Unit 3
Lesson
11

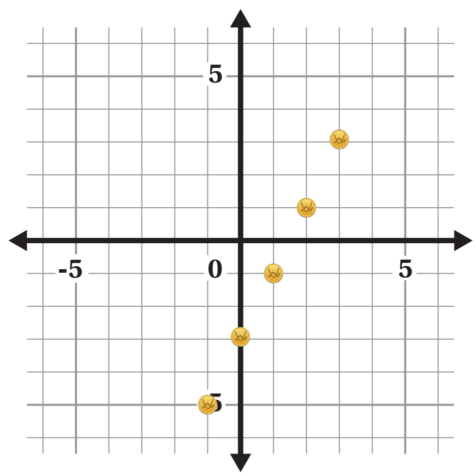
Name: ______ Date: ______ Period: ______

Why Intercepts?

Let's write an equation for a line that passes through two given points.

Warm-Up

1 Denali wants to "capture" these coins with just one line. ***Responses vary.***



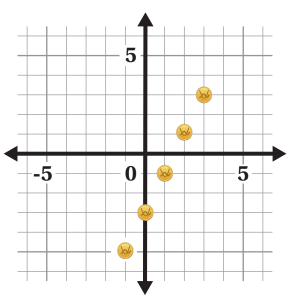
a What slope could Denali use?

2

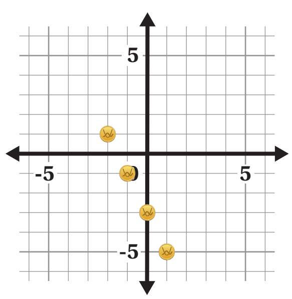
b What y-intercept could Denali use?

-3

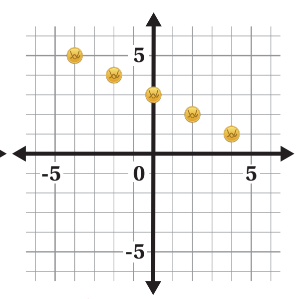
2 Write a single linear equation to capture all the coins for each challenge. ***Responses vary.***



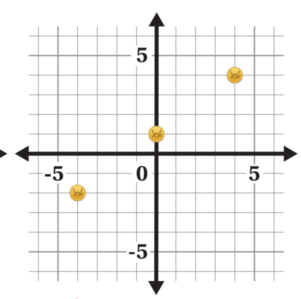
$y = 2x - 3$



$y = -2x - 3$



$y = -\frac{1}{2}x + 3$

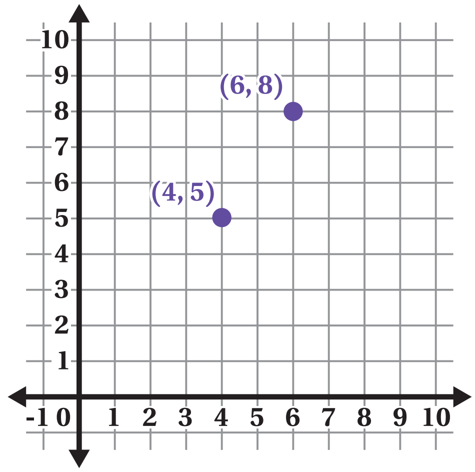


$y = \frac{3}{4}x + 1$

Name: .... Date: .... Period: ....

Determining the y-intercept

3 The points $(4, 5)$ and $(6, 8)$ represent the location of two coins.



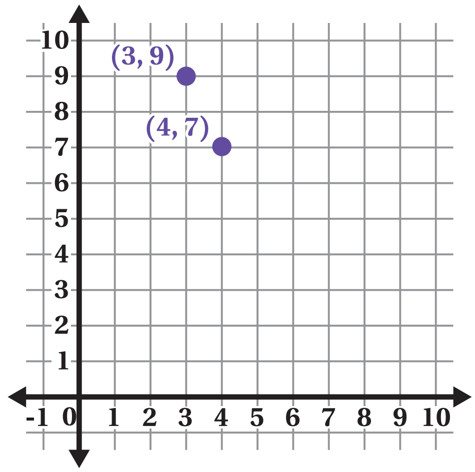
a Write an equation of the line that goes through both points.

$y = \frac{3}{2}x - 1$ **(or equivalent)**

b What was your strategy for determining the y-intercept?

***Responses vary*. I drew a line through the two points and saw that it crossed the y-axis at $(0, -1)$.**

4 Here are two new points: $(3, 9)$ and $(4, 7)$.



Describe how you could determine the y-intercept of the line going through these two points.

***Responses vary*.**

- **Draw a line through the two points and extend the grid.**
- **Use similar triangles to build up to the y-intercept.**
- **Write an equation using the slope, substitute in the x- and y-values of a point, then solve for the y-intercept.**

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Determining the y-intercept (continued)

5 Here are two students' strategies for determining the y-intercept in the previous problem.

Tariq

x	y
3	9
4	7

$+1$ -2

slope: $\frac{-2}{1} = -2$

$y = -2x + b$

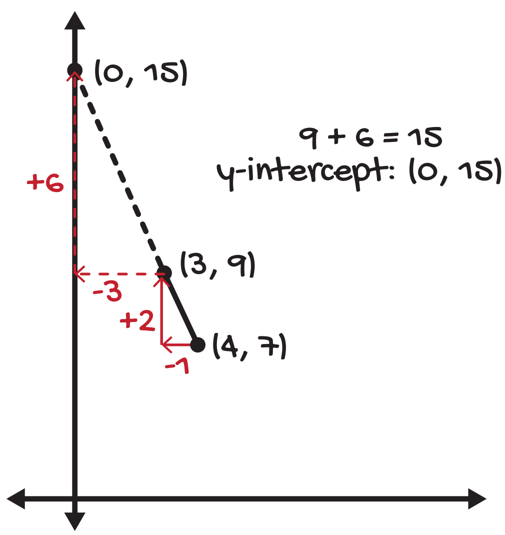
I'll substitute (3, 9) in for x and y!

$9 = -2(3) + b$

$9 = -6 + b$

$15 = b$

Nia



a Choose a student and explain their strategy to a classmate.

Responses vary.

- **Tariq wrote the ordered pairs in a table to determine the slope. Then he substituted the x- and y-values of one of the ordered pairs in the equation $y = mx + b$ to solve for b.**
- **Nia determined the difference in the x- and y-values of the two ordered pairs to determine the slope. Then she used similar triangles to determine the y-intercept.**

b **Discuss:** How are the students' strategies alike? How are they different?

Responses vary.

- **They both found that the y-intercept is at $(0, 15)$.**
- **Tariq set up an equation and solved for the y-intercept, while Nia used similar triangles to determine the y-intercept.**

Activity 2

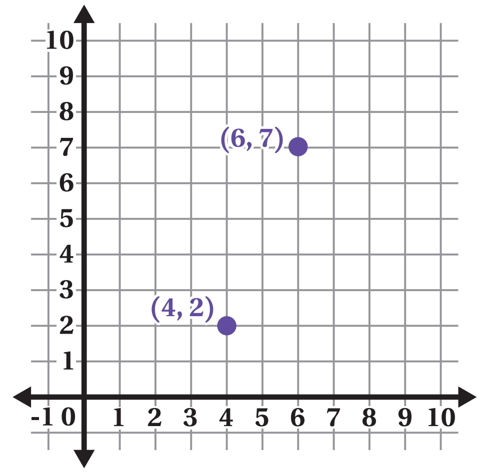
Name: ______ Date: ______ Period: ______

Critiquing, Correcting, and Clarifying

6 Here are two new points: $(4, 2)$ and $(6, 7)$.

Write an equation of the line that goes through both points.

$y = \frac{5}{2}x - 8$ (or equivalent)



7 Victor made a mistake while writing the equation for the line that goes through $(4, 2)$ and $(6, 7)$.

a **Discuss:** What did Victor do well?

***Responses vary.* Victor accurately calculated the slope.**

b Explain why Victor's work is incorrect.

***Responses vary.* Victor took the point $(4, 2)$ and substituted 4 in for y and 2 in for x, when he should have done the opposite.**

Victor

x	y
4	2
6	7

+2 (x), +5 (y)

slope: $\frac{5}{2}$

$$y = \frac{5}{2}x + b$$
$$4 = \frac{5}{2}(2) + b$$
$$4 = 5 + b$$
$$-1 = b$$
$$y = \frac{5}{2}x - 1$$

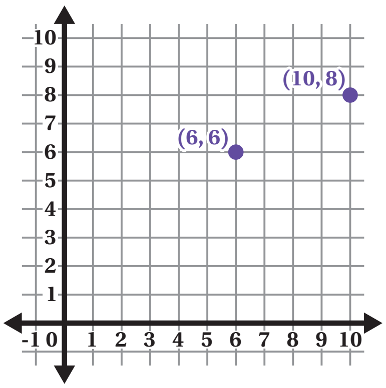
Name: ________ Date: ________ Period: ________

Repeated Challenges

8 For each problem, write an equation of the line that goes through both points.

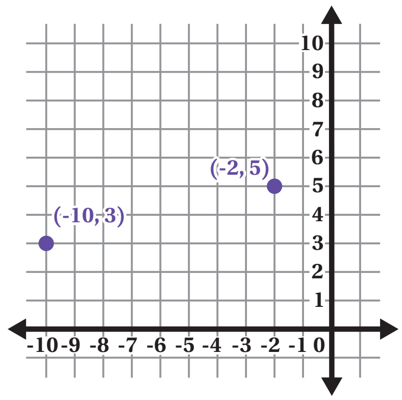
1. Points: (6, 6) and (10, 8).

Equation: $y = \frac{1}{2}x + 3$ **(or equivalent)**



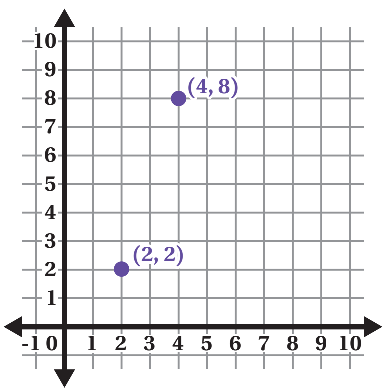
2. Points: (-10, 3) and (-2, 5).

Equation: $y = \frac{1}{4}x + 5.5$ **(or equivalent)**



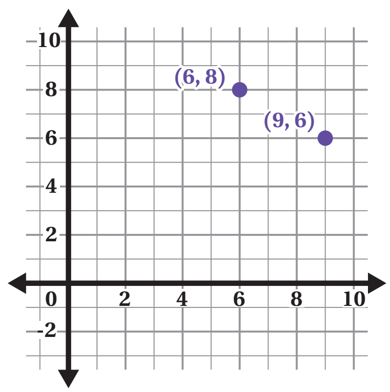
3. Points: (2, 2) and (4, 8).

Equation: $y = 3x - 4$ **(or equivalent)**



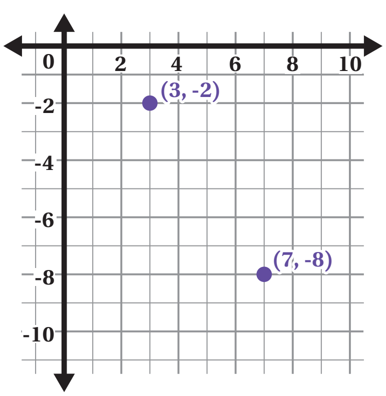
4. Points: (6, 8) and (9, 6).

Equation: $y = -\frac{2}{3}x + 12$ **(or equivalent)**



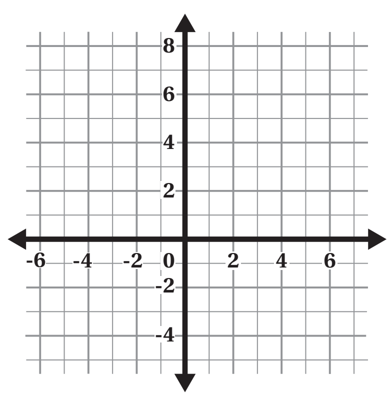
5. Points: (3, -2) and (7, -8).

Equation: $y = -\frac{3}{2}x + 2.5$ **(or equivalent)**



6. Points: (-1, 5) and (2, 2).

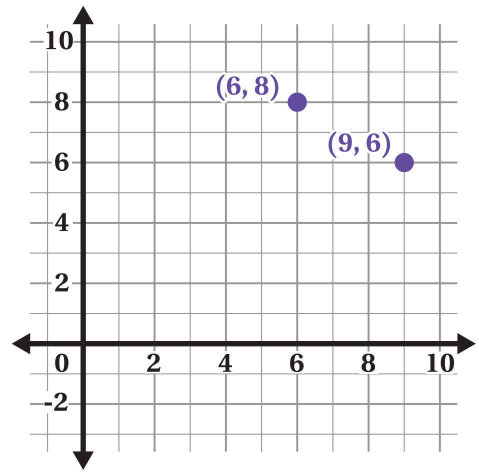
Equation: $y = -x + 4$ **(or equivalent)**



9 Synthesis

Describe how to write an equation of a line that goes through two points.

Use the example if it helps with your thinking.



Responses vary. **First you calculate the slope by determining the change in x and change in y. Then you determine the y-intercept by setting up the equation $y = mx + b$, where m is the slope. Substitute a point in for x and y and solve for b, which represents the y-intercept. Then you can write your final equation.**

Things to Remember:

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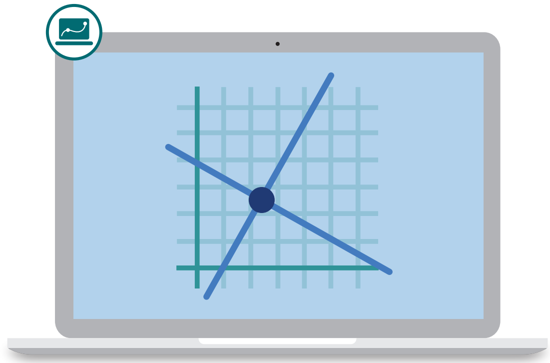

Solutions

Let's think about solutions to two-variable linear equations.

Warm-Up

1 Write *two* pairs of values for x and y that make the equation $x + 2y = 10$ true.

Responses vary.

x	y
-2	6
0	5

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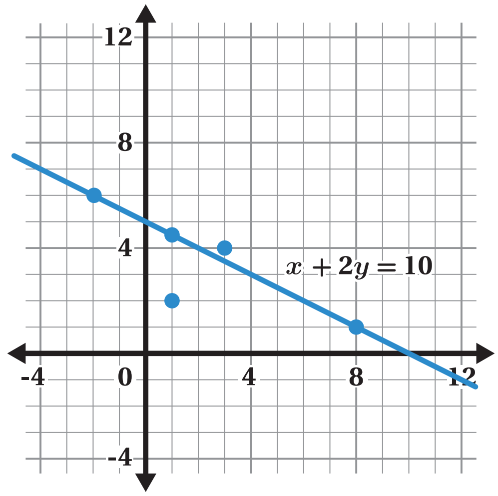
Solutions to Linear Equations

2 This graph shows the line $x + 2y = 10$, as well as some points that are **solutions** to the equation and some that are not.

Show or explain how you can tell from the graph if a point is *not* a solution to the equation.

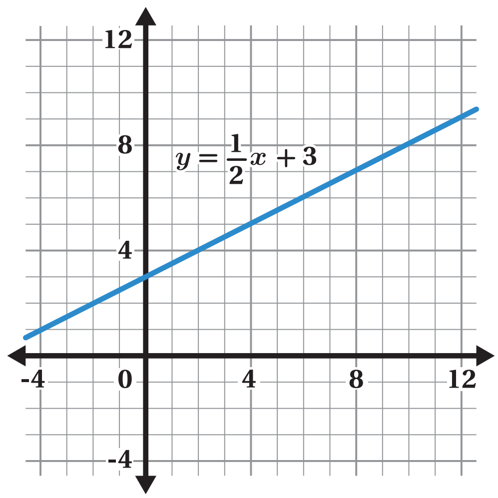
***Responses vary.* Solutions to the equation lie on the line. If a point is not on the line, it's not a solution.**



3 This graph shows the line $y = \frac{1}{2}x + 3$.

Complete the table so each point is a solution to the equation.

x	y
4	**5**
34	20



4 Describe your strategy for finding the solutions you wrote in the table.

***Responses vary.* First, I substituted 4 for x and solved for y. Next, I substituted 20 for y and solved for x.**

Activity 2

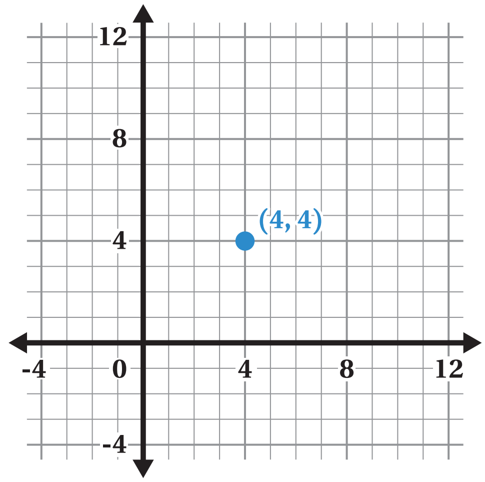
Name: .... Date: .... Period: ....

More Solutions

5 Write an equation for a line that has a solution of $(4, 4)$.

Responses vary.

- $x + y = 8$
- $x - y = 0$
- $y = \frac{1}{4}x + 3$
- $3x + y = 16$



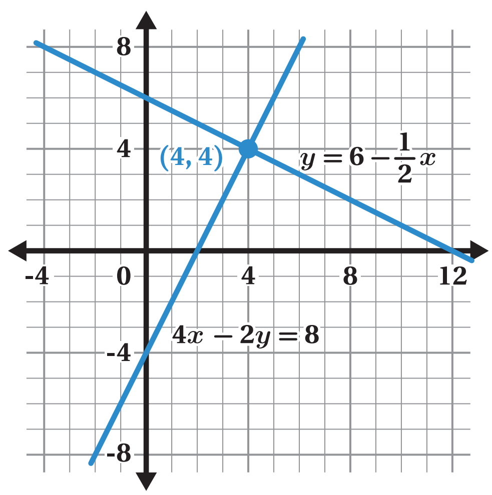
6 Here are the equations Hamza and Neo wrote for the previous problem:

Hamza: $4x - 2y = 8$

Neo: $y = 6 - \frac{1}{2}x$

Select *all* the statements that are true.

- ☑ **A.** $(4, 4)$ is a solution for both lines.
- ☑ **B.** $(7, 2.5)$ is a solution to $y = 6 - \frac{1}{2}x$.
- ☐ **C.** The lines have more than one solution in common.
- ☑ **D.** The line $4x - 2y = 8$ has more than one solution.



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Challenge Creator

7 Create your own solution challenge!

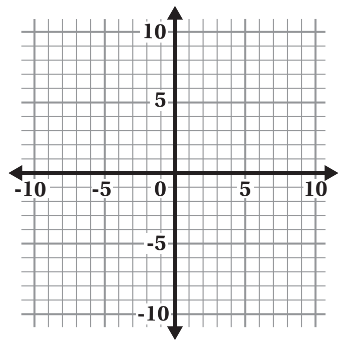
a **Make It!** Plot a point on the Activity 3 Sheet. Then write the equation of a line that the point is a solution for.

b **Solve It!** Then write another solution to your equation here.

Another solution: *Responses vary.*

c **Swap It!** Swap your challenge with one or more partners.

- Verify that the point plotted on your partner's graph is a solution to their equation. If it's not, allow your partner to revise their equation.
- Determine another solution to their equation. Use the graph if it helps with your thinking. *Responses vary.*

<table>
<tr><th></th><th>Equation</th><th>Solutions</th><th>Graph</th></tr>
<tr><td>Partner 1</td><td></td><td><table><tr><th>x</th><th>y</th></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td><td>
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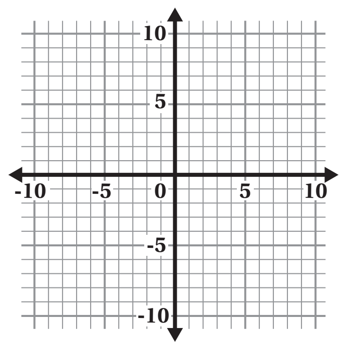
<tr><td>Partner 2</td><td></td><td><table><tr><th>x</th><th>y</th></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td><td>
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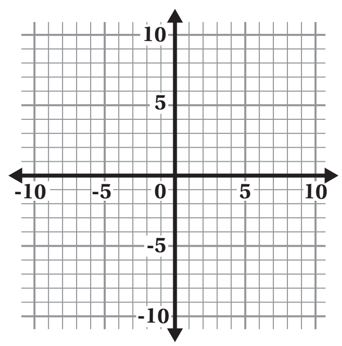
<tr><td>Partner 3</td><td></td><td><table><tr><th>x</th><th>y</th></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td><td>
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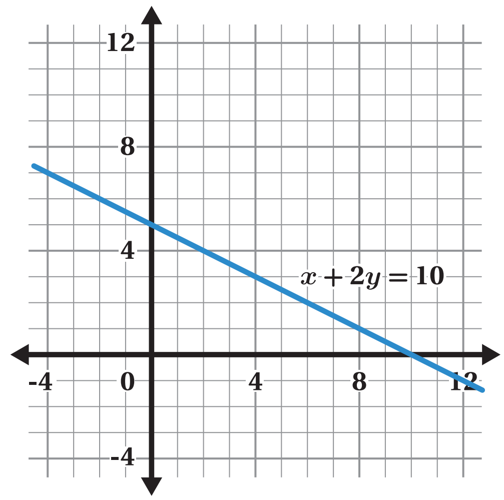
8 Synthesis

How can you determine whether a point is a solution to an equation?

Use the example if it helps with your thinking.



Responses vary.

- I know the point is a solution if substituting in the x- and y-values makes the equation true.
- I know the point is a solution if it lies on the line representing the equation.

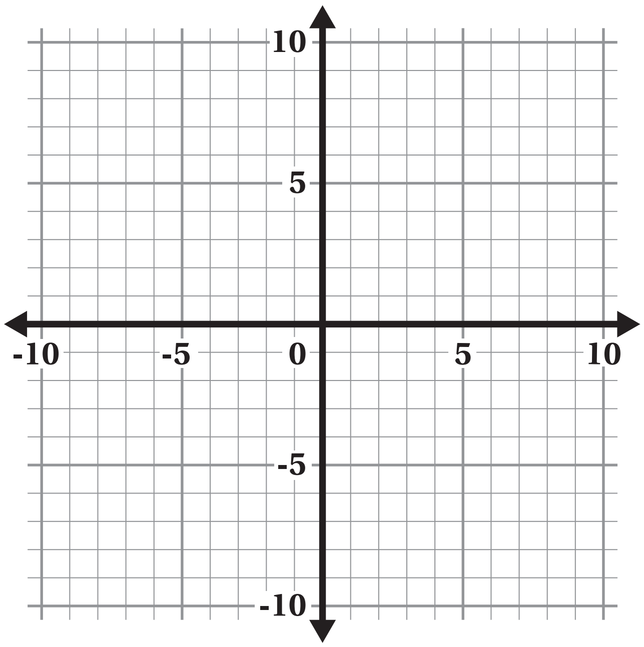
Things to Remember:

Name: .... Date: .... Period: ....

Challenge Creator

- Plot a point on any location on the graph. Then write its coordinates in the table.
- Write the equation of a line that your point is a solution for.

My Graph



My Solution and Equation

x	y

Equation: ....

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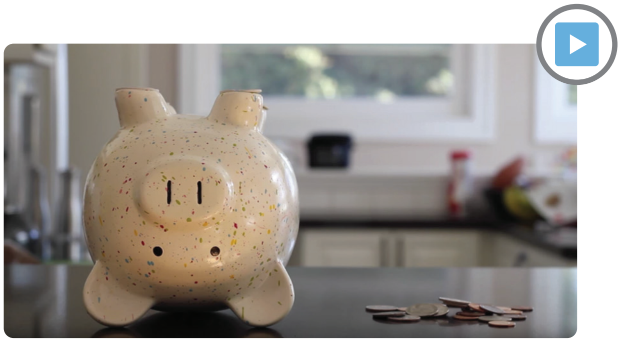
Name: Date: Period:

Pennies and Quarters

Let's determine solutions to real-world linear relationships.

Warm-Up

1 Let's watch a video.



What are some questions you could ask about this situation?

Responses vary.

- **What are all the different types of coins that went into the machine?**
- **How many pennies went in the machine?**
- **How many coins are there in total?**
- **How much is the total value of the coins?**

Activity 1

Name: Date: Period:

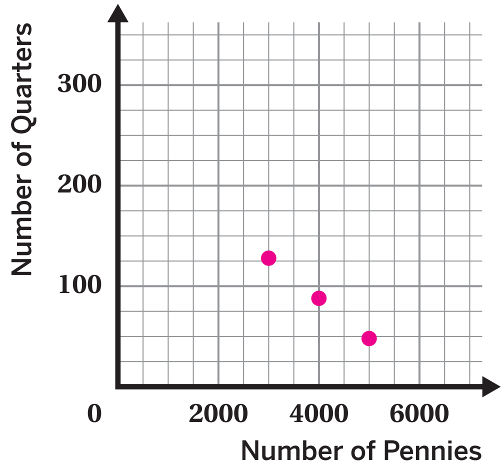
Pennies and Quarters

2 The piggy bank in the video was filled with pennies and quarters worth a total of $62.00.

Write *three* possible combinations of pennies and quarters that are worth $62.00.

Responses vary.

Number of Pennies	Number of Quarters
3000	**128**
4000	**88**
5000	**48**



Then graph the combinations of pennies and quarters you wrote.

3 Describe how you can tell if a combination of pennies and quarters is worth $62.00.

***Responses vary.* You can multiply the number of pennies by 0.01, multiply the number of quarters by 0.25, add the values together, and then see if it's equal to 62.**

4 Let's look at the graph of some students' combinations from Problem 2.

Write an equation that describes *all* the combinations of pennies, p, and quarters, q, that are worth $62.00.

$0.01p + 0.25q = 62$ **(or equivalent)**

Activity
1

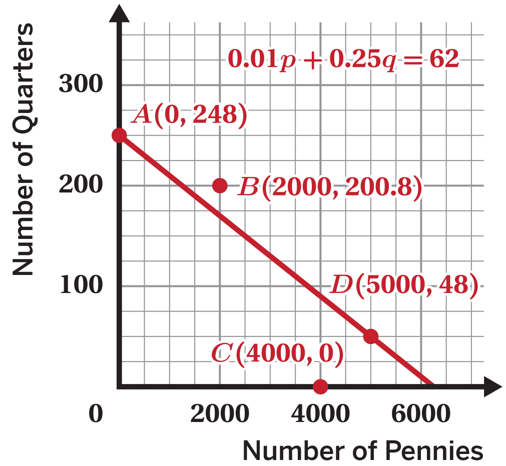
Name: .... Date: .... Period: ....

Pennies and Quarters (continued)

5 Ava wrote the equation $0.01p + 0.25q = 62$ to represent all the combinations of pennies, p, and quarters, q, that are worth \$62.00.

Select *all* the points that are solutions to the equation.

- ☑ **A.** Point A
- ☐ **B.** Point B
- ☐ **C.** Point C
- ☑ **D.** Point D



6 If there are 200 quarters, how many pennies do you need for a total of \$62.00?

Use the graph if it helps with your thinking.

1200 pennies

Explore More

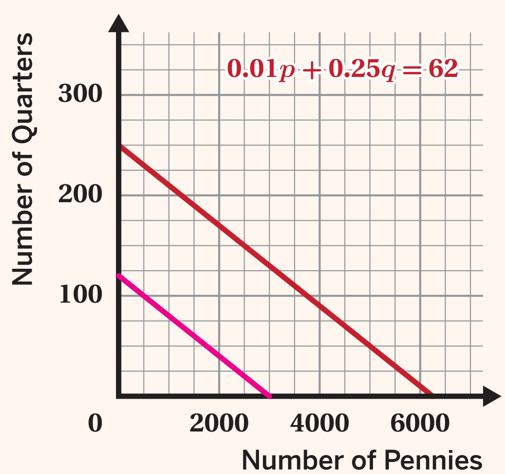
7 Here is a line and an equation describing all the combinations of pennies and quarters that are worth \$62.00.

A different pile of pennies and quarters is worth \$30.

a Draw the graph of this situation on the same plane.

b Describe the strategy you used.

***Responses vary.* I graphed both intercepts and then drew a line connecting the two points.**



Activity 2

Name: ______ Date: ______ Period: ______

Situation Sort

8 **a** Read each situation.

Situation 1

Sydney plans to buy 120 beverages for a picnic. Seltzers are sold in packs of 6. Waters are sold in packs of 8. x represents the number of packs of seltzers and y represents the number of packs of waters.

Situation 2

A coach has a \$120 budget to buy lunch for their team. They are ordering from a restaurant that charges \$8 per sandwich, plus a \$6 delivery fee. x represents the number of sandwiches and y represents the total cost of the lunch.

b **Discuss:**

- What is each situation about?
- What quantities or relationships do you see in each situation?

Responses vary.

- **Situation 1 is about buying two types of beverages for a picnic. Situation 2 is about buying sandwiches for lunch.**
- **In Situation 1, the quantities are the number of packs for each type of beverage and the amount of beverages in each pack. In Situation 2, the quantities are the cost of each sandwich, the delivery fee, and the coach's budget.**

9 Match each representation and possible solution with Situation 1 or Situation 2 from the previous problem. One representation has no match.

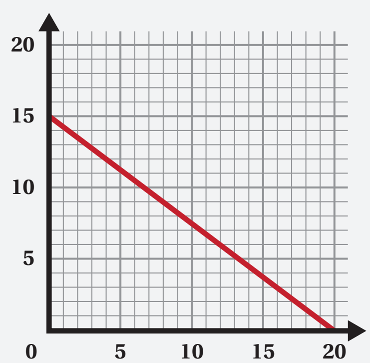
A

x	y
2	22
3	30
4	38

B

$120 = 6x + 8y$

C



D

$y = 8x + 6$

E

$y = 8x + 120$

F

$(5, 46)$

G

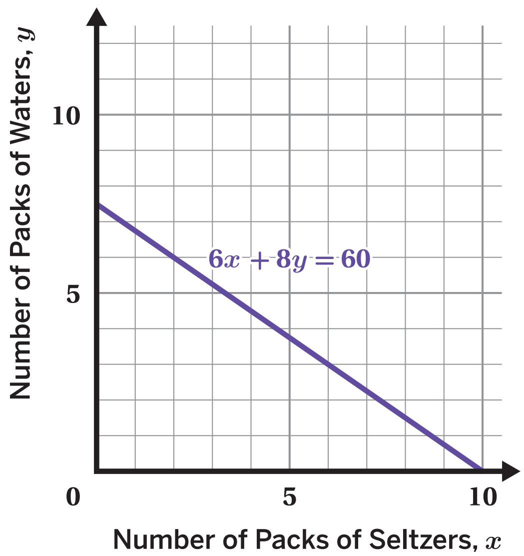
$(8, 9)$

Situation 1	Situation 2
B, C, G	A, D, F

10 Synthesis

Describe how you could use a graph or equation to determine whether a point is a solution to a linear relationship.

Use the example if it helps with your thinking.



Responses vary.

- **I can substitute 7 for x and 2 for y and see whether these values make the equation true.**
- **I can check whether the point (7, 2) is on the line of the graph.**

Things to Remember:

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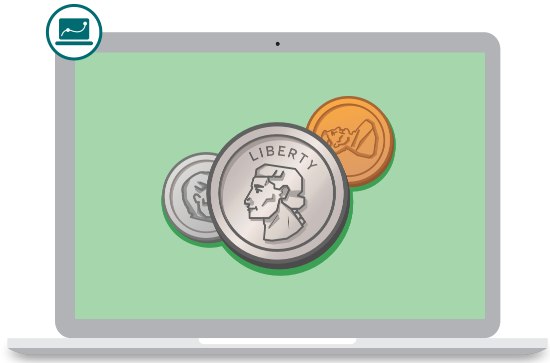
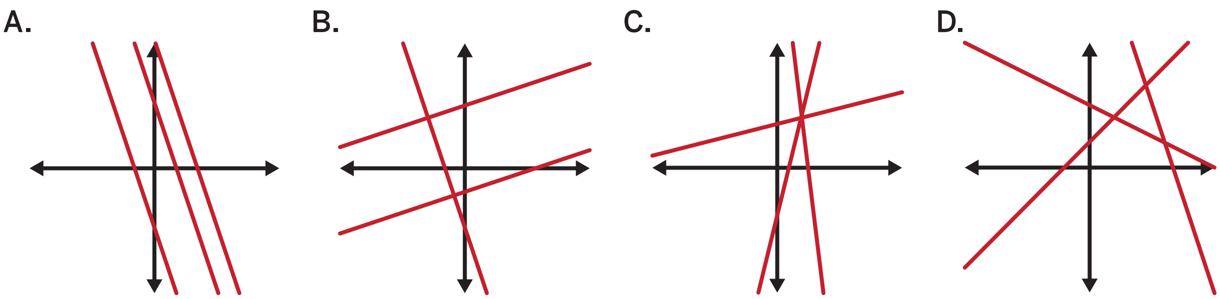


On or Off the Line?

Let's interpret the meaning of points on and off lines.

Warm-Up

1 Which one doesn't belong? Explain your thinking.



Responses and explanations vary.

- **Graph A is the only one where none of the lines cross (they are all parallel). Or, it is the only one that appears to be the same line translated vertically in two different ways.**
- **Graph B is the only one with an intersection point that has a negative coordinate or the only one with two parallel lines.**
- **Graph C is the only one with three lines through a single point.**
- **Graph D is the only one where there are three points where the lines cross.**

Activity 1

Name: ________ Date: ________ Period: ________

Two Dollars

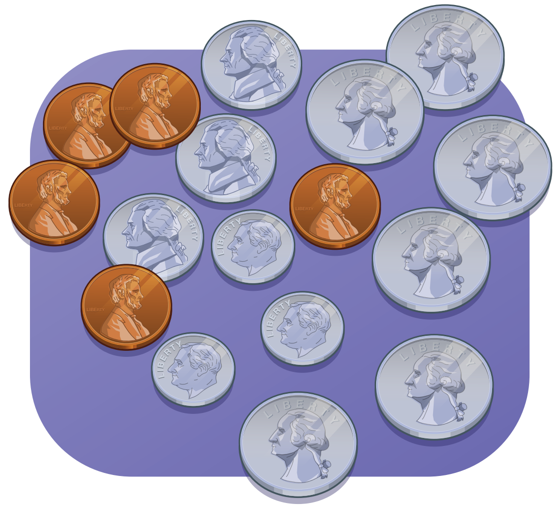
2 I have $2 worth of coins in my pocket.

What is a combination of any coins that I could have?

Try to think of a combination that no one else in the class will write.

Responses vary.

- **8 quarters**
- **2 dimes, 1 nickel, and 7 quarters**
- **200 pennies**

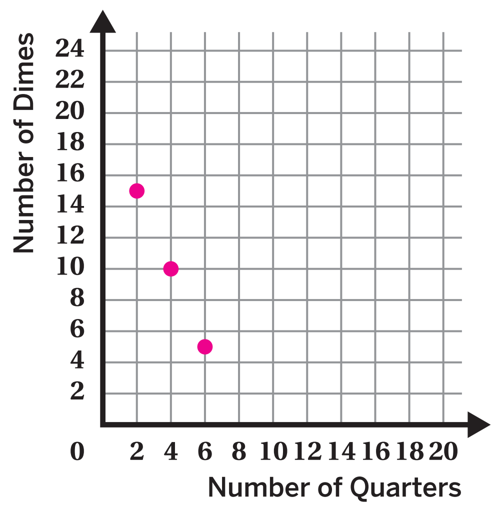


3 Here is more information about my coins:

- I only have quarters and dimes.

a Fill in three rows of possible combinations of quarters and dimes that are worth $2.
Responses vary.

Number of Quarters	Number of Dimes
2	15
4	10
6	5



b Plot your points on the graph.
Responses vary.

Name: .... Date: .... Period: ....

Two Dollars (continued)

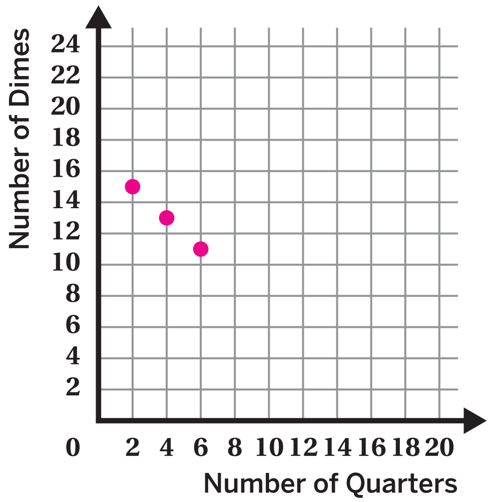
4 Here is some more information about my coins:

- I have a total of 17 coins.

a Fill in at least three rows of possible combinations of 17 coins.

Responses vary.

Number of Quarters	Number of Dimes
2	15
4	13
6	11



b Plot your points on the graph.

Responses vary.

5 Let's look at the graphs of these conditions on the same coordinate plane.

I have \$2 in my pocket: I only have quarters and dimes, and I have a total of 17 coins.

How many quarters and dimes must I have?

Number of Quarters	Number of Dimes
2	15

Explain your thinking.

Explanations vary. The point (2, 15) is the only solution that is on both lines.

Activity 2

Name: ______ Date: ______ Period: ______

Challenge Creator

6 Let's look again at the graphs of both of these conditions on the same coordinate plane.

I have \$2 in my pocket: I only have quarters and dimes, and I have a total of 17 coins.

In this situation, which conditions does the point $(9, 8)$ meet? Circle one.

I have \$2 worth of coins in my pocket | I have a total of 17 coins | Both | Neither

Explain your thinking.

***Explanations vary.* The point $(9, 8)$ is only on the line representing 17 coins combined.**

7 You will use the Activity 2 Sheet to create your own linear situations challenge.

a **Make it!** On the Activity 2 Sheet, create a linear situation challenge.

b **Solve it!** On this page, record the ordered pair that the statement represents and determine which line(s) your statement applies to. Write a description of the line, both, or neither. ***Responses vary.***

Situation	Ordered Pair	Line(s)

c **Swap it!** Swap your challenge with one or more partners. On this page, record the ordered pair that the statement represents and determine which line(s) their statement applies to. ***Responses vary.***

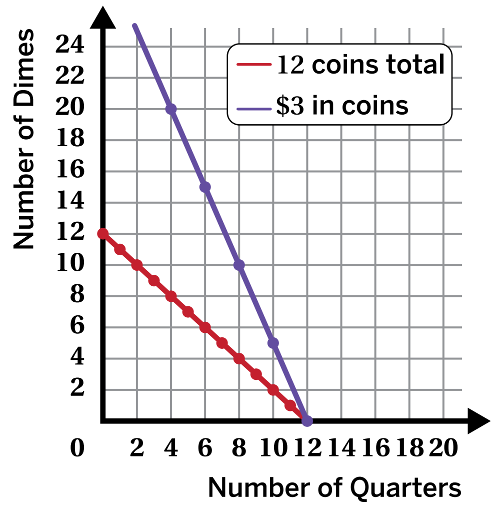
	Situation	Ordered Pair	Line(s)
Partner 1			
Partner 2			
Partner 3			
Partner 4			

8 Synthesis

Here is a graph that represents two conditions:

- I have $3 worth of coins in my pocket.
- I have a total of 12 coins.

How can you use the graph to determine whether a combination of dimes and quarters meets both conditions, one condition, or neither condition?



Responses vary.

- **A combination of dimes and quarters meets both conditions if the point is on both lines. (12, 0) is the only combination that meets both conditions here.**
- **A combination of dimes and quarters meets one condition if the point is on one line, but not the other. For example, (6, 6) represents a combination of dimes and quarters that is 12 coins but isn't worth $3.**
- **A combination of dimes and quarters meets neither condition if it isn't on either line. For example, (2, 2) represents a combination of dimes and quarters that is only 4 coins and isn't worth $3.**

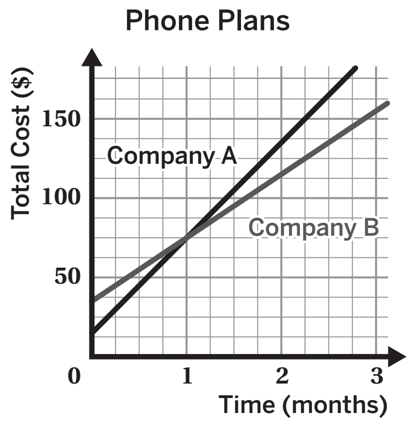
Things to Remember:

Name: .... Date: .... Period: ....

Lesson 4.09
Activity 2

Challenge Creator

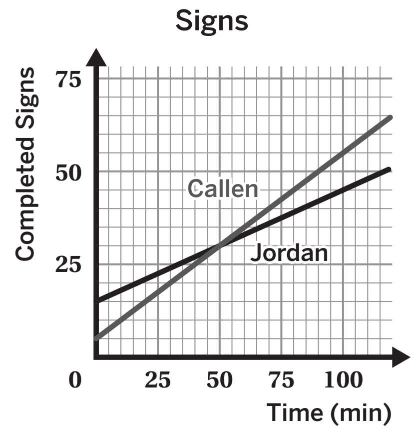
- Select *one* situation to focus on.
- Fill in the statement to describe a single point anywhere on the graph.



You are shopping for a new cell phone and a plan with unlimited data.

- Company A has a $15 setup fee, and then charges $60 per month.
- Company B has a $35 setup fee, and then charges $40 per month.

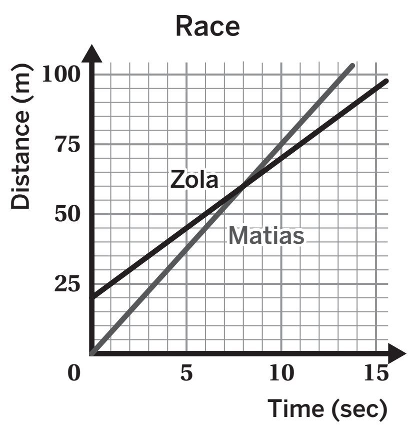
Statement: At .... months, the cost of my phone plan is ....



Callen and Jordan are making locker signs to decorate for spirit week. The coordinate plane shows each person's progress today.

- Callen made 5 signs yesterday and is making a new sign every 2 minutes today.
- Jordan made 15 signs yesterday and is making a new sign every 4 minutes today.

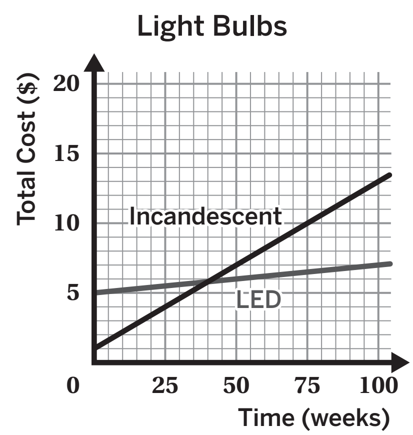
Statement: At .... minutes, ... signs were made.



Zola and Matias raced 100 meters. Both racers started at the same time and ran at a constant speed.

- Zola had a 20-meter head start and ran 5 meters per second.
- Matias ran 7.5 meters per second.

Statement: At .... seconds, the distance is .... meters.



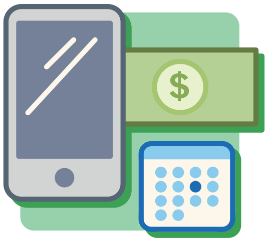
Traditional incandescent bulbs are cheaper to purchase, but they use more energy compared to LED bulbs.

- Traditional incandescent bulbs cost about $1 each, and 12 cents per week to use.
- LED bulbs cost about $5 each, and 2 cents per week to use.

Statement: At .... weeks, the total cost of the light bulbs is ....

Unit 4
Lesson
10

Name: .... Date: .... Period: ....



On Both Lines

Let's use lines to analyze real-world situations.

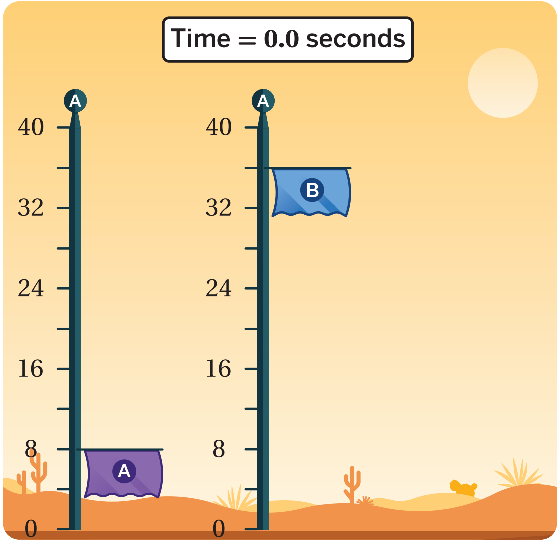
Warm-Up

Let's watch an animation with two flags.

1. **Discuss:** What do you notice? What do you wonder?

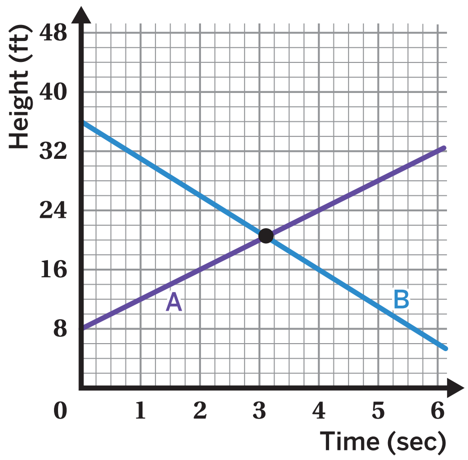
Responses vary.

- **I notice Flag A is moving up and Flag B is moving down.**
- **I notice Flag A and Flag B are each moving at a constant speed.**
- **I notice that the y-axis scale counts by 2 for the minor grid lines.**
- **I wonder when the two flags are at the same height?**
- **I wonder why the x-axis is counting by 1 and the y-axis is counting by 8?**
- **I wonder what the equation is for each line?**



2. A graph's **point of intersection** is a point where the lines meet. What does the point of intersection represent in this situation?

Responses vary. **Just after 3 seconds, both flags were at the same height: just over 18 feet.**



Activity
1

Name: ______ Date: ______ Period: ______

On Both Lines

You will use the Activity 1 Sheet and choose *two* different phone companies to compare.

3. Create a poster. Be sure to include these items in your poster:

- ☐ Your names
- ☐ The names of your selected companies
- ☐ A graph with both companies on the same set of axes
- ☐ An equation to represent each scenario
- ☐ Your conclusions about the situation
- ☐ The meaning of the point of intersection
- ☐ When each company has the better deal

Explore More

4. Select a third phone provider. On your poster, explain when this third plan would be a better and worse deal than your two previously selected plans.

***Responses vary*. Company #1 is a better deal than Company #2 or #3 for about the first 23 months. After that, Company #2 is the better deal.**

Activity
2

Name: .... Date: .... Period: ....

Gallery Tour

You will take a gallery tour as a class.

5. What features of your classmates' posters helped you understand their thinking?

Responses vary.

- **Seeing the different scales my classmates used to create their graphs.**
- **Seeing the points of intersection labeled on their graphs.**
- **Reading the different explanations my classmates wrote.**

6. Describe something you would change about your poster now that you have seen other groups' work.

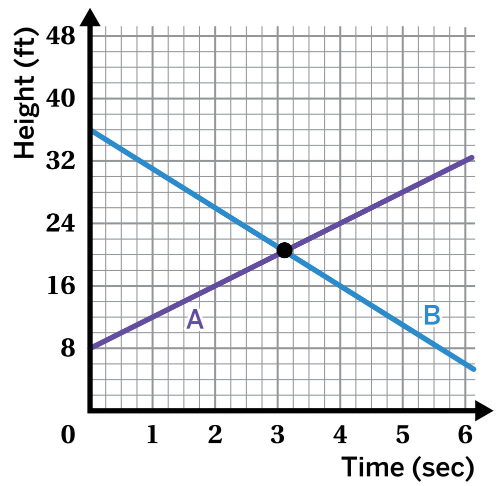
Responses vary.

- **I would use a different scale for the graph axes to show the information more precisely.**
- **I would add arrows and labels for the different regions of my graph.**

Synthesis

7. Describe how a graph can help you compare different linear relationships.

Use the example if it helps with your thinking.



Responses vary. **When I graph two linear relationships, I can visually compare the y-values of two linear relationships when the x-values are the same. The point of intersection shows the solution that they have in common.**

Things to Remember:

Name: ______________________ Date: __________ Period: __________

On Both Lines

You are shopping for a new cell phone and plan with unlimited data. Here are four companies and information about their set-up fee, phone cost, and service plan.

COMPANY #1

SET-UP FEE
$20

SERVICE PLAN
$25 /MONTH

PHONE PAYMENT
$40 /MONTH

COMPANY #2

SET-UP FEE
$30

NEW PHONE
$700

SERVICE PLAN
$35 /MONTH

COMPANY #3

PRICING INFORMATION TABLE (INCLUDES PLAN, PHONE, AND SET-UP FEE.)

Time (Months)	Total Cost ($)
0	$30
9	$885
12	$1170

COMPANY #4

$y=95x+30$

y = TOTAL COST
x = MONTHS

INCLUDES PLAN, NEW PHONE, AND SET-UP FEE.

Science Mom Lesson 67

Name: .... Date: .... Period: ....

On Both Lines (answers)

You are shopping for a new cell phone and plan with unlimited data. Here are four companies and information about their set-up fee, phone cost, and service plan.

COMPANY #1

SET-UP FEE
$20

SERVICE PLAN
$25 /MONTH

PHONE PAYMENT
$40 /MONTH

COMPANY #2

SET-UP FEE
$30

NEW PHONE
$700

SERVICE PLAN
$35 /MONTH

COMPANY #3

PRICING INFORMATION TABLE (INCLUDES PLAN, PHONE, AND SET-UP FEE.)

Time (Months)	Total Cost ($)
0	$30
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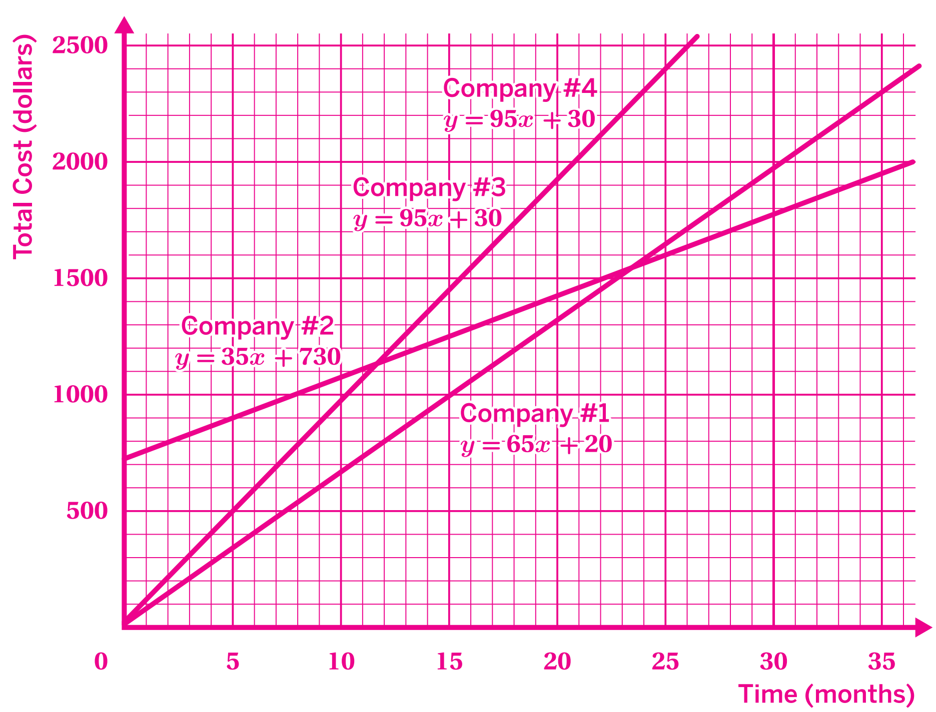
12	$1170

COMPANY #4

$y = 95x + 30$

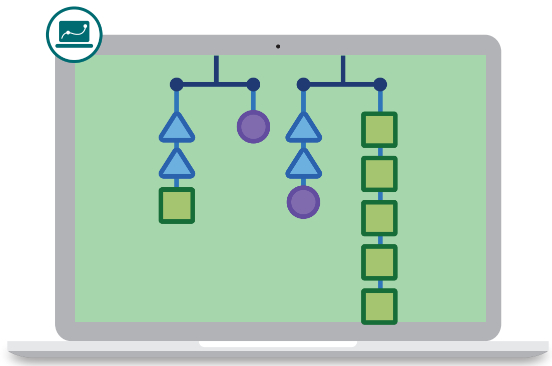
y = TOTAL COST
x = MONTHS

INCLUDES PLAN, NEW PHONE, AND SET-UP FEE.



Unit 4
Lesson
11

Name: .... Date: .... Period: ....



Make Them Balance

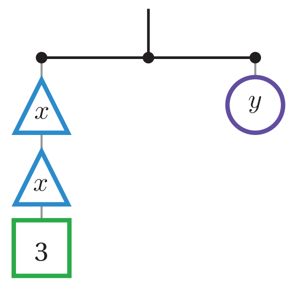
Let's explore solutions to more than one linear relationship.

Warm-Up

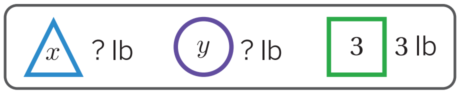
1 Determine values for x and y that will balance the hanger. ***Responses vary.***

x	y
2	**7**

Hanger A



$2x + 3 = y$



2 Let's look at a graph that shows points whose x- and y-values balance the hanger.

Discuss: What do you notice? What do you wonder?

Responses vary.

- **I notice that all of the points are on a line.**
- **I wonder if any set of numbers that make the hanger balance will be on that line.**

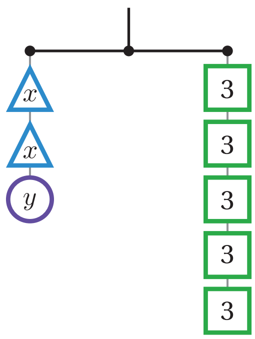
Activity 1

Name: ____________ Date: ________ Period: ________

Two Hangers

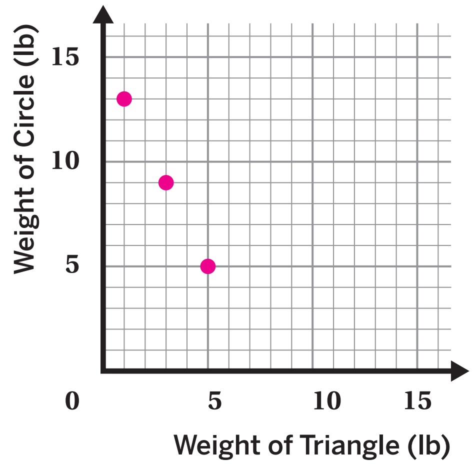
3 Here's another hanger. Plot three different pairs of x- and y-values that will balance this hanger. ***Responses vary.***

Hanger B



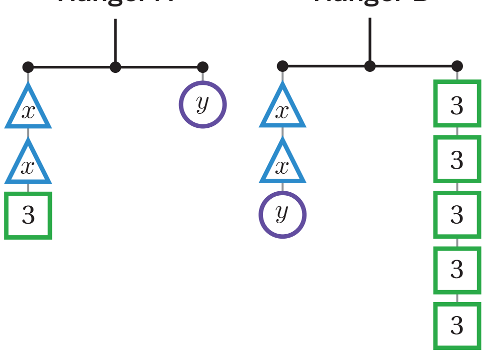
$2x + y = 15$

x ? lb y ? lb 3 3 lb



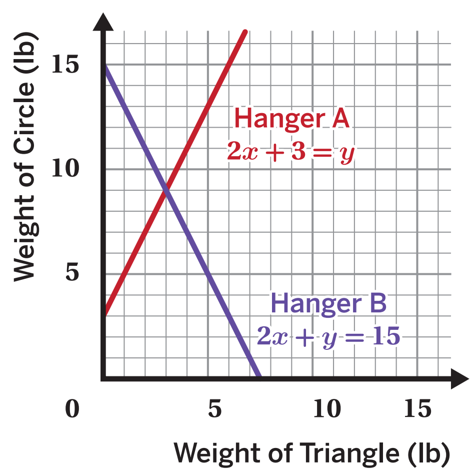
4 Here are both hangers' equations graphed on the same coordinate plane.

Hanger A Hanger B



$2x + 3 = y$ $2x + y = 15$

x ? lb y ? lb 3 3 lb



Discuss: Can you identify a point that balances just Hanger A? Just Hanger B? Both? Neither?

***Responses vary.* Both hangers balance when a point is on both of the lines. One hanger balances when a point is on one of the lines. Neither hanger balances when a point is off both lines.**

Name: ______ Date: ______ Period: ______

Two Hangers (continued)

5 You have been experimenting with two representations of this **system of equations**:

$2x + 3 = y$
$2x + y = 15$

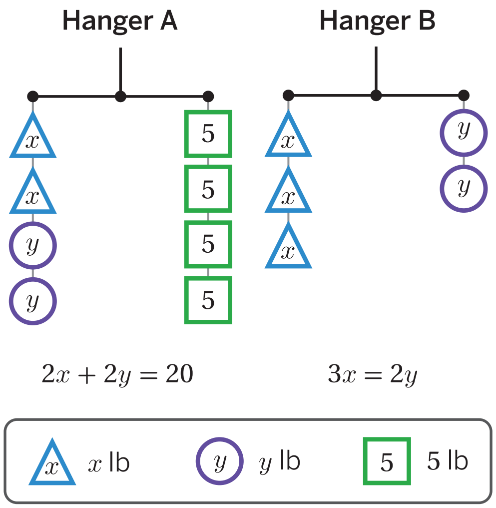
A **solution to a system of equations** is a set of values that makes both equations in that system true.

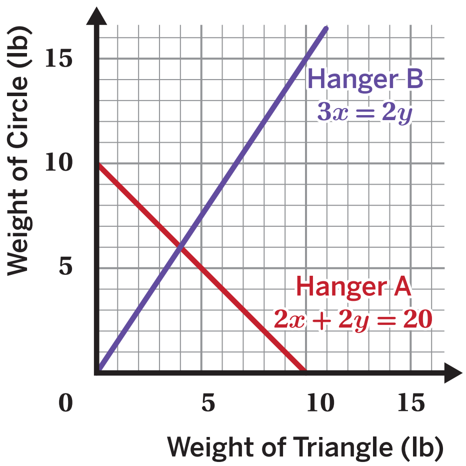
Write the solution to this system as an ordered pair. Explain your thinking.

$(3, 9)$. *Explanations vary.* $(3, 9)$ is the point of intersection of the two lines, which means that it is a solution to both equations.

6 This system represents another set of hangers:

$2x + 2y = 20$
$3x = 2y$





Which point is a solution to this system? Circle one.

$(1, 9)$ $(4, 6)$ $(6, 9)$ $(6, 4)$

Explain your thinking.

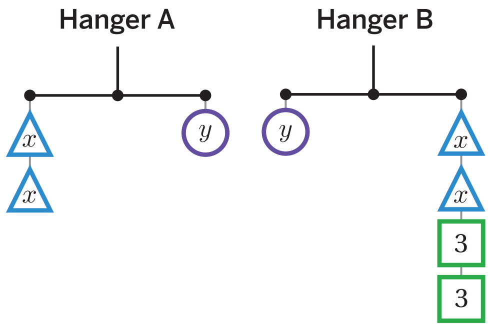
***Explanations vary.* I know that $(4, 6)$ is the solution to the system of equations because that is the point where the lines intersect. I can substitute $x = 4$ and $y = 6$ into each equation and the values make both equations true.**

Activity 2

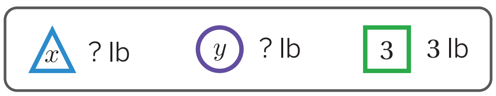
Name: ______ Date: ______ Period: ______

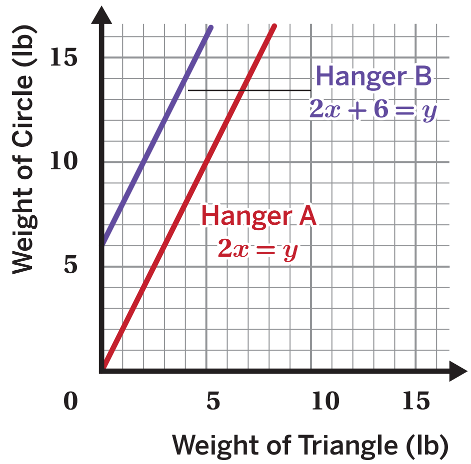
Hanger Solutions

7 Here's another system of equations.



$2x = y$ $\qquad$ $y = 2x + 6$





Discuss: Can you identify a point that balances just Hanger A? Just Hanger B? Both? Neither?

***Responses vary.* Any point that is on one of the lines will make that hanger balance. Any point that is not on either of the lines will make neither hanger balance. It is not possible to make both hangers balance for this system of equations.**

8 This system of equations from the previous problem has no solution:

$2x = y$

$y = 2x + 6$

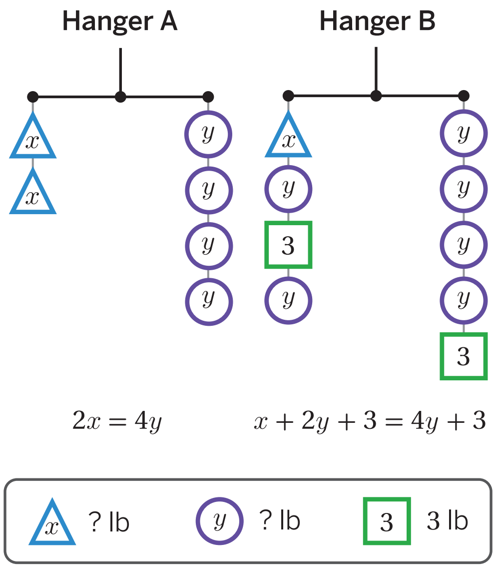
How can you use the hangers, graphs, or equations to tell that this system of equations has no solution?

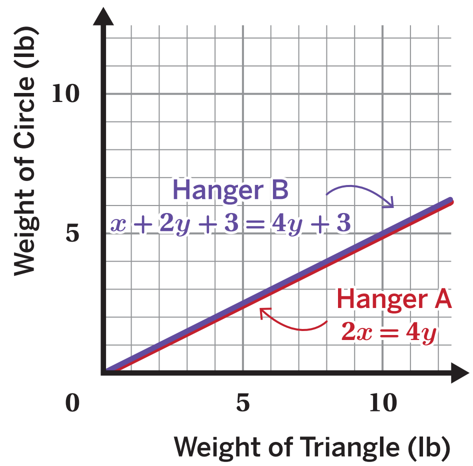
***Responses vary.* If I look at the graph, I can tell that the system has no solution because the lines are parallel and will never intersect. If I set the equations equal to each other and subtract $2x$ from each side, I get $0 = 6$, which is a false statement. This means that there is no solution to the equation $2x = 2x + 6$, so there is no solution to this system of equations.**

Name: .... Date: .... Period: ....

Hanger Solutions (continued)

9 Here's one more system.





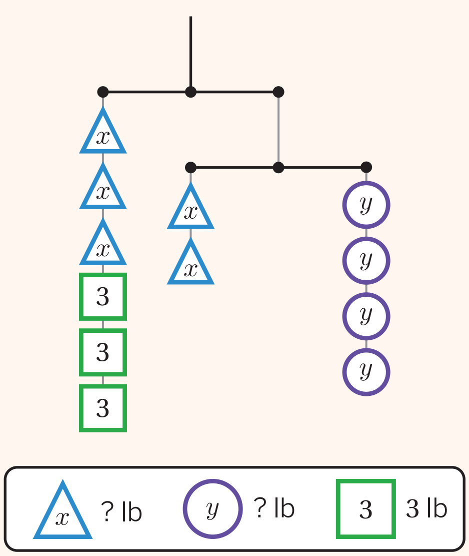
Discuss: How many solutions do you think this system has? How do you know?

Infinitely many solutions. *Explanations vary*. It is not possible to make one hanger balance and not the other because they are represented by the same line. Any pair of values where the x-value is double the y-value will make both equations true, and there are infinitely many of those.

Explore More

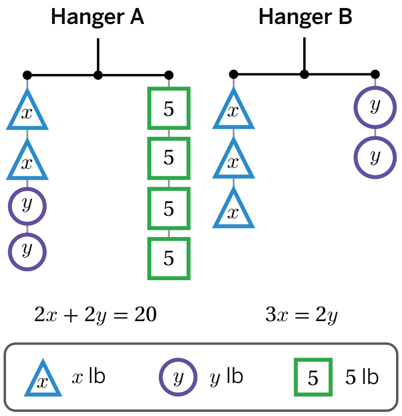
10 Determine values for x and y that will balance both hangers.

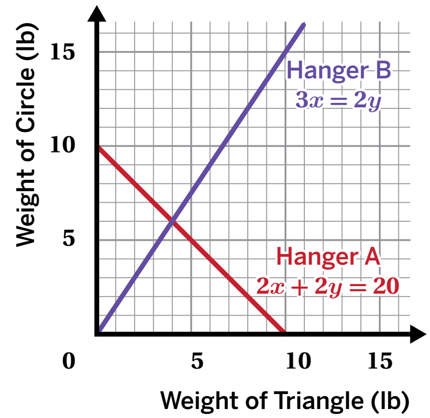
x	y
9	4.5



11 Synthesis

How can you tell if an ordered pair is a solution to a system of linear equations? Use the example if it helps with your thinking.





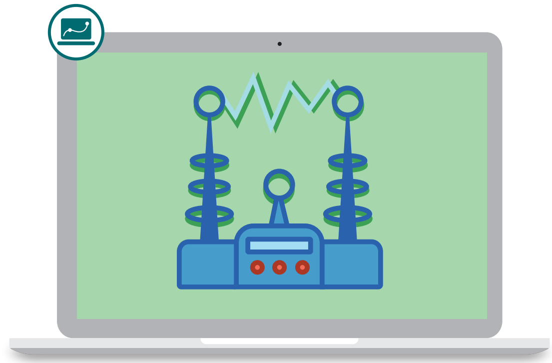
Responses vary.

- An ordered pair is a solution to a system if it is the point of intersection for the lines.
- An ordered pair is a solution to a system of equations if it makes both equations true when I substitute in the x- and y-values.

Things to Remember:

Unit 4
Lesson
12

Name: ________ Date: ________ Period: ________



Line Zapper

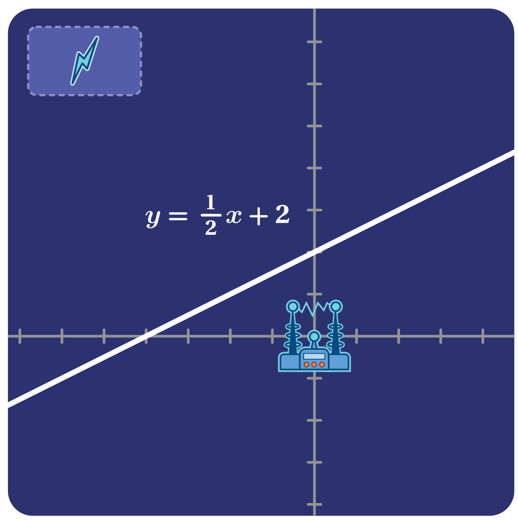
Let's solve systems of linear equations.

Warm-Up

1 Let's capture the line by writing the coordinates for a point that's on the line.

Zap	Point
Zap 1	

Responses vary.
- **(0, 2)**
- **(1.5, 2.75)**
- **(2, 3)**



Activity 1

Name: ______ Date: ______ Period: ______

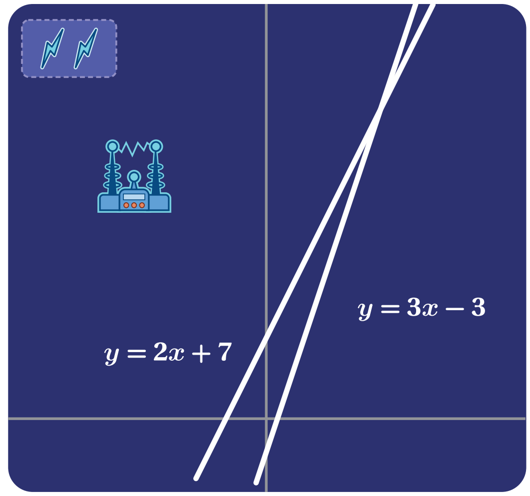
Two Lines, One Zap

2 Capture both of the lines by writing coordinates for points on these lines.

Use no more than two zaps!

Zap	Point
Zap 1	
Zap 2	

***Responses vary.* The coordinates of the intersection point are $(10, 27)$.**



3 Cameron wanted to capture both lines with one zap.

Cameron solved an equation to identify the point of intersection and determined that $x = 10$.

How could Cameron determine the y-value of the point of intersection?

***Responses vary.* Cameron could substitute the value of x into either equation and then simplify to find the value of y. Using either equation, Cameron can determine that y must be 27.**

Cameron

$y = 2x + 7$

$y = 3x - 3$

$2x + 7 = 3x - 3$

$-2x \qquad -2x$

$7 = 1x - 3$

$+3 \qquad +3$

$10 = 1x$

$\boxed{x = 10}$

Activity 2

Name: ______ Date: ______ Period: ______

Line Zapper

4 The following lines are hidden in a graph:

$y = -x + 10$
$y = 2x + 4$

Capture the lines by writing the coordinates for a point on both of these lines. You only have one zap!

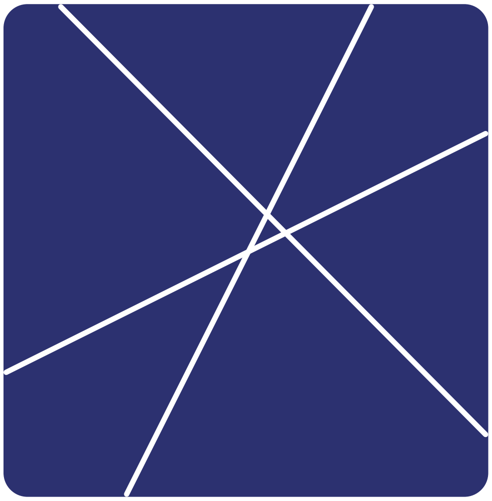
Zap	Point
Zap 1	$(2, 8)$

5 **a** Select *all* the lines that would be captured if the point $(2, 4)$ were zapped.

☐ **A.** Line A: $y = \frac{1}{2}x$

☑ **B.** Line B: $y = 2x$

☑ **C.** Line C: $y = -x + 6$



b Explain how you decided which lines to select.

Responses vary. **I know the point $(2, 4)$ is on these lines because if I substitute 2 for x in these equations, I get $y = 4$.**

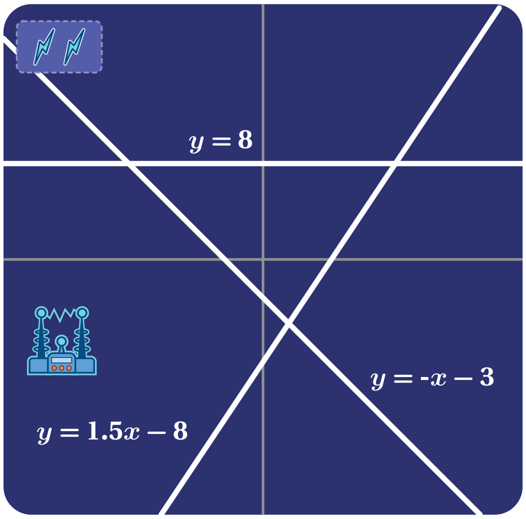
Name: .... Date: .... Period: ....

Line Zapper (continued)

6 Capture all of the lines by writing coordinates for points on the lines. You only have two zaps!

Zap	Point
Zap 1	
Zap 2	

***Responses vary.* The coordinates of the intersection points are $(-11, 8)$, $\left(\frac{32}{3}, 8\right)$, and $(2, -5)$.**



7 The following lines are hidden in a graph:

$y = -2x + 9$

$y = 3x - 1$

$y = \frac{1}{2}x - 1$

Capture all of the lines by writing the coordinates for points on the lines. You only have two zaps!

