

# Why Intercepts?

Let's write an equation for a line that passes through two given points.

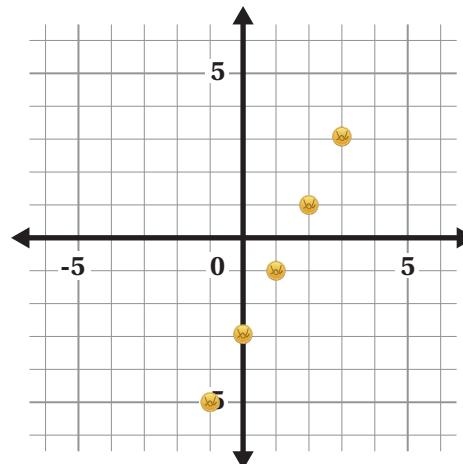


## Warm-Up

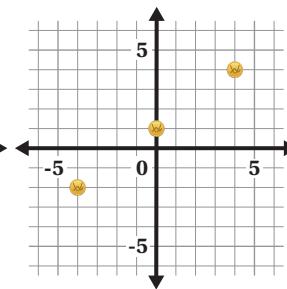
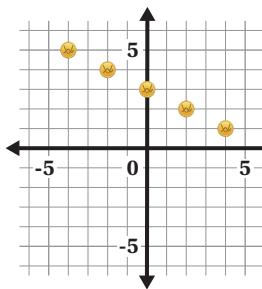
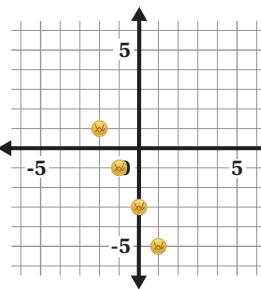
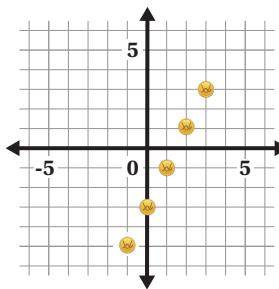
- 1** Denali wants to “capture” these coins with just one line.

**a** What slope could Denali use?

**b** What  $y$ -intercept could Denali use?



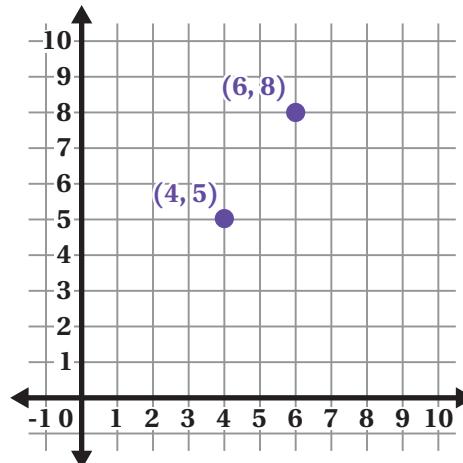
- 2** Write a single linear equation to capture all the coins for each challenge.



## Determining the $y$ -intercept

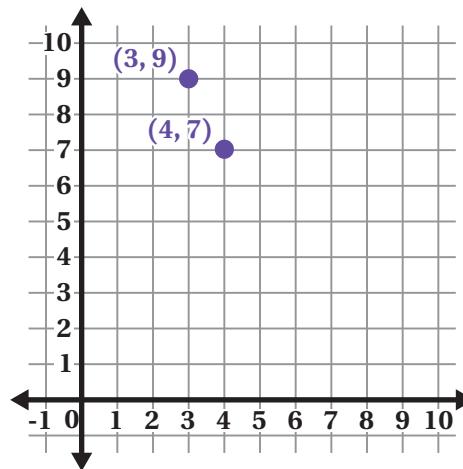
- 3** The points  $(4, 5)$  and  $(6, 8)$  represent the location of two coins.

- a** Write an equation of the line that goes through both points.
- b** What was your strategy for determining the  $y$ -intercept?



- 4** Here are two new points:  $(3, 9)$  and  $(4, 7)$ .

Describe how you could determine the  $y$ -intercept of the line going through these two points.



## Determining the $y$ -intercept (continued)

- 5** Here are two students' strategies for determining the  $y$ -intercept in the previous problem.

Tariq

$$\begin{array}{c|c} x & y \\ \hline 3 & 9 \\ 4 & ? \\ \hline & -2 \end{array}$$

slope:  
 $\frac{-2}{1} = -2$

$$y = -2x + b$$

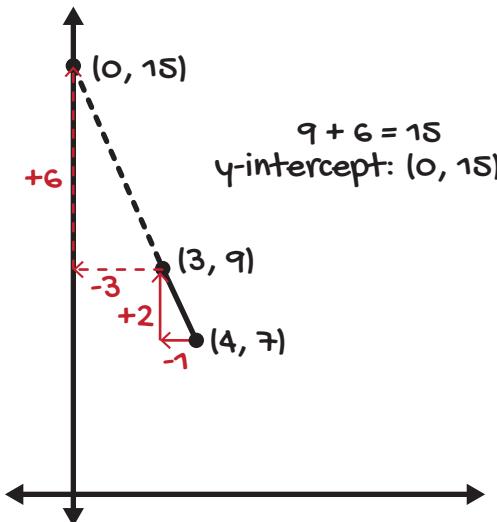
I'll substitute  $(3, 9)$  in for  $x$  and  $y$ !

$$9 = -2(3) + b$$

$$9 = -6 + b$$

$$15 = b$$

Nia



- a** Choose a student and explain their strategy to a classmate.

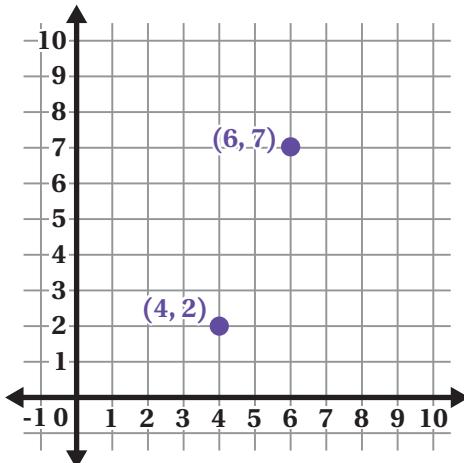


**Discuss:** How are the students' strategies alike? How are they different?

## Critiquing, Correcting, and Clarifying

- 6** Here are two new points: (4, 2) and (6, 7).

Write an equation of the line that goes through both points.



- 7** Victor made a mistake while writing the equation for the line that goes through (4, 2) and (6, 7).

- a** **Discuss:** What did Victor do well?

**Victor**

$$\begin{array}{c|c} x & y \\ \hline 4 & 2 \\ +2 & \\ \hline 6 & 7 \end{array} \quad \text{slope: } \frac{5}{2}$$

$$y = \frac{5}{2}x + b$$

$$4 = \frac{5}{2}(2) + b$$

$$4 = 5 + b$$

$$-1 = b$$

$$y = \frac{5}{2}x - 1$$

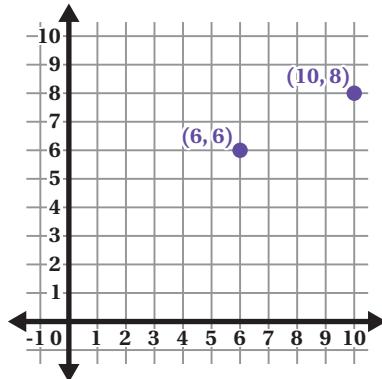
- b** Explain why Victor's work is incorrect.

## Repeated Challenges

- 8** For each problem, write an equation of the line that goes through both points.

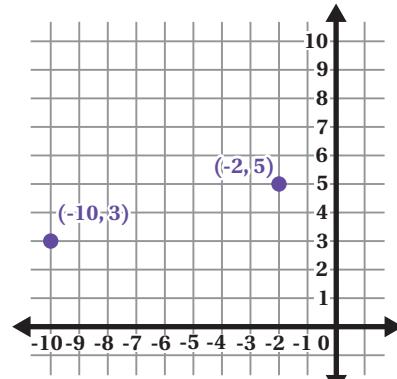
1. Points: (6, 6) and (10, 8).

Equation: \_\_\_\_\_



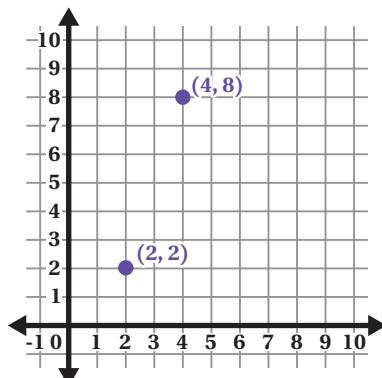
2. Points: (-10, 3) and (-2, 5).

Equation: \_\_\_\_\_



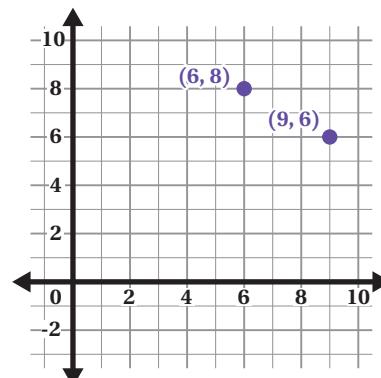
3. Points: (2, 2) and (4, 8).

Equation: \_\_\_\_\_



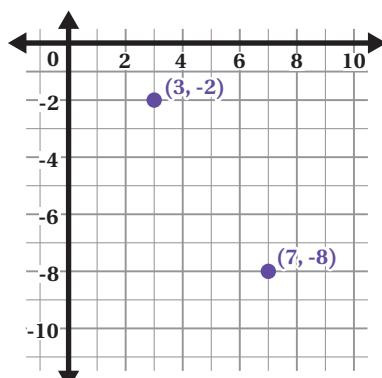
4. Points: (6, 8) and (9, 6).

Equation: \_\_\_\_\_



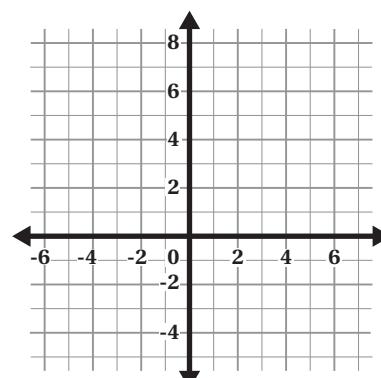
5. Points: (3, -2) and (7, -8).

Equation: \_\_\_\_\_



6. Points: (-1, 5) and (2, 2).

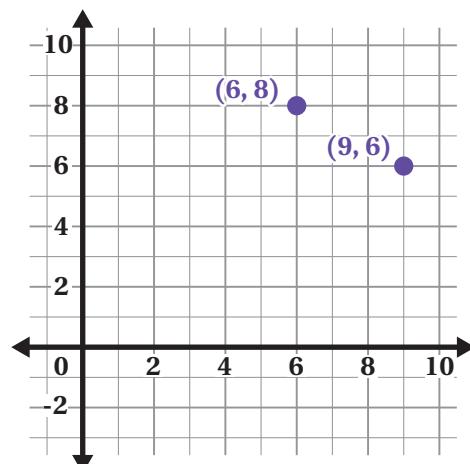
Equation: \_\_\_\_\_



## **9** Synthesis

Describe how to write an equation of a line that goes through two points.

Use the example if it helps with your thinking.

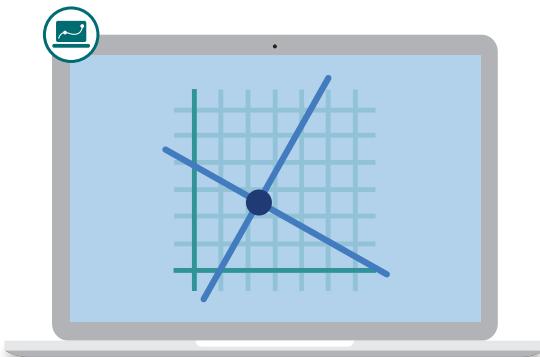


Things to Remember:

Name: ..... Date: ..... Period: .....

# Solutions

Let's think about solutions to two-variable linear equations.



## Warm-Up

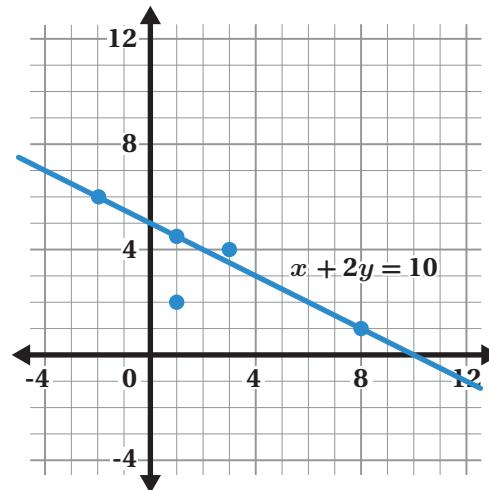
- 1 Write two pairs of values for  $x$  and  $y$  that make the equation  $x + 2y = 10$  true.

$x$	$y$
.....	.....
.....	.....

## Solutions to Linear Equations

- 2** This graph shows the line  $x + 2y = 10$ , as well as some points that are solutions to the equation and some that are not.

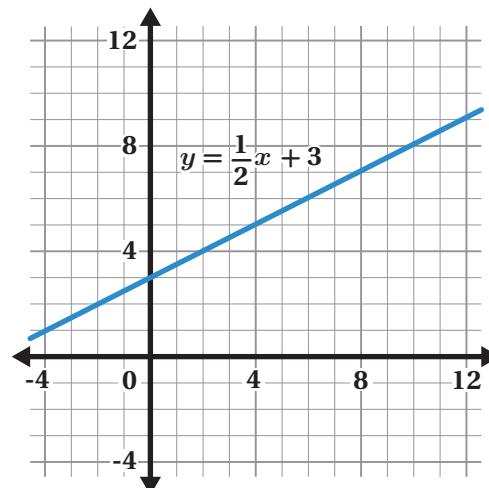
Show or explain how you can tell from the graph if a point is *not* a solution to the equation.



- 3** This graph shows the line  $y = \frac{1}{2}x + 3$ .

Complete the table so each point is a solution to the equation.

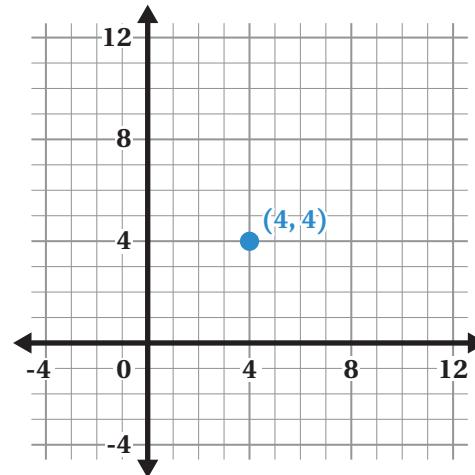
$x$	$y$
4	
	20



- 4** Describe your strategy for finding the solutions you wrote in the table.

## More Solutions

- 5** Write an equation for a line that has a solution of  $(4, 4)$ .



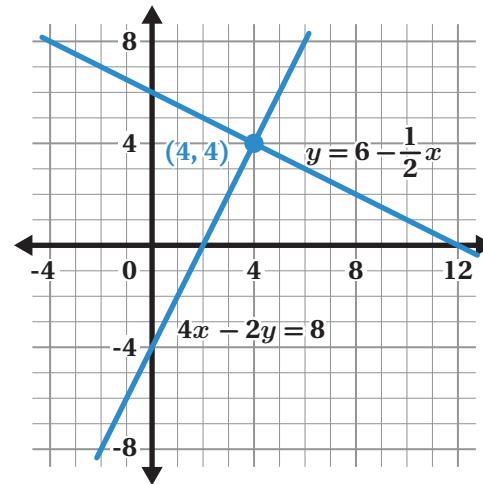
- 6** Here are the equations Hamza and Neo wrote for the previous problem:

Hamza:  $4x - 2y = 8$

Neo:  $y = 6 - \frac{1}{2}x$

Select *all* the statements that are true.

- A.  $(4, 4)$  is a solution for both lines.
- B.  $(7, 2.5)$  is a solution to  $y = 6 - \frac{1}{2}x$ .
- C. The lines have more than one solution in common.
- D. The line  $4x - 2y = 8$  has more than one solution.



## Challenge Creator

**7** Create your own solution challenge!

**a** **Make It!** Plot a point on the Activity 3 Sheet. Then write the equation of a line that the point is a solution for.

**b** **Solve It!** Then write another solution to your equation here.

Another solution: \_\_\_\_\_

**c** **Swap It!** Swap your challenge with one or more partners.

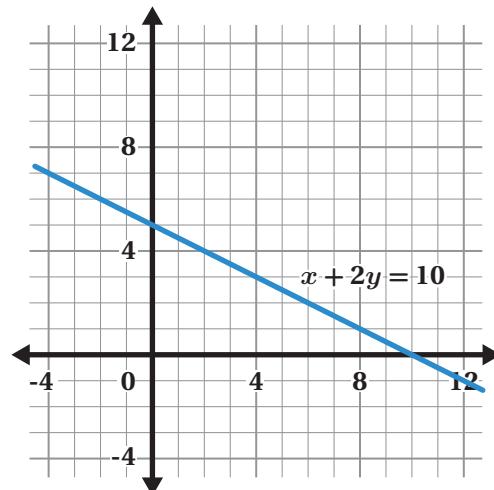
- Verify that the point plotted on your partner's graph is a solution to their equation. If it's not, allow your partner to revise their equation.
- Determine another solution to their equation. Use the graph if it helps with your thinking.

	Equation	Solutions	Graph										
Partner 1		<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><i>x</i></td> <td><i>y</i></td> </tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </table>	<i>x</i>	<i>y</i>									
<i>x</i>	<i>y</i>												
Partner 2		<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><i>x</i></td> <td><i>y</i></td> </tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </table>	<i>x</i>	<i>y</i>									
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Partner 3		<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><i>x</i></td> <td><i>y</i></td> </tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </table>	<i>x</i>	<i>y</i>									
<i>x</i>	<i>y</i>												

## 8 Synthesis

How can you determine whether a point is a solution to an equation?

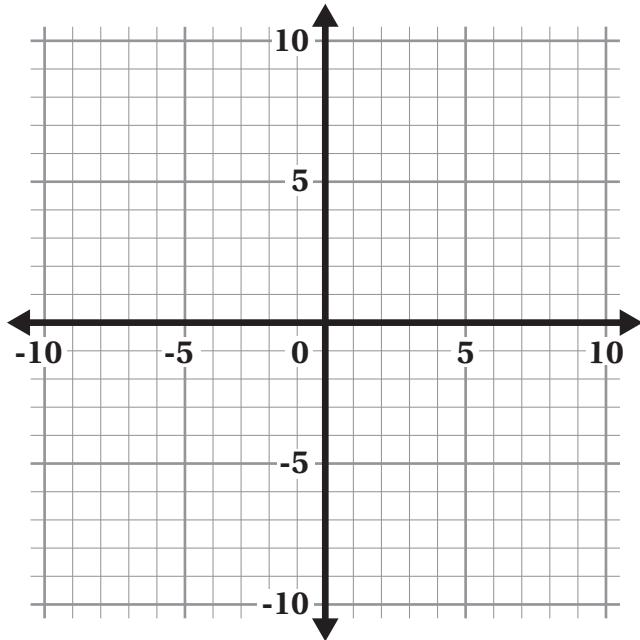
Use the example if it helps with your thinking.



Things to Remember:

# Challenge Creator

- Plot a point on any location on the graph. Then write its coordinates in the table.
- Write the equation of a line that your point is a solution for.

**My Graph****My Solution and Equation**

$x$	$y$
.....	.....

**Equation:** .....

Name: ..... Date: ..... Period: .....

# Pennies and Quarters

Let's determine solutions to real-world linear relationships.



## Warm-Up

- 1** Let's watch a video.

What are some questions you could ask about this situation?

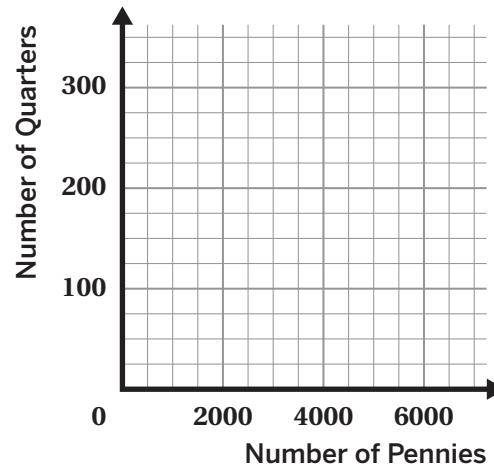


## Pennies and Quarters

- 2** The piggy bank in the video was filled with pennies and quarters worth a total of \$62.00.

Write *three* possible combinations of pennies and quarters that are worth \$62.00.

Number of Pennies	Number of Quarters
.....	.....
.....	.....
.....	.....



Then graph the combinations of pennies and quarters you wrote.

- 3** Describe how you can tell if a combination of pennies and quarters is worth \$62.00.

- 4** Let's look at the graph of some students' combinations from Problem 2.

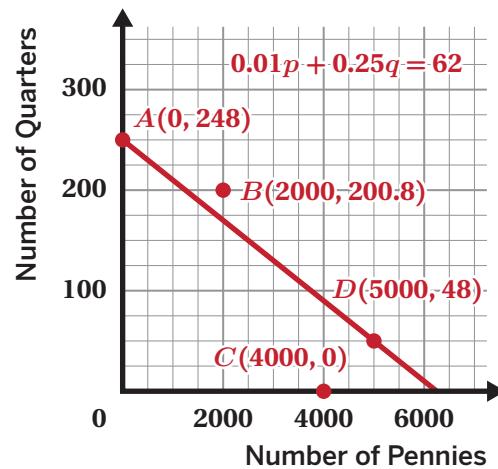
Write an equation that describes *all* the combinations of pennies,  $p$ , and quarters,  $q$ , that are worth \$62.00.

## Pennies and Quarters (continued)

- 5** Ava wrote the equation  $0.01p + 0.25q = 62$  to represent all the combinations of pennies,  $p$ , and quarters,  $q$ , that are worth \$62.00.

Select *all* the points that are solutions to the equation.

- A. Point  $A$
- B. Point  $B$
- C. Point  $C$
- D. Point  $D$



- 6** If there are 200 quarters, how many pennies do you need for a total of \$62.00?

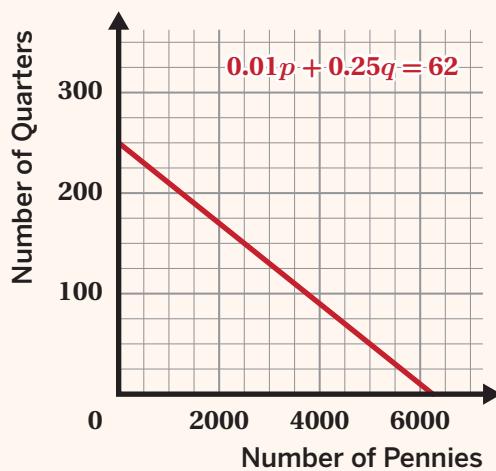
Use the graph if it helps with your thinking.

### Explore More

- 7** Here is a line and an equation describing all the combinations of pennies and quarters that are worth \$62.00.

A different pile of pennies and quarters is worth \$30.

- a** Draw the graph of this situation on the same plane.
- b** Describe the strategy you used.



**Situation Sort****8****a** Read each situation.**b****Discuss:**

- What is each situation about?
- What quantities or relationships do you see in each situation?

**Situation 1**

Sydney plans to buy 120 beverages for a picnic. Seltzers are sold in packs of 6. Waters are sold in packs of 8.  $x$  represents the number of packs of seltzers and  $y$  represents the number of packs of waters.

**Situation 2**

A coach has a \$120 budget to buy lunch for their team. They are ordering from a restaurant that charges \$8 per sandwich, plus a \$6 delivery fee.  $x$  represents the number of sandwiches and  $y$  represents the total cost of the lunch.

**9**

Match each representation and possible solution with Situation 1 or Situation 2 from the previous problem. One representation has no match.

**A**

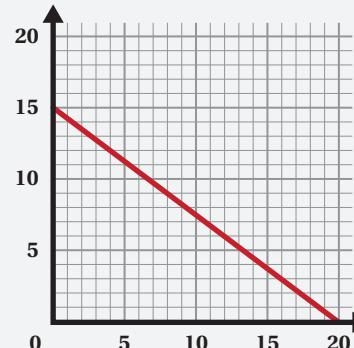
$x$	$y$
2	22
3	30
4	38

**B**

$$120 = 6x + 8y$$

**D**

$$y = 8x + 6$$

**C****E**

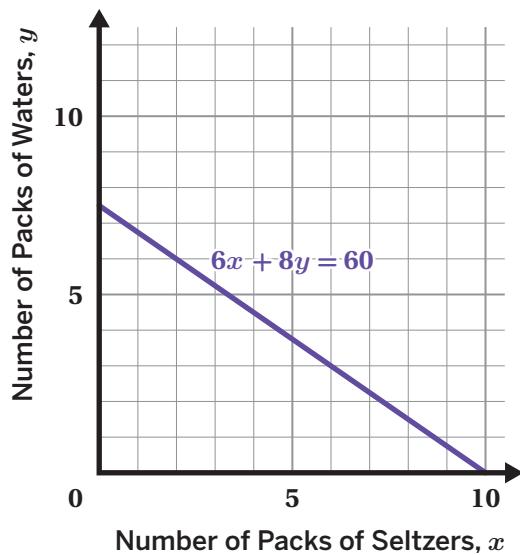
$$y = 8x + 120$$

**F**
 $(5, 46)$ 
**G**
 $(8, 9)$ 
**Situation 1****Situation 2**

## 10 Synthesis

Describe how you could use a graph or equation to determine whether a point is a solution to a linear relationship.

Use the example if it helps with your thinking.



Things to Remember:



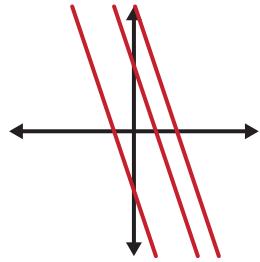
## On or Off the Line?

Let's interpret the meaning of points on and off lines.

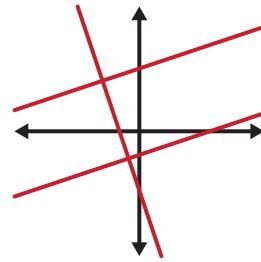
### Warm-Up

- 1** Which one doesn't belong? Explain your thinking.

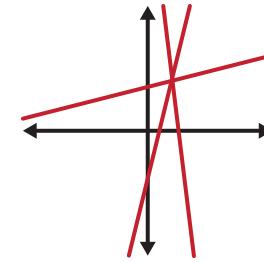
A.



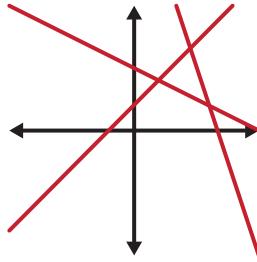
B.



C.



D.



## Two Dollars

- 2** I have \$2 worth of coins in my pocket.

What is a combination of any coins that I could have?

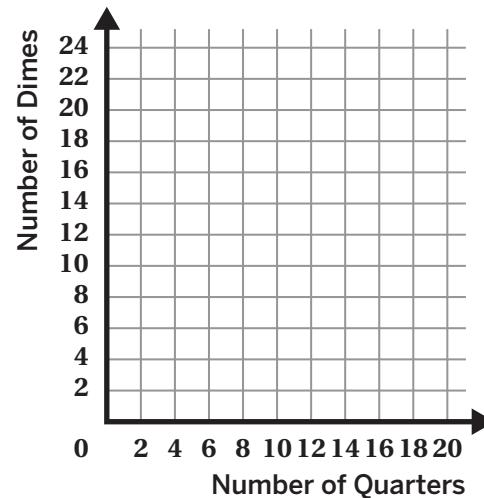
Try to think of a combination that no one else in the class will write.



- 3** Here is more information about my coins:

- I only have quarters and dimes.
- a Fill in three rows of possible combinations of quarters and dimes that are worth \$2.

Number of Quarters	Number of Dimes
.....	.....
.....	.....
.....	.....



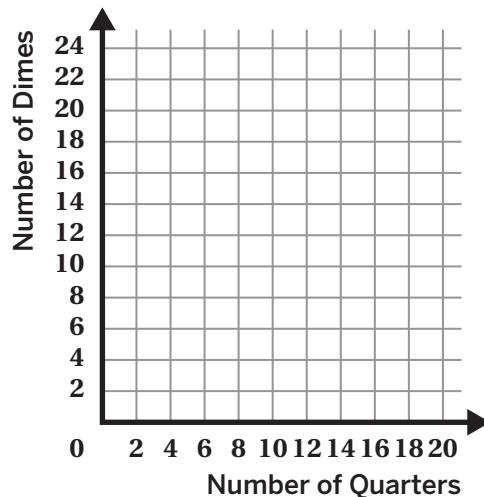
- b Plot your points on the graph.

**Two Dollars (continued)**

- 4** Here is some more information about my coins:

- I have a total of 17 coins.
- a** Fill in at least three rows of possible combinations of 17 coins.

Number of Quarters	Number of Dimes



- b** Plot your points on the graph.

- 5** Let's look at the graphs of these conditions on the same coordinate plane.

I have \$2 in my pocket: I only have quarters and dimes, and I have a total of 17 coins.

How many quarters and dimes must I have?

Number of Quarters	Number of Dimes

Explain your thinking.

## Challenge Creator

- 6** Let's look again at the graphs of both of these conditions on the same coordinate plane.

I have \$2 in my pocket: I only have quarters and dimes, and I have a total of 17 coins.

In this situation, which conditions does the point (9, 8) meet? Circle one.

I have \$2 worth of coins in my pocket

I have a total of 17 coins

Both

Neither

Explain your thinking.

- 7** You will use the Activity 2 Sheet to create your own linear situations challenge.

- a** **Make it!** On the Activity 2 Sheet, create a linear situation challenge.
- b** **Solve it!** On this page, record the ordered pair that the statement represents and determine which line(s) your statement applies to. Write a description of the line, both, or neither.

Situation	Ordered Pair	Line(s)

- c** **Swap it!** Swap your challenge with one or more partners. On this page, record the ordered pair that the statement represents and determine which line(s) their statement applies to.

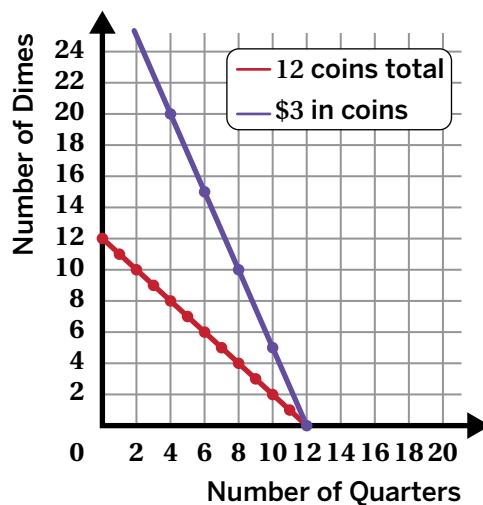
	Situation	Ordered Pair	Line(s)
Partner 1			
Partner 2			
Partner 3			
Partner 4			

## 8 Synthesis

Here is a graph that represents two conditions:

- I have \$3 worth of coins in my pocket.
- I have a total of 12 coins.

How can you use the graph to determine whether a combination of dimes and quarters meets both conditions, one condition, or neither condition?

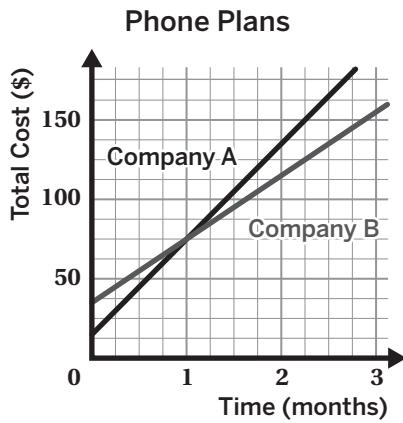


Things to Remember:

Name: ..... Date: ..... Period: .....

## Challenge Creator

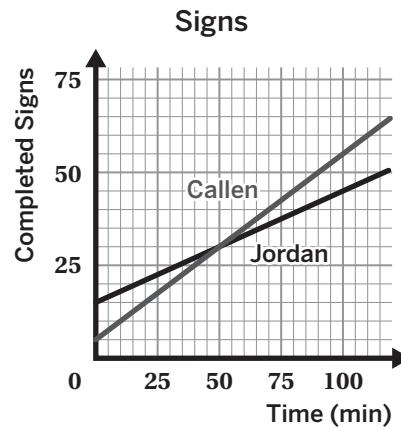
- Select one situation to focus on.
- Fill in the statement to describe a single point anywhere on the graph.



You are shopping for a new cell phone and a plan with unlimited data.

- Company A has a \$15 setup fee, and then charges \$60 per month.
- Company B has a \$35 setup fee, and then charges \$40 per month.

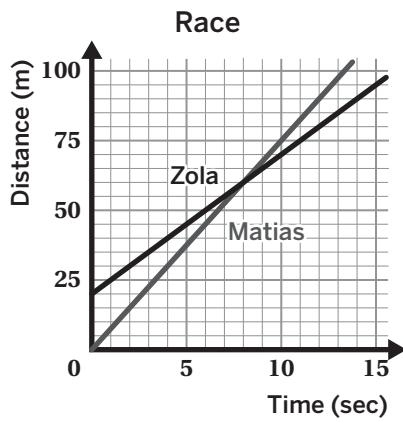
**Statement:** At ..... months, the cost of my phone plan is .....



Callen and Jordan are making locker signs to decorate for spirit week. The coordinate plane shows each person's progress today.

- Callen made 5 signs yesterday and is making a new sign every 2 minutes today.
- Jordan made 15 signs yesterday and is making a new sign every 4 minutes today.

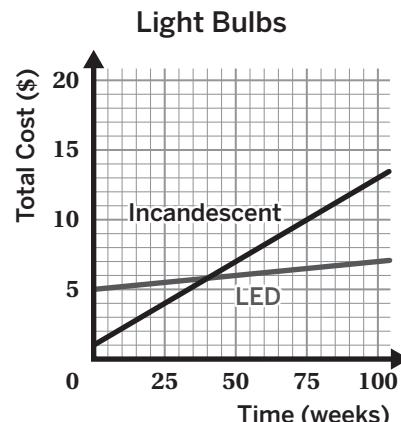
**Statement:** At ..... minutes, ..... signs were made.



Zola and Matias raced 100 meters. Both racers started at the same time and ran at a constant speed.

- Zola had a 20-meter head start and ran 5 meters per second.
- Matias ran 7.5 meters per second.

**Statement:** At ..... seconds, the distance is ..... meters.



Traditional incandescent bulbs are cheaper to purchase, but they use more energy compared to LED bulbs.

- Traditional incandescent bulbs cost about \$1 each, and 12 cents per week to use.
- LED bulbs cost about \$5 each, and 2 cents per week to use.

**Statement:** At ..... weeks, the total cost of the light bulbs is .....

Name: ..... Date: ..... Period: .....

# On Both Lines

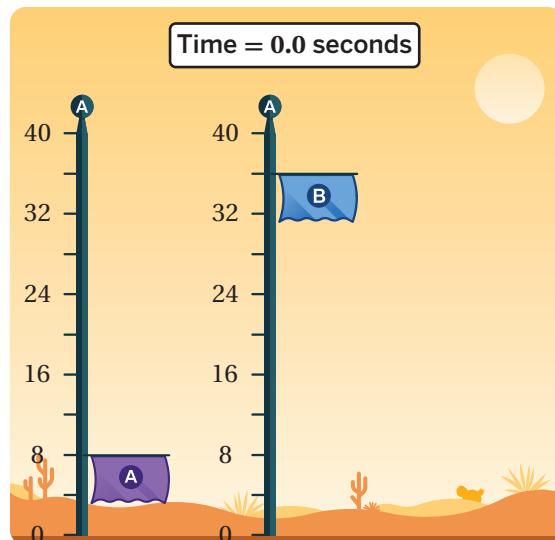
Let's use lines to analyze real-world situations.



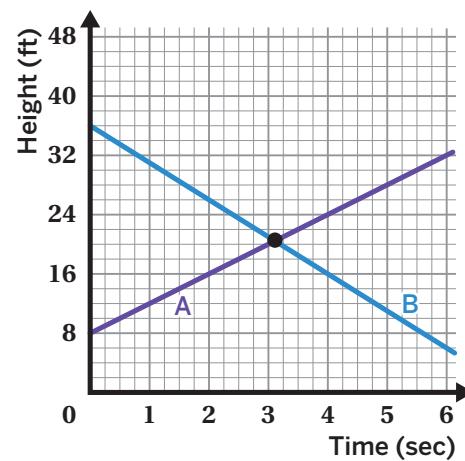
## Warm-Up

Let's watch an animation with two flags.

1. **Discuss:** What do you notice? What do you wonder?



2. A graph's **point of intersection** is a point where the lines meet. What does the point of intersection represent in this situation?



## On Both Lines

You will use the Activity 1 Sheet and choose *two* different phone companies to compare.

- 3.** Create a poster. Be sure to include these items in your poster:

- Your names
- The names of your selected companies
- A graph with both companies on the same set of axes
- An equation to represent each scenario
- Your conclusions about the situation
- The meaning of the point of intersection
- When each company has the better deal

### Explore More

- 4.** Select a third phone provider. On your poster, explain when this third plan would be a better and worse deal than your two previously selected plans.

# Gallery Tour

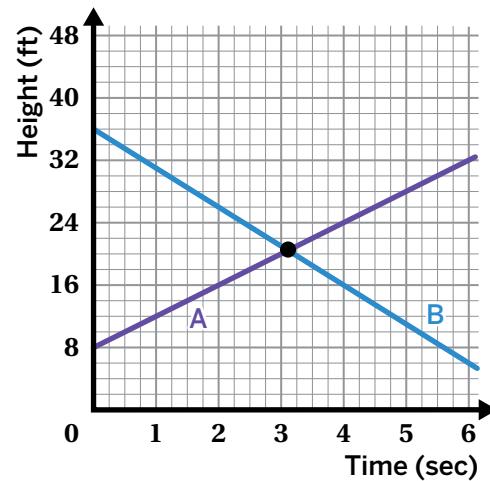
You will take a gallery tour as a class.

5. What features of your classmates' posters helped you understand their thinking?
  6. Describe something you would change about your poster now that you have seen other groups' work.

## Synthesis

7. Describe how a graph can help you compare different linear relationships.

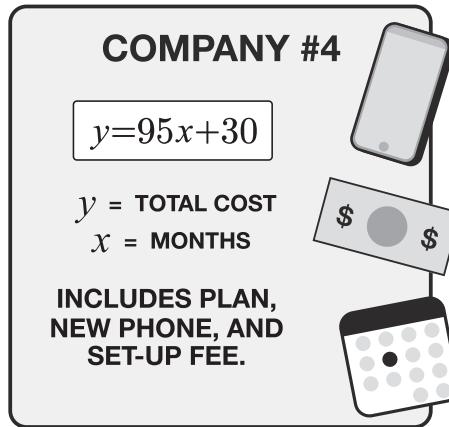
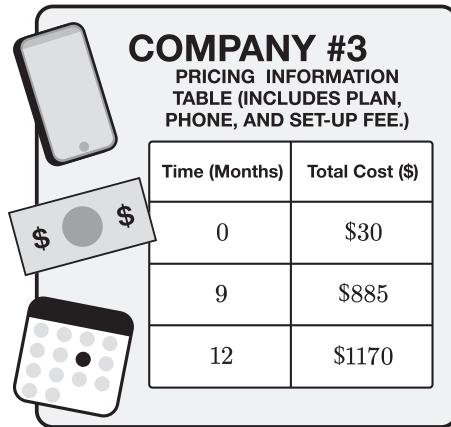
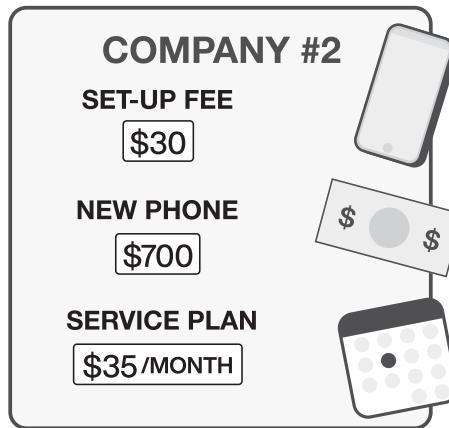
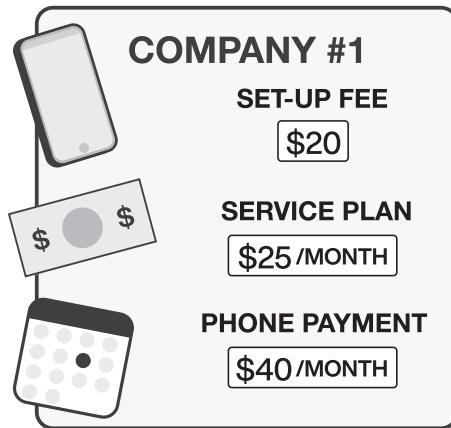
Use the example if it helps with your thinking.



Things to Remember:

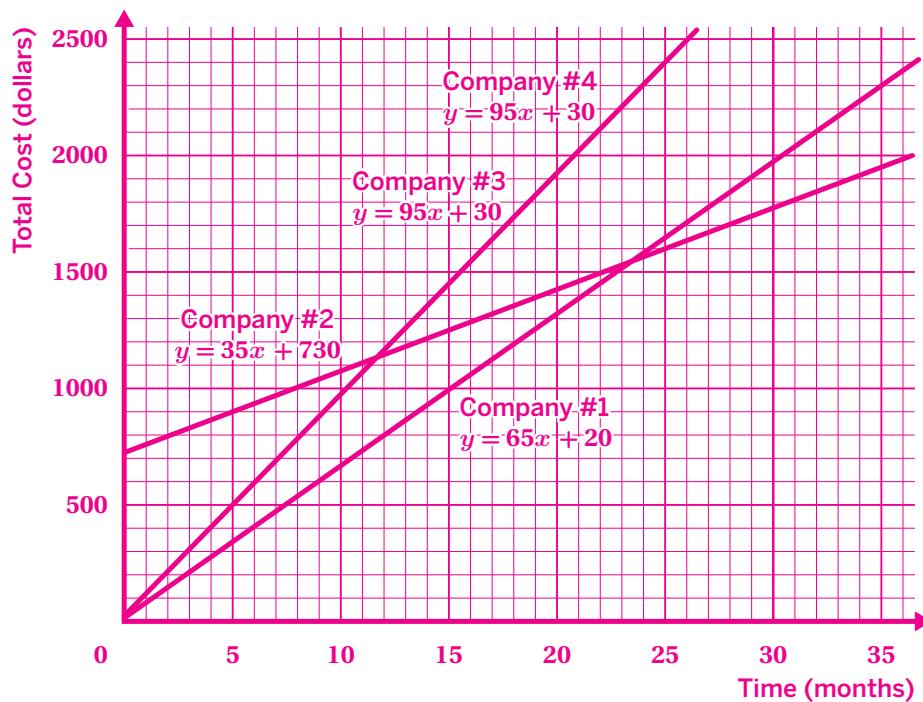
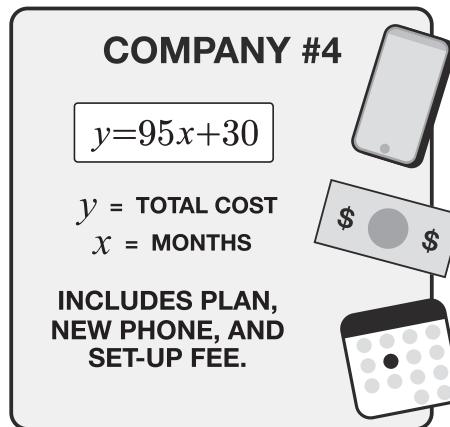
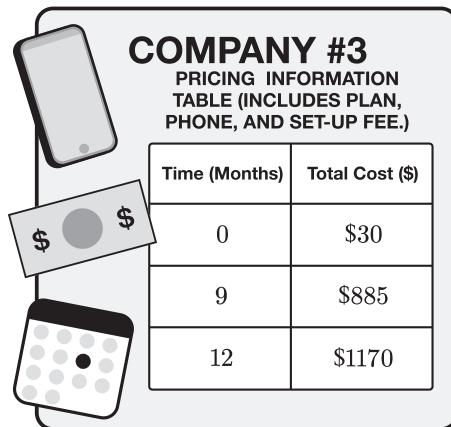
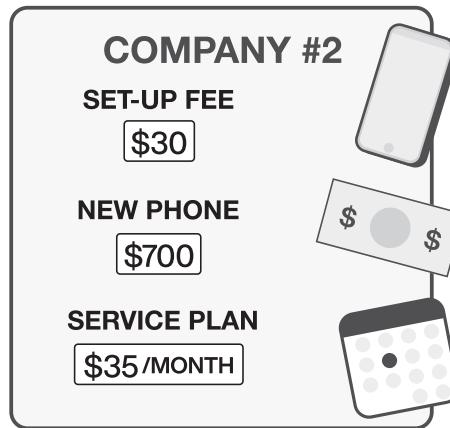
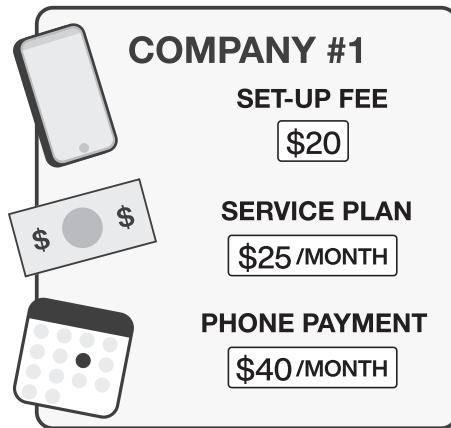
## On Both Lines

You are shopping for a new cell phone and plan with unlimited data. Here are four companies and information about their set-up fee, phone cost, and service plan.



## On Both Lines (answers)

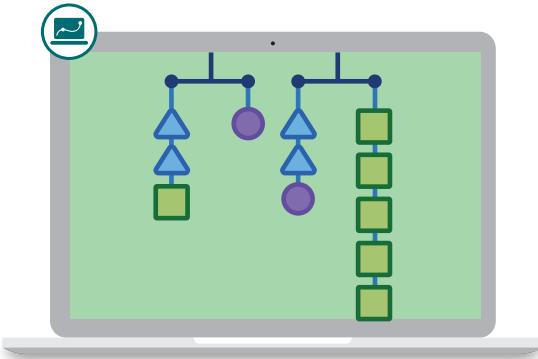
You are shopping for a new cell phone and plan with unlimited data. Here are four companies and information about their set-up fee, phone cost, and service plan.



Name: ..... Date: ..... Period: .....

# Make Them Balance

Let's explore solutions to more than one linear relationship.

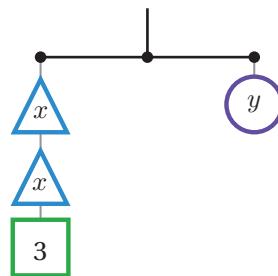


## Warm-Up

- 1** Determine values for  $x$  and  $y$  that will balance the hanger.

$x$	$y$

Hanger A



$$2x + 3 = y$$



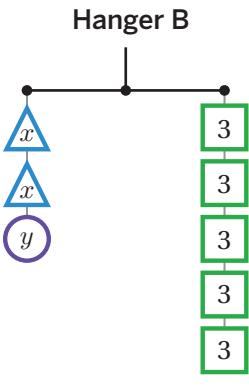
- 2** Let's look at a graph that shows points whose  $x$ - and  $y$ -values balance the hanger.



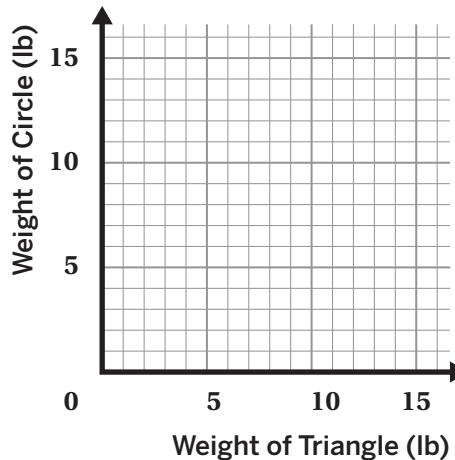
**Discuss:** What do you notice? What do you wonder?

**Two Hangers**

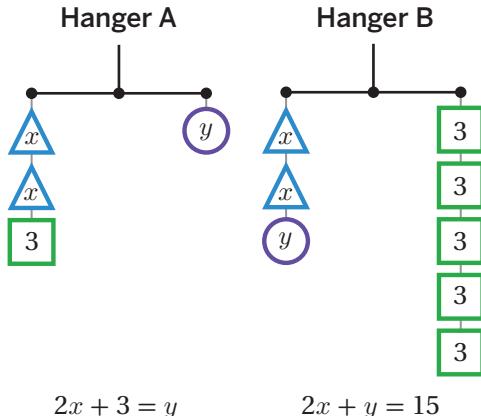
- 3** Here's another hanger. Plot three different pairs of  $x$ - and  $y$ -values that will balance this hanger.



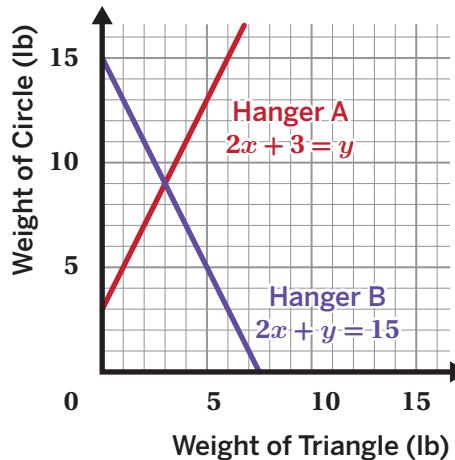
$x$  ? lb     $y$  ? lb    3 3 lb



- 4** Here are both hangers' equations graphed on the same coordinate plane.



? lb    ? lb    3 3 lb



**Discuss:** Can you identify a point that balances just Hanger A? Just Hanger B? Both? Neither?

**Two Hangers (continued)**

- 5** You have been experimenting with two representations of this **system of equations**:

$$2x + 3 = y$$

$$2x + y = 15$$

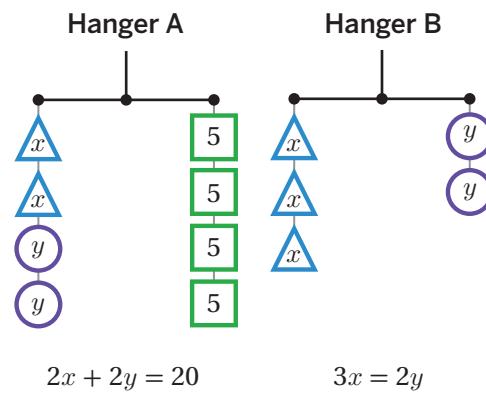
A **solution to a system of equations** is a set of values that makes both equations in that system true.

Write the solution to this system as an ordered pair. Explain your thinking.

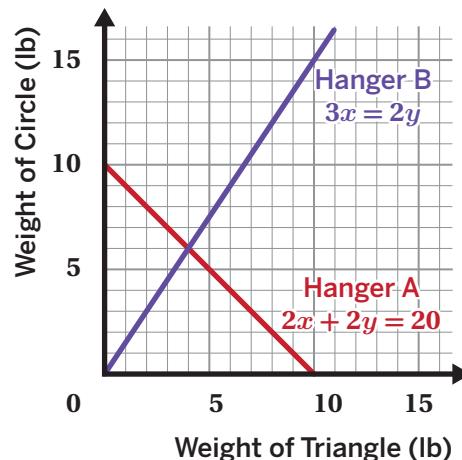
- 6** This system represents another set of hangers:

$$2x + 2y = 20$$

$$3x = 2y$$



$x$  lb     $y$  lb    5 lb



Which point is a solution to this system? Circle one.

(1, 9)

(4, 6)

(6, 9)

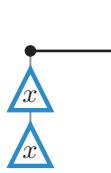
(6, 4)

Explain your thinking.

## Hanger Solutions

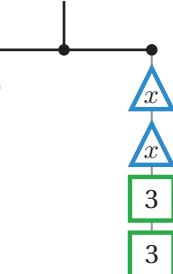
**7** Here's another system of equations.

Hanger A



$$2x = y$$

Hanger B

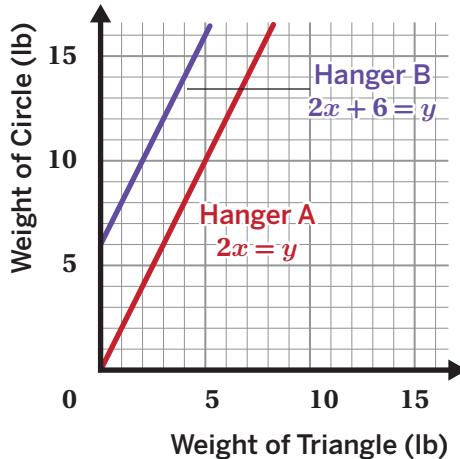


$$y = 2x + 6$$

? lb

? lb

3 lb



**Discuss:** Can you identify a point that balances just Hanger A? Just Hanger B? Both? Neither?

**8** This system of equations from the previous problem has no solution:

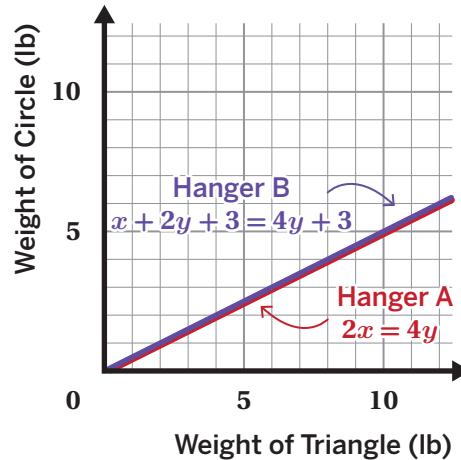
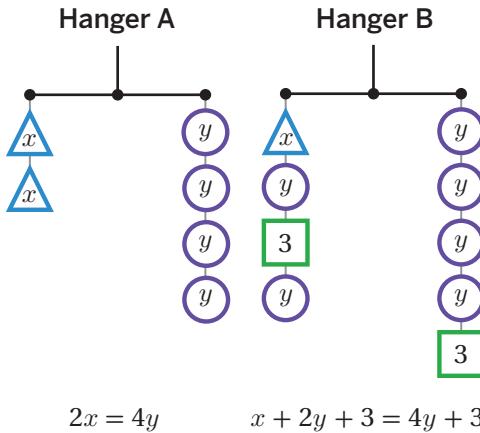
$$2x = y$$

$$y = 2x + 6$$

How can you use the hangers, graphs, or equations to tell that this system of equations has no solution?

**Hanger Solutions (continued)**

- 9** Here's one more system.



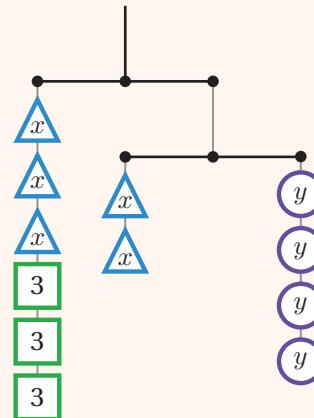
? lb	? lb	3 lb
------	------	------

**Discuss:** How many solutions do you think this system has? How do you know?

**Explore More**

- 10** Determine values for  $x$  and  $y$  that will balance both hangers.

$x$	$y$

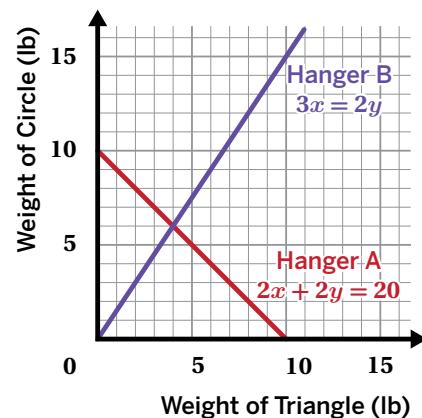
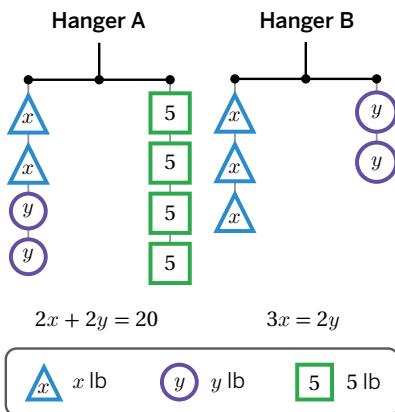


? lb	? lb	3 lb
------	------	------

## 11 Synthesis

How can you tell if an ordered pair is a solution to a system of linear equations?

Use the example if it helps with your thinking.

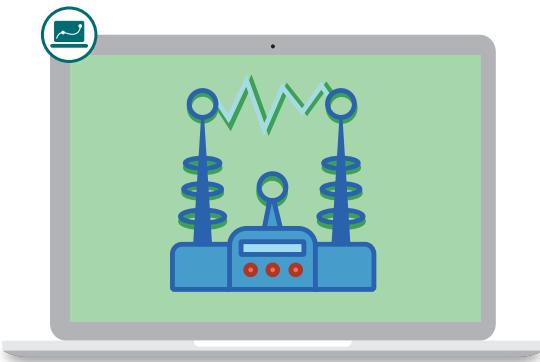


Things to Remember:

Name: ..... Date: ..... Period: .....

# Line Zapper

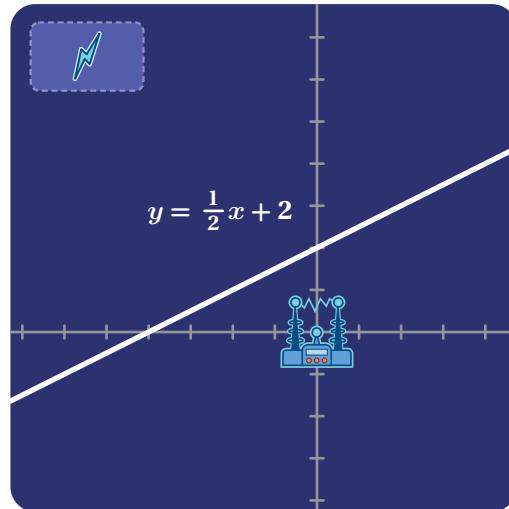
Let's solve systems of linear equations.



## Warm-Up

- 1** Let's capture the line by writing the coordinates for a point that's on the line.

Zap	Point
Zap 1	

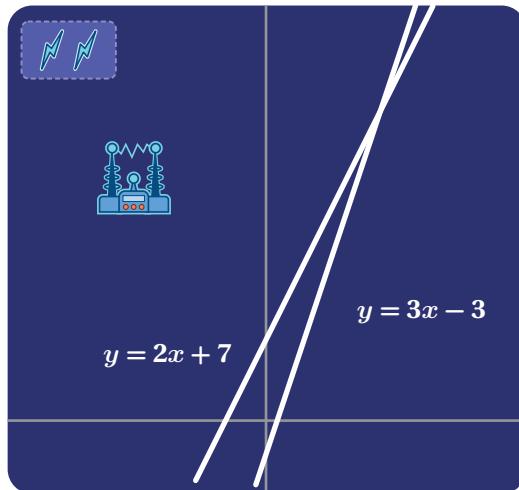


## Two Lines, One Zap

- 2** Capture both of the lines by writing coordinates for points on these lines.

