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Unit 8.1, Lesson 2: Notes

Name ______________________________

Learning Goal(s):

There are three phrases that describe the movements we have been exploring more precisely.

Match each phrase with a definition.

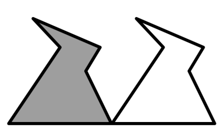
A. Moves every point to a point directly on the opposite side of a line. ____ Translation

B. Moves every point around a center by an angle in a specific direction. ____ Rotation

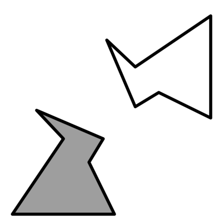
C. Moves every point in a figure a given distance in a given direction. ____ Reflection

Describe how to move each shaded polygon to the unshaded one using these new words.

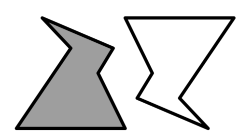
1.



2.



3.



Summary Question

How can you tell the difference between a reflection, a translation, and a rotation?

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Unit 8.1, Lesson 2: Notes Name ______________________

Learning Goal(s):

- **I know the difference between translations, rotations, and reflections.**

There are three phrases that describe the movements we have been exploring more precisely.

Match each phrase with a definition.

A. Moves every point to a point directly on the opposite side of a line.

B. Moves every point around a center by an angle in a specific direction.

C. Moves every point in a figure a given distance in a given direction.

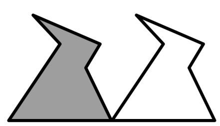
C. Translation

B. Rotation

A. Reflection

Describe how to move each shaded polygon to the unshaded one using these new words.

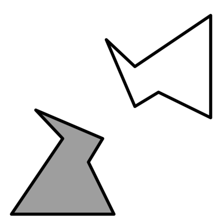
1.



Responses vary.

Translate the shaded polygon to the right.

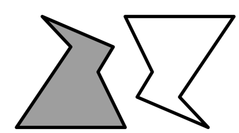
2.



Responses vary.

Reflect the shaded polygon over a diagonal line in between the two shapes.

3.



Responses vary.

Rotate the shaded polygon $180°$ around the point in the center between the two polygons.

Summary Question

How can you tell the difference between a reflection, a translation, and a rotation?

Responses vary. **A translation does not change the way a shape is facing at all. If it is upright in the original figure, then it is still upright in the new figure. A rotation looks like a turn. If you can turn the paper or computer and create the new figure, then it is a rotation. A reflection changes a figure so that it is facing the other direction, like looking in a mirror.**

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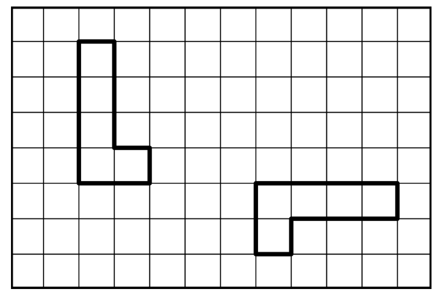
Unit 8.1, Lesson 3: Notes Name ________________________________

Learning Goal(s):

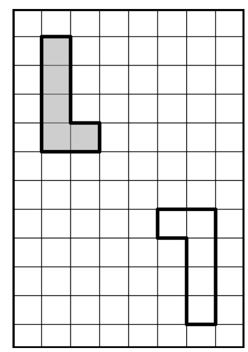
You often need a sequence of transformations to transform a pre-image onto an image.

The pre-image is translated to the right 5 units, then rotated $90°$ clockwise about its bottom-left corner.

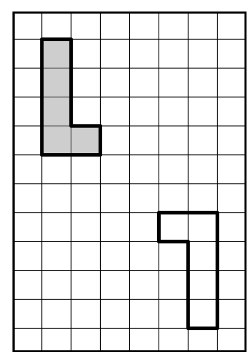
Label the pre-image and the image on the grid.



Draw or describe a series of **reflections** to transform the shaded pre-image onto the unshaded image.

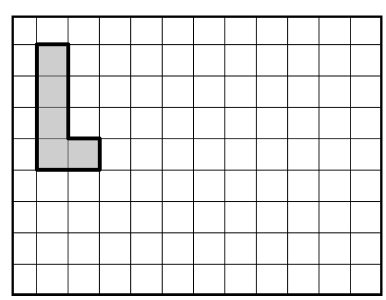


Draw or describe a series of **rotations** to transform the shaded pre-image onto the unshaded image.



Summary Question

Draw an image that you cannot create using only one transformation. Explain your thinking.



Unit 8.1, Lesson 3: Notes Name ____________________

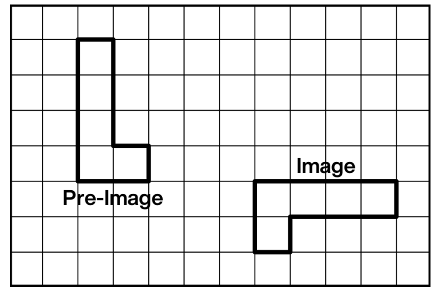
Learning Goal(s):

- **I can decide which type of transformation will work to move one figure to another.**

You often need a sequence of transformations to transform a pre-image onto an image.

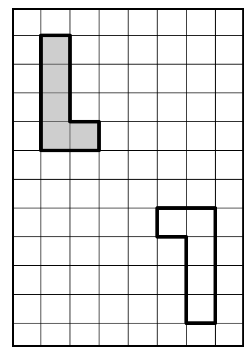
The pre-image is translated to the right 5 units, then rotated $90°$ clockwise about its bottom-left corner.

Label the pre-image and the image on the grid.



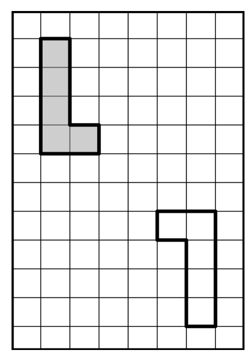
Describe a series of **reflections** to transform the shaded pre-image onto the unshaded image.

***Responses vary.* Reflect along the horizontal line one unit below the pre-image. Then reflect over the vertical line one unit to the right of the pre-image.**



Describe a series of **rotations** to transform the shaded pre-image onto the unshaded image.

***Responses vary.* Rotate the pre-image $180°$ about the point one unit to the right and one unit down from the bottom-right corner of the pre-image.**

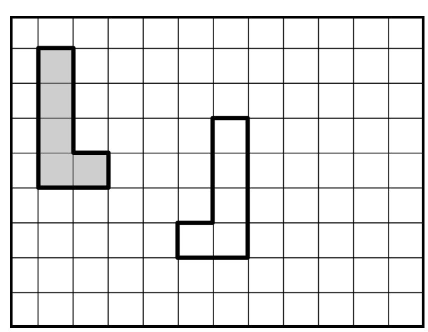


Summary Question

Draw an image that you cannot create using only one transformation. Explain your thinking.

Drawings and explanations vary.

In this example, it is not possible to reflect, rotate, or translate the pre-image once to become the image.



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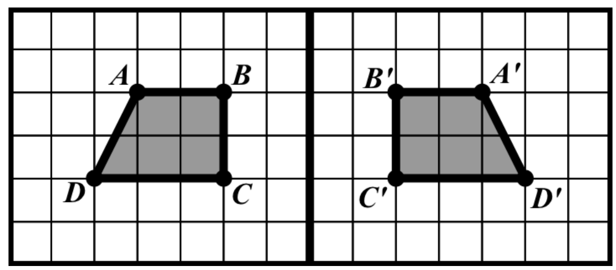
Unit 8.1, Lesson 4: Notes Name ______________________________

Learning Goal(s):

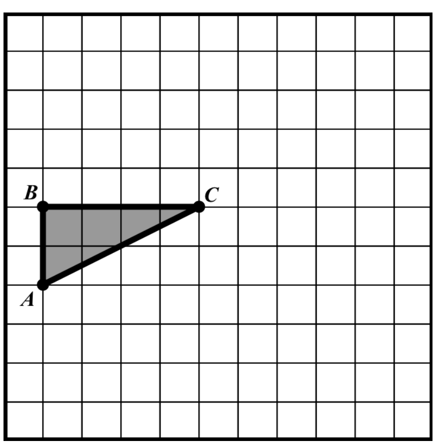
Grids can help us be more precise when describing translations, rotations, and reflections.

Explain what the prime symbol ($'$) means in terms of transformations.

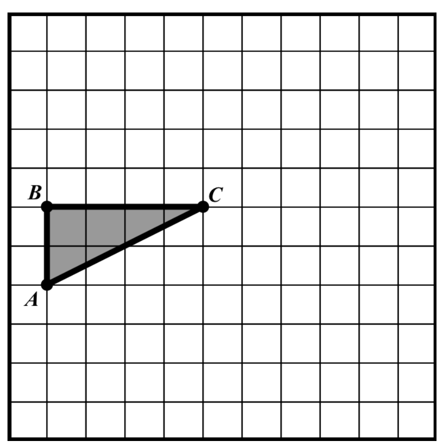


Draw the result of each transformation.

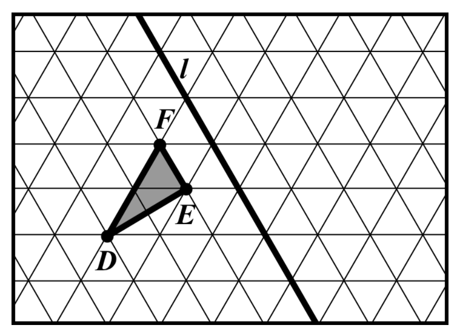
Translate triangle ABC 4 units up and 2 units to the right.



Rotate triangle ABC 90° clockwise using center C.



Reflect triangle DEF using line l.



Summary Question

What are some important things to remember when performing transformations on a grid?

Unit 8.1, Lesson 4: Notes Name ___________________________

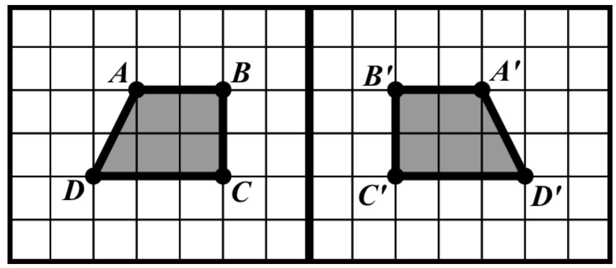
Learning Goal(s):

- **I can use the terms *translation*, *rotation*, and *reflection* to precisely describe transformations on a grid.**
- **I can use a grid to perform a translation, rotation, or reflection.**

Grids can help us be more precise when describing translations, rotations, and reflections.

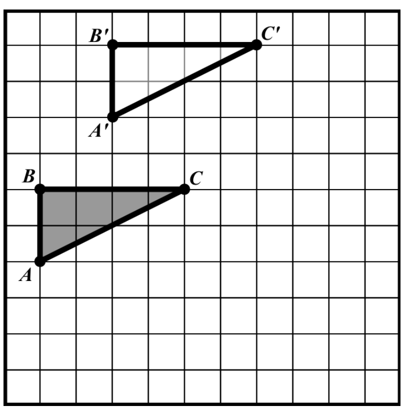
Explain what the prime symbol (′) means in terms of transformations. ***Responses vary.***

The prime is the point on the image that corresponds to a point on the pre-image. For example, A' is the result of transformations of point A.

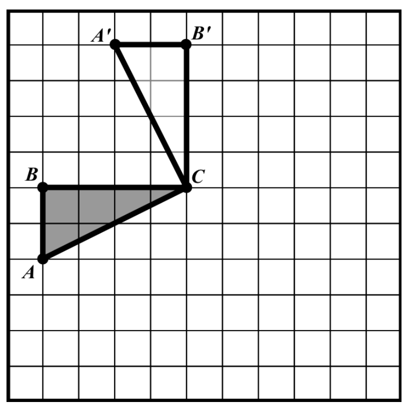


Draw the result of each transformation.

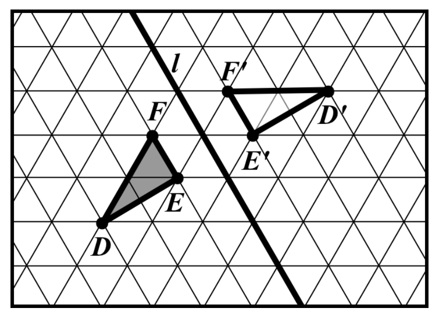
Translate triangle ABC 4 units up and 2 units to the right.



Rotate triangle ABC 90° clockwise using center C.



Reflect triangle DEF using line l.



Summary Question

What are some important things to remember when performing transformations on a grid?

Responses vary.

- **Count the number of units each point is away from the line of reflection.**
- **Pay attention to what the center of rotation is. Sometimes it is helpful to turn the paper to get an idea of what the rotation might look like.**
- **Double check at the end to make sure the shape still looks the same.**

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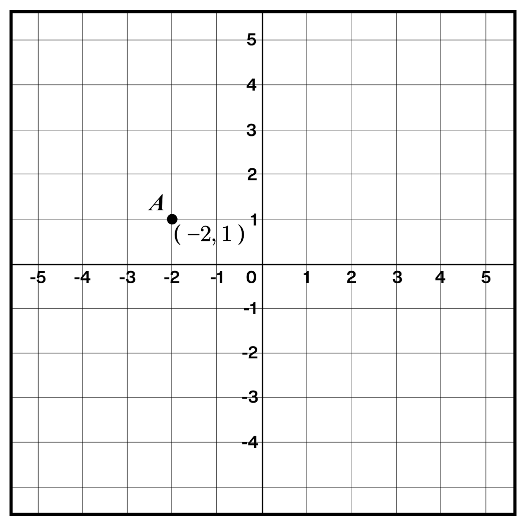
Unit 8.1, Lesson 5: Notes Name ______________________________

Learning Goal(s):

Definition	Facts/Characteristics
Corresponding Points	
Examples	Non-Examples

What are the coordinates of point A after each transformation?



Reflect across the x-axis:

Reflect across the y-axis:

Translate right 3 units and down 2 units:

Rotate $180°$ with center $(0,\ 0)$:

Rotate $90°$ with center $(0,\ 0)$:

Summary Question

What happens to the coordinates of a point after a rotation? A translation? A reflection?

Unit 8.1, Lesson 5: Notes Name ______________________________

Learning Goal(s):
- **I can apply transformations to points on a grid if I know their coordinates.**

Corresponding Points

Definition

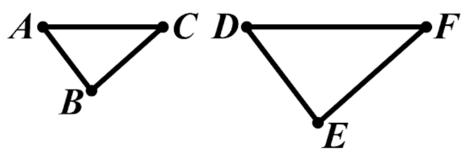
A point in an original figure that matches up with a point on a copy.

Facts/Characteristics

Sometimes labeled with the same letter but with a tick mark.

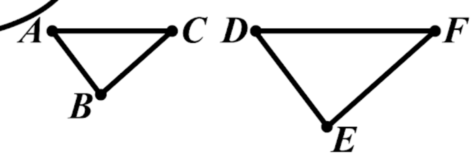
e.g., point A and point A'

Examples



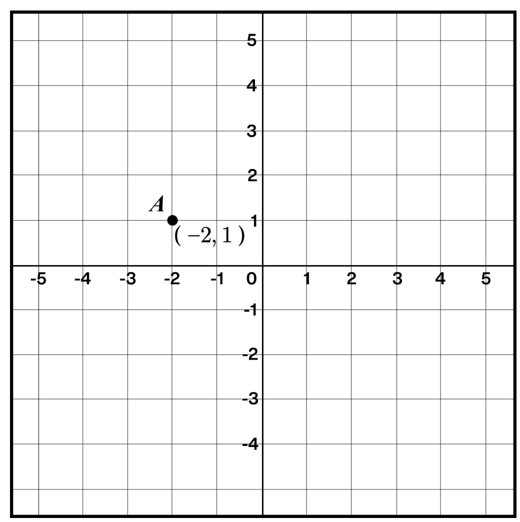
Point B corresponds to point E.

Non-Examples



A does not correspond to E.

What are the coordinates of point A after each transformation?



Reflect across the x-axis: $(-2, -1)$

Reflect across the y-axis: $(2, 1)$

Translate right 3 units and down 2 units: $(1, -1)$

Rotate $180°$ with center $(0, 0)$: $(2, -1)$

Rotate $90°$ with center $(0, 0)$: $(-1, -2)$

Summary Question

What happens to the coordinates of a point after a rotation? A translation? A reflection?

- **For a rotation, the change in coordinates depends on the angle of rotation. For a $180°$ rotation, the x- and y-coordinates are multiplied by -1.**
- **For a translation, a horizontal shift is added to or subtracted from the x-coordinate and a vertical shift is added to or subtracted from the y-coordinate.**
- **Reflecting a point across an axis changes the sign of one coordinate.**

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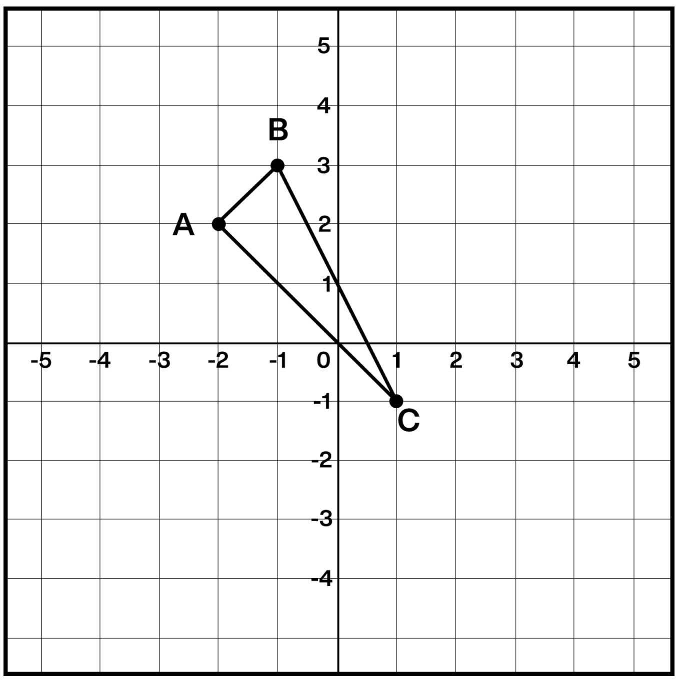
Unit 8.1, Lesson 6: Notes Name ______________________________

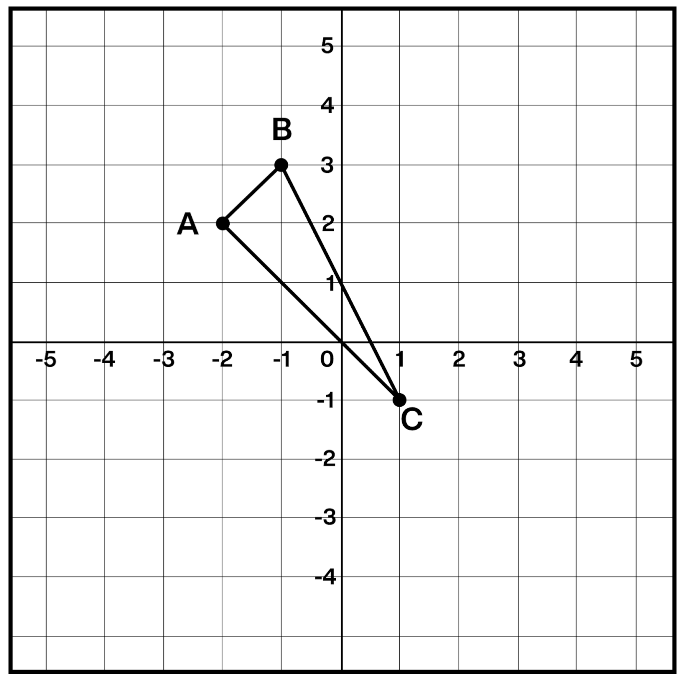
Learning Goal(s):

When we perform a sequence of transformations, the order of the transformations can be important.

1. Translate triangle ABC up 2 units and then reflect it over the x-axis.



2. Reflect triangle ABC over the x-axis and then translate it up 2 units.



Triangle $A'B'C'$ ends up in different places when the same transformations are applied in the opposite order!

Summary Question

How are coordinates useful when describing and drawing transformations?

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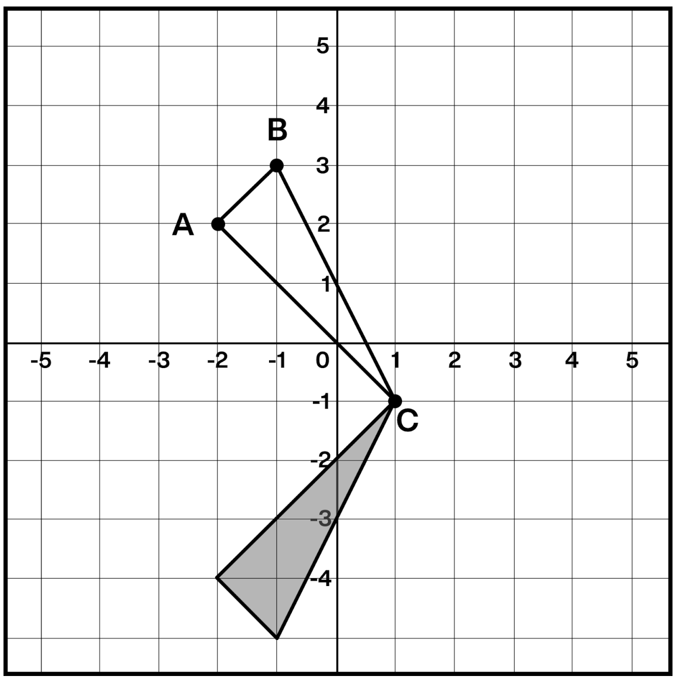
Unit 8.1, Lesson 6: Notes Name ___________________________________

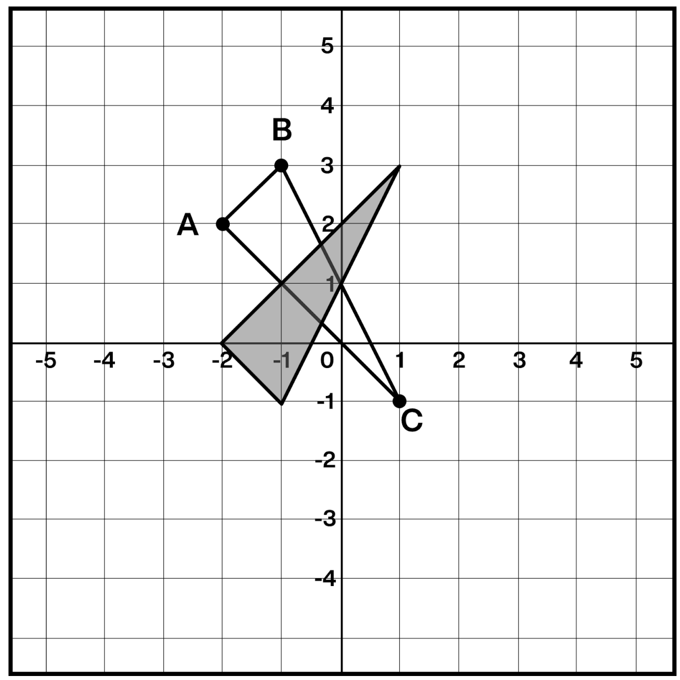
Learning Goal(s):
- **I can apply transformations to a polygon on a grid if I know the coordinates of its vertices.**

When we perform a sequence of transformations, the order of the transformations can be important.

1. Translate triangle ABC up 2 units and then reflect it over the x-axis.



2. Reflect triangle ABC over the x-axis and then translate it up 2 units.



Triangle $A'B'C'$ ends up in different places when the same transformations are applied in the opposite order!

Summary Question

How are coordinates useful when describing and drawing transformations?

Coordinates are useful when describing and drawing transformations because they communicate the location of the vertices. When describing a transformation, we can specify a line of reflection, a point of rotation, or the distance and direction of a translation and then use the grid to precisely identify and draw the vertices of the image.

Unit 8.1, Lesson 7: Notes

Name ___________________________________

Learning Goal(s):

Definition

Facts/Characteristics

Congruent

Examples

Non-Examples

Here are some facts about congruent figures:

- We ______________ need to check all the measurements to prove two figures are congruent; we have to find a ____ that match up the figures.
- Two figures that look like mirror images of each other can be congruent. There must be a ___________________ in the sequence of transformations that matches up the figures.
- Two polygons that have the same area and the same perimeter ___________ always congruent.

Summary Question

What are some ways to determine if two figures are congruent?

Unit 8.1, Lesson 7: Notes Name ______________________________

Learning Goal(s):

- **I can decide whether or not two figures are congruent just by looking.**

Congruent

Definition

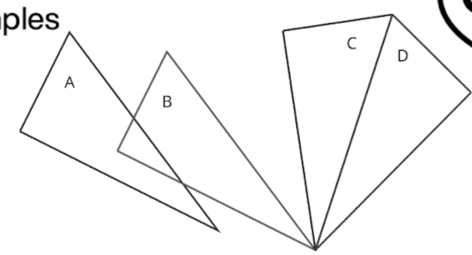
One figure is congruent to another if it can be moved with translations, rotations, and reflections to fit exactly over the other.

Facts/Characteristics

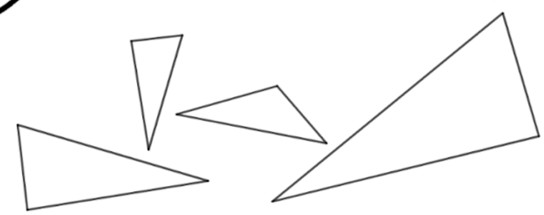
Congruent polygons have:

- **corresponding sides congruent.**
- **corresponding angles congruent.**
- **equal areas and perimeters.**

Examples



Non-Examples



Here are some facts about congruent figures:

- We **do not** need to check all the measurements to prove two figures are congruent; we have to find a **sequence of rotations, reflections, and translations** that match up the figures.
- Two figures that look like mirror images of each other can be congruent. There must be a **reflection** in the sequence of transformations that matches up the figures.
- Two polygons that have the same area and the same perimeter **are not** always congruent.

Summary Question

What are some ways to determine if two figures are congruent?

- **Use tracing paper and see if one figure fits on top of the other.**
- **Find a sequence of rotations, reflections, and translations that matches up the figures.**

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Unit 8.1, Lesson 8: Notes Name ______________________________

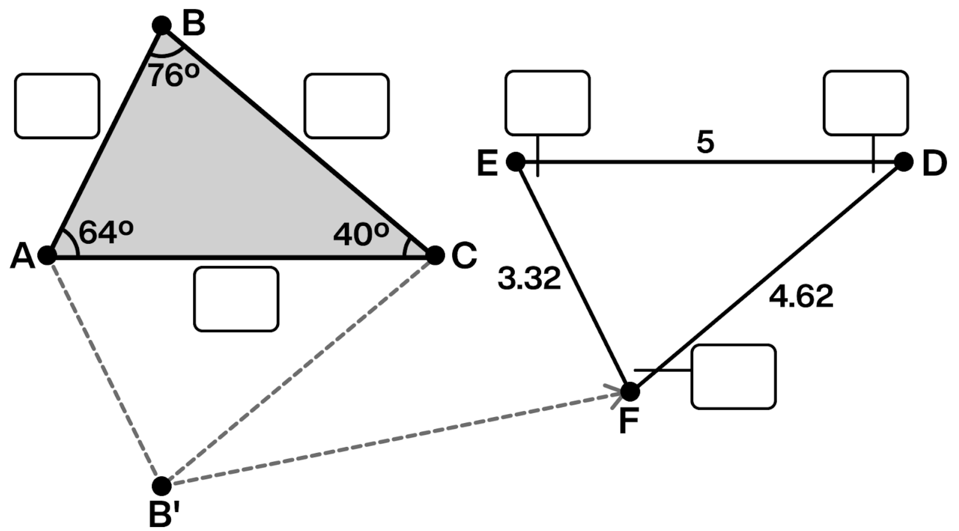
Learning Goal(s):

Definition	Facts/Characteristics
Rigid Transformation	
Examples	Non-Examples

With a rigid transformation, corresponding parts in polygons have the same ________________.

Triangle EFD was made by reflecting triangle ABC across a horizontal line and then translating.

Fill in all of the missing measurements in the diagram.



Summary Question

How can you use measurements to decide if two figures will match up after a sequence of rigid transformations?

Unit 8.1, Lesson 8: Notes Name ________________________________

Learning Goal(s):

- **I can describe the effects of a rigid transformation on the lengths and angles of a polygon.**

Definition

A move that does not change any measurements of a figure.

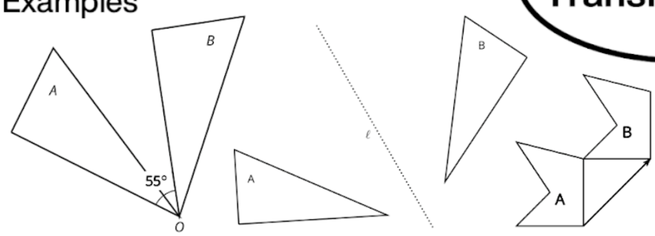
Facts/Characteristics

Rigid Transformations:

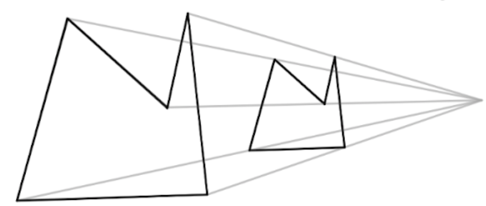
- **Translations**
- **Rotations**
- **Reflections**

Rigid Transformation

Examples



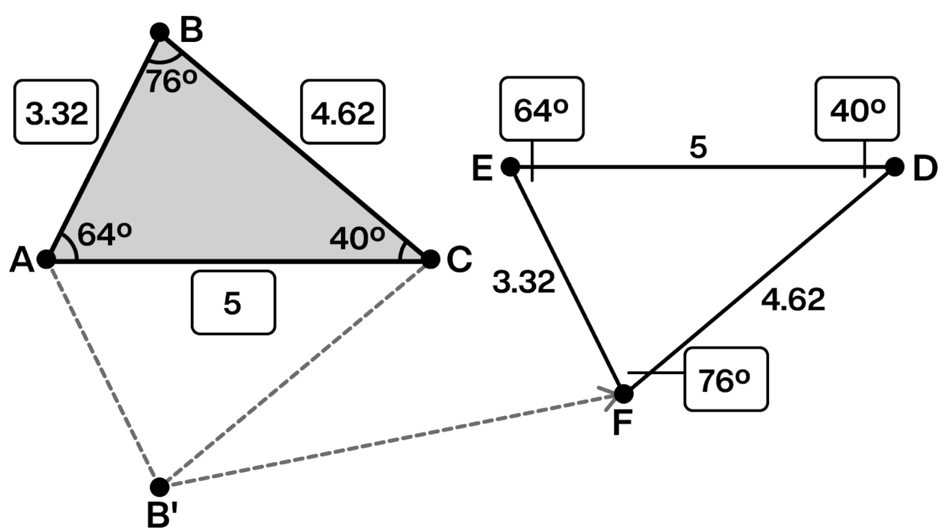
Non-Examples



With a rigid transformation, corresponding parts in polygons have the same **measurements.**

Triangle EFD was made by reflecting triangle ABC across a horizontal line and then translating.

Fill in all of the missing measurements in the diagram.



Summary Question

How can you use measurements to decide if two figures will match up after a sequence of rigid transformations?

If each of the corresponding parts in two figures have the same measurements, then the figures must match up through rigid transformations. This makes them congruent.

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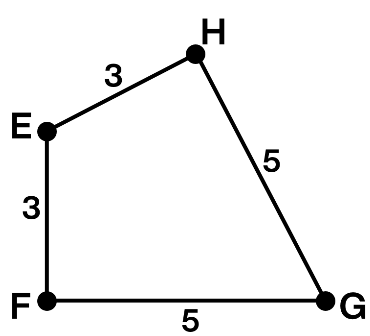
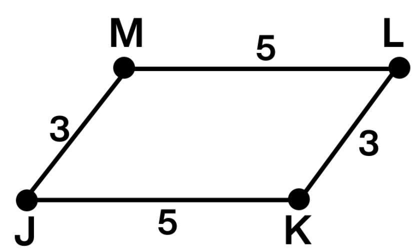
Unit 8.1, Lesson 9: Notes Name ______________________________

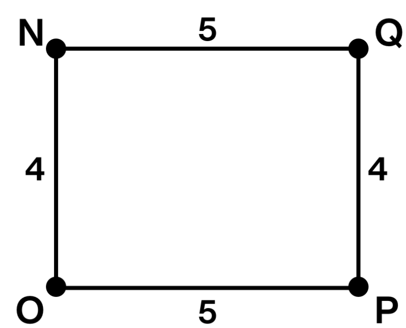
Learning Goal(s):

To show two figures are congruent, you align one with the other using a sequence of ______________________________. Distances between ____________________________ on congruent figures are always equal.

Explain why each figure below is **not** congruent to $ABCD$.







Summary Question

What are some ways you can check if two figures are congruent?

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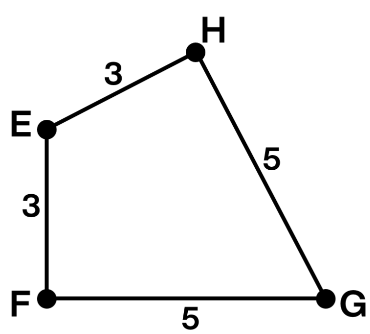
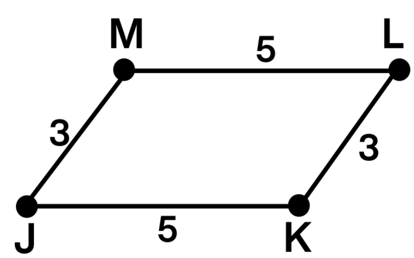
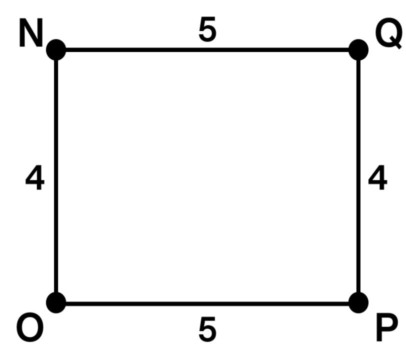
Unit 8.1, Lesson 9: Notes Name ____________________

Learning Goal(s):

- **I can decide whether or not two figures are congruent using rigid transformations.**
- **I understand whether or not congruent sides are enough to determine if two polygons are congruent.**

To show two figures are congruent, you align one with the other using a sequence of **rigid transformations.** Distances between **corresponding points** on congruent figures are always equal.

Explain why each figure below is **not** congruent to $ABCD$.

 **$ABCD$ and $EFGH$ have the same side lengths, but their orders can't be matched as you go around each polygon, so these polygons can't be congruent.**	 **$ABCD$ and $JKLM$ have the same side lengths in the same order but different corresponding angles, so these polygons can't be congruent.**	 **$ABCD$ and $NOPQ$ have the same angles but different corresponding side lengths, so these polygons can't be congruent.**

Summary Question

What are some ways you can check if two figures are congruent?

- **If we copy one figure on tracing paper and move the paper so the copy covers the other figure exactly, then that suggests they are congruent.**
- **We can prove that two figures are congruent by describing a sequence of translations, rotations, and reflections that moves one figure onto the other so they match up exactly.**

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Unit 7.7, Lesson 2: Notes

Name ____________________

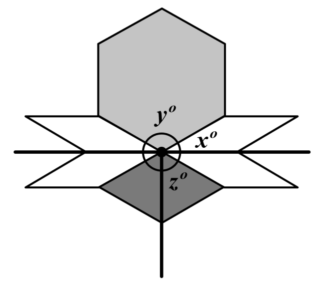
My Notes

1.1 Draw an example of *complementary angles*.

1.2 Draw an example of *supplementary angles*.

2.1 Select **all** of the true equations.

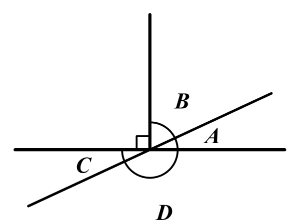
- ☐ $x + y = 180$
- ☐ $x + z = 90$
- ☐ $2x + y = 180$
- ☐ $2x + 2z = 180$
- ☐ $x + y + z = 180$



2.2 Choose one equation that is **not true.** Explain why it is not true.

3. Angle $A = 25°$. A and B are *complementary angles*.

What is the measure of angle B?



Summary

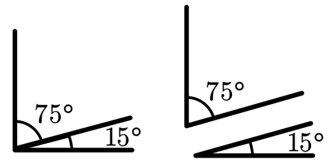
- ☐ I can describe what complementary and supplementary angles are.
- ☐ I can determine unknown angles using what I know about complementary and supplementary angles.
- ☐ I can connect an angle diagram with an equation that represents it.

desmos

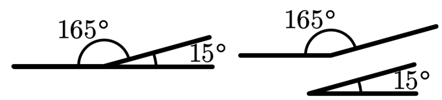
Unit 7.7, Lesson 2: Notes Name ______________________________

My Notes

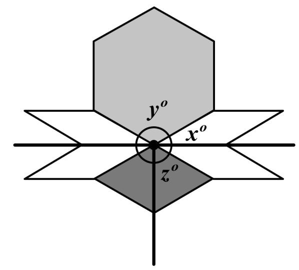
1.1 Draw an example of *complementary angles*.



1.2 Draw an example of *supplementary angles*.



2.1 Select **all** of the true equations.



- ☐ $x + y = 180$
- ✓ $x + z = 90$
- ✓ $2x + y = 180$
- ✓ $2x + 2z = 180$
- ☐ $x + y + z = 180$

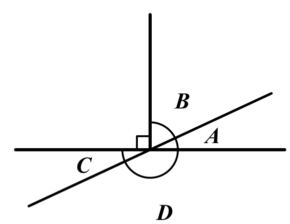
2.2 Choose one equation that is **not true.** Explain why it is not true.

- **The first equation is not true because x and y do not make a straight angle together.**
- **The last equation is not true because those angles together would be more than a straight angle.**

4. Angle $A = 25°$. A and B are *complementary angles*.

What is the measure of angle B?

$65°$



Summary

- ☐ I can describe what complementary and supplementary angles are.
- ☐ I can determine unknown angles using what I know about complementary and supplementary angles.
- ☐ I can connect an angle diagram with an equation that represents it.

desmos

Unit 7.7, Lesson 3: Notes

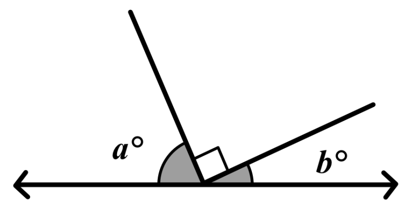
Name ____________________

My Notes

1.1 Draw an example of *vertical angles*.

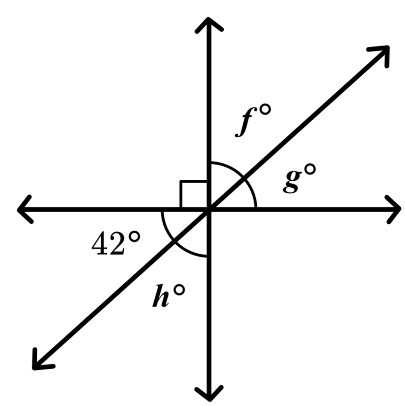
1.2 Label each angle with an estimate of its measure.

1.3 Explain why the shaded angles are not *vertical.*



2.1 Determine the values of f, g, and h.

2.2 Explain how you figured out the value of angle f.

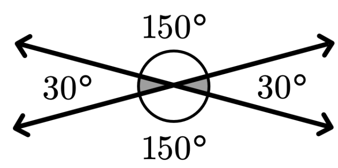


Summary

- [ ] I can describe what vertical angles are.
- [ ] I can write and use equations to determine unknown angles.

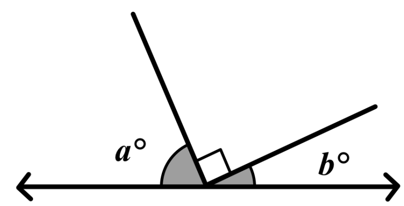
My Notes

1.1 Draw an example of *vertical angles*.



1.2 Label each angle with an estimate of its measure.

1.3 Explain why the shaded angles are not *vertical.*



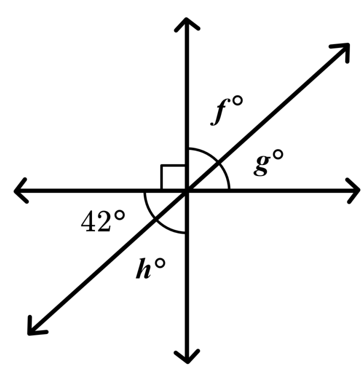
***Explanations vary.* The shaded angles are not opposite where two lines cross. Also, we don't know that they have the same measure. All we know is that $a + b = 90$.**

2.1 Determine the values of f, g, and h.

$f = 48°$

$g = 42°$

$h = 48°$



2.2 Explain how you figured out the value of angle f.

***Explanations vary.* I knew that the $42°$ angle and $h°$ were complementary, so h had to be $48°$. $f°$ and $h°$ are vertical angles, so their measures are equal. Also, angle $g°$ is vertical to $42°$ and $42 + 48 = 90$.**

Summary

☐ I can describe what vertical angles are.

☐ I can write and use equations to determine unknown angles.

desmos

Unit 8.1, Lesson 10: Notes Name ______________________________

Learning Goal(s):

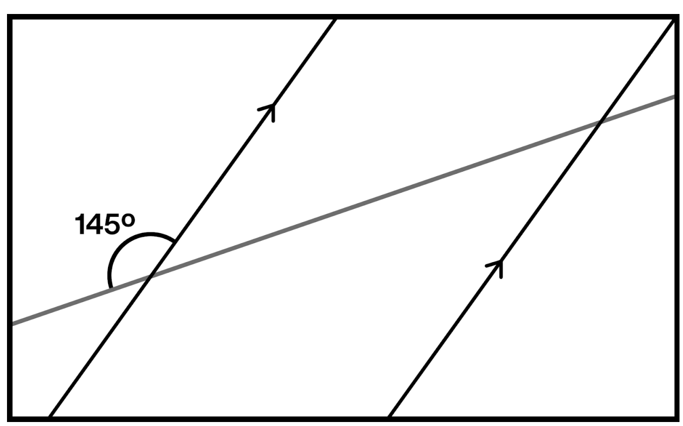
Definition	Facts/Characteristics
Vertical Angles	
Examples	Non-Examples

Definition	Facts/Characteristics
Transversal	
Examples	Non-Examples

Summary Task

Here is a pair of parallel lines and a transversal.

Use what you know about angle relationships to determine the measurements for all of the other angles in the diagram.



Unit 8.1, Lesson 10: Notes Name ______________________

Learning Goal(s):

- **I can describe the effects of a rigid transformation on a pair of parallel lines.**
- **If I have a pair of vertical angles and know the angle measure of one of them, I can use vertical angles to determine missing angle measurements.**
- **I can identify congruent angles on two parallel lines cut by a transversal and use that to determine missing angle measurements.**

Vertical Angles

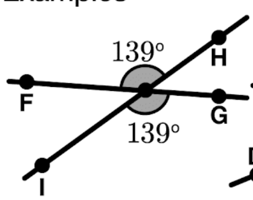
Definition

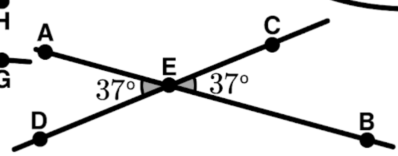
Vertical angles are opposite angles that share the same vertex. They are formed by a pair of intersecting lines.

Facts/Characteristics

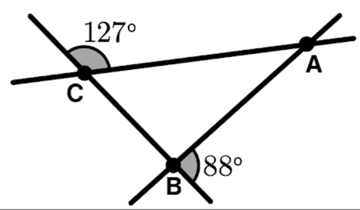
Vertical angle measures are always equal.

Examples





Non-Examples



Transversal

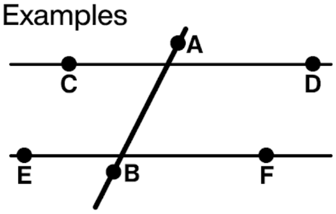
Definition

A transversal is a line that cuts across two lines.

Facts/Characteristics

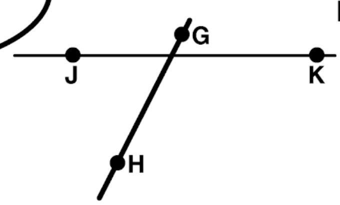
A pair of parallel lines intersected by a transversal forms pairs of congruent angles.

Examples



Line AB is a transversal intersecting parallel lines CD and EF.

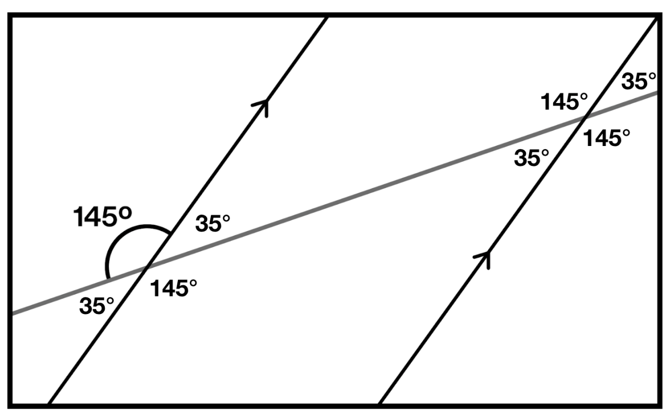
Non-Examples



Summary Task

Here is a pair of parallel lines and a transversal.

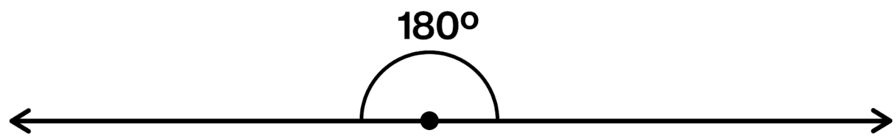
Use what you know about angle relationships to determine the measurements for all of the other angles in the diagram.



Unit 8.1, Lesson 11: Notes Name ________________________________

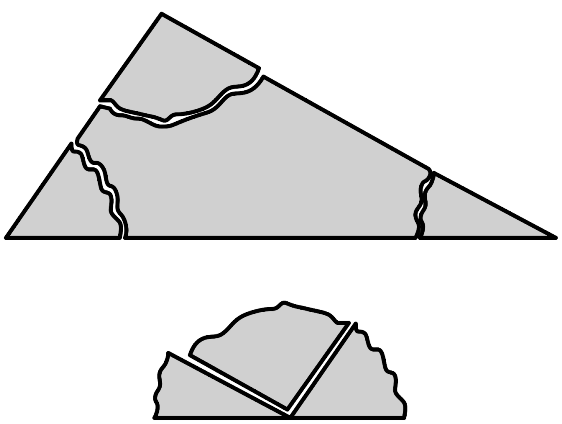
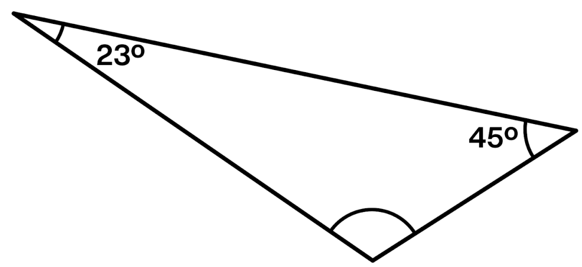
Learning Goal(s):

A $180°$ angle is called a **straight angle** because when it is made with two rays, the rays point in opposite directions and form a line.



If we experiment with angles in a triangle, we find that the sum of all three angle measures in a triangle also forms a line.

The angles in a triangle add up to ______° !

Mark the angles (or use color) to show the corresponding angles in each diagram.	Show how you can find the missing angle measure in this triangle.
	

Summary Question

How can you use the measures of two angles in a triangle to find the measure of the third angle?

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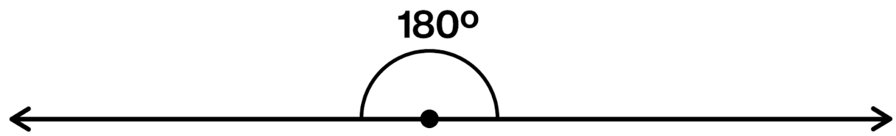
Unit 8.1, Lesson 11: Notes Name ______________________________

Learning Goal(s):

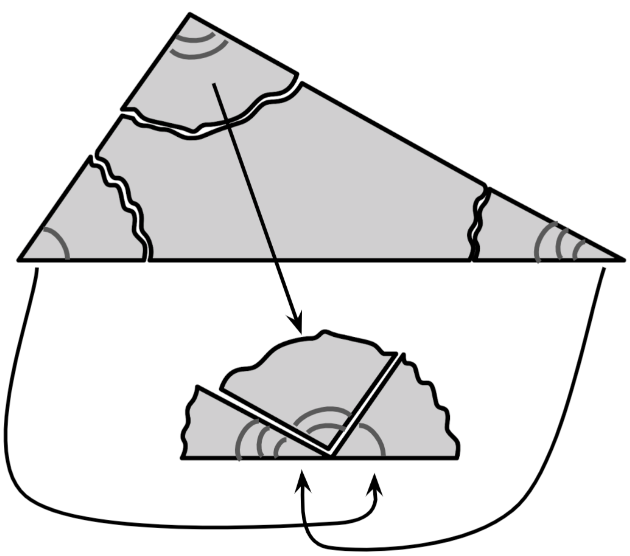
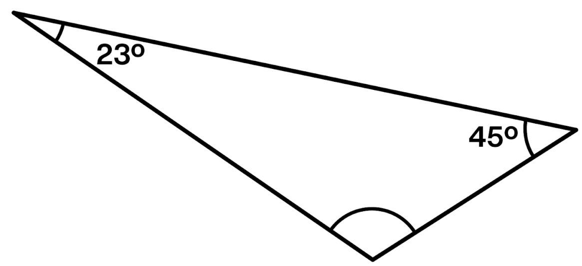
- **If I know two of the angle measures in a triangle, I can find the third angle measure.**

A $180°$ angle is called a **straight angle** because when it is made with two rays, the rays point in opposite directions and form a line.



If we experiment with angles in a triangle, we find that the sum of all three angle measures in a triangle also forms a line.

The angles in a triangle add up to $180°$!

Mark the angles (or use color) to show the corresponding angles in each diagram.	Show how you can find the missing angle measure in this triangle.
	 $23 + 45 + x = 180$ $68 + x = 180$ $x = 112$ **The missing angle measures $112°$.**

Summary Question

How can you use the measures of two angles in a triangle to find the measure of the third angle?

The three angles in a triangle must add up to $180°$, so if you know the measures of two angles, figure out what you need to add to make the sum of the three angle measures equal 180.

desmos

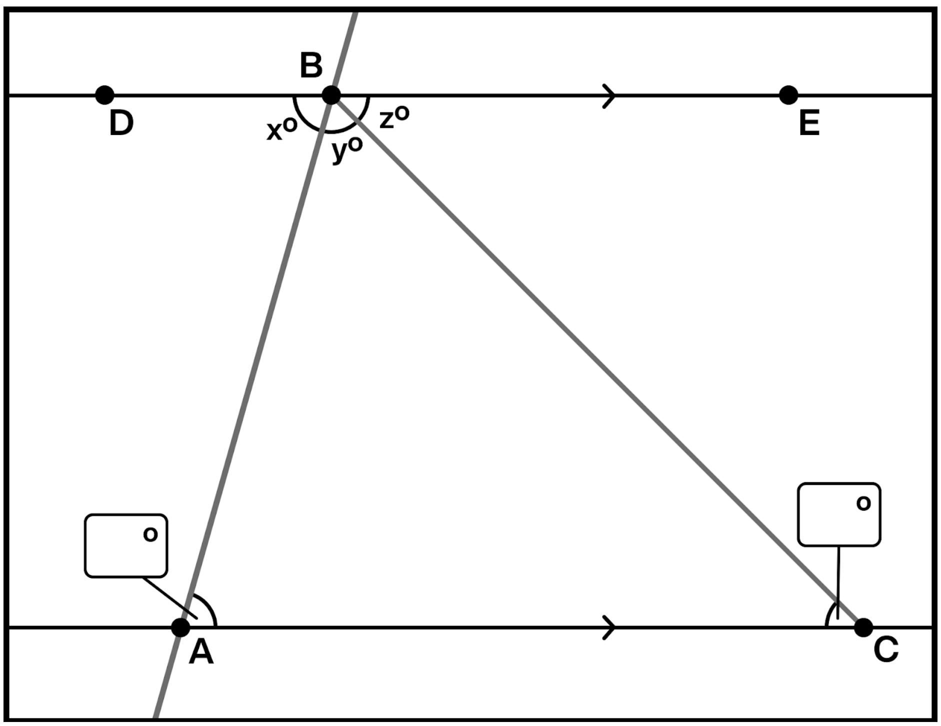
Unit 8.1, Lesson 12: Notes

Name ___________________________________

Learning Goal(s):

Using parallel lines and angle relationships, we can understand why the angles in a triangle always add to $180°$. Here is triangle ABC. Line DE is parallel to AC and contains B.

Label the missing angles using x, y, or z to show which angles have the same measures.



Summary Question

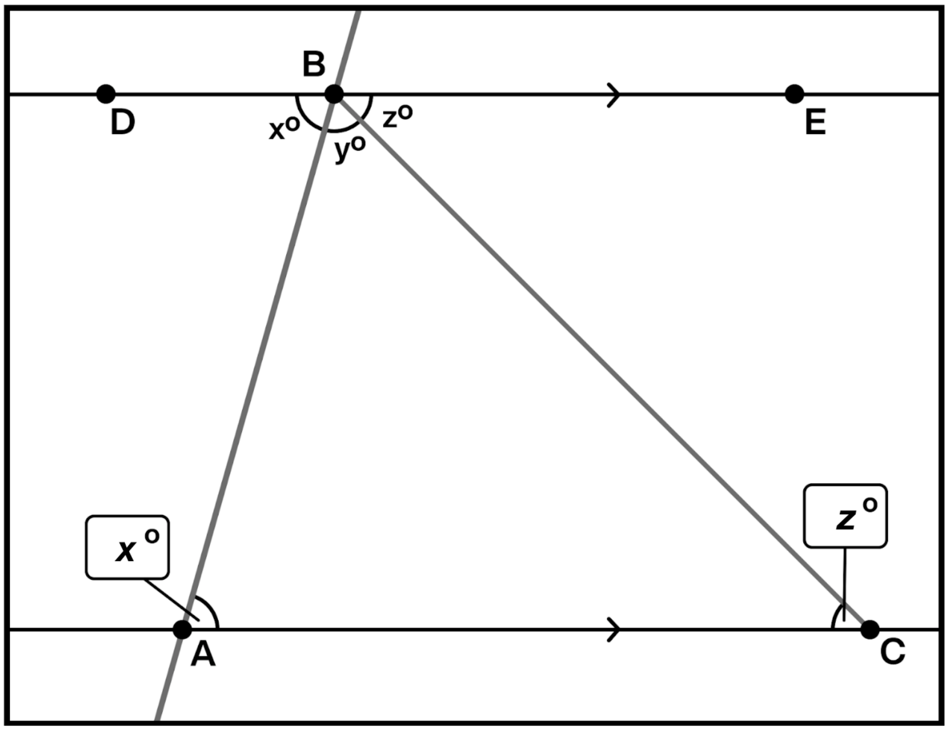
How does this diagram show that the angles in a triangle add up to $180°$?

Learning Goal(s):

- **I can explain using pictures why the sum of the angles in any triangle is 180 degrees.**

Using parallel lines and angle relationships, we can understand why the angles in a triangle always add to $180°$. Here is triangle ABC. Line DE is parallel to AC and contains B.

Label the missing angles using x, y, or z to show which angles have the same measures.



Summary Question

How does this diagram show that the angles in a triangle add up to $180°$?

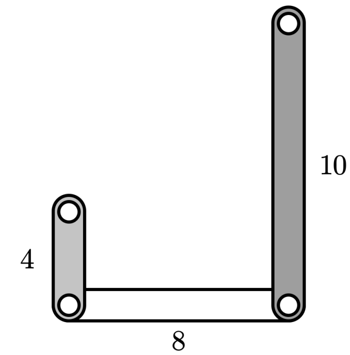
Angle ABD is congruent to angle BAC, so both angle measures are $x°$. Angle CBE is congruent to angle ACB, so both angle measures are $z°$. Angles $ABD\ (x°)$, $ABC\ (y°)$, and $CBE\ (z°)$ form a straight line, so we know $x + y + z = 180$. Since x, y, and z are also the measures of the angles in triangle ABC, we know the angles in a triangle must add up to $180°$.

desmos

Name ________________________________

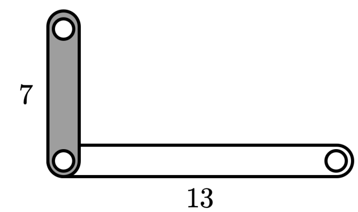
My Notes

1. Will these side lengths form a triangle? Explain your thinking.



2.1 What is one possible third length that would form a triangle?

Explain how you know.



2.2 What is a length that would be too long? Too short?

Too long: _______ Too short: _______

Explain one of your answers.

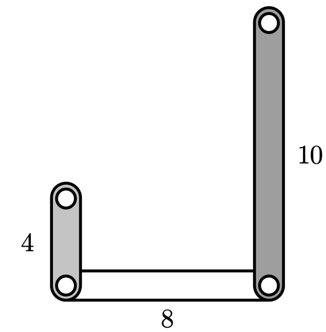
Summary

☐ I can decide whether or not three side lengths will make a triangle.

My Notes

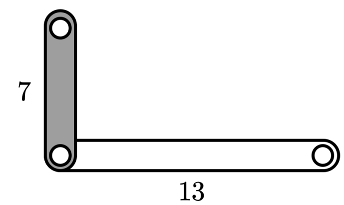
1. Will these side lengths form a triangle? **Yes.**

   ***Explanations vary.* If the two shorter sides put together are longer than the third side, then you will get a triangle. In this case, $4+8$ is greater than 10, so these three sides will form a triangle.**



2.1 What is one possible third length that would form a triangle?

***Responses and explanations vary.* A length of 8 units would create a triangle because if you connected that side to the other side of 11 units, it would be long enough to connect to the 7 unit side.**



2.2 What is a length that would be too long? Too short?

Responses vary. Too long: 25 Too short: 2.3

Explanations vary. 25 units is too long because if 7 and 13 were each attached to one side of 25, they would not be able to connect to create a triangle.

Summary

☐ I can decide whether or not three side lengths will make a triangle.

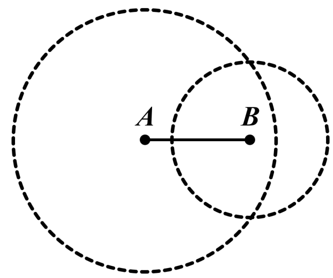
desmos

Unit 7.7, Lesson 6: Notes Name ______________________________

My Notes

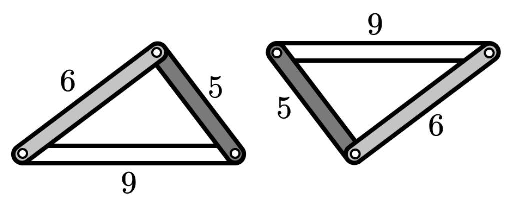
Amanda began to draw a triangle with side lengths 5, 2, and 6 units.

1.1 What does each circle in Amanda's drawing represent?



1.2 Explain or show how she can complete the triangle.

2. Emika built two triangles with side lengths 5, 6, and 9 units. Explain how you know these two triangles are *identical copies*.



How many nonidentical triangles can be made using these lengths:

3.1 4.5, 8, and 10 units

3.2 9, 11, and 21 units

Summary

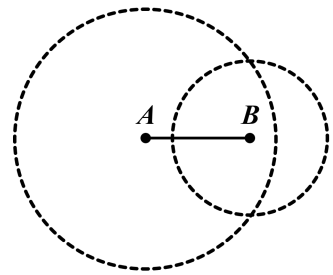
- ☐ I can explain what it means for shapes to be identical copies.
- ☐ I can determine whether you can make zero, one, or more than one shape given a set of side lengths.

Unit 7.7, Lesson 6: Notes Name ____________________

My Notes

Amanda began to draw a triangle with side lengths 5, 2, and 6 units.

1.1 What does each circle in Amanda's drawing represent?

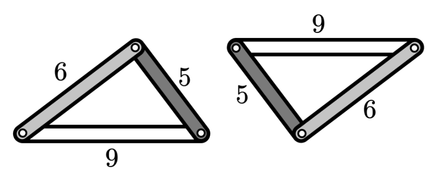


***Responses vary.* The left circle represents all the points that are 6 cm from A. The right circle is all the points that are 2 cm from B.**

1.2 Explain or show how she can complete the triangle. ***Responses vary.* Mark a point C where the two circles intersect. Draw lines from A to C and B to C.**

2. Emika built two triangles with side lengths 5, 6 and 9 units. Explain how you know these two triangles are *identical copies*.

***Explanations vary.* These are identical copies because the triangles are the same shape and size. If you turn the left triangle upside down and move it, it would fit right on top of the right triangle.**



How many nonidentical triangles can be made using these lengths:

3.1 4.5, 8, and 10 units	3.2 9, 11, and 21 units
1 **triangle**	0 **triangles**

Summary

- ☐ I can explain what it means for shapes to be identical copies.
- ☐ I can determine whether you can make zero, one, or more than one shape given a set of side lengths.

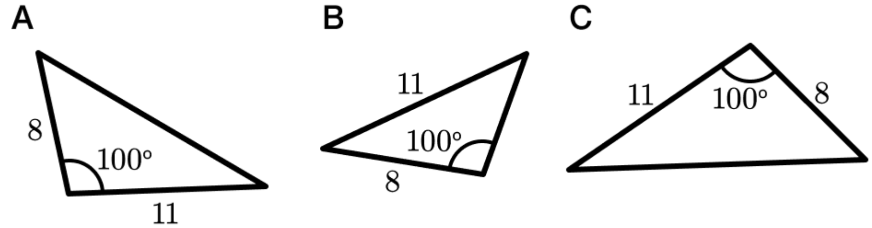
desmos

Unit 7.7, Lesson 7: Notes

Name ___________________________________

My Notes

1. Which of the triangles below are identical?



Explain your thinking.

2.1 Mariana and Jamir are both drawing triangles that have a 5 cm side, a $60°$ angle, and a $45°$ angle. Will Mariana's and Jamir's triangles be identical?

Show or explain your thinking.

2.2 What information would Jamir need about Mariana's triangle in order to be sure she was creating an identical triangle?

Summary

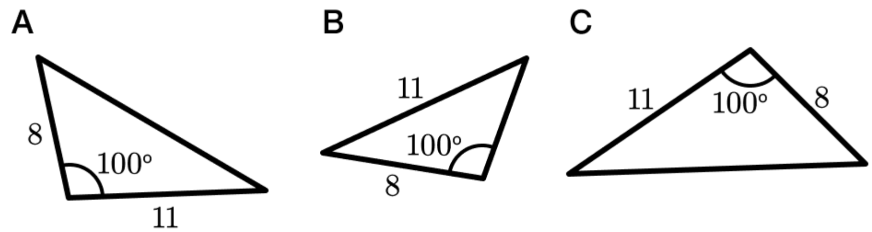
☐ I can build triangles given three measurements.

☐ I can explain why there is sometimes more than one possible triangle given three measurements.

Unit 7.7, Lesson 7: Notes Name ______________________________

My Notes

1. Which of the triangles below are identical?



Triangles A and C

Explanations vary. **In Triangles A and C, the $100°$ angle is between the two side lengths we know. In Triangle B, the angle is only next to the 8-unit side, not the 11-unit side. Also, Triangle B is smaller than the other two.**

2.1 Mariana and Jamir are both drawing triangles that have a 5 cm side, a $60°$ angle, and a $45°$ angle. Will Mariana's and Jamir's triangles be identical?

Maybe. ***Explanations vary.*** **The triangles could be identical, but Mariana and Jamir could have put the side in different places, which would create different triangles.**

2.2 What information would Jamir need about Mariana's triangle in order to be sure she was creating an identical triangle?

Responses vary. **Jamir would need to know if the 5 cm side is between the two angles, adjacent to only the $60°$ angle, or adjacent to only the $45°$ angle .**

Summary

- [ ] I can build triangles given three measurements.
- [ ] I can explain why there is sometimes more than one possible triangle given three measurements.

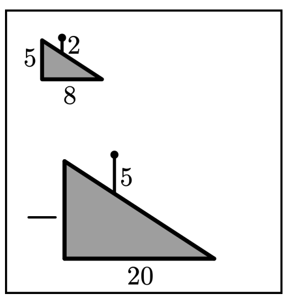
desmos

Unit 7.1, Lesson 2: Notes Name ____________________________________

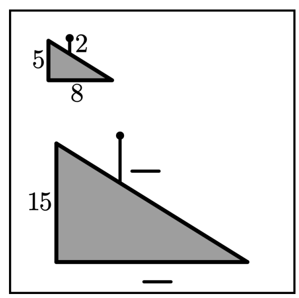
My Notes

1. What is a **scale factor**? Draw an example.

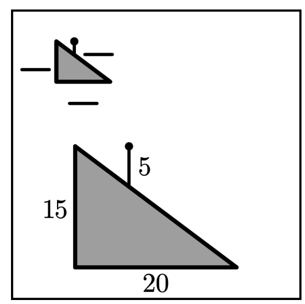
2. Fill in measurements so that the large robot is a scaled copy of the small robot. Then, identify the scale factor from the small robot to the large robot.



Scale factor: _____



Scale factor: _____



Scale factor: _____

Summary

☐ I can explain what *scale factor* is.

☐ I can state the relationship between lengths in an original figure and in a scaled copy.

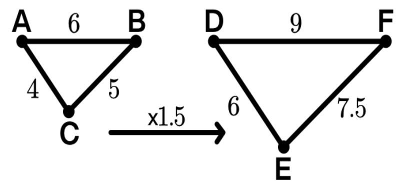
desmos

Unit 7.1, Lesson 2: Notes Name ______________________________

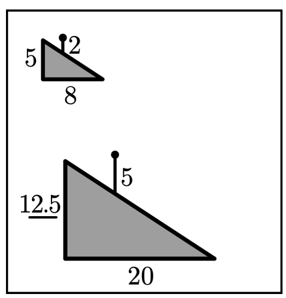
My Notes

1. What is a **scale factor**? Draw an example.

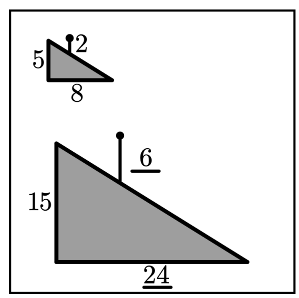
   A scale factor is the number by which we multiply all the lengths in the original figure to create a scaled copy. In the example, the scale factor from the smaller figure to the larger figure is 1.5**.**



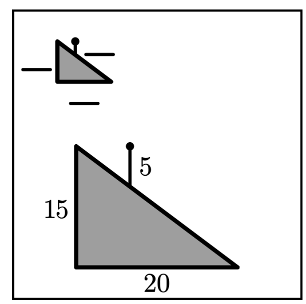
2. Fill in measurements so that the large robot is a scaled copy of the small robot. Then, identify the scale factor from the small robot to the large robot.



Scale factor: 2.5



Scale factor: 3



Scale factors vary.

Summary

☐ I can explain what *scale factor* is.

☐ I can state the relationship between lengths in an original figure and in a scaled copy.

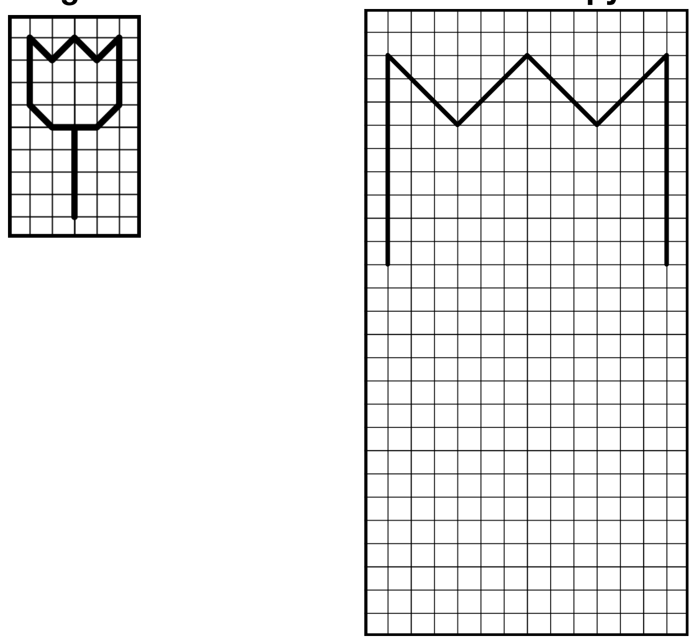
desmos

Unit 7.1, Lesson 3: Notes Name ______________________________

My Notes

1. Draw the rest of the figure using a scale factor of 3.

Original **Scaled Copy**



What is the length of the original stem? ______ grid units.

What is the length of the scaled stem? ______ grid units.

2. What do you keep in mind when drawing a scaled copy?

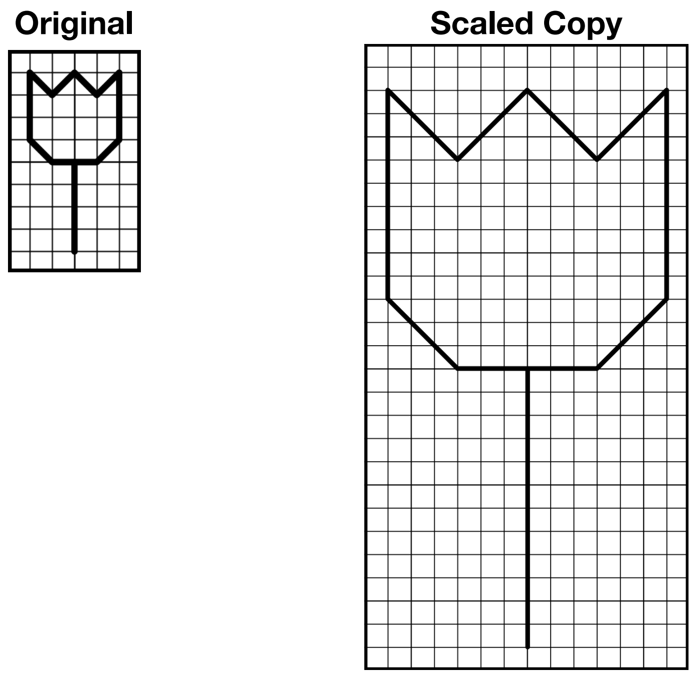
Summary

☐ I can draw a scaled copy of a figure using a given scale factor.

Unit 7.1, Lesson 3: Notes Name ______________________________

My Notes

1. Draw the rest of the figure using a scale factor of 3.



What is the length of the original stem? 4 grid units.

What is the length of the scaled stem? 12 grid units.

2. What do you keep in mind when drawing a scaled copy?

- **Check to make sure the angles of the scaled copy are the same as the original so the shape is correct.**
- **Multiply the number of units in the original figure by the scale factor to calculate the distance in the scaled copy.**

Summary

☐ I can draw a scaled copy of a figure using a given scale factor.

desmos

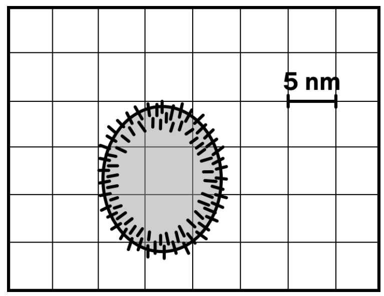
Unit 7.1, Lesson 6: Notes Name ___________________________

My Notes

1. In your own words, describe what a **scale** is.

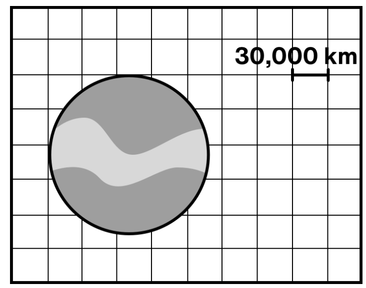
2. Estimate the diameter of the objects below.

Flu Virus



Diameter: ____________

Jupiter



Diameter: ____________

3. Choose one object from Problem 2 and explain how you estimated its diameter.

Summary

☐ I can explain what a scale is.

☐ I can interpret the scale of a drawing.

Unit 7.1, Lesson 6: Notes Name ____________________________

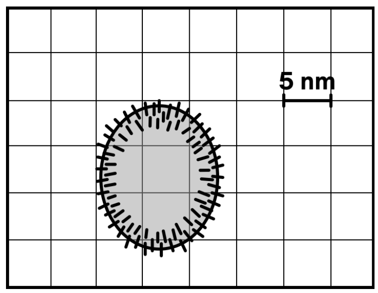
My Notes

1. In your own words, describe what a **scale** is.

   ***Responses vary.* A scale shows how much actual distance one measurement in a drawing represents. For example, the scale 1 cm to 3 miles means one centimeter in the drawing represents an actual distance of three miles.**

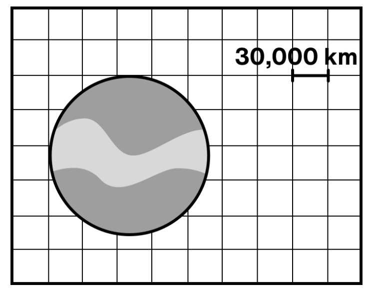
2. Calculate the diameter of the objects below.

Flu Virus



Diameter: **Approx.** 13 **nm**

Jupiter



Diameter: **Approx.** $140\,000$ **km**

3. Choose one object from Problem 2 and explain how you calculated the diameter. ***Responses vary.***

   - **The flu virus is about 2.6 units in the image. Each unit represents 5 nm, and $2.6 \cdot 5 = 13$ nm.**
   - **Jupiter is about 4.6 units in the image. Each unit represents $30\,000$ km, and $4.6 \cdot 30\,000 = 139\,800$ km.**

Summary

- [ ] I can explain what a scale is.
- [ ] I can interpret the scale of a drawing.

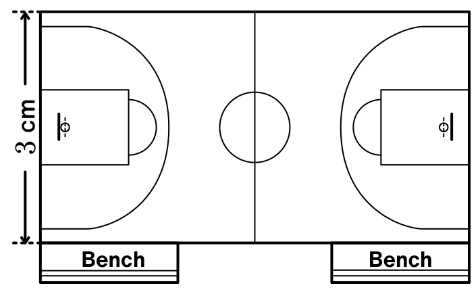
Name ___________________________________

My Notes

1. What are some characteristics of **scale drawings**?

2. Remy used the scale 2 cm to 10 m to create a scale drawing of a basketball court.

   Explain what the numbers in the scale mean.



3. The width of the court in Remy's scale drawing is 3 centimeters. Explain how to use the scale from Problem 2 to determine the width of the actual court.

Summary

- ☐ I can use a scale drawing and a scale to calculate actual and scaled distances.
- ☐ I can determine actual areas from a scale drawing.

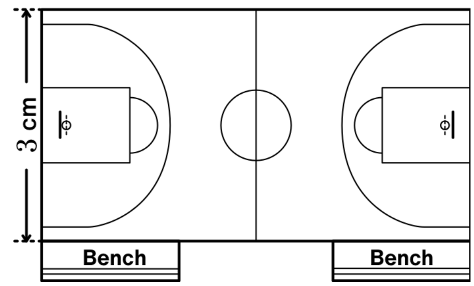
desmos

Unit 7.1, Lesson 7: Notes Name ________________

My Notes

1. What are characteristics of **scale drawings**? ***Responses vary.***
   - **Scale drawings represent an actual place or object.**
   - **The measurements in scale drawings correspond to the measurements of actual objects using the same scale.**

2. Remy used the scale 2 cm to 10 m to create a scale drawing of a basketball court.

   Explain what the numbers in the scale mean.

   Responses vary. **This scale means that every 2 cm on this drawing represents an actual distance of 10 m.**

3. Explain how to use the scale from Problem 2 to determine the actual width of the court.

   The length of the court is 15 m long. ***Explanations vary.***
   - **Each centimeter represents 5 meters, so $5 \cdot 3 = 15$ m.**
   - **3 cm $= 2$ cm $\cdot 1.5$, so 10 m $\cdot 1.5 = 15$ m.**

Summary

☐ I can use a scale drawing and a scale to calculate actual and scaled distances.

☐ I can determine actual areas from a scale drawing.

desmos

Unit 7.1, Lesson 8: Notes

Name ______________________________

My Notes

1. What is important to remember when you create a scale drawing?

2.1 Kyrie wants to create two scale drawings of Nevada using the scales below.

Scale A:	Scale B:
1 cm to 14 mi.	2 cm to 40 mi.

Which scale will produce a larger scale drawing?

Explain your thinking.

2.2 What will be the same in both scale drawings?

Summary

☐ I can create a scale drawing given a scale.

☐ I can describe how different scales affect lengths in a scale drawing.

desmos

Unit 7.1, Lesson 8: Notes Name ______________________________

My Notes

1. What is important to remember when you create a scale drawing? ***Responses vary.***

- **Calculate each scaled length using the same strategy.**
- **Make sure to draw the angles so they are the same in the actual object and the scale drawing.**

2.1 Kyrie wants to create two scale drawings of Nevada using the scales below.

Scale A:	Scale B:
1 cm to 14 mi.	2 cm to 40 mi.

Which scale will produce a larger scale drawing? **Scale A**

Explain your thinking. ***Explanations vary.***

- **In Scale A, 1 cm represents 14 mi. In Scale B, 14 mi. would be represented by less than 1 cm.**
- **Using Scale B, you would need 2 cm to draw 40 mi., but you would use more than 2 cm using Scale A.**

2.2 What will be the same in both scale drawings? ***Responses vary.***

- **Both drawings will represent the same distances and area.**
- **The angles on both drawings will be the same.**
- **The longest distance in one drawing will be the longest distance in the other.**

Summary

- ☐ I can create a scale drawing given a scale.
- ☐ I can describe how different scales affect lengths in a scale drawing.

desmos

Unit 8.2, Lesson 1: Notes

Name ___________________________________

Learning Goal(s):

Definition	Facts/Characteristics
Dilation	
Examples	Non-Examples

Definition	Facts/Characteristics
Similar Figures	
Examples	Non-Examples

Summary Question

How can you check if two figures are similar?

desmos

Unit 8.2, Lesson 1: Notes Name ____________________

Learning Goal(s):
- **I know what it means for two figures to be similar.**

Dilation

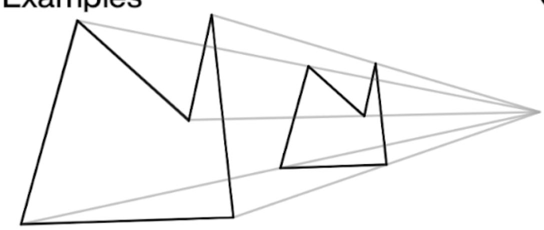
Definition

A transformation in which each point on a figure moves along a line and changes its distance from a fixed point.

Facts/Characteristics

The fixed point is the center of the dilation. All of the original distances are multiplied by the same scale factor.

Examples



Non-Examples

Similar Figures

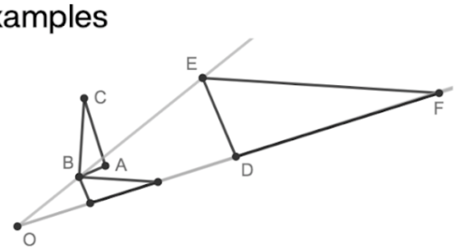
Definition

Two figures are similar if one can fit exactly over the other after rigid transformations and dilations.

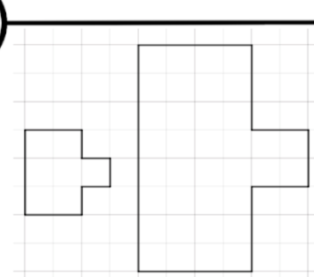
Facts/Characteristics

If ABC is rotated around point B and then dilated with center point O, then it will fit exactly over DEF. This means that they are similar.

Examples



Non-Examples



Summary Question

How can you check if two figures are similar?

Two figures are similar if you can make them congruent through a dilation.

desmos

Unit 8.2, Lesson 2: Notes Name ______________________________

Learning Goal(s):

Definition

Facts/Characteristics

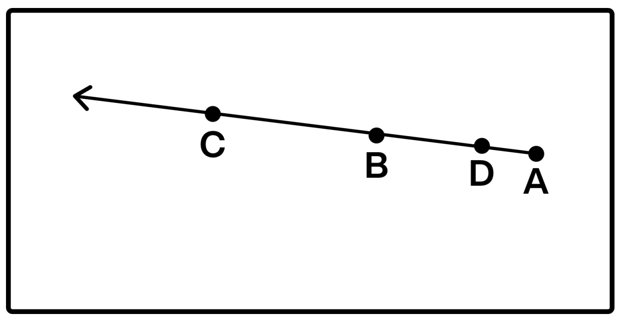
Scale Factor

Examples

Non-Examples

If A is the center of dilation, how can we find which point is the dilation of B . . .

- . . . with a scale factor of 2?



- . . . with a scale factor of $\frac{1}{3}$?

Summary Question

How do you use a ruler to apply a dilation?

desmos

Unit 8.2, Lesson 2: Notes Name ____________________

Learning Goal(s):

- **I can identify the center and scale factor used in a dilation.**
- **I can apply a dilation to figures using a ruler.**

Scale Factor

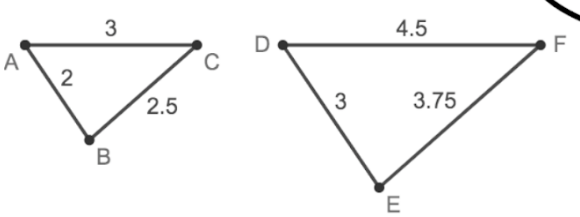
Definition

To create a scaled copy, we multiply all the lengths in the original figure by the same number. This number is called the scale factor.

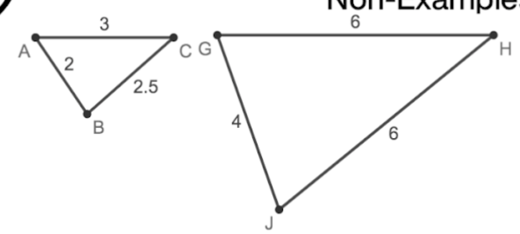
Facts/Characteristics

Every side must be multipled by the same number to make a scaled copy.

Examples

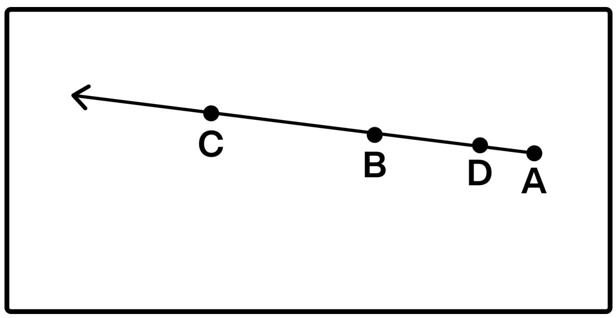


Non-Examples



If A is the center of dilation, how can we find which point is the dilation of B . . .

- . . . with a scale factor of 2?



Since the scale factor is larger than 1, the point must be farther away from A than B is, which makes C the point we are looking for. If we measure the distance between A and C, we would find that it is exactly twice the distance between A and B.

- . . . with a scale factor of $\frac{1}{3}$?

A dilation with scale factor less than 1 brings points closer. Point D is the dilation of B with center A and a scale factor of $\frac{1}{3}$.

Summary Question

How do you use a ruler to apply a dilation?

Measure the distance from the center to the point being dilated. Then multiply that distance by the scale factor. Plot the vertex of the image by measuring from the center and using the calculated distance.

desmos

Unit 8.2, Lesson 3: Notes Name ______________________________

Learning Goal(s):

Square grids can be useful for showing dilations. Instead of using a ruler to measure the distance between the points, we can count grid units.

Sometimes the square grid comes with coordinates. The coordinate grid gives us a convenient way to name points, and sometimes the coordinates of the image can be found using arithmetic.

Compared to the pre-image, a dilation with . . .

- . . . a scale factor **less than** 1 makes the image ____________________ .
- . . . a scale factor **greater than** 1 makes the image ____________________ .
- . . . a scale factor **equal to** 1 makes the image ____________________ .

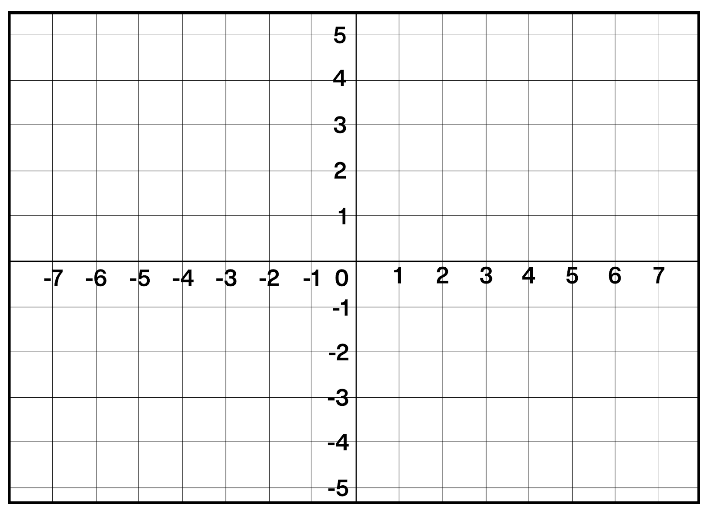
To make a dilation of triangle JKL with center $(0,\ 0)$ and a scale factor of 2:

- Graph triangle JKL with coordinates:

  $J\ (-1,\ -2)$, $K\ (3,\ 1)$, and $L\ (2,\ -1)$

- Use the scale factor to find the coordinates of the image:

$J'\ (\quad,\quad)$, $K'\ (\quad,\quad)$, and $L'\ (\quad,\quad)$



Summary Question

What stays the same between a pre-image and its dilations?

Unit 8.2, Lesson 3: Notes Name ____________________

Learning Goal(s):

- **Apply dilations to figures on a grid.**
- **Describe how scale factors greater than 1, between 0 and 1, or equal to 1 affect the size of an image, its angle measures, and its distance from the center of dilation.**

Square grids can be useful for showing dilations. Instead of using a ruler to measure the distance between the points, we can count grid units.

Sometimes the square grid comes with coordinates. The coordinate grid gives us a convenient way to name points, and sometimes the coordinates of the image can be found using arithmetic.

Compared to the pre-image, a dilation with . . .

- . . . a scale factor **less than** 1 makes the image **smaller**.
- . . . a scale factor **greater than** 1 makes the image **larger**.
- . . . a scale factor **equal to** 1 makes the image **congruent**.

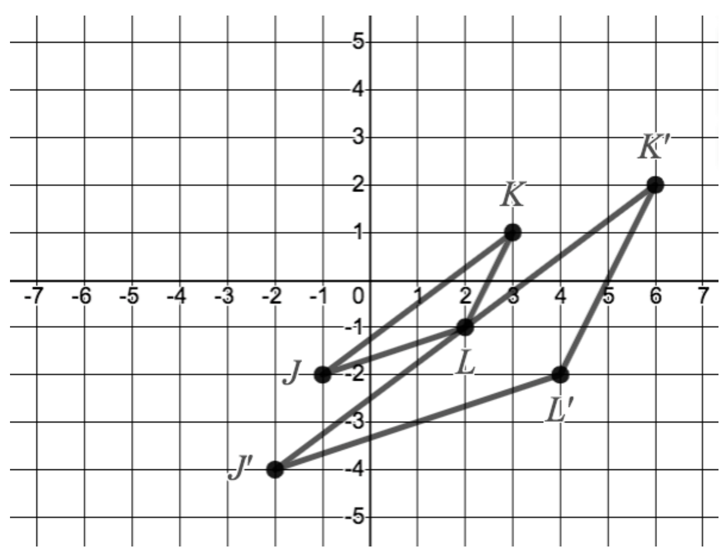
To make a dilation of triangle JKL with center $(0, 0)$ and a scale factor of 2:

- Graph triangle JKL with coordinates:

$J\,(-1, -2)$, $K\,(3, 1)$, and $L\,(2, -1)$

- Use the scale factor to find the coordinates of the image:

$J'\,(-2, -4)$, $K'\,(6, 2)$, and $L'\,(4, -2)$



Summary Question

What stays the same between a pre-image and its dilations?

Corresponding angle measures stay the same between a pre-image and its dilations.

desmos

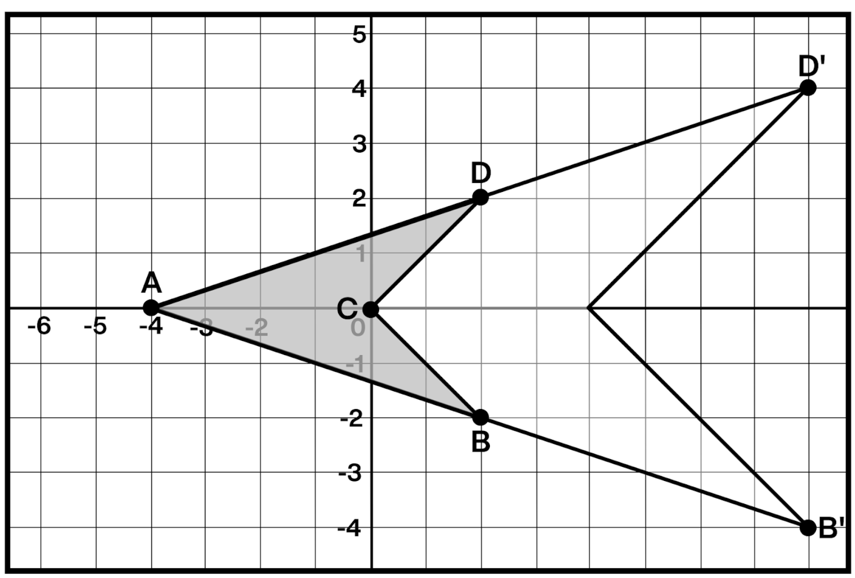
Unit 8.2, Lesson 4: Notes Name ____________________

Learning Goal(s):

One important use of coordinates is to communicate geometric information precisely. Let's consider a quadrilateral $ABCD$ in the coordinate plane. Performing a dilation of $ABCD$ requires three vital pieces of information:

1. The ____________________ of A, B, C, and D.
2. The coordinates of the ____________________.
3. The __________ ____________________ of the dilation.

With this information, we can dilate the vertices A, B, C, and D and then draw the corresponding segments to find the dilation of $ABCD$. Without coordinates, describing the location of the new points would likely require sharing a picture of the polygon and the center of dilation.



Summary Question

How are coordinates useful when describing and drawing dilations?

desmos

Unit 8.2, Lesson 4: Notes Name ___________________________________

Learning Goal(s):

- **I can describe and apply dilations to polygons on a grid if I know the coordinates of the vertices and the center of dilation.**

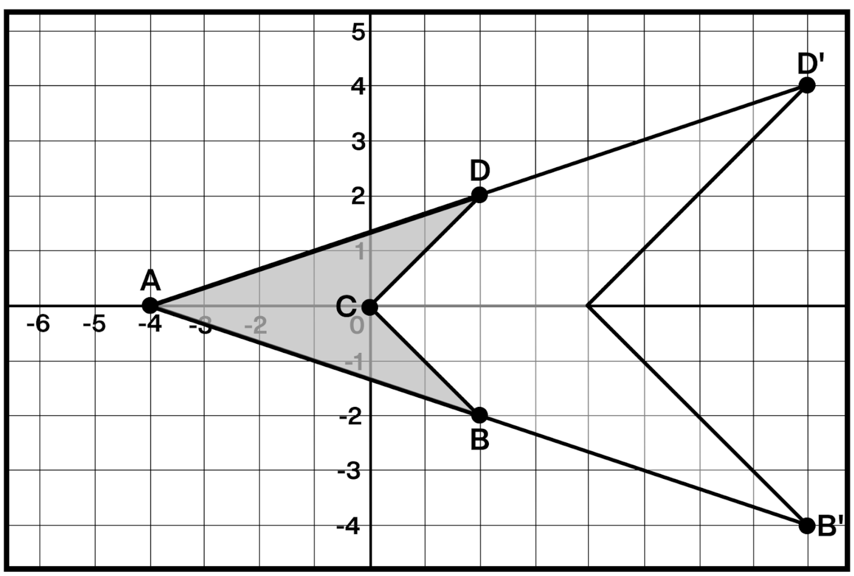
Let's consider a quadrilateral $ABCD$ in the coordinate plane. Performing a dilation of $ABCD$ requires three vital pieces of information:

1. The **coordinates** of A, B, C, and D.

2. The coordinates of the **center of dilation.**

3. The **scale factor** of the dilation.

With this information, we can dilate the vertices A, B, C, and D and then draw the corresponding segments to find the dilation of $ABCD$. Without coordinates, describing the location of the new points would likely require sharing a picture of the polygon and the center of dilation.



Summary Question

How are coordinates useful when describing and drawing dilations?

Coordinates allow us to communicate geometric information precisely. We can specify the exact location of the pre-image and the center of dilation. We can also use the grid to locate the corresponding points on the image.

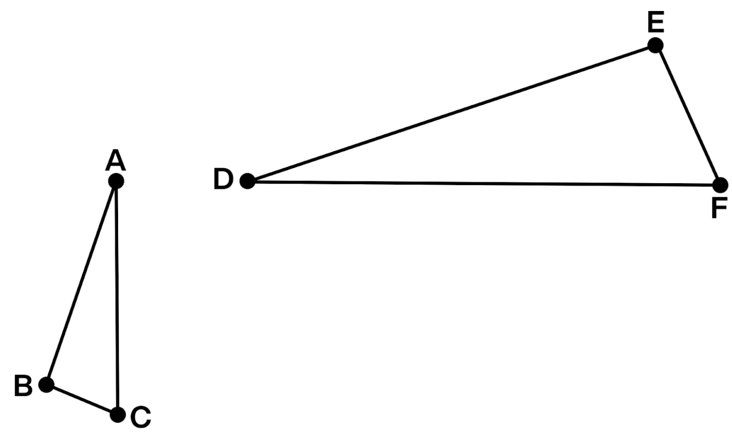
desmos

Unit 8.2, Lesson 5: Notes Name ______________________________

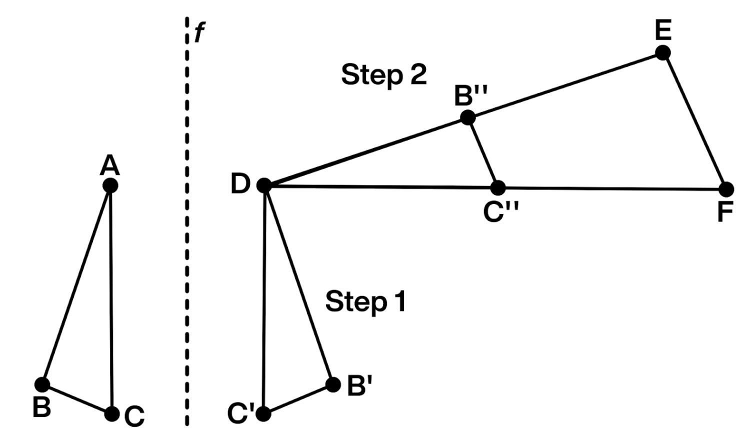
Learning Goal(s):

Two figures are similar if one figure can be transformed into the other by a sequence of translations, rotations, reflections, and dilations. There are many correct sequences of transformations, but we only need to describe one to show that two figures are similar.

Let's show that triangle ABC is similar to triangle DEF.



One way to get from ABC to DEF follows these steps:

Step 1: Reflect across line _________ . **Step 2**: Rotate ________° ____________________ around point ______ . **Step 3**: Dilate with center _________ and a scale factor of ________ .	

Summary Question

How can you apply a sequence of transformations to one figure to get a similar figure?

desmos

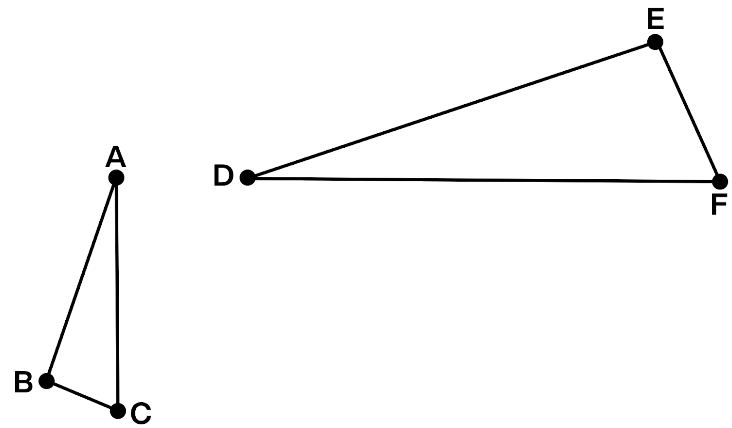
Unit 8.2, Lesson 5: Notes Name ____________________________________

Learning Goal(s):

- **I can show that two figures are similar by applying a sequence of transformations.**

Two figures are similar if one figure can be transformed into the other by a sequence of translations, rotations, reflections, and dilations. There are many correct sequences of transformations, but we only need to describe one to show that two figures are similar.

Let's show that triangle ABC is similar to triangle DEF.

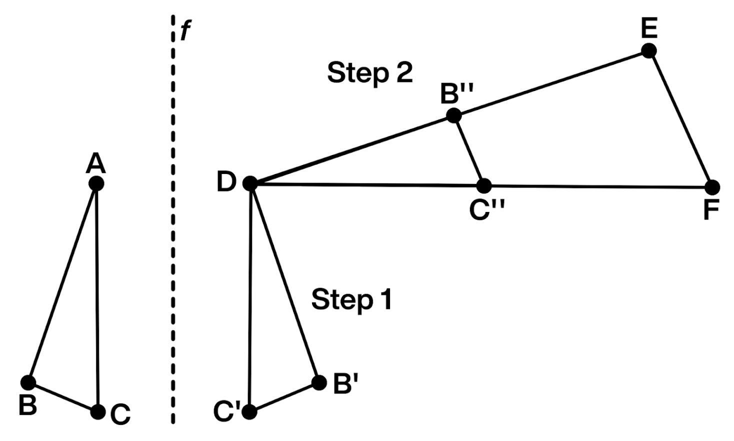


One way to get from ABC to DEF follows these steps:

Step 1: Reflect across line f.

Step 2: Rotate $90°$ **counterclockwise** around point D.

Step 3: Dilate with center D and a scale factor of 2.



Summary Question

How can you apply a sequence of transformations to a figure so that it aligns with a similar figure?

There are many different ways to apply a sequence of transformations to one figure to align with a similar figure. One strategy is to find a sequence of translations, rotations, and reflections to get a corresponding vertex to align. Then dilate with the center at the aligned vertex so that the figures match up.

desmos

Unit 8.2, Lesson 6: Notes Name ______________________________

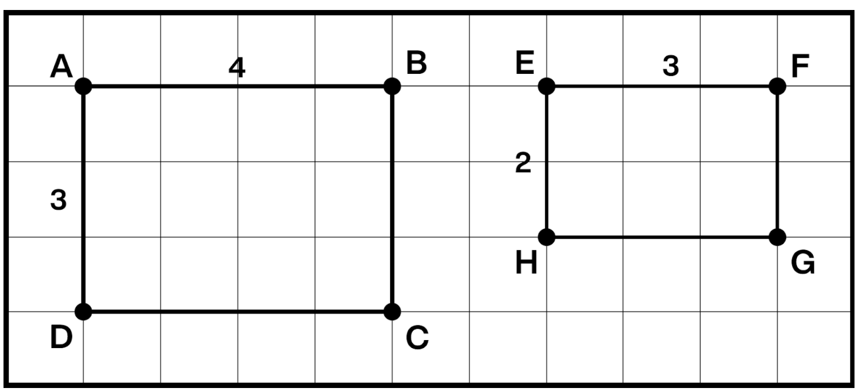
Learning Goal(s):

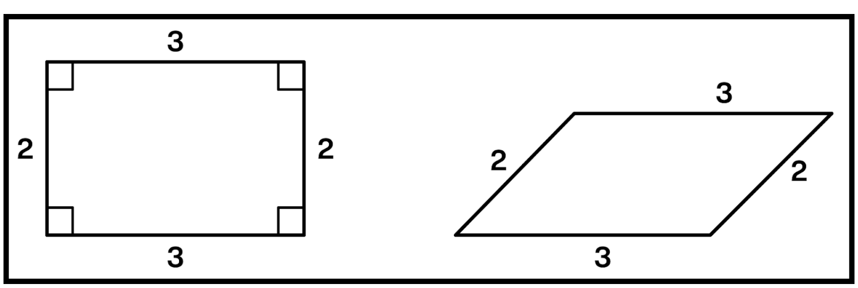
Two polygons are similar when:

- Every angle and side in one polygon has a _______________ part in the other polygon.
- All pairs of corresponding angles have the __________ _______________________.
- Corresponding sides are related by a single __________ ___________________. Each side length in one figure is multiplied by the scale factor to get the ___________________ side length in the other figure.

Summary Question

Are these pairs of polygons similar? Explain how you know.





desmos

Unit 8.2, Lesson 6: Notes Name ____________________

Learning Goal(s):

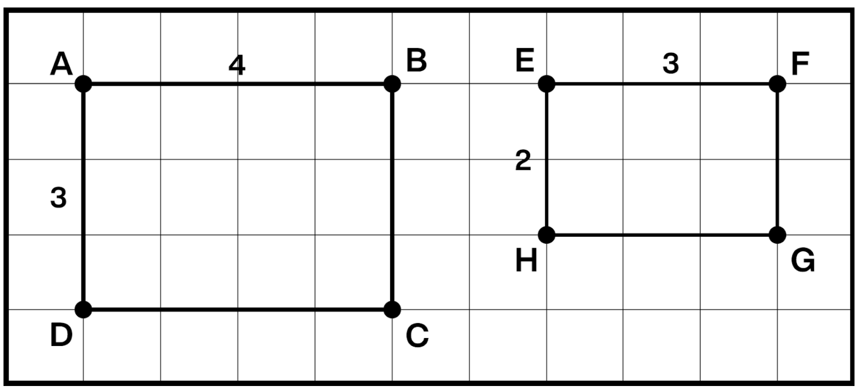
- **I can decide whether or not two polygons are similar based on their angle measures and side lengths.**
- **I can explain whether or not congruent corresponding angles are enough information to prove that two figures are similar.**

Two polygons are similar when:

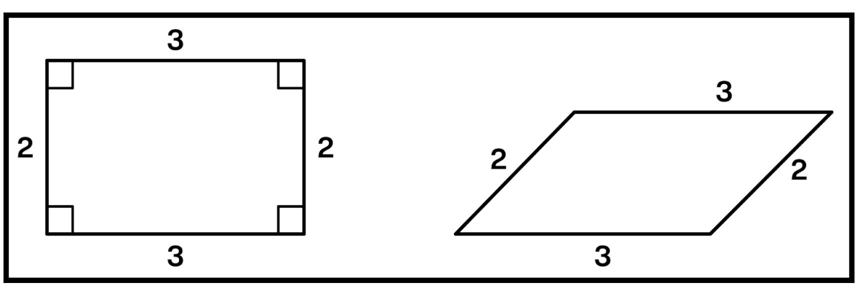
- Every angle and side in one polygon has a **corresponding** part in the other polygon.
- All pairs of corresponding angles have the **same measure**.
- Corresponding sides are related by a single **scale factor**. Each side length in one figure is multiplied by the scale factor to get the **corresponding** side length in the other figure.

Summary Question

Are these pairs of polygons similar? Explain how you know.



All of the corresponding angles are congruent because they are all right angles. To scale the long side AB to the long side EF, the scale factor must be $\frac{3}{4}$. But the scale factor that matches AD to EH is $\frac{2}{3}$. The rectangles are not similar because the scale factors for all the parts must be the same.



The sides all correspond (with a scale factor of 1), but the quadrilaterals are not similar because the corresponding angles do not have the same measure.

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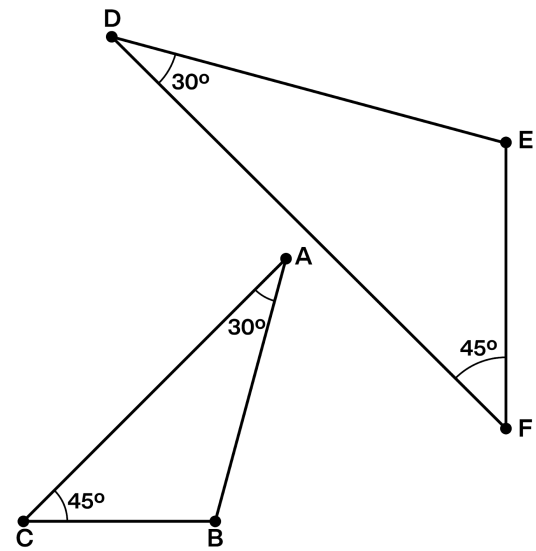
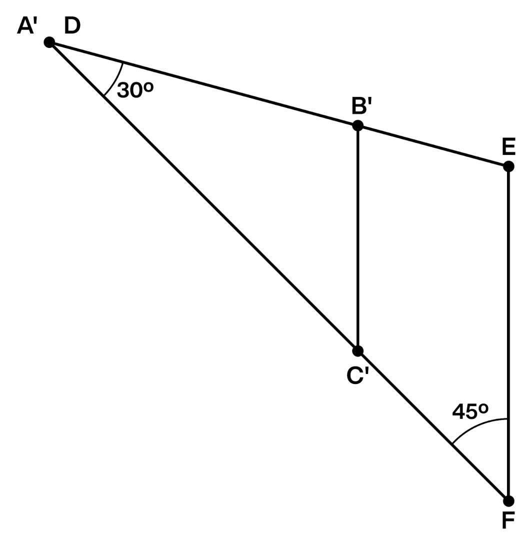
Unit 8.2, Lesson 7: Notes Name ______________________________

Learning Goal(s):

To show that two triangles are similar, we must show that there are two pairs of ______________________ ______________________ ______________.



Triangle ABC and triangle DEF both have a $30°$ angle and a $45°$ angle.

Step 1: Translate A to D and then ______________ so that the two $30°$ angles are aligned. **Step 2:** Dilating $A'B'C'$ with center ________ and the appropriate scale factor will move _____ to _____. This dilation also moves ______ to _____, showing that triangles ABC and DEF are similar.	

Summary Question

Why are two congruent corresponding angles enough to be able to tell if two triangles are similar?

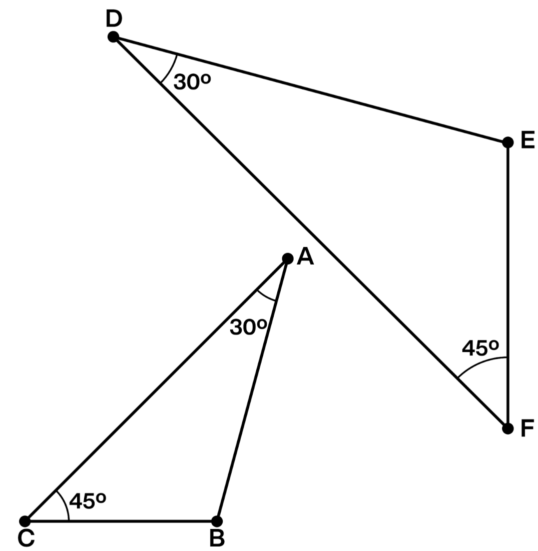
Unit 8.2, Lesson 7: Notes Name ______________________________

Learning Goal(s):

- **I know whether or not two triangles are similar just by looking at their angle measures.**

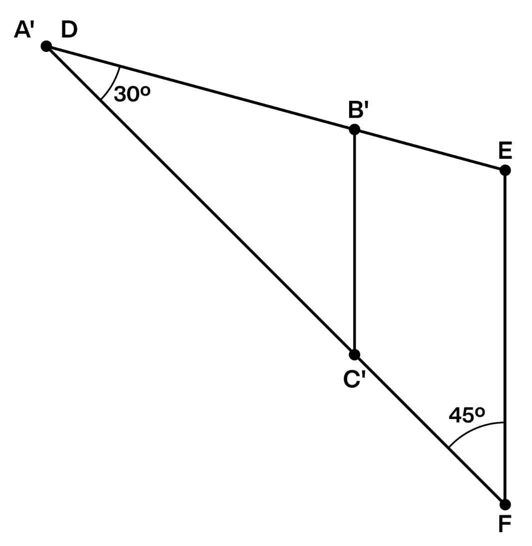
To show that two triangles are similar, we must show that there are two pairs of **congruent corresponding angles**.

Triangle ABC and triangle DEF both have a $30°$ angle and a $45°$ angle.



Step 1: Translate A to D and then **rotate** so that the two $30°$ angles are aligned.

Step 2: Dilating $A'B'C'$ with center D and the appropriate scale factor will move C' to F. This dilation also moves B' to E, showing that triangles ABC and DEF are similar.



Summary Question

Why are two congruent corresponding angles enough to be able to tell if two triangles are similar?

Two triangles with two pairs of congruent corresponding angles are similar because all triangle angles sum to $180°$, so the third pair of corresponding angles must also be congruent.

desmos

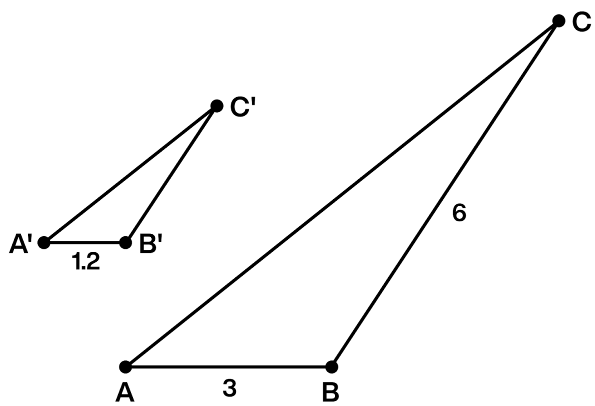
Unit 8.2, Lesson 8: Notes Name ______________________________

Learning Goal(s):

The ratio of two sides in one polygon is the same as the ratio of the corresponding sides in a similar polygon.

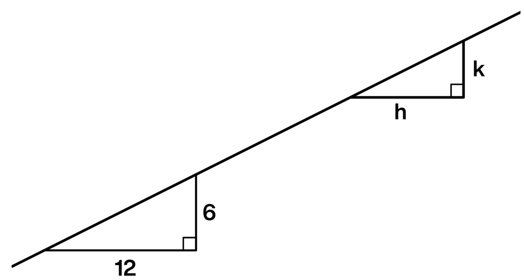
We can use this fact:

1. To calculate missing lengths in similar polygons.



Triangles $A'B'C'$ and ABC are similar. Find the length of segment $B'C'$.

2. To know the ratio of two sides of a triangle without measurements using the measurements of a similar triangle.



The two triangles shown are similar. Find the value of $\frac{k}{h}$.

Summary Question

How can the lengths of corresponding sides within similar triangles be helpful in finding unknown side lengths?

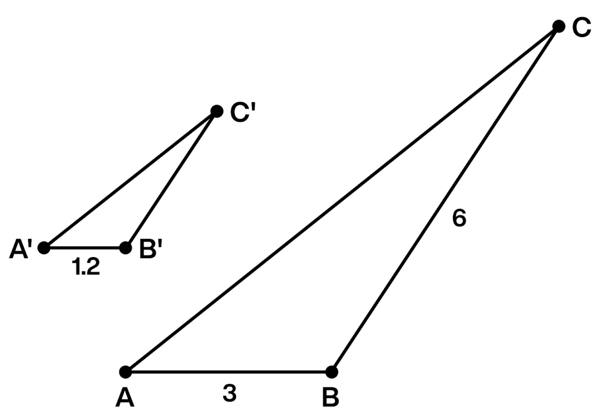
Unit 8.2, Lesson 8: Notes Name ____________________________________

Learning Goal(s):
- **I can decide if two triangles are similar by looking at quotients of lengths of corresponding sides.**
- **I can figure out missing side lengths in pairs of similar triangles using the quotients of their side lengths.**

The ratio of two sides in one polygon is the same as the ratio of the corresponding sides in a similar polygon.

We can use this fact:

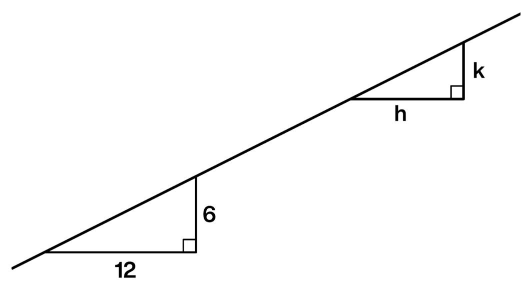
1. To calculate missing lengths in similar polygons.



Triangles $A'B'C'$ and ABC are similar. Find the length of segment $B'C'$.

2.4 **units**

2. To know the ratio of two sides of a triangle without measurements if we know the measurements of a similar triangle.



The two triangles are similar. Find the value of $\frac{k}{h}$.

$$\frac{k}{h} = \frac{6}{12} = \frac{1}{2}$$

Summary Question

How can lengths of corresponding sides within similar triangles be helpful in finding unknown side lengths?

There are many different relationships that can be used to find the side lengths of similar triangles. Sometimes it is more convenient to use the ratio of corresponding side lengths within the triangles depending on what information is missing and the numbers involved in the calculations.

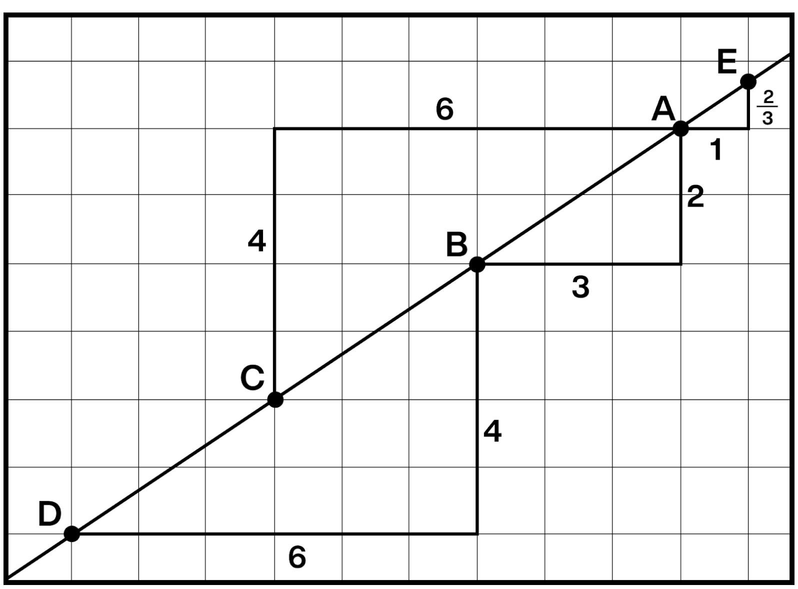
desmos

Unit 8.2, Lesson 9: Notes Name ______________________________

Learning Goal(s):

Definition	Facts/Characteristics
Slope	
Examples	Non-Examples

Here is a line drawn on a grid. There are also four right triangles.
Show how the slope is calculated using the slope triangles between each pair of points:



Points A and B:

Points D and B:

Points A and C:

Points A and E:

Summary Question

What do all of the right triangles drawn along the same line have in common?

Name ______________________________

Learning Goal(s):

- **I can show that all slope triangles on the same line are similar.**
- **I can figure out the slope of a line using slope triangles.**

Slope

Definition

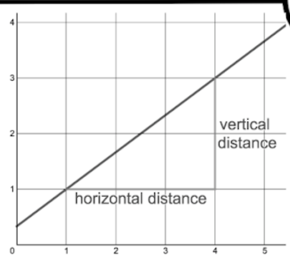
The slope of a line is a number we can calculate using any two points on the line. To find the slope, divide the vertical distance between the points by the horizontal distance.

Facts/Characteristics

You can use the coordinates of two points or two points on the graph of a line to calculate slope.

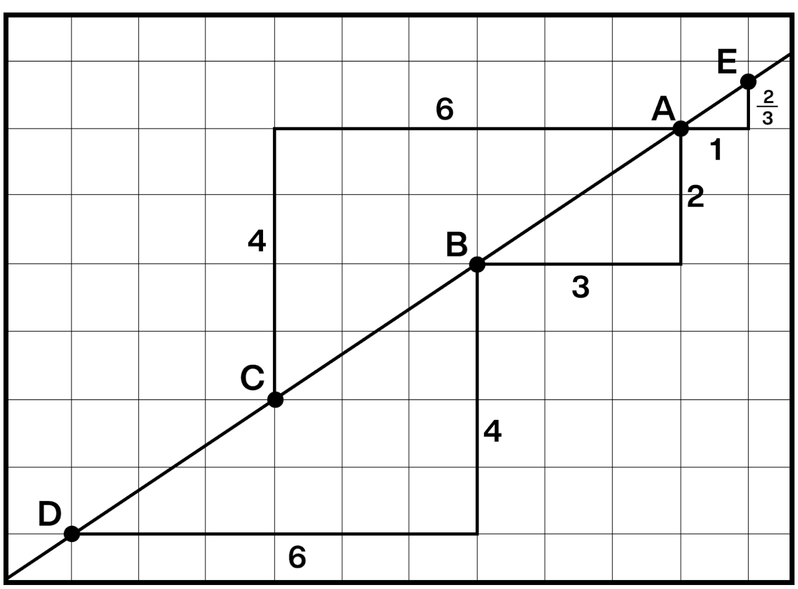
Examples

This line's slope is: $\frac{2}{3}$



Non-Examples

Here is a line drawn on a grid. There are also four right triangles.
Show how the slope is calculated using the slope triangles between each pair of points:



Points A and B: $2 \div 3 = \frac{2}{3}$	
Points D and B: $4 \div 6 = \frac{2}{3}$	
Points A and C: $4 \div 6 = \frac{2}{3}$	
Points A and E: $\frac{2}{3} \div 1 = \frac{2}{3}$	

Summary Question

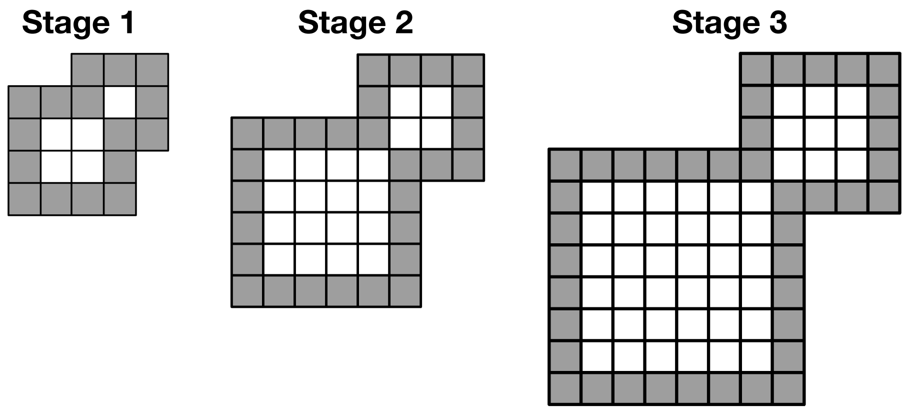
What do all of the right triangles drawn along the same line have in common?

They all have the same slope because all slope triangles for the same line are similar.

Name ______________________________

My Notes

Here is a pattern. The tiles around the edge are called border tiles.



1. Enter the missing information in the table.

Stage	Border Tiles
1	16
2	28
3	

2. Predict how many border tiles are used in Stage 4. Explain how you know.

3. Will there be a stage with 100 border tiles? Explain.

Summary

☐ I can use patterns to determine unknown values.

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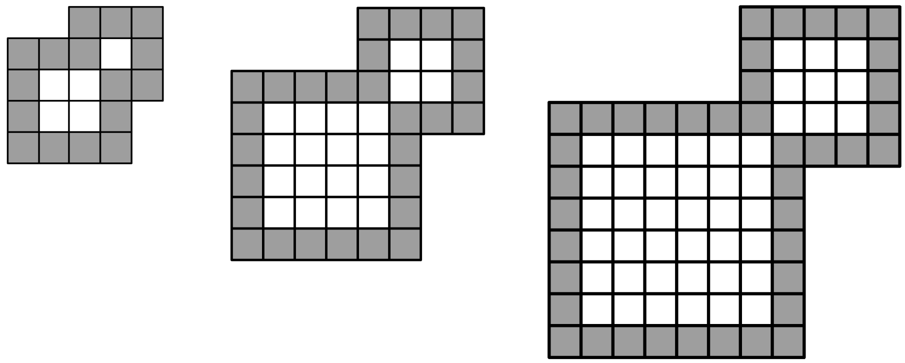
Unit 7.6, Lesson 1: Notes Name ____________________

My Notes

Here is a pattern. The tiles around the edge are called border tiles.

Stage 1 **Stage 2** **Stage 3**



1. Enter the missing information in the table.

Stage	Border Tiles
1	16
2	28
3	40

2. Predict how many border tiles are used in Stage 4. Explain how you know.

52 **border tiles.**

***Explanations vary*. The large square will have $4(8) + 3 = 35$ border tiles and the small square will have $4(4) + 3 = 19$. Two overlap though, so there are $35 + 19 - 2 = 52$ total border tiles.**

3. Will there be a stage with 100 border tiles? Explain.

Yes. *Explanations vary*. As the stage number increases by 1, there are 12 additional border tiles. To get 48 more border tiles, we need exactly 4 more stages, so Stage 8 will have 100 border tiles.

Summary

☐ I can use patterns to determine unknown values.

 Name ______________________

My Notes

Aba bought a loaf of bread and some apples. Her receipt is below.

Bread	(1 @ \$2.25)	\$2.25
Apples	(3 @	\$
Total:		\$6.00

1.1 Which tape diagram represents the receipt?

Diagram A

$\longleftarrow 6 \longrightarrow$

$y+2.25$	$y+2.25$	$y+2.25$

Diagram B

$\longleftarrow 6 \longrightarrow$

y	y	y	2.25

1.2 What is the price of an apple?

2. Tell a story that this diagram could represent.

$\longleftarrow 17.75 \longrightarrow$

5.75	x	x	x	x

Summary

- ☐ I can connect a tape diagram to a story.
- ☐ I can use a tape diagram to figure out an unknown value.

Name ______________________________

My Notes

Aba bought a loaf of bread and some apples. Her receipt is below.

Bread	(1 @ $2.25)	$2.25
Apples	(3 @	$
Total:		$6.00

1.1 Which tape diagram represents the receipt?

Diagram A

$\longleftarrow 6 \longrightarrow$

$y + 2.25$	$y + 2.25$	$y + 2.25$

Diagram B

$\longleftarrow 6 \longrightarrow$

y	y	y	2.25

1.2 What is the price of an apple?

$1.25

2. Tell a story that this diagram could represent.

$\longleftarrow 17.75 \longrightarrow$

5.75	x	x	x	x

***Responses vary.* Aba bought a sandwich for** $5.25 **and four bags of chips. Altogether she spent** $17.75**.**

Summary

☐ I can connect a tape diagram to a story.

☐ I can use a tape diagram to figure out an unknown value.

Unit 7.6, Lesson 3: Notes Name ______________________

My Notes

A drive-in movie theater charges $\$6.00$ per car, plus a fee for each person in the car. A family of 3 came in one car and paid $\$22.50$ total.

1. Select the tape diagram that best matches this situation.

Diagram A

22.5

$x+6$	$x+6$	$x+6$

Diagram B

22.5

x	x	x	6

2. Write an equation to represent this situation.

3. How much was the fee for each family member?

4. Describe how you can tell from the tape diagram that your solution makes sense.

5. Describe how you can tell from the equation that your solution makes sense.

Summary

- [ ] I can connect tape diagrams, equations, and stories.
- [ ] I can write an equation to represent a tape diagram or a story.

Unit 7.6, Lesson 3: Notes Name ____________________

My Notes

A drive-in movie theater charges $\$6.00$ per car, plus a fee for each person in the car. A family of 3 came in one car and paid $\$22.50$ total.

1. Select the tape diagram that best matches this situation.

Diagram A

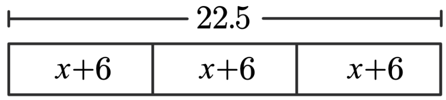
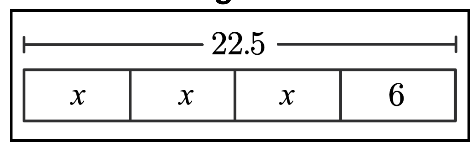


Diagram B



2. Write an equation to represent this situation.

   $6 + 3x = 22.50$ (or equivalent)

3. How much was the fee for each family member?

   $\$5.50$

4. Describe how you can tell from the tape diagram that your solution makes sense.

   ***Responses vary*. In the tape diagram, 5.5 is the length of the x segments because $5.5 + 5.5 + 5.5 + 6 = 22.5$.**

5. Describe how you can tell from the equation that your solution makes sense.

   ***Responses vary*. In the equation, 5.5 makes sense as the value for x because $6 + 3(5.5) = 22.50$.**

Summary

☐ I can connect tape diagrams, equations, and stories.

☐ I can write an equation to represent a tape diagram or a story.

Unit 7.6, Lesson 5: Notes

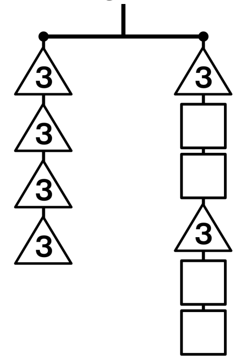
Name ______________________________

My Notes

1. Complete the table so Hanger A stays balanced.

Weight of Triangle (lb.)	Weight of Square (lb.)
3	

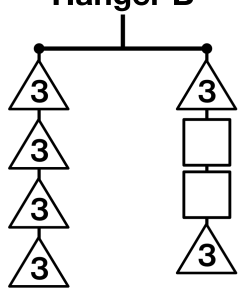
Hanger A



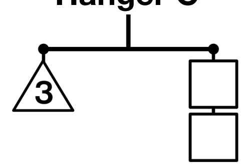
2. Describe how you figured out the weight of each square.

3.1 If Hanger A is balanced, which of these hangers will also be balanced?

Hanger B



Hanger C



3.2 Explain how you know.

Summary

- ☐ I can figure out an unknown value in a hanger diagram and explain my strategy.
- ☐ I can make moves to keep a hanger balanced.

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Unit 7.6, Lesson 5: Notes

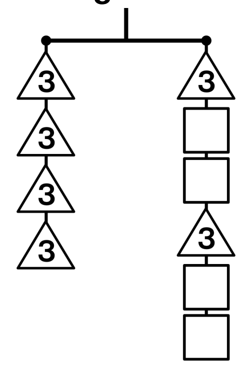
Name ______________________________

My Notes

1. Complete the table so Hanger A stays balanced.

Weight of Triangle (lb.)	Weight of Square (lb.)
3	1.5

Hanger A

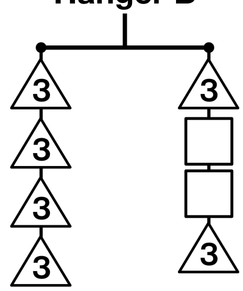


2. Describe how you figured out the weight of each square.

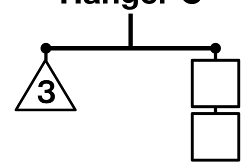
***Responses vary.* I removed two triangles from each side and found that four squares weigh 6 pounds. This means each square weighs 1.5 pounds.**

3.1 If Hanger A is balanced, which of these hangers will also be balanced? **Hanger C**

Hanger B



Hanger C



3.2 Explain how you know.

***Responses vary.* Each square weighs 1.5 pounds, so each side of Hanger C will be 3 pounds, which balances.**

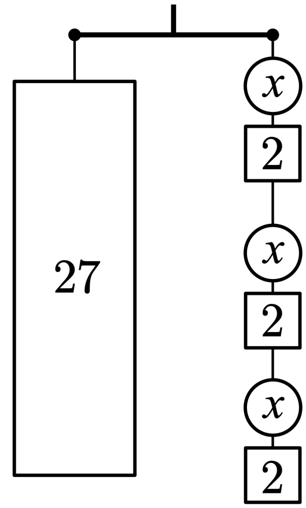
Summary

- [ ] I can figure out an unknown value in a hanger diagram and explain my strategy.
- [ ] I can make moves to keep a hanger balanced.

Unit 7.6, Lesson 6: Notes Name ____________________

My Notes

1.1 What is the value of x?



Anand and Darius used equations to figure out the value of x.

Anand	**Darius**
$27 = 3(x + 2)$	$27 = 3x + 6$
$9 = x + 2$	$21 = 3x$

1.2 Why did Anand write $9 = x + 2$?

1.3 Why did Darius write $21 = 3x$?

2. What is the value of x in the equation $4x + 11 = 14$?

Summary

☐ I can connect balancing moves on hangers to solving equations.

☐ I can solve equations with positive numbers.

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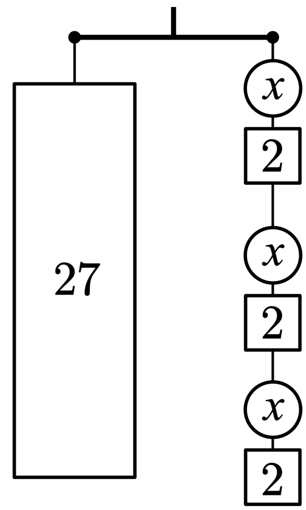
Unit 7.6, Lesson 6: Notes Name ______________________________

My Notes

1.1 What is the value of x?



Anand and Darius used equations to figure out the value of x.

Anand	**Darius**
$27 = 3(x+2)$	$27 = 3x + 6$
$9 = x + 2$	$21 = 3x$

1.2 Why did Anand write $9 = x + 2$?

Anand wrote $9 = x + 2$ because he divided both sides of the hanger by 3.

1.3 Why did Darius write $21 = 3x$?

Darius wrote $21 = 3x$ because he subtracted 6 from each side of the hanger.

2. What is the value of x in the equation $4x + 11 = 14$?

$x = \frac{3}{4}$

Summary

☐ I can connect balancing moves on hangers to solving equations.

☐ I can solve equations with positive numbers.

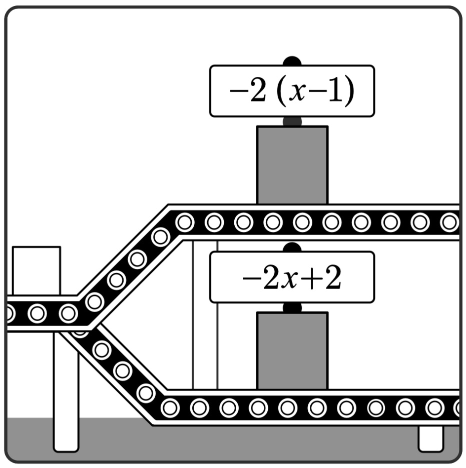
Unit 7.6, Lesson 9: Notes

Name ___________________________________

My Notes

1. Describe what an *equivalent expression* is in your own words.

Here are two number machines.



2.1 When will these number machines have equal outputs?

Always / **Sometimes** / **Never**

2.2 Explain your thinking.

3. Select **all** of the expressions equivalent to $10 - 25x$.

- ☐ $25x - 10$
- ☐ $5(2 - 5x)$
- ☐ $-25x + 10$
- ☐ $25x + (-10)$
- ☐ $-5(5x - 2)$

4. Write an equivalent expression for $-4x + 14$.

Summary

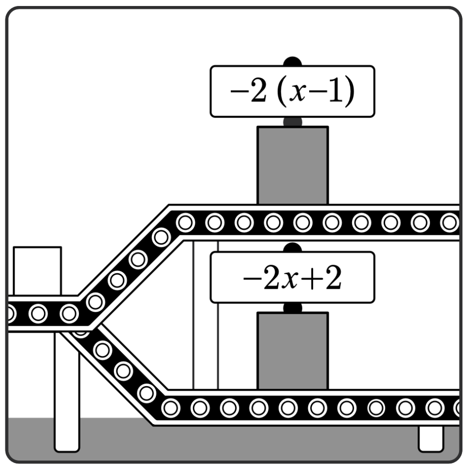
- ☐ I can write equivalent expressions.
- ☐ I can explain whether or not two expressions are equivalent.

My Notes

1. Describe what an *equivalent expression* is in your own words.

   ***Responses vary*. An equivalent expression is a different way of writing an expression by adding, expanding, or reordering. It will always have the same output as the original expression no matter what number I input.**

Here are two number machines.



2.1 When will these number machines have equal outputs?

Always / Sometimes / Never

2.2 Explain your thinking.

***Explanations vary*. If I expand the expression $-2\,(x-1)$ in the top machine, I get $-2x+2$. The expressions are equivalent.**

3. Select **all** of the expressions equivalent to $10 - 25x$.

   ☐ $25x - 10$

   ✓ $5\,(2 - 5x)$

   ✓ $-25x + 10$

   ☐ $25x + (-10)$

   ✓ $-5\,(5x - 2)$

4. Write an equivalent expression for $-4x + 14$.

   $-2\,(2x - 7)$
   (or equivalent)

Summary

☐ I can write equivalent expressions.

☐ I can explain whether or not two expressions are equivalent.

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Unit 7.6, Lesson 10: Notes

Name ____________________________

My Notes

1. How many **terms** does the expression $5x - 10 + 3x + 6$ have? Explain how you know.

2. Mai collected the squares by adding across each row. Write each of her sums using the fewest number of terms.

$5(x-2)$	$3x+6$
$-11x$	$-3(x+2)$

Top sum:

Bottom sum:

3. Ayaan collected the squares by adding down each column. Write each of his sums using the fewest number of terms.

$5(x-2)$	$3x+6$
$-11x$	$-3(x+2)$

Left sum:

Right sum:

Summary

☐ I can write equivalent expressions with fewer terms.

Name ___________________________

My Notes

1. How many **terms** does the expression $5x - 10 + 3x + 6$ have? Explain how you know.

   4 **terms**

   ***Explanations vary*. There are four different parts that are added or subtracted together, so there are 4 terms.**

2. Mai collected the squares by adding across each row. Write each of her sums using the fewest number of terms.

$5(x-2)$	$3x+6$
$-11x$	$-3(x+2)$

Top sum:

$8x - 4$

Bottom sum:

$-14x - 6$

3. Ayaan collected the squares by adding down each column. Write each of his sums using the fewest number of terms.

$5(x-2)$	$3x+6$
$-11x$	$-3(x+2)$

Left sum:

$-6x - 10$

Right sum:

0

Summary

☐ I can write equivalent expressions with fewer terms.

Unit 8.4, Lesson 3: Notes Name ____________________________________

Learning Goal(s):

An equation tells us that two expressions have equal value. For example, if $4x + 9$ and $-2x - 3$ have equal value, we can write the equation $4x + 9 = -2x - 3$.

In order to figure out what number x is so that $4x + 9$ is equal to $-2x - 3$, we can use moves that keep both sides balanced. Complete each step in the table:

$4x + 9 = -2x - 3$	We can subtract 9 from both sides of this equation and keep the equation balanced.
	We can add $2x$ to each side of the equation and maintain equality.
	If we divide the expressions on each side of the equation by 6, we will also maintain the equality.

We just figured out that when x is ____, $4x + 9$ is equal to $-2x - 3$.

Let's check if it works.

$4x + 9$ $4(\quad) + 9$	$-2x - 3$ $-2(\quad) - 3$

Summary Question
How are balanced moves on a hanger similar to solving an equation?

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Unit 8.4, Lesson 3: Notes Name ______________________________

Learning Goal(s): • **I can add, subtract, multiply, or divide each side of an equation by the same expression to get a new equation with the same solution.**

An equation tells us that two expressions have equal value. For example, if $4x + 9$ and $-2x - 3$ have equal value, we can write the equation $4x + 9 = -2x - 3$.

In order to figure out what number x is so that $4x + 9$ is equal to $-2x - 3$, we can use moves that keep both sides balanced. Complete each step in the table:

$4x + 9 = -2x - 3$ $-9 \qquad -9$	We can subtract 9 from both sides of this equation and keep the equation balanced.
$4x = -2x - 12$ $+2x \qquad +2x$	We can add $2x$ to each side of the equation and maintain equality.
$\frac{6x}{6} = \frac{-12}{6}$	If we divide the expressions on each side of the equation by 6, we will also maintain the equality.
$x = -2$	

We just figured out that when x is -2, $4x + 9$ is equal to $-2x - 3$.

Let's check if it works.

$4x + 9$	$-2x - 3$
$4(\ -2\ ) + 9$	$-2(\ -2\ ) - 3$
$-8 + 9$	$4 - 3$
1	1

Summary Question

How are balanced moves on a hanger similar to solving an equation?

They are similar because as long as you keep both sides balanced, you can make moves that help to determine the value of an unknown object.

Unit 8.4, Lesson 4: Notes Name ____________________________________

Learning Goal(s):

Each step we take when solving an equation results in a new equation with the _____________ solution as the original. This means we can check our work by ___________________ the value of the ________________ into the original equation.

Say we solve the following equation:	When we replace x with 3 in the original equation . . .
$2x =- 3(x+5)$ $2x =- 3x+15$ $5x = 15$ $x = 3$	$2x =- 3(x+5)$ $2(\quad) =- 3(\quad +5)$ . . . we get a statement that isn't true!

This tells us we must have made a mistake somewhere.
Checking our original steps carefully, we made a mistake when distributing -3.

After fixing it, we now have:	Let's replace x with ____ in the original equation to make sure we didn't make another mistake.
$2x = -3(x+5)$	$2x = -3(x+5)$ $2(\quad) = -3(\quad +5)$

This equation is true, so $x =$_____ is the solution to the equation $2x = -3(x+5)$.

Summary Question
What is the relationship between an equation and its solution?

Unit 8.4, Lesson 4: Notes Name ___________________________________

Learning Goal(s): • **I can make sense of multiple ways to solve an equation.**

Each step we take solving an equation results in a new equation with the **same** solution as the original. This means we can check our work by **substituting** the value of the **solution** into the original equation.

Say we solve the following equation:	When we replace x with 3 in the original equation . . .
$2x = -3(x+5)$ $2x = -3x + 15$ $5x = 15$ $x = 3$	$2x = -3(x+5)$ $2(\ 3\ ) = -3(\ 3\ + 5)$ $6 = -3\ (8)$ $6 = -24$ . . . we get a statement that isn't true!

This tells us we must have made a mistake somewhere.
Checking our original steps carefully, we made a mistake when distributing -3.

After fixing it, we now have:	Let's substitute -3 in place of x into the original equation to make sure we didn't make another mistake.
$2x = -3(x+5)$ $2x = -3x - 15$ $5x = -15$ $x = -3$	$2x = -3(x+5)$ $2(\ -3\ ) = -3(\ -3\ + 5)$ $-6 = -3(\ 2)$ $-6 = -6$

This equation is true, so $x = -3$ is the solution to the equation $2x = -3(x+5)$.

Summary Question
What is the relationship between an equation and its solution?

The solution is the value of the variable that makes the equation true.

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Unit 8.4, Lesson 5: Notes Name ___________________________

Learning Goal(s):

There are many different ways to solve equations with one variable. In general, we want to make moves that get us closer to an equation like: $variable = number$.

For example, ____ = ______ or ____ = ______.

If we have an equation like:	Or, if we have an equation like:
$$3t + 5 = 7$$	$$4(a - 5) = 12$$
subtracting 5 from each side will leave us with fewer terms. The equation then becomes:	dividing each side by 4 will leave us with fewer factors on the left:
Dividing each side of this equation by 3 will leave us with t by itself on the left:	Adding 5 to each side will leave us with a by itself on the left:

Some people use the following steps to solve a linear equation in one variable:

1. Use the ________________ property so that all the expressions no longer have parentheses.
2. Collect ______________ ____________________ on each side of the equation.
3. Add or subtract an expression so that there is a variable on just __________ side.
4. Add or subtract an expression so that there is just a ________________ on the other side.
5. _______________ or _______________ by a number so that you have an equation that looks like $variable = number$.

Following these steps will always work, but they may not always be the most efficient method. From lots of experience, we learn when to use different approaches.

Summary Question

What are some different first steps towards solving the equation $9 - 2b + 6 = -3(b + 5) + 4b$?

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Unit 8.4, Lesson 5: Notes Name ___________________________________

Learning Goal(s):

- **I can solve an equation in which the variable appears on both sides.**

There are many different ways to solve equations with one variable. In general, we want to make moves that get us closer to an equation like: $variable = number$.

For example, ____ =______ or ____ =______.

If we have an equation like:	Or, if we have an equation like:
$3t + 5 = 7$	$4(a - 5) = 12$
subtracting 5 from each side will leave us with fewer terms. The equation then becomes:	dividing each side by 4 will leave us with fewer factors on the left:
$3t = 2$	$a - 5 = 3$
Dividing each side of this equation by 3 will leave us with t by itself on the left:	Adding 5 to each side will leave us with a by itself on the left:
$t = \frac{2}{3}$	$a = 8$

Some people use the following steps to solve a linear equation in one variable:

1. Use the **distributive** property so that all the expressions no longer have parentheses.
2. Collect **like terms** on each side of the equation.
3. Add or subtract an expression so that there is a variable on just **one** side.
4. Add or subtract an expression so that there is just a **number** on the other side.
5. **Multiply** or **divide** by a number so that you have an equation that looks like $variable = number$.

Following these steps will always work, but they may not always be the most efficient method. From lots of experience, we learn when to use different approaches.

Summary Question

What are some different first steps towards solving the equation $9 - 2b + 6 = -3(b + 5) + 4b$?

Distribute -3 into the parentheses. Add $2b$ to each side. Combine 9 and 6 on the left.

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Unit 8.4, Lesson 6: Notes Name ___________________________________

Learning Goal(s):

Sometimes we are asked to solve equations with a lot of things going on. For example:	$x - 2(x+5) = \frac{3(2x-20)}{6}$
Before we start distributing, let's take a closer look at the fraction on the right side. The expression $2x - 20$ is being multiplied by ____ and divided by ____, which is the same as dividing by ____, so we can rewrite the equation as:	$x - 2(x+5) = \frac{2x-20}{2}$
Now it's easier to see that all the terms in the numerator on the right side are divisible by ___, which means we can rewrite the right side again:	$x - 2(x+5) =$ ___ - ___
At this point, we could _____________ and then collect like terms on each side of the equation. Another choice would be to **use the structure of the equation**. Both the left and the right side have something being subtracted from x. But, if the two sides are equal, that means the "something" being subtracted on each side must also be equal. The equation can be rewritten with less terms, like:	$2(x+5) = 10$
When we finish the steps, we have:	$2(x+5) = 10$

Summary Question

How does pausing and thinking about the structure of an equation help when solving the equation?

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Unit 8.4, Lesson 6: Notes Name ______________________________

Learning Goal(s):

- **I can solve linear equations with one variable.**

Sometimes we are asked to solve equations with a lot of things going on. For example:	$x - 2(x+5) = \frac{3(2x-20)}{6}$
Before we start distributing, let's take a closer look at the fraction on the right side. The expression $2x - 20$ is being multiplied by 3 and divided by 6, which is the same as dividing by 2, so we can rewrite the equation as:	$x - 2(x+5) = \frac{2x-20}{2}$
Now it's easier to see that all the terms in the numerator on the right side are divisible by 2, which means we can rewrite the right side again:	$x - 2(x+5) = x - 10$
At this point, we could **distribute** and then collect like terms on each side of the equation. Another choice would be to use the structure of the equation. Both the left and the right side have something being subtracted from x. But, if the two sides are equal, that means the "something" being subtracted on each side must also be equal. The equation can be re-written with less terms, like:	$2(x+5) = 10$
When we finish the steps, we have:	$2(x+5) = 10$ $x + 5 = 5$ $x = 0$

Summary Question

How does pausing and thinking about the structure of an equation help when solving the equation?

By thinking about the structure of an equation, I can avoid having a lot of steps and find an efficient method to get to a solution.

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Unit 8.4, Lesson 7: Notes Name ____________________________________

Learning Goal(s):

An equation is a statement that two expressions have an equal value. The equation $2x = 6$ . . .

. . . is a true statement if x is _____. . . . is a false statement if x is _____.

The equation $2x = 6$ has one and only one solution because there is only one number, 3, that you can double to get 6.

Some equations are *true no matter what* the value of the variable is. For example: $$2x = x + x$$ is always true because doubling a number will always be the same as adding the number to itself. Equations like $2x = x + x$ have an ______________ number of solutions. We say it is true for _______ values of x. Sometimes we make allowable moves and get an equation like this: $$8 = 8$$ This statement is true, so the original equation must be true no matter what value x has.	Some equations have *no solutions*. For example: $$x = x + 1$$ has no solutions because no matter what the value of x is, it can't equal one more than itself. Equations like $x = x + 1$ have ________ solutions. We say it is __________ true for any value of x. Sometimes we make allowable moves and get an equation like this: $$8 = 9$$ This statement is false, so the original equation must have no solutions at all.

Summary Question

What does it mean for an equation to have no solutions, one solution, or infinitely many solutions?

Learning Goal(s):

- **I can determine whether an equation has no solutions, one solution, or infinitely many solutions.**

An equation is a statement that two expressions have an equal value. The equation $2x = 6$ . . .

. . . is a true statement if x is 3. . . . is a false statement if x is 4.

The equation $2x = 6$ has one and only one solution because there is only one number, 3, that you can double to get 6.

Some equations are *true no matter what* the value of the variable is. For example: $$2x = x + x$$ is always true because doubling a number will always be the same as adding the number to itself. Equations like $2x = x + x$ have an **infinite** number of solutions. We say it is true for **all** values of x. Sometimes we make allowable moves and get an equation like this: $$8 = 8$$ This statement is true, so the original equation must be true no matter what value x has.	Some equations have *no solutions*. For example: $$x = x + 1$$ has no solutions because no matter what the value of x is, it can't equal one more than itself. Equations like $x = x + 1$ have **no** solutions. We say it is **never** true for any value of x. Sometimes we make allowable moves and get an equation like this: $$8 = 9$$ This statement is false, so the original equation must have no solutions at all.

Summary Question

What does it mean for an equation to have no solutions, one solution, or infinitely many solutions?

- **When an equation has no solutions, there is no value of x that will make the equation true.**
- **When an equation has one solution, a specific value for x will make the equation true.**
- **When an equation has infinitely many solutions, any value of x that will make the equation true.**

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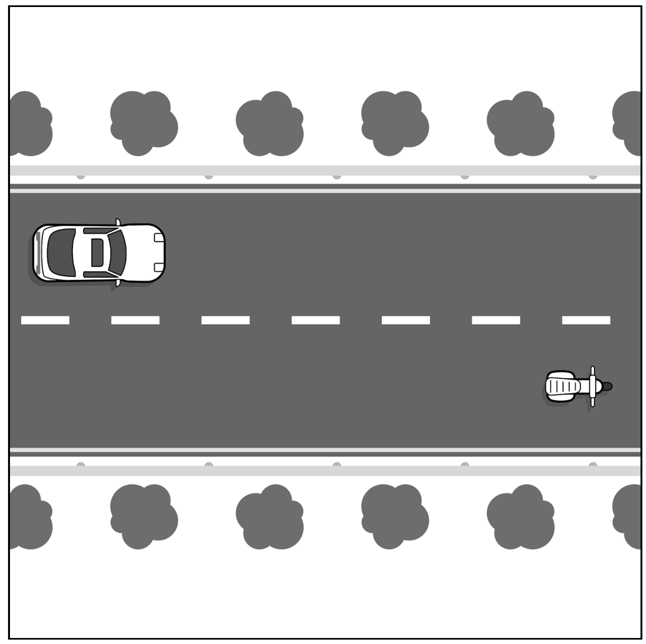
Unit 8.4, Lesson 8: Notes Name ______________________________________

Learning Goal(s):

Imagine a car traveling on a road at a constant speed of 16 meters per second. We can represent the distance the car travels with the expression ___________________, where t represents the number of seconds the car has been traveling.

Now imagine, at the same time, there is a scooter traveling at 9 meters per second and is 42 meters ahead of the car. We can represent the distance the scooter travels with the expression _______________, where t represents the number of seconds the scooter has been traveling.



Since the car is behind the scooter and is traveling at a faster rate, at some point, the vehicles will meet . . . but when? Asking when the two vehicles will meet is the same as asking when ___________________ is equal to ___________________.

$$16t = 9t + 42$$

Solving for t gives us ___________, which means ____.

Summary Question

If two quantities are changing, how can you determine when they will be the same?

Unit 8.4, Lesson 8: Notes

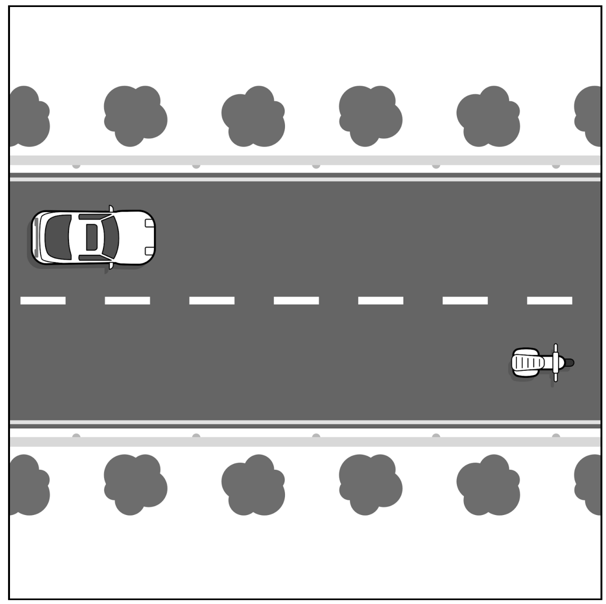
Name ___________________________________

Learning Goal(s):

- **I can use an expression to find when two things, like height, are the same in a real-world situation.**

Imagine a car traveling on a road at a constant speed of 16 meters per second. We can represent the distance the car travels with the expression $16t$, where t represents the number of seconds the car has been traveling.

Now imagine, at the same time, there is a scooter traveling at 9 meters per second and is 42 meters ahead of the car. We can represent the distance the scooter travels with the expression $9t + 42$, where t represents the number of seconds the scooter has been traveling.



Since the car is behind the scooter and is traveling at a faster rate, at some point, the vehicles will meet . . . but when? Asking when the two vehicles will meet is the same as asking when $16t$ is equal to $9t + 42$.

$$16t = 9t + 42$$

Solving for t gives us $t = 6$, which means **that the vehicles will meet after 6 seconds.**

Summary Question

If two quantities are changing, how can you determine when they will be the same?

You can set the expressions that represent the quantities equal to each other, then solve that equation.

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Unit 6.7, Lesson 6: Notes

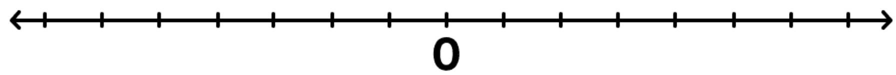
Name ______________________________

My Notes

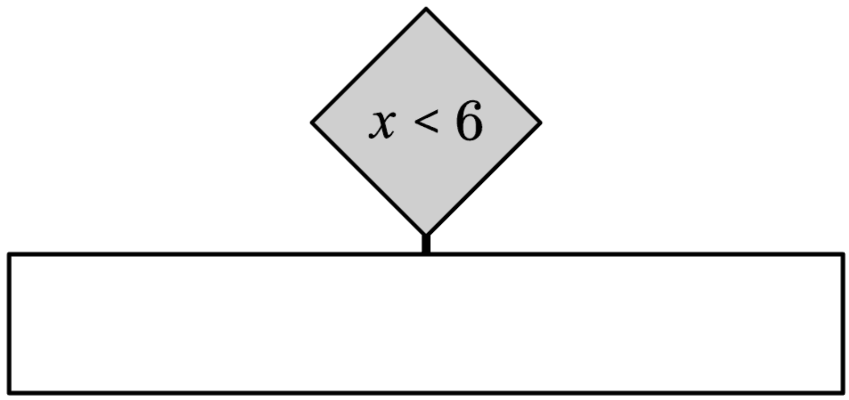
1.1 Circle all the values of x that make the inequality $x < 6$ true.

5.9   –6   6.1   0   100

1.2 Create a graph to represent the inequality $x < 6$.



1.3 Write a sign that could be represented by this inequality.



2. Match each sentence with an inequality.

______ I spent less than 3 hours on my homework.	A. $x > 3$
______ This game is for kids over 3 years old.	B. $x = 3$
______ This recipe uses 3 cups of flour.	C. $3 > x$

Summary

☐ I can show the same information about an inequality using words, symbols, and a number line.

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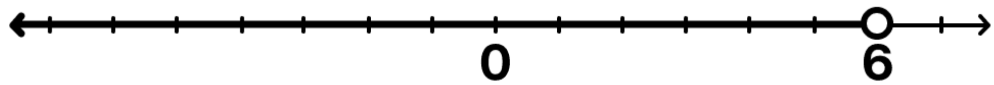
Unit 6.7, Lesson 6: Notes Name ______________________________

My Notes

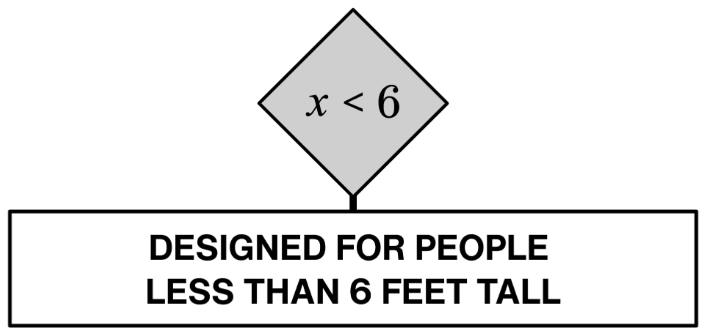
1.1 Circle all the values of x that make the inequality $x < 6$ true.

5.9 $\quad -6$ $\quad 6.1$ $\quad 0$ $\quad 100$

1.2 Create a graph to represent the inequality $x < 6$.



1.3 Write a sign that could be represented by this inequality.
Responses vary.



2. Match each sentence with an inequality.

C.	I spent less than 3 hours on my homework.	A. $x > 3$
A.	This game is for kids more than 3 years old.	B. $x = 3$
B.	This recipe uses 3 cups of flour.	C. $3 > x$

Summary

☐ I can show the same information about an inequality using words, symbols, and a number line.

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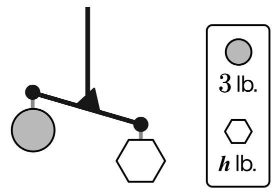
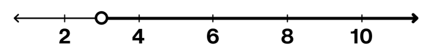
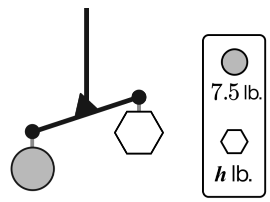
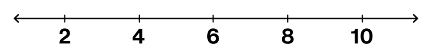
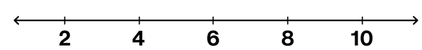
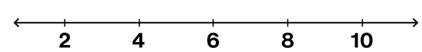
Unit 6.7, Lesson 7: Notes Name ______________________

My Notes

Complete the table so that each row shows the same relationship.

Hanger	Inequality	Graph
		
		
	$h < 6.5$	
	$6.5 < h$	

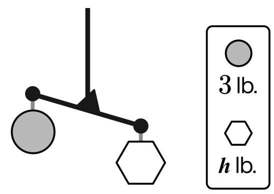
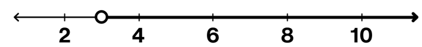
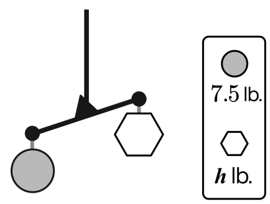
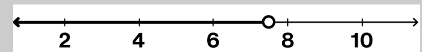
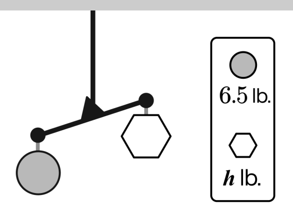
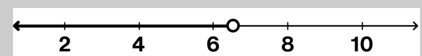
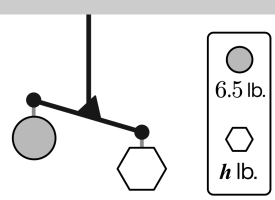
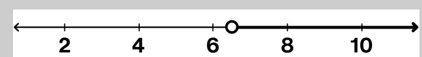
Summary

☐ I can write and interpret inequalities to describe unbalanced hangers.

Name ______________________________

My Notes

Complete the table so that each row shows the same relationship.

Hanger	Inequality	Graph
	$h > 3$	
	$h < 7.5$	
	$h < 6.5$	
	$6.5 < h$	

Summary

☐ I can write and interpret inequalities to describe unbalanced hangers.

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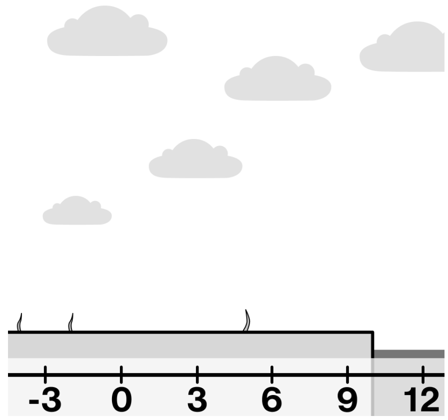
Unit 6.7, Lesson 8: Notes Name ____________________________

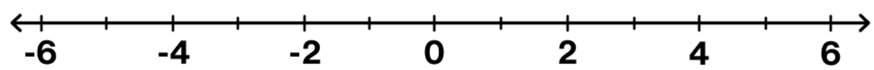
My Notes

1.1 Write an inequality so that all the blades of grass are solutions and the water has no solutions.



1.2 Write three other solutions to the inequality you wrote.

2.1 Plot all of the solutions to $-2 < x$ on the number line.



2.2 Is -2 a solution to $-2 < x$? ________ Explain how you know.

2.3 How many solutions does the inequality $-2 < x$ have? ______

Explain how you know.

Summary

- ☐ I can draw and label a number line diagram that represents the solutions to an inequality.
- ☐ I can explain how many solutions an inequality can have.
- ☐ I can justify whether or not a value is a solution to a given inequality.

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Unit 6.7, Lesson 8: Notes

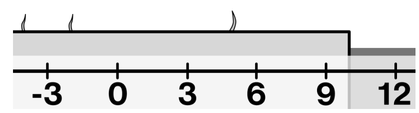
Name ______________________________

My Notes

1.1 Write an inequality so that all the blades of grass are solutions and the water has no solutions. ***Responses vary.***

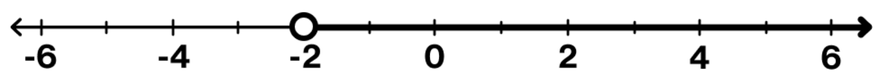
$x < 6$

1.2 Write three other solutions to the inequality you wrote. ***Responses vary.***



$0, -2, 5.5$

2.1 Plot all of the solutions to $-2 < x$ on the number line.



2.2 Is −2 a solution to $-2 < x$? **No.** Explain how you know.

Explanations vary. **Solutions to $-2 < x$ are numbers that are greater than -2. -2 isn't greater than itself.**

2.3 How many solutions does the inequality $-2 < x$ have? **Infinite.**

Explain how you know. ***Explanations vary.***

There are infinitely many numbers greater than -2, so there are infinite solutions to the inequality $-2 < x$.

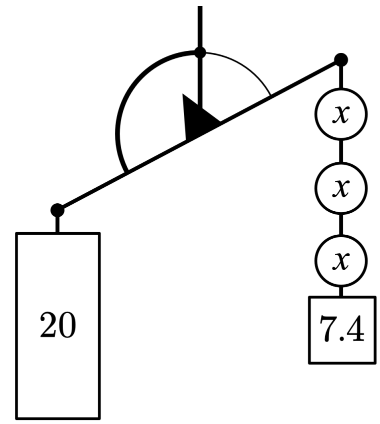
Summary

- ☐ I can draw and label a number line diagram that represents the solutions to an inequality.
- ☐ I can explain how many solutions an inequality can have.
- ☐ I can justify whether or not a value is a solution to a given inequality.

Name ______________________________

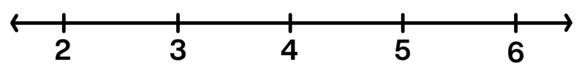
My Notes

This hanger represents the inequality $20 > 3x + 7.4$.



1.1 What are the solutions to this inequality?

1.2 Graph the solutions on the numberline.



1.3 Jasmine and Terrance tried to solve this inequality. Here are their solutions:

Jasmine
$x < 4.2$

Terrance
$4.2 < x$

Who is correct? Explain how you know.

2. Solve the inequality $2(x + 7.5) \leq 18$.

Summary

☐ I can figure out the solutions to an inequality.

☐ I can explain the difference between the solution to an equation and the solutions to an inequality.

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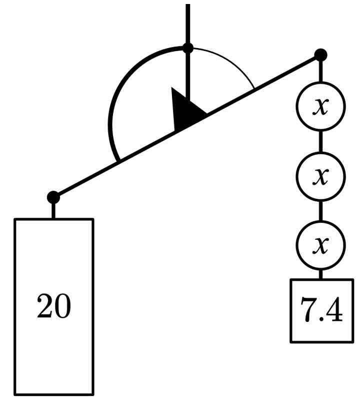
Unit 7.6, Lesson 14: Notes Name ____________________

My Notes

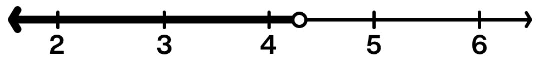
This hanger represents the inequality $20 > 3x + 7.4$.



1.1 What are the solutions to this inequality?

$4.2 > x$

1.2 Graph the solutions on the numberline.



1.3 Jasmine and Terrance tried to solve this inequality. Here are their solutions:

Jasmine	**Terrance**
$x < 4.2$	$4.2 < x$

Who is correct? Explain how you know.

Jasmine is correct.

Explanations vary. **Her solutions show that x could be any value less than 4.2.**

2. Solve the inequality $2(x + 7.5) \leq 18$.

$x \leq 1.5$

Summary

☐ I can figure out the solutions to an inequality.

☐ I can explain the difference between the solution to an equation and the solutions to an inequality.

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Unit 7.6, Lesson 16: Notes

Name ______________________________

My Notes

Here's an inequality: $3(10 - 2x) < 18$.

Ava solved the equation $3(10 - 2x) = 18$ and calculated $x = 2$.

1.1 Choose a value for x that is greater than 2 and substitute it into $3(10 - 2x) < 18$.

1.2 Choose a value for x that is less than 2 and substitute it into $3(10 - 2x) < 18$.

1.3 What are the solutions to the inequality?

1.4 Graph the solutions on this number line.

2. Tyrone is solving the inequality $5 - 0.5x \geq 3$.
He says that the solutions to the inequality are $x \leq 4$.

Is this correct? Explain how you know.

Summary

- ☐ I can solve an inequality with positive and negative numbers and graph the solutions.
- ☐ I can test values to decide which inequality symbol makes sense.

Name ____________________

My Notes

Here's an inequality: $3(10 - 2x) < 18$.

Ava solved the equation $3(10 - 2x) = 18$ and calculated $x = 2$.

1.1 Choose a value for x that is greater than 2 and substitute it into $3(10 - 2x) < 18$.

Responses vary.

$$3(10 - 2(5)) < 18$$
$$3(0) < 18$$
$0 < 18$ True!

1.2 Choose a value for x that is less than 2 and substitute it into $3(10 - 2x) < 18$.

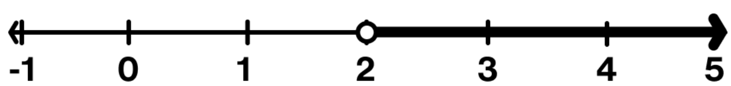
Responses vary.

$$3(10 - 2(0)) < 18$$
$$3(10) < 18$$
$30 < 18$ False!

1.3 What are the solutions to this inequality?

$$x > 2$$

1.4 Graph the solutions on this number line.



2. Tyrone is solving the inequality $5 - 0.5x \geq 3$.
He says that the solutions to the inequality are $x \leq 4$.

Is this correct? **Yes** Explain how you know.

***Responses vary.* The equation $5 - 0.5x = 3$ is true when $x = 4$. When I substitute 0 for x in the inequality, I see that $5 - 0.5(0) \geq 3$ is true, so this inequality is true when x is less than or equal to 4.**

Summary

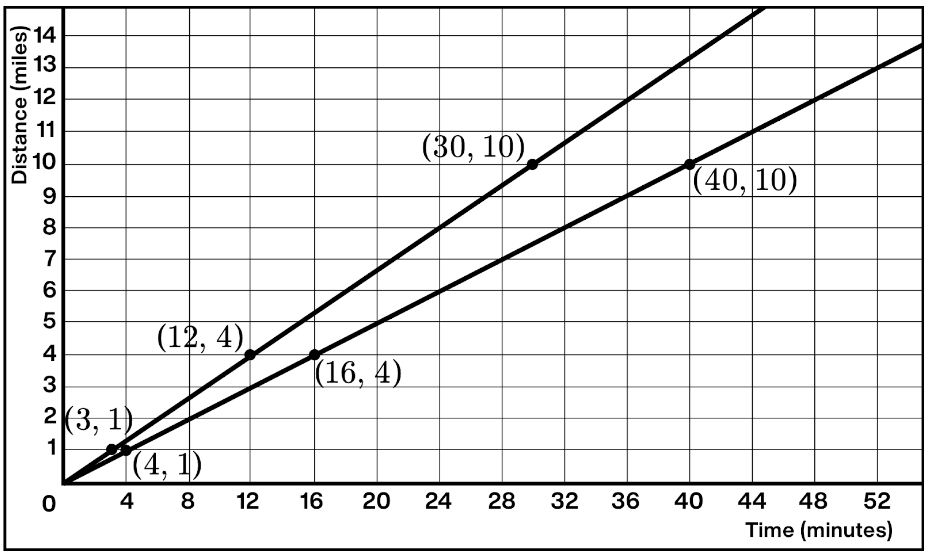
☐ I can solve an inequality with positive and negative numbers and graph the solutions.

☐ I can test values to decide which inequality symbol makes sense.

Unit 8.3, Lesson 1: Notes Name ____________________

Learning Goal(s):

Here are the graphs showing Jasmine and Sothy's distance on a long bike ride. Jasmine rides 4 miles every 16 minutes, and Sothy rides 4 miles every 12 minutes.



Which graph goes with which rider?	Who rides faster?
Jasmine and Sothy start a bike trip at the same time. How far have they traveled after 24 minutes?	How long will it take each of them to reach the end of the 12-mile bike path?

Summary Question

How can you graph a proportional relationship from a story?

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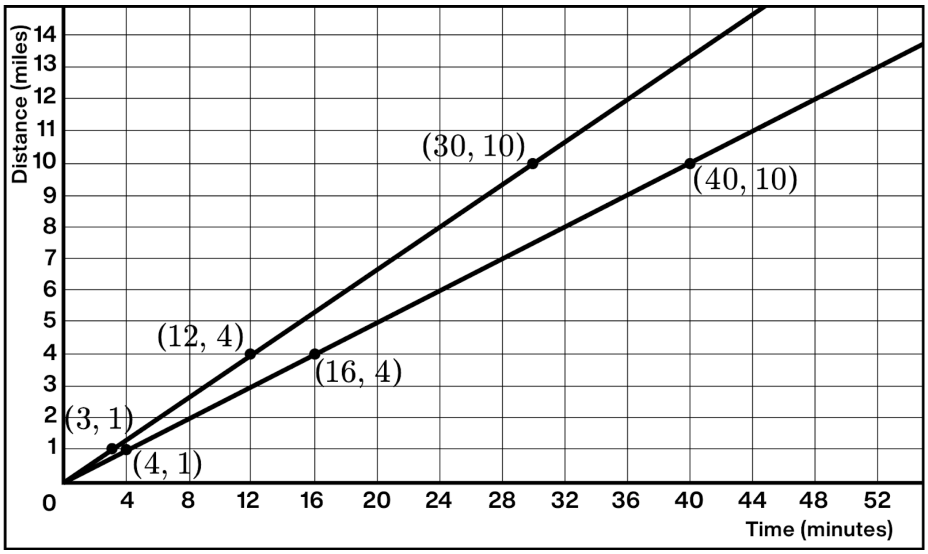
Unit 8.3, Lesson 1: Notes Name ________________________________

Learning Goal(s):

- **I can graph a proportional relationship from a story.**
- **I can use the constant of proportionality to compare the pace of different turtles.**

Here are the graphs showing Jasmine and Sothy's distance on a long bike ride. Jasmine rides 4 miles every 16 minutes, and Sothy rides 4 miles every 12 minutes.



Which graph goes with which rider?	Who rides faster?
The line that includes the points $(3,\ 1)$ **and** $(30,\ 10)$ **represents Sothy.** **The line that includes the points** $(4,\ 1)$ **and** $(40,\ 10)$ **represents Jasmine.**	**Sothy rides faster.**
Jasmine and Sothy start a bike trip at the same time. How far have they traveled after 24 minutes? **Jasmine traveled** 6 **miles and Sothy traveled** 8 **miles.**	How long will it take each of them to reach the end of the 12-mile bike path? **Jasmine will take** 48 **minutes and Sothy will take** 36 **minutes.**

Summary Question

How can you graph a proportional relationship from a story?

To graph a proportional relationship from a story, look for how one measurement is related to another measurement. Then you can graph that point and draw a line that includes both $(0,\ 0)$ **and the graphed point.**

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Unit 8.3, Lesson 2: Notes Name ____________________

Learning Goal(s):

Definition

Facts/Characteristics

Rate of Change

Examples

Non-Examples

Sketch the graph of the proportional relationship $y = 3x$ by scaling the axes three different ways.

Summary Question

How can you tell when two graphs have the same proportional relationship?

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Unit 8.3, Lesson 2: Notes Name ______________________________

Learning Goal(s):

- **I can graph a proportional relationship from an equation.**
- **I can tell when two graphs are of the same proportional relationship even if the scales are different.**

Rate of Change

Definition

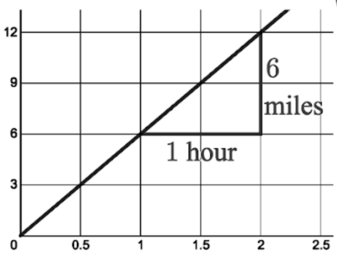
The amount y changes when x increases by 1.

Facts/Characteristics

The rate of change in a linear relationship is also the slope of its graph.

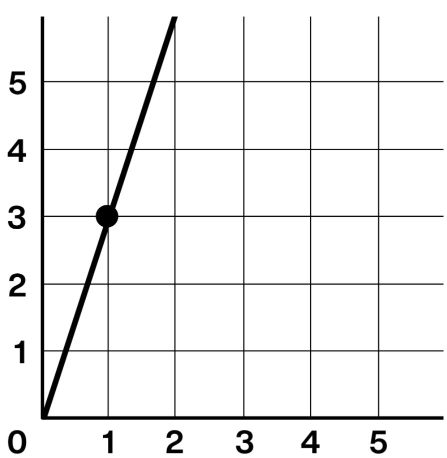
Examples

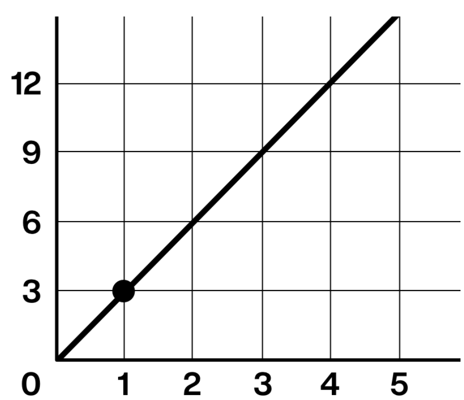
I bike 6 miles in 1 hour.

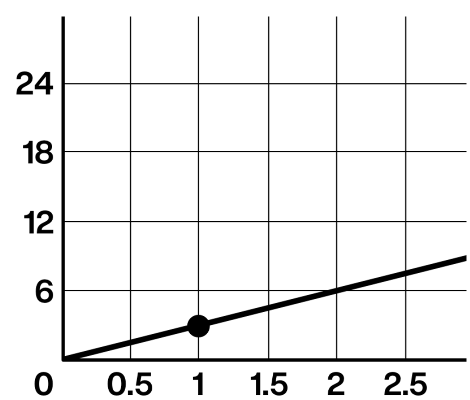


Non-Examples

Sketch the graph of the proportional relationship $y = 3x$ by scaling the axes three different ways.







Summary Question

How can you tell when two graphs have the same proportional relationship?

When the rate of change is the same for different graphs, then the graphs represent the same proportional relationship.

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Unit 8.3, Lesson 3: Notes Name ________________________________

Learning Goal(s):

When two proportional relationships are represented in different ways, we compare them by finding a common piece of information.

Terrance's earnings are represented by the equation $y = 14.5x$, where y is the amount of money he earns, in dollars, for working x hours.	The table shows some information about Jaylin's pay. (see table below)
How much does Terrance get paid per hour?	How much does Jaylin get paid per hour?
After 20 hours, how much more does the person who gets paid a higher rate have?	

Time Worked (hours)	**Earnings (dollars)**
7	92.75
4.5	59.63
37	490.25

Summary Question

How can you determine the rate of change of a proportional relationship from . . .

. . . a table? . . . a graph? . . . an equation?

Name ____________________________________

Learning Goal(s):
- **I can compare proportional relationships represented in different ways.**

When two proportional relationships are represented in different ways, we compare them by finding a common piece of information.

Terrance's earnings are represented by the equation $y = 14.5x$, where y is the amount of money he earns, in dollars, for working x hours.

The table shows some information about Jaylin's pay.

Time Worked (hours)	Earnings (dollars)
7	92.75
4.5	59.63
37	490.25

How much does Terrance get paid per hour?

$\$14.50$ **per hour**

How much does Jaylin get paid per hour?

$\$13.25$ **per hour**

After 20 hours, how much more does the person who gets paid a higher rate have?

Terrance is paid a higher rate than Jaylin. Terrance earns $\$1.25$ more per hour than Jaylin, which means that after 20 hours of work, Terrance has $\$25$ more than Jaylin.

Summary Question

How can you determine the rate of change of a proportional relationship from . . .

. . . a table? . . . a graph? . . . an equation?

From a table: Divide the y-value by the corresponding x-value. You should get the same value regardless of which pair of values is selected.

From a graph: Look at how much the y-value changes when the x-value increases by 1.

From an equation: The rate of change is the amount by which y changes when x increases by 1. For a proportional relationship, this can be found by setting $x = 1$ and solving for y.

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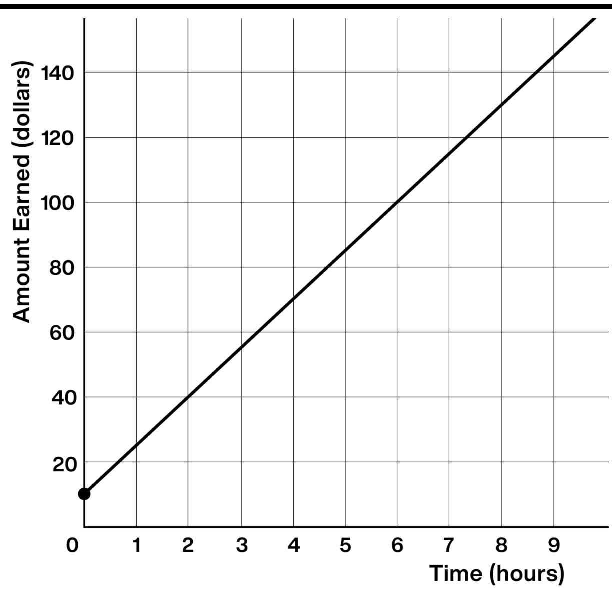
Unit 8.3, Lesson 4: Notes Name ______________________________

Learning Goal(s):

Definition	Facts/Characteristics
Linear Relationship	
Examples	Non-Examples

Aniyah starts babysitting. She charges $\$10$ for traveling to and from the job, and $\$15$ per hour. Here is a graph of Aniyah's earnings based on how long she works.



Show how to figure out the rate of change using the graph.

Summary Question

How can you find the rate of change of a linear relationship?

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Unit 8.3, Lesson 4: Notes Name ______________________________

Learning Goal(s):

- **I can find the rate of change of a linear relationship by figuring out the slope of the line representing the relationship.**

Linear Relationship

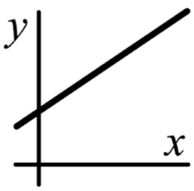
Definition

When one quantity changes by a certain amount, the other quantity always changes by a set amount.

Facts/Characteristics

In a linear relationship, one quantity has a constant rate of change with respect to the other. The relationship is called linear because its graph is a line.

Examples

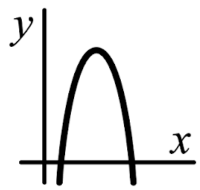


$y=2x+10$

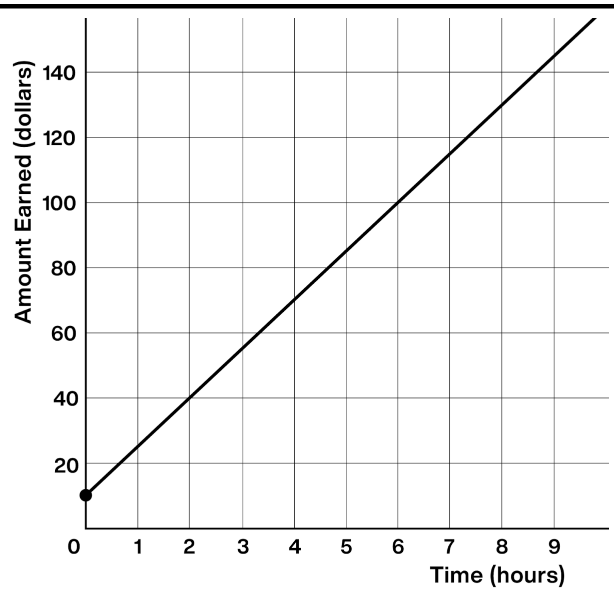
For each cup added, the total height increases 2 inches.

Non-Examples

$y=|x|+10$



Aniyah starts babysitting. She charges $\$10$ for traveling to and from the job, and $\$15$ per hour. Here is a graph of Aniyah's earnings based on how long she works.



Show how to figure out the rate of change using the graph.

Looking at the points $(2,\ 40)$ and $(6,\ 100)$, we can see that when Aniyah's hours increase by $6-2=4$, her earnings increased by $100-40=60$ dollars.

So the rate of change is $\frac{60}{4}$ ($\$15$ per hour).

Summary Question

How can you find the rate of change of a linear relationship?

The rate of change can be found by figuring out the slope of the line representing the relationship.

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Unit 8.3, Lesson 5: Notes Name ______________________________

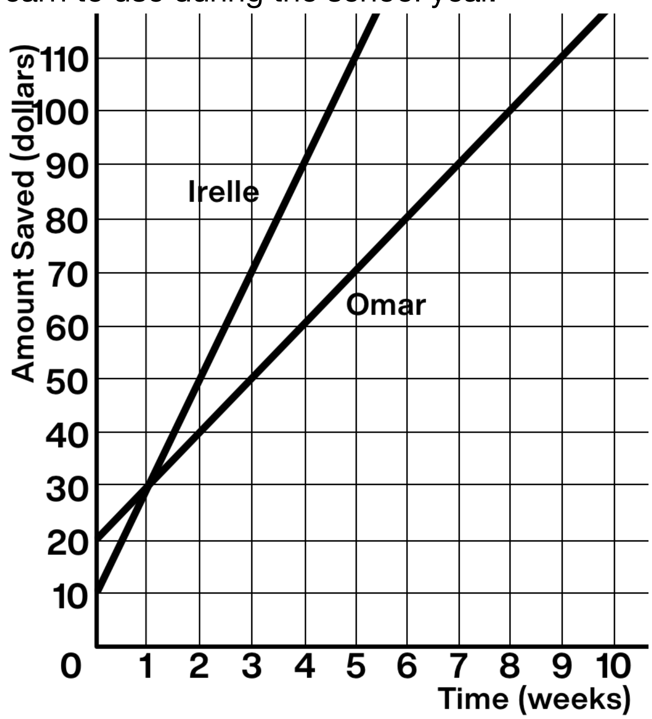
Learning Goal(s):

Definition	Facts/Characteristics
Vertical Intercept	
Examples	Non-Examples

Omar and Irelle decide to save some of the money they earn to use during the school year.

Here are graphs of how much money they will save after 10 weeks if they each follow their plans.

How much money does Omar have to start?	How much money does Irelle have to start?
How much money does Omar plan to save per week?	How much money does Irelle plan to save per week?



Summary Question

How can you find the vertical intercept and the slope from a graph?

Unit 8.3, Lesson 5: Notes Name ____________________________

Learning Goal(s):

- **I can interpret the vertical intercept of a graph of a real-world situation.**
- **I can match graphs to the real-world situations they represent by identifying the slope and the vertical intercept.**
- **I can use patterns to write a linear equation to represent a situation.**

Vertical Intercept

Definition

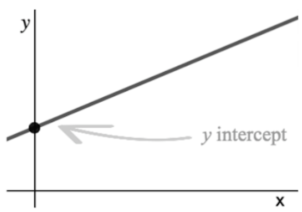
The vertical intercept is the point where the graph of a line crosses the vertical axis.

Facts/Characteristics

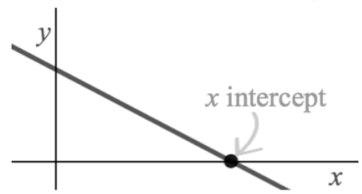
This is the y value when $x=0$.

Proportional relationships have a vertical intercept of 0.

Examples

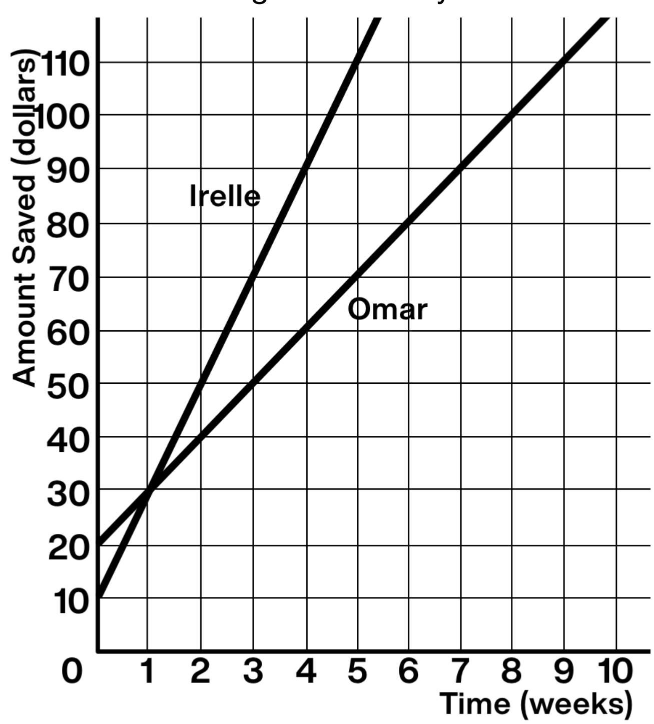


Non-Examples



Omar and Irelle decide to save some of the money they earn to use during the school year.

Here are graphs of how much money they will save after 10 weeks if they each follow their plans.



How much money does Omar have to start? $20	How much money does Irelle have to start? $10
How much money does Omar plan to save per week? $10 **per week**	How much money does Irelle plan to save per week? $20 **per week**

Summary Question

How can you find the vertical intercept and the slope from a graph?

To find the vertical intercept, look at the value where the graph intersects the y -axis.

To find the slope, look at how much y increases when x increases by 1 .

Unit 8.3, Lesson 6: Notes Name ____________________________________

Learning Goal(s):

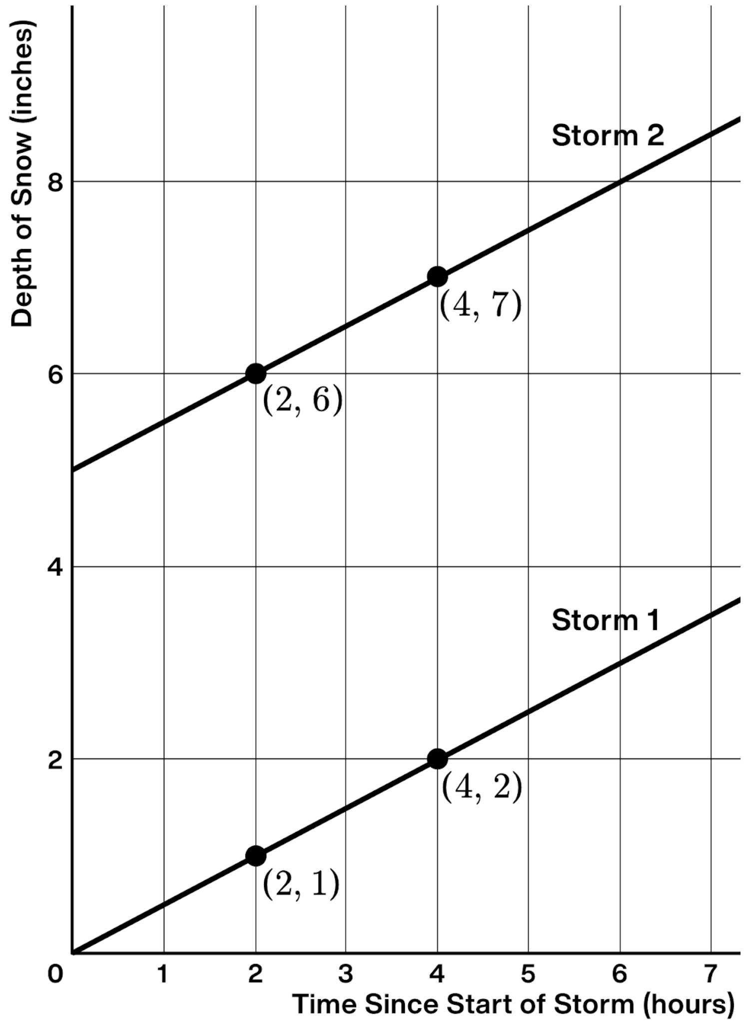
Snow fell at the same rate for two separate snow storms. During the storms, Raven measured the depth of snow on the ground for each hour.

The depth of snow on the ground for Storm 1 is a ____________________ relationship because there were 0 inches of snow on the ground at the start of the storm.

What is the equation representing Storm 1?

The depth of snow on the ground for Storm 2 is a ____________________ relationship because there were 5 inches of snow on the ground at the start of the storm.

What is the equation representing Storm 2?



Summary Question

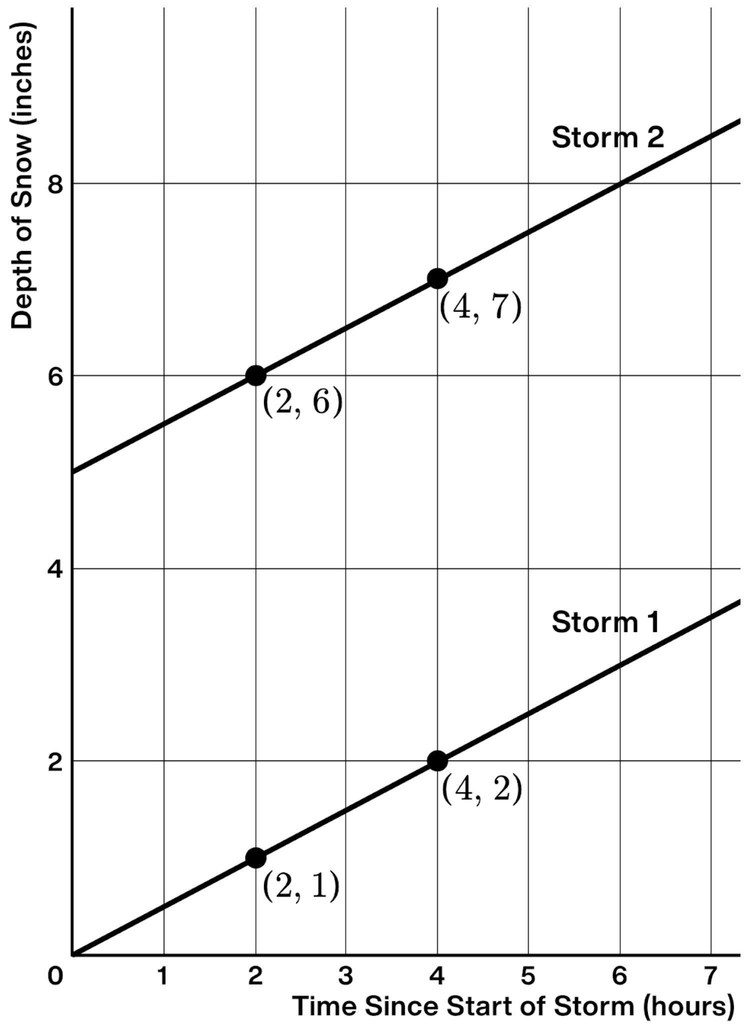
How do you use a graph to write the equation of a line using $y = mx + b$?

Unit 8.3, Lesson 6: Notes Name ____________________

Learning Goal(s):

- **I can explain where to find the slope and the vertical intercept in both an equation and its graph.**
- **I can write equations of lines using $y = mx + b$.**

Snow fell at the same rate for two separate snow storms. During the storms, Raven measured the depth of snow on the ground for each hour.



The depth of snow on the ground for Storm 1 is a **proportional** relationship because there were 0 inches of snow on the ground at the start of the storm.

What is the equation representing Storm 1?

$$y = \frac{1}{2}x$$

The depth of snow on the ground for Storm 2 is a **linear** relationship because there were 5 inches of snow on the ground at the start of the storm.

What is the equation representing Storm 2?

$$y = \frac{1}{2}x + 5$$

Summary Question

How do you use a graph to write the equation of a line using $y = mx + b$?

In the equation of a line $y = mx + b$, m represents the rate of change and the slope of the graph, and b is the vertical intercept of the line.

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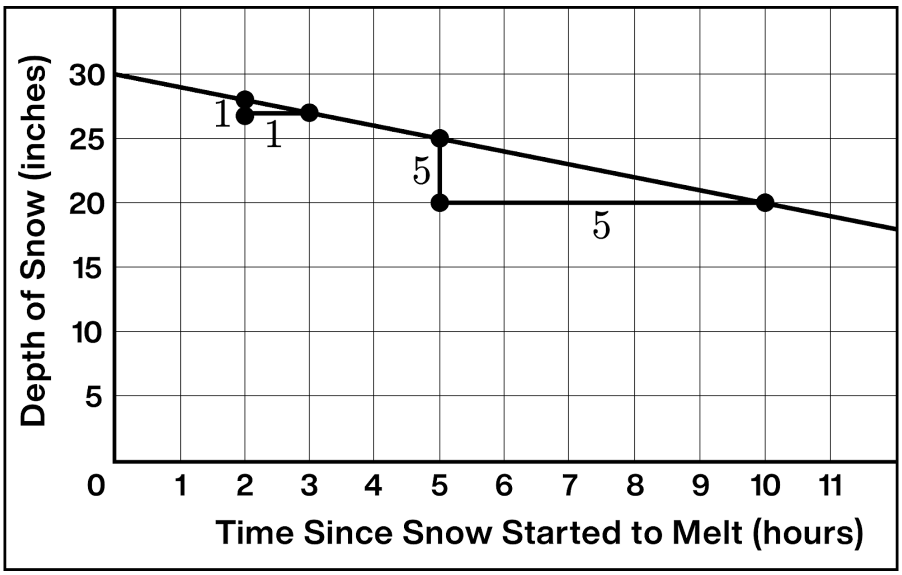
Unit 8.3, Lesson 7: Notes

Name ________________________________

Learning Goal(s):

The snow on the ground was 30 inches deep. On a warm day, the snow began to melt. The graph shows the relationship between the time since the snow started to melt and the depth of the snow.



The slope of this graph is ________ since the rate of change is ______ inches of snow per ________ .

This means that the depth of snow __________________________ at a rate of _____ inch per hour.

The vertical intercept is ______________.

This means that the snow was _____ inches deep when the time since snow started to melt was ______ hours.

Summary Task

Give an example of a different situation that would have a negative slope when graphed. Explain how you know the slope would be negative.

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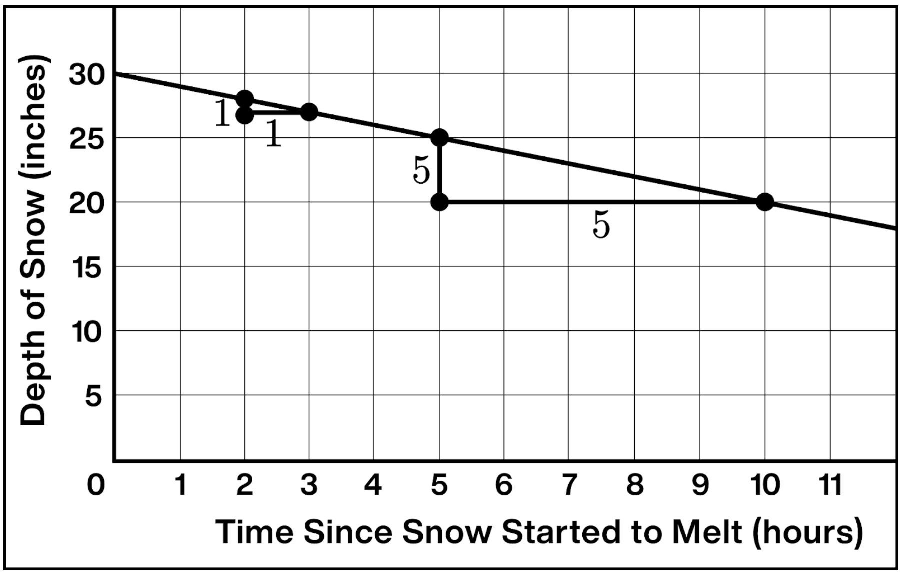
Unit 8.3, Lesson 7: Notes Name ____________________

Learning Goal(s):
- **I can give an example of a situation that would have a negative slope when graphed.**
- **I can look at a graph and tell if the slope is positive or negative and explain how I know.**

The snow on the ground was 30 inches deep. On a warm day, the snow began to melt. The graph shows the relationship between the time since the snow started to melt and the depth of the snow.



The slope of this graph is -1 since the rate of change is -1 inches of snow per **hour**.

This means that the depth of snow **decreased** at a rate of 1 inch per hour.

The vertical intercept is 30 **inches**.

This means that the snow was 30 inches deep when the time since snow started to melt was 0 hours.

Summary Task

Give an example of a different situation that would have a negative slope when graphed. Explain how you know the slope would be negative.

The amount of cereal in my bowl as I eat the cereal. This would have a negative slope because as time increases, the cereal in my bowl decreases.

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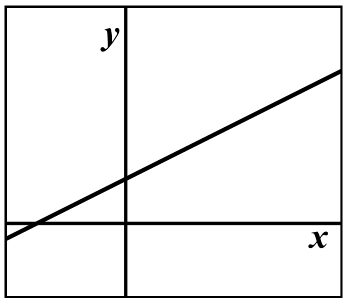
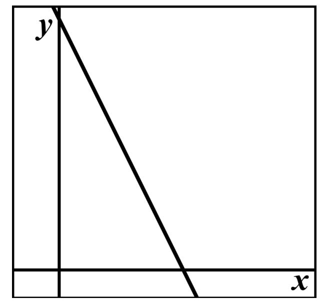
Unit 8.3, Lesson 8: Notes Name ____________________

Learning Goal(s):

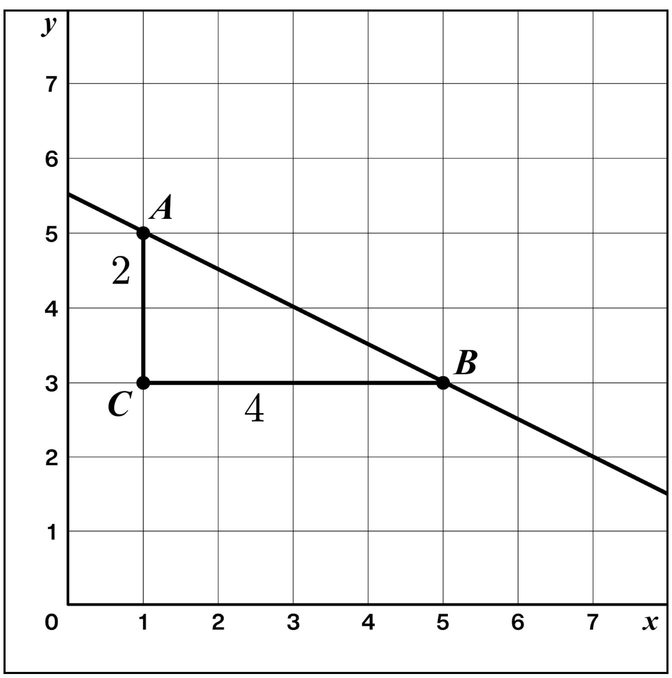
From left to right, if the graph increases, then the slope is ________________ .



From left to right, if the graph decreases, then the slope is ________________ .



Now we know two different ways to find the slope of a line:



1. We learned earlier that one way to find the slope of a line is by drawing a slope triangle.

Using the slope triangle shown here, the slope of the line is:

2. We can also compute the slope of this line using two points.

Using the points $A = (1,\ 5)$ and $B = (5,\ 3)$, the slope of the line is:

Summary Question

How can you calculate slope using two points on a line?

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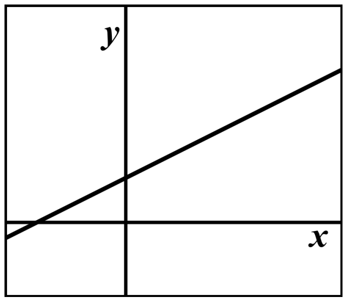
Unit 8.3, Lesson 8: Notes Name ______________________________

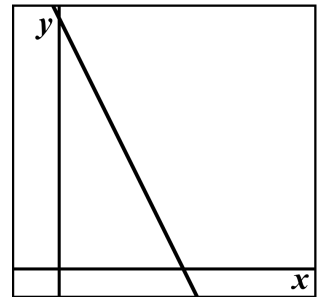
Learning Goal(s):

- **I can calculate positive and negative slopes given two points on the line.**
- **I can describe a line precisely enough that another student can draw it.**

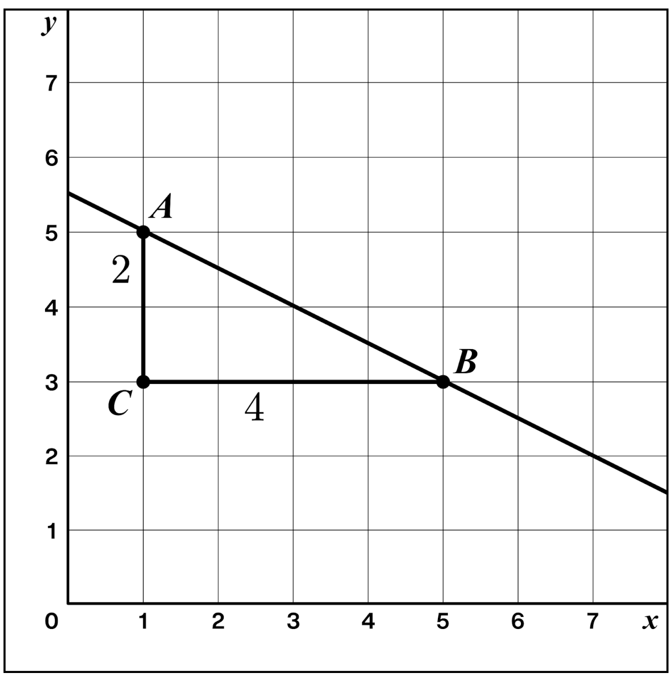
From left to right, if the graph increases, then the slope is **positive**.



From left to right, if the graph decreases, then the slope is **negative** .



Now we know two different ways to find the slope of a line:



1. We learned earlier that one way to find the slope of a line is by drawing a slope triangle.

Using the slope triangle shown here, the slope of the line is:

$$-\frac{2}{4} = -\frac{1}{2}$$

2. We can also compute the slope of this line using just two points.

Using the points $A = (1, 5)$ and $B = (5, 3)$, the slope of the line is:

$$\frac{3-5}{5-1} = \frac{-2}{4} = -\frac{1}{2} \text{ or } \frac{5-3}{1-5} = \frac{2}{-4} = -\frac{1}{2}$$

Summary Question

How can you calculate slope using two points on a line?

To calculate the slope of any two coordinates, find the vertical change by subtracting the y-coordinates and find the horizontal change by subtracting the x-coordinates (in the same order). Then divide the vertical change by the horizontal change.

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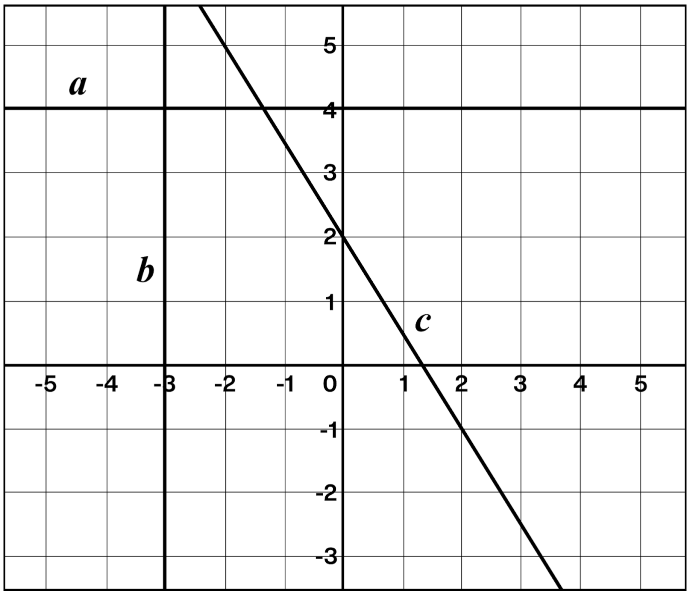
Unit 8.3, Lesson 9: Notes Name ____________________

Learning Goal(s):



Here are three lines on a coordinate grid.

Write an equation for each line.

Line	Equation
a	
b	
c	

	Description	Graph	Slope	Equation
Horizontal Lines				
Vertical Lines				

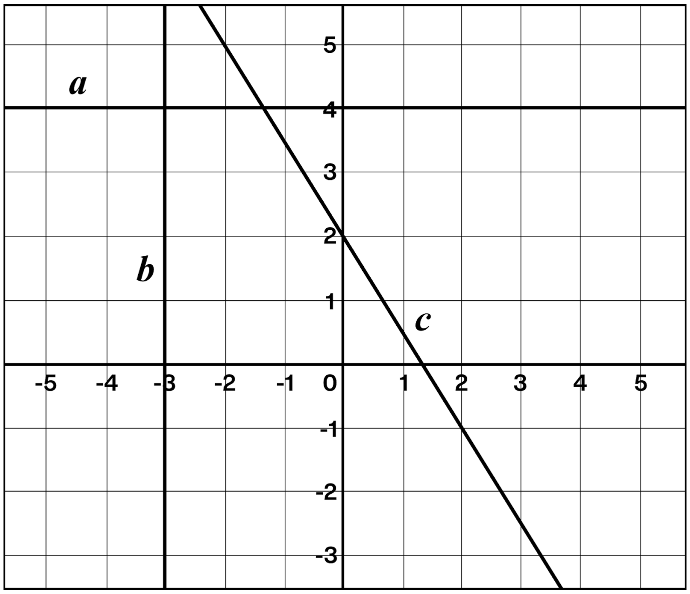
Summary Question

Write an example of an equation for a . . .

. . . horizontal line. . . . vertical line. . . . line with a negative slope.

Unit 8.3, Lesson 9: Notes Name ____________________________

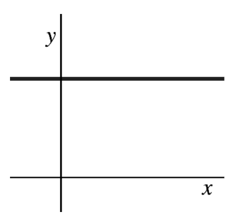
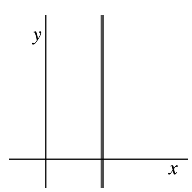
Learning Goal(s):
- **I can write equations of lines that have a positive or negative slope.**
- **I can write equations of vertical and horizontal lines.**



Here are three lines on a coordinate grid.

Write an equation for each line.

Line	Equation
a	$y = 4$
b	$x = -3$
c	$y = -\frac{3}{2}x + 2$

	Description	Graph	Slope	Equation Example
Horizontal Lines	**Lines where the y-value does not change, while the x-value changes.**		**Since the y-value does not change, the slope is 0.**	$y = 10$
Vertical Lines	**Lines where the x-value does not change, while the y-value changes.**		**Since the x-value does not change, the slope is undefined.**	$x = 2$

Summary Question

Write an example of an equation for a . . .

. . . horizontal line.

Responses vary.
$y = 3$

. . . vertical line.

Responses vary.
$x = -8$

. . . line with a negative slope.

Responses vary.
$y = -2x + 6$