Zap	Point
Zap 1	
Zap 2	

***Responses vary.* The coordinates of the intersection points are $(2, 5)$, $(4, 1)$, $(0, -1)$.**

Explore More

8 The following lines are hidden in a graph:

$y = 3x + 6$

$y = 3(x - 5)$

$y = -\frac{1}{2}x - 15$

$-x + y = 12$

Capture all of the lines by writing coordinates for points on the lines. You only have three zaps!

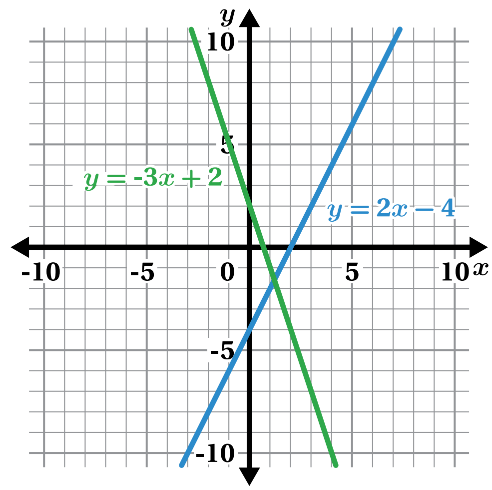
***Responses vary.* The coordinates of the intersection points are $(0, -15)$, $(-6, -12)$, $(-18, -6)$, $(3, 15)$, $(13.5, 25.5)$.**

Zap	Point
Zap 1	
Zap 2	
Zap 3	

9 Synthesis

Here is the graph of this system of equations:

$y = \text{-}3x + 2$
$y = 2x - 4$



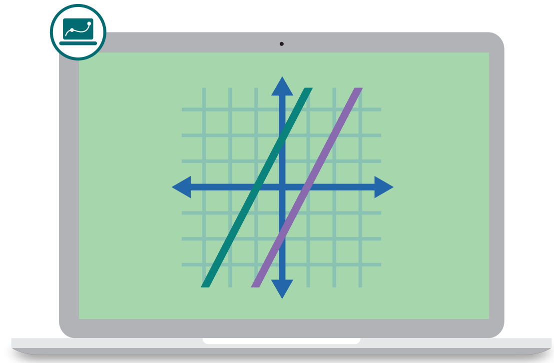
How can you determine the *exact* solution to this system of equations?

Responses vary. **I can find the x-value when the y-values are equal. I would write $\text{-}3x + 2 = 2x - 4$ and then solve for x. Once I find x, I can substitute the value in either equation to calculate the value for y. The solution to this system of equations is $(1.2, \text{-}1.6)$.**

Things to Remember:

Unit 4
Lesson
13

Name: ____________ Date: ________ Period: ________



All, Some, or None? Part 2

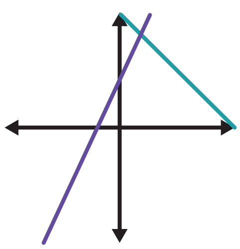
Let's solve systems with no solution and infinitely many solutions.

Warm-Up

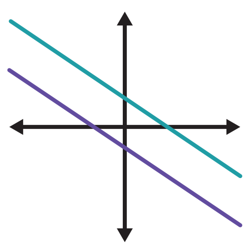
1 Which graph could represent this system of equations?

$y = 2x + 4$
$y = \text{-}x + 10$

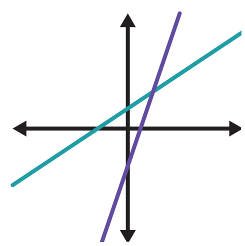
A.



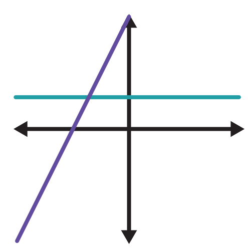
B.



C.



D.



Explain your thinking.

***Explanations vary.* If you look at the equations, one line has a negative slope and the other line has a positive slope. Graph A is the only one that has lines with a negative and positive slope.**

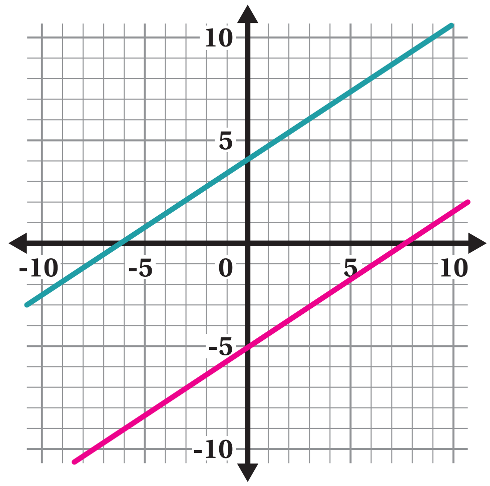
Activity 1

Name: ________ Date: ________ Period: ________

Connecting Graphs and Equations

2 Here is the graph of $y = \frac{2}{3}x + 4$.

Graph another line to create a system of equations that has *no* solution.



3 Here are the equations for a system that has no solution:

$y = \frac{2}{3}x + 4$

$y = \frac{2}{3}x - 5$

How can you determine from the equations that the system will have no solution?

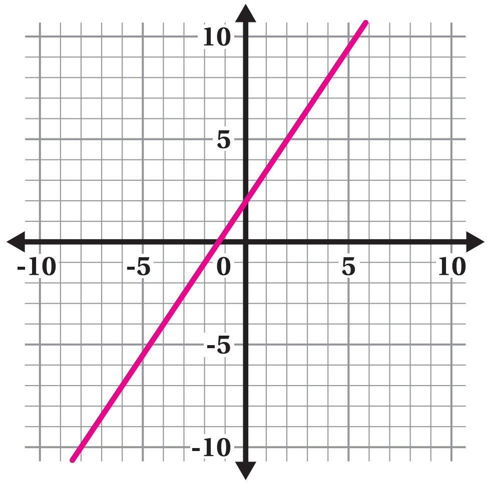
***Responses vary.* If the equations have the same slope but different y-intercepts, then the system will have no solution.**

Name: ______ Date: ______ Period: ______

Connecting Graphs and Equations (continued)

4 Here is the graph of $y = \frac{1}{2}(3x + 4)$.

Graph another line to create a system of equations that has *infinitely many* solutions.



5 Here are the equations for a system that has infinitely many solutions:

$y = \frac{1}{2}(3x + 4)$

$y = \frac{3}{2}x + 2$

How can you determine from the equations that the system will have infinitely many solutions?

***Responses vary.* If the equations are equivalent, then the system will have infinitely many solutions.**

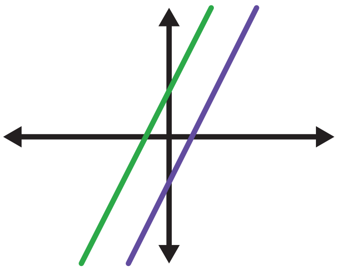
Activity
2

Name: ______ Date: ______ Period: ______

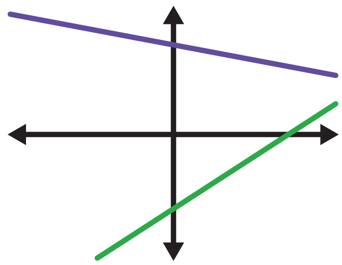
Sorting Systems

6 Group the systems of equations based on their number of solutions.

A.



B.



C. $\begin{cases} y = \frac{3}{4}x - 14 \\ y = \text{-}\frac{1}{4}x + 9 \end{cases}$

D. $\begin{cases} y = 2x + 1 \\ \text{-}2x + y = 1 \end{cases}$

E. $\begin{cases} x + y = 10 \\ y = \text{-}x - 3 \end{cases}$

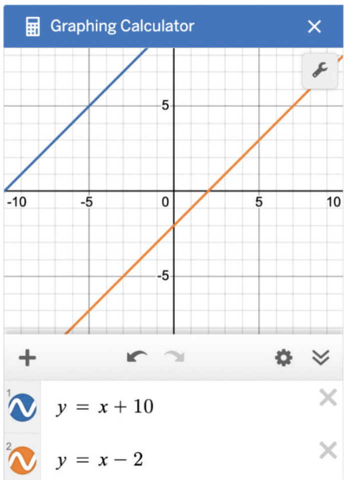
F. $\begin{cases} y = x + 10 \\ y = x - 2 \end{cases}$

No Solution	One Solution	Infinitely Many Solutions
A, E, F	B, C	D

Sorting Systems (continued)

7 Here are Oscars's and Melanie's strategies to check whether System F goes in the "No Solution" group.

Oscar



Melanie

F $\begin{cases} y = x + 10 \\ y = x - 2 \end{cases}$

$x + 10 = x - 2$
$-x \qquad -x$
$10 = -2$

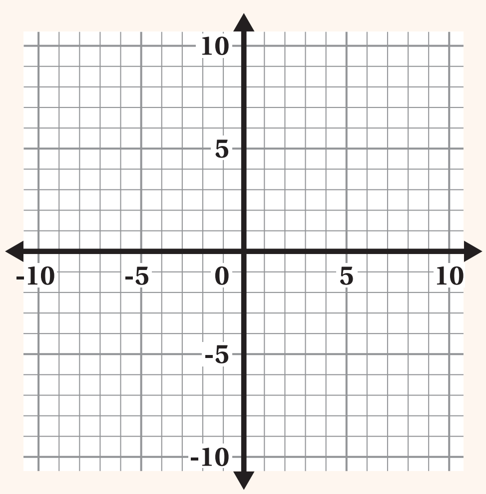
Discuss: How are their strategies alike? How are they different?

Responses vary. Oscar used a graphing calculator to graph the lines, and Melanie set the two equations equal to each other and tried to solve. These both show that the system has no solution because the lines never meet and there is no value for x that makes the equations true.

Explore More

8 The graphs of the equations $Ax + By = 15$ and $Ax - By = 9$ intersect at $(2, 1)$. Determine the values of A and B.

Use the grid if it helps with your thinking.
$A = 6$ and $B = 3$



9 Synthesis

How can you determine the number of solutions for a system of equations?

Responses vary.

- **I could graph the system and look to see how many points of intersection the lines have.**
- **I could determine the slope and y-intercept of each line. If the slopes are different, there will be one solution. If they are the same but the y-intercepts are different, there will be no solution. If the slopes are the same and the y-intercepts are the same, there will be infinitely many solutions.**
- **If the equations are both in $y =$ form, I could set them equal to each other and solve for x, which will tell me whether that equation has one, no, or infinitely many solutions.**

C
$$\begin{cases} y = \frac{3}{4}x - 14 \\ y = \text{-}\frac{1}{4}x + 9 \end{cases}$$

D
$$\begin{cases} y = 2x + 1 \\ \text{-}2x + y = 1 \end{cases}$$

E
$$\begin{cases} x + y = 10 \\ y = \text{-}x - 3 \end{cases}$$

Things to Remember:

Name: ________ Date: ________ Period: ________

Strategic Solving, Part 2

Let's explore strategies for solving systems of equations.

Warm-Up

Determine the solution of each system mentally. Be prepared to share your strategy.

1. $\begin{cases} x = 5 \\ y = x - 7 \end{cases}$

(5, -2)

2. $\begin{cases} y = 4 \\ y = x + 3 \end{cases}$

(1, 4)

3. $\begin{cases} x - y = 4 \\ x + y = 10 \end{cases}$

(7, 3)

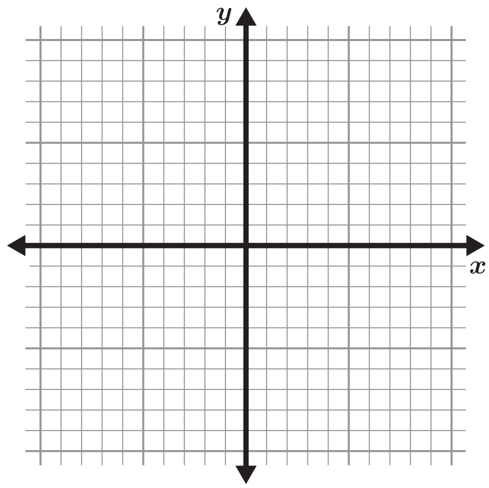
Name: .... Date: .... Period: ....

Thinking About Solutions

4. How many solutions does this system have? Circle one.

$$\begin{cases} x + y = 5 \\ x + y = 7 \end{cases}$$

One solution   No solution   Infinitely many solutions



Explain your thinking.

Explanations vary.

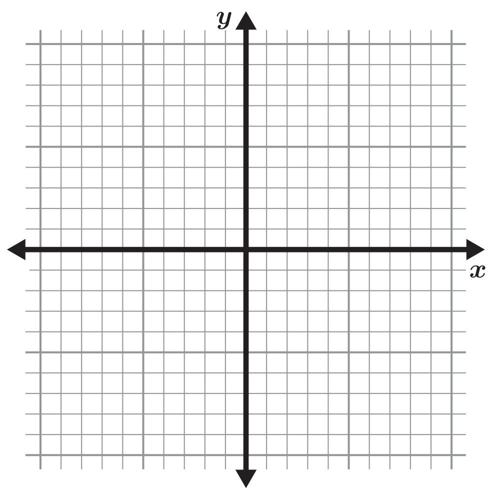
- **The sum of two numbers can't be equal to both 5 and 7.**
- **If you graph the equations, they have the same slope but different y-intercepts. There will be no point of intersection.**

5. Martina says that $(2, -4)$ is a solution to the system. Sai says there are infinitely many solutions.

$$\begin{cases} y = 4(x - 3) \\ y = 4x - 12 \end{cases}$$

Whose claim is correct? Circle one.

Martina's   Sai's   Both   Neither



Explain your thinking.

***Explanations vary.* The equations are equivalent so they share all the same solutions, including the point $(2, -4)$. Note: Students who select Martina, Sai, or Both are considered correct.**

Activity 2

Name: ________ Date: ________ Period: ________

The Choice Is Yours

System A

$$\begin{cases} y = 3x + 2 \\ 2x + y = 47 \end{cases}$$

System B

$$\begin{cases} y = 4 \\ y = \text{-}2 \end{cases}$$

System C

$$\begin{cases} y = \frac{1}{4}x + 7 \\ x = \text{-}4 \end{cases}$$

System D

$$\begin{cases} y = \text{-}3x + 10 \\ y = \text{-}2x + 6 \end{cases}$$

System E

$$\begin{cases} y = 3x + 5 \\ \text{-}3x + y = 5 \end{cases}$$

System F

$$\begin{cases} y = \text{-}3x - 5 \\ y = 4x + 30 \end{cases}$$

System G

$$\begin{cases} y = 3 \\ x = \text{-}2y + 56 \end{cases}$$

System H

$$\begin{cases} \text{-}4x + y = 30 \\ y = \text{-}3x - 5 \end{cases}$$

6. Examine these systems. Organize the equations into two or three groups based on the patterns you notice. ***Responses vary.***

Group 1	Group 2	Group 3
System B System C System G	System D System F	System A System E System H

7. **Discuss:** How did you group the equations?

Responses vary. **Group 1 has systems where at least one of the equations is a vertical or horizontal line. Group 2 includes systems where both equations are written in $y =$ form. Group 3 includes systems where one of the equations has both variables on the same side of the equation.**

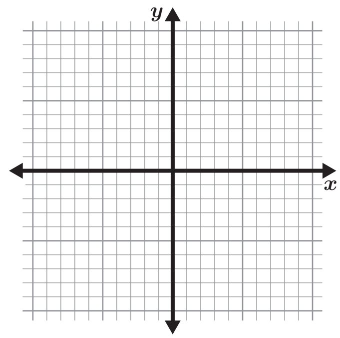
Name: ________ Date: ________ Period: ________

The Choice Is Yours (continued)

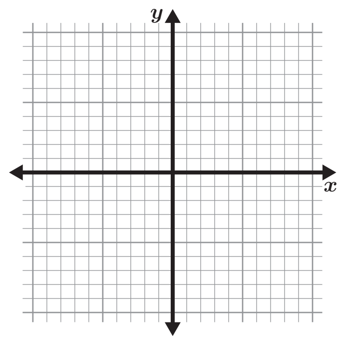
8. Choose *three* systems to solve. (Choose at least one from each group.)

Show your thinking and use the graphs if they help you to solve.

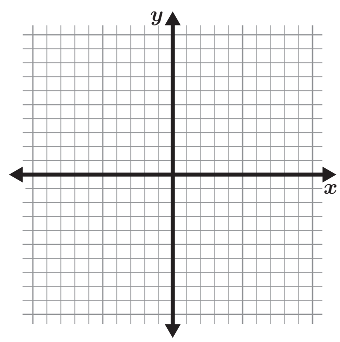
System 1:



System 2:



System 3:



Work varies.

- System A: (9, 29)
- System B: No solution
- System C: (-4, 6)
- System D: (4, -2)
- System E: Infinitely many solutions
- System F: (-5, 10)
- System G: (50, 3)
- System H: (-5, 10)

Synthesis

9. What are some strategies for solving systems like these?

Responses vary.

- Look at the structure to decide if the system has one solution, no solution, or infinitely many solutions.
- Graph the system and look for the point of intersection.
- Substitute part of one equation into the other in order to make an equation that has only one variable. Solve for that variable, then substitute that value back into one of the original equations to determine the value of the other variable. Those values will make both equations true.

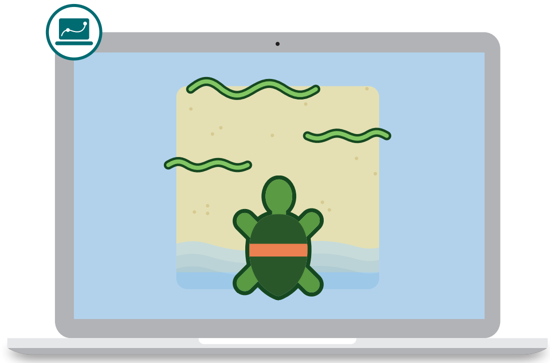
A $\begin{cases} y = 3x + 2 \\ 2x + y = 47 \end{cases}$

C $\begin{cases} y = \frac{1}{4}x + 7 \\ x = \text{-}4 \end{cases}$

F $\begin{cases} y = \text{-}3x - 5 \\ y = 4x + 30 \end{cases}$

Things to Remember:

Name: ______________________ Date: __________ Period: __________

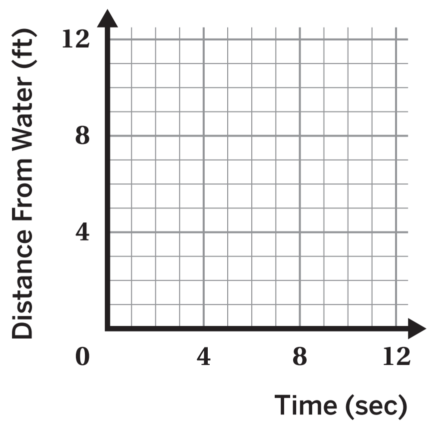


Turtle Crossing

Let’s make sense of graphs.

Warm-Up

1 a) Draw a distance vs. time graph to represent a turtle’s journey across the sand.



b) What story does your graph tell about the turtle’s journey?

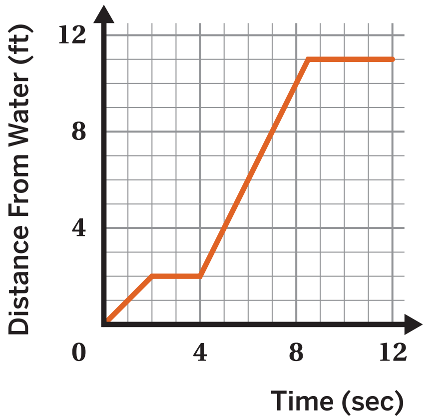
Activity 1

Name: ______ Date: ______ Period: ______

Turtle Graphs

2 Kris drew this graph to represent a turtle's journey.

What story does the graph tell about the turtle's journey?



3 Let's watch an animation to see what Kris's turtle did.

Discuss: How does this animation compare to your story?

4 Use Kris's graph to answer each question.

a. At 8 seconds, how far is the turtle from the water?

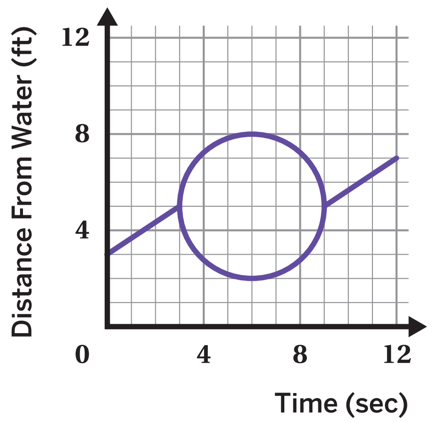
b. When is the turtle 4 feet from the water?

Name: .... Date: .... Period: ....

Turtle Graphs (continued)

5 Arnav drew this graph to represent a new turtle.

What story does the graph tell about the turtle's journey?



6 Let's watch an animation to see what Arnav's turtle did.

Discuss: How does this animation compare to your story?

Activity
2

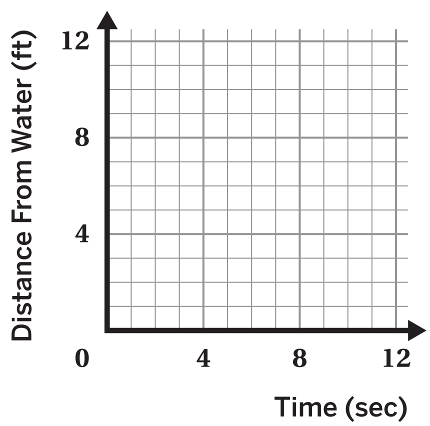
Name: .... Date: .... Period: ....

Dangerous Crossings

7 Let's watch the animation of another turtle crossing.

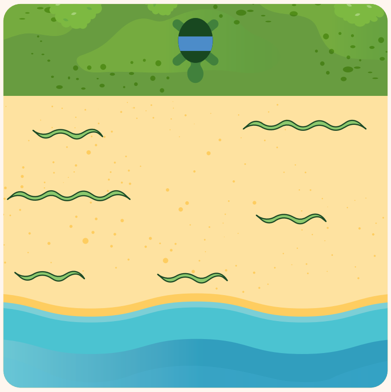
Discuss: What do you notice? What do you wonder?

8 Draw a graph of the turtle's distance from the water over time based on the animation from the previous problem.



Explore More

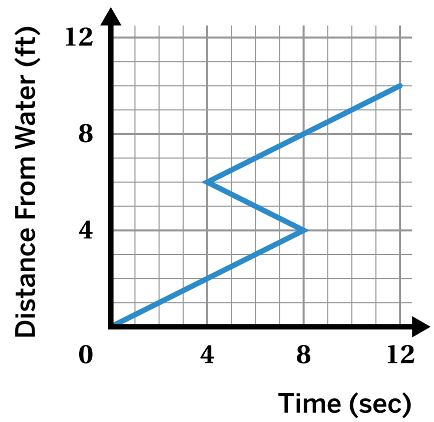
9 Use the digital activity to draw a new graph to help the turtle go from the grass to the water while avoiding the snakes.



10 Synthesis

Citlalli drew this graph to represent a new turtle.

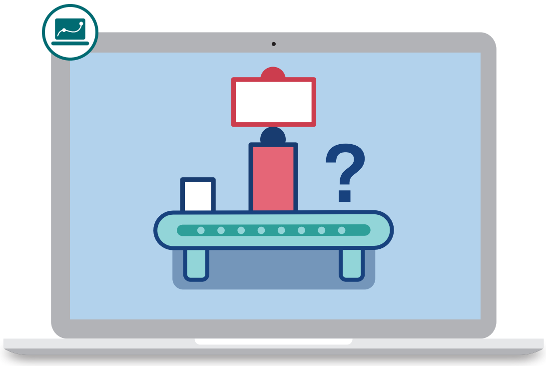
Explain what story the graph is telling at 6 seconds. Does the turtle's journey seem realistic?



Things to Remember:

Unit 5
Lesson
2

Name: ________ Date: ________ Period: ________



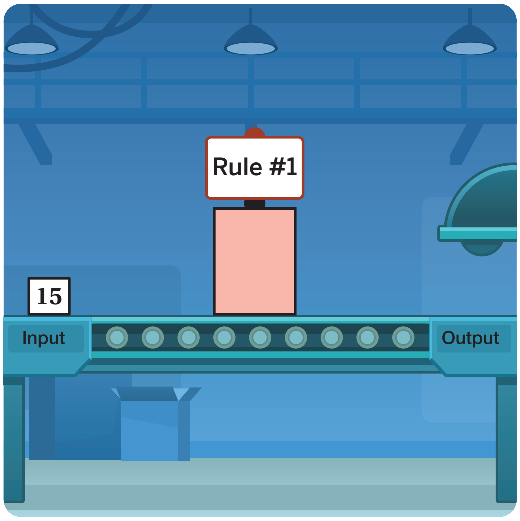
Guess My Rule

Let's explore rules to develop the concept of a function.

Warm-Up

1 This machine uses a secret rule, Rule #1, to turn inputs into outputs.

Rule #1 allows *all integers* as inputs.



a Let's watch this machine at work.

b What could Rule #1 be? Select *all* that apply.

- ☐ **A.** Divide by 2, then add 5.
- ☐ **B.** Divide by 3.
- ☐ **C.** Take the ones digit.
- ☐ **D.** Multiply by 3.
- ☐ **E.** Subtract 10.

2 Let's enter a different integer to help you decide what Rule #1 is.

What could Rule #1 be now? Select *all* that apply.

- ☐ **A.** Divide by 2, then add 5.
- ☐ **B.** Divide by 3.
- ☐ **C.** Take the ones digit.
- ☐ **D.** Multiply by 3.
- ☐ **E.** Subtract 10.

Activity 1

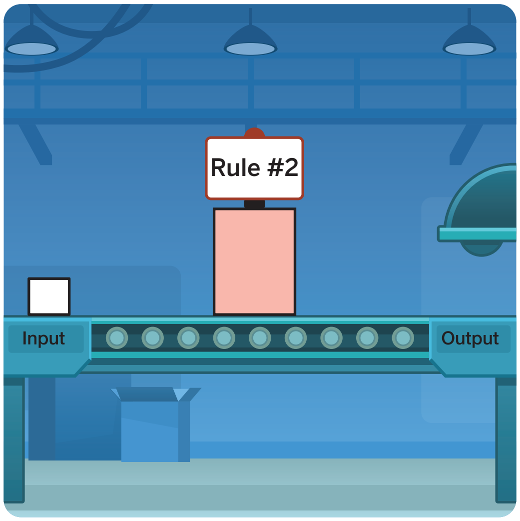
Name: ______ Date: ______ Period: ______

Guess My Rule

3 This machine uses a new rule called Rule #2.

Rule #2 allows *all numbers* as inputs.

Let's test several inputs to see how Rule #2 works.



4 Here are some inputs and outputs from other students.

If you input 6 into Rule #2, what do you think the output will be?

Explain your thinking.

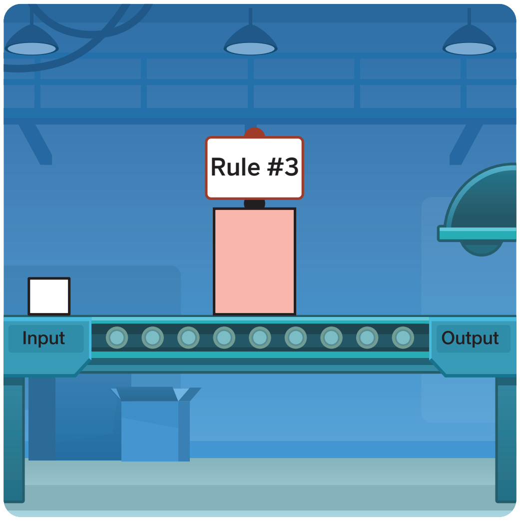
Rule #2

Input	Output
-13	7
-61	7
81	7
-100	7
60	7

5 Rule #3 allows *single words* as inputs.

a Let's test several inputs to see how Rule #3 works.

b **Discuss:** What do you think the rule might be?



Name: .... Date: .... Period: ....

Guess My Rule (continued)

6 Here are some inputs and outputs from other students.

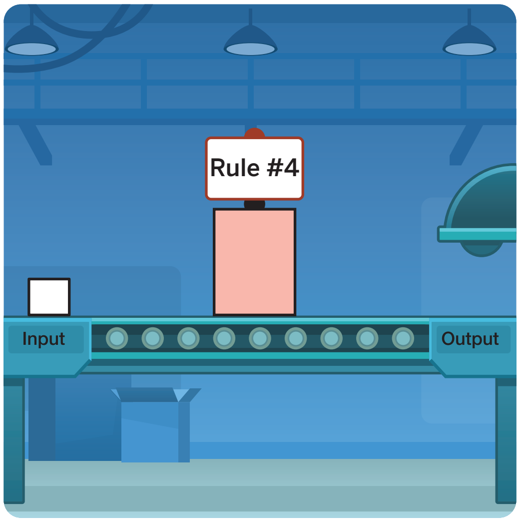
Test your understanding of Rule #3 by completing the table.

Rule #3

Input	Output
mint	u
hen	o
clear	s
friend	
wallet	
friend	
party	

7 Rule #4 allows *single letters* (like "A") as inputs.

Let's test several inputs to see how Rule #4 works.



8 Here are some inputs and outputs from other students.

What do you think the output will be if you input "A" into Rule #4? Explain your thinking.

Then compare your response with your classmates' responses.

Input	Output
W	William
A	Anand
A	Adam

Activity 2

Name: .... Date: .... Period: ....

What Is a Function?

9 Rules #1, #2, and #3 all represent **functions**.

Rule #4 does *not* represent a function.

a What do you think makes a rule a function?

b Compare your response with a classmate's. Then revise your response to make it stronger and clearer.

Rule #1: Function

Input	Output
35	25
723	713
-4	-14
53	43
723	713

Rule #2: Function

Input	Output
15	7
18	7
262	7
-3	7
82.3	7

Rule #3: Function

Input	Output
hi	J
my	Z
name	F
is	T
Arturo	P

Rule #4: Not a Function

Input	Output
H	Hailey
J	Jada
M	Mai
H	Hamza
M	Madison

10 Determine whether each table represents a function.

a

Button Selected (Input)	Drink Received (Output)
A	Water
B	Seltzer
C	Juice
D	Water

b

Money Spent (Input)	Number of Items (Output)
$1	2
$8	12
$7	1
$1	3

c

Height in Feet (Input)	Height in Inches (Output)
5	60
4.5	54
6	72
5.4	64.8

d

Time in Seconds (Input)	Musical Note (Output)
1.5	D
1.25	D
1.5	E
2	D

11 This table does *not* represent a function.

a Change the fewest numbers so the table could represent a function.

b Explain your thinking.

Input	Output
1	5
2	10
3	15
2	20
1	24

12 Synthesis

How can you determine whether a table could represent a function?

Use the examples if they help with your thinking.

Rule #1: Function

Input	Output
35	25
723	713
-4	-14
53	43
723	713

Rule #2: Function

Input	Output
15	7
18	7
262	7
-3	7
82.3	7

Rule #3: Function

Input	Output
hi	J
my	Z
name	F
is	T
Arturo	P

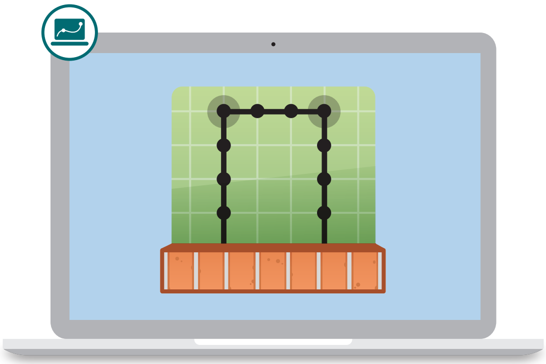
Rule #4: Not a Function

Input	Output
H	Hailey
J	Jada
M	Mai
H	Hamza
M	Madison

Things to Remember:

Unit 5
Lesson
3

Name: ______ Date: ______ Period: ______



Function or Not?

Let's determine whether a graph represents a function.

Warm-Up

1 We learned that a function is a rule that assigns exactly one output to each possible input.

That means each input determines a single output.

Complete the table so that y is *not* a function of x.

x	y
3	
5	4
	6

Activity
1

Name: .... Date: .... Period: ....

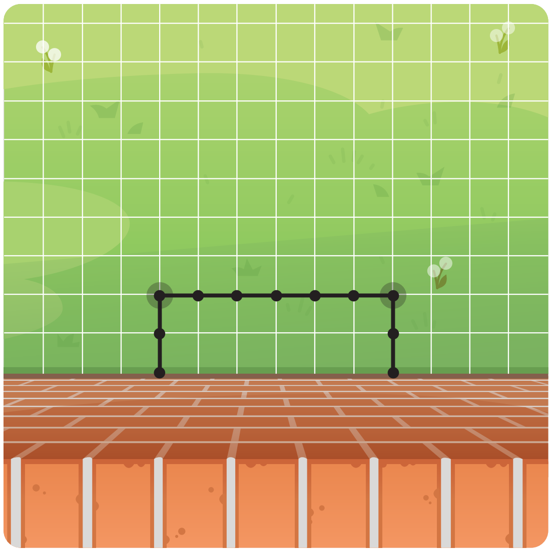
Rectangular Pen

2 Emma is hiring builders to create a rectangular pen, a closed space where animals can be kept. Emma wants to build three sides of fencing against a brick wall.

Let's observe the relationship between the area and the amount of fencing.

Enter an amount of fencing and the area for a pen that you've observed.

Amount of Fencing (m)	Area (sq. m)



3 Here is a table that shows the amount of fencing and the area for several pens.

Emma gave the builders 12 meters of fencing to build the pen.

Is it possible for the builders to determine the area of Emma's pen?

Yes No I'm not sure

Explain your thinking.

Amount of Fencing (m)	Area (sq. m)
8	8
12	10
14	24
12	16
16	24
20	32

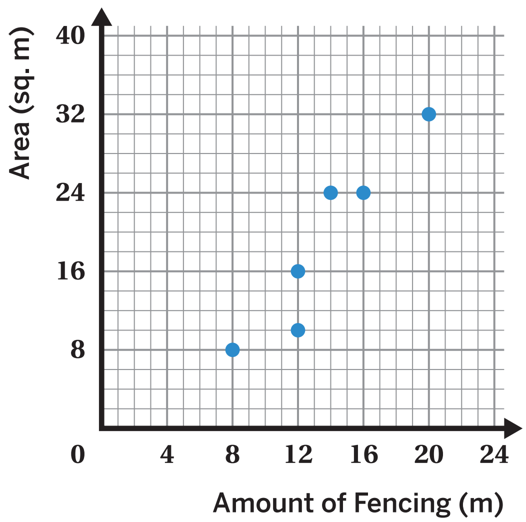
Name: .... Date: .... Period: ....

Rectangular Pen (continued)

4 Here is a graph that shows the relationship between area and amount of fencing for several pens.

How can you use the graph to quickly determine that area is not a function of the amount of fencing?

Area is not a function of the amount of fencing because . . .



Activity 2

Name: .... Date: .... Period: ....

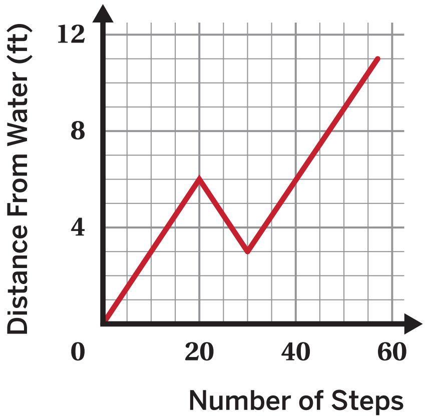
Turtle Crossing

5 Let's watch an animation of a turtle's journey across the sand.

Write the turtle's number of steps and distance from the water for one point in time.

Number of Steps	Distance from Water (ft)

6 How can you use this graph to decide whether distance from the water is a function of the number of steps?

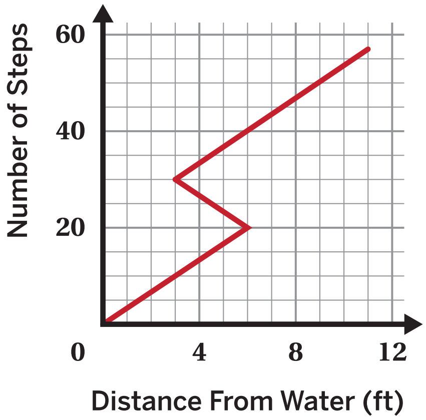


7 Here is the relationship in reverse.

Is the number of steps a function of distance? Circle one.

Yes No

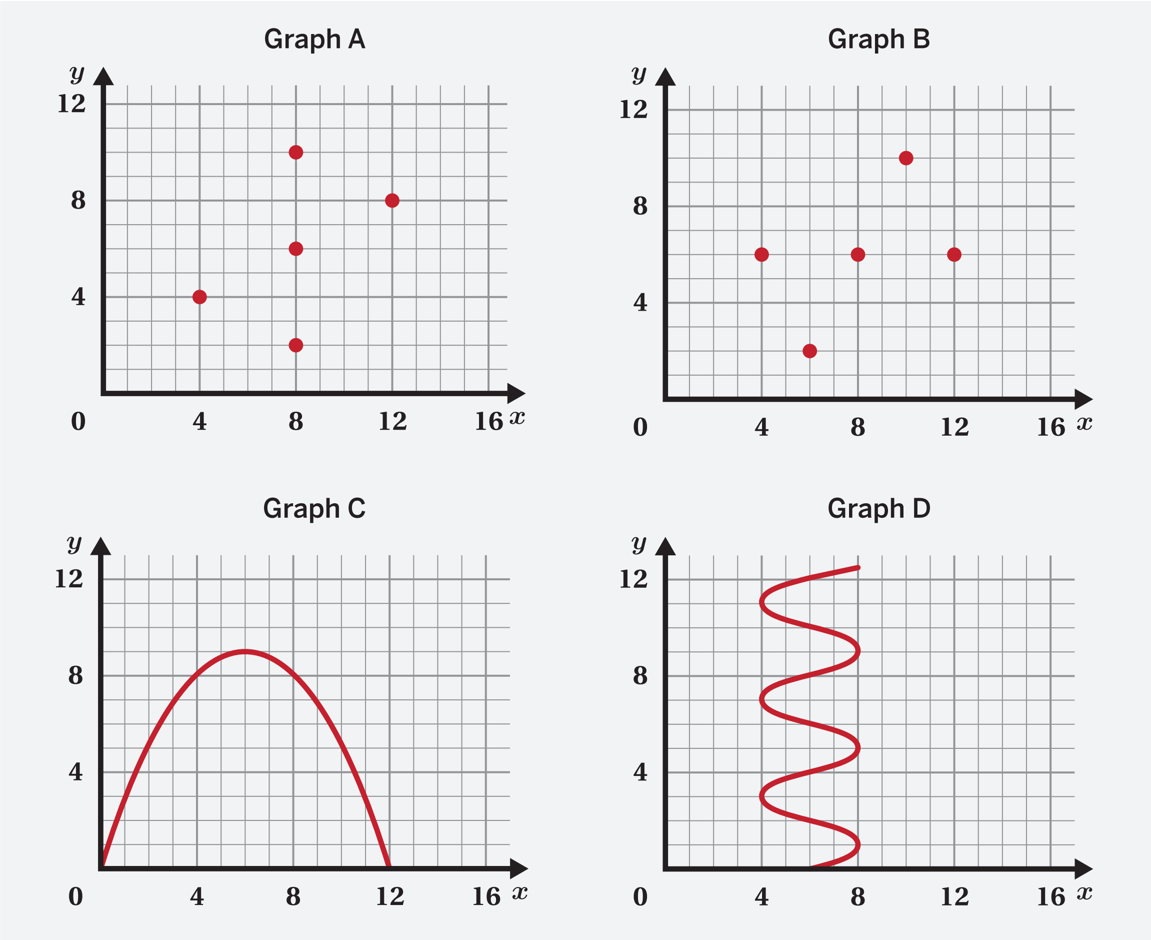
Explain your thinking.



Name: .... Date: .... Period: ....

Turtle Crossing (continued)

8 Determine whether each graph represents a function or not.



Function	Not a function

Explore More

9 **a** Use the digital activity to produce a graph that is surprising in some way.

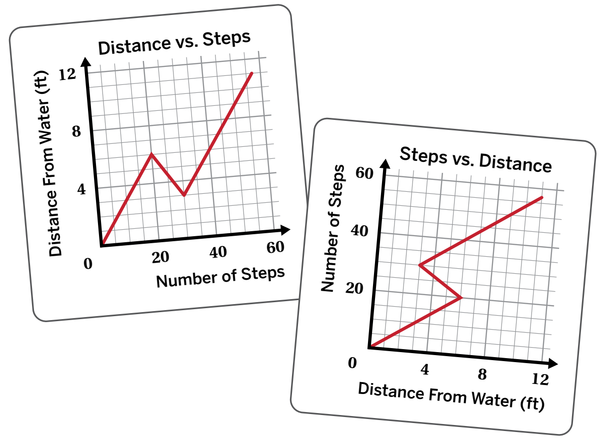
b **Discuss:**

- Does this graph represent a function?
- What does the surprising part of this graph represent in context?

10 Synthesis

How can you determine whether a graph represents a function?

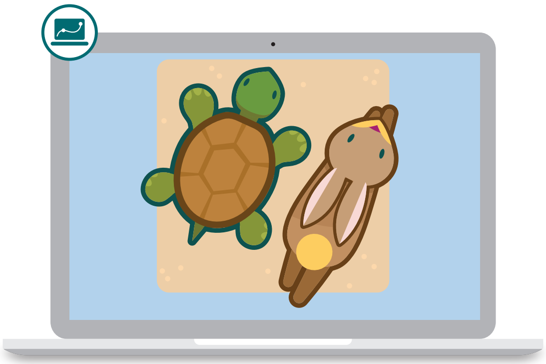
Use the examples if it helps with your thinking.



Things to Remember:

Unit 5
Lesson
5

Name: ______ Date: ______ Period: ______

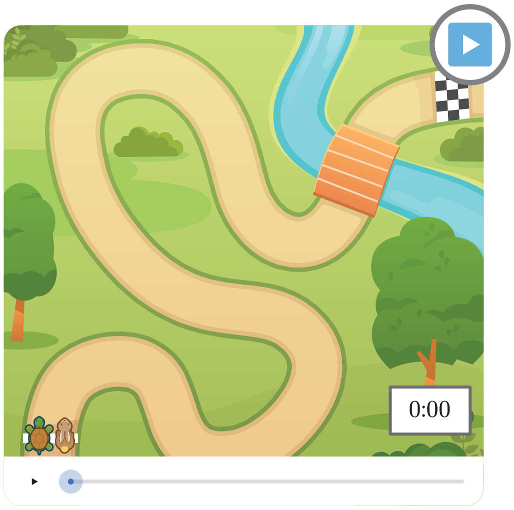


The Tortoise and the Hare

Let's interpret the graph of a function in context.

Warm-Up

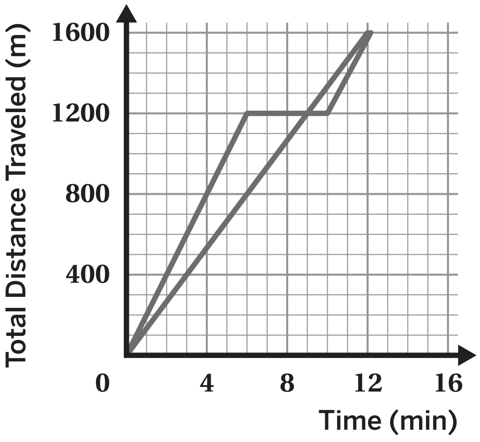
1 Let's watch an animation of the tortoise and the hare. Then tell a story based on what you see.



2 This graph shows two functions representing the relationship between distance and time: one for the tortoise and one for the hare.

a Which animal does each function represent? Label the graph with "Tortoise" and "Hare."

b Explain your thinking.



Activity

1

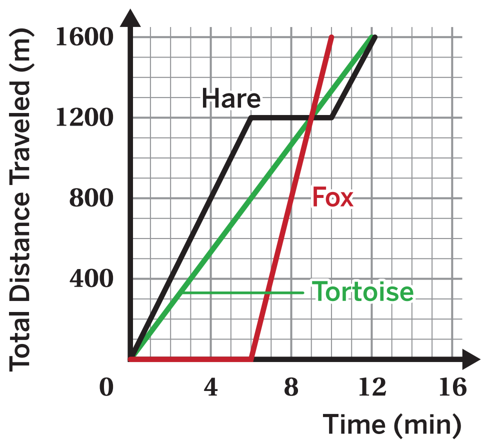
Name: .... Date: .... Period: ....

The Tortoise, the Hare, and the Fox

3 This graph shows the relationship between distance and time for a third animal, the fox.

Tell a story about the fox's journey during the race.

Include specific details about time and distance.



4 Let's watch an animation of the race.

Discuss: How does your story compare to the actual race?

5 Here are five statements about the race. Select *all* the true statements.

- ☐ **A.** The fox's distance was always increasing.
- ☐ **B.** Between 6 and 10 minutes, the fox was traveling faster than the other animals.
- ☐ **C.** When the hare reached 800 meters, the fox was still at the starting line.
- ☐ **D.** The graph of the tortoise represents a function, but the other two graphs do not represent functions.
- ☐ **E.** All three graphs represent functions.

Activity 2

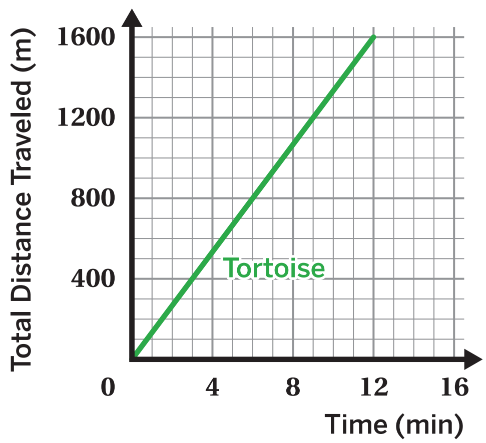
Name: ______ Date: ______ Period: ______

The Tortoise and the Dog

6 Next, the tortoise raced a dog.

Draw a graph representing the relationship between distance and time for a dog that makes *all* of these statements true:

- The dog got a head start but lost the race.
- The dog and tortoise were tied at 800 meters.
- The dog's distance was decreasing for 3 minutes.



7 Let's watch an animation of the race described in Problem 6 and a graph that was made to represent it.

Revise your graph for Problem 6, as needed.

8 The blue graph shows what Ash drew to represent the dog's race.

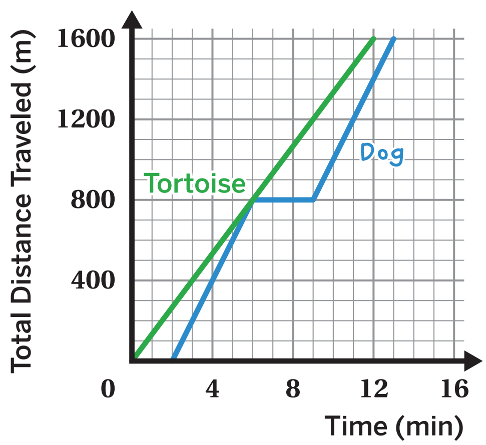
At least one of the following statements is false. Circle a false statement.

A. The dog got a head start but lost the race.

B. The dog and tortoise were tied at 800 meters.

C. The dog's distance was decreasing for 3 minutes.

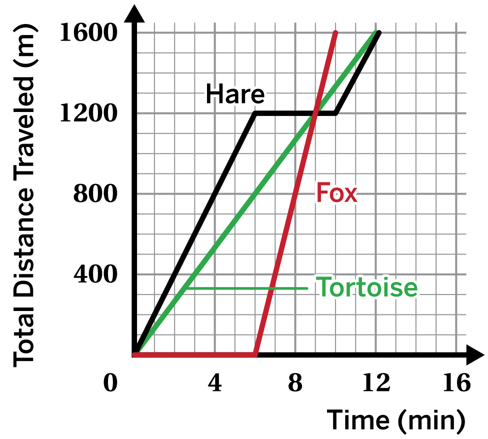
Explain your thinking.



9 Synthesis

What can a function's graph tell us about a situation?

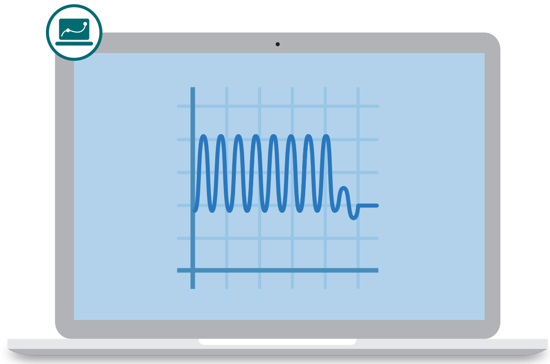
Use the example if it helps with your thinking.



Things to Remember:

Unit 5
Lesson
6

Name: .... Date: .... Period: ....



Graphing Stories

Let's make connections between scenarios and the graphs that represent them.

Warm-Up

1 Clem loves to play on the playground. Let's watch a short video of Clem on the swings. What different quantities are changing in this video?

2 Let's look at two possible graphs to represent this situation.

Discuss: How are these graphs similar? How are they different?

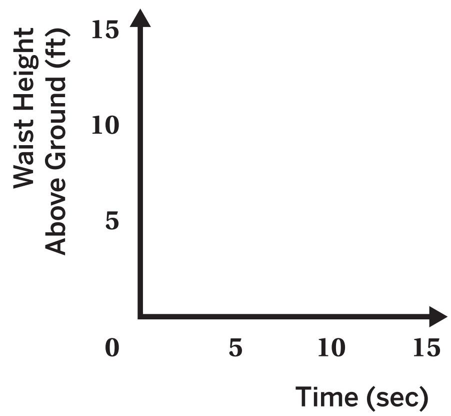
Activity 1

Name: Date: Period:

Tyler on the Slide

3 Let's watch an animation of Tyler on the slide.

Sketch a graph representing the relationship between Tyler's waist height and time.

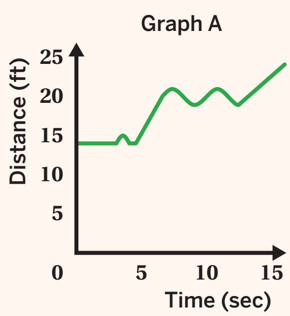


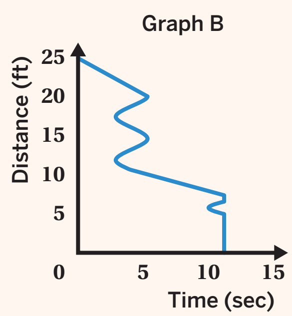
4 Let's watch an animation of a graph representing the relationship between Tyler's waist height above the ground and time.

Discuss: How does this graph represent the situation? How might you revise this graph?

Explore More

5 Use the digital activity to watch the video of Tyler again. Which graph could represent the relationship between Tyler's distance from the right edge of the screen and time?





Explain your thinking.

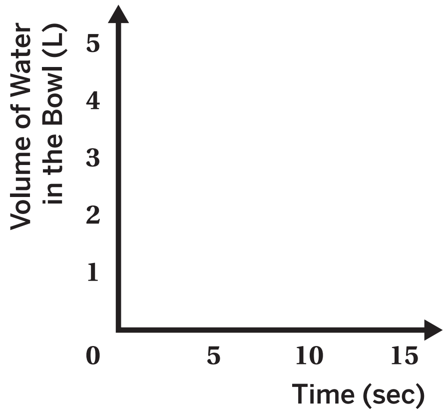
Activity
2

Name: .... Date: .... Period: ....

Water in a Bowl

6 Let's watch an animation of water in a bowl.

Sketch a graph representing the relationship between the volume of water in the 5-liter bowl and time.



7 Let's watch an animation of Cielo's graph representing the relationship between the volume of water in the bowl and time.

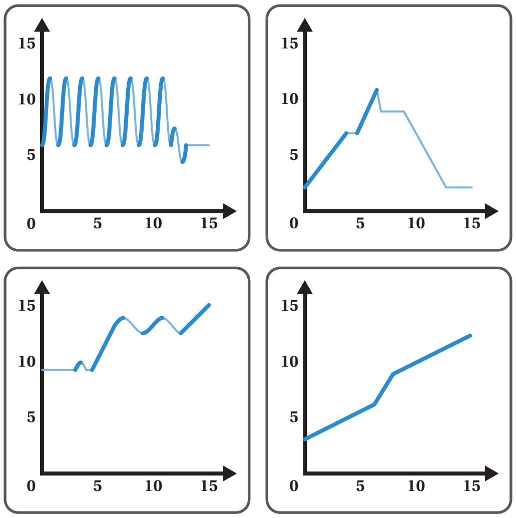
Discuss: How does Cielo's graph represent the situation? How might you revise this graph?

Name: .... Date: .... Period: ....

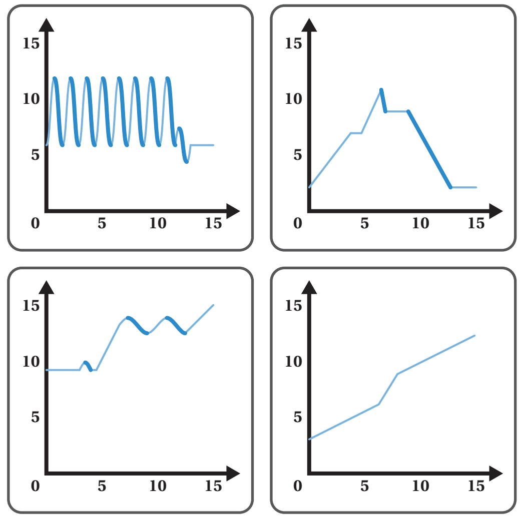
Describing Graphs

8 Here are some graphs from this lesson. Parts of the graphs are bolded to show where they are either increasing, decreasing, *linear*, or **non-linear**.

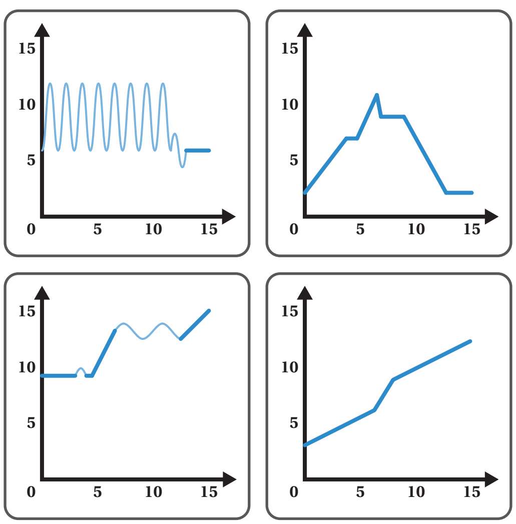
Increasing



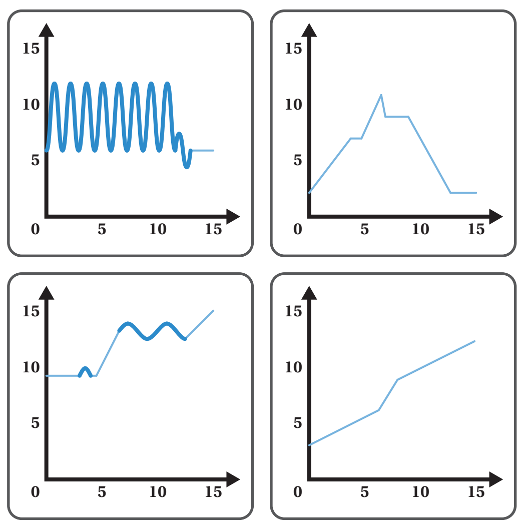
Decreasing



Linear



Non-Linear

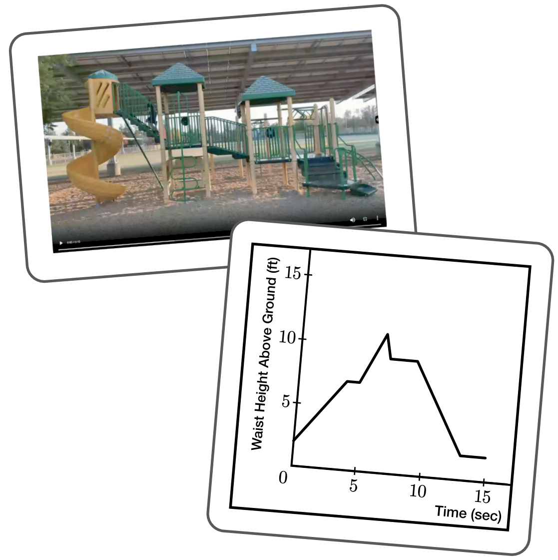


Discuss: What do you think each term means?

9 Synthesis

What can be helpful to consider when graphing a function that represents a real-world situation?

Use the example if it helps with your thinking.



Things to Remember:

Unit 5
Lesson
7

Name: .... Date: .... Period: ....

Comparing Linear Functions

Let's compare linear functions represented in different ways.

Warm-Up

Jada has $50 in a savings account and saves $7 per week.

1. What could the independent variable and dependent variable be in this situation? Explain your thinking.

2. Write an equation representing this situation. Be sure to define the variables that you use.

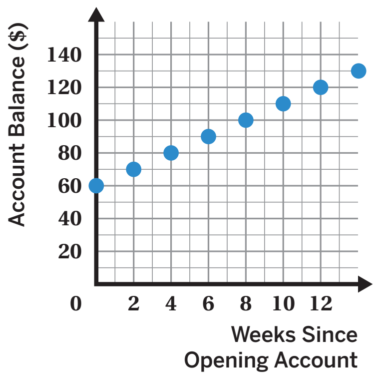
3. Is your relationship from Problem 2 a function? Explain your thinking.

Name: ____________ Date: ________ Period: ________

Which Is Growing Faster?

Noah and Azul both opened a savings account on the same day. Each person's account balance is a function of the weeks since the account was opened. Here is some information about each account.

Noah's Account



Azul's Account

$a = 8w + 60$, where w is the number of weeks since the account was opened, and a is the account balance

4. Who started with more money in their account? Circle one.

Noah Azul They started with the same amount.

Explain your thinking.

5. Who is saving money at a faster rate? Circle one.

Noah Azul They are saving at the same rate.

Explain your thinking.

6. How much will Noah save over the course of a year if he does not make any withdrawals?

(Note: There are 52 weeks in a year.) Show or explain your thinking.

7. How long will it take Azul to save the same amount? Show or explain your thinking.

Activity 2

Name: ______ Date: ______ Period: ______

Making Deposits vs. Withdrawals

Take a look at the accounts of four customers. They each have an account balance, a, measured over w weeks.

Account A

The account balance, a, is represented by the function $a = 65 + 10w$, where w represents the number of weeks since the account was opened.

Account B

The account balance starts at $\$40$ and decreases by $\$8.50$ per week.

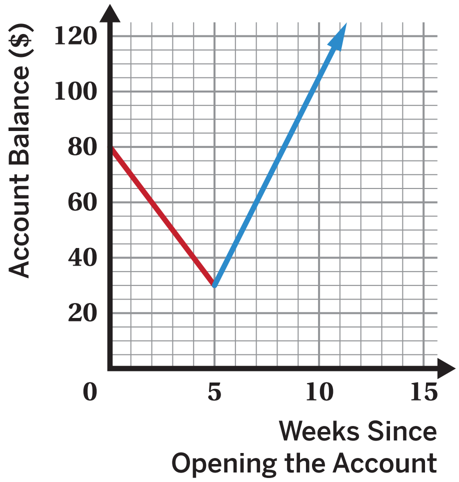
Account C

The account balance is represented by this table:

Number of Weeks	Account Balance ($)
1	71
3	48
7	2

Account D

The account balance is represented by this graph:



8. **Discuss:**

a Which account(s) show customers making deposits? Explain your thinking.

b Which account(s) show customers making withdrawals? Explain your thinking.

Name: .... Date: .... Period: ....

Making Deposits vs. Withdrawals (continued)

9. Kiana says that all four relationships are **linear functions** and that each situation can be represented with one equation in the form $y = mx + b$. Is Kiana's claim correct? Explain your thinking.

10. Which account will have the most money at the end of a year? Explain your thinking.

11. **a** How much money did Account A have after one year?

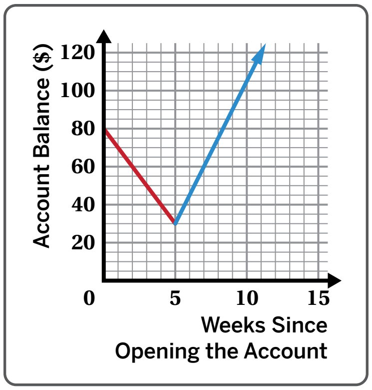
b How long did it take Account D to have the same amount?

Synthesis

12. How can you compare linear functions shown using different representations?

Number of Weeks	Account Balance ($)
1	71
3	48
7	2

$a = 65 + 10w$



Things to Remember:

Unit 5
Lesson
9

Name: ________ Date: ________ Period: ________

Piecing It Together

Let's create functions to model data sets.

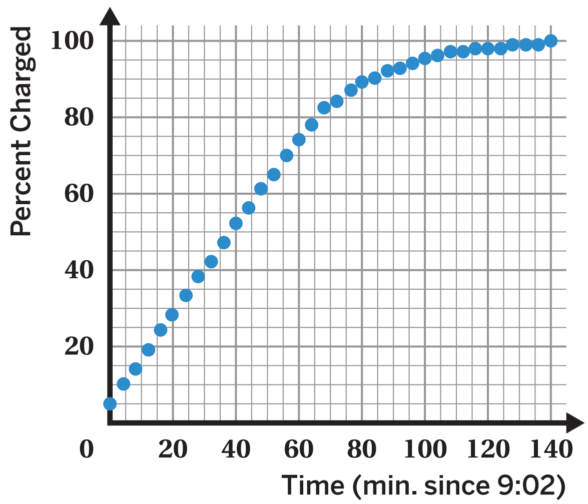
Warm-Up

1 Here is a data set for a phone charging over time.

A single linear function does not model this relationship very well.

a Sketch two connected line segments to model the relationship better.

b When was the phone charging the slowest? Explain your thinking.



Activity 1

Name: .... Date: .... Period: ....

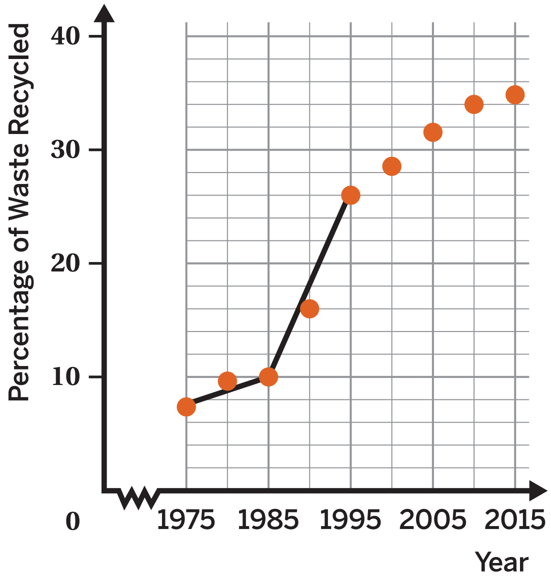
Recycling

2 This data set shows the percentage of waste produced in the United States that gets recycled over time.

A student started sketching a function to model this data.

a Sketch one more linear segment to complete the function.

b Approximate the slope of the segment you created.



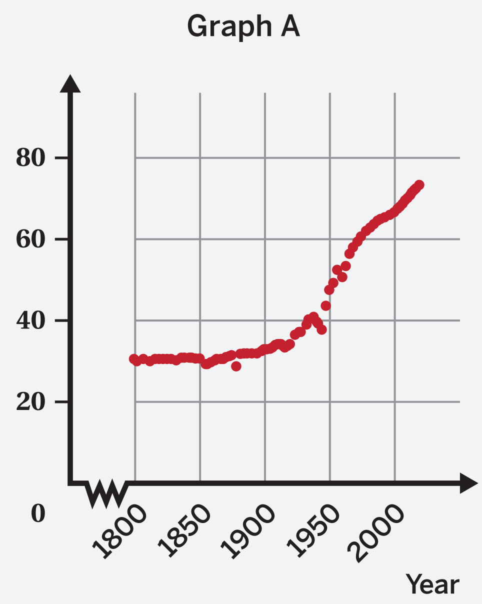
Explain what this number means about the percentage of waste recycled.

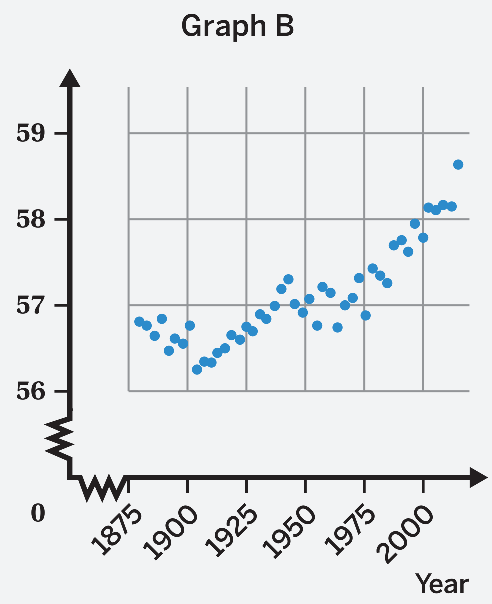
Activity 2

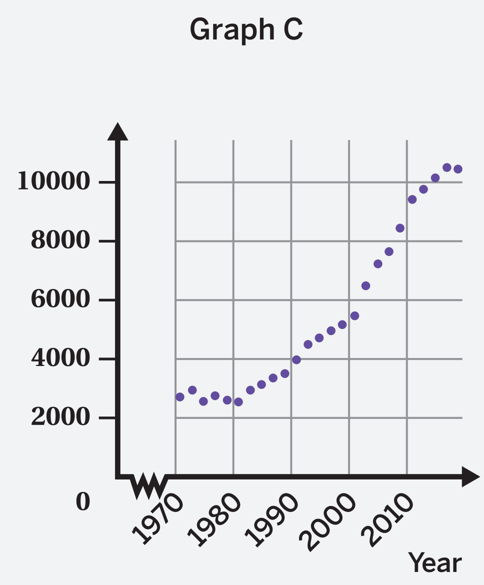
Name: ______ Date: ______ Period: ______

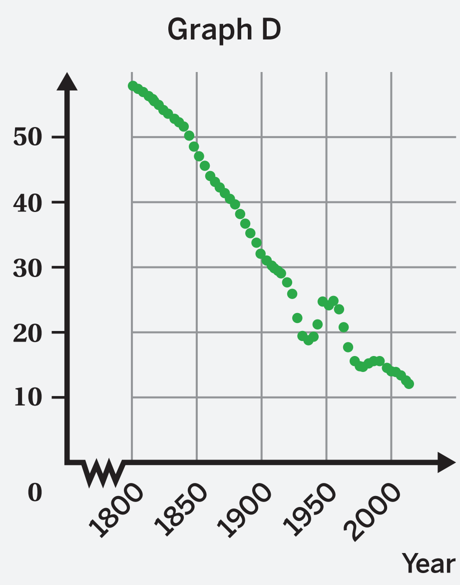
Four Data Sets

3 Here are four new data sets. Match each graph to the description you think it represents.









U.S. College Cost ($)	U.S. Births (per 1000 people)	Global Life Expectancy (age in years)	Global Temperature (°F)

Activity 3

Name: .... Date: .... Period: ....

Analyzing Data Sets

4 Look at the four data sets on the Activity 3 Sheet. Pick one that interests you.

Which data set are you choosing?

What do you notice? What do you wonder?

I notice:

I wonder:

5 **a** Using at least two line segments, sketch a function on the graph of the data set you chose on the Activity 3 Sheet to model the data.

b Describe your function using vocabulary from this unit.

linear	increasing
non-linear	decreasing

6 During which interval of time did the data seem to change the most?

Activity 3

Name: ______ Date: ______ Period: ______

Analyzing Data Sets (continued)

7 a Use your function to make a prediction for the year 2030.

b Do you think your function can be used to make an accurate prediction for the year 2050? Explain your thinking.

8 Share your findings with a student or a group that examined a different data set.

Discuss:

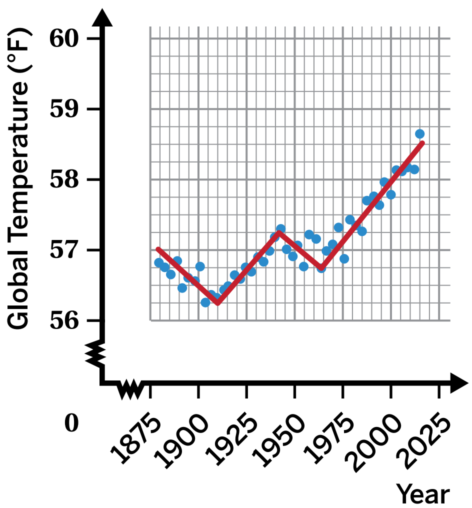
- What choices did you make as you sketched your function?
- What is your prediction for 2030, and how did you arrive at your prediction?
- Do you think your function can be used to make an accurate prediction for 2050?

9 Synthesis

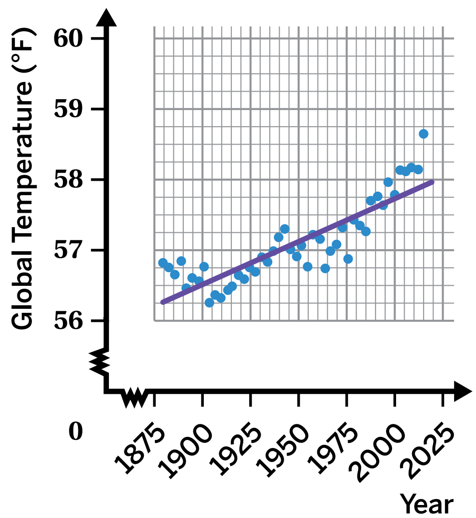
When is it helpful to model a data set with multiple linear segments instead of a single linear segment?

Use the examples if they help with your thinking.

Function A: Multiple Segments



Function B: Single Segment



Things to Remember:

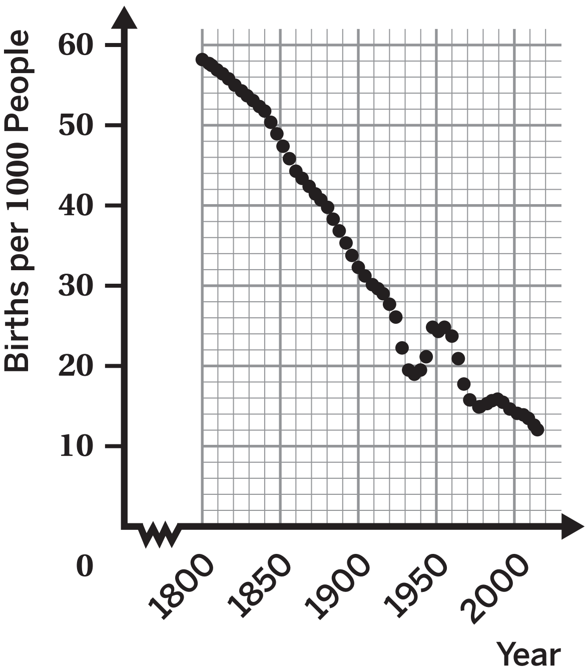
Name: .... Date: .... Period: ....

Lesson 5.09
Activity 3

Analyzing Data Sets

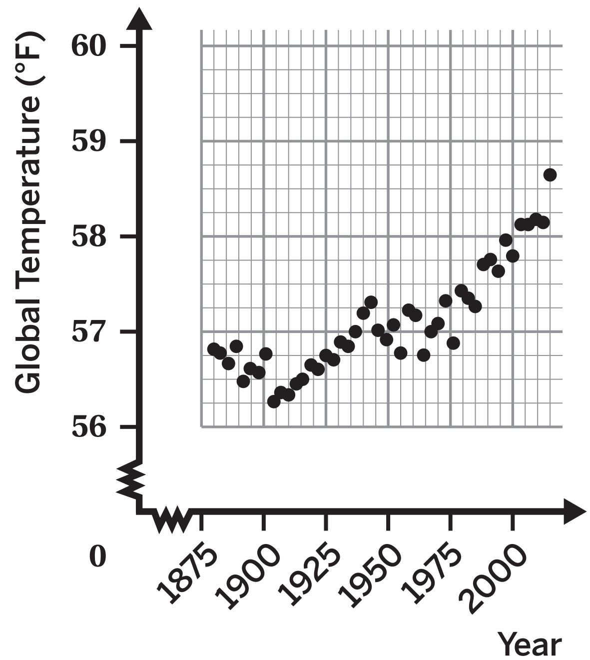
Choose one of the data sets to analyze and record your findings in your Student Edition.

Data Set A: U.S. Births



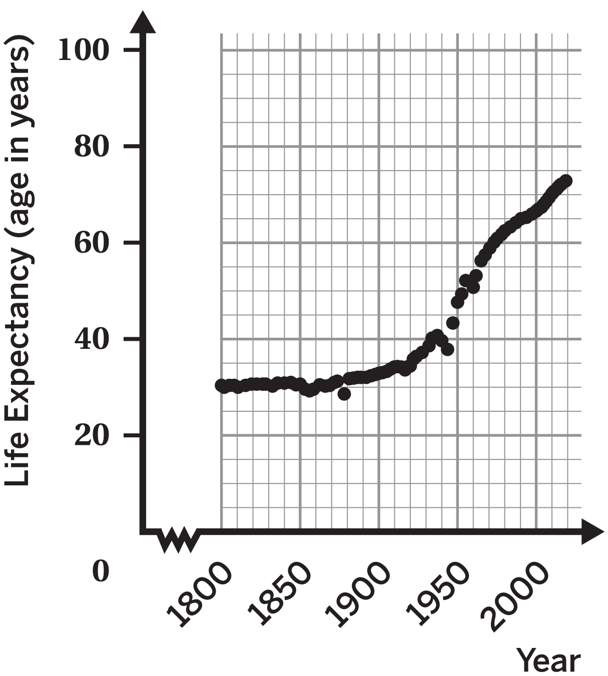
Source: *United Nations, World Population Prospects*

Data Set B: Global Temperature



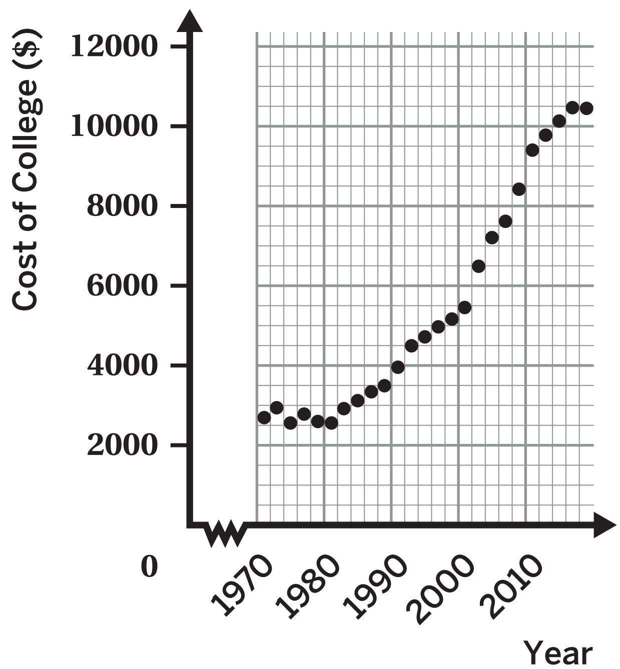
Source: *NOAA, Climate at a Glance*

Data Set C: Global Life Expectancy



Source: *Gapminder, Life Expectancy at Birth*

Data Set D: U.S. College Cost



Source: *CollegeBoard, Trends in Higher Education*

Unit 6
Lesson
1

Name: Date: Period:

Click Battle

Let's find ways to show patterns in data.

Warm-Up

1 Tap your pencil on your desk as many times as you can for 2 seconds. Record your number of taps.

Responses vary.

2 Tap your pencil on your desk as many times as you can for 6 seconds. Record your number of taps.

Responses vary.

Activity 1

Name: .... Date: .... Period: ....

Organizing Data

3 Let's look at some class data about button clicks, which are similar to pencil taps.

a Organize the data in a way that makes sense to you.

Responses vary.

b **Discuss:** What patterns do you see in the data?

Responses vary. I organized the data in a two-column table with time (in seconds) in the left column and number of clicks in the right column. I noticed that, for the most part, students who had more time made more clicks.

4 Let's look at one way to represent the data. What do you notice?

Responses vary.

- I notice that the times are organized from least to greatest.
- I notice that some people had the same amount of time but made different numbers of clicks.

5 Let's look at another way to represent the data.

Discuss: What connections do you see between the scatter plot and the table?

Responses vary.

- Each row in the table appears as a point in the scatter plot.
- The table headers (Time and Number of Clicks) appear as the axis labels.
- I see that values in the left column are between 2.5 and 8 and the points in the scatter plot are between these same numbers when looking along the x-axis.

Activity 2

Name: .... Date: .... Period: ....

Make a Prediction

Here is click data organized as a list, table, and scatter plot.

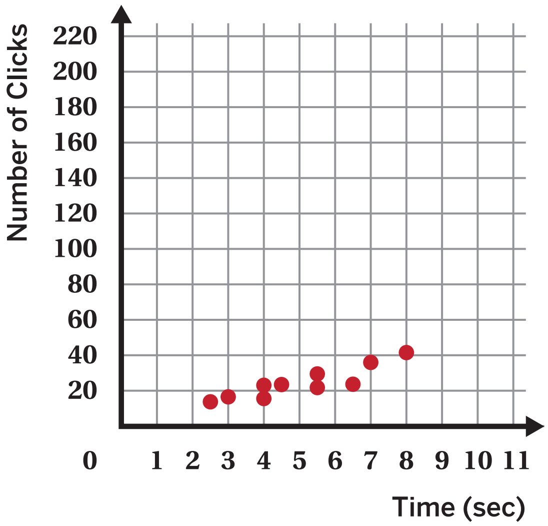
List

14 clicks in 2.5 seconds	24 clicks in 6.5 seconds
17 clicks in 3 seconds	30 clicks in 5.5 seconds
16 clicks in 4 seconds	24 clicks in 4.5 seconds
42 clicks in 8 seconds	36 clicks in 7 seconds
22 clicks in 5.5 seconds	23 clicks in 4 seconds

Table

Time (sec)	Number of Clicks
2.5	14
3	17
4	16
4	23
4.5	24
5.5	22
5.5	30
6.5	24
7	36
8	42

Scatter Plot



6 Select a representation and use it to answer this question:

How many clicks do you think a typical student in your class would make in 10 seconds?

***Responses vary.* 55 clicks**

Explain your thinking.

***Explanations vary.* I looked at the scatter plot. If I imagine a line that goes through roughly the middle of the points, there will be a point at approximately (10, 55).**

7 Test your prediction by counting the number of pencil taps you can make in 10 seconds.

Responses vary.

8 Synthesis

What are some advantages of using a list, a table, or a scatter plot to organize data?

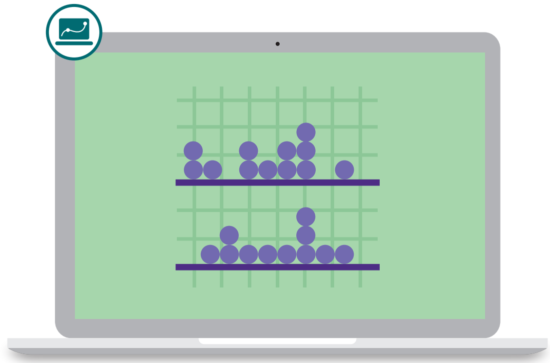
Responses vary.

- **List: This is the easiest representation to create.**
- **Table: This makes it easier to see some numerical patterns (e.g., as students get more time, they tend to make more clicks).**
- **Scatter plot: This makes it easier to see some visual patterns (e.g., the data falls roughly on a line) and to use those patterns to make a prediction (e.g., the number of clicks in 10 seconds).**

Things to Remember:

Unit 6
Lesson
2

Name: ________ Date: ________ Period: ________



Wingspan

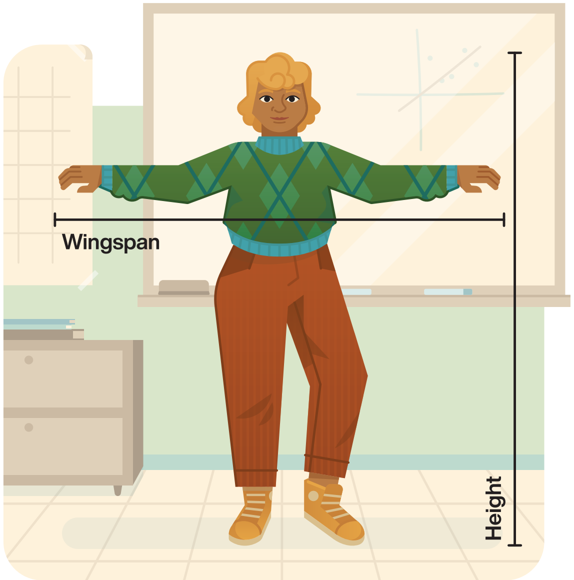
Let's compare dot plots and scatter plots.

Warm-Up

1 With a partner, measure your height and wingspan to the nearest inch.

Responses vary.

Height (in.)	Wingspan (in.)
68	70



2 Let's look at a table of height and wingspan data.

Discuss:
- How could we reorganize the data to make it more useful for analyzing?
- What are some questions this data could help answer?

Responses vary.
- **List the values for height in order from shortest to tallest.**
- **List the values for wingspan in order from shortest to longest.**
- **Do taller students always have longer wingspans?**
- **What is the difference in inches between the largest and smallest wingspans?**

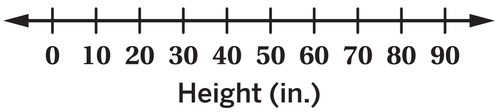
Activity
1

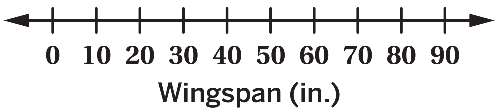
Name: .... Date: .... Period: ....

Visualizing Data

3 Plot points on the *dot plots* to represent your height and wingspan.

Responses vary based on students' heights and wingspans.





4 Let's look at dot plots of height and wingspan data.

a What is a question you *can* answer based on the dot plots?

Responses vary.

- **How many people are 62 inches tall?**
- **What is the range of heights in the class?**

b What is a question you *cannot* answer based on the dot plots?

Responses vary.

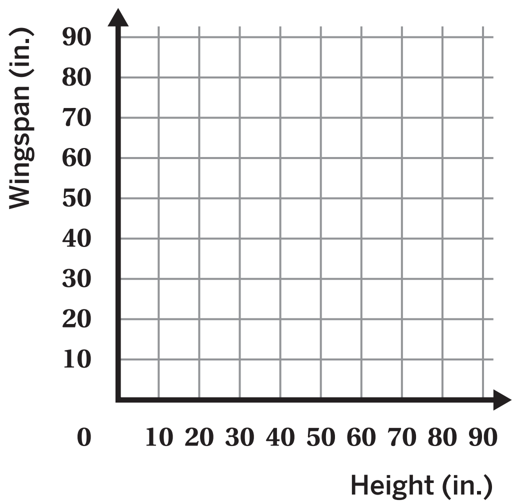
- **What is the wingspan of the person who is 60 inches tall?**
- **Is there a relationship between height and wingspan?**

Name: ______ Date: ______ Period: ______

Visualizing Data (continued)

5 Plot a point to represent your height and wingspan.

Responses vary based on students' heights and wingspans.



6 Let's look at a scatter plot that represents height and wingspan data.

a What is a question you *can* answer based on the scatter plot?

Responses vary.

- **Do people's heights and wingspans tend to be close?**
- **Is there a relationship between height and wingspan?**

b What is a question you *cannot* answer based on the scatter plot?

Responses vary.

- **Who is the tallest person?**
- **Are there other factors that influence a person's height?**

Explore More

7 Use the digital activity to examine the heights, weights, wingspans, and hand lengths of professional basketball players.

What do you notice? What do you wonder?

Responses vary.

- **I notice that the graph of height on the x-axis and weight on the y-axis looks different than the graph of weight on the x-axis and height on the y-axis.**
- **I wonder how the graphs would be different if I added non-professional players to this data set.**

8 Synthesis

The height and wingspan data from a different class are shown in the Summary below. What are some advantages of using a dot plot or a scatter plot to represent data? Use the examples if they help with your thinking.

Responses vary.

- **Dot plot: This shows how many people share a value for one quantity. A dot plot would be helpful if you wanted to know how many people are the same height.**
- **Scatter plot: Each point represents the height and the wingspan for one person, as opposed to showing the two variables separately.**

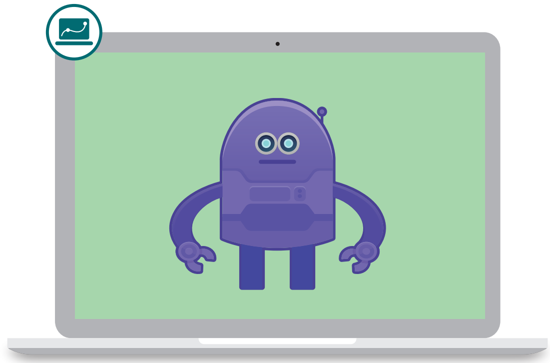
Things to Remember:

Unit 6

Lesson

3

Name: .... Date: .... Period: ....



Robots

Let's investigate points on a scatter plot.

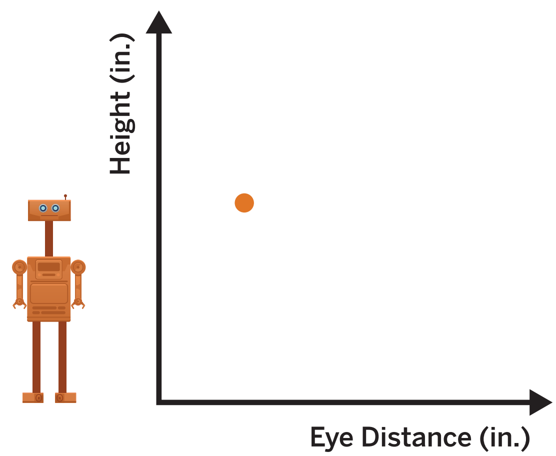
Warm-Up

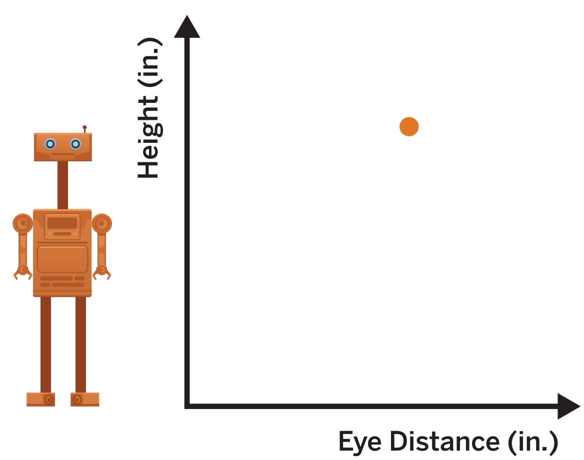
1 Here are two graphs and images of a robot.

Discuss: What do you notice?

Responses vary.

- I notice that the robot's height is related to how high up the point is.
- I notice that the distance between the robot's eyes is related to where the point is horizontally.





Activity 1

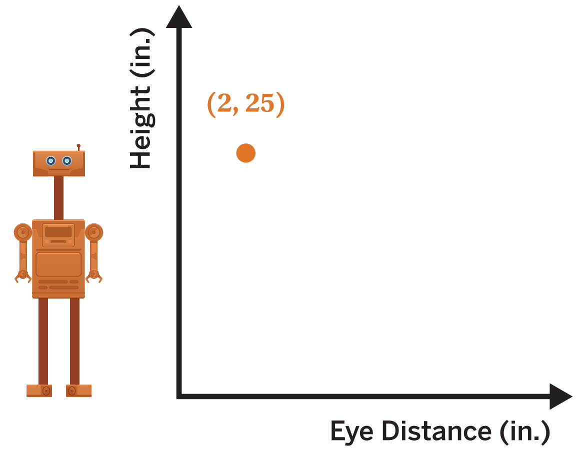
Name: Date: Period:

Robots

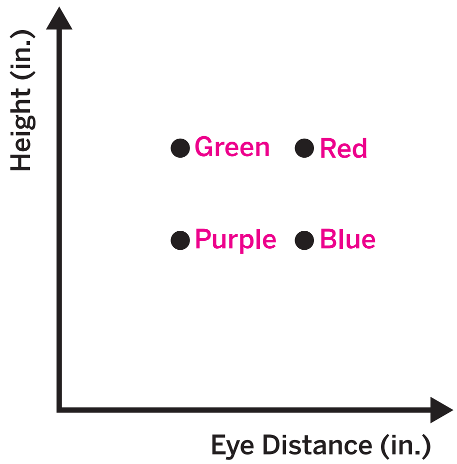
2 Describe something you know about the robot based on the graph.

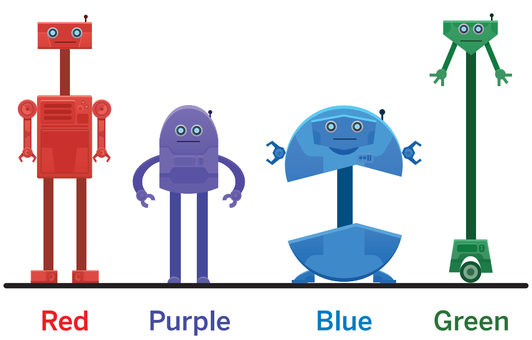
Responses vary.

- The robot's height is 25 inches.
- The robot's eyes are 2 inches apart.



3 Here are four different robots. Label each point on the graph with the color robot it represents.





Name: .... Date: .... Period: ....

Robots (continued)

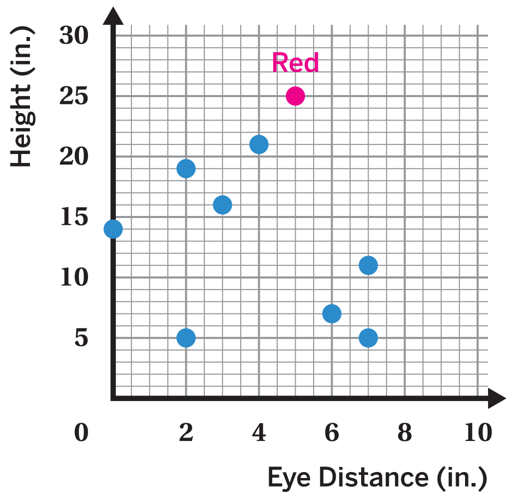
4 The table shows the heights and eye distances for five different robots. Plot a point to represent the pink robot.

Robot Color	Eye Distance (in.)	Height (in.)
Teal	2	30
Black	4	10
Gray	8	10
Orange	6	20
Pink	8	20

5 The graph shows the heights and eye distances for eight blue robots.

Plot a point for a red robot to make this statement true: *The red robot is taller than all the blue robots, and its eye distance is 5 inches.*

***Responses vary.* Sample shown on graph. The point should be on the line $x = 5$ with a y-coordinate greater than 21.**



Activity
2

Name: .... Date: .... Period: ....

Challenge Creator

6 You will use a set of cards with scatter plots to create your own challenge.

a **Make It!** Choose one card that interests you and plot a point somewhere you think is interesting.

b **Solve It!** On this page, write the point as an ordered pair. Then tell a story about this point.

Responses vary.

My Point	My Story

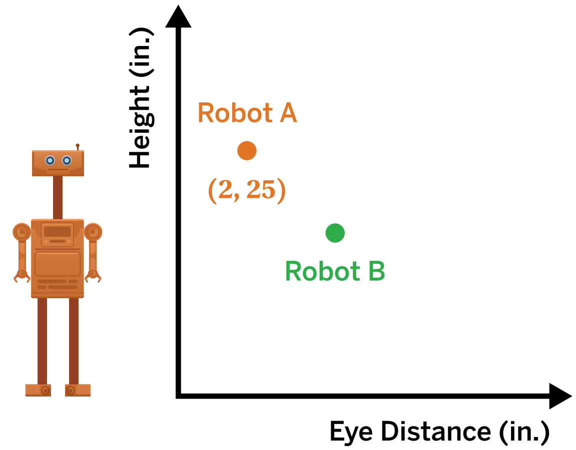
c **Swap It!** Swap your challenge with one or more partners. Write the point they plotted as an ordered pair and tell a story about it.

	Point	Story
Partner 1		
Partner 2		
Partner 3		

7 Synthesis

This graph shows the height and eye distance for two robots.

Describe some things you know about Robot B given the information about Robot A.



Responses vary.

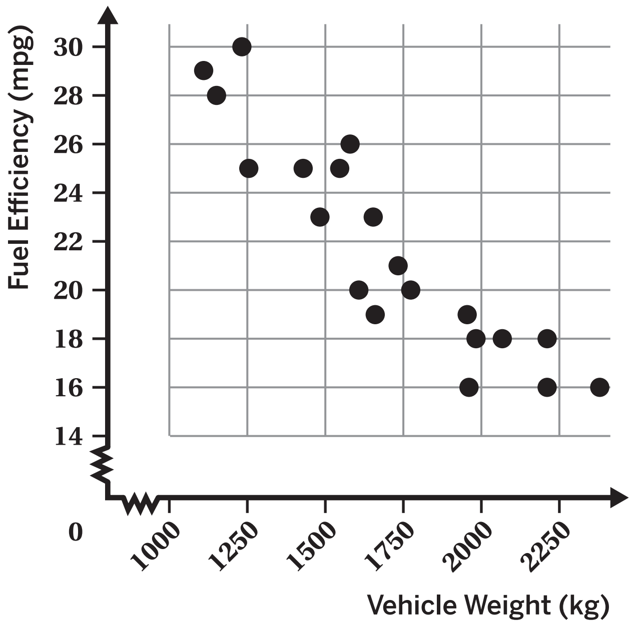
- Robot B's height is less than 25 inches.
- Robot B's eyes are more than 2 inches apart.
- Robot B is shorter than Robot A.
- Robot B's eyes are farther apart than Robot A's eyes.

Things to Remember:

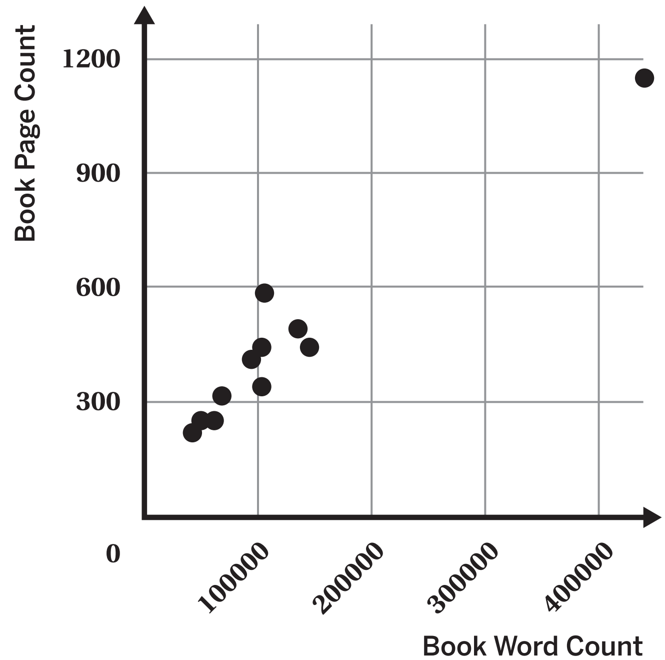
Challenge Creator

Directions: Make one copy for each group of students. Then pre-cut the cards and give each group one set.

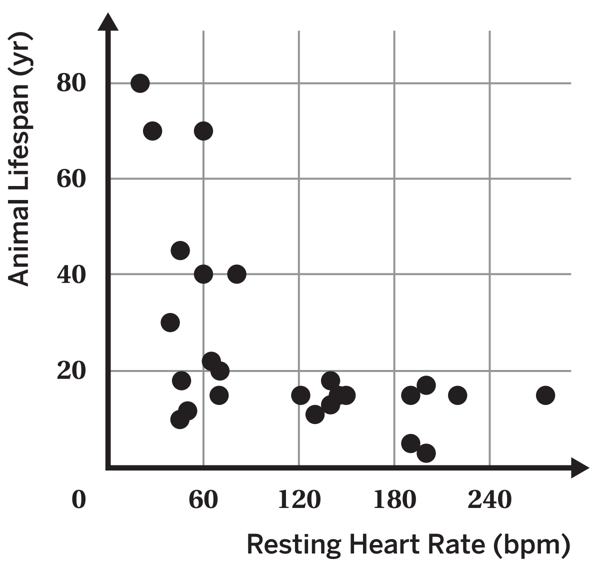
Fuel Efficiency



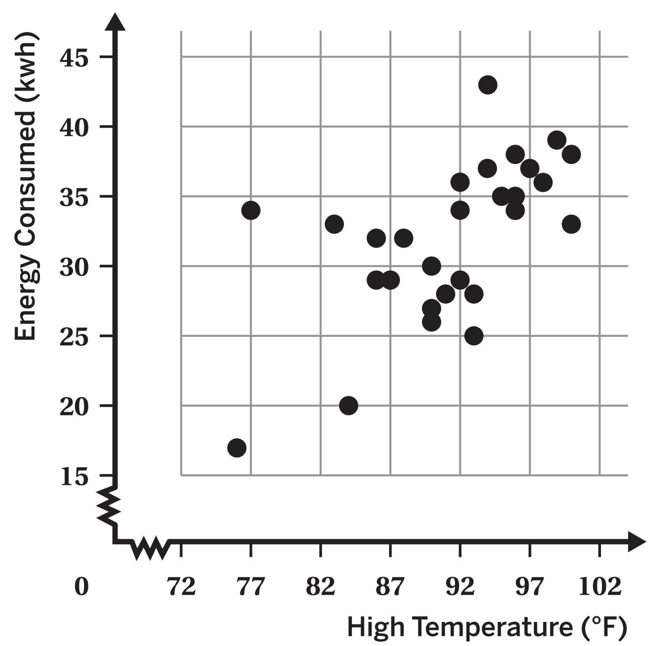
Book Page Count



Animal Lifespan

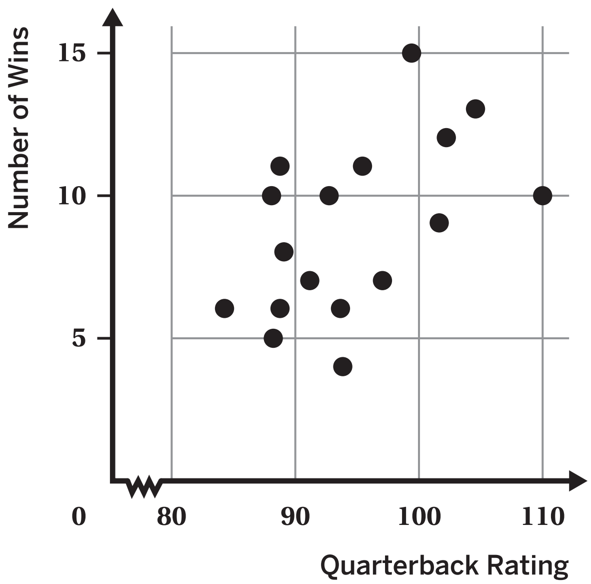


Energy Use

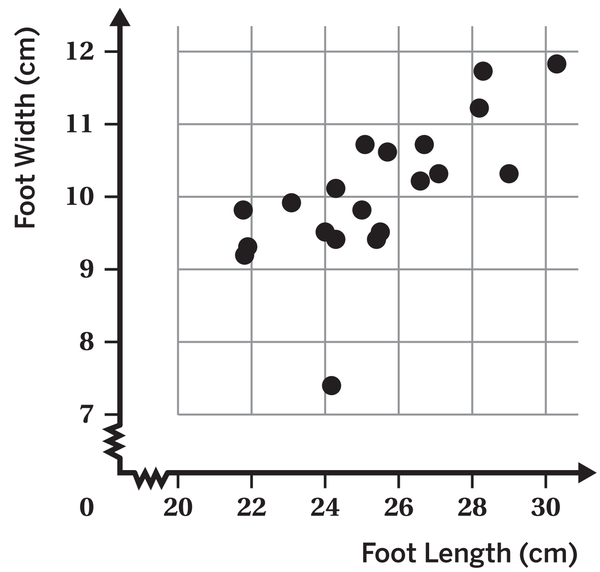


Challenge Creator

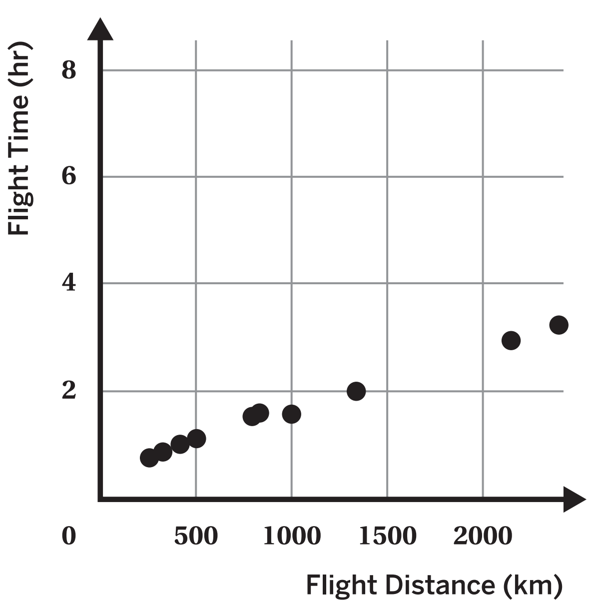
Quarterback Wins



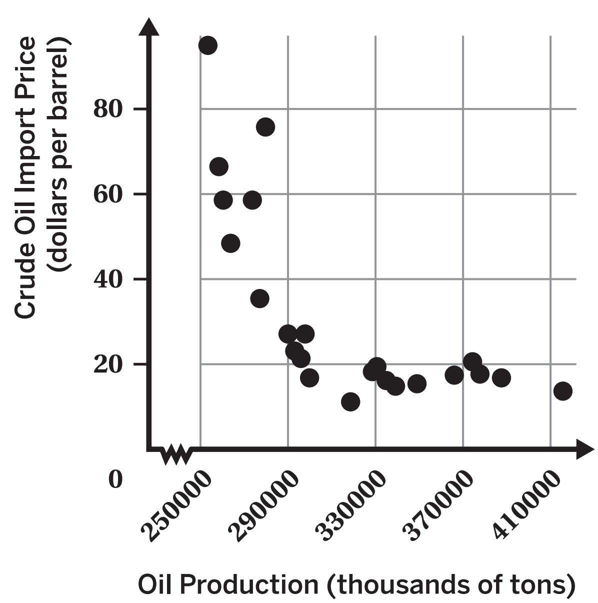
Foot Width



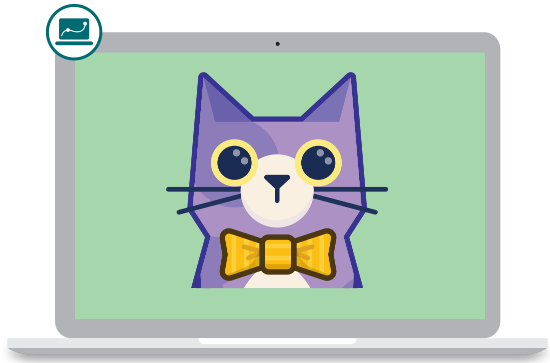
Flight Time



Oil Imports



Name: ______ Date: ______ Period: ______



Dapper Cats

Let's identify potential outliers and use a linear model to predict values.

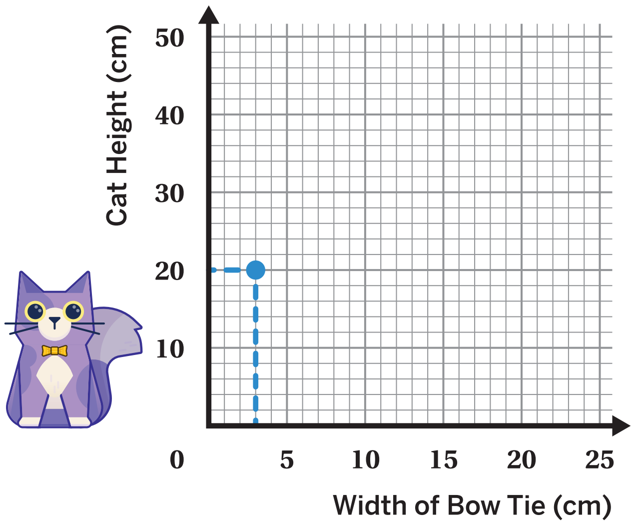
Warm-Up

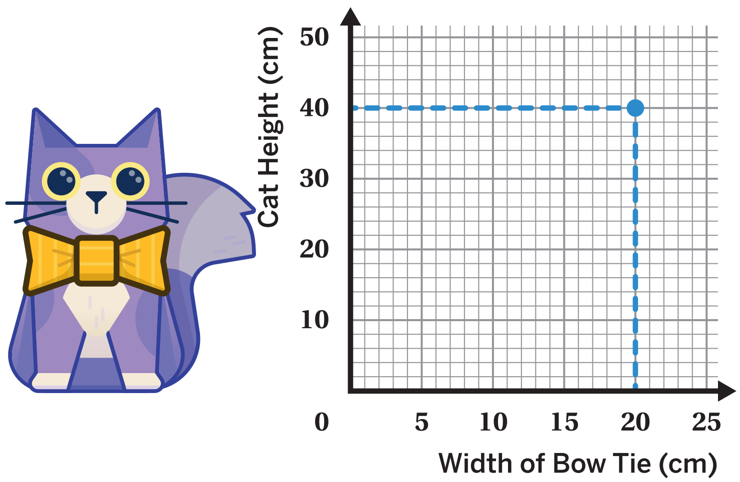
1 Here are two toy cats built at the Build-a-Cat workshop.

Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that the cats and their bow ties are different sizes depending on where the point is.
- I notice that the higher the point, the taller the cat is and the further the point is to the right, the wider the bow tie is.
- I notice that both of the points are on whole number values, not decimals.
- I wonder how big the biggest bow tie is.
- I wonder If there is a "correct" size bow tie.



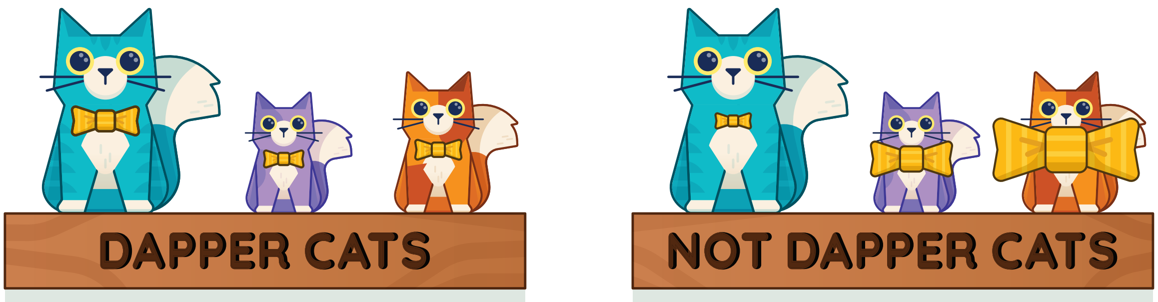


Activity 1

Name: .... Date: .... Period: ....

Dapper Cats

2 Some of these toy cats are dapper and some are not.



What do you think makes a toy cat "dapper"? What makes a toy cat "not dapper"?

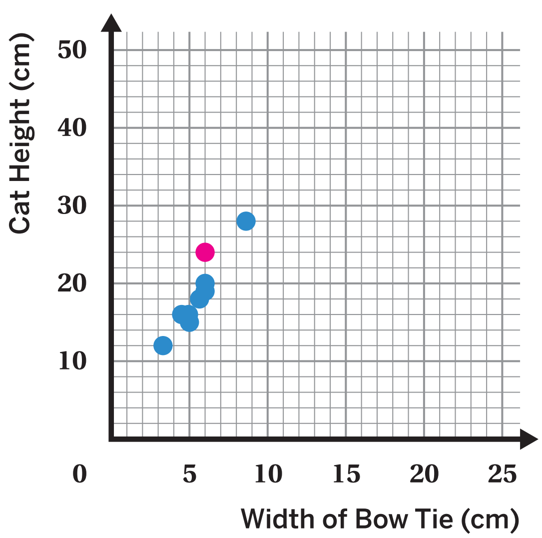
Responses vary.

- **The toy cats are "dapper" when their bow ties are the right width for their size.**
- **Cats are not dapper when they have a bow tie that is too big or too small.**
- **The size of the bow tie directly corresponds with the size of the cat.**

3 This scatter plot shows many "dapper cat" orders.

A customer just ordered a dapper cat that is 24 centimeters tall. Plot a point on the scatter plot to represent this cat.

***Responses vary.* Sample shown on graph.**



4 Describe what you notice about the scatter plot.

Responses vary.

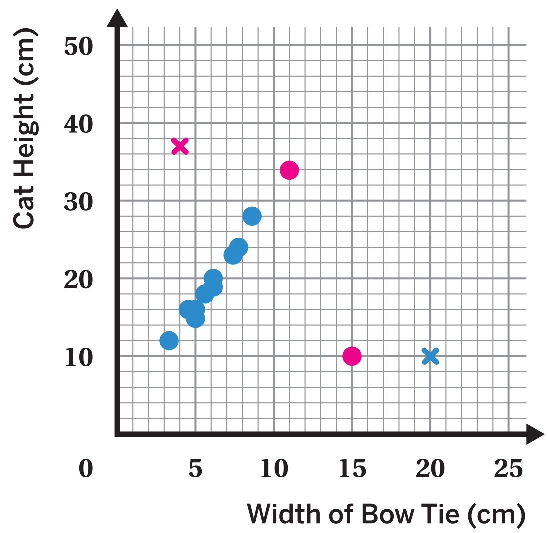
- **I notice that there are different-sized dapper cats.**
- **I notice that the points fall roughly along a line.**
- **I notice that the points could lie on a line with a positive slope and a y-intercept of 0.**
- **I notice that the points are all clumped together.**

Name: .... Date: .... Period: ....

Dapper Cats (continued)

5 Plot a point to represent a dapper cat that is taller than any of the other cats in the scatter plot.

***Responses vary.* Sample shown on graph at (11, 34).**



6 Plot a point to represent a not-dapper cat that is very short with a very large bow tie.

***Responses vary.* Sample shown on graph at (15, 10).**

7 A student plotted the point (20, 10). It is shown on the scatter plot with an X. This point is an **outlier** because it is far from the rest of the data.

a Plot another outlier on the scatter plot.

***Responses vary.* Sample shown on graph at (4, 37).**

b Describe the cat your point represents.

Responses vary.

- **The cat is dapper, but it's much taller than the other cats.**
- **The cat is not dapper. Its bow tie is too large for its height.**
- **The cat is not dapper. Its bow tie is too small for its height.**
- **The cat's bow tie is 5 centimeters wide and the cat is 40 centimeters tall.**

Activity 2

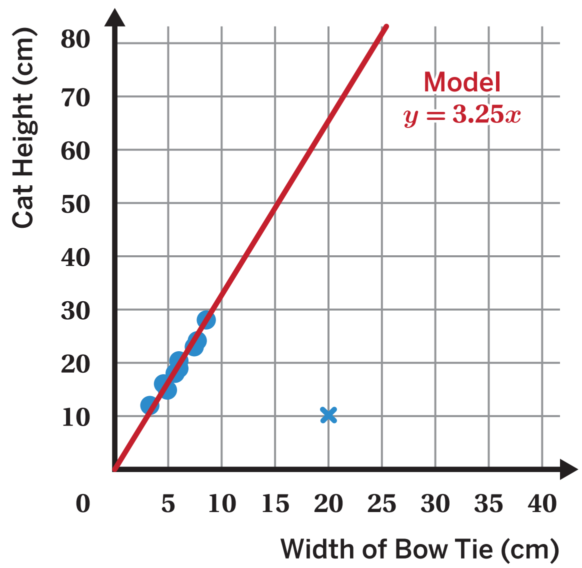
Name: .... Date: .... Period: ....

Using a Linear Model to Predict Data

A **linear model** (also called a *line of fit*) can be used to help identify trends in data and to make predictions. The line and equation model the relationship between bow tie width, x, and cat height, y.

8–9 Use the linear model to predict the height of a dapper cat with a 12-centimeter bow tie.

Responses between 37 and 41 centimeters are considered correct.



10–11 Another dapper cat is 65 centimeters tall. Use the linear model to predict the width of its bow tie.

Responses between 18.5 and 21.5 centimeters are considered correct.

12 Chey's cat has a 60-centimeter bow tie width. What does the linear model predict for its height?

195 centimeters

Explain your thinking.

***Explanations vary.* The linear model predicts that the height of Chey's cat will be 195 centimeters. I know this because $3.25 \cdot 60 = 195$.**

13 Synthesis

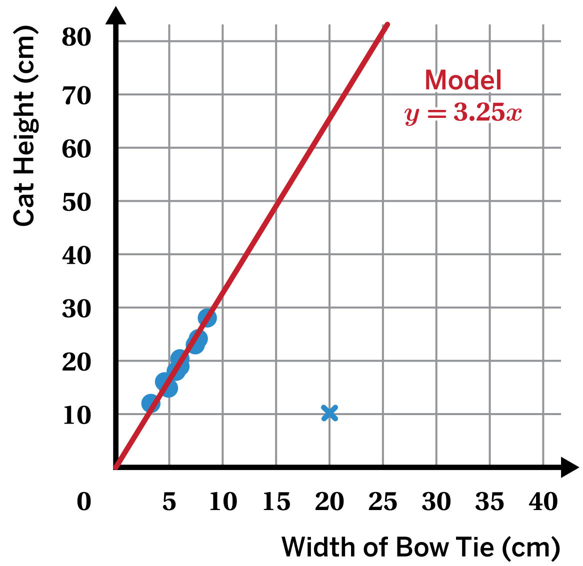
A line of fit and an equation are two ways to represent a linear model.

a How can a linear model be helpful?

***Responses vary.* A linear model is helpful because it helps us see the trend in the data more clearly so we can make predictions.**

b How can you identify an outlier on a scatter plot?

***Responses vary.* I can identify an outlier by looking for points that are far from the other points on the scatter plot.**



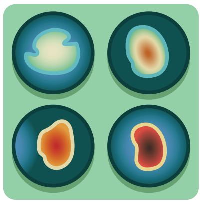
Things to Remember:

Unit 6
Lesson
5

Name: .... Date: .... Period: ....

Interpreting Scatter Plots

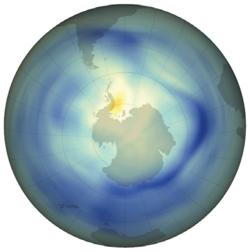
Let's find ways to show and identify patterns in data.



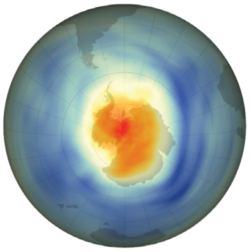
Warm-Up

1. The images show the hole in the ozone layer over Antarctica. The size and shape of the hole are monitored yearly. What do you notice? What do you wonder?

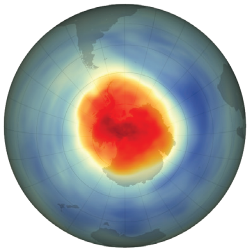
October 16, 1980



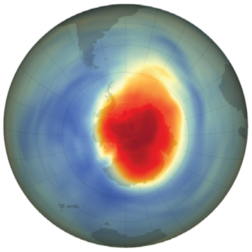
October 3, 1984



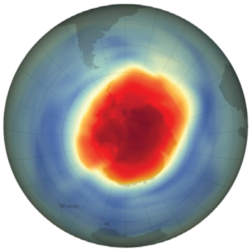
October 7, 1989



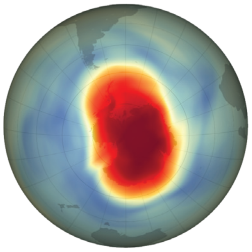
October 11, 1992



October 6, 1998



October 6, 2006



"World of Change: Antarctic Ozone Hole", NASA Earth Observatory.

I notice:

***Responses vary.* I notice that the spot gets more red over time.**

I wonder:

***Responses vary.* I wonder how the hole in the ozone layer is measured and how large it is today.**

Activity 1

Name: .... Date: .... Period: ....

Creating a Scatter Plot

NASA records the area of the hole in the ozone layer between September and October every year.

The Australian government records the number of new cases of skin cancer in Australia.

This table shows some of the data these groups collected between 1982 and 2008.

Year	Ozone Hole Area (million sq. km)	Number of Skin Cancer Cases in Australia
1982	5	3,541
1986	11	4,712
1988	10	6,013
1991	19	5,970
1997	22	8,444
2001	25	9,000
2005	24	10,832
2006	27	10,427
2007	22	10,450
2008	25	11,135

Sources: *NASA Earth Observatory and Cancer Australia*

2. **Discuss:** What are some questions you can ask about this situation?

Responses vary.

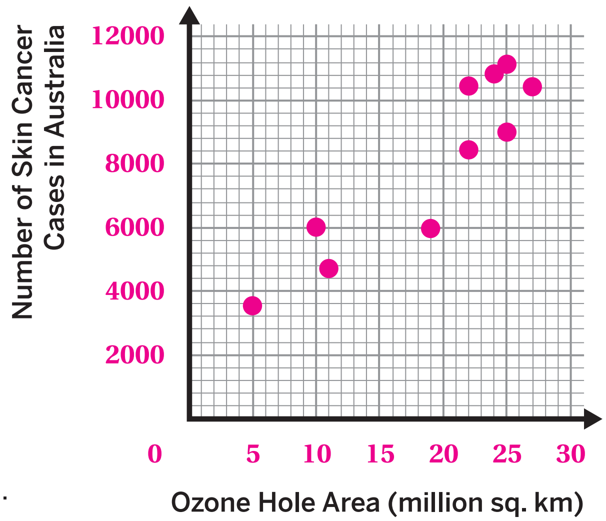
3. Let's make a scatter plot of this data.

a. Create a scale for the graph so it fits all of the data.

Responses vary.
Sample shown on graph.

b. Create a scatter plot of the data.

Sample shown on graph.



4. In 2021, the hole in the ozone layer was 23 million square kilometers. Use the graph to estimate the number of new skin cancer cases in Australia in 2021.

Responses vary. **Approximately 9,000 new cases.**

5. A linear model for this data is $y = 334x + 1706$. Use the model to predict the number of new skin cancer cases in Australia in 2021. Show your thinking.

9,388 cases. ***Work varies.***
$334(23) + 1706 = 9388$

Activity 2

Name: .... Date: .... Period: ....

Scavenger Hunt

6. Start at any of the Scavenger Hunt Sheets.

- Record the sheet shape, solve the problem, and write your answer.
- Look for your answer at the top of another scavenger hunt sheet. Solve that problem.
- Repeat until you make it back to your starting sheet.

The sheet students starts with varies.

Sheet: Trapezoid *Work varies.* **Answer** 12,000 →	**Sheet:** Circle *Work varies.* ↙ **Answer** C
Sheet: Pentagon *Work varies.* **Answer** B and D →	**Sheet:** Star *Work varies.* **Answer** 30 ↗

continued on next page...

Activity
2

Name: ______ Date: ______ Period: ______

Scavenger Hunt (continued)

Sheet: Rectangle *Work varies.* **Answer** A →	Sheet: Triangle *Work varies.* ↙ **Answer** 25
Sheet: Octagon *Work varies.* **Answer** D →	Sheet: Crescent *Work varies.* ↙ **Answer** D and E
Sheet: Hexagon *Work varies.* **Answer** 9.16 →	Sheet: Oval *Work varies.* **Answer** B

Synthesis

7. How can a scatter plot help make sense of the relationship between two variables? Use the scatter plot you created in Activity 1 if it helps with your thinking.

***Responses vary.* Scatter plots help visualize how one variable generally affects the other. For example, the scatter plot in Activity 1 shows that as the ozone hole area increases, the number of skin cancer cases also increases.**

Things to Remember:

Scavenger Hunt

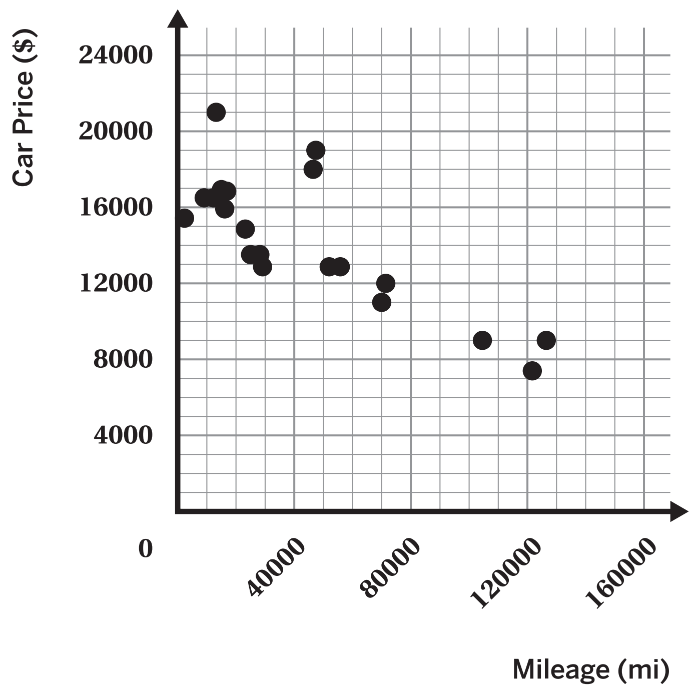
Trapezoid Sheet

Answer

B

Problem:

Based on this data, approximately what is the cost of a car with 60,000 miles?



Scavenger Hunt

Circle Sheet

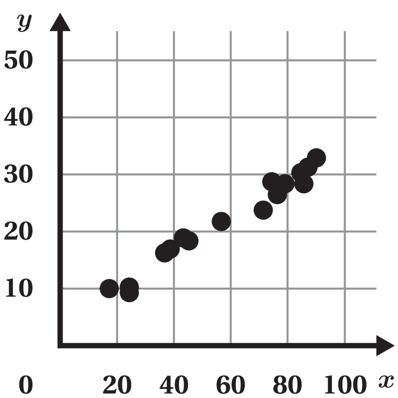
Answer

12,000

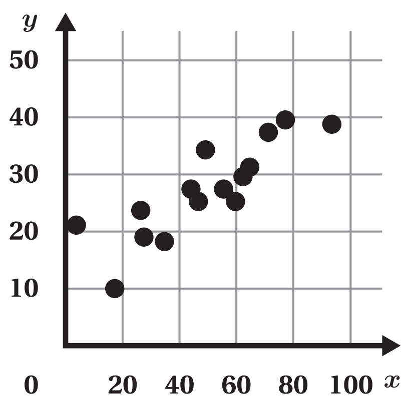
Problem:

Which scatter plot matches the table?

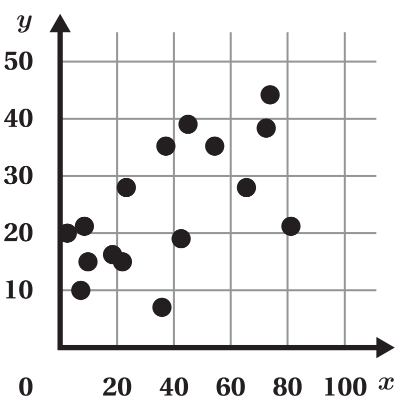
A.



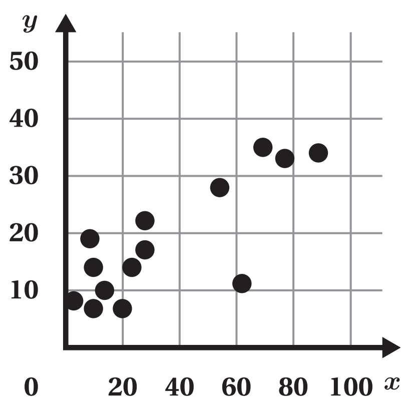
B.



C.



D.



x	y
66	28
43	18.5
3	19.5
24	28
22	15
38	35
19	15.5
74	44
10	15
81	20.5
73	37.5
55	34.5
45	38.5
36	7
9	20.5
8	10

Scavenger Hunt

Pentagon Sheet

Answer

C

Problem:

Select *all* the representations that are appropriate for comparing top speed to engine size for five different cars.

- ☐ **A.** Histogram
- ☐ **B.** Scatter plot
- ☐ **C.** Dot plot
- ☐ **D.** Table
- ☐ **E.** Box plot

Scavenger Hunt

Star Sheet

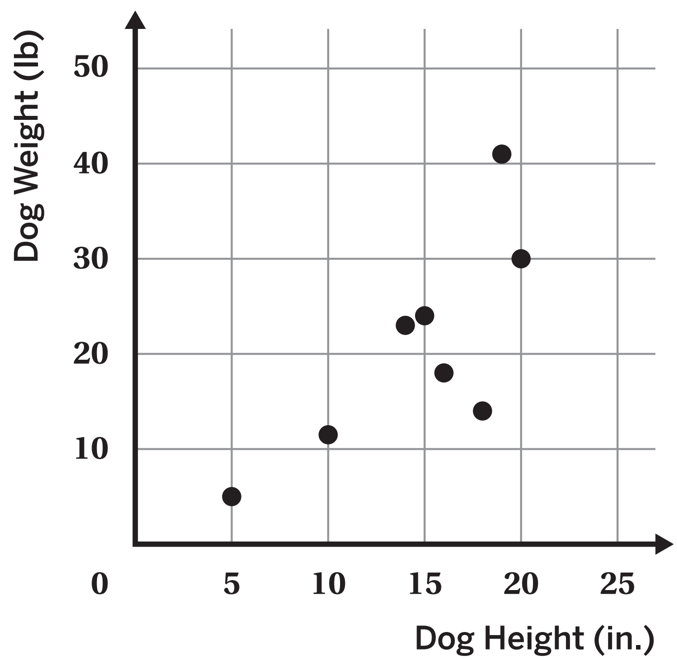
Answer

B and D

Problem:

Here is a scatter plot showing dog heights and weights.

What is the weight of the tallest dog on this scatter plot?



Scavenger Hunt

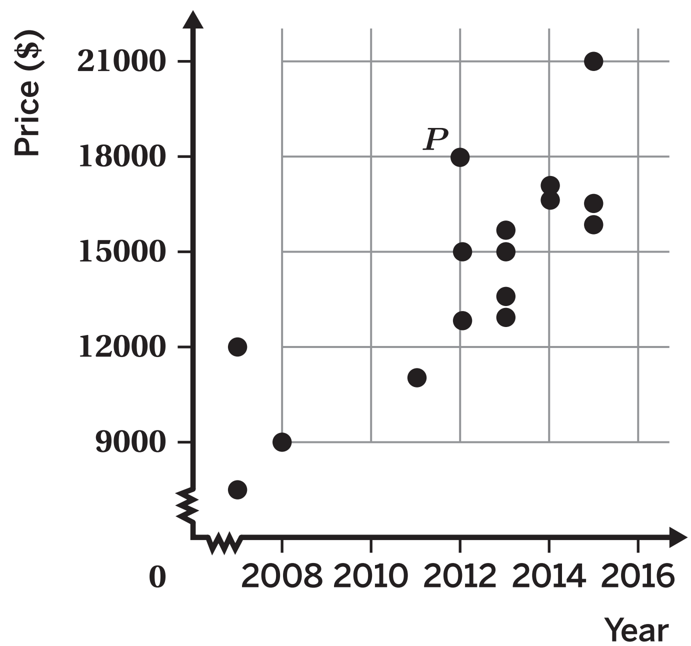
Rectangle Sheet

Answer

30

Problem:

Describe point P.



A. In the year 2012, the price was $18,000.

B. The price was $2012 in the year 18,000.

C. In the year 2012, the price was $15,000.

D. The price was $18,000 in the year 2010.

Scavenger Hunt

Triangle Sheet

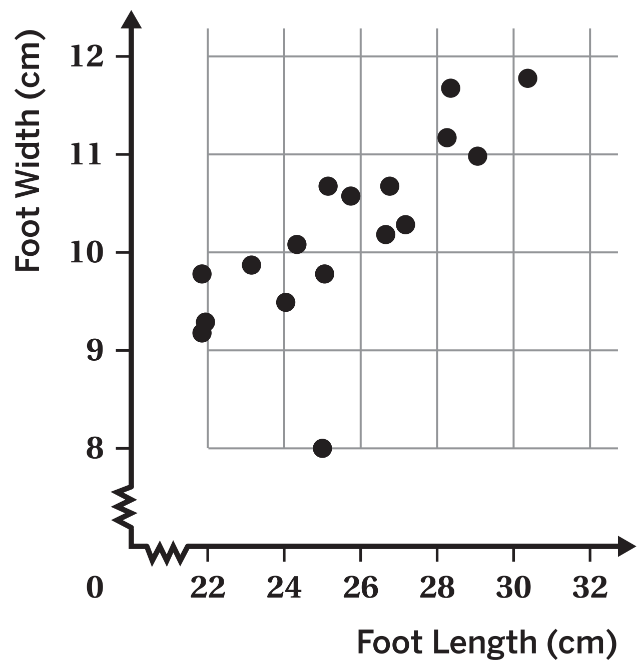
Answer

A

Problem:

Here is a scatter plot that compares the length and width of different people's feet.

What is the foot length of the person represented by the point that is an outlier?



Scavenger Hunt

Octagon Sheet

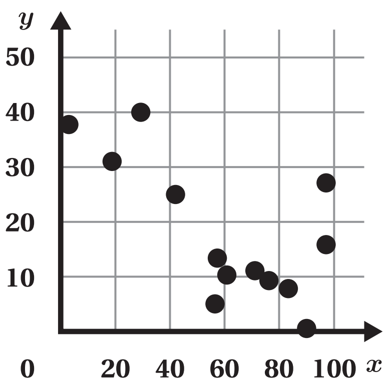
Answer

25

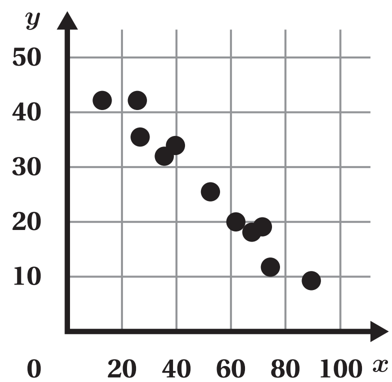
Problem:

Which scatter plot matches the table?

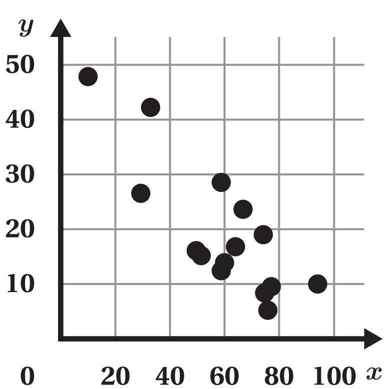
A.



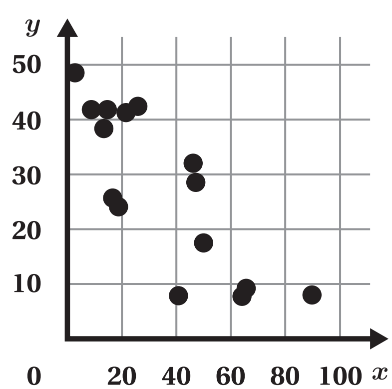
B.



C.



D.



Height (cm)	Foot Length (cm)
90	8
66	9
22	41.3
15	42
41	7.7
46	32.3
14	38.7
19	24.3
3	49
26	42.7
64	7.7
17	25.7
50	17.7
47	28.7
9	42

Scavenger Hunt

Crescent Sheet

Answer

D

Problem:

Select *all* of the representations that are appropriate for comparing the amount of time spent studying and the score earned for 10 quizzes.

- ☐ **A.** Box plot
- ☐ **B.** Dot plot
- ☐ **C.** Histogram
- ☐ **D.** Scatter plot
- ☐ **E.** Table

Scavenger Hunt

Hexagon Sheet

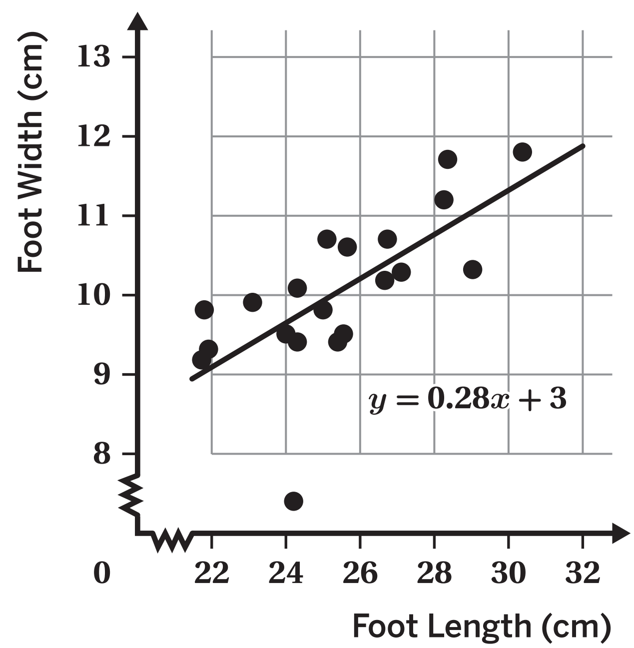
Answer

D and E

Problem:

This scatter plot shows the lengths and widths of 20 left feet, together with the graph of a model of the relationship between foot length and width.

Use the model to predict the width of a foot with a length of 22 centimeters.



Scavenger Hunt

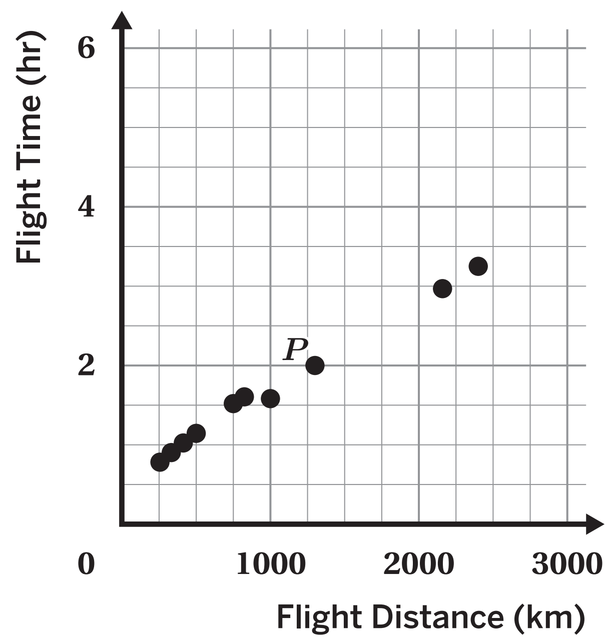
Oval Sheet

Answer

9.16

Problem:

Describe point P.



A. A flight distance of 1,500 kilometers is 2 hours long.

B. A flight distance of 1,300 kilometers is 2 hours long.

C. A flight distance of 1,500 kilometers is 2.5 hours long.

D. A flight distance of 1,300 kilometers is 2.5 hours long.

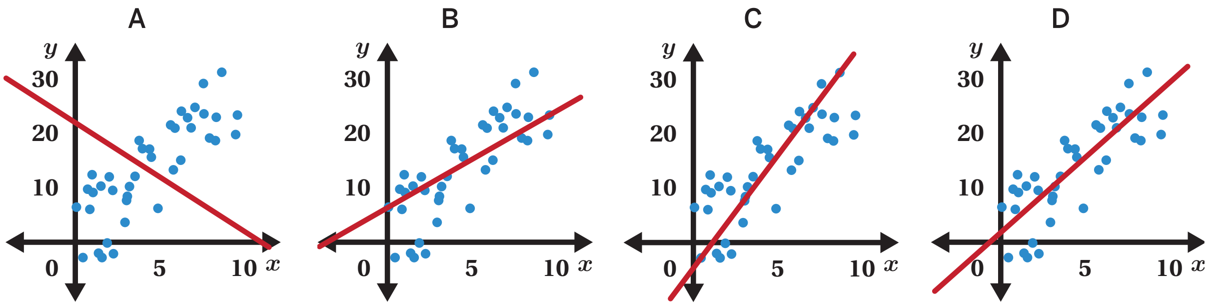
Name: .... Date: .... Period: ....

Find the Fit

Let's fit a line to data on a scatter plot.

Warm-Up

1 Which one doesn't belong? Explain your thinking.



Responses and explanations vary.

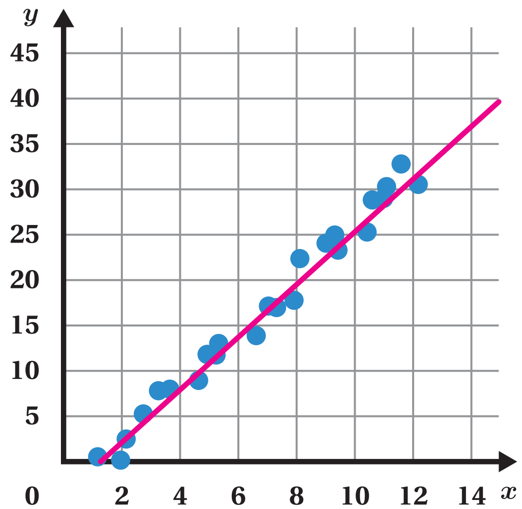
- **Graph A doesn't belong because it is the only one where the line has a negative slope.**
- **Graph B doesn't belong because it is the only one where the line goes through the points that are furthest left and furthest right.**
- **Graph C doesn't belong because it is the only one where the line goes through the points that are lowest and highest.**
- **Graph D doesn't belong because it is the only one where the line goes right through the middle of the points and follows the trend of the data.**

Activity 1

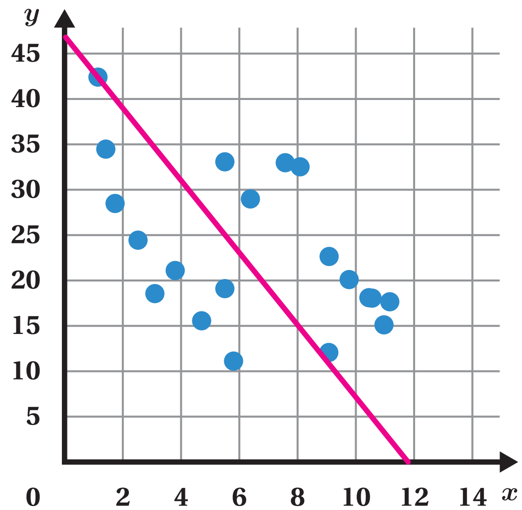
Name: .... Date: .... Period: ....

Lines of Fit

2 Create a line that is a good fit for each data set.



Responses vary. **Sample shown on graph. An appropriate line created by students will have a positive slope and follow the trend of the data set.**



Responses vary. **Sample shown on graph. An appropriate line created by students will have a negative slope and follow the trend of the data set.**

3 Let's look at some lines that other students created.

Discuss: How could you decide if a line is a good fit for the data?

Responses vary.

- **A line is a good fit for the data if it's as close as possible to all the points.**
- **The slope should follow the trend of the data.**

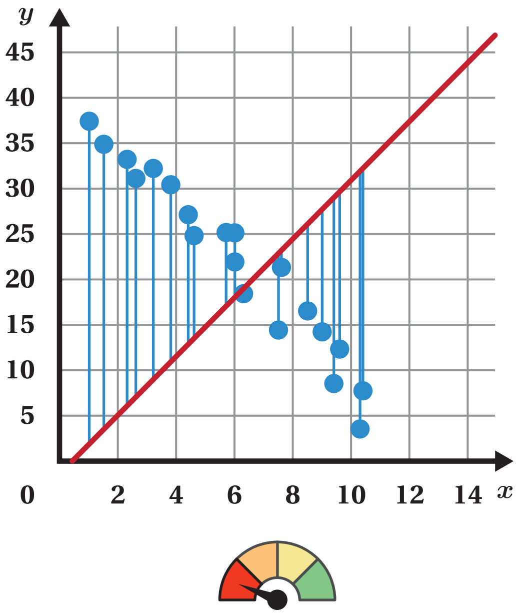
Activity 1

Name: .... Date: .... Period: ....

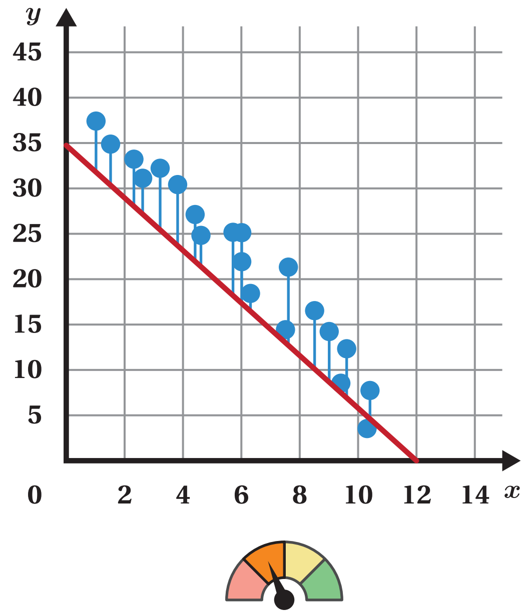
Lines of Fit (continued)

Here are four different lines for the same scatter plot. The meter shows a score for each line.

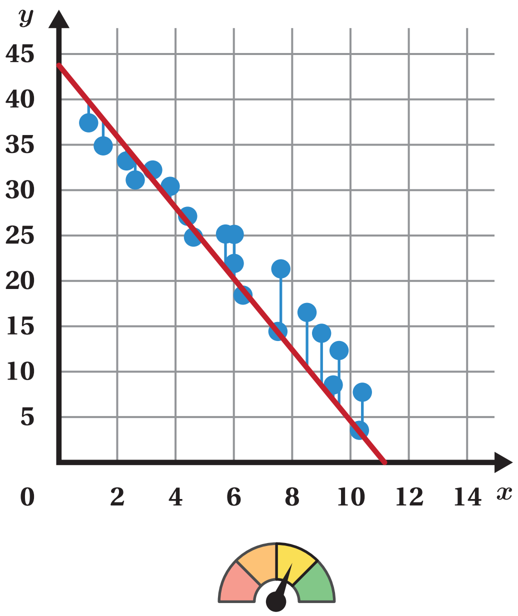
Graph A



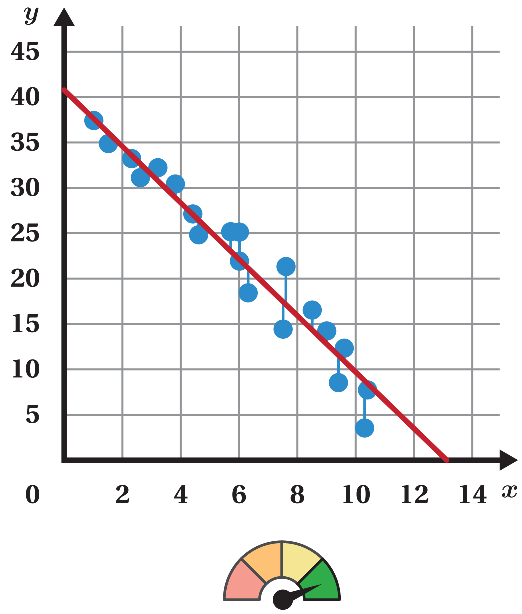
Graph B



Graph C



Graph D



4 Describe how to get a high score (green) on the meter.

Responses vary.

- Make a line that is as close to as many points as possible.
- The line should follow the trend of the data.
- The line should have a good balance of points that are above and below it.

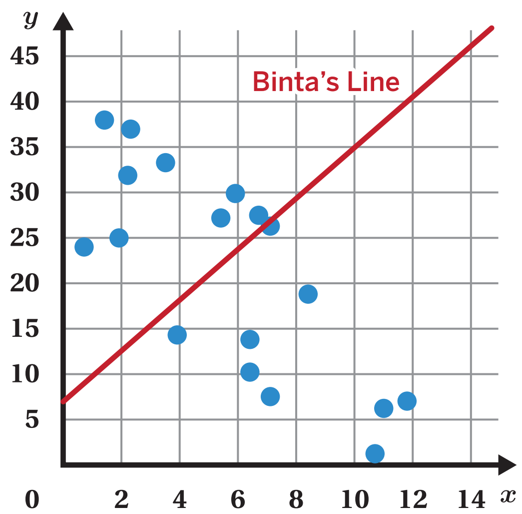
Name: ______ Date: ______ Period: ______

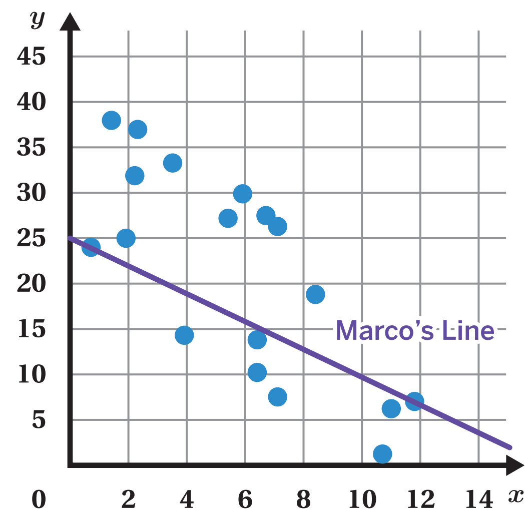
Find the Fit

5 Binta and Marco each sketched a *line of fit* on this scatter plot.

Binta says: *My line is a good fit because half of the points are on each side of the line.*

Marco says: *My line is a good fit because it passes through the leftmost and rightmost points.*





Whose line is a good fit for the data? Circle one.

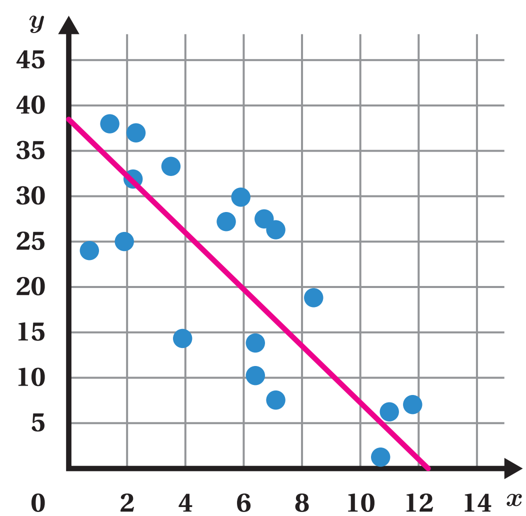
Binta's Marco's Both Neither

Explain your thinking.

***Explanations vary.* The data looks like it needs a line with a negative slope, but Binta's line has a positive slope. Marco's line is not a good fit either because most of the points are above it.**

6 Sketch a line that fits the data from the previous problem.

***Responses vary.* Sample shown on graph.**



Activity 3

Name: ____ Date: ____ Period: ____

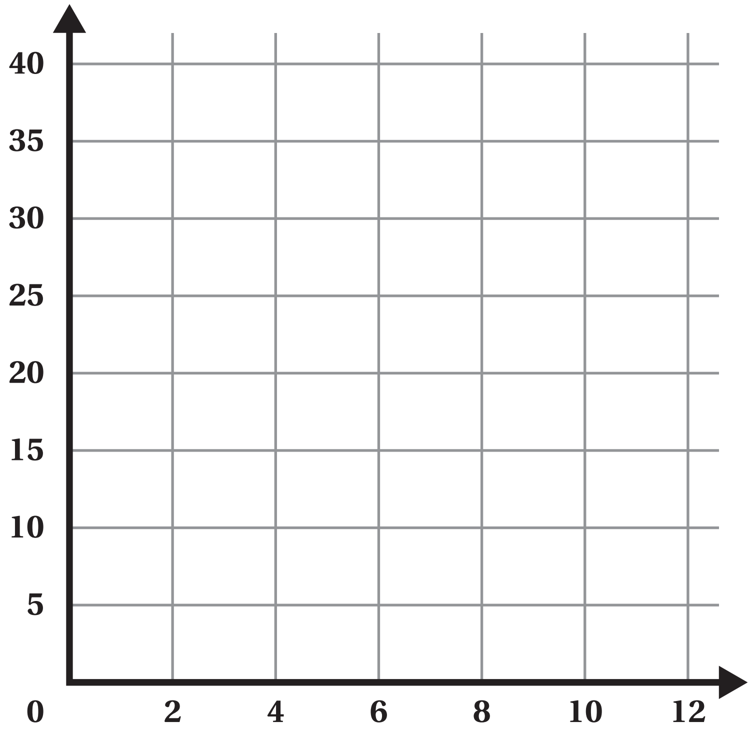
Challenge Creator

7 Follow these instructions to create scatter plots and solve your classmates' challenges.

a **Make It!** Create a scatter plot with at least ten points.

b **Solve It!** Use tracing paper to draw a line of fit for your data.

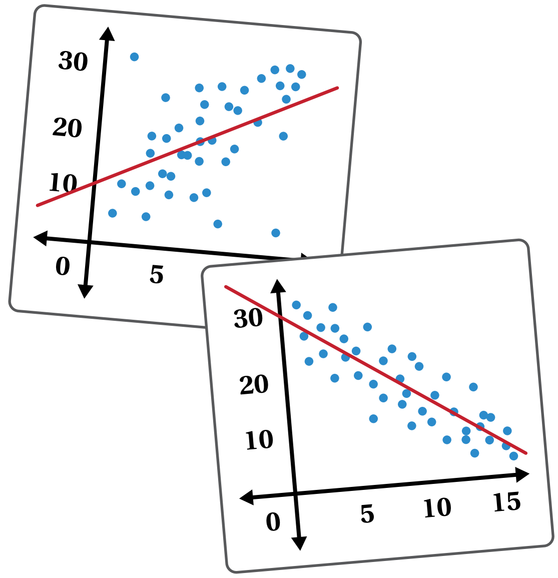
c **Swap It!** Pass your scatter plot to another student and use tracing paper to draw a line of fit for the data you received.



8 Synthesis

What are some things to consider when creating a line of fit? Use the examples if they help with your thinking.

Responses vary. **Try to make the line go through the middle of the points and follow the trend of the data.**

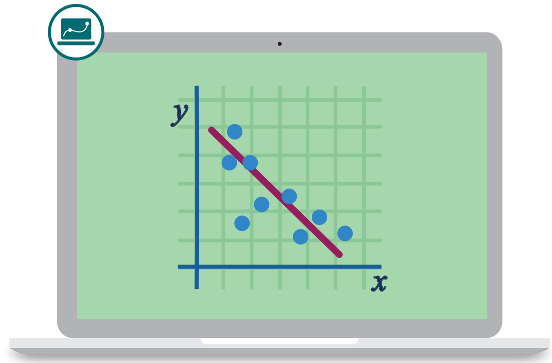


Things to Remember:

Name: ______________ Date: ________ Period: ________

Interpreting Slopes

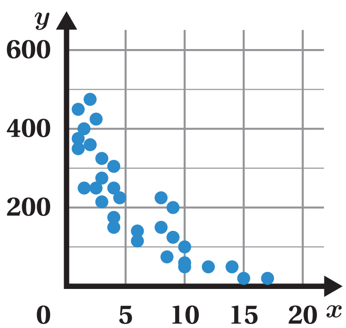
Let's identify different types of associations.



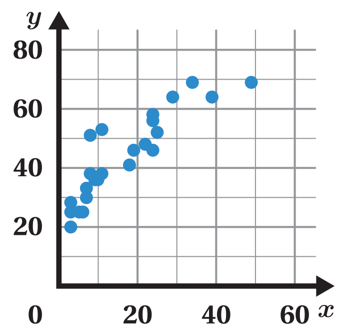
Warm-Up

1 Which one doesn't belong? Explain your thinking.

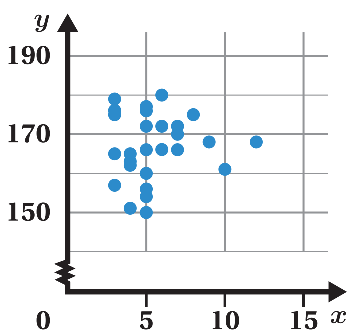
A.



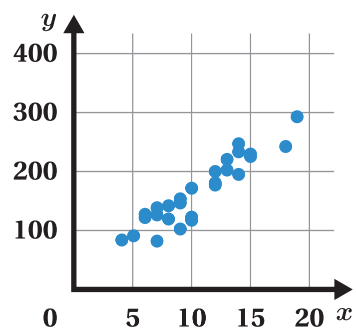
B.



C.



D.



Responses and explanations vary.

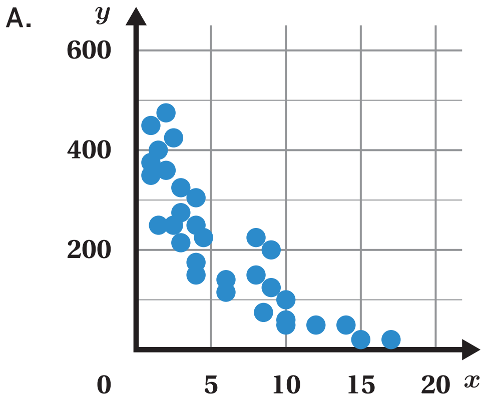
- Graph A doesn't belong because it is the only one where the points seem to decrease from left to right.
- Graph B doesn't belong because it is the only one where the x and y scales go by the same increment.
- Graph C doesn't belong because it is the only one where there is a break in the axis numbers, between 0 and 140 on the y-axis.
- Graph D doesn't belong because it is the only one where the points could be modeled by a line going through the origin.

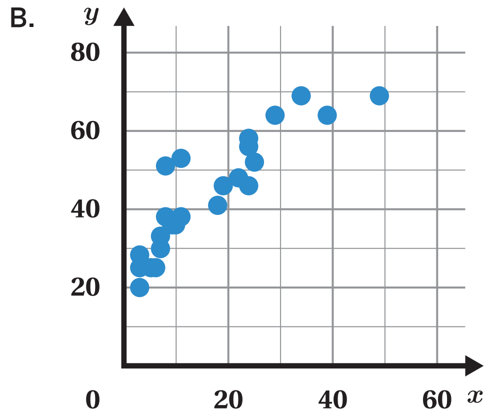
Activity 1

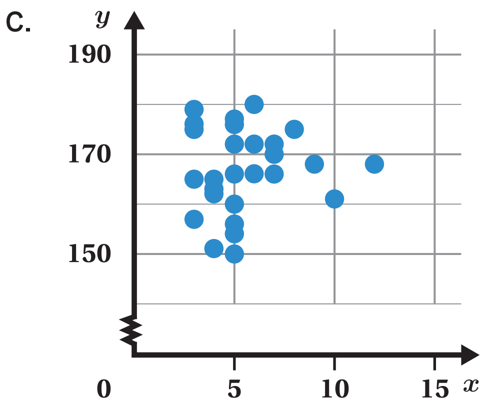
Name: Date: Period:

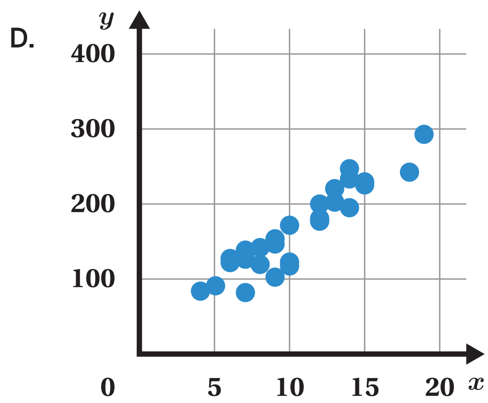
Associations

2 Match each scatter plot with the variables that it most likely represents.









x: Number of Floors y: Building Height (ft)	x: Age of Bike (yr) y: Bike Price ($)	x: Dog Weight (kg) y: Dog Height (cm)	x: Letters in Name y: Height (cm)
D	A	B	C

3 How did you decide which scatter plot matches these variables?

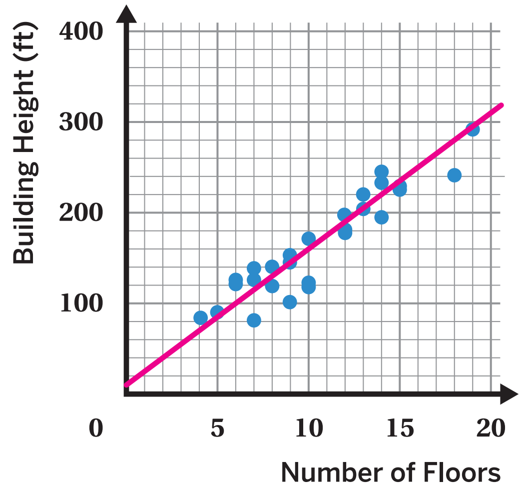
x: Number of Floors
y: Building Height (ft)

***Responses vary.* I assumed that buildings with more floors are taller than those with fewer floors, so I chose a scatter plot where the points trend upward from left to right.**

Name: ______ Date: ______ Period: ______

Associations (continued)

4 Here is the scatter plot that shows data from some buildings.



a) Sketch a linear model to fit the data.

***Responses vary.* Sample shown on graph.**

b) Esi sketched a linear model whose equation is $y = 13x + 30$.

Discuss:

- What is the *slope* of their line?
- What does the slope represent in this situation?

13. *Responses vary.* For every additional floor, the building's height increases by 13 feet.

5 An **association** is a relationship between two variables. There is a positive association if both variables increase together and a negative association if one variable decreases as the other increases.

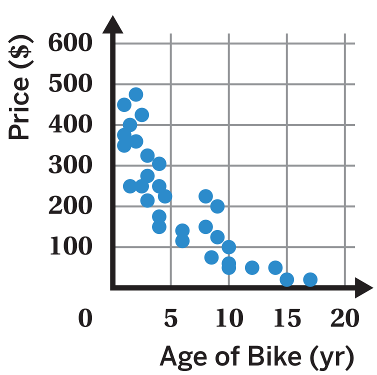
What type of association is there between building height and number of floors? Circle one.

Positive association   Negative association   No association

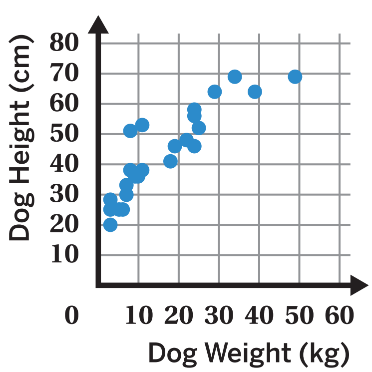
Explain your thinking.

***Explanations vary.* As the number of floors increases, the height of the building increases.**

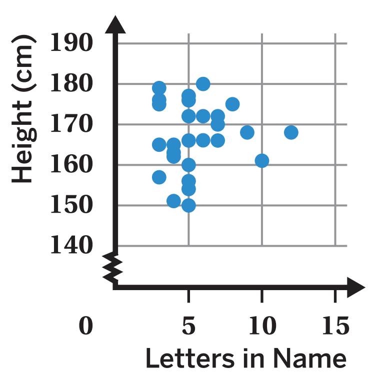
6 Determine what type of association each scatter plot shows. Discuss your thinking.



Negative association



Positive association

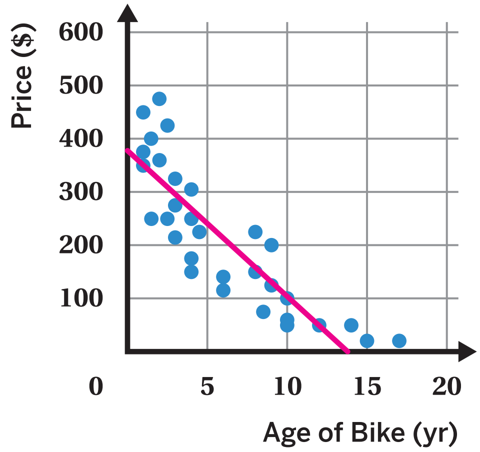


No association

Name: .... Date: .... Period: ....

Interpretations

7 Here is the scatter plot that shows the prices and ages of some used bikes.



a Sketch a linear model that fits the data.

Responses vary. **Sample shown on graph.**

b **Discuss:** How can you tell from the linear model that there is a negative association between bike age and price?

Responses vary. **The linear model suggests a negative association because the line has a negative slope.**

8 Tay drew a linear model for the bike data using the equation $y = -25x + 375$. Use Tay's model to finish this sentence:

The model predicts that as the age of a bike increases by 1 year:

A. The price will increase by $25.

B. The price will decrease by $25.

C. The price will increase by $375.

D. The price will decrease by $375.

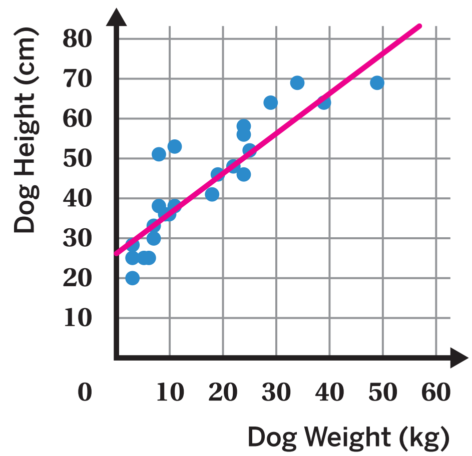
Explain your thinking.

Explanations vary. **The slope of the linear model is -25. The model predicts that, as the age of a bike increases by 1 year, the price of the bike decreases by $25.**

Name: .... Date: .... Period: ....

Interpretations (continued)

9 Here is the scatter plot that shows the heights and weights of some dogs.



a Sketch a linear model that fits the data.

***Responses vary.* Sample shown on graph.**

b Nikolai sketched a linear model whose equation is $y = 1.3x + 28$.

Identify the slope of Nikolai's model and describe what it means in this situation.

1.3. *Responses vary.* This means that as a dog's weight increases by 1 kilogram, the dog's height is predicted to increase by 1.3 centimeters.

Explore More

10 Fuel efficiency measures the number of miles a car can go using one gallon of gas (miles per gallon). This scatter plot shows the relationship between fuel efficiency and weight for 20 vehicles.

Write three statements about this scatter plot — two that are true and one that is a lie.

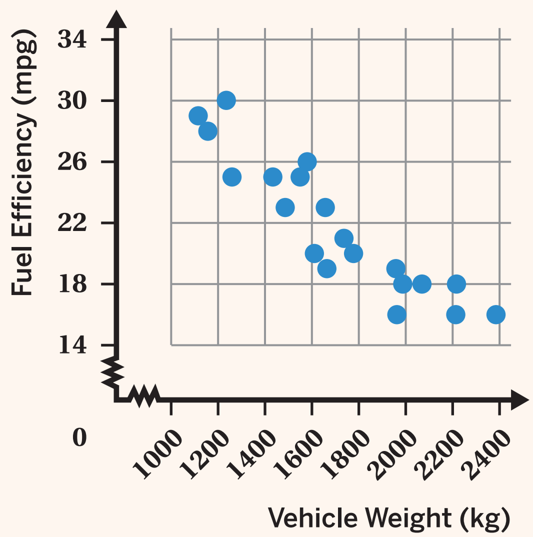
Sketch a line of fit if it helps with your thinking.

Responses vary.

Truth: As vehicle weight increases, fuel efficiency decreases.

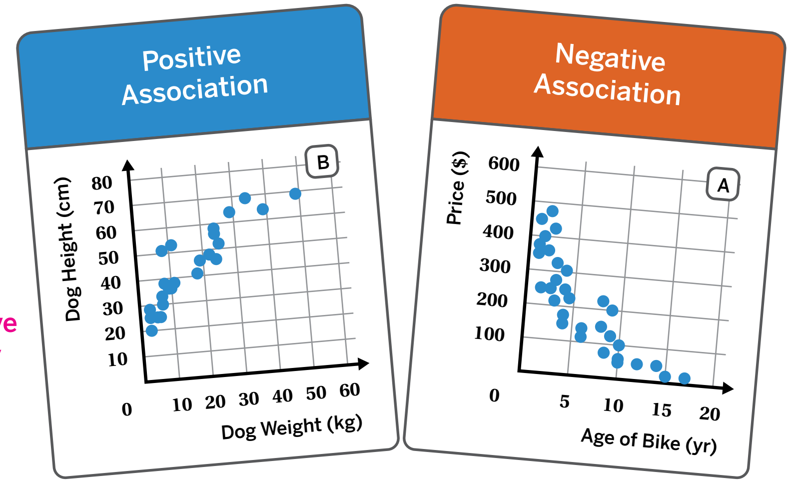
Truth: There is a negative association between fuel efficiency and vehicle weight.

Lie: If the weight of the vehicle increases by 1 kilogram, the fuel efficiency decreases by 2 miles per gallon.



11 Synthesis

Use the example scatterplots if they help to answer these questions:



a What are some clues that a scatter plot might have a positive or negative association?

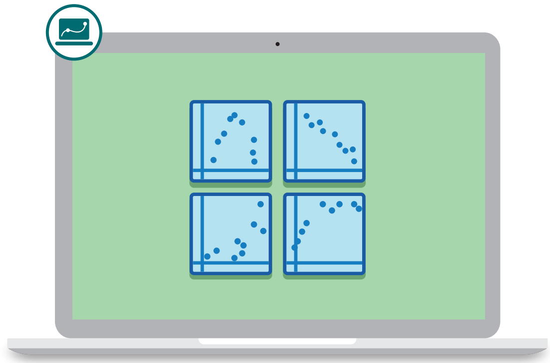
Responses vary.
A scatter plot has a positive association if one quantity increases as the other increases. The data points will trend up and to the right, and a linear model for the data would have a positive slope. A scatter plot has a negative association if one quantity decreases as the other increases. The data points will trend down and to the right, and a linear model for the data would have a negative slope.

b What does the slope of a linear model tell you about the data?

Responses vary. **The slope of a linear model tells you what the model predicts as the quantity on the x-axis increases by 1.**

Things to Remember:

Name: .... Date: .... Period: ....



Scatter Plot City

Let's use precise language to describe the trends in a scatter plot.

Warm-Up

1 Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet with scatter plots for four rounds. For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a scatter plot from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating scatter plots until you're ready to guess which scatter plot the Picker chose.

Record helpful questions from each round in the space below.

Responses vary.

Activity
1

Name: ______________________ Date: __________ Period: __________

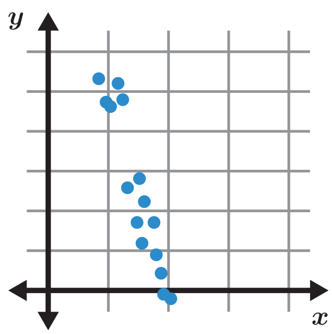
Scatter Plot City

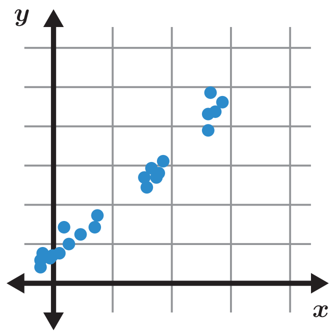
2 You will use the Activity 1 Sheet to see some scatter plots from the Polygraph and some terms that describe them.

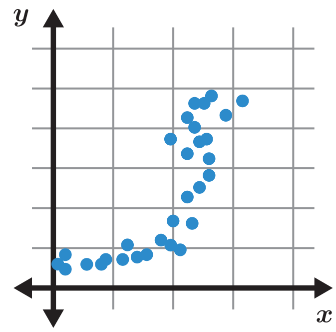
Discuss: What does each term mean? ***Responses vary.***

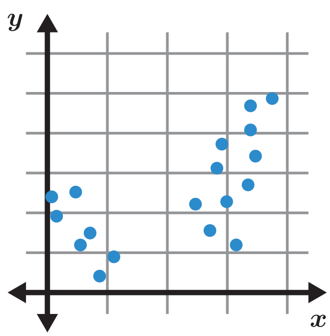
- Linear association
  When a straight line can model the data on a scatter plot.
- Non-linear association
  When a straight line cannot model the data on a scatter plot.
- With **clusters**
  When there is a grouping of data points around the same value.
- Without clusters
  When there is no grouping of data points around the same value.

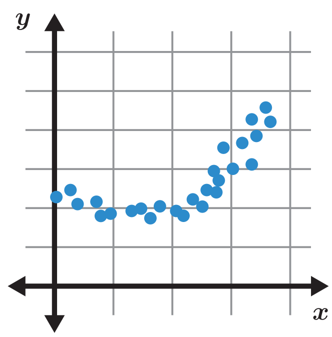
Here are six new scatter plots.

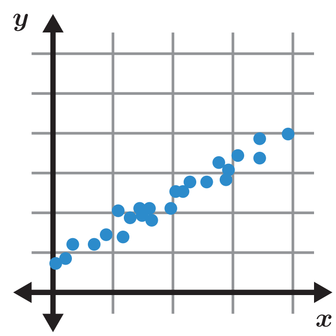
A.


B.


C.


D.


E.


F.


3 Sort them according to their type of association.

Linear Association	Non-Linear Association
A, B, F	**C, D, E**

4 Sort them in a different way: those with clusters and those without.

With Clusters	Without Clusters
A, B, D	**C, E, F**

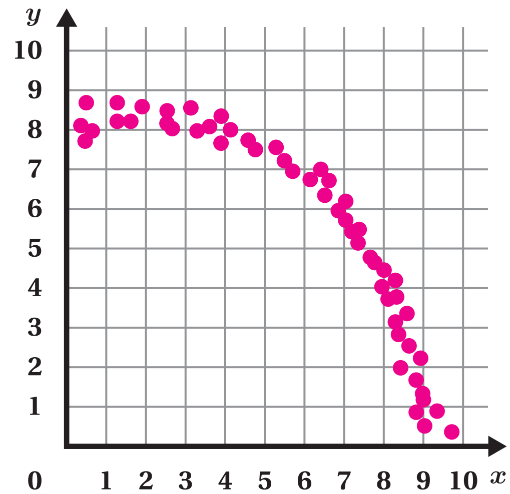
Activity 2

Name: .... Date: .... Period: ....

Putting It All Together

5 **a** Create a scatter plot that has a negative non-linear association, without clusters.

Responses vary. Sample shown on graph.



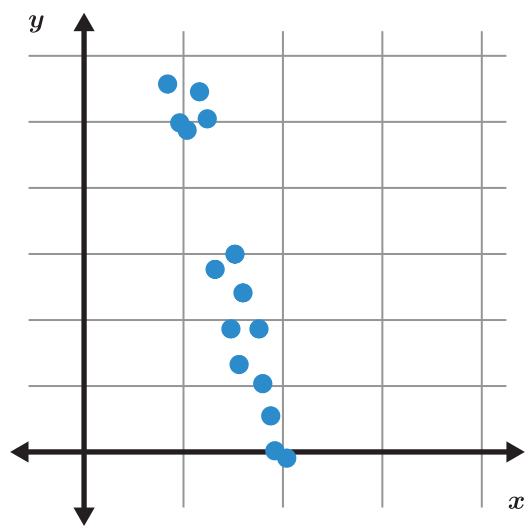
b Compare your scatter plot with your partner's.

6 Here is one of the scatter plots from before.

Describe the scatter plot using vocabulary from this unit.

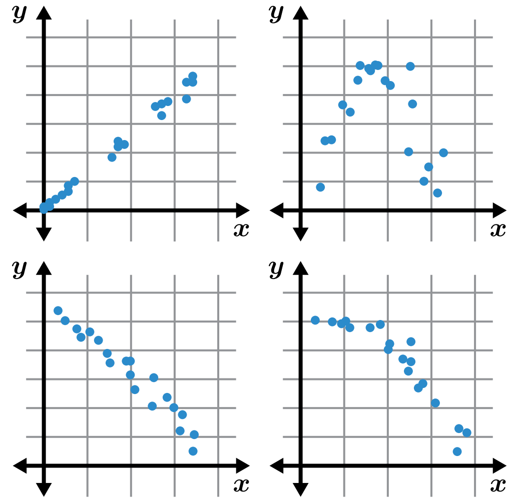
positive association	negative association	clusters
linear association	non-linear association	outlier

Responses vary. This scatter plot has a negative linear association, with two clusters of points.



7 Synthesis

How can you identify a non-linear association or clusters in a scatter plot? Use the examples if they help with your thinking.



Responses vary.

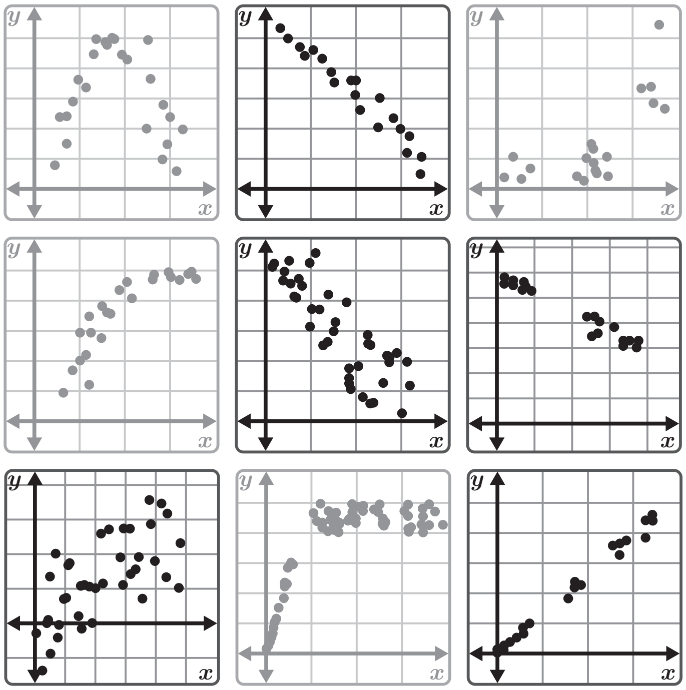
- Non-linear association: A scatter plot has a non-linear association when there isn't a line that is a good fit for the data.
- Clusters: A scatter plot has clusters if the points are gathered together in groups.

Things to Remember:

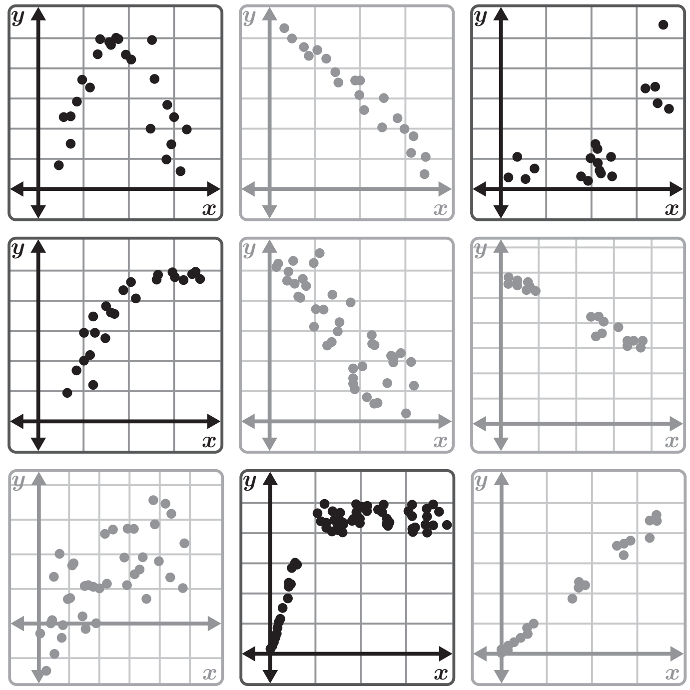
Name: .... Date: .... Period: ....

Scatter Plot City

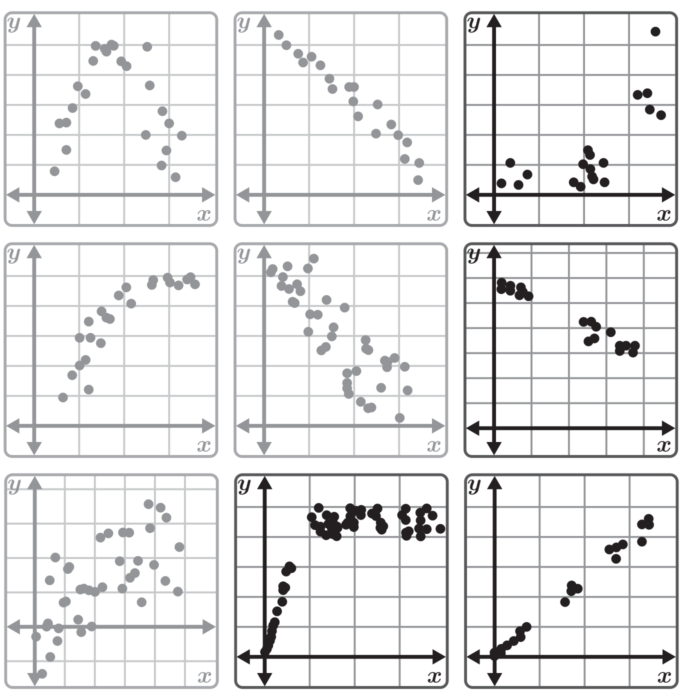
Linear Association



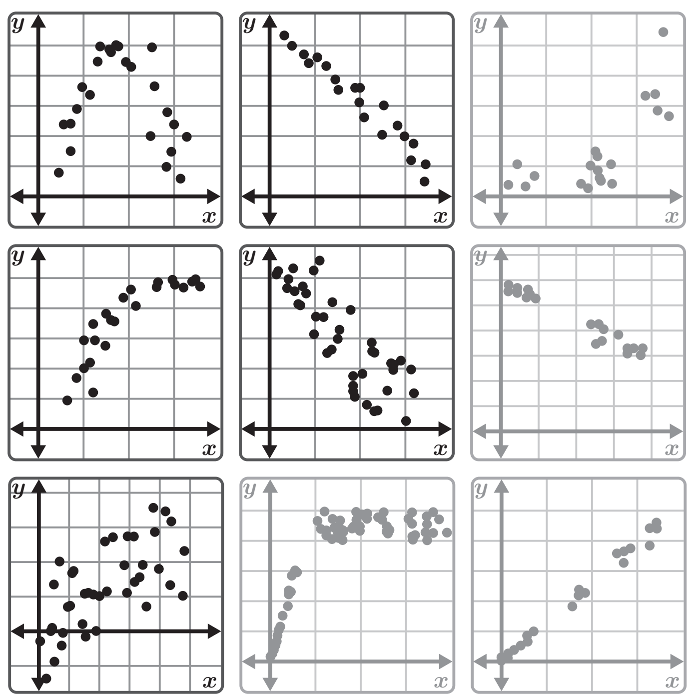
Non-Linear Association



With Clusters



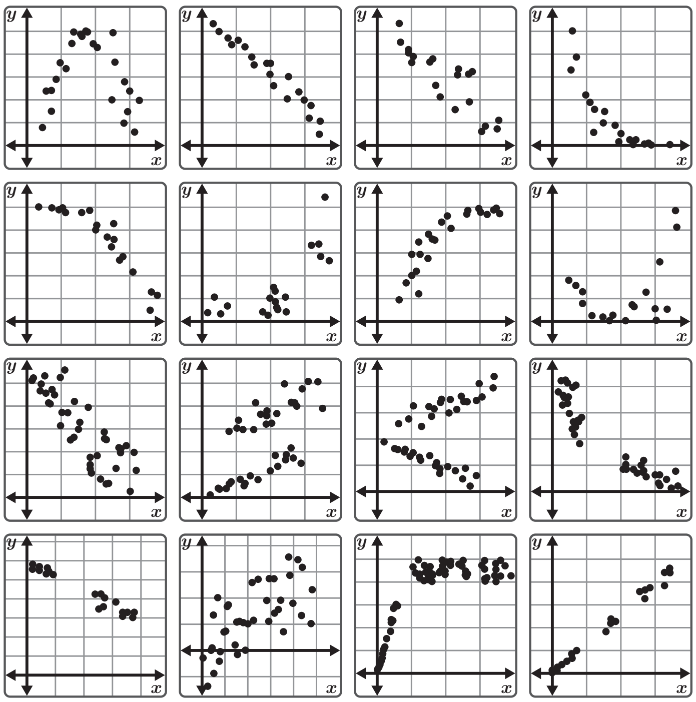
Without Clusters



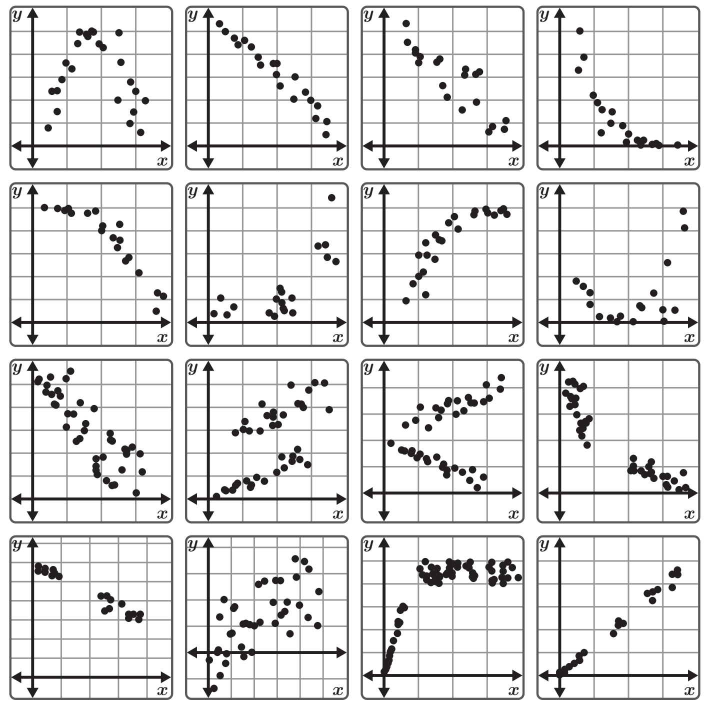
Name: ______________________ Date: __________ Period: __________

Polygraph Set A

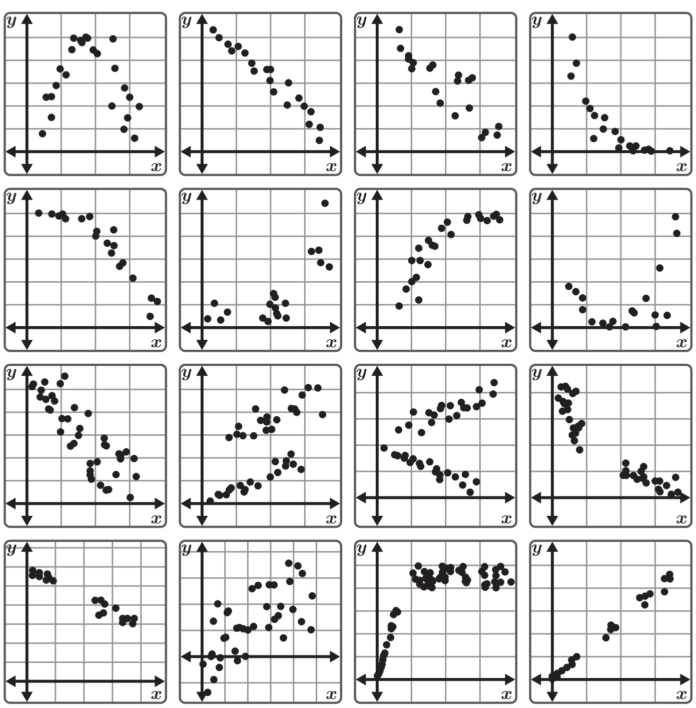
Round 1



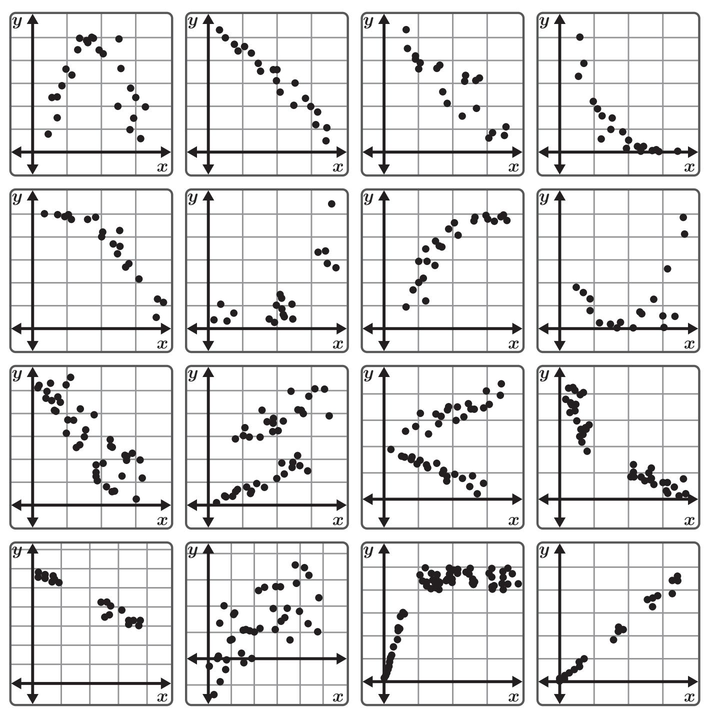
Round 2



Round 3



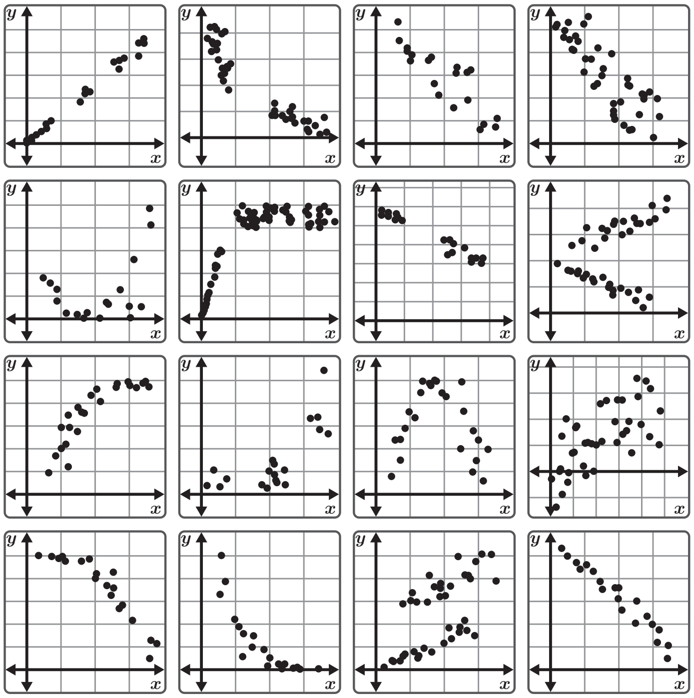
Round 4



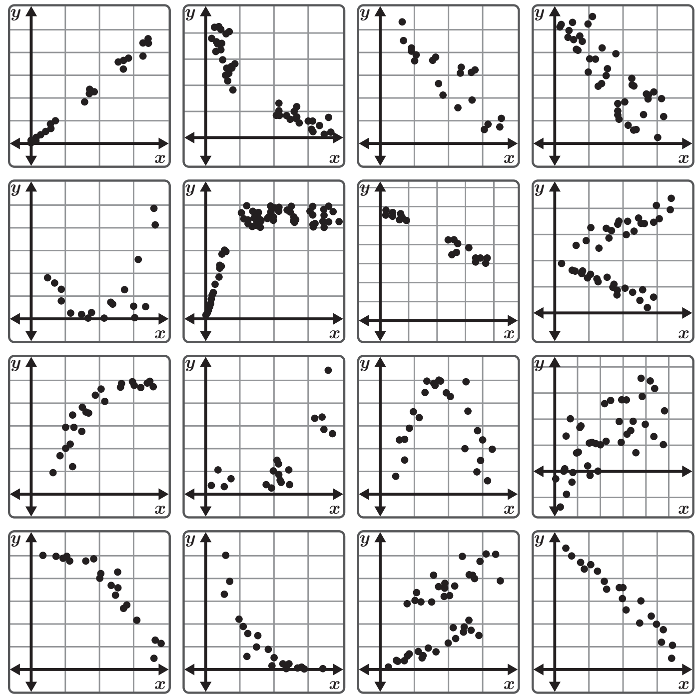
Name: .... Date: .... Period: ....

Polygraph Set B

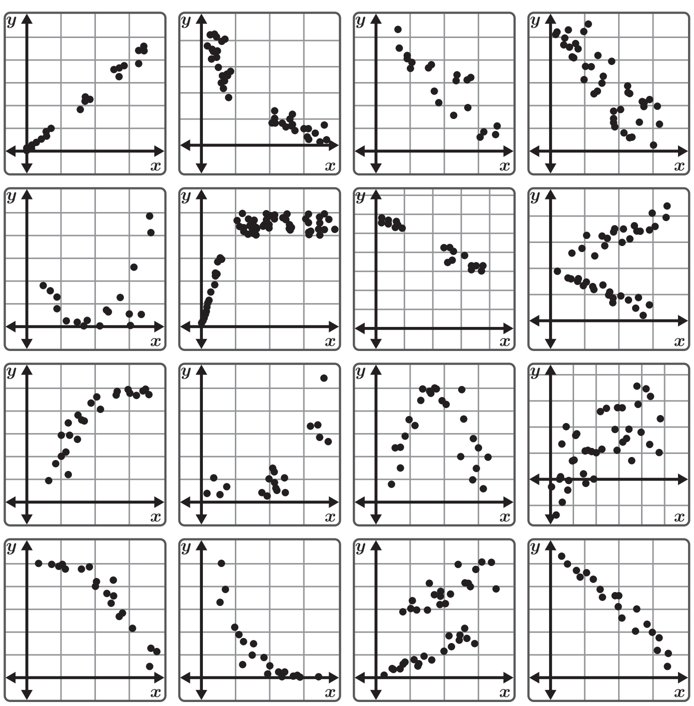
Round 1



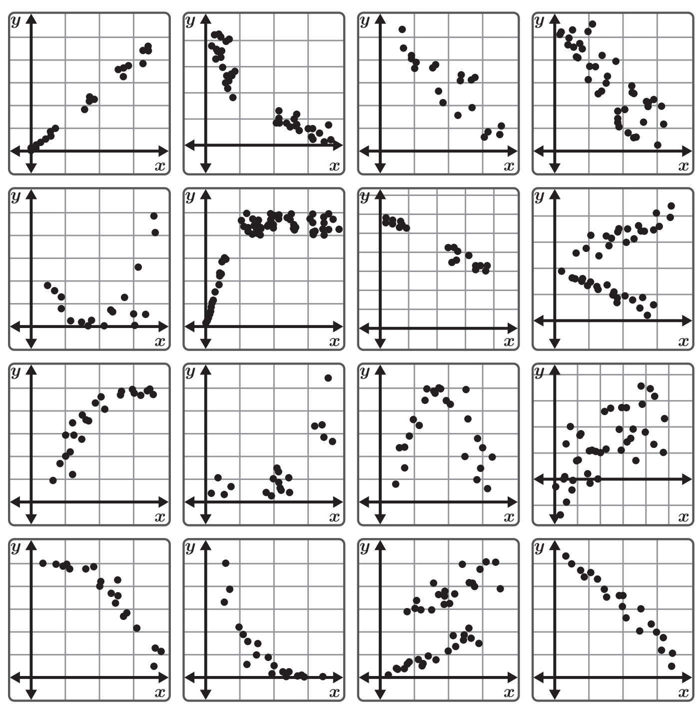
Round 2



Round 3

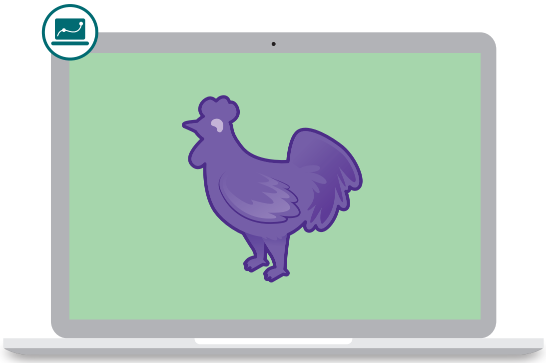


Round 4



Unit 6
Lesson
9

Name: .... Date: .... Period: ....



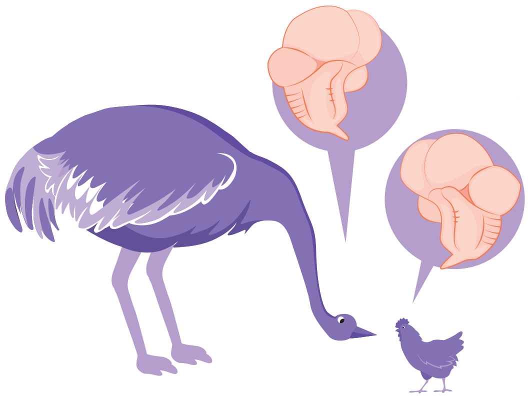
Animal Brains

Let's analyze bivariate data.

Warm-Up

1 Do you think heavier animals have heavier brains? Explain your thinking.

Responses vary.
- **Yes, bigger animals will have bigger brains.**
- **No, sometimes smaller animals can have bigger brains than animals that weigh more.**



2 What information would help you determine whether heavier animals have heavier brains?

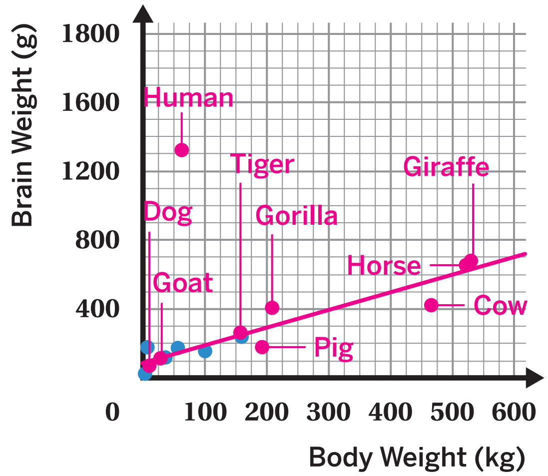
Responses vary. **Having data about the body weight and brain weight of several different animals would be helpful.**

Activity 1

Name: Date: Period:

Creating a Scatter Plot

3 The scatter plot shows the body weight and brain weight of 6 different animals. The table shows the weights of 4 more animals. Plot and label a point on the graph to represent each animal's data from the table.



Animal	Body Weight (kg)	Brain Weight (g)
Giraffe	529	680
Tiger	157	264
Goat	28	115
Cow	465	423

Response shown on graph for Screens 3, 5, 6, and 8.

4 Look back at your prediction from the Warm-Up. Based on the scatter plot, what type of association does there appear to be between brain weight and body weight? Circle one.

Positive association   Negative association   No association

Explain your thinking.

***Explanations vary.* There appears to be a linear association between brain and body weight, and a linear model with a positive slope would fit the data.**

5 The table shows the body weight of three more animals.

Plot and label points on the graph to predict the brain weight of each animal.

Complete the table with your predictions.

***Responses vary.* Actual values shown on graph and in table.**

Animal	Body Weight (kg)	Brain Weight (g)
Dog	10	**72**
Pig	192	**180**
Horse	521	**655**

Activity 2

Name: .... Date: .... Period: ....

Line of Fit

Fitting a line to data can help make predictions more accurate.

6 Draw a line that fits the data on the scatter plot on the previous page.

***Responses vary.* Sample shown on graph on the previous page.**

7 The equation for Inola's line of fit is $y = 0.9x + 79$.

a What is the slope of the line? What is the y-*intercept*?

Slope: **0.9** y-intercept: **79**

b **Discuss:**

- What does each number mean in this situation?
- Do these values make sense in this situation?

Responses vary.

- **The slope of the line is 0.9, which means that for every additional kilogram of body weight, the predicted brain weight increases by 0.9 grams. This makes sense because larger animals tend to have larger brains.**
- **The y-intercept of the line is 79, which means that an animal with a body weight of 0 kilograms will have a predicted brain weight of 79 grams. This doesn't make sense because if the body weighs nothing, how could the brain weigh anything?**

8 Use your line of fit to predict the brain weight for a gorilla and a human. Plot and label points on the previous page to show your predictions.

***Responses vary.* Actual values shown in table.**

Animal	Body Weight (kg)	Brain Weight (g)
Gorilla	207	**406**
Human	62	**1320**

Name: .... Date: .... Period: ....

Line of Fit (continued)

9 Let's look at a scatter plot that shows body and brain weight data, including the data for the gorilla and human.

What do you notice? What do you wonder?

Responses vary.

I notice:

- **I notice that the point for the human is an outlier.**
- **I notice there are two different clusters of points and one point that's really far away from the others.**

I wonder:

- **I wonder what animals' brains might be outliers in the direction opposite to humans.**

Explore More

10 *Tyrannosaurus rex* (T. rex) is a dinosaur with an estimated body weight of 8,000 kilograms.

a Based on your line of fit from Activity 2, how much might a T. rex's brain weigh?

***Responses vary.* 7,735 grams**

b Do you think the point representing the actual brain weight of a T. rex will be above or below the line of fit?

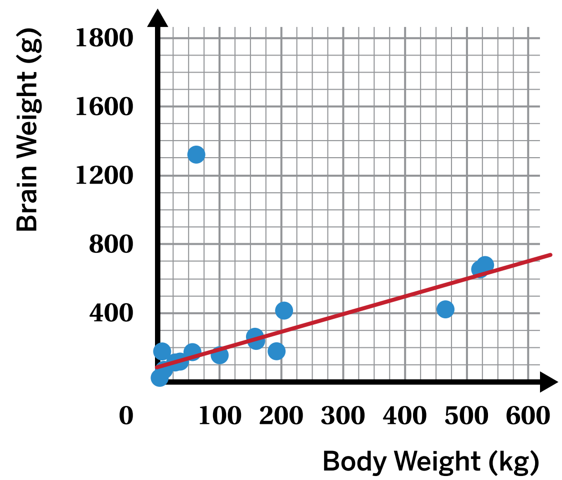
***Responses vary.* I think the T. rex's intelligence is below average, so the point would fall below the line.**

11 Synthesis

Describe an advantage and disadvantage of using a line of fit to make predictions.

Use the graph if it helps with your thinking.

Responses vary. An advantage of using a line of fit is that it helps me make reasonable predictions about where new points might be. A disadvantage is that it's not precise. Even if my prediction is reasonable, it might be wrong.



Things to Remember:

Unit 6
Lesson
10

Name: ______ Date: ______ Period: ______

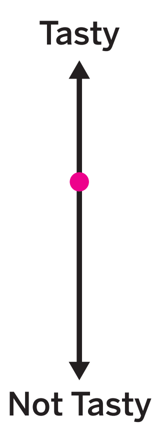
Tasty Fruit

Let's explore two-way tables and bar graphs.

Warm-Up

1 Draw a point to show how tasty you think red apples are.

Responses vary.



2 Draw a point to show how easy you think red apples are to eat.

Responses vary.

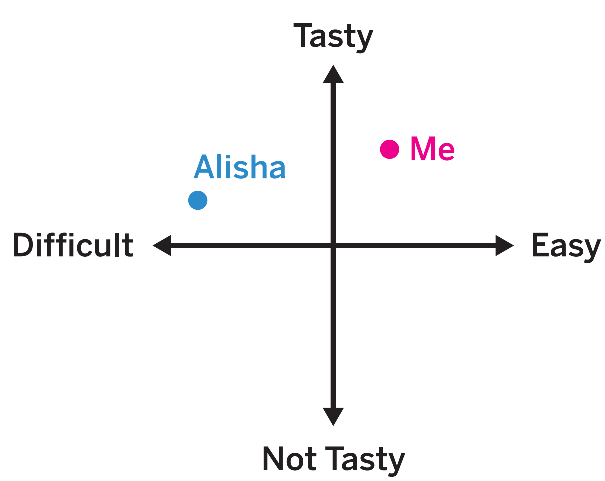
3 A scatter plot is one way to show how people feel about red apples.

How does Alisha feel about red apples?

***Responses vary.* Alisha says that red apples are tasty but difficult to eat.**

Plot a point on the graph to represent *your* feelings about red apples.

Responses vary.



Activity 1

Name: .... Date: .... Period: ....

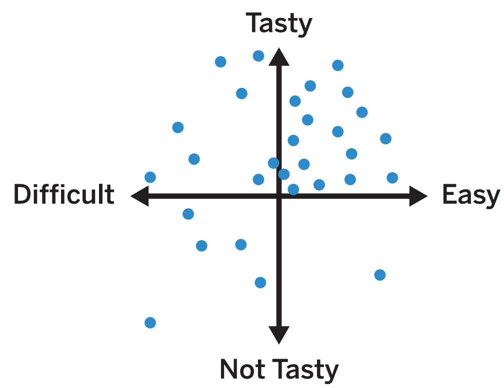
Displaying Categorical Data

Mr. Diaz's students are analyzing their class's opinions about red apples.

4 A student made a scatter plot and began to make a **two-way table**. The table shows **frequency**, which is the number of times each category appears in the data.

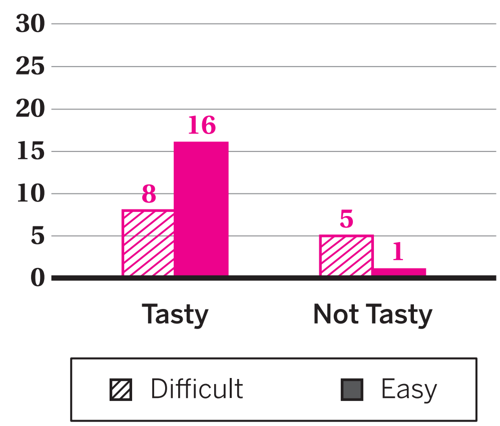
Complete the table to match the scatter plot.

	Difficult	Easy	Total
Tasty	8	16	24
Not Tasty	5	1	6
Total	13	17	30



5 Another student wanted to make a bar graph. Create a bar graph to match the table.

Response shown on graph.



6 The scatter plot, two-way table, and bar graph all represent *categorical data* about red apples.

How many students in total shared their opinions about red apples?

30

Explain your thinking.

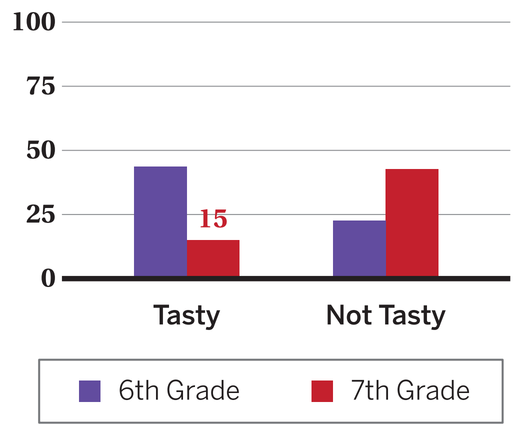
***Explanations vary.* The four bars in the bar graph represent everyone who was surveyed. If I add the totals of the four bars, the sum is 30.**

Activity 2

Name: .... Date: .... Period: ....

Analyzing Categorical Data

Abena surveyed the 6th and 7th graders at school about grapes. The bar graph shows the results from Abena's survey.



7 The two-way table shows partial results from the survey. Complete the table.

	6th Grade	7th Grade	Total
Tasty	**42**	15	**57**
Not Tasty	**22**	**41**	63
Total	64	56	120

8 Based on the data, do 6th and 7th graders feel the same about grapes? Circle one.

Yes No I'm not sure

Explain your thinking.

***Explanations vary.* Among 6th graders, there are more students who think grapes are tasty. In 7th grade, there are more students who think grapes are not tasty. It seems what grade you are in is related to whether or not you think grapes are tasty.**

Explore More

9 150 students were asked what grade they are in and whether they play a sport.

The two-way table shows partial results from this survey.

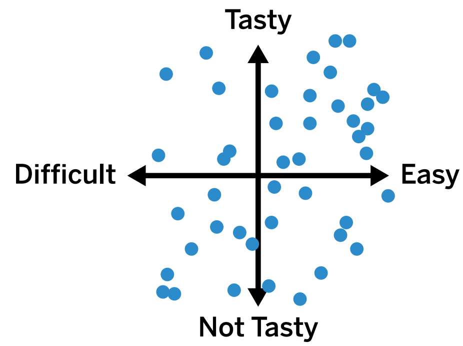
Complete the table.

	Plays a Sport	Does Not Play a Sport	Total
6th Grade	46	11	**57**
7th Grade	**19**	**1**	20
8th Grade	16	**5**	**21**
9th Grade	**29**	23	52
Total	110	**40**	150

10 Synthesis

A school surveyed the 8th graders about how tasty bananas are, and how easy they are to eat.

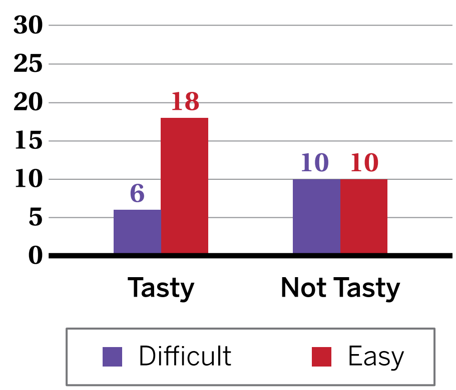
	Difficult	Easy	Total
Tasty	6	18	24
Not Tasty	10	10	20
Total	16	28	44



What are some advantages of using a scatter plot, a two-way table, or a bar graph to represent data? Use the examples if they help with your thinking.

Responses vary.

- **A scatter plot gives you more information about the degree to which someone thinks bananas are tasty.**
- **A two-way table organizes data so that it can be counted easily.**
- **A bar graph helps you visualize differences between the different groups.**



Things to Remember:

Name: ____________ Date: ________ Period: ________

Finding Associations

Let's use data displays to find associations.

Warm-Up

Shanice needs to book a flight from one of three nearby airports. Shanice researched each airport to determine how many flights were delayed last month.

1 Of the 20,175 flights that left the three airports near Shanice last month, 4,465 were delayed.

What percent of flights were delayed?

Responses between 22% and 22.14% are considered correct.

DEPARTURES

Time	Destination	Status
11:05	BOSTON	DELAYED
11:25	MIAMI	DELAYED
12:05	LONDON	ON TIME
13:15	CHICAGO	DELAYED
13:30	NEW YORK	ON TIME
13:48	DUBAI	ON TIME
14:00	TOKYO	ON TIME
14:20	HOUSTON	DELAYED
15:05	TORONTO	ON TIME

2 This two-way table shows some data about flights that departed in the last month from the three airports near Shanice.

	On Time	Delayed	Total
Burbank	?	?	2,110
L.A.	?	?	13,765
Santa Ana	?	?	4,300
Total	15,710	4,465	20,175

Discuss: Do you think the data will show an association between airport and flight status?

Responses vary.

- **Yes, I think the airport will be associated with flight status. Los Angeles probably has a big airport, which could mean more congestion and delays.**
- **No, I think delays are more associated with things like weather or mechanical problems than with specific airports.**

Activity 1

Name: .... Date: .... Period: ....

Frequency and Relative Frequency

3 This table shows the frequencies of on-time and delayed flights from the three airports.

Based on the data, is there evidence of an association between airport and flight status? Circle one.

Yes No I'm not sure

Explain your thinking.

Frequencies

	On Time	Delayed	Total
Burbank	1,520	590	2,110
L.A.	11,530	2,235	13,765
Santa Ana	2,660	1,640	4,300
Total	15,710	4,465	20,175

Responses and explanations vary.

- **Yes. I calculated what percent of flights are on time for each airport. The percentages varied a lot between airports.**
- **I'm not sure. It's hard to say, because Los Angeles has more delays than the other airports but also more on-time flights.**

4 One way to look for associations is to calculate **relative frequencies**. The relative frequency of a category is the percentage of data that is in that category.

The relative frequency of on-time flights from Burbank is about 72% because $\frac{1520}{2110} \approx 0.72$.

Relative Frequencies

	On Time	Delayed	Total
Burbank	72%	**28%**	100%
L.A.	**84%**	**16%**	100%
Santa Ana	**62%**	**38%**	100%

Complete the table. Round each percent to the nearest whole number.

5 Let's compare the two tables.

a Write a question you can answer using frequencies.

***Responses vary.* Which airport has the most total flights?**

b Write a question you can answer using relative frequencies.

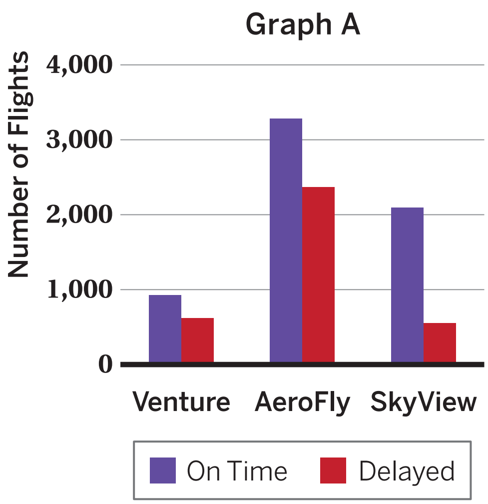
***Responses vary.* Which airport has the highest percentage of on-time flights?**

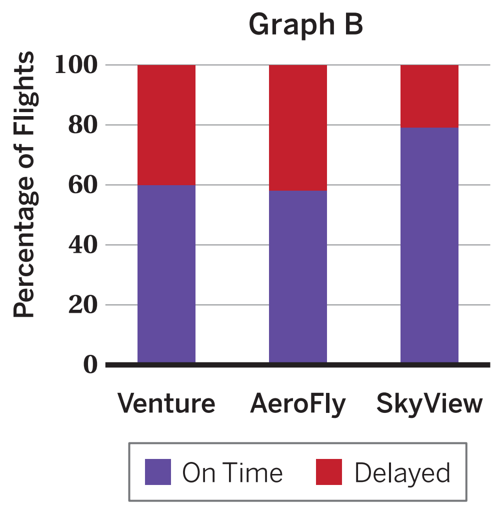
Name: .... Date: .... Period: ....

Analyzing Data Representations

6 Shanice also researched three airlines that she can fly with for her trip.

Discuss: Where do you see the same information in each representation?





Frequencies

	On Time	Delayed	Total
Venture	914	605	1,519
AeroFly	3,288	2,369	5,657
SkyView	2,100	542	2,642
Total	6,308	3,516	9,818

Relative Frequencies

	On Time	Delayed	Total
Venture	60%	40%	100%
AeroFly	58%	42%	100%
SkyView	79%	21%	100%

***Responses vary.* The data is the same across the two tables, but the table on the left shows the number of flights and the table on the right shows the percent of flights, or frequencies versus relative frequencies.**

7 Graph B is called a **segmented bar graph**. Use the bar graph or segmented bar graph to help you answer: *Is there evidence of an association between airline and flight status?* **Yes**

Which graph was more helpful for determining if there is an association? Explain your thinking.

***Responses vary.* The bar graph shows how many on-time and delayed flights there are for each airline, but relative frequencies are more helpful for comparing airlines to one another.**

8 Consider the claim, "AeroFly Airlines has the highest rate of on-time flights because it has more on-time flights than Venture and SkyView combined." Is this claim correct? Explain your thinking.

No. *Explanations vary.* AeroFly has more on-time flights than the other airlines because it has more flights overall. When making a claim about rate, it's better to look at relative frequencies. AeroFly has a lower percentage of on-time flights than the other airlines.

Name: .... Date: .... Period: ....

Analyzing Data Representations (continued)

9 Based on the data, which airport and airline would you recommend for Shanice's flight? Explain your thinking.

***Responses vary.* I would recommend flying out of Los Angeles and using SkyView Airlines, if possible. Los Angeles has the highest percentage of on-time flights compared to other airports, and SkyView has the highest percentage of on-time flights compared to other airlines.**

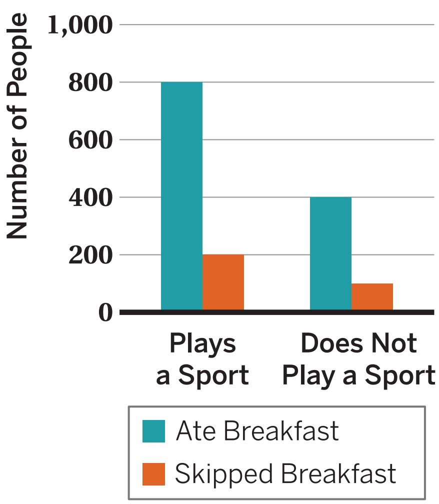
10 Here are four new data sets. For each data set, decide where there is evidence of an association between the variables.

	Left-Handed	Right-Handed
Has a Pet	83%	81%
No Pet	17%	19%
Total	100%	100%

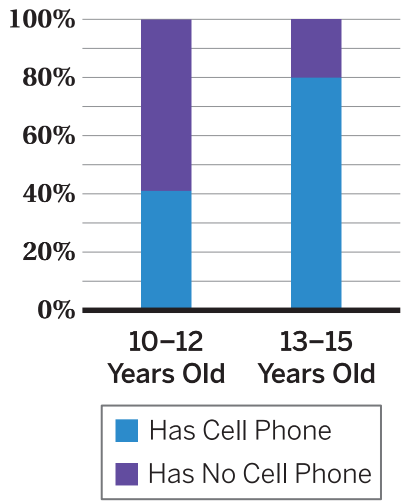
Association / No association

	Completed Course	Did Not Complete	Total
Free Online Course	6%	94%	100%
In-Person Course	85%	15%	100%

Association / No association



Association / No association



Association / No association

Explore More

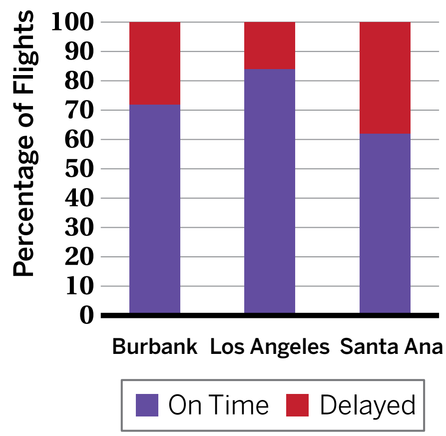
11 Use the digital activity to look at representations of data collected by the National Household Travel Survey on whether people took public transit over a one-month period.

Responses vary.

11 Synthesis

Here are two representations of relative frequencies.

	On Time	Delayed	Total
Burbank	72%	28%	100%
L.A.	84%	16%	100%
Santa Ana	62%	38%	100%

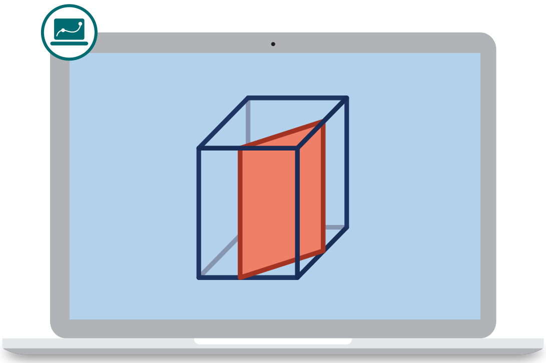


How can you use relative frequencies to identify possible associations between variables?

***Responses vary.* If variables aren't associated, then I'd expect their relative frequencies to be about the same. In this example, if there was no association, the percent of on-time flights would be the same for each airport. Since the relative frequencies vary by airport, that suggests there is an association.**

Things to Remember:

Name: ______ Date: ______ Period: ______

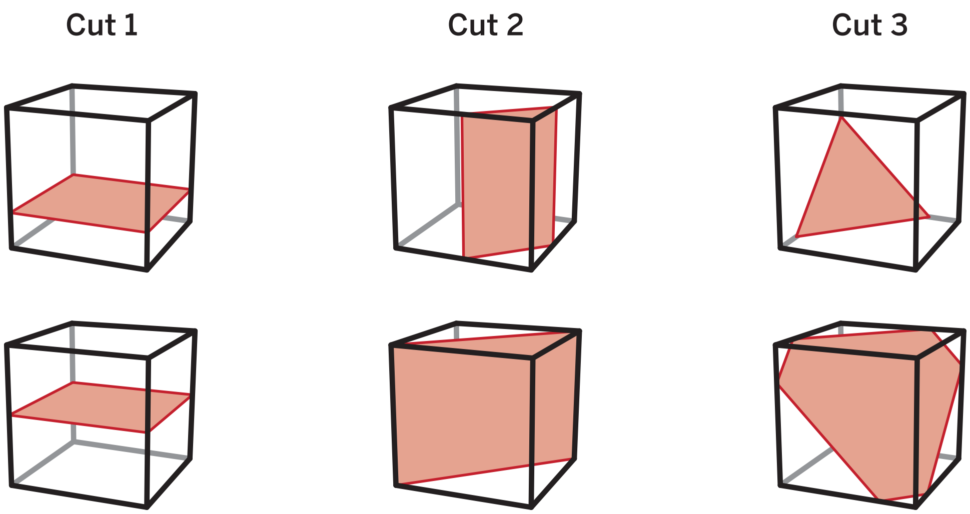


Slicing Solids

Let's explore and describe cross sections of solids.

Warm-Up

1 A plane cuts a cube into two pieces. When the plane cuts the cube parallel to the *base*, the **cross section** is a square. Here are different ways of cutting the cube and their cross sections.



What do you notice? What do you wonder?

I notice:

Responses vary.

- I notice that the cross sections are different shapes with different numbers of sides.
- I notice that one cross section is a long and skinny rectangle.
- I notice that one cross section is a hexagon, but not all the sides are the same length.

I wonder:

Responses vary.

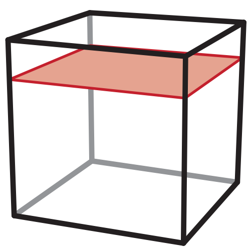
- I wonder if there is a cross section for a cube with more than 6 sides.
- I wonder if there are any other “cuts” that can be used to create cross sections.
- I wonder if all three-dimensional shapes have this many cross sections that can be made.

Name: .... Date: .... Period: ....

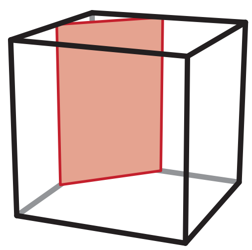
Creating Cross Sections

2 Different cuts create different cross sections.

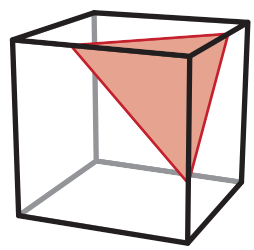
Cut 1



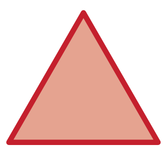
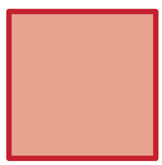
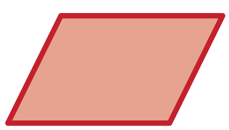
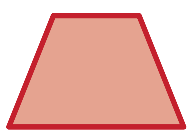
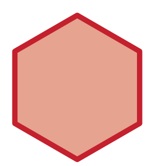
Cut 2



Cut 3

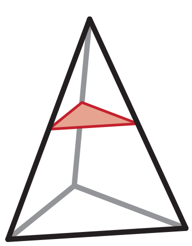


Select *all* of the cross sections you think you can make from a cube.

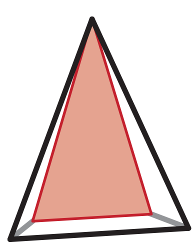
- [x] **A.** 
- [x] **B.** 
- [x] **C.** 
- [ ] **D.** 
- [ ] **E.** 
- [x] **F.** 

3 Here is a triangular *pyramid*.

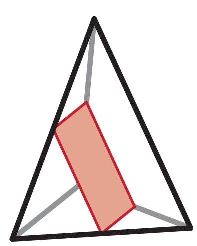
Cut 1



Cut 2



Cut 3



Select *all* of the shapes you think you can make.

- [x] **A.** Equilateral triangle
- [x] **B.** Isosceles triangle
- [x] **C.** Rectangle
- [x] **D.** Trapezoid
- [ ] **E.** Hexagon

Name: .... Date: .... Period: ....

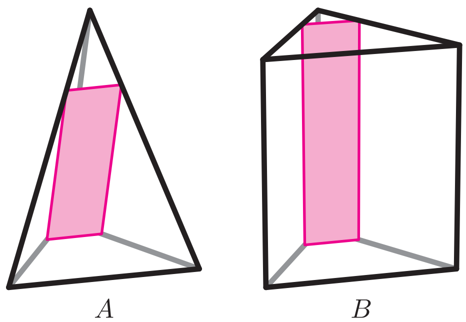
Creating Cross Sections (continued)

4 Amir says it's possible to cut a rectangular cross section of figure A.

Peter says it's possible to cut a rectangular cross section of figure B.

Whose claim is correct? Circle one.

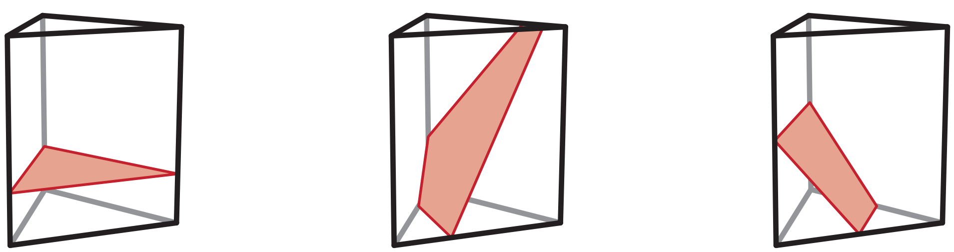
Amir Peter Both Neither



Show or explain your thinking.

***Explanations vary.* Amir can create a rectangular cross section of figure A using Cut 3 from the previous problem. Peter can create a rectangular cross section of figure B using a vertical cut.**

5 Here is a new *prism*.



What is the greatest number of sides a cross section could have?

A. 4 **B.** 5 **C.** 6 **D.** More than 6

Explain your thinking.

***Explanations vary.* Cut 2 creates a 5-sided cross section. I don't think you can create a 6-sided cross section because the prism only has 5 faces.**

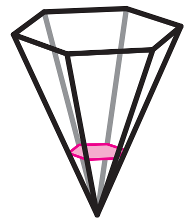
Activity 2

Name: ________ Date: ________ Period: ________

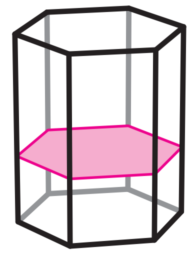
Prisms and Pyramids

6 Here is a pyramid and a prism. They have identical hexagon bases.

Pyramid



Prism



If you cut the solids parallel to their bases, how would the cross sections be alike? ***Responses vary.***

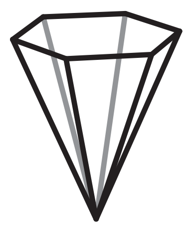
- **Both cross sections are hexagons.**
- **Both are the same shape as the base.**
- **They are scaled copies of each other.**

How would they be different? ***Responses vary.***

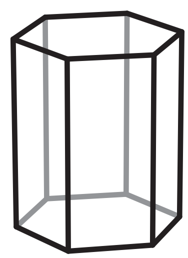
- **The cross section of the pyramid is almost always smaller than the cross section of the prism.**
- **The cross section of the prism is the same size as its base. The cross section of the pyramid is smaller than its base.**

7 Imagine cutting each solid with a vertical cut.

Pyramid



Prism



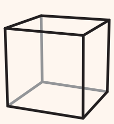
What might the cross section of the pyramid look like?
***Responses vary.* The cross section is a triangle that is taller than it is wide.**

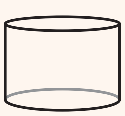
What might the cross section of the prism look like?
***Responses vary.* The cross section is a rectangle as tall as the prism.**

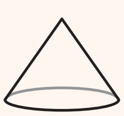
Explore More

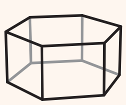
8 Match each solid with exactly one possible cross section.

Solid

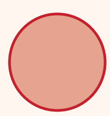
a. 

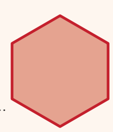
b. 

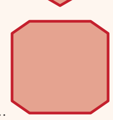
c. 

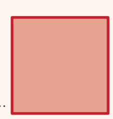
d. 

Cross Section

c 

a 

d 

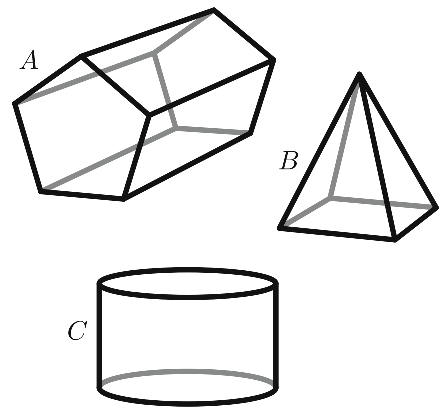
b 

Note: While there is only one full solution, some of the individual solids can be cut to create more than one cross section. Here is a complete list: Cube: square, hexagon; Cylinder: circle, square; Cone: circle; Hexagonal prism: square, hexagon, octagon.

9 Synthesis

Describe why different cuts of a solid create different possible cross sections. Use the solids if they help with your thinking.

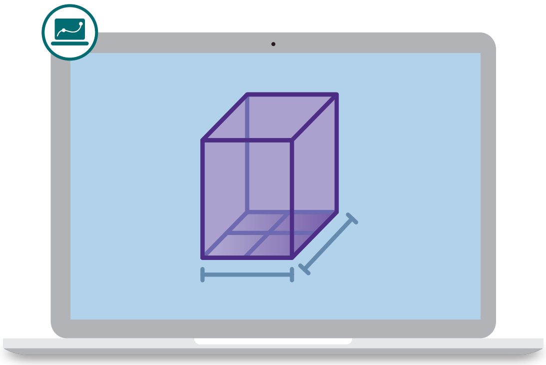
Responses vary. **Different cross sections are possible because I can perform different cuts at different angles. Some solids have more cross sections than others. For example, in figure A, I can create a triangle, rectangle, pentagon, hexagon, and heptagon. In figure B, I can only create a triangle, rectangle, and pentagon.**



Things to Remember:

Unit 7
Lesson
10

Name: ________ Date: ________ Period: ________



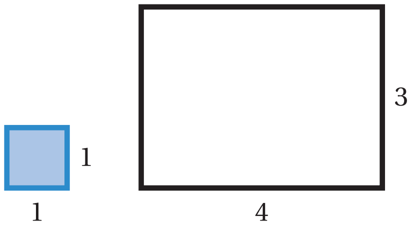
Simple Prisms

Let's calculate the volume of prisms.

Warm-Up

1 How many unit squares will it take to cover this rectangle?

12 unit squares



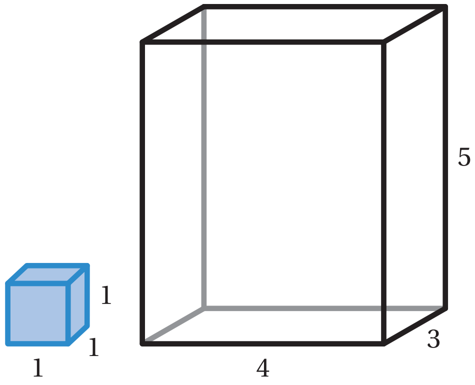
2 Here is a rectangular prism.

a How many unit cubes will it take to fill this rectangular prism?

60 unit cubes

b **Discuss:** What strategy did you use?

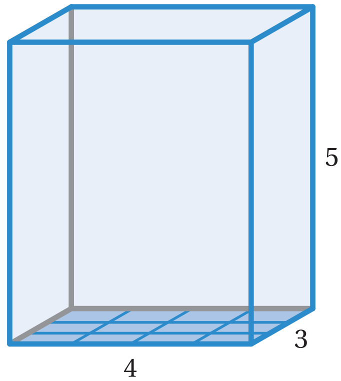
***Responses vary*. I thought about how many unit cubes would fill up the bottom layer of the prism. Then, I thought about how many layers of cubes I would need in order to fill the whole prism.**



Name: .... Date: .... Period: ....

Volume of Prisms

3 Here is how Mio calculated the *volume* of the prism:



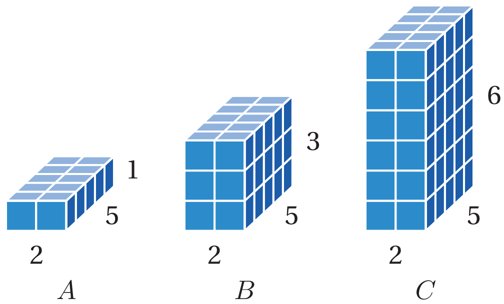
$$V = 12 \cdot 5$$

Explain what 12 and 5 represent.

12 represents . . . **the area of the base (3×4).**

5 represents . . . **the height.**

4 Here are three rectangular prisms: A, B, and C. They all have the same base.



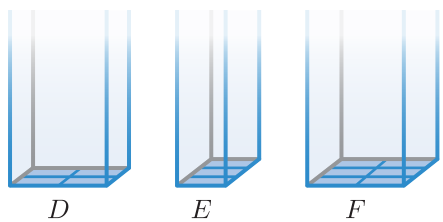
Use Mio's strategy to determine the missing volumes.

$A = 10$ cubic units

$B = 30$ cubic units

$C = 60$ cubic units

5 Here are the bases of prisms D, E, and F. They all have the same volume.



Which prism has the greatest height? Circle one.

D E F Not enough information

Explain your thinking.

Explanations vary.

- **E has the greatest height because it has the smallest base area.**
- **E has only 3 cubes per layer, while D and F have more cubes per layer. So E would need to have more layers in order to have the same volume as D and F.**

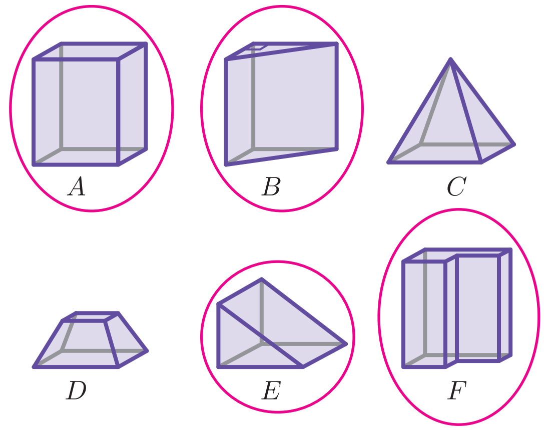
Activity 2

Name: ______ Date: ______ Period: ______

Volume Calculation Strategies

6 Here's how Mio described her strategy for determining the volume:

Step 1: Calculate the area of the base.
Step 2: Multiply that area by the height of the object.

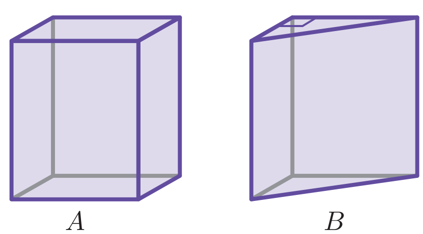


a Circle *all* of the objects that Mio's strategy will work for.

b **Discuss:** How did you decide which objects to select?

***Responses vary*. I chose the four objects that had two congruent bases.**

7 Here are two objects from the previous problem. One is a rectangular prism. The other is a triangular prism.



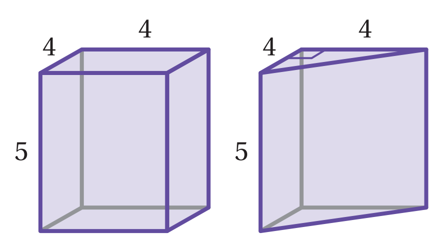
What information would you need to calculate each of their volumes?

Responses vary.

- **If I knew all the dimensions of each object (length, width, and height), I could calculate their volumes.**
- **I need to know the base area and height of each prism.**

8 Calculate the volume of each prism.

	Volume (cubic units)
Rectangular Prism	80
Triangular Prism	40

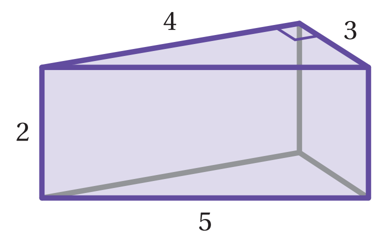


Name: ________ Date: ________ Period: ________

Volume Calculation Strategies (continued)

9 Here is a new triangular prism. Calculate its volume.

12 cubic units



10 Nasir and Omari also calculated the volume of this triangular prism. They each made a mistake.

Nasir	Omari
$\frac{1}{2} \cdot 4 \cdot 3 \cdot 5 = 30$ cubic units	$4 \cdot 3 \cdot 2 = 24$ cubic units

Choose one student and consider their calculations.

a **Discuss:** What did the student do well?

Responses vary.

- Nasir used the correct approach for calculating the area of the triangular base.
- Omari correctly identified the height of the object.

b What would you recommend the student change about their calculations?

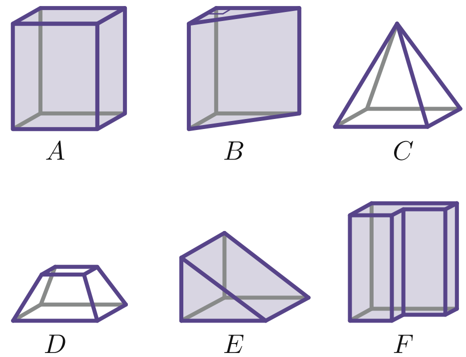
Responses vary.

- I would recommend that Nasir think carefully about what the height of the object is. The height is the side length that connects one base to another, which in this prism is 2 and not 5.
- I would recommend Omari calculate the area of a triangle as half of the base times the height.

11 Synthesis

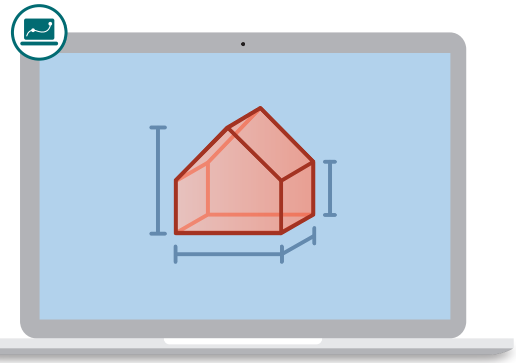
Describe how to determine the volume of a prism. Draw if it helps to show your thinking.

Responses vary. **I can determine the volume of a prism by calculating the area of the base first, then multiplying that number by the height.**



Things to Remember:

Name: .... Date: .... Period: ....

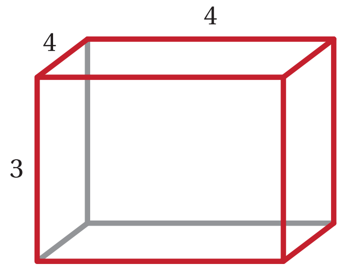


Complex Prisms

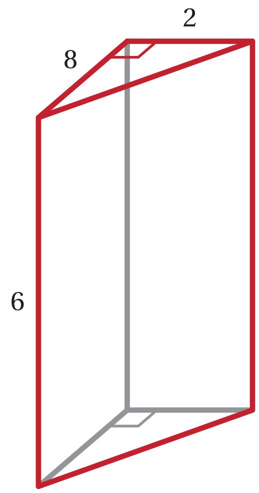
Let's determine the volume of prisms
with other bases.

Warm-Up

1 Calculate the volume of each prism.



48 cubic units



48 cubic units

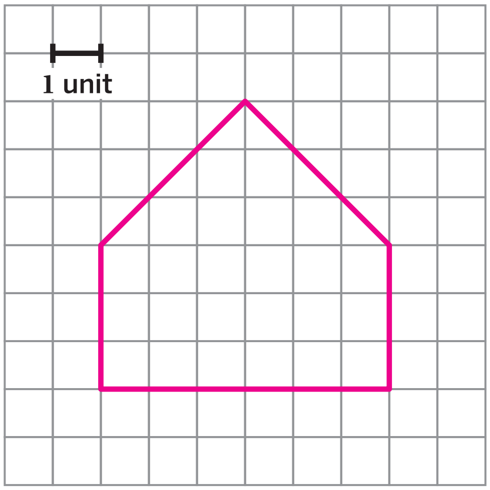
Activity 1

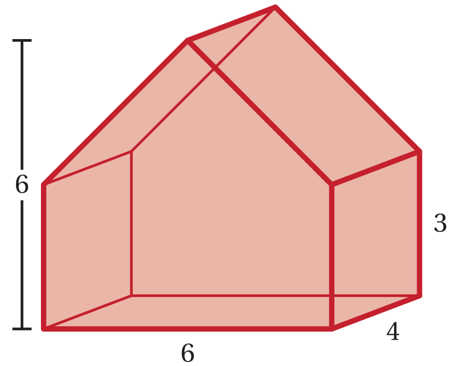
Name: .... Date: .... Period: ....

Using the Base to Calculate Volume

2 Sketch the base of this prism on the grid.

Sample shown on grid.





3 What is the area of the prism's base?

27 square units

Show or explain how you determined the area.

Explanations vary.

- **I broke the shape up into a rectangle and a triangle. The area of the rectangle is $6 \cdot 3 = 18$ square units. The area of the triangle is $\frac{1}{2} \cdot 6 \cdot 3 = 9$ square units. In total, the area is $18 + 9 = 27$ square units.**
- **I made a big 6-by-6 unit square. There are two triangles not included in the square that each have an area of 4.5 square units, so the total is $36 - 4.5 - 4.5 = 27$ square units.**

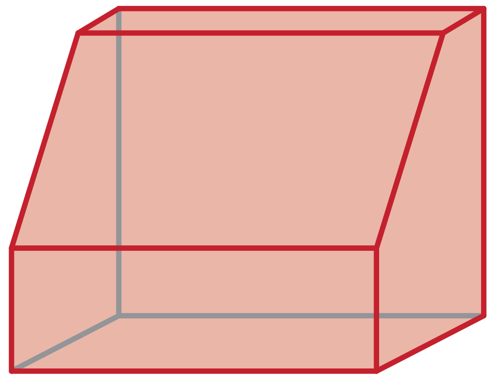
4 What is the volume of the prism?

108 cubic units

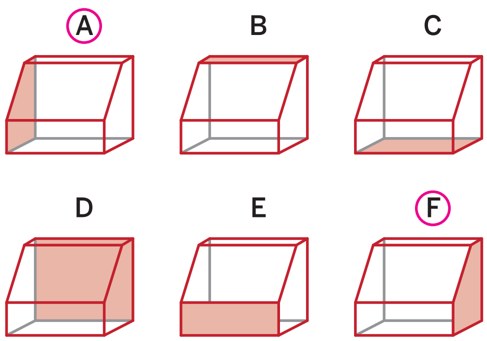
Name: ____________ Date: ________ Period: ________

Prisms With Other Bases

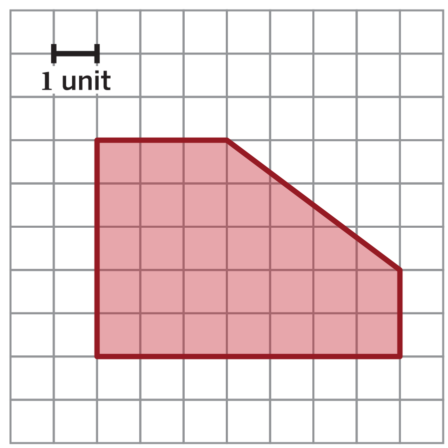
5 Here is a new prism.



Circle *all* the faces that are bases you could use to calculate the prism's volume.

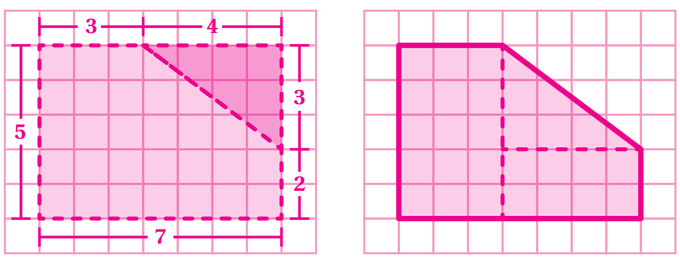


6 Here is one face of the prism.

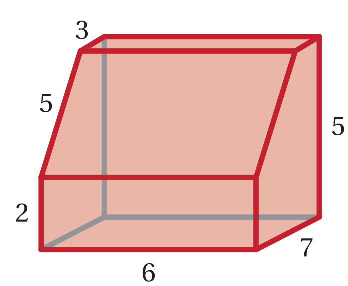


What is its area? Show your thinking.

29 square units. Sample sketches shown.



7 Jaylin says the volume of this prism is $29 \cdot 5$, or 145 cubic units.



Kimaya claims that it's $29 \cdot 6$, or 174 cubic units.

Whose claim is correct? Circle one.

Jaylin Kimaya Both Neither

Explain your thinking.

***Explanations vary.* Even though a vertical distance measures 5 units, the height of a prism is how far apart the two bases are. The bases are the pentagons, and the distance between them is 6 units.**

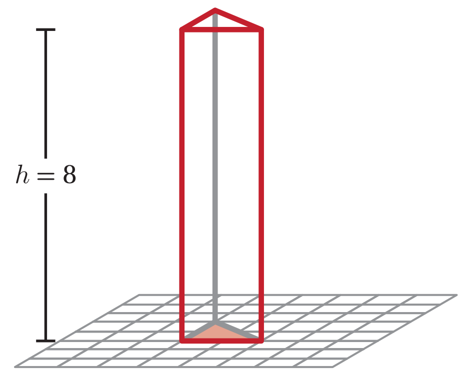
Activity
2

Name: .... Date: .... Period: ....

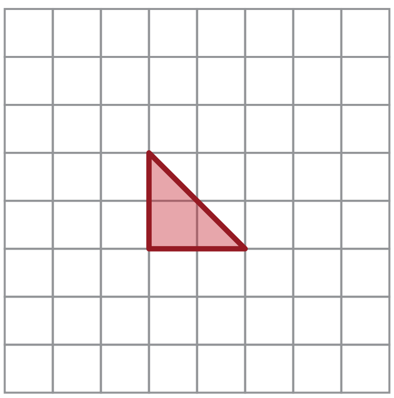
Prisms With Other Bases (continued)

8 This triangular prism has a height of 8 units and a volume of 16 cubic units. Draw a new base so its volume is 48 cubic units.

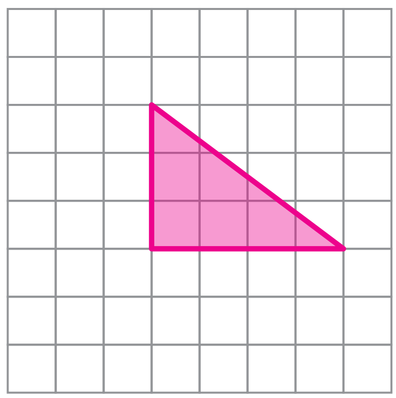
***Responses vary.* Any triangle whose area is 6 square units. Sample sketch shown.**



Base



New Base

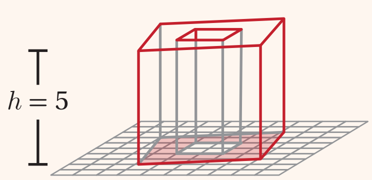


Explore More

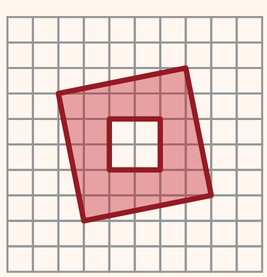
9 A prism has a height of 5 units and a volume of 110 cubic units. Its base is made of two squares. Draw a new base, made of two squares, that creates a prism with a volume of 80 cubic units.

Responses vary.

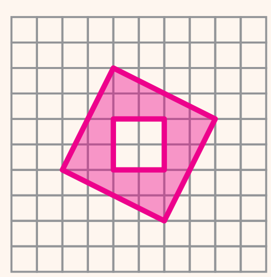
Prism



Base

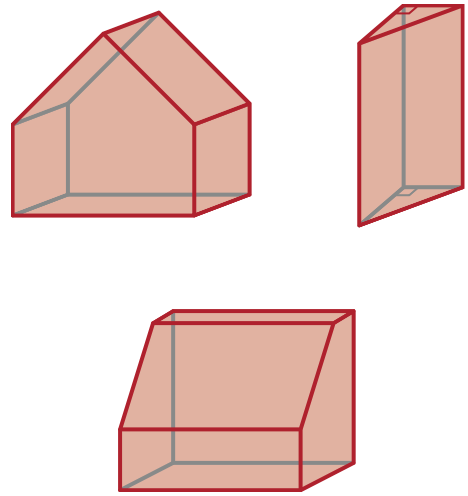


New Base



10 Synthesis

Here are several prisms you've seen in this lesson.



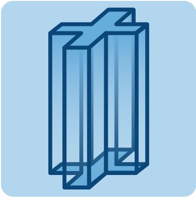
Describe a general strategy for determining the volume of any prism.

Responses vary. **First, determine which faces are the bases. It isn't always the face on the bottom. Then calculate the area of the base. You can either break the shape into triangles and rectangles or make a bigger shape and take away any parts that are not there. Once you have the area of the base, multiply by the height, which is how far apart the bases are.**

Things to Remember:

Name: .... Date: .... Period: ....

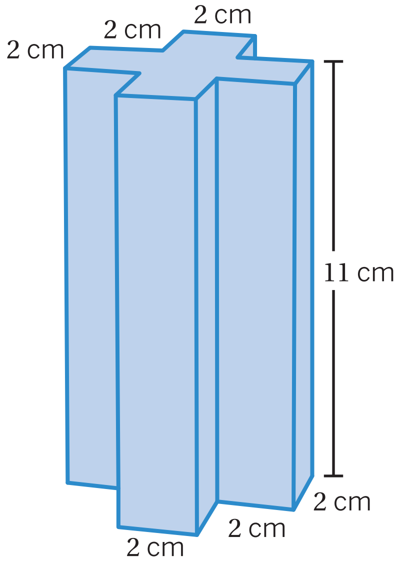
Surface Area Strategies



Let's calculate the surface area of different prisms.

Warm-Up

1. Here is a prism.



a How many *faces* does this prism have? Explain your thinking.

14 faces. *Explanations vary*. There are 2 bases and 12 sides. I counted the number of sides by counting the number of edges of the plus sign.

b Calculate the area of the prism's base.

20 square centimeters

Activity 1

Name: .... Date: .... Period: ....

Examining Different Strategies

2. Here are Amoli's, Nyanna's, and Miko's strategies for calculating the *surface area* of the prism from the Warm-Up.

Amoli

I have to draw each of the 14 different faces, find their areas, and add them up.

Nyanna

There are only two different shapes: the plus sign and the rectangle. I can find the area of each shape and use a calculator to multiply by the number there are of each shape.

Miko

I see another way! Imagine unfolding the prism into a net. I can use one large rectangle instead of 12 smaller ones.

a **Discuss:** How would you describe each student's strategy in your own words?

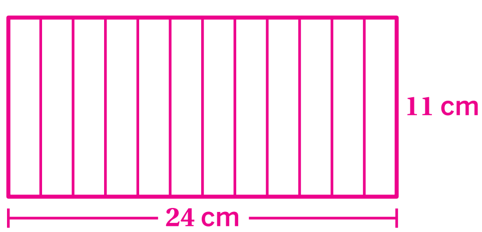
Responses vary.

b How are Amoli's and Nyanna's strategies alike? How are they different?

Responses vary.

3. Let's look at Miko's strategy more closely.

a Sketch the "one large rectangle" Miko is talking about.



b What are the dimensions of this rectangle? Show or explain your thinking.

11 centimeters by 24 centimeters. *Explanations vary.* The large rectangle is made up of 12 smaller rectangles. Each of the smaller rectangles has a length of 2 centimeters, so the large rectangle must have a length of $2 \cdot 12 = 24$.

4. Use any strategy to calculate the surface area of this solid. Organize your thinking and calculations so that others can follow them.

304 square centimeters. *Strategies vary.*

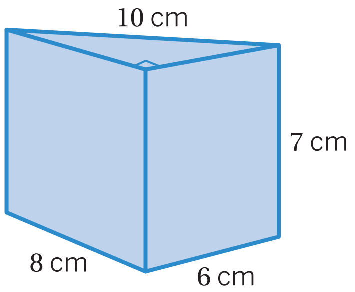
Name: .... Date: .... Period: ....

Calculating Surface Area

Here are three prisms. For each prism:

- Determine how many faces the prism has.
- Use any strategy to calculate the surface area. Organize your thinking and calculations so that others can follow them.
- Trade papers with your partner. Work together to reach an agreement.

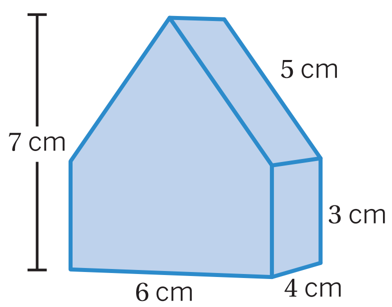
5.



Number of faces: 5

Surface area: 216 square centimeters

6.



Number of faces: 7

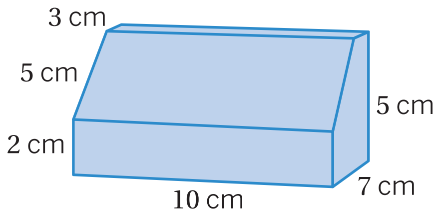
Surface area: 148 square centimeters

Activity
2

Name: .... Date: .... Period: ....

Calculating Surface Area (continued)

7.



Number of faces: *7*

Surface area: *278 square centimeters*

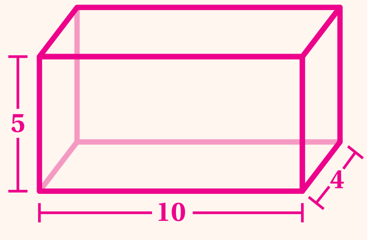
8. **Discuss:** Is your strategy more similar to Amoli's, Nyanna's, or Miko's? What about your partner's strategy?

Responses vary.

Explore More

9. Sketch a prism with a surface area of 220 square units.

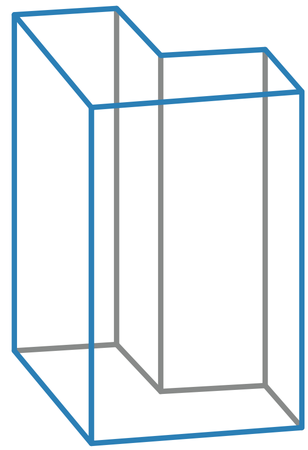
Responses vary.



Synthesis

10. Describe your favorite method for calculating the surface area of a prism.

Use this prism if it helps you with your explanation.



Responses vary. I would first find the area of the base by cutting the L shape into two rectangles. Then I would calculate the perimeter of the L shape and multiply it by the height of the prism. That would give me the surface area of all the sides. Finally, I would add the surface area of the sides plus two times the area of the base.

Things to Remember:

Unit 5

Lesson

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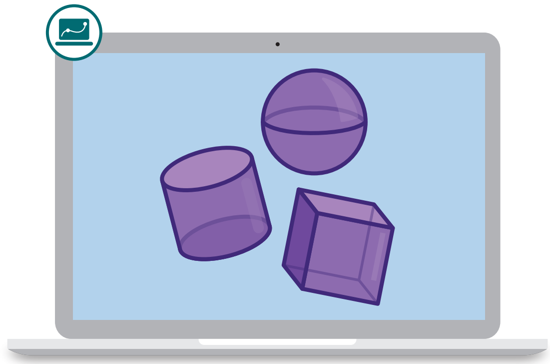


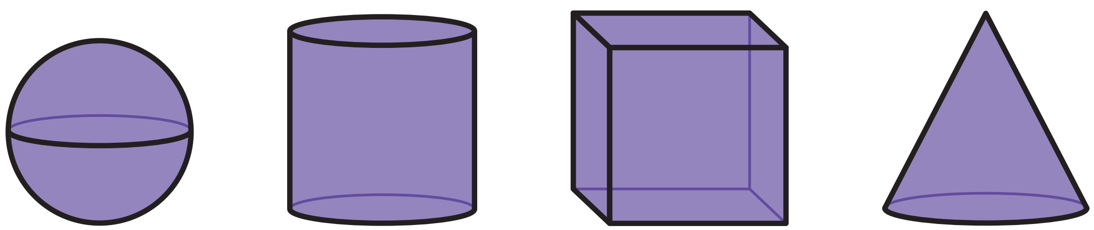
Volume Lab

Let's estimate the volume of three-dimensional solids.

Warm-Up

1 Which one doesn't belong? Explain your thinking.

Figure A Figure B Figure C Figure D



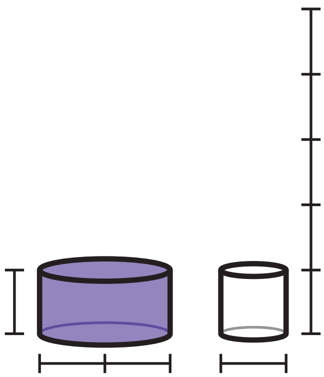
Activity
1

Name: .... Date: .... Period: ....

Comparing Volumes

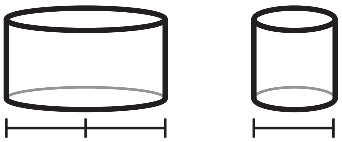
2 Here are two **cylinders**.

Draw a new height for the cylinder on the right so that both cylinders have the same *volume*.



3 Here are two cylinders with the same height.

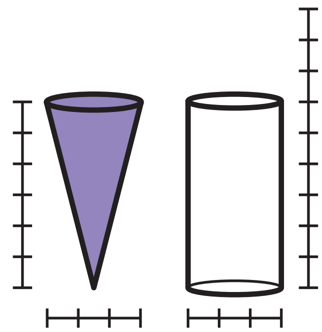
How many small cylinders do you think it would take to fill the large cylinder?



Name: .... Date: .... Period: ....

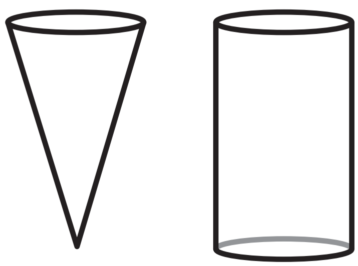
Comparing Volumes (continued)

4 Draw a new height for the cylinder so that both objects have the same volume.



5 Here is a **cone** and a cylinder with the same height and *diameter*.

How many cones would it take to fill the cylinder?



Activity 2

Name: .... Date: .... Period: ....

Volume Lab

6 Let's use Screen 6 of the digital activity to explore some volume relationships!

a Select any *two* objects and adjust their dimensions. Then press "Compare." Repeat this with several pairs of objects. Draw or record something that you found interesting or surprising.

b Two cones have equal diameters. The height of one cone is 2 times as large as the height of the other cone. How are the volumes of the cones related?

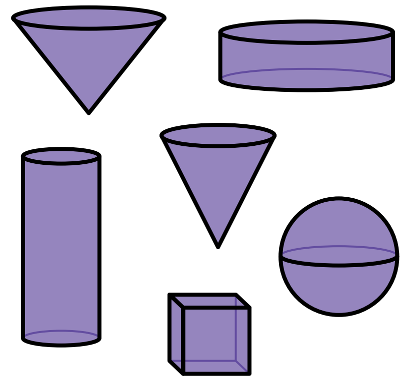
c Two cylinders have the same height. The diameter of one cylinder is 3 times as large as the diameter of the other cylinder. How are the volumes of the cylinders related?

d Describe the relationship between two different objects, where one has twice the volume of the other.

e Describe another interesting relationship between the volumes of two different objects.

7 Synthesis

Describe one volume relationship you discovered during this lesson.



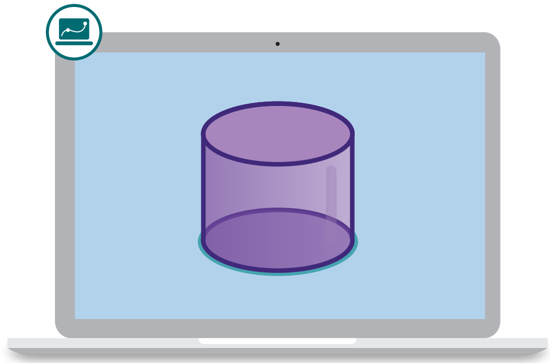
Things to Remember:

Unit 5

Lesson

11

Name: .... Date: .... Period: ....

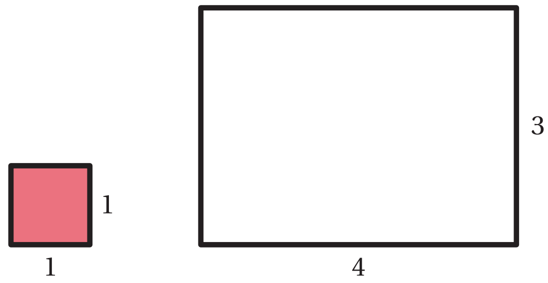


Cylinders

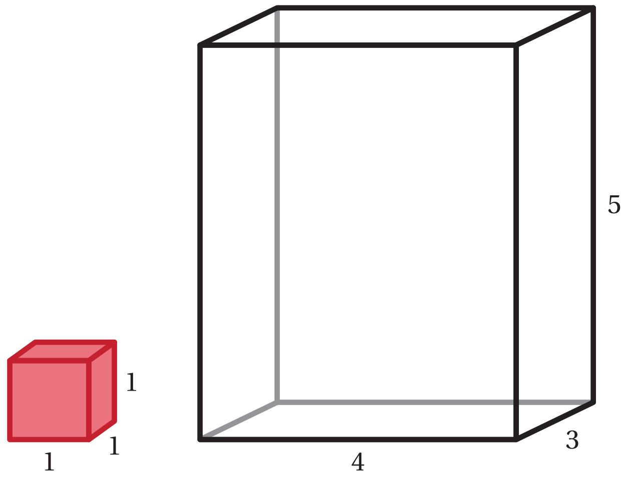
Let's calculate the volume of cylinders.

Warm-Up

1 How many unit squares are needed to cover this rectangle?



2 How many unit cubes are needed to fill this rectangular prism?



Activity 1

Name: .... Date: .... Period: ....

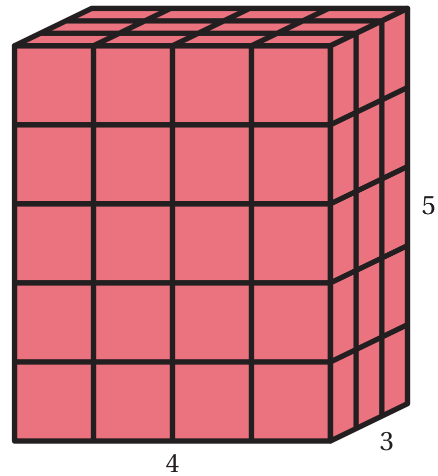
Volume Strategies

3 DeAndre calculated the volume in three ways:

- $V = 12 \bullet 5$
- $V = 15 \bullet 4$
- $V = 20 \bullet 3$

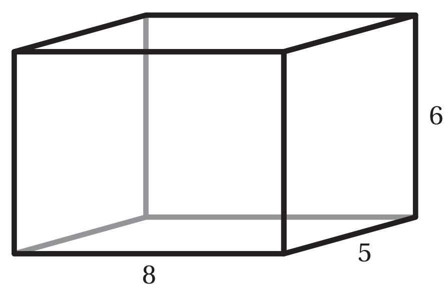
Let's focus on DeAndre's first equation.

Explain what 12 and 5 represent in the diagram.



4 Here is a new rectangular prism.

Use DeAndre's strategy to write an expression for its volume.

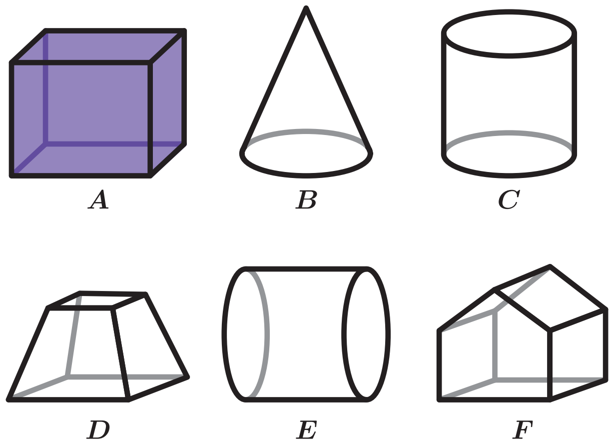


Name: ________ Date: ________ Period: ________

Volume Strategies (continued)

5 DeAndre described his strategy for calculating the volume of a solid.

- First, find the area of the base.
- Then multiply that area by the height of the object.

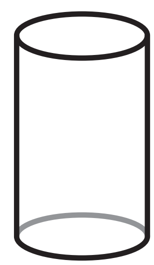


a Circle *all* of the objects for which DeAndre's strategy will work.

b **Discuss:** How did you decide which objects to choose?

6 DeAndre's strategy for calculating volume works for cylinders.

What information would you need to calculate the volume of this cylinder?



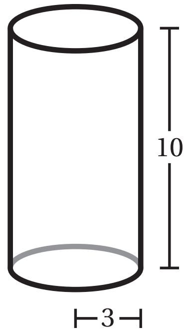
Activity
2

Name: ________ Date: ________ Period: ________

Cylinder Volumes

7 This cylinder has a height of 10 units and a *radius* of 3 units.

Calculate the volume of the cylinder.



8 Caasi incorrectly determined the volume of the cylinder with this calculation:

$V = 3 \cdot 3 \cdot 10 = 90$

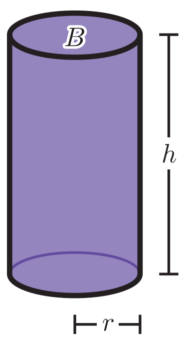
What did Caasi do well and what mistake did she make?

9 Here are two formulas for the volume of a cylinder:

$V = \pi r^2 \cdot h$

$V = B \cdot h$

Describe how the formulas are related.



Activity 3

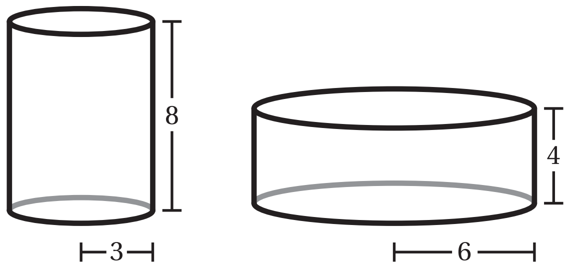
Name: .... Date: .... Period: ....

Calculating Cylindrical Volume

10 Calculate the volume of each cylinder.

Tall cylinder:

Short cylinder:



11 Sketch two different cylinders that have the same volume. What is the volume of each cylinder?

Cylinder A **Cylinder B**

Explore More

12 **a** Write dimensions for a rectangular prism and a cylinder with volumes that are close to equal.

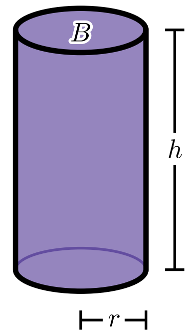
Rectangular prism:

Cylinder:

b Explain why they have close to equal volumes.

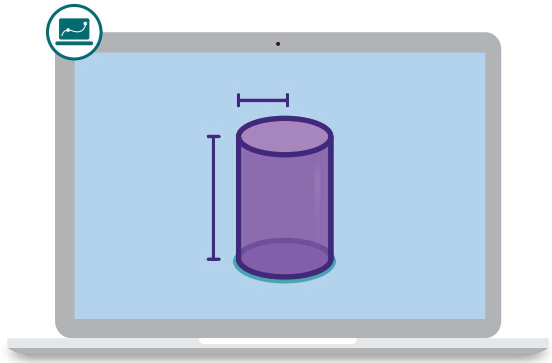
13 Synthesis

Describe a strategy for determining the volume of a cylinder given its radius and height.



Things to Remember:

Name: .... Date: .... Period: ....



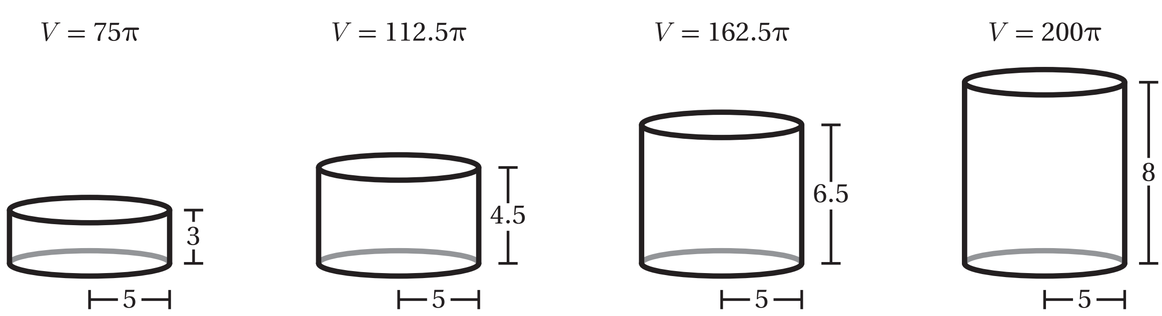
Scaling Cylinders

Let's see how changing a cylinder's radius or height impacts its volume.

Warm-Up

1 We learned that in a function, the independent variable represents the input and the dependent variable represents the output.

Here are several cylinders:



In this situation, what could the independent and dependent variables be?

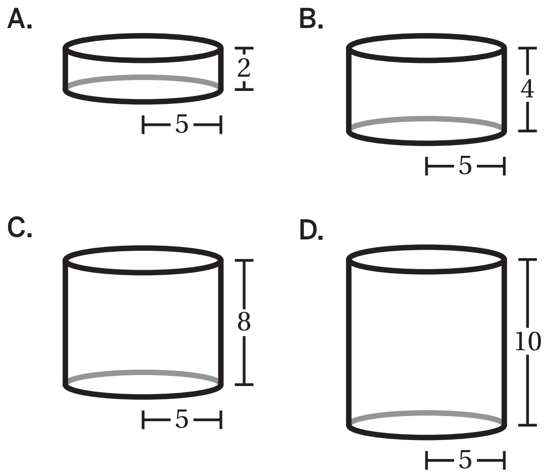
Independent variable: ....

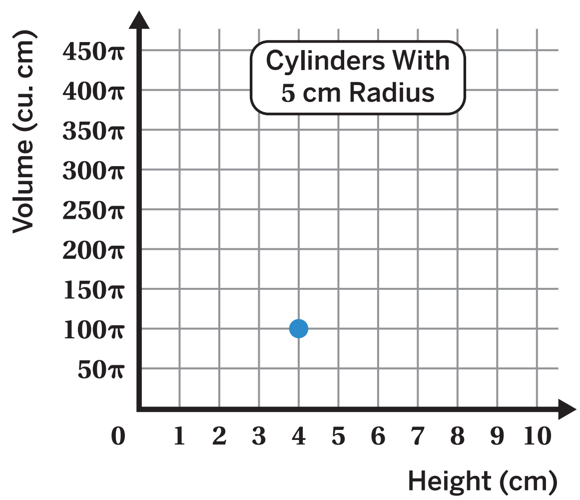
Dependent variable: ....

Name: .... Date: .... Period: ....

Changing the Height

2 Select the cylinder that represents the plotted point. Explain to a classmate how you chose.





3 Let's watch an animation graphing the relationship between a cylinder's height and its volume.

Discuss: What do you notice? What do you wonder?

4 Let's look at a graph that represents the relationship between the height and the volume for cylinders with a radius of 5 centimeters.

Use the graph and the table to help you find the volume of each of the four cylinders.

Express each volume in terms of π.

Object	Height	Volume (cu. cm)
Cylinder A	2	
Cylinder B	4	100π
Cylinder C	8	
Cylinder D	16	

Activity 2

Name: .... Date: .... Period: ....

Changing the Radius

5 Let's see what happens if we keep the height of a cylinder constant but change the radius.

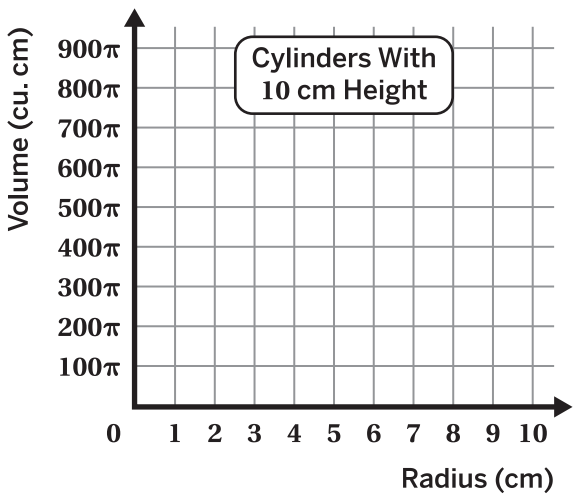
a Choose and record a radius for two different cylinders that each have a height of 10 centimeters.

	Radius (cm)	Volume (cu. cm)
Cylinder 1		
Cylinder 2		

b Calculate the volume for each cylinder. Express each volume in terms of π.

6 **a** Plot points to represent your two cylinders.

b Make a sketch of what you think the graph looks like for *all* cylinders with a height of 10 centimeters.



7 Let's watch an animation graphing the relationship between a cylinder's radius and its volume.

Discuss: Is this relationship a linear function? Explain your thinking.

Name: ________ Date: ________ Period: ________

Changing the Radius (continued)

8 Let's look at a graph that represents the relationship between the radius and the volume for cylinders with a height of 10 centimeters.

Use the graph and the table to help you find the volume of each of the four cylinders.

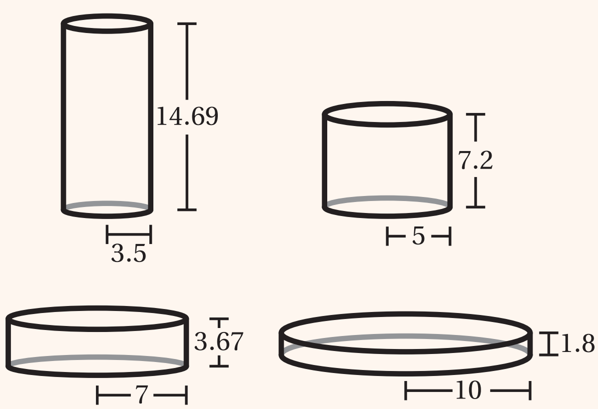
Express each volume in terms of π.

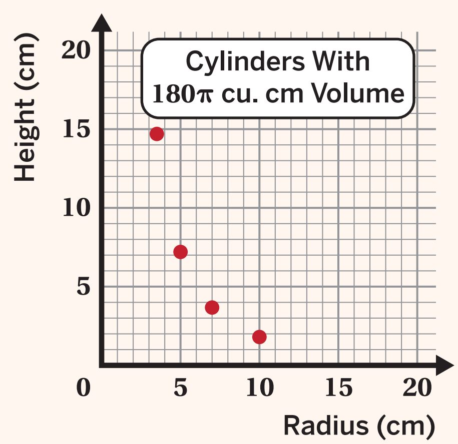
Object	Radius (cm)	Volume (cu. cm)
Cylinder E	2	
Cylinder F	4	
Cylinder G	8	
Cylinder H	16	

Explore More

9 Explore the relationship between radius and height when the volume of a cylinder is fixed. Here are several cylinders that all have a volume of 180π.

Discuss: What do you notice? What do you wonder?

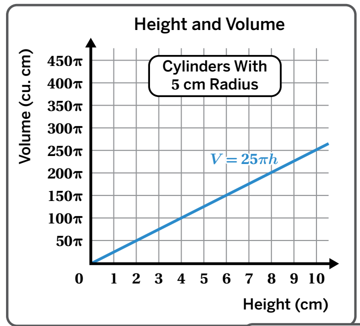


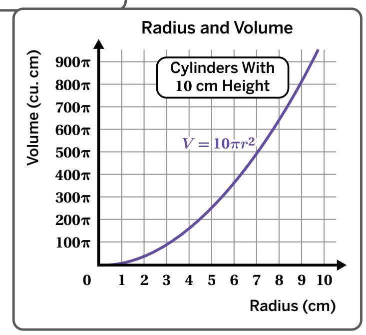


10 Synthesis

Here are the relationships we explored today.

How can you tell if the relationship between height and volume and the relationship between radius and volume are linear or non-linear?





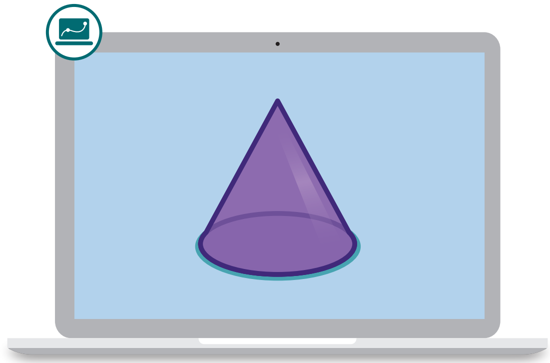
Things to Remember:

Unit 5

Lesson

13

Name: .... Date: .... Period: ....



Cones

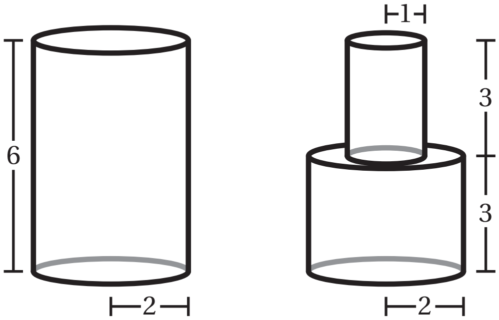
Let's explore cones and their volumes.

Warm-Up

1 Determine the volume of figure A (a cylinder) and figure B (which is composed of two cylinders).

Write their volumes in terms of π.

Figure	Volume (cu. cm)
A	
B	



A B

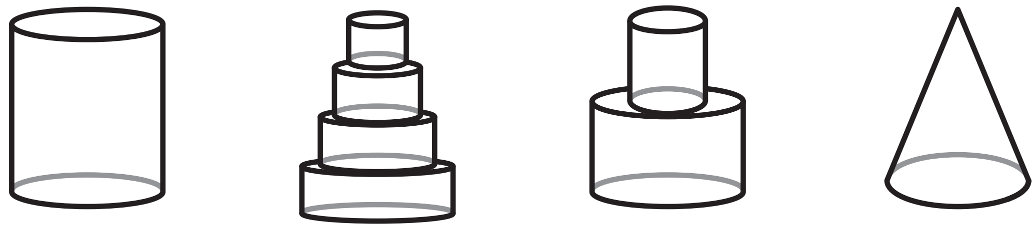
Activity 1

Name: ______ Date: ______ Period: ______

Estimating the Volume of a Cone

2 Here are four figures with the same height and the same radius for their largest bases.

Cylinder | **Four cylinders** | **Two cylinders** | **Cone**



Order the figures by volume from *least* to *greatest*.

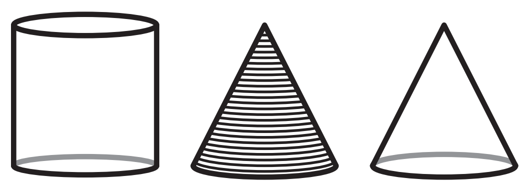
Least **Greatest**

3 Let's see what happens when we increase the number of cylinders.

What do you think the exact volume of the cone might be?

Explain your thinking.

$V = 24\pi$ $V = 8.49\pi$ $V = ?$



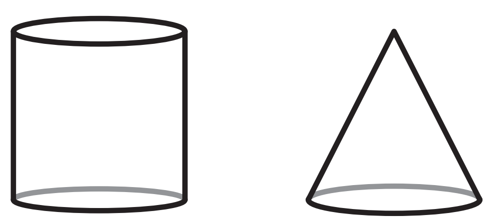
Cylinders: 25

4 Here is another set of figures.

a Estimate the volume of the cone.

b Let's see what happens when we increase the number of cylinders.

$V = 30\pi$ $V = ?$



Discuss: What do you notice about the relationship between cylinders and cones?

Activity 2

Name: ____ Date: ____ Period: ____

Volume of a Cone

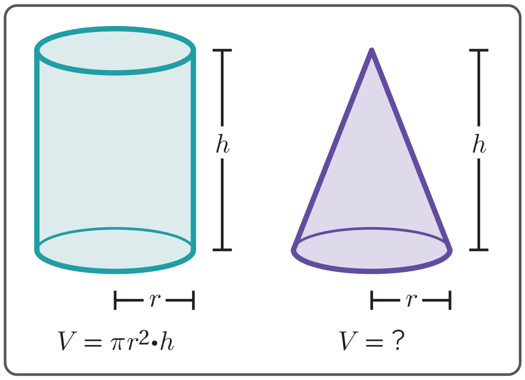
5 Each row of the table shows the volumes of a cylinder and a cone with the same height and radius.

Fill in the unknown values.

Volume of Cylinder (cu. cm)	Volume of Cone (cu. cm)
24π	8π
30π	10π
120π	
60π	
	15π

6 One way of writing a formula for the volume of a cylinder with radius r and height h is $V = \pi r^2 \cdot h$.

Write a formula for the volume of a cone.

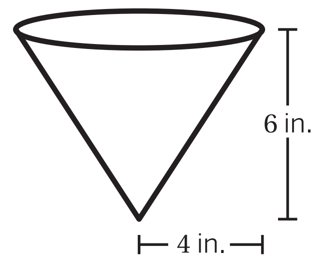


Name: ______ Date: ______ Period: ______

Comparing the Volume of a Cone

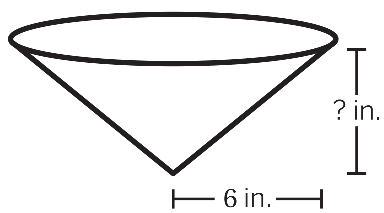
7 Let's look at one way of writing a formula for the volume of a cone with radius r and height h.

Calculate the volume of this cone.



8 The volume of this cone is 60π cubic inches.

What is the height of the cone?

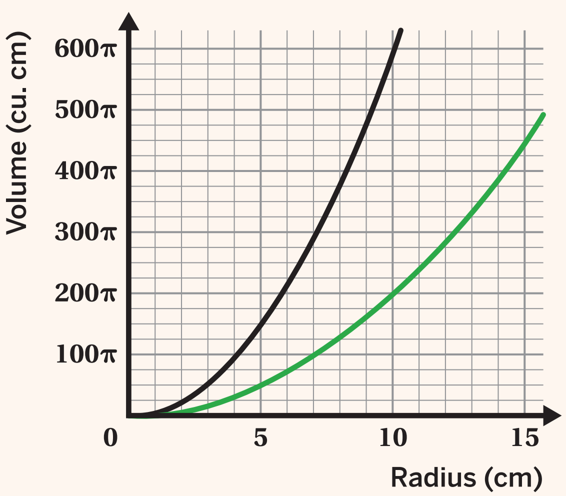


Explore More

9 This graph shows the relationship between radius and volume for cones and cylinders that have the same height.

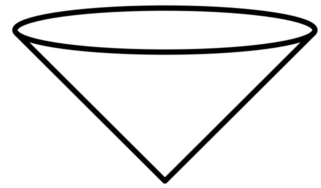
a. Label each curve to show which shape it represents.

b. What is the height of these cylinders and cones? Explain your thinking.



10 Synthesis

One way of writing the formula for volume of a cone is $V = \frac{1}{3}\pi r^2 \cdot h$.



What does each part of the formula represent?

Things to Remember:

Unit 5
Lesson
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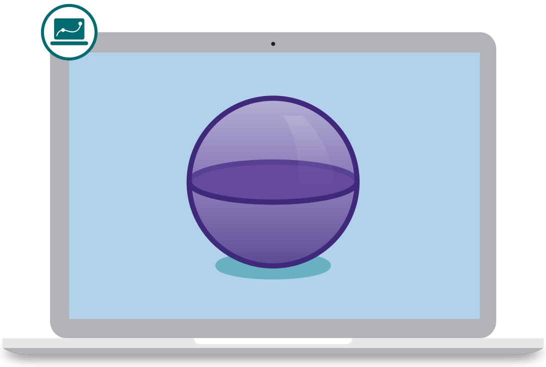
Name: ______ Date: ______ Period: ______



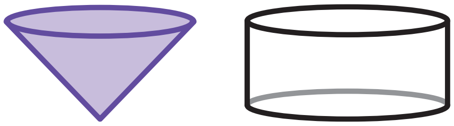
Spheres

Let's develop a formula for the volume of a sphere.

Warm-Up

1 A cone and a cylinder have the same height and radius.

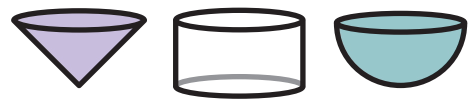
What fraction of the cylinder will be filled by the cone?



Name: .... Date: .... Period: ....

Hemispheres

2 What if we pour both a cone *and* a hemisphere into the cylinder?



Let's see what fraction of the cylinder will be filled.

Describe how the three volumes are related.

3 The volume of the cylinder is 27π cubic units.

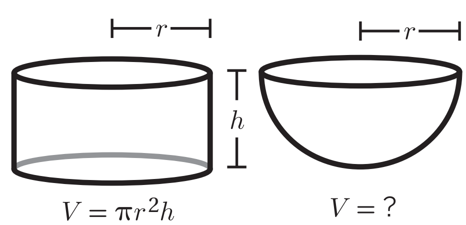
Write the volume of the cone and hemisphere.

Express the volumes in terms of π.

Object	Volume (cu. units)
Cone	
Cylinder	27π
Hemisphere	

4 One way of writing a formula for the volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h is the height.

Write a formula for the volume of a hemisphere. Explain your thinking.



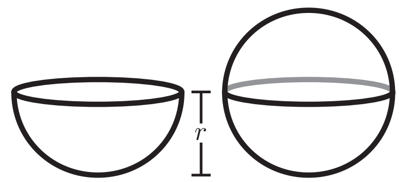
Activity 2

Name: ________ Date: ________ Period: ________

Finding Sphere Dimensions

5 One way of writing a formula for the volume of a hemisphere is $V = \frac{2}{3}\pi r^3$, where r is the radius.

Write a formula for the volume of a sphere. Explain your thinking.



$V = \frac{2}{3}\pi r^3$ $\quad$ $V = ?$

6 Karima and Nasir are calculating the volume of a sphere with a radius of 2 units.

Karima

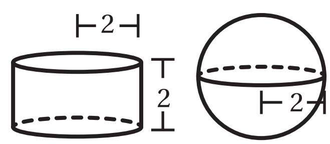
$V = \frac{4}{3}\pi r^3$ $\quad$ $r = 2$

$V = \frac{4}{3}\pi (2)^3$

$V = \frac{4}{3}\pi \cdot 8$

$V = \frac{32}{3}\pi$

Nasir



$V = \pi r^2 h$

$V = \pi (2)^2 \cdot 2$

$V = 8\pi$

$8\pi \cdot \frac{4}{3}$

$V = \frac{32}{3}\pi$

Discuss: How did each student determine the volume of a sphere?

Name: .... Date: .... Period: ....

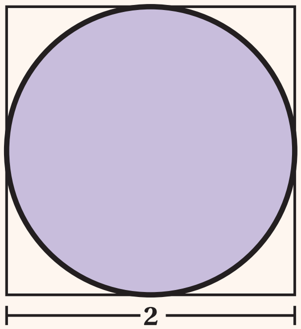
Finding Sphere Dimensions (continued)

7 Complete the table with the unknown dimensions of each sphere. Express your answers in terms of π.

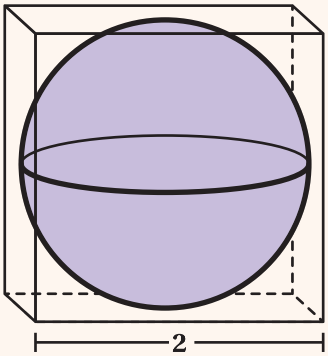
Diameter (units)	Radius (units)	Sphere Volume (cu. units)
	9	
8		
	3	
	6	
9		

Explore More

8 **a** What fraction of the square is filled by the circle?

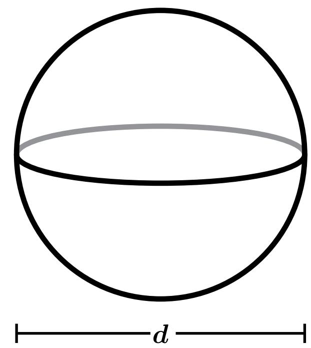


b What fraction of the cube is filled by the sphere?



9 Synthesis

Describe a strategy for determining the volume of a sphere given its diameter.



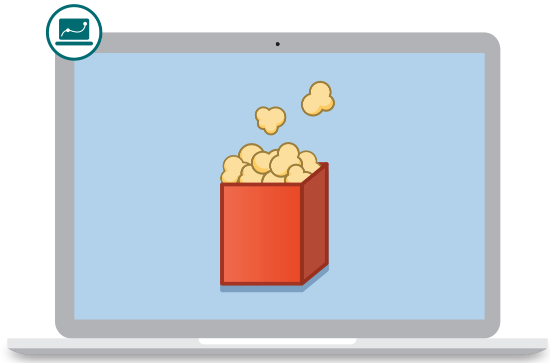
Things to Remember:

Unit 7

Lesson

13

Name: Date: Period:

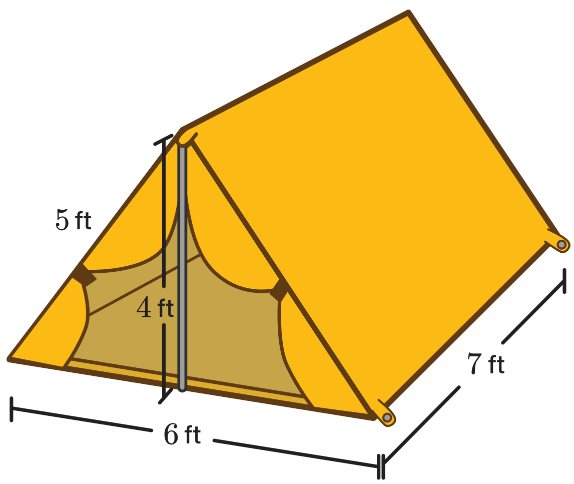


Popcorn Possibilities

Let's apply surface area and volume to real-world situations.

Warm-Up

1 Daeja is trying to determine how much fabric it took to create this tent.



a What would be more useful for Daeja to calculate? Circle one.

Surface area Volume

b Calculate the measurement you selected.

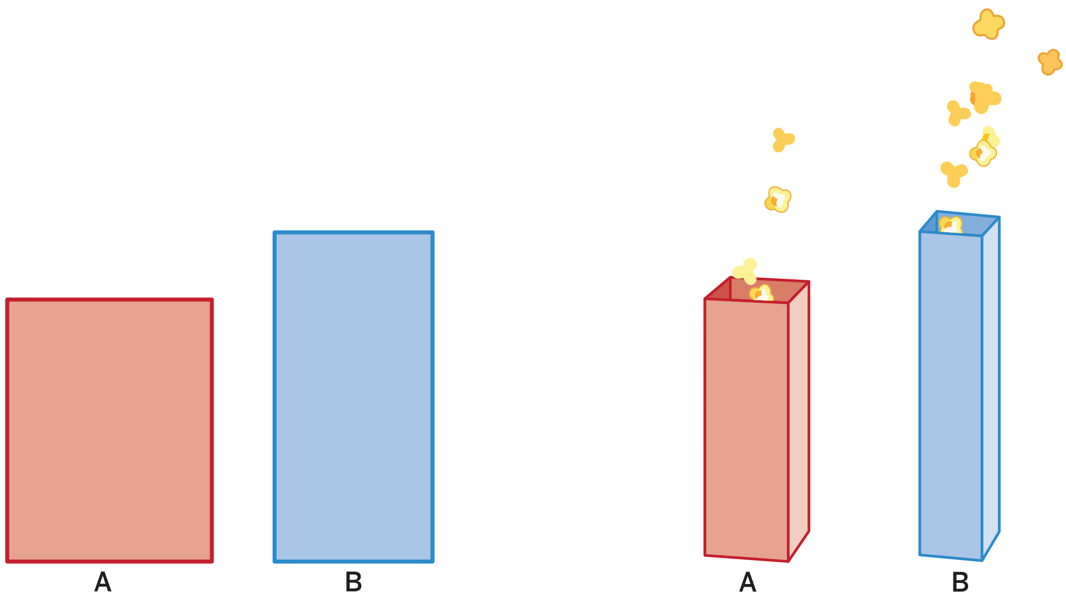
Responses vary.

- **Surface area excluding the bottom rectangle: 94 square feet**
- **Surface area including the bottom rectangle: 136 square feet**

Name: .... Date: .... Period: ....

Which Holds More?

2 Let's watch two 8.5-by-11-inch sheets of paper fold into containers.



a Which container do you think will hold more popcorn? Circle one.

Responses vary.

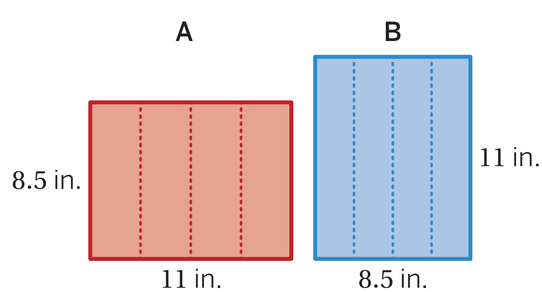
Container A   Container B   They will hold the same amount

b What information would help you know for sure?

Responses vary.

- **It would be helpful to know the length, width, and height of the prisms.**
- **All we need to know are the dimensions of the sheet of paper. We can figure out everything else from those.**

3 Determine the amount of popcorn each container can hold. Show your thinking.



	Width (in.)	Height (in.)	Popcorn (cu. in.)
A	11	8.5	**64.28**
B	8.5	11	**49.67**

Container A: $\frac{11}{4} = 2.75$ **and** $2.75 \cdot 2.75 \cdot 8.5 \approx 64.28$

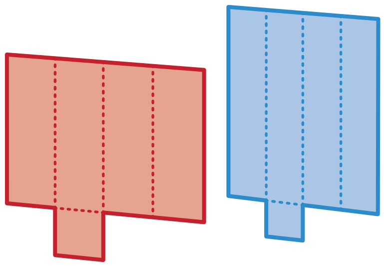
Container B: $\frac{8.5}{4} = 2.125$ **and** $2.125 \cdot 2.125 \cdot 11 \approx 49.67$

Activity 2

Name: .... Date: .... Period: ....

Which Uses More Paper?

4 An extra piece of paper has been added to the bottom of each container (so it can actually hold popcorn).



Which container do you think uses more paper? Circle one.

Responses vary.

Container A   Container B   They use the same amount

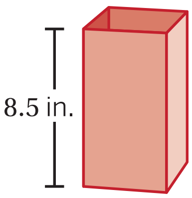
Explain your thinking.

Explanations vary.

- **Container A uses more paper because it has a larger bottom, and the amount of paper to build the rest of the container is the same for both containers.**
- **They use the same amount because they're made of the same materials: one piece of paper and an extra piece for the bottom.**

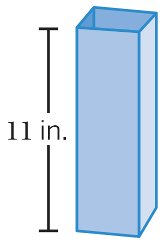
5 Determine the amount of paper each container uses (including the bottom). Show your thinking.

Container A



101.06 square inches
Work varies.

Container B



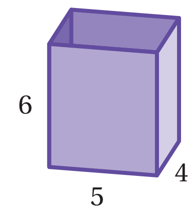
98.02 square inches
Work varies.

Activity 3

Name: .... Date: .... Period: ....

More Popcorn Containers

6 Here is a new container.

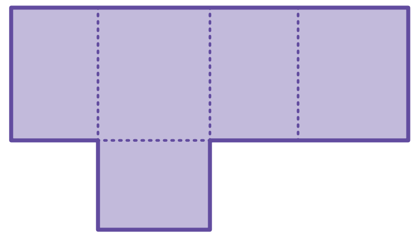


a How many cubic inches of popcorn can it hold?

120 cubic inches

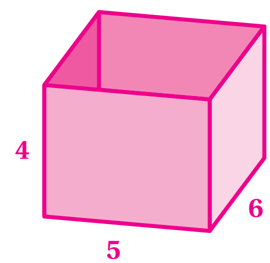
b How many square inches of paper does it use?

128 square inches



7 **a** Draw a *different* container that can hold 120 cubic inches of popcorn.

***Drawings vary.* Sample figure shown.**



b Calculate the amount of paper your container uses.

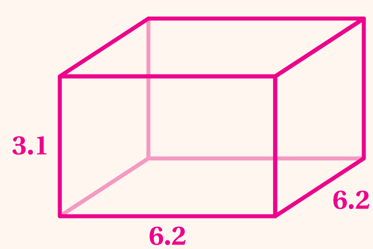
Responses vary.

Explore More

8 **a** Draw a new container that can hold approximately 120 cubic inches of popcorn using as little paper as possible. Label your container's dimensions.

Sample figure shown.

Sketch

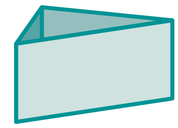


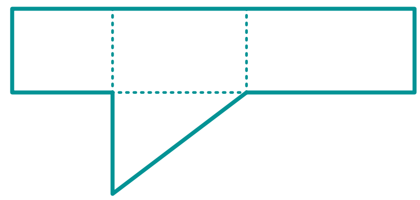
b Calculate the amount of paper your container uses.

***Responses vary.* If the length, width, and height of the prism are approximately 6.2, 6.2, and 3.1 inches, the surface area of the container will be approximately 115 square inches.**

9 Synthesis

What is important to remember when calculating the surface area or volume of a prism? Use the example if it helps with your thinking.





Responses vary.

- **Volume is a helpful calculation when you are trying to figure out "how much fits" into a solid. Surface area is a helpful calculation when you are trying to figure out "how much material" is needed to make a solid.**
- **When calculating the volume, make sure you know which face is the base. In this prism, the base is the triangle.**
- **When calculating the volume, remember that the height is always the distance between the bases, not necessarily a vertical distance.**
- **When calculating the surface area, first count how many faces you have. If the prism is a closed container, you will have two bases.**

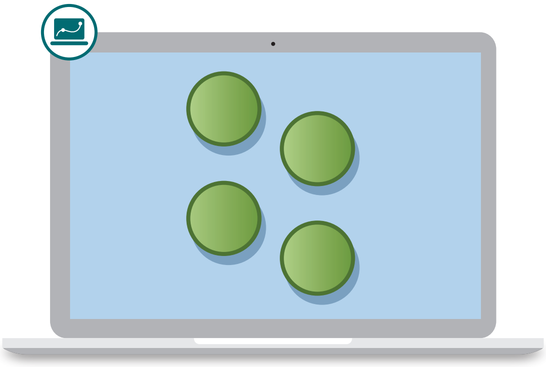
Things to Remember:

Unit 7

Lesson

1

Name: ________ Date: ________ Period: ________



Circles

Let's revisit exponents.

Warm-Up

Evaluate each expression mentally. Try to think of more than one strategy.

1. $5 \cdot 2$

   10

2. $5 \cdot 2 \cdot 2$

   20

3. $5 \cdot 2 \cdot 2 \cdot 2$

   40

4. $5 \cdot 2^4$

   80

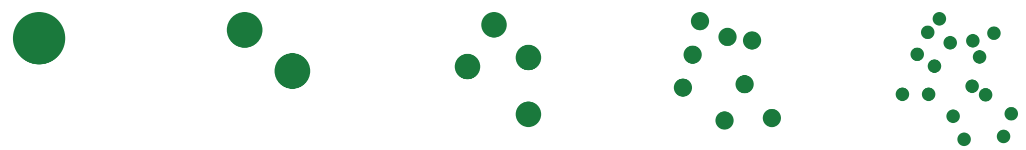
Activity
1

Name: ________________ Date: ________ Period: ________

Lots of Circles

5 Let's look at a pattern.

Stage 0 **Stage 1** **Stage 2** **Stage 3** **Stage 4**



Discuss: What do you notice? What do you wonder?

Responses vary.

- **I notice that the number of circles doubles with each stage.**
- **I wonder how many circles there will be at Stage 100.**

6 How many circles will there be in Stage 5?

32 circles

Explain your thinking.

***Explanations vary.* I noticed that the pattern doubles after each stage. Stage 4 has 16 circles, so if I double that I get 32 circles in Stage 5.**

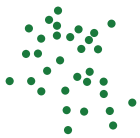
Activity
1

Name: .... Date: .... Period: ....

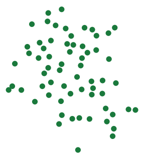
Lots of Circles (continued)

7 Here are Stages 5–12 of the same pattern.

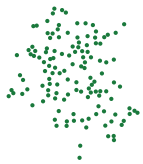
Stage 5



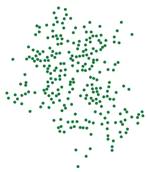
Stage 6



Stage 7

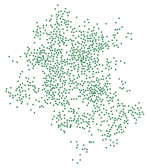


Stage 8

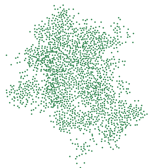


Stage 9

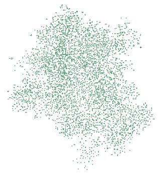
Stage 10



Stage 11



Stage 12



How many circles are there in Stage 12?

4,096 circles

8 Adah and Jamal were calculating the number of circles in Stage 12.

Adah wrote: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Jamal wrote: 2^{12}

Whose expression is correct? Circle one.

Adah's Jamal's Both Neither

Explain your thinking.

Explanations vary. **Both expressions are correct because $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ has the same value as 2^{12}.**

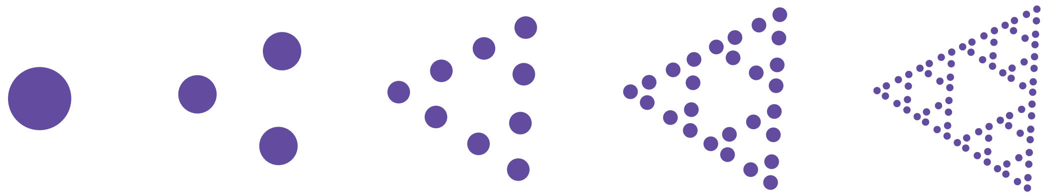
Activity 2

Name: .... Date: .... Period: ....

A New Pattern

9 Here is a new pattern.

Stage 0 **Stage 1** **Stage 2** **Stage 3** **Stage 4**



How many circles are there in Stage 4?

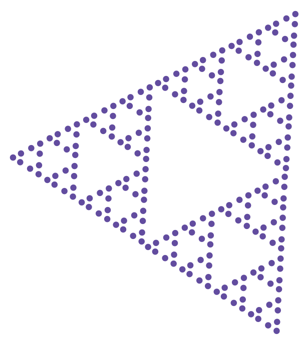
81 circles

Explain your thinking.

***Explanations vary*. Each stage has the number of circles in the previous stage multiplied by 3. Stage 1 has 3 circles, Stage 2 has 9 circles, Stage 3 has 27 circles, and Stage 4 has 81 circles.**

10 There are 243 circles in Stage 5.

Stage 5



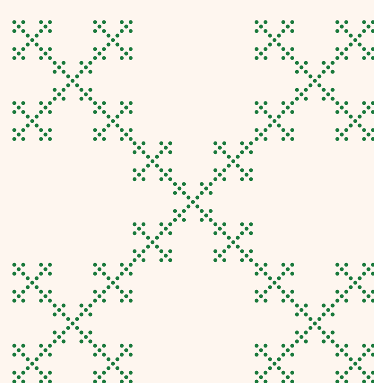
Select *all* the expressions that represent the number of circles in Stage 7.

- [x] **A.** 3^7
- [x] **B.** $243 \cdot 3^2$
- [ ] **C.** $243 \cdot (3 \cdot 2)$
- [ ] **D.** $243 + 243 + 243$
- [x] **E.** $243 \cdot 3 \cdot 3$

Explore More

11 How many circles are in this image?

625 circles



Explain your thinking.

***Explanations vary*. There are 25 circles in one X shape, and there are 5 of those in each of the bigger X shapes, so each of the 5 bigger X shapes has $25 \cdot 5 = 125$ circles. Because there are 5 of these in the image, there are $125 \cdot 5 = 625$ circles total.**

12 Synthesis

When might it be helpful to write values or expressions using *exponents*?

Use the examples if they help with your thinking.

$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

$6 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 5$

$10 + 10 + 10 + 10 + 10$

***Responses vary*. When an expression has repeated multiplication, it can be helpful to write the values using exponents. For example, $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ can be written as 7^5 because 7 is multiplied 5 times. But I can't use exponents for $10 + 10 + 10 + 10 + 10$ because there isn't any repeated multiplication.**

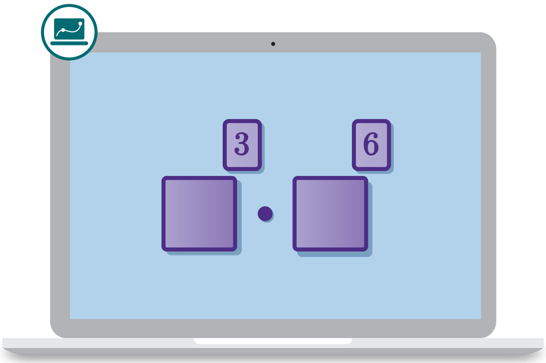
Things to Remember:

Unit 7
Lesson
2

Name: Date: Period:

Combining Exponents

Let's explore equivalent expressions with exponents.



Warm-Up

1 Which one doesn't belong? Explain your thinking.

A. $(2^2)^3$

B. $2^3 \cdot 2 \cdot 2^2$

C. $2 \cdot 32$

D. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Responses and explanations vary.

- Expression A doesn't belong because it's the only expression with parentheses.
- Expression B doesn't belong because it's the only expression that contains numbers with and without exponents.
- Expression C doesn't belong because it's the only expression that is the product of two numbers.
- Expression D doesn't belong because it's the longest expression.

Activity 1

Name: .... Date: .... Period: ....

Combining Exponents

2 Here are some different ways to build a billion, 10^9, by multiplying two **powers of ten**.

$(10 \cdot 10)\ (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$
$10^2 \cdot 10^7 = 10^9$

$(10 \cdot 10 \cdot 10 \cdot 10)\ (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$
$10^4 \cdot 10^5 = 10^9$

$(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)\ (10 \cdot 10 \cdot 10)$
$10^6 \cdot 10^3 = 10^9$

$(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)\ (10)$
$10^8 \cdot 10^1 = 10^9$

Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that nine 10s are being grouped in different ways.
- I notice that the exponents of both powers of ten always add up to 9.
- I wonder if the pattern will continue for negative exponents, like $10^{11} \cdot 10^{-2}$.
- I wonder if this will work for more than two powers of ten.
- I wonder if there is a similar pattern for other powers that don't have a base of ten.

3 Group the expressions by whether they are equivalent to 8^5, 8^6, or neither. Some expressions will have no match.

$8 + 8 + 8 + 8 + 8 + 8$	40	$(8 \cdot 8 \cdot 8)\ (8 \cdot 8 \cdot 8)$	$8^3 \cdot 8^3$
$8^3 \cdot 8^2$	$(8^3)^2$	$8^4 + 8$	$(8 \cdot 8 \cdot 8)\ (8 \cdot 8)$

Equivalent to 8^5	Equivalent to 8^6	Neither
$8^3 \cdot 8^2$ $(8 \cdot 8 \cdot 8)\ (8 \cdot 8)$	$(8 \cdot 8 \cdot 8)\ (8 \cdot 8 \cdot 8)$ $8^3 \cdot 8^3$ $(8^3)^2$	$8 + 8 + 8 + 8 + 8 + 8$ 40 $8^4 + 8$

4 Here is a new expression: $4^5 \cdot 2^5$. Is this expression equivalent to 8^5? Show or explain your thinking.

Yes. *Explanations vary.* $4^5 \cdot 2^5$ means there are five 4s multiplied by five 2s. If I group each 4 with a 2, then I get five groups of $4 \cdot 2$, which is the same as five 8s.

Activity 2

Name: ____________ Date: ________ Period: ________

Odd One Out

5 **a** Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

$3 + 3 + 3 + 3 + 3$ $\quad$ $3^2 \cdot 3 \cdot 3 \cdot 3$ $\quad$ 3^5

b How could you change this expression so that it has the same value as the others?

***Responses vary*. If I change the pluses (+) to multiplication dots (•), the expressions would all have the same value.**

6 **a** Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

$5 \cdot 5 \cdot 5 \cdot 4 \cdot 4 \cdot 4$ $\quad$ 20^3 $\quad$ $(5 \cdot 3) \cdot (4 \cdot 3)$

b How could you change this expression so that it has the same value as the others?

***Responses vary*. If I change the expression to $(5^3) \cdot (4^3)$, the expressions would all have the same value.**

7 **a** Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

$(2 \cdot 2 \cdot 2)\,(2 \cdot 2 \cdot 2)$ $\quad$ 6^2 $\quad$ $(2^3)^2$

b How could you change this expression so that it has the same value as the others?

***Responses vary*. If I change the expression to 2^6, the expressions would all have the same value.**

Activity
2

Name: .... Date: .... Period: ....

Odd One Out (continued)

8 Here are some more pairs of equivalent expressions.

For each row, write one more equivalent expression. ***Responses vary.***

Expression 1	Expression 2	Expression 3
$4 \cdot 4 \cdot 4^3$	4^5	$\mathbf{4^2 \cdot 4^3}$
6^4	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	$\mathbf{2^4 \cdot 3^4}$
$(7 \cdot 7) \cdot (7 \cdot 7) \cdot (7 \cdot 7)$	$(7^2)^3$	$\mathbf{7^6}$
5^{18}	$(5^3)^6$	$\mathbf{(5^2)^9}$

Explore More

9 Using whole numbers 0 through 9 without repeating, fill in the blanks so that all expressions have the same value.

Responses vary.

- $(5^2)^4$
- $5^1 \cdot 5^7$
- $5^3 \cdot 5^5 \cdot 5^0$
- 5^8

$$(5^{\square})^{\square}$$

$$5^{\square} \cdot 5^{\square}$$

$$5^{\square} \cdot 5^{\square} \cdot 5^{\square}$$

$$5^{\square}$$

10 Synthesis

Describe some strategies for writing equivalent expressions involving exponents.

Use the examples if they help with your thinking.

$3^4 \cdot 3^2 = 3^6$

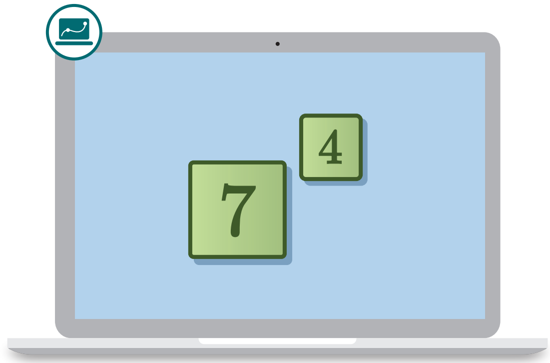
$3^5 \cdot 4^5 = 12^5$

$(3^2)^4 = 3^8$

***Responses vary.* I can expand and regroup expressions with exponents. For example, in 3^6 there are six 3s, so I can regroup the 3s into four 3s and two 3s, which is the same as $3^4 \cdot 3^2$. I could use this same strategy to expand $3^5 \cdot 4^5$ into a group of five 3s times a group of five 4s, then I can regroup to make five groups of $3 \cdot 4$, which is the same as five 12s or 12^5.**

Things to Remember:

Name: .... Date: .... Period: ....



Rewriting Powers

Let's rewrite expressions with exponents as a single power.

Warm-Up

1 Order the expressions from what you think is *least complicated* to *most complicated*.

$$\frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7 \qquad \frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7}$$

$$\frac{7^3 \cdot 7^3}{7^2} \qquad \frac{7^7}{7^3} \qquad 7^4$$

Responses vary.

Least Complicated

Most Complicated

Activity 1

Name: ________ Date: ________ Period: ________

Single Powers

2 Circle one expression.

$\frac{7^3 \cdot 7^3}{7^2}$ $\frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7$ $\frac{7^7}{7^3}$ $\frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7}$

Show or explain how to rewrite this expression as the single power 7^4.

Responses vary. Using $\frac{7^3 \cdot 7^3}{7^2}$, I wrote out all the factors of 7 and looked for parts of the expression that could be rewritten as 1. Then I rewrote the remaining factors using exponential notation: $\frac{7^3 \cdot 7^3}{7^2} = \frac{(7 \cdot 7 \cdot 7) \cdot (7 \cdot 7 \cdot 7)}{7 \cdot 7} = \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 1 \cdot 1 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^4$.

3 Here is how Jayla rewrote $\frac{7^5 \cdot 7^2}{7^3}$ as a single power.

Jayla

$$\frac{7^5 \cdot 7^2}{7^3} = \frac{(7 \cdot 7 \cdot 7 \cdot 7 \cdot 7) \cdot (7 \cdot 7)}{7 \cdot 7 \cdot 7}$$
$$= \frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7$$
$$= 1 \cdot 1 \cdot 1 \cdot 7 \cdot 7 \cdot 7 \cdot 7$$
$$= 7^4$$

single power of 7

Discuss: How could you rewrite $\frac{4^9}{4^2 \cdot 4^4}$ as a single power?

Responses vary. I can expand the expression so there are nine 4s multiplied together in the numerator and six 4s multiplied together in the denominator. After dividing, I get $4 \cdot 4 \cdot 4$, which is the same as 4^3.

4 Rewrite each expression as a single power.

Expression	Single Power
$\frac{7^5 \cdot 7^2}{7^3}$	7^4
$\frac{4^9}{4^2 \cdot 4^4}$	4^3
$\frac{2^3 \cdot 2^3 \cdot 2^3}{2 \cdot 2 \cdot 2}$	2^6

Expression	Single Power
$2^4 \cdot 3^4$	6^4
$\frac{6^7}{2^7}$	3^7
$\frac{(8^4)^2}{8}$	8^7

Activity 1

Name: ______ Date: ______ Period: ______

Single Powers (continued)

5 Sort the expressions based on whether they are equivalent to 6^8.

$2^8 \cdot 3^8$ $\quad$ $2^3 \cdot 3^5$ $\quad$ $\frac{12^8}{2^8}$

$\frac{6^4 \cdot 6^4}{6^1}$ $\quad$ $\frac{6^7 \cdot 6^7}{(6^3)^2}$

Equivalent to 6^8	Not Equivalent to 6^8
$2^8 \cdot 3^8$	$\frac{6^4 \cdot 6^4}{6^1}$
$\frac{6^7 \cdot 6^7}{(6^3)^2}$	$2^3 \cdot 3^5$
$\frac{12^8}{2^8}$	

6 Create one (or more) expressions that are equivalent to 4^5. Write something as unique and as complicated as you want!

Responses vary.

- $\frac{4^8}{4^3}$
- $4^2 \cdot 4^3$
- $\frac{(4^3)^2}{4^1}$
- $2^5 \cdot 2^5$

Activity 2

Name: ________ Date: ________ Period: ________

Challenge Creator

7 You will use a separate sheet of paper to create your own single power challenge.

a **Make It!** On your sheet of paper, write down a single power, using a positive integer for the base and any integer for the exponent. (For example, 4^3 or 5^{16} or 2^{100}.)

b **Solve It!** On this page, record your single power, then create an expression that is equivalent to it. (For example, $5^{10} \cdot 5^6$ is equivalent to 5^{16}.)

My Single Power	Equivalent Expression

c **Swap It!** Swap your challenge with one or more partners. Record your partner's single power, then create an expression that is equivalent to it. *Responses vary.*

	Single Power	Equivalent Expression
Partner 1		
Partner 2		
Partner 3		
Partner 4		

8 Synthesis

Describe a strategy for rewriting an expression as a single power.

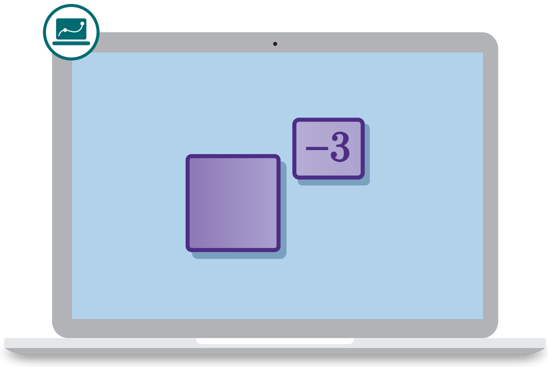
Use these expressions if they help with your thinking.

$$\frac{6^7 \cdot 6^7}{(6^3)^2} \qquad 2^8 \cdot 3^8 \qquad \frac{12^8}{2^8}$$

***Responses vary.* First, write out all the exponents using repeated multiplication. Then look for things that can be combined (e.g., $2 \cdot 3 = 6$) or rewritten as 1 (e.g., $\frac{12}{12} = 1$). At the end, rewrite everything you can using exponents again.**

Things to Remember:

Name: .... Date: .... Period: ....



Negative and Zero Exponents

Let's explore exponents that are not positive.

Warm-Up

1 Order the expressions by value from *least* to *greatest*.

$\frac{2^6}{2^3}$	$\frac{2^5}{2^5}$	2^6	0	$\frac{2^3}{2^4}$

Least

0

$\frac{2^3}{2^4}$

$\frac{2^5}{2^5}$

$\frac{2^6}{2^3}$

2^6

Greatest

Name: .... Date: .... Period: ....

Negative and Zero Exponents

2 Complete as much of the table as you can.

Exponent Form	Expanded Form	Value
10^4	$10 \cdot 10 \cdot 10 \cdot 10$	10,000
10^3	$10 \cdot 10 \cdot 10$	**1,000**
10^2	**$10 \cdot 10$**	100
10^1	10	10
10^0	**1**	**1**
10^{-1}	**$\frac{1}{10}$**	**0.1 or $\frac{1}{10}$**
10^{-2}	**$\frac{1}{10 \cdot 10}$ or $\frac{1}{10} \cdot \frac{1}{10}$**	$\frac{1}{100}$

3 What patterns do you see in the table? Describe as many as you can.

Responses vary.

- **As you move down the left column, the exponents decrease by 1 for each row.**
- **Positive exponents describe the number of factors of 10.**
- **Negative exponents describe the number of factors of 10 in the denominator.**
- **Negative exponents describe the number of factors of $\frac{1}{10}$.**
- **As you move down the table, the values get closer to 0.**
- **There is a form of mirror symmetry in each column of the table.**

4 Cameron wanted to investigate more about negative and zero exponents. So Cameron decided to write some expressions and apply exponent properties.

Cameron

$10^5 \cdot 10^2 = 10^7$

$10^5 \cdot 10^1 = 10^6$

$10^5 \cdot 10^0 = 10^5$

$10^5 \cdot 10^{-1} = 10^4$

Discuss:

- What patterns do you notice in these expressions?
- How could you use Cameron's work to determine the values of 10^0 and 10^{-1}?

Responses vary.

- **Each line on Cameron's paper decreases by a factor of 10.**
- **The third line suggests that 10^0 must be 1 because 1 is the only number that you can multiply by and get what you started with. The fourth line suggests that 10^{-1} must be $\frac{1}{10}$. because to get from 10^5 to 10^4 you must divide by 10 or multiply by $\frac{1}{10}$.**

Activity 2

Name: ______ Date: ______ Period: ______

Beyond 10

5 Complete this new table about powers of 3.

Exponent Form	Expanded Form	Value
3^3	$3 \cdot 3 \cdot 3$	27
3^2	$3 \cdot 3$	**9**
3^1	3	3
3^0	**1**	**1**
3^{-1}	$\frac{1}{3}$	$\frac{1}{3}$
3^{-2}	$\frac{1}{3 \cdot 3}$ or $\frac{1}{3} \cdot \frac{1}{3}$	$\frac{1}{9}$ (or equivalent)
3^{-3}	$\frac{1}{3 \cdot 3 \cdot 3}$ or $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$	$\frac{1}{27}$ (or equivalent)

6 The value of $3^6 = 729$. Predict the value of 3^{-6}.

$\frac{1}{729}$ **(or equivalent)**

Name: .... Date: .... Period: ....

Beyond 10 (continued)

7 Group the equivalent expressions. Some expressions may have no match.

$\left(\frac{1}{3}\right)^5$	$\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$	3^{-5}	$\frac{1}{5 \cdot 5 \cdot 5}$
5^{-3}	-15	$\frac{1}{3^5}$	$\frac{1}{15}$

Group 1	Group 2	No match
3^{-5}	5^{-3}	-15
$\frac{1}{3^5}$	$\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$	$\frac{1}{15}$
$\left(\frac{1}{3}\right)^5$	$\frac{1}{5 \cdot 5 \cdot 5}$	

8 Here are two expressions from earlier.

3^{-5} $\quad$ -15

Are these equivalent? Circle one.

Yes $\quad$ No $\quad$ I'm not sure.

Explain your thinking.

***Explanations vary.* These are not equivalent because the expression 3^{-5} is the same as $\frac{1}{3^5}$, which is a positive number, whereas -15 is a negative number.**

Explore More

9 Write as many different expressions that are equivalent to $\left(\frac{2}{3}\right)^{-3}$ as you can.

Here is one example: $\left(\frac{3}{2}\right)^3$

Responses vary.

- $\frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2}$
- $\left(\frac{2}{3}\right)^{-5} \cdot \left(\frac{2}{3}\right)^2$
- $\frac{1}{\left(\frac{2}{3}\right)^3}$
- $\frac{27}{8}$
- $\frac{2^{-3}}{3^{-3}}$

10 Synthesis

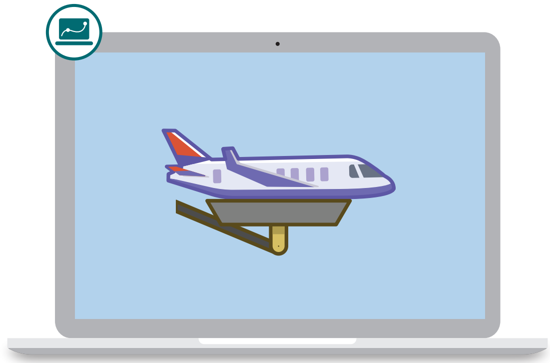
How could you use the table to convince someone that $6^0 = 1$ and $6^{-1} = \frac{1}{6}$?

Exponent Form	Value
6^3	216
6^2	36
6^1	6
6^0	
6^{-1}	

Responses vary. **Each time the exponent decreases by 1, the value gets divided by 6. Based on this pattern and the fact that $6^1 = 6$, we can determine that $6^0 = 1$. We can use the same logic for 6^{-1}. If we divide both sides of $6^0 = 1$ by 6, we get $6^{-1} = \frac{1}{6}$.**

Things to Remember:

Name: .... Date: .... Period: ....

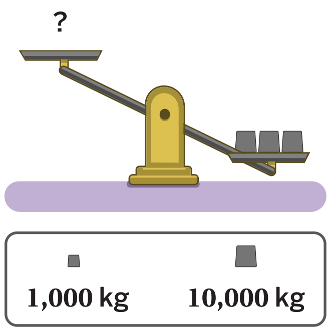


Scales and Weights, Part 1

Let's explore ways to represent large numbers.

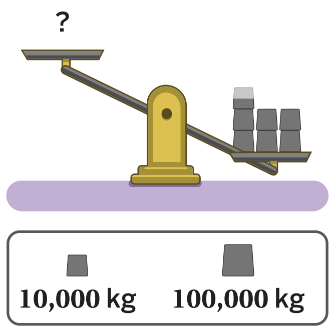
Warm-Up

1 **a**



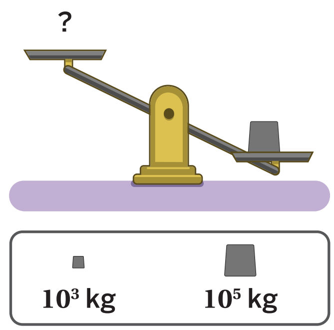
How many 1,000 kg weights are needed to balance with three 10,000 kg weights? **30**

b



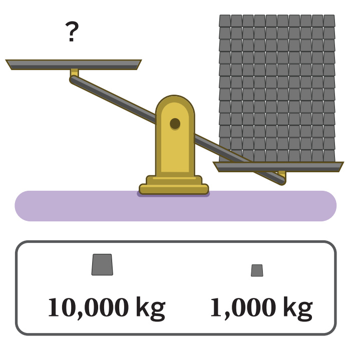
How many 10,000 kg weights are needed to balance with 6.5 100,000 kg weights? **65**

c



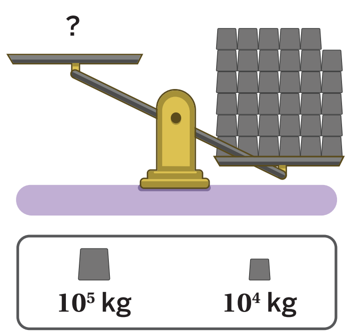
How many 10^3 kg weights are needed to balance with one 10^5 kg weight? **100**

d



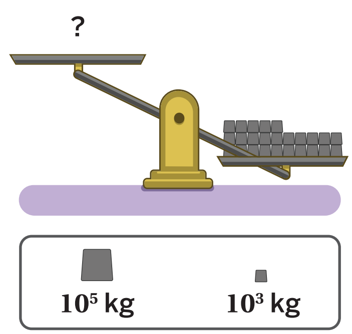
How many 10,000 kg weights are needed to balance with 120 1,000 kg weights? **12**

e



How many 10^5 kg weights are needed to balance with 35 10^4 kg weights? **3.5**

f



How many 10^5 kg weights are needed to balance with 25 10^3 kg weights? **0.25**

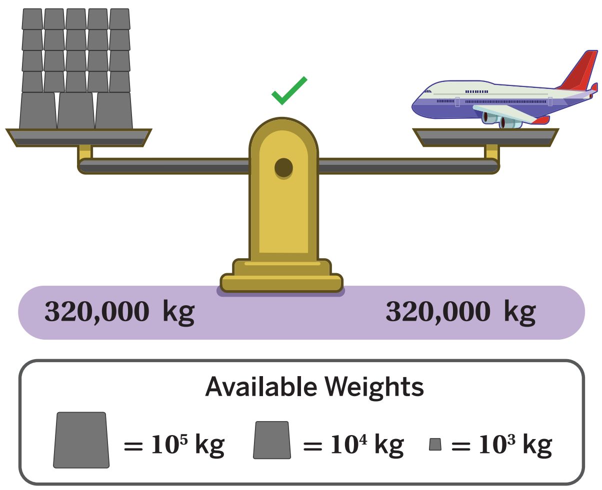
Activity 1

Name: .... Date: .... Period: ....

Scales and Weights

2 A plane weighs 320,000 kilograms.

The table shows how a student balanced the scale using the weights provided.

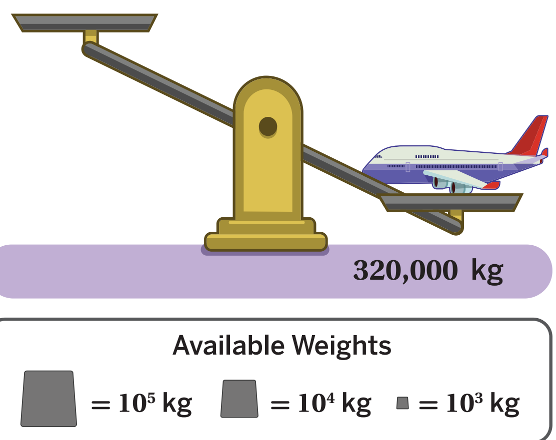


3 Write two other combinations of weights that will balance the scale.

Responses vary.

10^5 kg Weights	10^4 kg Weights	10^3 kg Weights
3	0	20
3	**2**	**0**
0	**30**	**20**

4 Another way to write 320,000 kilograms is to use a combination of powers of 10. For example: $3 \cdot 10^5 + 2 \cdot 10^4$ kilograms.



a **Discuss:** What does each part of the expression represent in terms of weights?

Responses vary. **$3 \cdot 10^5$ represents three 100,000 kg weights, or 300,000 kg. The $2 \cdot 10^4$ represents two 10,000 kg weights, or 20,000 kg.**

b Write an expression to represent a *different* combination of available weights that will balance the scale.

Responses vary.

- **$3 \cdot 10^5 + 20 \cdot 10^3$ kilograms**
- **$32 \cdot 10^4$ kilograms**
- **$320 \cdot 10^3$ kilograms**

Activity 2

Name: ______ Date: ______ Period: ______

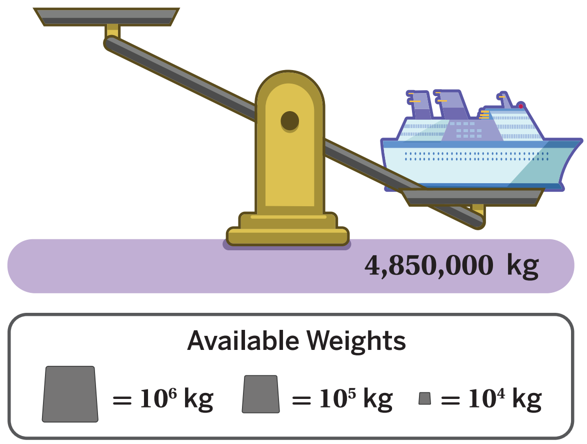
Ships and Shuttles

5 A ship weighs 4,850,000 kilograms.

Write an expression to represent a combination of available weights that will balance the scale.

Responses vary.

- $4 \cdot 10^6 + 8 \cdot 10^5 + 5 \cdot 10^4$ kilograms
- $485 \cdot 10^4$ kilograms
- $48.5 \cdot 10^5$ kilograms



6 Rishi and Anh tried to balance the scale with a 2,030,000-kilogram space shuttle.

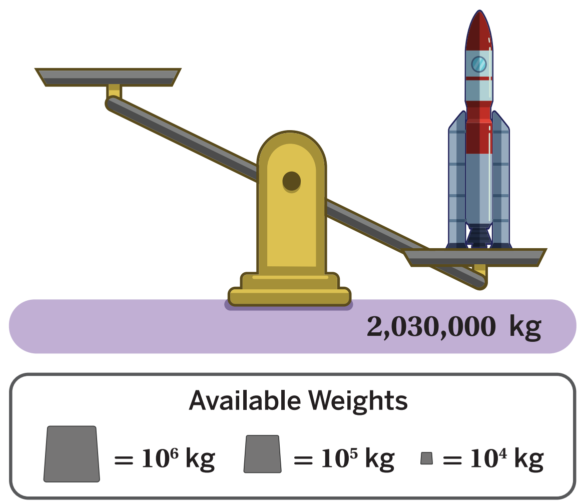
- Rishi wrote: $2.03 \cdot 10^6$ kilograms
- Anh wrote: $20 \cdot 10^5 + 3 \cdot 10^4$ kilograms

Whose expression represents a combination of weights that will balance the scale? Circle one.

Rishi's   Anh's   Both   Neither

Explain your thinking.

Explanations vary. I know that Rishi's expression will balance the scale because $2.03 \cdot 10^6 = 203 \cdot 10^4 = 2030000$. I know that Anh's expression will also balance the scale because $20 \cdot 10^5 = 2000000$ and $3 \cdot 10^4 = 30000$, which makes 2,030,000 altogether. Note: Students will be marked correct on this screen for selecting either Rishi or Anh, or for selecting the "Both" option.



7 Write an expression to represent a *different* combination of available weights that will balance the scale.

Responses vary.

- $2 \cdot 10^6 + 3 \cdot 10^4$ kilograms
- $203 \cdot 10^4$ kilograms
- $1 \cdot 10^6 + 103 \cdot 10^4$ kilograms

Activity 2

Name: ________ Date: ________ Period: ________

Ships and Shuttles (continued)

8 Match each expression with the value it is equivalent to. One expression will have no match.

$4.7 \cdot 10^5$ $4.7 \cdot 10^4$ $7 \cdot 10^4 + 400 \cdot 10^3$

$4 \cdot 10^4 + 7 \cdot 10^2$ $4 \cdot 10^5 + 7 \cdot 10^3$ $40 \cdot 10^3 + 70 \cdot 10^2$

$4 \cdot 10^5 + 5 \cdot 10^4 + 20 \cdot 10^3$

407,000	47,000	470,000
$4 \cdot 10^5 + 7 \cdot 10^3$	$4.7 \cdot 10^4$	$4.7 \cdot 10^5$
	$40 \cdot 10^3 + 70 \cdot 10^2$	$7 \cdot 10^4 + 400 \cdot 10^3$
		$4 \cdot 10^5 + 5 \cdot 10^4 + 20 \cdot 10^3$

9 In the previous problem, how did you decide which value this expression is equivalent to?
$7 \cdot 10^4 + 400 \cdot 10^3$

Responses vary. I matched this expression with 470,000 because $7 \cdot 10^4 = 7 \cdot 10000 = 70000$ and $400 \cdot 10^3 = 400 \cdot 1000 = 400000$. When combined, $70000 + 400000 = 470000$.

10 Synthesis

What are some strategies for writing a number as a combination of powers of 10? Use the examples if they help with your thinking.

- $2{,}560$
- $2 \cdot 10^3 + 5 \cdot 10^2 + 6 \cdot 10^1$
- $256 \cdot 10^1$
- $2.56 \cdot 10^3$
- $25 \cdot 10^2 + 6 \cdot 10^1$
- $25.6 \cdot 10^2$

Responses vary.

- **I think about the place value of each digit. For example, in $2{,}560$ the 2 represents $2{,}000$, so I could write it as $2 \cdot 10^3$. I can apply similar logic to the 5 and 6.**
- **I can write a number as a single multiple of a power of 10 using exponent rules. For example, $2560 = 2560 \cdot 10^0$, because $10^0 = 1$. Then I can borrow powers of 10 like this: $2560 \cdot 10^0 = 256 \cdot 10^1 = 25.6 \cdot 10^2 = 2.56 \cdot 10^3$.**

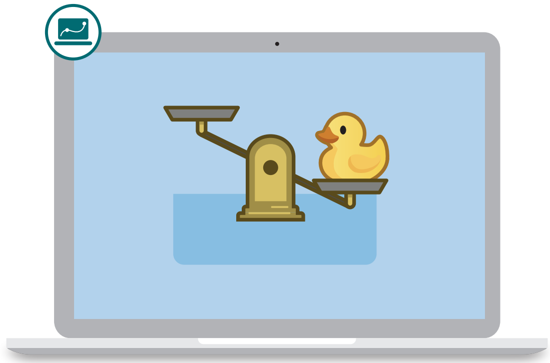
Things to Remember:

Unit 7
Lesson
8

Name: ______ Date: ______ Period: ______

Scales and Weights, Part 2

Let's explore ways to represent small numbers with powers of 10.



Warm-Up

1 Match each number to a verbal description. One number will have no match.

1,000	0.000001	10^{-6}
10^{-3}	0.001	
0.00001	10^{3}	

One thousand	One thousandth	One millionth
1,000 10^{3}	0.001 10^{-3}	0.000001 10^{-6}

Activity 1

Name: Date: Period:

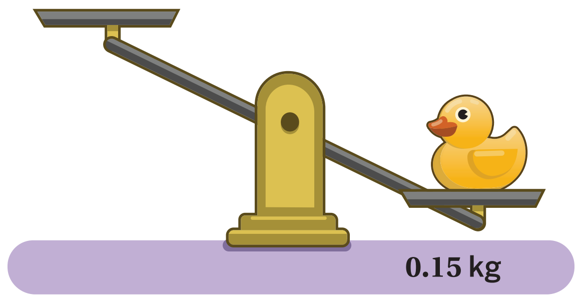
Light Weights

2-3 A rubber duck weighs 0.15 kilograms. A student balanced the scale using the weights shown in the top row of the table.

Write two other combinations of weights that will balance the scale.

Responses vary.

10^{-1} kg	10^{-2} kg	10^{-3} kg
1	5	0
0	**15**	**0**
0	**0**	**150**



Available Weights

= 10^{-1} kg   = 10^{-2} kg   = 10^{-3} kg

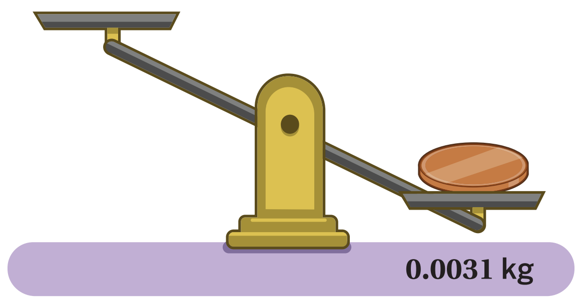
4 A penny weighs 0.0031 kilograms. Here's one way to write its weight using a combination of powers of 10:

$3 \cdot 10^{-3} + 1 \cdot 10^{-4}$

Write an expression to represent a *different* combination of available weights that will balance the scale.

Responses vary.

- **$3.1 \cdot 10^{-3}$ kilograms**
- **$30 \cdot 10^{-4} + 10 \cdot 10^{-5}$ kilograms**
- **$30 \cdot 10^{-4} + 0.1 \cdot 10^{-3}$ kilograms**



Available Weights

= 10^{-3} kg   = 10^{-4} kg   = 10^{-5} kg

Name: ________ Date: ________ Period: ________

Light Weights (continued)

5 Two students made mistakes writing 0.0031 using combinations of powers of 10.

Arturo

$3.1 \cdot 10^{-2}$

Kimaya

$3 \cdot 10^{-4} + 1 \cdot 10^{-3}$

Circle your favorite mistake. ***Responses vary.***

Arturo's mistake　　　　Kimaya's mistake

a What is correct about this student's work?

Responses vary.

- **Arturo correctly identified that 0.0031 could be written as 3.1 times a negative power of 10.**
- **Kimaya correctly identified that 0.0031 could be made by adding 3 times a power of 10 and 1 times a power of 10.**

b What could you add or change to make *all* of their work correct?

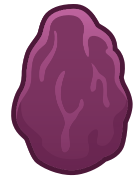
Responses vary.

- **In Arturo's work, I could change the power of 10 from -2 to -3.**
- **In Kimaya's work, I could swap the -4 and -3 in the powers of 10.**

Name: .... Date: .... Period: ....

Getting Smaller and Smaller

6 The weight of a raisin is 0.000572 kilograms.



0.000572 kg

Here is how two students rewrote 0.000572.

Alina

$5 \cdot 10^{-4} + 7 \cdot 10^{-5} + 2 \cdot 10^{-6}$

Lukas

$57.2 \cdot 10^{-5}$

Discuss:

- What are some advantages of Alina's strategy?
- What are some advantages of Lukas's strategy?

Responses vary.

- **Alina's strategy multiplies each digit by a power of 10 that represents the digit's place value. This makes it clearer what value each digit represents.**
- **Lukas's strategy uses fewer numbers and symbols because there is only one power of 10.**

7 Lukas's strategy was to rewrite the raisin's weight, 0.000572 kilograms, as a number times a single power of 10:

$57.2 \cdot 10^{-5}$

Write the same weight as:

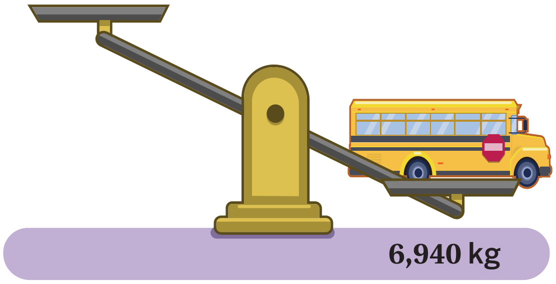
- A number times 10^{-6}: **$572 \cdot 10^{-6}$ kg**
- A number times 10^{-4}: **$5.72 \cdot 10^{-4}$ kg**

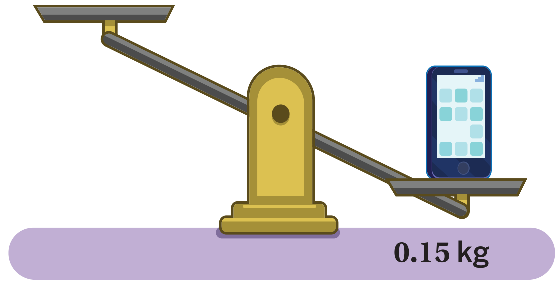
Activity 3

Name: ______ Date: ______ Period: ______

Repeated Challenges

8 Write the weight of each object using a number times a single power of 10.



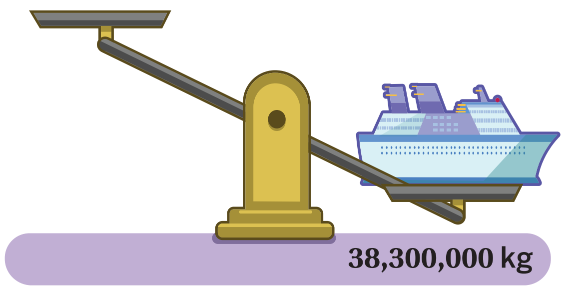


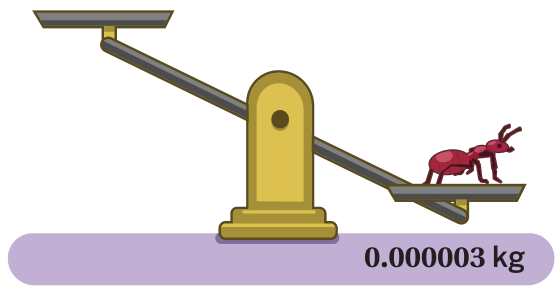
a Write the weight of the bus ($6{,}940$ kg) as a number times 10^3.

$6.94 \cdot 10^3$

b Write the weight of the cell phone (0.15 kg) as a number times 10^{-2}.

$15 \cdot 10^{-2}$





c Write the weight of the cruise ship ($38{,}300{,}000$ kg) as a number times 10^7.

$3.83 \cdot 10^7$

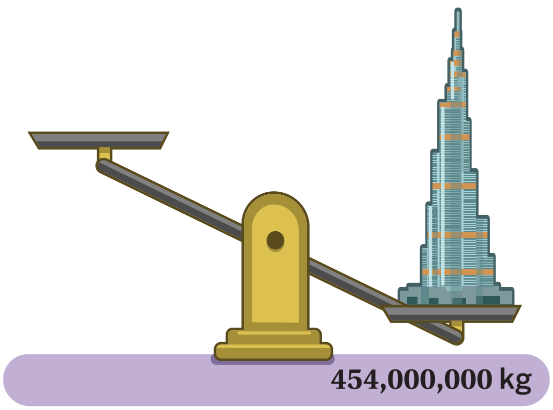
d Write the weight of the ant (0.000003 kg) as a number times 10^{-5}.

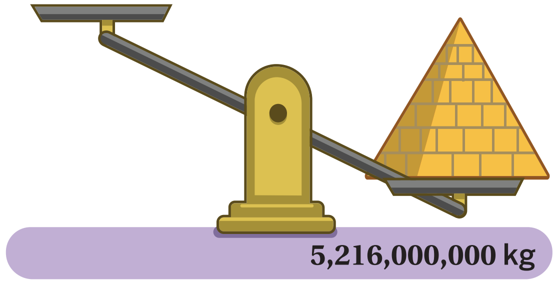
$0.3 \cdot 10^{-5}$

Activity
3

Name: .... Date: .... Period: ....

Repeated Challenges (continued)



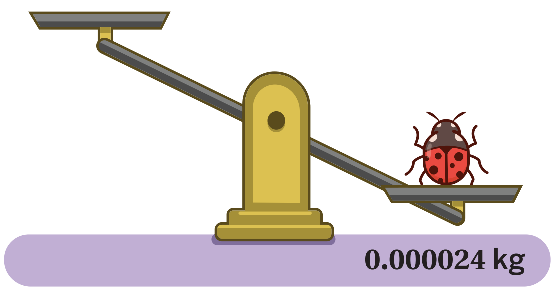


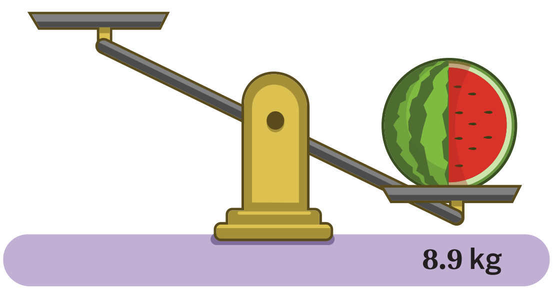
e Write the weight of the Burj Khalifa (454,000,000 kg) as a number times 10^7.

$45.4 \cdot 10^7$

f Write the weight of the pyramid (5,216,000,000 kg) as a number times 10^9.

$5.216 \cdot 10^9$





g Write the weight of the ladybug (0.000024 kg) as a number times 10^{-6}.

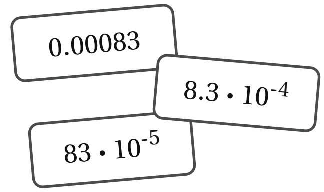
$24 \cdot 10^{-6}$

h Write the weight of the watermelon (8.9 kg) as a number times 10^1.

$0.89 \cdot 10^1$

9 Synthesis

What are some strategies for writing very small values as a number times a single power of 10? Use the examples if they help with your thinking.



Responses vary.

- **I can use the place value of each digit to rewrite the number using powers of 10. For example, in 0.00083, the 83 represents $\frac{83}{100000}$ or $\frac{83}{10^5}$, so I could write it as $83 \cdot 10^{-5}$. I can apply similar logic if I think of it as $\frac{8.3}{10000}$ to write it as $8.3 \cdot 10^{-4}$.**
- **I can use exponent rules to write the number as a combination of powers of 10. For example, $0.00083 = 0.00083 \cdot 10^0$ because $10^0 = 1$. Then I can borrow powers of 10 like this: $0.00083 \cdot 10^0 = 0.0083 \cdot 10^{-1} = 0.083 \cdot 10^{-2} = 0.83 \cdot 10^{-3} = 8.3 \cdot 10^{-4}$, and so on.**

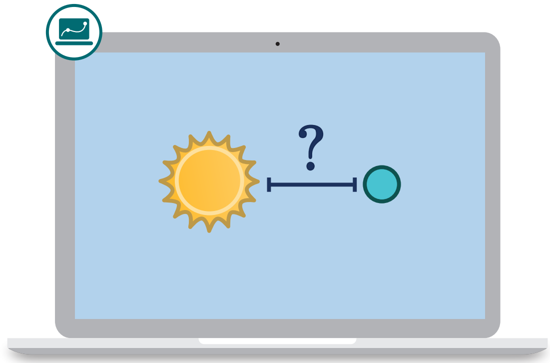
Things to Remember:

Unit 7
Lesson
9

Name: ________ Date: ________ Period: ________

Specific and Scientific

Let's explore scientific notation.



Warm-Up

1 Order these numbers from *least* to *greatest*.

$75 \cdot 10^5$	4,000,000	$0.6 \cdot 10^7$	$5 \cdot 10^5$

$5 \cdot 10^5$	4,000,000	$0.6 \cdot 10^7$	$75 \cdot 10^5$

Least Greatest

2 Order these numbers from *least* to *greatest*.

$4 \cdot 10^6$	$6 \cdot 10^6$	$5 \cdot 10^5$	$7.5 \cdot 10^6$

$5 \cdot 10^5$	$4 \cdot 10^6$	$6 \cdot 10^6$	$7.5 \cdot 10^6$

Least Greatest

3 Which list was easier to sort? Explain your thinking.

Responses vary.

- List 1: My strategy for ordering the lists was to do the multiplication for every number that was written as a product. List 1 only had three of these, compared to List 2 which had four.
- List 2: Powers of 10 helped guide my sorting of List 2. For instance, I could see that three numbers were in the millions (10^6) and one was not. To order the three numbers in the millions, I could use the leading digit, because I know 7 million is greater than 6 million is greater than 4 million.

Activity 1

Name: ______ Date: ______ Period: ______

Scientific Notation

Scientific notation is a specific way of writing very large or very small numbers that can help us compare numbers.

4 Some of these numbers are written in scientific notation and some are not.

What do you think it means for a number to be written in scientific notation?

Responses vary. **A number is written in scientific notation if it's written as a product of two numbers, where the first part is a number between 1 and 10 and the second part is an integer power of 10.**

In Scientific Notation	Not in Scientific Notation
$3 \cdot 10^9$	$3{,}000{,}000{,}000$
$1.257 \cdot 10^5$	$125.7 \cdot 10^3$
$2 \cdot 10^{-1}$	0.2
$5.1 \cdot 10^{-4}$	$0.51 \cdot 10^{-3}$

5 Sort the numbers based on whether they are written in scientific notation.

0.00099	$48{,}200$	$0.78 \cdot 10^{-3}$
$5.23 \cdot 10^8$	$8.7 \cdot 10^{-12}$	$36 \cdot 10^5$

In Scientific Notation	Not in Scientific Notation
$5.23 \cdot 10^8$ $8.7 \cdot 10^{-12}$	$0.78 \cdot 10^{-3}$ $48{,}200$ $36 \cdot 10^5$ 0.00099

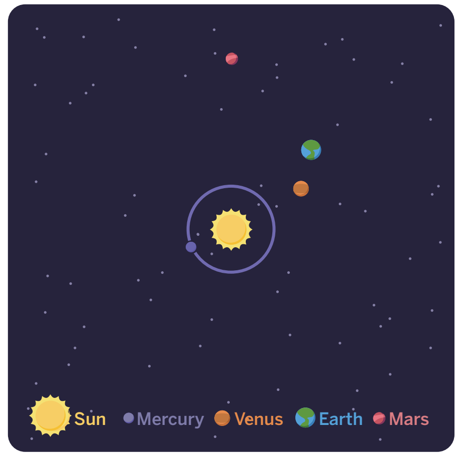
Activity 2

Name: ______ Date: ______ Period: ______

Solar System and Test Tubes

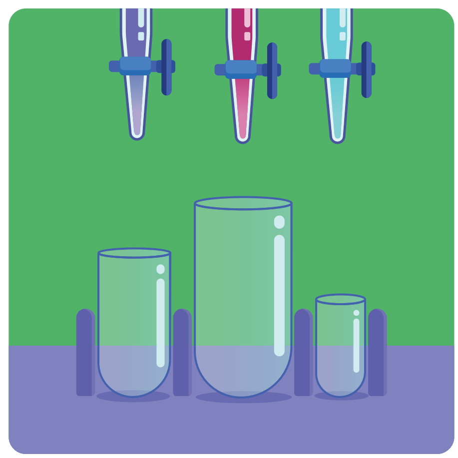
6 Here are the distances of four planets from the Sun. Write each distance in scientific notation. Mercury has been done for you.

Planet	Distance From Sun (mi)	Scientific Notation (mi)
Mercury	36,000,000	$3.6 \cdot 10^7$
Venus	67,000,000	$6.7 \cdot 10^7$
Earth	92,960,000	$9.296 \cdot 10^7$
Mars	$1417 \cdot 10^5$	$1.417 \cdot 10^8$



7 We can use scientific notation to represent small numbers, too! Write each volume in scientific notation.

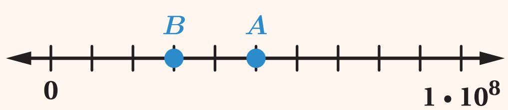
Liquid Color	Volume (L)	Scientific Notation (L)
Purple	0.000125	$1.25 \cdot 10^{-4}$
Red	0.0002	$2 \cdot 10^{-4}$
Blue	$325 \cdot 10^{-8}$	$3.25 \cdot 10^{-6}$



Explore More

8 What are the values of points A and B? Write your answers in scientific notation.

Point	Value
A	$5 \cdot 10^7$
B	$3 \cdot 10^7$



Explain your thinking.

Explanations vary. First, I rewrote the given tick mark as $10 \cdot 10^7$. Because there are ten tick marks, I can divide up $10 \cdot 10^7$ into ten equal chunks, which means each tick mark is a multiple of 10^7. The first tick mark is $1 \cdot 10^7$, the second is $2 \cdot 10^7$, and so on.

9 Synthesis

Describe a strategy for writing a number in scientific notation. Use the table if it helps with your thinking.

Responses vary. **To write a number in scientific notation, I can use exponent rules. For example, $6700000 = 6700000 \cdot 10^0$, because $10^0 = 1$. Then I can borrow powers of 10 to shift the place values, like this: $6700000 \cdot 10^0 = 670000 \cdot 10^1$. I can continue this until the first number is between 1 and 10. Large numbers will end up with positive integer powers of 10 and small numbers less than 1 will have negative integer powers of 10.**

Not in Scientific Notation	In Scientific Notation
36,000,000	$3.6 \cdot 10^7$
6,700,000	
0.00024	
$417 \cdot 10^3$	

Things to Remember:

Unit 7
Lesson
11

Name: .... Date: .... Period: ....

Balance the Scale

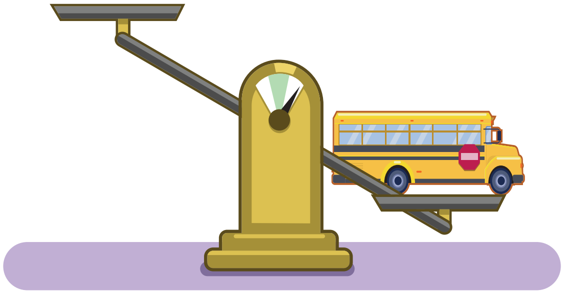
Let's use multiplication and division to compare large and small numbers in scientific notation.

Warm-Up

1 A school bus sits on one side of the scale.

How many jelly beans do you think it would take to balance the scale? Use scientific notation.

Responses vary.



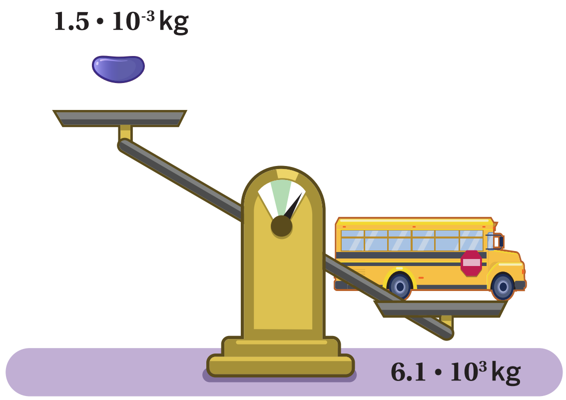
Activity 1

Name: .... Date: .... Period: ....

Balance the Scale, Part 1

2 One jelly bean weighs $1.5 \cdot 10^{-3}$ kilograms. A school bus weighs $6.1 \cdot 10^{3}$ kilograms.

Describe a strategy for determining about how many jelly beans weigh as much as a bus.



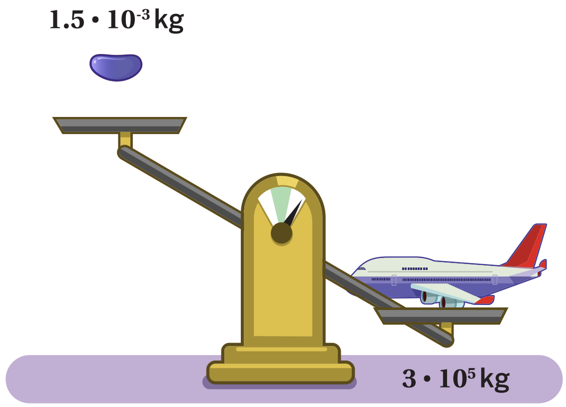
Responses vary.

- **Start with the lighter weight. Multiply its first part so that it reaches the bigger weight's first part ($1.5 \cdot 4 \approx 6.1$). Then multiply by the power of 10 needed to reach the bigger weight's power of 10 ($10^{-3} \cdot 10^{6} = 10^{3}$).**
- **Begin by rounding the first parts to the nearest whole or half number. Then divide the larger quantity by the smaller quantity. This will result in dividing the first parts and subtracting the exponents.**
- **I expanded each number out, so the weight of a jelly bean is 0.0015 kilograms and the weight of the school bus is 6,100 kilograms. Then I divided 6,100 by 0.0015 to get about 4,066,667 jelly beans.**

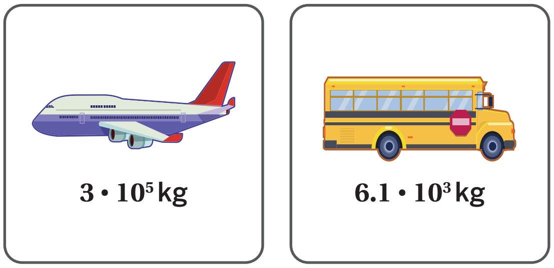
3 One jelly bean weighs $1.5 \cdot 10^{-3}$ kilograms. A jumbo jet weighs $3 \cdot 10^{5}$ kilograms.

How many jelly beans will it take to balance the scale?

$2 \cdot 10^{8}$ jelly beans



4 Basheera says the jumbo jet weighs about 200 times as much as the school bus. Elena says it weighs about 50 times as much as the school bus.



Whose claim is correct? Circle one.

Basheera's   Elena's   Neither

Explain your thinking.

Explanations vary.

- **I rounded 6.1 to 6. When I multiplied $6 \cdot 10^{3}$ by 50, I got $300 \cdot 10^{3}$, which is the same as $3 \cdot 10^{5}$, the jumbo jet's weight.**
- **I rounded 6.1 to 6. If I divide $3 \cdot 10^{5}$ by $6 \cdot 10^{3}$, I get $0.5 \cdot 10^{2}$, which is the same as 50.**

Name: .... Date: .... Period: ....

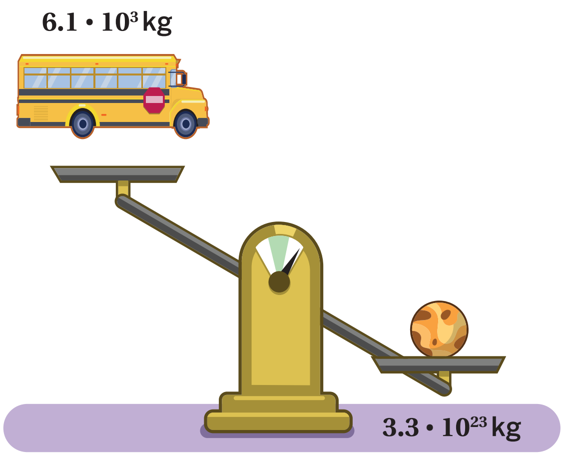
Balance the Scale, Part 2

5 A school bus weighs $6.1 \cdot 10^3$ kilograms. Mercury weighs $3.3 \cdot 10^{23}$ kilograms.

About how many school buses will it take to balance the scale? Write your answer in scientific notation.

Responses vary.

- **$5 \cdot 10^{19}$ school buses**
- **$5.4 \cdot 10^{19}$ school buses**

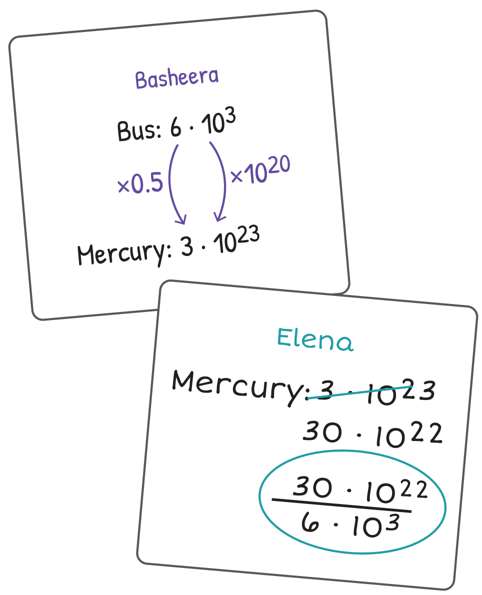


6 Here are two students' strategies for the previous problem. Examine their work.

Explain how Basheera and Elena each arrived at the answer $5 \cdot 10^{19}$ buses. ***Responses vary.***

Basheera: **She rounded each number first. Then she analyzed the weights of the bus and Mercury in pieces to see how much to multiply. To get from 6 to 3, she multiplied by 0.5. To get from 10^3 to 10^{23}, she multiplied by 10^{20}. Then she rewrote $0.5 \cdot 10^{20}$ in scientific notation.**

Elena: **She also rounded each number first. Then she rewrote Mercury's weight by borrowing a power of 10 to make the division easier. Then she used division: $\frac{30}{6} = 5$ and $\frac{10^{22}}{10^3} = 10^{19}$.**

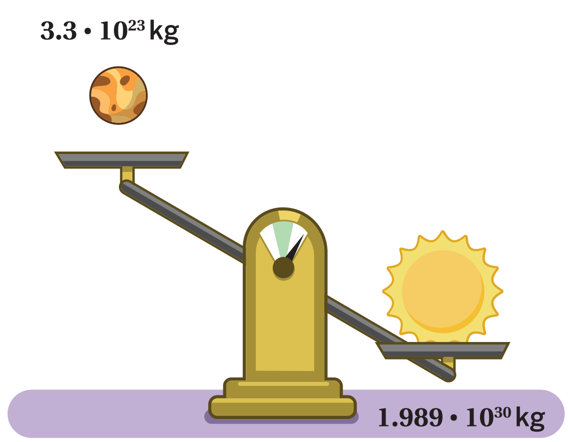


7 Mercury weighs $3.3 \cdot 10^{23}$ kilograms. The Sun weighs $1.989 \cdot 10^{30}$ kilograms.

About how many Mercurys will it take to balance the scale? Write your answer in scientific notation.

Responses vary.

- **$6.03 \cdot 10^6$ Mercurys**
- **$6.7 \cdot 10^6$ Mercurys**



Name: ______ Date: ______ Period: ______

Balance the Scale, Part 2 (continued)

8

- Choose an object from each row and then compare them.
- Use these weights to determine how many of your first object weighs as much as your second object.
- Complete as many comparisons as you have time for.

Watermelon $8.9 \cdot 10^{0}$ kilograms	**Horse** $7.1 \cdot 10^{2}$ kilograms	**Ant** $3 \cdot 10^{-6}$ kilograms	**Cell Phone** $1.5 \cdot 10^{-1}$ kilograms	**Penny** $3.1 \cdot 10^{-3}$ kilograms
Bus $7.81 \cdot 10^{3}$ kilograms	**Moon** $7.348 \cdot 10^{22}$ kilograms	**Rocket** $2.03 \cdot 10^{6}$ kilograms	**Cruise Ship** $3.83 \cdot 10^{7}$ kilograms	**Pyramid** $5.216 \cdot 10^{9}$ kilograms

Responses vary.

Comparison 1

How many ______ weigh about as much as the ______?

Write your answer in scientific notation.

Comparison 2

How many ______ weigh about as much as the ______?

Write your answer in scientific notation.

Comparison 3

How many ______ weigh about as much as the ______?

Write your answer in scientific notation.

Comparison 4

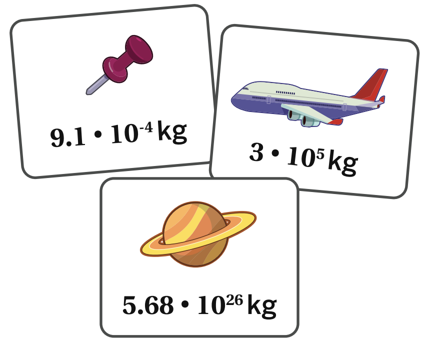
How many ______ weigh about as much as the ______?

Write your answer in scientific notation.

9 Synthesis

Describe a strategy for determining how many times as large one number is compared to another.

Responses vary. **Divide the larger quantity by the smaller quantity. Sometimes it's helpful to round the numbers first. I can also make the division easier by borrowing powers of 10. After dividing, sometimes the result needs more borrowing of powers of 10 so that it's in scientific notation.**



Things to Remember:

Unit 7

Lesson

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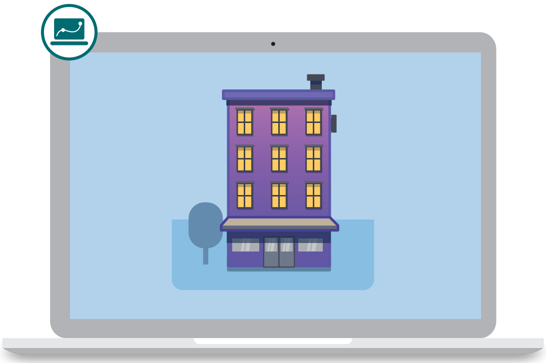
Name: ______ Date: ______ Period: ______



City Lights

Let's apply our understanding of place value to add and subtract with scientific notation.

Warm-Up

1 Ariel says: $2 \cdot 10^2 + 3 \cdot 10^3 = 5 \cdot 10^5$.

Is Ariel's claim correct? Circle one.

Yes　　　No　　　I'm not sure

Explain your thinking.

Explanations vary. **Ariel's claim is not correct because $2 \cdot 10^2 = 200$ and $3 \cdot 10^3 = 3000$. 3,200 is not equal to $5 \cdot 10^5$.**

Activity 1

Name: .... Date: .... Period: ....

City Lights, Part 1

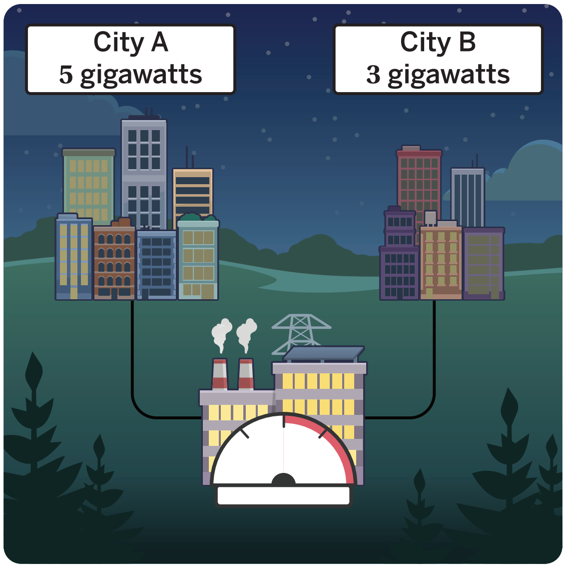
2 City A and City B get electricity from the same source.

City A needs 5 gigawatts of electricity.

City B needs 3 gigawatts.

How many gigawatts are needed to power both cities?

8 gigawatts



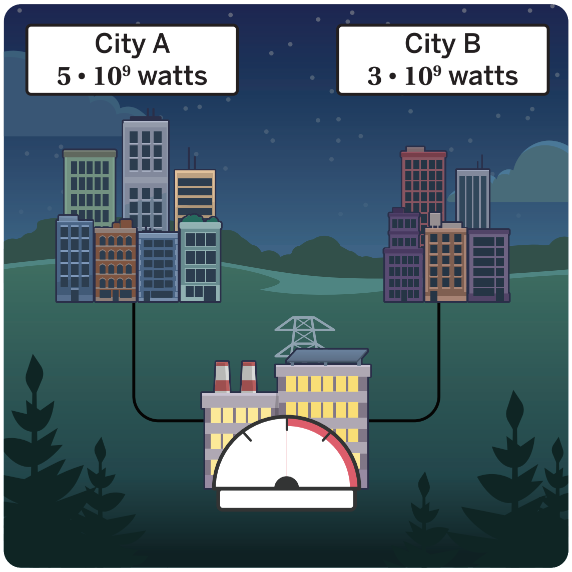
3 1 gigawatt is equal to 10^9 watts.

City A needs $5 \cdot 10^9$ watts of electricity.

City B needs $3 \cdot 10^9$ watts.

How many watts are needed to power both cities? Write your answer in scientific notation.

$8 \cdot 10^9$ watts



Name: .... Date: .... Period: ....

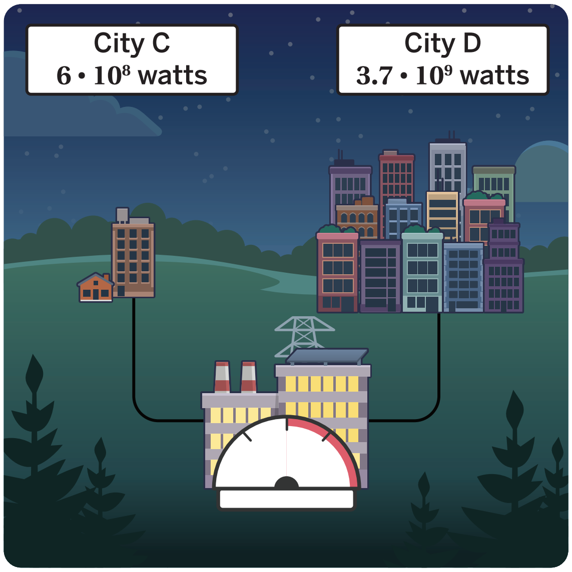
City Lights, Part 1 (continued)

4 Here are two new cities: City C and City D.

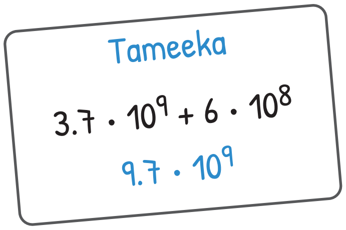
City C needs $6 \cdot 10^8$ watts of electricity.
City D needs $3.7 \cdot 10^9$ watts.

How many watts are needed to power both cities? Write your answer in scientific notation.

$4.3 \cdot 10^9$ watts



5 Tameeka made a mistake on the previous problem.



a What do you think Tameeka did well?

Responses vary.

- **I think Tameeka understood that the first part of each number needs to be combined and that the power of 10 will stay the same.**
- **Tameeka used the larger power of 10 as the common power for both numbers.**

b What would you recommend Tameeka change in her work?

Responses vary. **The terms have different powers of 10. I would recommend that Tameeka rewrite $3.7 \cdot 10^9$ as $37 \cdot 10^8$. From there, she can add $37 \cdot 10^8$ and $6 \cdot 10^8$ because they both have the same power of 10: $37 \cdot 10^8 + 6 \cdot 10^8 = 43 \cdot 10^8$, or $4.3 \cdot 10^9$.**

Name: .... Date: .... Period: ....

City Lights, Part 2

6 Here are two new cities: City E and City F.

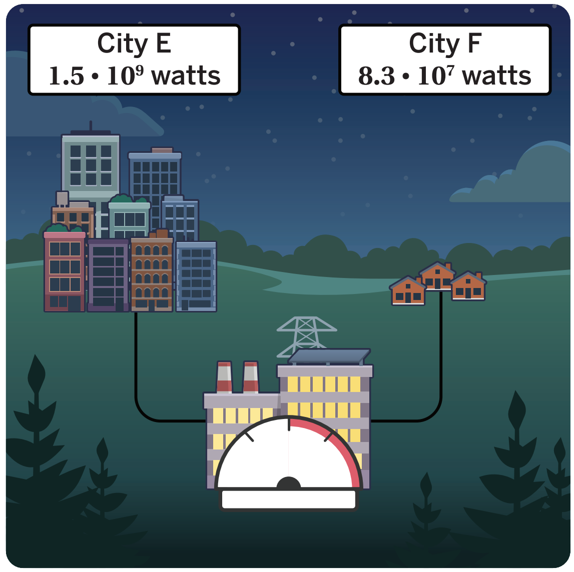
City E needs $1.5 \cdot 10^9$ watts of electricity.

City F needs $8.3 \cdot 10^7$ watts.

How many watts are needed to power both cities? Write your answer in scientific notation.

$1.583 \cdot 10^9$ watts. ***Work varies.***

$$1.5 \cdot 10^9 + 8.3 \cdot 10^7 = 1.5 \cdot 10^9 + 0.083 \cdot 10^9$$
$$= 1.583 \cdot 10^9$$



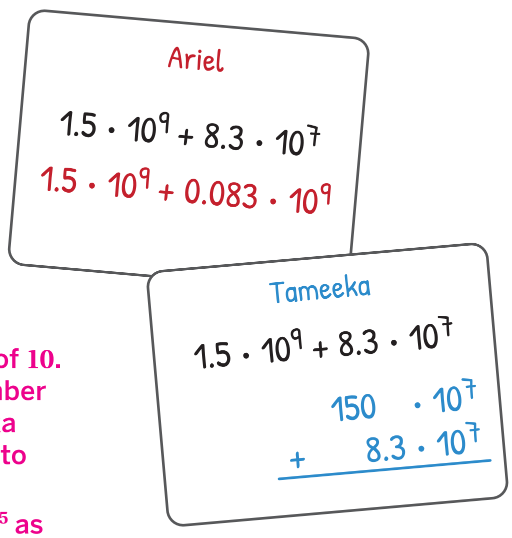
7 Here are Ariel's and Tameeka's strategies for the previous problem.

Discuss:

- How might each student finish the problem?
- After seeing both strategies, how would you add $3.6 \cdot 10^6 + 2.5 \cdot 10^5$?

Responses vary.

- **Both students rewrote the problem so that the numbers they are adding have the same power of 10. Then they would add the first parts of each number together. Ariel would get $1.583 \cdot 10^9$ and Tameeka would get $158.3 \cdot 10^7$. Tameeka would then have to rewrite her answer in scientific notation.**
- **To add $3.6 \cdot 10^6 + 2.5 \cdot 10^5$, I would rewrite $2.5 \cdot 10^5$ as $0.25 \cdot 10^6$. Then I could add $3.6 \cdot 10^6$ and $0.25 \cdot 10^6$ to get $3.85 \cdot 10^6$.**



Name: Date: Period:

City Lights, Part 2 (continued)

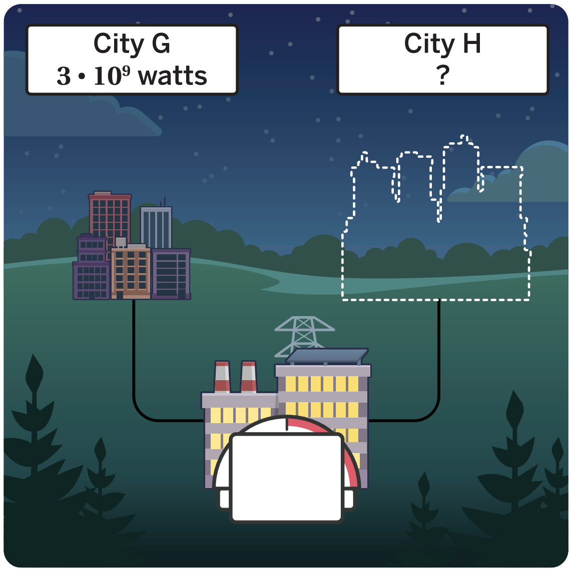
8 Here are two new cities: City G and City H.

The power plant provides exactly enough electricity for both cities: $4.75 \cdot 10^{10}$ watts.

City G uses $3 \cdot 10^9$ watts.

How many watts does City H use? Write your answer in scientific notation.

$4.45 \cdot 10^{10}$ watts



9 Match each value with the correct situation. Note: All units are in watts.

$3.76 \cdot 10^{10}$	$1.03 \cdot 10^{12}$	$1.6 \cdot 10^{10}$	$1.03 \cdot 10^{11}$

Situation A	Situation B	Situation C	Situation D
City 1: $8 \cdot 10^{10}$	City 1: $4.5 \cdot 10^{10}$	City 1: $9.6 \cdot 10^{10}$	City 1: $3.76 \cdot 10^{10}$
City 2: $9.5 \cdot 10^{11}$	City 2: $1.6 \cdot 10^{10}$	City 2: $7 \cdot 10^9$	City 2: $2.4 \cdot 10^9$
Total: $1.03 \cdot 10^{12}$	Total: $6.1 \cdot 10^{10}$	Total: $1.03 \cdot 10^{11}$	Total: $4 \cdot 10^{10}$

10 Synthesis

What are some important things to remember when adding or subtracting numbers written in scientific notation?

$4.6 \cdot 10^7 + 3.2 \cdot 10^6$

$1.57 \cdot 10^8 - 4 \cdot 10^6$

Use the examples if they help with your thinking.

***Responses vary*. Make sure the powers of 10 are the same. To rewrite a number with a different power of 10, multiply the first part of the number by 10 to make the exponent smaller by 1, or divide the first part by 10 to make the exponent larger by 1.**

Things to Remember:

Name: .... Date: .... Period: ....

Nothing but Net Worth

Let's compare the net worths of different celebrities and billionaires.

Warm-Up

You will use the Warm-Up Card to complete this activity.

1. Choose *four* of the celebrities listed on the card. Record their names and their net worths in the table.

2. Record each celebrity's net worth written in scientific notation. ***Responses vary.***

Name	Net Worth ($)	Net Worth Written in Scientific Notation ($)

3. Order the celebrities in your table from lowest to highest net worth. ***Responses vary.***

Lowest Net Worth **Highest Net Worth**

Activity 1

Name: Date: Period:

Star Power

Let's look at some of the richest people in the world.

4. Choose one billionaire to focus on. Write their name and their net worth in scientific notation. ***Responses vary.* Sample response shown.**

Name: Jeff Bezos **Net Worth:** $2.233 * 10^{11}$

5. You will use the Warm-Up Card to help you answer: *Who has more money?*

A. The billionaire you chose. **B.** All 10 celebrities combined. **C.** I'm not sure.

Explain your thinking.

***Explanations vary.* The net worth of the 10 celebrities combined is about $6.1 \cdot 10^9$ dollars, which is 6.1 billion dollars. This is less than Jeff Bezos's net worth of $2.233 \cdot 10^{11}$ dollars.**

Note: The combined net worth is less than the net worth of each billionaire on the list.

6. As of 2023, the median salary of a full-time worker in the U.S. was around $60,000 per year. How long would someone with this salary need to work to earn the net worth of the person you chose? Write your answer in scientific notation.

***Responses vary based on which person students chose.* Jeff Bezos: $3.7 \cdot 10^6$ years**

7. Which unit do you think is most appropriate to use for your response to Problem 5?

A. Days **B.** Years **C.** Centuries **D.** Millennia

Explain your thinking.

***Responses and explanations vary.* I think using millennia as the unit emphasizes how long it would take for someone with a salary of $60,000 to earn the equivalent of Jeff Bezos's net worth. It would take $3.7 \cdot 10^3$ (or 3,700) millennia, which is a very long time considering we're only living in the third millennium.**

Name: .... Date: .... Period: ....

What Would You Buy?

8. If you had a billion dollars and you wanted to spend it all, what would you buy?

Responses vary.

9. You will use the Activity 2 Sheet that lists the costs of various items. Purchase at least four different items. Spend as close to the net worth of the billionaire you chose without going over it. Record each item you chose in the table. Then calculate how much money you spent.

Responses vary.

Item	Price Per Item	Quantity	Total Cost
	Total Spent		

10. How many dollars away were you from spending the net worth of the billionaire you chose?

Responses vary.

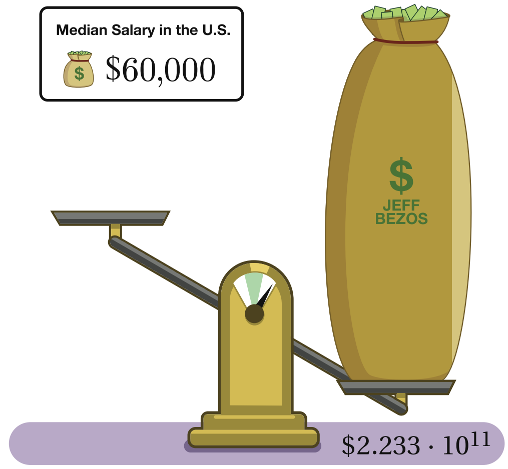
Explain your thinking.

Explanations vary. I took the total cost I spent and wrote it as a power of 10^{11}. Then I subtracted the total cost from $2.233 \cdot 10^{11}$ to figure out how close I was to spending all of the money.

Synthesis

11. What are some important things to remember about adding, subtracting, multiplying, and dividing numbers written in scientific notation?

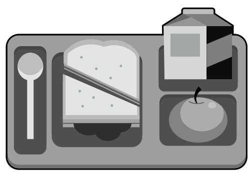
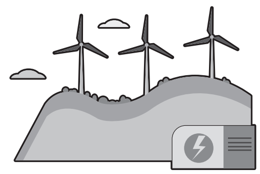
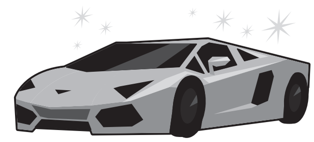
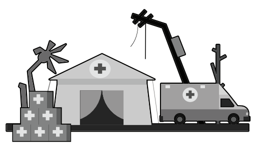
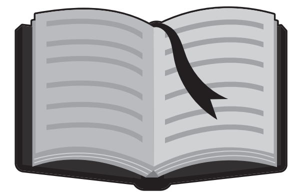
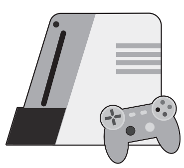
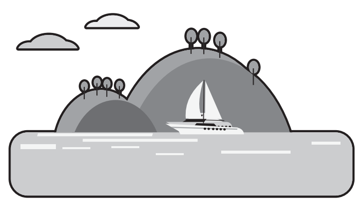
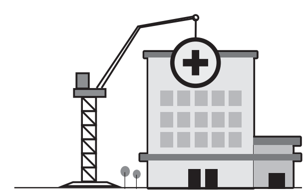
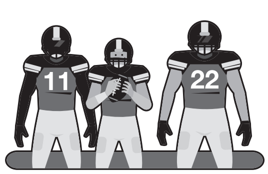
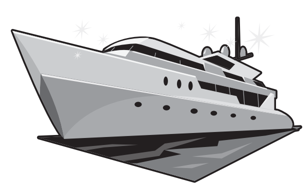
Responses vary. **When adding or subtracting numbers written in scientific notation, it's helpful to have the same power of 10 for both expressions. When multiplying or dividing numbers written in scientific notation, you can multiply or divide the first parts of the expressions and then multiply or divide the powers of 10.**



Things to Remember:

Name: ______________________ Date: ____________ Period: ____________

Costs of Items

 Pay student lunch debt for one year $\$1.76 \cdot 10^8$	 Build a wind farm $\$3 \cdot 10^6$	 Buy a luxury car $\$1.3 \cdot 10^5$
 Provide relief for one natural disaster $\$3.44 \cdot 10^8$	 Pay one student's college loan debt $\$4 \cdot 10^4$	 Buy a book $\$2 \cdot 10^1$
 Buy a gaming console $\$5 \cdot 10^2$	 Buy a private island $\$2 \cdot 10^7$	 Build a hospital $\$3.3 \cdot 10^8$
 Buy a professional football team $\$5 \cdot 10^9$	 Buy a luxury yacht $\$1 \cdot 10^8$	 Feed a family of five for one year $\$1.2 \cdot 10^4$

Net Worths of Celebrities

Directions: Make one copy per pair of students. Then pre-cut the cards and give each pair one set.

Net Worths of Celebrities

The table shows the net worths of 10 celebrities as of 2023.

Name	Net Worth ($)
Beyonce	500,000,000
Barack Obama	70,000,000
Lady Gaga	320,000,000
Billie Eilish	30,000,000
LeBron James	600,000,000
Kylie Jenner	700,000,000
Cardi B	80,000,000
Post Malone	45,000,000
Oprah Winfrey	3,500,000,000
Ariana Grande	240,000,000

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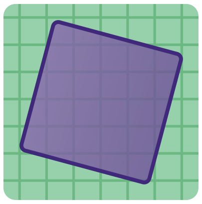
Ariana Grande	240,000,000

Unit 8
Lesson
1

Name: .... Date: .... Period: ....

Tilted Squares



Let's explore finding the areas of tilted squares.

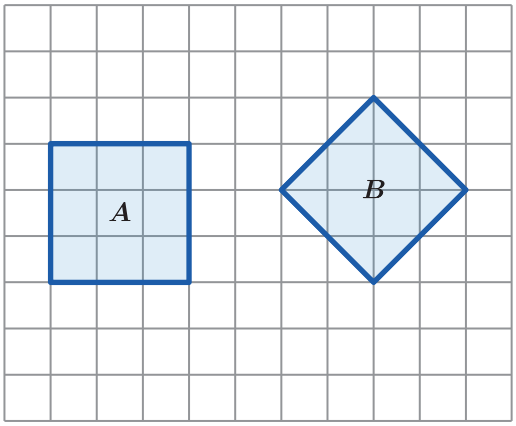
Warm-Up

1. Which shaded region is larger?

Square A

Explain your thinking.

Explanations vary. Square A has an area of 9 square units, which I found by multiplying the base length by the height length: $3 \cdot 3$. To find the area of square B, I divided the figure into four congruent triangles and calculated the area of each triangle. $4 \cdot \frac{1}{2}(2 \cdot 2) = 8$ square units.

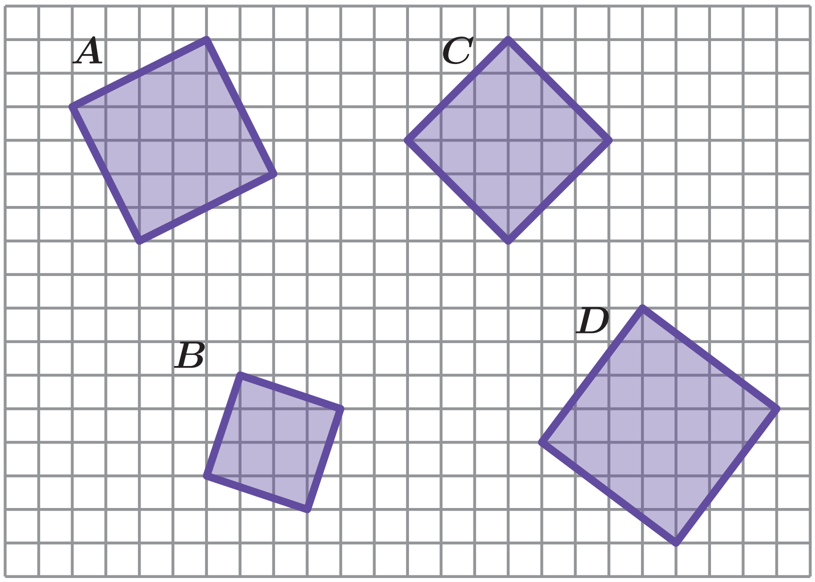


Activity 1

Name: .... Date: .... Period: ....

Area of Tilted Squares

2. Determine the area of each tilted square (in square units). Record the areas in the table.



Square	A	B	C	D
Area (sq. units)	**20**	**10**	**18**	**25**

3. What strategies did you use to determine the areas of the tilted squares?

Responses vary.

- **First, I drew a larger square around a tilted square. The area of the large square is equal to the area of the tilted square plus the area of the four congruent triangles. I found the area of the large square and then subtracted the area of each triangle.**
- **The area of a tilted square can be divided into four congruent triangles and a square. I calculated the area of one triangle, multiplied that by four, and then added the area of the square.**
- **I counted the number of unit squares within a tilted square. Since the square is tilted, there were a number of partial square units. I counted those partial square units and estimated how many full square units they would equal.**

4. What is the side length of square D?

5 units

Explain your thinking.

Explanations vary. **The area of square D is 25 square units. The side length of the square must be 5 units because 5^2 is equal to 25.**

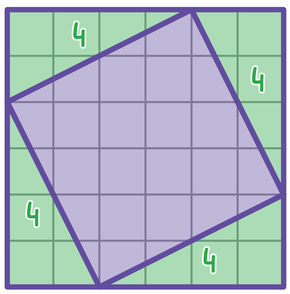
Activity 2

Name: .... Date: .... Period: ....

Different Strategies

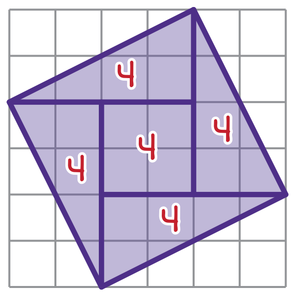
Here are Trevon's and Zahra's strategies for finding the area of tilted square A.

Trevon



$6 \cdot 6 - 4 \cdot 4 = 20$ square units

Zahra



$4 \cdot 4 + 4 = 20$ square units

5. How are Trevon's and Zahra's strategies alike? How are they different?

***Responses vary.* Both strategies require finding the area of congruent triangles and squares. In Trevon's strategy, the original tilted square is surrounded by a larger square, the area of the larger square is calculated, and then the area of the four congruent triangles are subtracted. In Zahra's strategy, the tilted square is decomposed into four congruent triangles and one square. The area of one triangle is found, multiplied by four, and then added to the area of the center square.**

6. How does each strategy compare to your own?

Responses vary.

- **I used the same strategy as Trevon to find the area by surrounding and subtracting.**
- **I used the same strategy as Zahra to find the area by decomposing the tilted square.**
- **I used something different than Trevon and Zahra and found an approximate area by counting the unit squares and estimating.**

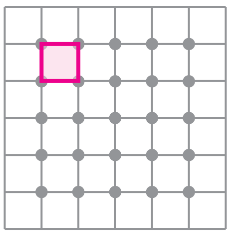
Activity 3

Name: .... Date: .... Period: ....

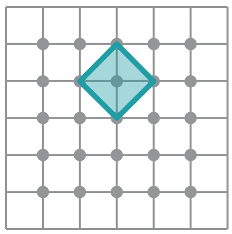
Building Squares With Different Areas

7. Here are squares with areas of 2 square units and 9 square units. On each dot grid, try to draw a square with the given area. Then circle "P" for any area that is possible to draw and "N" for any area that's not possible to draw. **Samples shown on grids.**

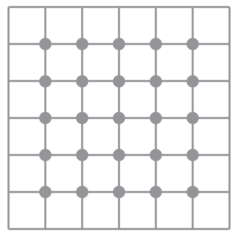
Area:
1 square unit
P / N



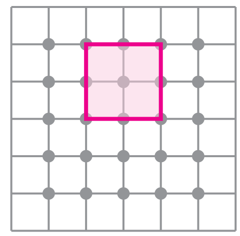
Area:
2 square units
P / N



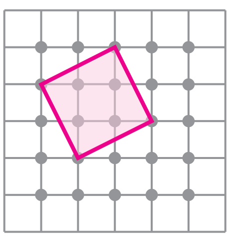
Area:
3 square units
P / N



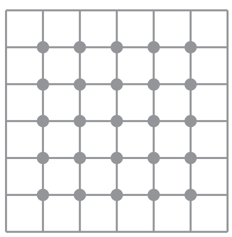
Area:
4 square units
P / N



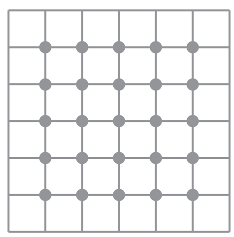
Area:
5 square units
P / N



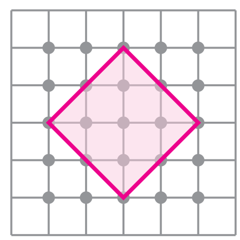
Area:
6 square units
P / N



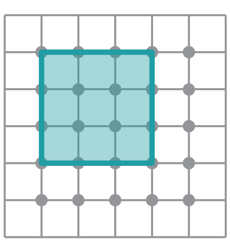
Area:
7 square units
P / N



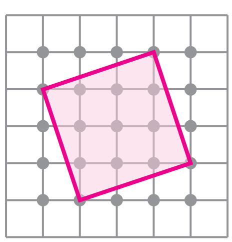
Area:
8 square units
P / N



Area:
9 square units
P / N



Area:
10 square units
P / N



8. Choose one of the squares and determine its side length.

Responses vary.

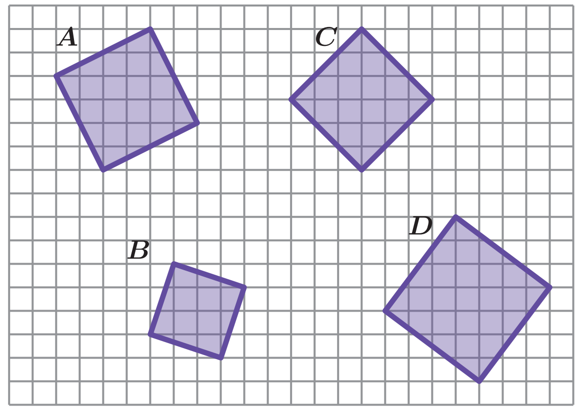
- **The square with 4 square units has a side length of 2 units.**
- **The square with 9 square units has a side length of 3 units.**

Synthesis

9. Describe a strategy for determining the area of a tilted square.

Use the examples if they help with your thinking.

Responses vary. **Draw a larger square around the tilted square. Then find the areas of the triangles and subtract them from the area of the larger square.**



Things to Remember:

Unit 8

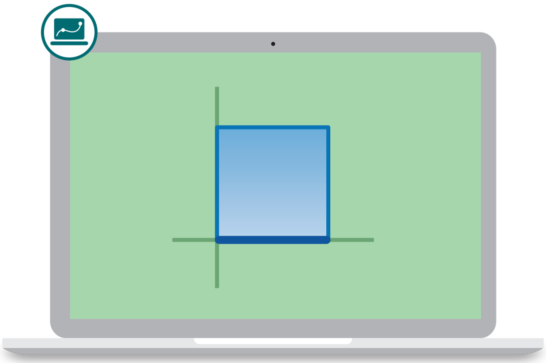
Lesson

2

Name: .... Date: .... Period: ....

From Squares to Roots

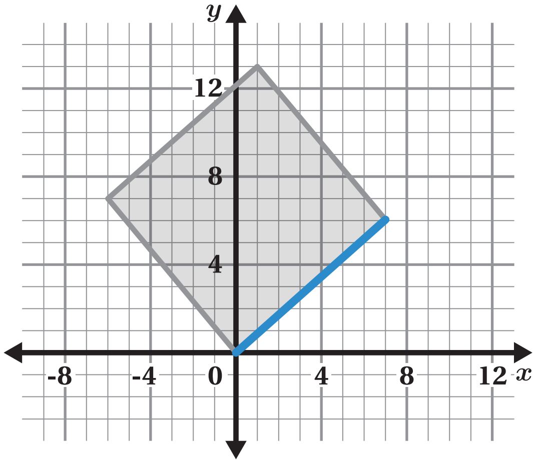
Let's explore the connection between the area and side length of a square.



Warm-Up

1 Estimate the side length of the square.

Responses vary. 9 units

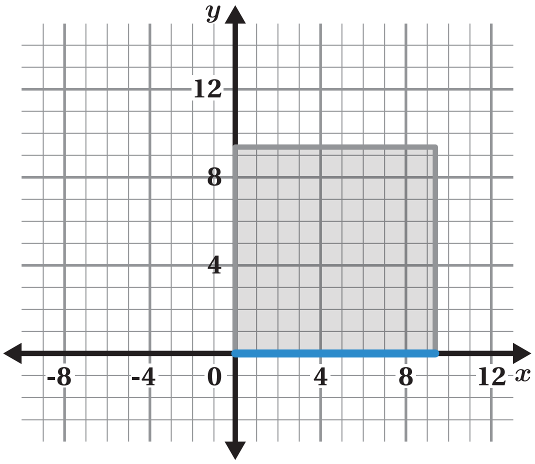


2 You can *approximate* the side length of a tilted square by rotating it onto an axis.

a Here is the square from the previous problem rotated so that the highlighted side length is along the x-axis.

b Write a new estimate for the side length of the square.

Responses vary. 9.2 units

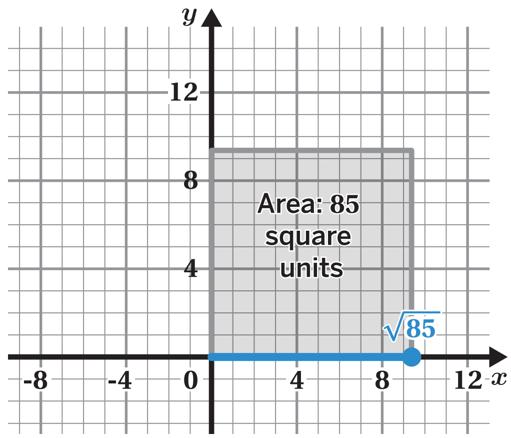


Activity 1

Name: ______ Date: ______ Period: ______

Square Roots

3 The *exact* side length of this square is the **square root** of 85, written as $\sqrt{85}$.

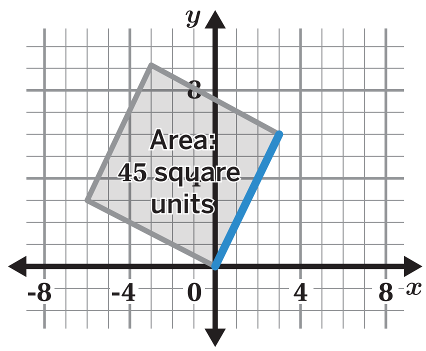


a Take a look at the square and how the side length is written.

b **Discuss:** Explain what you think a *square root* is in your own words.

***Responses vary.* A square root, $\sqrt{n}$, has the same value as a side length of a square with an area of n square units.**

4 The area of this square is 45 square units.



Write the exact value of the side length.

$\sqrt{45}$ units

5 Determine the unknown side lengths and areas for each square.

Square	Side Length (units)	Area (sq. units)
A	$\sqrt{55}$	55
B	**$\sqrt{81}$ or 9**	81
C	2.5	**6.25**
D	**$\sqrt{14}$**	14
E	$\sqrt{44}$	**44**
F	**$\sqrt{32}$**	32

Name: ______ Date: ______ Period: ______

Square Roots (continued)

6 Order the squares from *smallest* to *largest* area.

A	B	C	D
Area of 50 square units	Side length of $\sqrt{81}$ units	Side length of $\sqrt{55}$ units	Side length of 8 units

A	C	D	B

Smallest **Largest**

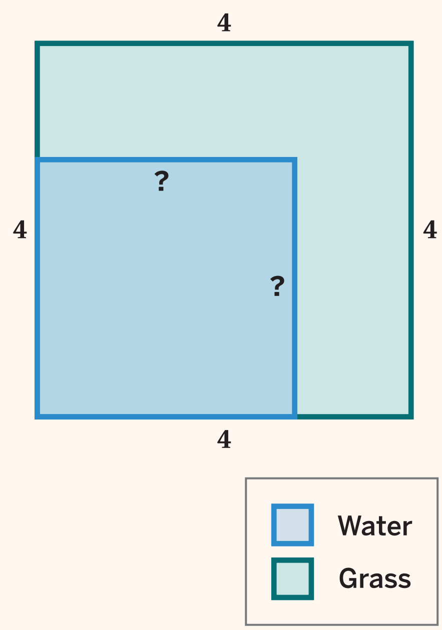
Explore More

7 Metropolis has a park surrounded by a square fence with 4-meter side lengths.

The city would like to build a square pool as shown in the figure.

What should the side length of the pool be so that half of the area is grass and half is water? Explain your thinking.

$\sqrt{8}$ meters. *Explanations vary*. The area surrounded by the fence is 16 square meters, so we want the area of both the grassy region and the water region to be 8 square meters. For the blue square in the figure to have an area of 8 square meters, the side length needs to be $\sqrt{8}$ meters.

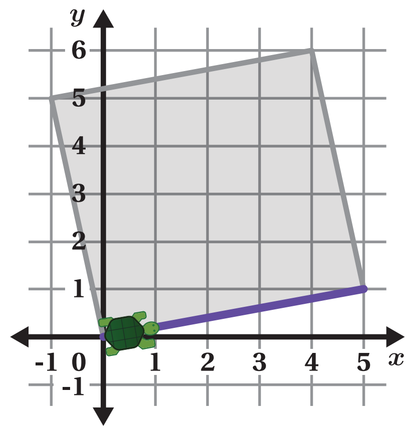


Activity 2

Name: .... Date: .... Period: ....

Turtle Tracing

Tiam the turtle is walking on one side of a square.

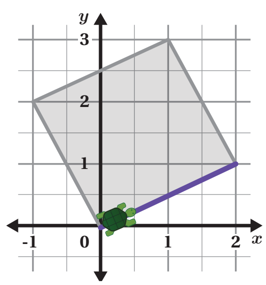
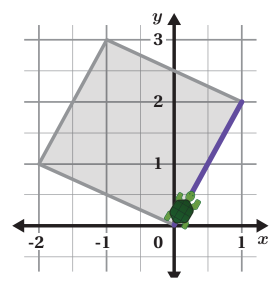
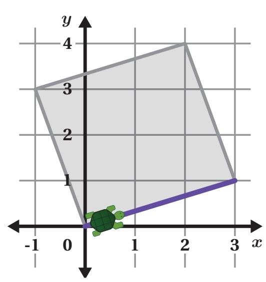
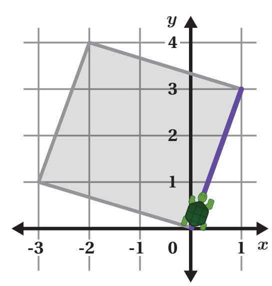
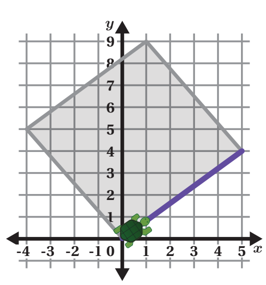
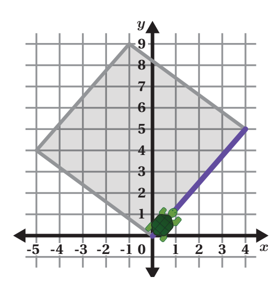


8 Exactly how far does Tiam need to travel?

$\sqrt{26}$ units

9 Complete this chart with a partner.

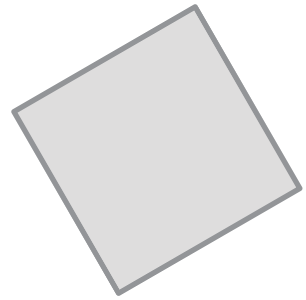
- Decide with your partner who will complete Column A and who will complete Column B.
- Determine how far Tiam the turtle needs to travel, and then compare your solutions. The solutions in each row should be the same. Discuss and resolve any differences.
- Determine the side lengths of as many squares as you have time for.

	Column A	Column B
a	$\sqrt{5}$ units 	$\sqrt{5}$ units 
b	$\sqrt{10}$ units 	$\sqrt{10}$ units 
c	$\sqrt{41}$ units 	$\sqrt{41}$ units 

10 Synthesis

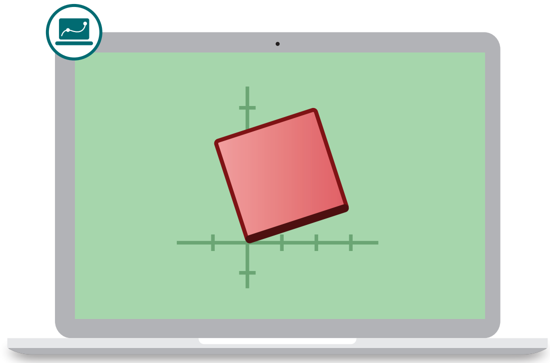
Describe the relationship between the side length and the area of a square using the term *square root*.

***Responses vary.* The square root of a square's area will give you the exact value of the square's side length.**



Things to Remember:

Name: .... Date: .... Period: ....

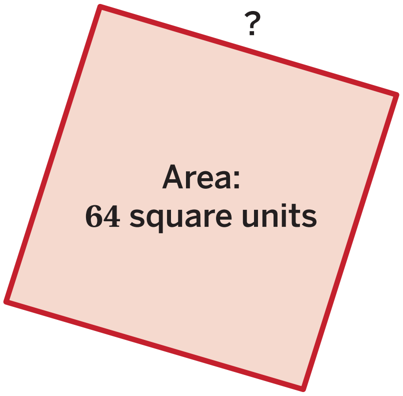


Between Squares

Let's approximate the value of square roots.

Warm-Up

1 Select *one* correct expression for the side length of this square.



- ☐ A. $\frac{64}{2}$
- ☑ B. $\sqrt{64}$
- ☑ C. 8
- ☐ D. $\sqrt{8}$
- ☐ E. 4
- ☐ F. $\frac{64}{4}$

Explain your thinking.

Explanations vary.

- $\sqrt{64}$. The side length of a square is the square root of the area.
- 8. You can determine the area of a square by squaring the side length, and 8^2 is 64 square units.

Activity 1

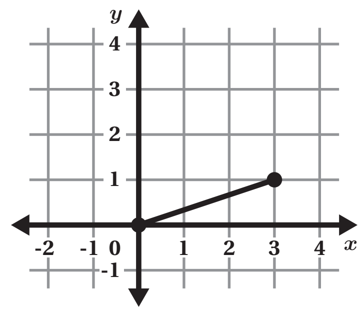
Name: .... Date: .... Period: ....

Squaring Lines

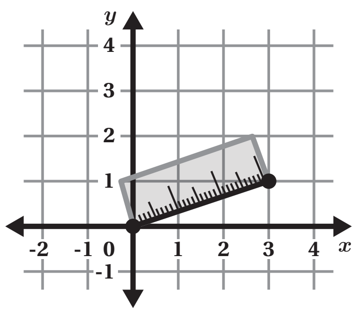
2 What do you think is the length of this segment?

Use the ruler, circle, or square to help with your thinking.

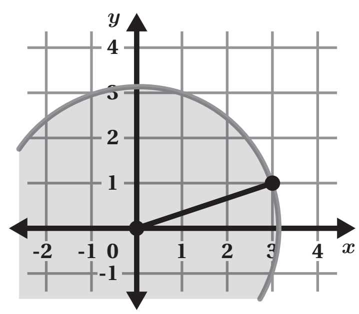
Responses vary.
- $\sqrt{10}$ units
- 3.2 units



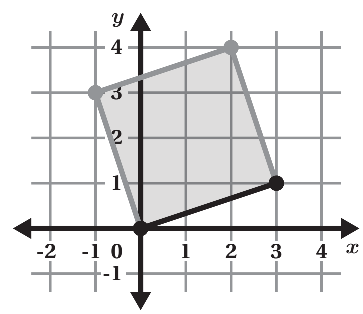
Ruler



Circle



Square



3 Ava says that the segment length is $\sqrt{10}$ units because the area of the square is 10 square units.

Raine says that the segment length is about 3.2 units because that's the approximate length of the circle's radius.

Discuss:
- How are Ava's and Raine's strategies alike? How are they different?
- What is helpful about each strategy?

Responses vary.
- Ava's and Raine's strategies are alike in that they both use shapes to find the length of the line segment. Their strategies are different because Ava used the area of a square to determine the exact length of the segment, and Raine used the radius of a circle to approximate the length of the segment.
- Ava's strategy is helpful for determining the exact value of a line segment. Raine's strategy is helpful for determining the approximate value of a line segment.

Activity 2

Name: ____________ Date: ______ Period: ______

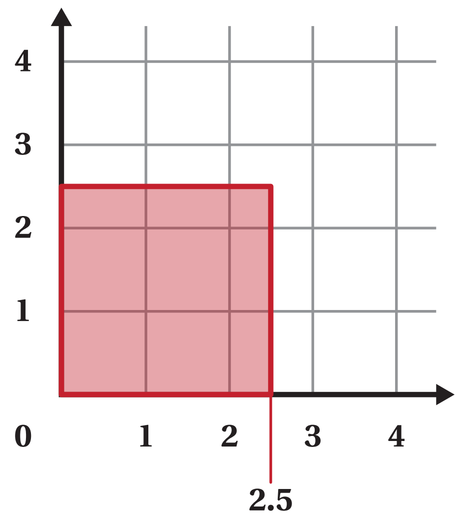
Using Squares to Estimate

4 How does $\sqrt{5}$ compare to 2.5? Circle one.
Use the square to help with your thinking.

$\sqrt{5}$ is less than 2.5 | $\sqrt{5}$ is greater than 2.5 | $\sqrt{5}$ is equal to 2.5

Explain your thinking.

***Explanations vary.* When the side length of a square is 2.5 units, its area is 6.25 square units. For the area of a square to be 5 square units (or $\sqrt{5} \cdot \sqrt{5}$), its side length must be less than 2.5 units.**

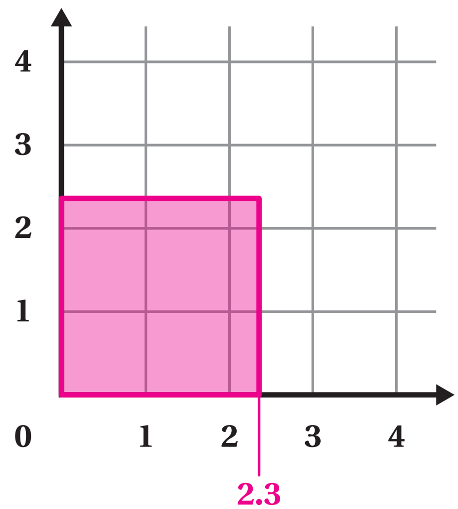


5 **a** Sketch a square to help you estimate $\sqrt{5}$.

***Responses vary*. Sample shown on graph.**

b Write your estimate for $\sqrt{5}$.

Responses between 2 and 2.5 are considered correct.



Activity 3

Name: ______ Date: ______ Period: ______

Closest Decimal Approximation

6 $\sqrt{5}$ is a number that equals 5 when squared.

Use a calculator to approximate the value of $\sqrt{5}$ as closely as you can. Record each guess, n, and its square, n^2, in the table.

Responses vary. **The exact value of n is between 2.236 and 2.237.**

n	n^2
2.0	$(2.0)^2 = 4.00$
2.2	**4.84**
2.3	**5.29**
2.25	**5.0625**
2.23	**4.9729**

7 Describe your strategy for finding a decimal approximation that is as close as possible to $\sqrt{5}$.

Responses vary. **I started with 2.1, but its square was too low. So then I tried 2.4, but its square was too high. I kept using each approximation to make a better approximation the next time.**

8 Use a calculator to approximate the value of $\sqrt{30}$ as closely as you can. Record each guess, n, and its square, n^2, in the table.

Responses vary. **The exact value of n is between 5.477 and 5.478.**

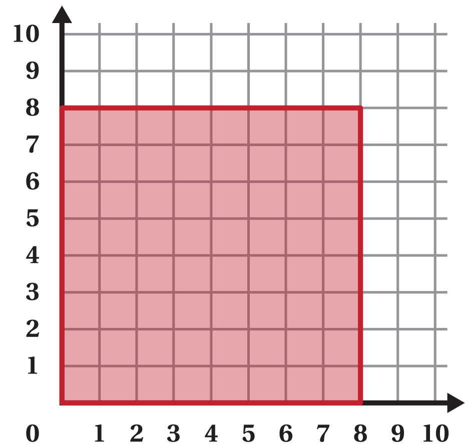
n	n^2
5.2	$(5.2)^2 = 27.04$
5.3	**28.09**
5.4	**29.16**
5.5	**30.25**
5.45	**29.7025**

9 Synthesis

What are some strategies for approximating a square root, such as $\sqrt{75}$?

Responses vary.

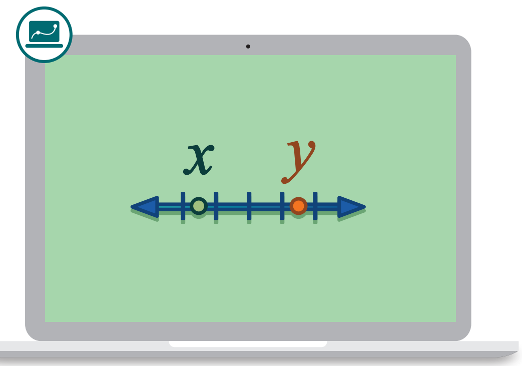
- I could make a square with one side along the x-axis and an area of approximately 75 square units. Then I could use the x-axis scale to estimate the square's side length, which would approximate $\sqrt{75}$.
- I could use a calculator to square different approximations of the square root, trying to get as close to the target as possible. With $\sqrt{75}$, I might start with 8.5. This is because $8^2 = 64$ and $9^2 = 81$, so $\sqrt{75}$ must be somewhere in between. Then depending on whether 8.5^2 is greater than or less than 75, I would revise my estimate and try again.



n	n^2
8.0	64

Things to Remember:

Name: Date: Period:



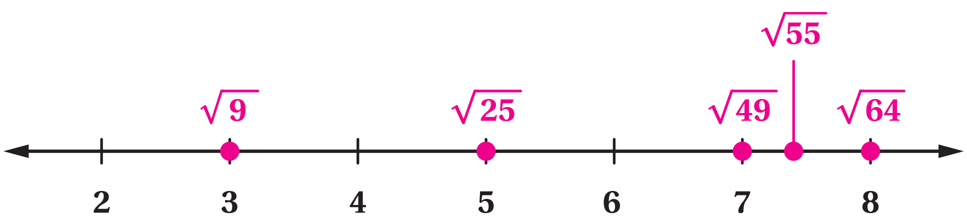
Root Down

Let's estimate the value of square roots and represent them on a number line.

Warm-Up

1 Plot these values on the number line.

$\sqrt{49}$	$\sqrt{64}$	$\sqrt{9}$	$\sqrt{25}$	$\sqrt{55}$



2 9, 25, 49, and 64 are **perfect squares**. 55 is not.

What do you think a perfect square is?

***Responses vary.* A perfect square is a number that is the square of an integer. For example, 9 is a perfect square because $3 \cdot 3 = 9$, but 55 is not a perfect square because there is no square of an integer that equals 55.**

Activity 1

Name: Date: Period:

Between Whole Numbers

3 Match each value to the whole numbers it is between. Two values will not have a match. Note: The numbers x, y, and z are positive numbers.

The value of x when $x^2 = 50$	The value of y when $y^2 = 20$	The value of z when $z^2 = 80$	$\sqrt{24}$
$\sqrt{62}$	$\sqrt{60}$	$\sqrt{17}$	$\sqrt{15}$

Between 4 and 5	Between 7 and 8	No Match
The value of y when $y^2 = 20$ $\sqrt{24}$ $\sqrt{17}$	$\sqrt{62}$ $\sqrt{60}$ The value of x when $x^2 = 50$	The value of z when $z^2 = 80$ $\sqrt{15}$

4 Esi thinks that the description "The value of z when $z^2 = 80$" doesn't belong in either category.

Between 7 and 8	Between 4 and 5

The value of z when $z^2 = 80$

a Which two whole numbers is the value of z between?

A. 4 and 5

B. 6 and 7

C. 7 and 8

D. 8 and 9

b Of those two numbers, which would z be closer to?

9

Explain your thinking.

***Explanations vary.* Since $9^2 = 81$, $\sqrt{80}$ is slightly less than 9.**

Name: ________ Date: ________ Period: ________

Between Whole Numbers (continued)

5 Order the numbers from *least* to *greatest*.

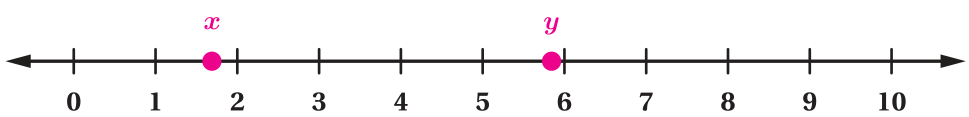
$\sqrt{99}$	$\sqrt{75}$	9	9.5	10

$\sqrt{75}$	9	9.5	$\sqrt{99}$	10

Least **Greatest**

6 The numbers x and y are positive. $x^2 = 3$ and $y^2 = 35$.

a Plot x and y on the number line.



b **Discuss:** How did you decide where to plot each point?

Responses vary. I know $\sqrt{4} = 2$, and since 3 is less than 4, x should be slightly less than 2. I know $\sqrt{36} = 6$, and since 35 is less than 36, y should be slightly less than 6.

Name: Date: Period:

Challenge Creator

7 Use blank paper to create your own challenge. ***Responses vary.***

a **Make It!** On the paper, write five numbers in a random order. Include at least one square root.

b **Solve It!** On this page, order the numbers from *least* to *greatest.*

My Challenge

Least **Greatest**

c **Swap It!** Swap your challenge on the blank paper with one or more partners. For each partner's challenge, order the numbers from *least* to *greatest*.

Partner 1's Challenge

Least **Greatest**

Partner 2's Challenge

Least **Greatest**

Partner 3's Challenge

Least **Greatest**

Partner 4's Challenge

Least **Greatest**

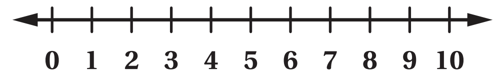
8 Synthesis

What are some strategies for plotting square roots on a number line?

Use the number line and examples if they help to show your thinking.

Responses vary. **To figure out where to plot a square root on a number line, determine the two whole numbers it is between. For example, $\sqrt{40}$ is between 6 and 7 because $\sqrt{36} = 6$, $\sqrt{49} = 7$, and 40 is between 36 and 49.**

$\sqrt{25}$ $\sqrt{36}$ $\sqrt{31}$ $\sqrt{40}$



Things to Remember:

Unit 8
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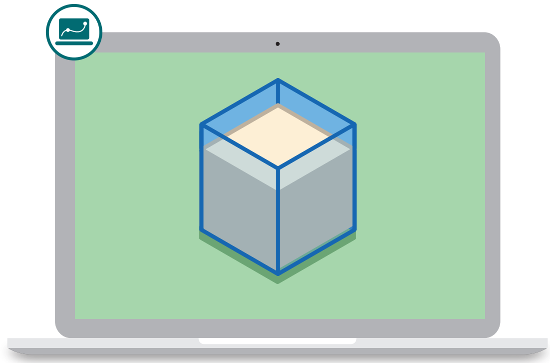
5

Name: ________ Date: ________ Period: ________



Filling Cubes

Let's explore the relationship between the edge length and the volume of a cube.

Warm-Up

1 Order each value from *least* to *greatest*. Note: Let a, b, c, and d be positive numbers.

a when $a^2 = 9$	b when $b^3 = 8$	c when $c^2 = 8$	d when $d^3 = 9$

b when $b^3 = 8$	**d when $d^3 = 9$**	**c when $c^2 = 8$**	**a when $a^2 = 9$**

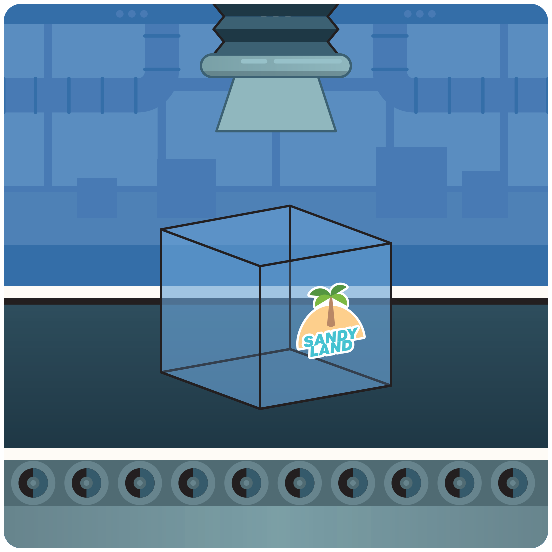
Least **Greatest**

Activity 1

Name: Date: Period:

Filling Cubes

Your job is to make sure the right amount of sand ends up in each cube. Use a calculator if it helps with your thinking.



2 This cube has an edge length of 6 inches.

How much sand is needed to fill it?

Edge Length (in.)	Amount of Sand (cu. in.)
6	**216**

3 Four new orders just came in. Complete the table for each order.

Edge Length (in.)	Amount of Sand (cu. in.)
3	**27**
2.1	**9.261**
4	64
5	125

4 Describe a strategy you used to find the unknown edge lengths.

Responses vary. **To determine the edge lengths, I thought about what number raised to the third power would result in that amount of sand. For example, to determine the edge length of a box with 64 cubic inches, I know that $4^3 = 64$, so the edge length would be 4 inches.**

Activity 1

Name: ______ Date: ______ Period: ______

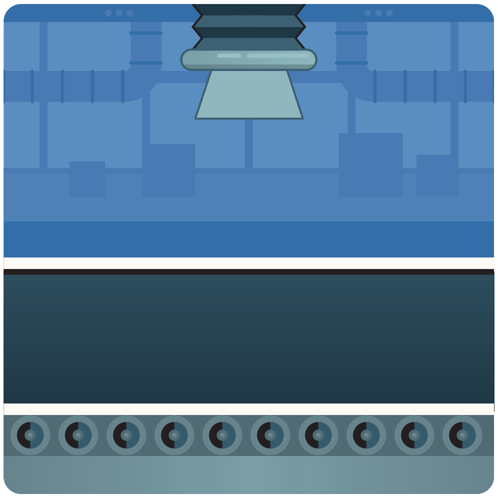
Filling Cubes (continued)

5 A customer wants a cube filled with 100 cubic inches of sand.

Let's try to find the *exact* edge length of this cube.

Choose an edge length and use your calculator to determine how much sand will fill that cube. Keep revising your estimate to get as close to the target as possible.

***Responses vary*. The exact edge length is between 4.641 and 4.642 inches.**



Edge Length (in.)	Amount of Sand (cu. in.)
4.5	$(4.5)^3 = 91.125$
4.7	**103.823**
4.65	**100.544625**
4.64	**99.897344**
4.642	**100.026577288**

6 The equation $x^3 = 100$ can help you determine the edge length of a cube that holds 100 cubic inches of sand. The exact solution to this equation is a **cube root**: $x = \sqrt[3]{100}$.

a. Enter $\sqrt[3]{100}$ on your calculator to see its approximate value. $\sqrt[3]{100} \approx 4.64159$

b. **Discuss:** What is the relationship between the edge length and the volume of a cube?

***Responses vary*. The edge length of a cube is equal to the cube root of its volume.**

7 Determine the *exact* unknown value for each cube.

Edge Length (in.)	Amount of Sand (cu. in.)
$\sqrt[3]{200}$	200
$\sqrt[3]{150}$	150
$\sqrt[3]{91.125}$ **or** 4.5	91.125
$\sqrt[3]{42}$	**42**

Activity 2

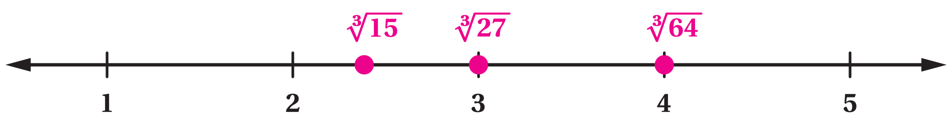
Name: ______ Date: ______ Period: ______

The Number Line

8 Here are three cube roots.

$\sqrt[3]{27}$	$\sqrt[3]{64}$	$\sqrt[3]{15}$

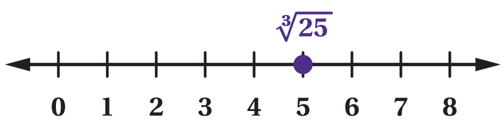
a Plot the cube roots on the number line.



b Describe your strategy for plotting $\sqrt[3]{15}$.

Responses vary. **15 is between the perfect cubes 8 and 27. Since $\sqrt[3]{8} = 2$ and $\sqrt[3]{27} = 3$, $\sqrt[3]{15}$ must be between 2 and 3.**

9 Nia incorrectly plotted $\sqrt[3]{25}$.



Discuss:

- What mistake could Nia have made?
- What question could you ask Nia to help her correct her work?

Responses vary.

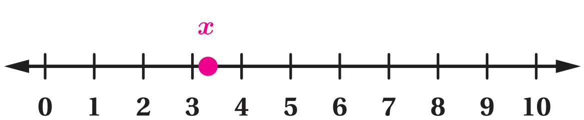
- **Nia might have been thinking about $\sqrt{25}$ instead of $\sqrt[3]{25}$.**
- **What does the cube root symbol mean?**
- **What two whole numbers is $\sqrt[3]{25}$ between?**

10 Here is an equation: $x^3 = 30$.

a Write the exact solution to the equation.

$x = \sqrt[3]{30}$

b Plot the solution on the number line.

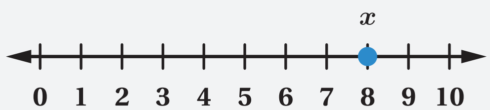
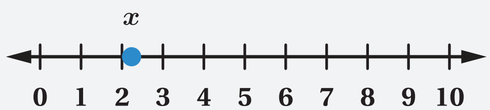
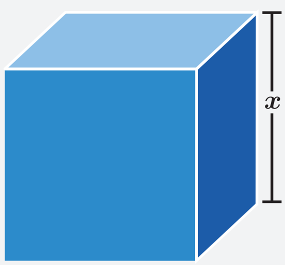
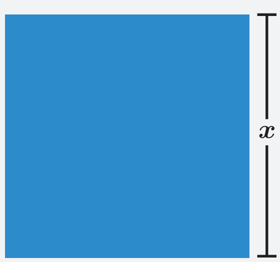


Name: ______ Date: ______ Period: ______

The Number Line (continued)

11 Match each equation to the visual that represents the same value of x.

$x = \sqrt{10}$	$x = \sqrt{64}$	$x = \sqrt[3]{10}$	$x = \sqrt[3]{64}$

Visual	Equation
	$x = \sqrt{64}$
	$x = \sqrt[3]{10}$
Volume: 64 cu. in. 	$x = \sqrt[3]{64}$
Area: 10 sq. in. 	$x = \sqrt{10}$

Explore More

12 **a** If you double the edge length of a cube, what happens to the volume?

Responses vary. **If you double the edge length, the volume is multiplied by 2^3, or 8.**

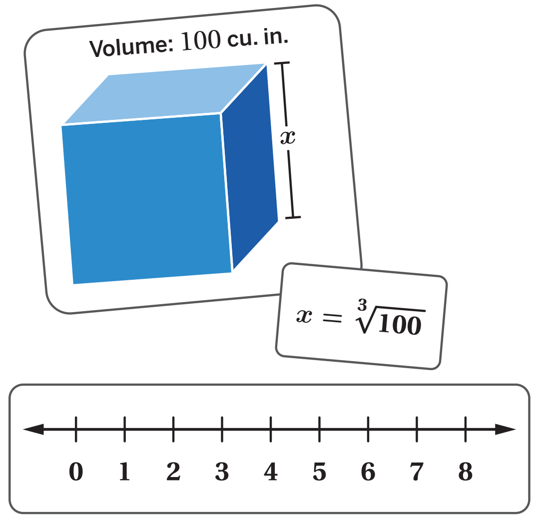
b If you double the volume of a cube, what happens to the edge length?

Responses vary. **If you double the volume, the edge length is multiplied by $\sqrt[3]{2}$.**

13 Synthesis

Explain a strategy for determining where to plot a cube root on the number line.

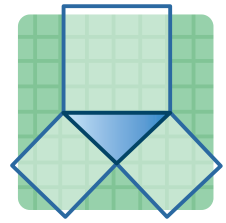
Responses vary. To determine where to plot a cube root on the number line, start by identifying the whole numbers around it. For example, $\sqrt[3]{100}$ is between 4 and 5 because $4^3 = 64$ and $5^3 = 125$. 100 is a little bit closer to 125 than 64, so $\sqrt[3]{100}$ should be just over halfway between 4 and 5.



Things to Remember:

Name: Date: Period:

The Pythagorean Theorem

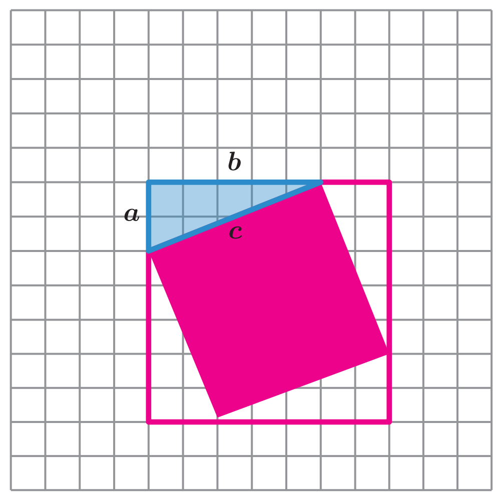


Let's explore the relationship between the squares of side lengths in triangles.

Warm-Up

1. Use any strategy to determine the value of c^2.

29 square units



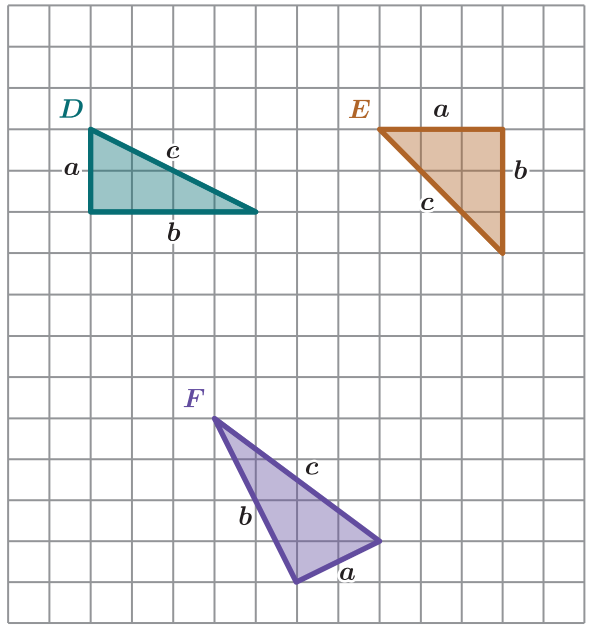
Explain your thinking.

Explanations vary. I created a tilted square with side length c inside a larger square with an area of 49 square units. I then determined the area of each triangle outside the tilted square to be 5 square units. Then I subtracted the area of the four congruent triangles from the area of the large square to find the area of just the tilted square, which is the value of c^2.

Name: ______ Date: ______ Period: ______

Squares of Side Lengths

2. Use these triangles to complete the table.



Triangle	a^2	b^2	c^2
D	**4**	**16**	**20**
E	**9**	**9**	**18**
F	**5**	**20**	**25**

3. What do you notice? What do you wonder? ***Responses vary.***

I notice:

- **For each of these triangles, c^2 is larger than a^2 or b^2.**
- **If you add a^2 and b^2 together, the result is equal to c^2.**

I wonder:

- **If $a^2 + b^2 = c^2$ is true for all triangles?**
- **What type of triangles are these?**

Activity 2

Name: ______ Date: ______ Period: ______

True for Every Triangle?

4. You will use a set of cards for this problem.

a Work with a partner to create groups where $a^2 + b^2 = c^2$ and $a^2 + b^2 \neq c^2$. Complete the table with the card numbers.

$a^2 + b^2 = c^2$	$a^2 + b^2 \neq c^2$
Cards 1, 4, and 6	**Cards 2, 3, and 5**

b Revisit your noticings and wonderings from Activity 1. What do you notice and wonder now?

I notice:

***Responses vary.* It looks like the triangles where $a^2 + b^2 = c^2$ are right triangles.**

I wonder:

***Responses vary.* For what type of triangles is $a^2 + b^2 > c^2$?**

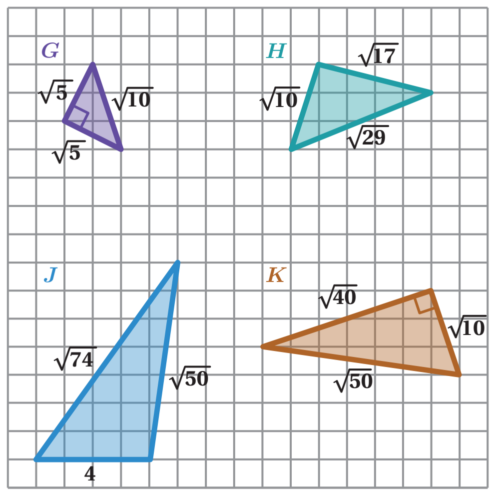
For each triangle, let a and b represent the two shorter sides and c represent the longest side.

5. Circle one triangle where the equation $a^2 + b^2 = c^2$ is true.

Triangle G Triangle H Triangle J Triangle K

Show or explain your thinking.

***Explanations vary.* Triangle G. The sum of the squares of the two shorter sides is $(\sqrt{5})^2 + (\sqrt{5})^2 = 10$. This is the same measure as the square of the longest side, $(\sqrt{10})^2 = 10$, which shows that $a^2 + b^2 = c^2$ is true for this triangle. This triangle is also a right triangle, which matches what I saw in the card sort.**

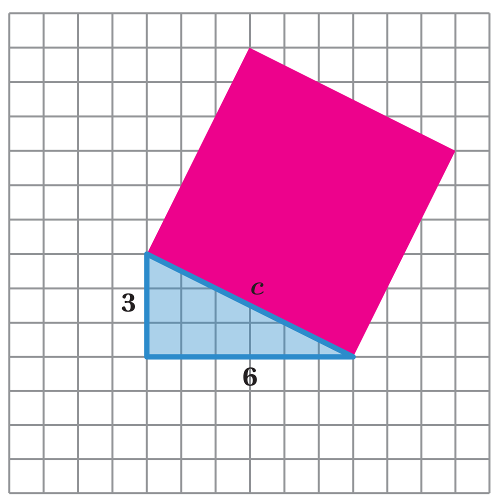


6. Sai says that the value of c is $\sqrt{45}$. Do you agree? Circle one.

Agree Disagree There's not enough information

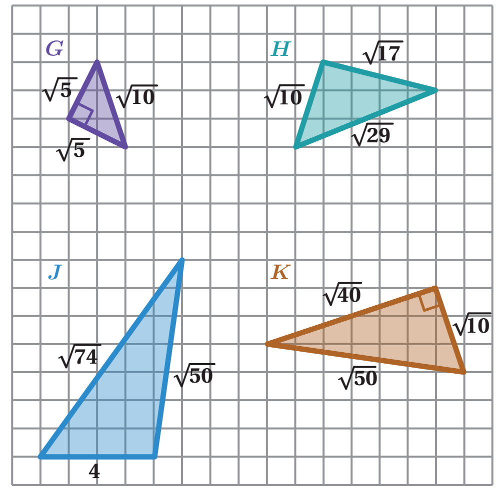
Show or explain your thinking.

***Explanations vary.* I drew a square with c as one of its sides and determined its area to be 45 square units. To find the value of c, I took the square root of the area to determine $c = \sqrt{45}$.**



Synthesis

7. The **Pythagorean theorem** says that for right triangles, $a^2 + b^2 = c^2$. The date of the first discovery is unknown, but the Babylonians used the Pythagorean theorem over 3,500 years ago (1,000 years before Pythagoras was born).



Explain the Pythagorean theorem in your own words. Use the triangles if they help with your thinking.

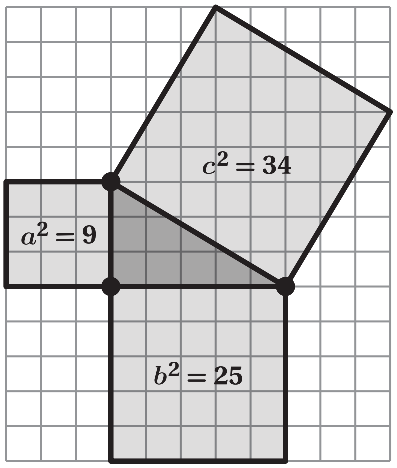
***Responses vary.* The Pythagorean theorem tells us that for right triangles, the sum of the squares of the shorter side lengths is equal to the square of the longest side length.**

Things to Remember:

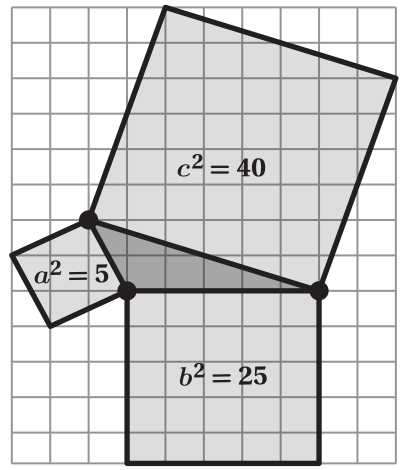
True for Every Triangle?

Directions: Make one copy per pair. Then pre-cut the cards and give each pair of students one set.

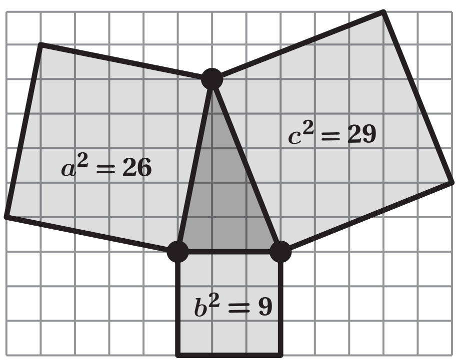
Card 1



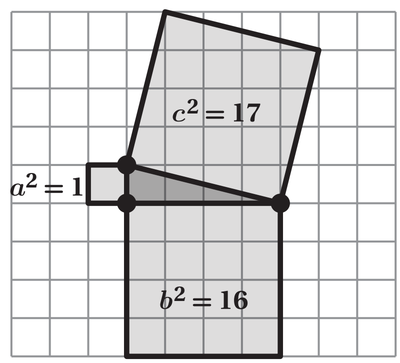
Card 2



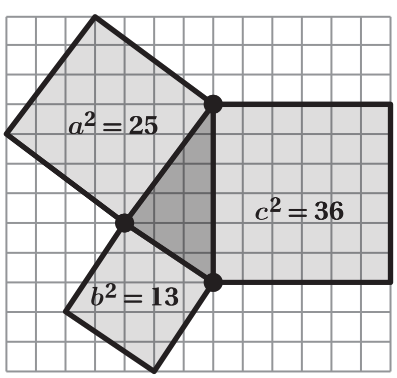
Card 3



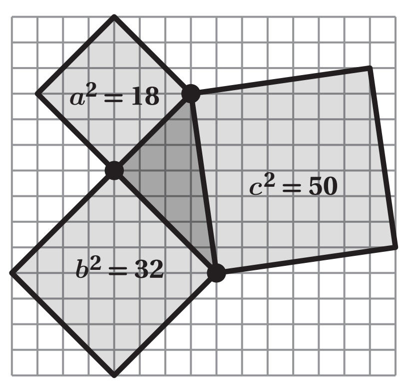
Card 4



Card 5

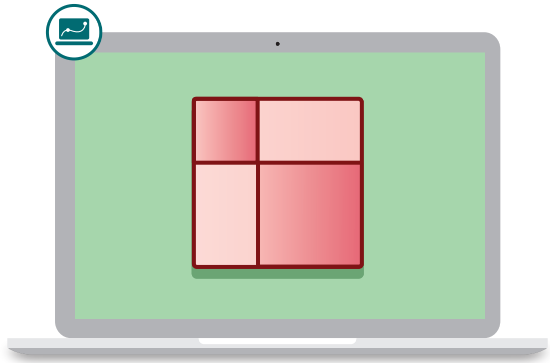


Card 6



Unit 8
Lesson
7

Name: ________ Date: ________ Period: ________



Pictures to Prove It

Let's prove the Pythagorean theorem.

Warm-Up

1 Here are two figures.

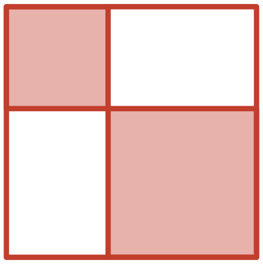


Figure A

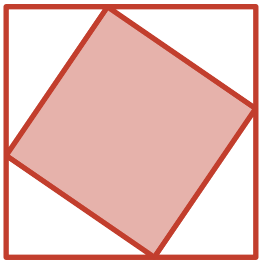


Figure B

Which one do you think has a larger *shaded* area? Circle one.

Figure A Figure B The areas are equal I'm not sure

Explain your thinking.

Responses and explanations vary.

- **Figure A. There are two shaded squares, whereas figure B only has one.**
- **Figure B. There is one big tilted square.**
- **The areas are equal. The triangles in figure B have the same area as the non-shaded rectangles in figure A.**
- **I'm not sure. There are no measurements or units on the squares. I need more information.**

Activity 1

Name: ____________ Date: ________ Period: ________

Pictures to Prove It

Note: For this lesson, assume figures that look like squares are squares.

2 Determine the total area of the shaded region in each figure. Mark the diagram if it helps with your thinking.

	Figure C	Figure D
Total Shaded Area (sq. units)	13	13

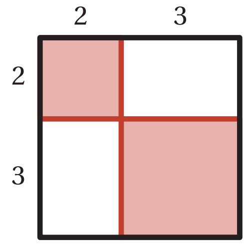


Figure C

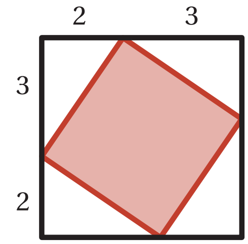


Figure D

3 Here are three different dimensions for a pair of new figures.

a Choose one of the options and circle your choice.

Option 1

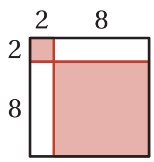


Figure E

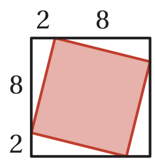


Figure F

Option 2

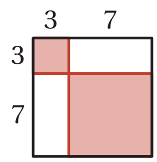


Figure E

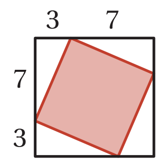


Figure F

Option 3

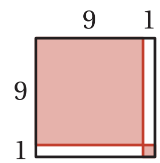


Figure E

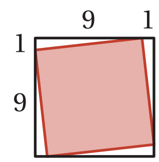


Figure F

b Determine the total area of the shaded region in each figure.

	Figure E	Figure F
Total Shaded Area (sq. units)	Option 1: 68 Option 2: 58 Option 3: 82	Option 1: 68 Option 2: 58 Option 3: 82

4 Do you think the total shaded area will *always* be equal between figures like E and F, even if their outer dimensions change?

Yes

Explain your thinking.

Explanations vary. The two figures have the same total area, and the two white rectangles have the same area as the four white triangles in the other figure, so the shaded areas must also be equal.

Name: .... Date: .... Period: ....

Thinking More Generally

Let's generalize using variables instead of numbers.

5 Determine the area of the shaded region in each figure using the variables a, b, or c.

	Figure G	Figure H
Total Shaded Area (sq. units)	$a^2 + b^2$	c^2

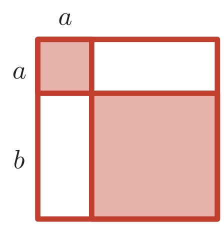


Figure G

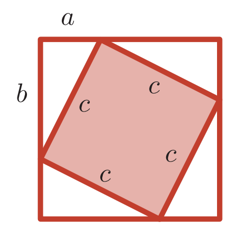


Figure H

6 The Pythagorean theorem states that for any *right* triangle, $a^2 + b^2 = c^2$.

How can we use these figures to prove that the Pythagorean theorem is true?

Mark the diagram if it helps to show your thinking.

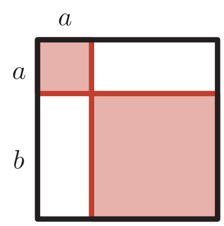


Figure G

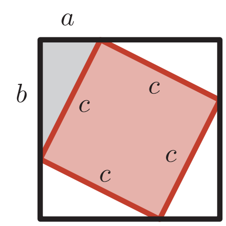


Figure H

***Responses vary.* Figure H shows a shaded right triangle with legs a and b and a hypotenuse c. Figures G and H both have a total area of $(a + b)^2$, so they are equal. We also know that the unshaded areas are equal. In figure G, the unshaded area is $2ab$ and in figure H, the unshaded area is $(4)\left(\frac{1}{2}\ ab\right)$, which simplifies to $2ab$. This means the remaining shaded areas in each figure also have to be equal. The shaded area in figure G can be expressed as $a^2 + b^2$ and the shaded area in figure H can be expressed as c^2, so $a^2 + b^2 = c^2$.**

Activity 3

Name: .... Date: .... Period: ....

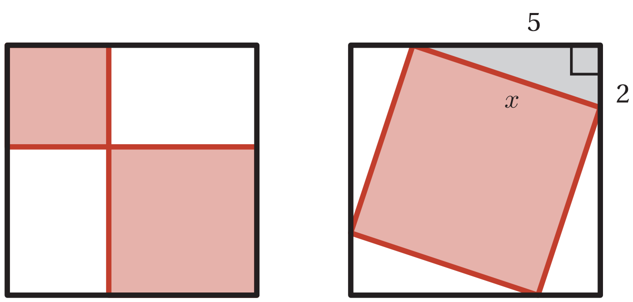
Let's Put It to Work

Let's put the Pythagorean theorem to work.

7 Calculate the value of x.

Draw on the diagram if it helps with your thinking.

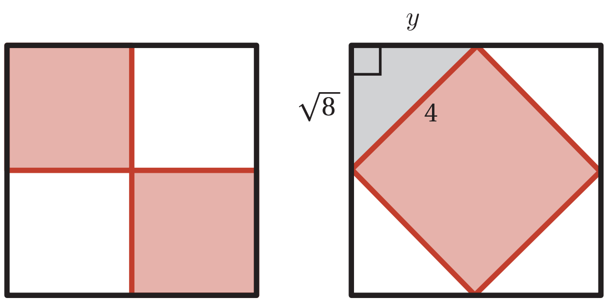
$\sqrt{29}$



8 Calculate the value of y.

Draw on the diagram if it helps with your thinking.

$\sqrt{8}$



Explore More

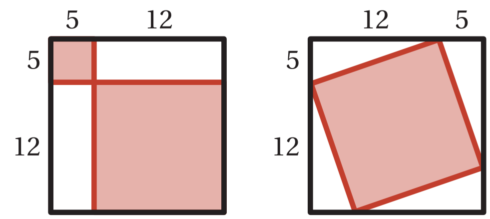
9 You will need the Explore More Sheet. Use scissors to cut along the dashed lines.

a Arrange the pieces in the smaller squares to fit in the large square.

b Describe what you notice about the relationship between the areas of the two smaller squares and the area of the large square.

10 Synthesis

Explain how the equation $5^2 + 12^2 = 13^2$ is related to the figures on the right and to the Pythagorean theorem.



***Responses vary.* In the left figure, I see $5^2 + 12^2$ as the total area of the two shaded squares. The Pythagorean theorem says that the area of the shaded square in the right figure will be equal to the sum of the shaded squares in the left figure, so its area is 169 square units, or 13^2 square units.**

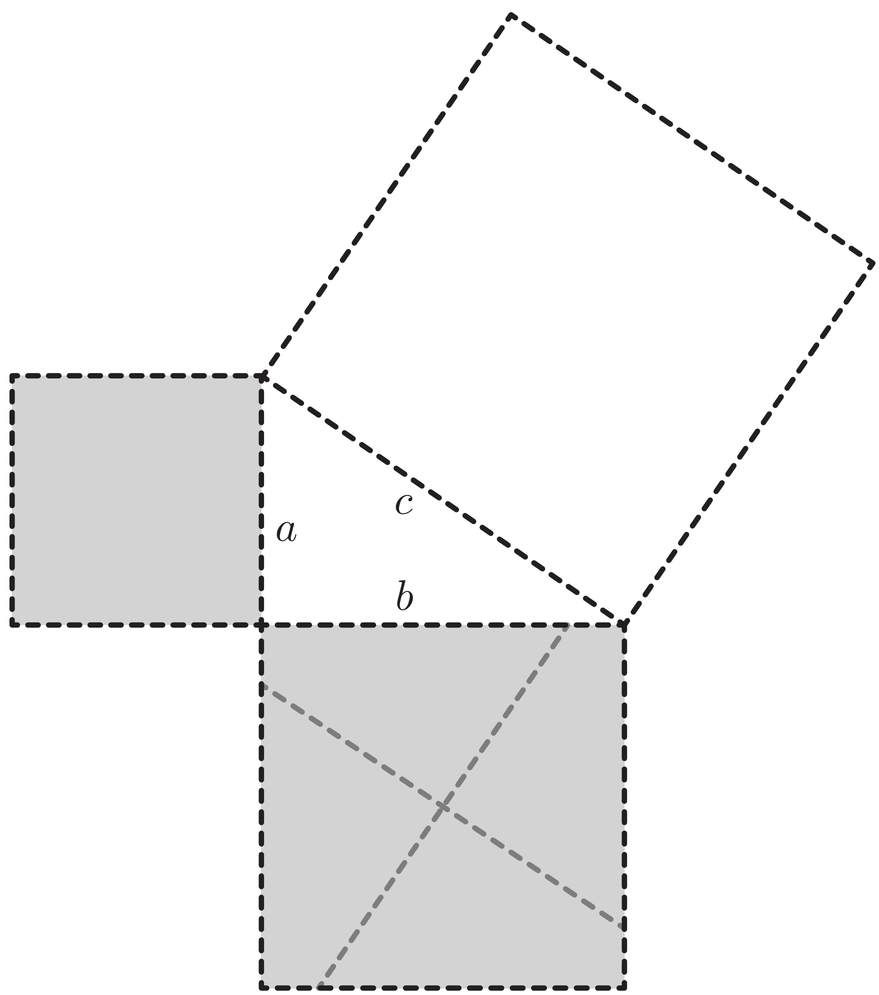
Things to Remember:

Name: ________________________ Date: ____________ Period: ____________

Explore More

a Use scissors to cut along the dashed lines. Arrange the pieces in the smaller squares to fit in the large square.

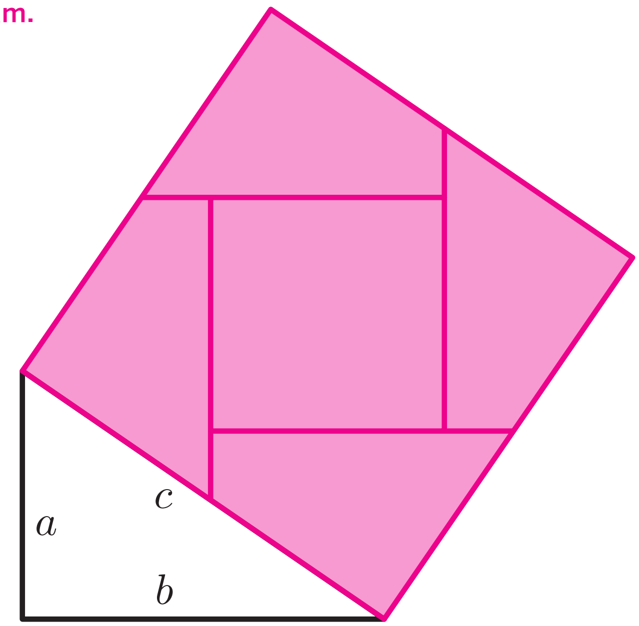
b Describe what you notice about the relationship between the areas of the two smaller squares and the area of the large square.



Explore More (answers)

a) Use scissors to cut along the dashed lines. Arrange the pieces in the smaller squares to fit in the large square.

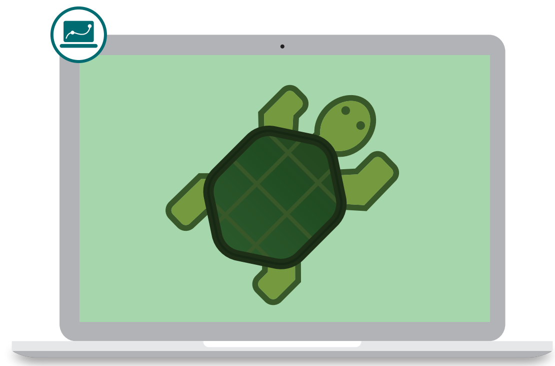
Response shown in diagram.



b) Describe what you notice about the relationship between the areas of the two smaller squares and the area of the large square.

The areas of the two smaller squares can always be arranged to fit inside the large square, showing that $a^2 + b^2 = c^2$.

Name: .... Date: .... Period: ....

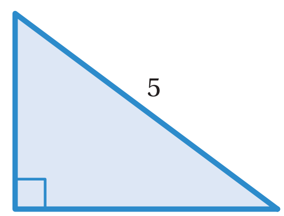


Triangle-Tracing Turtle

Let's calculate unknown side lengths in right triangles.

Warm-Up

1 Select *all* the equations that could help you calculate the unknown side length of this triangle.



☑ **A.** $a^2 + 4^2 = 5^2$

☐ **B.** $a = \sqrt{4^2 + 5^2}$

☐ **C.** $b^2 = 5^2 + 4^2$

☑ **D.** $4^2 + b^2 = 5^2$

☑ **E.** $b = \sqrt{5^2 - 4^2}$

Activity 1

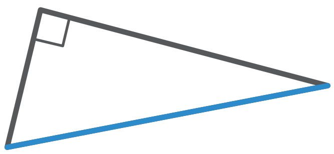
Name: .... Date: .... Period: ....

Hypotenuse

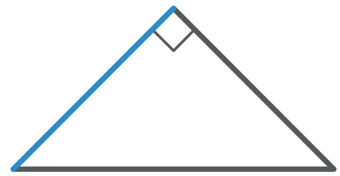
2 The **hypotenuse** is the side of a right triangle that is opposite the right angle. The **legs** of a right triangle are the sides that make the right angle.

Select *all* the triangles where a hypotenuse is highlighted.

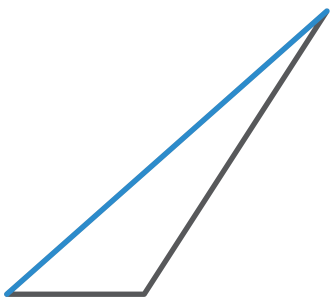
☑ **A.**



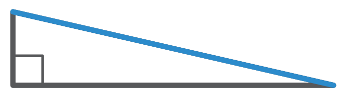
☐ **B.**



☐ **C.**



☑ **D.**



3 Melissa incorrectly thinks that triangles A, C, and D from the previous problem have a highlighted hypotenuse.

a What do you think Melissa did well?

***Responses vary.* Melissa selected A and D, both of which are right triangles with the longest side highlighted.**

b What mistake might Melissa have made?

***Responses vary.* Melissa selected a triangle that is not a right triangle. Hypotenuses can only be part of right triangles.**

Activity 2

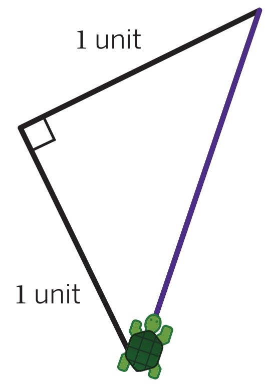
Name: ________ Date: ________ Period: ________

Turtle Tracing

Tiam the turtle is walking on one side of a triangle.

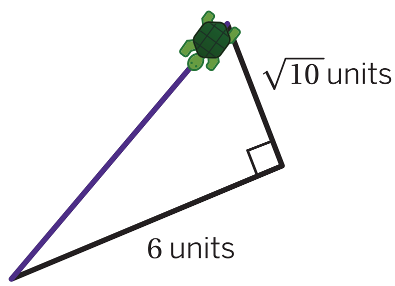
4 Exactly how far does Tiam need to travel?

$\sqrt{2}$ units



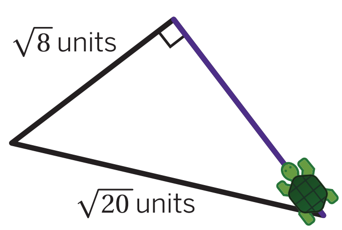
5 Exactly how far does Tiam need to travel?

$\sqrt{46}$ units



6 Exactly how far does Tiam need to travel?

$\sqrt{12}$ units



Activity 3

Name: ____ Date: ____ Period: ____

Challenge Creator

7 You will use a separate sheet of paper to create your own triangle challenge.

a **Make It!** On the paper, sketch a right triangle. Label two of the sides with their approximate lengths.

b **Solve It!** On this page, write the two side lengths you labeled on your triangle. Then calculate the length of the third side.

Responses vary.

My Sides	My Lengths (units)
Leg 1	
Leg 2	
Hypotenuse	

c **Swap It!** Swap your challenge with one or more partners. Calculate the unknown side length for each partner's triangle.

Responses vary.

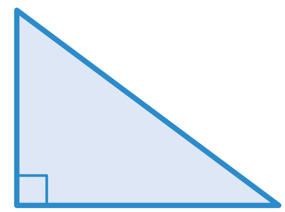
	Side	Length (units)
Partner 1	Leg 1	
	Leg 2	
	Hypotenuse	
Partner 2	Leg 1	
	Leg 2	
	Hypotenuse	
Partner 3	Leg 1	
	Leg 2	
	Hypotenuse	

8 Synthesis

If you know two side lengths of a right triangle, how can you calculate the third side length?

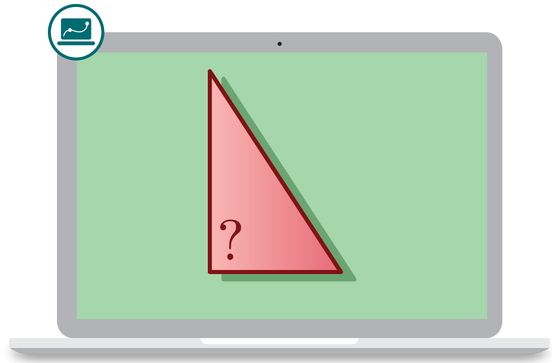
Use the image if it helps to show your thinking.

***Responses vary.* I can calculate the third side of a right triangle by substituting the two known side lengths into the Pythagorean theorem and solving for the unknown side.**



Things to Remember:

Name: .... Date: .... Period: ....



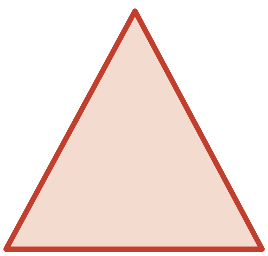
Make It Right

Let's determine if a triangle is a right triangle.

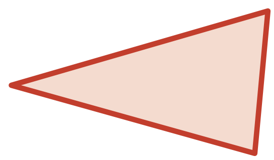
Warm-Up

1 Which one doesn't belong?

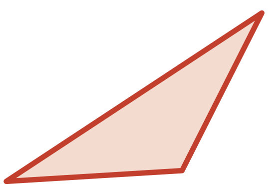
A.



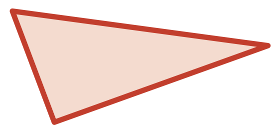
B.



C.



D.



Explain your thinking.

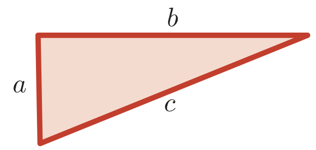
Responses and explanations vary.

- **Choice A doesn't belong because it's the only one that looks like it has three equal sides and three equal angles.**
- **Choice B doesn't belong because it's the only one that has all acute angles, and no sides look like they are the same length.**
- **Choice C doesn't belong because it's the only one that has an obtuse angle.**
- **Choice D doesn't belong because it's the only one that looks like a right triangle.**

Name: Date: Period:

Is the Converse True?

2 Mathematicians sometimes think about a statement's *converse*, which is a statement in the opposite direction.



The converse of the Pythagorean theorem says: *If a triangle has side lengths such that* $a^2 + b^2 = c^2$, *it is a right triangle.*

Do you think this statement is always, sometimes, or never true? Circle one.

Responses vary.

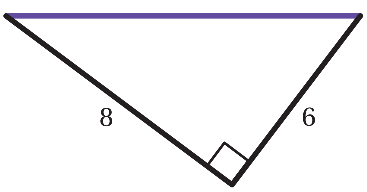
Always Sometimes Never

Explain your thinking.

Explanations vary.

- **I think this is always true because I know there are right triangles with side lengths that make $a^2 + b^2 = c^2$ true.**
- **I think this is sometimes true because you could have lots of different triangles with side lengths a, b, and c, but only one of them would be a right triangle.**

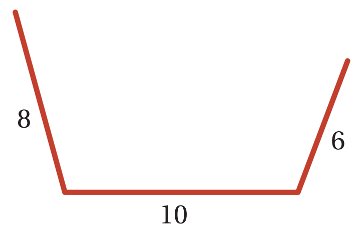
3 Let's explore the converse of the Pythagorean theorem by focusing on a specific example. Here is a right triangle with legs that are 6 and 8 units long.



What is the length of its hypotenuse?

10 units

4 You will use the cutouts from the Activity 1 Card to create several triangles.



a Experiment with making two other triangles with sides lengths measuring 6, 8, and 10 units.

Responses vary.

b **Discuss:** What do you notice? What do you wonder?

Responses vary.

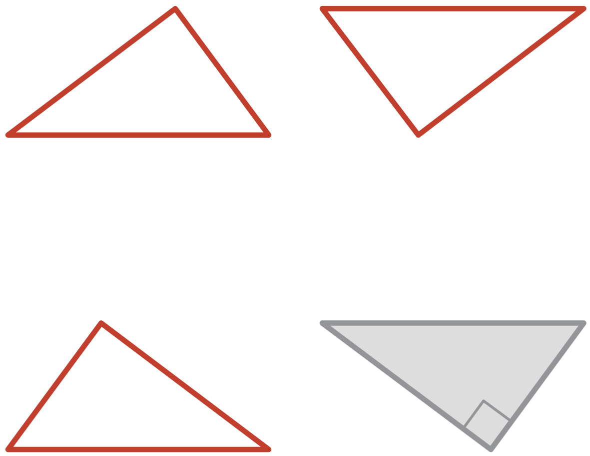
- **I notice that all the triangles appear to have a right angle.**
- **I notice that the triangles are reflections or rotations of each other.**
- **I wonder if the triangles are all right triangles.**
- **I wonder if the triangles are all the same triangle.**

Activity 1

Name: Date: Period:

Is the Converse True? (continued)

5 Yosef says that every triangle with sides lengths measuring 6, 8, and 10 units *must* be a right triangle, because the triangles with these lengths are all *congruent*.



Do you agree? Circle one. Use tracing paper to help with your thinking.

Yes No I'm not sure

Explain your thinking.

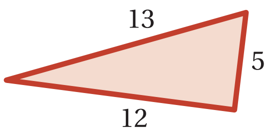
Explanations vary. **The Pythagorean theorem tells us that a right triangle with leg lengths 6 and 8 has a hypotenuse of length 10. We can use rigid transformations to show that any other triangle with those side lengths is congruent to the right triangle.**

Activity
2

Name: Date: Period:

Make It Right

6 What type of triangle is this? Circle one.



A right triangle

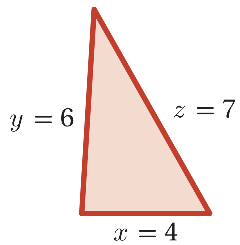
Not a right triangle

Explain your thinking.

***Explanations vary.* This is a right triangle because $5^2 + 12^2 = 13^2$.**

7 Change *one* of the values to make this triangle a right triangle.

There are many different solutions. Try to find *at least* four.



Responses vary.

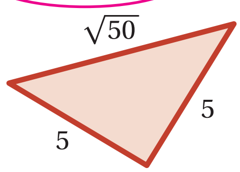
x	y	z
4	6	$\sqrt{52}$
4	6	$\sqrt{20}$
4	$\sqrt{65}$	7
4	$\sqrt{33}$	7
$\sqrt{13}$	6	7
$\sqrt{85}$	6	7

Name: .... Date: .... Period: ....

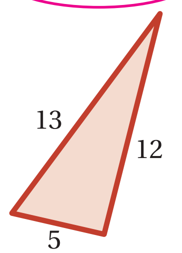
Make It Right (continued)

8 Circle whether each triangle is a right triangle.

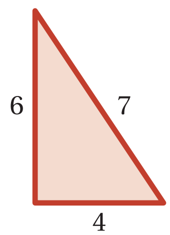
a Right Triangle   Not a Right Triangle



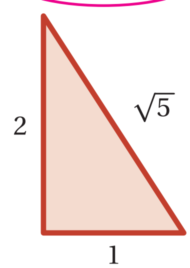
b Right Triangle   Not a Right Triangle



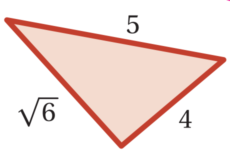
c Right Triangle   Not a Right Triangle



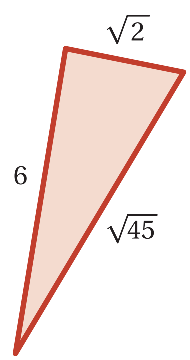
d Right Triangle   Not a Right Triangle



e Right Triangle   Not a Right Triangle



f Right Triangle   Not a Right Triangle



Explore More

9 Here is an obtuse triangle, an acute triangle, and a right triangle. *All* triangles are one of these three types.

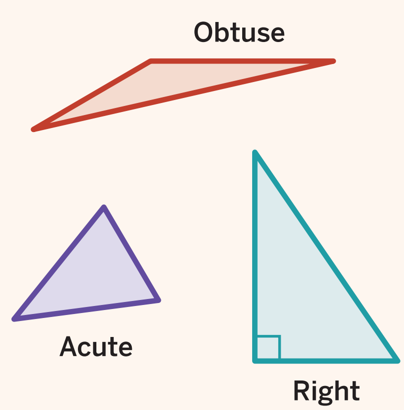
Decide whether triangles X, Y, and Z are acute, right, or obtuse based on their side lengths.

Triangle X, side lengths: 15, 20, 8
Triangle Y, side lengths: 8, 15, 13
Triangle Z, side lengths: 17, 8, 15

Triangle X: Obtuse. *Work varies.* $8^2 + 15^2 < 20^2$.

Triangle Y: Acute. *Work varies.* $8^2 + 13^2 > 15^2$.

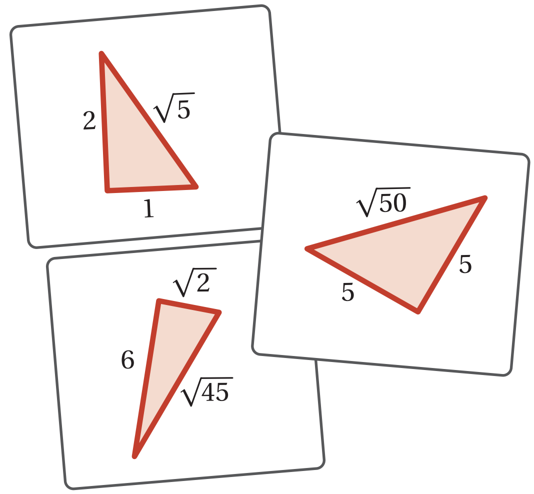
Triangle Z: Right. *Work varies.* $8^2 + 15^2 = 17^2$.



10 Synthesis

How can you tell from just the side lengths if a triangle is a right triangle?

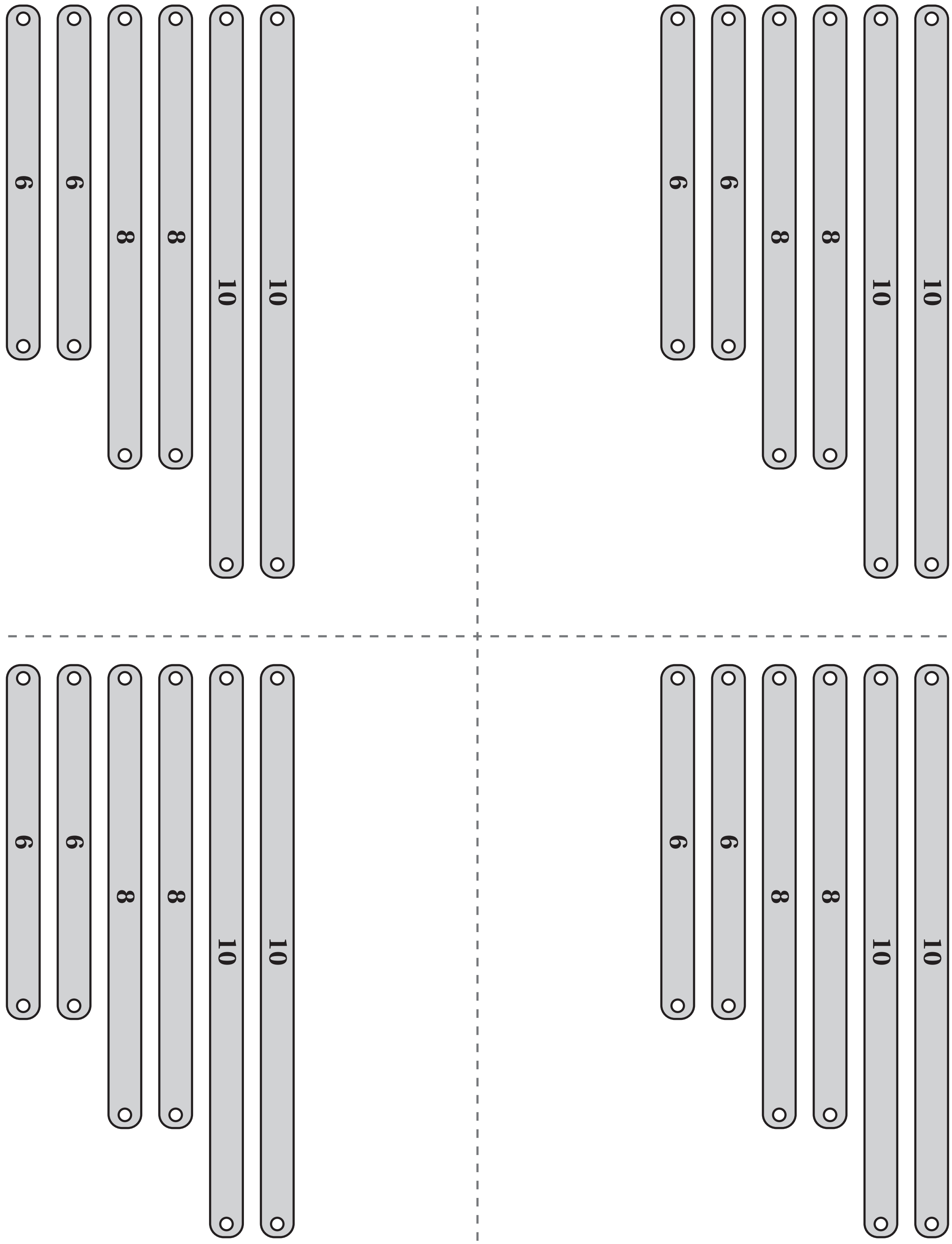
Responses vary. **Square the lengths of the two shorter sides and add them together. If the sum is equal to the square of the length of the longest side, then the triangle is a right triangle.**



Things to Remember:

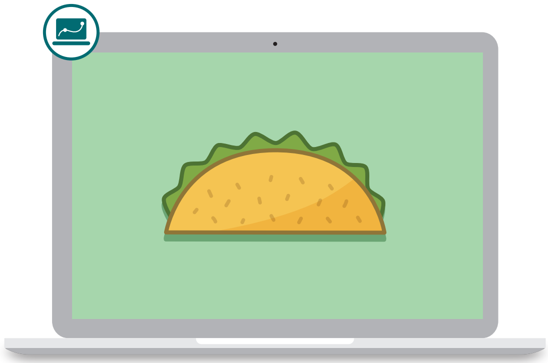
Is the Converse True?

Directions: Make one copy for every four students. Then pre-cut the cards and give each student one set of line segments. Have students cut out the line segments.



Unit 8
Lesson
10

Name: .... Date: .... Period: ....



Taco Truck

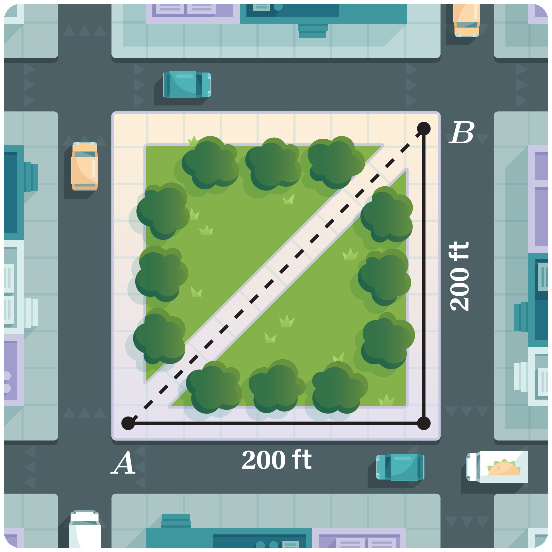
Let's solve problems with the Pythagorean theorem.

Warm-Up

1 Alma is going to walk through the park from point A to point B.

What distance will she walk?

$\sqrt{80000} \approx 282.8$ **feet (or equivalent)**



2 If Alma walks at a speed of 4 feet per second, how long will it take for her to walk across the park?

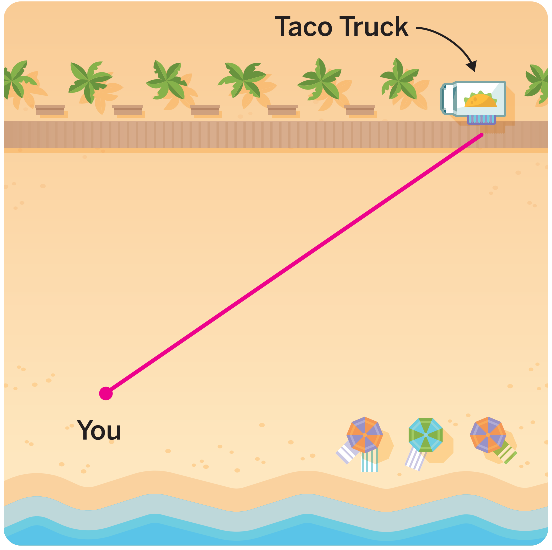
About 70.7 seconds (or equivalent)

Activity 1

Name: ________ Date: ________ Period: ________

Taco Truck

3 Imagine you're on the beach, and you're getting hungry.



Sketch the route you would take to the taco truck.

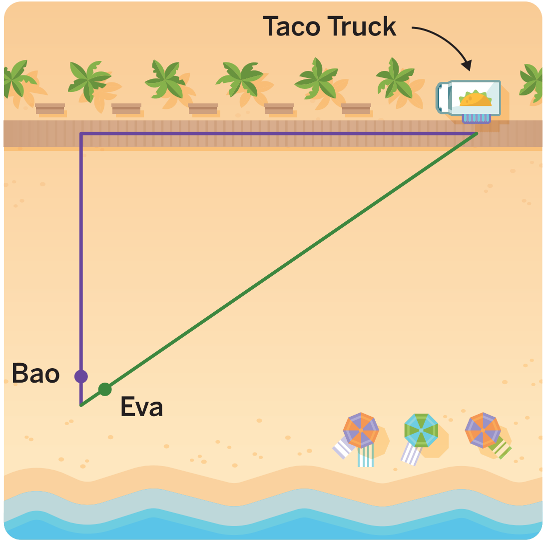
***Responses vary.* Sample shown in image.**

Explain your thinking.

Explanations vary.

- **The quickest way to get from one point to another is to follow a straight line between them.**
- **I hate walking on sand, so I would walk to the boardwalk as quickly as possible and walk the rest of the way on the boardwalk.**

4 Bao and Eva choose different routes to get to the taco truck.



a Who do you think will reach the taco truck first? ***Responses vary.***

Bao   Eva   They'll arrive at the same time

b What information would help you know for sure?

Responses vary.

- **Both Bao's and Eva's speeds.**
- **The distance to the taco truck.**
- **The distance to the boardwalk from the starting point and the distance along the boardwalk to the taco truck.**

Activity 2

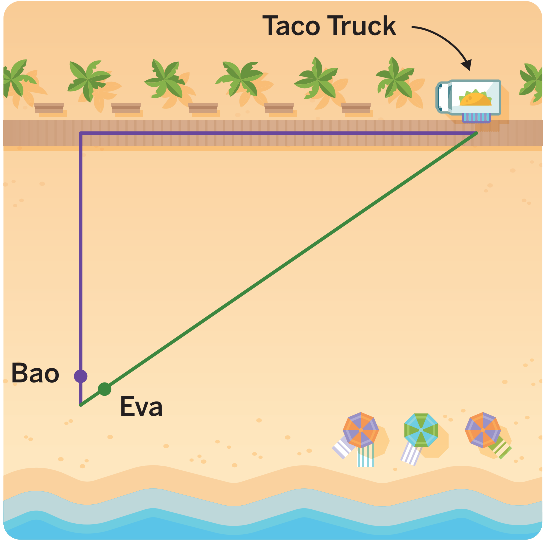
Name: .... Date: .... Period: ....

Let's Eat

5 Let's look at some additional information about Bao's and Eva's routes on the screen.

Use the information to calculate how long it will take for each person to get to the taco truck. Show your thinking.

- **Bao's time: 207 seconds (or equivalent).** ***Work varies.***
  $\frac{327.6}{3} + \frac{489}{5} = 207$
- **Eva's time: About 196.2 seconds (or equivalent).** ***Work varies.***
  $\frac{\sqrt{327.6^2 + 489^2}}{3} \approx 196.2$



Explore More

6 Determine the speed on the boardwalk that would make Eva and Bao arrive at the same time.

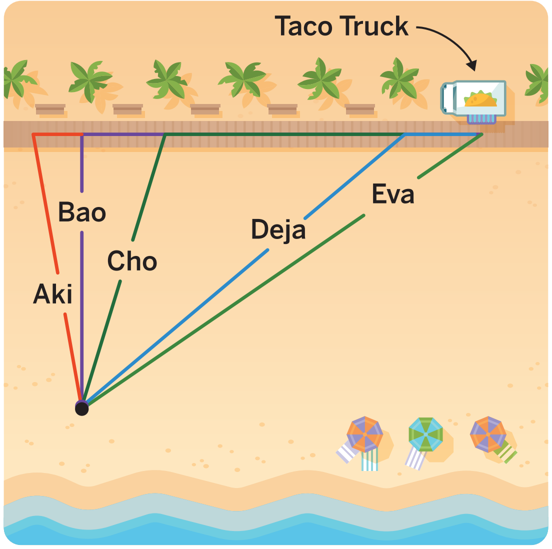
Speed on Sand	Speed on Boardwalk
3 feet per second	**About 5.6 feet per second**

Activity 3

Name: .... Date: .... Period: ....

Three More Paths

7 Here are three more possible paths.



Who do you think will reach the taco truck first? ***Responses vary.***

- ☐ Aki
- ☐ Bao
- ☐ Cho
- ☐ Deja
- ☐ Eva

Explain your thinking.

Explanations vary.

- **Cho will get there first because Cho spends more time walking faster on the boardwalk.**
- **Deja will arrive first because she has almost a direct path to the truck, but she saves some time on the boardwalk.**
- **Eva will get to the truck first because she walks in a straight line.**

8 You will watch a race between Aki, Bao, Cho, Deja, and Eva.

Discuss: What do you notice? What do you wonder?

Responses vary.

- **Bao is the first person to the boardwalk, but he doesn't get to the truck first.**
- **Eva is the last person to the boardwalk, but she isn't the last person to the truck.**
- **Why would Aki take a route that moves away from the truck at first?**
- **Did Deja find the fastest route to the truck?**

9 Let's look at some information about the winning path.

Remember that:

- The speed on the *boardwalk* is 5 feet per second.
- The speed on the *sand* is 3 feet per second.

Use this information to calculate the amount of time it took the winner to get to the taco truck. Show your thinking.

About 189.5 seconds (or equivalent). ***Work varies.***

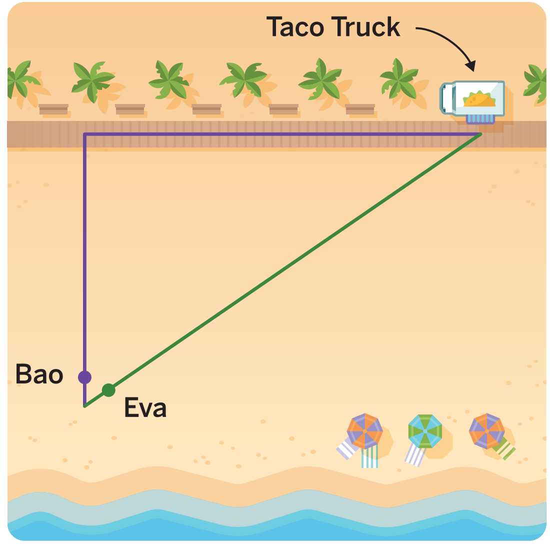
$$\frac{\sqrt{327.6^2 + 389^2}}{3} + \frac{100}{5} \approx 189.52$$

10 Synthesis

What are some important things to remember when using the Pythagorean theorem to solve problems?

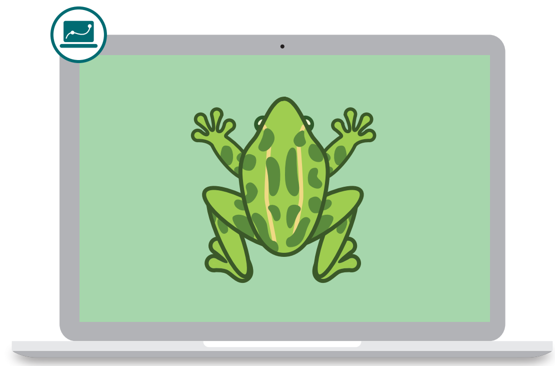
Responses vary.

- **The Pythagorean theorem can only be applied to right triangles.**
- **I need to use the squares of the side lengths rather than the lengths themselves.**
- **I need to identify which sides are the legs (vs. the hypotenuse) of the right triangle.**



Things to Remember:

Name: ________ Date: ________ Period: ________



Pond Hopper

Let's calculate distances between points on the coordinate plane.

Warm-Up

1 Order the pairs of points from closest together to farthest apart.

Use the graph if it helps with your thinking.

Pair A: $(-8, 1)$ and $(-8, 8)$

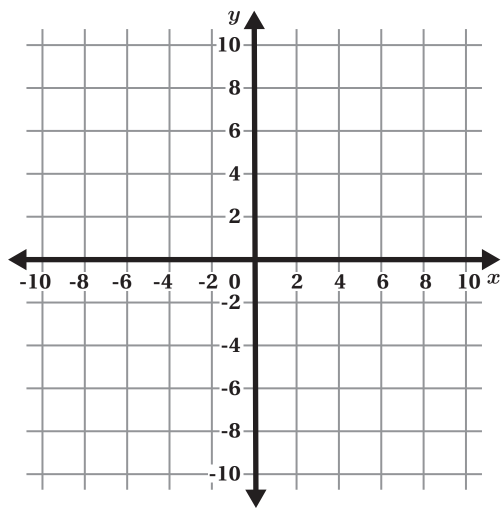
Pair B: $(7, 0)$ and $(7, -9)$

Pair C: $(2, 3)$ and $(2, 9)$

Pair D: $(-3, 6)$ and $(5, 6)$

C	A	D	B

Closest Together — **Farthest Apart**



Activity 1

Name: ______ Date: ______ Period: ______

Pond Hopper

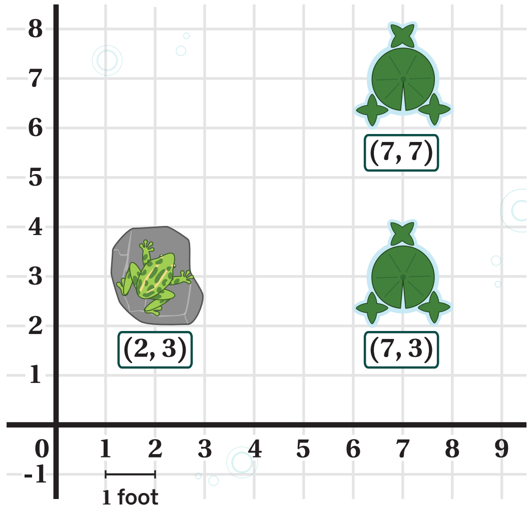
2 Let's help the frog hop to all the lily pads.

Pick a lily pad. Write its coordinates in the table.

Then write the distance between the frog and the lily pad.

Repeat until you have hopped to all the lily pads.

Responses vary.

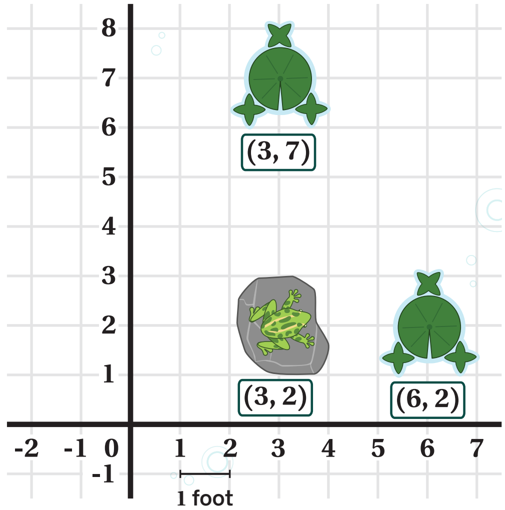


	Lily Pad Coordinates	Distance (ft)
Hop 1	$(7, 3)$	5
Hop 2	$(7, 7)$	4
Hop 3		
Hop 4		

Another route:
- Hop $\sqrt{41}$ feet to $(7, 7)$
- Hop 4 feet to $(7, 3)$

3 Help the frog hop to all the lily pads in *as few hops as possible*.

Responses vary.



	Lily Pad Coordinates	Distance (ft)
Hop 1	$(6, 2)$	3
Hop 2	$(3, 7)$	$\sqrt{34}$
Hop 3		
Hop 4		

Another route:
- Hop 5 feet to $(3, 7)$
- Hop $\sqrt{34}$ feet to $(6, 2)$

Name: .... Date: .... Period: ....

Pond Hopper (continued)

4 Which expression represents the distance between the frog and the lily pad?

A. $\sqrt{7^2 - 3^2}$

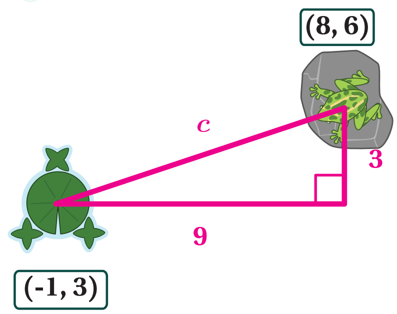
B. $\sqrt{7^2 + 3^2}$

C. $\sqrt{9^2 - 3^2}$

D. $\sqrt{9^2 + 3^2}$

Show or explain your thinking.

Explanations vary. I can create a right triangle by drawing a line segment straight down from $(8, 6)$ and another line segment straight left to $(-1, 3)$. Then I can use the Pythagorean theorem to calculate the hypotenuse, which is the same as the distance between the frog and the lily pad. The legs of the triangle are 9 feet and 3 feet, so $9^2 + 3^2 = c^2$ and $c = \sqrt{9^2 + 3^2}$.



All measurements in feet

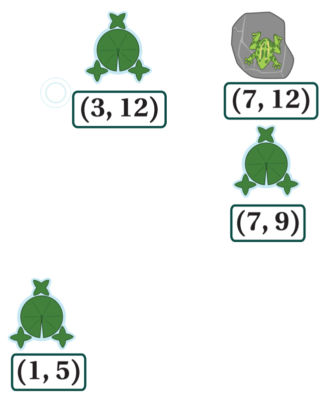
5 Help the frog hop to all the lily pads in *as few hops as possible*.

Responses vary. Samples shown below.

	Lily Pad Coordinates	Distance (ft)
Hop 1	$(3, 12)$	4
Hop 2	$(7, 9)$	5
Hop 3	$(1, 5)$	$\sqrt{52}$
Hop 4		
Hop 5		

Another route:

- Hop 3 feet to $(7, 9)$
- Hop $\sqrt{52}$ feet to $(1, 5)$
- Hop $\sqrt{53}$ feet to $(3, 12)$



All measurements in feet

Activity
2

Name: .... Date: .... Period: ....

Challenge Creator

6 You will use the Activity 2 Sheet to complete this activity.

a **Make It!** On the Activity 2 Sheet, create a lily pad challenge by sketching lily pads and a rock.

b **Solve It!** On this page, help the frog hop to all the lily pads in your challenge in *as few hops as possible*.

Responses vary.

My Challenge

	Hop 1	Hop 2	Hop 3	Hop 4	Hop 5	Hop 6	Hop 7	Hop 8
Lily Pad Coordinates								
Distance (ft)								

c **Swap It!** Swap your challenge with one or more partners. Help the frog hop to all the lily pads in each of your partners' challenges in *as few hops as possible*.

Responses vary.

Partner 1

	Hop 1	Hop 2	Hop 3	Hop 4	Hop 5	Hop 6	Hop 7	Hop 8
Lily Pad Coordinates								
Distance (ft)								

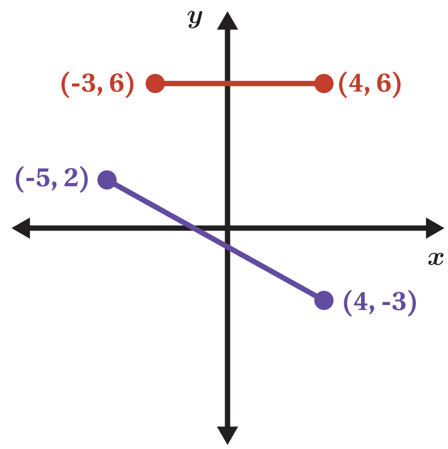
Partner 2

	Hop 1	Hop 2	Hop 3	Hop 4	Hop 5	Hop 6	Hop 7	Hop 8
Lily Pad Coordinates								
Distance (ft)								

7 Synthesis

What are some strategies to calculate the distance between two points on the coordinate plane?

Use the examples in the graph if they help to show your thinking.



Responses vary.

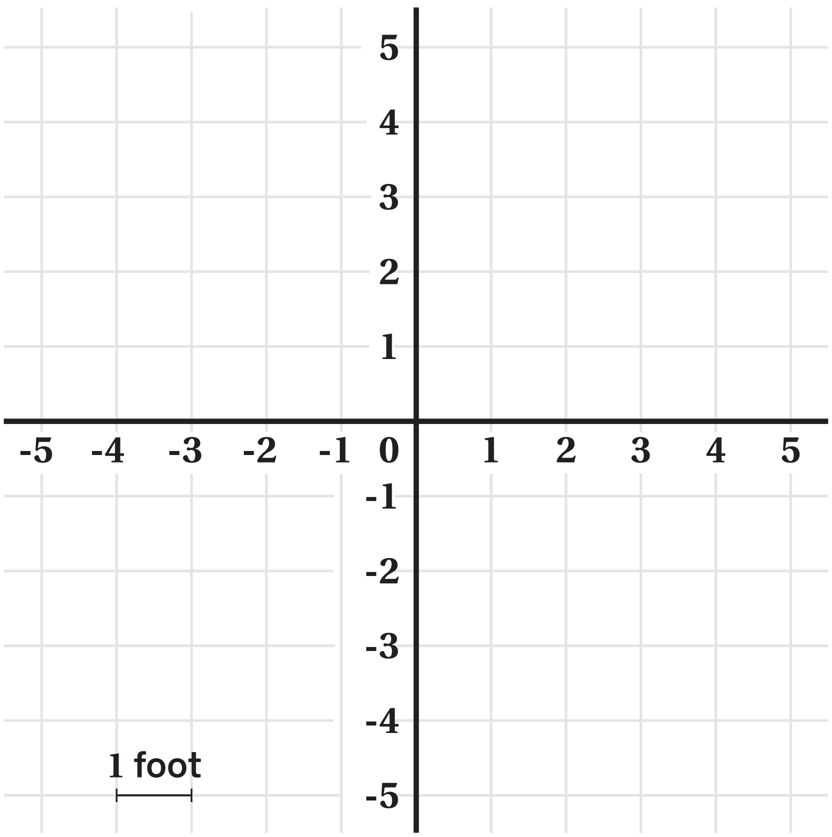
- **When points have the same x- or y-coordinate, they will be perfectly horizontal or vertical. I can determine the distance between them by just subtracting the coordinates that are different. If the value is negative, I will take the opposite because distances are always positive.**
- **When points are not aligned horizontally or vertically, I can make a right triangle and use the Pythagorean theorem to calculate the distance between the two points. I can determine the length of one leg by subtracting the points' x-coordinates, and the other leg by subtracting the y-coordinates. If either value is negative, I will take the opposite because lengths are positive.**

Things to Remember:

Name: ______________________ Date: __________ Period: __________

Challenge Creator

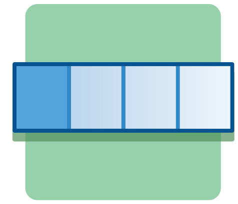
- Sketch five lily pads and a rock on the coordinate plane. The rock and lily pads must be at least 2 feet apart from each other.
- Label the coordinates of the rock and lily pads, then sketch a frog on top of the rock or label it with an F.
- Do *not* show any distances between the rock and lily pads. You and your classmates will solve each other's challenges on the lesson page.



Unit 8
Lesson
12

Name: ______ Date: ______ Period: ______

Fractions to Decimals

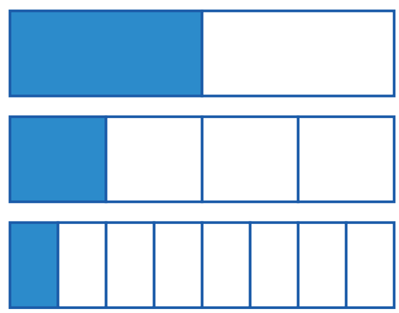


Let's explore connections between fractions and their decimal representations.

Warm-Up

1. Here are three rectangles.

A
B
C



a What fraction of each rectangle is shaded?

b Write your answer as a fraction and as a decimal.

Rectangle	Fraction	Decimal
A	$\frac{1}{2}$	0.5
B	$\frac{1}{4}$	0.25
C	$\frac{1}{8}$	0.125

Name: .... Date: .... Period: ....

Terminating or Repeating

2. Every number can be written as a decimal. Here are some fractions written as decimals.

Fraction	Decimal	Terminating	Repeating	Neither
$\frac{1}{8}$	0.125	✓		
$\frac{3}{5}$	0.6	✓		
$\frac{341}{100}$	3.41	✓		
$\frac{1}{3}$	$0.333...$		✓	
$\frac{243}{99}$	$2.454545...$		✓	
$\frac{121}{15}$	$8.0666...$		✓	

What do you notice? What do you wonder? ***Responses vary.***

I notice:

- **All of the examples either terminate or repeat.**
- **For repeating decimals, the denominator is a multiple of 3.**

I wonder:

- **Are there any decimal representations that would be in the neither category?**
- **What is the easiest way to convert a fraction to its decimal representation?**

3. We can also use **bar notation** to write repeating decimals.

For example, $0.333... = 0.\overline{3}$ and $2.454545... = 2.\overline{45}$.

Order these numbers from *least* to *greatest*: $8.06, 8.0\overline{63}, 8.0\overline{6}, 8.063$.

$\mathbf{8.06}$	$\mathbf{8.063}$	$\mathbf{8.0\overline{63}}$	$\mathbf{8.0\overline{6}}$

Least **Greatest**

4. Write $\frac{11}{50}$ as a decimal and decide whether it is *terminating*, *repeating*, or *neither*.

Fraction	Decimal	Terminating	Repeating	Neither
$\frac{11}{50}$	$\mathbf{0.22}$	✓		

Activity 2

Name: ____ Date: ____ Period: ____

Converting Unit Fractions

5. Use long division to write each *unit fraction* as a decimal. Use the workspace or blank paper if it helps with your thinking.

Unit Fraction	Decimal	Terminating	Repeating	Neither
$\frac{1}{2}$	0.5	✓		
$\frac{1}{3}$	$0.\overline{3}$		✓	
$\frac{1}{4}$	**0.25**	✓		
$\frac{1}{5}$	**0.2**	✓		
$\frac{1}{6}$	$\mathbf{0.1\overline{6}}$		✓	
$\frac{1}{7}$	$\mathbf{0.\overline{142857}}$		✓	
$\frac{1}{8}$	**0.125**	✓		
$\frac{1}{9}$	$\mathbf{0.\overline{1}}$		✓	
$\frac{1}{10}$	**0.1**	✓		
$\frac{1}{11}$	$\mathbf{0.\overline{09}}$		✓	
$\frac{1}{12}$	$\mathbf{0.08\overline{3}}$		✓	

Workspace:

6. Write another unit fraction that terminates when written as a decimal.

Responses vary.

- $\frac{1}{20}$
- $\frac{1}{40}$
- $\frac{1}{100}$
- $\frac{1}{125}$
- $\frac{1}{800}$

7. Write another unit fraction that repeats when written as a decimal.

Responses vary.

- $\frac{1}{13}$
- $\frac{1}{17}$
- $\frac{1}{30}$
- $\frac{1}{41}$
- $\frac{1}{101}$

Activity 2

Name: ________ Date: ________ Period: ________

Converting Unit Fractions (continued)

8. How can you predict whether a unit fraction will terminate, repeat, or neither when written as a decimal?

Responses vary.

- **Write the denominator in factored form. If the factors consist only of 2s and 5s, then the decimal representation will terminate. Otherwise, it will repeat.**
- **If you can write an equivalent fraction with a power of 10 as the denominator, the decimal representation will terminate. If not, it will repeat.**

Explore More

9. Complete the table. Then answer these questions:

Fraction	Decimal
$\frac{1}{7}$	$0.\overline{142857}$
$\frac{2}{7}$	$0.\overline{285714}$
$\frac{3}{7}$	$0.\overline{428571}$
$\frac{4}{7}$	$0.\overline{571428}$
$\frac{5}{7}$	$0.\overline{714285}$
$\frac{6}{7}$	$0.\overline{857142}$

a How are the decimal representations in the table similar to each other?

Each decimal contains the same six repeating digits (1, 4, 2, 8, 5, 7) in the same order.

b How are the decimal representations in the table similar to each other?

Each decimal starts at a different digit in the cycle.

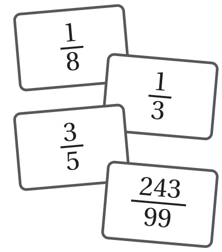
c Add the decimal representations of $\frac{3}{7}$ and $\frac{4}{7}$. What is the result? How does this compare to when you add the fractions $\frac{3}{7}$ and $\frac{4}{7}$?

$0.\overline{428571} + 0.\overline{571428} = 0.\overline{999999} = 0.\overline{9}$;

$\frac{3}{7} + \frac{4}{7} = \frac{7}{7}$; **This suggests that $0.\overline{9} = 1$.**

Synthesis

10. Explain a strategy for writing fractions as decimals.



Responses vary.

- **To write a fraction as a decimal, use long division.**
- **If the denominator is a factor of 100, you can multiply the numerator and denominator to make the fraction out of 100, then write the number of hundredths as a decimal.**

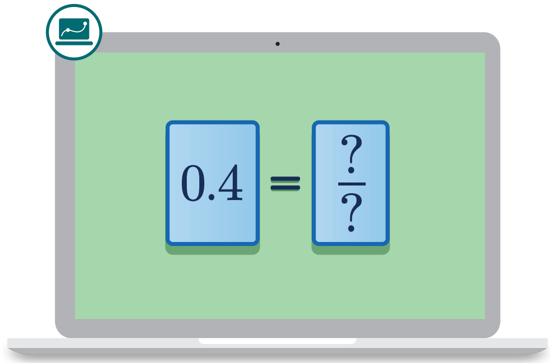
Things to Remember:

Unit 8
Lesson
13

Name: ______ Date: ______ Period: ______

Decimals to Fractions

Let's develop a strategy for rewriting repeating decimals as fractions.



Warm-Up

Determine the value of each expression mentally. Try to think of more than one strategy.

1. $234 - 34$

   200

2. $9.7 - 0.7$

   9

3. $100.\overline{25} - 99.\overline{25}$

   1

4. $18.8\overline{3} - 1.4\overline{3}$

   17.4

Activity
1

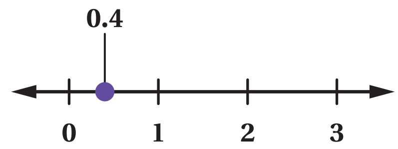
Name: ______ Date: ______ Period: ______

Terminating Decimals to Fractions

5 Write a fraction as close to 0.4 as you can.

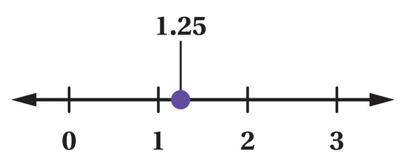
Use a calculator to check how close you are and revise as needed.

$\frac{2}{5}$ (or equivalent)



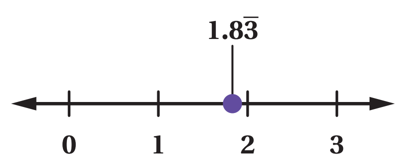
6 Write a fraction as close to 1.25 as you can.

$\frac{5}{4}$ (or equivalent)



7 Write a fraction as close to $1.8\overline{3}$ as you can.

$\frac{11}{6}$ (or equivalent)



Activity 2

Name: ______ Date: ______ Period: ______

Repeating Decimals to Fractions

8 Here is some of the work Mai did to write $1.8\overline{3}$ as a fraction.

Mai

$$x = 1.8\overline{3}$$
$$10x = 18.\overline{3}$$
$$100x = 183.\overline{3}$$

$$100x = 183.\overline{3}$$
$$-(10x = 18.\overline{3})$$
$$90x = 165$$
$$x = ?$$

a **Discuss:** What did Mai do? Why do you think she chose these steps?

Responses vary.

- **Mai set x equal to $1.8\overline{3}$ and created two equivalent equations by multiplying each side by 10 and 100. Mai then subtracted the two equations.**
- **I think Mai chose to multiply by 10 and 100 to create equations that were easy to subtract from each other, so she could get rid of the repeating decimal.**

b Write $1.8\overline{3}$ as a fraction.

$\frac{165}{90}$ **(or equivalent).** ***Work varies.***

$$\frac{90x}{90} = \frac{165}{90}$$
$$x = \frac{165}{90}$$

9 Use Mai's strategy to write $2.7\overline{4}$ as a fraction. Show your thinking.

$\frac{247}{90}$ **(or equivalent).** ***Work varies.***

$$x = 2.7\overline{4}$$
$$10x = 27.\overline{4}$$
$$100x = 274.\overline{4}$$

$$100x = 274.\overline{4}$$
$$-(10x = 27.\overline{4})$$
$$90x = 247$$
$$x = \frac{247}{90}$$

Activity
3

Name: .... Date: .... Period: ....

Repeated Challenges

10 Write each decimal as a fraction. Complete as many problems as you have time for.

a 0.05

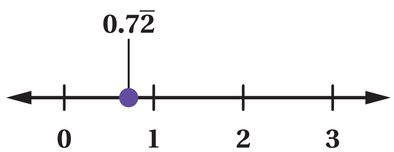
$\frac{1}{20}$ (or equivalent)

b $0.3\overline{1}$

$\frac{14}{45}$ (or equivalent)

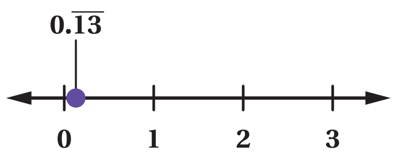
c $0.7\overline{2}$

$\frac{13}{18}$ (or equivalent)



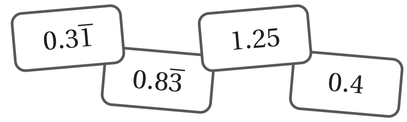
d $0.\overline{13}$

$\frac{13}{99}$ (or equivalent)



11 Synthesis

How is writing a repeating decimal as a fraction like writing a terminating decimal as a fraction? How is it different?



Responses vary.

- **Both repeating and terminating decimals can be written as fractions by creating an equation where x equals the decimal. For repeating decimals, you can multiply the equation by 10 and again by 100 to create two new equations, then subtract the two equations to get rid of the repetition. Then you can solve for x to determine how to write the decimal in fraction form. For terminating decimals, you only need one equation because there is no repetition.**
- **A terminating decimal can be expressed as a fraction using place value, while a repeating decimal can't. For example, 0.4 can be read as "four tenths" and can be written as the fraction $\frac{4}{10}$.**

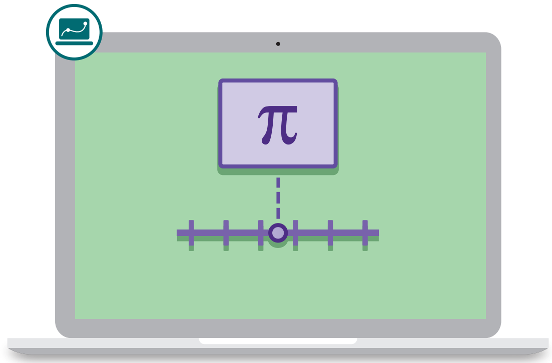
Things to Remember:

Unit 8
Lesson
14

Name: ______ Date: ______ Period: ______

Hit the Target

Let's build an understanding of two new types of numbers.



Warm-Up

1 Which number is greater? Circle one.

$\sqrt{13}$ π I'm not sure

Explain your thinking.

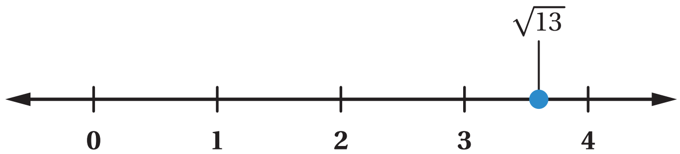
Explanations vary. π is approximately equal to 3.14. $\sqrt{13}$ is greater than that because it's closer to $\sqrt{16} = 4$ than $\sqrt{9} = 3$.

Activity 1

Name: ______ Date: ______ Period: ______

Hit the Target

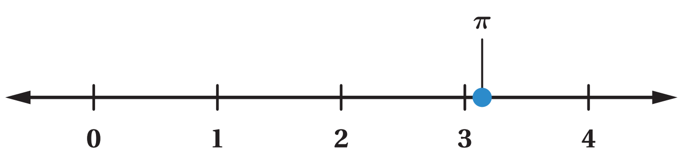
2 Write a fraction as close to $\sqrt{13}$ as you can (without using the square root symbol). Use a calculator to check how close you are, then revise your fraction to get as close to the target as possible.



Responses vary.

- $\frac{18}{5}$
- $\frac{361}{100}$
- $\frac{360555}{100000}$

3 Write a fraction as close to π as you can (without using the π symbol). Use a calculator to check how close you are, then revise your fraction to get as close to the target as possible.



Responses vary.

- $\frac{314}{100}$
- $\frac{22}{7}$
- $\frac{31415926}{10000000}$

Activity 2

Name: ______ Date: ______ Period: ______

Irrational Numbers

4 A **rational number** is a number that can be written as a fraction of two integers, where the denominator is not zero.

It is impossible to write $\sqrt{13}$ as a fraction with non-zero integers, which makes it an **irrational number**.

Discuss: What are some other numbers you think might be irrational?

Responses vary.

- π
- $\sqrt{5}$
- $\sqrt[3]{10}$

5 Is $\sqrt{\frac{9}{4}}$ rational or irrational? Circle one.

Rational   Irrational   I'm not sure

Explain your thinking.

Explanations vary. $\sqrt{\frac{9}{4}} = \frac{3}{2}$ because $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$. Since $\sqrt{\frac{9}{4}}$ can be written as a fraction with non-zero integers for the numerator and denominator, it must be rational.

6 Sort the numbers into groups based on whether they are rational or irrational.

$\frac{8}{4}$	$2\frac{3}{20}$	$1.\overline{73}$	$\sqrt{2}$
$2 \cdot \sqrt{13}$	$\sqrt{10}$	2π	$\sqrt{\frac{1}{4}}$
$\sqrt[3]{9}$	$\sqrt[3]{8}$	$1.73205080757...$	1.73

Rational	Irrational	I'm Not Sure
$\sqrt{\frac{1}{4}}$	$\sqrt{2}$	There is no expectation yet that students can prove that the numbers in the irrational category are irrational, so they may place them here instead.
1.73	$2 \cdot \sqrt{13}$	
$1.\overline{73}$	2π	
$\frac{8}{4}$	$\sqrt[3]{9}$	
$\sqrt[3]{8}$	$\sqrt{10}$	
$2\frac{3}{20}$	$1.73205080757...$	

Name: ______ Date: ______ Period: ______

Irrational Numbers (continued)

7 Jada claims that any number written with a square root or a cube root is irrational. Is Jada correct? Circle one.

Yes

No

Explain your thinking.

***Explanations vary.* Many numbers written with a square root or a cube root are rational. For example, $\sqrt{16}$ is rational, as it equals $\frac{4}{1}$. So is $\sqrt[3]{\frac{8}{125}}$, as it equals $\frac{2}{5}$.**

Explore More

8 Here are some problems to explore why $\sqrt{2}$ is irrational. Use a calculator if it helps with your thinking.

a $\left(\frac{577}{408}\right)^2$ is very close to 2, but is it exactly equal to 2?

No. Note: While some simpler calculators display 2 as the result for $\left(\frac{577}{408}\right)^2$, more accurate tools show that $\left(\frac{577}{408}\right)^2 > 2$.

b If $\left(\frac{577}{408}\right)^2 = 2$, then $408^2 \cdot 2 = 577^2$. Diya says that's not true without computing any of these numbers. How can Diya know that?

***Responses vary.* $408^2 \cdot 2$ is even. 577^2 is odd. Therefore, $408^2 \cdot 2$ cannot be equal to 577^2.**

c How does this show that $\frac{577}{408} \neq \sqrt{2}$?

***Responses vary.* $\sqrt{2}$ is the solution to the equation $x^2 = 2$. Since $\frac{577}{408}$ is not a solution to that equation, $\frac{577}{408} \neq \sqrt{2}$.**

d Is $\frac{1414213562375}{1000000000000} = \sqrt{2}$? Explain your thinking.

No. *Explanations vary.* $\sqrt{2}$ is the solution to the equation $x^2 = 2$. Since $\frac{1414213562375}{1000000000000}$ is not a solution to that equation, $\frac{1414213562375}{1000000000000} \neq \sqrt{2}$.

9 Synthesis

What is an irrational number? Give at least one example.

Responses vary. **An irrational number is a number that is not rational. In other words, an irrational number *cannot* be written as a fraction using non-zero integers for the numerator and denominator. $\sqrt{13}$ is an example of an irrational number.**

Things to Remember: