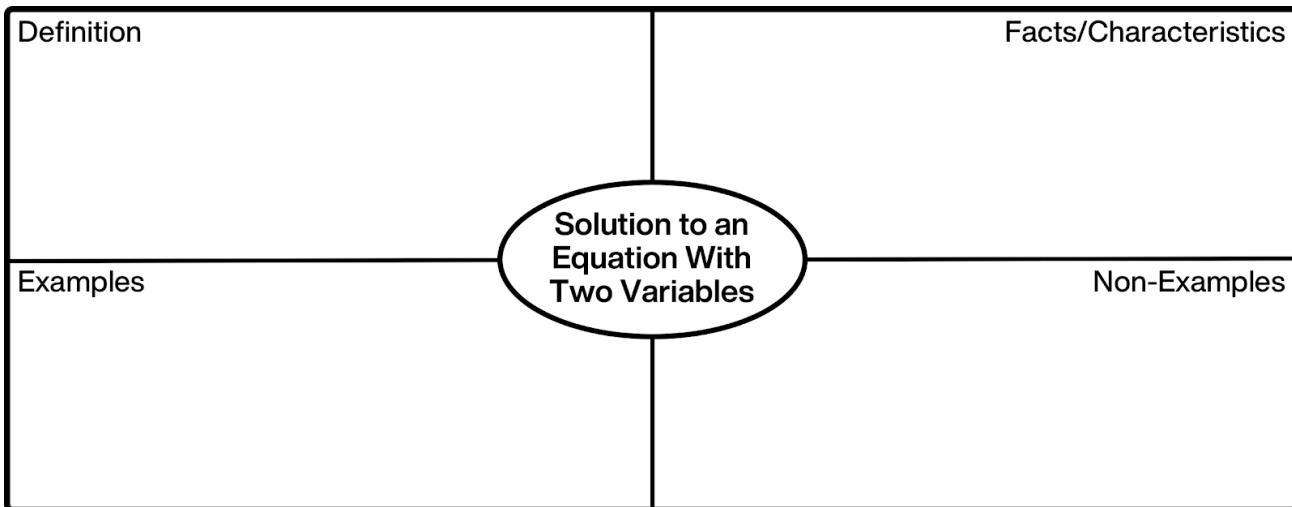


Learning Goal(s):



Here are some facts about solutions in two variables:

1. A solution to a linear equation is a pair of values that makes the equation _____ .
2. Solutions can be found by _____ a value for one of the variables and solving the equation for the other.
3. The set of all the solutions to a linear equation can be shown in the coordinate plane and is called the _____ of the equation.
4. The graph of a linear equation is _____ .
5. Any points in the coordinate plane that **do not** lie on the graph of the linear equation are _____ to the equation.
6. The number of solutions might be limited in a real-world situation even though the equation has _____ solutions.

Summary Question

How can you find solutions to linear equations? How do you know when you've found a solution?

Learning Goal(s):

No matter the form of a linear equation, we can always find solutions to the equation by starting with one value and then solving for the other value.

Let's think about the linear equation $2x - 4y = 12$.

Find the y -intercept by making $x = 0$.

Find the x -intercept by making $y = 0$.

Based on your work above, what are the coordinates of two points on the line $2x - 4y = 12$?

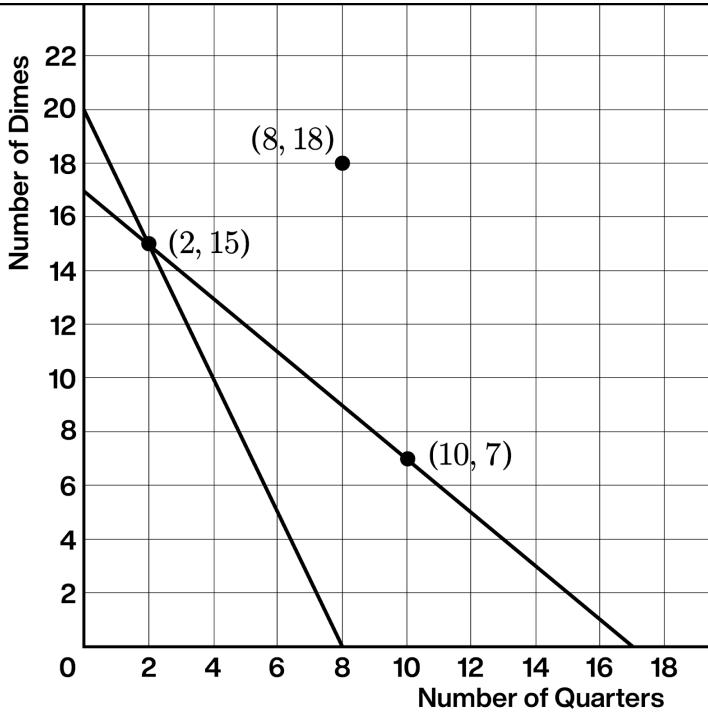
Summary Question

Once you have identified one solution to your equation, what are some ways you can find others?

Learning Goal(s):

Values of x and y that make an equation _____ correspond to points (x, y) on the graph. For example, if we have x number of quarters and y number of dimes and the total cost is \$2.00, then we can write an equation like this to represent the relationship between x and y : $0.25x + 0.10y = 2$.

Since 2 quarters is \$_____ and 15 dimes is \$_____, we know that $x = 2$, $y = 15$ is a _____ to the equation, and the point $(\underline{\quad}, \underline{\quad})$ is a point on the graph. The line shown is the graph of the equation.



We also know that the quarters and dimes together total 17 coins. That means that : $x + y = 17$

1. Label the graph of each equation on the coordinate plane.
2. Pick another point on the coordinate plane and explain what it means in context:

In general, if we have two lines in the coordinate plane:

- The coordinates of a point that is on both lines make _____ equations true.
- The coordinates of a point on only one line make _____ equation true.
- The coordinates of a point on neither line make _____ equation true.

Summary Question

If you are given two linear relationships, how can you determine x - and y -values that will make both relationships true?

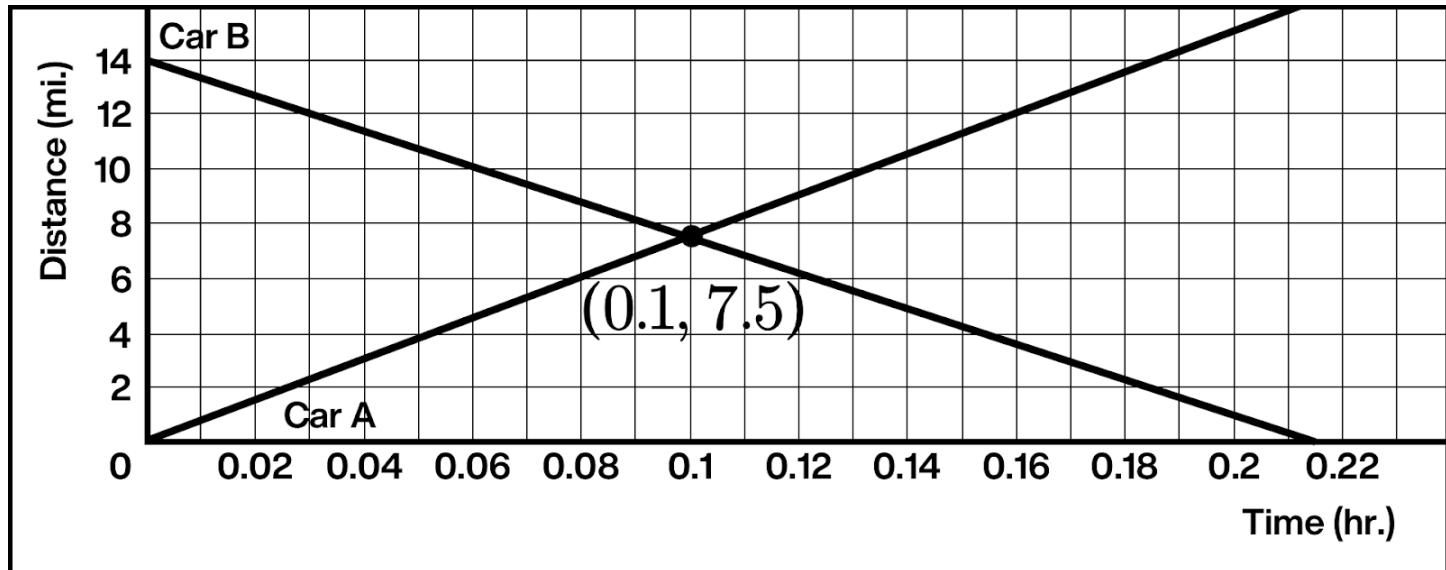
Learning Goal(s):

The solutions to an equation correspond to _____ on its graph. For example, if Car A is traveling 75 miles per hour and passes a rest area when $t = 0$, then the distance in miles it has traveled from the rest area after t hours can be represented by the equation _____.

1. What is one point that will be on this graph? How do you know?

If you have **two** equations, you can ask whether there is an ordered pair that is a solution to both equations simultaneously.

For example, if Car B is traveling toward the rest area and its distance from the rest area is $d = 14 - 65t$, we can ask if there is ever a time when the distance of Car A from the rest area is the same as the distance of Car B from the rest area. If the answer is yes, then the solution will correspond to a point _____.

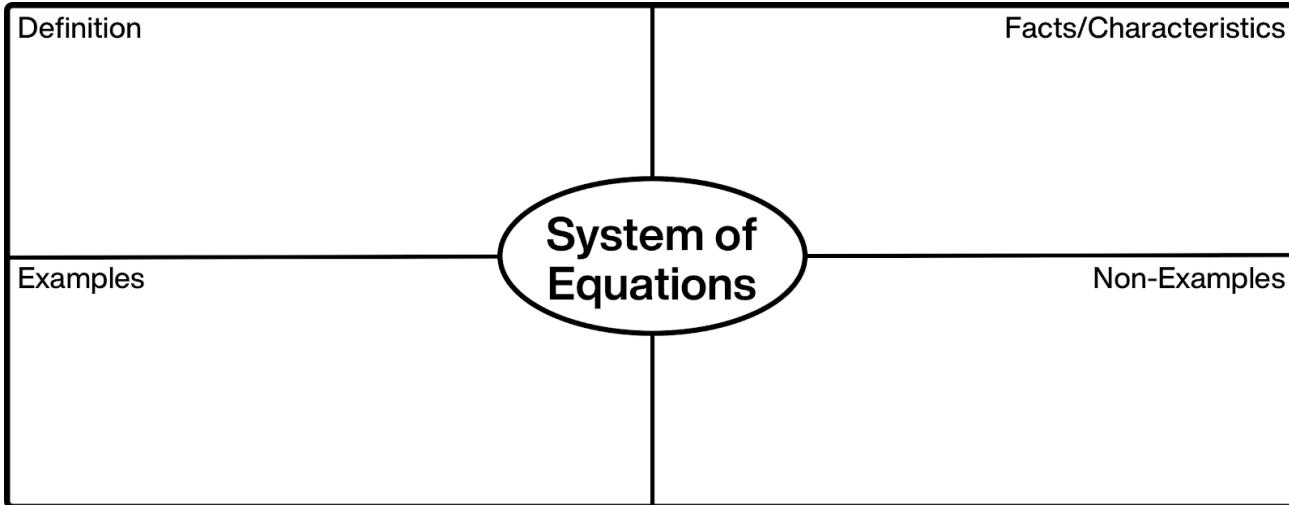


Looking at the coordinates of the intersection point, we see that Car A and Car B will both be _____ miles from the rest area after _____ hours.

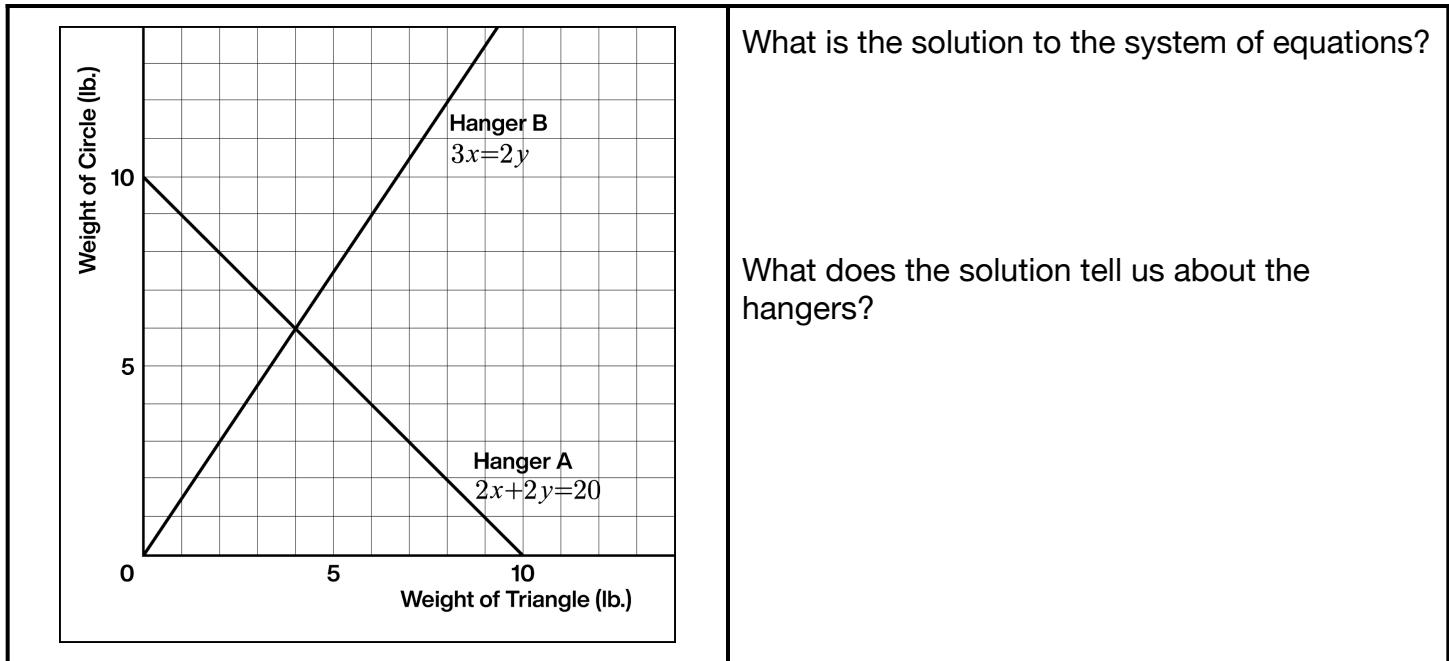
Summary Question

How can you tell by looking at a graph when two linear relationships will be the same?

Learning Goal(s):



The system of equations below represents the weights of two balanced hangers.



Summary Question

What does it mean to solve a system of equations?

Learning Goal(s):

Sometimes it is easier to solve a system of equations without having to graph the equations and look for an _____.

In general, whenever we are solving a system of equations we know that we are looking for a pair of (x, y) values that makes both equations _____. In particular, we know that the value for y will be the _____ in both equations.

If we have a system like this:

$$\begin{aligned}y &= 2x + 6 \\y &= -3x - 4\end{aligned}$$

we know the _____ of the solution is the same in both equations, so we can write the following:

$$2x + 6 = -3x - 4$$

and we can solve this equation for x :

$$2x + 6 = -3x - 4$$

Solving for x is only half of what we are looking for; we know the value for x , but we need the corresponding value for y .

Since both equations have the same y -value, we can use either equation to find the y -value:

$$\begin{aligned}y &= 2(-2) + 6 \\y &= -3(-2) - 4\end{aligned}$$

In both cases, we find that $y = 2$. So the solution to the system is _____.

We can verify this by graphing both equations in the coordinate plane.

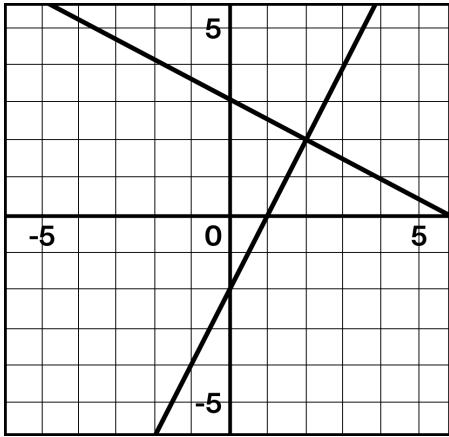
Summary Question

What are the first steps you can take when solving the following system of equations?

$$\begin{aligned}y &= 2x \\y &= -3x + 10\end{aligned}$$

Learning Goal(s):

The x - and y - values that make both equations true are known as the _____ to a system of equations. Depending on the equations, a system can have _____, _____, or _____ solutions.

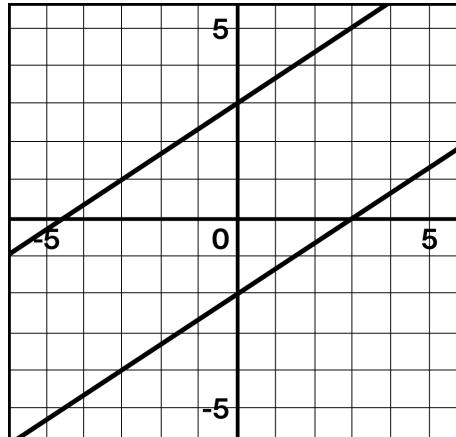


$$y = 2x - 2$$

$$y = -\frac{1}{2}x + 3$$

If the two lines of a system intersect at a point, there is _____ solution.

If the two lines have _____ slopes, there is one solution.

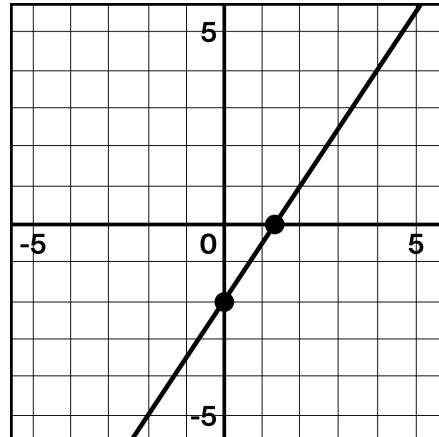


$$y = \frac{2}{3}x - 2$$

$$y = \frac{2}{3}x + 3$$

If the two lines of a system do not intersect at a point, there are _____ solutions.

If the two lines have _____ slope and different y -intercepts, there are no solutions.



$$y = 1.5x - 2$$

$$y = \frac{3}{2}x - 2$$

If the two equations have the **same** slope and the **same** y -intercept, the system has _____ solutions.

Summary Question

How can you tell from the structure of the equations if a system has no solutions, one solution, or infinite solutions?

Learning Goal(s):

When we have a system of linear equations where one of the equations is of the form $y = [stuff]$ or $x = [stuff]$, we can solve it algebraically by using _____.

$$\begin{cases} y=5x \\ 2x-y=9 \end{cases}$$

The basic idea is to replace a variable with an equivalent _____.

Since $y = 5x$, we can substitute _____ for y in $2x - y = 9$.

$$2x - () = 9$$

And then solve the equation for x .

$$x =$$

We can calculate y using either equation. Let's use the first one:

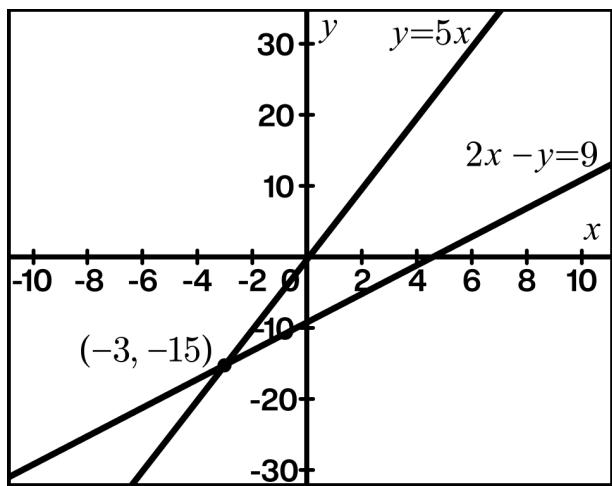
$$\begin{aligned} y &= 5x \\ y &= 5 \cdot \underline{\hspace{2cm}} \\ y &= \underline{\hspace{2cm}} \end{aligned}$$

The x - and y -values that make both equations true are known as the _____ to the system.

Solution
 $(-3, \underline{\hspace{2cm}})$

We can check this by looking at the graphs of the equations in the system:

They intersect at $(-3, -15)$.



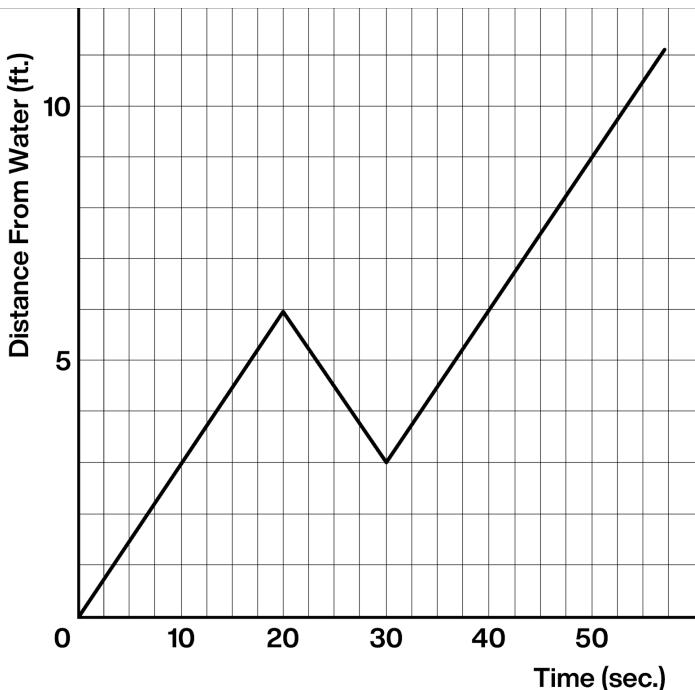
Summary Question

Describe one strategy you can use for solving a system of equations algebraically.

Making Sense of Graphs

Learning Goal(s):

Here is the graph of a turtle's journey.



What story does the graph tell about the turtle's journey?

What is the turtle's distance from the water after 40 seconds?

When is the turtle's distance from the water 3 feet?

Summary Question

How does a point on a graph represent part of a story? Give at least one example.

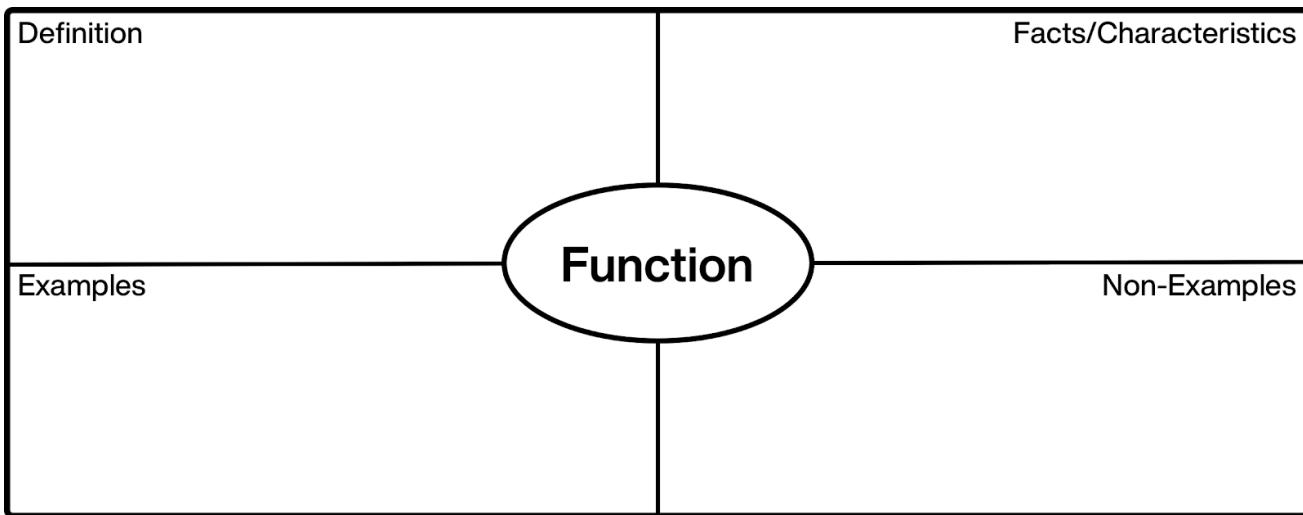
desmos

Unit 8.5, Lesson 2: Notes

Name _____

Introduction to Functions

Learning Goal(s):



For each rule, decide if the rule represents a function or not. Explain your thinking.

Possible Inputs: Any person

Rule: Output the month the person was born in.

Function? Yes No

Possible Inputs: Any month

Rule: Output a person born in that month.

Function? Yes No

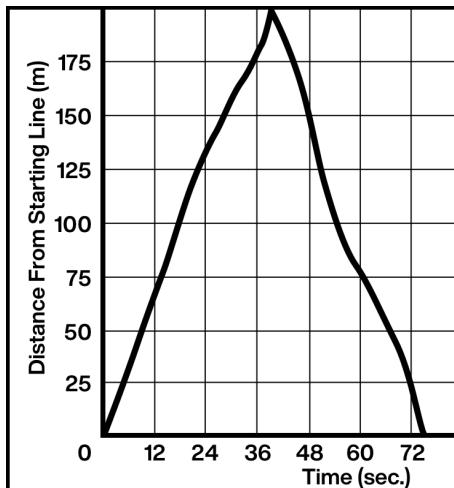
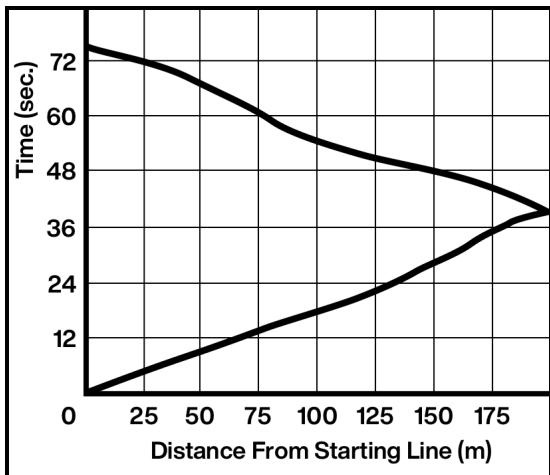
Summary Question

Why might it be useful to know whether a rule is a function?

Graphs of Functions and Non-Functions

Learning Goal(s):

Ariana is running once around the track. The graphs below show the relationship between her time and her distance from the starting point.



Estimate when Ariana was 100 meters from her starting point.

Estimate how far Ariana was from the starting line after 60 seconds.

Is time a function of Ariana's distance from the starting point? Explain how you know.

Is Ariana's distance from the starting point a function of time? Explain how you know.

Summary Question

What is something you won't see on the graph of a function?

Functions and Equations

Learning Goal(s):

In each situation, complete the table with a possible *independent variable* or *dependent variable*.

Question or Equation	Independent Variable	Dependent Variable
How many pickles can I make?	The number of cucumbers	The number of pickles
How much does my ice cream cost if I get different amounts of toppings?		Cost of my ice cream cone
How does sleep affect performance on tests?		
$y = 3x + 5$		

What is the *independent variable*? How is it represented on a graph?

What is the *dependent variable*? How is it represented on a graph?

Brown rice costs \$2 per pound and beans cost \$1.60 per pound. Rudra has \$10 to spend on these items. The amount of brown rice, r , is related to the amount of beans, b , Rudra can buy.

Rudra wrote the equation $r = \frac{10 - 1.60b}{2}$. What is the dependent variable? How do you know?

Summary Question

How does the choice of independent and dependent variables affect the equation of a function?

Interpreting Graphs of Functions

Learning Goal(s):

This graph shows the temperature between noon and midnight on one day.



Tell the story of the temperature on this day.

Did the temperature change more between 1 p.m. and 3 p.m. or between 7 p.m. and 9 p.m.? Explain your thinking.

Was it warmer at 3 p.m. or 9 p.m.?

Summary Question

How can you tell from a graph whether a function is increasing or decreasing?

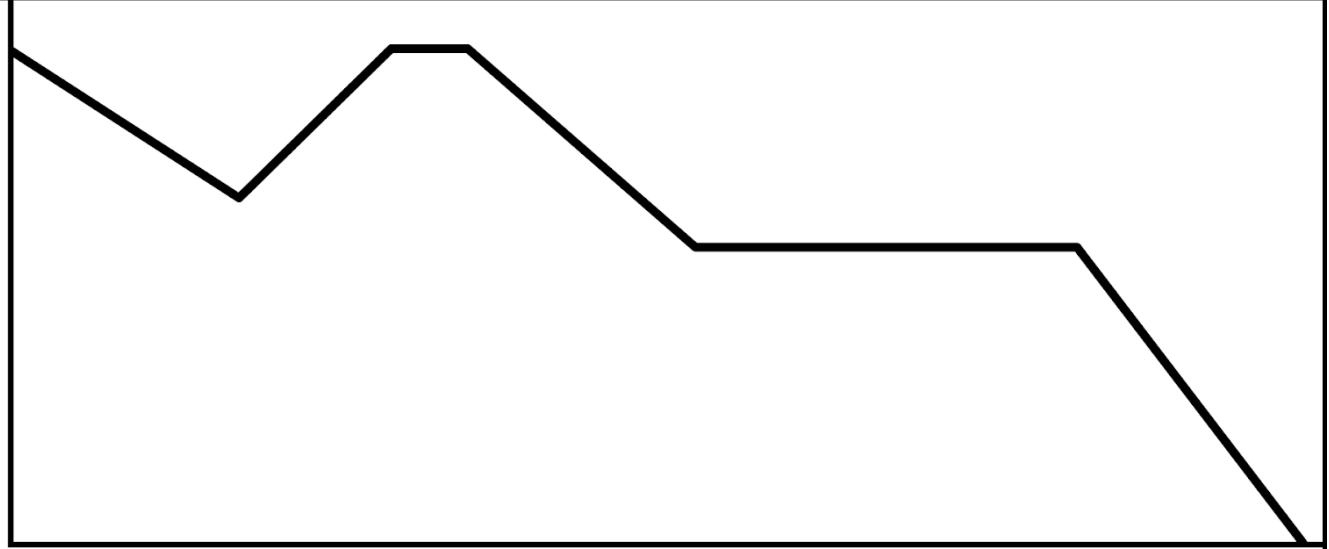
Creating Graphs of Functions

Learning Goal(s):

Elena starts to walk home from school. She turns around and goes back to school because she left something in her locker. At school, she runs into a friend who invites her to the library to do homework. She goes to the library, reads a book, then heads home to do her chores.

Label both axes so that the graph accurately represents the situation.

Label each segment with what is happening in the story during that time. (E.g., in the first segment, she is **walking home from school**).

**Summary Question**

What is important to pay attention to when drawing the graph of a function from a story?

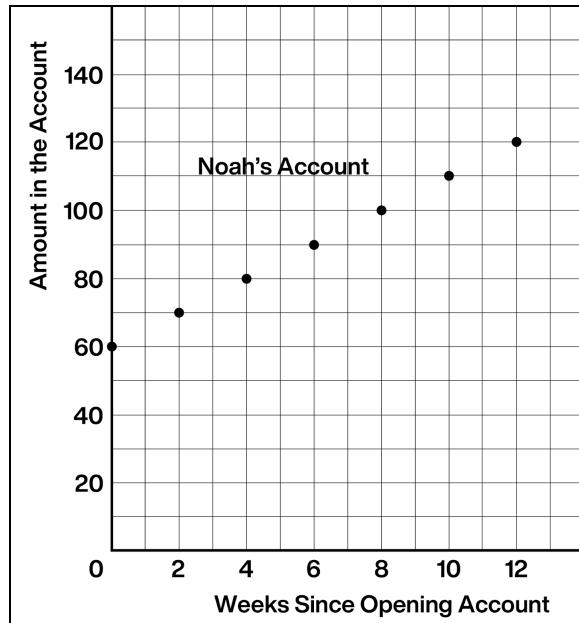
Comparing Representations of Functions

Learning Goal(s):

Elena opened an account on the same day as Noah. The amount of money, E , in Elena's account is given by the function $E = 8w + 70$, where w is the number of weeks since the account was opened. The graph below shows some data about the amount of money in Noah's account.

Who started out with more money in their account? Explain how you know.

Who is saving money at a faster rate? Explain how you know.



Write one question that might be easier to answer using the equation than using the graph.

Write one question that might be easier to answer using the graph than using the equation.

Summary Question: What are the strengths of using . . .

. . . a table?

. . . a graph?

. . . an equation?

Modeling With Piecewise Linear Functions

Learning Goal(s):

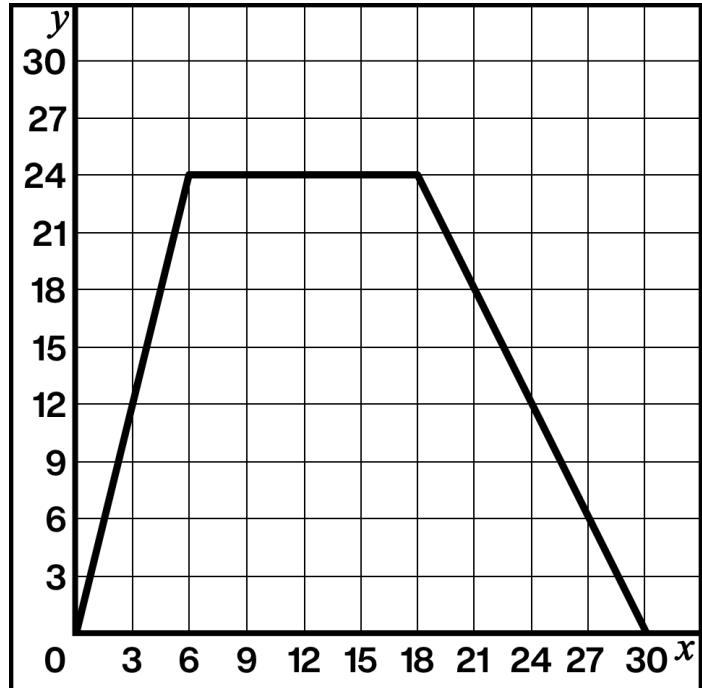
Deiondre gave their dog a bath in a bathtub. This graph shows the volume of water in the tub, in gallons, as a function of time, in minutes.

Why do you think this function is called a piecewise linear function?

At what rate did the water in the tub fill up?
Explain how you know.

At what rate did the water in the tub drain?
Explain how you know.

Select one linear piece of this function. Then write an equation for that piece in the form $y = mx + b$.



x represents the time (min.).

y represents the water in the bath (gal.).

Summary Question

How would you describe a piecewise linear function to someone who has never seen one?

Organizing Data

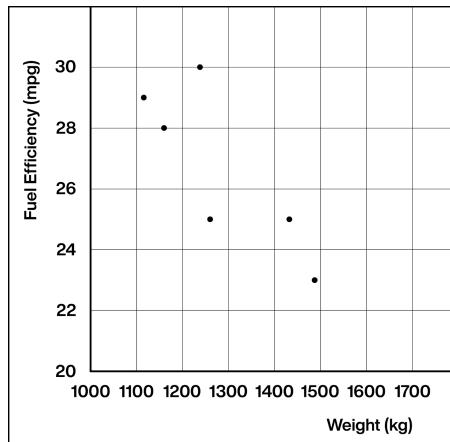
Learning Goal(s):

We can organize and display data with two variables in different ways.

Latifa is curious about different cars and their fuel efficiency (miles driven for each gallon of gas).

Table

Car Weight (kg)	Fuel Efficiency (mpg)
1116	28.93
1160	27.88
1238	29.94
1260	24.95
1432	25
1487	22.96

Scatter Plot

Predict the fuel efficiency of a typical car that weighs 1600 kilograms.

A teacher asked her students how many hours of sleep they had the night before a test.

How might you organize or display this data?

Why might someone want to organize it this way?

Student	Hours of Sleep	Score
Ayaan	7	74
Emika	6	76
Inola	8	88
Kwasi	5	63
Zoe	7	90

Summary Questions

What is one advantage of representing data in . . .

. . . a scatter plot?

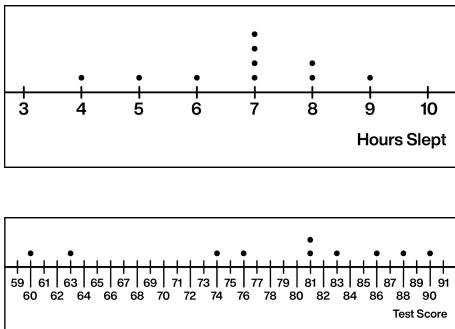
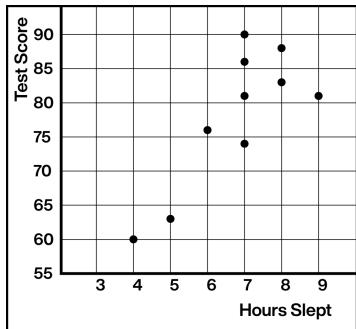
. . . a table?

Plotting Data

Learning Goal(s):

Representing data with a scatter plot is different from ways we have represented data before.

A teacher asked her students how many hours of sleep they had the night before a test.

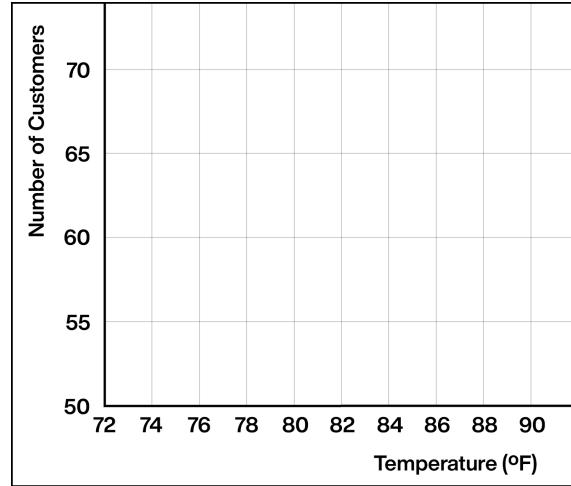
Dot Plots**Scatter Plot**

What is different about the two ways of representing the data?

One week, an ice cream stand collected data on the temperature and the number of customers.

Create a scatter plot of this data.

Day	Temperature (°F)	Number of Customers
Monday	85	58
Tuesday	83	55
Wednesday	90	63
Thursday	75	50
Friday	85	72

**Summary Question**

Scatterplots allow us to investigate possible connections between two numerical variables.

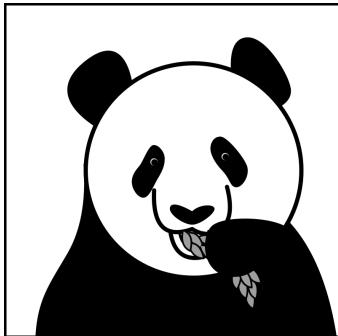
Explain what this sentence means in your own words.

What a Point in a Scatter Plot Means

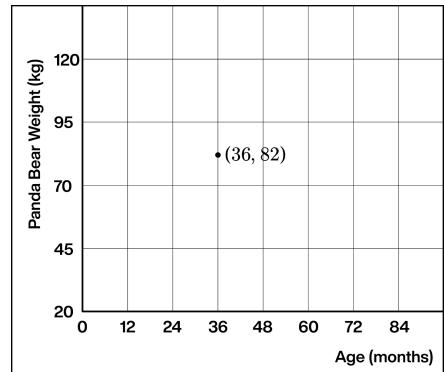
Learning Goal(s):

Scatter plots are made up of many individual data points. What do each of those points represent?

A giant panda lives in a zoo. What does the point on the graph tell you about the panda?



The panda is _____ months old and weighs _____ kilograms.

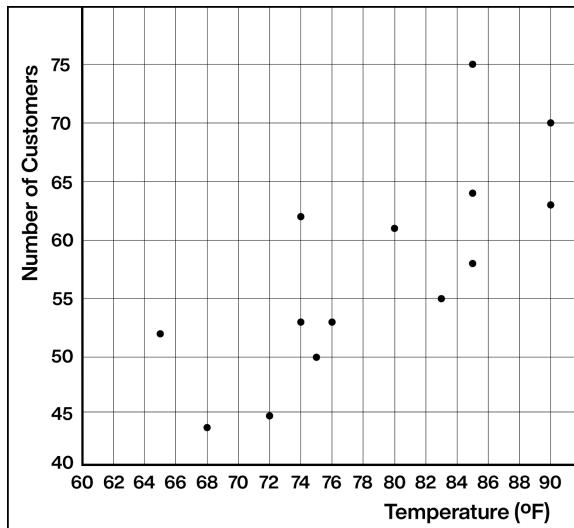


An ice cream stand collected data on the temperature and the number of customers over time.

Put a circle around the data point that represents the day it was 72 °F outside.

Put a square around the day when the stand had the most number of customers.

Why might the ice cream stand want to collect and visualize this data?

**Summary Question**

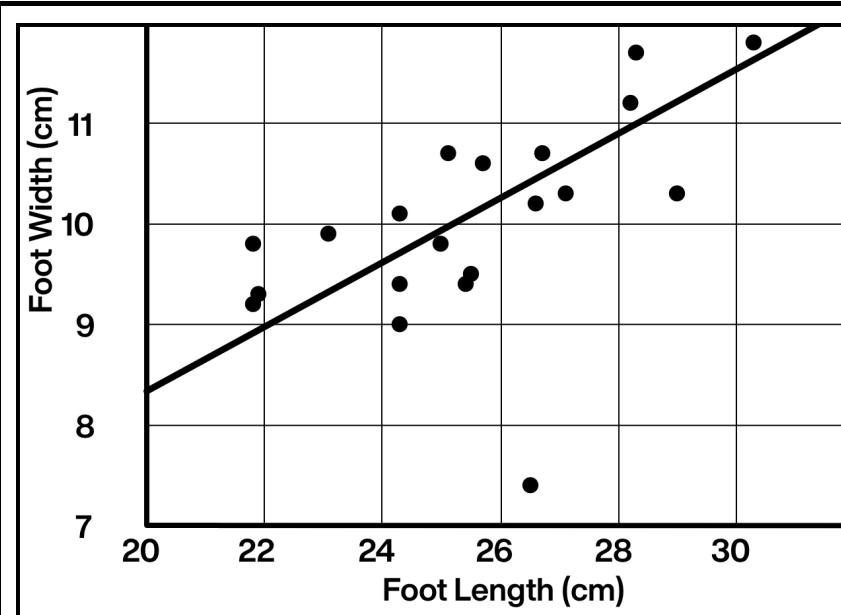
Describe a strategy to determine what a single point on a scatter plot means.

Lines of Fit and Outliers

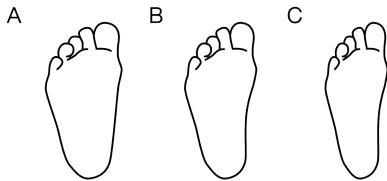
Learning Goal(s):

What if we want to make predictions about data not in the original data set? We can use linear functions to model data on a scatter plot. Models typically fit some data points well and not others.

This is data collected about different feet's length and width.



These three feet are all size 8 (about 24 centimeters long).



Label each foot on the scatter plot.

Which foot does the linear model fit best? _____ Explain your thinking.

Circle the outlier on this graph. On the right, draw what the outlier foot might look like.

Is the outlier wider or less wide than predicted for its length?

Summary Question

What does it mean for a data point to be an outlier?

Fitting a Line to Data

Learning Goal(s):

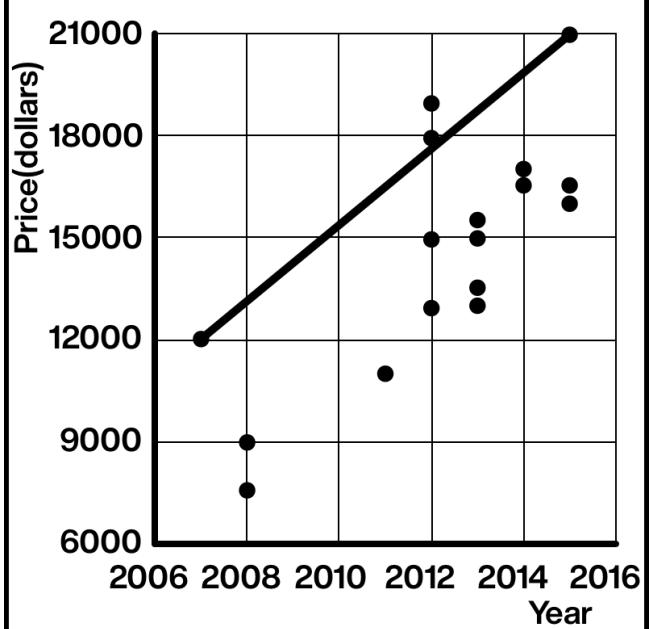
For any given data set with a linear association, there are infinite linear models we can draw. How do we decide what linear models are good fits for the data?

Here is data about the price of a used car and the year it was manufactured.

Saanvi drew this line of fit for the data.

Why might she have chosen this line?

Explain why this model is not a good fit for the data.



Draw a linear model that fits the data better. Explain how you chose your model.

Summary Question

Describe some characteristics of a line that is a good fit for the data in a scatter plot.

The Slope of a Fitted Line

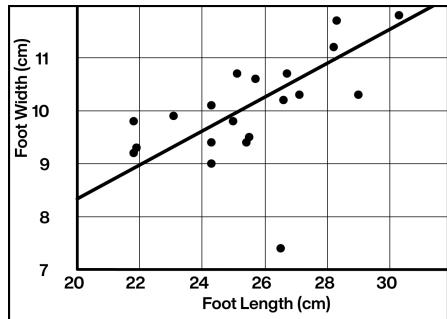
Learning Goal(s):

Sometimes we want to know how two variables are related. In this case, we can use the slope of a linear model to explain how increasing one variable typically changes the other.

Here is a scatter plot of foot length and width for various feet.

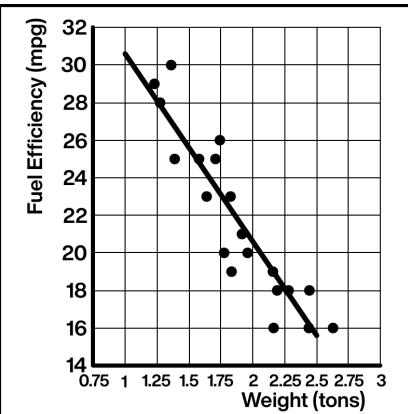
As foot length increases, foot width tends to _____.

This means there is a positive association/a negative association/no association between foot length and foot width.



The slope of the fitted line is about 0.32 . If the length of a foot increases by _____, the model predicts that its width will increase/decrease by _____.

Here is data on the weight of 21 cars and their fuel efficiency (miles driven for each gallon of gas).



Describe the relationship between weight and fuel efficiency.

The slope of the fitted line is about -10 . What does this number mean for the weight of a car and its predicted fuel efficiency?

Summary Question

When looking at a scatter plot of data, how can we tell if there is . . .

. . . a positive association?

. . . a negative association?

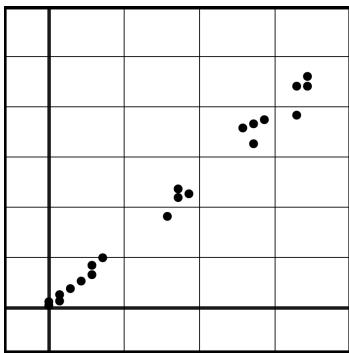
. . . no association?

Observing More Patterns in Scatter Plots

Learning Goal(s):

Sometimes the points in a scatter plot show an association, and sometimes there is no association.

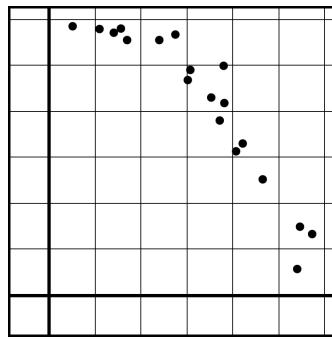
Circle the terms that describe the association in each scatter plot.



Positive / negative / no association

Linear / non-linear association

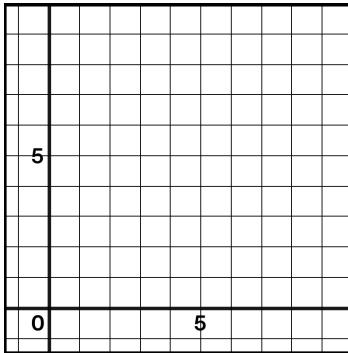
With / without clustering



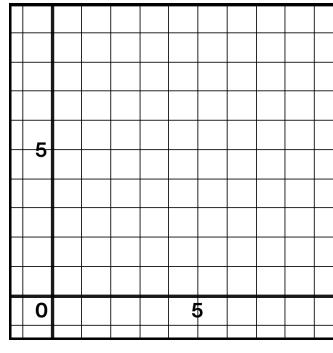
Positive / negative / no association

Linear / non-linear association

With / without clustering



Draw a scatter plot
that shows no
association.



Draw a scatter plot
that shows a negative
linear association with
clustering.

Summary Question

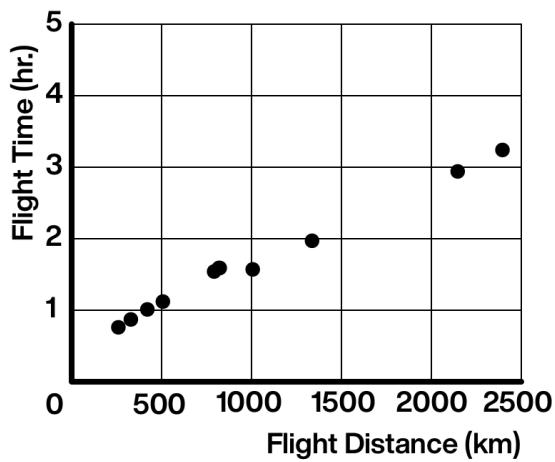
What is a strategy you can use to decide if two variables have a linear association?

Analyzing Bivariate Data

Learning Goal(s):

People often collect data to investigate possible associations between two numerical variables and use the connections that they find to predict more values of the variables.

The scatter plot shows flight distances and times for a set of flights.



Sketch a line on the scatter plot that fits the data well.

Add a point to the scatter plot that shows a 1,500 - kilometer flight with a flight time of 2 hours.

Add an outlier to the scatter plot.

Explain why this point is an outlier.

Describe the association between flight distance and flight time.

Use your model to predict the y -value of a point on the scatter plot with $x = 2000$. _____

What does this point tell you about the flight distance and flight time for the airplane?

Summary Question

What are some things that are important to remember when analyzing a scatter plot?

Two-Way Tables and Bar Graphs

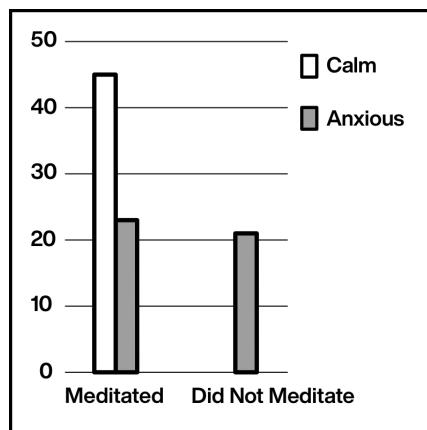
Learning Goal(s):

When we collect data by measuring attributes, such as height, we call that numerical data. When we collect data by counting things in various categories, such as tall or short, we call that categorical data. To help organize categorical data, we can use two-way tables and bar graphs.

These are the results of a study on meditation and athletes' state of mind before a track meet.

Two-Way Table

	Meditated	Did Not Meditate	Total
Calm	45		53
Anxious	23	21	
Total		29	97

Bar Graph

Fill in the missing values in the table.

Add a bar above to represent the number of people who did not meditate and were calm.

Add a star where 21 appears in the bar graph. What does 21 mean in this scenario?

Circle 44 in the two-way table. What does 44 mean in this scenario?

Summary Question

What are some advantages to displaying information in . . .

. . . a two-way table?

. . . a bar graph?

Using Data Displays to Find Associations

Learning Goal(s):

Flu Treatment Results

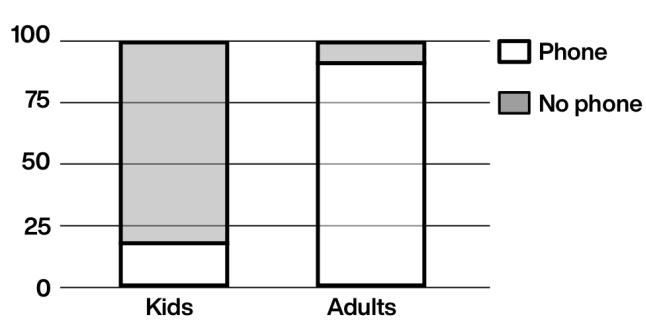
	Treatment A	Treatment B
Improved Health	57.5%	41.7%
No Improvement	42.5%	58.3%
Total	100%	100%

_____ of people who took Treatment A had improved health, whereas _____ of those who took Treatment B had improved health.

This means there **is / is not** an association between treatment and improved health.

For each situation, decide if there is an association. Explain your thinking.

Cell Phone Ownership



(Circle one) There **is / is not** an association between age and cell phone ownership.

Explain your thinking:

Lucky Socks and Winning

	Winners	Losers	Total
Lucky Socks	80%	20%	100%
Regular Socks	79%	21%	100%

(Circle one) There **is / is not** an association between wearing lucky socks and winning.

Explain your thinking:

Summary Question

How can you tell when there is a possible association between variables?

desmos

Unit 8.6, Lesson 11: Notes

Name _____

Creating Data Representations

Learning Goal(s):

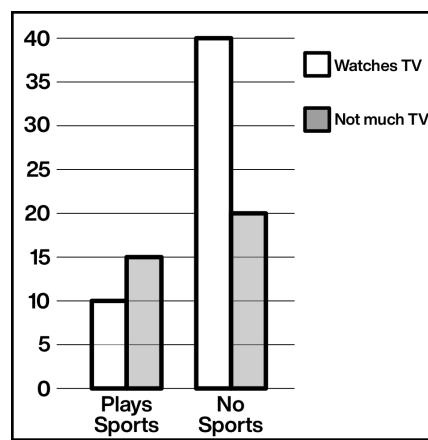
These data displays show the results of a survey of sports playing and TV watching of a group of students.

Fill in the missing information so that all of the data displays represent the same information.

Two-Way Table

	Watches TV	Not Much TV
Plays Sports	10	15
No Sports	40	20

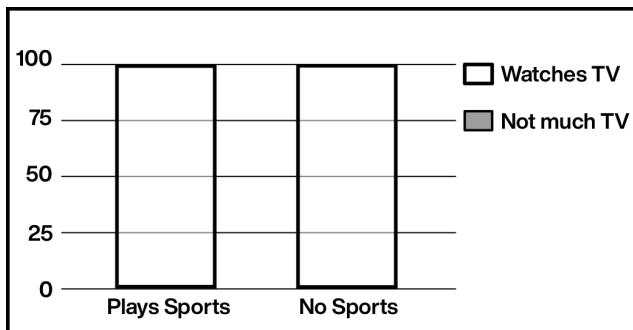
Bar Graph



Relative Frequency Table

	Watches TV	Not Much TV	Total
Plays Sports			
No Sports			

Segmented Bar Graph



Is there an association between playing sports and watching TV? Explain your thinking.

Summary Question

What are some things to remember when making relative frequency tables or segmented bar charts?

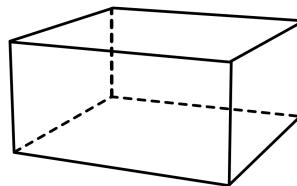
My Notes

1. Explain in your own words what a *cross section* is.

Here is a rectangular prism.

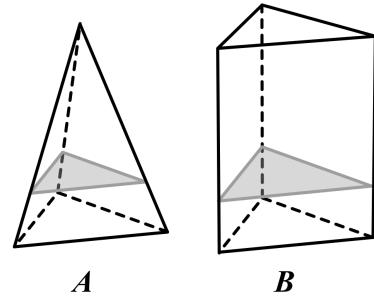
2. Select **all** the possible cross sections of this prism.

- Triangle
- Rectangle
- Pentagon
- Hexagon
- Octagon



Here is a triangular pyramid and a triangular prism.

- 3.1 If you cut both the pyramid and the prism parallel to their bases, how would the cross sections be **similar**?



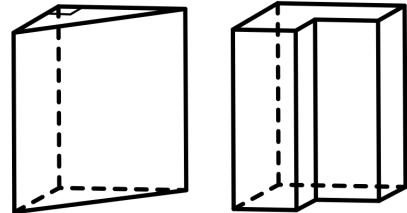
- 3.2 How would they be **different**?

Summary

- I can describe cross sections of a solid.
- I can compare and contrast cross sections of prisms and pyramids.

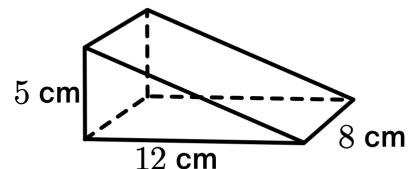
My Notes

1. Describe a strategy for calculating the volume of a prism.



- 2.1 Shade in a base of this prism.

- 2.2 Calculate the volume.
Show all of your calculations.



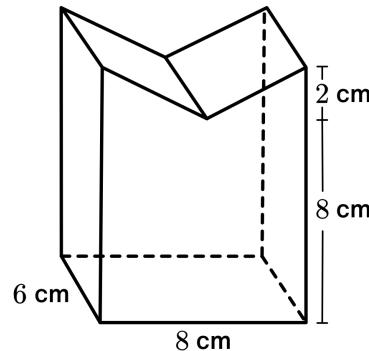
- 2.3 Sketch and label a **rectangular** prism with the same volume.

Summary

- I can explain how the volume of a prism is related to the area of its base and its height.
- I can calculate the volume of rectangular and triangular prisms.

My Notes

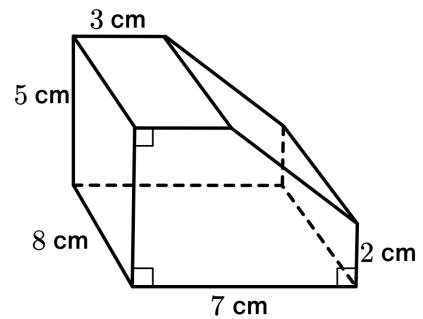
- 1.1 Sketch the base of this prism and label its dimensions.



- 1.2 What is the area of the base? Explain or show your reasoning.

- 1.3 What is the volume of the prism?

2. Use any strategy to calculate the volume of this prism. Show all of your thinking.

**Summary**

I can calculate the volume of more complicated prisms.

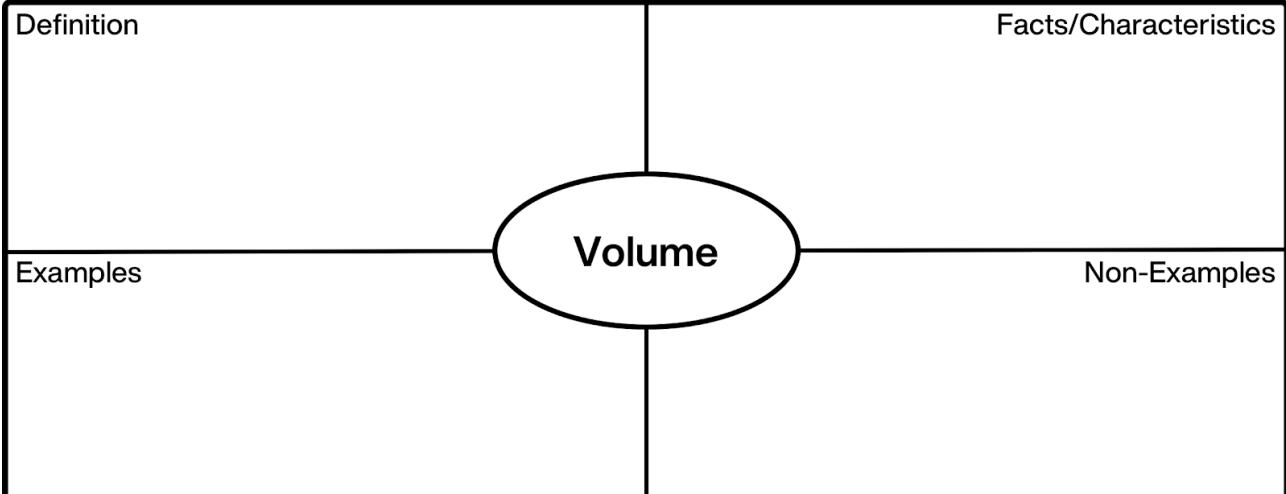
desmos

Unit 8.5, Lesson 10: Notes

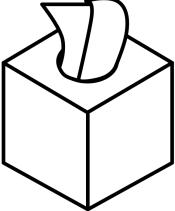
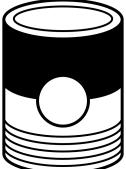
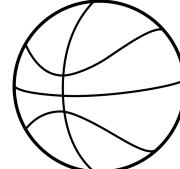
Name _____

Exploring Volume

Learning Goal(s):



For each household object, name the 3-D solid it most resembles and a fact you learned today.

 Name: Fact:	 Name: Fact:
 Name: Fact:	 Name: Fact:

Summary Question

How would you describe volume to a 3rd grader?

The Volume of a Cylinder

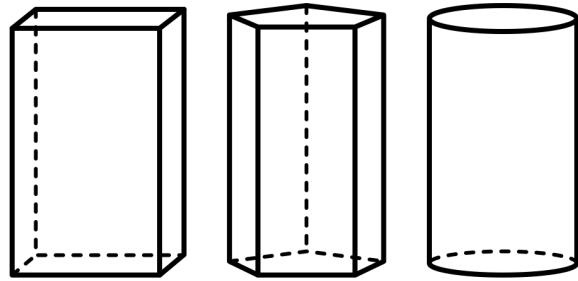
Learning Goal(s):

Here is the formula for the volume of a prism.

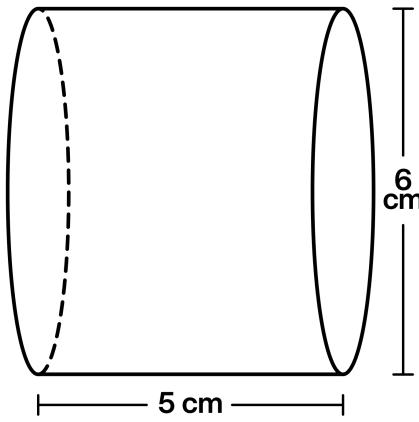
$$V = Bh$$

Explain what each of the variables mean.

Use these figures if it helps you explain your thinking.



Find the volume of the cylinder (exactly or rounded to the nearest tenth).

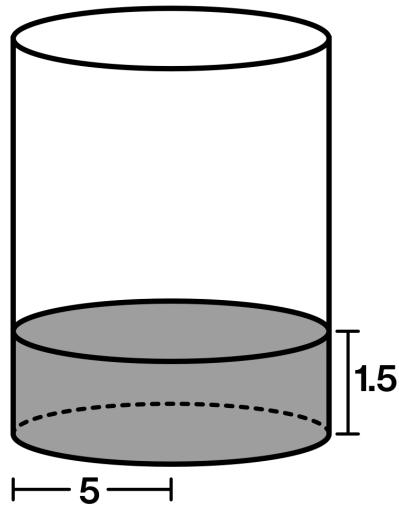
**Summary Question**

How is finding the volume of a cylinder like finding the volume of a prism?

Scaling Cylinders Using Functions

Learning Goal(s):

Imagine a water tank that is shaped like a cylinder.



If you triple the height of the water, will you triple the volume inside the container?

Yes

No

Explain your thinking.

If you triple the radius of the water tank, will you triple the volume inside the container?

Yes

No

Explain your thinking.

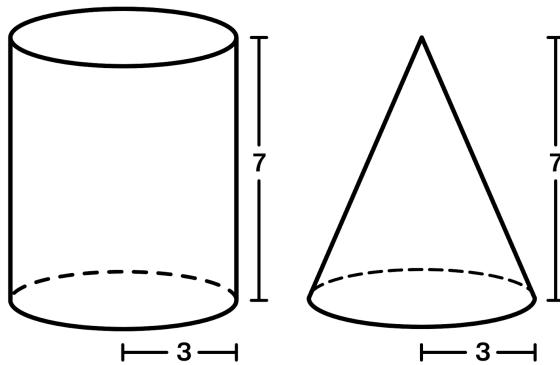
What are all of the ways you could change the water or the tank so that its volume is 4 times its current amount?

Summary Question

Why is the relationship between radius and volume non-linear?

Volumes of Cones

Learning Goal(s):



Find the volume of the cylinder above.

Find the volume of the cone above.

Sketch a cone. Label the diameter 8 units and the height 5 units.

Find the volume of the cone whose diameter is 8 units and height is 5 units.

Summary Question

How would you explain the relationship between the volume of a cone and the volume of a cylinder to a 3rd grader?



Unit 8.5, Lesson 14: Notes

Name _____

Finding Cylinder and Cone Dimensions

Learning Goal(s):

Use the space below to find the missing dimensions of each object (rounded to the nearest tenth). Use 3.14 as an approximation for π . Circle your answers.

1. A cylinder has a radius of 4 cm and a volume of 80π cm³. What is the height of the cylinder?
 2. A cylinder with a volume of 405 in.³ has a diameter of 10 in. What is its height?
 3. A cone with a volume of 135π in.³ has a height of 5 in. What must the radius of the cone be?

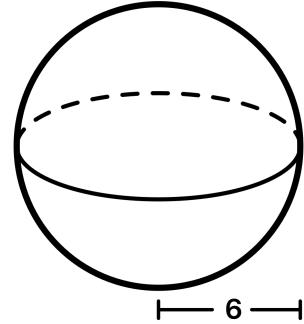
Summary Question

How are the strategies for finding the missing dimensions of a cone and a cylinder . . .
. . . similar? . . . different?

Volumes of Spheres

Learning Goal(s):

Darryl, Na'ilah, and Maia calculated the volume of the sphere on the right. Each of them made an error in their calculations. Identify their errors and explain what they might have been thinking.



$$\text{Darryl: } V = \frac{4}{3} \pi (6)^3 = \frac{4}{3} \pi (36) = 48\pi \text{ in.}^3$$

$$\text{Maia: } V = \frac{4}{3} \pi (6)^3 = \frac{4}{3} (18.84)^3 \approx 8196 \text{ in.}^3$$

$$\text{Na'ilah: } V = \frac{4}{3} \pi (6)^3 = \frac{4}{3} \pi (18) = 24\pi \text{ in.}^3$$

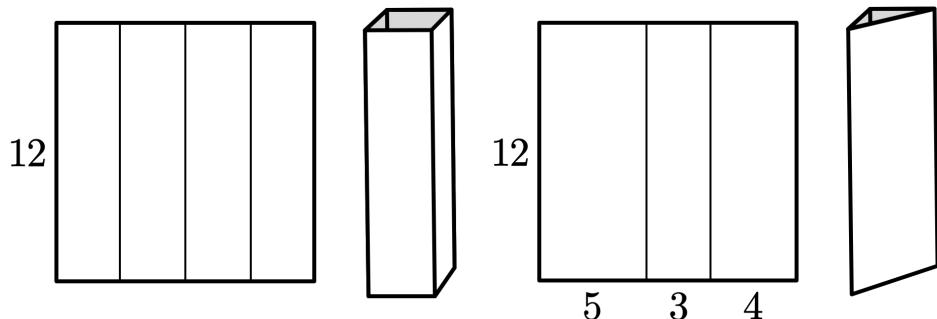
Find the volume of the sphere.

Summary Question

What advice would you give a student to help them find the volume of a sphere?

My Notes

Imani is folding origami paper to make pencil holders for their room. They fold two holders: a square prism and a right triangular prism.



1. Which container holds more pencils? Calculate the amount each container can hold to support your claim.

2. Imani added a bottom and a top to each container. Which container do you think uses more paper? Calculate the amount of paper each container uses to support your claim.

Summary

- I can decide whether volume or surface area is more useful to answer a question about a situation.
 - I can answer a question about a real-world situation using my knowledge of surface area and volume.

Learning Goal(s):

Exponents make it easy to show repeated multiplication. It is easier to write 2^6 than to write $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. Imagine writing 2^{100} using multiplication!

For each expression below, write an equivalent expression that uses exponents:

A. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

B. $5 \cdot 8 \cdot 5 \cdot 8 \cdot 5 \cdot 8 \cdot 5$

C. $10 \cdot 10 \cdot 10 + 10 \cdot 10$

Consider this situation: Each day, the number of grains of rice you have triples. On day one, you have 3 grains of rice. On day two, you have 9 grains of rice.

- On what day will you have 243 grains of rice?
- On what day will you have 3^{13} grains of rice?
- How many grains of rice will you have *two days after* you have 3^{13} grains of rice?

Summary Question

When is it useful to express a number or expression with exponents?

Learning Goal(s):

Sometimes writing an expression in an equivalent way can help us compare it to other expressions. The fact that exponents represent repeated multiplication can help us write equivalent expressions.

Decide if Expression 1 is equivalent to Expression 2 for each pair. Consider “expanding” each expression, as shown in Pair A.

	Expression 1	Expression 2	Equivalent?
Pair A	$(12^2)^3$ $(12 \cdot 12)(12 \cdot 12)(12 \cdot 12)$	$12^4 \cdot 12^2$ $(12 \cdot 12 \cdot 12 \cdot 12)(12 \cdot 12)$	YES NO
Pair B	$7^3 \cdot 2^3$	$(7 \cdot 2)^3$	YES NO
Pair C	$16^3 + 16^2 + 16$	16^6	YES NO
Pair D	15^6	$(5 \cdot 3 \cdot 3 \cdot 5)^4$	YES NO

Summary Question

Show or explain why $6^5 \cdot 6^3$ is equivalent to $(6^4)^2$. Then write another expression that is equivalent to both of them.

Learning Goal(s):

Sometimes, we want to investigate whether two expressions are equivalent. In those instances, it can be helpful to convert between exponents and repeated multiplication.

For each pair, decide if Expression 1 is equivalent to Expression 2.

	Expression 1	Expression 2	Equivalent?
Pair A	$(5^5)^2$	$5^4 \cdot 5^3$	YES NO
Pair B	$4^3 \cdot 2^5$	8^8	YES NO
Pair C	$15^3 \cdot 2^3$	$(5 \cdot 2)^3 \cdot 3^3$	YES NO

Decide whether each expression below is equivalent to 10^6 . For any that are not, change the expression so that it *is* equivalent to 10^6 .

A. $10 \cdot 10^3 \cdot 10^2$	B. 100^5	C. $10^3 + 10^3$	D. $(10^2)^3$
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Summary Question

What are some important things to remember when determining whether expressions with exponents are equivalent?

Learning Goal(s):

Expressions that have a single base and a single exponent (like 7^3) are sometimes preferable to expressions with more parts because they can help us easily compare numbers to each other.

For each expression below, fill in the blanks. The first row has been done for you.

Expression	Expanded Expression	Single Power
$(12^2)^3$	$(12 \cdot 12)(12 \cdot 12)(12 \cdot 12)$	12^6
A. $\frac{6^5 \cdot 6^2}{6^4}$		
B. $7^3 \cdot 2^3$		
C. $\frac{(3^3)^2}{3^4}$		
D. $\frac{9^2 \cdot 3^5}{3^3}$		

Which of the four above expressions (A, B, C, or D) is greatest? Explain your reasoning.

Summary Question

Describe a strategy for rewriting an expression like $\frac{(6^{30})^3}{6^{40}}$ as a single power.

Learning Goal(s):

Our concept of “exponents as repeated multiplication” is less helpful when the exponent is zero or a negative number. Patterns can help us discover what zero or negative numbers mean as exponents.

Powers of 8		
8^3	$1 \cdot 8 \cdot 8 \cdot 8$	512
8^2	$1 \cdot 8 \cdot 8$	64
8^1	$1 \cdot 8$	8
8^0	1	1
8^{-1}	$1 \div 8$	$\frac{1}{8}$
8^{-2}	$1 \div 8 \div 8$	$\frac{1}{8^2}$ or $\frac{1}{64}$
8^{-3}	$1 \div 8 \div 8 \div 8$	$\frac{1}{8^3}$ or $\frac{1}{512}$

Examine the **Powers of 8** table. How do the numbers change as you look *down* the table from 8^3 to 8^2 to 8^1 ?

Based on the patterns in the table, what is another way to represent 8^{-5} ?

Why does it make sense that $8^0 = 1$?

Write each expression as a single power:

A.
$$\frac{7^4 \cdot 7^{-2}}{7^{12}}$$

B.
$$\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$$

C.
$$\frac{2^{-4}}{(2^{-5})^2}$$

Summary Questions

What is the relationship between 10^5 and 10^{-5} ?

What is the value of $10^5 \cdot 10^{-5}$?

Learning Goal(s):

Patterns emerge when we rewrite expressions with exponents. We can generalize these patterns into exponent rules.

Fill in the blanks. Then write why the rule makes sense.

Symbolic Rule	Example	Why It Makes Sense
$x^m \cdot x^n = x^{m+n}$	$8^5 \cdot 8^2 = 8^7$	Both sides of the equal sign have seven factors of 8.
$(x^m)^n = (x^n)^m = x^{m \cdot n}$	$(11^2)^3 = (11^3)^2 = 11^6$	
	$6^3 \cdot 5^3 = 30^3$	
$\frac{x^m}{x^n} = x^{m-n}$		
$x^{-n} = \frac{1}{x^n}$		
	$188^0 = 1$	

Summary Question

Explain why $15^{10} \cdot 2^{13}$ is equivalent to $30^{10} \cdot 2^3$.

Learning Goal(s):

The United States Mint has made over 500, 000, 000, 000 pennies. Exactly how many pennies is that? One way to make sense of that number is by considering how many thousands, millions, or billions of pennies that is. Another way of making sense is to rewrite it using powers of 10.

Number	In Billions	In Millions	In Thousands	Rewrite as a Multiple of a Power of 10
500, 000, 000, 000	billion (10^9)			
500, 000, 000, 000		million (10^6)		
500, 000, 000, 000			thousand (10^3)	

Write two different expressions that represent the weight of the object using a power of ten.

Object and Weight	Expression #1	Expression #2
Bus: 7, 810 kg	$781 \cdot 10^1$	
Ship: 4, 850, 000kg		
Cell Phone: 0.13 kg		

Summary Question

What does it mean to write a number using a single multiple of a power of 10?

Learning Goal(s):

For each example below, write the number shown on the number line diagram.

	<p>Write the number shown on the number line diagram.</p>
	<p>What is another way to write this number?</p>
	<p>Write the number shown on the number line diagram.</p>
	<p>What is another way to write this number?</p>
	<p>Write the number shown on the number line diagram.</p>
	<p>What is another way to write this number?</p>

Summary Question

When a number is given as a multiple of a power of 10, what is a strategy for writing an equivalent number?

Learning Goal(s):

Powers of 10 and exponent rules can be helpful for making calculations with large or small numbers. The table below shows the number of people in the United States in 2014 and how much total oil they used for energy.

	Estimated Amount	Write Using a Power of 10
Population of United States in 2014	300,000,000 people	
Total Oil Used	2,000,000,000,000 kilograms	

Approximately how many kilograms of oil did the average person in the United States use in 2014?

The table shows the total number of creatures as well as the approximate masses of each creature.

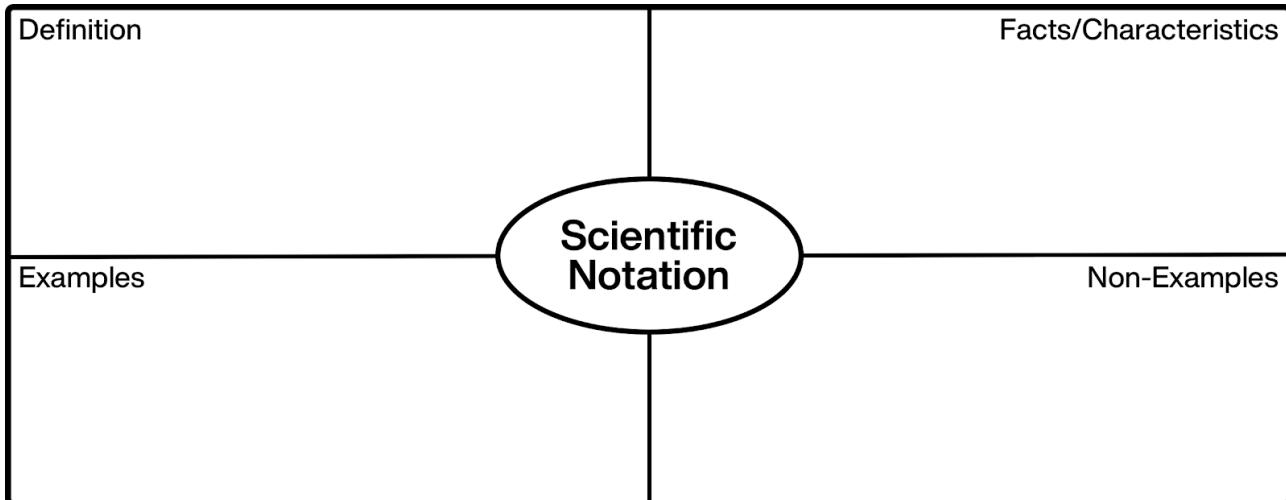
Creature	Total	Mass of One Individual (kg)
Humans	$7.5 \cdot 10^9$	$6 \cdot 10^1$
Ants	$5 \cdot 10^{16}$	$3 \cdot 10^{-6}$

Which is more massive: the total mass of all humans or the total mass of all the ants? About how many times more massive is it?

Summary Question

If you have two very large numbers, how can you tell which is larger?

Learning Goal(s):



Write each number using scientific notation, or say if it is already written using scientific notation.

Number	Scientific Notation
540,000	
0.003	
$6.8 \cdot 10^9$	
$12 \cdot 10^{-2}$	
$97 \cdot 10^5$	

Summary Question

What is important to pay attention to when writing a number in scientific notation?

Learning Goal(s):

Comparing the relative sizes of very large or very small numbers is easier with scientific notation. The table shows the total numbers of humans and ants.

	Approximate Number	Scientific Notation
Humans	7,500,000,000	
Ants	50,000,000,000,000	

About how many ants are there for every human?

Ants weigh about $3 \cdot 10^{-6}$ kilograms each. Humans weigh about $6.2 \cdot 10^1$ kilograms each.
About how many ants weigh the same as one human?

There are about $3.9 \cdot 10^7$ residents in California. The average Californian uses about 180 gallons of water per day. About how many gallons of water total do Californians use in a day?

Summary Question

Describe a strategy you used to divide two numbers given in scientific notation.

Learning Goal(s):

The table below shows the diameters for objects in our solar system.

Object	Diameter (km)
Sun	$1.392 \cdot 10^6$
Mars	$6.785 \cdot 10^3$
Jupiter	$1.428 \cdot 10^5$
Neptune	$4.95 \cdot 10^4$
Saturn	$1.2 \cdot 10^5$

If we place Mars and Neptune next to each other, are they wider than Saturn?

First, add the diameters of Mars and Neptune:

$$6.785 \cdot 10^3 + 4.95 \cdot 10^4$$

To add these numbers, we can either rewrite them as multiples of 10^3 or as multiples of 10^4 .

Method 1: Rewrite each number as a multiple of 10^3 .

Method 2: Rewrite each number as a multiple of 10^4 .

If we place Jupiter and Neptune next to each other, are they wider than the Sun?

About how much wider is Jupiter than Neptune?

Summary Question

What are some important things to remember when adding numbers written in scientific notation?

Learning Goal(s):

Use the table to answer questions about different life forms on our planet.

Creature	Number	Mass of One Individual (kg)
Humans	$7.5 \cdot 10^9$	$6.2 \cdot 10^1$
Sheep	$1.75 \cdot 10^9$	$6 \cdot 10^1$
Chickens	$2.4 \cdot 10^{10}$	$2 \cdot 10^0$
Antarctic Krill	$7.8 \cdot 10^{14}$	$4.86 \cdot 10^{-4}$
Bacteria	$5 \cdot 10^{30}$	$1 \cdot 10^{-12}$

Which is larger: the total mass of all humans or of all the Antarctic krill?

How can you tell which creature has the greatest total mass?

About how many more chickens are there than sheep?

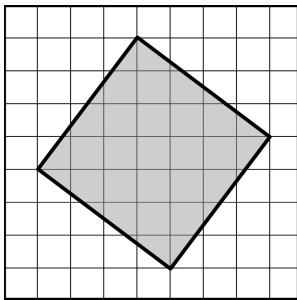
Summary Question

What are some important things to remember about adding, subtracting, multiplying, and dividing numbers written in scientific notation?

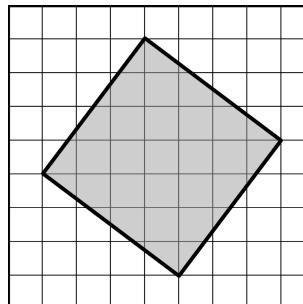
Learning Goal(s):

Sometimes we want to find the area of a square, but we don't know the side length. When this is true, we can use strategies such as "decompose and rearrange" and "surround and subtract."

Decompose and Rearrange



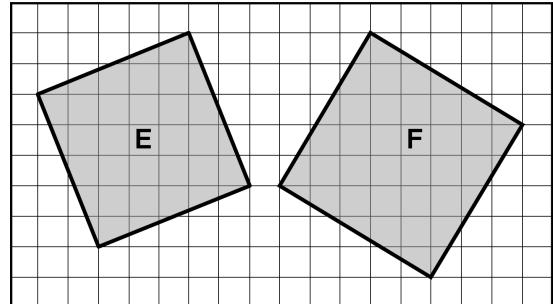
Surround and Subtract



Use any strategy to calculate the area of each square.

Square E

Square F



Which of these squares must have a side length that is greater than 5 but less than 6? _____
Explain how you know.

Summary Question

If you don't know the side length of a square, how can you find its area?

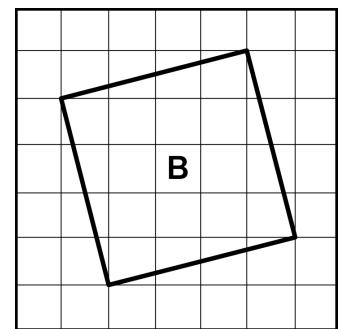
Learning Goal(s):

Sometimes we want to know the side length of a square whose length is not countable using a grid. When this is true, we can take the square root of the area in order to find the side length.

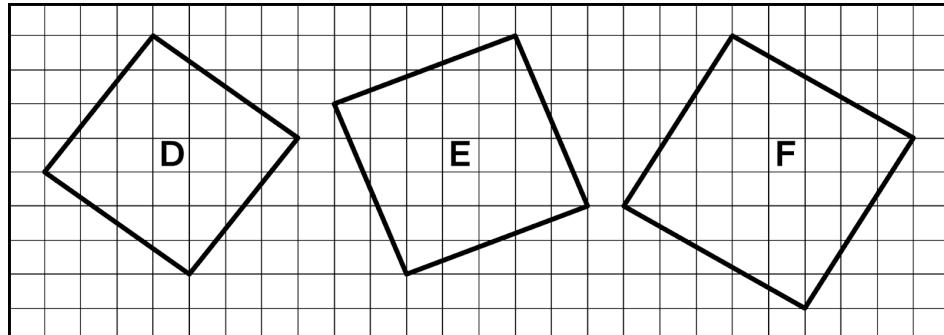
Square B has an area of 17.

We say the side length of a square with an area of 17 units is $\sqrt{17}$ units.

This means that $(\quad)^2 = \underline{\hspace{2cm}}$.



Find each missing value.



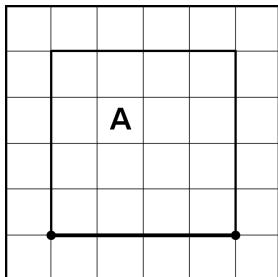
Square	Side Length of Square (units)	Area of Square (square units)
D		25
E	$\sqrt{29}$	
F		

Summary Question

Explain the meaning of $(\sqrt{9})^2 = 9$ using squares and side lengths.

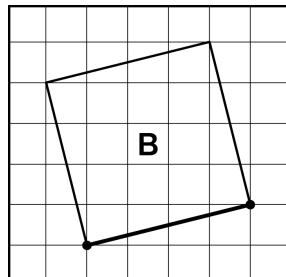
Learning Goal(s):

Determine the side length of each square. Use a square root if the value is not exact.



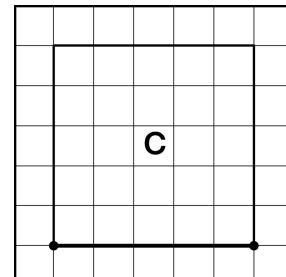
Area: 16 square units

Side length: _____



Area: 17 square units

Side length: _____



Area: 25 square units

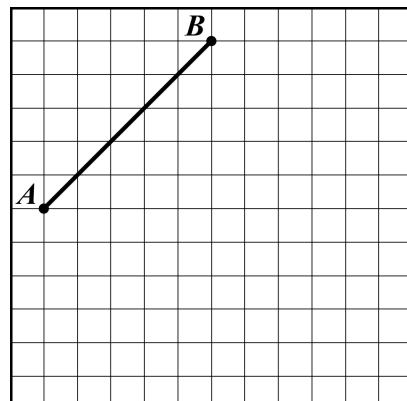
Side length: _____

Square B has a side length of _____ units. In order to approximate numbers like _____, we can find two integer values that the number lies between. Square B has an area between _____ and _____ square units, so its side length must be between _____ and _____ units.

Draw a square so that segment AB is along one side of the square.

Exact length of AB (as a square root): _____

Approximate length of AB : _____



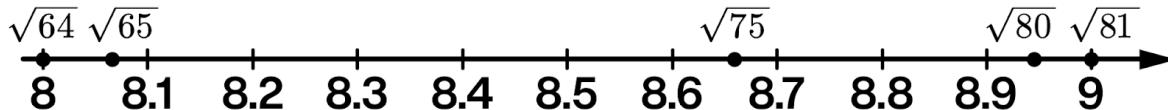
Summary Question

What two integers does $\sqrt{60}$ lie between? Explain how you know. Then use a calculating device to approximate $\sqrt{60}$ as closely as possible.

Learning Goal(s):

We can approximate the values of square roots by looking for whole numbers nearby.

- $\sqrt{65}$ is a little more than _____, because $\sqrt{65}$ is a little more than $\sqrt{64} = _____$.
- $\sqrt{80}$ is a little less than _____, because $\sqrt{80}$ is a little less than $\sqrt{81} = _____$.
- $\sqrt{75}$ is between _____ and _____, because 75 is between 64 and 81.
- $\sqrt{75}$ is approximately _____. We can check this by calculating _____.



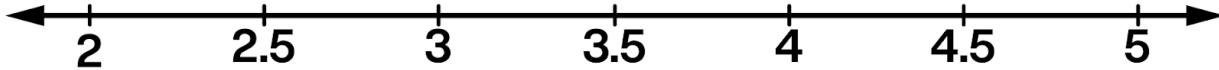
Under each description, write the square root(s) that lie between the integers described.

- $\sqrt{6}$
- $\sqrt{12}$
- $\sqrt{24}$
- x when $x^2 = 8$

Between 2 and 3

Between 4 and 5

Add each number above to the number line below.



Summary Question

Where would $\sqrt{17}$ belong on the number line above? Explain how you know.

Learning Goal(s):

Sometimes we are interested in the edge length of a cube instead of the side length of a square.

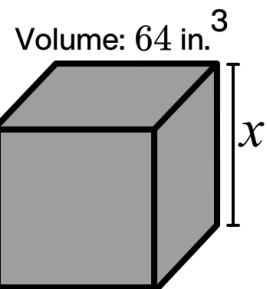
The number $\sqrt[3]{17}$, read as “cube root of 17,” is the edge length of a cube that has a volume of 17.

We can approximate the value of a cube root in a similar way to approximating a square root:

$\sqrt[3]{17}$ is more than _____, because $\sqrt[3]{17}$ is more than $\sqrt[3]{8} =$ _____.

$\sqrt[3]{17}$ is less than _____, because $\sqrt[3]{17}$ is less than $\sqrt[3]{27} =$ _____.

$\sqrt[3]{17}$ is approximately _____, because $(2.57)^3 = 16.9746$.



Find each missing value without using a calculator.

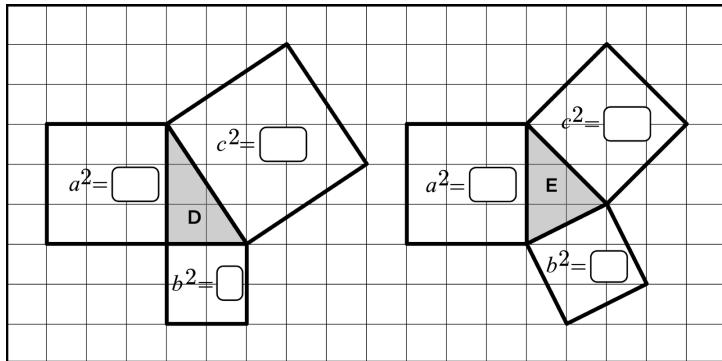
Exact Edge Length of Cube (units)	Approximate Edge Length of Cube (units)	Volume of Cube (cubic units)
	Between _____ and _____	60
$\sqrt[3]{4}$	Between _____ and _____	
	Between _____ and _____	25

Summary Question

Approximate the value of x when $x^3 = 81$. Explain your thinking.

Learning Goal(s):

Find the missing values. Record what you notice and wonder.



I notice . . .

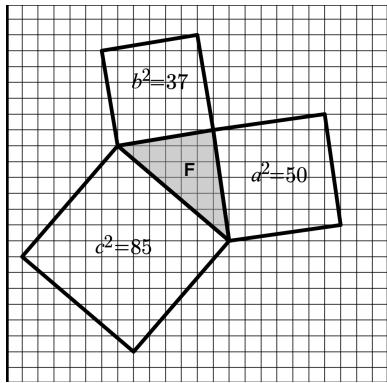
I wonder . . .

In Triangle D, the square of the hypotenuse is equal to the sum of the squares of the legs.

This relationship is true for **all** right triangles. It is often known as the **Pythagorean theorem**.

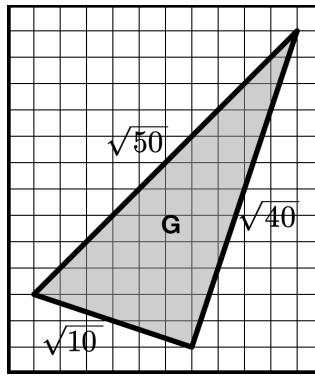
Another way to describe this relationship is $a^2 + b^2 = c^2$, where a and b are the lengths of the legs and c is the length of the hypotenuse of a right triangle.

Decide if the Pythagorean theorem is true for each triangle. Show your thinking.



Yes / No

Your thinking:



Yes / No

Your thinking:

Summary Question

What does the Pythagorean theorem tell us about the side lengths of a right triangle?

Learning Goal(s):

We observed that $a^2 + b^2 = c^2$ is true for many right triangles with legs of a and b . How do we know this relationship is **always** true? Proofs help us know when a relationship will always be true.

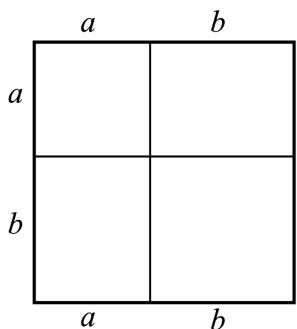


Figure G

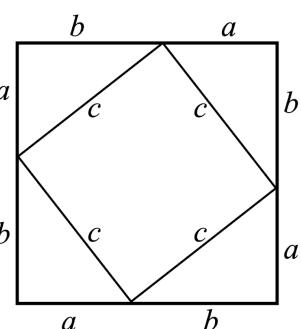


Figure H

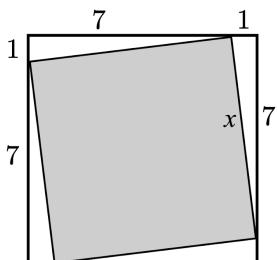
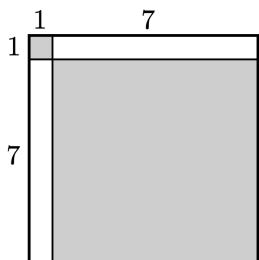
Habib wrote the following proof of the Pythagorean theorem based on the diagram:

$$a^2 + b^2 + ab + ab = c^2 + 4 \cdot \frac{1}{2} ab$$

$$a^2 + b^2 + 2ab = c^2 + 2ab$$

$$a^2 + b^2 = c^2$$

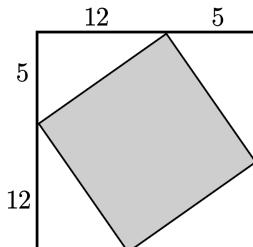
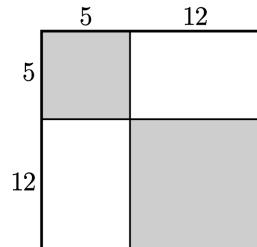
Describe Habib's strategy for proving the Pythagorean theorem. Use the diagrams if that helps to support your thinking.



Find the value of x .

Summary Question

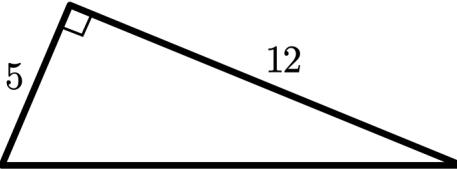
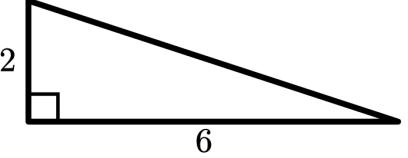
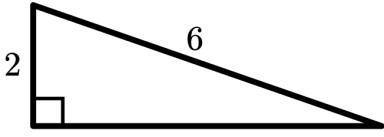
Show how you can see the equation $5^2 + 12^2 = 13^2$ in the figures on the right. Explain how this relates to the Pythagorean theorem.



Learning Goal(s):

Sometimes we know the length of two sides of a right triangle and want to find the third. In this situation, we can use the Pythagorean theorem.

Highlight the hypotenuse of each triangle. Then find the length of the missing side of the triangle.

Triangle	Missing Side Length
 A right triangle with a vertical leg labeled 5, a horizontal leg labeled 12, and a hypotenuse labeled with a question mark.	
 A right triangle with a vertical leg labeled 2, a horizontal leg labeled 6, and a hypotenuse labeled with a question mark.	
 A right triangle with a vertical leg labeled 2, a horizontal leg labeled 6, and a hypotenuse labeled with a question mark.	

Summary Question

How can you use the Pythagorean theorem to find an unknown side length in a right triangle?

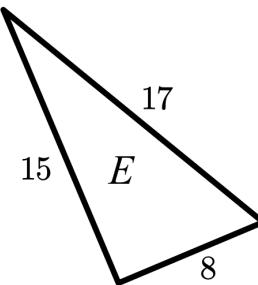
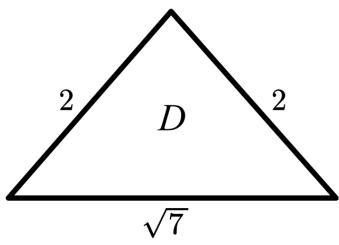
Learning Goal(s):

Sometimes it's hard to tell if a triangle is a right triangle just by looking. In this situation, we can use what is called the converse of the Pythagorean theorem to help us decide.

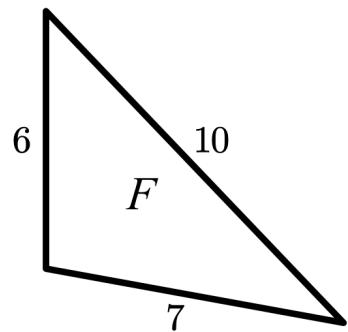
If _____, the triangle is a right triangle.

If _____, the triangle is not a right triangle.

Use the converse of the Pythagorean theorem to decide which of the following are right triangles.



Change **one** of the values to make triangle *F* into a right triangle.



Summary Question

Explain how to tell if a triangle is a right triangle using its side lengths.

Learning Goal(s):

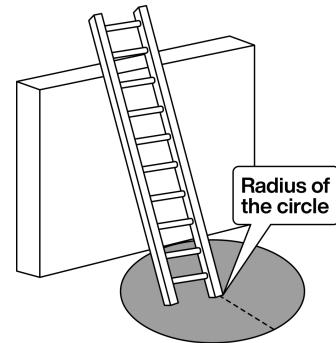
Name some situations in your world that might involve right triangles.

A 17 -foot ladder is leaning against a wall. The ladder can reach a window 15 feet up the wall. How far from the wall should the base of the ladder be so that it reaches the window?

Draw a picture of the situation.

Write your answer to the question.
Show all of your thinking.

To avoid accidents, the fire department wants to create a circular no-walk zone under the ladder with a radius that is the distance between the ladder and the wall. What is the area of the no-walk zone?



Summary Question

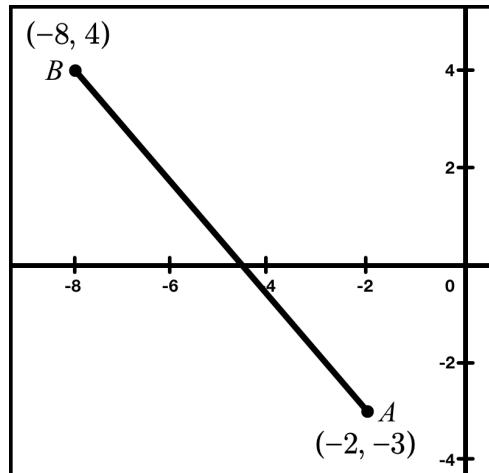
What are some things that are important to remember when using the Pythagorean theorem?

Learning Goal(s):

Sometimes we want to find the distance between two points that are not easily countable on a grid.

Draw a right triangle whose hypotenuse is \overline{AB} .

Use the tools you have to calculate the length of \overline{AB} .



Calculate the distances between each pair of points on the graph.

$$\overline{PR} = \underline{\hspace{2cm}}$$

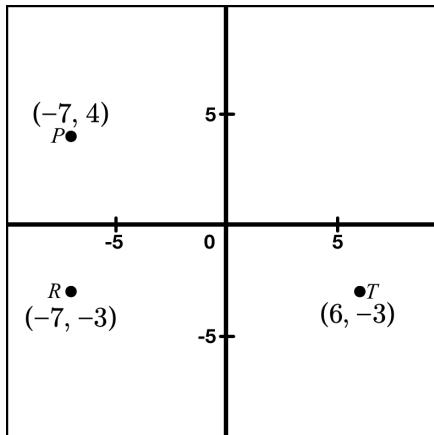
units

$$\overline{RT} = \underline{\hspace{2cm}}$$

units

$$\overline{PT} = \underline{\hspace{2cm}}$$

units



Ready for more?

Plot a point that is exactly $\sqrt{29}$ units away from point R .

Summary Question

How is using the Pythagorean theorem on a grid similar to or different from using it on a triangle?

Learning Goal(s):

Sometimes it's helpful to rewrite fractions as decimals. Can you think of times this might be true?

Decimals can either terminate (stop) or continue infinitely. When the decimal repeats indefinitely, we draw a line over the repeating digits.

Expand $0.\overline{5673} =$ _____ ...

Describe how Kwame calculated that $\frac{2}{11} = \overline{.18}$ in your own words.

$$\begin{array}{r} 0.1818\dots \\ 11 \overline{)2.00000} \\ -11 \\ \hline 90 \\ -88 \\ \hline 20 \\ -11 \\ \hline 90 \\ -88 \\ \hline 20 \end{array}$$

Use any strategy to write each fraction as a decimal. Decide whether it is terminating or repeating.

$$\frac{3}{8}$$

$$\frac{3}{11}$$

$$\frac{98}{6}$$

Terminating or repeating?

Terminating or repeating?

Terminating or repeating?

Summary Question

What are some clues you can use to predict if a fraction will be a terminating or a repeating decimal?

Learning Goal(s):

Some decimals terminate, while others repeat. However, **all** terminating and repeating decimals can be written as fractions. Look at the example below to see what we mean.

Describe each step of Adhira's process for converting $4.\overline{85}$ to $\frac{481}{99}$. $x = 4.\overline{85}$

- | | |
|----|-------------------------------|
| 1. | 1. $100x = 485.\overline{85}$ |
| 2. | 2. $-x = -4.\overline{85}$ |
| 3. | 3. $99x = 481$ |
| 4. | 4. $x = \frac{481}{99}$ |

Use any strategy to write each decimal as a fraction.

$$5.\overline{37}$$

$$5.\overline{3}$$

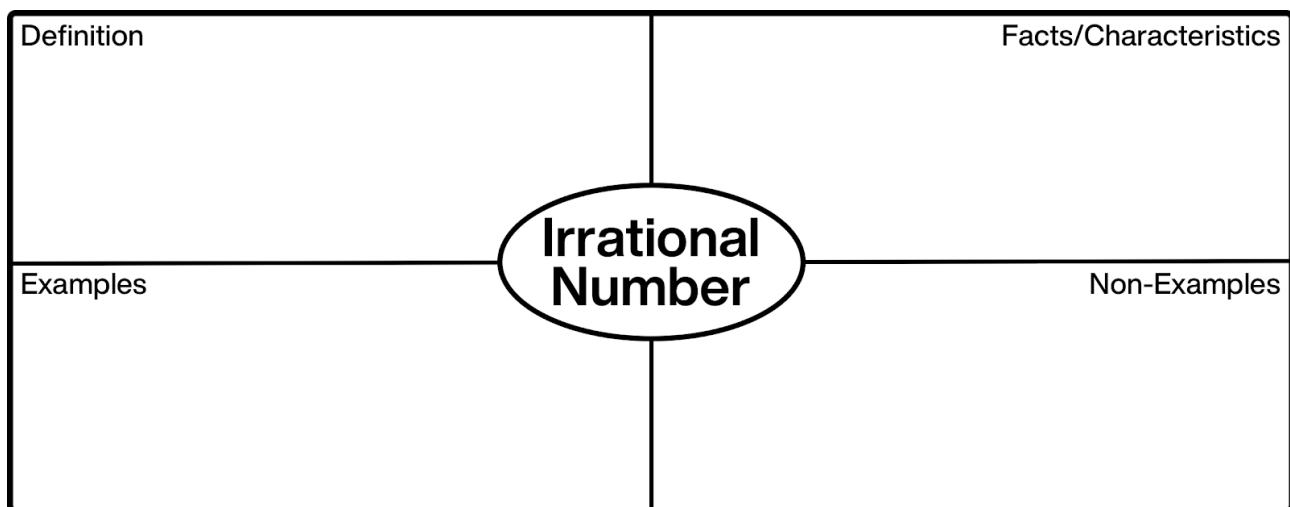
$$0.\overline{37}$$

Summary Question

What question(s) do you have about converting repeating decimals into fractions? (You can also record a question you imagine someone else having about this topic.)

Learning Goal(s):

Rational numbers are numbers that can be written as a fraction of two integers. What if a number cannot be written as a fraction of two integers? We call this type of number an irrational number.



Write each number as a rational number. If it is impossible, write “irrational.”

$$0.16$$

$$\frac{\sqrt{16}}{\sqrt{100}}$$

$$\sqrt{8}$$

$$x \text{ when } x^3 = 64$$

$$\sqrt[3]{16}$$

Summary Question

What does it mean when someone says that $\sqrt{3}$ is irrational?