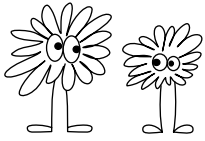


# FRACTION NINJA BOOTCAMP LEVEL 3





# MULTIPLYING FRACTIONS



Simplify each fractional expression.

$$\frac{5}{8} \times \frac{3}{7} = \frac{5 \times 3}{8 \times 7} \\ = \frac{15}{56}.$$

$$\frac{11}{12} \times \frac{3}{4} = \frac{11 \times 3}{12 \times 4} \\ = \frac{11 \times \cancel{3}}{\cancel{3} \times 4 \times 4} \\ = \frac{11}{16}.$$

$$\frac{5}{24} \times \frac{8}{9} = \frac{5 \times 8}{24 \times 9} \\ = \frac{5 \times \cancel{8}}{\cancel{8} \times 3 \times 9} \\ = \frac{5}{27}.$$

$$\frac{4}{14} \times \frac{7}{18} = \frac{4 \times 7}{14 \times 18} \\ = \frac{\cancel{2} \times \cancel{2} \times 7}{\cancel{7} \times 2 \times 2 \times 9} \\ = \frac{1}{9}.$$

$$\frac{11}{12} \times \frac{18}{20} = \frac{11 \times 18}{12 \times 20} \\ = \frac{11 \times \cancel{2} \times \cancel{3} \times 3}{\cancel{2} \times 2 \times \cancel{3} \times 2 \times 2 \times 5} \\ = \frac{33}{40}.$$

$$\frac{3}{8} \times \frac{4}{15} = \frac{3 \times 4}{8 \times 15} \\ = \frac{\cancel{3} \times \cancel{4}}{\cancel{4} \times 2 \times 5 \times \cancel{3}} \\ = \frac{1}{10}.$$

$$\frac{6}{5} \times \frac{20}{21} = \frac{6 \times 20}{5 \times 21} \\ = \frac{\cancel{2} \times 2 \times \cancel{5} \times 4}{\cancel{5} \times 7 \times \cancel{3}} \\ = \frac{8}{7}.$$

$$\frac{12}{25} \times \frac{10}{9} = \frac{12 \times 10}{25 \times 9} \\ = \frac{\cancel{2} \times 4 \times \cancel{5} \times 2}{5 \times \cancel{5} \times \cancel{3} \times 3} \\ = \frac{8}{15}.$$

$$\frac{20}{32} \times \frac{4}{5} = \frac{20 \times 4}{32 \times 5} \\ = \frac{\cancel{2} \times \cancel{4} \times \cancel{4} \times \cancel{4}}{2 \times \cancel{4} \times \cancel{4} \times \cancel{4} \times 5} \\ = \frac{1}{2}.$$

$$\frac{13}{16} \times \frac{2}{3} = \frac{13 \times 2}{16 \times 3} \\ = \frac{13 \times \cancel{2}}{\cancel{2} \times 8 \times 3} \\ = \frac{13}{24}.$$

$$\frac{9}{28} \times \frac{80}{27} = \frac{9 \times 80}{28 \times 27} \\ = \frac{\cancel{9} \times \cancel{4} \times 20}{\cancel{4} \times 7 \times \cancel{9} \times 3} \\ = \frac{20}{21}.$$

$$\frac{15}{16} \times \frac{24}{25} = \frac{15 \times 24}{16 \times 25} \\ = \frac{\cancel{3} \times 3 \times \cancel{8} \times 3}{\cancel{8} \times 2 \times \cancel{5} \times 5} \\ = \frac{9}{10}.$$

$$\begin{aligned}
 2\frac{1}{2} \times 2\frac{2}{3} &= \frac{5}{2} \times \frac{8}{3} \\
 &= \frac{5 \times 8}{2 \times 3} \\
 &= \frac{5 \times 4 \times \cancel{2}}{\cancel{2} \times 3} \\
 &= \frac{20}{3} \\
 &= 6\frac{2}{3}.
 \end{aligned}$$

$$\begin{aligned}
 3\frac{3}{5} \times 2\frac{2}{3} &= \frac{18}{5} \times \frac{8}{3} \\
 &= \frac{18 \times 8}{5 \times 3} \\
 &= \frac{6 \times \cancel{3} \times 8}{5 \times \cancel{3}} \\
 &= \frac{48}{5} \\
 &= 9\frac{3}{5}.
 \end{aligned}$$

$$\begin{aligned}
 \frac{9}{16} \times 3\frac{1}{21} &= \frac{9}{16} \times \frac{64}{21} \\
 &= \frac{9 \times 64}{16 \times 21} \\
 &= \frac{3 \times \cancel{3} \times \cancel{16} \times 4}{\cancel{16} \times 3 \times 7} \\
 &= \frac{12}{7} \\
 &= 1\frac{5}{7}.
 \end{aligned}$$

$$\begin{aligned}
 3\frac{3}{10} \times 1\frac{4}{11} &= \frac{33}{10} \times \frac{15}{11} \\
 &= \frac{33 \times 15}{10 \times 11} \\
 &= \frac{3 \times \cancel{11} \times 5 \times 3}{\cancel{5} \times 2 \times \cancel{11}} \\
 &= \frac{9}{2} \\
 &= 4\frac{1}{2}.
 \end{aligned}$$

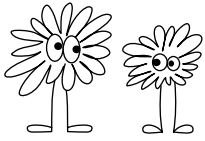
$$\begin{aligned}
 \frac{12}{17} \times 6\frac{3}{8} &= \frac{12}{17} \times \frac{51}{8} \\
 &= \frac{12 \times 51}{17 \times 8} \\
 &= \frac{\cancel{4} \times 3 \times \cancel{17} \times 3}{\cancel{17} \times \cancel{4} \times 2} \\
 &= \frac{9}{2} \\
 &= 4\frac{1}{2}.
 \end{aligned}$$

$$\begin{aligned}
 6\frac{1}{4} \times 1\frac{1}{15} &= \frac{25}{4} \times \frac{16}{15} \\
 &= \frac{25 \times 16}{4 \times 15} \\
 &= \frac{\cancel{5} \times 5 \times 4 \times \cancel{4}}{\cancel{4} \times \cancel{5} \times 3} \\
 &= \frac{20}{3} \\
 &= 6\frac{2}{3}.
 \end{aligned}$$

Paul had  $\frac{4}{5}$  of a cake. He gave  $\frac{2}{3}$  of his portion to John. John then gave  $\frac{3}{4}$  of his cake to Ringo. Ringo ate half of his cake and gave the rest to George. What fraction of the original cake did George get? To take a fraction of an amount, multiply by that fraction.

$$\begin{aligned}
 \frac{4}{5} \times \frac{2}{3} \times \frac{3}{4} \times \frac{1}{2} &= \frac{\cancel{4} \times 2 \times \cancel{3} \times 1}{5 \times \cancel{3} \times \cancel{4} \times 2} \\
 &= \frac{1}{5}.
 \end{aligned}$$

George got  $\frac{1}{5}$  of the original cake.



# DIVIDING FRACTIONS



Simplify each fractional expression.

$$\begin{aligned}\frac{2}{9} \div \frac{8}{3} &= \frac{2}{9} \times \frac{3}{8} \\ &= \frac{2 \times 3}{9 \times 8} \\ &= \frac{\cancel{2} \times \cancel{3}}{\cancel{3} \times 3 \times 4 \times \cancel{2}} \\ &= \frac{1}{12}\end{aligned}$$

$$\begin{aligned}\frac{15}{8} \div \frac{8}{3} &= \frac{15}{8} \times \frac{3}{8} \\ &= \frac{15 \times 3}{8 \times 8} \\ &= \frac{45}{64}\end{aligned}$$

$$\begin{aligned}\frac{10}{9} \div \frac{5}{3} &= \frac{10}{9} \times \frac{3}{5} \\ &= \frac{10 \times 3}{9 \times 5} \\ &= \frac{2 \times \cancel{5} \times \cancel{3}}{3 \times \cancel{3} \times \cancel{5}} \\ &= \frac{2}{3}\end{aligned}$$

$$\begin{aligned}\frac{13}{24} \div \frac{3}{4} &= \frac{13}{24} \times \frac{4}{3} \\ &= \frac{13 \times 4}{24 \times 3} \\ &= \frac{13 \times \cancel{4}}{\cancel{4} \times 6 \times 3} \\ &= \frac{13}{18}\end{aligned}$$

$$\begin{aligned}\frac{25}{18} \div \frac{10}{27} &= \frac{25}{18} \times \frac{27}{10} \\ &= \frac{25 \times 27}{18 \times 10} \\ &= \frac{\cancel{5} \times 5 \times \cancel{3} \times 3}{\cancel{3} \times 2 \times 2 \times \cancel{5}} \\ &= \frac{15}{4}\end{aligned}$$

$$\begin{aligned}\frac{16}{15} \div \frac{12}{5} &= \frac{16}{15} \times \frac{5}{12} \\ &= \frac{16 \times 5}{15 \times 12} \\ &= \frac{\cancel{4} \times 4 \times \cancel{5}}{\cancel{5} \times 3 \times \cancel{4} \times 3} \\ &= \frac{4}{9}\end{aligned}$$

$$\begin{aligned}\frac{8}{15} \div \frac{4}{3} &= \frac{8}{15} \times \frac{3}{4} \\ &= \frac{8 \times 3}{15 \times 4} \\ &= \frac{\cancel{4} \times 2 \times \cancel{3}}{5 \times \cancel{3} \times \cancel{4}} \\ &= \frac{2}{5}\end{aligned}$$

$$\begin{aligned}\frac{6}{25} \div \frac{3}{20} &= \frac{6}{25} \times \frac{20}{3} \\ &= \frac{6 \times 20}{25 \times 3} \\ &= \frac{2 \times \cancel{3} \times \cancel{4} \times 5}{5 \times \cancel{5} \times \cancel{3}} \\ &= \frac{8}{5}\end{aligned}$$

$$\begin{aligned}\frac{7}{18} \div \frac{14}{15} &= \frac{7}{18} \times \frac{15}{14} \\ &= \frac{7 \times 15}{18 \times 14} \\ &= \frac{\cancel{7} \times 5 \times \cancel{3}}{\cancel{3} \times 6 \times 2 \times \cancel{7}} \\ &= \frac{5}{12}\end{aligned}$$

$$\begin{aligned}\frac{22}{7} \div \frac{11}{14} &= \frac{22}{7} \times \frac{14}{11} \\ &= \frac{22 \times 14}{7 \times 11} \\ &= \frac{2 \times \cancel{11} \times 2 \times \cancel{7}}{\cancel{7} \times \cancel{11}} \\ &= 4\end{aligned}$$

$$\begin{aligned}\frac{8}{21} \div \frac{12}{35} &= \frac{8}{21} \times \frac{35}{12} \\ &= \frac{8 \times 35}{21 \times 12} \\ &= \frac{\cancel{4} \times 2 \times \cancel{7} \times 5}{\cancel{7} \times 3 \times \cancel{4} \times 3} \\ &= \frac{10}{9}\end{aligned}$$

$$\begin{aligned}\frac{9}{49} \div \frac{15}{14} &= \frac{9}{49} \times \frac{14}{15} \\ &= \frac{9 \times 14}{49 \times 15} \\ &= \frac{\cancel{3} \times 3 \times 2 \times \cancel{7}}{\cancel{7} \times \cancel{7} \times \cancel{3} \times 5} \\ &= \frac{6}{35}\end{aligned}$$

$$\begin{aligned}
 2\frac{1}{2} \div 2\frac{2}{3} &= \frac{5}{2} \div \frac{8}{3} \\
 &= \frac{5}{2} \times \frac{3}{8} \\
 &= \frac{5 \times 3}{2 \times 8} \\
 &= \frac{15}{16}
 \end{aligned}$$

$$\begin{aligned}
 5\frac{5}{8} \div 1\frac{2}{3} &= \frac{45}{8} \div \frac{5}{3} \\
 &= \frac{45}{8} \times \frac{3}{5} \\
 &= \frac{9 \times \cancel{5} \times 3}{8 \times \cancel{5}} \\
 &= \frac{27}{8} \\
 &= 3\frac{3}{8}
 \end{aligned}$$

$$\begin{aligned}
 4\frac{2}{7} \div 1\frac{1}{14} &= \frac{30}{7} \div \frac{15}{14} \\
 &= \frac{30}{7} \times \frac{14}{15} \\
 &= \frac{30 \times 14}{7 \times 15} \\
 &= \frac{\cancel{15} \times 2 \times \cancel{7} \times 2}{\cancel{7} \times \cancel{15}} \\
 &= \frac{4}{1} \\
 &= 4
 \end{aligned}$$

$$\begin{aligned}
 2\frac{7}{9} \div 5 &= \frac{25}{9} \div \frac{5}{1} \\
 &= \frac{25}{9} \times \frac{1}{5} \\
 &= \frac{25 \times 1}{9 \times 5} \\
 &= \frac{5 \times 5}{9 \times 5} \\
 &= \frac{5}{9}
 \end{aligned}$$

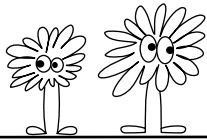
$$\begin{aligned}
 21 \div 4\frac{2}{3} &= \frac{21}{1} \div \frac{14}{3} \\
 &= \frac{21}{1} \times \frac{3}{14} \\
 &= \frac{21 \times 3}{1 \times 14} \\
 &= \frac{\cancel{7} \times 3 \times 3}{2 \times \cancel{7}} \\
 &= \frac{9}{2} \\
 &= 4\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 1\frac{2}{3} \div 4\frac{5}{6} &= \frac{5}{3} \div \frac{29}{6} \\
 &= \frac{5}{3} \times \frac{6}{29} \\
 &= \frac{5 \times 6}{3 \times 29} \\
 &= \frac{5 \times \cancel{3} \times 2}{\cancel{3} \times 29} \\
 &= \frac{10}{29}
 \end{aligned}$$

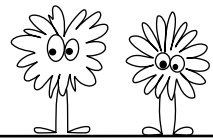
Molly has a board that is  $18\frac{1}{2}$  feet long. How many of pieces of length  $\frac{3}{4}$  ft can Molly make if she cuts up the board?

$$\begin{aligned}
 18\frac{1}{2} \div \frac{3}{4} &= \frac{37}{2} \div \frac{3}{4} \\
 &= \frac{37}{2} \times \frac{4}{3} \\
 &= \frac{37 \times 4}{2 \times 3} \\
 &= \frac{37 \times 2 \times \cancel{2}}{\cancel{2} \times 3} \\
 &= \frac{74}{3} \\
 &= 24\frac{2}{3}
 \end{aligned}$$

Molly will get 24 pieces with part left over.

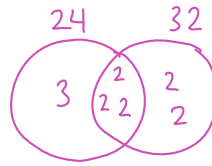
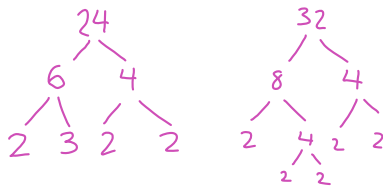


# COMMON MULTIPLES



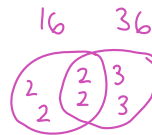
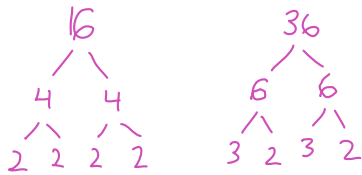
For each pair of numbers, find the least common multiple

24, 32



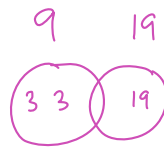
$$\begin{aligned} \text{LCM}(24, 32) &= 3 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 96 \end{aligned}$$

16, 36



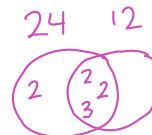
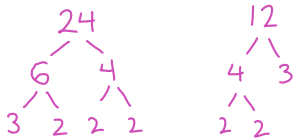
$$\begin{aligned} \text{LCM}(16, 36) &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \\ &= 144 \end{aligned}$$

9, 19



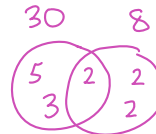
$$\begin{aligned} \text{LCM}(9, 19) &= 3 \times 3 \times 19 \\ &= 171 \end{aligned}$$

24, 12



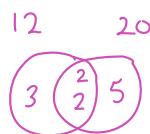
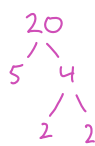
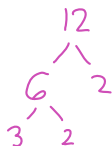
$$\begin{aligned} \text{LCM}(24, 12) &= 2 \times 2 \times 2 \times 3 \\ &= 24 \end{aligned}$$

30, 8



$$\begin{aligned} \text{LCM}(30, 8) &= 5 \times 3 \times 2 \times 2 \times 2 \\ &= 120 \end{aligned}$$

12, 20

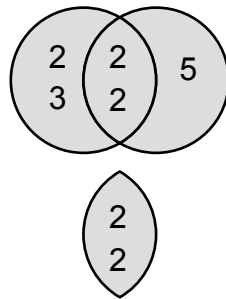


$$\begin{aligned} \text{LCM}(12, 20) &= 3 \times 2 \times 2 \times 5 \\ &= 60 \end{aligned}$$

The lesson showed a cool way to find the least common multiple (LCM) of two numbers and their greatest common factor (GCF) using their prime factorizations and a Venn diagram. Below is an example for the numbers 20 and 24.

$$24 = 2 \times 2 \times 2 \times 3$$

$$20 = 2 \times 2 \times 5$$



$$\text{LCM}(24, 20) = 2 \times 2 \times 2 \times 3 \times 5 = 120$$

$$\text{GCF}(24, 20) = 2 \times 2 = 4$$

The least common multiple came from multiplying each prime factor in the Venn diagram of factors. The greatest common factor came from multiplying the prime factors that were common to both circles. The four factors of 24 and three factors of 20 each showed up exactly once between the two diagrams above, so that means  $24 \times 20 = \text{LCM}(24, 20) \times \text{GCF}(24, 20)$ . A similar fact is true for all pairs of numbers,  $a$  and  $b$ .  $a \times b = \text{LCM}(a, b) \times \text{GCF}(a, b)$

Using that formula, you can find the LCM if you know the GCF. We could find

$$\text{LCM}(24, 20) = \frac{24 \times 20}{\text{GCF}(24, 20)} = \frac{24 \times 20}{4} = 120$$

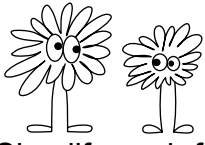
For each pair of numbers, find the least common multiple. Can you do it using the technique described above (multiplying the two numbers and dividing by their greatest common factor)?

19, 21      $\text{GCD}(19, 21) = 1$

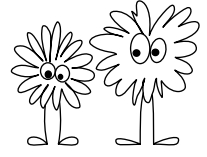
$$\begin{aligned} \text{LCM}(19, 21) &= \frac{19 \times 21}{\text{GCD}(19, 21)} \\ &= \frac{399}{1} \\ &= 399. \end{aligned}$$

55, 45

$$\begin{aligned} \text{GCD}(55, 45) &= 5 & \text{LCM}(55, 45) &= \frac{55 \times 45}{\text{GCD}(55, 45)} \\ & & &= \frac{55 \times 45}{5} \\ & & &= \frac{5 \times 11 \times 45}{5} \\ & & &= 495. \end{aligned}$$



# ADDING AND SUBTRACTING FRACTIONS



Simplify each fractional expression.

$$\begin{aligned}\frac{2}{9} + \frac{2}{3} &= \frac{2}{9} + \frac{2 \times 3}{3 \times 3} \\ &= \frac{2}{9} + \frac{6}{9} \\ &= \frac{2+6}{9} \\ &= \frac{8}{9}.\end{aligned}$$

$$\begin{aligned}\frac{4}{3} - \frac{3}{4} &= \frac{4 \times 4}{4 \times 3} - \frac{3 \times 3}{4 \times 3} \\ &= \frac{16}{12} - \frac{9}{12} \\ &= \frac{16-9}{12} \\ &= \frac{7}{12}.\end{aligned}$$

$$\begin{aligned}\frac{3}{7} + \frac{5}{9} &= \frac{3 \times 3}{7 \times 3} + \frac{5 \times 7}{9 \times 7} \\ &= \frac{27}{63} + \frac{35}{63} \\ &= \frac{62}{63}.\end{aligned}$$

$$\begin{aligned}\frac{11}{24} - \frac{1}{4} &= \frac{11}{24} - \frac{1 \times 6}{4 \times 6} \\ &= \frac{11}{24} - \frac{6}{24} \\ &= \frac{5}{24}.\end{aligned}$$

$$\begin{aligned}\frac{5}{6} + \frac{3}{10} &= \frac{5 \times 5}{6 \times 5} + \frac{3 \times 3}{10 \times 3} \\ &= \frac{25}{30} + \frac{9}{30} \\ &= \frac{34}{30} \\ &= \frac{17}{15}.\end{aligned}$$

$$\begin{aligned}\frac{7}{8} - \frac{2}{7} &= \frac{7 \times 7}{8 \times 7} - \frac{2 \times 8}{7 \times 8} \\ &= \frac{49}{56} - \frac{16}{56} \\ &= \frac{33}{56}.\end{aligned}$$

$$\begin{aligned}\frac{5}{8} + \frac{5}{16} &= \frac{5 \times 2}{8 \times 2} + \frac{5}{16} \\ &= \frac{10}{16} + \frac{5}{16} \\ &= \frac{15}{16}.\end{aligned}$$

$$\begin{aligned}\frac{17}{20} - \frac{3}{8} &= \frac{17 \times 2}{20 \times 2} - \frac{3 \times 5}{8 \times 5} \\ &= \frac{34}{40} - \frac{15}{40} \\ &= \frac{19}{40}.\end{aligned}$$

$$\begin{aligned}\frac{16}{25} + \frac{3}{10} &= \frac{16 \times 2}{25 \times 2} + \frac{3 \times 5}{10 \times 5} \\ &= \frac{32}{50} + \frac{15}{50} \\ &= \frac{47}{50}.\end{aligned}$$

$$\begin{aligned}\frac{33}{40} - \frac{3}{4} &= \frac{33}{40} - \frac{3 \times 10}{4 \times 10} \\ &= \frac{33}{40} - \frac{30}{40} \\ &= \frac{3}{40}.\end{aligned}$$



Note that when adding mixed numbers, you could choose to first add the whole number parts and fractional parts separately.

$$\begin{aligned} 2\frac{1}{2} + 2\frac{2}{3} &= \frac{3}{3} \times \frac{5}{2} + \frac{8}{3} \times \frac{2}{2} \\ &= \frac{15}{6} + \frac{16}{6} \\ &= \frac{31}{6} \\ &= 5\frac{1}{6}. \end{aligned}$$

$$\begin{aligned} 2\frac{5}{8} - 1\frac{2}{3} &= \frac{3}{3} \times \frac{21}{8} - \frac{5}{3} \times \frac{8}{8} \\ &= \frac{63}{24} - \frac{40}{24} \\ &= \frac{23}{24}. \end{aligned}$$

$$\begin{aligned} 5\frac{3}{10} + 3\frac{5}{6} &= \frac{3}{3} \times \frac{53}{10} + \frac{23}{6} \times \frac{5}{5} \\ &= \frac{159}{30} + \frac{115}{30} \\ &= \frac{274}{30} \\ &= 9\frac{4}{30} \\ &= 9\frac{2}{15}. \end{aligned}$$

$$\begin{aligned} 12\frac{1}{8} - 5\frac{1}{3} &= \frac{3}{3} \times \frac{97}{8} - \frac{16}{3} \times \frac{8}{8} \\ &= \frac{291}{24} - \frac{128}{24} \\ &= \frac{163}{24} \\ &= 6\frac{19}{24}. \end{aligned}$$

$$\begin{aligned} 4\frac{5}{12} + 1\frac{7}{8} &= \frac{53}{12} + \frac{15}{8} \\ &= \frac{53}{12} \times \frac{2}{2} + \frac{15}{8} \times \frac{3}{3} \\ &= \frac{106}{24} + \frac{45}{24} \\ &= \frac{151}{24} \\ &= 6\frac{7}{24}. \end{aligned}$$

$$\begin{aligned} 9\frac{3}{4} - 3\frac{7}{8} &= \frac{2}{2} \times \frac{39}{4} - \frac{31}{8} \\ &= \frac{78}{8} - \frac{31}{8} \\ &= \frac{47}{8} \\ &= 5\frac{7}{8}. \end{aligned}$$

The final problem asks you to add and subtract multiple fractions at once. You can deal with them two-at-a-time or tackle the whole problem in one attempt by getting a common denominator for all four fractions. Take your time and write out all the steps so you can check your work.

$$\begin{aligned} 1\frac{1}{6} + 2\frac{3}{5} + 1\frac{1}{4} - 3\frac{3}{4} &= \underbrace{1+2+1-3} + \frac{1}{6} + \frac{3}{5} + \frac{1}{4} - \frac{3}{4} \\ &= 1 + \frac{10}{60} + \frac{36}{60} + \frac{15}{60} - \frac{45}{60} \\ &= 1 + \frac{10+36+15-45}{60} \\ &= 1 + \frac{16}{60} \\ &= 1 + \frac{4}{15} \\ &= 1\frac{4}{15}. \end{aligned}$$

# PUTTING IT ALL TOGETHER

Simplify each expression. Be careful to do neat work so you avoid mistakes.

$$\frac{4 - \frac{3+3}{2}}{5} = \frac{4 - \frac{6}{2}}{5}$$

$$= \frac{4 - 3}{5}$$

$$= \frac{1}{5}$$

$$\frac{\frac{3}{5} + \frac{2}{3}}{3 - \frac{1}{4}} = \frac{\frac{\frac{3}{5} + \frac{2}{3}}{\frac{3}{1} - \frac{1}{4}}}{\frac{\frac{9}{15} + \frac{10}{15}}{\frac{12}{4} - \frac{1}{4}}} = \frac{\frac{19}{15}}{\frac{11}{4}} = \frac{19}{15} \times \frac{4}{11} = \frac{76}{165}$$

$$3 + \frac{1}{2 + \frac{3}{1 + \frac{2}{3}}} = 3 + \frac{1}{2 + \frac{3}{\frac{3}{3} + \frac{2}{3}}} = 3 + \frac{1}{2 + \frac{3}{1 + \frac{2}{3}}} = 3 + \frac{1}{2 + \frac{3}{\frac{5}{3}}} = 3 + \frac{1}{2 + \frac{9}{5}} = 3 + \frac{1}{\frac{19}{5}} = 3 + \frac{5}{19}$$

$$3 - \frac{1}{2 - \frac{3}{1 - \frac{2}{3}}} = 3 - \frac{1}{2 - \frac{3}{\frac{3}{3} - \frac{2}{3}}} = 3 - \frac{1}{2 - \frac{3}{\frac{1}{3}}} = 3 - \frac{1}{2 - 9} = 3 - \frac{1}{-7} = 3 + \frac{1}{7} = 3\frac{1}{7}$$

$$\frac{\frac{\frac{1}{4}}{\frac{2}{3}}}{\frac{5}{8} - \frac{6}{7}} = \frac{\frac{\frac{1}{4} \times \frac{3}{2}}{\frac{5}{8} \times \frac{7}{6}}}{\frac{3}{5 \times 7} - \frac{6}{8 \times 6}} = \frac{\frac{3}{8} \times \frac{6}{5 \times 7}}{\frac{3}{35} - \frac{6}{48}} = \frac{18}{35}$$

$$\frac{2}{3} + \frac{4}{5} + \frac{3}{4} + \frac{1}{2} = \frac{40}{60} + \frac{48}{60} + \frac{45}{60} + \frac{30}{60}$$

$$= \frac{163}{60}$$

$$= 2\frac{43}{60}$$

$$\frac{2 \cdot \frac{5}{8} + 1}{3 - \frac{3}{8}} = \frac{\frac{2 \times 5}{1 \times 8} + 1}{\frac{24}{8} - \frac{3}{8}} = \frac{18}{21} \times \frac{8}{8}$$

$$= \frac{18}{21}$$

$$= \frac{6}{7}$$

$$\frac{\frac{1}{2} + \frac{3}{5} + \frac{1+3}{2+5}}{3} = \frac{\frac{1}{2} + \frac{3}{5} + \frac{4}{7}}{3} = \frac{117}{70} \times \frac{1}{3}$$

$$= \frac{39 \times 3 \times 1}{70 \times 3}$$

$$= \frac{39}{70}$$

$$\frac{\frac{5+5+5+5}{3+3+3}}{5+5} = \frac{\frac{20}{9}}{\frac{10}{3}} = \frac{20}{9} \times \frac{1}{10}$$

$$= \frac{2}{9}$$

$$= \frac{2}{27}$$

$$9 - \frac{4 - \frac{5 - \frac{3}{2}}{3}}{5} = 9 - \frac{4 - \frac{\frac{10-3}{2}}{3}}{5} = 9 - \frac{24}{6} - \frac{7}{6}$$

$$= 9 - \frac{17}{6}$$

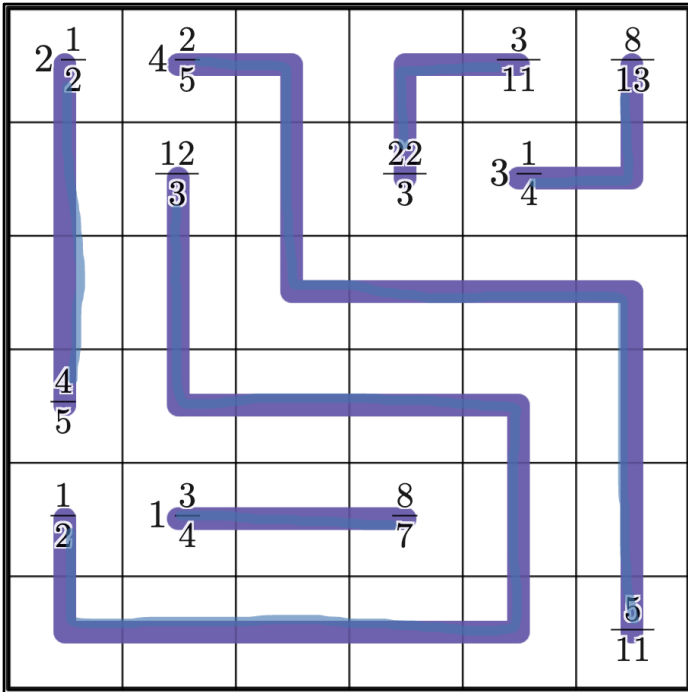
$$= \frac{270}{30} - \frac{17}{30}$$

$$= \frac{253}{30}$$

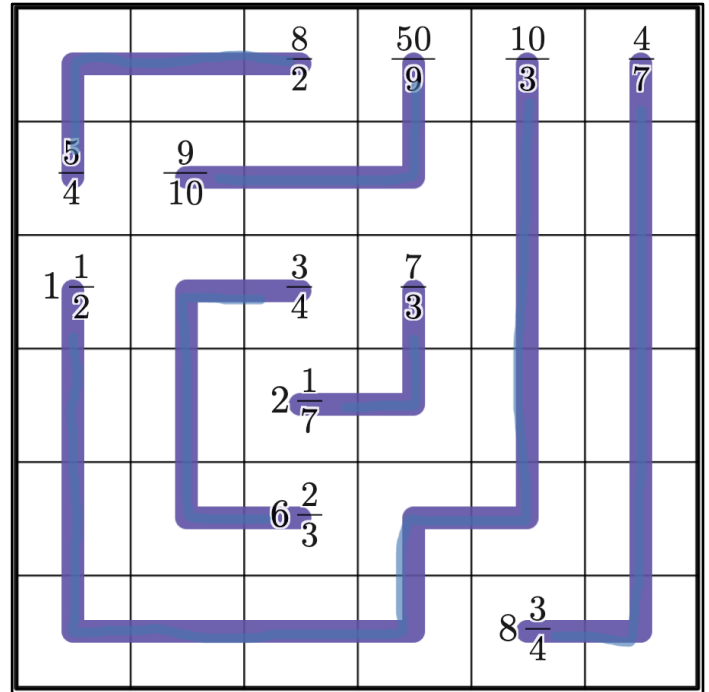
$$= 8\frac{13}{30}$$

Some of the quests have pipeflow puzzles. These can be quite challenging, so solutions are included below. Only peek if absolutely necessary.

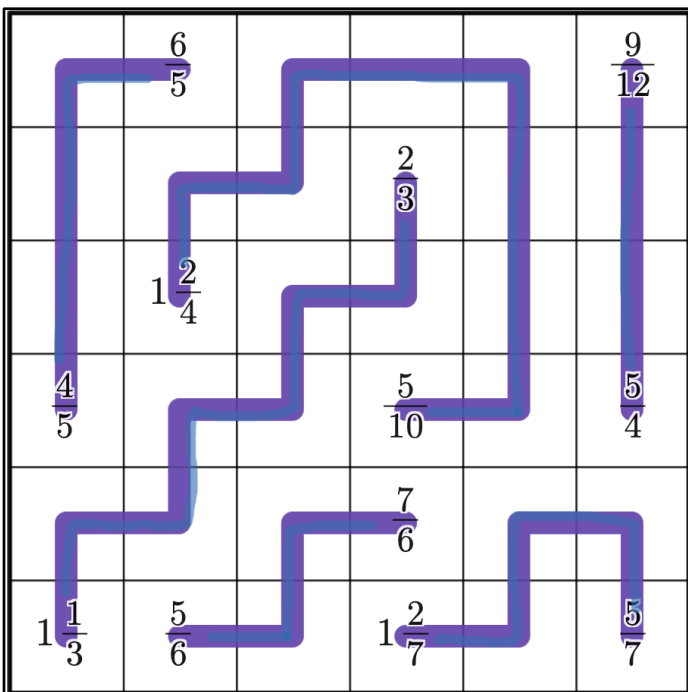
3.1



3.1



3.4



3.4

