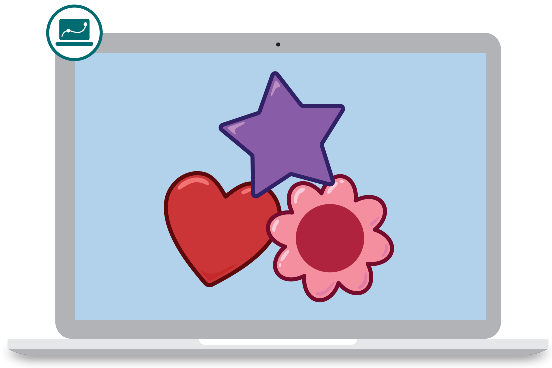
Unit 5
Lesson
1

Name: ______ Date: ______ Period: ______

Shape It Up

Let's use reasoning to solve shape puzzles.

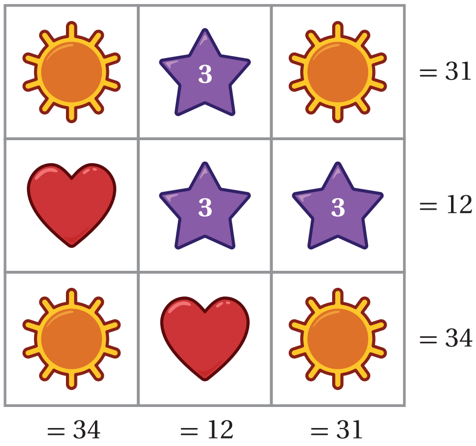


Warm-Up

1 Here is a shape puzzle. The *sum* of each row and column is shown.

Determine the value of the heart and the sun.

Shape	Value
Heart	
Star	3
Sun	



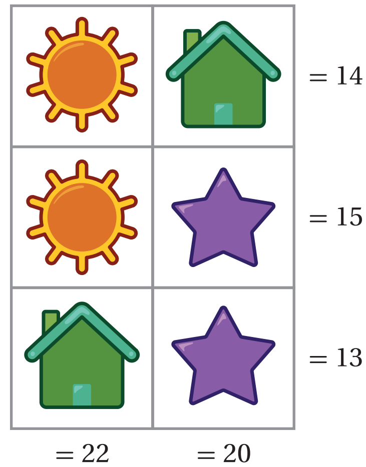
Name: ______________ Date: ______ Period: ______

Shape Puzzle Strategies

2 Here is a different shape puzzle.

Jayden thinks that each sun has a value of 10.

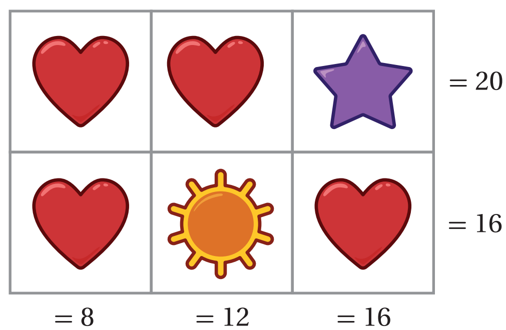
Show or explain why that is not possible.



3 Here is a shape puzzle.

Determine the *solution* for this puzzle.

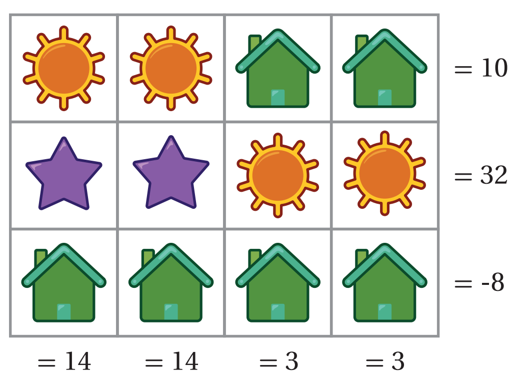
Shape	Value
Heart	
Star	
Sun	



4 Here is a shape puzzle.

Determine the solution for this puzzle.

Shape	Value
Star	
Sun	
House	



Activity
1

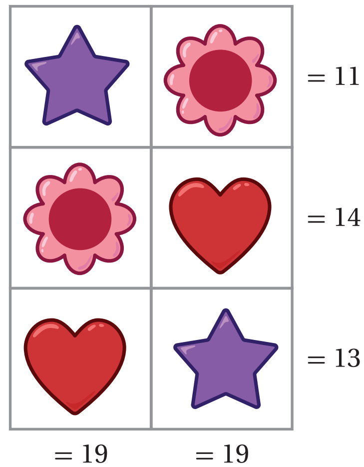
Name: ____________ Date: ____________ Period: ____________

Shape Puzzle Strategies (continued)

5 Here is a shape puzzle.

Determine the solution for this puzzle.

Shape	Value
Heart	
Star	
Flower	



6 Let's take a look at Jayden's first step for solving the puzzle on the previous problem.

How is this helpful in solving the puzzle?

Activity 2

Name: ______ Date: ______ Period: ______

Make Your Own Puzzle

7 In the digital activity, create your shape puzzle. Use this page to support your thinking.

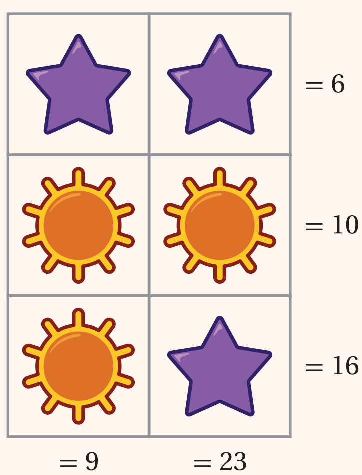
a. **Make It!** Fill your puzzle with shapes! You can use as many as four different shapes.

b. **Solve It!** Determine the value of each shape in your puzzle.

Explore More

8 Here is a different shape puzzle.

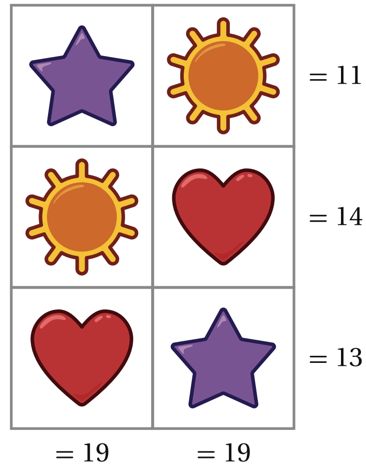
Does this puzzle have a solution? Explain your thinking.



9 Synthesis

Describe some strategies for solving shape puzzles.

Use the puzzle if it helps with your thinking.



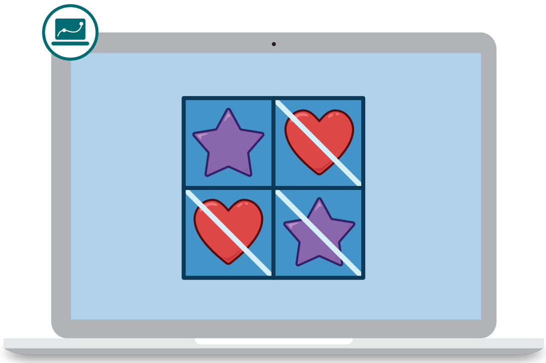
Things to Remember:

Unit 5
Lesson
2

Name: ______ Date: ______ Period: ______

Eliminating Shapes

Let's solve systems of equations by adding or subtracting the equations to eliminate a variable.



Warm-Up

Determine an expression that makes each equation true for any value of x and y.

1. $3x + \underline{\qquad\qquad} = 0$

2. $3x - \underline{\qquad\qquad} = 0$

3. $(3x + y) - (\underline{\qquad\qquad}) = 0$

4. $(3x + y) + (\underline{\qquad\qquad}) = 0$

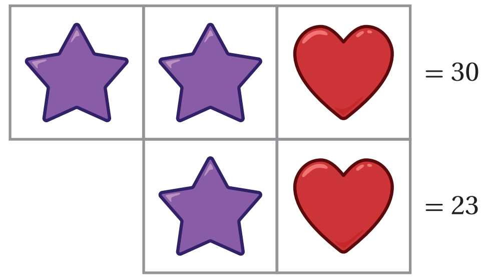
Name: .... Date: .... Period: ....

Adding and Subtracting Equations

5 Here is a shape puzzle. The sum of each row is shown.

Determine the solution for this puzzle.

Shape	Value
Heart	
Star	

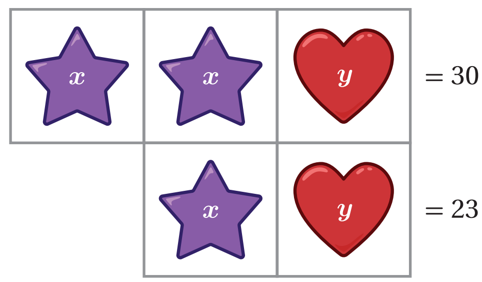


6 This shape puzzle could be written as a *system of equations*, where x is the value of each star and y is the value of each heart.

$$2x + y = 30$$

$$x + y = 23$$

Explain how this system of equations is like the puzzle.

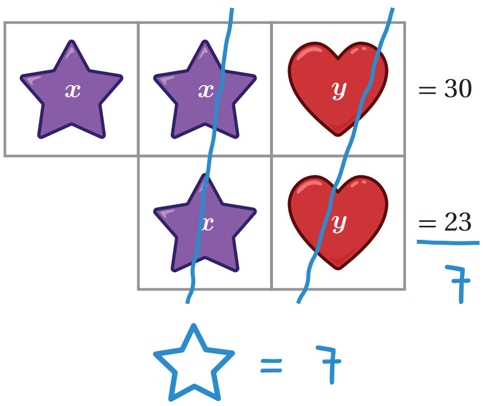


Name: ____________ Date: ________ Period: ________

Adding and Subtracting Equations (continued)

7 Here is how Ebony and Nia each determined the value of a star.

Ebony



Nia

$$2x + y = 30$$
$$-(x + y = 23)$$
$$x + 0 = 7$$
$$x = 7$$

Discuss: Where do you see subtraction in each strategy?

Name: .... Date: .... Period: ....

Elimination

8 Here is a new system of equations.

$$x + 2y = 10$$
$$x + y = 7$$

Determine the values of x and y that make both equations true (the *solution to the system*).

Draw a puzzle if it helps with your thinking.

$x =$ ...., $y =$ ....

9 Ebony and Nia want to eliminate the y's in this system of equations.

$$-2x + y = 9$$
$$8x - y = 3$$

- Ebony says to *add* the equations.
- Nia says to *subtract* the equations.

Whose strategy will eliminate the y's? Circle one.

Ebony's Nia's Both Neither

Name: .... Date: .... Period: ....

Elimination (continued)

10 Determine the *solution to the system of equations* from the previous problem:

$-2x + y = 9$

$8x - y = 3$

$x =$ ...., $y =$ ....

11 The strategy of adding or subtracting equations to eliminate a variable is called **elimination**.

Nia says elimination works because it's like adding or subtracting the same value from each side of an equation.

Explain what Nia is saying in your own words.

Nia

$$\begin{array}{r} 2x + y = 30 \\ -(x + y = 23) \\ \hline x + 0 = 7 \\ x = 7 \end{array}$$

12 Determine the solution to this system of equations:

$-7x - 5y = 15$

$7x + 3y = 12$

$x =$ ...., $y =$ ....

Activity 3

Name: ______ Date: ______ Period: ______

Elimination Repeated Challenges

13 Choose four of the systems of equations and solve them using elimination.

A. $5x + 3y = 21$
$2x + 3y = 12$

B. $8x + 5y = 12$
$8x + 3y = 4$

C. $2x + 3y = 14$
$-2x + 7y = 6$

D. $9x + 3y = -3$
$4x - 3y = -23$

E. $2x + 3y = 4$
$2x + 7y = -12$

F. $y = 4x - 1$
$y = 6x - 7$

14 Synthesis

How can you determine whether to add or subtract equations in order to eliminate a variable?

$$\begin{aligned} 2x + y &= 30 \\ x + y &= 23 \end{aligned}$$

$$\begin{aligned} x + 2y &= 10 \\ x + y &= 7 \end{aligned}$$

$$\begin{aligned} -2x + y &= 9 \\ 8x - y &= 3 \end{aligned}$$

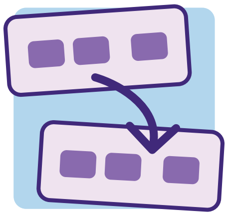
$$\begin{aligned} -7x - 5y &= 15 \\ 7x + 3y &= 12 \end{aligned}$$

Things to Remember:

Unit 5
Lesson
3

Name: ______ Date: ______ Period: ______

Process of Elimination



Let's create equivalent equations to eliminate a variable.

Warm-Up

Here are some linear equations.

Equation A	Equation B	Equation C
$x + 2y = 11$	$4x + y = 2$	$5x + 10y = 55$
Equation D	**Equation E**	**Equation F**
$y = 2 - 4x$	$2x + \frac{1}{2}y = 1$	$x + 2y - 11 = 0$

1. Sort the equivalent linear equations into two groups. Record your groupings.

Group 1	Group 2

2. Choose one equation. Write a new *equivalent equation* that would belong in that group.

______ *is equivalent to* ______ *because . . .*

Activity 1

Name: ______ Date: ______ Period: ______

First Steps of Elimination

Caasi is solving this system of equations, but she got stuck.

Here's how Caasi started.

Caasi

$$\begin{array}{r} x + 2y = 11 \\ -(4x + y = 2) \\ \hline -3x + y = 9 \end{array}$$

3. **Discuss:**

- What was Caasi's first step?
- Why do you think she got stuck?

Diego is trying to solve this system of equations. Here's how Diego started.

Diego

$$\begin{array}{l} x + 2y = 11 \\ 4x + y = 2 \end{array} \rightarrow \begin{array}{r} x + 2y = 11 \\ 8x + 2y = 4 \end{array}$$

$$\begin{array}{r} x + 2y = 11 \\ -(8x + 2y = 4) \\ \hline -7x + 0 = 7 \\ x = -1 \end{array}$$

4. What was Diego's first step?

5. Diego got stuck using his method after solving for $x = -1$. What do you think he should do next?

6. Ariel thinks that Diego can solve this system:

$$\begin{array}{r} x = -1 \\ x + 2y = 11 \end{array}$$

Discuss: Do you think this system will have the same solution as Diego's original system?

7. Finish Diego's work to solve the system.

$x =$ ______ and $y =$ ______

Name: ____________ Date: ________ Period: ________

More Than One Way?

8. Caasi and Kwabena started solving this system in different ways.

$$\begin{aligned} 4x - y &= 5 \\ x + 2y &= 8 \end{aligned}$$

With a partner, solve the system both ways. Compare your solutions.

Caasi: *Multiply the first equation by* 2.   Kwabena: *Multiply the second equation by* -4.

9. **Discuss:**

- What is similar about Caasi’s and Kwabena’s methods for solving the linear system?
- What is different about their methods?

Name: ________ Date: ________ Period: ________

Prepare to Be Eliminated

You will use a set of cards for this activity.

10. Here are the instructions for each round.

Select a card from A–F.

- Discuss two possible first steps you could take to solve the system.
- Choose a different first step from your partner. Solve your system individually.
- Compare your solutions and support each other to make adjustments as needed.

Round 1, Card ________

Equation 1: ________

Equation 2: ________

Solution: $x =$ ________ and $y =$ ________

Round 2, Card ________

Equation 1: ________

Equation 2: ________

Solution: $x =$ ________ and $y =$ ________

Activity 3

Name: ______ Date: ______ Period: ______

Prepare to Be Eliminated (continued)

Round 3, Card ______

Equation 1: ______

Equation 2: ______

Solution: $x =$ ______ and $y =$ ______

Round 4, Card ______

Equation 1: ______

Equation 2: ______

Solution: $x =$ ______ and $y =$ ______

Explore More

11. The solution to this system of equations is $x = 5$ and $y = 2$.

$$Ax - By = 24$$
$$Ax + By = 16$$

What are possible values for A and B?

$A =$ ______ and $B =$ ______

Synthesis

12. Describe how writing equivalent equations can help you solve systems of equations.

Use this system if it helps you explain your thinking.

$$\begin{aligned} x + 3y &= 6 \\ 2x + y &= 7 \end{aligned}$$

Things to Remember:

Prepare to Be Eliminated

Directions: Make one copy per pair of students. Then pre-cut the cards and give each pair of students one set.

Card A

$-x + 4y = 1$

$2x + y = 7$

Card B

$x + y = 12$

$3x - 5y = 4$

Card C

$4x - 4y = 44$

$6x + 3y = 12$

Card D

$4y = 4 - 2x$

$x + 5y = -7$

Card E

$\frac{1}{3}x + 2y = 4$

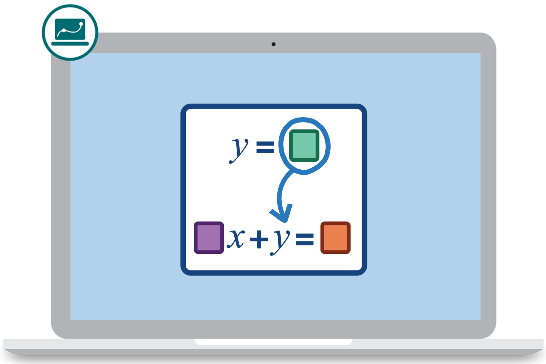
$x + y = -3$

Card F

$4x + 2y = 8$

$5x = 5y + 55$

Name: ________________ Date: ________ Period: ________



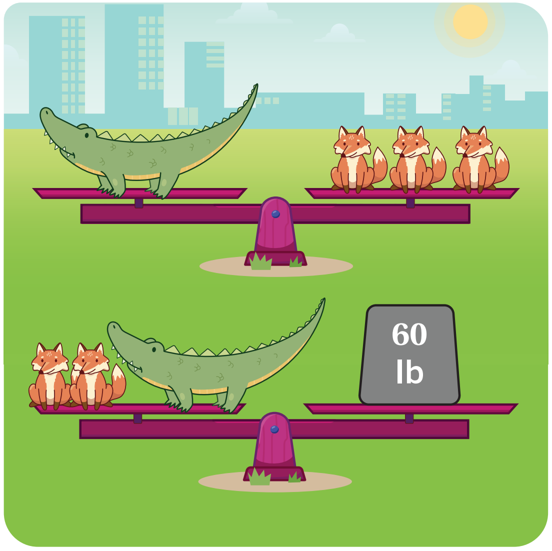
Solution by Substitution

Let's use substitution to solve systems of equations.

Warm-Up

1 Here are two scales showing the weights of foxes and alligators.

What do you notice? What do you wonder?



2 Let's watch an animation.

a **Discuss:** What happened in the animation?

b Determine the weight of each animal.

Fox: ________ pounds   Alligator: ________ pounds

Activity 1

Name: ________ Date: ________ Period: ________

Introducing Substitution

3 Riya determined the weight of a fox by writing a system of equations and doing these steps.

Then she needed to solve a new system of equations.

Show or explain what Riya's first step might be as she solves the new system.

Riya

$y = 3x$ $\quad 2x + y = 60$

$2x + (3x) = 60$

$5x = 60$

$x = 12$

New $\quad y = 2x + 3 \quad 4x + y = 15$

4 Here is the system from the previous problem:

$y = 2x + 3$

$4x + y = 15$

Determine the solution.

$x =$ ________ , $y =$ ________

5 Riya's strategy is called solving by **substitution**.

Substitution is when a variable is replaced with an expression that is equal to it.

Show or explain the first step to solving the new system of equations with substitution.

$y = 3x$ $\quad 2x + y = 60$

$2x + (3x) = 60$

$y = 2x + 3$ $\quad 4x + y = 15$

$4x + (2x + 3) = 15$

New $\quad 5x - 2y = 15 \quad y = 3x - 12$

Name: ______ Date: ______ Period: ______

Introducing Substitution (continued)

6 Natalia made a mistake as she solved the system of equations from the previous problem.

What did Natalia do well? What should she fix?

Natalia

$5x - 2y = 15$ $\quad y = 3x - 12$

5x — 2(3x — 12) = 15

5x — 6x — 24 = 15

-x = 39

x = -39

7 Determine the solution to the previous problem.

$5x - 2y = 15$

$y = 3x - 12$

$x =$ ______ , $y =$ ______

Activity 2

Name: .... Date: .... Period: ....

Practicing Substitution

8 Here are three systems of equations.

Discuss:

- What would be your first step in solving each of these systems using substitution?
- Would you prefer to solve each system using substitution or elimination? Why?

A
$$-2x + y = 9$$
$$x = 14$$

B
$$x + 2y = 10$$
$$x + y = 7$$

C
$$y = 4x + 63$$
$$y = 7x + 15$$

9 Determine the solution to each system of equations.

a
$$y = 7x + 12$$
$$y = -3x + 2$$

$x =$ ...., $y =$ ....

b
$$2x + 2y = 8$$
$$x = 4 + 3y$$

$x =$ ...., $y =$ ....

c
$$-2x + 4y = 9$$
$$y = x - 1$$

$x =$ ...., $y =$ ....

d
$$3x - 2y = 14$$
$$x + 3y = 1$$

$x =$ ...., $y =$ ....

Explore More

10 Solve this system of *four* equations. All values in the solution are integers.

$$3x + 2y - z + 5w = 20$$

$$y = 2z - 3w$$

$$z = w + 1$$

$$2w = 8$$

$w =$ ...., $x =$ ...., $y =$ ...., $z =$ ....

11 Synthesis

Substitution and *elimination* are two strategies for solving systems of equations.

How are these strategies alike? How are they different?

Use the examples if they help with your thinking.

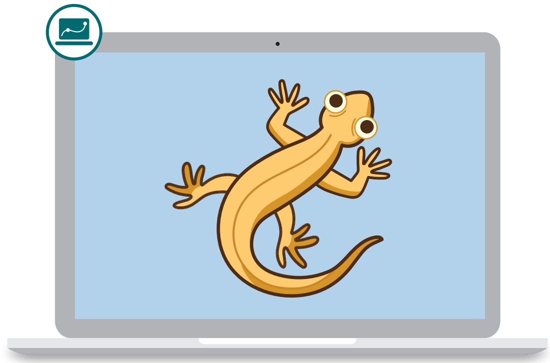
A
$-2x + y = 9$
$x = 14$

B
$x + 2y = 10$
$x + y = 7$

C
$y = 4x + 63$
$y = 7x + 15$

Things to Remember:

Name: .... Date: .... Period: ....

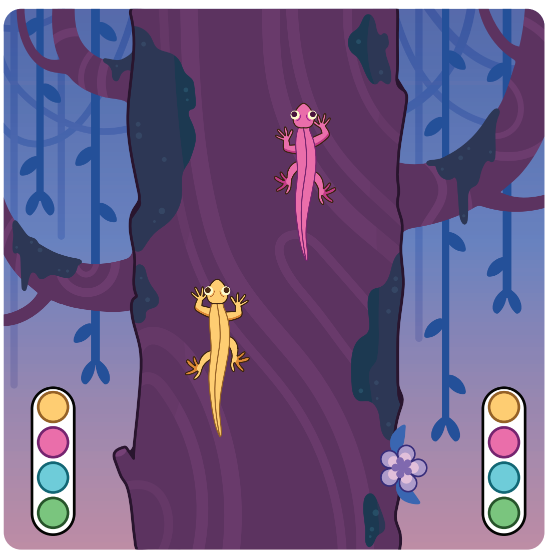


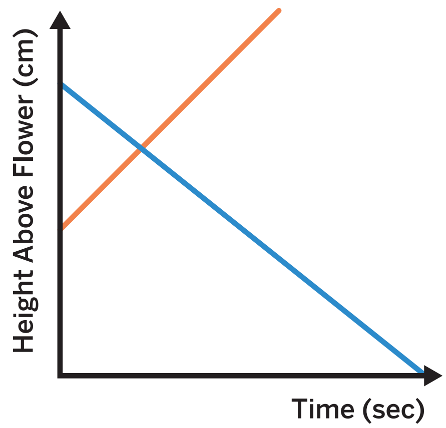
Lizard Lines

Let's explore systems of equations using graphs.

Warm-Up

1 Let's watch different pairs of lizards walk along a tree trunk together.





Discuss: What do you notice about the lizards and the graphs? What do you wonder?

Activity 1

Name: ____________ Date: ________ Period: ________

Making Connections

2 Here are equations for each lizard's height above the flower, y, as a function of time, x, in seconds:

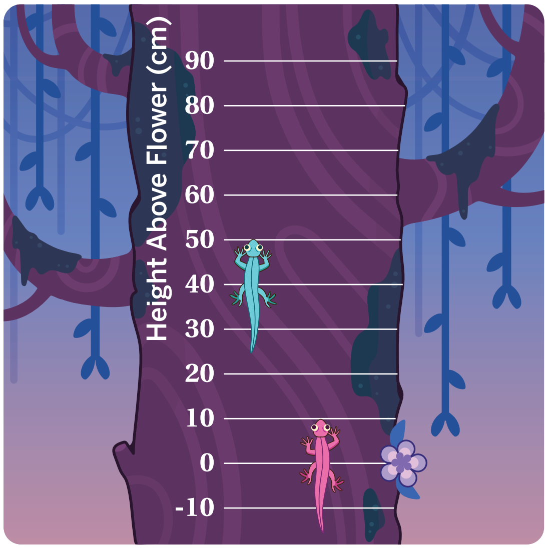
$$y = -5x + 50$$

$$y = 5x + 10$$

When and where will the lizards have the same position?

Time (sec), x: ____________

Height (cm), y: ____________



3 Let's look at Jin's and Nasir's strategies for figuring out when the lizards will be in the same position.

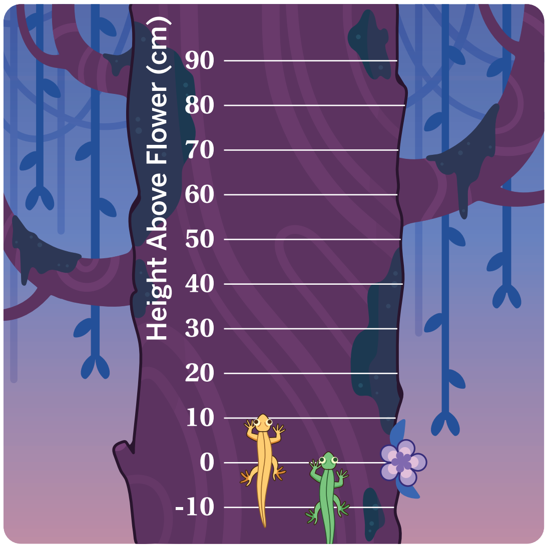
Discuss: Where do you see the solution in each strategy?

Activity
2

Name: ______ Date: ______ Period: ______

Will They Meet?

You will use a graphing calculator for this activity.



4 Here are equations for each lizard's height above the flower, y, as a function of time, x, in seconds:

$$y = \text{-}2x + 11$$

$$y = 4x + 2$$

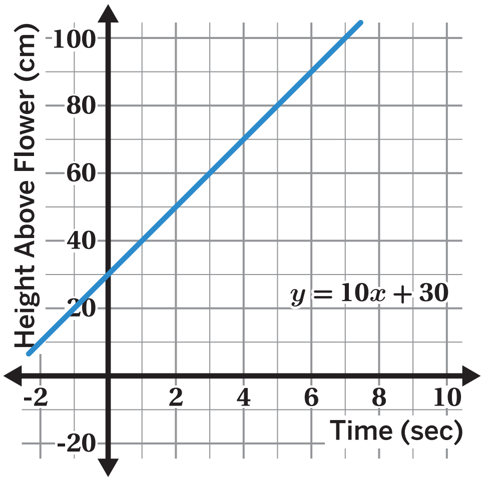
When and where will the lizards have the same position?

Use a graphing calculator if it helps with your thinking.

Time (sec), x: ______

Height (cm), y: ______

5 The blue line represents the graph of a blue lizard.



Create a line for a green lizard so that the lizards meet at exactly 5 seconds.

Try to make a line that none of your classmates will make.

$y =$ ______

6 Here are equations for two lizards' heights above the flower, y, as a function of time, x, in seconds.

$$y = 8x + 60$$

$$y = 8x + 35$$

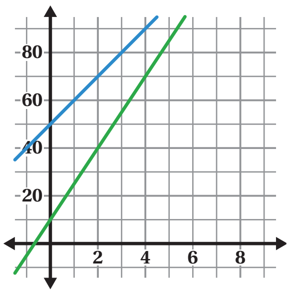
Will these lizards meet? Explain your thinking.

Name: ______________ Date: ________ Period: ________

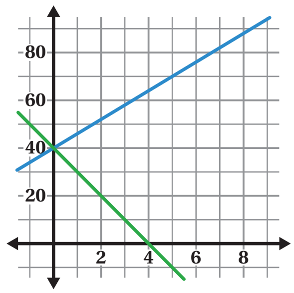
Graphing Systems

7 Here are some systems of equations.

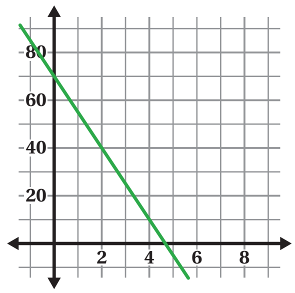
$y = 10x + 50$
$y = 15x + 10$



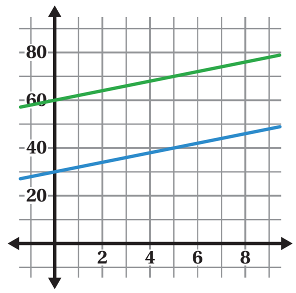
$y = 6x + 40$
$y = -10x + 40$



$y = -15x + 70$
$y = -15x + 70$



$y = 2x + 60$
$y = 2x + 30$



Select each type of system that is possible to make.

- ☐ **A.** No solution
- ☐ **B.** Exactly one solution
- ☐ **C.** Exactly two solutions
- ☐ **D.** Infinitely many solutions

8 For each system of equations, circle the number of solutions that it has. If there is one solution, what is the solution?

a $y = \frac{1}{2}x - 1$, $y = \frac{1}{2}x + 2$ — No solution — One solution (______, ______) — Infinitely many solutions

b $y = x + 2$, $y = -3x - 2$ — No solution — One solution (______, ______) — Infinitely many solutions

c $y = 2x + 6$, $y = 2(x + 3)$ — No solution — One solution (______, ______) — Infinitely many solutions

d $y - 5x = -7$, $y = 5x$ — No solution — One solution (______, ______) — Infinitely many solutions

e $y = 20x$, $20y = x$ — No solution — One solution (______, ______) — Infinitely many solutions

Name: .... Date: .... Period: ....

Graphing Systems (continued)

9 Group each system of equations based on the number of solutions it has.

A

$2x + 4y = 16$

$y = \frac{1}{2}x + 2$

B

$y = \frac{2}{3}x + 10$

$y = \frac{2}{3}x - 7$

C

$y = 2x + \frac{1}{4}$

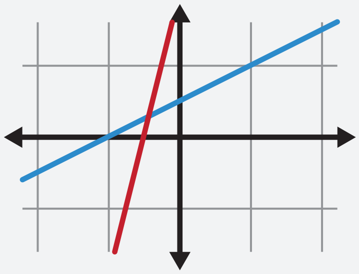
$y = 4x + \frac{1}{4}$

D

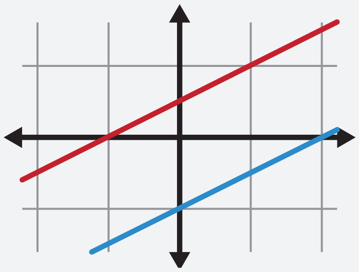
$y = \frac{1}{2}x + 3$

$2y = x + 6$

E



F



No Solution	One Solution	Infinitely Many Solutions

10 Jaleel and Irene are trying to decide when a system of equations may have *no solution*.

A system of equations may have no solution when . . .

Jaleel: . . . the *slopes* are the same.

Irene: . . . the y-*intercepts* are the same.

Whose claim is correct? Circle one and explain your thinking.

Jaleel's   Irene's   Both   Neither

No Solution

$y = \frac{2}{3}x + 10$

$y = \frac{2}{3}x - 7$

$y = 2x + \frac{1}{4}$

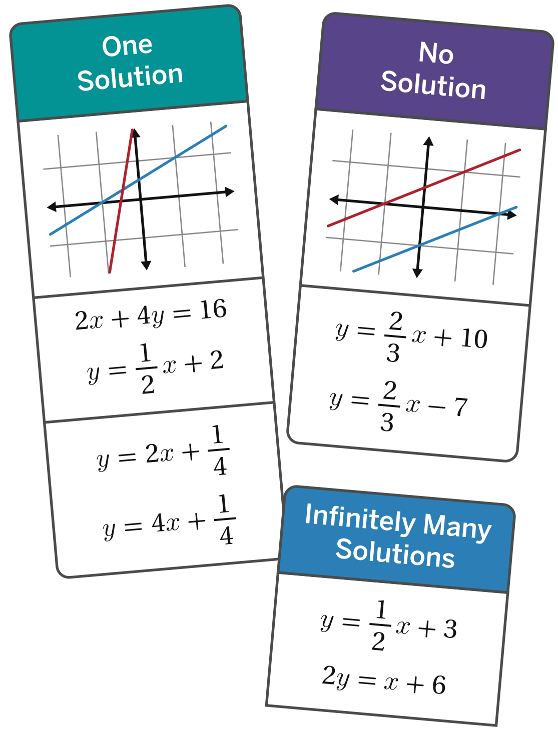
$y = 4x + \frac{1}{4}$

11 Synthesis

Select one and explain.

How can you tell if a system of equations has:

- ☐ **A.** No solution?
- ☐ **B.** One solution?
- ☐ **C.** Infinitely many solutions?



Things to Remember:

Unit 5
Lesson
8

Name: ________ Date: ________ Period: ________

Electric Line Zapper

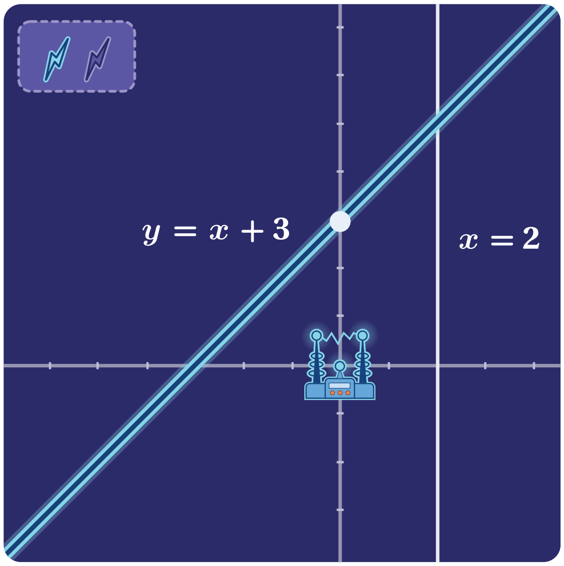
Let's solve systems of equations strategically.

Warm-Up

1 Use the digital activity to zap a point on each of these lines to light it up.

$y = x + 3$
$x = 2$

Zap	Ordered Pair
Zap 1	$(0, 3)$
Zap 2	



Activity

1

Name: .... Date: .... Period: ....

Zapping Two Lines

You'll use the digital activity for Problems 2–4.

2 These two lines are hidden in the graph:

$3x + 4y = 3$

$-3x + 3y = 18$

Zap a point on each line to light it up.

3 These two lines are hidden in the graph:

$y = 2x - 4$
$y = 0.5x + 5$

Adah zapped the point $(-4, 5)$ but did not light up either line.

Help her light up both lines with *one* zap.

4 Let's look at two ways to start solving the system of equations from the previous problem.

Discuss: Which strategy would you use? Why?

Name: ________ Date: ________ Period: ________

Zapping Many Lines

5 These three lines are hidden in a graph:

$y = 3x + 6$

$2x + 2y = 20$

$x - y = 10$

Use the digital activity to zap *two* points to light up all three lines.

6 Adah wants to light up two lines with one zap by using *elimination* to solve a system of equations.

Which two lines from the previous problem might she choose to zap? Circle two.

Line A: $y = 3x + 6$

Line B: $2x + 2y = 20$

Line C: $x - y = 10$

Explain your thinking.

Activity 3

Name: ______ Date: ______ Period: ______

Repeated Challenges

7 You'll use the digital activity to play a few rounds of Line Zapper. Use this page to show your thinking.

a $y = 2x + 10$

$y = 7 + 3x$

b $2x - y = 12$

$x + 4y = 15$

c $y = 4x + 5$

$4x + 2y = 16$

$x - y = 7$

8 Adah tried to light up these lines with one zap:

$y = 3x + 4$

$y = 3x - 2$

What does her work say about this system of equations?

Adah

$y = 3x + 4$ $y = 3x - 2$

$3x + 4 = 3x - 2$

$3x + 6 = 3x$

$6 = 0$

9 Synthesis

What are some ways you can decide what strategy to use when solving a system of equations?

Use the examples if they help with your thinking.

$$x = 2$$
$$y = x + 3$$

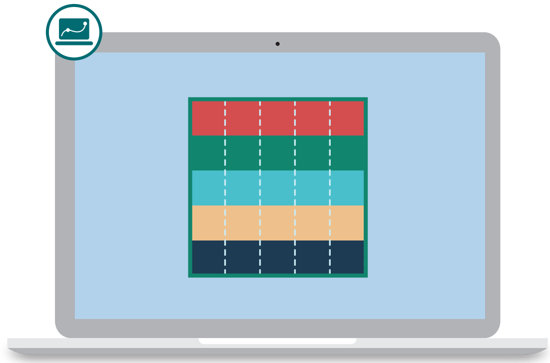
$$3x + 4y = 3$$
$$-3x + 3y = 18$$

$$y = 3x + 6$$
$$2x + 2y = 20$$

$$y = 2x - 4$$
$$y = 0.5x + 5$$

Things to Remember:

Name: .... Date: .... Period: ....



Quilts

Let's explore what solutions to systems of inequalities mean.

Warm-Up

1 People across many different cultures make quilts. They can be used for warmth, storytelling, political involvement, income, and more.

a Let's look at a variety of different quilts.

b What are some decisions people might make when designing a quilt?

2 There is a longstanding patchwork quilt tradition in Gee's Bend, Alabama. Spend a few minutes researching and learning about Gee's Bend and quilters like Annie Mae Young.

Discuss:

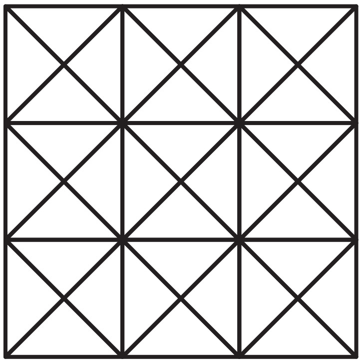
- What *constraints* might quilters like Annie Mae Young have experienced when creating quilts?
- How has quilting been a source of income and political involvement in this community?

Name: ________ Date: ________ Period: ________

Sai and Jordan's Quilt

3 **a** Let's make a quilt square together.

b **Discuss:** Why did you choose to design your quilt the way you did?



4 Sai and Jordan are making a quilt using solid and patterned fabric.

They need *at least* 35 sq. ft of fabric to cover their bed.

Fabric Constraint

$$x + y \geq 35$$

What are some combinations of fabric Sai and Jordan could use?

Solid Fabric (sq. ft), x	Patterned Fabric (sq. ft), y

5 Sai and Jordan want to spend no more than $\$30$ on fabric.

- Solid fabric costs $\$0.50$ per sq. ft.
- Patterned fabric costs $\$1$ per sq. ft.

Cost Constraint

$$0.50x + y \leq 30$$

What are some combinations of fabric they could use?

Solid Fabric (sq. ft), x	Patterned Fabric (sq. ft), y

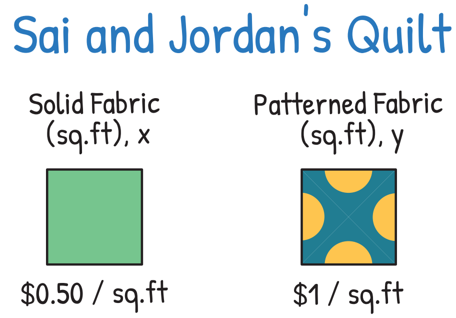
Name: .... Date: .... Period: ....

Sai and Jordan's Quilt (continued)

6 Sai and Jordan want their quilt to have at least 35 sq. ft of fabric and cost no more than $\$30$. They wrote a **system of inequalities** to represent these two constraints.

$$x + y \geq 35$$

$$0.50x + y \leq 30$$



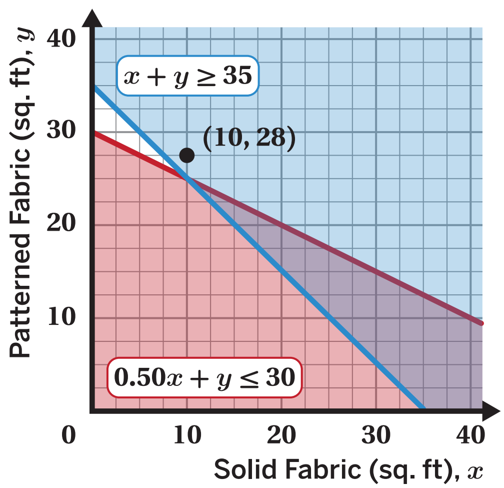
They designed a quilt using 10 sq. ft of solid fabric and 28 sq. ft of patterned fabric.

Does their design meet both constraints?

7 This graph represents their system of inequalities.

The fabric used in their design is represented by the point $(10, 28)$.

How can the graph help Sai and Jordan decide whether their design meets both constraints?



8 Let's test several fabric combinations.

Discuss: How can you see which constraints a point meets by looking at the graph?

Activity 2

Name: .... Date: .... Period: ....

Evan's Quilt

9 Evan is making a quilt using different fabrics.

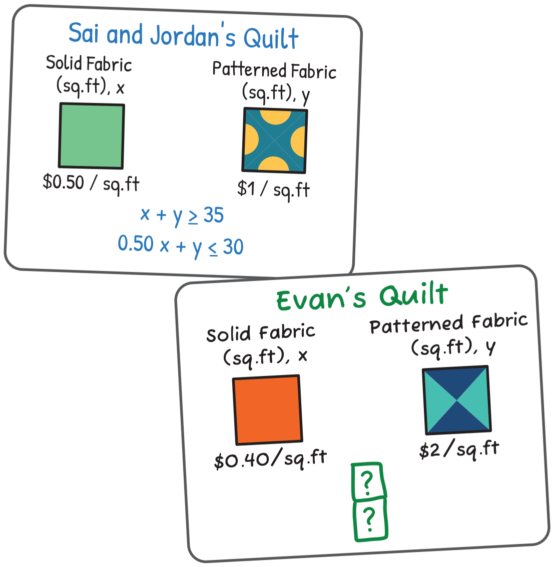
- Solid fabric costs $\$0.40$ per sq. ft.
- Patterned fabric costs $\$2$ per sq. ft.

Evan wants his quilt to have *at least* 35 sq. ft of fabric and cost *no more than* $\$30$.

Write a system of inequalities to represent Evan's quilt.

Fabric inequality: ....

Cost inequality: ....

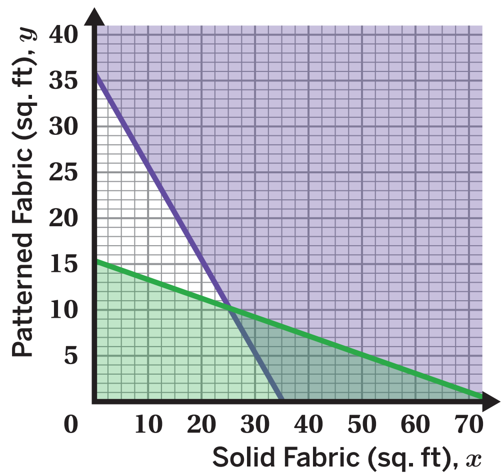


10 This graph represents Evan's system of inequalities.

Determine a combination of solid and patterned fabric that meets both constraints.

Solid Fabric (sq. ft), x: ....

Patterned Fabric (sq. ft), y: ....



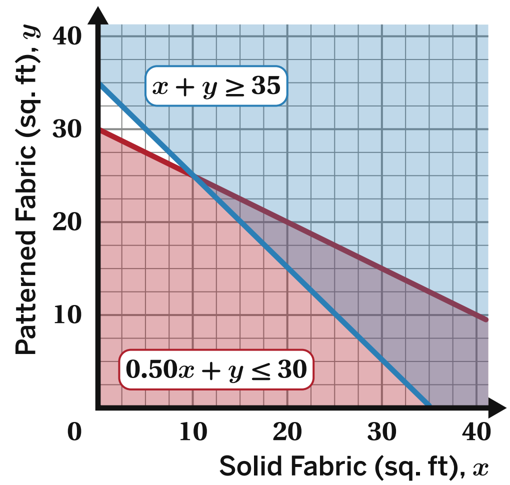
11 Titus says $(10, 10)$ *is* a solution to Evan's system of inequalities. Alma says it *is not* a solution.

Whose thinking is correct? Explain your thinking.

12 Synthesis

How can you determine if a point is a solution to a system of inequalities?

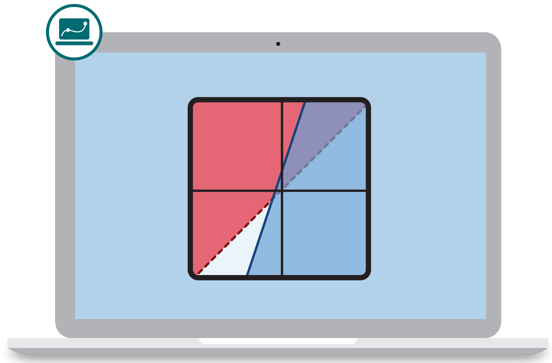
Draw on the graph if it helps to show your thinking.



Things to Remember:

Unit 5
Lesson
10

Name: ______ Date: ______ Period: ______



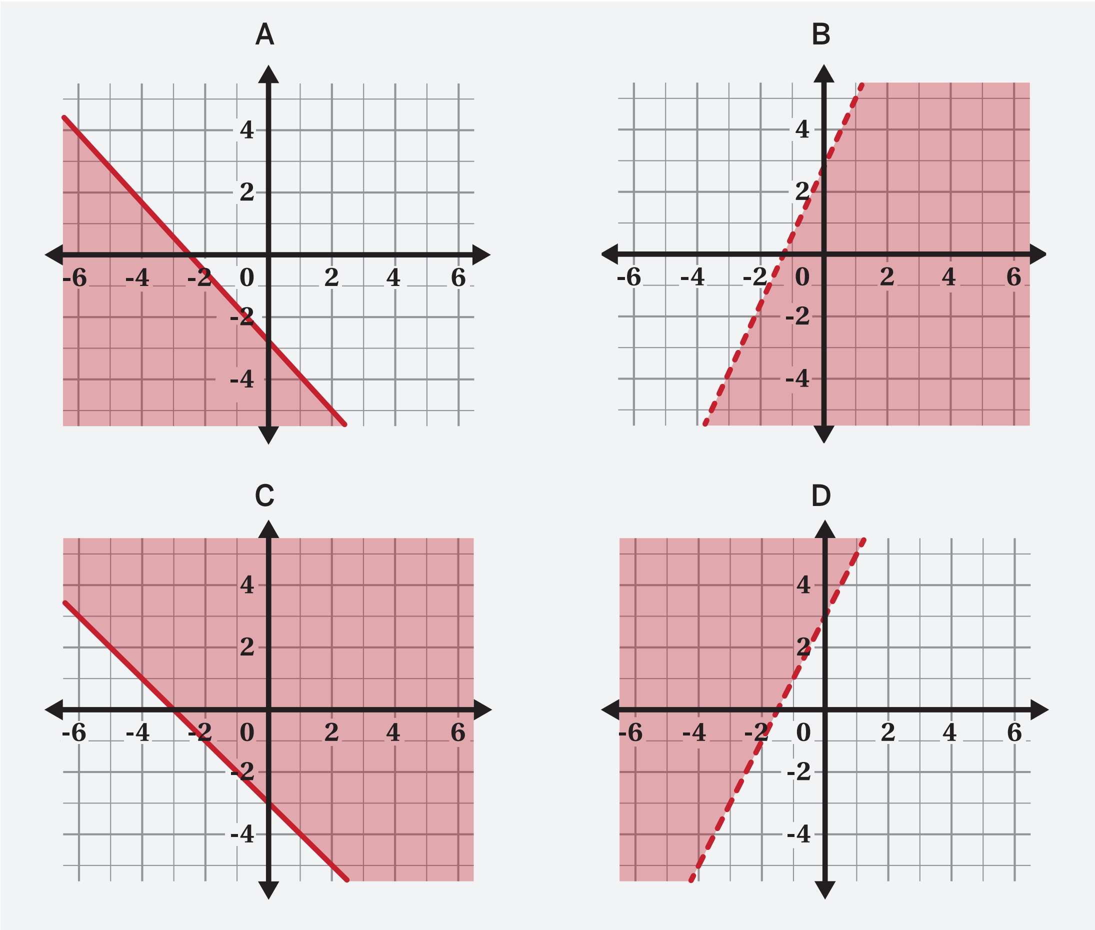
Seeking Solutions

Let's explore strategies for determining the solution region for a system of inequalities.

Warm-Up

1 Match each inequality to its graph. There will be two graphs without a match.

$x + y \geq -3$	$y > 2x + 3$



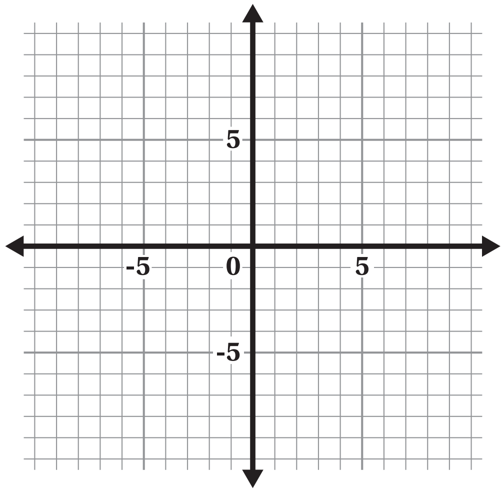
Activity 1

Name: ______ Date: ______ Period: ______

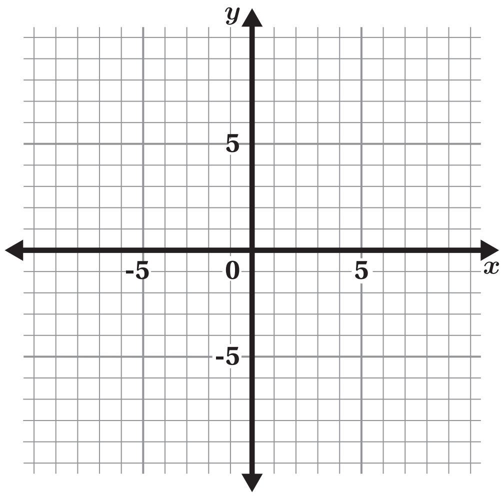
The Overlap

2 Graph the solution to each inequality.

$y > x$



$-3x + y \leq 2$



3 Let's look at a graph that shows the system of inequalities from the previous question.

a Watch as the point is moved to different *regions* of the graph.

b **Discuss:**

- How many regions do you see?
- When is each inequality highlighted? When are both highlighted?
- What happens when the point is on the dashed line? On the solid line?

Activity 1

Name: ________ Date: ________ Period: ________

The Overlap (continued)

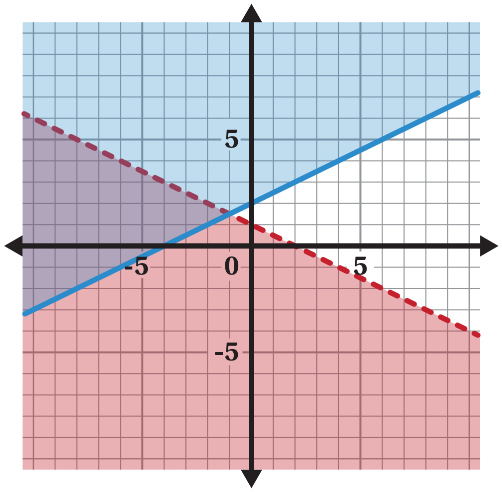
4 The **solutions to a system of inequalities** are all the points that make both inequalities true.

On a graph, the solutions are located in the same region.

Draw a point in the **solution region** of this system of inequalities:

$$\frac{1}{2}x + y < 1$$

$$-x + 2y \geq 4$$



Name: ________________ Date: ________ Period: ________

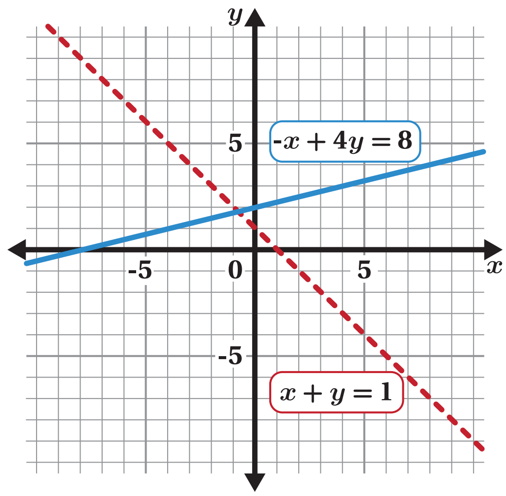
Where Is the Solution Region?

5 This graph shows the *boundary lines* and their equations for this system of inequalities:

$$x + y > 1$$

$$-x + 4y \leq 8$$

How can you determine where the solution region is?

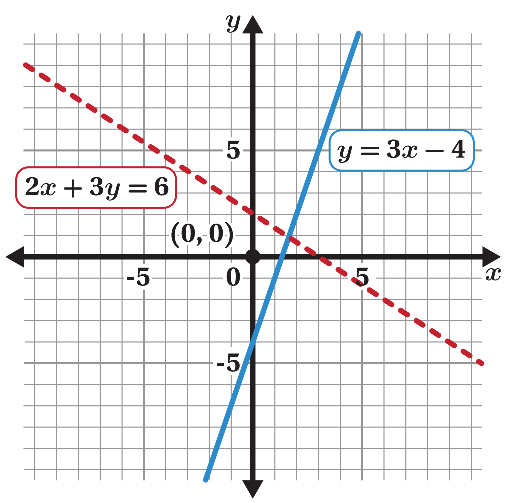


6 Plot a point on the solution region for the system of inequalities in the previous problem.

7 Terrance is trying to graph the solutions to this system of inequalities. First, he tests the point $(0, 0)$.

$$2x + 3y > 6$$
$$y \geq 3x - 4$$

Dashed Line	Solid Line
$2(0) + 3(0) > 6$	$0 \geq 3(0) - 4$
$0 + 0 > 6$	$0 \geq 0 - 4$
$0 > 6$	$0 \geq -4$
False!	True!



Discuss: What can Terrence do next to determine the solution region?

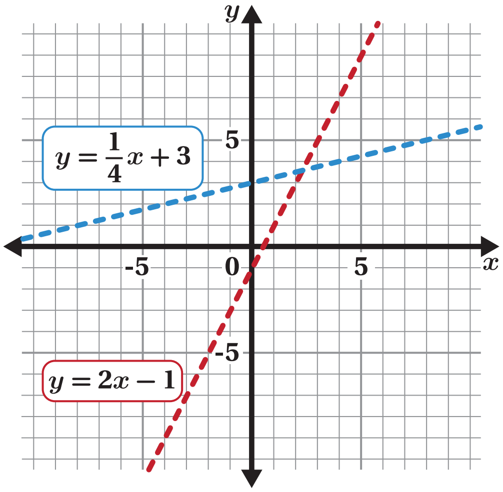
Name: .... Date: .... Period: ....

Solution Region Practice

8 Plot a point in the solution region for each system of inequalities.

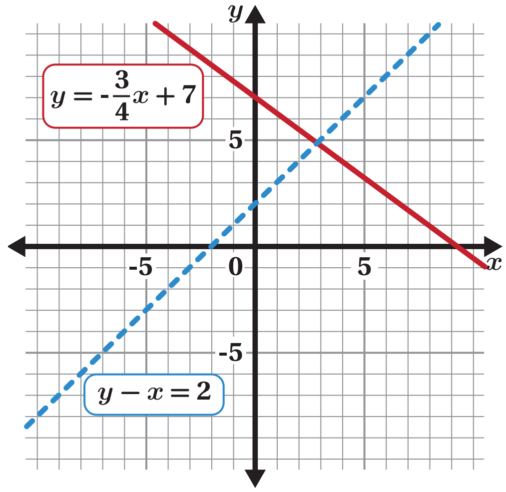
a $y > 2x - 1$

$y < \frac{1}{4}x + 3$



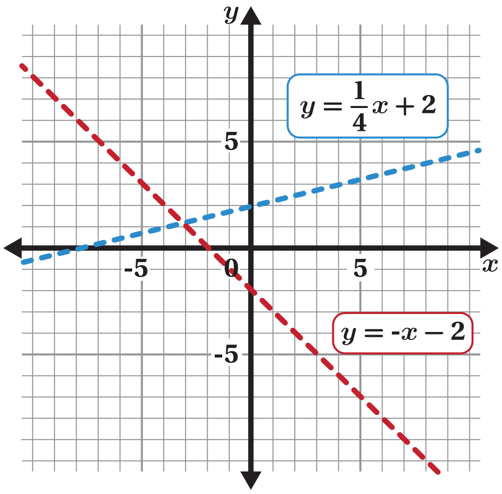
b $y \leq -\frac{3}{4}x + 7$

$y - x < 2$



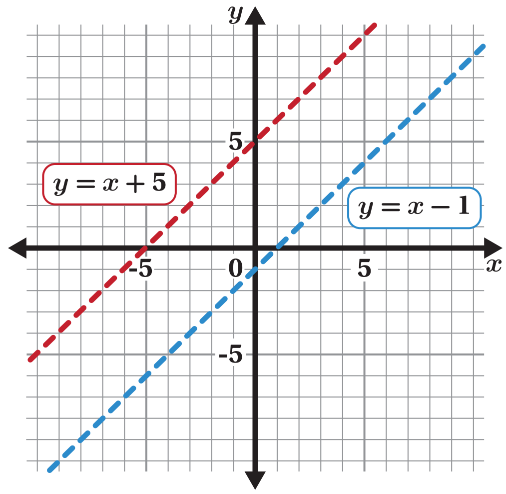
c $y < -x - 2$

$y < \frac{1}{4}x + 2$



d $y > x + 5$

$y > x - 1$

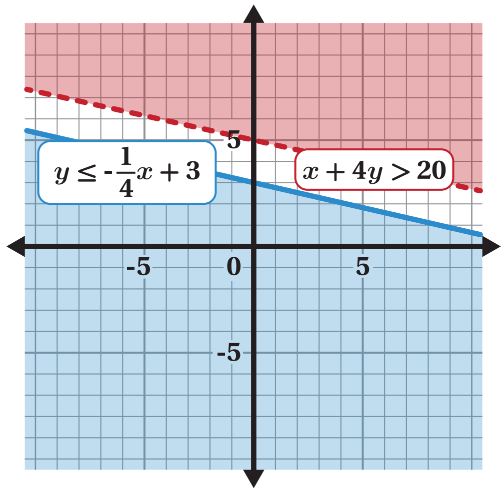


Name: ______ Date: ______ Period: ______

Solution Region Practice (continued)

9 This system of inequalities has *no solutions*.

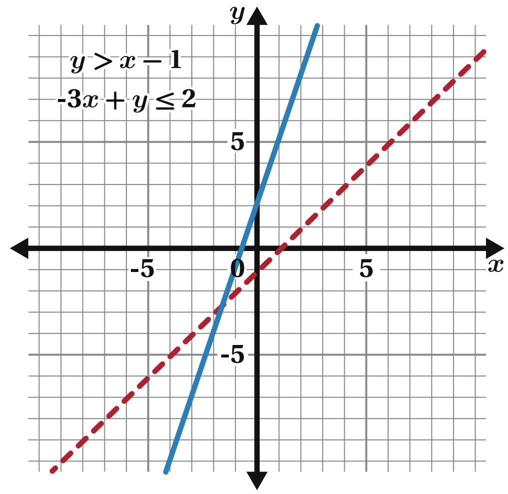
How would you convince a classmate that there are no solutions to this system?



10 Synthesis

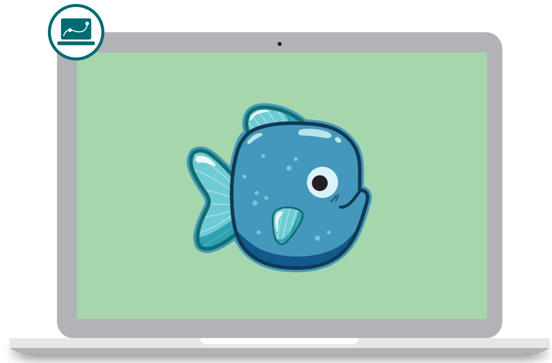
What are some things you should keep in mind when determining the solution region of a system of inequalities?

Use the graph if it helps with your thinking.



Things to Remember:

Name: .... Date: .... Period: ....



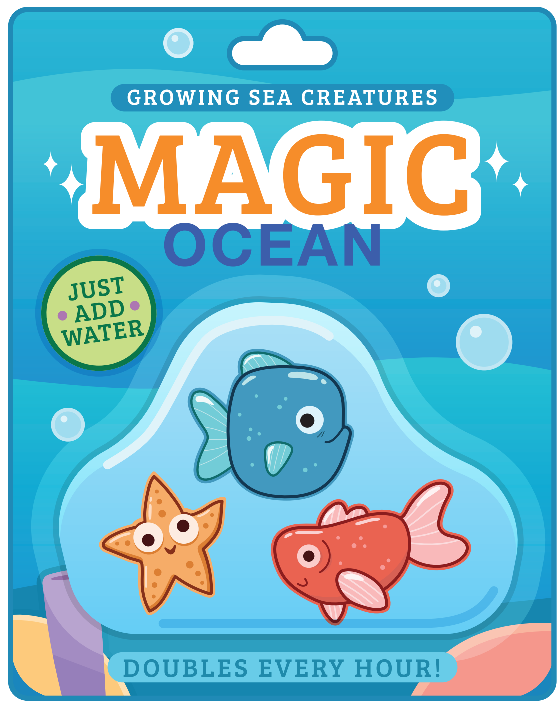
Carlos's Fish

Let's make connections between exponential equations and the situations they represent.

Warm-Up

1 Carlos's apartment doesn't allow pets, so he decided to buy toy fish.

What do you notice? What do you wonder?



Activity 1

Name: .... Date: .... Period: ....

Doubles Every Hour

2 Let's watch Carlos's toy fish grow when you drop it in water.

Discuss: What patterns do you see?

3 This fish grows by a constant ratio.

What will the mass of the toy fish be after 5 hours?

4 What was the mass of the fish before it was in the water?

Time (hr)	Mass (g)
0	
1	50
2	100
3	200
4	400
5	

Name: ____ Date: ____ Period: ____

Doubles Every Hour (continued)

5 Carlos wrote this equation to model the fish's mass: $m = 25 \cdot 2^t$.

He used m for mass and t for time.

Explain what the 25 and 2 mean in this situation.

- The 25 means. . .
- The 2 means. . .

Time (hr)	Mass (g)
0	25
1	50
2	100
3	200
4	400
5	800

6 If the fish continues growing this way, what will its mass be after 7 hours?

7 Let's look at how Angel and Sora figured out the mass of the fish after 7 hours.

Discuss: What strategies do you see each student using?

Name: ________ Date: ________ Period: ________

Fish Growing and Shrinking

8 Here is a new toy fish.

Carlos wrote this equation to model the fish's mass:

$$m = 30 \cdot 1.5^t$$

Explain what the 30 and 1.5 mean in this situation.

- The 30 means. . .
- The 1.5 means. . .

9 What is the mass of the fish when $t = 0$?

10 Carlos wrote this equation to model the starfish's mass:

$$m = 270\left(\frac{1}{3}\right)^t$$

He used m for mass and t for time.

What will its mass be 3 hours after taking it out of water?

Name: Date: Period:

Fish Growing and Shrinking (continued)

11 Match the cards with an equation. Two cards will have no match.

Card A

This fish's mass is multiplied by $\frac{1}{2}$ each hour.

Card B

After 2 hours, this fish has a mass of 1.5 grams.

Card C

After 2 hours, this fish has a mass of 24 grams.

Card D

After 2 hours, this fish has a mass of 18 grams.

Card E

This fish has a mass of $\frac{1}{2}$ gram before it is put in water.

Card F

This fish has a mass of 6 grams before it is put in water.

Card G

The fish's mass increases by $\frac{1}{2}$ gram every hour.

$m = 6 \cdot \left(\frac{1}{2}\right)^t$	$m = \frac{1}{2} \cdot 6^t$

12 Synthesis

Here are two strategies that can be used to solve problems with exponential models.

Describe the benefits of each strategy.

Angel

Time (hr)	Mass (g)
1	50
2	100
3	200
4	400
5	800
6	1,600
7	3,200

Sora

$m = 25 \cdot 2^t$

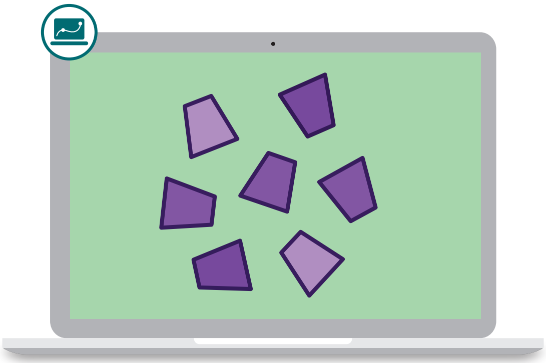
$m = 25 \cdot 2^7$

$m = 25 \cdot 128$

$m = 3200$

Things to Remember:

Name: .... Date: .... Period: ....



Growing Globs

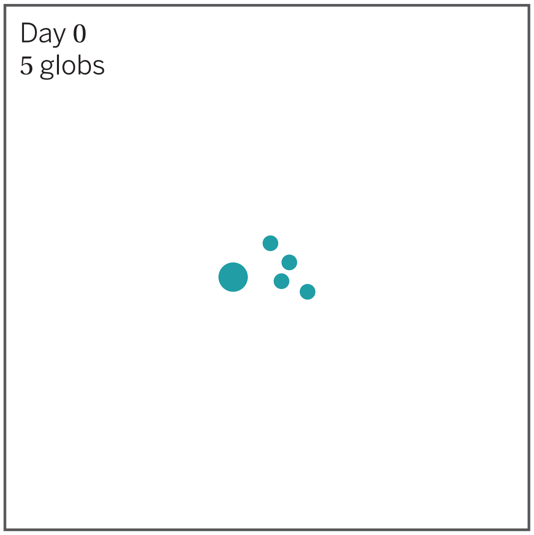
Let's identify and compare two different patterns of growth.

Warm-Up

1 Let's look at some teal globs.

a Watch how the number of globs grows.

b Write a story about these globs.



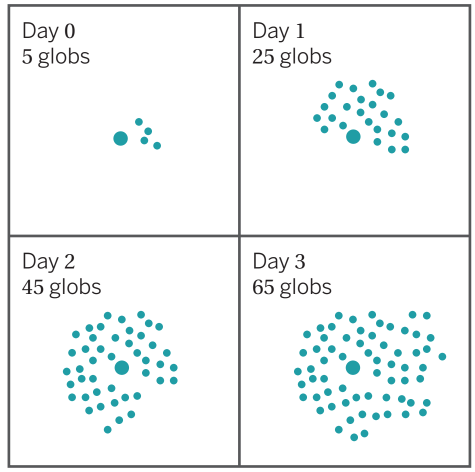
Activity 1

Name: ______________ Date: ______ Period: ______

Purple vs. Teal

2 How many teal globs will there be on day 4?

Day	Teal Globs
0	5
1	25
2	45
3	65
4	



3 Here is a new group of globs.

a Let's watch how the number of globs grows.

b How many purple globs will there be on day 4?

Day	Purple Globs
0	2
1	6
2	18
3	54
4	

4 The graph shows the number of each type of glob for the first 3 days.

Will there be more teal globs or purple globs on day 10? Circle one.

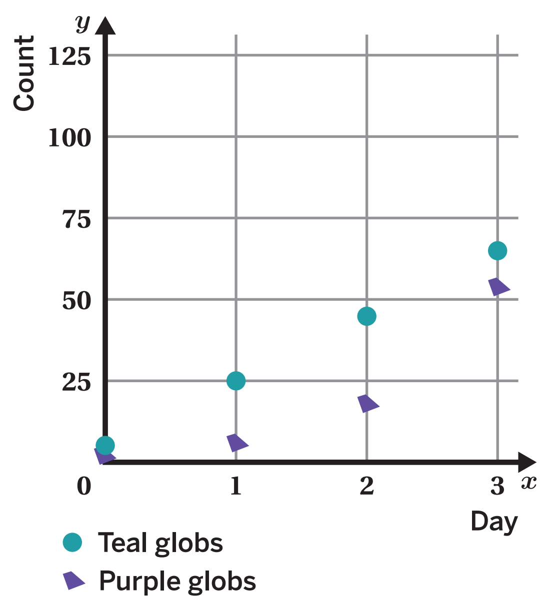
Teal Globs

Purple Globs

There will be the same

Not enough information

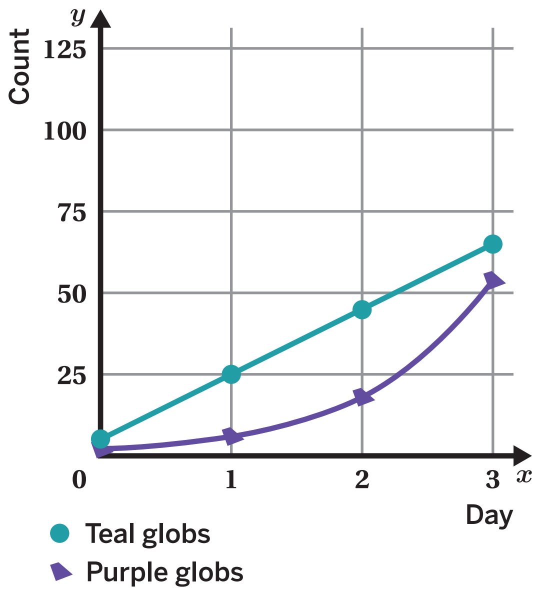
Explain your thinking.



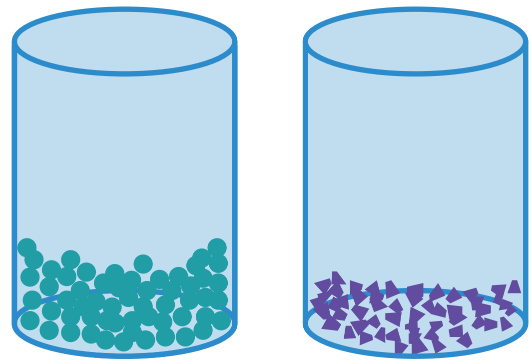
Name: ______ Date: ______ Period: ______

Purple vs. Teal (continued)

5 Let's watch how the number of teal and purple globs grows.



Day 3



Count: 65 **Count: 54**

6 Here are tables for the teal and purple growing globs.

- Teal globs grow are modeled by a **linear function** and have a constant *rate of change*.
- Purple globs are modeled by an **exponential function** and have a constant **growth factor**.

Discuss:

- How are *rate of change* and *growth factor* alike?
- How are they different?

Linear

Constant rate of change

Day	Teal Globs
0	5
1	25
2	45
3	65

+1, +1, +1 (Day); +20, +20, +20 (Teal Globs)

Exponential

Constant growth factor

Day	Purple Globs
0	2
1	6
2	18
3	54

+1, +1, +1 (Day); ×3, ×3, ×3 (Purple Globs)

Activity 2

Name: ______ Date: ______ Period: ______

Comparing Growth

7 Let's make two new species of globs and compare their growth.

8 Let's compare globs with different starting amounts and a constant rate of change or a constant growth factor.

Fabiana says: *Globs that grow by a constant growth factor will always eventually outnumber globs that grow by a constant rate of change*.

Lukas says: *If the constant rate of change is large enough, then this won't be true*.

Whose idea do you agree with? Circle one.

Fabiana's | Lukas's | Both | Neither

Explain your thinking.

9 Group these cards by their function type.

Card A

x	y
0	2
1	4
2	6
3	8
4	10

Card B

x	y
0	0
1	1
2	4
3	9
4	16

Card C

x	y
0	1
1	2
2	4
3	8
4	16

Card D

x	y
0	0
1	4
2	8
3	12
4	16

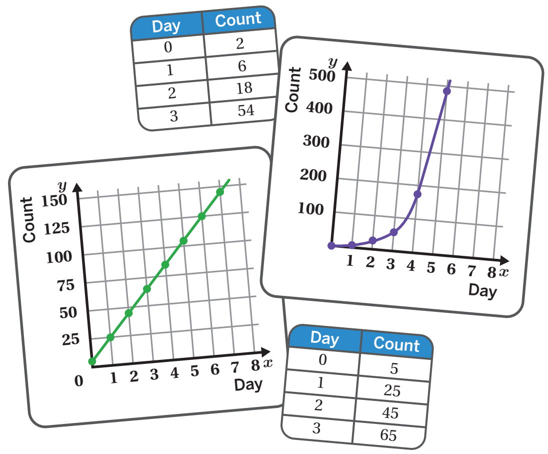
Linear	Exponential	Neither

10 Synthesis

Quantities that grow by a constant rate of change can be modeled by linear functions.

Quantities that grow by a constant growth factor can be modeled by exponential functions.

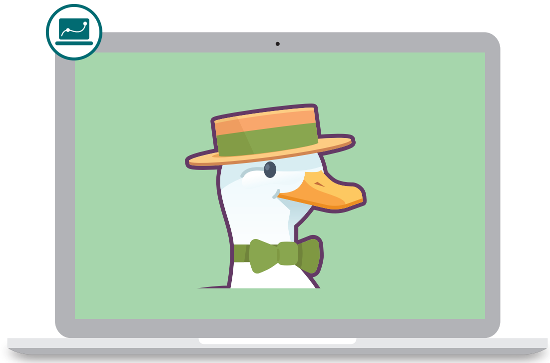
Describe strategies for determining whether a function is linear or exponential.



Things to Remember:

Unit 6
Lesson
3

Name: ______ Date: ______ Period: ______



Going Viral

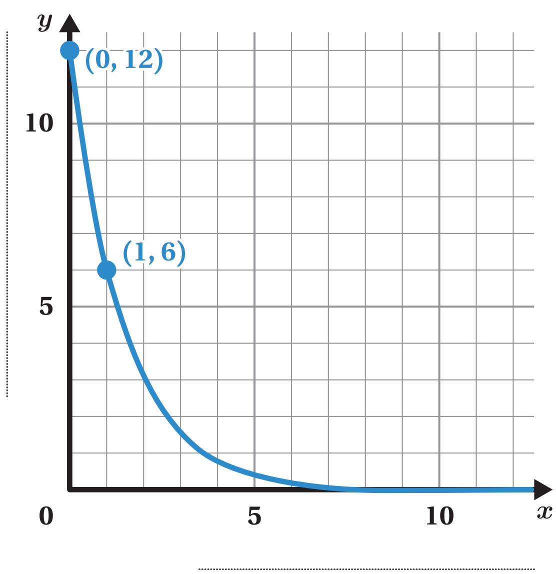
Let's describe connections between graphs and equations, and use graphs to write exponential functions.

Warm-Up

1 Here is a graph of an exponential relationship.

a Label the axes with any units you'd like.

b Write a story about the quantities based on the graph.



Activity 1

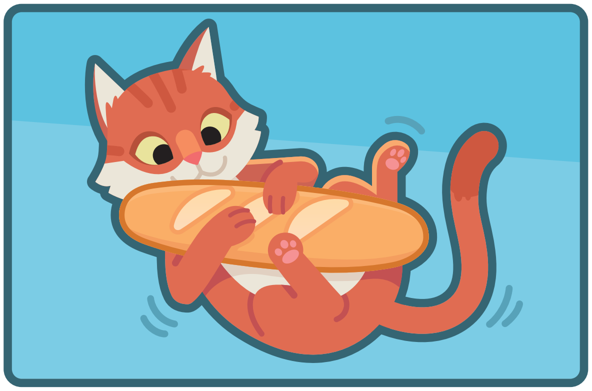
Name: ______ Date: ______ Period: ______

Three Memes

2 Let's watch this meme go viral.

Time (hr)	Likes
0	65
1	130
2	260
3	520
4	1,040
5	2,080

Hours since noon: 0:00



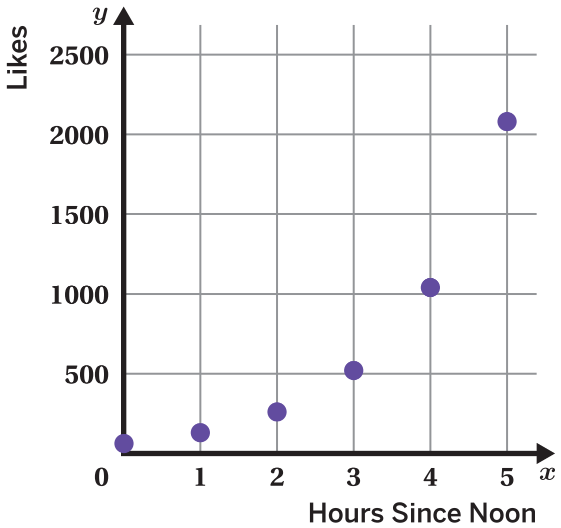
65

What type of relationship is this?

A. Linear **B.** Exponential **C.** Something else

3 There is an exponential relationship between the number of likes and the hours since noon.

Hours Since Noon, x	Likes, $f(x)$
0	65
1	130
2	260
3	520
4	1,040
5	2,080



Write an exponential function for this relationship.

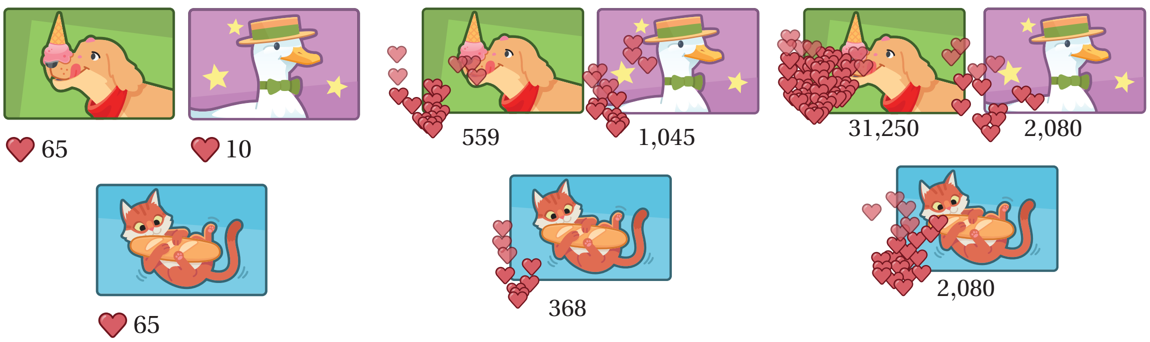
$f(x) =$ ______

Name: .... Date: .... Period: ....

Three Memes (continued)

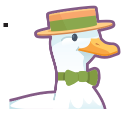
4 Let's look at the likes for these memes at different times.

Hours Since Noon: 0:00 **Hours Since Noon: 2:30** **Hours Since Noon: 5:00**

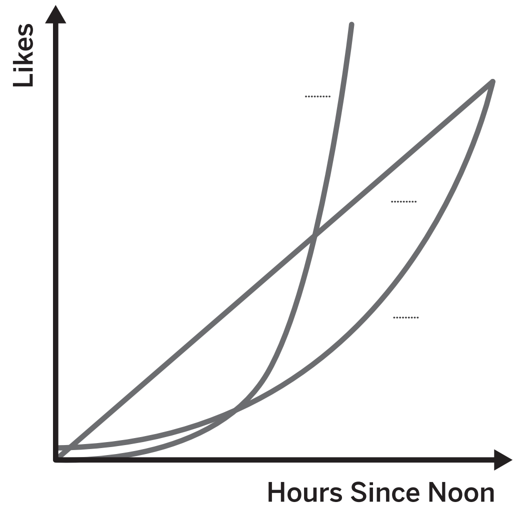


Match each meme to the graph that represents it.

A. 

B. 

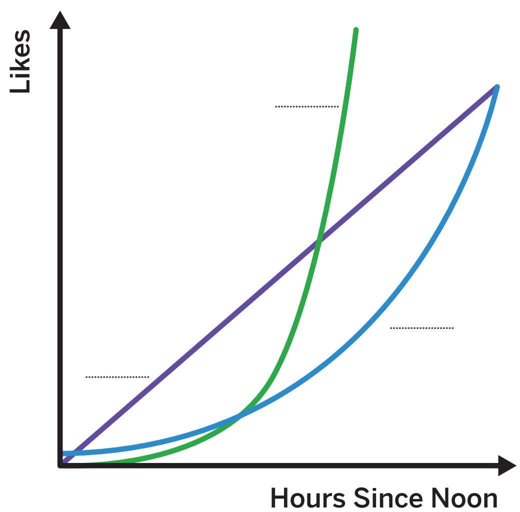
C. 



5 Match each function to its graph.

- $f(x) = 10 + 414x$
- $g(x) = 65 \cdot 2^x$
- $h(x) = 10 \cdot 5^x$

Explain your thinking.

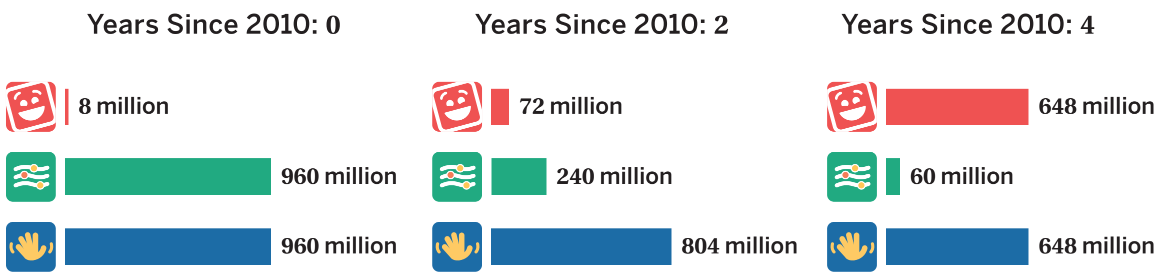


Activity 2

Name: .... Date: .... Period: ....

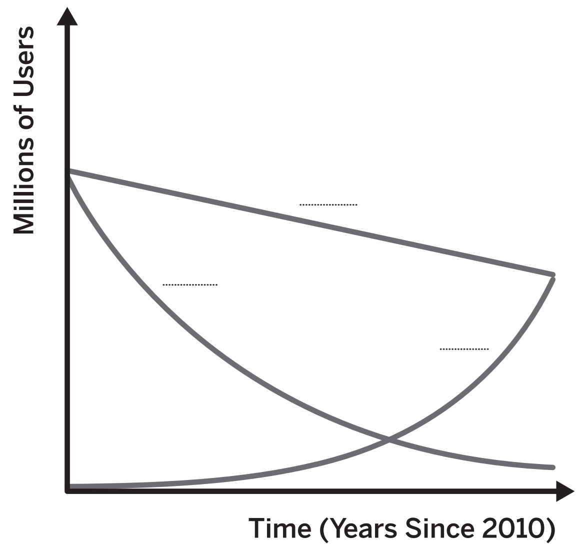
Take It Further

6 Let's look at the number of app users at different times.



Match each app to the graph that represents it.

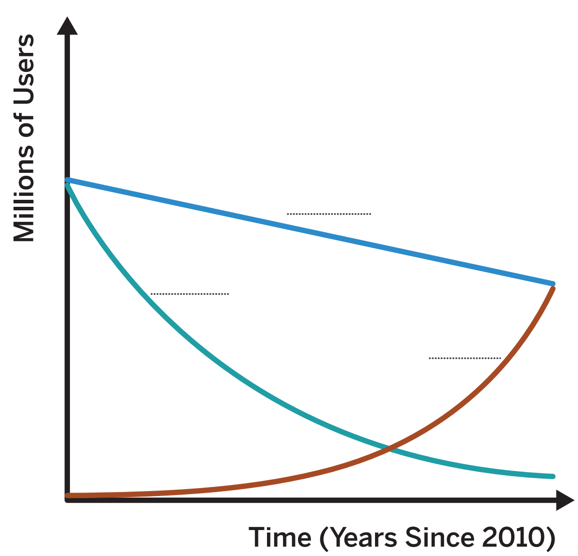
A.  B.  C. 



7 Match each function to its graph.

- $m(x) = 960 - 78x$
- $n(x) = 960 \cdot \left(\frac{1}{2}\right)^x$
- $p(x) = 8 \cdot 3^x$

Explain your thinking.



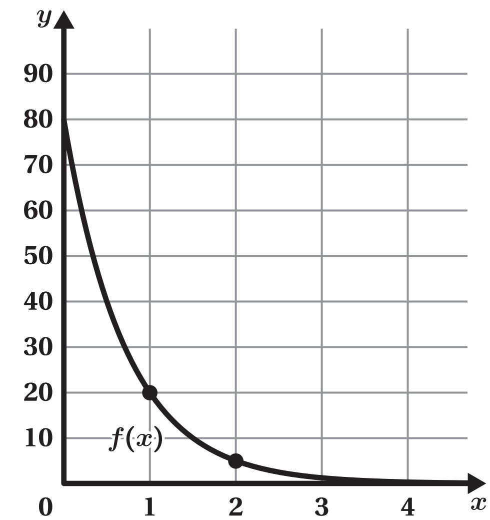
Name: ____________ Date: ________ Period: ________

Take It Further (continued)

8 Here is the graph of $f(x) = 80 \bullet \left(\frac{1}{4}\right)^x$.

What might $g(x) = 80 \bullet \left(\frac{1}{2}\right)^x$ look like?

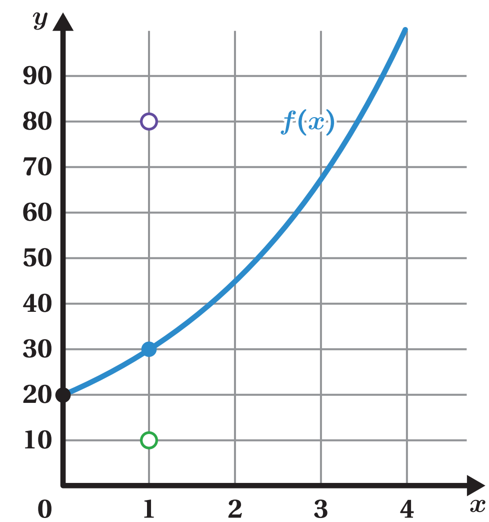
Show or explain your thinking.



9 Here are three different exponential relationships.

Each relationship includes the point $(0, 20)$ and one other point shown on the graph.

One function has been written for you. Write the other two functions.



	Includes the Point	Function
Graph 1	$(1, 80)$	$f(x) =$
Graph 2	$(1, 30)$	$g(x) = 20 \bullet \left(\frac{3}{2}\right)^x$
Graph 3	$(1, 10)$	$h(x) =$

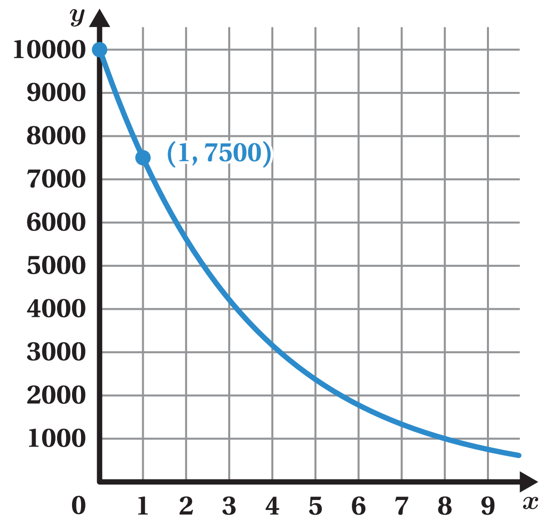
Explore More

10 Use the Explore More Sheet to answer a question about a pattern.

11 Synthesis

Here is a graph of $f(x) = 10000 \cdot \left(\frac{3}{4}\right)^x$.

Explain where you can see $10,000$ and $\frac{3}{4}$ on the graph.

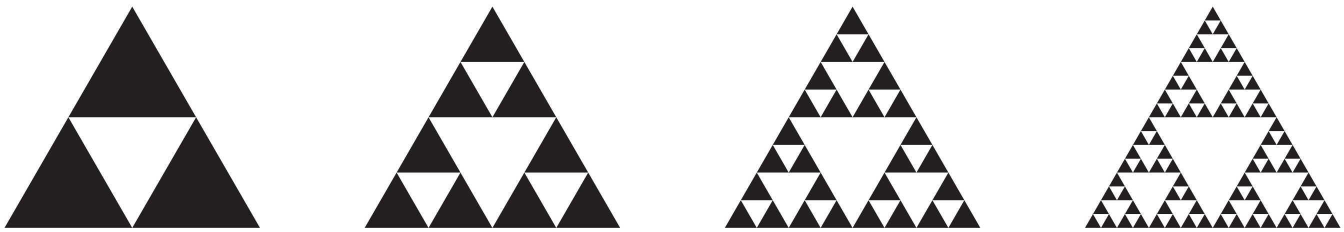


Things to Remember:

Explore More

Here are Figures 1–4 of a pattern.

Figure 1 Figure 2 Figure 3 Figure 4

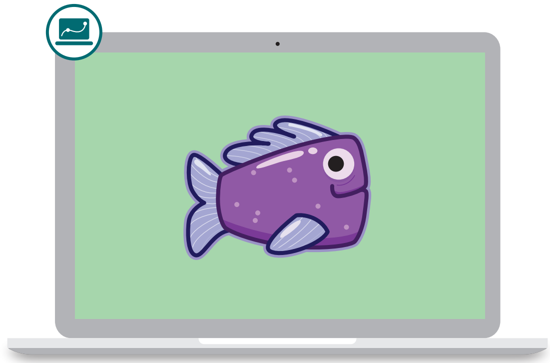


a Complete the table.

Figure	Number of Black Triangles	Fraction of Shaded Area
1	3	$\frac{3}{4}$
2	9	$\frac{9}{16}$
3	27	$\frac{27}{64}$
4		

b How could you figure out the number of black triangles and the fraction of shaded area for Figure 10?

Name: ________ Date: ________ Period: ________



Carlos and Corals

Let's evaluate exponential functions with positive, negative, and zero inputs.

Warm-Up

1 Select *all* the expressions that are equivalent to $2^{(-3)}$.

- ☐ **A.** $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
- ☐ **B.** $\frac{1}{2 \cdot 2 \cdot 2}$
- ☐ **C.** $2 \cdot -3$
- ☐ **D.** $-2 \cdot -2 \cdot -2$
- ☐ **E.** $8^{(-1)}$

Name: .... Date: .... Period: ....

Carlos's Fish

2 Carlos's apartment still does not allow pets, so he decided to buy a new toy fish.

The mass of the fish doubles every hour.

What type of function do you think will model the mass of this fish over time?

Linear Exponential Neither

Explain your thinking.

3 Carlos's new toy fish has a constant growth factor when placed in water.

What is the mass of the toy fish after 4 hours?

Time (hr)	Mass (g)
1	20
2	40
3	80
4	

4 Carlos writes $m(t)$ to model the fish's mass over time.

$$m(t) = 10 \cdot 2^t$$

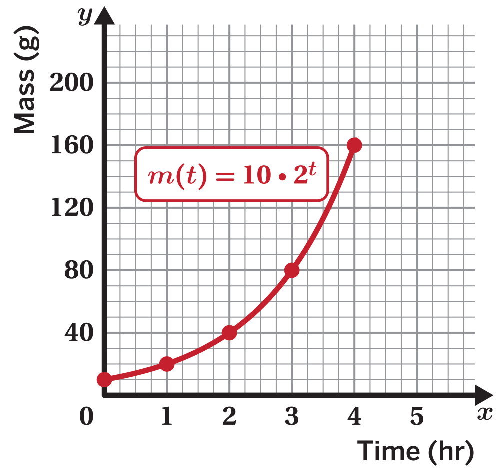
What is the value of $m(0)$? Explain your thinking.

Name: .... Date: .... Period: ....

Carlos's Fish (continued)

5 Explain where you see the growth factor in the graph, the table, or the function.

Time, t	Mass, $m(t)$
0	10
1	20
2	40
3	80
4	160



6 a What is the value of $m(5)$?

b What is the value of $m(\text{-}1)$?

c **Discuss:**

- What does each value say about the fish's mass?
- How would you describe the *domain* of $m(t)$?

Activity 2

Name: ______ Date: ______ Period: ______

Coral Reefs

Screens 7–9: A marine biologist is studying a coral reef. In 2010, she estimated that its volume was 320 cubic meters.

She wrote the function $v(t)$ to represent the volume of the coral reef t years after 2010:

$$v(t) = 320\left(\frac{4}{5}\right)^t$$

7 Based on $v(t)$, what was the reef's volume in 2011?

A. Less than 320 cubic meters

B. Equal to 320 cubic meters

C. Greater than 320 cubic meters

Explain your thinking.

8 **a** Determine the value of $v(2)$.

b **Discuss:** What domain could make sense for $v(t)$?

9 Determine the missing values.

Years Since 2010	-3	-2	-1	0	1	2
Volume (cu. m)				320	256	204.8

Name: .... Date: .... Period: ....

Coral Reefs (continued)

10 Here is how Angel and Sora determined the volume of the coral reef in 2007 (3 years before 2010).

Angel

Year Since 2010	Volume (cubic meters)
-3	625
-2	$\cdot \frac{4}{5}$ 500
-1	400
0	320 $\cdot \frac{5}{4}$

Sora

$$V(-3) = 320\left(\frac{4}{5}\right)^{-3}$$
$$V(-3) = 320 \cdot \left(\frac{5}{4}\right)^{3}$$
$$V(-3) = 320 \cdot \frac{125}{64}$$
$$V(-3) = 625$$

Discuss: What is each student's strategy?

11 Here is a new function: $f(x) = 18 \cdot 3^x$.

a. **Discuss:** Will $f(\text{-}2)$ be less than or greater than 18?

b. Determine the value of $f(\text{-}2)$.

12 Synthesis

Describe a strategy for evaluating exponential functions for negative inputs.

Use the examples if they help with your thinking.

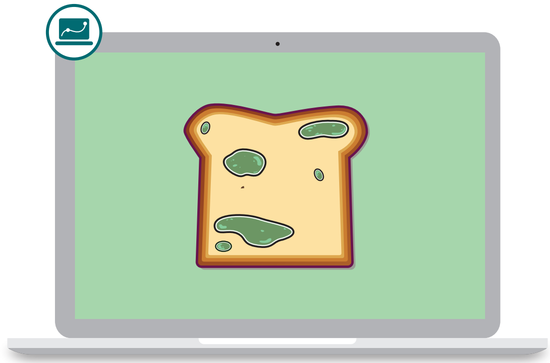
$$a(-4) = 5 \cdot 10^{(-4)}$$

$$b(-3) = 10 \cdot \left(\frac{1}{2}\right)^{(-3)}$$

Things to Remember:

Unit 6
Lesson
7

Name: .... Date: .... Period: ....



Growing Mold

Let's explore how to model situations that change by a percent increase with exponential functions.

Warm-Up

Determine the value of each statement.

1 10% of 30

2 100% of 30

3 110% of 30

4 110% of 50

Activity 1

Name: ______ Date: ______ Period: ______

Growing Mold

5 A piece of bread is left out on a counter.

Let's watch an animation to see what happens over time.

What do you notice? What do you wonder?

6 This mold grows by 75% each day. How much mold will there be on day 4?

7 How much mold will there be on days 5 and 6?

Days	Area of Mold (sq. cm)
3	16
4	
5	
6	

Name: ____________ Date: ________ Period: ________

Growing Mold (continued)

8 Arnav made a table to help him write a function to represent the area of mold, $m(x)$, after x days.

Where do you see the 75% increase represented in the function's equation? In the table?

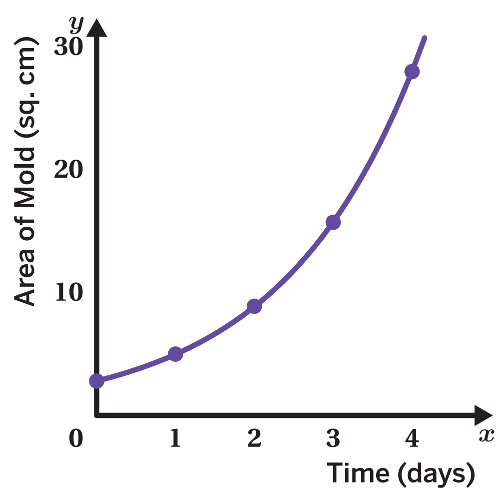
$$m(x) = 2.985(1.75)^x$$

Days	Area of Mold (sq. cm)
0	2.985
1	5.224
2	9.14
3	16
4	28

Equation:

Table:

9 Here is the graph of $m(x) = 2.985(1.75)^x$.



a Determine how much mold there will be after 10 days.

b **Discuss:** Do you think the mold could grow according to $m(x)$ forever?

Name: .... Date: .... Period: ....

Growing with Percents

10 Tyler made potato salad and forgot to put it in the refrigerator.

The amount of bacteria in the potato salad increases by 4% every minute.

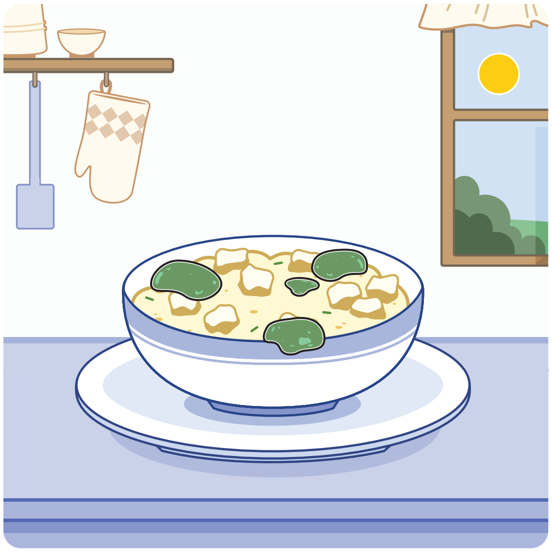
Which function, $p(t)$, represents the amount of bacteria in the potato salad after t minutes?

A. $p(t) = 5 \cdot 0.4^t$

B. $p(t) = 5 \cdot 1.04^t$

C. $p(t) = 5 \cdot 1.4^t$

Explain your thinking.



11 Let's look at which function Tyler selected.

What does the 1 represent in this situation?

Name: .... Date: .... Period: ....

Growing with Percents (continued)

12 Match each function with the situation that represents the same relationship. One function will have no match.

Functions

a. $a(x) = 20 \cdot 0.85^x$

b. $b(x) = 20 \cdot 1.85^x$

c. $c(x) = 85 \cdot 1.2^x$

d. $d(x) = 85 \cdot 1.02^x$

Situations

.... A population of bacteria starts with 20 cells and grows by 85%.

.... A population of frogs starts with 85 frogs and grows by 20%.

....

Days	Amount of Money ($)
0	85
1	86.7
2	88.4

Explore More

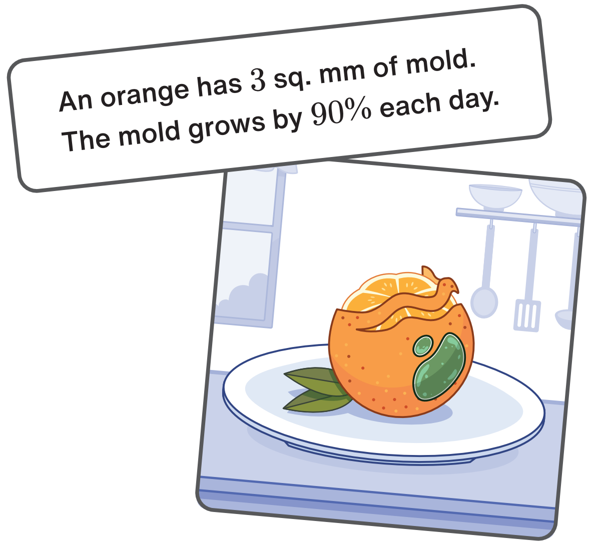
13 Heat and humidity can cause some types of bacteria to grow quickly. Imagine that in a humid room the amount of bacteria in a potato salad *triples* every hour.

By what percent is the bacteria growing per hour? Explain your thinking.

14 Synthesis

How do you write an exponential function that represents growing by a percentage?

Use the example if it helps with your thinking.



Things to Remember:

Unit 6
Lesson
10

Name: Date: Period:

Marbleslides: Exponentials

Let's practice translating exponential functions by playing a game.

Warm-Up

You'll use the digital activity for the Warm-Up.

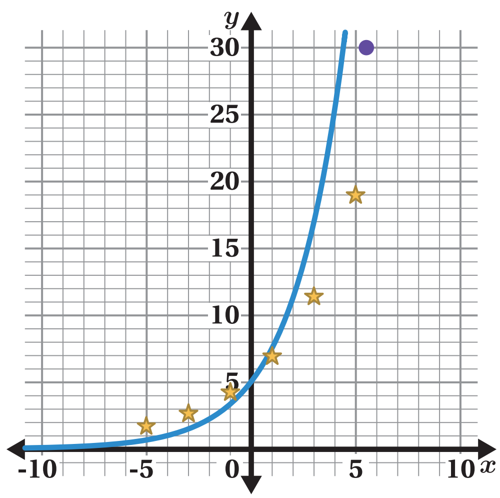
1 Your goal is to capture all the stars.

Change the function to capture all the stars.

Original function:

$f(x) = 5 \cdot 1.5^x$

Your function:

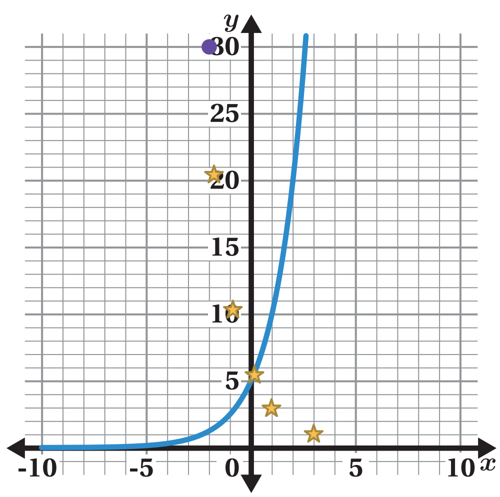


2 Change the function to capture all the stars.

Original function:

$f(x) = 5 \cdot 2^x$

Your function:



Activity

1

Name: ______ Date: ______ Period: ______

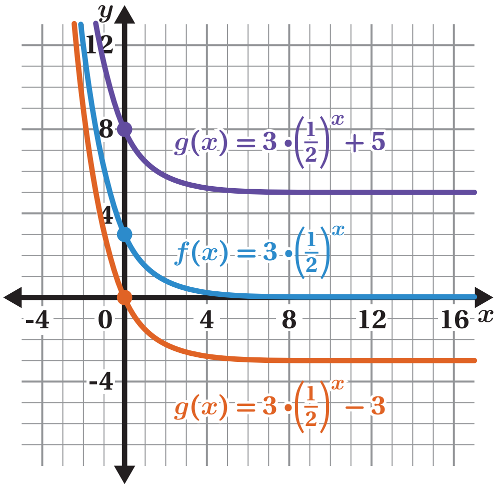
Translations

You'll use the digital activity for Problems 3–6.

3 To capture all of the stars, you may want to translate your function up or down.

Use the activity to translate $f(x)$ vertically.

What do you notice? What do you wonder?

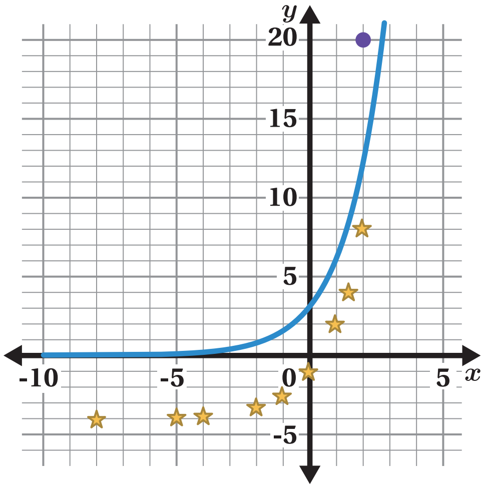


4 Change the function to capture all the stars.

Original function:

$f(x) = 3 \cdot 2^x$

Your function:



Activity 1

Name: ____________ Date: ________ Period: ________

Translations (continued)

5 To capture all of the stars, you may want to translate your function left or right.

Use the activity to translate $f(x)$ horizontally.

Which function will translate $f(x)$ 12 units to the right?

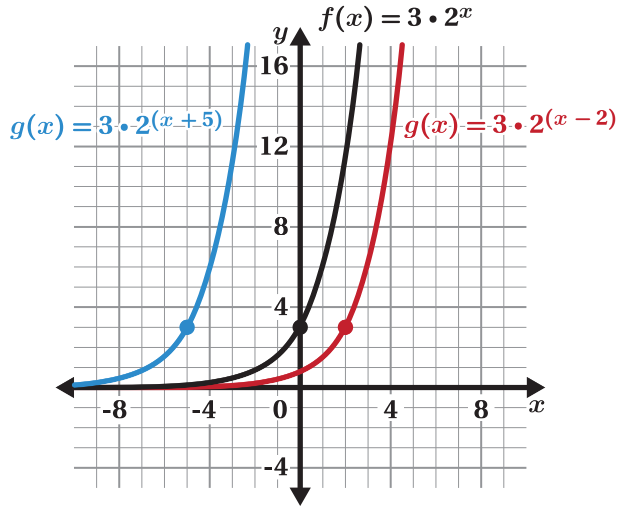
A. $g(x) = 3 \cdot 2^{x} - 12$

B. $g(x) = 3 \cdot 2^{(x-12)}$

C. $g(x) = 3 \cdot 2^{(x+12)}$

D. $g(x) = 3 \cdot 2^{x} + 12$

Explain your thinking.

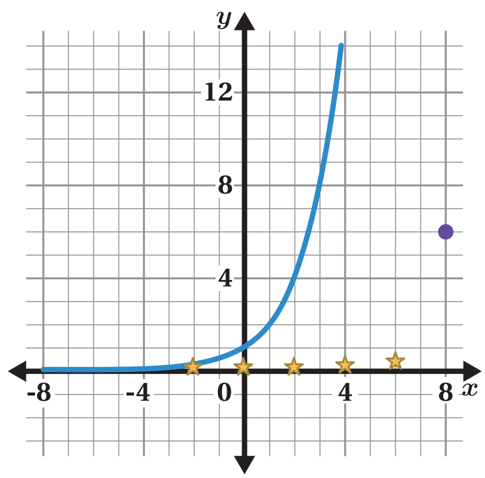


6 Change the function to capture all the stars.

Original function:

$f(x) = 2^{(x)}$

Your function:



Activity 2

Name: .... Date: .... Period: ....

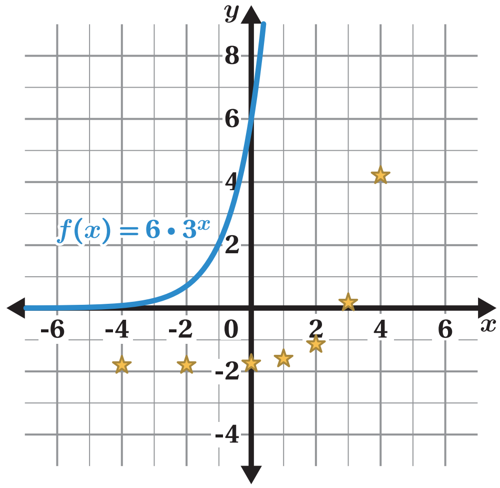
Challenges

7 Kiri wrote the function $g(x) = 6 \cdot 3^{(x+4)} - 2$ to capture all the stars.

Will Kiri's function capture all of the stars? Circle one.

Yes No I'm not sure

Explain your thinking.



You'll use the digital activity for Problems 8–12.

8 Here is Kiri's function from the previous screen. Change the function to capture all the stars.

$g(x) = 6 \cdot 3^{(x+4)} - 2$

9 Create as many exponential functions as you need to capture all the stars.

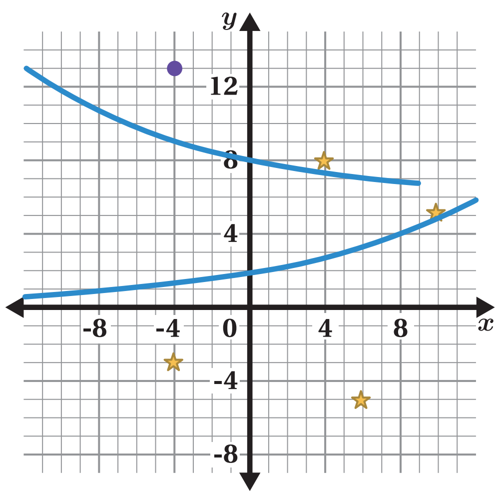
We have included functions that may help you start.

Original functions:

$f(x) = 2 \cdot (0.9)^x + 6 \; \{x < 9\}$

$g(x) = 3 \cdot (1.1)^{(x-5)}$

Your functions:



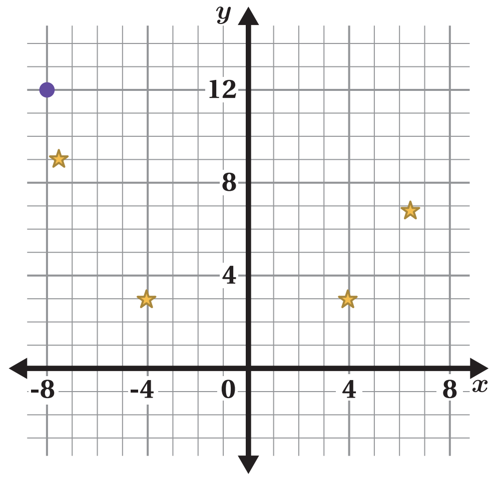
Activity
2

Name: .... Date: .... Period: ....

Challenges (continued)

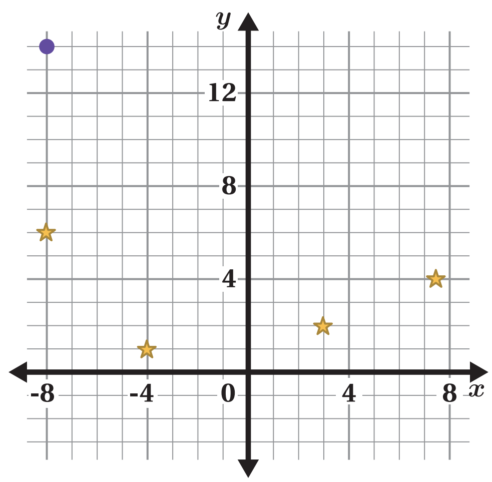
10 Create as many exponential functions as you need to capture all the stars.

Your functions:



11 Create as many exponential functions as you need to capture all the stars.

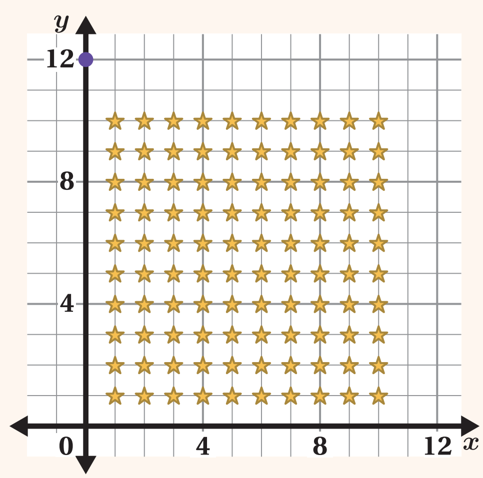
Your functions:



Explore More

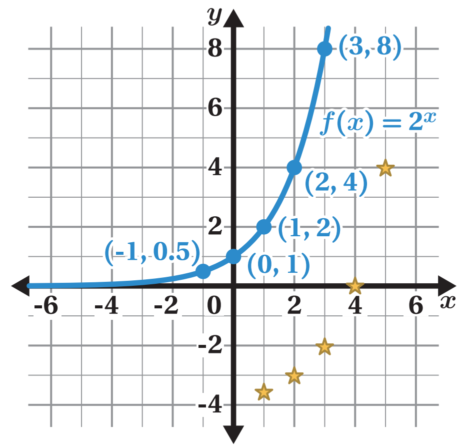
12 Challenge yourself to capture as many stars as you can!

Your functions:



13 Synthesis

Describe how to use vertical and horizontal *translations* to write a function that would capture all the stars.



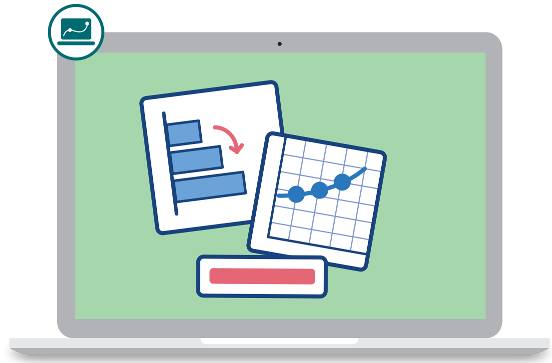
Things to Remember:

Unit 6

Lesson

13

Name: .... Date: .... Period: ....



Bank Accounts

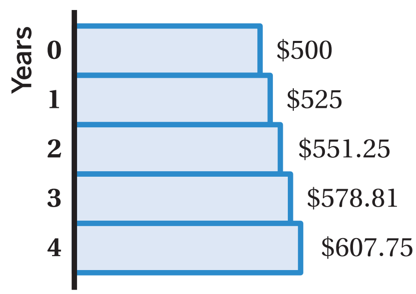
Let's learn how to model situations involving simple and compound interest.

Warm-Up

1-2 Mauricio has $500 to invest. He is researching different kinds of investment accounts. Here are the values of Accounts A and B over time.

Account A

Account B



Show or describe how the value of each account grows over time.

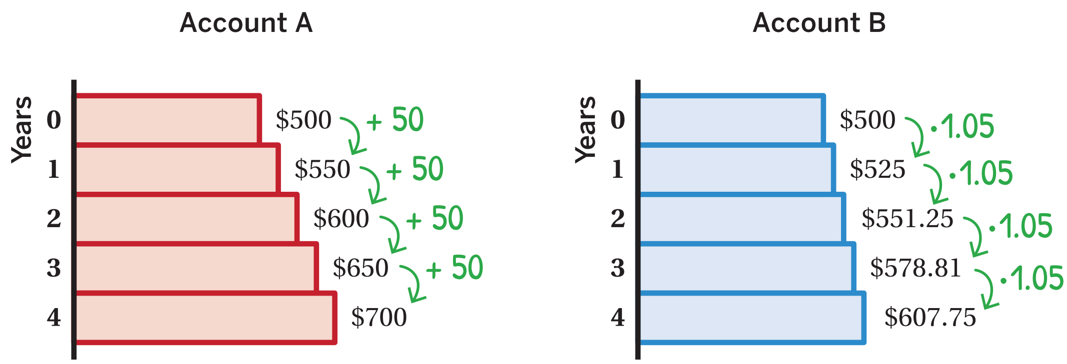
Activity 1

Name: .... Date: .... Period: ....

Earning Interest

3 Here is Mauricio's work to show how each account is growing.

Discuss: Describe his work to a partner. Which account would you recommend he invest in?

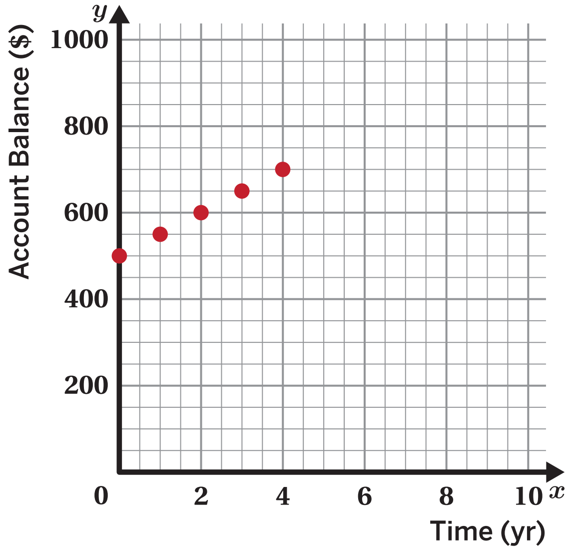


4 Account A earns 10% **simple interest** per year.

a **Discuss:** How do you think *simple interest* works?

b Determine the account balance after 5 years.

Time (yr)	Account Balance ($)
0	500
1	550
2	600
3	650
4	700
5	



Activity 1

Name: ______ Date: ______ Period: ______

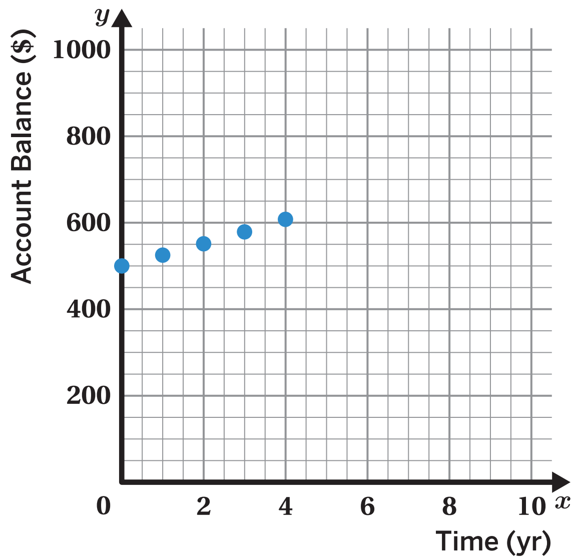
Earning Interest (continued)

5 Account B earns 5% **compound interest** per year.

a **Discuss:** How do you think *compound interest* works?

b Determine the account balance after 5 years.

Time (yr)	Account Balance ($)
0	500
1	525
2	551.25
3	578.81
4	607.75
5	

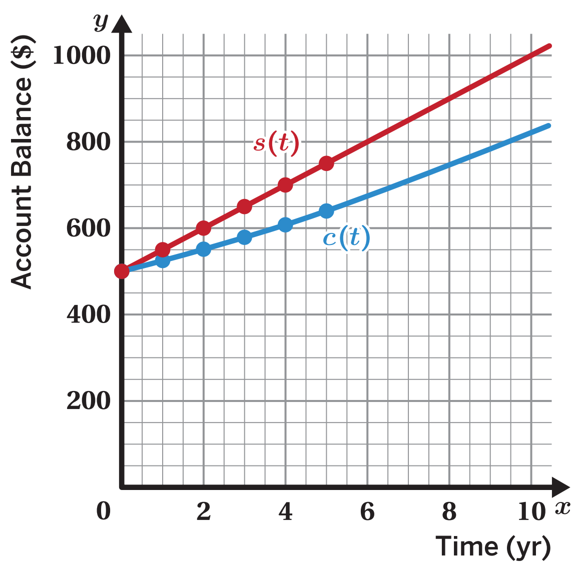


6 We can use functions to describe the account balances after t years.

- Simple interest: $s(t) = 500 + 50t$
- Compound interest: $c(t) = 500(1.05)^t$

How are these functions alike?

How are they different?



7 **a** Let's watch the account balances grow.

b **Discuss:** Which account would you recommend Mauricio invest in? Why?

Activity 2

Name: ________ Date: ________ Period: ________

Simple and Compound Interest

8 Mauricio decided to invest in an account that offers 6% compound interest per year.

$a(t) = 500(1.06)^t$ represents its balance after t years.

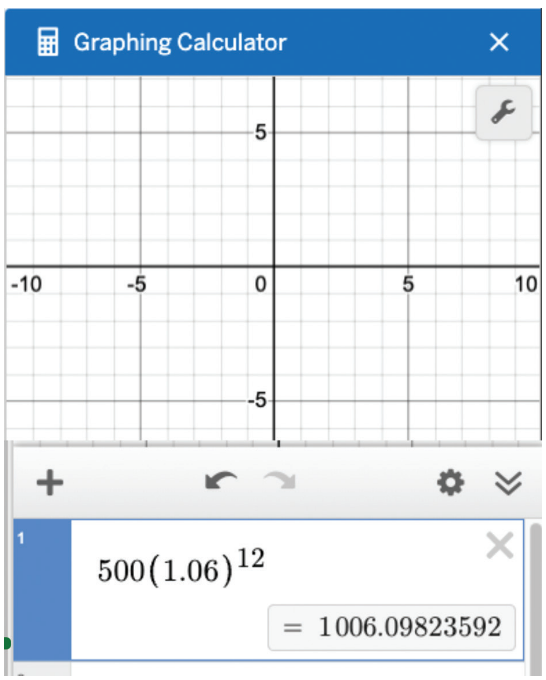
About how many years will it take for the balance to reach $\$1{,}000$? Explain your thinking.

Use this space or the Desmos Graphing Calculator to help with your thinking.

9 Let's watch each video to see how two students determined the time it would take to reach $\$1{,}000$.

Discuss: Which method helps get a more precise answer?

Fabiana



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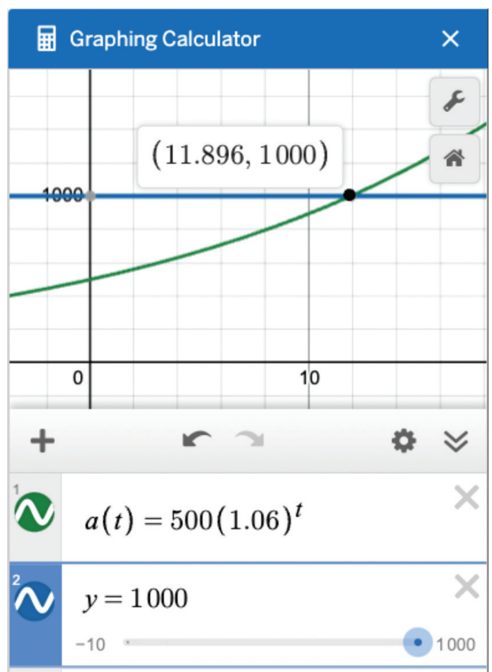


Name: .... Date: .... Period: ....

Simple and Compound Interest (continued)

10 Solve as many challenges as you have time for.

a A $\$1{,}000$ investment earns 4% compound interest.

The function $f(t) = 1000(1.04)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $\$2{,}500$?

b A $\$200$ investment earns 7% compound interest.

The function $f(t) = 200(1.07)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $\$450$?

c A $\$1{,}700$ investment earns 3% compound interest.

The function $f(t) = 1700(1.03)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $\$4{,}150$?

d A $\$1{,}150$ investment earns 2% compound interest.

The function $f(t) = 1150(1.02)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $\$2{,}700$?

Explore More

11 Use the Explore More Sheet to answer questions about an account balance.

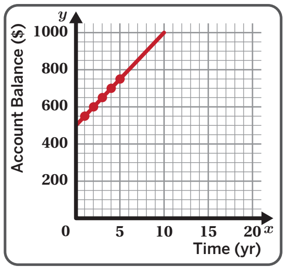
12 Synthesis

Here are some examples of simple and compound interest.

Simple Interest

$s(t) = 500 + 50t$

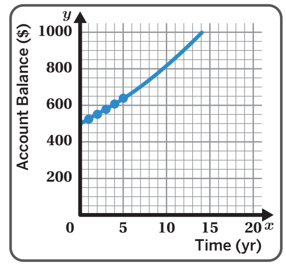
Time (yr)	Account Balance ($)
0	500
1	550
2	600
3	650
4	700



Compound Interest

$c(t) = 500(1.05)^t$

Time (yr)	Account Balance ($)
0	500
1	525
2	551.25
3	578.81
4	607.75



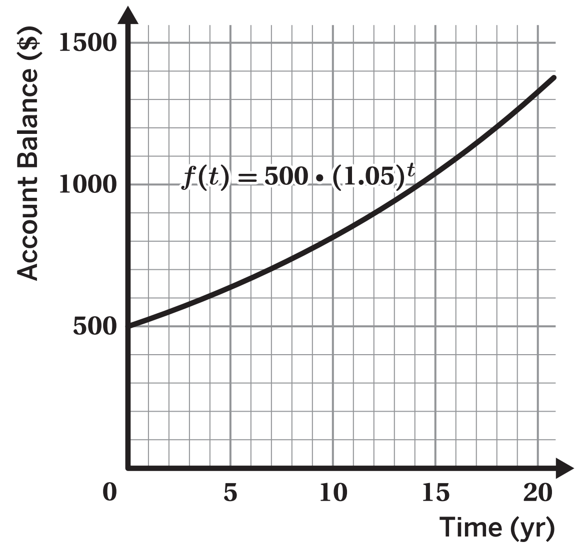
How do investments grow with simple interest?

How do investments grow with simple interest?

Things to Remember:

Explore More

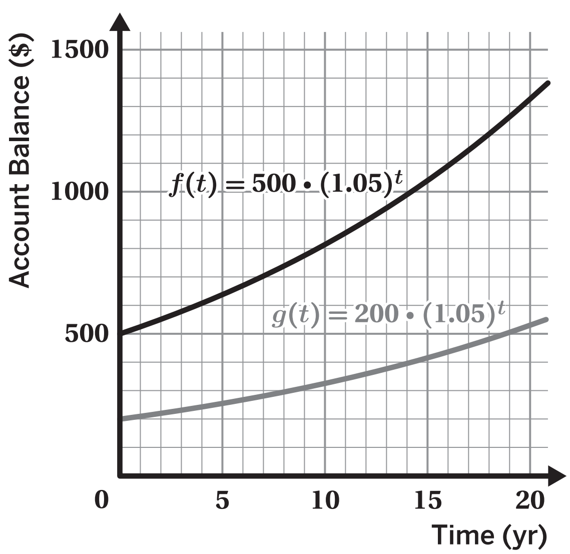
An account with \$500 that earns 5% compound interest doubles its value in about 14 years. The function $f(t)$ gives the account balance after t years.

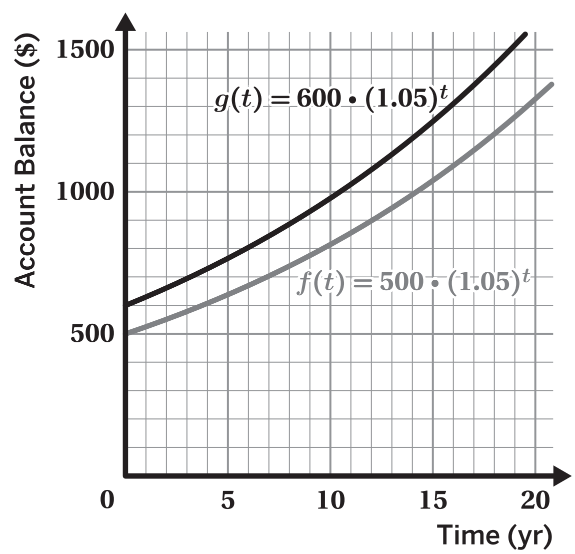


a **Discuss:**

- What does it mean to double in value?
- Where do you see that in the graph?

b Here are two different graphs with different initial account balances.





c **Discuss:** How long does it take the initial account balance to double? Compare with a partner. Is the amount of time it takes to double the same or different?

Unit 6
Lesson
14

Name: ________ Date: ________ Period: ________

Payday Loan

Let's analyze exponential functions that represent different compound interest scenarios.

Warm-Up

1 Zola says that $x^{12} = (x^4)^3$.

The diagram shows why that is true.

$$\underbrace{(x \cdot x \cdot x \cdot x)}_{x^4}\ (x \cdot x \cdot x \cdot x)\ (x \cdot x \cdot x \cdot x)$$

Write three other expressions equivalent to x^{12}.

Activity 1

Name: Date: Period:

Payday Loan

2 A payday loan is a short-term loan designed to be paid back within a month.

Here is an advertisement for a payday loan.

Discuss: What do you notice? What do you wonder?

3 FastCash offers payday loans that charge 15% compound interest per month.

Marc borrows $\$100$ to help pay his heating bill.

How much will Marc owe after one month?

Explain your thinking.

Name: ______ Date: ______ Period: ______

Payday Loan (continued)

4 The function $f(m)$ represents how much Marc will owe if he doesn't pay back the loan for m months. Write an equation to represent $f(m)$.

$f(m) =$ ______

Months, m	Amount Owed, $f(m)$
0	100
1	115
2	132.25
3	152.09
4	174.90

5 Marc wonders how much money he would owe if he doesn't pay back the loan after 3 years.

He wrote two expressions to represent this situation.

Expression A	Expression B
$100 \cdot 1.15^{36}$	$100 \cdot (1.15^{12})^3$

Discuss: How are the expressions alike? How are they different?

6 Marc wrote a third equivalent expression to represent this situation.

Expression C

$100 \cdot (5.35)^3$

What interest rate does the 5.35 represent?

A. 435% per year **B.** 535% per year **C.** Neither

Explain your thinking.

Activity 2

Name: ______ Date: ______ Period: ______

Credit Cards and Other Loans

7 Marc sees an advertisement for a credit card that charges a 2% monthly interest rate.

How much would he owe for a $100 charge on the credit card after 3 years of no payments?

Payday Loan

- $100 loan
- 15% monthly interest

Amount owed after 3 years of no payments: $15315.19

Credit Card

- $100 charge
- 2% monthly interest

Amount owed after 3 years of no payments: ?

8 Here are three equivalent functions that represent the amount owed on a credit card charge of $100 after t years of 2% monthly interest.

- $g(t) = 100 \cdot 1.02^{12t}$
- $g(t) = 100 \cdot (1.02^{12})^t$
- $g(t) = 100 \cdot 1.2682^t$

Use one or more of the functions to determine the interest rate *per year*.

Explain your thinking.

Activity 3

Name: ________ Date: ________ Period: ________

Comparing Rates

9 Marc wants to compare interest rates on different types of loans.

a Complete the table.

	Monthly Interest Rate (%)	Monthly Growth Factor	Growth Factor per Year	Interest Rate per Year (%)
Payday Loan	15.00	1.15	5.3503	435.03
Credit Card	2.00	1.02	1.2682	
Private Loan	1.21	1.0121		
30-year Mortgage	0.53			
Federal Student Loan	0.41			

b **Discuss:** In what situations might people take out each of these different types of loans?

Name: .... Date: .... Period: ....

Comparing Rates (continued)

10 Here's the information about federal student loans that another student entered on the previous screen.

Monthly Interest Rate (%)	Monthly Growth Factor	Growth Factor per Year	Interest Rate per Year (%)
0.41	1.0041	1.0503	5.03

Annika takes out a $20,000 federal student loan.

Write a function, $h(t)$, to calculate the amount Annika owes after making no payments for t years.

Explore More

11 Tyler charges $3,000 to a credit card with a 2% monthly interest rate.

Many credit cards require a monthly minimum payment.

a Let's see how long will it take Tyler to pay off the charges with different monthly payment amounts.

b What do you think is important to remember when getting and using a credit card?

12 Synthesis

What can different equivalent expressions tell us about the same situation involving compound interest?

Expression A	Expression B
$100 \cdot 1.15^{36}$	$100 \cdot (1.15^{12})^3$

Expression C
$100 \cdot (5.35)^3$

Things to Remember:

Unit 6
Lesson
15

Name: ______________________ Date: __________ Period: __________

Credit Card Compounding

Let's explore how to calculate and compare account balances with interest rates that compound at different intervals.

Warm-Up

1 Group each card with the word that it describes.

Card A	Card B	Card C	Card D
2 times per year	4 times per year	$\frac{1}{12}$ of a year	$\frac{1}{4}$ of a year

Card E	Card F	Card G
Every 3 months	Every 6 months	Every 12 months

Monthly	Quarterly	Semi-Annually	Annually

Activity
1

Name: ________ Date: ________ Period: ________

PayLater

2 Alejandro is considering charging $\$1,000$ to this credit card.

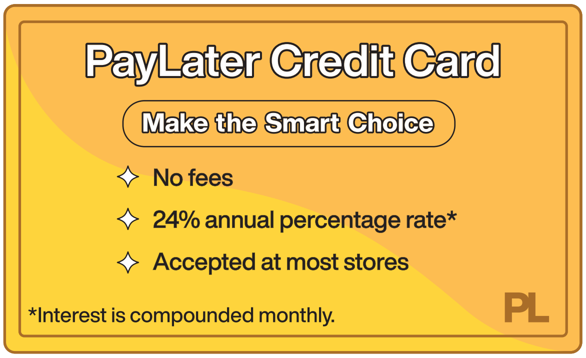
He wrote $1000(1 + 0.24)^5$ to determine the balance after 5 years with no payments or additional charges.

Explain what each part of the expression means.

1000:

$1 + 0.24$:

5:



3 The fine print says interest is compounded monthly.

This means the interest is $\frac{24}{12} = 2$, or 2% per month.

Compared to compounding annually, how do you think compounding monthly will affect the total Alejandro owes after 5 years? Circle one.

A. He will owe more

B. He will owe less

C. He will owe the same

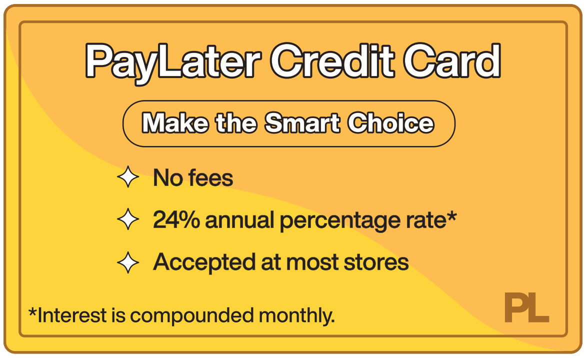
Explain your thinking.

Name: ________ Date: ________ Period: ________

PayLater (continued)

4 Alejandro is considering charging $\$1{,}000$ to this credit card.

If the interest is compounded at 2% monthly, how much would he owe after 5 years?



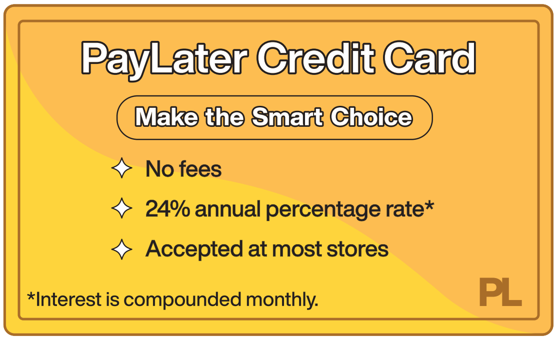
5 Alejandro wrote $1000\left(1+\frac{0.24}{12}\right)^5$ to determine the balance after 5 years, but he made an error.

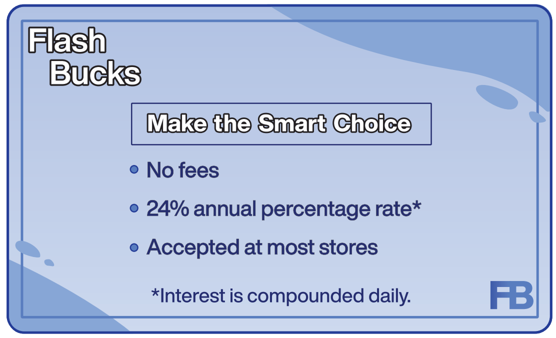
Find the error and explain why it is incorrect.

Name: ______ Date: ______ Period: ______

PayLater and Flash Bucks

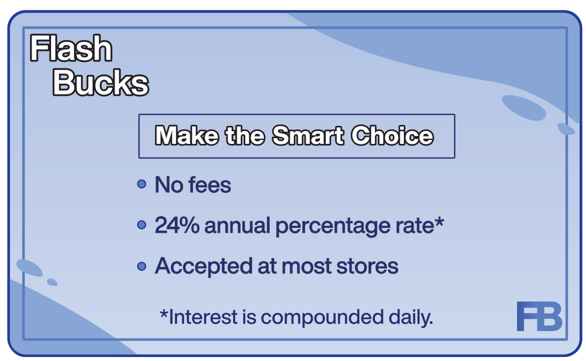
6 Alejandro is considering charging $\$1{,}000$ to a different credit card.





Discuss: Compared to compounding monthly, how do you think compounding daily will affect the total amount owed?

7 Alejandro is considering charging $\$1{,}000$ to this credit card.



a **Discuss:** How would you determine the daily interest rate?

b If interest is compounded daily, how much would Alejandro owe after 5 years with no payments or additional charges?

Activity 3

Name: .... Date: .... Period: ....

Compounding Differently

8 Here are some expressions to calculate the total amount for \$800 and a 12% annual interest rate compounded using different *intervals*.

Match each expression with its compounding period and length. One card will have no match.

Card A

$$800\left(1 + \frac{0.12}{4}\right)^{(4 \cdot 3)}$$

Card B

$$800(1 + 0.01)^{24}$$

Card C

$$800\left(1 + \frac{0.12}{12}\right)^{(12 \cdot 2)}$$

Card D

$$800(1 + 0.04)^{(3 \cdot 2)}$$

Card E

$$800(1 + 0.03)^{12}$$

Compounded Quarterly for 3 Years	Compounded Monthly for 2 Years

9 Compound interest expressions can be represented using this formula:

$$P\left(1 + \frac{r}{n}\right)^{nt}$$

Circle one variable and describe what it represents.

P r n t

Name: .... Date: .... Period: ....

Compounding Differently (continued)

10 Solve as many challenges as you have time for.

a A person puts $\$500$ into an account with a 10% annual interest rate compounded quarterly.

What is the balance in the account after 4 years?

b A person puts $\$800$ into an account with a 5% annual interest rate compounded daily.

What is the balance in the account after 3 years?

c A person puts $\$3,000$ into an account with a 5% annual interest rate compounded daily.

What is the balance in the account after 7 years?

d A person puts $\$1,000$ into an account with a 20% annual interest rate compounded yearly.

What is the balance in the account after 8 years?

11 Synthesis

How can you use this formula to calculate the total value of an account or loan with compound interest?

$$P\left(1+\frac{r}{n}\right)^{nt}$$

Things to Remember:

Unit 6
Lesson
16

Name: ________ Date: ________ Period: ________

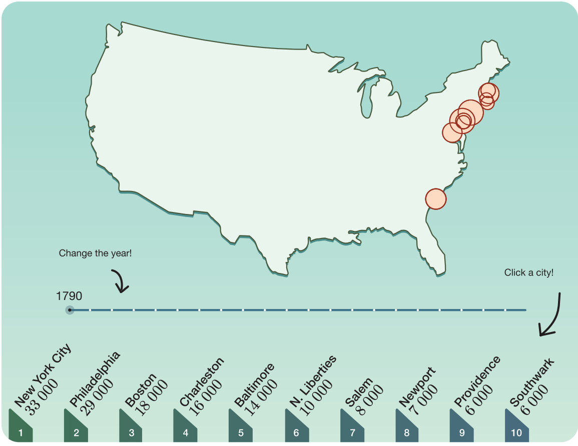
Detroit's Population, Part 1

Let's use functions to model the population growth of Detroit.

Warm-Up

1 Use the digital activity to see the ten U.S. cities with the largest populations from 1790–2000.

Discuss: What do you notice? What do you wonder?



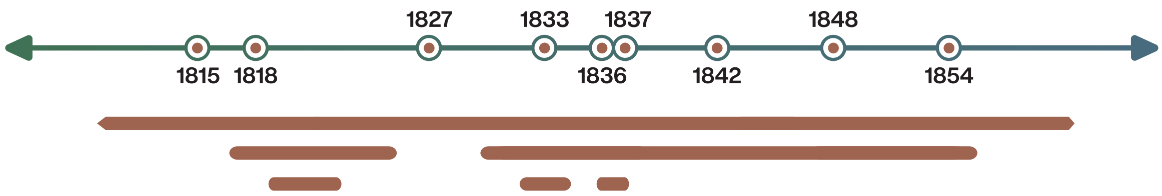
Source: *United States Census Bureau*

Name: Date: Period:

Early History of Detroit

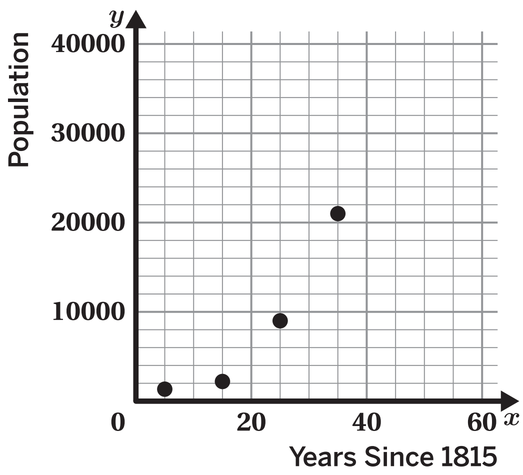
You'll use the digital activity for Problems 2–6.

2 Explore the timeline to learn about one of the largest U.S. cities from 1815 to 1855.



Discuss: How do you think the population of Detroit changed during that time?

3 Detroit became a U.S. city in 1815. This scatter plot shows the census data for Detroit's population from 1820 to 1850.



a Which type of function do you think better fits the data?

Linear

Exponential

b Use the digital activity to fit the function to the data.

c Explain how you decided which function to use.

Activity 1

Name: ______ Date: ______ Period: ______

Early History of Detroit (continued)

4 Write the function you created to model the population of Detroit x years since 1815.

$p(x) =$ ______

Explain what each part of the *model* represents.

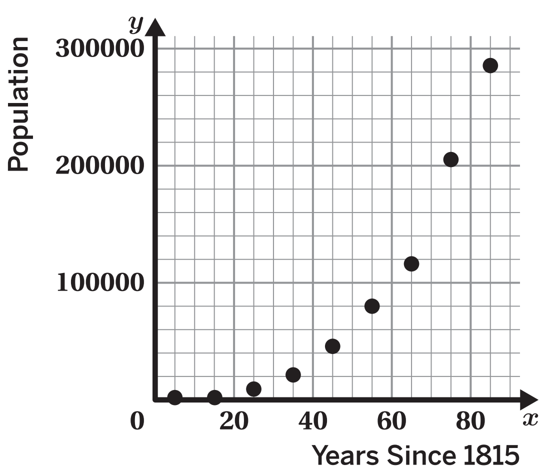
5 **a** Determine the value of $p(30)$.

b What does $p(30)$ represent in this situation?

6 The U.S. census data for Detroit's population from 1860 to 1900 has been added to the graph.

Discuss:

- What do you notice? What do you wonder?
- How does your model compare to the actual data?



Activity 2

Name: ____________ Date: ________ Period: ________

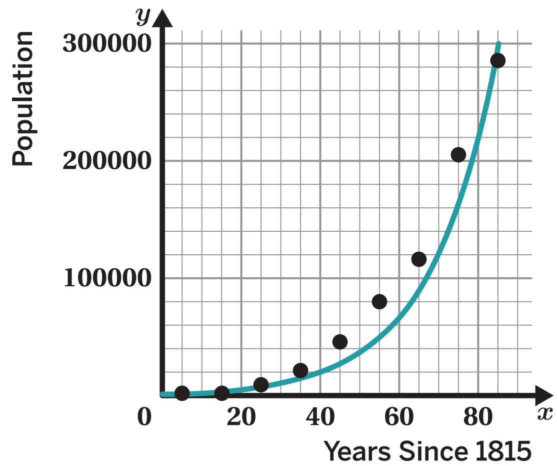
Predicting the Future

7 Zwena chose to revise her model to better fit the data from 1820 to 1900.

$$q(x) = 1904(1.0611)^x$$

According to Zwena's model, by what percent is Detroit's population increasing each year?

Explain your thinking.



8 Use the model to predict Detroit's population in 1910.

Years Since 1815, x	Population, $q(x)$
95	

9 Zwena wondered what the model would predict for Detroit's population in 2000, 185 years after 1815.

She used her model to calculate that the population of Detroit was about 111 million people in 2000.

Do you think this number is realistic? Explain your thinking.

Zwena

$q(x) = 1904(1.0611)^x$

$q(185) = 1904(1.0611)^{185}$

$q(185) = 110811576$

Activity 3

Name: .... Date: .... Period: ....

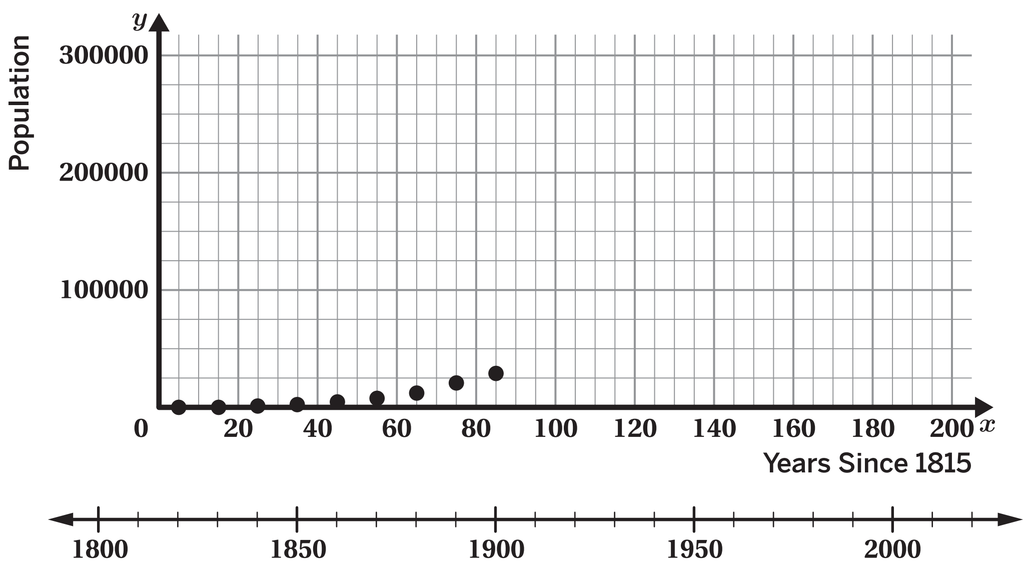
Based on Historical Events

You'll use the digital activity for Problems 10–13.

10 Examine the timeline in the digital activity to learn more about industry and migration throughout Detroit's history.

Discuss: How would you describe the change in population of Detroit in the years since 1900?

11 Sketch a prediction for the population of Detroit in the years after 1900.



12 Use the digital activity to reveal the population data after 1900.

Discuss: What do you notice? What do you wonder?

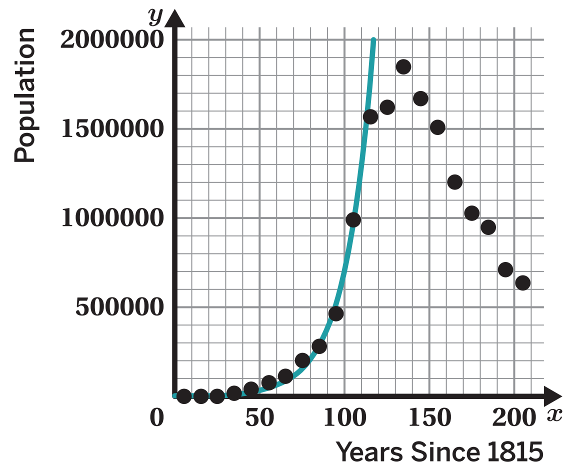
Activity 3

Name: .... Date: .... Period: ....

Based on Historical Events (continued)

13 Here is Zwena's model.

a. Use the digital activity to highlight a *domain* in which this model would be useful for making predictions.



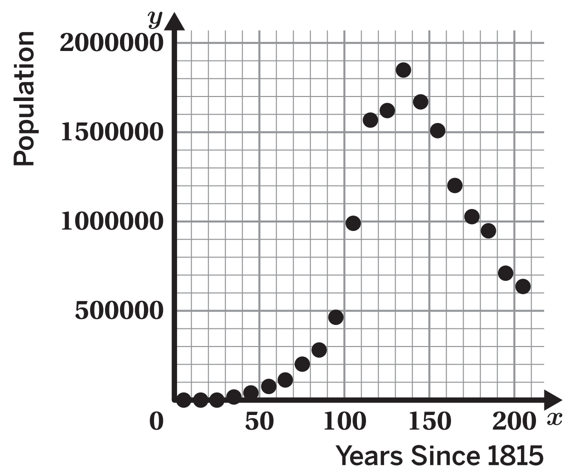
b. What might be some issues with using this model outside of the highlighted domain?

14 Synthesis

Select one of the questions to answer:

A. What is something you learned about using exponential functions to model population change over time?

B. What is a question you have about the population or history of Detroit?



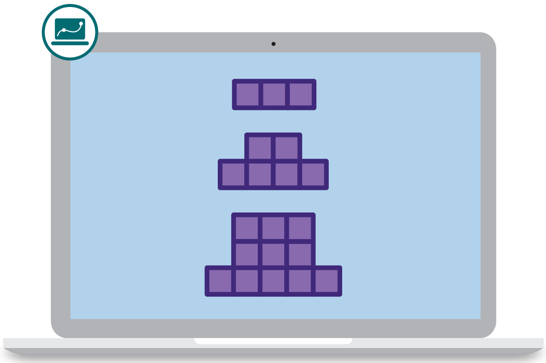
Things to Remember:

Unit 7
Lesson
1

Name: Date: Period:

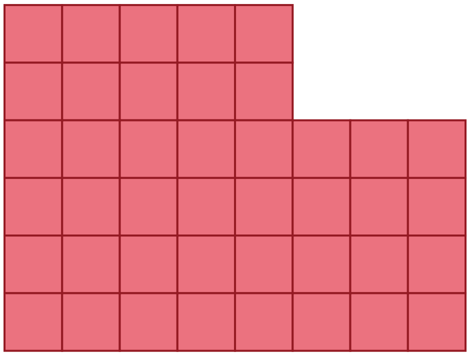
Revisiting Visual Patterns

Let's explore a new type of visual pattern.



Warm-Up

1 Here are different ways of counting the tiles.



Select one expression.

A. $5 + 5 + 8 + 8 + 8 + 8$

B. $4^2 + 4^2 + 10$

C. $6 \cdot 8 - 2 \cdot 3$

D. $5 \cdot 6 + 3 \cdot 4$

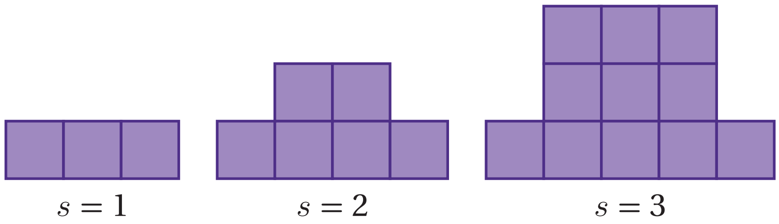
Show or explain how you see this expression in the diagram.

Activity 1

Name: ________ Date: ________ Period: ________

A New Type of Pattern

2 Here are the first three steps of a pattern.

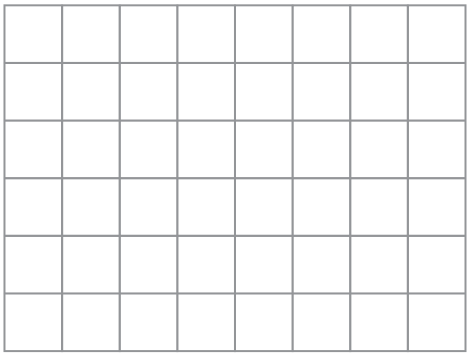


What about the pattern is changing? What is staying the same?

Things that are changing:

Things that are staying the same:

3 Draw the pattern when $s = 4$.



4 How many tiles will there be when $s = 10$?

Activity 1

Name: .... Date: .... Period: ....

A New Type of Pattern (continued)

5 Abdullah used a table to figure out how many tiles there will be when $s = 10$.

s	Number of Tiles
1	3
2	6
3	11
4	18
5	27
6	38
7	51
8	66
9	83
10	?

What type of relationship is there between s and the number of tiles?

A. Linear

B. Exponential

C. Something else

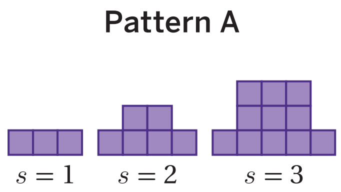
Explain your thinking.

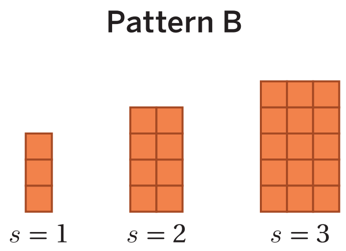
Activity 2

Name: ______ Date: ______ Period: ______

Comparing Patterns

6 Take a look at Pattern A and Pattern B.





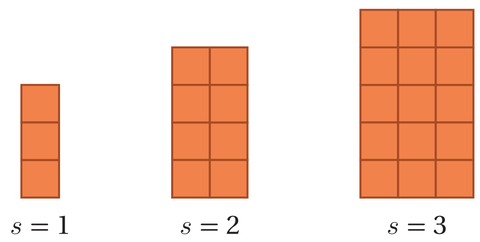
How are the two patterns alike? How are they different?

Alike:

Different:

7 Abdullah said: *I see a square plus two rows.*

Deja said: *I see a rectangle where the length is two more than the width.*



a Show how one student saw the pattern.

b **Discuss:** How might this student describe how to draw the image when $s = 4$?

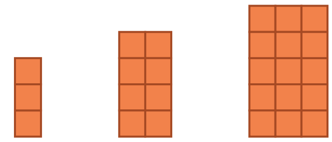
Activity 2

Name: .... Date: .... Period: ....

Comparing Patterns (continued)

8 Determine the number of tiles when $s = 4$.

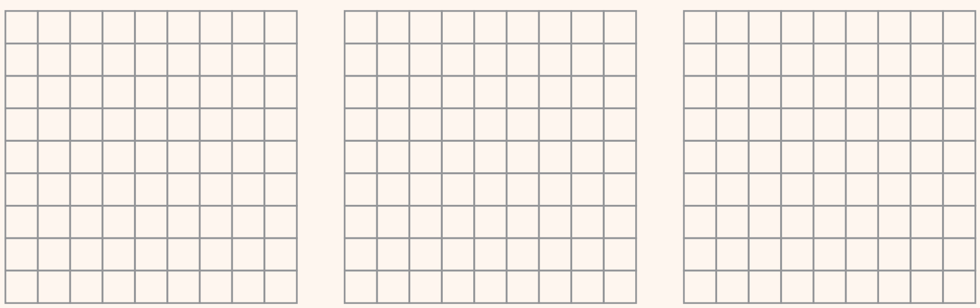
Determine the number of tiles when $s = 10$.



$s = 1$ $s = 2$ $s = 3$

Explore More

9 Create the first three steps of your own visual pattern.



$s = 1$ $s = 2$ $s = 3$

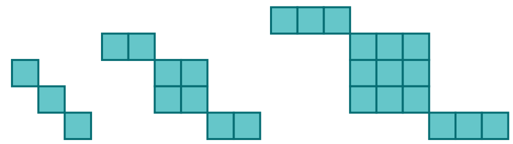
Discuss:

- What about your pattern is changing? What is staying the same?
- How many tiles will there be when $s = 4$? When $s = 10$?

10 Synthesis

What would you say to help a classmate who is trying to describe a pattern?

Use the example if it helps with your thinking.



$s = 1$ $s = 2$ $s = 3$

Things to Remember:

Unit 7

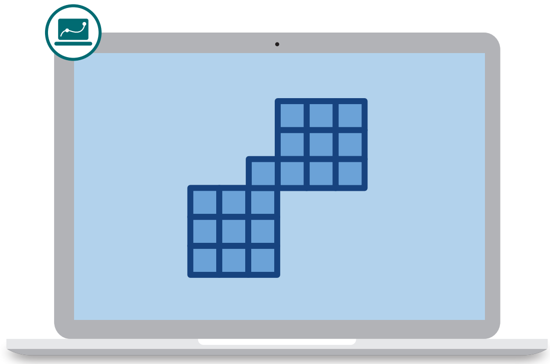
Lesson

2

Name: .... Date: .... Period: ....

Quadratic Visual Patterns

Let's describe a new type of pattern using expressions.



Warm-Up

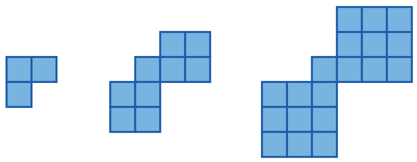
1 Here are Pattern A and Pattern B.

How are the two patterns alike? How are they different?

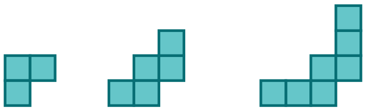
Alike:

Different:

Pattern A



Pattern B



Activity 1

Name: ______ Date: ______ Period: ______

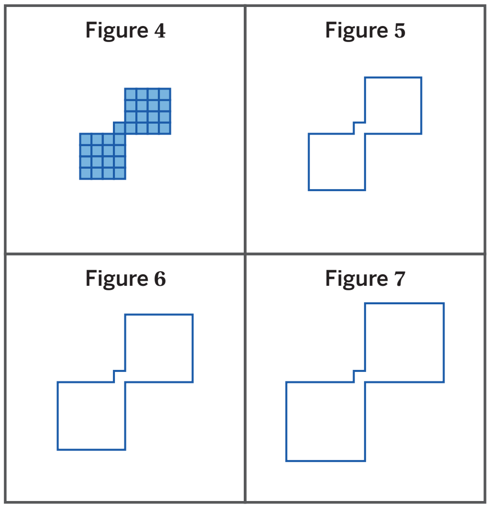
Figure n

2 Let's take a closer look at Pattern A.

Calculate the number of tiles for each figure.

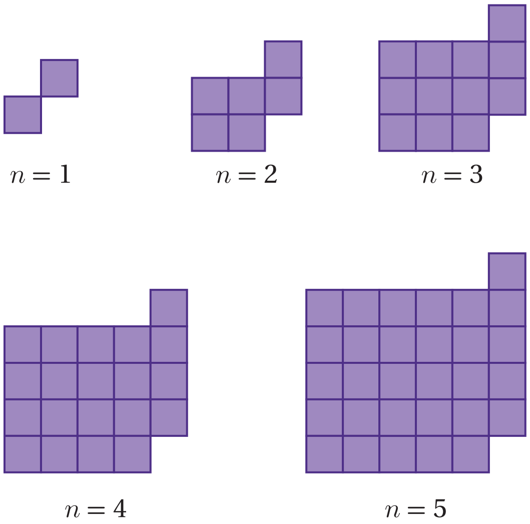
Use an expression if it helps with your thinking.

Figure	Number of Tiles
4	33
5	
6	
7	



3 A student created a table from the previous problem.

Write an expression for the number of tiles in Figure n.



Activity 2

Name: ________ Date: ________ Period: ________

Writing Expressions

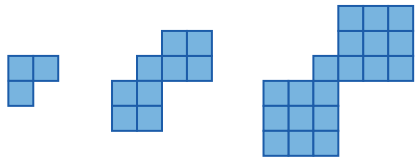
4 Take a look at Pattern A and Pattern C.

How are the two patterns alike? How are they different?

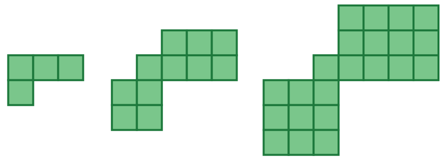
Alike:

Different:

Pattern A



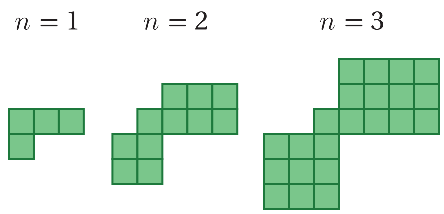
Pattern C



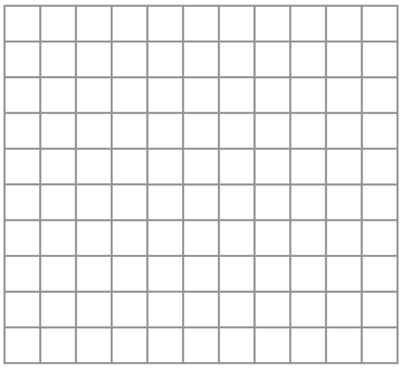
5 Here is Pattern C.

a Sketch the pattern for $n = 4$.

b Calculate the number of tiles when $n = 4$. Use an expression if it helps with your thinking.



$n = 4$

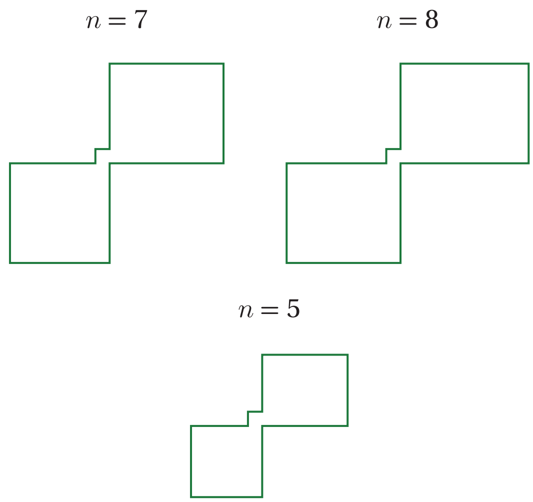


Name: .... Date: .... Period: ....

Writing Expressions (continued)

6 Here are three more figures of Pattern C.

Write an expression in terms of n to evaluate them all at once.



7 Let's look at expressions that two students wrote for Pattern C.

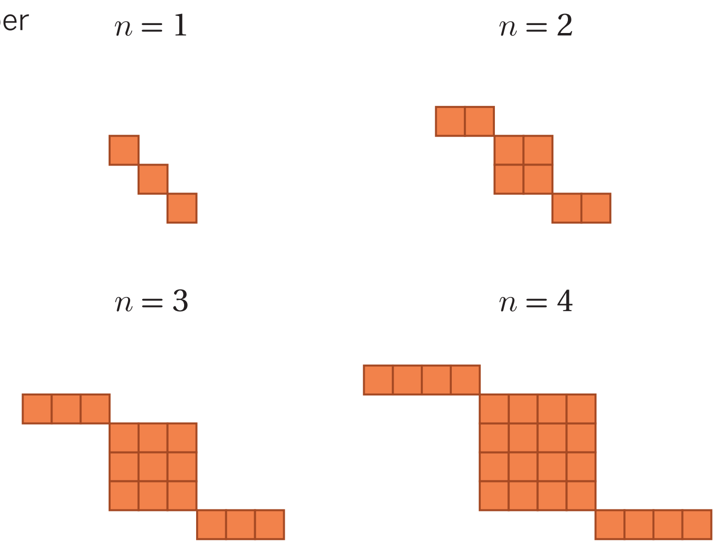
Discuss: Where do you see each part of their expression in their sketch?

Activity 3

Name: ______ Date: ______ Period: ______

Quadratic Relationships

8 Write an expression for the number of tiles in terms of n.



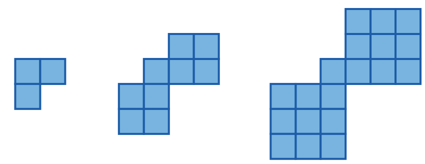
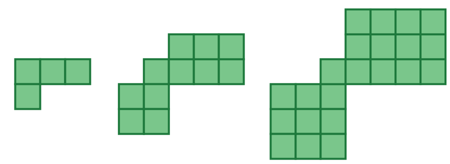
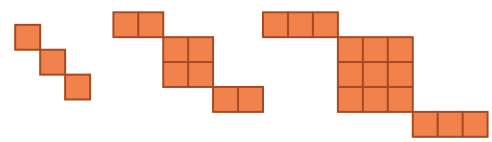
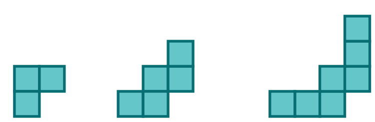
9 Select *all* the expressions that represent the number of tiles in this pattern.

- ☐ **A.** $5n$
- ☐ **B.** $n^2 + 2n$
- ☐ **C.** $(n \bullet n) + (n + n)$
- ☐ **D.** $3n^2$

Name: ________ Date: ________ Period: ________

Quadratic Relationships (continued)

10 Here are some of the relationships we've explored in this lesson.

	Expressions	Patterns
Quadratic	$2n^2 + 1$	
	$2n^2 + n + 1$	
	$n^2 + 2n$	
Linear	$2n + 1$	

What do you think **quadratic relationships** all have in common?

11 Synthesis

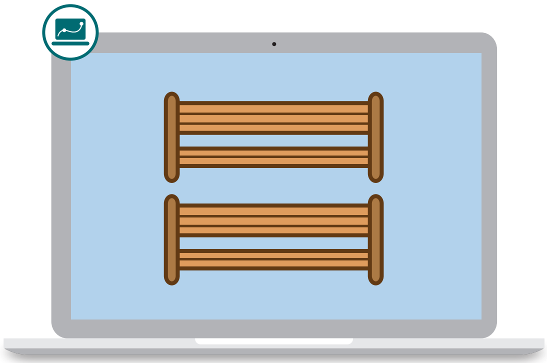
How does creating a table with expressions help someone trying to write an expression to represent a quadratic relationship

Use this table if it helps with your thinking.

Figure	Number of Tiles
4	33
5	$5^2 + 5^2 + 1$
6	$6^2 + 6^2 + 1$
7	$7^2 + 7^2 + 1$
n	?

Things to Remember:

Name: .... Date: .... Period: ....



On the Fence

Let's use the context of building fences to explore symmetry in quadratic relationships.

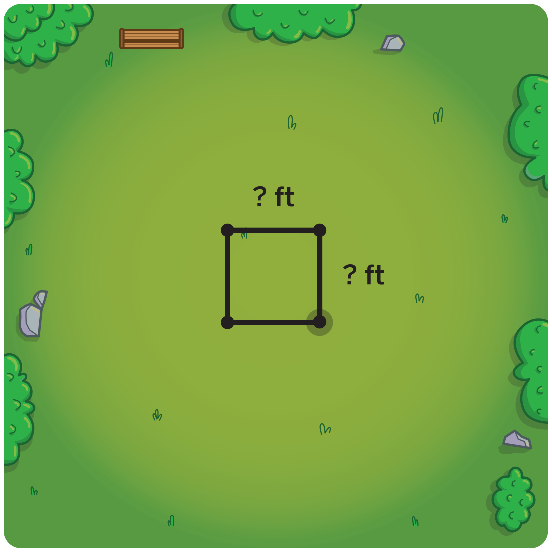
Warm-Up

1 Farmer Farah is building a fence for her sheep.

Each panel of fencing is 5 feet long and she has 100 feet of fencing total.

Build three different fences in the pasture that each use *exactly* 100 feet of fencing.

Width (ft)	Length (ft)



Activity 1

Name: .... Date: .... Period: ....

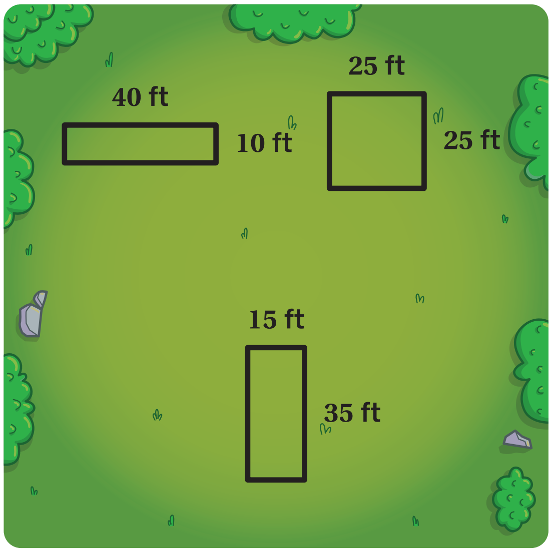
Farmer Farah's Fencing

2 Let's look at three fences.

How are these fences alike? How are they different?

Alike:

Different:



3 Farah noticed that for each fence, the perimeter stays the same but the area changes.

Here are Farah's fences.

Calculate the areas of the three fences.

Width (ft)	Length (ft)	Area (sq. ft)
40	10	
15	35	
5	45	

Name: .... Date: .... Period: ....

Farmer Farah's Fencing (continued)

4 The table represents all the possible fences Farmer Farah can build in the pasture.

a. Complete the missing values in the table.

b. **Discuss:** What do you notice? What do you wonder?

Width (ft)	Length (ft)	Area (sq. ft)
5	45	225
10	40	400
15	35	525
20	30	
	25	625
		600
35	15	
	10	
	5	

5 Here is a graph of the areas of all the possible fences.

What type of relationship is represented?

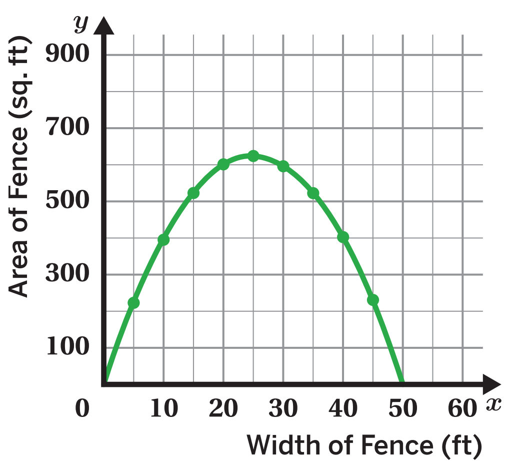
A. Linear

B. Exponential

C. Quadratic

D. Something else

Explain your thinking.



6 The graph of a quadratic function is called a **parabola**.

Parabolas have a **line of symmetry**. If you fold a parabola along this line, you get two identical halves.

a. Draw the line of symmetry on the graph of possible fences.

b. **Discuss:** What does this line mean in the context of the sheep fence?

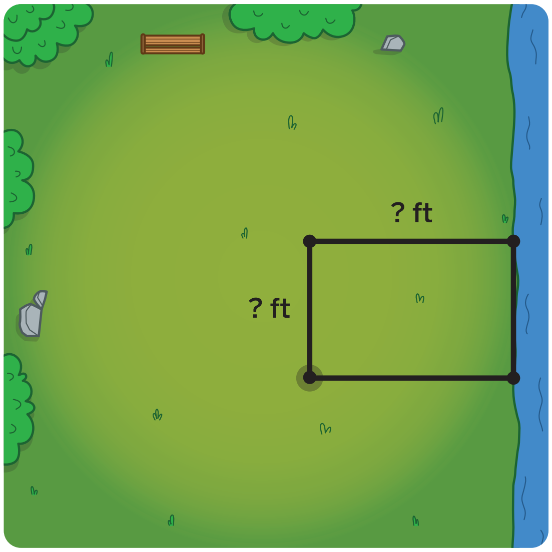
Name: ____________ Date: ____________ Period: ____________

By the Stream

7 Farmer Farah's sheep don't like to swim. If she builds her fence by the stream, it will only need three sides.

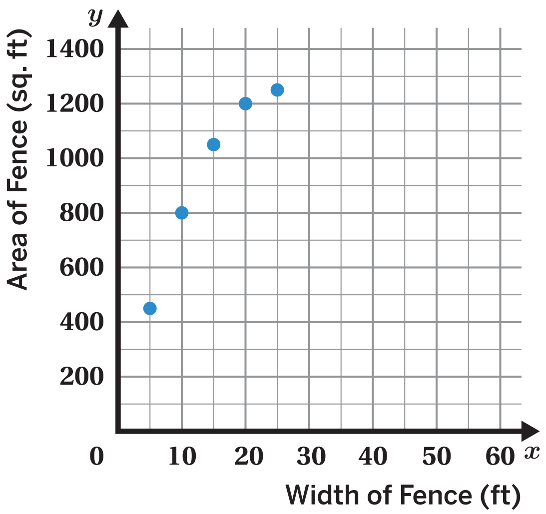
Using 100 feet of fencing, build three possible fences.

Width (ft)	Length (ft)



8 Here are a few of the possible fences Farmer Farah can build by the stream.

Width (ft)	Area (sq. ft)
5	450
10	800
15	1050
20	1200
25	1250



Is this relationship quadratic? Circle one.

Yes   No   I'm not sure

Explain your thinking.

Name: .... Date: .... Period: ....

By the Stream (continued)

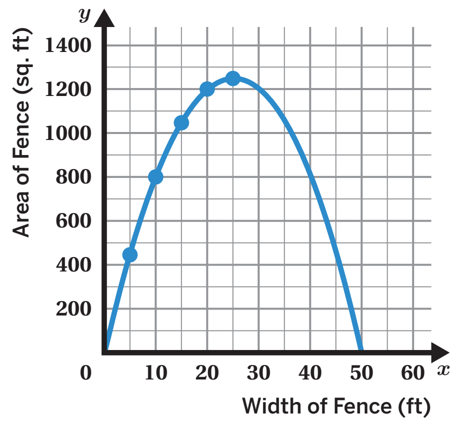
9 What are the areas of all the possible fences this parabola could represent?

Complete the table to show all the possible areas of fences Farmer Farah could make by the stream.

Width (ft)	Area (sq. ft)
5	450
10	800
15	1050
20	1200
25	1250
30	
35	
40	
45	

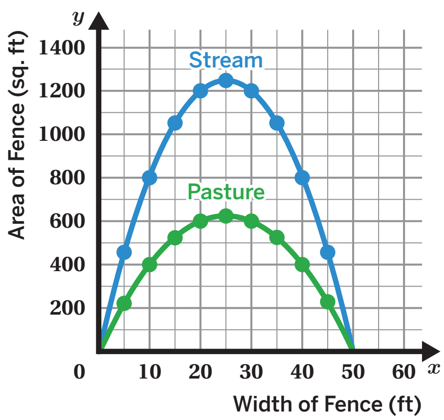
10 Here is a graph of a parabola that includes all the possible areas of fences along the stream.

Write the equation for the line of symmetry for this parabola.



11 Here are the graphs of parabolas that include all the possible fences Farmer Farah could build in the pasture and by the stream.

How are these relationships alike? How are they different?



Alike:

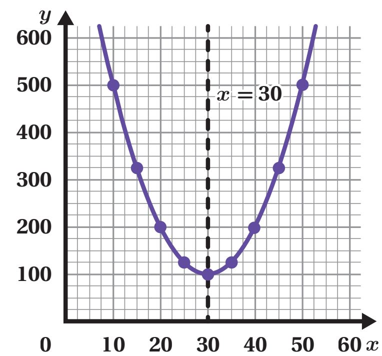
Different:

12 Synthesis

The table and the graph represent the same relationship.

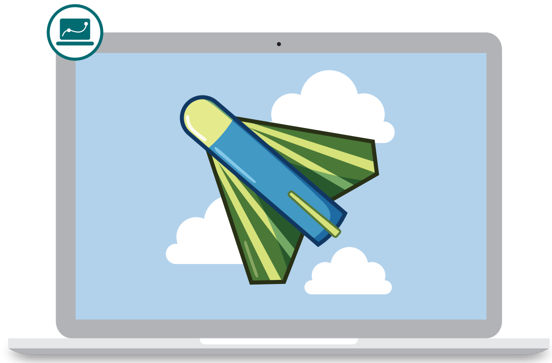
Describe two ways you know that the relationship in the table and graph is quadratic.

x	y
15	325
20	200
25	125
30	100
35	125
40	200
45	325



Things to Remember:

Name: .... Date: .... Period: ....



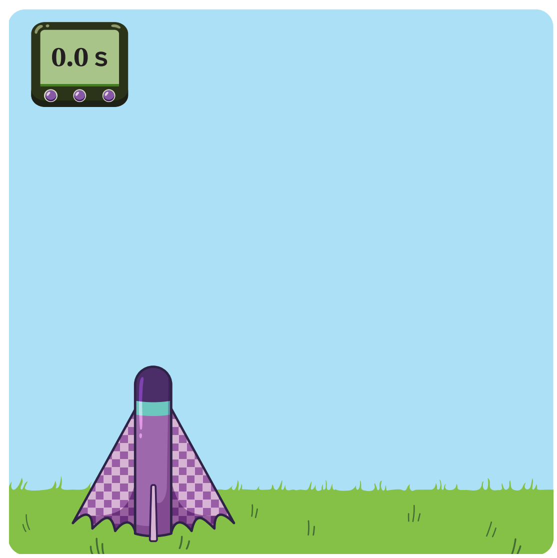
Stomp Rockets

Let's use tables and graphs to make predictions about quadratic relationships in the context of launching stomp rockets.

Warm-Up

1 A stomp rocket is a toy rocket with no engine that is launched by a quick burst of compressed air.

- **a** Let's watch a stomp rocket launch.
- **b** **Discuss:** What do you notice? What do you wonder?



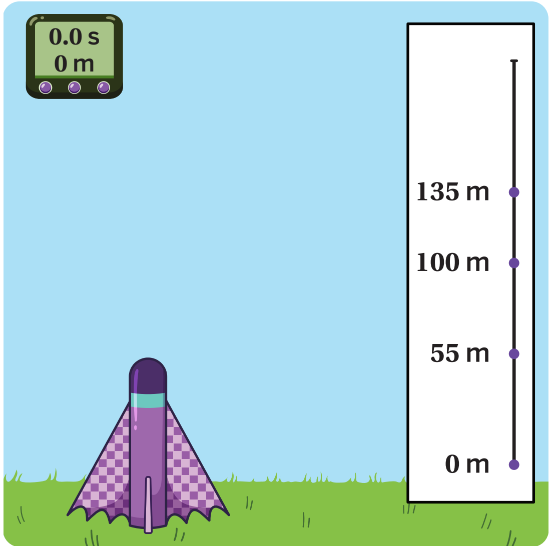
Activity
1

Name: .... Date: .... Period: ....

Predicting With Tables

2 Make a prediction: How high do you think the rocket will be after 4 and 5 seconds?

Time (sec)	Height (m)
0	0
1	55
2	100
3	135
4	
5	



3 Let's look at how Maia made her prediction.

Describe what she did to find the heights at 4 and 5 seconds.

Name: Date: Period:

Predicting With Tables (continued)

4 Here is a new rocket. The table shows its height at various times.

How high will this rocket go?

Use the table if it helps with your thinking.

Time (sec)	Height (m)
0	0
1	45
2	80

5 How many seconds will it take for the rocket to touch the ground?

Use the table if it helps with your thinking.

Activity 2

Name: ________ Date: ________ Period: ________

Predicting With Tables and Graphs

6 A new stomp rocket is launched from the top of a building.

About how many seconds will it take for the rocket to touch the ground?

Use the table if it helps with your thinking.

A. 6 seconds

B. 8 seconds

C. Between 6 and 7 seconds

D. Between 7 and 8 seconds

Explain your thinking.

Time (sec)	Height (m)
0	20
1	45
2	60

7 Let's look at Ivan's graph.

a Why do you think Ivan drew a parabola?

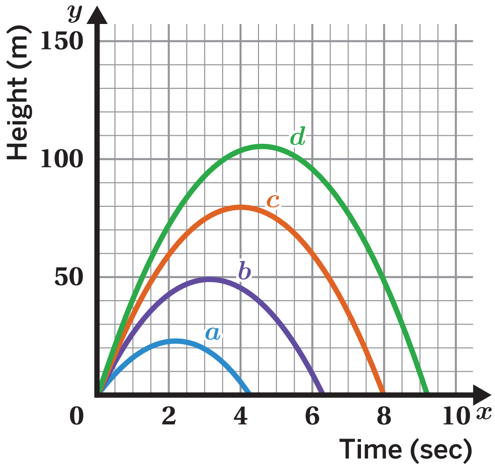
b How do you think he used his graph to determine when the rocket landed?

Name: ____________ Date: ____________ Period: ____________

Predicting With Tables and Graphs (continued)

8 Let's watch each of the stomp rockets launch.

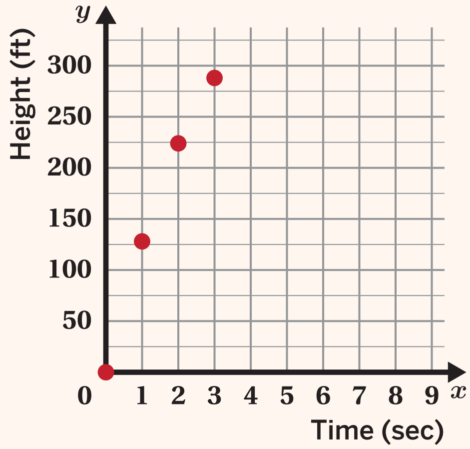
Discuss: What is the same and what is different about each rocket launch?



Explore More

9 The table and graph show the height of a stomp rocket at various times. How many seconds will it take for this rocket to reach its maximum height?

Time (sec)	Height (m)
0	0
1	128
2	224
3	288



Explain how you know.

10 Synthesis

The table and graph show the height of a stomp rocket at various times.

Write one question about the rocket that you can answer using the table and one you can answer using the graph.

Time (sec)	Height (m)
0	0
1	52
2	94
3	126

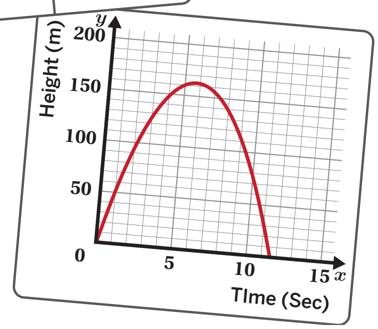
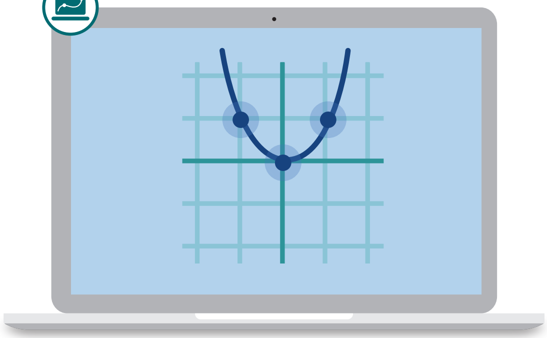


Table:

Graph:

Things to Remember:

Name: ______________ Date: ______ Period: ______



Plenty of Parabolas

Let's describe the key features of a parabola.

Warm-Up

1 Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet with parabolas for four rounds.

For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a parabola from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating parabolas until you're ready to guess which parabola the Picker chose.

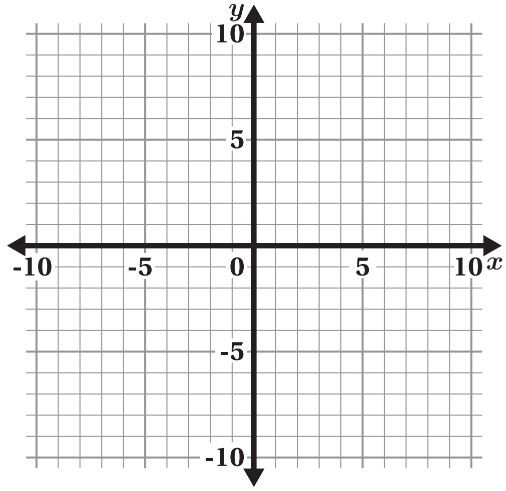
Record helpful questions from each round in the space below.

Activity 1

Name: .... Date: .... Period: ....

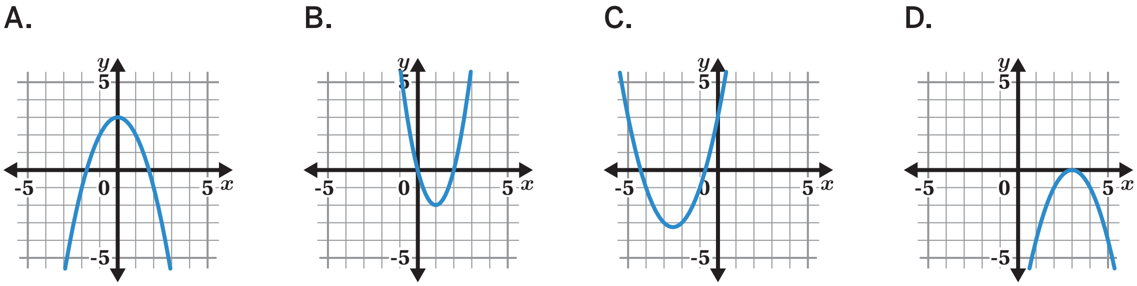
Describing Parabolas

2 Now it's your turn to graph a parabola.



3 Ama says her parabola turns around at 3.

Select a parabola that could be Ama's.

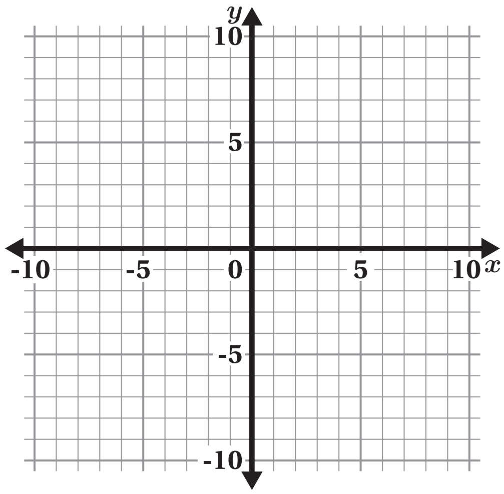


Explain your thinking.

4 The turning point of a parabola is called the **vertex**. This is also its *maximum* or *minimum*.

Draw a parabola with a vertex at (-5, 1).

Try to make a parabola you think none of your classmates will make.

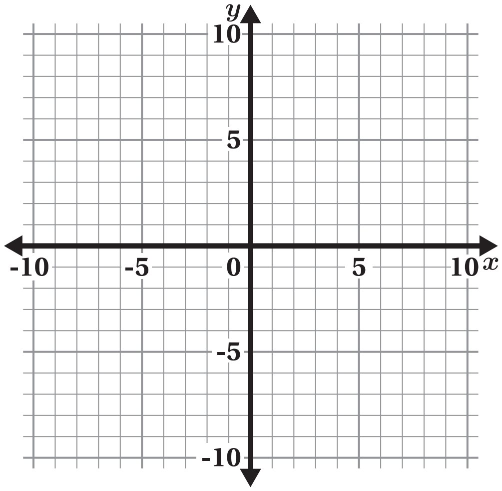


Name: .... Date: .... Period: ....

Describing Parabolas (continued)

5 Katie says her parabola has an x-intercept at -2 and looks like a smile.

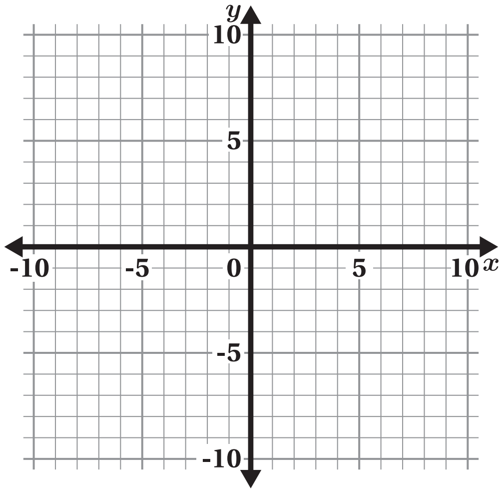
Draw what her parabola could look like.



6 Parabolas that look like a smile are **concave up**.

Parabolas that look like a frown are **concave down**.

Draw a concave down parabola with vertex $(4, -2)$.

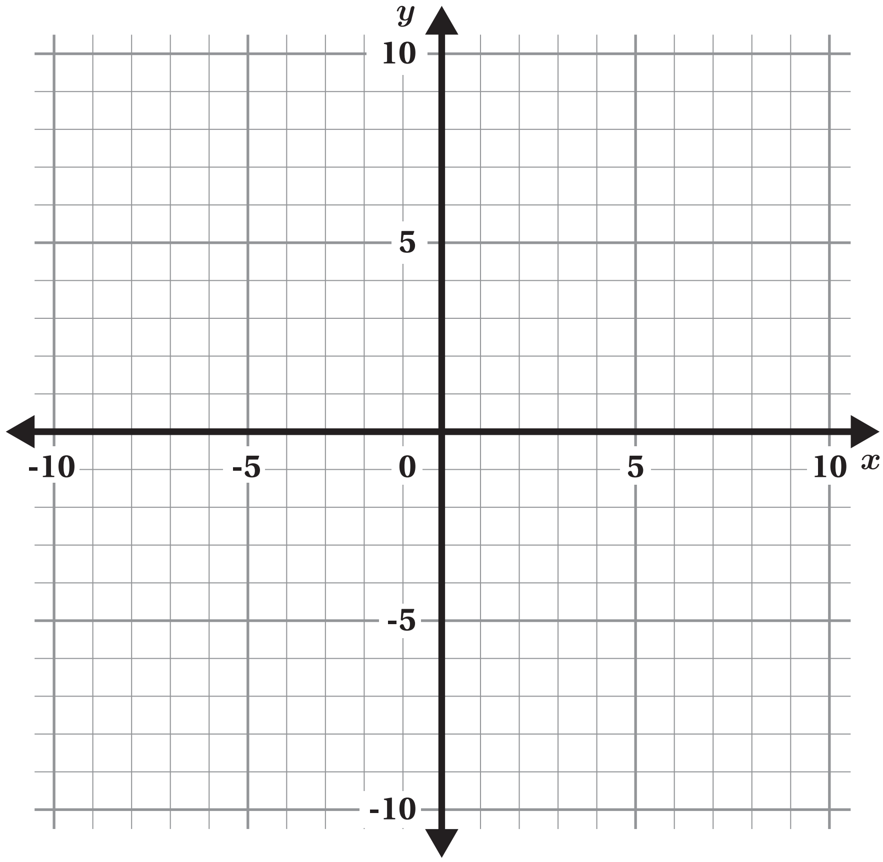


Name: ______ Date: ______ Period: ______

Parabola Art

7 It's time to make some parabola art!

Create some art by drawing multiple parabolas.



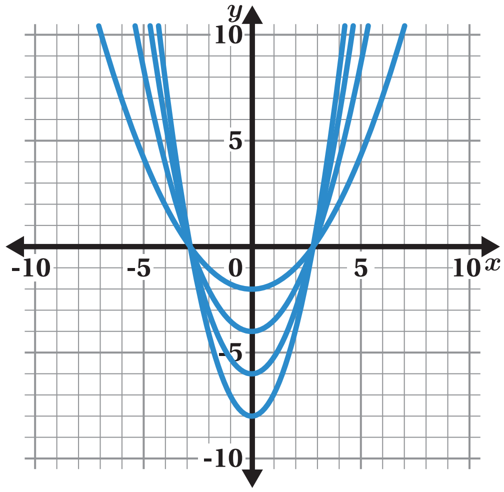
Name: ________ Date: ________ Period: ________

Parabola Art (continued)

8 Manuel made this design using parabolas.

How are these parabolas alike?

Describe as many similarities as you can.



Explore More

9 Try to make concave up and concave down parabolas with different numbers of intercepts.

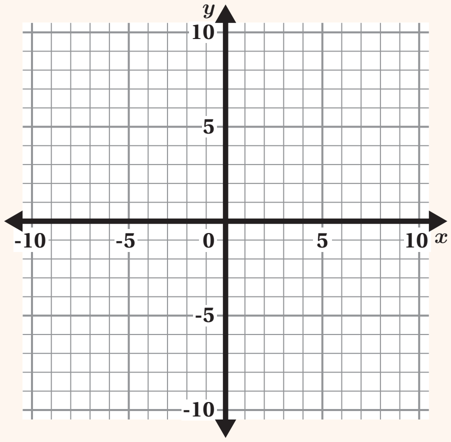
Then select the number of x-intercepts and y-intercepts that are possible.

Number of x-intercepts:

☐ 0 ☐ 1 ☐ 2

Number of y-intercepts:

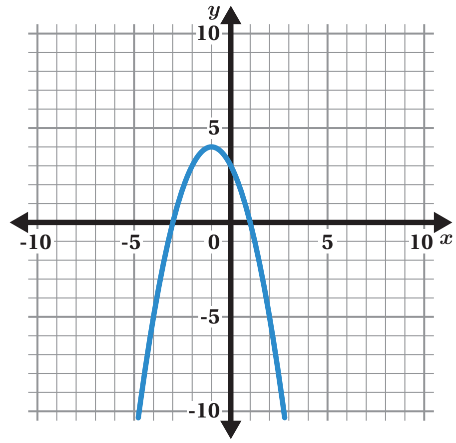
☐ 0 ☐ 1 ☐ 2



10 Synthesis

Describe the graph using vocabulary from this lesson.

Draw on the graph if it helps with your thinking.

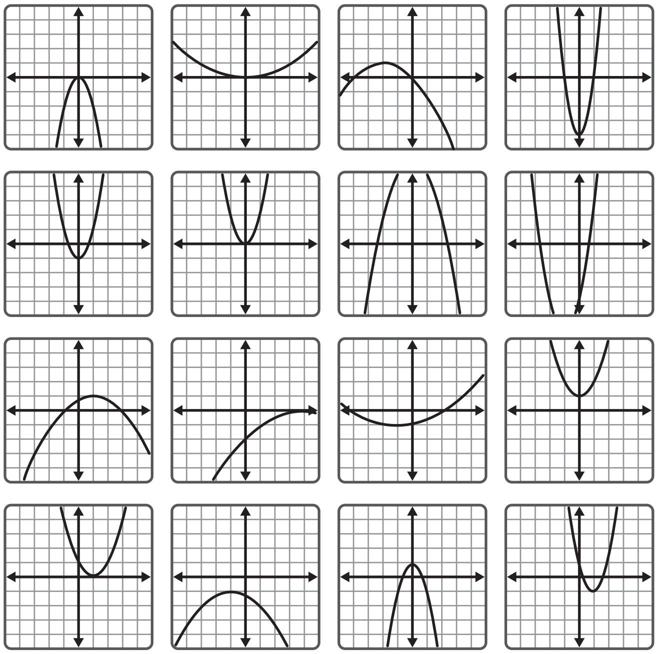


Things to Remember:

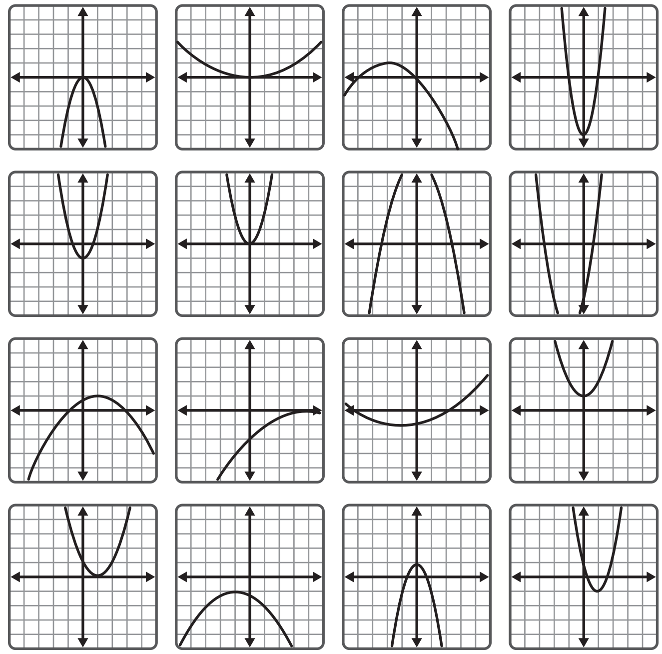
Name: .... Date: .... Period: ....

Polygraph Set A

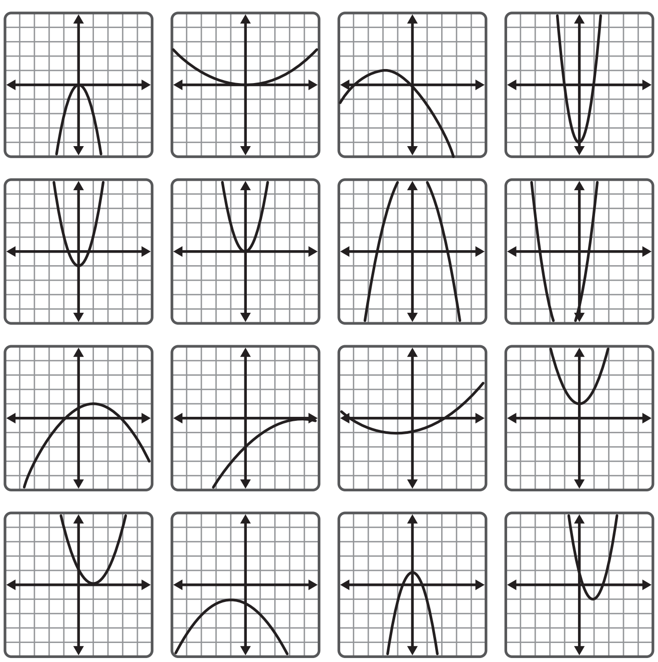
Round 1



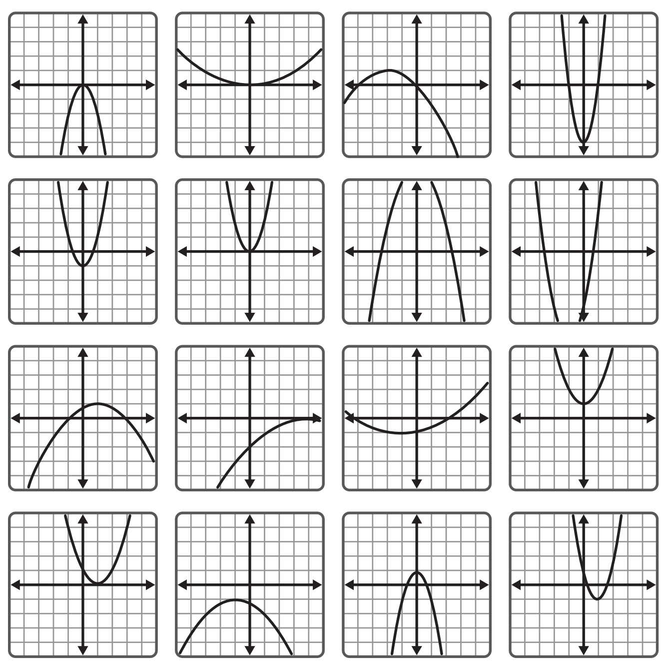
Round 2



Round 3



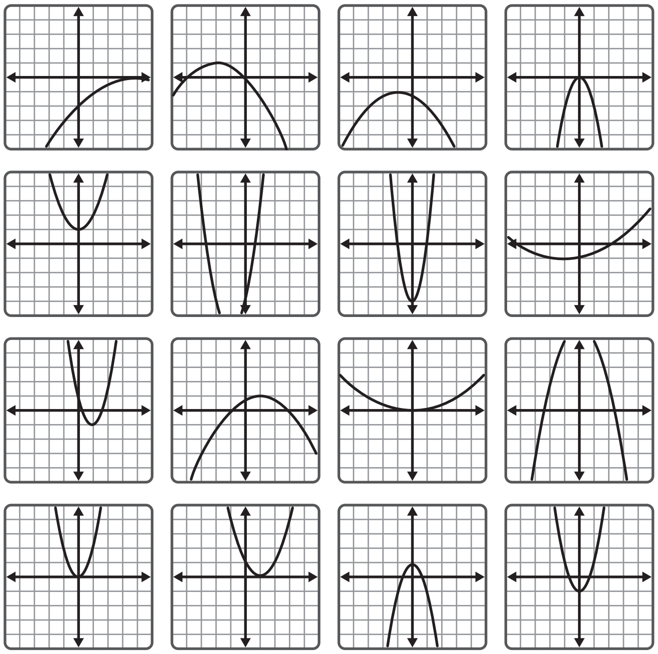
Round 4



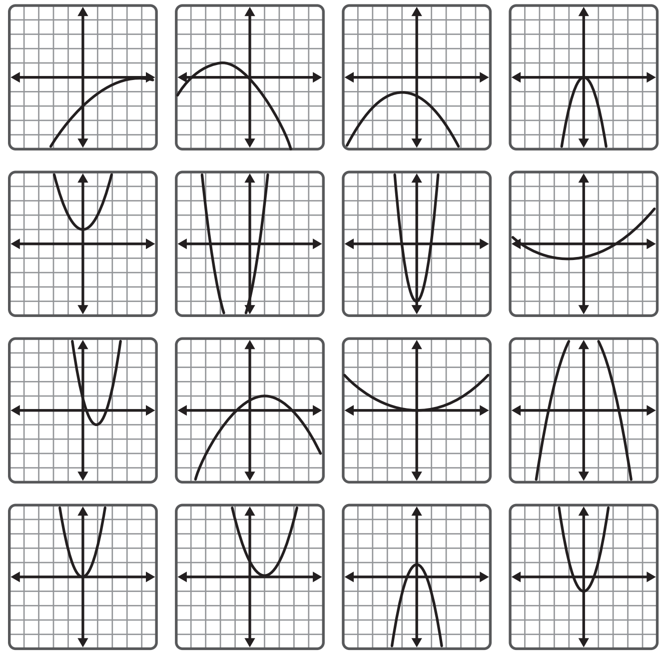
Name: .... Date: .... Period: ....

Polygraph Set B

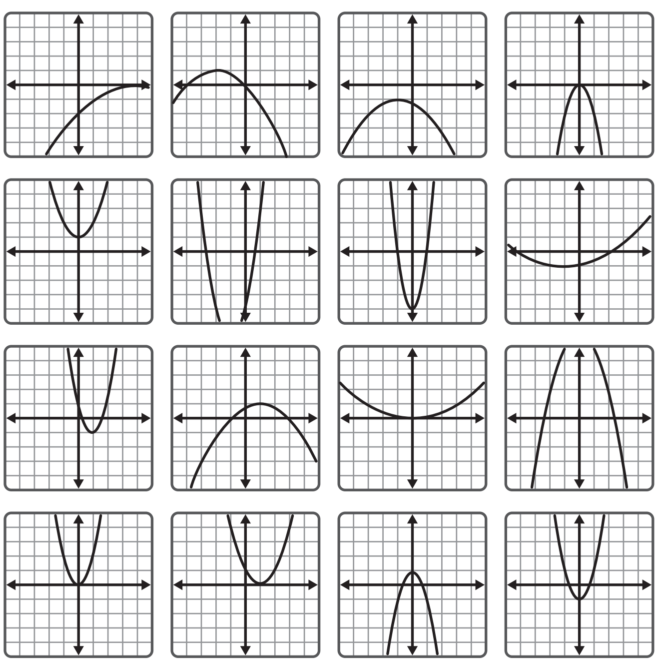
Round 1



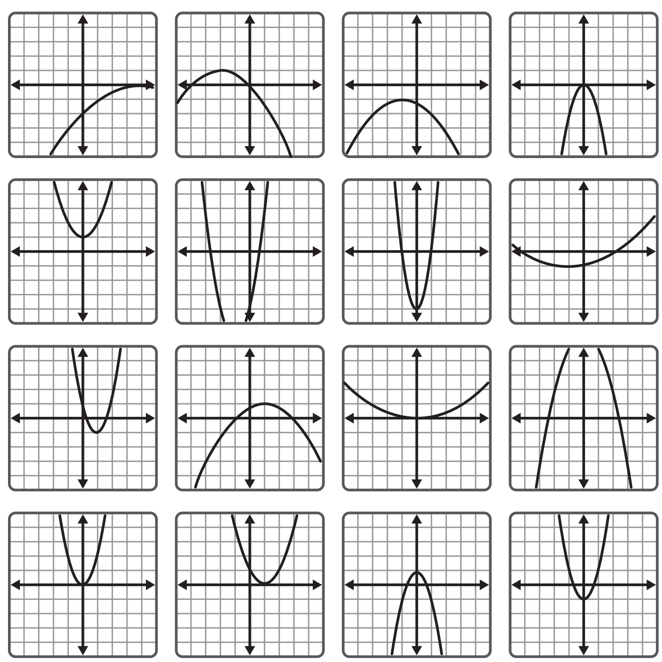
Round 2



Round 3



Round 4



Unit 7

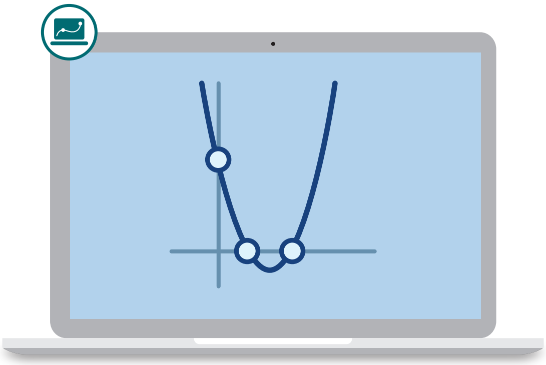
Lesson

10

Name: ______ Date: ______ Period: ______

Interesting Intercepts

Let's make connections between the intercepts of a parabola and the structure of its equation.



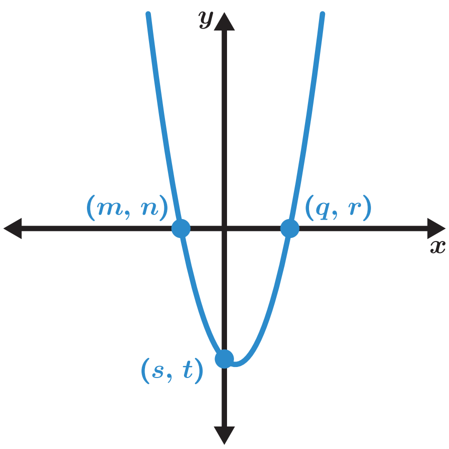
Warm-Up

1 Here is the graph of a function.

Select *all* the values that are equal to 0.

- ☐ m
- ☐ n
- ☐ q
- ☐ r
- ☐ s
- ☐ t

Explain your thinking.



Activity
1

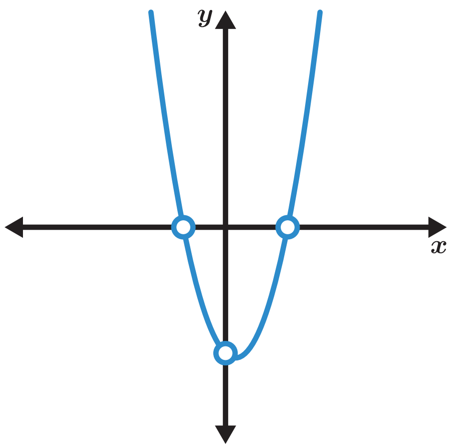
Name: ______ Date: ______ Period: ______

Intercepts in Factored Form

2 Here is the function from the Warm-Up.

Its equation is $f(x) = (x + 2)(x - 3)$.

Graph the x- and y-*intercepts* in the digital activity.



3 Look at the x- and y-intercepts of the previous function: $f(x) = (x + 2)(x - 3)$.

What do you notice about the intercepts?

x-intercepts:

y-intercepts:

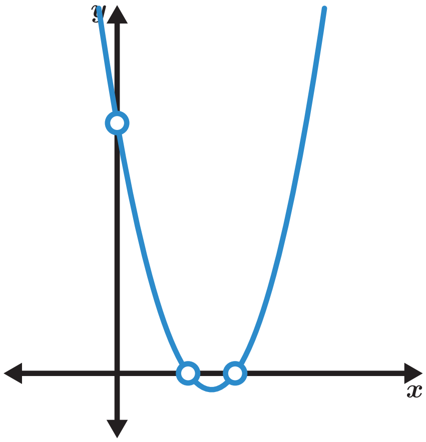
Activity 2

Name: ______ Date: ______ Period: ______

Determine the Intercepts

4 Here is a new function: $g(x) = (2x - 6)(x - 5)$.

Graph the x- and y-*intercepts* in the digital activity.



5 Let's look at Raven's work from the previous challenge.

Explain why Raven's thinking is incorrect.

Name: ______ Date: ______ Period: ______

Determine the Intercepts (continued)

6 Here is a new function:

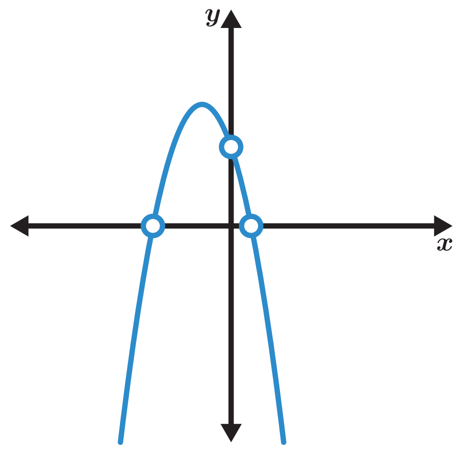
$h(x) = (\text{-}x + 0.5)(4x + 8)$

What are the intercepts of h?

x-intercept: ______

x-intercept: ______

y-intercept: ______



7 Two students were asked to determine the x-intercepts of $p(x) = 2x(x + 9)$.

Yolanda says the x-intercepts are at -2 and -9.
Julian says the x-intercepts are at 0 and -9.

Whose thinking is correct? Circle one.

Julian's Yolanda's Both Neither

Explain your thinking.

Activity 3

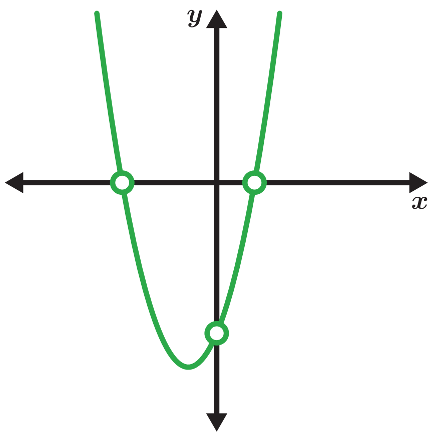
Name: .... Date: .... Period: ....

Intercepts in Standard Form

8 Here is a new function:

$w(x) = x^2 + 3x - 10$

Graph the x- and y-*intercepts* in the digital activity.



9 Look at the x- and y-intercepts of the previous function.

Which intercepts were easier for you to determine? Circle one.

x-intercepts y-intercept

Explain your thinking.

10 Match each equation with its y-intercept. One equation will have no match.

$a(x) = x^2 - 3x + 5$	$b(x) = x^2 + 5x - 3$	$c(x) = x^2 - 5x + 3$
$d(x) = \text{-}3x^2 + 5$	$e(x) = 5x^2 - 3$	$f(x) = 3x^2 + 5 + x^2$

(0, 5)	(0, -3)

11 Synthesis

The same function is written in factored and standard form. What does each form tell you about the graph of $f(x)$?

Factored Form

$$f(x) = (2x - 1)(x + 3)$$

Standard Form

$$f(x) = 2x^2 + 5x - 3$$

Factored form:

Standard form:

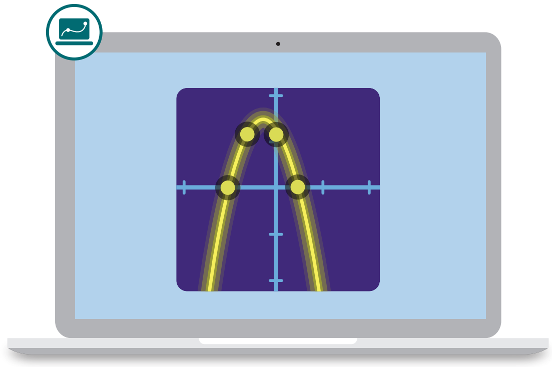
Things to Remember:

Unit 7
Lesson
11

Name: Date: Period:

Parabola Zapper

Let's graph quadratic equations in factored form.

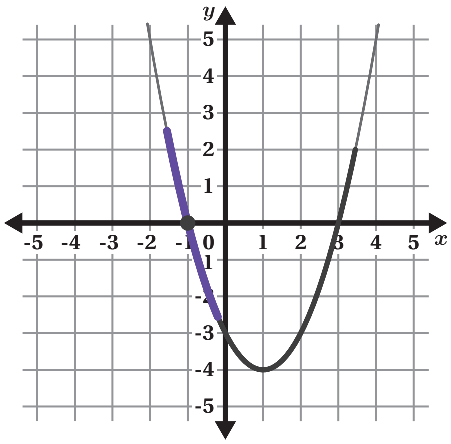


Warm-Up

1 Use the digital activity to light up the parabola by zapping a few points on it.

Its equation is $f(x) = (x - 3)(x + 1)$.

Zap	Coordinate
Zap #1	(-1, 0)
Zap #2	
Zap #3	



Activity 1

Name: ______ Date: ______ Period: ______

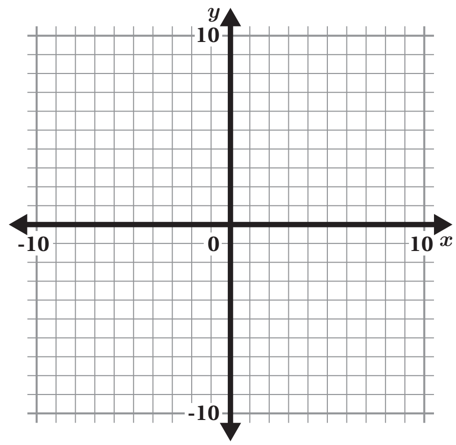
Graphing by Zapping

You'll use the digital activity for Problems 2–5.

2 There is a hidden parabola on a graph.

Its equation is $h(x) = (x + 4)(\text{-}x + 2)$.

Light up the entire parabola by zapping points on it.



3 Maria and Laila compared their strategies on the previous challenge.

Maria said: *I used four zaps! Take a look at my graph*.

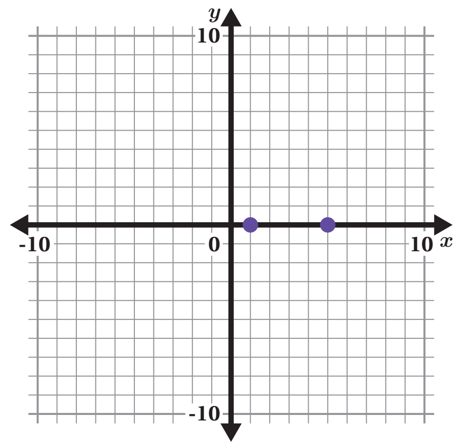
Laila said: *We used some of the same points, but I only used three zaps*.

What three points do you think Laila zapped?

4 Here are the x-intercepts of a new parabola.

a Draw a point to show where the *vertex* could be.

b Let's look at some possible vertices. What do you notice and wonder about the vertex?



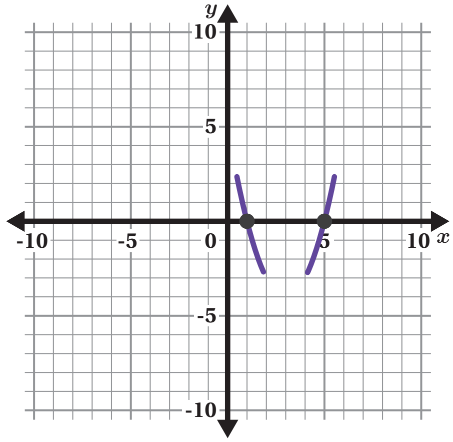
Name: .... Date: .... Period: ....

Graphing by Zapping (continued)

5 Here is the parabola from the previous problem.

Its equation is $g(x) = (x - 1)(x - 5)$.

Complete this parabola by zapping its vertex.



6 Here is Pilar's work to determine the vertex of $g(x)$.

She completed part of the table before getting stuck.

Pilar

$g(x) = (x - 1)(x - 5)$

x	x - 1	x - 5	g(x)
1	0	-4	0
5	4	0	0
3			

a What does the 3 in Pilar's table represent?

b How can Pilar find the y-coordinate of the vertex?

Name: .... Date: .... Period: ....

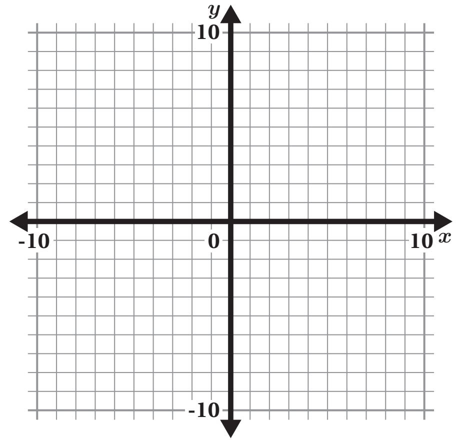
Repeated Challenges

You'll use the digital activity for problems 7–9.

7 There is a hidden parabola on the graph.

Its equation is $p(x) = (2x + 4)(x + 6)$.

Light up the entire parabola by zapping points on it. One way to light up the entire parabola is to zap both x-intercepts and the vertex.



8 Describe the strategy you used to light up $p(x) = (2x + 4)(x + 6)$.

9 For each challenge, light up the entire parabola by zapping points on it.

a $f(x) = (x - 3)(x + 1)$

b $f(x) = (\text{-}x + 4)(x + 2)$

c $f(x) = (1 - x)(x + 3)$

d $f(x) = (x - 2.5)(x - 6.5)$

e $f(x) = (x + 1)(x - 6)$

f $f(x) = (2x + 6)(x - 1)$

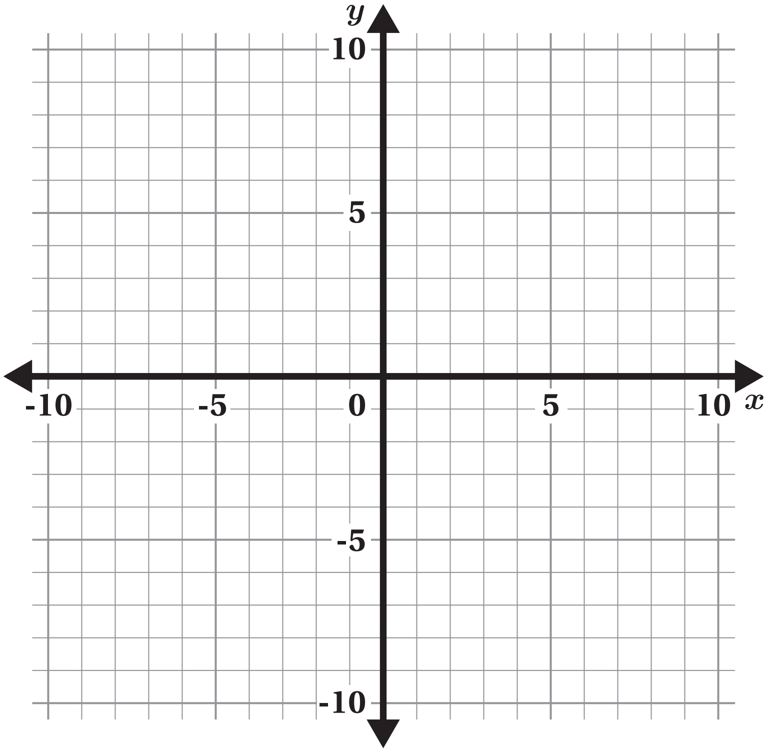
Activity
3

Name: ______ Date: ______ Period: ______

Graphing by Hand

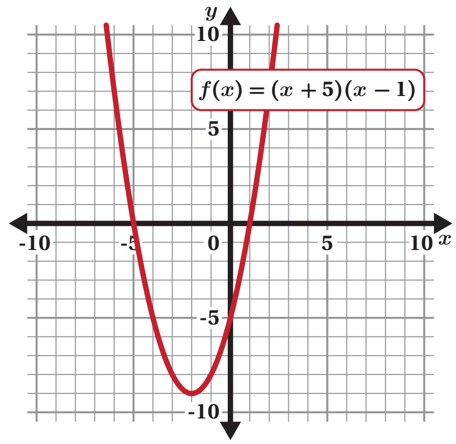
10 Without graphing, describe as much as you can about the graph of $w(x) = (x + 2)(x - 3)$.

11 Draw the graph of $w(x) = (x + 2)(x - 3)$.



12 Synthesis

Identifying key features is a helpful strategy for graphing parabolas. Describe how to identify the key features of a parabola when given an equation in factored form. Use this example if it helps with your thinking.



Things to Remember:

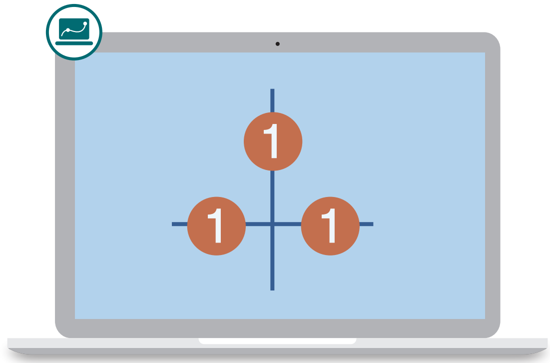
Unit 7

Lesson

12

Name: ________ Date: ________ Period: ________

Break Through: Parabolas



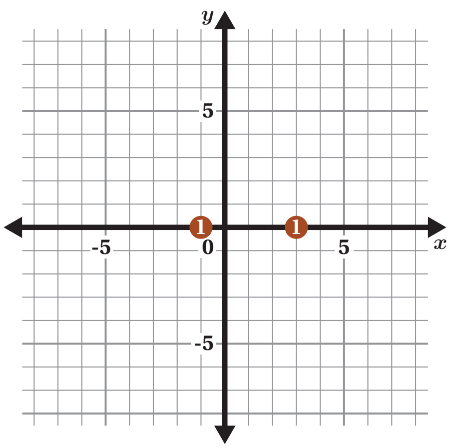
Let's write equations of parabolas in factored form.

Warm-Up

You'll use the digital activity for the Warm-Up.

1 Welcome to Break Through: Parabolas!

Your goal is to write equations of parabolas that will break the targets.



a Press "Try It" to see what happens.

b Change the equation to break all the targets.
Original equation:

$y = (x + 1)(x - 2)$

Your equation:

Name: .... Date: .... Period: ....

Building Quadratic Functions

You'll use the digital activity for Problems 2–6.

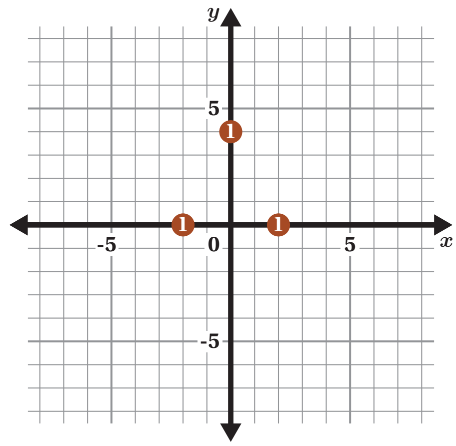
2 Here's another challenge.

Change the equation to break *all* the targets.

Original equation:

$y = (x + 2)(x - 2)$

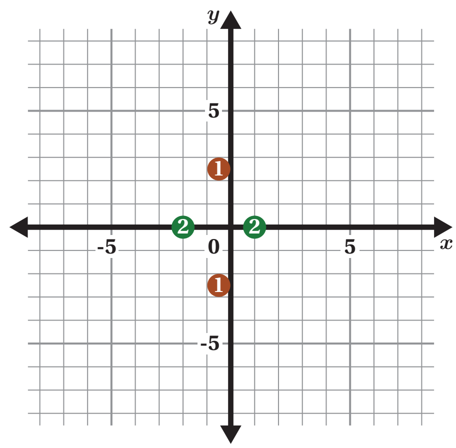
Your equation:



3 Some of the targets will require multiple parabolas.

Write another equation to break the remaining targets.

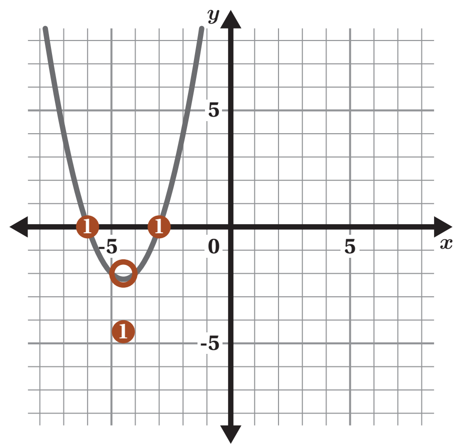
Equation
$y = (x - 1)(x + 2)$



4 Carlos started this challenge.

Write another equation to finish it.

Equation
$y = (x + 6)(x + 3)$

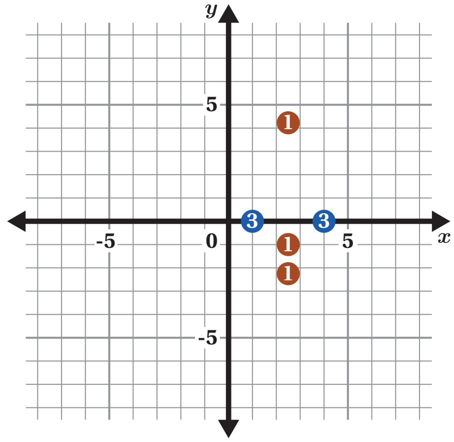


Name: .... Date: .... Period: ....

Building Quadratic Functions (continued)

5 Break the targets using three equations.

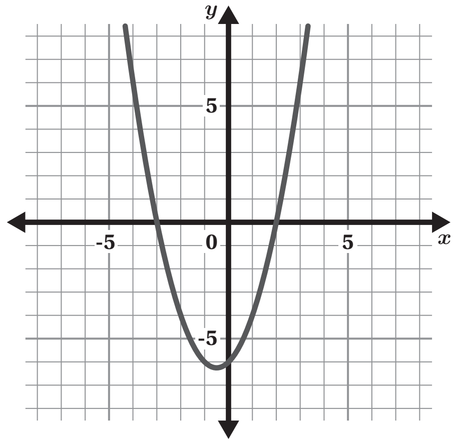
Equation



6 Here is the graph of $y = (x + 3)(x - 2)$.

How might $y = \frac{1}{2}(x + 3)(x - 2)$ look similar? Different?

Show or explain your thinking.



Activity 2

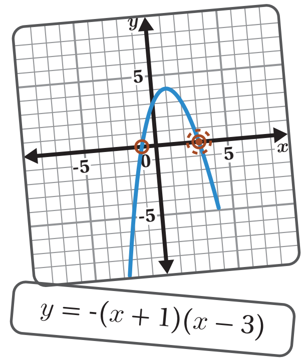
Name: ______ Date: ______ Period: ______

Lots of Challenges

You'll use the digital activity for Problems 7–12.

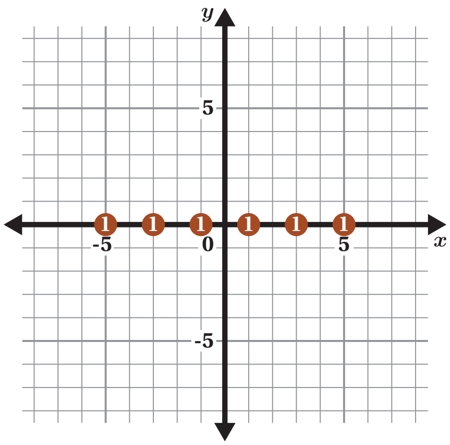
7 Move on to the final set of challenges.

Use what you know about writing quadratic equations to break all the targets in fun and creative ways!



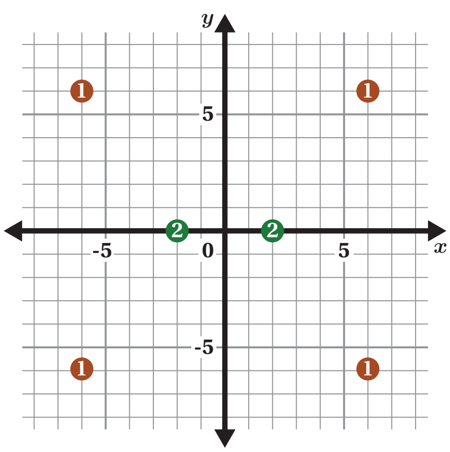
8 Break the targets using as few equations as you can.

Equation



9 Break the targets using as few equations as you can.

Equation

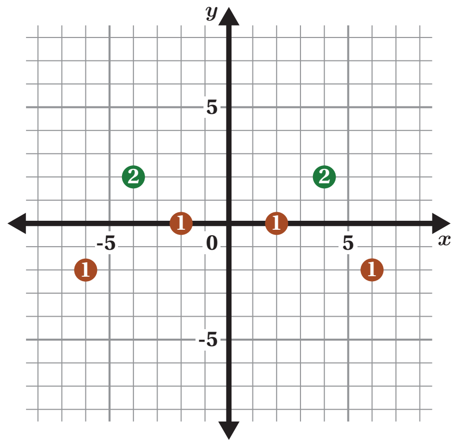


Name: ________________ Date: ________ Period: ________

Lots of Challenges (continued)

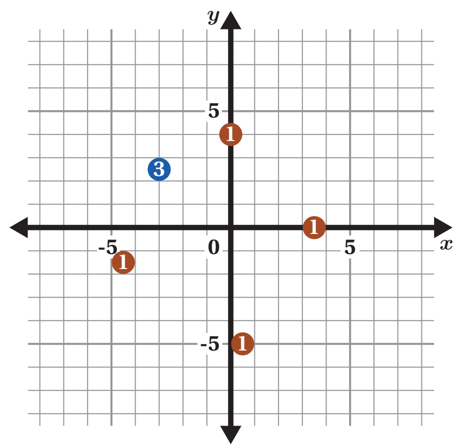
10 Break the targets using as few equations as you can.

Equations



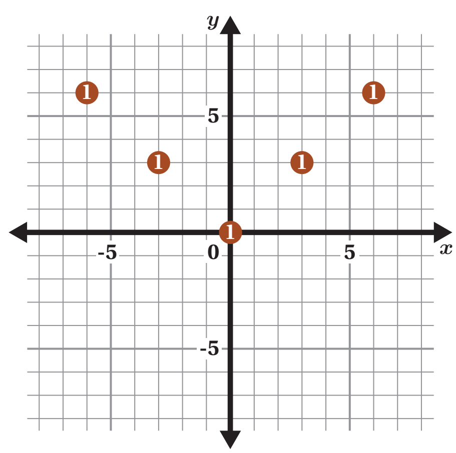
11 Break the targets using as few equations as you can.

Equations



12 Break the targets using as few equations as you can.

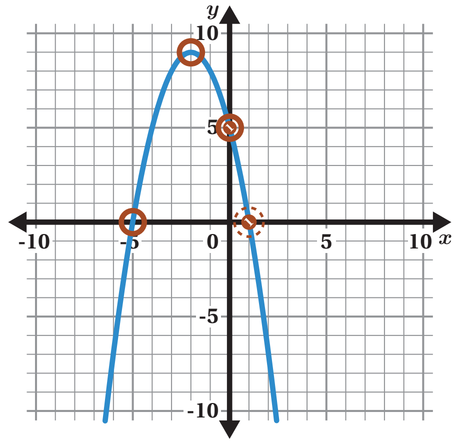
Equations



13 Synthesis

Describe a strategy for writing a quadratic equation in factored form that matches a graph.

Use the example if it helps with your thinking.



Things to Remember:

Unit 7

Lesson

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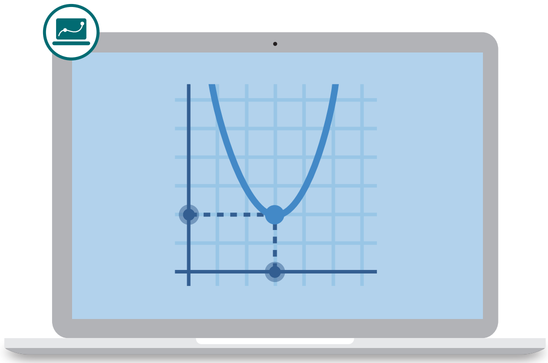
Name: Date: Period:



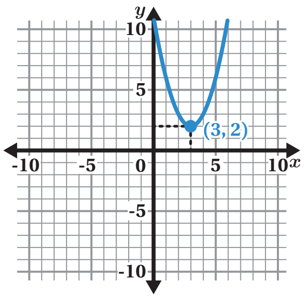
Vertex Form

Let's transform quadratic functions using translations and write their equations in a new form.

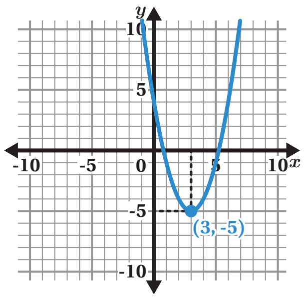
Warm-Up

1 Here are a few transformations of a parabola.

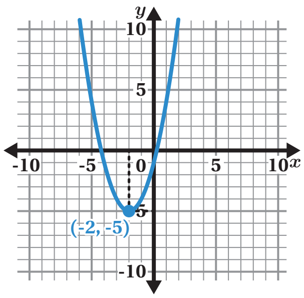
$y = (x - 3)^2 + 2$



$y = (x - 3)^2 - 5$



$y = (x + 2)^2 - 5$



What changes? What stays the same?

Changes:

Same:

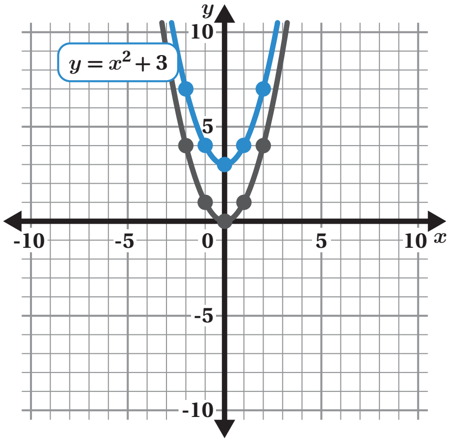
Activity 1

Name: .... Date: .... Period: ....

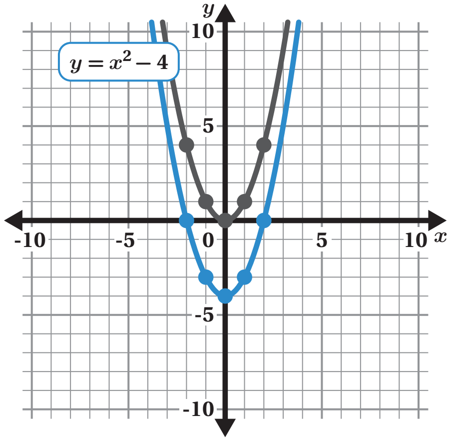
Translating Parabolas

2 Here are two different vertical *translations* of $y = x^2$.

Graph A



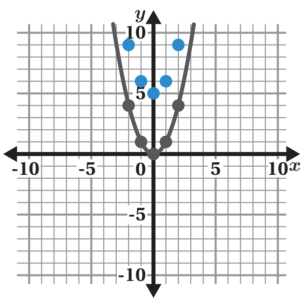
Graph B



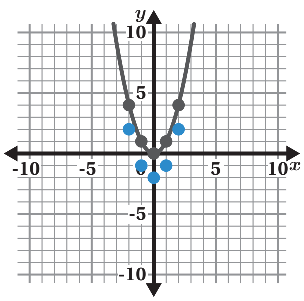
Discuss: What do you notice? What do you wonder?

3 For each challenge, write the equation for the vertical translation of $y = x^2$.

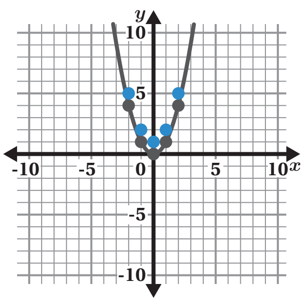
a $y =$ ....



b $y =$ ....



c $y =$ ....

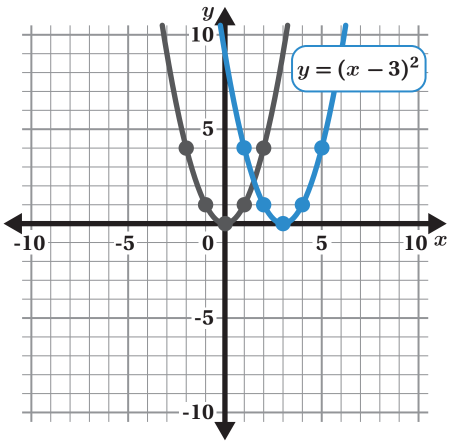


Name: ______ Date: ______ Period: ______

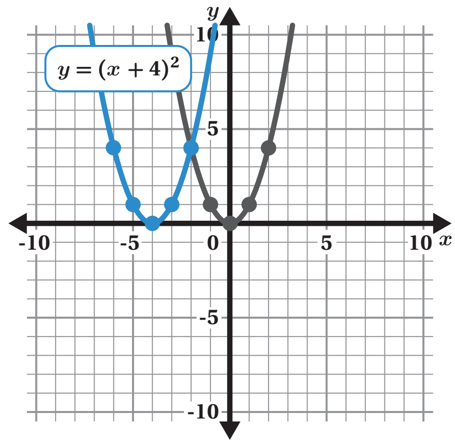
Translating Parabolas (continued)

4 Here are two different horizontal translations of $y = x^2$.

Graph C



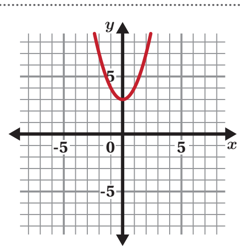
Graph D



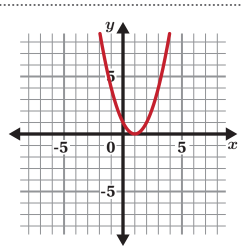
How can you see the translation in the equation?

5 Match each equation to a graph. One graph will have no match.

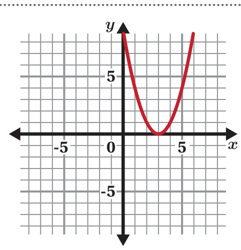
a. $y = (x + 3)^2$ **b.** $y = x^2 + 3$ **c.** $y = (x - 1)^2$



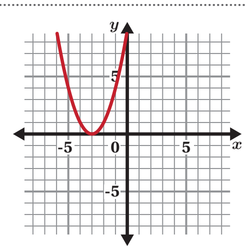
Equation: ______



Equation: ______



Equation: ______



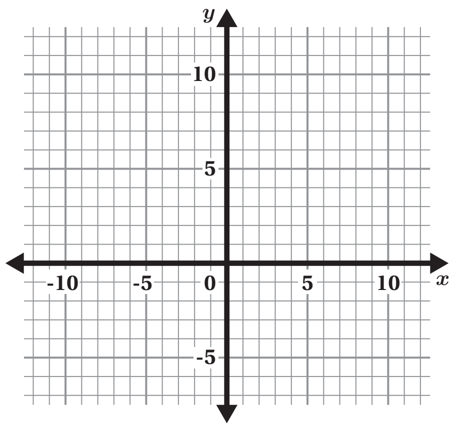
Equation: ______

Name: ______ Date: ______ Period: ______

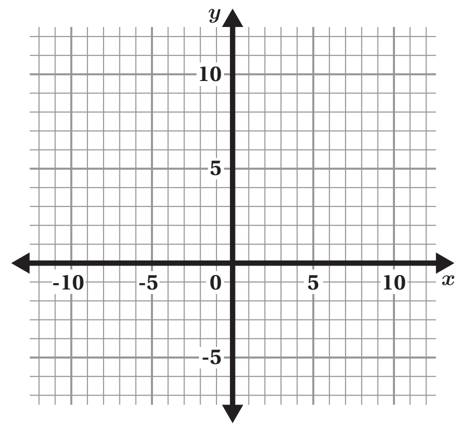
Translating Parabolas (continued)

6 Draw the graph of each parabola.

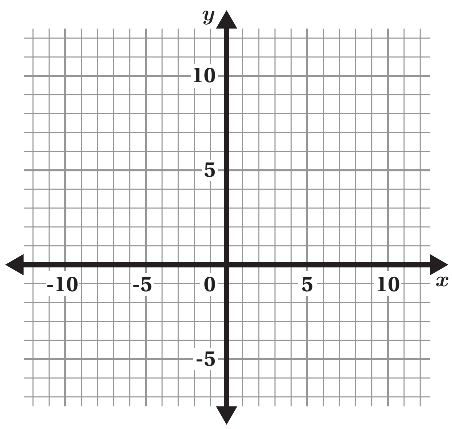
a $y = (x + 2)^2$



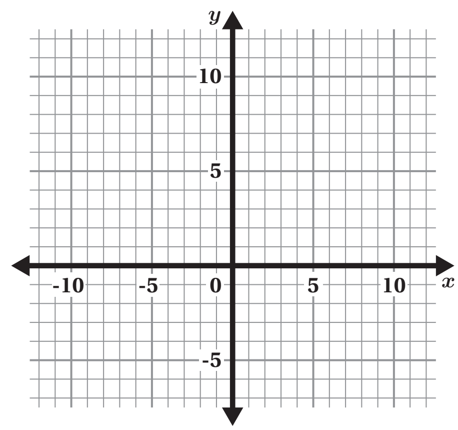
b $y = x^2 - 6$



c $y = (x + 1)^2 + 4$



d $y = (x + 8)^2 - 6$

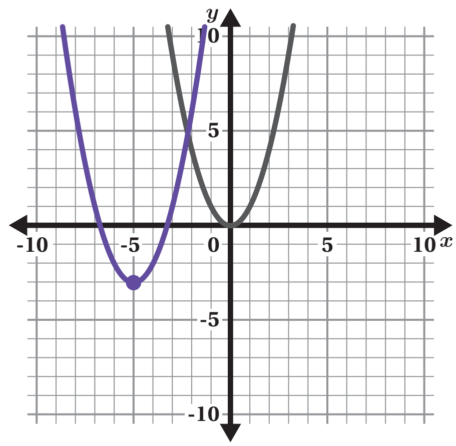


Activity 2

Name: ________ Date: ________ Period: ________

Vertex Form

7 Let's watch a translation of $f(x) = x^2$ left 5 units and down 3 units to $g(x) = (x + 5)^2 - 3$.



Discuss: Why do you think this type of equation is called **vertex form**?

8 Here is Liam's function for a parabola with a vertex at (-3, -4).

$f(x) = (x - 3)^2 - 4$

a What did Liam do well?

b What was Liam's mistake?

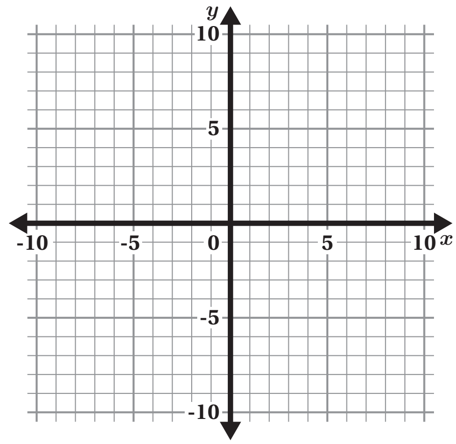
Activity 2

Name: ____________ Date: ________ Period: ________

Vertex Form (continued)

9 Write a quadratic function with its vertex at at (-6, -2).

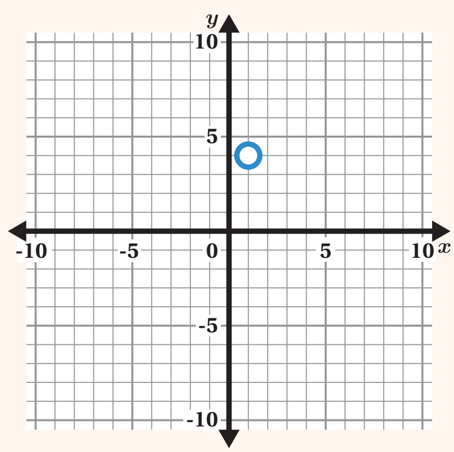
Use the graph if it helps with your thinking.



Explore More

10 Write the function of as many different parabolas as you can that go through the point $(1, 4)$.

Use the graph if it helps with your thinking.

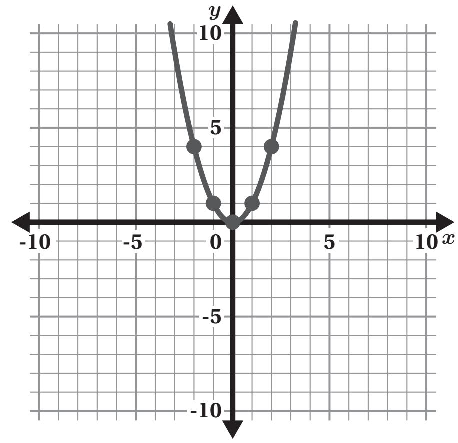


11 Synthesis

Here is a quadratic function written in vertex form: $g(x) = (x - 2)^2 + 3$.

Describe how the graph of $g(x)$ compares to $f(x) = x^2$.

Use the graph if it helps your thinking.



Things to Remember:

Unit 7

Lesson

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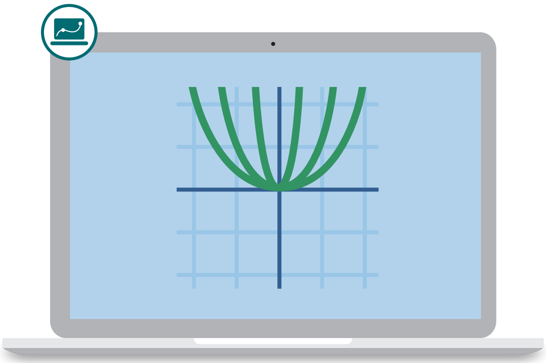
Name: .... Date: .... Period: ....

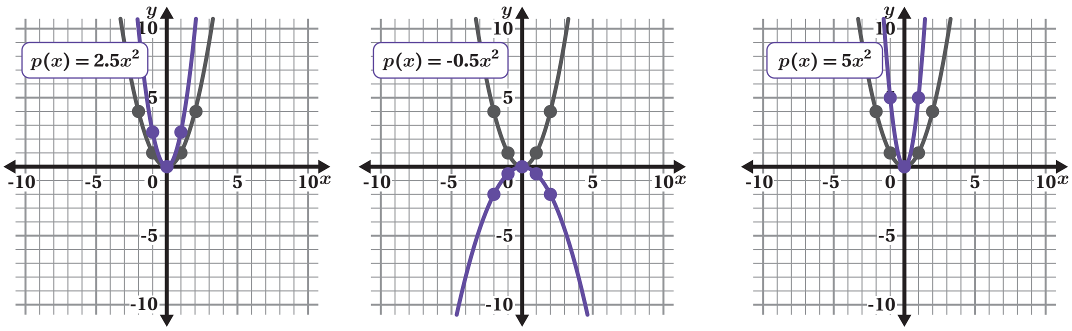


Scaled It!

Let's transform quadratic functions by scaling vertically.

Warm-Up

1 Here is a new kind of transformation.



What changes? What stays the same?

Changes:

Stays the same:

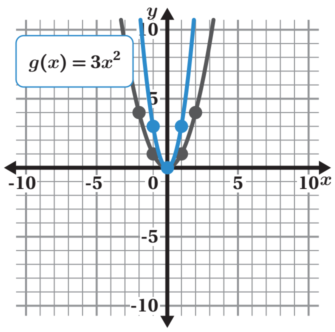
Activity 1

Name: ______ Date: ______ Period: ______

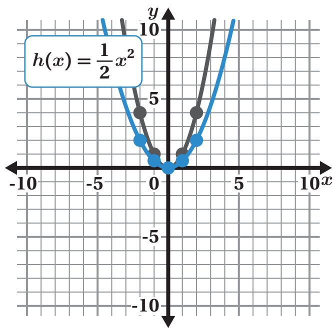
Scaling Vertically

2 Here are three different ways that $f(x) = x^2$ can be scaled vertically.

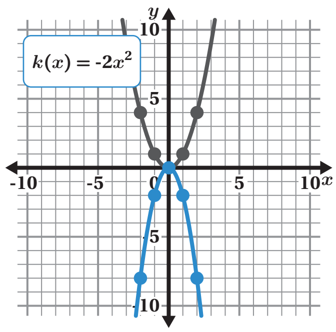
Graph A



Graph B



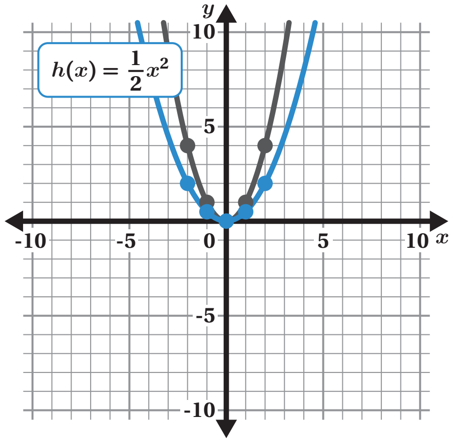
Graph C



Discuss: What do you notice? What do you wonder?

3 Here is Graph B from the previous problem.

Show or explain where you see the **vertical scale** by a factor of $\frac{1}{2}$ in the graph of $h(x) = \frac{1}{2}x^2$.

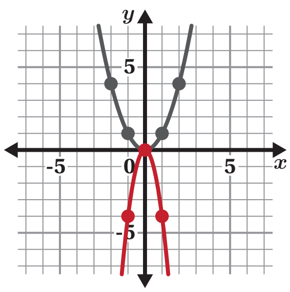


Name: .... Date: .... Period: ....

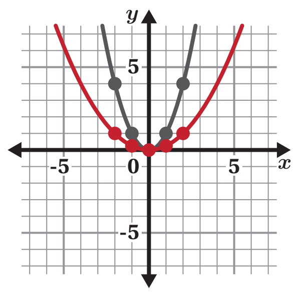
Scaling Vertically (continued)

4 Match the graph of each function with its equation. One equation will have no match.

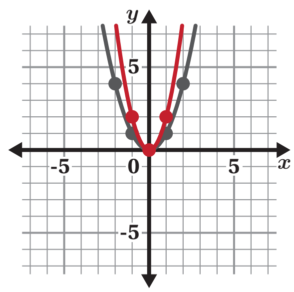
a. $a(x) = \text{-}4x^2$ **b.** $b(x) = \text{-}2x^2$ **c.** $c(x) = \frac{1}{4}x^2$ **d.** $d(x) = 2x^2$



Equation: ....



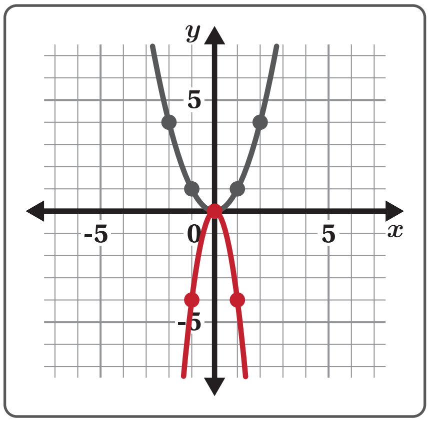
Equation: ....



Equation: ....

5 How did you decide which of these equations matches this graph?

$a(x) = \text{-}4x^2$ $b(x) = \text{-}2x^2$



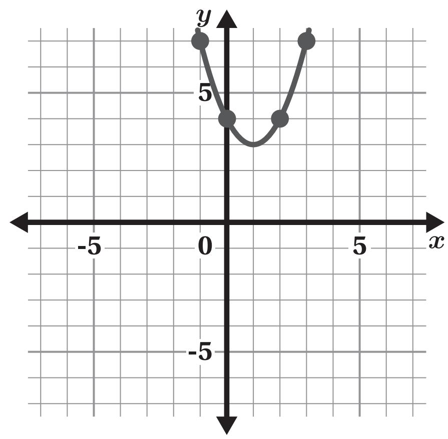
Activity
2

Name: ________ Date: ________ Period: ________

A Bit More Precision

6 The parabola $f(x) = (x - 1)^2 + 3$ has a vertex at $(1, 3)$ and is scaled vertically by a factor of 1.

Draw the graph of $g(x) = \text{-}2(x - 1)^2 + 3$.



7 Here is Kai's work to graph $g(x) = \text{-}2(x - 1)^2 + 3$.

How did this strategy help Kai graph the parabola?

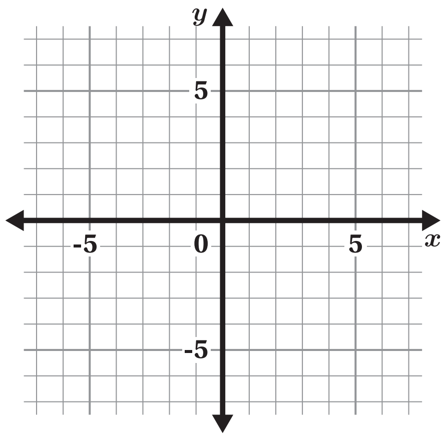
$g(x) = -2(x - 1)^2 + 3$
$g(2) = -2(2 - 1)^2 + 3$
$g(2) = -2(1)^2 + 3$
$g(2) = -2 + 3$
$g(2) = 1$

Name: ______________ Date: ______ Period: ______

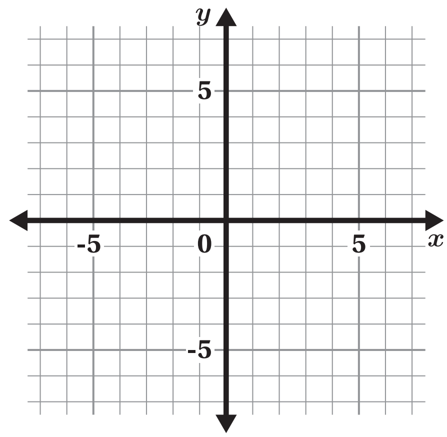
A Bit More Precision (continued)

8 Draw the graph of each quadratic function.

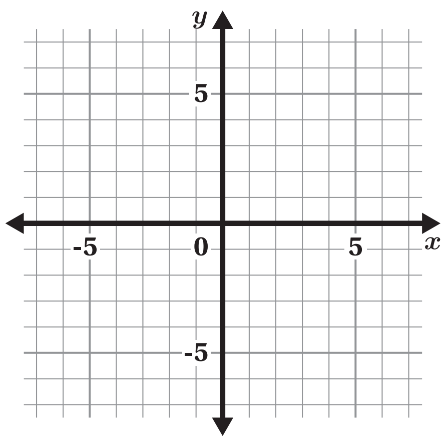
a $a(x) = 4x^2$



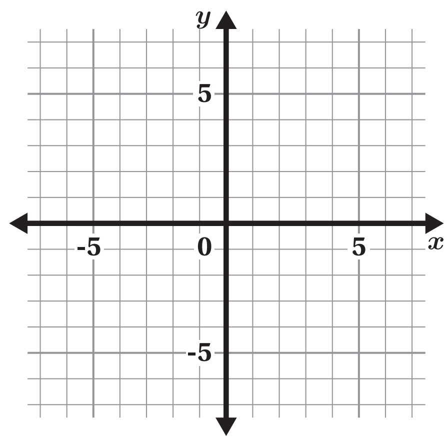
b $b(x) = 0.5x^2 - 3$



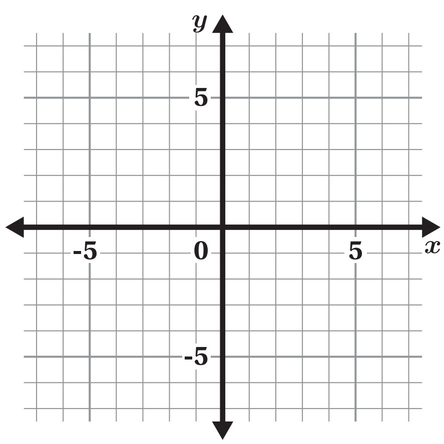
c $c(x) = -2(x - 5)^2$



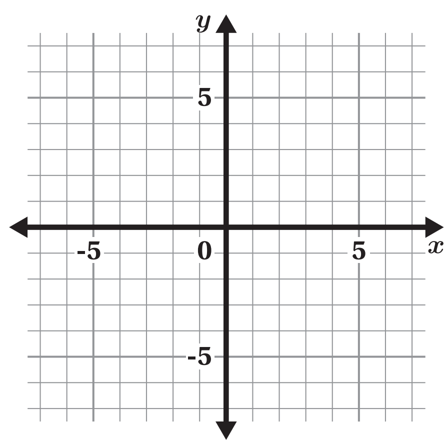
d $d(x) = (x + 1)^2 - 6$



e $e(x) = -0.5(x - 4)^2 + 2$



f $f(x) = -x^2 - 1$

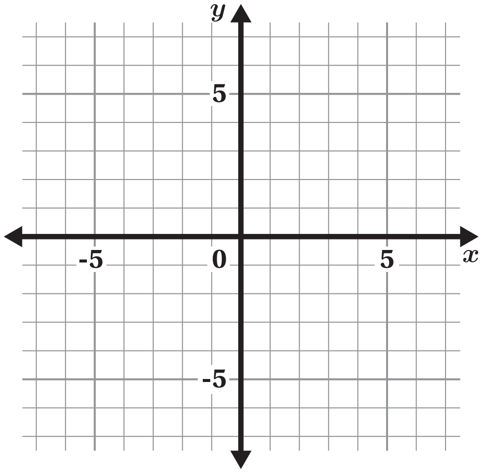


Name: .... Date: .... Period: ....

Parabola Art

9 Create a design by graphing parabolas. Record the functions you use.

Functions



10 Synthesis

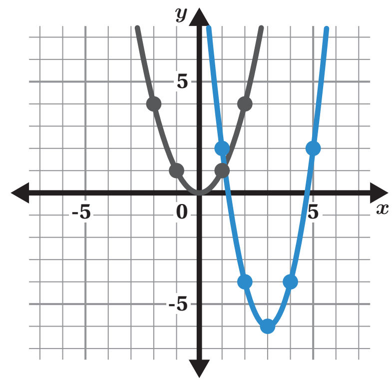
Here is the graph of $g(x) = 2(x - 3)^2 - 6$.

$g(x)$ is a transformation of $f(x) = x^2$.

Explain how you can determine the vertical scale from the equation and the graph.

Equation:

Graph:



Things to Remember:

Unit 7

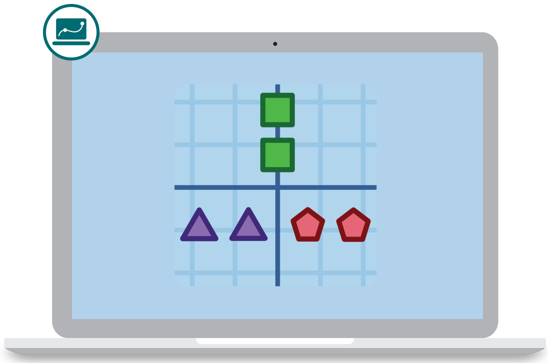
Lesson

16

Name: .... Date: .... Period: ....

Through the Gates

Let's write equations of parabolas given their key features.

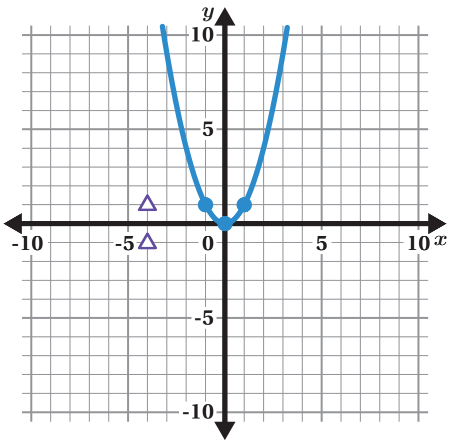


Warm-Up

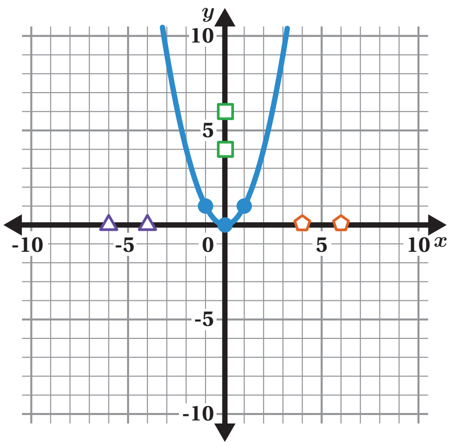
You'll use the digital activity for Problems 1–2.

1 In this lesson, a gate is the space between two points that look the same.

Adjust the parabola so that it goes through the gate between the purple triangles.



2 Adjust the parabola to go through all three gates.



Name: ______ Date: ______ Period: ______

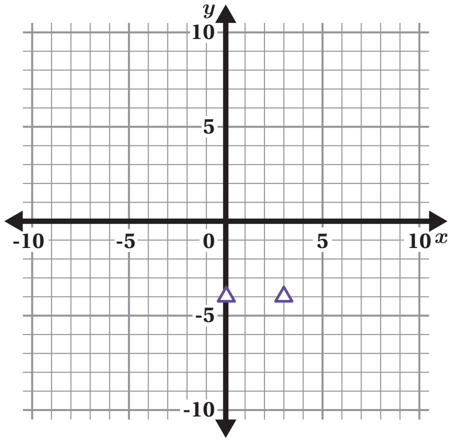
Going Through Gates

You'll use the digital activity for Problems 3–4.

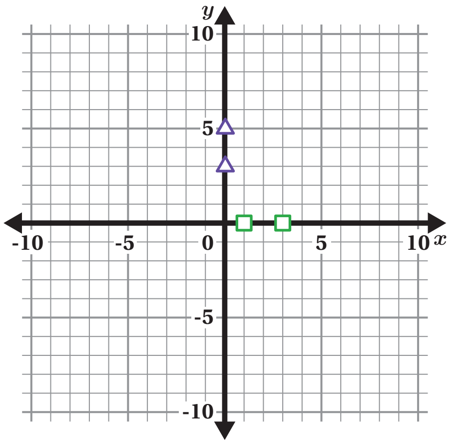
3 Change the equation so that the parabola goes through the gate.

Original equation: $y = x^2 - 1$

New equation:



4 Write an equation for a parabola that goes through each set of gates.



5 Renata and Mateo each wrote an equation for a new challenge.

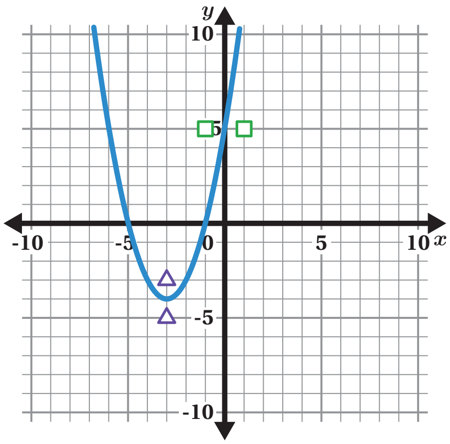
Renata wrote $y = (x + 1)(x + 5)$.

Mateo wrote $y = (x + 3)^2 - 4$.

Whose equation created this parabola?

Renata's Mateo's Both Neither

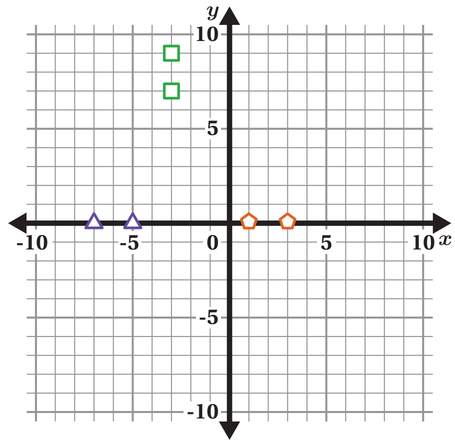
Explain your thinking.



Name: ______ Date: ______ Period: ______

Going Through Gates (continued)

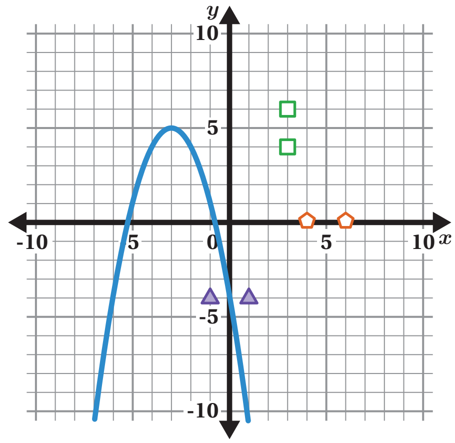
6 In the digital activity, write an equation for a parabola that goes through each set of gates.



7 Here is the equation Haru entered for the previous challenge: $y = -(x + 3)^2 + 5$.

a What did Haru do well?

b How would you change Haru's equation so that it goes through all of the gates?



Activity
2

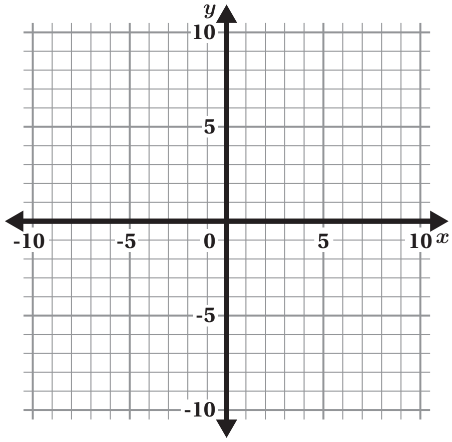
Name: ________ Date: ________ Period: ________

Challenge Creator

8 Use the digital activity to make a challenge for your classmates to solve.

- Select the number of gates you want to include.
- Drag the movable points in the activity to create a challenge.
- Write an equation for a parabola that goes through each set of gates.

My Challenge

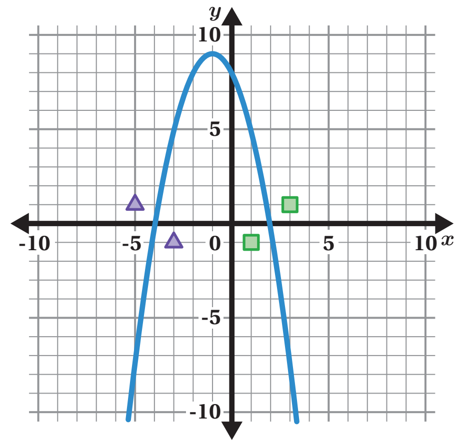


Equation: ________

9 Synthesis

Describe a strategy for writing a quadratic equation that matches a graph.

Use the example if it helps with your thinking.



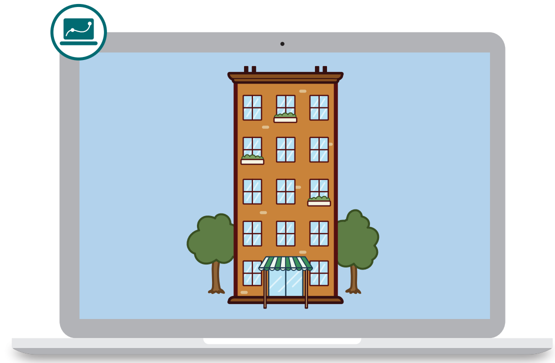
Things to Remember:

Unit 7

Lesson

17

Name: ________ Date: ________ Period: ________



Reasonable Rent

Let's use quadratic functions and revenue to make sense of an issue in society: the cost of housing.

Warm-Up

1 The median income for a family in Metropolis is $2,000 a month.

a **Discuss:** What do you think median income means?

b Create a diagram to show how much you think a family should budget for rent, food, savings, and other expenses. Discuss your choices with a partner.

|←—— $2,000 ——→|

2 The median rent in Metropolis is $1,300 a month. Let's look at the budget of a typical Metropolis family. Residents claim that rent is too expensive in Metropolis.

a Why might they feel that rent is too expensive?

b How might rent prices affect the community?

Activity 1

Name: ______ Date: ______ Period: ______

Rent vs. Units

3 Many families in Metropolis are struggling to afford rent.

City Roots (C.R.) is an organization working to establish affordable housing in Metropolis. They buy apartments and rent them at an affordable price.

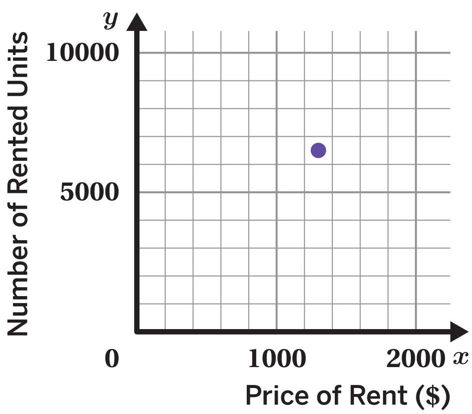
Discuss: How could C.R. decide what to charge for rent?

4 Metropolis has a total of $13{,}000$ housing units.

Every time rent increases by $\$100$, C.R. estimates that they can rent 500 fewer units.

Use this information to complete the table and graph.

Price of Rent ($)	Number of Rented Units
1,000	
1,100	
1,200	
1,300	6,500
1,400	



5 Let's look at the table and graph. Write an expression to represent the number of units C.R. can rent at *any* price, x.

Activity 2

Name: .... Date: .... Period: ....

Rent vs. Revenue

6 City Roots Collective used their model to determine how much revenue they would make.

a. Complete the table.

b. **Discuss:** What do you notice? What do you wonder?

Price of Rent ($)	Number of Rented Units	Revenue ($)
1,000	8,000	8,000,000
1,100	7,500	
1,200	7,000	
1,300	6,500	
1,400	6,000	

7 Let's look at some revenues from the previous problem.

Write an expression to represent the revenue City Roots Collective could make at *any* price, x.

Name: .... Date: .... Period: ....

Rent vs. Revenue (continued)

8 Here are four considerations City Roots Collective might care about.

a. Making the most revenue

b. Making housing affordable to the most people

c. Making enough money to equal the cost of development

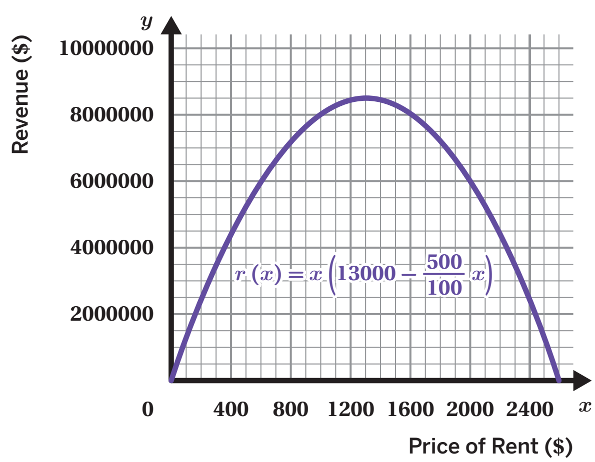
d. Building enough housing to meet the demand

Order them from *most important to you* to *least important to you*.

Most important **Least important**

9 This project will cost City Roots Collective $\$8{,}000{,}000$ a month.

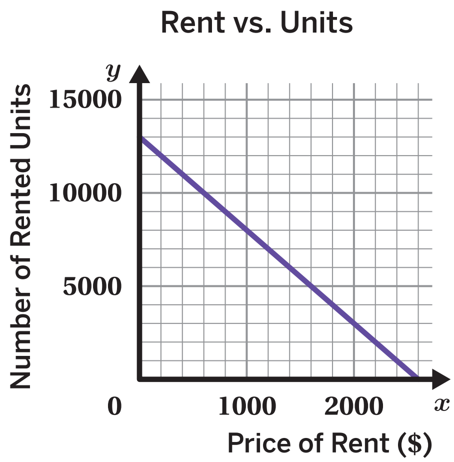
What do you think is a fair price for rent? Explain your thinking.

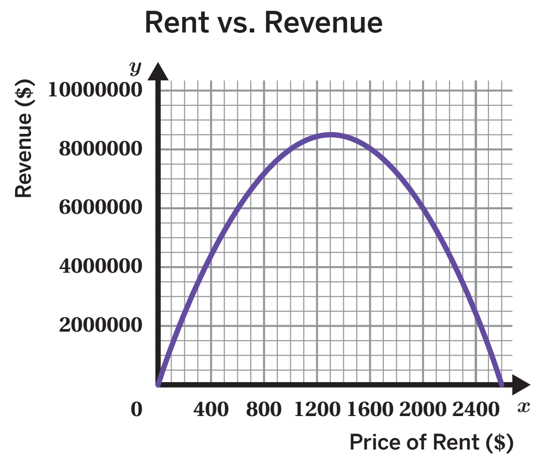


10 Synthesis

The British statistician George Box once said: *All models are wrong, but some are useful*.

a Choose a model we've explored today.





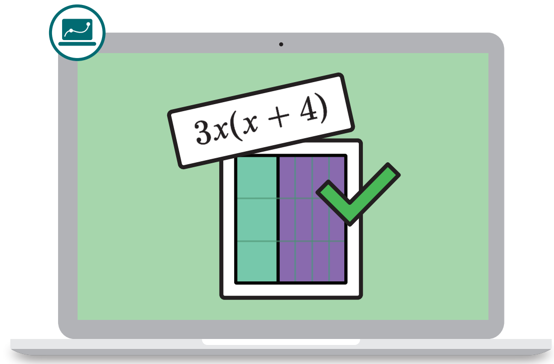
b Explain how that model is wrong and how it is useful.

Things to Remember:

Name: ________ Date: ________ Period: ________

Two-Factor Multiplication

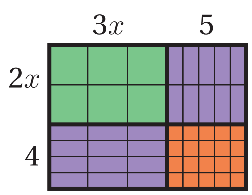
Let's rewrite factored-form quadratic expressions in standard form.



Warm-Up

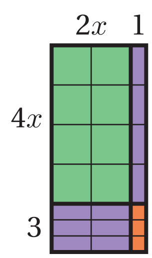
1 An area model shows equivalent quadratic expressions.

a Here are three area models.



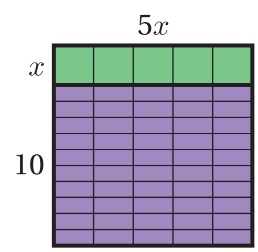
Factored Form
$(2x + 4)(3x + 5)$

Standard Form
$6x^2 + 22x + 20$



Factored Form
$(4x + 3)(2x + 1)$

Standard Form
$8x^2 + 10x + 3$



Factored Form
$(x + 10)(5x)$

Standard Form
$5x^2 + 50x$

b Where on the area model do you see the *factored form*? Where do you see the *standard form*?

Factored Form:

Standard Form:

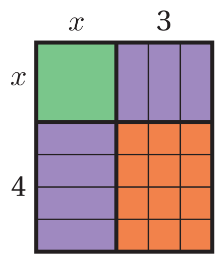
Activity 1

Name: ______ Date: ______ Period: ______

Multiplying With Area Models

2 Match each expression with an equivalent area model. One area model will have no match.

$x(x + 4)$	$3x(x + 4)$	$(x + 4)(x + 3)$



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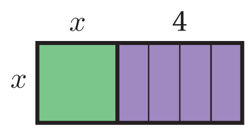

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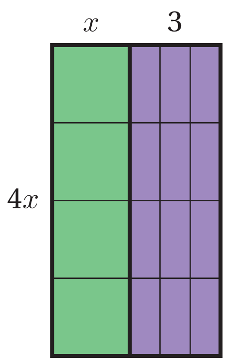

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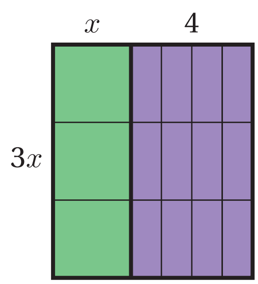


______

3 Let's look at two cards that Sahana correctly matched.

She wrote this standard-form expression: $3x^2 + 12x$.

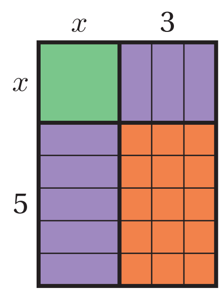
Show or explain where you see $3x^2 + 12x$ in the area model or in the factored-form expression.

Activity 1

Name: ________ Date: ________ Period: ________

Multiplying With Area Models (continued)

4 Here is a list of equivalent expressions.



Circle *one* expression.

A. $x^2 + 3x + 5x + 15$

B. $x^2 + 8x + 15$

C. $x(x + 3) + 5(x + 3)$

Show or explain how you see it represented in the area model.

5 **a** Draw an area model to represent the expression $(x + 7)(x + 3)$.

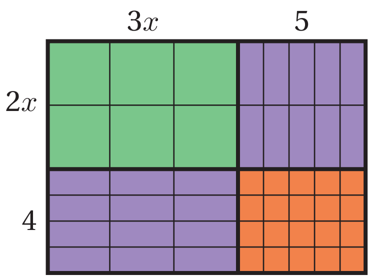
b Rewrite $(x + 7)(x + 3)$ in standard form.

Activity 2

Name: .... Date: .... Period: ....

Multiplying With Diagrams

6 The diagram on the right shows a different way to represent the area model.



	$3x$	5
$2x$	$6x^2$	$10x$
4	$12x$	20

Discuss: How are the area model and the diagram alike? How are they different?

7 Multiply to rewrite $(2x + 5)(x + 8)$ in standard form.

Use the diagram if it helps with your thinking.

	x	8
$2x$		
5		

Name: ______ Date: ______ Period: ______

Multiplying With Diagrams (continued)

8 Karima tried to rewrite $(5x - 3)(2x + 5)$ in standard form and made an error.

What did Karima do well? What could she improve?

	5x	-3
2x	$10x^2$	-6x
5	25x	-15

$10x^2 + 31x - 15$

Something Karima did well:

Something Karima could improve:

9 Multiply to rewrite each expression in standard form. Use the diagrams if they help with your thinking.

a $(x + 6)(x + 10)$

______

b $(3x + 1)(x + 6)$

______

c $(2x - 6)(3x + 1)$

______

d $(4x + 5)(x - 7)$

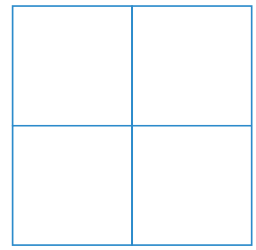
______

12 Synthesis

Describe how to write a factored-form expression in standard form.

Use the diagram if it helps with your thinking.

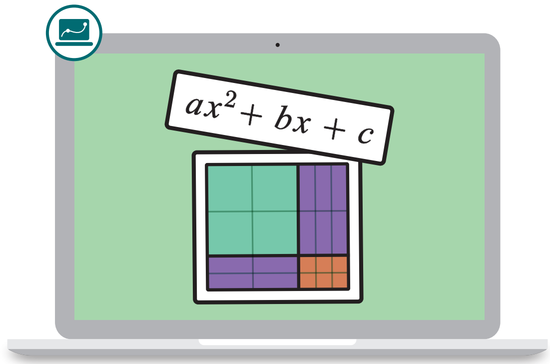
$$(2x-3)(x+4)$$



Things to Remember:

Unit 8
Lesson
3

Name: .... Date: .... Period: ....



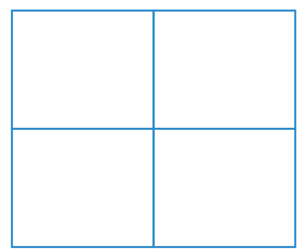
Standard Feature

Let's look for patterns that help us rewrite factored-form expressions in standard form.

Warm-Up

1 Which expression is equivalent to $(x + 5)^2$?

Use the diagram if it helps with your thinking.



A. $x^2 + 25$

B. $x^2 + 10x + 25$

C. Both

D. Neither

Explain your thinking.

2 Let's look at how two students determined the expression equivalent to $(x + 5)^2$.

Discuss: How are their strategies alike? How are they different?

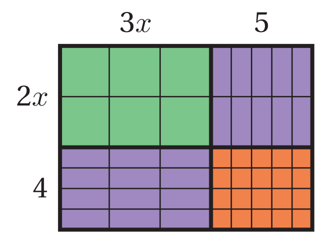
Activity 1

Name: ______ Date: ______ Period: ______

Standard Form

3 We often use $ax^2 + bx + c$ to represent a quadratic expression in *standard form*.

Where do you see a, b, and c in the area model?

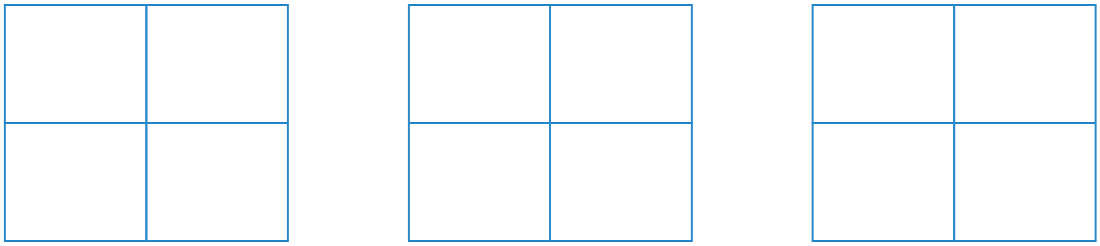


$$\underbrace{6}_{a}x^2 + \underbrace{22}_{b}x + \underbrace{20}_{c}$$

4 Group the equivalent expressions. One expression will have no match. Use the diagrams if they help with your thinking.

$(x + 3)(x + 3)$	$3x^2 + 9x$	$x^2 - 9$	$(x + 3)^2$
$(x - 3)(x + 3)$	$3x(x + 3)$	$x^2 + 9$	$x^2 + 6x + 9$

Group 1	Group 2	Group 3



Activity 1

Name: .... Date: .... Period: ....

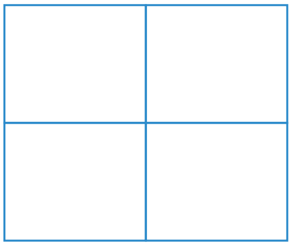
Standard Form (continued)

5 Here are two equivalent expressions:

$$(x - 3)(x + 3) \text{ and } x^2 - 9$$

Discuss: Why does $(x - 3)(x + 3)$ have a b-value of 0 when written in standard form?

Use the diagram if it helps with your thinking.

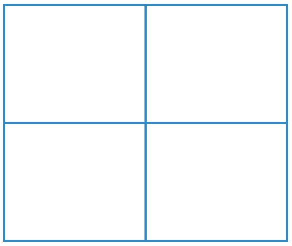


6 Here are two equivalent expressions:

$$3x(x + 3) \text{ and } 3x^2 + 9x$$

Why is the c-value 0?

Use the diagram if it helps with your thinking.



7 Select *all* the expressions that have a b- or c-value of 0 when written in standard form.

☐ **A.** $(3x - 1)(3x + 1)$

☐ **B.** $(x - 4)(x - 4)$

☐ **C.** $x(x + 4)$

☐ **D.** $(3x + 1)(x - 1)$

☐ **E.** $(x + 10)(x - 10)$

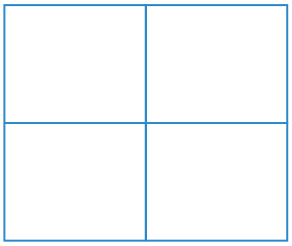
Activity
2

Name: .... Date: .... Period: ....

Now I Know My ABC's

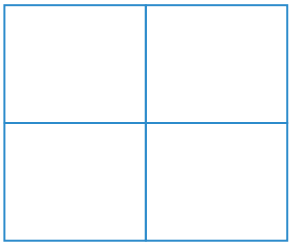
8 Write an expression in *factored form* that has a b*-value of* 0 when written in standard form.

Use the diagram if it helps with your thinking.



9 Write an expression in factored form that has a *positive* b*-value* and a *negative* c*-value* when written in standard form.

Use the diagram if it helps with your thinking.



10 Here are four expressions that have a *positive* b*-value* and a *negative* c*-value*.

Describe any patterns you notice.

Factored Form	Standard Form
$(x-2)(x+9)$	$x^2+7x-18$
$(x+10)(x-8)$	$x^2+2x-80$
$(3x+7)(x-2)$	$3x^2+1x-14$
$(4x-9)(x+3)$	$4x^2+3x-27$

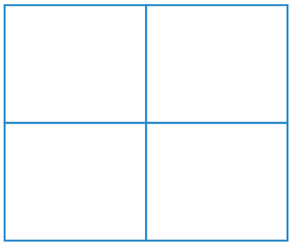
Activity 2

Name: Date: Period:

Now I Know My ABC's (continued)

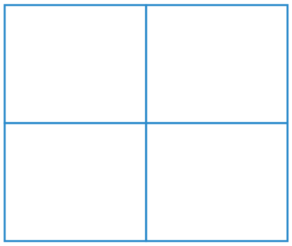
11 Write an expression in factored form that has a b-*value greater than* 5 and a c-*value of* 1 when written in standard form.

Use the diagram if it helps with your thinking.



12 **a** Write an expression in factored form that has a *negative* a-*value*, a *negative* b-*value*, and a *negative* c-*value* when written in standard form.

Use the diagram if it helps with your thinking.



b Compare your expression with another group's expression.

Discuss: What patterns do you notice?

Explore More

13 Do you think it's possible to write an expression in factored form that has a b-value of 0 *and* a positive c-value when written in standard form? Circle one.

Yes No Not enough information

Explain your thinking.

14 Synthesis

Describe 2–3 patterns you noticed between equivalent expressions in factored form and in standard form.

Use the examples if they help with your thinking.

$(3x - 2)(3x + 2)$

$(x - 6)(x - 3)$

$(x + 5)(x + 5)$

$(5x + 2)(x - 5)$

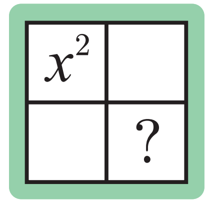
$2x(x - 4)$

Things to Remember:

Name: .... Date: .... Period: ....

X-Factor

Let's rewrite standard-form quadratic expressions in factored form.



Warm-Up

1. Match each expression in *factored form* with its equivalent expression in *standard form*.

Factored Form	**Standard Form**
a. $(5x+6)(x-3)$	.... $5x^2+43x-18$
b. $(5x-3)(x+6)$	.... $5x^2-9x-18$
c. $(5x-2)(x+9)$	.... $5x^2-43x-18$
d. $(5x+2)(x-9)$	.... $5x^2+27x-18$

Name: ____________ Date: ________ Period: ________

Diagram Puzzles

Complete each diagram puzzle, standard-form expression, and factored-form expression.

<table>
<tr><th>Diagram</th><th>Standard Form</th><th>Factored Form</th></tr>
<tr><td>

2.

<table>
<tr><td></td><td>$3x$</td><td>-5</td></tr>
<tr><td>$4x$</td><td></td><td></td></tr>
<tr><td></td><td>-9x</td><td>15</td></tr>
</table>

</td><td>______ $+ 15$</td><td>$(3x - 5)(4x$ ______$)$</td></tr>
<tr><td>

3.

<table>
<tr><td></td><td>$2x$</td><td>3</td></tr>
<tr><td></td><td>$4x^2$</td><td>$6x$</td></tr>
<tr><td></td><td></td><td>-9</td></tr>
</table>

</td><td>$4x^2$ ______</td><td>$(2x + 3)($______$)$</td></tr>
<tr><td>

4.

<table>
<tr><td></td><td></td><td>-3</td></tr>
<tr><td></td><td>$2x^2$</td><td>-3x</td></tr>
<tr><td></td><td>$8x$</td><td></td></tr>
</table>

</td><td>$2x^2 + 5x$ ______</td><td></td></tr>
<tr><td>

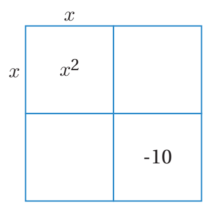
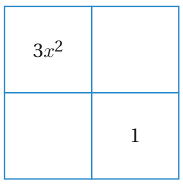
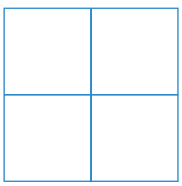
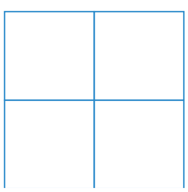
5.

<table>
<tr><td>$3x^2$</td><td>$4x$</td></tr>
<tr><td>$15x$</td><td>20</td></tr>
</table>

</td><td></td><td></td></tr>
</table>

Name: ______ Date: ______ Period: ______

Diagram Puzzles (continued)

Diagram	Standard Form	Factored Form
6. 	$x^2 - 3x - 10$	
7. 	$3x^2 + 4x + 1$	
8. 	$x^2 + 9x + 20$	
9. 	$6x^2 + 7x + 2$	

Name: .... Date: .... Period: ....

Next Steps

Nicolas is trying to factor $2x^2 + 9x + 7$.

	$2x$	1
x	$2x^2$	x
7	$14x$	7

10. **Discuss:**

- What did Nicolas do well?
- Explain what you think is incorrect about Nicolas's work.
- What could he try next?

Sneha is trying to factor $2x^2 + 23x - 12$. She started by creating this diagram.

	$2x$	
x	$2x^2$	
		-12

11. List pairs of constants Sneha could try in order to complete the outside of the diagram.

Sneha tried the numbers -6 and 2.

	$2x$	2
x	$2x^2$	$2x$
-6	$-12x$	-12

12. **Discuss:**

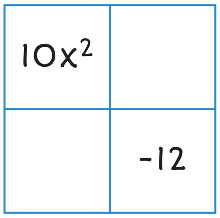
- How can you tell Sneha's work is incorrect?
- What did Sneha do well?
- What could she try next?

13. Rewrite $2x^2 + 23x - 12$ in factored form.

Name: Date: Period:

Next Steps (continued)

14. Ariana is trying to factor $10x^2 - 7x - 12$. She starts by creating this diagram.



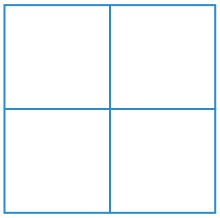
Ariana says: *I have to use factors of* 10. *I also need to use factors of* -12.

What do you think she means?

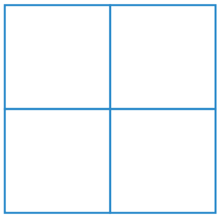
15. Rewrite $10x^2 - 7x - 12$ in factored form.

16. Here are three other expressions with a c-value of -12. Rewrite each expression in factored form. Use the diagrams if they help with your thinking.

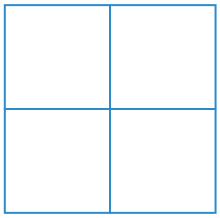
a $x^2 + x - 12$



b $3x^2 - 16x - 12$



c $6x^2 - x - 12$

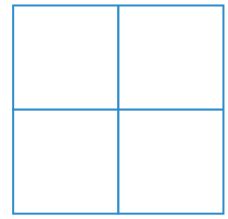


Synthesis

17. Describe how to rewrite a standard-form expression in factored form.

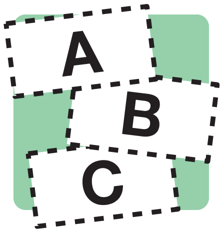
Use the example if it helps with your thinking.

$5x^2 - 31x - 28$



Things to Remember:

Name: ______________________ Date: __________ Period: __________



Form Up

Let's factor some special quadratic expressions.

Warm-Up

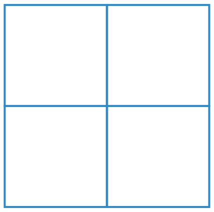
Eliza is trying to factor $x^2 + x - 56$. She started by listing pairs of numbers that multiply to -56.

1 and -56	2 and -28	4 and -14	7 and -8
-1 and 56	-2 and 28	-4 and 14	-7 and 8

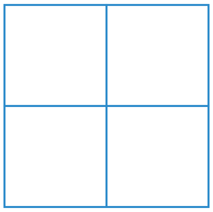
1. **Discuss:** Which pairs might Eliza try first? Why?

2. Factor each expression. Use the diagrams if they help with your thinking.

a. $x^2 + x - 56$



b. $x^2 + 26x - 56$



Activity 1

Name: .... Date: .... Period: ....

Spotting Similarities

Here are three groups of expressions.

Group 1	Group 2	Group 3
$4x^2 - 25$	$8x^2 + 32x + 24$	$x^2 - 6x - 27$
$x^2 - 36$	$-4x^2 + 8x + 32$	$x^2 + 2x - 80$
$x^2 - 100$	$-10x^2 - 20x - 10$	$x^2 - 13x + 30$
$25x^2 - 49$	$2x^2 - 22x + 60$	$x^2 + 2x - 63$

3. Explain how the expressions in each group are alike.

Group 1:

Group 2:

Group 3:

4. Factor one expression from each group. Use the diagrams if they help with your thinking.

Group 1: .... Group 2: .... Group 3: ....

Activity 1

Name: ________ Date: ________ Period: ________

Spotting Similarities (continued)

Deiondre factored the expression $7x^2 + 28x + 21$.

Deiondre

$7x^2 + 28x + 21$

$7(x^2 + 4x + 3)$

$7(x + 3)(x + 1)$

5. **Discuss:**

- Are $7x^2 + 28x + 21$ and $7(x^2 + 4x + 3)$ equivalent? How do you know?
- Why might Deiondre have written $7(x^2 + 4x + 3)$ as a first step?

6. Does Deiondre's expression belong in Group 1, 2, or 3? Explain your thinking.

Yasmine factored the expression $9x^2 - 49$.

Yasmine

$9x^2 - 49$

$9x^2 + 0x - 49$

$(3x - 7)(3x + 7)$

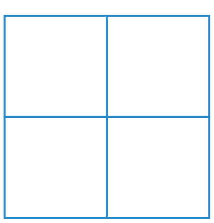
7. **Discuss:** Does Yasmine's expression belong in Group 1, 2, or 3? Explain your thinking.

8. Write a new expression in *standard form* that belongs in the same group as Yasmine's.

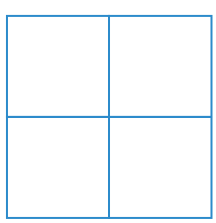
9. Factor the expression you wrote in the previous problem.

10. Factor each expression. Use the diagrams if they help with your thinking.

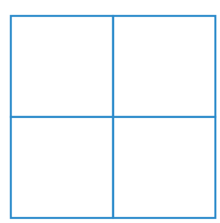
a $3x^2 - 6x - 105$



b $16x^2 - 49$



c $4x^2 + 52x + 120$



Activity 2

Name: ______ Date: ______ Period: ______

Solve and Swap

You will get a card.

- Factor the expression on your card. Draw a diagram if it helps with your thinking.
- Find a partner and swap cards. Factor your new expression, then check with your partner.
- Find a new partner and repeat this process.

Card ______

Card ______

Card ______

Card ______

Card ______

Card ______

Card ______

Card ______

Synthesis

What do you think is important to remember when factoring an expression in standard form?

Use the expressions if they help with your thinking.

$5x^2 - 18x - 8$

$9x^2 - 16$

$6x^2 - 24x - 30$

Things to Remember:

Lesson 8.05
Activity 2
(p. 1 of 2)

Trading Cards

Directions: Make one copy per 32 students. Then pre-cut the cards and give each student one card.

Card 1 $x^2 + 5x - 6$	**Card 2** $2x^2 - 13x - 24$	**Card 3** $-2x^2 + 2x + 4$	**Card 4** $x^2 + 18x + 80$
Card 5 $x^2 + 3x - 10$	**Card 6** $100x^2 - 9$	**Card 7** $6x^2 - 6x - 36$	**Card 8** $4x^2 + 13x + 10$
Card 9 $4x^2 - 8x - 5$	**Card 10** $9x^2 - 1$	**Card 11** $x^2 - 15x + 56$	**Card 12** $3x^2 + 8x - 16$
Card 13 $x^2 - 25$	**Card 14** $x^2 - 100$	**Card 15** $x^2 + 5x - 14$	**Card 16** $2x^2 + 20x + 18$

Lesson 8.05 Activity 2 (p. 2 of 2)

Trading Cards

Card 17

$25x^2 - 64$

Card 18

$5x^2 - 15x - 20$

Card 19

$3x^2 + 13x + 12$

Card 20

$x^2 - 4$

Card 21

$x^2 - 16$

Card 22

$x^2 - 6x - 40$

Card 23

$2x^2 + 15x + 18$

Card 24

$15x^2 - 5x - 20$

Card 25

$-6x^2 + 21x$

Card 26

$x^2 + 11x + 18$

Card 27

$x^2 - 36$

Card 28

$10x^2 - 60x + 80$

Card 29

$x^2 + 8x - 9$

Card 30

$5x^2 - 45$

Card 31

$2x^2 - 17x - 9$

Card 32

$10x^2 + 30x + 20$

Unit 8
Lesson
6

Name: Date: Period:

Shooting Stars

Let's determine the x-intercepts of quadratic functions written in factored form and standard form.

Warm-Up

1 Determine whether each coordinate pair is an x-intercept or a y-intercept.

Ordered Pair	x-intercept	y-intercept	Neither
$(1.7, 0)$			
$(1, 1)$			
$(0, 4)$			
$\left(-\frac{3}{2}, 0\right)$			
$(5, 0)$			
$(0, -6)$			

Activity 1

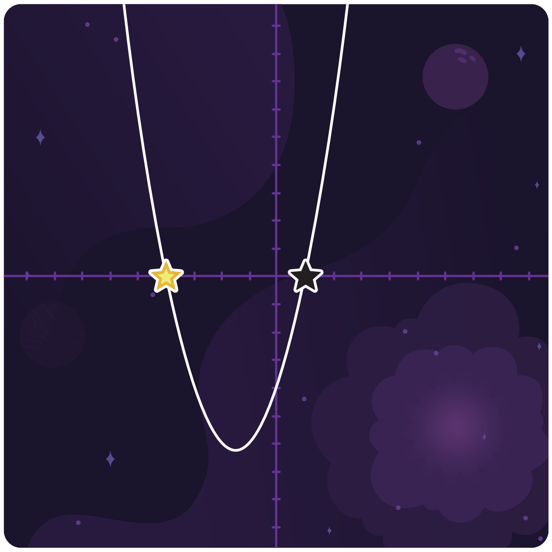
Name: ______________ Date: ________ Period: ________

Star Mail

2 Send stars to the x-intercepts of this function:

$$f(x) = (x + 4)(x - 1)$$

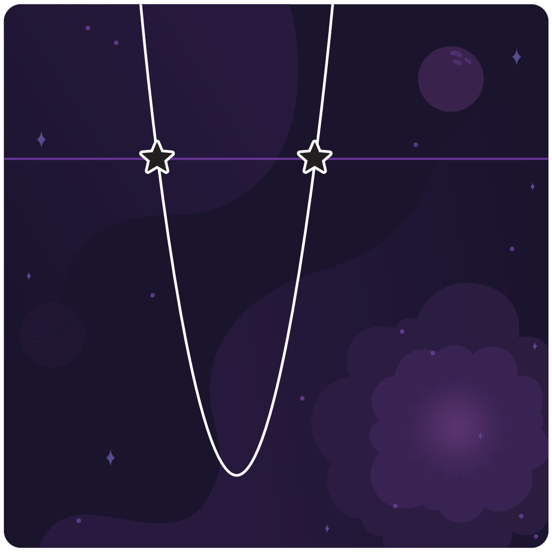
Star	Ordered Pair
Star #1	(-4, 0)
Star #2	



3 Send stars to the x-intercepts of this function:

$$g(x) = (x + 1)(2x - 6)$$

Star	Ordered Pair
Star #1	
Star #2	



4 Let's look at Aba's strategy from the previous problem.

Discuss:

- Why did Aba replace $g(x)$ with 0?
- How did Aba figure out the coordinates of the x-intercepts?

Activity 2

Name: ______ Date: ______ Period: ______

Standard Space Mail

5 Send stars to the x-intercepts of this function:

$$h(x) = x^2 + 3x - 10$$

Star	Ordered Pair
Star #1	
Star #2	

6 Aba, Darius, and Rishi factored the function $a(x) = 4x^2 + 20x + 24$ in three different ways.

Aba	Darius	Rishi
$a(x) = 4(x+3)(x+2)$	$a(x) = (4x+8)(x+3)$	$a(x) = (2x+6)(2x+4)$

a **Discuss:** How can you see that each equation has the same x-intercepts?

b Write the x-intercepts in the table below.

x-intercepts	Ordered Pair
x-intercept #1	
x-intercept #2	

7 Send stars to the x-intercepts of this function:

$$b(x) = 2x^2 - 11x + 12$$

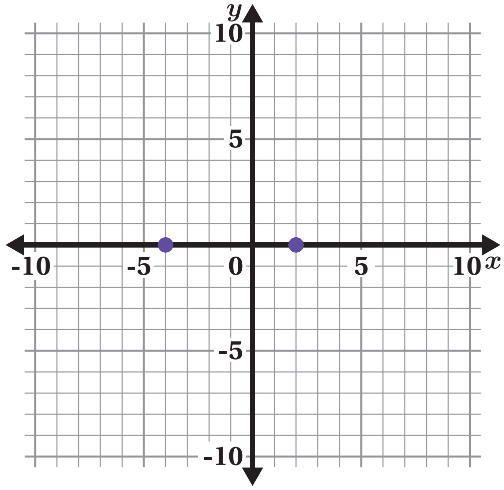
Star	Ordered Pair
Star #1	
Star #2	

Activity 3

Name: ________ Date: ________ Period: ________

Zero, My Hero

8 A term related to x-intercepts is **zeros**. The zeros of a function are the x-values that make $f(x) = 0$.



a **Discuss:** How are *zeros* related to x-intercepts?

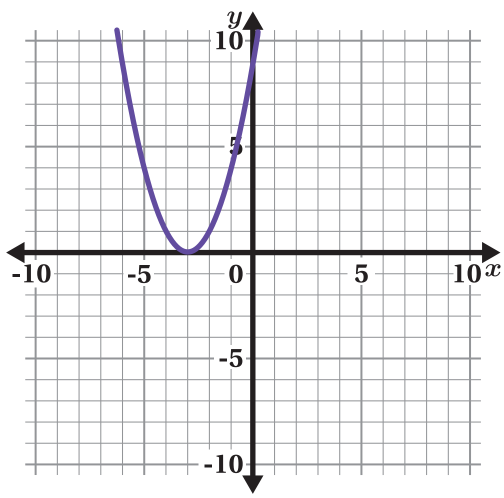
b Write a function whose zeros are $x = -4$ and $x = 2$.

$f(x) =$ ________

9 The function $f(x) = x^2 + 6x + 9$ has exactly one zero.

Write a new quadratic function that has exactly one zero.

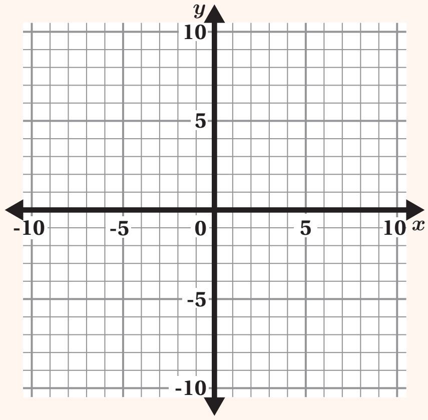
$g(x) =$ ________



Explore More

10 Show or describe as much as you can about the graph of this function:

$$f(x) = (x + 1)(x - 2)(x + 3)$$



11 Synthesis

Describe a strategy for determining the x-intercepts or zeros of a quadratic function.

Use the examples if they help with your thinking.

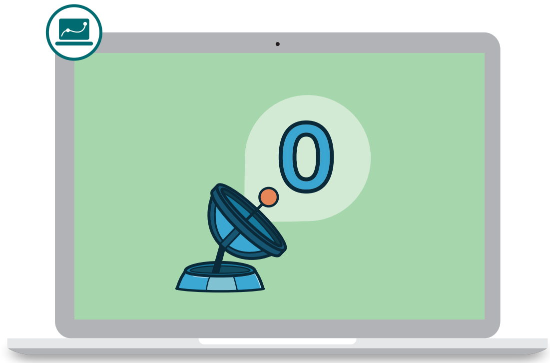
$g(x) = (x + 1)(2x - 4)$

$h(x) = x^2 + 3x - 10$

Things to Remember:

Unit 8
Lesson
7

Name: Date: Period:



Make It Zero

Let's use the zero-product property to solve quadratic equations.

Warm-Up

1 Determine the *solution* to each equation.

$4a = 0$ $0 = 2\pi b$ $6(c - 5) = 0$ $7 \cdot (d + 8) \cdot 9 = 0$

2 The **zero-product property** states: If the product of two or more *factors* is 0, then at least one of the factors is 0.

We can use this to help solve equations like $4a = 0$ or $6(c - 5) = 0$.

Write a new equation using the variable x that the zero-product property could help solve.

Activity 1

Name: ________ Date: ________ Period: ________

Solve It

Use the zero-product property to solve the following equations.

3 $(x - 4)(2x + 3) = 0$

$x =$ ______ $x =$ ______

4 $x^2 + 5x + 4 = 0$

$x =$ ______ $x =$ ______

5 $3x^2 - 18x + 15 = 0$

$x =$ ______ $x =$ ______

Name: .... Date: .... Period: ....

Solve It (continued)

6 Here is Hamza's work from the previous problem.

$$3x^2 - 18x + 15 = 0$$
$$(3x - 3)(x - 5) = 0$$
$$x = 3 \text{ or } x = 5$$

a What is something Hamza did well?

b What is something Hamza can improve?

Name: ________ Date: ________ Period: ________

Zeroing In

7 Inola says you can't use the zero-product property to solve the equation $x^2 - 4 = 3x$.

a. **Discuss:** Why might Inola think that?

b. Describe how you could rewrite the equation so that the zero-product property *can* be used.

8 Solve Inola's equation: $x^2 - 4 = 3x$.

$x =$ ________ $x =$ ________

Name: ______ Date: ______ Period: ______

Zeroing In (continued)

9 The equation $9x^2 = 12x - 4$ has one solution.

What's the solution? Show or explain your thinking.

Explore More

10 Write at least one equation with $x = 2$ and $x = 3$ as solutions.

Try to write some equations you think none of your classmates will write.

11 Synthesis

How can you use the zero-product property to solve a quadratic equation?

Consider the examples if they help with your thinking.

A $(2x + 4)(x + 3) = 0$

B $3x^2 - 18x + 15 = 0$

C $x^2 - 4 = 3x$

D $(x - 5)(x + 1) = 7$

Things to Remember:

Unit 8
Lesson
8

Name: .... Date: .... Period: ....

Zero, One, or Two?

Let's determine whether quadratic equations have zero, one, or two solutions.

Warm-Up

1-2 Determine the value of each expression using mental math.

a. 8^2

b. -8^2

c. $(-8)^2$

d. Solve $x^2 = 64$.

Activity 1

Name: ________ Date: ________ Period: ________

How Many?

3 For each equation, put a check for the number of solutions.

Equation	No Solutions	One Solution	Two Solutions
$(x-3)^2 = 1$			
$(x-3)^2 = 0$			
$(x-3)^2 = -1$			
$(x-3) = 1$			
$(x-3)(x-3) = 1$			

4 Diya says that $x = 4$ is the only solution to $(x-3)^2 = 1$.

a **Discuss:** How do you know that $x = 4$ is a solution to $(x-3)^2 = 1$?

b Write a hint to help Diya determine *another* solution.

5 Here is a new equation: $x^2 - 16 = 9$.

How many solutions does this equation have? Circle one.

No solutions One solution Two solutions

Record any solution(s) here:

$x =$ ________

$x =$ ________

Name: .... Date: .... Period: ....

How Many? (continued)

6 Rewrite the equation $x^2 - 16 = 9$ so that it has no solutions.

Show or explain your thinking.

7 Here is a new equation: $(x - 5)^2 = 36$.

How many solutions does this equation have? Circle one.

No solutions One solution Two solutions

Record any solution(s) here:

$x =$ ....

$x =$ ....

Activity 2

Name: ________ Date: ________ Period: ________

How Many and More

8 For each equation, put a check for the number of solutions.

Equation	No Solutions	One Solution	Two Solutions
$x(x-6)=0$			
$2x^2=50$			
$x^2=-9$			
$x^2+4=0$			
$(x+2)(x+2)=0$			

9 Here are two equations from the previous problem.

$$(x+2)(x+2)=0 \qquad\qquad x^2=-9$$

Explain how you decided on the number of solutions for $(x+2)(x+2)=0$.

Explain how you decided on the number of solutions for $x^2=-9$.

Name: ____________ Date: ________ Period: ________

How Many and More (continued)

10 Solve as many challenges as you have time for.

- Circle how many solutions each equation has.
- Record any solutions.

	Equation	Number of Solutions			Solution(s)
a	$30 = x^2 - 6$	No solutions	One solution	Two solutions	$x =$ ____ $x =$ ____
b	$7x^2 + 1 = 1$	No solutions	One solution	Two solutions	$x =$ ____ $x =$ ____
c	$(x - 4)^2 = -12$	No solutions	One solution	Two solutions	$x =$ ____ $x =$ ____
d	$x(x + 2) = 15$	No solutions	One solution	Two solutions	$x =$ ____ $x =$ ____
e	$(x - 4)(x - 4) = 16$	No solutions	One solution	Two solutions	$x =$ ____ $x =$ ____
f	$3x^2 - 3 = -3$	No solutions	One solution	Two solutions	$x =$ ____ $x =$ ____

11 Synthesis

a Discuss these questions:

- How can you determine the number of solutions to a quadratic equation?
- How can you solve a quadratic equation?

b Select *one question* and record your response.

$(x + 2)(x + 2) = 0$

$x^2 = -9$

$(x - 5)^2 = 36$

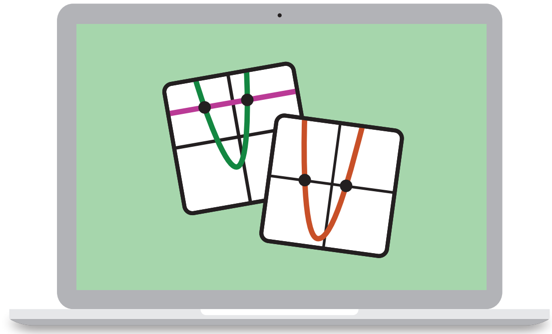
Things to Remember:

Unit 8

Lesson

9

Name: .... Date: .... Period: ....



Graph to Solve

Let's use graphs to solve quadratic equations.

Warm-Up

1 Determine whether each statement is true or false.

Statement	True	False
$x = 2$ and $x = 3$ are the solutions to $(x - 2)(x - 3) = 6$.		
$x = 3$ is the only solution to $x^2 - 9 = 0$.		
$x(x - 7) = 0$ has two solutions.		
$x =$ -5 is a solution to $x^2 + 25 = 0$.		

2 Let's look at the statement Saanvi incorrectly said was true.

a What might have Saanvi been thinking?

b What would you say to Saanvi to help her see that this statement is false?

Activity 1

Name: .... Date: .... Period: ....

When in Doubt, Graph It Out

3 Malik used the Desmos Graphing Calculator to determine the solutions to $(x-2)(x-3)=6$.

a Let's watch the animation to see Malik's strategy.

b **Discuss:** Where in the graph can you see that the solutions are $x=0$ and $x=5$?

4 Use a graphing calculator and Malik's strategy to solve $x^2+2x+1=4$.

How many solutions does this equation have? Circle one.

No solutions   One solution   Two solutions

Record any solution(s) here:

$x=$ ....

$x=$ ....

If you circled *No solutions*, complete the statement:

The equation has no solutions because . . .

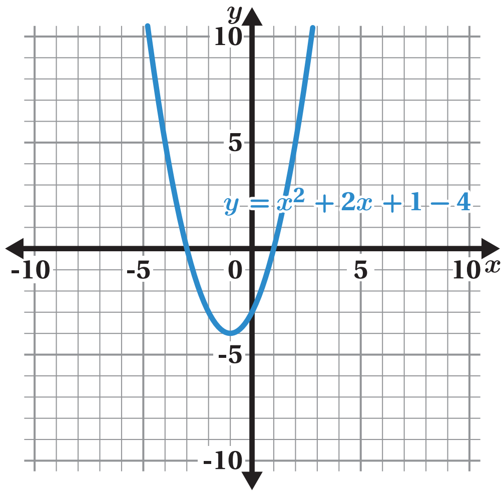
Name: .... Date: .... Period: ....

When in Doubt, Graph It Out (continued)

5 Saanvi also solved $x^2 + 2x + 1 = 4$ by graphing.

She graphed the equation $y = x^2 + 2x + 1 - 4$.

Show or describe where you see the solutions to $x^2 + 2x + 1 = 4$ in Saanvi's graph.



6 Use any strategy to solve $(x - 3)^2 = -1$

How many solutions does this equation have? Circle one.

No solutions One solution Two solutions

Record any solution(s) here:

$x =$ ....

$x =$ ....

If you circled *No solutions*, complete the statement:

The equation has no solutions because . . .

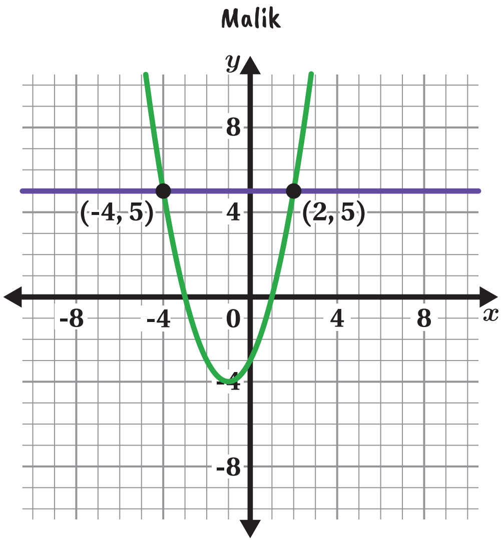
Activity
2

Name: .... Date: .... Period: ....

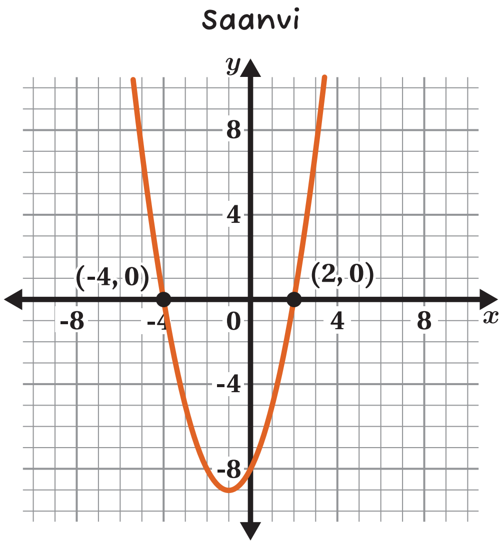
None, One, or Some

7 Malik and Saanvi each used graphing to solve $(x + 3)(x - 1) = 5$.

a Take a look at each student's strategy.



Graph $y = (x + 3)(x - 1)$ and $y = 5$.
Where do the graphs intersect?



Graph $y = (x + 3)(x - 1) - 5$.
What are the x-intercepts?

b **Discuss:**

- How are their strategies alike? How are they different?
- When might you use one strategy or the other?

Name: ________ Date: ________ Period: ________

None, One, or Some (continued)

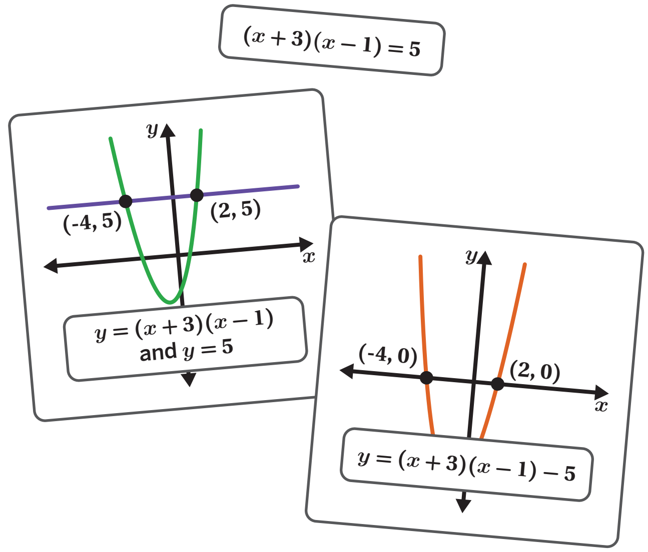
8 Use a graphing calculator to solve as many challenges as you have time for.

- Circle how many solutions each equation has.
- Record any solutions.

	Equation	Number of Solutions			Solution(s)
a	$-4x^2 + 5 = 1$	No solutions	One solution	Two solutions	$x =$ ____ $x =$ ____
b	$(x - 4)(x - 2) = -5$	No solutions	One solution	Two solutions	$x =$ ____ $x =$ ____
c	$2x^2 - x - 4 = 2$	No solutions	One solution	Two solutions	$x =$ ____ $x =$ ____
d	$x(x - 2) = -1$	No solutions	One solution	Two solutions	$x =$ ____ $x =$ ____
e	$7 = x(x - 6)$	No solutions	One solution	Two solutions	$x =$ ____ $x =$ ____
f	$(x + 6)(x + 8) = -1$	No solutions	One solution	Two solutions	$x =$ ____ $x =$ ____

9 Synthesis

Describe a strategy for using a graphing calculator to solve a quadratic equation.



Things to Remember:

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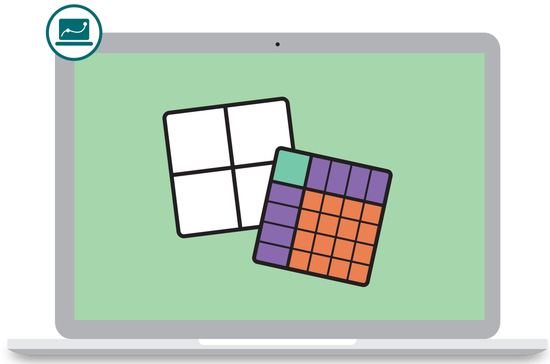


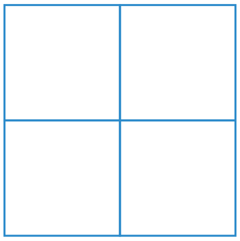
Square Dance

Let's build squares using tiles and algebra.

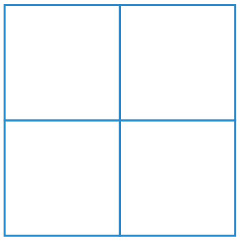
Warm-Up

1 **a** Write each expression in factored form. Use the diagrams if they help with your thinking.

$x^2 + 8x + 16$



$x^2 + 8x + 12$



b **Discuss:** How are $x^2 + 8x + 16$ and $x^2 + 8x + 12$ alike? How are they different?

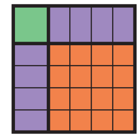
Activity 1

Name: ______ Date: ______ Period: ______

Perfect Squares

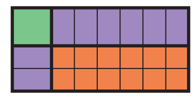
2 $(x+4)^2$ and $x^2+8x+16$ are **perfect squares**.

	x	4
x	x^2	$4x$
4	$4x$	16



$(x+6)(x+2)$ and $x^2+8x+12$ are not perfect squares.

	x	6
x	x^2	$6x$
2	$2x$	12

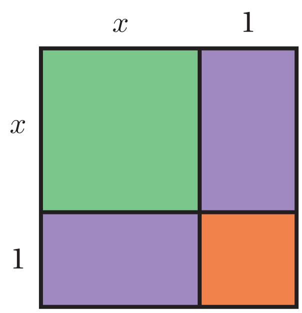


a **Discuss:** What do you think makes an expression a perfect square?

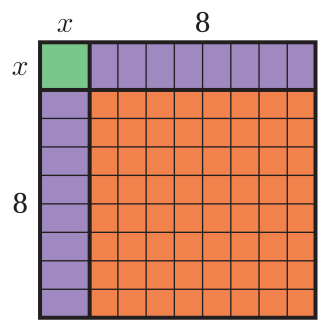
b Write a different expression that is a perfect square.

3 Here are more perfect square expressions written in factored and standard form.

Factored form: $(x+1)^2$
Standard form: x^2+2x+1



Factored form: $(x+8)^2$
Standard form: $x^2+16x+64$



a **Discuss:** What do you notice? What do you wonder?

b Is $x^2+12x+144$ a perfect square? Circle one. Yes No Not enough information

Explain your thinking.

Activity 1

Name: .... Date: .... Period: ....

Perfect Squares (continued)

4 Sort the expressions based on whether they are perfect squares.

$x^2 + 10x + 100$	$x^2 - 24x - 144$	$x^2 + 4$	$x^2 + 5x + 6.25$
$x^2 - 24x + 144$	$x^2 + 10x + 25$	$(x - 4)^2$	

Perfect Square	Not a Perfect Square

5 How did you decide whether the expression $x^2 + 5x + 6.25$ was a perfect square?

Name: .... Date: .... Period: ....

Completing Squares

6 This perfect square is written in factored and standard form. Some numbers are smudged.

Factored Form	**Standard Form**
$(x + \bullet)^2$	$x^2 + 6x + \bullet$

Is there enough information to determine the smudged numbers? Explain your thinking.

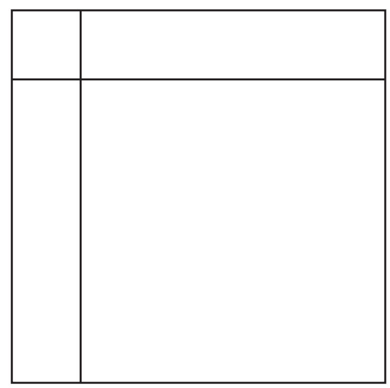
Use algebra tiles if they help with your thinking.

7 Here is a new expression with a smudge.

$x^2 + 22x + \bullet$

If the expression is a perfect square, what number is smudged?

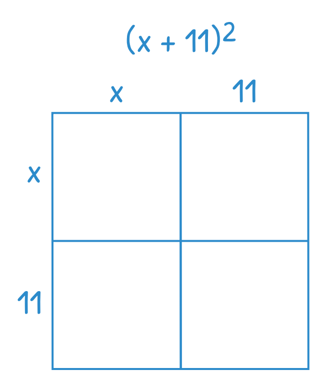
Use the diagram if it helps with your thinking.



8 Sadia wrote the expression $(x + 11)^2$ to help determine the smudged number.

Explain why this makes sense and how it can help Sadia figure out the smudged number.

$x^2 + 22x + \bullet$



Name: ______ Date: ______ Period: ______

Completing Squares (continued)

9 Solve as many challenges as you have time for.

If each expression is a perfect square, what number is missing?

a $x^2 - 10x +$ ______

b $x^2 + 20x +$ ______

c $x^2 +$ ______ $x + 36$

d $x^2 +$ ______ $x + 100$

e $x^2 - 50x +$ ______

f $x^2 +$ ______ $x + 144$

g $x^2 + \frac{1}{5}x +$ ______

h $x^2 + 7x +$ ______

i $x^2 +$ ______ $x + 4$

Explore More

10 Use the Explore More sheet to explore the graph of perfect square equations.

11 Synthesis

How can you determine whether an expression is a perfect square?

$(x+4)^2$

x^2+4

$x^2-14x+49$

$x^2+6x+36$

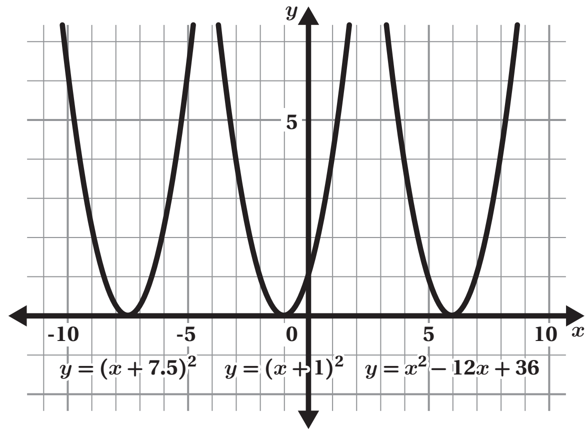
Things to Remember:

Explore More

Sothy noticed that when he graphs perfect square equations, each parabola's vertex is on the x-axis. Will this *always* be true?

Explain your thinking.

Use a graphing calculator if it helps with your thinking.



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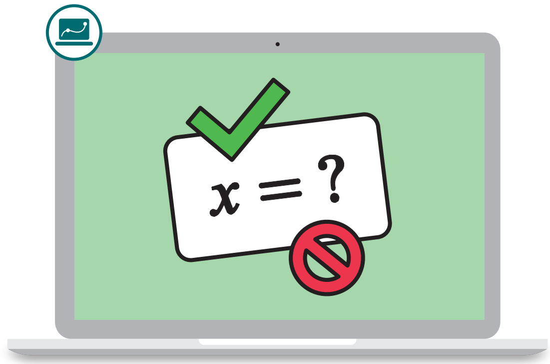


Square Tactic

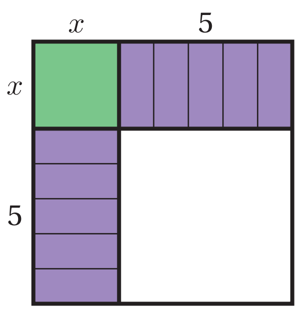
Let's develop a new strategy for solving quadratic equations called "completing the square."

Warm-Up

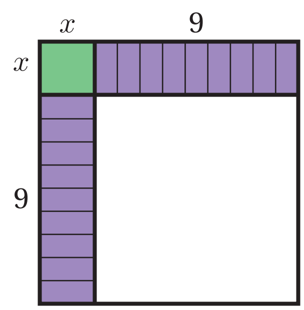
1 Here are seven expressions. The first four are represented with algebra tiles.

For each expression, how many unit tiles do you need to add to make it a perfect square?

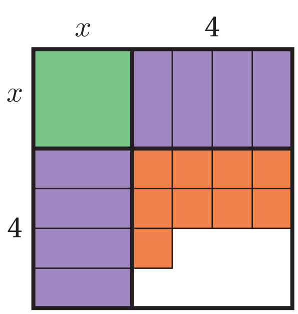
a $x^2 + 10x +$ ______



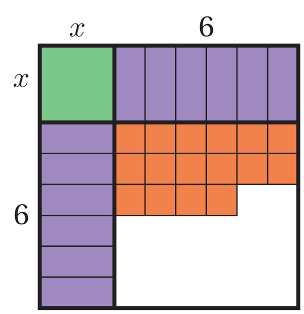
b $x^2 + 18x +$ ______



c $x^2 + 8x + 9 +$ ______



d $x^2 + 12x + 16 +$ ______



e $x^2 + 20x + 70 +$ ______

f $x^2 + 12x + 2 +$ ______

g $x^2 + 16x + 11 +$ ______

Activity 1

Name: .... Date: .... Period: ....

Ancient Equations

2 Here are three equations.

Equation A	Equation B	Equation C
$(x+3)^2 = 25$	$x^2 + 6x + 9 = 25$	$x^2 + 6x = 16$

Discuss:

- What do you notice about each equation's structure?
- Which equations can be solved by taking the square root? Explain how you know.

3 Solve the equation $x^2 - 8x + 16 = 9$.

4 Let's look at how Deven and Tay each solved the previous equation.

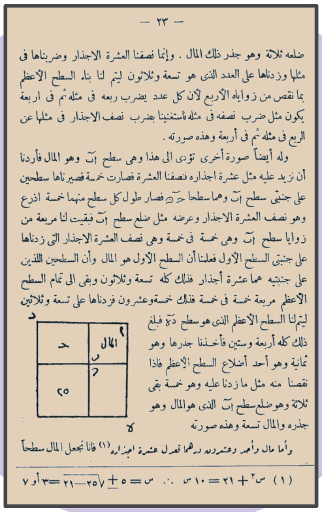
Discuss: What was each student's strategy? When might you use one strategy or the other?

Name: ______ Date: ______ Period: ______

Ancient Equations (continued)

5 The word *algebra* comes from the title of the book *Hisab al-jabr w'al-Muqabala, "The Compendious Book on Calculation by Completion and Balancing."*

Original



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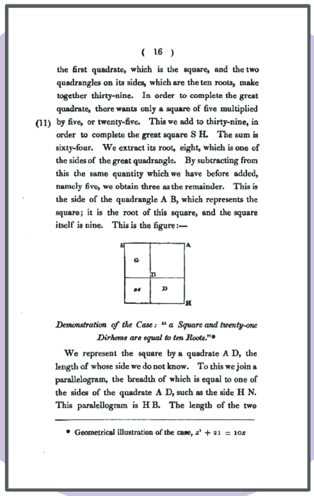
English Translation



(16)

the first quadrate, which is the square, and the two quadrangles on its sides, which are the ten roots, make together thirty-nine. In order to complete the great quadrate, there wants only a square of five multiplied
(11) by five, or twenty-five. This we add to thirty-nine, in order to complete the great square S H. The sum is sixty-four. We extract its root, eight, which is one of the sides of the great quadrangle. By subtracting from this the same quantity which we have before added, namely five, we obtain three as the remainder. This is the side of the quadrangle A B, which represents the square; it is the root of this square, and the square itself is nine. This is the figure:—

*Demonstration of the Case: "a Square and twenty-one Dirhems are equal to ten Roots."**

We represent the square by a quadrate A D, the length of whose side we do not know. To this we join a parallelogram, the breadth of which is equal to one of the sides of the quadrate A D, such as the side H N. This paralellogram is H B. The length of the two

* Geometrical illustration of the case, $x^2 + 21 = 10x$

The book was written in 830 CE by Muhammad ibn Mūsā al-Khwarizmi, the mathematician who many scholars believe began the study of algebra.

The focus of the book is solving equations, including: $x^2 + 10x = 39$.

Discuss: What are some different ways you could solve this equation?

6 Here is a translated version of the author's first step in solving the equation.

How does this help solve the equation?

al-Khwarizmi

$$x^2 + 10x = 39$$

$$x^2 + 10x + 25 = 39 + 25$$

Activity
2

Name: ______ Date: ______ Period: ______

Completing the Square

7 al-Khwarizmi's process is called **completing the square**.

Solve $x^2 + 14x = 31$ by completing the square.

8 Solve the equation $x^2 + 6x + 4 = -3$.

Name: .... Date: .... Period: ....

Completing the Square (continued)

9 Roberto made a mistake while solving the equation $x^2 - 12x + 6 = 14$.

What did Roberto do well? What should he fix?

Roberto

$x^2 - 12x + 6 = 14$

$x^2 - 12x + 36 = 14 + 36$

$(x - 6)^2 = 50$

$x - 6 = \pm\sqrt{50}$

$x = 6 \pm \sqrt{50}$

Explore More

10 a **Discuss:** How many solutions does the equation $x^2 + 10x = -60$ have?

b Can you write another equation of the form $x^2 + 10x =$ .... that has . . .

No solutions: $x^2 + 10x =$ ....

One solutions: $x^2 + 10x =$ ....

Two solutions: $x^2 + 10x =$ ....

11 Synthesis

Here are the solving strategies you've seen in this unit: factoring, graphing, and completing the square.

$x^2 + 8x + 2 = 0$

$x^2 + 10x = 39$

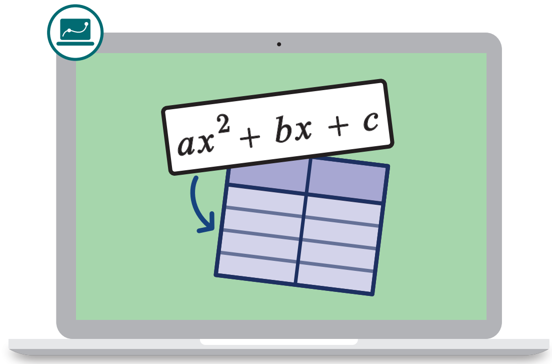
$x^2 + 5x + 6 = 2$

Discuss:

- Which strategy would you use for each equation?
- What are the advantages and disadvantages of completing the square?

Things to Remember:

Name: .... Date: .... Period: ....



Formula Foundations

Let's explore how the quadratic formula can be derived from the process of completing the square.

Warm-Up

1 Alma wants to solve $5x^2 + 9x + 3 = 0$ by completing the square.

Here is her first step.

Alma

$$5x^2 + 9x + 3 = 0$$

$$x^2 + \frac{9x}{5} + \frac{3}{5} = 0$$

Discuss:

- What did Alma do?
- Why do you think this is her first step?
- What would you do next?

Activity 1

Name: ________ Date: ________ Period: ________

Completing Any Square

2 Here are the rest of the steps Alma took to solve $5x^2 + 9x + 3 = 0$. Describe what she does in each step.

Steps	Description
$5x^2 + 9x + 3 = 0$	Original equation
$x^2 + \frac{9}{5}x + \frac{3}{5} = 0$	Divide by 5
$x^2 + \frac{9}{5}x = -\frac{3}{5}$	
$x^2 + \frac{9}{5}x + \left(\frac{9}{2 \cdot 5}\right)^2 = -\frac{3}{5} + \left(\frac{9}{2 \cdot 5}\right)^2$	
$\left(x + \frac{9}{10}\right)^2 = -\frac{3}{5} + \left(\frac{9}{10}\right)^2$	
$x + \frac{9}{10} = \pm\sqrt{-\frac{3}{5} + \left(\frac{9}{10}\right)^2}$	
$x = -\frac{9}{10} \pm \sqrt{-\frac{3}{5} + \left(\frac{9}{10}\right)^2}$	
$x = \frac{-9 \pm \sqrt{9^2 - 4 \cdot 5 \cdot 3}}{2 \cdot 5}$	Rewrite

3 Let's look at how we can use Alma's steps to solve other quadratic equations.

Discuss: What do you notice? What do you wonder?

Name: .... Date: .... Period: ....

Completing Any Square (continued)

4 Felipe notices that you can write the solutions to an equation without completing the square.

How would the solutions change if the original equation were $3x^2 + 8x - 15 = 0$?

Steps	Description
$10x^2 + 7x + 1 = 0$	Original equation
$x^2 + \frac{7}{10}x + \frac{1}{10} = 0$	Divide by 10
$x^2 + \frac{7}{10}x = -\frac{1}{10}$	Subtract $\frac{1}{10}$
$x^2 + \frac{7}{10}x + \left(\frac{7}{2 \cdot 10}\right)^2 = -\frac{1}{10} + \left(\frac{7}{2 \cdot 10}\right)^2$	Complete the square
$\left(x + \frac{7}{20}\right)^2 = -\frac{1}{10} + \left(\frac{7}{20}\right)^2$	Rewrite the left side as a perfect square
$x + \frac{7}{20} = \pm\sqrt{-\frac{1}{10} + \left(\frac{7}{20}\right)^2}$	Take the square root
$x = -\frac{7}{20} \pm \sqrt{-\frac{1}{10} + \left(\frac{7}{20}\right)^2}$	Subtract $\frac{7}{20}$
$x = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 10 \cdot 1}}{2 \cdot 10}$	Rewrite

Activity 2

Name: ____________ Date: ________ Period: ________

Formula-izing

5 We just discovered a way to write the solutions for any quadratic equation without completing the square!

Use the variables a, b, and c to represent the solutions to $ax^2 + bx + c = 0$.

Equation:

$$7x^2 - 9x + 5 = 0$$

Solutions:

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \cdot 7 \cdot 5}}{2 \cdot 7}$$

Equation:

$$\boxed{a}\, x^2 + \boxed{b}\, x + \boxed{c} = 0$$

Solutions:

$$x = \frac{-\square \pm \sqrt{\square^2 - 4 \cdot \square \cdot \square}}{2 \cdot \square}$$

6 **For any quadratic equation:** $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This equation is known as the **quadratic formula**.

Discuss:

- What do the a, b, and c in the formula represent?
- Why is there a $\pm$ symbol in the formula?
- What new things could this formula help you do?

Step	Description
$ax^2 + bx + c = 0$	Original equation
$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	Divide by a
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Subtract $\frac{c}{a}$
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Complete the square
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Rewrite the left side as a perfect square
$\left(x + \frac{b}{2a}\right) = \pm\sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Take the square root
$x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Subtract $\frac{b}{2a}$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Rewrite

Activity 3

Name: ____________ Date: ________ Period: ________

Solution Search

7 You will use the digital activity to explore solutions to quadratic equations.

a Here is an equation that has *two integer solutions*. Find two more equations with two integer solutions.

Equation	$1x^2 - 5x + 6 = 0$		
Solutions	$x = \dfrac{5 \pm \sqrt{1}}{2}$	$x =$	$x =$

b Here is an equation that has *one solution*. Find two more equations with one solution.

Equation	$1x^2 + 4x + 4 = 0$		
Solutions	$x = \dfrac{-4 \pm \sqrt{0}}{2}$	$x =$	$x =$

c Here is an equation that has *no solution*. Find two more equations with no solution.

Equation	$7x^2 + 2x + 5 = 0$		
Solutions	$x = \dfrac{-2 \pm \sqrt{-136}}{14}$	$x =$	$x =$

d Examine the equations and solutions you found.

Discuss: What patterns do you notice?

8 Synthesis

A classmate who is absent today asks for your help.

What would you say to help them understand where the quadratic formula came from?

Step	Description
$ax^2 + bx + c = 0$	Original equation
$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	Divide by a
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Subtract $\frac{c}{a}$
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Complete the square
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Rewrite the left side as a perfect square
$\left(x + \frac{b}{2a}\right) = \pm\sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Take the square root
$x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Subtract $\frac{b}{2a}$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Rewrite

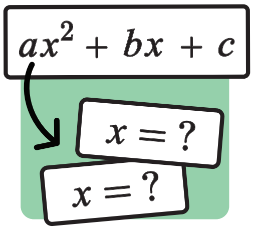
Things to Remember:

Unit 8
Lesson
15

Name: ______ Date: ______ Period: ______

Formula Fluency

Let's use the quadratic formula to solve quadratic equations.



Warm-Up

1. The quadratic formula can be used to find the solutions to any quadratic equation in the form $ax^2 + bx + c = 0$.

Determine the a-, b-, and c-values of the following equations.

a $3x^2 - 8x + 15 = 0$

$a =$ ______ $b =$ ______ $c =$ ______

b $x^2 + 4 + 3x = 0$

$a =$ ______ $b =$ ______ $c =$ ______

c $5x^2 - 20 = 0$

$a =$ ______ $b =$ ______ $c =$ ______

d $-x^2 + 2x = -12$

$a =$ ______ $b =$ ______ $c =$ ______

Activity 1

Name: ______ Date: ______ Period: ______

Form Over Function

2. Here are four quadratic equations and their solutions.

Use the quadratic formula to show that the solutions are correct.

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a $x^2 - 8x + 15 = 0$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$$

Solutions: $x = 5$ and $x = 3$

b $x^2 + 10x + 18 = 0$

Solutions: $x = -5 \pm \frac{\sqrt{28}}{2}$

c $9x^2 - 6x = -1$

Solution: $x = \frac{1}{3}$

d $2x^2 + 6x + 5 = 0$

No solutions

3. **Discuss**: Do you think that the quadratic formula is the best strategy for solving each of these equations? Explain your thinking.

Activity
2

Name: .... Date: .... Period: ....

Error Analysis

You will use a sheet for this activity with the same equations from the previous activity.

There is an error in each attempt to solve the equation.

4. 💬 **Discuss:**

- What is the error in each attempt?
- How would you correct the error?
- Why might someone make this error?

5. Solve the following equation using the quadratic formula, but include an error that you think would be common.

$3x^2 - 6x - 1 = 0$

6. Swap equations with a classmate. Identify and describe the error in each other's work.

Name: .... Date: .... Period: ....

Error Analysis (continued)

7. **Reflect:** What kinds of errors do you think you are most likely to make when using the quadratic formula?

8. Write two pieces of advice that will help your future self correctly use the quadratic formula. Include examples if they help with your thinking.

Synthesis

9. What are some advantages of using the quadratic formula to solve quadratic equations?

What are some disadvantages?

Use the examples if they help with your thinking.

$x^2 - 6x + 8 = 0$

$x^2 + 4x - 1 = 0$

$2x^2 + 7x - 10 = 0$

Things to Remember:

Name: .... Date: .... Period: ....

Error Analysis

a $x^2 - 8x + 15 = 0$

$a = 1$, $b = $ -8, $c = 15$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{-64 - 60}}{2}$$

$$x = \frac{8 \pm \sqrt{-124}}{2}$$

No solutions

b $x^2 + 10x + 18 = 0$

$a = 1$, $b = 10$, $c = 18$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(18)}}{2(1)}$$

$$x = \frac{-10 \pm \sqrt{100 - 72}}{2}$$

$$x = \frac{-10 \pm \sqrt{28}}{2}$$

$$x = -5 \pm \sqrt{14}$$

c $9x^2 - 6x = -1$

$a = 9$, $b = -6$, $c = -1$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(-1)}}{2(9)}$$

$$x = \frac{6 \pm \sqrt{36 + 36}}{18}$$

$$x = \frac{6 \pm \sqrt{72}}{18}$$

d $2x^2 + 6x + 5 = 0$

$a = 2$, $b = 6$, $c = 5$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{-6 \pm \sqrt{36 - 40}}{4}$$

$$x = \frac{-6 \pm \sqrt{-4}}{4}$$

$$x = \frac{-6 \pm 2}{4}$$

$x = -2$ and $x = -1$

Unit 8
Lesson
16

Name: .... Date: .... Period: ....

Stomp Rockets in Space

Let's solve quadratic equations and explain what the solutions mean for a situation.

Warm-Up

1 Here is a stomp rocket on another planet.

The function $h(t) = -3t^2 + 20t + 4$ represents the height, in meters, of the stomp rocket t seconds after it has been launched.

a Let's look at a stomp rocket launch.

b Write a question about the stomp rocket that $h(t)$ could help you answer.

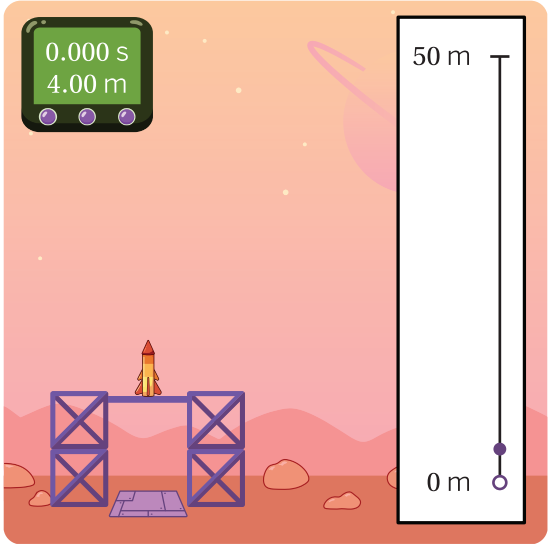
Activity 1

Name: .... Date: .... Period: ....

Rocket Time

2 The function $h(t) = -3t^2 + 20t + 4$ represents the height, in meters, of the stomp rocket t seconds after it has been launched.

When will the rocket touch the ground? Round to three decimal places if necessary.



3 Let's look at Makayla's work from the previous problem.

Makayla says that the rocket will touch the ground at about -0.194 seconds and 6.861 seconds.

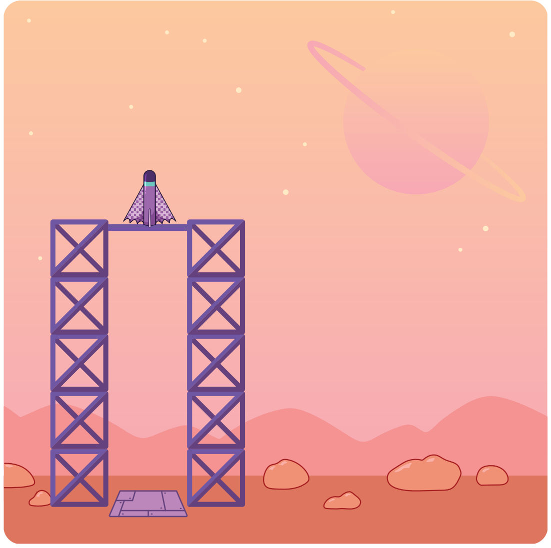
Discuss:

- Why did Makayla substitute 0 for $h(t)$?
- What is correct about Makayla's response?
- What is incorrect? Why?

Name: ________ Date: ________ Period: ________

Beam Me Up

4 Here is a new rocket.



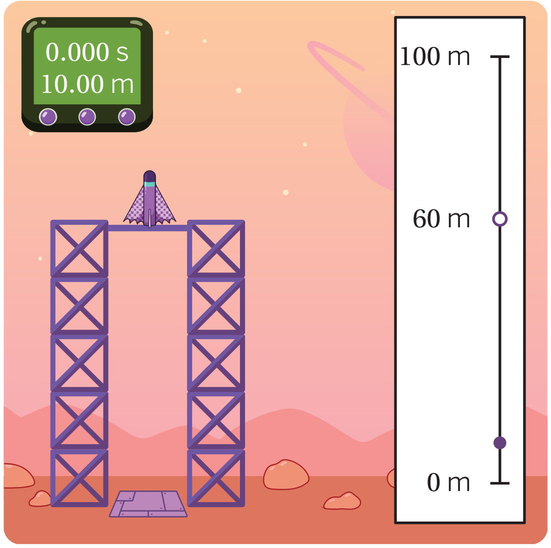
The function $h(t) = \text{-}8t^2 + 40t + 10$ represents the height, in meters, of the rocket t seconds after it has been launched.

The rocket reaches a maximum height of 60 meters.

Write an equation that can be solved to determine when the rocket is at its maximum height.

Explain your thinking.

5 Here is an equation someone wrote on the previous screen: $60 = \text{-}8t^2 + 40t + 10$.



How many seconds will it take for the rocket to reach its maximum height?

Round to three decimal places if necessary.

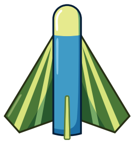
Name: ______ Date: ______ Period: ______

Beam Me Up (continued)

6 Here is a new rocket.

$h(t) = \text{-}4t^2 + 30t + 10$

The function $h(t) = \text{-}4t^2 + 30t + 10$ represents the height, in meters, of the rocket t seconds after it has been launched.



Does this rocket ever reach a height of 100 meters? Circle one.

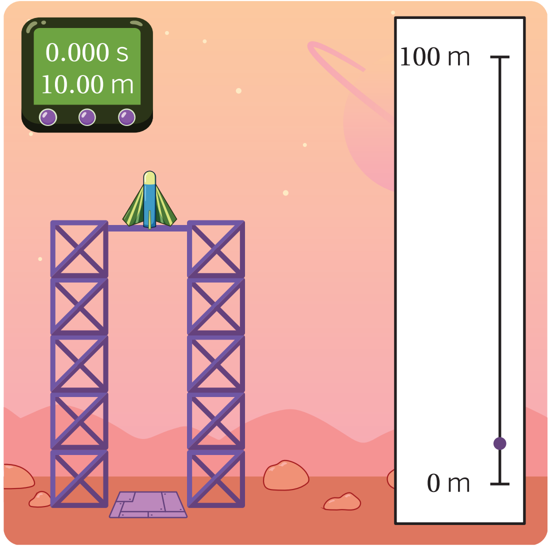
Yes　　　No

Show or explain your thinking.

7 The function $h(t) = \text{-}4t^2 + 30t + 10$ represents the height, in meters, of the stomp rocket t seconds after it has been launched.

a Let's watch the stomp rocket launch.

b Write a question about the height of the rocket that will have two answers.



Name: ______ Date: ______ Period: ______

Rocket Scientist

8 You will use the Activity 3 Sheet to choose your own stomp rocket.

a Choose It!

- On the activity sheet, choose a stomp rocket.
- In this table, write down the function, the question you selected, and the solution to the question.

Round your solution to three decimal places if necessary.

Function:	$h(t) =$
Question:	
Solution:	

b Swap It!

- Share your stomp rocket with a partner who has a different rocket.
- Solve the question they chose for their stomp rocket.

Round your solution to three decimals places if necessary.

Partner 1's Rocket

Function:	$h(t) =$
Question:	
Solution:	

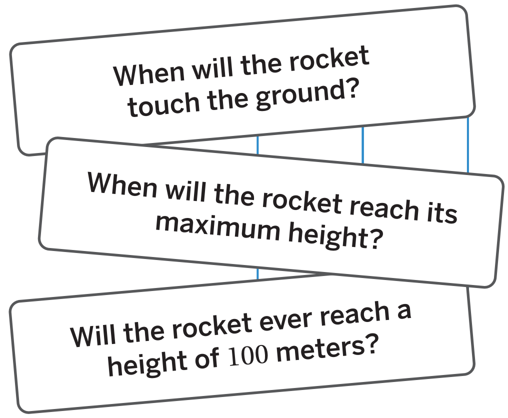
Partner 2's Rocket

Function:	$h(t) =$
Question:	
Solution:	

9 Synthesis

Describe a strategy that helped you answer the question in today's lesson.

If you learned it from another student, give them a shout-out!



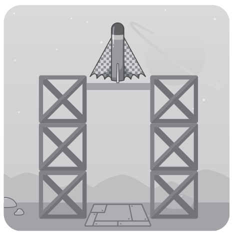
Things to Remember:

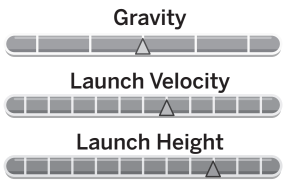
Name: ______ Date: ______ Period: ______

Rocket Scientist

- Choose one stomp rocket.
- Select one of the questions about your stomp rocket.
- Solve the question you chose on the lesson page.

Rocket 1

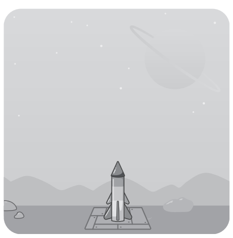


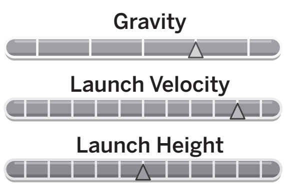


$h(t) = -2t^2 + 6t + 6$

- When will the rocket touch the ground?
- When will the rocket reach its maximum height of 10.5 meters?
- When will the rocket return to its original height of 6 meters?

Rocket 2

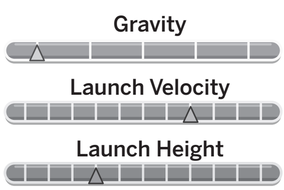




$h(t) = -2.5t^2 + 24t$

- When will the rocket land on the ground?
- When will the rocket reach its maximum height of 57.6 meters?

Rocket 3



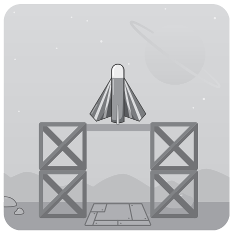
$h(t) = -t^2 + 12t - 4$

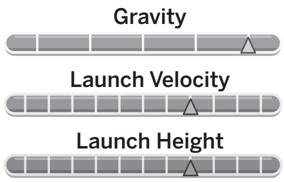
- When will the rocket touch the ground?
- When will the rocket reach its maximum height of 32 meters?

Name: .... Date: .... Period: ....

Rocket Scientist

Rocket 4

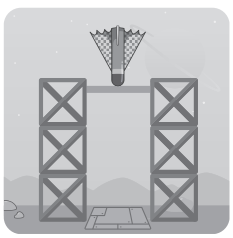


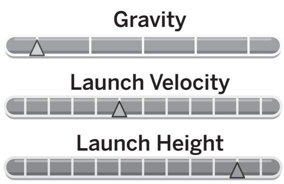


$h(t) = -3t^2 + 12t + 4$

- When will the rocket touch the ground?
- When will the rocket reach its maximum height of 16 meters?
- When will the rocket return to its original height of 4 meters?

Rocket 5

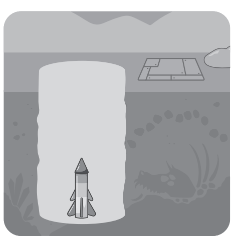


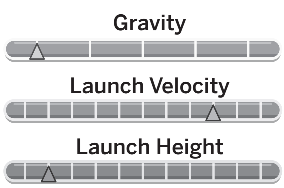


$h(t) = -t^2 - 6t + 8$

- When will the rocket touch the ground?

Rocket 6

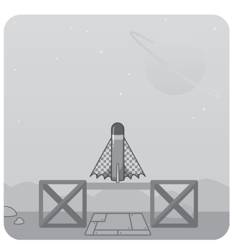


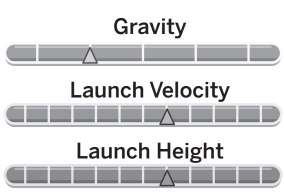


$h(t) = -t^2 + 18t - 8$

- When will the rocket touch the ground?
- When will the rocket reach its maximum height of 73 meters?

Rocket 7





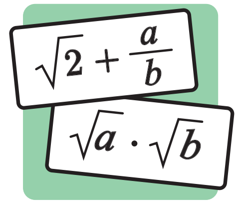
$h(t) = -1.5t^2 + 6t + 2$

- When will the rocket touch the ground?
- When will the rocket reach its maximum height of 8 meters?
- When will the rocket return to its original height of 2 meters?

Unit 8
Lesson
17

Name: ________ Date: ________ Period: ________

Sums and Products



Let's make arguments about why the sums and products of rational and irrational numbers are always, sometimes, or never rational.

Warm-Up

1. Here are four claims.

   Discuss: Is each claim *always*, *sometimes*, or *never* true?

   a. The sum of two even numbers is an even number.

   b. The product of two negative numbers is negative.

   c. The sum of an odd number and an even number is even.

   d. The product of two numbers is larger than either number.

2. For the last claim, how would you convince someone that it is always, sometimes, or never true?

Name: .... Date: .... Period: ....

Rational and Irrational Numbers

Here are some examples of *rational numbers* and *irrational numbers*.

Rational Numbers			Irrational Numbers		
2	-2.3	$\frac{2}{3}$	2	π (or $3.14159\ldots$)	$\sqrt{-5}$
$\sqrt{9}$	$\sqrt{\frac{9}{4}}$		$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{3}{4}}$	

3. **Discuss:** What do you notice about the numbers in the table? What do you wonder?

4. Use the examples in the table to help you define rational and irrational numbers.

Rational number:

Irrational number:

5. Determine whether each number is *rational* or *irrational*. Circle one.

a. $-\sqrt{4}$ Rational Irrational

b. $-\sqrt{5}$ Rational Irrational

c. 0 Rational Irrational

d. $0.\overline{3}$ Rational Irrational

e. $\frac{7}{3}$ Rational Irrational

f. $\sqrt{\frac{7}{3}}$ Rational Irrational

Name: ________ Date: ________ Period: ________

Sums and Products of Rational Numbers

Let's explore the *sums* of two rational numbers.

6. With a partner, determine whether each sum is *rational* or *irrational*.

a $2 + (-2.3)$ Rational Irrational

b $2 + \frac{2}{3}$ Rational Irrational

c $\frac{2}{3} + 0.\overline{1}$ Rational Irrational

d $\sqrt{9} + (-2.3)$ Rational Irrational

e $\sqrt{9} + \sqrt{4}$ Rational Irrational

f $2 + (-2.3)$ Rational Irrational

7. Tyani says: *I know that the value of* $\frac{11}{43} + \frac{273}{101}$ *is rational without even calculating it.*

Discuss: Do you agree or disagree? Why?

8. Fill in the blank with *always*, *sometimes*, or *never*. Then explain your thinking.

The sum of two rational numbers is ________ rational.

Name: ________ Date: ________ Period: ________

Sums and Products of Rational Numbers (continued)

Let's explore the *products* of two rational numbers.

9. With a partner, determine whether each product is *rational* or *irrational*.

a $0 \cdot -2.3$ Rational Irrational

b $\frac{2}{3} \cdot 2$ Rational Irrational

c $\frac{3}{4} \cdot 0.\overline{1}$ Rational Irrational

d $\sqrt{9} \cdot (-1)$ Rational Irrational

e $\sqrt{9} \cdot \sqrt{4}$ Rational Irrational

f $0 \cdot -2.3$ Rational Irrational

10. Tyani says: *I know that* $\frac{11}{43} \cdot \frac{273}{101}$ *is rational without even calculating it.*

Discuss: Do you agree or disagree? Why?

11. Fill in the blank with *always*, *sometimes*, or *never*. Then explain your thinking.

The product of two rational numbers is ________ rational.

Activity
3

Name: ________ Date: ________ Period: ________

Sums and Products of Rational and Irrational Numbers

Let's explore the *sums* of rational and irrational numbers.

12. Determine whether $\sqrt{2} + \frac{a}{b}$ is rational or irrational when:

$a = 3$ and $b = 1$ $a =$ -5 and $b = 2$

13. Choose any values for a and b so that $\frac{a}{b}$ is rational.

$a =$ ________ $b =$ ________

14. For the values you chose, determine whether $\sqrt{2} + \frac{a}{b}$ is rational or irrational.

15. Are there any integers a and b that make $\sqrt{2} + \frac{a}{b}$ a rational number? Explain your thinking.

16. Fill in the blank with *always*, *sometimes*, or *never*.

The sum of a rational number and an irrational number is ________ rational.

Let's explore the *products* of rational and irrational numbers.

17. Determine whether $\sqrt{5} \cdot \frac{c}{d}$ is rational or irrational when $c = 5$ and $b = 9$.

18. Are there any non-zero integers c and d that make $\sqrt{5} \cdot \frac{c}{d}$ a rational number? Explain your thinking.

19. Fill in the blank with *always*, *sometimes*, or *never*.

The product of a non-zero rational number and an irrational number is ________ rational.

Synthesis

20. Choose one of the example expressions.

A. $3 \cdot \pi$ **B.** $\frac{1}{2} + \pi$ **C.** $\sqrt{16} + \sqrt{25}$ **D.** $\frac{3}{4} \cdot \sqrt{36}$

Determine whether the sum or product is rational or irrational. Explain how you know.

Things to Remember: