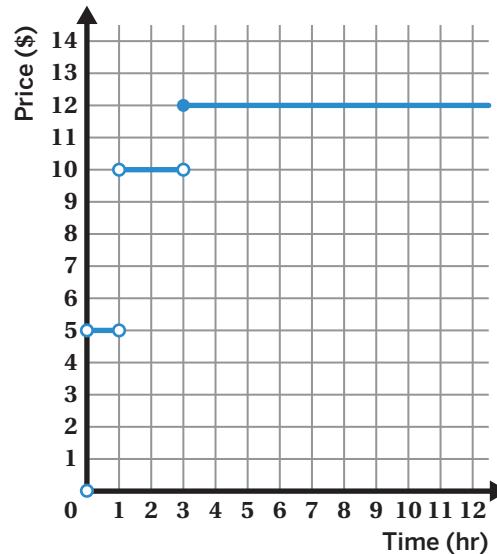
4. Complete the table.

$t$ (hr)	0	0.5	1	2	3	5
$c(t)$ (\$)	0	5	10	10	10	12

5. The parking garage tried to represent their pricing with this graph. What is correct and what should change to make the graph more accurate?

*Responses vary.*

- The location of the horizontal lines correctly represent the different amounts in the pricing structure.
- Some of the endpoints don't correctly reflect this pricing. The points at  $(0, 0)$ ,  $(1, 10)$ , and  $(3, 10)$  should be filled in and the point at  $(3, 12)$  should be an open circle.



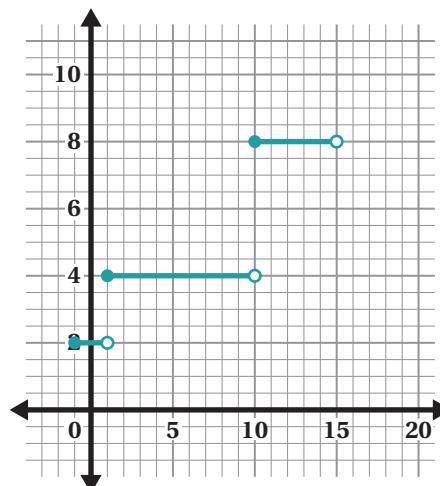
**Problems 6–8:** Here is the graph of function  $h(x)$ .

6. Complete  $h(x)$  so that it matches the graph.

$$h(x) = \begin{cases} \boxed{2} & -1 \leq x < 1 \\ 4 & \boxed{1} \leq x < \boxed{10} \\ 8 & 10 \leq x < 15 \end{cases}$$

7. What is the value of  $h(2)$ ?  $\textcolor{red}{4}$

8. What is the value of  $h(10)$ ?  $\textcolor{red}{8}$



# Lesson Practice

A1.4.11

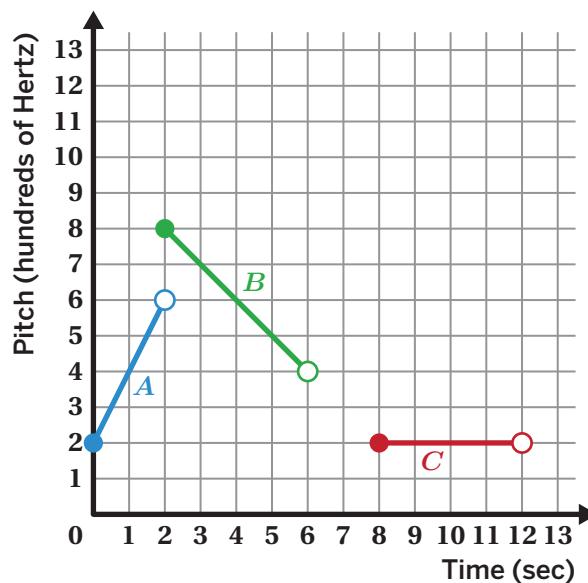
Name: ..... Date: ..... Period: .....

9. The pitch of a singer's recorded voice is represented by the piecewise function  $p(t)$  and its graph consists of pieces A, B, and C.

$$p(t) = \begin{cases} 2t + 2 & 0 \leq t < 2 \\ -t + 10 & 2 \leq t < 6 \\ 2 & 8 \leq t < 12 \end{cases}$$

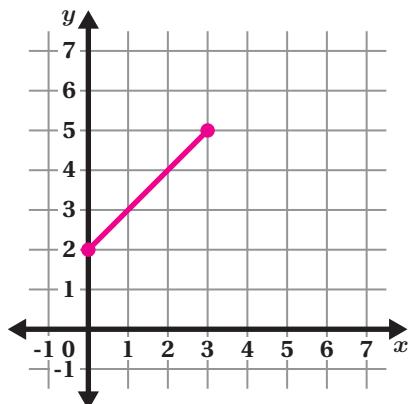
Explain how you could change the function to eliminate any breaks or jumps in the recording.

*Responses vary. I would change the expression for piece A to  $2t + 4$  to connect it with piece B. Then I would change the domain of piece B to  $2 \leq t < 8$  to connect it with piece C.*

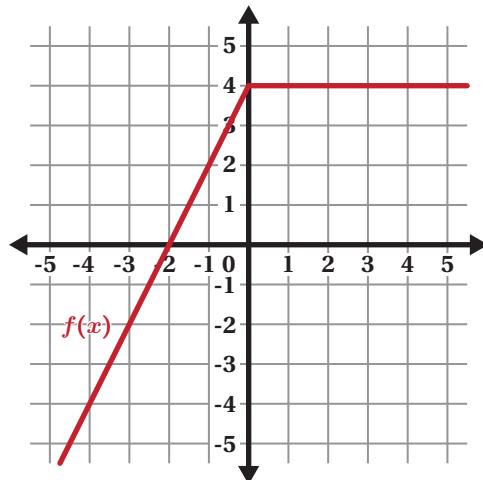


## Spiral Review

10. Graph the function  $f(x) = x + 2$  over the domain  $0 \leq x \leq 3$ .



**Problems 11–13:** Use the graph of  $f(x)$  to determine the value of each function notation statement.



11.  $f(-3)$       12.  $f(0)$       13.  $f(3)$

-2

4

4

## Reflection

- Circle the problem you think will help you most on the End-of-Unit Assessment.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can use *piecewise-defined functions* to represent situations. A **step function** is one special kind of piecewise-defined function, where every section of the graph is a point or a horizontal line at a constant value.

Here are some helpful ways to write the equations of a piecewise-defined function:

- The number of conditions in an equation is equal to the number of pieces in the function and graph.
- In the piecewise equation, each piece represents one condition and has its own domain.
- You can write the domain as an inequality.
  - $\geq$  or  $\leq$  means to include that value.
  - $>$  or  $<$  means to exclude that value.
- You can represent each condition by a section of the graph with open boundary points ( $>$  or  $<$ ) or closed boundary points ( $\geq$  or  $\leq$ ).

Graphing your piecewise-defined function might also help in making sense of the situation.

**Things to Remember:**

# Lesson Practice

A1.4.12

Name: ..... Date: ..... Period: .....

**Problems 1–2:** An online makeup store charges shipping fees depending on the weight of the order.

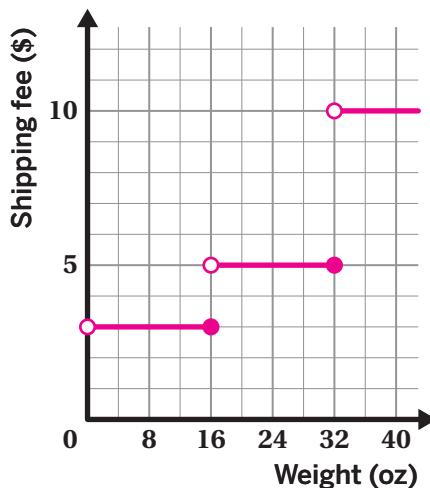
- Orders up to and including 16 ounces have a \$3 shipping fee.
- Orders greater than 16 ounces but less than or equal to 32 ounces have a \$5 shipping fee.
- Orders greater than 32 ounces have a \$10 shipping fee.

Let  $c(x)$  represent the shipping fee for an order that weighs  $x$  ounces.

1. Complete the table.

Weight (oz)	Shipping Fee (\$)
5	3
16	3
20	5
30	5
50	10

2. Make a graph that represents the function  $c(x)$ .



**Problems 3–5:** Citlalli wrote a piecewise-defined function,  $f(x)$ , that models the price of using a recording studio for  $x$  minutes:

- \$1 per minute for sessions of 20 minutes or less.
  - \$0.75 per minute for sessions between 20 and 40 minutes.
  - \$0.50 per minute for sessions of 40 minutes or longer.
3. Describe one thing that is incorrect in Citlalli's function. *Responses vary.*
- The inequality symbol in the first restriction is incorrect. It should be  $0 < x \leq 20$ .
  - Both inequality symbols in the second restriction are incorrect. It should be  $20 < x < 40$ .
  - The third equation is incorrect. It should be  $0.50x$ .

4. Write the correct function.

$$f(x) = \begin{cases} 1.00x & 0 < x \leq 20 \\ 0.75x & 20 < x < 40 \\ 0.50 & x \geq 40 \end{cases}$$

5. What would the price be if Citlalli used the recording studio for a 36-minute session?  
**The total cost of a 36-minute recording session would be \$27.**

$$f(x) = \begin{cases} 1.00x & 0 < x < 20 \\ 0.75x & 20 \leq x < 40 \\ 0.50 & x \geq 40 \end{cases}$$

# Lesson Practice

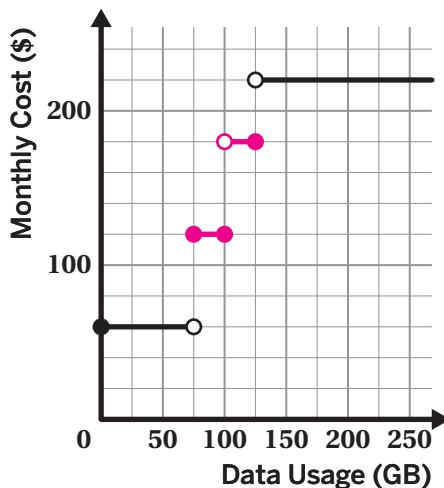
A1.4.12

Name: ..... Date: ..... Period: .....

**Problems 6–7:** An internet company sells internet based on the number of gigabytes of data used,  $x$ .  $a(x)$  represents the monthly cost.

6. Here is part of the piecewise function,  $a(x)$ , and the graph that models this situation. Fill in the missing numbers and symbols. Then complete the graph to represent this situation.

$$a(x) = \begin{cases} 60 & 0 \leq x < 75 \\ 120 & 75 \leq x \leq 100 \\ 180 & 100 < x \leq 125 \\ 220 & x > 125 \end{cases}$$



7. Diya thinks her business will use 110 gigabytes of data each month. How much would this data cost Diya per month?  
\$180

## Spiral Review

**Problems 8–11:** Determine the values of the piecewise-defined function  $p(x)$ .

$$p(x) = \begin{cases} 2x & 0 \leq x \leq 3 \\ 10 & x > 3 \end{cases}$$

8.  $p(10) = \underline{\hspace{2cm}}$  10

9.  $p(3) = \underline{\hspace{2cm}}$  6

10.  $p(1) = \underline{\hspace{2cm}}$  2

11.  $p(22) = \underline{\hspace{2cm}}$  10

## Reflection

- Put a star next to a problem you want to understand better.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

There are several ways we can define, or describe, a sequence. When you define a sequence recursively, you are determining each term using the previous term.

You can define a sequence recursively by identifying the first term of the sequence and writing a rule for how the sequence changes between terms by either a *constant ratio* or a *constant difference*. When writing the rule in function notation, you can write the recursive definition by referencing the previous term, which can be written as  $f(n - 1)$ .

Here is an example of a recursive definition for the sequence

$$32, 16, 8, 4, 2, 1 \text{ written in function notation.} \quad f(n) = \begin{cases} 32 & n = 1 \\ 0.5 f(n - 1) & n \geq 2 \end{cases}$$

The first term is 32, and the sequence changes by a constant ratio of 0.5. The rule is to multiply the previous term  $f(n - 1)$  by 0.5.

The expression  $32(0.5)^{n-1}$  could also be used to define this sequence.

**Things to Remember:**

## Lesson Practice

### A1.4.13

Name: ..... Date: ..... Period: .....

**Problems 1–3:** Let  $h(n)$  represent the value of term  $n$  in this sequence: 2, 6, 10, 14, 18, .... Write the number that makes each equation true.

- 1.**  $h(1) = \underline{\quad 2 \quad}$       **2.**  $h(8 - 1) = \underline{\quad 26 \quad}$       **3.**  $h(n) = h(n - 1) + \underline{\quad 4 \quad}$

- 4.** Match each sequence with one of the definitions.

Sequence	Definition
a. $2, 8, 14, 20, 26, \dots$	.....b..... $f(1) = 2$ $f(n) = 7 \bullet f(n - 1)$
b. $2, 14, 98, 686, \dots$	.....c..... $g(1) = 2$ $g(n) = \frac{1}{2} \bullet g(n - 1)$
c. $2, 1, 0.5, 0.25, 0.125, \dots$	.....a..... $h(1) = 2$ $h(n) = h(n - 1) + 0$

**Problems 5–6:** Write the first four terms of each sequence.

- |  |  |
|--|--|
| <b>5.</b> $a(1) = 1$<br>$a(n) = 10 \cdot a(n - 1)$ | <b>6.</b> $b(1) = 1$<br>$b(n) = b(n - 1) + 10$ |
| <b>1</b> , <b>10</b> , <b>100</b> , <b>1000</b>    | <b>1</b> , <b>11</b> , <b>21</b> , <b>31</b>   |

**Problems 7–8:** Here are the first five terms of some sequences. Write a recursive definition for each one.

- 7.** 2, 8, 32, 128, 512      **8.** 240, 180, 120, 60, 0

$g(1) = \underline{\quad 2 \quad}$        $h(1) = \underline{\quad 240 \quad}$

$q(n) = \underline{\quad q(n-1) \cdot 4 \quad}$        $h(n) = \underline{\quad h(n-1) - 60 \quad}$

## Lesson Practice

A1.4.13

Name: ..... Date: ..... Period: .....

9. An arithmetic sequence  $a(n)$  and geometric sequence  $g(n)$  both have the same first and fourth term. Determine a recursive definition for each. **Responses vary.**

**Arithmetic Sequence**

$$a(1) = \underline{\quad 3 \quad}$$

$$a(n) = \underline{a(n-1) + 7}$$

**Geometric Sequence**

$$g(1) = \underline{\quad 3 \quad}$$

$$g(n) = \underline{g(n-1) \cdot 2}$$

## Spiral Review

**Problems 10–12:** Determine whether each sequence is arithmetic, geometric, or neither.

10. 162, 54, 18, 6 . . .

Arithmetic

Geometric

Neither

11. 2, 4, 8, 14 . . .

Arithmetic

Geometric

Neither

12. 6, 12, 18, 24 . . .

Arithmetic

Geometric

Neither

## Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

The output of an **absolute value function** is the distance of its input from a given value. The equation of an absolute value function is defined using absolute value symbols, and its graph forms the shape of a V. We can write absolute value functions in the form  $f(x) = |x - h|$ , where  $f(x)$  gives the distance of any input,  $x$ , from  $h$ . Let's look at an example.

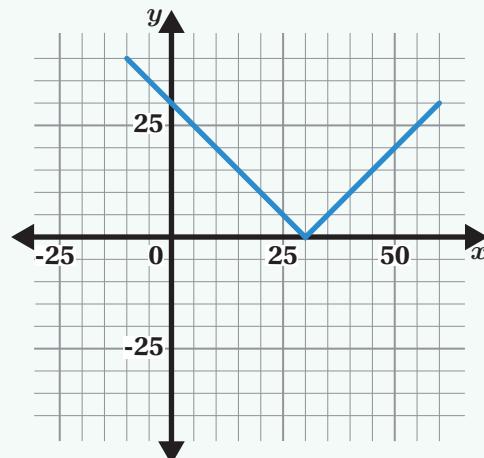
Mr. DeAndre asked his students to guess a mystery number and gave each student a score. Their score was how far away their guess was from his mystery number, 30.

Here is the graph of the function  $f(x) = |x - 30|$ , which gives the score for each guess,  $x$ .

We can use the equation to determine the value of  $f(25)$  and interpret its meaning.

$$\begin{aligned}f(25) &= |25 - 30| \\&= |-5| \\&= 5\end{aligned}$$

This means a student who guessed 25 was 5 away from the mystery number.

**Things to Remember:**

## Lesson Practice

A1.4.15

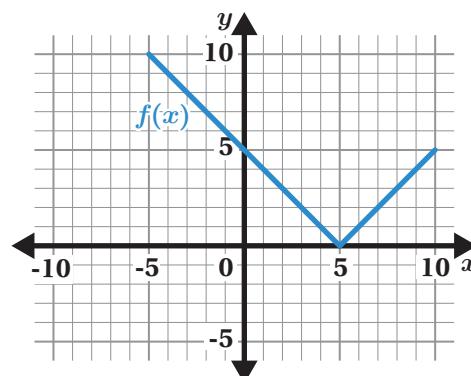
Name: ..... Date: ..... Period: .....

**Problems 1–3:** Use the graph of  $f(x)$  to determine each value.

1.  $f(0) = \underline{\hspace{1cm}} 5 \underline{\hspace{1cm}}$

2.  $f(8) = \underline{\hspace{1cm}} 3 \underline{\hspace{1cm}}$

3.  $f(5) = \underline{\hspace{1cm}} 0 \underline{\hspace{1cm}}$



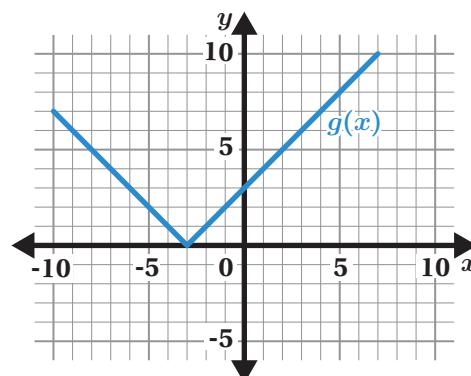
4. Which equation represents the graph of  $g(x)$ ? Circle your choice.

A.  $g(x) = |x| - 3$

B.  $g(x) = |x - 3|$

C.  $g(x) = |x| + 3$

D.  $g(x) = |x + 3|$



**Problems 5–7:** Ricardo's teacher challenged his class to guess how many marbles were in a jar. Each student was given a score equal to how far away their guess was from the actual number of marbles in the jar. The graph shows each students' guess,  $x$ , and score,  $m(x)$ .

5. How many marbles are in the jar?

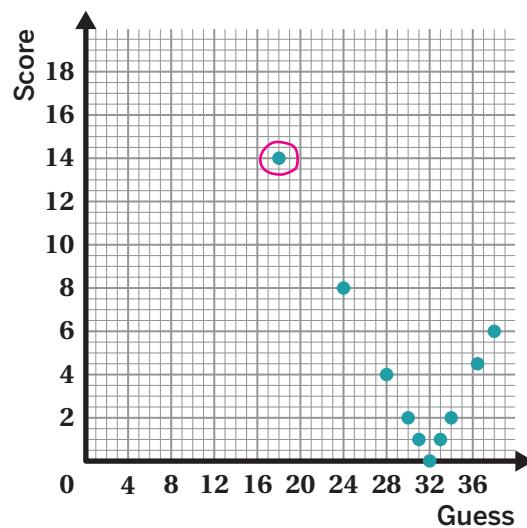
**32**

6. Circle the point that represents the furthest guess from the actual number of marbles.

7. Ricardo writes the equation  $m(25) = 7$ .

What does his equation mean?

**Responses vary.** Ricardo's equation means that a guess of 25 would give you a score of 7 because it is 7 numbers away from the correct number of marbles in the jar.



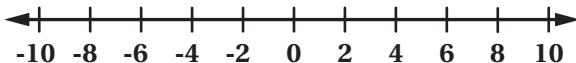
# Lesson Practice

A1.4.15

Name: ..... Date: ..... Period: .....

## Spiral Review

8. Select all the true statements. Use the number line if it helps with your thinking.



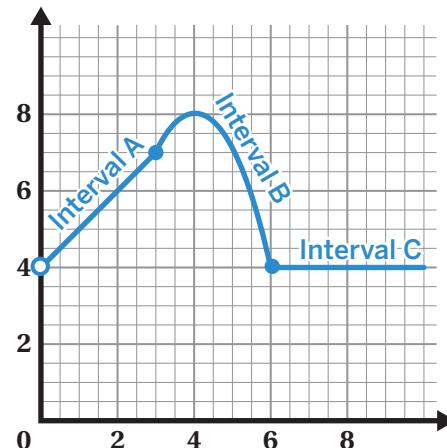
- A.  $-4 > -2$        B. The distance from -2 to 2 is equal to the distance from 6 to 10.  
 C.  $-4 \geq -6$        D. 8 is the only number 2 units away from 6.  
 E.  $|3| = |-3|$

**Problems 9–11:** Match each domain to an interval of the piecewise function on the graph.

9.  $3 \leq x \leq 6$  ..... Interval B

10.  $x \geq 6$  ..... Interval C

11.  $0 < x \leq 3$  ..... Interval A



12. The California Department of Fish and Wildlife estimated there were 460,420 deer in the state in 2021. They estimated that the deer population in 2018 was 470,000.

Calculate the average rate of change during this time interval and explain what it tells us about the deer population.

**About -3,193.3 deer per year. Explanations vary. The population of deer in California is decreasing at an average of approximately 3,193 deer per year.**

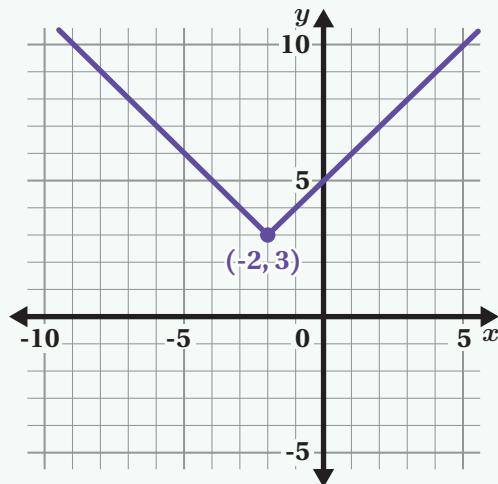
## Reflection

1. Put a smiley face next to a problem you were stuck on and then figured out.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can determine key features of the graph of an *absolute value function* by analyzing its table or equation, which are both helpful in sketching its graph.

Here are the graph and table for the absolute value function  $f(x) = |x + 2| + 3$ .



$x$	$f(x) =  x + 2  + 3$
-4	5
-2	3
0	5
2	7

Evaluating  $f(x)$  at  $x = -2$  makes the equation equal to 3. This means that when the input of the function is  $-2$ , the output is 3 and a point on the graph of  $f(x)$  is  $(-2, 3)$ . The values in the table show that there is symmetry around the point  $(-2, 3)$ . This tells us that  $(-2, 3)$  is the minimum value of the function, which we can see by looking at the graph.

**Things to Remember:**

# Lesson Practice

A1.4.16

Name: ..... Date: ..... Period: .....

**Problems 1–3:** Write each expression as a single integer.

1.  $|-4|$

4

2.  $|7| - 2$

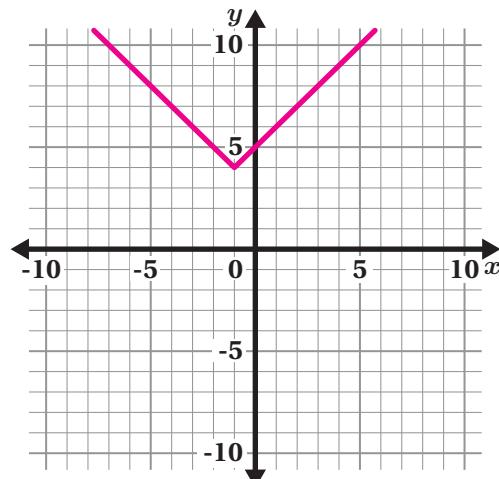
5

3.  $|-8| + 1$

9

4. Graph  $g(x) = |x + 1| + 4$ . Use the table if it helps with your thinking.

$x$	$g(x)$
-3	6
-1	4
0	5
2	7
3	8

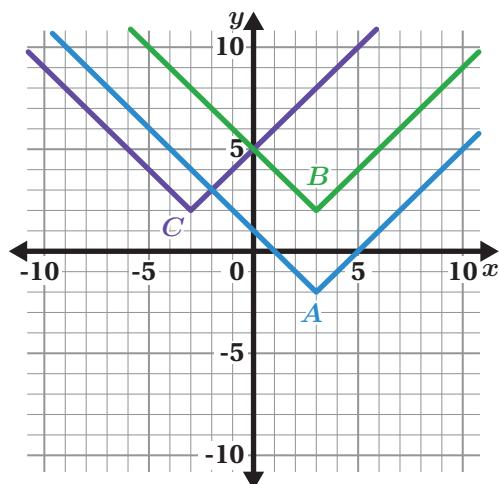


5. Match each function with its graph.

$f(x) = |x + 3| + 2$  ..... C

$g(x) = |x - 3| - 2$  ..... A

$h(x) = |x - 3| + 2$  ..... B



## Lesson Practice

A1.4.16

Name: ..... Date: ..... Period: .....

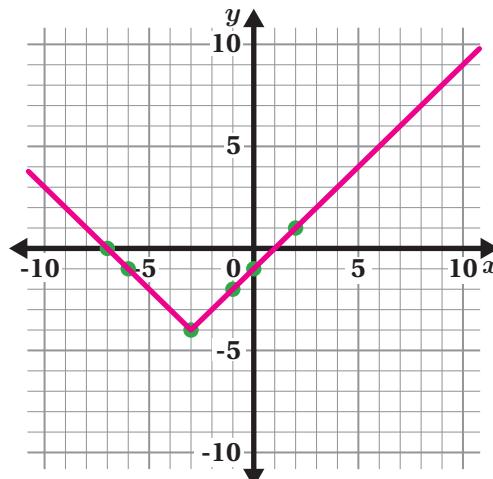
6. Here are some points on the graph of  $h(x) = |x + 3| - 4$ .

- Sketch a graph of  $h(x)$ .
- Describe the graph using some of these terms:

positive maximum increasing domain

negative minimum decreasing range

symmetry piecewise-defined function



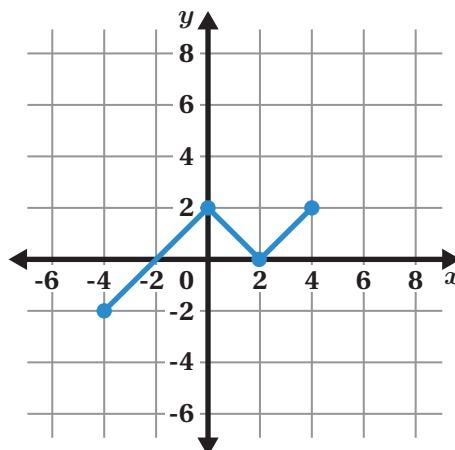
**Responses vary.** This graph has a minimum of -4 when  $x = -3$ . The domain is all numbers and the range is all numbers greater than or equal to -4. The graph is increasing when  $x > -3$  and decreasing when  $x < -3$ .

7. Determine two different piecewise-defined functions that could represent this graph.

**Responses vary.**

$$f(x) = \begin{cases} -|x| + 2 & -4 \leq x \leq 2 \\ x - 2 & 2 < x \leq 4 \end{cases}$$

$$g(x) = \begin{cases} x + 2 & -4 \leq x \leq 0 \\ |x - 2| & 0 < x \leq 4 \end{cases}$$



## Spiral Review

**Problems 8–10:** For each square root, write which two consecutive whole numbers the value is between.

8.  $\sqrt{14}$

3 and 4

9.  $\sqrt{60}$

7 and 8

10.  $\sqrt{88}$

9 and 10

## Reflection

- Put a heart next to a problem you understand well.
- Use this space to ask a question or share something you're proud of.

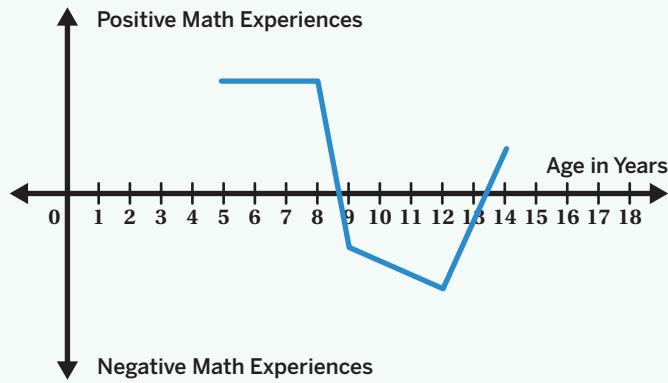
**Lesson Summary**

Storytelling is a powerful way to learn more about others and to reflect on your own journey. Graphing a story can help us see interesting self-discoveries and have deeper discussions.

When you use equations, tables, words, graphs, and their key features to represent real-world relationships, pay close attention to the scale and units. You can look for the maximums or minimums; intercepts; intervals where the graph increases, decreases, or remains constant; and domain and range to make sense of the situation or someone's story. Let's look at an example.

Here's a graph of someone's math experiences over time. From the graph, we can learn that:

- Most of their positive math experiences were from ages 5 to 8, and their most negative math experience was at age 12.
- Their math experiences decreased from age 8 to 12 and increased after age 12.
- This person graphed their experiences from age 5 to age 14. It's possible they drew this graph at age 14.



There is also a lot we can't tell from the graph of someone's story. For example, we don't know what the positive or negative math experiences were, or what emotions they were feeling at the time. The graph gives us only a window into someone else's story, not the full image.

**Things to Remember:**

# Lesson Practice

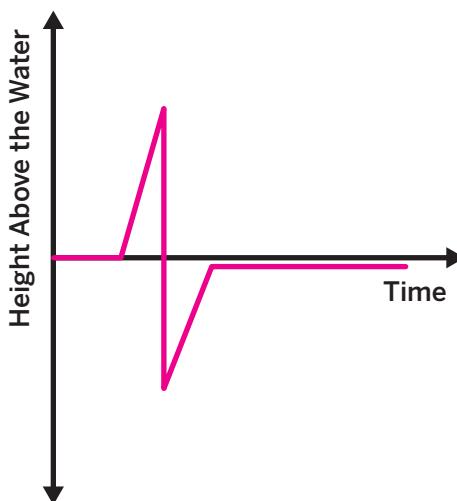
A1.4.17

Name: ..... Date: ..... Period: .....

1. Sketch a graph to represent this situation:

Pablo went swimming at the city pool. When he first arrived, he climbed to the top of the high diving board where he dived into the deep end of the pool. He swam to the shallow end where he stayed for several minutes.

**Responses vary.**



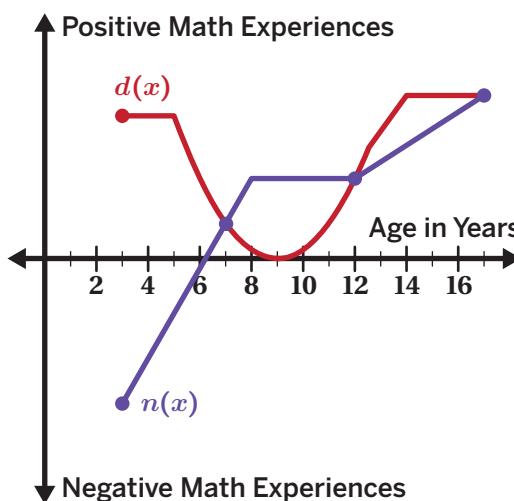
**Problems 2–6:** Adhira and Anand are twins.

This graph represents their math stories.

$d(x)$  represents Adhira's math experience as a function of age and  $n(x)$  represents Anand's math experience as a function of age.

2. What does  $n(17) > n(12)$  say about Anand's math experience?

**Responses vary.** Anand's math experience was better at age 17 than at age 12.



3. What does  $d(3) = d(5)$  say about Adhira's math experience?

**Responses vary.** Adhira's math experience was the same at age 3 as age 5.

4. True or false: The domain for Adhira's graph is the same as the domain for Anand's graph.

**True**

5. True or false: The range for Adhira's graph is the same as the range for Anand's graph.

**False**

6. Who had a greater average rate of change between ages 3 and 17: Adhira or Anand? Explain how you know.

**Anand.** *Explanations vary.* Since the value of their functions is the same at age 17, the question is about which function grew the most from age 3. Anand's value at age 3 was much lower than Adhira's.

## Lesson Practice

A1.4.17

Name: ..... Date: ..... Period: .....

7. Create a story that could be modeled by this piecewise function.

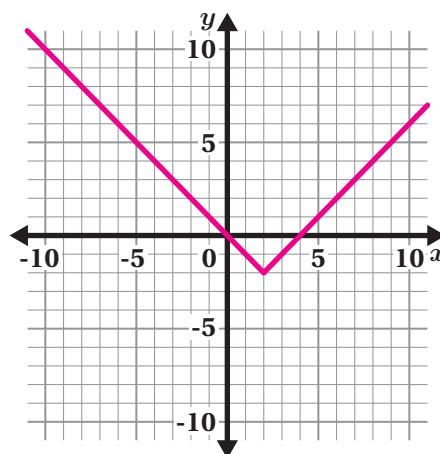
Create a graph on graph paper if it helps with your thinking.

**Responses vary.** I stood on top of a rock 4 meters above the water for 8 seconds. I then jumped into the water. I immediately climbed out of the water and stood on a higher rock for 3 seconds.

$$f(x) = \begin{cases} 4 & 0 \leq x < 8 \\ -4x + 36 & 8 \leq x < 9 \\ 2x - 18 & 9 \leq x \leq 12 \\ 6 & 12 < x \leq 15 \end{cases}$$

## Spiral Review

8. Sketch the graph of  $g(x) = |x - 2| - 2$ .



**Problems 9–11:** Evaluate each expression.

9.  $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

10.  $\left(\frac{1}{2}\right)^{-3} = 8$

11.  $\left(\left(\frac{1}{2}\right)^3\right)^{-2} = 64$

## Reflection

1. Circle a problem you want to talk to a classmate about.
2. Use this space to ask a question or share something you're proud of.