Lesson Summary

When you're trying to visualize how a pattern will grow, it helps to have a toolbox of different strategies.

Here are the first three figures in a pattern and a table showing the number of tiles at each stage. You could determine how many tiles there will be in Figure 7 by:

- Drawing the next four figures and adding a row of 3 each time.
- Continuing the table, increasing the number of tiles by 3 in each new row.
- Noticing that the number of tiles is 3 times the figure number, plus 2, and then calculating the tiles for Figure 7.

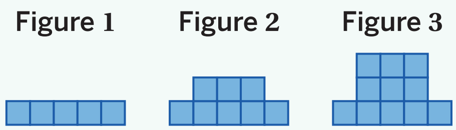
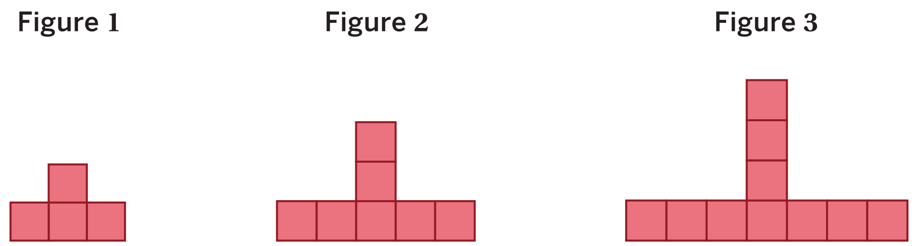


Figure	Number of Tiles
1	5
2	8
3	11

Things to Remember:

Name: ______ Date: ______ Period: ______

Problems 1–3: Here is a visual pattern.



1. Draw Figure 4.

2. Complete the table with the number of tiles in each figure.

Figure	1	2	3	4	...	10
Number of Tiles					...	

3. Which expression represents the number of tiles in Figure 11?

A. $1 + 3(11)$

B. $1 + 4(11)$

C. $4 + 3(11)$

D. $4 + 1(11)$

For Problems 4–5: Create your own pattern.

4. Draw Figures 1 and 3. Then complete the table.

Figure 1 **Figure 2** **Figure 3**

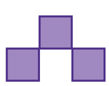


Figure	Number of Tiles
1	
2	3
3	

5. How many tiles will Figure 6 of your pattern have?

Lesson Practice
A1.1.01

Name: ______ Date: ______ Period: ______

Spiral Review

6. Create two different patterns that begin with the numbers 5 and then 10.

Pattern A: $5, 10,$ ____, ____, ____

Pattern B: $5, 10,$ ____, ____, ____

Describe how each pattern is changing.

Pattern A:

Pattern B:

7. Create two different patterns that each include the numbers 2 and 12.

Pattern C:

Pattern D:

8. Determine the value of each expression.

Expression	Value
$4 + 4 + 4 + 4$	
$4 \bullet 4 \bullet 4$	
$4(3)$	
3^4	

9. Select *all* the expressions that are equal to 16.

☐ **A.** 8^2 ☐ **B.** 2^4 ☐ **C.** 2^8 ☐ **D.** 4^2 ☐ **E.** 16^1

Reflection

1. Put a smiley face next to a problem you were stuck on and then figured out.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

A **sequence** is a list of numbers in a particular order.

Sequences can change in predictable ways. Two examples of predictable change are constant differences and constant ratios.

A sequence has a **constant difference** when the difference between any two consecutive terms is the same.

$$5 \xrightarrow{+6} 11 \xrightarrow{+6} 17 \xrightarrow{+6} 23$$

$5,\ 11,\ 17,\ 23$

A sequence has a **constant ratio** when the ratio between any two consecutive terms is the same.

$$2 \xrightarrow{\times 4} 8 \xrightarrow{\times 4} 32 \xrightarrow{\times 4} 128$$

$2,\ 8,\ 32,\ 128$

Sequences can also change in predictable ways that are neither a constant difference or ratio.

$$13 \xrightarrow{-1} 12 \xrightarrow{-2} 10 \xrightarrow{-3} 7$$

$13,\ 12,\ 10,\ 7$

Once you know how a sequence changes, you can use that change to determine unknown terms.

Here is an example:

- The known terms in this sequence have a constant ratio of 0.5.
- The value of the unknown term should be 0.5 times the term before, 40, or 2 times the term after, 10.
- That means the unknown term is 20.

$$80 \xrightarrow{\times 0.5} 40,\ ?,\ 10 \xrightarrow{\times 0.5} 5$$

Things to Remember:

Lesson Practice
A1.1.02

Name: .... Date: .... Period: ....

1. Determine the *constant ratio* for each sequence.

Sequence	Constant Ratio
$256, 128, 64$	
$18, 54, 162$	
$0.8, 0.08, 0.008$	

2. Determine the *constant difference* for each sequence.

Sequence	Constant Difference
$12, 17, 22$	
$102, 85, 68$	
$\frac{1}{4}, 1, \frac{7}{4}$	

3. Here's the start of a sequence: $1, -1, \ldots$

a Write two different rules the sequence could follow.

Rule 1: .... **Rule 2:** ....

b Determine the next three terms for each rule.

Rule 1: $1, -1,$ ......, ..., ... **Rule 2:** $1, -1,$ ......, ..., ...

Problems 4–7: Fill in the blanks to complete each sequence. Each sequence has a constant difference.

4. $-3, -2,$ ......, ..., 1

5. ......, $13, 25,$ ......, ...

6. $1, 0.25,$ ......, $-1.25,$ ......

7. $92,$ ......, ..., ..., 80

Problems 8–9: Here are Sequence A and Sequence B. One sequence has a constant difference and one has a constant ratio.

8. Complete the table.

Sequence A	$70, 90, 110, \ldots$	Constant ratio or Constant difference	Fourth term:
Sequence B	$10, 20, 40, \ldots$	Constant ratio or Constant difference	Fourth term:

9. Which sequence will have the greater 10th term? Show or explain your thinking.

Name: ______ Date: ______ Period: ______

10. A sequence has a first term of 6 and a constant ratio of -2. What are the first four terms of the sequence?

A. 6, 4, 2, 0 **B.** -2, 4, 10, 16 **C.** 6, 8, 10, 12 **D.** 6, -12, 24, -48

Spiral Review

11. Square A has an area of 64 square feet. Select *all* the expressions that are equal to the side length of this square in feet.

- ☐ **A.** $\sqrt{8}$
- ☐ **B.** $\sqrt{64}$
- ☐ **C.** 4
- ☐ **D.** 8
- ☐ **E.** $\frac{64}{2}$

12. The points (7, 21) and (-5, 17) lie on a line. What is the slope of the line?

13. Determine the value of each expression when $n = 4$.

Expression	$n^2 - 5$	$n(n + 6)$	$3n^2$
Value When $n = 4$			

Reflection

1. Put a heart next to the problem you found most interesting.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

There are several ways to define, or describe, a sequence. When you define a sequence recursively, you're determining each term using the previous term.

A **recursive definition** of a sequence includes a first term and a rule for finding every term that follows.

Here are some examples of recursive definitions for this sequence: $32, 16, 8, 4, 2, 1, 0.5$.

First term: 32

Rule: Half of the previous term

First term: 32

Rule: Constant ratio of $\frac{1}{2}$

First term: 32

Rule: Multiply the previous term by 0.5

Things to Remember:

Lesson Practice
A1.1.03

Name: .... Date: .... Period: ....

Problems 1–2: Use the sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

1. Write a recursive definition for this sequence.

First term:

Rule:

2. Write the next three terms of the sequence.

3. Complete the recursive definition for each sequence.

	Sequence A	Sequence B	Sequence C	Sequence D
Sequence	$2, 4, 6, 8, \ldots$	$5, 7, 9, 11, \ldots$	$50, 25, 0, -25, \ldots$	$\frac{1}{3}, 1, 3, 9, \ldots$
First Term				
Constant Difference or Constant Ratio?				
Rule				

4. This sequence has a constant difference of 5. Fill in the missing terms.

......, ..., 7, ..., ...

Problems 5–6: Here is the start of a sequence: $1, 5, \ldots$

5. Write a rule and the next three terms the sequence could follow.

Rule:

Terms:

6. Write a *different* rule and the next three terms the sequence could follow.

Rule:

Terms:

7. Remy is studying this sequence: 20, ?, 80. Remy thinks the missing term could be less than the first and third terms. Is Remy correct? Explain your thinking.

Name: ______ Date: ______ Period: ______

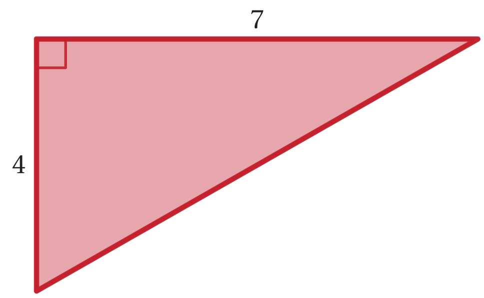
8. A swimming pool is filling with water at a rate of 17 gallons per minute. The pool started with 100 gallons of water. Write a recursive definition to model the situation.

First term: Rule:

Spiral Review

9. Explain how you know that $\sqrt{42}$ is between 6 and 7.

10. Determine the exact length of the unlabeled side.



11. Write each expression as a single power of 10.

Expression	$\frac{10^7}{10^3}$	$10^2 \cdot 10^5$	$10^0 \cdot 10^9$	$\frac{10^5}{10^0}$
Single Power of 10				

Reflection

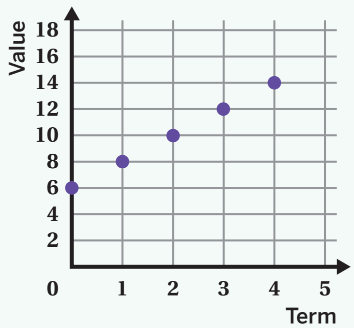
1. Put a star next to a problem you could explain to a classmate.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

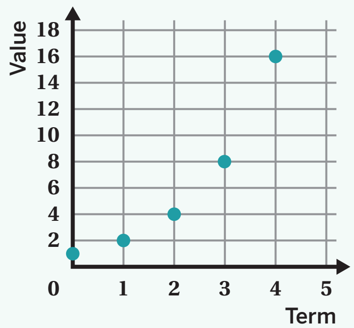
Sequences that change by a constant difference are called **arithmetic sequences**, while sequences that change by a constant ratio are called **geometric sequences**.

Sequences can be represented in multiple ways: as a list of numbers, in a table, on a graph, or with a recursive definition. In each representation, there are ways to identify if the sequence is arithmetic, geometric, or neither. Here are some examples of graphs of different kinds of sequences.

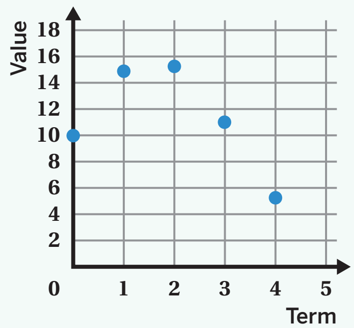
Arithmetic



Geometric



Neither



Things to Remember:

Lesson Practice
A1.1.04

Name: .... Date: .... Period: ....

1. Determine whether each sequence is arithmetic, geometric, or neither.

Sequence	1000, 200, 40, 8	2, 4, 16, 256	10, 20, 30, 40	500, 100, 20, 4
Arithmetic, Geometric, or Neither				

2. Complete each arithmetic sequence with its missing terms.

-2, 4, ________, 16, ________

11, 111, ________, ________, 411

________, 7.5, 10, ________, ________

5, ________, -13, -22, ________

3. Complete each geometric sequence with its missing terms.

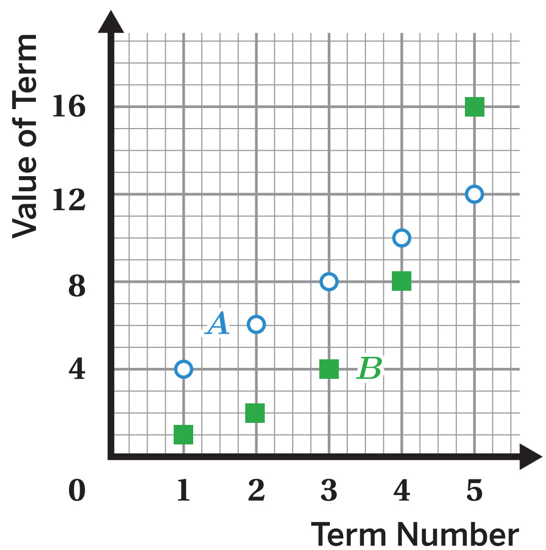
________, 5, 25, ________, 625

-1, ________, -36, 216, ________

10, 5, ________, ________, 0.625

________, ________, -36, 108, ________

Problems 4–6: Here are the graphs of two sequences.



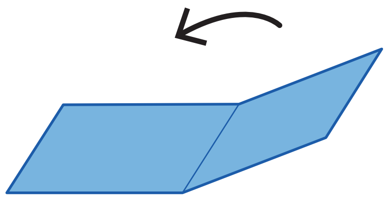
4. For Sequence A, describe a way to produce a new term from the previous term.

5. For Sequence B, describe a way to produce a new term from the previous term.

6. Which of these is a geometric sequence? Explain your thinking.

Name: .... Date: .... Period: ....

7. Ahmed is trying to see how many times he can fold a piece of paper in half. The thickness of the piece of paper he starts with is 0.001 centimeters. How thick will the paper be after 10 folds?



Spiral Review

8. Select *all* the sets of side lengths that form a right triangle.

- ☐ **A.** $2, 3, 5$
- ☐ **B.** $\sqrt{7}, 9, \sqrt{88}$
- ☐ **C.** $\sqrt{12}, 6, \sqrt{48}$
- ☐ **D.** $4, 5, \sqrt{41}$
- ☐ **E.** $4, 5, 9$

9. If a line has a negative slope and contains the point $(4, 6)$, which of these points could it also contain?

A. $(7, 6)$ **B.** $(3, 1)$ **C.** $(6, 5)$ **D.** $(5, 8)$

10. Evaluate each expression.

Expression	$\left(\frac{1}{3}\right)^2$	$\left(\frac{1}{3}\right)^{-2}$
Value		

Reflection

1. Circle a problem you're still curious about.
2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 5

Lesson Summary

An **explicit definition** is a formula that determines the value of any term in a sequence using the term number. An explicit definition of a sequence can be written as an *expression* or in words.

Here are two situations and the explicit expressions that define them.

Situation: Julian's family is 250 miles away from home. They drive 50 miles toward their home for every n hours.

Expression: $250 - 50n$
The expression represents the number of miles Julian's family is from home after n hours.

- 250 represents the starting distance.
- -50 represents the distance traveled each hour.
- -$50n$ represents the total distance traveled.
- n represents the number of hours driven.

Situation: A scientist tracks a group of 250 bugs. Each month, the number of bugs increases by 1.5 times, for n months.

Expression: $250 \cdot 1.5^n$
The expression represents the number of bugs after n months.

- 250 represents the initial number of bugs.
- 1.5 represents the ratio that the bugs grow by each month.
- n represents the number of months.

Things to Remember:

Lesson Practice
A1.1.05

Name: .... Date: .... Period: ....

1. Jamar had 80 followers on social media. His number of followers *tripled* every month for 4 months. Select *all* the expressions that represent Jamar's followers after 4 months.

- ☐ **A.** $80 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
- ☐ **B.** $80 + 4^3$
- ☐ **C.** $80 \cdot 4 \cdot 4 \cdot 4$
- ☐ **D.** $80 + 3 + 3 + 3 + 3$
- ☐ **E.** $80 \cdot 3^4$

2. The weather forecast predicts a snowfall rate of 0.75 inches of snow per hour overnight. There are already 9 inches of snow on the ground in Anushka's neighborhood.

Complete the table to show the amount of snow on the ground, in inches, after n hours.

Hours, n	0	1	2	3	...	8
Amount of Snow (in.)					...	

3. The population of a city was $100{,}000$ in 1970 and has doubled 3 times since then. Select *all* the expressions that represent the population today.

- ☐ **A.** 300000
- ☐ **B.** 800000
- ☐ **C.** $100000 \cdot 2 \cdot 2 \cdot 2$
- ☐ **D.** $100000 \cdot 3^2$
- ☐ **E.** $100000 \cdot 2^3$

Problems 4–5: Here is a table representing a pattern.

x	y
0	60
1	30
2	15
3	7.5
4	3.75

4. Circle the equation that represents the table.

A. $y = 60 + \frac{1}{2}x$

B. $y = 60 \cdot \left(\frac{1}{2}\right)^x$

C. $y = 60 \cdot 2^x$

D. $y = 60 - \frac{1}{2}x$

5. Explain your thinking.

Lesson Practice
A1.1.05

Name: ______ Date: ______ Period: ______

Problems 6–7: Maia earns money by shoveling snow in her neighborhood. She starts the winter with $\$52$ in her bank account and will deposit $\$10$ for every driveway she shovels.

6. Complete the table with Maia's account balance, in dollars, after she shovels n driveways.

Number of Driveways, n	0	1	2	3
Account Balance ($)	52			

7. Write an explicit expression for this situation.

Problems 8–9: A group of students tracked the number of likes on a social media post. They represented this number using the equation $n = 40 \bullet 1.5^t$, where n is the total number of likes and t is the number of days since the students started counting.

8. Explain what the 40 and 1.5 mean in this situation.

9. How many likes do the students predict there will be 2 days after they started counting?

Spiral Review

Problems 10–11: Write each expression using an exponent.

10. $\left(\frac{4}{5}\right) \bullet \left(\frac{4}{5}\right) \bullet \left(\frac{4}{5}\right)$

11. $9.3 \bullet 9.3 \bullet 9.3 \bullet 9.3 \bullet 9.3$

12. An arithmetic sequence starts with $10, 5, \ldots$
Explain how you would calculate the value of the 100th term.

Reflection

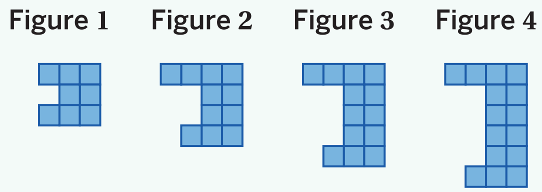
1. Put a star next to a problem you spent the most time on.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can write an explicit expression for a pattern or sequence in multiple equivalent ways, just by referencing different term numbers.

Here's an example of a pattern and its matching sequence.



$7 + 2(1)$ $7 + 2(2)$ $7 + 2(3)$ $7 + 2(4)$

Sequence

9, 11, 13, 15

Term 1

$9 + 2(n - 1)$

One expression that represents this sequence is $9 + 2(n - 1)$; this expression uses Term 1 as the starting value and $n - 1$ to calculate the change.

The expression $7 + 2n$ also represents this sequence, using Term 0 as the starting value, and n to calculate the change.

Things to Remember:

Lesson Practice

A1.1.06

Name: .... Date: .... Period: ....

1. Match each sequence to its explicit expression.

Sequence		Explicit Expression
a. $4, 10, 16, 22$	....	$320 \cdot \left(\frac{1}{2}\right)^n$
b. $4, 12, 36, 108$	....	$4 + 6(n - 1)$
c. $160, 80, 40, 20$	....	$320 + \frac{1}{2}(n - 1)$
d. $320, 320.5, 321, 321.5$	....	$4 \cdot 3^{(n - 1)}$

2. Select *all* the expressions that could represent the number of tiles in Figure n of this pattern.

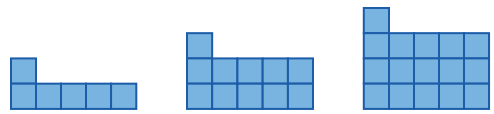


Figure 1 Figure 2 Figure 3

- ☐ **A.** $5 + n$
- ☐ **B.** $5n + 1$
- ☐ **C.** $6n - 1$
- ☐ **D.** $6 + 5(n - 1)$
- ☐ **E.** $10n - 4$

Problems 3–4: Here is a visual pattern.

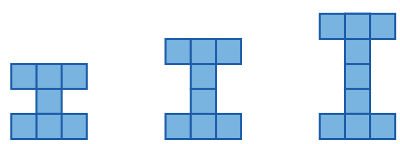


Figure 1 Figure 2 Figure 3

3. Complete the table with the number of tiles in each figure.

4. Write an explicit expression for the number of tiles in Figure n.

Figure	# of Tiles
1	
2	
3	
4	
...	...
15	

5. The first four terms of a geometric sequence are: $6, 18, 54, 162$. Which explicit expression can be used to describe this sequence?

A. $2 + 3n$ **B.** 3^n **C.** $2 \cdot 3^n$ **D.** $6 \cdot 3^n$

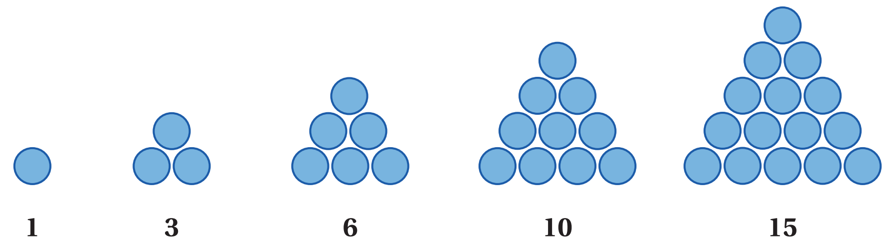
Name: .... Date: .... Period: ....

6. The first four terms in a sequence are $15, 22, 29, 36$. Write two explicit expressions for Term n of this sequence.

Expression 1: ....

Expression 2: ....

7. A *triangular number* can be represented as the number of dots in an equilateral triangle.



What is the 8th triangular number?

Spiral Review

Problems 8–10: Determine the value of each expression when $x = 3$.

8. $7x - 8$

9. $5(x + 9)$

10. $\frac{x-6}{4x}$

Problems 11–13: Decide whether each sequence is arithmetic, geometric, or neither. Circle one.

11. $25, 5, 1, \ldots$ Arithmetic Geometric Neither

12. $25, 19, 13, \ldots$ Arithmetic Geometric Neither

13. $25, 52, 25, 52, \ldots$ Arithmetic Geometric Neither

Reflection

1. Put a heart next to the problem you feel most confident about.

2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 7

Lesson Summary

You can solve one-variable equations by creating *equivalent equations*. To create equivalent equations, use moves that keep the equation balanced, such as combining like terms or using inverse operations to move a variable from one side of the equation to the other.

Here is an example of a set of moves that keep an equation balanced:

$-3m + 5 + m = 2(6m + 3)$	This is the original equation.
$-2m + 5 = 12m + 6$	Combine like terms on the left and distribute on the right.
$5 = 14m + 6$	Add $2m$ to each side of the equation.
$-1 = 14m$	Subtract 6 from each side of the equation.
$\frac{-1}{14} = m$	Divide each side of the equation by 14.

All of the equations created at each step of this solution process are *equivalent equations* because they have the same solution: $m = -\frac{1}{14}$.

Things to Remember:

Lesson Practice
A1.2.02

Name: ________ Date: ________ Period: ________

1. Which equation is equivalent to $6x + 9 = 12$?

A. $x + 9 = 6$

B. $2x + 3 = 4$

C. $3x + 9 = 6$

D. $6x + 12 = 9$

2. Write another equation that is equivalent to $6x + 9 = 12$.

3. Select *all* the equations that are equivalent to $\frac{-8x - 6}{2} = 15$.

☐ A. $4x + 3 = 15$

☐ B. $\frac{1}{2}(-4x - 3) = 15$

☐ C. $-4x - 3 = 15$

☐ D. $-8x - 6 = 30$

☐ E. $8x + 6 = 30$

Problems 4–6: Solve each equation.

4. $26 - 2x = 3(x + 2)$

5. $\frac{4x - 6}{2} = x - 8$

6. $\frac{1}{4}x - 5 = x - 14$

Problems 7–9: Paz made a mistake when solving $-3(x + 7) = 24$ for x.

Polina

Step 1: $-3(x + 7) = 24$

Step 2: $x + 7 = 27$

Step 3: $x = 20$

7. What is one thing that Paz did well?

8. What is one thing that Paz did incorrectly?

Lesson Practice
A1.2.02

Name: ________ Date: ________ Period: ________

9. Which equation is equivalent to $0.05n + 0.1d = 3.65$?

A. $5n + d = 365$

B. $0.5n + d = 365$

C. $5n + 10d = 365$

D. $0.05d + 0.1n = 365$

10. Select *all* the moves that could be the first step to solving the equation $4(x + 3) = 8x - 4 + 12x$.

- ☐ **A.** Divide each side by 4.
- ☐ **B.** Take away $12x$ on the right side.
- ☐ **C.** Distribute 4 on the left side.
- ☐ **D.** Combine like terms on the right side.
- ☐ **E.** Add 4 to the right side.

11. Determine a value for x that makes this equation true: $9x - 4(x - 3) = 27 + 2x$. Show or explain your thinking.

Spiral Review

12. Rio scored 409 points in a video game. This was 223 more points than Sadia scored, s. Which equation does *not* represent this situation?

A. $223 = 409 - s$ **B.** $s = 409 - 223$ **C.** $s = 409 + 223$ **D.** $223 + s = 409$

Reflection

1. Put a question mark next to a response you'd like to compare with a classmate's.
2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 8

Lesson Summary

Not all one-variable linear equations have a single solution. Some linear equations have *infinitely many solutions*, and some have *no solution*.

In the process of solving, there is a difference between equations with one solution, no solution, or infinitely many solutions:

- In an equation with one solution, *a single value* of x will make the equation true.
- In an equation with no solution, *no value* of x will make the equation true.
- In an equation with infinitely many solutions, *any value* of x will make the equation true.

Here are examples of equations with one solution, no solution, and infinitely many solutions.

One Solution	No Solution	Infinitely Many Solutions
$3x + 4 = 2x + 10$	$2x + 4 = 2x + 10$	$2(x + 5) = 2x + 10$
$3x = 2x + 6$	$4 = 10$	$2x + 10 = 2x + 10$
$x = 6$	This is *never* true!	$10 = 10$
		This is *always* true!

If the variable in an equation is eliminated during the solving process, the equation has either no solution or infinitely many solutions. If the statement remaining is false, the equation has no solution. If the statement remaining is true, the equation has infinitely many solutions.

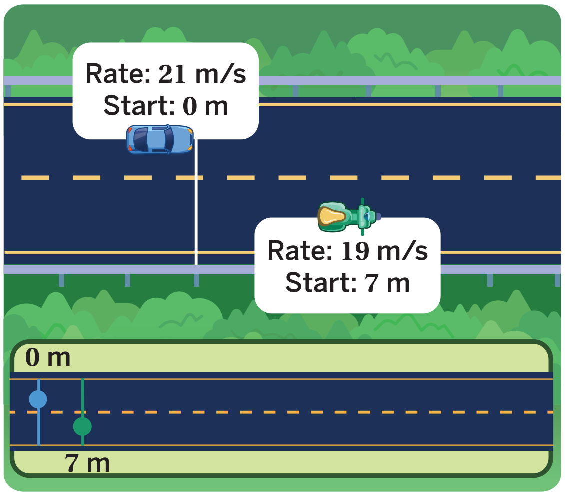
Things to Remember:

Lesson Practice
A1.2.03

Name: .... Date: .... Period: ....

1. The car and scooter are moving at constant speeds. The equation $21t = 19t + 7$ represents the time, t, when they will be in the same position.

When will the car and scooter be in the same position?



Problems 2–3: The equation $10t = 2.5t$ represents the time, t, when two vehicles will be in the same position.

2. When will these two vehicles be in the same position? Circle one.

Once Never Always

3. Explain how you know.

4. Here is Kiandra's work to solve $16x = 10x$. She says there is no solution.

Is this correct? Show or explain your thinking.

Kiandra

$$\frac{16x}{x} = \frac{10x}{x}$$

$$16 = 10$$

There is no solution.

5. Group the equations based on their number of solutions.

$5t = 3t$	$2t = 10 - 2t$	$15 - 3(t + 5) = -3t$
$4t + 7 = 4(t + 2)$	$6t + 2 = -3 + 6t$	

One Solution	No Solution	Infinitely Many Solutions

Name: ________ Date: ________ Period: ________

Problems 6–7: Create two different equations that each have a solution of $x = 1$.

6. Fill in each blank using the whole numbers 0 to 9 only once each.

$\square x + \square = \square x + \square$

$\square x + \square = \square x + \square$

7. Explain what you notice about your equations.

Spiral Review

8. Select *all* the equations where $x = 2$ is a solution.

- ☐ **A.** $\frac{x}{4} = 8$
- ☐ **B.** $19 = 2(x + 6) + 3$
- ☐ **C.** $2x + 10 = 2x + 8$
- ☐ **D.** $5 - 3x = \text{-}1$
- ☐ **E.** $4 - x = x$

9. Select *all* the expressions that are equivalent to $2(x + 3)$.

- ☐ **A.** $(x + 3) \cdot 2$
- ☐ **B.** $2x + 5$
- ☐ **C.** $2x + 3 \cdot 2$
- ☐ **D.** $2x + 3$
- ☐ **E.** $2x + 6$

10. Select the expression that is equivalent to $6 - 2(x + 1)$.

A. $4(x + 1)$ **B.** $7 - 2x$ **C.** $4 - 2x$ **D.** $4x + 4$

Reflection

1. Put a heart next to the problem you're most proud of.

2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 9

Lesson Summary

Two-variable linear equations can be written in different forms. Sometimes the different forms of an equation can reveal information that is useful for solving problems. Depending on what information you are looking for, you might choose to use one form or the other.

Here is an example of two equivalent equations that represent the number of seats and handholds in possible in a subway car that has 300 square feet of floor space. They are each written in different forms, and they each reveal different information about the situation. In each equation, t is the seating capacity and d is the standing capacity.

$$4t + 2d = 300$$

- Each seat takes up 4 square feet of floor space.
- Each standing spot requires 2 square feet of floor space.
- The total amount of floor space is 300 square feet.

$$d = 150 - 2t$$

- When there are no seats ($t = 0$), 150 passengers can stand in the car.
- For every seat that is added, the number of spots for standing decreases by 2.

Things to Remember:

Lesson Practice
A1.2.04

Name: .... Date: .... Period: ....

Problems 1–4: Adriana spent $\$24$ on fruit punch and apple juice. Fruit punch costs $\$3$ per bottle. Apple juice costs $\$2$ per bottle.

1. How many bottles of *fruit punch* could Adriana buy if she didn't get any apple juice?

2. How many bottles of *apple juice* could Adriana buy if she did not get any fruit punch?

Adriana wrote the equation $3f + 2a = 24$ to represent the situation.

3. Use the equation to help you complete the table.

f	a
2	
6	

4. Which equation represents the same relationship?

A. $a = 8 - \frac{2}{3}f$

B. $a = 8 - \frac{3}{2}f$

C. $a = 12 - \frac{2}{3}f$

D. $a = 12 - \frac{3}{2}f$

Problems 5–6: Here is an equation: $2x + 4y = 80$.

5. Use the equation to help you complete the table.

x	y
6	
12	

6. Which equation represents the same relationship?

A. $y = 20 - 2x$

B. $y = 40 - 2x$

C. $y = 20 - \frac{1}{2}x$

D. $y = 40 - \frac{1}{2}x$

Problems 7–8: Nia is buying bananas and apples at the farmer's market. Bananas cost $\$0.50$ each. Apples cost $\$1.00$ each.

7. Select *all* the combinations of bananas and apples that Nia could buy for exactly $\$3.50$.

☐ **A.** 1 banana and 3 apples

☐ **B.** 5 bananas and 1 apple

☐ **C.** 1 banana and 2 apples

☐ **D.** 3 bananas and 2 apples

☐ **E.** 5 bananas and 2 apples

8. The equation $0.5b + 1a = 3.50$ represents the number of bananas and apples that Nia can buy for $\$3.50$. Solve this equation for a.

Lesson Practice
A1.2.04

Name: ________________ Date: ________ Period: ________

Spiral Review

9. Select *all* the expressions that are equivalent to $8 - 12 - (6 + 4)$.

- ☐ **A.** $(6 + 4) - 8 - 12$
- ☐ **B.** $8 - 6 - 12 + 4$
- ☐ **C.** $8 - 12 - 6 - 4$
- ☐ **D.** $8 - (6 + 4) - 12$
- ☐ **E.** $(8 - 12) - 6 + 4$

10. Which equation is equivalent to $\frac{1}{3}m + \frac{1}{2}n = 9$?

A. $3m + 2n = 9$

B. $2m + 3n = 54$

C. $m + 3n = 27$

D. $2m + n = 18$

11. Explain how you know that Equation A and Equation B are equivalent.

Equation A	Equation B
$48 - 5x = 13$	$5x = 35$

12. Create two equivalent equations by filling in the blanks using the whole numbers 0 to 9 only once.

$$\square\, x + \square\, y = \square$$

$$y = \square - \square\, x$$

Reflection

1. Circle the problem you feel most confident about.

2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 10

Lesson Summary

Equations, tables, and graphs are all different ways to model a situation. The graph of a linear equation represents all the pairs of values that are solutions to the equation (make the equation true).

Linear equations can be written in different but equivalent forms. Rearranging equations helps reveal new information, such as the x-*intercept* and y-*intercept*, which we can see in a graph, table, or description of a situation.

Let's say a lemonade stand sold lemonade for \$3 per cup and cookies for \$2 each. The stand made \$12. ℓ represents the number of cups of lemonade sold and c represents the number of cookies sold. This situation can be represented in many different ways:

Equation

$$3\ell + 2c = 12$$

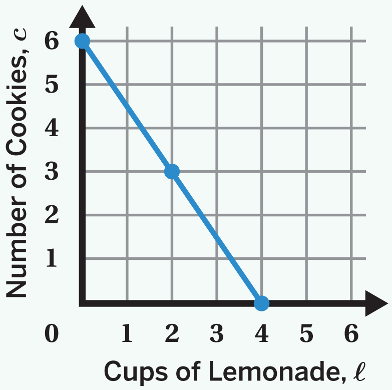
Equation Solved for c

$$c = 6 - \frac{3}{2}\ell$$

Table

ℓ	0	2	4
c	6	3	0

Graph



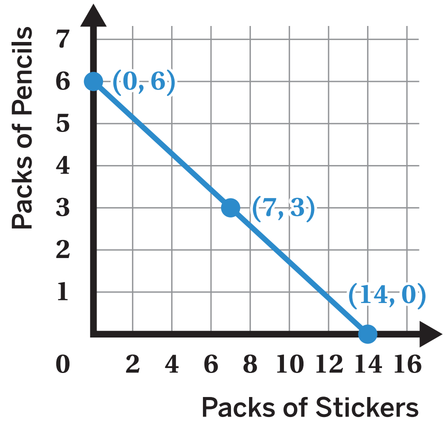
Things to Remember:

Lesson Practice

A1.2.06

Name: .... Date: .... Period: ....

Problems 1–2: Here is a graph about school supplies.



1. A teacher spent $\$21$ on packs of stickers and packs of pencils for her class.

- Stickers cost $\$1.50$ per pack.
- Pencils cost $\$3.50$ per pack.

Show or explain how you know that this graph represents this situation.

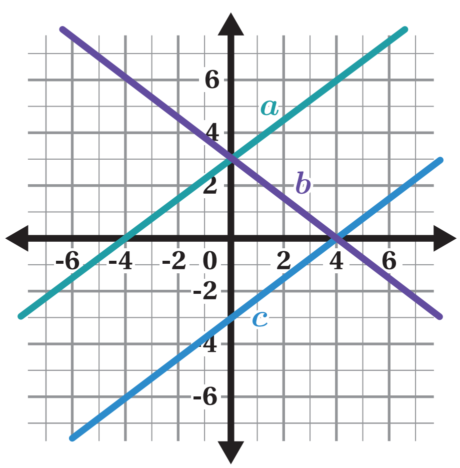
2. Circle a coordinate pair and explain what it means in this situation.

$(0, 6)$ $(7, 3)$ $(14, 0)$

3. Circle the line that represents $12 = 3x + 4y$.

Line a Line b Line c

Show or explain how you know.



4. Which equation is equivalent to $15x + 3y = 2$?

A. $y = \frac{2}{3} + 5x$ **B.** $y = \frac{2}{3} - 5x$ **C.** $y = 2 - 15x$ **D.** $y = 2 - 5x$

5. Match each equation with its equivalent equation.

a. $4x + 6y = 20$ **b.** $3x - 6y = 16$ **c.** $2x - 3y = 10$ **d.** $-3x + 6y = 16$

.... $y = \frac{8}{3} + \frac{1}{2}x$ .... $y = -\frac{10}{3} + \frac{2}{3}x$ .... $y = \frac{10}{3} - \frac{2}{3}x$ .... $y = -\frac{8}{3} + \frac{1}{2}x$

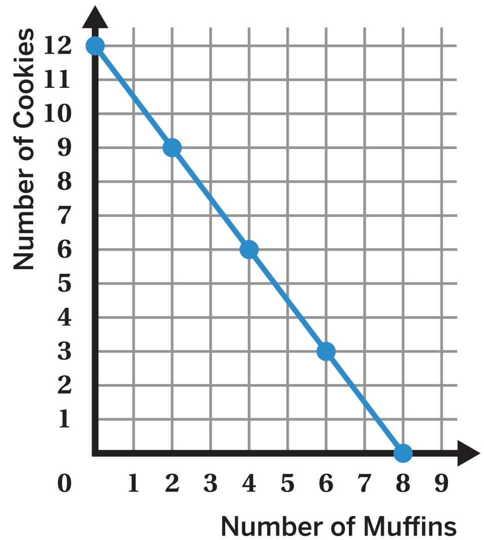
Name: .... Date: .... Period: ....

6. Jordan had a bake sale. Muffins were \$3 each and cookies were \$2 each. Jordan earned \$24.

Here is a graph of Jordan's situation.

Select *all* the combinations of muffins and cookies Jordan could have sold.

- ☐ **A.** 0 muffins and 8 cookies
- ☐ **B.** 9 muffins and 2 cookies
- ☐ **C.** 2 muffins and 9 cookies
- ☐ **D.** 6 muffins and 4 cookies
- ☐ **E.** 4 muffins and 6 cookies



Spiral Review

7. A tourist is trying to predict the price of their next taxi ride.

The tourist recorded their last three rides in the table.

How much can they expect to pay to travel 10 miles?

Distance (mi)	Price (\$)
2	10
4	17
7	27.50

8. Select *all* the equations where $x = -2$ is a solution.

- ☐ **A.** $4x = 4 + 2x$
- ☐ **B.** $2(x + 5) = x + 8$
- ☐ **C.** $3x - 5 = 1$
- ☐ **D.** $19 = 2(x - 6) + 3$
- ☐ **E.** $5 + 3x = -1$

9. Solve $-3x + 4y = 28$ for y.

10. Solve $6x - 3y = 36$ for y.

Reflection

1. Put a star next to a problem you could explain to a classmate.

2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 11

Lesson Summary

We can use *inequalities* to model situations with constraints. A **constraint** is a limitation on what values are possible in a model or situation.

Here is an example of a situation, the constraint, and the inequality that models them.

Situation	Constraint	Inequality
Tasia is ordering pizza for a party. Each plain pizza costs \$12 and there is a delivery fee of \$8.	Tasia can spend *up to* \$140.	$12p + 8 \leq 140$ Where p represents the number of pizzas.

When writing inequalities to model situations, you can use the symbols $>$, $\geq$, $<$, and $\leq$ to represent the nature of the constraint. Terms like *greater than, less than, at most, at least*, or *up to* can help you determine which inequality symbol to use.

Constraints modeled by inequalities produce a set of solutions. When using inequalities to model constraints, it is important to note that not all solutions to inequalities are viable solutions or make sense for the situation.

Things to Remember:

Lesson Practice
A1.2.08

Name: .... Date: .... Period: ....

1. For each constraint, write the letter of the matching inequality.

a. $x \leq 10$

b. $x > 10$

c. $x \geq 10$

.... x is less than or equal to 10.

.... x is at most 10.

.... x is greater than or equal to 10.

.... x is greater than 10.

.... x is at least 10.

2. Marquis wants to work at least 20 hours a week to earn enough money to go to a concert. Which inequality represents x, the number of hours Marquis wants to work?

A. $x > 20$ **B.** $x < 20$ **C.** $x \leq 20$ **D.** $x \geq 20$

3. Demetrius can spend as much as $\$50$ on shirts. Shirts, s, cost $\$16$ each at a nearby store. Which inequality represents this situation?

A. $16\text{s} \geq 50$ **B.** $16s \leq 50$ **C.** $50s \geq 16$ **D.** $50s \leq 16$

Explain your thinking.

4. List *three* values for x that would make $8 + 2x \leq 20$ true.

$x =$ .... $x =$ .... $x =$ ....

Problems 5–7: Write an inequality for each constraint. Use t for time (in hours).

5. Trevon practices the clarinet at least 1 hour each day.

6. At some colleges, students must work 20 hours or less per week.

7. The American Academy of Pediatrics recommends teenagers play video games for no more than 2 hours each day.[1]

[1] Source: *American Academy of Pediatrics*

Name: .... Date: .... Period: ....

Problems 8–9: Tell a story about each inequality. Specify the constraint and what the variable represents.

8. $x \geq 3$

9. $5 > y$

10. Fatima makes $\$9.25$ per hour, h, plus $\$150$ in commissions. She created an inequality that could be used to find the number of hours she needs to work to make at least $\$510$ on her next paycheck.

Her inequality is $9.25 + 150h \leq 510$. Find the mistakes in Fatima's inequality and correct it.

Spiral Review

Problems 11–13: Solve each equation.

11. $4x - 6 = 12 - 2x$

12. $\frac{1}{3}x - 8 = 12 - 3x$

13. $2x + 7 - 3x = \frac{5}{2}$

Reflection

1. Circle a problem you want to talk to a classmate about.
2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 12

Lesson Summary

The *solution set* of an inequality contains all the values that make the inequality true. You can represent a solution set on a number line by marking the *boundary point* and then shading the region of values that make the inequality true. To identify the boundary point, you can solve the equation that corresponds to the inequality. Then you can test one or more values to determine whether the *solution set* is greater than or less than the boundary point.

Here is an example of how you can determine and represent the solution set for $2x - 4 \geq 8$:

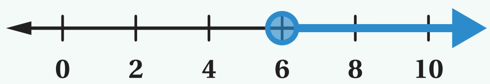
Determine the boundary point:

$$2x - 4 = 8$$
$$2x = 12$$
$$x = 6$$

Determine all the solutions:

$$2x - 4 \geq 8$$
$$2(0) - 4 \geq 8$$
$$4 \geq 8$$ ***False!***

- 6 is the boundary point and the $\geq$ symbol means it is included in the solution set.
- Since this statement is false when we substitute 0, we know 0 is not in the solution set.

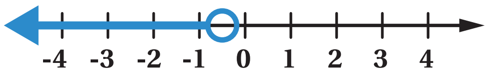


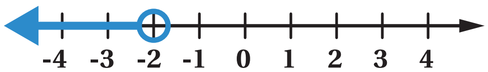
When a boundary point is not included in the solution set, this is represented with an open circle on the number line. Here is an example of the solution $-0.5 > x$ graphed on a number line.

Things to Remember:

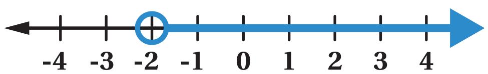
Name: ______________ Date: ______ Period: ______

1. Which graph represents the solutions to $2x < -4$?

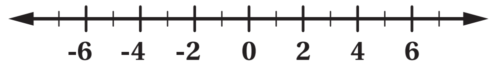
A. 

B. 

C. 

D. 

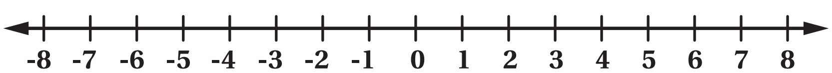
2. Create a graph of the solutions to $2x \leq 10$.



3. Leo is trying to solve $-15 + x < -14$. He knows the boundary point is at $x = 1$. How can he determine whether the solutions are $x < 1$ or $x > 1$?

4. Diego says that $x = 5$ is a solution to $-3x > 9$ because when you divide both sides by -3, you get $x > -3$. Is this correct? Explain your thinking.

5. Solve and graph the inequality $5x + 7 > 22$.



6. What are the solutions to $-2(2 + c) + 4 < 6$?

A. $c > -3$

B. $c < -3$

C. $c > -2$

D. $c < -2$

Lesson Practice
A1.2.09

Name: ________ Date: ________ Period: ________

7. Create an inequality and the graph of its solutions by filling in each blank using the numbers 0 to 9 only once.

$\square x + \square > \square x$

$x < \square$

Spiral Review

Problems 8–10: A community pool offers two different membership plans.

Moon wants to spend no more than $\$48$ at the community pool this month.

Plan A	Plan B
$\$4$ per visit	An initial $\$12$ fee, then $\$2$ per visit

8. What is the most number of visits Moon can make with Plan A?

9. What is the most number of visits Moon can make with Plan B?

10. After how many visits will the cost of both plans be the same?

Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 13

Lesson Summary

You can solve any one-variable inequality by solving its corresponding equation to determine the boundary point, then testing values to determine the rest of the solutions.

Here's an example of how you could solve the inequality $10 > -3x - 2$.

Step 1: Solve the corresponding equation: $10 = -3x - 2$.

$$10 = -3x - 2$$
$$12 = -3x$$
$$-4 = x$$

The solution to this equation is -4, so the boundary point is $x = -4$.

Step 2: Test values to determine the rest of the solutions.

In this example, we tested $x = -5$ and $x = 0$.

When $x = -5$, the inequality is false, so $x = -5$ is not a solution. This means that the solutions are greater than -4.

$x = -5$	$x = 0$
$10 > -3(-5) - 2$	$10 > -3(0) - 2$
$10 > 13$	$10 > -2$
False!	***True!***

The solutions to an inequality do not always have the same inequality symbol as the original inequality. Since the solutions are *greater than* -4, the solution set can be written as $-4 < x$ or $x > -4$.

Things to Remember:

Lesson Practice
A1.2.10

Name: .... Date: .... Period: ....

1. Write three values of n that make this inequality true: $-5(n+1) > 10$.

2. Here is an inequality: $7x + 6 < 3x + 2$. Select *all* values that are solutions.

- ☐ **A.** $x = 1$
- ☐ **B.** $x = 0$
- ☐ **C.** $x = -1$
- ☐ **D.** $x = -2$
- ☐ **E.** $x = -8$

Problems 3–6: Solve each inequality.

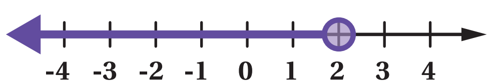
3. $4x + 5 \geq 37$

4. $-3x + 4 \geq 12$

5. $-6 + \frac{x}{2} < 7$

6. $-8x - 6 > 2x - 26$

7. Which inequality is represented by the graph?



A. $2x + 6 < 10$

B. $2x + 6 \geq 10$

C. $-2x + 6 \geq 10$

D. $-2x - 6 \geq -10$

Name: .... Date: .... Period: ....

8. Make the two inequalities equivalent by filling in each blank using the numbers 0 to 9 only once.

$\square\, x + \square < \square\, x + \square$

$x > \square$

Spiral Review

Problems 9–11: Rewrite each expression as a single power of 10.

9. $\dfrac{10^2 \cdot 10^5}{10^4}$

10. $(10^4)^2 \cdot (10^2)^2$

11. Imani is going shopping with a budget of $125. Which inequality represents the amount of dollars, x, that Imani can spend while shopping?

A. $x \leq 125$ **B.** $x \geq 125$ **C.** $x > 125$ **D.** $x < 125$

Reflection

1. Put a star next to the problem you think is the most important.
2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 14

Lesson Summary

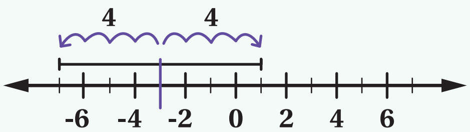
The *absolute value* of a number is its distance from 0 on a number line. Equations with absolute values often have two solutions because there are often two numbers that are the same distance from a number.

For example, the equation $|x + 3| = 4$ has two solutions, $x = 1$ and $x = -7$, because 1 and -7 are both 4 units away from -3.

You can determine the solutions using a number line or by solving two equations.

To determine the solutions to an inequality with an absolute value, you can:

- Step 1: Solve for the boundary points.
- Step 2: Determine whether the boundary points are included in the solution set. 1 and -7 are *not* included.
- Step 3: Test a value between the two boundary points to decide which values make the inequality true.
- Step 4: Graph all the solutions.



$$|x + 3| = 4$$

$x + 3 = 4$ $\quad\quad$ $-(x + 3) = 4$

$x = 1$ $\quad\quad$ $x = -7$

$$|x + 3| > 4$$

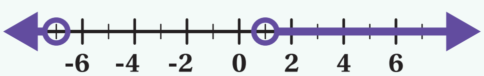
Boundary points:

$x = 1$ $\quad\quad$ $x = -7$

Test $x = 0$:

$$|(0) + 3| > 4$$

False!

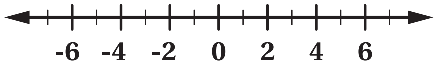


Things to Remember:

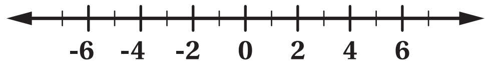
Name: ________ Date: ________ Period: ________

Problems 1–4: Graph all the solutions to each inequality or equation.

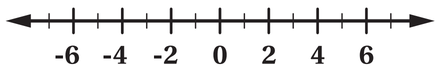
1. $|x| \leq 3$



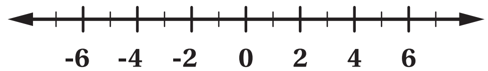
2. $|x + 2| > 4$



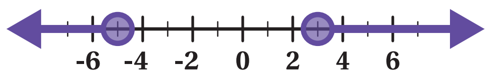
3. $|x - 5| = 1$



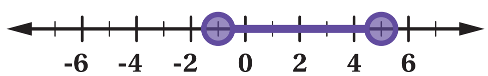
4. $2 > |x - 3|$



5. What value of p would make $|x - p| \geq 4$ match the graph?



6. Which inequality's solutions are shown by the graph?



A. $|x + 5| < -1$

B. $|x - 2| \leq 3$

C. $|x - 2| \geq 3$

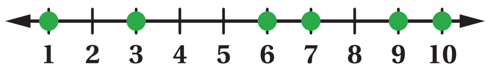
D. $|x + 3| \leq 2$

7. List three numbers that are not solutions to $|x + 1| \geq 2$.

Lesson Practice
A1.2.11

Name: .... Date: .... Period: ....

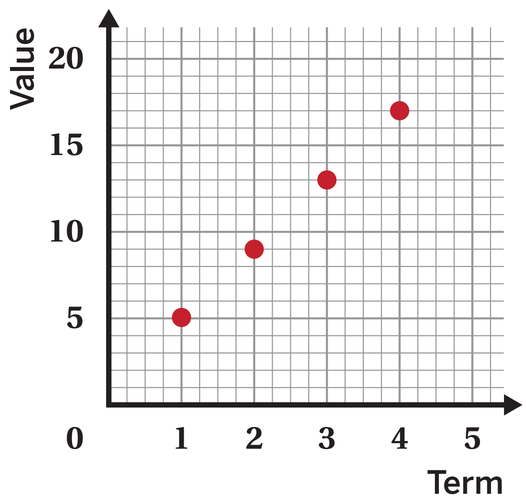
8. Omar is thinking of a number somewhere on this number line. Here are some guesses from Omar's friends.



No guess was more than 5 away. Only one guess was correct. What is Omar's number? Explain your thinking.

Spiral Review

Problems 9–11: Here is the graph of a sequence.



9. Is this sequence arithmetic, geometric, or something else? Explain your thinking.

10. What is the value of the 5th term?

11. What is the value of the nth term?

Reflection

1. Circle the problem you enjoyed doing the most.

2. Use this space to ask a question or share something you're proud of.

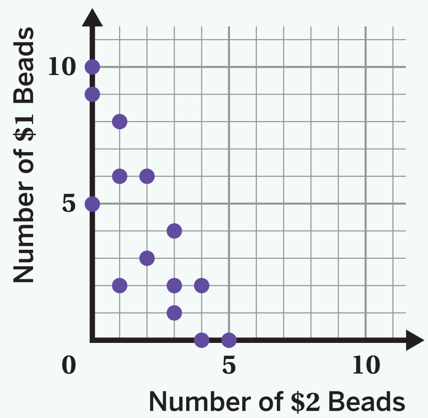
Science Mom Lesson 15

Lesson Summary

The solutions to a two-variable inequality are all of the ordered pairs that make the inequality true.

Here's a situation that can be modeled by an inequality: Marco is making bracelets. Each bracelet needs to cost no more than $10. Planet beads cost $1 and oval beads cost $2. Marco wants to know if he can make a bracelet with 3 planet beads and 4 oval beads.

Marco is wondering if $(4, 3)$ is a solution to the inequality $2x + y \le 10$, where x represents the number of $2 beads and y represents the number of $1 beads.



To check, Marco substitutes $x = 4$ and $y = 3$ into the inequality:

$$2(4) + (3) \le 10$$

$$8 + (3) \le 10$$

$$11 \le 10$$ **False!**

That means that $(4, 3)$ is *not* a solution and Marco *cannot* make a bracelet with 3 planet beads and 4 oval beads while staying within his budget.

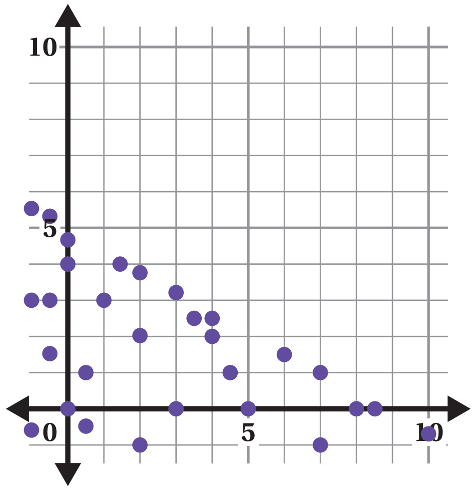
Things to Remember:

Lesson Practice
A1.2.13

Name: ________ Date: ________ Period: ________

1. This graph shows some solutions to $5x + 9y < 45$. Select *all* of the points that are also solutions.

- ☐ **A.** $(1, 1)$
- ☐ **B.** $(4, 0)$
- ☐ **C.** $(10, 4)$
- ☐ **D.** $(0, 10)$
- ☐ **E.** $(6, -1)$



2. Write at least *three* coordinate pairs that are solutions to the inequality $x \le y$.

Problems 3–4: Tyler can spend up to \$45 on hats and socks. A hat costs \$10 and a pair of socks costs \$2.50.

- h is the number of hats.
- s is the number of pairs of socks.

3. Which inequality represents this situation?

A. $10h + 2.50s > 45$

B. $10h + 2.50s < 45$

C. $10h + 2.50s \ge 45$

D. $10h + 2.50s \le 45$

4. Explain how you know that $h = 2$ and $s = 1$ are solutions to this situation.

5. Marc is making a bracelet for his sister using beads that cost \$0.50 and \$0.75. He cannot spend more than \$8.00 on the bracelet.

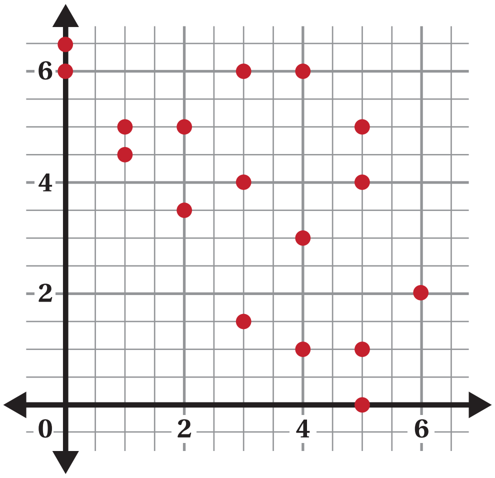
- x is the number of \$0.50 beads.
- y is the number of \$0.75 beads.

Marc says that the inequality $0.5x + 0.75y \ge 8$ represents all the bracelets he can make. Do you agree with him? Explain your thinking.

Name: .... Date: .... Period: ....

6. This graph shows some solutions to $3x + 2y \geq 12$.

   Write an ordered pair for *another* point that is a solution to this inequality.



7. Write an ordered pair for a point that is *not* a solution to $3x + 2y \geq 12$.

Spiral Review

Problems 8–9: Write an inequality for each situation.

8. Duri will stay warm in a sleeping bag when the temperature is at least 30°F. Use t to represent temperatures at which Duri will stay warm.

9. Duri's backpack needs to weigh less than 45 pounds. Use w to represent the weight of the backpack.

Problems 10–11: Here is an equation: $6x + 2y = 36$.

10. For each value of x, determine the value of y.

x	y
2	
4	

11. Which equation represents the same relationship?

   A. $y = 6 - 3x$ **B.** $y = 18 - 3x$ **C.** $y = 18 - \frac{1}{3}x$ **D.** $y = 6 - \frac{1}{3}x$

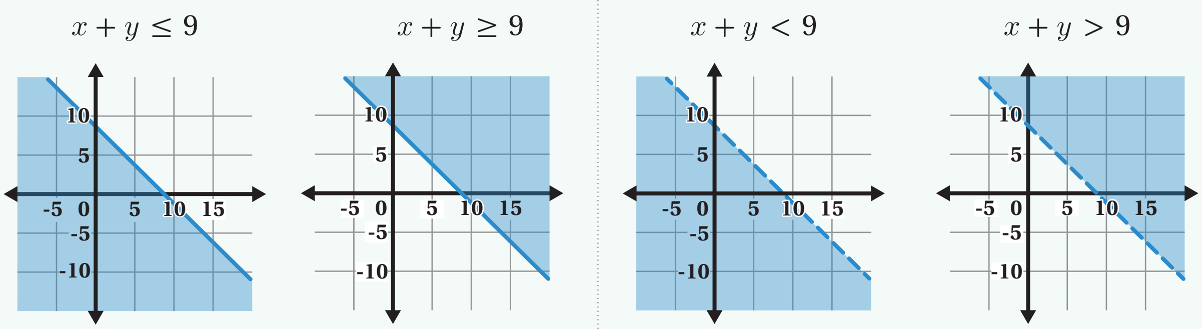
Reflection

1. Circle the problem you enjoyed doing the most.
2. Use the space to ask a question or share something you're proud of.

Science Mom Lesson 16

Lesson Summary

The solutions to a two-variable linear inequality can be represented on a graph as a *half-plane*. A **boundary line** separates the plane into the region that contains solutions and the region that does not. The shaded area represents all of the solutions, which are the values of (x, y) that make the inequality true.



A solid line means that the points on the boundary line *are* included in the solutions. This is represented by the $\leq$ and $\geq$ symbols.

A dashed line means the points on the boundary line are *not* included in the solutions. This is represented by the $<$ and $>$ symbols.

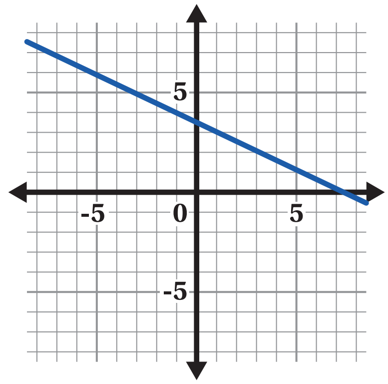
To determine which of the half-planes is the **solution region**, you can test points on either side of the *boundary line* to see whether they make the inequality true or false.

Things to Remember:

Lesson Practice
A1.2.14

Name: .... Date: .... Period: ....

1. Here is a graph of the equation $x + 2y = 7$.



Which of these points is a solution to the inequality $x + 2y < 7$?

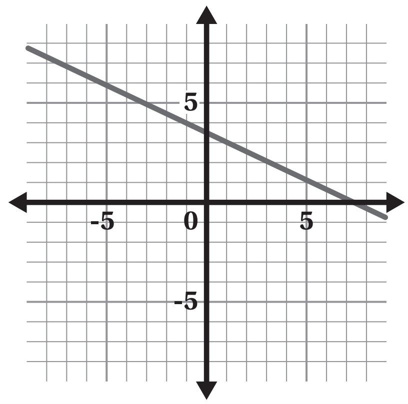
A. $(0, 0)$

B. $(10, 0)$

C. $(7, 0)$

D. $(0, 7)$

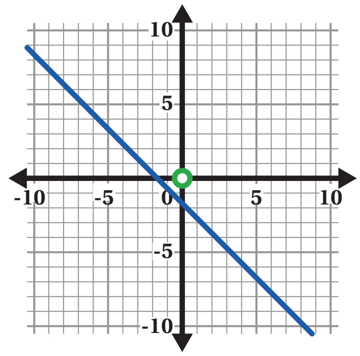
2. Graph all the solutions to $x + 2y < 7$.
Explain your thinking.



3. Here is an inequality: $x + y \leq -2$.

- Ada graphed the equation $x + y = -2$.
- Ada noticed that $(0, 0)$ is not a solution to $x + y \leq -2$.

How can Ada use this information to graph the solutions to this inequality?



4. Brianna is creating a graph for the inequality $2x - 5y > 10$. She says that since the inequality has a greater-than symbol, she should shade the region above the line $2x - 5y = 10$. Is Brianna correct? Explain your thinking.

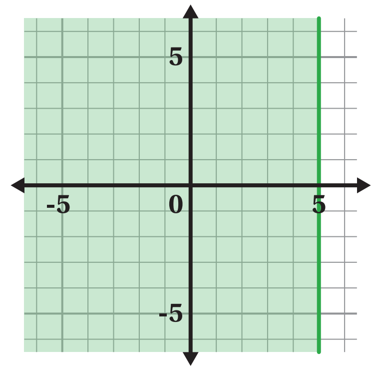
5. Which inequality is shown on the graph?

A. $y \leq 5$

B. $y \geq 5$

C. $x \geq 5$

D. $x \leq 5$

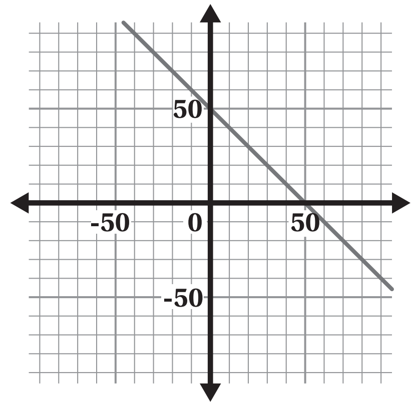


Lesson Practice
A1.2.14

Name: ______________ Date: ________ Period: ________

Problems 6–7: A food truck only sells hot dogs and hamburgers. They want to sell 50 items or more each day.

- x represents the number of hot dogs sold.
- y represents the number of hamburgers sold.



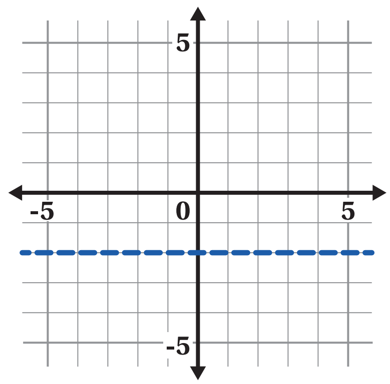
6. Which inequality represents this situation?

A. $x + y > 50$ **B.** $x + y < 50$

C. $x + y \geq 50$ **D.** $x + y \leq 50$

7. Complete the graph so that it represents all the solutions to the inequality for this situation.

8. Leah-James started to graph the inequality $y >$ -2 by graphing a dashed line at $y =$ -2. How might Leah-James decide where to shade?



Spiral Review

9. The equation of line a is $y =$ -2$x - 1$. Select *all* the points that are on line a.

☐ **A.** (-2, 3) ☐ **B.** (0, 4) ☐ **C.** (1, -3)

☐ **D.** (-1, -3) ☐ **E.** (2, 5)

10. Lola can spend up to \$15 on pens and notebooks. A pen costs \$2 and a notebook costs \$1.50. Using p for the number of pens and n for the number of notebooks, write an inequality that represents this situation.

11. A golf ball weighs 1.6 ounces and an empty metal bucket weighs 12 ounces. Neel adds golf balls one at a time to the empty metal bucket. How many golf balls will be in the bucket when the total weight is 20 ounces?

Reflection

1. Put a smiley face next to the problem you learned from most.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Different types of questions lead to different types of data:

- **Quantitative data** has values that are numbers, measurements, or quantities instead of words. It's sometimes called *numerical data*.

  How many pets do you have? is a question that produces quantitative data.

- *Categorical data* has values that are categories, such as colors, words, or zip codes.

  What's your favorite animal? is a question that produces categorical data.

It's important to be specific when writing survey questions, so you can gather the exact type of data you need.

Things to Remember:

Lesson Practice
A1.3.01

Name: ______ Date: ______ Period: ______

1. Which would *not* be a good survey question?

A. What grade are you in?

B. How many books did you read last year?

C. How many inches are in 1 foot?

D. How many pets do you have?

2. Determine which type of data these questions produce.

Question	Categorical or Quantitative?
How old are you?	
Do you have any pets?	
How many siblings do you have?	

3. Select *all* the questions that would produce quantitative data.

- ☐ **A.** How many people live in your home?
- ☐ **B.** What is your favorite breakfast food?
- ☐ **C.** How did you travel to school this morning?
- ☐ **D.** How many minutes did it take you to get ready this morning?
- ☐ **E.** What is the last thing you ate or drank?

4. Callen claims that players on the school basketball teams are taller than players on the soccer teams. Write two survey questions that Callen could ask to investigate the claim.

Question 1: ______

Question 2: ______

5. Deven and Erendirani want to know about the types of food their classmates prefer. Write a survey question that would give them *categorical* data about their classmates' food preferences.

Problems 6–7: Here are some responses to the question: *What is your birthday?*

January 7 March 18 December 23

6. Jaylin is not sure whether the data is categorical or quantitative. Explain why this type of data is unclear.

7. What is another question that might generate data that is unclear?

Name: ________ Date: ________ Period: ________

8. Think about your community. What information would you like to know about the people in it? Write an example of one question that will produce *quantitative* data and one question that will produce *categorical* data about your community.

Quantitative	Categorical

Spiral Review

9. Determine the value of x that makes the equation $5(3x - 2) = -55$ true.

Problems 10–11: A scientist is studying how two different types of bacteria grow. This table represents the number of bacteria cells in the days since her experiment began.

	Day 0	Day 1	Day 2	Day 3	Day 4
Bacteria A (number of cells)	1	3	9	27	?
Bacteria B (number of cells)	80	100	120	140	?

10. If the patterns continue, which type of bacteria will there be more of after 4 days?

11. If the patterns continue, which type of bacteria will there be more of after 10 days?

Problems 12–13: Tamiya is organizing two sets of marbles.

	Green Marbles	Marbles That Are Not Green
Set A	17	23
Set B	13	12

12. How many marbles are in each set?

Set A: ________

Set B: ________

13. Across both sets, how many marbles does Tamiya have in total that are *not* green?

Reflection

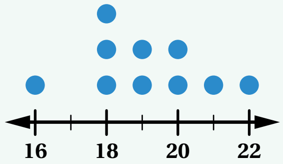
1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

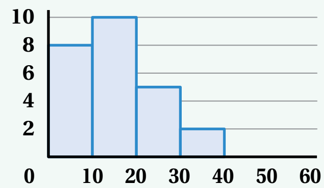
Lesson Summary

It can be helpful to visualize quantitative data using dot plots or histograms.

Dot plots present each data point as a dot at its value on a number line. Dots with the same value are stacked on top of one another.



Histograms group data into rectangular bins. The height of each rectangle shows how many values are in that bin.



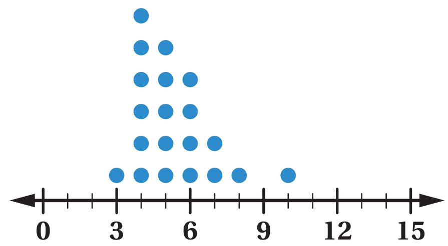
A dot plot is useful for observing the frequency of individual data, and the number of points in a data set. Histograms are useful for representing large data sets and observing the shape of a distribution.

Things to Remember:

Lesson Practice
A1.3.04

Name: .... Date: .... Period: ....

Problems 1–2: A class made this dot plot to explore how many letters are in their first names.

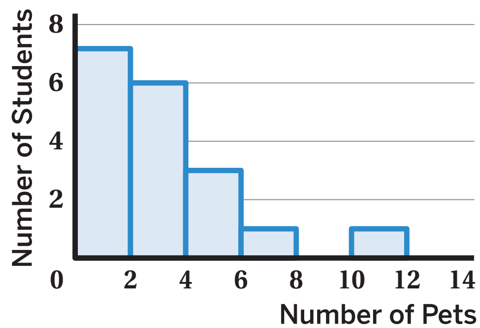


1. How many letters are in the shortest first name?

2. Dyani says that most students have 10 letters in their first name.

Is Dyani correct? Explain your thinking.

Problems 3–5: Jayden created a histogram about the number of pets students have.

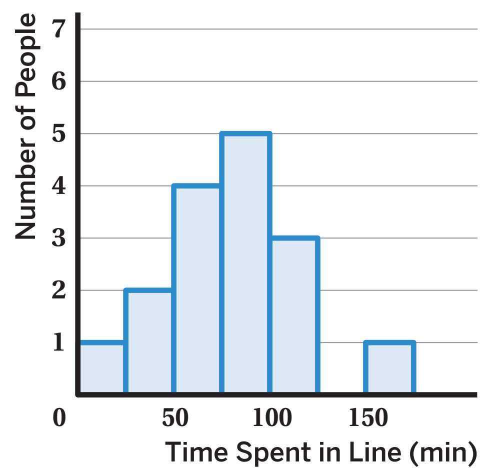


3. What is the bin width of the histogram?

4. How many students have less than 4 pets?

5. What is a number of pets that no students have?

6. A group of 15 friends went to a theme park for the day. Each person kept track of how many minutes they spent waiting in line. This histogram shows their data. Select *all* the statements that must be true.



- ☐ **A.** 1 person was in line for at least 150 minutes.
- ☐ **B.** 3 people were in line for less than 50 minutes.
- ☐ **C.** Most people spent over 100 minutes in line.
- ☐ **D.** 1 person spent 0 minutes in line.
- ☐ **E.** No one spent exactly 130 minutes in line.

7. Imani collected this data about the outdoor temperature, in degrees Fahrenheit, at noon over the last two weeks:

50.0	55.4	57.2	60.8	64.4	59.0	60.9
68.9	60.8	60.8	64.4	55.9	55.9	57.9

Would this data be best displayed using a dot plot or histogram? Explain your thinking.

Lesson Practice
A1.3.04

Name: ______________ Date: ________ Period: ________

Spiral Review

Problems 8–9: Vihaan is investigating whether there is an association between wearing sneakers and participating on an athletic team. This table summarizes the data Vihaan collected from a survey of students.

	On Athletic Team	Not on Athletic Team	Total
Wear Sneakers	16%	17%	33%
Don't Wear Sneakers	32%	35%	67%
Total	48%	52%	100%

8. Interpret what 32% means in this situation.

9. Is there evidence of an association between wearing sneakers and participating on an athletic team?

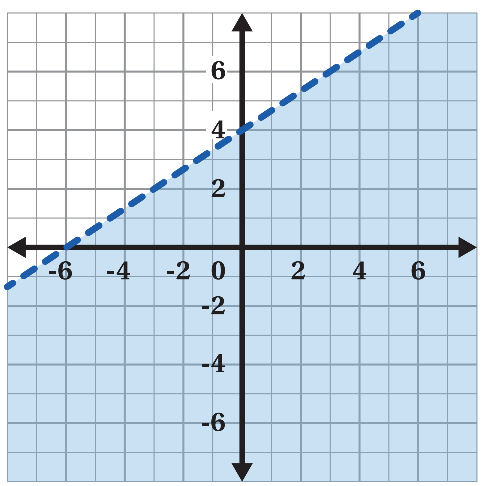
10. Which inequality is shown on the graph?

A. $y > \frac{2}{3}x - 6$

B. $y \leq -\frac{2}{3}x + 4$

C. $y > \frac{2}{3}x + 4$

D. $y < \frac{2}{3}x + 4$



11. What is the median of this data set? 2, 4, 5, 5, 7, 9, 10, 11, 11, 11, 21

Reflection

1. Put a smiley face next to the problem you learned from most.

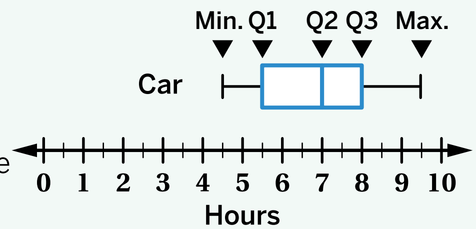
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

A *box plot* is one way to visualize quantitative data. The data is divided into four sections using five values: the *minimum*, the *maximum*, and three *quartiles*. Quartiles divide a data set into four sections, or quarters. Each quarter represents 25% of the data.

Quartile 1 (Q1) is the median of the lower half of the data. Quartile 2 (Q2) is the median of the entire data set, which divides the data into two halves. Quartile 3 (Q3) is the median of the upper half of the data.

The minimum, maximum, median, and quartiles are all examples of statistics. A statistic is a single number that measures something about a data set.



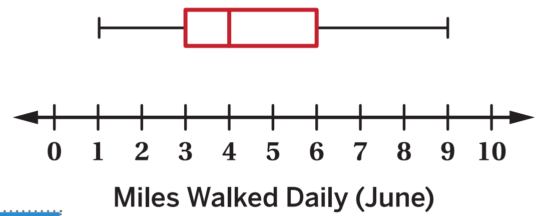
Box plots are useful for representing large data sets, especially those with extreme values. They are less
helpful for seeing individual data points.

Box plots can be used to make statements about percentages of data. This box plot represents data on road trip travel times. According to the box plot, 50% of travelers had a travel time of under 7 hours. However, 75% of the recorded travel times were less than 8 hours. The longest road trip recorded was 9.5 hours.

Things to Remember:

Name: .... Date: .... Period: ....

Problems 1–2: Ricardo used a fitness app to track how many miles he walked each day in June. Here is a box plot of Ricardo's data.



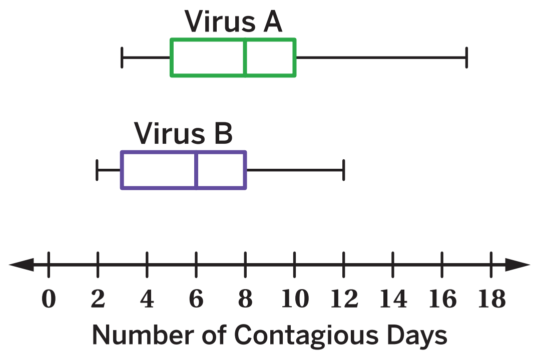
1. Determine each of these statistics:

Min	Q1	Median	Q3	Max

2. Select *all* of the questions that the box plot can answer.

- ☐ **A.** Did Ricardo walk more than 8 miles any day in June?
- ☐ **B.** How many times did Ricardo walk more than 8 miles in a day?
- ☐ **C.** Did Ricardo walk more on weekends or weekdays?
- ☐ **D.** About how often did Ricardo walk 4 miles or more in a day?
- ☐ **E.** Did Ricardo walk exactly 5 miles on any day in June?

Problems 3–5: A team of scientists wanted to know how long people sick with certain viruses were contagious. They studied 2 groups of 500 people and measured how many days each person was contagious.



3. Approximately what percentage of people with Virus B were contagious for more than 8 days?

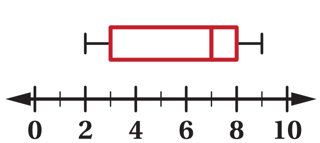
A. 25% **B.** 50% **C.** 75% **D.** 33%

4. How do the two viruses compare?

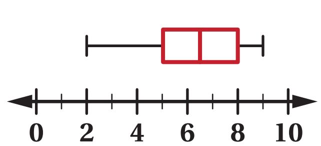
5. Citlali says that the box plot for Virus A tells us that at least one person was contagious for exactly 14 days. Is Citlali correct? Explain your thinking.

6. Determine which box plot represents the data set: 2, 3, 5, 5, 6, 6, 6, 7, 7, 7, 8, 8, 8, 9

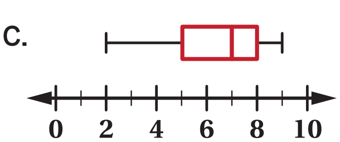
A.



B.



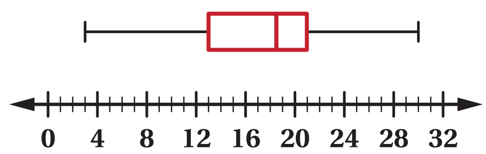
C.

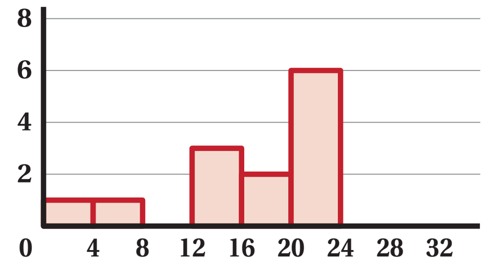


Lesson Practice
A1.3.05

Name: .... Date: .... Period: ....

7. Here is a box plot and a histogram. Could these two representations have come from the same set of data? Explain your thinking.

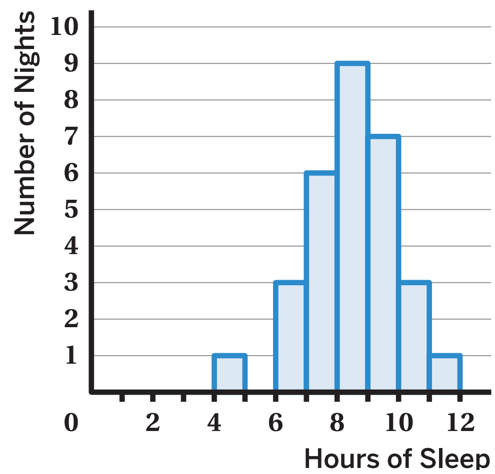




Spiral Review

8. This histogram shows how many hours of sleep Dakota got each night in September.

How many nights did Dakota sleep less than 7 hours?



9. Select *all* the ordered pairs that are solutions to the inequality, $2x - 3y > 18$.

- ☐ **A.** (6, -4)
- ☐ **B.** (-4, 6)
- ☐ **C.** (6, -2)
- ☐ **D.** (0, 0)
- ☐ **E.** (0, -8)

10. Calculate the mean of the data set: 1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15

Reflection

1. Put a star next to a problem where you revised your thinking.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Here are some terms that can help us describe the *shape* of a data set.

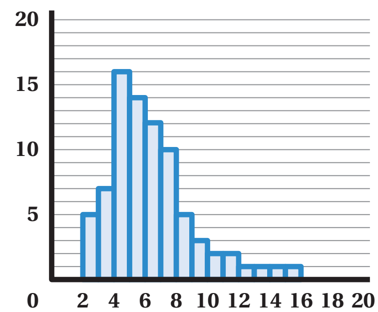
- A data set is **bell-shaped** when most of the data is at the center and there are fewer points farther from the center. When presented in a dot plot or histogram, the data looks like a bell.
- A data set is **bimodal** when it has two very common data values. These appear in a dot plot or histogram as two peaks.
- A data set is **skewed** when more values are concentrated on one end of the data than the other.
- A data set is **symmetric** when you can draw a vertical line of symmetry through it.
- A data set is **uniform** when the data values are evenly distributed.

Things to Remember:

Lesson Practice
A1.3.06

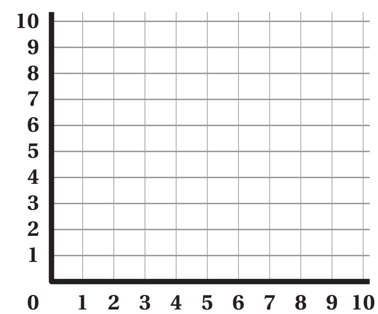
Name: .... Date: .... Period: ....

1. Which term best describes this histogram?

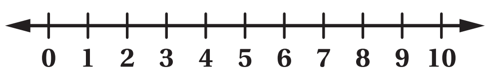


- **A.** Symmetric
- **B.** Skewed
- **C.** Uniform
- **D.** Bimodal

2. Create a histogram that is bell-shaped.

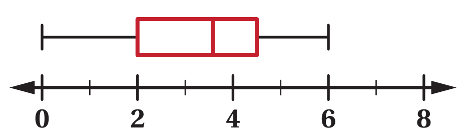


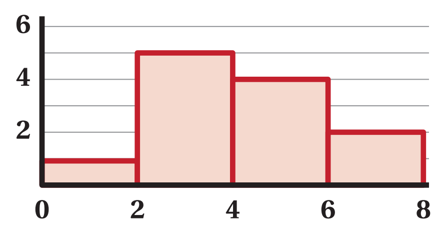
3. Create a dot plot that is *symmetric but not uniform*. Plot at least eight points.



Spiral Review

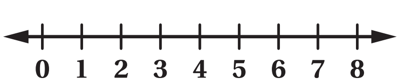
Problems 4–5: Here is a box plot and a histogram of the same data set.





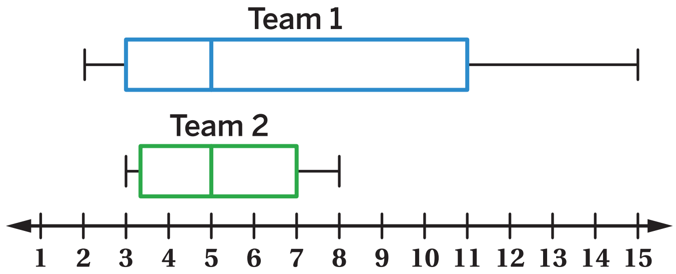
4. What is the value of Q1?.

5. Create a dot plot that could also represent this data set.



Name: .... Date: .... Period: ....

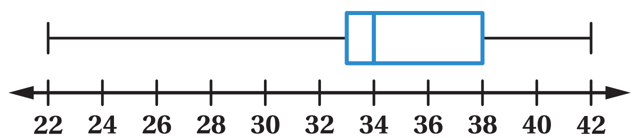
Problems 6–7: The players on two different football teams were surveyed about the number of hours they spend in the weight room each week. Here are the box plots for each team's data.



6. What is one *similarity* between the two data sets?

7. What is one *difference* between the two data sets?

8. This box plot shows the number of points scored by a cross country team at 12 different meets.

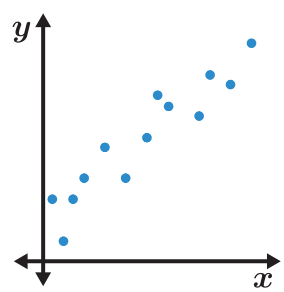


What percentage of meets did the team score between 33 and 38 points?

9. Which equation is equivalent to $5x + 3y = 30$?

A. $y = 10 - 5x$ **B.** $y = 10 - \frac{5}{3}x$ **C.** $y = 30 - 5x$ **D.** $y = 10 + \frac{5}{3}x$

10. Select *all* the terms that describe the association in this scatter plot.



- ☐ **A.** Linear association
- ☐ **B.** Non-linear association
- ☐ **C.** Positive association
- ☐ **D.** Negative association
- ☐ **E.** No association

Reflection

1. Put a question mark next to a response you'd like to compare with a classmate's.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice A1.3.07

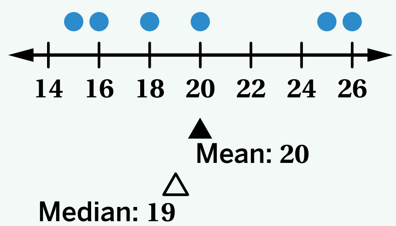
Science Mom Lesson 21

Lesson Summary

A **measure of center** is a single number that summarizes all of the values in a data set. *Mean* and *median* are measures of center that are used to describe a typical value of a data set.

- The mean is also called the average of a data set. To calculate the mean, you can add up all the data values, and divide by the number of data points.
- The median is the middle value of a data set when the values are in numerical order. If there are two values in the middle of the data set, then the median is the middle of those two values.

Here is one way to determine the mean of the data in this dot plot.



$15 + 16 + 18 + 20 + 25 + 26 = 120$

$120 \div 6 = 20$

The median is the value halfway between 18 and 20, so the median is 19.

When a data set includes extreme values that are much larger or smaller than most of the data, the value of the mean and median can be very different. Extreme values impact the mean more than they impact the median.

Things to Remember:

Lesson Practice
A1.3.07

Name: ______________ Date: ______ Period: ______

1. Determine the mean and median of each data set.

Data Set	Mean	Median
27, 30, 33		
0, 100, 100, 100, 100		
3, 5, 7, 15		

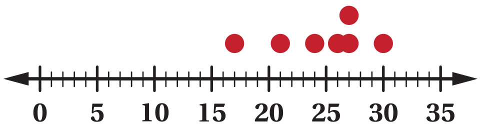
2. Seven people estimated how many marbles there were in a jar. Determine the mean and median of the estimates.

Mean: ______

Median: ______

Marble Jar Estimates

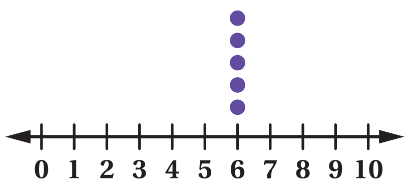
17	21	27	27	24	26	30



3. Here is a dot plot. If you added 4 and 9 to the data set, which statistic would change? Circle one.

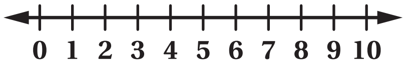
Mean Median Both Neither

Explain your thinking.

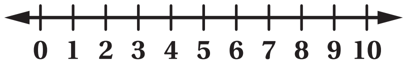


Problems 4–5: Create a dot plot that matches each description.

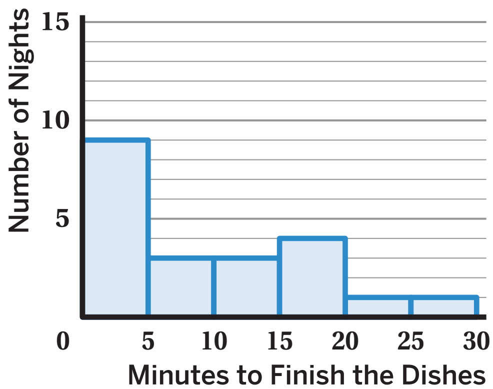
4. A median of 3 and a mean that is greater than the median.



5. Bell-shaped, with a median of 6.



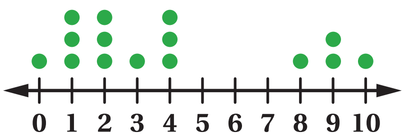
Problems 6–7: For the past few weeks, Anand kept track of how long it took him to do the dishes each night. This histogram shows the results organized in 5-minute bins.



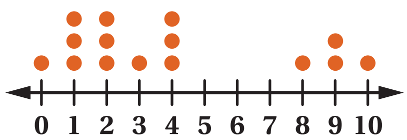
6. Circle the bin that contains the median.

7. Explain how you know.

8. Remove one value to change the *mean* but not the median.



9. Remove one value to change the *median* but not the mean.



Spiral Review

10. Select all the expressions that are equivalent to $2(x + 3)$.

☐ **A.** $2x + 6$

☐ **B.** $2 \cdot x + 3$

☐ **C.** $x + 6$

☐ **D.** $(x + 3) \cdot 2$

☐ **E.** $2 \cdot x + 3 \cdot 2$

Problems 11–12: Evaluate each expression. Write your solution in scientific notation.

11. $(2 \cdot 10^4)(6 \cdot 10^5)$

12. $\dfrac{3 \cdot 10^{-8}}{2 \cdot 10^{-3}}$

Reflection

1. Put a star next to the problem you understood best.

2. Use this space to ask a question or share something you're proud of.

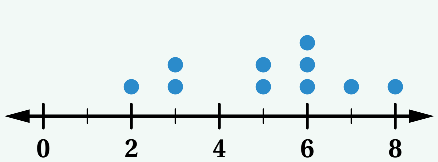
Lesson Practice A1.3.08

Lesson Summary

The **standard deviation** of a data set is a **measure of spread**, or a way to measure how spread out the values in a data set are from its mean. Measures of spread can help you describe how consistent a data set is. If a data set is very consistent and much of the data clustered together, the standard deviation will be small. If the data is spread out, the standard deviation will be larger.

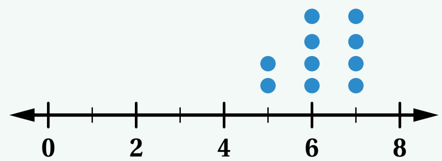
For example, compare these two dot plots.

Dot Plot A



Standard deviation: 1.81

Dot Plot B



Standard deviation: 0.75

The data in Dot Plot A has a greater standard deviation than the data in Dot Plot B because the data in A is more spread out, or variable. Since the data values in Dot Plot B are more consistent, B has a lower standard deviation.

One way to calculate the standard deviation of a data set is to use the Desmos Graphing Calculator. In the calculator, use the functions menu or type $\mathrm{stdevp}(\ )$ and then insert a list with the data values. For example, to calculate the standard deviation of Data Set A, you can type $\mathrm{stdevp}([\ 2, 3, 3, 5, 5, 6, 6, 6, 7, 8])$.

Things to Remember:

Lesson Practice
A1.3.08

Name: .... Date: .... Period: ....

Problems 1–2: Use a graphing calculator to determine the mean and standard deviation of each data set. Round your answer to the nearest hundredth.

1. 14, 14, 15, 16, 16, 18 ,18, 19

Mean: ....

Standard Deviation: ....

2. 2, 3, 6, 6, 8, 8, 10, 11

Mean: ....

Standard Deviation: ....

Problems 3–4: Moon and Alejandro are both on the track team. Each day in May, they recorded the average distance they threw the javelin.

	Mean (ft)	Standard Deviation (ft)
Alejandro	128	1.2
Moon	130	2.6

3. Based on the data, who is the more consistent thrower?

4. If a throw of 160 feet was added to each data set, whose standard deviation would change more?

5. Order the data sets from smallest to largest standard deviation.

Use a graphing calculator to help with your thinking.

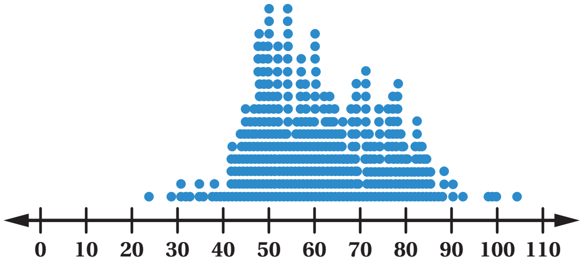
Set A	Set B	Set C
7, 10, 12, 14, 17, 19	2, 2, 5, 22, 25, 25	4, 8, 9 ,10, 12, 13

Smallest Standard Deviation **Largest Standard Deviation**

6. Here are the high temperatures in Seattle for each day of 2021. If the data points for the hottest and coldest temperatures were removed, would the standard deviation increase, decrease, or stay the same? Circle one.

Increase Decrease Stay the same

High Temperatures in Seattle in 2021 (°F)



Source: *National Weather Service*

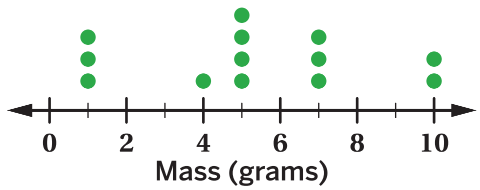
Name: ________ Date: ________ Period: ________

7. Here is a data set: $4, 5, 5, 6, 7, 9$.

Select *all* of the changes that will make the standard deviation *smaller*.

- ☐ **A.** Add a 2 to the data set.
- ☐ **B.** Add a 6 to the data set.
- ☐ **C.** Remove the 4 from the data set.
- ☐ **D.** Remove the 9 from the data set.
- ☐ **E.** Increase each value by 10.

Problems 8–9: Here is a dot plot showing the weights of fish.



8. Circle the weight of fish that would change the standard deviation the *most* if added to the data set.

1 gram 4 grams 5 grams 7 grams 10 grams

9. Circle the weight that would change the standard deviation the *least* if added to the data set.

1 gram 4 grams 5 grams 7 grams 10 grams

Spiral Review

Problems 10–12: Rewrite each expression as a single power.

10. $(12^3)^5$

11. $\frac{6^4}{6^5}$

12. $5^7 \cdot 5^2 \cdot 5$

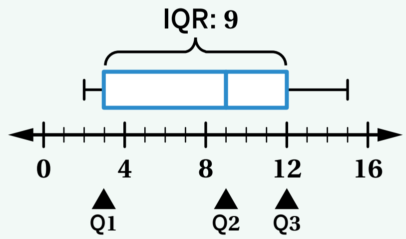
Reflection

1. Put a star next to a problem you could explain to a classmate.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

The *interquartile range* (IQR) is a statistic that measures the spread of a data set. In a box blot, the IQR is the width of the box. Measures of spread, like IQR, help us determine how consistent the data within a set is. The IQR represents the middle half of the data set, and we calculate it by determining the distance from Q1 to Q3. This makes IQR a more resistant measure of spread for skewed data than standard deviation.

Here is an example of a data set where Q1 $= 3$ and Q3 $= 12$. The IQR of this data set is 9, because $12 - 3 = 9$.



Things to Remember:

Lesson Practice
A1.3.10

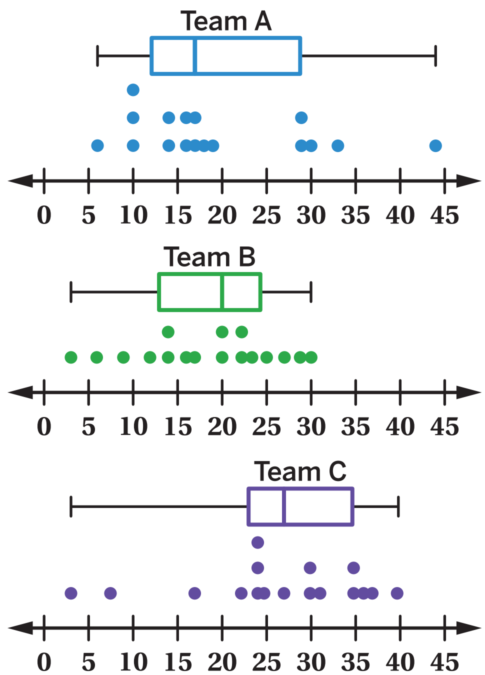
Name: .... Date: .... Period: ....

1. Create two different data sets, each with 5 numbers or less and a median of 13.

Data Set 1: ....

Data Set 2: ....

Problems 2–4: These box plots represent the points scored per game for three different football teams over the course of a season.

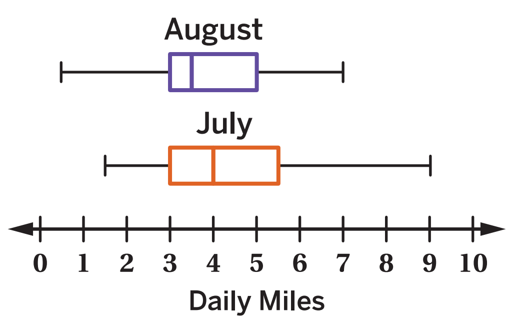


2. Which team generally scored the most points?

3. Which team scored the most points in a single game?

4. Which team was the most consistent? Explain your thinking.

Problems 5–6: Mariana used a fitness app to track how many miles she walked each day in July and August. Here is a box plot of her data.



5. Which month has a smaller IQR?

6. What does the IQR tell you about Mariana's walking habits in July and August?

Name: .... Date: .... Period: ....

7. Here is a data set: $[2, 2, 4, 4, 5, 5, 6, 7, 9, 15]$.

   If you added 24 as a new data point, how would the IQR change? Explain your thinking.

8. Here is a different data set: $[3, 6, 7, 23]$.

   Determine a number you could add to make an IQR of 15.

   or

Spiral Review

Problems 9–11: For each equation, write the positive solution as a whole number or by using square root or cube root notation.

9. $t^3 = 729$

10. $a^2 = 27$

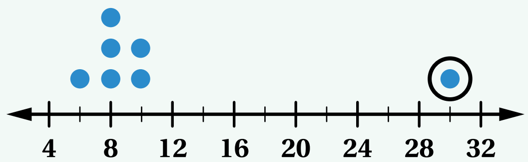
11. $m^3 = 12$

Reflection

1. Put a star next to the problem you understood best.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Outliers are data values that are far from the other values in the data set. In this data set, the circled data point is an outlier.



You can identify outliers using dot plots, box plots, and technology tools such as graphing calculators. You can also identify outliers using the *IQR*. Outliers are values further than 1.5 times the IQR below Q1 or above Q3.

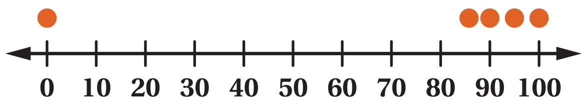
When deciding which measure of center is appropriate to represent a data set, it's important to identify any outliers. Outliers have a big impact on the *mean,* but they don't impact the *median* very much.

Things to Remember:

Lesson Practice
A1.3.11

Name: ______________ Date: ______ Period: ______

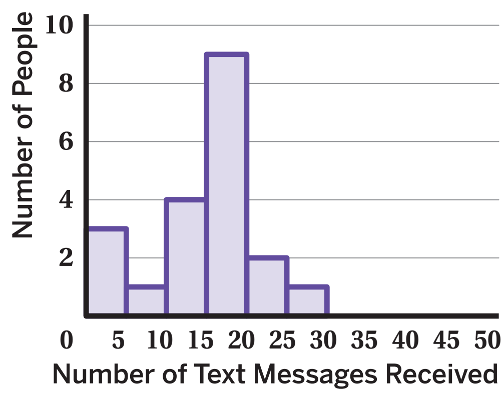
Problems 1–2: Ricardo got the following scores on five class assignments: 87, 90, 0, 95, and 100. Ricardo's teacher lets students decide whether their final score will be the mean or the median of those five scores.



1. Would using the mean or the median give Ricardo the higher final score?

2. Explain your thinking.

Problems 3–4: This histogram represents the number of text messages that 20 people received in one day.



3. How many people received 15 or more text messages in one day?

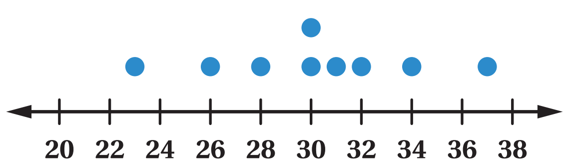
4. If you added a person who received 52 text messages to the data set, would the mean or median change more? Circle one.

Mean Median

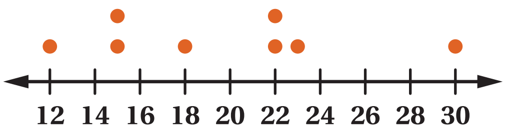
Explain your thinking.

5. Here is a data set:
26, 30, 31, 32, 28, 30, 34, 37, 23.

Julian says that 37 is an outlier because it is the maximum value. What could you say to Julian to help him understand his mistake?

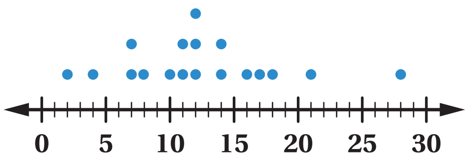


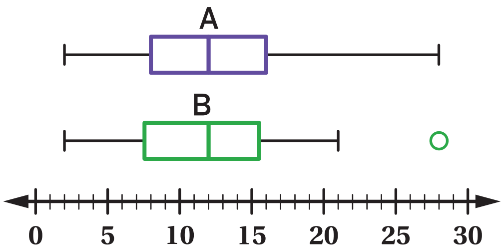
6. This dot plot represents the ages of different people in a bike shop. Which data point would be an outlier if it were added to the data set?



A. 44 **B.** 8 **C.** Both **D.** Neither

7. The dot plot represents the data in box plot A. Circle one point to remove from the dot plot so that it represents box plot B instead.





Spiral Review

Problems 8–10: Evaluate each expression. Write your answer as a fraction.

8. $\frac{2}{3} + \frac{1}{5}$

9. $\frac{2}{5} - \frac{3}{8}$

10. $\frac{6}{15} + \frac{8}{20}$

Reflection

1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

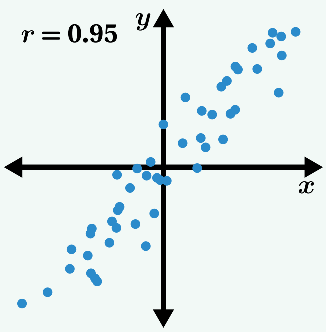
Lesson Summary

A *scatter plot* is a graph of plotted points that shows the relationship, or association, between two variables. When there is a linear association between the variables, you can describe the *strength* and *direction* of the association using the **correlation coefficient** (also called the **r-value**). The correlation coefficient is a number between -1 and 1 that describes the strength and direction of a linear association between two variables in a scatter plot.

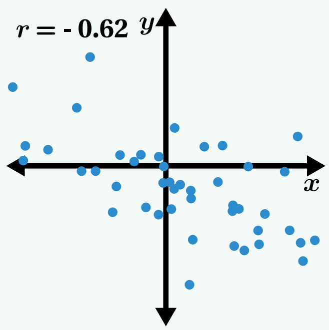
The closer the r-value is to 0, the weaker the linear association. The closer the r value is to -1 or 1, the stronger the linear association.

These graphs show examples of different associations and their correlation coefficients.

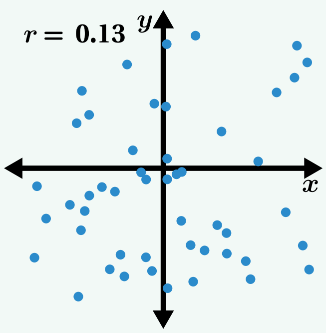
Positive Association
Strong Association



Negative Association



Positive Association
Weak Association

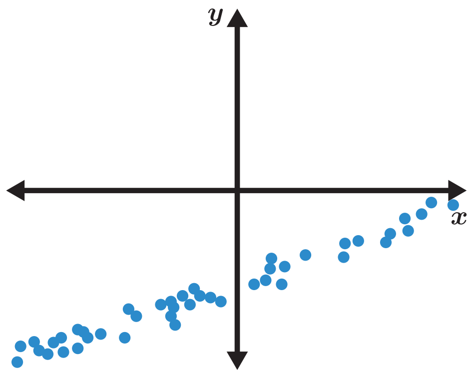


Things to Remember:

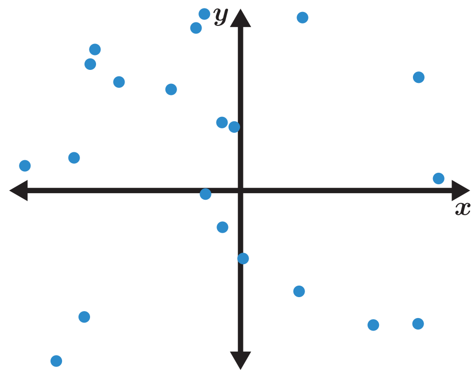
Name: .... Date: .... Period: ....

Problems 1–4: Determine whether each scatter plot has a strong linear relationship, weak linear relationship, or no linear relationship.

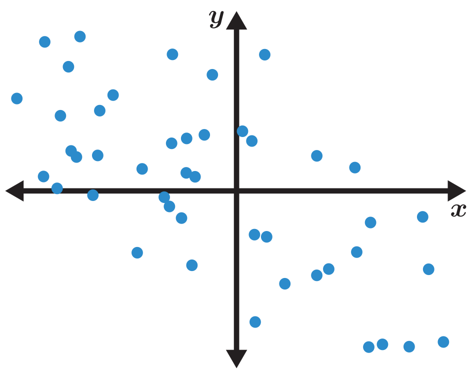
1. Strong Weak None



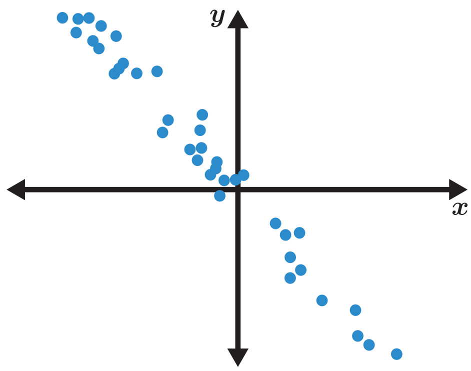
2. Strong Weak None



3. Strong Weak None



4. Strong Weak None

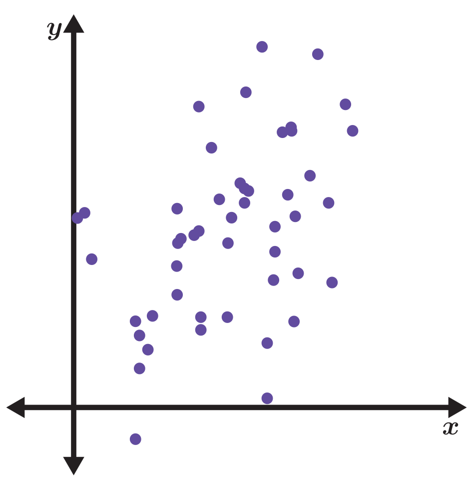


5. Which number could be the correlation coefficient for this scatter plot?

A. 0.4 **B.** -0.4

C. 0.9 **D.** -0.9

Explain your thinking.



6. A scatter plot has a correlation coefficient of $r = 0.85$. What does this tell you about the data?

Lesson Practice
A1.3.13

Name: ________ Date: ________ Period: ________

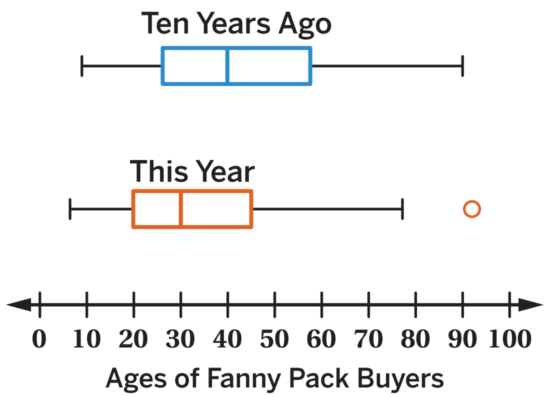
7. Fill in each blank using the numbers 0 to 9 only once to create a set of four points that have a correlation coefficient between -0.1 and 0.1. Use the a graphing calculator to help with your thinking.

$(\square, \square), (\square, \square)$
$(\square, \square), (\square, \square)$

Spiral Review

8. An online store is curious about who buys the fanny packs they sell.

Use the box plots to make a claim about the ages of fanny pack buyers this year compared to ten years ago.



Problems 9–10: Evaluate each expression. Write your answer as a fraction.

9. $\frac{3}{16} \cdot 8$

10. $\frac{5}{6} \cdot \frac{3}{4} \cdot \frac{14}{10}$

Reflection

1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

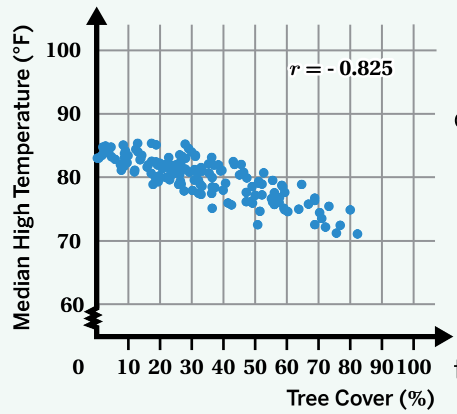
Lesson Summary

You can use a correlation coefficient to analyze the relationship between two variables and determine whether there is an association between them. The correlation coefficient, or r-value, describes the strength and direction of the relationship that may exist between two variables.

- A positive r-value means that as one variable increases, the other variable also increases.
- A negative r-value means that as one variable increases, the other variable decreases.
- The closer the r-value is to 1 or -1, the stronger the correlation.

People may use correlations in data to understand and address issues in their community.

For example, this scatter plot shows data on tree cover and temperature for 150 blocks in Detroit, Michigan.



The r-value is -0.825. This means there is a negative and strong relationship between the amount of tree cover and median high temperature in Detroit neighborhoods. Community members may use this correlation to advocate for more trees to be planted in different neighborhoods across the city.

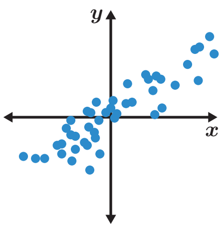
Things to Remember:

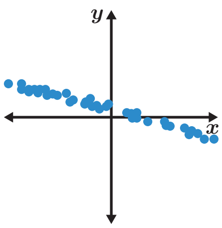
Lesson Practice
A1.3.14

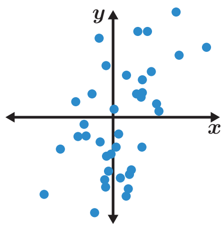
Name: ____________ Date: ________ Period: ________

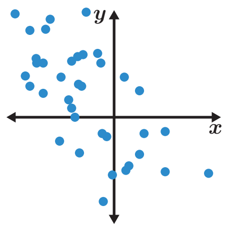
1. Match each scatter plot to its r-value.

a. $r = -0.7$ **b.** $r = -0.99$ **c.** $r = 0.7$ **d.** $r = 0.9$







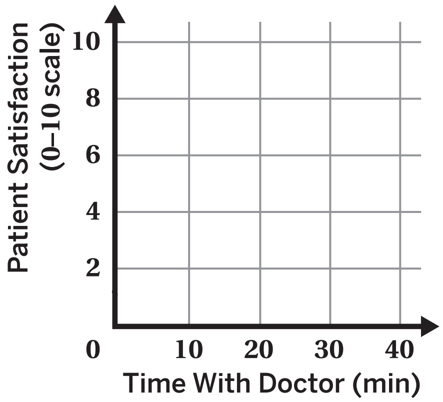


2. A medical clinic wanted to know about the experiences of their patients. They looked at the following variables:

- The number of minutes spent with a doctor.
- Patient satisfaction (on a 0-10 scale).

They found the variables have a *weak*, *positive* relationship.

Make a scatter plot that could represent this data.



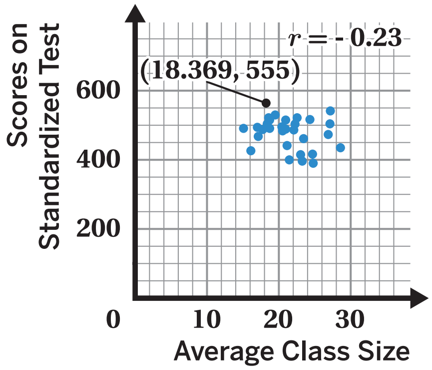
3. Noah creates a scatter plot showing the relationship between the number of free throws taken in a basketball game and the final score. The correlation coefficient for the line of best fit is 0.76.

Are the number of free throws and the final score correlated? Explain your thinking.

Problems 4–6: Martina is interested in learning about different education systems around the world. Martina found data about two variables:

- The average class size.
- The scores on a standardized test.

Here is a scatter plot for the data.



Source: *OECD*

4. The point (18.369, 555) represents Slovakia. What do the coordinates tell you about Slovakia?

5. Based on the r-value, what relationship is there between the variables? Circle one.

Positive Negative None

6. What is the strength of the relationship? Circle one. Weak Strong

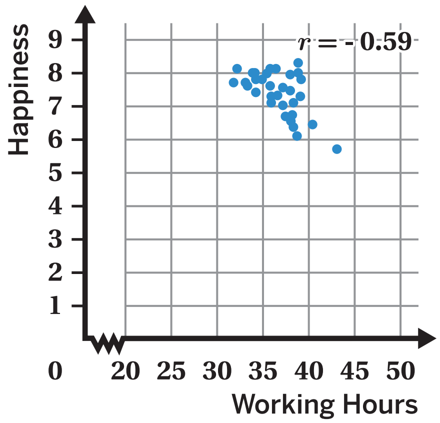
7. The correlation coefficient for a given set of data is $r = 0.962$. Select *all* of the conclusions you can make about the data.

- ☐ **A.** There is a strong association between the two variables.
- ☐ **B.** There is a weak association between the two variables.
- ☐ **C.** There is no association between the two variables.
- ☐ **D.** As one variable *increases*, the other variable *increases*.
- ☐ **E.** As one variable *increases*, the other variable *decreases*.

8. Saanvi is interested in learning more about life in different countries. She found data about two variables:

- Average happiness (on a 1–10 scale).
- The average number of hours people worked in a week.

What does the r-value tell you about the relationship



Sources: *ILOSTAT* and *OECD*

Spiral Review

Problems 9–10: This table shows the statistics for Rio's quiz scores in math class and science class.

	Median (%)	Mean (%)	Standard Deviation (%)	IQR (%)
Math Class	85	85	7.5	6
Science Class	80	85.5	13.4	30

9. Were Rio's quiz scores more consistent in math or science class? Explain your thinking.

10. If Rio scored 65% on a math quiz, how would the standard deviation change? Circle your choice and explain your thinking.

Increase   Decrease   No change

Reflection

1. Put a smiley face next to a problem you were stuck on and then figured out.

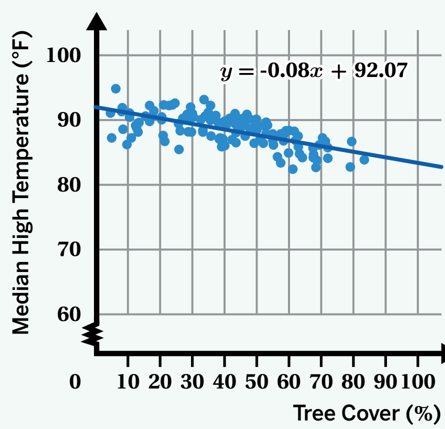
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

While the correlation coefficient can help you understand the general relationship between two variables, a *line of fit* can help you make predictions about specific values in a data set. Points along the line of fit represent the likely value of unknown data in the data set.

You can use the equation of the line of fit to better understand the data. The y-intercept represents a potential initial value and the slope of the line describes the rate that the variables change in relationship to one another.

For example, this scatter plot shows data on tree cover and temperature for 150 blocks in Austin, Texas. The equation of the line of fit is $y = -0.08x + 92.07$.



The slope is -0.08. This means that when the tree cover increases by 1% in Austin, the predicted temperature decreases by 0.08°F.

The y-intercept is 92.07. This means that if the tree cover in Austin is 0%, the predicted temperature is 92.07°F.

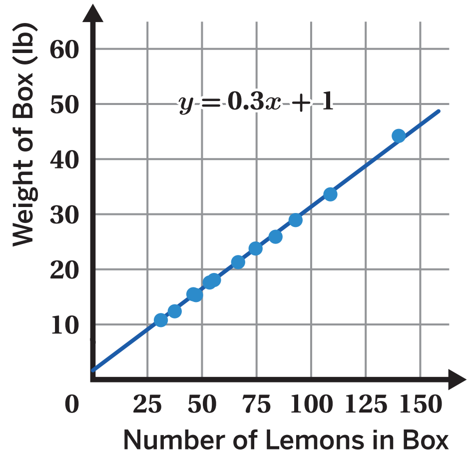
You can use the line to predict that if a block in Austin has 80% tree cover, the temperature will be about 85°F.

Things to Remember:

Lesson Practice
A1.3.15

Name: ________ Date: ________ Period: ________

Problems 1–2: A store receives 12 boxes that contain many different numbers of lemons. They weigh each box and then count the lemons. Here is a scatter plot of that data.



1. A new box weighs 30 pounds. Approximately how many lemons are in the box?

2. The equation for the line that best fits this data is $y = 0.3x + 1$. What do the numbers 0.3 and 1 mean in this context?

Problems 3–4: The store collects data about the weight of other types of fruit boxes. The table shows equations for the line of best fit for each type of fruit box, where y represents the weight of a box and x represents the number of fruits in a box.

Fruit	Line of Best Fit
Orange	$y = 0.45x + 0.95$
Lemon	$y = 0.3x + 1$
Pomegranate	$y = 1.05x + 1.15$
Mango	$y = 0.85x + 1.25$

3. Which fruit is heavier? Circle your choice.

Lemons Pomegranates They are the same

Explain your thinking.

4. Order these fruits by their weight.

	Lightest
	Heaviest

5. Sora collects data about the number of bananas she buys at the store, x, and the total weight of the bananas in pounds, y. If Sora graphed her data, which value would be closest to the slope of the line of best fit?

A. -4

B. -0.4

C. 0.4

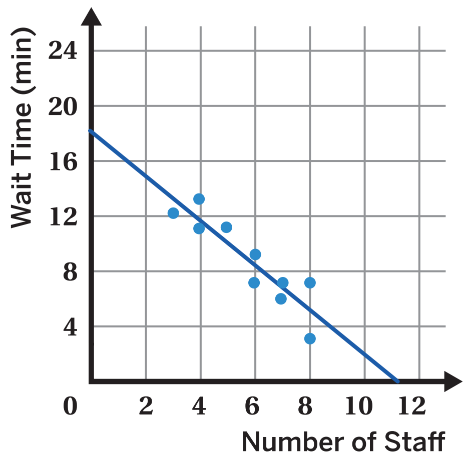
D. 4

Lesson Practice

A1.3.15

Name: ______________________ Date: ________ Period: ________

Problems 6–8: A restaurant gathered data about how long customers had to wait and how many staff members were working. The slope of the line of fit is -1.62. The r-value is -0.9.



6. What does the slope of -1.62 mean in this situation?

7. What does the r-value of -0.9 mean in this situation?

8. Madison says that for any scatter plot, if the r-value is negative, then the slope must also be negative. Is this true or false? Circle one.

True False

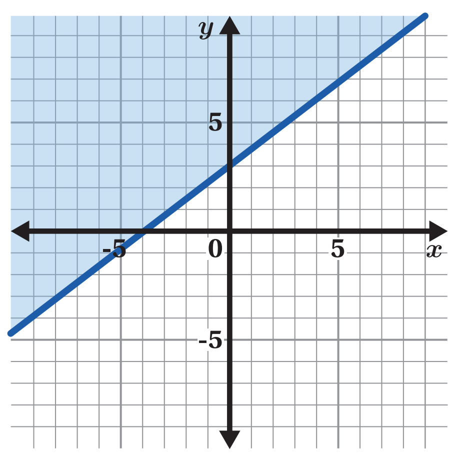
Explain your thinking.

Spiral Review

9. Select *all* the coordinate pairs that are solutions to $y = -\frac{1}{2}x + 10$.

☐ **A.** $\left(0, \frac{2}{3}\right)$ ☐ **B.** $(-10, 15)$ ☐ **C.** $(4, 12)$ ☐ **D.** $(2, 9)$ ☐ **E.** $(5, 8)$

10. Here is the graph of the inequality $y \geq 0.75x + 3$. Select *all* the coordinate pairs that are solutions to this inequality.



☐ **A.** $(-9, -5)$

☐ **B.** $(-3, 2)$

☐ **C.** $(0, 3)$

☐ **D.** $(2, -3)$

☐ **E.** $(6, 8)$

Reflection

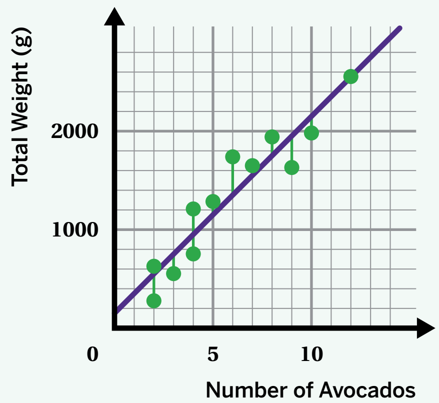
1. Put a heart next to the problem you feel most confident about.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

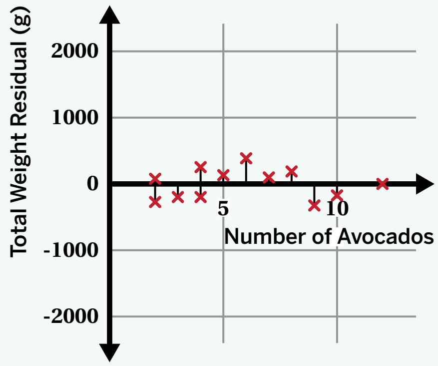
You can use residuals to determine how well a line fits a data set. A **residual** is the difference between the actual y-value of a data point and the value predicted by the line of best fit.

Here is a scatter plot with data on the number of avocados and their weights. The residuals are represented with lines connecting each point to the line of best fit.



You can also create a residual plot to analyze how well a line fits a data set. A **residual plot** is a scatter plot of residual values for a data set.

Here is the residual plot of the graph of avocado weights. The closer a point is to the x-axis, the closer that point is to the line of best fit. A line is a good fit for the data if the points on the residual plot are close to the *x*-axis and are randomly dispersed above and below the axis.

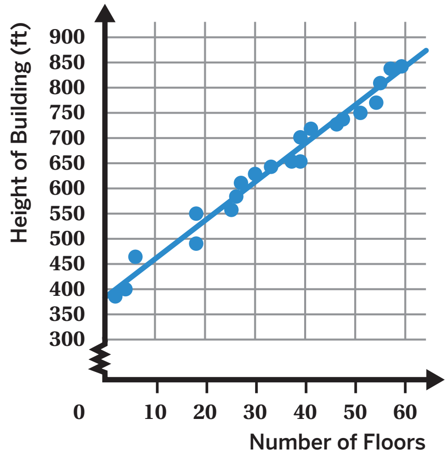


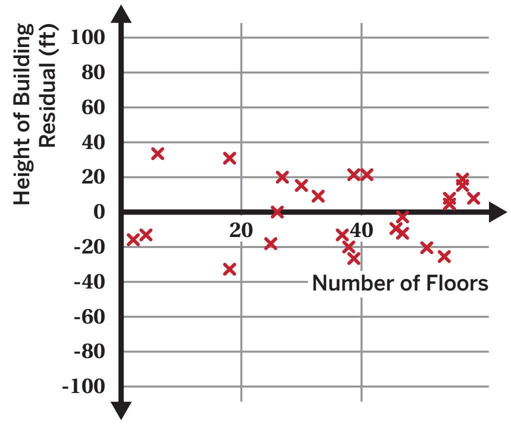
Things to Remember:

Lesson Practice
A1.3.16

Name: .... Date: .... Period: ....

Problems 1–2: The scatter plot shows the heights of a group of buildings, the number of floors in each building, and a line that best fits the data. The residual plot is also shown.



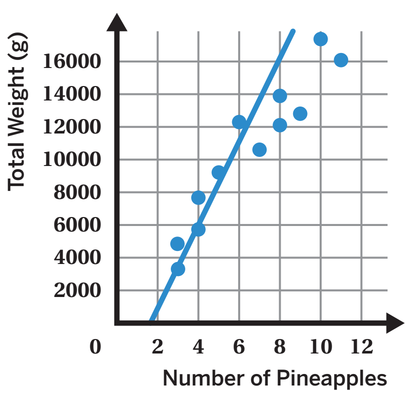


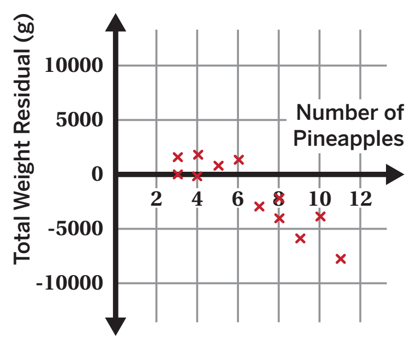
1. Predict the height of a building that has 50 floors.

2. How can you tell that the graphed line is a good fit for the data? Use the residual plot if it helps with your thinking.

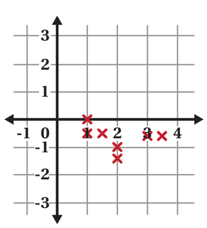
3. Here is a scatter plot and its corresponding residual plot.

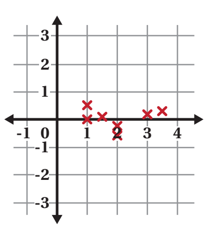
Draw a better line of fit on the scatter plot.

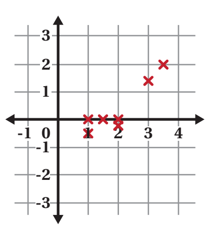


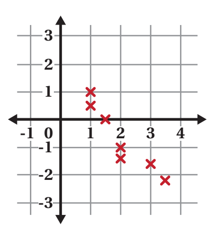


4. These residual plots are from the same set of data, but each one represents a different line of fit. Which residual plot shows the best line of fit?

A.


B.


C.


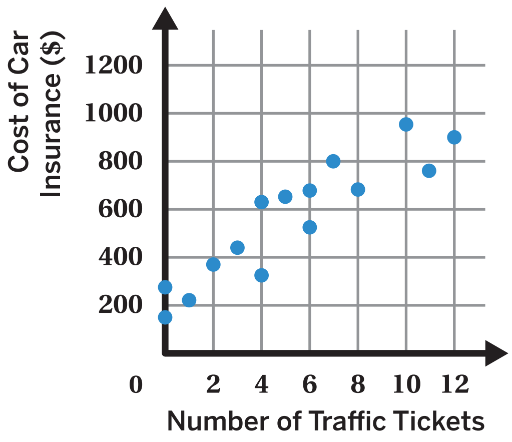
D.


Explain your thinking.

Lesson Practice
A1.3.16

Name: .... Date: .... Period: ....

Problems 5–6: This scatter plot shows the number of traffic tickets and the cost of car insurance for 16 people.



5. Which r-value could represent the correlation coefficient for this data?

A. 0.25 **B.** -0.25

C. 0.9 **D.** -0.9

6. Which equation could represent the line of best fit?

A. $y = -62x + 220$ **B.** $y = 62x + 220$ **C.** $y = -220x + 62$ **D.** $y = 220x + 62$

Explain your thinking.

Spiral Review

Problems 7–9: Solve each equation for x.

7. $y = 3x + 5$

8. $4y - 2x = 10$

9. $y = \frac{1}{6}x$

10. Are there any outliers in this data set? Use a calculator if it helps with your thinking.

$$[1, 1, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 20]$$

Circle one. Yes No

Explain your thinking.

Reflection

1. Put a star next to the problem you think is the most important.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

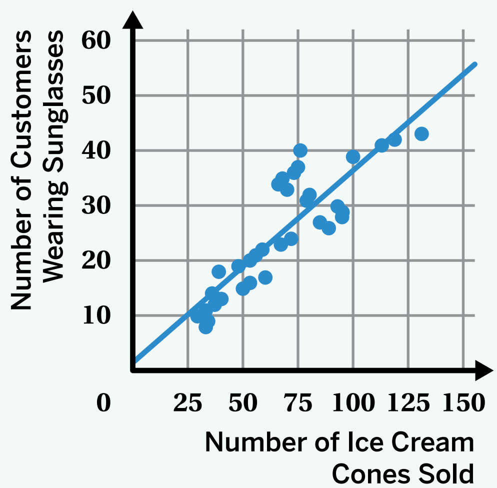
The **line of best** fit is the line on a scatter plot that best represents the trend created by the points in a data set.

Instead of sketching a line of fit, you can use a graphing calculator to precisely generate the equation of the line of best fit from a scatter plot. The equation of the line of best fit can help you interpret information about a situation, or allow you to substitute values into the equation to make predictions. You can also use a graphing calculator to calculate the correlation coefficient for a given data set.

Here is an example of a line of best fit generated by a graphing calculator, and the information about the line that a graphing calculator will show you.

$y = 0.351312x + 1.31984$

STATISTICS	RESIDUALS
$r_2 = 0.7642$	plot residuals
$r = 0.8742$	plot residuals

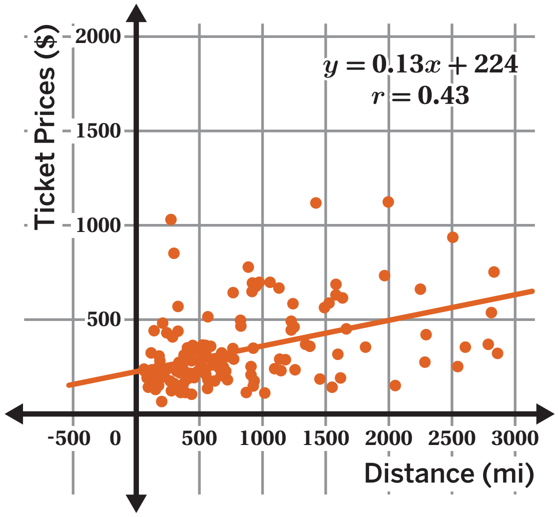


Things to Remember:

Lesson Practice
A1.3.17

Name: .... Date: .... Period: ....

Problems 1–3: This scatter plot shows the distances of some one-way flights and their ticket prices.



1. The equation for the line of best fit is $y = 0.13x + 224$.

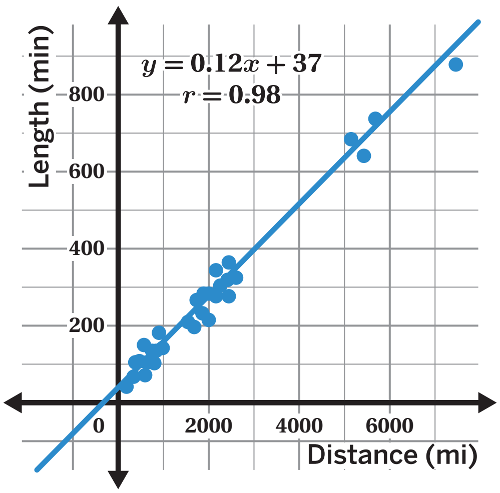
What does 0.13 mean in this situation?

2. The distance from Phoenix, Arizona, to Jacksonville, Florida, is $1{,}795$ miles.

Use the equation of line of best fit to predict the cost of a plane ticket from Phoenix to Jacksonville.

3. Do you think the prediction is accurate? Use the r-value to explain your thinking.

Problems 4–6: This scatter plot shows the distances of some non-stop flights and their lengths in minutes.



4. The equation for the line of best fit is $y = 0.12x + 37$.

What does 0.12 mean in this situation?

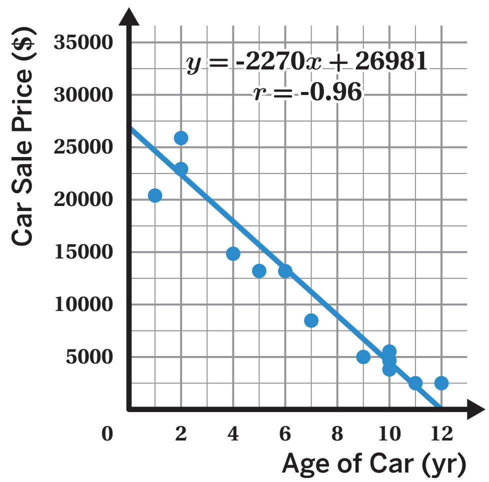
5. Use the equation of line of best fit to predict the length, in minutes, of a direct flight from Phoenix to Jacksonville ($1{,}795$ miles).

6. Do you think the prediction is accurate? Use the r-value to explain your thinking.

Lesson Practice
A1.3.17

Name: ________ Date: ________ Period: ________

Problems 7–9: Kwasi wanted to know the relationship between the ages of cars and their values. He found some data on the ages of several cars (in years) and their sale prices (in dollars).



7. Describe the relationship between the age of a car and its sale price.

8. Do you think one of the variables causes the other? Explain your thinking.

9. What else might affect this relationship? Explain your thinking.

Spiral Review

Problems 10–11: Here is a data set: $[1, 1, 2, 2, 3, 3, 7, 8, 9, 10, 11, 35]$

10. Complete the table. Use a graphing calculator if it helps with your thinking.

Min.	Q1	Median	Q3	Max.

11. Are there any outliers in this data set? Circle one. Yes No

Problems 12–14: Solve each equation for y.

12. $7 = 6x - y$

13. $3y + 15x = 24$

14. $4y - x = 44$

Reflection

1. Circle the question that you are least confident about.

2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 30

Lesson Summary

We can represent rules with verbal descriptions or as a table of inputs and outputs. All sets of inputs and outputs are called relations. A *function* is a special kind of relation that assigns exactly one output to each possible input.

You can determine whether a rule is a function by organizing the inputs and outputs into a table. If one input has multiple possible outputs, then the rule is not a function.

Here are two examples.

Rule A takes an integer and outputs an integer that is one less.

Input	Output
1	0
2	1
2	1
4	3

In this relationship, Rule A is a function because each input has exactly one output.

Rule B takes a number and outputs a random number that is greater.

Input	Output
0	2
0	10
-2	0
-1.6	-1.2

In this relationship, Rule B is *not* a function because each input has multiple outputs.

Things to Remember:

Lesson Practice
A1.4.01

Name: .... Date: .... Period: ....

1. Rule A takes any word as an input and writes the word backwards as an output.

Is Rule A a function?
Explain your thinking.

Rule A

Input	hat	sock	racecar
Output	tah	kcos	racecar

Problems 2–3: Here is Rule B.

Rule B

Input	4	6	6	5
Output	blue	purple	yellow	white

2. Is Rule B a function?
Explain your thinking.

3. Predict what the output could be when the input is 3.

Problems 4–5: Here is Rule C.

Rule C

Input	6	8	12	14
Output	4	6	10	?

4. Is Rule C a function?
Explain your thinking.

5. Predict the missing output for Rule C.

6. This table shows the total number of days in each month of a given year.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
28		×										
29		×										
30				×		×			×		×	
31	×		×		×		×	×		×		×

Imagine a rule where the input is a month and the output is the number of days in that month. Does this rule represent a function? Explain your thinking.

7. A machine uses Rule D to turn inputs into outputs. The table shows two inputs and their outputs.

Hoang tried the input 4 again and the output was not 27. He claims that this is enough information to determine whether or not Rule D is a function.

Is Hoang correct? Explain how you know.

Rule D

Input	Output
3	21
4	27

Spiral Review

8. Complete the table using this rule: Add 2 to the input, then multiply by 3 to get the output.

Input	-5	0	4
Output	-9		

Problems 9–10: A school sells two types of tickets for a play: adult tickets and student tickets. Adult tickets are $\$5$ each and student tickets are $\$2$ each. The school collects $\$400$ total.

9. Write an equation where x represents the number of adult tickets sold and y represents the number of student tickets sold.

10. How many of each ticket type could the school sell to collect $\$400$?

Adult tickets: .... Student tickets: ....

Reflection

1. Put a star next to the problem you spent the most time on.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Function notation is a way to write the inputs and outputs of a function. For example, $f(4) = 9$ is a statement written in function notation. It says that when the input of the function f is 4, the output is 9. In other words, when the value of the *independent variable* is 4, the value of the *dependent variable* is 9.

This table shows some input-output pairs for the function $s(t)$, that determine the price of a slice of pizza based on the number of toppings

$s(2) = 2.75$ is a statement written in function notation.

- $s(2)$ can be read as "s of two."
- For this situation, the number of toppings is the *independent variable*, t, and the price of a slice of pizza is the *dependent variable*, $s(t)$.
- $s(2) = 2.75$ means the price of a slice of pizza with 2 toppings is $\$2.75$.

Menu

Slice of Pizza
$\$1.75$ plus $\$0.50$ per topping

Number of Toppings	Price ($)
0	1.75
1	2.25
2	2.75

Things to Remember:

Lesson Practice
A1.4.02

Name: .... Date: .... Period: ....

Problems 1–2: The function $f(t)$ models the temperature, in degrees Celsius, t hours after midnight.

1. Select the equation that represents the statement: *At 1 AM, the temperature was* 20°*C*.

A. $f(100) = 20$ **B.** $f(20) = 100$ **C.** $f(1) = 20$ **D.** $f(20) = 1$

2. Use function notation to represent each statement.

The temperature at 2 AM. ....

The temperature was the same at 9 AM and at 11 AM. ....

The temperature was higher at 9 AM than at 2 AM. ....

t hours after midnight, the temperature was 24°C. ....

3. A restaurant sells three different salads. They use the functions $c(x)$, $g(x)$, and $p(x)$ to represent the cost of their caesar, garden, and pasta salads in dollars, with x additional ingredients added. Explain the meaning of each statement.

$g(0) = 10$	
$g(3) > c(1)$	
$p(2) < g(3)$	

Problems 4–6: Use the table to determine the missing values in the function statements.

x	$f(x)$
-5	17
-2	-5
5	23

4. $f($..........$) = 23$

5. $f(-5) =$ ....

6. $f($..........$) = -5$

Name: ____________ Date: ________ Period: ________

Problems 7–9: Desmos Pizza's online menu offers small, medium, and large pizzas. Fill in each blank to make each equation true.

MENU
Small: $12 plus $1 per topping
Medium: $15 plus $2 per topping
Large: $18 plus $3 per topping

7. $s(7) = m$ (________)

8. m (________) $= l(5)$

9. l(________) $= s$ (________)

Spiral Review

10. Here are Rules A and B.

Which rule is a function? Circle one.

Rule A Rule B Both Neither

Rule A

Input	Output
4	2
9	-3
9	3

Rule B

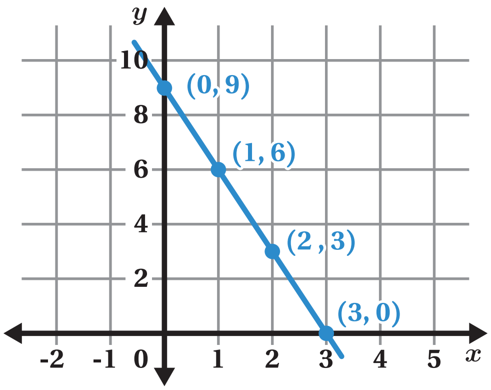
Input	Output
1	5
2	9
2	9

11. Here are two data sets. Which one has the larger standard deviation? Circle one.

Data Set A: 6, 7, 8, 8, 8, 8.5, 9

Data Set B: 4, 7, 8, 8, 9, 12, 12

12. Here's a graph of a relationship and its table of values.



x	y
0	9
1	6
2	3
3	0

Write an equation to represent this relationship.

Reflection

1. Put a heart next to a problem you understand well.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can represent function rules with equations, verbal descriptions, and tables.

For example, the function $s(t)$ describes the relationship between the cost of a slice of pizza and the number of toppings, t.

Description

Menu
Slice of Pizza
$1.75 plus $0.50
per topping

Table

Number of Toppings	Price ($)
0	1.75
1	2.25
2	2.75

Equation

$s(t) = 1.75 + 0.50t$

You can use the equation to determine different values of the function.

Let's determine the value of $s(4)$:

$s(4) = 1.75 + 0.50(4)$

$s(4) = 3.75$

This means the price of a slice of pizza with 4 toppings is $3.75.

Things to Remember:

Lesson Practice
A1.4.03

Name: ________ Date: ________ Period: ________

1. Let $f(x) = 2x + 5$.

Calculate the value of each function notation expression.

The first value is already completed.

Expression	Value
$f(0)$	5
$f(4)$	
$f(6)$	
$f(-3)$	

Problems 2–3: A toy factory makes toy bunnies. Each toy bunny holds a carrot. A bunny's height, $h(x)$, is three times the length of the carrot, x.

2. Complete the table.

x	1	2	3	4	5	6
$h(x)$						

3. Write an equation for the function $h(x)$.

4. Here are two functions: $f(x) = -15x + 80$ and $g(x) = 10x + 25$. Which is greater? Circle one.

$f(2)$ or $g(2)$

Explain your thinking.

Problems 5–6: The function $p(s)$ models the perimeter of a square of side length s. The perimeter is represented by the equation $p(s) = 4s$.

5. What is the value of $p(20)$?

6. What does your answer mean in this situation?

Name: .... Date: .... Period: ....

7. Model rockets are created in various sizes. The height of a rocket in inches, $h(x)$, depends on the radius of the base of the rocket in inches, x.

Use the table to write an equation for $h(x)$ that outputs the height of the rocket with a base radius of x.

Radius (in.), x	Height (in.), $h(x)$
1	5
3	13
5	21
10	41

Problems 8–9: The function $w(t)$ models the weight of a pumpkin, in pounds, as a function of how many months, t, it has been growing. Explain the meaning of each statement.

8. $w(2) = 5$

9. $w(6) > w(4)$

Spiral Review

Problems 10–11: Here are the first four figures in a pattern.

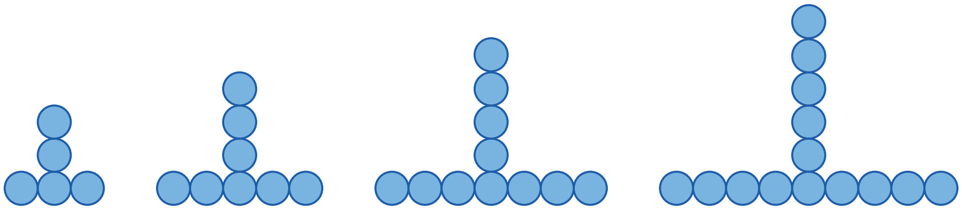


Figure 1 **Figure 2** **Figure 3** **Figure 4**

10. How many dots will be in Figure 5?

11. Write an equation for the number of dots, $d(n)$, in Figure n.

Reflection

1. Circle a problem you want to talk to a classmate about.

2. Use this space to ask a question or share something you're proud of.

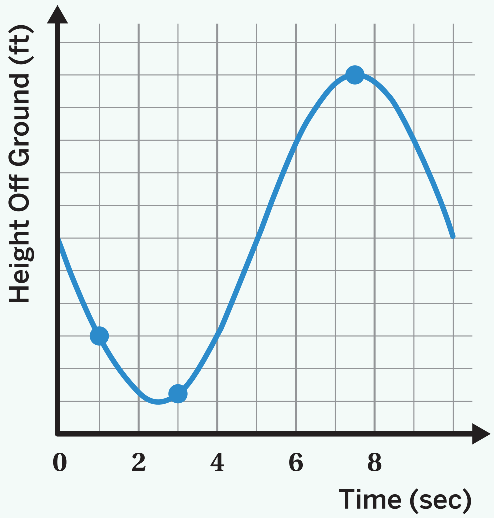
Lesson Summary

A graph can reveal in more detail what is happening during a situation. Here is an example.

The function $h(t)$ represents the height of the cart on the Ferris wheel at time t.

We can use the graph to describe many parts of the situation. For example:

- At around 7.5 seconds, the Ferris wheel cart is at its maximum height.
- $h(1)$ is greater than $h(3)$. This means the Ferris wheel cart was higher off the ground at 1 second than at 3 seconds.



While we can use the graph to describe many things, there are lots of things the graph cannot describe. For example:

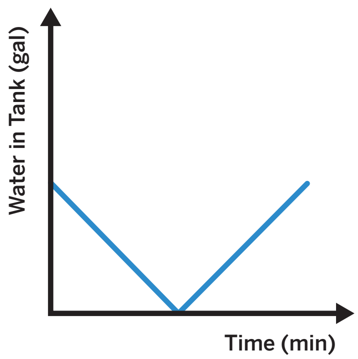
- How much fun the people are having
- How many people are riding the Ferris wheel

Things to Remember:

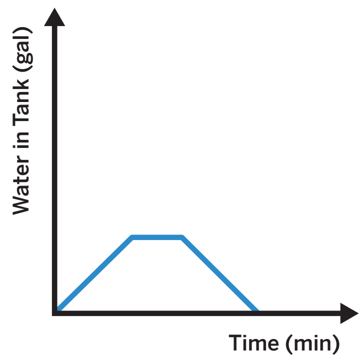
Name: .... Date: .... Period: ....

1. An empty water tank is filled until it is half full. Two minutes later, it drains until it is empty again. Which graph could represent this situation? Circle your choice.

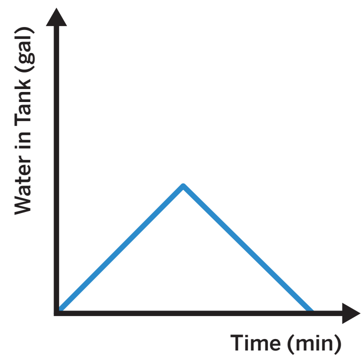
Graph A



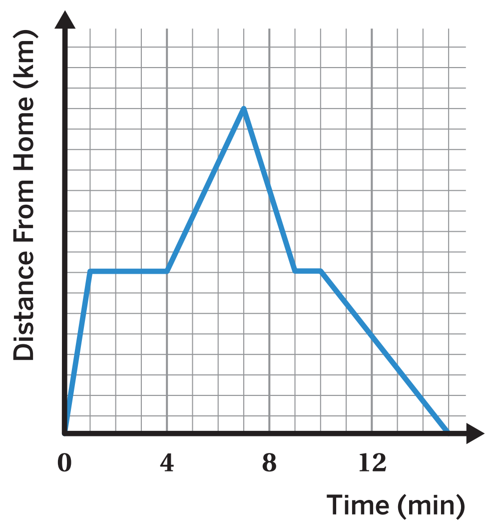
Graph B



Graph C



Problems 2–4: Prisha rode her bike around town. Her fitness tracker made a graph to represent the distance she was from her home at any given time during her ride.



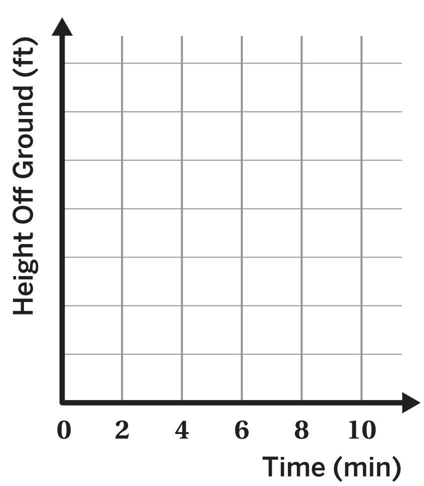
2. How many minutes was Prisha's bike ride?

3. At what time was Prisha the farthest distance from her home?

4. How long did Prisha rest during her ride?

5. Here is some information about a hot air balloon ride.
   - Ascends (goes up) quickly for 2 minutes.
   - Ascends slowly for another minute until it reaches its maximum height.
   - Maintains its maximum height for 3 minutes.
   - Descends (goes down) for the next 4 minutes until it lands on the ground.

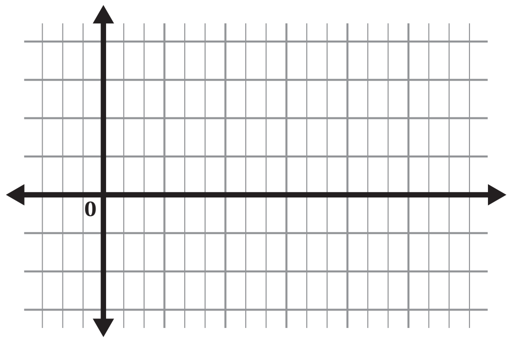
   Make a graph that could represent the height of a hot air balloon over time.



Name: .... Date: .... Period: ....

Problems 6–7: Aba described yesterday morning at school like this:

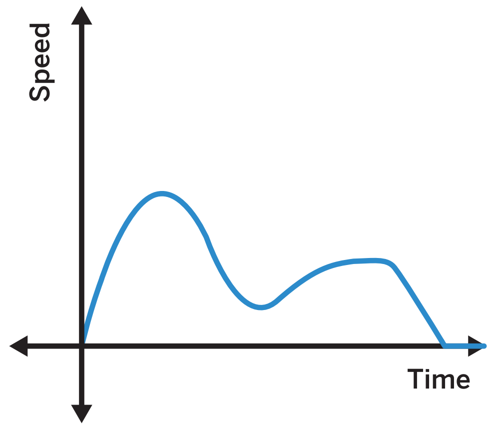
I entered the school on the ground floor, then walked up the stairs to the third floor to attend an hour-long class. Afterward, I had an hour-long class in the basement, then I went up to the ground floor and sat outside to eat my lunch.



6. Label both axes.

7. Sketch a possible graph of Aba's height from the ground floor as a function of time.

Problems 8–9: Here is a graph of speed and time.



8. Which sport could this graph represent?

A. Fishing
B. Skydiving
C. 100-yard sprint
D. Golf
E. Soccer

9. Describe how you think that sport fits the graph.

Spiral Review

10. The function $p(t)$ represents the height of water in a bathtub, in inches, after t minutes. Match each sentence to its equation.

a. After 20 minutes, the bathtub is empty. .... $p(10) = 4$

b. The bathtub starts out with no water. .... $p(t) = w$

c. After 10 minutes, the height of the water is 4 inches. .... $p(20) = 0$

d. The height of the water is 10 inches after 4 minutes. .... $p(0) = 0$

e. The height of the water is w inches after t minutes. .... $p(4) = 10$

Reflection

1. Put a question mark next to a problem you're feeling unsure of.

2. Use this space to ask a question or share something you're proud of.

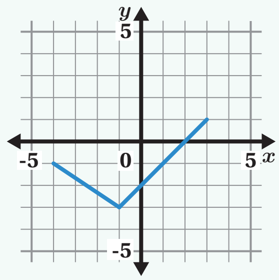
Lesson Practice A1.4.05

Science Mom Lesson 34

Lesson Summary

We can use the key features of a graph to help us describe a function or sketch a possible graph of a function. Here is an example. Let's analyze the graph of this function.

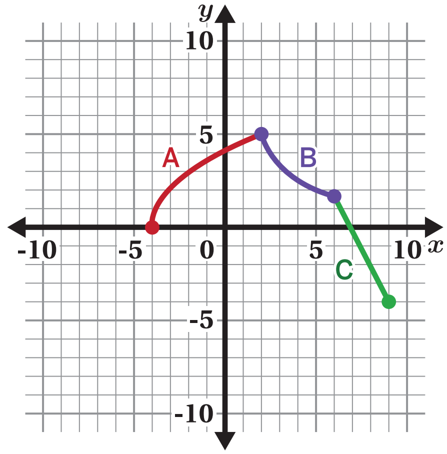
Minimum: The lowest point on a graph.	(-1, -3)
Maximum: The highest point on a graph.	(3, 1)
Positive: The x-values where the function has positive outputs; the graph is *above* the x-axis.	$x > 2$
Negative: The x-values where the function has negative outputs; the graph is *below* the x-axis.	$x < 2$
Increasing: The x-values where the graph is sloping upward as you read the graph from left to right. As the inputs increase, the outputs also increase.	$x > -1$
Decreasing: The x-values where the graph is sloping downward as you read the graph from left to right. As the inputs increase, the outputs decrease.	$x < -1$



Things to Remember:

Name: .... Date: .... Period: ....

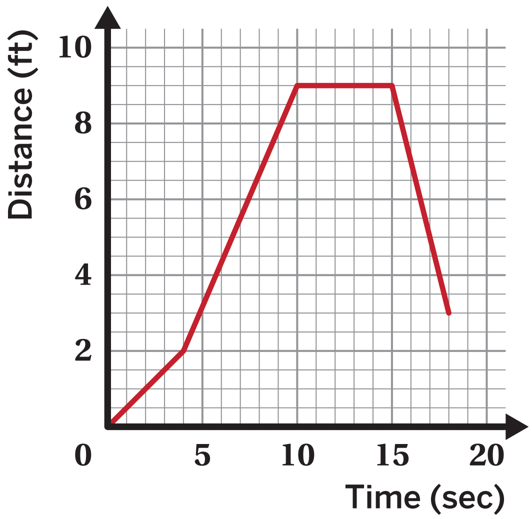
1. Select *all* the true statements about this graph.



- ☐ **A.** This graph is a function.
- ☐ **B.** Part A is decreasing.
- ☐ **C.** Part B is decreasing.
- ☐ **D.** The maximum is at $(2, 5)$.
- ☐ **E.** The minimum is at $(-4, 0)$.

2. Manuel is watching his younger sibling at the park. The graph represents the distance Manuel is from his sibling as a function of time.

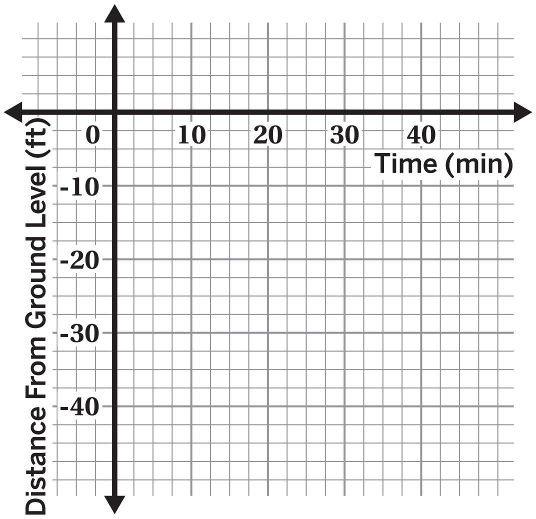
Describe Manuel's distance from his sibling over time. Use terms that you learned in this lesson.



3. Ivory goes on a tour of a cave. The tour starts at ground level.

- The tour stays at ground level for 15 minutes.
- Then the tour descends (goes down) for 15 minutes to a depth of 20 feet below ground level.
- The tour stays at this level for 10 minutes.
- The tour spends the last 5 minutes ascending (going up) to ground level.

Sketch a graph describing Ivory's elevation as a function of time.



Lesson Practice
A1.4.05

Name: .... Date: .... Period: ....

Problems 4–6: Here is a table that lists Seattle's temperatures for one day.

Hours After Midnight	Temperature (°F)
1	32
2	33
3	35
4	36
5	35

4. How many hours after midnight is the minimum temperature?

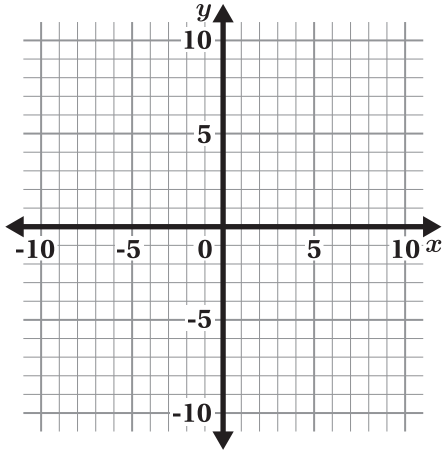
5. How many hours after midnight is the maximum temperature?

6. Between what hours is the temperature decreasing?

7. Here are three statements:

- The function is always positive.
- The function is always increasing.
- The function is always decreasing.

Sketch a graph of a function so that two of the statements are true and one is false.



Spiral Review

Problems 8–10: Determine each quotient.

8. $\frac{5}{6} \div \frac{2}{3}$

9. $\frac{8}{6} \div \frac{4}{3}$

10. $\frac{12}{20} \div \frac{18}{16}$

Reflection

1. Put a star next to your favorite problem.

2. Use this space to ask a question or share something you're proud of.

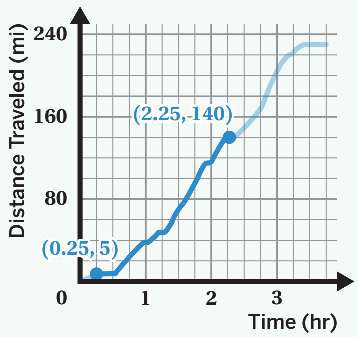
Science Mom Lesson 35

Lesson Summary

Functions can have different rates of change over different intervals. The **average rate of change** is equivalent to the *slope* of the line between two points.

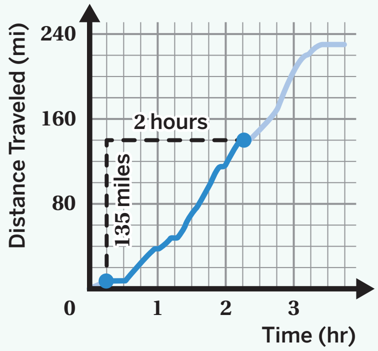
This graph represents Troy's car trip.

We can calculate the average rate of change over an **interval**, a specific length between two points, like the interval from 0.25 to 2.25 hours.

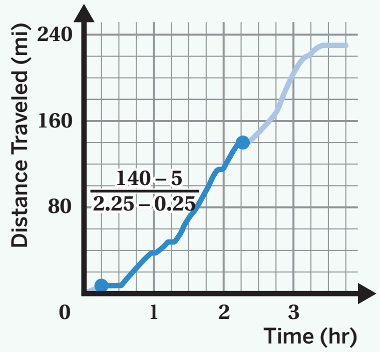


Here are two different strategies.

Strategy 1



Strategy 2



The average rate of change for the interval 0.25 to 2.25 hours is $\frac{135}{2} = 67.5$.

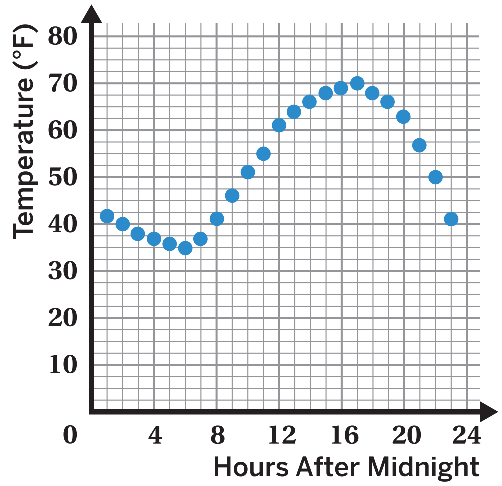
That means that Troy's average speed was 67.5 miles per hour in that interval.

Things to Remember:

Name: ____________ Date: ________ Period: ________

1. The temperature was recorded at several times in a 24-hour period. Function $t(n)$ gives the temperature in degrees Fahrenheit n hours after midnight. Use the graph to determine if the average rate of change for each interval is *positive* (+), *negative* (–), or *zero*. Place a check in the appropriate column.

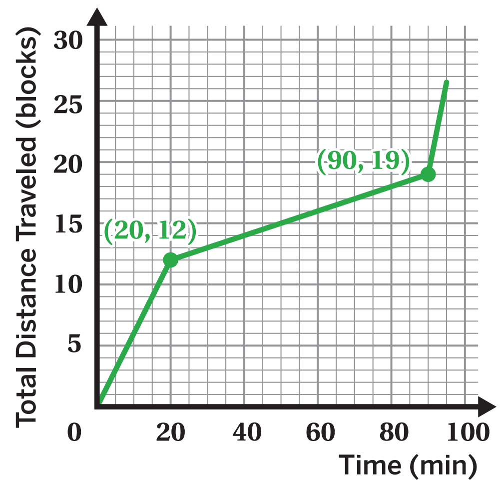
	+	–	0
$n = 1$ to $n = 5$			
$n = 5$ to $n = 7$			
$n = 10$ to $n = 20$			



Problems 2–3: This graph shows the total distance in city blocks, $d(t)$, that Pilar walked as a function of time in minutes, t.

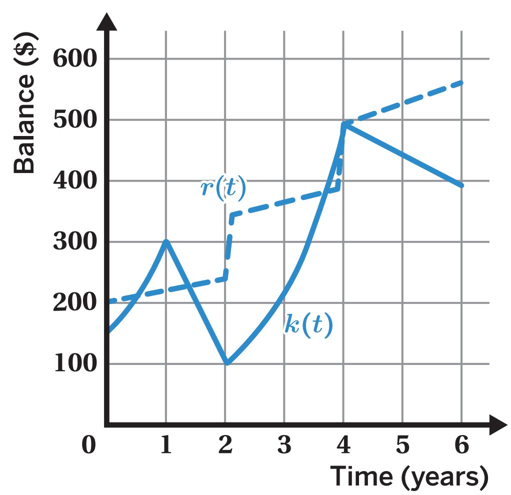
2. Determine the average rate of change between $t = 20$ and $t = 90$.

3. What do you think the average rate of change you calculated means in this situation?



4. $r(t)$ and $k(t)$ model the savings account balances of Rafael and Katie after t years. Select *all* the statements that are true.

- ☐ **A.** Katie has a lower average rate of change in the last two years.
- ☐ **B.** Katie's balance is always less than Rafael's.
- ☐ **C.** $r(2) = 100$
- ☐ **D.** Rafael's balance is increasing from year 0 to year 6.
- ☐ **E.** Rafael has a higher average rate of change in the first four years.



Lesson Practice
A1.4.06

Name: .... Date: .... Period: ....

Problems 5–7: This table shows the population of a city from 1988 to 2016.

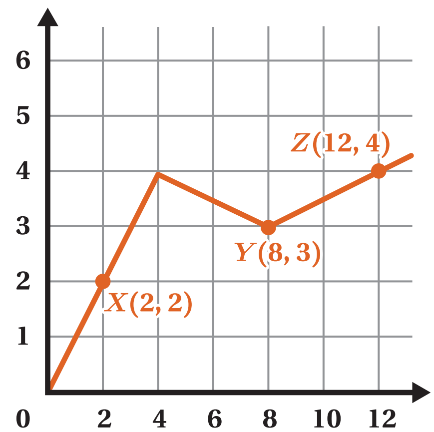
Year, t	Population, $p(t)$
1988	35,700
1992	42,700
1996	33,100
2000	33,700
2004	45,000
2008	48,400
2012	40,900
2016	43,000

5. Determine the average rate of change for $p(t)$ between 1992 and 2000.

6. State two values of t that create an interval with a *negative* rate of change.

7. State two values of t that create an interval with a *positive* rate of change.

8. Match each interval to its average rate of change.



Interval	Average Rate of Change
a. X to Y	.... $\frac{1}{5}$
b. Y to Z	.... $\frac{1}{4}$
c. X to Z	.... $\frac{1}{6}$

Spiral Review

9. Jada is walking to school. The function $d(t)$ gives the distance from school, in meters, t minutes since Jada left home. Which equation represents this statement? *Jada is* 600 *meters from school after* 5 *minutes*.

A. $d(5) = 600$ **B.** $d(600) = 5$ **C.** $t(5) = 600$ **D.** $t(600) = 5$

10. Complete the arithmetic sequence with the missing terms: ...., 6, ..., 22, 30

Reflection

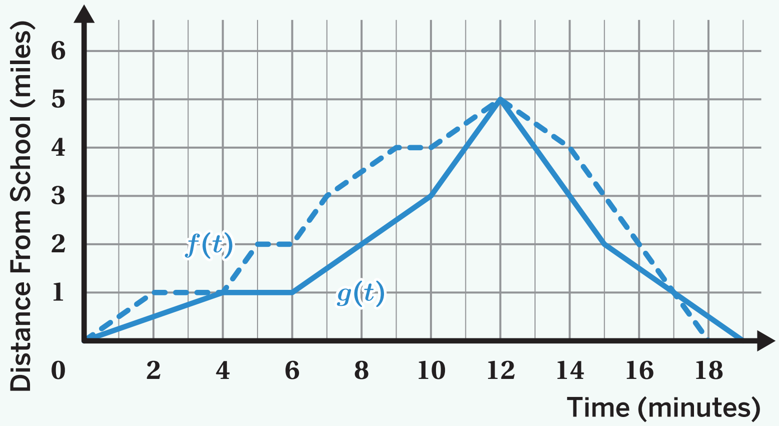
1. Put a heart next to the problem you're most proud of.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

We can analyze functions by comparing their key features at different intervals and then use function notation to describe them.

For instance, here are some true statements about these two graphs:



- When $t = 4$, $f(t) = g(t)$.
- $f(8) > g(8)$
- $f(12) = g(12)$
- $f(15) > g(15)$
- $f(t)$ and $g(t)$ have the same maximum.
- $f(t)$ and $g(t)$ are both decreasing from 12 to 15 minutes.
- $f(t)$ and $g(t)$ have the same average rate of change from 5 to 6 minutes.

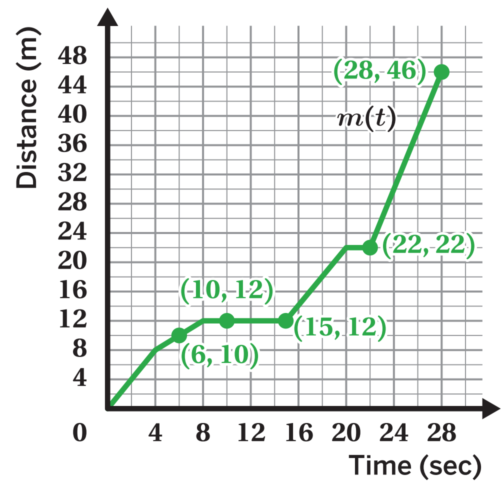
Things to Remember:

Lesson Practice
A1.4.07

Name: .... Date: .... Period: ....

Problems 1–2: Mai built a model race car for a school competition.

$m(t)$ represents the distance of Mai's car, in meters, after t seconds.



1. Use the graph to determine the missing value in each function statement.

$m(\ldots\ldots) = 10$

$m(10) = \ldots\ldots$

$m(22) = \ldots\ldots$

$m(\ldots\ldots) = 46$

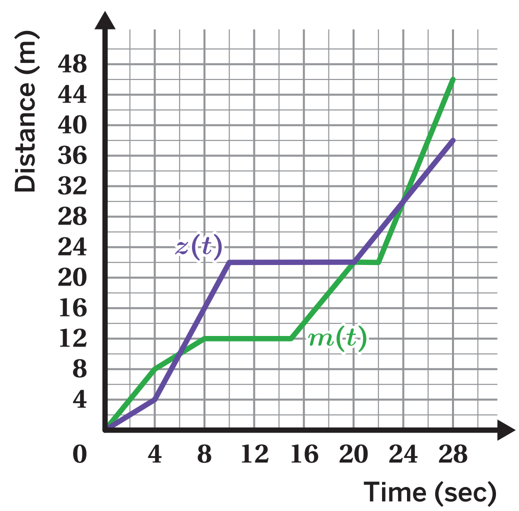
2. Over what interval did Mai's car travel the slowest?

A. 0 to 4 seconds **B.** 4 to 8 seconds **C.** 8 to 15 seconds **D.** 15 to 20 seconds

Problems 3–6: Zion also built a model race car for the school competition.

$z(t)$ represents the distance of Zion's car, in meters, after t seconds.

Did Zion or Mai have the greater average rate of change over the following intervals? Explain your thinking.



3. $t = 4$ to $t = 8$

4. $t = 10$ to $t = 20$

5. Name a time when Zion's and Mai's cars had traveled the same distance.

6. Select *all* the true statements.

- ☐ **A.** $m(t)$ has a greater maximum than $z(t)$.
- ☐ **B.** $z(t)$ and $m(t)$ have the same minimum at $(6, 10)$.
- ☐ **C.** $z(20) = m(20)$
- ☐ **D.** $m(15) > z(15)$
- ☐ **E.** $m(t)$ and $z(t)$ both increase from 22 to 28 seconds.

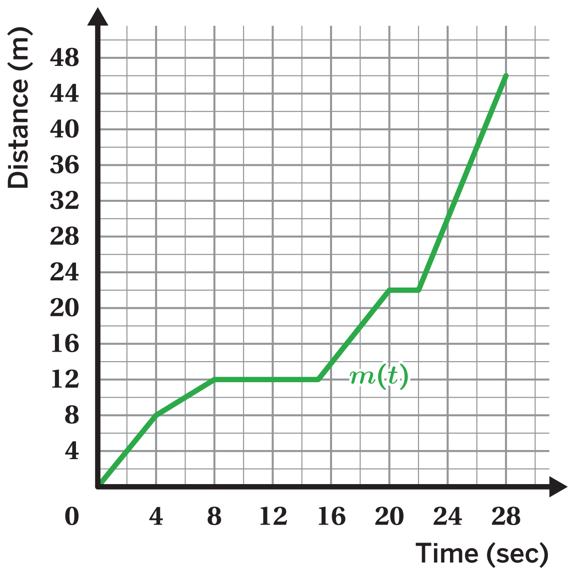
Lesson Practice
A1.4.07

Name: .... Date: .... Period: ....

7. Parv built a race car to race against Mai.

Use this information to make a graph that could represent the distance of Parv's race car, $p(t)$, after t seconds:

- $p(8) < m(8)$
- $p(12) = m(12)$
- The average rate of change of $p(t)$ and $m(t)$ is the same from $t = 22$ to $t = 28$.
- $m(t)$ has a greater maximum than $p(t)$.



Spiral Review

Problems 8–10: Nekeisha goes for a bike ride. $d(t)$ represents Nekeisha's distance from home, in miles, t minutes after she leaves.

Explain the meaning of each statement in context:

$d(0) = 0$	
$d(30) = d(60)$	
$d(90) = 0$	

Reflection

1. Put a star next to the problem you understood best.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

The *domain* and *range* of a function can each be described using a **compound inequality**, which is two or more inequalities joined together. You can write a compound inequality using symbols or using the words "and" or "or."

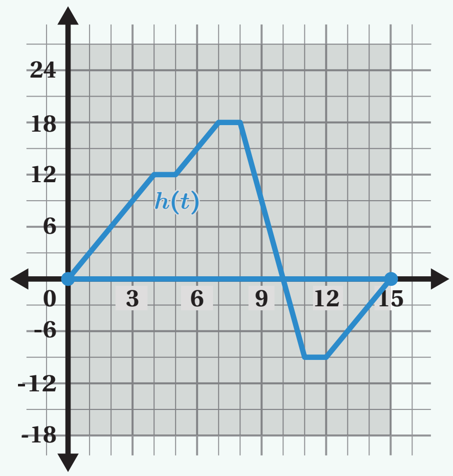
A graph can help you visualize the domain and range of a function, making it easier to describe them using compound inequalities.

Domain: The width of the function, or how far left and right the function goes. The domain is also the set of all inputs to the function, or all values of the independent variable. The domain of this function is all the values of t from 0 to 15.

$t \geq 0$ and $t \leq 15$

$0 \leq t$ and $t \leq 15$

$$0 \leq t \leq 15$$

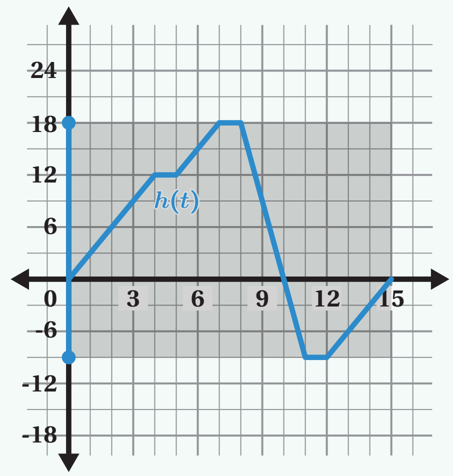


Range: The height of the function, or how far up and down the function goes. The range is also the set of all outputs of the function, or all values of the dependent variable. The range of this function is all the values of $h(t)$ from -9 to 18.

$h(t) \geq -9$ and $h(t) \leq 18$

$-9 \leq h(t)$ and $h(t) \leq 18$

$$-9 \leq h(t) \leq 18$$



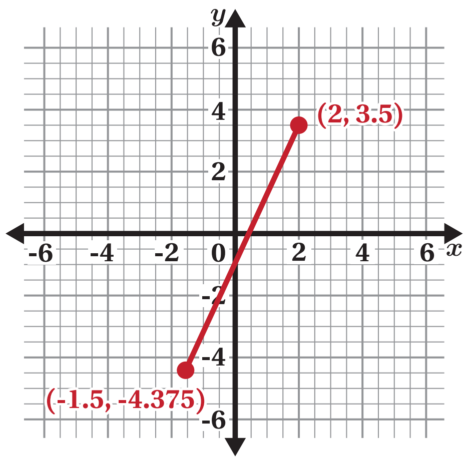
Things to Remember:

Lesson Practice
A1.4.09

Name: ____________ Date: ________ Period: ________

Problems 1–2: Valeria and Thiago disagree about the domain of $f(x)$.

- Valeria says the domain is $-1.5 \le x \le 2$
- Thiago says the domain is $-4.375 \le x \le 3.5$

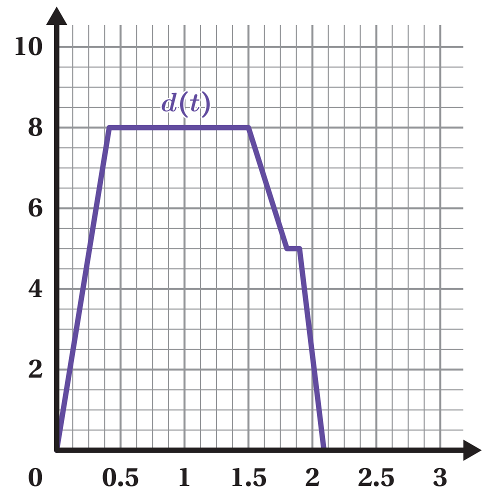


1. Whose answer is correct? Circle one.

Valeria's Thiago's

2. Explain why the other person's answer is incorrect.

Problems 3–5: Haru bikes to his friend's house. After a while, he heads home. On the way, he stops at the store to buy a bottle of water. $d(t)$ represents Haru's distance from his house, in kilometers, after t hours. This graph shows Haru's distance over time.



3. Which inequality describes the domain of $d(t)$?

A. $0 \le d(t) \le 2.1$ **B.** $0 \le d(t) \le 8$

C. $0 \le t \le 2.1$ **D.** $0 \le t \le 8$

4. Which inequality describes the range of $d(t)$?

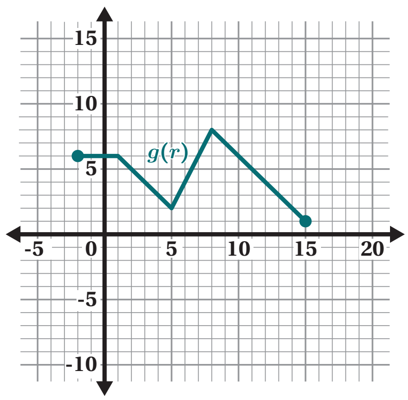
A. $0 \le d(t) \le 2.1$ **B.** $0 \le d(t) \le 8$

C. $0 \le t \le 2.1$ **D.** $0 \le t \le 8$

5. If Haru had not stopped at the store, would that change the domain or the range? Circle one.

Domain Range Both Neither

Problems 6–7 Here is the graph of $g(r)$.



6. Write a compound inequality to describe the domain.

7. Write a compound inequality to describe the range.

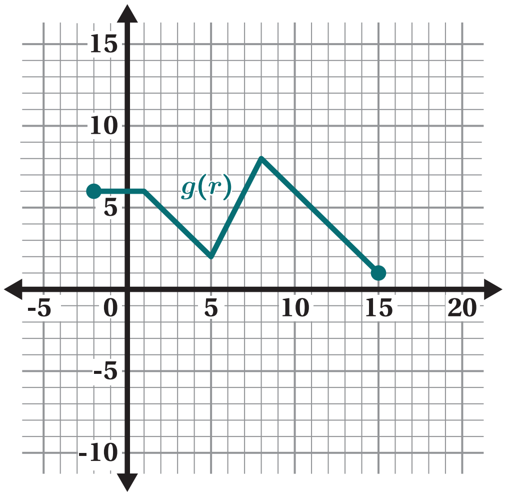
Lesson Practice
A1.4.09

Name: ________ Date: ________ Period: ________

Spiral Review

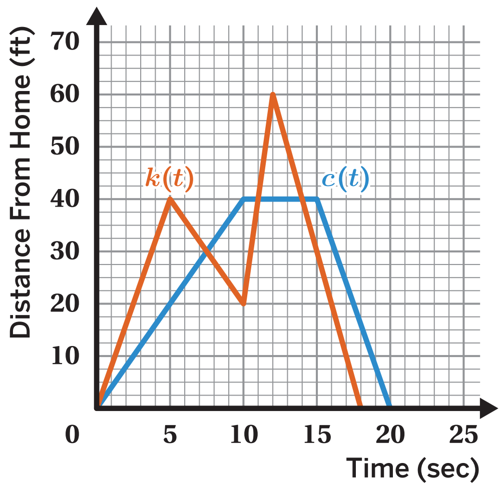
8. Determine the average rate of change for each interval on the graph of $g(r)$.

Interval	Average Rate of Change
$r = -2$ to $r = 1$	
$r = -2$ to $r = 5$	
$r = -2$ to $r = 8$	



9. Functions $c(t)$ and $k(t)$ represent the distance of two cats from home after t seconds. Select *all* the true statements.

- ☐ **A.** $k(5) > c(5)$
- ☐ **B.** $k(t)$ and $c(t)$ have the same domain and range.
- ☐ **C.** $k(t)$ keeps increasing from 0 to 13 seconds.
- ☐ **D.** $k(11) = c(11)$
- ☐ **E.** Both cats return home.



10. Tickets to the state fair cost \$10 each. The function $c(t) = 10t$ gives the total cost in dollars, $c(t)$, for the number of tickets purchased, t. Select *all* the values that are possible outputs for $c(t)$.

☐ **A.** 0 ☐ **B.** 70 ☐ **C.** 105 ☐ **D.** 880 ☐ **E.** 963

Reflection

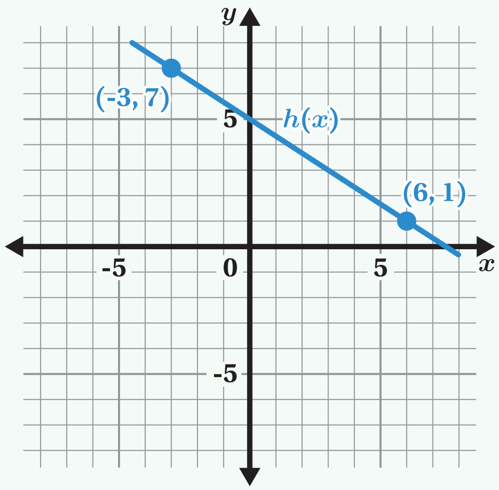
1. Put a star next to a problem where you revised your thinking.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can restrict a function's domain or range to highlight specific portions of a graph. Inequalities are one way to represent these restrictions symbolically.

Here is an example. Let's restrict the domain and range of $h(x)$ to highlight the interval from (-3, 7) to (6, 1).



To restrict the domain, use the x-values of each ordered pair:

Domain: $-3 \le x \le 6$

When you restrict the domain, you are restricting the x-values, which should be included when you write the inequality.

To restrict the range, use the y-values of each ordered pair:

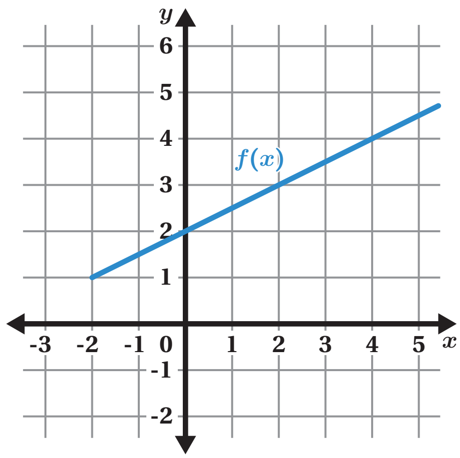
Range: $1 \le h(x) \le 7$

When you restrict the range, you are restricting the y-values, or the output values of $h(x)$.

Things to Remember:

Name: ____________ Date: ________ Period: ________

Problems 1–2: Here is the graph of $f(x)$.



1. What is the domain of $f(x)$?

A. $x \geq 0$ B. $x \geq -2$

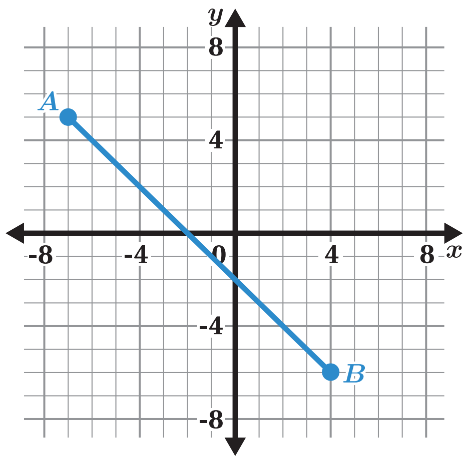
C. $x \leq -2$ D. $x \geq 1$

2. What is the range of $f(x)$?

A. $f(x) \geq 0$ B. $f(x) \geq -2$

C. $f(x) \leq 1$ D. $f(x) \geq 1$

3. Fill in the blanks for the domain and range of $y = -x - 2$ from point A to point B.



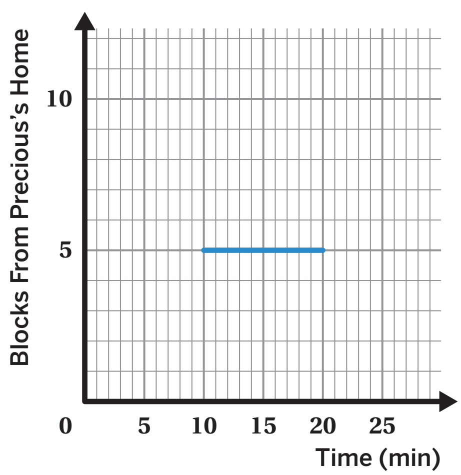
________ $\leq x \leq$ ________

________ $\leq y \leq$ ________

Problems 4–5: Precious leaves her home to go to the grocery store. This is her path:

- She walks to the store, which is 5 blocks away, at a speed of half a block per minute.
- She is in the store for 10 minutes.
- She runs back home at a speed of 1 block per minute.

The graph shows part of her path.



4. Sketch the missing pieces of the graph of Precious's path.

5. Which equation represents the beginning piece of Precious's path? Circle your choice.

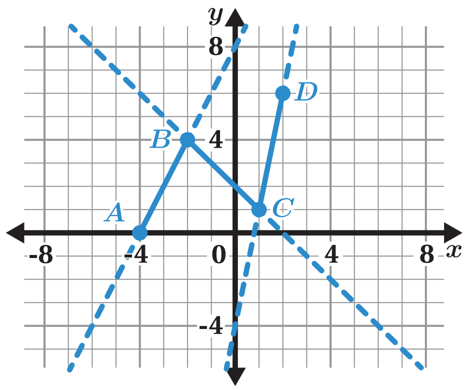
A. $y = 0.5x$ $\{0 < x < 5\}$ B. $y = 0.5x$ $\{0 < x < 10\}$ C. $y = -x$ $\{20 < x < 25\}$ D. $y = -x + 15$ $\{10 < x < 20\}$

Explain your thinking.

Lesson Practice
A1.4.10

Name: .... Date: .... Period: ....

Problems 6–8: Sneha tries to connect points A to B to C to D with three segments. She graphs three lines but needs help restricting each line's domain. Determine the domain restriction for each of the lines.

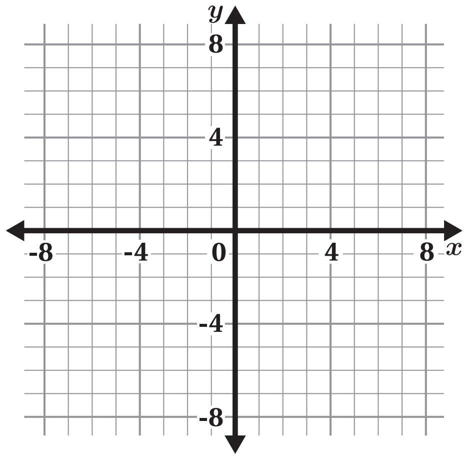


6. Domain restriction for A to B:

7. Domain restriction for B to C:

8. Domain restriction for C to D:

9. Create a design on the graph using at least six lines. Then write the equations with their domain or range restriction to represent each line segment in your design.



Spiral Review

10. Elena and Alexis are deciding between two cafeteria meal plans. They estimate that they will buy 15 meals from the cafeteria each month.

Which meal plan would cost them less? Circle one.

Plan A Plan B

Explain your choice.

Meal Plans

Plan A: $2.50 per meal
Plan B: $30 per month

Reflection

1. Circle the problem you enjoyed doing the most.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice A1.4.11

Science Mom Lesson 39

Lesson Summary

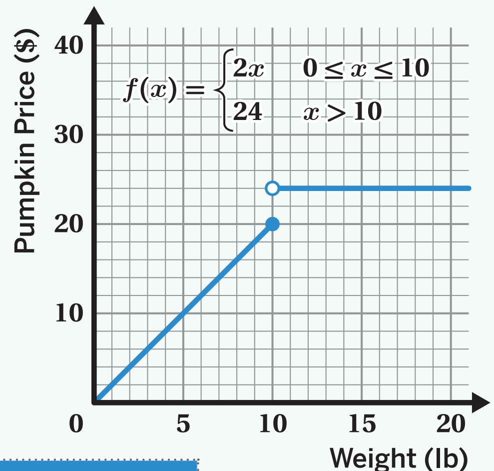
A **piecewise-defined function** is a function in which different rules apply to different intervals in its domain. You can use a graph or an equation to evaluate a piecewise-defined function.

Let's look at an example. The function $f(x)$ represents the price of a pumpkin with a weight of x pounds.

You can use the graph to evaluate $f(4)$ and $f(15)$.

- The point $(4, 8)$ is on the graph, so $f(4) = 8$.
- The point $(15, 24)$ is on the graph, so $f(15) = 24$.

You can also use the equation to evaluate $f(4)$ and $f(15)$.



Value	Domain Interval	Equation	Evaluate
$f(4)$	$x = 4$ is in $0 \le x \le 10$	$f(x) = 2x$	$f(4) = 2(4)$ $= 8$
$f(15)$	$x = 15$ is in $x > 10$	$f(x) = 24$	$f(15) = 24$

Things to Remember:

Lesson Practice

A1.4.11

Name: .... Date: .... Period: ....

Problems 1–3: Here is the piecewise-defined function, $g(x)$. Determine each value.

$$g(x) = \begin{cases} -3x & 0 \le x < 2 \\ 12 & x \ge 2 \end{cases}$$

1. $g(0) =$ ....

2. $g(2) =$ ....

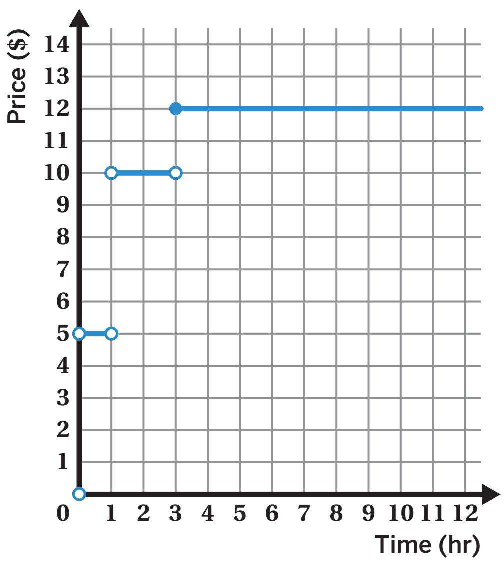
3. $g(4) =$ ....

Problems 4–5: A parking garage charges \$5 to park for less than 1 hour, \$10 to park for 1 to 3 hours, and \$12 to park for more than 3 hours. Let $c(t)$ represent the price of parking, in dollars, for t hours.

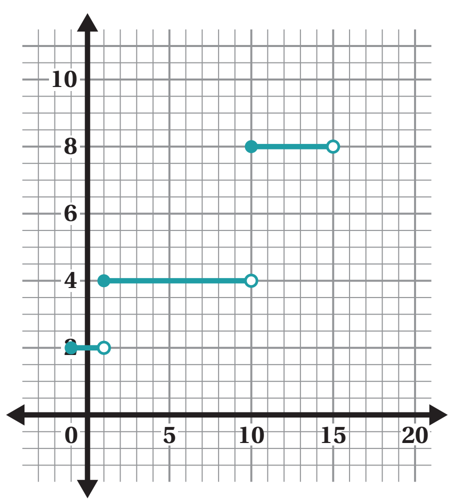
4. Complete the table.

t (hr)	0	0.5	1	2	3	5
$c(t)$ (\$)						

5. The parking garage tried to represent their pricing with this graph. What is correct and what should change to make the graph more accurate?



Problems 6–8: Here is the graph of function $h(x)$.



6. Complete $h(x)$ so that it matches the graph.

$$h(x) = \begin{cases} \square & -1 \le x < 1 \\ 4 & \square \le x < \square \\ 8 & 10 \le x < 15 \end{cases}$$

7. What is the value of $h(2)$?

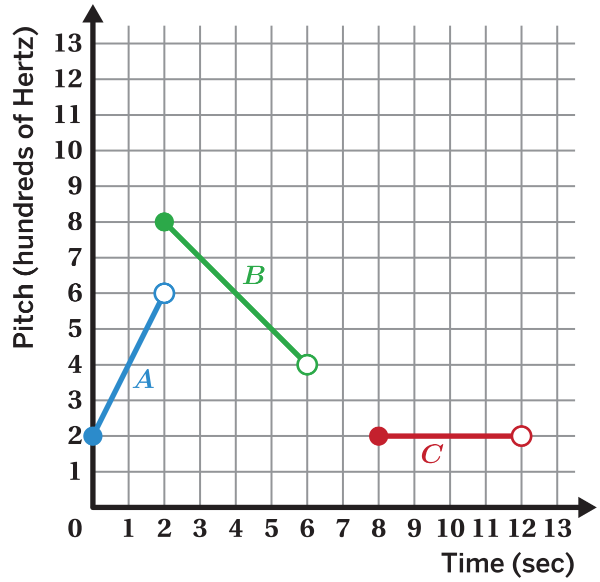
8. What is the value of $h(10)$?

Name: .... Date: .... Period: ....

9. The pitch of a singer's recorded voice is represented by the piecewise function $p(t)$ and its graph consists of pieces A, B, and C.

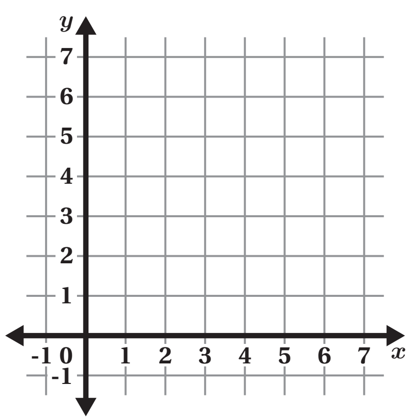
$$p(t) = \begin{cases} 2t + 2 & 0 \le t < 2 \\ -t + 10 & 2 \le t < 6 \\ 2 & 8 \le t < 12 \end{cases}$$

Explain how you could change the function to eliminate any breaks or jumps in the recording.

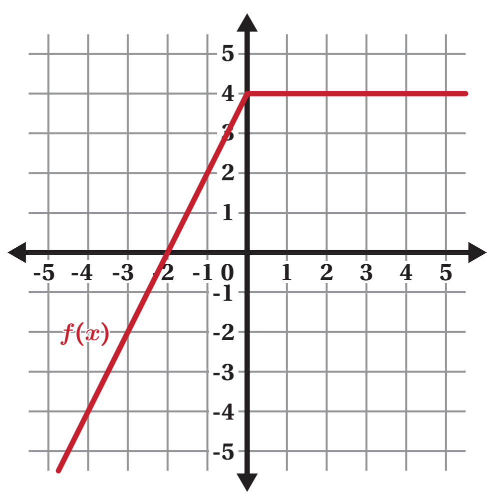


Spiral Review

10. Graph the function $f(x) = x + 2$ over the domain $0 \le x \le 3$.



Problems 11–13: Use the graph of $f(x)$ to determine the value of each function notation statement.



11. $f(-3)$ **12.** $f(0)$ **13.** $f(3)$

Reflection

1. Circle the problem you think will help you most on the End-of-Unit Assessment.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice A1.4.12

Science Mom Lesson 40

Lesson Summary

You can use *piecewise-defined functions* to represent situations. A **step function** is one special kind of piecewise-defined function, where every section of the graph is a point or a horizontal line at a constant value.

Here are some helpful ways to write the equations of a piecewise-defined function:

- The number of conditions in an equation is equal to the number of pieces in the function and graph.
- In the piecewise equation, each piece represents one condition and has its own domain.
- You can write the domain as an inequality.
  - $\geq$ or $\leq$ means to include that value.
  - $>$ or $<$ means to exclude that value.
- You can represent each condition by a section of the graph with open boundary points ($>$ or $<$) or closed boundary points ($\geq$ or $\leq$).

Graphing your piecewise-defined function might also help in making sense of the situation.

Things to Remember:

Lesson Practice
A1.4.12

Name: ______ Date: ______ Period: ______

Problems 1–2: An online makeup store charges shipping fees depending on the weight of the order.

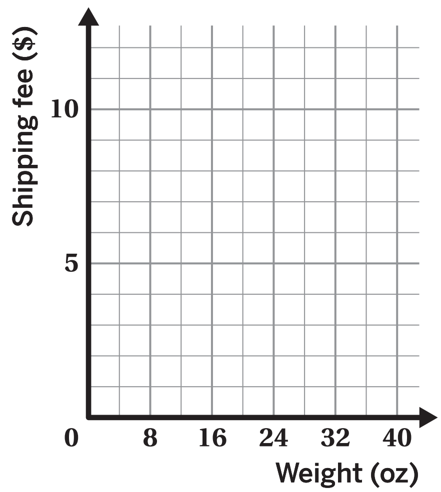
- Orders up to and including 16 ounces have a $\$3$ shipping fee.
- Orders greater than 16 ounces but less than or equal to 32 ounces have a $\$5$ shipping fee.
- Orders greater than 32 ounces have a $\$10$ shipping fee.

Let $c(x)$ represent the shipping fee for an order that weighs x ounces.

1. Complete the table.

Weight (oz)	Shipping Fee ($)
5	
16	
20	
30	
50	

2. Make a graph that represents the function $c(x)$.



Problems 3–5: Citlalli wrote a piecewise-defined function, $f(x)$, that models the price of using a recording studio for x minutes:

$$f(x) = \begin{cases} 1.00x & 0 < x < 20 \\ 0.75x & 20 \le x \le 40 \\ 0.50 & x \ge 40 \end{cases}$$

- $\$1$ per minute for sessions of 20 minutes or less.
- $\$0.75$ per minute for sessions between 20 and 40 minutes.
- $\$0.50$ per minute for sessions of 40 minutes or longer.

3. Describe one thing that is incorrect in Citlalli's function.

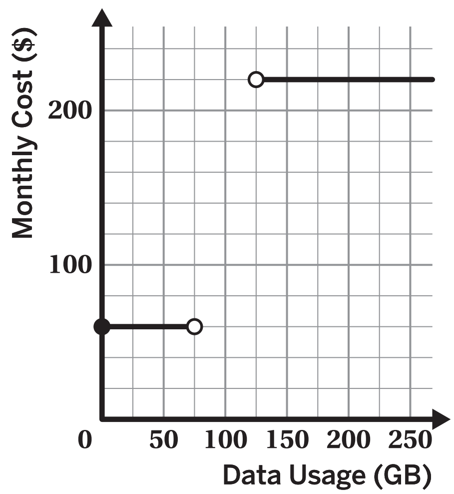
4. Write the correct function.

5. What would the price be if Citlalli used the recording studio for a 36-minute session?

Problems 6–7: An internet company sells internet based on the number of gigabytes of data used, x. $a(x)$ represents the monthly cost.

6. Here is part of the piecewise function, $a(x)$, and the graph that models this situation. Fill in the missing numbers and symbols. Then complete the graph to represent this situation.

$$a(x) = \begin{cases} \boxed{} & 0 \le x \ \boxed{} \ 75 \\ 120 & 75 \le x \le 100 \\ 180 & 100 < x \le 125 \\ 220 & \boxed{} \end{cases}$$



7. Diya thinks her business will use 110 gigabytes of data each month. How much would this data cost Diya per month?

Spiral Review

Problems 8–11: Determine the values of the piecewise-defined function $p(x)$.

$$p(x) = \begin{cases} 2x & 0 \le x \le 3 \\ 10 & x > 3 \end{cases}$$

8. $p(10) =$ ....

9. $p(3) =$ ....

10. $p(1) =$ ....

11. $p(22) =$ ....

Reflection

1. Put a star next to a problem you want to understand better.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

There are several ways we can define, or describe, a *sequence*. When you define a sequence recursively, you are determining each term using the previous term.

You can define a sequence recursively by identifying the first term of the sequence and writing a rule for how the sequence changes between terms by either a *constant ratio* or a *constant difference.* When writing the rule in function notation, you can write the recursive definition by referencing the previous term, which can be written as $f(n-1)$.

Here is an example of a recursive definition for the sequence $32, 16, 8, 4, 2, 1$ written in function notation.

$$f(n) = \begin{cases} 32 & n = 1 \\ 0.5\, f(n-1) & n \geq 2 \end{cases}$$

The first term is 32, and the sequence changes by a constant ratio of 0.5. The rule is to multiply the previous term $f(n-1)$ by 0.5.

The expression $32(0.5)^{n-1}$ could also be used to define this sequence.

Things to Remember:

Lesson Practice
A1.4.13

Name: ________ Date: ________ Period: ________

Problems 1–3: Let $h(n)$ represent the value of term n in this sequence: $2, 6, 10, 14, 18, \ldots$. Write the number that makes each equation true.

1. $h(1) =$ ______

2. $h(8-1) =$ ______

3. $h(n) = h(n-1) +$ ______

4. Match each sequence with one of the definitions.

Sequence	Definition
a. $2, 8, 14, 20, 26, \ldots$	______ $f(1) = 2$ $f(n) = 7 \cdot f(n-1)$
b. $2, 14, 98, 686, \ldots$	______ $g(1) = 2$ $g(n) = \frac{1}{2} \cdot g(n-1)$
c. $2, 1, 0.5, 0.25, 0.125, \ldots$	______ $h(1) = 2$ $h(n) = h(n-1) + 6$

Problems 5–6: Write the first four terms of each sequence.

5. $a(1) = 1$

$a(n) = 10 \cdot a(n-1)$

______, ______, ______, ______

6. $b(1) = 1$

$b(n) = b(n-1) + 10$

______, ______, ______, ______

Problems 7–8: Here are the first five terms of some sequences. Write a recursive definition for each one.

7. $2, 8, 32, 128, 512$

$g(1) =$ ______

$g(n) =$ ______

8. $240, 180, 120, 60, 0$

$h(1) =$ ______

$h(n) =$ ______

Name: .... Date: .... Period: ....

9. An arithmetic sequence $a(n)$ and geometric sequence $g(n)$ both have the same first and fourth term. Determine a recursive definition for each.

Arithmetic Sequence	**Geometric Sequence**
$a(1) =$	$g(1) =$ ....
$a(n) =$	$g(n) =$ ....

Spiral Review

Problems 10–12: Determine whether each sequence is arithmetic, geometric, or neither.

10. $162, 54, 18, 6 \ldots$ Arithmetic Geometric Neither

11. $2, 4, 8, 14 \ldots$ Arithmetic Geometric Neither

12. $6, 12, 18, 24 \ldots$ Arithmetic Geometric Neither

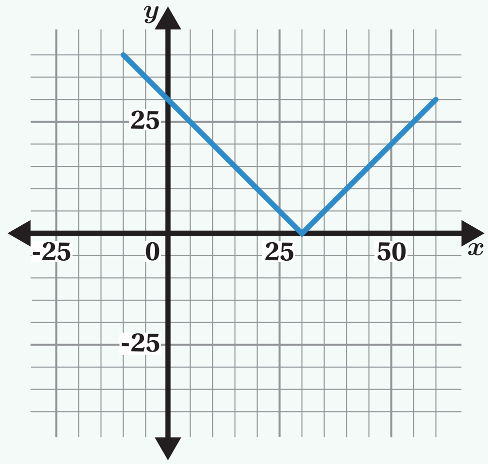
Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 42

Lesson Summary

The output of an **absolute value function** is the distance of its input from a given value. The equation of an absolute value function is defined using absolute value symbols, and its graph forms the shape of a V. We can write absolute value functions in the form $f(x) = |x - h|$, where $f(x)$ gives the distance of any input, x, from h. Let's look at an example.



Mr. DeAndre asked his students to guess a mystery number and gave each student a score. Their score was how far away their guess was from his mystery number, 30.

Here is the graph of the function $f(x) = |x - 30|$, which gives the score for each guess, x.

We can use the equation to determine the value of $f(25)$ and interpret its meaning.

$$\begin{aligned} f(25) &= |25 - 30| \\ &= |\text{-}5| \\ &= 5 \end{aligned}$$

This means a student who guessed 25 was 5 away from the mystery number.

Things to Remember:

Lesson Practice
A1.4.15

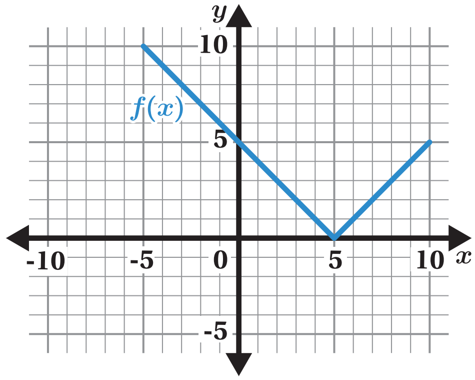
Name: .... Date: .... Period: ....

Problems 1–3: Use the graph of $f(x)$ to determine each value.

1. $f(0) =$ ______

2. $f(8) =$ ______

3. $f(5) =$ ______



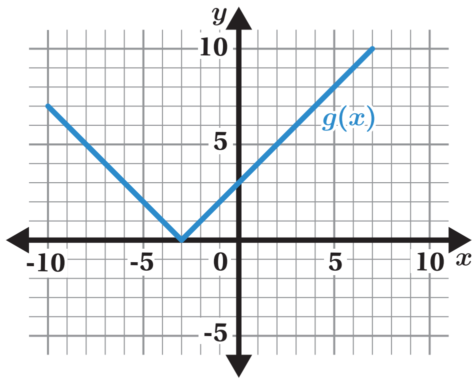
4. Which equation represents the graph of $g(x)$? Circle your choice.

   A. $g(x) = |x| - 3$

   B. $g(x) = |x - 3|$

   C. $g(x) = |x| + 3$

   D. $g(x) = |x + 3|$



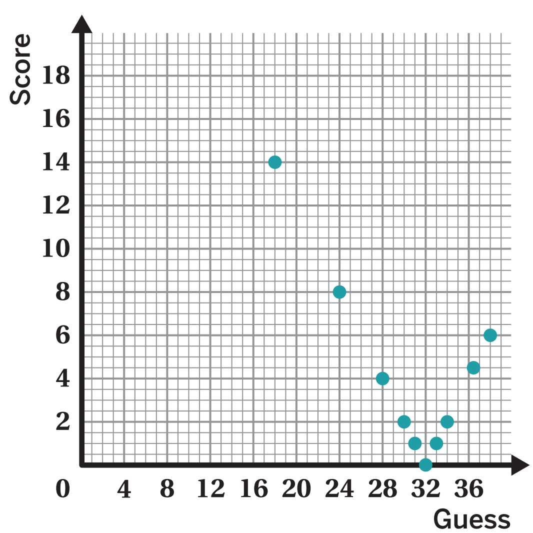
Problems 5–7: Ricardo's teacher challenged his class to guess how many marbles were in a jar. Each student was given a score equal to how far away their guess was from the actual number of marbles in the jar. The graph shows each students' guess, x, and score, $m(x)$.

5. How many marbles are in the jar?

6. Circle the point that represents the furthest guess from the actual number of marbles.

7. Ricardo writes the equation $m(25) = 7$.

   What does his equation mean?

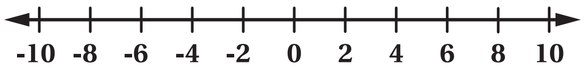


Lesson Practice
A1.4.15

Name: ______ Date: ______ Period: ______

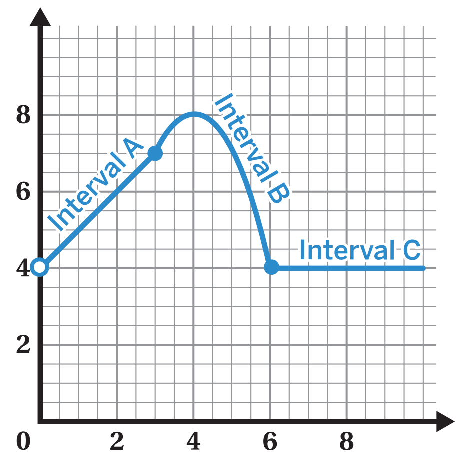
Spiral Review

8. Select *all* the true statements. Use the number line if it helps with your thinking.



- ☐ **A.** $-4 > -2$
- ☐ **B.** The distance from -2 to 2 is equal to the distance from 6 to 10.
- ☐ **C.** $-4 \geq -6$
- ☐ **D.** 8 is the only number 2 units away from 6.
- ☐ **E.** $|3| = |-3|$

Problems 9–11: Match each domain to an interval of the piecewise function on the graph.



9. $3 \leq x \leq 6$ ______

10. $x \geq 6$ ______

11. $0 < x \leq 3$ ______

12. The California Department of Fish and Wildlife estimated there were 460,420 deer in the state in 2021. They estimated that the deer population in 2018 was 470,000.

Calculate the average rate of change during this time interval and explain what it tells us about the deer population.

Reflection

1. Put a smiley face next to a problem you were stuck on and then figured out.

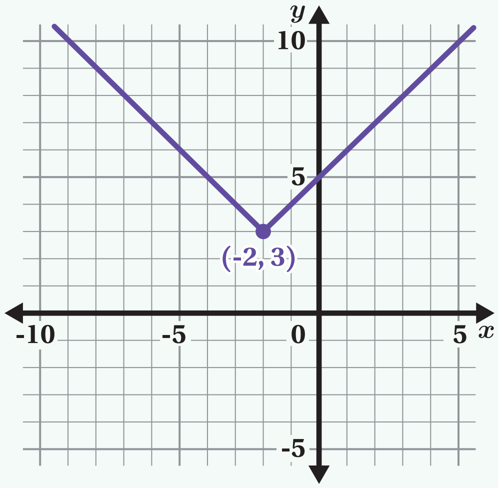
2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 43

Lesson Summary

You can determine key features of the graph of an *absolute value function* by analyzing its table or equation, which are both helpful in sketching its graph.

Here are the graph and table for the absolute value function $f(x) = |x + 2| + 3$.



x	$f(x) = \lvert x + 2\rvert + 3$
-4	5
-2	3
0	5
2	7

Evaluating $f(x)$ at x = -2 makes the equation equal to 3. This means that when the input of the function is -2, the output is 3 and a point on the graph of $f(x)$ is (-2, 3). The values in the table show that there is symmetry around the point (-2, 3). This tells us that (-2, 3) is the minimum value of the function, which we can see by looking at the graph.

Things to Remember:

Lesson Practice
A1.4.16

Name: .... Date: .... Period: ....

Problems 1–3: Write each expression as a single integer.

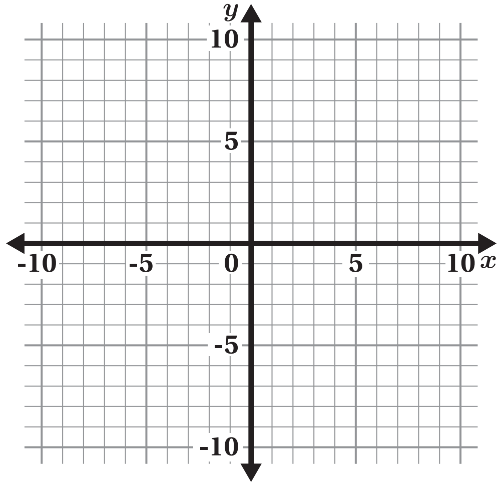
1. $|-4|$

2. $|7| - 2$

3. $|-8| + 1$

4. Graph $g(x) = |x + 1| + 4$. Use the table if it helps with your thinking.

x	$g(x)$
-3	
-1	
0	

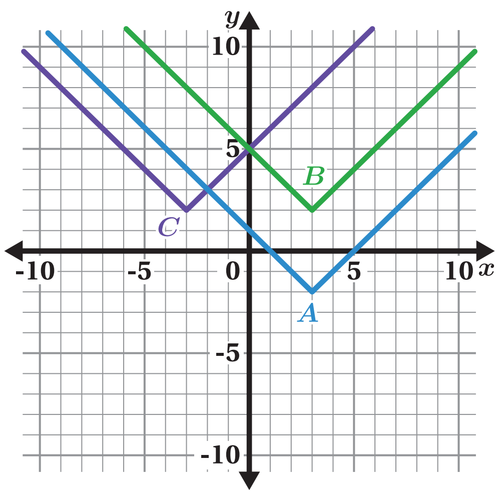


5. Match each function with its graph.

$f(x) = |x + 3| + 2$ ....

$g(x) = |x - 3| - 2$ ....

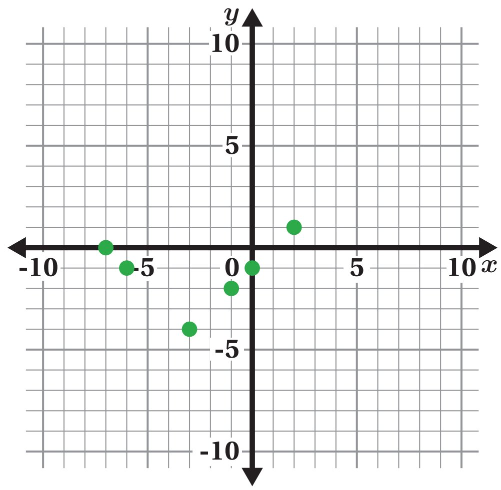
$h(x) = |x - 3| + 2$ ....



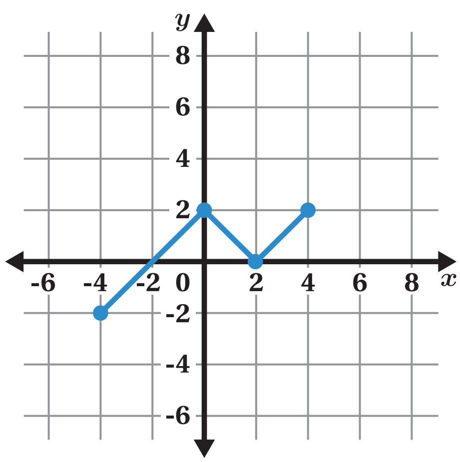
6. Here are some points on the graph of $h(x) = |x + 3| - 4$.

- Sketch a graph of $h(x)$.
- Describe the graph using some of these terms:

positive	maximum	increasing	domain
negative	minimum	decreasing	range
symmetry		piecewise-defined function	



7. Determine two different piecewise-defined functions that could represent this graph.



Spiral Review

Problems 8–10: For each square root, write which two consecutive whole numbers the value is between.

8. $\sqrt{14}$

9. $\sqrt{60}$

10. $\sqrt{88}$

Reflection

1. Put a heart next to a problem you understand well.
2. Use this space to ask a question or share something you're proud of.

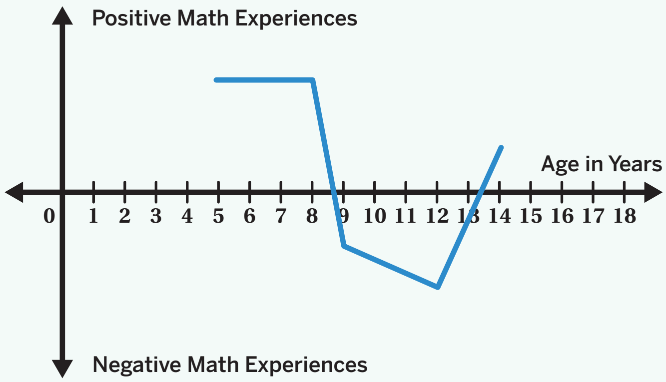
Lesson Summary

Storytelling is a powerful way to learn more about others and to reflect on your own journey. Graphing a story can help us see interesting self-discoveries and have deeper discussions.

When you use equations, tables, words, graphs, and their key features to represent real-world relationships, pay close attention to the scale and units. You can look for the maximums or minimums; intercepts; intervals where the graph increases, decreases, or remains constant; and domain and range to make sense of the situation or someone's story. Let's look at an example.

Here's a graph of someone's math experiences over time. From the graph, we can learn that:

- Most of their positive math experiences were from ages 5 to 8, and their most negative math experience was at age 12.
- Their math experiences decreased from age 8 to 12 and increased after age 12.
- This person graphed their experiences from age 5 to age 14. It's possible they drew this graph at age 14.



There is also a lot we can't tell from the graph of someone's story. For example, we don't know what the positive or negative math experiences were, or what emotions they were feeling at the time. The graph gives us only a window into someone else's story, not the full image.

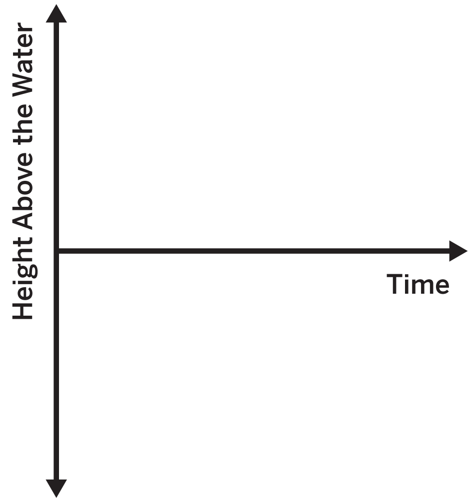
Things to Remember:

Lesson Practice
A1.4.17

Name: .... Date: .... Period: ....

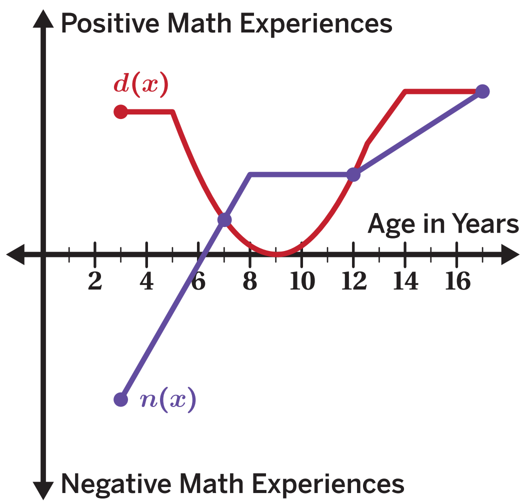
1. Sketch a graph to represent this situation:

Pablo went swimming at the city pool. When he first arrived, he climbed to the top of the high diving board where he dived into the deep end of the pool. He swam to the shallow end where he stayed for several minutes.



Problems 2–6: Adhira and Anand are twins. This graph represents their math stories.

$d(x)$ represents Adhira's math experience as a function of age and $n(x)$ represents Anand's math experience as a function of age.



2. What does $n(17) > n(12)$ say about Anand's math experience?

3. What does $d(3) = d(5)$ say about Adhira's math experience?

4. True or false: The domain for Adhira's graph is the same as the domain for Anand's graph.

5. True or false: The range for Adhira's graph is the same as the range for Anand's graph.

6. Who had a greater average rate of change between ages 3 and 17: Adhira or Anand? Explain how you know.

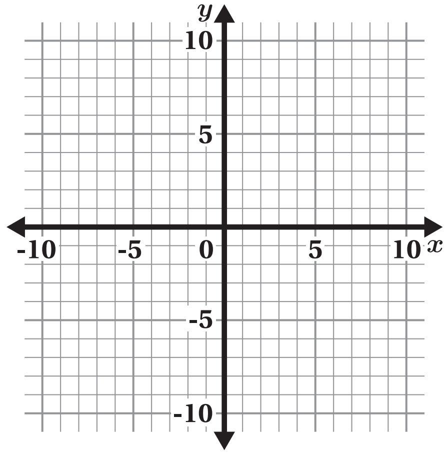
7. Create a story that could be modeled by this piecewise function.

Create a graph on graph paper if it helps with your thinking.

$$f(x) = \begin{cases} 4 & 0 \leq x < 8 \\ -4x + 36 & 8 \leq x < 9 \\ 2x - 18 & 9 \leq x \leq 12 \\ 6 & 12 < x \leq 15 \end{cases}$$

Spiral Review

8. Sketch the graph of $g(x) = |x - 2| - 2$.



Problems 9–11: Evaluate each expression.

9. $\left(\frac{1}{2}\right)^{3}$

10. $\left(\frac{1}{2}\right)^{-3}$

11. $\left(\left(\frac{1}{2}\right)^{3}\right)^{-2}$

Reflection

1. Circle a problem you want to talk to a classmate about.
2. Use this space to ask a question or share something you're proud of.