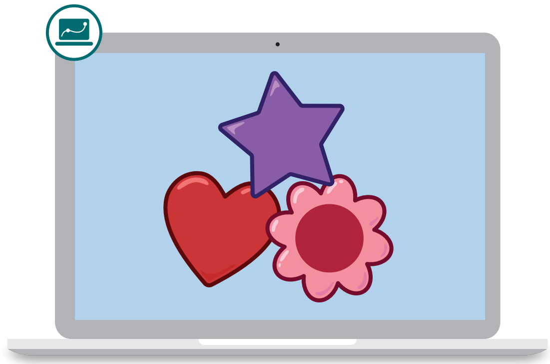
Name: .... Date: .... Period: ....



Shape It Up

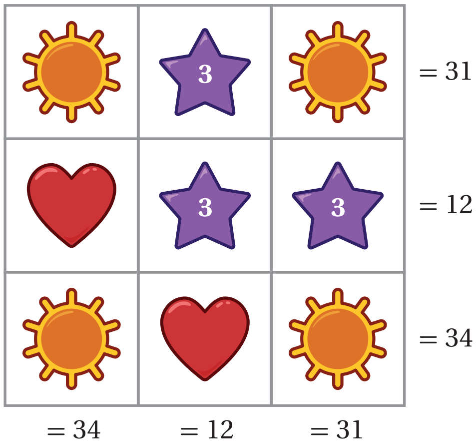
Let’s use reasoning to solve shape puzzles.

Warm-Up

1 Here is a shape puzzle. The *sum* of each row and column is shown.

Determine the value of the heart and the sun.

Shape	Value
Heart	6
Star	3
Sun	14



Activity
1

Name: ______ Date: ______ Period: ______

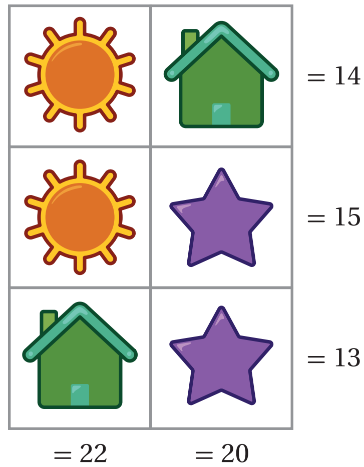
Shape Puzzle Strategies

2 Here is a different shape puzzle.

Jayden thinks that each sun has a value of 10.

Show or explain why that is not possible.

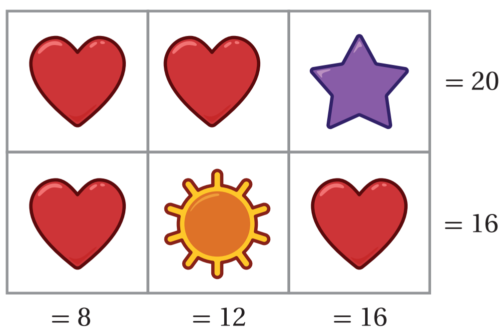
***Responses vary.* In the first row, if a sun has a value of 10, then a house would have a value of 4, but those values don't make 22 in the first column. The solution needs to work for every row and column.**



3 Here is a shape puzzle.

Determine the *solution* for this puzzle.

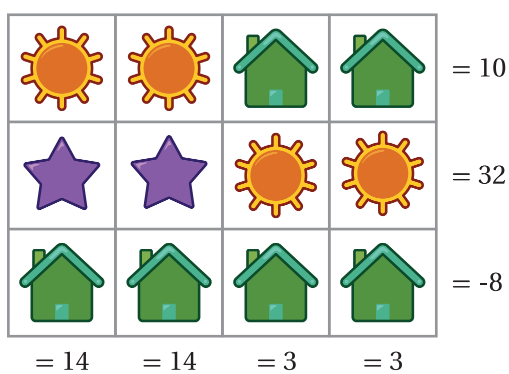
Shape	Value
Heart	**4**
Star	**12**
Sun	**8**



4 Here is a shape puzzle.

Determine the solution for this puzzle.

Shape	Value
Star	**9**
Sun	**7**
House	**-2**

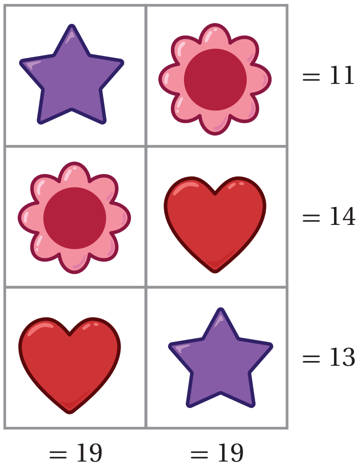


Name: .... Date: .... Period: ....

Shape Puzzle Strategies (continued)

5 Here is a shape puzzle.

Determine the solution for this puzzle.



Shape	Value
Heart	**8**
Star	**5**
Flower	**6**

6 Let's take a look at Jayden's first step for solving the puzzle on the previous problem.

How is this helpful in solving the puzzle?

***Responses vary.* Knowing that a star and a flower together equal 11 is helpful because you can use the left column to determine that the value of the heart is 8. Knowing the value of the heart lets you determine the value of the other shapes.**

Activity
2

Name: ______ Date: ______ Period: ______

Make Your Own Puzzle

7 In the digital activity, create your shape puzzle. Use this page to support your thinking. ***Puzzles and responses vary.***

a **Make It!** Fill your puzzle with shapes! You can use as many as four different shapes.

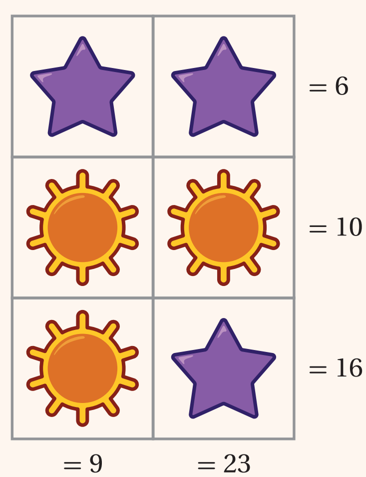
b **Solve It!** Determine the value of each shape in your puzzle.

Explore More

8 Here is a different shape puzzle.

Does this puzzle have a solution? Explain your thinking.

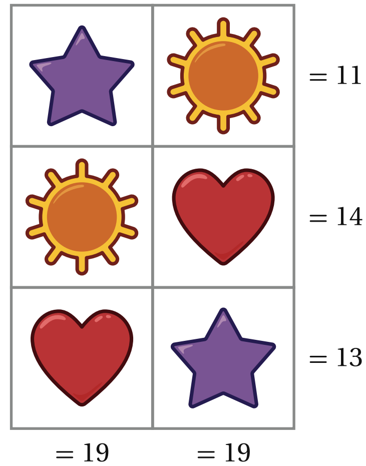
This puzzle has no solutions. *Explanations vary.* There's no one value for a sun and no one value for a star that could make the correct total values for each row and column.



9 Synthesis

Describe some strategies for solving shape puzzles.

Use the puzzle if it helps with your thinking.



Responses vary.

- **If you have a row or column with all the same shape, then you can divide to find the value of that shape.**
- **Take the sum of two shapes and replace those two shapes with their value in another row or column.**
- **Once you know the value of one shape, it gets easier to find the value of the other shapes.**

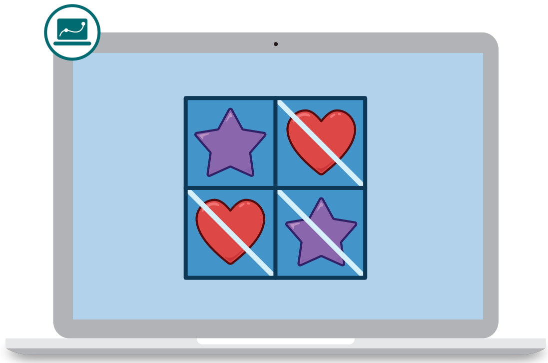
Things to Remember:

Unit 5
Lesson
2

Name: ______ Date: ______ Period: ______

Eliminating Shapes

Let's solve systems of equations by adding or subtracting the equations to eliminate a variable.



Warm-Up

Determine an expression that makes each equation true for any value of x and y.

1. $3x +$ ___ -$3x$ ___ $= 0$

2. $3x -$ ___ $3x$ ___ $= 0$

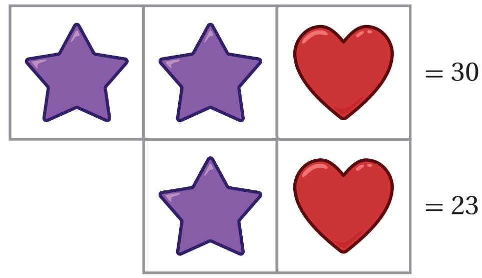
3. $(3x + y) - ($ ___ $3x + y$ ___ $) = 0$

4. $(3x + y) + ($ ___ $-3x - y$ ___ $) = 0$

Name: .... Date: .... Period: ....

Adding and Subtracting Equations

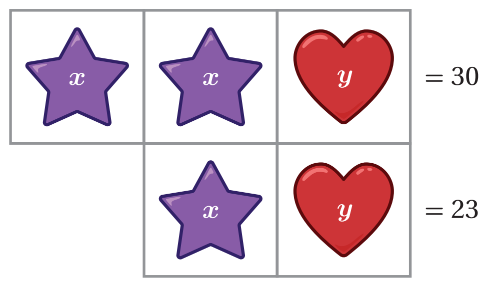
5 Here is a shape puzzle. The sum of each row is shown.



Determine the solution for this puzzle.

Shape	Value
Heart	**16**
Star	**7**

6 This shape puzzle could be written as a *system of equations*, where x is the value of each star and y is the value of each heart.



$$2x + y = 30$$

$$x + y = 23$$

Explain how this system of equations is like the puzzle.

Responses vary.

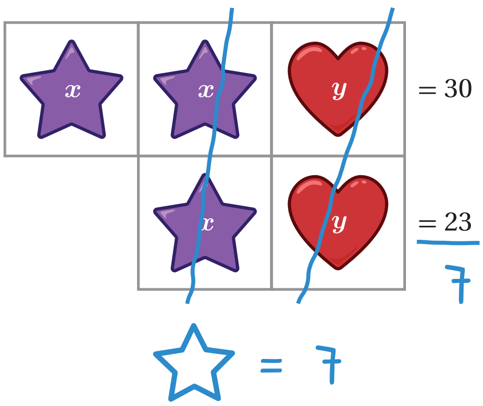
- **The puzzle has two rows and the system has two equations.**
- **You can see how many of each shape there is by looking at the coefficients in the equations.**

Name: ______________________ Date: __________ Period: __________

Adding and Subtracting Equations (continued)

7 Here is how Ebony and Nia each determined the value of a star.

Ebony



Nia

$$\begin{array}{r} 2x + y = 30 \\ -(x + y = 23) \\ \hline x + 0 = 7 \\ x = 7 \end{array}$$

Discuss: Where do you see subtraction in each strategy?

Responses vary.

- I can see subtraction in Ebony's strategy where she canceled out a pair of stars and a pair of hearts.
- Nia subtracted the second equation from the first one. She used parentheses to show that she is subtracting every part of the equation.
- They both subtracted 23 from 30 to get 7.

Activity 2

Name: ______ Date: ______ Period: ______

Elimination

8 Here is a new system of equations.

$x + 2y = 10$

$x + y = 7$

Determine the values of x and y that make both equations true (the *solution to the system*).

Draw a puzzle if it helps with your thinking.

$x =$ **4**, $y =$ **3**

9 Ebony and Nia want to eliminate the y's in this system of equations.

$$-2x + y = 9$$

$$8x - y = 3$$

- Ebony says to *add* the equations.
- Nia says to *subtract* the equations.

Whose strategy will eliminate the y's? Circle one.

Ebony's | Nia's | Both | Neither

Explanations vary.

- **The y's are already opposites (one is positive and one is negative), so adding them will make 0 y's.**
- **If you subtract a negative y, that is the same as adding, which would give you $2y$, not 0.**

Name: .... Date: .... Period: ....

Elimination (continued)

10 Determine the *solution to the system of equations* from the previous problem:

$-2x + y = 9$

$8x - y = 3$

$x =$ 2, $y =$ 13

11 The strategy of adding or subtracting equations to eliminate a variable is called **elimination**.

Nia says elimination works because it's like adding or subtracting the same value from each side of an equation.

Explain what Nia is saying in your own words.

Nia

$$\begin{array}{r} 2x + y = 30 \\ -(x + y = 23) \\ \hline x + 0 = 7 \\ x = 7 \end{array}$$

Responses vary.

- **It's like hanger diagrams: to keep them balanced, you can do something of equal value to both sides.**
- **In the example, $x + y$ is equal to 23, so subtracting $x + y$ from one side is the same as subtracting 23 from the other.**

12 Determine the solution to this system of equations:

$-7x - 5y = 15$

$7x + 3y = 12$

$x =$ 7.5, $y =$ -13.5

Activity
3

Name: Date: Period:

Elimination Repeated Challenges

13 Choose four of the systems of equations and solve them using elimination.

A. $5x + 3y = 21$
$2x + 3y = 12$
$x = 3, y = 2$

B. $8x + 5y = 12$
$8x + 3y = 4$
$x = -1, y = 4$

C. $2x + 3y = 14$
$-2x + 7y = 6$
$x = 4, y = 2$

D. $9x + 3y = -3$
$4x - 3y = -23$
$x = -2, y = 5$

E. $2x + 3y = 4$
$2x + 7y = -12$
$x = 8, y = -4$

F. $y = 4x - 1$
$y = 6x - 7$
$x = 3, y = 11$

14 Synthesis

How can you determine whether to add or subtract equations in order to eliminate a variable?

***Responses vary*. To eliminate a variable, you should add equations when you see a pair of terms that have opposite signs and subtract equations when you see terms that have the same sign in each equation.**

$$2x + y = 30$$
$$x + y = 23$$

$$x + 2y = 10$$
$$x + y = 7$$

$$-2x + y = 9$$
$$8x - y = 3$$

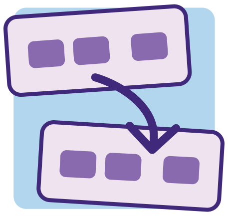
$$-7x - 5y = 15$$
$$7x + 3y = 12$$

Things to Remember:

Unit 5
Lesson
3

Name: ________ Date: ________ Period: ________

Process of Elimination



Let's create equivalent equations to eliminate a variable.

Warm-Up

Here are some linear equations.

Equation A	Equation B	Equation C
$x + 2y = 11$	$4x + y = 2$	$5x + 10y = 55$
Equation D	**Equation E**	**Equation F**
$y = 2 - 4x$	$2x + \frac{1}{2}y = 1$	$x + 2y - 11 = 0$

1. Sort the equivalent linear equations into two groups. Record your groupings.

Group 1	Group 2
Equations A, C, and F	**Equations B, D, and E**

2. Choose one equation. Write a new *equivalent equation* that would belong in that group.

________ *is equivalent to* ________ *because . . .*

Responses vary.
- **$3x + 6y = 33$ is equivalent to $x + 2y = 11$ because if you multiply each term in the second equation by 3, you get the first equation.**
- **-$8x - 2y =$ -4 is equivalent to $4x + y = 2$ because if you multiply each term in the second equation by -2, you get the first equation.**
- **$5x = 55 - 10y$ is equivalent to $5x + 10y = 55$ because if you subtract $10y$ from each side in the second equation, you get the first equation.**

Activity 1

Name: .... Date: .... Period: ....

First Steps of Elimination

Caasi is solving this system of equations, but she got stuck.

Here's how Caasi started.

Caasi

$$\begin{array}{r} x + 2y = 11 \\ -(4x + y = 2) \\ \hline -3x + y = 9 \end{array}$$

3. **Discuss:**

- What was Caasi's first step?
- Why do you think she got stuck?

Responses vary.
- **Caasi subtracted the equations.**
- **Neither of the variables were eliminated, so she still can't solve the equation.**

Diego is trying to solve this system of equations.
Here's how Diego started.

Diego

$$\begin{array}{r} x + 2y = 11 \\ 4x + y = 2 \end{array} \rightarrow \begin{array}{r} x + 2y = 11 \\ 8x + 2y = 4 \end{array}$$

$$\begin{array}{r} x + 2y = 11 \\ -(8x + 2y = 4) \\ \hline -7x + 0 = 7 \\ x = -1 \end{array}$$

4. What was Diego's first step?

***Responses vary.* Diego multiplied each term in the second equation by 2. Now Diego can eliminate the y-variable from the system.**

5. Diego got stuck using his method after solving for $x = -1$. What do you think he should do next?

***Responses vary.* He needs to use $x = -1$ to determine the value of y.**

6. Ariel thinks that Diego can solve this system:

$$\begin{array}{r} x = -1 \\ x + 2y = 11 \end{array}$$

Discuss: Do you think this system will have the same solution as Diego's original system?

Yes. *Responses vary.* Diego was able to find that $x = -1$ by changing $4x + y = 2$ to $8x + 2y = 4$, which is an equivalent equation. Since the two equations are the same, the solutions should be the same.

7. Finish Diego's work to solve the system.

$x =$ **-1** and $y =$ **6**

Activity 2

Name: ________ Date: ________ Period: ________

More Than One Way?

8. Caasi and Kwabena started solving this system in different ways.

$$\begin{aligned} 4x - y &= 5 \\ x + 2y &= 8 \end{aligned}$$

With a partner, solve the system both ways. Compare your solutions.

Caasi: *Multiply the first equation by* 2.

$$\begin{array}{r} 8x - 2y = 10 \\ + \quad x + 2y = 8 \\ \hline 9x + 0 = 18 \\ x = 2 \end{array} \qquad \begin{aligned} (2) + 2y &= 8 \\ 2y &= 6 \\ y &= 3 \end{aligned}$$

Kwabena: *Multiply the second equation by* -4.

$$\begin{array}{r} 4x - y = 5 \\ + \quad -4x - 8y = -32 \\ \hline 0 + -9y = -27 \\ y = 3 \end{array} \qquad \begin{aligned} x + 2(3) &= 8 \\ x + 6 &= 8 \\ x &= 2 \end{aligned}$$

9. **Discuss:**

- What is similar about Caasi's and Kwabena's methods for solving the linear system?
- What is different about their methods?

Responses vary.

- **Similar: They both used elimination. They both solved for one variable and then had to plug it back into the equation to get the other.**
- **Different: Caasi solved for x first. Kwabena solved for y first. Caasi multiplied by a positive number, Kwabena multiplied by a negative number.**

Activity 3

Name: ________ Date: ________ Period: ________

Prepare to Be Eliminated

You will use a set of cards for this activity.

10. Here are the instructions for each round.

Select a card from A–F.

- Discuss two possible first steps you could take to solve the system.
- Choose a different first step from your partner. Solve your system individually.
- Compare your solutions and support each other to make adjustments as needed.

Round 1, Card ________

Equation 1: ________

Equation 2: ________

Solution: $x =$ ________ and $y =$ ________

Round 2, Card ________

Equation 1: ________

Equation 2: ________

Solution: $x =$ ________ and $y =$ ________

Name: .... Date: .... Period: ....

Prepare to Be Eliminated (continued)

Round 3, Card ....

Equation 1: ....

Equation 2: ....

Solution: $x =$ .... and $y =$ ....

Round 4, Card ....

Equation 1: ....

Equation 2: ....

Solution: $x =$ .... and $y =$ ....

Card A: $x = 3$ and $y = 1$
Card B: $x = 8$ and $y = 4$
Card C: $x = 5$ and $y =$ -6
Card D: $x = 8$ and $y =$ -3
Card E: $x =$ -6 and $y = 3$
Card F: $x = 5$ and $y =$ -6

Explore More

11. The solution to this system of equations is $x = 5$ and $y = 2$.

$$Ax - By = 24$$
$$Ax + By = 16$$

What are possible values for A and B?

$A =$ 4 and $B =$ -2

Synthesis

12. Describe how writing equivalent equations can help you solve systems of equations.

Use this system if it helps you explain your thinking.

$$x + 3y = 6$$
$$2x + y = 7$$

Responses vary.

- **If you want to use elimination to solve a system of equations, you need to be able to add or subtract the equations and have one of the variables be eliminated. If a variable can't be eliminated, then you can write an equivalent equation to create a pair that has coefficients with the same or opposite signs.**
- **In this system, you can multiply all the terms in the first equation by 2, so that both x-terms have a coefficient of 2, and then you can subtract the equations to eliminate x.**

Things to Remember:

Prepare to Be Eliminated

Directions: Make one copy per pair of students. Then pre-cut the cards and give each pair of students one set.

Card A

$$-x + 4y = 1$$
$$2x + y = 7$$

Card B

$$x + y = 12$$
$$3x - 5y = 4$$

Card C

$$4x - 4y = 44$$
$$6x + 3y = 12$$

Card D

$$4y = 4 - 2x$$
$$x + 5y = -7$$

Card E

$$\frac{1}{3}x + 2y = 4$$
$$x + y = -3$$

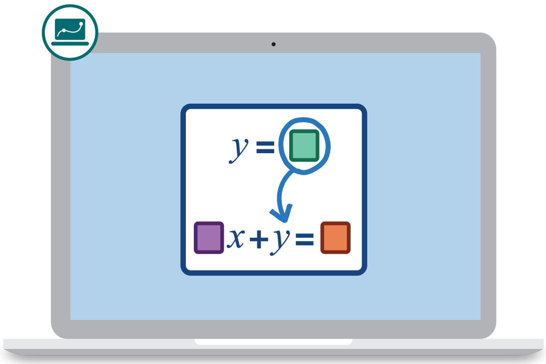
Card F

$$4x + 2y = 8$$
$$5x = 5y + 55$$

Name: ______ Date: ______ Period: ______

Solution by Substitution

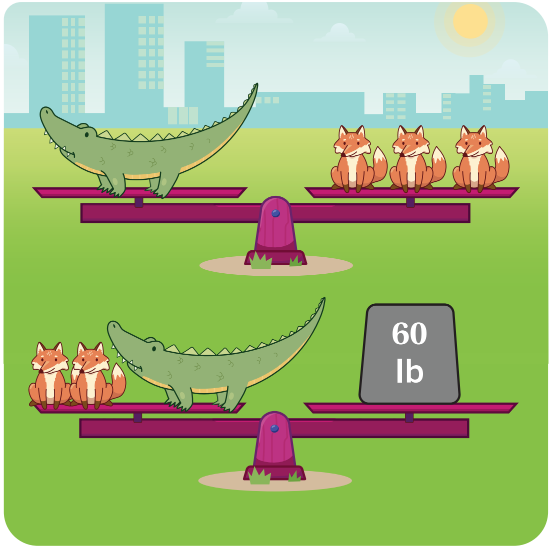
Let's use substitution to solve systems of equations.



Warm-Up

1 Here are two scales showing the weights of foxes and alligators.

What do you notice? What do you wonder?



Responses vary.

- I notice that an alligator weighs the same as three foxes.
- I notice that two foxes and an alligator together weigh 60 pounds.
- I wonder if all the foxes weigh the same amount.
- I wonder if both see-saws are balanced.
- I wonder how much a fox weighs. I wonder how much an alligator weighs.

2 Let's watch an animation.

a **Discuss:** What happened in the animation? *Responses vary.* Three foxes replaced the alligator on one of the see-saws. Since they have equal weights, the see-saw stayed balanced.

b Determine the weight of each animal.

Fox: 12 pounds   Alligator: 36 pounds

Activity 1

Name: ______ Date: ______ Period: ______

Introducing Substitution

3 Riya determined the weight of a fox by writing a system of equations and doing these steps.

Then she needed to solve a new system of equations.

Show or explain what Riya's first step might be as she solves the new system.

Responses vary.

- She can substitute $2x+3$ in for y in the second equation.
- $4x+(2x+3)=15$
- Identify which variable is isolated, so you know what to substitute.

Riya

$y=3x \qquad 2x+y=60$

$2x+(3x)=60$

$5x=60$

$x=12$

New

$y=2x+3 \qquad 4x+y=15$

4 Here is the system from the previous problem:

$y=2x+3$

$4x+y=15$

Determine the solution.

$x=$ 2, $y=$ 7

5 Riya's strategy is called solving by **substitution**.

Substitution is when a variable is replaced with an expression that is equal to it.

Show or explain the first step to solving the new system of equations with substitution.

Responses vary.

- Substitute $3x-12$ in for y in the first equation.
- $5x-2(3x-12)=15$

$y=3x \qquad 2x+y=60$

$2x+(3x)=60$

$y=2x+3 \qquad 4x+y=15$

$4x+(2x+3)=15$

New

$5x-2y=15 \qquad y=3x-12$

Name: ________ Date: ________ Period: ________

Introducing Substitution (continued)

6 Natalia made a mistake as she solved the system of equations from the previous problem.

What did Natalia do well? What should she fix?

Responses vary.

- Natalia did so many things well! She substituted an equivalent expression for y into the first equation, she remembered to use parentheses, and she solved for x almost all correctly.
- Natalia did not distribute the -2 correctly to the 12. The -24 should be positive 24. She also still needs to solve for y.

Natalia

$5x - 2y = 15 \qquad y = 3x - 12$

5x — 2(3x — 12) = 15

5x — 6x — 24 = 15

-x = 39

x = -39

7 Determine the solution to the previous problem.

$5x - 2y = 15$

$y = 3x - 12$

$x =$ **9** , $y =$ **15**

Activity 2

Name: .... Date: .... Period: ....

Practicing Substitution

8 Here are three systems of equations.

Discuss:

- What would be your first step in solving each of these systems using substitution?
- Would you prefer to solve each system using substitution or elimination? Why?

A
$$-2x + y = 9$$
$$x = 14$$

B
$$x + 2y = 10$$
$$x + y = 7$$

C
$$y = 4x + 63$$
$$y = 7x + 15$$

Responses vary.

- **System A: Since $x = 14$ is the second equation, I would plug 14 in for x in the first equation.**
- **System B: I would have to isolate a variable before I could use substitution, so I might prefer to use elimination and just subtract the two equations.**
- **System C: Both y's are isolated. I can substitute $4x + 63$ in for y in the second equation.**
- **I would use substitution for systems A and C because they already have isolated variables.**

9 Determine the solution to each system of equations.

a $y = 7x + 12$

$y = -3x + 2$

$x =$ **-1**, $y =$ **5**

b $2x + 2y = 8$

$x = 4 + 3y$

$x =$ **4**, $y =$ **0**

c $-2x + 4y = 9$

$y = x - 1$

$x =$ **6.5**, $y =$ **5.5**

d $3x - 2y = 14$

$x + 3y = 1$

$x =$ **4**, $y =$ **-1**

Explore More

10 Solve this system of *four* equations. All values in the solution are integers.

$3x + 2y - z + 5w = 20$

$y = 2z - 3w$

$z = w + 1$

$2w = 8$

$w =$ **4**, $x =$ **3**, $y =$ **-2**, $z =$ **5**

11 Synthesis

Substitution and *elimination* are two strategies for solving systems of equations.

How are these strategies alike? How are they different?

Use the examples if they help with your thinking.

A
$$-2x + y = 9$$
$$x = 14$$

B
$$x + 2y = 10$$
$$x + y = 7$$

C
$$y = 4x + 63$$
$$y = 7x + 15$$

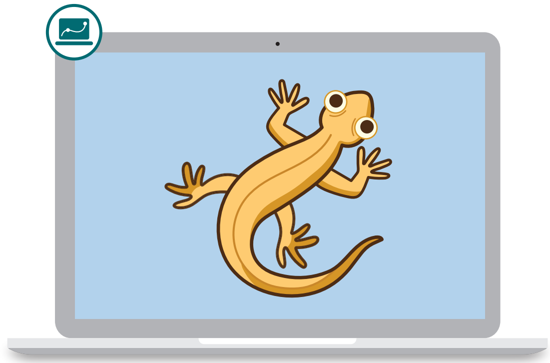
Responses vary.

- **Substitution and elimination are alike because they both lead to a situation where there is one equation and one variable. Once that one variable is solved for, it can be used to solve for the other variable.**
- **They are different because elimination requires adding or subtracting the two equations in the system.**
- **In elimination, it's important for the equations to have opposite terms. For substitution, it's important to have an isolated variable, like in systems A and C.**

Things to Remember:

Unit 5
Lesson
5

Name: .... Date: .... Period: ....

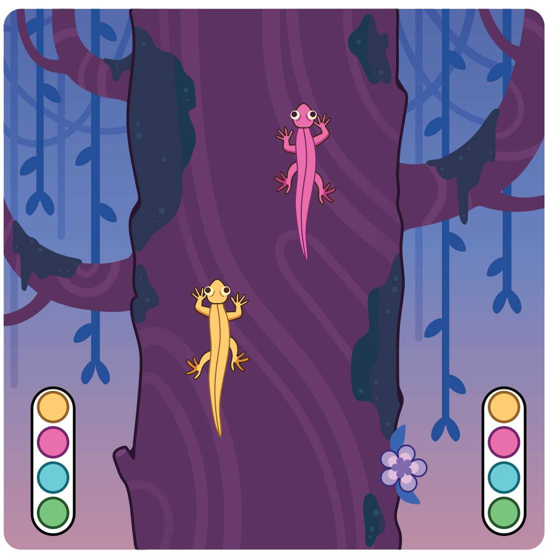


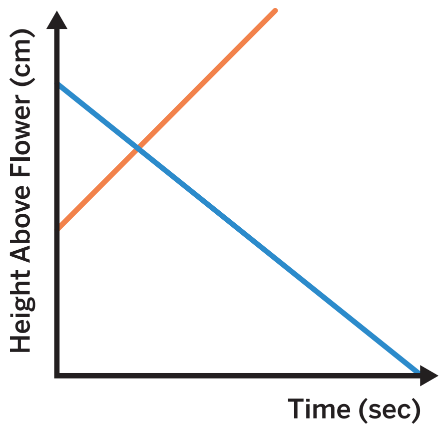
Lizard Lines

Let's explore systems of equations using graphs.

Warm-Up

1 Let's watch different pairs of lizards walk along a tree trunk together.





Discuss: What do you notice about the lizards and the graphs? What do you wonder?

Responses vary.

- I notice that you can compare the starting heights of the lizards on the y-axis of the graph.
- I notice that the green and yellow lizards will never be in the same place at the same time. Their graphs never intersect.
- I notice that the blue lizard is going down the tree, so the blue line on the graph is decreasing.
- I notice that when you pair two lizards of the same color, there is only one line on the graph.
- I wonder why there is only one line on the graph when there are two lizards of the same color.
- I wonder which lizard is moving the fastest.

Name: .... Date: .... Period: ....

Making Connections

2 Here are equations for each lizard's height above the flower, y, as a function of time, x, in seconds:

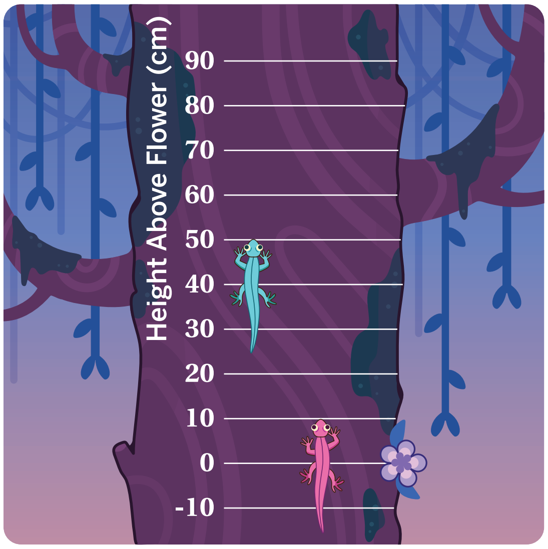
$$y = -5x + 50$$

$$y = 5x + 10$$

When and where will the lizards have the same position?

Time (sec), x: **4**

Height (cm), y: **30**



3 Let's look at Jin's and Nasir's strategies for figuring out when the lizards will be in the same position.

Discuss: Where do you see the solution in each strategy?

Responses vary.

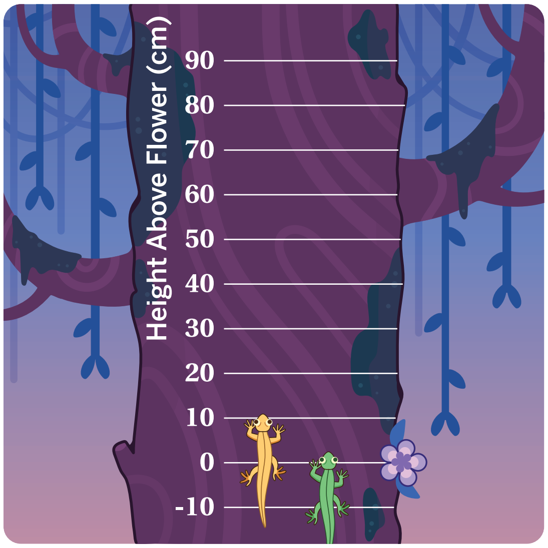
- **In Jin's strategy, I can see the solution where it says $x =$ and $y =$: $x = 4$ and $y = 30$.**
- **On the graph, I can see the solution as the intersection point of the two lines. The first value in the coordinate pair is the x-value and the second is the y-value.**

Activity 2

Name: .... Date: .... Period: ....

Will They Meet?

You will use a graphing calculator for this activity.



4 Here are equations for each lizard's height above the flower, y, as a function of time, x, in seconds:

$$y = -2x + 11$$

$$y = 4x + 2$$

When and where will the lizards have the same position?

Use a graphing calculator if it helps with your thinking.

Time (sec), x: **1.5**

Height (cm), y: **8**

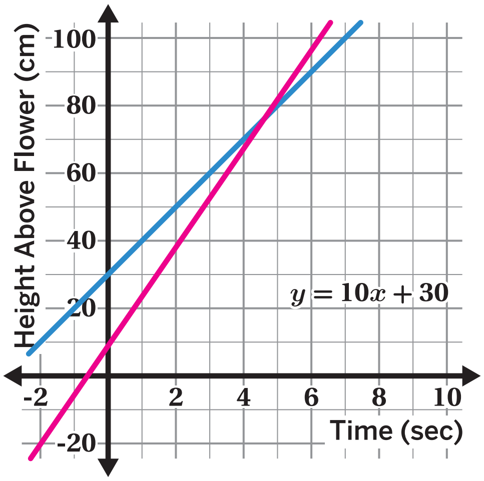
5 The blue line represents the graph of a blue lizard.

Create a line for a green lizard so that the lizards meet at exactly 5 seconds.

Try to make a line that none of your classmates will make.

Lines vary.

$y =$ **$14x + 10$**



6 Here are equations for two lizards' heights above the flower, y, as a function of time, x, in seconds.

$$y = 8x + 60$$

$$y = 8x + 35$$

Will these lizards meet? Explain your thinking.

No. ***Explanations vary.***

- **The lizards will never meet because they start in different places and move at the same speed.**
- **The lizards will never meet because these lines have the same slope but a different y-intercept, so on a graph these lines would be parallel.**
- **The lizards will never meet because when I try to solve this system, I get a false statement.**

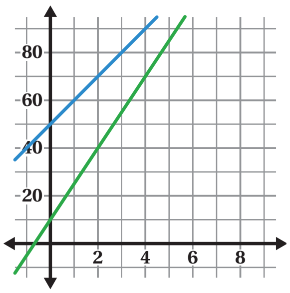
Activity
3

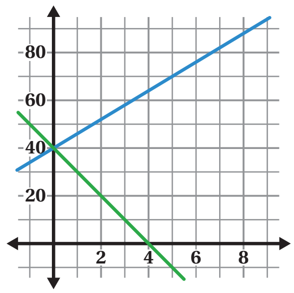
Name: ______ Date: ______ Period: ______

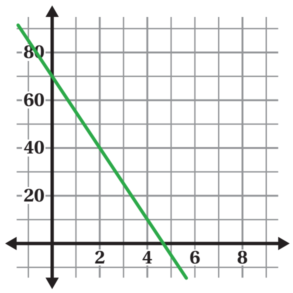
Graphing Systems

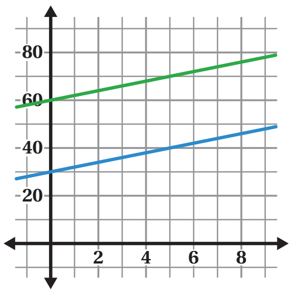
7 Here are some systems of equations.

$y = 10x + 50$ $y = 15x + 10$	$y = 6x + 40$ $y = -10x + 40$	$y = -15x + 70$ $y = -15x + 70$	$y = 2x + 60$ $y = 2x + 30$









Select each type of system that is possible to make.

- ☑ **A.** No solution
- ☑ **B.** Exactly one solution
- ☐ **C.** Exactly two solutions
- ☑ **D.** Infinitely many solutions

8 For each system of equations, circle the number of solutions that it has. If there is one solution, what is the solution?

a	$y = \frac{1}{2}x - 1$ $y = \frac{1}{2}x + 2$	No solution	One solution (______ , ______)	Infinitely many solutions
b	$y = x + 2$ $y = -3x - 2$	No solution	One solution (-1 , 1)	Infinitely many solutions
c	$y = 2x + 6$ $y = 2(x + 3)$	No solution	One solution (______ , ______)	Infinitely many solutions
d	$y - 5x = -7$ $y = 5x$	No solution	One solution (______ , ______)	Infinitely many solutions
e	$y = 20x$ $20y = x$	No solution	One solution (0 , 0)	Infinitely many solutions

Name: ______ Date: ______ Period: ______

Graphing Systems (continued)

9 Group each system of equations based on the number of solutions it has.

A

$2x + 4y = 16$

$y = \frac{1}{2}x + 2$

B

$y = \frac{2}{3}x + 10$

$y = \frac{2}{3}x - 7$

C

$y = 2x + \frac{1}{4}$

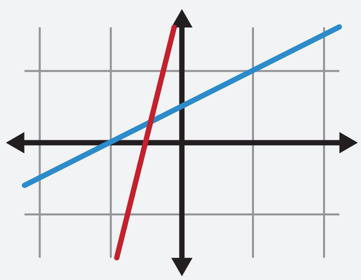
$y = 4x + \frac{1}{4}$

D

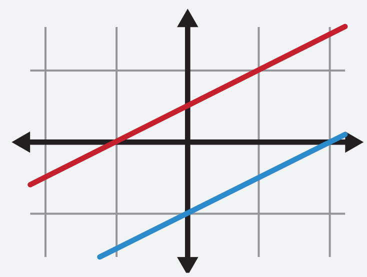
$y = \frac{1}{2}x + 3$

$2y = x + 6$

E



F



No Solution	One Solution	Infinitely Many Solutions
B, F	A, C, E	D

10 Jaleel and Irene are trying to decide when a system of equations may have *no solution*.

A system of equations may have no solution when . . .

Jaleel: . . . the *slopes* are the same.

Irene: . . . the y-*intercepts* are the same.

Whose claim is correct? Circle one and explain your thinking.

No Solution

$y = \frac{2}{3}x + 10$

$y = \frac{2}{3}x - 7$

$y = 2x + \frac{1}{4}$

$y = 4x + \frac{1}{4}$

Jaleel's Irene's Both Neither

Responses and explanations vary. **Note: Students who select Jaleel or neither are considered correct.**

- **Jaleel's. Jaleel's claim is correct when the y-intercepts are different because then the lines would be parallel.**
- **Neither. The lines must have the same slopes but different y-intercepts.**

11 Synthesis

Select one and explain.

How can you tell if a system of equations has:

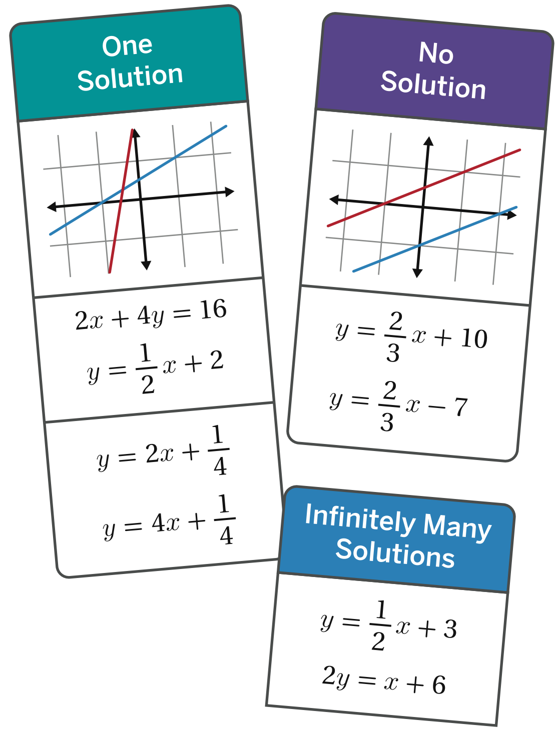
☐ **A.** No solution?

☐ **B.** One solution?

☐ **C.** Infinitely many solutions?

Responses vary.

- **A system has no solution if the graphs never intersect. The equations would have different y-intercepts and the same slope.**
- **A system has one solution if the graphs intersect at one point. The equations would have different slopes.**
- **A system has infinitely many solutions if the equations are equivalent. The lines have the same y-intercept, the same slope, and the same graph.**



Things to Remember:

Unit 5
Lesson
8

Name: .... Date: .... Period: ....

Electric Line Zapper

Let's solve systems of equations strategically.

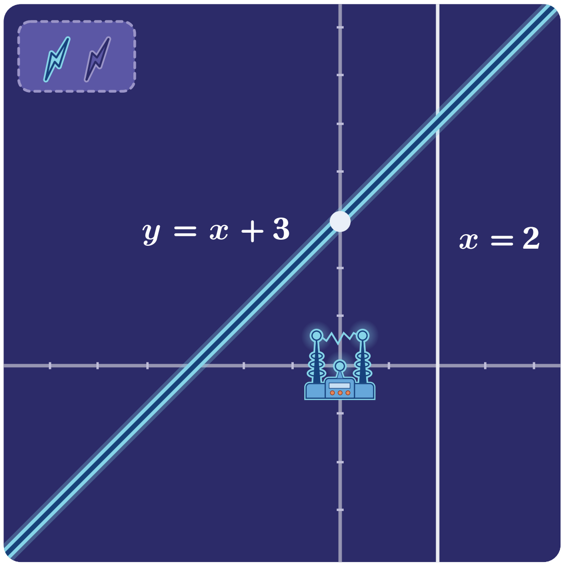
Warm-Up

1 Use the digital activity to zap a point on each of these lines to light it up.

$y = x + 3$
$x = 2$

Zap	Ordered Pair
Zap 1	(0, 3)
Zap 2	

Responses vary.
(2, 0), (2, 5), (2, -1)



Activity 1

Name: ______ Date: ______ Period: ______

Zapping Two Lines

You'll use the digital activity for Problems 2–4.

2 These two lines are hidden in the graph:

$3x + 4y = 3$

$-3x + 3y = 18$

Zap a point on each line to light it up.

***Responses vary.* (-3, 3) is on both lines.**

3 These two lines are hidden in the graph:

$y = 2x - 4$
$y = 0.5x + 5$

Adah zapped the point (-4, 5) but did not light up either line.

Help her light up both lines with *one* zap.

(6, 8)

4 Let's look at two ways to start solving the system of equations from the previous problem.

Discuss: Which strategy would you use? Why?

Responses vary.

- **I would use either strategy because they both help me find the point on both lines.**
- **I would use substitution because I get confused when I get a 0 by itself in elimination.**
- **I would use elimination because I could avoid making integer mistakes using the distributive property in the substitution strategy.**

Activity 2

Name: Date: Period:

Zapping Many Lines

5 These three lines are hidden in a graph:

$y = 3x + 6$

$2x + 2y = 20$

$x - y = 10$

Use the digital activity to zap *two* points to light up all three lines.

Responses vary.

- **The first two lines intersect at $(1, 9)$.**
- **The second and third lines intersect at $(10, 0)$.**
- **The first and third lines intersect at $(-8, -18)$.**

6 Adah wants to light up two lines with one zap by using *elimination* to solve a system of equations.

Which two lines from the previous problem might she choose to zap? Circle two.

Responses vary.

Line A: $y = 3x + 6$

Line B: $2x + 2y = 20$

Line C: $x - y = 10$

Explain your thinking.

Explanations vary. **She would choose lines B and C because they are both in standard form and she can multiply line C by 2 to make opposite y-terms.**

Name: ________ Date: ________ Period: ________

Repeated Challenges

7 You'll use the digital activity to play a few rounds of Line Zapper. Use this page to show your thinking.

Responses vary. The points of intersection in the first three challenges are shown.

a $y = 2x + 10$

$y = 7 + 3x$

(3, 16)

b $2x - y = 12$

$x + 4y = 15$

(7, 2)

c $y = 4x + 5$

$4x + 2y = 16$

$x - y = 7$

(0.5, 7), (5, -2), (-4, -11)

8 Adah tried to light up these lines with one zap:

$y = 3x + 4$

$y = 3x - 2$

Adah

$y = 3x + 4 \qquad y = 3x - 2$

$3x + 4 = 3x - 2$

$3x + 6 = 3x$

$6 = 0$

What does her work say about this system of equations?

Responses vary.

- When you solve correctly and run into a false statement, it means there are no solutions to the system.
- Her work means these two lines will never intersect. They are parallel because they have the same slope but different y-intercepts.

9 Synthesis

What are some ways you can decide what strategy to use when solving a system of equations?

Use the examples if they help with your thinking.

$x = 2$
$y = x + 3$

$3x + 4y = 3$
$-3x + 3y = 18$

$y = 3x + 6$
$2x + 2y = 20$

$y = 2x - 4$
$y = 0.5x + 5$

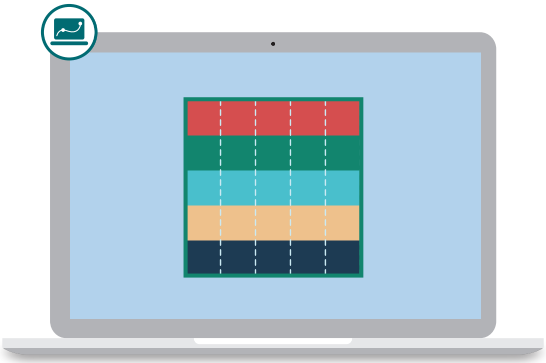
Responses vary.

- **I like to use substitution when I see an isolated variable.**
- **When both equations are in standard form, I like to use elimination.**
- **When I see the same or opposite terms in both equations, I can normally do elimination pretty easily.**
- **If I see two equations with the same slope, then I can compare the y-intercepts.**

Things to Remember:

Unit 5
Lesson
9

Name: ____________ Date: ________ Period: ________



Quilts

Let's explore what solutions to systems of inequalities mean.

Warm-Up

1 People across many different cultures make quilts. They can be used for warmth, storytelling, political involvement, income, and more.

a Let's look at a variety of different quilts.

b What are some decisions people might make when designing a quilt?

Responses vary.
- **The size of the quilt.**
- **The pattern of the quilt.**
- **The type of fabric to use.**
- **Who will use the quilt and for what purpose.**

2 There is a longstanding patchwork quilt tradition in Gee's Bend, Alabama. Spend a few minutes researching and learning about Gee's Bend and quilters like Annie Mae Young.

Discuss:

- What *constraints* might quilters like Annie Mae Young have experienced when creating quilts?
- How has quilting been a source of income and political involvement in this community?

Responses vary.
- **Quilt makers like Annie Mae Young may have dealt with constraints related to poverty and racism. This may have impacted the amount of time they had to quilt, the fabric they had access to, what they used the quilts for, and more.**
- **Many of the quilts from Gee's Bend are now in museums, making these residents famous across the country and providing additional income to support the community.**

Name: .... Date: .... Period: ....

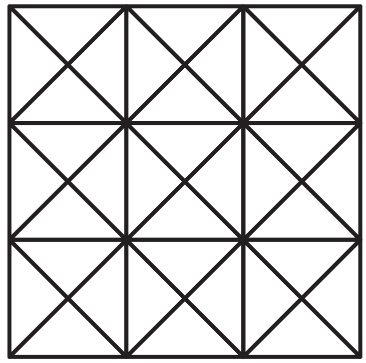
Sai and Jordan's Quilt

3 **a** Let's make a quilt square together.

b **Discuss:** Why did you choose to design your quilt the way you did?

Responses vary.

- **Our design uses more patterned fabric because we thought the pattern was fun.**
- **We made our design symmetrical.**



4 Sai and Jordan are making a quilt using solid and patterned fabric.

They need *at least* 35 sq. ft of fabric to cover their bed.

Fabric Constraint

$$x + y \geq 35$$

What are some combinations of fabric Sai and Jordan could use?

Responses vary.

Solid Fabric (sq. ft), x	Patterned Fabric (sq. ft), y
25	10
20	20
12	41

5 Sai and Jordan want to spend no more than \$30 on fabric.

- Solid fabric costs \$0.50 per sq. ft.
- Patterned fabric costs \$1 per sq. ft.

Cost Constraint

$$0.50x + y \leq 30$$

What are some combinations of fabric they could use?

Responses vary.

Solid Fabric (sq. ft), x	Patterned Fabric (sq. ft), y
25	10
20	20
23	13

Name: ______ Date: ______ Period: ______

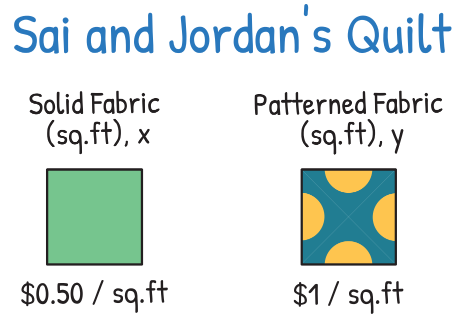
Sai and Jordan's Quilt (continued)

6 Sai and Jordan want their quilt to have at least 35 sq. ft of fabric and cost no more than $30. They wrote a **system of inequalities** to represent these two constraints.

$$x + y \geq 35$$

$$0.50x + y \leq 30$$

They designed a quilt using 10 sq. ft of solid fabric and 28 sq. ft of patterned fabric.



Does their design meet both constraints?

No. ***Explanations vary.***

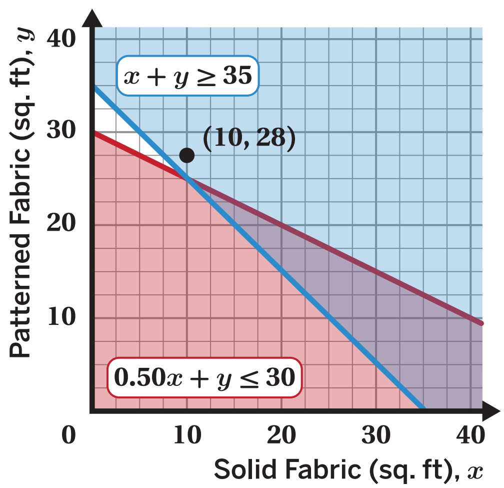
- **I plugged 10 in for x and 28 in for y into the inequalities. It only made one of them true.**
- **These values meet the fabric constraint but not the cost constraint. Their quilt would cost $33, which is too expensive.**

7 This graph represents their system of inequalities.

The fabric used in their design is represented by the point $(10, 28)$.

How can the graph help Sai and Jordan decide whether their design meets both constraints?

Responses vary. **Sai and Jordan could check if their design is in both of the shaded regions.**



8 Let's test several fabric combinations.

Discuss: How can you see which constraints a point meets by looking at the graph?

Responses vary.

- **If a point is only in the red-shaded region or only in the blue-shaded region, it only meets one constraint.**
- **Only points in the region where the blue and red overlap would meet both constraints.**

Activity
2

Name: ______ Date: ______ Period: ______

Evan's Quilt

9 Evan is making a quilt using different fabrics.

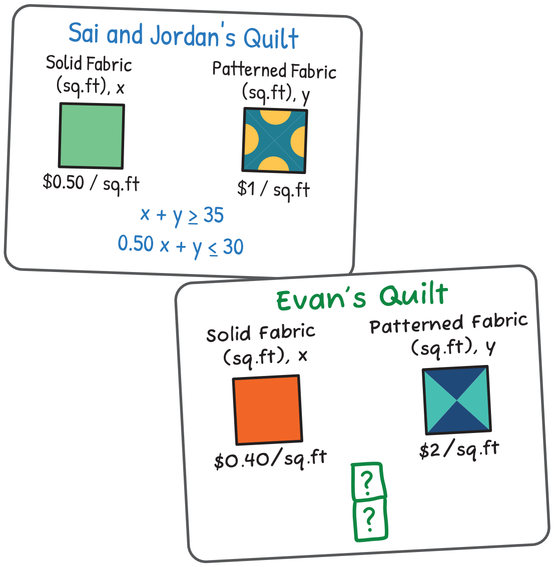
- Solid fabric costs $0.40 per sq. ft.
- Patterned fabric costs $2 per sq. ft.

Evan wants his quilt to have *at least* 35 sq. ft of fabric and cost *no more than* $30.

Write a system of inequalities to represent Evan's quilt.

Fabric inequality: **$x + y \geq 35$ (or equivalent)**

Cost inequality: **$0.4x + 2y \leq 30$ (or equivalent)**



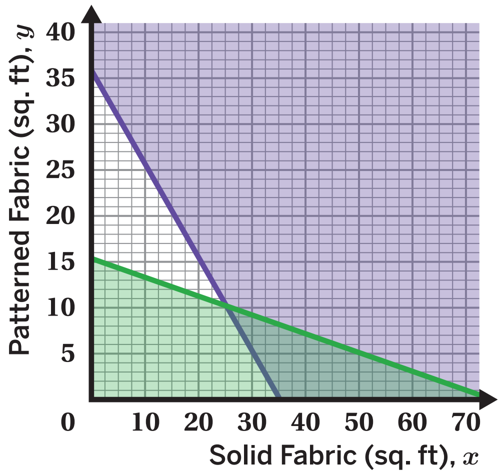
10 This graph represents Evan's system of inequalities.

Determine a combination of solid and patterned fabric that meets both constraints.

Responses vary.

Solid Fabric (sq. ft), x: **40**

Patterned Fabric (sq. ft), y: **5**



11 Titus says (10, 10) *is* a solution to Evan's system of inequalities. Alma says it *is not* a solution.

Whose thinking is correct? Explain your thinking.

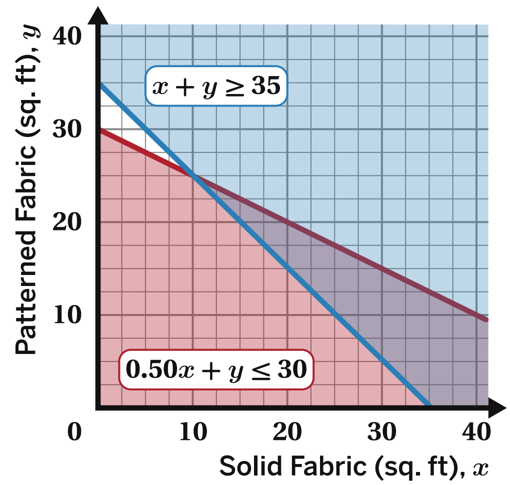
Alma's. *Explanations vary.*

- **(10, 10) is only a solution to one of the inequalities, so it is not a solution to the system.**
- **(10, 10) is only in the blue-shaded region. It needs to be in both the blue- and red-shaded regions to be a solution to the system of inequalities.**

12 Synthesis

How can you determine if a point is a solution to a system of inequalities?

Draw on the graph if it helps to show your thinking.

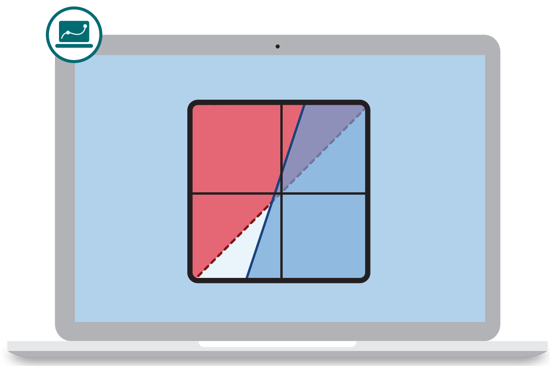


Responses vary.

- On a graph, you can check to make sure the point is in the shaded region for both inequalities.
- You can plug the point into both inequalities and check if it makes both inequalities true.

Things to Remember:

Name: ______________ Date: ________ Period: ________



Seeking Solutions

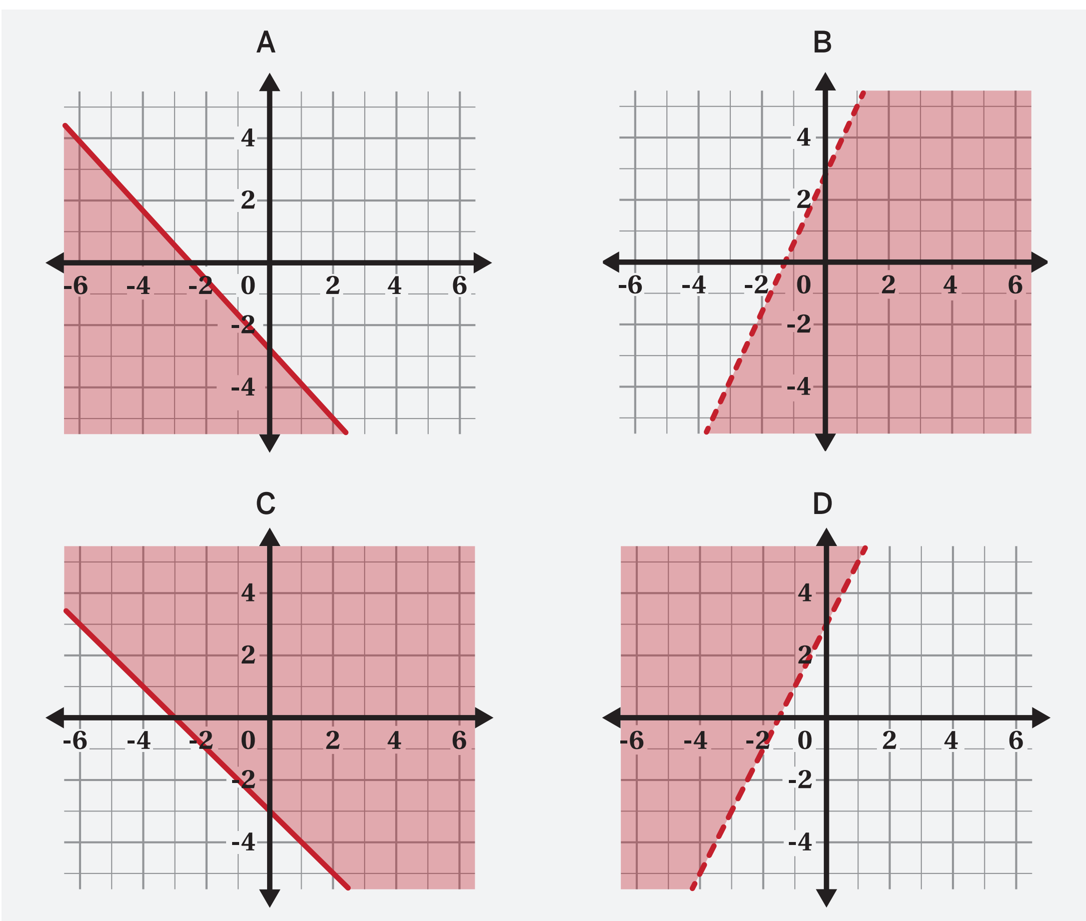
Let's explore strategies for determining the solution region for a system of inequalities.

Warm-Up

1 Match each inequality to its graph. There will be two graphs without a match.

$x + y \geq -3$	$y > 2x + 3$
C	D

No match: A, B

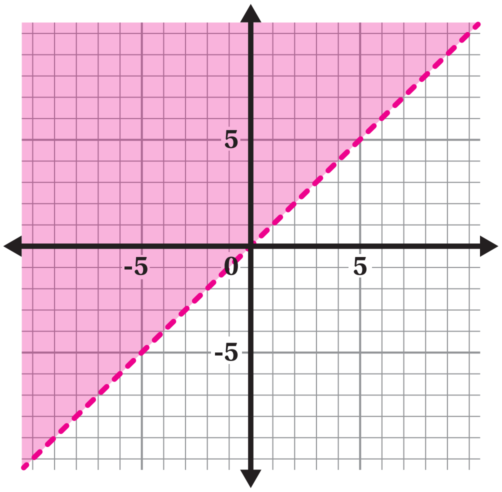


Name: ______ Date: ______ Period: ______

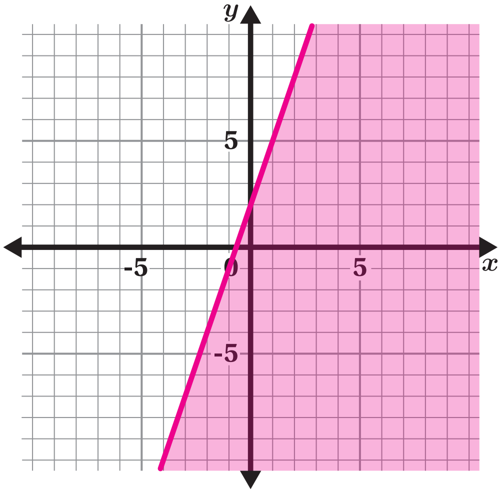
The Overlap

2 Graph the solution to each inequality.

$y > x$



$-3x + y \leq 2$



3 Let's look at a graph that shows the system of inequalities from the previous question.

a Watch as the point is moved to different *regions* of the graph.

b **Discuss**:

- How many regions do you see?
- When is each inequality highlighted? When are both highlighted?
- What happens when the point is on the dashed line? On the solid line?

Responses vary.

- **I see four regions: a white region, a blue region, a red region, and a region where the blue and red overlap.**
- **If the inequality is highlighted, that means the point is a solution to that inequality.**
- **The red inequality is not highlighted when the point is on the dashed line because dashed boundary lines are not included in the solution. The blue inequality is highlighted when the point is on the solid line because solid boundary lines are included in the solution.**

Name: ______ Date: ______ Period: ______

The Overlap (continued)

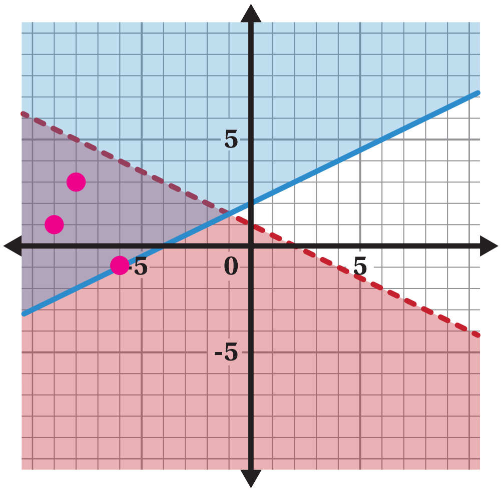
4 The **solutions to a system of inequalities** are all the points that make both inequalities true.

On a graph, the solutions are located in the same region.

Draw a point in the **solution region** of this system of inequalities:

$$\frac{1}{2}x + y < 1$$

$$-x + 2y \geq 4$$



Points vary.

Name: .... Date: .... Period: ....

Where Is the Solution Region?

5 This graph shows the *boundary lines* and their equations for this system of inequalities:

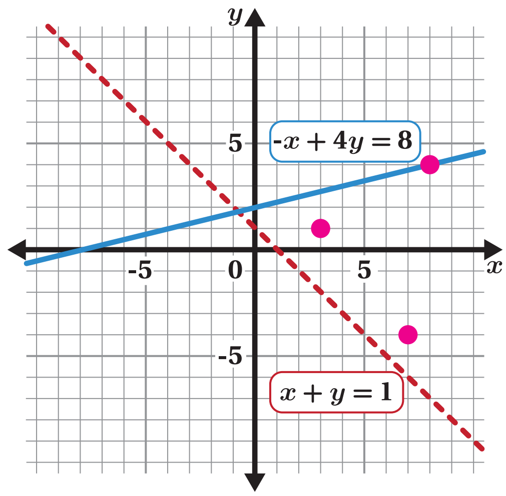
$$x + y > 1$$

$$-x + 4y \le 8$$

How can you determine where the solution region is?

Responses vary.

- **I can plug in (0, 0) to both inequalities.**
- **I can test a point in each region to see which one makes both inequalities true.**
- **For each inequality, I can test a point that's not on the boundary line, then shade both inequalities and see where they overlap.**



6 Plot a point on the solution region for the system of inequalities in the previous problem.

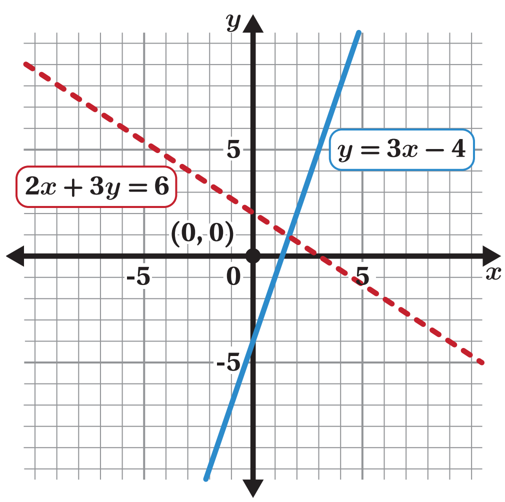
Points vary.

7 Terrance is trying to graph the solutions to this system of inequalities. First, he tests the point (0, 0).

$2x + 3y > 6$

$y \ge 3x - 4$

Dashed Line	Solid Line
$2(0) + 3(0) > 6$	$0 \ge 3(0) - 4$
$0 + 0 > 6$	$0 \ge 0 - 4$
$0 > 6$	$0 \ge -4$
False!	True!



Discuss: What can Terrence do next to determine the solution region?

***Responses vary.* Since (0, 0) makes the first inequality false, Terrence could shade the side of the dashed boundary line that doesn't include (0, 0). Since (0, 0) makes the second inequality true, Terrence could shade the side of the solid boundary line that does include (0, 0). Then he could look for where the shading overlaps.**

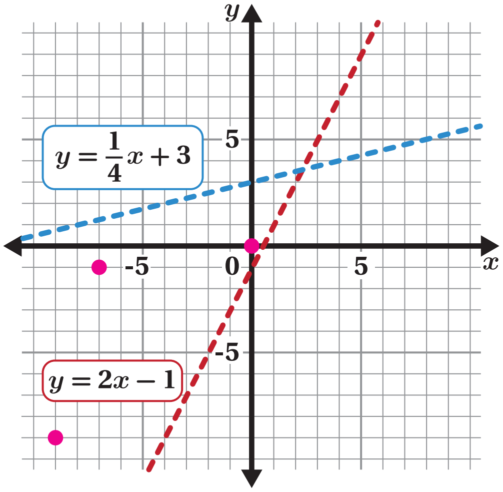
Name: .... Date: .... Period: ....

Solution Region Practice

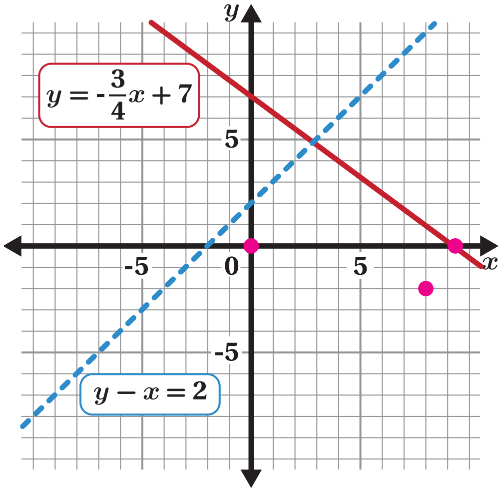
8 Plot a point in the solution region for each system of inequalities.

Points vary.

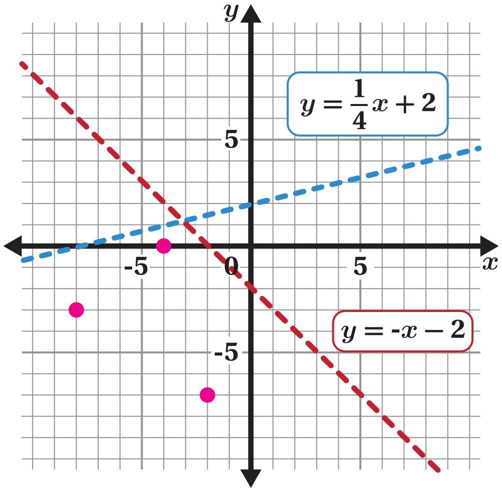
a $y > 2x - 1$
$y < \frac{1}{4}x + 3$



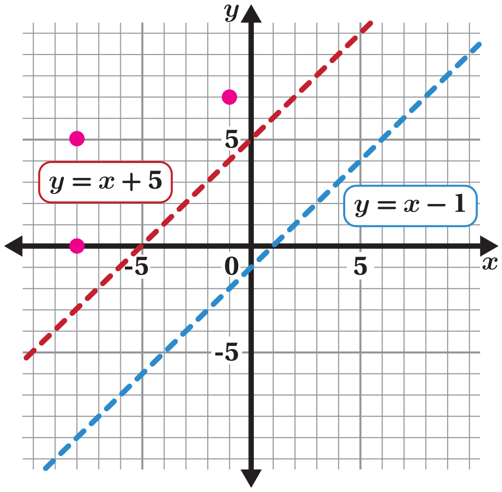
b $y \le -\frac{3}{4}x + 7$
$y - x < 2$



c $y < -x - 2$
$y < \frac{1}{4}x + 2$



d $y > x + 5$
$y > x - 1$

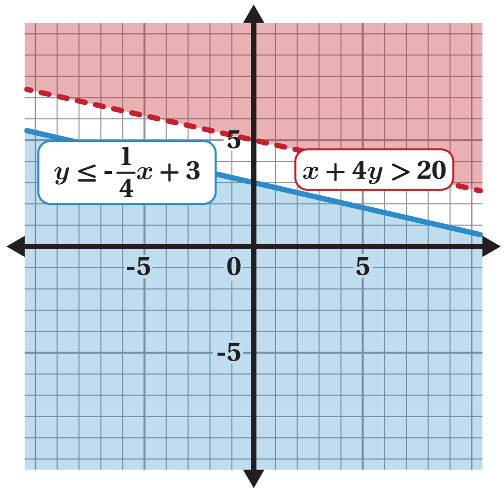


Name: .... Date: .... Period: ....

Solution Region Practice (continued)

9 This system of inequalities has *no solutions*.

How would you convince a classmate that there are no solutions to this system?



Responses vary.

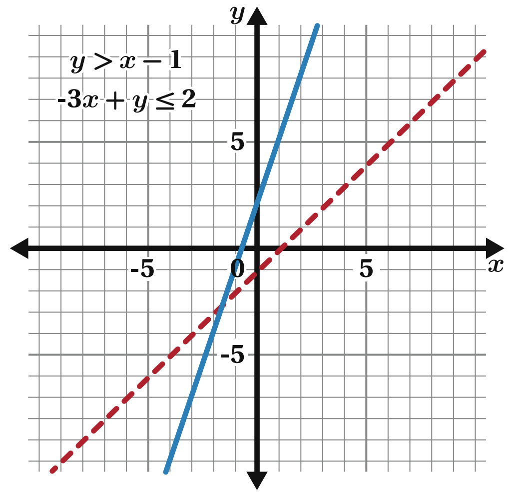
- **The solution regions for the inequalities will never overlap.**
- **These two lines are parallel and are shaded on opposite sides, so there are no points that will make both inequalities true.**
- **I could ask my classmate to try to find a point that is in both the blue- and red-shaded regions.**

10 Synthesis

What are some things you should keep in mind when determining the solution region of a system of inequalities?

Use the graph if it helps with your thinking.

Responses vary. **Test a point that is not on the boundary line. It can be helpful to pick one that will have simple calculations. Shade both inequalities based on the test. Where the shaded regions overlap is the solution region.**



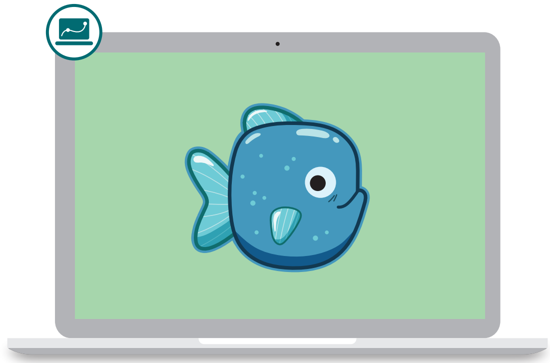
Things to Remember:

Unit 6
Lesson
1

Name: ______ Date: ______ Period: ______

Carlos's Fish

Let's make connections between exponential equations and the situations they represent.



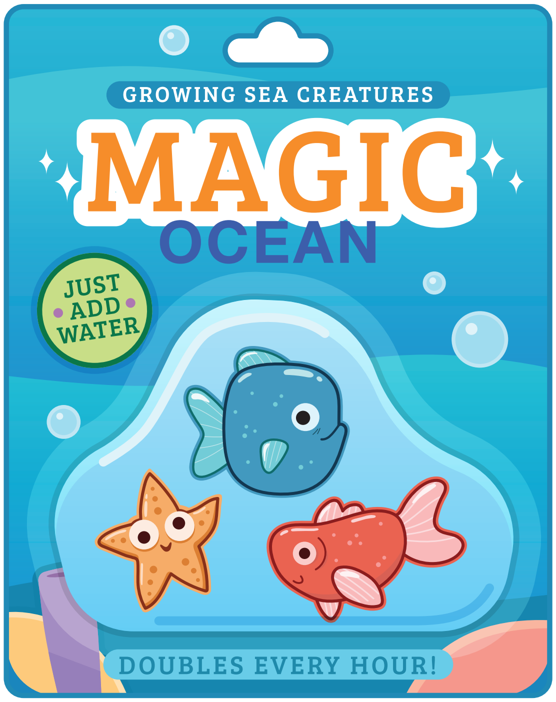
Warm-Up

1 Carlos's apartment doesn't allow pets, so he decided to buy toy fish.

What do you notice? What do you wonder?

Responses vary.

- I notice that there are three fish.
- I notice that they grow over time.
- I wonder how big they can possibly get.
- I wonder how big their tank is.
- I wonder how big they are now.
- I wonder what would happen if you put them in oil instead of water.



Activity 1

Name: ______ Date: ______ Period: ______

Doubles Every Hour

2 Let's watch Carlos's toy fish grow when you drop it in water.

Discuss: What patterns do you see?

***Responses vary.* The mass of the fish doubles every hour.**

3 This fish grows by a constant ratio.

What will the mass of the toy fish be after 5 hours?

800 grams

4 What was the mass of the fish before it was in the water?

25 grams

Time (hr)	Mass (g)
0	**25**
1	50
2	100
3	200
4	400
5	**800**

Name: .... Date: .... Period: ....

Doubles Every Hour (continued)

5 Carlos wrote this equation to model the fish's mass: $m = 25 \cdot 2^t$.

He used m for mass and t for time.

Explain what the 25 and 2 mean in this situation. ***Responses vary.***

- The 25 means. . . **the mass of the toy fish started at 25 grams.**
- The 2 means. . . **the fish's mass doubles every hour.**

Time (hr)	Mass (g)
0	25
1	50
2	100
3	200
4	400
5	800

6 If the fish continues growing this way, what will its mass be after 7 hours?

3,200 grams

7 Let's look at how Angel and Sora figured out the mass of the fish after 7 hours.

Discuss: What strategies do you see each student using?

Responses vary.

- **Angel made a table and continued the pattern to 7 hours.**
- **Sora used the equation and calculated the value of m by substituting 7 for t.**

Activity 2

Name: .... Date: .... Period: ....

Fish Growing and Shrinking

8 Here is a new toy fish.

Carlos wrote this equation to model the fish's mass:

$$m = 30 \cdot 1.5^t$$

Explain what the 30 and 1.5 mean in this situation. ***Responses vary.***

- The 30 means. . . **the fish had a mass of 30 grams when Carlos bought it.**
- The 1.5 means. . . **that the fish's mass grows 1.5 times bigger every hour.**

9 What is the mass of the fish when $t = 0$?

30 grams

10 Carlos wrote this equation to model the starfish's mass:

$$m = 270\left(\frac{1}{3}\right)^t$$

He used m for mass and t for time.

What will its mass be 3 hours after taking it out of water?

10 grams

Name: .... Date: .... Period: ....

Fish Growing and Shrinking (continued)

11 Match the cards with an equation. Two cards will have no match.

Card A

This fish's mass is multiplied by $\frac{1}{2}$ each hour.

Card B

After 2 hours, this fish has a mass of 1.5 grams.

Card C

After 2 hours, this fish has a mass of 24 grams.

Card D

After 2 hours, this fish has a mass of 18 grams.

Card E

This fish has a mass of $\frac{1}{2}$ gram before it is put in water.

Card F

This fish has a mass of 6 grams before it is put in water.

Card G

The fish's mass increases by $\frac{1}{2}$ gram every hour.

$m = 6 \cdot \left(\frac{1}{2}\right)^t$	$m = \frac{1}{2} \cdot 6^t$
Cards A, B, F	Cards D, E

Cards with no match: C, G

12 Synthesis

Here are two strategies that can be used to solve problems with exponential models.

Describe the benefits of each strategy.

Responses vary. Angel's strategy is useful for finding the next values that would appear in the table. Sora's strategy is useful for finding values that are much larger or father away than the values that appear in the table.

Angel

Time (hr)	Mass (g)
1	50
2	100
3	200
4	400
5	800
6	1,600
7	3,200

Sora

$m = 25 \cdot 2^t$

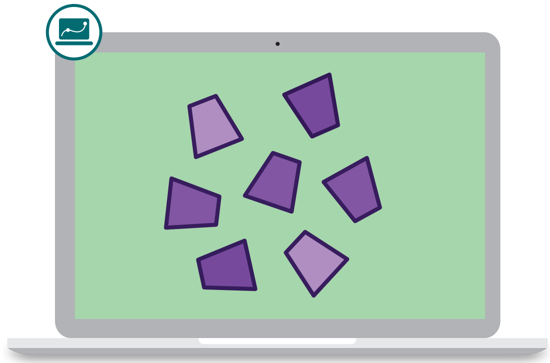
$m = 25 \cdot 2^7$

$m = 25 \cdot 128$

$m = 3200$

Things to Remember:

Name: .... Date: .... Period: ....



Growing Globs

Let's identify and compare two different patterns of growth.

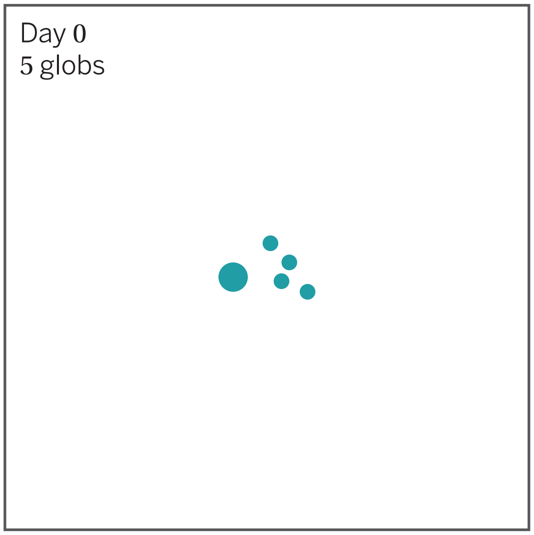
Warm-Up

1 Let's look at some teal globs.

a Watch how the number of globs grows.

b Write a story about these globs.

***Responses vary.* At first, there were 5 globs. Each day, 20 more globs arrived. Soon they will take over the world.**



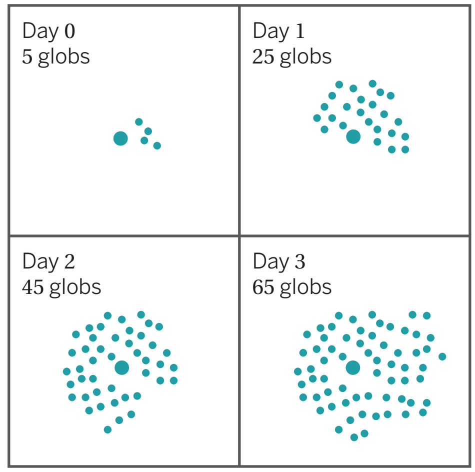
Activity 1

Name: ________ Date: ________ Period: ________

Purple vs. Teal

2 How many teal globs will there be on day 4?

Day	Teal Globs
0	5
1	25
2	45
3	65
4	**85**



3 Here is a new group of globs.

a Let's watch how the number of globs grows.

b How many purple globs will there be on day 4?

Day	Purple Globs
0	2
1	6
2	18
3	54
4	**162**

4 The graph shows the number of each type of glob for the first 3 days.

Will there be more teal globs or purple globs on day 10? Circle one.

Teal Globs

Purple Globs

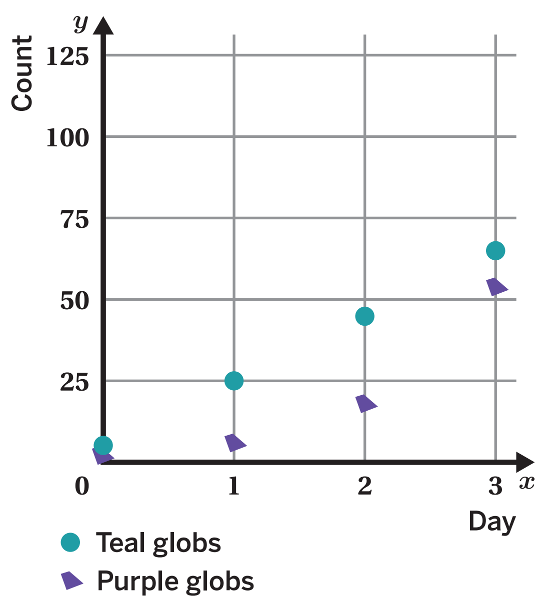
There will be the same

Not enough information

Explain your thinking.

Responses and explanations vary.

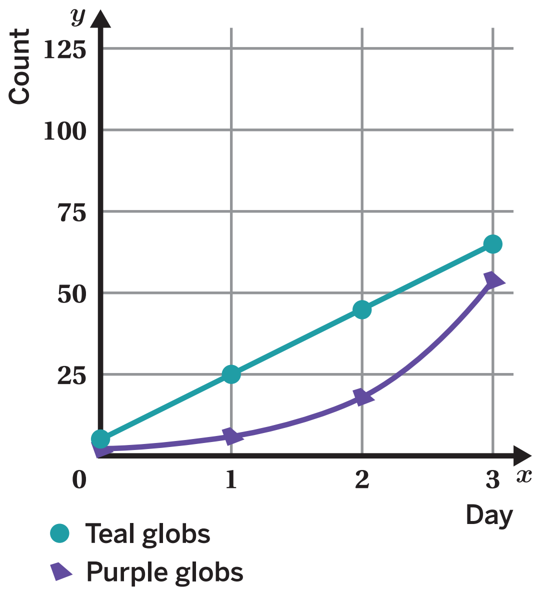
- **Purple globs. They are growing faster. They are already almost caught up to the teal globs by day 3.**
- **Not enough information. I don't know if the globs will keep growing in the same way every day. What if the purple globs stop growing after day 3?**

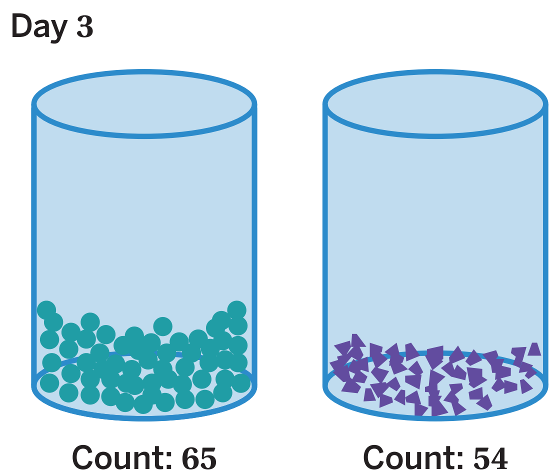


Name: .... Date: .... Period: ....

Purple vs. Teal (continued)

5 Let's watch how the number of teal and purple globs grows.





6 Here are tables for the teal and purple growing globs.

- Teal globs grow are modeled by a **linear function** and have a constant *rate of change*.
- Purple globs are modeled by an **exponential function** and have a constant **growth factor**.

Discuss:

- How are *rate of change* and *growth factor* alike?
- How are they different?

Linear

Constant rate of change

Day	Teal Globs
0	5
1	25
2	45
3	65

+1 +1 +1

+20 +20 +20

Exponential

Constant growth factor

Day	Purple Globs
0	2
1	6
2	18
3	54

+1 +1 +1

×3 ×3 ×3

Responses vary.

- **Rate of change and growth factor are alike because they both tell you how the y-values of a function are changing for a constant change in x-values.**
- **They are different because rate of change is adding to each previous value of a function and growth factor is multiplying each previous value in a function.**

Name: Date: Period:

Comparing Growth

7 Let's make two new species of globs and compare their growth.

8 Let's compare globs with different starting amounts and a constant rate of change or a constant growth factor.

Fabiana says: *Globs that grow by a constant growth factor will always eventually outnumber globs that grow by a constant rate of change*.

Lukas says: *If the constant rate of change is large enough, then this won't be true*.

Whose idea do you agree with? Circle one.

Fabiana's Lukas's Both Neither

Explain your thinking.

***Explanations vary*. Multiplying by a number greater than 1 always eventually gets larger than adding a number greater than 1. You just keep getting bigger even if you start with a smaller number.**

9 Group these cards by their function type.

Card A

x	y
0	2
1	4
2	6
3	8
4	10

Card B

x	y
0	0
1	1
2	4
3	9
4	16

Card C

x	y
0	1
1	2
2	4
3	8
4	16

Card D

x	y
0	0
1	4
2	8
3	12
4	16

Linear	Exponential	Neither
Cards A, D	Card C	Card B

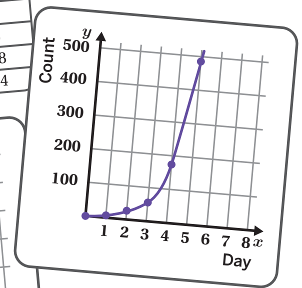
10 Synthesis

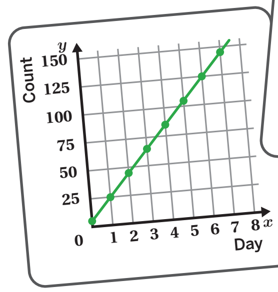
Quantities that grow by a constant rate of change can be modeled by linear functions.

Quantities that grow by a constant growth factor can be modeled by exponential functions.

Describe strategies for determining whether a function is linear or exponential.

Day	Count
0	2
1	6
2	18
3	54





Day	Count
0	5
1	25
2	45
3	65

Responses vary.

- **In a table where the change in the x-values is constant, I would look at the y-values to see how they change. If they add or subtract by a constant value, it's linear, but if they multiply or divide by a constant value, it's exponential.**
- **In a graph, linear functions look like a straight line, but exponential functions look like a curve.**

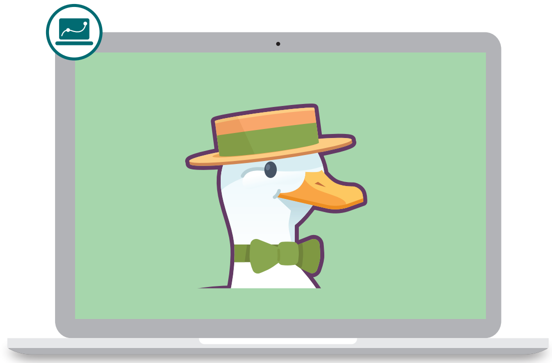
Things to Remember:

Unit 6
Lesson
3

Name: ______ Date: ______ Period: ______

Going Viral

Let's describe connections between graphs and equations, and use graphs to write exponential functions.



Warm-Up

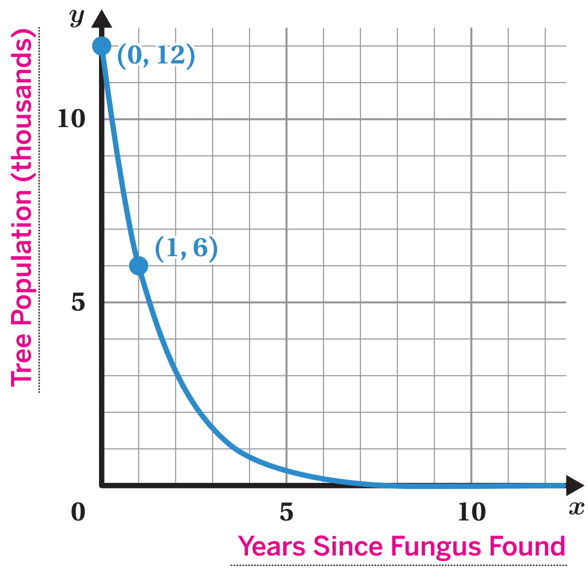
1 Here is a graph of an exponential relationship.

a Label the axes with any units you'd like.

Labels vary.

b Write a story about the quantities based on the graph.

***Responses vary*. The number of native trees in a state park is decreasing by half each year after a fungus was introduced in the park. There were 12,000 trees at the time the fungus was first discovered, and each year half of the trees die out.**



Activity 1

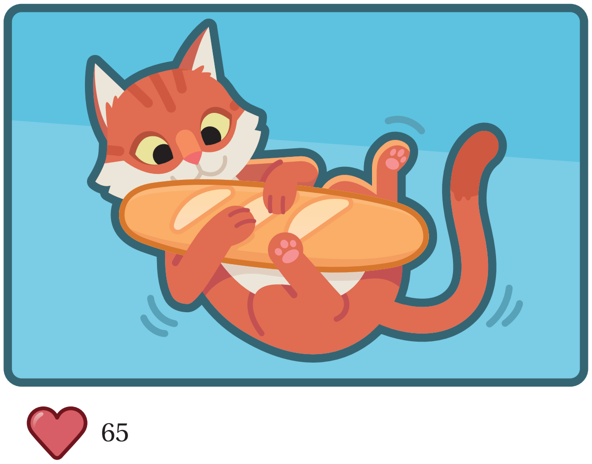
Name: ______ Date: ______ Period: ______

Three Memes

2 Let's watch this meme go viral.

Time (hr)	Likes
0	65
1	130
2	260
3	520
4	1,040
5	2,080

Hours since noon: 0:00



What type of relationship is this?

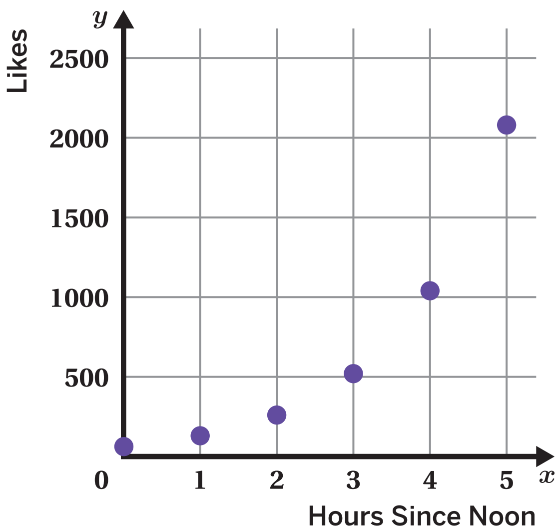
A. Linear

B. Exponential

C. Something else

3 There is an exponential relationship between the number of likes and the hours since noon.

Hours Since Noon, x	Likes, $f(x)$
0	65
1	130
2	260
3	520
4	1,040
5	2,080



Write an exponential function for this relationship.

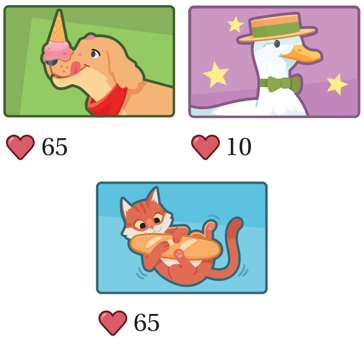
$f(x) =$ $65 \cdot 2^x$ (or equivalent)

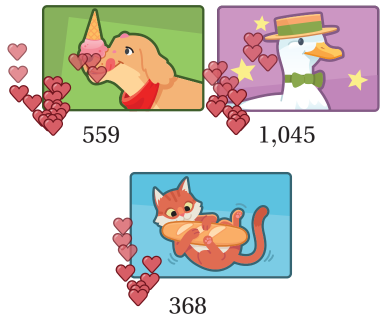
Name: ______ Date: ______ Period: ______

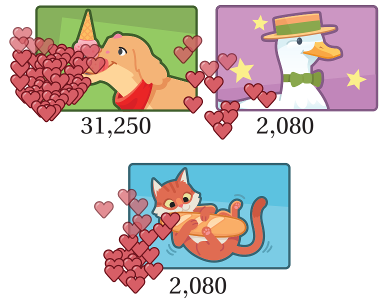
Three Memes (continued)

4 Let's look at the likes for these memes at different times.

Hours Since Noon: 0:00 | **Hours Since Noon: 2:30** | **Hours Since Noon: 5:00**

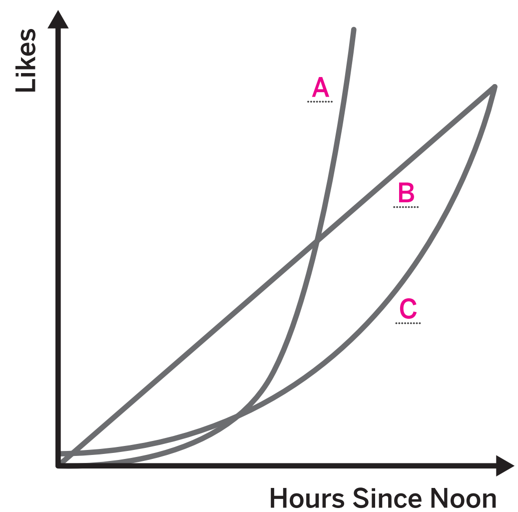






Match each meme to the graph that represents it.

A. **B.** **C.**

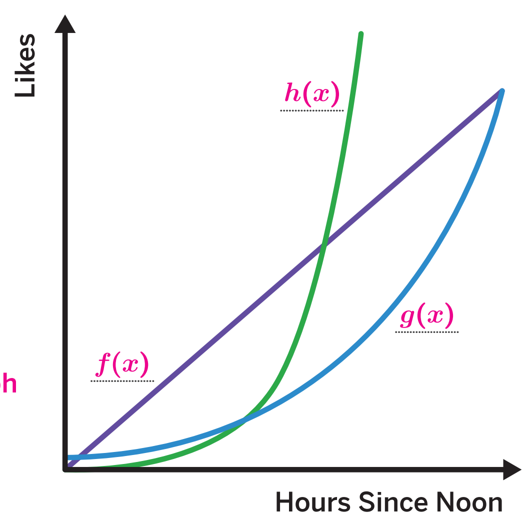


5 Match each function to its graph.

- $f(x) = 10 + 414x$
- $g(x) = 65 \cdot 2^x$
- $h(x) = 10 \cdot 5^x$

Explain your thinking.

Explanations vary. **I know that the linear graph matches with $f(x) = 10 + 414x$ because that is a linear equation. For the two exponential equations, I looked at the initial value. The graph with the fewer number of likes at the beginning matches with $h(x) = 10 \cdot (5)^x$ and the other one matches with $g(x) = 65 \cdot (2)^x$ because 10 is less than 65.**

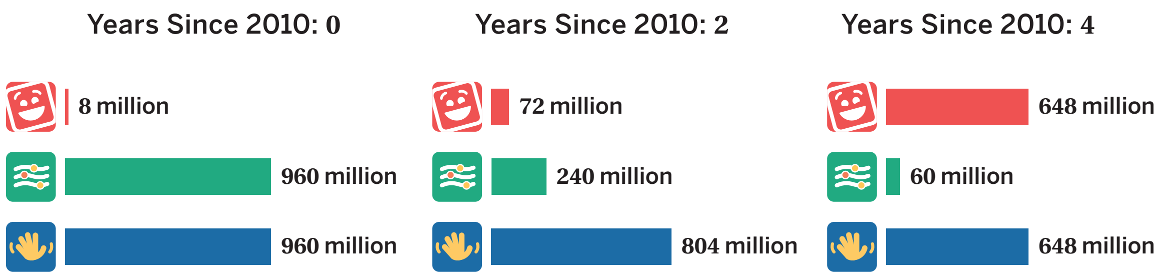


Activity 2

Name: ________ Date: ________ Period: ________

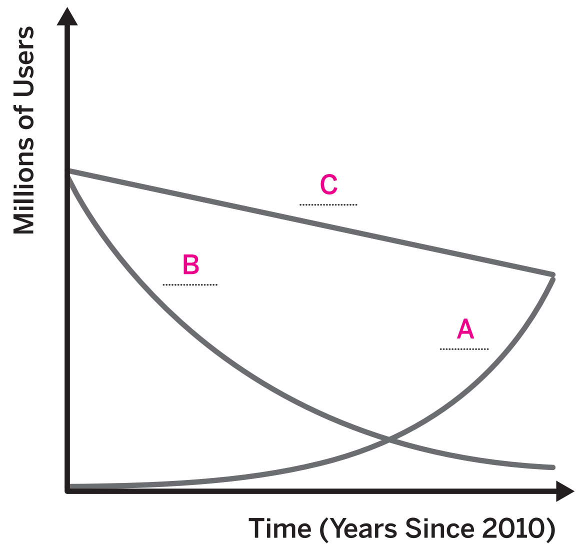
Take It Further

6 Let's look at the number of app users at different times.



Match each app to the graph that represents it.

A.  B.  C. 

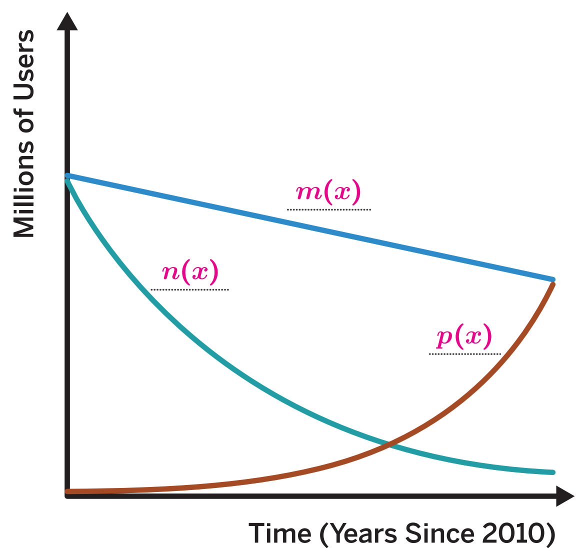


7 Match each function to its graph.

- $m(x) = 960 - 78x$
- $n(x) = 960 \cdot \left(\frac{1}{2}\right)^x$
- $p(x) = 8 \cdot 3^x$

Explain your thinking.

Explanations vary. **The equation $n(x)$ matches the exponential graph that decreases over time because the growth factor is $\frac{1}{2}$. The equation $m(x)$ matches the linear graph. The equation $p(x)$ matches the exponential graph that increases because the growth factor is 3.**



Name: .... Date: .... Period: ....

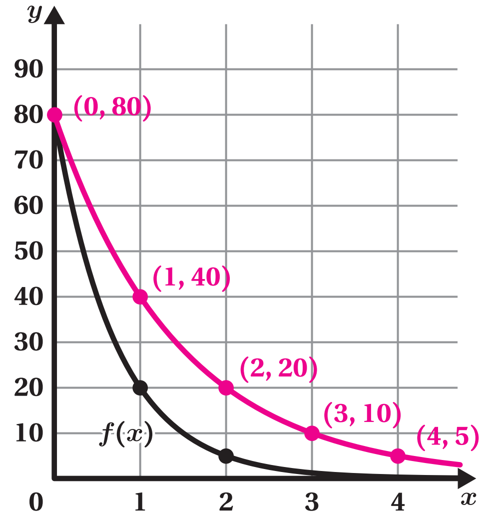
Take It Further (continued)

8 Here is the graph of $f(x) = 80 \bullet \left(\frac{1}{4}\right)^x$.

What might $g(x) = 80 \bullet \left(\frac{1}{2}\right)^x$ look like?

Show or explain your thinking.

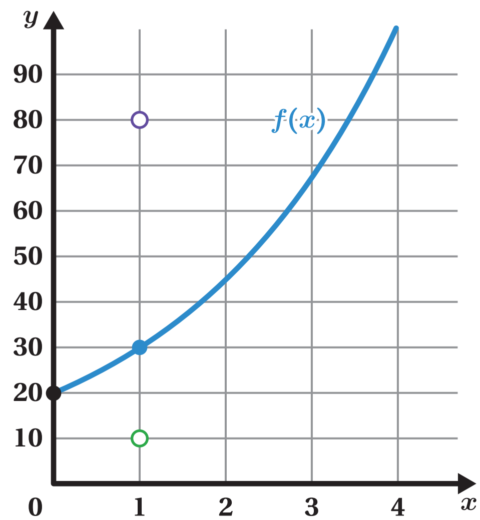
***Responses vary.* It will start at the same point but go down more slowly. For example, the y-value will only drop from 80 to 40 instead of 80 to 20.**



9 Here are three different exponential relationships.

Each relationship includes the point $(0, 20)$ and one other point shown on the graph.

One function has been written for you. Write the other two functions.



	Includes the Point	Function
Graph 1	$(1, 80)$	$f(x) = \mathbf{20 \bullet (4)^x}$
Graph 2	$(1, 30)$	$g(x) = 20 \bullet \left(\frac{3}{2}\right)^x$
Graph 3	$(1, 10)$	$h(x) = \mathbf{20 \bullet \left(\frac{1}{2}\right)^x}$

Explore More

10 Use the Explore More Sheet to answer a question about a pattern.

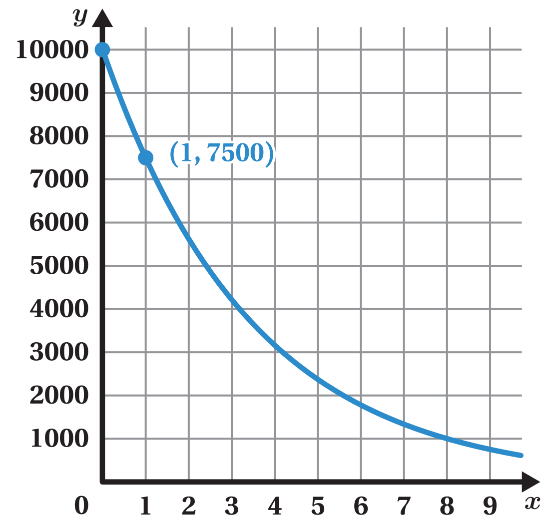
***Responses vary.* See Teacher Edition for sample responses.**

11 Synthesis

Here is a graph of $f(x) = 10000 \cdot \left(\frac{3}{4}\right)^x$.

Explain where you can see $10,000$ and $\frac{3}{4}$ on the graph.

***Responses vary.* 10,000 is the starting value. You can see it in the point (0, 10000) on the graph. $\frac{3}{4}$ is the constant ratio. You can see that because 7,500 is $\frac{3}{4}$ of 10,000.**

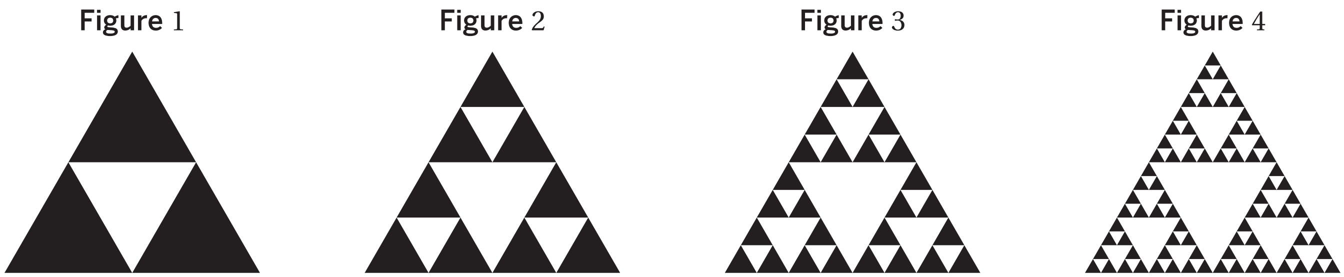


Things to Remember:

Name: .... Date: .... Period: ....

Explore More

Here are Figures 1–4 of a pattern.

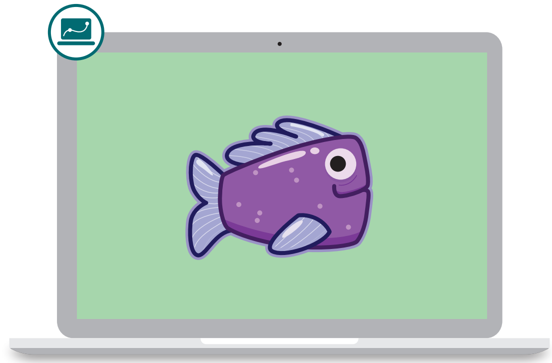


a Complete the table.

Figure	Number of Black Triangles	Fraction of Shaded Area
1	3	$\frac{3}{4}$
2	9	$\frac{9}{16}$
3	27	$\frac{27}{64}$
4		

b How could you figure out the number of black triangles and the fraction of shaded area for Figure 10?

Name: .... Date: .... Period: ....



Carlos and Corals

Let's evaluate exponential functions with positive, negative, and zero inputs.

Warm-Up

1 Select *all* the expressions that are equivalent to $2^{(-3)}$.

☑ **A.** $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

☑ **B.** $\frac{1}{2 \cdot 2 \cdot 2}$

☐ **C.** $2 \cdot -3$

☐ **D.** $-2 \cdot -2 \cdot -2$

☑ **E.** $8^{(-1)}$

Activity 1

Name: Date: Period:

Carlos's Fish

2 Carlos's apartment still does not allow pets, so he decided to buy a new toy fish.

The mass of the fish doubles every hour.

What type of function do you think will model the mass of this fish over time?

Linear Exponential Neither

Explain your thinking.

Explanations vary. **I think the function that models the mass of the fish is exponential because the packaging says "doubles in size." To double means to multiply by 2, and we can use an exponent to show multiplying over and over again.**

3 Carlos's new toy fish has a constant growth factor when placed in water.

What is the mass of the toy fish after 4 hours?

Time (hr)	Mass (g)
1	20
2	40
3	80
4	**160**

4 Carlos writes $m(t)$ to model the fish's mass over time.

$$m(t) = 10 \cdot 2^t$$

What is the value of $m(0)$? Explain your thinking.

10. ***Explanations vary.***

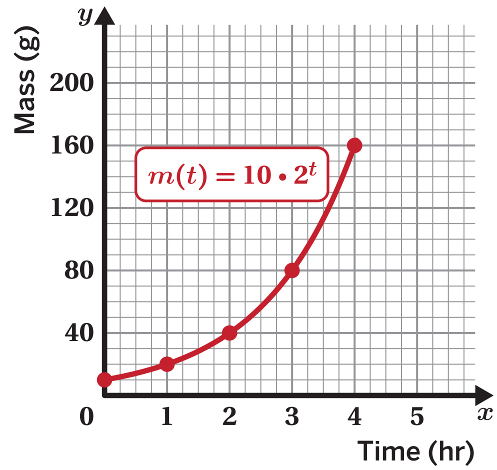
- **I substituted 0 into the equation: $m(0) = 10 \cdot 2^{(0)}$. This equals 10.**
- **The toy fish was 10 grams when Carlos first put it in the water or when he first opened the package.**

Name: Date: Period:

Carlos's Fish (continued)

5 Explain where you see the growth factor in the graph, the table, or the function.

Time, t	Mass, $m(t)$
0	10
1	20
2	40
3	80
4	160



Responses vary.

- I see that the growth factor of 2 is the base in the function.
- I see the growth factor of 2 in between the rows of $m(t)$ because you have to multiply by 2 to get to the next output.
- I see the growth factor of 2 when I look at the points on the graph and count the blocks on the grid and notice they double for every input.

6 a What is the value of $m(5)$?

320

b What is the value of $m(\text{-}1)$?

5

c **Discuss:**

- What does each value say about the fish's mass?
- How would you describe the *domain* of $m(t)$?

Responses vary.

- $m(5)$ tells me that the fish weighed 320 grams after 5 hours in the water. $m(\text{-}1)$ tells me that the fish would weigh 5 grams 1 hour before it was put in the water. This doesn't make sense in this situation because the fish's mass does not change before going in the water.
- The domain of this function is all positive numbers and zero because the fish isn't getting smaller when it is in the package. The domain can't be larger than the number of hours it takes the fish to grow to the size of the bowl. Using that information, I estimate the domain to be $0 \leq t \leq 8$.

Activity 2

Name: .... Date: .... Period: ....

Coral Reefs

Screens 7–9: A marine biologist is studying a coral reef. In 2010, she estimated that its volume was 320 cubic meters.

She wrote the function $v(t)$ to represent the volume of the coral reef t years after 2010:

$$v(t) = 320\left(\frac{4}{5}\right)^t$$

7 Based on $v(t)$, what was the reef's volume in 2011?

A. Less than 320 cubic meters

B. Equal to 320 cubic meters

C. Greater than 320 cubic meters

Explain your thinking.

***Explanations vary.* The amount of coral is decreasing because $\frac{4}{5}$ is less than 1, which means the original amount is getting smaller. This means that the reef's volume in 2011 was less than it was the year before.**

8 **a** Determine the value of $v(2)$.

204.8 cubic meters

b **Discuss:** What domain could make sense for $v(t)$?

Responses vary.

- **The domain for $v(t)$ could be all real numbers because coral reefs have been around for a long time, so it makes sense to think about how big they were before 2010.**
- **The domain could be from $-10 \leq t \leq 10$ because I don't think the coral reef will keep changing this way. This domain is from the years 2000 to 2020.**

9 Determine the missing values.

Years Since 2010	-3	-2	-1	0	1	2
Volume (cu. m)	**625**	**500**	**400**	320	256	204.8

Name: .... Date: .... Period: ....

Coral Reefs (continued)

10 Here is how Angel and Sora determined the volume of the coral reef in 2007 (3 years before 2010).

Angel

Year Since 2010	Volume (cubic meters)
-3	625
-2	$\cdot \frac{4}{5}$ 500
-1	400 $\cdot \frac{5}{4}$
0	320

Sora

$$V(-3) = 320\left(\frac{4}{5}\right)^{-3}$$

$$V(-3) = 320 \cdot \left(\frac{5}{4}\right)^{3}$$

$$V(-3) = 320 \cdot \frac{125}{64}$$

$$V(-3) = 625$$

Discuss: What is each student's strategy?

Responses vary. **I noticed that Angel moved backwards in the table by multiplying by $\frac{5}{4}$ and forwards in the table by multiplying by $\frac{4}{5}$. I noticed that Sora substituted $t = $ -3 into the function and evaluated the expression.**

11 Here is a new function: $f(x) = 18 \cdot 3^x$.

a **Discuss:** Will $f(\text{-}2)$ be less than or greater than 18?

Less than 18. ***Explanations vary.***

- **When I evaluate an expression with a negative exponent that has a base that is greater than 1, the value decreases. When I evaluate an expression with a negative exponent that has a base that is between 0 and 1, the value comes out larger than the initial value. So $f(\text{-}2)$ will be less than 18 because the base, 3, is greater than 1 and represents exponential growth.**
- **$f(\text{-}2)$ will be less than 18 because the base, 3, represents exponential growth.**

b Determine the value of $f(\text{-}2)$.

2

12 Synthesis

Describe a strategy for evaluating exponential functions for negative inputs.

Use the examples if they help with your thinking.

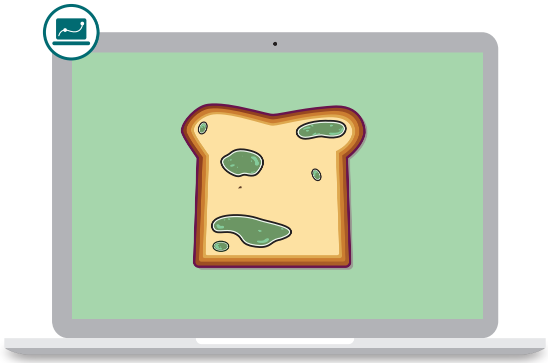
$a(\text{-}4) = 5 \cdot 10^{(\text{-}4)}$

$b(\text{-}3) = 10 \cdot \left(\frac{1}{2}\right)^{(\text{-}3)}$

Responses vary. **When I evaluate $5 \cdot 10^{(\text{-}4)}$, I first need to remember to rewrite the expression as $5 \cdot \left(\frac{1}{10}\right)^{(4)}$ because of the negative in the exponent. Then I rewrite the expression $5 \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$ to show the number of times I need to multiply 5 by the fraction $\frac{1}{10}$. This is equal to $\frac{5}{10000}$.**

Things to Remember:

Name: ____ Date: ____ Period: ____



Growing Mold

Let's explore how to model situations that change by a percent increase with exponential functions.

Warm-Up

Determine the value of each statement.

1. 10% of 30 **3**

2. 100% of 30 **30**

3. 110% of 30 **33**

4. 110% of 50 **55**

Name: Date: Period:

Growing Mold

5 A piece of bread is left out on a counter.

Let's watch an animation to see what happens over time.

What do you notice? What do you wonder?

Responses vary.

- **I notice that the mold on the bread is growing as time goes by.**
- **I notice that the mold is growing in several different spots on the bread.**
- **I wonder what type of mold is growing on the bread.**
- **I wonder when the bread will be completely covered by mold.**

6 This mold grows by 75% each day. How much mold will there be on day 4?

7 How much mold will there be on days 5 and 6?

Days	Area of Mold (sq. cm)
3	16
4	**28**
5	**49**
6	**85.75**

Name: .... Date: .... Period: ....

Growing Mold (continued)

8 Arnav made a table to help him write a function to represent the area of mold, $m(x)$, after x days.

$$m(x) = 2.985(1.75)^x$$

Days	Area of Mold (sq. cm)
0	2.985
1	5.224
2	9.14
3	16
4	28

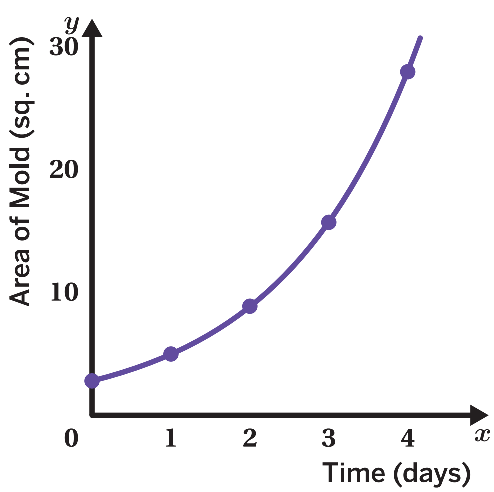
Where do you see the 75% increase represented in the function's equation? In the table?

Responses vary.

Equation: **I see the 75% represented as the decimal 0.75 after the 1 in the base of the exponential equation.**

Table: **I found 1.75 in the table by dividing the amount of mold on day 4 by the amount of mold on day 3. The growth factor is constant when I continue to divide an amount by the previous amount.**

9 Here is the graph of $m(x) = 2.985(1.75)^x$.



a Determine how much mold there will be after 10 days.

Approximately 804.127 sq. cm

b **Discuss:** Do you think the mold could grow according to $m(x)$ forever?

***Responses vary.* No, I think eventually there won't be any more area on the bread, so the graph will just be horizontal.**

Activity 2

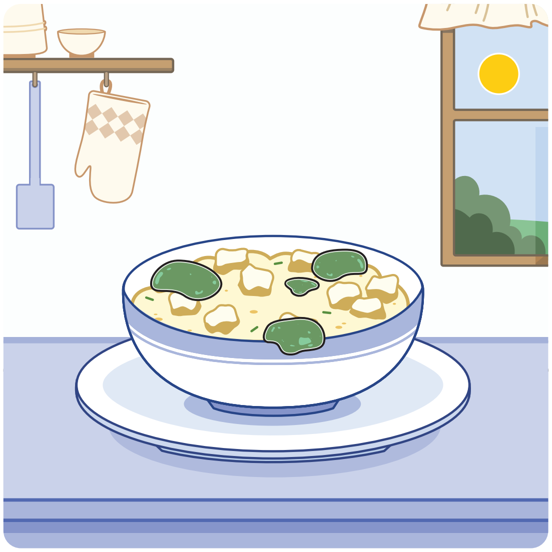
Name: ______ Date: ______ Period: ______

Growing with Percents

10 Tyler made potato salad and forgot to put it in the refrigerator.

The amount of bacteria in the potato salad increases by 4% every minute.

Which function, $p(t)$, represents the the amount of bacteria in the potato salad after t minutes?



A. $p(t) = 5 \cdot 0.4^t$

B. $p(t) = 5 \cdot 1.04^t$

C. $p(t) = 5 \cdot 1.4^t$

Explain your thinking.

***Explanations vary.* I know that 4 written as a percent is 0.04 and I know that the amount of bacteria is growing, so we need to add a 1 in front to represent 100% of the amount of bacteria that exists in the bowl at the beginning.**

11 Let's look at which function Tyler selected.

What does the 1 represent in this situation?

***Explanations vary.* In order to represent exponential growth, we have to multiply by a value greater than 1. The 1 represents 100%, which is the same as the original amount. Multiplying by 100% gives back the original value.**

Name: .... Date: .... Period: ....

Growing with Percents (continued)

12 Match each function with the situation that represents the same relationship. One function will have no match.

Functions

a. $a(x) = 20 \cdot 0.85^x$

b. $b(x) = 20 \cdot 1.85^x$

c. $c(x) = 85 \cdot 1.2^x$

d. $d(x) = 85 \cdot 1.02^x$

Situations

b A population of bacteria starts with 20 cells and grows by 85%.

c A population of frogs starts with 85 frogs and grows by 20%.

d

Days	Amount of Money ($)
0	85
1	86.7
2	88.4

Explore More

13 Heat and humidity can cause some types of bacteria to grow quickly. Imagine that in a humid room the amount of bacteria in a potato salad *triples* every hour.

By what percent is the bacteria growing per hour? Explain your thinking.

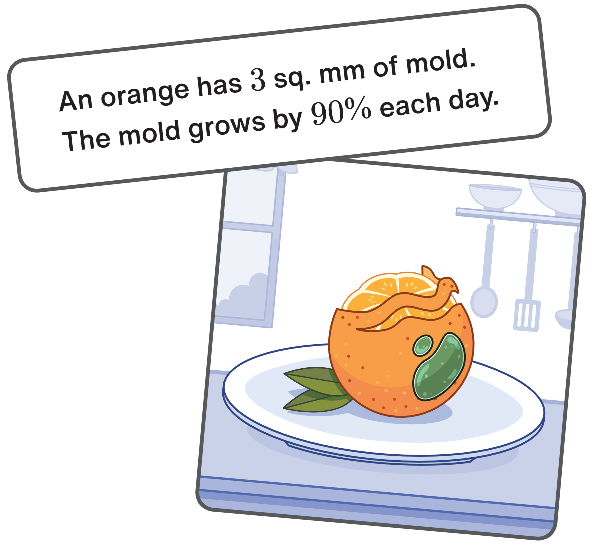
200%. *Explanations vary.* I think the mold is growing by 200% every hour because tripling means multiplying by 3. $1 + 2 = 3$, and the 2 would represent the amount of growth of 200%.

14 Synthesis

How do you write an exponential function that represents growing by a percentage?

Use the example if it helps with your thinking.

Responses vary. When writing an exponential function like the one for the orange, I need to remember to include a 1 to represent the 100% and then to change the percentage to a decimal, so 90% is 0.90. So the function would be $f(x) = 3(1 + 0.90)^x$, or I could write the base as 1.90.



Things to Remember:

Unit 6
Lesson
10

Name: ______ Date: ______ Period: ______

Marbleslides: Exponentials

Let's practice translating exponential functions by playing a game.

Warm-Up

You'll use the digital activity for the Warm-Up.

1 Your goal is to capture all the stars.

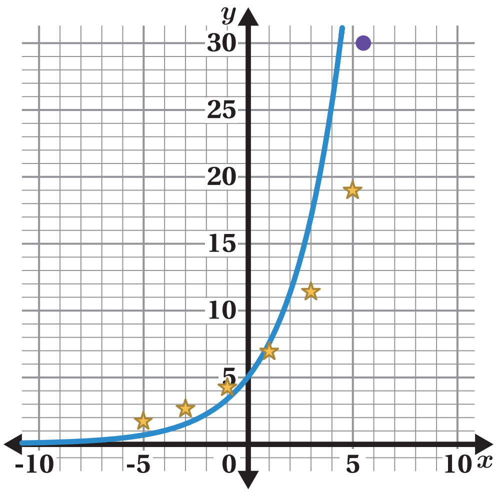
Change the function to capture all the stars.

Original function:

$f(x) = 5 \cdot 1.5^x$

Your function:

Responses vary.
$f(x) = 5 \cdot 1.3^x$



2 Change the function to capture all the stars.

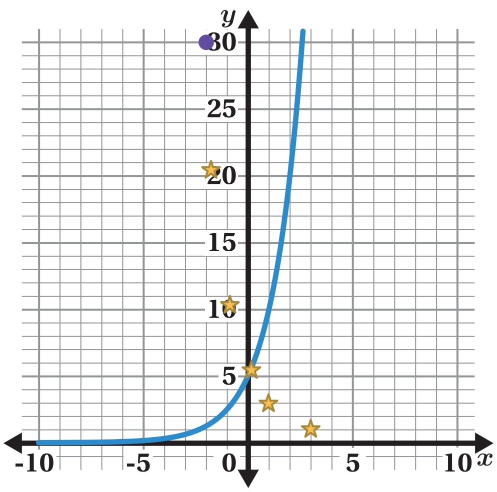
Original function:

$f(x) = 5 \cdot 2^x$

Your function:

Responses vary.

$f(x) = 5 \cdot \left(\frac{1}{2}\right)^x$



Activity 1

Name: ______ Date: ______ Period: ______

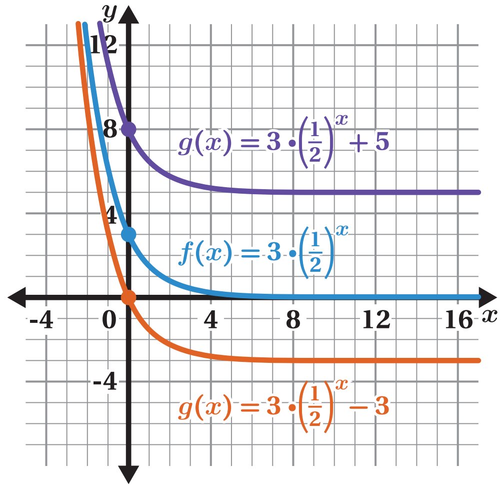
Translations

You'll use the digital activity for Problems 3–6.

3 To capture all of the stars, you may want to translate your function up or down.

Use the activity to translate $f(x)$ vertically.

What do you notice? What do you wonder?



Responses vary.
- **I notice that the purple point is the y-intercept for the functions.**
- **I notice that when you translate the graph down 6 units the equation has a -6 in it. When you translate the graph up 3 units the equation has a +3 in it.**
- **I notice that the function stays the same shape, but moves to different locations on the grid.**
- **I wonder if you can translate a function in other ways.**

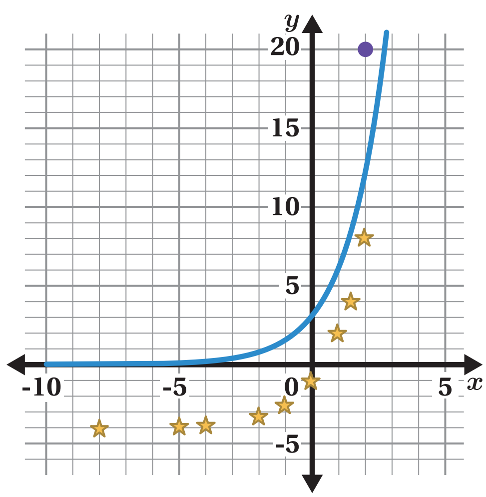
4 Change the function to capture all the stars.

Original function:

$f(x) = 3 \cdot 2^x$

Your function:

Responses vary.
$f(x) = 3 \cdot 2^x - 5$



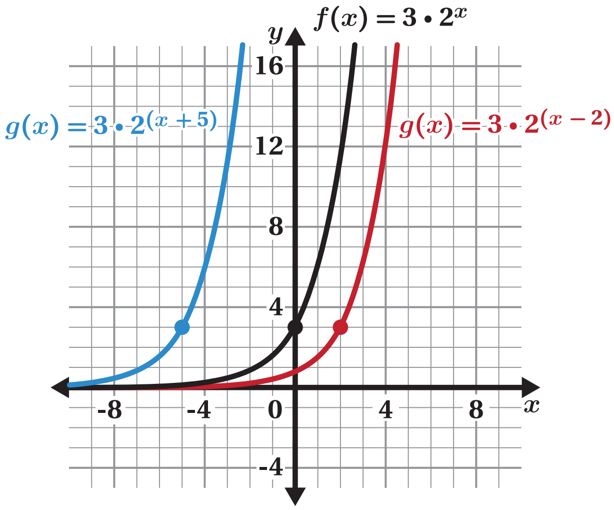
Name: ______ Date: ______ Period: ______

Translations (continued)

5 To capture all of the stars, you may want to translate your function left or right.

Use the activity to translate $f(x)$ horizontally.

Which function will translate $f(x)$ 12 units to the right?



A. $g(x) = 3 \cdot 2^x - 12$

B. $g(x) = 3 \cdot 2^{(x-12)}$

C. $g(x) = 3 \cdot 2^{(x+12)}$

D. $g(x) = 3 \cdot 2^x + 12$

Explain your thinking.

Responses vary. **The function will need to have $x - 12$ in the exponent to move 12 units to the right because x minus a number moves the function to the right and x plus a number moves the function to the left.**

6 Change the function to capture all the stars.

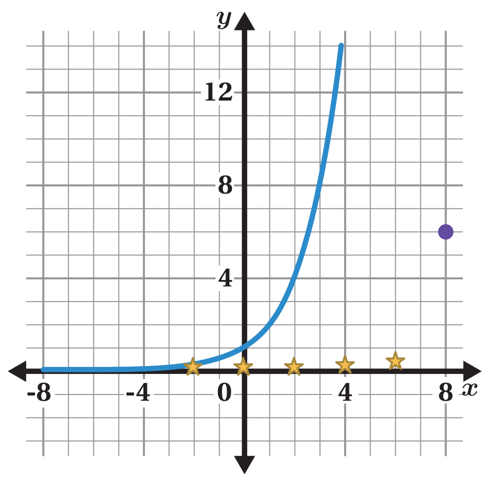
Original function:

$f(x) = 2^{(x)}$

Your function:

Responses vary.

$f(x) = 2^{(x-8)}$

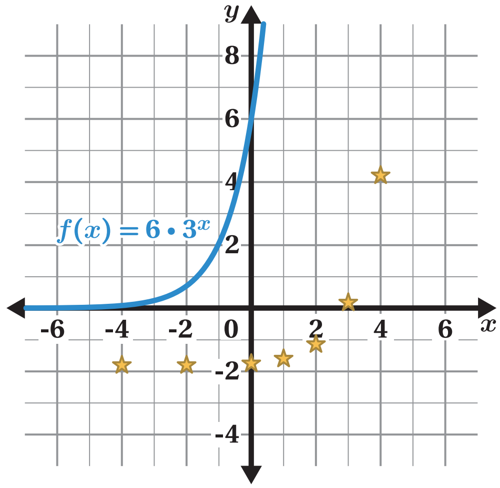


Activity 2

Name: ________ Date: ________ Period: ________

Challenges

7 Kiri wrote the function $g(x) = 6 \cdot 3^{(x+4)} - 2$ to capture all the stars.



Will Kiri's function capture all of the stars? Circle one.

Yes No I'm not sure

Explain your thinking.

***Explanations vary.* Having $x + 4$ in the exponent translates the function 4 units to the left. $g(x)$ needs to translate 4 units to the right and 2 units down to capture the stars.**

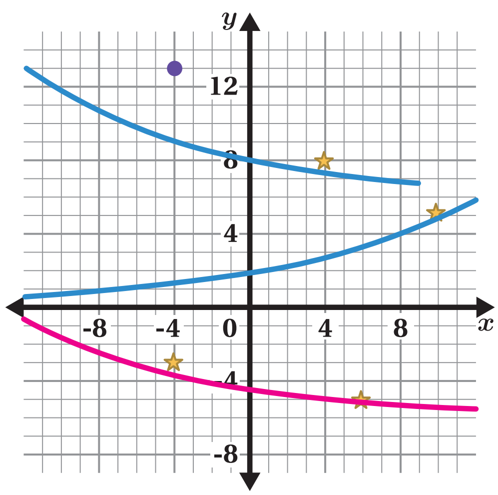
You'll use the digital activity for Problems 8–12.

8 Here is Kiri's function from the previous screen. Change the function to capture all the stars.

$g(x) = 6 \cdot 3^{(x+4)} - 2$

***Responses vary.* $f(x) = 6 \cdot 3^{(x-4)} - 2$**

9 Create as many exponential functions as you need to capture all the stars.



We have included functions that may help you start.

Original functions:

$f(x) = 2 \cdot (0.9)^x + 6 \{x < 9\}$

$g(x) = 3 \cdot (1.1)^{(x-5)}$

Your functions:

Responses vary.

$f(x) = 2 \cdot (0.9)^x + 6 \{x < 9\}$

$g(x) = 3 \cdot (1.1)^{(x-5)} \{-8 < x\}$

$h(x) = 1.5 \cdot (0.9)^x - 6$

Name: ____________ Date: ________ Period: ________

Challenges (continued)

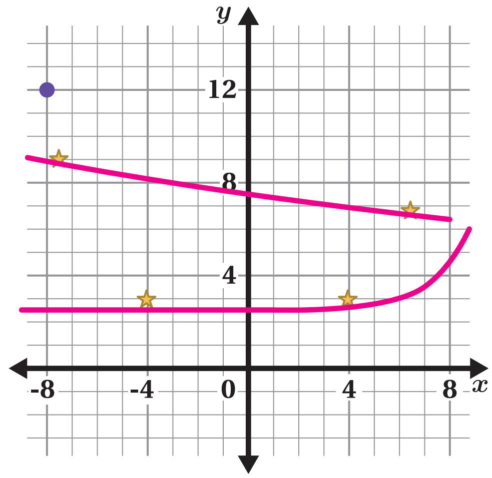
10 Create as many exponential functions as you need to capture all the stars.

Your functions:

Responses vary.

$f(x) = 5 \cdot (0.97)^x + 2.5 \{x < 8\}$

$g(x) = 2^{(x-7)} + 2.5$



11 Create as many exponential functions as you need to capture all the stars.

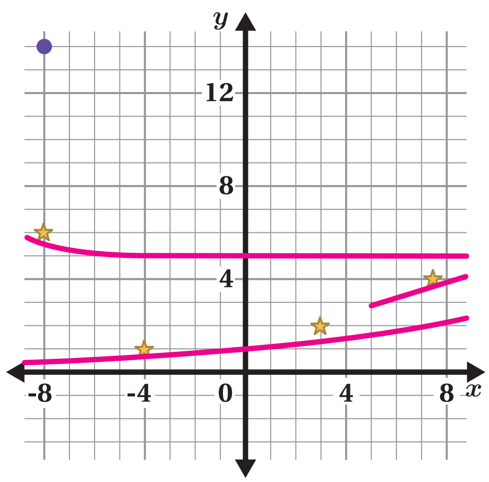
Your functions:

Responses vary.

$f(x) = 0.5^{(x-9)} + 5 \{x < 8\}$

$g(x) = 1.1^{(x+6)} \{5 < x\}$

$h(x) = 1.1^x$



Explore More

12 Challenge yourself to capture as many stars as you can!

Your functions:

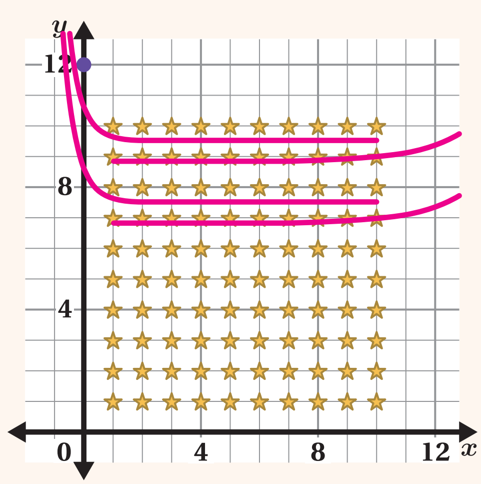
Responses vary.

$f(x) = 0.1^x + 9.5 \{x < 10\}$

$g(x) = 2^{(x-13)} 8.8 \{x > 1\}$

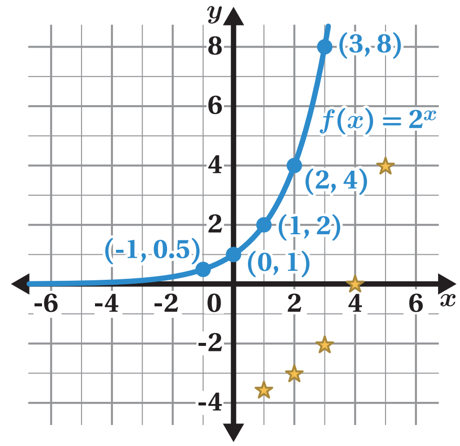
$h(x) = 0.1^x + 7.5 \{x < 10\}$

$j(x) = 2^{(x-13)} + 6.8 \{x > 1\}$



13 Synthesis

Describe how to use vertical and horizontal *translations* to write a function that would capture all the stars.



***Responses vary.* I could translate the function to the right 2 units and down 4 units. The function for the translation would be $g(x) = 2^{(x-2)} - 4$.**

Things to Remember:

Unit 6
Lesson
13

Name: .... Date: .... Period: ....

Bank Accounts

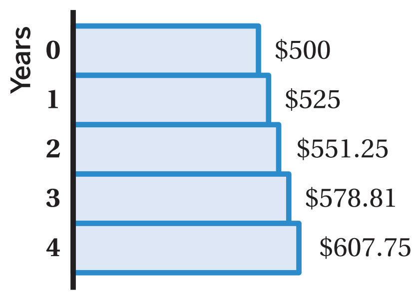
Let's learn how to model situations involving simple and compound interest.

Warm-Up

1-2 Mauricio has $500 to invest. He is researching different kinds of investment accounts. Here are the values of Accounts A and B over time.

Account A

Account B



Show or describe how the value of each account grows over time. ***Responses vary.***

- **The value of Account A increases by $50 each year.**
- **Account A has a constant rate of change of $50 and is growing linearly.**
- **The value of Account B is increasing by a growth factor of 1.05.**
- **Account B is growing exponentially since each year gets bigger by a factor of 1.05.**

Name: .... Date: .... Period: ....

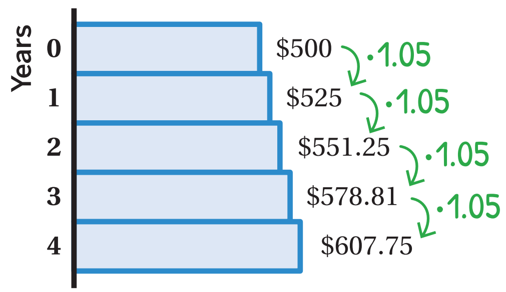
Earning Interest

3 Here is Mauricio's work to show how each account is growing.

Discuss: Describe his work to a partner. Which account would you recommend he invest in?

Account A

Account B



Responses vary.

- **Mauricio noticed that Account A grows by a constant rate of change of $50, and Account B changes by a constant growth factor of 1.05.**
- **I recommend Mauricio invest in Account A because he will get more interest quickly.**
- **I recommend Mauricio invest in Account B. Since Account B is growing exponentially, eventually the balance will earn more interest than Account A and grow very quickly.**

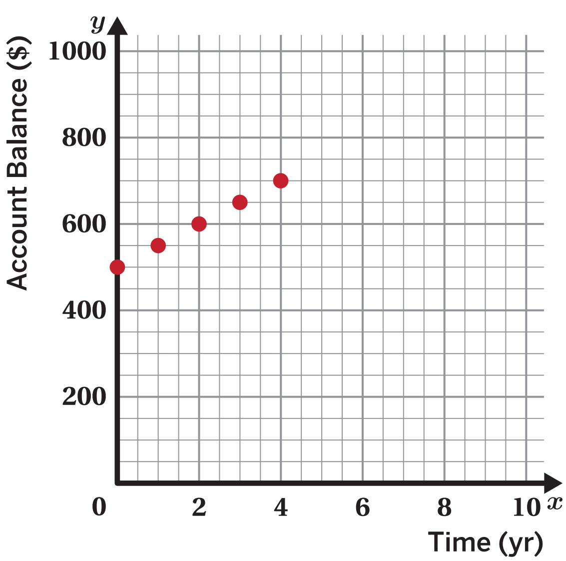
4 Account A earns 10% **simple interest** per year.

a **Discuss:** How do you think *simple interest* works?

Responses vary. **Simple interest adds a constant amount to the account each year.**

b Determine the account balance after 5 years.

Time (yr)	Account Balance ($)
0	500
1	550
2	600
3	650
4	700
5	**750**



Name: ________ Date: ________ Period: ________

Earning Interest (continued)

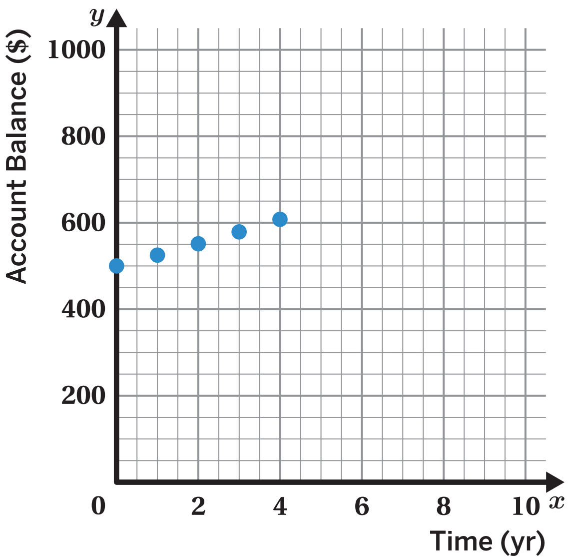
5 Account B earns 5% **compound interest** per year.

a **Discuss:** How do you think *compound interest* works?

Responses vary. Compound interest adds a percentage of the previous balance to the account each year.

b Determine the account balance after 5 years.

Time (yr)	Account Balance ($)
0	500
1	525
2	551.25
3	578.81
4	607.75
5	638.14



6 We can use functions to describe the account balances after t years.

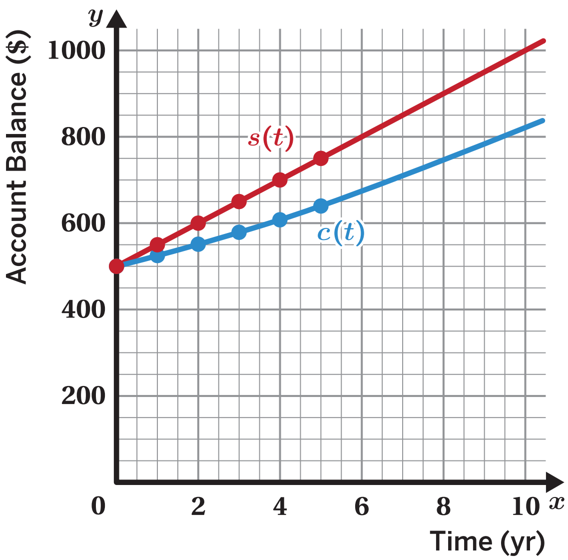
- Simple interest: $s(t) = 500 + 50t$
- Compound interest: $c(t) = 500(1.05)^t$

How are these functions alike?

Responses vary. Both of the functions are increasing and have an initial value of $500.

How are they different?

Responses vary. One function is linear, and the other is exponential.



7 **a** Let's watch the account balances grow.

b **Discuss:** Which account would you recommend Mauricio invest in? Why?

Responses vary. I would ask Mauricio how long he wants to invest his money for. If he wants to invest for less than 26 years, then the simple interest account is a better investment. Otherwise, he will make more money with the compound interest account over a longer investment period.

Activity 2

Name: .... Date: .... Period: ....

Simple and Compound Interest

8 Mauricio decided to invest in an account that offers 6% compound interest per year.

$a(t) = 500(1.06)^t$ represents its balance after t years.

About how many years will it take for the balance to reach \$1,000? Explain your thinking.

Use this space or the Desmos Graphing Calculator to help with your thinking.

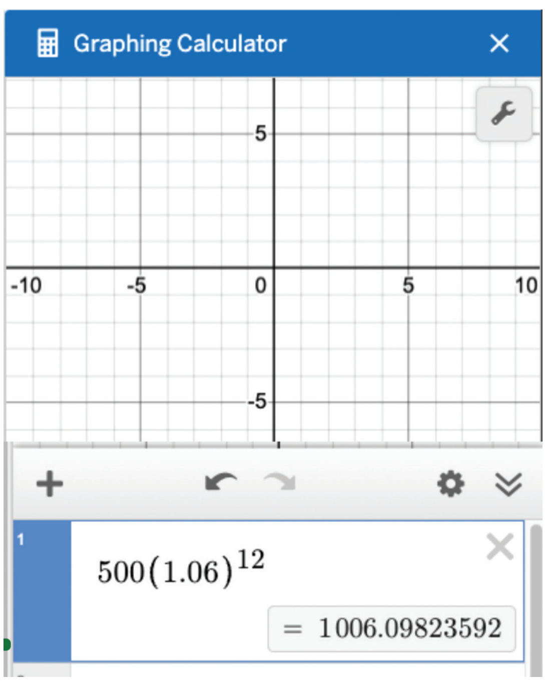
Responses between 11 and 12 years are considered correct. *Explanations vary.*

- **I plugged in different numbers for t until the balance got close to \$1,000. The t-value represents the number of years until the account balance is about \$1,000.**
- **I graphed the function $a(t) = 500(1.06)^t$ and found the point where the y-value is \$1,000. This happens after about 11.9 years**

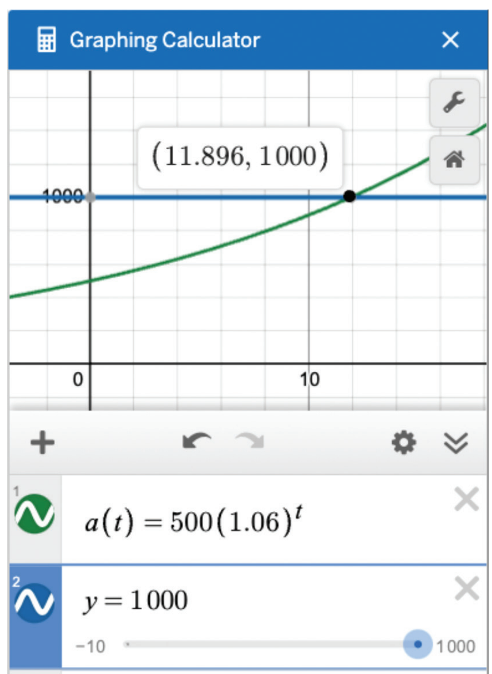
9 Let's watch each video to see how two students determined the time it would take to reach \$1,000.

Discuss: Which method helps get a more precise answer?

Fabiana



Antwon



***Responses vary.* Antwon's method is more accurate. Graphing the function $a(t) = 500(1.06)^t$ and the line $y = 1000$ and determining the x-coordinate where they intersect will help you get a more precise answer.**

Activity 2

Name: Date: Period:

Simple and Compound Interest (continued)

10 Solve as many challenges as you have time for.

a A $\$1,000$ investment earns 4% compound interest.

The function $f(t) = 1000(1.04)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $\$2,500$?

Responses between 23 and 24 are considered correct.

b A $\$200$ investment earns 7% compound interest.

The function $f(t) = 200(1.07)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $\$450$?

Responses between 11 and 12 are considered correct.

c A $\$1,700$ investment earns 3% compound interest.

The function $f(t) = 1700(1.03)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $\$4,150$?

Responses between 30 and 31 are considered correct.

d A $\$1,150$ investment earns 2% compound interest.

The function $f(t) = 1150(1.02)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $\$2,700$?

Responses between 43 and 44 are considered correct.

Explore More

11 Use the Explore More Sheet to answer questions about an account balance.

***Responses vary*. See the Teacher Edition for sample responses.**

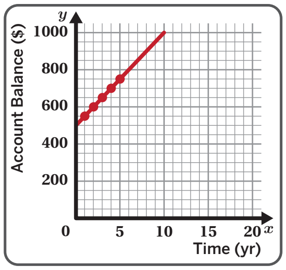
12 Synthesis

Here are some examples of simple and compound interest.

Simple Interest

$$s(t) = 500 + 50t$$

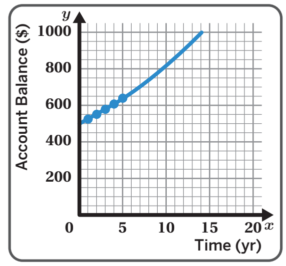
Time (yr)	Account Balance ($)
0	500
1	550
2	600
3	650
4	700



Compound Interest

$$c(t) = 500(1.05)^t$$

Time (yr)	Account Balance ($)
0	500
1	525
2	551.25
3	578.81
4	607.75



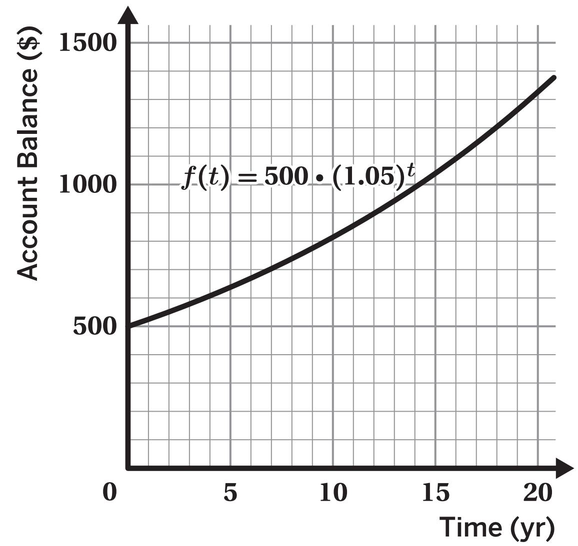
How do investments grow with simple interest? ***Responses vary.* The investments grow linearly by a constant difference each year.**

How do investments grow with simple interest? ***Responses vary.* The investments grow exponentially by a constant ratio each year.**

Things to Remember:

Explore More

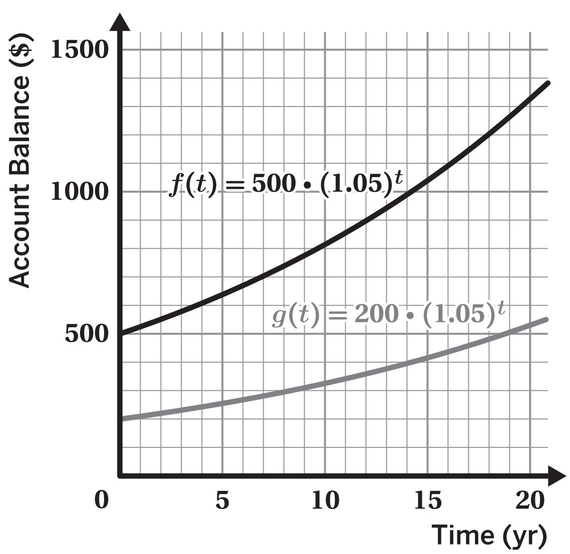
An account with \$500 that earns 5% compound interest doubles its value in about 14 years. The function $f(t)$ gives the account balance after t years.

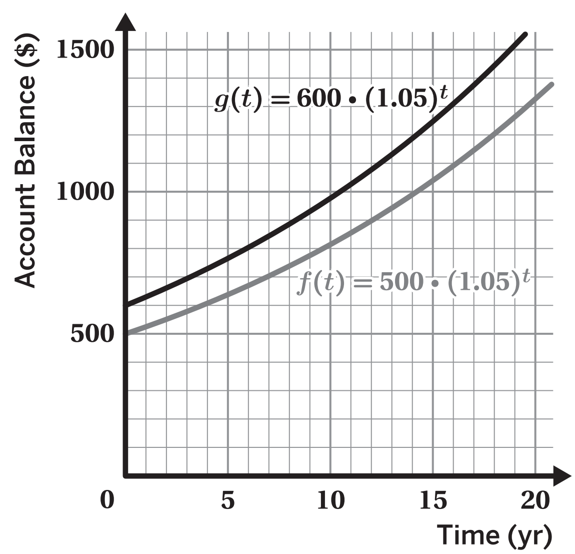


a **Discuss:**

- What does it mean to double in value?
- Where do you see that in the graph?

b Here are two different graphs with different initial account balances.





c **Discuss:** How long does it take the initial account balance to double? Compare with a partner. Is the amount of time it takes to double the same or different?

Name: .... Date: .... Period: ....

Payday Loan

Let's analyze exponential functions that represent different compound interest scenarios.

Warm-Up

1 Zola says that $x^{12} = (x^4)^3$.

The diagram shows why that is true.

$$\underbrace{(x \cdot x \cdot x \cdot x)}_{x^4}\ (x \cdot x \cdot x \cdot x)\ (x \cdot x \cdot x \cdot x)$$

Write three other expressions equivalent to x^{12}.

Responses vary.

- $(x^2)^6$
- $(x^{12})^1$
- $(x^3)^4$

Activity 1

Name: ______ Date: ______ Period: ______

Payday Loan

2 A payday loan is a short-term loan designed to be paid back within a month.

Here is an advertisement for a payday loan.

Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that you can get up to $1,000.
- I notice that you need proof of a regular paycheck to apply.
- I wonder what the highest amount is that someone could take out in a payday loan.
- I wonder what happens if you cannot settle your loan by your next payday.

3 FastCash offers payday loans that charge 15% compound interest per month.

Marc borrows $100 to help pay his heating bill.

How much will Marc owe after one month?

$115

Explain your thinking.

Explanations vary. After one month he will owe the original $100 plus 15% of $100, so $15 of interest. $\$100 + \$15 = \$115$.

Name: ________ Date: ________ Period: ________

Payday Loan (continued)

4 The function $f(m)$ represents how much Marc will owe if he doesn't pay back the loan for m months. Write an equation to represent $f(m)$.

$f(m) =$ $100 \cdot 1.15^m$ (or equivalent)

Months, m	Amount Owed, $f(m)$
0	100
1	115
2	132.25
3	152.09
4	174.90

5 Marc wonders how much money he would owe if he doesn't pay back the loan after 3 years.

He wrote two expressions to represent this situation.

Expression A	Expression B
$100 \cdot 1.15^{36}$	$100 \cdot (1.15^{12})^3$

Discuss: How are the expressions alike? How are they different?

Responses vary.

- **The expressions are similar because $12 \cdot 3 = 36$ using the powers of powers property, and they both have an interest rate of 15%.**
- **The expressions are different because Expression A represents the payday loan for 36 months of interest and Expression B represents the payday loan for 3 years of 12 months of interest.**

6 Marc wrote a third equivalent expression to represent this situation.

Expression C
$100 \cdot (5.35)^3$

What interest rate does the 5.35 represent?

A. 435% per year **B.** 535% per year **C.** Neither

Explain your thinking.

Explanations vary.

- **The account represents percent increase, so the percent must be larger than 100%. 5.35 represents 535%. When I subtract the 100% that represents the initial value, I can find the growth rate.**
- **$5.35 - 1 = 4.35$. 4.35 represents 435%.**

Activity
2

Name: .... Date: .... Period: ....

Credit Cards and Other Loans

7 Marc sees an advertisement for a credit card that charges a 2% monthly interest rate.

How much would he owe for a $100 charge on the credit card after 3 years of no payments?

$203.99

Payday Loan

- $100 loan
- 15% monthly interest

Amount owed after
3 years of no payments:
$15315.19

Credit Card

- $100 charge
- 2% monthly interest

Amount owed after
3 years of no payments:
?

8 Here are three equivalent functions that represent the amount owed on a credit card charge of $100 after t years of 2% monthly interest.

- $g(t) = 100 \cdot 1.02^{12t}$
- $g(t) = 100 \cdot (1.02^{12})^t$
- $g(t) = 100 \cdot 1.2682^t$

Use one or more of the functions to determine the interest rate *per year*.

26.82%

Explain your thinking.

Explanations vary.

- **$1.02^{12} = 1.26824$. Because of the powers of powers property, the interest rate per year is 26.824% in the first two functions.**
- **In $g(t) = 100 \cdot 1.2682^t$, 1.2682 is the growth factor per year, which represents 26.82%.**

Activity 3

Name: ________ Date: ________ Period: ________

Comparing Rates

9 Marc wants to compare interest rates on different types of loans.

a Complete the table.

	Monthly Interest Rate (%)	Monthly Growth Factor	Growth Factor per Year	Interest Rate per Year (%)
Payday Loan	15.00	1.15	5.3503	435.03
Credit Card	2.00	1.02	1.2682	**26.82**
Private Loan	1.21	1.0121	**1.1553**	**15.53**
30-year Mortgage	0.53	**1.0053**	**1.0655**	**6.55**
Federal Student Loan	0.41	**1.0041**	**1.0503**	**5.03**

b **Discuss:** In what situations might people take out each of these different types of loans?

Responses vary.

- **A college student might take out a private or a federal student loan to pay for college.**
- **A person or family might take out a mortgage when they have a down payment and can't immediately pay the full price for a house.**

Name: .... Date: .... Period: ....

Comparing Rates (continued)

10 Here's the information about federal student loans that another student entered on the previous screen.

Monthly Interest Rate (%)	Monthly Growth Factor	Growth Factor per Year	Interest Rate per Year (%)
0.41	1.0041	1.0503	5.03

Annika takes out a $20,000 federal student loan.

Write a function, $h(t)$, to calculate the amount Annika owes after making no payments for t years.

$h(t) = 20000 \cdot 1.0041^{12t}$

Explore More

11 Tyler charges $3,000 to a credit card with a 2% monthly interest rate.

Many credit cards require a monthly minimum payment.

a Let's see how long will it take Tyler to pay off the charges with different monthly payment amounts.

b What do you think is important to remember when getting and using a credit card?

Responses vary.

- **Make more than the minimum payment because the payment might be less than the interest applied each month.**
- **Find out what the interest rate is.**
- **Don't make a lot of charges without a plan to pay them off.**
- **Pay off the balance as soon as possible.**

12 Synthesis

What can different equivalent expressions tell us about the same situation involving compound interest?

Responses vary.

- **Raising the monthly growth factor to the power of 12 gives an equivalent growth factor per year.**
- **The powers of powers property allows us to manipulate the monthly growth factor into a growth factor per year.**

Expression A	Expression B	Expression C
$100 \cdot 1.15^{36}$	$100 \cdot (1.15^{12})^3$	$100 \cdot (5.35)^3$

Things to Remember:

Unit 6
Lesson
15

Name: ________ Date: ________ Period: ________

Credit Card Compounding

Let's explore how to calculate and compare account balances with interest rates that compound at different intervals.

Warm-Up

1 Group each card with the word that it describes.

Card A	Card B	Card C	Card D
2 times per year	4 times per year	$\frac{1}{12}$ of a year	$\frac{1}{4}$ of a year

Card E	Card F	Card G
Every 3 months	Every 6 months	Every 12 months

Monthly	Quarterly	Semi-Annually	Annually
C	B, D, E	A, F	G

Activity 1

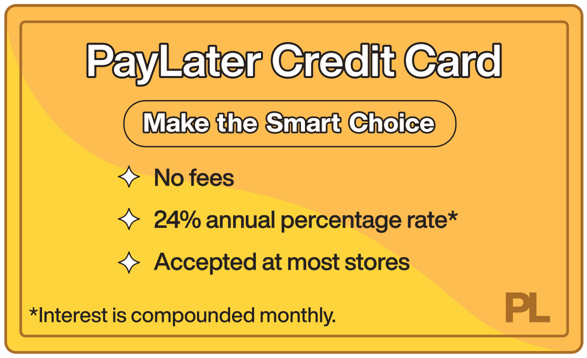
Name: ______ Date: ______ Period: ______

PayLater

2 Alejandro is considering charging $\$1,000$ to this credit card.

He wrote $1000(1 + 0.24)^5$ to determine the balance after 5 years with no payments or additional charges.

Explain what each part of the expression means. ***Responses vary.***



1000: **The initial charge to the credit card**

$1 + 0.24$: **The growth factor per year**

5: **The number of years Alejandro will have the credit card without making a payment**

3 The fine print says interest is compounded monthly.

This means the interest is $\frac{24}{12} = 2$, or 2% per month.

Compared to compounding annually, how do you think compounding monthly will affect the total Alejandro owes after 5 years? Circle one. ***Responses vary.***

A. He will owe more **B.** He will owe less **C.** He will owe the same

Explain your thinking.

Explanations vary.

- **He will owe more. Compounding monthly will make the total amount more than compounding annually because instead of multiplying the rate 5 times, you multiply the rate 60 times.**
- **He will owe the same. The annual interest rate is the same regardless of the compounding period.**

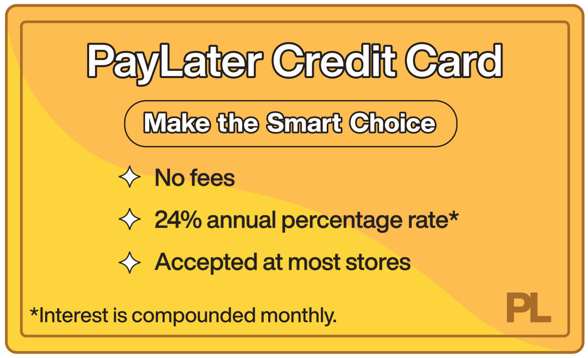
Name: ________ Date: ________ Period: ________

PayLater (continued)

4 Alejandro is considering charging $\$1{,}000$ to this credit card.

If the interest is compounded at 2% monthly, how much would he owe after 5 years?

$\$3{,}281.03$



5 Alejandro wrote $1000\left(1+\frac{0.24}{12}\right)^5$ to determine the balance after 5 years, but he made an error.

Find the error and explain why it is incorrect.

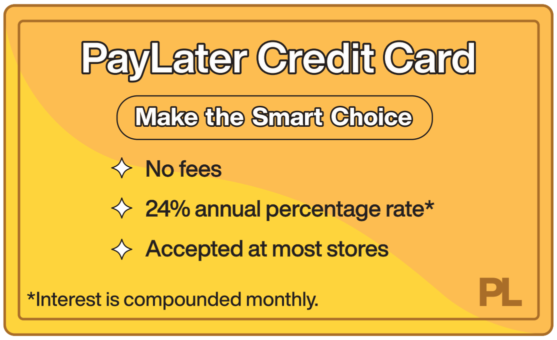
***Responses vary.* Alejandro needs to multiply the rate 60 times instead of 5 times because there are 12 months in a year, and he is keeping the card for 5 years, which is 60 compounding intervals.**

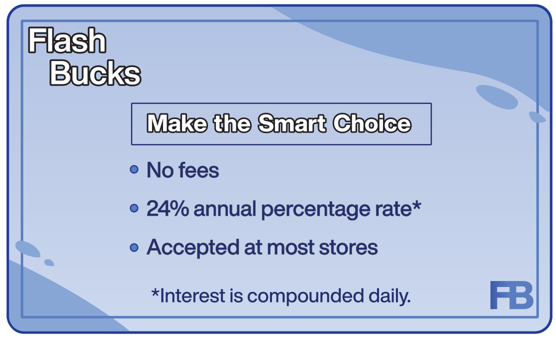
Activity
2

Name: .... Date: .... Period: ....

PayLater and Flash Bucks

6 Alejandro is considering charging $\$1{,}000$ to a different credit card.

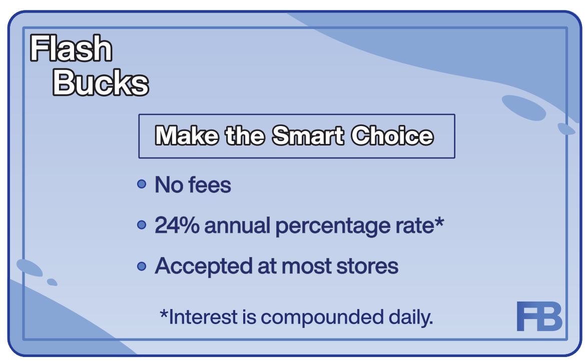




Discuss: Compared to compounding monthly, how do you think compounding daily will affect the total amount owed?

***Responses vary.* One card is compounding monthly, or 12 times a year. The other card is compounding daily, or 365 times a year. They both have no additional fees and a 24% annual interest rate. I think compounding daily means applying the rate 365 • 5 or 1825 times, so I think it will make the total amount owed increase.**

7 Alejandro is considering charging $\$1{,}000$ to this credit card.



a **Discuss:** How would you determine the daily interest rate?

***Responses vary.* I would divide the 24% annual rate by the number of days in a year. So $0.24 \div 365$.**

b If interest is compounded daily, how much would Alejandro owe after 5 years with no payments or additional charges?

$$1000\left(1+\frac{0.24}{365}\right)^{(5 \cdot 365)} \approx \$3{,}318.81$$

Name: ______ Date: ______ Period: ______

Compounding Differently

8 Here are some expressions to calculate the total amount for $800 and a 12% annual interest rate compounded using different *intervals*.

Match each expression with its compounding period and length. One card will have no match.

Card A

$$800\left(1+\frac{0.12}{4}\right)^{(4 \cdot 3)}$$

Card B

$$800(1+0.01)^{24}$$

Card C

$$800\left(1+\frac{0.12}{12}\right)^{(12 \cdot 2)}$$

Card D

$$800(1+0.04)^{(3 \cdot 2)}$$

Card E

$$800(1+0.03)^{12}$$

Compounded Quarterly for 3 Years	Compounded Monthly for 2 Years
A, E	B, C

Card D does not have a match.

9 Compound interest expressions can be represented using this formula:

$$P\left(1+\frac{r}{n}\right)^{nt}$$

Circle one variable and describe what it represents. *Responses vary.*

P r n t

- P is the initial balance of an account.
- r is the annual interest rate.
- n is the number of compounding intervals in one year.
- t is the time, in years.

Name: ______ Date: ______ Period: ______

Compounding Differently (continued)

10 Solve as many challenges as you have time for.

a A person puts $500 into an account with a 10% annual interest rate compounded quarterly.

What is the balance in the account after 4 years?

$742.25

b A person puts $800 into an account with a 5% annual interest rate compounded daily.

What is the balance in the account after 3 years?

$929.46

c A person puts $3,000 into an account with a 5% annual interest rate compounded daily.

What is the balance in the account after 7 years?

$4,257.10

d A person puts $1,000 into an account with a 20% annual interest rate compounded yearly.

What is the balance in the account after 8 years?

$4,299.82

11 Synthesis

How can you use this formula to calculate the total value of an account or loan with compound interest?

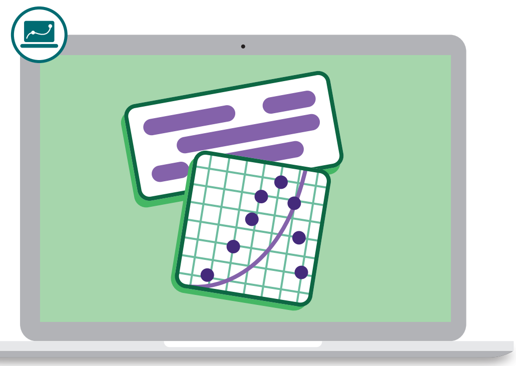
$$P\left(1+\frac{r}{n}\right)^{nt}$$

***Responses vary*. If I know the starting account balance, the annual interest rate, the number of years, and how many times the interest compounds, I can use the formula to calculate the final account balance.**

Things to Remember:

Unit 6
Lesson
16

Name: .... Date: .... Period: ....



Detroit's Population, Part 1

Let's use functions to model the population growth of Detroit.

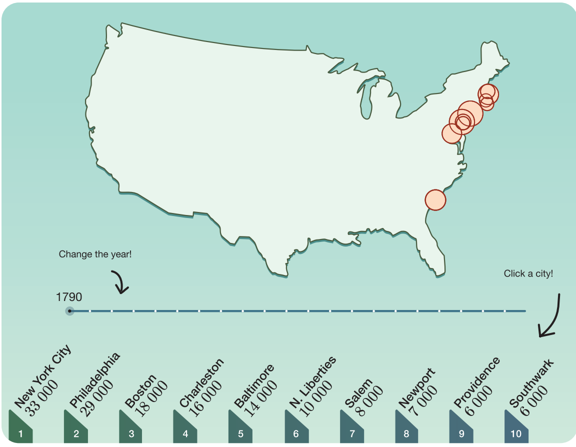
Warm-Up

1 Use the digital activity to see the ten U.S. cities with the largest populations from 1790–2000.

Discuss: What do you notice? What do you wonder?

Responses vary.

- **I notice that all of the circles begin on the East Coast and expand westward over time.**
- **I notice that New York City was the largest U.S. city from 1790–2000.**
- **I notice that all of the circles seem to get larger until 1930 when some of them start to decrease slightly.**
- **I wonder how populations were growing in the green areas over these two centuries.**



Source: *United States Census Bureau*

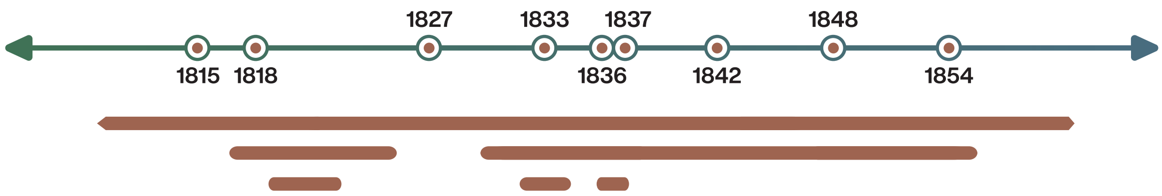
Activity 1

Name: ________ Date: ________ Period: ________

Early History of Detroit

You'll use the digital activity for Problems 2–6.

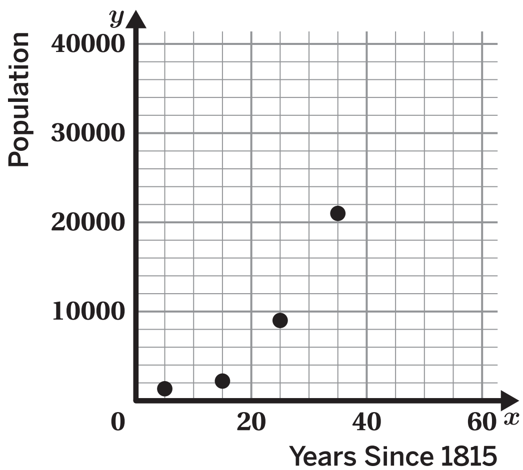
2 Explore the timeline to learn about one of the largest U.S. cities from 1815 to 1855.



Discuss: How do you think the population of Detroit changed during that time?

Responses vary. **I think the population of Detroit was not growing steadily from 1815–1855 because there were several events that caused the population to decrease (e.g., cholera epidemics and treaties that displaced the Anishinaabe Nation) and increase (e.g., wave of Irish immigration and train connection to New York City).**

3 Detroit became a U.S. city in 1815. This scatter plot shows the census data for Detroit's population from 1820 to 1850.



a Which type of function do you think better fits the data?

Choices vary.

Linear Exponential

b Use the digital activity to fit the function to the data.

Functions vary.

c Explain how you decided which function to use.

Responses vary.

- **I think a linear function best fits the data because the events in the timeline showed a steady growth in population.**
- **I think an exponential function best fits the data because the points are beginning to curve upwards, and if the train was just built to connect to New York City at this point, we might expect the population to increase more after 1855.**

Name: ________ Date: ________ Period: ________

Early History of Detroit (continued)

4 Write the function you created to model the population of Detroit x years since 1815.

Responses vary based on the model students created on Screen 3. The sample responses for Problems 4–6 are for the model $p(x) = 733(1.1019)^x$.

$p(x) =$ $733(1.1019)^x$

Explain what each part of the *model* represents.

- **The 733 in my exponential model represents the number of people that lived in Detroit in 1815, or year 0.**
- **The 1.1019 in my exponential model represents that the population of Detroit was growing by 10.19% each year.**

5 **a** Determine the value of $p(30)$.

Responses between 13,470 and 13,473 are considered correct.

b What does $p(30)$ represent in this situation?

***Responses vary.* In 1845 (30 years since 1815), there were 13,470 people in the city of Detroit.**

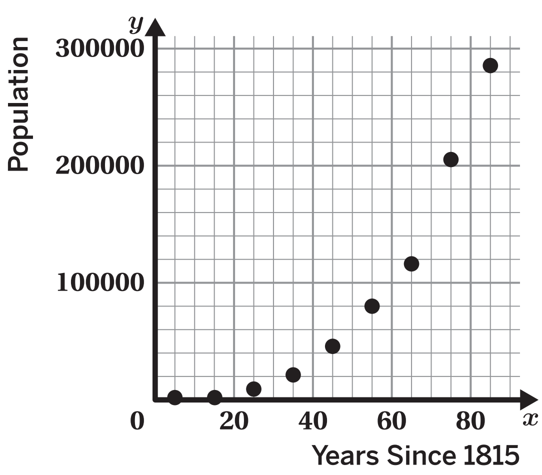
6 The U.S. census data for Detroit's population from 1860 to 1900 has been added to the graph.

Discuss:

- What do you notice? What do you wonder?
- How does your model compare to the actual data?

Responses vary.

- **I notice that my model is increasing too quickly compared to the data points, meaning the population of Detroit wasn't growing as fast as my model predicts.**
- **I wonder if the data is still best modeled by an exponential function.**



Activity 2

Name: .... Date: .... Period: ....

Predicting the Future

7 Zwena chose to revise her model to better fit the data from 1820 to 1900.

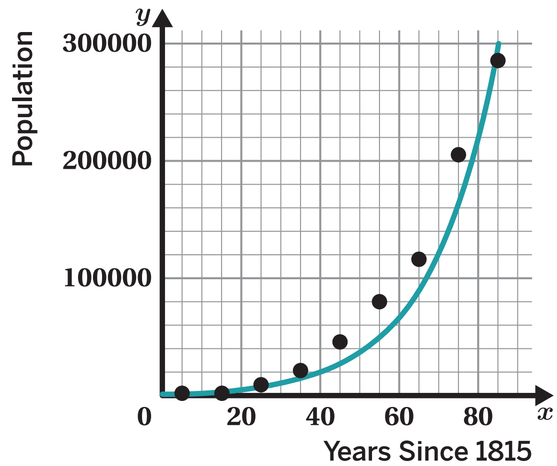
$$q(x) = 1904(1.0611)^x$$

According to Zwena's model, by what percent is Detroit's population increasing each year?

6.11%

Explain your thinking.

Explanations vary. **This model shows that the growth factor is 1.0611. When I rewrite this as a percent, it converts to 106.11%. 100% means the population is staying the same, so the part above 100% is the percent increase each year.**



8 Use the model to predict Detroit's population in 1910.

Years Since 1815, x	Population, $q(x)$
95	

Responses between 532,739 and 532,742 are considered correct.

9 Zwena wondered what the model would predict for Detroit's population in 2000, 185 years after 1815.

She used her model to calculate that the population of Detroit was about 111 million people in 2000.

Do you think this number is realistic? Explain your thinking.

Zwena

$q(x) = 1904(1.0611)^x$

$q(185) = 1904(1.0611)^{185}$

$q(185) = 110811576$

Responses vary.

- **The prediction of 111 million people in 2000 is not realistic or accurate because it is more than the number of people who actually live in Detroit now, so the model overpredicted the population.**
- **I'm not sure, because Zwena's calculations are accurate, but we don't know how good the model will be that far into the future.**

Activity 3

Name: .... Date: .... Period: ....

Based on Historical Events

You'll use the digital activity for Problems 10–13.

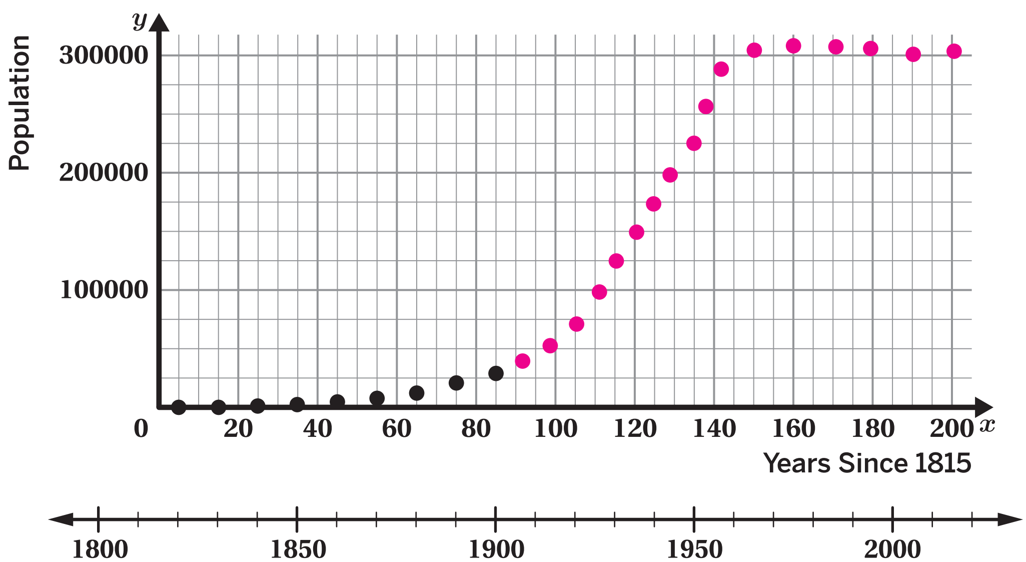
10 Examine the timeline in the digital activity to learn more about industry and migration throughout Detroit's history.

Discuss: How would you describe the change in population of Detroit in the years since 1900?

***Responses vary.* I think the population of Detroit grew exponentially until 1970. After 1970, I think the population of Detroit decreased linearly.**

11 Sketch a prediction for the population of Detroit in the years after 1900.

Responses vary.



12 Use the digital activity to reveal the population data after 1900.

Discuss: What do you notice? What do you wonder?

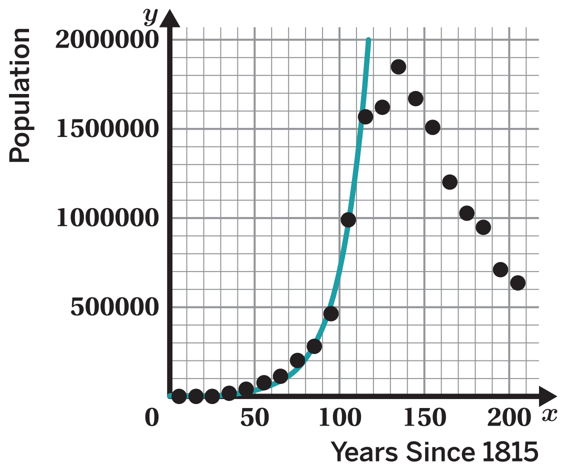
Responses vary.

- **I notice that my sketch assumed the population would continue to grow and then stay constant, but the population actually started decreasing in the 1960s, which is very different than what I expected.**
- **I wonder what happened in the 1960s to cause the population to start decreasing.**

Name: .... Date: .... Period: ....

Based on Historical Events (continued)

13 Here is Zwena's model.



a Use the digital activity to highlight a *domain* in which this model would be useful for making predictions.

Responses vary. $0 \le x \le 115$

b What might be some issues with using this model outside of the highlighted domain?

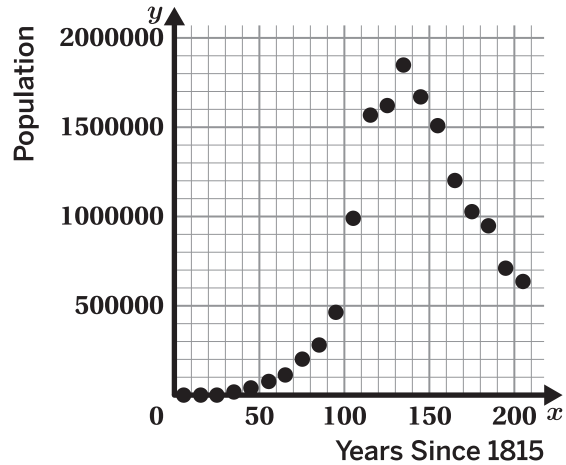
Responses vary. If I use this model to predict the population of Detroit past 1930, the model no longer fits the data well and my prediction will be inaccurate and too high.

14 Synthesis

Select one of the questions to answer:

A. What is something you learned about using exponential functions to model population change over time?

B. What is a question you have about the population or history of Detroit?

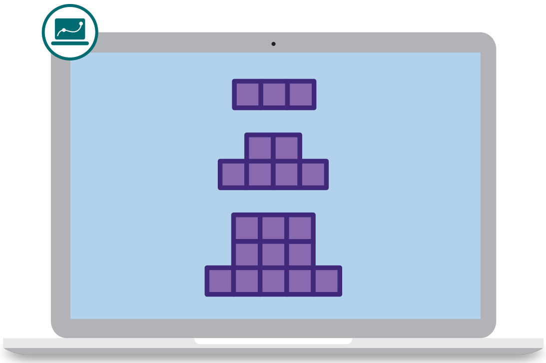


Responses vary.

- I learned that exponential functions will only be accurate in modeling population data up to a certain point because the number of people cannot continue to grow forever.
- What happened in the 1960s that caused the population of Detroit to decrease?

Things to Remember:

Name: ______ Date: ______ Period: ______

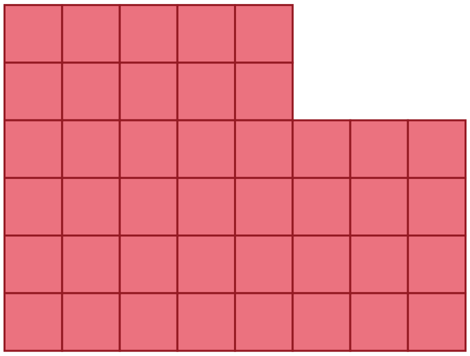


Revisiting Visual Patterns

Let's explore a new type of visual pattern.

Warm-Up

1 Here are different ways of counting the tiles.



Select one expression.

A. $5+5+8+8+8+8$

B. 4^2+4^2+10

C. $6 \cdot 8 - 2 \cdot 3$

D. $5 \cdot 6 + 3 \cdot 4$

Show or explain how you see this expression in the diagram.

Responses vary.

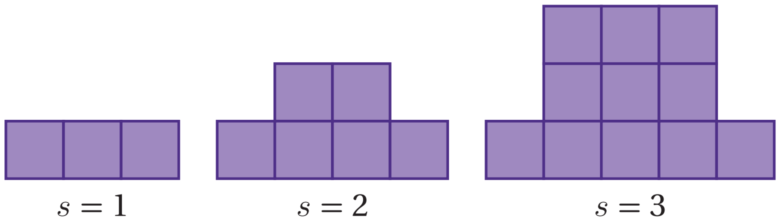
- **Choice A: I see two rows of 5 at the top and four rows of 8 at the bottom.**
- **Choice B: I see two 4-by-4 squares at the bottom of the shape, plus 10 small squares on the top.**
- **Choice C: I can draw a large 6-by-8 rectangle around the entire shape, then subtract a 2-by-3 rectangle from the corner.**
- **Choice D: I see two rectangles put together. One 5-by-6 rectangle on the left and a smaller 3-by-4 rectangle on the right.**

Activity 1

Name: .... Date: .... Period: ....

A New Type of Pattern

2 Here are the first three steps of a pattern.



What about the pattern is changing? What is staying the same? ***Responses vary.***

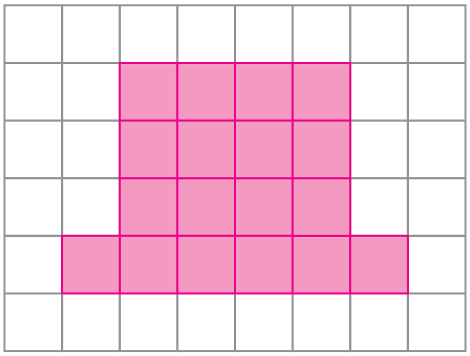
Things that are changing:

- **The width of the shape**
- **The value of s**
- **The size of the square in the middle**

Things that are staying the same:

- **One square on each side**
- **The squares are purple**
- **Each figure is made up of smaller squares**

3 Draw the pattern when $s = 4$.



4 How many tiles will there be when $s = 10$?

102 tiles

Name: .... Date: .... Period: ....

A New Type of Pattern (continued)

5 Abdullah used a table to figure out how many tiles there will be when $s = 10$.

What type of relationship is there between s and the number of tiles?

A. Linear

B. Exponential

C. Something else

Explain your thinking.

***Explanations vary*. The number of tiles is not increasing by the same number from row to row. Since I can't add or multiply over and over, I know this isn't a linear or exponential relationship.**

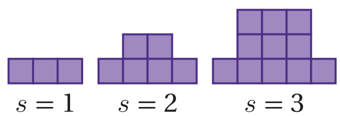
S	Number of Tiles
1	3
2	6
3	11
4	18
5	27
6	38
7	51
8	66
9	83
10	?

Name: ________ Date: ________ Period: ________

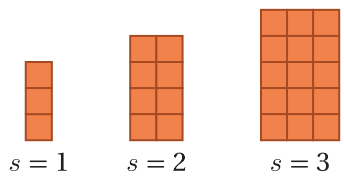
Comparing Patterns

6 Take a look at Pattern A and Pattern B.

Pattern A



Pattern B



How are the two patterns alike? How are they different? ***Responses vary.***

Alike:

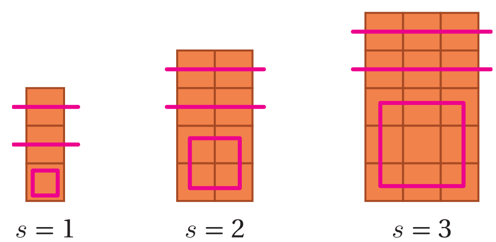
- **Both are getting taller and wider.**
- **Both have a square in the pattern.**
- **I can see a width of 3 in both.**

Different:

- **Pattern B is always taller than Pattern A.**
- **Pattern B doesn't have any squares that are constant.**

7 Abdullah said: *I see a square plus two rows.*

Deja said: *I see a rectangle where the length is two more than the width.*



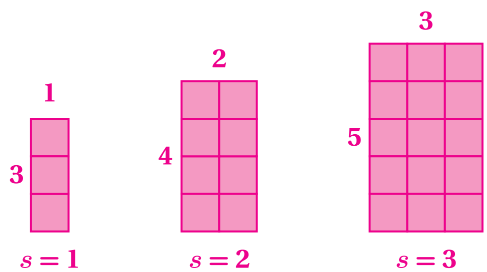
a Show how one student saw the pattern.

Sketches vary.

b **Discuss:** How might this student describe how to draw the image when $s = 4$?

Responses vary.

- **Abdullah might draw a 4-by-4 square and then two rows below it.**
- **Deja might draw a 5-by-6 rectangle because 6 is two more than 4.**



Name: ______ Date: ______ Period: ______

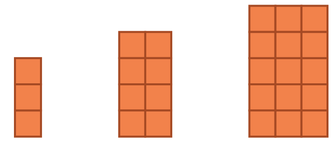
Comparing Patterns (continued)

8 Determine the number of tiles when $s = 4$.

24, $4^2 + (2 \cdot 4)$, $4 \cdot 6$ (or equivalent)

Determine the number of tiles when $s = 10$.

120 (or equivalent)

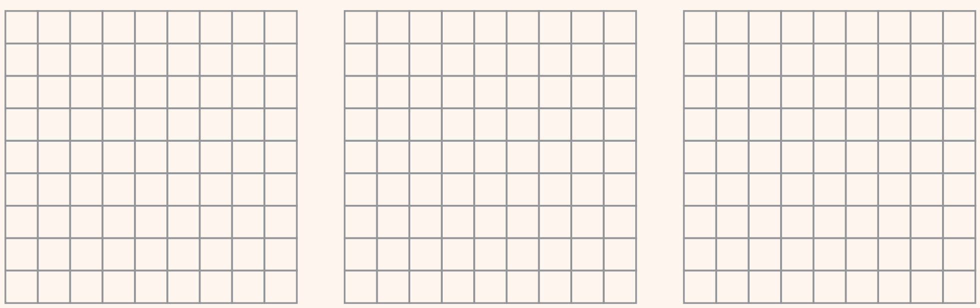


$s = 1$ $s = 2$ $s = 3$

Explore More

9 Create the first three steps of your own visual pattern.

Patterns vary.



$s = 1$ $s = 2$ $s = 3$

Discuss:

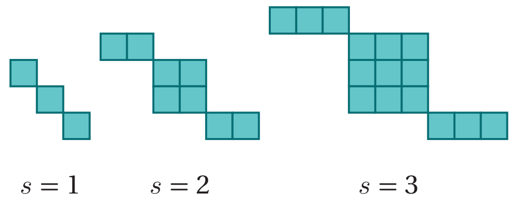
- What about your pattern is changing? What is staying the same?
- How many tiles will there be when $s = 4$? When $s = 10$?

Responses vary.

10 Synthesis

What would you say to help a classmate who is trying to describe a pattern?

Use the example if it helps with your thinking.

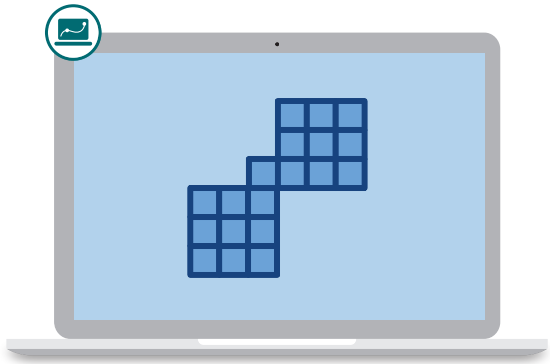


Responses vary. **First, I would look for what parts of the pattern change and what parts stay the same. Then, I would try to break the pattern into shapes, like squares or rectangles, that grow.**

Things to Remember:

Unit 7
Lesson
2

Name: .... Date: .... Period: ....



Quadratic Visual Patterns

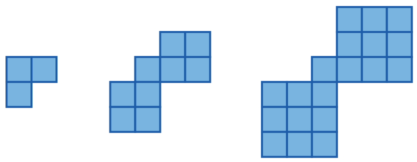
Let's describe a new type of pattern using expressions.

Warm-Up

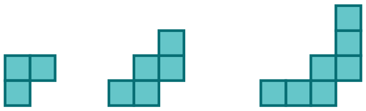
1 Here are Pattern A and Pattern B.

How are the two patterns alike? How are they different? ***Responses vary.***

Pattern A



Pattern B



Alike:

- **They both start with 3 tiles.**
- **They both grow in kind of a W pattern.**

Different:

- **Pattern A has more tiles than Pattern B when you drag the slider.**
- **Pattern A keeps making squares every time and Pattern B keeps making lines. For example, in the third figure, Pattern A is two 3-by-3 squares and Pattern B has two lines 3 tiles long.**

Name: ________ Date: ________ Period: ________

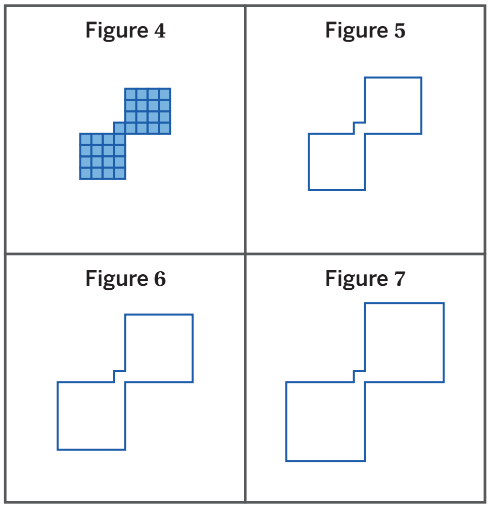
Figure n

2 Let's take a closer look at Pattern A.

Calculate the number of tiles for each figure.

Use an expression if it helps with your thinking.

Figure	Number of Tiles
4	33
5	**51**
6	**73**
7	**99**



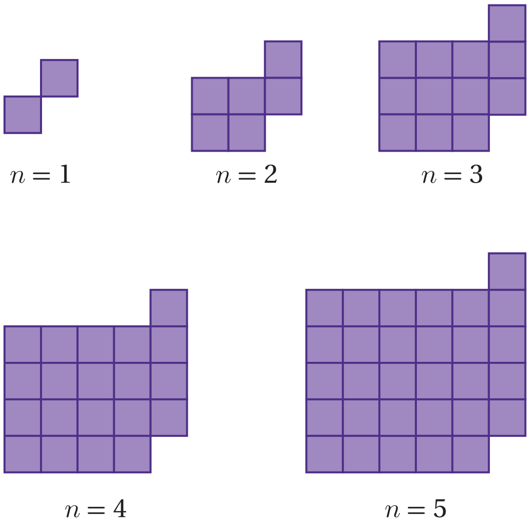
Note: Students may write expressions such as $5^2 + 5^2 + 1$ *or* $25 + 25 + 1$.

3 A student created a table from the previous problem.

Write an expression for the number of tiles in Figure n.

- $n^2 + n^2 + 1$
- $2n^2 + 1$

(or equivalent)



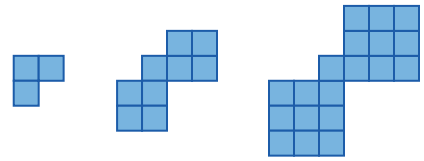
Activity 2

Name: ________ Date: ________ Period: ________

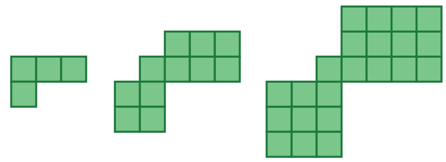
Writing Expressions

4 Take a look at Pattern A and Pattern C.

Pattern A



Pattern C



How are the two patterns alike? How are they different? ***Responses vary.***

Alike:

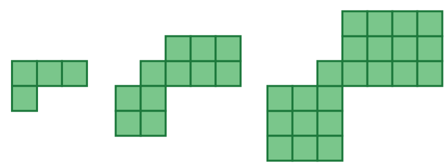
- **They both have a growing square on the bottom.**
- **They both have one tile in the middle between the two other shapes.**

Different:

- **Pattern A starts with 3 tiles and Pattern C starts with 4 tiles.**
- **Pattern A has two squares that keep growing. Pattern C has one square and one rectangle.**

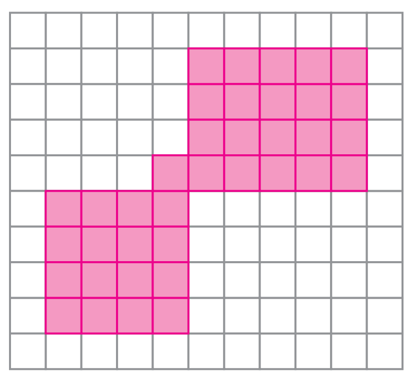
5 Here is Pattern C.

$n = 1$ $n = 2$ $n = 3$



a Sketch the pattern for $n = 4$.

$n = 4$



b Calculate the number of tiles when $n = 4$. Use an expression if it helps with your thinking.

37 tiles

Name: Date: Period:

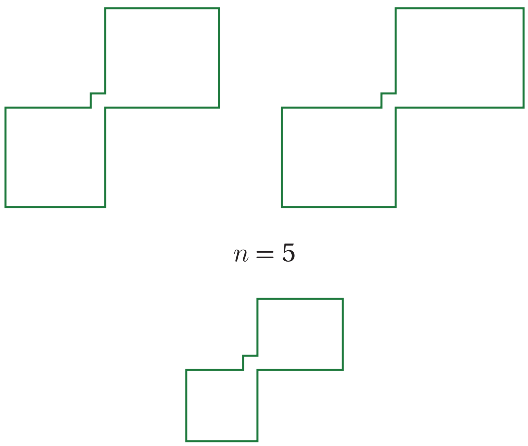
Writing Expressions (continued)

6 Here are three more figures of Pattern C.

Write an expression in terms of n to evaluate them all at once.

- $n^2 + n^2 + n + 1$
- $n^2 + n(n+1) + 1$
  (or equivalent)

$n = 7$ $n = 8$ $n = 5$



7 Let's look at expressions that two students wrote for Pattern C.

Discuss: Where do you see each part of their expression in their sketch?

Responses vary. Luis sees two squares that are n-by-n and then a line that is n tiles tall and then another 1. Ishaan sees one square, one rectangle that is 1 tile wider than it is tall, and then another 1.

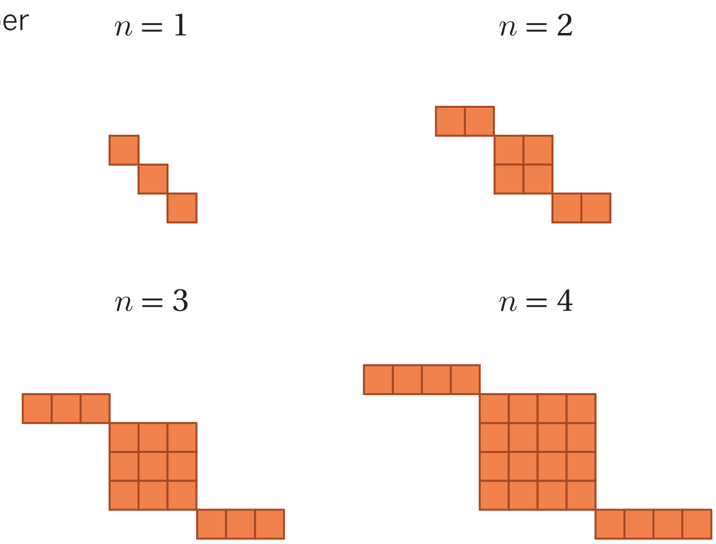
Activity
3

Name: ______ Date: ______ Period: ______

Quadratic Relationships

8 Write an expression for the number of tiles in terms of n.

$n^2 + 2n$ **(or equivalent)**



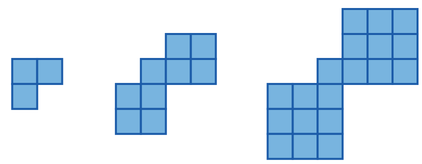
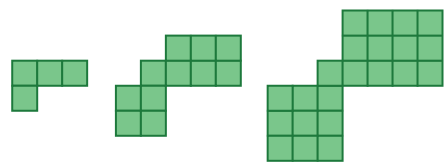
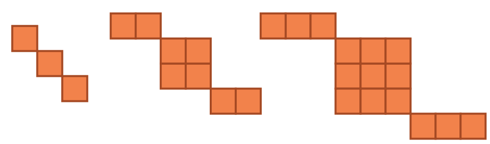
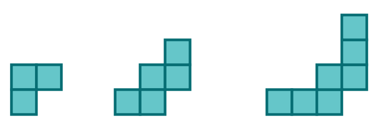
9 Select *all* the expressions that represent the number of tiles in this pattern.

- ☐ **A.** $5n$
- ☑ **B.** $n^2 + 2n$
- ☑ **C.** $(n \bullet n) + (n + n)$
- ☐ **D.** $3n^2$

Name: ________ Date: ________ Period: ________

Quadratic Relationships (continued)

10 Here are some of the relationships we've explored in this lesson.

	Expressions	Patterns
Quadratic	$2n^2 + 1$	
	$2n^2 + n + 1$	
	$n^2 + 2n$	
Linear	$2n + 1$	

What do you think **quadratic relationships** all have in common?

Responses vary.

- **They all have something to do with squares.**
- **There's an n^2 in all of the expressions.**
- **They don't grow by the same number of tiles every time like linear relationships do. They have a growing square in them.**

11 Synthesis

How does creating a table with expressions help someone trying to write an expression to represent a quadratic relationship

Use this table if it helps with your thinking.

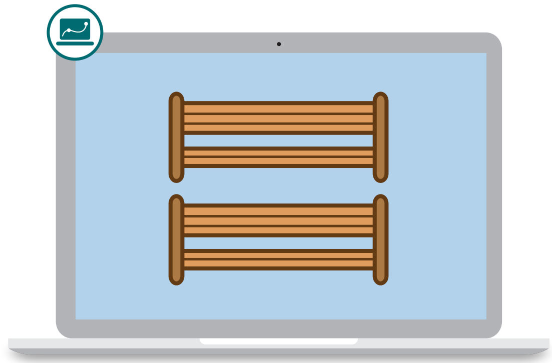
Responses vary. A table makes it easy to keep track of the changes in a pattern. Putting expressions in the table help to capture the shapes or pieces that make up the pattern. The changes in the expressions in the table make it easy to identify which part of the expression will be constant and which parts will be represented by a variable in the expression that represents the quadratic relationship.

For example, in this table, I see that the $+ 1$ is constant. I also see that the squared term changes in each row of the table. This means that the squared term should be represented by n. The expression for this table should be $2n^2 + 1$.

Figure	Number of Tiles
4	33
5	$5^2 + 5^2 + 1$
6	$6^2 + 6^2 + 1$
7	$7^2 + 7^2 + 1$
n	?

Things to Remember:

Name: ______ Date: ______ Period: ______



On the Fence

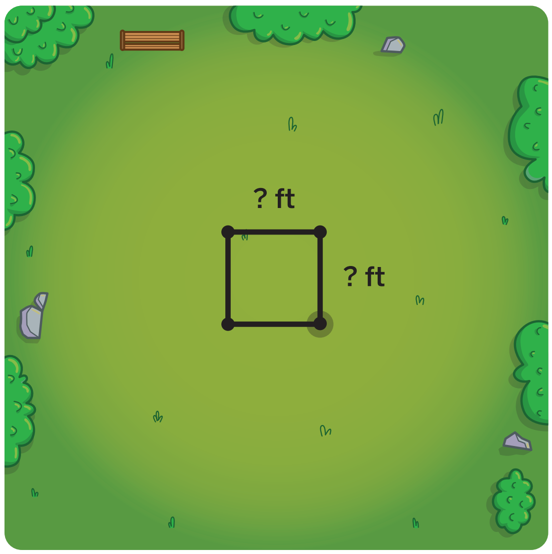
Let's use the context of building fences to explore symmetry in quadratic relationships.

Warm-Up

1 Farmer Farah is building a fence for her sheep.

Each panel of fencing is 5 feet long and she has 100 feet of fencing total.

Build three different fences in the pasture that each use *exactly* 100 feet of fencing.



Width (ft)	Length (ft)
40	10
25	25
15	35

Fences vary. In all correct fences, the sum of the width and the length is 50 feet.

Activity 1

Name: .... Date: .... Period: ....

Farmer Farah's Fencing

2 Let's look at three fences.

How are these fences alike? How are they different?

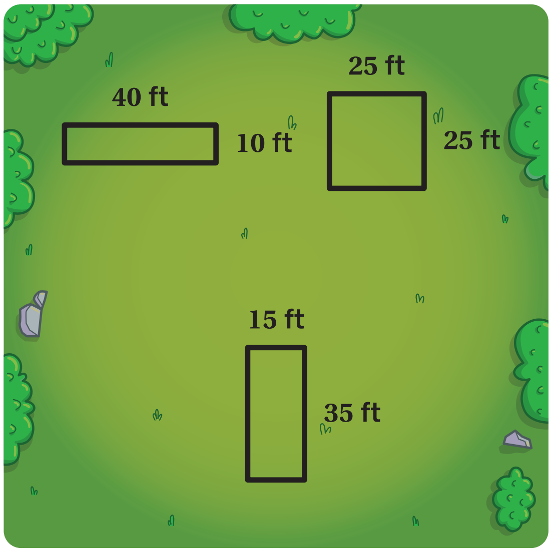
Responses vary.

Alike:

- All the fences are rectangles.
- All the numbers in the fences are multiples of 5.
- The length and the width add to 50.
- They all have the same perimeter.

Different:

- Some fences are skinnier than others.
- They have different areas.
- Some fences are more wide than long. Others are more long than wide.



3 Farah noticed that for each fence, the perimeter stays the same but the area changes.

Here are Farah's fences.

Calculate the areas of the three fences.

Note: Students using digital will see the widths and lengths of the fences they made in the Warm-Up.

Width (ft)	Length (ft)	Area (sq. ft)
40	10	400
15	35	525
5	45	225

Name: Date: Period:

Farmer Farah's Fencing (continued)

4 The table represents all the possible fences Farmer Farah can build in the pasture.

a Complete the missing values in the table.

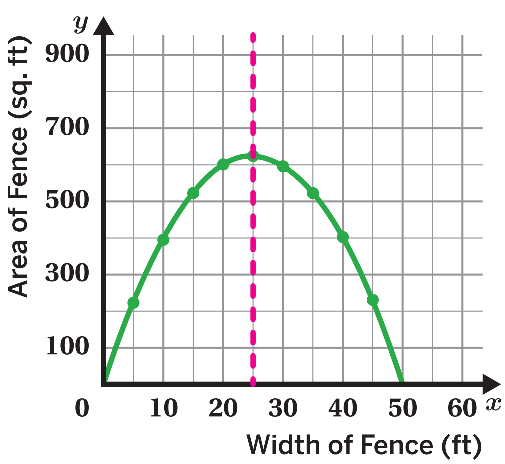
b **Discuss:** What do you notice? What do you wonder?

Responses vary.

- **I notice the widths are going up by 5 and the lengths are going down by 5.**
- **I notice that there are a lot of the same numbers in the table.**
- **I wonder if Farah could have a width of 22 feet.**
- **I wonder if 625 is the biggest area you could make.**

Width (ft)	Length (ft)	Area (sq. ft)
5	45	225
10	40	400
15	35	525
20	30	**600**
25	25	625
30	**20**	600
35	15	**525**
40	10	**400**
45	5	**225**

5 Here is a graph of the areas of all the possible fences.



What type of relationship is represented?

A. Linear

B. Exponential

C. Quadratic

D. Something else

Explain your thinking. *Explanations vary.*

- **$400 - 225 = 175$, $525 - 400 = 125$, and $600 - 525 = 75$, so there is a constant second difference of -50.**
- **The graph is shaped like a quadratic. The difference between the y-values decreases by 50 between each point.**

6 The graph of a quadratic function is called a **parabola**.

Parabolas have a **line of symmetry**. If you fold a parabola along this line, you get two identical halves.

a Draw the line of symmetry on the graph of possible fences. $x = 25$

b **Discuss:** What does this line mean in the context of the sheep fence?

Responses vary. **It is the width of the fence when the area is the largest. The width of the fence is 25 feet.**

Name: ______ Date: ______ Period: ______

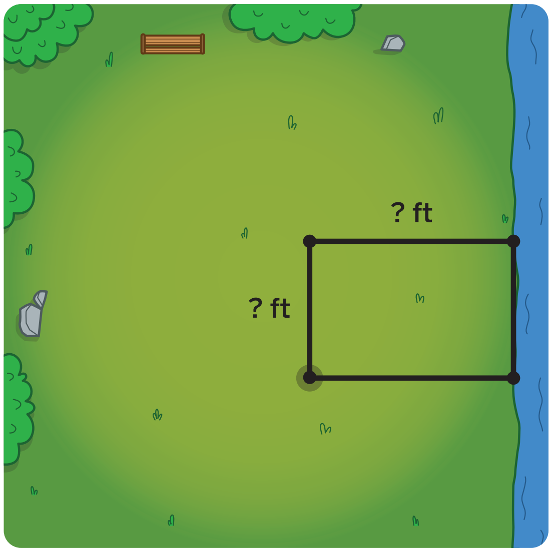
By the Stream

7 Farmer Farah's sheep don't like to swim. If she builds her fence by the stream, it will only need three sides.

Using 100 feet of fencing, build three possible fences.

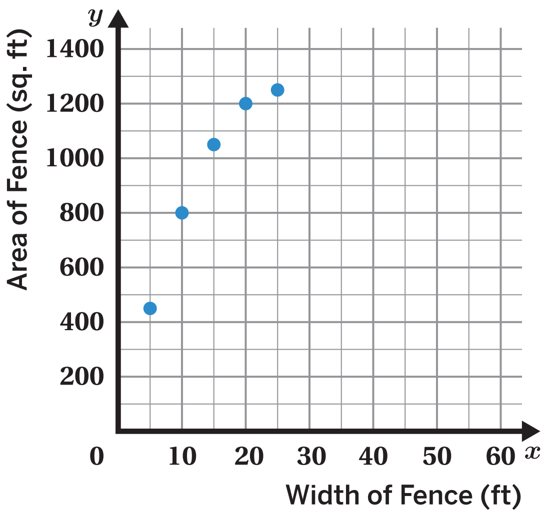
Width (ft)	Length (ft)
40	**20**
20	**60**
45	**10**

Fences vary.



8 Here are a few of the possible fences Farmer Farah can build by the stream.

Width (ft)	Area (sq. ft)
5	450
10	800
15	1050
20	1200
25	1250



Is this relationship quadratic? Circle one.

Yes   No   I'm not sure

Explain your thinking.

***Explanations vary.* There is a constant second difference of -100.**

Name: .... Date: .... Period: ....

By the Stream (continued)

9 What are the areas of all the possible fences this parabola could represent?

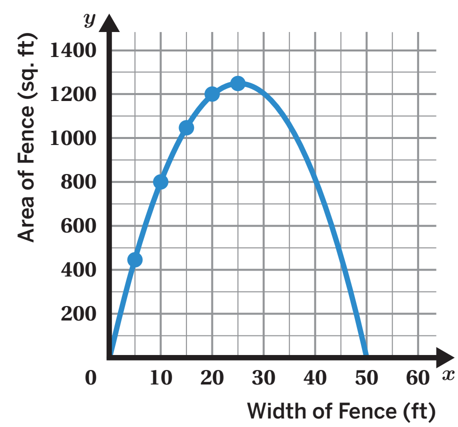
Complete the table to show all the possible areas of fences Farmer Farah could make by the stream.

Width (ft)	Area (sq. ft)
5	450
10	800
15	1050
20	1200
25	1250
30	**1200**
35	**1050**
40	**800**
45	**450**

10 Here is a graph of a parabola that includes all the possible areas of fences along the stream.

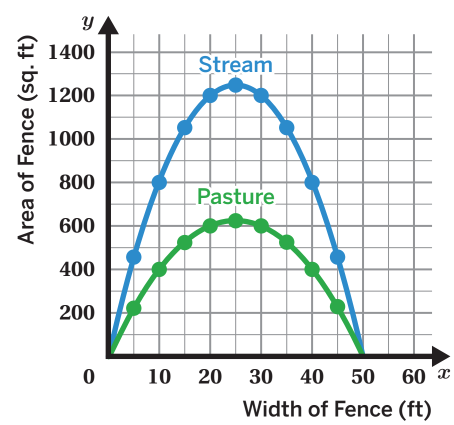
Write the equation for the line of symmetry for this parabola.

$x = 25$



11 Here are the graphs of parabolas that include all the possible fences Farmer Farah could build in the pasture and by the stream.

How are these relationships alike? How are they different? ***Responses vary.***



Alike:

- **They are both parabolas that have a highest point.**
- **They both start at 0 feet and go until 50 feet.**
- **They both have a line of symmetry at $x = 25$.**

Different:

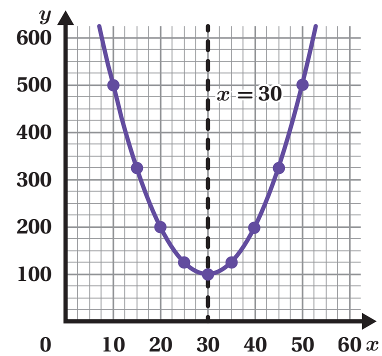
- **The stream has a higher maximum than the pasture. It looks about twice as high.**

12 Synthesis

The table and the graph represent the same relationship.

Describe two ways you know that the relationship in the table and graph is quadratic.

x	y
15	325
20	200
25	125
30	100
35	125
40	200
45	325

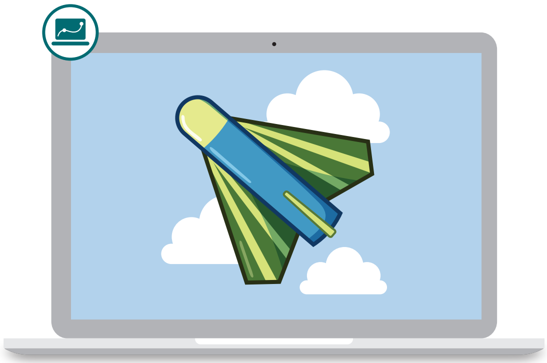


Responses vary.

1. The graph is a parabola and has a line of symmetry down the middle.
2. The table shows a constant second difference of 50.

Things to Remember:

Name: .... Date: .... Period: ....

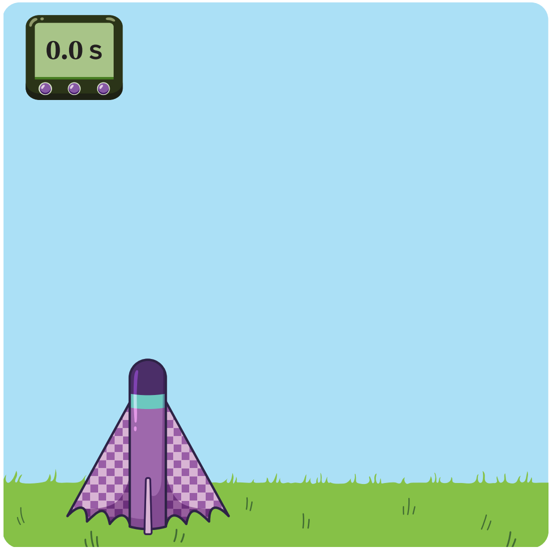


Stomp Rockets

Let's use tables and graphs to make predictions about quadratic relationships in the context of launching stomp rockets.

Warm-Up

1 A stomp rocket is a toy rocket with no engine that is launched by a quick burst of compressed air.



a Let's watch a stomp rocket launch.

b **Discuss:** What do you notice? What do you wonder?

Responses vary.

- **I notice this stomp rocket is going very high.**
- **I notice the rocket moves faster at the start and then slows down at the top.**
- **I notice the rocket lands after 12 seconds.**

- **I wonder how high the rocket goes.**
- **I wonder if the rocket always takes the same amount of time to go up as it does to come down.**
- **I wonder how high the highest stomp rocket ever went.**

Activity 1

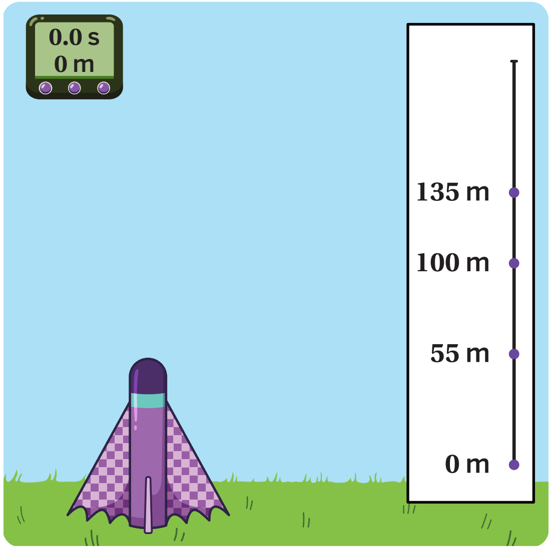
Name: .... Date: .... Period: ....

Predicting With Tables

2 Make a prediction: How high do you think the rocket will be after 4 and 5 seconds?

Predictions vary.

Time (sec)	Height (m)
0	0
1	55
2	100
3	135
4	**160**
5	**175**



3 Let's look at how Maia made her prediction.

Describe what she did to find the heights at 4 and 5 seconds.

***Responses vary.* Maia noticed the first difference was going down by 10 each time. That means the height at 4 seconds is 25 more than the height at 3 seconds, and the height at 5 seconds is 15 more than the height at 4 seconds.**

Name: ________ Date: ________ Period: ________

Predicting With Tables (continued)

4 Here is a new rocket. The table shows its height at various times.

How high will this rocket go?

Use the table if it helps with your thinking.

125 meters

Time (sec)	Height (m)
0	0
1	45
2	80

5 How many seconds will it take for the rocket to touch the ground?

Use the table if it helps with your thinking.

10 seconds

Activity 2

Name: Date: Period:

Predicting With Tables and Graphs

6 A new stomp rocket is launched from the top of a building.

About how many seconds will it take for the rocket to touch the ground?

Use the table if it helps with your thinking.

Time (sec)	Height (m)
0	20
1	45
2	60

A. 6 seconds

B. 8 seconds

C. Between 6 and 7 seconds

D. Between 7 and 8 seconds

Explain your thinking.

Explanations vary. **If I use the second difference to extend the table, I get 20 meters for 6 seconds and -15 meters for 7 seconds. This means the stomp rocket must have touched the ground and stopped moving sometime between 6 and 7 seconds.**

7 Let's look at Ivan's graph.

a Why do you think Ivan drew a parabola?

Responses vary.

- **Ivan probably plotted the points and noticed that when you connect them, it makes a parabola.**
- **The table showed a constant second difference, so the relationship is quadratic. That means the graph is a parabola.**
- **All graphs of projectiles like the stomp rocket make a parabola.**

b How do you think he used his graph to determine when the rocket landed?

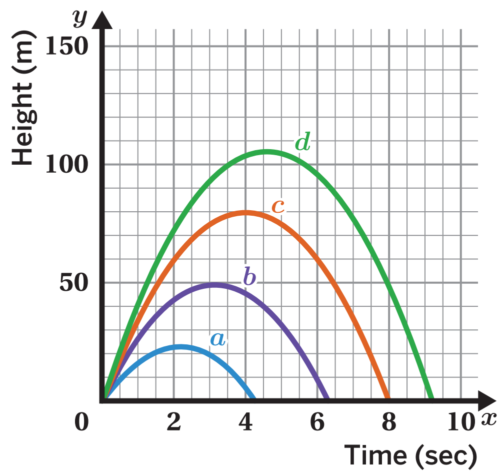
Responses vary.

- **Ivan looked for when the parabola touches the x-axis.**
- **The x-intercept will show when the rocket landed.**
- **Look for where the y-value equals 0. That's how long it takes for the rocket to land.**

Name: .... Date: .... Period: ....

Predicting With Tables and Graphs (continued)

8 Let's watch each of the stomp rockets launch.



Discuss: What is the same and what is different about each rocket launch?

Responses vary.

Same

- Each graph goes through the origin.
- The graphs all have a domain and range that are positive.
- All of the graphs are above the x-axis because the height of the stomp rocket is always 0 or larger.

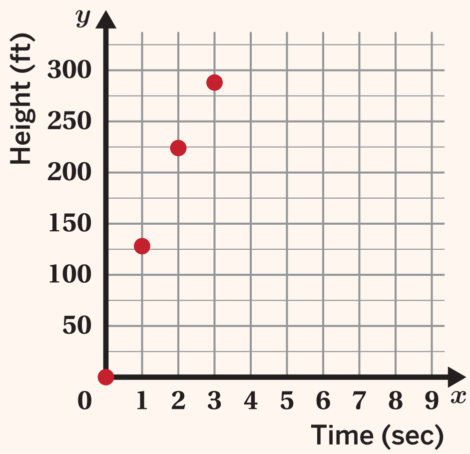
Different

- All of the graphs have different x-intercepts for the landing time.
- All of the rockets have a different maximum height, so each parabola has a different maximum height at the top.
- Some of the parabolas are wider because some of the rockets take longer to land.

Explore More

9 The table and graph show the height of a stomp rocket at various times. How many seconds will it take for this rocket to reach its maximum height? **4.5 seconds**

Time (sec)	Height (m)
0	0
1	128
2	224
3	288



Explain how you know.

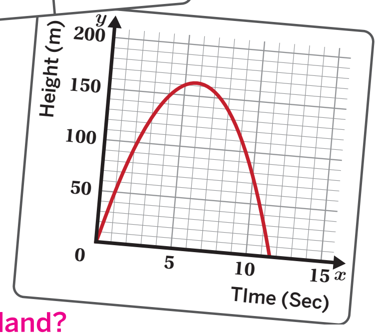
Explanations vary. At 4 seconds, the height is 320 feet. The first difference between 4 and 5 seconds will be 0. The rocket must be at 320 feet at 5 seconds as well, which means the axis of symmetry is $x = 4.5$. That represents the time when the rocket is at its maximum height.

10 Synthesis

The table and graph show the height of a stomp rocket at various times.

Write one question about the rocket that you can answer using the table and one you can answer using the graph.

Time (sec)	Height (m)
0	0
1	52
2	94
3	126



Responses vary.

Table:

- **How high was the stomp rocket after 1 second?**
- **How long did it take for the stomp rocket to reach a height of 126 meters?**

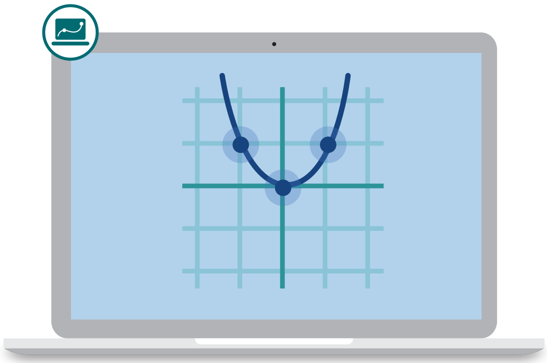
Graph:

- **About how high did the stomp rocket go?**
- **About how many seconds did it take for the stomp rocket to land?**
- **How long did it take for the stomp rocket to reach its maximum height?**

Things to Remember:

Unit 7
Lesson
6

Name: Date: Period:



Plenty of Parabolas

Let's describe the key features of a parabola.

Warm-Up

1 Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet with parabolas for four rounds.

For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a parabola from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating parabolas until you're ready to guess which parabola the Picker chose.

Record helpful questions from each round in the space below.

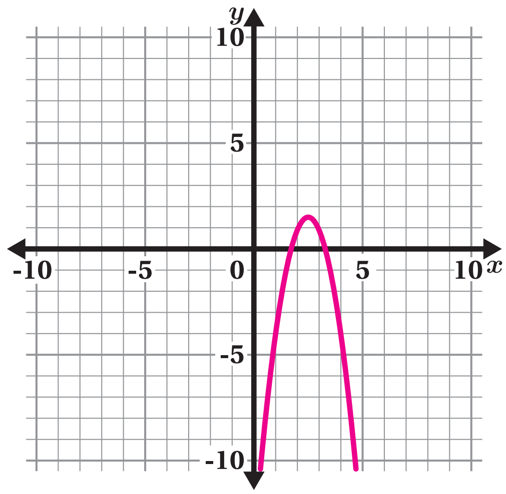
Activity
1

Name: .... Date: .... Period: ....

Describing Parabolas

2 Now it's your turn to graph a parabola.

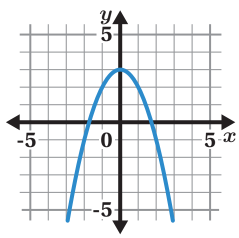
Graphs vary.



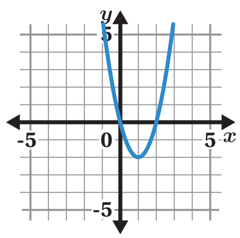
3 Ama says her parabola turns around at 3.

Select a parabola that could be Ama's.

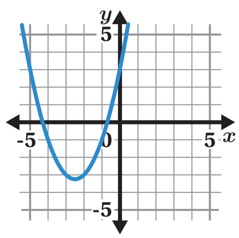
A.



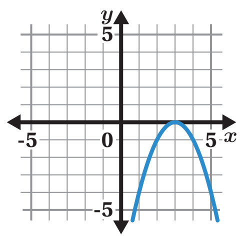
B.



C.



D.



Explain your thinking.

Explanations vary.

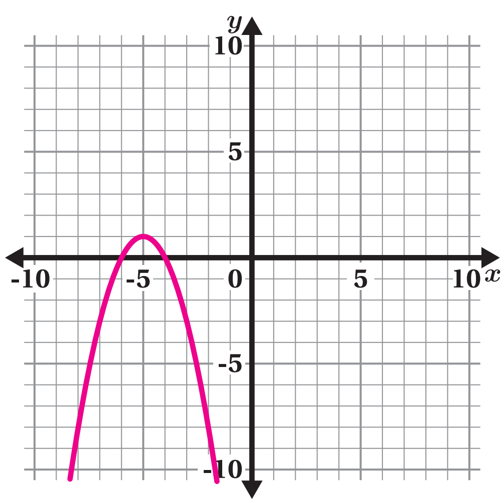
- **Parabola A turns around when the y-intercept is 3.**
- **Parabola D turns around when the x-value is 3.**

4 The turning point of a parabola is called the **vertex**. This is also its *maximum* or *minimum*.

Draw a parabola with a vertex at (-5, 1).

Try to make a parabola you think none of your classmates will make.

Responses vary.



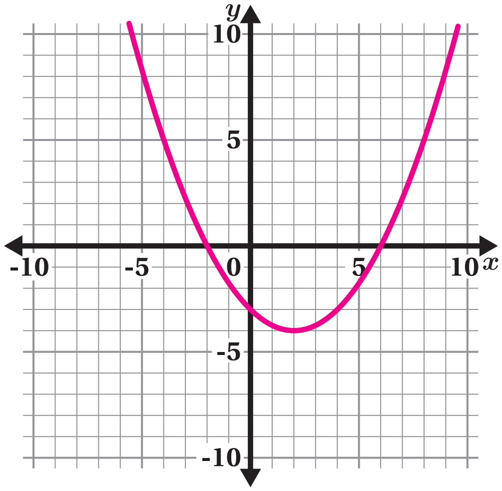
Name: .... Date: .... Period: ....

Describing Parabolas (continued)

5 Katie says her parabola has an x-intercept at -2 and looks like a smile.

Draw what her parabola could look like.

Responses vary. All graphs pass through (-2, 0).

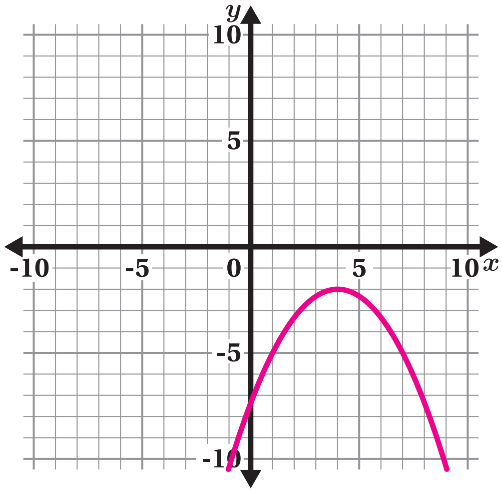


6 Parabolas that look like a smile are **concave up**.

Parabolas that look like a frown are **concave down**.

Draw a concave down parabola with vertex (4, -2).

Responses vary. All graphs have a vertex at (4, -2).



Activity 2

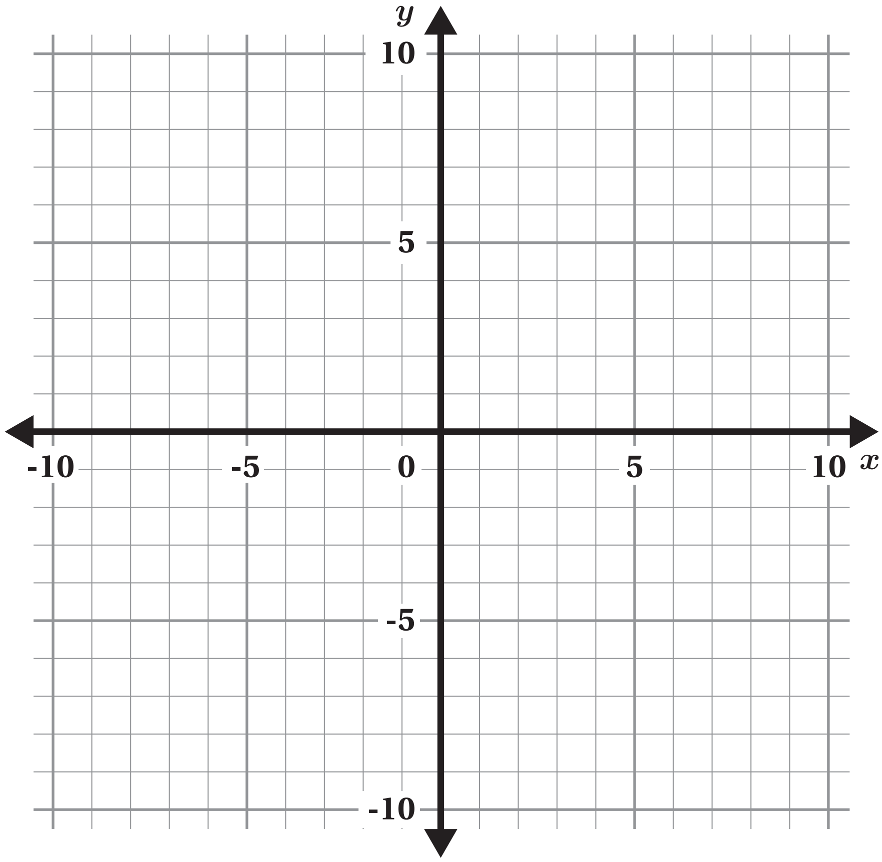
Name: Date: Period:

Parabola Art

7 It's time to make some parabola art!

Create some art by drawing multiple parabolas.

Designs vary.



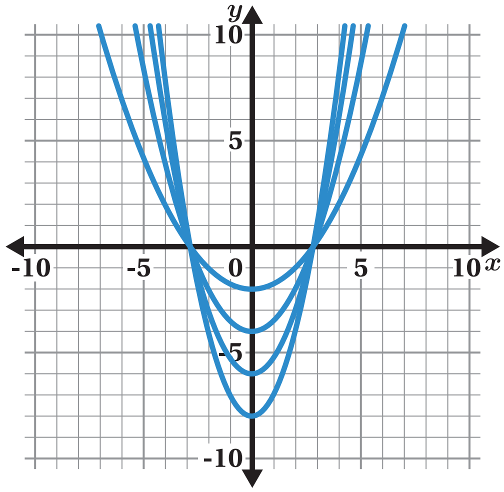
Name: .... Date: .... Period: ....

Parabola Art (continued)

8 Manuel made this design using parabolas.

How are these parabolas alike?

Describe as many similarities as you can.



Explanations vary.

- **All of the parabolas are concave up.**
- **They all have the same x-intercepts.**
- **The axis of symmetry is at $x = 0$.**

Explore More

9 Try to make concave up and concave down parabolas with different numbers of intercepts.

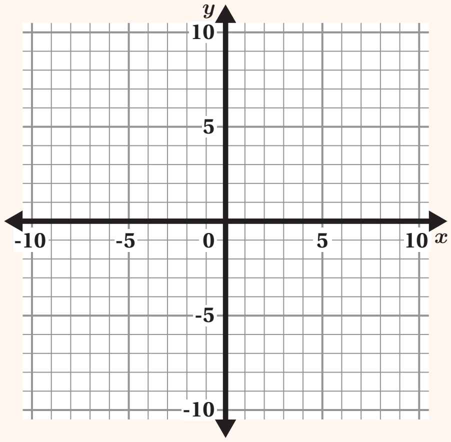
Then select the number of x-intercepts and y-intercepts that are possible.

Number of x-intercepts:

- [x] 0
- [x] 1
- [x] 2

Number of y-intercepts:

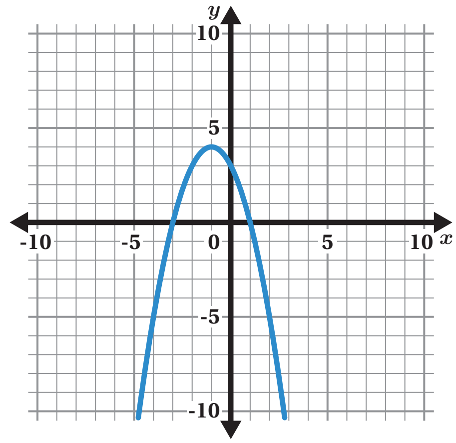
- [ ] 0
- [x] 1
- [ ] 2



10 Synthesis

Describe the graph using vocabulary from this lesson.

Draw on the graph if it helps with your thinking.

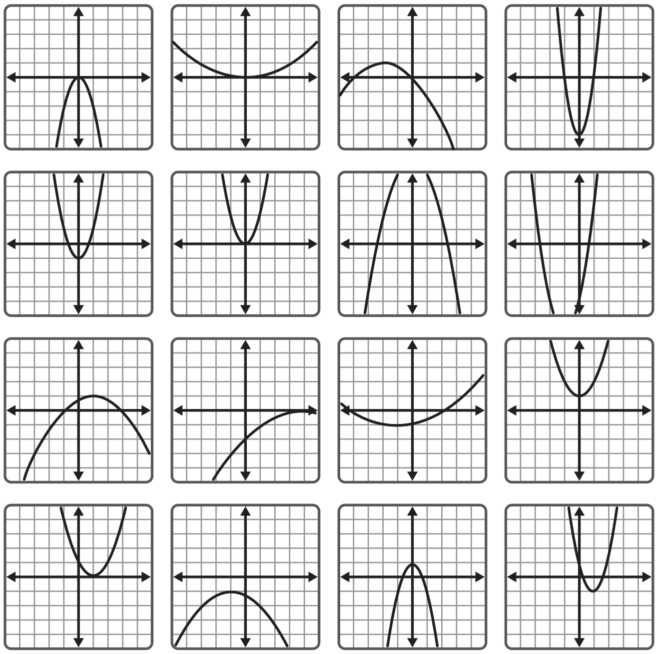


***Responses vary*. The parabola has a vertex at $(-1, 4)$ and is concave down. It also has x-intercepts at $(-3, 0)$ and $(1, 0)$.**

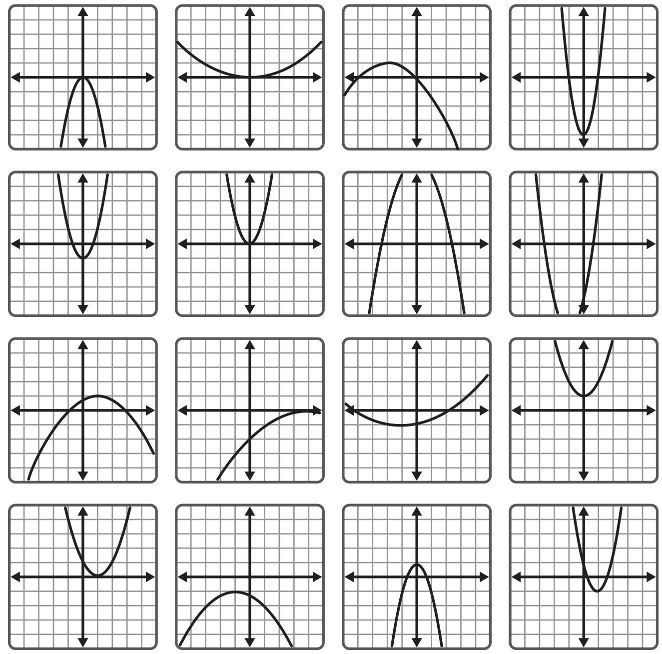
Things to Remember:

Polygraph Set A

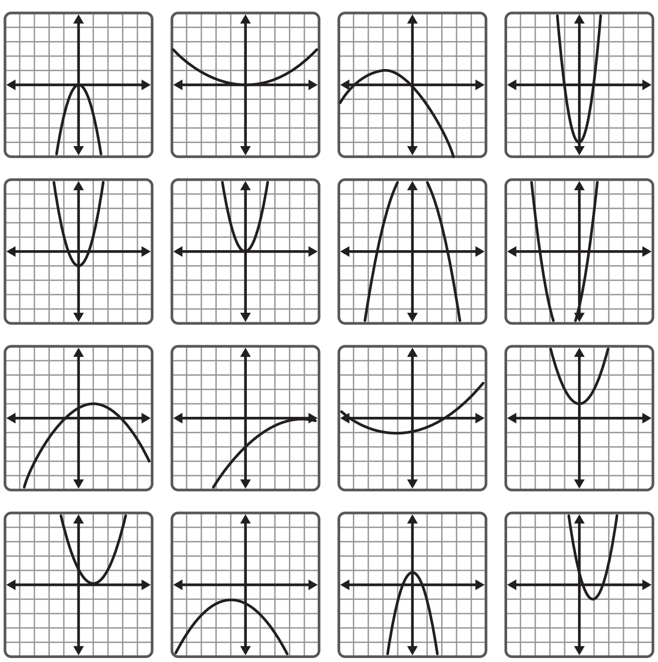
Round 1



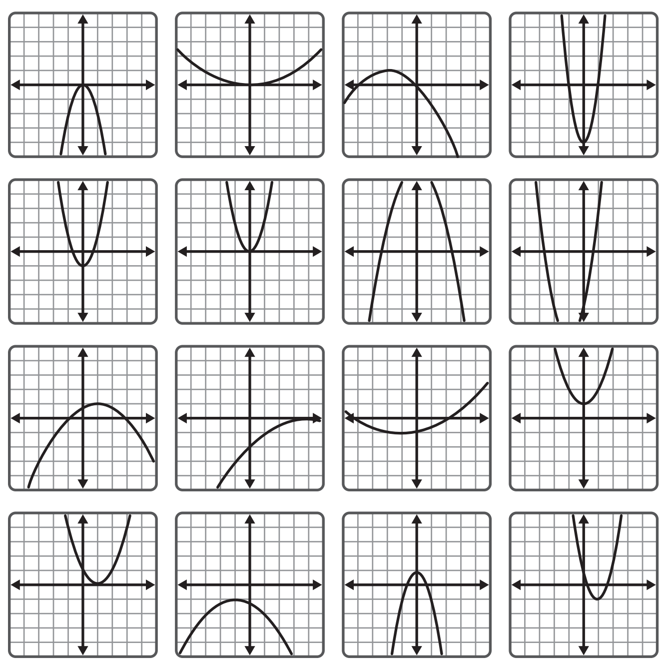
Round 2



Round 3



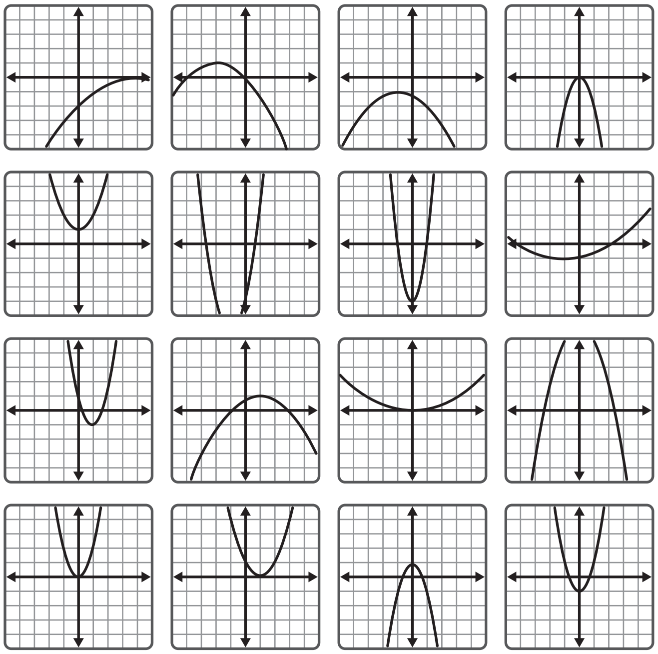
Round 4



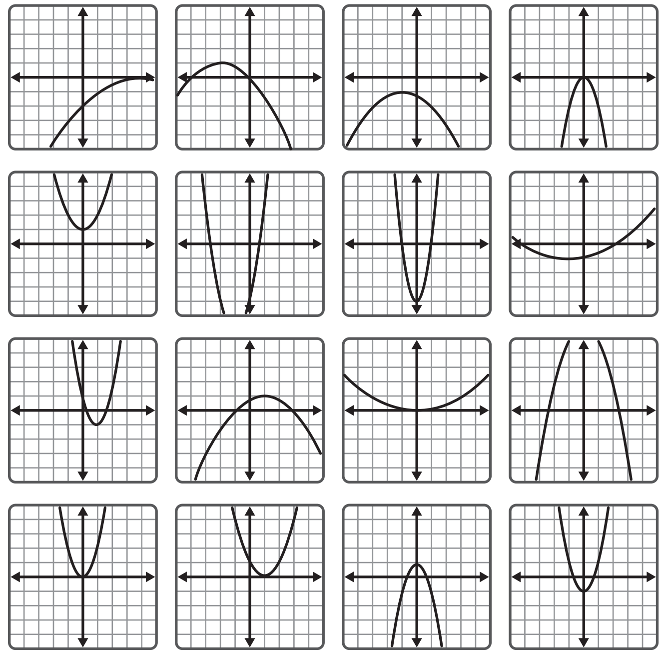
Name: ______________________ Date: __________ Period: __________

Polygraph Set B

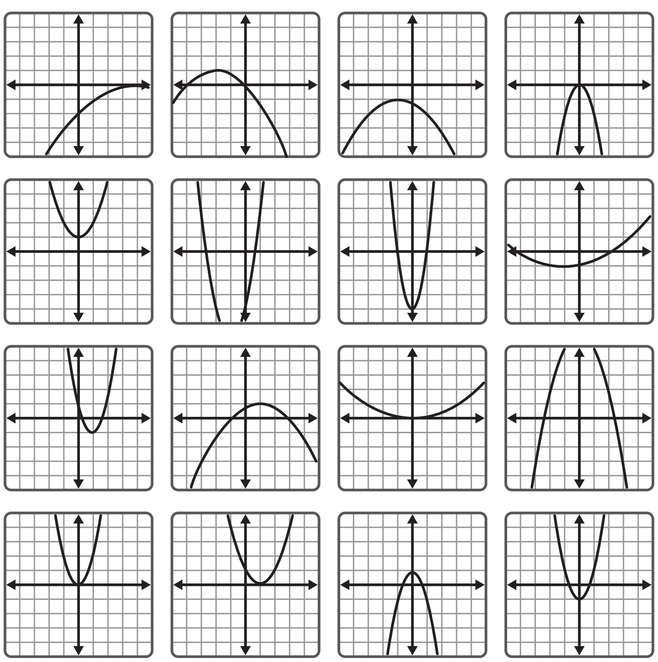
Round 1



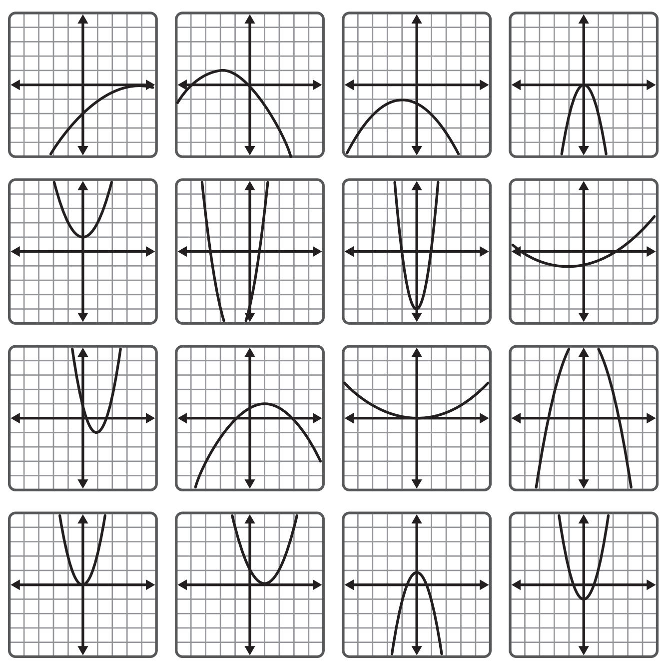
Round 2



Round 3

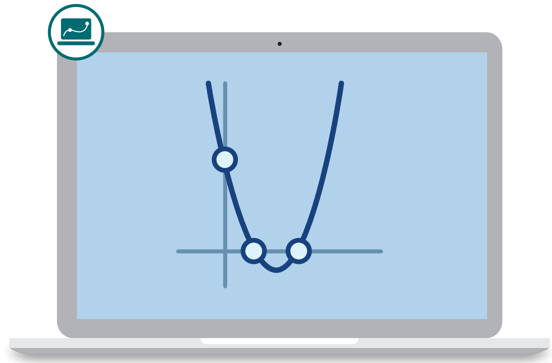


Round 4



Unit 7
Lesson
10

Name: ______ Date: ______ Period: ______

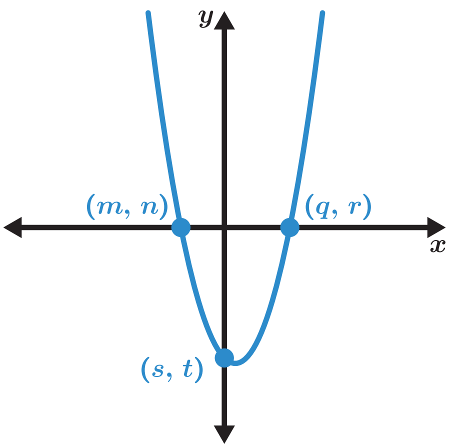


Interesting Intercepts

Let's make connections between the intercepts of a parabola and the structure of its equation.

Warm-Up

1 Here is the graph of a function.



Select *all* the values that are equal to 0.

- [ ] m
- [x] n
- [ ] q
- [x] r
- [x] s
- [ ] t

Explain your thinking.

Explanations vary.

- **If a point is on the x-axis, that means the y-value is 0, so n and r have to be 0. I can use the same idea to figure out that s has to be 0 on the y-axis.**
- **(m, n) and (q, r) are x-intercepts and all x-intercepts have a y-coordinate that is 0. (s, t) is a y-intercept, so the opposite has to be true.**
- **m and t are negative numbers. q is a positive number. The rest are equal to 0.**

Activity
1

Name: .... Date: .... Period: ....

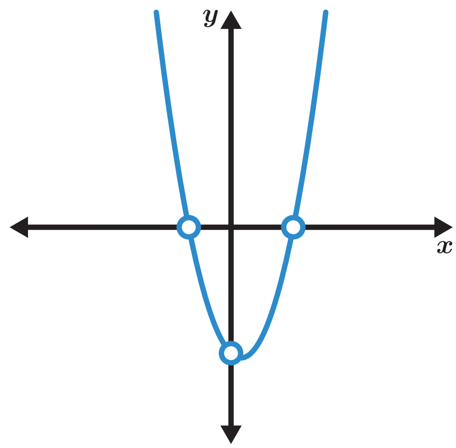
Intercepts in Factored Form

2 Here is the function from the Warm-Up.

Its equation is $f(x) = (x + 2)(x - 3)$.

Graph the x- and y-*intercepts* in the digital activity.

x-intercepts: (-2, 0) and (3, 0)
y-intercept: (0, -6)



3 Look at the x- and y-intercepts of the previous function: $f(x) = (x + 2)(x - 3)$.

What do you notice about the intercepts? ***Responses vary.***

x-intercepts:

- **They always have a 0 at the end in the last column.**
- **There's always a 0 in one of the other columns.**
- **They are the opposite of the numbers in the parentheses in the equation.**

y-intercepts:

- **It has a 0 in the first column.**
- **It's equal to the two numbers in the parentheses multiplied together.**

Activity 2

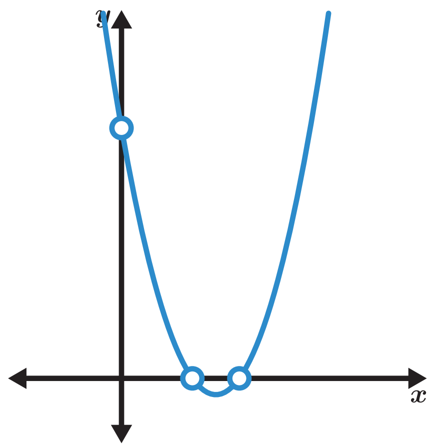
Name: ________ Date: ________ Period: ________

Determine the Intercepts

4 Here is a new function: $g(x) = (2x - 6)(x - 5)$.

Graph the x- and y-*intercepts* in the digital activity.

x-intercepts: $(3, 0)$ and $(5, 0)$
y-intercept: $(0, 30)$



5 Let's look at Raven's work from the previous challenge.

Explain why Raven's thinking is incorrect.

***Responses vary.* $2 \cdot 6 - 6 = 6$ and not 0. If we want the output to be 0, then x has to be 3 because $2 \cdot 3 - 6 = 0$.**

Name: Date: Period:

Determine the Intercepts (continued)

6 Here is a new function:

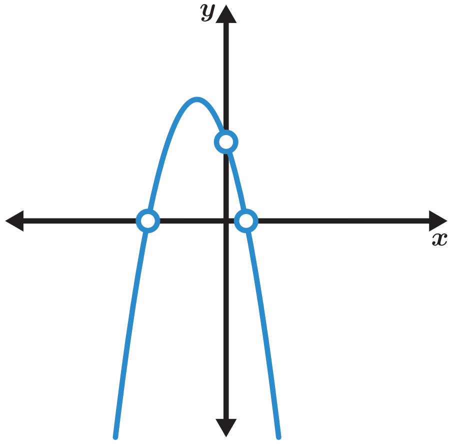
$h(x) = (\text{-}x + 0.5)(4x + 8)$

What are the intercepts of h?

x-intercept: **(0.5, 0)**

x-intercept: **(-2, 0)**

y-intercept: **(0, 4)**



7 Two students were asked to determine the x-intercepts of $p(x) = 2x(x + 9)$.

Yolanda says the x-intercepts are at -2 and -9.
Julian says the x-intercepts are at 0 and -9.

Whose thinking is correct? Circle one.

Julian's Yolanda's Both Neither

Explain your thinking.

***Explanations vary.* 2 is being multiplied by x, so $x = 0$ would make the output 0.**

Name: .... Date: .... Period: ....

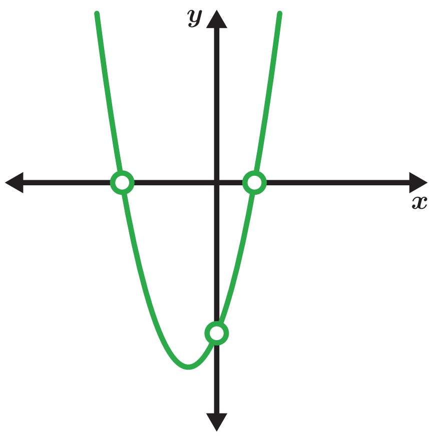
Intercepts in Standard Form

8 Here is a new function:

$w(x) = x^2 + 3x - 10$

Graph the x- and y-*intercepts* in the digital activity.

x-intercepts: (2, 0) and (-5, 0)
y-intercept: (0, -10)



9 Look at the x- and y-intercepts of the previous function.

Which intercepts were easier for you to determine? Circle one. ***Choices vary.***

x-intercepts y-intercept

Explain your thinking.

Responses vary.

- **y-intercept. When you plug in 0 for x, everything multiplies to 0 except for the -10.**
- **x-intercepts. I tried different numbers for x and knew I was getting close when the output got close to 0.**

10 Match each equation with its y-intercept. One equation will have no match.

$a(x) = x^2 - 3x + 5$	$b(x) = x^2 + 5x - 3$	$c(x) = x^2 - 5x + 3$
$d(x) = \text{-}3x^2 + 5$	$e(x) = 5x^2 - 3$	$f(x) = 3x^2 + 5 + x^2$

(0, 5)	(0, -3)
$a(x)$, $d(x)$, $f(x)$	**$b(x)$, $e(x)$**

Equation with no match: $c(x)$

11 Synthesis

The same function is written in factored and standard form. What does each form tell you about the graph of $f(x)$? ***Responses vary.***

Factored Form

$$f(x) = (2x - 1)(x + 3)$$

Standard Form

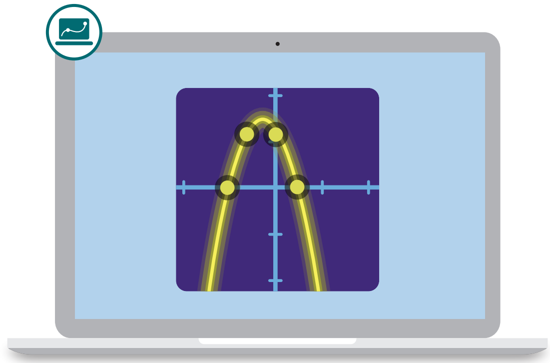
$$f(x) = 2x^2 + 5x - 3$$

Factored form: **Factored form can tell you the x-intercepts if you figure out what number will make each factor equal to 0.**

Standard form: **Standard form can tell you the y-intercept. It's the constant term because when x is equal to 0, the only part that is not 0 is -30.**

Things to Remember:

Name: .... Date: .... Period: ....



Parabola Zapper

Let's graph quadratic equations in factored form.

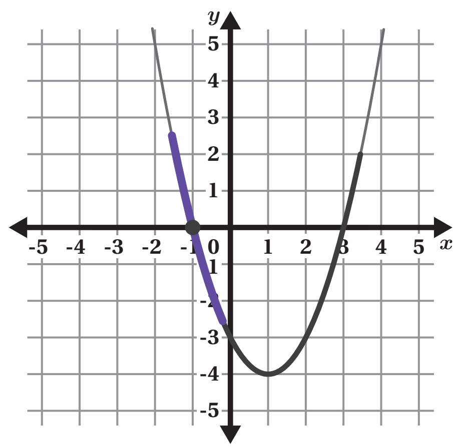
Warm-Up

1 Use the digital activity to light up the parabola by zapping a few points on it.

Its equation is $f(x) = (x - 3)(x + 1)$.

Zap	Coordinate
Zap #1	(-1, 0)
Zap #2	**(3, 0)**
Zap #3	**(1, -4)**

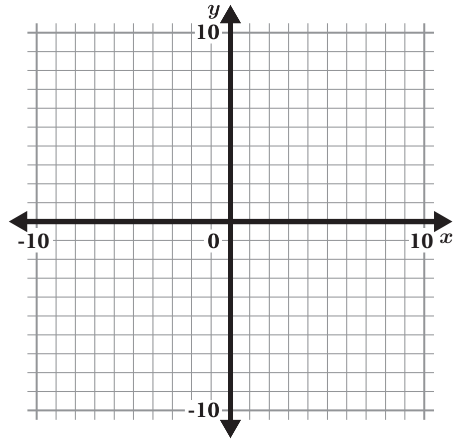
Responses vary.



Name: .... Date: .... Period: ....

Graphing by Zapping

You'll use the digital activity for Problems 2–5.



2 There is a hidden parabola on a graph.

Its equation is $h(x) = (x + 4)(-x + 2)$.

Light up the entire parabola by zapping points on it.

Responses vary. **(-5, -7), (-4, 0), (-3, 5), (-1, 9), (0, 8), (1, 5), (2, 0), or (3, -7).**

3 Maria and Laila compared their strategies on the previous challenge.

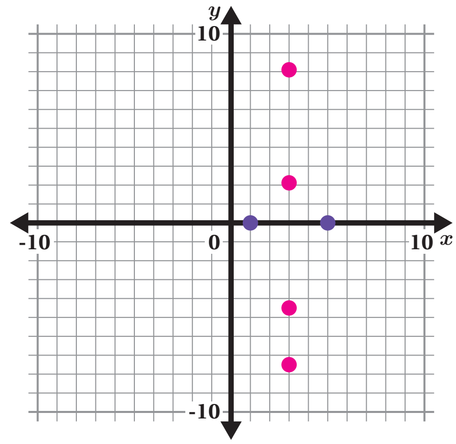
Maria said: *I used four zaps! Take a look at my graph*.

Laila said: *We used some of the same points, but I only used three zaps*.

What three points do you think Laila zapped?

Responses vary. **I think Laila zapped the two x-intercepts and the point at the top of the parabola.**

4 Here are the x-intercepts of a new parabola.



a Draw a point to show where the *vertex* could be.

b Let's look at some possible vertices. What do you notice and wonder about the vertex?

Responses vary.

- **I notice that the vertex will always lie on the axis of symmetry.**
- **I notice that some of the parabolas would be concave up and some would be concave down.**
- **I notice that all the points are on the line $x = 3$.**
- **I wonder why the number 3 is important.**
- **I wonder which point is the correct vertex.**

Name: ________ Date: ________ Period: ________

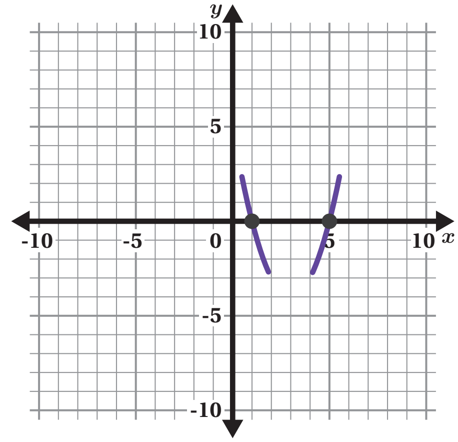
Graphing by Zapping (continued)

5 Here is the parabola from the previous problem.

Its equation is $g(x) = (x - 1)(x - 5)$.

Complete this parabola by zapping its vertex.

(3, -4)



6 Here is Pilar's work to determine the vertex of $g(x)$.

She completed part of the table before getting stuck.

a What does the 3 in Pilar's table represent?

***Responses vary.* 3 is the x-coordinate of the vertex because it's in the middle of 1 and 5.**

b How can Pilar find the y-coordinate of the vertex?

***Responses vary.* Pilar could continue the table until she gets to $g(x)$, which is like the y-coordinate.**

Pilar

$g(x) = (x - 1)(x - 5)$

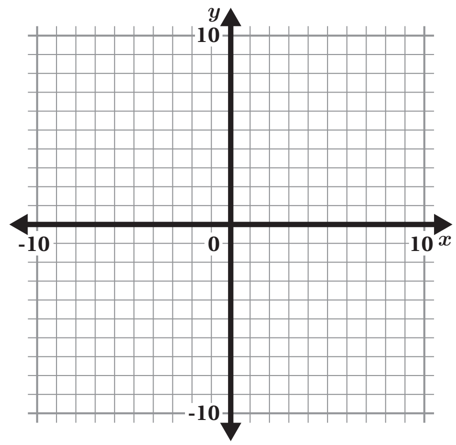
x	x - 1	x - 5	g(x)
1	0	-4	0
5	4	0	0
3			

Activity
2

Name: .... Date: .... Period: ....

Repeated Challenges

You'll use the digital activity for problems 7–9.



7 There is a hidden parabola on the graph.

Its equation is $p(x) = (2x + 4)(x + 6)$.

Light up the entire parabola by zapping points on it. One way to light up the entire parabola is to zap both x-intercepts and the vertex.

Responses vary. (-1, 10), (-2, 0), (-3, -6), (-4, -8), (-5, -6), (-6, 0), (-7, 10)

8 Describe the strategy you used to light up $p(x) = (2x + 4)(x + 6)$.

Responses vary. First, I figured out where the x-intercepts were by figuring out when each factor was equal to 0. These were (-2, 0) and (-6, 0). -4 is between -2 and -6, so I substituted -4 into $p(x)$ and got -8, so the vertex is at (-4, -8).

9 For each challenge, light up the entire parabola by zapping points on it.

a $f(x) = (x - 3)(x + 1)$

Responses vary. (3, 0), (-1, 0), (1, -4)

b $f(x) = (-x + 4)(x + 2)$

Responses vary. (4, 0), (-2, 0), (1, 9)

c $f(x) = (1 - x)(x + 3)$

Responses vary. (1, 0), (-3, 0), (-1, 4)

d $f(x) = (x - 2.5)(x - 6.5)$

Responses vary. (2.5, 0), (6.5, 0), (4.5, -4)

e $f(x) = (x + 1)(x - 6)$

Responses vary. (-1, 0), (6, 0), (2.5, -12.25)

f $f(x) = (2x + 6)(x - 1)$

Responses vary. (-3, 0), (1, 0), (-1, -8)

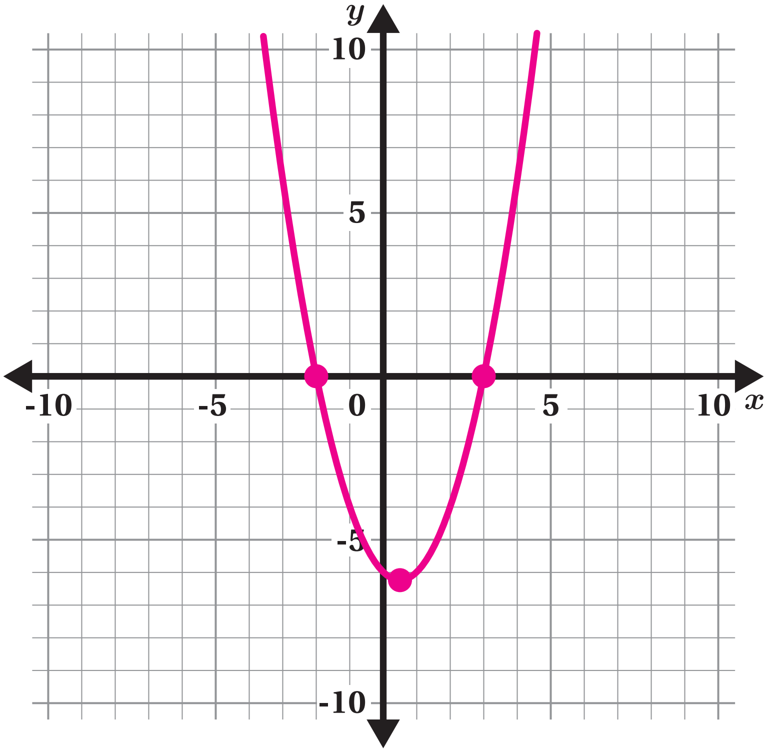
Name: .... Date: .... Period: ....

Graphing by Hand

10 Without graphing, describe as much as you can about the graph of $w(x) = (x + 2)(x - 3)$.

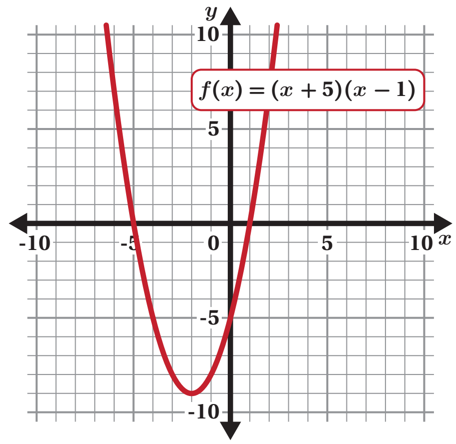
Responses vary. **Its x-intercepts are at $(-2, 0)$ and $(3, 0)$. Its y-intercept is at $(0, -6)$. Its vertex is at $(0.5, -6.25)$. It's concave up. The axis of symmetry is at $x = 0.5$.**

11 Draw the graph of $w(x) = (x + 2)(x - 3)$.



12 Synthesis

Identifying key features is a helpful strategy for graphing parabolas. Describe how to identify the key features of a parabola when given an equation in factored form. Use this example if it helps with your thinking.



Responses vary.

- I can determine the x-value that makes each factor 0 to find the x-intercepts at $x = -5$ and $x = 1$.
- The axis of symmetry is $x = -2$ because -2 is halfway between the x-intercepts of $x = -5$ and $x = 1$.
- The y-intercept occurs when $x = 0$. I can substitute $x = 0$ in the function to get a y-intercept at $(0, -5)$.
- The vertex is always on the axis of symmetry. I can substitute $x = -2$ into the function to find the y-coordinate. $y = (-2 + 5)(-2 - 1) = (3)(-3) = -9$. The vertex is at $(-2, -9)$.

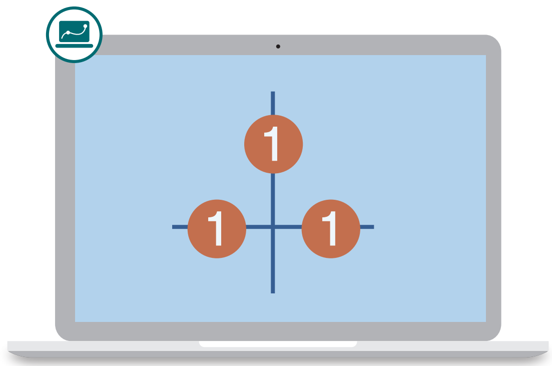
Things to Remember:

Unit 7
Lesson
12

Name: ______ Date: ______ Period: ______

Break Through: Parabolas

Let's write equations of parabolas in factored form.



Warm-Up

You'll use the digital activity for the Warm-Up.

1 Welcome to Break Through: Parabolas!

Your goal is to write equations of parabolas that will break the targets.

a Press "Try It" to see what happens.

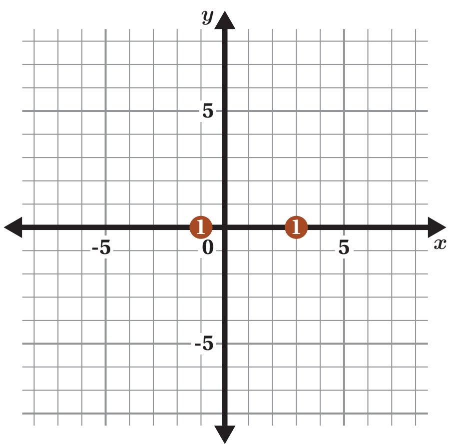
b Change the equation to break all the targets.

Original equation:

$y = (x + 1)(x - 2)$

Your equation:

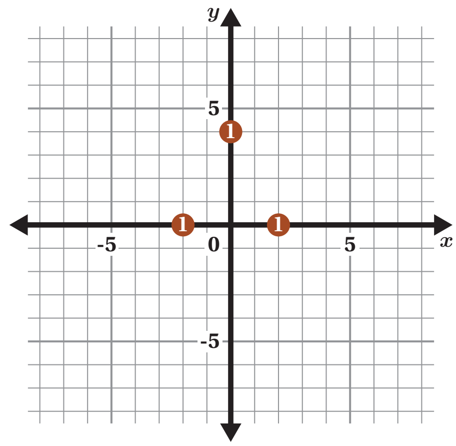
Responses vary.
$\boldsymbol{y = (x + 1)(x - 3)}$



Name: ______ Date: ______ Period: ______

Building Quadratic Functions

You'll use the digital activity for Problems 2–6.



2 Here's another challenge.

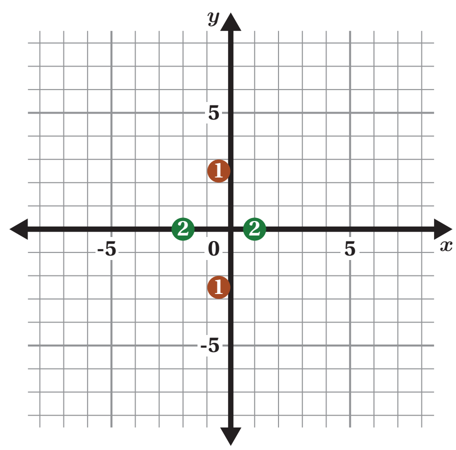
Change the equation to break *all* the targets.

Original equation:

$y = (x + 2)(x - 2)$

Your equation:

Responses vary.
$y = \text{-}1(x + 2)(x - 2)$

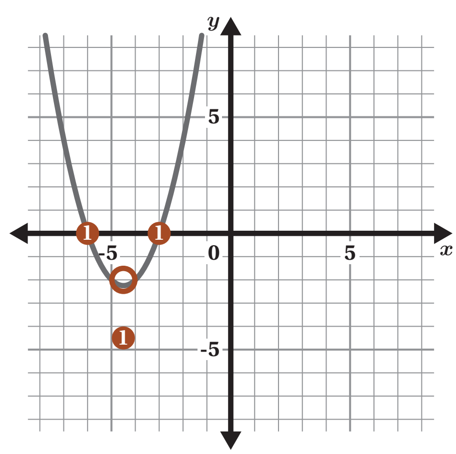


3 Some of the targets will require multiple parabolas.

Write another equation to break the remaining targets.

Responses vary.

Equation
$y = (x - 1)(x + 2)$
$y = \text{-}1(x - 1)(x + 2)$



4 Carlos started this challenge.

Write another equation to finish it.

Responses vary.

Equation
$y = (x + 6)(x + 3)$
$y = 2(x + 6)(x + 3)$

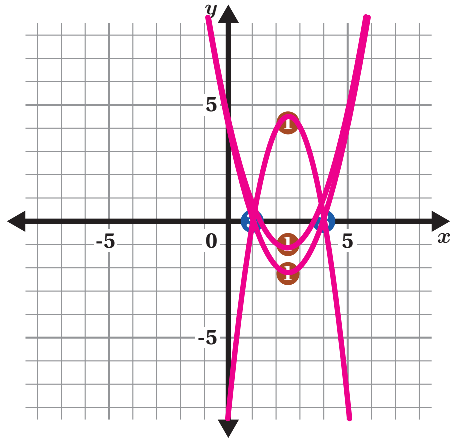
Name: ______ Date: ______ Period: ______

Building Quadratic Functions (continued)

5 Break the targets using three equations.

Responses vary.

Equation
$y = (x-1)(x-4)$
$y = \frac{1}{2}(x-1)(x-4)$
$y = -2(x-1)(x-4)$

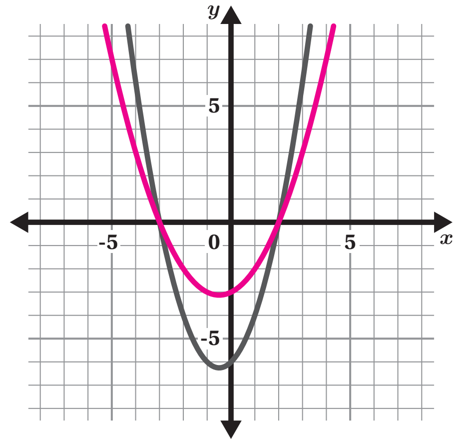


6 Here is the graph of $y = (x+3)(x-2)$.

How might $y = \frac{1}{2}(x+3)(x-2)$ look similar? Different?

Show or explain your thinking.

Responses vary. The x-intercepts would be the same because the only thing changed was the multiplier outside the factors, so they would be at (-3, 0) and (2, 0). The vertex and y-intercept would be half as far away from the x-axis as before.



Activity

2

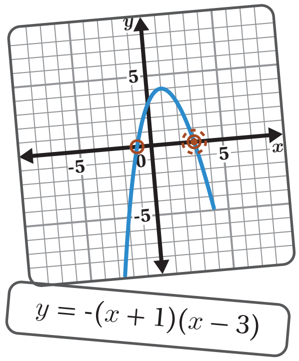
Name: ______ Date: ______ Period: ______

Lots of Challenges

You'll use the digital activity for Problems 7–12.

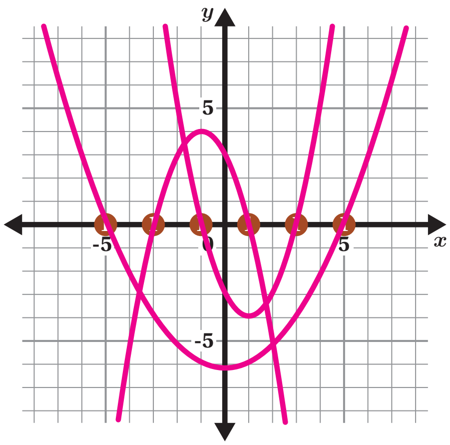
7 Move on to the final set of challenges.

Use what you know about writing quadratic equations to break all the targets in fun and creative ways!



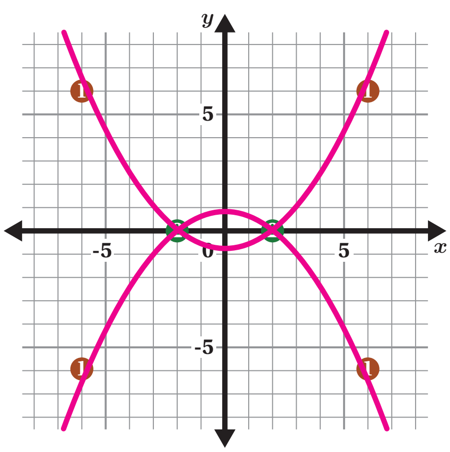
8 Break the targets using as few equations as you can.
Responses vary.

Equation
$y = \frac{1}{4}(x-5)(x+5)$
$y = -(x+3)(x-1)$
$y = (x+1)(x-3)$



9 Break the targets using as few equations as you can.
Responses vary.

Equation
$y = \frac{1}{5}(x+2)(x-2)$
$y = -\frac{1}{5}(x+2)(x-2)$



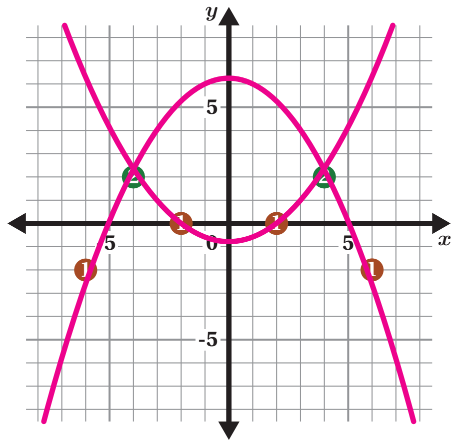
Name: .... Date: .... Period: ....

Lots of Challenges (continued)

10 Break the targets using as few equations as you can.

Responses vary.

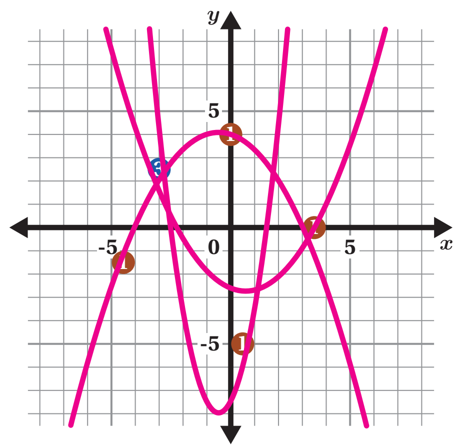
Equations
$y = \frac{1}{5}(x+2)(x-2)$
$y = -\frac{1}{4}(x+5)(x-5)$



11 Break the targets using as few equations as you can.

Responses vary.

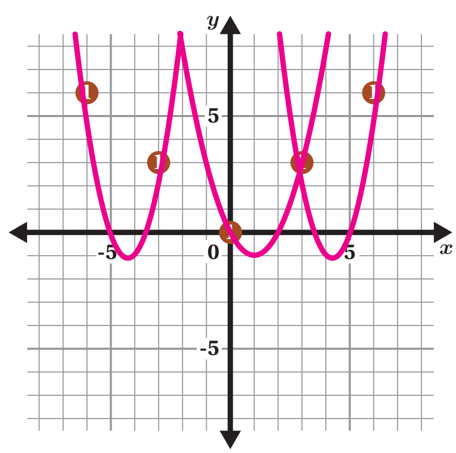
Equations
$y = \frac{1}{3}(x+2.25)(x-3.5)$
$y = -\frac{1}{3}(x+4)(x-3)$
$y = 2(x+2.5)(x-1.5)$



12 Break the targets using as few equations as you can.

Responses vary.

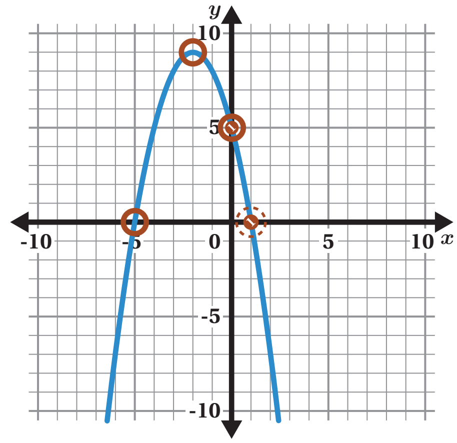
Equations
$y = x(x-2)$
$y = 2(x-5)(x-3.5)$
$y = 2(x+5)(x+3.5)$



13 Synthesis

Describe a strategy for writing a quadratic equation in factored form that matches a graph.

Use the example if it helps with your thinking.

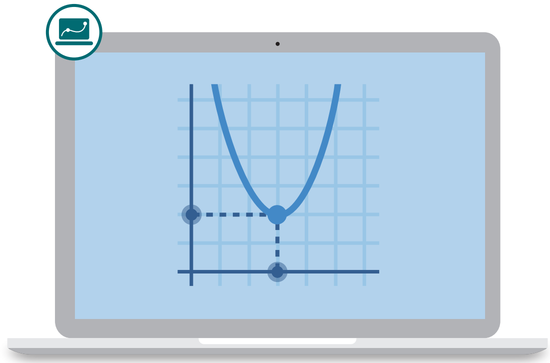


Responses vary. **First, use the x-intercepts to write part of the equation. In this situation, $(x+5)(x-1)$ would be part of the equation because the x-intercepts are at (-5, 0) and $(1, 0)$. Since the parabola is concave down we need to multiply by a negative number. The equation is $y = \text{-}1(x+5)(x-1)$, or $y = \text{-}(x+5)(x-1)$.**

Things to Remember:

Unit 7
Lesson
14

Name: .... Date: .... Period: ....



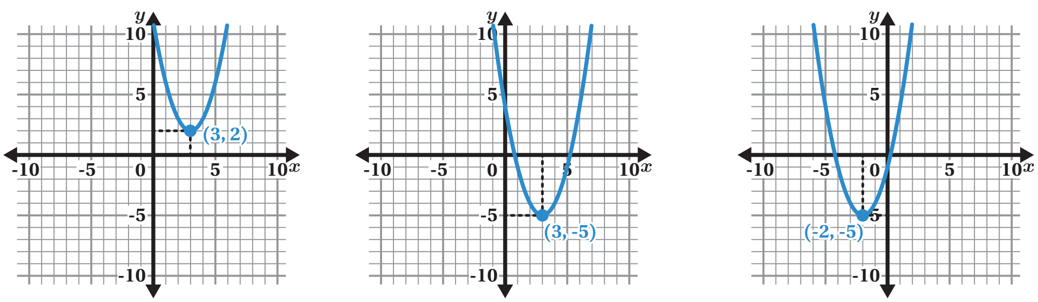
Vertex Form

Let's transform quadratic functions using translations and write their equations in a new form.

Warm-Up

1 Here are a few transformations of a parabola.

$y = (x - 3)^2 + 2$ $\quad$ $y = (x - 3)^2 - 5$ $\quad$ $y = (x + 2)^2 - 5$



What changes? What stays the same?

Responses vary.

Changes:

- **The vertex is in a different location in each graph.**
- **The numbers in the equations**
- **Depending on the transformation, you see more or less of the graph.**

Same:

- **The equations all have the same structure.**
- **The numbers in the equation are the same as the numbers in the vertex.**
- **The shape/curve of the parabola stays the same.**

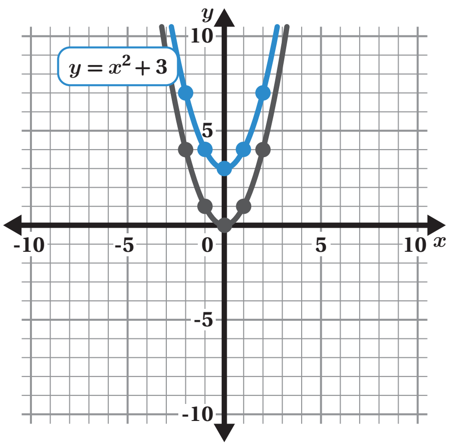
Activity 1

Name: ________ Date: ________ Period: ________

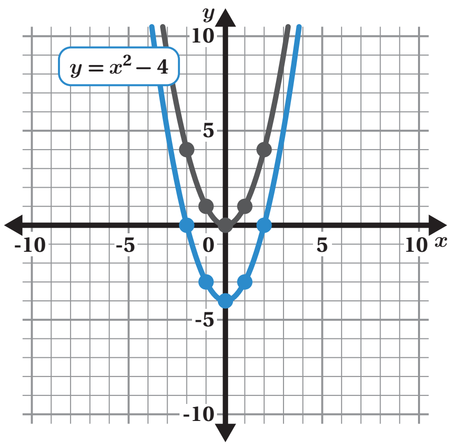
Translating Parabolas

2 Here are two different vertical *translations* of $y = x^2$.

Graph A



Graph B



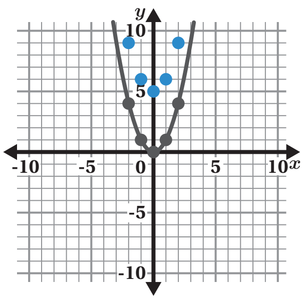
Discuss: What do you notice? What do you wonder?

Responses vary.

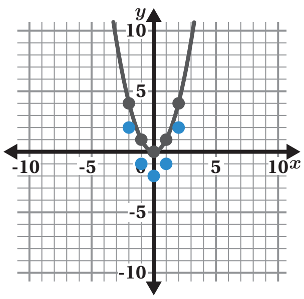
- I notice the vertex of $y = x^2 + 3$ is at $(0, 3)$.
- I notice that $y = x^2 - 4$ is shifted 4 units down from $y = x^2$.
- I wonder if $y = x^2 + 10$ will be shifted 10 units up.
- I wonder if the number after x^2 in the equation will always tell you the vertical shift.
- I wonder what the equation is like for a parabola that has a vertex to the left or right.

3 For each challenge, write the equation for the vertical translation of $y = x^2$.

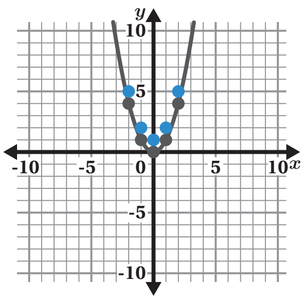
a $y =$ $x^2 + 5$



b $y =$ $x^2 - 2$



c $y =$ $x^2 + 1$



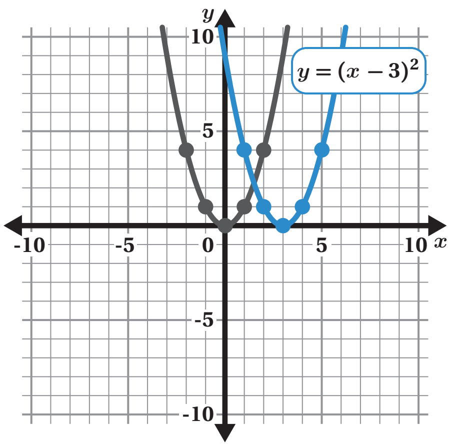
Activity 1

Name: .... Date: .... Period: ....

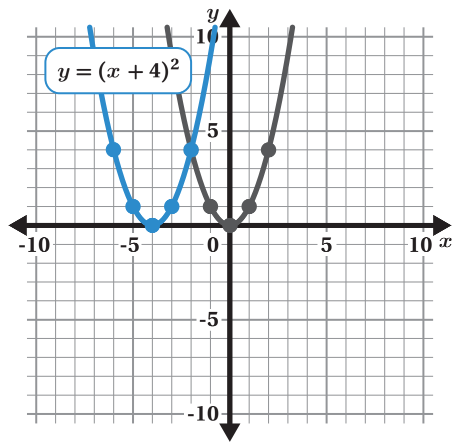
Translating Parabolas (continued)

4 Here are two different horizontal translations of $y = x^2$.

Graph C



Graph D

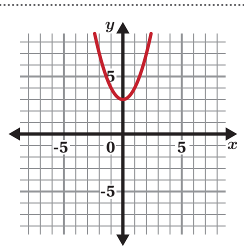
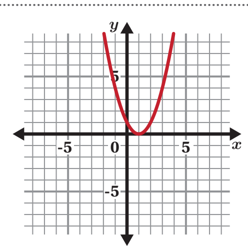
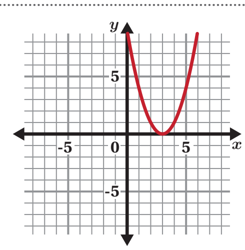
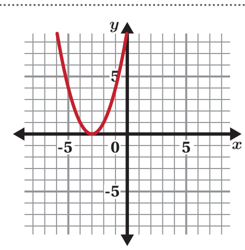


How can you see the translation in the equation? ***Responses vary.***

- **The number in the parentheses tells you how many units left or right the parabola moves.**
- **$y = (x - 3)^2$ moves the graph of $y = x^2$ to the right 3 units.**
- **$y = (x + 4)^2$ moves the graph of $y = x^2$ to the left 4 units.**

5 Match each equation to a graph. One graph will have no match.

a. $y = (x + 3)^2$ **b.** $y = x^2 + 3$ **c.** $y = (x - 1)^2$

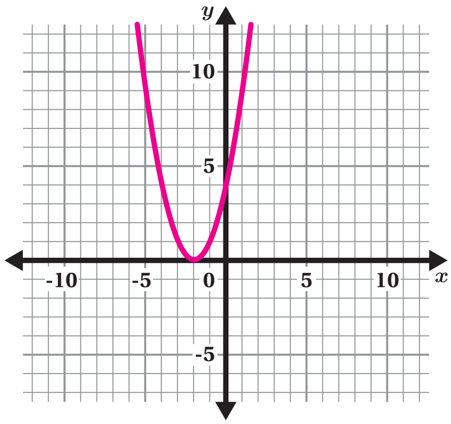
			
Equation: **b**	Equation: **c**	Equation:	Equation: **a**

Name: ______ Date: ______ Period: ______

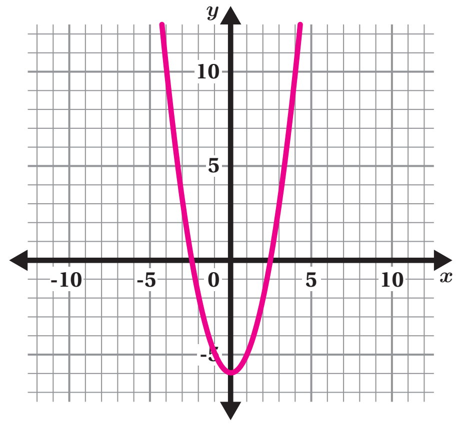
Translating Parabolas (continued)

6 Draw the graph of each parabola.

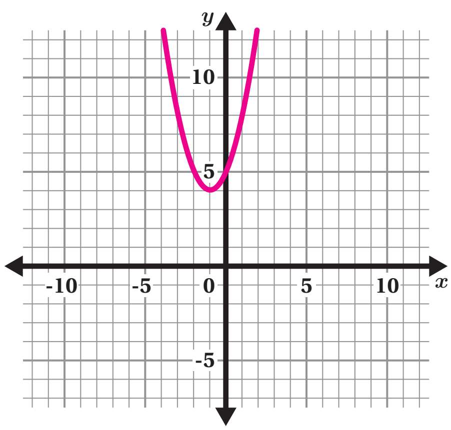
a $y = (x + 2)^2$



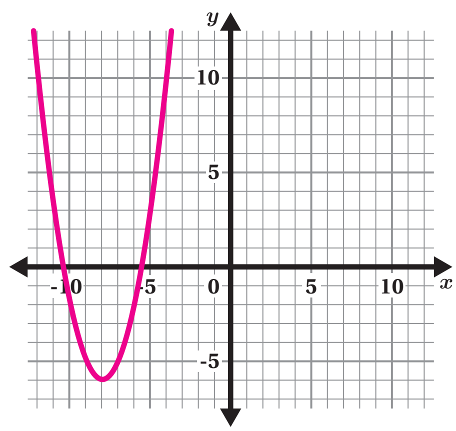
b $y = x^2 - 6$



c $y = (x + 1)^2 + 4$



d $y = (x + 8)^2 - 6$



Activity 2

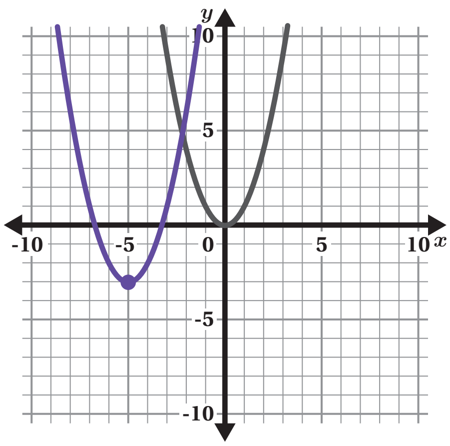
Name: .... Date: .... Period: ....

Vertex Form

7 Let's watch a translation of $f(x) = x^2$ left 5 units and down 3 units to $g(x) = (x + 5)^2 - 3$.

Discuss: Why do you think this type of equation is called **vertex form**?

Responses vary. I can see the vertex at (-5, -3) in the equation. The x-value of the vertex is the number that makes the value in the parentheses equal to 0. The y-value of the vertex is the number outside of the parenthesis.



8 Here is Liam's function for a parabola with a vertex at (-3, -4).

$f(x) = (x - 3)^2 - 4$

a. What did Liam do well?

Responses vary. Liam used the structure of an equation for a parabola. He also correctly wrote the vertical shift of -4.

b. What was Liam's mistake?

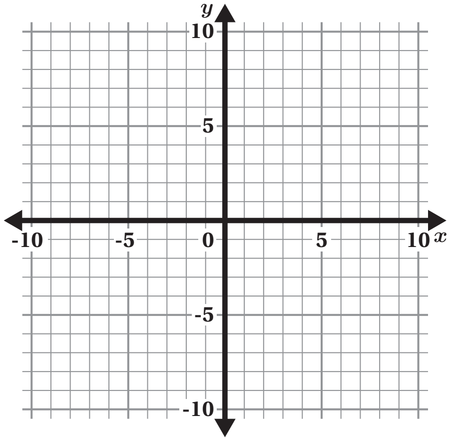
Responses vary. Liam wrote the equation for a parabola whose vertex is at (3, -4). He used the incorrect sign in the parentheses.

Name: ______ Date: ______ Period: ______

Vertex Form (continued)

9 Write a quadratic function with its vertex at at (-6, -2).

Use the graph if it helps with your thinking.



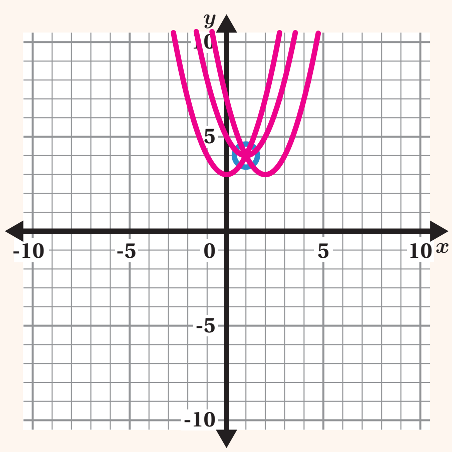
Responses vary.

- $f(x) = (x+6)^2 - 2$
- $f(x) = -(x+6)^2 - 2$
- $f(x) = -\frac{1}{2}(x+6)^2 - 2$

Explore More

10 Write the function of as many different parabolas as you can that go through the point $(1, 4)$.

Use the graph if it helps with your thinking.



Responses vary.

$y = (x-1)^2 + 4$

$y = (x-2)^2 + 3$

$y = x^2 + 3$

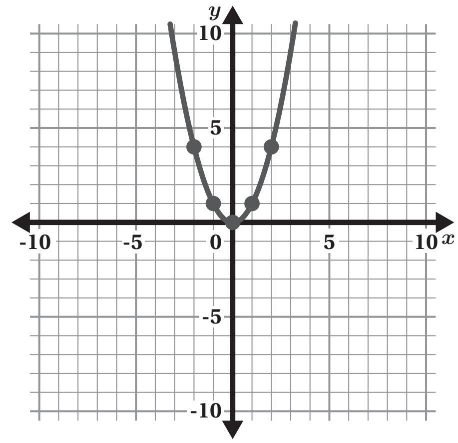
11 Synthesis

Here is a quadratic function written in vertex form: $g(x) = (x - 2)^2 + 3$.

Describe how the graph of $g(x)$ compares to $f(x) = x^2$.

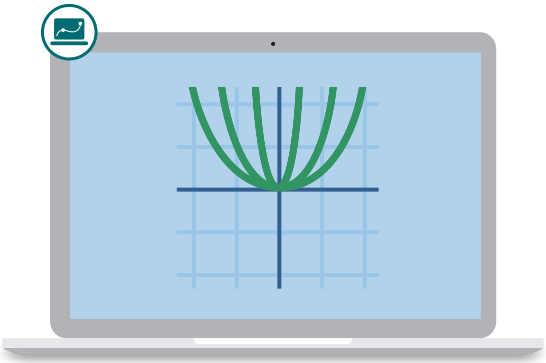
Use the graph if it helps your thinking.

***Responses vary.* Both graphs have the same shape. $g(x)$ is translated 2 units to the right and 3 units up from $y = x^2$.**



Things to Remember:

Name: .... Date: .... Period: ....

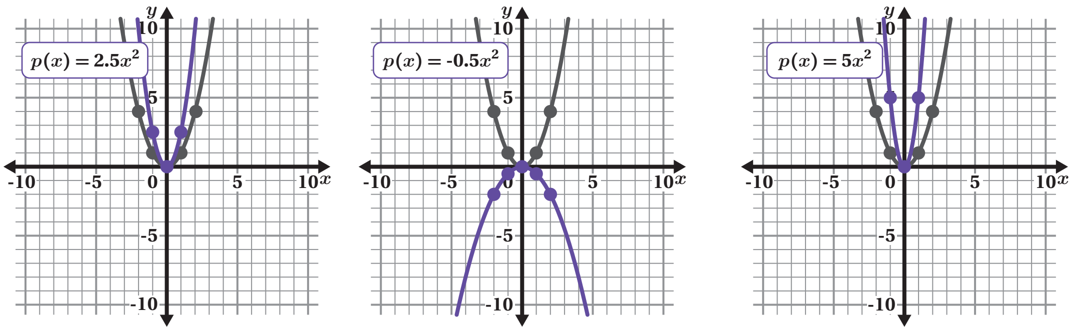


Scaled It!

Let's transform quadratic functions by scaling vertically.

Warm-Up

1 Here is a new kind of transformation.



What changes? What stays the same?

Responses vary.

Changes:

- **The number in front of the x^2.**
- **The width of the parabola.**
- **The concavity.**

Stays the same:

- **They are all parabolas.**
- **They all have a line of symmetry at $x = 0$.**
- **They all have a vertex at $(0, 0)$.**

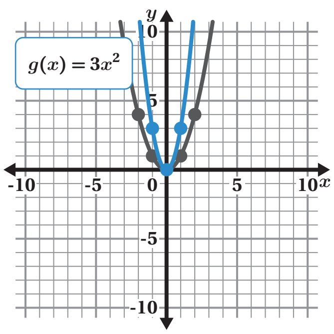
Activity 1

Name: ______ Date: ______ Period: ______

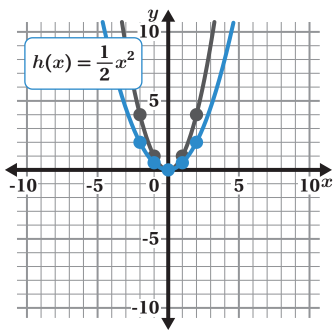
Scaling Vertically

2 Here are three different ways that $f(x) = x^2$ can be scaled vertically.

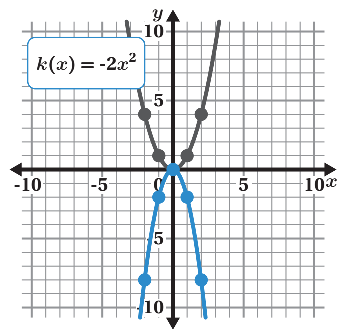
Graph A



Graph B



Graph C



Discuss: What do you notice? What do you wonder?

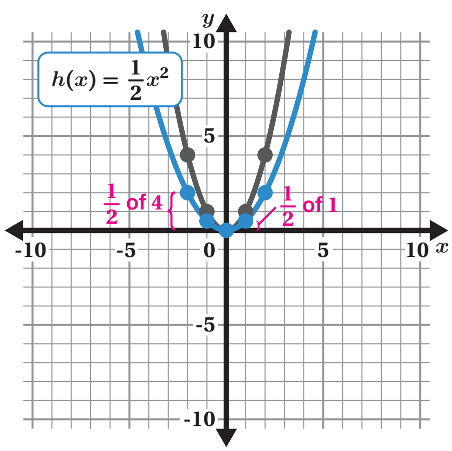
Responses vary.

- **I notice that the plotted points on $f(x) = x^2$ and the scaled graphs all have the same set of x-values. Only the y-values change.**
- **I notice that when the number in front of the x^2 is greater than 1, the parabola looks more narrow. When the number in front of the x^2 is less than 1, the parabola looks wider.**
- **I wonder why multiplying by 2 makes the graph look narrower.**
- **I wonder if there is a way to determine the number in front of x^2 by looking for a pattern in the graph.**

3 Here is Graph B from the previous problem.

Show or explain where you see the **vertical scale** by a factor of $\frac{1}{2}$ in the graph of $h(x) = \frac{1}{2}x^2$.

Responses vary. **The vertical distance from the vertex to the next point went from 1 unit in the $f(x) = x^2$ graph to $\frac{1}{2}$ of a unit in the $h(x) = \frac{1}{2}x^2$ graph. Each of the outputs on $h(x) = \frac{1}{2}x^2$ are half of the value of the outputs on $f(x) = x^2$.**

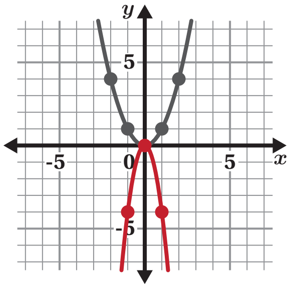


Name: ______ Date: ______ Period: ______

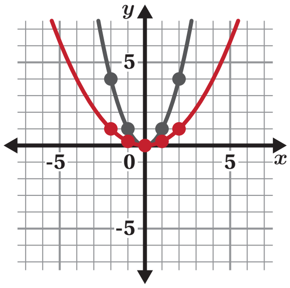
Scaling Vertically (continued)

4 Match the graph of each function with its equation. One equation will have no match.

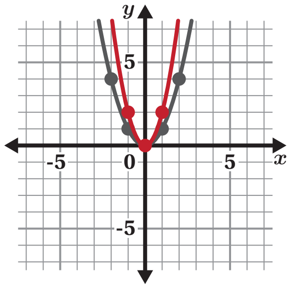
a. $a(x) = -4x^2$ **b.** $b(x) = -2x^2$ **c.** $c(x) = \frac{1}{4}x^2$ **d.** $d(x) = 2x^2$



Equation: a



Equation: c

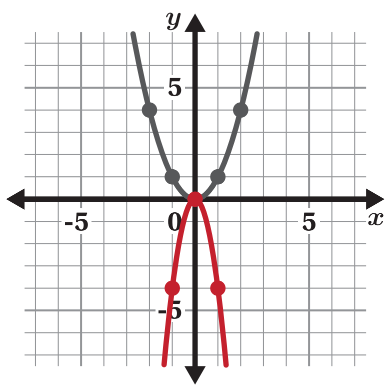


Equation: d

5 How did you decide which of these equations matches this graph?

Responses vary. I noticed this graph includes the point $(1, -4)$ which corresponds to the equation $a(x) = -4x^2$.

$a(x) = -4x^2$ $b(x) = -2x^2$



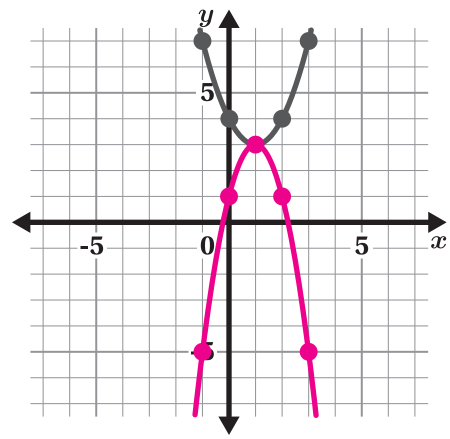
Activity 2

Name: ______ Date: ______ Period: ______

A Bit More Precision

6 The parabola $f(x) = (x - 1)^2 + 3$ has a vertex at $(1, 3)$ and is scaled vertically by a factor of 1.

Draw the graph of $g(x) = \text{-}2(x - 1)^2 + 3$.



7 Here is Kai's work to graph $g(x) = \text{-}2(x - 1)^2 + 3$.

How did this strategy help Kai graph the parabola?

***Responses vary.* Kai noticed that the next point on the parabola would be an x-value of 2, so Kai evaluated $y = \text{-}2(x - 1)^2 + 3$ at $x = 2$ to determine the y-value for the point.**

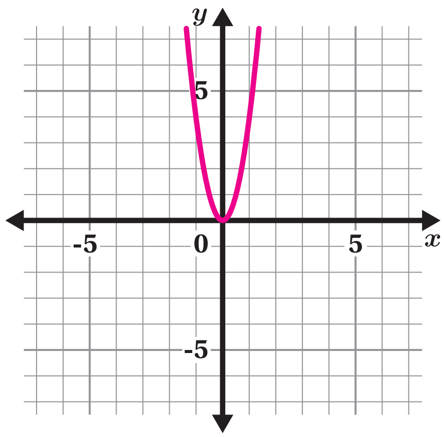
$g(x) = -2(x - 1)^2 + 3$
$g(2) = -2(2 - 1)^2 + 3$
$g(2) = -2(1)^2 + 3$
$g(2) = -2 + 3$
$g(2) = 1$

Name: ______ Date: ______ Period: ______

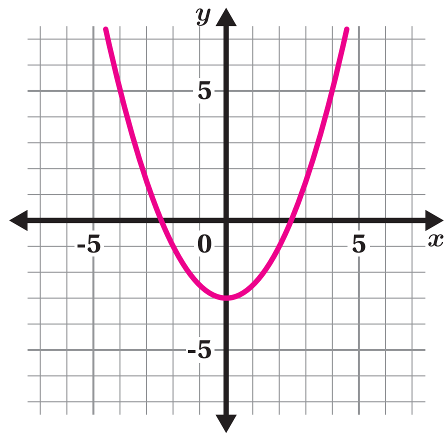
A Bit More Precision (continued)

8 Draw the graph of each quadratic function.

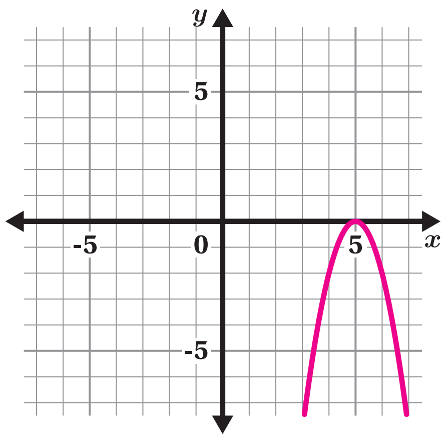
a $a(x) = 4x^2$



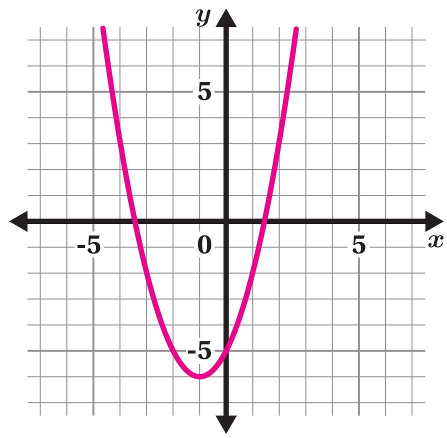
b $b(x) = 0.5x^2 - 3$



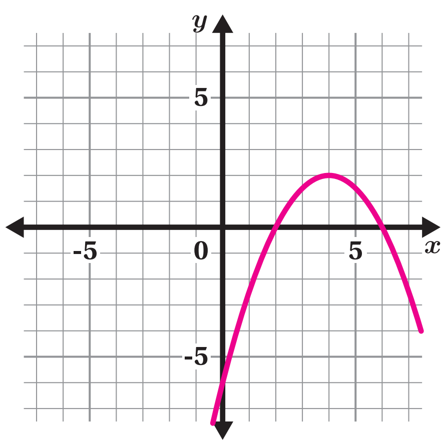
c $c(x) = -2(x - 5)^2$



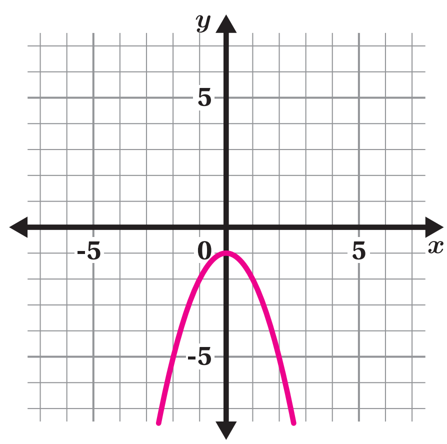
d $d(x) = (x + 1)^2 - 6$



e $e(x) = -0.5(x - 4)^2 + 2$



f $f(x) = -x^2 - 1$



Activity 3

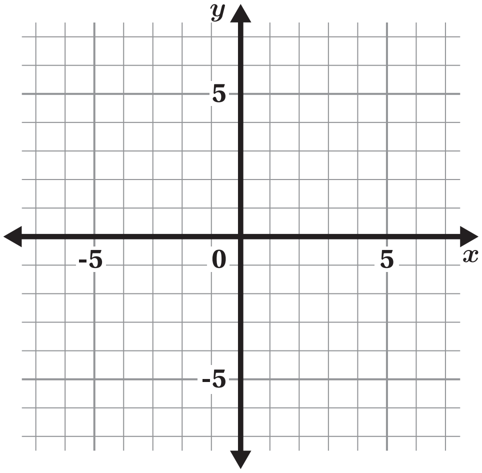
Name: .... Date: .... Period: ....

Parabola Art

9 Create a design by graphing parabolas. Record the functions you use.

Designs vary.

Functions

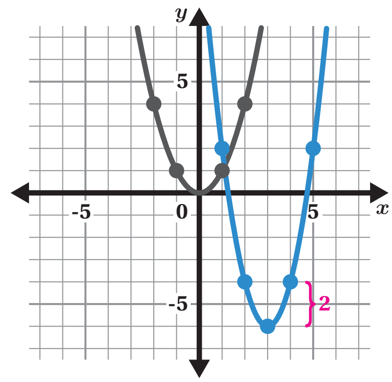


10 Synthesis

Here is the graph of $g(x) = 2(x - 3)^2 - 6$.

$g(x)$ is a transformation of $f(x) = x^2$.

Explain how you can determine the vertical scale from the equation and the graph. ***Responses vary.***



Equation: **The 2 that multiplies the $(x - 3)^2$ term means that the function has been scaled vertically.**

Graph: **The vertical distance between the vertex and the next point is 2 units.**

Things to Remember:

Unit 7

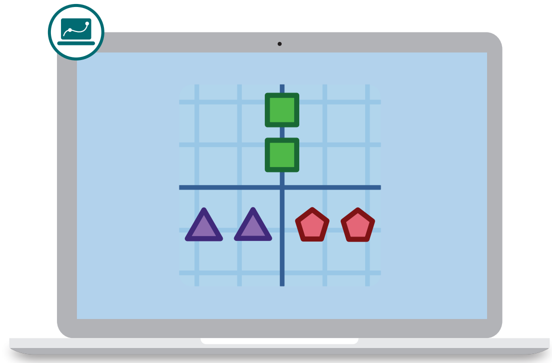
Lesson

16

Name: .... Date: .... Period: ....

Through the Gates

Let's write equations of parabolas given their key features.



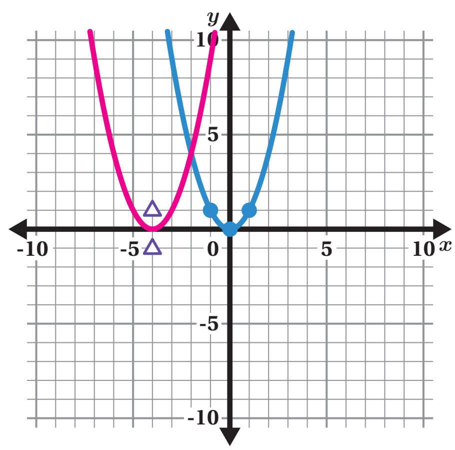
Warm-Up

You'll use the digital activity for Problems 1–2.

1 In this lesson, a gate is the space between two points that look the same.

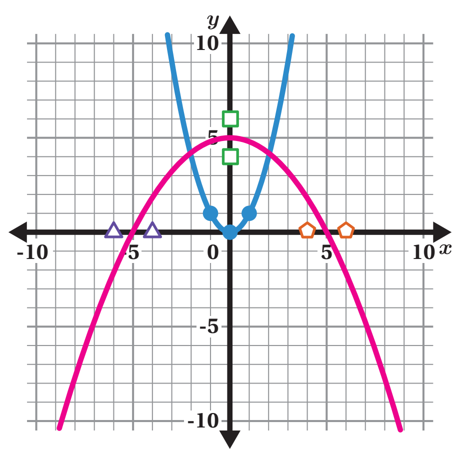
Adjust the parabola so that it goes through the gate between the purple triangles.

Responses vary.



2 Adjust the parabola to go through all three gates.

Responses vary.



Activity 1

Name: .... Date: .... Period: ....

Going Through Gates

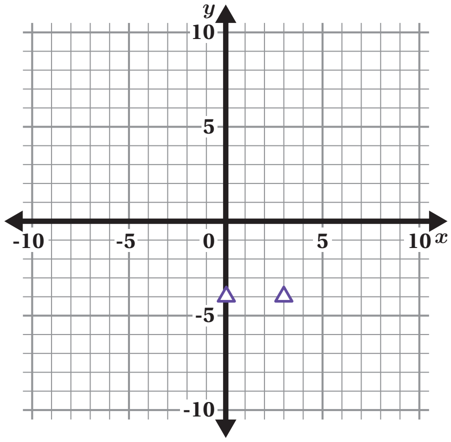
You'll use the digital activity for Problems 3–4.

3 Change the equation so that the parabola goes through the gate.

Responses vary.

Original equation: $y = x^2 - 1$

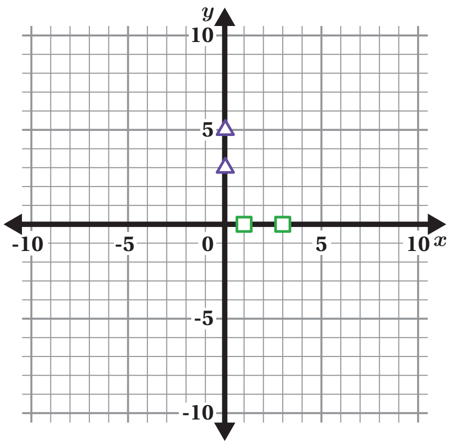
New equation: $y = x^2 - 5$



4 Write an equation for a parabola that goes through each set of gates.

Responses vary.

$y = -x^2 + 4$



5 Renata and Mateo each wrote an equation for a new challenge.

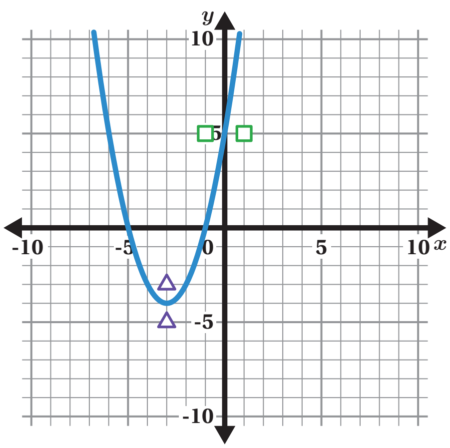
Renata wrote $y = (x + 1)(x + 5)$.

Mateo wrote $y = (x + 3)^2 - 4$.

Whose equation created this parabola?

Renata's Mateo's Both Neither

Explain your thinking.



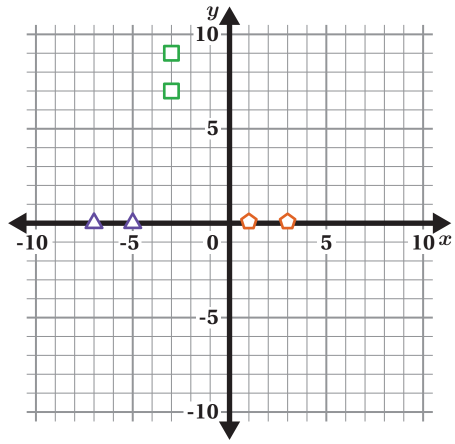
Explanations vary. Students who select Renata, Mateo, or Both are considered correct. Renata's equation created this parabola because it has x-intercepts at -1 and -5. If I plug in -3, I get -4 which is the vertex of the parabola. Mateo's equation created the parabola because it has a vertex at (-3, -4). It also has a y-intercept at (0, 5) because if I plug in 0, the output is 5.

Name: ______ Date: ______ Period: ______

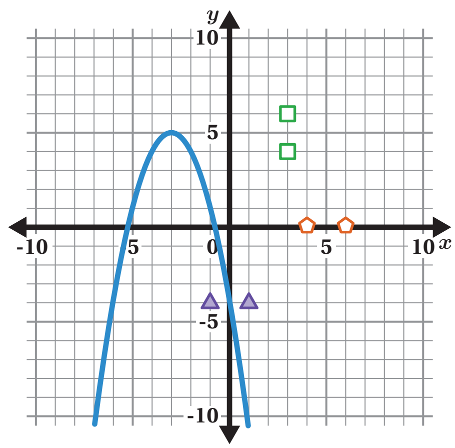
Going Through Gates (continued)

6 In the digital activity, write an equation for a parabola that goes through each set of gates.

Responses vary.
$y = -0.5(x+6)(x-2)$



7 Here is the equation Haru entered for the previous challenge: $y = -(x+3)^2 + 5$.



a What did Haru do well?

***Responses vary.* Haru wrote an equation of a parabola that opens down and is shifted 5 units up.**

b How would you change Haru's equation so that it goes through all of the gates?

***Responses vary.* I would change Haru's equation so the vertex of the parabola is at (3, 4) instead of (-3, 4). An equation that could work is $y = -(x-3)^2 + 5$.**

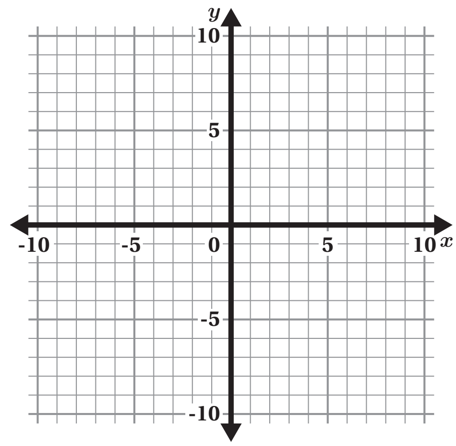
Name: .... Date: .... Period: ....

Challenge Creator

8 Use the digital activity to make a challenge for your classmates to solve.

- Select the number of gates you want to include.
- Drag the movable points in the activity to create a challenge.
- Write an equation for a parabola that goes through each set of gates.

My Challenge

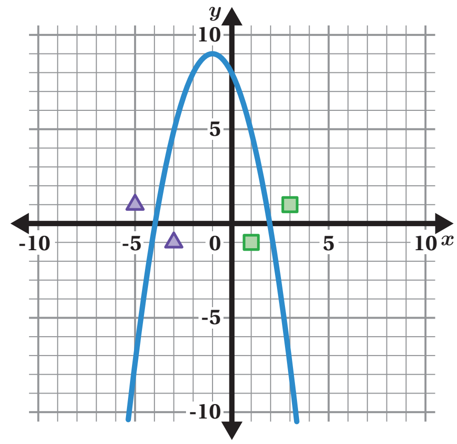


Equation: ....

9 Synthesis

Describe a strategy for writing a quadratic equation that matches a graph.

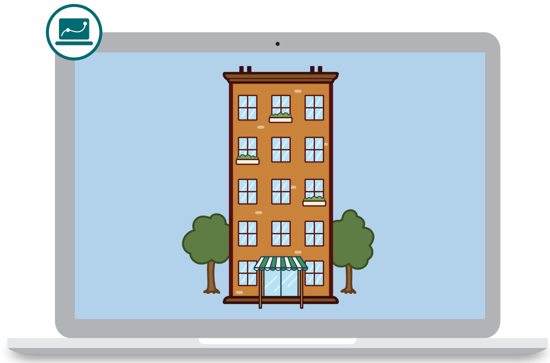
Use the example if it helps with your thinking.



Responses vary. **Identify the key features of the parabola. Use the x-intercepts to start an equation in factored form or use the vertex to start an equation in vertex form. Then adjust the a-value to match the other key features. For example, if the parabola is concave down, make the a-value negative or adjust the value to vertically stretch or compress the parabola.**

Things to Remember:

Name: .... Date: .... Period: ....



Reasonable Rent

Let's use quadratic functions and revenue to make sense of an issue in society: the cost of housing.

Warm-Up

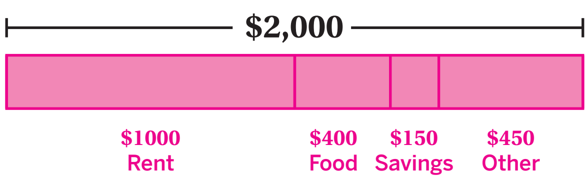
1 The median income for a family in Metropolis is $2,000 a month.

a **Discuss:** What do you think median income means?

Responses vary. **Median income means how much money a typical family makes every month.**

b Create a diagram to show how much you think a family should budget for rent, food, savings, and other expenses. Discuss your choices with a partner.

Diagrams and explanations vary.



2 The median rent in Metropolis is $1,300 a month. Let's look at the budget of a typical Metropolis family. Residents claim that rent is too expensive in Metropolis.

a Why might they feel that rent is too expensive?

Responses vary. **Rent is more than half of the budget. It is more than double any other category and more than 5 times as much as the budget for food.**

b How might rent prices affect the community?

Responses vary. **If people are paying so much in rent and don't have savings, they could lose their homes if something goes wrong. Also, this might affect the health of the community because you might not be able to buy enough food with $200 per month.**

Activity 1

Name: ______ Date: ______ Period: ______

Rent vs. Units

3 Many families in Metropolis are struggling to afford rent.

City Roots (C.R.) is an organization working to establish affordable housing in Metropolis. They buy apartments and rent them at an affordable price.

Discuss: How could C.R. decide what to charge for rent?

Responses vary.

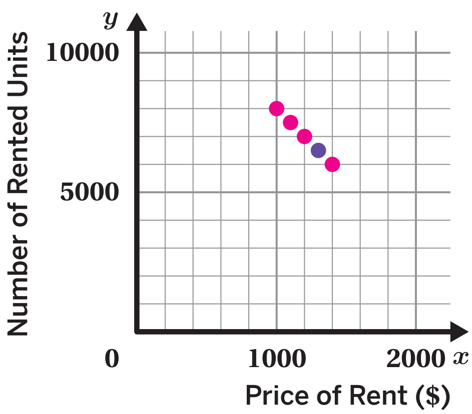
- **C.R. could ask different people what they can afford to pay and then charge that amount.**
- **They could charge a percentage of whatever the median rent was, like 50% or 75%.**
- **They could give everyone a discount of a certain amount. For example, every apartment could be $500 cheaper per month.**

4 Metropolis has a total of 13,000 housing units.

Every time rent increases by $100, C.R. estimates that they can rent 500 fewer units.

Use this information to complete the table and graph.

Price of Rent ($)	Number of Rented Units
1,000	**8,000**
1,100	**7,500**
1,200	**7,000**
1,300	6,500
1,400	**6,000**



5 Let's look at the table and graph. Write an expression to represent the number of units C.R. can rent at *any* price, x.

$13000 - \frac{500}{100}x$ **(or equivalent)**

Activity 2

Name: ________ Date: ________ Period: ________

Rent vs. Revenue

6 City Roots Collective used their model to determine how much revenue they would make.

Price of Rent ($)	Number of Rented Units	Revenue ($)
1,000	8,000	8,000,000
1,100	7,500	**8,250,000**
1,200	7,000	**8,400,000**
1,300	6,500	**8,450,000**
1,400	6,000	**8,400,000**

a. Complete the table.

b. **Discuss:** What do you notice? What do you wonder?

Responses vary.

- **C.R.C. makes the most revenue when the price of rent is $1,300.**
- **C.R.C. makes the same revenue for renting 7,000 and 6,000 units.**
- **I wonder if the pattern continues.**
- **I wonder what C.R.C. will charge for rent.**

7 Let's look at some revenues from the previous problem.

Write an expression to represent the revenue City Roots Collective could make at *any* price, x.

$x\left(13000 - \frac{500}{100}x\right)$ **(or equivalent)**

Name: .... Date: .... Period: ....

Rent vs. Revenue (continued)

8 Here are four considerations City Roots Collective might care about.

a. Making the most revenue

b. Making housing affordable to the most people

c. Making enough money to equal the cost of development

d. Building enough housing to meet the demand

Order them from *most important to you* to *least important to you*. ***Orderings vary.***

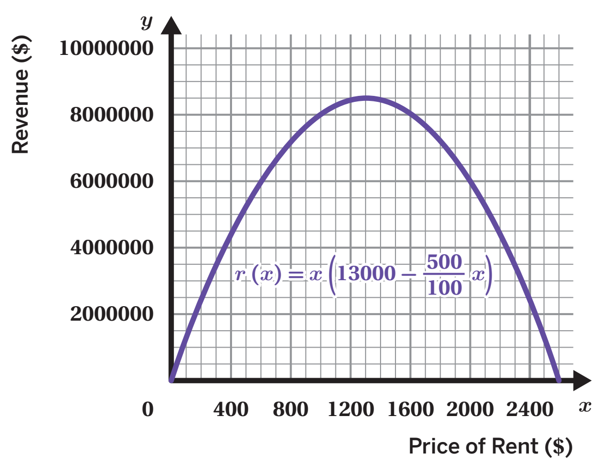
Most important **Least important**

9 This project will cost City Roots Collective $\$8{,}000{,}000$ a month.

What do you think is a fair price for rent? Explain your thinking.

Responses and explanations vary.

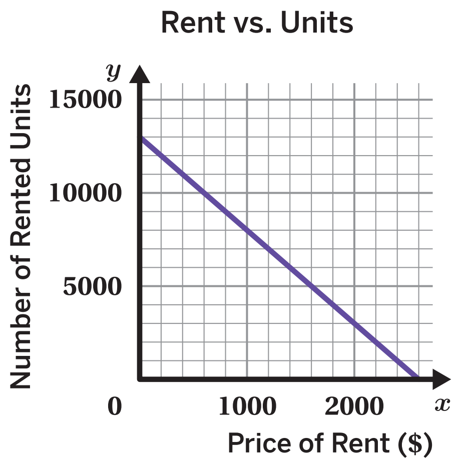
- **\$1000 because that is the lowest amount of money C.R.C. can charge and still make back the money they invested.**
- **\$1300 because if C.R.C. makes more money than they spend, they could spend the extra money on other important projects.**
- **\$800 because many more people could afford rent and C.R.C. can raise the rest of the money for the project through donations.**

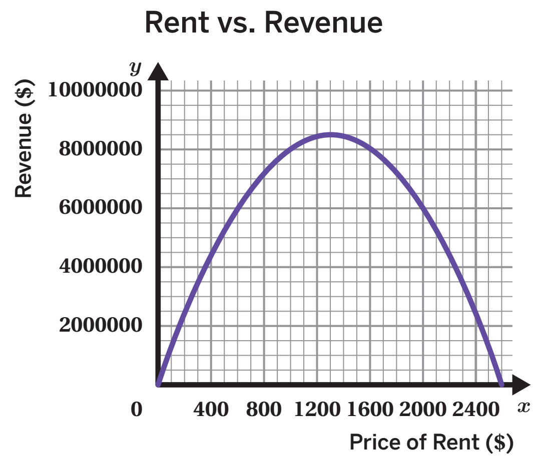


10 Synthesis

The British statistician George Box once said: *All models are wrong, but some are useful*.

a Choose a model we've explored today.





b Explain how that model is wrong and how it is useful.

- **The rent vs. units model is wrong because the intercepts are not real possibilities.**
- **The rent vs. revenue model is wrong because it assumes that all the units cost the same amount of money, which probably isn't true.**

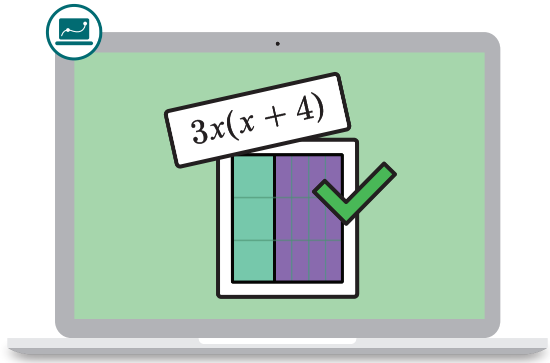
- **The rent vs. units model is useful because it shows the trend that if rent is more expensive, then fewer families can afford the rent.**
- **The rent vs. revenue model is useful because it can help people make decisions when the actual information is really complicated and messy.**

Things to Remember:

Name: .... Date: .... Period: ....

Two-Factor Multiplication

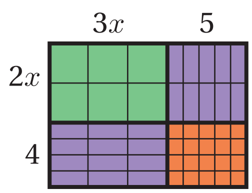
Let's rewrite factored-form quadratic expressions in standard form.



Warm-Up

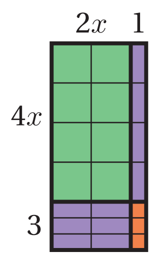
1 An area model shows equivalent quadratic expressions.

a Here are three area models.



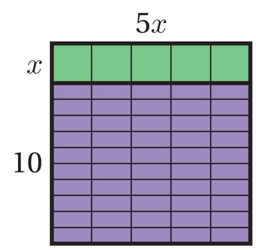
Factored Form
$(2x + 4)(3x + 5)$

Standard Form
$6x^2 + 22x + 20$



Factored Form
$(4x + 3)(2x + 1)$

Standard Form
$8x^2 + 10x + 3$



Factored Form
$(x + 10)(5x)$

Standard Form
$5x^2 + 50x$

b Where on the area model do you see the *factored form*? Where do you see the *standard form*? ***Responses vary.***

Factored Form: **I see factored form on the outside of the model. It is the length and width of the rectangle.**

Standard Form: **I see standard form on the inside of the model. For example, when the standard form is $6x^2 + 22x + 20$, I see $6x^2$ in the green tiles, $22x$ in the purple tiles, and 20 in the orange tiles.**

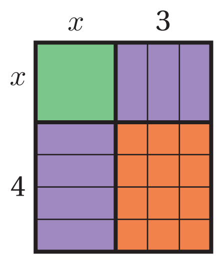
Activity 1

Name: ________ Date: ________ Period: ________

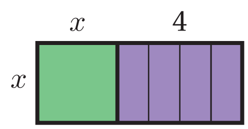
Multiplying With Area Models

2 Match each expression with an equivalent area model. One area model will have no match.

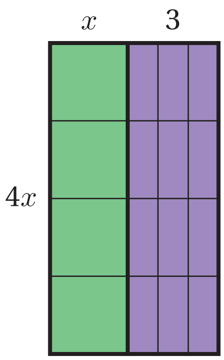
$x(x + 4)$	$3x(x + 4)$	$(x + 4)(x + 3)$



$(x+4)(x+3)$



$x(x+4)$



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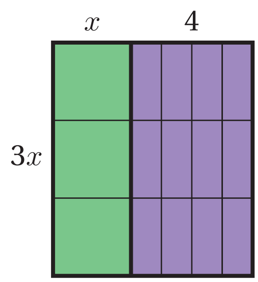


$3x(x+4)$

3 Let's look at two cards that Sahana correctly matched.

She wrote this standard-form expression: $3x^2 + 12x$.

Show or explain where you see $3x^2 + 12x$ in the area model or in the factored-form expression.

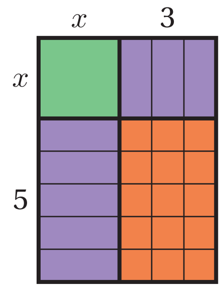
Responses vary.

- **I see $3x^2 + 12x$ in the tiles because I counted 3 green tiles and 12 purple tiles. Each green tile represents an x^2 and each purple tile represents $1x$. Together, this makes $3x^2 + 12x$.**
- **I see $3x^2 + 12x$ in the expression because if you distribute $3x$ to $(x + 4)$, you end up with $3x^2 + 12x$.**

Name: ______ Date: ______ Period: ______

Multiplying With Area Models (continued)

4 Here is a list of equivalent expressions.



Circle *one* expression.

A. $x^2 + 3x + 5x + 15$

B. $x^2 + 8x + 15$

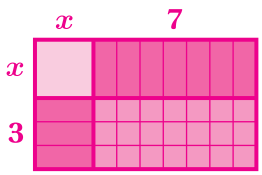
C. $x(x + 3) + 5(x + 3)$

Show or explain how you see it represented in the area model.

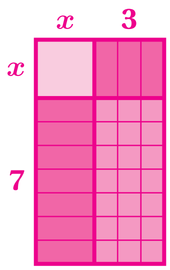
Responses vary.

- **If I look at all of the individual sections, I can see the expression $x^2 + 3x + 5x + 15$.**
- **If I break the model into two rectangles, I can see the expression $x(x + 3) + 5(x + 3)$.**

5 **a** Draw an area model to represent the expression $(x + 7)(x + 3)$.



or



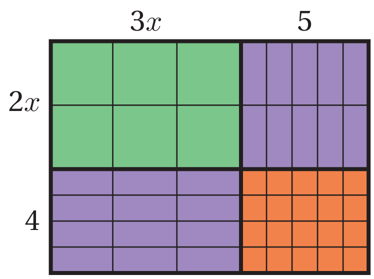
b Rewrite $(x + 7)(x + 3)$ in standard form.

$\mathbf{x^2 + 10x + 21}$

Name: ______ Date: ______ Period: ______

Multiplying With Diagrams

6 The diagram on the right shows a different way to represent the area model.



	$3x$	5
$2x$	$6x^2$	$10x$
4	$12x$	20

Discuss: How are the area model and the diagram alike? How are they different?

Responses vary.

- The area model and the diagram are similar because they both represent two forms of the same quadratic expression. In both representations, the x^2 term is in the top-left corner.
- The area model and the diagram are different because the area model shows each individual piece, while the diagram just shows terms. The area model is a scale representation, while the size of the diagram doesn't change.

7 Multiply to rewrite $(2x + 5)(x + 8)$ in standard form.

Use the diagram if it helps with your thinking.

$2x^2 + 21x + 40$

	x	8
$2x$		
5		

Name: ________ Date: ________ Period: ________

Multiplying With Diagrams (continued)

8 Karima tried to rewrite $(5x - 3)(2x + 5)$ in standard form and made an error.

	$5x$	-3
$2x$	$10x^2$	$-6x$
5	$25x$	-15

$10x^2 + 31x - 15$

What did Karima do well? What could she improve?

Responses vary.

Something Karima did well: **She correctly multiplied in each of the four smaller boxes.**

Something Karima could improve: **She incorrectly combined like terms. $-6x + 25x = 19x$, not $31x$.**

9 Multiply to rewrite each expression in standard form. Use the diagrams if they help with your thinking.

a $(x + 6)(x + 10)$

$x^2 + 16x + 60$

b $(3x + 1)(x + 6)$

$3x^2 + 19x + 6$

c $(2x - 6)(3x + 1)$

$6x^2 - 16x - 6$

d $(4x + 5)(x - 7)$

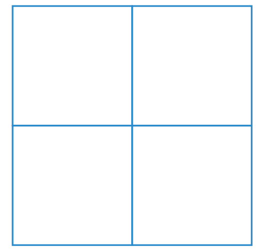
$4x^2 - 23x - 35$

12 Synthesis

Describe how to write a factored-form expression in standard form.

$(2x - 3)(x + 4)$

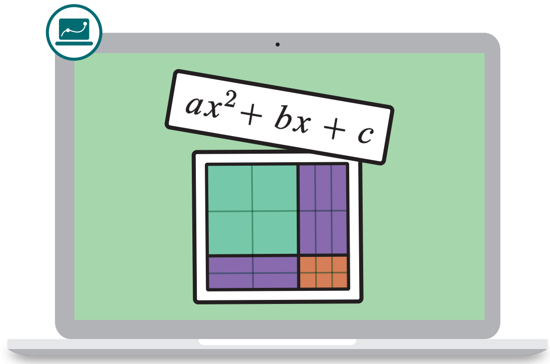
Use the diagram if it helps with your thinking.



***Responses vary.* To write a factored-form expression in standard form, you can use a diagram. You write factored form on the outside of the diagram and standard form on the inside. Then you can find the area of the smaller sections and combine like terms to write the expression in standard form.**

Things to Remember:

Name: Date: Period:



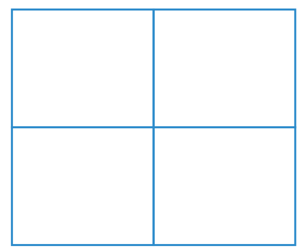
Standard Feature

Let's look for patterns that help us rewrite factored-form expressions in standard form.

Warm-Up

1 Which expression is equivalent to $(x+5)^2$?

Use the diagram if it helps with your thinking.



A. $x^2 + 25$

B. $x^2 + 10x + 25$

C. Both

D. Neither

Explain your thinking.

Explanations vary.

- **I know that $(x+5)^2 = (x+5)(x+5)$. When I use the diagram to multiply, I get $x^2 + 10x + 25$.**
- **I get the same parabola when I graph $y = x^2 + 10x + 25$ and $y = (x+5)^2$.**
- **When I plug in different values for x into both expressions, I get the same results.**

2 Let's look at how two students determined the expression equivalent to $(x+5)^2$.

Discuss: How are their strategies alike? How are they different?

Responses vary.

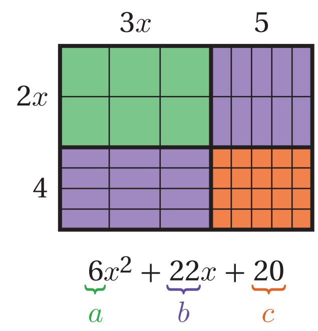
- **The strategies are similar because they both show four pairs of terms being multiplied. They also show how the $10x$ in standard form came from $5x + 5x$.**
- **The strategies are different because Tyani's strategy doesn't involve drawing a diagram. She drew arrows for each section of the diagram instead.**

Name: ______ Date: ______ Period: ______

Standard Form

3 We often use $ax^2 + bx + c$ to represent a quadratic expression in *standard form*.

Where do you see a, b, and c in the area model?



Responses vary.

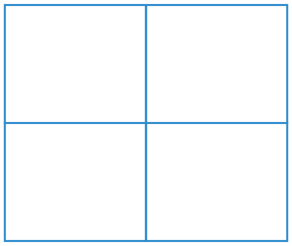
- **I see the a-value in the green tiles, the b-value in the purple tiles, and the c-value in the orange tiles.**
- **The a-value is the number of large square tiles in the top left; the b-value is the number of rectangular tiles in the top right and bottom left; and the c-value is the number of small square tiles in the bottom right.**

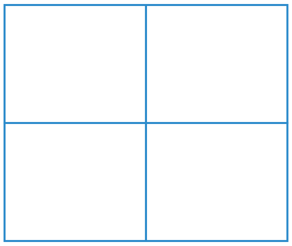
4 Group the equivalent expressions. One expression will have no match. Use the diagrams if they help with your thinking.

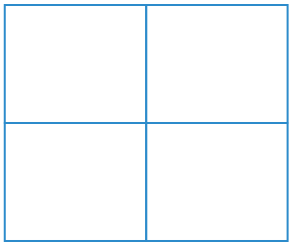
$(x+3)(x+3)$	$3x^2 + 9x$	$x^2 - 9$	$(x+3)^2$
$(x-3)(x+3)$	$3x(x+3)$	$x^2 + 9$	$x^2 + 6x + 9$

Group 1	Group 2	Group 3
$x^2 + 6x + 9$ $(x+3)^2$ $(x+3)(x+3)$	$3x^2 + 9x$ $3x(x+3)$	$x^2 - 9$ $(x-3)(x+3)$

Expression with no match: $x^2 + 9$







Name: ________ Date: ________ Period: ________

Standard Form (continued)

5 Here are two equivalent expressions:

$$(x - 3)(x + 3) \text{ and } x^2 - 9$$

Discuss: Why does $(x - 3)(x + 3)$ have a b-value of 0 when written in standard form?

Use the diagram if it helps with your thinking.

Responses vary.

- **The b-value is 0 because when you multiply you get $x^2 - 3x + 3x - 9$. Adding -$3x$ and $3x$ makes 0.**
- **-$3x$ and $3x$ are opposites, so they cancel out.**

6 Here are two equivalent expressions:

$$3x(x + 3) \text{ and } 3x^2 + 9x$$

Why is the c-value 0?

Use the diagram if it helps with your thinking.

Responses vary.

- **The c-value has no variable with it. Since I distributed an x to both terms, there is no term without a variable.**
- **The c-value is 0 because the first expression only has one term, so I can think of it as $3x + 0$. Then when I multiply, I get $0 \cdot 3$, and that is 0.**

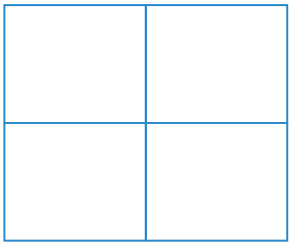
7 Select *all* the expressions that have a b- or c-value of 0 when written in standard form.

- ☑ **A.** $(3x - 1)(3x + 1)$
- ☐ **B.** $(x - 4)(x - 4)$
- ☑ **C.** $x(x + 4)$
- ☐ **D.** $(3x + 1)(x - 1)$
- ☑ **E.** $(x + 10)(x - 10)$

Name: ____________ Date: ________ Period: ________

Now I Know My ABC's

8 Write an expression in *factored form* that has a b*-value of* 0 when written in standard form.

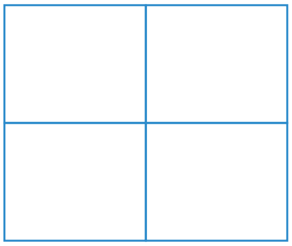


Use the diagram if it helps with your thinking.

Responses vary.

- $(x - 6)(x + 6)$
- $(5x - 1)(5x + 1)$

9 Write an expression in factored form that has a *positive* b*-value* and a *negative* c*-value* when written in standard form.



Use the diagram if it helps with your thinking.

Responses vary.

- $(x + 9)(x - 7)$
- $(3x - 1)(x + 4)$

10 Here are four expressions that have a *positive* b*-value* and a *negative* c*-value*.

Describe any patterns you notice.

***Responses vary.* In order to have a negative c-value, my two constants in factored form need to have opposite signs. To have a positive b-value, usually the larger constant needs to be the positive number.**

Factored Form	Standard Form
$(x - 2)(x + 9)$	$x^2 + 7x - 18$
$(x + 10)(x - 8)$	$x^2 + 2x - 80$
$(3x + 7)(x - 2)$	$3x^2 + 1x - 14$
$(4x - 9)(x + 3)$	$4x^2 + 3x - 27$

Name: ________ Date: ________ Period: ________

Now I Know My ABC's (continued)

11 Write an expression in factored form that has a b-*value greater than* 5 and a c-*value of* 1 when written in standard form.

Use the diagram if it helps with your thinking.

Responses vary.

- $(5x + 1)(6x + 1)$
- $(7x + 1)(x + 1)$

12 **a** Write an expression in factored form that has a *negative* a-*value*, a *negative* b-*value*, and a *negative* c-*value* when written in standard form.

Use the diagram if it helps with your thinking.

Responses vary.

- $(-x - 3)(x + 7)$
- $(2x + 1)(-5x - 3)$

b Compare your expression with another group's expression.

Discuss: What patterns do you notice?

Responses vary.

- For every pair of terms that gets multiplied, one term has to be positive and one term has to be negative.
- I noticed that I needed to make one factor have two negative terms and the other have no negative terms so that I never end up with a negative number times a negative number.

Explore More

13 Do you think it's possible to write an expression in factored form that has a b-value of 0 *and* a positive c-value when written in standard form? Circle one.

Yes   No   Not enough information

Explain your thinking.

Explanations vary. I can write the factored form $(-3x + 6)(3x + 6)$, which is $-9x^2 + 36$ in standard form.

14 Synthesis

Describe 2–3 patterns you noticed between equivalent expressions in factored form and in standard form.

Use the examples if they help with your thinking.

$(3x - 2)(3x + 2)$

$(x - 6)(x - 3)$

$(x + 5)(x + 5)$

$(5x + 2)(x - 5)$

$2x(x - 4)$

Responses vary.

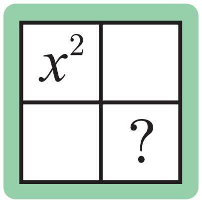
- **Constants with the same signs in factored form will make a positive c-value in standard form.**
- **If factored form has the same coefficients but opposite constants, the b-value will be 0 in standard form.**
- **If factored form is one term multiplied by two terms, standard form will only have two terms.**

Things to Remember:

Name: ________ Date: ________ Period: ________

X-Factor

Let's rewrite standard-form quadratic expressions in factored form.



Warm-Up

1. Match each expression in *factored form* with its equivalent expression in *standard form*.

Factored Form	Standard Form
a. $(5x+6)(x-3)$	c $5x^2+43x-18$
b. $(5x-3)(x+6)$	a $5x^2-9x-18$
c. $(5x-2)(x+9)$	d $5x^2-43x-18$
d. $(5x+2)(x-9)$	b $5x^2+27x-18$

Activity
1

Name: ________ Date: ________ Period: ________

Diagram Puzzles

Complete each diagram puzzle, standard-form expression, and factored-form expression.

Diagram	Standard Form	Factored Form
2. $\begin{array}{c\|c\|c} & 3x & -5 \\ \hline 4x & 12x^2 & -20x \\ \hline -3 & -9x & 15 \end{array}$	$12x^2 - 29x + 15$	$(3x - 5)(4x - 3)$
3. $\begin{array}{c\|c\|c} & 2x & 3 \\ \hline 2x & 4x^2 & 6x \\ \hline -3 & -6x & -9 \end{array}$	$4x^2 - 9$ or $4x^2 + 0x - 9$	$(2x + 3)(2x - 3)$ (or equivalent)
4. $\begin{array}{c\|c\|c} & 2x & -3 \\ \hline x & 2x^2 & -3x \\ \hline 4 & 8x & -12 \end{array}$	$2x^2 + 5x - 12$	$(2x - 3)(x + 4)$ (or equivalent)
5. $\begin{array}{c\|c\|c} & 3x & 4 \\ \hline x & 3x^2 & 4x \\ \hline 5 & 15x & 20 \end{array}$	$3x^2 + 19x + 20$	$(3x + 4)(x + 5)$ (or equivalent)

Activity 1

Name: ________ Date: ________ Period: ________

Diagram Puzzles (continued)

Diagram	Standard Form	Factored Form
6. $\begin{array}{c\|c\|c} & x & 2 \\ \hline x & x^2 & 2x \\ \hline -5 & -5x & -10 \end{array}$ (or equivalent)	$x^2 - 3x - 10$	$(x + 2)(x - 5)$ (or equivalent)
7. $\begin{array}{c\|c\|c} & 3x & 1 \\ \hline x & 3x^2 & x \\ \hline 1 & 3x & 1 \end{array}$ (or equivalent)	$3x^2 + 4x + 1$	$(3x + 1)(x + 1)$ (or equivalent)
8. $\begin{array}{c\|c\|c} & x & 5 \\ \hline x & x^2 & 5x \\ \hline 4 & 4x & 20 \end{array}$ (or equivalent)	$x^2 + 9x + 20$	$(x + 5)(x + 4)$ (or equivalent)
9. $\begin{array}{c\|c\|c} & 2x & 1 \\ \hline 3x & 6x^2 & 3x \\ \hline 2 & 4x & 2 \end{array}$ (or equivalent)	$6x^2 + 7x + 2$	$(2x + 1)(3x + 2)$ (or equivalent)

Activity 2

Name: .... Date: .... Period: ....

Next Steps

Nicolas is trying to factor $2x^2 + 9x + 7$.

	$2x$	1
x	$2x^2$	x
7	$14x$	7

10. **Discuss:**

- What did Nicolas do well?
- Explain what you think is incorrect about Nicolas's work.
- What could he try next?

Responses vary.

- **Nicolas did well choosing factors of 7, and he multiplied correctly.**
- **Nicolas's work is incorrect because $14x + x = 15x$, not $9x$.**
- **Next, he could try to switch the places of the 1 and 7.**

Sneha is trying to factor $2x^2 + 23x - 12$. She started by creating this diagram.

	$2x$	
x	$2x^2$	
		-12

11. List pairs of constants Sneha could try in order to complete the outside of the diagram.

***Responses vary.* -1 and 12, -12 and 1, 2 and -6, -2 and 6, -3 and 4, -4 and 3**

Sneha tried the numbers -6 and 2.

	$2x$	2
x	$2x^2$	$2x$
-6	$-12x$	-12

12. **Discuss:**

- How can you tell Sneha's work is incorrect?
- What did Sneha do well?
- What could she try next?

Responses vary.

- **Sneha's work is incorrect because $2x - 12x = -10x$, not $23x$.**
- **Sneha did well choosing factors of -12, and she multiplied correctly.**
- **Next, she could try to switch the places of the -6 and 2, switch which constant is positive, or try a different pair of numbers that multiply to -12.**

13. Rewrite $2x^2 + 23x - 12$ in factored form.

$(2x - 1)(x + 12)$ (or equivalent)

Activity
2

Name: ______ Date: ______ Period: ______

Next Steps (continued)

14. Ariana is trying to factor $10x^2 - 7x - 12$. She starts by creating this diagram.

$10x^2$	
	-12

Ariana says: *I have to use factors of* 10*. I also need to use factors of* -12.

What do you think she means?

***Responses vary.* Because the a-value is not prime, Ariana needs to think about possible pairs of numbers that multiply to get both the a- and the c-value. There is more than one pair of numbers, so Ariana has multiple factors of 10 and multiple factors of -12 to try.**

15. Rewrite $10x^2 - 7x - 12$ in factored form.

$(5x + 4)(2x - 3)$ (or equivalent)

16. Here are three other expressions with a c-value of -12. Rewrite each expression in factored form. Use the diagrams if they help with your thinking.

a $x^2 + x - 12$

$(x + 4)(x - 3)$ (or equivalent)

b $3x^2 - 16x - 12$

$(3x + 2)(x - 6)$ (or equivalent)

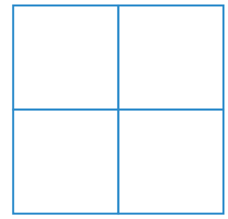
c $6x^2 - x - 12$

$(2x - 3)(3x + 4)$ (or equivalent)

Synthesis

17. Describe how to rewrite a standard-form expression in factored form.

$5x^2 - 31x - 28$

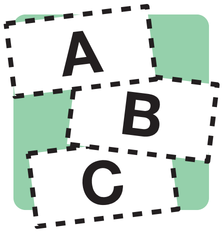


Use the example if it helps with your thinking.

***Responses vary.* You can use a diagram to help you by putting the ax^2 in the top-left corner of the diagram and c in the bottom right. Then try different pairs of numbers that multiply to the terms on the inside of the diagram until you get an expression equivalent to the given standard-form expression.**

Things to Remember:

Name: .... Date: .... Period: ....



Form Up

Let's factor some special quadratic expressions.

Warm-Up

Eliza is trying to factor $x^2 + x - 56$. She started by listing pairs of numbers that multiply to -56.

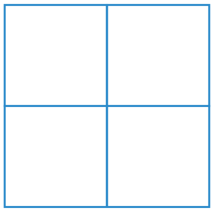
1 and -56	2 and -28	4 and -14	7 and -8
-1 and 56	-2 and 28	-4 and 14	-7 and 8

1. **Discuss:** Which pairs might Eliza try first? Why?

***Responses vary.* She should try 7 and -8, because these numbers are close together.**

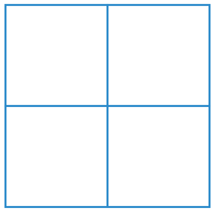
2. Factor each expression. Use the diagrams if they help with your thinking.

a $x^2 + x - 56$



$(x+8)(x-7)$ (or equivalent)

b $x^2 + 26x - 56$



$(x+28)(x-2)$ (or equivalent)

Activity 1

Name: .... Date: .... Period: ....

Spotting Similarities

Here are three groups of expressions.

Group 1	Group 2	Group 3
$4x^2 - 25$	$8x^2 + 32x + 24$	$x^2 - 6x - 27$
$x^2 - 36$	$-4x^2 + 8x + 32$	$x^2 + 2x - 80$
$x^2 - 100$	$-10x^2 - 20x - 10$	$x^2 - 13x + 30$
$25x^2 - 49$	$2x^2 - 22x + 60$	$x^2 + 2x - 63$

3. Explain how the expressions in each group are alike. ***Responses vary.***

Group 1: **Each of these expressions only has two terms. Both terms are perfect squares.**

Group 2: **Each of these expressions has an a-value that is not 1. They all have three terms. All of the terms share a common factor.**

Group 3: **Each of these expressions has three terms. All of the a-values are 1.**

4. Factor one expression from each group. Use the diagrams if they help with your thinking. ***Choices vary.***

Group 1: .... Group 2: .... Group 3: ....

Group 1	Group 2	Group 3
$(2x + 5)(2x - 5)$	$8(x + 3)(x + 1)$	$(x - 9)(x + 3)$
$(x - 6)(x + 6)$	$-4(x - 4)(x + 2)$	$(x + 10)(x - 8)$
$(x + 10)(x - 10)$	$-10(x + 1)(x + 1)$	$(x - 3)(x - 10)$
$(5x - 7)(5x + 7)$	$2(x - 6)(x - 5)$	$(x + 9)(x - 7)$
	(or equivalent)	

Name: ____ Date: ____ Period: ____

Spotting Similarities (continued)

Deiondre factored the expression $7x^2 + 28x + 21$.

Deiondre
$7x^2 + 28x + 21$
$7(x^2 + 4x + 3)$
$7(x + 3)(x + 1)$

5. **Discuss:**

- Are $7x^2 + 28x + 21$ and $7(x^2 + 4x + 3)$ equivalent? How do you know? **Yes. Explanations vary. They are equivalent because multiplying all three terms by 7 will give you back $7x^2 + 28x + 21$.**
- Why might Deiondre have written $7(x^2 + 4x + 3)$ as a first step? ***Responses vary.* Dieondre may have written $7(x^2 + 4x + 3)$ because he might find smaller a-, b-, and c-values easier to work with.**

6. Does Deiondre's expression belong in Group 1, 2, or 3? Explain your thinking.
Group 2. *Responses vary.* It belongs in Group 2 because it has three terms, they have a common factor, and $a \neq 1$.

Yasmine factored the expression $9x^2 - 49$.

Yasmine
$9x^2 - 49$
$9x^2 + 0x - 49$
$(3x - 7)(3x + 7)$

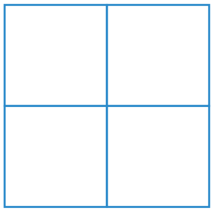
7. **Discuss:** Does Yasmine's expression belong in Group 1, 2, or 3? Explain your thinking. **Group 1. *Explanations vary.* In the beginning, there are only two terms and they are both perfect squares.**

8. Write a new expression in *standard form* that belongs in the same group as Yasmine's.
***Responses vary.* $x^2 - 81$**

9. Factor the expression you wrote in the previous problem.
***Responses vary.* $(x - 9)(x + 9)$**

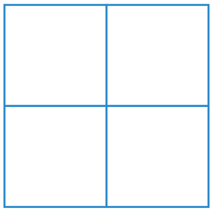
10. Factor each expression. Use the diagrams if they help with your thinking.

a $3x^2 - 6x - 105$



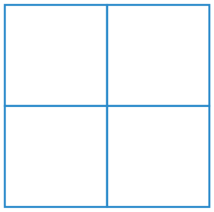
$3(x - 7)(x + 5)$ (or equivalent)

b $16x^2 - 49$



$(4x - 7)(4x + 7)$ (or equivalent)

c $4x^2 + 52x + 120$



$4(x + 10)(x + 3)$ (or equivalent)

Activity 2

Name: ____ Date: ____ Period: ____

Solve and Swap

You will get a card.

- Factor the expression on your card. Draw a diagram if it helps with your thinking.
- Find a partner and swap cards. Factor your new expression, then check with your partner.
- Find a new partner and repeat this process.

Card ____

1. $(x+6)(x-1)$
2. $(2x+3)(x-8)$
3. $-2(x-2)(x+1)$
4. $(x+10)(x+8)$

Card ____

5. $(x+5)(x-2)$
6. $(10x-3)(10x+3)$
7. $6(x+2)(x-3)$
8. $(4x+5)(x+2)$

Card ____

9. $(2x+1)(2x-5)$
10. $(3x-1)(3x+1)$
11. $(x-7)(x-8)$
12. $(3x-4)(x+4)$

Card ____

13. $(x+5)(x-5)$
14. $(x+10)(x-10)$
15. $(x-2)(x+7)$
16. $2(x+1)(x+9)$ (or equivalent)

Card ____

17. $(5x-8)(5x+8)$
18. $5(x-4)(x+1)$ (or equivalent)
19. $(3x+4)(x+3)$
20. $(x-2)(x+2)$

Card ____

21. $(x-4)(x+4)$
22. $(x-10)(x+4)$
23. $(2x+3)(x+6)$
24. $5(3x-4)(x+1)$ (or equivalent)

Card ____

25. $-3x(2x-7)$ (or equivalent)
26. $(x+9)(x+2)$
27. $(x-6)(x+6)$
28. $10(x-4)(x-2)$ (or equivalent)

Card ____

29. $(x-1)(x+9)$
30. $5(x+3)(x-3)$ (or equivalent)
31. $(2x+1)(x-9)$
32. $10(x+1)(x+2)$ (or equivalent)

Synthesis

What do you think is important to remember when factoring an expression in standard form?

$5x^2 - 18x - 8$

$9x^2 - 16$

$6x^2 - 24x - 30$

Use the expressions if they help with your thinking.

Responses vary.

- Remember to try 1 and c when testing pairs of numbers that multiply to c.
- Try to divide out a common factor first if possible.
- Stick with it! Factoring often takes multiple tries.
- When standard form only has two terms, rewrite the expression with a 0 for the missing term.
- If the c-value is negative, the signs of the constants are different.

Things to Remember:

Lesson 8.05
Activity 2
(p. 1 of 2)

Trading Cards

Directions: Make one copy per 32 students. Then pre-cut the cards and give each student one card.

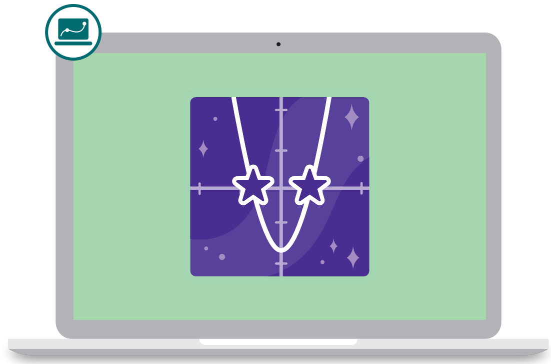
Card 1 $x^2 + 5x - 6$	**Card 2** $2x^2 - 13x - 24$	**Card 3** $-2x^2 + 2x + 4$	**Card 4** $x^2 + 18x + 80$
Card 5 $x^2 + 3x - 10$	**Card 6** $100x^2 - 9$	**Card 7** $6x^2 - 6x - 36$	**Card 8** $4x^2 + 13x + 10$
Card 9 $4x^2 - 8x - 5$	**Card 10** $9x^2 - 1$	**Card 11** $x^2 - 15x + 56$	**Card 12** $3x^2 + 8x - 16$
Card 13 $x^2 - 25$	**Card 14** $x^2 - 100$	**Card 15** $x^2 + 5x - 14$	**Card 16** $2x^2 + 20x + 18$

Lesson 8.05 **Activity 2** *(p. 2 of 2)*

Trading Cards

Card 17 $25x^2 - 64$	**Card 18** $5x^2 - 15x - 20$	**Card 19** $3x^2 + 13x + 12$	**Card 20** $x^2 - 4$
Card 21 $x^2 - 16$	**Card 22** $x^2 - 6x - 40$	**Card 23** $2x^2 + 15x + 18$	**Card 24** $15x^2 - 5x - 20$
Card 25 $-6x^2 + 21x$	**Card 26** $x^2 + 11x + 18$	**Card 27** $x^2 - 36$	**Card 28** $10x^2 - 60x + 80$
Card 29 $x^2 + 8x - 9$	**Card 30** $5x^2 - 45$	**Card 31** $2x^2 - 17x - 9$	**Card 32** $10x^2 + 30x + 20$

Name: ________________ Date: ________ Period: ________



Shooting Stars

Let's determine the x-intercepts of quadratic functions written in factored form and standard form.

Warm-Up

1 Determine whether each coordinate pair is an x-intercept or a y-intercept.

Ordered Pair	x-intercept	y-intercept	Neither
$(1.7, 0)$	✓		
$(1, 1)$			✓
$(0, 4)$		✓	
$\left(-\frac{3}{2}, 0\right)$	✓		
$(5, 0)$	✓		
$(0, -6)$		✓	

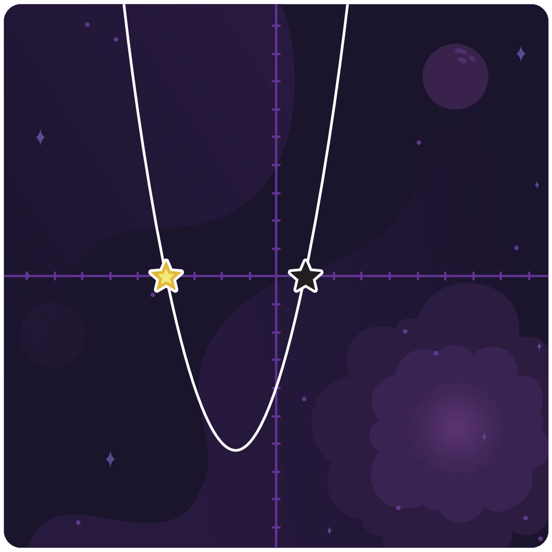
Name: ______ Date: ______ Period: ______

Star Mail

2 Send stars to the x-intercepts of this function:

$$f(x) = (x + 4)(x - 1)$$

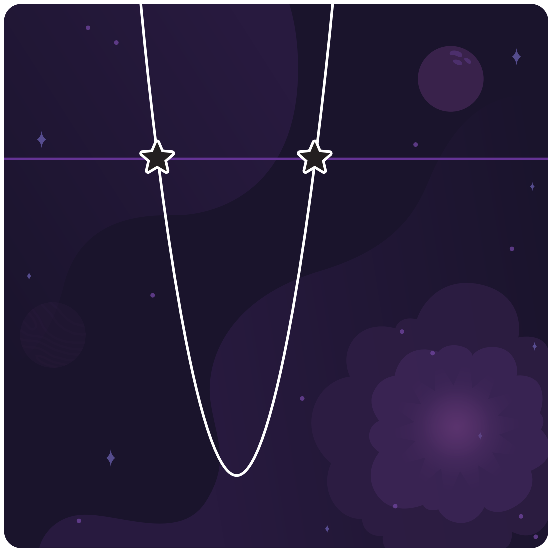
Star	Ordered Pair
Star #1	(-4, 0)
Star #2	**(1, 0)**



3 Send stars to the x-intercepts of this function:

$$g(x) = (x + 1)(2x - 6)$$

Star	Ordered Pair
Star #1	**(-1, 0)**
Star #2	**(3, 0)**



4 Let's look at Aba's strategy from the previous problem.

Discuss:

- Why did Aba replace $g(x)$ with 0?
- How did Aba figure out the coordinates of the x-intercepts?

Explanations vary.

- **Aba replaced $g(x)$ with 0 because an x-intercept of a function is a point where the output is 0.**
- **Once Aba found the inputs that make $g(x) = 0$, she wrote them as ordered pairs in the form $(x, 0)$ since x-intercepts are where the graph of the function crosses the x-axis.**

Activity 2

Name: .... Date: .... Period: ....

Standard Space Mail

5 Send stars to the x-intercepts of this function:

$$h(x) = x^2 + 3x - 10$$

Star	Ordered Pair
Star #1	**(-5, 0)**
Star #2	**(2, 0)**

6 Aba, Darius, and Rishi factored the function $a(x) = 4x^2 + 20x + 24$ in three different ways.

Aba	Darius	Rishi
$a(x) = 4(x+3)(x+2)$	$a(x) = (4x+8)(x+3)$	$a(x) = (2x+6)(2x+4)$

a **Discuss:** How can you see that each equation has the same x-intercepts?

Responses vary.

- Aba's equation: Since there are no coefficients, I take the opposite signs of the constant in each factor to solve for x. That gives me $x =$ -3 and $x =$ -2.
- Darius's equation: When I set $(4x + 8) = 0$ and solve for x, I get $x =$ -2. When I set $(x + 3) = 0$ and solve for x, I get $x =$ -3.
- Rishi's equation: When I substitute $x =$ -3 or $x =$ -2, I get an output value of 0. That means all three of the equations have the same x-intercepts.

b Write the x-intercepts in the table below.

x-intercepts	Ordered Pair
x-intercept #1	**(-3, 0)**
x-intercept #2	**(-2, 0)**

7 Send stars to the x-intercepts of this function:

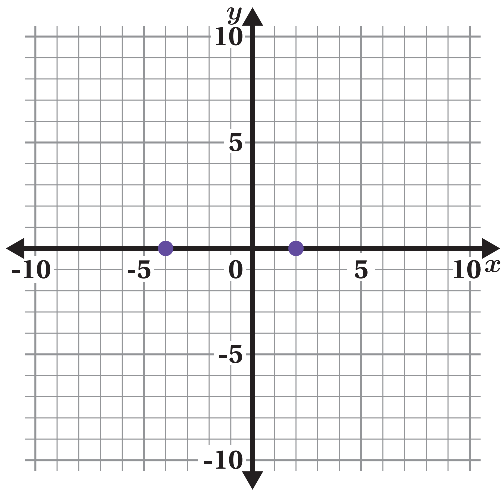
$$b(x) = 2x^2 - 11x + 12$$

Star	Ordered Pair
Star #1	$\left(\frac{3}{2}, 0\right)$
Star #2	**(4, 0)**

Name: ________ Date: ________ Period: ________

Zero, My Hero

8 A term related to x-intercepts is **zeros**. The zeros of a function are the x-values that make $f(x) = 0$.



a **Discuss:** How are *zeros* related to x-intercepts?

Responses vary. **An x-intercept is where the graph of a function crosses the x-axis, so it's written as an ordered pair, $(x, 0)$. A zero is only the x-value of the x-intercept.**

b Write a function whose zeros are $x = -4$ and $x = 2$.

$f(x) =$ ________

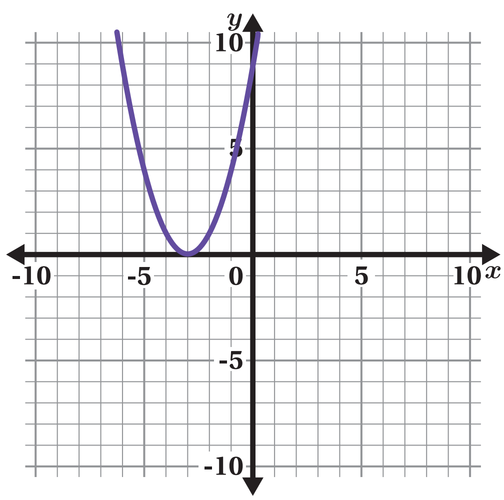
Responses vary. $f(x) = (x + 4)(x - 2)$, $f(x) = x^2 + 2x - 8$, $f(x) = 10x^2 + 20x - 80$

9 The function $f(x) = x^2 + 6x + 9$ has exactly one zero.

Write a new quadratic function that has exactly one zero.

$g(x) =$ ________

Responses vary. $g(x) = (x - 3)^2$,
$g(x) = (x - 7)(x - 7)$, $g(x) = 2(x - 2)(x - 2)$,
$g(x) = 5x^2 + 10x + 5$

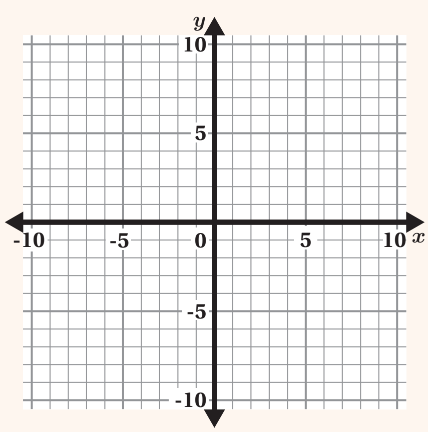


Explore More

10 Show or describe as much as you can about the graph of this function:

$$f(x) = (x + 1)(x - 2)(x + 3)$$

Responses vary. **$f(x)$ has three x-intercepts at $(-1, 0)$, $(2, 0)$, and $(-3, 0)$. The function could be rewritten in standard form as $f(x) = x^3 + 2x^2 - 5x - 6$. It also has a y-intercept at $(0, -6)$. It is not a quadratic function.**



11 Synthesis

Describe a strategy for determining the x-intercepts or zeros of a quadratic function.

$g(x) = (x + 1)(2x - 4)$

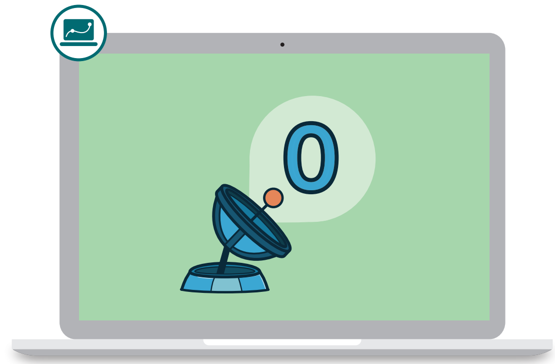
$h(x) = x^2 + 3x - 10$

Use the examples if they help with your thinking.

Responses vary. **First, I need to make sure the function is written in factored form. Then I can determine the x-values that make the function equal to 0.**

Things to Remember:

Name: ____________ Date: ________ Period: ________



Make It Zero

Let's use the zero-product property to solve quadratic equations.

Warm-Up

1 Determine the *solution* to each equation.

$4a = 0$	$0 = 2\pi b$	$6(c - 5) = 0$	$7 \bullet (d + 8) \bullet 9 = 0$
$a = 0$	$b = 0$	$c = 5$	$d = -8$

2 The **zero-product property** states: If the product of two or more *factors* is 0, then at least one of the factors is 0.

We can use this to help solve equations like $4a = 0$ or $6(c - 5) = 0$.

Write a new equation using the variable x that the zero-product property could help solve.

Responses vary.

- $17x = 0$
- $100(x + 5) = 0$
- $0 = 8(x + 2)$

Activity 1

Name: .... Date: .... Period: ....

Solve It

Use the zero-product property to solve the following equations.

3 $(x - 4)(2x + 3) = 0$

$x =$ 4 $x =$ $-\frac{3}{2}$ (or equivalent)

4 $x^2 + 5x + 4 = 0$

$x =$ -4 $x =$ -1

5 $3x^2 - 18x + 15 = 0$

$x =$ 5 $x =$ 1

Name: .... Date: .... Period: ....

Solve It (continued)

6 Here is Hamza's work from the previous problem.

$$3x^2 - 18x + 15 = 0$$
$$(3x - 3)(x - 5) = 0$$
$$x = 3 \text{ or } x = 5$$

a What is something Hamza did well?

Responses vary.

- **Hamza knew writing the equation in factored form could help to solve it.**
- **Hamza factored correctly.**
- $x = 5$ **is one of the correct solutions.**

b What is something Hamza can improve?

Responses vary.

- **Hamza could plug the solutions into the original equation to check the work.**
- **Hamza could adjust** $x = 3$ **to** $x = 1$ **because** $3(1) - 3 = 0$.
- **Hamza could factor out the greatest common factor to more easily see the solutions:** $3(x - 1)(x - 5) = 0$.

Activity 2

Name: .... Date: .... Period: ....

Zeroing In

7 Inola says you can't use the zero-product property to solve the equation $x^2 - 4 = 3x$.

a. **Discuss:** Why might Inola think that?

Responses vary.

- The equation is not in factored form.
- The equation is not set equal to zero.

b. Describe how you could rewrite the equation so that the zero-product property *can* be used.

Responses vary.

I could set the equation equal to zero and factor it.

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

8 Solve Inola's equation: $x^2 - 4 = 3x$.

$x =$ 4 $x =$ -1

Activity 2

Name: ______ Date: ______ Period: ______

Zeroing In (continued)

9 The equation $9x^2 = 12x - 4$ has one solution.

What's the solution? Show or explain your thinking.

$x = \frac{2}{3}$ (or equivalent). *Explanations vary.*

- When I set the equation equal to zero and factor it, the factors are the same, so only one number will make either of the factors zero.
- The equation is equivalent to $(3x - 2)(3x - 2) = 0$. The zero-product property says $3x - 2$ has to equal 0, and that only has one solution.

Explore More

10 Write at least one equation with $x = 2$ and $x = 3$ as solutions.

Try to write some equations you think none of your classmates will write.

Responses vary.

- $(x - 2)(x - 3) = 0$
- $0 = x^2 - 5x + 6$
- $-5(x - 2)(x - 3) = 0$
- $(x - 2)(-4x + 12) = 0$
- $-2x^2 = -10x + 12$
- $-66 = -55x + 11x^2$

11 Synthesis

How can you use the zero-product property to solve a quadratic equation?

Consider the examples if they help with your thinking.

A $(2x+4)(x+3)=0$

B $3x^2-18x+15=0$

C $x^2-4=3x$

D $(x-5)(x+1)=7$

Responses vary.

- **First make sure that the equation is set equal to zero and factored. Then determine what number would make each factor zero.**
- **Rewrite the equation as a product that equals zero. Then you can set each factor equal to zero and solve to find the solutions.**

Things to Remember:

Name: .... Date: .... Period: ....

Zero, One, or Two?

Let's determine whether quadratic equations have zero, one, or two solutions.

Warm-Up

1-2 Determine the value of each expression using mental math.

a 8^2 $= \mathbf{64}$

b -8^2 $= \mathbf{-64}$

c $(-8)^2$ $= \mathbf{64}$

d Solve $x^2 = 64$.

$x = \mathbf{8}$, $x = \mathbf{-8}$

Activity 1

Name: ________ Date: ________ Period: ________

How Many?

3 For each equation, put a check for the number of solutions.

Equation	No Solutions	One Solution	Two Solutions
$(x-3)^2 = 1$			✓
$(x-3)^2 = 0$		✓	
$(x-3)^2 = -1$	✓		
$(x-3) = 1$		✓	
$(x-3)(x-3) = 1$			✓

4 Diya says that $x = 4$ is the only solution to $(x-3)^2 = 1$.

a. **Discuss:** How do you know that $x = 4$ is a solution to $(x-3)^2 = 1$?

Responses vary.

- **When I substitute 4 for x, I get $(4-3)^2$, which is equal to 1.**
- **$(x-3)^2$ is the same as $(x-3)(x-3)$. Plugging in 4 for x, I get $(4-3)(4-3) = 1$, which is true.**

b. Write a hint to help Diya determine *another* solution.

Responses vary.

- **There are two numbers that you could square to get 1. What are they?**
- **Since $x^2 = (x)(x)$, it's important to remember that you can multiply two positives or two negatives to get a positive. So you need to check $(x-3) = 1$ and $(x-3) = -1$.**

5 Here is a new equation: $x^2 - 16 = 9$.

How many solutions does this equation have? Circle one.

No solutions One solution Two solutions

Record any solution(s) here:

$x =$ 5

$x =$ -5

Name: .... Date: .... Period: ....

How Many? (continued)

6 Rewrite the equation $x^2 - 16 = 9$ so that it has no solutions.

Responses vary.

- $x^2 - 16 = -17$
- $x^2 - 16 = -20$
- $x^2 + 16 = 9$

Show or explain your thinking.

Explanations vary. **After adding 16 to both sides of the equation, I know x^2 has to be greater than or equal to 0 to have solutions. To have no solutions, after adding 16 to both sides of the equation, x^2 has to be equal to a negative number.**

7 Here is a new equation: $(x - 5)^2 = 36$.

How many solutions does this equation have? Circle one.

No solutions   One solution   Two solutions

Record any solution(s) here:

$x =$ **11**

$x =$ **-1**

Activity 2

Name: ________ Date: ________ Period: ________

How Many and More

8 For each equation, put a check for the number of solutions.

Equation	No Solutions	One Solution	Two Solutions
$x(x-6)=0$			✓
$2x^2 = 50$			✓
$x^2 = -9$	✓		
$x^2 + 4 = 0$	✓		
$(x+2)(x+2)=0$		✓	

9 Here are two equations from the previous problem.

$$(x+2)(x+2)=0 \qquad\qquad x^2 = -9$$

Explain how you decided on the number of solutions for $(x+2)(x+2)=0$.

***Responses vary*. To be true, $(x+2)$ must be 0, which means there is only one value for x.**

Explain how you decided on the number of solutions for $x^2 = -9$.

***Responses vary*. There are no values for x that would lead to a negative value when squared. Therefore there are no solutions.**

Name: ________ Date: ________ Period: ________

How Many and More (continued)

10 Solve as many challenges as you have time for.

- Circle how many solutions each equation has.
- Record any solutions.

	Equation	Number of Solutions			Solution(s)
a	$30 = x^2 - 6$	No solutions	One solution	Two solutions	$x =$ -6 $x =$ 6
b	$7x^2 + 1 = 1$	No solutions	One solution	Two solutions	$x =$ 0 $x =$ ____
c	$(x - 4)^2 = -12$	No solutions	One solution	Two solutions	$x =$ ____ $x =$ ____
d	$x(x + 2) = 15$	No solutions	One solution	Two solutions	$x =$ -5 $x =$ 3
e	$(x - 4)(x - 4) = 16$	No solutions	One solution	Two solutions	$x =$ 8 $x =$ 0
f	$3x^2 - 3 = -3$	No solutions	One solution	Two solutions	$x =$ 0 $x =$ ____

11 Synthesis

a Discuss these questions:

- How can you determine the number of solutions to a quadratic equation?
- How can you solve a quadratic equation?

$(x+2)(x+2)=0$

$x^2 = -9$

$(x-5)^2 = 36$

b Select *one question* and record your response.

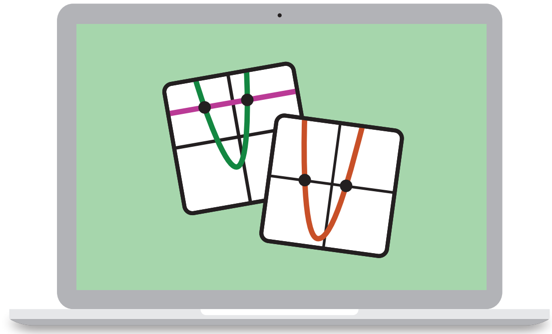
Responses vary.

- **To determine the number of solutions, you can look at the structure of the equation. If the structure is something like $(x + \_\_\_)^2 = 0$, then the equation only has one solution. If you see $(x + \_\_\_)^2 =$ *a negative number*, then that has no solutions. Structures like $(x + \_\_\_)^2 =$ *a positive number* have two solutions.**
- **To solve a quadratic equation like $(x-5)^2 = 36$, you need to check for both the positive and negative values that can make this equation true. For example, both $(-6)^2$ and $(6)^2$ equal 36.**
- **To solve a quadratic equation like $(x+2)(x+2)=0$, only one value for x can give you 0, and that is -2.**

Things to Remember:

Unit 8
Lesson
9

Name: ______ Date: ______ Period: ______



Graph to Solve

Let's use graphs to solve quadratic equations.

Warm-Up

1 Determine whether each statement is true or false.

Statement	True	False
$x = 2$ and $x = 3$ are the solutions to $(x - 2)(x - 3) = 6$.		✓
$x = 3$ is the only solution to $x^2 - 9 = 0$.		✓
$x(x - 7) = 0$ has two solutions.	✓	
$x = -5$ is a solution to $x^2 + 25 = 0$.		✓

2 Let's look at the statement Saanvi incorrectly said was true.

a What might have Saanvi been thinking?

Responses vary.

- **Saanvi might have thought she could use the zero-product property.**
- **Saanvi might have thought that $x = 2$ and $x = 3$ could be substituted like this: $(2)(3) = 6$.**

b What would you say to Saanvi to help her see that this statement is false?

Responses vary.

- **Try substituting $x = 2$ or $x = 3$ to check if the left side of the equation would equal 6.**
- **If you want to use the zero-product property, make sure your equation is equal to 0.**

Activity 1

Name: ______ Date: ______ Period: ______

When in Doubt, Graph It Out

3 Malik used the Desmos Graphing Calculator to determine the solutions to $(x-2)(x-3)=6$.

a Let's watch the animation to see Malik's strategy.

b **Discuss:** Where in the graph can you see that the solutions are $x=0$ and $x=5$?

Responses vary.

- **$x=0$ and $x=5$ are the input values that give an output of $y=6$ for the parabola $y=(x-2)(x-3)$.**
- **The x-coordinates of the points $(0,0)$ and $(5,0)$ are where the graphs of $y=(x-2)(x-3)$ and $y=6$ intersect.**

4 Use a graphing calculator and Malik's strategy to solve $x^2+2x+1=4$.

How many solutions does this equation have? Circle one.

No solutions One solution Two solutions

Record any solution(s) here:

$x=$ -3

$x=$ 1

If you circled *No solutions*, complete the statement:

The equation has no solutions because . . .

Name: ________ Date: ________ Period: ________

When in Doubt, Graph It Out (continued)

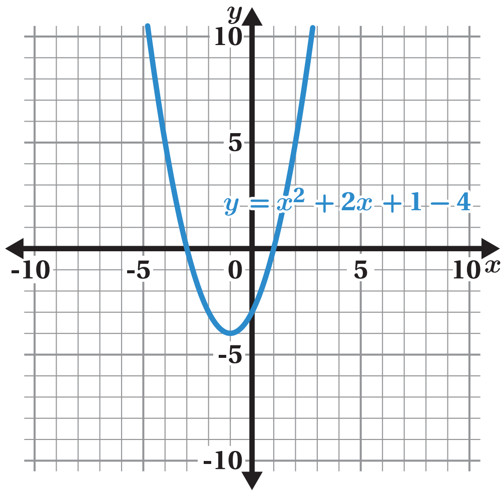
5 Saanvi also solved $x^2 + 2x + 1 = 4$ by graphing.

She graphed the equation $y = x^2 + 2x + 1 - 4$.

Show or describe where you see the solutions to $x^2 + 2x + 1 = 4$ in Saanvi's graph.

Responses vary.

- **At the x-intercepts, $x = -3$ and $x = 1$**
- **At the zeros of the function, $(-3, 0)$ and $(1, 0)$**



6 Use any strategy to solve $(x - 3)^2 = -1$

How many solutions does this equation have? Circle one.

No solutions   One solution   Two solutions

Record any solution(s) here:

$x =$ ________

$x =$ ________

If you circled *No solutions*, complete the statement:

The equation has no solutions because . . .

Responses vary.

- **The graphs of $y = (x - 3)^2$ and $y = -1$ never intersect.**
- **The equation has no solutions because whatever number you square, the result cannot be negative.**

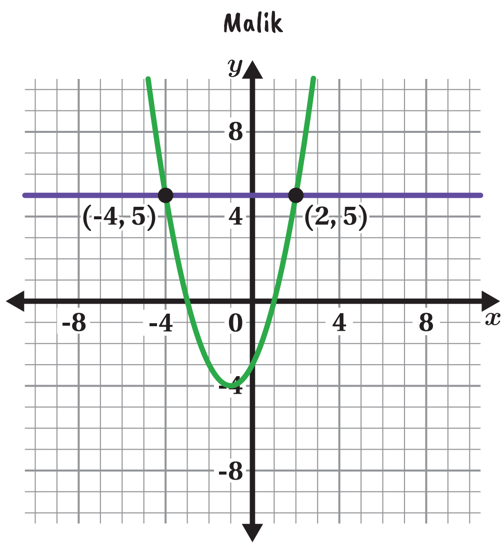
Activity 2

Name: Date: Period:

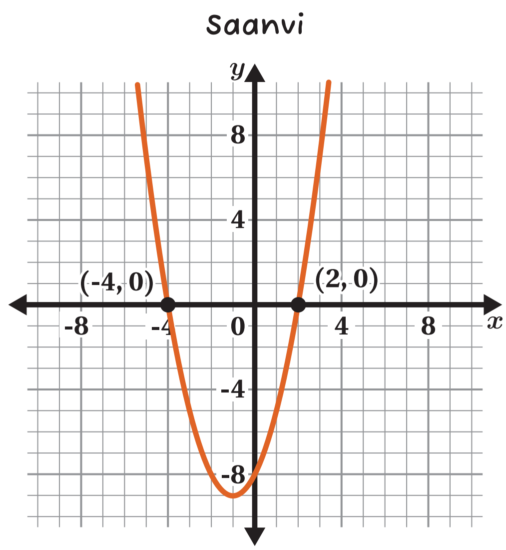
None, One, or Some

7 Malik and Saanvi each used graphing to solve $(x + 3)(x - 1) = 5$.

a Take a look at each student's strategy.



Graph $y = (x + 3)(x - 1)$ and $y = 5$.
Where do the graphs intersect?



Graph $y = (x + 3)(x - 1) - 5$.
What are the x-intercepts?

b **Discuss:**

- How are their strategies alike? How are they different?
- When might you use one strategy or the other?

Responses vary.

- **Both strategies use x-values as their solutions.**
- **Both strategies graph a parabola.**
- **Malik's strategy graphs two equations, while Saanvi's strategy graphs one equation.**
- **Saanvi's strategy rearranges the equation so that it's equal to 0.**
- **I would use Saanvi's strategy when the equation is already equal to 0.**
- **I would use Malik's strategy when the left side and the right side of the equations are both not zero.**

Name: ________ Date: ________ Period: ________

None, One, or Some (continued)

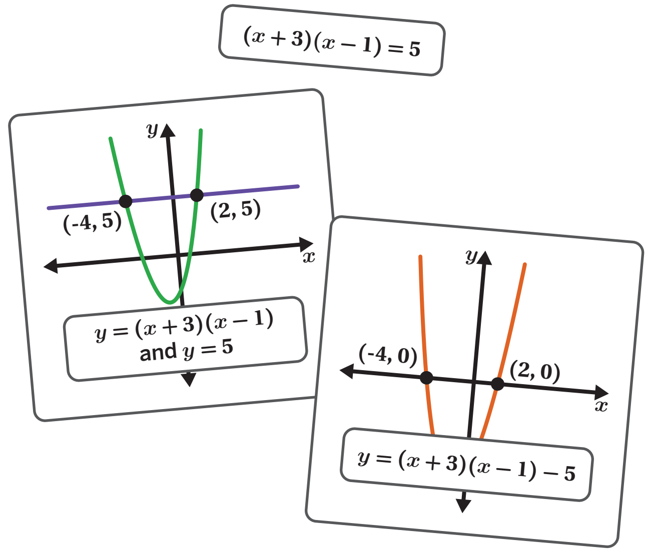
8 Use a graphing calculator to solve as many challenges as you have time for.

- Circle how many solutions each equation has.
- Record any solutions.

	Equation	Number of Solutions			Solution(s)
a	$-4x^2 + 5 = 1$	No solutions	One solution	Two solutions	$x =$ -1 $x =$ 1
b	$(x - 4)(x - 2) = -5$	No solutions	One solution	Two solutions	$x =$ ____ $x =$ ____
c	$2x^2 - x - 4 = 2$	No solutions	One solution	Two solutions	$x =$ -1.5 $x =$ 2
d	$x(x - 2) = -1$	No solutions	One solution	Two solutions	$x =$ 1 $x =$ ____
e	$7 = x(x - 6)$	No solutions	One solution	Two solutions	$x =$ -1 $x =$ 7
f	$(x + 6)(x + 8) = -1$	No solutions	One solution	Two solutions	$x =$ -7 $x =$ ____

9 Synthesis

Describe a strategy for using a graphing calculator to solve a quadratic equation.

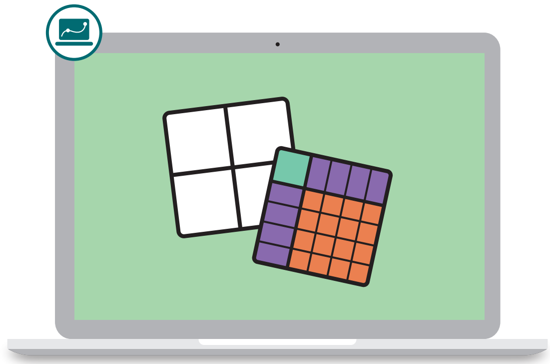


Responses vary.

- Graph each side of the equation ("$y =$ left hand side" and "$y =$ right hand side"). Find the solution at the x-coordinates where the parabola intersects with the horizontal line.
- Rearrange the equation so that it equals 0. Then graph the equation and find the solution at the x-intercepts.

Things to Remember:

Name: ________ Date: ________ Period: ________



Square Dance

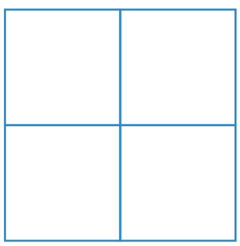
Let's build squares using tiles and algebra.

Warm-Up

1 **a** Write each expression in factored form. Use the diagrams if they help with your thinking.

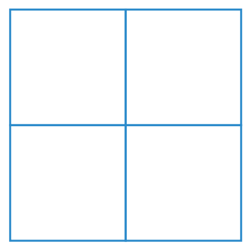
$x^2 + 8x + 16$

$(x + 4)(x + 4)$



$x^2 + 8x + 12$

$(x + 2)(x + 6)$



b **Discuss:** How are $x^2 + 8x + 16$ and $x^2 + 8x + 12$ alike? How are they different?

Responses vary.

- **The expressions are alike because they are both in standard form, and they both have the same first two terms: $x^2 + 8x$.**
- **Their third terms are different, which means that the constants needed to factor the expressions are different.**
- **When I factor $x^2 + 8x + 16$, both factors are the same, but not when I factor $x^2 + 8x + 12$.**

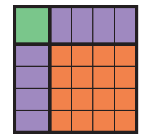
Activity 1

Name: ____________ Date: ________ Period: ________

Perfect Squares

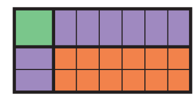
2 $(x + 4)^2$ and $x^2 + 8x + 16$ are **perfect squares**.

	x	4
x	x^2	$4x$
4	$4x$	16



$(x + 6)(x + 2)$ and $x^2 + 8x + 12$ are not perfect squares.

	x	6
x	x^2	$6x$
2	$2x$	12



a **Discuss:** What do you think makes an expression a perfect square? ***Responses vary.***

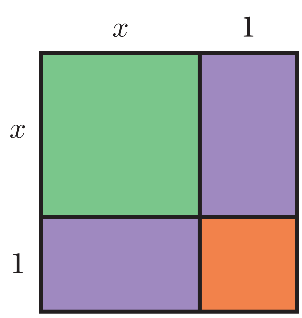
- **An expression is a perfect square if you can represent it as a square using an area model.**
- **An expression is a perfect square if the factors as the same, like $(x + 4)(x + 4)$.**
- **An expression is a perfect square if it can be written like $(x + \_\_\_\_)^2$.**

b Write a different expression that is a perfect square.

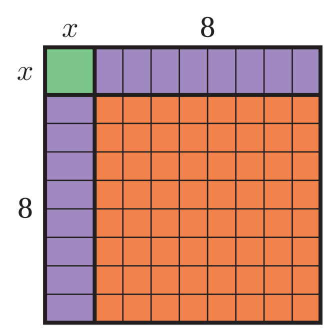
Responses vary. **$(x + 6)(x + 6)$; $(x + 10)^2$; $x^2 + 6x + 9$; $x^2 + 10x + 25$**

3 Here are more perfect square expressions written in factored and standard form.

Factored form: $(x + 1)^2$
Standard form: $x^2 + 2x + 1$



Factored form: $(x + 8)^2$
Standard form: $x^2 + 16x + 64$



a **Discuss:** What do you notice? What do you wonder?

- **I notice that the factored-form expressions all look like $(x + \_\_\_\_)^2$.**
- **I notice that the c-value of the standard form is a perfect square number.**
- **I notice that the b-value of the standard form is twice the value of the constant in factored form.**
- **I wonder if perfect square expressions can have negative values.**
- **I wonder if there are perfect squares that have fractional or decimal values.**
- **I wonder if there are ways to tell if something is a perfect square without drawing a diagram.**

b Is $x^2 + 12x + 144$ a perfect square? Circle one. Yes No Not enough information

Explain your thinking.

Explanations vary.

- **If the b-value is 12, that would mean that the factors would be $(x + 6)(x + 6)$, leading to a c-value of 36.**
- **The c-value is 144. Based on that, the factors for a perfect square expression would be $(x + 12)(x + 12)$, but that would have a b-value of 24.**

Name: .... Date: .... Period: ....

Perfect Squares (continued)

4 Sort the expressions based on whether they are perfect squares.

$x^2 + 10x + 100$	$x^2 - 24x - 144$	$x^2 + 4$	$x^2 + 5x + 6.25$
$x^2 - 24x + 144$	$x^2 + 10x + 25$	$(x - 4)^2$	

Perfect Square	Not a Perfect Square
$x^2 + 5x + 6.25$	$x^2 - 24x - 144$
$x^2 - 24x + 144$	$x^2 + 4$
$x^2 + 10x + 25$	$x^2 + 10x + 100$
$(x - 4)^2$	

5 How did you decide whether the expression $x^2 + 5x + 6.25$ was a perfect square?

Responses vary.

- **I decided this was a perfect square because I could draw a diagram with factors of $(x + 2.5)(x + 2.5)$.**
- **I decided this was a perfect square because I tried taking half of 5 to get 2.5, and then I squared 2.5, which is 6.25.**

Activity 2

Name: ______ Date: ______ Period: ______

Completing Squares

6 This perfect square is written in factored and standard form. Some numbers are smudged.

Factored Form	**Standard Form**
$(x + \bullet)^2$	$x^2 + 6x + \bullet$

Is there enough information to determine the smudged numbers? Explain your thinking.

Use algebra tiles if they help with your thinking.

Yes. *Explanations vary*. The only way to make a square using the rectangular tiles is to split them into two groups of $3x$. That means that the factored-form expression must be $(x + 3)^2$ and that the unknown c-value is $3 \cdot 3 = 9$.

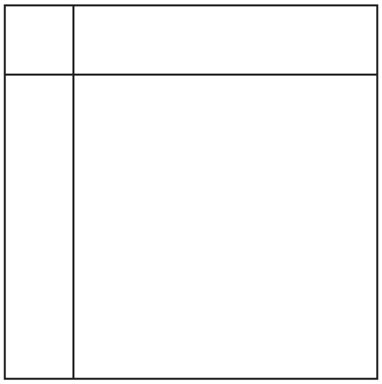
7 Here is a new expression with a smudge.

$x^2 + 22x + \bullet$

If the expression is a perfect square, what number is smudged?

Use the diagram if it helps with your thinking.

121 (or equivalent)



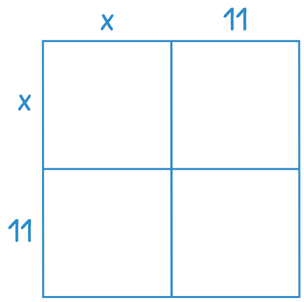
8 Sadia wrote the expression $(x + 11)^2$ to help determine the smudged number.

Explain why this makes sense and how it can help Sadia figure out the smudged number.

***Responses vary*. In perfect square expressions, the b-value is always double the constant in the factored form. Since the b-value is 22, the factored-form expression must be $(x + 11)^2$. Sadia can use this to determine the unknown c-value: $11 \cdot 11 = 121$.**

$x^2 + 22x + \bullet$

$(x + 11)^2$



Name: ________ Date: ________ Period: ________

Completing Squares (continued)

9 Solve as many challenges as you have time for.

If each expression is a perfect square, what number is missing?

a. $x^2 - 10x +$ **25**

b. $x^2 + 20x +$ **100**

c. $x^2 +$ **12 or -12** $x + 36$

d. $x^2 +$ **20 or -20** $x + 100$

e. $x^2 - 50x +$ **625**

f. $x^2 +$ **24 or -24** $x + 144$

g. $x^2 + \frac{1}{5}x +$ ____ **$\frac{1}{100}$ (or equivalent)**

h. $x^2 + 7x +$ ____ **12.25 (or equivalent)**

i. $x^2 +$ **4 or -4** $x + 4$

Explore More

10 Use the Explore More sheet to explore the graph of perfect square equations.
***Responses vary*. See sample response on Screen 10.**

11 Synthesis

How can you determine whether an expression is a perfect square?

$(x+4)^2$

x^2+4

$x^2-14x+49$

$x^2+6x+36$

Responses vary.

- **If an expression can be written like $(x + \_\_\_)^2$, it is a perfect square.**
- **$(x^2 - 14x + 49)$ is a perfect square because it can be factored and written as $(x - 7)^2$.**
- **$x^2 + 4$ and $x^2 + 6x + 36$ cannot be written like $(x + \_\_\_)^2$ so they are not perfect squares.**

Things to Remember:

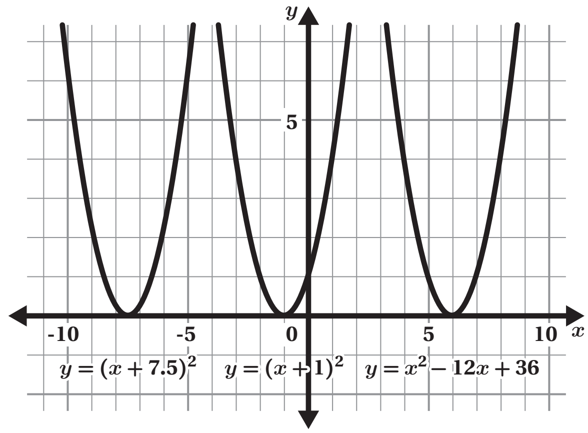
Name: .... Date: .... Period: ....

Explore More

Sothy noticed that when he graphs perfect square equations, each parabola's vertex is on the x-axis. Will this *always* be true?

Explain your thinking.

Use a graphing calculator if it helps with your thinking.



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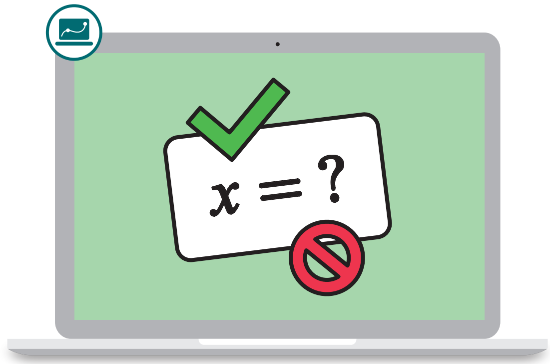


Square Tactic

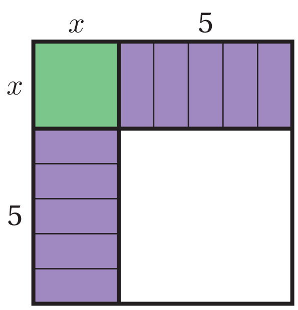
Let's develop a new strategy for solving quadratic equations called "completing the square."

Warm-Up

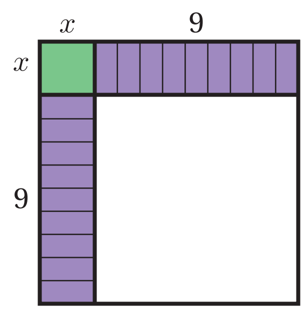
1 Here are seven expressions. The first four are represented with algebra tiles.

For each expression, how many unit tiles do you need to add to make it a perfect square?

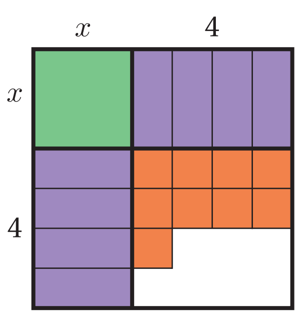
a $x^2 + 10x +$ 25



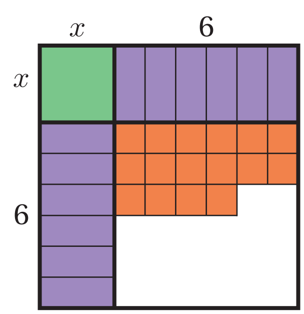
b $x^2 + 18x +$ 81



c $x^2 + 8x + 9 +$ 7



d $x^2 + 12x + 16 +$ 20



e $x^2 + 20x + 70 +$ 30

f $x^2 + 12x + 2 +$ 34

g $x^2 + 16x + 11 +$ 53

Activity
1

Name: .... Date: .... Period: ....

Ancient Equations

2 Here are three equations.

Equation A	Equation B	Equation C
$(x+3)^2 = 25$	$x^2 + 6x + 9 = 25$	$x^2 + 6x = 16$

Discuss:

- What do you notice about each equation's structure?
- Which equations can be solved by taking the square root? Explain how you know.

Responses vary.

- **I notice that Equations A and B are equivalent. I also notice that both equations contain a perfect square on the left-hand side, but only Equation B is written in standard form.**
- **I notice that Equation C is also equivalent to Equations A and B, but 9 has been subtracted from both sides.**

- **After I rewrite Equation B as a perfect square like Equation A, both equations can be solved by taking the square root.**
- **Equation C can also be rewritten as a perfect square, although I would have to add 9 to both sides to keep the solutions to the equation the same.**

3 Solve the equation $x^2 - 8x + 16 = 9$.

- $x = 1, x = 7$
- $x = 4 \pm 3$

4 Let's look at how Deven and Tay each solved the previous equation.

Discuss: What was each student's strategy? When might you use one strategy or the other?

Responses vary.

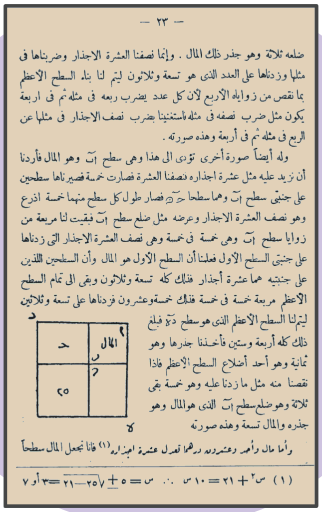
- **Deven set the equation equal to 0, then factored and used the zero-product property to solve.**
- **Tay rewrote the left side as a perfect square, then solved by taking the square root of both sides to isolate x.**
- **I might solve by factoring if the equation is already equal to 0 and is factorable. I could also try to set the equation equal to 0 to see if the equation is factorable.**
- **I might solve by taking the square root if the quadratic can be written as a perfect square.**

Name: .... Date: .... Period: ....

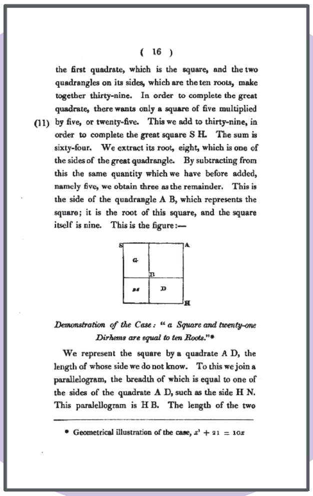
Ancient Equations (continued)

5 The word *algebra* comes from the title of the book *Hisab al-jabr w'al-Muqabala, "The Compendious Book on Calculation by Completion and Balancing."*

Original



English Translation



(16)

the first quadrate, which is the square, and the two quadrangles on its sides, which are the ten roots, make together thirty-nine. In order to complete the great quadrate, there wants only a square of five multiplied
(11) by five, or twenty-five. This we add to thirty-nine, in order to complete the great square S H. The sum is sixty-four. We extract its root, eight, which is one of the sides of the great quadrangle. By subtracting from this the same quantity which we have before added, namely five, we obtain three as the remainder. This is the side of the quadrangle A B, which represents the square; it is the root of this square, and the square itself is nine. This is the figure:—

*Demonstration of the Case: "a Square and twenty-one Dirhems are equal to ten Roots."**

We represent the square by a quadrate A D, the length of whose side we do not know. To this we join a parallelogram, the breadth of which is equal to one of the sides of the quadrate A D, such as the side H N. This paralellogram is H B. The length of the two

* Geometrical illustration of the case, $x^2 + 21 = 10x$

The book was written in 830 CE by Muhammad ibn Mūsā al-Khwarizmi, the mathematician who many scholars believe began the study of algebra.

The focus of the book is solving equations, including: $x^2 + 10x = 39$.

Discuss: What are some different ways you could solve this equation?

Responses vary.

- **I could set the equation equal to 0 and then factor and use the zero-product property to solve.**
- **I could graph $y = x^2 + 10x$ and $y = 39$ and find the x-values of the points of intersection.**
- **I could add 25 to both sides of the equation to keep it balanced and rewrite the left-hand side as a perfect square. Then solve by taking the square root of both sides.**

6 Here is a translated version of the author's first step in solving the equation.

al-Khwarizmi

$$x^2 + 10x = 39$$

$$x^2 + 10x + 25 = 39 + 25$$

How does this help solve the equation?

***Responses vary.* The left side is now a perfect square and can be written as $(x+5)^2$. You can then solve by taking the square root of both sides.**

Activity 2

Name: Date: Period:

Completing the Square

7 al-Khwarizmi's process is called **completing the square**.

Solve $x^2 + 14x = 31$ by completing the square.

$x = -7 \pm \sqrt{80}$ (or equivalent)

8 Solve the equation $x^2 + 6x + 4 = -3$.

$x = -3 \pm \sqrt{2}$ (or equivalent)

Name: ________ Date: ________ Period: ________

Completing the Square (continued)

9 Roberto made a mistake while solving the equation $x^2 - 12x + 6 = 14$.

What did Roberto do well? What should he fix?

Responses vary.

- **Roberto knew to add the same value to both sides to keep the equation balanced. He also knew that 36 was the value that would allow him to rewrite the quadratic equation as a perfect square.**
- **The equation already had a constant value of 6, so he should have added 30 to make the total 36. Roberto could have subtracted 6 from both sides first if he wanted to add 36.**

Roberto

$$x^2 - 12x + 6 = 14$$

$$x^2 - 12x + 36 = 14 + 36$$

$$(x - 6)^2 = 50$$

$$x - 6 = \pm\sqrt{50}$$

$$x = 6 \pm \sqrt{50}$$

Explore More

10 **a** **Discuss:** How many solutions does the equation $x^2 + 10x = -60$ have?

No solutions. ***Explanations vary.***

- **This equation has no solutions because adding 25 to both sides of the equation will result in $(x + 5)^2 = -35$, and you can't take the square root of a negative value.**
- **This equation has no solutions because when graphed, $y = -60$ doesn't intersect with $y = x^2 + 10x$.**

b Can you write another equation of the form $x^2 + 10x =$ ______ that has . . .

Responses vary.

No solutions: $x^2 + 10x =$ **-30**

One solutions: $x^2 + 10x =$ **-25**

Two solutions: $x^2 + 10x =$ **-20**

11 Synthesis

Here are the solving strategies you've seen in this unit: factoring, graphing, and completing the square.

$x^2 + 8x + 2 = 0$

$x^2 + 10x = 39$

$x^2 + 5x + 6 = 2$

Discuss:

- Which strategy would you use for each equation?
- What are the advantages and disadvantages of completing the square?

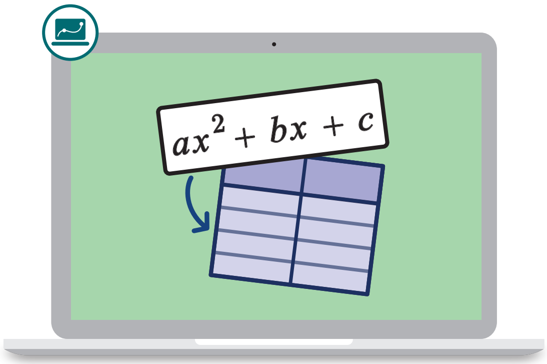
Responses vary.

- Factoring is the easiest strategy for me, so I would choose to factor as often as I can by setting equations equal to 0. That strategy works for the second and third equations, but not the first. For the first equation, I would choose completing the square. If I solve the first equation by graphing the solutions won't be exact.
- One advantage to solving by completing the square is that it works for any quadratic equation and gives exact solutions when we write the answers using the plus or minus symbol. One disadvantage is that it's easy to make mistakes because there are several steps to the process, and when the b-value is odd, it can be hard to think of what value will create a perfect square.

Things to Remember:

Unit 8
Lesson
14

Name: ______ Date: ______ Period: ______



Formula Foundations

Let's explore how the quadratic formula can be derived from the process of completing the square.

Warm-Up

1 Alma wants to solve $5x^2 + 9x + 3 = 0$ by completing the square.

Here is her first step.

Alma

$$5x^2 + 9x + 3 = 0$$

$$x^2 + \frac{9x}{5} + \frac{3}{5} = 0$$

Discuss:

- What did Alma do?
- Why do you think this is her first step?
- What would you do next?

Responses vary.

- **Alma divided the entire equation by 5.**
- **Alma may have been trying to get $a = 1$ to make it easier to complete the square.**
- **Subtract $\frac{3}{5}$ from each side of the equation.**

Activity 1

Name: ______ Date: ______ Period: ______

Completing Any Square

2 Here are the rest of the steps Alma took to solve $5x^2 + 9x + 3 = 0$. Describe what she does in each step. ***Responses vary.***

Steps	Description
$5x^2 + 9x + 3 = 0$	Original equation
$x^2 + \frac{9}{5}x + \frac{3}{5} = 0$	Divide by 5
$x^2 + \frac{9}{5}x = -\frac{3}{5}$	**Subtract $\frac{3}{5}$**
$x^2 + \frac{9}{5}x + \left(\frac{9}{2 \cdot 5}\right)^2 = -\frac{3}{5} + \left(\frac{9}{2 \cdot 5}\right)^2$	**Complete the square**
$\left(x + \frac{9}{10}\right)^2 = -\frac{3}{5} + \left(\frac{9}{10}\right)^2$	**Rewrite the left side as a perfect square**
$x + \frac{9}{10} = \pm\sqrt{-\frac{3}{5} + \left(\frac{9}{10}\right)^2}$	**Take the square root**
$x = -\frac{9}{10} \pm\sqrt{-\frac{3}{5} + \left(\frac{9}{10}\right)^2}$	**Subtract $\frac{9}{10}$**
$x = \frac{-9 \pm \sqrt{9^2 - 4 \cdot 5 \cdot 3}}{2 \cdot 5}$	Rewrite

3 Let's look at how we can use Alma's steps to solve other quadratic equations.

Discuss: What do you notice? What do you wonder?

Responses vary.

- **I notice that when I change the a-values in the equation, the denominator changes.**
- **I notice that the steps remain the same. It's just the a-, b-, and c-values that change.**

- **I wonder if we can use these steps to solve any quadratic equation.**
- **I wonder if there are steps you can skip.**

Name: .... Date: .... Period: ....

Completing Any Square (continued)

4 Felipe notices that you can write the solutions to an equation without completing the square.

How would the solutions change if the original equation were $3x^2 + 8x - 15 = 0$?

Steps	Description
$10x^2 + 7x + 1 = 0$	Original equation
$x^2 + \frac{7}{10}x + \frac{1}{10} = 0$	Divide by 10
$x^2 + \frac{7}{10}x = -\frac{1}{10}$	Subtract $\frac{1}{10}$
$x^2 + \frac{7}{10}x + \left(\frac{7}{2 \cdot 10}\right)^2 = -\frac{1}{10} + \left(\frac{7}{2 \cdot 10}\right)^2$	Complete the square
$\left(x + \frac{7}{20}\right)^2 = -\frac{1}{10} + \left(\frac{7}{20}\right)^2$	Rewrite the left side as a perfect square
$x + \frac{7}{20} = \pm\sqrt{-\frac{1}{10} + \left(\frac{7}{20}\right)^2}$	Take the square root
$x = -\frac{7}{20} \pm \sqrt{-\frac{1}{10} + \left(\frac{7}{20}\right)^2}$	Subtract $\frac{7}{20}$
$x = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 10 \cdot 1}}{2 \cdot 10}$	Rewrite

Responses vary. **All the 10s would become 3s, all the 7s would become 8s, and the 1 would become -15.**

Activity 2

Name: .... Date: .... Period: ....

Formula-izing

5 We just discovered a way to write the solutions for any quadratic equation without completing the square!

Use the variables a, b, and c to represent the solutions to $ax^2 + bx + c = 0$.

Equation:	Equation:
$7x^2 - 9x + 5 = 0$	$\boxed{a}\,x^2 + \boxed{b}\,x + \boxed{c} = 0$
Solutions:	Solutions:
$x = \dfrac{-(-9) \pm \sqrt{(-9)^2 - 4 \cdot 7 \cdot 5}}{2 \cdot 7}$	$x = \dfrac{-\boxed{b} \pm \sqrt{\boxed{b}^2 - 4 \cdot \boxed{a} \cdot \boxed{c}}}{2 \cdot \boxed{a}}$

(or equivalent)

6 For any quadratic equation: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This equation is known as the quadratic formula.

Step	Description
$ax^2 + bx + c = 0$	Original equation
$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	Divide by a
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Subtract $\frac{c}{a}$
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Complete the square
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Rewrite the left side as a perfect square
$\left(x + \frac{b}{2a}\right) = \pm\sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Take the square root
$x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Subtract $\frac{b}{2a}$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Rewrite

Discuss:

- What do the a, b, and c in the formula represent?
- Why is there a $\pm$ symbol in the formula?
- What new things could this formula help you do?

Responses vary.

- a, b, and c represent the coefficients and the constant in the standard-form quadratic equation.
- There is a $\pm$ symbol because when you take the square root, there is a positive and a negative value.
- I can use this formula to solve quadratic equations that may be hard to solve by factoring or completing the square.

Activity 3

Name: .... Date: .... Period: ....

Solution Search

7 You will use the digital activity to explore solutions to quadratic equations.

a Here is an equation that has *two integer solutions*. Find two more equations with two integer solutions.

Responses vary.

Equation	$1x^2 - 5x + 6 = 0$	$2x^2 + 8x + 6 = 0$	$1x^2 + 0x - 9 = 0$
Solutions	$x = \frac{5 \pm \sqrt{1}}{2}$	$x = \frac{-8 \pm \sqrt{16}}{4}$	$x = \frac{0 \pm \sqrt{36}}{2}$

b Here is an equation that has *one solution*. Find two more equations with one solution.

Responses vary.

Equation	$1x^2 + 4x + 4 = 0$	$9x^2 + 0x + 0 = 0$	$1x^2 + 6x + 9 = 0$
Solutions	$x = \frac{-4 \pm \sqrt{0}}{2}$	$x = \frac{0 \pm \sqrt{0}}{18}$	$x = \frac{-6 \pm \sqrt{0}}{2}$

c Here is an equation that has *no solution*. Find two more equations with no solution.

Responses vary.

Equation	$7x^2 + 2x + 5 = 0$	$-2x^2 + 5x - 4 = 0$	$1x^2 + 0x + 9 = 0$
Solutions	$x = \frac{-2 \pm \sqrt{-136}}{14}$	$x = \frac{-5 \pm \sqrt{-7}}{-4}$	$x = \frac{0 \pm \sqrt{-36}}{2}$

d Examine the equations and solutions you found.

Discuss: What patterns do you notice?

Responses vary.

- I notice that the number under the square root tells me how many solutions the equation has.
- I notice that when there is a square root of a negative number, there are no solutions.

8 Synthesis

A classmate who is absent today asks for your help.

What would you say to help them understand where the quadratic formula came from?

***Responses vary.* Complete the square, but instead of using numbers, use the variables a, b, and c and solve for x.**

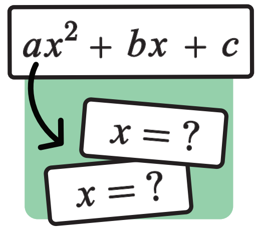
Step	Description
$ax^2 + bx + c = 0$	Original equation
$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	Divide by a
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Subtract $\frac{c}{a}$
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Complete the square
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Rewrite the left side as a perfect square
$\left(x + \frac{b}{2a}\right) = \pm\sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Take the square root
$x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Subtract $\frac{b}{2a}$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Rewrite

Things to Remember:

Name: .... Date: .... Period: ....

Formula Fluency

Let's use the quadratic formula to solve quadratic equations.



Warm-Up

1. The quadratic formula can be used to find the solutions to any quadratic equation in the form $ax^2 + bx + c = 0$.

Determine the a-, b-, and c-values of the following equations.

a $3x^2 - 8x + 15 = 0$

$a =$ 3 $b =$ -8 $c =$ 15

b $x^2 + 4 + 3x = 0$

$a =$ 1 $b =$ 3 $c =$ 4

c $5x^2 - 20 = 0$

$a =$ 5 $b =$ 0 $c =$ -20

d $-x^2 + 2x = -12$

$a =$ -1 $b =$ 2 $c =$ 12

Activity 1

Name: .... Date: .... Period: ....

Form Over Function

2. Here are four quadratic equations and their solutions.

Use the quadratic formula to show that the solutions are correct. ***Work varies.***

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a $x^2 - 8x + 15 = 0$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 - 60}}{2}$$

$$x = \frac{8 \pm \sqrt{4}}{2}$$

$$x = \frac{8 \pm 2}{2}$$

$x = \frac{10}{2}$ and $x = \frac{6}{2}$

Solutions: $x = 5$ and $x = 3$

b $x^2 + 10x + 18 = 0$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(18)}}{2(1)}$$

$$x = \frac{-10 \pm \sqrt{100 - 72}}{2}$$

$$x = \frac{-10 \pm \sqrt{28}}{2}$$

Solutions: $x = -5 \pm \frac{\sqrt{28}}{2}$

c $9x^2 - 6x = -1$

$9x^2 - 6x + 1 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)}$$

$$x = \frac{6 \pm \sqrt{36 - 36}}{18}$$

$$x = \frac{6 \pm \sqrt{0}}{18}$$

$$x = \frac{6 \pm 0}{18}$$

Solution: $x = \frac{1}{3}$

d $2x^2 + 6x + 5 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{-6 \pm \sqrt{36 - 40}}{4}$$

$$x = \frac{-6 \pm \sqrt{-4}}{4}$$

No solutions

3. **Discuss**: Do you think that the quadratic formula is the best strategy for solving each of these equations? Explain your thinking.

Responses vary. **The quadratic formula is the best strategy for solving $9x^2 - 6x = -1$ and $2x^2 + 6x + 5 = 0$ because I didn't know how to factor either equation. I would rather solve $x^2 - 8x + 15 = 0$ by factoring because I can rewrite it as $(x - 5)(x - 3) = 0$.**

Activity
2

Name: .... Date: .... Period: ....

Error Analysis

You will use a sheet for this activity with the same equations from the previous activity.

There is an error in each attempt to solve the equation.

4. **Discuss:**

- What is the error in each attempt?
- How would you correct the error?
- Why might someone make this error?

Responses vary.

a. $(-8)^2$ is equal to 64, not -64.

b. $\frac{-10 \pm \sqrt{28}}{2}$ is equivalent to $\frac{-10}{2} \pm \frac{\sqrt{28}}{2}$.
$\frac{-10}{2}$ can be reduced to -5 but $\frac{\sqrt{28}}{2}$ cannot be reduced to $\sqrt{14}$.

c. The c-value should be 1 because the equation is $9x^2 - 6x + 1 = 0$ when written in standard form.

d. There are no numbers that would give you $\sqrt{-4}$ when squared, so $2x^2 + 6x + 5 = 0$ has no solutions.

5. Solve the following equation using the quadratic formula, but include an error that you think would be common. *Errors vary.*

$3x^2 - 6x - 1 = 0$

$3x^2 - 6x - 1 = 0$

$a = 3,\ b = -6,\ c = -1$

$x = \frac{-6 \pm \sqrt{(-6)^2 - 4(3)(-1)}}{2(3)}$

$x = \frac{-6 \pm \sqrt{36 + 12}}{6}$

$x = \frac{-6 \pm \sqrt{48}}{6}$

$x = -1 \pm \frac{\sqrt{48}}{6}$

6. Swap equations with a classmate. Identify and describe the error in each other's work.

Responses vary. $b = -6$, so when you substitute b into the quadratic formula, $-b$ is $-(-6)$, which is equal to 6.

Name: .... Date: .... Period: ....

Error Analysis (continued)

7. **Reflect:** What kinds of errors do you think you are most likely to make when using the quadratic formula?

Responses vary.

- **I think it is easy to get my positive and negative values confused.**
- **I could forget to make my equation equal to 0 before using the quadratic formula.**

8. Write two pieces of advice that will help your future self correctly use the quadratic formula. Include examples if they help with your thinking.

Responses vary.

- **When you substitute, use parentheses to keep track of what numbers should be negative and positive.**
- **Check your work by using a calculator to plug your solutions back into your equation.**

Synthesis

9. What are some advantages of using the quadratic formula to solve quadratic equations?

What are some disadvantages?

Use the examples if they help with your thinking.

$$x^2 - 6x + 8 = 0$$

$$x^2 + 4x - 1 = 0$$

$$2x^2 + 7x - 10 = 0$$

Responses vary.

- **The quadratic formula can be used to solve any quadratic equation, so you don't need to think about which strategy to use.**
- **You could make a mistake while calculating your answers, so it might be a good idea to always check your work.**
- **Some equations might be easier to solve by factoring.**

Things to Remember:

Error Analysis

a $x^2 - 8x + 15 = 0$

$a = 1$, $b = $ -8, $c = 15$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{-64 - 60}}{2}$$

$$x = \frac{8 \pm \sqrt{-124}}{2}$$

No solutions

b $x^2 + 10x + 18 = 0$

$a = 1$, $b = 10$, $c = 18$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(18)}}{2(1)}$$

$$x = \frac{-10 \pm \sqrt{100 - 72}}{2}$$

$$x = \frac{-10 \pm \sqrt{28}}{2}$$

$$x = -5 \pm \sqrt{14}$$

c $9x^2 - 6x = -1$

$a = 9$, $b = -6$, $c = -1$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(-1)}}{2(9)}$$

$$x = \frac{6 \pm \sqrt{36 + 36}}{18}$$

$$x = \frac{6 \pm \sqrt{72}}{18}$$

d $2x^2 + 6x + 5 = 0$

$a = 2$, $b = 6$, $c = 5$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{-6 \pm \sqrt{36 - 40}}{4}$$

$$x = \frac{-6 \pm \sqrt{-4}}{4}$$

$$x = \frac{-6 \pm 2}{4}$$

$x = -2$ and $x = -1$

Unit 8
Lesson
16

Name: .... Date: .... Period: ....

Stomp Rockets in Space

Let's solve quadratic equations and explain what the solutions mean for a situation.

Warm-Up

1 Here is a stomp rocket on another planet.

The function $h(t) = -3t^2 + 20t + 4$ represents the height, in meters, of the stomp rocket t seconds after it has been launched.

a Let's look at a stomp rocket launch.

b Write a question about the stomp rocket that $h(t)$ could help you answer.

Responses vary.

- What is the starting height of the rocket?
- When does the rocket hit the ground?
- When does the rocket reach its maximum height?
- Will the rocket ever reach a height of 15 meters?
- How high will the rocket be at 3 seconds?

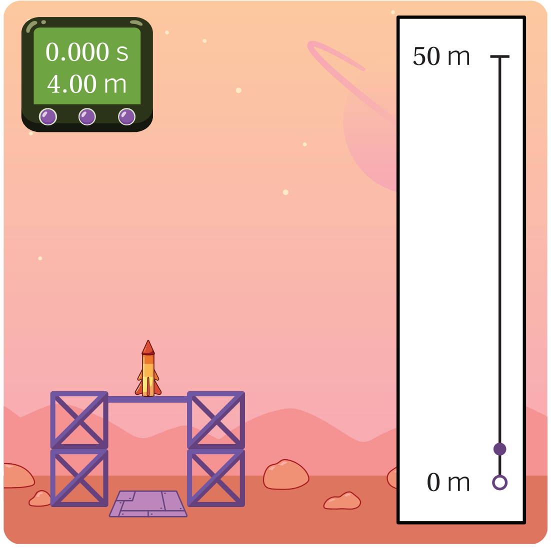
Name: .... Date: .... Period: ....

Rocket Time

2 The function $h(t) = -3t^2 + 20t + 4$ represents the height, in meters, of the stomp rocket t seconds after it has been launched.

When will the rocket touch the ground? Round to three decimal places if necessary.

$\frac{-20-\sqrt{448}}{-6}$ or 6.861 seconds



3 Let's look at Makayla's work from the previous problem.

Makayla says that the rocket will touch the ground at about -0.194 seconds and 6.861 seconds.

Discuss:

- Why did Makayla substitute 0 for $h(t)$?
- What is correct about Makayla's response?
- What is incorrect? Why?

Responses vary.

- In this function, $h(t)$ represents height. To determine when the rocket will touch the ground, Makayla set $h(t)$ equal to 0.
- Makayla correctly used the quadratic formula to solve $0 = -3t^2 + 20t + 4$.
- Makayla incorrectly interpreted -0.194 as a time when the rocket hits the ground. In this situation, it doesn't make sense for time to be negative.

Name: .... Date: .... Period: ....

Beam Me Up

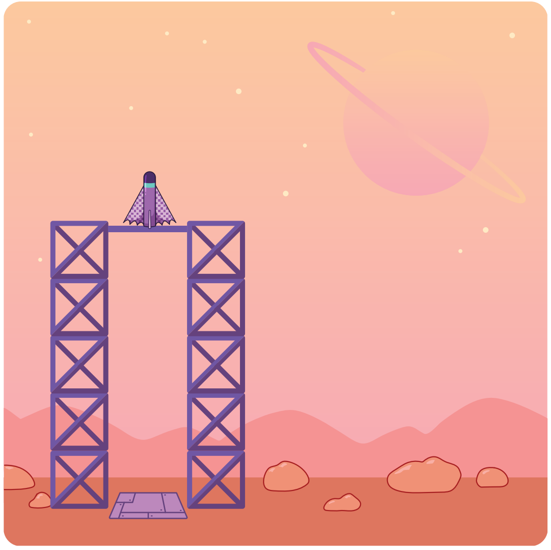
4 Here is a new rocket.

The function $h(t) = \text{-}8t^2 + 40t + 10$ represents the height, in meters, of the rocket t seconds after it has been launched.

The rocket reaches a maximum height of 60 meters.

Write an equation that can be solved to determine when the rocket is at its maximum height.

$60 = \text{-}8t^2 + 40t + 10$ (or equivalent)



Explain your thinking.

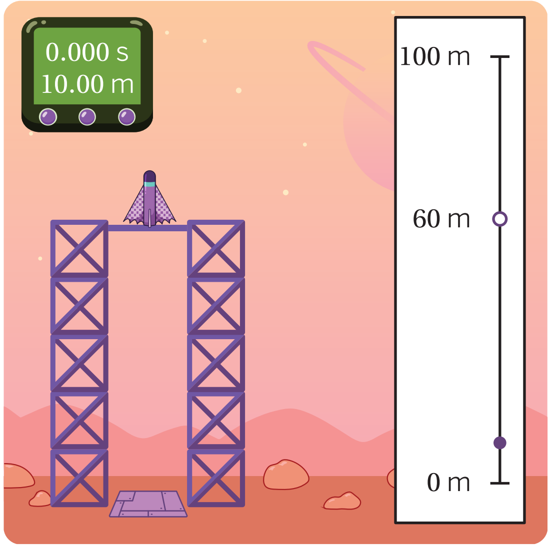
***Explanations vary.* $h(t)$ represents the height of the rocket, so I set it equal to 60.**

5 Here is an equation someone wrote on the previous screen: $60 = \text{-}8t^2 + 40t + 10$.

How many seconds will it take for the rocket to reach its maximum height?

Round to three decimal places if necessary.

2.5 seconds



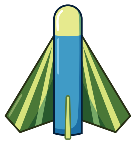
Name: ______ Date: ______ Period: ______

Beam Me Up (continued)

6 Here is a new rocket.

$h(t) = -4t^2 + 30t + 10$

The function $h(t) = -4t^2 + 30t + 10$ represents the height, in meters, of the rocket t seconds after it has been launched.



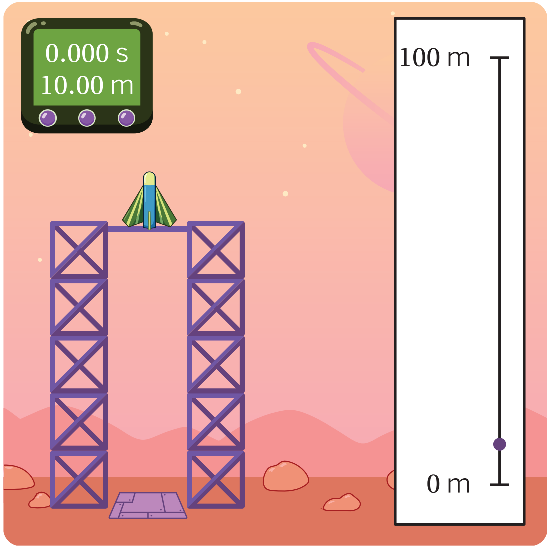
Does this rocket ever reach a height of 100 meters? Circle one.

Yes   No

Show or explain your thinking.

***Explanations vary.* Since 100 represents the height, I substituted 100 for $h(t)$. Then I set the equation equal to 0 to use the quadratic formula. Using the quadratic formula, I got a square root of a negative value. So there are no solutions, which means there is no time when the rocket is at 100 meters.**

7 The function $h(t) = -4t^2 + 30t + 10$ represents the height, in meters, of the stomp rocket t seconds after it has been launched.



a Let's watch the stomp rocket launch.

b Write a question about the height of the rocket that will have two answers.

Responses vary.
- **When will the rocket be at a height of 20 meters?**
- **At what times will the rocket be 45 meters above the ground?**

Activity 3

Name: ______ Date: ______ Period: ______

Rocket Scientist

8 You will use the Activity 3 Sheet to choose your own stomp rocket.

a Choose It!

- On the activity sheet, choose a stomp rocket.
- In this table, write down the function, the question you selected, and the solution to the question.

Round your solution to three decimal places if necessary.

Function:	$h(t) =$
Question:	
Solution:	

Responses vary.

b Swap It!

- Share your stomp rocket with a partner who has a different rocket.
- Solve the question they chose for their stomp rocket.

Round your solution to three decimals places if necessary.

Responses vary.

Partner 1's Rocket

Function:	$h(t) =$
Question:	
Solution:	

Partner 2's Rocket

Function:	$h(t) =$
Question:	
Solution:	

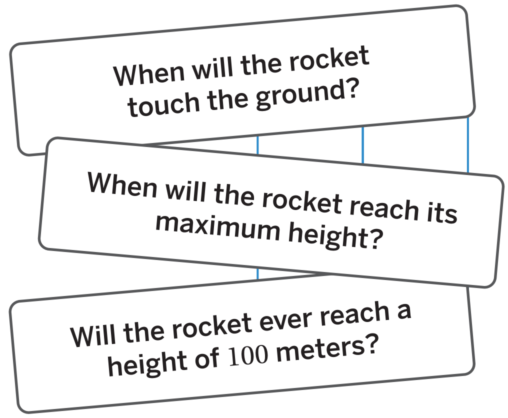
9 Synthesis

Describe a strategy that helped you answer the question in today's lesson.

If you learned it from another student, give them a shout-out!

Responses vary.

- **Start by setting the equation equal to a height.**
- **Try to solve your equation by factoring, since it's a simpler strategy. If you can't factor the equation, use the quadratic formula instead because it will always work.**
- **When interpreting the solutions of a quadratic function, think about whether the solutions make sense in the situation.**



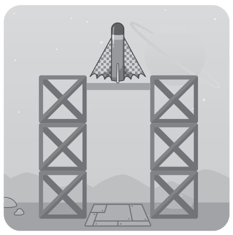
Things to Remember:

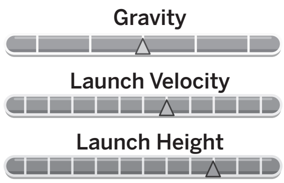
Name: ______________________ Date: __________ Period: __________

Rocket Scientist

- Choose one stomp rocket.
- Select one of the questions about your stomp rocket.
- Solve the question you chose on the lesson page.

Rocket 1

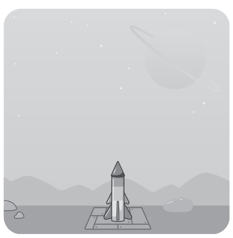


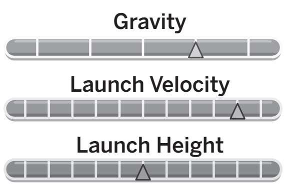


$h(t) = -2t^2 + 6t + 6$

- When will the rocket touch the ground?
- When will the rocket reach its maximum height of 10.5 meters?
- When will the rocket return to its original height of 6 meters?

Rocket 2

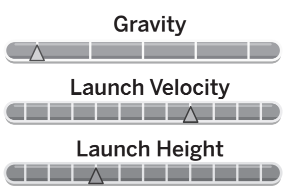




$h(t) = -2.5t^2 + 24t$

- When will the rocket land on the ground?
- When will the rocket reach its maximum height of 57.6 meters?

Rocket 3

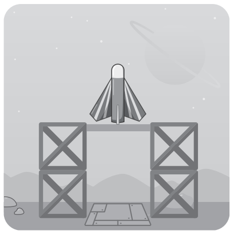
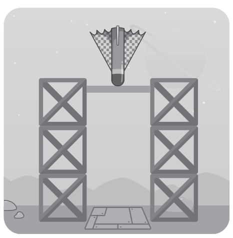
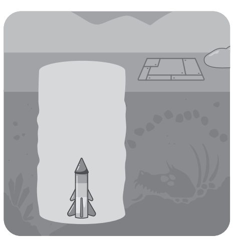
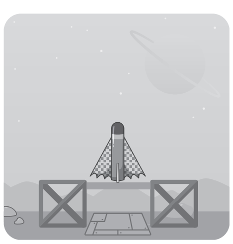


$h(t) = -t^2 + 12t - 4$

- When will the rocket touch the ground?
- When will the rocket reach its maximum height of 32 meters?

Name: ______________________ Date: __________ Period: __________

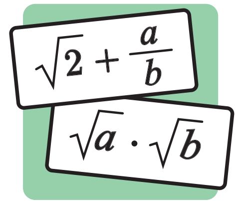
Rocket Scientist

Rocket 4		
	Gravity Launch Velocity Launch Height $h(t) = -3t^2 + 12t + 4$	• When will the rocket touch the ground? • When will the rocket reach its maximum height of 16 meters? • When will the rocket return to its original height of 4 meters?
Rocket 5		
	Gravity Launch Velocity Launch Height $h(t) = -t^2 - 6t + 8$	• When will the rocket touch the ground?
Rocket 6		
	Gravity Launch Velocity Launch Height $h(t) = -t^2 + 18t - 8$	• When will the rocket touch the ground? • When will the rocket reach its maximum height of 73 meters?
Rocket 7		
	Gravity Launch Velocity Launch Height $h(t) = -1.5t^2 + 6t + 2$	• When will the rocket touch the ground? • When will the rocket reach its maximum height of 8 meters? • When will the rocket return to its original height of 2 meters?

Unit 8
Lesson
17

Name: ________ Date: ________ Period: ________

Sums and Products



Let's make arguments about why the sums and products of rational and irrational numbers are always, sometimes, or never rational.

Warm-Up

1. Here are four claims.

Discuss: Is each claim *always*, *sometimes*, or *never* true?

a The sum of two even numbers is an even number. **Always true.**

b The product of two negative numbers is negative. **Never true.**

c The sum of an odd number and an even number is even. **Never true.**

d The product of two numbers is larger than either number. **Sometimes true.**

2. For the last claim, how would you convince someone that it is always, sometimes, or never true?

Responses vary. You could brainstorm different pairs of numbers to multiply in order to test whether or not the product is larger or smaller than the original numbers. For example, the product of 3 and 4 is 12, which is larger than the original numbers. But the product of $\frac{1}{2}$ and $\frac{1}{2}$ is $\frac{1}{4}$, which is smaller than the original numbers.

Activity 1

Name: ______ Date: ______ Period: ______

Rational and Irrational Numbers

Here are some examples of *rational numbers* and *irrational numbers*.

Rational Numbers			Irrational Numbers		
2	-2.3	$\frac{2}{3}$	2	π (or $3.14159\ldots$)	$\sqrt{-5}$
$\sqrt{9}$		$\sqrt{\frac{9}{4}}$	$\sqrt{\frac{2}{3}}$		$\sqrt{\frac{3}{4}}$

3. **Discuss:** What do you notice about the numbers in the table? What do you wonder?

Responses vary.

- **I notice that some square roots are rational and some are irrational.**
- **I notice that $\sqrt{9} = 3$ and that is a rational number.**
- **I wonder if all fractions are rational.**
- **I wonder how irrational and rational numbers are related.**

4. Use the examples in the table to help you define rational and irrational numbers.

Responses vary.

Rational number:

Rational numbers can be written as a fraction of two integers.

Irrational number:

Irrational numbers cannot be written as a fraction of two integers.

5. Determine whether each number is *rational* or *irrational*. Circle one.

a. $-\sqrt{4}$ Rational Irrational

b. $-\sqrt{5}$ Rational Irrational

c. 0 Rational Irrational

d. $0.\overline{3}$ Rational Irrational

e. $\frac{7}{3}$ Rational Irrational

f. $\sqrt{\frac{7}{3}}$ Rational Irrational

Activity
2

Name: .... Date: .... Period: ....

Sums and Products of Rational Numbers

Let's explore the *sums* of two rational numbers.

6. With a partner, determine whether each sum is *rational* or *irrational*.

a $2 + (-2.3)$ Rational Irrational

b $2 + \frac{2}{3}$ Rational Irrational

c $\frac{2}{3} + 0.\overline{1}$ Rational Irrational

d $\sqrt{9} + (-2.3)$ Rational Irrational

e $\sqrt{9} + \sqrt{4}$ Rational Irrational

f $2 + (-2.3)$ Rational Irrational

7. Tyani says: *I know that the value of* $\frac{11}{43} + \frac{273}{101}$ *is rational without even calculating it.*

Discuss: Do you agree or disagree? Why?

***Responses vary.* I agree because 11, 43, 273, and 101 are all integers, so when I create a common denominator and then add the numerators, I know that the numerator and denominator will be integers even if I don't know what their values will be.**

8. Fill in the blank with *always*, *sometimes*, or *never*. Then explain your thinking.

The sum of two rational numbers is **always** rational.

***Explanations vary.* Rational numbers can be written as fractions of integers. When you add two fractions, you create a common denominator and add the numerators. This will create a new fraction, so the sum is rational.**

Activity 2

Name: Date: Period:

Sums and Products of Rational Numbers (continued)

Let's explore the *products* of two rational numbers.

9. With a partner, determine whether each product is *rational* or *irrational*.

a	$0 \cdot -2.3$	Rational	Irrational
b	$\frac{2}{3} \cdot 2$	Rational	Irrational
c	$\frac{3}{4} \cdot 0.\overline{1}$	Rational	Irrational
d	$\sqrt{9} \cdot (-1)$	Rational	Irrational
e	$\sqrt{9} \cdot \sqrt{4}$	Rational	Irrational
f	$0 \cdot -2.3$	Rational	Irrational

10. Tyani says: *I know that* $\frac{11}{43} \cdot \frac{273}{101}$ *is rational without even calculating it.*

Discuss: Do you agree or disagree? Why?

Responses vary. **I agree because when I multiply fractions, I multiply the numerators and the denominators. I know that $11 \cdot 273$ and $43 \cdot 101$ are integers, so $\frac{11}{43} \cdot \frac{273}{101}$ must be rational.**

11. Fill in the blank with *always*, *sometimes*, or *never*. Then explain your thinking.

The product of two rational numbers is **always** rational.

Explanations vary. **Rational numbers can be written as fractions of integers. When you multiply two fractions, you multiply the numerators and the denominators. This will create a new fraction, so the product is rational.**

Activity **3**

Name: ________ Date: ________ Period: ________

Sums and Products of Rational and Irrational Numbers

Let's explore the *sums* of rational and irrational numbers.

12. Determine whether $\sqrt{2} + \frac{a}{b}$ is rational or irrational when:

$a = 3$ and $b = 1$

Irrational

$a = -5$ and $b = 2$

Irrational

13. Choose any values for a and b so that $\frac{a}{b}$ is rational. **Responses vary.**

$a =$ **4**

$b =$ **7**

14. For the values you chose, determine whether $\sqrt{2} + \frac{a}{b}$ is rational or irrational.

Irrational

15. Are there any integers a and b that make $\sqrt{2} + \frac{a}{b}$ a rational number? Explain your thinking.

No. *Explanations vary.* If $\sqrt{2} + \frac{a}{b}$ were rational, then $\left(\sqrt{2} + \frac{a}{b}\right) - \frac{a}{b}$ should be rational, based on Activity 2. $\left(\sqrt{2} + \frac{a}{b}\right) - \frac{a}{b} = \sqrt{2}$ and I know that $\sqrt{2}$ is irrational, so $\sqrt{2} + \frac{a}{b}$ is irrational.

16. Fill in the blank with *always*, *sometimes*, or *never*.

The sum of a rational number and an irrational number is **never** rational.

Let's explore the *products* of rational and irrational numbers.

17. Determine whether $\sqrt{5} \cdot \frac{c}{d}$ is rational or irrational when $c = 5$ and $b = 9$.

$\sqrt{5} \cdot \frac{5}{9}$ is irrational.

18. Are there any non-zero integers c and d that make $\sqrt{5} \cdot \frac{c}{d}$ a rational number? Explain your thinking.

No. *Explanations vary.* If $\sqrt{5} \cdot \frac{c}{d}$ were rational, then $\left(\sqrt{5} \cdot \frac{c}{d}\right) \cdot \frac{d}{c}$ should be rational, based on Activity 2. $\left(\sqrt{5} \cdot \frac{c}{d}\right) \cdot \frac{d}{c} = \sqrt{5}$ and I know that $\sqrt{5}$ is irrational, so $\sqrt{5} \cdot \frac{c}{d}$ is irrational.

19. Fill in the blank with *always*, *sometimes*, or *never*.

The product of a non-zero rational number and an irrational number is **never** rational.

Synthesis

20. Choose one of the example expressions.

A. $3 \cdot \pi$ B. $\frac{1}{2} + \pi$ C. $\sqrt{16} + \sqrt{25}$ D. $\frac{3}{4} \cdot \sqrt{36}$

Determine whether the sum or product is rational or irrational.
Explain how you know.

A. **Irrational.** ***Explanations vary.*** **$3 \cdot \pi$ is irrational because π is an irrational number, and the product of a rational number and an irrational number is irrational.**

B. **Irrational.** ***Explanations vary.*** **$\frac{1}{2} + \pi$ is irrational because π is an irrational number, and the sum of an irrational number and a rational number is irrational.**

C. **Rational.** ***Explanations vary.*** **$\sqrt{16} + \sqrt{25}$ is rational because $\sqrt{16} = 4$ and $\sqrt{25} = 5$. $4 + 5 = 9$, and 9 is a rational number.**

D. **Rational.** ***Explanations vary.*** **$\frac{3}{4} \cdot \sqrt{36}$ is rational because $\sqrt{36} = 6$, and the product of $\frac{3}{4}$ and 6 is a rational number.**

Things to Remember: