

Lessons 2–3: Visual Patterns

Summary

Here is a pattern of tiles.

Figure 1 **Figure 2** **Figure 3**

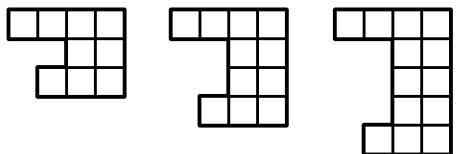
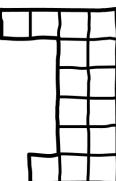


Figure 4



Sketch or describe figure 4.

See sketch next to figure 3.

Descriptions vary. Figure 4 would be a row of 4 tiles on top, then a 4-by-2 rectangle, and then a row of 3 tiles on the bottom.

Complete the table. Show or explain how to write an expression for the number of tiles in figure n .

Explanations vary. Figure n would have 7 tiles (the top and the bottom), and then the middle part would be an n -by-2 rectangle. This means the total number of tiles is $7 + 2 \cdot n$.

Figure	Number of Tiles
1	9
2	11
3	13
4	15
10	27
n	$7 + 2 \cdot n$

Things I Want to Remember

desmos

Unit A1.1, Representing Relationships: Notes Name _____

Lessons 2–3: Visual Patterns

Try This!

- 1.1 Sketch or describe figure 10.

See sketch below figure 2.

Descriptions vary. Figure 10 would be
3 columns of 10 tiles, with 1 tile on
either side of the bottom row.

- 1.2 How many tiles would figure 10 have?

32 tiles

- 1.3 How many tiles would figure n have?

$2 + 3n$ tiles (or equivalent)

Figure 1

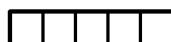


Figure 2

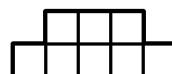


Figure 3

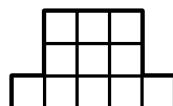
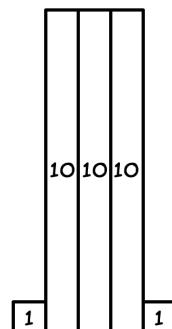


Figure 10



- 2.1 Sketch or describe figure 10.

See sketch below figure 2.

Descriptions vary. Figure 10 would be
all the tiles around a 10-by-10 square.

Figure 1

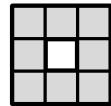


Figure 2

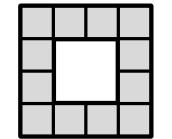


Figure 3

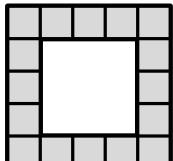
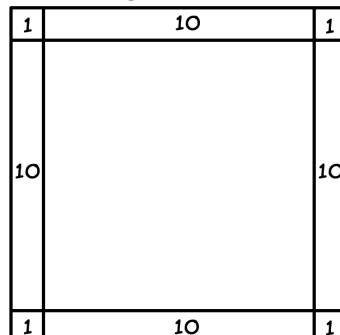


Figure 10



- 2.2 How many tiles would figure 10 have?

44 tiles

- 2.3 How many tiles would figure n have?

$+ 4n$ tiles (or equivalent)

- I can describe patterns in a table and patterns in an image.
- I can use tables and images to make predictions about a pattern.
- I can use expressions with variables to describe and make predictions about a pattern.

Lesson 4: Solving Problems With Graphs**Summary**

We can use graphs to better understand mathematical relationships and to solve problems.

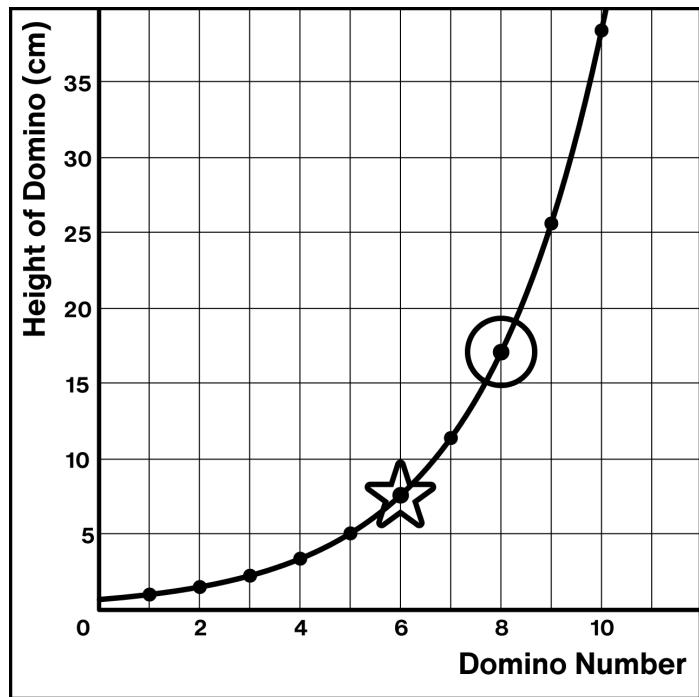
This graph shows the height of different dominoes in a domino chain.

Circle the point that represents the 8th domino in the chain. **See the graph.**

Star the point that represents the domino whose height is closest to 8 cm tall. **See the graph.**

List some advantages of using graphs to solve problems. **Responses vary.**

- **Graphs can help us see trends over time.**
- **Each point on a graph represents something about a situation, so finding the correct point can help answer lots of different questions.**



Things I Want to Remember

Lesson 4: Solving Problems With Graphs**Try This!**

Brianna is keeping track of how much money she saves. Use the graph to answer each question.

- 1.1 How much money will Brianna have saved after 4 weeks?

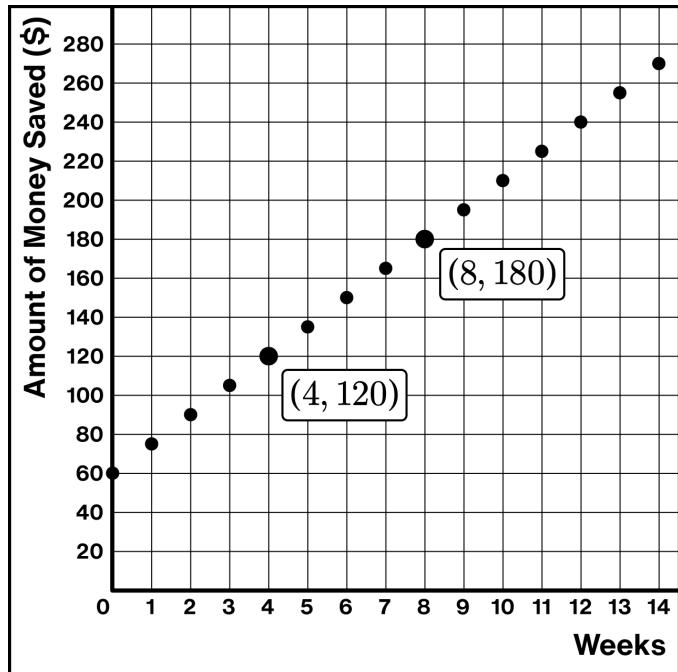
\$120

- 1.2 How long will it take Brianna to save enough to buy a pair of \$180 sneakers?

8 weeks

- 1.3 Write a question about Brianna's situation that this graph could help answer.

Responses vary. How long will it take until Brianna doubles the amount she had after 4 weeks?



Croissants (a French pastry) are made by putting a layer of butter between two layers of dough, then folding repeatedly to make more layers. Every time the dough is folded, the number of layers triples.

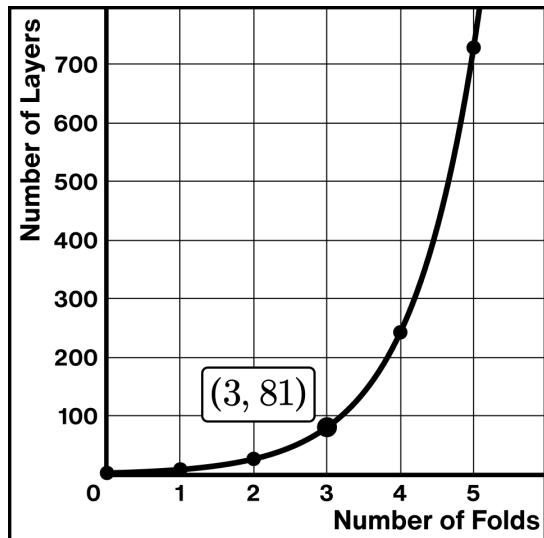
- 2.1 A traditional croissant is folded 3 times. About how many layers does it have?

Responses vary. Slightly less than 100 layers.

- 2.2 How many folds do you need if you want a croissant with over 500 layers? **5 folds**

Show or explain your thinking.

Explanations vary. The first y -value that is more than 500 matches with the x -value of 5, so it takes 5 folds to get more than 500 layers.



I can interpret points on a graph to solve a problem.

Lessons 5–6: Linear and Exponential Relationships

Summary

In this unit, we explore two types of relationships: linear and exponential.

Complete the table with an example of each relationship.

	Story	Table	Graph										
Linear <i>Constant Difference</i>	<p>After 1 month, my plant was 3 inches tall.</p> <p>Every month, the height of the plant . . .</p> <p>. . . increases by 2 inches.</p>	<table border="1"> <thead> <tr> <th>x</th><th>y</th></tr> </thead> <tbody> <tr> <td>1</td><td>3</td></tr> <tr> <td>2</td><td>5</td></tr> <tr> <td>3</td><td>7</td></tr> <tr> <td>4</td><td>9</td></tr> </tbody> </table> <p>+2 +2 +2</p>	x	y	1	3	2	5	3	7	4	9	
x	y												
1	3												
2	5												
3	7												
4	9												
Exponential <i>Constant Ratio</i>	<p>After 1 hour, there were 3 mold cells on a piece of bread.</p> <p>Every hour, the number of mold cells . . .</p> <p>. . . doubles.</p>	<table border="1"> <thead> <tr> <th>x</th><th>y</th></tr> </thead> <tbody> <tr> <td>1</td><td>3</td></tr> <tr> <td>2</td><td>6</td></tr> <tr> <td>3</td><td>12</td></tr> <tr> <td>4</td><td>24</td></tr> </tbody> </table> <p>×2 ×2 ×2</p>	x	y	1	3	2	6	3	12	4	24	
x	y												
1	3												
2	6												
3	12												
4	24												

Things I Want to Remember

desmos

Unit A1.1, Representing Relationships: Notes Name _____

Lessons 5–6: Linear and Exponential Relationships

Try This!

Decide whether each relationship is linear, exponential, or something else.

Show or explain how you know. ***Explanations vary.***

1.1

x	y
0	1
1	3
2	9
3	27

Exponential

There is a constant ratio of 3.

1.2

x	y
0	15
1	21
2	27
3	33

Linear

There is a constant difference of 6.

1.3

x	y
0	3
1	6
2	10
3	15

Something Else

There is no constant difference ($6 - 3 = 3$, but $10 - 6 = 4$).
There is no constant ratio ($6 \div 3 = 2$, but $10 \div 6 \neq 2$).

1.4

x	y
0	20
1	10
2	5
3	2.5

Exponential

There is a constant ratio of $\frac{1}{2}$.

2. Which of these four relationships will have the greatest y -value when $x = 5$?

Relationship 1.1

Explanations vary. Even though the value is smallest when $x = 0$, growing by a constant ratio is faster than growing by a constant difference.

- I can compare relationships that grow by constant differences and by constant ratios.
- I can explain that quantities that grow by a constant ratio (exponential) eventually exceed those that grow by a constant difference (linear).
- I can compare and contrast linear and exponential relationships.
- I can determine if a situation, table, or graph shows a linear or exponential relationship.

Lesson 7: Equations of Linear and Exponential Relationships**Summary**

Here are tables for three different relationships: one linear and two exponential.

Linear

x	y
0	3
1	$3 + 2$
2	$3 + 2 + 2$
3	$3 + 2 + 2 + 2$

Exponential #1

x	y
0	3
1	$3 \cdot 2$
2	$3 \cdot 2 \cdot 2$
3	$3 \cdot 2 \cdot 2 \cdot 2$

Exponential #2

x	y
0	4
1	12
2	36
3	108

Starting value: 3

Constant difference: 2

Equation: $y = 3 + 2x$
(or equivalent)

Starting value: 3

Constant ratio: 2

Equation: $y = 3 \cdot 2^x$
(or equivalent)

Starting value: 4

Constant ratio: 3

Equation: $y = 4 \cdot 3^x$
(or equivalent)

Things I Want to Remember

Lesson 7: Equations of Linear and Exponential Relationships

Try This!

For each table:

- Decide if it represents a linear or exponential relationship.
- Determine the starting value and constant difference/ratio.
- Write an equation to represent the relationship.

1.1

x	y
0	1
1	3
2	9
3	27

Linear or exponential?
Exponential

Starting value:

1

Constant difference/ratio:
3

Equation:

$$y = 1 \cdot 3^x$$

(or equivalent)

1.2

x	y
0	15
1	21
2	27
3	33

Linear or exponential?
Linear

Starting value:

15

Constant **difference**/ratio:
6

Equation:

$$y = 15 + 6x$$

(or equivalent)

1.3

x	y
0	20
1	15
2	10
3	5

Linear or exponential?
Linear

Starting value:

20

Constant **difference**/ratio:
-5

Equation:

$$y = 20 - 5x$$

(or equivalent)

1.4

x	y
0	20
1	10
2	5
3	2.5

Linear or exponential?
Exponential

Starting value:

20

Constant difference/ratio:
 $\frac{1}{2}$

Equation:

$$y = 20 \cdot \left(\frac{1}{2}\right)^x$$

(or equivalent)

I can write equations for linear and exponential situations given descriptions or tables.

Lesson 8: Exponential Equations in Context**Summary**

Carlos bought a new mega-growing fish and recorded its mass for each hour.

Use the table to help Carlos create a model for its growth.

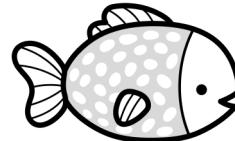
When Carlos bought the fish, it weighed 4 grams.

The relationship between hours and mass is exponential because ...

... there is a constant ratio of 1.5. The mass of the fish grows 1.5 times larger every hour.

Time (hours)	Mass (grams)
0	4
1	6
2	9
3	13.5

An equation to model the fish's mass over time is $m = 4 \cdot 1.5^t$.



m represents the mass of Carlos's fish in grams.

t represents the time in hours.

If the fish continues growing this way, after 7 hours its mass will be about 68 grams.

Things I Want to Remember

Lesson 8: Exponential Equations in Context**Try This!**

An invasive species of plants (plants from another region of the world that don't belong in their new environment) is growing all over Metropolis.

The number of invasive plants in Metropolis is modeled by the function: $p = 10 \cdot 3^t$.

p is the total number of invasive plants.

t is the number of years since the plants were brought to Metropolis.

- According to the model, how many plants were brought to Metropolis originally?

10 plants

- Is the number of plants growing linearly, exponentially, or something else? How do you know?

Exponentially.

Explanations vary. The equation has a variable in an exponent, which represents a constant ratio of 3.

- If the plants continue spreading this way, how many of these plants will there be in Metropolis 8 years after they were brought?

$$10 \cdot 3^8 = 65\,610 \text{ plants}$$

- Write and answer another question you could ask about these plants using the model.

Questions and responses vary

Will there be more than 1 million plants after 10 years?

No, there will only be $10 \cdot 3^{10} = 590\,490$ plants. That is still a lot.

- | |
|--|
| <input type="checkbox"/> I can interpret each part of an exponential function in context. |
| <input type="checkbox"/> I can use equations of the form $y = a \cdot b^x$ to solve problems in context. |

Lesson 11: Introduction to Modeling

Summary

We can use exponential and linear relationships to create models of situations in the world.

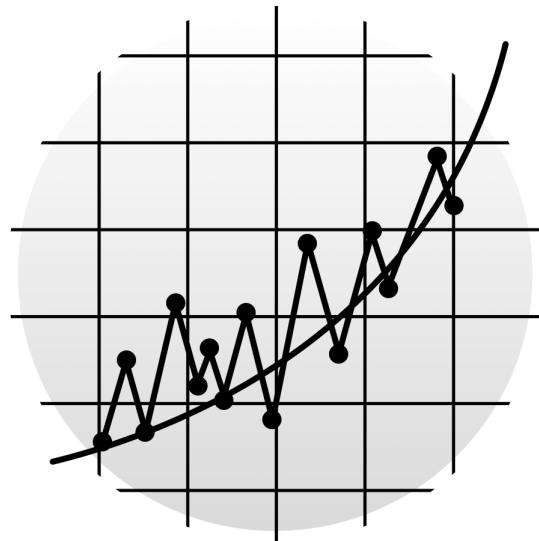
The British statistician George Box once said:

All models are wrong, but some are useful.

Explain what each part of this quote means.

Responses vary.

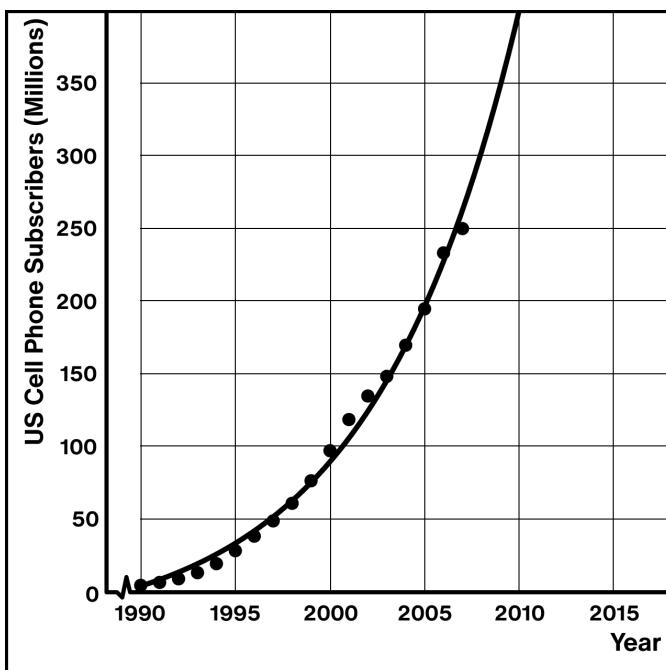
- All models are wrong: **If you are dealing with real-life data, then the data will not fit a model perfectly. There will be points that are above or below the model. Also, the world is unpredictable, so models are wrong because they can't perfectly predict the future.**
- Some models are useful: **We can use models to help see trends and make predictions, even if those predictions aren't perfect. Noticing how something is changing can help us understand it and sometimes having a best guess is useful.**



Things I Want to Remember

Lesson 11: Introduction to Modeling**Try This!**

Here is data about the number of U.S. cell phone subscribers by year from 1990 to 2007.



Source: Cellular Telecommunications Industry Association, 2007

1. What model do you think is a best fit for the data: linear, exponential, or something else?

Exponential. Explanations vary. The data does not form a straight line. The growth starts off slowly and then grows faster, similar to an exponential.

2. Sketch a model on top of the graph. **See the graph.**
3. Write one or more predictions you are confident about from your model. **Predictions vary.**

There were more than 300 million cell phone subscribers in 2010.

4. Write one prediction you are less confident about. Why are you less confident?

There were more than 500 million cell phone subscribers in 2020. This is just too far away from the data that I have. Also, there are only about 330 million people in the U.S.

- I can model situations with linear or exponential relationships and use the models to make predictions.
 - I can use models to analyze an issue in society.

Lesson 1: Solving Equations With Balanced Moves**Summary**

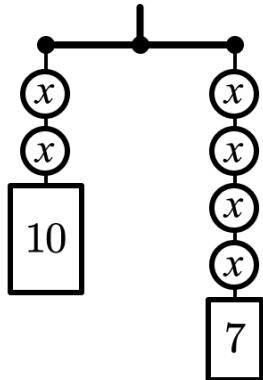
Solving an equation means determining all the values that make an equation true.

Hanger diagrams can be useful to represent and help solve equations.

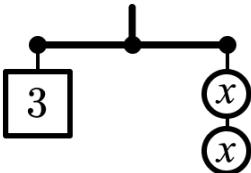
Here is Ayaan's work to solve the equation $2x + 10 = 4x + 7$.

Write what Ayaan did under each arrow.

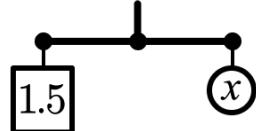
$$2x + 10 = 4x + 7$$



$$3 = 2x$$



$$1.5 = x$$



**Subtract
2x and 7
from
both
sides.**

**Divide
both
sides
by 2.**

$x = 1.5$ is the *solution* to $2x + 10 = 4x + 7$. Explain what *solution* means in your own words.

Explanations vary. If $x = 1.5$ is a solution, then when 1.5 is substituted for x in the equation, the left and right sides are equal.

Things I Want to Remember

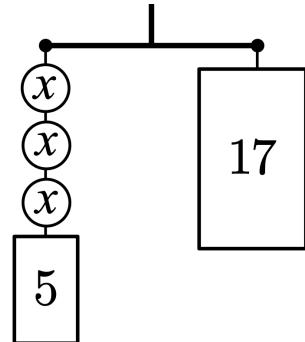
Lesson 1: Solving Equations With Balanced Moves

Try This!

1. Solve the equation $3x + 5 = 17$.

Use the balanced hanger if it helps with your thinking.

$$x = 4$$



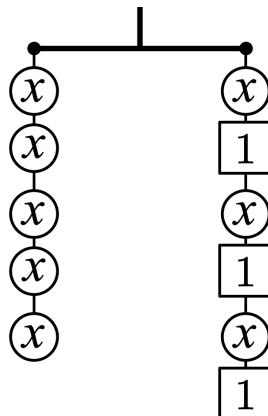
2. Write an equation that this balanced hanger represents.

Equations vary.

$$5x = 3(x + 1)$$

Solve the equation that you wrote.

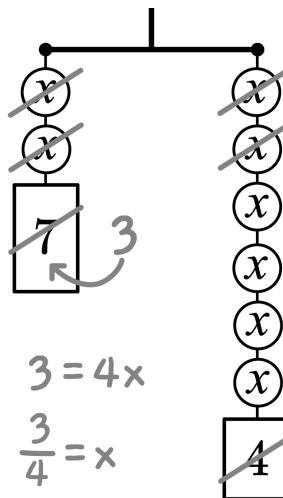
$$x = \frac{3}{2}$$



3. Solve the equation $2x + 7 = 6x + 4$.

Draw a hanger if it helps with your thinking. **Hangers vary.**

$$x = \frac{3}{4}$$



- I can determine a solution to an equation by modeling it with a hanger diagram.
 I can describe balanced moves and use them to solve an equation.

Lesson 2: Solving Equations With Inverse Operations**Summary**

Working backwards can help solve equations.

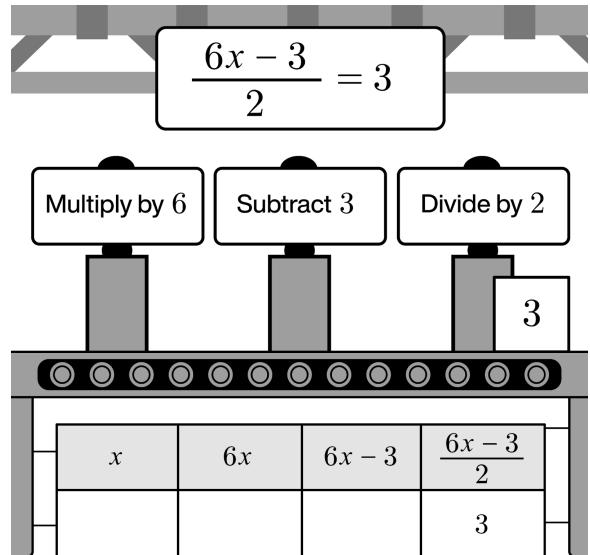
A table and a number machine are two strategies for solving the equation $\frac{6x-3}{2} = 3$.

Show or explain how the table or number machine are each connected to the equation.

Explanations vary. The machine and table show the order of the operations that happen to the input x to create $\frac{6x-3}{2}$. The machine and the table also show that the output is equal to 3.

Solve the equation $\frac{6x-3}{2} = 3$.

Use the table or machine if it helps with your thinking.



Show that your solution is correct.

I substituted $x = \frac{3}{2}$ into the original equation and got a true equation.

$$\frac{6\left(\frac{3}{2}\right)-3}{2} = 3$$

$$3 = 3 \checkmark$$

Things I Want to Remember

Lesson 2: Solving Equations With Inverse Operations**Try This!**

1. Solve $-30 = -5(x + 2)$.

$$x = 4$$

Use the table if it helps with your thinking.

x	$x + 2$	$-5(x + 2)$
		-30

Show that your solution is correct.

Responses vary.

$$-30 = -5((4) + 2)$$

$$-30 = -5(6)$$

$$-30 = -30 \checkmark$$

2. Solve $\frac{3x + 9}{2} = 12$.

$$x = 5$$

Use the table if it helps with your thinking.

x			$\frac{3x + 9}{2}$
			12

Show that your solution is correct.

Responses vary.

$$\frac{3(5) + 9}{2} = 12$$

$$\frac{15 + 9}{2} = 12$$

$$\frac{24}{2} = 12$$

$$12 = 12 \checkmark$$

- I can describe and use inverse operations to solve an equation

Lesson 5: No Solution and Infinite Solutions**Summary**

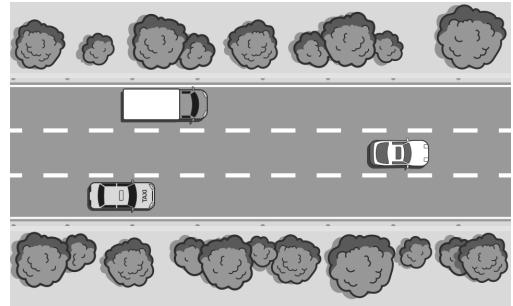
Linear equations can have *no solutions*, *one solution*, or *infinitely many solutions*.

- In an equation with *no solutions*, no value of x makes the equation true.
- In an equation with *infinitely many solutions*, every value of x makes the equation true.

The equation $t = t + 2$ has **no** solution(s).

If this equation represents the time, t , that two vehicles would be in the same position, then:

- A. They will never be in the same position.**
- B. They will be in the same position after 2 seconds.
- C. They will always be in the same position.



The equation $2t = 8t$ has **one** solution(s).

If this equation represents the time, t , that two vehicles would be in the same position, then . . .

... they will be in the same position after 0 seconds.

The equation $2t + 6 = 2(t + 3)$ has **infinite** solution(s).

If this equation represents the time, t , that two vehicles would be in the same position, then . . .

... they will always be in the same position.

Things I Want to Remember

Lesson 5: No Solution and Infinitely Many Solutions**Try This!**

Solve each equation and determine how many solutions it has.

1. $10x + 4 = 2(5x + 4)$

Strategies vary.

$$5x + 2 = 5x + 4$$

$$2 = 4$$

Circle one:	No solution	One solution	Infinite solutions
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2. $10x = 5x - 12$

Strategies vary.

$$5x = -12$$

$$x = -\frac{12}{5}$$

Circle One:	No solution	One solution	Infinite solutions
--------------------	-------------	--------------	--------------------

3. $10x = 5x$

Strategies vary.

$$5x = 0$$

$$x = 0$$

Circle one:	No solution	One solution	Infinite solutions
--------------------	-------------	--------------	--------------------

4. $\frac{10x + 8}{2} = 5x + 4$

Strategies vary.

$$10x + 8 = 10x + 8$$

$$8 = 8$$

Circle one:	No solution	One solution	Infinite solutions
--------------------	-------------	--------------	--------------------

- I can describe the effect of dividing by a variable when solving an equation.
- I can justify whether a one-variable equation has one solution, no solution, or infinite solutions.

Lesson 6: Representing Situations With Two-Variable Equations**Summary**

Sometimes equations have more than one variable in them. Different forms of the equation can be helpful in different situations.

Here are two equivalent equations about a subway car's capacity (i.e., the number of people who fit inside).

$$6t + 2d = 600$$

$$d = 300 - 3t$$

- t is the number of seats (seating capacity).
- d is the standing capacity.

Show the steps to solve $6t + 2d = 600$ for d .

$$\begin{array}{r} 6t + 2d = 600 \\ \underline{-6t} \quad \underline{-6t} \\ 2d = 600 - 6t \end{array}$$

$$\frac{2d}{2} = \frac{600 - 6t}{2}$$

$$d = 300 - 3t$$



When would it be useful to use the equation solved for d ? **Responses vary.**

- It would be useful if you know how many seats are in a subway car and you want to know how many people standing can fit.
- It would be useful if you wanted to know how the standing capacity changes based on the number of seats.

Things I Want to Remember

Lesson 6: Representing Situations With Two-Variable Equations**Try This!**

Tiara is saving \$240 for a new gaming console. To earn the money she needs, she works at the pool for \$8 an hour and earns \$12 an hour tutoring Spanish.

Tiara wrote the equation $8p + 12t = 240$ to represent her situation.

1. Explain what each part of $8p + 12t = 240$ represents in Tiara's situation. **Responses vary.**

- The 8 represents how much Tiara makes per hour at the pool, and p represents the number of hours she spends working at the pool.
- The 12 represents how much Tiara makes per hour tutoring and t represents the number of hours she spends tutoring.
- The 240 represents the total amount of money Tiara wants to save.

2. Complete the table for the missing values of t .

3. Which equation solved for t is equivalent to $8p + 12t = 240$?

- A. $t = 240 - 8p$ B. $t = 20 - \frac{2}{3}p$
 C. $t = 30 - \frac{3}{2}p$ D. $t = -\frac{2}{3}p + 30$

p	t
3	18
15	10
18	8

Show or explain how you know. **Explanations vary. See the work on the right.**

$$8p + 12t = 240$$

$$12t = 240 - 8p$$

$$t = 20 - \frac{2}{3}p$$

4. When might the equation that you chose in problem 3 be helpful to Tiara? **Responses vary.**

It could be helpful if Tiara already knows how many hours she will be working at the pool and needs to know how many hours she has to tutor to reach her goal.

- I can represent constraints using two-variable equations and interpret their solutions.
 I understand that different forms of a linear equation can be useful for different purposes.

Lessons 8–9: Linear Relationships in Equations, Tables, and Graphs**Summary**

Equations, tables, and graphs are different ways to model a situation.

Situation: A lemonade stand sells lemonade for \$3 per cup and cookies for \$2 each. They made \$12. Let l be the number of cups of lemonade sold and c be the number of cookies sold.

Show the steps to solve

$$3l + 2c = 12 \text{ for } c.$$

$$3l + 2c = 12$$

$$2c = 12 - 3l$$

$$c = 6 - \frac{3}{2}l$$

Equation in Standard Form

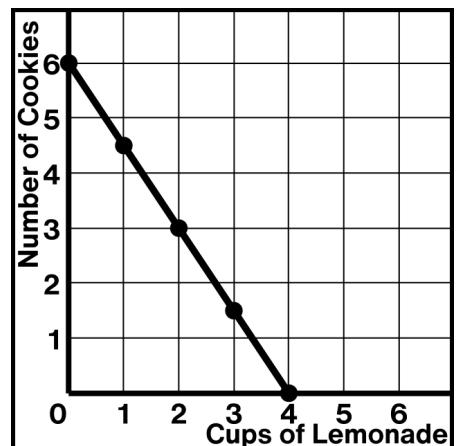
$$3l + 2c = 12$$

Equation Solved for c

$$c = 6 - \frac{3}{2}l$$

Table

l	0	2	4
c	6	3	0

Graph

Explain how **each** form of the equation is connected to the situation, table, or graph.

Responses vary.

The equation $3l + 2c = 12$ is connected to the **situation** because . . .

. . . you can see each part of the situation in the equation. For example, $3l$ is the money that the stand gets for selling l cups of lemonade for \$3 each.

The equation $c = 6 - \frac{3}{2}l$ is connected to the **table/graph** because . . .

. . . you can see the slope and vertical intercept in this equation. The graph includes the point $(0, 6)$ and the slope is $-\frac{3}{2}$.

Things I Want to Remember

Lessons 8–9: Rewriting Two-Variable Equations

Try This!

Here is an equation in standard form: $4x + 2y = 24$.

1. Solve $4x + 2y = 24$ for y .

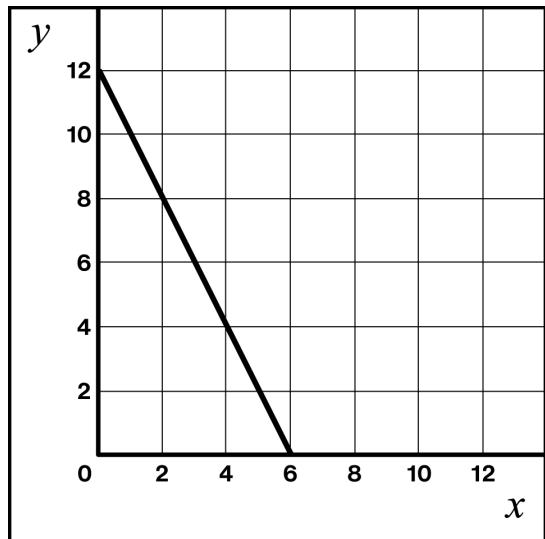
$$2y = 24 - 4x$$

$$y = 12 - 2x \text{ (or equivalent)}$$

2. Graph the equation $4x + 2y = 24$.

Make a table if it helps with your thinking.

x	y
0	12
2	8
4	4



3. Write a situation that $4x + 2y = 24$ could represent.

Write what x and y represent in your situation.

Responses vary. This situation could represent a snail trying to cross a gap that is 24 mm tall using 4 mm and 2 mm blocks. x represents the number of 4 mm blocks that are used and y represents the number of 2 mm blocks used.

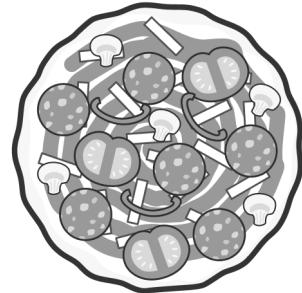
- I understand that the graph of a linear equation represents all the solutions to the equation.
- I can solve an equation for one of its variables and connect my new equation to its graph.
- I can make connections between equations, tables, descriptions, and graphs.
- I can write two linear equations to represent the same situation.

Lesson 10: Representing Situations With One-Variable Inequalities**Summary**

Writing and solving inequalities can help us make sense of *constraints*.

Here is one example of a constraint:

- Tasia is planning a pizza party and can spend up to \$140. Each plain pizza costs \$12 and there is a delivery fee of \$8.



Write an inequality to represent the constraint in this situation.

Use p to represent the number of pizzas Tasia can buy with her budget.

$$12p + 8 \leq 140 \text{ (or equivalent)}$$

What are 2–3 other constraints people might consider when planning a party?

Responses vary.

- I can fit up to 35 people in the room I am using for the party.
- I need more than 50 cupcakes so everyone can choose their favorite flavor.

Write inequalities to represent each constraint. **Responses vary.**

$a \leq 35$ where a represents the number of people at the party.

$c > 50$ where c represents the number of cupcakes purchased.

Things I Want to Remember

Lesson 10: Representing Situations With One-Variable Inequalities**Try This!**

Valeria wants to donate at least \$120 to her local food bank. She has already saved \$64 and is planning to save \$8 each week.

1. Why is Valeria's situation an example of a constraint? **Responses vary.**

Valeria's situation is an example of a constraint because there is a specific goal that she wants to meet or exceed.

2. Write an inequality to match Valeria's situation.

Use w to represent the number of weeks Valeria will save \$8.

$$64 + 8w \geq 120 \text{ (or equivalent)}$$

3. Write some solutions to the inequality you wrote in problem 2. **Responses vary.**

Some possible values for w are 7, 18.5, and 100.

4. What are some other constraints that Valeria could have in her situation? **Responses vary.**

The food bank could ask for a minimum donation. Valeria could need to get her donation into the food bank by a specific deadline.

- | |
|--|
| <input type="checkbox"/> I understand what a solution to an inequality is. |
| <input type="checkbox"/> I can interpret and write one-variable inequalities that represent constraints. |

Lessons 11 and 12: Graphing and Solving One-Variable Inequalities**Summary**

Solutions to one-variable inequalities can be represented on a number line.

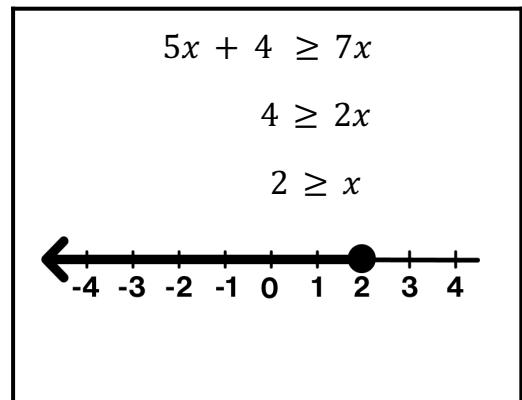
List some solutions to $5x + 4 \geq 7x$.

Responses vary. $x = 2, x = -1, x = -3.7, x = -20$

Is $x = 2$ a solution to $5x + 4 \geq 7x$? **Yes.**

Explain how you know. **Explanations vary.**

2 is a solution because the graph has a closed circle at $x = 2$ and $2 \geq 2$.



Strategies for solving equations can help solve inequalities.

Let's solve the inequality $10 - 5x < 0$.

1. Show that the solution to its corresponding equation $10 - 5x = 0$ is $x = 2$.

$$10 - 5(2) = 0$$

$$0 = 0 \checkmark$$

2. Test values of x that are less than and greater than 2 in the inequality $10 - 5x < 0$.

$$\begin{aligned} x &= 5 \\ 10 - 5(5) &< 0 \\ -15 &< 0 \checkmark \end{aligned}$$

$$\begin{aligned} x &= 0 \\ 10 - 5(0) &< 0 \\ 10 &< 0 \text{ False.} \end{aligned}$$

3. What are the solutions to $10 - 5x < 0$?

$x > 2$ because $x = 5$ made the inequality true and $5 > 2$.

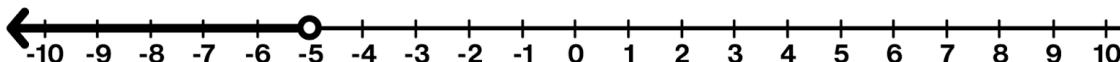
Things I Want to Remember

Lessons 11 and 12: Graphing and Solving One-Variable Inequalities**Try This!**

- 1.1 Select **all** the values of x that are solutions to $-8x > 40$.

$x = 10$ $x = 5$ $x = -10$ $x = -5$ $x = -6$

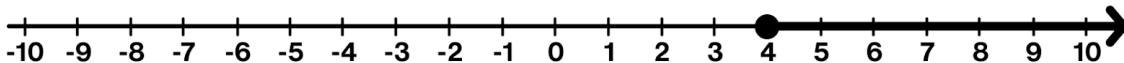
- 1.2 Graph all the solutions to $-8x > 40$ on the number line.



- 2.1 Solve $11 - 2x \leq 3$. Use its corresponding equation if it helps with your thinking. **Steps vary.**

$$\begin{aligned} 11 - 2x &\leq 3 \\ -2x &\leq -8 \\ x &\geq 4 \end{aligned}$$

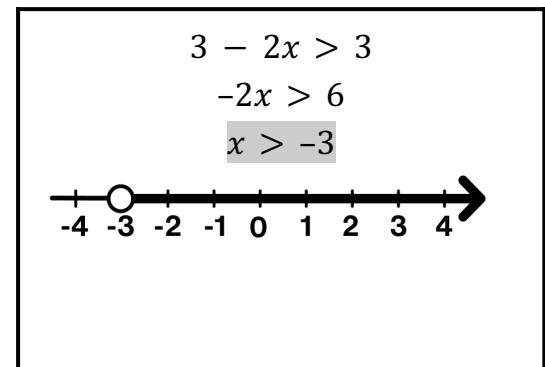
- 2.2 Graph the solutions to $11 - 2x \leq 3$ on the number line.



3. Here is Marco's work to solve and graph $3 - 2x > 3$.

Explain the error Marco made in his work.

Responses vary. Marco should test values less than and greater than -3 . The values greater than -3 create false inequalities so the solutions are $x < -3$.



- I can solve one-variable inequalities by reasoning.
- I can graph solutions to a one-variable inequality on the number line.
- I can solve a one-variable linear inequality using its corresponding equation.

Lesson 13: Introduction to Two-Variable Inequalities**Summary**

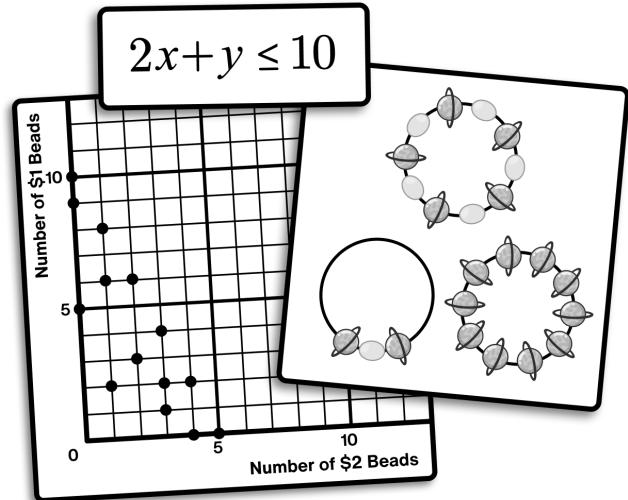
Graphs can help us visualize the solutions to two-variable inequalities.

Marco is making bracelets.

Planet beads cost \$1 and oval beads cost \$2.

Show or explain what each part of $2x + y \leq 10$ represents in Marco's situation.

- 2 represents the cost of each oval bead.
- x represents the number of oval beads Marco buys.
- y represents the number of planet beads.
- There is a secret 1 in front of the y , which represents how much a planet bead costs.
- 10 represents how much money Marco has for buying beads for his bracelets.



Choose a point shown on the graph. **Points vary.** (2, 3)

Show that this point is a solution to $2x + y \leq 10$.

Responses vary based on the point chosen.

$$2(2) + 3 \leq 10$$

$$4 + 3 \leq 10$$

$$7 \leq 10 \checkmark$$

Choose a point that is **not** shown that you think is also a solution. **Points vary.** (1, 4)

Show that this point is a solution to $2x + y \leq 10$.

Responses vary based on the point chosen.

$$2(1) + 4 \leq 10$$

$$2 + 4 \leq 10$$

$$6 \leq 10 \checkmark$$

Things I Want to Remember

Lesson 13: Introduction to Two-Variable Inequalities**Try This!**

The Theater Club makes \$5 for every student ticket they sell, x , and \$7 for every adult ticket, y . They want to make at least \$180 to buy costumes for their next show.

- 1.1 Explain how you know this situation is an example of a constraint.

Responses vary. This is an example of a constraint because the Theater Club has a minimum amount of money they want to make in order to buy costumes.

- 1.2 Which inequality or equation represents this situation?

- A. $5x + 7y \leq 180$ B. $5x + 7y = 180$ C. $5x + 7y \geq 180$ D. $7y = 5x + 180$

This graph shows some solutions to the Theater Club's situation.

- 2.1 Choose one solution: **Points vary.**

(20, 15)

Explain what it means in the situation.

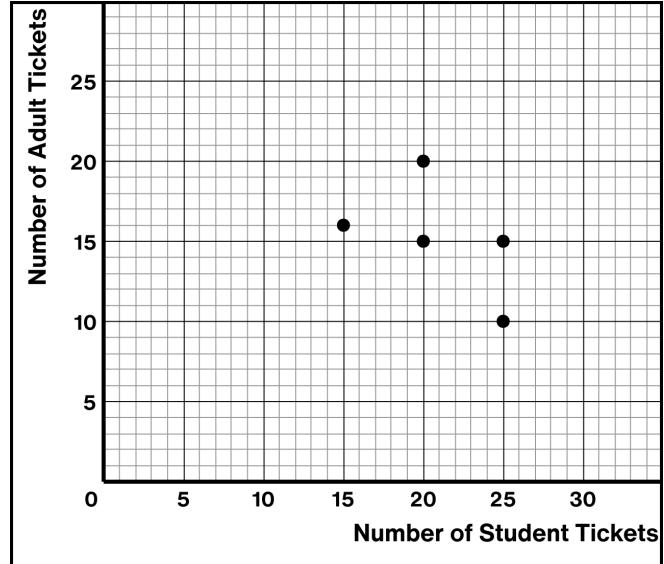
Explanations vary. If the theater club sells 20 student tickets and 15 adult tickets, they will reach their goal.

- 2.2 Show that this point is a solution to the inequality you chose in problem 2.

Responses vary.

$$5(20) + 7(15) \geq 180$$

$$205 \geq 180 \checkmark$$



- 2.3 Choose another solution that is **not** shown on the graph. **Points vary.** (30, 15)

Show or explain how you know it is a solution.

Explanations vary. I know that (25, 15) is a solution, so if they meet the goal selling only 25 student tickets, then they should also meet it if they sell 30 student tickets.

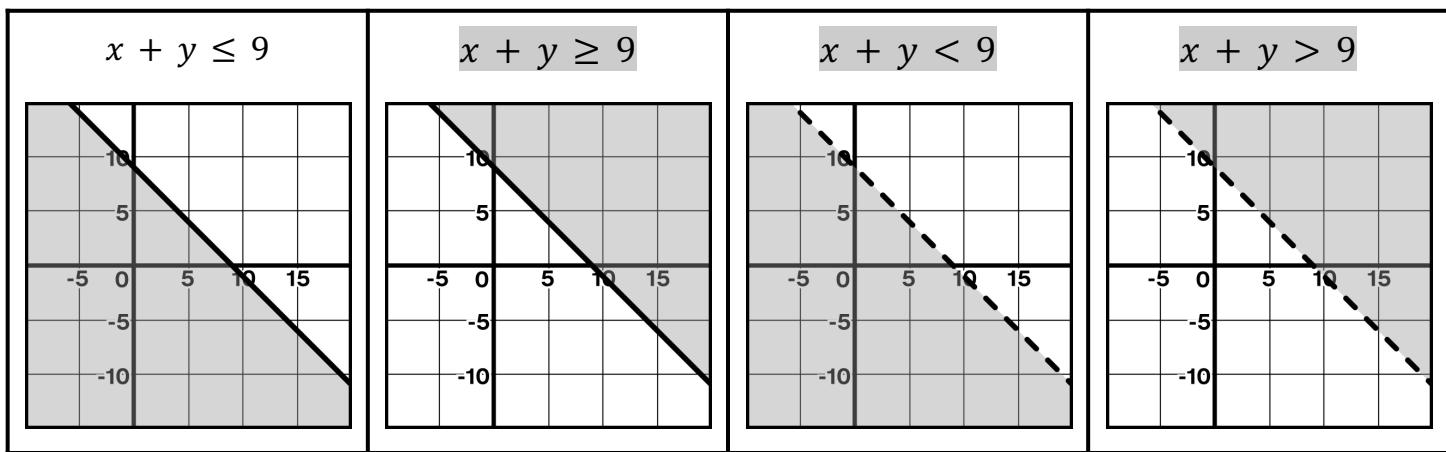
- I can interpret what two-variable inequalities represent in a situation.
- I can show and explain what it means to be a solution to a two-variable inequality.

Lesson 14: Graphing Solutions to Two-Variable Inequalities**Summary**

All the solutions to a two-variable linear inequality are represented on a graph as a half-plane.

The graph on the left represents **all** the solutions to the inequality $x + y \leq 9$.

Write inequalities to match each of the remaining three graphs.



Juliana is graphing the solutions to $x - y < 5$.

Why is her line dashed? **Responses vary.**

Juliana's line is dashed because the points on the boundary line are not included in the solution region.

Graph the solutions to $x - y < 5$.

Test points in the inequality to help with your thinking.

$$(0, 0)$$

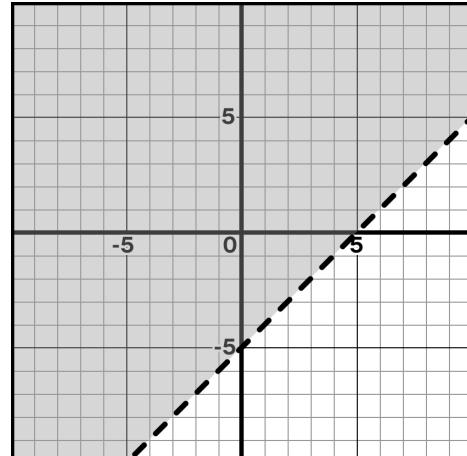
$$0 - 0 < 5$$

$$0 < 5 \checkmark$$

$$(7, 0)$$

$$7 - 0 < 5$$

$$7 < 5 \text{ False.}$$

**Things I Want to Remember**

Lesson 14: Graphing Solutions to Two-Variable Inequalities

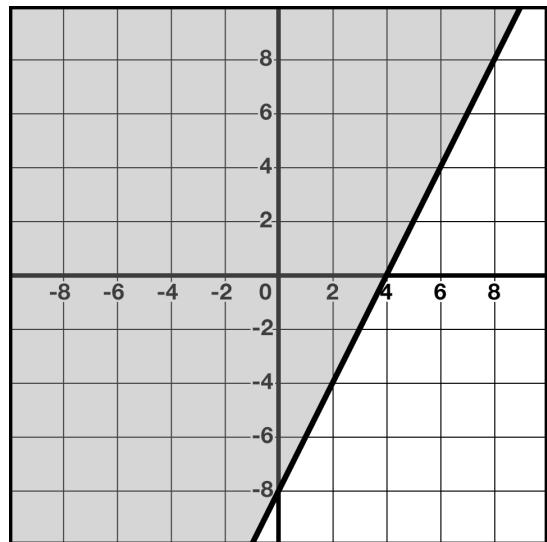
Try This!

1. Which inequality does this graph represent?

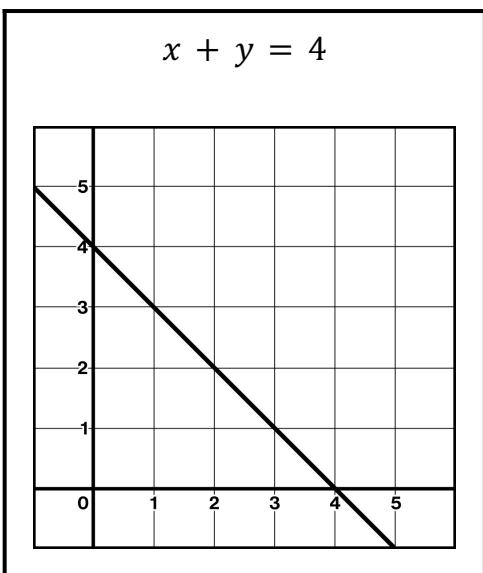
- A. $2x - y > 8$ B. $2x - y \geq 8$
 C. $2x - y < 8$ D. $2x - y \leq 8$

Show or explain your thinking. **Responses vary.**

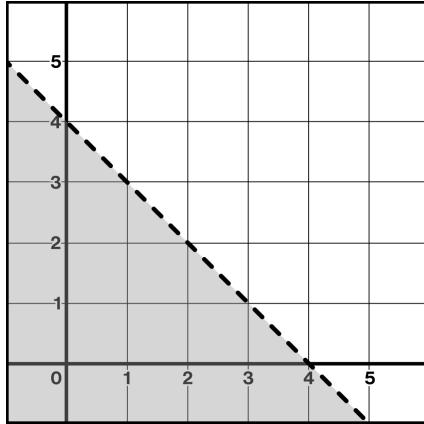
The solid line means the points on the boundary line are included in the solution, so the inequality symbol must be either \leq or \geq . $(0, 0)$ is a solution. $2(0) - 0 = 0$ and since $0 \leq 8$, the inequality symbol should be \leq .



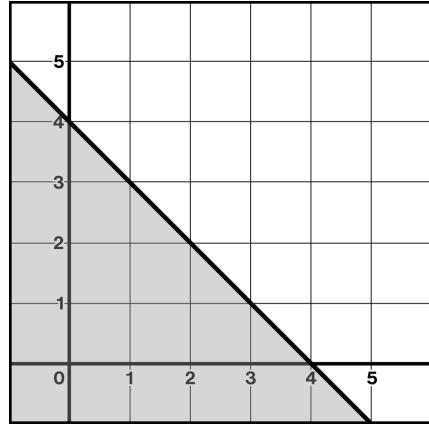
Here is a graph of $x + y = 4$. Graph the solutions to each of the following inequalities:



2.1 $x + y < 4$



2.2 $x + y \leq 4$



- I understand how solutions to a two-variable linear inequality are represented on a graph.
 I can graph the solutions to a linear two-variable inequality given the graph of its corresponding line.

Lesson 1: What Kinds of Data Can I Collect?**Summary**

Survey data can be classified as *categorical* or *quantitative*.

	Categorial	Quantitative
Example Survey Questions and Data	<p>Question: What is your favorite color? <i>Responses: red, blue, yellow, green</i></p> <p>Question: Do you usually sleep more than 8 hours a night? <i>Responses: yes, no, no, yes</i></p>	<p>Question: How many pairs of shoes do you own? <i>Responses: 3, 1, 5, 7</i></p> <p>Question: How many hours of sleep did you get last night? <i>Responses: 8, 9, 7, 8</i></p>
Your Example Survey Questions and Data	<p>Question: <i>Responses vary.</i> What is your favorite meal? Responses: <i>Responses vary. Breakfast, Lunch, Dinner, Lunch</i></p>	<p>Question: <i>Responses vary.</i> How many seasons does your favorite TV show have? Responses: <i>Responses vary. 8, 2, 15, 1</i></p>

In your own words, what is the difference between categorical and quantitative data?

Explanations vary. Categorical data has values that are categories or words, and quantitative data has values that are numbers, measurements, or quantities.

Things I Want to Remember

Lesson 1: What Kinds of Data Can I Collect?

Try This!

Decide whether each survey question will produce categorical or quantitative data.

1.1 How many languages do you speak?

Quantitative

1.2 Are you left- or right-handed?

Categorical

1.3 Do you have any pets?

Categorical

1.4 What is your height?

Quantitative

1.5 How many pets do you have?

Quantitative

1.6 Which month were you born in?

Categorical

Write a question that could produce each data set.

2.1 Responses: swimming, running, walking

Responses vary. What is your favorite type of exercise?

2.2 Responses: 10 min., 15 min., 5 min.

Responses vary. How long did it take you to get to school today?

3.1 Write a question about music that will produce quantitative data.

Responses vary. How many music concerts have you gone to?

3.2 Write a question about music that will produce categorical data.

Responses vary. Who is your favorite musical artist?

I can explain the difference between quantitative and categorical data.

Lesson 2: Revisiting Dot Plots and Histograms

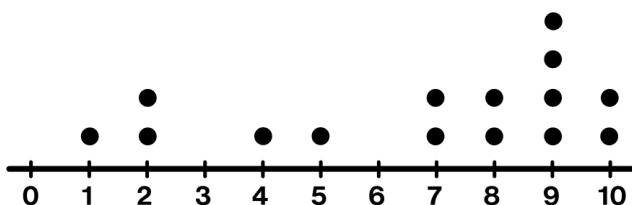
Summary

A *dot plot* and a *histogram* are two ways to visualize quantitative data.

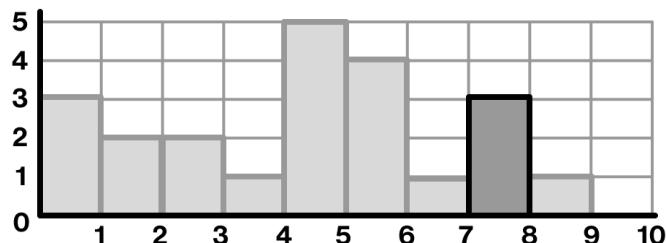
A class played *Love It or Hate It* and rated each season on a scale from 0 to 10.

Here are two representations of their ratings.

Dot Plot of Summer Ratings



Histogram of Winter Ratings



There were 15 ratings for Summer. There were 21 ratings for Winter.

The highest rating for Summer was 10. For Winter, it was between 8 and 9.

A new student gave winter a 7.7. Add this data point to the histogram above.

What are some advantages of representing data with a histogram? A dot plot?

Explanations vary. One advantage of representing data with a histogram is that you can use bins, which make it easier to organize data that includes decimals. One advantage of representing data on a dot plot is that you can see all of the individual data points.

Things I Want to Remember

Lesson 2: Revisiting Dot Plots and Histograms

Try This!

Here is a histogram of students' ratings for the fall season.

Decide if each statement is true, false, or cannot be determined.

- 1.1 There are 29 total ratings.

True	False	Cannot be determined
-------------	-------	----------------------

- 1.2 The highest rating included was a 9.9.

True	False	Cannot be determined
------	-------	-----------------------------

- 1.3 The lowest rating was less than 2.

True	False	Cannot be determined
-------------	-------	----------------------

- 1.4 There are 10 ratings higher than 6.

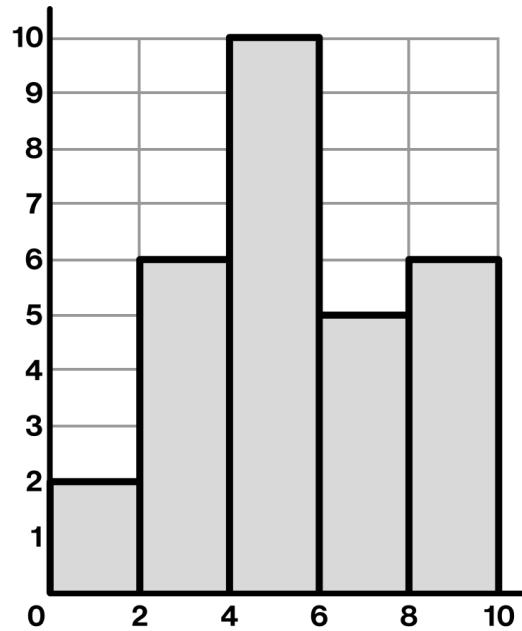
True	False	Cannot be determined
------	--------------	----------------------

2. Here are students' ratings for spring: 4.5, 5.1, 5.6, 6.5, 6.9, 7.1, 7.4, 7.9, 8.4.

Why might someone make a histogram over a dot plot to visualize this data set?

Explanations vary. Someone might make a histogram over a dot plot because a histogram can allow us to organize decimals in bins that can be easier to see. For example, grouping all of the ratings between 7 and 8.

Histogram of Fall Ratings



- I can use technology to represent data with a dot plot or histogram.
- I can describe the advantages and disadvantages of using a dot plot or a histogram to represent data.

Unit A1.3, Describing Data: Notes**Answer Key****Lesson 3: Revisiting Box Plots****Summary**

A *box plot* can be used to visualize a one-variable quantitative data set.

Zahra used a fitness app to track how many miles she walked on foot. Here is a box plot of daily miles traveled on foot each day by Zahra in June.

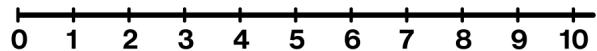
Complete the definitions and identify the statistics for Zahra's data.

Minimum: The smallest value

Quartile 1: The middle of the lower half of the data



Median: The middle of the entire data set



Quartile 3: The middle of the upper half of the data

Min.	Q1	Median	Q3	Max.
1	2	4	5	8

Maximum: The largest value

Select **all** the statements that are true according to the box plot.

- Zahra's mean miles walked in June was 5 miles.
- The middle 50% of miles walked were between 1 and 8.
- Zahra never walked 9 miles in June.
- There was one day Zahra walked 3 miles.
- Zahra walked 4 miles or less for half of the days in June.

Things I Want to Remember

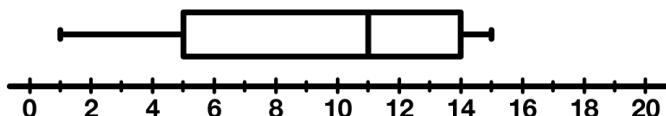
Lesson 3: Revisiting Box Plots

Try This!

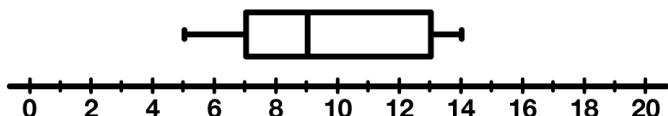
Two basketball players recorded their points for each game in the season.

Use the box plots of their data to identify each statistic.

1.1 Basketball Player A



1.2 Basketball Player B



Min.	Q1	Median	Q3	Max.
1	5	11	14	15

Minimum	Median	Maximum
5	9	14

Decide if each statement is true, false, or cannot be determined.

2.1 Player A played 15 games this season.

True

False

Cannot be determined

2.2 In half of Player B's games, they scored 9 points or fewer.

True

False

Cannot be determined

2.3 Player A scored 13 points in at least one game.

True

False

Cannot be determined

2.4 Player A scored 0 points in a game.

True

False

Cannot be determined

- I can interpret the parts of a box plot and use technology to represent data with a box plot.
- I can use box plots to compare data sets.

Unit A1.3, Describing Data: Notes

Answer Key

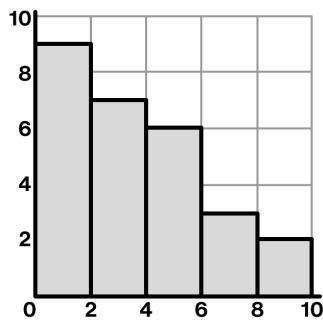
Lesson 4: Describing Data Sets

Summary

The shapes of data can be described as *bimodal*, *uniform*, *symmetric*, *skewed*, and *bell-shaped*.

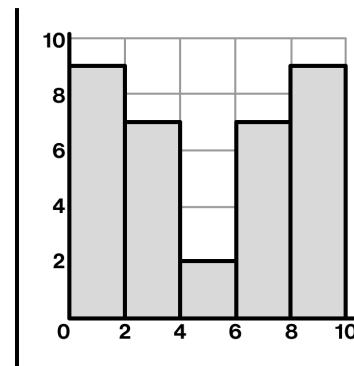
Create the missing definitions or sketches.

Shape Description	Dot Plot	Definition
Bimodal		There are two peaks in the data.
Uniform		Data values are evenly distributed.
Symmetric		The data has a line of symmetry.
Skewed		One side of the data has more values than the other.
Bell-Shaped		Most of the data is at the center with fewer points farther from the center.



Shape Description:

Skewed



Shape Description:

Bimodal and
Symmetric

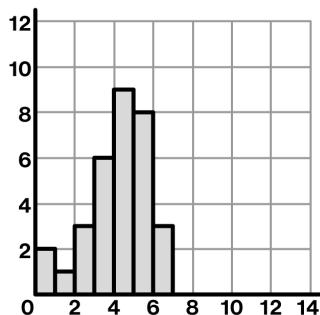
Things I Want to Remember

Lesson 4: Describing Data Sets

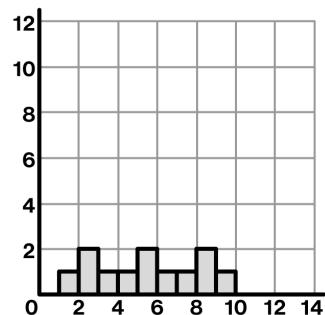
Try This!

Match each histogram with the best description of its shape.

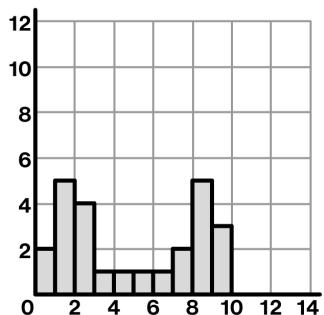
1.1 C) Skewed



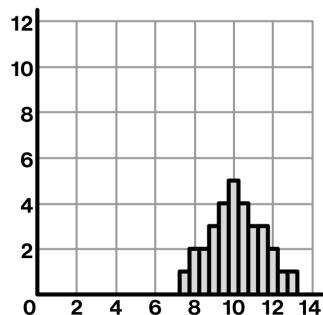
1.2 D) Symmetric



1.3 A) Bimodal



1.4 B) Bell-shaped



A. Bimodal

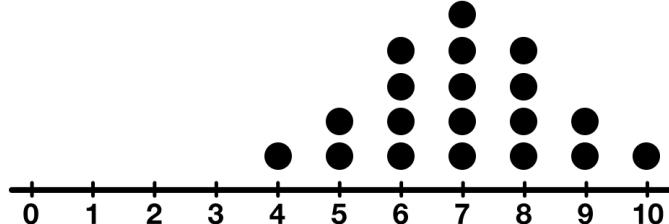
B. Bell-shaped

C. Skewed

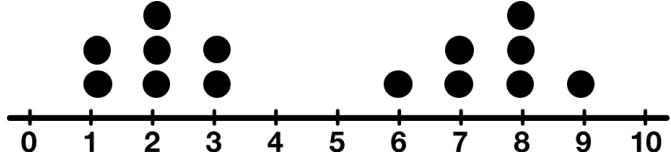
D. Symmetric

Sketch a dot plot or histogram that matches each description.

2.1 Bell-shaped



2.2 Bimodal



I can describe the shape of data sets represented with dot plots, histograms, and box plots.

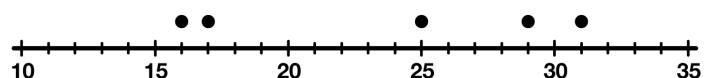
Unit A1.3, Describing Data: Notes**Answer Key****Lesson 5: Revisiting Measures of Center****Summary**

Mean and *median* are two *measures of center* used to describe data sets.

The shape of the data can influence which measure of center to use.

Here is a dot plot of Oscar's scores from a video game. Calculate the mean and median. Use the Unit 3 Calculator Guide if it helps with your thinking.

Mean	Median
23.6	25



Here is a histogram of starting salaries (in thousands of dollars) at Des-Cafe.

Mean	Median
33.5	29.5



What is the shape of the data? **Skewed**

Explain why someone might say the *median* is more representative of a typical starting salary.

Explanations vary. Someone might say the median is more representative of the typical starting salary because the shape is skewed. The higher values above 40 thousand have a big impact on the mean compared to the median.

Things I Want to Remember

Lesson 5: Revisiting Measures of Center

Try This!

Use the Desmos Graphing Calculator to create a dot plot or histogram of each data set and calculate the mean and median. Use the Unit 3 Calculator Guide if it helps with your thinking.

1.1 DesWash n' Go hourly wages (in dollars)

12	13	13
14	14	14
15	15	16

Mean: 14

Median: 14

Which is larger? **They are the same**

Shape: **Symmetric and bell-shaped**

1.2 DesTunes Music hourly wages (in dollars)

12	12	13
13	13	15
17	18	19

Mean: 14.67

Median: 13

Which is larger? **Mean**

Shape: **Skewed**

The worker making \$16 an hour is promoted to \$22 an hour. Which measure would increase?

Circle One: **mean / median / both / neither**

Explain your thinking.

Explanations vary. The mean will increase because one of the values increased. The median will stay the same because changing the maximum value in this data set does not affect the middle of the data set.

A new worker is hired and will make \$20 an hour. Which measure would increase?

Circle One: **mean / median / both / neither**

Explain your thinking.

Explanations vary. The mean would increase because adding a value higher than the mean will increase the mean. The median will increase because we are including a new wage that is changing the middle of the data set.

- I can explain how to calculate the mean and median and what these tell us about a data set.
- I can use technology to calculate the measure of centers (mean and median) for a data set.
- I can explain the effect of extreme values on the mean and median.

Unit A1.3, Describing Data: Notes

Answer Key

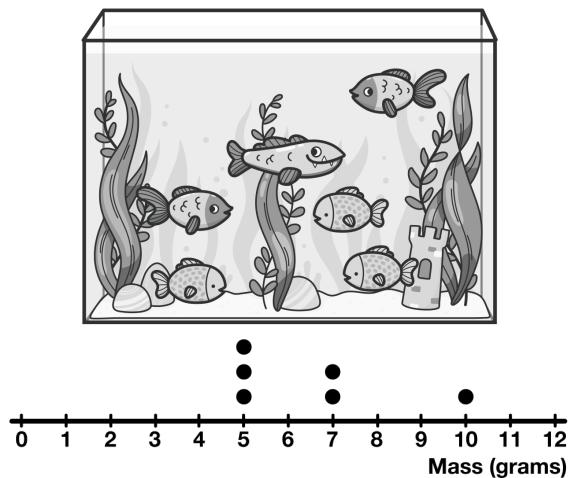
Lesson 6: Introduction to Standard Deviation

Summary

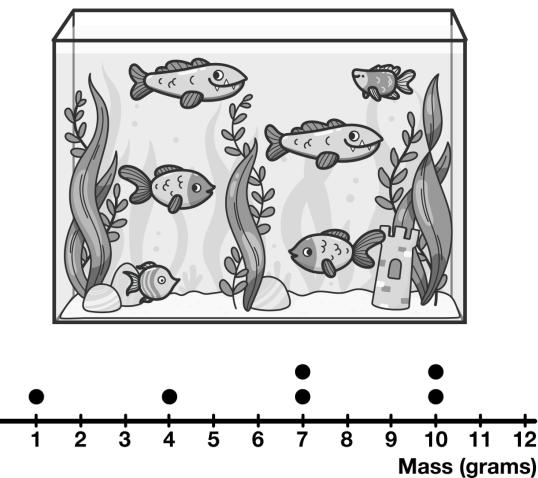
One way to measure the consistency or *spread of data* is to calculate its *standard deviation*. Data with a larger standard deviation is more spread out than data with a smaller standard deviation.

Here are the masses (in grams) of the fish in two new tanks. Calculate the statistics for Tank B.

Tank A: 5, 5, 5, 7, 7, 10



Tank B: 1, 4, 7, 7, 10, 10



Mean	Standard Deviation
$A = [5, 5, 5, 7, 7, 10]$	$A = [5, 5, 5, 7, 7, 10]$
$\text{mean}(A) = 6.5$	$\text{stdevp}(A) \approx 1.8$

Mean	Standard Deviation
$B = [1, 4, 7, 7, 10, 10]$	$B = [1, 4, 7, 7, 10, 10]$
$\text{mean}(B) = 6.5$	$\text{stdevp}(B) \approx 3.2$

Describe what the mean and standard deviation say about how the fish in Tanks A and B compare.

Explanations vary. On average, fish in Tank A will weigh about the same as fish in Tank B since their means are the same. Tank A's fish have a more consistent weight than Tank B's fish because Tank A has a smaller standard deviation.

Things I Want to Remember

Lesson 6: Introduction to Standard Deviation

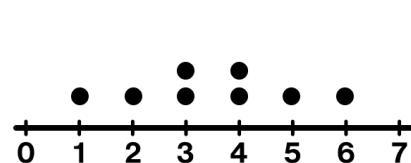
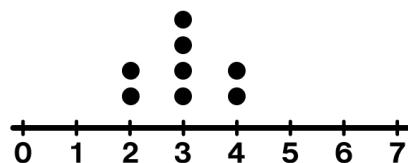
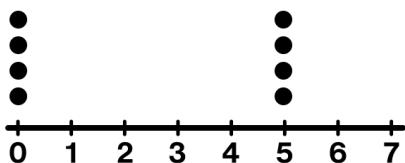
Try This!

- 1.1 Which dot plot do you think has the greatest standard deviation? **A**
- 1.2 Which dot plot do you think has the lowest standard deviation? **B**

A.

B.

C.



Calculate the mean and standard deviation for each of the data sets above.

Use a calculator to help you with your thinking.

2.1 Data Set A

Mean	Standard Deviation
2.5	2.5

2.2 Data Set B

Mean	Standard Deviation
3	0.71

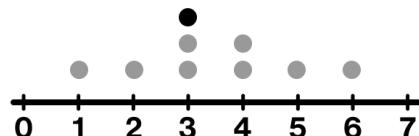
2.3 Data Set C

Mean	Standard Deviation
3.5	1.5

3. Add a data point to this dot plot that will lower the standard deviation.

Explain your thinking. ***Explanations vary.***

I added a data point at 3 to lower the standard deviation because I can tell from this graph that the mean is about 3.5. When I add a point near the mean, it lowers the standard deviation.



<input type="checkbox"/> I understand that standard deviation is a measure of spread and can use it to compare data sets.
<input type="checkbox"/> I can use technology to calculate the standard deviation of a data set.

Unit A1.3, Describing Data: Notes

Answer Key

Lesson 8: Comparing Data Using Median and IQR

Summary

The *interquartile range* (or *IQR*) measures the middle half of a data set, or the distance between the first and third quartiles.

Here are box plots of the distances traveled by three racecars. Identify the statistics for each car.

Car A

Q1	Q3	IQR	Median
16	23	7	19

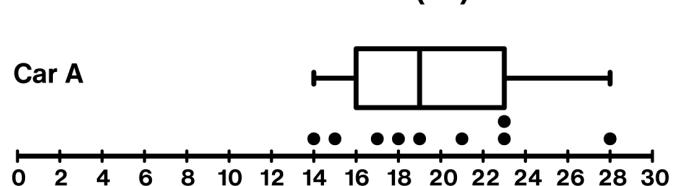
Car B

Q1	Q3	IQR	Median
13	15	2	14

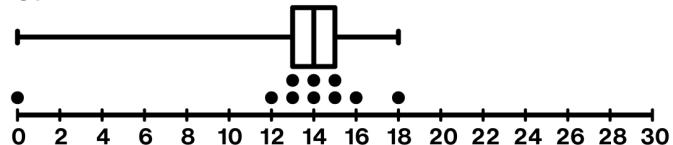
Car C

Q1	Q3	IQR	Median
14	23.5	9.5	21

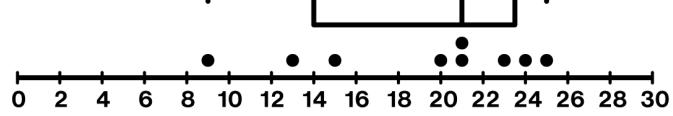
Car Distances (in.)



Car B



Car C



Which car is the most consistent? **Responses vary. Car B**

Explain which statistics you used to decide.

Responses vary. Car B is the most consistent because the IQR is the smallest.

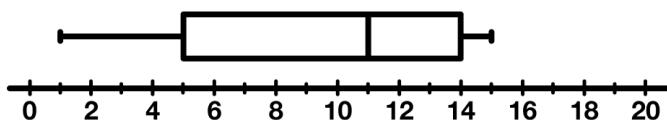
Things I Want to Remember

Lesson 8: Comparing Data Using Median and IQR**Try This!**

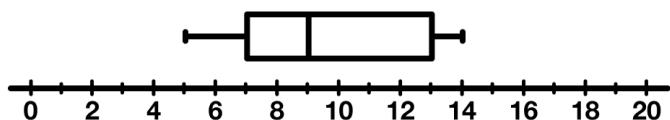
Two basketball players recorded their points for each game in the season.

Use the box plots of their data to identify each statistic.

1.1 Basketball Player A



1.2 Basketball Player B



Q1	Q3	IQR	Median
5	14	9	11

Q1	Q3	IQR	Median
7	13	6	9

2.1 Which player was more consistent in their points scored? Explain how you know.

Explanations vary. Player B was more consistent because their IQR was smaller.

2.2 Which player generally scored more points? Explain how you know.

Explanations vary. Player A generally scored more points because their median was higher.

- | |
|---|
| <input type="checkbox"/> I can calculate the IQR of a data set and understand that it is a measure of spread. |
| <input type="checkbox"/> I can use medians and IQRs to compare skewed data sets. |

Unit A1.3, Describing Data: Notes

Answer Key

Lesson 9: Identifying Outliers

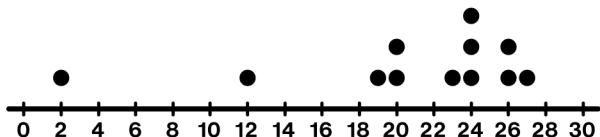
Summary

Data points that are far from other values in a data set are called *outliers*.

Here are Koharu's scores from a different game.

The mean is 20.58 and the median is 23.5.

Do you think there are any outliers? Why or why not?

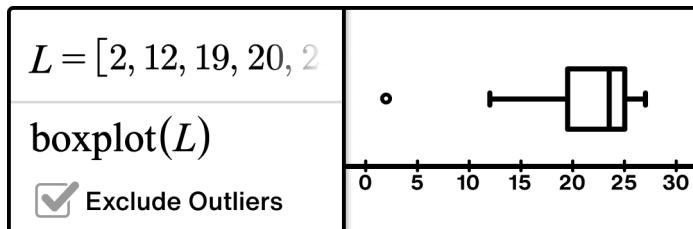


Explanations vary. Yes, because the data point of 2 is far from the rest of the points in the data set.

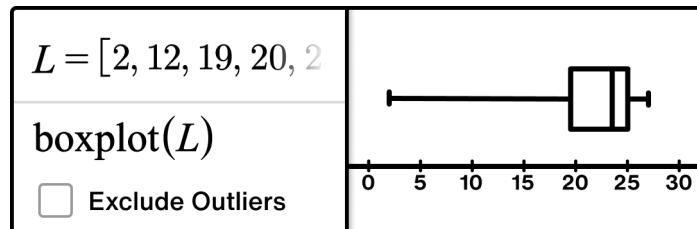
A box plot can help confirm whether or not values in a data set are outliers.

1. Enter the data as a list in the Desmos Calculator.
2. Create a box plot. Select “Exclude Outliers” to see each outlier as its own point.

Box Plot With Outliers Excluded



Box Plot With Outliers Included



Are there any outliers in Koharu's data? Explain your thinking.

Explanations vary. Yes, at 2 because the box plot shows an open circle, which represents an outlier.

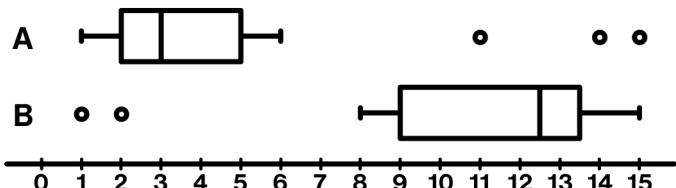
Things I Want to Remember

Lesson 9: Identifying Outliers

Try This!

Use the box plot to identify any outliers in each data set.

1. Data Set A outliers: 11, 14, 15



2. Data Set B outliers: 1, 2

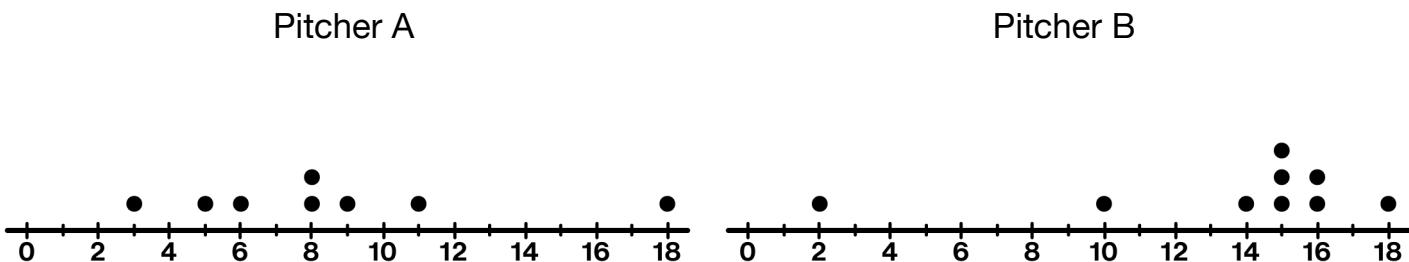
Here are dot plots that show the number of strikeouts thrown by two pitchers.

Use a calculator to make a box plot and identify any outliers in each data set.

Use the Unit 3 Calculator Guide if it helps with your thinking.

- 2.1 Pitcher A outliers: 18

- 2.2 Pitcher B outliers: 2



- I can determine whether or not a data point is an outlier.
 I can explain how outliers impact the mean or median of a data set.

Unit A1.3, Describing Data: Notes

Answer Key

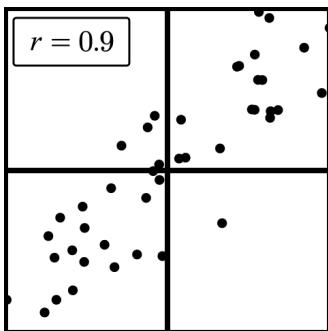
Lessons 11–12: Interpreting Correlation Coefficient in Context

Summary

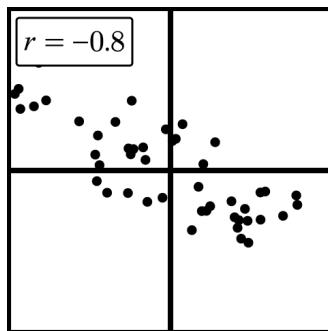
When the points on a scatter plot follow a line, we say there is a *linear association* between x and y .

The r -value, also called the *correlation coefficient*, describes the strength (weak, strong) and direction (negative, positive) of an association.

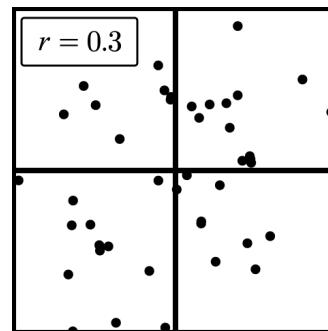
Strong and Positive



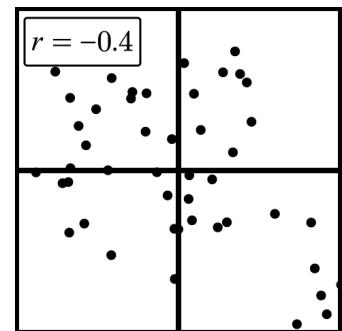
Strong and Negative



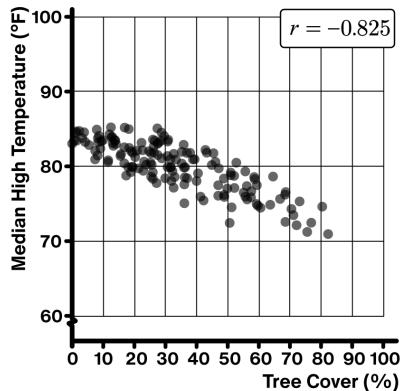
Weak and Positive



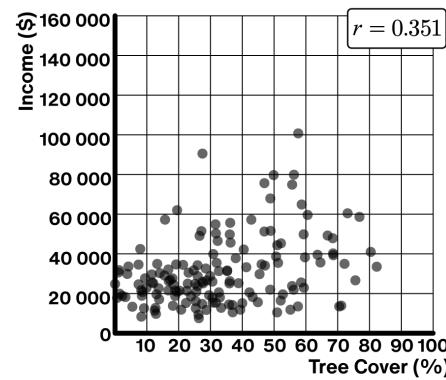
Weak and Negative



Here are two scatter plots with data recorded for 150 blocks in Detroit, Michigan.

**Description:**

The r -value is -0.825 . This means there is a negative and strong relationship between tree cover % and median high temperature in Detroit, Michigan.

**Description:**

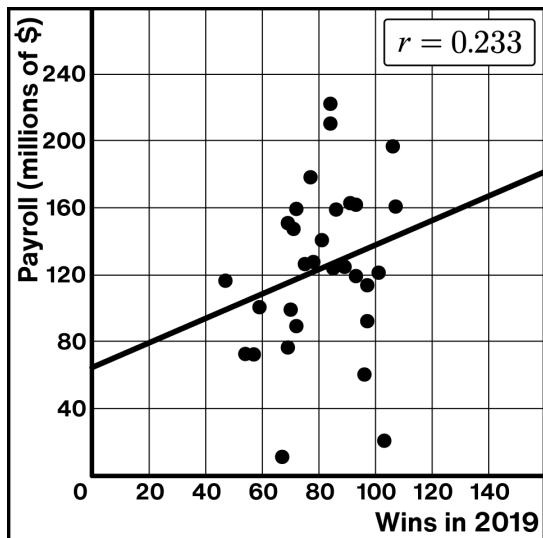
The r -value is 0.351 . This means...
there is a positive and weak relationship between tree cover % and income in Detroit, Michigan.

Things I Want to Remember

Lessons 11–12: Interpreting Correlation Coefficient in Context

Try This!

Use the correlation coefficient to describe the association shown in each scatter plot.

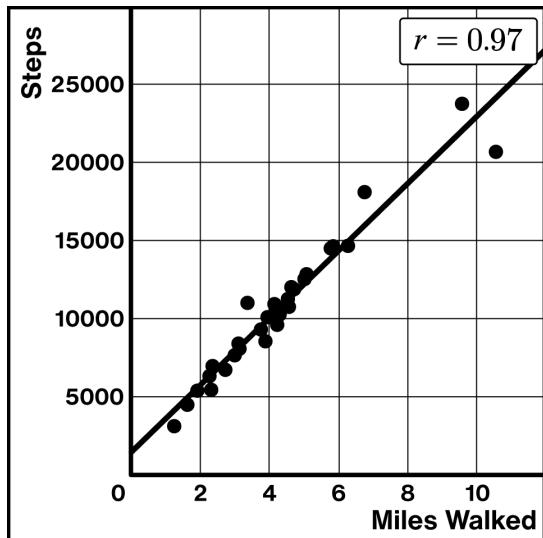


- 1.1 Lucy was curious about the relationship between money and wins in baseball.

She found data about:

- Payroll (in millions of dollars)
- Wins in 2019

Description: The r -value is 0.233. This means . . .
there is a positive and weak relationship between wins in 2019 and payroll in millions of dollars.



- 1.2 Daeja tracks her fitness data on her watch.

She recorded data about:

- Steps
- Miles walked

Description: The r -value is 0.97. This means . . .
there is a positive and strong relationship between Daeja's miles walked and steps.

- I can use a correlation coefficient to describe the strength and sign of the relationship between variables on a scatter plot.
- I can use technology to calculate the correlation coefficient of data on a scatter plot.
- I can use a correlation coefficient to describe the strength and direction of a linear association.
- I can interpret correlation coefficients in context.

Unit A1.3, Describing Data: Notes

Answer Key

Lesson 13: Interpreting Slope and Vertical Intercept in Context

Summary

Mathematicians use lines of fit to describe linear associations and make predictions.

Here are the median high temperatures and tree covers (%) for 150 blocks in two different cities.

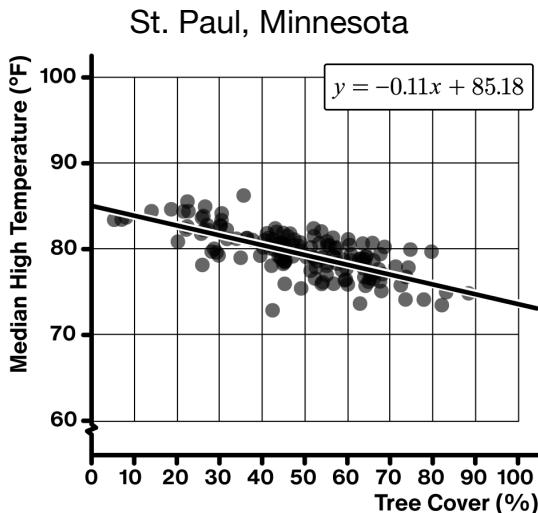
Slope interpretation:

When the tree cover increases by 1% in St. Paul, the predicted temperature decreases by 0.11°F .

 y -intercept interpretation:

If the tree cover in St. Paul is 0%, the predicted temperature is 85.18°F .

Prediction: If a block in St. Paul has 80% tree cover, the predicted median high temperature will be about 75°F .

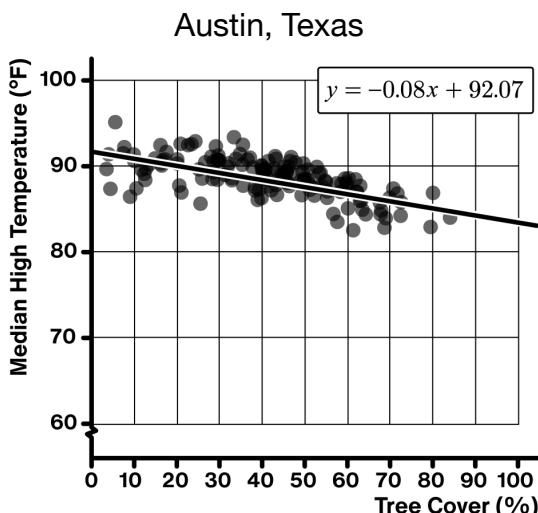
**Slope interpretation: Explanations vary.**

When the tree cover increases by 1% in Austin, the predicted temperature decreases by 0.08°F .

 y -intercept interpretation: Explanations vary.

If the tree cover in Austin is 0%, the predicted temperature is 92.07°F .

Prediction: If a block in Austin has 80% tree cover, . . .
the predicted temperature will be about 85°F .



Things I Want to Remember

Lesson 13: Interpreting Slope and Vertical Intercept in Context

Try This!

- 1.1 Nyanna noticed a trend at an ice cream shop. She recorded the number of ice cream cones sold and the customers wearing sunglasses one day.

Slope interpretation: *Explanations vary.*

When the number of ice cream cones sold increases by 1 cone, the predicted number of customers wearing sunglasses increases by 0.35 customers.

y-intercept interpretation: *Explanations vary.*

If the number of ice cream cones sold is 0, the predicted number of customers wearing sunglasses would be 1.32 customers.

Prediction: *Predictions vary.* If 30 cones are sold, the predicted number of customers wearing sunglasses would be about 12 customers.

- 1.2 Kwasi was curious about the relationship between the ages of cars and their values. He found data on the ages of several cars (in years) and their sale prices (in dollars).

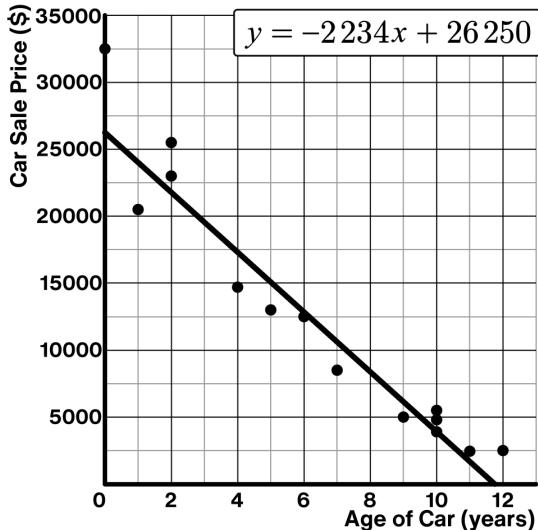
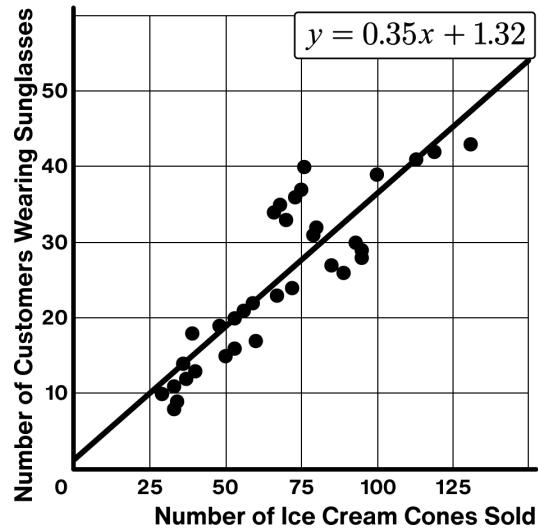
Slope interpretation: *Explanations vary.*

When the age of the car increases by 1 year, the predicted car sale price decreases by \$2234.

y-intercept interpretation: *Explanations vary.*

If the age of the car is 0 years old, the predicted car sale price is \$26 250.

Prediction: *Predictions vary.* If a car is 3 years old, . . . the predicted sale price will be about \$20 000.



- I can describe the slope and vertical intercept for a linear model in everyday language.
 I can estimate unknown values using a line of fit on a graph.

Unit A1.3, Describing Data: Notes

Answer Key

Lesson 14: Residuals and Residual Plots

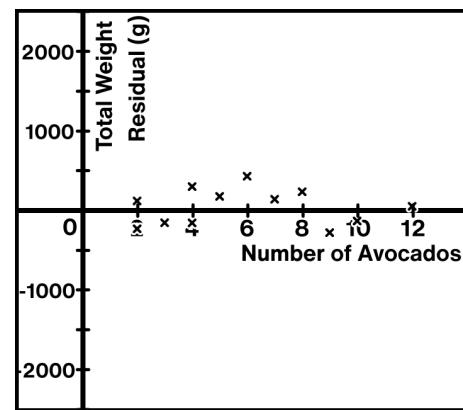
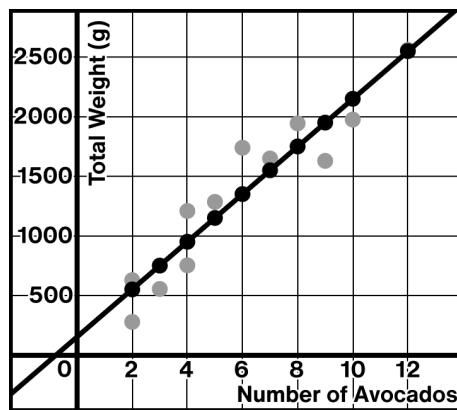
Summary

A *residual* is the difference between the y -value of a data point and the value predicted by the line of best fit. A scatter plot of all the residuals (a *residual plot*) can help us decide if a line fits the data well.

On the left is data and a line of fit for several orders of avocados. On the right is its residual plot.

Use the residual plot to explain how you know this line is a good fit for the data.

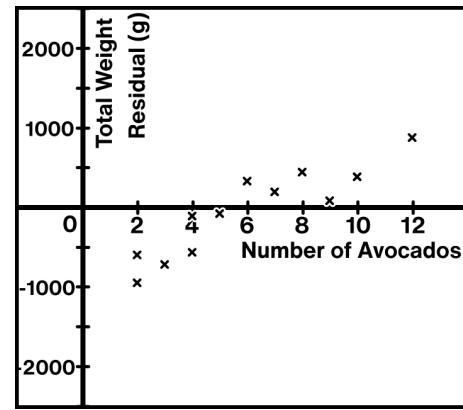
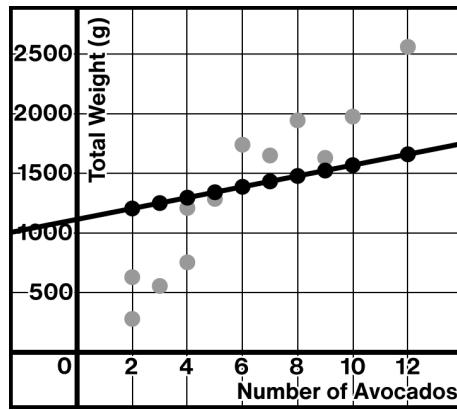
Explanations vary. The line is a good fit for the data because the residuals are close to the x -axis and there are random points below and above the x -axis.



Here is a different line of fit for the data and its residual plot.

Use the residual plot to explain how you know this line is **not** a good fit for the data.

Explanations vary. The line is not a good fit for the data because many of the points are far from the x -axis. The residuals start off all negative and then turn positive, which shows that the line does not follow the pattern of the data.



Things I Want to Remember

Lesson 14: Residuals and Residual Plots

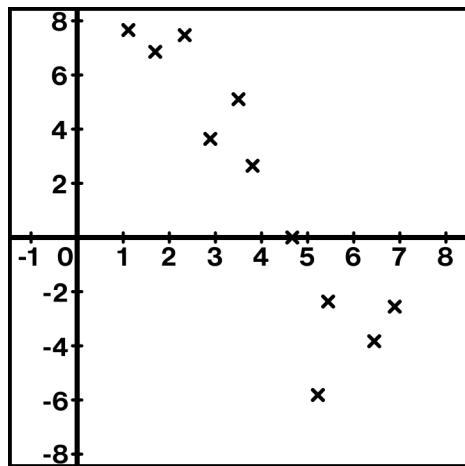
Try This!

1. Describe what the residual plot for a good line of fit looks like.

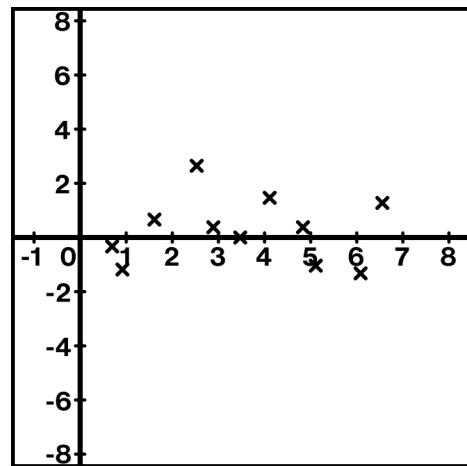
Explanations vary. The residual plot for a good line of fit will have point values that are close to the x -axis and will have random values above and below the x -axis.

Here are residual plots for lines that are not shown. Describe how you think each line fits the data.

2.1



2.2



Circle one:

The line fits the data **well** / **not well**.

Explain your thinking.

Explanations vary. The residual plot has points that are very far from the x -axis. Also, the residual values go from positive to negative, and are not random.

Circle one:

The line fits the data **well** / **not well**.

Explain your thinking.

Explanations vary. The residual plot has points that are close to the x -axis and has random points above and below the x -axis.

- I can make connections between a residual plot and residuals on a graph.
 I can recognize when a residual plot indicates a better or worse fit.

Unit A1.3, Describing Data: Notes

Answer Key

Lessons 15–17: Using Technology to Analyze Two-Variable Data

Summary

A calculator can compute the *line of best fit* and the correlation coefficient to help describe the relationship (or correlation) between two variables. *Causation* is one type of *correlation*.

In a causal relationship, a change in one variable causes a change in the other variable.

Nyanna noticed a trend at an ice cream shop. She recorded the number of ice cream cones sold and the customers wearing sunglasses one day.

Nyanna used a calculator to generate a line of best fit.

Line of best fit equation:

$$y = 0.35x + 1.32$$

The r -value is 0.87. This means . . .

Explanations vary. There is a positive and strong relationship between the number of ice cream cones and the number of customers wearing sunglasses.

Use Nyanna's model to predict the number of people wearing sunglasses if there are 150 ice cream cones sold.

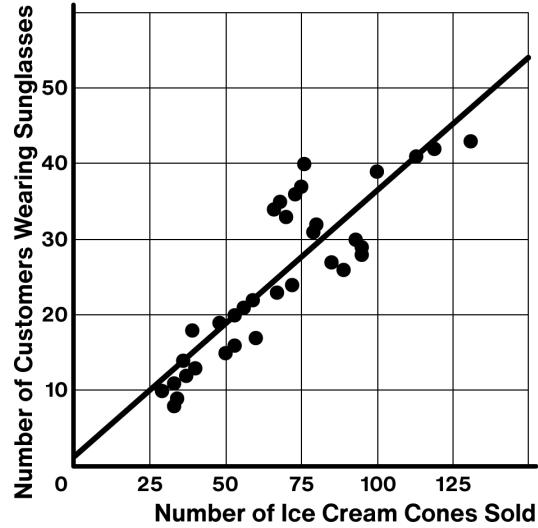
$$y = 0.35(150) + 1.32$$

$$y = 53.82$$

~54 customers wearing sunglasses

Do you think one of the variables causes the other? If not, what else could be affecting the relationship?

Responses vary. I do not think one of the variables causes the other. If it is sunny out, people might be more likely to wear sunglasses and to buy ice cream.



$$y_1 \sim mx_1 + b$$

STATISTICS

$$r^2 = 0.7642$$

$$r = 0.8742$$

PARAMETERS

$$m = 0.351312$$

$$b = 1.31984$$

Things I Want to Remember

Lessons 15–17: Using Technology to Analyze Two-Variable Data

Try This!

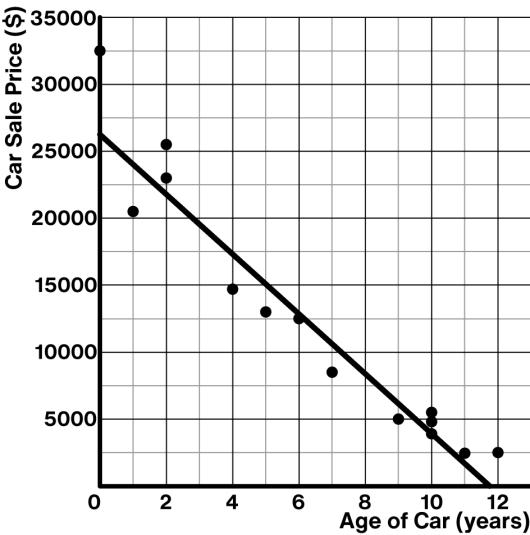
Kwasi was curious about the relationship between the ages of cars and their values. He found data on the ages of several cars (in years) and their sale prices (in dollars).

- Line of best fit equation:

$$y = -2270.38x + 26886.7$$

- The r -value is -0.96 . This means . . .

Explanations vary. There is a negative and strong relationship between the age of the car and the car sale price.



- What does the model predict the price would be for a car that was 8 years old?

$$y = -2270.38(8) + 26886.7$$

$$y = 8723.66$$

The predicted price will be \$8 723. 66

$$y_1 \sim mx_1 + b$$

STATISTICS

$$r^2 = 0.9215$$

$$r = -0.96$$

PARAMETERS

$$m = -2270.38$$

$$b = 26886.7$$

- Do you think one of the variables causes the other?

If not, what else could be affecting the relationship? Explain your thinking.

Explanations vary. Yes. I believe that the age of the car causes the price to go down because an older car is more likely to have more mileage or other mechanical issues.

- I can use technology to generate the line of best fit for data on a scatter plot.
- I can use the equation of the best fit line to make predictions.
- I can determine if the relationship between two variables represents correlation or causation.
- I can analyze the relationship between two variables in context using scatter plots, lines of best fit, and correlation coefficients.

Lesson 1: What Is a Function?

Summary

A *function* is a rule that assigns exactly one output to each possible input.

When determining if a rule is a function, a table can be used to organize inputs and outputs. If one input has multiple possible outputs, then the rule is not a function.

Rule H takes any measurement in meters and converts it to centimeters.

Input	Output
3 m	300 cm
2.6 m	260 cm
5.5 m	550 cm
3 m	300 cm

In this relationship, each input has exactly one output, so it's a function.

For instance, when inputting 3 m, the output will always be 300 cm.

Rule J takes whole numbers from 1 to 15 and outputs a word of that length.

Input	Output
5	watch
9	vegetable
9	classroom
1	a

In this relationship, inputs have multiple possible outputs, so it's not a function.

For instance, the table shows that the input 9 has two different outputs ("vegetable" and "classroom").

Rule K takes any year and returns the last two digits.

Input	Output
2009	09
1915	15
2015	15
2012	12

Decide if **Rule K** is a function. Explain your thinking.

Explanations vary. In this relationship, each input has exactly one output, so it's a function. For instance, when inputting the year 2022, the output will always be 22.

Things I Want to Remember

Lesson 1: What Is a Function?

Try This!

Rules L and M are functions. Complete the remaining inputs and outputs.

- 1.1 **Rule L** takes a value and outputs that value plus one.

Input	Output
-5	-4
6	7
2	3
-5	-4
5*	6*

- 1.2 **Rule M** takes a value and outputs that value multiplied by 10.

Input	Output
2	20
60	600
0.8	8
7	70
3*	30*

*Responses vary.

2. Circle the rule that is **not** a function.

Rule P takes any word and outputs the number of letters.

Input	Output
at	2
tree	4
simple	6
the	3

Rule Q takes any month and outputs its calendar order.

Input	Output
January	1
March	3
April	4
March	3

Rule R takes any value and either multiplies or divides it by 2.

Input	Output
2	4
10	20
3	6
2	1

Rule S takes a letter and shifts it one place forward in the alphabet.

Input	Output
C	D
M	N
Z	A
O	P

- | |
|---|
| <input type="checkbox"/> I can decide whether or not a rule is a function. |
| <input type="checkbox"/> I can explain that a function has only one output for every input. |

Lessons 2–3: Function Notation and Equations

Summary

Function notation is a way of writing the inputs and outputs of a function.

For example, suppose we made a function for determining the price of a medium pizza.

The table shows some inputs and outputs.

MENU	
Small:	\$13.50 plus \$1.25 per topping
Medium:	\$15.50 plus \$1.50 per topping
Large:	\$17.75 plus \$2.25 per topping

$m(2)$ is an example of a statement in function notation and is pronounced “m of two.”

$m(2) = 18.50$ means:

The price of a medium pizza with 2 toppings is \$18.50.

$m(1) = 17.00$ means:

Number of Toppings	Price
0	\$15.50
1	\$17.00
2	\$18.50

The price of a medium pizza with 1 topping costs \$17.00

We can define the function $m(t)$ using an equation.

$m(t) = 15.5 + 1.5t$ represents the cost of a medium pizza with t toppings.

What is the value of $m(4)$?

$$m(4) = 15.5 + 1.5(4)$$

$$m(4) = 21.5$$

$s(t) = 13.50 + 1.25x$ represents the cost of a small pizza with t toppings.

What is the value of $s(2)$?

$$s(2) = 16$$

Things I Want to Remember

Lessons 2–3: Function Notation and Equations

Try This!

An ice cream shop serves ice cream in either a waffle cone or a bowl. Customers also decide how many scoops of ice cream they want.

$w(x) = 1.25x + 2.5$ represents the cost of a waffle cone order, where x represents the number of scoops of ice cream.

1.1 What is the value of $w(2)$?

$$w(2) = 5$$

1.2 What does $w(4) = 7.5$ mean in the context of the ice cream shop?

Responses vary. A waffle cone with 4 scoops of ice cream costs \$7.50

1.3 $b(x)$ represents the cost of a bowl order, where x represents the number of scoops of ice cream. The bowl costs \$1 plus \$1.50 for each scoop of ice cream. Write an equation for $b(x)$.

$$b(x) = 1 + 1.50x$$

1.4 Fatima compares her ice cream order to her brother's order. What does the statement $w(2) < b(4)$ mean?

Responses vary. A waffle cone with 2 scoops of ice cream costs less than a bowl with 4 scoops of ice cream.

- I can interpret a statement that uses function notation in context.
- I can evaluate functions written in function notation.
- I can write equations of functions using function notation.

Lessons 5–6: Key Features of Graphs

Summary

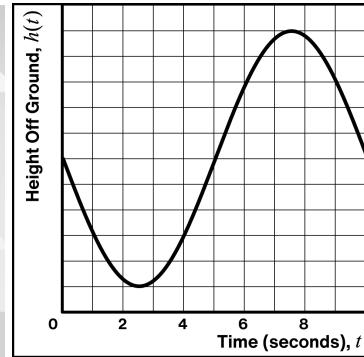
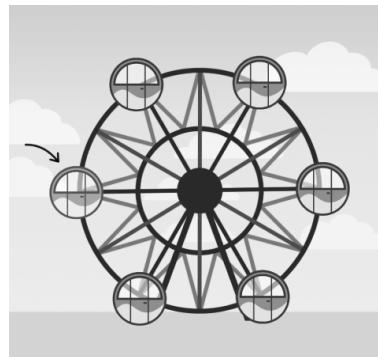
A graph can reveal in more detail what is happening to the inputs and outputs during a situation.

Here $h(t)$ represents the height of the cart on the Ferris wheel at time t .

When is the Ferris wheel cart at its lowest height?

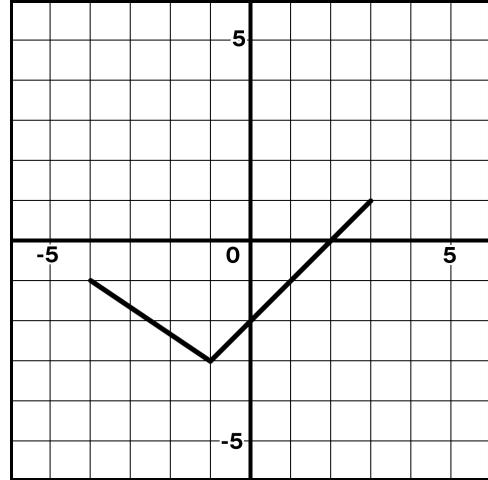
Around 2.5 seconds

Which is greater: $h(1)$ or $h(3)$? $h(1)$



The terms *maximum*, *minimum*, *positive*, *negative*, *increasing*, and *decreasing* can be used to describe parts of a graph. Complete the table.

Key Features	When
Minimum: Coordinates of the lowest point of the graph	(-1, -3)
Maximum: Coordinates of the highest point of the graph	(3, 1)
Positive: When the function has positive outputs; graph is above the x -axis	$x > 2$
Negative: When the function has negative outputs. The graph is below the x -axis.	$x < 2$
Increasing: When the function's outputs increase as the inputs increase; graph is upward-sloping, left to right	$x > -1$
Decreasing: When the function's outputs decrease as the inputs increase; graph is downward-sloping, left to right	$x < -1$

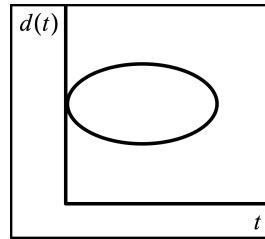
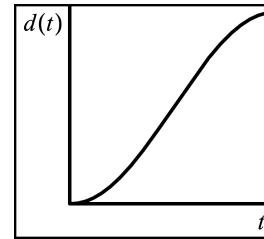
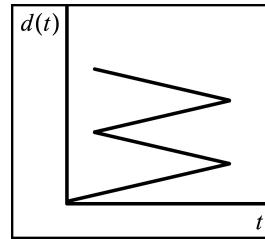
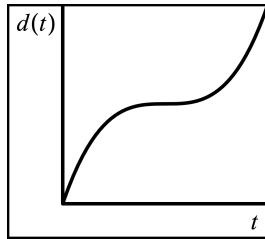
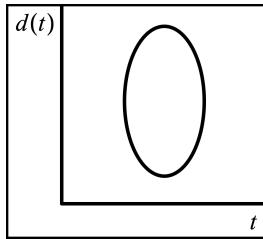


Things I Want to Remember

Lessons 5–6: Key Features of Graphs

Try This!

1. Jasmine races around an oval track. $d(t)$ represents how many meters were run at time t . Select all possible graphs that could represent Jasmine's race.

 Graph 1 Graph 2 Graph 3 Graph 4 Graph 5

Circle **all** descriptions that apply to the specified interval of $f(x)$.

2.1 $x < -2$

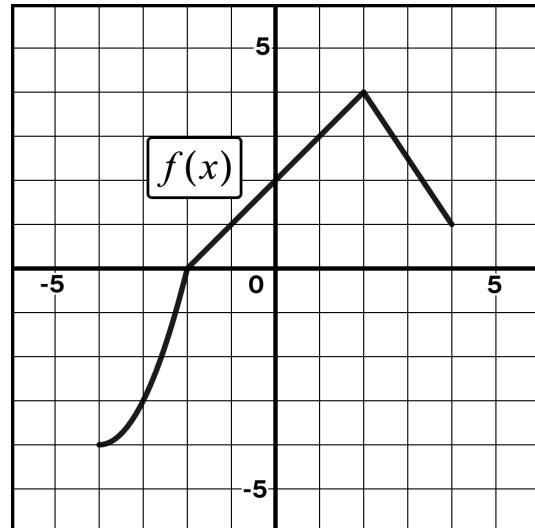
positive **negative** **increasing** **decreasing**

2.2 $x > 2$

positive **negative** **increasing** **decreasing**

3.1 What is the maximum? (2, 4)

3.2 What is the minimum? (-4, -4)



- I can sketch a graph of a function to match a situation.
- I can make connections between situations and graphs.
- I can describe the key features of a graph using words like *positive*, *negative*, *maximum*, *minimum*, *increasing*, and *decreasing*.
- I can use the key features of a function to build a graph of a function.

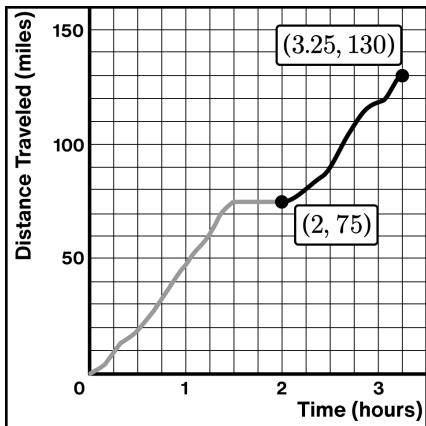
Lesson 7: Average Rate of Change

Summary

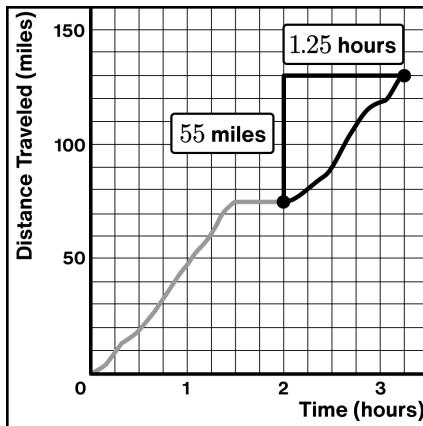
The *average rate of change* is equivalent to the slope of the line between two points.

Let's look at an interval of Arjun's car trip below. To determine the average rate of change between 2 to 3.25 hours, divide the change in distance (55 miles) by the change in time (1.25 hours). Two approaches for doing that are shown below.

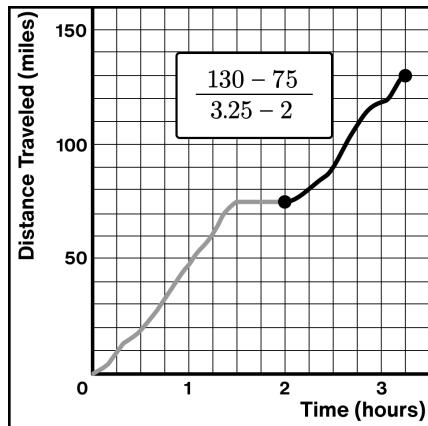
Arjun's Car Trip



Approach 1



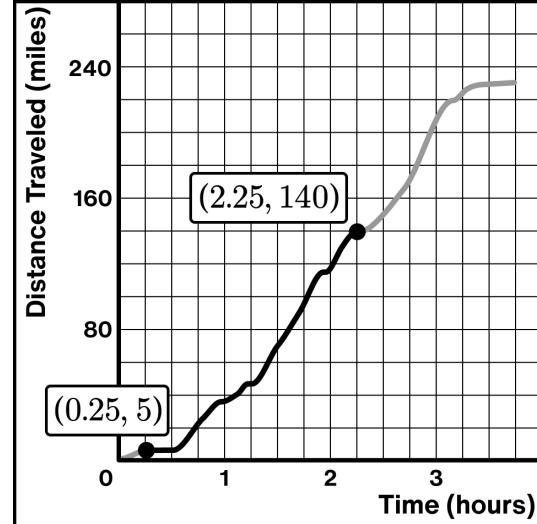
Approach 2



The average rate of change for the interval 2 to 3.25 is 44. That means Arjun's average speed was 44 miles per hour in that interval.

Here is Troy's trip. Determine Troy's average rate of change from 0.25 to 2.25 hours.

67.5 miles per hour.



Things I Want to Remember

Lesson 7: Average Rate of Change

Try This!

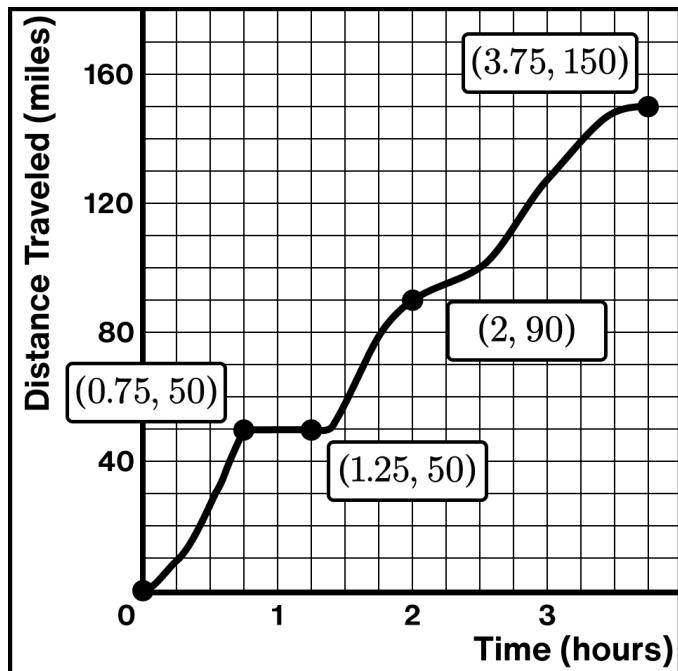
Oscar took the train to attend his friend's birthday. This graph represents his trip.

- 1.1 Which interval had the greater average rate of change?

0 to 0.75 hours 0 to 1.25 hours

- 1.2 Calculate the average rate of change for each interval.

Interval	Average Rate of Change (mph)
0 to 3.75 hours	40
0.75 to 2 hours	32
0.75 to 1.25 hours	0



- 1.3 What could have happened during the interval 0.75 to 1.25 hours?
- The train was traveling at a constant speed of 50 miles per hour.
 - The train was traveling on a straight track during that time.
 - The train stopped completely to wait for the track to clear.
 - The train traveled east for 30 minutes.

I can calculate the average rate of change over an interval on a graph.

- I can interpret the average rate of change in context.

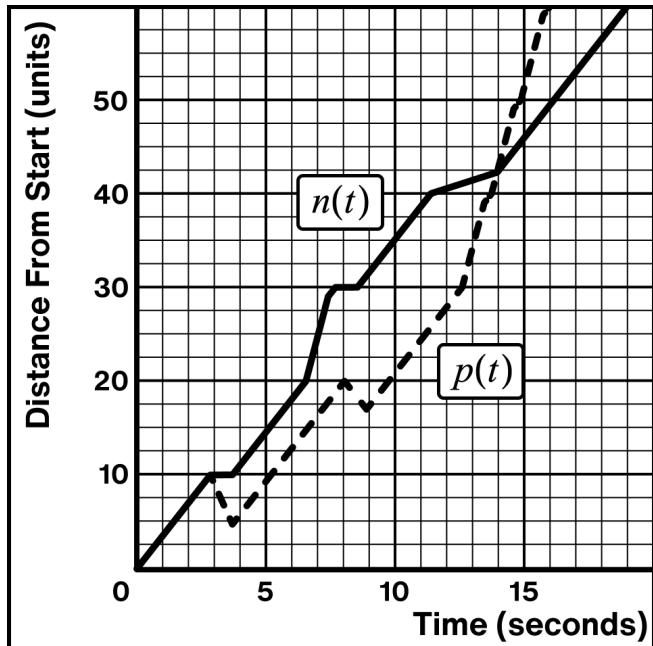
Lesson 8: Comparing Graphs

Summary

When analyzing two or more functions, you can compare the key features and behavior of different parts of their graphs.

Nekeisha and Polina raced their spaceships. Functions $n(t)$ and $p(t)$ give their spaceships' distance after t seconds.

Statement	Meaning
$n(6) > p(6)$	At 6 seconds, Nekeisha was ahead of Polina.
$n(16) < p(16)$	At 16 seconds, Nekeisha was behind Polina.
$n(3) = p(3)$	At 3 seconds, Nekeisha and Polina both traveled the same distance.
$n(14) = p(14)$	At 14 seconds, Nekeisha and Polina both traveled the same distance.



Decide if each statement is true, false, or cannot be determined.

$n(t)$ and $p(t)$ are both decreasing from 8 to 9 seconds.

True

False

Cannot be determined

$n(t)$ and $p(t)$ have the same average rate of change from 0 to 14 seconds.

True

False

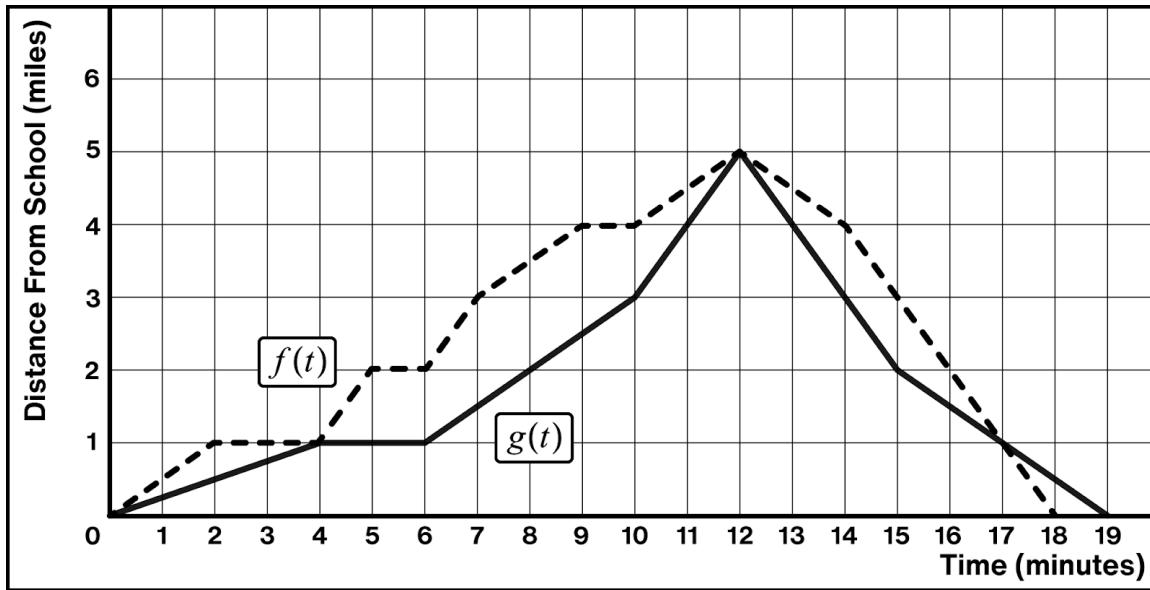
Cannot be determined

Things I Want to Remember

Lesson 8: Comparing Graphs

Try This!

A school has two buses that take different routes to drop students off. They leave at the same time. $f(t)$ and $g(t)$ represent the distance of each bus from school (in miles) after t minutes.



- 1.1 Select **all** the true statements.

- $f(2) = g(2)$
- $f(15) > g(15)$
- $f(10) = g(10)$
- $f(2) = 6$
- $f(8) > g(8)$

- 1.3 Select **all** the true statements.

- $f(t)$ and $g(t)$ have the same maximum
- $f(t)$ and $g(t)$ are both increasing from 4 to 5 minutes
- $f(t)$ and $g(t)$ are both decreasing from 12 to 15 minutes
- $f(t)$ and $g(t)$ have the same average rate of change from 5 to 6 minutes
- $f(t)$ and $g(t)$ have the same average rate of change from 6 to 12 minutes

- 1.2 Write one value of t where $f(t) = g(t)$.

Possible correct solutions.

- $t = 0$
- $t = 4$
- $t = 12$
- $t = 17$

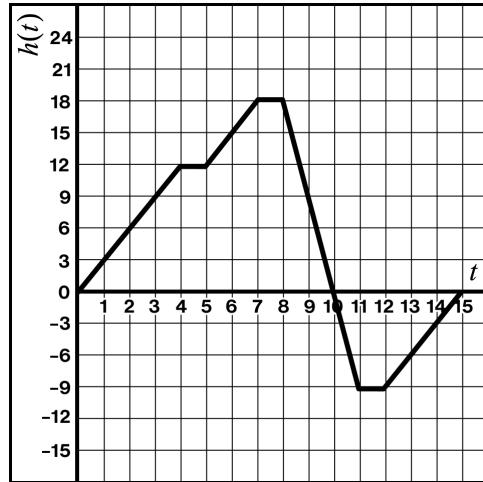
- I can compare two graphs of functions using their key features.
- I can use function notation to compare two graphs of functions.

Lessons 10–11: Domain and Range of Graphs

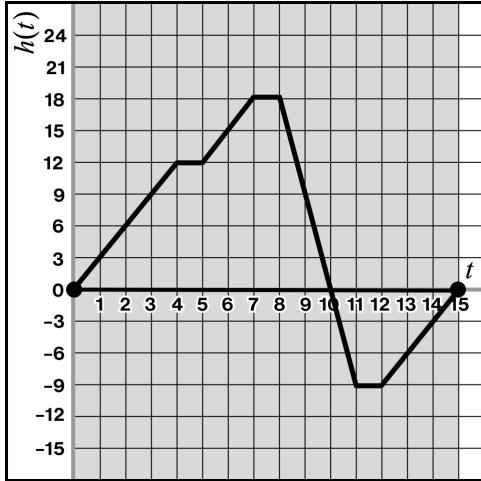
Summary

The domain and range of functions can be described using a *compound inequality*, which is two or more inequalities joined together.

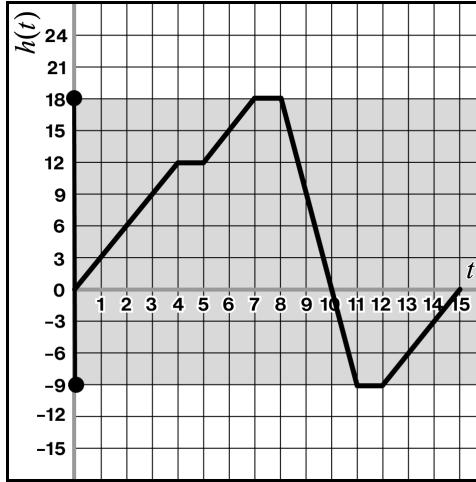
Let's look at a guest's elevator ride at the Four Quadrants Hotel. The graph shows $h(t)$, the height of the elevator in meters, t seconds into the guest's ride.



Sketch where you see the domain of $h(t)$.



Sketch where you see the range of $h(t)$.



Complete the compound inequality to describe the domain of $h(t)$.

$$0 \leq t \leq 15$$

Complete the compound inequality to describe the range of $h(t)$.

$$-9 \leq h(t) \leq 18$$

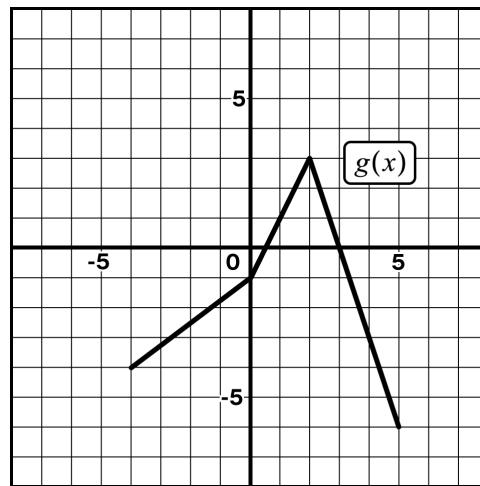
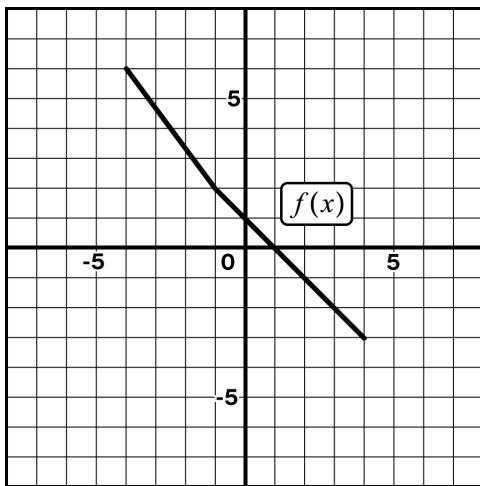
Things I Want to Remember

Lessons 10–11: Domain and Range of Graphs

Try This!

Complete the compound inequalities to describe the domain and range of each function.

1.1 Domain	1.2 Range	2.1 Domain	2.2 Range
$-4 \leq x \leq 4$	$-3 \leq f(x) \leq 6$	$-4 \leq x \leq 5$	$-6 \leq g(x) \leq 3$

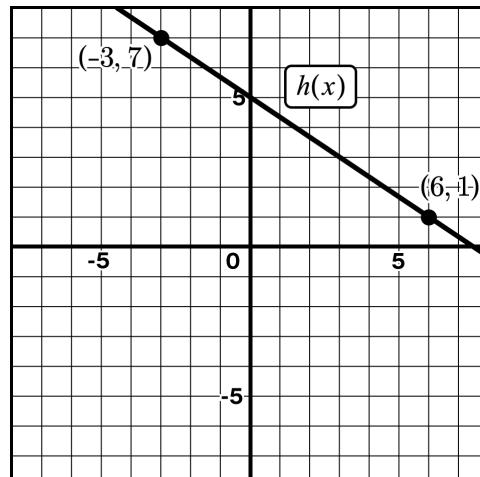


- 3.1 Write a domain that could restrict the graph of $h(x)$ from $(-3, 7)$ to $(6, 1)$.

$$-3 \leq x \leq 6$$

- 3.2 Write a range that could restrict the graph of $h(x)$ from $(-3, 7)$ to $(6, 1)$.

$$1 \leq h(x) \leq 7$$



- I can write the domain and range of a function using inequalities.
- I can interpret the meaning of the domain and range in context.
- I can restrict the domain and range of a function using inequalities.

Lesson 12: Functions in Context

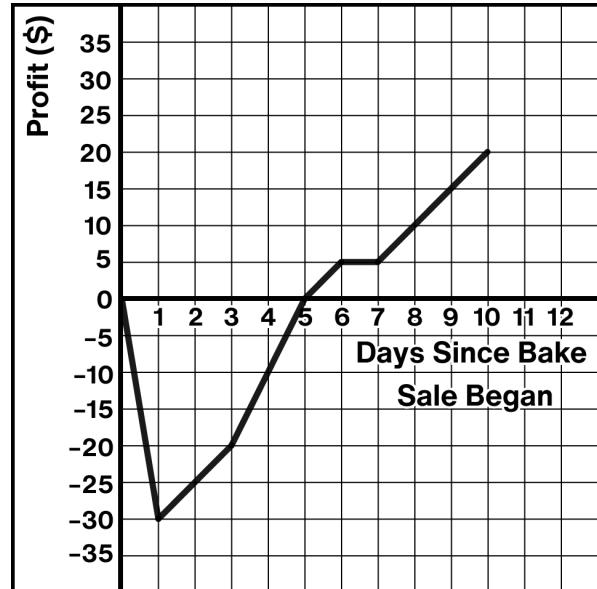
Summary

A function's graph can be described using key features, which can be interpreted when provided a context.

Let's look at Kayleen's bake sale experience at her school. Kayleen decided to make cakes for her school's bake sale and tracked her profits.

Complete the table with interpretations of each term in this context.

Term	Meaning
maximum	Kayleen makes the most profit.
negative interval	Kayleen has not made her money back.
positive interval	Kayleen has made her money back and more.
decreasing interval	Kayleen spends money on materials to make cakes.
increasing interval	Kayleen is selling cakes.



Tell a story about Kayleen's bake sale experience that makes sense based on the graph.

Responses vary. To prepare for the bake sale, Kayleen bought materials to make cakes. After a day of making cakes, she starts selling them and sells enough cakes by day 5 to make her money back. On day 6, she does not sell any cakes, but starts selling more cakes after day 7. By day 10, she has made her highest profit.

Things I Want to Remember

Lesson 12: Functions in Context

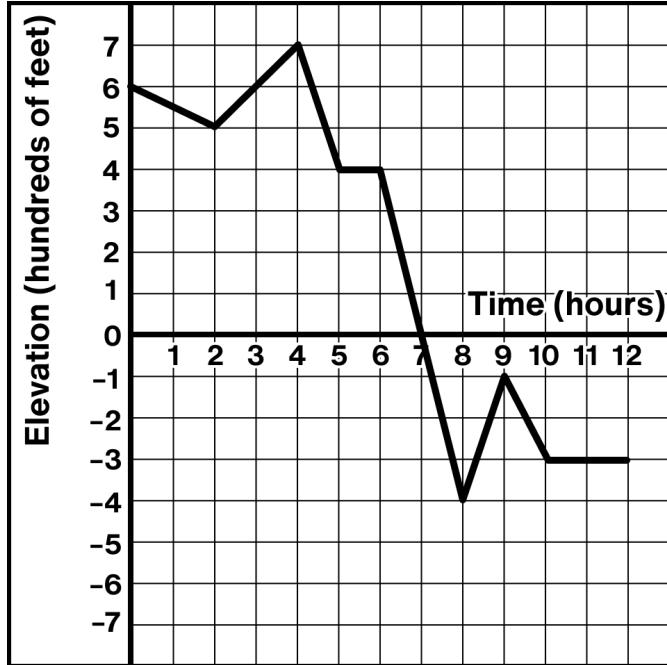
Try This!

Parv hiked down to the bottom of a canyon and tracked his elevation above and below sea level. The graph shows $p(t)$, Parv's elevation after t hours.

- 1.1 Calculate the average rate of change of Parv's hike from 0 to 12 hours.

0.75

- 1.2 Complete the table.



Term	Meaning
minimum	Parv's lowest elevation
increasing interval	When Parv's elevation is getting higher
decreasing interval	When Parv's elevation is getting lower
domain	How long Parv hiked for
range	All of the elevations that Park hiked at

I can interpret the key features of a function in context.

Lessons 13–14: Piecewise-Defined Functions

Summary

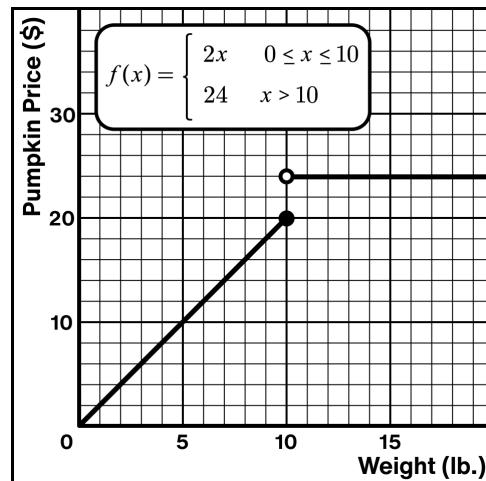
A *piecewise-defined function* is a function in which different rules apply to different intervals in its domain.

At Omar's Farm, the function $f(x)$ represents the price of a pumpkin with a weight of x pounds. Pumpkins 10 pounds or less cost \$2 per pound, and pumpkins greater than 10 pounds cost \$24.

When $0 \leq x \leq 10$, $f(x) = 2x$.

When $x > 10$, $f(x) = 24$.

Evaluate	Interval	Equation	Calculate
$f(4)$	4 is in $0 \leq x \leq 10$	$f(x) = 2x$	$f(4) = 2(4) = 8$
$f(15)$	15 is in $x > 10$	$f(x) = 24$	$f(15) = 24$
$f(10)$	10 is in $0 \leq x \leq 10$	$f(x) = 2x$	$f(10) = 2(10) = 20$
$f(11)$	11 is in $x > 10$	$f(x) = 24$	$f(11) = 24$

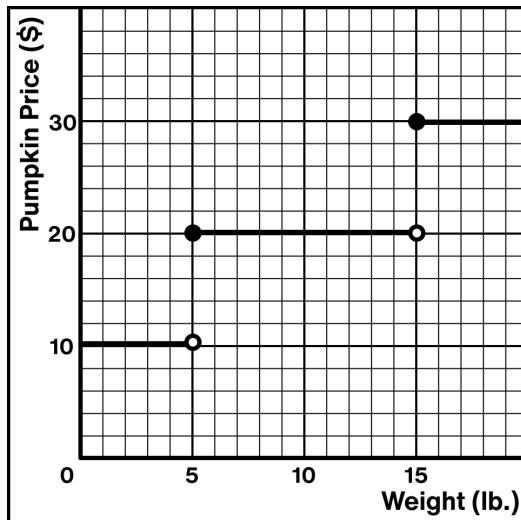


The farm is changing their prices to the following:

- Pumpkins less than 5 pounds: \$10
- Pumpkins greater than or equal to 15 pounds: \$30
- All other pumpkins: \$20

Complete the piecewise-defined function and the graph.

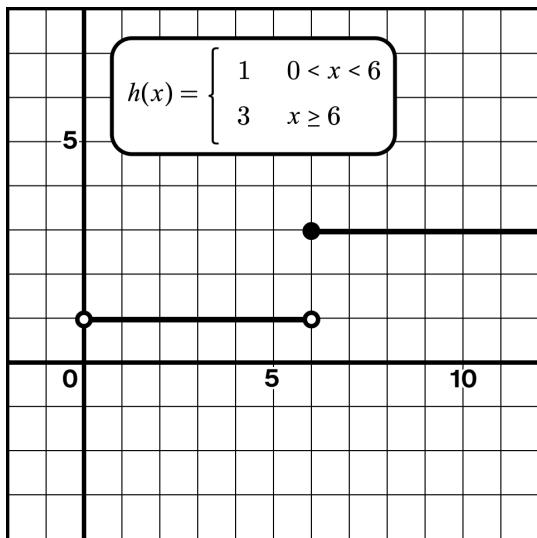
$$f(x) = \begin{cases} 10 & x < 5 \\ 20 & 5 \leq x < 15 \\ 30 & x \geq 15 \end{cases}$$



Things I Want to Remember

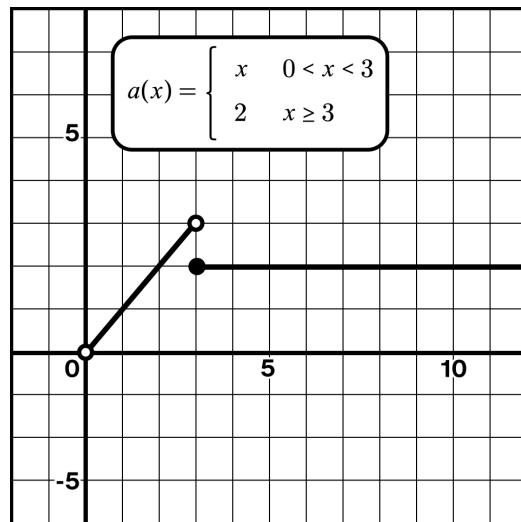
Lessons 13–14: Piecewise-Defined Functions

Try This!



1.1 What is $h(4)$? $h(4) = 1$

1.2 What is $h(6)$? $h(6) = 3$



2.1 What is $a(1)$? $a(1) = 1$

2.2 What is $a(10)$? $a(10) = 2$

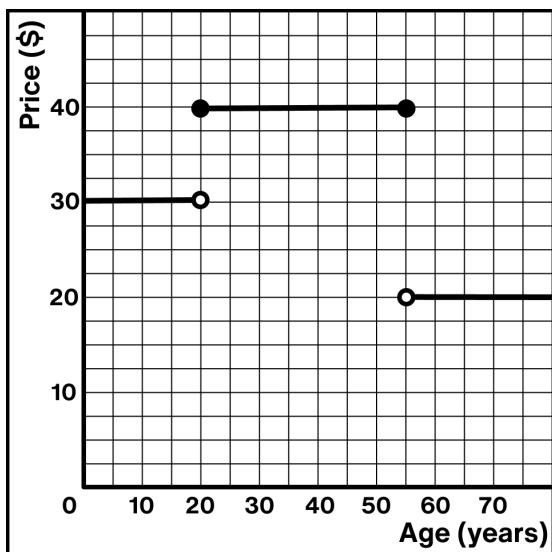
The Desmopolis Transportation Agency is considering using a passenger's age to determine the price of a train ticket. The function $c(x)$ gives the price of a ticket for a person who is x years old.

One plan suggested the following ticket prices.

- Younger than 20 years old: \$30
- Older than 55 years old: \$20
- All other ages: \$40

- 3.1 Complete the piecewise-defined function and the graph.

$$p(x) = \begin{cases} 30 & \boxed{x < 20} \\ \boxed{40} & \boxed{20 \leq x \leq 55} \\ 20 & \boxed{x > 55} \end{cases}$$



- I can read and understand a piecewise-defined function.
 I can explain how a piecewise-defined function represents a situation.
 I can evaluate a piecewise-defined function in function notation.
 I can use information from a situation to write equations of piecewise-defined functions.
 I can sketch a graph of a piecewise-defined function.

Lessons 15–16: Absolute Value Functions

Summary

The output of an *absolute value function* is the distance of its input from a given value.

For example, Mr. DeAndre asked his students to guess a mystery number and gave each student a score. In this game, their score was how far away their guess was from the mystery number. The function $f(x) = |x - 30|$ gave the score for each guess, x .

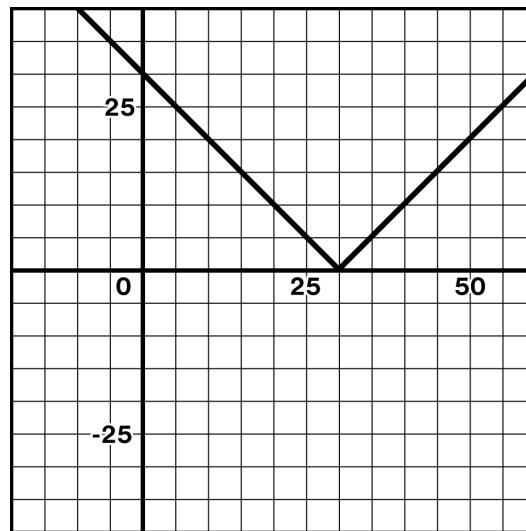
What is the value of $f(25)$? What does it mean?

$$\begin{aligned}f(25) &= |25 - 30| \\&= |-5| \\&= 5\end{aligned}$$

A student who guessed 25 was 5 away from the mystery number.

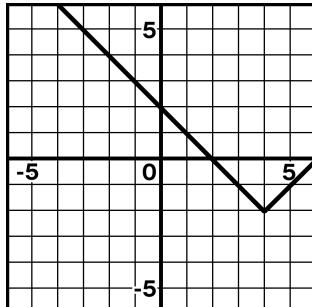
What is the value of $f(40)$ and what is its meaning?

$f(40) = 10$. It means that the student was 10 away from the mystery number.



Identifying the minimum or making a table can be helpful in making a graph or writing an equation of an absolute value function.

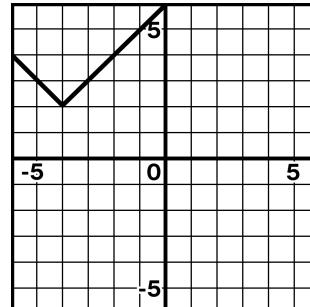
$$g(x) = |x - 4| - 2$$



Minimum: $(4, -2)$

x	$g(x)$
4	-2
3	-1
5	-1
0	2

$$h(x) = |x + 4| + 2$$



Minimum: $(-4, 2)$

x	$h(x)$
-6*	4*
-4*	2*
-2*	4*
0*	6*

*Responses vary.

Things I Want to Remember

Lessons 15–16: Absolute Value Functions

Try This!

$$a(x) = |x - 6|$$

$$b(x) = |x + 4| - 1$$

1.1 What is $a(8)$?

$$a(8) = 2$$

1.2 What is $a(-2)$?

$$a(-2) = 8$$

2.1 What is $b(6)$?

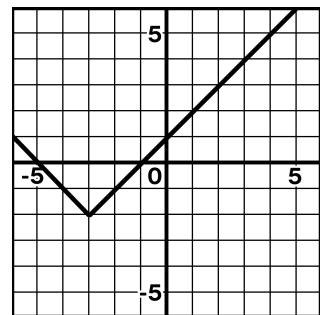
$$b(6) = 9$$

2.2 What is $b(-4)$?

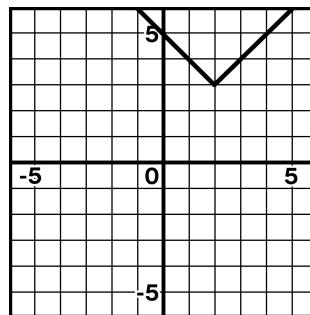
$$b(-4) = -1$$

Complete the missing graphs and minimums. Use the tables if they help with your thinking.

3.1 $c(x) = |x + 3| - 2$

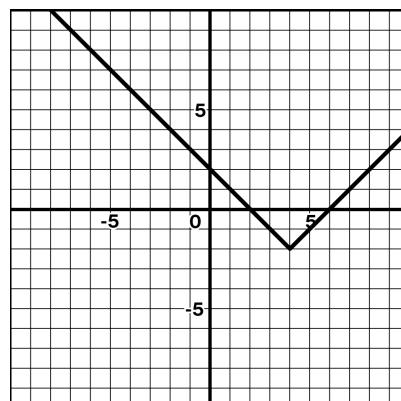
Minimum: $(-3, -2)$

3.2 $d(x) = |x - 2| + 3$

Minimum: $(2, 3)$ ***Responses vary.**

4. Which equation represents this function?

- A. $f(x) = |x| - 2$
- B. $f(x) = |x + 4| - 2$
- C. $f(x) = |x - 2| + 4$
- D. $f(x) = |x - 4| - 2$



- I can explain how an absolute value function is like the distance from a number.
- I can calculate and interpret outputs for absolute value functions.
- I can graph an absolute value function.
- I can analyze the key features of an absolute value function.