Use no more than two zaps!

Zap	Point
Zap 1	
Zap 2	



- 3** Cameron wanted to capture both lines with one zap.

Cameron solved an equation to identify the point of intersection and determined that  $x = 10$ .

How could Cameron determine the  $y$ -value of the point of intersection?

**Cameron**

$$\begin{aligned} y &= 2x + 7 \\ y &= 3x - 3 \\ 2x + 7 &= 3x - 3 \\ -2x \quad -2x \\ 7 &= 1x - 3 \\ +3 \quad +3 \\ 10 &= 1x \\ x &= 10 \end{aligned}$$

## Line Zapper

- 4** The following lines are hidden in a graph:

$$y = -x + 10$$

$$y = 2x + 4$$

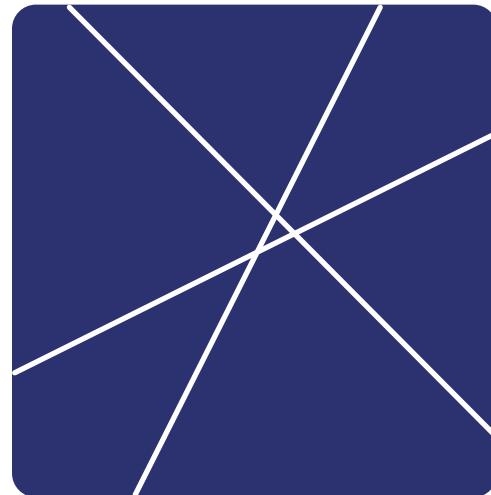
Zap	Point
Zap 1	

Capture the lines by writing the coordinates for a point on both of these lines. You only have one zap!

- 5** **a** Select *all* the lines that would be captured if the point (2, 4) were zapped.

- A. Line A:  $y = \frac{1}{2}x$
- B. Line B:  $y = 2x$
- C. Line C:  $y = -x + 6$

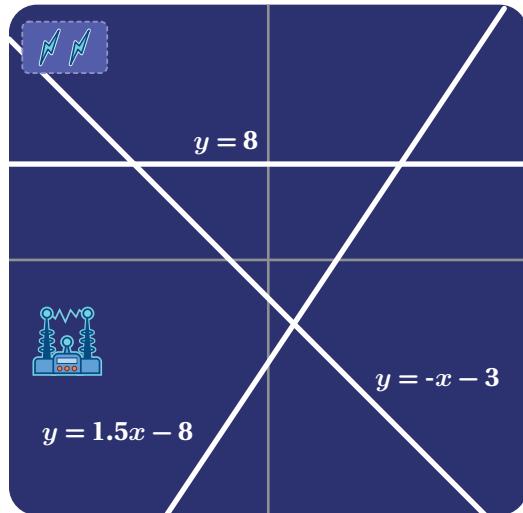
- b** Explain how you decided which lines to select.



**Line Zapper (continued)**

- 6** Capture all of the lines by writing coordinates for points on the lines. You only have two zaps!

Zap	Point
Zap 1	
Zap 2	



- 7** The following lines are hidden in a graph:

$$y = -2x + 9$$

$$y = 3x - 1$$

$$y = \frac{1}{2}x - 1$$

Capture all of the lines by writing the coordinates for points on the lines.  
You only have two zaps!

Zap	Point
Zap 1	
Zap 2	

**Explore More**

- 8** The following lines are hidden in a graph:

$$y = 3x + 6$$

$$y = 3(x - 5)$$

$$y = -\frac{1}{2}x - 15$$

$$-x + y = 12$$

Capture all of the lines by writing coordinates for points on the lines. You only have three zaps!

Zap	Point
Zap 1	
Zap 2	
Zap 3	

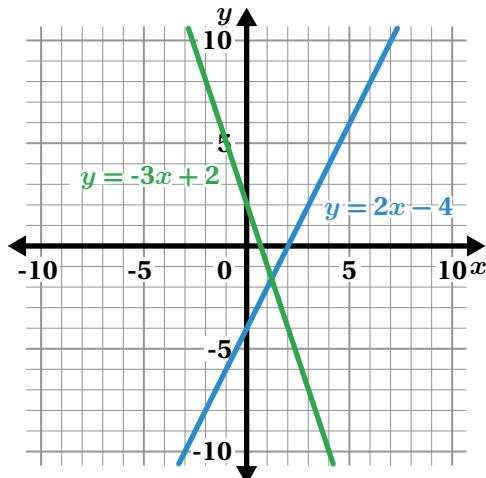
## 9 Synthesis

Here is the graph of this system of equations:

$$y = -3x + 2$$

$$y = 2x - 4$$

How can you determine the exact solution to this system of equations?



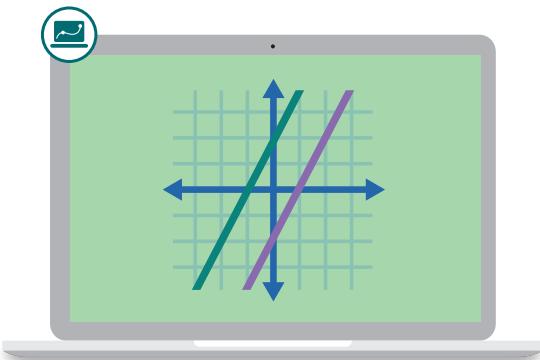
Things to Remember:

Name: ..... Date: ..... Period: .....

# All, Some, or None?

## Part 2

Let's solve systems with no solution and infinitely many solutions.

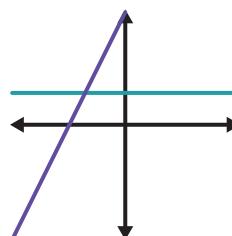
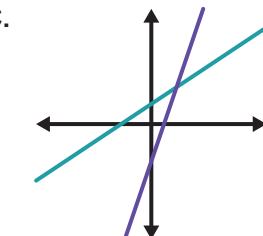
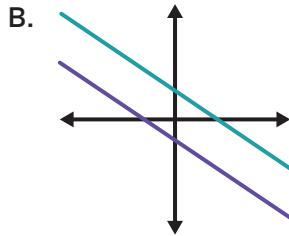
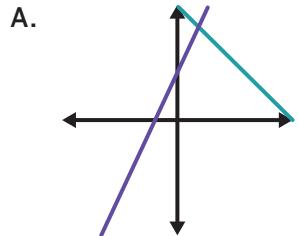


### Warm-Up

- 1 Which graph could represent this system of equations?

$$y = 2x + 4$$

$$y = -x + 10$$

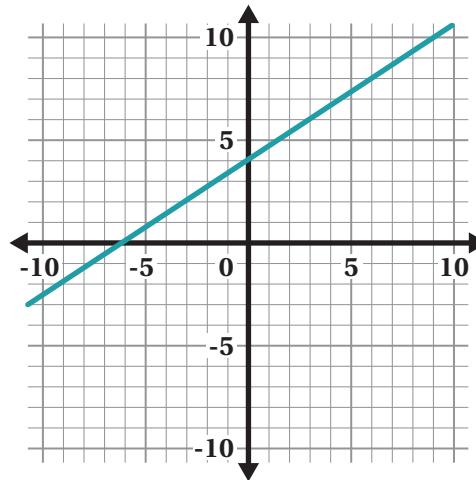


Explain your thinking.

## Connecting Graphs and Equations

- 2** Here is the graph of  $y = \frac{2}{3}x + 4$ .

Graph another line to create a system of equations that has *no* solution.



- 3** Here are the equations for a system that has no solution:

$$y = \frac{2}{3}x + 4$$

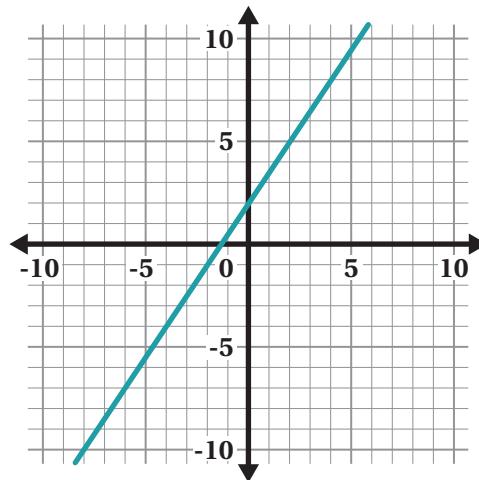
$$y = \frac{2}{3}x - 5$$

How can you determine from the equations that the system will have no solution?

## Connecting Graphs and Equations (continued)

- 4** Here is the graph of  $y = \frac{1}{2}(3x + 4)$ .

Graph another line to create a system of equations that has *infinitely many* solutions.



- 5** Here are the equations for a system that has infinitely many solutions:

$$y = \frac{1}{2}(3x + 4)$$

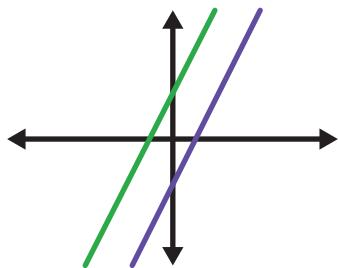
$$y = \frac{3}{2}x + 2$$

How can you determine from the equations that the system will have infinitely many solutions?

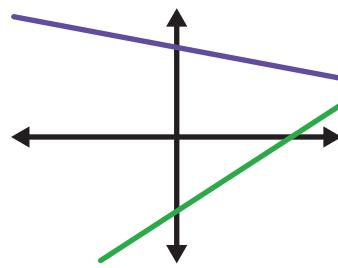
## Sorting Systems

**6** Group the systems of equations based on their number of solutions.

A.



B.



C. 
$$\begin{cases} y = \frac{3}{4}x - 14 \\ y = -\frac{1}{4}x + 9 \end{cases}$$

D. 
$$\begin{cases} y = 2x + 1 \\ -2x + y = 1 \end{cases}$$

E. 
$$\begin{cases} x + y = 10 \\ y = -x - 3 \end{cases}$$

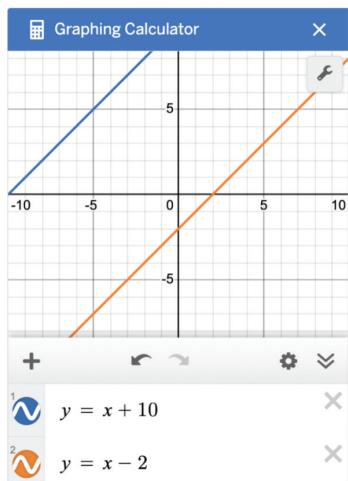
F. 
$$\begin{cases} y = x + 10 \\ y = x - 2 \end{cases}$$

No Solution	One Solution	Infinitely Many Solutions

## Sorting Systems (continued)

- 7** Here are Oscars's and Melanie's strategies to check whether System F goes in the "No Solution" group.

Oscar



Melanie

F

$$\begin{cases} y = x + 10 \\ y = x - 2 \end{cases}$$

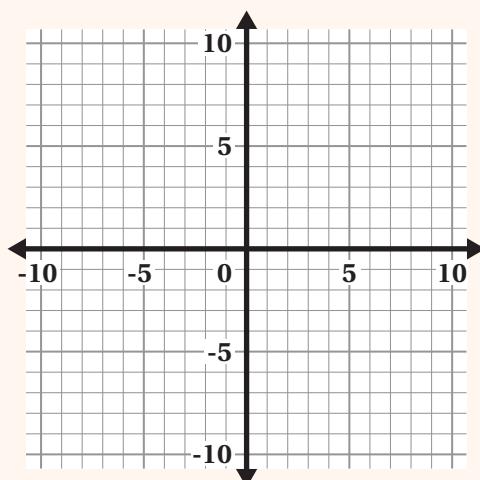
$$\begin{aligned} x + 10 &= x - 2 \\ -x &\quad -x \\ 10 &= -2 \end{aligned}$$

**Discuss:** How are their strategies alike? How are they different?

### Explore More

- 8** The graphs of the equations  $Ax + By = 15$  and  $Ax - By = 9$  intersect at  $(2, 1)$ . Determine the values of  $A$  and  $B$ .

Use the grid if it helps with your thinking.



## 9 Synthesis

How can you determine the number of solutions for a system of equations?

C

$$\begin{cases} y = \frac{3}{4}x - 14 \\ y = -\frac{1}{4}x + 9 \end{cases}$$

D

$$\begin{cases} y = 2x + 1 \\ -2x + y = 1 \end{cases}$$

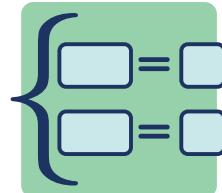
E

$$\begin{cases} x + y = 10 \\ y = -x - 3 \end{cases}$$

Things to Remember:

# Strategic Solving, Part 2

Let's explore strategies for solving systems of equations.



## Warm-Up

Determine the solution of each system mentally. Be prepared to share your strategy.

1.  $\begin{cases} x = 5 \\ y = x - 7 \end{cases}$

2.  $\begin{cases} y = 4 \\ y = x + 3 \end{cases}$

3.  $\begin{cases} x - y = 4 \\ x + y = 10 \end{cases}$

## Thinking About Solutions

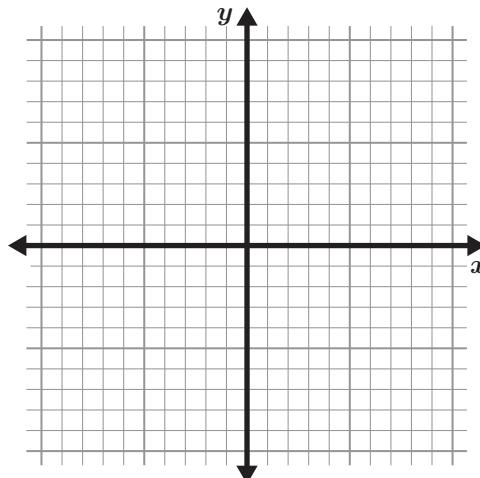
4. How many solutions does this system have?

Circle one.

$$\begin{cases} x + y = 5 \\ x + y = 7 \end{cases}$$

One solution   No solution   Infinitely many  
solutions

Explain your thinking.



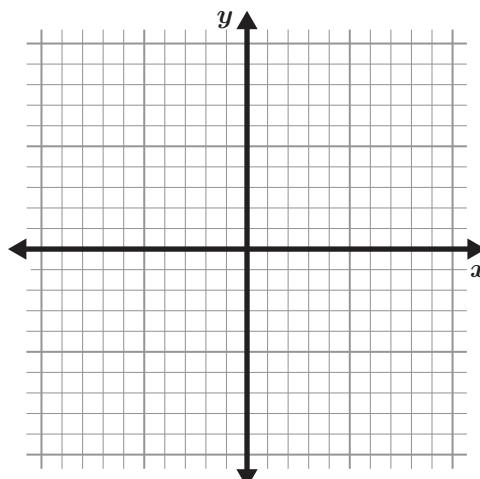
5. Martina says that  $(2, -4)$  is a solution to the system. Sai says there are infinitely many solutions.

$$\begin{cases} y = 4(x - 3) \\ y = 4x - 12 \end{cases}$$

Whose claim is correct? Circle one.

Martina's    Sai's    Both    Neither

Explain your thinking.



## The Choice Is Yours

**System A**

$$\begin{cases} y = 3x + 2 \\ 2x + y = 47 \end{cases}$$

**System B**

$$\begin{cases} y = 4 \\ y = -2 \end{cases}$$

**System C**

$$\begin{cases} y = \frac{1}{4}x + 7 \\ x = -4 \end{cases}$$

**System D**

$$\begin{cases} y = -3x + 10 \\ y = -2x + 6 \end{cases}$$

**System E**

$$\begin{cases} y = 3x + 5 \\ -3x + y = 5 \end{cases}$$

**System F**

$$\begin{cases} y = -3x - 5 \\ y = 4x + 30 \end{cases}$$

**System G**

$$\begin{cases} y = 3 \\ x = -2y + 56 \end{cases}$$

**System H**

$$\begin{cases} -4x + y = 30 \\ y = -3x - 5 \end{cases}$$

6. Examine these systems. Organize the equations into two or three groups based on the patterns you notice.

Group 1	Group 2	Group 3

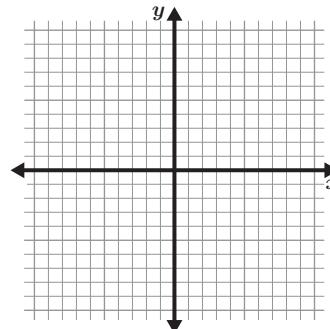
7.  **Discuss:** How did you group the equations?

## The Choice Is Yours (continued)

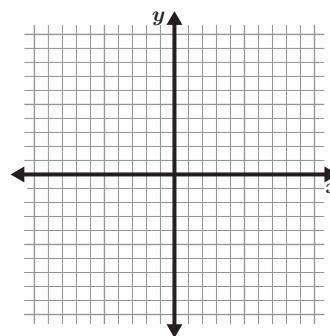
8. Choose three systems to solve. (Choose at least one from each group.)

Show your thinking and use the graphs if they help you to solve.

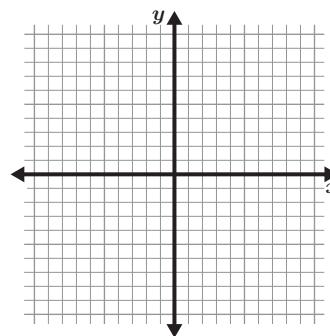
System 1:



System 2:



System 3:



## Synthesis

9. What are some strategies for solving systems like these?

A 
$$\begin{cases} y = 3x + 2 \\ 2x + y = 47 \end{cases}$$

C 
$$\begin{cases} y = \frac{1}{4}x + 7 \\ x = -4 \end{cases}$$

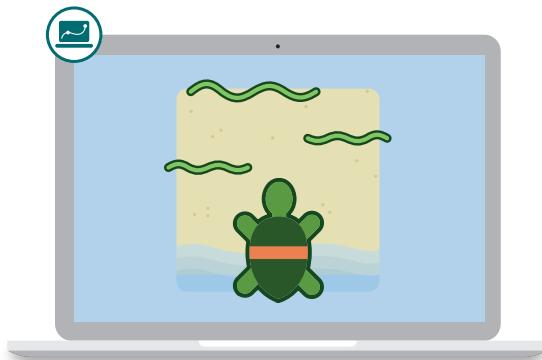
F 
$$\begin{cases} y = -3x - 5 \\ y = 4x + 30 \end{cases}$$

Things to Remember:

Name: ..... Date: ..... Period: .....

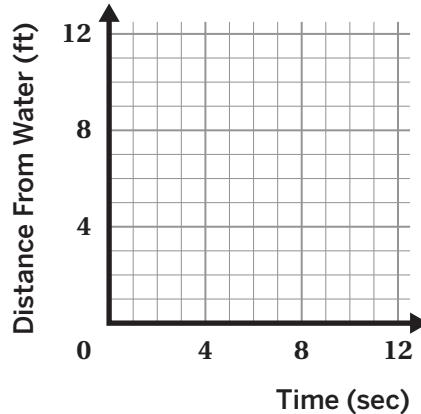
# Turtle Crossing

Let's make sense of graphs.



## Warm-Up

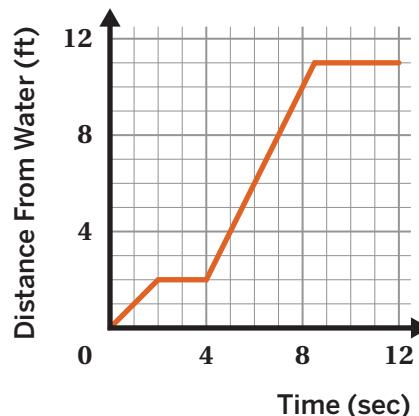
- 1** **a** Draw a distance vs. time graph to represent a turtle's journey across the sand.
- b** What story does your graph tell about the turtle's journey?



## Turtle Graphs

- 2** Kris drew this graph to represent a turtle's journey.

What story does the graph tell about the turtle's journey?



- 3** Let's watch an animation to see what Kris's turtle did.



**Discuss:** How does this animation compare to your story?

- 4** Use Kris's graph to answer each question.

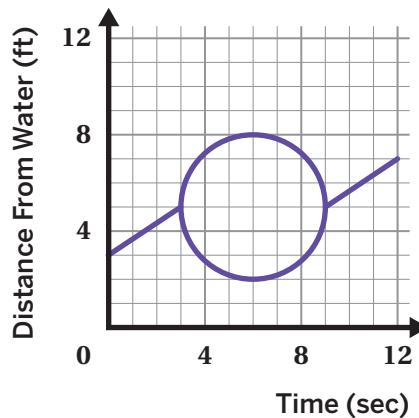
**a** At 8 seconds, how far is the turtle from the water?

**b** When is the turtle 4 feet from the water?

## Turtle Graphs (continued)

- 5** Arnav drew this graph to represent a new turtle.

What story does the graph tell about the turtle's journey?



- 6** Let's watch an animation to see what Arnav's turtle did.

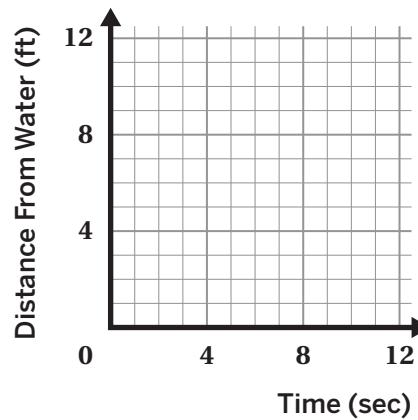
 **Discuss:** How does this animation compare to your story?

## Dangerous Crossings

- 7 Let's watch the animation of another turtle crossing.

 **Discuss:** What do you notice? What do you wonder?

- 8 Draw a graph of the turtle's distance from the water over time based on the animation from the previous problem.



### Explore More

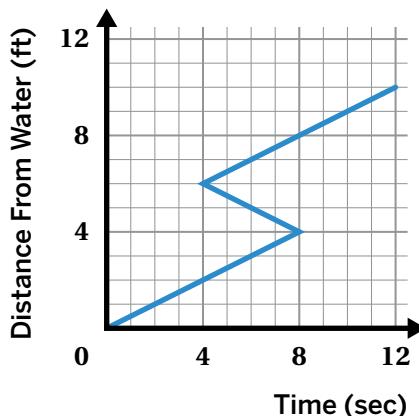
- 9 Use the digital activity to draw a new graph to help the turtle go from the grass to the water while avoiding the snakes.



## 10 Synthesis

Citlalli drew this graph to represent a new turtle.

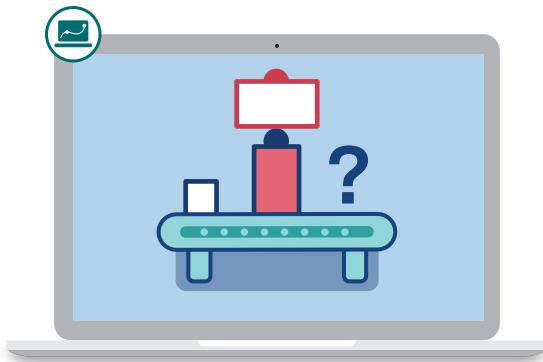
Explain what story the graph is telling at 6 seconds.  
Does the turtle's journey seem realistic?



Things to Remember:

# Guess My Rule

Let's explore rules to develop the concept of a function.

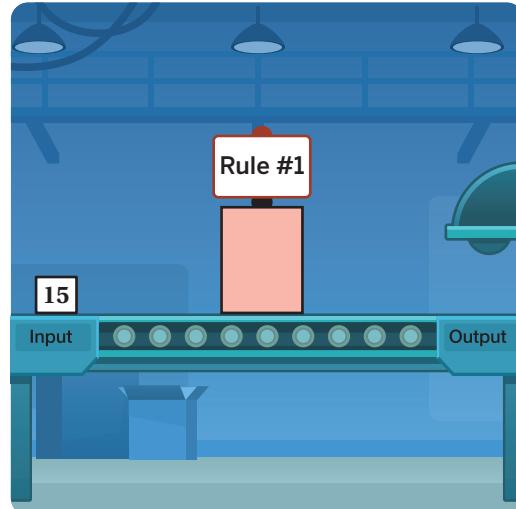


## Warm-Up

- 1** This machine uses a secret rule, Rule #1, to turn inputs into outputs.

Rule #1 allows *all integers* as inputs.

- a** Let's watch this machine at work.
- b** What could Rule #1 be? Select *all* that apply.
  - A. Divide by 2, then add 5.
  - B. Divide by 3.
  - C. Take the ones digit.
  - D. Multiply by 3.
  - E. Subtract 10.



- 2** Let's enter a different integer to help you decide what Rule #1 is.

What could Rule #1 be now? Select *all* that apply.

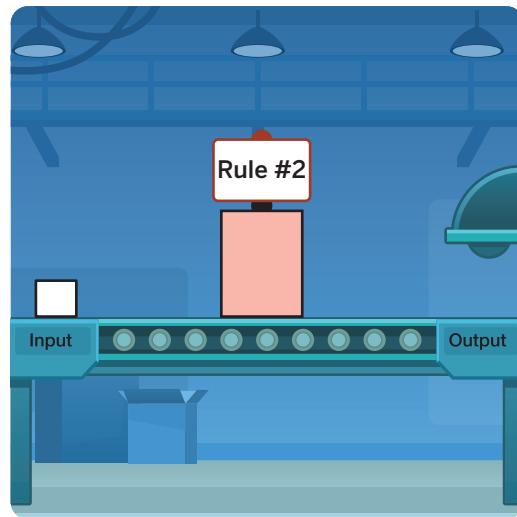
- A. Divide by 2, then add 5.
- B. Divide by 3.
- C. Take the ones digit.
- D. Multiply by 3.
- E. Subtract 10.

## Guess My Rule

- 3** This machine uses a new rule called Rule #2.

Rule #2 allows *all numbers* as inputs.

Let's test several inputs to see how Rule #2 works.



- 4** Here are some inputs and outputs from other students.

If you input 6 into Rule #2, what do you think the output will be?

Explain your thinking.

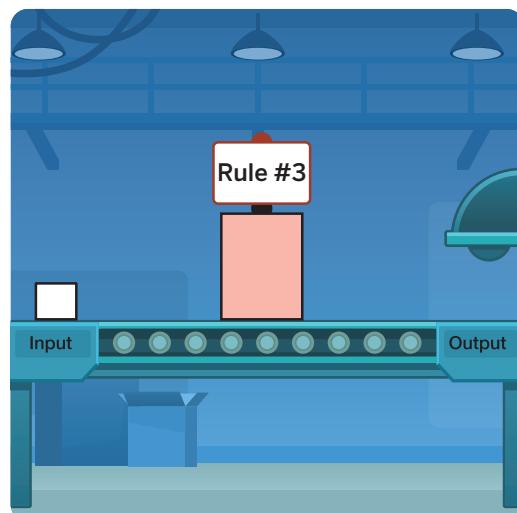
**Rule #2**

Input	Output
-13	7
-61	7
81	7
-100	7
60	7

- 5** Rule #3 allows *single words* as inputs.

- a** Let's test several inputs to see how Rule #3 works.

- b**  **Discuss:** What do you think the rule might be?



## Guess My Rule (continued)

- 6** Here are some inputs and outputs from other students.

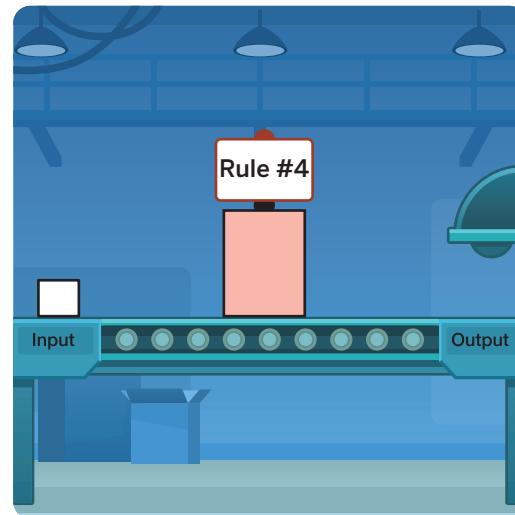
Test your understanding of Rule #3 by completing the table.

**Rule #3**

Input	Output
mint	u
hen	o
clear	s
friend	
wallet	
friend	
party	

- 7** Rule #4 allows *single letters* (like "A") as inputs.

Let's test several inputs to see how Rule #4 works.



- 8** Here are some inputs and outputs from other students.

What do you think the output will be if you input "A" into Rule #4? Explain your thinking.

Then compare your response with your classmates' responses.

Input	Output
W	William
A	Anand
A	Adam

## What Is a Function?

- 9** Rules #1, #2, and #3 all represent **functions**.

Rule #4 does *not* represent a function.

- a** What do you think makes a rule a function?

- b** Compare your response with a classmate's. Then revise your response to make it stronger and clearer.

Rule #1: Function

Input	Output
35	25
723	713
-4	-14
53	43
723	713

Rule #2: Function

Input	Output
15	7
18	7
262	7
-3	7
82.3	7

Rule #3: Function

Input	Output
hi	J
my	Z
name	F
is	T
Arturo	P

Rule #4: Not a Function

Input	Output
H	Hailey
J	Jada
M	Mai
H	Hamza
M	Madison

- 10** Determine whether each table represents a function.

**a**

Button Selected (Input)	Drink Received (Output)
A	Water
B	Seltzer
C	Juice
D	Water

**b**

Money Spent (Input)	Number of Items (Output)
\$1	2
\$8	12
\$7	1
\$1	3

**c**

Height in Feet (Input)	Height in Inches (Output)
5	60
4.5	54
6	72
5.4	64.8

**d**

Time in Seconds (Input)	Musical Note (Output)
1.5	D
1.25	D
1.5	E
2	D

- 11** This table does *not* represent a function.

- a** Change the fewest numbers so the table could represent a function.

- b** Explain your thinking.

Input	Output
1	5
2	10
3	15
2	20
1	24

## 12 Synthesis

How can you determine whether a table could represent a function?

Use the examples if they help with your thinking.

**Rule #1: Function**

Input	Output
35	25
723	713
-4	-14
53	43
723	713

**Rule #2: Function**

Input	Output
15	7
18	7
262	7
-3	7
82.3	7

**Rule #3: Function**

Input	Output
hi	J
my	Z
name	F
is	T
Arturo	P

**Rule #4: Not a Function**

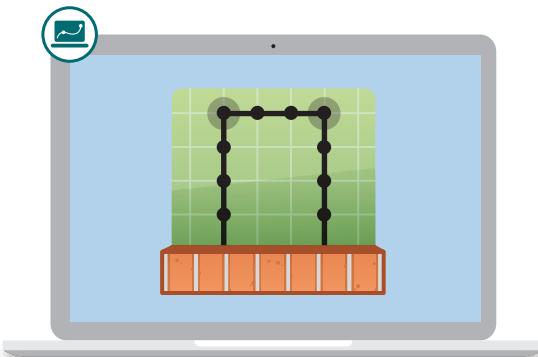
Input	Output
H	Hailey
J	Jada
M	Mai
H	Hamza
M	Madison

Things to Remember:

Name: ..... Date: ..... Period: .....

## Function or Not?

Let's determine whether a graph represents a function.



### Warm-Up

- 1 We learned that a function is a rule that assigns exactly one output to each possible input.

That means each input determines a single output.

Complete the table so that  $y$  is *not* a function of  $x$ .

$x$	$y$
3	
5	4
	6

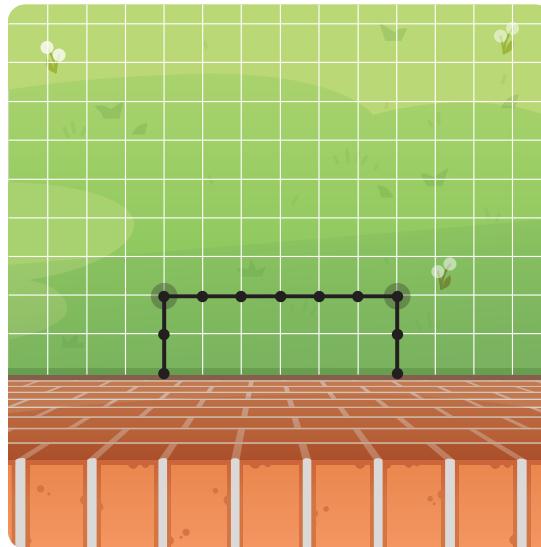
## Rectangular Pen

- 2** Emma is hiring builders to create a rectangular pen, a closed space where animals can be kept. Emma wants to build three sides of fencing against a brick wall.

Let's observe the relationship between the area and the amount of fencing.

Enter an amount of fencing and the area for a pen that you've observed.

Amount of Fencing (m)	Area (sq. m)



- 3** Here is a table that shows the amount of fencing and the area for several pens.

Emma gave the builders 12 meters of fencing to build the pen.

Is it possible for the builders to determine the area of Emma's pen?

Yes      No      I'm not sure

Explain your thinking.

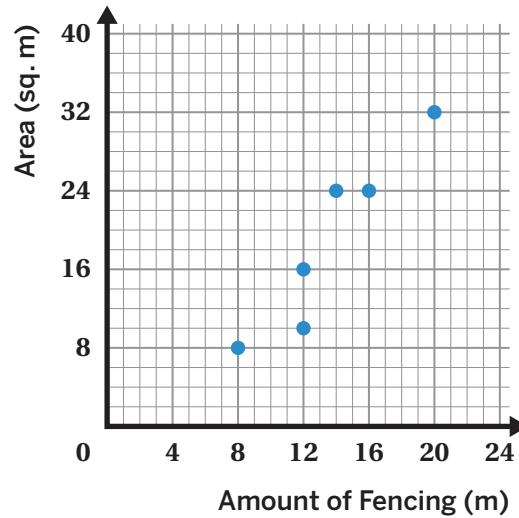
Amount of Fencing (m)	Area (sq. m)
8	8
12	10
14	24
12	16
16	24
20	32

## Rectangular Pen (continued)

- 4** Here is a graph that shows the relationship between area and amount of fencing for several pens.

How can you use the graph to quickly determine that area is not a function of the amount of fencing?

Area is not a function of the amount of fencing because ...



## Turtle Crossing

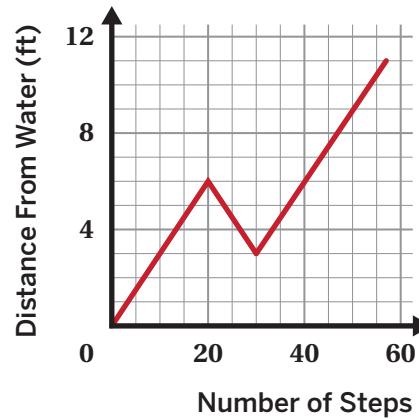
- 5** Let's watch an animation of a turtle's journey across the sand.

Write the turtle's number of steps and distance from the water for one point in time.

Number of Steps	Distance from Water (ft)



- 6** How can you use this graph to decide whether distance from the water is a function of the number of steps?



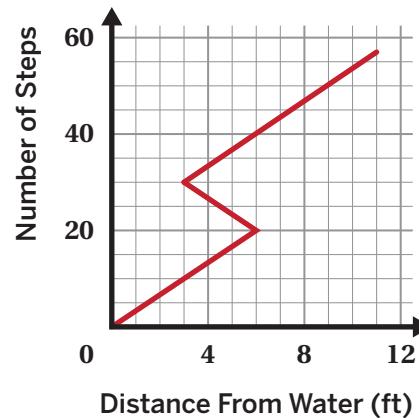
- 7** Here is the relationship in reverse.

Is the number of steps a function of distance?  
Circle one.

Yes

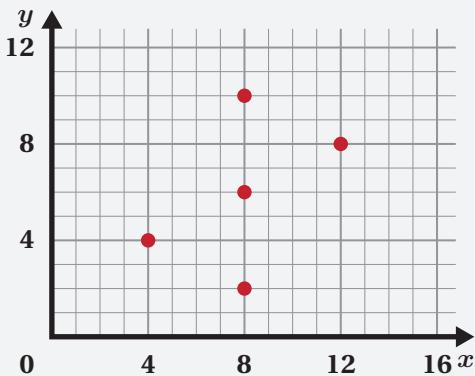
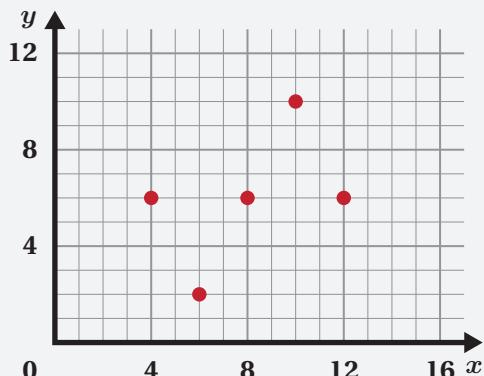
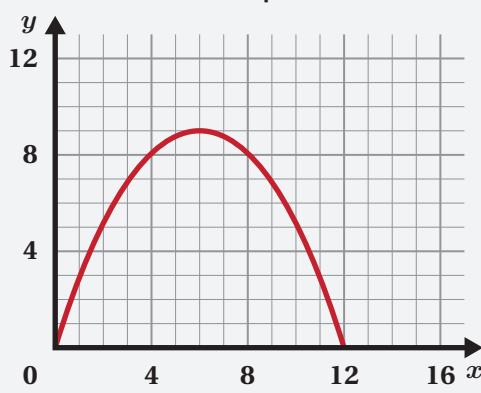
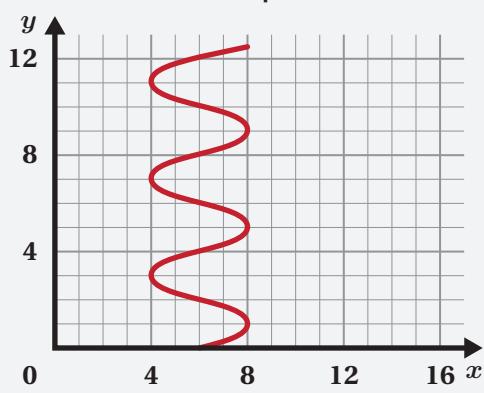
No

Explain your thinking.



**Turtle Crossing** (continued)

- 8** Determine whether each graph represents a function or not.

**Graph A****Graph B****Graph C****Graph D****Function****Not a function****Explore More**

- 9** **a** Use the digital activity to produce a graph that is surprising in some way.

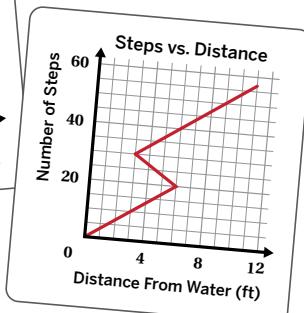
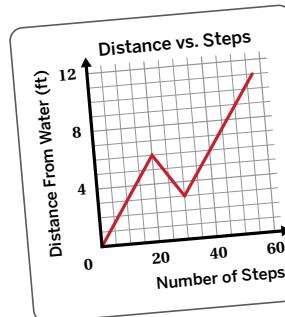
**b** **Discuss:**

- Does this graph represent a function?
- What does the surprising part of this graph represent in context?

## **10** Synthesis

How can you determine whether a graph represents a function?

Use the examples if it helps with your thinking.



Things to Remember:

# Dependence Day

Let's explore the relationships between variables of functions.



## Warm-Up

The Metropolis Events Committee is in charge of planning events for the city.

1. What might they need to consider as part of the planning process?
  
  
  
  
  
  
2. The Events Committee wants to hire a taco truck to provide food for an event.  
The taco truck charges \$150 for set-up and \$10 for each person.  
  
How much would the taco truck cost if 80 people attend the event?  
Show or explain your thinking.

## Party Planning

- 3. Situation A:** The Events Committee is thinking about hiring the taco truck again. The taco truck always charges \$150 for set-up and \$10 for each person.

- a** Complete the table to show the cost of the taco truck for different numbers of attendees.

Number of Attendees	Food Cost (\$)
80	
120	
180	
200	

- b** Write an equation to calculate the food cost,  $f$ , of any number of attendees,  $a$ .

- c** Ari thinks the relationship between food cost and attendees represents a function because each input has only one possible output.

Do you agree with Ari's claim? Explain your thinking.

## Party Planning (continued)

- 4. Situation B:** The Events Committee has a food budget, which is the maximum food cost, for each event.

**a** The budget for the next event is \$1,000. What is the maximum number of people they can feed?

**b** Complete the table to show the number of possible attendees for other budgets.

Food Cost (\$)	Number of Attendees
1,000	
1,200	
1,500	
2,000	

**c** Describe in words how to determine the number of attendees for any food cost.

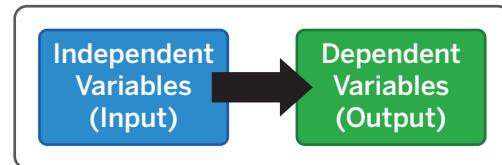
**d** Write an equation to calculate the number of attendees,  $a$ , based on the budget for the food cost,  $f$ .

**e** Does the relationship between food cost and attendees represent a function? Explain your thinking.

## Independent and Dependent Variables

In some relationships between two variables, one is called the **independent variable** and the other is called the **dependent variable**.

The independent variable is an input. The dependent variable, or output, depends on the input.



- 5. Situation A:** The Events Committee is using the equation  $f = 10 \cdot a + 150$  to calculate possible food costs for an event with  $a$  attendees. Explain why it would make sense to call the food cost,  $f$ , the dependent variable.

- 6. Situation B:** The Events Committee has raised money for the food cost for their next event and wants to know how many attendees they can host.

- a** In this situation, what do you think they should use as the independent and dependent variable?

Independent variable: .....

Dependent variable: .....

Explain your thinking.

- b** What equation might they use? Circle one.

$$a = \frac{f - 150}{10} \quad f = 10 \cdot a + 150$$

- c** **Discuss:** How did you decide which equation they might use?

## Winter Carnival

The Events Committee is planning a Winter Carnival. At the carnival, attendees will get tickets that they can exchange to play games. Each game uses 3 tickets.

- 7.** Use the bank to fill in the table. Not all variables and equations will be used and some may be used twice.

Number of Tickets, $t$	Ticket Cost, $c$	Number of Games, $g$
$c = 3 + t$	$t = 3g$	$g = \frac{t}{3}$

	How many games can you play with 28 tickets?	How many tickets should you purchase if you want to play 9 games?
Independent Variable		
Dependent Variable		
Equation		

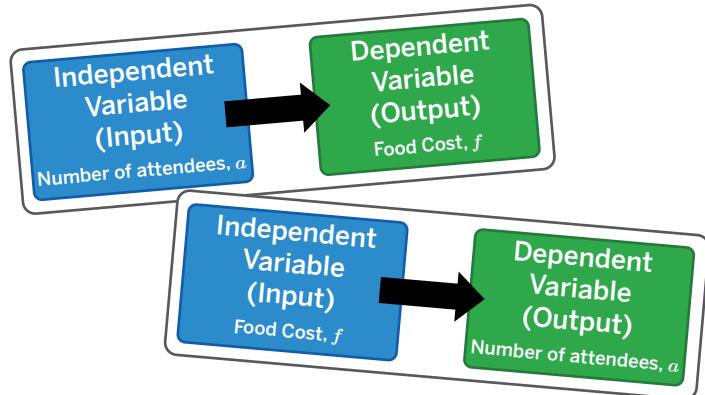
- 8.**  **Discuss:** How did you choose which variables were independent and dependent?

### Explore More

- 9.** Create your own situation where the dependent variable is the amount of space needed to hold an event.

## Synthesis

10. How can you determine which variable is independent in a situation? Which is dependent? Use the example if it helps with your thinking.

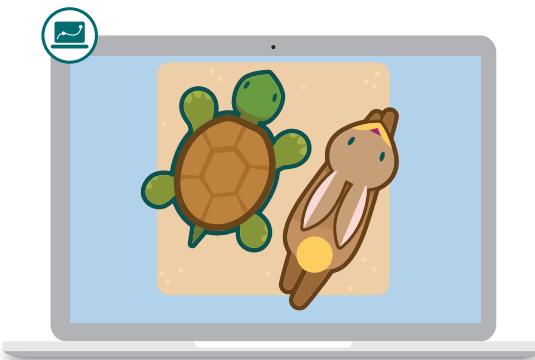


Things to Remember:

Name: ..... Date: ..... Period: .....

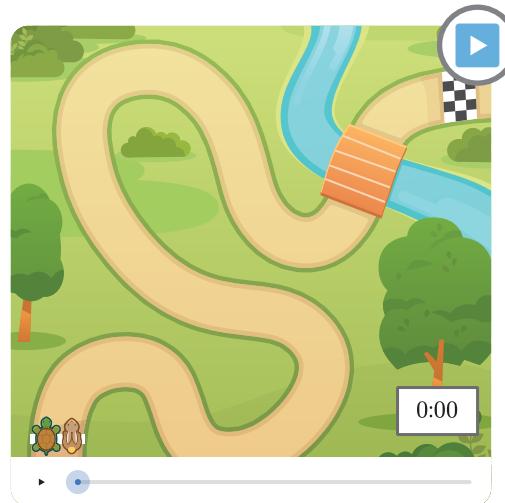
# The Tortoise and the Hare

Let's interpret the graph of a function in context.

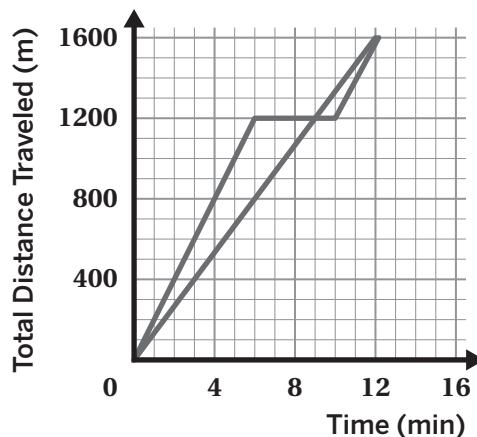


## Warm-Up

- 1** Let's watch an animation of the tortoise and the hare. Then tell a story based on what you see.



- 2** This graph shows two functions representing the relationship between distance and time: one for the tortoise and one for the hare.
- Which animal does each function represent? Label the graph with "Tortoise" and "Hare."
  - Explain your thinking.

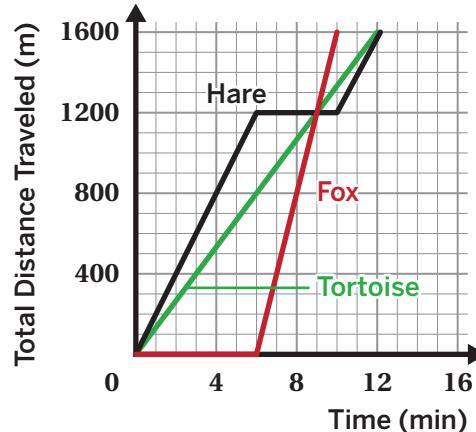


## The Tortoise, the Hare, and the Fox

- 3** This graph shows the relationship between distance and time for a third animal, the fox.

Tell a story about the fox's journey during the race.

Include specific details about time and distance.



- 4** Let's watch an animation of the race.



**Discuss:** How does your story compare to the actual race?

- 5** Here are five statements about the race. Select *all* the true statements.

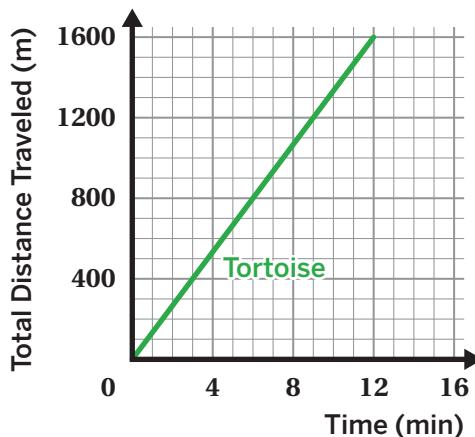
- A. The fox's distance was always increasing.
- B. Between 6 and 10 minutes, the fox was traveling faster than the other animals.
- C. When the hare reached 800 meters, the fox was still at the starting line.
- D. The graph of the tortoise represents a function, but the other two graphs do not represent functions.
- E. All three graphs represent functions.

## The Tortoise and the Dog

- 6** Next, the tortoise raced a dog.

Draw a graph representing the relationship between distance and time for a dog that makes *all* of these statements true:

- The dog got a head start but lost the race.
- The dog and tortoise were tied at 800 meters.
- The dog's distance was decreasing for 3 minutes.



- 7** Let's watch an animation of the race described in Problem 6 and a graph that was made to represent it.

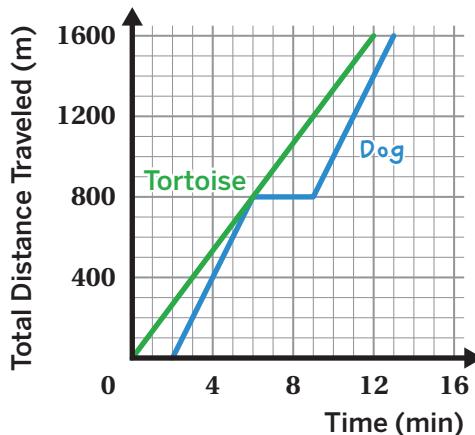
Revise your graph for Problem 6, as needed.

- 8** The blue graph shows what Ash drew to represent the dog's race.

At least one of the following statements is false. Circle a false statement.

- The dog got a head start but lost the race.
- The dog and tortoise were tied at 800 meters.
- The dog's distance was decreasing for 3 minutes.

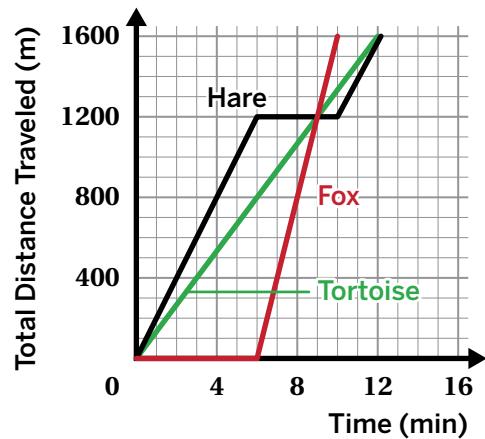
Explain your thinking.



## **9** Synthesis

What can a function's graph tell us about a situation?

Use the example if it helps with your thinking.

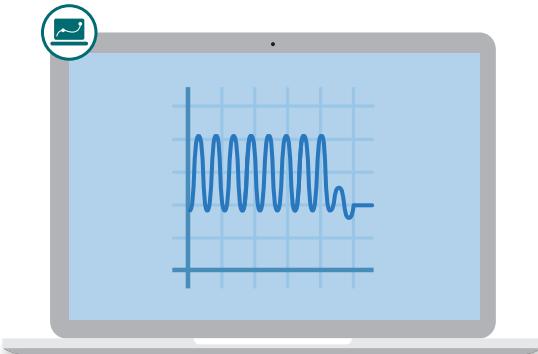


Things to Remember:

Name: ..... Date: ..... Period: .....

# Graphing Stories

Let's make connections between scenarios and the graphs that represent them.



## Warm-Up

- 1** Clem loves to play on the playground. Let's watch a short video of Clem on the swings. What different quantities are changing in this video?

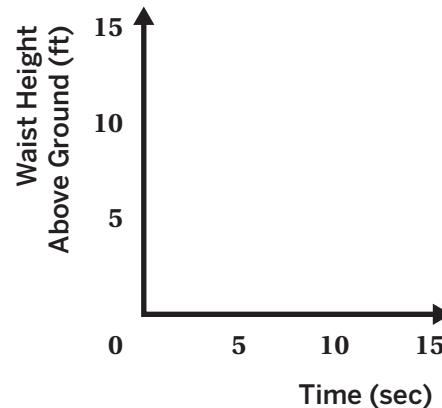
- 2** Let's look at two possible graphs to represent this situation.

 **Discuss:** How are these graphs similar? How are they different?

## Tyler on the Slide

- 3** Let's watch an animation of Tyler on the slide.

Sketch a graph representing the relationship between Tyler's waist height and time.

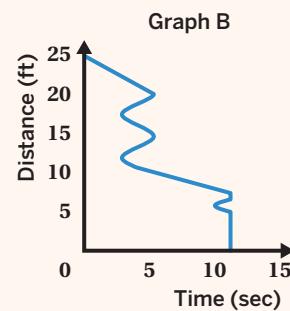
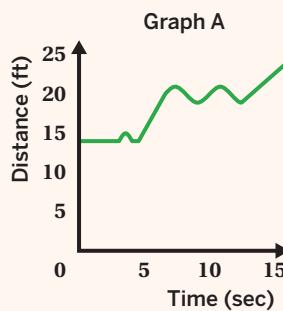


- 4** Let's watch an animation of a graph representing the relationship between Tyler's waist height above the ground and time.

**Discuss:** How does this graph represent the situation? How might you revise this graph?

### Explore More

- 5** Use the digital activity to watch the video of Tyler again. Which graph could represent the relationship between Tyler's distance from the right edge of the screen and time?

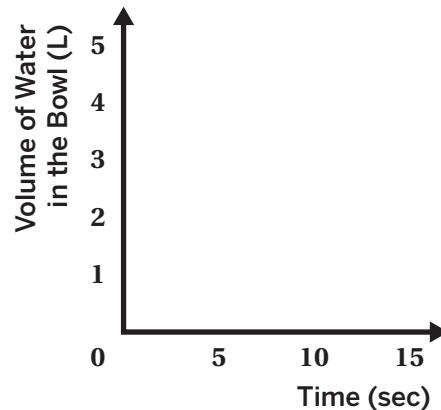


Explain your thinking.

## Water in a Bowl

- 6** Let's watch an animation of water in a bowl.

Sketch a graph representing the relationship between the volume of water in the 5-liter bowl and time.



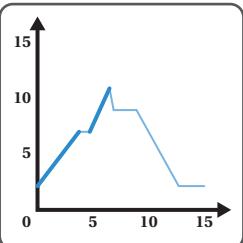
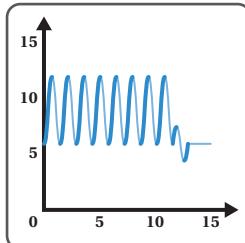
- 7** Let's watch an animation of Cielo's graph representing the relationship between the volume of water in the bowl and time.

 **Discuss:** How does Cielo's graph represent the situation? How might you revise this graph?

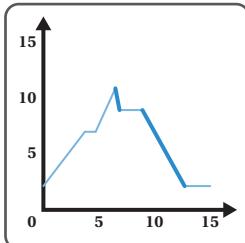
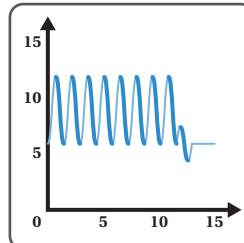
## Describing Graphs

- 8** Here are some graphs from this lesson. Parts of the graphs are bolded to show where they are either increasing, decreasing, *linear*, or non-linear.

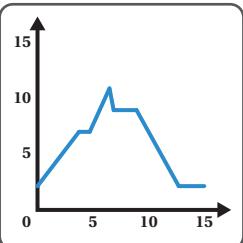
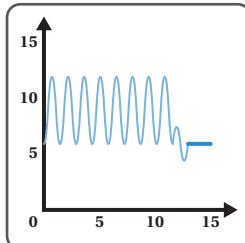
Increasing



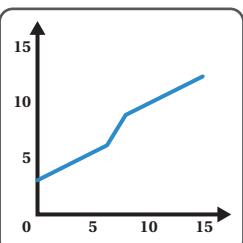
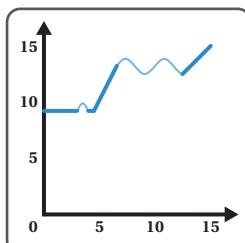
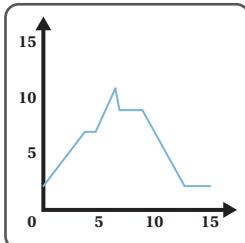
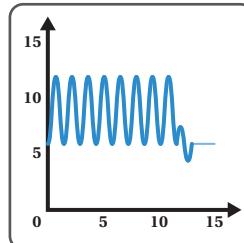
Decreasing



Linear



Non-Linear

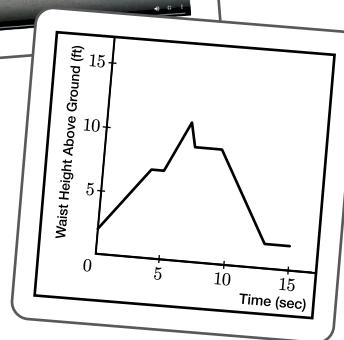


**Discuss:** What do you think each term means?

## **9** Synthesis

What can be helpful to consider when graphing a function that represents a real-world situation?

Use the example if it helps with your thinking.



Things to Remember:

# Comparing Linear Functions

Let's compare linear functions represented in different ways.



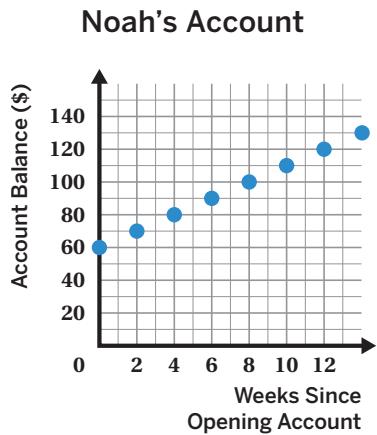
## Warm-Up

Jada has \$50 in a savings account and saves \$7 per week.

1. What could the independent variable and dependent variable be in this situation? Explain your thinking.
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
2. Write an equation representing this situation. Be sure to define the variables that you use.
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
3. Is your relationship from Problem 2 a function? Explain your thinking.

## Which Is Growing Faster?

Noah and Azul both opened a savings account on the same day. Each person's account balance is a function of the weeks since the account was opened. Here is some information about each account.



### Azul's Account

$a = 8w + 60$ , where  $w$  is the number of weeks since the account was opened, and  $a$  is the account balance

4. Who started with more money in their account? Circle one.

Noah

Azul

They started with the same amount.

Explain your thinking.

5. Who is saving money at a faster rate? Circle one.

Noah

Azul

They are saving at the same rate.

Explain your thinking.

6. How much will Noah save over the course of a year if he does not make any withdrawals?

(Note: There are 52 weeks in a year.) Show or explain your thinking.

7. How long will it take Azul to save the same amount? Show or explain your thinking.

## Making Deposits vs. Withdrawals

Take a look at the accounts of four customers. They each have an account balance,  $a$ , measured over  $w$  weeks.

### Account A

The account balance,  $a$ , is represented by the function  $a = 65 + 10w$ , where  $w$  represents the number of weeks since the account was opened.

### Account B

The account balance starts at \$40 and decreases by \$8.50 per week.

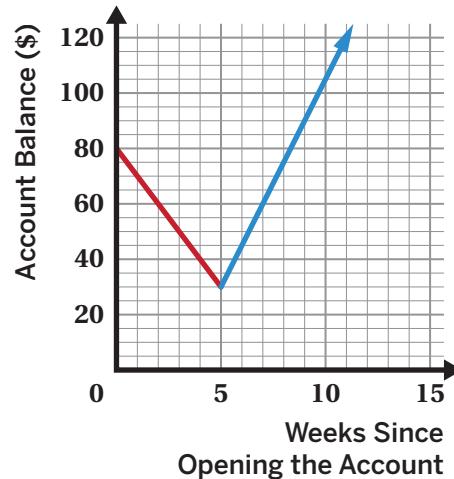
### Account C

The account balance is represented by this table:

Number of Weeks	Account Balance (\$)
1	71
3	48
7	2

### Account D

The account balance is represented by this graph:



8. **Discuss:**

- a** Which account(s) show customers making deposits? Explain your thinking.
  
  
  
  
- b** Which account(s) show customers making withdrawals? Explain your thinking.

## Making Deposits vs. Withdrawals (continued)

**9.** Kiana says that all four relationships are **linear functions** and that each situation can be represented with one equation in the form  $y = mx + b$ . Is Kiana's claim correct? Explain your thinking.

**10.** Which account will have the most money at the end of a year? Explain your thinking.

**11. a** How much money did Account A have after one year?

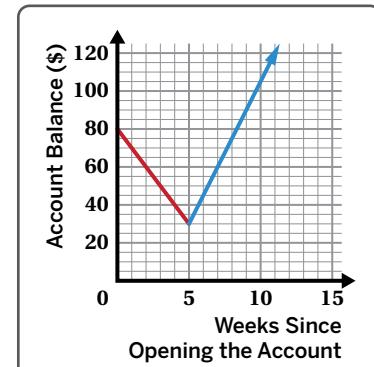
**b** How long did it take Account D to have the same amount?

## Synthesis

12. How can you compare linear functions shown using different representations?

Number of Weeks	Account Balance (\$)
1	71
3	48
7	2

$$a = 65 + 10w$$



Things to Remember:

Name: ..... Date: ..... Period: .....

## Piecing It Together

Let's create functions to model data sets.



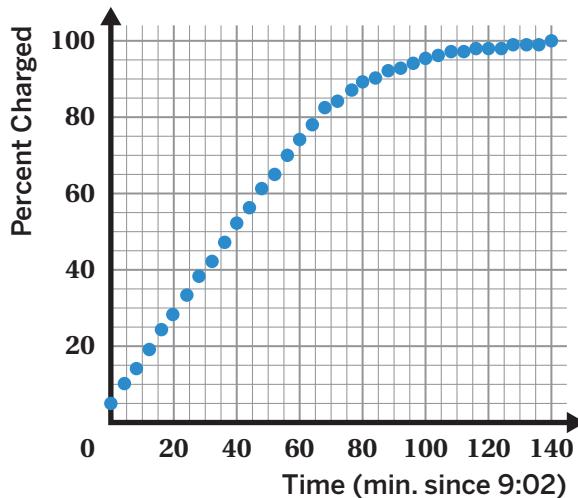
### Warm-Up

- 1** Here is a data set for a phone charging over time.

A single linear function does not model this relationship very well.

- a** Sketch two connected line segments to model the relationship better.

- b** When was the phone charging the slowest? Explain your thinking.

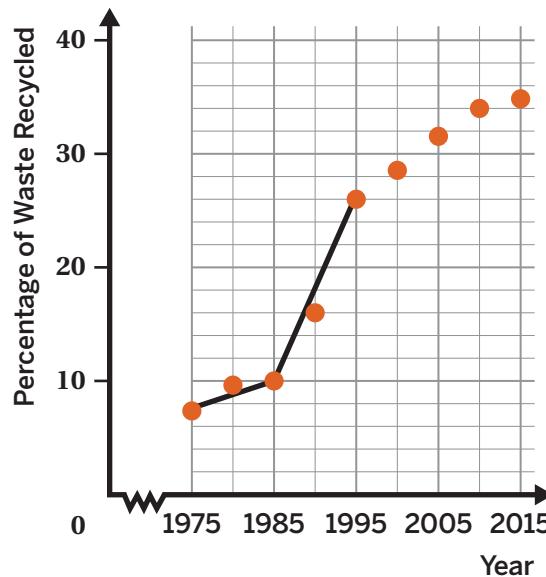


## Recycling

- 2** This data set shows the percentage of waste produced in the United States that gets recycled over time.

A student started sketching a function to model this data.

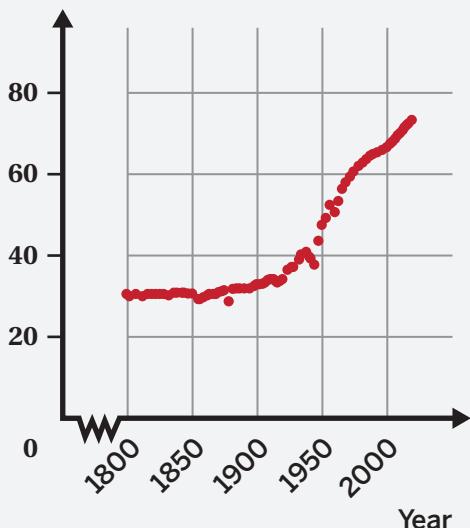
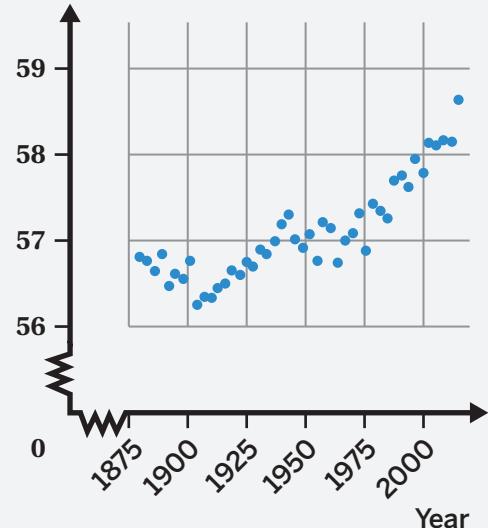
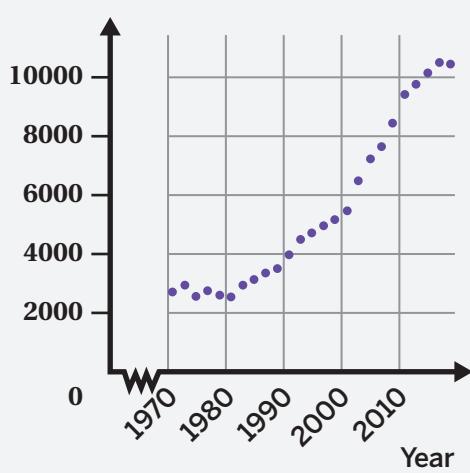
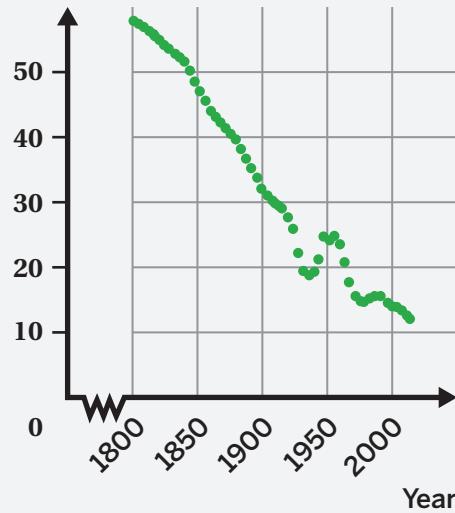
- a** Sketch one more linear segment to complete the function.
- b** Approximate the slope of the segment you created.



Explain what this number means about the percentage of waste recycled.

## Four Data Sets

- 3** Here are four new data sets. Match each graph to the description you think it represents.

**Graph A****Graph B****Graph C****Graph D**

U.S. College Cost (\$)

U.S. Births (per 1000 people)

Global Life Expectancy (age in years)

Global Temperature (°F)

## Analyzing Data Sets

- 4** Look at the four data sets on the Activity 3 Sheet. Pick one that interests you.

Which data set are you choosing?

What do you notice? What do you wonder?

I notice:

I wonder:

- 5** **a** Using at least two line segments, sketch a function on the graph of the data set you chose on the Activity 3 Sheet to model the data.

- b** Describe your function using vocabulary from this unit.

linear	increasing
non-linear	decreasing

- 6** During which interval of time did the data seem to change the most?

## Analyzing Data Sets (continued)

**7**

- a** Use your function to make a prediction for the year 2030.

- b** Do you think your function can be used to make an accurate prediction for the year 2050? Explain your thinking.

**8**

- Share your findings with a student or a group that examined a different data set.

**Discuss:**

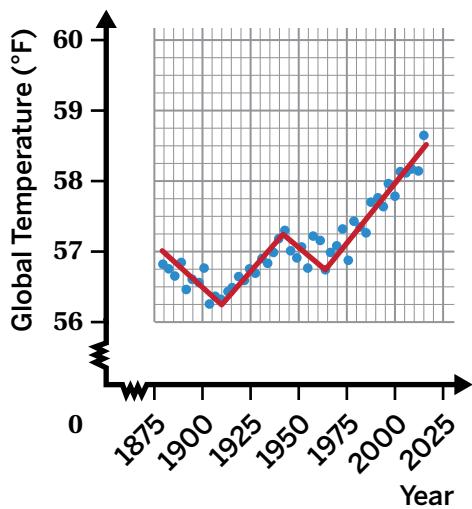
- What choices did you make as you sketched your function?
- What is your prediction for 2030, and how did you arrive at your prediction?
- Do you think your function can be used to make an accurate prediction for 2050?

## **9** Synthesis

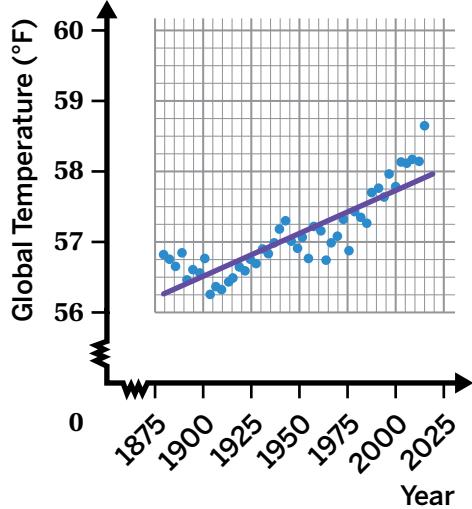
When is it helpful to model a data set with multiple linear segments instead of a single linear segment?

Use the examples if they help with your thinking.

**Function A:** Multiple Segments



**Function B:** Single Segment



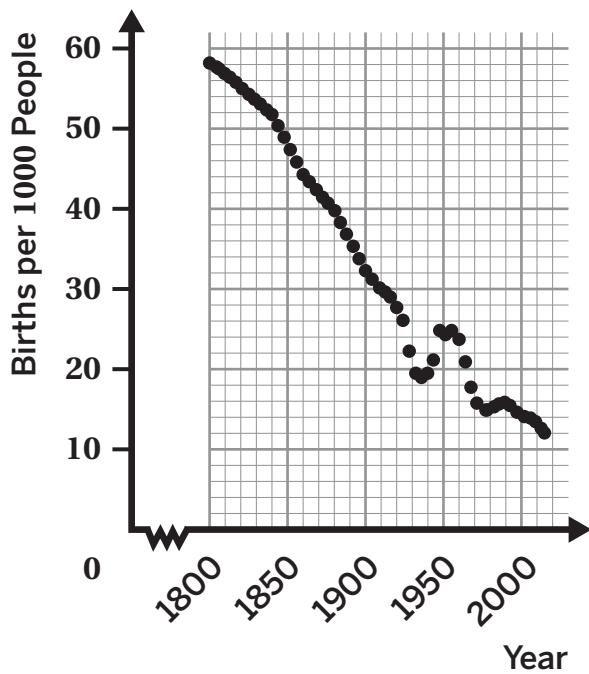
Things to Remember:

Name: ..... Date: ..... Period: .....

# Analyzing Data Sets

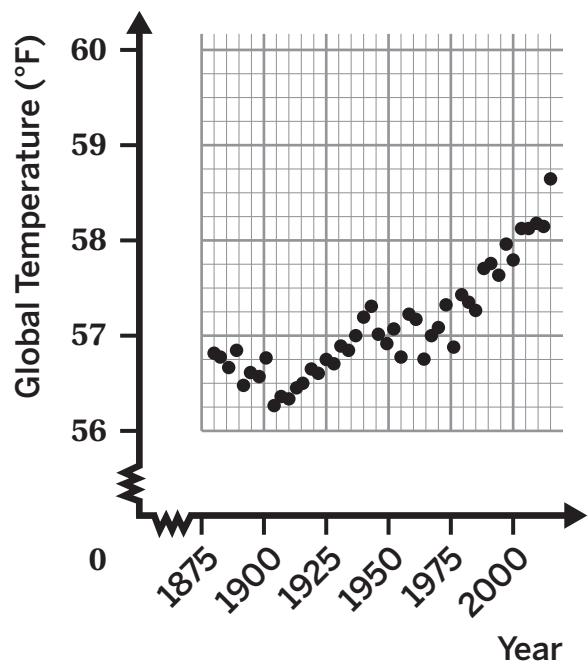
Choose one of the data sets to analyze and record your findings in your Student Edition.

Data Set A: U.S. Births



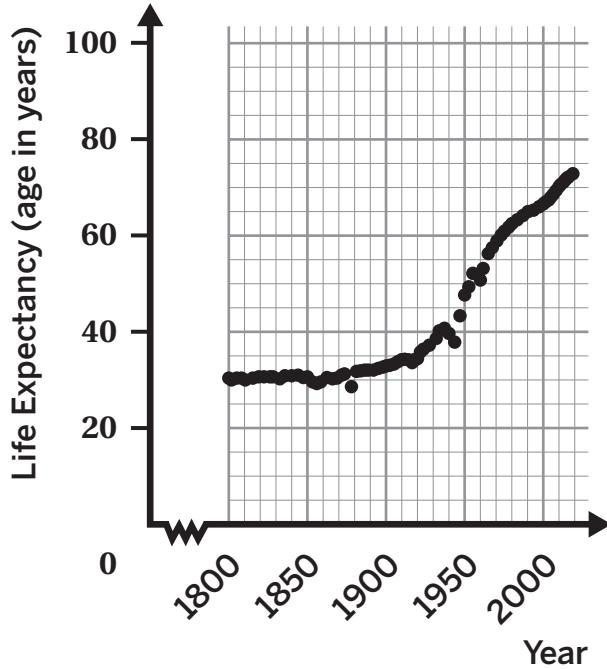
Source: United Nations, World Population Prospects

Data Set B: Global Temperature



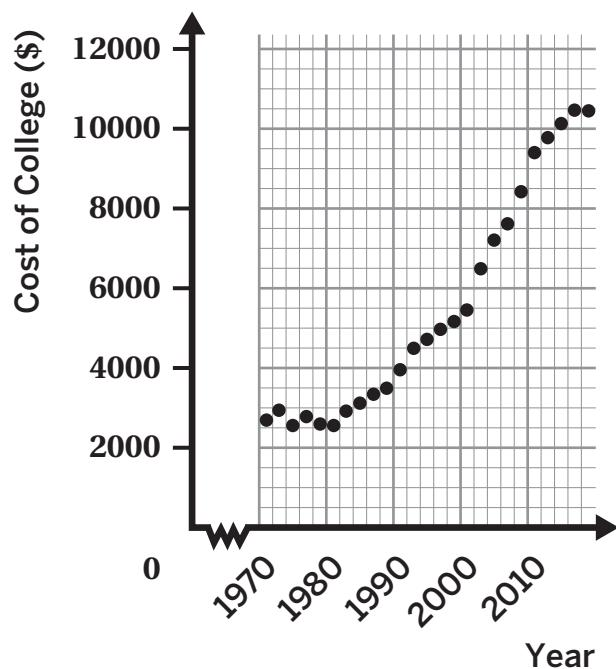
Source: NOAA, Climate at a Glance

Data Set C: Global Life Expectancy



Source: Gapminder, Life Expectancy at Birth

Data Set D: U.S. College Cost



Source: CollegeBoard, Trends in Higher Education

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_



# Click Battle

Let's find ways to show patterns in data.

## **Warm-Up**

- 1** Tap your pencil on your desk as many times as you can for 2 seconds.  
Record your number of taps.
  - 2** Tap your pencil on your desk as many times as you can for 6 seconds.  
Record your number of taps.

## Organizing Data

**3** Let's look at some class data about button clicks, which are similar to pencil taps.

**a** Organize the data in a way that makes sense to you.

**b**  **Discuss:** What patterns do you see in the data?

**4** Let's look at one way to represent the data. What do you notice?

**5** Let's look at another way to represent the data.

 **Discuss:** What connections do you see between the scatter plot and the table?

## Make a Prediction

Here is click data organized as a list, table, and scatter plot.

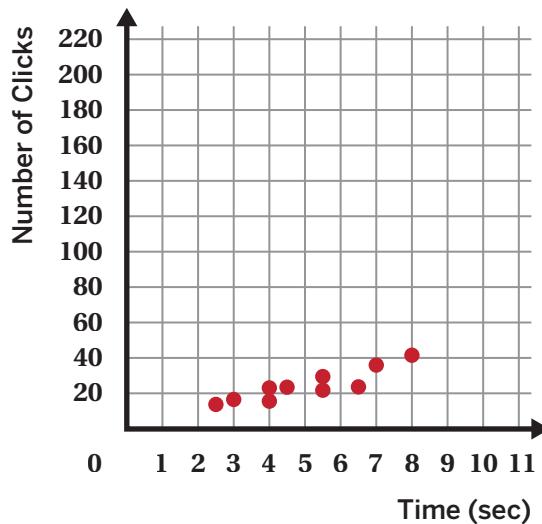
**List**

- |                          |                          |
|--------------------------|--------------------------|
| 14 clicks in 2.5 seconds | 24 clicks in 6.5 seconds |
| 17 clicks in 3 seconds   | 30 clicks in 5.5 seconds |
| 16 clicks in 4 seconds   | 24 clicks in 4.5 seconds |
| 42 clicks in 8 seconds   | 36 clicks in 7 seconds   |
| 22 clicks in 5.5 seconds | 23 clicks in 4 seconds   |

**Table**

Time (sec)	Number of Clicks
2.5	14
3	17
4	16
4	23
4.5	24
5.5	22
5.5	30
6.5	24
7	36
8	42

**Scatter Plot**



- 6** Select a representation and use it to answer this question:  
*How many clicks do you think a typical student in your class would make in 10 seconds?*

Explain your thinking.

- 7** Test your prediction by counting the number of pencil taps you can make in 10 seconds.

## **8** Synthesis

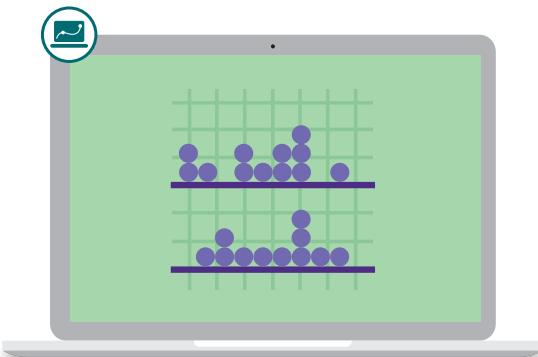
What are some advantages of using a list, a table, or a scatter plot to organize data?

Things to Remember:

Name: ..... Date: ..... Period: .....

# Wingspan

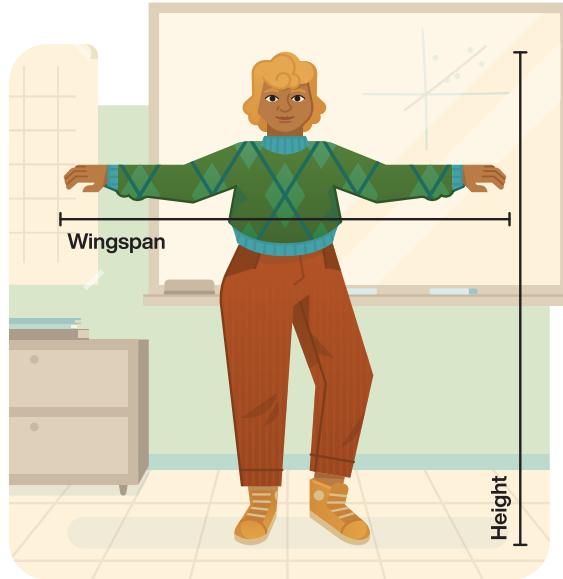
Let's compare dot plots and scatter plots.



## Warm-Up

- 1** With a partner, measure your height and wingspan to the nearest inch.

Height (in.)	Wingspan (in.)



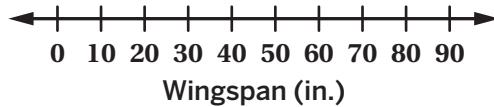
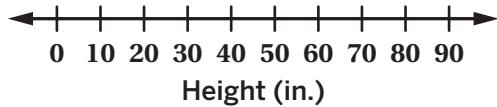
- 2** Let's look at a table of height and wingspan data.

**Discuss:**

- How could we reorganize the data to make it more useful for analyzing?
- What are some questions this data could help answer?

# Visualizing Data

- 3** Plot points on the *dot plots* to represent your height and wingspan.



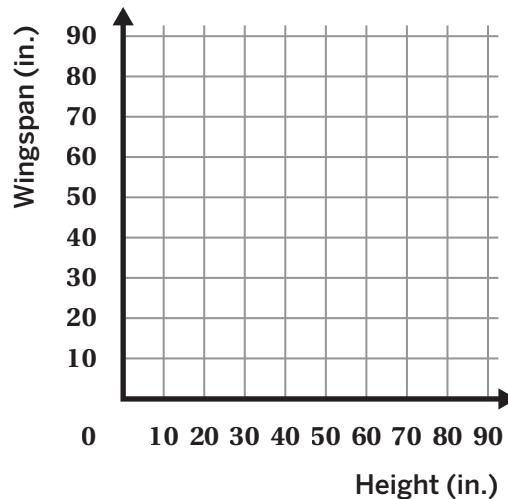
- ## 4 Let's look at dot plots of height and wingspan data.

- a) What is a question you can answer based on the dot plots?

- b** What is a question you cannot answer based on the dot plots?

## Visualizing Data (continued)

- 5** Plot a point to represent your height and wingspan.



- 6** Let's look at a scatter plot that represents height and wingspan data.

**a** What is a question you can answer based on the scatter plot?

**b** What is a question you cannot answer based on the scatter plot?

### Explore More

- 7** Use the digital activity to examine the heights, weights, wingspans, and hand lengths of professional basketball players.

What do you notice? What do you wonder?

## **8** Synthesis

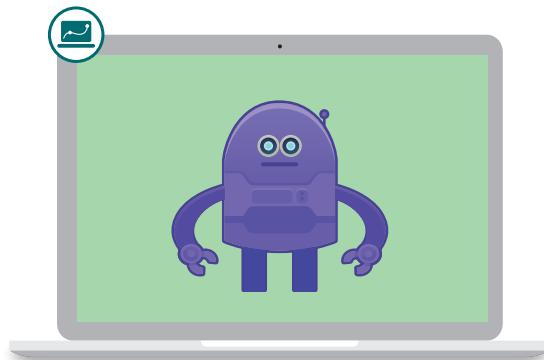
The height and wingspan data from a different class are shown in the Summary below. What are some advantages of using a dot plot or a scatter plot to represent data? Use the examples if they help with your thinking.

Things to Remember:

Name: ..... Date: ..... Period: .....

# Robots

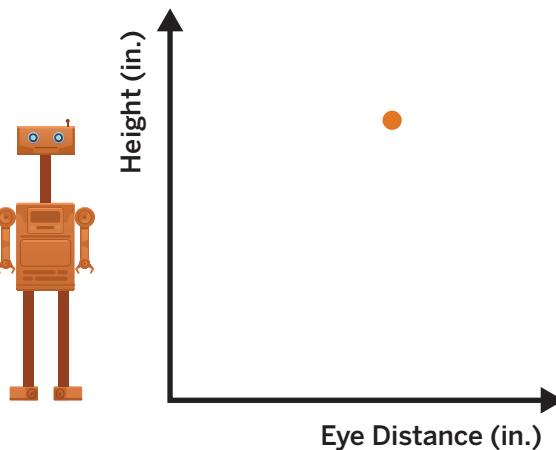
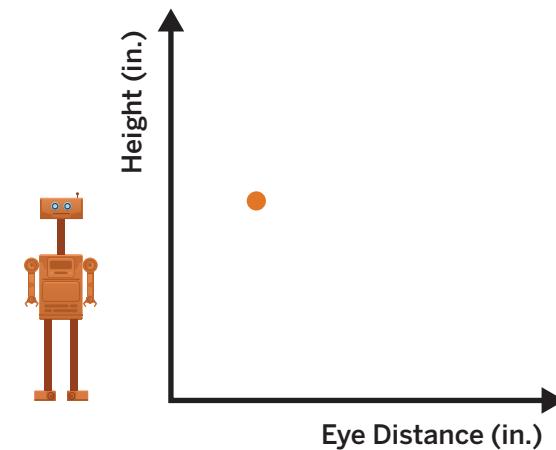
Let's investigate points on a scatter plot.



## Warm-Up

- 1 Here are two graphs and images of a robot.

**Discuss:** What do you notice?

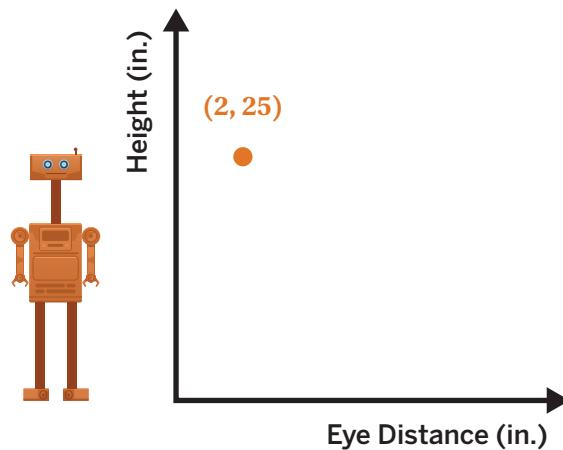


Activity  
**1**

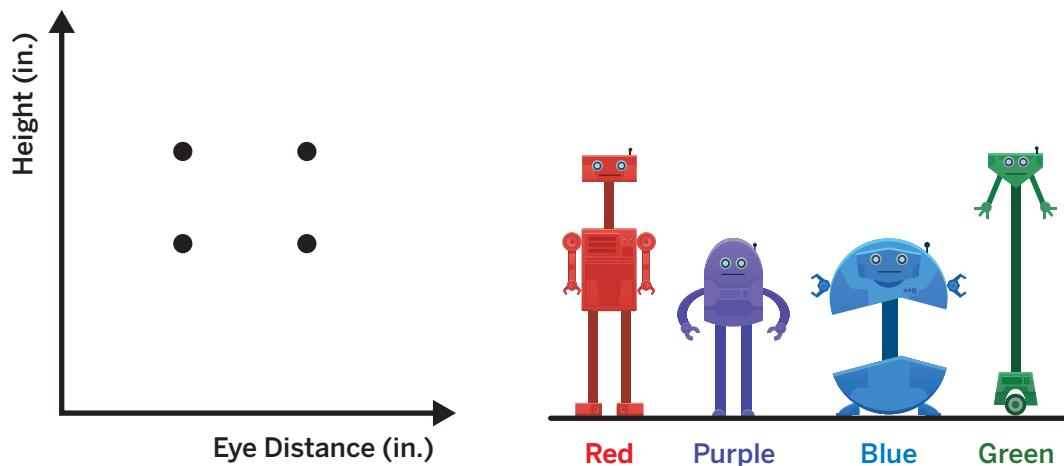
Name: ..... Date: ..... Period: .....

## Robots

- 2** Describe something you know about the robot based on the graph.



- 3** Here are four different robots. Label each point on the graph with the color robot it represents.



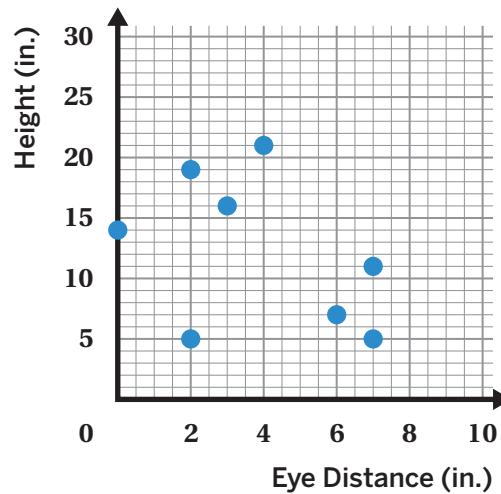
**Robots (continued)**

- 4** The table shows the heights and eye distances for five different robots.  
Plot a point to represent the pink robot.



Robot Color	Eye Distance (in.)	Height (in.)
Teal	2	30
Black	4	10
Gray	8	10
Orange	6	20
Pink	8	20

- 5** The graph shows the heights and eye distances for eight blue robots.  
Plot a point for a red robot to make this statement true: *The red robot is taller than all the blue robots, and its eye distance is 5 inches.*



Activity  
**2**

Name: ..... Date: ..... Period: .....

## Challenge Creator

- 6** You will use a set of cards with scatter plots to create your own challenge.

- a** **Make It!** Choose one card that interests you and plot a point somewhere you think is interesting.
- b** **Solve It!** On this page, write the point as an ordered pair. Then tell a story about this point.

My Point	My Story

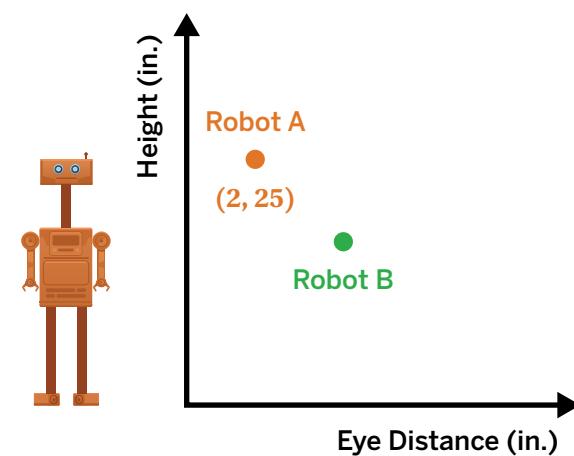
- c** **Swap It!** Swap your challenge with one or more partners. Write the point they plotted as an ordered pair and tell a story about it.

	Point	Story
Partner 1		
Partner 2		
Partner 3		

## 7 Synthesis

This graph shows the height and eye distance for two robots.

Describe some things you know about Robot B given the information about Robot A.



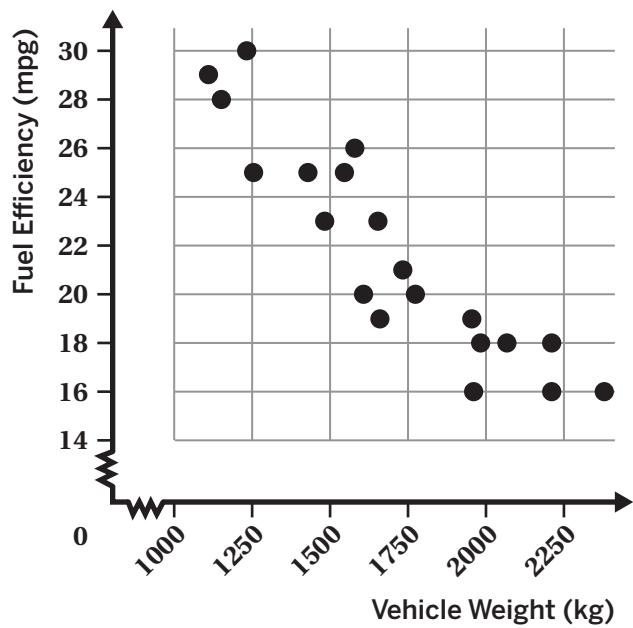
Things to Remember:

# Challenge Creator

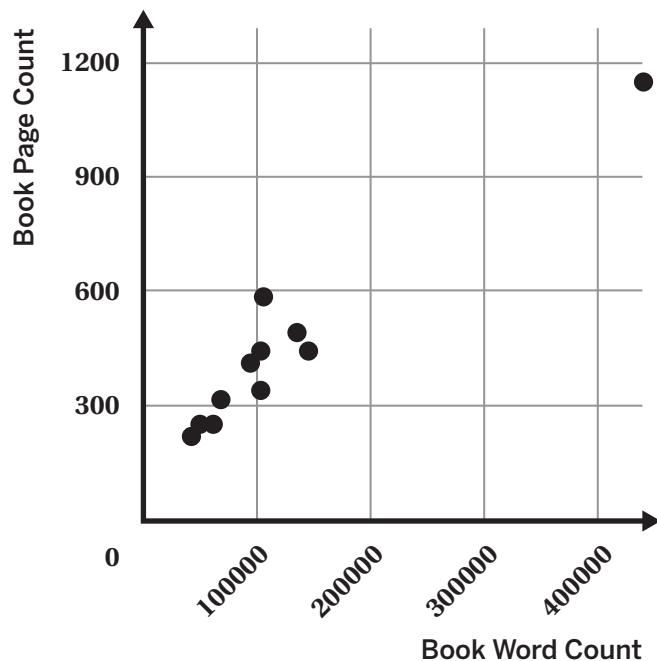
 **Directions:** Make one copy for each group of students. Then pre-cut the cards and give each group one set.

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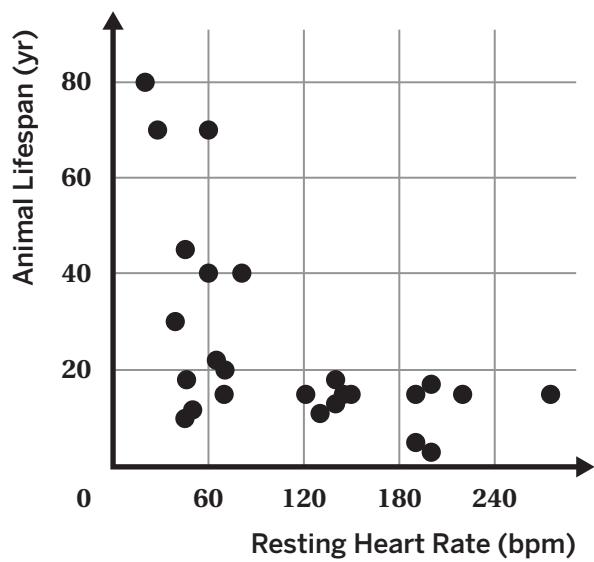
## Fuel Efficiency



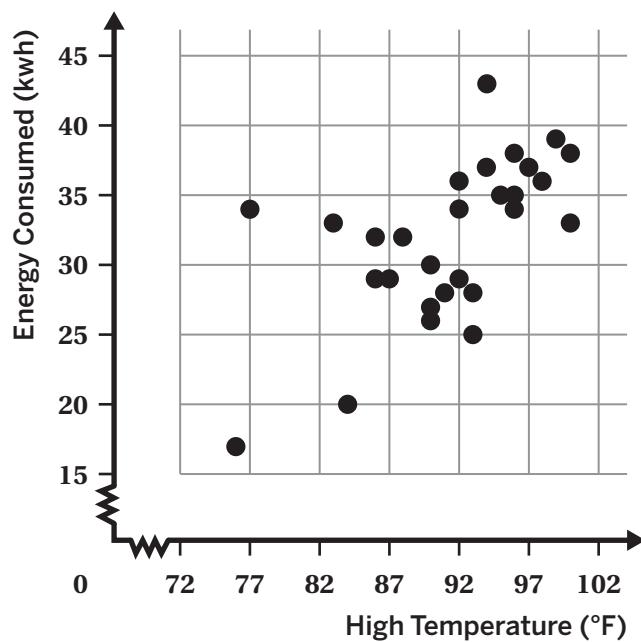
## Book Page Count



## Animal Lifespan



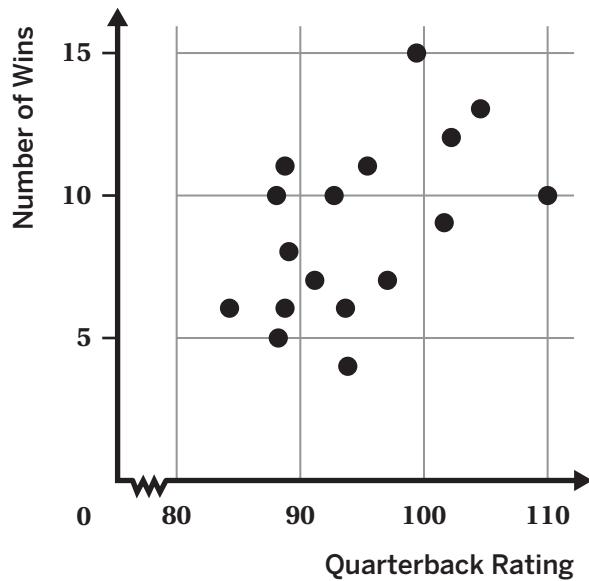
## Energy Use



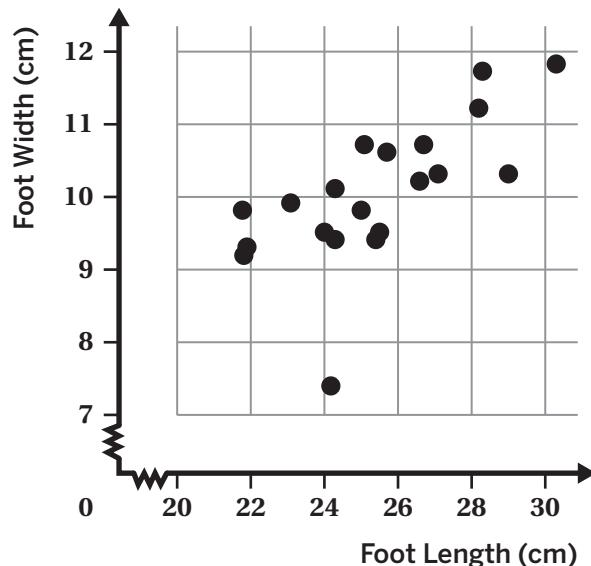
# Challenge Creator

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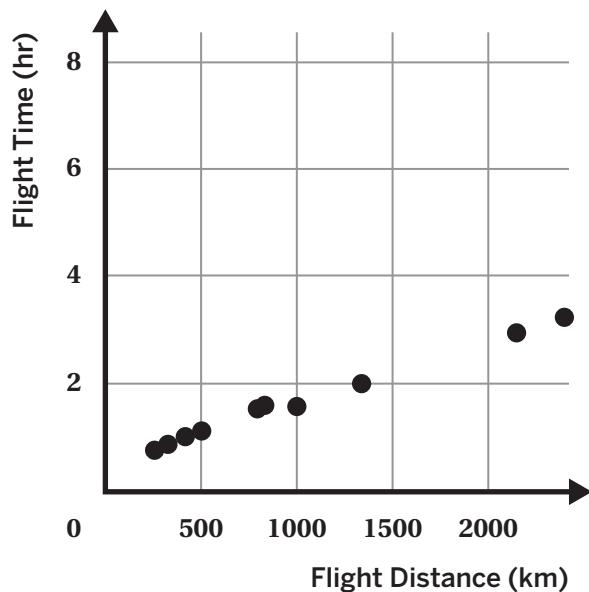
## Quarterback Wins



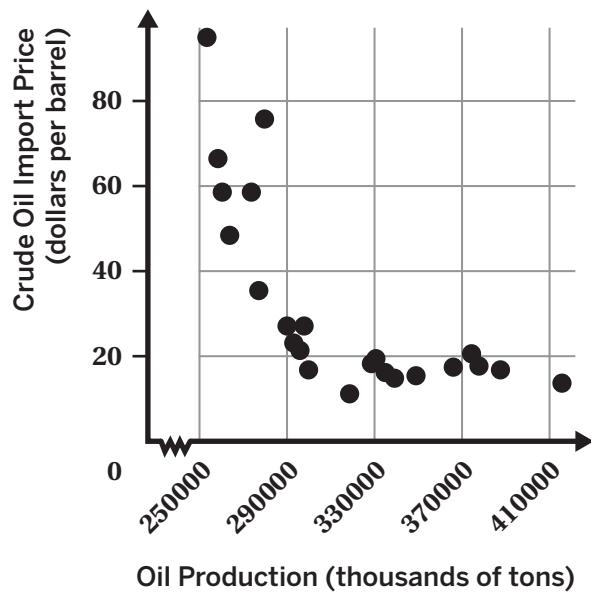
## Foot Width



## Flight Time



## Oil Imports



Name: ..... Date: ..... Period: .....

# Dapper Cats

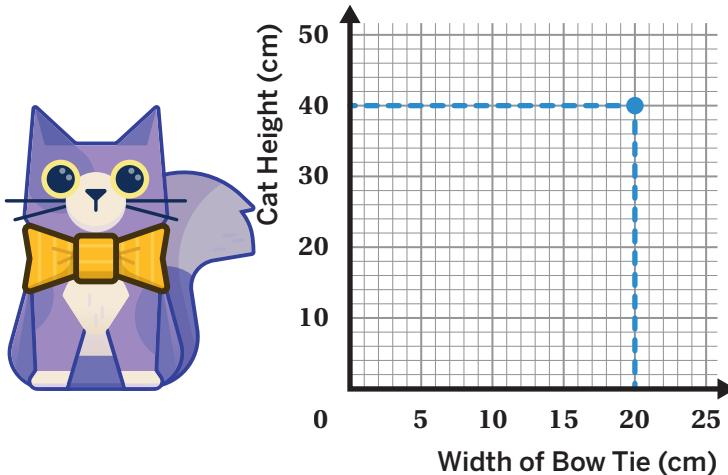
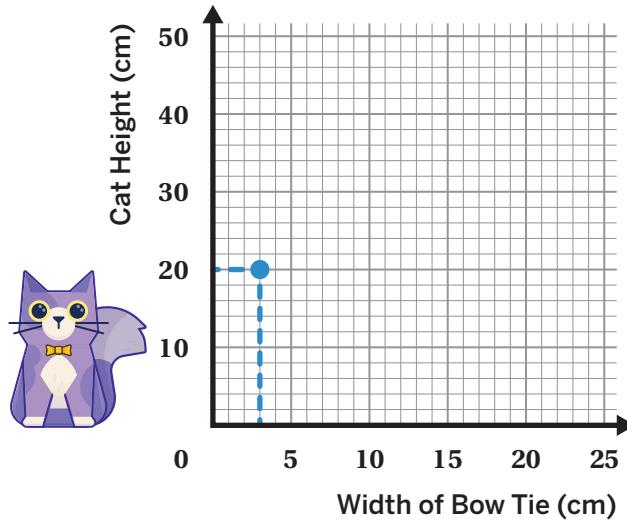
Let's identify potential outliers and use a linear model to predict values.



## Warm-Up

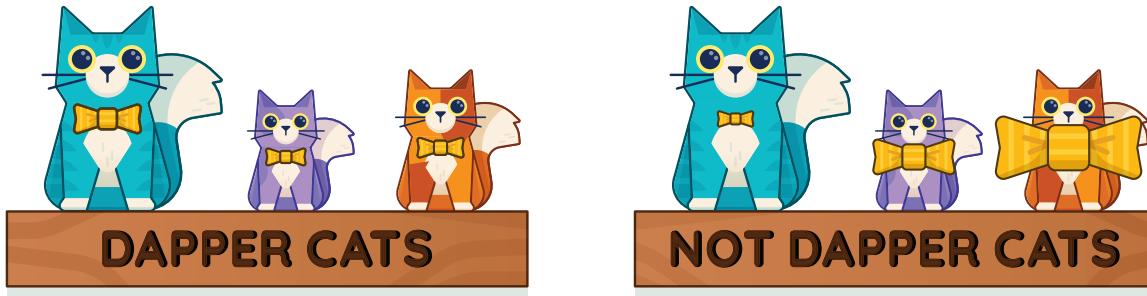
- 1 Here are two toy cats built at the Build-a-Cat workshop.

 **Discuss:** What do you notice?  
What do you wonder?



## Dapper Cats

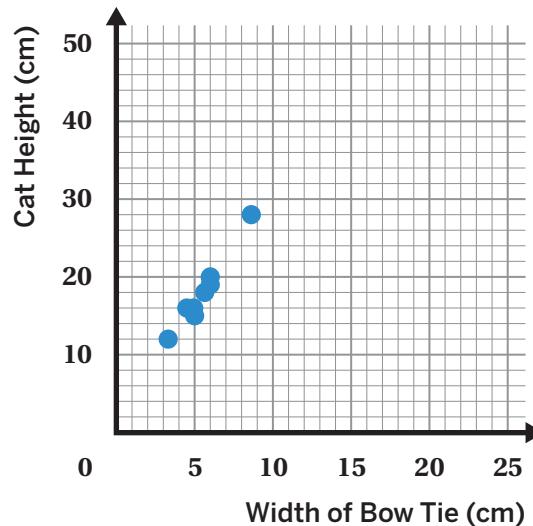
- 2** Some of these toy cats are dapper and some are not.



What do you think makes a toy cat “dapper”? What makes a toy cat “not dapper”?

- 3** This scatter plot shows many “dapper cat” orders.

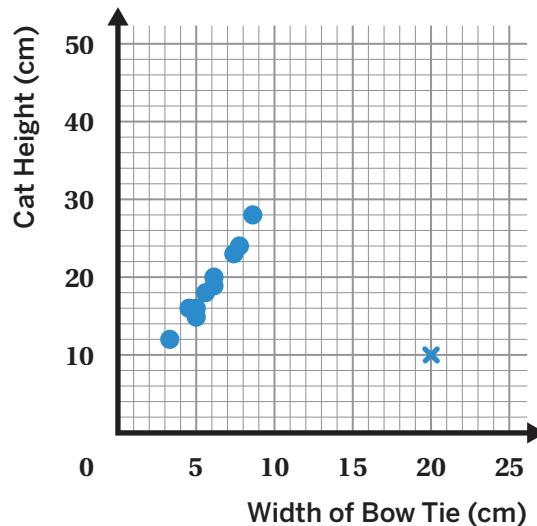
A customer just ordered a dapper cat that is 24 centimeters tall. Plot a point on the scatter plot to represent this cat.



- 4** Describe what you notice about the scatter plot.

## Dapper Cats (continued)

- 5** Plot a point to represent a dapper cat that is taller than any of the other cats in the scatter plot.



- 6** Plot a point to represent a not-dapper cat that is very short with a very large bow tie.

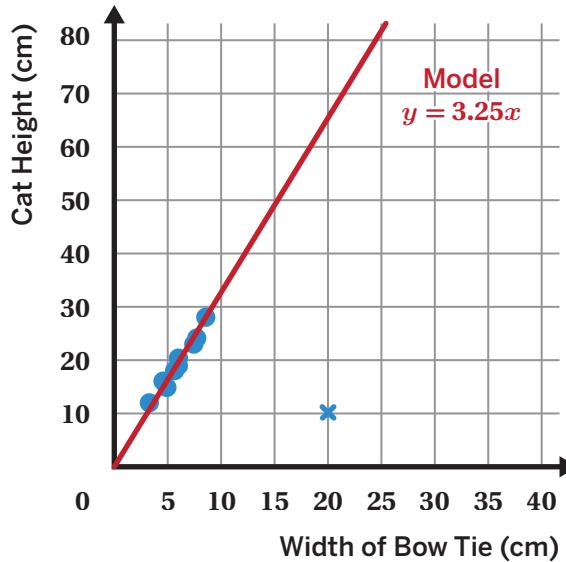
- 7** A student plotted the point  $(20, 10)$ . It is shown on the scatter plot with an X. This point is an **outlier** because it is far from the rest of the data.

- a** Plot another outlier on the scatter plot.
- b** Describe the cat your point represents.

## Using a Linear Model to Predict Data

A **linear model** (also called a *line of fit*) can be used to help identify trends in data and to make predictions. The line and equation model the relationship between bow tie width,  $x$ , and cat height,  $y$ .

- 8–9** Use the linear model to predict the height of a dapper cat with a 12-centimeter bow tie.



- 10–11** Another dapper cat is 65 centimeters tall. Use the linear model to predict the width of its bow tie.

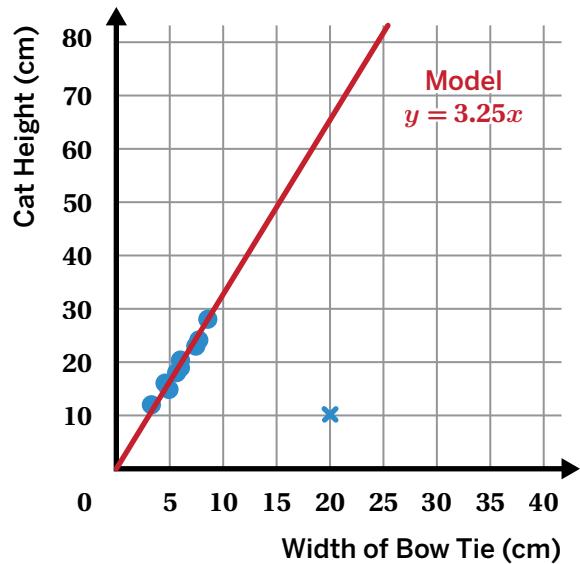
- 12** Chey's cat has a 60-centimeter bow tie width. What does the linear model predict for its height?

Explain your thinking.

## 13 Synthesis

A line of fit and an equation are two ways to represent a linear model.

- a How can a linear model be helpful?
  
- b How can you identify an outlier on a scatter plot?



Things to Remember:

# Interpreting Scatter Plots

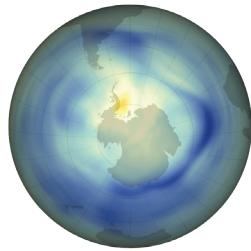
Let's find ways to show and identify patterns in data.



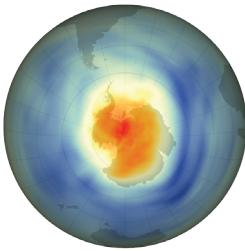
## Warm-Up

1. The images show the hole in the ozone layer over Antarctica. The size and shape of the hole are monitored yearly. What do you notice? What do you wonder?

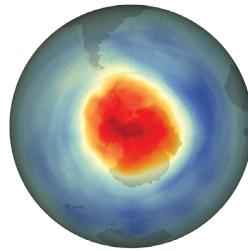
October 16, 1980



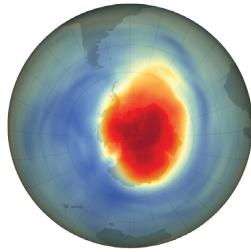
October 3, 1984



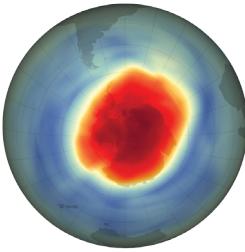
October 7, 1989



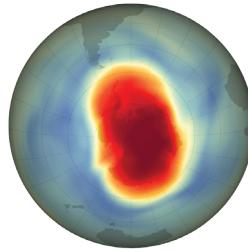
October 11, 1992



October 6, 1998



October 6, 2006



"World of Change: Antarctic Ozone Hole", NASA Earth Observatory.

I notice:

I wonder:

## Creating a Scatter Plot

NASA records the area of the hole in the ozone layer between September and October every year.

The Australian government records the number of new cases of skin cancer in Australia.

This table shows some of the data these groups collected between 1982 and 2008.

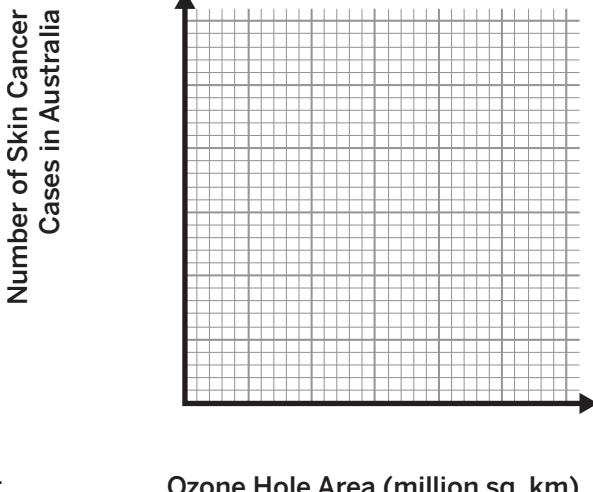
- 2.**  **Discuss:** What are some questions you can ask about this situation?

Year	Ozone Hole Area (million sq. km)	Number of Skin Cancer Cases in Australia
1982	5	3,541
1986	11	4,712
1988	10	6,013
1991	19	5,970
1997	22	8,444
2001	25	9,000
2005	24	10,832
2006	27	10,427
2007	22	10,450
2008	25	11,135

Sources: NASA Earth Observatory and Cancer Australia

- 3.** Let's make a scatter plot of this data.

- a** Create a scale for the graph so it fits all of the data.
- b** Create a scatter plot of the data.



- 4.** In 2021, the hole in the ozone layer was 23 million square kilometers. Use the graph to estimate the number of new skin cancer cases in Australia in 2021.

- 5.** A linear model for this data is  $y = 334x + 1706$ . Use the model to predict the number of new skin cancer cases in Australia in 2021. Show your thinking.

## Scavenger Hunt

### 6. Start at any of the Scavenger Hunt Sheets.

- Record the sheet shape, solve the problem, and write your answer.
- Look for your answer at the top of another scavenger hunt sheet. Solve that problem.
- Repeat until you make it back to your starting sheet.

<b>Sheet:</b> .....	<b>Sheet:</b> .....
<b>Answer</b> <input type="text"/>	<b>Answer</b> <input type="text"/>
<b>Sheet:</b> .....	<b>Sheet:</b> .....
<b>Answer</b> <input type="text"/>	<b>Answer</b> <input type="text"/>

*continued on next page...*

## Scavenger Hunt (continued)

Sheet: .....

Sheet: .....

**Answer****Answer**

Sheet: .....

Sheet: .....

**Answer****Answer**

Sheet: .....

Sheet: .....

**Answer****Answer**

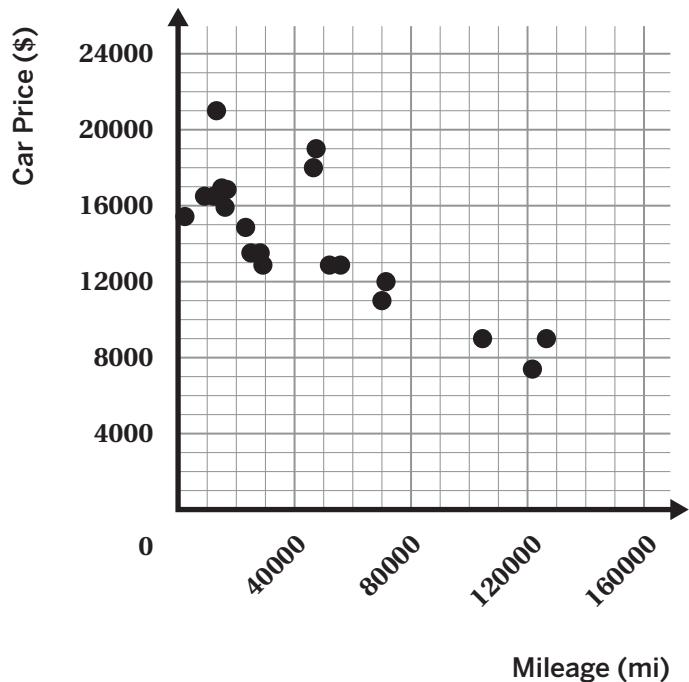
## Synthesis

7. How can a scatter plot help make sense of the relationship between two variables?  
Use the scatter plot you created in Activity 1 if it helps with your thinking.

Things to Remember:

**Scavenger Hunt**  Trapezoid Sheet**Answer****B****Problem:**

Based on this data, approximately what is the cost of a car with 60,000 miles?



# Scavenger Hunt

Circle Sheet

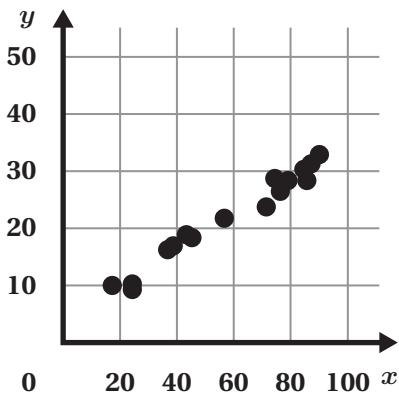
Answer

12,000

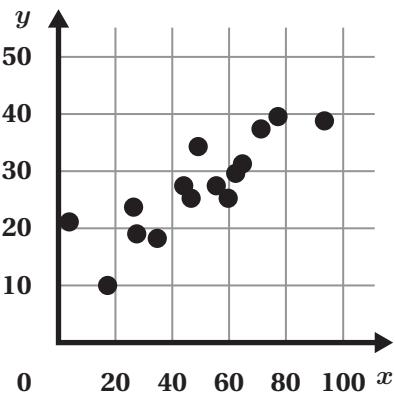
## Problem:

Which scatter plot matches the table?

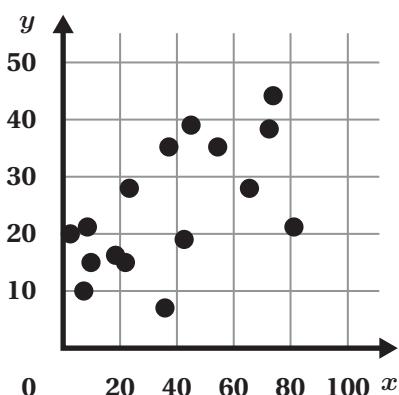
A.



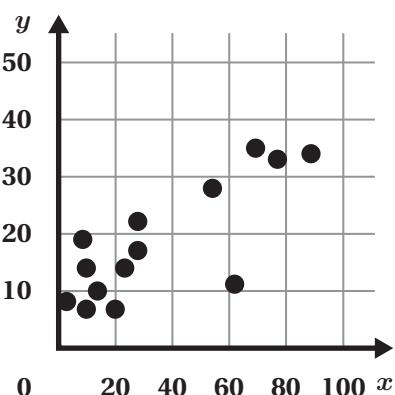
B.



C.



D.



$x$	$y$
66	28
43	18.5
3	19.5
24	28
22	15
38	35
19	15.5
74	44
10	15
81	20.5
73	37.5
55	34.5
45	38.5
36	7
9	20.5
8	10

## Scavenger Hunt Pentagon Sheet

Answer

C

### Problem:

Select *all* the representations that are appropriate for comparing top speed to engine size for five different cars.

- A. Histogram
- B. Scatter plot
- C. Dot plot
- D. Table
- E. Box plot

## Scavenger Hunt

### ★ Star Sheet

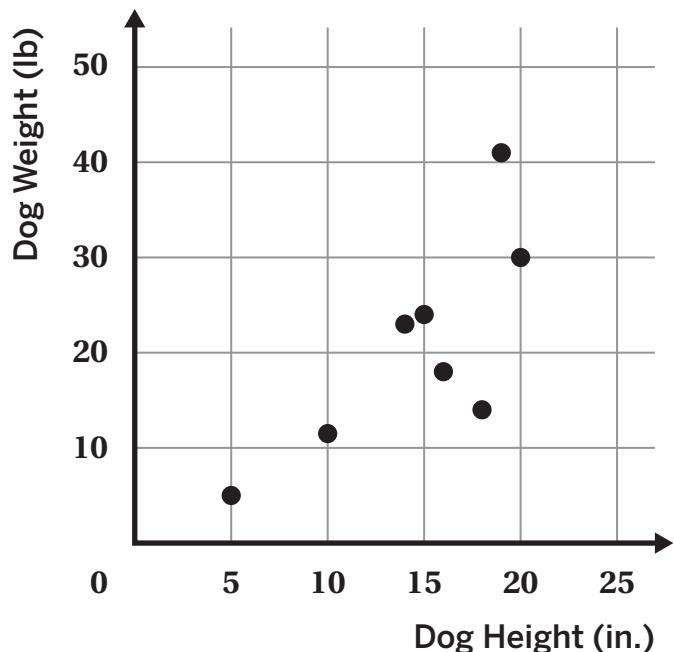
Answer

# B and D

### Problem:

Here is a scatter plot showing dog heights and weights.

What is the weight of the tallest dog on this scatter plot?



## Scavenger Hunt

Rectangle Sheet

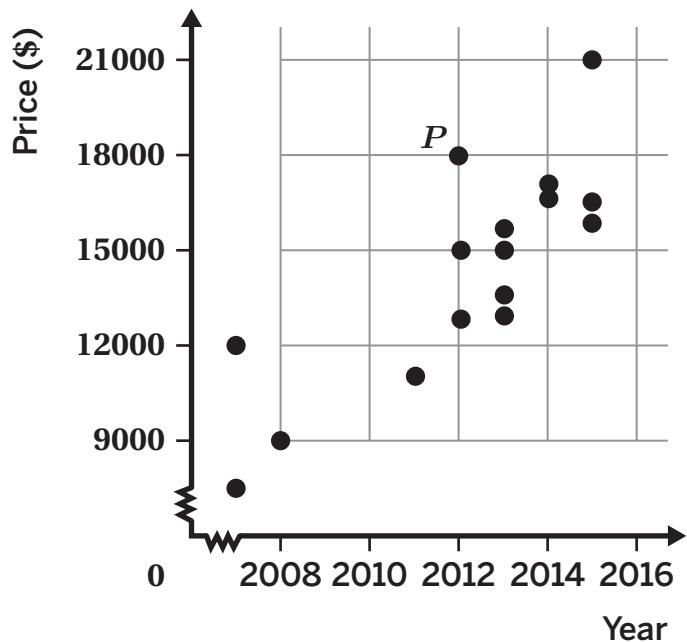
Answer

30

### Problem:

Describe point  $P$ .

- A. In the year 2012, the price was \$18,000.
- B. The price was \$2012 in the year 18,000.
- C. In the year 2012, the price was \$15,000.
- D. The price was \$18,000 in the year 2010.



## Scavenger Hunt



Triangle Sheet

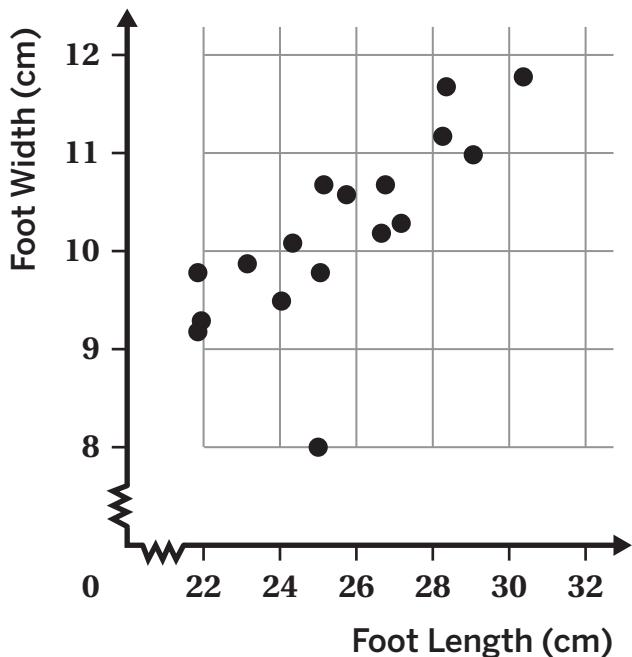
Answer

A

### Problem:

Here is a scatter plot that compares the length and width of different people's feet.

What is the foot length of the person represented by the point that is an outlier?



# Scavenger Hunt

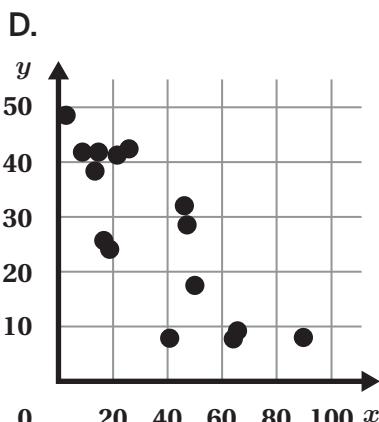
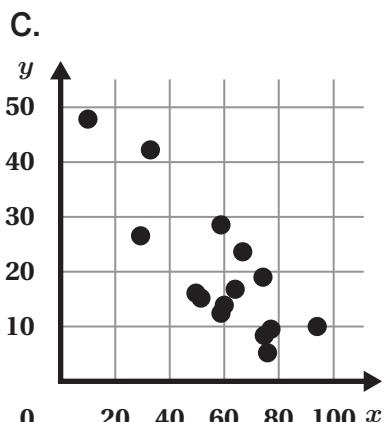
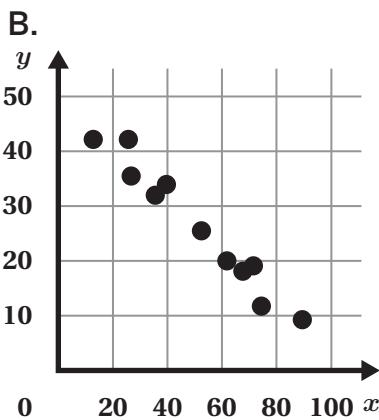
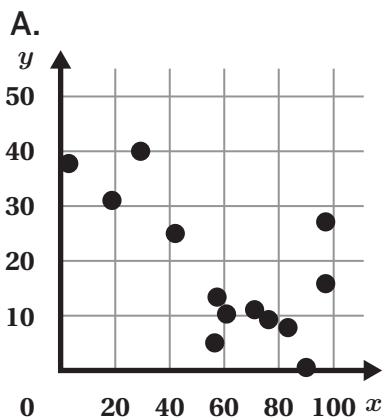
 Octagon Sheet

Answer

25

## Problem:

Which scatter plot matches the table?



Height (cm)	Foot Length (cm)
90	8
66	9
22	41.3
15	42
41	7.7
46	32.3
14	38.7
19	24.3
3	49
26	42.7
64	7.7
17	25.7
50	17.7
47	28.7
9	42

## Scavenger Hunt Crescent Sheet

Answer

D

### Problem:

Select *all* of the representations that are appropriate for comparing the amount of time spent studying and the score earned for 10 quizzes.

- A. Box plot
- B. Dot plot
- C. Histogram
- D. Scatter plot
- E. Table

## Scavenger Hunt



Hexagon Sheet

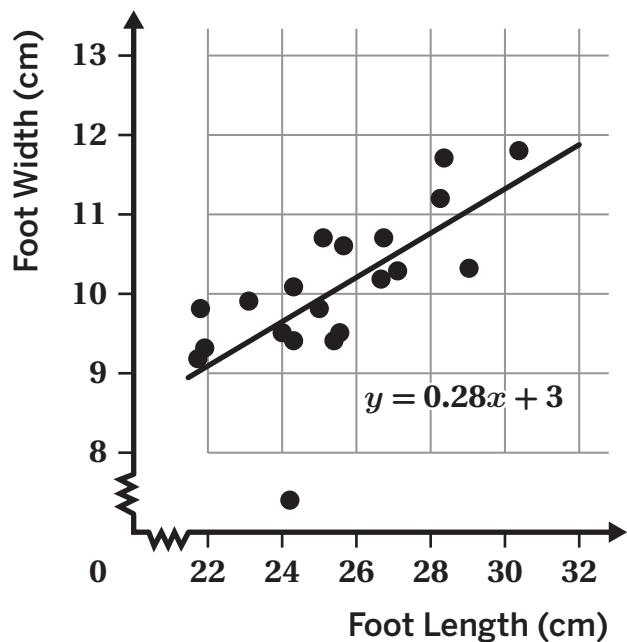
Answer

# D and E

### Problem:

This scatter plot shows the lengths and widths of 20 left feet, together with the graph of a model of the relationship between foot length and width.

Use the model to predict the width of a foot with a length of 22 centimeters.



## Scavenger Hunt

Oval Sheet

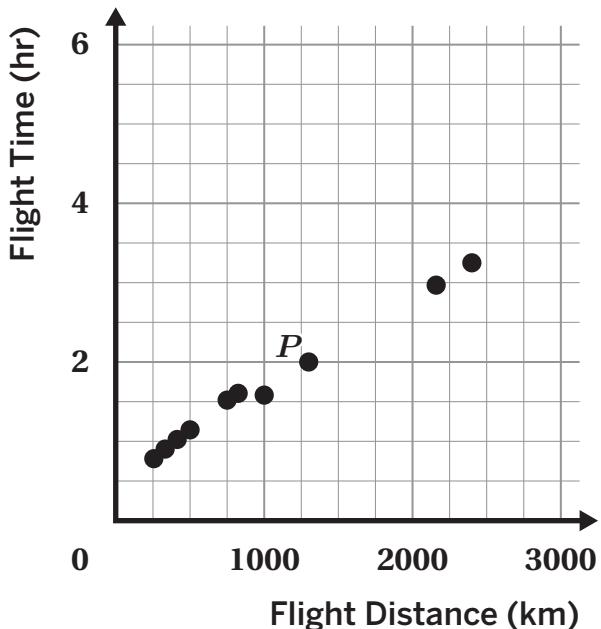
Answer

# 9.16

### Problem:

Describe point  $P$ .

- A. A flight distance of 1,500 kilometers is 2 hours long.
- B. A flight distance of 1,300 kilometers is 2 hours long.
- C. A flight distance of 1,500 kilometers is 2.5 hours long.
- D. A flight distance of 1,300 kilometers is 2.5 hours long.



Name: ..... Date: ..... Period: .....

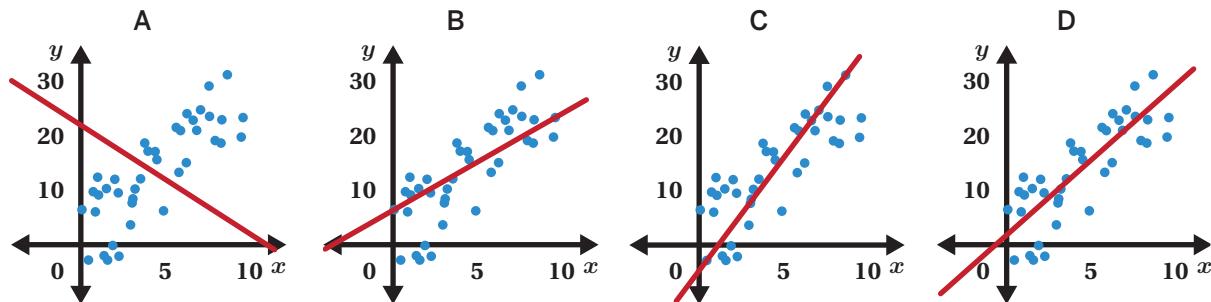
# Find the Fit

Let's fit a line to data on a scatter plot.



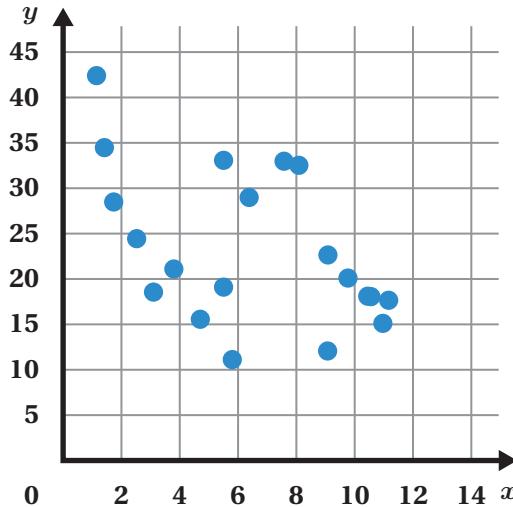
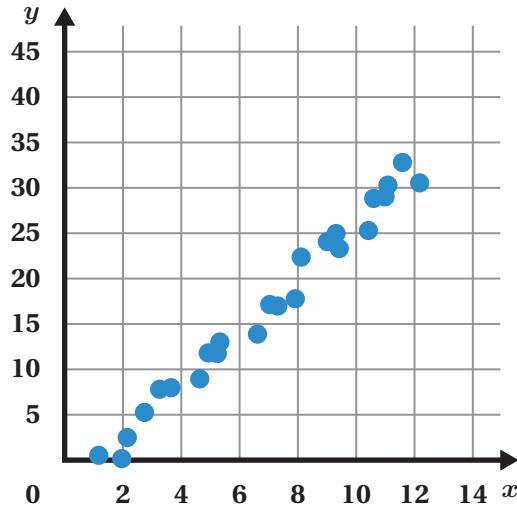
## Warm-Up

- 1 Which one doesn't belong? Explain your thinking.



## Lines of Fit

- 2** Create a line that is a good fit for each data set.



- 3** Let's look at some lines that other students created.

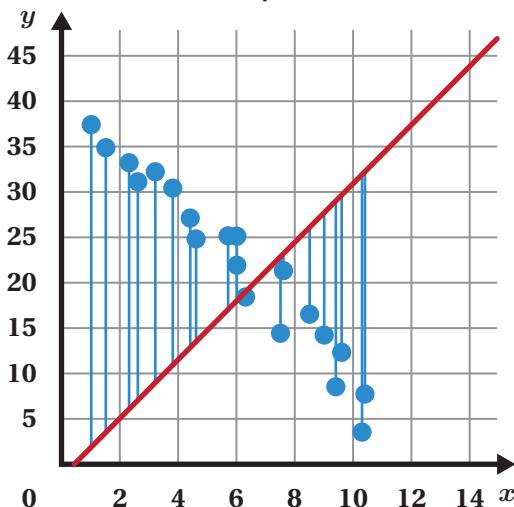


**Discuss:** How could you decide if a line is a good fit for the data?

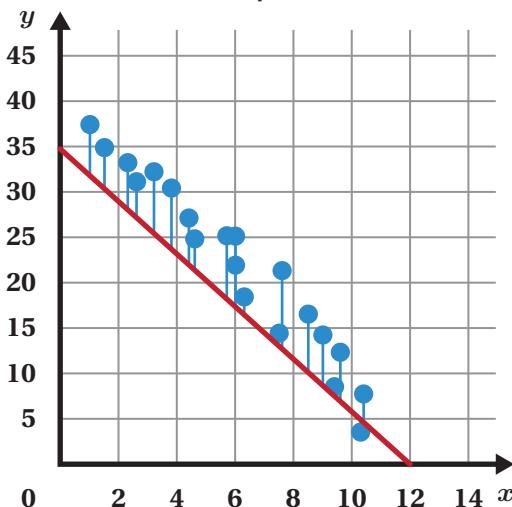
## Lines of Fit (continued)

Here are four different lines for the same scatter plot. The meter shows a score for each line.

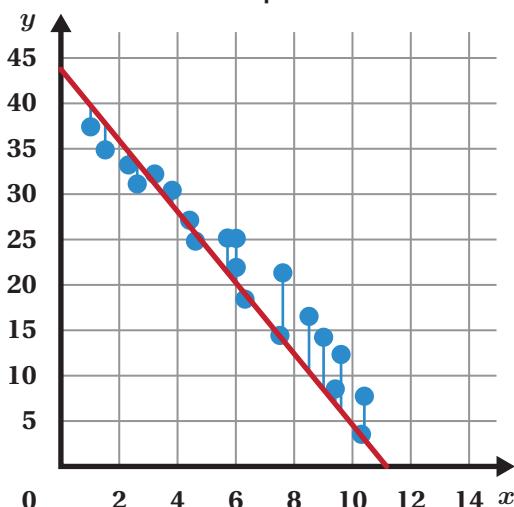
Graph A



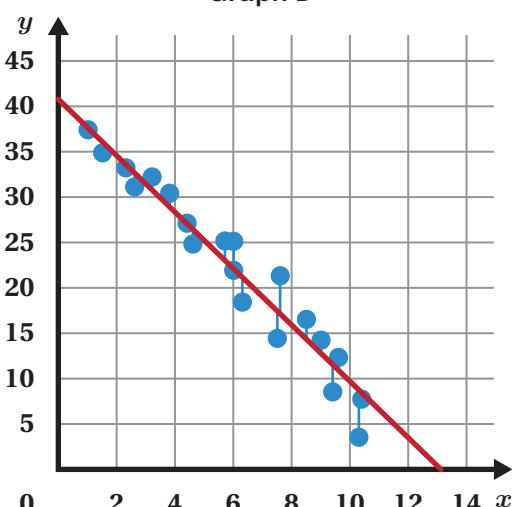
Graph B



Graph C



Graph D



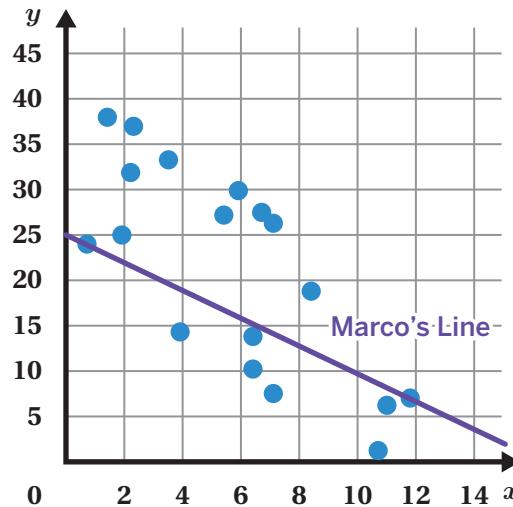
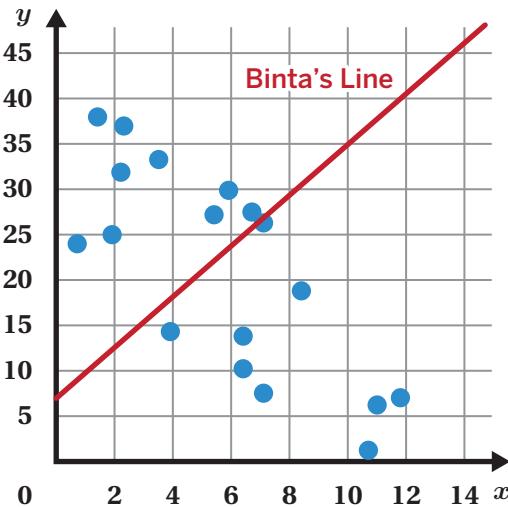
- 4** Describe how to get a high score (green) on the meter.

**Find the Fit**

- 5** Binta and Marco each sketched a *line of fit* on this scatter plot.

Binta says: *My line is a good fit because half of the points are on each side of the line.*

Marco says: *My line is a good fit because it passes through the leftmost and rightmost points.*



Whose line is a good fit for the data? Circle one.

Binta's

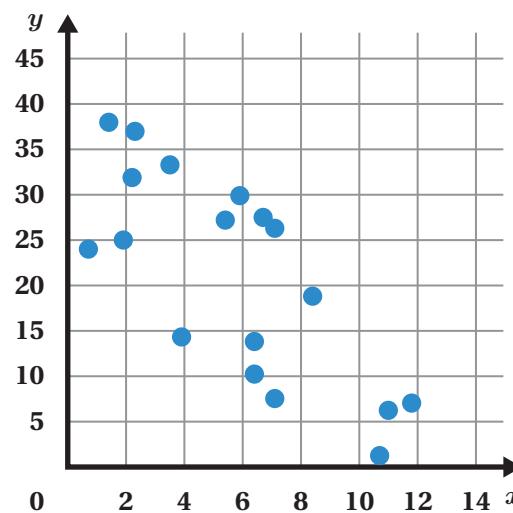
Marco's

Both

Neither

Explain your thinking.

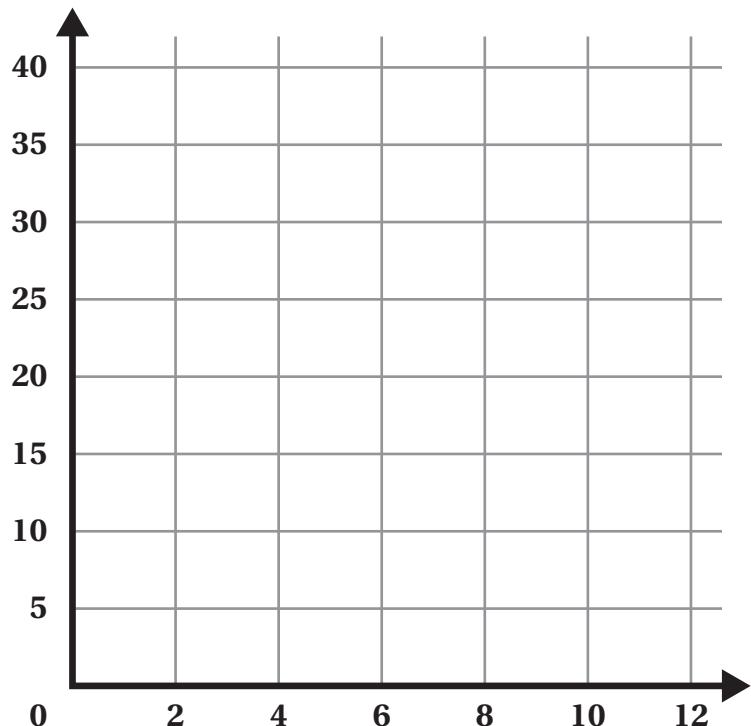
- 6** Sketch a line that fits the data from the previous problem.



## Challenge Creator

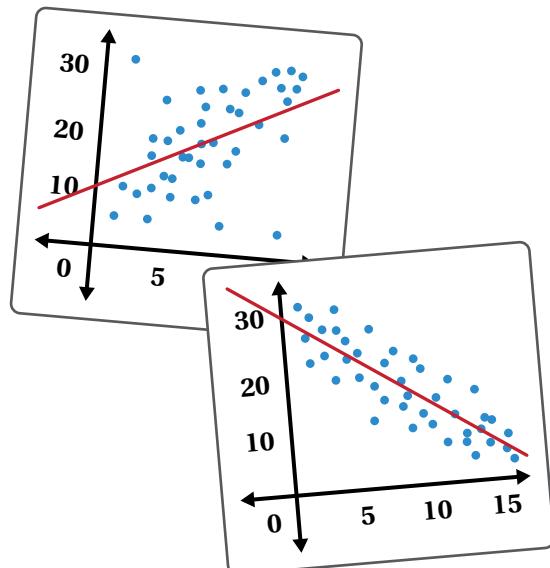
- 7** Follow these instructions to create scatter plots and solve your classmates' challenges.

- a** **Make It!** Create a scatter plot with at least ten points.
- b** **Solve It!** Use tracing paper to draw a line of fit for your data.
- c** **Swap It!** Pass your scatter plot to another student and use tracing paper to draw a line of fit for the data you received.



## 8 Synthesis

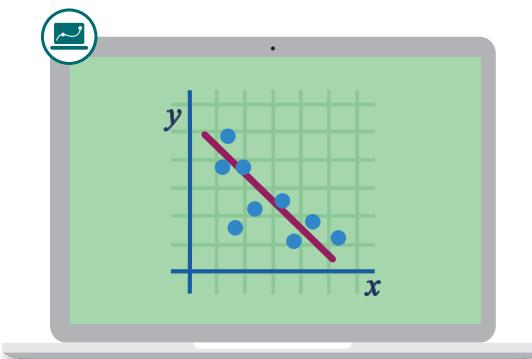
What are some things to consider when creating a line of fit? Use the examples if they help with your thinking.



Things to Remember:

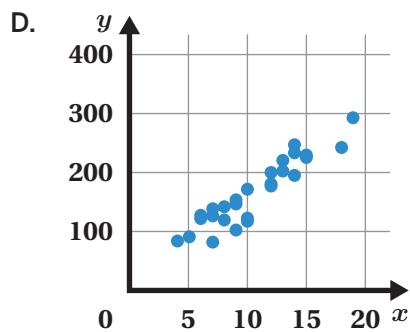
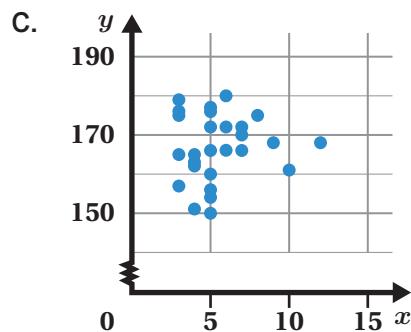
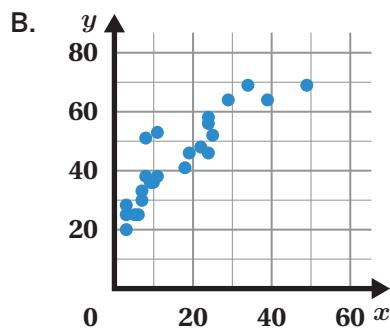
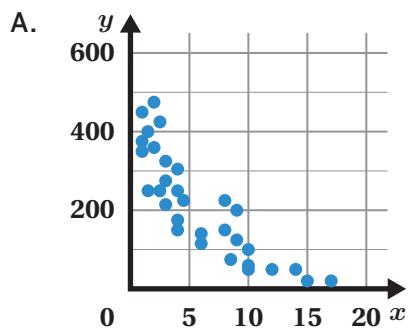
# Interpreting Slopes

Let's identify different types of associations.



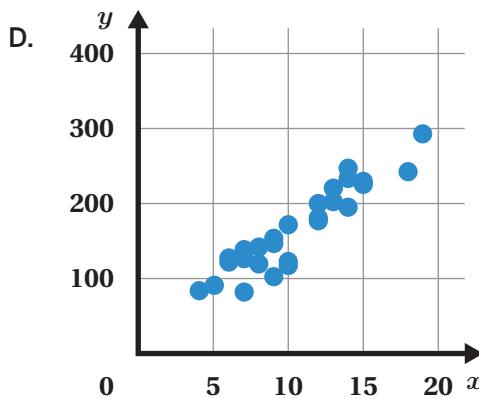
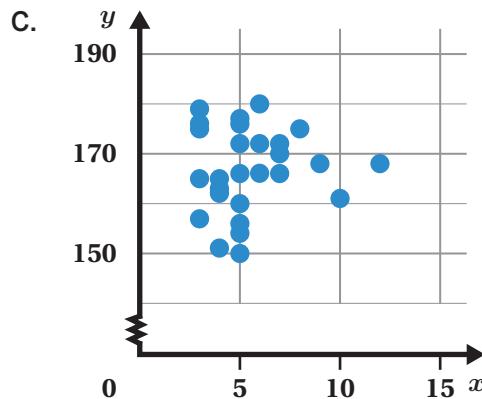
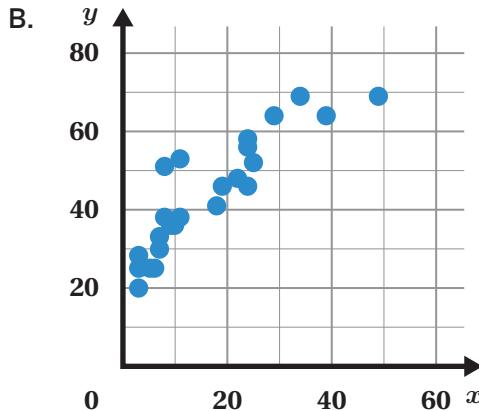
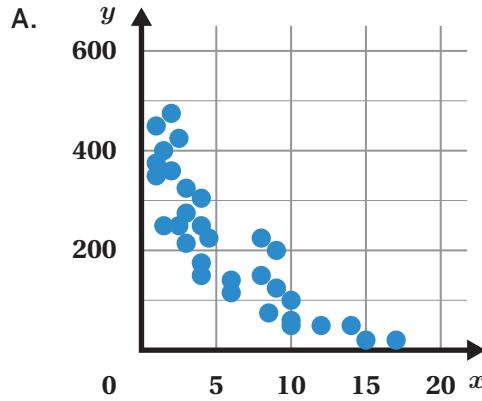
## Warm-Up

- 1 Which one doesn't belong? Explain your thinking.



**Associations**

- 2** Match each scatter plot with the variables that it most likely represents.



*x*: Number of Floors  
*y*: Building Height (ft)

*x*: Age of Bike (yr)  
*y*: Bike Price (\$)

*x*: Dog Weight (kg)  
*y*: Dog Height (cm)

*x*: Letters in Name  
*y*: Height (cm)

- 3** How did you decide which scatter plot matches these variables?

*x*: Number of Floors  
*y*: Building Height (ft)

**Associations (continued)**

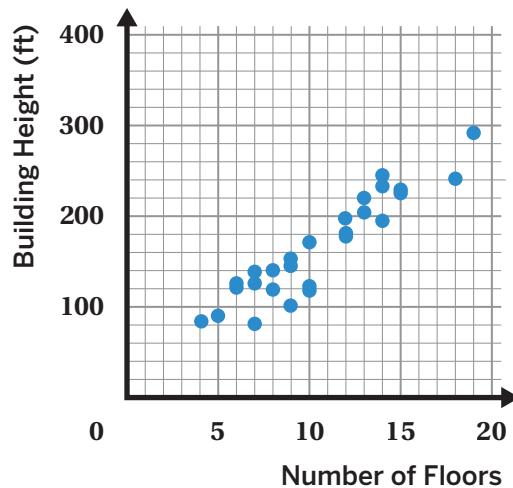
- 4** Here is the scatter plot that shows data from some buildings.

**a** Sketch a linear model to fit the data.

**b** Esi sketched a linear model whose equation is  $y = 13x + 30$ .

 **Discuss:**

- What is the *slope* of their line?
- What does the slope represent in this situation?



- 5** An **association** is a relationship between two variables. There is a positive association if both variables increase together and a negative association if one variable decreases as the other increases.

What type of association is there between building height and number of floors?  
Circle one.

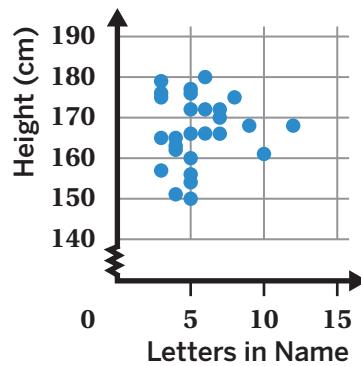
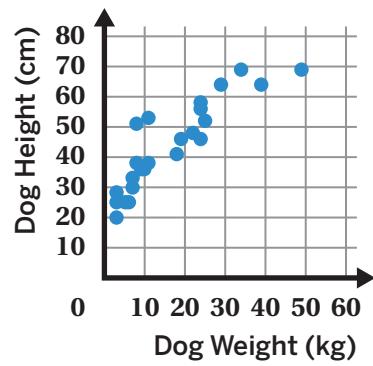
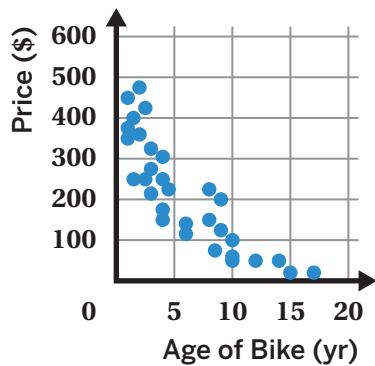
Positive association

Negative association

No association

Explain your thinking.

- 6** Determine what type of association each scatter plot shows. Discuss your thinking.

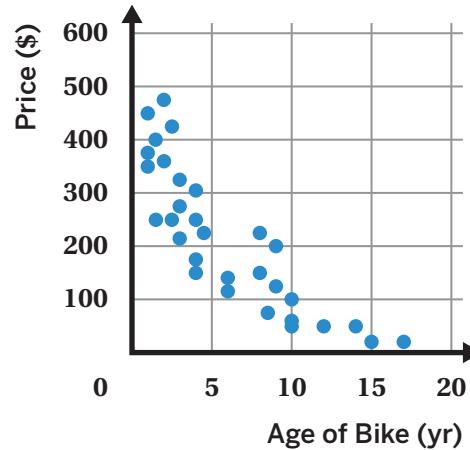


## Interpretations

- 7** Here is the scatter plot that shows the prices and ages of some used bikes.

**a** Sketch a linear model that fits the data.

**b**  **Discuss:** How can you tell from the linear model that there is a negative association between bike age and price?



- 8** Tay drew a linear model for the bike data using the equation  $y = -25x + 375$ . Use Tay's model to finish this sentence:

The model predicts that as the age of a bike increases by 1 year:

- A. The price will increase by \$25.
- B. The price will decrease by \$25.
- C. The price will increase by \$375.
- D. The price will decrease by \$375.

Explain your thinking.

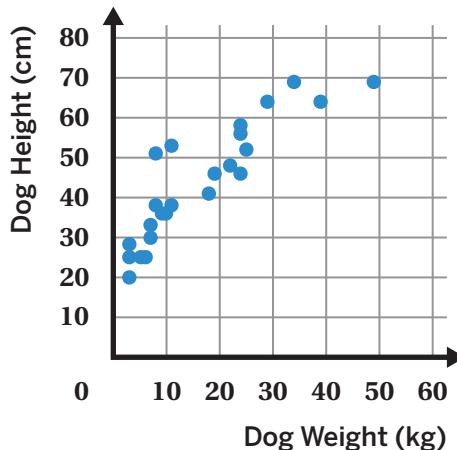
**Interpretations** (continued)

- 9** Here is the scatter plot that shows the heights and weights of some dogs.

**a** Sketch a linear model that fits the data.

**b** Nikolai sketched a linear model whose equation is  $y = 1.3x + 28$ .

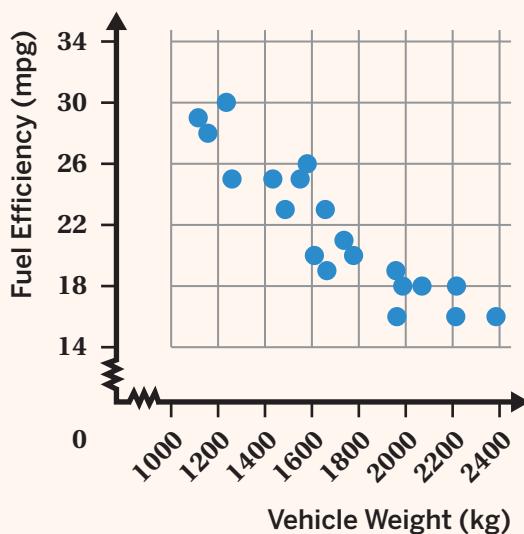
Identify the slope of Nikolai's model and describe what it means in this situation.

**Explore More**

- 10** Fuel efficiency measures the number of miles a car can go using one gallon of gas (miles per gallon). This scatter plot shows the relationship between fuel efficiency and weight for 20 vehicles.

Write three statements about this scatter plot — two that are true and one that is a lie.

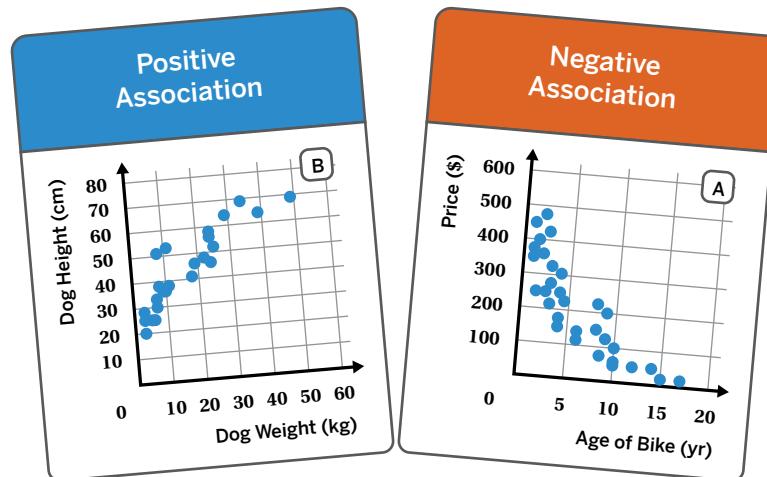
Sketch a line of fit if it helps with your thinking.



## 11 Synthesis

Use the example scatterplots if they help to answer these questions:

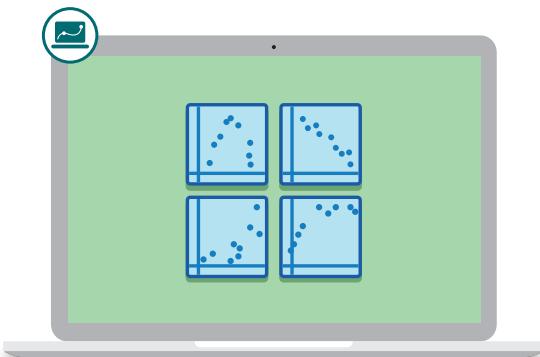
- a) What are some clues that a scatter plot might have a positive or negative association?



- b) What does the slope of a linear model tell you about the data?

Things to Remember:

Name: ..... Date: ..... Period: .....



## Scatter Plot City

Let's use precise language to describe the trends in a scatter plot.

### Warm-Up

- 1 Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet with scatter plots for four rounds.

For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a scatter plot from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating scatter plots until you're ready to guess which scatter plot the Picker chose.

Record helpful questions from each round in the space below.

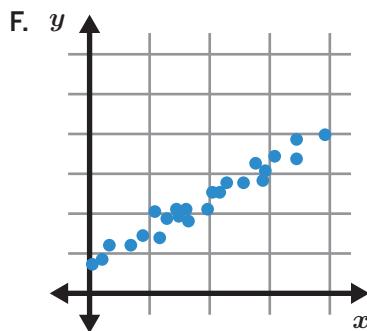
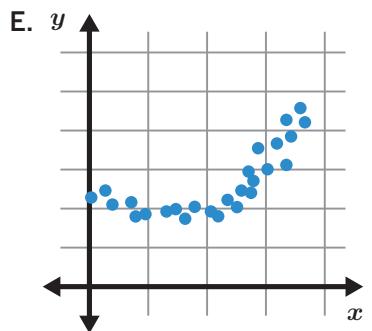
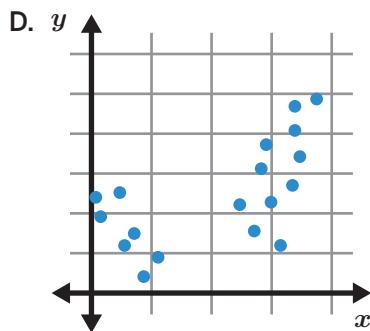
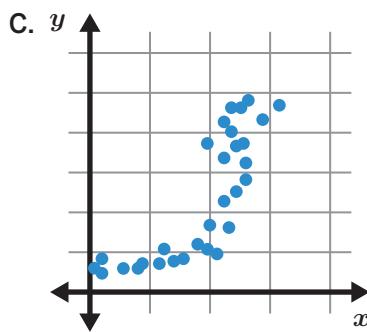
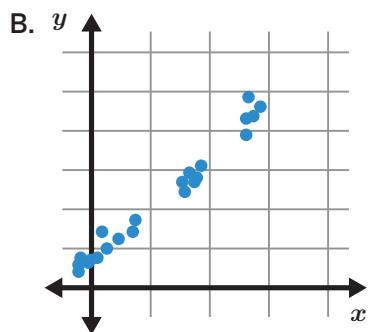
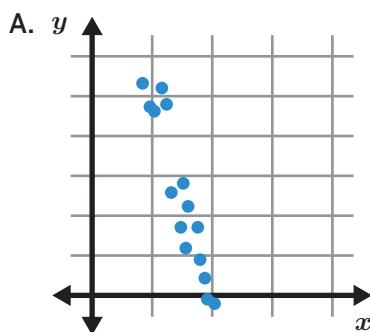
## Scatter Plot City

- 2** You will use the Activity 1 Sheet to see some scatter plots from the Polygraph and some terms that describe them.

 **Discuss:** What does each term mean?

- Linear association
- Non-linear association
- With **clusters**
- Without clusters

Here are six new scatter plots.



- 3** Sort them according to their type of association.

Linear Association	Non-Linear Association

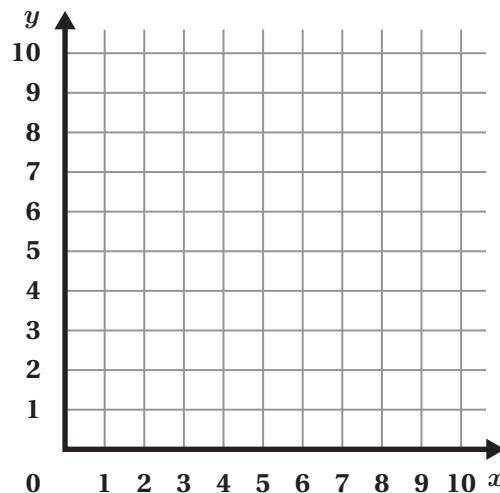
- 4** Sort them in a different way: those with clusters and those without.

With Clusters	Without Clusters

## Putting It All Together

- 5** **a** Create a scatter plot that has a negative non-linear association, without clusters.

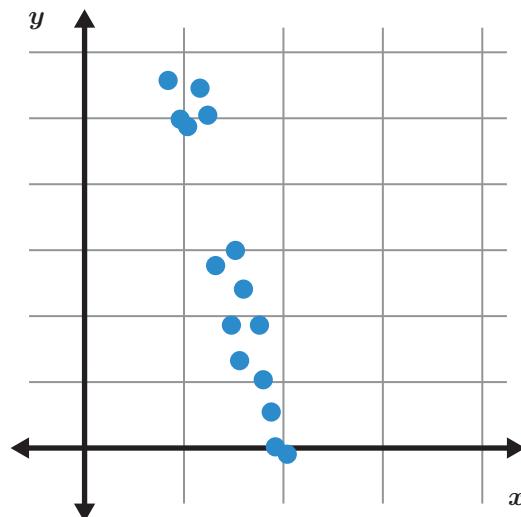
- b** Compare your scatter plot with your partner's.



- 6** Here is one of the scatter plots from before.

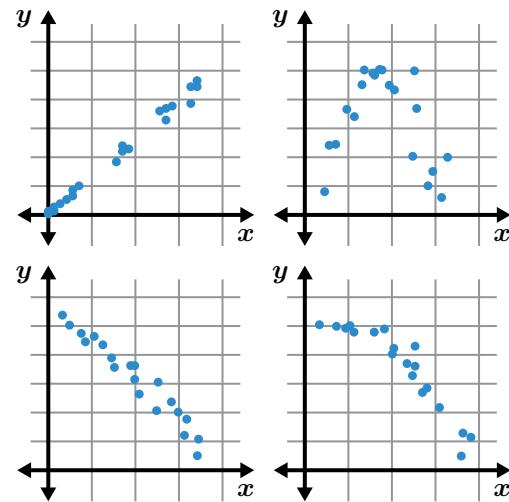
Describe the scatter plot using vocabulary from this unit.

positive association	negative association	clusters
linear association	non-linear association	outlier



## 7 Synthesis

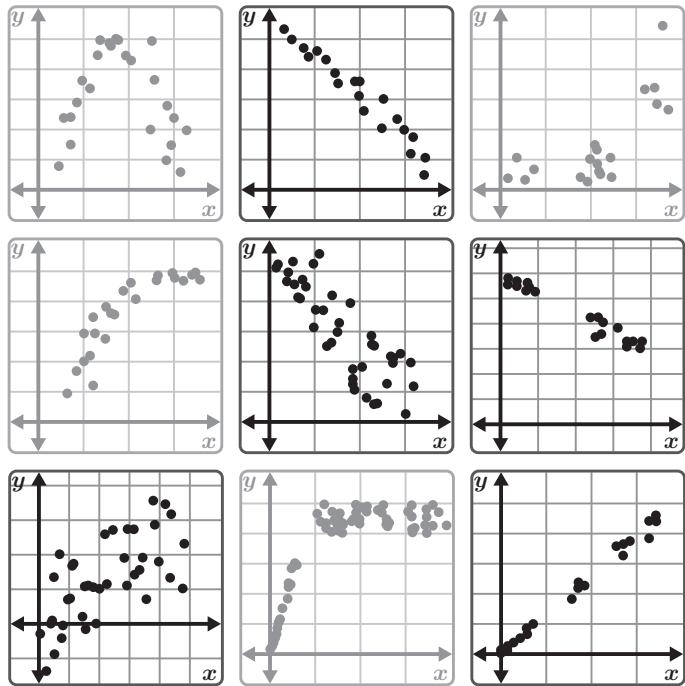
How can you identify a non-linear association or clusters in a scatter plot? Use the examples if they help with your thinking.



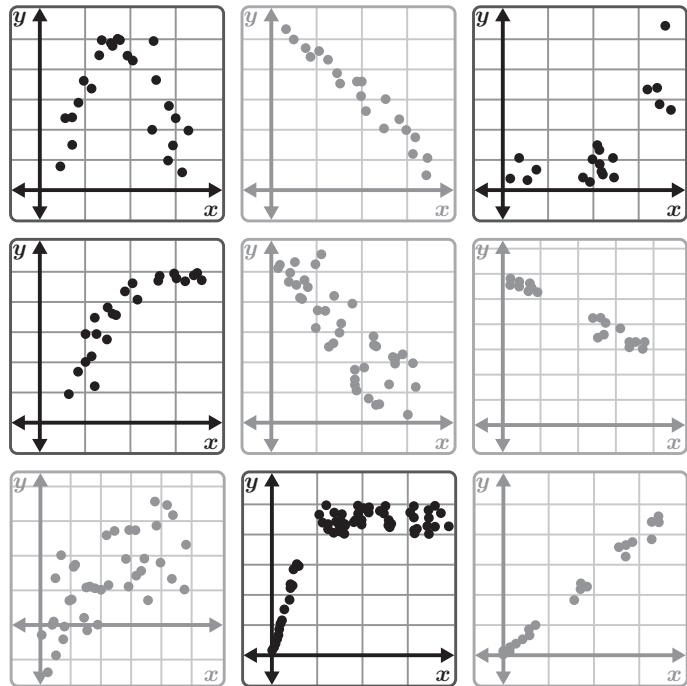
Things to Remember:

# Scatter Plot City

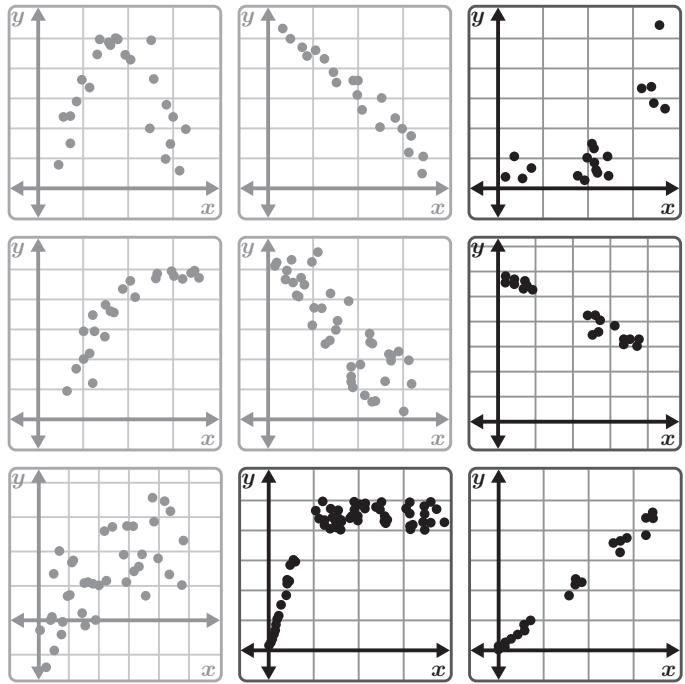
## Linear Association



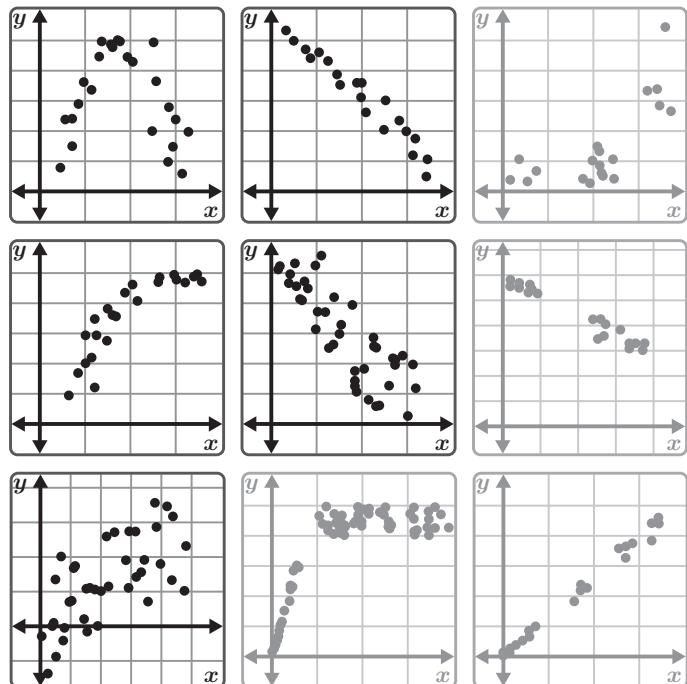
## Non-Linear Association



## With Clusters



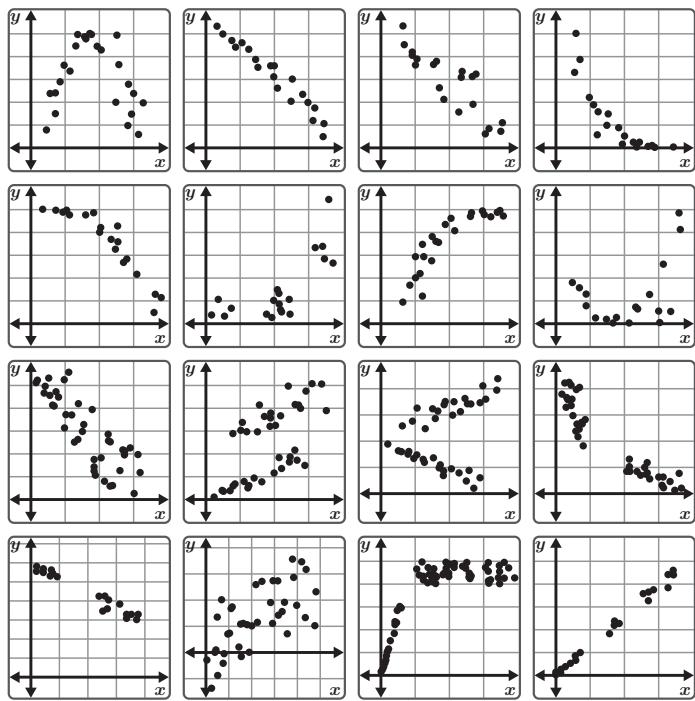
## Without Clusters



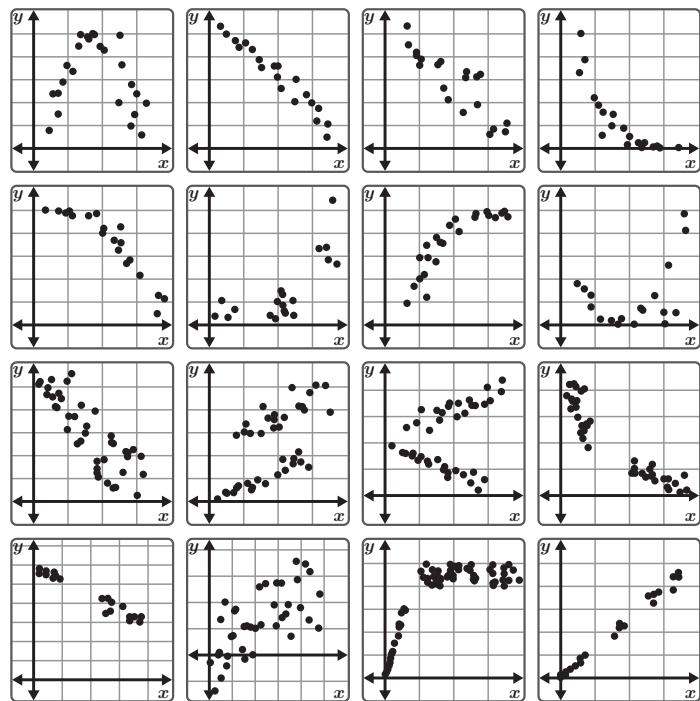
Name: ..... Date: ..... Period: .....

# Polygraph Set A

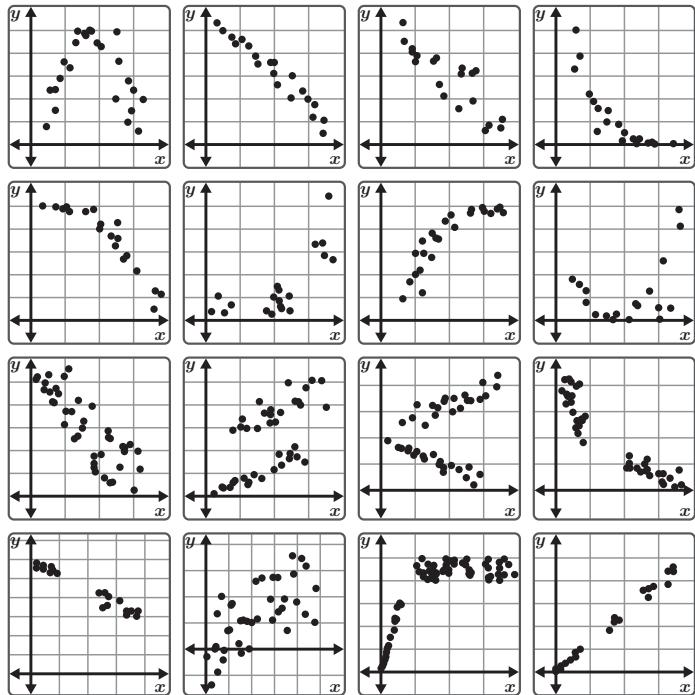
## Round 1



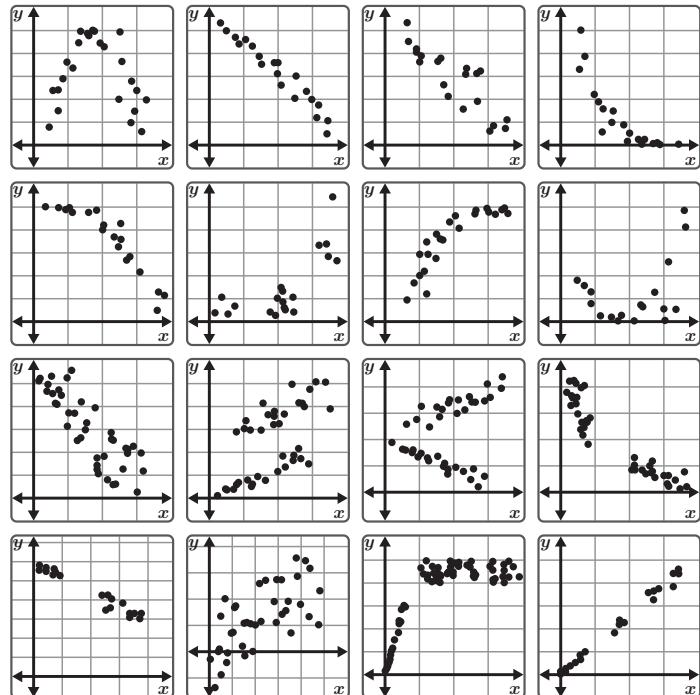
## Round 2



## Round 3



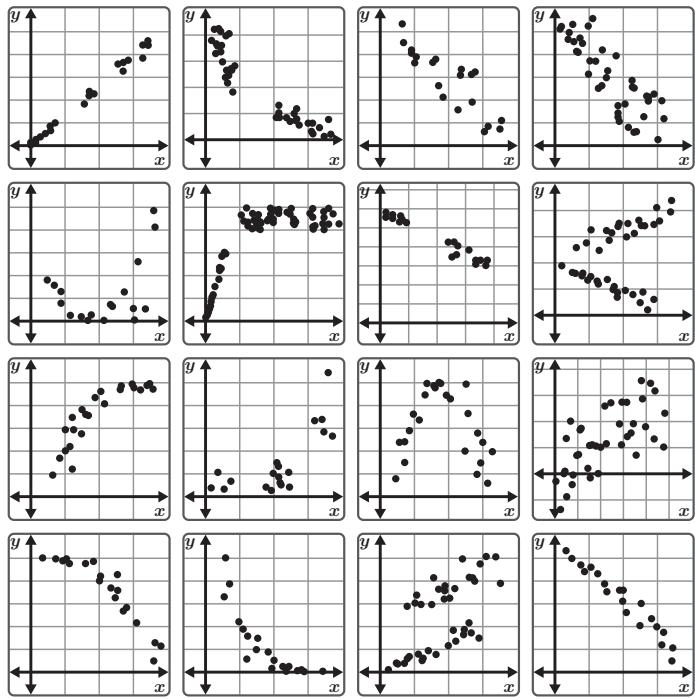
## Round 4



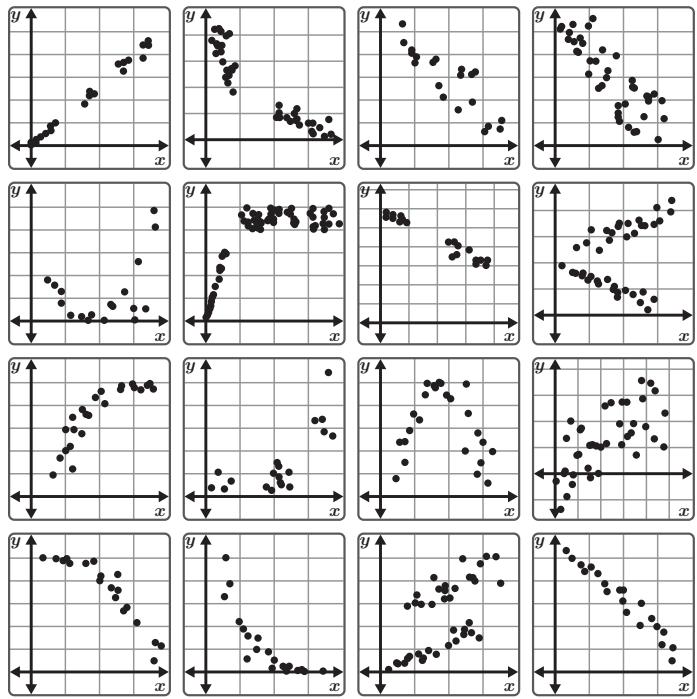
Name: ..... Date: ..... Period: .....

# Polygraph Set B

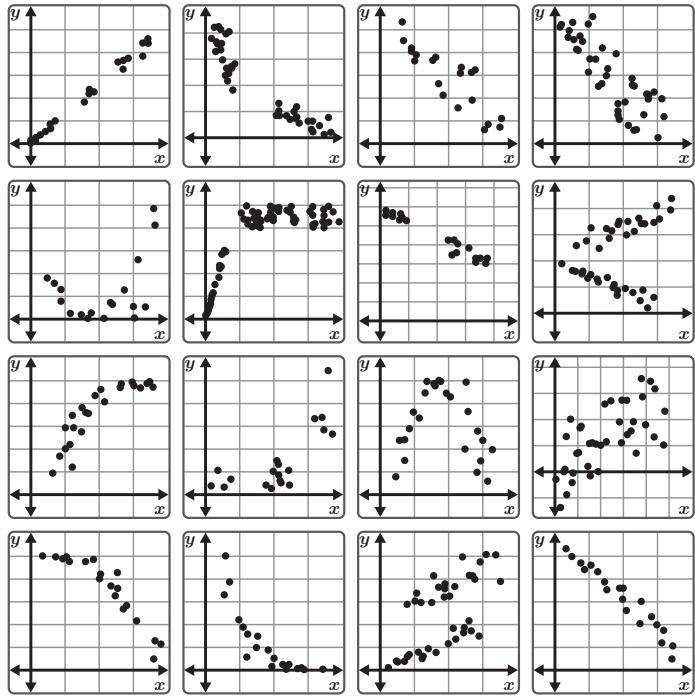
## Round 1



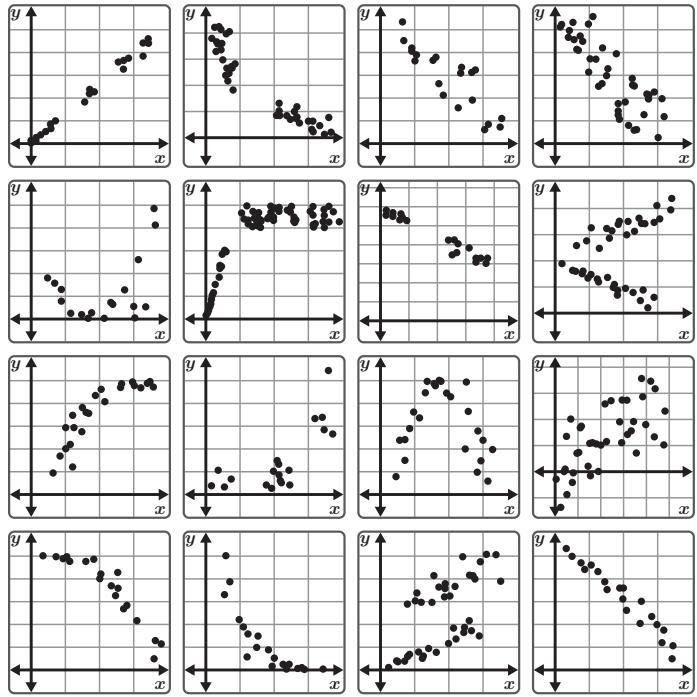
## Round 2



## Round 3



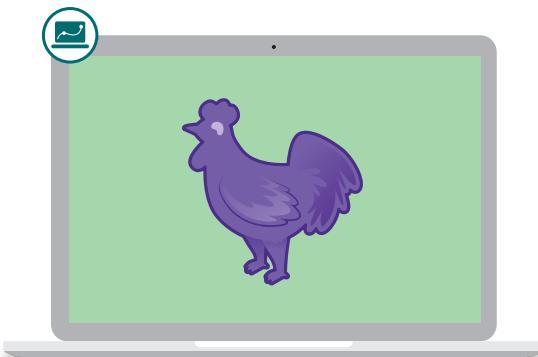
## Round 4



Name: ..... Date: ..... Period: .....

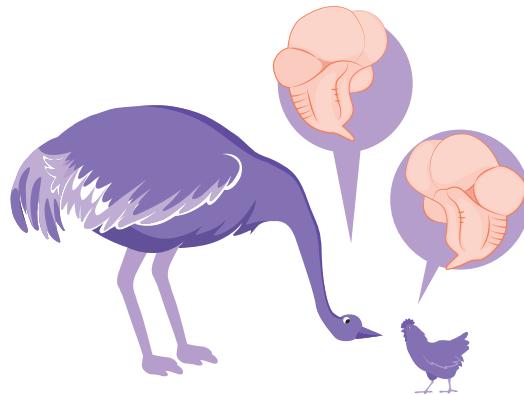
# Animal Brains

Let's analyze bivariate data.



## Warm-Up

- 1** Do you think heavier animals have heavier brains? Explain your thinking.

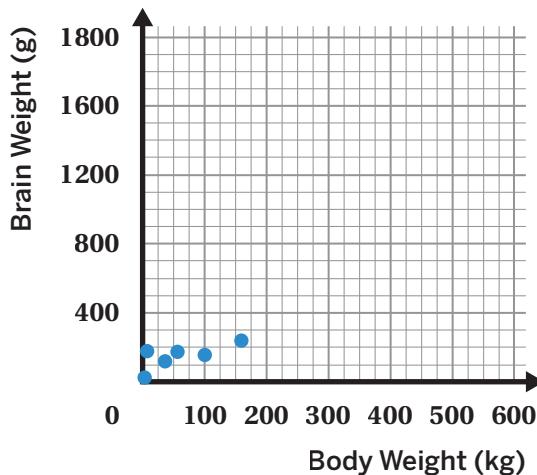


- 2** What information would help you determine whether heavier animals have heavier brains?

## Creating a Scatter Plot

- 3** The scatter plot shows the body weight and brain weight of 6 different animals. The table shows the weights of 4 more animals. Plot and label a point on the graph to represent each animal's data from the table.

Animal	Body Weight (kg)	Brain Weight (g)
Giraffe	529	680
Tiger	157	264
Goat	28	115
Cow	465	423



- 4** Look back at your prediction from the Warm-Up. Based on the scatter plot, what type of association does there appear to be between brain weight and body weight? Circle one.

Positive association

Negative association

No association

Explain your thinking.

- 5** The table shows the body weight of three more animals.

Plot and label points on the graph to predict the brain weight of each animal.

Complete the table with your predictions.

Animal	Body Weight (kg)	Brain Weight (g)
Dog	10	
Pig	192	
Horse	521	

## Line of Fit

Fitting a line to data can help make predictions more accurate.

- 6** Draw a line that fits the data on the scatter plot on the previous page.

- 7** The equation for Inola's line of fit is  $y = 0.9x + 79$ .

- a** What is the slope of the line? What is the  $y$ -intercept?

Slope: .....  $y$ -intercept: .....

- b**  **Discuss:**

- What does each number mean in this situation?
- Do these values make sense in this situation?

- 8** Use your line of fit to predict the brain weight for a gorilla and a human.  
Plot and label points on the previous page to show your predictions.

Animal	Body Weight (kg)	Brain Weight (g)
Gorilla	207	
Human	62	

## Line of Fit (continued)

- 9** Let's look at a scatter plot that shows body and brain weight data, including the data for the gorilla and human.

What do you notice? What do you wonder?

I notice:

I wonder:

### Explore More

- 10** *Tyrannosaurus rex* (T. rex) is a dinosaur with an estimated body weight of 8,000 kilograms.

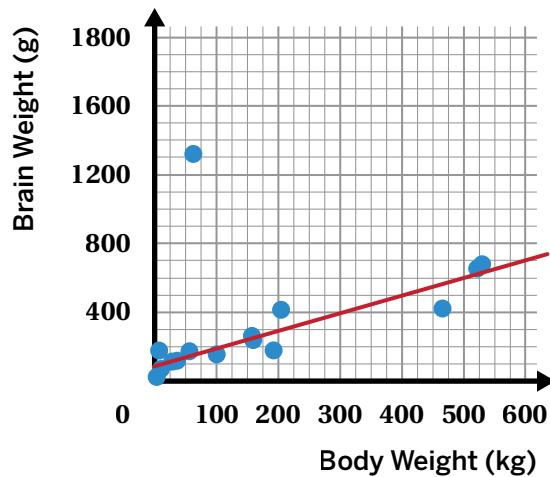
**a** Based on your line of fit from Activity 2, how much might a T. rex's brain weigh?

**b** Do you think the point representing the actual brain weight of a T. rex will be above or below the line of fit?

## 11 Synthesis

Describe an advantage and disadvantage of using a line of fit to make predictions.

Use the graph if it helps with your thinking.



Things to Remember:

Name: ..... Date: ..... Period: .....

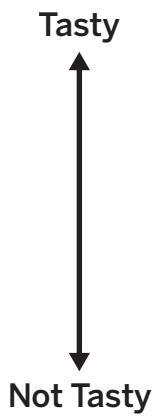
# Tasty Fruit

Let's explore two-way tables and bar graphs.



## Warm-Up

- 1** Draw a point to show how tasty you think red apples are.



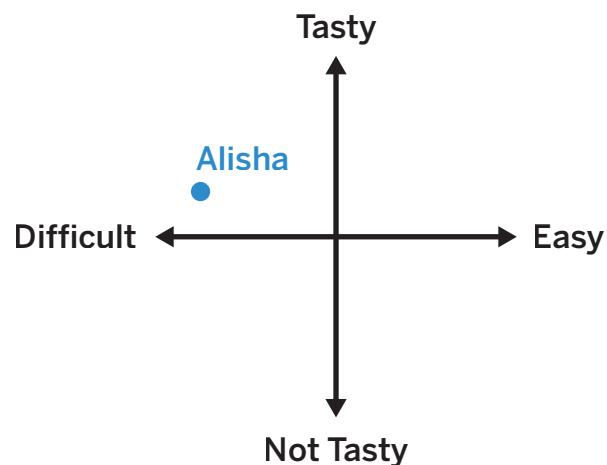
- 2** Draw a point to show how easy you think red apples are to eat.

Difficult ← → Easy

- 3** A scatter plot is one way to show how people feel about red apples.

How does Alisha feel about red apples?

Plot a point on the graph to represent your feelings about red apples.



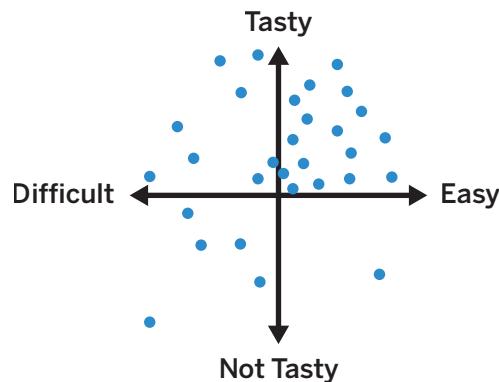
## Displaying Categorical Data

Mr. Diaz's students are analyzing their class's opinions about red apples.

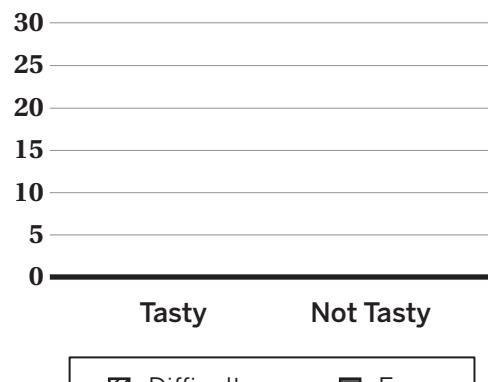
- 4** A student made a scatter plot and began to make a two-way table. The table shows frequency, which is the number of times each category appears in the data.

Complete the table to match the scatter plot.

	Difficult	Easy	Total
Tasty			24
Not Tasty			6
Total	13	17	30



- 5** Another student wanted to make a bar graph. Create a bar graph to match the table.



- 6** The scatter plot, two-way table, and bar graph all represent *categorical data* about red apples.

How many students in total shared their opinions about red apples?

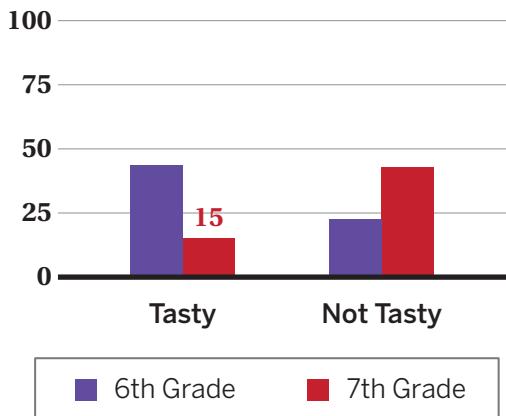
Explain your thinking.

## Analyzing Categorical Data

Abena surveyed the 6th and 7th graders at school about grapes. The bar graph shows the results from Abena's survey.

- 7** The two-way table shows partial results from the survey. Complete the table.

	6th Grade	7th Grade	Total
Tasty		15	
Not Tasty			63
Total	64	56	120



- 8** Based on the data, do 6th and 7th graders feel the same about grapes? Circle one.

Yes                          No                          I'm not sure

Explain your thinking.

### Explore More

- 9** 150 students were asked what grade they are in and whether they play a sport.

The two-way table shows partial results from this survey.

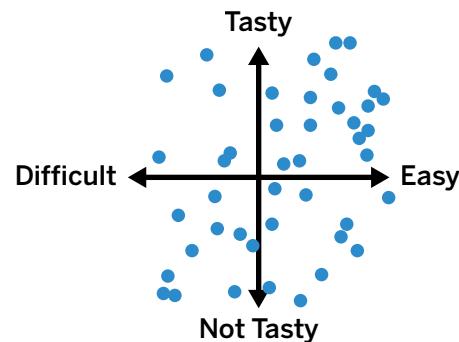
Complete the table.

	Plays a Sport	Does Not Play a Sport	Total
6th Grade	46	11	
7th Grade			20
8th Grade	16		
9th Grade		23	52
Total	110		150

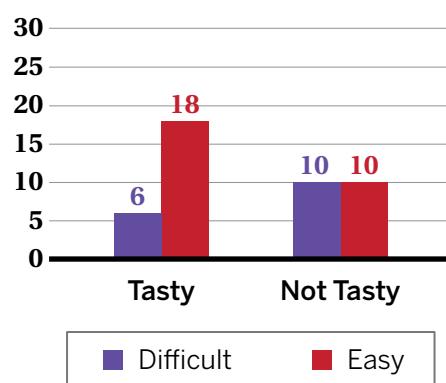
## 10 Synthesis

A school surveyed the 8th graders about how tasty bananas are, and how easy they are to eat.

	Difficult	Easy	Total
Tasty	6	18	24
Not Tasty	10	10	20
Total	16	28	44



What are some advantages of using a scatter plot, a two-way table, or a bar graph to represent data? Use the examples if they help with your thinking.



Things to Remember:

# Finding Associations

Let's use data displays to find associations.



## Warm-Up

Shanice needs to book a flight from one of three nearby airports.

Shanice researched each airport to determine how many flights were delayed last month.

- 1** Of the 20,175 flights that left the three airports near Shanice last month, 4,465 were delayed.

What percent of flights were delayed?

DEPARTURES		
Time	Destination	Status
11:05	BOSTON	DELAYED
11:25	MIAMI	DELAYED
12:05	LONDON	ON TIME
13:15	CHICAGO	DELAYED
13:30	NEW YORK	ON TIME
13:48	DUBAI	ON TIME
14:00	TOKYO	ON TIME
14:20	HOUSTON	DELAYED
15:05	TORONTO	ON TIME

- 2** This two-way table shows some data about flights that departed in the last month from the three airports near Shanice.

**Discuss:** Do you think the data will show an association between airport and flight status?

	On Time	Delayed	Total
Burbank	?	?	2,110
L.A.	?	?	13,765
Santa Ana	?	?	4,300
<b>Total</b>	15,710	4,465	20,175

## Frequency and Relative Frequency

- 3** This table shows the frequencies of on-time and delayed flights from the three airports.

Based on the data, is there evidence of an association between airport and flight status? Circle one.

Yes      No      I'm not sure

Explain your thinking.

Frequencies			
	On Time	Delayed	Total
Burbank	1,520	590	2,110
L.A.	11,530	2,235	13,765
Santa Ana	2,660	1,640	4,300
<b>Total</b>	15,710	4,465	20,175

- 4** One way to look for associations is to calculate **relative frequencies**. The relative frequency of a category is the percentage of data that is in that category.

The relative frequency of on-time flights from Burbank is about 72% because  $\frac{1520}{2110} \approx 0.72$ .

Relative Frequencies			
	On Time	Delayed	Total
Burbank	72%		100%
L.A.			100%
Santa Ana			100%

Complete the table. Round each percent to the nearest whole number.

- 5** Let's compare the two tables.

**a** Write a question you can answer using frequencies.

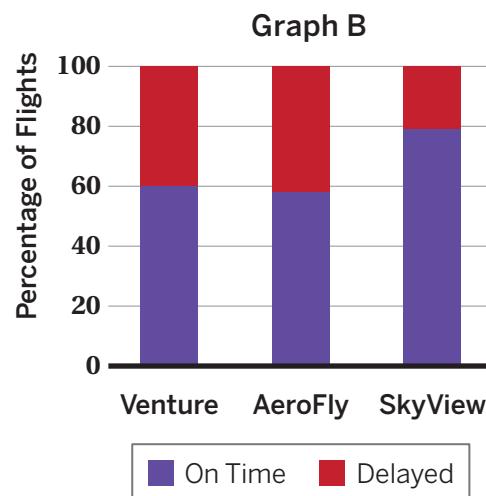
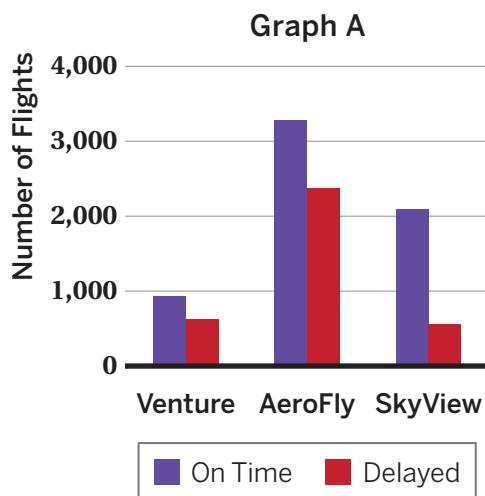
**b** Write a question you can answer using relative frequencies.

## Analyzing Data Representations

- 6** Shanice also researched three airlines that she can fly with for her trip.



**Discuss:** Where do you see the same information in each representation?



**Frequencies**

	On Time	Delayed	Total
Venture	914	605	1,519
AeroFly	3,288	2,369	5,657
SkyView	2,100	542	2,642
Total	6,308	3,516	9,818

**Relative Frequencies**

	On Time (%)	Delayed (%)	Total
Venture	60%	40%	100%
AeroFly	58%	42%	100%
SkyView	79%	21%	100%

- 7** Graph B is called a **segmented bar graph**. Use the bar graph or segmented bar graph to help you answer: *Is there evidence of an association between airline and flight status?*

Which graph was more helpful for determining if there is an association? Explain your thinking.

- 8** Consider the claim, “AeroFly Airlines has the highest rate of on-time flights because it has more on-time flights than Venture and SkyView combined.” Is this claim correct? Explain your thinking.

## Analyzing Data Representations (continued)

- 9** Based on the data, which airport and airline would you recommend for Shanice's flight? Explain your thinking.

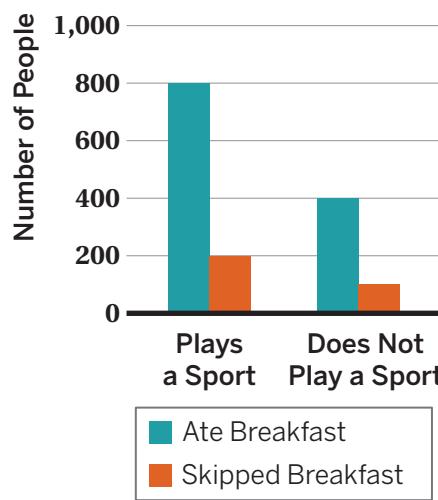
- 10** Here are four new data sets. For each data set, decide where there is evidence of an association between the variables.

	Left-Handed	Right-Handed
Has a Pet	83%	81%
No Pet	17%	19%
Total	100%	100%

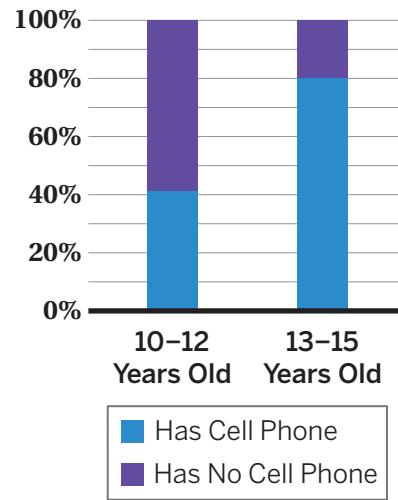
Association / No association

	Completed Course	Did Not Complete	Total
Free Online Course	6%	94%	100%
In-Person Course	85%	15%	100%

Association / No association



Association / No association



Association / No association

### Explore More

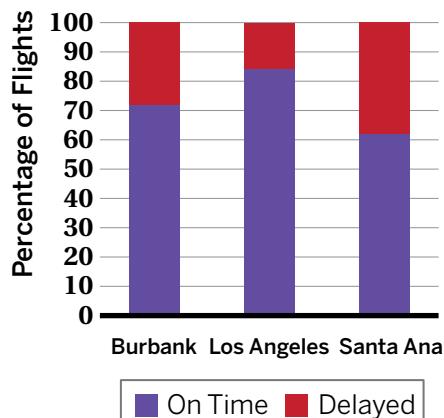
- 11** Use the digital activity to look at representations of data collected by the National Household Travel Survey on whether people took public transit over a one-month period.

## 11 Synthesis

Here are two representations of relative frequencies.

	On Time	Delayed	Total
Burbank	72%	28%	100%
L.A.	84%	16%	100%
Santa Ana	62%	38%	100%

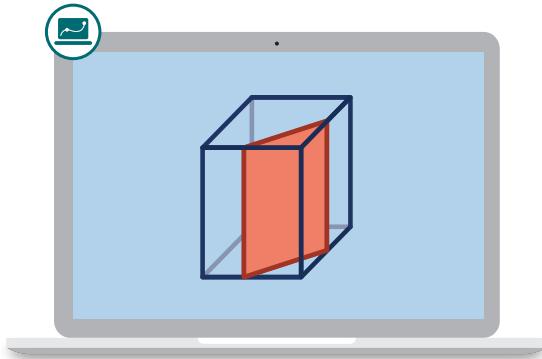
How can you use relative frequencies to identify possible associations between variables?



Things to Remember:

# Slicing Solids

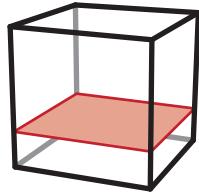
Let's explore and describe cross sections of solids.



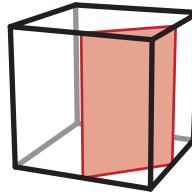
## Warm-Up

- 1** A plane cuts a cube into two pieces. When the plane cuts the cube parallel to the *base*, the **cross section** is a square. Here are different ways of cutting the cube and their cross sections.

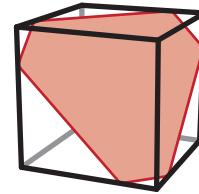
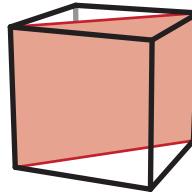
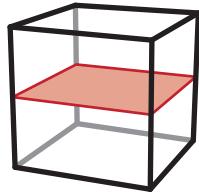
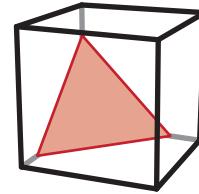
Cut 1



Cut 2



Cut 3



What do you notice? What do you wonder?

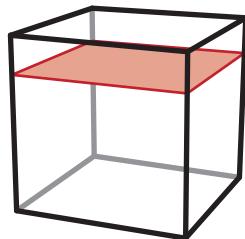
I notice:

I wonder:

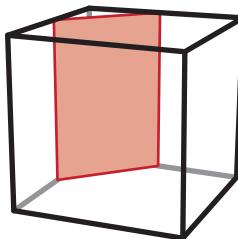
## Creating Cross Sections

- 2** Different cuts create different cross sections.

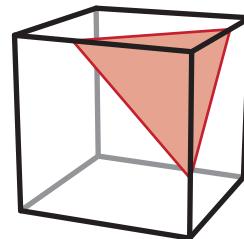
Cut 1



Cut 2

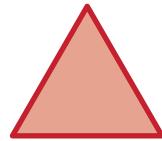


Cut 3

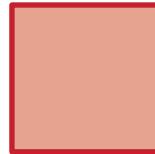


Select *all* of the cross sections you think you can make from a cube.

A.



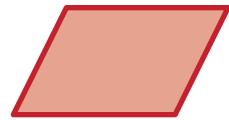
B.



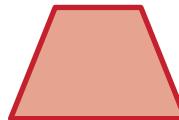
C.



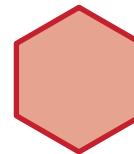
D.



E.

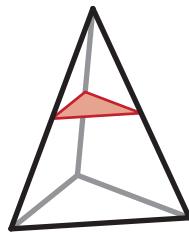


F.

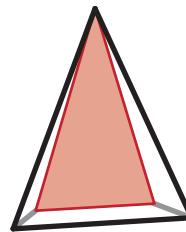


- 3** Here is a triangular pyramid.

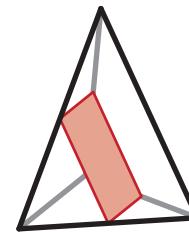
Cut 1



Cut 2



Cut 3



Select *all* of the shapes you think you can make.

A. Equilateral triangle

B. Isosceles triangle

C. Rectangle

D. Trapezoid

E. Hexagon

## Creating Cross Sections (continued)

- 4** Amir says it's possible to cut a rectangular cross section of figure A.

Peter says it's possible to cut a rectangular cross section of figure B.

Whose claim is correct? Circle one.

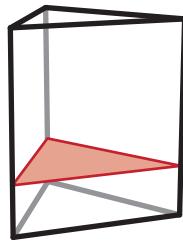
Amir      Peter      Both      Neither

Show or explain your thinking.

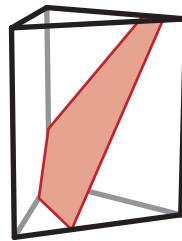


- 5** Here is a new *prism*.

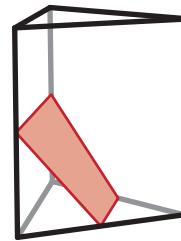
Cut 1



Cut 2



Cut 3



What is the greatest number of sides a cross section could have?

- A. 4      B. 5      C. 6      D. More than 6

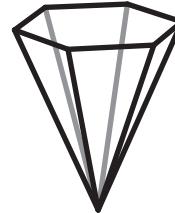
Explain your thinking.

## Prisms and Pyramids

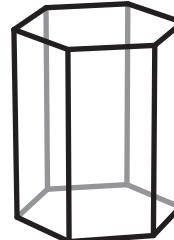
- 6** Here is a pyramid and a prism. They have identical hexagon bases.

If you cut the solids parallel to their bases, how would the cross sections be alike?

Pyramid



Prism



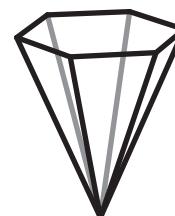
How would they be different?

- 7** Imagine cutting each solid with a vertical cut.

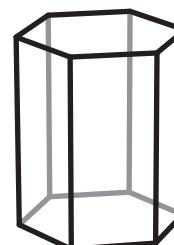
What might the cross section of the pyramid look like?

What might the cross section of the prism look like?

Pyramid



Prism



### Explore More

- 8** Match each solid with exactly one possible cross section.

Solid



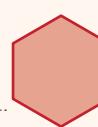
a.

Cross Section



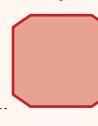
.....

b.



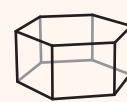
.....

c.



.....

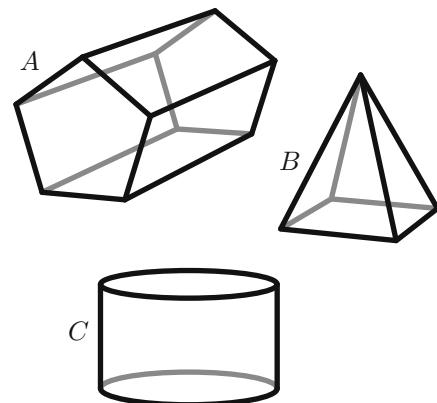
d.



.....

## 9 Synthesis

Describe why different cuts of a solid create different possible cross sections. Use the solids if they help with your thinking.

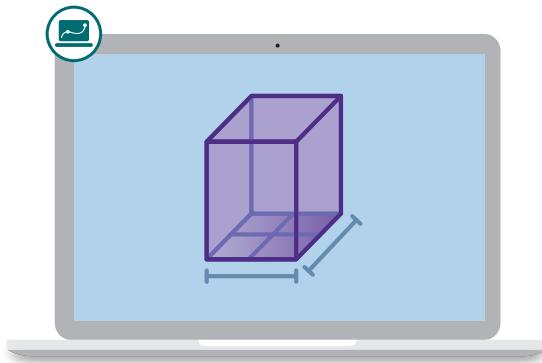


Things to Remember:

Name: ..... Date: ..... Period: .....

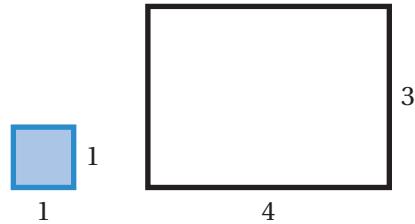
## Simple Prisms

Let's calculate the volume of prisms.



### Warm-Up

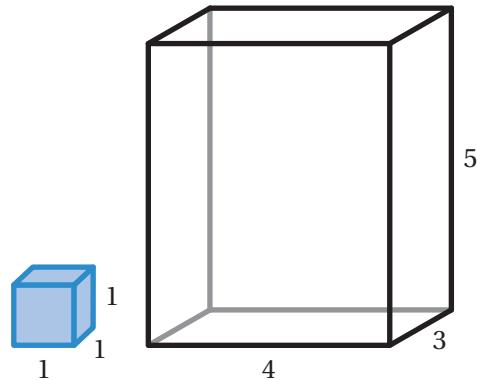
- 1** How many unit squares will it take to cover this rectangle?



- 2** Here is a rectangular prism.

- a** How many unit cubes will it take to fill this rectangular prism?

- b** **Discuss:** What strategy did you use?



## Volume of Prisms

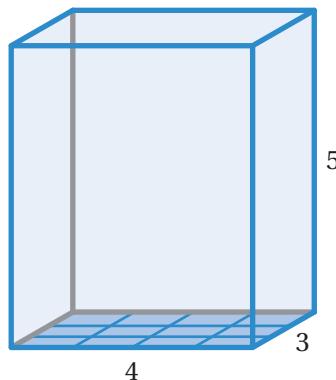
- 3** Here is how Mio calculated the *volume* of the prism:

$$V = 12 \cdot 5$$

Explain what 12 and 5 represent.

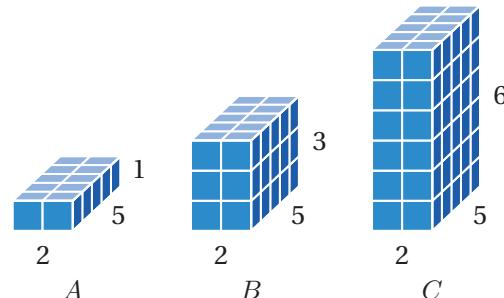
12 represents . . .

5 represents . . .

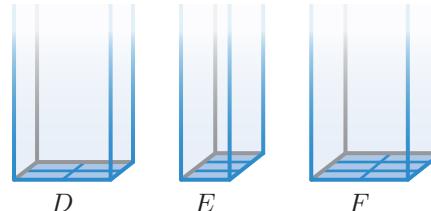


- 4** Here are three rectangular prisms: *A*, *B*, and *C*. They all have the same base.

Use Mio's strategy to determine the missing volumes.



- 5** Here are the bases of prisms *D*, *E*, and *F*. They all have the same volume.



Which prism has the greatest height? Circle one.

*D*

*E*

*F*

Not enough information

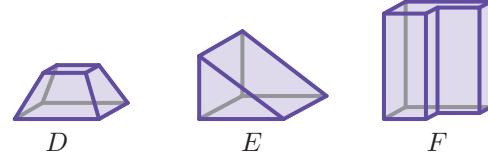
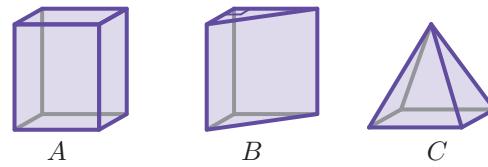
Explain your thinking.

## Volume Calculation Strategies

- 6** Here's how Mio described her strategy for determining the volume:

**Step 1: Calculate the area of the base.**  
**Step 2: Multiply that area by the height of the object.**

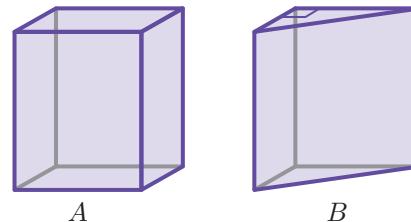
- a** Circle all of the objects that Mio's strategy will work for.



- b** **Discuss:** How did you decide which objects to select?

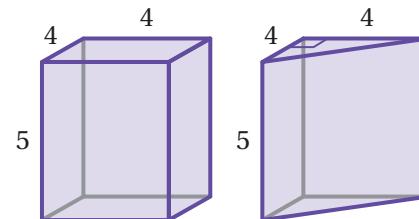
- 7** Here are two objects from the previous problem. One is a rectangular prism. The other is a triangular prism.

What information would you need to calculate each of their volumes?



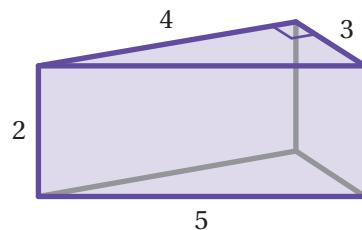
- 8** Calculate the volume of each prism.

	Volume (cubic units)
Rectangular Prism	
Triangular Prism	



## Volume Calculation Strategies (continued)

- 9** Here is a new triangular prism. Calculate its volume.



- 10** Nasir and Omari also calculated the volume of this triangular prism. They each made a mistake.

Nasir

$$\frac{1}{2} \cdot 4 \cdot 3 \cdot 5 = 30 \text{ cubic units}$$

Omari

$$4 \cdot 3 \cdot 2 = 24 \text{ cubic units}$$

Choose one student and consider their calculations.

a



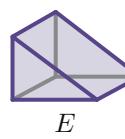
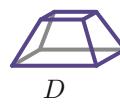
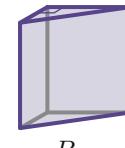
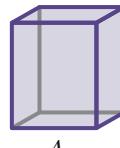
**Discuss:** What did the student do well?

b

What would you recommend the student change about their calculations?

## 11 Synthesis

Describe how to determine the volume of a prism. Draw if it helps to show your thinking.

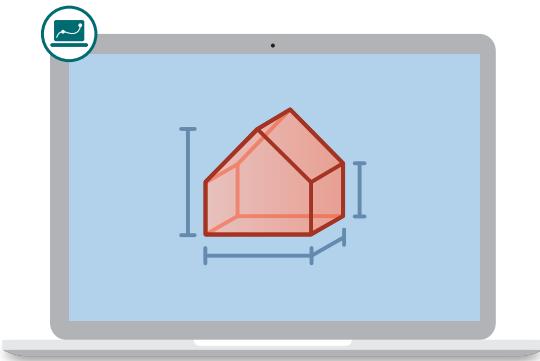


Things to Remember:

Name: ..... Date: ..... Period: .....

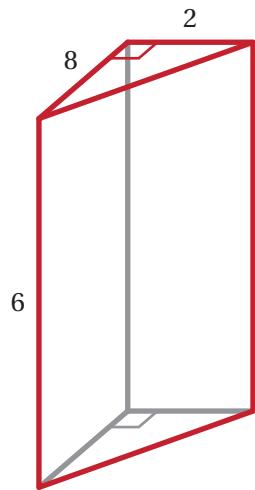
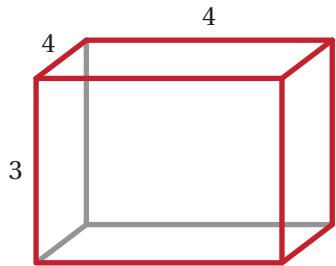
# Complex Prisms

Let's determine the volume of prisms with other bases.



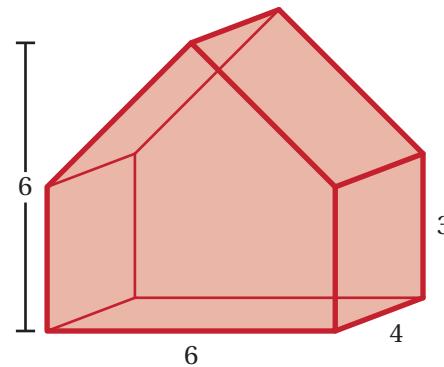
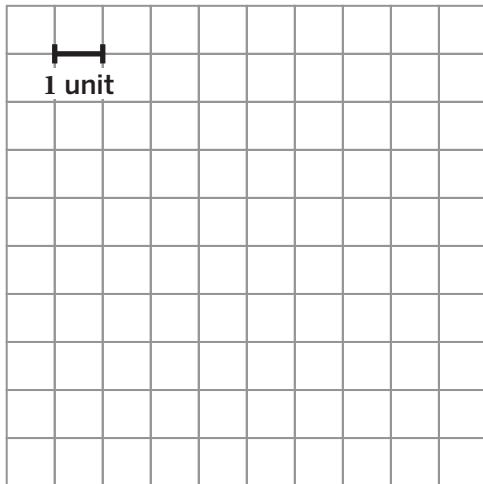
## Warm-Up

- 1** Calculate the volume of each prism.



## Using the Base to Calculate Volume

- 2** Sketch the base of this prism on the grid.



- 3** What is the area of the prism's base?

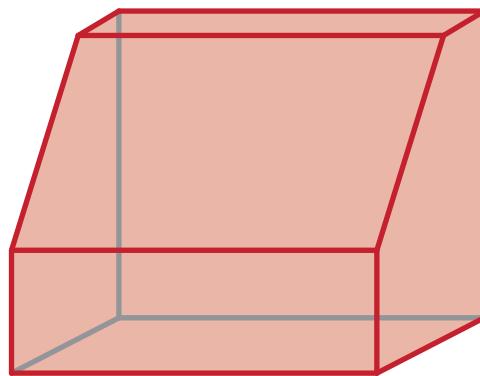
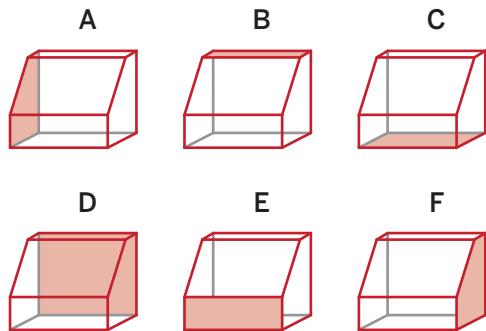
Show or explain how you determined the area.

- 4** What is the volume of the prism?

## Prisms With Other Bases

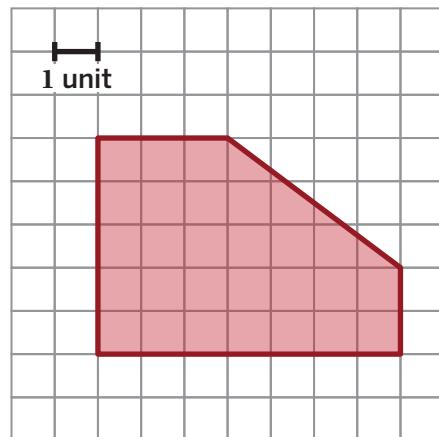
- 5** Here is a new prism.

Circle *all* the faces that are bases you could use to calculate the prism's volume.



- 6** Here is one face of the prism.

What is its area? Show your thinking.



- 7** Jaylin says the volume of this prism is  $29 \cdot 5$ , or 145 cubic units.

Kimaya claims that it's  $29 \cdot 6$ , or 174 cubic units.

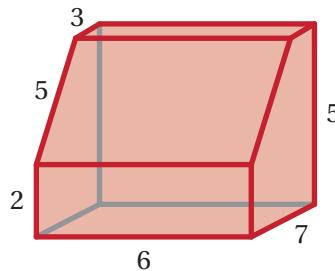
Whose claim is correct? Circle one.

Jaylin

Kimaya

Both

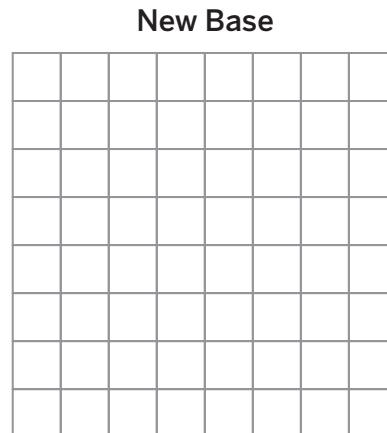
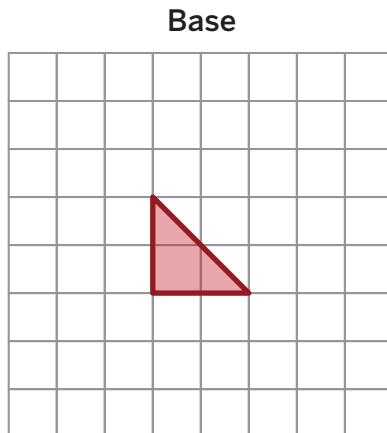
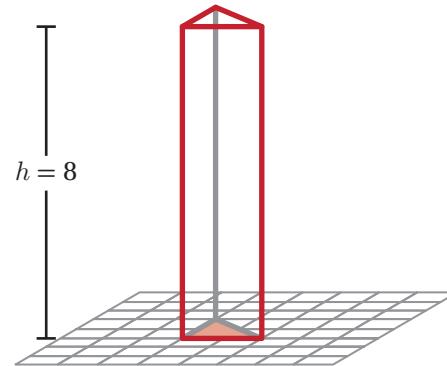
Neither



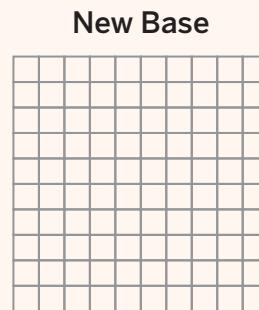
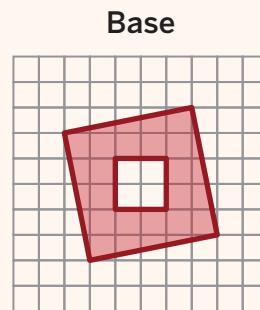
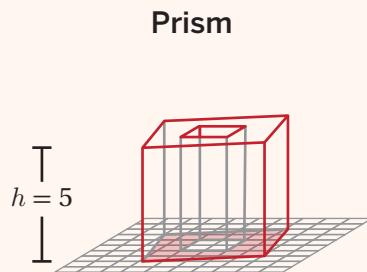
Explain your thinking.

**Prisms With Other Bases (continued)**

- 8** This triangular prism has a height of 8 units and a volume of 16 cubic units. Draw a new base so its volume is 48 cubic units.

**Explore More**

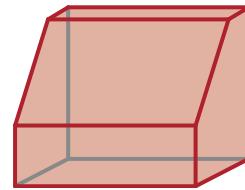
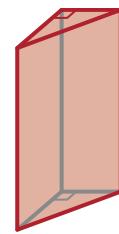
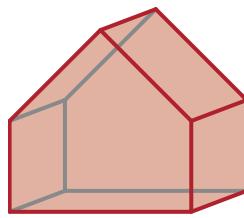
- 9** A prism has a height of 5 units and a volume of 110 cubic units. Its base is made of two squares. Draw a new base, made of two squares, that creates a prism with a volume of 80 cubic units.



## 10 Synthesis

Here are several prisms you've seen in this lesson.

Describe a general strategy for determining the volume of any prism.

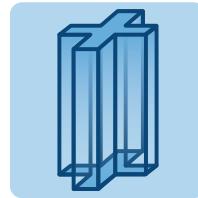


Things to Remember:

Name: ..... Date: ..... Period: .....

# Surface Area Strategies

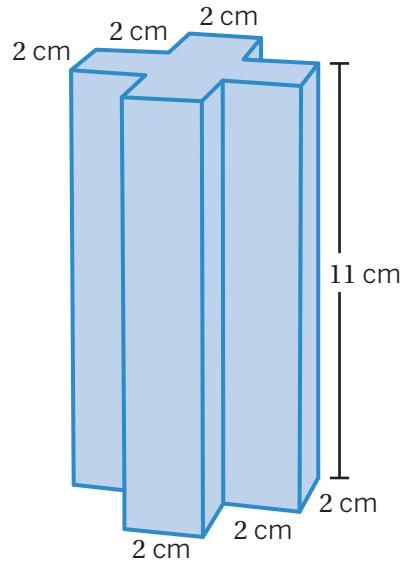
Let's calculate the surface area of different prisms.



## Warm-Up

1. Here is a prism.

- a How many *faces* does this prism have?  
Explain your thinking.
  
- b Calculate the area of the prism's base.



## Examining Different Strategies

2. Here are Amoli's, Nyanna's, and Miko's strategies for calculating the surface area of the prism from the Warm-Up.

**Amoli**

I have to draw each of the 14 different faces, find their areas, and add them up.

**Nyanna**

There are only two different shapes: the plus sign and the rectangle. I can find the area of each shape and use a calculator to multiply by the number there are of each shape.

**Miko**

I see another way! Imagine unfolding the prism into a net. I can use one large rectangle instead of 12 smaller ones.

**a**

 **Discuss:** How would you describe each student's strategy in your own words?

**b**

How are Amoli's and Nyanna's strategies alike? How are they different?

3. Let's look at Miko's strategy more closely.

**a**

Sketch the "one large rectangle" Miko is talking about.

**b**

What are the dimensions of this rectangle? Show or explain your thinking.

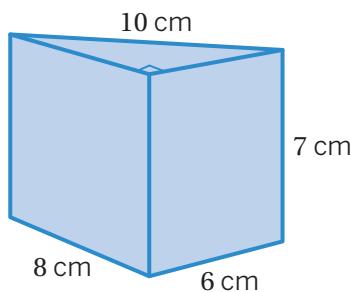
4. Use any strategy to calculate the surface area of this solid. Organize your thinking and calculations so that others can follow them.

## Calculating Surface Area

Here are three prisms. For each prism:

- Determine how many faces the prism has.
- Use any strategy to calculate the surface area. Organize your thinking and calculations so that others can follow them.
- Trade papers with your partner. Work together to reach an agreement.

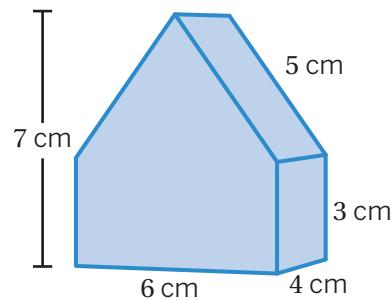
5.



Number of faces: .....

Surface area: .....

6.

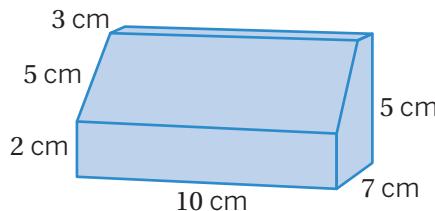


Number of faces: .....

Surface area: .....

## Calculating Surface Area (continued)

7.



Number of faces: .....

Surface area: .....

8. **Discuss:** Is your strategy more similar to Amoli's, Nyanna's, or Miko's?  
What about your partner's strategy?

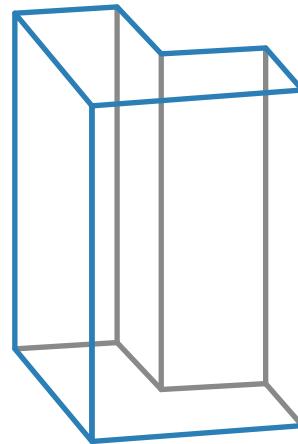
### Explore More

9. Sketch a prism with a surface area of 220 square units.

## Synthesis

10. Describe your favorite method for calculating the surface area of a prism.

Use this prism if it helps you with your explanation.

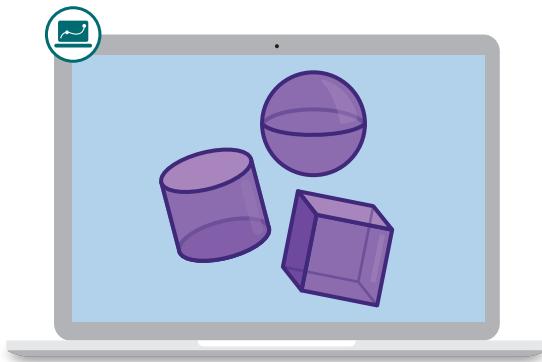


Things to Remember:

Name: ..... Date: ..... Period: .....

# Volume Lab

Let's estimate the volume of three-dimensional solids.



## Warm-Up

- 1** Which one doesn't belong? Explain your thinking.

Figure A

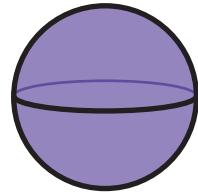


Figure B

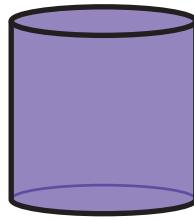


Figure C

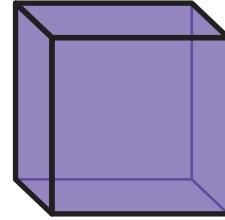
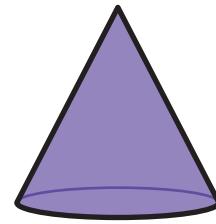


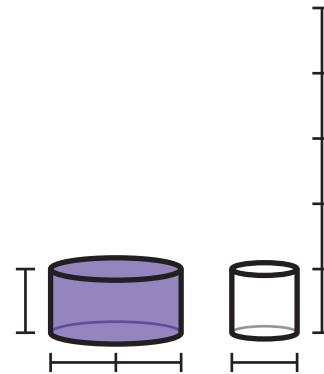
Figure D



## Comparing Volumes

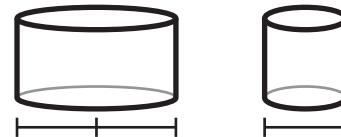
- 2** Here are two cylinders.

Draw a new height for the cylinder on the right so that both cylinders have the same *volume*.



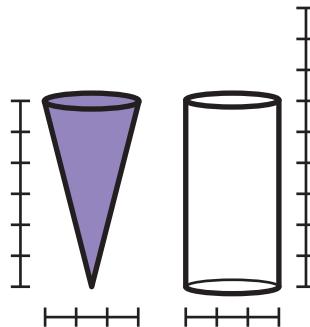
- 3** Here are two cylinders with the same height.

How many small cylinders do you think it would take to fill the large cylinder?



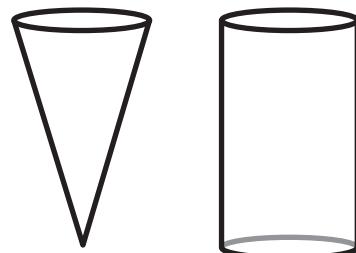
## Comparing Volumes (continued)

- 4** Draw a new height for the cylinder so that both objects have the same volume.



- 5** Here is a **cone** and a cylinder with the same height and *diameter*.

How many cones would it take to fill the cylinder?



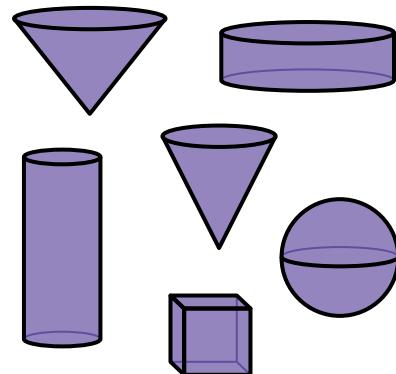
## Volume Lab

**6** Let's use Screen 6 of the digital activity to explore some volume relationships!

- a** Select any two objects and adjust their dimensions. Then press "Compare." Repeat this with several pairs of objects. Draw or record something that you found interesting or surprising.
  
  
  
  
  
  
- b** Two cones have equal diameters. The height of one cone is 2 times as large as the height of the other cone. How are the volumes of the cones related?
  
  
  
  
  
  
- c** Two cylinders have the same height. The diameter of one cylinder is 3 times as large as the diameter of the other cylinder. How are the volumes of the cylinders related?
  
  
  
  
  
  
- d** Describe the relationship between two different objects, where one has twice the volume of the other.
  
  
  
  
  
  
- e** Describe another interesting relationship between the volumes of two different objects.

## 7 Synthesis

Describe one volume relationship you discovered during this lesson.

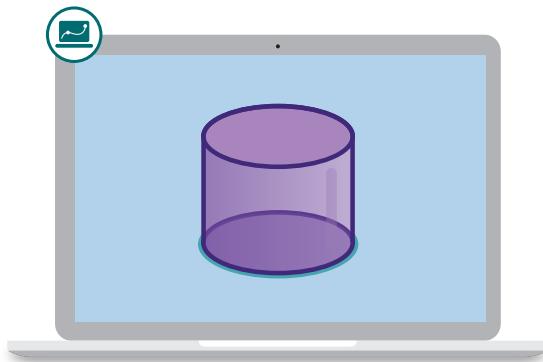


Things to Remember:

Name: ..... Date: ..... Period: .....

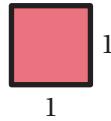
# Cylinders

Let's calculate the volume of cylinders.



## Warm-Up

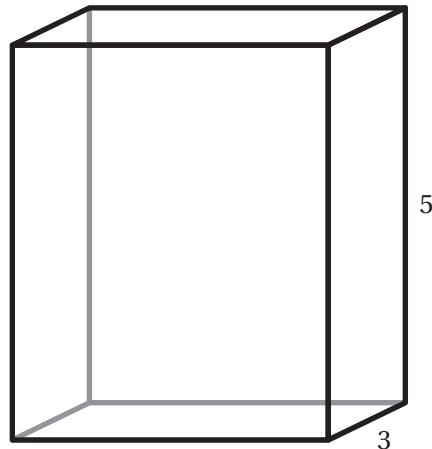
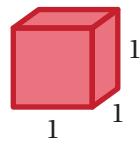
- 1** How many unit squares are needed to cover this rectangle?



3

4

- 2** How many unit cubes are needed to fill this rectangular prism?



5

3

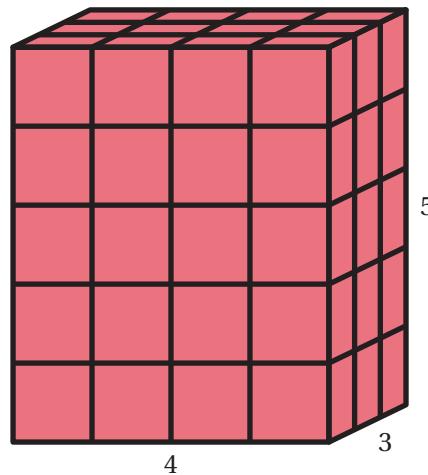
## Volume Strategies

- 3** DeAndre calculated the volume in three ways:

- $V = 12 \cdot 5$
- $V = 15 \cdot 4$
- $V = 20 \cdot 3$

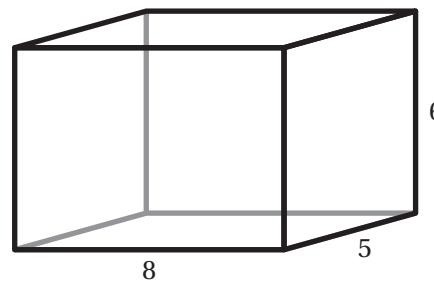
Let's focus on DeAndre's first equation.

Explain what 12 and 5 represent in the diagram.



- 4** Here is a new rectangular prism.

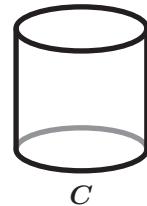
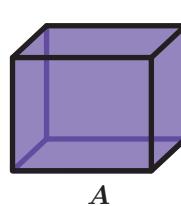
Use DeAndre's strategy to write an expression for its volume.



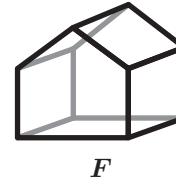
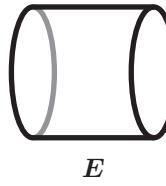
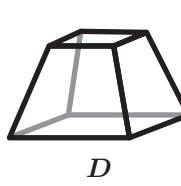
## Volume Strategies (continued)

- 5** DeAndre described his strategy for calculating the volume of a solid.

- First, find the area of the base.
- Then multiply that area by the height of the object.



- a** Circle *all* of the objects for which DeAndre's strategy will work.



- b** **Discuss:** How did you decide which objects to choose?

- 6** DeAndre's strategy for calculating volume works for cylinders.

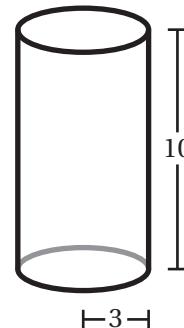
What information would you need to calculate the volume of this cylinder?



## Cylinder Volumes

- 7** This cylinder has a height of 10 units and a *radius* of 3 units.

Calculate the volume of the cylinder.



- 8** Caasi incorrectly determined the volume of the cylinder with this calculation:

$$V = 3 \cdot 3 \cdot 10 = 90$$

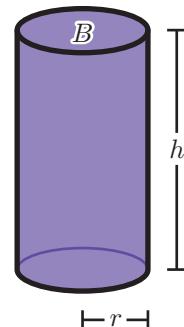
What did Caasi do well and what mistake did she make?

- 9** Here are two formulas for the volume of a cylinder:

$$V = \pi r^2 \cdot h$$

$$V = B \cdot h$$

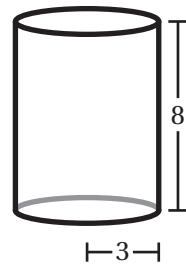
Describe how the formulas are related.



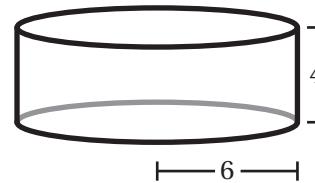
## Calculating Cylindrical Volume

- 10** Calculate the volume of each cylinder.

Tall cylinder:



Short cylinder:



- 11** Sketch two different cylinders that have the same volume. What is the volume of each cylinder?

**Cylinder A**

**Cylinder B**

### Explore More

- 12** **a** Write dimensions for a rectangular prism and a cylinder with volumes that are close to equal.

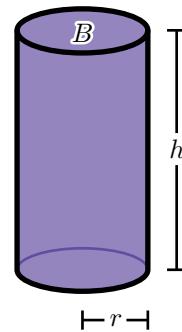
Rectangular prism:

Cylinder:

- b** Explain why they have close to equal volumes.

### **13** Synthesis

Describe a strategy for determining the volume of a cylinder given its radius and height.

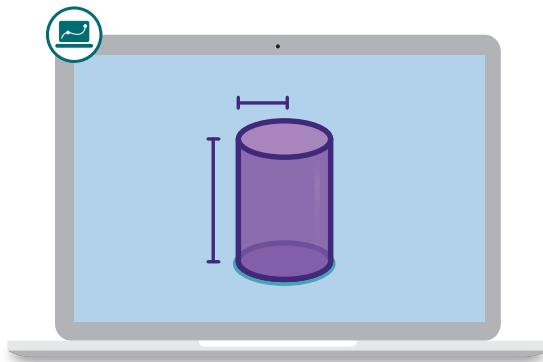


Things to Remember:

Name: ..... Date: ..... Period: .....

## Scaling Cylinders

Let's see how changing a cylinder's radius or height impacts its volume.

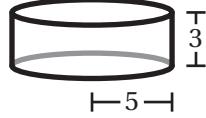


### Warm-Up

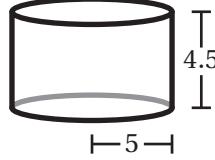
- 1** We learned that in a function, the independent variable represents the input and the dependent variable represents the output.

Here are several cylinders:

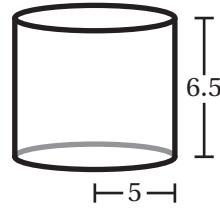
$$V = 75\pi$$



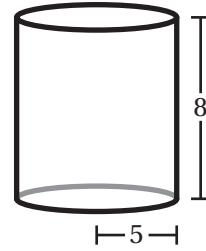
$$V = 112.5\pi$$



$$V = 162.5\pi$$



$$V = 200\pi$$



In this situation, what could the independent and dependent variables be?

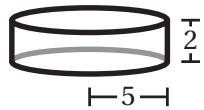
Independent variable: .....

Dependent variable: .....

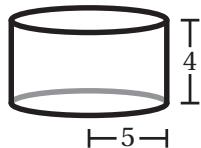
## Changing the Height

- 2** Select the cylinder that represents the plotted point. Explain to a classmate how you chose.

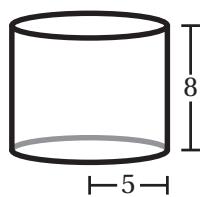
A.



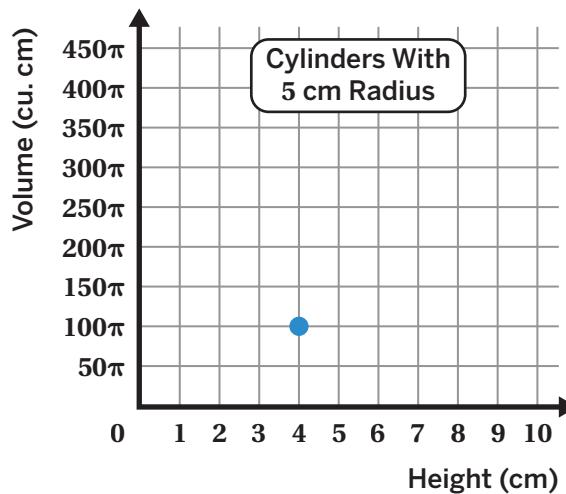
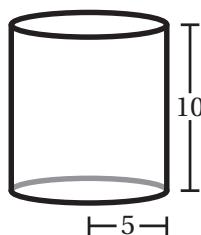
B.



C.



D.



- 3** Let's watch an animation graphing the relationship between a cylinder's height and its volume.



**Discuss:** What do you notice? What do you wonder?

- 4** Let's look at a graph that represents the relationship between the height and the volume for cylinders with a radius of 5 centimeters.

Use the graph and the table to help you find the volume of each of the four cylinders.

Express each volume in terms of  $\pi$ .

Object	Height	Volume (cu. cm)
Cylinder A	2	
Cylinder B	4	$100\pi$
Cylinder C	8	
Cylinder D	16	

## Changing the Radius

- 5** Let's see what happens if we keep the height of a cylinder constant but change the radius.

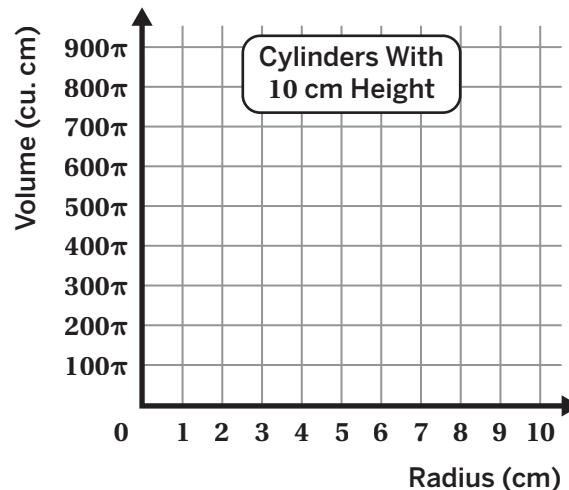
a Choose and record a radius for two different cylinders that each have a height of 10 centimeters.

	Radius (cm)	Volume (cu. cm)
Cylinder 1		
Cylinder 2		

b Calculate the volume for each cylinder. Express each volume in terms of  $\pi$ .

- 6** a Plot points to represent your two cylinders.

b Make a sketch of what you think the graph looks like for all cylinders with a height of 10 centimeters.



- 7** Let's watch an animation graphing the relationship between a cylinder's radius and its volume.

**Discuss:** Is this relationship a linear function? Explain your thinking.

## Changing the Radius (continued)

- 8** Let's look at a graph that represents the relationship between the radius and the volume for cylinders with a height of 10 centimeters.

Use the graph and the table to help you find the volume of each of the four cylinders.

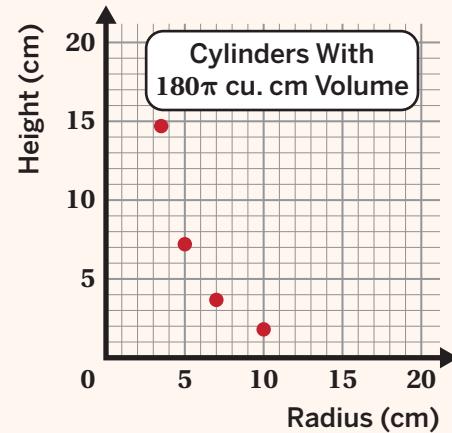
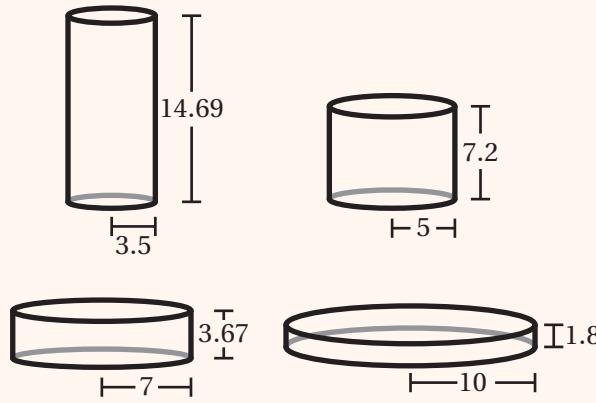
Express each volume in terms of  $\pi$ .

Object	Radius (cm)	Volume (cu. cm)
Cylinder E	2	
Cylinder F	4	
Cylinder G	8	
Cylinder H	16	

### Explore More

- 9** Explore the relationship between radius and height when the volume of a cylinder is fixed. Here are several cylinders that all have a volume of  $180\pi$  cu. cm.

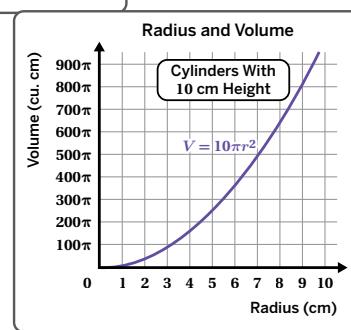
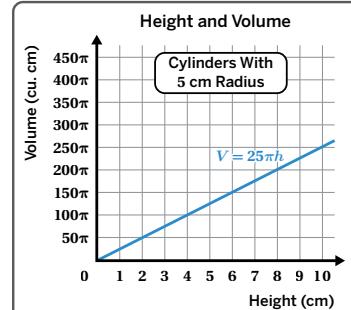
**Discuss:** What do you notice? What do you wonder?



## 10 Synthesis

Here are the relationships we explored today.

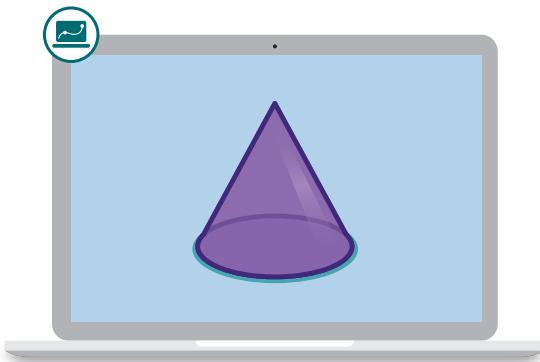
How can you tell if the relationship between height and volume and the relationship between radius and volume are linear or non-linear?



Things to Remember:

# Cones

Let's explore cones and their volumes.

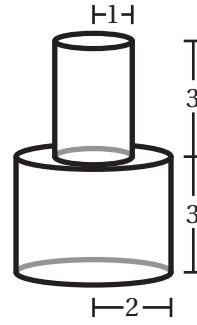
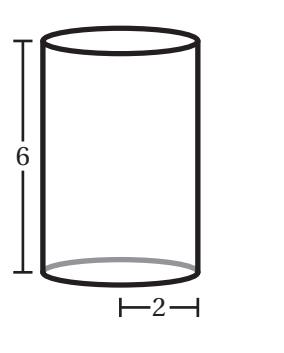


## Warm-Up

- 1** Determine the volume of figure *A* (a cylinder) and figure *B* (which is composed of two cylinders).

Write their volumes in terms of  $\pi$ .

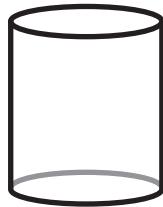
Figure	Volume (cu. cm)
<i>A</i>	
<i>B</i>	



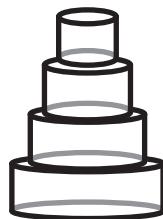
## Estimating the Volume of a Cone

- 2** Here are four figures with the same height and the same radius for their largest bases.

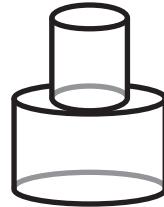
Cylinder



Four cylinders



Two cylinders



Cone



Order the figures by volume from *least* to *greatest*.

--	--	--	--	--

**Least**

**Greatest**

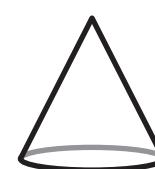
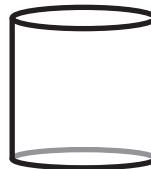
- 3** Let's see what happens when we increase the number of cylinders.

What do you think the exact volume of the cone might be?

$$V = 24\pi$$

$$V = 8.49\pi$$

$$V = ?$$



Explain your thinking.

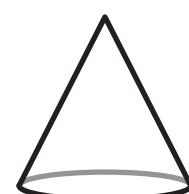
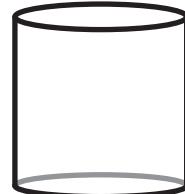
Cylinders: 25

- 4** Here is another set of figures.

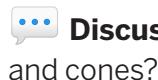
- a** Estimate the volume of the cone.

$$V = 30\pi$$

$$V = ?$$



- b** Let's see what happens when we increase the number of cylinders.



**Discuss:** What do you notice about the relationship between cylinders and cones?

## Volume of a Cone

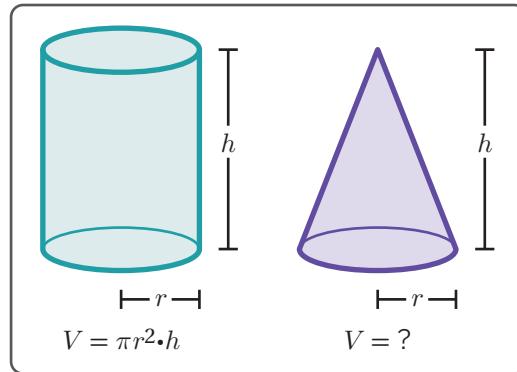
- 5** Each row of the table shows the volumes of a cylinder and a cone with the same height and radius.

Fill in the unknown values.

Volume of Cylinder (cu. cm)	Volume of Cone (cu. cm)
$24\pi$	$8\pi$
$30\pi$	$10\pi$
$120\pi$	
$60\pi$	
	$15\pi$

- 6** One way of writing a formula for the volume of a cylinder with radius  $r$  and height  $h$  is  
 $V = \pi r^2 \cdot h$ .

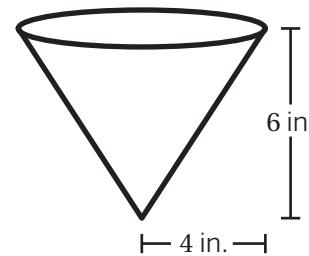
Write a formula for the volume of a cone.



## Comparing the Volume of a Cone

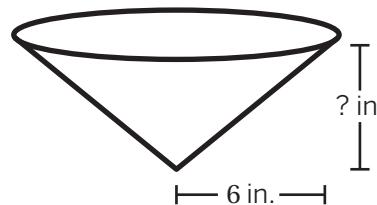
- 7** Let's look at one way of writing a formula for the volume of a cone with radius  $r$  and height  $h$ .

Calculate the volume of this cone.



- 8** The volume of this cone is  $60\pi$  cubic inches.

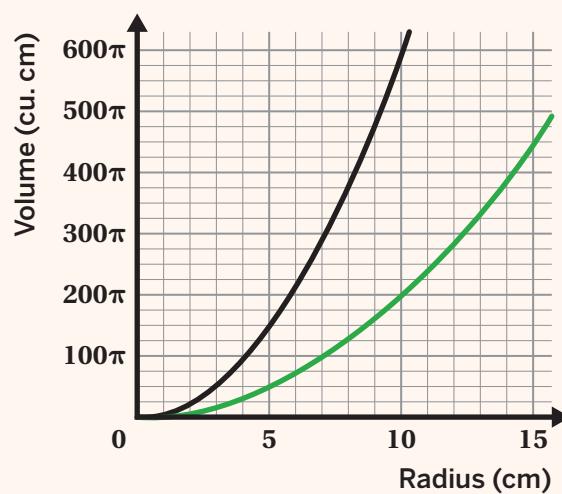
What is the height of the cone?



### Explore More

- 9** This graph shows the relationship between radius and volume for cones and cylinders that have the same height.

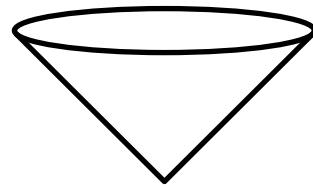
- a** Label each curve to show which shape it represents.
- b** What is the height of these cylinders and cones? Explain your thinking.



## 10 Synthesis

One way of writing the formula for volume of a cone is  $V = \frac{1}{3}\pi r^2 \cdot h$ .

What does each part of the formula represent?

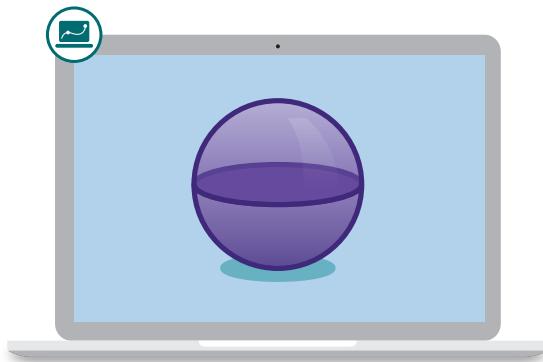


Things to Remember:

Name: ..... Date: ..... Period: .....

# Spheres

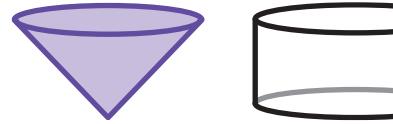
Let's develop a formula for the volume of a sphere.



## Warm-Up

- 1** A cone and a cylinder have the same height and radius.

What fraction of the cylinder will be filled by the cone?



## Hemispheres

- 2** What if we pour both a cone *and* a hemisphere into the cylinder?

Let's see what fraction of the cylinder will be filled.

Describe how the three volumes are related.



- 3** The volume of the cylinder is  $27\pi$  cubic units.

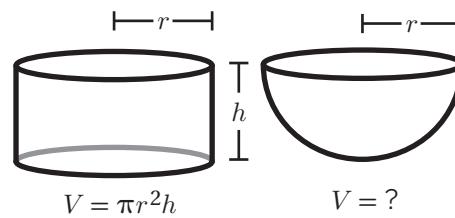
Write the volume of the cone and hemisphere.

Express the volumes in terms of  $\pi$ .

Object	Volume (cu. units)
Cone	
Cylinder	$27\pi$
Hemisphere	

- 4** One way of writing a formula for the volume of a cylinder is  $V = \pi r^2 h$ , where  $r$  is the radius and  $h$  is the height.

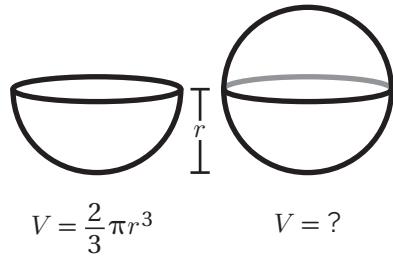
Write a formula for the volume of a hemisphere. Explain your thinking.



## Finding Sphere Dimensions

- 5** One way of writing a formula for the volume of a hemisphere is  $V = \frac{2}{3}\pi r^3$ , where  $r$  is the radius.

Write a formula for the volume of a sphere. Explain your thinking.



- 6** Karima and Nasir are calculating the volume of a sphere with a radius of 2 units.

Karima

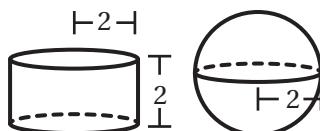
$$V = \frac{4}{3}\pi r^3 \quad r = 2$$

$$V = \frac{4}{3}\pi(2)^3$$

$$V = \frac{4}{3}\pi \cdot 8$$

$$V = \frac{32}{3}\pi$$

Nasir



$$V = \pi r^2 h$$

$$V = \pi(2)^2 \cdot 2$$

$$V = 8\pi$$

$$8\pi \cdot \frac{4}{3}$$

$$V = \frac{32}{3}\pi$$



**Discuss:** How did each student determine the volume of a sphere?

## Finding Sphere Dimensions (continued)

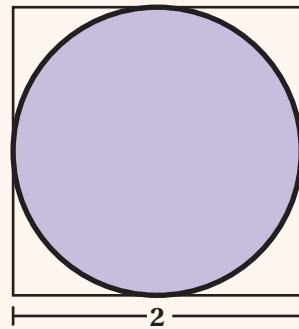
- 7** Complete the table with the unknown dimensions of each sphere. Express your answers in terms of  $\pi$ .

Diameter (units)	Radius (units)	Sphere Volume (cu. units)
	9	
8		
	3	
	6	
9		

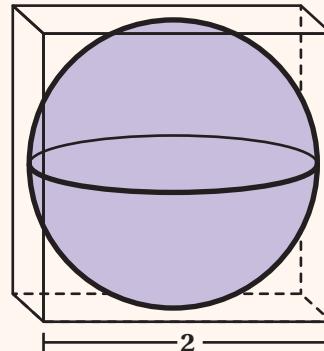
### Explore More

**8**

- a** What fraction of the square is filled by the circle?

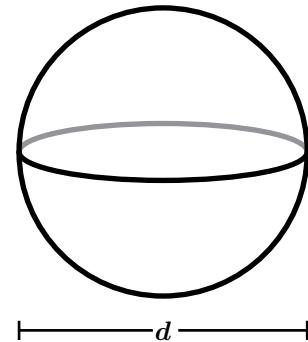
**b**

- What fraction of the cube is filled by the sphere?



## 9 Synthesis

Describe a strategy for determining the volume of a sphere given its diameter.

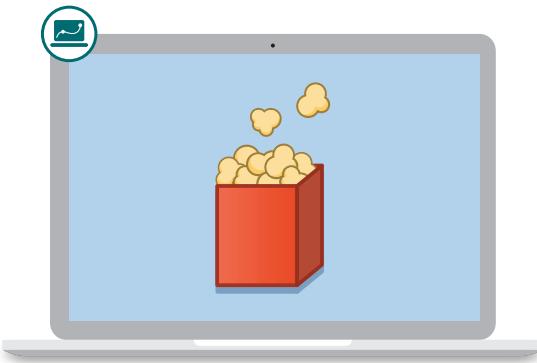


Things to Remember:

Name: ..... Date: ..... Period: .....

# Popcorn Possibilities

Let's apply surface area and volume to real-world situations.



## Warm-Up

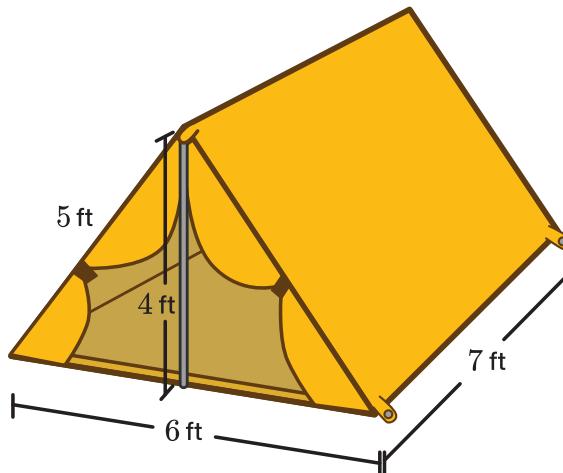
- 1** Daeja is trying to determine how much fabric it took to create this tent.

- a** What would be more useful for Daeja to calculate? Circle one.

Surface area

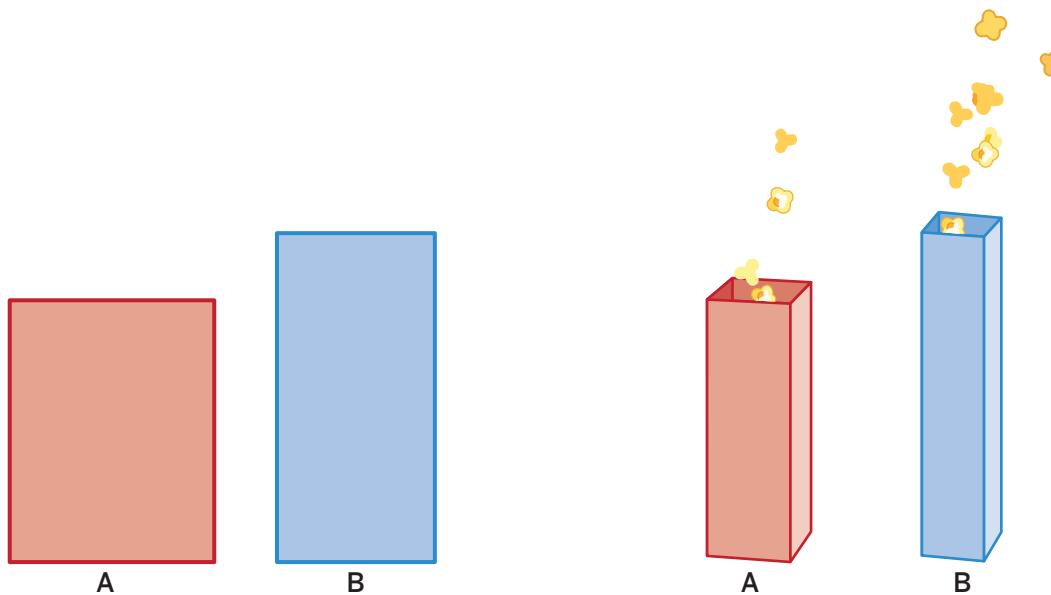
Volume

- b** Calculate the measurement you selected.



## Which Holds More?

- 2** Let's watch two 8.5-by-11-inch sheets of paper fold into containers.



- a** Which container do you think will hold more popcorn? Circle one.

Container A

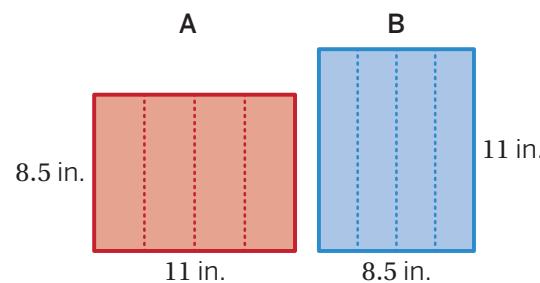
Container B

They will hold the same amount

- b** What information would help you know for sure?

- 3** Determine the amount of popcorn each container can hold. Show your thinking.

	Width (in.)	Height (in.)	Popcorn (cu. in.)
A	11	8.5	
B	8.5	11	



## Which Uses More Paper?

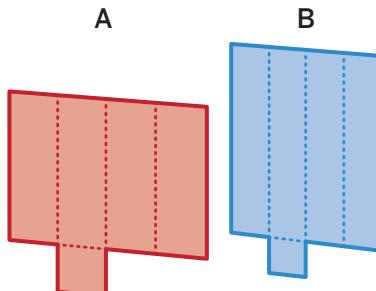
- 4** An extra piece of paper has been added to the bottom of each container (so it can actually hold popcorn).

Which container do you think uses more paper?  
Circle one.

Container A

Container B

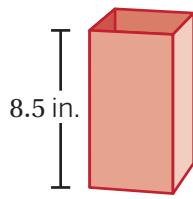
They use the same amount



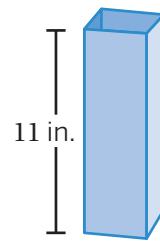
Explain your thinking.

- 5** Determine the amount of paper each container uses (including the bottom).  
Show your thinking.

Container A



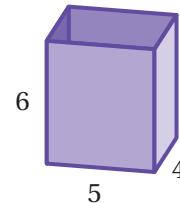
Container B



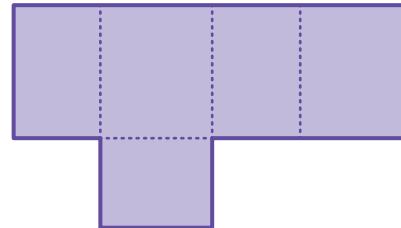
## More Popcorn Containers

**6** Here is a new container.

- a** How many cubic inches of popcorn can it hold?



- b** How many square inches of paper does it use?



**7** **a** Draw a *different* container that can hold 120 cubic inches of popcorn.

- b** Calculate the amount of paper your container uses.

### Explore More

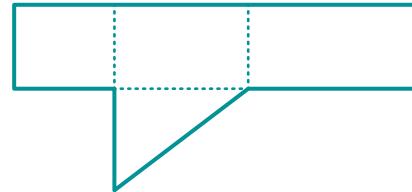
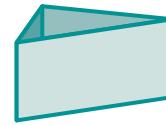
**8** **a** Draw a new container that can hold approximately 120 cubic inches of popcorn using as little paper as possible. Label your container's dimensions.

#### Sketch

- b** Calculate the amount of paper your container uses.

## 9 Synthesis

What is important to remember when calculating the surface area or volume of a prism? Use the example if it helps with your thinking.

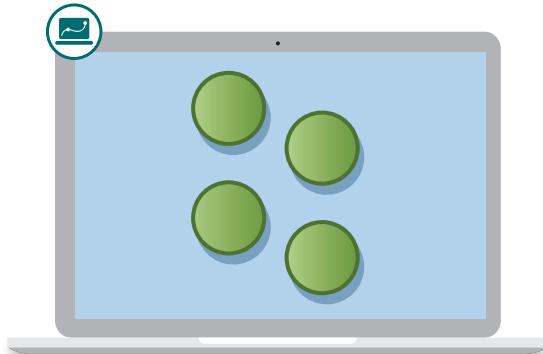


Things to Remember:

Name: ..... Date: ..... Period: .....

# Circles

Let's revisit exponents.



## Warm-Up

Evaluate each expression mentally. Try to think of more than one strategy.

**1**  $5 \cdot 2$

**2**  $5 \cdot 2 \cdot 2$

**3**  $5 \cdot 2 \cdot 2 \cdot 2$

**4**  $5 \cdot 2^4$

## Lots of Circles

**5** Let's look at a pattern.

Stage 0



Stage 1



Stage 2



Stage 3



Stage 4



**Discuss:** What do you notice? What do you wonder?

**6** How many circles will there be in Stage 5?

Explain your thinking.

## Lots of Circles (continued)

- 7** Here are Stages 5–12 of the same pattern.

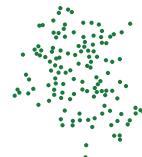
Stage 5



Stage 6



Stage 7



Stage 8



Stage 9



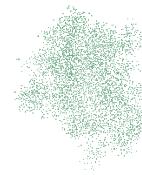
Stage 10



Stage 11



Stage 12



How many circles are there in Stage 12?

- 8** Adah and Jamal were calculating the number of circles in Stage 12.

Adah wrote:  $2 \cdot 2 \cdot 2$

Jamal wrote:  $2^{12}$

Whose expression is correct? Circle one.

Adah's

Jamal's

Both

Neither

Explain your thinking.

**A New Pattern**

- 9** Here is a new pattern.

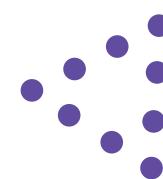
Stage 0



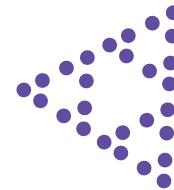
Stage 1



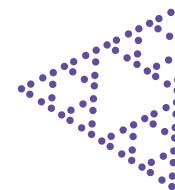
Stage 2



Stage 3



Stage 4



How many circles are there in Stage 4?

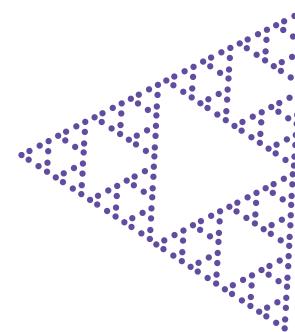
Explain your thinking.

- 10** There are 243 circles in Stage 5.

Select *all* the expressions that represent the number of circles in Stage 7.

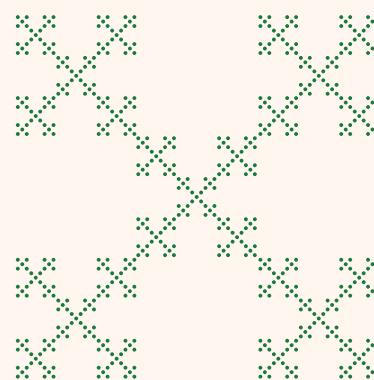
- A.  $3^7$
- B.  $243 \cdot 3^2$
- C.  $243 \cdot (3 \cdot 2)$
- D.  $243 + 243 + 243$
- E.  $243 \cdot 3 \cdot 3$

Stage 5

**Explore More**

- 11** How many circles are in this image?

Explain your thinking.



## 12 Synthesis

When might it be helpful to write values or expressions using *exponents*?

Use the examples if they help with your thinking.

$$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$$

$$6 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 5$$

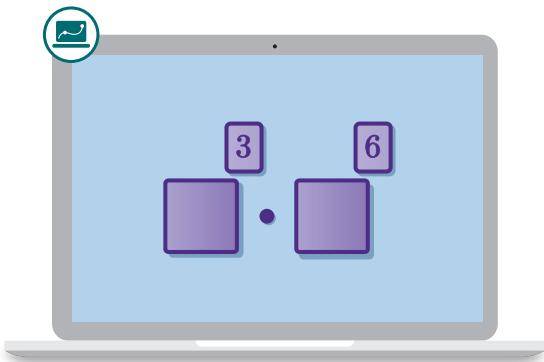
$$10 + 10 + 10 + 10 + 10$$

Things to Remember:

Name: ..... Date: ..... Period: .....

# Combining Exponents

Let's explore equivalent expressions with exponents.



## Warm-Up

- 1** Which one doesn't belong? Explain your thinking.
- A.  $(2^2)^3$
  - B.  $2^3 \cdot 2 \cdot 2^2$
  - C.  $2 \cdot 32$
  - D.  $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

## Combining Exponents

**2** Here are some different ways to build a billion,  $10^9$ , by multiplying two **powers of ten**.

$$(10 \cdot 10) (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$$

$$10^2 \cdot 10^7 = 10^9$$

$$(10 \cdot 10 \cdot 10 \cdot 10) (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$$

$$10^4 \cdot 10^5 = 10^9$$

$$(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) (10 \cdot 10 \cdot 10)$$

$$10^6 \cdot 10^3 = 10^9$$

$$(10 \cdot 10 \cdot 10) (10)$$

$$10^8 \cdot 10^1 = 10^9$$



**Discuss:** What do you notice? What do you wonder?

**3** Group the expressions by whether they are equivalent to  $8^5$ ,  $8^6$ , or neither.

Some expressions will have no match.

$$8 + 8 + 8 + 8 + 8 + 8$$

$$40$$

$$(8 \cdot 8 \cdot 8) (8 \cdot 8 \cdot 8)$$

$$8^3 \cdot 8^3$$

$$8^3 \cdot 8^2$$

$$(8^3)^2$$

$$8^4 + 8$$

$$(8 \cdot 8 \cdot 8) (8 \cdot 8)$$

Equivalent to $8^5$	Equivalent to $8^6$	Neither

**4** Here is a new expression:  $4^5 \cdot 2^5$ . Is this expression equivalent to  $8^5$ ?

Show or explain your thinking.

## Odd One Out

**5**

- a Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

$$3 + 3 + 3 + 3 + 3$$

$$3^2 \cdot 3 \cdot 3 \cdot 3$$

$$3^5$$

- b How could you change this expression so that it has the same value as the others?

**6**

- a Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

$$5 \cdot 5 \cdot 5 \cdot 4 \cdot 4 \cdot 4$$

$$20^3$$

$$(5 \cdot 3) \cdot (4 \cdot 3)$$

- b How could you change this expression so that it has the same value as the others?

**7**

- a Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

$$(2 \cdot 2 \cdot 2) (2 \cdot 2 \cdot 2)$$

$$6^2$$

$$(2^3)^2$$

- b How could you change this expression so that it has the same value as the others?

**Odd One Out (continued)**

**8** Here are some more pairs of equivalent expressions.

For each row, write one more equivalent expression.

Expression 1	Expression 2	Expression 3
$4 \cdot 4 \cdot 4^3$	$4^5$	
$6^4$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	
$(7 \cdot 7) \cdot (7 \cdot 7) \cdot (7 \cdot 7)$	$(7^2)^3$	
$5^{18}$	$(5^3)^6$	

**Explore More**

**9** Using whole numbers 0 through 9 without repeating, fill in the blanks so that all expressions have the same value.

$$\begin{array}{c}
 (\underline{\hspace{1cm}} \underline{\hspace{1cm}}) \underline{\hspace{1cm}} \\
 \underline{\hspace{1cm}} \underline{\hspace{1cm}} \cdot \underline{\hspace{1cm}} \underline{\hspace{1cm}} \\
 \underline{\hspace{1cm}} \underline{\hspace{1cm}} \cdot \underline{\hspace{1cm}} \underline{\hspace{1cm}} \underline{\hspace{1cm}} \\
 \underline{\hspace{1cm}} \\
 \end{array}$$

## 10 Synthesis

Describe some strategies for writing equivalent expressions involving exponents.

Use the examples if they help with your thinking.

$$3^4 \cdot 3^2 = 3^6$$

$$3^5 \cdot 4^5 = 12^5$$

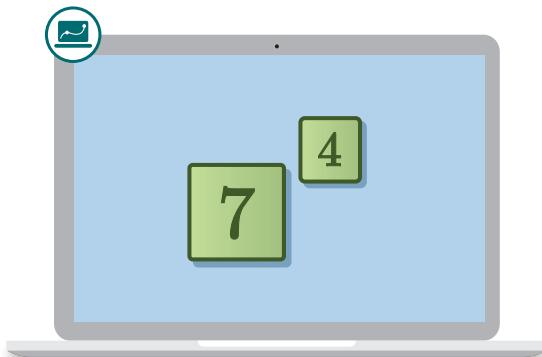
$$(3^2)^4 = 3^8$$

Things to Remember:

Name: ..... Date: ..... Period: .....

# Rewriting Powers

Let's rewrite expressions with exponents as a single power.



## Warm-Up

- 1** Order the expressions from what you think is *least complicated* to *most complicated*.

$$\frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7$$

$$\frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7}$$

$$\frac{7^3 \cdot 7^3}{7^2}$$

$$\frac{7^7}{7^3}$$

$$7^4$$


**Least Complicated**

**Most Complicated**

**Activity****1**

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Single Powers**

- 2** Circle one expression.

$$\frac{7^3 \cdot 7^3}{7^2}$$

$$\frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7$$

$$\frac{7^7}{7^3}$$

$$\frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7}$$

Show or explain how to rewrite this expression as the single power  $7^4$ .

- 3** Here is how Jayla rewrote  $\frac{7^5 \cdot 7^2}{7^3}$  as a single power.

 **Discuss:** How could you rewrite  $\frac{4^9}{4^2 \cdot 4^4}$  as a single power?

Jayla

$$\begin{aligned}
 \frac{7^5 \cdot 7^2}{7^3} &= \frac{(7 \cdot 7 \cdot 7 \cdot 7 \cdot 7) \cdot (7 \cdot 7)}{7 \cdot 7 \cdot 7} \\
 &= \frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \\
 &= 1 \cdot 1 \cdot 1 \cdot 7 \cdot 7 \cdot 7 \\
 &= 7^4
 \end{aligned}$$

 single power of 7

- 4** Rewrite each expression as a single power.

Expression	Single Power
$\frac{7^5 \cdot 7^2}{7^3}$	$7^4$
$\frac{4^9}{4^2 \cdot 4^4}$	
$\frac{2^3 \cdot 2^3 \cdot 2^3}{2 \cdot 2 \cdot 2}$	

Expression	Single Power
$2^4 \cdot 3^4$	
$\frac{6^7}{2^7}$	
$\frac{(8^4)^2}{8}$	

## Single Powers (continued)

- 5** Sort the expressions based on whether they are equivalent to  $6^8$ .

$$2^8 \cdot 3^8$$

$$2^3 \cdot 3^5$$

$$\frac{12^8}{2^8}$$

$$\frac{6^4 \cdot 6^4}{6^1}$$

$$\frac{6^7 \cdot 6^7}{(6^3)^2}$$

Equivalent to $6^8$	Not Equivalent to $6^8$

- 6** Create one (or more) expressions that are equivalent to  $4^5$ . Write something as unique and as complicated as you want!

## Challenge Creator

**7** You will use a separate sheet of paper to create your own single power challenge.

- a** **Make It!** On your sheet of paper, write down a single power, using a positive integer for the base and any integer for the exponent. (For example,  $4^3$  or  $5^{16}$  or  $2^{100}$ .)
- b** **Solve It!** On this page, record your single power, then create an expression that is equivalent to it. (For example,  $5^{10} \cdot 5^6$  is equivalent to  $5^{16}$ .)

My Single Power	Equivalent Expression

- c** **Swap It!** Swap your challenge with one or more partners. Record your partner's single power, then create an expression that is equivalent to it.

	Single Power	Equivalent Expression
Partner 1		
Partner 2		
Partner 3		
Partner 4		

## 8 Synthesis

Describe a strategy for rewriting an expression as a single power.

Use these expressions if they help with your thinking.

$$\frac{6^7 \cdot 6^7}{(6^3)^2}$$

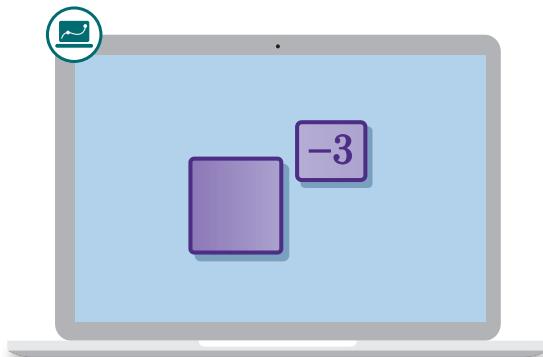
$$2^8 \cdot 3^8$$

$$\frac{12^8}{2^8}$$

Things to Remember:

# Negative and Zero Exponents

Let's explore exponents that are not positive.



## Warm-Up

- 1** Order the expressions by value from *least* to *greatest*.

$$\frac{2^6}{2^3} \quad \frac{2^5}{2^5} \quad 2^6 \quad 0 \quad \frac{2^3}{2^4}$$

	Least
	Greatest

## Negative and Zero Exponents

**2** Complete as much of the table as you can.

Exponent Form	Expanded Form	Value
$10^4$	$10 \cdot 10 \cdot 10 \cdot 10$	10,000
$10^3$	$10 \cdot 10 \cdot 10$	
$10^2$		100
	10	10
$10^0$		
$10^{-1}$		
$10^{-2}$		$\frac{1}{100}$

**3** What patterns do you see in the table? Describe as many as you can.

**4** Cameron wanted to investigate more about negative and zero exponents. So Cameron decided to write some expressions and apply exponent properties.



**Discuss:**

- What patterns do you notice in these expressions?
- How could you use Cameron's work to determine the values of  $100$  and  $10^{-1}$ ?

Cameron

$$10^5 \cdot 10^2 = 10^7$$

$$10^5 \cdot 10^1 = 10^6$$

$$10^5 \cdot 10^0 = 10^5$$

$$10^5 \cdot 10^{-1} = 10^4$$

## Beyond 10

- 5** Complete this new table about powers of 3.

Exponent Form	Expanded Form	Value
$3^3$	$3 \cdot 3 \cdot 3$	27
$3^2$	$3 \cdot 3$	
$3^1$	3	3
$3^0$		
	$\frac{1}{3}$	$\frac{1}{3}$
$3^{-2}$		
$3^{-3}$		

- 6** The value of  $3^6 = 729$ . Predict the value of  $3^{-6}$ .

**Beyond 10** (continued)

- 7** Group the equivalent expressions. Some expressions may have no match.

$$\left(\frac{1}{3}\right)^5$$

$$\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$$

$$3^{-5}$$

$$\frac{1}{5 \cdot 5 \cdot 5}$$

$$5^{-3}$$

$$-15$$

$$\frac{1}{3^5}$$

$$\frac{1}{15}$$

- 8** Here are two expressions from earlier.

Are these equivalent? Circle one.

Yes

No

I'm not sure.

$$3^{-5}$$

$$-15$$

Explain your thinking.

**Explore More**

- 9** Write as many different expressions that are equivalent to  $\left(\frac{2}{3}\right)^{-3}$  as you can.  
Here is one example:  $\left(\frac{3}{2}\right)^3$

## 10 Synthesis

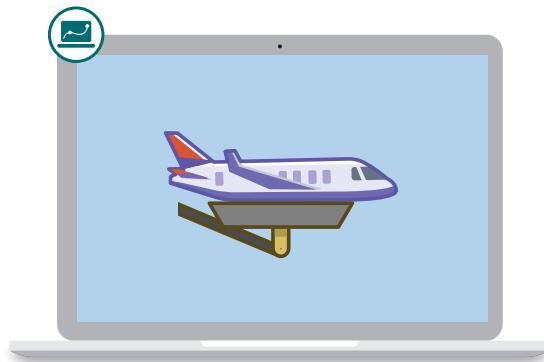
How could you use the table to convince someone that  $6^0 = 1$  and  $6^{-1} = \frac{1}{6}$ ?

Exponent Form	Value
$6^3$	216
$6^2$	36
$6^1$	6
$6^0$	
$6^{-1}$	

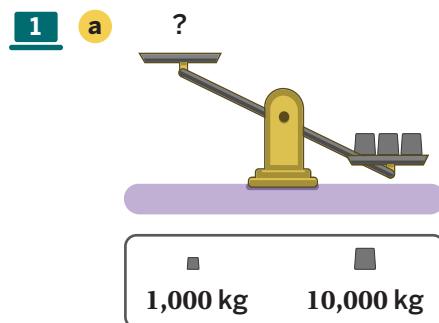
Things to Remember:

# Scales and Weights, Part 1

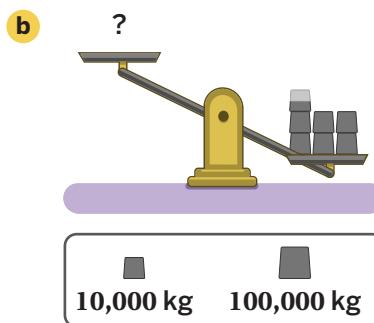
Let's explore ways to represent large numbers.



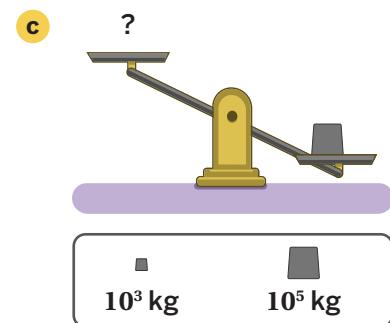
## Warm-Up



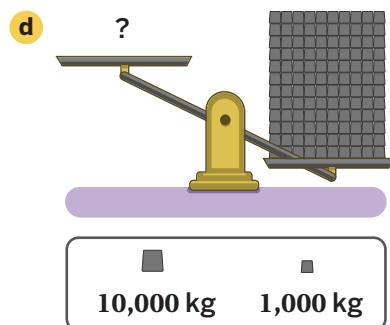
How many 1,000 kg weights are needed to balance with three 10,000 kg weights?



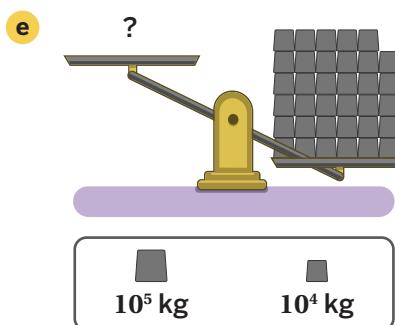
How many 10,000 kg weights are needed to balance with 6.5 100,000 kg weights?



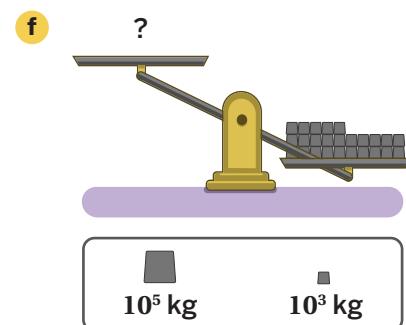
How many  $10^3$  kg weights are needed to balance with one  $10^5$  kg weight?



How many 10,000 kg weights are needed to balance with 120 1,000 kg weights?



How many  $10^5$  kg weights are needed to balance with 35  $10^4$  kg weights?



How many  $10^5$  kg weights are needed to balance with 25  $10^3$  kg weights?

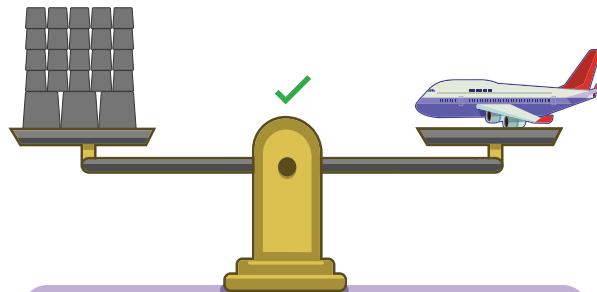
## Scales and Weights

- 2** A plane weighs 320,000 kilograms.

The table shows how a student balanced the scale using the weights provided.

- 3** Write two other combinations of weights that will balance the scale.

$10^5$ kg Weights	$10^4$ kg Weights	$10^3$ kg Weights
3	0	20



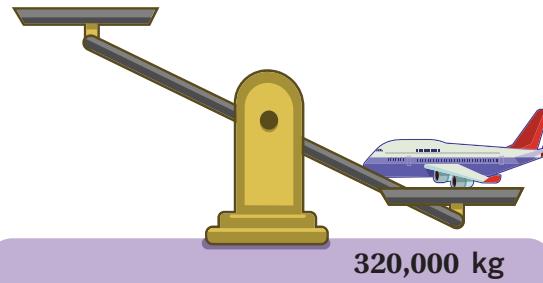
320,000 kg                    320,000 kg

### Available Weights

=  $10^5$  kg   =  $10^4$  kg   =  $10^3$  kg

- 4** Another way to write 320,000 kilograms is to use a combination of powers of 10. For example:  $3 \cdot 10^5 + 2 \cdot 10^4$  kilograms.

- a** **Discuss:** What does each part of the expression represent in terms of weights?



### Available Weights

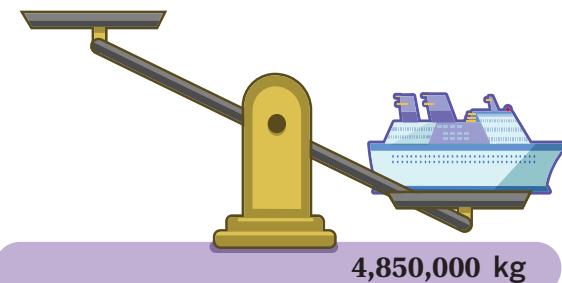
=  $10^5$  kg   =  $10^4$  kg   =  $10^3$  kg

- b** Write an expression to represent a *different* combination of available weights that will balance the scale.

## Ships and Shuttles

- 5** A ship weighs 4,850,000 kilograms.

Write an expression to represent a combination of available weights that will balance the scale.



### Available Weights

$$\blacksquare = 10^6 \text{ kg} \quad \blacksquare = 10^5 \text{ kg} \quad \blacksquare = 10^4 \text{ kg}$$

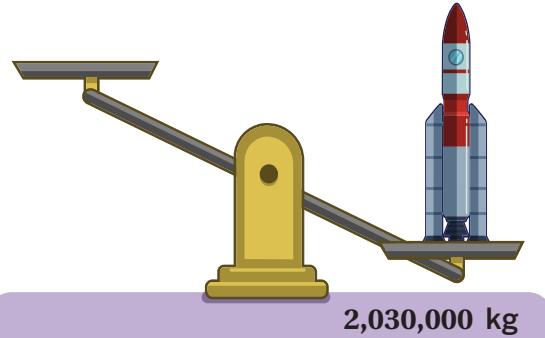
- 6** Rishi and Anh tried to balance the scale with a 2,030,000-kilogram space shuttle.

- Rishi wrote:  $2.03 \cdot 10^6$  kilograms
- Anh wrote:  $20 \cdot 10^5 + 3 \cdot 10^4$  kilograms

Whose expression represents a combination of weights that will balance the scale? Circle one.

Rishi's      Anh's      Both      Neither

Explain your thinking.



### Available Weights

$$\blacksquare = 10^6 \text{ kg} \quad \blacksquare = 10^5 \text{ kg} \quad \blacksquare = 10^4 \text{ kg}$$

- 7** Write an expression to represent a *different* combination of available weights that will balance the scale.

## Ships and Shuttles (continued)

- 8** Match each expression with the value it is equivalent to. One expression will have no match.

$$4.7 \cdot 10^5$$

$$4.7 \cdot 10^4$$

$$7 \cdot 10^4 + 400 \cdot 10^3$$

$$4 \cdot 10^4 + 7 \cdot 10^2$$

$$4 \cdot 10^5 + 7 \cdot 10^3$$

$$40 \cdot 10^3 + 70 \cdot 10^2$$

$$4 \cdot 10^5 + 5 \cdot 10^4 + 20 \cdot 10^3$$

407,000	47,000	470,000

- 9** In the previous problem, how did you decide which value this expression is equivalent to?

$$7 \cdot 10^4 + 400 \cdot 10^3$$

## 10 Synthesis

What are some strategies for writing a number as a combination of powers of 10? Use the examples if they help with your thinking.

The diagram illustrates five different ways to represent the number 2,560 using powers of 10. The first way is the standard form: 2,560. The second way shows it as a sum:  $2 \cdot 10^3 + 5 \cdot 10^2 + 6 \cdot 10^1$ . The third way shows it as a single multiplication:  $256 \cdot 10^1$ . The fourth way shows it as another single multiplication:  $2.56 \cdot 10^3$ . The fifth way shows it as another sum:  $25 \cdot 10^2 + 6 \cdot 10^1$ .

2,560

$2 \cdot 10^3 + 5 \cdot 10^2 + 6 \cdot 10^1$

$256 \cdot 10^1$

$2.56 \cdot 10^3$

$25 \cdot 10^2 + 6 \cdot 10^1$

$25.6 \cdot 10^2$

Things to Remember:

Name: ..... Date: ..... Period: .....

# Scales and Weights, Part 2

Let's explore ways to represent small numbers with powers of 10.



## Warm-Up

- 1** Match each number to a verbal description. One number will have no match.

1,000

0.000001

 $10^{-6}$  $10^{-3}$ 

0.001

0.00001

 $10^3$ 

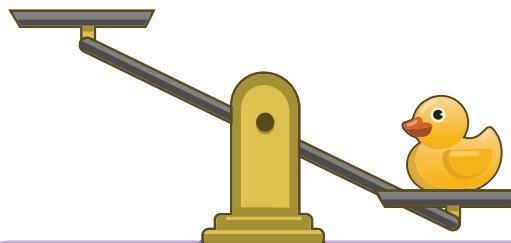
One thousand	One thousandth	One millionth

## Light Weights

- 2-3** A rubber duck weighs 0.15 kilograms. A student balanced the scale using the weights shown in the top row of the table.

Write two other combinations of weights that will balance the scale.

$10^{-1}$ kg	$10^{-2}$ kg	$10^{-3}$ kg
1	5	0



0.15 kg

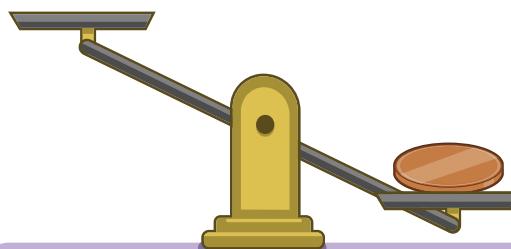
### Available Weights

=  $10^{-1}$  kg   =  $10^{-2}$  kg   =  $10^{-3}$  kg

- 4** A penny weighs 0.0031 kilograms. Here's one way to write its weight using a combination of powers of 10:

$$3 \cdot 10^{-3} + 1 \cdot 10^{-4}$$

Write an expression to represent a *different* combination of available weights that will balance the scale.



0.0031 kg

### Available Weights

=  $10^{-3}$  kg   =  $10^{-4}$  kg   =  $10^{-5}$  kg

## Light Weights (continued)

- 5** Two students made mistakes writing 0.0031 using combinations of powers of 10.

Circle your favorite mistake.

Arturo's mistake

Kimaya's mistake

Arturo  
 $3.1 \cdot 10^{-2}$

Kimaya  
 $3 \cdot 10^{-4} + 1 \cdot 10^{-3}$

- a** What is correct about this student's work?

- b** What could you add or change to make *all* of their work correct?

## Getting Smaller and Smaller

- 6** The weight of a raisin is 0.000572 kilograms.



0.000572 kg

Here is how two students rewrote 0.000572.



**Discuss:**

- What are some advantages of Alina's strategy?
- What are some advantages of Lukas's strategy?

Alina

$$5 \cdot 10^{-4} + 7 \cdot 10^{-5} + 2 \cdot 10^{-6}$$

Lukas

$$57.2 \cdot 10^{-5}$$

- 7** Lukas's strategy was to rewrite the raisin's weight, 0.000572 kilograms, as a number times a single power of 10:

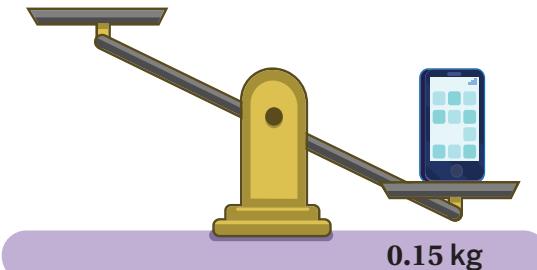
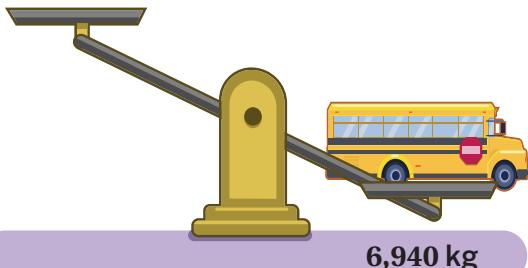
$$57.2 \cdot 10^{-5}$$

Write the same weight as:

- A number times  $10^{-6}$ : .....
- A number times  $10^{-4}$ : .....

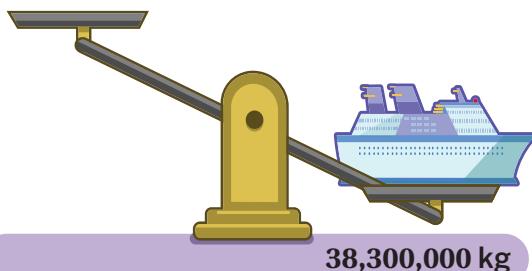
## Repeated Challenges

- 8** Write the weight of each object using a number times a single power of 10.

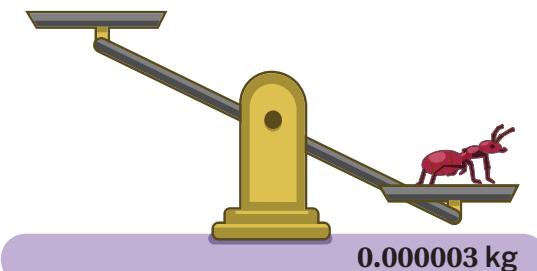


**a** Write the weight of the bus (6,940 kg) as a number times  $10^3$ .

**b** Write the weight of the cell phone (0.15 kg) as a number times  $10^{-2}$ .

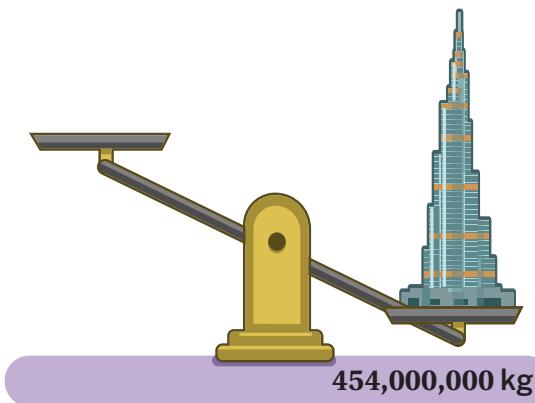


**c** Write the weight of the cruise ship (38,300,000 kg) as a number times  $10^7$ .



**d** Write the weight of the ant (0.000003 kg) as a number times  $10^{-5}$ .

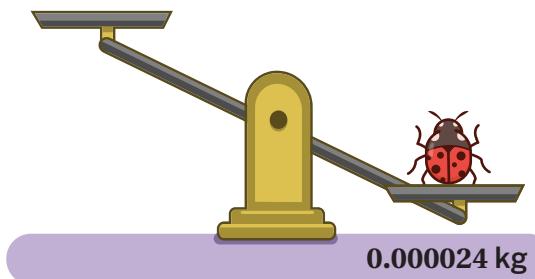
## Repeated Challenges (continued)



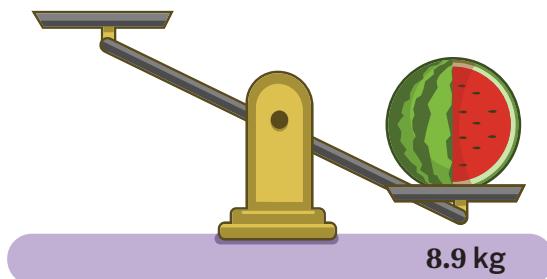
- e Write the weight of the Burj Khalifa (454,000,000 kg) as a number times  $10^7$ .



- f Write the weight of the pyramid (5,216,000,000 kg) as a number times  $10^9$ .



- g Write the weight of the ladybug (0.000024 kg) as a number times  $10^{-6}$ .



- h Write the weight of the watermelon (8.9 kg) as a number times  $10^1$ .

## 9 Synthesis

What are some strategies for writing very small values as a number times a single power of 10? Use the examples if they help with your thinking.

0.00083

$8.3 \cdot 10^{-4}$

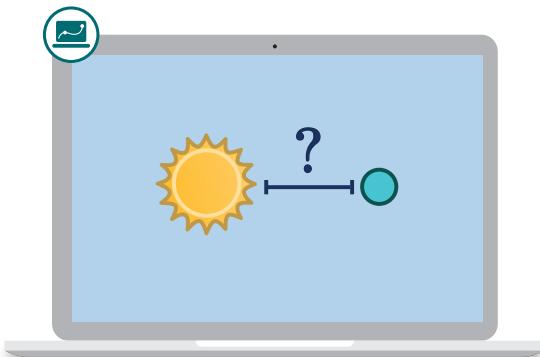
$83 \cdot 10^{-5}$

Things to Remember:

Name: ..... Date: ..... Period: .....

# Specific and Scientific

Let's explore scientific notation.



## Warm-Up

- 1** Order these numbers from *least* to *greatest*.

$$75 \cdot 10^5$$

$$4,000,000$$

$$0.6 \cdot 10^7$$

$$5 \cdot 10^5$$

--	--	--	--	--	--

Least

Greatest

- 2** Order these numbers from *least* to *greatest*.

$$4 \cdot 10^6$$

$$6 \cdot 10^6$$

$$5 \cdot 10^5$$

$$7.5 \cdot 10^6$$

--	--	--	--	--	--

Least

Greatest

- 3** Which list was easier to sort? Explain your thinking.

## Scientific Notation

**Scientific notation** is a specific way of writing very large or very small numbers that can help us compare numbers.

- 4** Some of these numbers are written in scientific notation and some are not.

What do you think it means for a number to be written in scientific notation?

In Scientific Notation	Not in Scientific Notation
$3 \cdot 10^9$	3,000,000,000
$1.257 \cdot 10^5$	$125.7 \cdot 10^3$
$2 \cdot 10^{-1}$	0.2
$5.1 \cdot 10^{-4}$	$0.51 \cdot 10^{-3}$

- 5** Sort the numbers based on whether they are written in scientific notation.

0.00099

48,200

$0.78 \cdot 10^{-3}$

$5.23 \cdot 10^8$

$8.7 \cdot 10^{-12}$

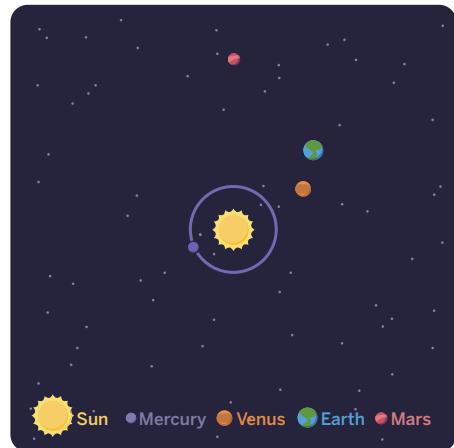
$36 \cdot 10^5$

In Scientific Notation	Not in Scientific Notation

## Solar System and Test Tubes

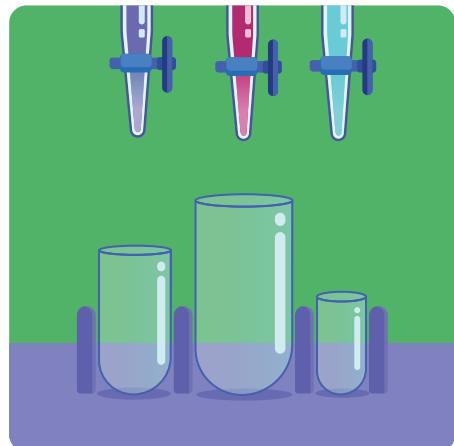
- 6** Here are the distances of four planets from the Sun. Write each distance in scientific notation. Mercury has been done for you.

Planet	Distance From Sun (mi)	Scientific Notation (mi)
Mercury	36,000,000	$3.6 \cdot 10^7$
Venus	67,000,000	
Earth	92,960,000	
Mars	$1417 \cdot 10^5$	



- 7** We can use scientific notation to represent small numbers, too! Write each volume in scientific notation.

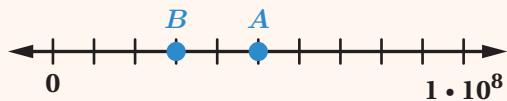
Liquid Color	Volume (L)	Scientific Notation (L)
Purple	0.000125	
Red	0.0002	
Blue	$325 \cdot 10^{-8}$	



### Explore More

- 8** What are the values of points A and B? Write your answers in scientific notation.

Point	Value
A	
B	



Explain your thinking.

## 9 Synthesis

Describe a strategy for writing a number in scientific notation. Use the table if it helps with your thinking.

Not in Scientific Notation	In Scientific Notation
36,000,000	$3.6 \cdot 10^7$
6,700,000	
0.00024	
$417 \cdot 10^3$	

Things to Remember:

# Balance the Scale

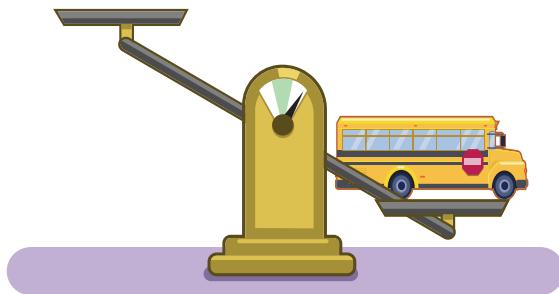
Let's use multiplication and division to compare large and small numbers in scientific notation.



## Warm-Up

- 1** A school bus sits on one side of the scale.

How many jelly beans do you think it would take to balance the scale?  
Use scientific notation.

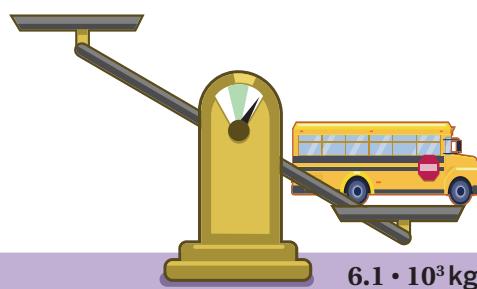


## Balance the Scale, Part 1

- 2** One jelly bean weighs  $1.5 \cdot 10^{-3}$  kilograms.  
A school bus weighs  $6.1 \cdot 10^3$  kilograms.

Describe a strategy for determining about how many jelly beans weigh as much as a bus.

$$1.5 \cdot 10^{-3} \text{ kg}$$

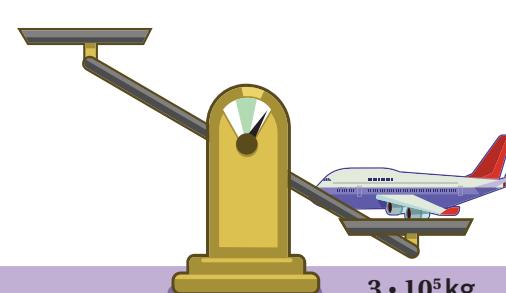


$$6.1 \cdot 10^3 \text{ kg}$$

- 3** One jelly bean weighs  $1.5 \cdot 10^{-3}$  kilograms.  
A jumbo jet weighs  $3 \cdot 10^5$  kilograms.

How many jelly beans will it take to balance the scale?

$$1.5 \cdot 10^{-3} \text{ kg}$$



$$3 \cdot 10^5 \text{ kg}$$

- 4** Basheera says the jumbo jet weighs about 200 times as much as the school bus. Elena says it weighs about 50 times as much as the school bus.

Whose claim is correct? Circle one.

Basheera's      Elena's      Neither

Explain your thinking.



$$3 \cdot 10^5 \text{ kg}$$



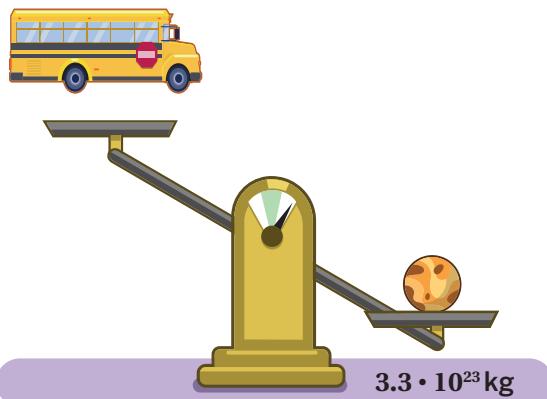
$$6.1 \cdot 10^3 \text{ kg}$$

**Balance the Scale, Part 2**

- 5** A school bus weighs  $6.1 \cdot 10^3$  kilograms.  
Mercury weighs  $3.3 \cdot 10^{23}$  kilograms.

About how many school buses will it take to balance the scale? Write your answer in scientific notation.

$$6.1 \cdot 10^3 \text{ kg}$$



- 6** Here are two students' strategies for the previous problem. Examine their work.

Explain how Basheera and Elena each arrived at the answer  $5 \cdot 10^{19}$  buses.

Basheera:

Basheera

$$\begin{aligned} \text{Bus: } & 6 \cdot 10^3 \\ & \times 0.5 \quad \left( \frac{\text{Mercury}}{\text{Bus}} \right) \times 10^{20} \\ \text{Mercury: } & 3 \cdot 10^{23} \end{aligned}$$

Elena:

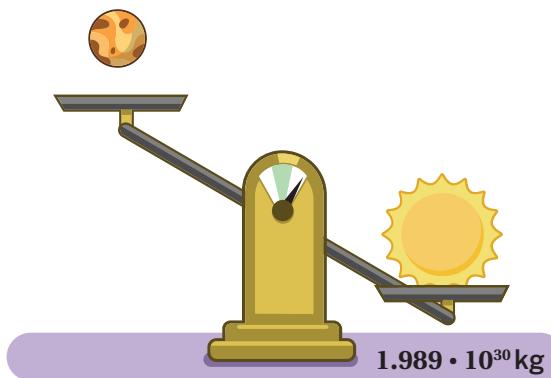
Elena

$$\begin{aligned} \text{Mercury: } & 3 \cdot 10^{23} \\ & 30 \cdot 10^{22} \\ \hline & \frac{30 \cdot 10^{22}}{6 \cdot 10^3} \end{aligned}$$

- 7** Mercury weighs  $3.3 \cdot 10^{23}$  kilograms.  
The Sun weighs  $1.989 \cdot 10^{30}$  kilograms.

About how many Mercurys will it take to balance the scale? Write your answer in scientific notation.

$$3.3 \cdot 10^{23} \text{ kg}$$



**Balance the Scale, Part 2 (continued)****8**

- Choose an object from each row and then compare them.
- Use these weights to determine how many of your first object weighs as much as your second object.
- Complete as many comparisons as you have time for.

<b>Watermelon</b> $8.9 \cdot 10^0$ kilograms	<b>Horse</b> $7.1 \cdot 10^2$ kilograms	<b>Ant</b> $3 \cdot 10^{-6}$ kilograms	<b>Cell Phone</b> $1.5 \cdot 10^{-1}$ kilograms	<b>Penny</b> $3.1 \cdot 10^{-3}$ kilograms
<b>Bus</b> $7.81 \cdot 10^3$ kilograms	<b>Moon</b> $7.348 \cdot 10^{22}$ kilograms	<b>Rocket</b> $2.03 \cdot 10^6$ kilograms	<b>Cruise Ship</b> $3.83 \cdot 10^7$ kilograms	<b>Pyramid</b> $5.216 \cdot 10^9$ kilograms

**Comparison 1**

How many ..... weigh about  
as much as the ..... ?

Write your answer in scientific notation.

**Comparison 2**

How many ..... weigh  
about as much as the ..... ?

Write your answer in scientific notation.

**Comparison 3**

How many ..... weigh  
about as much as the ..... ?

Write your answer in scientific notation.

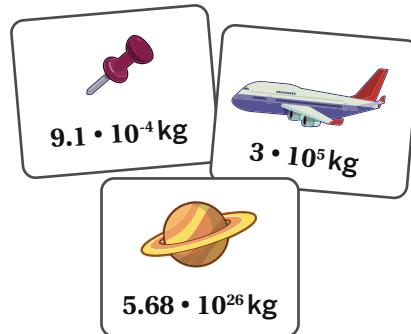
**Comparison 4**

How many ..... weigh  
about as much as the ..... ?

Write your answer in scientific notation.

## 9 Synthesis

Describe a strategy for determining how many times as large one number is compared to another.



Things to Remember:

Name: ..... Date: ..... Period: .....

# City Lights

Let's apply our understanding of place value to add and subtract with scientific notation.



## Warm-Up

- 1** Ariel says:  $2 \cdot 10^2 + 3 \cdot 10^3 = 5 \cdot 10^5$ .

Is Ariel's claim correct? Circle one.

Yes

No

I'm not sure

Explain your thinking.

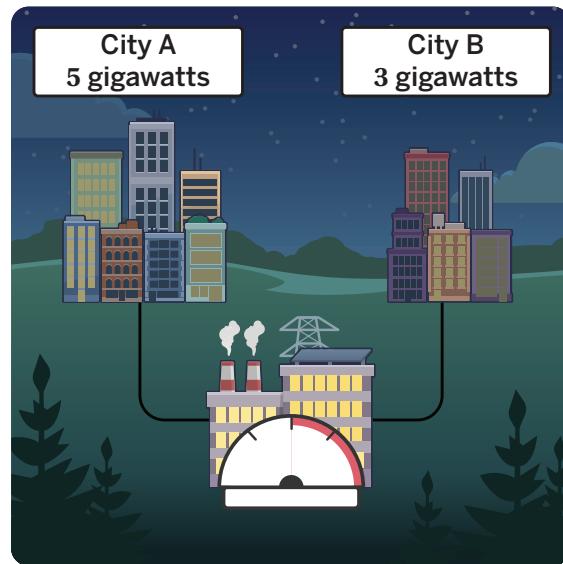
## City Lights, Part 1

- 2** City A and City B get electricity from the same source.

City A needs 5 gigawatts of electricity.

City B needs 3 gigawatts.

How many gigawatts are needed to power both cities?

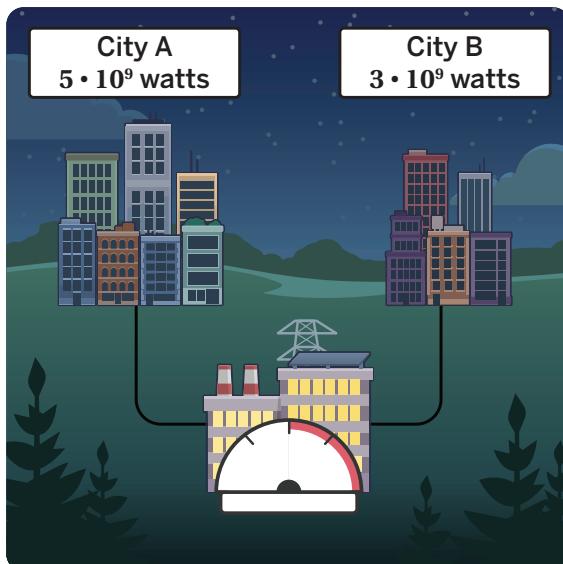


- 3** 1 gigawatt is equal to  $10^9$  watts.

City A needs  $5 \cdot 10^9$  watts of electricity.

City B needs  $3 \cdot 10^9$  watts.

How many watts are needed to power both cities? Write your answer in scientific notation.



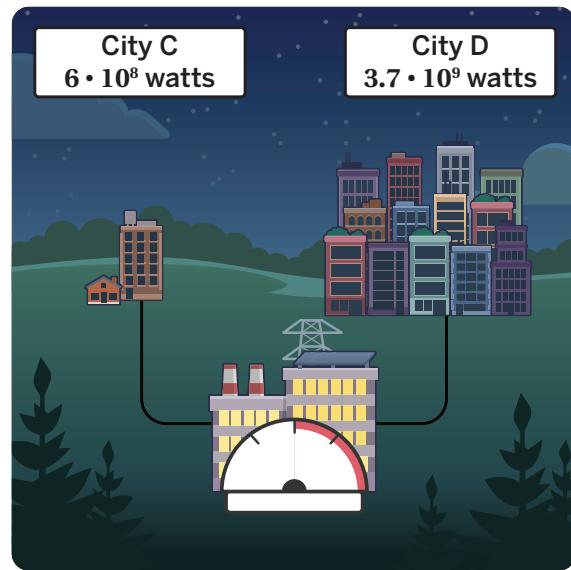
## City Lights, Part 1 (continued)

- 4** Here are two new cities: City C and City D.

City C needs  $6 \cdot 10^8$  watts of electricity.

City D needs  $3.7 \cdot 10^9$  watts.

How many watts are needed to power both cities? Write your answer in scientific notation.



- 5** Tameeka made a mistake on the previous problem.

**a** What do you think Tameeka did well?

Tameeka  
 $3.7 \cdot 10^9 + 6 \cdot 10^8$   
 $9.7 \cdot 10^9$

**b** What would you recommend Tameeka change in her work?

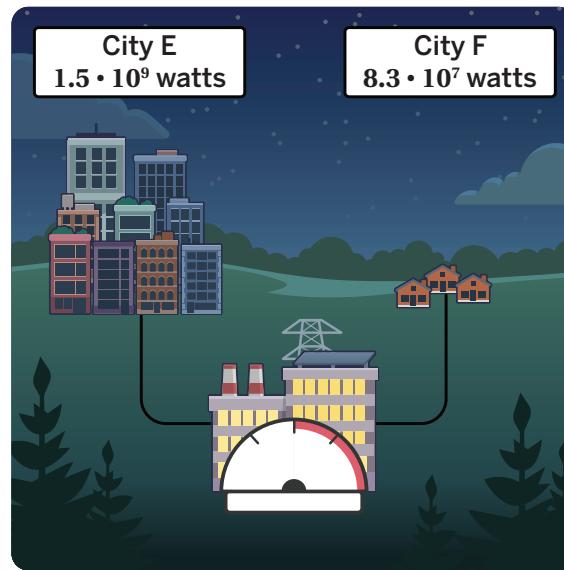
## City Lights, Part 2

- 6** Here are two new cities: City E and City F.

City E needs  $1.5 \cdot 10^9$  watts of electricity.

City F needs  $8.3 \cdot 10^7$  watts.

How many watts are needed to power both cities? Write your answer in scientific notation.



- 7** Here are Ariel's and Tameeka's strategies for the previous problem.

**Discuss:**

- How might each student finish the problem?
- After seeing both strategies, how would you add  $3.6 \cdot 10^6 + 2.5 \cdot 10^5$ ?

*Ariel*

$$1.5 \cdot 10^9 + 8.3 \cdot 10^7$$

$$1.5 \cdot 10^9 + 0.083 \cdot 10^9$$

*Tameeka*

$$1.5 \cdot 10^9 + 8.3 \cdot 10^7$$

$$\begin{array}{r} 150 \cdot 10^7 \\ + 8.3 \cdot 10^7 \\ \hline \end{array}$$

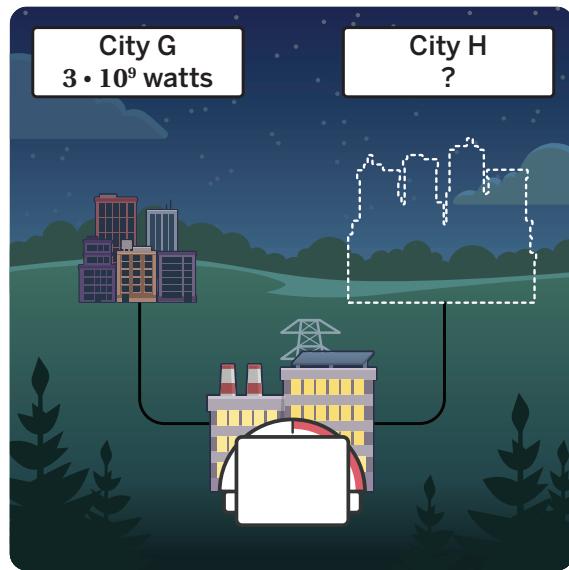
## City Lights, Part 2 (continued)

- 8** Here are two new cities: City G and City H.

The power plant provides exactly enough electricity for both cities:  $4.75 \cdot 10^{10}$  watts.

City G uses  $3 \cdot 10^9$  watts.

How many watts does City H use? Write your answer in scientific notation.



- 9** Match each value with the correct situation. Note: All units are in watts.

$3.76 \cdot 10^{10}$

$1.03 \cdot 10^{12}$

$1.6 \cdot 10^{10}$

$1.03 \cdot 10^{11}$

Situation A	Situation B	Situation C	Situation D
City 1: $8 \cdot 10^{10}$	City 1: $4.5 \cdot 10^{10}$	City 1: $9.6 \cdot 10^{10}$	City 1: _____
City 2: $9.5 \cdot 10^{11}$	City 2: _____	City 2: $7 \cdot 10^9$	City 2: $2.4 \cdot 10^9$
Total: _____	Total: $6.1 \cdot 10^{10}$	Total: _____	Total: $4 \cdot 10^{10}$

## 10 Synthesis

What are some important things to remember when adding or subtracting numbers written in scientific notation?

Use the examples if they help with your thinking.

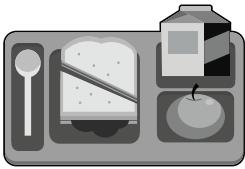
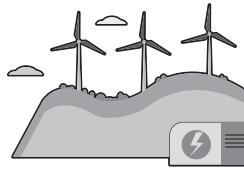
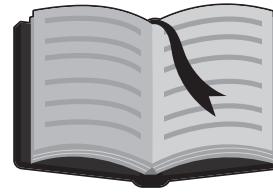
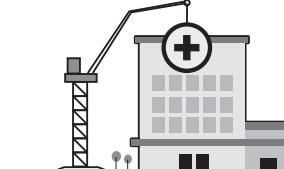
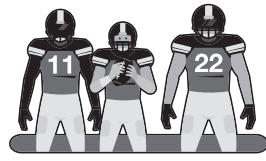
$$4.6 \cdot 10^7 + 3.2 \cdot 10^6$$

$$1.57 \cdot 10^8 - 4 \cdot 10^6$$

Things to Remember:

Name: ..... Date: ..... Period: .....

# Costs of Items

 <p><b>Pay student lunch debt for one year</b>  <math>\\$1.76 \cdot 10^8</math></p>	 <p><b>Build a wind farm</b>  <math>\\$3 \cdot 10^6</math></p>	 <p><b>Buy a luxury car</b>  <math>\\$1.3 \cdot 10^5</math></p>
 <p><b>Provide relief for one natural disaster</b>  <math>\\$3.44 \cdot 10^8</math></p>	 <p><b>Pay one student's college loan debt</b>  <math>\\$4 \cdot 10^4</math></p>	 <p><b>Buy a book</b>  <math>\\$2 \cdot 10^1</math></p>
 <p><b>Buy a gaming console</b>  <math>\\$5 \cdot 10^2</math></p>	 <p><b>Buy a private island</b>  <math>\\$2 \cdot 10^7</math></p>	 <p><b>Build a hospital</b>  <math>\\$3.3 \cdot 10^8</math></p>
 <p><b>Buy a professional football team</b>  <math>\\$5 \cdot 10^9</math></p>	 <p><b>Buy a luxury yacht</b>  <math>\\$1 \cdot 10^8</math></p>	 <p><b>Feed a family of five for one year</b>  <math>\\$1.2 \cdot 10^4</math></p>

# Net Worths of Celebrities

 **Directions:** Make one copy per pair of students. Then pre-cut the cards and give each pair one set.

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## Net Worths of Celebrities

The table shows the net worths of 10 celebrities as of 2023.

Name	Net Worth (\$)
Beyonce	500,000,000
Barack Obama	70,000,000
Lady Gaga	320,000,000
Billie Eilish	30,000,000
LeBron James	600,000,000
Kylie Jenner	700,000,000
Cardi B	80,000,000
Post Malone	45,000,000
Oprah Winfrey	3,500,000,000
Ariana Grande	240,000,000

## Net Worths of Celebrities

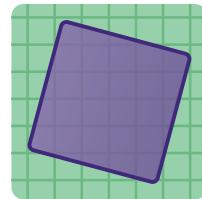
The table shows the net worths of 10 celebrities as of 2023.

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Cardi B	80,000,000
Post Malone	45,000,000
Oprah Winfrey	3,500,000,000
Ariana Grande	240,000,000

Name: ..... Date: ..... Period: .....

# Tilted Squares

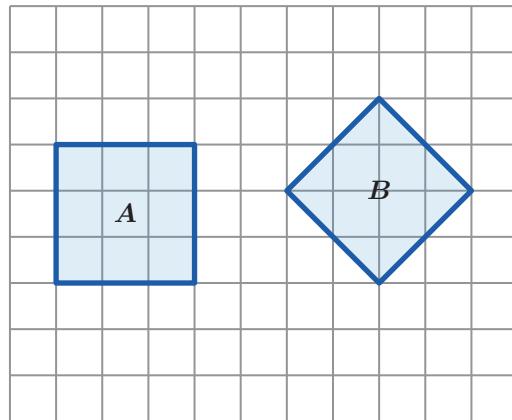
Let's explore finding the areas of tilted squares.



## Warm-Up

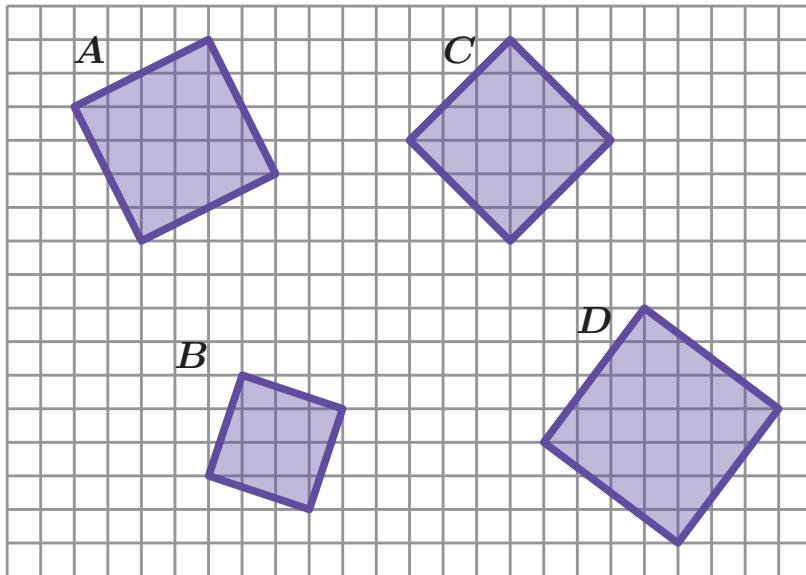
1. Which shaded region is larger?

Explain your thinking.



## Area of Tilted Squares

2. Determine the area of each tilted square (in square units). Record the areas in the table.



Square	A	B	C	D
Area (sq. units)				

3. What strategies did you use to determine the areas of the tilted squares?

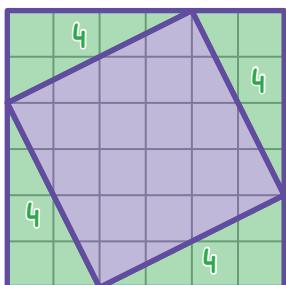
4. What is the side length of square D?

Explain your thinking.

## Different Strategies

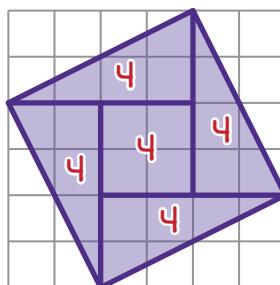
Here are Trevon's and Zahra's strategies for finding the area of tilted square  $A$ .

Trevon



$$6 \cdot 6 - 4 \cdot 4 = 20 \text{ square units}$$

Zahra



$$4 \cdot 4 + 4 = 20 \text{ square units}$$

5. How are Trevon's and Zahra's strategies alike? How are they different?

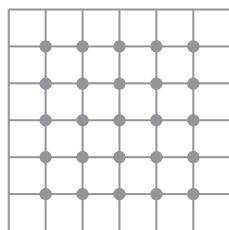
6. How does each strategy compare to your own?

## Building Squares With Different Areas

7. Here are squares with areas of 2 square units and 9 square units. On each dot grid, try to draw a square with the given area. Then circle “P” for any area that is possible to draw and “N” for any area that’s not possible to draw.

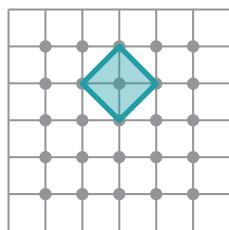
**Area:**  
**1 square unit**

P / N



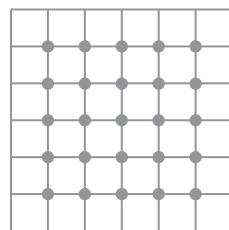
**Area:**  
**2 square units**

(P) / N



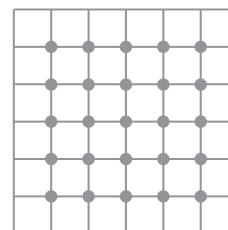
**Area:**  
**3 square units**

P / N



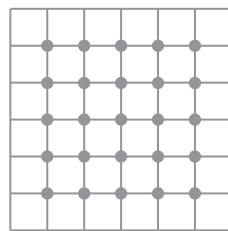
**Area:**  
**4 square units**

P / N



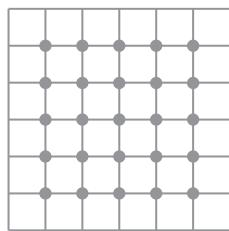
**Area:**  
**5 square units**

P / N



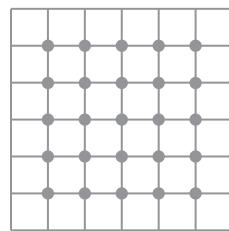
**Area:**  
**6 square units**

P / N



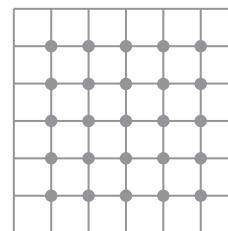
**Area:**  
**7 square units**

P / N



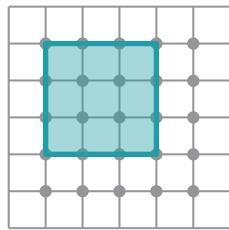
**Area:**  
**8 square units**

P / N



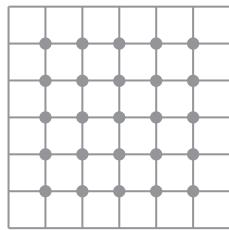
**Area:**  
**9 square units**

(P) / N



**Area:**  
**10 square units**

P / N

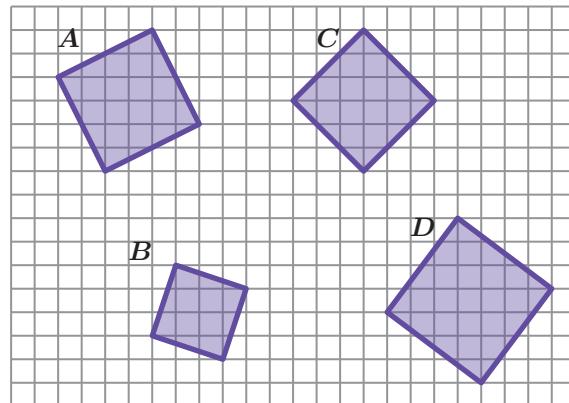


8. Choose one of the squares and determine its side length.

## Synthesis

9. Describe a strategy for determining the area of a tilted square.

Use the examples if they help with your thinking.

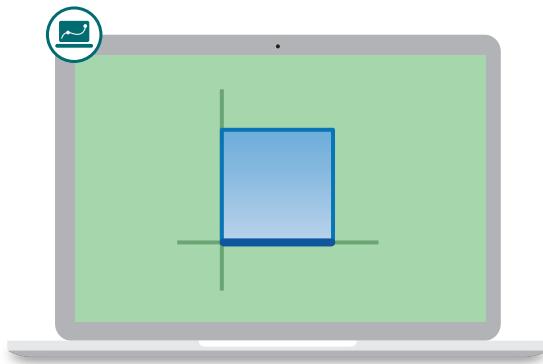


Things to Remember:

Name: ..... Date: ..... Period: .....

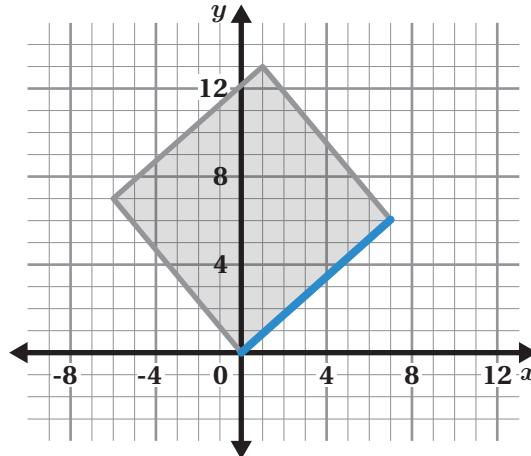
# From Squares to Roots

Let's explore the connection between the area and side length of a square.



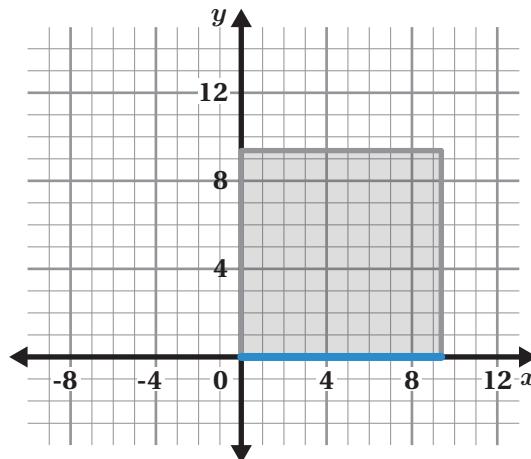
## Warm-Up

- 1** Estimate the side length of the square.



- 2** You can approximate the side length of a tilted square by rotating it onto an axis.

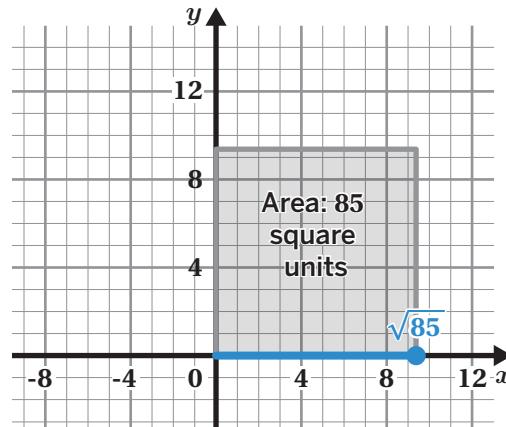
- a** Here is the square from the previous problem rotated so that the highlighted side length is along the  $x$ -axis.
- b** Write a new estimate for the side length of the square.



## Square Roots

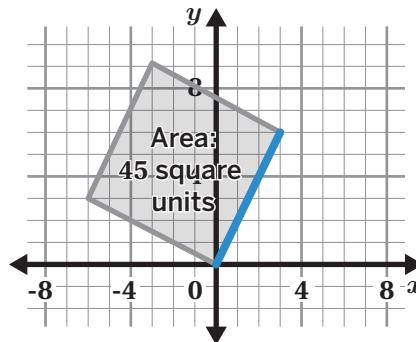
- 3** The exact side length of this square is the **square root** of 85, written as  $\sqrt{85}$ .

- a** Take a look at the square and how the side length is written.
- b**  **Discuss:** Explain what you think a square root is in your own words.



- 4** The area of this square is 45 square units.

Write the exact value of the side length.



- 5** Determine the unknown side lengths and areas for each square.

Square	Side Length (units)	Area (sq. units)
A	$\sqrt{55}$	55
B		81
C	2.5	
D		14
E	$\sqrt{44}$	
F		32

**Square Roots (continued)**

- 6** Order the squares from *smallest* to *largest* area.

**A**  
Area of 50  
square units

**B**  
Side length of  
 $\sqrt{81}$  units

**C**  
Side length of  
 $\sqrt{55}$  units

**D**  
Side length of  
8 units



**Smallest**

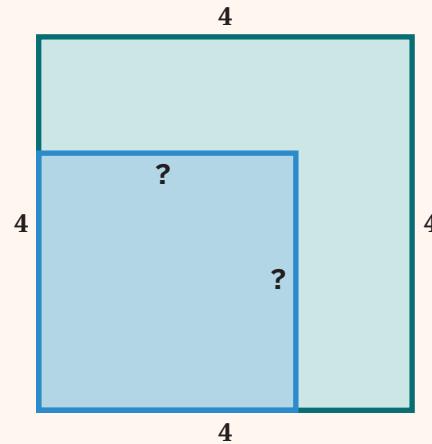
**Largest**

**Explore More**

- 7** Metropolis has a park surrounded by a square fence with 4-meter side lengths.

The city would like to build a square pool as shown in the figure.

What should the side length of the pool be so that half of the area is grass and half is water? Explain your thinking.



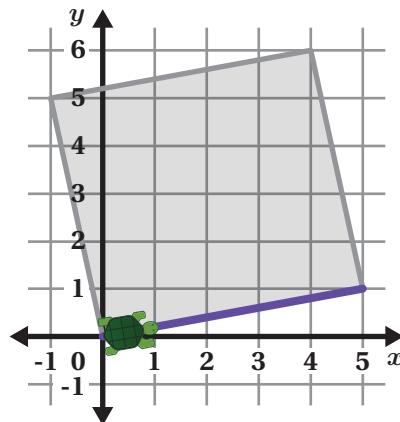
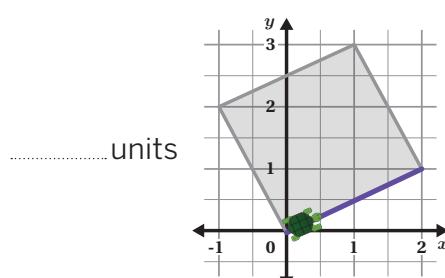
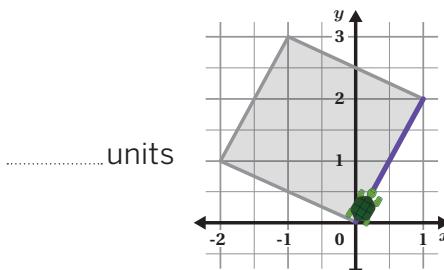
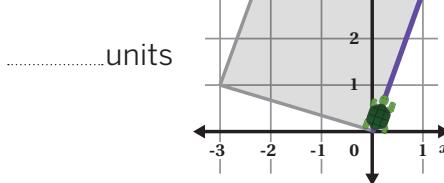
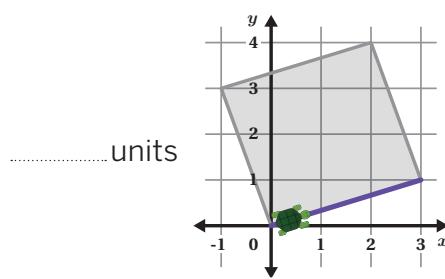
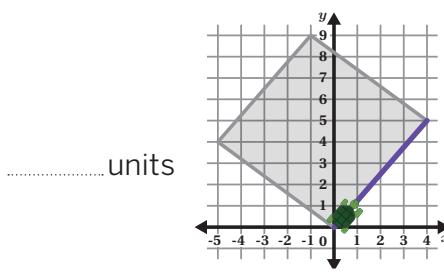
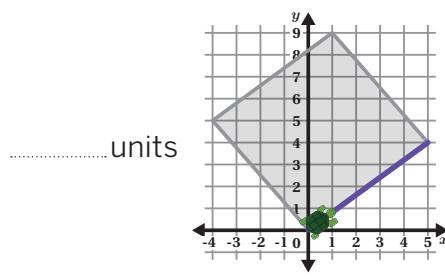
## Turtle Tracing

Tiam the turtle is walking on one side of a square.

- 8** Exactly how far does Tiam need to travel?

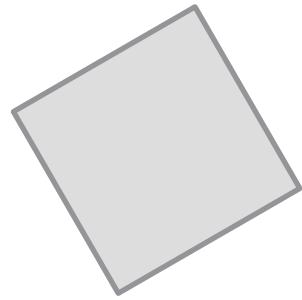
- 9** Complete this chart with a partner.

- Decide with your partner who will complete Column A and who will complete Column B.
- Determine how far Tiam the turtle needs to travel, and then compare your solutions. The solutions in each row should be the same. Discuss and resolve any differences.
- Determine the side lengths of as many squares as you have time for.

**a****Column B****b****c**

## 10 Synthesis

Describe the relationship between the side length and the area of a square using the term *square root*.

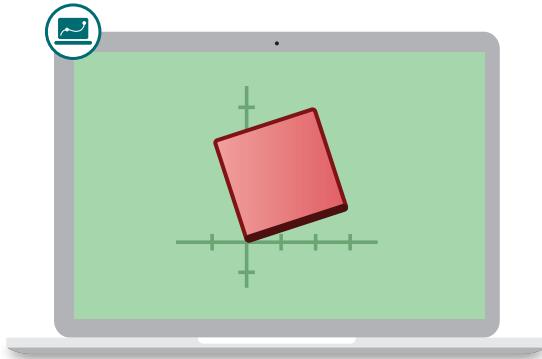


Things to Remember:

Name: ..... Date: ..... Period: .....

## Between Squares

Let's approximate the value of square roots.

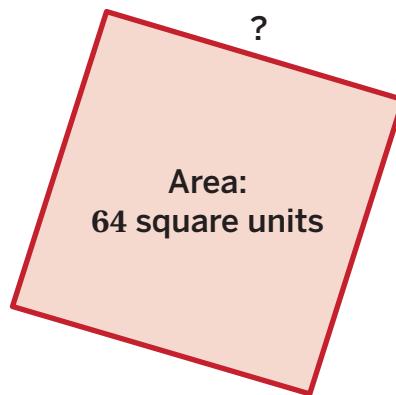


### Warm-Up

- 1** Select one correct expression for the side length of this square.

- A.  $\frac{64}{2}$        B.  $\sqrt{64}$   
 C. 8       D.  $\sqrt{8}$   
 E. 4       F.  $\frac{64}{4}$

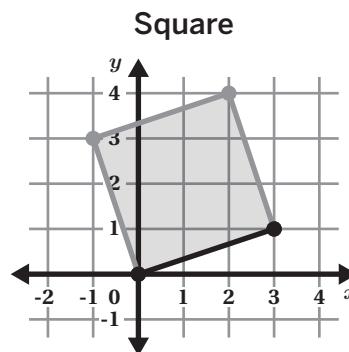
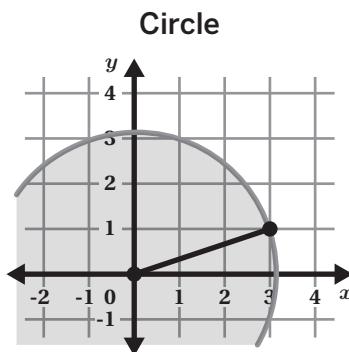
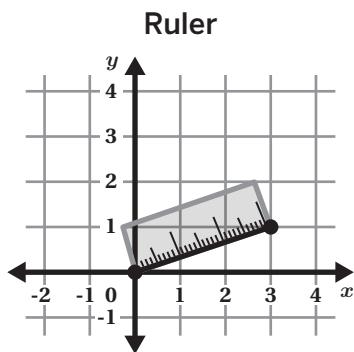
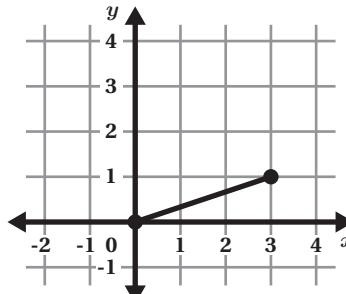
Explain your thinking.



## Squaring Lines

**2** What do you think is the length of this segment?

Use the ruler, circle, or square to help with your thinking.



**3** Ava says that the segment length is  $\sqrt{10}$  units because the area of the square is 10 square units.

Raine says that the segment length is about 3.2 units because that's the approximate length of the circle's radius.



### Discuss:

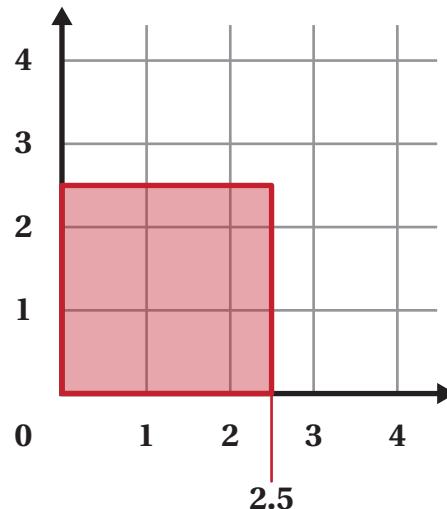
- How are Ava's and Raine's strategies alike? How are they different?
- What is helpful about each strategy?

## Using Squares to Estimate

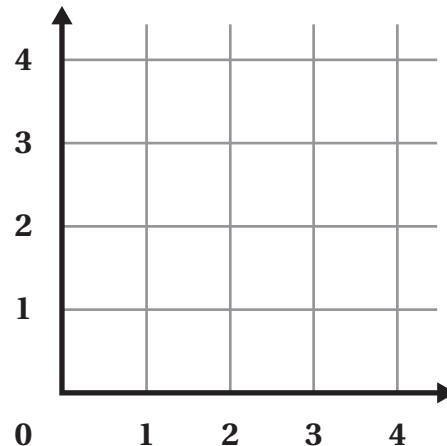
- 4** How does  $\sqrt{5}$  compare to 2.5? Circle one.  
Use the square to help with your thinking.

$\sqrt{5}$  is less than 2.5       $\sqrt{5}$  is greater than 2.5       $\sqrt{5}$  is equal to 2.5

Explain your thinking.



- 5** **a** Sketch a square to help you estimate  $\sqrt{5}$ .
- b** Write your estimate for  $\sqrt{5}$ .



## Closest Decimal Approximation

- 6**  $\sqrt{5}$  is a number that equals 5 when squared.

Use a calculator to approximate the value of  $\sqrt{5}$  as closely as you can. Record each guess,  $n$ , and its square,  $n^2$ , in the table.

$n$	$n^2$
2.0	$(2.0)^2 = 4.00$

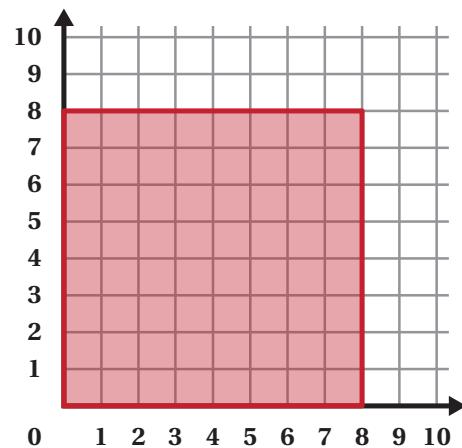
- 7** Describe your strategy for finding a decimal approximation that is as close as possible to  $\sqrt{5}$ .

- 8** Use a calculator to approximate the value of  $\sqrt{30}$  as closely as you can. Record each guess,  $n$ , and its square,  $n^2$ , in the table.

$n$	$n^2$
5.2	$(5.2)^2 = 27.04$

## 9 Synthesis

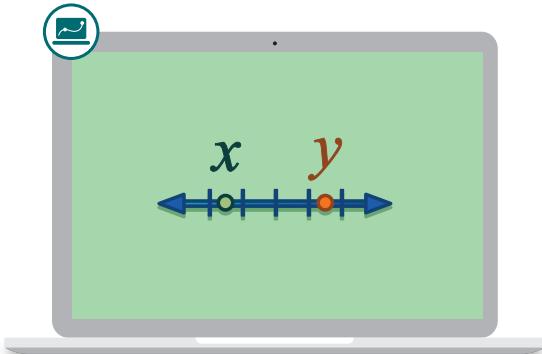
What are some strategies for approximating a square root, such as  $\sqrt{75}$ ?



$n$	$n^2$
8.0	64

Things to Remember:

Name: ..... Date: ..... Period: .....



## Root Down

Let's estimate the value of square roots and represent them on a number line.

### Warm-Up

- 1** Plot these values on the number line.

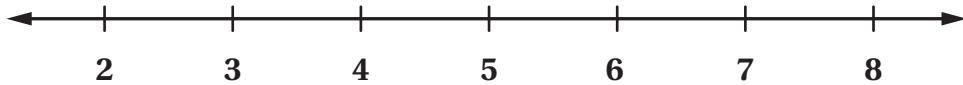
$\sqrt{49}$

$\sqrt{64}$

$\sqrt{9}$

$\sqrt{25}$

$\sqrt{55}$



- 2** 9, 25, 49, and 64 are perfect squares. 55 is not.

What do you think a perfect square is?

## Between Whole Numbers

- 3** Match each value to the whole numbers it is between. Two values will not have a match.

Note: The numbers  $x$ ,  $y$ , and  $z$  are positive numbers.

The value of  $x$   
when  $x^2 = 50$

$$\sqrt{62}$$

The value of  $y$   
when  $y^2 = 20$

$$\sqrt{60}$$

The value of  $z$   
when  $z^2 = 80$

$$\sqrt{24}$$

$$\sqrt{17}$$

$$\sqrt{15}$$

Between 4 and 5	Between 7 and 8	No Match

- 4** Esi thinks that the description “The value of  $z$  when  $z^2 = 80$ ” doesn’t belong in either category.

**a** Which two whole numbers is the value of  $z$  between?

- A. 4 and 5
- B. 6 and 7
- C. 7 and 8
- D. 8 and 9

Between 7 and 8

Between 4 and 5

The value of  $z$  when  
 $z^2 = 80$

**b** Of those two numbers, which would  $z$  be closer to?

Explain your thinking.

## Between Whole Numbers (continued)

- 5** Order the numbers from *least* to *greatest*.

$\sqrt{99}$

$\sqrt{75}$

9

9.5

10

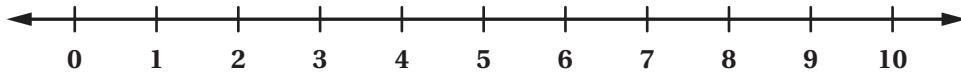
--	--	--	--	--	--	--

Least

Greatest

- 6** The numbers  $x$  and  $y$  are positive.  $x^2 = 3$  and  $y^2 = 35$ .

- a** Plot  $x$  and  $y$  on the number line.

**b**

- Discuss:** How did you decide where to plot each point?

## Challenge Creator

**7** Use blank paper to create your own challenge.

- a** **Make It!** On the paper, write five numbers in a random order. Include at least one square root.
- b** **Solve It!** On this page, order the numbers from *least* to *greatest*.

### My Challenge

--	--	--	--	--

**Least**

**Greatest**

- c** **Swap It!** Swap your challenge on the blank paper with one or more partners. For each partner's challenge, order the numbers from *least* to *greatest*.

### Partner 1's Challenge

--	--	--	--	--

**Least**

**Greatest**

### Partner 2's Challenge

--	--	--	--	--

**Least**

**Greatest**

### Partner 3's Challenge

--	--	--	--	--

**Least**

**Greatest**

### Partner 4's Challenge

--	--	--	--	--

**Least**

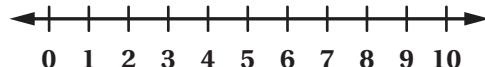
**Greatest**

## 8 Synthesis

What are some strategies for plotting square roots on a number line?

Use the number line and examples if they help to show your thinking.

$$\sqrt{25} \quad \sqrt{36} \quad \sqrt{31} \quad \sqrt{40}$$

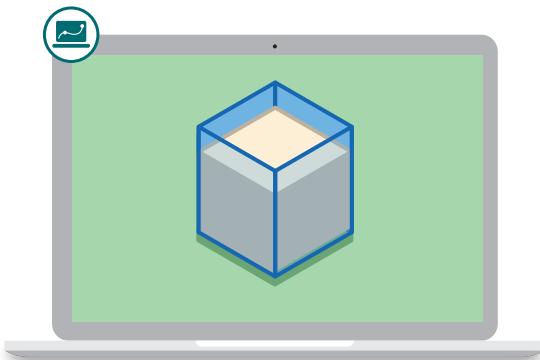


Things to Remember:

Name: ..... Date: ..... Period: .....

# Filling Cubes

Let's explore the relationship between the edge length and the volume of a cube.



## Warm-Up

- 1** Order each value from *least* to *greatest*. Note: Let  $a$ ,  $b$ ,  $c$ , and  $d$  be positive numbers.

$$a \text{ when } a^2 = 9$$

$$b \text{ when } b^3 = 8$$

$$c \text{ when } c^2 = 8$$

$$d \text{ when } d^3 = 9$$

.....      .....

**Least****Greatest**

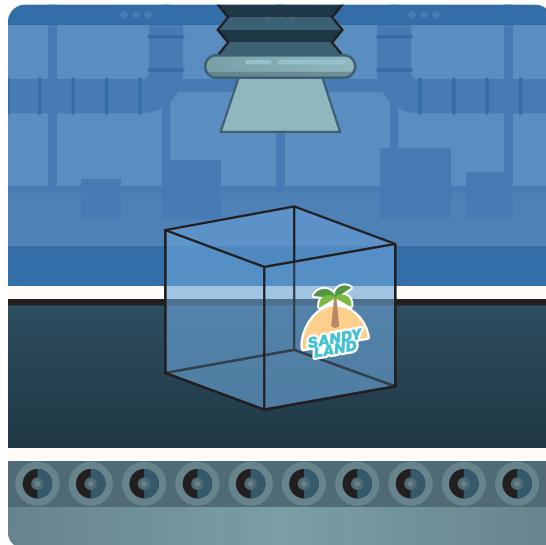
## Filling Cubes

Your job is to make sure the right amount of sand ends up in each cube. Use a calculator if it helps with your thinking.

- 2** This cube has an edge length of 6 inches.

How much sand is needed to fill it?

Edge Length (in.)	Amount of Sand (cu. in.)
6	



- 3** Four new orders just came in. Complete the table for each order.

Edge Length (in.)	Amount of Sand (cu. in.)
3	
2.1	
	64
	125

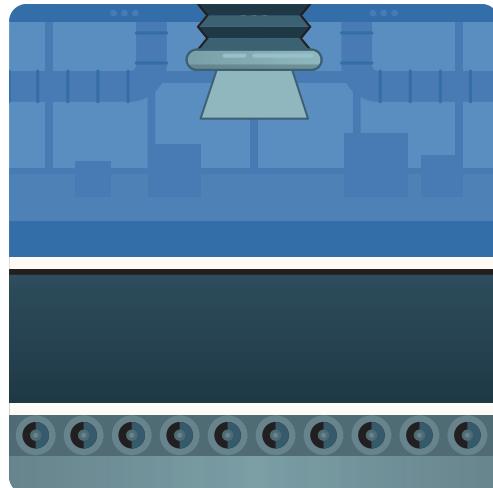
- 4** Describe a strategy you used to find the unknown edge lengths.

## Filling Cubes (continued)

- 5** A customer wants a cube filled with 100 cubic inches of sand.

Let's try to find the exact edge length of this cube.

Choose an edge length and use your calculator to determine how much sand will fill that cube. Keep revising your estimate to get as close to the target as possible.



Edge Length (in.)	Amount of Sand (cu. in.)
4.5	$(4.5)^3 = 91.125$

- 6** The equation  $x^3 = 100$  can help you determine the edge length of a cube that holds 100 cubic inches of sand. The exact solution to this equation is a **cube root**:  $x = \sqrt[3]{100}$ .

- a** Enter  $\sqrt[3]{100}$  on your calculator to see its approximate value.
- b** **Discuss:** What is the relationship between the edge length and the volume of a cube?

- 7** Determine the exact unknown value for each cube.

Edge Length (in.)	Amount of Sand (cu. in.)
	200
	150
	91.125
$\sqrt[3]{42}$	

## The Number Line

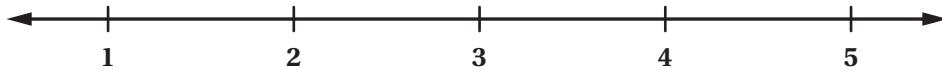
**8** Here are three cube roots.

$\sqrt[3]{27}$

$\sqrt[3]{64}$

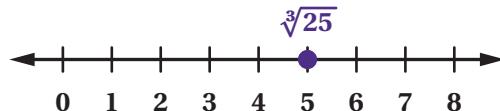
$\sqrt[3]{15}$

- a) Plot the cube roots on the number line.



- b) Describe your strategy for plotting  $\sqrt[3]{15}$ .

**9** Nia incorrectly plotted  $\sqrt[3]{25}$ .



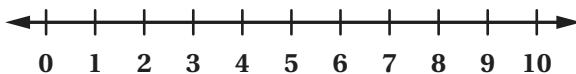
**Discuss:**

- What mistake could Nia have made?
- What question could you ask Nia to help her correct her work?

**10** Here is an equation:  $x^3 = 30$ .

- a) Write the exact solution to the equation.

- b) Plot the solution on the number line.



## The Number Line (continued)

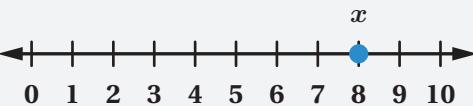
- 11** Match each equation to the visual that represents the same value of  $x$ .

$$x = \sqrt{10}$$

$$x = \sqrt{64}$$

$$x = \sqrt[3]{10}$$

$$x = \sqrt[3]{64}$$

Visual	Equation
	
	
<b>Volume: 64 cu. in.</b> 	
<b>Area: 10 sq. in.</b> 	

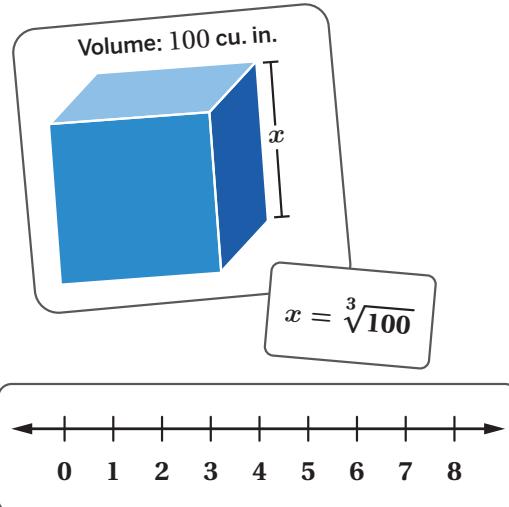
### Explore More

- 12** **a** If you double the edge length of a cube, what happens to the volume?

- b** If you double the volume of a cube, what happens to the edge length?

## 13 Synthesis

Explain a strategy for determining where to plot a cube root on the number line.

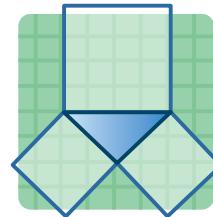


Things to Remember:

Name: ..... Date: ..... Period: .....

# The Pythagorean Theorem

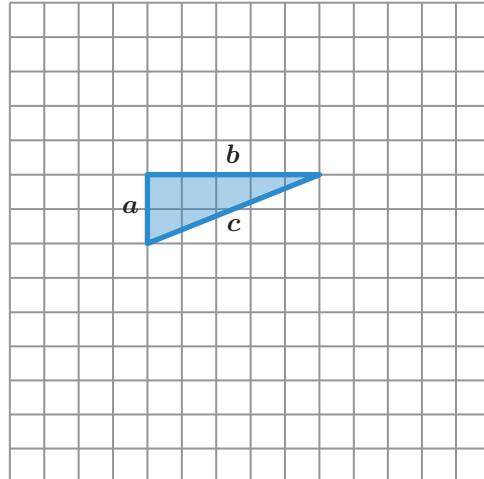
Let's explore the relationship between the squares of side lengths in triangles.



## Warm-Up

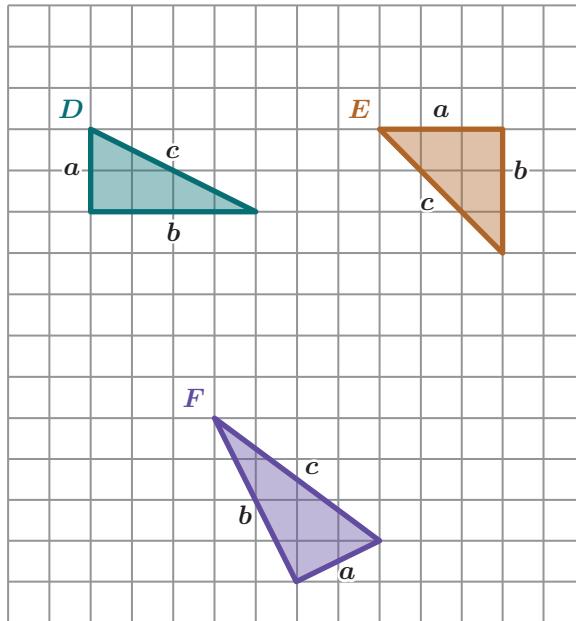
1. Use any strategy to determine the value of  $c^2$ .

Explain your thinking.



## Squares of Side Lengths

2. Use these triangles to complete the table.



Triangle	$a^2$	$b^2$	$c^2$
D			
E			
F			

3. What do you notice? What do you wonder?

I notice:

I wonder:

## True for Every Triangle?

- 4.** You will use a set of cards for this problem.

- a** Work with a partner to create groups where  $a^2 + b^2 = c^2$  and  $a^2 + b^2 \neq c^2$ . Complete the table with the card numbers.

$a^2 + b^2 = c^2$	$a^2 + b^2 \neq c^2$

- b** Revisit your noticing and wonderings from Activity 1. What do you notice and wonder now?

I notice:

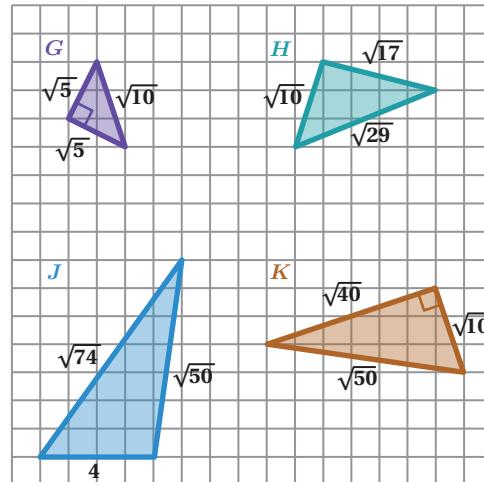
I wonder:

For each triangle, let  $a$  and  $b$  represent the two shorter sides and  $c$  represent the longest side.

- 5.** Circle one triangle where the equation  $a^2 + b^2 = c^2$  is true.

Triangle G   Triangle H   Triangle J   Triangle K

Show or explain your thinking.

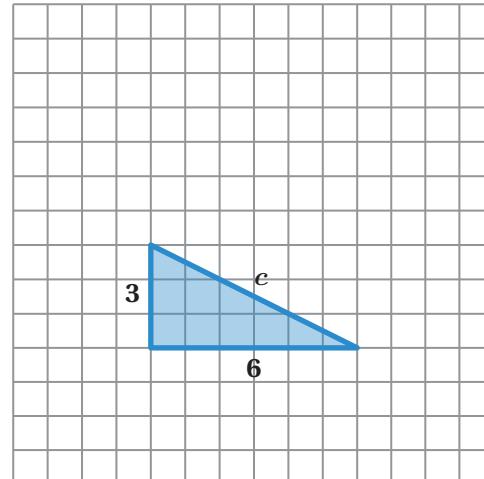


- 6.** Sai says that the value of  $c$  is  $\sqrt{45}$ .

Do you agree? Circle one.

Agree   Disagree   There's not enough information

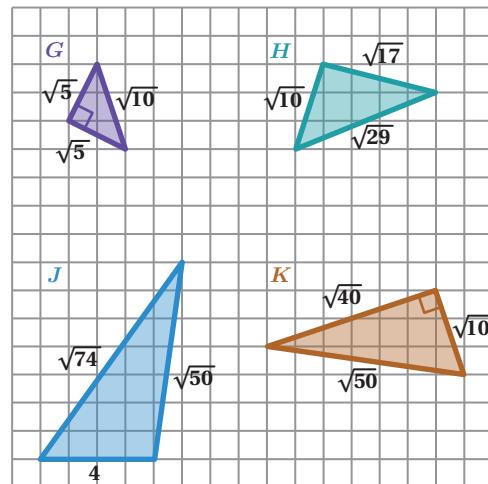
Show or explain your thinking.



## Synthesis

7. The **Pythagorean theorem** says that for right triangles,  $a^2 + b^2 = c^2$ . The date of the first discovery is unknown, but the Babylonians used the Pythagorean theorem over 3,500 years ago (1,000 years before Pythagoras was born).

Explain the Pythagorean theorem in your own words. Use the triangles if they help with your thinking.



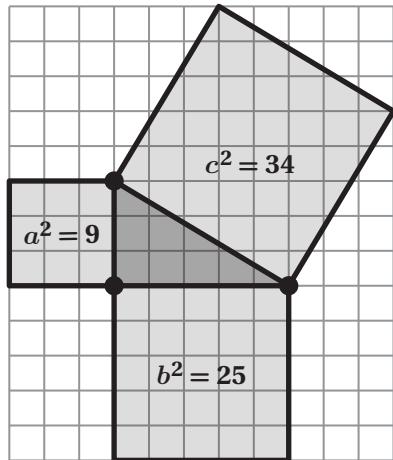
Things to Remember:

# True for Every Triangle?

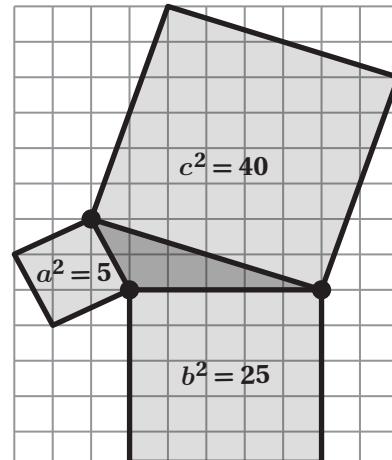
 **Directions:** Make one copy per pair. Then pre-cut the cards and give each pair of students one set.

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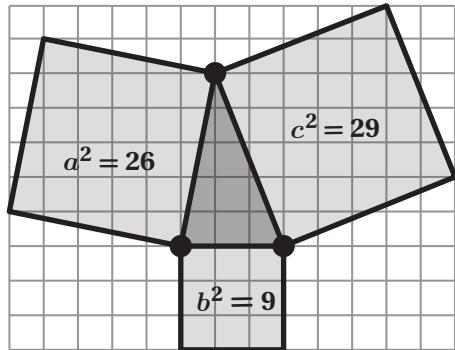
**Card 1**



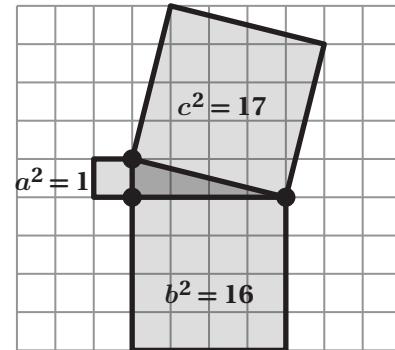
**Card 2**



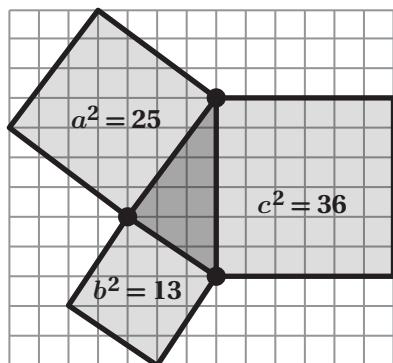
**Card 3**



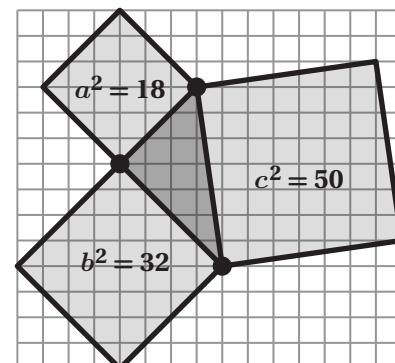
**Card 4**



**Card 5**



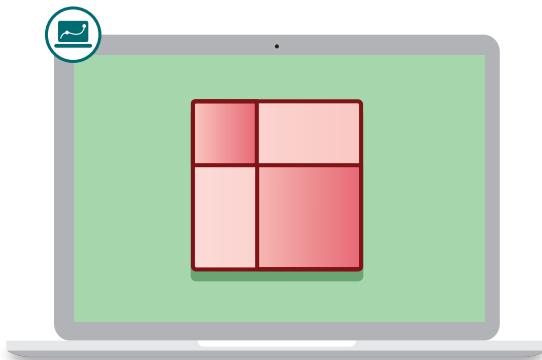
**Card 6**



Name: ..... Date: ..... Period: .....

## Pictures to Prove It

Let's prove the Pythagorean theorem.



### Warm-Up

- 1 Here are two figures.

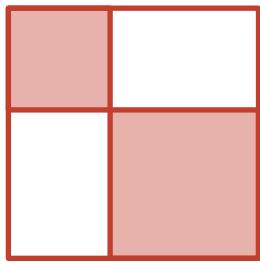


Figure A

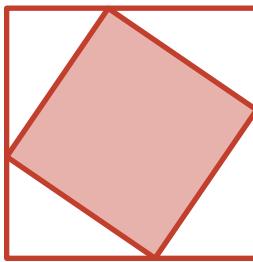


Figure B

Which one do you think has a larger shaded area? Circle one.

Figure A

Figure B

The areas are equal

I'm not sure

Explain your thinking.

## Pictures to Prove It

Note: For this lesson, assume figures that look like squares are squares.

- 2** Determine the total area of the shaded region in each figure. Mark the diagram if it helps with your thinking.

Total Shaded Area (sq. units)	Figure C	Figure D

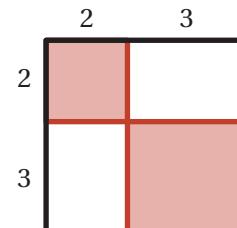


Figure C

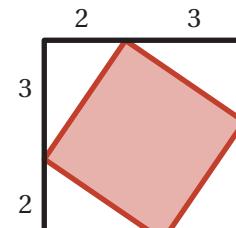
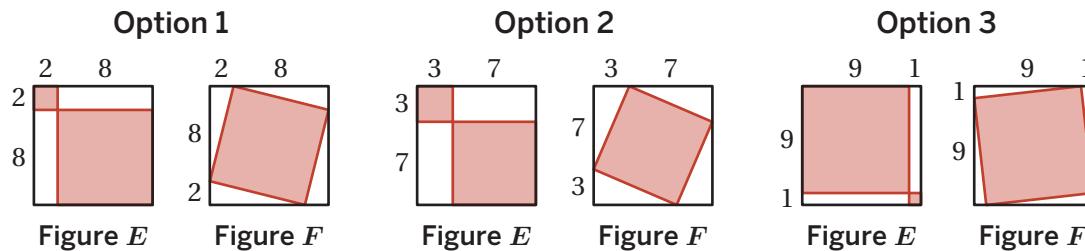


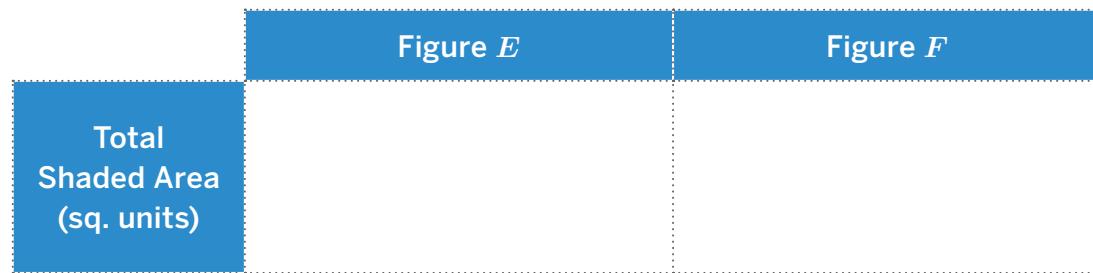
Figure D

- 3** Here are three different dimensions for a pair of new figures.

- a** Choose one of the options and circle your choice.



- b** Determine the total area of the shaded region in each figure.



- 4** Do you think the total shaded area will *always* be equal between figures like *E* and *F*, even if their outer dimensions change?

Explain your thinking.

## Thinking More Generally

Let's generalize using variables instead of numbers.

- 5** Determine the area of the shaded region in each figure using the variables  $a$ ,  $b$ , or  $c$ .

	Figure G	Figure H
Total Shaded Area (sq. units)		

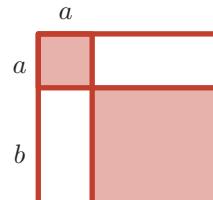


Figure G

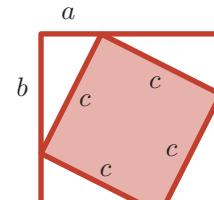


Figure H

- 6** The Pythagorean theorem states that for any right triangle,  $a^2 + b^2 = c^2$ .

How can we use these figures to prove that the Pythagorean theorem is true?

Mark the diagram if it helps to show your thinking.

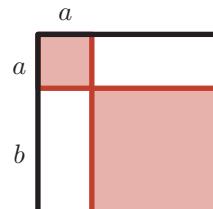


Figure G

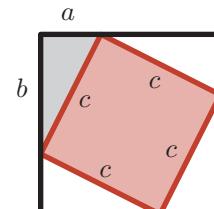


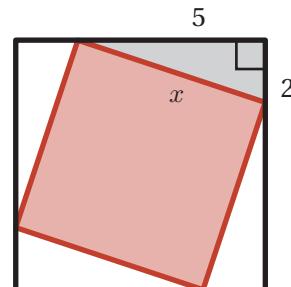
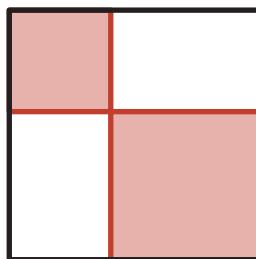
Figure H

## Let's Put It to Work

Let's put the Pythagorean theorem to work.

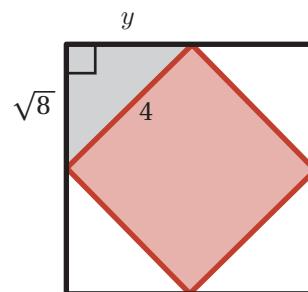
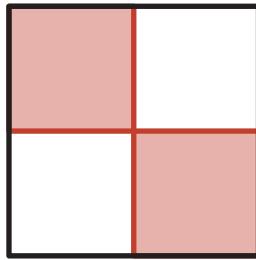
- 7** Calculate the value of  $x$ .

Draw on the diagram if it helps with your thinking.



- 8** Calculate the value of  $y$ .

Draw on the diagram if it helps with your thinking.



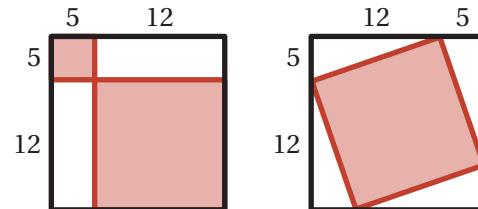
### Explore More

- 9** You will need the Explore More Sheet. Use scissors to cut along the dashed lines.

- Arrange the pieces in the smaller squares to fit in the large square.
- Describe what you notice about the relationship between the areas of the two smaller squares and the area of the large square.

## 10 Synthesis

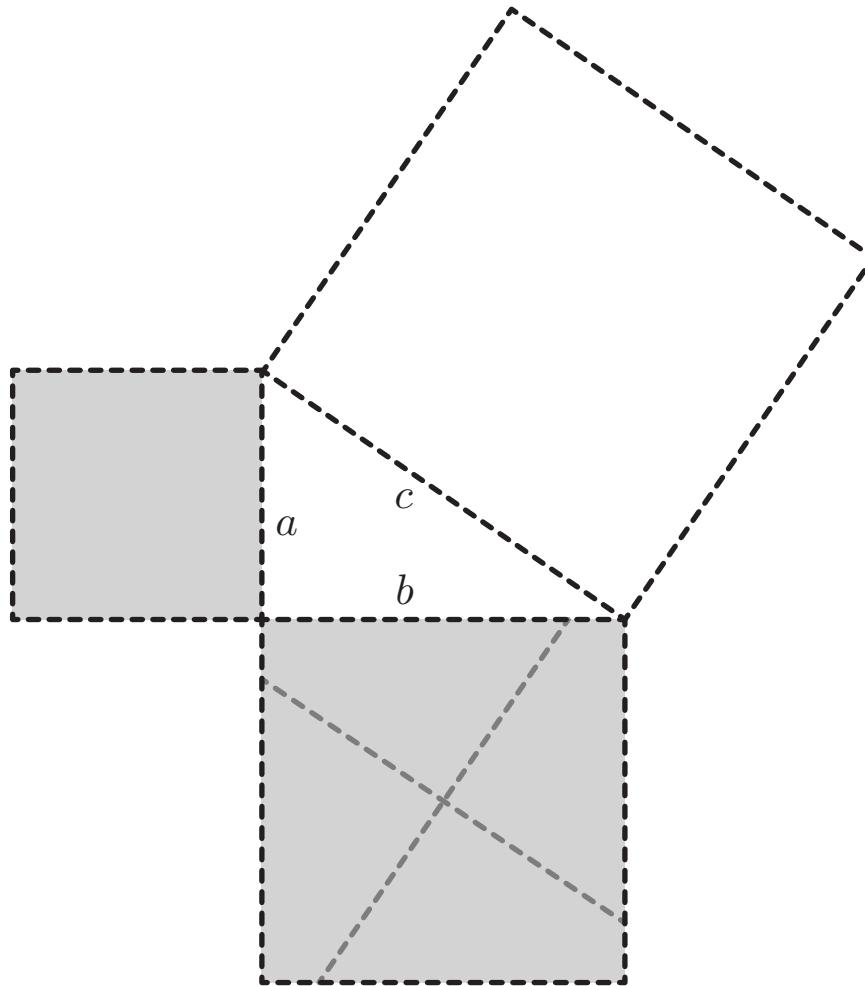
Explain how the equation  $5^2 + 12^2 = 13^2$  is related to the figures on the right and to the Pythagorean theorem.



Things to Remember:

## Explore More

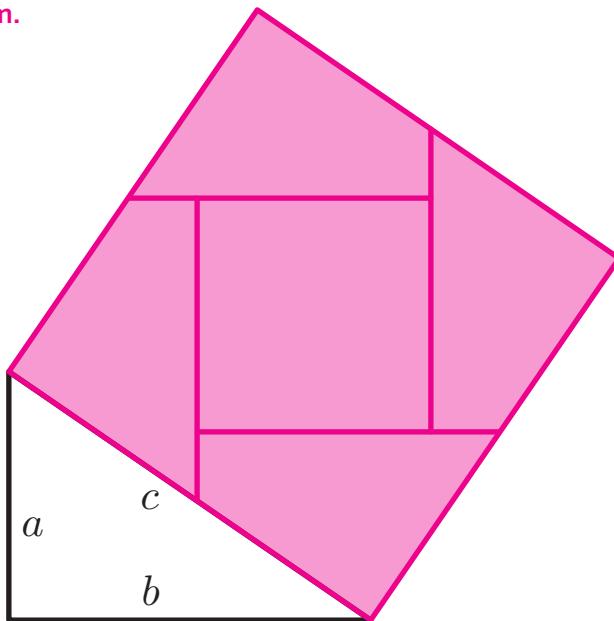
- a Use scissors to cut along the dashed lines. Arrange the pieces in the smaller squares to fit in the large square.
- b Describe what you notice about the relationship between the areas of the two smaller squares and the area of the large square.



## Explore More (answers)

- a Use scissors to cut along the dashed lines. Arrange the pieces in the smaller squares to fit in the large square.

Response shown in diagram.



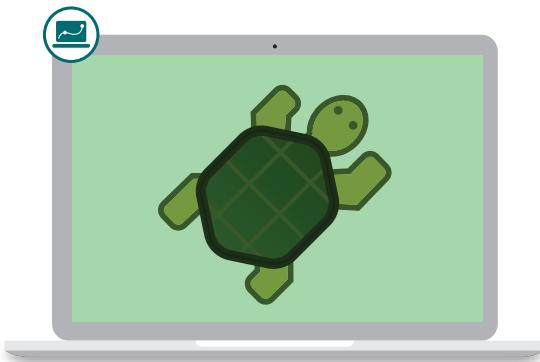
- b Describe what you notice about the relationship between the areas of the two smaller squares and the area of the large square.

The areas of the two smaller squares can always be arranged to fit inside the large square, showing that  $a^2 + b^2 = c^2$ .

Name: ..... Date: ..... Period: .....

# Triangle-Tracing Turtle

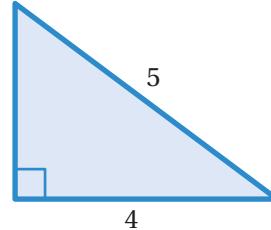
Let's calculate unknown side lengths in right triangles.



## Warm-Up

- 1** Select *all* the equations that could help you calculate the unknown side length of this triangle.

- A.  $a^2 + 4^2 = 5^2$
- B.  $a = \sqrt{4^2 + 5^2}$
- C.  $b^2 = 5^2 + 4^2$
- D.  $4^2 + b^2 = 5^2$
- E.  $b = \sqrt{5^2 - 4^2}$



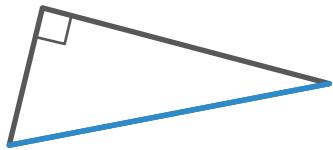
## Hypotenuse

**2** The **hypotenuse** is the side of a right triangle that is opposite the right angle.

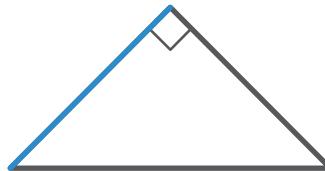
The **legs** of a right triangle are the sides that make the right angle.

Select *all* the triangles where a hypotenuse is highlighted.

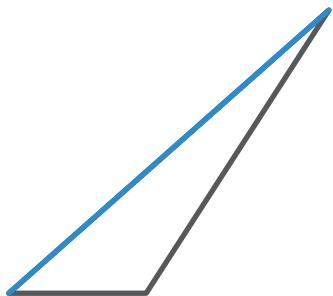
A.



B.



C.



D.



**3** Melissa incorrectly thinks that triangles A, C, and D from the previous problem have a highlighted hypotenuse.

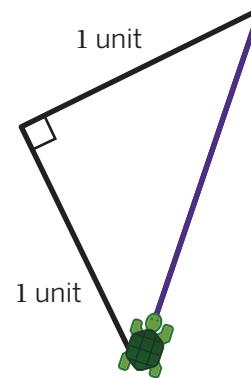
**a** What do you think Melissa did well?

**b** What mistake might Melissa have made?

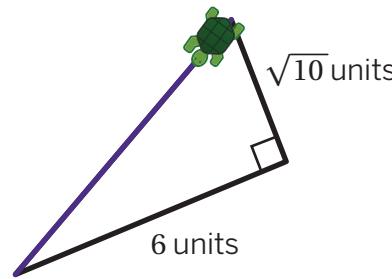
## Turtle Tracing

Tiam the turtle is walking on one side of a triangle.

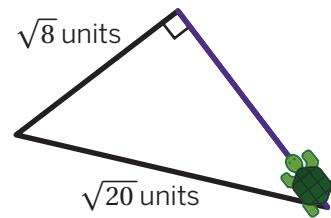
- 4** Exactly how far does Tiam need to travel?



- 5** Exactly how far does Tiam need to travel?



- 6** Exactly how far does Tiam need to travel?



## Challenge Creator

**7** You will use a separate sheet of paper to create your own triangle challenge.

- a** **Make It!** On the paper, sketch a right triangle. Label two of the sides with their approximate lengths.
- b** **Solve It!** On this page, write the two side lengths you labeled on your triangle. Then calculate the length of the third side.

My Sides	My Lengths (units)
Leg 1	
Leg 2	
Hypotenuse	

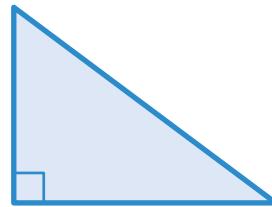
- c** **Swap It!** Swap your challenge with one or more partners. Calculate the unknown side length for each partner's triangle.

	Side	Length (units)
Partner 1	Leg 1	
	Leg 2	
	Hypotenuse	
Partner 2	Leg 1	
	Leg 2	
	Hypotenuse	
Partner 3	Leg 1	
	Leg 2	
	Hypotenuse	

## **8** Synthesis

If you know two side lengths of a right triangle,  
how can you calculate the third side length?

Use the image if it helps to show your thinking.

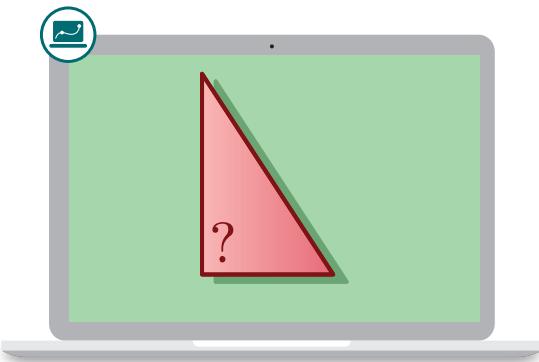


Things to Remember:

Name: ..... Date: ..... Period: .....

## Make It Right

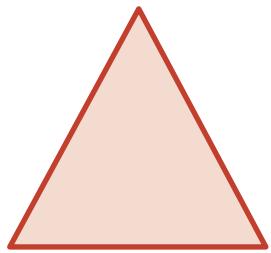
Let's determine if a triangle is a right triangle.



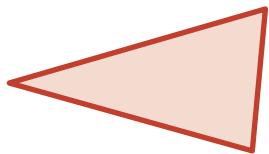
### Warm-Up

- 1 Which one doesn't belong?

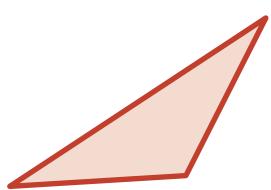
A.



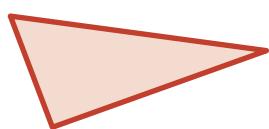
B.



C.



D.

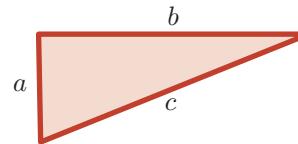


Explain your thinking.

## Is the Converse True?

- 2** Mathematicians sometimes think about a statement's converse, which is a statement in the opposite direction.

The converse of the Pythagorean theorem says: *If a triangle has side lengths such that  $a^2 + b^2 = c^2$ , it is a right triangle.*



Do you think this statement is always, sometimes, or never true? Circle one.

Always

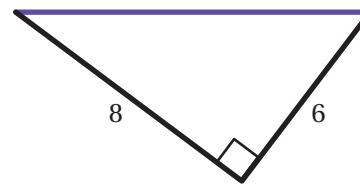
Sometimes

Never

Explain your thinking.

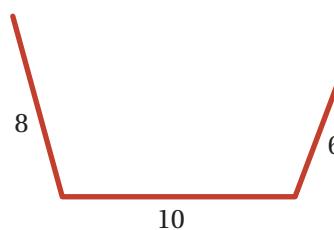
- 3** Let's explore the converse of the Pythagorean theorem by focusing on a specific example. Here is a right triangle with legs that are 6 and 8 units long.

What is the length of its hypotenuse?



- 4** You will use the cutouts from the Activity 1 Card to create several triangles.

- a** Experiment with making two other triangles with sides lengths measuring 6, 8, and 10 units.

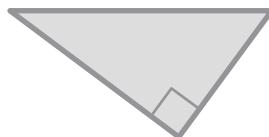
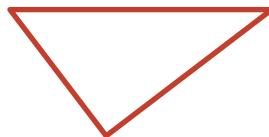


- b** **Discuss:** What do you notice? What do you wonder?

## Is the Converse True? (continued)

**5**

Yosef says that every triangle with sides lengths measuring 6, 8, and 10 units *must* be a right triangle, because the triangles with these lengths are all congruent.



Do you agree? Circle one. Use tracing paper to help with your thinking.

Yes

No

I'm not sure

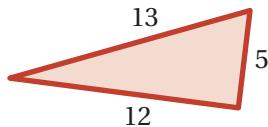
Explain your thinking.

## Make It Right

- 6** What type of triangle is this? Circle one.

A right triangle

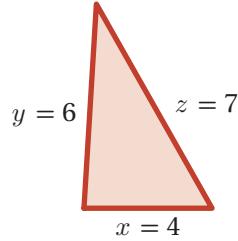
Not a right triangle



Explain your thinking.

- 7** Change one of the values to make this triangle a right triangle.

There are many different solutions. Try to find *at least* four.

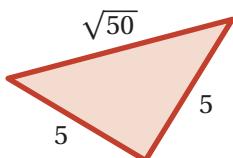


$x$	$y$	$z$

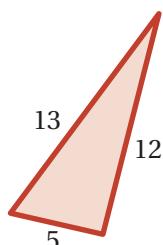
## Make It Right (continued)

- 8** Circle whether each triangle is a right triangle.

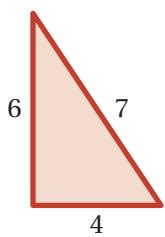
**a** Right Triangle Not a Right Triangle



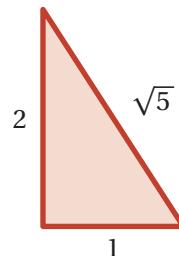
**b** Right Triangle Not a Right Triangle



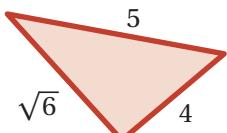
**c** Right Triangle Not a Right Triangle



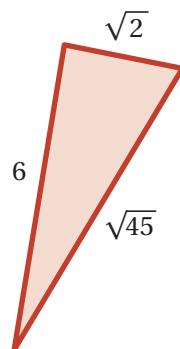
**d** Right Triangle Not a Right Triangle



**e** Right Triangle Not a Right Triangle



**f** Right Triangle Not a Right Triangle



### Explore More

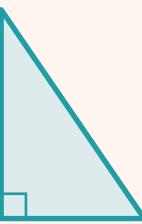
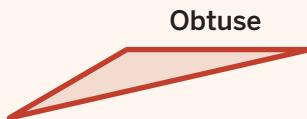
- 9** Here is an obtuse triangle, an acute triangle, and a right triangle. All triangles are one of these three types.

Decide whether triangles X, Y, and Z are acute, right, or obtuse based on their side lengths.

Triangle X, side lengths: 15, 20, 8

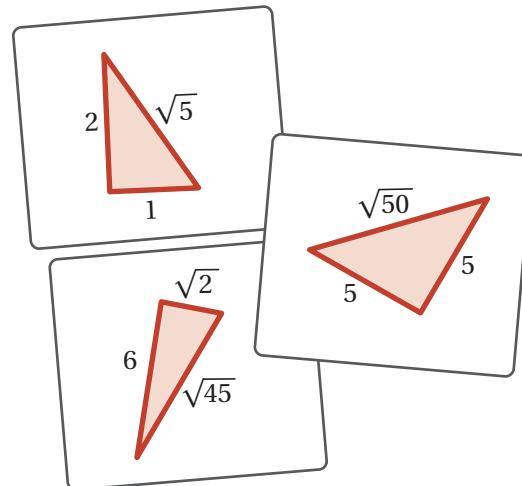
Triangle Y, side lengths: 8, 15, 13

Triangle Z, side lengths: 17, 8, 15



## 10 Synthesis

How can you tell from just the side lengths if a triangle is a right triangle?

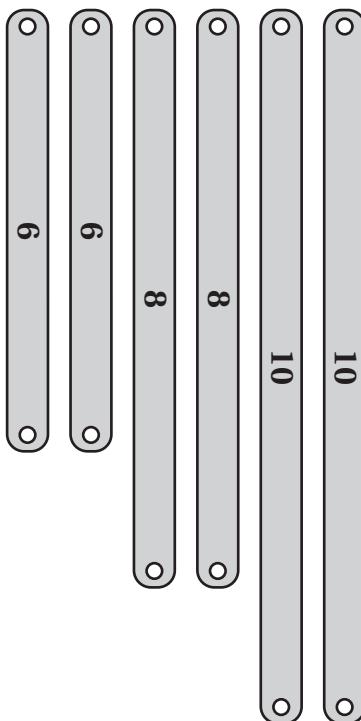
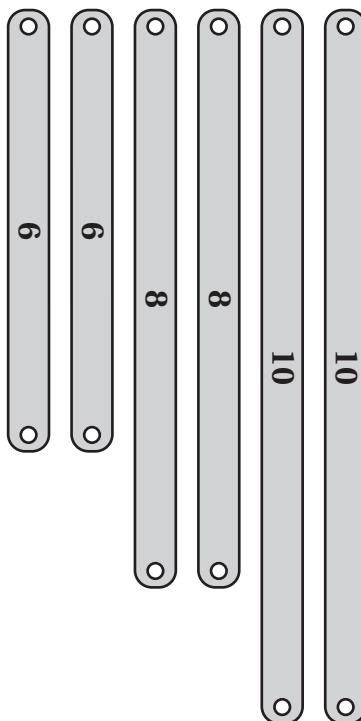
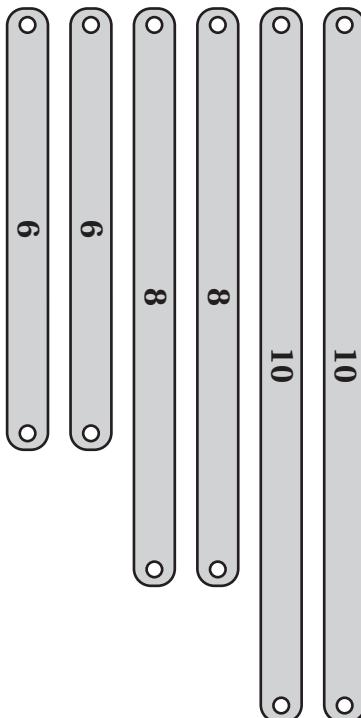
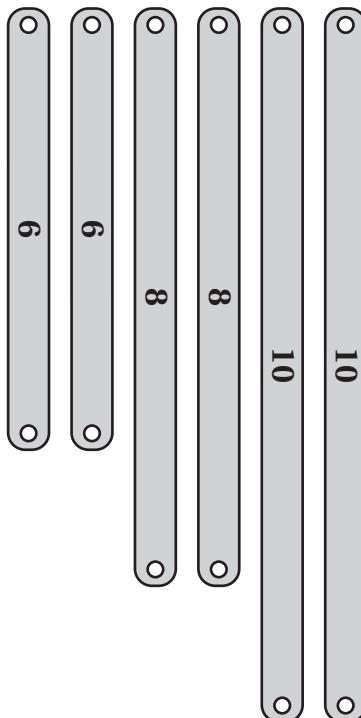


Things to Remember:

# Is the Converse True?

 **Directions:** Make one copy for every four students. Then pre-cut the cards and give each student one set of line segments. Have students cut out the line segments.

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Name: ..... Date: ..... Period: .....

# Taco Truck

Let's solve problems with the Pythagorean theorem.



## Warm-Up

- 1** Alma is going to walk through the park from point *A* to point *B*.

What distance will she walk?

- 2** If Alma walks at a speed of 4 feet per second, how long will it take for her to walk across the park?



# Taco Truck

- 3** Imagine you're on the beach, and you're getting hungry.

Sketch the route you would take to the taco truck.

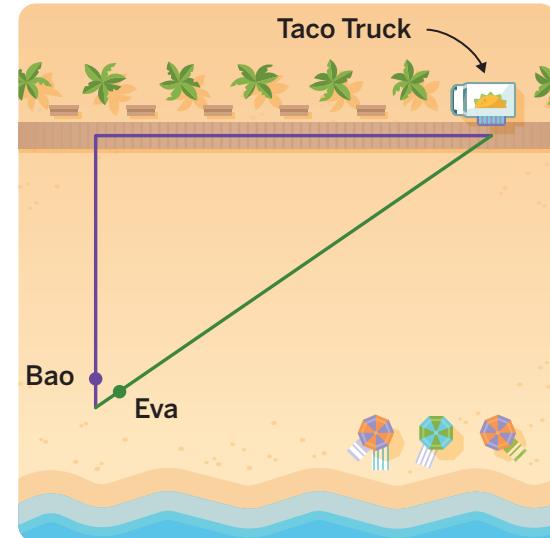
Explain your thinking.



- 4** Bao and Eva choose different routes to get to the taco truck.

- a** Who do you think will reach the taco truck first?

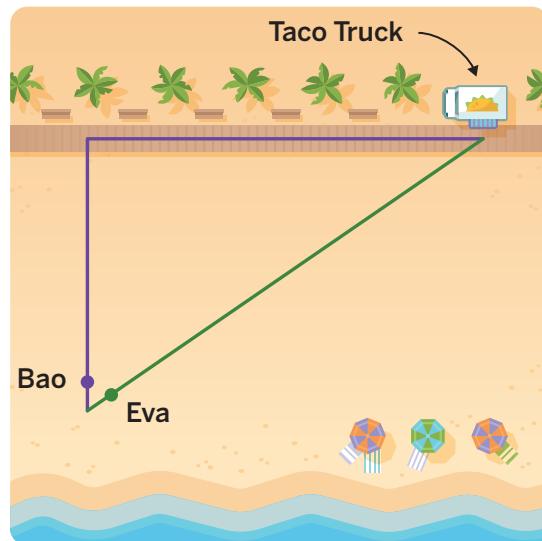
- b** What information would help you know for sure?



## Let's Eat

- 5** Let's look at some additional information about Bao's and Eva's routes on the screen.

Use the information to calculate how long it will take for each person to get to the taco truck. Show your thinking.



### Explore More

- 6** Determine the speed on the boardwalk that would make Eva and Bao arrive at the same time.

Speed on Sand	Speed on Boardwalk
3 feet per second	

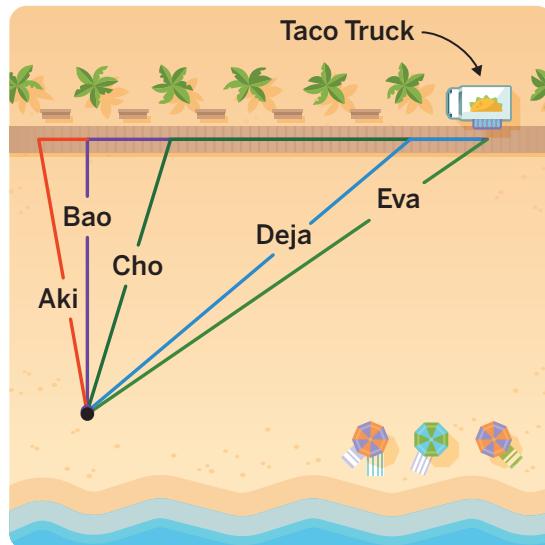
## Three More Paths

**7** Here are three more possible paths.

Who do you think will reach the taco truck first?

- Aki
- Bao
- Cho
- Deja
- Eva

Explain your thinking.



**8** You will watch a race between Aki, Bao, Cho, Deja, and Eva.

 **Discuss:** What do you notice? What do you wonder?

**9** Let's look at some information about the winning path.

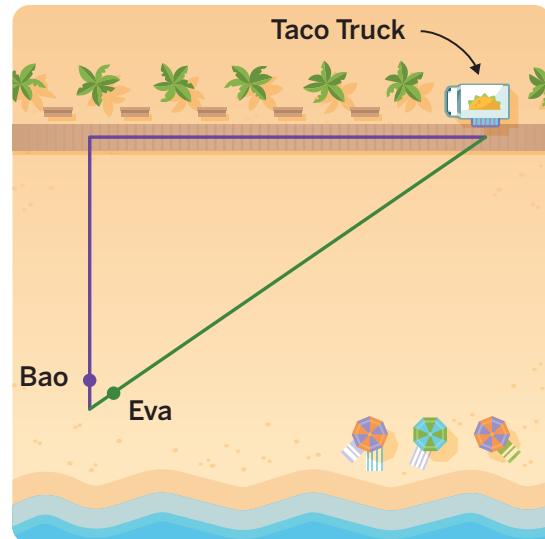
Remember that:

- The speed on the *boardwalk* is 5 feet per second.
- The speed on the *sand* is 3 feet per second.

Use this information to calculate the amount of time it took the winner to get to the taco truck. Show your thinking.

## 10 Synthesis

What are some important things to remember when using the Pythagorean theorem to solve problems?

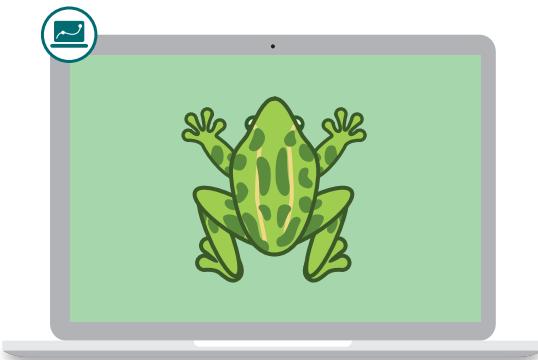


Things to Remember:

Name: ..... Date: ..... Period: .....

# Pond Hopper

Let's calculate distances between points on the coordinate plane.



## Warm-Up

- 1** Order the pairs of points from closest together to farthest apart.

Use the graph if it helps with your thinking.

Pair A: (-8, 1) and (-8, 8)

Pair B: (7, 0) and (7, -9)

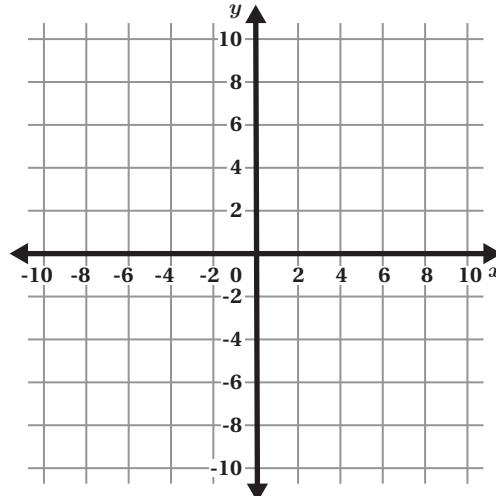
Pair C: (2, 3) and (2, 9)

Pair D: (-3, 6) and (5, 6)

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**Closest  
Together**

**Farthest  
Apart**



## Pond Hopper

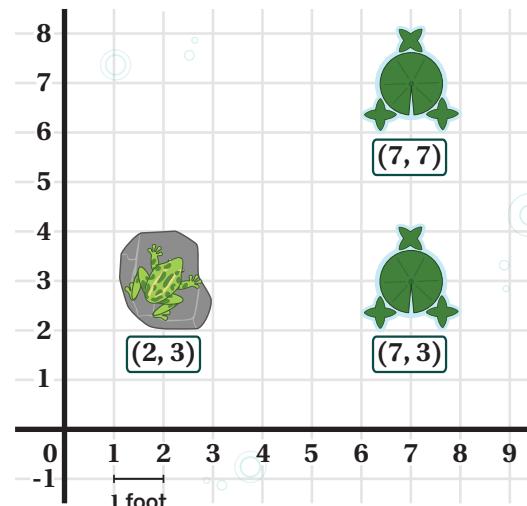
- 2** Let's help the frog hop to all the lily pads.

Pick a lily pad. Write its coordinates in the table.

Then write the distance between the frog and the lily pad.

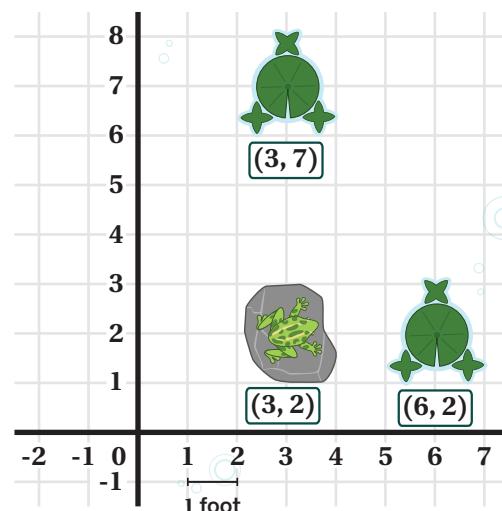
Repeat until you have hopped to all the lily pads.

	Lily Pad Coordinates	Distance (ft)
Hop 1		
Hop 2		
Hop 3		
Hop 4		



- 3** Help the frog hop to all the lily pads in as few hops as possible.

	Lily Pad Coordinates	Distance (ft)
Hop 1		
Hop 2		
Hop 3		
Hop 4		



**Pond Hopper (continued)**

- 4** Which expression represents the distance between the frog and the lily pad?

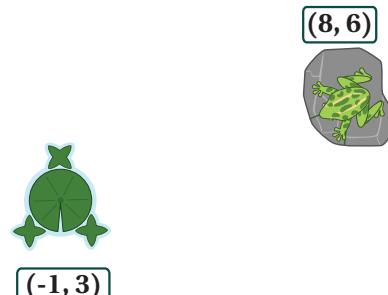
A.  $\sqrt{7^2 - 3^2}$

B.  $\sqrt{7^2 + 3^2}$

C.  $\sqrt{9^2 - 3^2}$

D.  $\sqrt{9^2 + 3^2}$

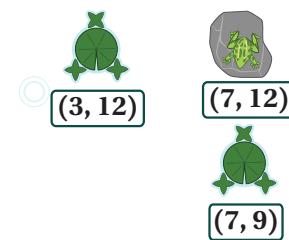
Show or explain your thinking.



All measurements in feet

- 5** Help the frog hop to all the lily pads in as few hops as possible.

	Lily Pad Coordinates	Distance (ft)
Hop 1		
Hop 2		
Hop 3		
Hop 4		
Hop 5		



All measurements in feet

## Challenge Creator

**6** You will use the Activity 2 Sheet to complete this activity.

- a** **Make It!** On the Activity 2 Sheet, create a lily pad challenge by sketching lily pads and a rock.
- b** **Solve It!** On this page, help the frog hop to all the lily pads in your challenge in as few hops as possible.

### My Challenge

	Hop 1	Hop 2	Hop 3	Hop 4	Hop 5	Hop 6	Hop 7	Hop 8
Lily Pad Coordinates								
Distance (ft)								

- c** **Swap It!** Swap your challenge with one or more partners. Help the frog hop to all the lily pads in each of your partners' challenges in as few hops as possible.

### Partner 1

	Hop 1	Hop 2	Hop 3	Hop 4	Hop 5	Hop 6	Hop 7	Hop 8
Lily Pad Coordinates								
Distance (ft)								

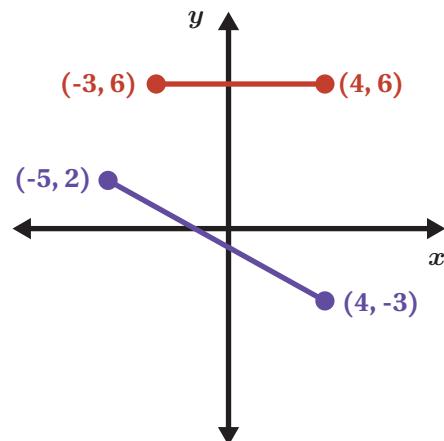
### Partner 2

	Hop 1	Hop 2	Hop 3	Hop 4	Hop 5	Hop 6	Hop 7	Hop 8
Lily Pad Coordinates								
Distance (ft)								

## 7 Synthesis

What are some strategies to calculate the distance between two points on the coordinate plane?

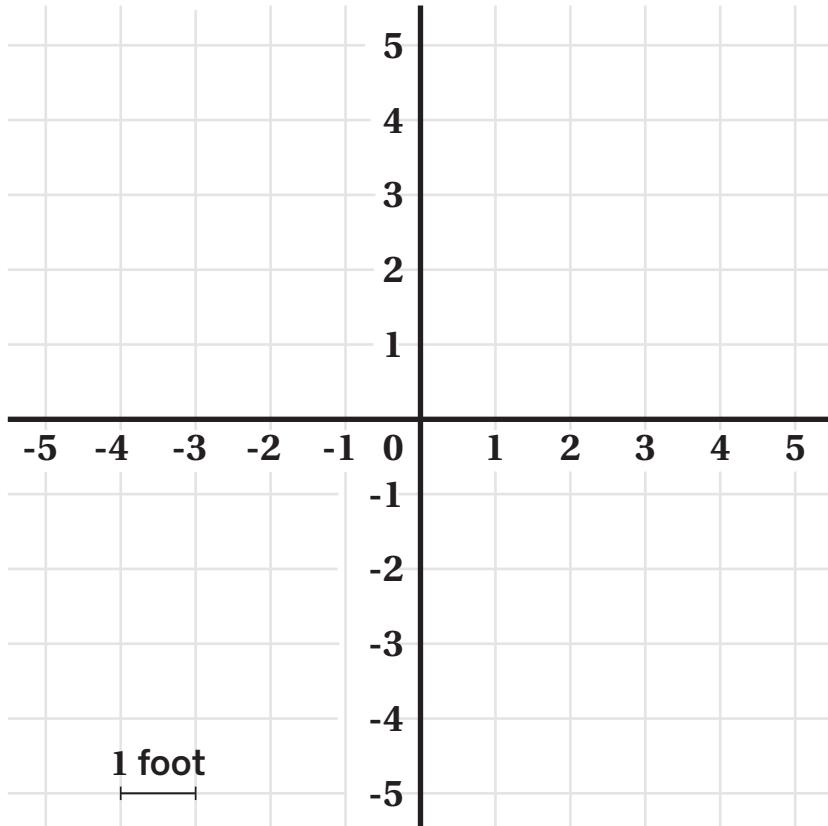
Use the examples in the graph if they help to show your thinking.



Things to Remember:

## Challenge Creator

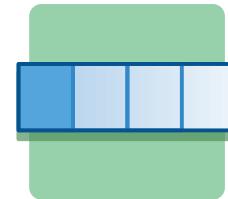
- Sketch five lily pads and a rock on the coordinate plane. The rock and lily pads must be at least 2 feet apart from each other.
- Label the coordinates of the rock and lily pads, then sketch a frog on top of the rock or label it with an *F*.
- Do *not* show any distances between the rock and lily pads. You and your classmates will solve each other's challenges on the lesson page.



Name: ..... Date: ..... Period: .....

# Fractions to Decimals

Let's explore connections between fractions and their decimal representations.

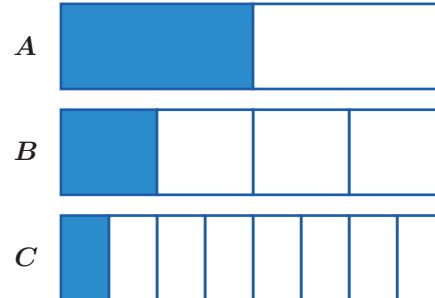


## Warm-Up

1. Here are three rectangles.

- a What fraction of each rectangle is shaded?
- b Write your answer as a fraction and as a decimal.

Rectangle	Fraction	Decimal
A		
B		
C		



## Terminating or Repeating

2. Every number can be written as a decimal. Here are some fractions written as decimals.

Fraction	Decimal	Terminating	Repeating	Neither
$\frac{1}{8}$	0.125	✓		
$\frac{3}{5}$	0.6	✓		
$\frac{341}{100}$	3.41	✓		
$\frac{1}{3}$	0.333...		✓	
$\frac{243}{99}$	2.454545...		✓	
$\frac{121}{15}$	8.0666...		✓	

What do you notice? What do you wonder?

I notice:

I wonder:

3. We can also use **bar notation** to write repeating decimals.

For example,  $0.333\dots = 0\bar{3}$  and  $2.454545\dots = 2.\bar{4}\bar{5}$ .

Order these numbers from *least* to *greatest*:  $8.06$ ,  $8.0\bar{6}3$ ,  $8.0\bar{6}$ ,  $8.063$ .

--	--	--	--

**Least**

**Greatest**

4. Write  $\frac{11}{50}$  as a decimal and decide whether it is *terminating*, *repeating*, or *neither*.

Fraction	Decimal	Terminating	Repeating	Neither
$\frac{11}{50}$				

## Converting Unit Fractions

5. Use long division to write each *unit fraction* as a decimal. Use the workspace or blank paper if it helps with your thinking.

Unit Fraction	Decimal	Terminating	Repeating	Neither
$\frac{1}{2}$	0.5	✓		
$\frac{1}{3}$	$0.\overline{3}$		✓	
$\frac{1}{4}$				
$\frac{1}{5}$				
$\frac{1}{6}$				
$\frac{1}{7}$				
$\frac{1}{8}$				
$\frac{1}{9}$				
$\frac{1}{10}$				
$\frac{1}{11}$				
$\frac{1}{12}$				

Workspace:

6. Write another unit fraction that terminates when written as a decimal.

7. Write another unit fraction that repeats when written as a decimal.

## Converting Unit Fractions (continued)

8. How can you predict whether a unit fraction will terminate, repeat, or neither when written as a decimal?

### Explore More

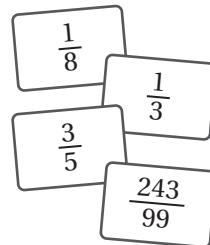
9. Complete the table. Then answer these questions:

- a How are the decimal representations in the table similar to each other?
  
- b How are the decimal representations in the table similar to each other?
  
- c Add the decimal representations of  $\frac{3}{7}$  and  $\frac{4}{7}$ . What is the result? How does this compare to when you add the fractions  $\frac{3}{7}$  and  $\frac{4}{7}$ ?

Fraction	Decimal
$\frac{1}{7}$	
$\frac{2}{7}$	
$\frac{3}{7}$	
$\frac{4}{7}$	
$\frac{5}{7}$	
$\frac{6}{7}$	

## Synthesis

10. Explain a strategy for writing fractions as decimals.

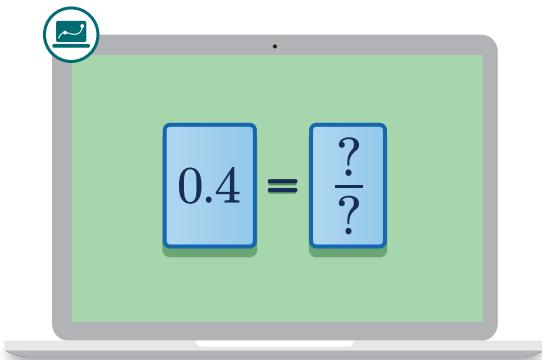


Things to Remember:

Name: ..... Date: ..... Period: .....

# Decimals to Fractions

Let's develop a strategy for rewriting repeating decimals as fractions.



## Warm-Up

Determine the value of each expression mentally. Try to think of more than one strategy.

**1**  $234 - 34$

**2**  $9.7 - 0.7$

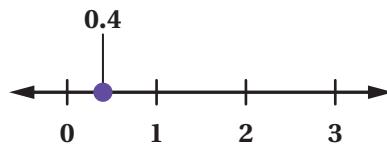
**3**  $100.\overline{25} - 99.\overline{25}$

**4**  $18.8\bar{3} - 1.4\bar{3}$

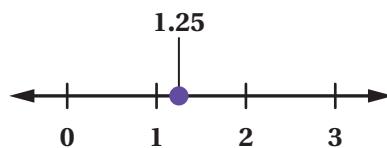
## Terminating Decimals to Fractions

- 5** Write a fraction as close to 0.4 as you can.

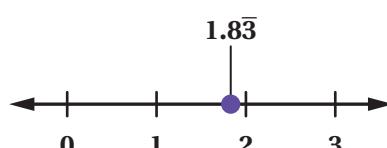
Use a calculator to check how close you are and revise as needed.



- 6** Write a fraction as close to 1.25 as you can.



- 7** Write a fraction as close to 1.83 as you can.



## Repeating Decimals to Fractions

- 8** Here is some of the work Mai did to write  $1.\bar{83}$  as a fraction.

- a**  **Discuss:** What did Mai do? Why do you think she chose these steps?

Mai

$$x = 1.\bar{83}$$

$$10x = 18.\bar{3}$$

$$100x = 183.\bar{3}$$

$$100x = 183.\bar{3}$$

$$-(10x = 18.\bar{3})$$

---

$$90x = 165$$

$$x = ?$$

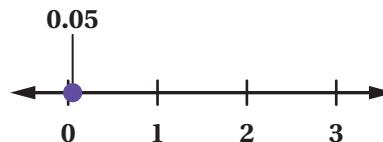
- b** Write  $1.\bar{83}$  as a fraction.

- 9** Use Mai's strategy to write  $2.\bar{74}$  as a fraction. Show your thinking.

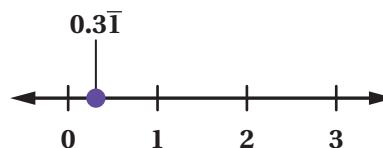
## Repeated Challenges

**10** Write each decimal as a fraction. Complete as many problems as you have time for.

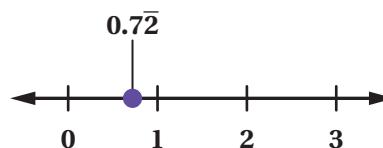
**a** 0.05



**b**  $0.\overline{3}$



**c**  $0.7\overline{2}$



**d**  $0.\overline{13}$



## 11 Synthesis

How is writing a repeating decimal as a fraction like writing a terminating decimal as a fraction? How is it different?

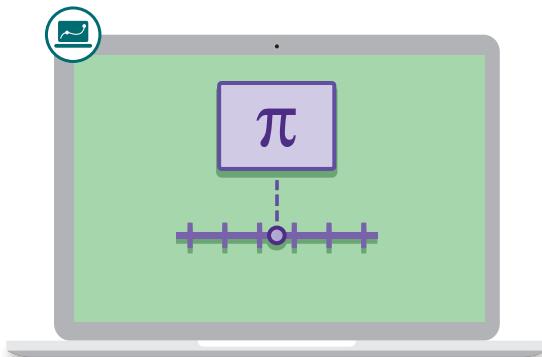


Things to Remember:

Name: ..... Date: ..... Period: .....

## Hit the Target

Let's build an understanding of two new types of numbers.



### Warm-Up

- 1** Which number is greater? Circle one.

$\sqrt{13}$

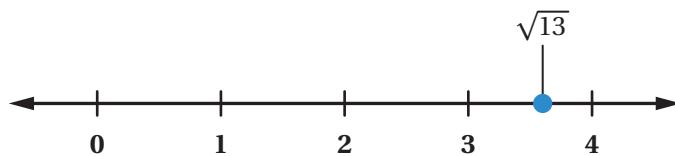
$\pi$

I'm not sure

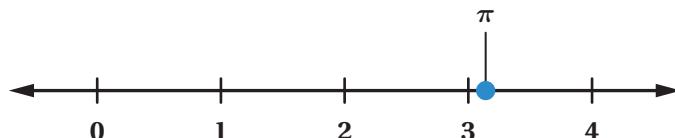
Explain your thinking.

## Hit the Target

- 2** Write a fraction as close to  $\sqrt{13}$  as you can (without using the square root symbol). Use a calculator to check how close you are, then revise your fraction to get as close to the target as possible.



- 3** Write a fraction as close to  $\pi$  as you can (without using the  $\pi$  symbol). Use a calculator to check how close you are, then revise your fraction to get as close to the target as possible.



## Irrational Numbers

- 4** A **rational number** is a number that can be written as a fraction of two integers, where the denominator is not zero.

It is impossible to write  $\sqrt{13}$  as a fraction with non-zero integers, which makes it an **irrational number**.

 **Discuss:** What are some other numbers you think might be irrational?

- 5** Is  $\sqrt{\frac{9}{4}}$  rational or irrational? Circle one.

Rational

Irrational

I'm not sure

Explain your thinking.

- 6** Sort the numbers into groups based on whether they are rational or irrational.

$$\frac{8}{4}$$

$$2\frac{3}{20}$$

$$1.\overline{73}$$

$$\sqrt{2}$$

$$2 \cdot \sqrt{13}$$

$$\sqrt{10}$$

$$2\pi$$

$$\sqrt{\frac{1}{4}}$$

$$\sqrt[3]{9}$$

$$\sqrt[3]{8}$$

$$1.73205080757\dots$$

$$1.73$$

Rational	Irrational	I'm Not Sure

## Irrational Numbers (continued)

- 7** Jada claims that any number written with a square root or a cube root is irrational. Is Jada correct? Circle one.

Yes                          No

Explain your thinking.

### Explore More

- 8** Here are some problems to explore why  $\sqrt{2}$  is irrational. Use a calculator if it helps with your thinking.

**a**  $\left(\frac{577}{408}\right)^2$  is very close to 2, but is it exactly equal to 2?

**b** If  $\left(\frac{577}{408}\right)^2 = 2$ , then  $408^2 \cdot 2 = 577^2$ . Diya says that's not true without computing any of these numbers. How can Diya know that?

**c** How does this show that  $\frac{577}{408} \neq \sqrt{2}$ ?

**d** Is  $\frac{1414213562375}{1000000000000} = \sqrt{2}$ ? Explain your thinking.

## **9** Synthesis

What is an irrational number? Give at least one example.

Things to Remember: