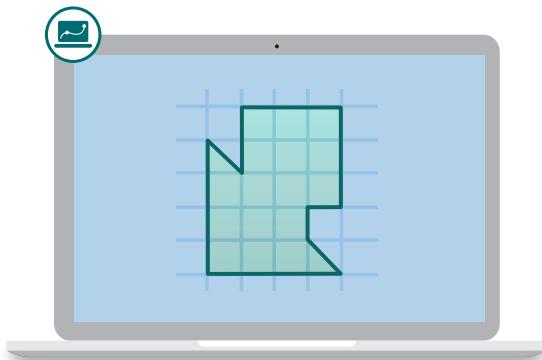


Name: Date: Period:

Shapes on a Plane

Let's play with shapes and find their areas.

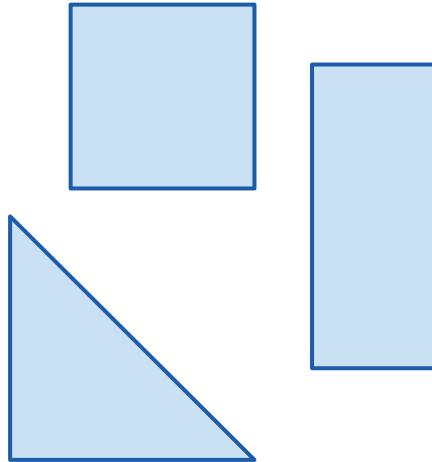


Warm-Up

- 1** Trevon made a shape with these pieces.

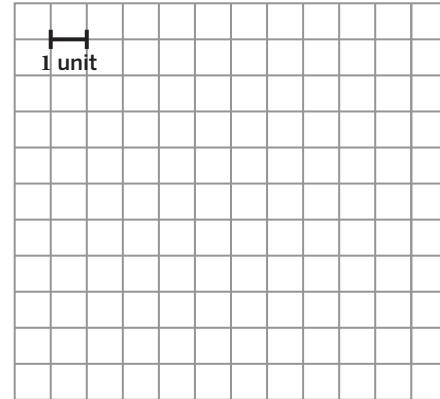
a Let's watch the shape come together.

b Describe what the shape reminds you of.



Areas of Non-Rectangular Shapes

- 2** **a** You will use a set of shape cutouts to make your own fun shape.



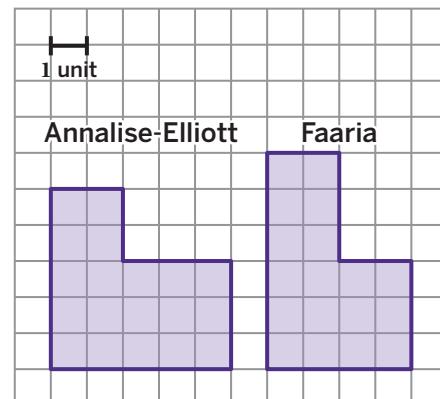
- b** Describe your shape and what it reminds you of.

- 3** Annalise-Elliott and Faaria both made shapes that look like boots.

Whose boot shape is larger? Circle one.

Annalise-Elliott Faaria I'm not sure

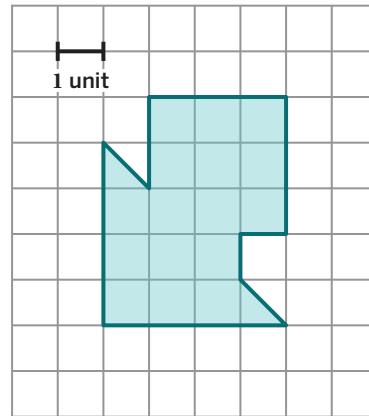
Explain your thinking.



Areas of Non-Rectangular Shapes (continued)

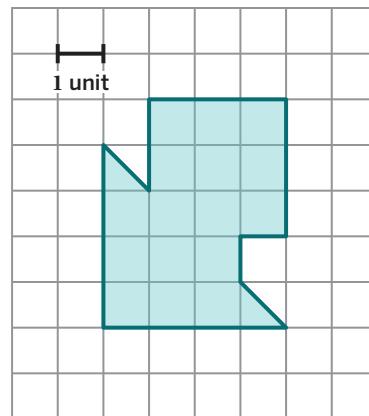
- 4** The area of a shape is one way to describe its size.

Determine the area of Trevon's shape.



- 5** There is often more than one way to determine area.

- a** Draw another way to determine the area of Trevon's shape.
- b** Describe your strategy.



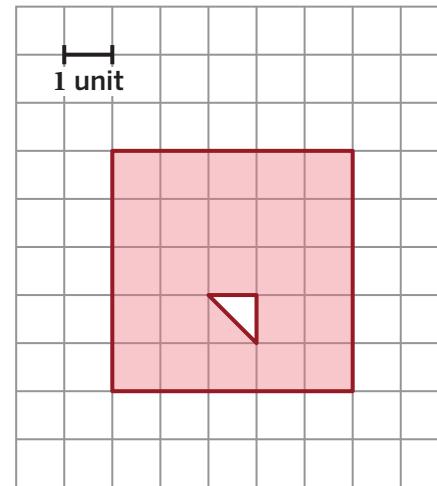
Area Challenges

6**a**

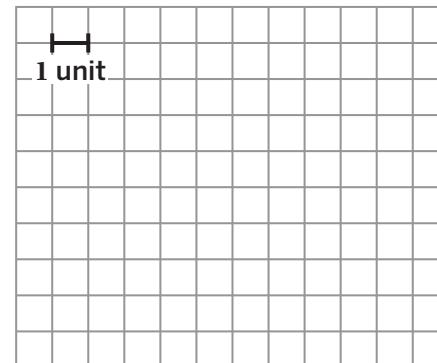
Discuss: What does Nur's shape remind you of?

b

Determine the area of Nur's shape. Draw on the shape if it helps with your thinking.

**7**

How many different areas can you make with your shape cutouts? Use the grid if it helps with your thinking.

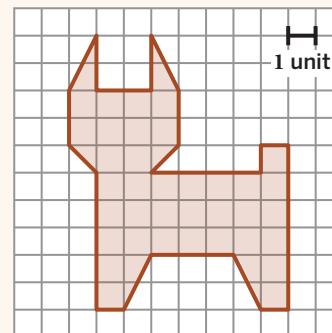


Explore More

8

Determine the area of the dog.

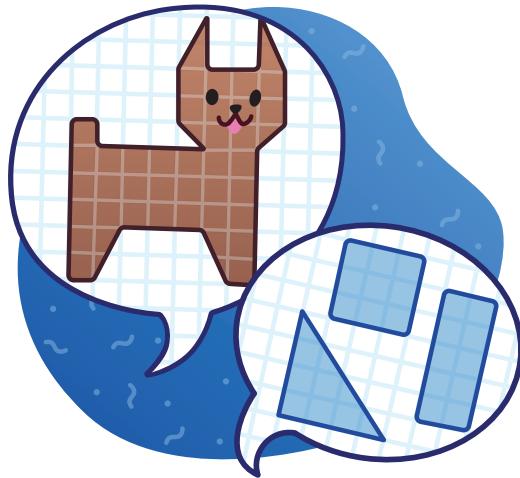
Explain your thinking.



9 Synthesis

Discuss both questions, then select one and write your response.

- What's something you learned today?
- What do you want to learn more about?



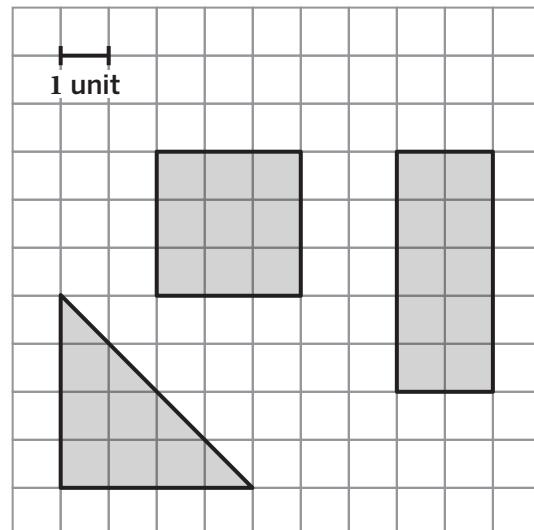
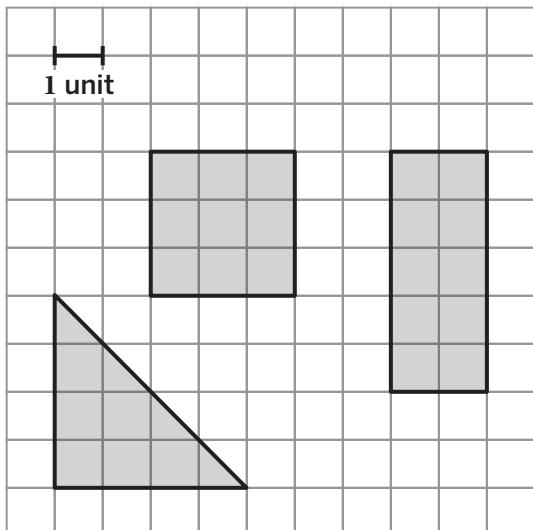
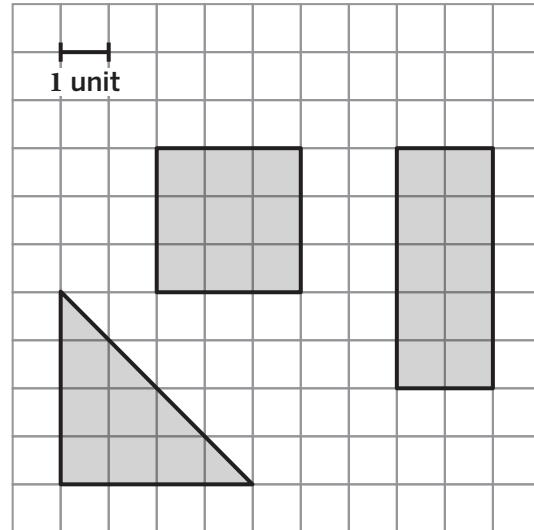
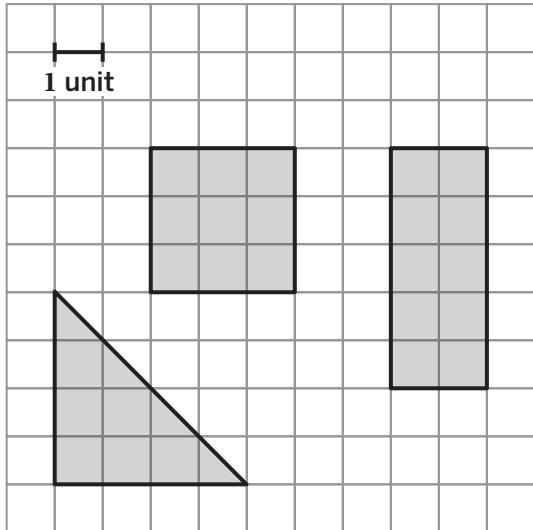
Things to Remember:

Areas of Non-Rectangular Shapes

 **Directions:** Make one copy per four students. Then pre-cut the cards and give each student one set of shapes.

Have students cut out the shapes.

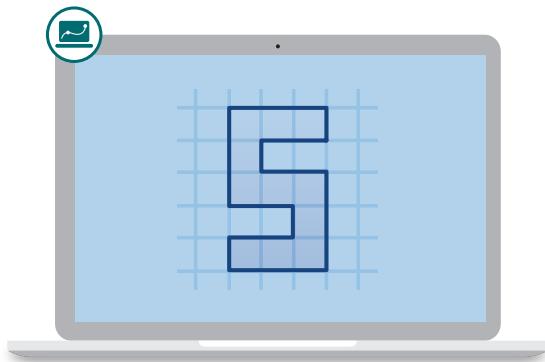
© Amplify Education, Inc. and its licensors. Amplify Desmos Math is based on curricula from Illustrative Mathematics (IM) and Open Up Resources.



Name: Date: Period:

Letters

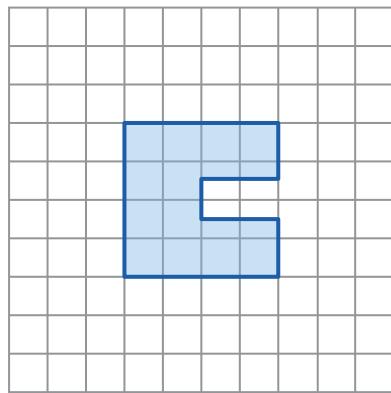
Let's explore the area of shapes.



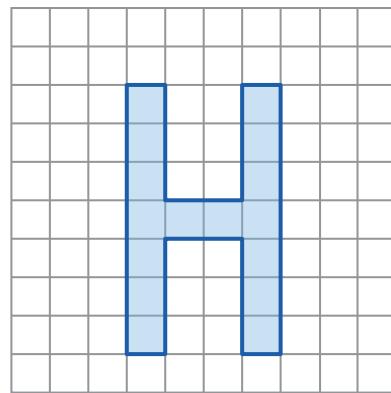
Warm-Up

- 1** Which figure doesn't belong? Explain your thinking.

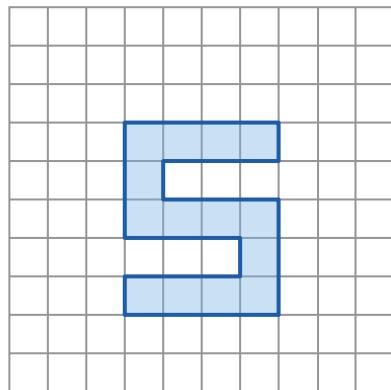
A.



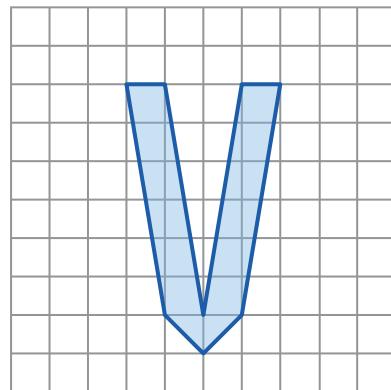
B.



C.

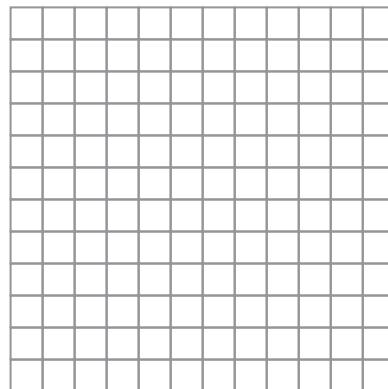


D.



Rearranging Shapes

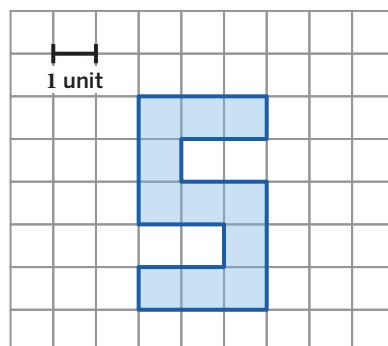
- 2** **a** Draw the first letter of your name on the grid.



- b** Tell a story about your name.

- 3** Saanvi sketched an “S” and colored it in.

What is the area of the shape Saanvi colored?

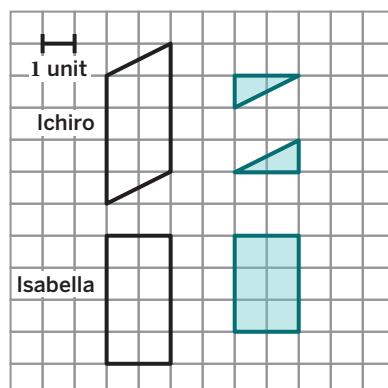


- 4** Ichiro and Isabella each drew an “I.” Ichiro cut his “I” into pieces to see how much area to color.

Whose letter has a greater area? Circle one.

Ichiro Isabella They are the same

Explain your thinking.

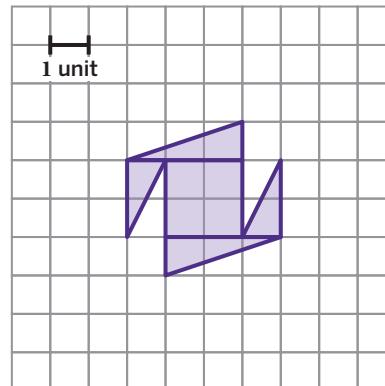


Rearranging Shapes (continued)

- 5** Zahra also cut up her “Z” to see how much area it covered.

What is the area of the shape she colored?

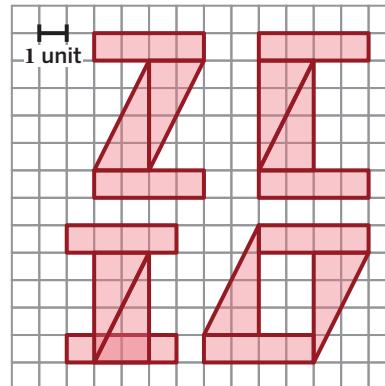
Use arrows to show how you could rearrange the pieces, if it helps with your thinking.



- 6** Zola cut a “Z” into pieces and rearranged it to make new letters.

Select *all* the new letters that have the same area as the “Z.”

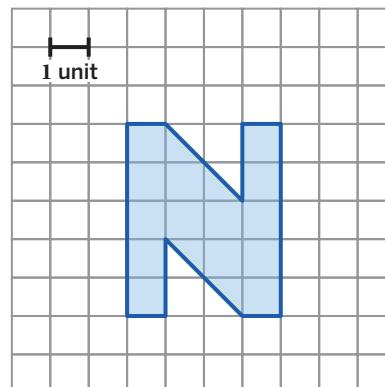
C I O



- 7** Nathan made an “N.”

- a** What is the area of the shape Nathan colored?
Sketch on the grid if it helps to show your thinking.

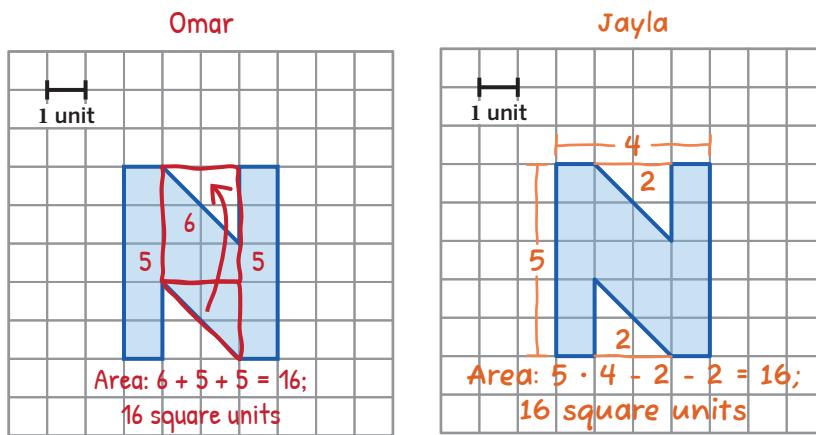
- b** **Discuss:** What strategy did you use?



Area Strategies

- 8** Omar and Jayla used different strategies to determine the area of “N.”

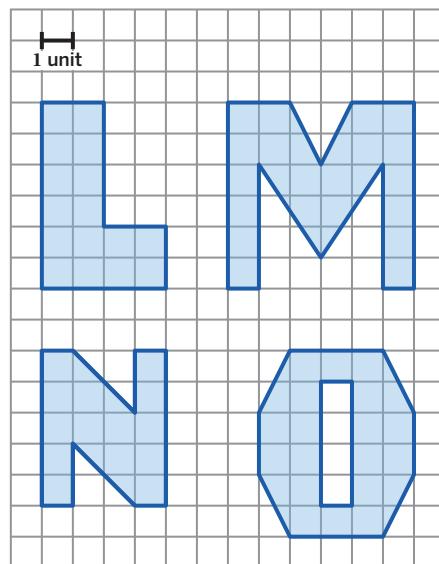
- a** Take a look at each student’s work.



- b** Pick one student and explain how you think they determined the area.

- 9** Complete the table.

Letter	Area (sq. units)
L	
M	
N	16
O	

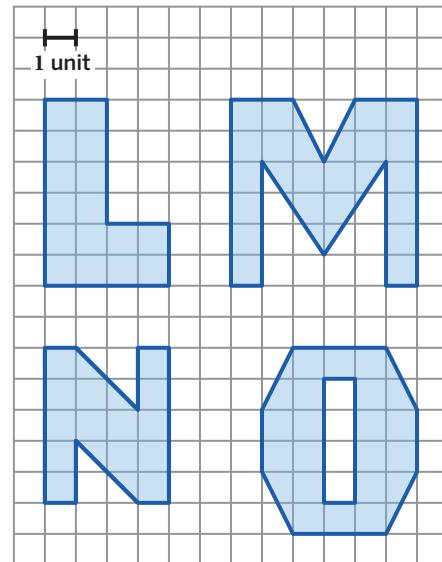


10 Synthesis

- a** Which area calculation are you most proud of?
Circle one.

L M N O

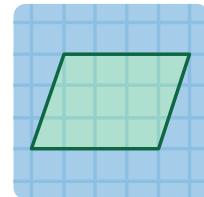
- b** Write some advice for someone determining
this area.



Things to Remember:

Name: Date: Period:

Exploring Parallelograms, Part 1

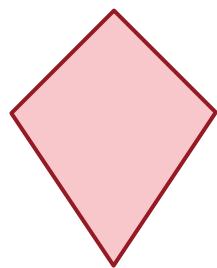


Let's investigate features of parallelograms.

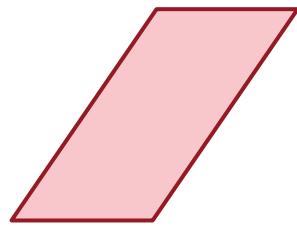
Warm-up

1. Which one doesn't belong? Explain your thinking.

A.



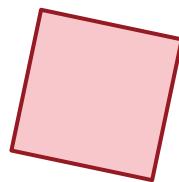
B.



C.



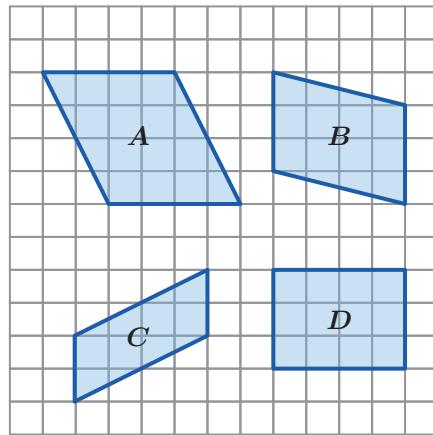
D.



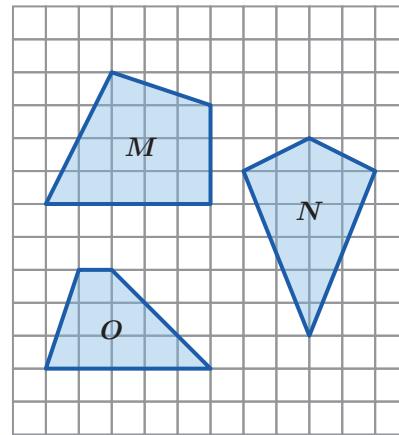
Parallelograms

2. Figures *A*, *B*, *C*, and *D* are *parallelograms*. Figures *M*, *N*, and *O* are *quadrilaterals* that are *not* parallelograms. What do you notice? What do you wonder?

Parallelograms



Not Parallelograms



I notice:

I wonder:

3. What do you think makes a shape a parallelogram?

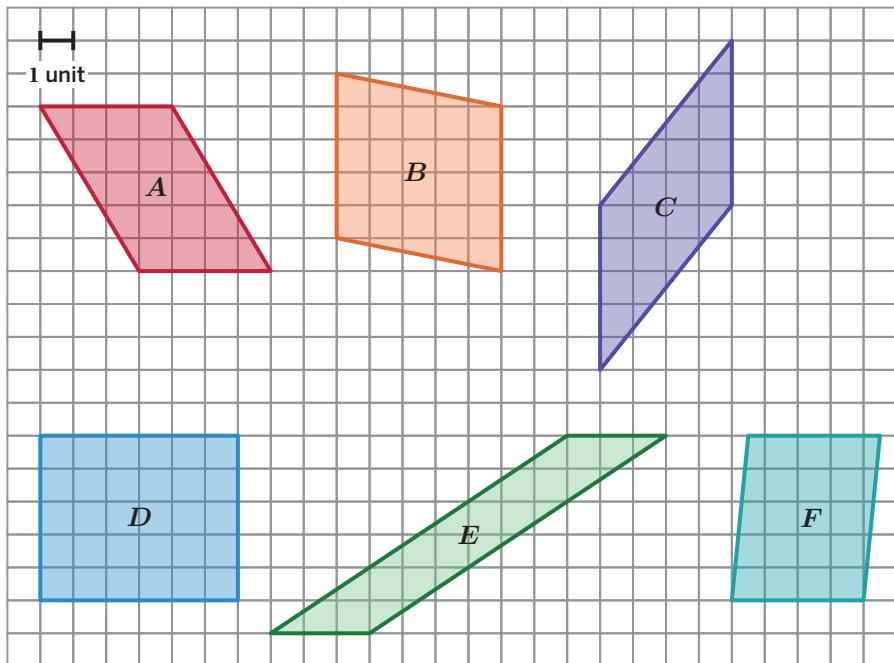
- a Write a first draft of your definition.

- b Meet with a partner to discuss your first drafts. Use the questions on the screen to help you provide feedback to each other.

- c Write a second draft that is stronger and clearer.

Area Strategies

4. Use any strategy to determine the area of these parallelograms.



Parallelogram	A	B	C	D	E	F
Area (sq. units)						

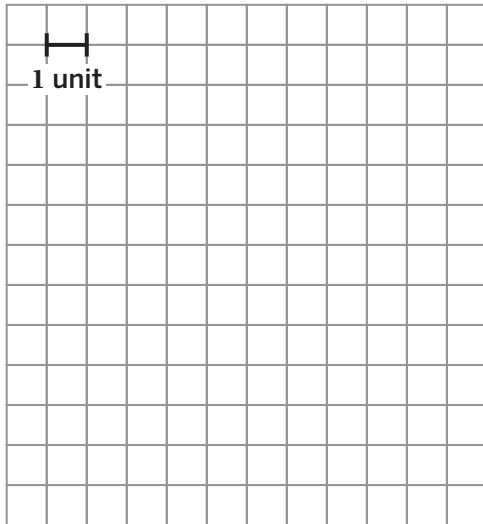
5. Describe your strategy for determining the area of parallelogram C.

6. What other parallelograms would your strategy work for? Explain your thinking.

Area Strategies (continued)

7. Show or describe a classmate's strategy that was different from your own.

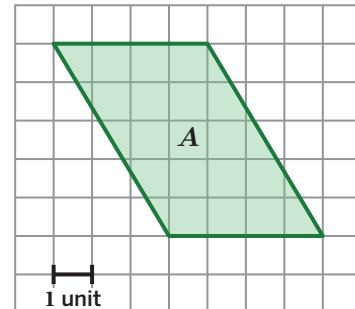
8. Draw a parallelogram with an area of 36 square units that is *not* a rectangle.



9. Explain how you know your parallelogram has an area of 36 square units.

Synthesis

10. Show or describe a strategy for calculating the area of a parallelogram. Use parallelogram A if it helps with your thinking.

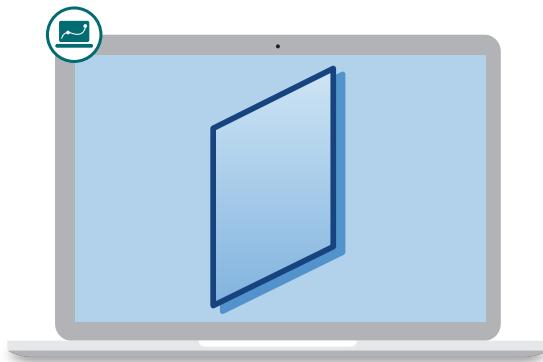


Things to Remember:

Name: Date: Period:

Off the Grid, Part 1

Let's practice determining the area of parallelograms.



Warm-Up

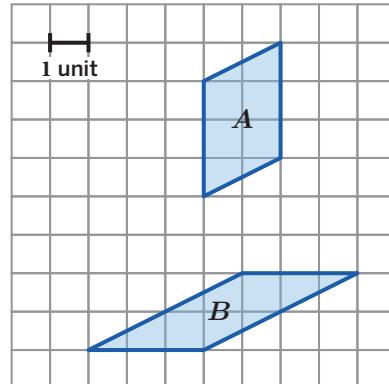
- 1** Which parallelogram has a greater area? Circle one.

A

B

They have the same area

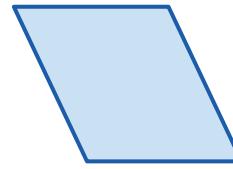
Show or explain your thinking.



Measuring to Determine Area

- 2** In this lesson, you'll measure different parts of parallelograms.

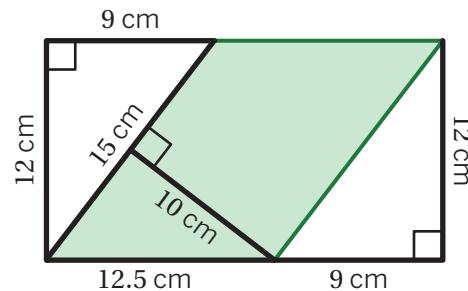
a Let's watch how the measuring tool works.



b Label as many measurements on this parallelogram as you want.

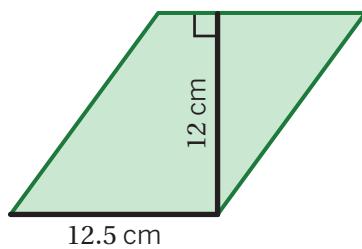
- 3** What is the area of this parallelogram?

Use as many measurements as you need to calculate the area.

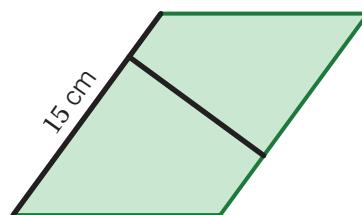


- 4** Here are some measurements that Angel and Ebony took. Sketch a line that Ebony can measure next to help calculate the area.

Angel



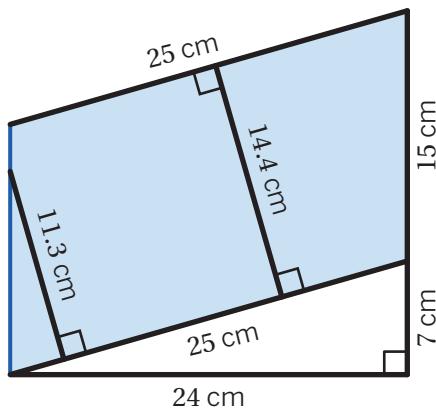
Ebony



Measuring to Determine Area (continued)

- 5** What is the area of this parallelogram?

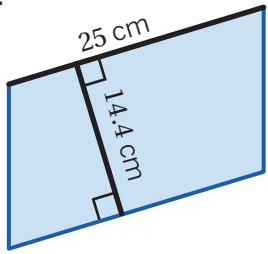
Use as many measurements as you need to help with your thinking.



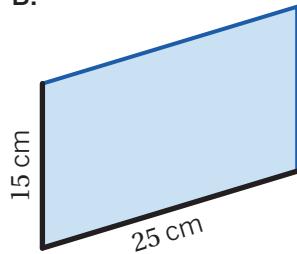
- 6** Here are some measurements taken by four different students.

Select *all* the parallelograms with measurements that can be used as a base and height pair.

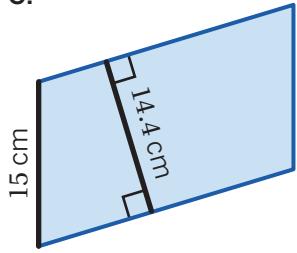
A.



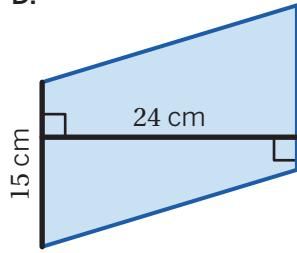
B.



C.



D.



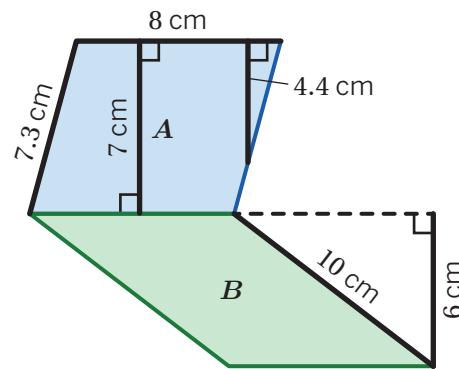
More Parallelograms

- 7** Which parallelogram has a greater area?

Use as few measurements as you can to help you decide.

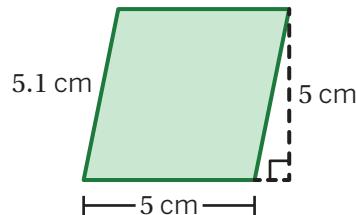
A B They have the same area

Show or explain your thinking.

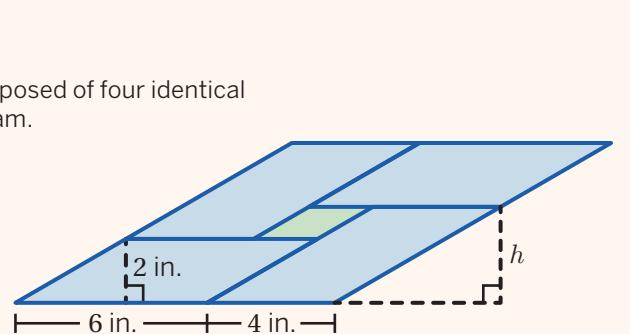


- 8** Draw a new parallelogram with the same base and a different height, so the area measures 40 square centimeters.

Original Parallelogram



New Parallelogram



Explore More

- 9** The shaded region in this diagram is composed of four identical parallelograms and a smaller parallelogram.

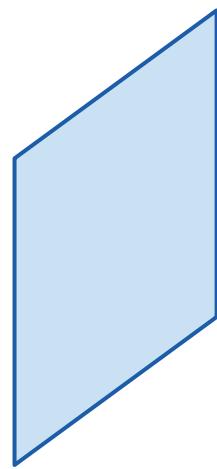
a What is the value of h ?

b What is the total shaded area?

10 Synthesis

Describe how you can determine the area of any parallelogram.

Draw on this image if it helps to show your thinking.

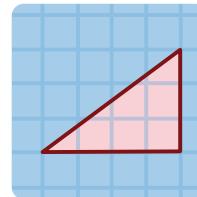


Things to Remember:

Name: Date: Period:

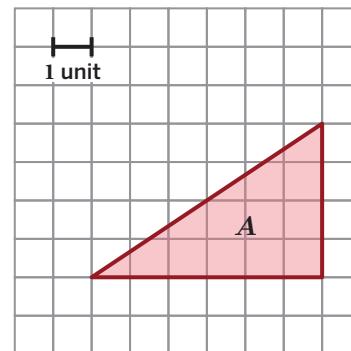
Exploring Triangles

Let's explore the area of triangles.



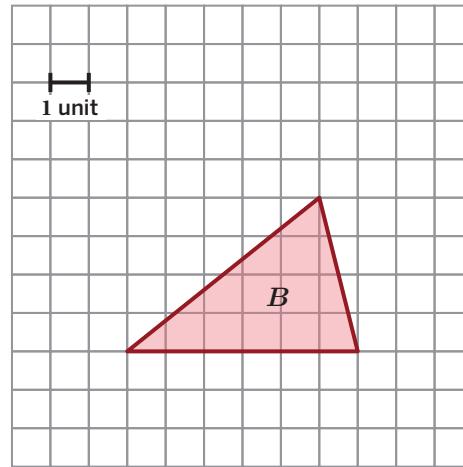
Warm-Up

1. Determine the area of triangle A. Show or describe your thinking.

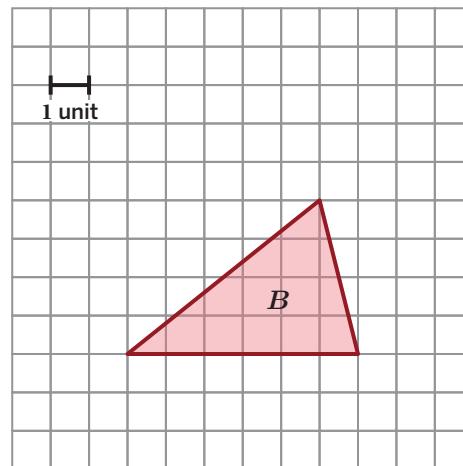


Area Strategies

- 2.** Determine the area of triangle *B*. Show or describe your thinking.



- 3.** Find a classmate who calculated the area of triangle *B* using a different strategy. Show or describe how your partner calculated the area.



- 4.** Let's look at two strategies for calculating the area of triangle *B*.



Discuss: How are these two strategies alike? How are they different?

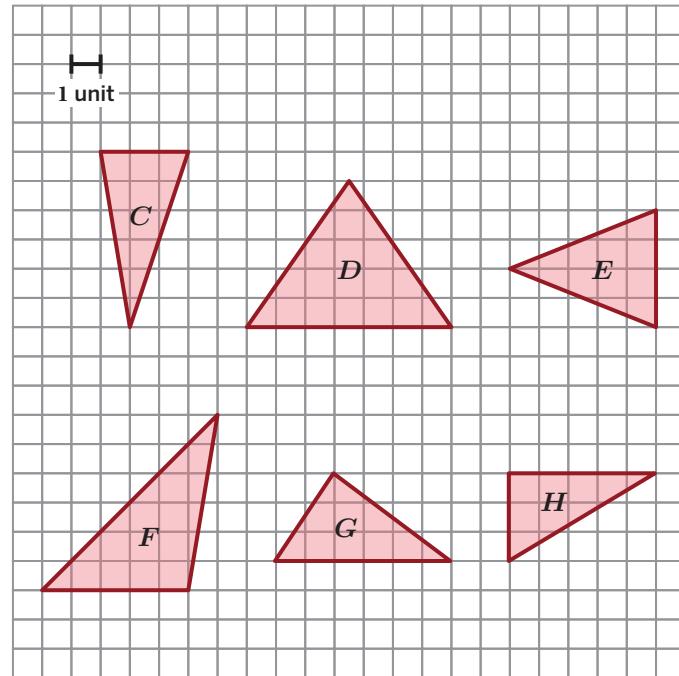
Activity**2**

Name: _____ Date: _____ Period: _____

Lots of Triangles

5. Determine the area of as many of these triangles as you can.

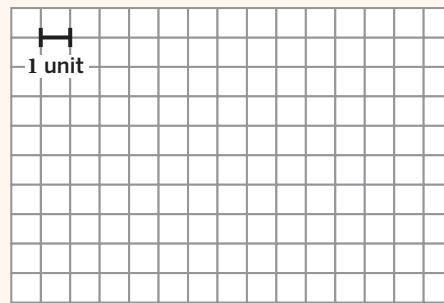
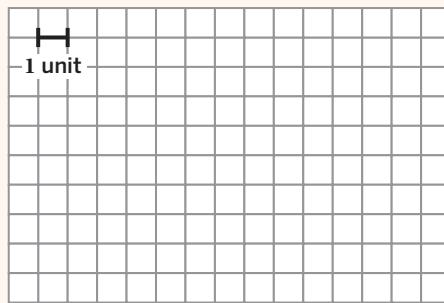
Triangle	Area (sq. units)
C	
D	
E	
F	
G	
H	



6. Describe the strategy that was most helpful to you. Did this strategy work for *all* the triangles?

Explore More

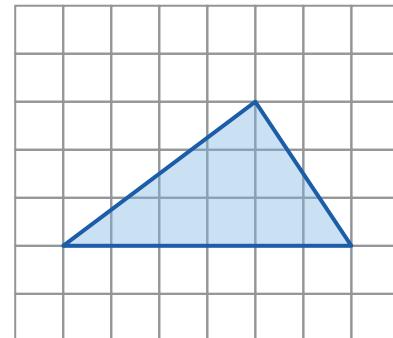
7. Draw two different triangles that both have an area of 18 square units.



Synthesis

8. Describe a strategy to determine the area of a triangle.

Use the example if it helps with your thinking.

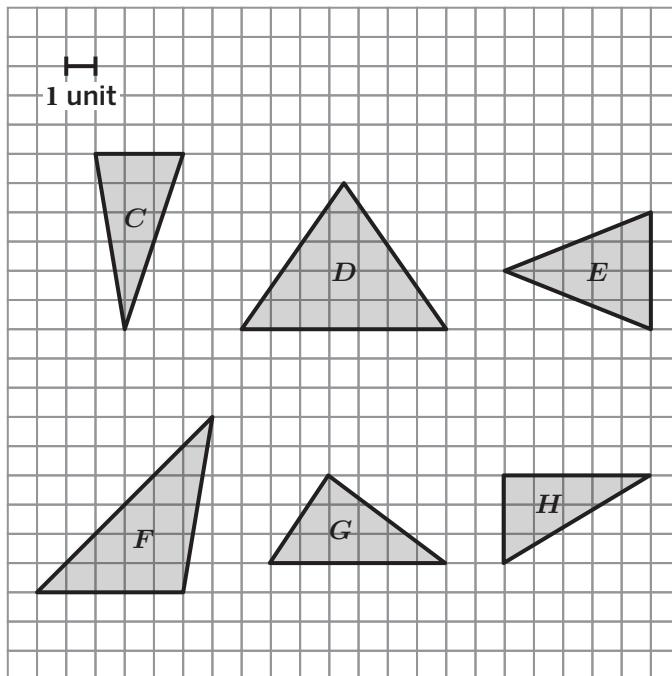
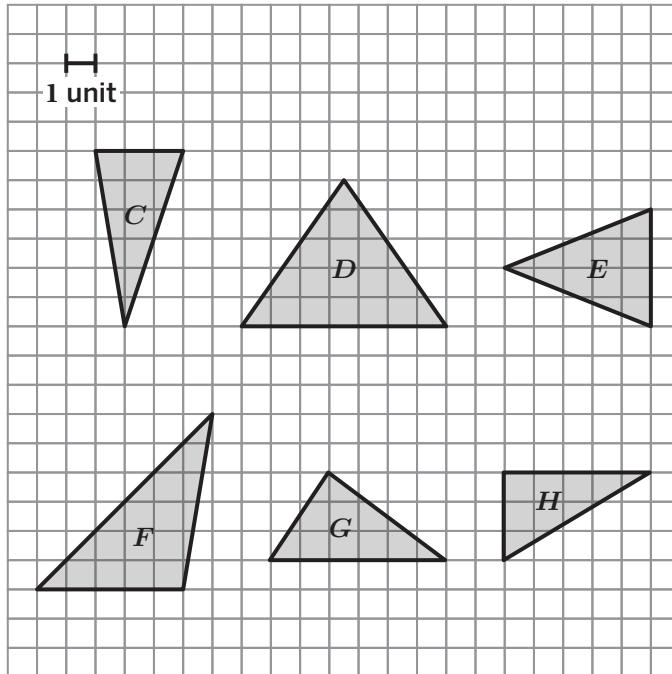


Things to Remember:

Lots of Triangles

Cut out the bottom set of triangles and use them to form parallelograms with the top set of triangles. As you create parallelograms, consider discussing these questions with your classmates:

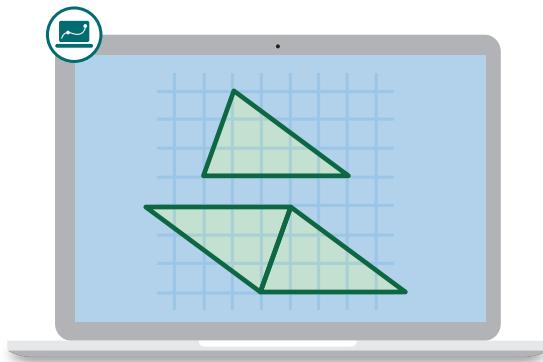
- Which triangle did you start with?
- Where did you get stuck?
- What did you try?
- Which strategies were most helpful to you?



Name: Date: Period:

Triangles and Parallelograms

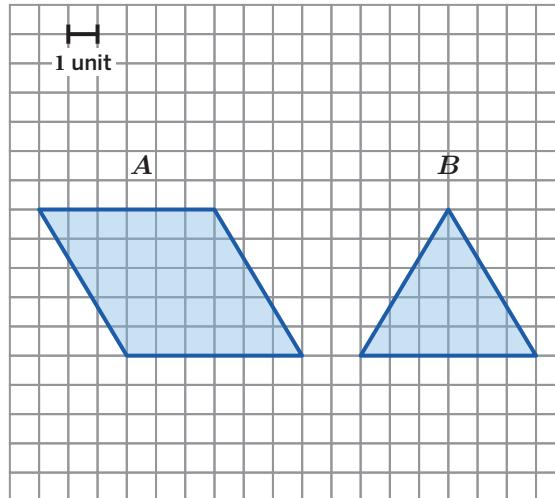
Let's explore the relationship between triangles and parallelograms.



Warm-Up

- 1** List two things that are the same about these figures.

List two things that are different.

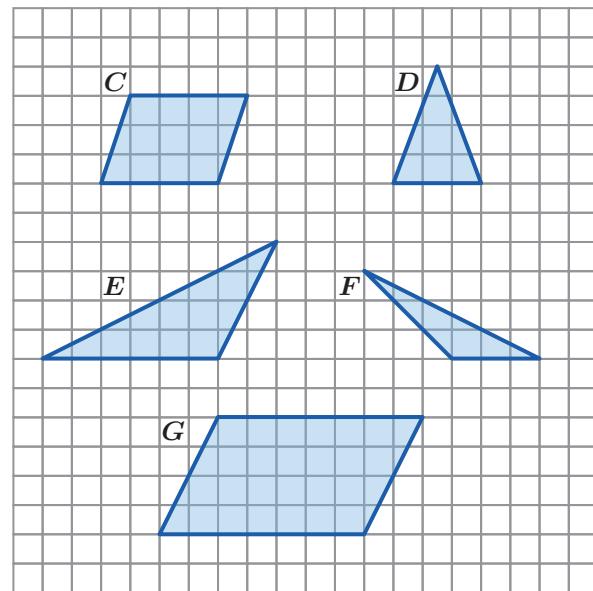


Triangles and Parallelograms

- 2** You can determine the heights of triangles, just like you can with parallelograms.

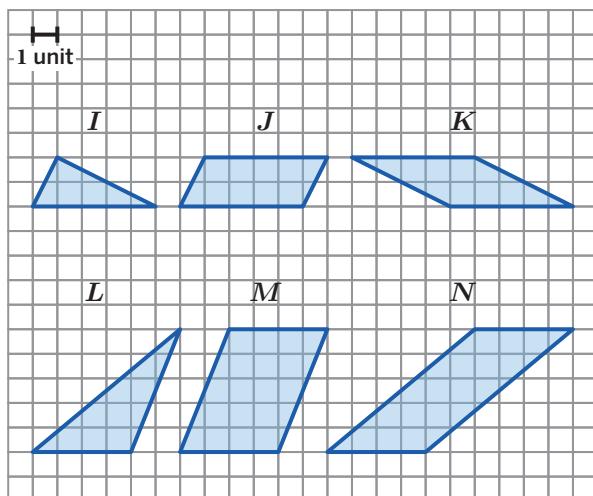
Select a set of shapes that have the same height.

- C
- D
- E
- F
- G



- 3** **a** Determine the base, height, and area of each shape.

Shape	Base (units)	Height (units)	Area (sq. units)
I			
J			
K			
L			
M			
N			

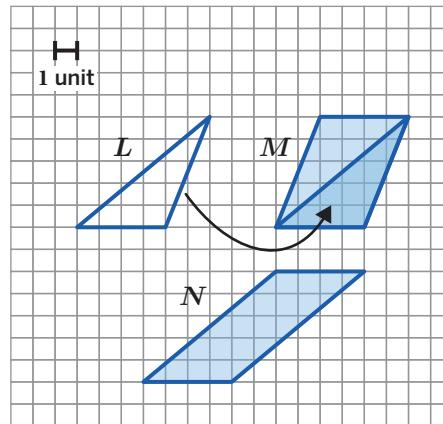
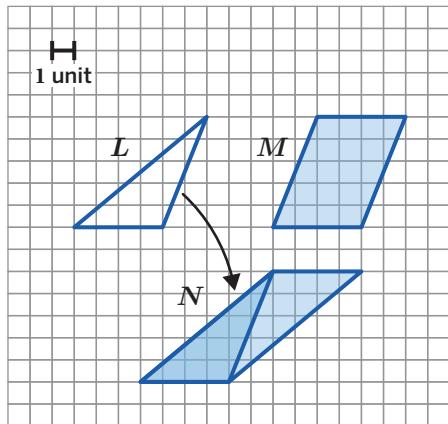


- b**  **Discuss:** What patterns do you notice?

Triangles and Parallelograms (continued)

4 Here is a triangle and two parallelograms from the previous problem.

- a** Take a look at how the shapes compare.



- b** What is the relationship between the areas of these three shapes?

Generalizing Triangle Area

- 5** Let's see if we can always combine two copies of a triangle to form a parallelogram.

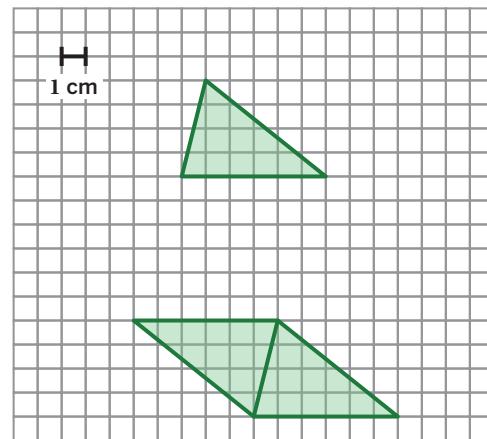
a Let's create a triangle.

b  **Discuss:** How many different parallelograms can you create using two copies of your triangle?

- 6** Here is a triangle and a parallelogram.

a What is the area of the triangle?

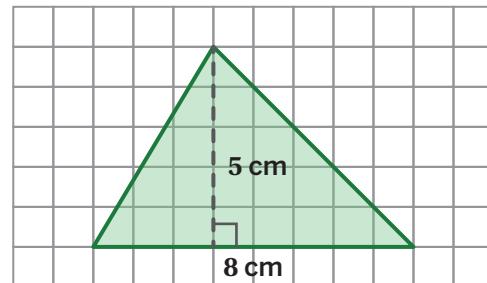
b What is the area of the parallelogram?



- 7** Here's the expression that Alisha entered to find the area of this triangle.

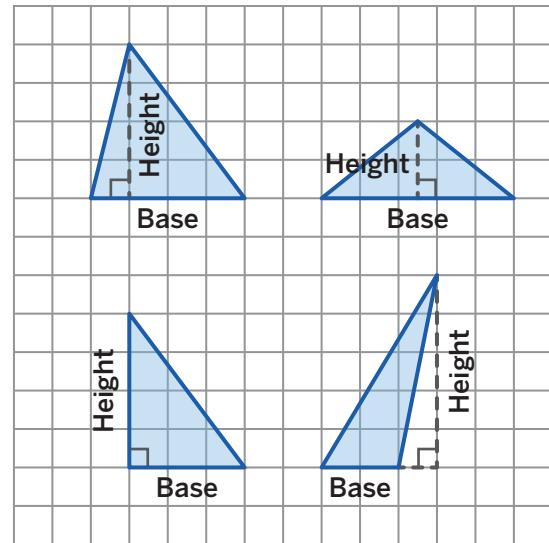
$$8 \cdot 5 \cdot \frac{1}{2}$$

Explain what each number represents in the expression.



8 Synthesis

How can you use the base and height of any triangle to calculate its area?

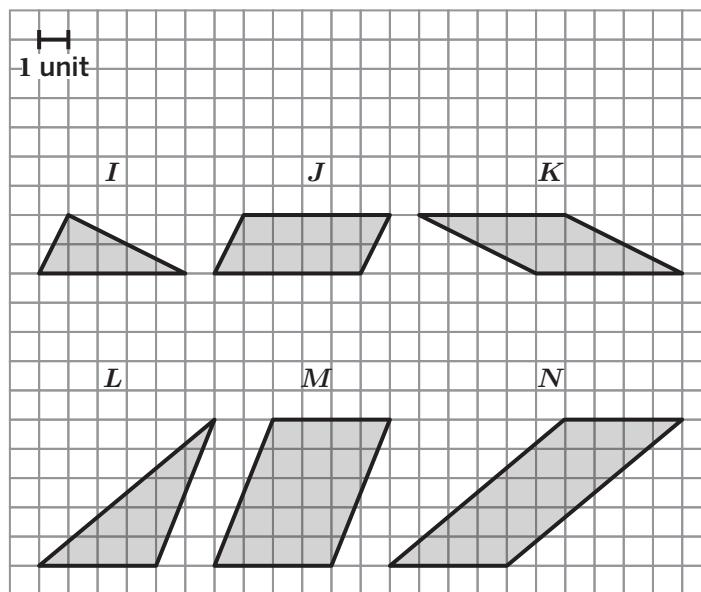
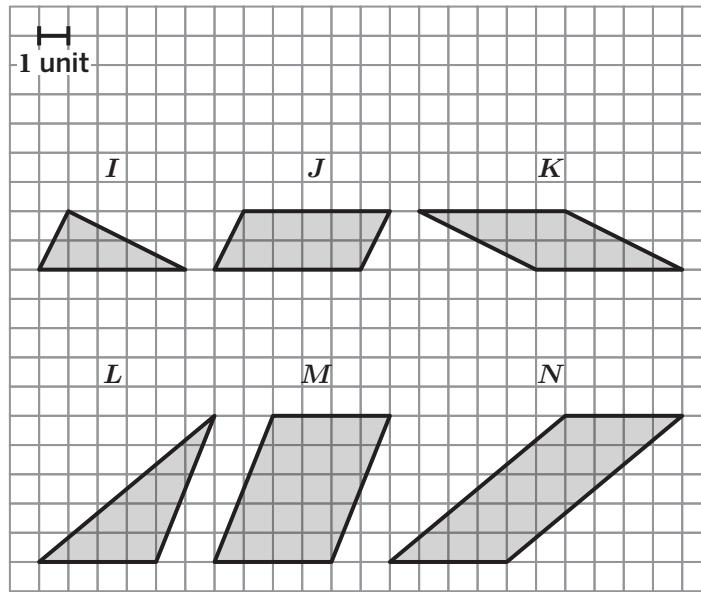


Things to Remember:

Triangles and Parallelograms

 **Directions:** Make one copy per pair of students. Then pre-cut the cards and give each student one set of shapes.

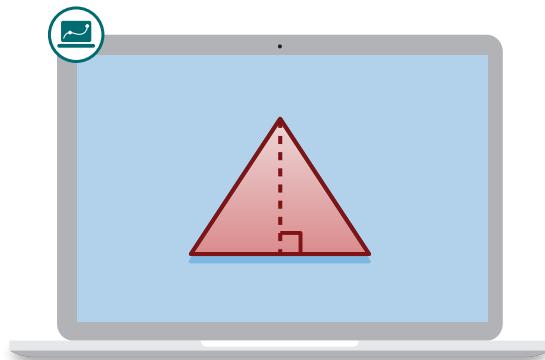
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Name: Date: Period:

Off the Grid, Part 2

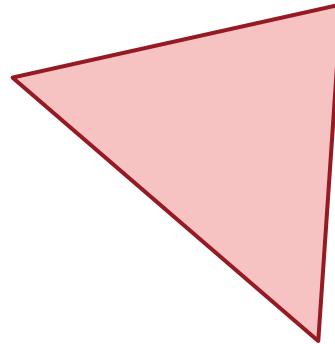
Let's practice calculating the area of triangles.



Warm-Up

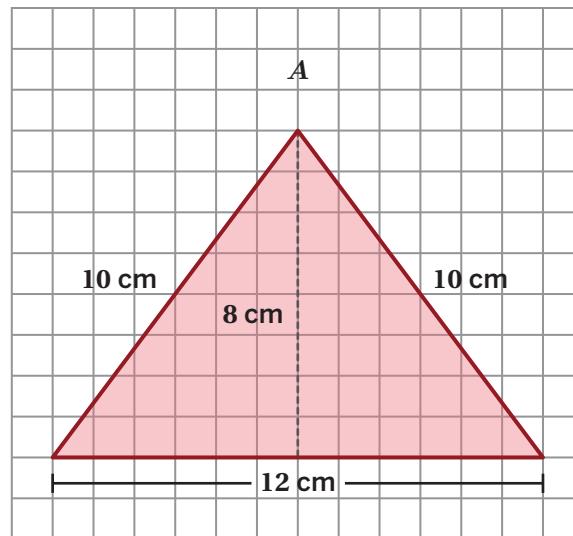
- 1** Let's look at the different sides of a triangle.

What is one thing that changes? What is one thing that stays the same?



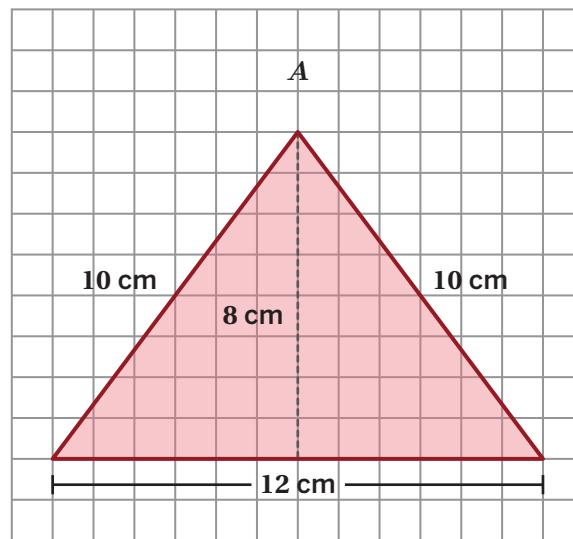
Base, Height, and Area

- 2** Use any strategy to determine the area of triangle A.



- 3** Select *all* the expressions that could represent the area of this triangle. Draw on the triangle if it helps with your thinking.

- A. $\frac{1}{2} \cdot 12 \cdot 8$
- B. $\frac{12 \cdot 10}{2}$
- C. $12 \cdot 8 \div 2$
- D. $6 \cdot 8$
- E. $6 \cdot 4$



- 4** Here is triangle A from the previous problem, along with a new triangle. Which triangle has the greater area? Circle one.

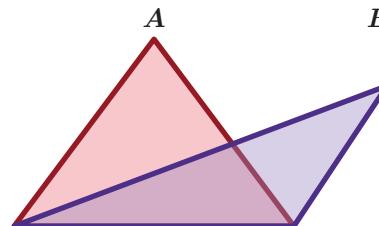
Triangle A

They are the same

Triangle B

Not enough information

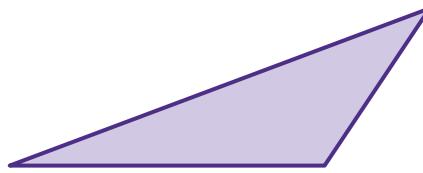
Explain your thinking.



Choose Your Measurements

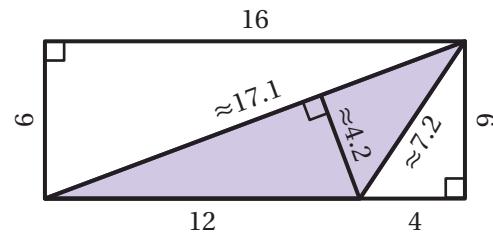
- 5** Ishaan wants to calculate the area of this triangle, but the measurements are not labeled.

Draw on the triangle to show what Ishaan should measure to calculate the area.



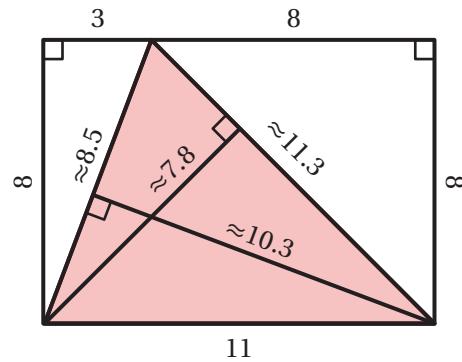
- 6** Use as many measurements as you want to calculate the area of the triangle.

All measurements are in centimeters.



- 7** Use as many measurements as you want to calculate the area of the triangle.

All measurements are in centimeters.

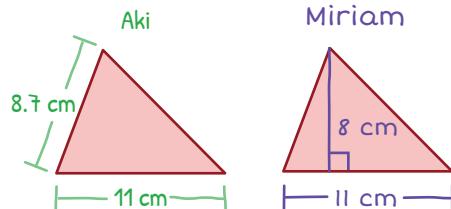


Choose Your Measurements (continued)

- 8** Aki and Miriam found different areas for the same triangle.

Here are the measurements they took.

Whose measurements lead to the correct area?
Circle one.

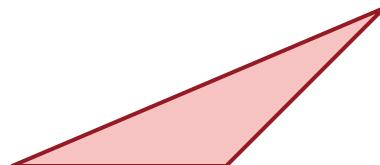


Aki's Miriam's Both Neither

Explain your thinking.

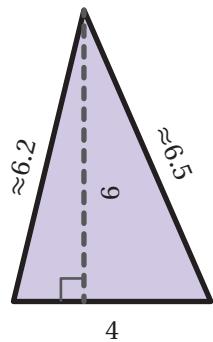
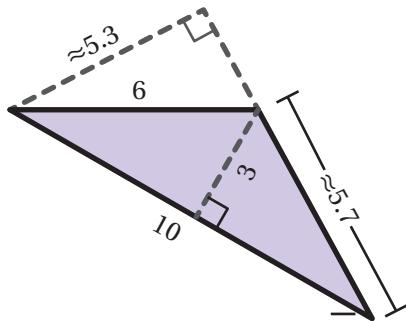
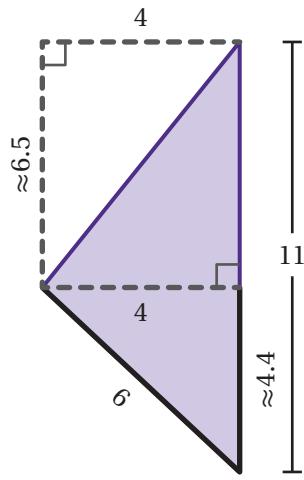
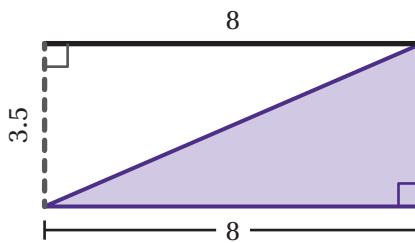
- 9** Sketch as many different triangles as you can with the same area as triangle C.

Triangle C
Area = 24 sq. cm



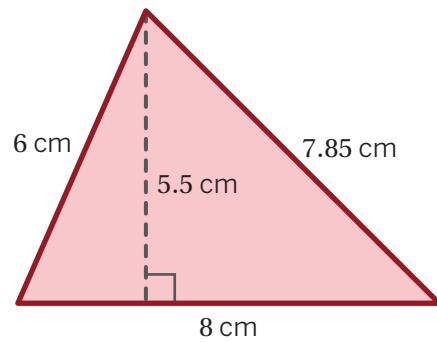
Repeated Challenges

- 10** Calculate the area of each triangle. Use as many measurements as you need. All measurements are in centimeters.

a**b****c****d**

11 Synthesis

Describe how to calculate the area of a triangle.
Use this example if it helps with your thinking.

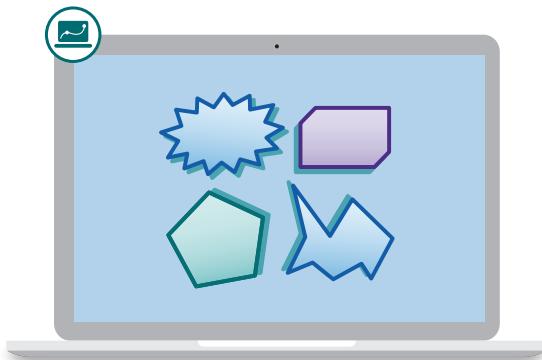


Things to Remember:

Name: Date: Period:

Pile of Polygons

Let's play with polygons.



Warm-Up

- 1** Play a few rounds of Polygraph with your classmates!

You will use the Warm-Up Sheet with shapes for four rounds. In each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a shape from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating shapes until you're ready to guess which shape the Picker chose.

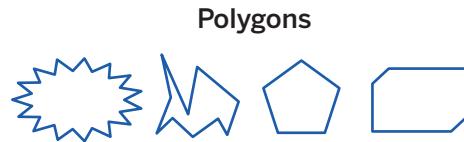
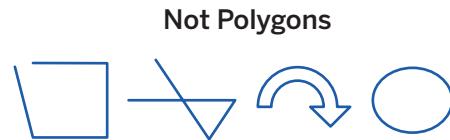
Record helpful questions from each round in this workspace:

Activity**1**

Name: Date: Period:

Polygons and Not Polygons

- 2** How are **polygons** different from shapes that are not polygons?

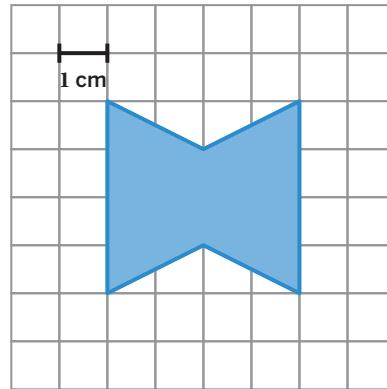
**Polygons****Not Polygons**

- 3** Sketch a shape that is a polygon and a shape that is not a polygon.

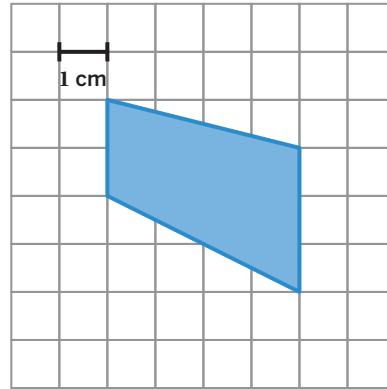
Polygon**Not a Polygon**

What is the Area?

- 4** What is the area of this polygon? Draw on the shape if it helps with your thinking.

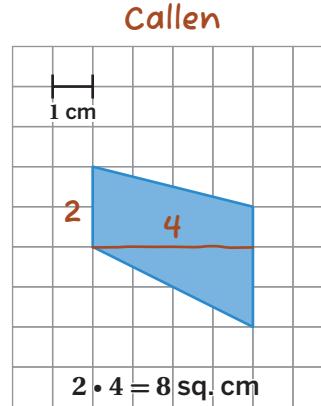
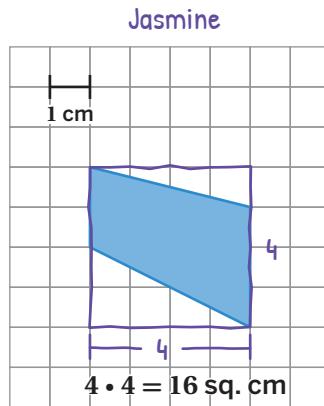


- 5** Use any strategy to determine the area of this polygon.



What is the Area? (continued)

- 6** Jasmine and Callen both made mistakes when they calculated the area for this polygon.



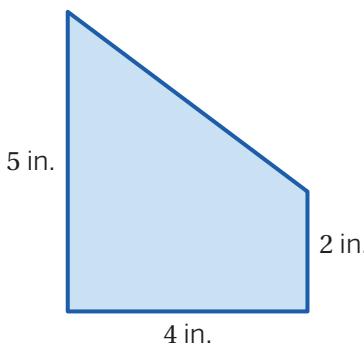
- a** Choose your favorite mistake. Circle one.

Jasmine

Callen

- b** Explain what you think is incorrect about the work you chose.

- 7** Determine the area of this polygon.



Challenge Creator

8 You will use the Challenge Creator Sheet to create your own area challenge.

- a** **Make It!** On the Challenge Creator Sheet, create your own area challenge.
- b** **Solve It!** On this page, determine the area of your polygon.

My Area

- c** **Swap It!** Swap your challenge with one or more partners. Determine the area of each partner's polygon.

Partners' Areas

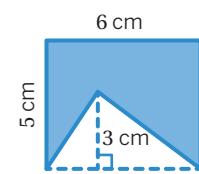
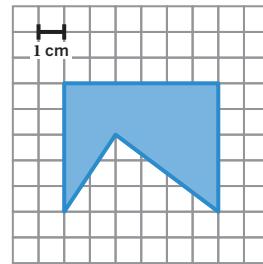
Partner 1

Partner 2

Partner 3

9 Synthesis

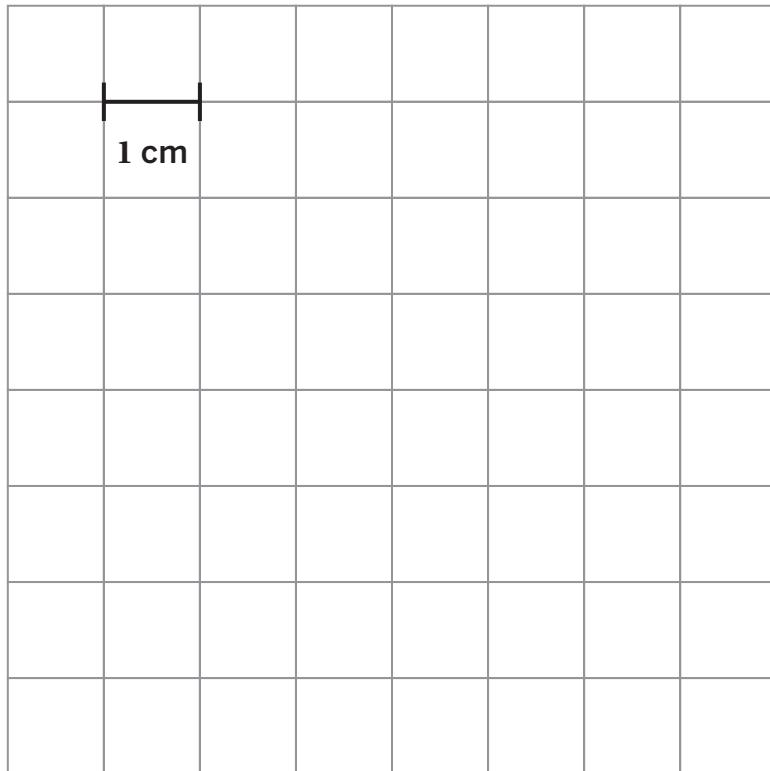
Describe a strategy for calculating the area of this polygon.



Things to Remember:

Challenge Creator

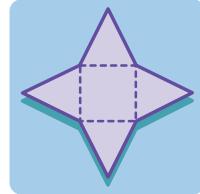
- Create a 4-sided, 5-sided, or 6-sided polygon on this grid.
- Calculate the area of your polygon in your Student Edition. (Don't write it on this page!)



Name: Date: Period:

Nothing But Nets

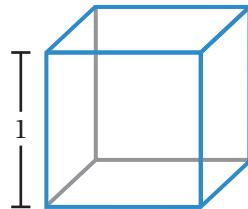
Let's make connections between polyhedra and their nets.



Warm-Up

Here is a polyhedron.

1. What could you call this type of polyhedron?



2. Draw its net.

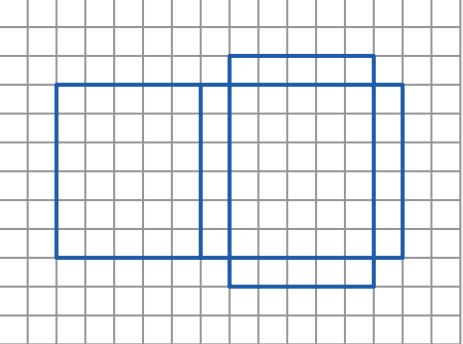
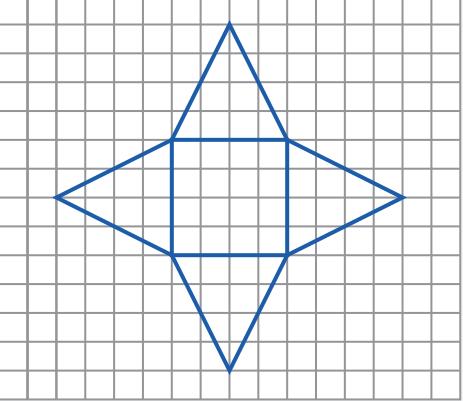
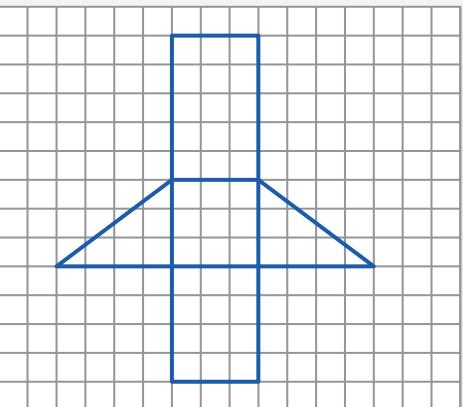
Nets and Polyhedra

3. You will use a set of cards for this activity. Match each polyhedron to its net. Record your matches in the table below and circle the name of the polyhedron.

Polyhedron	Net	Name		
		Triangular pyramid	Triangular prism	Rectangular prism
		Triangular pyramid	Triangular prism	Rectangular prism
		Triangular pyramid	Triangular prism	Rectangular prism
		Triangular pyramid	Triangular prism	Rectangular prism

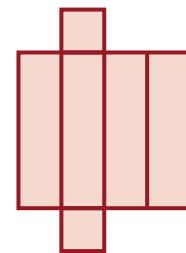
Make Polyhedra

4. You will use the Activity 2 Sheet for this activity. Take a look at the nets for polyhedra A, B, and C. Cut out, assemble, and name each polyhedron. Then calculate its surface area. Record your responses and show your thinking in the table below.

Net	Name	Surface Area
<p>Polyhedron A</p> 		
<p>Polyhedron B</p> 		
<p>Polyhedron C</p> 		

Synthesis

5. How can a net help you calculate surface area?

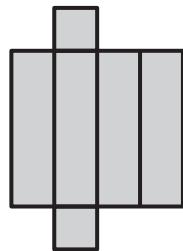
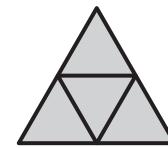
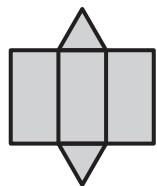
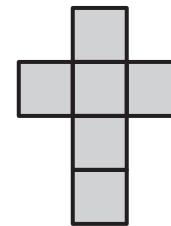
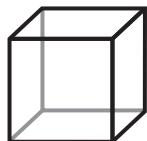
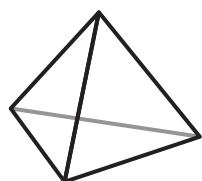
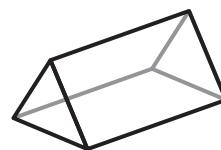


Things to Remember:

Nets and Polyhedra

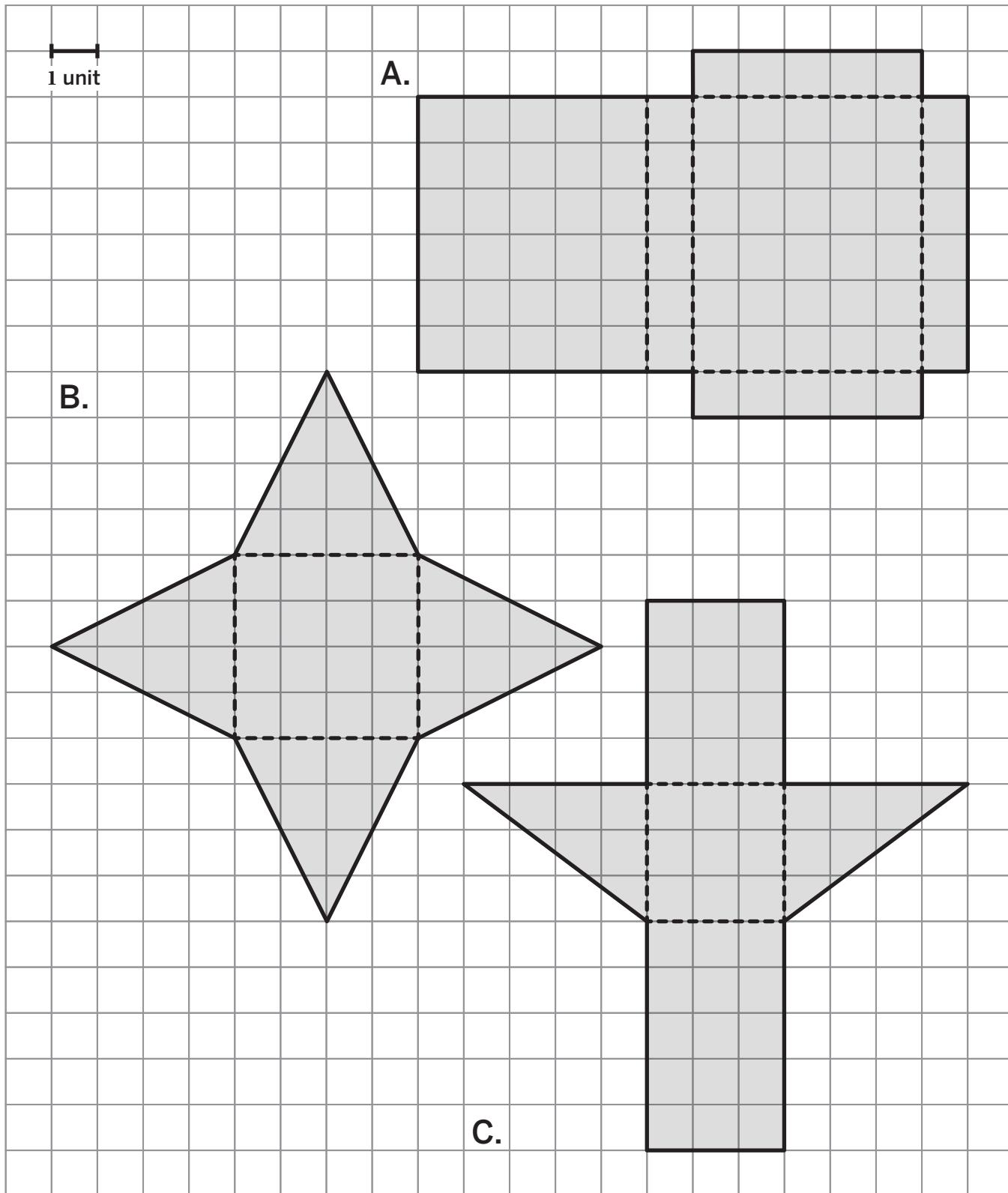
 **Directions:** Make one copy per pair of students. Then pre-cut the cards and give each pair of students one set.

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Card 1**Card 2****Card 3****Card 4****Card 5****Card 6****Card 7****Card 8**

Make Polyhedra

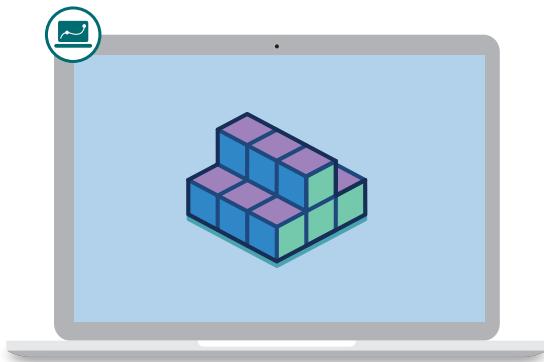
Cut out each of these nets along the solid lines and fold them along the dotted lines to assemble the polyhedra.



Name: Date: Period:

Face Value

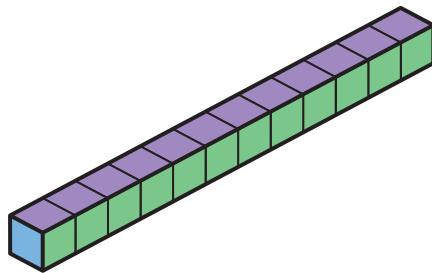
Let's determine the surface area of prisms and pyramids.



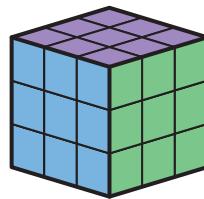
Warm-Up

- 1** Which one doesn't belong? Explain your thinking.

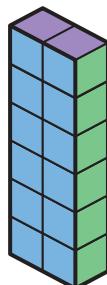
A.



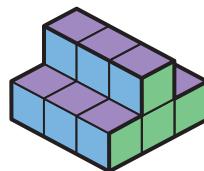
B.



C.



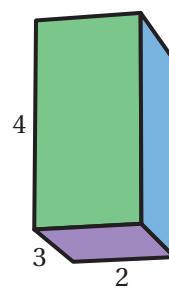
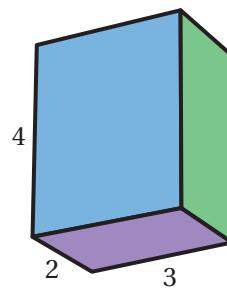
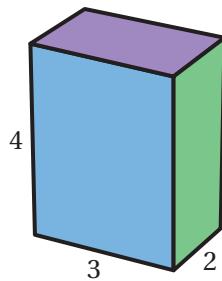
D.



Surface Area Without a Grid

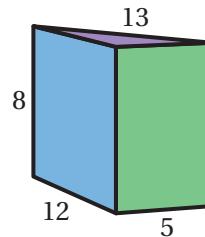
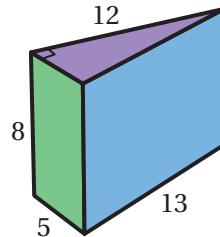
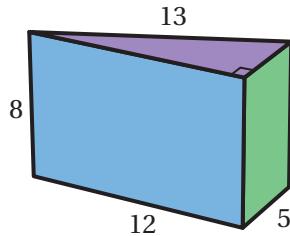
2 Note: All measurements in this lesson are in units.

- a Take a look at all the faces of this rectangular prism in square units.



- b Calculate its surface area.

3 Calculate the surface area of this triangular prism.

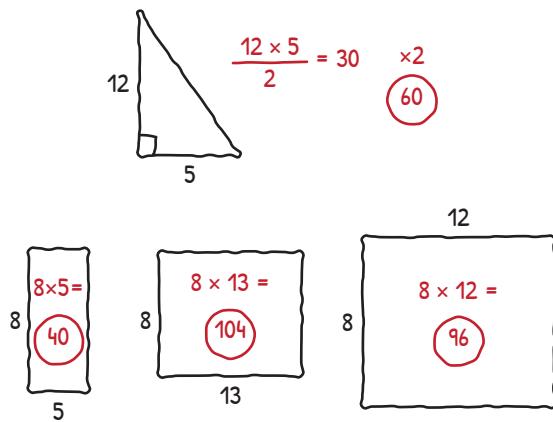


Surface Area Without a Grid (continued)

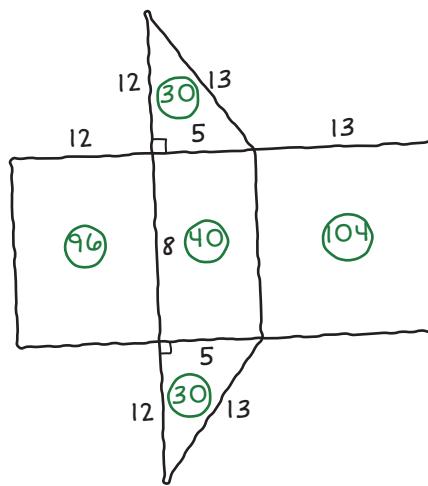
- 4** Here are two different strategies for finding the surface area of the prism from the previous problem.



Discuss: What did each student do? How are their strategies alike? How are they different?

Ebele

$$60 + 40 + 104 + 96 = 300 \text{ square units}$$

Kiri

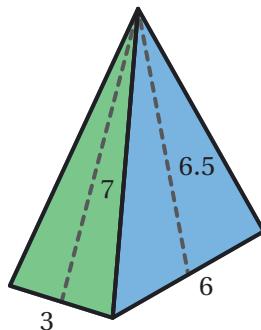
$$300$$

From Polyhedra to Nets

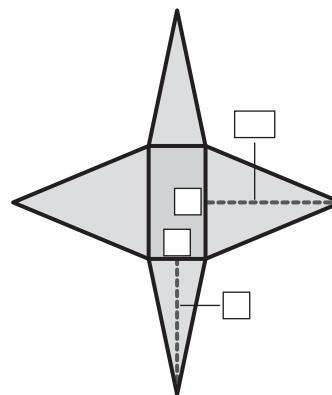
- 5** Here is a rectangular pyramid. The dotted lines represent the heights of the triangles.

Fill in the lengths to make a net that can be folded to create this pyramid.

Rectangular Pyramid



Net



- 6** Calculate the surface area of the rectangular pyramid from the previous problem.

Use the pyramid's net if it helps with your thinking.

- 7** Which of these polyhedra has a greater surface area? Circle one.

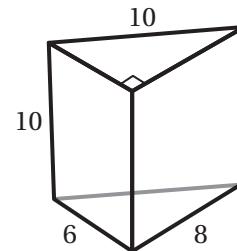
Prism A

Prism B

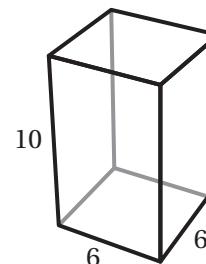
They are
the same

Explain your thinking.

Prism A

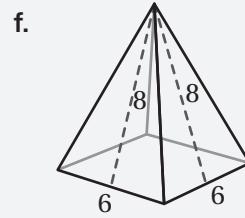
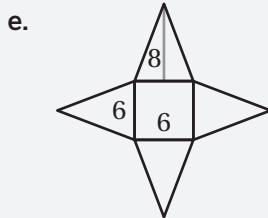
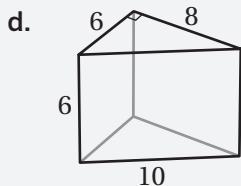
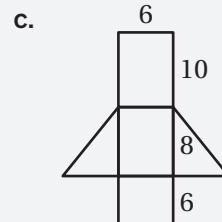
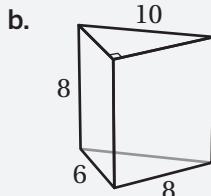
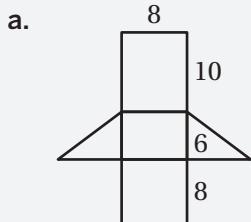


Prism B



Nets of Prisms and Pyramids (continued)

- 8** Match each polyhedron and net to their surface area.



The surface area is
132 square units.

The surface area is
240 square units.

The surface area is
192 square units.

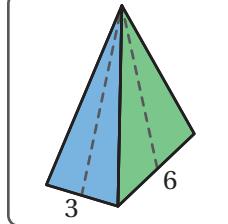
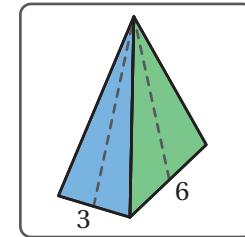
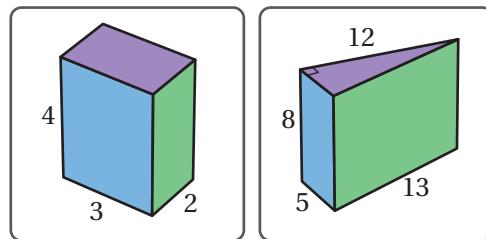
Explore More

- 9** Select one question and write your response.

- a** What are the dimensions of two prisms that have the same surface area but different volumes?
- b** What are the dimensions of two prisms that have the same volume but different surface areas?

10 Synthesis

How can you calculate the surface area of a prism or a pyramid from a picture?

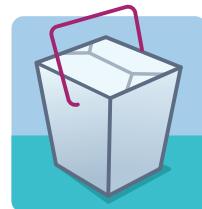


Things to Remember:

Name: Date: Period:

Take It To Go

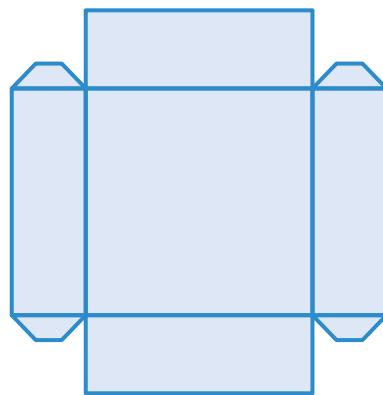
Let's design a to-go container.



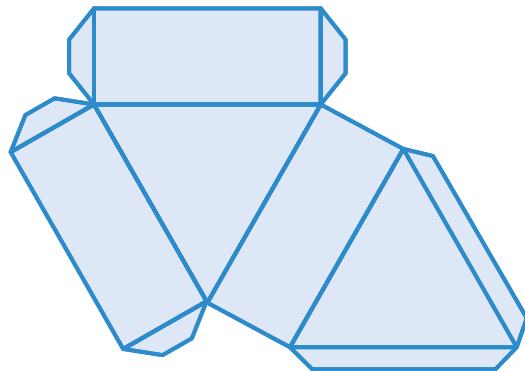
Warm-Up

1. DeAndre is opening a new restaurant. He is making patterns that can be folded into to-go containers for the different foods he will sell.

Pattern A



Pattern B



Discuss:

- a** How are the patterns alike? How are they different?

- b** Which container would you prefer to hold a bagel?

- c** Which pattern do you think requires more material?

Lots of Triangles

DeAndre's restaurant will serve sandwiches, salads, and single slices of pizza. He needs to design to-go containers for each item.

Your task is to design a to-go container for one of the food items and calculate the amount of material you need to make it.

Use this information to help you create your design.

- A sandwich is roughly 4 inches by 4 inches by 2 inches.
- A salad is roughly 120 cubic inches.
- A slice is roughly the shape of a triangle with a height of 8 inches and a base of 5 inches.

2. Which food item are you designing a container for? Circle one.

Sandwich

Salad

Slice of pizza

3. What is the shape of the base of your container?

4. How many faces does your container have?

5. Draw or describe how you want your container to look. Be sure to include all the necessary measurements.

6. Calculate how much material you need to make your container.



Make It!

7. Share your design with a partner. Discuss how you might improve your design and write down what adjustments you want to make.
 8. Draw your revised pattern on blank paper using the measurements you designed.
 9. Cut out and fold your pattern to create your container.

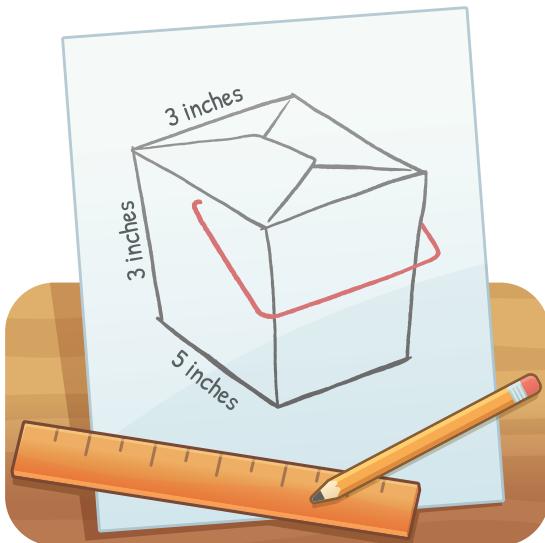
Explore More

- 10.** To-go containers can be made out of different materials, like cardboard, styrofoam, or aluminum. Research two different types of materials. Then write a pitch to DeAndre about which material(s) he should use for his containers, and why.

Synthesis

11. a How was surface area related to the work you did today?

- b Now that you have seen your classmates' designs, what would you have done differently if you had more time?



Things to Remember:

Name: Date: Period:

Ratio Relationships

Let's describe how to compare pizza toppings.



Warm-Up

Evaluate each expression mentally.

1. $2 \cdot 15$

2. $4 \cdot 15$

3. $6 \cdot 15$

4. $12 \cdot 15$

Ratio Rounds

5. You will use pizza cards to complete Rounds 1–3.

Round 1: Write down as many **ratios** as you can about your pizza card.

My Ratios

Complete as many statements as you can about your pizza card.

For every , there are

Round 2: Find a classmate whose card has a pizza that is *exactly the same* as one of your pizzas. Then write down the ratio relationship between the two toppings on each of your cards.

My Ratio

The ratio of to mushrooms is to

's Ratio (Classmate)

The ratio of to mushrooms is to

What is the same about your ratios?

What is different about your ratios?

Ratio Rounds (continued)

Round 3: Form a group with 2–3 classmates whose cards each have the same *total number of mushrooms* as your card. Then write down the ratio relationship between the two toppings on each of your cards.

My Ratios

What is the ratio of mushrooms : ? :

What is the ratio of : mushrooms? :

's Ratio (Classmate)

What is the ratio of mushrooms : ? :

What is the ratio of : mushrooms? :

's Ratio (Classmate)

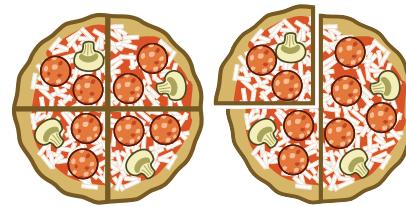
What is the ratio of mushrooms : ? :

What is the ratio of : mushrooms? :

Two Truths and a Lie

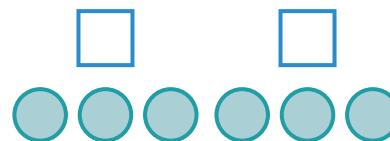
6. Which statement is false?

- A. The ratio of mushrooms to pepperonis is 2 : 1.
- B. For every 4 mushrooms, there are 8 pepperonis.
- C. The ratio of pepperonis to mushrooms is 12 to 6.



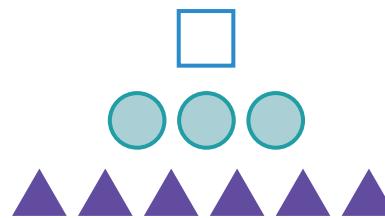
7. Which statement is false?

- A. The ratio of circles to squares is 1 : 3.
- B. There are 2 squares for every 6 circles.
- C. For every square, there are 3 circles.



8. Which statement is false?

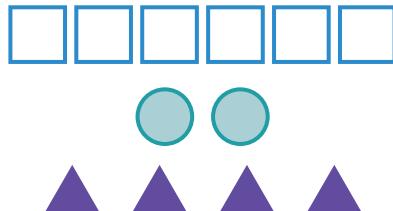
- A. For every circle, there are 2 triangles.
- B. The ratio of circles to squares is 3 : 1.
- C. The ratio of squares to triangles is 1 to 2.



Two Truths and a Lie (continued)

9. Here is another set of shapes.

- a** Write three statements about these shapes: two that are true and one that is false.



- b** Trade your statements with a classmate. Which of their statements is false?

10. Now create your own challenge!

- a** Draw your own set of shapes.

- b** Write three statements about your drawing: two that are true and one that is false.

- c** Trade your challenge with a classmate. Which of their statements is false?

Synthesis

11. **a** Describe the ratio between these moons and stars in as many different ways as you can.



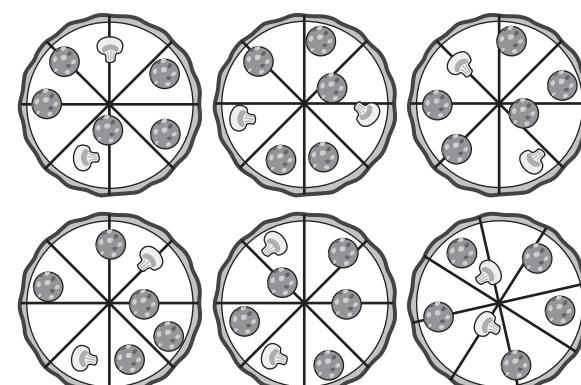
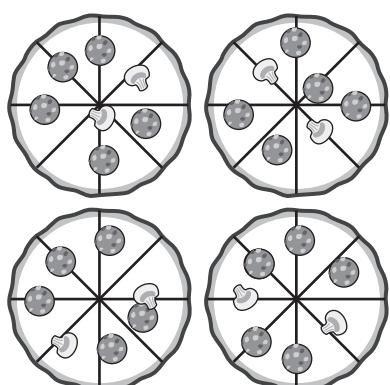
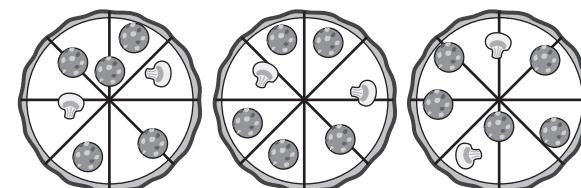
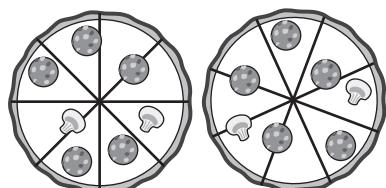
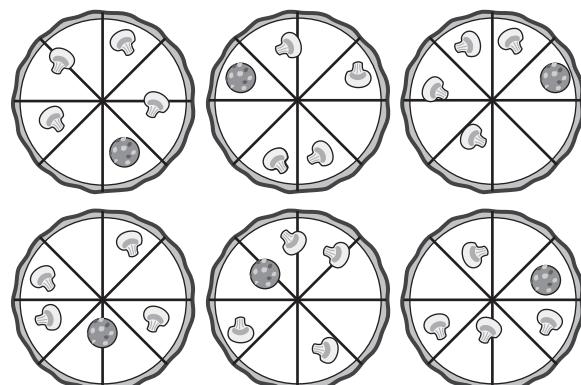
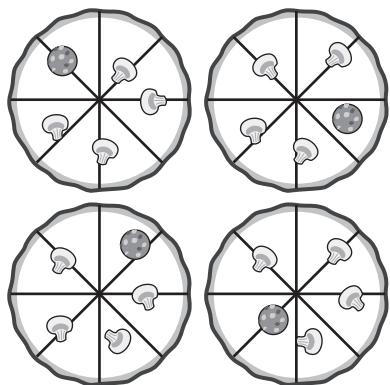
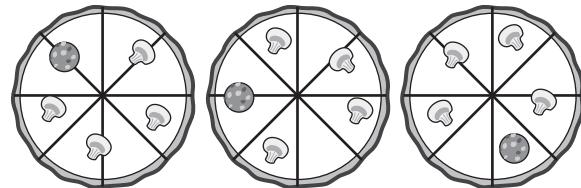
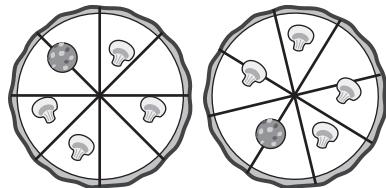
- b** Which way of describing a ratio is your favorite? Explain your reasoning.

Things to Remember:

Ratio Rounds

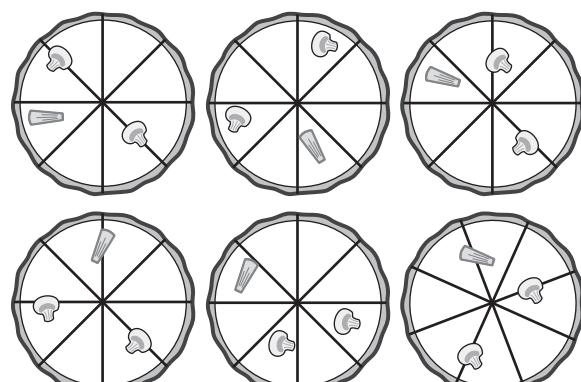
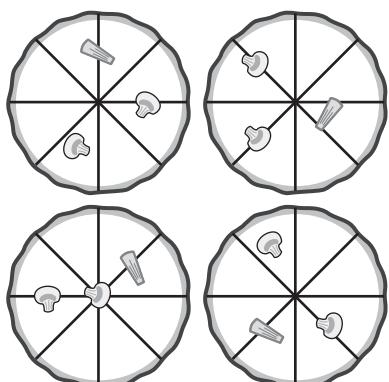
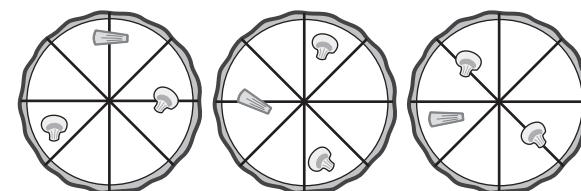
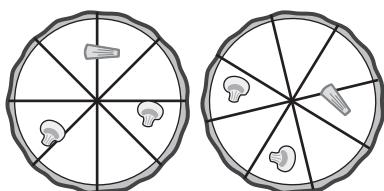
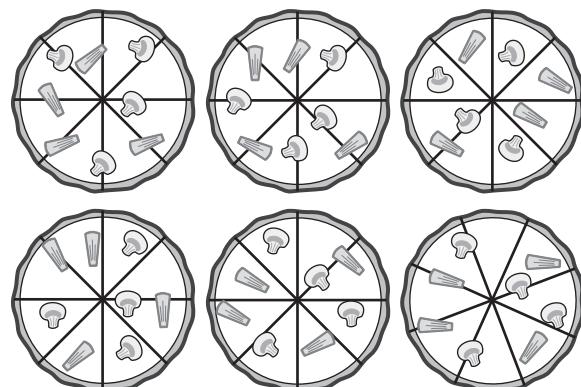
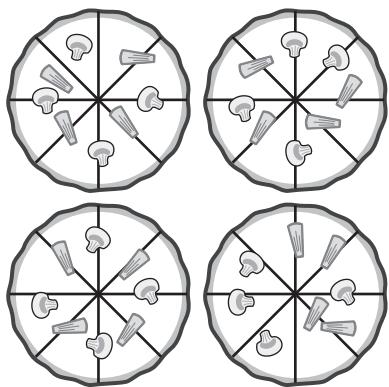
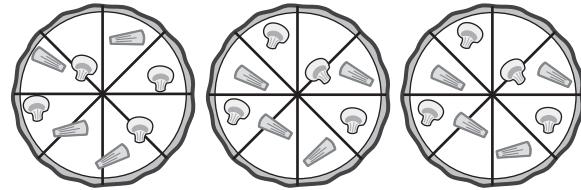
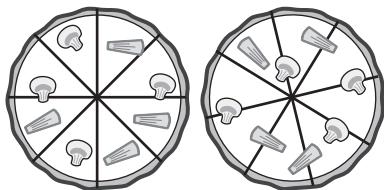
 **Directions:** Make one copy for the whole class. Then pre-cut the cards and give each student one card.

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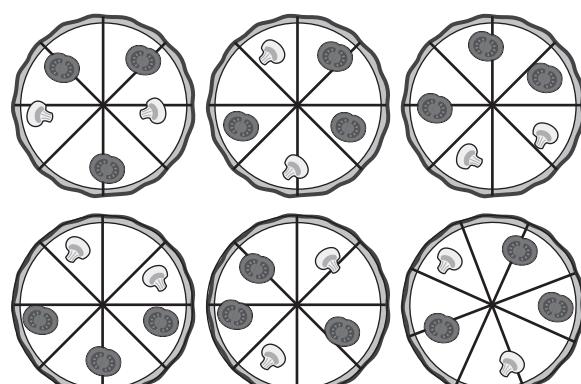
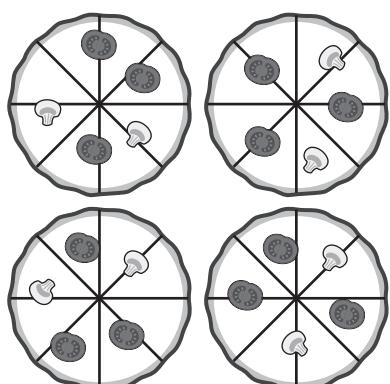
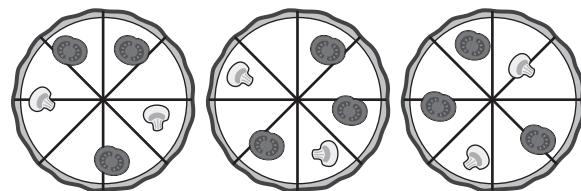
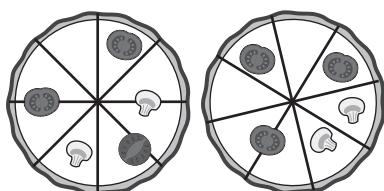
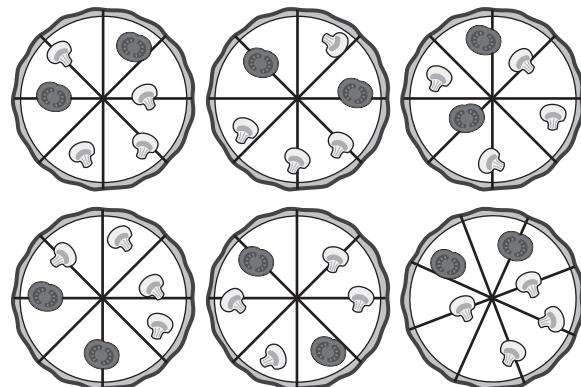
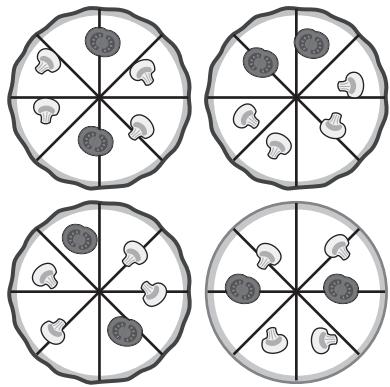
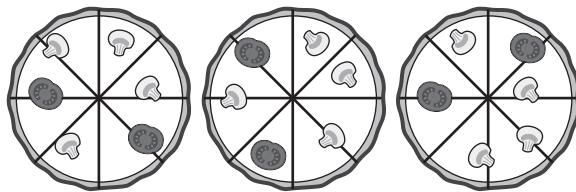
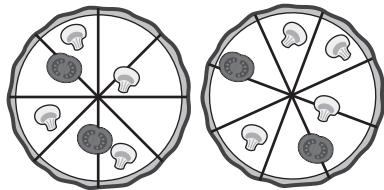
Ratio Rounds

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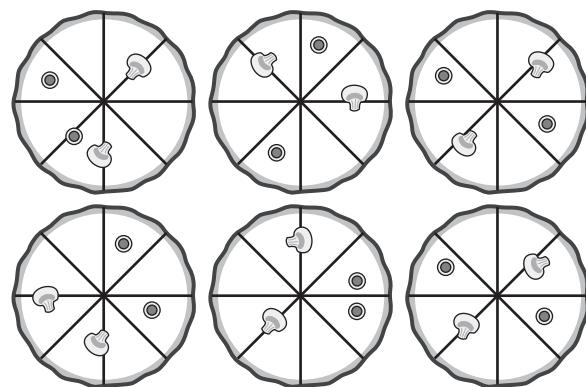
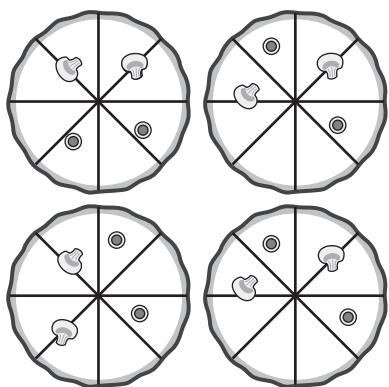
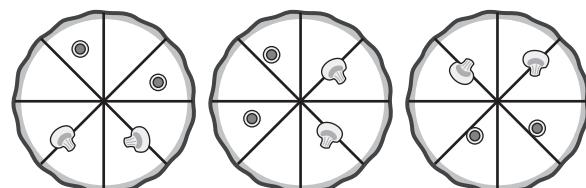
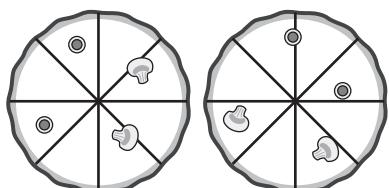
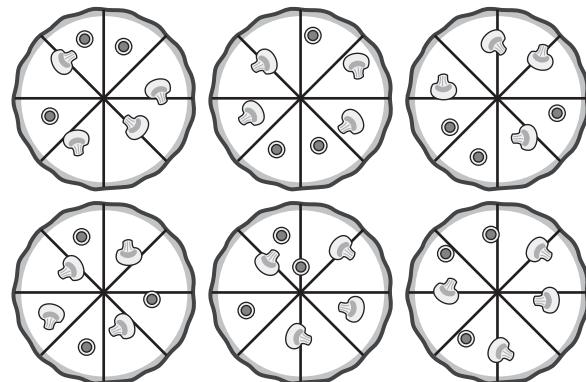
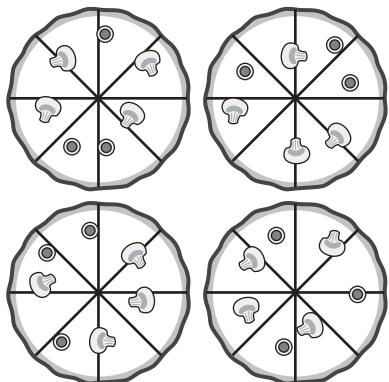
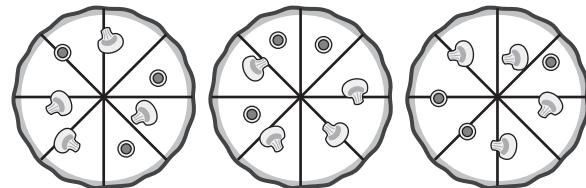
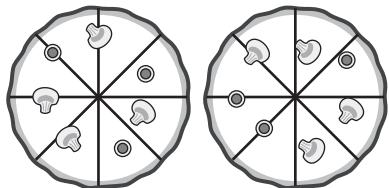
Ratio Rounds

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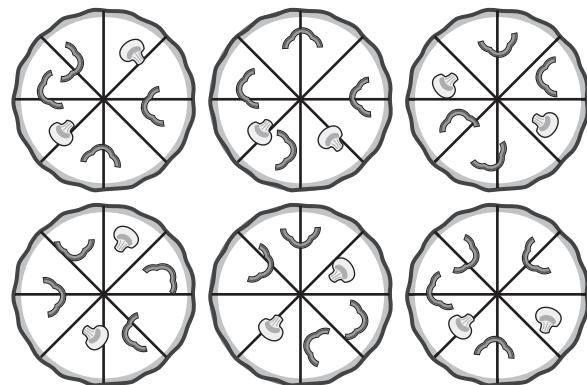
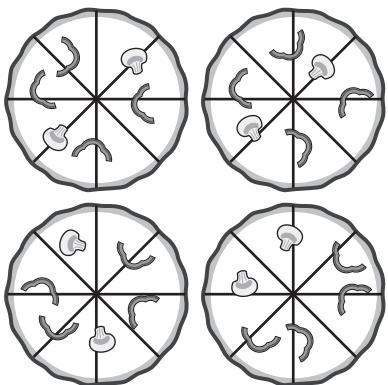
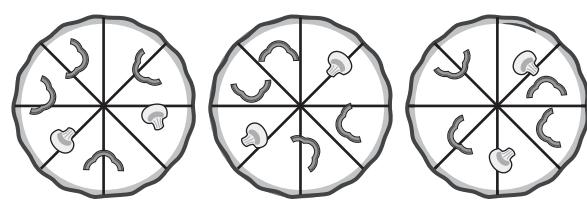
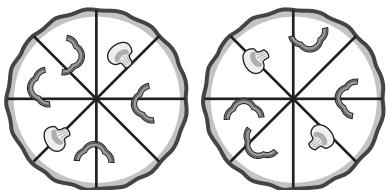
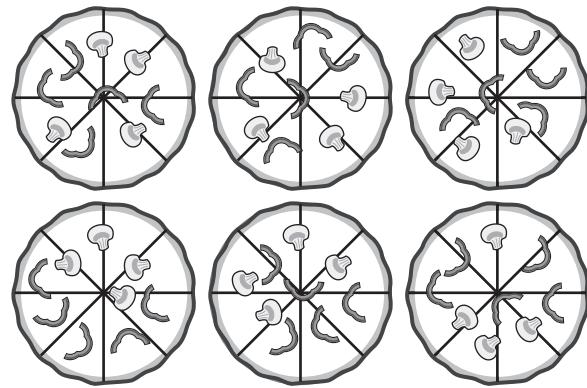
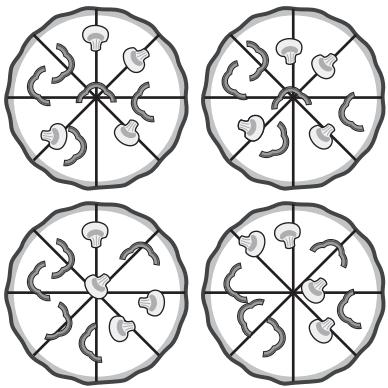
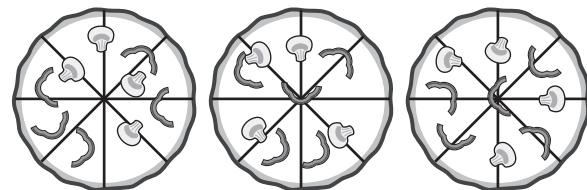
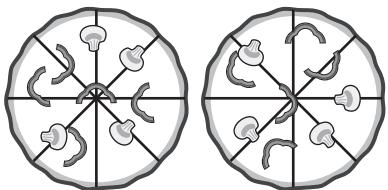
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Name: Date: Period:

Rice Ratios

Let's explore ratios in recipes..



Warm-Up

Evaluate each expression mentally.

1. $4 \cdot 8$

2. $4 \cdot 10$

3. $4 \cdot 18$

4. $4 \cdot 30$

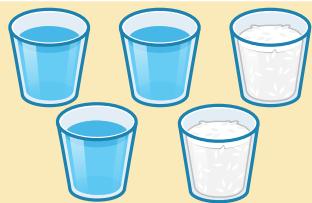
5. $4 \cdot 38$

Rice Advice

6. Here are the cooking instructions for three different bags of basmati rice.

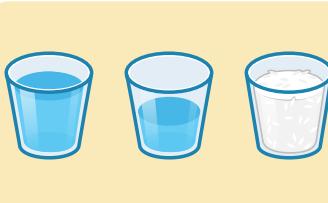
Bag A

Boil 3 cups of water for every 2 cups of rice.



Bag B

Boil $1\frac{1}{2}$ cups of water for every 1 cup of rice.



Bag C

Boil 4 cups of water for every 2 cups of rice.



- a** The ratios for Bag A and Bag B are called **equivalent ratios**. Why do you think they're called that?
- b** Marco wants to follow the directions for Bag A but use more rice. What is another ratio of water to rice that Marco could use? Explain your thinking.
- c** The recipe for Bag A says it makes rice for 6 people. What ratio of water to rice would you use to feed 18 people?

Rice Around the World

Here are the recipes for four rice dishes from around the world.

- 7.** Jamar invited a friend over for dinner. How much of each ingredient does Jamar need to make 2 large bowls of jollof rice?

..... cups of rice

..... tablespoons of tomato paste

..... bell peppers

..... tomatoes

..... onions

..... cups of oil

Jollof Rice



Jollof rice is a tomato-based rice dish from Senegal, Ghana, and Nigeria.

Ingredients

Makes one large bowl

- 4 cups of rice
- 3 tablespoons of tomato paste
- 1 bell pepper
- 5 tomatoes
- 2 onions
- $\frac{1}{3}$ cups of oil

- 8.** Nia wants to cook arroz con leche for 12 people.

- a** How much of each ingredient does Nia need?

..... cups of rice

..... cups of milk

..... cups of sugar

..... handfuls of raisins

..... cinnamon sticks

Arroz Con Leche



Arroz con leche is a creamy dessert from Mexico and Spain.

Ingredients

Serves 4 people

- 2 cups of rice
- 4 cups of milk
- $\frac{1}{3}$ cups of sugar
- 1 handful of raisins
- 1 cinnamon stick

- b** Valeria wrote that Nia needs 9 cinnamon sticks. Why might Valeria think this?

- c** What advice would you give Valeria?

Rice Around the World (continued)

- 9.** Julian has 1 cup of sugar and wants to use all of it to make champorado.

- a** How much of the other ingredients does he need?

..... cups of rice

..... cups of water

..... cans of coconut milk

..... cups of cocoa powder

Champorado



Champorado is a chocolate rice porridge eaten in the Philippines.

Ingredients

Serves 4 people

- 1 cup of rice
- 4 cups of water
- 2 cans of coconut milk
- $\frac{1}{2}$ cups of cocoa powder
- 2 cups of sugar

- b** How many people will Julian's champorado serve?

- 10.** Ariana says this recipe makes too much risotto.

- a** How much of each ingredient would it take to make a smaller amount of risotto?

..... cups of rice

..... cups of chicken broth

..... tablespoons of olive oil

..... tablespoons of butter

..... ounces of Parmesan cheese

Risotto



Risotto is an Italian rice dish that uses broth to create a creamy texture.

Ingredients

Serves 8 people

- 3 cups of rice
- 10 cups of chicken broth
- 4 tablespoons of olive oil
- 2 tablespoons of butter
- 8 ounces of Parmesan cheese

- b** How many people will this serve?

Synthesis

11. The cooking instructions on Bag A and Bag B call for equivalent ratios of water to rice.

Bag A

Boil 4 cups of water for every 2 cups of rice.



Bag B

Boil 2 cups of water for every 1 cup of rice.



- a** Explain what equivalent ratios are in your own words.

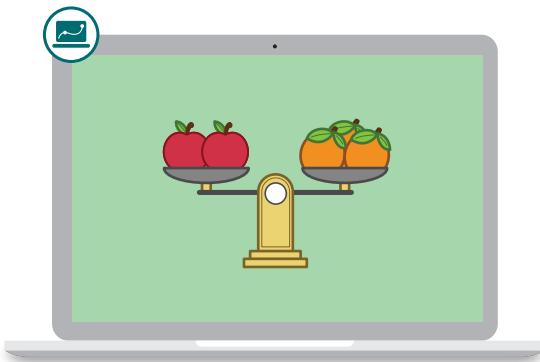
- b** Create a new ratio of water to rice that is equivalent to the ratios for Bag A and Bag B.

Things to Remember:

Name: Date: Period:

Fruit Lab

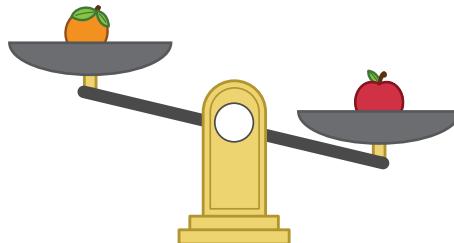
Let's investigate equivalent ratios by balancing fruit on scales.



Warm-Up

- 1** Let's watch apples and oranges balance on a scale.

- a** When the scale balances, record the values in the table.
- b** Find as many ways as you can to balance the scale.



Number of Oranges	Number of Apples
.....
.....
.....
.....
.....

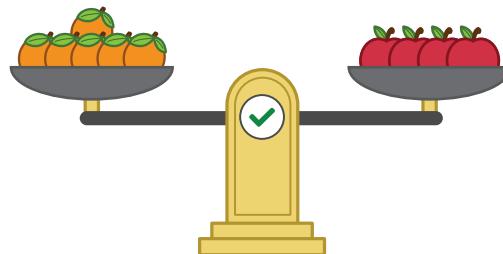
Apples to Oranges

- 2 Here is Victor's table from the Warm-Up. What do you notice about the table? What do you wonder?

I notice:

Number of Oranges	Number of Apples
15	10
3	2
6	4

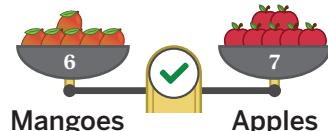
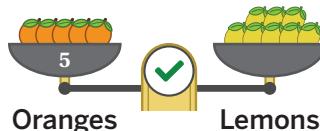
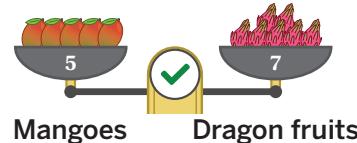
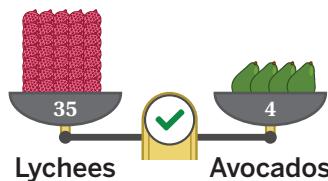
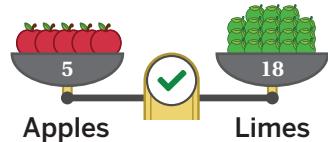
I wonder:



- 3 Write another equivalent ratio in the last row. Try to find one that you think no one else will think of.

Fruit Lab

- 4** You will use the Activity 2 Sheet to complete this activity. Choose a pair of fruits to see how they balance. Then record several equivalent ratios for that pair of fruits. Repeat with different combinations of fruits.



- 5** A student knows that 15 grapes balance with 1 dragon fruit. They say that 16 grapes will balance with 2 dragon fruits. Will this $16 : 2$ ratio balance the scale? Circle one.

Yes

No

I'm not sure

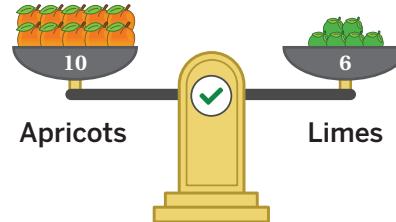
Explain your thinking.



Fruit Lab (continued)

- 6** The scale balances with a ratio of 10 apricots to 6 limes. Select *all* of the equivalent ratios.

- A. 20 apricots to 16 limes
- B. 50 apricots to 30 limes
- C. 7 apricots to 3 limes
- D. 5 apricots to 3 limes
- E. 11 apricots to 7 limes



- 7** The table shows some ratios of limes to lychees that balance the scale. Dyani says that 22 limes will balance with 55 lychees. Will the $22 : 55$ ratio balance? Circle one.

Yes No I'm not sure

Explain your thinking.

Number of Limes	Number of Lychees
2	5
20	50

Explore More

- 8** A ratio of 11 kiwis : 4 peaches balances. So does a ratio of 15 pears : 6 peaches.

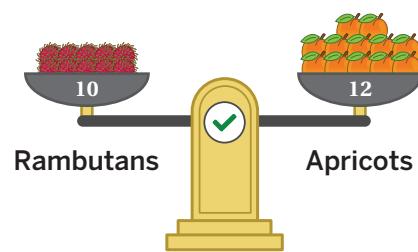
Write a ratio of kiwis to pears that would balance.
Explain your thinking.



9 Synthesis

When you know a ratio balances a scale, how can you create equivalent ratios that also balance the scale?

Use the example if it helps with your thinking.



Things to Remember:

Name: Date: Period:

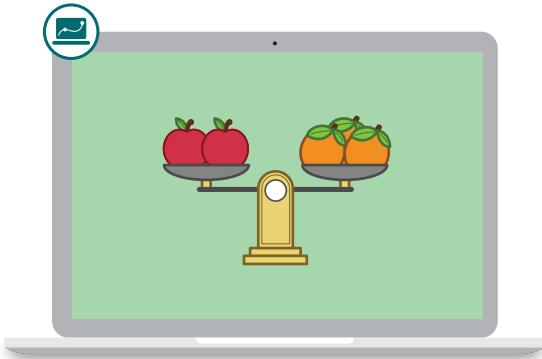
Fruit Lab

- Choose a pair of fruits. Record their names in the first row of a table.
- Record several equivalent ratios for those fruits in the table.
- Repeat for as many pairs of fruits as you like!

Name: Date: Period:

Fruit Lab

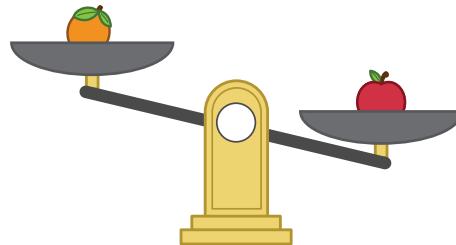
Let's investigate equivalent ratios by balancing fruit on scales.



Warm-Up

- 1** Let's watch apples and oranges balance on a scale.

- a** When the scale balances, record the values in the table.
- b** Find as many ways as you can to balance the scale.



Number of Oranges	Number of Apples
.....
.....
.....
.....
.....

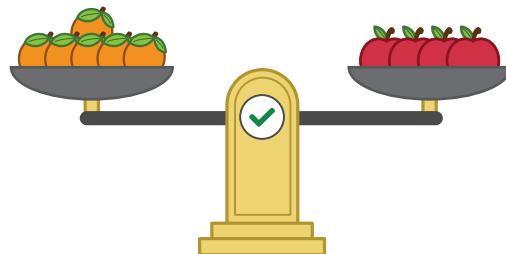
Apples to Oranges

- 2 Here is Victor's table from the Warm-Up. What do you notice about the table? What do you wonder?

I notice:

Number of Oranges	Number of Apples
15	10
3	2
6	4

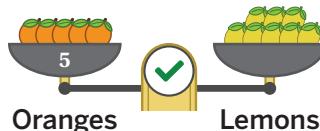
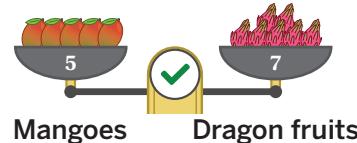
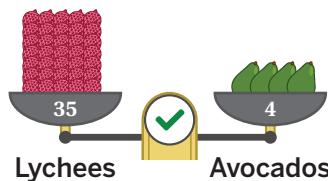
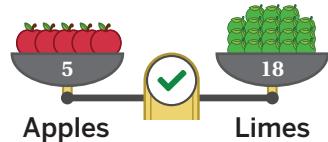
I wonder:



- 3 Write another equivalent ratio in the last row. Try to find one that you think no one else will think of.

Fruit Lab

- 4** You will use the Activity 2 Sheet to complete this activity. Choose a pair of fruits to see how they balance. Then record several equivalent ratios for that pair of fruits. Repeat with different combinations of fruits.



- 5** A student knows that 15 grapes balance with 1 dragon fruit. They say that 16 grapes will balance with 2 dragon fruits. Will this $16 : 2$ ratio balance the scale? Circle one.

Yes

No

I'm not sure

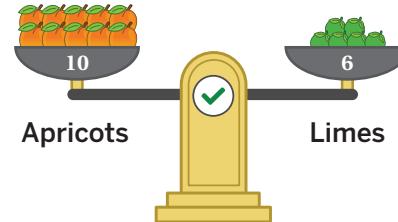
Explain your thinking.



Fruit Lab (continued)

- 6** The scale balances with a ratio of 10 apricots to 6 limes. Select *all* of the equivalent ratios.

- A. 20 apricots to 16 limes
- B. 50 apricots to 30 limes
- C. 7 apricots to 3 limes
- D. 5 apricots to 3 limes
- E. 11 apricots to 7 limes



- 7** The table shows some ratios of limes to lychees that balance the scale. Dyani says that 22 limes will balance with 55 lychees. Will the $22 : 55$ ratio balance? Circle one.

Yes No I'm not sure

Explain your thinking.

Number of Limes	Number of Lychees
2	5
20	50

Explore More

- 8** A ratio of 11 kiwis : 4 peaches balances. So does a ratio of 15 pears : 6 peaches.

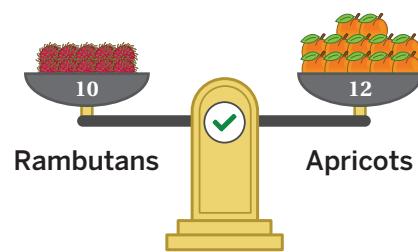
Write a ratio of kiwis to pears that would balance.
Explain your thinking.



9 Synthesis

When you know a ratio balances a scale, how can you create equivalent ratios that also balance the scale?

Use the example if it helps with your thinking.



Things to Remember:

Name: Date: Period:

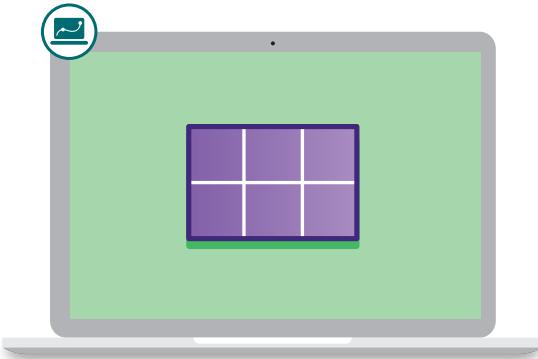
Fruit Lab

- Choose a pair of fruits. Record their names in the first row of a table.
- Record several equivalent ratios for those fruits in the table.
- Repeat for as many pairs of fruits as you like!

Name: Date: Period:

Common Factors

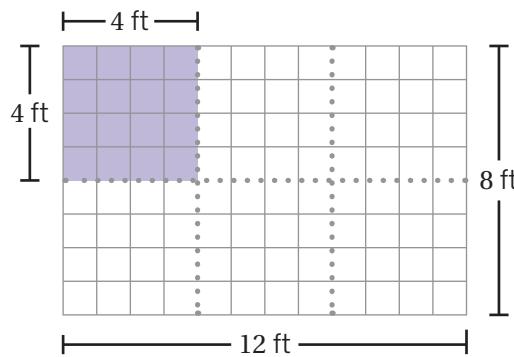
Let's explore factors.



Warm-Up

- 1** A 4-by-4-foot square will tile the floor of this 8-by-12-foot room.

- a** Take a look at the diagram to see what we mean.
- b** Find other square sizes that tile the room.



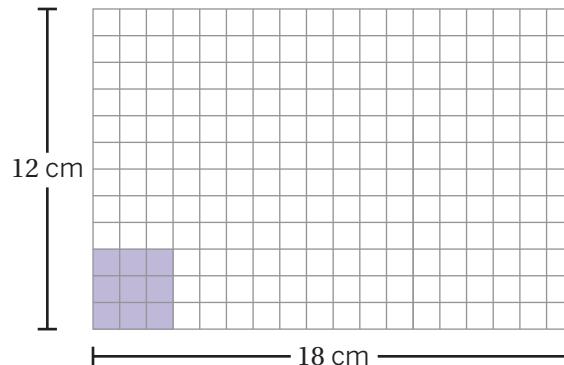
- c** **Discuss:** What does it mean to *tile*?

Greatest Common Factors

- 2** A 3-by-3-centimeter square will tile this 12-by-18-centimeter rectangle. This means that 3 is a **common factor** of 12 and 18.

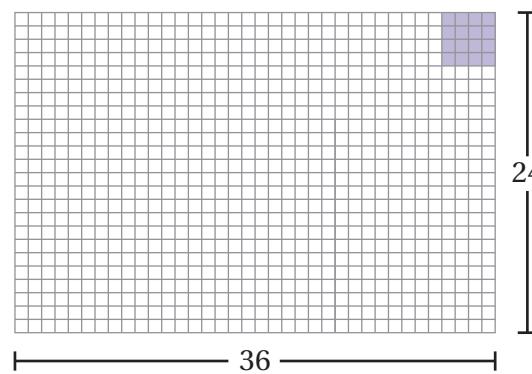
Select *all* the other common factors of 12 and 18.

- A. 1
- B. 2
- C. 4
- D. 6
- E. 12



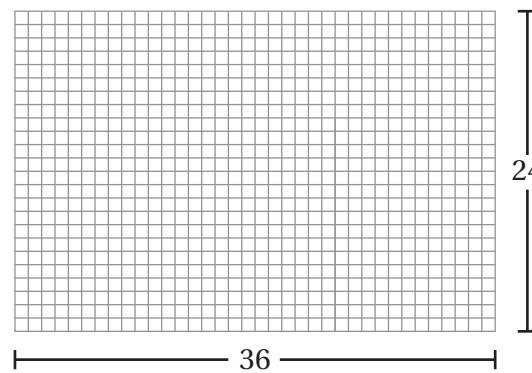
- 3** 4 is a common factor of 24 and 36.

Determine as many common factors of 24 and 36 as you can.



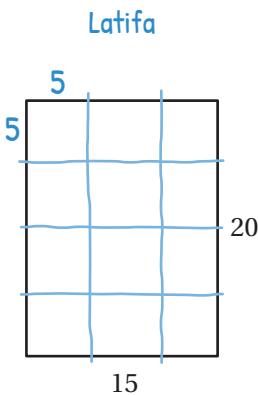
- 4** The **greatest common factor (GCF)** is the greatest number that is a common factor of two numbers.

What is the greatest common factor of 24 and 36?



Greatest Common Factors (continued)

- 5** Here are Latifa's and Tameeka's strategies for determining the greatest common factor of 20 and 15.



Tameeka

Factors of 15: 1, 3, **5**, 15

Factors of 20: 1, 2, 4, **5**, 10, 20



Discuss: What are the advantages and disadvantages of each strategy?

- 6** What is the greatest common factor of 27 and 36? Explain your thinking.

Common Factors and Multiples

7 The greatest common factor of 20 and 15 is 5.

What is the *least common multiple (LCM)* of 20 and 15?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

8 How are *GCF* and *LCM* alike? How are they different?

Alike:

Different:

Repeated Challenges

9 Determine the least common multiple or greatest common factor. Use a 100-grid or draw a diagram if it helps with your thinking.

- a** What is the least common multiple of 6 and 4?
- b** What is the greatest common factor of 9 and 12?
- c** What is the least common multiple of 10 and 6?
- d** What is the least common multiple of 2 and 16?
- e** What is the greatest common factor of 16 and 2?
- f** What is the greatest common factor of 18 and 27?

Explore More

10 Jamir and Kimaya each wrote a question about greatest common factor (GCF) and least common multiple (LCM).

Jamir

Does every pair of numbers have a GCF and a LCM?

Kimaya

Is the GCF of two numbers always smaller than the LCM?

Discuss your answer to at least one question with a classmate.

11 Synthesis

Discuss these questions with a classmate. Sketch a diagram if it helps with your thinking.

- a What does *greatest common factor* mean?

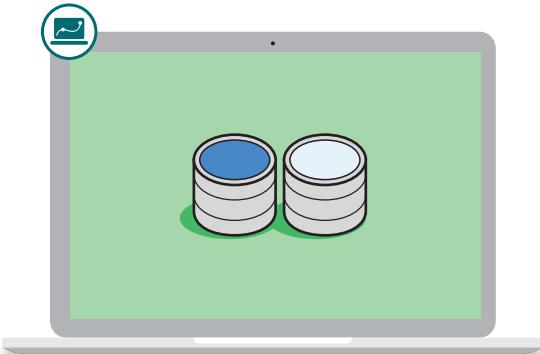
- b Why do you think we don't study the *least common factor*?

Things to Remember:

Name: Date: Period:

Mixing Paint, Part 1

Let's see how mixing colors relates to ratios.



Warm-Up

Mentally determine the missing value that makes each pair of fractions equivalent.

$$\underline{1} \quad \frac{1}{5} = \frac{\square}{10}$$

$$\underline{2} \quad \frac{2}{5} = \frac{6}{\square}$$

$$\underline{3} \quad \frac{4}{\square} = \frac{12}{15}$$

$$\underline{4} \quad \frac{\square}{8} = \frac{15}{12}$$

Comparing Ratios

- 5** Paint stores create different colors by using different ratios of white paint to tint.

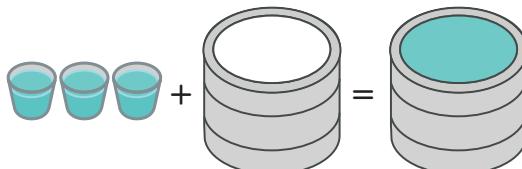
a Choose one tint color.

b Circle the amount of tint you want to add to 2 gallons of white paint.

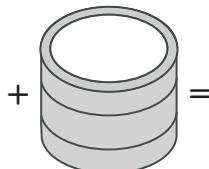
Use the digital activity to see your paint mix.



Example Paint Mix



My Paint Mix



- 6** Write the ratio you created.

..... ounces tint : 2 gallons white paint

a Can you find two different ways to make a *darker* color? Use the digital activity to check your work.

..... ounces tint : gallons white

..... ounces tint : gallons white

b Can you find two different ways to make a *lighter* color? Use the digital activity to check your work.

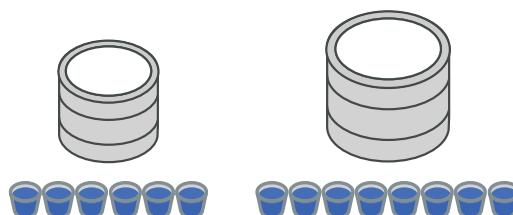
..... ounces tint : gallons white

..... ounces tint : gallons white

Comparing Ratios (continued)

- 7** Here are Luca's and Marc's ratios. Which will make a darker blue? Circle one.

Luca's ratio Marc's ratio They'll make the same blue



Explain your thinking.

Luca's Ratio
6 ounces blue
2 gallons white

Marc's Ratio
8 ounces blue
4 gallons white

- 8** **a** Let's watch to see which ratio makes a darker blue.

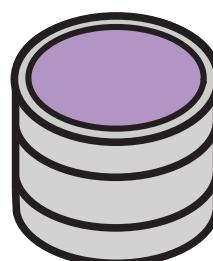
- b** **Discuss:** What is a different strategy you could use to compare the ratios?

- 9** Here is Amoli's ratio:

8 ounces purple : 4 gallons white

Select *all* of the choices that will result in a darker purple.

- A. Adding white paint
- B. Using less white paint
- C. Adding purple tint
- D. Using less purple tint
- E. Adding 2 ounces of purple tint and 2 gallons of white paint



Lighter or Darker Paint

- 10** Order the ratios from *darkest blue* to *lightest blue*.

- A. 5 ounces blue : 4 gallons white
- B. 4 ounces blue : 3 gallons white
- C. 10 ounces blue : 6 gallons white
- D. 9 ounces blue : 6 gallons white



- 11** Luca says that these two ratios make the same shade of blue.

$\equiv 4 \text{ ounces blue} : 3 \text{ gallons white}$

What would you recommend Luca change?

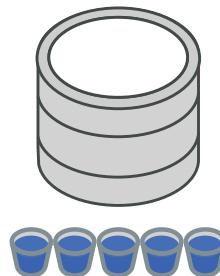
$\equiv 5 \text{ ounces blue} : 4 \text{ gallons white}$

- 12** Solve all six challenges. For each pair of ratios, choose which ratio makes a darker blue.

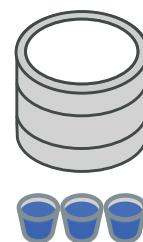
Ratio A	Ratio B	Ratio A	Ratio B	They'll make the same blue.
2 oz blue : 4 gal white	3 oz blue : 4 gal white			
4 oz blue : 3 gal white	4 oz blue : 5 gal white			
3 oz blue : 2 gal white	5 oz blue : 4 gal white			
5 oz blue : 2 gal white	15 oz blue : 6 gal white			
7 oz blue : 3 gal white	5 oz blue : 2 gal white			
5 oz blue : 4 gal white	9 oz blue : 7 gal white			

13 Synthesis

Describe a strategy for comparing two ratios.
Use the example if it helps with your thinking.



Ratio A
5 ounces blue
4 gallons white



Ratio B
3 ounces blue
2 gallons white

Things to Remember:

Name: Date: Period:

Disaster Preparation

Let's use ratio tables to help prepare for disasters.



Warm-Up

- 1** Cities need to prepare for possible disasters.

What are *three* things a city should have for its people in case of a disaster?

- 1.
- 2.
- 3.

- 2** There are national recommendations for the items that cities should stock up on in case of a disaster. Here are three of those items.¹

How many of each item do you think a city with a population of 100 should have?

Handheld Shower



Power Strip



Air Pump



Population	Handheld Showers	Power Strips	Air Pumps
100			

¹ Source: *Commonly Used Sheltering Items Catalog*

Shower, Power, and Air

- 3 Here are the recommendations for a city of 100 people.

How many of each item would you recommend for Lucas, Wisconsin?

	Population	Handheld Showers	Power Strips	Air Pumps
Recommendations	100	4	5	1
Lucas, Wisconsin	700			

- 4 How many of each item would you recommend for Blue Ridge, Georgia?

	Population	Handheld Showers	Power Strips	Air Pumps
Recommendations	100	4	5	1
Blue Ridge, Georgia	1,200			

- 5 How many of each item would you recommend for Hamlin City, Kansas?

	Population	Handheld Showers	Power Strips	Air Pumps
Recommendations	100	4	5	1
Hamlin City, Kansas	25			

Shower, Power, and Air (continued)**6**

Taylor recommended that Hamlin City, Kansas should buy 1 handheld shower, 1 power strip, and 1 air pump.

	Population	Handheld Showers	Power Strips	Air Pumps
Recommendations	100	4	5	1
Lucas, Wisconsin	700	28	35	7
Blue Ridge, Georgia	1,200	48	60	12
Hamlin City, Kansas	25	1	1	1

a

 **Discuss:** Why do you think Taylor made this recommendation?

b

What do you agree with about Taylor's recommendations? What do you disagree with?

7

Here are the actual recommendations for Lucas, Wisconsin; Blue Ridge, Georgia; and Hamlin City, Kansas.



Discuss: What was their strategy for calculating the number of each item? Are there any recommendations you disagree with?

	Population	Handheld Showers	Power Strips	Air Pumps
Lucas, Wisconsin	700	28	35	7
Blue Ridge, Georgia	1,200	48	60	12
Hamlin City, Kansas	25	1	2	1

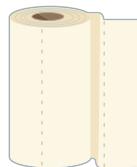
Make a Poster

8

Here are the national recommendations about other items to stock up on in case of disaster.

Paper Towels

For every 5 people, have 1 roll of paper towels.



Duct Tape

Have 3 rolls of duct tape for every 25 people.



Magnifying Glass

Have 1 magnifying glass for every 50 people.



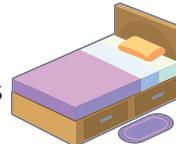
Cotton Balls

For every 100 people, have 4 bags of 50 cotton balls each.



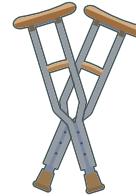
Beds

Have 1 bed for each person, plus 10 extra beds for volunteers.



Crutches

Have 6 pairs of crutches.



a Use these recommendations to make a disaster preparedness proposal for these 3 cities.

	Population	Rolls of Paper Towels	Magnifying Glasses	Cotton Balls	Pairs of Crutches
Branch City, Arkansas	300				
Bennington City, Nebraska	2,000				
Harrisburg, Pennsylvania	50,200				

b Is there anything you disagree with about these recommendations? If so, explain which numbers you think should change and why. If not, explain why not.

Make a Poster (continued)

c Complete these steps and make a poster of your work:

- Choose a city or town that is meaningful to you and look up its population.

City, State

Population (to the nearest 10 people)

.....

- Make a proposal for items that this city should stock up on. Choose *at least* four different supplies from the national recommendations list. Then determine how many of each item the city should have on hand in case of a disaster.

Item 1:	Item 2:
Item 3:	Item 4:

- Show or explain how you determined the amount of each item your city will need.
- Explain *at least* two changes or additions you would make to the national recommendations.

9 Synthesis

Explain how to use a table of equivalent ratios to determine unknown values. Use the example if it helps with your thinking.

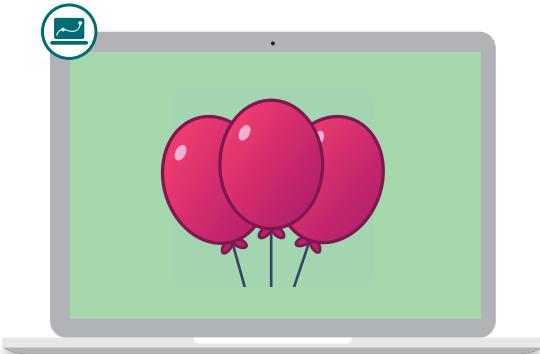
	Population	Handheld Showers	Power Strips
Recommendations	100	4	5
Lucas, Wisconsin	700	28	35
Blue Ridge, Georgia	1,200	48	60

Things to Remember:

Name: Date: Period:

Balloons

Let's develop and use tools to solve problems involving equivalent ratios.



Warm-Up

Evaluate each expression mentally.

1 $2 \cdot 31$

2 $8 \cdot 31$

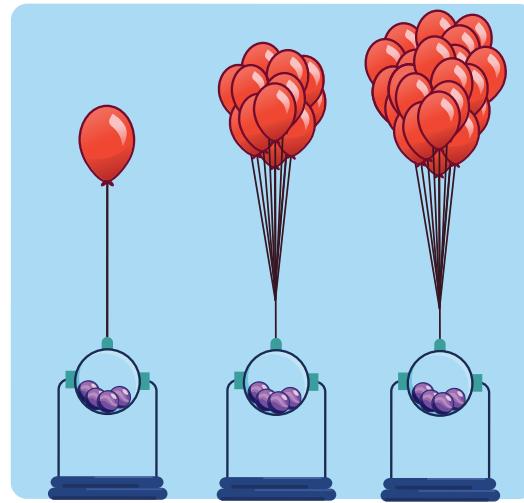
3 $9 \cdot 31$

4 $11 \cdot 31$

Balloon Float

- 5** Helium balloons can make objects float, but too many balloons will make objects fly away!

Let's watch an animation to see the *middle* container float.

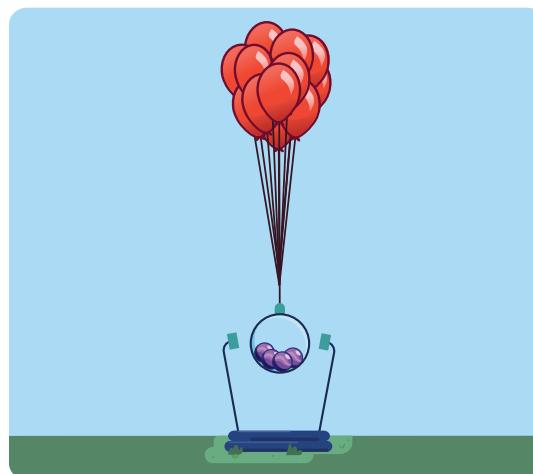


- 6** Red balloons float purple marbles at a ratio of 12 : 4.

What will happen to the marbles if we add 1 balloon and 1 marble? Circle one.

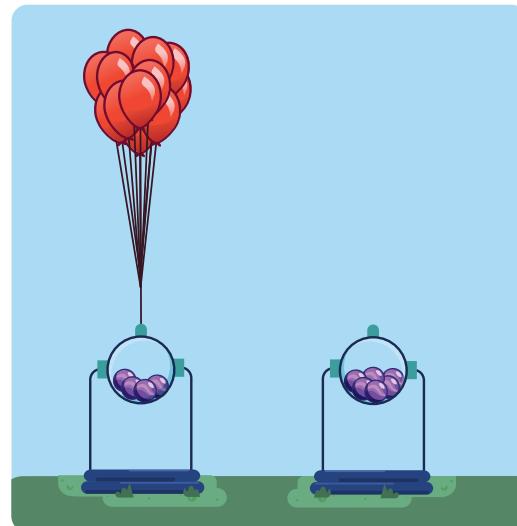
Sink down Float in place Fly up

Explain your thinking.



- 7** Red balloons float purple marbles at a ratio of 12 : 4.

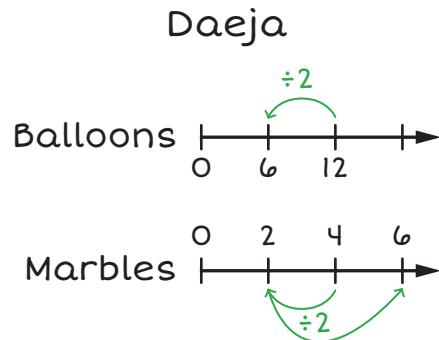
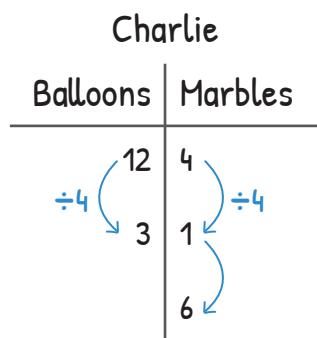
How many red balloons will float 6 purple marbles?



Balloon Float (continued)

- 8** Here are Charlie's and Daeja's strategies for determining how many red balloons will float 6 purple marbles.

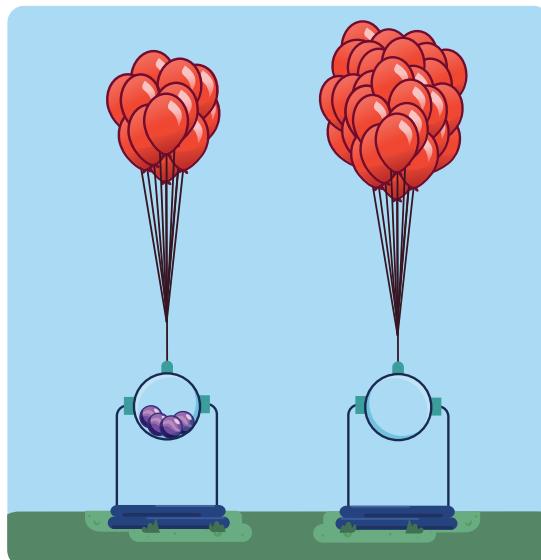
- a** Look at each student's strategy.



- b** Select one student by circling their name, then explain how they could finish their strategy to solve the problem.

- 9** Red balloons float purple marbles at a ratio of 12 : 4.

How many purple marbles will 30 red balloons float?



Marble Float

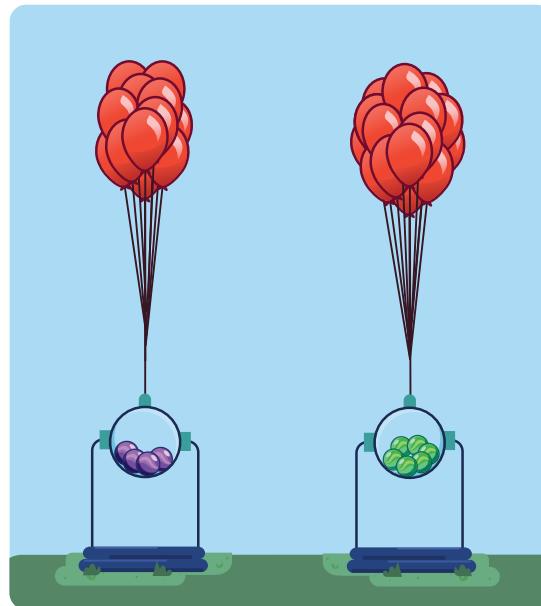
- 10** Red balloons float purple marbles at a ratio of 12 : 4.

Red balloons float green marbles at a ratio of 15 : 6.

Which is heavier: a purple marble or a green marble?

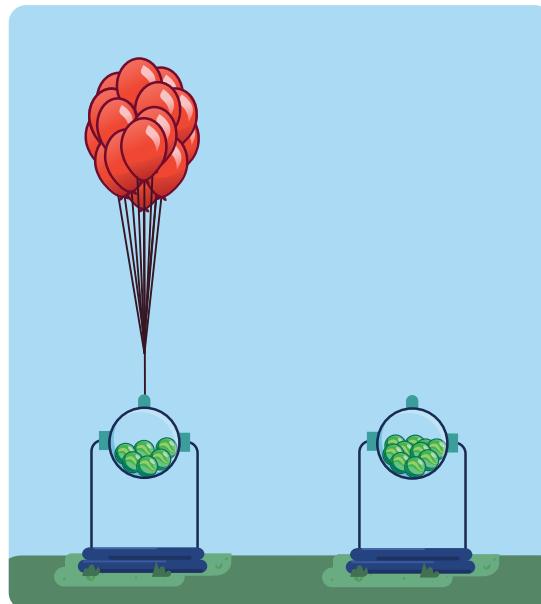
Purple Green They're the same

Explain your thinking.



- 11** Red balloons float green marbles at a ratio of 15 : 6.

How many red balloons will float 10 green marbles?



Marble Float (continued)

- 12** Here are Charlie's and Daeja's strategies for determining how many red balloons will float 10 green marbles.



Discuss: How are their strategies alike? How are they different?

Charlie

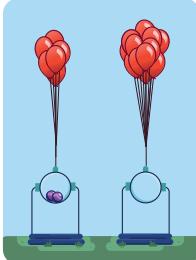
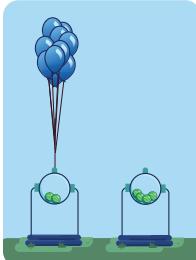
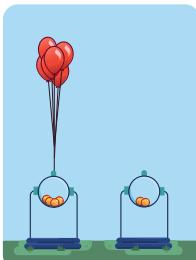
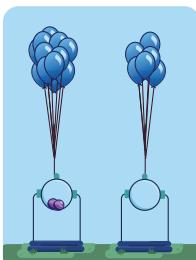
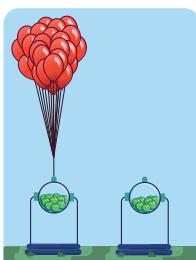
Balloons	Marbles
$\times \frac{1}{6}$	15
2.5	$\times \frac{1}{6}$
25	$\times 10$

Daeja

Balloons	Marbles
$\times 5$	15
75	$\times 5$
25	$\div 3$
	10

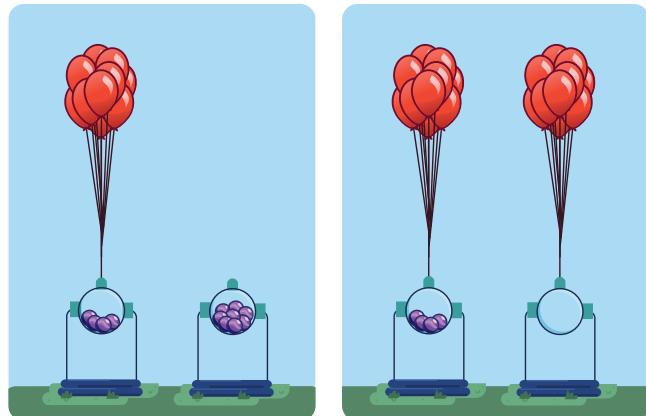
Repeated Challenges

- 13** For each ratio, create an equivalent ratio to make the balloons float.

Ratio	Number of Balloons	Number of Marbles
 <p>Red balloons float purple marbles at a ratio of 6 : 2.</p>	12 red balloons	
 <p>Blue balloons float green marbles at a ratio of 10 : 2.</p>		4 green marbles
 <p>Red balloons float orange marbles at a ratio of 6 : 4.</p>		2 orange marbles
 <p>Blue balloons float purple marbles at a ratio of 12 : 2.</p>	6 blue balloons	
 <p>Red balloons float green marbles at a ratio of 25 : 10.</p>		8 green marbles

14 Synthesis

Describe a strategy for determining missing values in equivalent ratios, like an unknown number of balloons or marbles..

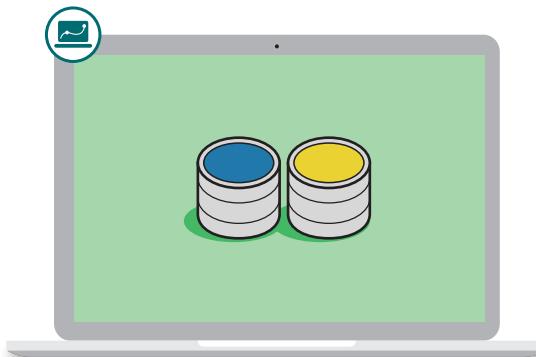


Things to Remember:

Name: Date: Period:

Mixing Paint, Part 2

Let's use tape diagrams to represent ratios.

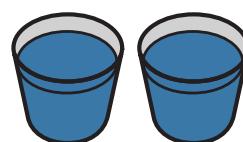


Warm-Up

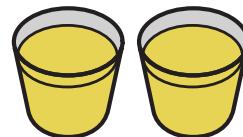
- 1** Let's watch how to make a new color.

What do you notice? What do you wonder?

I notice:



I wonder:

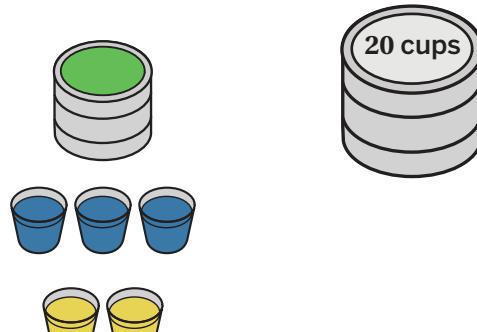


How Much of Each?

- 2** Tyrone makes a green paint by mixing 3 cups of blue with 2 cups of yellow.

He needs 20 more cups of green paint to finish painting a mural.

How much of each color should he mix?



Blue (cups)	Yellow (cups)	Total (cups)
		20

- 3** Tyrone drew a *tape diagram* to help determine that he needs 12 cups of blue and 8 cups of yellow to make 20 cups of green paint.



Where do you see the 3 : 2 ratio, 20, and 12 represented in Tyrone's diagram?

The 3 : 2 ratio is shown by . . .

The 20 total cups are shown by . . .

The 12 cups of blue are shown by . . .

- 4** Kayla needs 35 gallons of the same green paint.

She used this tape diagram to determine how much of each paint color she needs.

How many gallons should go into each box in the tape diagram?



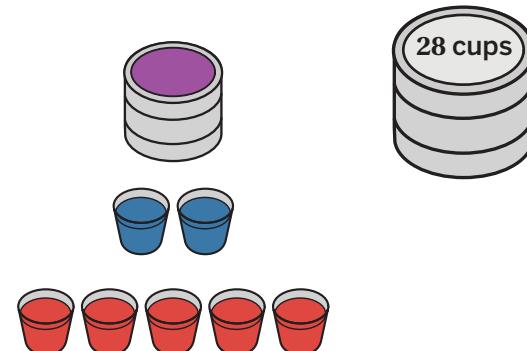
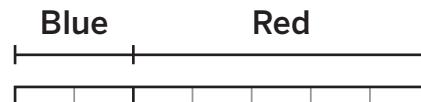
Same Color?

- 5** Sai makes purple paint by mixing 2 cups of blue and 5 cups of red.

How much of each color should Sai mix to get 28 cups of purple paint?

Use the tape diagram if it helps with your thinking.

Blue (cups)	Red (cups)	Total (cups)
		28



- 6** Select *all* of the combinations that would make the same color.

- A. 1 cup blue and 2.5 cups red
- B. 2.5 cups blue and 1 cup red
- C. 2 quarts blue and 5 quarts red
- D. 2 cups blue and 5 gallons red
- E. 1 gallon blue and 1 cup red

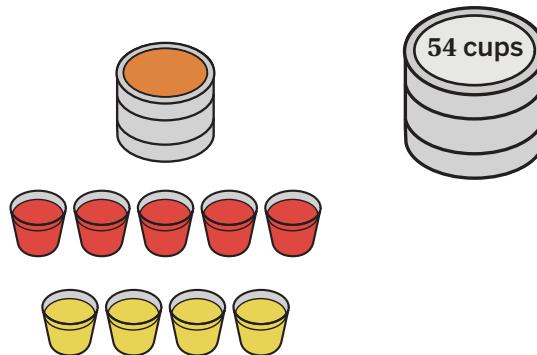


- 7** Dylan has a recipe for orange paint that mixes 5 parts red paint and 4 parts yellow paint.

How much of each color should Dylan mix to get 54 cups of orange paint?

Draw your own tape diagram if it helps with your thinking.

Red (cups)	Yellow (cups)	Total (cups)
		54



Same Color? (continued)

- 8** Here are Ethan's and Zion's recipes for orange paint.

Which student made more paint? Circle one.

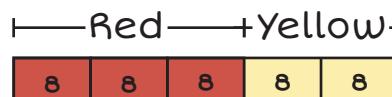
Ethan Zion Same amount

Explain your thinking.

Ethan's Orange



Zion's Orange

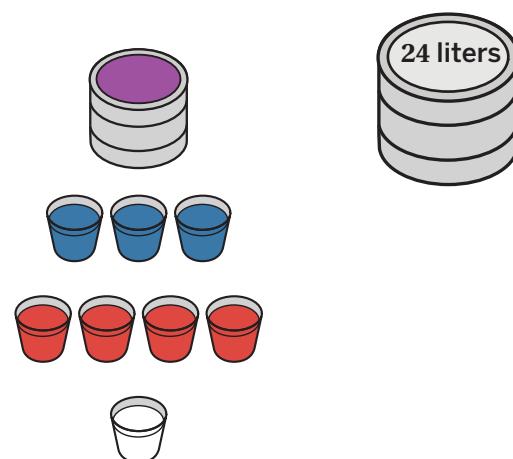


- 9** A recipe for purple paint calls for 3 parts blue, 4 parts red, and 1 part white paint.

Kimora needs 24 liters of purple paint to paint a mural. How much of each color will Kimora need?

Draw your own tape diagram if it helps with your thinking.

Blue (L)	Red (L)	White (L)	Total (L)
			24

**Explore More**

- 10** Create *three* equivalent ratios by filling in each blank using the numbers 0 to 9 only once.

$\square : \square$

$\square : \square$

$\square : 1\square$

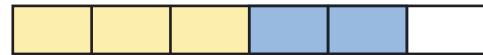
11 Synthesis

Here are the ingredients for a mango lassi drink.

Explain how the tape diagram represents this situation.

Ingredients for Mango Lassi

- 6 cups of mango
- 4 cups of yogurt
- 2 cups of milk



Things to Remember:

Name: Date: Period:

City Planning

Let's explore city planning using ratios.



Warm-Up

- 1** Imagine that you're moving to a new city.

What would be important to you when looking for a place to live?



Affordable and Market-Rate Housing

- 2** Many cities have a shortage of housing affordable enough for residents to have money left over for other necessities, like food and healthcare.

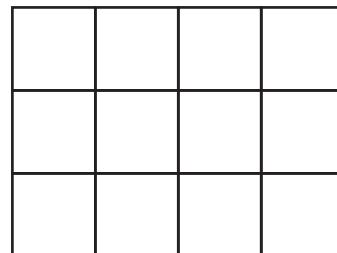
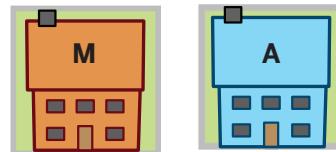
One approach cities use is to create *affordable housing units* that have cost limits.

Housing with no cost limit is called *market-rate*.

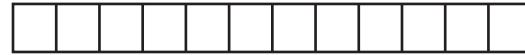
Design a neighborhood.

Write **M** in each square that you want to represent a market-rate house and **A** in each square that you want to represent an affordable house.

Market-Rate Affordable



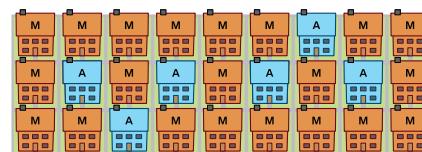
Create a tape diagram to represent your neighborhood.



- 3** Metropolis requires a 7 : 2 ratio of market-rate housing units to affordable housing units.

How does your neighborhood compare to Metropolis's requirement?

Metropolis Neighborhood



Market-Rate



Affordable

Affordable and Market-Rate Housing (continued)

- 4** Imagine you are a member of a city council, part of whose job is to help develop new neighborhoods. The city has a law that says each neighborhood must have a 7 : 2 ratio of market-rate units to affordable units.

The city wants to develop a new neighborhood with 36 units of land. How many units of each type are needed to meet the requirement?

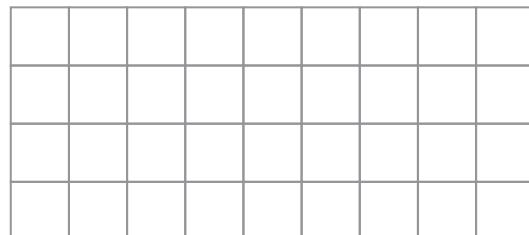
Use the tape diagram or the grid if it helps with your thinking.

Market-Rate	Affordable	Total
		36

Market-Rate



Affordable



- 5** Metropolis requires a 7 : 2 ratio of market-rate units to affordable units.

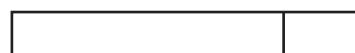
Does this neighborhood meet the requirement? Circle one.

Yes No I'm not sure

Explain your thinking. If you're not sure, what would help you be more sure?

Market-Rate	Affordable	Total
62	10	72

Market-Rate

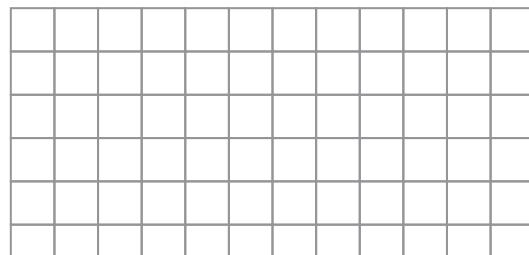


Affordable

- 6** How many units of each type of housing does this neighborhood need to meet Metropolis's requirement?

Use the tape diagram or the grid if it helps with your thinking.

Market-Rate	Affordable	Total
		72



Green Space

- 7** Urban green spaces, such as parks and gardens, give people space for physical activity, relaxation, peace, and an escape from the heat.

Here are two neighborhoods in Evergreen City.

Where would you prefer to live?
Circle one.

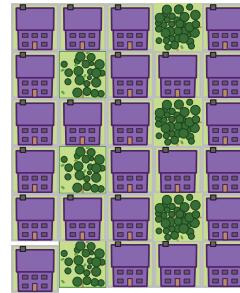
Neighborhood A

Neighborhood B

Explain your thinking.



Neighborhood A



Neighborhood B

- 8** Evergreen City requires a 3 : 5 ratio of units of green space to units of building space.

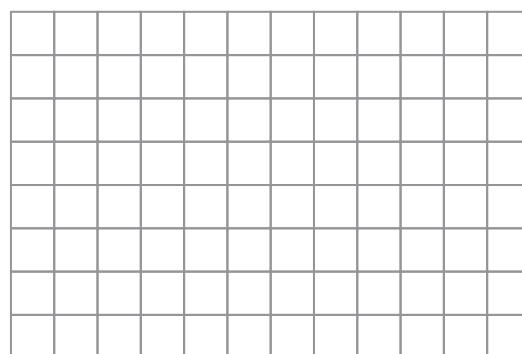
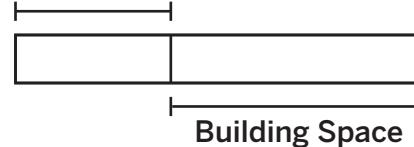
The city is developing 96 units of land for a new neighborhood.

How many of each type of space should the city plan for?

Use the tape diagram or the grid if it helps with your thinking.

Green Space	Building Space	Total
		96

Green Space



Green Space (continued)

- 9** Overall, Evergreen City requires a 4 : 1 : 3 ratio of market-rate housing to affordable housing to green space.

Here are 24 units of land.

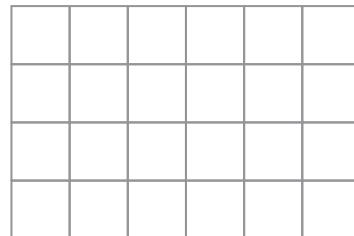
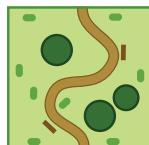
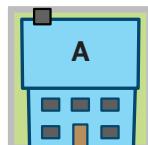
Design a neighborhood that meets Evergreen City's requirements.

Check your work using the digital activity.

Market-Rate

Affordable

Green



- 10** This neighborhood in Evergreen City meets the requirement of a 4 : 1 : 3 ratio of market-rate housing to affordable housing to green space.

However, residents claim the neighborhood is not fair.

- a** Why might the residents feel it's not fair?

Market-Rate

Green Space

Affordable



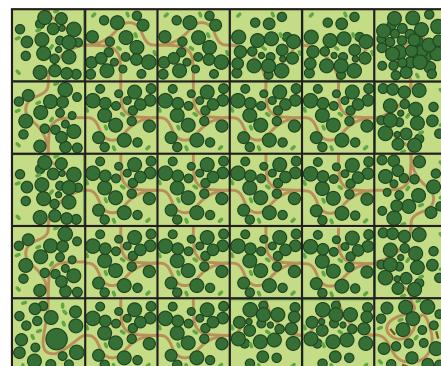
- b** What changes do you think should be made?

11 Synthesis

Des-Town requires a 3 : 2 ratio of building space to green space.

Explain how a city planner might determine how many units of building space can be developed in this neighborhood.

Draw on the diagram if it helps with your thinking.



Things to Remember:

Many Measurements

Let's connect units of measure with everyday objects.



Warm-Up

1. Which do you think is taller? Circle one.

A coconut

A pineapple

2. Which do you think is larger? Circle one.

A grapefruit

A plum

3. Which do you think is heavier? Circle one.

A cherry

A grape

Activity**1**

Name: Date: Period:

Describe It

4.  **Discuss:** Use words, drawings, hand gestures, familiar objects, or other strategies to answer the question: *How much is?*

1 foot 1 meter 1 gallon 1 millimeter
 1 cup 1 square foot 1 yard 1 pound

5. Which measurements were *less* complicated to describe? Which measurements were *more* complicated to describe?

Less Complicated	More Complicated

6. Sort the Activity 1 Cards based on whether they measure length, volume, or weight. There will be four cards in each group.

Length	Volume	Weight

7. Sort the measurements in each group from the *smallest* unit to the *largest* unit.

	Smallest Unit		Largest Unit
Length			
Volume			
Weight			

Match It

- 8.** Match each Activity 2 Card with the unit of measurement that best represents it.

1 Kilogram

Card

1 Ounce

Card

1 Millimeter

Card

1 Mile

Card

1 Liter

Card

1 Gram

Card

1 Kilometer

Card

1 Pound

Card

1 Cup

Card

1 Milliliter

Card

1 Gallon

Card

1 Centimeter

Card

- 9.**  **Discuss:** Choose one of the measurements from Problem 8. What else could you measure with this unit of measurement?

Explore More

- 10.** Here are four unit conversions arranged by what they're measuring. Add any other unit conversions you can think of for each category.

Length	Weight
1 foot = 12 inches	1 kilogram = 1000 grams
Volume	Time
1 gallon = 4 quarts	1 hour = 60 minutes

Synthesis

11. a List several things you could measure about this can.

- b What units would you use to measure each of those things?



Things to Remember:

Describe It

 **Directions:** Make one copy per pair of students. Then pre-cut the 12 cards and give each pair of students one set.

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Card 1

1 kilogram (kg)

Card 2

1 ounce (oz)

Card 3

1 millimeter (mm)

Card 4

1 mile (mi)

Card 5

1 liter (L)

Card 6

1 gram (g)

Card 7

1 kilometer (km)

Card 8

1 pound (lb)

Card 9

1 cup

Card 10

1 milliliter (mL)

Card 11

1 gallon (gal)

Card 12

1 centimeter (cm)

Match It

 **Directions:** Make one copy per pair of students. Then pre-cut the 12 cards and give each pair of students one set.

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Card A

Distance walked in 10 minutes

Card B

Weight of a paper clip

Card C

Thickness of a dime

Card D

Volume of milk in a large milk jug

Card E

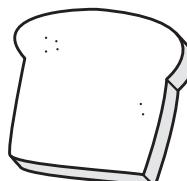
Weight of a hooded sweatshirt

Card F

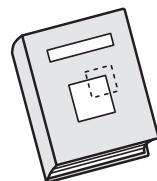
Volume of soda in a large soda bottle that is half full

Card G

Distance ran in 10 minutes

Card H

Weight of a slice of bread

Card I

Weight of a textbook

Card J

Volume of water in a raindrop



Width of a pinky finger

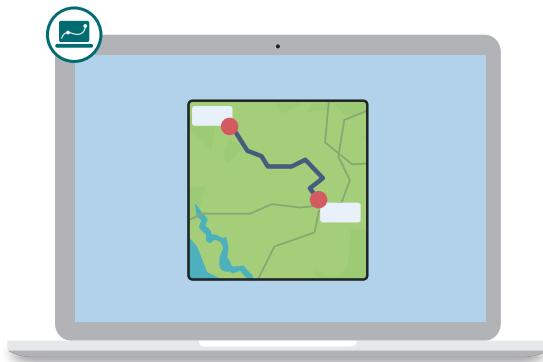
Card L

Volume of milk in a school milk carton

Name: Date: Period:

Pen Pals

Let's compare measurements in different units.



Warm-Up

- 1** Four pen pals share letters with each other.

Discuss: What do you notice? What do you wonder?

Name	Eva	Ayaan	Thiago	Binta
Country	United States	India	Brazil	Liberia
Favorite Food				
Bubble tea	Mango lassi	Quindim	Spaghetti	
Favorite Animal				
Horse	Dog	Horse	Bird	
Favorite Sport				
Football	Cricket	Futebol	Football	

Traveling to School

- 2** The pen pals discuss how far they each live from school. Use your best estimates to order the pen pals from *closest* to *farthest* from school.

Thiago: Brazil



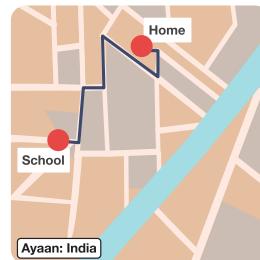
20 kilometers

Eva: United States



2,000 feet

Ayaan: India



900 meters

Binta: Liberia



15 miles

Closest**Farthest**

- 3** Binta lives 15 miles from her school in Liberia. Thiago lives 20 kilometers from his school in Brazil.

Who lives closer to their school?
Circle one and explain your thinking.

Binta

Thiago

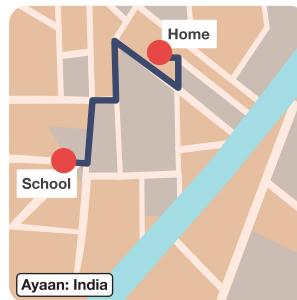
About the
same distance**8 kilometers ≈ 5 miles**

- 4** Ayaan lives 900 meters from his school in India. Eva lives 2,000 feet from her school in the United States.

Who lives closer to their school?
Circle one and explain your thinking.

Ayaan

Eva

About the
same distance**3 meters ≈ 10 feet**

Weighing Strategies

- 5** Thiago's horse eats about 6 kilograms of hay per day.

Eva wants to know how many pounds that is.

About how many pounds is 6 kilograms?

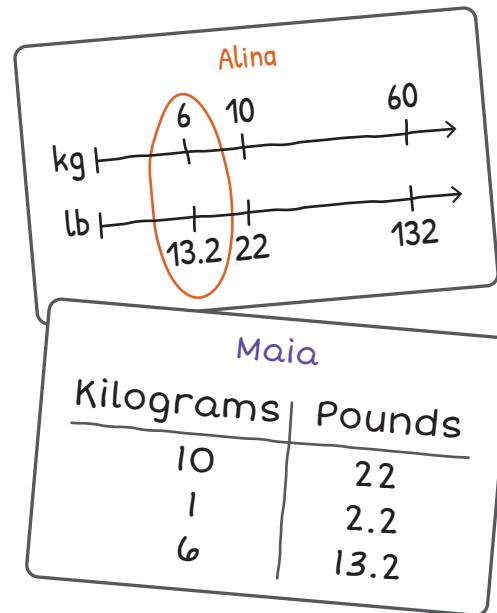


10 kilograms \approx 22 pounds

- 6** Alina and Maia both determined how many pounds of hay Thiago's horse eats per day.

Alina used a double number line and Maia used a table.

Choose a student and explain their thinking.



Favorite Things

- 7** Binta decides to make Ayaan's recipe for mango lassi.

The recipe calls for 135 milliliters of milk.

About how many tablespoons of milk should Binta use?

MANGO LASSI

Ingredients

Serves 2 people

- 250 milliliters mango pulp
- 240 milliliters yogurt
- 135 milliliters milk
- 20 grams sugar
- 1 gram cardamom powder

30 milliliters ≈ 2 tablespoons

- 8** Thiago decides to make Binta's recipe for spaghetti.

The recipe calls for 450 grams of bell pepper.

About how many ounces of bell pepper should Thiago use?

SPAGHETTI

Ingredients

- 1 box of spaghetti
- 450 grams of bell pepper
- 200 grams of tomato
- 150 grams of onion
- 3 bouillon cubes
- 400 grams of ground beef
- 800 grams of Italian sausage
- Habanero peppers, curry powder, ginger, salt, oil

200 grams ≈ 7 ounces

Explore More

- 9** People have known for over 2,000 years that Earth is round, but it took a long time to discover how big it is.

A Greek mathematician named Eratosthenes was the first known person to calculate the distance around Earth's equator. In about 240 BCE, he calculated the distance around Earth's equator to be about 250,000 stadia using an estimated distance from Alexandria to Syene, along with the lengths of shadows.

The actual distance is about 24,901 miles.

What is the difference, in miles, between Eratosthenes's calculation and the actual distance? Explain your thinking.



10 Synthesis

Describe a strategy for converting a measurement from one unit to another.

Use the examples if they help you with your thinking.

8 kilometers \approx 5 miles

200 grams \approx 7 ounces

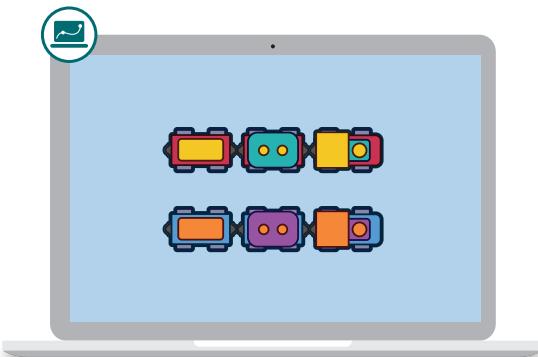
30 milliliters \approx 2 tablespoons

Things to Remember:

Name: Date: Period:

Model Trains

Let's use ratios to compare speeds.



Warm-Up

- 1** Which one doesn't belong? Circle one.

5 miles in
15 minutes

20 miles
per 1 hour

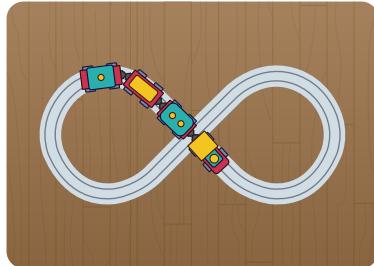
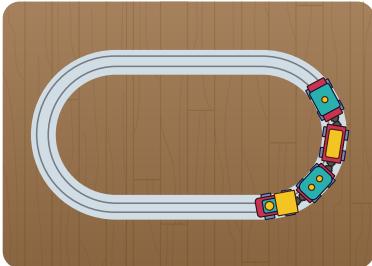
3 minutes
per mile

32 kilometers
per 1 hour

Explain your thinking.

How Fast?

- 2** A children's museum has three types of model train sets for students to build and play with. Let's watch how the train moves on each track.

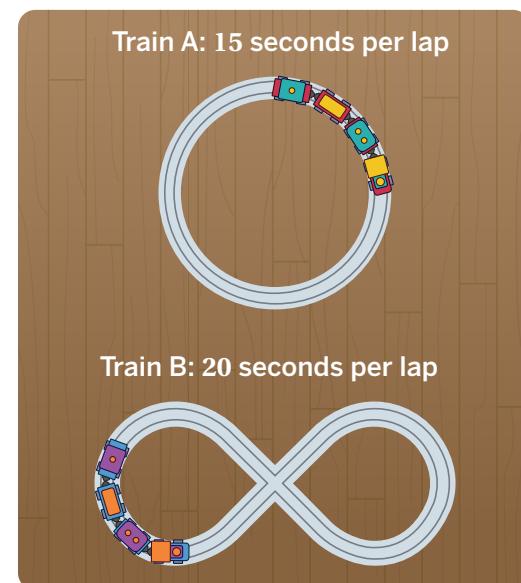


- 3** Here are trains from two students.

Which train is faster? Circle one.

Train A Train B Not enough information

Explain your thinking. If you don't have enough information, what information would help you determine which train travels faster?



- 4** Here is a track. It is 325 centimeters long.

This train takes 10 seconds per lap.

What is its speed in centimeters per second?



How Fast? (continued)

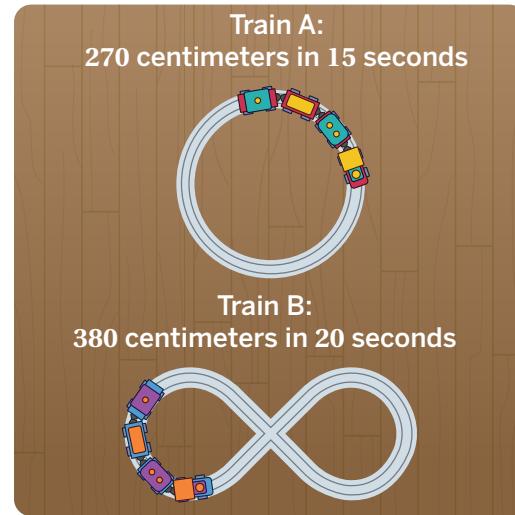
- 5** Which train is faster? Circle one.

Train A

Train B

They go the same speed

Explain your thinking.



- 6** Amoli and Tiam used different strategies to determine which train was faster.



Discuss: How are their strategies alike? How are they different?

Amoli

Train A

$$270 \div 15 = 18 \text{ cm per sec}$$

Train B

$$380 \div 20 = 19 \text{ cm per sec}$$

Train B is faster.

Tiam

Train A Train B

cm	Sec	cm	Sec
270	15	380	20
1080	60	1140	60

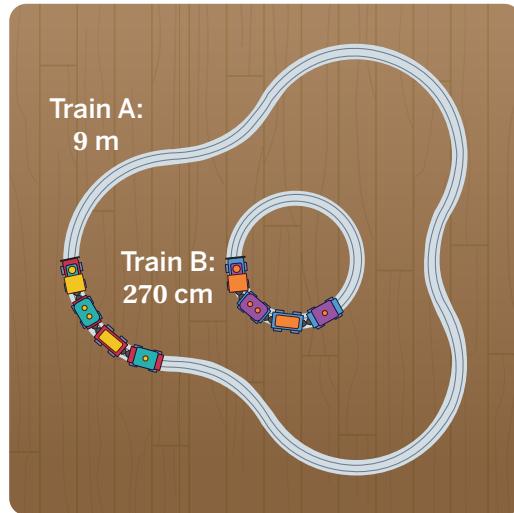
Train B is faster.

Which is Faster?

- 7** Here are two trains. They each complete a lap in 20 seconds.

What is each train's speed in centimeters per second?

Speed (centimeters per second)	
Train A	
Train B	



- 8** Here are distances and times for four model trains.

Order the trains by speed.

- A. 3.25 meters in 1 minute
- B. 3.25 meters in 20 seconds
- C. 270 centimeters in 20 seconds
- D. 325 centimeters in 30 seconds

$$1 \text{ meter} = 100 \text{ centimeters}$$

$$1 \text{ minute} = 60 \text{ seconds}$$

--	--	--	--

Slowest

Fastest

Explore More

- 9** A train's speed is 60 centimeters per second.

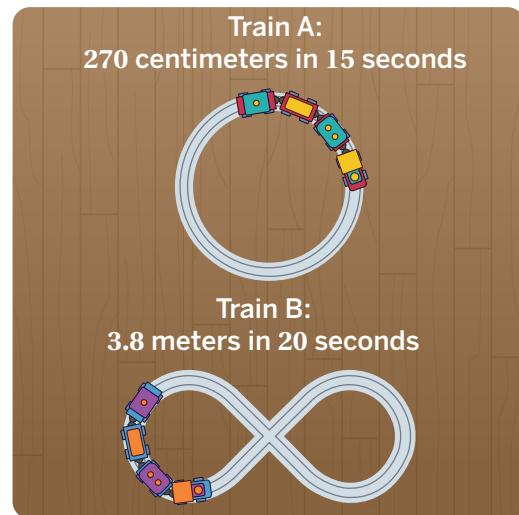
Write a track length. Then determine the number of laps the train can complete in 10 seconds.

Track Length (cm)	Laps in 10 Seconds

10 Synthesis

Describe two strategies for determining which of two trains is faster.

Use the examples if they help with your thinking.



Things to Remember:

Name: Date: Period:

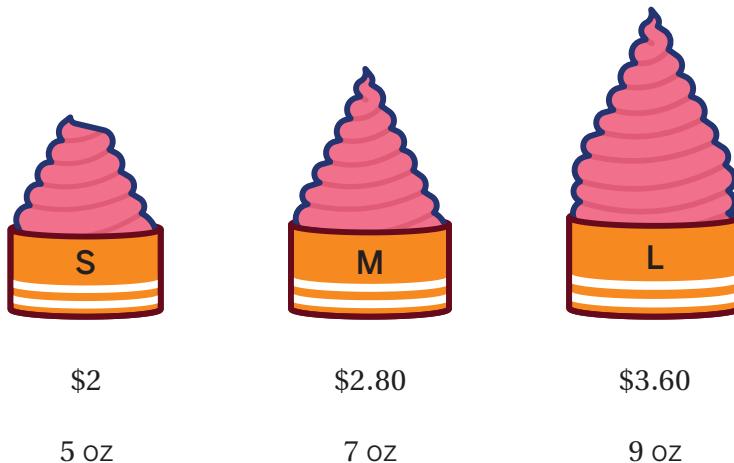
Soft Serve

Let's compare soft serve prices using unit rates.



Warm-Up

- 1** Take a look at the prices for different sizes of soft serve sold at a store.



- b** **Discuss:** Which size offers the best deal?

Two Unit Rates

- 2** Kala notices that soft serve costs the same per ounce no matter what size you get.

She suggests that the store put the rate on the menu.

How much does soft serve cost per ounce?

 S	 M	 L
\$2.00 5 oz	\$2.80 7 oz	\$3.60 9 oz
Make Your Own \$ __ . __ per oz		



- 3** The store added the price per ounce, or **unit price**, to the menu.

A customer asks for 8 ounces of soft serve.

How much will this cost?

- 4** A new customer comes in with \$3 and wants to spend it all on soft serve.

How many ounces can they get for \$3?

Two Unit Rates (continued)

- 5** Here is how Neena figured out how much soft serve you can get for \$3.

a  **Discuss:** What was Neena's strategy?

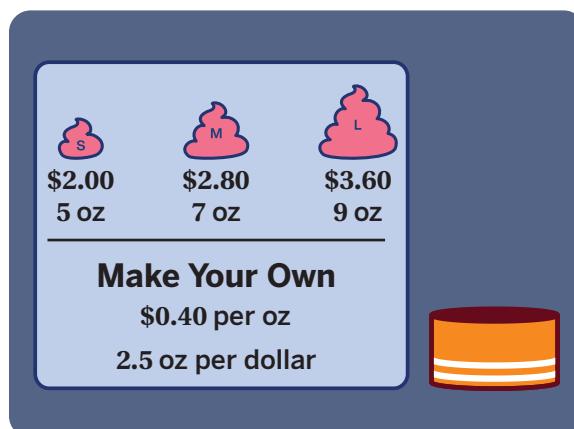
Cost (dollars)	Weight (ounces)
2	5
$\div 2$	$\div 2$
1	2.5
$\times 3$	$\times 3$
3	7.5

- b** Explain or show where you can see *ounces per dollar* in Neena's work.

- 6** The store's menu now includes *both* unit rates.

A new customer comes in with \$7 and wants to spend it all on soft serve.

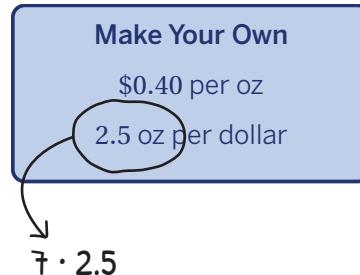
How much soft serve can they get for \$7?



The image shows a soft serve menu with three options: Small (S) for \$2.00 and 5 oz, Medium (M) for \$2.80 and 7 oz, and Large (L) for \$3.60 and 9 oz. Below the menu, there is a section titled "Make Your Own" with the price "\$0.40 per oz" and the rate "2.5 oz per dollar". To the right of this text is a drawing of a soft serve container with orange and white stripes.

- 7** Here is how Tamiya figured out how much soft serve you can get for \$7.

How do you think Tamiya knew which unit rate to use?



The diagram shows a box containing the "Make Your Own" information from the previous menu. An arrow points from the text "2.5 oz per dollar" down to the multiplication expression $7 \cdot 2.5$.

New Flavors

- 8** The store offers a new flavor, Swirl, with this pricing: \$5 for every 4 ounces.

a How much does Swirl cost per ounce?



b How many ounces can you get per dollar?

- 9** How much does 7 ounces of Swirl cost?

Explain your thinking.

- 10** Match each rate with either chocolate or vanilla.

\$2
per ounce

\$0.50
per ounce

2 ounces
per dollar

$\frac{1}{2}$ ounces per
dollar

\$9 for
4.5 ounces

Chocolate



2 oz for \$4

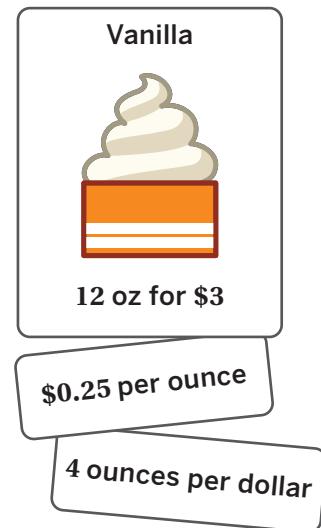
Vanilla



8 oz for \$4

11 Synthesis

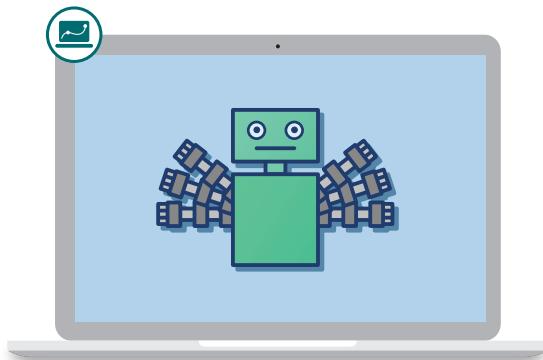
Explain how to calculate the two unit rates for vanilla soft serve.



Things to Remember:

Welcome to the Robot Factory

Let's determine unknown values using unit rates.



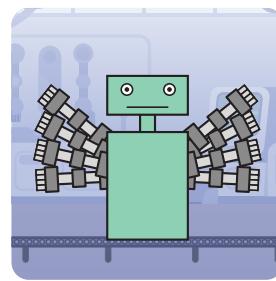
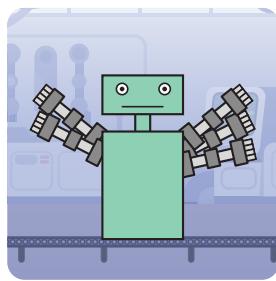
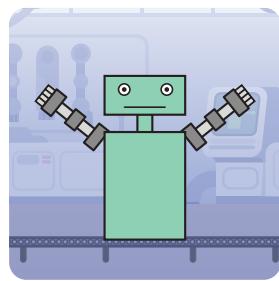
Warm-Up

- 1** This table shows some lengths in both inches and feet.

What are *three* things you notice about the table?

Length (ft)	Length (in.)
1	12
3	36
5	60
10	120

- 2** Welcome to the Robot Factory! Take a look at how many arms the robot has in each image.



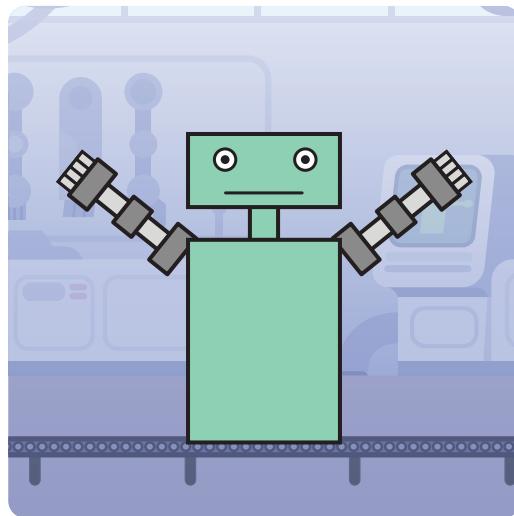
Arms and Fingers

- 3** This robot has 2 arms and 8 fingers.

Here are some other robots with different numbers of arms.

Complete the table to show the number of fingers on each robot.

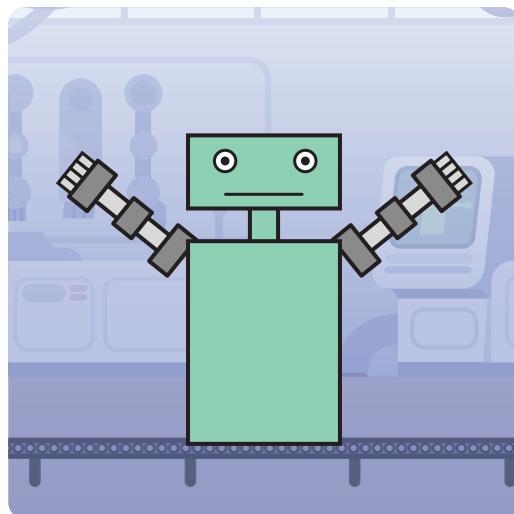
Arms	Fingers
2	8
7	
3	
9	



- 4** A new row has been added to the table.

How many arms go with this many fingers?

Arms	Fingers
2	8
	44

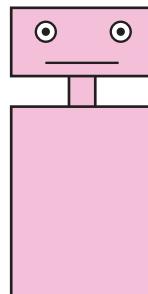
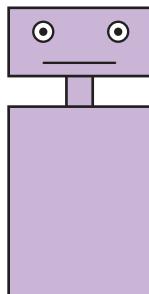
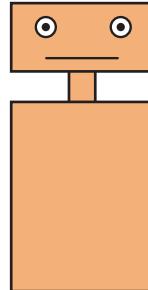
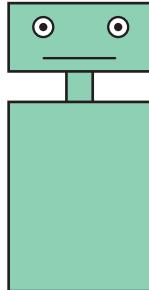


- 5** Choose one question and write your response.

- If you know the number of *fingers*, how can you determine the number of arms?
- If you know the number of *arms*, how can you determine the number of fingers?

Painting Robots

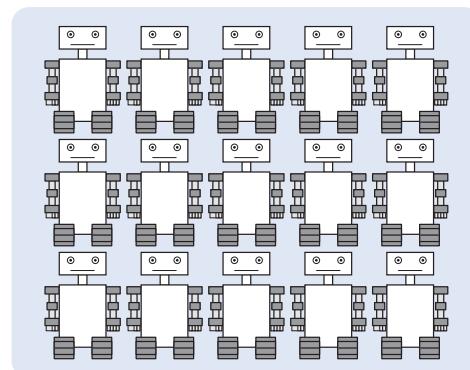
- 6** Choose a color to paint your robot.



- 7** 6 robots need 2 gallons of paint.

Complete the table.

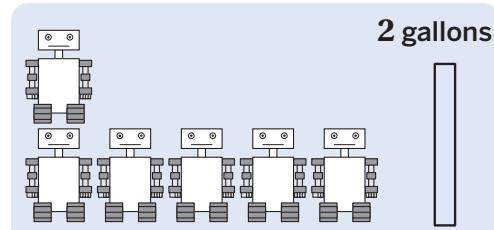
Number of Robots	Amount of Paint (gal)
6	2
15	
21	
11	



Painting Robots (continued)

- 8** Write instructions for how you could determine the amount of paint needed for *any* number of robots.

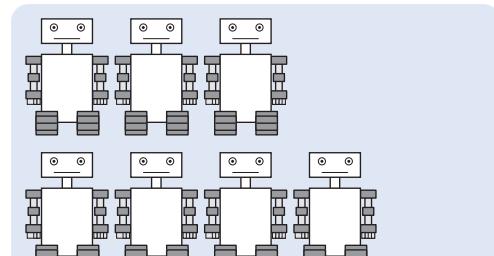
Use your table from the previous problem if it helps with your thinking.



- 9** Here are some extra-large robots. 4 robots need 10 gallons of paint.

Complete the table.

Number of Robots	Amount of Paint (gal)
4	10
7	
9	
13	

**Explore More**

- 10** Joud wrote down the amount of paint and the painting time needed for different numbers of robots. Some of the values are missing. Complete the table.

Number of Robots	Amount of Paint (gal)	Painting Time (min)
5	2	
	5	10
15		12
	1	

11 Synthesis

Explain how you can use a table of equivalent ratios to determine unknown values, like the amount of paint needed for different numbers of robots.

Use this table if it helps with your thinking.

Number of Robots	Amount of Paint (gal)
1	$\frac{1}{3}$
6	2
33	11
18	6
9	3

Things to Remember:

Name: Date: Period:

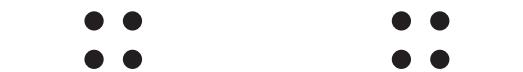


Flour Planner

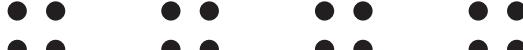
Let's think about fractions by using drawings and diagrams to ask, "How many groups?"

Warm-Up

- 1** **a** How many dots are in this image?



- b** Explain or show how you saw them.

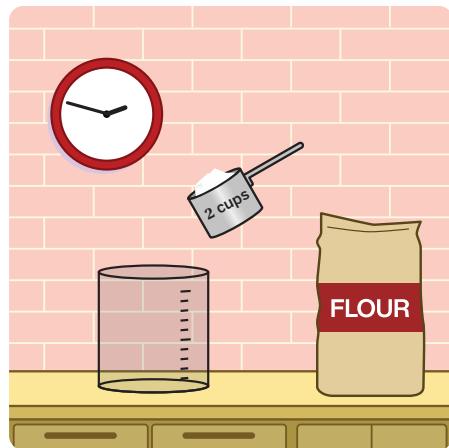


Fractional Scoops

- 2** Tres leches cake is a popular dessert in Mexico and Central America that's made with three kinds of milk.

Alexis needs 6 cups of flour to make tres leches cake but only has a 2-cup measuring scoop.

How many scoops does Alexis need?



- 3** Circle an equation that you could use to determine how many 2-cup scoops make 6 cups of flour.

$$6 \cdot ? = 2$$

$$6 \div 2 = ?$$

$$2 \div 6 = ?$$

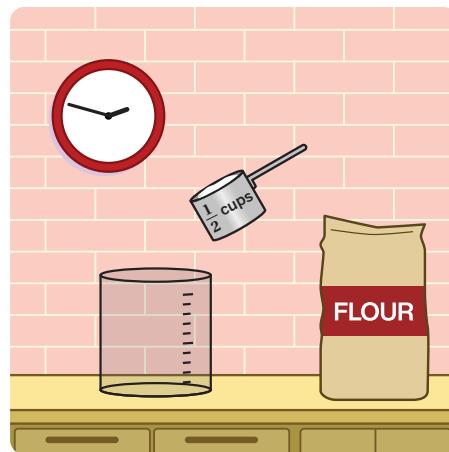
$$2 \cdot ? = 6$$

Explain your thinking.

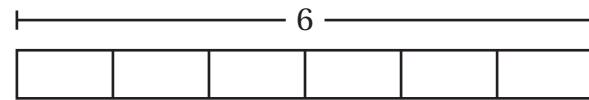
Fractional Scoops (continued)

- 4** LaShawn also needs 6 cups of flour to make tres leches cake but only has a $\frac{1}{2}$ -cup measuring scoop.

How many scoops does LaShawn need?



- 5** **a** Let's watch how LaShawn determined the number of $\frac{1}{2}$ -cup scoops needed for 6 cups of flour.



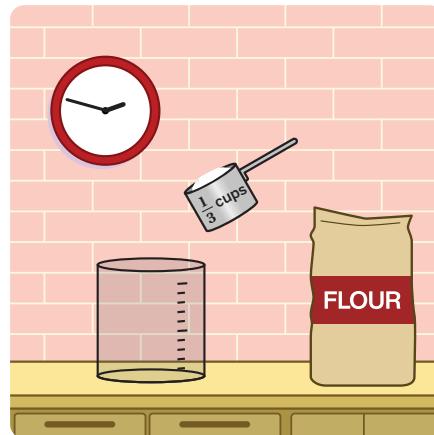
- b** Explain how this tape diagram helped LaShawn decide to use 12 scoops.

A Bigger Scoop

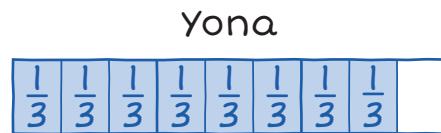
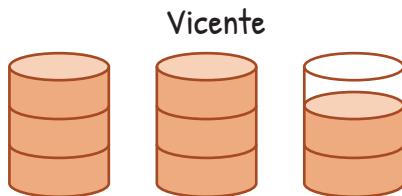
- 6** Sirnee is a sweet dish that is often made for Islamic celebratory feasts.

Hamza needs $2\frac{2}{3}$ cups of flour to make sirnee but only has a $\frac{1}{3}$ -cup measuring scoop.

How many scoops does Hamza need?



- 7** Vicente and Yona each sketched a diagram to determine how many $\frac{1}{3}$ -cup scoops they need to measure $2\frac{2}{3}$ cups of flour.



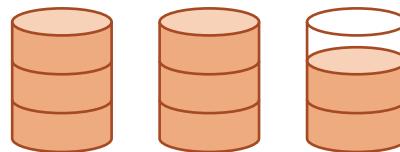
Discuss: How could each diagram help us calculate the number of scoops needed?

A Bigger Scoop (continued)

- 8** Hamza found a $\frac{2}{3}$ -cup measuring scoop to use to make sirnee.

How many of these scoops would Hamza need to measure $2\frac{2}{3}$ cups of flour?

Use the cups diagram and the tape diagram if they help with your thinking.



Explain your thinking.

- 9** Group together the choices that represent the same situation. Two choices will have no match.

$$3 \div \frac{3}{4} = ?$$

$$\frac{3}{4} \div 3 = ?$$

$$\frac{3}{4} \cdot ? = 3$$

$$3 \cdot ? = \frac{3}{4}$$

$$? \div \frac{3}{4} = 3$$

$$\frac{3}{4} \cdot 3 = ?$$

4 scoops

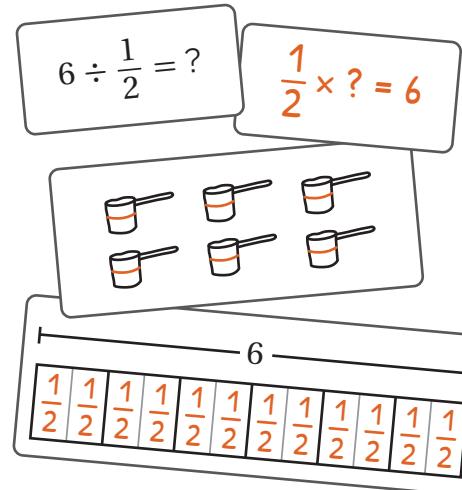
$\frac{1}{4}$ scoops

Alexis needs 3 cups of flour and has a $\frac{3}{4}$ -cup measuring scoop.

LaShawn needs $\frac{3}{4}$ cups of flour and has a 3-cup measuring scoop.

10 Synthesis

How can you use an equation or a diagram to determine how many $\frac{1}{2}$ -cup scoops you need to make 6 cups?



Things to Remember:

Name: Date: Period:



Flower Planters

Let's use flower planters to answer the question
"How many in one group?"

Warm-Up

- 1** Order these expressions from *least* to *greatest* by the value of the quotient.

$12 \div 12$

$12 \div \frac{2}{3}$

$12 \div 1$

$12 \div 3$

$12 \div \frac{1}{4}$

.....
.....
.....
.....
.....

Least

Greatest

Plenty of Planters

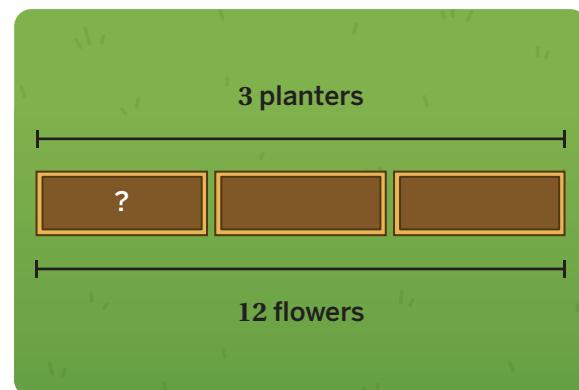
- 2** Write a story that could be represented by the expression $12 \div \frac{1}{3}$.

Draw a sketch if it helps you illustrate your story.

- 3** Brianna is planting flowers in the school garden.

12 flowers fill 3 small planters.

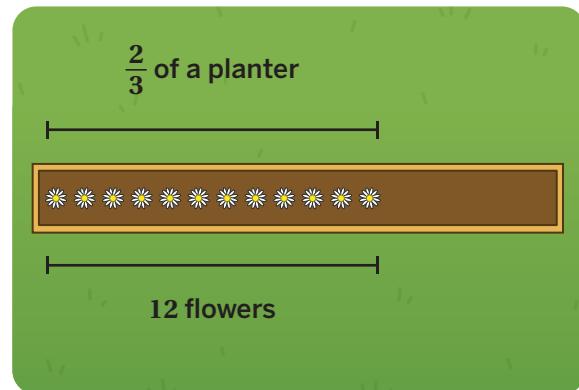
How many flowers fill 1 small planter?



- 4** Brianna also put flowers in a big planter.

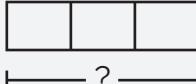
12 flowers fill $\frac{2}{3}$ of a big planter.

How many flowers fill 1 big planter?



Plenty of Planters (continued)

- 5** Match each representation with a question.

	12 flowers fill 3 planters. How many flowers fill 1 planter?	12 flowers fill $\frac{2}{3}$ of a planter. How many flowers fill 1 planter?
$12 \div 3 = ?$		
$12 \div \frac{2}{3} = ?$		
$\frac{2}{3} \cdot ? = 12$		
$3 \cdot ? = 12$		
$\overbrace{\hspace{1cm}}^{12}$  $\overbrace{\hspace{1cm}}^{?}$		
$\overbrace{\hspace{1cm}}^{12}$  $\overbrace{\hspace{1cm}}^{?}$		

- 6** How are these expressions alike?

How are they different?

Alike:

12 flowers fill 3 planters.
How many flowers
fill 1 planter?

$12 \div 3 = ?$

12 flowers fill $\frac{2}{3}$ planters.
How many flowers
fill 1 planter?

$12 \div \frac{2}{3} = ?$

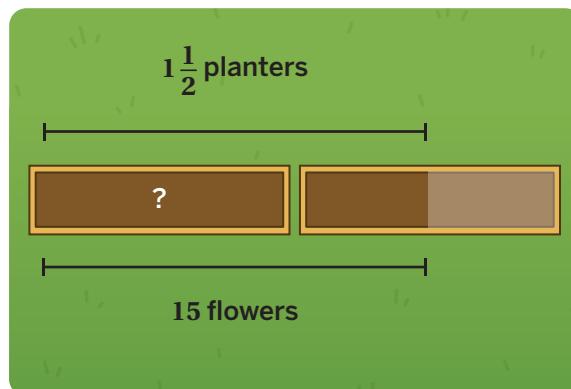
Different:

Practicing With Planters

- 7** Brianna has 15 flowers to put in these planters.

The flowers fill $1\frac{1}{2}$ planters.

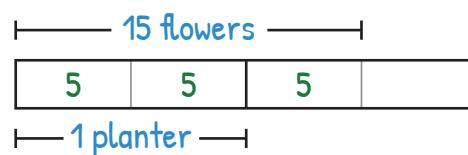
How many flowers fill 1 planter?



- 8** Here is a diagram Brianna made to calculate how many flowers fill 1 planter when 15 flowers fill $1\frac{1}{2}$ planters.

Explain how Brianna can use this diagram to help her answer the question.

$$15 \div 1\frac{1}{2} = ?$$



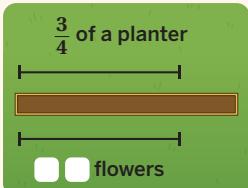
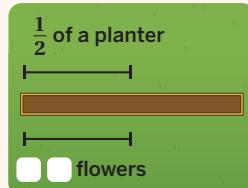
Practicing With Planters (continued)

- 9** Solve as many challenges as you have time for.

Situation	Diagram	How many flowers fill 1 planter?
a 8 flowers fill 4 planters.	<p>4 planters</p> <p>8 flowers</p>	
b 8 flowers fill $\frac{1}{3}$ of a planter.	<p>$\frac{1}{3}$ of a planter</p> <p>8 flowers</p>	
c 12 flowers fill $\frac{3}{4}$ of a planter.	<p>$\frac{3}{4}$ of a planter</p> <p>12 flowers</p>	
d 18 flowers fill $1\frac{1}{2}$ planters.	<p>$1\frac{1}{2}$ planters</p> <p>18 flowers</p>	
e 26 flowers fill $2\frac{8}{9}$ of a planter.	<p>$2\frac{8}{9}$ planters</p> <p>26 flowers</p>	

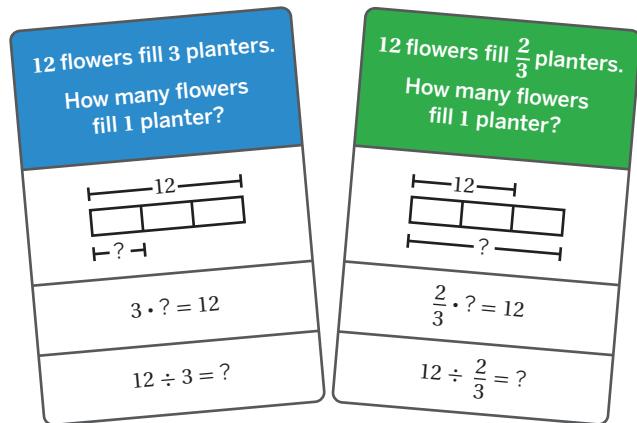
Explore More

- 10** Fill in each blank using the digits 0 to 9 only once, so that the same number of flowers fill each planter.



11 Synthesis

Describe how a tape diagram can represent a division problem.



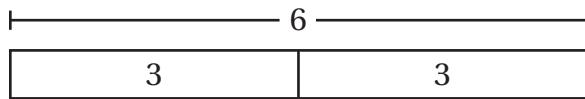
Things to Remember:

Connecting Tape Diagrams

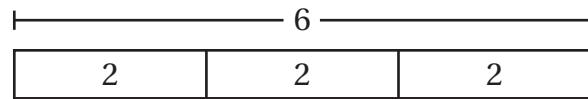
 **Directions:** Make one copy for the whole class. For classes with more than 36 students, create multiple copies. Then pre-cut the cards and give each student one card. For Round 3, both cards in each row must be distributed.

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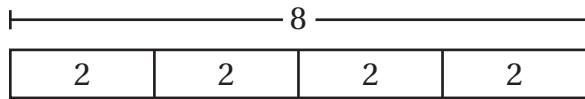
Card 1



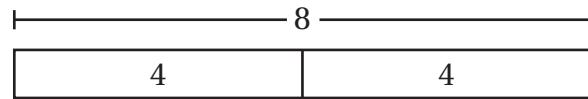
Card 2



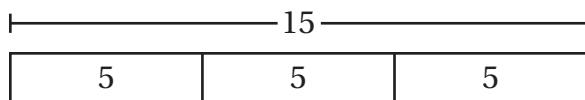
Card 3



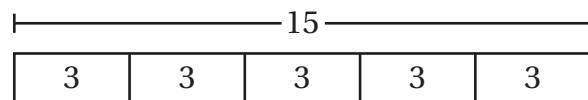
Card 4



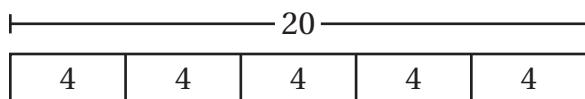
Card 5



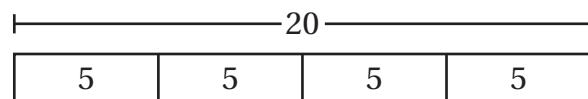
Card 6



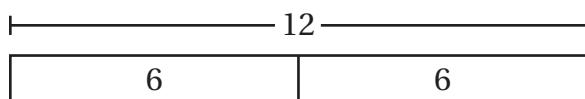
Card 7



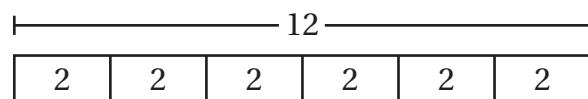
Card 8



Card 9



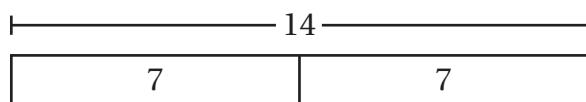
Card 10



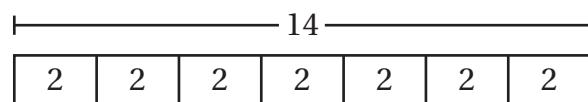
Connecting Tape Diagrams

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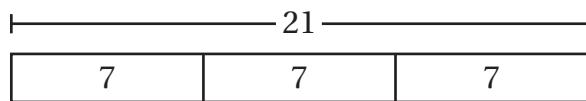
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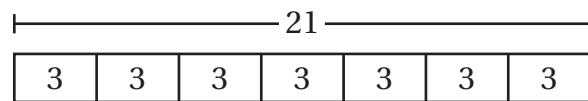
Card 12



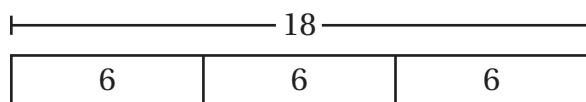
Card 13



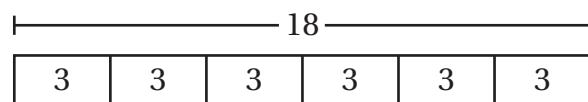
Card 14



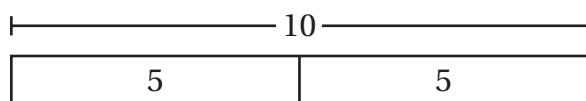
Card 15



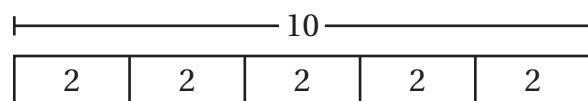
Card 16



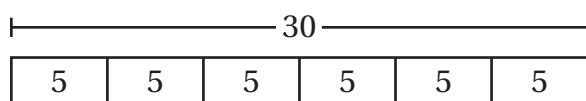
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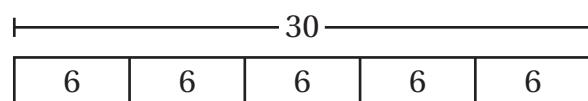
Card 18



Card 19



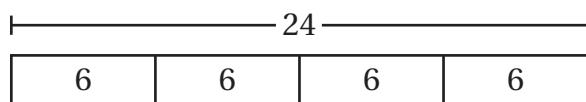
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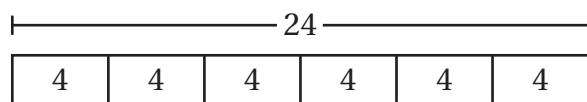
Connecting Tape Diagrams

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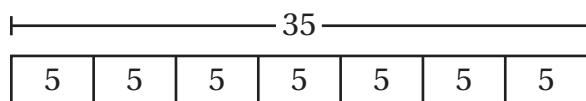
Card 21



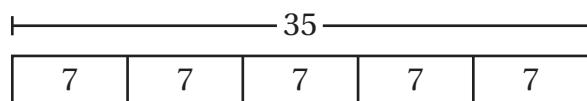
Card 22



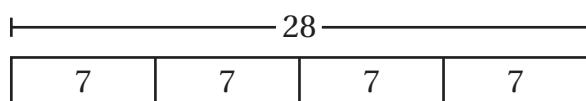
Card 23



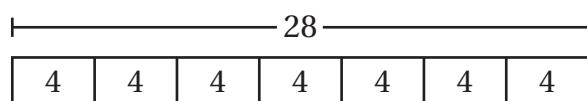
Card 24



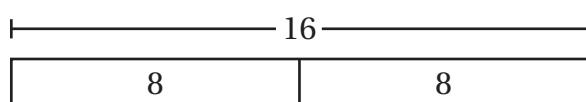
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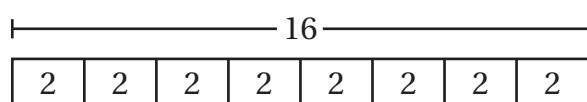
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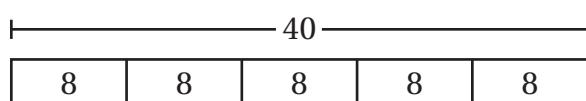
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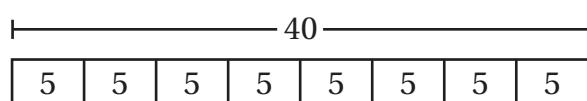
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Card 29



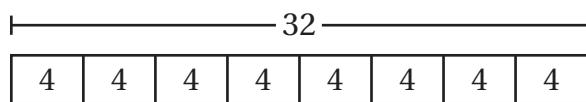
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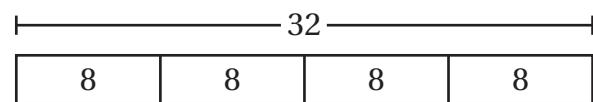
Connecting Tape Diagrams

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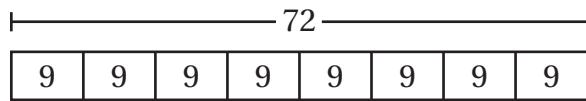
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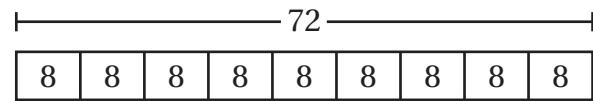
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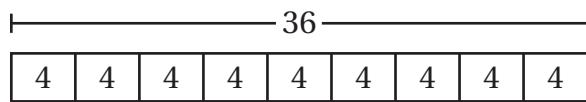
Card 33



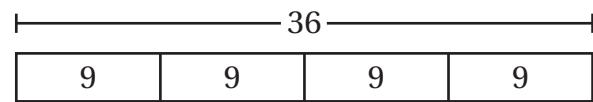
Card 34



Card 35



Card 36



Garden Bricks

Let's use tape diagrams to think about,
"How many groups?"



Warm-Up

Question

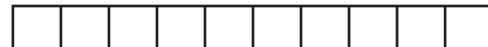
How many groups of $2\frac{1}{2}$ are in 10?

Expression

$$10 \div 2\frac{1}{2}$$

1. **Discuss:** How do you know that the expression represents the question?

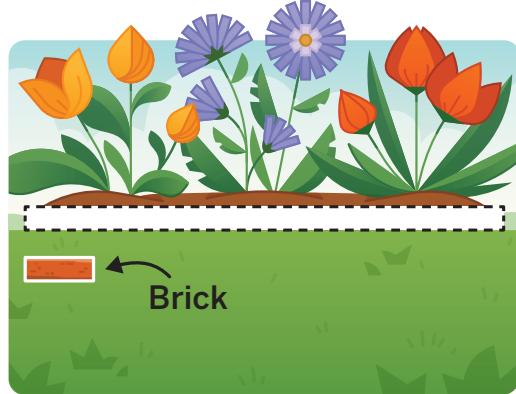
2. Use the tape diagram to answer the question.



How Many Bricks?

Deja and CK are upgrading their class gardens by placing bricks along the front of each garden.

3. The first garden is 4 feet long. Deja is using small bricks, which are $\frac{1}{3}$ of a foot long. How many small bricks does Deja need? Draw a tape diagram to show your thinking.



4. The second garden is also 4 feet long. CK is using large bricks, which are $\frac{2}{3}$ of a foot long. How many large bricks does CK need? Draw a tape diagram to show your thinking.

5. The third garden is 5 feet long. How many large bricks do Deja and CK need? Draw a tape diagram to show your thinking.

How Many Bricks? (continued)

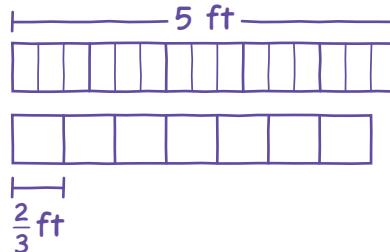
6. Deja and CK are working on Problem 5. Each student's work contains accurate and inaccurate parts.

Deja

$$5 \div \frac{2}{3}$$

" $5 \div \frac{2}{3}$ is less than 5
because I'm dividing."

CK



"I need $7\frac{1}{3}$ bricks because
there are 7 whole bricks
and $\frac{1}{3}$ left over."

a

Discuss: How are their methods alike? How are they different?

b

Pick one student's work. What do you think they did well? What question could you ask to help them understand their mistake?

7. CK wrote $4\frac{1}{4} \div \frac{3}{4}$ to help answer a different question about bricks and gardens.

a

Explain what $4\frac{1}{4}$ and $\frac{3}{4}$ mean in this situation.

b

Draw a tape diagram and use it to determine the value of $4\frac{1}{4} \div \frac{3}{4}$.

**Activity
2**

Name: _____ Date: _____ Period: _____

What's Missing?

8. Complete each row in the table.

Expression	Tape Diagram	Quotient
a $6 \div \frac{3}{4}$		
b		
c $2 \div \frac{3}{5}$		
d		7

Explore More

9. a Write a division expression.
 b On a separate piece of paper, draw a tape diagram that represents your expression.
 c Trade tape diagrams with a partner. Determine their division expression and calculate its quotient.

Synthesis

10. a) Draw a tape diagram to represent $3 \div \frac{2}{3}$.
- b) Describe how you can use the tape diagram to help determine the value of $3 \div \frac{2}{3}$.

Things to Remember:

Name: Date: Period:



Fill the Gap

Let's use garden bricks to determine whether the number of groups is greater or less than 1.

Warm-Up

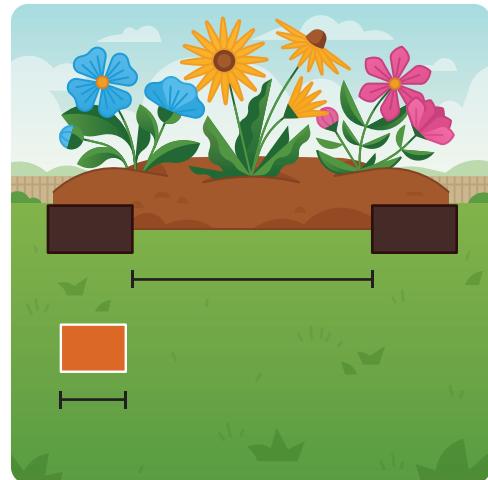
1-2 Complete the table.

Tape Diagram	Fraction	Mixed Number
	$\frac{9}{4}$	
		$5\frac{1}{2}$
	$\frac{12}{5}$	
		$4\frac{1}{9}$

More or Less Than One Group

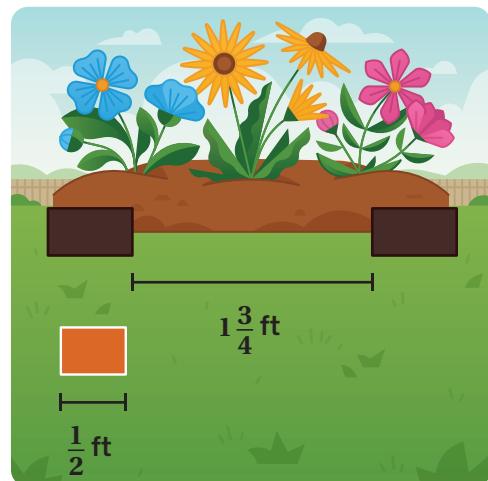
- 3** Deja is filling a gap along the front of this garden.

About how many bricks does Deja need?



- 4** The gap in Deja's garden is $1\frac{3}{4}$ feet long. Each brick is $\frac{1}{2}$ of a foot long.

How many bricks does Deja need to fill the gap?



- 5** Deja and CK each wrote an expression to represent the number of bricks needed to fill the gap.

Deja wrote $1\frac{3}{4} \div \frac{1}{2}$. CK wrote $\frac{1}{2} \div 1\frac{3}{4}$. Whose expression is correct? Circle one.

Deja's

CK's

Both

Neither

Explain your thinking.

More or Less Than One Group (continued)

6 Here is CK's expression: $\frac{1}{2} \div 1\frac{3}{4}$.

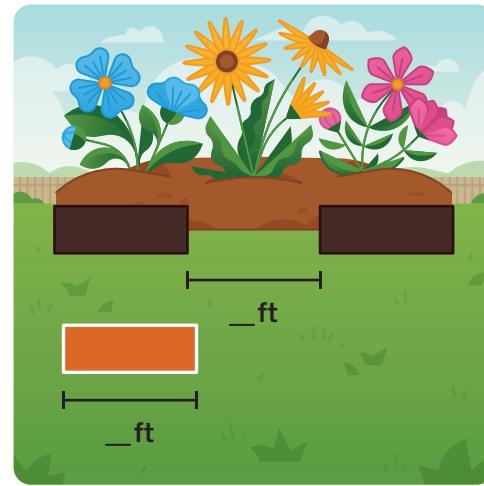
- a) Draw a sketch to represent this expression in the garden situation.

- b) The value of $\frac{1}{2} \div 1\frac{3}{4}$ is:

Less than 1

Greater than 1

Equal to 1



7 Sort these expressions by the value of their quotient.

$$2\frac{1}{4} \div \frac{3}{4}$$

$$\frac{1}{4} \div \frac{3}{8}$$

$$\frac{3}{8} \div \frac{1}{4}$$

$$1 \div \frac{1}{4}$$

$$\frac{5}{4} \div 1\frac{1}{4}$$

$$\frac{3}{8} \div \frac{3}{8}$$

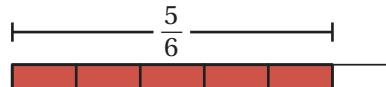
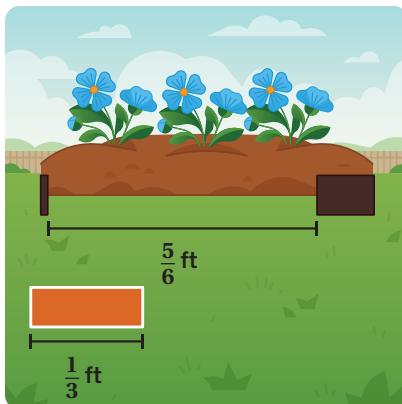
$$1 \div 4$$

Less than 1	Greater than 1	Equal to 1

Equal-Sized Pieces

- 8** Here is a new expression: $\frac{5}{6} \div \frac{1}{3}$.

Use the garden or tape diagram to determine its value.



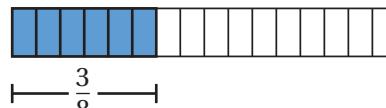
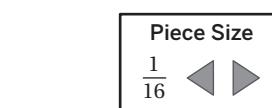
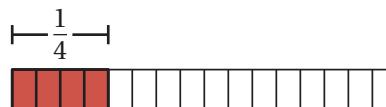
- 9** What is $\frac{1}{4} \div \frac{3}{8}$?



- 10** Deja broke $\frac{1}{4}$ and $\frac{3}{8}$ into $\frac{1}{16}$ -sized pieces.

- a** **Discuss:** How does Deja's strategy show that $\frac{1}{4} \div \frac{3}{8} = \frac{4}{6}$?

- b** Let's determine other helpful ways to break up $\frac{1}{4}$ and $\frac{3}{8}$.

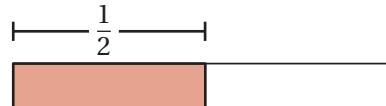


The Return of Common Denominators

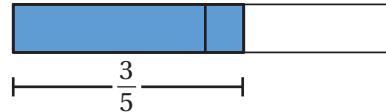
11

- a** Let's look at how we can break these fractions into equal pieces and set up a *common denominator*.

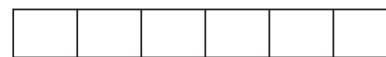
- b** Draw on the diagram to break the fractions into equal pieces. Then calculate $\frac{1}{2} \div \frac{3}{5}$.



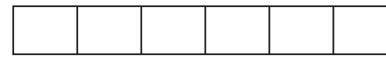
Piece Size
 $\frac{1}{2}$

**12** Calculate $\frac{2}{3} \div \frac{1}{2}$.

Use the diagram if it helps you with your thinking.

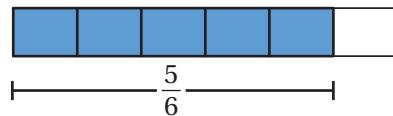


Piece Size
 $\frac{1}{6}$



13 Synthesis

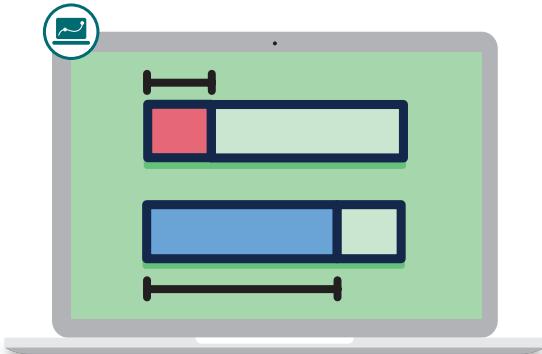
Explain how you can show that $\frac{2}{3} \div \frac{5}{6} = \frac{4}{5}$. Use the tape diagrams if they help with your thinking.



Things to Remember:

Break It Down

Let's divide fractions by rewriting with common denominators.



Warm-Up

1 Calculate the following:

a $12 \div 3$

b $\frac{12}{5} \div \frac{3}{5}$

How are these problems alike? How are they different?

Alike:

Different:

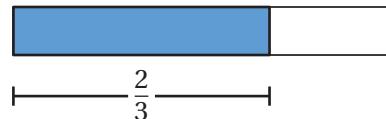
Common Denominators

- 2** The value of $\frac{1}{6} \div \frac{2}{3}$ is:

Less than 1 Greater than 1 Equal to 1

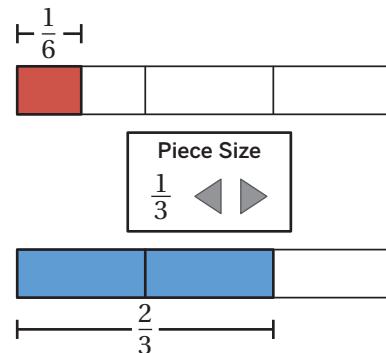


Explain your thinking.



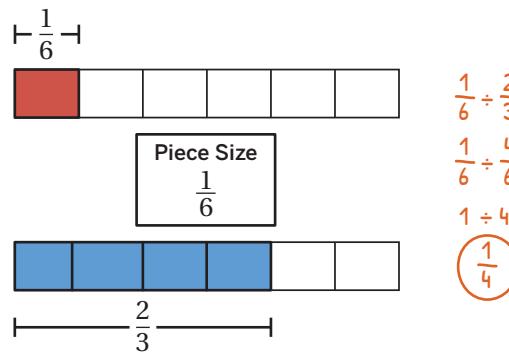
- 3** **a** Let's look at how we can break both of these fractions into equal pieces and make common denominators.

- b** Draw on the diagram to break the fractions into equal pieces. Then calculate $\frac{1}{6} \div \frac{2}{3}$.



- 4** Here's how Ahmed calculated $\frac{1}{6} \div \frac{2}{3}$.

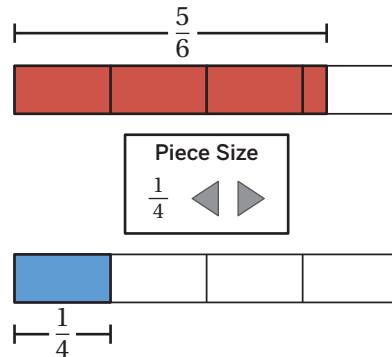
Discuss: Why do you think Ahmed used $\frac{1}{6}$ -sized pieces?



Common Denominators (continued)

- 5** **a** Let's look at how to make common denominators.

- b** Draw on the diagram to break the fractions into equal pieces. Then calculate $\frac{5}{6} \div \frac{1}{4}$.



- 6** Ahmed and Zoe calculated the previous problem without a diagram. Their calculations are both correct.

How are their strategies alike? How are they different?

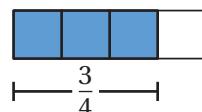
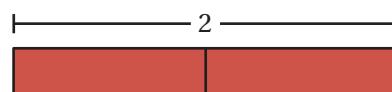
Alike:

Ahmed	Zoe
$\frac{5}{6} \div \frac{1}{4}$	$\frac{5}{6} \div \frac{1}{4}$
$\frac{10}{12} \div \frac{3}{12}$	$\frac{20}{24} \div \frac{6}{24}$
$10 \div 3$	$20 \div 6$
$(\frac{10}{3})$	$(\frac{20}{6})$

Different:

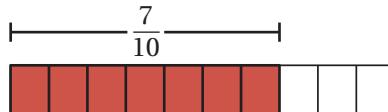
- 7** Zoe says you can't use common denominators to calculate $2 \div \frac{3}{4}$ because 2 is a whole number.

What advice would you give Zoe?

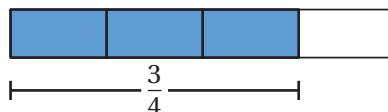


Dividing With Common Denominators

8 **a** Calculate $\frac{7}{10} \div \frac{3}{4}$.



b **Discuss:** What was your strategy?



9 Solve as many challenges as you have time for.

a $\frac{4}{3} \div \frac{2}{3}$

b $\frac{1}{6} \div \frac{5}{6}$

c $\frac{3}{8} \div \frac{1}{4}$

d $2 \div \frac{1}{3}$

e $\frac{3}{10} \div \frac{2}{5}$

f $\frac{5}{6} \div \frac{3}{4}$

g $4 \div \frac{3}{4}$

h $\frac{11}{4} \div \frac{2}{3}$

i $2\frac{1}{2} \div \frac{2}{3}$

j $1\frac{4}{5} \div \frac{1}{2}$

10 Synthesis

Describe how finding a common denominator can help you divide a fraction with another fraction.

$$\frac{4}{3} \div \frac{2}{3} \quad \frac{5}{2} \div \frac{4}{3}$$

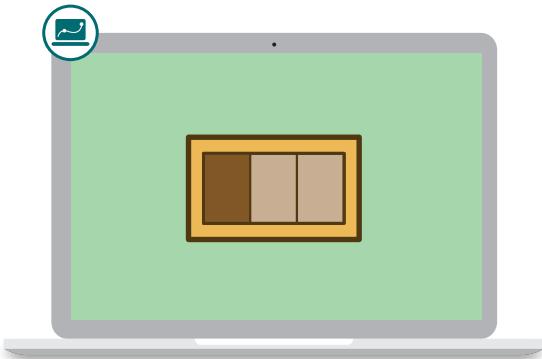
Use these examples if they help you explain your thinking.

Things to Remember:

Name: Date: Period:

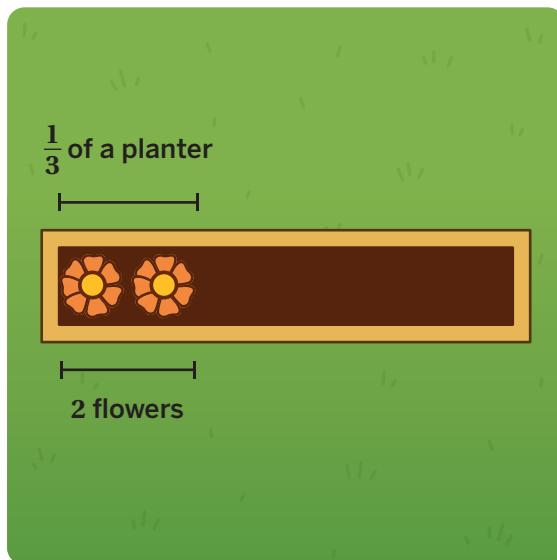
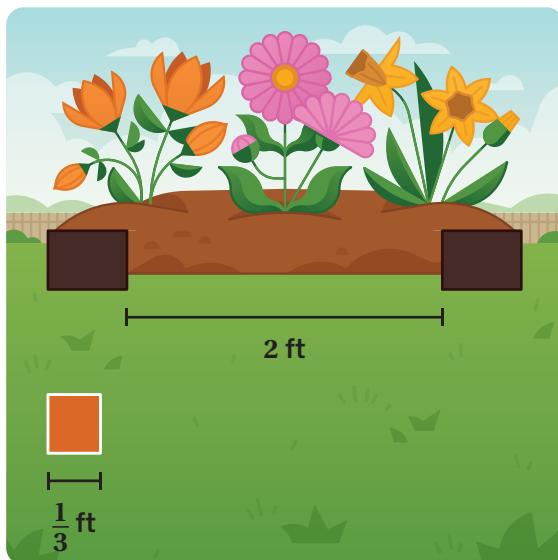
Potting Soil

Let's explore another strategy for dividing fractions.



Warm-Up

- 1 Habib says $2 \div \frac{1}{3}$ represents the brick situation. Inola says $2 \div \frac{1}{3}$ represents the flower situation.



Discuss: Why are they both correct?

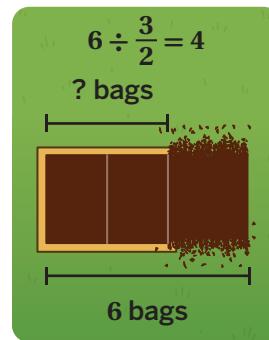
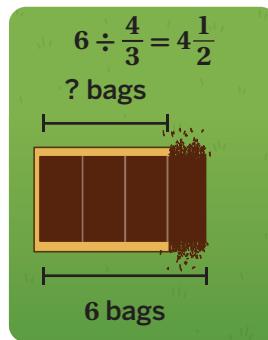
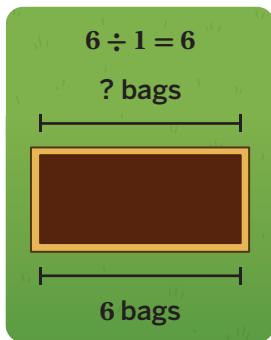
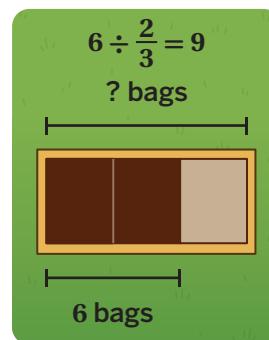
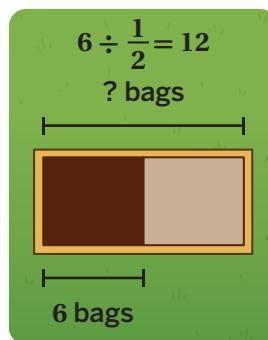
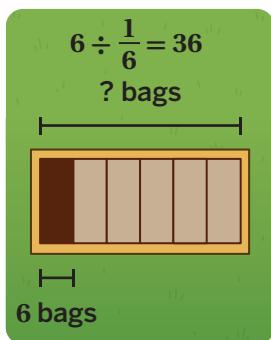
Digging Into Fraction Division

- 2** Habib and Inola are filling planters with potting soil so that their class can grow vegetables.

- a** Let's take a look at how many bags of soil fill $\frac{1}{2}$ of a planter.
- b** How many bags does it take to fill 1 planter?



- 3** **a** Take a look at six different soil situations.



- b** What do you notice? What do you wonder?

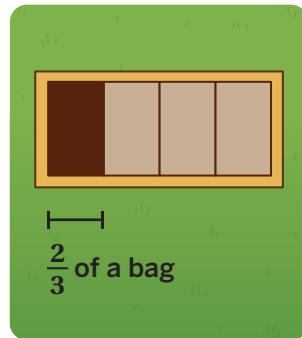
I notice:

I wonder:

Digging Into Fraction Division (continued)

- 4** It takes $\frac{2}{3}$ of a bag of soil to fill $\frac{1}{4}$ of this planter.

How many bags does it take to fill 1 planter?



- 5** Habib wrote $\frac{2}{3} \div \frac{1}{4} = \frac{8}{3}$ to solve the previous problem.

What does each fraction mean in this situation?

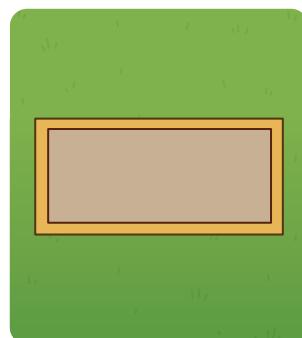
$\frac{2}{3}$ means . . .

$\frac{1}{4}$ means . . .

$\frac{8}{3}$ means . . .

- 6** Inola wrote $5\frac{1}{3} \div \frac{1}{2}$ to solve a new problem.

Draw or describe a situation about planters and potting soil that represents Inola's expression.

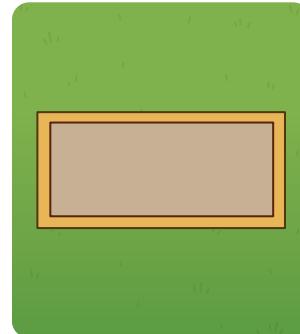


Different Operation, Same Value

7

 **Discuss:** How could you think about the expression $\frac{9}{2} \div \frac{1}{3}$ in terms of a planter?

Draw a diagram if it helps you with your thinking.

**8**

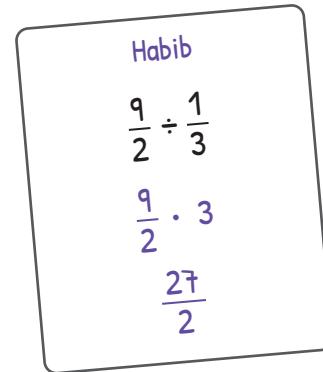
What is $\frac{9}{2} \div \frac{1}{3}$?

9

Habib says that $\frac{9}{2} \div \frac{1}{3}$ has the same value as $\frac{9}{2} \cdot 3$.

a

 **Discuss:** How would you show Habib's strategy using a tape diagram?

**b**

Use Habib's strategy to calculate $\frac{2}{3} \div \frac{1}{7}$.

Different Operation, Same Value (continued)

10

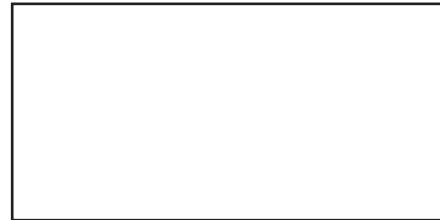
- a Calculate the value of each expression.

Expression	Value
$\frac{4}{3} \div \frac{1}{3}$	
$\frac{4}{3} \div \frac{1}{6}$	
$\frac{4}{3} \div \frac{1}{5}$	
$1\frac{2}{3} \div \frac{1}{4}$	

- b Discuss your answers and strategies with a classmate.

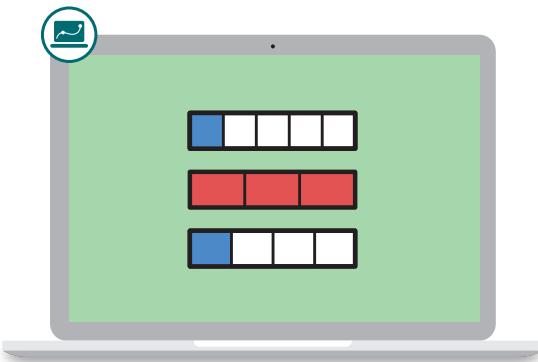
11 Synthesis

Describe a strategy for dividing a number by a unit fraction, such as $2\frac{1}{3} \div \frac{1}{5}$.



Things to Remember:

Name: Date: Period:



Division Challenges

Let's compare strategies for dividing fractions with and without tape diagrams.

Warm-Up

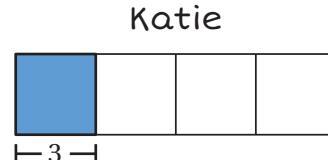
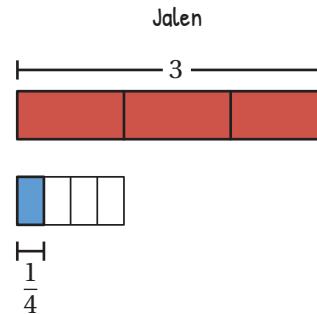
- 1** Solve as many challenges as you have time for. Try some problems of each type.

Multiplying	Dividing	Surprise Me!
 What is the value of $5 \cdot \frac{1}{2}$?	 What is the value of $\frac{2}{3} \div 2$?	 What is the value of $4 \cdot \frac{1}{3}$?
 What is the value of $2 \cdot \frac{3}{4}$?	 What is the value of $\frac{3}{5} \div 3$?	 What is the value of $\frac{3}{4} \div 3$?
 What is the value of $5 \cdot \frac{2}{5}$?	 What is the value of $\frac{9}{10} \div 4$?	 What is the value of $3 \cdot \frac{2}{5}$?

Two Strategies With Tape Diagrams

- 2** Jalen and Katie drew diagrams to calculate $3 \div \frac{1}{4}$.

a Take a look at each student's diagram.



b Calculate $3 \div \frac{1}{4}$.

- 3** Here is a new expression: $\frac{4}{3} \div \frac{1}{5}$.

Jalen says the quotient is $\frac{20}{3}$. Katie says the quotient is $\frac{4}{15}$.

Diagram 1

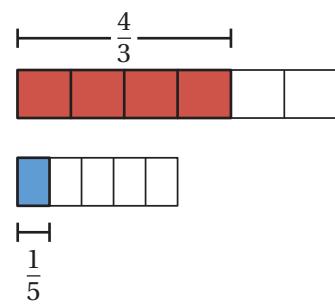
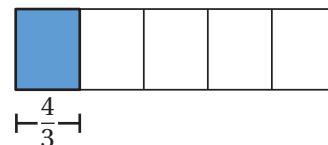


Diagram 2



a Whose quotient is correct? Circle one.

Jalen's

Katie's

Both

Neither

b Use one of the diagrams to help explain your thinking.

Two Strategies Revisited

4 Here are four expressions.

- a** Order these expressions by value from *least* to *greatest*.

$$\frac{4}{3} \div \frac{2}{5}$$

$$\frac{4}{3} \div 1$$

$$\frac{4}{3} \div \frac{1}{5}$$

$$\frac{4}{3} \div 2$$

Least				Greatest

- b**  **Discuss:** How are these expressions alike? How are they different?

5 Here is an expression from the previous problem:

$$\frac{4}{3} \div \frac{2}{5}$$

Calculate its value.

Two Strategies Revisited (continued)

- 6** Here is how Jalen and Katie calculated $\frac{4}{3} \div \frac{2}{5}$.

a Take a look at each of their strategies.

Jalen

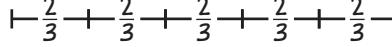
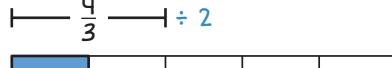
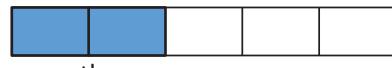
$$\frac{4}{3} \div \frac{2}{5}$$

$$\frac{20}{15} \div \frac{6}{15}$$

$$20 \div 6$$

$$\frac{10}{3}$$

Katie



$$\frac{4}{3} \div \frac{2}{5}$$

$$\frac{4}{3} \div \frac{2}{5}$$

$$\frac{2}{3} \div \frac{1}{5}$$

$$\frac{10}{3}$$

- b** Which strategy would you use to calculate $\frac{9}{10} \div \frac{3}{4}$? Circle one.

Jalen's

Katie's

My own

- c** If you chose Jalen's or Katie's strategy, what would your first step be for the strategy you chose? Otherwise, describe your own strategy.

**Activity
3**

Name: Date: Period:

Fraction Fluency

- 7** Here is an expression from the previous problem:

$$\frac{9}{10} \div \frac{3}{4}$$

Calculate its value.

- 8** Here is a new expression: $\frac{6}{5} \div \frac{2}{3}$.

The three answers below are *not* correct.

$$\frac{12}{15}$$

$$\frac{6}{5}$$

3

Circle your favorite (wrong) answer and explain why it cannot be correct.

Fraction Fluency (continued)

9 Solve as many challenges as you have time for. Calculate each expression.

a $5 \div \frac{2}{3}$

b $\frac{1}{2} \div \frac{3}{4}$

c $\frac{6}{5} \div \frac{2}{3}$

d $\frac{1}{2} \div \frac{5}{3}$

e $\frac{3}{4} \div \frac{3}{5}$

f $\frac{9}{4} \div \frac{7}{10}$

g $\frac{5}{9} \div \frac{5}{3}$

h $\frac{1}{9} \div \frac{3}{5}$

i $\frac{9}{7} \div \frac{1}{7}$

j $\frac{6}{7} \div \frac{4}{7}$

k $2 \div \frac{8}{9}$

l $\frac{6}{5} \div \frac{2}{5}$

10 Synthesis

Describe a strategy for calculating the quotient of two fractions, such as $\frac{2}{5} \div \frac{3}{4}$.

Draw a diagram if it helps you with your thinking.

Things to Remember:

Name: Date: Period:

Classroom Comparisons

Let's compare the size of familiar objects by asking, "How many times as long?"



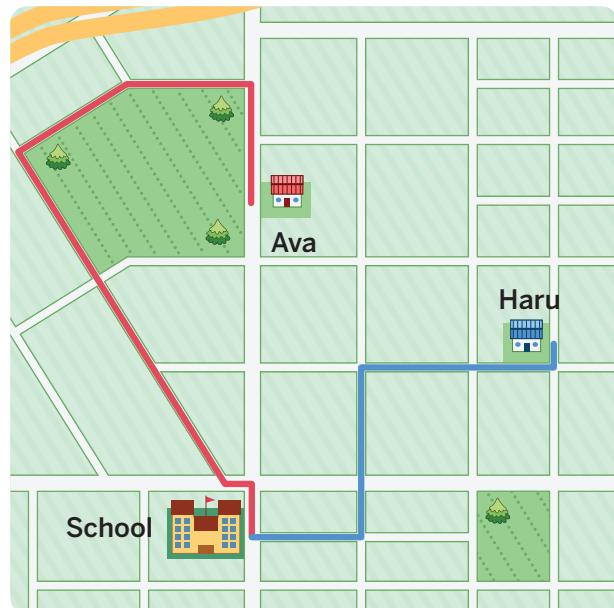
Warm-Up

- 1** Here is how Ava and Haru walked to school on Monday.

What do you notice? What do you wonder?

I notice:

I wonder:



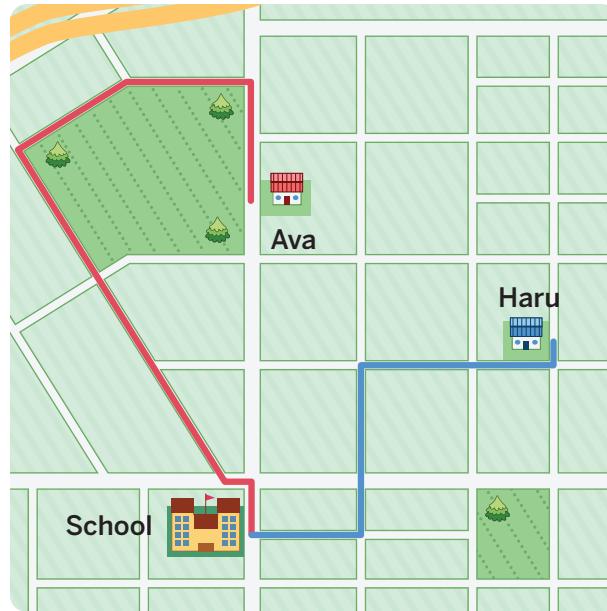
Comparing Distances

- 2** Ava walked farther than Haru.

About how many times as far do you think Ava walked?

- 3** Ava walked $1\frac{1}{4}$ miles. Haru walked $\frac{3}{4}$ of a mile.

How many times as far did Ava walk?



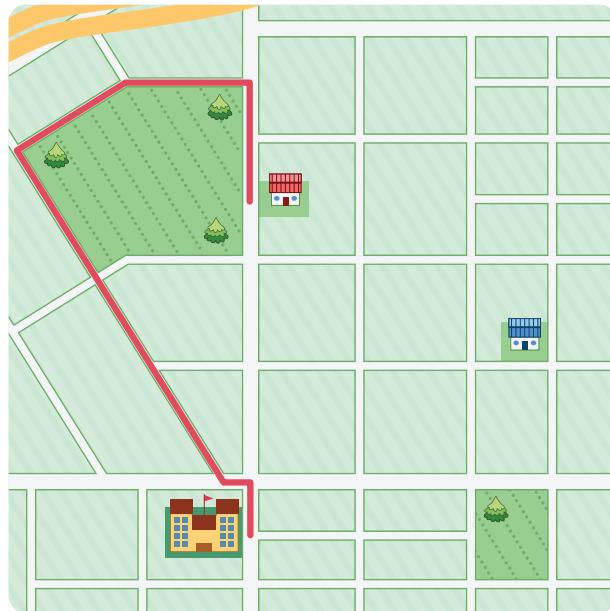
- 4** Select *all* the expressions that represent how many times as far Ava walked compared to Haru.

- A. $\frac{5}{4} - \frac{3}{4}$
- B. $1\frac{1}{4} \cdot \frac{3}{4}$
- C. $1\frac{1}{4} \div \frac{3}{4}$
- D. $\frac{3}{4} \div 1\frac{1}{4}$
- E. $\frac{5}{4} \div \frac{3}{4}$

Comparing Distances (continued)

- 5** Ava decides to walk a different path to school on Tuesday.

Draw a path Ava could walk.

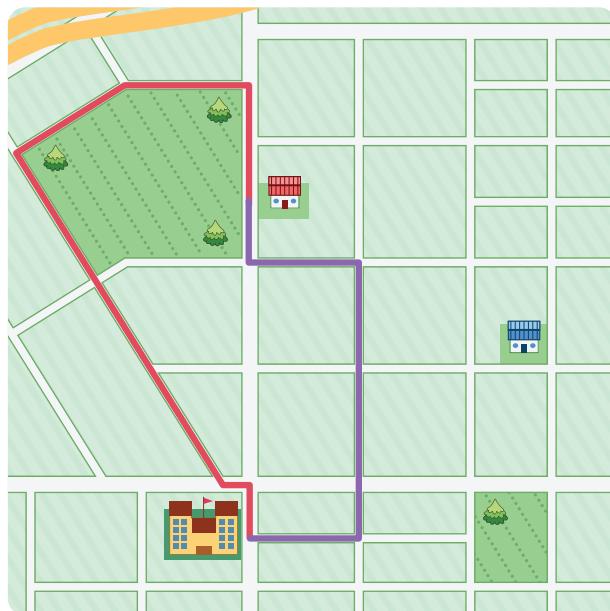


- 6** The map shows Ava's paths on Monday and Tuesday.

Monday: Ava walked $1\frac{1}{4}$ miles.

Tuesday: Ava walked $\frac{7}{8}$ of a mile.

How many times as far did Ava walk on Tuesday than on Monday?



Comparing Classroom Objects

- 7** Ava and Haru are comparing two objects in their classroom.

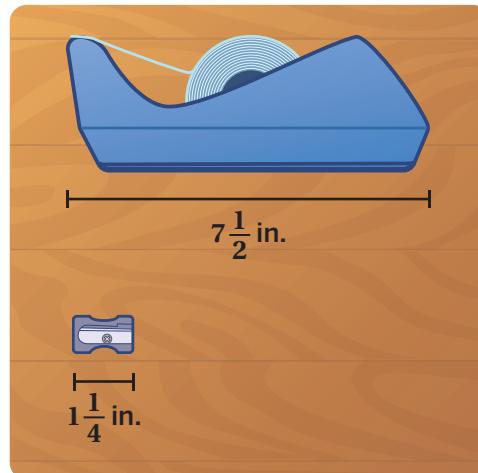
Ava says: *The tape dispenser is 6 times as long.*

Haru says: *The pencil sharpener is $\frac{1}{6}$ times as long.*

Whose thinking is correct? Circle one.

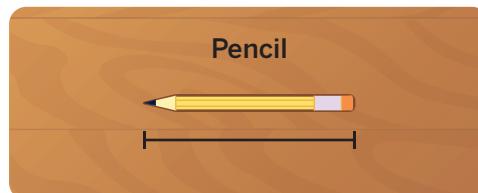
Ava's Haru's Both Neither

Explain your thinking.

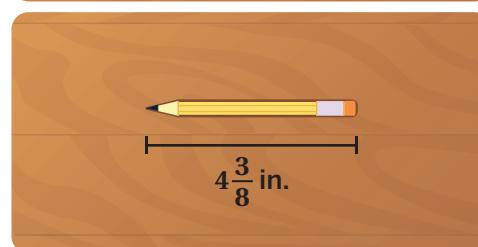
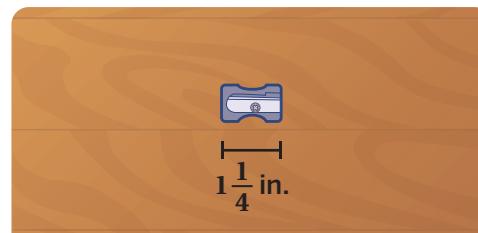


- 8** The sharpener is times as long as the pencil.

Estimate a value.



- 9** The sharpener is how many times as long as the pencil?



Comparing Classroom Objects (continued)

- 10** Here is a collection of classroom objects, along with their lengths (in inches).

Scissors	$6\frac{1}{4}$	Eraser	$1\frac{7}{8}$
Marker	$5\frac{5}{8}$	Stapler	5
Red pen	$6\frac{7}{8}$	Pencil sharpener	$1\frac{1}{4}$
Tape dispenser	$7\frac{1}{2}$	Glue bottle	$3\frac{1}{8}$
Highlighter	$3\frac{3}{4}$	Calculator	$2\frac{1}{2}$
Large pencil	$8\frac{1}{8}$	Small pencil	$4\frac{3}{8}$

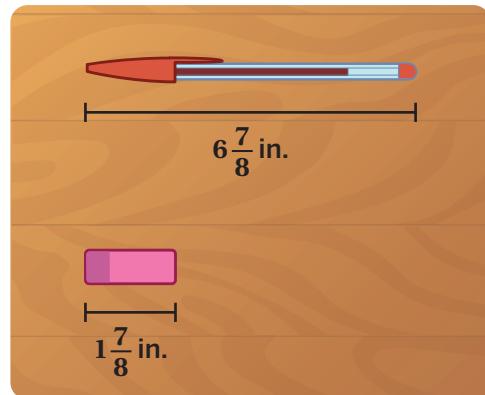


Choose pairs of objects to compare. Write an expression and a statement comparing their lengths. Solve as many challenges as you have time for!

	Objects	Expression	Statement
a	_____ and _____		The _____ is _____ times as long as the _____.
b	_____ and _____		The _____ is _____ times as long as the _____.
c	_____ and _____		The _____ is _____ times as long as the _____.
d	_____ and _____		The _____ is _____ times as long as the _____.

11 Synthesis

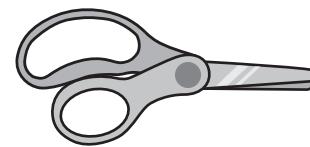
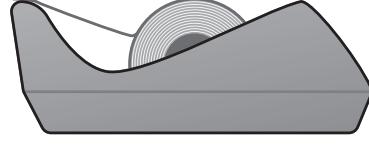
Describe a strategy for solving problems like this:
The pen is how many times as long as the eraser?



Things to Remember:

Comparing Classroom Objects

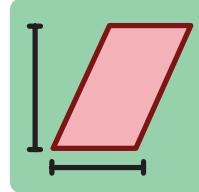
Select two classroom items to compare using the table in Problem 10.

Highlighter  $3\frac{3}{4}$ in.	Eraser  $1\frac{7}{8}$ in.	Small pencil  $4\frac{3}{8}$ in.
Marker  $5\frac{5}{8}$ in.	Pencil sharpener  $1\frac{1}{4}$ in.	Red pen  $6\frac{7}{8}$ in.
Large pencil  $8\frac{1}{8}$ in.	Stapler  5 inches	Scissors  $6\frac{1}{4}$ in.
Glue bottle  $3\frac{1}{8}$ in.	Calculator  $2\frac{1}{2}$ in.	Tape dispenser  $7\frac{1}{2}$ in.

Name: Date: Period:

Puzzling Areas

Let's explore the areas of rectangles and triangles with fractional side lengths.



Warm-Up

Evaluate each expression mentally.

1. $3 \cdot 4$

2. $\frac{1}{3} \cdot 4$

3. $\frac{1}{3} \cdot \frac{1}{4}$

4. $2 \cdot \frac{1}{3} \cdot \frac{1}{4}$

5. $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4}$

Activity

1

Name: Date: Period:

Areas

- 6.** Use any strategy to determine the area of as many figures as you can. Use the workspace below if it helps with your thinking.

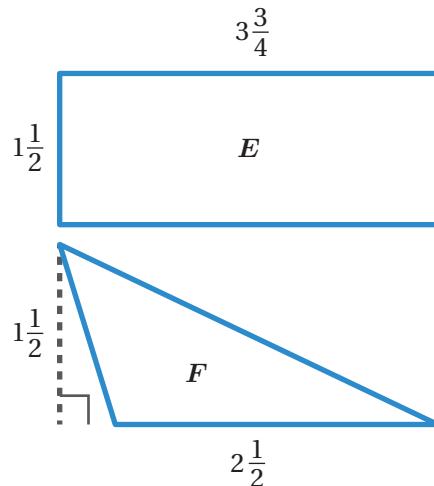
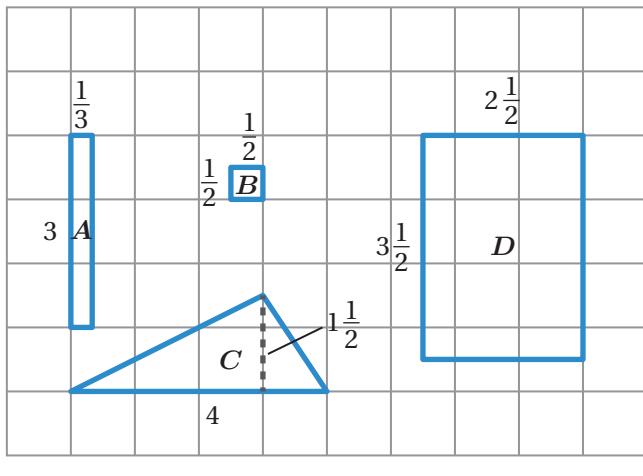
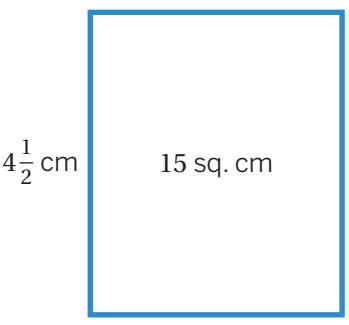
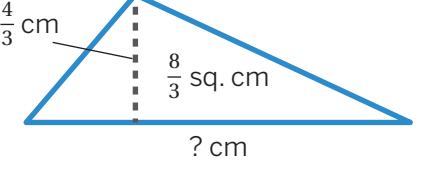
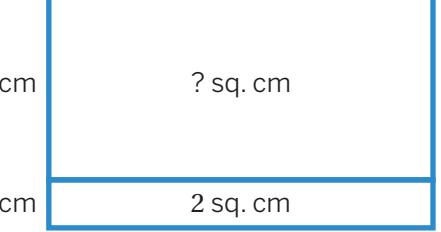


Figure	A	B	C	D	E	F
Area (sq. units)						

Workspace:

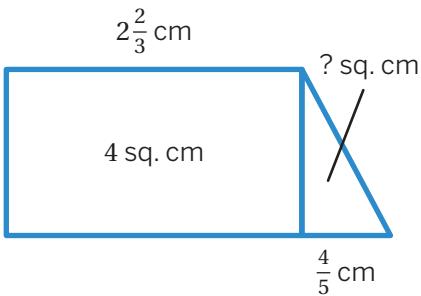
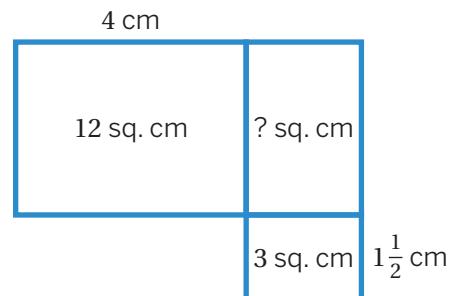
Level Up Area Puzzles

7. Use any strategy to determine the unknown side length or area.

Puzzle	Workspace
a 	Workspace $? = \dots$ centimeters
b 	Workspace $? = \dots$ centimeters
c 	Workspace $? = \dots$ centimeters
d 	Workspace $? = \dots$ square centimeters

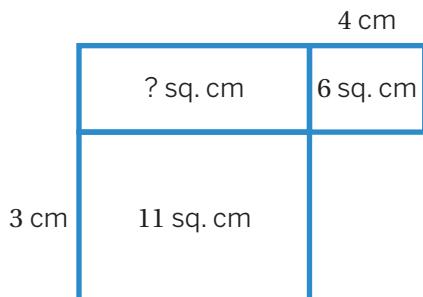
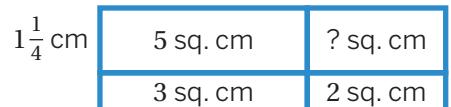
Level Up Area Puzzles (continued)

- 8.** Solve as many puzzles as you have time for. You can work on them in any order.

Puzzle A**Puzzle B**

? = square centimeters

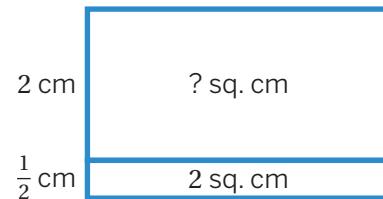
? = square centimeters

Puzzle C**Puzzle D**

? = square centimeters

Synthesis

9. a When is multiplication helpful for solving problems about areas?



- b When is division helpful for solving problems about areas?

Things to Remember:

Decimal Diagrams and Algorithms

Let's add and subtract decimals.

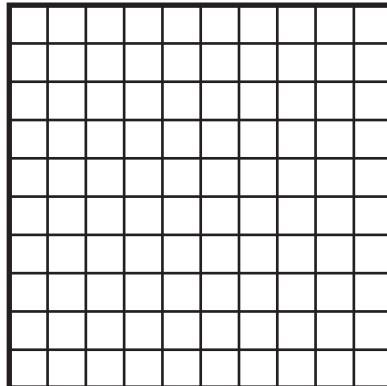


Warm-Up

- 1** This large square represents 1.

Write 0.425 using different quantities of tenths, hundredths, and/or thousandths.

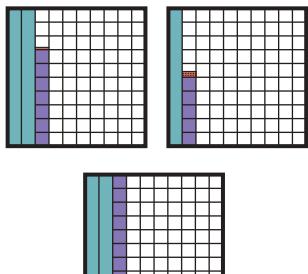
Write as many combinations as you can think of.



More Than One Way to Add

- 2** Prisha and Omar used different strategies to add $0.271 + 0.154$.

Prisha



$$= 0.271 + 0.154$$

Omar

$$\begin{array}{r}
 0.271 \\
 + 0.154 \\
 \hline
 0.3 \\
 0.12 \\
 0.005 \\
 \hline
 0.425
 \end{array}$$

- a** Whose calculation is correct? Circle one.

Prisha's

Omar's

Both

Neither

- b** Explain your thinking.

More Than One Way to Add (continued)

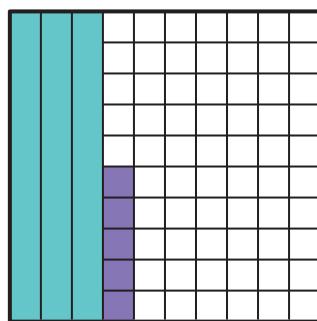
- 3** Group the representations that have the same value. You will have three groups.

a.
$$\begin{array}{r} 0.048 \\ + 0.302 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 0.04 \\ + 0.010 \\ \hline \end{array}$$

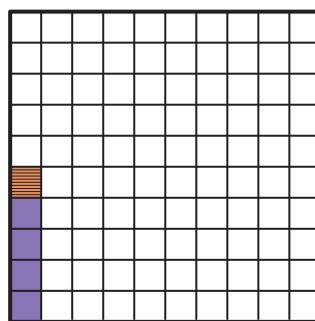
c.
$$\begin{array}{r} 0.499 \\ + 0.001 \\ \hline \end{array}$$

d.



e. 50 hundredths

f.



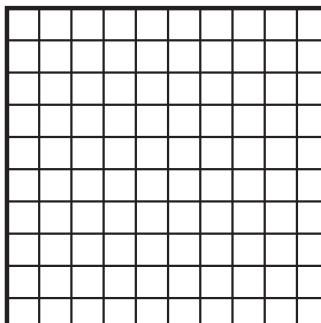
g. 0.5

h. 4 hundredths and
10 thousandths

i. 0.35

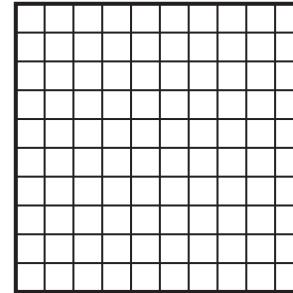
Group 1	Group 2	Group 3

- 4** Determine the value of $0.275 + 0.135$.

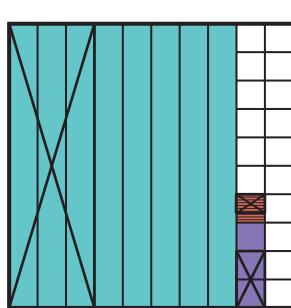


Decimal Differences

- 5** Determine the value of $0.2 - 0.11$.



- 6** Adhara and Malik used different strategies to calculate $0.84 - 0.327$.



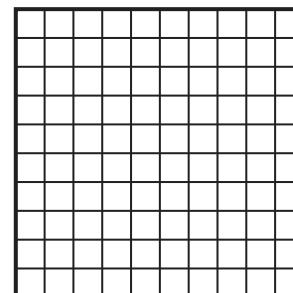
Malik

$$\begin{array}{r}
 3 \textcolor{green}{1} \textcolor{blue}{0} \\
 0.8 \textcolor{red}{4} \textcolor{blue}{0} \\
 - 0.3 \textcolor{red}{2} \textcolor{blue}{7} \\
 \hline
 0.5 \textcolor{red}{1} \textcolor{blue}{3}
 \end{array}$$

Discuss:

- How are these strategies alike? How are they different?
- Where do you see place value and regrouping in each strategy?

- 7** Determine the value of $0.562 - 0.17$.



Repeated Challenges

8 Determine the value for each sum or difference. Solve as many challenges as you have time for.

a $0.203 + 0.105$

b $0.15 - 0.1$

c $0.155 + 0.015$

d $0.5 - 0.151$

e $0.25 + 0.15$

f $0.01 + 0.002$

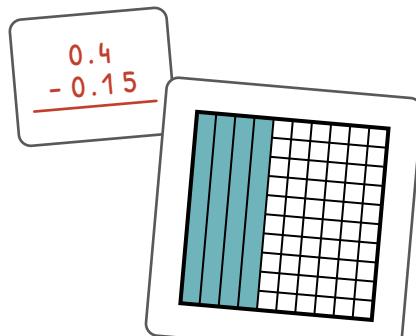
g $0.946 - 0.041$

h $0.589 - 0.187$

9 Synthesis

Describe a strategy for subtracting decimals.

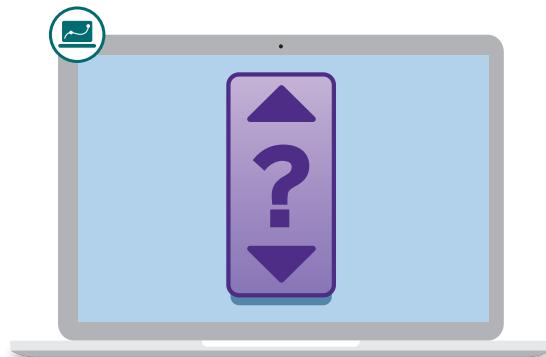
Use the example $0.4 - 0.15$ if it helps with your thinking.



40 hundredths minus 15 hundredths

Things to Remember:

Name: Date: Period:



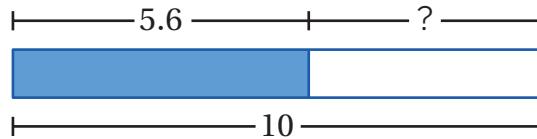
Missing Digits

Let's solve number puzzles.

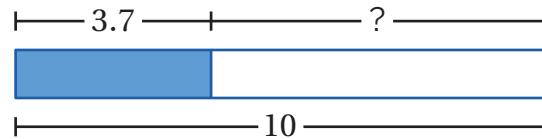
Warm-Up

- 1** Determine the missing number for each challenge.

a 5.6 plus what number equals 10?



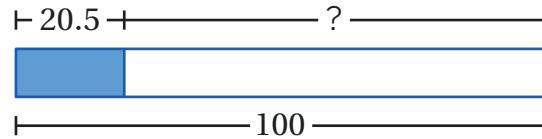
b 3.7 plus what number equals 10?



c 0.35 plus what number equals 1?

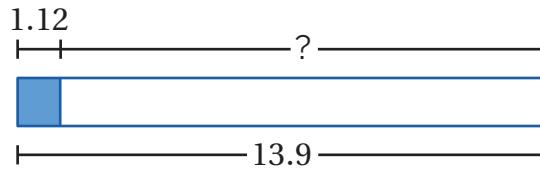


d 20.5 plus what number equals 100?



Vertical Calculations

- 2** 1.12 plus what number equals 13.9?



- 3** Here is how Kayleen calculated $13.9 - 1.12$.

What could you say to her to help her understand her mistake?

$$\begin{array}{r} 13.9 \\ -1.12 \\ \hline 2.7 \end{array}$$

Number Puzzles

- 4** Kayleen made a number puzzle.

Fill in the missing digit that makes the statement true.

$$\begin{array}{r} 2.5 \\ + 7. \square \\ \hline 9.8 \end{array}$$

- 5** Here is Yosef's number puzzle.

Fill in the missing digits that make the statement true.

$$\begin{array}{r} 0.0\square 85 \\ - 0.00\square 9 \\ \hline 0.005 \end{array}$$

- 6** Match each number puzzle to its missing digit. Two digits will have no match.

1

2

3

5

6

a.
$$\begin{array}{r} 0.58 \\ - 0.322 \\ \hline 0.2\square 8 \end{array}$$

b.
$$\begin{array}{r} 59.4\square ? \\ - 40.6 \\ \hline 18.83 \end{array}$$

c.
$$\begin{array}{r} 9.8\square 4 \\ + 0.\square 26 \\ \hline 10 \end{array}$$

Number Puzzles (continued)

7 Here is Aba's puzzle.

Fill in the missing digits to make the statement true.

$$\begin{array}{r} 5.8 - \underline{\quad}.3 \\ = 3.\underline{\quad}3 \end{array}$$

8 Aba used addition to check her solution to her number puzzle.

Does this sum prove that her solution is correct? Circle one.

Yes

No

I'm not sure

$$\begin{array}{r} & 1 \\ 3.43 & \\ + 2.37 & \\ \hline 5.80 & \end{array}$$

Explain your thinking.

Repeated Challenges

9 Fill in the missing digits to make each statement true. Solve as many challenges as you have time for.

a $2.5 + \underline{\quad} .6 = 9.\underline{\quad}$

b $9.5 - 5.\underline{\quad} 5 = \underline{\quad} .15$

c $0.404 + \underline{\quad} .\underline{\quad} \underline{\quad} 6 = 1.000$

d $\underline{\quad} 4.\underline{\quad} 3 + 224.17 = 318.80$

e $7 - \underline{\quad} .\underline{\quad} \underline{\quad} 33 = 3.4567$

f $0.7 - 0.68\underline{\quad} = 0.\underline{\quad} 12$

Explore More

10 How many different solutions are there to this problem? Explain your thinking.

$$\begin{array}{r} 5. ? \\ - ? . ? 3 \\ \hline 2.2 ? \end{array}$$

11 Synthesis

Describe a strategy for determining the missing digits in this subtraction problem.

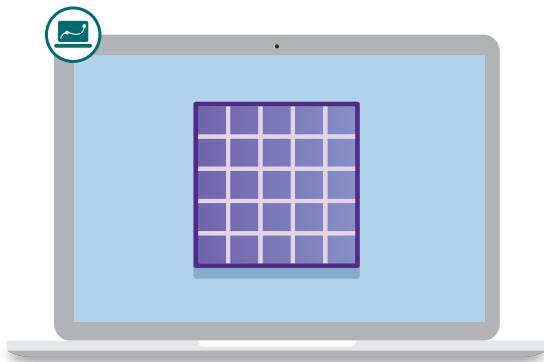
$$\begin{array}{r} - 1.0 ? \\ \hline ? . ? 8 \end{array}$$

Things to Remember:

Name: Date: Period:

Decimal Multiplication

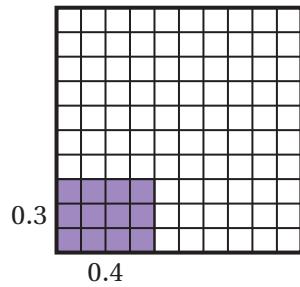
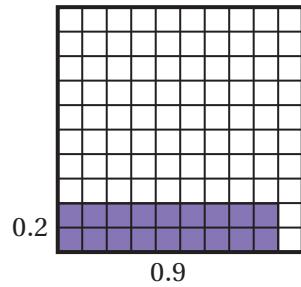
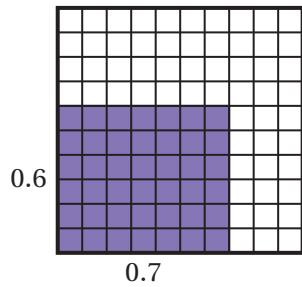
Let's explore decimal multiplication using place value.



Warm-Up

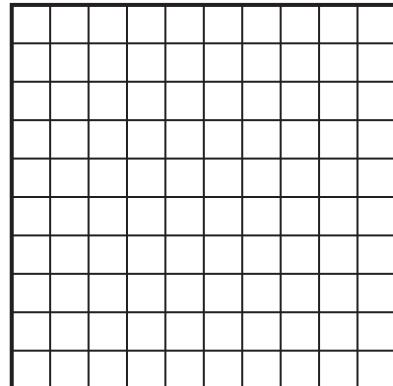
- 1** The large square has an area of 1.

- a** Take a look at these shaded rectangles with different lengths and widths.



- b** Try to create shaded rectangles with the following areas:

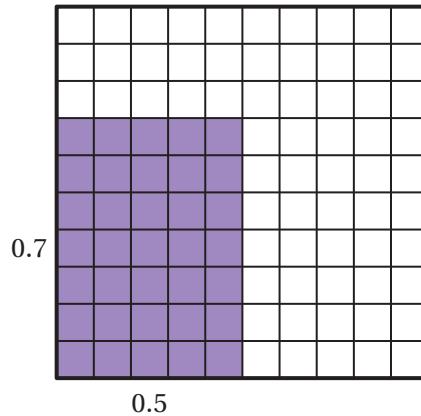
Length	Width	Area
		0.16
		0.24
		0.30



Keepin' It One Hundredth

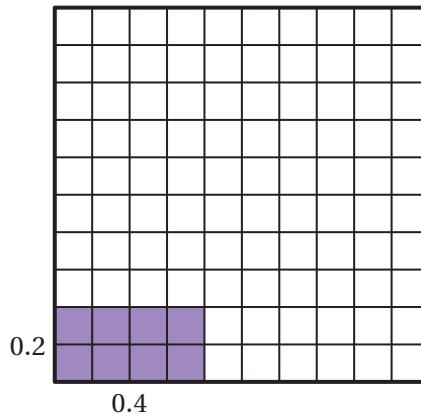
- 2** Jamya says that $0.5 \cdot 0.7 = 0.35$.

Use the diagram to show or explain why this makes sense.



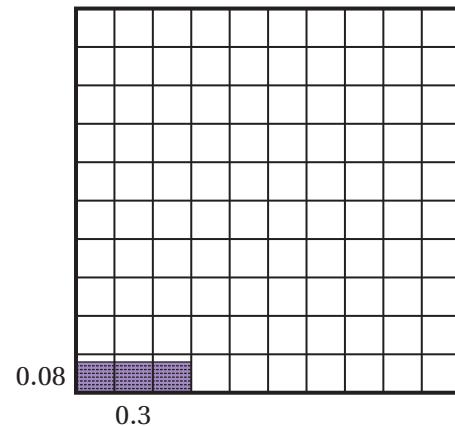
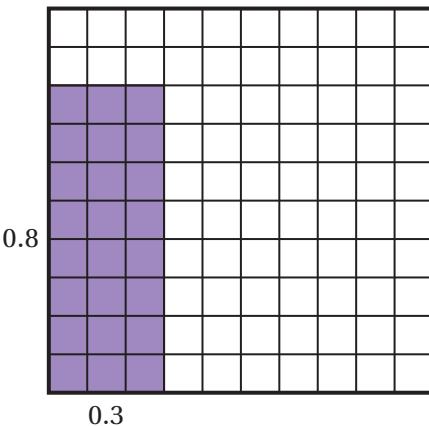
- 3** Multiply $0.4 \cdot 0.2$.

Write your answer as a decimal.



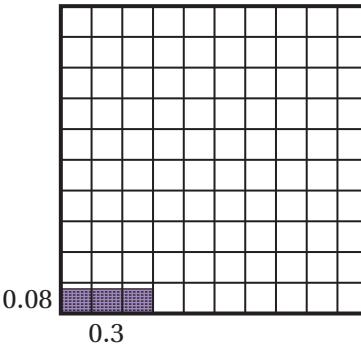
Keepin' It One Hundredth (continued)

- 4** Here are two diagrams that represent different multiplication problems.



Discuss: How are the diagrams alike? How are they different?

- 5** What is $0.3 \cdot 0.08$?



- 6** This is Jayden's work for multiplying $0.3 \cdot 0.08$.

$$0.3 \cdot 0.08$$

Discuss: What is Jayden's strategy?

$$\frac{3}{10} \cdot \frac{8}{100}$$

Show or explain how you would use this strategy to multiply $0.02 \cdot 0.9$.

$$\frac{24}{1000} = 0.024$$

Clicking Into Place Value

- 7** Match each area with its equivalent expressions.

$$\frac{15}{100}$$

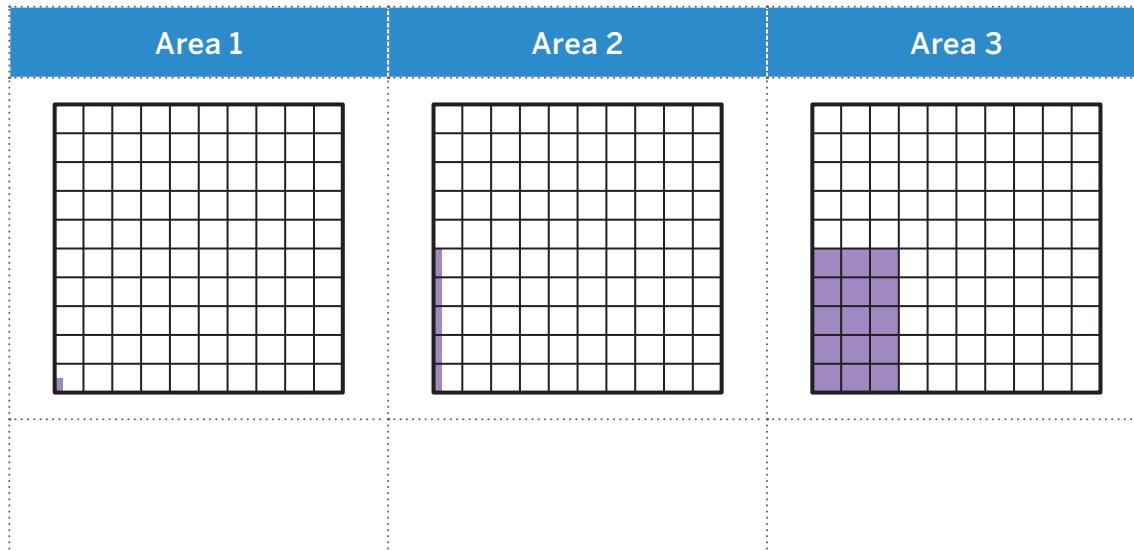
$$0.3 \cdot 0.5$$

$$0.015$$

$$0.03 \cdot 0.05$$

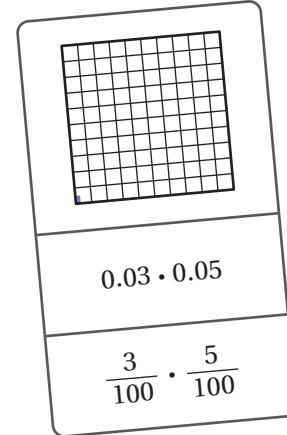
$$\frac{3}{100} \cdot \frac{5}{100}$$

$$0.03 \cdot 0.5$$



- 8** Jayden matched these choices in the previous problem and noticed that the product was missing.

Calculate $0.03 \cdot 0.05$.



Repeated Challenges

9 Solve as many challenges as you have time for.

a $0.04 \cdot 0.6$

b $0.03 \cdot 0.02$

c $0.2 \cdot 0.007$

d $0.003 \cdot 0.3$

e $0.5 \cdot 0.4$

f $0.001 \cdot 0.08$

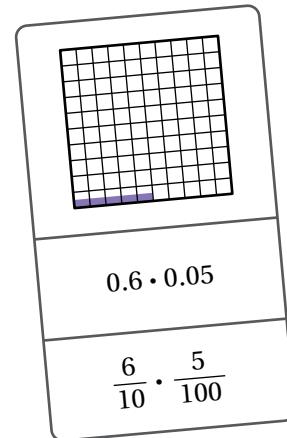
g $1 \cdot 0.05$

h $0.4 \cdot 0.005$

10 Synthesis

Show or describe how decimals, fractions, and the hundredths chart are related.

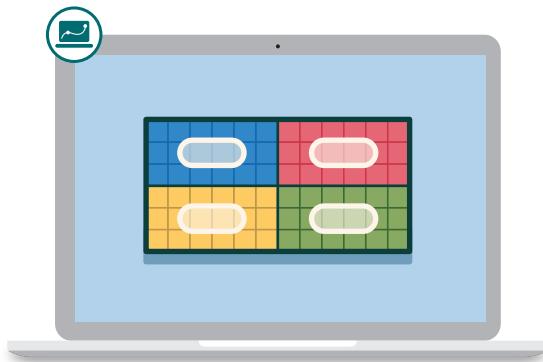
Use the example if it helps with your thinking.



Things to Remember:

Multiplying With Areas

Let's use area models to multiply decimals.

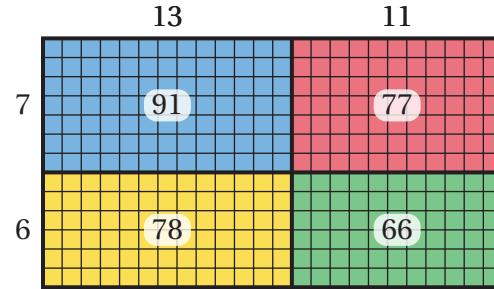


Warm-Up

- 1** Diego likes using area models to multiply whole numbers.

- a** Let's look at different ways to split this 24-by-13 rectangle.
- b** What do you notice? What do you wonder?

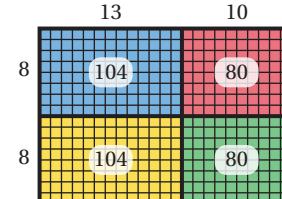
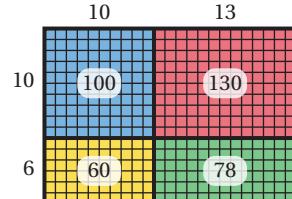
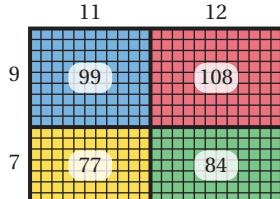
I notice:



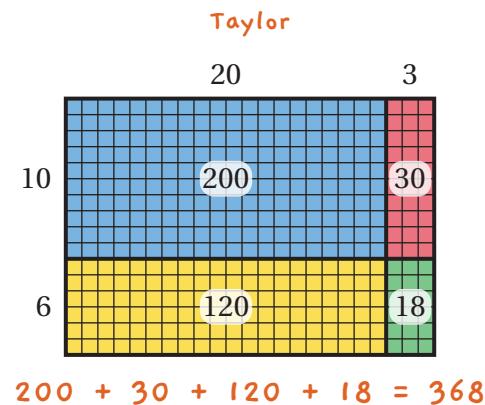
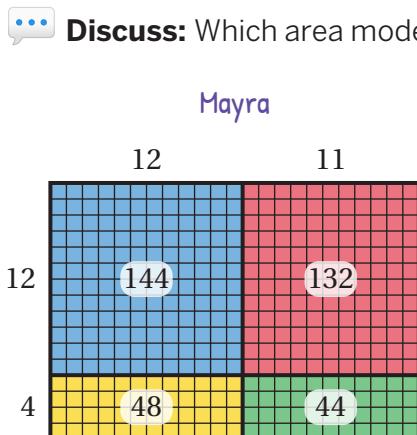
I wonder:

Creating an Area Model

- 2** Here are some new area models. Use them to multiply $23 \cdot 16$.



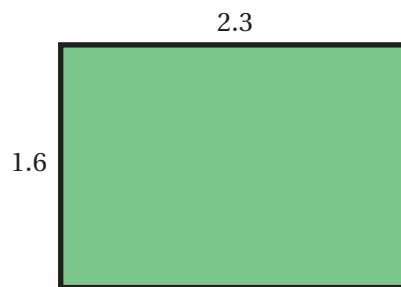
- 3** Mayra and Taylor made different area models to multiply $23 \cdot 16$.



Creating an Area Model (continued)

- 4** Diego wonders if area models could also help him multiply decimals like $2.3 \cdot 1.6$.

- a** Show how you would split this rectangle into smaller parts to multiply $2.3 \cdot 1.6$.
- b** Explain why you split it that way.



- 5** Use your area model to calculate $2.3 \cdot 1.6$.

Calculating With an Area Model

6

- a** Use the digital activity to create an area model that helps you multiply $4.5 \cdot 2.9$.

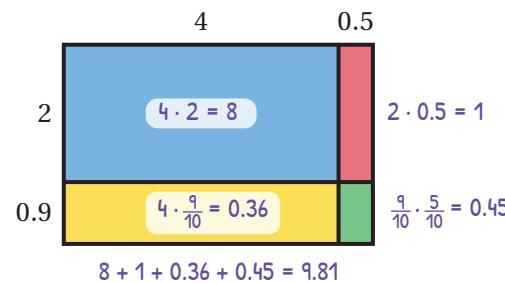
- b** Calculate the area of each part of your area model. The total area will be calculated for you.

**7**

- Diego made an error while multiplying $4.5 \cdot 2.9$.

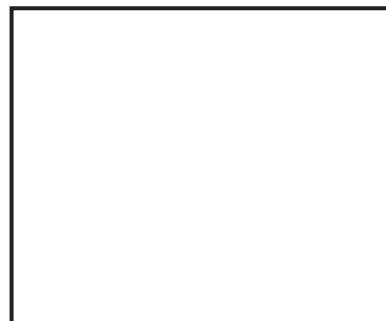
- a** Circle the error in Diego's work.

- b** What would you say to help him understand his mistake?

**8**

- Multiply $3.4 \cdot 2.8$.

Use the diagram if it helps with your thinking.



Calculating With an Area Model (continued)

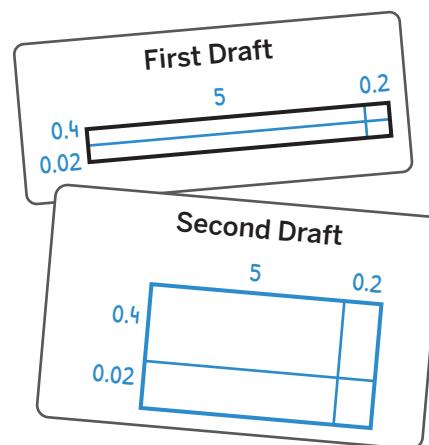
- 9** Multiply $5.2 \cdot 0.42$.

Use the diagram if it helps with your thinking.

- 10** Paz made two drafts to calculate $5.2 \cdot 0.42$.

What are the advantages and disadvantages of Paz's second draft?

Advantages:

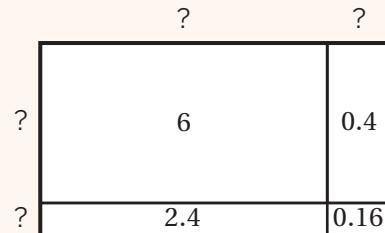


Disadvantages:

Explore More

- 11** Here is an area model that is missing some labels.

What multiplication problem could this help you solve?
Explain your thinking.



12 Synthesis

Describe how you can use an area model to multiply decimals like $2.7 \cdot 1.4$.

Draw on the diagram if it helps with your thinking.

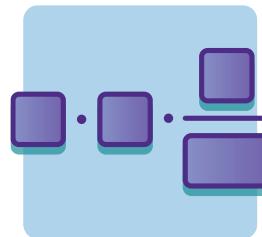


Things to Remember:

Name: Date: Period:

Multiplication Methods

Let's use place value to multiply decimals vertically.



Warm-Up

1. Calculate $84 \cdot 13$. Show your thinking.
2. Find someone who used a different strategy than you did. Discuss how you each calculated $84 \cdot 13$.

Activity

1

Name: Date: Period:

Multiple Methods

Kwame and Tiara used different strategies to multiply $8.4 \cdot 1.3$

Kwame

8	0.4
1	
0.3	2.4

$$8 + 0.4 + 2.4 + 0.12 = 10.92$$

Tiara

$$8.4 \cdot 1.3 \\ 84 \cdot 13 \cdot \frac{1}{10} \cdot \frac{1}{10}$$

$$\begin{array}{r} 84 \\ \times 13 \\ \hline 12 \\ 240 \\ 40 \\ \hline +800 \\ \hline 1092 \end{array} \cdot \frac{1}{100}$$

$$10.92$$

3.  **Discuss:** How are their strategies alike? How are they different?

4. Show how each student might set up $8.4 \cdot 0.13$.

Kwame**Tiara**

5. Use either strategy to finish calculating $8.4 \cdot 0.13$.

6. Calculate $0.352 \cdot 0.25$.

Multiple Multiplication Methods

7. Tiara wrote this expression to try and calculate $2.9 \cdot 0.015$.

$$29 \cdot 15 \cdot \frac{1}{10} \cdot \frac{1}{1000}$$

If $29 \cdot 15 = 435$, then what is $2.9 \cdot 0.015$?

- A. 4.35 B. 0.435 C. 0.0435 D. 0.00435
8. If $165 \cdot 12 = 1980$, then what is $16.5 \cdot 1.2$? Explain your thinking.

9. Select all the expressions that have a product of 0.024.

- A. $0.06 \cdot 0.4$
- B. $0.6 \cdot 0.04$
- C. $0.04 \cdot 0.06$
- D. $2 \cdot 0.012$
- E. $1.2 \cdot 0.02$

10. Write another expression that has a product of 0.024.

Scavenger Hunt

11. Start with any of the scavenger hunt sheets.

- Record the sheet shape, solve the problem, and write your answer.
- Look for your answer at the top of another scavenger hunt sheet. Solve that problem.
- Repeat until you make it back to your starting sheet.

Sheet:	Sheet:
Answer <input type="text"/>	Answer <input type="text"/>
Sheet:	Sheet:
Answer <input type="text"/>	Answer <input type="text"/>
Sheet:	Sheet:
Answer <input type="text"/>	Answer <input type="text"/>

Synthesis

12. Describe a strategy that helps you multiply decimals like $0.039 \cdot 3.2$.

Things to Remember:

Scavenger Hunt  Triangle Sheet**Answer****12.28****Problem:**Calculate $2.8 \cdot 0.41$.

Scavenger Hunt



Star Sheet

Answer

1.148

Problem:

Multiply $19 \cdot 0.72$.

Scavenger Hunt

Oval Sheet

Answer

13.68

Problem:

Calculate $4 \bullet 0.307$.

Scavenger Hunt



Trapezoid Sheet

Answer

1.228

Problem:

Determine the value of $0.0019 \bullet 7.2$.

Scavenger Hunt



Crescent Sheet

Answer

0.01368

Problem:

Calculate $4.1 \cdot 2.8$.

Scavenger Hunt Pentagon Sheet

Answer

11.48

Problem:

Determine the value of $30.7 \cdot 0.4$.

Just Keep Dividing

Let's use long division to divide whole numbers when there is a remainder.



Warm-Up

1. Here is the work that Peter and Alina did to calculate $584 \div 4$.

Peter	Alina
$ \begin{array}{r} 6 \\ 4 \overline{) 584} \\ -4 \quad \text{0} \\ \hline 1 \quad 8 \quad 4 \\ -4 \quad \leftarrow \text{4 groups of } 100 \\ \hline 1 \quad 8 \quad 4 \\ -1 \quad 6 \quad \leftarrow \text{4 groups of } 40 \\ \hline 2 \quad 4 \\ -2 \quad 4 \quad \leftarrow \text{4 groups of } 6 \\ \hline 0 \end{array} $	$ \begin{array}{r} 146 \\ 4 \overline{) 584} \\ -4 \\ \hline 184 \\ -16 \\ \hline 24 \\ -24 \\ \hline 0 \end{array} $



Discuss:

- Where do you see Peter's work in Alina's strategy?
- How is Alina's strategy different from Peter's partial quotients strategy?

Digit-by-Digit

- 2.** Alina calculates $685 \div 5$ using a strategy called **long division**.

$$\begin{array}{r} 1 \\ 5) 685 \\ -5 \\ \hline 1 \end{array}$$

← 5 groups
of 1 (hundred)

$$\begin{array}{r} 13 \\ 5) 685 \\ -5 \\ \hline 18 \\ -15 \\ \hline 3 \end{array}$$

← 5 groups
of 3 (tens)

$$\begin{array}{r} 13 \\ 5) 685 \\ -5 \\ \hline 18 \\ -15 \\ \hline 3 \end{array}$$

a

 **Discuss:** Alina explains this long division strategy as “dividing one digit at a time, from left to right.” What do you think Alina means by this?

b

Complete the work using this strategy.

- 3.** Use long division to complete each of these division problems.

a $792 \div 6$

$$\begin{array}{r} 1 \\ 6) 792 \\ -6 \\ \hline 19 \end{array}$$

b $1736 \div 8$

$$\begin{array}{r} \\ 8) 1736 \\ -16 \\ \hline \end{array}$$

c $2933 \div 7$

$$\begin{array}{r} \\ 7) 2933 \\ \hline \end{array}$$

Dicey Decimals

4. Here is Peter's work for calculating $318 \div 4$.

Peter says that 2 is the same as 20 tenths.



Discuss:

- Where do you see the 20 tenths represented in this work?
- Why do you think Peter might find it more helpful to divide 20 tenths by 4 instead of dividing 2 by 4?

$$\begin{array}{r}
 & 79.5 \\
 4) & 318.0 \\
 -28 & \downarrow \\
 \hline
 & 38 \\
 -36 & \downarrow \\
 \hline
 & 20 \\
 -20 & \leftarrow 4 \text{ groups of } 5 \text{ (tenths)} \\
 \hline
 & 0
 \end{array}$$

5. Use Peter's strategy to find the quotient of each expression.

a $162 \div 5$

$$5) \overline{162}$$

b $70 \div 8$

$$8) \overline{70.}$$

6. Nur is working on $2369 \div 8$ using Peter's strategy.

Nur wrote the remainder of 1 as 10 tenths, but that still leaves a remainder of 2 (tenths).

What could Nur do to complete the division? Show or explain your thinking.

$$\begin{array}{r}
 & 296.1 \\
 8) & 2369.0 \\
 -16 & \downarrow | \\
 \hline
 & 76 | \\
 -72 & \downarrow | \\
 \hline
 & 49 | \\
 -48 & \downarrow | \\
 \hline
 & 10 | \\
 -8 & \downarrow | \\
 \hline
 & 2
 \end{array}$$

**Activity
3**

Name: Date: Period:

Division Detectives

7. Decide with your partner who will complete Column A and who will complete Column B. The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

Column A	Column B
$13 \div 4$	$26 \div 8$
$2166 \div 12$	$722 \div 4$
$11 \div 8$	$33 \div 24$
$4 \div 16$	$2 \div 8$

Division Detectives (continued)

Column A	Column B
$102 \div 25$	$204 \div 50$
$36 \div 32$	$18 \div 16$

Explore More

8. a Calculate the quotient for $46 \div 3$.
- b Do you think this pattern will continue? Explain your thinking.

Synthesis

9. What are some things you think are important to remember when dividing whole numbers that have remainders?

Use the examples if they help with your thinking.

$$62 \div 5$$

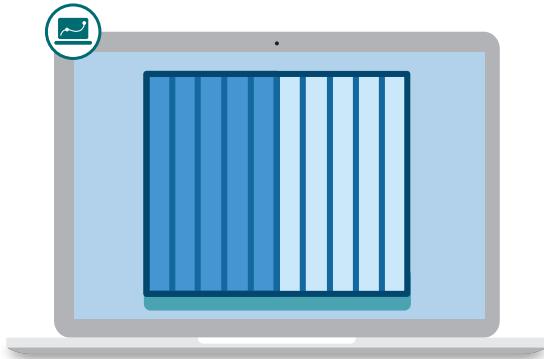
$$183 \div 15$$

$$1 \div 8$$

Things to Remember:

Division Diagrams

Let's divide decimals using hundredths charts and new expressions.

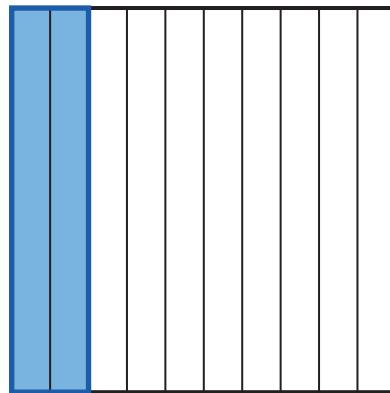


Warm-Up

- 1** This large square represents 1.

- a** Select *all* the equations you could use to determine how many blue pieces you need to fill the large square.

- A. $1 \div 0.2 = ?$
- B. $0.2 \div 1 = ?$
- C. $10 \div 2 = ?$
- D. $0.2 \cdot ? = 1$
- E. $2 \div 10 = ?$



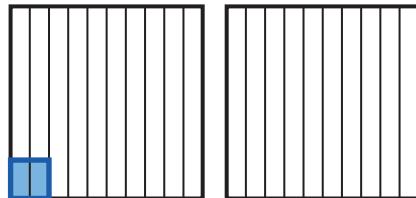
- b** Pick one equation and explain how it represents the diagram.

Decimal Division Strategies

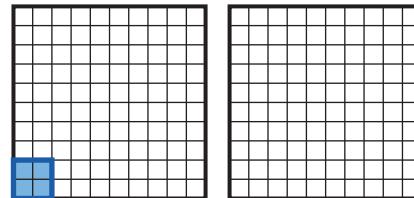
- 2** Each large square represents 1.

Determine the value of $2 \div 0.04$. Use the tenths or hundredths chart if it helps with your thinking.

Tenths

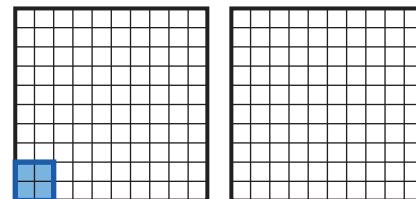


Hundredths



- 3** Diamond claims that $2 \div 0.04$ has the same value as $200 \div 4$.

Explain why this makes sense.

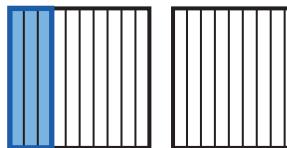


Decimal Division Strategies (continued)

- 4** Each large square represents 1.

Determine the value of $6 \div 0.3$. Use the tenths or hundredths chart if it helps with your thinking.

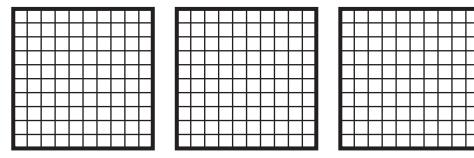
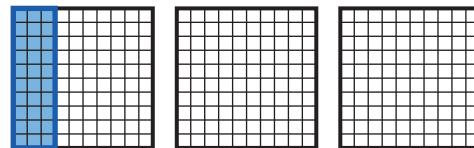
Tenths



Tenths



Hundredths



- 5** Here is how Arjun determined the value of $6 \div 0.3$.

a

Discuss: Describe Arjun's strategy.

$$\begin{aligned} 6 \div 0.3 &= \frac{6}{1} \div \frac{3}{10} \\ &= \frac{60}{10} \div \frac{3}{10} \\ &= 60 \div 3 \\ &= 20 \end{aligned}$$

b

Use his strategy to determine the value of $5 \div 0.02$.

Different Expression, Same Value

- 6** Select *all* the expressions that have the same value as $1.2 \div 0.05$.

Use the tenths or hundredths chart if it helps with your thinking.

A. $12 \div 5$

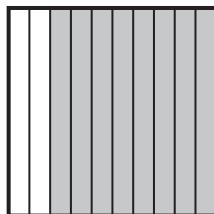
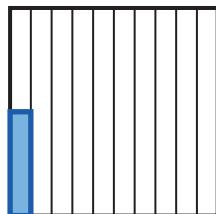
B. $12 \div 0.5$

C. $120 \div 5$

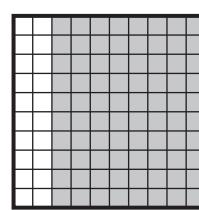
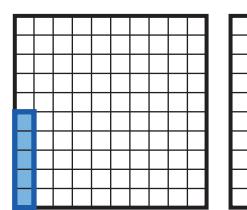
D. $\frac{120}{100} \div \frac{5}{100}$

E. $\frac{12}{100} \div \frac{5}{100}$

Tenths



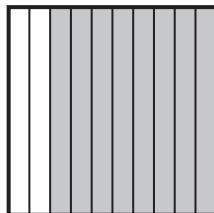
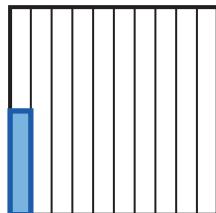
Hundredths



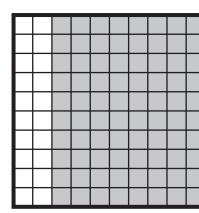
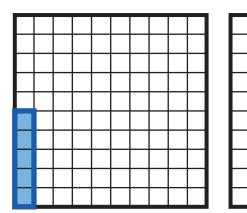
- 7** Calculate $1.2 \div 0.05$.

Use the tenths or hundredths chart if it helps with your thinking.

Tenths



Hundredths



Different Expression, Same Value (continued)

8 Write an expression with the same value as $3.6 \div 0.012$ using fractions.

9 Here is Raven's work for $3.6 \div 0.012$.

What would you say to help her understand her mistake?

$$\begin{aligned}3.6 \div 0.012 &= \frac{36}{10} \div \frac{12}{1000} \\&= 36 \div 12 \\&= 3\end{aligned}$$

Card Sort

- 10** Match each division problem with its equivalent representations.

a. $\frac{240}{100} \div \frac{8}{100}$

b. $\frac{240}{100} \div \frac{80}{100}$

c. $\frac{2400}{1000} \div \frac{8}{1000}$

d. 30

e. 300

f. $\frac{2400}{1000} \div \frac{800}{1000}$

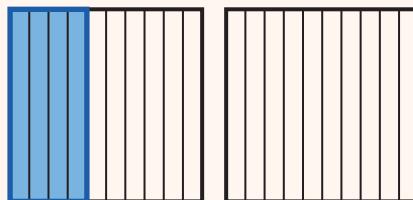
2.4 ÷ 0.8	2.4 ÷ 0.08	2.4 ÷ 0.008

Explore More

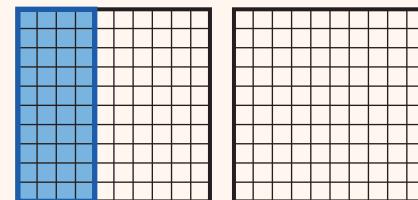
- 11** The value of $2 \div 0.4$ is 5.

How many other decimal division expressions can you write that have a value of 5?

Tenths



Hundreds



Record a dividend and divisor for each expression.

Dividend	2				
Divisor	0.4				

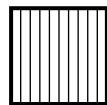
12 Synthesis

Circle an expression that has the same value as
 $1.9 \div 0.1$.

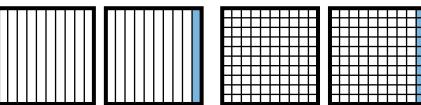
$$\frac{19}{10} \div \frac{1}{10} \quad 19 \div 1 \quad 190 \div 10$$

Explain how you know it has the same value.
Use the diagrams if they help with your thinking.

Tenths



Hundredths

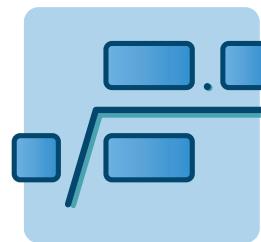


Things to Remember:

Name: Date: Period:

Return of the Long Division

Let's use long division to divide decimals.



Warm-Up

1. **Discuss:** What is the value of each expression?

a) $0.8 \div 4$

b) $0.24 \div 3$

c) $0.035 \div 5$

Nur's and Shep's Strategies

Round 1

2. Use any strategy to calculate $26.5 \div 5$.

3. Let's look at Nur's and Shep's strategies for calculating $26.5 \div 5$.



Discuss: How are their strategies alike? How are they different?

Round 2

4. Use any strategy to calculate $106 \div 0.8$.

5. Let's look at Nur's and Shep's strategies for calculating $106 \div 0.8$.



Discuss: Why was it helpful to rewrite this expression as $1060 \div 8$?

Nur's and Shep's Strategies (continued)

Round 3

6. Calculate $19.8 \div 1.5$.
7. Let's look at Nur's and Shep's strategies for calculating $19.8 \div 1.5$.



Discuss: Why did they each write $198 \div 15$ instead of $1980 \div 15$?

8. Use any strategy to show that these equations are true.

a $0.7 \div 0.4 = 1.75$

b $22.5 \div 0.04 = 562.5$

Finding Expressions

You will use the Activity 2 Cards to complete this table. You can use each card more than once.

9. Write down at least one expression that ...

... includes division by a number:	... has a quotient:
Greater than 1.	Less than 1.
Less than 1.	Greater than 15.
In the hundredths place.	Close to 10.

10. Work with a partner to calculate the value of at least three expressions each. Show all of your thinking. Make sure you and your partner select different expressions. When you're finished, compare your thinking with your partner.

Expression: Expression: Expression:

My work: My work: My work:

Synthesis

11. What are some things you think are important to remember when dividing with decimals?

Use the examples if they help with your thinking.

$$26.5 \div 5$$

$$57 \div 1.5$$

$$5.11 \div 0.05$$

Things to Remember:

Finding Expressions

 **Directions:** Make one copy per pair of students. Then pre-cut the cards and give each pair of students one set.

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Card A

$$62 \div 5$$

Card B

$$41 \div 4$$

Card C

$$1 \div 25$$

Card D

$$12.6 \div 0.08$$

Card E

$$5.125 \div 0.05$$

Card F

$$3.7 \div 0.4$$

Card G

$$9 \div 1.2$$

Card H

$$18.6 \div 1.5$$

Card I

$$7 \div 8$$

Card J

$$53.825 \div 5$$

Card K

$$77.4 \div 5$$

Card L

$$7.353 \div 0.3$$

Name: Date: Period:

Grocery Prices

Let's explore the cost of groceries in different cities.



Warm-Up

In 2021, the average household in Austin, Texas, made about \$1,500.00 per week.¹

1. **Discuss:** What percentage of their income do you think a family might spend to buy the groceries on this list?

2. Complete this statement using the screen:

These groceries would cost an average household in Austin % of their income.

Groceries

Milk (1 gal)	Apples (1 lb)
Bread (1 loaf)	Bananas (1 lb)
Rice (1 lb)	Oranges (1 lb)
Eggs (1 dozen)	Tomatoes (1 lb)
Cheese (1 lb)	Potatoes (1 lb)
Chicken (1 lb)	Onion (1 lb)
Beef (1 lb)	Lettuce (1 head)

3. Would this list of groceries cost more than \$100.00 or less than \$100.00? Explain your thinking.

¹Source: U.S. Bureau of Labor Statistics

Activity**1**

Name: Date: Period:

Grocery Prices

Tyani and Anika were asked to calculate approximately how much these groceries would cost for the average household in Austin, Texas. They each wrote expressions to help them. Here is their work.

Tyani

$$\begin{aligned} & \text{2.2\% of 1500} \\ & = 0.22 \cdot 1500 \end{aligned}$$

Anika

$$\begin{aligned} & \text{2.2\% of 1500} \\ & = 0.022 \cdot 1500 \end{aligned}$$

4. Whose work is correct? Explain your thinking.
5. Calculate 2.2% of \$1,500.00. Does this number make sense for this situation?

Here is the approximate average weekly household income in 2021 for three other cities.¹

Honolulu, Hawaii

\$1,700.00

Seattle, Washington

\$2,100.00

Jackson, Mississippi

\$700.00

6. If the average household in each city spent 2.2% of their weekly income on these groceries, how much would they spend? Complete the table and show your thinking.

	Honolulu	Seattle	Jackson
Expression for 2.2% of Income			
Money Spent			

¹Source: U.S. Bureau of Labor Statistics

Grocery Prices (continued)

You'll use the Activities 1 & 2 Sheet to complete this activity.

7. In which cities can someone buy all the groceries on the list using 2.2% or less of their weekly income? In which cities can they not?

8. Discuss:

- a** Why do you think the cost of these groceries is different in different places?

- b) What do you think the impact might be on families who spend more than 2.2% of their weekly income on these groceries?

Bought Milk?

You'll use the Activities 1 & 2 Sheet to complete this activity.

9. How much does a gallon of milk cost in Austin, Texas?

10. What percentage of the total cost of the grocery list is that? Show your thinking.
A. 0.0967% B. 0.967% C. 9.67% D. 96.7%

Yunuen says: *Milk is too expensive in Jackson. It's 11% of the total cost of that grocery list!*

11. Show or explain where the 11% comes from.

12. Do you agree with Yunuen? If you do, explain what you think would be a fair price for milk in Jackson. If you do not, explain why you think milk in Jackson is priced fairly.

13. Choose a different city. What percentage of the total grocery bill is milk? Do you think milk is too expensive in this city? Show or explain your thinking.

Explore More

14. Choose a different food that has a price that you think is too high. Use percentages, decimals, rates, or ratios to propose a fairer price for that food.

Synthesis

15. Choose one question and write your response.

- a What do you think is a fair way to determine the price for groceries in different places?

- b What can people do if they live in a place where the price of groceries is high?

- c What new questions do you have about food costs around the world?

Things to Remember:

Grocery Prices

All prices are based on data from the USDA in 2021, as collected by Balancing Everything.¹

Seattle, Washington

Milk (1 gal)	\$3.60
Bread (1 loaf)	\$3.06
Rice (1 lb)	\$2.03
Eggs (1 dozen)	\$3.01
Cheese (1 lb)	\$7.29
Chicken (1 lb)	\$5.58
Beef (1 lb)	\$6.67
Apples (1 lb)	\$2.30
Bananas (1 lb)	\$0.81
Oranges (1 lb)	\$2.11
Tomatoes (1 lb)	\$2.59
Potatoes (1 lb)	\$1.06
Onion (1 lb)	\$1.19
Lettuce (1 head)	\$2.09
Total	\$43.39

Honolulu, Hawaii

Milk (1 gal)	\$6.77
Bread (1 loaf)	\$4.82
Rice (1 lb)	\$2.65
Eggs (1 dozen)	\$4.52
Cheese (1 lb)	\$7.21
Chicken (1 lb)	\$6.37
Beef (1 lb)	\$7.55
Apples (1 lb)	\$3.30
Bananas (1 lb)	\$1.70
Oranges (1 lb)	\$3.13
Tomatoes (1 lb)	\$3.16
Potatoes (1 lb)	\$2.52
Onion (1 lb)	\$2.62
Lettuce (1 head)	\$3.63
Total	\$59.95

Austin, Texas

Milk (1 gal)	\$3.19
Bread (1 loaf)	\$2.21
Rice (1 lb)	\$1.34
Eggs (1 dozen)	\$2.77
Cheese (1 lb)	\$5.00
Chicken (1 lb)	\$3.88
Beef (1 lb)	\$5.86
Apples (1 lb)	\$1.57
Bananas (1 lb)	\$0.61
Oranges (1 lb)	\$1.49
Tomatoes (1 lb)	\$1.38
Potatoes (1 lb)	\$1.12
Onion (1 lb)	\$1.02
Lettuce (1 head)	\$1.56
Total	\$33.00

Jackson, Mississippi

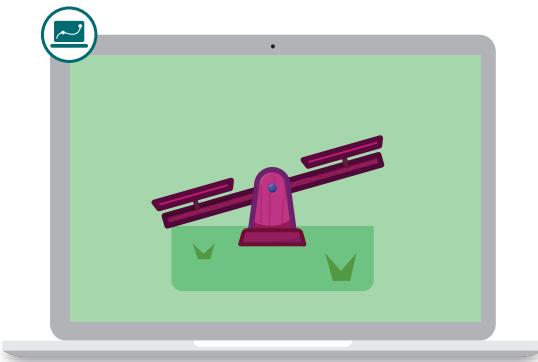
Milk (1 gal)	\$4.68
Bread (1 loaf)	\$3.66
Rice (1 lb)	\$2.00
Eggs (1 dozen)	\$1.63
Cheese (1 lb)	\$5.99
Chicken (1 lb)	\$4.75
Beef (1 lb)	\$3.98
Apples (1 lb)	\$3.66
Bananas (1 lb)	\$0.69
Oranges (1 lb)	\$2.41
Tomatoes (1 lb)	\$2.82
Potatoes (1 lb)	\$2.03
Onion (1 lb)	\$1.82
Lettuce (1 head)	\$2.17
Total	\$42.29

¹ Source: *Balancing Everything*

Name: Date: Period:

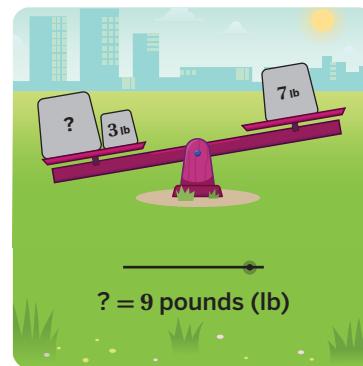
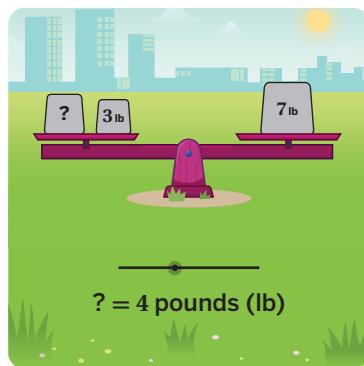
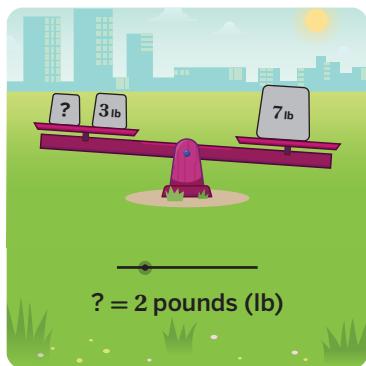
Weight for It

Let's use a seesaw to determine the weights of different animals.



Warm-Up

- 1** **a** Take a look at some weights on a seesaw.



- b** What do you notice? What do you wonder?

I notice:

I wonder:

Equations and Tape Diagrams

- 2** This dog and a 5-pound weight balance a 17-pound weight.

How much does the dog weigh?



- 3** Tariq wrote an *equation* to represent the situation. He used the **variable** d to represent the dog's weight.

Tariq

$$d + 5 = 17$$

Explain how Tariq's equation is like the seesaw situation.

Equations and Tape Diagrams (continued)

- 4** These 3 foxes balance with an 18-pound weight. Each fox weighs the same amount.

a Choose an equation that represents this situation.

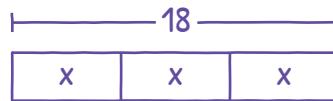
- A. $3 + x = 18$
- B. $3 \cdot x = 18$
- C. $x + x + x = 18$
- D. $3 + 18 = x$



b How much does each fox weigh?

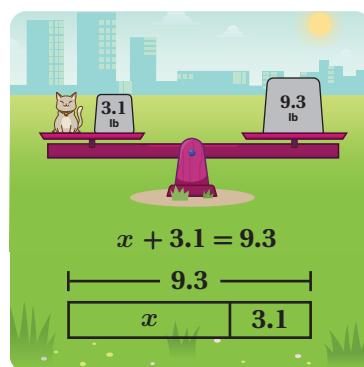
- 5** Tariq drew a tape diagram to determine the weight of each fox.

How are the tape diagram and the equation you chose in the previous problem alike?



- 6** This cat and a 3.1-pound weight balance a 9.3-pound weight.

How much does the cat weigh? Use the tape diagram if it helps with your thinking.



Determining Unknown Weights

- 7** For each equation or tape diagram, put a check mark under the balanced seesaw it represents.

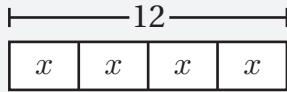


$$4 \cdot x = 12$$

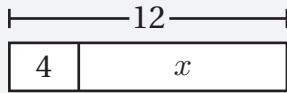
$$4 + x = 12$$

$$\begin{array}{c} x + x + x + x = \\ 12 \end{array}$$

$$x = 8$$

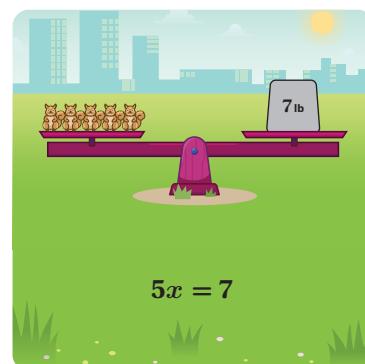


$$x = 3$$



- 8** These 5 squirrels balance with a 7-pound weight. Each squirrel weighs the same amount.

How much does each squirrel weigh? Draw a tape diagram if it helps with your thinking.



Challenge Creator

9 You will use the Activity 3 Sheet to create your own seesaw challenge.

- a** **Make It!** On the Activity 3 Sheet, create a balanced seesaw challenge.
- b** **Solve It!** On this page, write an equation that represents your balanced seesaw problem. Then determine the weight of your animal. Draw a tape diagram if it helps with your thinking.

My Equation	Weight of My Animal

- c** **Swap It!** Swap your challenge with one or more partners. Write your partner's equation, then determine the weight of their animal. Draw a tape diagram if it helps with your thinking.

Equation	Weight of One Animal

Partner 1

Partner 2

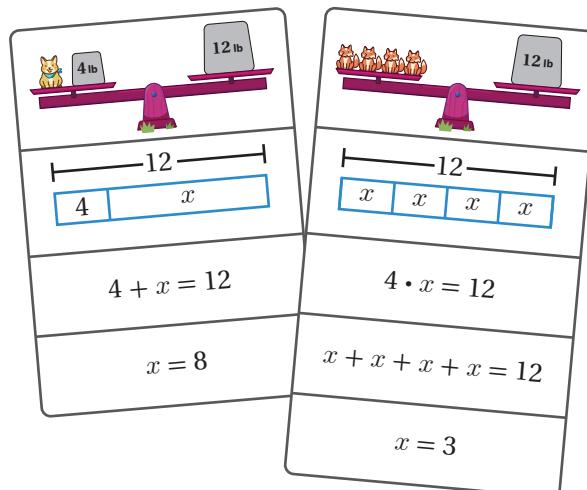
Partner 3

Partner 4

10 Synthesis

How can you tell if an equation and a tape diagram match?

I can tell if an equation and a tape diagram match . . .



Things to Remember:

Challenge Creator

- Choose one animal from the pictures or make up your own.
- Create a balanced seesaw. Draw copies of your animal on the left side of the seesaw. If you want to, add extra weight on the left side with your animal. Then fill in the weight on the right side.
- Write an equation that represents your balanced seesaw.
- Do not determine the weight of the animal on this page. You and your classmates will determine the weight of each other's animals in your Student Edition.



Weight



Cat



Dog



Squirrel



Fox



Raccoon

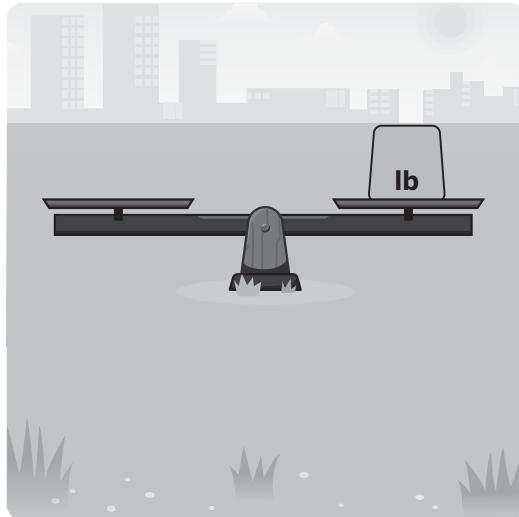


Alligator



Frog

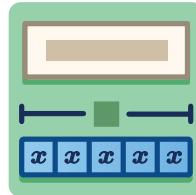
My seesaw:



My equation:

Five Equations

Let's represent situations with equations and tape diagrams.



Warm-Up

1. Here is a situation. Let's make sense of it together as a class.

_____ cats weigh _____ pounds total.

Each cat weighs the same, so they each weigh _____ pounds.

a



Discuss: What is this situation about?

b

Let's look at the missing information. What does the variable c represent?

c

Create a tape diagram or sketch that represents this situation.

d

Use your tape diagram or sketch to determine the value of c .

Equations and Tape Diagrams

Here are five equations.

$$x + 5 = 20$$

$$20 = x - 5$$

$$5 \cdot 20 = x$$

$$5x = 20$$

$$20x = 5$$

- 2.** Circle two equations that have something in common.



Discuss: How are these equations alike? How are they different?

- 3.** Match each tape diagram with one of the equations. Two equations will not have matches.

Tape Diagram	Equation

- 4.** Draw a tape diagram for an equation that did not have a match.

Tape Diagram	Equation

Which Equation?

4 cats weigh 48 pounds total. Each cat weighs the same, so they each weigh c pounds.

Equation	Solution to the Equation	Solution's Meaning
$4c = 48$	$c = 12$	Each cat weighs 12 pounds.

5. What do you think a solution to an equation is?

You will use a set of description and situation cards.

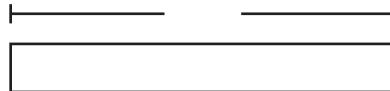
6. Match each card with the equation that represents it.
7. Determine the solution to each equation and write the solution's meaning for each situation.

	$x - 9 = 22$	$9x = 22$	$9 + x = 22$
Matching Cards			
Solution to the Equation			
Solution's Meaning			

Synthesis

8. How can you tell which equation represents a situation? Use the example if it helps with your thinking.

Kwasi rides the subway 20 stops to get to work. After x stops, he has 5 stops left.



$$x + 5 = 20$$

$$5x = 20$$

Things to Remember:

Which Equation?

 **Directions:** Make one copy per two pairs of students. Then pre-cut the cards and give each pair of students one set of Cards 1–6.

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Card 1

Kwasi rides the bus for a total of 22 stops. After 9 stops, there are x stops left.

Card 2

22 is 9 more than x .

Card 3

Mohamed made $\$x$ from mowing lawns. He spent $\$9$ on a new video game and has $\$22$ left.

Card 4

The product of 9 and x is 22.

Card 5

9 less than x is 22.

Card 6

Ren has $\$22$ to spend on day passes to ride the subway. Each day pass costs $\$9$, and Ren can buy x of them.

Card 1

Kwasi rides the bus for a total of 22 stops. After 9 stops, there are x stops left.

Card 2

22 is 9 more than x .

Card 3

Mohamed made $\$x$ from mowing lawns. He spent $\$9$ on a new video game and has $\$22$ left.

Card 4

The product of 9 and x is 22.

Card 5

9 less than x is 22.

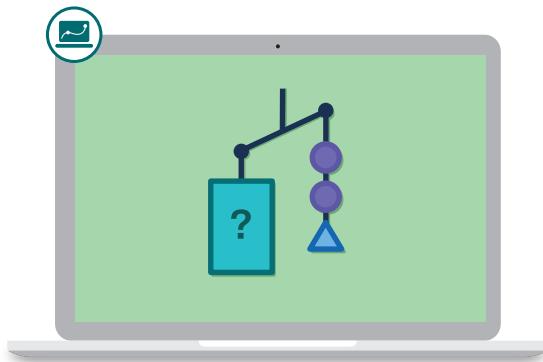
Card 6

Ren has $\$22$ to spend on day passes to ride the subway. Each day pass costs $\$9$, and Ren can buy x of them.

Name: Date: Period:

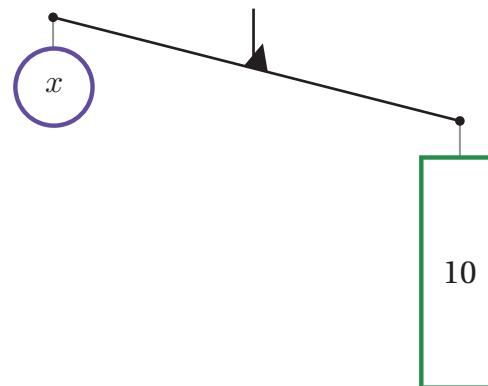
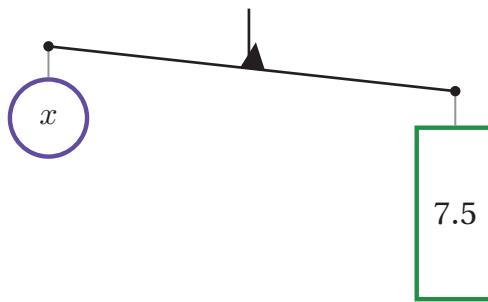
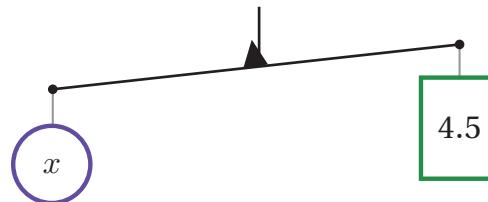
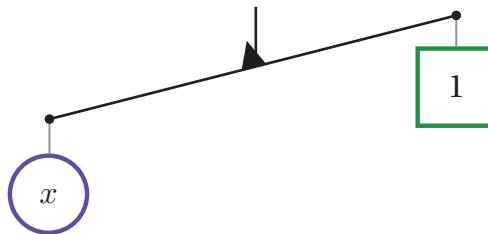
Hanging Around

Let's use balanced hangers to solve equations.



Warm-Up

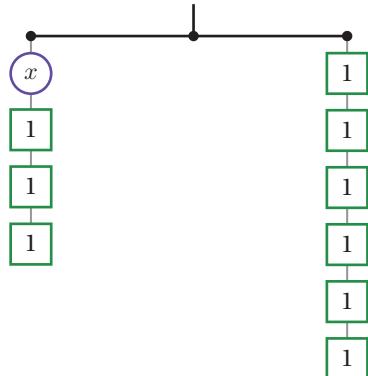
- 1** **a** Take a look at the hangers with a circle of weight x on one side and a rectangle of different weights on the other side.



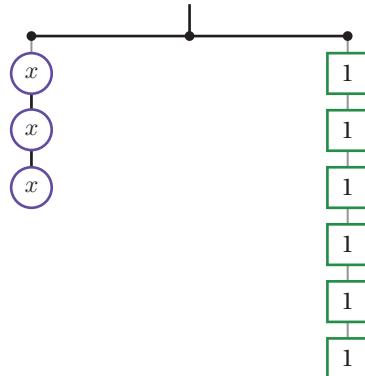
Discuss: What is the weight of the circle? Explain your thinking.

Connect It

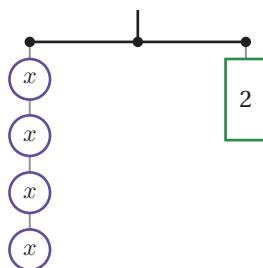
- 2** What value of x balances the hanger?



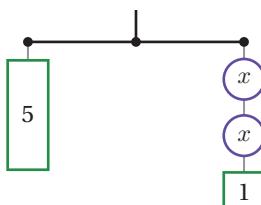
- 3** What value of x balances the hanger?



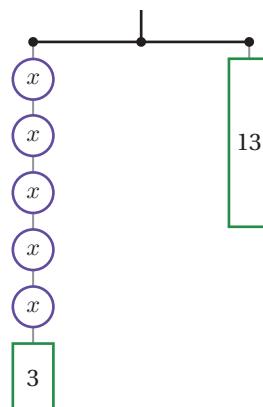
- 4** **a** Take a look at these hangers and the equations that represent them.



$$\text{Equation: } 4x = 2$$



$$\text{Equation: } 5 = 2x + 1$$



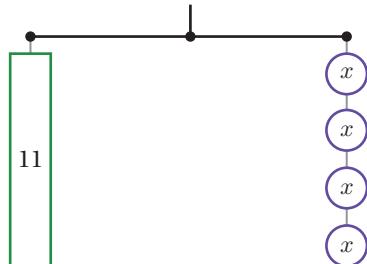
$$\text{Equation: } 5x + 3 = 13$$

- b** Explain how an equation is like a hanger.

Make It, Solve It

- 5** Select an equation that represents this hanger.

- A. $11 + x = 4$
- B. $11 = 4x$
- C. $11 = x + 4$
- D. $11 = x + x + x + x$



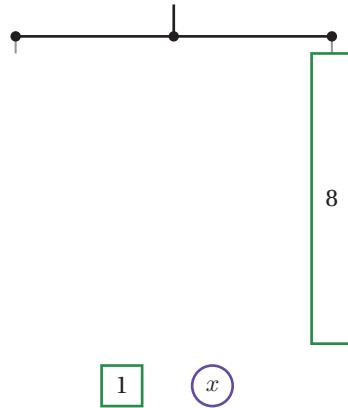
Explain your thinking.

- 6** Use the hanger or the equation from the previous problem to determine the value of x that balances the hanger.

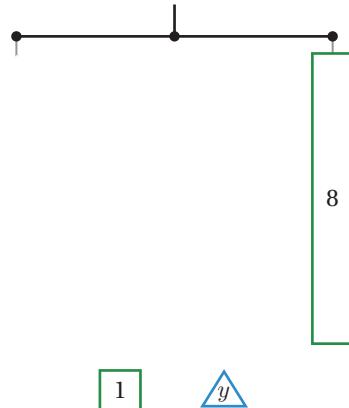
Make It, Solve It (continued)

- 7** Make a balanced hanger to represent each equation.

$$5x = 8$$



$$5 + y = 8$$



- 8** **a** Make a balanced hanger that represents $6 = x + 2$.



- b** What is the value of x that balances the hanger?

Challenge Creator

9 You will use the Activity 3 Sheet to create your own hanger challenge.

- a** **Make It!** On the Activity 3 Sheet, create your own balanced hanger challenge.
- b** **Solve It!** On this page, write the equation that represents your hanger and then determine the value of x that balances your hanger.

My Equation	Solution to My Equation

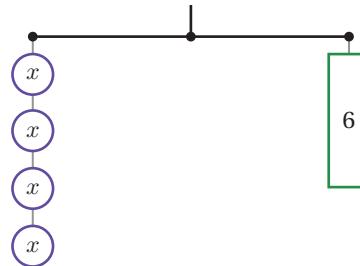
- c** **Swap It!** Swap your challenge with one or more partners. Write their equation, then determine the value of x that balances their hanger.

	Equation	Solution to Their Equation
Partner 1		
Partner 2		
Partner 3		
Partner 4		

10 Synthesis

How can a balanced hanger help determine the solution to an equation?

Use the hanger and equation if that helps you with your thinking.

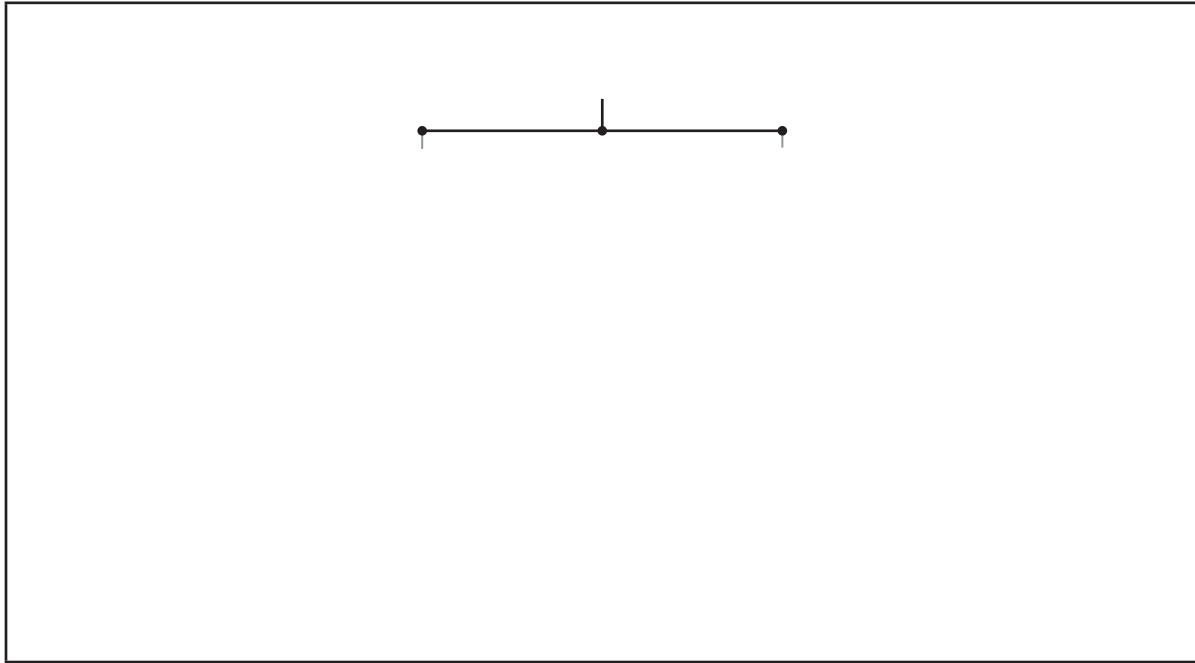


$$4x = 6$$

Things to Remember:

Challenge Creator

Create a balanced hanger using circles and squares. Write an equation that represents your hanger. Do not determine the solution to the equation on this page.



Equation:

Challenge Creator

- Create your own rectangle. Try to create a rectangle none of your classmates will.
- Write one expression to represent its area.
- Your classmates will write an expression that is equivalent to the one you wrote.

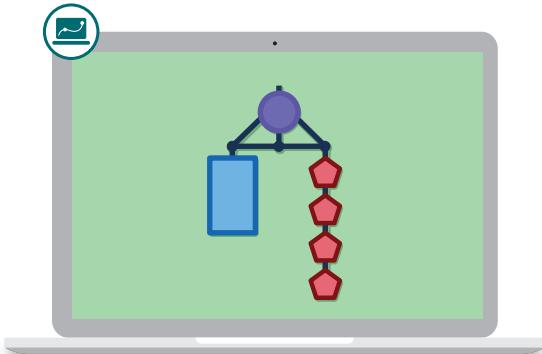
Rectangle:

Expression:

Name: Date: Period:

Hanging It Up

Let's use a variety of strategies to solve equations.



Warm-Up

Determine the value of each expression mentally. Try to think of more than one strategy.

1 $5 - 2$

2 $5 - 2.1$

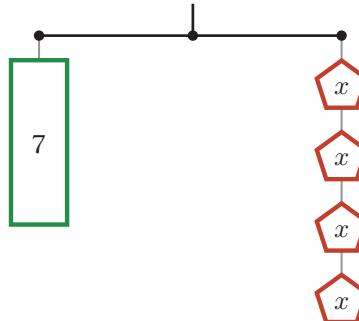
3 $5 - 2.17$

4 $5 - 2.017$

Methods for Solving

5

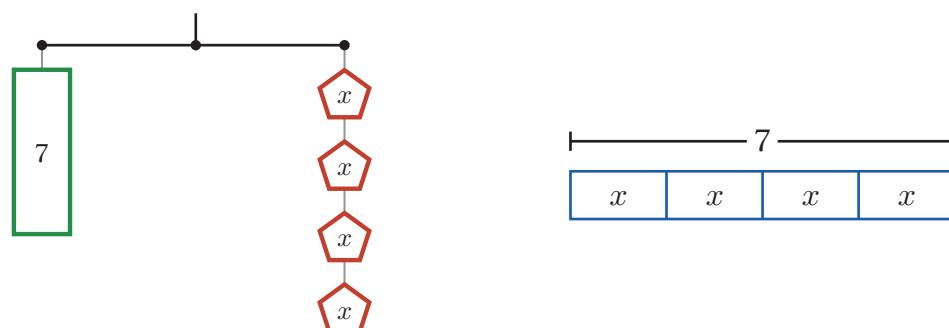
- a** Write an equation that matches the hanger.



- b** What is the *solution* to this equation?

6

- Here are two different representations of the equation $7 = 4x$.



Discuss: How can you determine the solution using each representation?

7

- Here is how Fabiana solved the equation $7 = 4x$.

Describe Fabiana's strategy.

Fabiana

$$\frac{7}{4} = \frac{4x}{4}$$

Solving and Solutions

- 8** Match each solution to its equation. One solution will not have a match.

8.5

0.1

6.1

 $\frac{3}{5}$ $\frac{1}{10}$ $\frac{5}{3}$

$$\frac{2}{3}d = \frac{10}{9}$$

$$12.6 = b + 4.1$$

$$10c = 1$$

$$10 + a = 16.1$$

- 9** Imani and Demari solved this equation.

Imani said the solution is $d = \frac{3}{5}$.

Demari said the solution is $d = \frac{5}{3}$.

Whose solution is correct? Circle one.

Imani's

Demari's

Both

Neither

Explain your thinking.

$$\frac{2}{3}d = \frac{10}{9}$$

 $\frac{3}{5}$ $\frac{5}{3}$

Solving and Solutions (continued)

- 10** Here are Fabiana's and Alejandro's strategies for solving $10 + a = 16.1$.

Fabiana

$$\begin{aligned}10 + a &= 16.1 \\ -10 &\quad -10 \\ a &= 6.1\end{aligned}$$

Alejandro

$$\begin{aligned}10 + a &= 16.1 \\ 10 + 6 &= 16 \\ \text{so . . .} & \\ 10 + 6.1 &= 16.1\end{aligned}$$



Discuss: How are their strategies alike? How are they different?

- 11** Use Fabiana's strategy to solve the equation $3.5 = x + 2.01$.

Repeated Challenges

12

- Decide with your partner who will complete Column A and who will complete Column B.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.
- Solve as many equations as you have time for. Sense-making is more important than speed.

	Column A	Column B
a	$36 = 4x$ $x = \dots$	$7x = 63$ $x = \dots$
b	$13 = x + 5$ $x = \dots$	$21 = x + 13$ $x = \dots$
c	$\frac{1}{3} = 2x$ $x = \dots$	$3x = \frac{1}{2}$ $x = \dots$
d	$x + 6.17 = 9$ $x = \dots$	$12.22 = x + 9.39$ $x = \dots$

Repeated Challenges (continued)

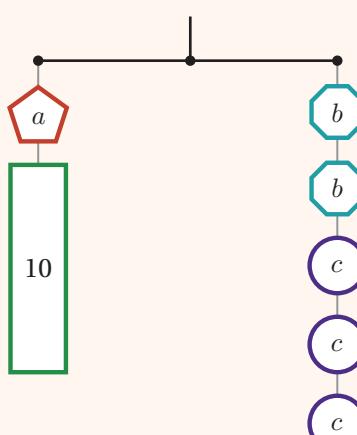
	Column A	Column B
e	$x + 1.8 = 14.7$ $x = \dots$	$x + 5.3 = 18.2$ $x = \dots$
f	$\frac{1}{2}x = 16$ $x = \dots$	$4 = \frac{1}{8}x$ $x = \dots$
g	$\frac{7}{8} = x + \frac{1}{4}$ $x = \dots$	$x + \frac{1}{16} = \frac{11}{16}$ $x = \dots$

Explore More

- 13 Complete the table with values for a , b , and c that will make the hanger balance.

Try to determine as many different sets of values as you can.

a	b	c



14 Synthesis

Describe a strategy for solving an equation.

Use the examples if they help with your thinking.

$$3x = 18$$

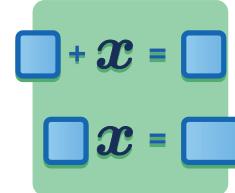
$$3 + x = 15.6$$

Things to Remember:

Name: Date: Period:

Swap and Solve

Let's write and solve equations.



Warm-Up

1. Here is a situation with hidden information. Let's make sense of it as a class.

Takeshi has [] to spend on laundry. It costs [] to wash and dry each load. Takeshi can wash [] loads of laundry.

2. Dhruv and Nyanna each wrote an equation to represent this situation.

Dhruv
 $p = 21 \cdot 3.50$

Nyanna
 $3.50p = 21$

Whose equation is correct? How do you know?

Stronger and Clearer Each Time

Here is a set of equations we will use throughout this lesson.

$$x + \frac{3}{4} = 6$$

$$\frac{3}{4}x = 6$$

$$6x = \frac{3}{4}$$

$$x - \frac{3}{4} = 6$$

$$0.25 + x = 20$$

$$0.25x = 20$$

$$20 \cdot 0.25 = x$$

$$x - 20 = 0.25$$

3. Select an equation and solve it for x .

Equation	Solution

4. Write a first draft of a situation to match this equation. Make sure to include what the variable represents in your situation.
5. Meet with a partner to discuss your first drafts. Use the questions on the screen to help you provide feedback to each other.
6. Write a second draft that is stronger and clearer.

Trade and Solve

Takeshi has \$21 to spend on laundry. It costs \$3.50 to wash and dry each load. Takeshi can wash p loads of laundry.

Equation	Solution	Solution Check	Solution's Meaning
$3.50p = 21$	$p = 6$	$3.50 \cdot 6 = 21$	Takeshi can wash and dry 6 loads of laundry for \$21.

7. What do you think a solution's meaning is?

8. You will need several different partners for this activity. With each partner, trade the slips of paper with your situations and complete the table for their situation.

	Partner A	Partner B	Partner C
Partner's Name			
Equation			
Solution			
Solution Check			
Solution's Meaning			

Synthesis

9. What do you think is important to remember when writing equations to represent situations?

Takeshi has \$10 to spend on laundry.
It costs \$2.50 to wash and dry each load.
Takeshi can wash p loads of laundry.

$$2.50p = 10$$

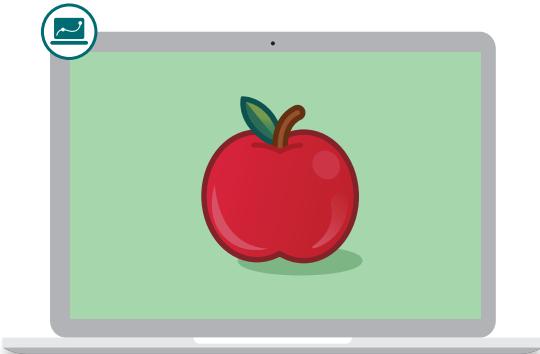


Things to Remember:

Name: Date: Period:

Vari-apples

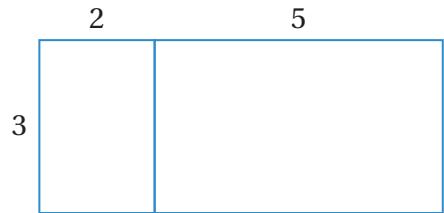
Let's use variable expressions to represent situations.



Warm-Up

- 1 Here are two rectangles.

Rectangle A



Rectangle B



Which rectangle has a greater area? Circle one.

Rectangle A

Rectangle B

They have the same area

Explain your thinking.

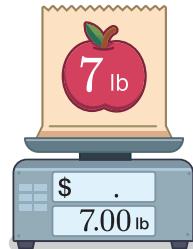
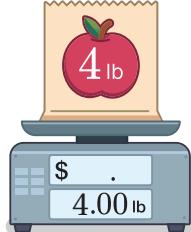
Intro to Variable Expressions

Apples at your store cost \$1.50 per pound.

- 2** A customer orders 3 pounds of apples. How much should you charge them?



- 3** Here are three new orders. How much should you charge for each order?



Apples (lb)	Cost (\$)
4	
7	
8	

- 4** Describe how to determine the cost of *any* number of pounds of apples.

Intro to Variable Expressions (continued)

- 5** Rudra and Sai each wrote an expression to describe the cost of p pounds of apples.

Rudra: $p + 1.50$

Sai: $1.50p$

Whose expression is correct? Circle one.

Rudra's

Sai's

Both

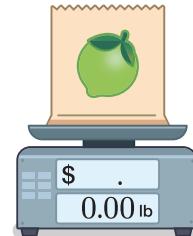
Neither

Explain your thinking.

Apples (lb)	Cost (\$)
3	4.50
4	6.00
7	10.50
8	12.00
p	

- 6** Limes at your store cost \$2.40 per pound.

How much should you charge for p pounds of limes?



Comparing Variable Expressions

- 7** For \$5, you can get your groceries delivered. What is the total cost for each of these grocery deliveries?

Cost of Groceries (\$)	Total Cost (\$)
37.95	42.95
50.86	
72.11	
87.94	



Grocery cost: \$37.95
 Delivery fee: \$5.00
Total: \$42.95

- 8** Write an expression for how much you should charge for g dollars worth of groceries, including delivery.

- 9** Match each situation with an expression that represents its cost. Two expressions will have no match.

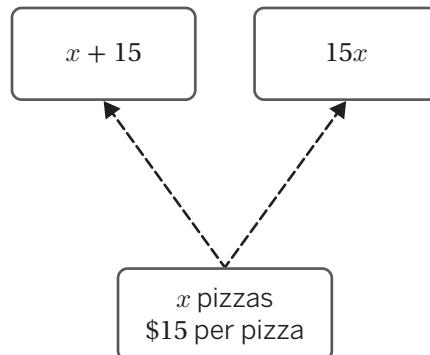
10 x 15 x $x + 10$ $x + 15$

x pizzas
\$15 per pizza

x dollars of groceries
\$10 for delivery

Comparing Variable Expressions (continued)

- 10** Which expression represents this situation?



- 11** The expression $15x$ has one term. The expression $15 + x$ has two terms.

Select *all* the expressions that also have two terms.

- A. $y - 6$
- B. $\frac{1}{2}x$
- C. $3x + 2$
- D. $x + y$
- E. $3 \cdot 2$

Explore More

- 12** Describe a situation that could be represented by the expression $2p + 6$. Create a table if it helps with your thinking.

$$2p + 6$$

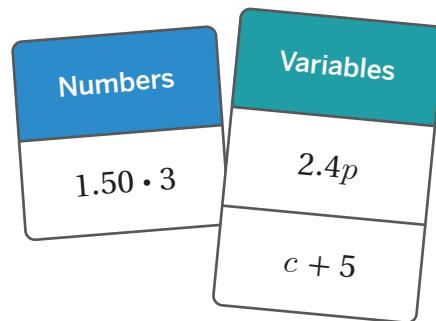


13 Synthesis

Here are two types of expressions: expressions with numbers and expressions with variables.

When might each kind of expression be useful?

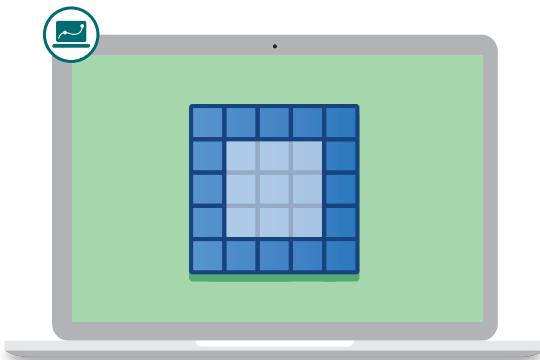
Expressions with numbers are useful when . . .



Expressions with variables are useful when . . .

Things to Remember:

Name: Date: Period:



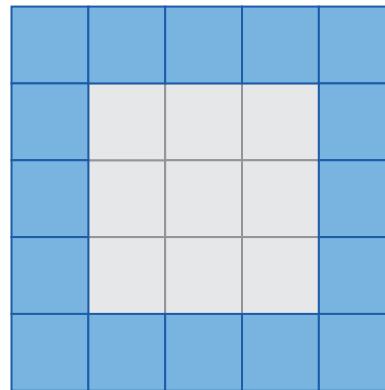
Border Tiles

Let's use diagrams to determine which expressions are equivalent.

Warm-Up

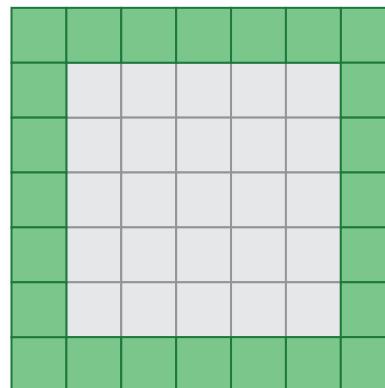
1 Here is a 3-by-3 square surrounded by border tiles.

- a** Without counting one by one, how many border tiles are there?
- b** Explain how you see it.



2 Here is a 5-by-5 square surrounded by border tiles.

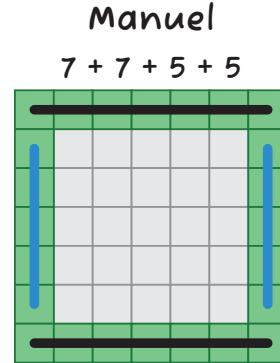
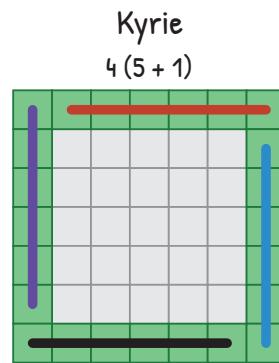
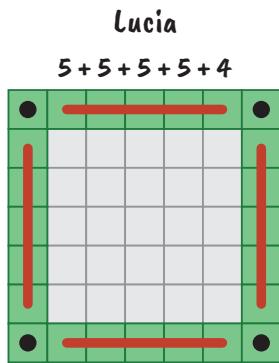
Without counting one by one, how many border tiles are there?



Border Tiles

3

- a** Take a look at Lucia's, Kyrie's, and Manuel's expressions for the 5-by-5 square.

**b**

- Discuss:** How are all of their expressions alike?

4

- Here are three new squares. Determine the number of border tiles for each square.

Model	Square	Border Tiles
	6-by-6	
	9-by-9	
	10-by-10	

5

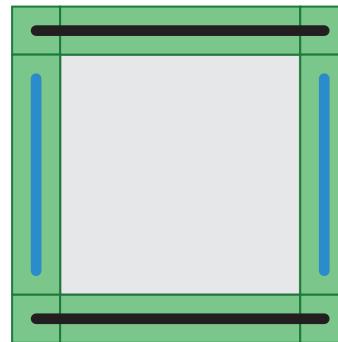
- How can you determine the number of border tiles for an n -by- n square? Use your table if it helps with your thinking.

Equivalent Expressions

- 6** Manuel wrote this expression for the number of border tiles in an n -by- n square:

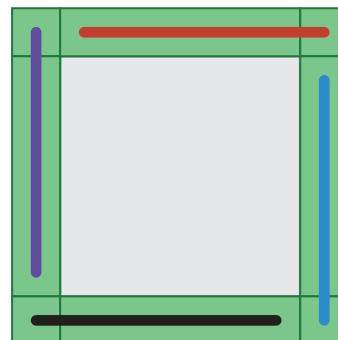
$$(n + 2) + (n + 2) + n + n$$

Show or explain how Manuel's expression is connected to his sketch.



- 7** Here is Kyrie's sketch for the same square.

What might Kyrie's expression be for the number of border tiles in an n -by- n square?



Equivalent Expressions (continued)

- 8** **Equivalent expressions** are expressions that are equal for every value of a variable.

Here are two expressions.

Kyrie: $4(n + 1)$

Manuel: $(n + 2) + (n + 2) + n + n$

- a** Use each expression to calculate the number of border tiles when $n = 8$.

Kyrie

$$4(n + 1)$$

Manuel

$$(n + 2) + (n + 2) + n + n$$

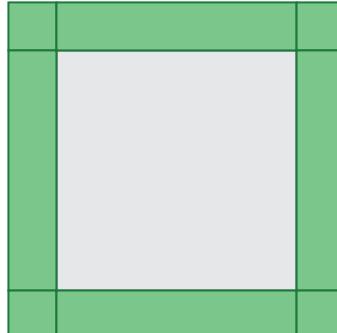
- b**



Discuss: How can you tell that Kyrie's and Manuel's expressions are equivalent?

- 9** Which expression is also equivalent to Kyrie's and Manuel's expressions?

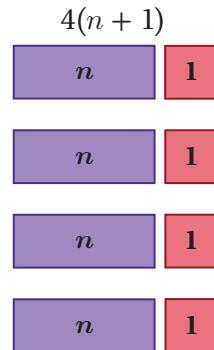
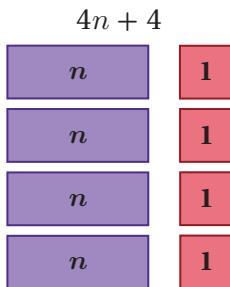
- A. $(n + 1)(n + 1)$
- B. $4n + 1$
- C. $4n + 4$



?

A New Diagram

- 10** Here is a new way of visualizing expressions.



Discuss: How do these diagrams show that $4n + 4$ and $4(n + 1)$ are equivalent?

- 11** Match each expression with the diagram it represents. One expression will have no match.

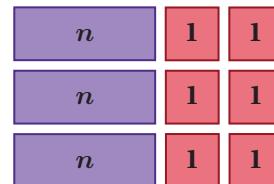
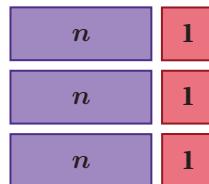
$$3(n + 1)$$

$$3(n + 3)$$

$$3n + 6$$

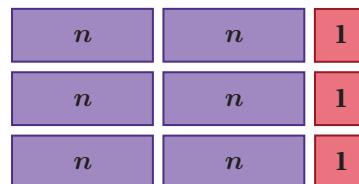
$$(n + 2) + (n + 2) + (n + 2)$$

$$3n + 3$$



- 12** Write an expression to represent this new diagram.

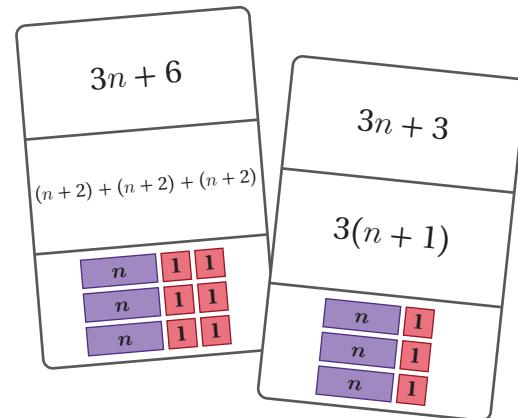
Try to write an expression you think none of your classmates will.



13 Synthesis

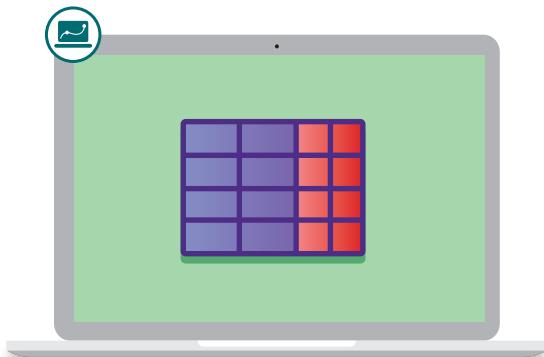
How can you decide if two expressions are equivalent?

Use the examples if they help you explain your thinking.



Things to Remember:

Name: Date: Period:

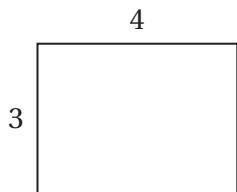


Products and Sums

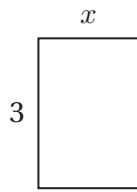
Let's explore equivalent expressions using rectangle areas.

Warm-Up

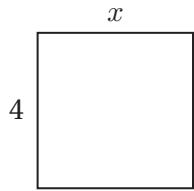
- 1** Write an expression for the area of each rectangle.

a

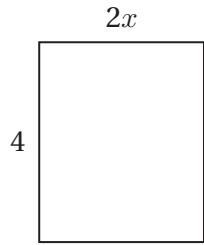
Expression:

b

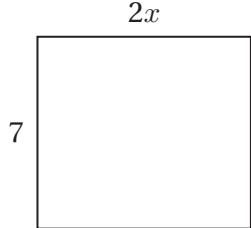
Expression:

c

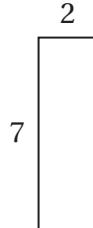
Expression:

d

Expression:

e

Expression:

f

Expression:

Rectangles and Equivalent Expressions

- 2** Here are four rectangles and the *product* and *sum* expressions that represent their areas.

Rectangle	$2x$	2	$2x$	1	$3x$	5	x	3	5
Product	$4(2x + 2)$		$2(2x + 1)$		$1(3x + 5)$		$5(x + 3)$		
Sum	$8x + 8$		$4x + 2$		$3x + 5$		$5x + 15$		

What do you notice about the product expressions?

What do you notice about the sum expressions?

- 3** **a** Create a rectangle with an area of $2(3x + 4)$. Label the sides of your rectangle.

- b** Write an equivalent expression for the area.

- 4** How would you convince someone that $2(3x + 4)$ is *not* equivalent to $6x + 4$?

More Rectangles and Equivalent Expressions

5 Create a rectangle with an area of $6x + 12$.

6 Select *all* the expressions that are equivalent to $6x + 12$. Use the rectangle you created or draw a new rectangle if it helps with your thinking.

- A. $6(x + 2)$
- B. $6(x + 12)$
- C. $3(2x + 4)$
- D. $3(x + 4)$
- E. $2(3x + 6)$

7 **a** Here are three new expressions. Select the two expressions that are equivalent.

- A. $4(x + 2)$
- B. $4(x + 8)$
- C. $4x + 8$

b Create a drawing to convince someone that the two expressions you selected are equivalent.

Challenge Creator

8 You will use the Activity 3 Sheet to create your own rectangle challenge.

- a** **Make It!** On the Activity 3 Sheet, create a rectangle challenge.
- b** **Solve It!** On this page, write two expressions that represent your rectangle. Record one expression on your Activity 3 Sheet.
- c** **Swap It!** Swap your challenge with one or more partners. Sketch your partner's rectangle and record their expression. Then create an equivalent expression.

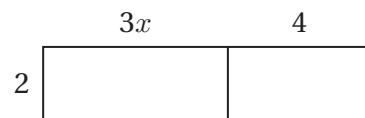
My First Expression	My Second Expression

	Sketch of Rectangle	Partner's Expression	Equivalent Expression
Partner 1			
Partner 2			
Partner 3			
Partner 4			

9 Synthesis

Describe how you can use the area of a rectangle to write two or more equivalent expressions.

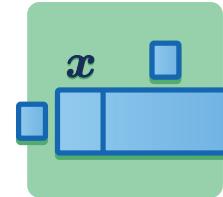
Use the example if it helps to show your thinking.



Things to Remember:

Equivalent Expressions

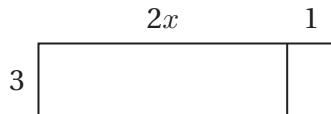
Let's explore equivalent expressions using area models and properties of operations.



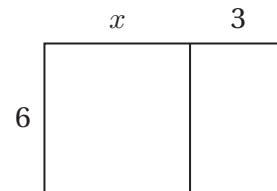
Warm-Up

1. Here are two rectangles.

Rectangle A



Rectangle B



Explain your thinking.

- b** What is the area of the other rectangle?

2. The expression $6x + 3$ has two terms. The coefficient of the term $6x$ is 6.

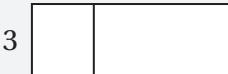
Select *all* the expressions that also have a coefficient of 6.

- A. $2(2x)$
- B. $1(6x)$
- C. $6x - 4$
- D. $x - 6$
- E. $6 + x$

Card Sort

- 3.** You will use a set of cards for this activity.

- Match each product or sum to its representation. Three expressions will be missing.
- Write in each missing expression.

	Representation	Product	Sum
a	x 6 		
b	x 2 		
c	3 2 		
d	a 3 		
e	a $3b$ 		
f	The product of 3 and the sum of a and b .		

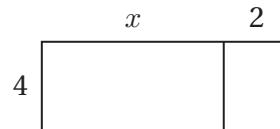
- 4.** Explain how you determined one of the missing expressions.

Two Truths and a Lie

5. Two of these expressions represent the area of this rectangle.

a Which are they?

- A. $3(x + 2) + 1(x + 2)$
- B. $1 + 3(x + 2)$
- C. $4(x + 2)$



b

Discuss: Which expression is not equivalent to the others? How do you know?

6. In each row, two choices are equivalent and one is not. Circle the one that is *not* equivalent.

	Expression A	Expression B	Expression C
a	$6(2 + x)$	$2(6 + x)$	$6(x + 2)$
b	$16x$	$1(8 + x)$	$x + 8$
c	$3x + 4(x + 2)$	$7(x + 2)$	$7x + 8$
d	$4(6x + 3x)$	$36x$	$24x + 3x$

7. Pick one problem and explain how you decided which choice was not equivalent.

8. Jazz says: $24x$ and $3x$ are **like terms**, but 9 and $2x$ are not.

What do you think Jazz means?

Synthesis

9. Explain how you can show that two expressions are equivalent.

Use these expressions if they help with your thinking.

Equivalent Expressions

$$\begin{aligned}3x + 4(x + 2) \\7x + 8\end{aligned}$$

Not Equivalent

$$7(x + 2)$$

Things to Remember:

Card Sort

 **Directions:** Make one copy per two pairs of students. Then pre-cut the cards and give each pair of students one set of Cards A–I.

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Card A

$$3(x + 6)$$

Card B

$$3x + 6$$

Card C

$$3(x + 2)$$

Card D

$$3x + 2x$$

Card E

$$3a + 3b$$

Card F

$$3a + 9$$

Card G

$$3a + 9b$$

Card H

$$3(a + 3)$$

Card I

$$3(a + b)$$

Card A

$$3(x + 6)$$

Card B

$$3x + 6$$

Card C

$$3(x + 2)$$

Card D

$$3x + 2x$$

Card E

$$3a + 3b$$

Card F

$$3a + 9$$

Card G

$$3a + 9b$$

Card H

$$3(a + 3)$$

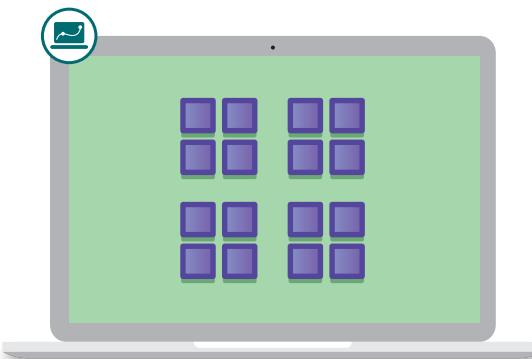
Card I

$$3(a + b)$$

Name: Date: Period:

Powers

Let's see how exponents show repeated multiplication.



Warm-Up

- 1** Here are some images and their matching expressions.

2^1

2^2

2^3

2^4



Discuss: What do you notice? What do you wonder?

Powers of 2

- 2** The expression 2^4 ("2 to the power of 4") is equivalent to $2 \cdot 2 \cdot 2 \cdot 2$.

$$2^4$$

How could you determine the value of 2^5 ?

- 3** Write a number or expression that is equivalent to 2^5 .

- 4** Group the equivalent expressions. One expression will not have a match.

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

$$2^5$$

$$2 + 2 + 2 + 2 + 2$$

$$5 + 5$$

$$2^4 \cdot 2$$

$$5^2$$

Group 1	Group 2

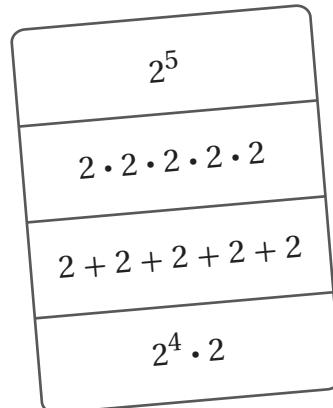
Powers of 2 (continued)

- 5** One expression in this group is not equivalent to the others.

Which expression is it?

- A. 2^5
- B. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
- C. $2 + 2 + 2 + 2 + 2$
- D. $2^4 \cdot 2$

Explain your thinking.



Exponents With Whole Number Bases

- 6** Select one expression that is equivalent to 3^4 .

A. 12

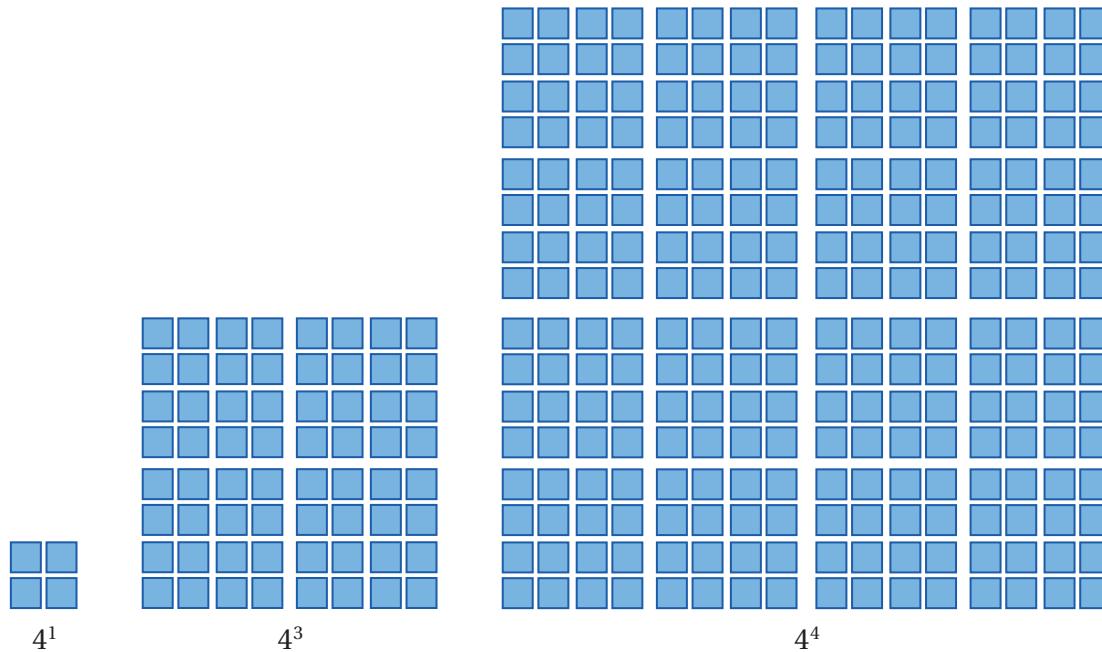
B. $4 \cdot 4 \cdot 4$

C. $3 \cdot 3 \cdot 3 \cdot 3$

D. 81

Show or explain your thinking.

- 7** $4 \cdot 4$ is equivalent to 4^2 , where 2 is the **exponent** and 4 is the **base**.



Write $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ using at least one exponent.

- 8** Write a number or an expression that is equivalent to 4^3 .

Exponents With Fractional Bases

- 9** Here are some images and their matching expressions.



$$\left(\frac{1}{2}\right)^1$$

$$\left(\frac{1}{2}\right)^2$$

$$\left(\frac{1}{2}\right)^3$$

Write two things you know about $\left(\frac{1}{2}\right)^3$.

- 1.
- 2.

- 10** Quinn wrote two expressions equivalent to $\left(\frac{1}{3}\right)^4$.

Write a different expression equivalent to $\left(\frac{1}{3}\right)^4$.

Quinn

$$\left(\frac{1}{3}\right)^4$$

$$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$$

$$\swarrow \qquad \searrow$$

$$\frac{1}{9} \cdot \frac{1}{9}$$

Explain your thinking.

- 11** Write an expression that is equivalent to $\left(\frac{1}{2}\right)^5$.

Activity
3

Name: Date: Period:

Exponents With Fractional Bases (continued)

12 Determine the value of each expression. Complete as many as you have time for.

a 2^3

b 4^2

c $\left(\frac{1}{3}\right)^2$

d 3^4

e 5^1

f $\left(\frac{1}{2}\right)^3$

g 1^8

h $\left(\frac{1}{4}\right)^3$

13 Synthesis

Without calculating, how can you tell whether expressions with exponents are equivalent?

$$11 + 11 + 11 + 11 + 11 + 11$$

$$11^4 \cdot 11$$

$$11 \cdot 5$$

$$11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$$

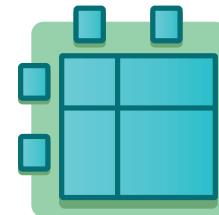
$$5^{11}$$

$$11^5$$

Things to Remember:

Name: Date: Period:

Exponent Expressions



Let's evaluate expressions with exponents.

Warm-Up

1. 4, 9, and 16 are examples of **perfect squares**. Here are some representations of these perfect squares

Perfect Square	4	9	16
Diagram			
Exponent Expression	2^2	3^2	4^2

a

Discuss: What do you notice? What do you wonder?

b

Do you think 49 is a perfect square? Explain your thinking.

What's Missing?

2. You will use a set of cards for this activity.

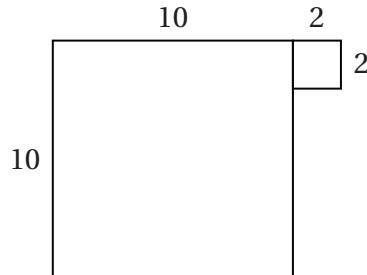
- Work with a partner to group the matching diagrams, expressions, and values. There will be three blank spaces.
- Complete the table with the missing representations.

Diagram	Expression	Value
$\begin{array}{ccccc} 3 & 3 & 3 & 3 & 3 \\ 3 & \boxed{} & \boxed{} & \boxed{} & \boxed{} \end{array}$	$5 \cdot 3^2$	
$\begin{array}{cc} 3 & 5 \\ 3 & \boxed{} \\ 5 & \boxed{} \end{array}$		
$\begin{array}{ccc} 5 & 5 & 5 \\ 5 & \boxed{} & \boxed{} \\ 5 & \boxed{} & \boxed{} \end{array}$		

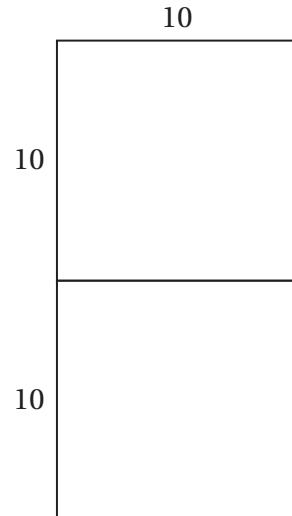
Evaluating Expressions

3.  **Discuss:** Which diagram represents $2 + 10^2$?

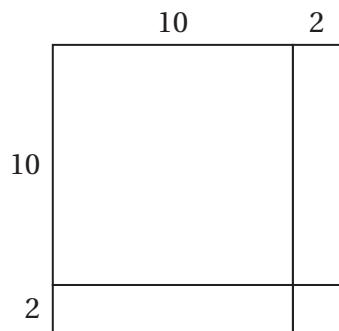
A.



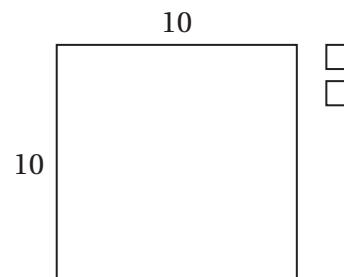
B.



C.



D.



4. Latifa and Nicolas each got a different value for $2 + 10^2$.

Latifa

$$\begin{aligned}2 + 10^2 \\ 12^2 \\ 144\end{aligned}$$

Nicolas

$$\begin{aligned}2 + 10^2 \\ 2 + 100 \\ 102\end{aligned}$$

Whose work is correct? Circle one.

Nicolas's

Latifa's

Both

Neither

Explain your thinking.

Partner Problems

- 5.** Decide with your partner who will complete Column A and who will complete Column B.

- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

	Column A	Column B
a	$5^2 + 4$	$2^2 + 25$
b	$5^2 - 4$	$25 - 2^2$
c	$\frac{4^2}{2}$	$\frac{2^5}{4}$
d	$(7 - 2)^2 - 3^2$	$\frac{1}{4}(1 + 3)^3$
e	$\frac{8 - 2^2}{4}$	$\frac{8 - 2(3)}{2}$
f	$\frac{2^4 - (3^2 - 5)}{2^2}$	$\frac{1^5 + (6^2 - 10)}{3^2}$

Explore More

- 6.** Write an expression for your partner to evaluate. Swap problems, then write an expression with an exponent that has the same value as your partner's but uses different numbers.

Synthesis

7. What are some things to remember when determining the value of expressions with exponents?

Use these examples if they help with your thinking.

$$5 \cdot 3^2$$

$$(3 + 5)^2$$

$$(3 \cdot 5)^2$$

$$5^2 + 3^2$$

Things to Remember:

What's Missing?

 **Directions:** Make one copy per two pairs of students. Then pre-cut the cards and give each pair of students one set of Cards A–E.

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Card A

$$5^2 + 3^2$$

Card B

$$(3 + 5)^2$$

Card C

$$34$$

Card D

$$(3 \cdot 5)^2$$

Card E

$$45$$

Card A

$$5^2 + 3^2$$

Card B

$$(3 + 5)^2$$

Card C

$$34$$

Card D

$$(3 \cdot 5)^2$$

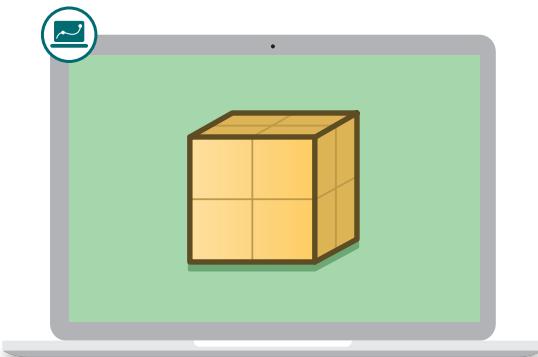
Card E

$$45$$

Name: Date: Period:

Squares and Cubes

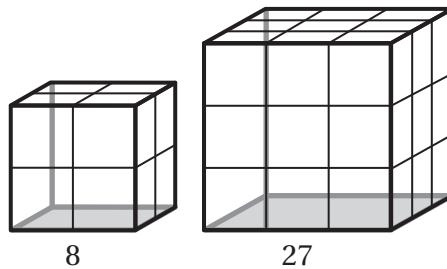
Let's evaluate variable expressions.



Warm-Up

- 1 8 and 27 are examples of **perfect cubes**.

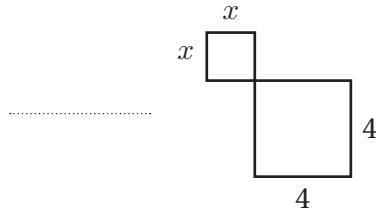
What other numbers do you think might be perfect cubes?



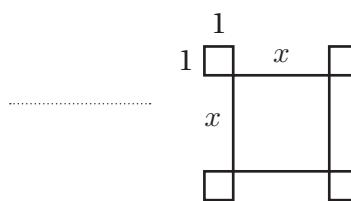
Variable Expressions With Area

2 Match each expression with its area model.

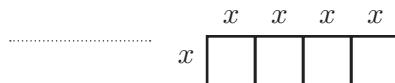
a. $(x + 4)^2$



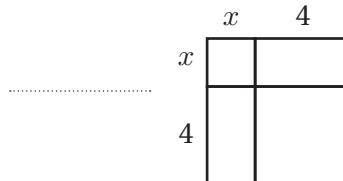
b. $x^2 + 4$



c. $4x^2$



d. $x^2 + 4^2$

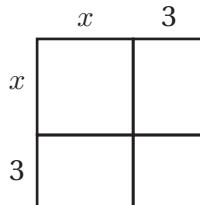


Variable Expressions With Area (continued)

- 3** The area of figure *A* is $(x + 3)^2$ square units.

What is its area when $x = 4$?

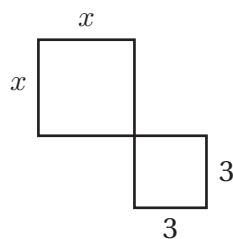
Figure A



- 4** The area of figure *B* is $x^2 + 3^2$ square units.

What is its area when $x = 4$?

Figure B

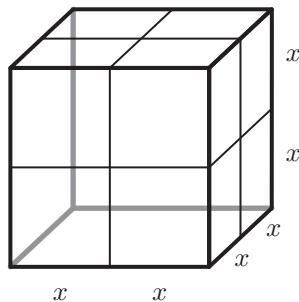


- 5** Amir says that $(x + 3)^2$ and $x^2 + 3^2$ are equivalent expressions. Help him understand why they are *not* equivalent.

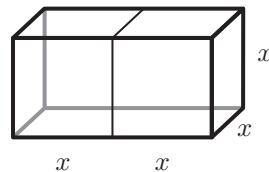
Use figures *A* and *B* if they help you show your thinking.

Cubes and Squares

- 6** Which prism has a volume of $(2x)^3$ cubic units?

Prism C

Prism C

Prism D

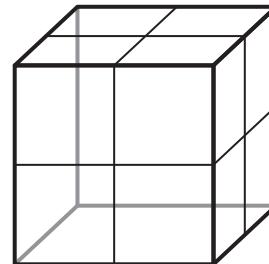
Prism D

Both

Neither

Explain your thinking.

- 7** What is the volume of prism C when $x = 3$?

Prism C

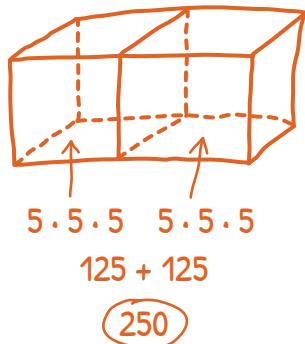
- 8** Here is a new expression: $2x^3$.

Evaluate $2x^3$ when $x = 5$. Show or explain your thinking.

Cubes and Squares (continued)

- 9** Here are Amir's and Chloe's strategies for evaluating $2x^3$ when $x = 5$.

Amir



Chloe

$$\begin{aligned}2x^3 \text{ when } x = 5 \\ 2(5)^3 &= 2(5 \cdot 5 \cdot 5) \\ &= 2 \cdot 125 \\ &= 250\end{aligned}$$



Discuss: How are their strategies alike? How are they different?

- 10** Use Chloe's strategy to evaluate $2x^3$ when $x = \frac{1}{2}$.

- 11** Use Chloe's strategy to determine the value of $5x^2 + 4x + 3$ when $x = 3$.

Repeated Challenges

- 12** Evaluate each expression for the given value of x .

Expression	Value of x	Value of the Expression
a $(x + 2)^2$	5	
b $5x^2$	3	
c $4x^3$	2	
d $4 + x^2$	6	
e $(2x)^3$	2	

Explore More

- 13** Here are two expressions: $(x + 5)^2$ and $x^2 + 5^2$.

Amir says they *always* have the same value.
 Chloe says they *never* have the same value.

Who is correct? Circle one.

Amir

Chloe

Both

Neither

Explain your thinking.

14 Synthesis

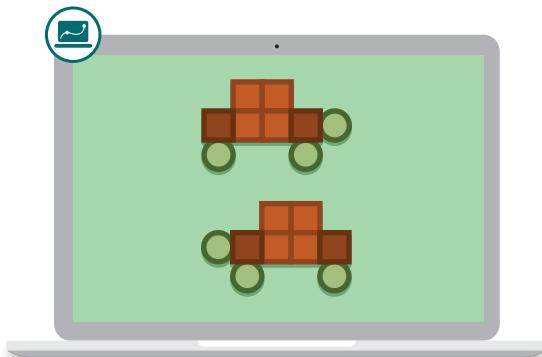
Describe how to evaluate $(x + 1)^3$ when $x = 3$.

Things to Remember:

Name: Date: Period:

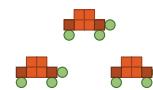
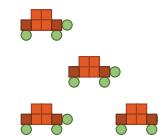
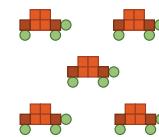
Turtles All the Way

Let's explore relationships between two variables.



Warm-Up

- 1** Here is a pattern of turtles. What different things can you count in this pattern?

 $t = 1$  $t = 2$  $t = 3$  $t = 4$  $t = 5$ 

Turtles, Turtles, Turtles

- 2** The variable t represents the number of turtles.

a Here are three other variables. Circle one.

s = number of square tiles h = height of a turtle g = number of green circles

b Describe how changing t affects the value of the variable you chose.

- 3** Saanvi made a table to help make sense of the relationship between t and g .

a Complete the table.

b  **Discuss:** What patterns do you see?

Number of Turtles, t	Number of Green Circles, g
1	3
2	
3	
4	
5	

- 4** Saanvi and Kadeem wrote equations to represent the relationship between t and g .

Saanvi

$$g = 3t$$

Kadeem

$$t = 3g$$

Whose equation is correct? Circle one.

Saanvi's

Kadeem's

Both

Neither

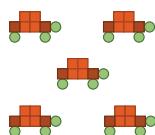
Explain your thinking.

Dependent or Independent

- 5** The **independent variable** is the variable in a relationship that is the cause. The **dependent variable** is the effect or result.

In Saanvi's example, the independent variable is the number of turtles, t , and the dependent variable is the number of green circles, g .

Image



Table

Number of Turtles, t	Number of Green Circles, g
1	3
2	6
3	9
4	12

Equation

$$g = 3t$$

 **Discuss:** If the independent variable is t , what other dependent variables could you explore?

- 6** Here is a pattern you may have seen before. The independent variable is n , the side length of the lighter square.

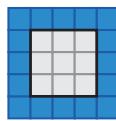
$$n = 1$$



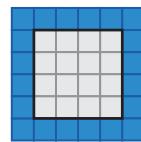
$$n = 2$$



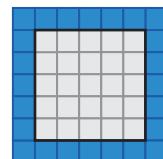
$$n = 3$$



$$n = 4$$



$$n = 5$$



- a** Circle one dependent variable to explore. Consider choosing one that's different than your partner's.

t = total area of the tiles g = area of the lighter tiles p = perimeter of the lighter square

- b** Describe how changing n affects the value of the dependent variable you chose.

Border Tiles Revisited

Here is the pattern from the previous screen.

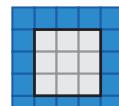
$$n = 1$$



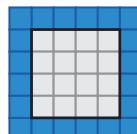
$$n = 2$$



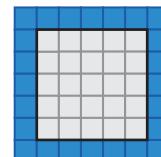
$$n = 3$$



$$n = 4$$



$$n = 5$$

**7**

- a** Complete the table for the variable you chose.

**t = total area
of the tiles**

n	t
1	
2	
3	
4	
5	

**g = area
of the gray tiles**

n	g
1	
2	
3	
4	
5	

**p = perimeter
of the gray square**

n	p
1	
2	
3	
4	
5	

b

- Discuss:** What patterns do you see?

8

- Which equation represents your pattern? Explain your thinking.

**t = total area
of the tiles**

- A. $t = (n + 2)^2$
- B. $n = (t + 2)^2$
- C. $t = n^2 + 2$

**g = area
of the gray tiles**

- A. $g = n^2$
- B. $n = g^2$
- C. $g = 2n$

**p = perimeter
of the gray square**

- A. $p = 4n$
- B. $p = 4(n + 2)$
- C. $n = 4p$

9

- What is the total area of the tiles when $n = 10$?

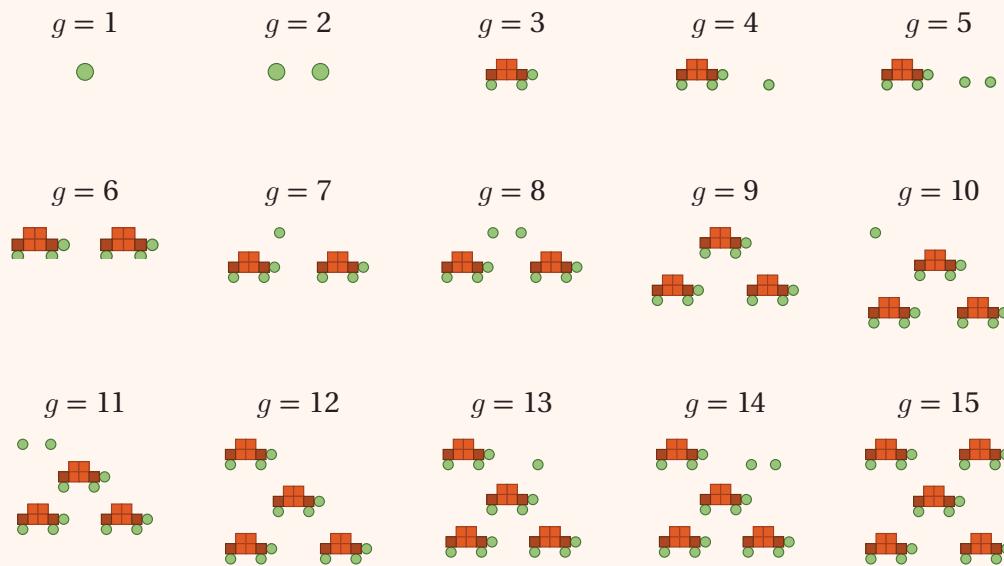
- What is the area of the gray tiles when $n = 10$?

- What is the perimeter of the gray square when $n = 10$?

Border Tiles Revisited (continued)

Explore More

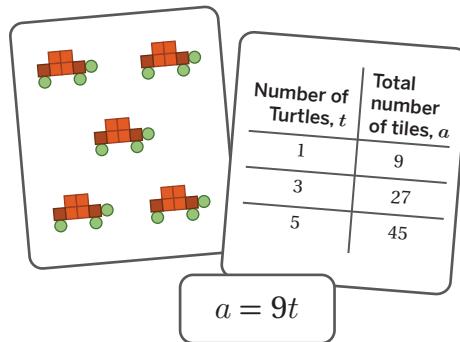
- 10** Here is a new pattern where g represents the number of green circles.



- What is the independent variable?
- Choose a dependent variable to explore.
- Describe the relationship between the independent variable and dependent variable you chose.

11 Synthesis

How can we see relationships between independent and dependent variables in tables and equations?

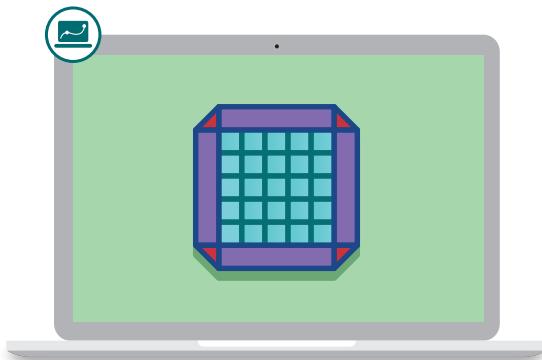


Things to Remember:

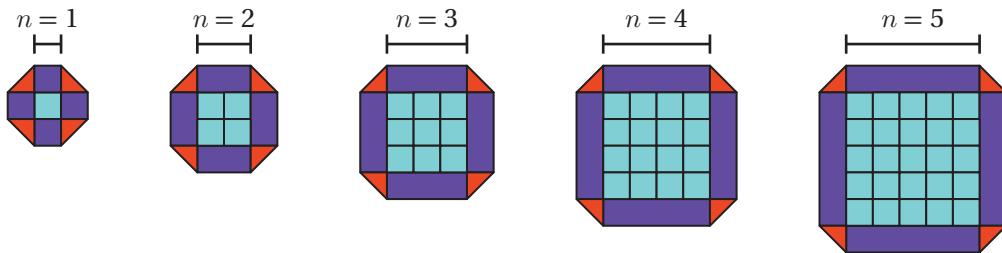
Name: Date: Period:

Representing Relationships

Let's use graphs to represent relationships between two variables.



Warm-Up



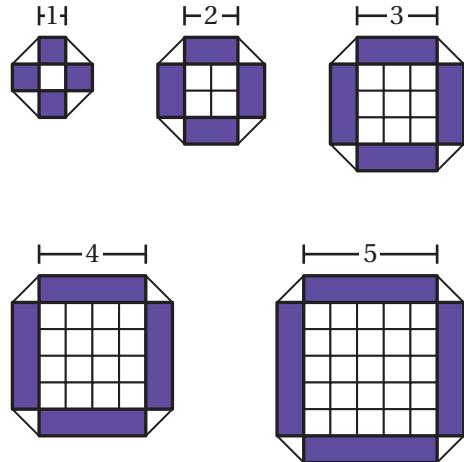
- 1** **Discuss:** What do you notice? What do you wonder?

- 2** In this pattern, the independent variable is the side length of the purple rectangles, n . What dependent variables could you explore?

Introducing Graphs

- 3** Jayden chose to explore the relationship between the side length of the purple rectangles, n , and the total area of the purple rectangles, p .

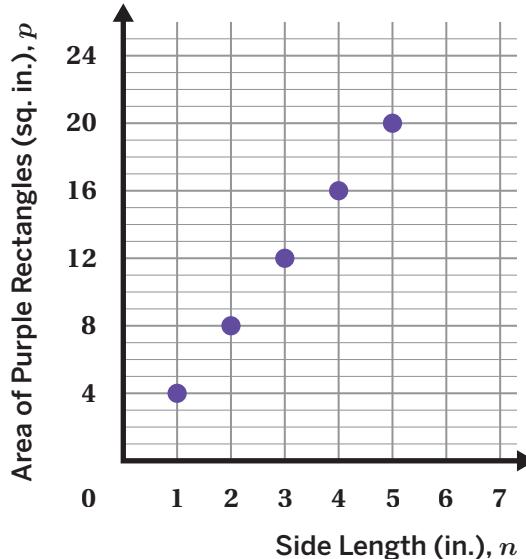
Complete the table to represent this relationship.



Side Length (in.), n	Area of Purple Rectangles (sq. in.), p
1	4
2	
3	
4	
5	

- 4** Rebecca represented the relationship between n and p with a graph.

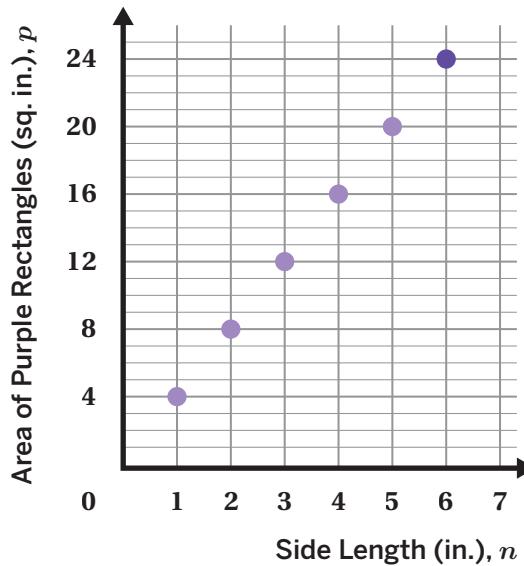
 **Discuss:** How does the graph show the same information as the images?



- 5** Explain how you know Jayden's table and Rebecca's graph represent the same relationship.

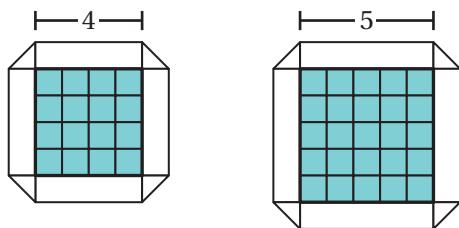
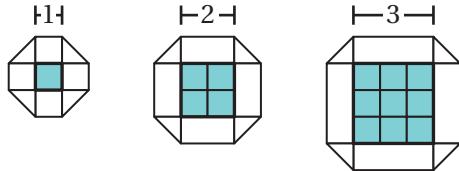
Introducing Graphs (continued)

- 6** Rebecca added the *ordered pair* (6, 24) to the graph. Explain what each value represents in this context.



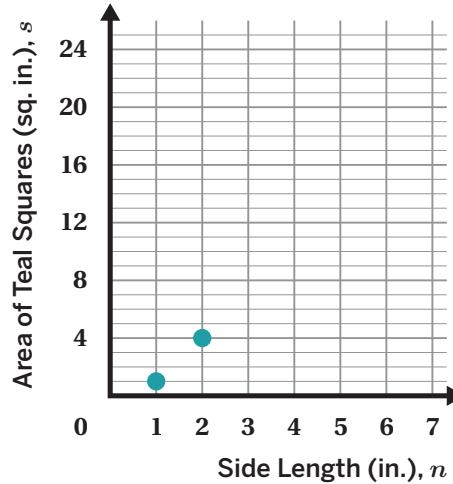
Plotting Points

- 7** Precious explored the relationship between the side length, n , and the total area of the teal squares, s . Complete the table to represent this relationship.



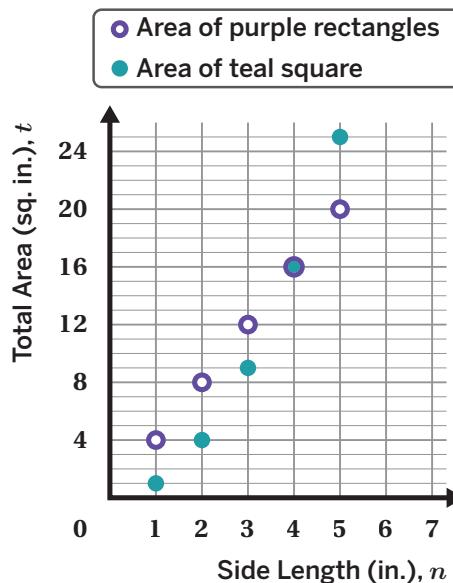
Side Length (in.), n	Area of Teal Squares (sq. in.), s
1	1
2	
3	
4	
5	

- 8** Plot points that represent the third and fourth rows in the table.



- 9** Precious noticed the point $(4, 16)$ was on both graphs.

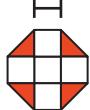
Discuss: What does this say about the area of purple rectangles and the area of the teal squares?



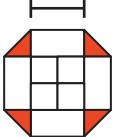
Comparing Relationships

- 10** Graph four points to represent the relationship between the side length, n , and the total area of the red triangles, r .

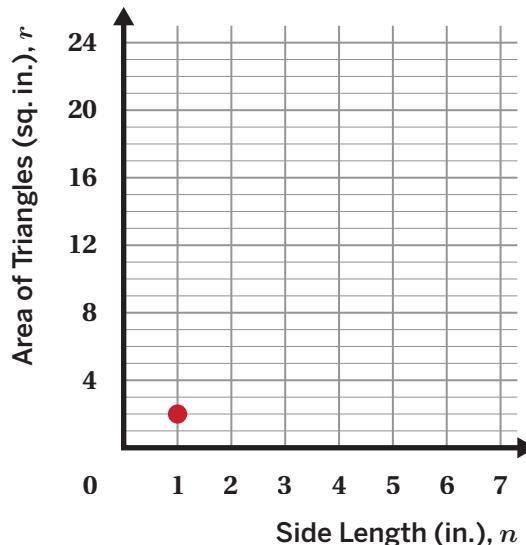
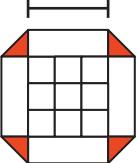
$$n = 1$$



$$n = 2$$



$$n = 3$$



- 11** Let's look at a graph that shows all three relationships.



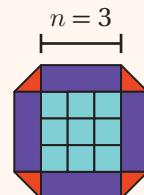
Discuss: How is the triangle relationship different from the other two relationships? How is it alike?

Explore More

- 12** Draw an image to represent when:

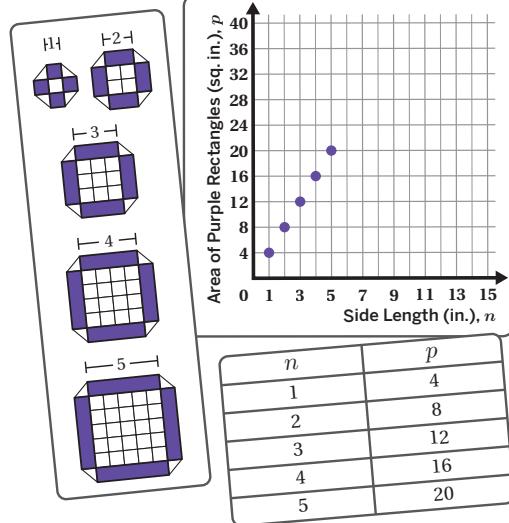
a $n = 0$

b $n = 10$



13 Synthesis

How can you tell that a table, a graph, and an image show the same relationship?



Things to Remember:

Connecting Representations

Let's make connections between different representations of the same relationship.



Warm-Up

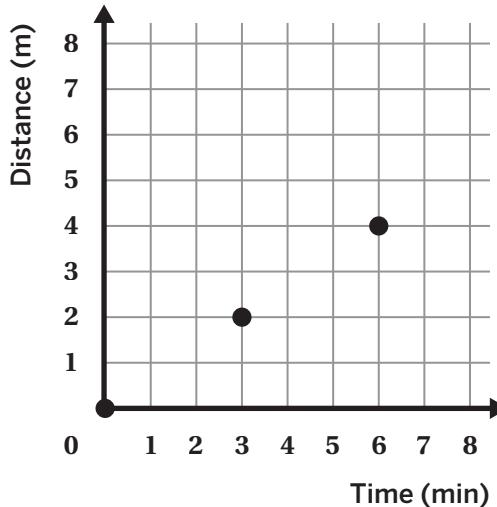
1. Which table represents this graph?

Table A

Time (min)	Distance (m)
0	0
2	3
4	6

Table B

Time (min)	Distance (m)
0	0
3	2
6	4



Explain your thinking.

What's Missing?

You will use a set of cards for this activity.



- With your partner, match each situation with the tables, graphs, and equations that show the same relationship.
- Complete the missing representations.

Situation	Table	Graph	Equation								
<p>Amanda sells paletas, p, for \$2 each.</p> <p>What is the total amount of money, m, Amanda can earn?</p>	<table border="1"> <thead> <tr> <th>p</th> <th>m</th> </tr> </thead> <tbody> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> </tbody> </table>	p	m							<p>Money Earned (\$)</p> <p>Number of Paletas</p>	
p	m										
<p>Tameeka sells paletas, p, for \$2.50 each.</p> <p>What is the total amount of money, m, Tameeka can earn?</p>	<table border="1"> <thead> <tr> <th>p</th> <th>m</th> </tr> </thead> <tbody> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> </tbody> </table>	p	m							<p>Money Earned (\$)</p> <p>Number of Paletas</p>	
p	m										
<p>Esteban sells piraguas, p, for \$3.50 each.</p> <p>What is the total amount of money, m, Esteban can earn?</p>	<table border="1"> <thead> <tr> <th>p</th> <th>m</th> </tr> </thead> <tbody> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> </tbody> </table>	p	m							<p>Money Earned (\$)</p> <p>Number of Piraguas</p>	
p	m										

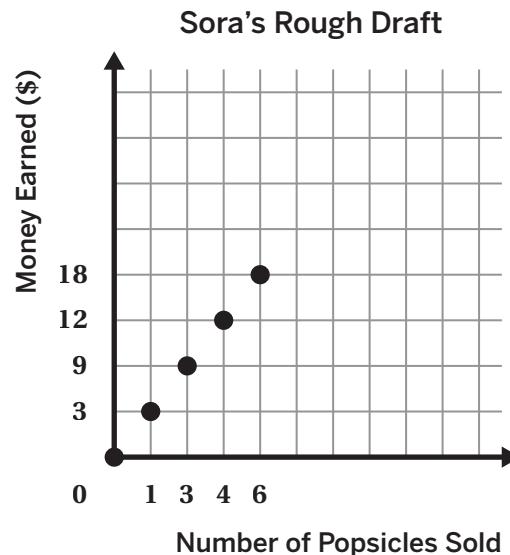
What's Missing? (continued)

4. Choose one of the previous situations. Show or explain where you see the price per item in the matching table, graph, and equation.
 5. Ángel sells piraguas for \$4.50 each. How will Ángel's graph be different from Esteban's?

Critique, Correct, Clarify

Sora sells popsicles for \$3 each. He made a table and a graph to help him understand the relationship between the number of popsicles he sells and the money he earns. His table is correct, but his graph is not quite correct.

Number of Popsicles Sold	Money Earned (\$)
0	0
1	3
3	9
4	12
6	18



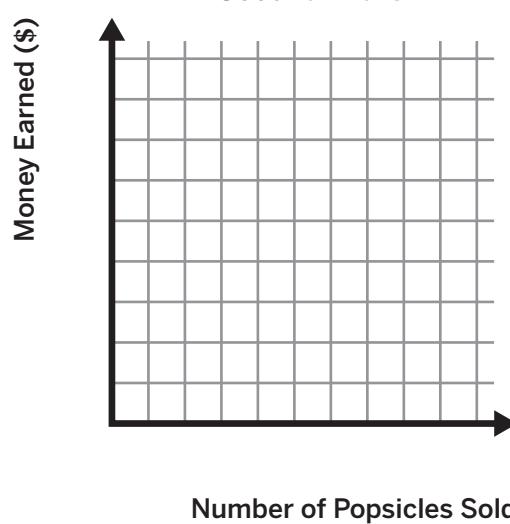
6. What do you think Sora did well in his graph?

7. What would you recommend Sora change about his graph?

8. Use Sora's table to create a second draft of the graph of the relationship between popsicles sold and money earned.

9. Circle one point on your graph. Explain what that point means in Sora's situation.

10. What are some other mistakes a person might make when they are creating a graph?



Synthesis

11. Explain how tables, equations, and graphs represent the same relationship.

Use the example if it helps with your explanation.

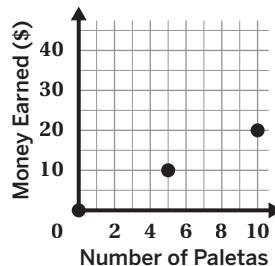
Situation

Amanda sells paletas for \$2 each.

Table

<i>p</i>	<i>m</i>
0	0
5	10
10	20

Graph



Equation

$$m = 2p$$

Things to Remember:

What's Missing?

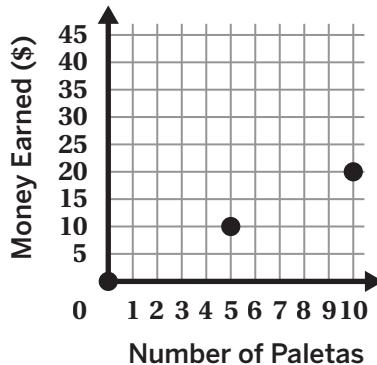
 **Directions:** Make one copy per pair of students. Then pre-cut the cards and give each pair of students one set.

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Card A

p	m
2	5
4	10
10	25

Card B



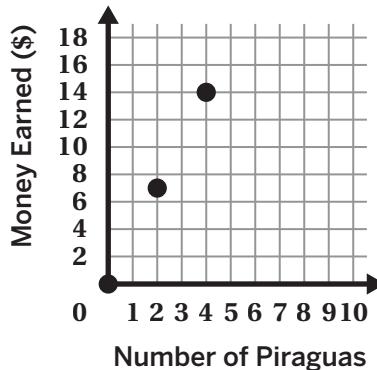
Card C

$$m = 3.50p$$

Card D

p	m
0	0
5	10
10	20

Card E



Card F

$$m = 2.50p$$

Name: Date: Period:

Subway Fares

Let's use tables, graphs, and equations to help customers compare subway fares.



Warm-Up

In Metropolis, there are three ticket options to ride the bus or subway.

Option 1

Regular Fare



Option 2

Unlimited 7-Day Pass



Option 3

Reduced Fare



For people who have low income, are 65 or older, or who have a qualifying disability.

1. For each option, how much will it cost to ride the subway 3 times in the same week?

Consider the Costs

2. The Metropolis Transit Association (MTA) is in charge of the public buses and subways in Metropolis. Your task is to help an MTA employee show customers how much each ticket option from the Warm-Up costs based on the number of rides.

- a As a group, work together to create a table, graph, and equation for each option.

Option 1

Regular Fare

Table

Number of Rides, r	Total Cost, c (\$)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Option 2

Unlimited 7-Day Pass

Table

Number of Rides, r	Total Cost, c (\$)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

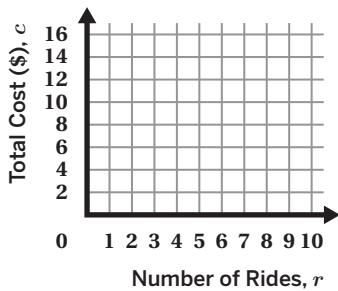
Option 3

Reduced Fare

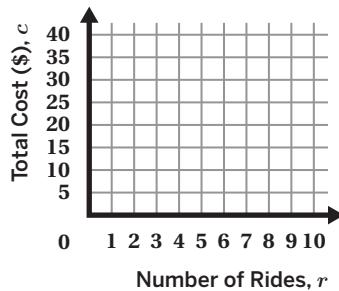
Table

Number of Rides, r	Total Cost, c (\$)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

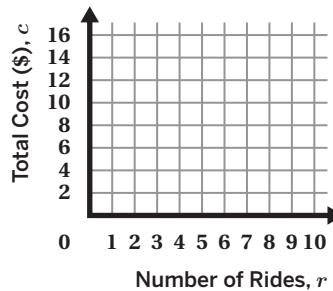
Graph



Graph



Graph



Equation

Equation

Equation

- b Which variable is the independent variable? Which is the dependent variable?

- c Write two to three sentences comparing and contrasting the graphs for the three options.

Helping Customers

- 3.** Read about four subway customers and choose one to help. Make sure each person in your group chooses a different customer.

Eliza	Nikhil	Sydney	Bao
Eliza is 70 years old. She works at a daycare about 1.5 miles away from her house. Sometimes she walks to work and sometimes she takes the subway. She rides the subway between 2–8 times per week.	Nikhil is 23 years old. He uses a wheelchair and it takes him 20 minutes to get to the closest wheelchair-accessible subway station from his house. Nikhil works as a chef and uses the subway to get to and from work 5 days a week.	Sydney is a 20 year old college student who works part time. Sydney uses the subway to get to school and work and usually rides between 15–20 times per week.	Bao is 16 years old. He walks to school during the week and only uses the subway on the weekends to visit friends.

- a** Which fare option should your customer choose? Circle one.

Regular fare

Unlimited 7-day pass

Reduced fare

- b** Use the tables, graphs, and equations you made in Activity 1 to support your argument.

Increased Fares

The MTA needs more money to help maintain the subway service. The MTA leadership is thinking about raising the regular fare by \$0.50.

- 4.** Describe one advantage and one disadvantage of raising the regular fare. Explain your thinking for each.

One advantage of raising the regular fare is . . .

One disadvantage of raising the regular fare is . . .

- 5.** Look back at your work in Activity 1 for the regular fare. How would raising the fare by \$0.50 change the table, graph, and equation?

Table	Graph	Equation

- 6.** Which of the four customers would be most impacted by the fare increase? Explain your thinking.
- 7.** If you were part of the MTA leadership, how would you adjust the fares to get the money you need to maintain service while also charging customers fairly?

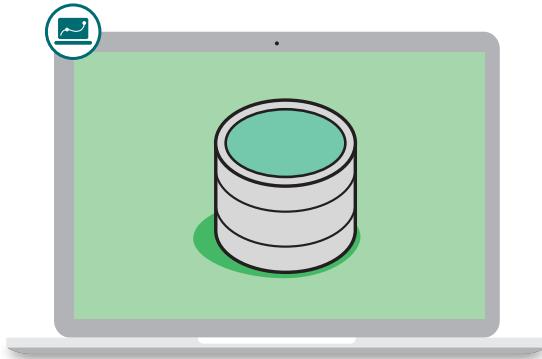
Synthesis

8. How can making a graph and a table help us understand relationships in the world, such as subway fares?

Things to Remember:

Paint

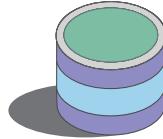
Let's explore equivalent ratios.



Warm-Up

- 1** Here is a color made from 5 cups of white paint and 7 cups of green paint.

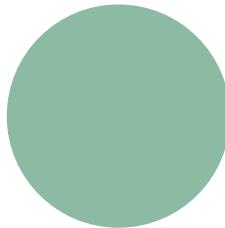
 **Discuss:** What would you name this color? Why?



- 2** Brielle wants to match this color.

How many cups of green paint should she mix with 10 cups of white paint to make the same color?

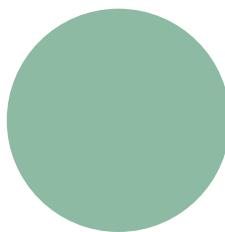
White Paint (cups)	Green Paint (cups)
5	7
10	



5 white cups



7 green cups



10 white cups



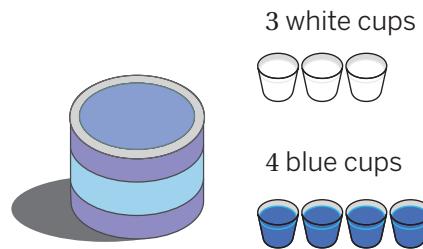
? green cups

Color Match

3 Here are two paint mixtures:

- 3 white cups and 4 blue cups
- 6 white cups and 8 blue cups

Both mixtures make the same color because they are *equivalent ratios*.



Which mixture would also make the same color?

- A. 5 white cups and 6 blue cups
- B. 4 white cups and 3 blue cups
- C. 1 white cup and $1\frac{1}{2}$ blue cups
- D. 1.5 white cups and 2 blue cups

Explain your thinking.

Paint Palooza

- 4** Darryl mixed 4 cups of white paint with 6 cups of red paint, but he didn't have enough to finish painting his wall.

How much red paint would he need to add to 1 cup of white paint to match the color?

White Paint (cups)	Red Paint (cups)
4	6
1	



- 5** Brielle ran out of paint for her room.

Complete the table so that the new mixture matches the original paint color.

Blue Paint (cups)	Red Paint (cups)	White Paint (cups)
12	9	14
4		



Colorful Challenge

6 You will use the Activity 3 Sheet to create your own paint color challenge!

- a** **Make It!** Create your challenge on the Activity 3 Sheet.
- b** **Solve It!** On this page, record the number of cups of paint used in both your original mixture and the new mixture.

	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				

- c** **Swap It!**

- Swap your challenge with one or more partners.
- Record the information about your partner's original mixture and their new mixture.
- Fill in the missing amounts to complete the new mixture.

Partner 1	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				

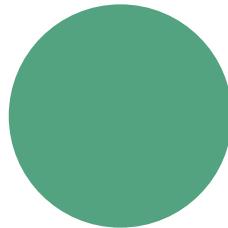
Partner 2	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				

Partner 3	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				

7 Synthesis

Explain how equivalent ratios can help make matching paint colors.

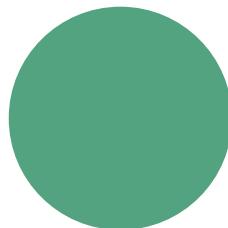
Use the example if it helps with your thinking.



1 white cup



4 green cups



2 white cups



8 green cups



Things to Remember:

Name: Date: Period:

Colorful Challenge

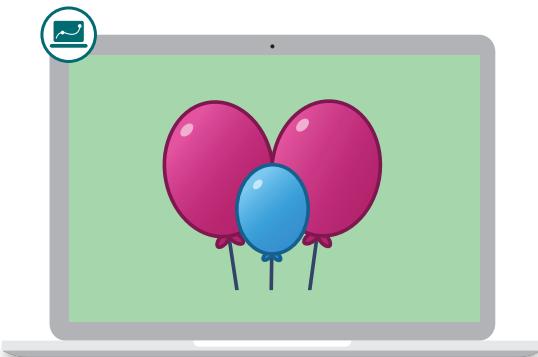
- Create your own paint color by filling in the amounts for *at least* two colors.

	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				

- Name your paint color and describe what you think it looks like.
- Add a new amount of *one* color that you used in your original mixture to the table. Then challenge your classmates to fill in the missing amounts and match your original paint color.

	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
New Mixture That Matches				

Name: Date: Period:



Balloon Float

Let's explore proportional relationships in tables.

Warm-Up

- 1 This table shows how many rolls of paper towels a store receives when they order different numbers of cases.

What do you notice? What do you wonder?

I notice:

I wonder:

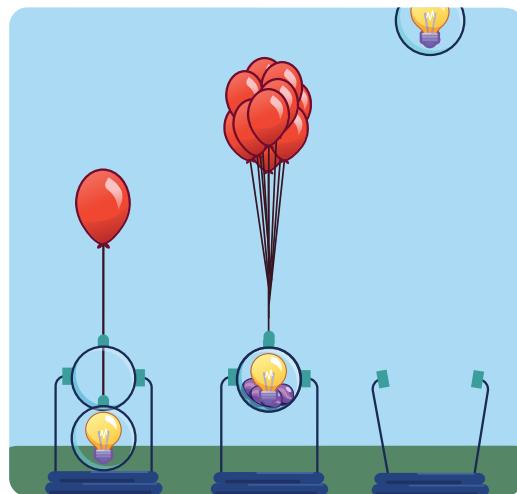
Number of Cases Ordered	Number of Rolls of Paper Towels
1	12
3	36
5	60
10	120

Balloon Float

Helium balloons can make objects float, but too many balloons will make objects fly away!

- 2** **a** Let's watch the *middle* light bulb float.

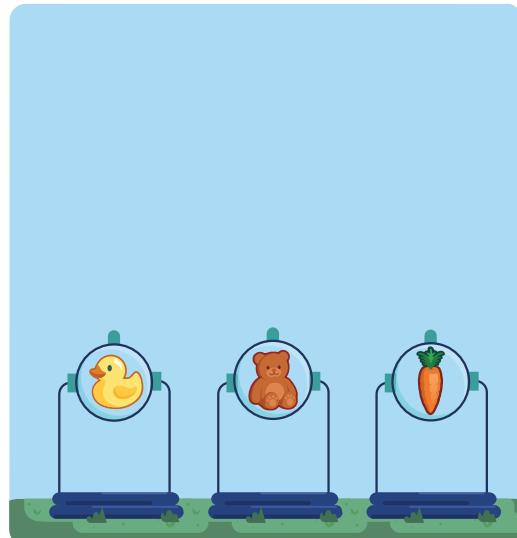
- b**  **Discuss:** Why do you think the number of balloons matters?



- 3** In the previous problem, the light bulb weighed 2 ounces and needed 6 balloons to float.

If each balloon carries the same weight, how many balloons would you need to float each object?

Object	Weight (oz)	Number of Balloons
Light bulb	2	6
Rubber duck	10	
Toy bear	6	
Carrot	3	



Balloon Float (continued)

- 4** Here are two strategies for determining the number of balloons needed to make the rubber duck float.

Ariel

Object	Weight (oz)	Number of Balloons
Light bulb	2	6
Rubber duck	$\times 5$ → 10	$30 \rightarrow \times 5$
Toy bear	6	
Carrot	3	

Emma

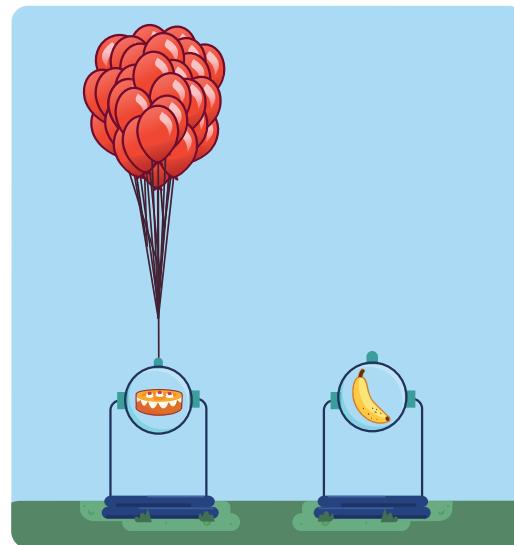
Object	Weight (oz)	Number of Balloons
Light bulb	2	$\xrightarrow{\times 3} 6$
Rubber duck	10	$\xrightarrow{\times 3} 30$
Toy bear	6	
Carrot	3	



Discuss: How might Ariel and Emma use their strategies to finish their tables?

- 5** Here are some new objects. Complete the table so that each object floats.

Object	Weight (oz)	Number of Balloons
Light bulb	2	6
Cake		60
Banana	$3\frac{1}{3}$	



Proportional Relationships

When two quantities are always in an equivalent ratio, they have what's called a **proportional relationship**.

- 6** Here are two more tables.

Which of these two tables represents a proportional relationship? Circle one.

Table 1 Table 2 Both Neither

Explain your thinking.

Table 1		Table 2	
Weight (oz)	Number of Balloons	Weight (oz)	Number of Balloons
3	6	4	12
7	10	6	18
9	12	42	126
30	33	8	24

- 7** Sort the tables into two groups based on whether they represent proportional relationships.

Table A

x	y
0	0
4	5
8	10
12	15

Table B

x	y
0	0
2	4
4	16
6	36

Table C

x	y
0	2
3	5
6	8
9	11

Table D

x	1	2	3	4
y	10	8	6	4

Table E

x	2	8	1	20
y	5	20	2.5	50

Proportional Relationship

Not a Proportional Relationship

Proportional Relationships (continued)

- 8** How did you decide whether this table represents a proportional relationship?

Proportional Relationship

Not a Proportional Relationship

x	y
0	0
2	4
4	16
6	36

- 9** Select *all* the relationships you think are proportional.

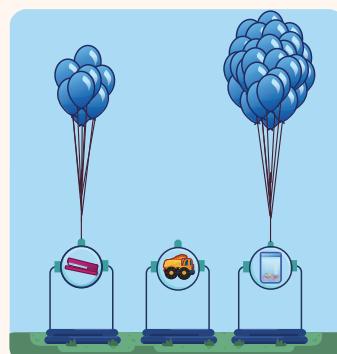
- A. A person's height in feet and their height in inches
- B. The number of cookies baked and the number of minutes they were in the oven
- C. The amount of bread baked and the number of grams of flour needed to bake it
- D. A person's time as they run a marathon and their total distance covered
- E. The gallons of gasoline purchased and their total cost

Explore More

- 10** Blue balloons are different from red balloons.
8 blue balloons can float a 10-ounce stapler.

Complete the table so that each object floats.

Object	Weight (oz)	Blue Balloons
Stapler	10	8
Toy truck	15	
Jelly beans		28



11 Synthesis

Here are some relationships, some of which are proportional and some of which are not.

What determines whether a relationship is proportional?

Use the examples if they help with your thinking.

- A person's height in feet and their height in inches
- The number of cookies baked and the number of minutes they were in the oven
- The amount of bread baked and the number of grams of flour needed to bake it
- A person's time as they run a marathon and their total distance covered
- The gallons of gasoline purchased and their total cost

Things to Remember: