Lesson Practice 7.2.04

Lesson Summary

Proportional relationships can be represented using the equation $y = kx$, where k is the constant of proportionality.

For example, the table shows the proportional relationship between the number of pounds of soybeans and the cost at a certain store.

- The cost of the soybeans is proportional to the weight with a constant of proportionality of 2.
- If c represents the cost and w represents the weight, then you can represent the proportional relationship with the equation $c = 2w$.

Weight (lb), w	Cost (\$), c
$\frac{1}{2}$	1.00
1	2.00
2	4.00
w	$2w$

Things to Remember:

Lesson Practice
7.2.04

Name: ______ Date: ______ Period: ______

1. The ceilings in many basements are made up of rectangular tiles. For one basement, each square meter of ceiling requires 10.75 tiles. Complete the table.

Area of Ceiling (sq. m)	Number of Tiles
1	
10	
	53.75
x	

Problems 2–3: Each table represents a proportional relationship. Determine the constant of proportionality that completes each equation.

2.

s	P
2	8
3	12
5	20
10	40

$P =$ ______ s

3.

d	C
2	6.28
3	9.42
5	15.7
10	31.4

$C =$ ______ d

Problems 4–6: While traveling in 2023, Mai received 342.50 Norwegian kroner in exchange for 50 Australian dollars.

4. How many Norwegian kroner would Mai have received in exchange for 1 Australian dollar?

5. Write an equation to represent the amount of Norwegian kroner, n, received in exchange for a Australian dollars.

6. Determine the number of Norwegian kroner Mai would receive in exchange for 120 Australian dollars.

Lesson Practice
7.2.04

Name: .... Date: .... Period: ....

Problems 7–8: A plane flew at a constant speed between Denver and Chicago. It took the plane 1.5 hours to fly 915 miles.

7. Complete the table.

Time (hr)	Distance (mi)
1	
1.5	915
2	
2.5	
t	

8. How far would the plane fly in 10 hours at this speed?

Spiral Review

Problems 9–10: A bicycle travels 21 meters in 3 seconds.

9. Complete the table.

Time (sec)	Distance (m)
3	21
$1\frac{1}{2}$	
	$6\frac{3}{10}$

10. What is a constant of proportionality in this relationship?

What does this represent in the situation?

Reflection

1. Put a star next to a problem that looked more difficult than it really was.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

When a vehicle is traveling at a constant speed, there is a proportional relationship between the time traveled and the distance traveled. This is true for any person, animal, or object traveling at a constant speed.

For example, imagine someone running 5 meters per second. The table shows the distance they travel over different periods of time. The table also shows their speed, in meters per second.

The last row in the table shows that we can multiply the amount of time, t, with the constant of proportionality, 5, to determine the distance traveled, d.

The equation $d = 5t$ represents this relationship.

Time (sec)	Distance Traveled (m)	Speed (m per sec)
1	5	5
2	10	5
3	15	5
7	35	5
t	$5t$	5

Things to Remember:

Lesson Practice
7.2.05

Name: ______ Date: ______ Period: ______

Problems 1–3: A performer expects to sell $5{,}000$ tickets for an upcoming concert. They plan to make $\$311{,}000$ from the sales of these tickets. Assume that all tickets have the same price.

1. What is the price for one ticket? Show or explain your thinking.

2. Write an equation to represent the relationship between the number of tickets sold, x, and the total amount of money (in dollars) that they make, y.

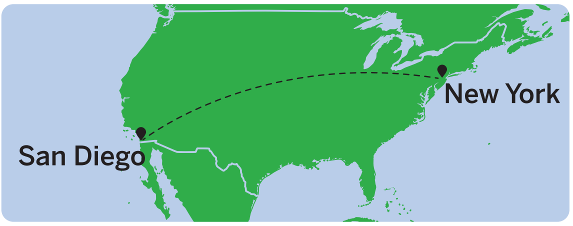
3. If they make $\$379{,}420$, how many tickets have they sold?

Problems 4–5: A car is traveling on a highway at a constant speed. The equation that represents the distance traveled in miles, d, for t hours is $d = 65t$.

4. What does the value 65 represent in this situation?

5. At this rate, how many miles will the car travel in 1.5 hours? Show or explain your thinking.

Problems 6–7: On its way from New York to San Diego, a plane flew at a constant speed over Pittsburgh, Saint Louis, Albuquerque, and Phoenix.



6. This table shows the flight time and distance traveled for each segment of the flight. Complete the table.

Segment	Time (hr)	Distance (mi)	Speed (mph)
Pittsburgh to Saint Louis	1	550	
Saint Louis to Albuquerque	1.7		
Albuquerque to Phoenix		330	

7. Let t represent the time in hours and d represent the distance in miles. Write an equation that represents the distance traveled for t hours.

Name: .... Date: .... Period: ....

8. A train travels at a constant speed between Springfield and Chicago. The train travels $100\frac{1}{2}$ miles in $\frac{3}{4}$ hours. How far does the train travel in one hour at this same speed?

9. Na'ilah is making a pitcher of a lemon-flavored sports drink. The drink mix container says to mix $\frac{1}{4}$ cups of powdered drink mix with 2 quarts of water. She prefers her sports drink to taste *more* lemony than the recipe on the container. Complete the equation to represent a mixture of cups of drink mix, c, and quarts of water, w, that would be *more* lemony than the original mixture.

$c =$ .... w

Spiral Review

10. Select *all* the tables that represent a proportional relationship between x and y.

☐ **A.**

x	0	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
y	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

☐ **B.**

x	0	$\frac{1}{2}$	2	3
y	0	$\frac{1}{4}$	4	9

☐ **C.**

x	0	4	8	12
y	0	2	4	6

☐ **D.**

x	0	1	2	3
y	0	4	8	12

11. Is 4.5 a solution to the equation $1.5 + x = 6$? Circle one.

Yes No

Reflection

1. Put a heart next to the problem you found most interesting.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

The structure of an equation representing the relationship between two quantities can tell us whether that relationship is proportional. An equation in the form of $y = kx$ has a constant of proportionality, k, which means it represents a proportional relationship.

Equations like $y = 3x + 1$ and $y = x^2$ do not have a constant of proportionality, so they do *not* represent proportional relationships.

Rewriting an equation in another form can help make a proportional relationship easier to see. For example, $y = \frac{x}{3}$ and $y = \frac{1}{3}x$ both represent the same proportional relationship.

Tables can help you determine whether an equation can be rewritten in the form of $y = kx$.

Things to Remember:

Lesson Practice
7.2.07

Name: .... Date: .... Period: ....

Problems 1–2: The relationship between a distance in yards, y, and the same distance in miles, x, is represented by the equation $y = 1760x$.

1. Complete the table.

Distance (mi), x	Distance (yd), y
1	
5	
	3,520
	17,600

2. Is there a proportional relationship between a distance in yards and the same distance in miles? Circle one.

Yes   No

Explain your thinking.

Problems 3–6: Determine whether or not each relationship is proportional.

	Proportional	Not Proportional
3. The remaining length, L, of a 120-inch rope after x inches have been cut off: $120 - x = L$	☐	☐
4. The total cost, t, after 8% sales tax is added to an item's price p: $1.08p = t$	☐	☐
5. The number of marbles, x, each sister gets after m marbles are shared equally among four sisters: $x = \frac{m}{4}$	☐	☐
6. The volume, V, of a rectangular prism whose height is 12 centimeters and whose base is a square with side lengths of s centimeters: $V = 12s^2$	☐	☐

Problems 7–8: Determine whether or not each relationship is proportional. Explain your thinking.

7.

x	y
2	5
3	7.5
6	15

8. $y = 3.2x + 5$

Name: ________ Date: ________ Period: ________

Problems 9–10: Determine whether each relationship is proportional or not proportional. Explain your thinking.

9. The weight of a stack of standard 8.5-by-11-inch paper and the number of sheets of paper.

10. The weight of a stack of different-sized books (where each book weighs a different amount) and the number of books in a stack.

11. Liam and Samar are running a 60-meter race. Each of their distances can be represented by an equation in the form $y = kx$ where y is the distance in meters and x is the time in seconds.

Use this information to complete the table.

- Liam's distance is represented with the equation $y = 6x$.
- At 8 seconds, Liam is 16 meters ahead of Samar.

Time (sec)	Liam's Distance (m)	Samar's Distance (m)
0		
2		
4		
6		
8		

Spiral Review

12. The equation $y = 3.5x$ can be used to determine the total cost, y, in dollars, of x ounces of blueberries. What does the number 3.5 represent in the equation?

A. The number of blueberries that $\$1$ can buy.

B. The number of blueberries in x ounces.

C. The cost of 1 ounce of blueberries.

D. The cost of x ounces of blueberries.

Reflection

1. Circle the problem that was the most challenging for you.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

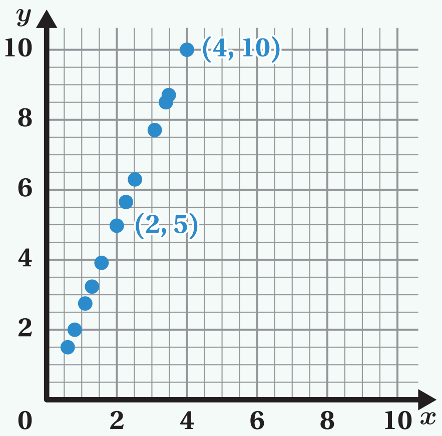
On a *coordinate plane*, if points all fall on a line that passes through $(0, 0)$, the relationship is proportional. The point $(0, 0)$ is known as the **origin**.

If it's unclear if the points form a line, you can test if the ratios of the coordinates are equivalent.

For example, the coordinates of two points on this line are $(2, 5)$ and $(4, 10)$.

$5 \div 2 = 2.5$ and $10 \div 4 = 2.5$

Since the ratio of the coordinates for both of these points is 2.5, these points are part of a proportional relationship.



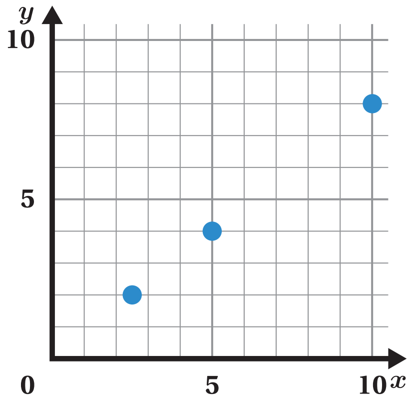
Things to Remember:

Lesson Practice
7.2.08

Name: ______________ Date: ________ Period: ________

Problems 1–4: Determine if each graph represents a proportional relationship. Explain your thinking.

1.



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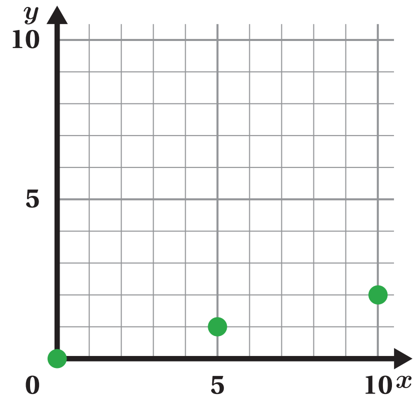
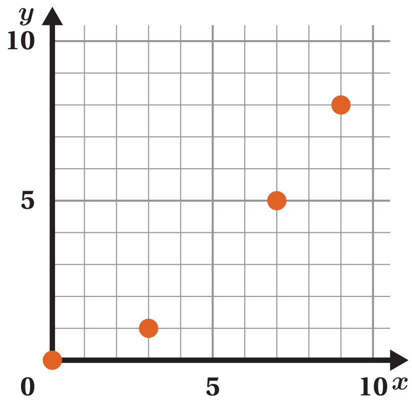
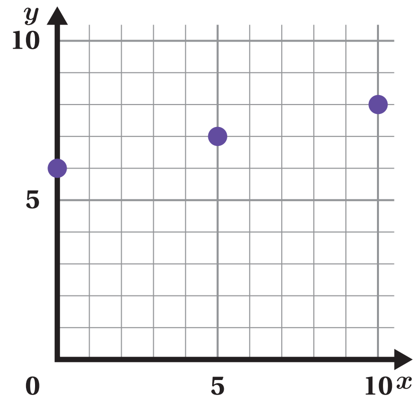


3.

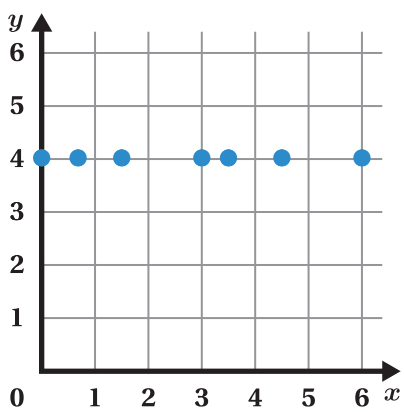


4.

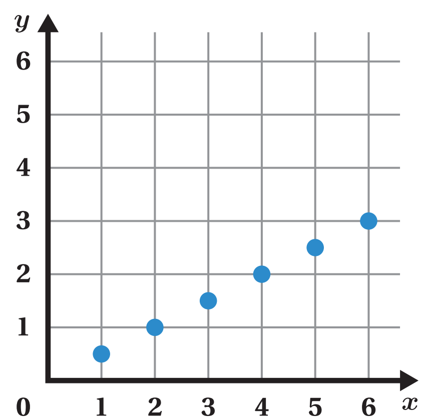


5. Select *all* the representations that show a proportional relationship.

☐ **A.**



☐ **B.**



☐ **C.**

x	0	2	4	6
y	0	3	9	27

☐ **D.**

x	0	2	4	6
y	0	12	24	36

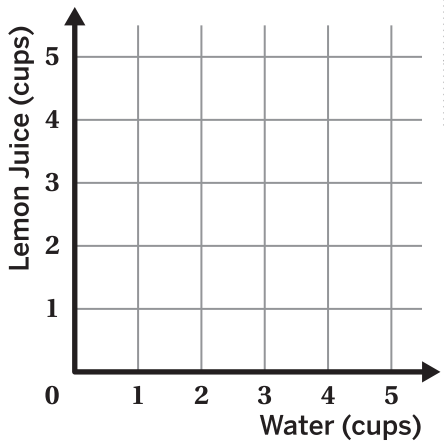
Name: .... Date: .... Period: ....

6. Arturo earns $\$33.00$ for babysitting for 4 hours. At this rate, how much will Arturo earn if he babysits for 7 hours? Explain your thinking.

7. A lemonade recipe calls for $\frac{1}{4}$ cups of lemon juice for every cup of water. The table shows different amounts of water and lemon juice you can use to make this recipe. Graph the ordered pairs to determine whether the relationship between water and lemon juice is proportional.

Water (cups)	Lemon Juice (cups)
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1

Explain your thinking.



Spiral Review

8. A turtle is walking away from a rock. x represents the time in minutes that the turtle is walking. y represents the distance in meters between the rock and the turtle. If x and y are in a proportional relationship, select *all* the true statements.

- ☐ **A.** The equation $y = 3x$ could represent the distance that the turtle walks.
- ☐ **B.** The turtle walks for a bit and then stops for a minute before walking again.
- ☐ **C.** The turtle walks away from the rock at a constant rate.
- ☐ **D.** The equation $y = x + 3$ could represent the distance that the turtle walks.
- ☐ **E.** After 6 minutes, the turtle walks 18 meters, and after 10 minutes, the turtle walks 20 meters.

Reflection

1. Put a star next to the problem you think is the most important.
2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 67

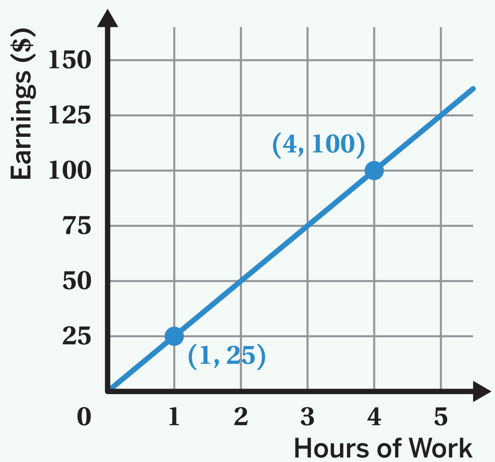
Lesson Summary

Each point on a graph of a proportional relationship tells a story using the quantities represented by x and y. You can determine a constant of proportionality from a graph by using:

- The value of y when x is equal to 1.
- The ratio of $\frac{y}{x}$ for any ordered pair.

For example, this graph shows a proportional relationship between hours worked, x, and money earned in dollars, y. One constant of proportionality is 25 because \$25 is earned for working 1 hour.

The ordered pair $(4, 100)$ shows that \$100 is earned for 4 hours of work, which is an equivalent ratio to earning \$25 per hour.

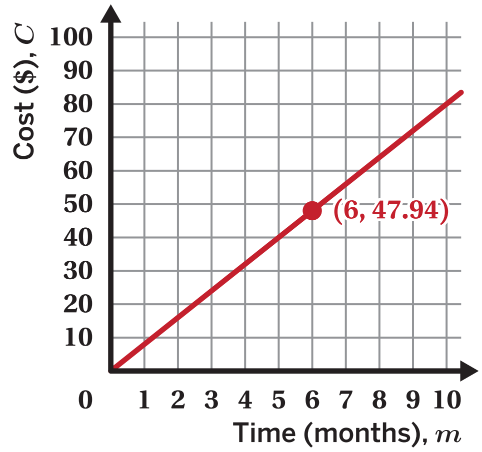


Things to Remember:

Lesson Practice
7.2.09

Name: .... Date: .... Period: ....

Problems 1–2: Here is a graph that shows a proportional relationship between the number of months Tiara had a streaming service subscription and the total amount of money she paid for the subscription.

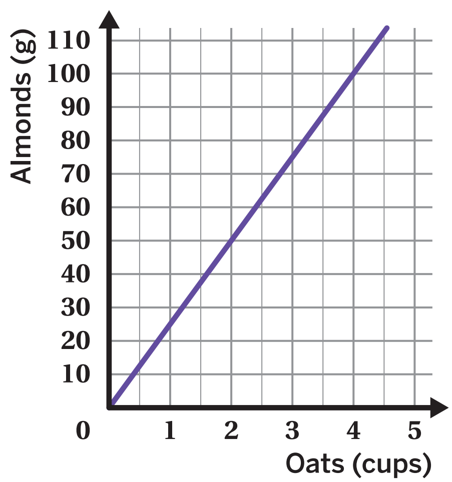


After 6 months, Tiara paid $47.94.

1. What is a constant of proportionality in this relationship?

2. Write an equation that represents the relationship between C, the total cost of the subscription, and m, the number of months.

Problems 3–4: A recipe for granola calls for a mix of almonds and oats. The graph shows the amount of almonds, in grams, for different amounts of oats, in cups.



3. Determine a constant of proportionality for this relationship, then explain its meaning.

4. Label one place you see that constant of proportionality on the graph.

5. The graph shows the cost for two different varieties of apples. Which variety of apples has a higher cost per pound? Circle one.

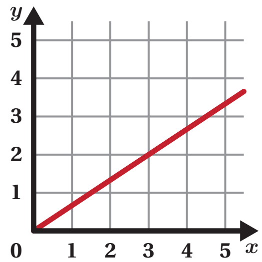
   Gala   Red Delicious

   Explain your thinking.

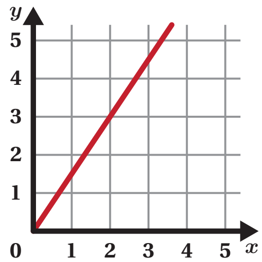
Name: .... Date: .... Period: ....

6. Which of these relationships has the same constant of proportionality as the equation $y = \frac{2}{3}x$? Select *all* that apply.

☐ **A.**



☐ **B.**



☐ **C.**

x	y
0	0
3	2
6	4
9	6

☐ **D.**

x	y
0	0
2	3
4	6
6	9

Spiral Review

7. Write an expression that is equivalent to $8(n + 6)$.

Problems 8–10: Haru and Irene were running laps around the track. Their coach recorded their times at the end of Laps 2, 4, and 6.

Haru's Run

Distance (laps)	Time (min)	Minutes Per Lap
2	4	
4	9	
6	15	

Irene's Run

Distance (laps)	Time (min)	Minutes Per Lap
2	5	
4	10	
6	15	

8. Complete the tables with the minutes per lap for each run.

9. Based on the table, is Haru running at a constant speed? Explain your thinking.

10. Based on the table, is Irene running at a constant speed? Explain your thinking.

Reflection

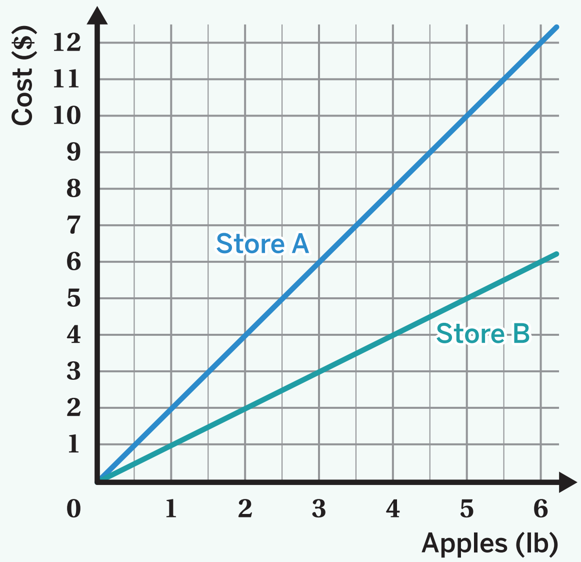
1. Put a smiley face next to the problem you learned from most.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can compare graphs of proportional relationships when they're on the same coordinate plane. The steeper the line, the greater the constant of proportionality.

For example, you can use this graph to compare the cost of apples at two different stores.



- The line representing Store A is steeper than the line representing Store B, so it has a greater constant of proportionality. This means Store A charges more per pound than Store B.
- Store A charges \$2 for one pound ($k = 2$), while Store B charges \$1 for one pound ($k = 1$). This is another way you can determine that the constant of proportionality at Store A is greater than that at Store B.

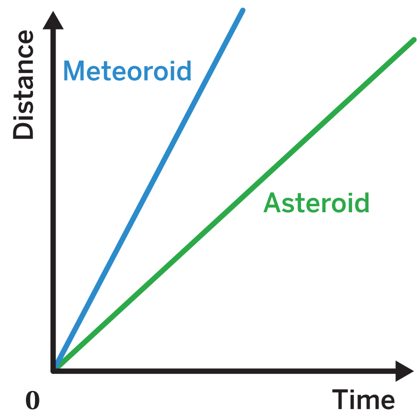
Things to Remember:

Name: .... Date: .... Period: ....

1. A meteoroid and an asteroid travel through the solar system. This graph shows how much distance they travel over time.

Does the asteroid travel faster or slower than the meteoroid?

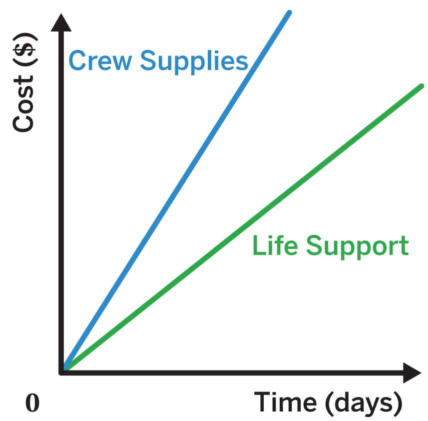
Explain your thinking.



2. Having a crew on the International Space Station requires life support and crew supplies. The graph shows the relationship between the cost of life support and crew supplies, and the number of days spent in space.[1]

Which costs less, life support or crew supplies?

Explain your thinking.



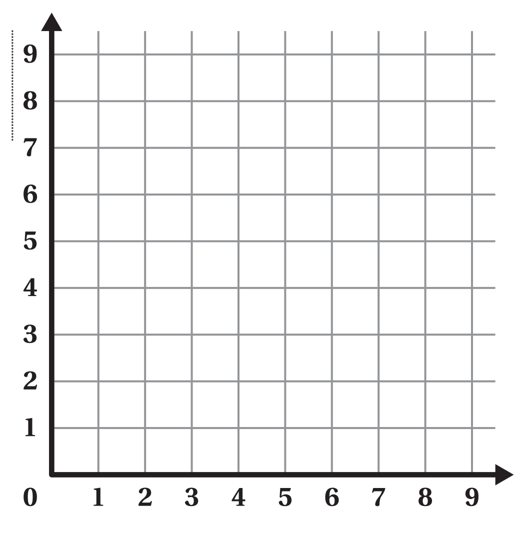
Problems 3–6: At a supermarket, you can fill your own honey container and pay by the ounce. A customer buys 12 ounces of honey for \$5.40.

3. How much does the honey cost per ounce? Show or explain your thinking.

4. How much honey can be bought per dollar? Show or explain your thinking.

5. Write *two* different equations representing this situation. Use h for ounces of honey and c for cost in dollars.

6. Choose *one* equation and draw its graph on the coordinate plane. Be sure to label the axes.

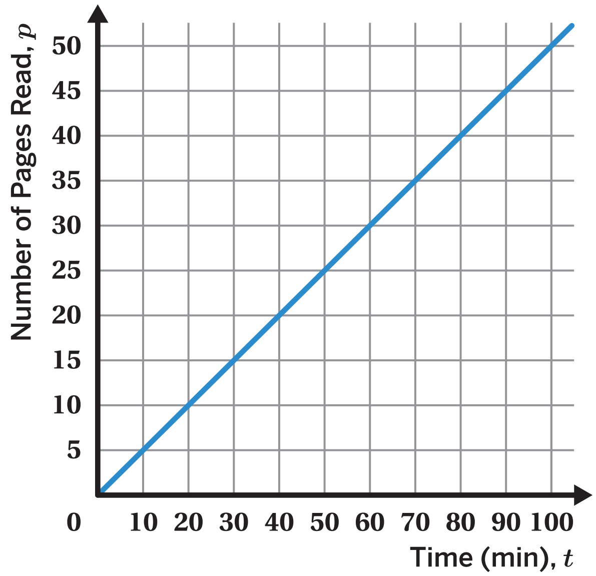


[1] Source: *NASA*

Lesson Practice
7.2.10

Name: ______________ Date: ________ Period: ________

Problems 7–10: The graph shows the relationship between time in minutes, t, and the total number of pages Joel has read, p.



7. Is this relationship proportional?

Explain your thinking.

8. What is a constant of proportionality in this relationship?

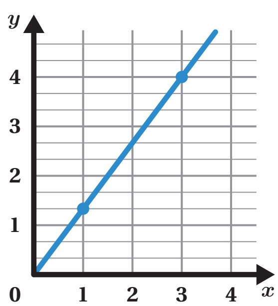
9. What does this number mean in this situation?

10. Write an equation that represents the relationship between the time in minutes, t, and the total number of pages Joel has read, p.

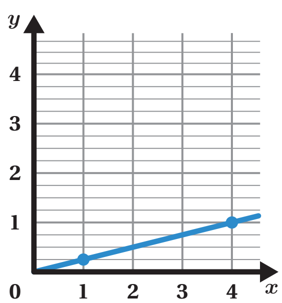
Spiral Review

Problems 11–14: Match each equation with its graph.

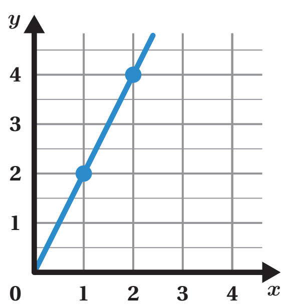
Graph A



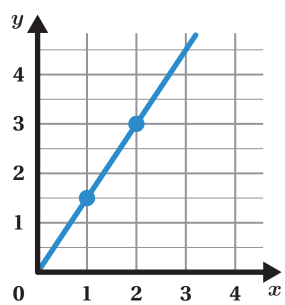
Graph B



Graph C



Graph D



11. $y = \frac{1}{4}x$ ________

12. $y = \frac{3}{2}x$ ________

13. $y = 2x$ ________

14. $y = \frac{4}{3}x$ ________

Reflection

1. Put a heart next to the problem you're most proud of.

2. Use this space to ask a question or share something you're proud of.

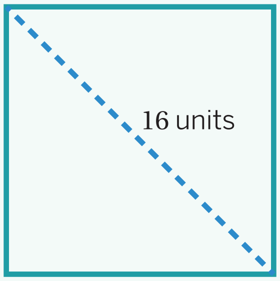
Lesson Summary

Proportional relationships can appear in geometry when you look at scaled copies of shapes.

Some examples of these proportional relationships include:

- The side lengths of a square and its perimeter.
- The diagonal length of a square and its perimeter.
- The diagonal length of an octagon and its perimeter.

Because the relationship between the diagonal length and the perimeter of a square is proportional, you can determine one of the measurements if you know the other.

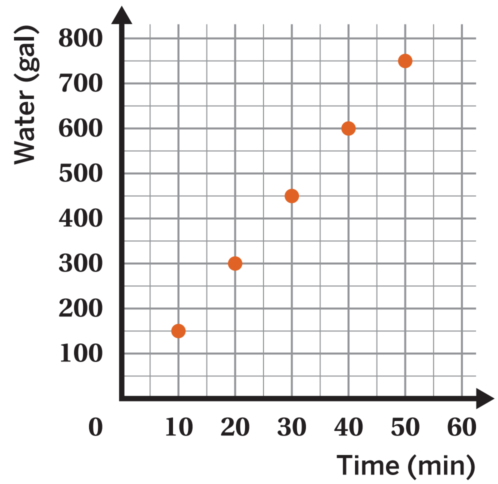


For example, to determine the perimeter of a square whose diagonal length is 16 units, you can multiply by the constant of proportionality, approximately 2.83. $16 \cdot 2.83 \approx 45$ units

Things to Remember:

Name: .... Date: .... Period: ....

Problems 1–3: This graph shows the amount of water in a swimming pool as it fills up.



1. Explain how the graph shows that the relationship between time and the amount of water is proportional.

2. Calculate a constant of proportionality for this relationship.

3. This pool is safe to swim in when it contains 9,000 gallons of water. How long will it take to fill the pool to this level?

Problems 4–6: An equilateral triangle is a triangle where all three sides have the same length.

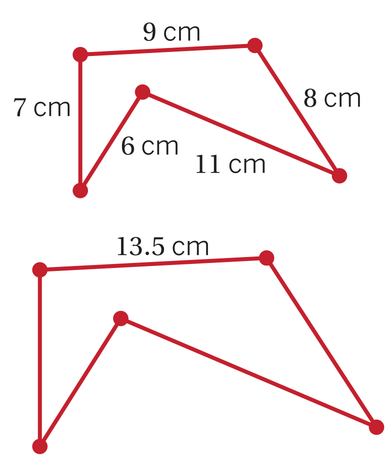
4. Complete the table to show the side lengths and perimeters for several equilateral triangles.

Side Length	Perimeter
3	
10	
	81
	315

5. Explain why the relationship between the perimeter and side length for an equilateral triangle is proportional.

6. What is a constant of proportionality for the relationship?

7. These polygons are scaled copies. Determine the perimeter of the larger polygon. Show or explain your thinking.



Lesson Practice
7.3.01

Name: ________ Date: ________ Period: ________

Spiral Review

Problems 8–9: Here are two recipes for making sparkling lemonade. For each recipe, determine how many tablespoons of lemonade mix are required *per cup* of sparkling water. Show or explain your thinking.

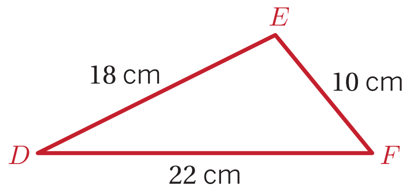
8. 4 tablespoons of lemonade mix and 12 cups of sparkling water

9. 4 tablespoons of lemonade mix and 6 cups of sparkling water

10. Select *all* of the coordinate pairs that could be part of a proportional relationship with a constant of proportionality of 4.

- ☐ **A.** $(3, 12)$
- ☐ **B.** $(4, 8)$
- ☐ **C.** $(3.5, 14)$
- ☐ **D.** $(6, 10)$
- ☐ **E.** $\left(\frac{1}{2}, 2\right)$

11. Here is triangle DEF and its dimensions. Which measurements represent the dimensions of a triangle that is a scaled copy of triangle DEF?



A. 20 cm, 12 cm, 24 cm

B. 9 cm, 5 cm, 10 cm

C. 27 cm, 16 cm, 33 cm

D. 13.5 cm, 7.5 cm, 16.5 cm

Reflection

1. Put a heart next to a problem you understand well.

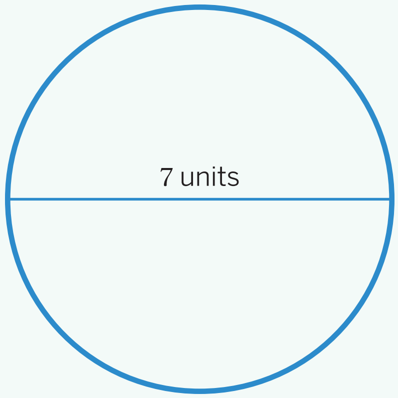
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

The distance around a circle is called the **circumference**. There is a proportional relationship between the diameter of a circle and its circumference. The *constant of proportionality* for that relationship is π (**pi**). π is often approximated as 3.14 or $\frac{22}{7}$.

For any circle, we can calculate the circumference, C, using the equation $C = \pi d$.

For example, if the diameter of a circle is 7 units, the circumference of the circle can be calculated approximately as $7 \cdot 3.14 = 21.98$. More accurately, $C = 7\pi$ units.



If we know the radius of a circle, we can calculate the circumference by first determining the diameter, then using the equation $C = \pi d$.

Things to Remember:

Lesson Practice
7.3.03

Name: .... Date: .... Period: ....

1. A student measured the diameter and circumference of several circular objects and recorded their measurements in a table.

Which object's circumference measurement is the least accurate? Explain your thinking.

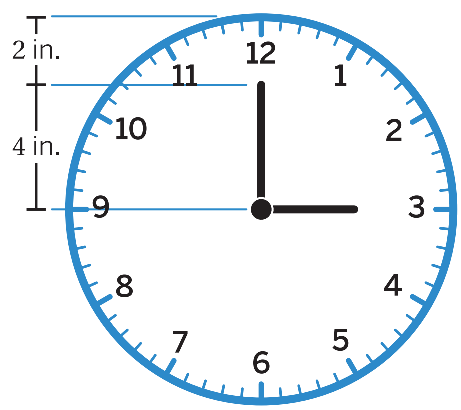
Object	Diameter (cm)	Circumference (cm)
Half dollar coin	3	10
Flying disc	23	50
Jar lid	8	25
Flower pot	15	48

2. Complete the table to determine the missing measurements for each object.

Object	Radius	Diameter	Circumference
Hula hoop		36 inches	
Circular pond			556 feet
Magnifying glass		5.2 centimeters	
Car tire			71.6 inches

3. The circumference of a circle is approximately 50.27 inches. Determine the length of the radius of the circle in inches. Round your answer to the nearest whole number.

Problems 4–5: The minute hand of a circular clock measures 4 inches. The distance from the end of the minute hand to the outer edge of the clock is 2 inches.



4. What is the length of the radius of the clock?

5. What is the circumference of the clock?

Lesson Practice
7.3.03

Name: .... Date: .... Period: ....

Problems 6–7: The size of a drum affects its pitch. When comparing drums, the one with a larger diameter has a lower pitch because the frequency of its sound waves is slower.

6. If a drummer wants a drum with a lower pitch, should she choose a drum with a circumference of 18π inches or 22.5π inches?

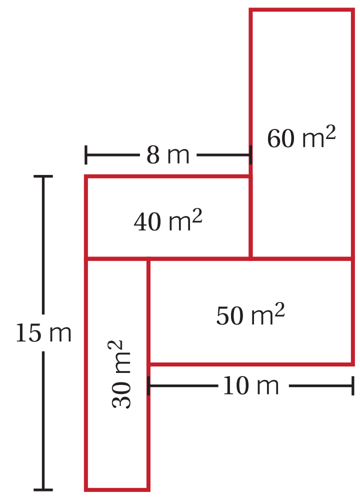
7. Determine the diameter of the drum she should choose.

Problems 8–10: Decide whether the measurement described is the diameter, radius, or circumference. Circle one.

8. The tires on a tractor are 4.5 feet tall.	Diameter	Radius	Circumference
9. The distance from the tip of a slice of pizza to the crust is 6 inches.	Diameter	Radius	Circumference
10. The length of the metal rim around a glass lens is 190 millimeters.	Diameter	Radius	Circumference

Spiral Review

11. This shape is made up of four rectangles. Determine the total perimeter of the shape.



Reflection

1. Put a heart next to the problem you feel most confident about.

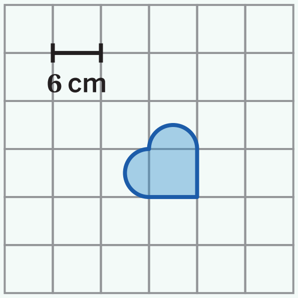
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can use what you know about the perimeter of squares and rectangles and the circumference of circles to find the perimeter of complex shapes.

- Determine what pieces it is made up of such as semicircles, quarter-circles, and straight pieces.
- Determine the length of each straight piece and the radius or diameter of each partial circle.
- Determine the perimeter or circumference of each piece.
- Add them together to get the total perimeter.

The perimeter of the heart shape is made up of 2 semicircles and 2 straight pieces. The *scale* is 6 centimeters.



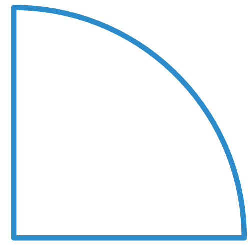
Each semicircle has a diameter of 6 centimeters. Together they make up one entire circle with a diameter of 6 centimeters and a circumference of $6 \cdot \pi = 6\pi$ centimeters. Each straight edge is 6 centimeters long.

The total perimeter is $6\pi + 12$ centimeters.

Things to Remember:

Name: ________ Date: ________ Period: ________

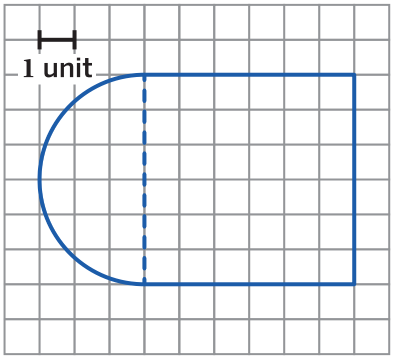
Problems 1–2: Here is a quarter-circle. It was created using a circle with a 12-inch diameter. The circle was folded in half and then folded in half again.



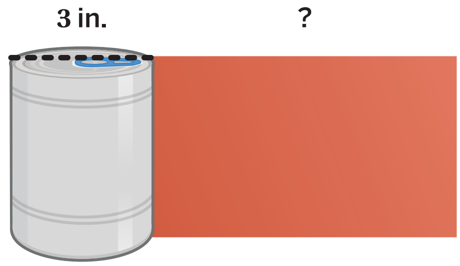
1. Label the quarter-circle with any important measurements.

2. What is the perimeter of this quarter-circle?

3. Here is a shape made of a semicircle and a square. What is the perimeter of this shape?

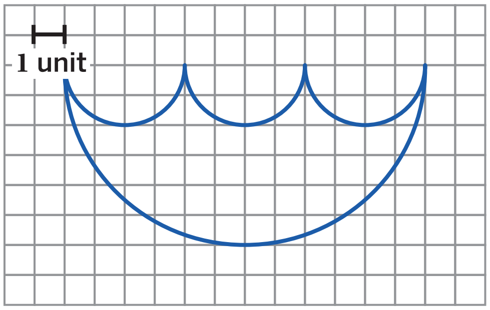


4. A soup can with a diameter of 3 inches has a wrapper that wraps exactly once around its outside. If the wrapper is rolled out, what is its approximate length? Show your thinking.



5. Determine the exact perimeter of this shape made from parts of circles.

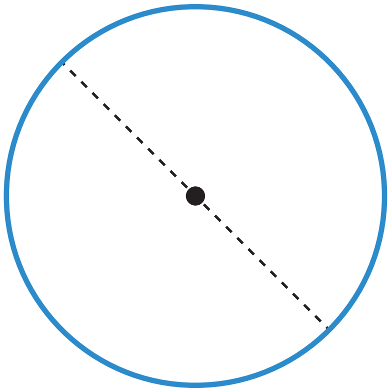
   Show or explain your thinking



Name: ______ Date: ______ Period: ______

Spiral Review

6. This circle has a diameter measurement of 5 centimeters. Which measurement is closest to the circumference of the circle in centimeters?



A. 6.25 centimeters

B. 7.85 centimeters

C. 15.7 centimeters

D. 25 centimeters

Problems 7–9: Determine whether each measurement represents the radius, diameter, or circumference, and record it in the appropriate column of the table. Then determine the exact lengths of the other two measurements of the circle.

	Measurement	Radius	Diameter	Circumference
7.	The tires of a mining truck are 14 feet tall.			
8.	The fence around a circular pool is 76 feet long.			
9.	The center to the edge of a small plate measures 60 millimeters.			

Reflection

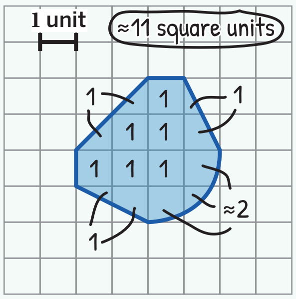
1. Put a smiley face next to a problem you were stuck on and then figured out.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

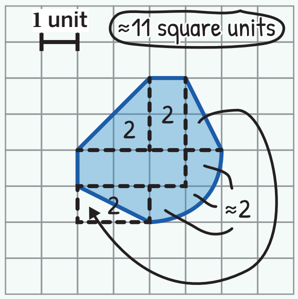
There are many ways you can find the *area* of a complex shape on a grid.

Here are some strategies you can use:

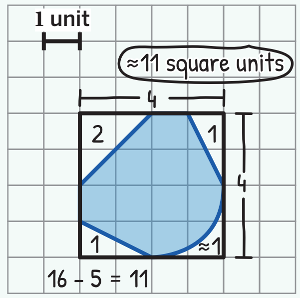
Counting Whole and Partial Squares



Decomposing and Rearranging



Enclosing and Subtracting

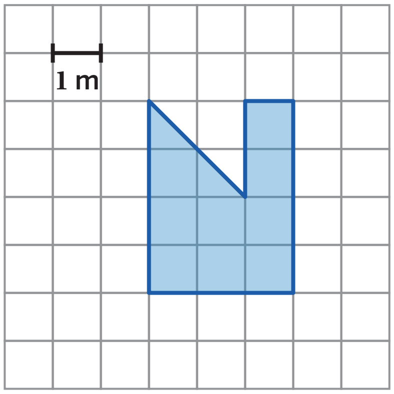


Things to Remember:

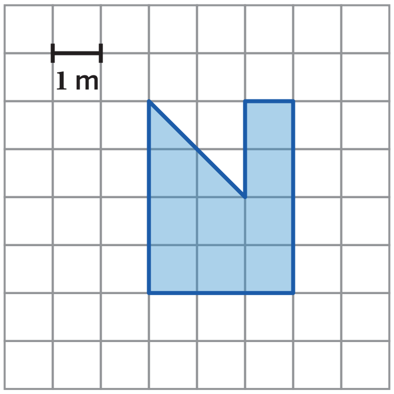
Lesson Practice
7.3.05

Name: .... Date: .... Period: ....

1. What is the area of this shape? Show your thinking.

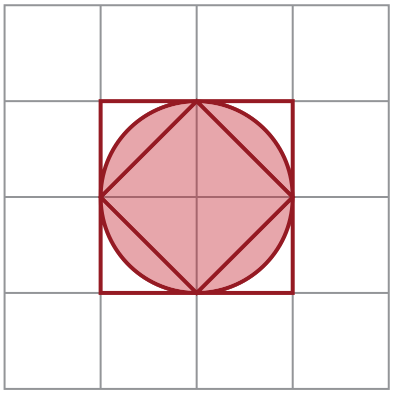


2. Show or describe another way to determine the area of this shape.



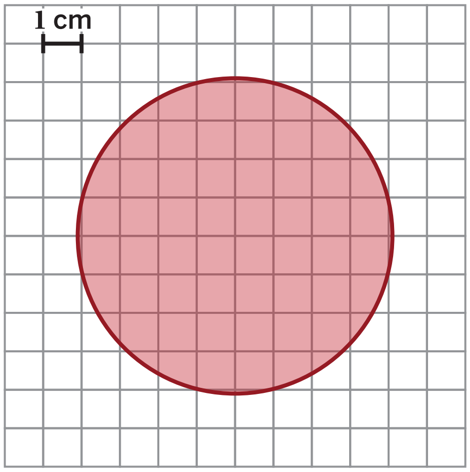
3. Here is a diagram of two squares and a circle.

Explain why the area of the circle is more than 2 square units but less than 4 square units.

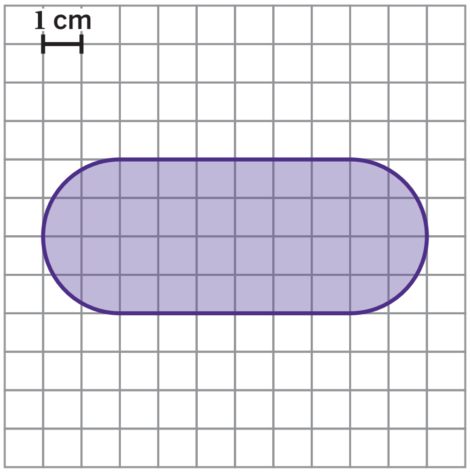


Problems 4–5: Estimate the area of each shape.

4.



5.

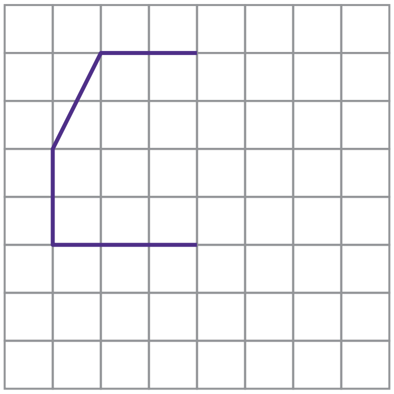


Lesson Practice
7.3.05

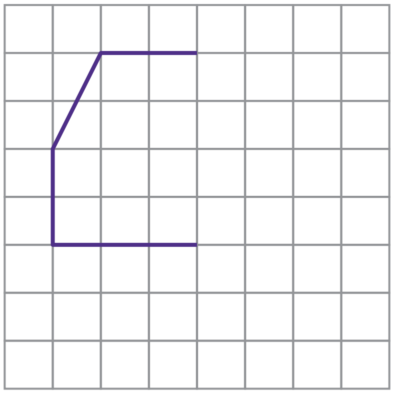
Name: .... Date: .... Period: ....

Problems 6–7: Basheera started drawing a polygon.

6. Complete Basheera's drawing so that the polygon has an area of 18 square units.



7. Complete Basheera's drawing in a different way so that the polygon has an area of 18 square units.



Spiral Review

8. Select *all* the expressions that are equivalent to 48.

- ☐ **A.** $3 \bullet 4^2$
- ☐ **B.** $6 + 6 \bullet 4$
- ☐ **C.** 50% of 96
- ☐ **D.** $(3 \bullet 4)^2$
- ☐ **E.** $4(6 + 6)$

9. What is the greatest common factor of 48 and 64?

10. What is the least common multiple of 4 and 6?

Reflection

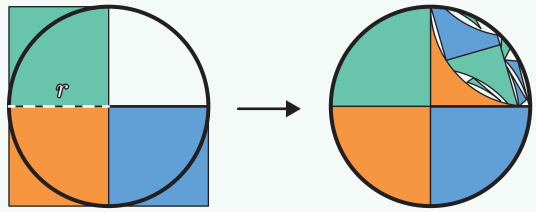
1. Circle a problem you want to talk to a classmate about.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

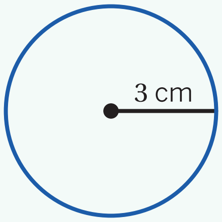
You can find the area of a circle if you know the length of its radius, r.

The *approximate* area of a circle is equal to the area of a little more than 3 radius squares. Each radius square has an area of r^2.



The *exact* area of a circle is equal to the area of π radius squares. You can express this with the formula $A = \pi \cdot r^2$.

For example, to find the area of a circle with a radius of 3 centimeters, you can calculate 3^2, then multiply the result by π. The area of the circle is 9π square centimeters.

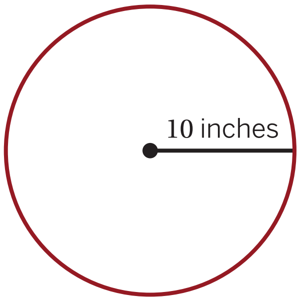


Things to Remember:

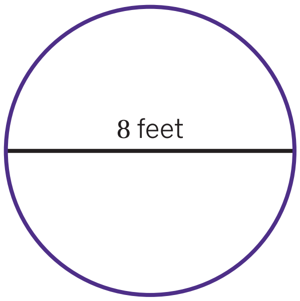
Name: ______ Date: ______ Period: ______

Problems 1–2: Determine the exact area of each circle.

1.



2.



Problems 3–4: The table shows the diameters of 4 different coins.

Coin	Diameter (cm)	Area (sq. cm)
Penny	1.9	
Nickel	2.1	
Dime	1.8	
Quarter	2.4	

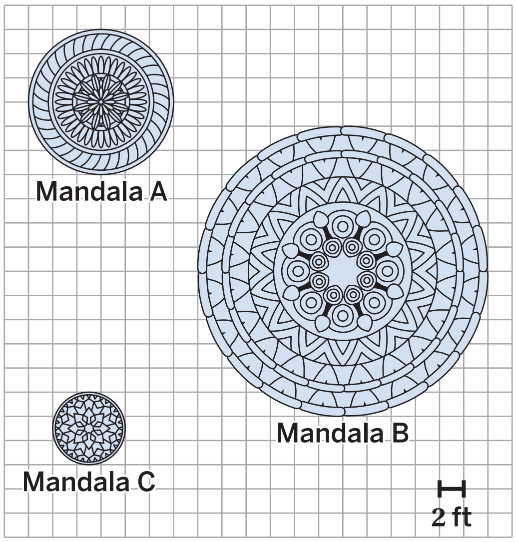
3. To determine how much metal is on one face of a coin, it is more useful to use the area rather than the circumference. Explain why this is the case.

4. Complete the table by calculating the exact area of each coin.

5. A mandala is a geometric figure that has spiritual relevance in many religions, including Hinduism and Buddhism. The word *mandala* is Sanskrit for *circle*.

Saanvi designed three mandalas. Determine the exact circumference and area for each mandala.

	Circumference (ft)	Area(sq. ft)
A		
B		
C		



Lesson Practice
7.3.07

Name: .... Date: .... Period: ....

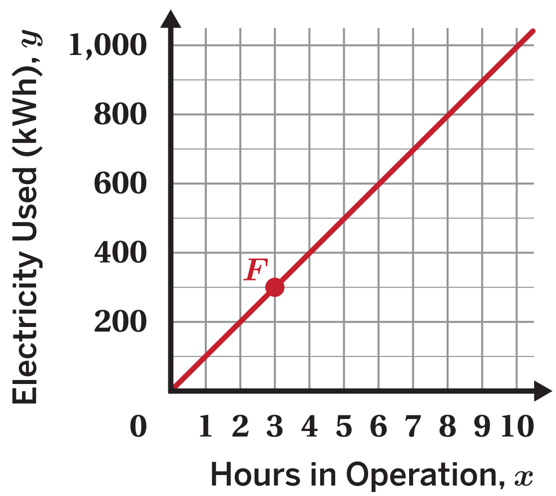
Problems 6–7: The radius of Earth is approximately $6{,}400$ kilometers. The equator is the circle around Earth that divides it into the northern and southern hemispheres.

6. Is the circumference of a circle or the area of a circle more useful for finding the length of the equator?

7. What is the length of the equator?

Spiral Review

Problems 8–10: This graph shows a proportional relationship between the number of hours a manufacturing factory is in operation and the number of kilowatt-hours (kWh) of electricity used. Use the graph to determine whether each statement is true or false.



8. Point F represents the number of kilowatt-hours of electricity used when the factory is in operation for 3 hours.

9. The factory uses 6 kilowatt-hours of electricity when it is in operation for 600 hours.

10. The factory uses 700 kilowatt-hours of electricity when it is in operation for 7 hours.

Reflection

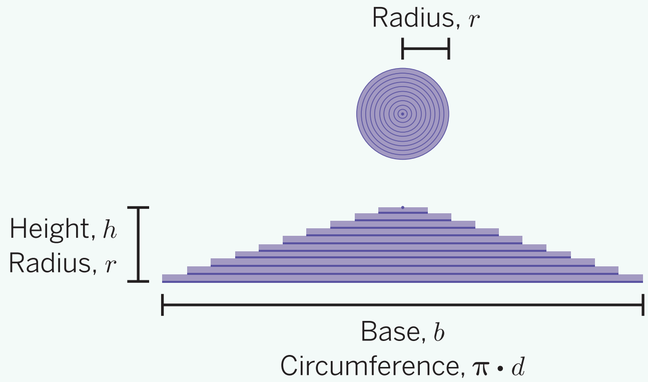
1. Put a heart next to the problem you found most interesting.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

If you take apart a circle and rearrange it to resemble a triangle, the formula for the area of a circle can be related to the formula for the area of the triangle.

This helps us make sense of each part of the formula for the area of a circle.



- The radius of a circle is r and the circumference is $\pi \cdot d$, so you can substitute those values into the equation for the area of a triangle.
- You can replace d with $2 \cdot r$.
- $\frac{1}{2} \cdot 2 = 1$, which leaves $A = \pi \cdot r^2$.

$$A = \frac{1}{2} \cdot b \cdot h$$

$$A = \frac{1}{2} \cdot (\pi \cdot d) \cdot r$$

$$A = \frac{1}{2} \cdot \pi \cdot (2 \cdot r) \cdot r$$

$$A = \pi \cdot r \cdot r$$

$$A = \pi \cdot r^2$$

Things to Remember:

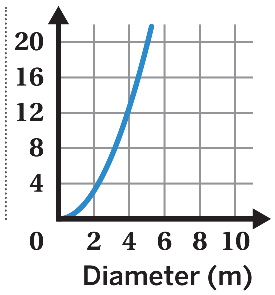
Lesson Practice
7.3.08

Name: .... Date: .... Period: ....

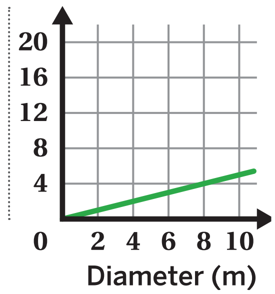
Problems 1–3: Label each y-axis with a term from the word bank to describe the relationship within a circle that the graph represents.

Radius	Circumference	Area

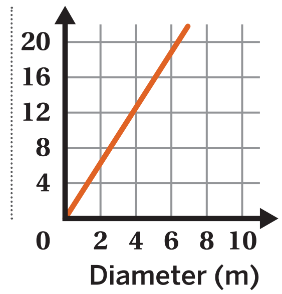
1.



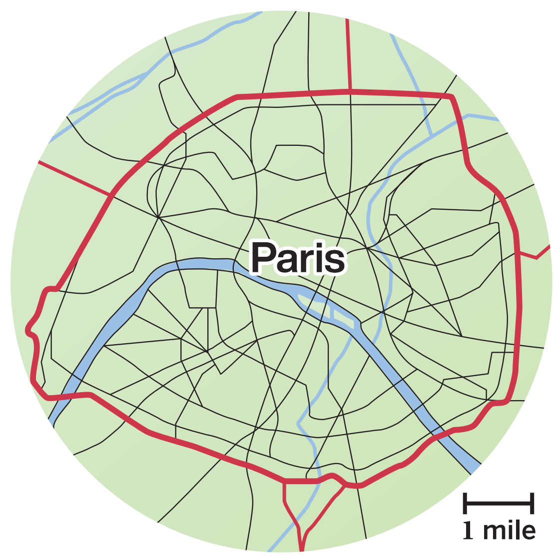
2.



3.



Problems 4–5: The city of Paris, France, is surrounded by an almost circular road called the Périphérique.



4. Use the map and the scale to estimate the length of the road. Explain your thinking.

5. Use the map and the scale to estimate the total area enclosed by the road. Explain your thinking.

6. Select *all* the pairs of quantities that are proportional to each other. For the quantities that are proportional, write an equation that relates them.

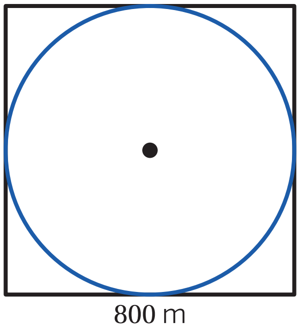
- ☐ **A.** The radius and diameter measurements of a circle
- ☐ **B.** The radius and circumference measurements of a circle
- ☐ **C.** The radius and area measurements of a circle
- ☐ **D.** The diameter and circumference measurements of a circle
- ☐ **E.** The diameter and area measurements of a circle

Lesson Practice
7.3.08

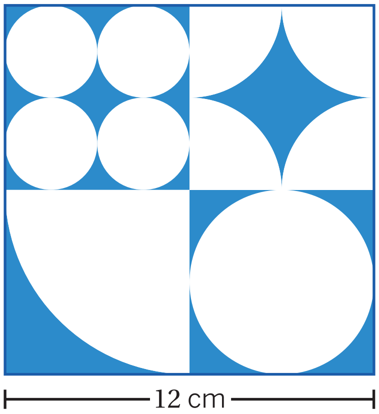
Name: ______ Date: ______ Period: ______

Spiral Review

7. Determine the exact area of the circle. Explain your thinking.



8. This 12-centimeter square contains squares, circles, and quarter-circles. Determine the exact area of the shaded region. Show or explain your thinking.



9. A circle has a diameter of 13 units. What is the area of the circle to the nearest hundredth of a square unit?

A. 40.84 **B.** 132.73 **C.** 530.93 **D.** 2,123.72

Problems 10–11: Solve each equation. Show your thinking.

10. $x - 2\frac{2}{3} = 6\frac{2}{3}$

11. $\frac{1}{2}x = 12$

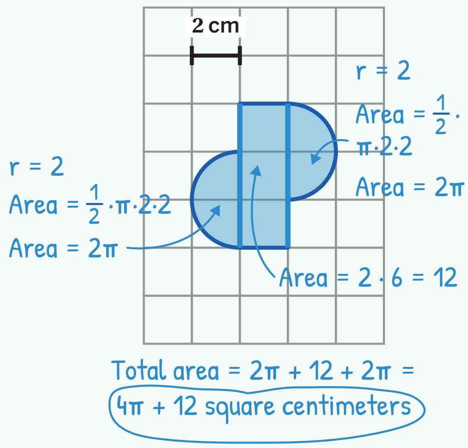
Reflection

1. Put a question mark next to a problem you're feeling unsure of.
2. Use this space to ask a question or share something you're proud of.

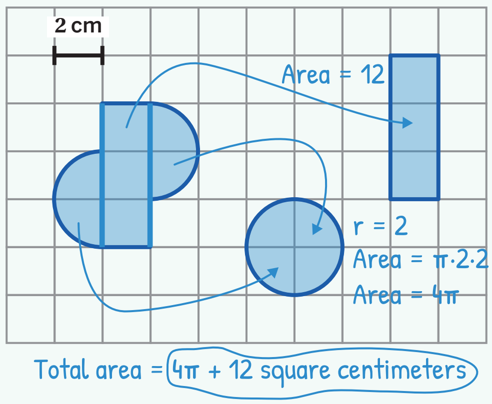
Lesson Summary

We can use different strategies and different ways of thinking to calculate the area of complex figures. Here are two ways we can determine the area of this shape:

Strategy 1



Strategy 2



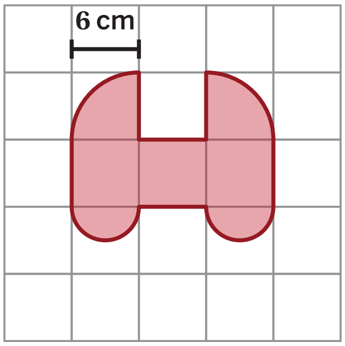
The path to the solution may not be obvious at first, but by breaking the shape down into squares, circles, and parts of circles, we can figure things out!

Things to Remember:

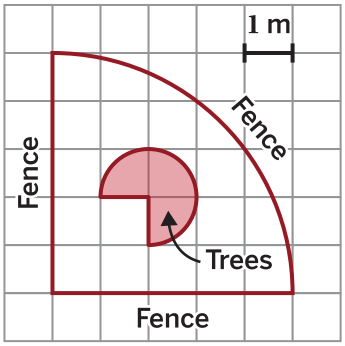
Lesson Practice
7.3.09

Name: .... Date: .... Period: ....

1. Determine the area of this shape.



Problems 2–4: Here is the architectural plan for a fenced-in garden. The shaded region represents an area for trees.



2. What is the area of the tree region?

3. The rest of the fenced-in garden is for flowers. How much area will be covered in flowers? Show or explain your thinking.

4. How long is the fence around the edge of the garden? (Note: The edge around the tree region is not a part of the fence.)

5. Each of these squares has a side length of 12 units. Compare the areas of the shaded regions in these three figures. Which figure has the largest shaded region? Show or explain your thinking.

Figure A

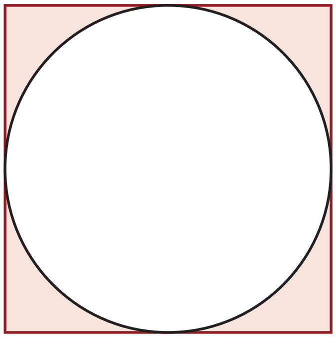


Figure B

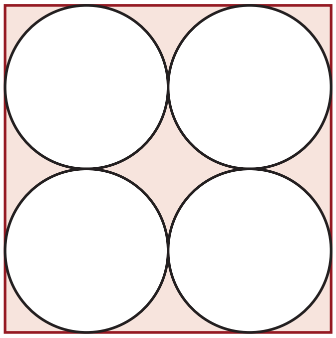
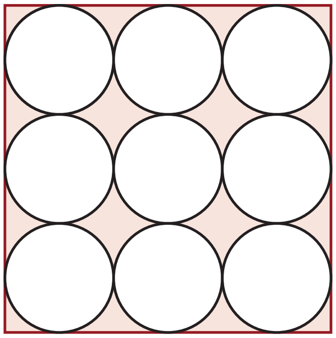


Figure C



Name: .... Date: .... Period: ....

6. A circle with a 12-inch long diameter is cut in half and then cut in half again. What is the area of the new shape? Show or explain your thinking.

Spiral Review

7. A graffiti artist is spray painting a circular mural onto a wall. The mural will have a diameter that is 10 feet long. Each can of spray paint covers about 24 square feet. How many cans of spray paint will the artist need to create the mural? Explain your thinking.

Problems 8–9: Determine each value.

8. What is the length of the radius of a circle with a diameter that measures 10 units?

9. What is the length of the diameter of a circle with a radius that measures 10 units?

10. Abena used wire fencing to form a border around a circular garden. If the radius of the circular garden was 10 yards long, what was the total length of the border, rounded to the nearest tenth of a yard?

A. 31.4 yards **B.** 62.8 yards **C.** 314.2 yards **D.** 628.3 yards

Reflection

1. Put a star next to a problem where you revised your thinking.

2. Use this space to ask a question or share something you're proud of.

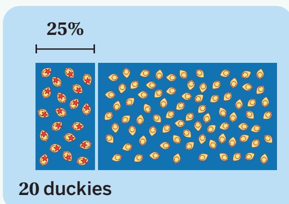
Lesson Summary

Percent means for every 100. It's represented by the percent symbol, %.

Each of the different ducky games in this lesson had a certain percentage of ducks with stars: 10%, 25%, 50%, or 75%. Fractions and tape diagrams can help us interpret these percentage problems.

Example Problem

25% of the 80 duckies have stars.

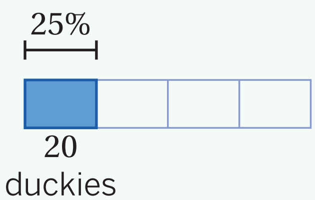


Using Fractions

25% of something means $\frac{25}{100}$ or $\frac{1}{4}$.

$\frac{1}{4}$ of 80 duckies is 20 duckies.

80 total duckies



Using Tape Diagrams

There are four 25s in 100, so the tape diagram can be split into 4 pieces. The total number of duckies can also be split into 4 parts, so there are 20 duckies in each section.

80 total duckies

20	20	20	20

Things to Remember:

Lesson Practice
6.3.09

Name: ________ Date: ________ Period: ________

1. Here are 24 stars. Circle 25% of these stars.

2. Shep made 40 muffins. 50% of the muffins are chocolate. How many muffins are chocolate?

3. Which is greater? Show or explain your thinking.

   A. 75% of 8

   B. 25% of 32

   C. They are the same.

Problems 4–5: Complete each statement. Make a tape diagram if it helps with your thinking.

4. 10% of 20 is ________.

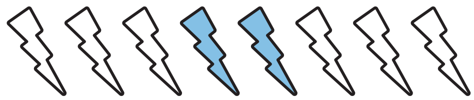
5. 25% of 60 is ________.

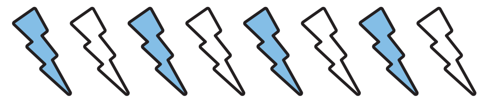
6. Explain how you could mentally calculate 10% of any number.

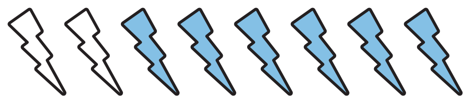
Lesson Practice
6.3.09

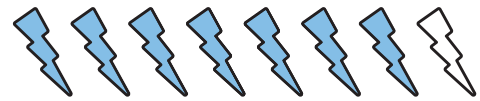
Name: ____________ Date: ________ Period: ________

7. Which group shows 75% of the lightning bolts shaded?

A. 

B. 

C. 

D. 

Spiral Review

8. Abdel paid \$13 for 3 books. Jayden bought 12 books priced at the same rate. How much did Jayden pay for the 12 books? Explain your thinking.

Problems 9–11: Determine whether each product will be *less than*, *greater than*, or *equal to* 40.

9. $\left(\frac{6}{4}\right) \cdot 40$

10. $\left(\frac{8}{8}\right) \cdot 40$

11. $\left(\frac{1}{2}\right) \cdot 40$

Reflection

1. Put a question mark next to a problem you're feeling unsure of.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice 6.3.10

Science Mom Lesson 77

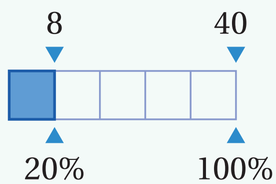
Lesson Summary

You can represent percentages using tape diagrams, double number lines, and tables. The strategies you've already used to solve ratio problems can help you think about and solve percentage problems, too!

Let's say a biker traveled 8 kilometers, which is 20% of their goal distance. What's their goal distance?

Here are three ways to represent and solve this percentage problem.

Tape Diagram

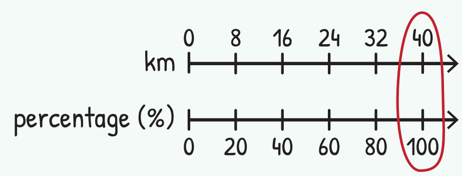


Table

km	%
8	20
40	100

×5 ×5

Double Number Line

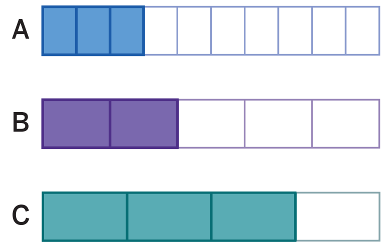


So the biker's goal distance is 40 kilometers.

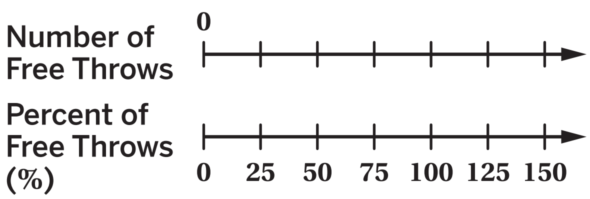
Things to Remember:

1. What percent of each figure is shaded? Record your answers in the table.

Figure A	
Figure B	
Figure C	



2. Martina shot 40 free throws at basketball practice. 25% of her free throws went into the basket. How many of them went into the basket? Use the double number line if it helps with your thinking.



3. On Tuesday, Parv made 12 cookies. On Wednesday, he made 150% as many cookies as he made on Tuesday. How many cookies did Parv make on Wednesday?

Problems 4–5: Leonardo works as a server in a restaurant. He gets tipped 20% of the cost of each order.

4. What tip will he get if the food costs $60?

5. Leonardo got an $18 tip. What was the cost of the food for this order?

6. Nikhil says that to determine 20% of a number, you can divide the number by 5. For example, 20% of 60 is 12 because $60 \div 5 = 12$. Does Nikhil's method always work? Explain your thinking.

Name: ________ Date: ________ Period: ________

Spiral Review

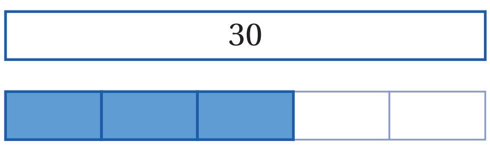
Problems 7–8: Light travels about $180{,}000{,}000$ kilometers in 10 minutes.

7. How many kilometers per minute is that?

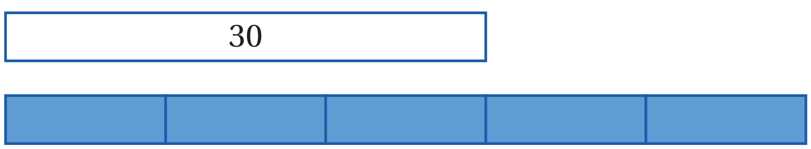
8. How many kilometers per *second* is that?

9. Match each expression with the tape diagram that represents it.

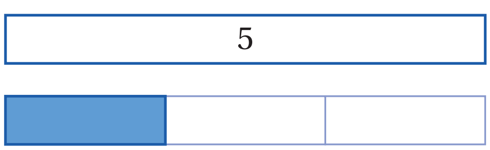
a.



b.



c.



________ $\frac{3}{5} \cdot 30$

________ $\frac{1}{3} \cdot 5$

________ $\frac{5}{3} \cdot 30$

Reflection

1. Put a question mark next to a problem you're feeling unsure of.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice 6.3.11

Lesson Summary

You can use tables, tape diagrams, and double number lines to solve percentage problems.

There are three main types of percentage problems.

- Determine the whole when you're given the part and the percentage.
- Determine the percentage when you're given the part and the whole.
- Determine the part when you're given the percentage and the whole.

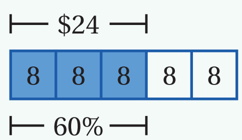
Let's say the sale price of a sweater is $24. The sweater is on sale for 60% of the original price. How much did the sweater cost before the sale?

Here are three representations that you can use to find the whole (the original price of the sweater) given the part and the percentage.

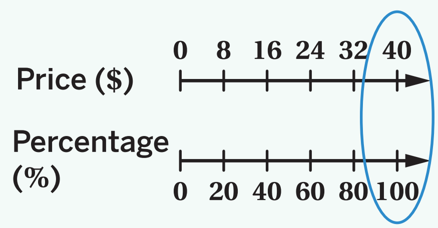
Table

	Price ($)	Percentage (%)	
	24	60	
÷3	8	20	÷3
×5	40	100	×5

Tape Diagram



Double Number Line



Things to Remember:

Lesson Practice
6.3.11

Name: ______ Date: ______ Period: ______

Problems 1–4: Evaluate each percentage problem.

1. 100% of 40

2. 50% of 40

3. 150% of 40

4. 10% of 40

Problems 5–6: A hardware store offers customers a coupon for $\$25$ off.

5. The original price of a power drill is $\$125$. If a customer uses the coupon, what percent will they save?

6. The original price of a ladder is $\$250$. If a customer uses the coupon, what percent will they save?

Problems 7–8: Kiri is curious how many people think aliens exist. She asks 30 students in her class.

7. 12 students in Kiri's class say they think aliens exist. What percent of the class is that?

8. Kiri's older sibling also asks the 25 students in his class. 11 students say they think aliens exist. Which class has a greater percent of students who think aliens exist?

A. Kiri's class **B.** Kiri's sibling's class **C.** Same percent

Explain your thinking.

Lesson Practice
6.3.11

Name: .... Date: .... Period: ....

Spiral Review

Problems 9–10: Afia is 56 inches tall. Note: 100 inches $= 254$ centimeters

9. What is Afia's height in centimeters? Show your thinking.

10. What is her height in meters? Show your thinking.

Reflection

1. Circle one problem, word, or concept that you want to know more about.
2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 79

Lesson Summary

When solving percentage problems related to money, you can:

- Determine the value of 1% and multiply that by the percentage you're looking for.
- Determine how many cents per dollar a given percentage represents.

Let's say a pair of pants costs $\$42$. If the factory makes a *profit* of 14% on the price of a pair of pants, how many dollars of profit does the factory make from each sale?

Strategy 1

	Cost (dollars)	Percentage (%)	
	42	100	
÷100	$\frac{42}{100}$	1	÷100
×14	$\frac{42}{100} \cdot 14$	14	×14

$$\frac{42}{100} \cdot 14 = 5.88$$

The factory makes $\$5.88$ of profit from each sale.

Strategy 2

- 14% profit means 14 cents of each dollar is profit.

  $\frac{14}{100} = 0.14$
- The price of the pants is $\$42$.
- $\frac{14}{100} \cdot 42 = 5.88$

The factory makes $\$5.88$ of profit from each sale.

Things to Remember:

Lesson Practice
6.3.12

Name: ______________________ Date: __________ Period: __________

Problems 1–4: Evaluate each expression.

1. 50% of 70

2. 10% of 70

3. 1% of 70

4. 2% of 70

5. A store is having a 30% off sale. The original price for a pair of headphones is $\$150$. How much would a customer save with this sale?

6. Order the following expressions from *least* to *greatest* value.

55% of 180 $\quad$ 300% of 26 $\quad$ 12% of 700

Least **Greatest**

7. To find 40% of 75, Jamal calculates $\frac{2}{5} \cdot 75$. Does Jamal's calculation give the correct value for 40% of 75? Explain your thinking.

8. Emika has a monthly budget for her cell phone bill. Last month, she spent 120% of her budget, and the bill was $\$60$. What is Emika's monthly budget?

9. Kyrie spent 75 minutes practicing the piano over the weekend. Yasmine practiced the violin for 152% as much time as Kyrie practiced the piano. How long did Yasmine practice?

Name: ________ Date: ________ Period: ________

10. Select *all* the expressions that could be used to calculate 45% of 60.

- ☐ **A.** $\frac{100}{45} \cdot 60$
- ☐ **B.** $\frac{60}{45} \cdot 100$
- ☐ **C.** $\frac{45}{100} \cdot 60$
- ☐ **D.** $\frac{100}{60} \cdot 45$
- ☐ **E.** $\frac{0.45}{100} \cdot 60$
- ☐ **F.** $\frac{60}{100} \cdot 45$

11. Fill in each blank using the digits 0 to 9 only once, so that the expression on the left is greater than the expression on the right.

Left	**Right**
☐☐ **% of 50**	**50% of** ☐☐

Spiral Review

12. Two stores sell identical sandwich rolls in different-sized packages. Store A sells a six-pack for $\$5.28$. Store B sells a four-pack for $\$3.40$. Which store offers the better price per roll?

A. Store A **B.** Store B **C.** They are the same

Show or explain your thinking.

Reflection

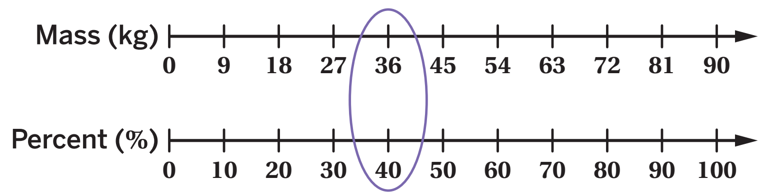
1. Put a star next to a problem you want to understand better.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can use ratios to determine what percent one amount is compared to another amount.

Let's say an adult giant panda weighs 90 kilograms and a giant panda cub weighs 36 kilograms. You can determine the cub's weight as a percent of the adult's weight using several strategies.

Double Number Line



Ratio Tables

Mass (kg)	Percent (%)
90	100
1	$\frac{1}{90} \times 100$
36	$\frac{36}{90} \times 100$

$\times \frac{1}{90}$, $\times 36$

- Determine the unit rate (what percent matches 1 kilogram).
- Use the unit rate to determine what percent 36 kilograms is of 90 kilograms.

Expressions

$$36 \div 90 \cdot 100 = \frac{36}{90} \cdot 100 = 40$$

Evaluate $\frac{36}{90} \cdot 100$ to determine what percent the cub's weight, 36, is of the adult's weight, 90.

Things to Remember:

Lesson Practice
6.3.13

Name: .... Date: .... Period: ....

1. Select *all* the expressions that represent what percent 19 is of 20.

- ☐ **A.** $\frac{19}{20} \cdot 100$
- ☐ **B.** $\frac{19}{20} \div 100$
- ☐ **C.** $\frac{20}{19} \cdot 100$
- ☐ **D.** $19 \cdot \frac{100}{20}$
- ☐ **E.** $\frac{19}{100} \cdot 20$

2. At a hardware store, a tool set normally costs $\$80$. During a sale this week, the tool set costs $\$12$ less than normal. What percent of the original price can a customer save? Show or explain your thinking.

Problems 3–5: A 6th grade class did a weekend fitness challenge. Each student set a goal for 75 minutes of exercise.

3. Luca exercised for 54 minutes. What percent of the goal did Luca complete?

4. Brianna completed 64% of her goal. How many minutes did she exercise for?

5. Jada exercised for 78 minutes. What percent of the goal did Jada complete?

Lesson Practice
6.3.13

Name: .... Date: .... Period: ....

Spiral Review

Problems 6–9: Determine each product.

6. $0.72 \bullet 15$

7. $0.72 \bullet 1.5$

8. $0.72 \bullet 0.15$

9. $72 \bullet 0.15$

Reflection

1. Put a heart next to the problem you feel most confident about.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Tape diagrams and tables can help us make sense of problems involving **percent increase** and **percent decrease**.

The terms *percent increase* and *percent decrease* describe an increase or decrease of a quantity as a percentage of the starting amount.

One method to solve these types of problems is to start with the original amount and then add or subtract the amount that matches the percent of increase or decrease.

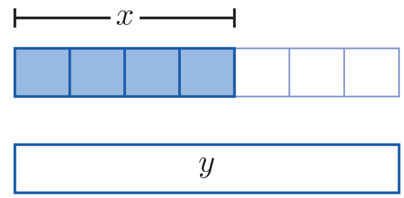
Things to Remember:

Lesson Practice
7.4.02

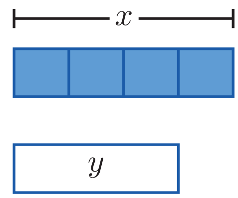
Name: .... Date: .... Period: ....

1. Match each situation with a diagram.

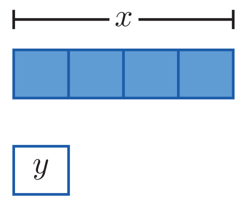
a. Hoang drinks x ounces of juice. Nekeisha drinks $\frac{1}{4}$ less than that. ....



b. Hoang runs x miles. Nekeisha runs $\frac{3}{4}$ more than that. ....



c. Hoang buys x pounds of almonds. Nekeisha buys $\frac{1}{4}$ of that. ....



Problems 2–3: Draw a diagram to represent each situation.

2. The amount of flour that the bakery used this month increased 40% compared to last month.

3. The amount of milk that the bakery used this month decreased 75% compared to last month.

4. At the beginning of the month, there were 80 ounces of peanut butter in the pantry. Since then, a family has eaten 30% of it. Which expression represents the ounces of peanut butter left in the pantry?

A. $0.7 \cdot 80$

B. $0.3 \cdot 80$

C. $8 - 0.30$

D. $(1 + 0.3) \cdot 80$

Lesson Practice
7.4.02

Name: ______ Date: ______ Period: ______

Problems 5–7: Fill in the blanks to describe each increase or decrease as a percentage of the original amount.

5. This year, there was 40% more snow than last year.

The amount of snow this year is ______% of the amount last year.

6. This year, there were 25% fewer sunny days than last year.

The number of sunny days this year is ______% of the number of sunny days last year.

7. A restaurant adds a 93% markup to the price of the ingredients to set the menu price.

The menu price is ______% of the price of ingredients.

Spiral Review

Problems 8–9: A store sells strawberries for $\$1.38$ per pound.

8. Write an equation relating the cost, c, and the pounds of strawberries, p.

9. One strawberry order costs $\$8.97$. How many pounds did the person order?

Problems 10–11: Solve each equation.

10. $x \cdot \frac{7}{3} = 1$

11. $1 \div \frac{11}{2} = x$

Reflection

1. Put a star next to a problem you're still wondering about.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice 7.4.03

Science Mom Lesson 82

Lesson Summary

We can use equations to help us make sense of situations involving *percent increase* or *percent decrease*.

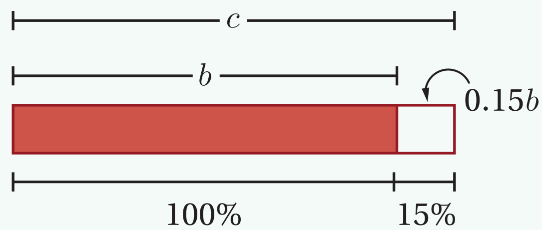
For example, c is 15% more than b.

Three equations can be written to model the relationship between b and c:

$c = b + 0.15b$

$c = (1 + 0.15)b$

$c = 1.15b$



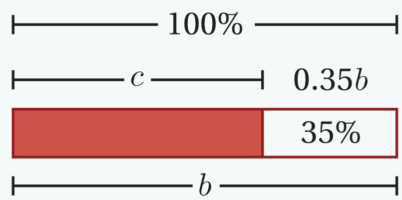
In this example, c is 35% less than b.

Three equations can be written to model the relationship between b and c:

$c = b - 0.35b$

$c = (1 - 0.35)b$

$c = 0.65b$



Things to Remember:

Lesson Practice
7.4.03

Name: .... Date: .... Period: ....

1. Draw a diagram that represents this situation:

   The number of people in a town has increased by 50% in the past decade.

2. Write a story that matches the diagram.

3. Draw a diagram that represents this situation:

   The amount of paper that the copy shop used this month decreased by 20% compared to what they used last month.

4. A new video game costs $\$60$ on the day it gets released. The price of a new game generally drops by about 12% within a few weeks. What will the price of this new game be a few weeks after release? Show or explain your thinking.

5. Jada and Braylen both wrote an equation to represent an increase of 3%, where x represents the amount before the increase and y represents the new amount.

   Jada wrote the equation $y = 1x + 0.03x$. Braylen wrote the equation $y = 1.03x$.

   Whose equation is correct? Circle one.

   Jada   Braylen   Both   Neither

   Explain your thinking.

Lesson Practice
7.4.03

Name: .... Date: .... Period: ....

Problems 6–7: A sneaker store raised its prices 15% compared to last year.

6. If x is the price before the increase and y is the price after the increase, which equations are correct? Select *all* that apply.

☐ **A.** $y = 1.15x$

☐ **B.** $y = x + 0.15$

☐ **C.** $y = x + 0.15x$

☐ **D.** $y = 15 + x$

☐ **E.** $y = (1 + 0.15)x$

7. A pair of designer sneakers was $\$120$ last year. What is the price this year?

8. Two stores each advertise a discount on the same type of sweatshirt. At both stores, the original price of the sweatshirt is $\$25$.

- Store A discounts the price of the sweatshirt by 20%.
- Store B discounts the price of the sweatshirt by 15%.

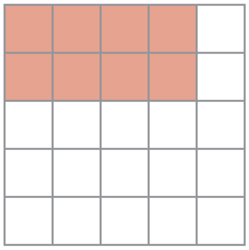
How much less is the discounted price of the sweatshirt at Store A than the discounted price of the sweatshirt at Store B?

A. $\$0.75$ **B.** $\$1.00$ **C.** $\$1.25$ **D.** $\$1.75$

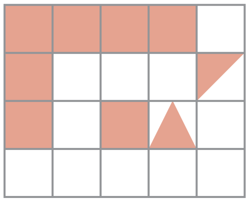
Spiral Review

Problems 9–10: Determine the percentage of each grid that is shaded.

9.



10.



Reflection

1. Put a smiley face next to a problem you were stuck on and then figured out.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice 7.4.04

Lesson Summary

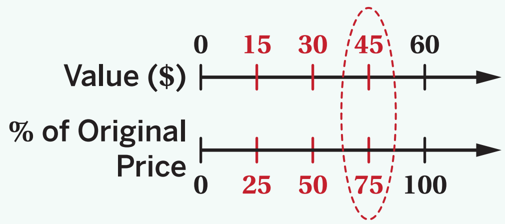
A double number line is a helpful tool for understanding percentage problems.

When using double number lines, it helps to first identify which value aligns with which percentage. It may be helpful to think of the values as a new amount and an original, or old, amount.

Once the known values and percentages are aligned, filling in more values on both number lines can help you solve the problem. For percentage problems, the 0 on each number line should be aligned.

Example:

A furniture store offers 25% off every piece of furniture to make room in the warehouse. If a chair normally sells for $60, what is its sale price?

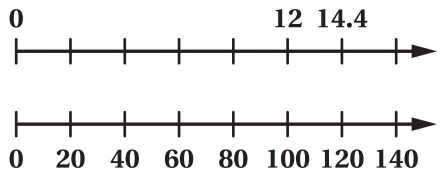


Things to Remember:

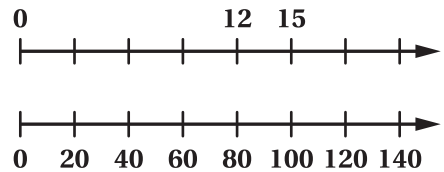
1. Abdullah and Kanna are working on this problem:

A juice box has 20% more juice in its new packaging. The original packaging held 12 fluid ounces. How many fluid ounces of juice does the new packaging hold?

Abdullah's Double Number Line



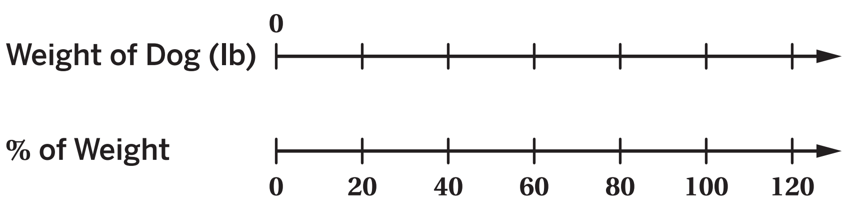
Kanna's Double Number Line



Whose double number line represents the problem? Explain your thinking.

Problems 2–3: Read each scenario and then answer the question. Complete the double number line if it helps with your thinking.

2. A dog weighs 20% more than it did three months ago. It weighs 36 pounds now. How much did the dog weigh three months ago?



3. A bakery used 25% less butter this month than last month. The bakery used 240 kilograms of butter this month. How much did it use last month?

Lesson Practice
7.4.04

Name: ____________ Date: ____________ Period: ____________

Problems 4–5: Next week, the price of oranges at the farmer's market will decrease by 20%.

4. Select *all* the equations that represent the relationship between the price of oranges this week, x, and the price of oranges next week, y.

- ☐ **A.** $y = \frac{1}{5}x$
- ☐ **B.** $y = \frac{4}{5}x$
- ☐ **C.** $y = x - \frac{1}{5}x$
- ☐ **D.** $y = 0.2x$
- ☐ **E.** $y = 0.8x$

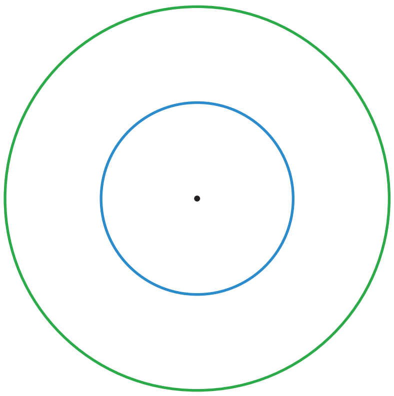
5. The price of oranges this week is $\$4.50$. What will the price be next week?

6. The number of fish in a lake decreased by 15% between last year and this year. This year, there are 51 fish in the lake. What was the population last year?

7. Without determining the actual percent increase, explain why it is not reasonable to say that a price increase from $\$4.00$ to $\$5.00$ for a bag of clementines represents a 125% increase.

Spiral Review

Problems 8–9: Here is a circle and a scaled copy of the circle with a scale factor of 2.



8. How does the circumference of the scaled copy compare to the circumference of the original circle?

9. How does the area of the scaled copy compare to the area of the original circle?

Reflection

1. Put a heart next to the problem you're most proud of.
2. Use this space to ask a question or share something you're proud of.

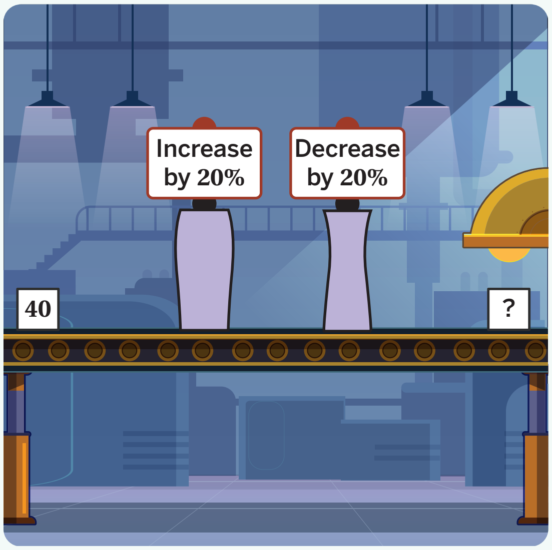
Lesson Summary

Percent machines take an input value and increase or decrease that value by a percentage to produce an output.

Increasing and then decreasing by the same percentage will not produce the original input value.

For example:

- An input value of 40 is increased by 20%.
  $40 \cdot 1.2 = 48$ or $40 \cdot 0.2 + 40 = 48$
- The new value is decreased by 20%.
  $48 \cdot 0.8 = 38.4$ or $48 - (48 \cdot 0.2) = 38.4$



It may surprise you to discover the final result is *not* the original input value of 40! This is because the input value represents the whole when you determine 20% of 40. But in the second calculation, the input value changes. You are now determining 20% of 48 and then subtracting that value from 48.

Things to Remember:

Lesson Practice
7.4.05

Name: .... Date: .... Period: ....

1. Match each situation with a diagram that represents it.

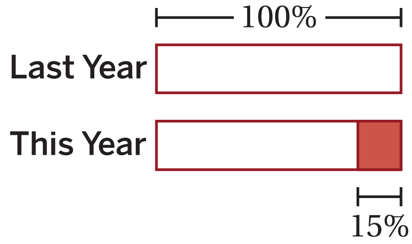
a. The amount of apples this year decreased by 15% compared to last year's amount.

b. The amount of cherries this year increased by 15% compared to last year's amount.

c. The amount of pears this year is 85% of last year's amount.

d. The amount of oranges this year is 115% of last year's amount.

....



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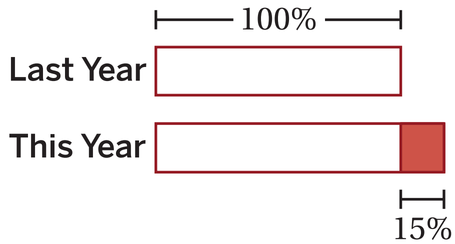


2. Mar's aunt bought a trading card many years ago. The value of the card increased by 80%. Its value is now \$270. What was the value of the card when Mar's aunt bought it?

Problems 3–5: A small town had a population of 4,000 in 1990.

3. By 2000, the population increased to 5,000. What was the percent change from 1990 to 2000? Show or explain your thinking.

4. In 2010, the population decreased back to 4,000. What was the percent change from 2000 to 2010? Show or explain your thinking.

5. Explain why the percent change in Problem 3 is not equal to the percent change in Problem 4.

Name: .... Date: .... Period: ....

6. A restaurant pays all of its servers by the hour. Tyler earned $\$136$ for 8 hours of work last week and got a 5% raise this week. What is Tyler's new hourly wage?

Spiral Review

Problems 7–8: Write an equation to represent each situation.

7. A worker is paid the minimum wage in their state, which is $\$10.00$ per hour. What is their total earnings, t, for h hours worked?

8. What is the value of y if it represents 40% of a given value x?

9. Eliza walked 12 miles. Then she walked $\frac{1}{4}$ of that distance. How many miles did she walk altogether? Select *all* that apply.

- ☐ **A.** $12 + \frac{1}{4}$
- ☐ **B.** $12 \bullet \frac{1}{4}$
- ☐ **C.** $12 + 12 \bullet \frac{1}{4}$
- ☐ **D.** $12\left(1 + \frac{1}{4}\right)$
- ☐ **E.** $12 \bullet \frac{3}{4}$
- ☐ **F.** $12 \bullet \frac{5}{4}$

Reflection

1. Circle the problem you're most interested in knowing more about.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

The listed price and the total a customer ends up paying are often two different quantities. Tax, tip, and discounts are some of the reasons why the listed price and final price are different.

These changes are often calculated as percentages of the listed price.

For example, a tip is an amount of money that a person gives someone who provides a service, such as restaurant servers, hairdressers, and delivery drivers. If a person plans to leave a 20% tip, then the total cost with tip will be 120% of the bill.

Things to Remember:

Lesson Practice
7.4.07

Name: .... Date: .... Period: ....

1. Wessam orders a meal that costs $\$15$. There is no tax on the meal, and Wessam decides to tip 20%. Select the expression that represents the total cost, in dollars, including the tip.

A. $15 + 0.20$

B. $15 + 1.20$

C. $15 \cdot 0.20$

D. $15 \cdot 1.20$

2. In a city in Ohio, the sales tax rate is 7.25%. Complete the table.

Item	Price Before Tax ($)	Sales Tax ($)	Price Including Tax ($)
Pillow	8.00		
Blanket	24.00		
Trash can		1.16	

Problems 3–4: A family eats at a restaurant. The bill is $\$42$. There is no tax on the meal, and the family leaves a tip and spends $\$49.77$ total.

3. How much money does the family tip?

4. How much is the tip as a percent of the bill?

Problems 5–7: A music store buys instruments and then sells them for 30% more than they paid.

5. If the store buys a guitar for $\$45$, what will the store sell it for?

6. If the price tag on a trumpet says $\$104$, how much did the store pay for it?

7. During a 20% off sale, the store offers a clarinet for $\$93.60$. How much did the store pay for the clarinet?

Name: .... Date: .... Period: ....

8. A clothing store is having a sale: 25% off shirts and 30% off jeans.

- Crow bought a shirt with an original price of \$24.
- Crow bought a pair of jeans with an original price of \$32.

Determine how much money Crow saved.

Spiral Review

9. In 1969, the annual global mean temperature was 14°C. In 2019, the annual global mean temperature was 14.8°C. Two different news websites reported this change.

Website A	Website B
"In the last 50 years, the annual global mean temperature has increased by almost 6%."	"There has been an increase in the annual global mean temperature of approximately 5% during the last half century."

Which website more accurately reported the change? Explain your thinking.

Reflection

1. Put a question mark next to a problem you're feeling unsure of.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

There are two different federal minimum wages — one for workers who receive tips and one for workers who don't.

For tipped workers, such as restaurant servers, pay depends not only on their hourly wage and the number of hours they work, but also the number of tables served, the average bill at those tables, and the average percent tip.

Let's say a restaurant server earns $\$2.13$ per hour and works 30 hours per week. They serve about 40 tables in a week, where the typical bill is $\$75$ and the tip is 18%. Here's an equation that shows how much this server earns in a typical week:

$$2.13 \cdot 30 + 40 \cdot 75 \cdot 0.18 = \$603.90$$

We can compare this amount to other ways of paying servers, such as a simple rate of $\$15$ per hour ($15 \cdot 30 = \$450$ in a week), with no tips. Switching to this way would mean about a 25% decrease in pay for the restaurant server in our example because $\frac{603.90 - 450}{603.90} \approx 0.2548$.

Things to Remember:

Lesson Practice
7.4.08

Name: ________ Date: ________ Period: ________

Problems 1–2: A customer leaves a 15% tip on a $\$20$ meal.

1. Select the expression that represents the value of the tip.

- **A.** $15 \cdot 20$
- **B.** $20 + 1.5 \cdot 20$
- **C.** $1.15 \cdot 20$
- **D.** $\frac{15}{100} \cdot 20$

2. Select the expression that represents the *total* bill.

- **A.** $15 \cdot 20$
- **B.** $20 + 1.5 \cdot 20$
- **C.** $1.15 \cdot 20$
- **D.** $\frac{15}{100} \cdot 20$

Problems 3–4: Mauricio is a server at a restaurant. In an average 8-hour work day, he serves 10 tables, with an average bill of $\$45$ per table. He typically receives a 20% tip on each bill and earns $\$7.25$ per hour.

3. How much money does Mauricio earn in a typical day?

4. Let's say the typical tip increased to 23% of the bill. By what percent would Mauricio's earnings increase? Show or explain your thinking.

Problems 5–7: Here is some information about three parks. Complete each sentence.

Park	Area (acres)
Central Park (New York City)	843
Golden Gate Park (San Francisco)	1,017
Longview Lake Park (Kansas City)	2,381

5. Golden Gate Park is about ________ % larger than Central Park.

6. Longview Lake Park is about ________ % larger than Golden Gate Park.

7. If the size of Central Park increased by 200%, would it be larger than Longview Lake Park? Explain your thinking.

Lesson Practice
7.4.08

Name: ________ Date: ________ Period: ________

Spiral Review

8. Match each situation with an equation.

a. Tay sleeps for x hours. Omar sleeps for 20% less than that. .... $y = 0.6x$

b. Tay practices piano for x hours. Omar practices for 40% less than that. .... $y = 0.75x$

c. Tay drinks x ounces of juice. Omar drinks 130% more than that. .... $y = 0.8x$

d. Tay spends x dollars. Omar spends 25% less than that. .... $y = 2.3x$

e. Tay completes x puzzles. Omar completes 60% more than that. .... $y = 1.4x$

f. Tay eats x grams of almonds. Omar eats 40% more than that. .... $y = 1.6x$

9. The radius of circle S is half the radius of circle L. The radius of circle L is 10 millimeters. Which measurement is closest to the area of circle S?

A. 78.54 square millimeters

B. 31.42 square millimeters

C. 314.16 square millimeters

D. 15.71 square millimeters

Problems 10–12: Solve each equation.

10. $\frac{2}{3}x = \frac{8}{15}$

11. $1.8 + x = 7.2$

12. $5\frac{4}{5} = 3\frac{2}{3} + x$

Reflection

1. Put a star next to a problem where you revised your thinking.

2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 87

Lesson Summary

Percent error describes the difference between a desired value and the actual value, expressed as a percent of the desired value.

To determine the percent error, the amount of the error is compared to the desired value. You can use this formula:

$$\text{Percent error} = \frac{\text{(difference between actual value and the desired value)}}{\text{desired value}} \cdot 100$$

For example, a milk carton is supposed to contain 16 fluid ounces, but it only contains 15 fluid ounces.

- The error is 1 fluid ounce.
- The percent error is 6.25% because $\frac{1}{16} \cdot 100 = 6.25$.

In some situations, there is no clear "desired" value. In those cases, the denominator is the value that has no error. Here are some examples:

- For a thermometer that reads $73°$, if the real temperature is $70°$, the percent error for that thermometer's reading is $\frac{3}{70}$, or 4.3%.
- For an estimate of 800 jelly beans in a jar, if the jar actually has 947 jelly beans, the percent error of the estimate is $\frac{147}{947}$, or 15.5%.

Things to Remember:

Name: .... Date: .... Period: ....

1. A baker needs 500 grams of flour for a batter, but accidentally only puts 300 grams of flour into the mixing bowl. What is the percent error?

2. Any can of seltzer should have 12 fluid ounces of liquid to be appropriately carbonated. One can on the assembly line contains 13.5 fluid ounces. What is the percent error?

3. A radar gun measured the speed of a baseball at 93 miles per hour. If the baseball was actually going 90 miles per hour, what was the percent error in this measurement?

Problems 4–5: In Burlington, all city departments get an annual budget. They are allowed to spend 1.5% over their budget. If they spend more than that, the mayor conducts a review.

4. The Department of Health has a budget of $\$90,000$. What is the largest acceptable amount for their spending? Show or explain your thinking.

5. The Department of Parks and Recreation has a budget of $\$30,000$, and they spent $\$31,000$. Will the mayor conduct a review of their spending? Show or explain your thinking.

Name: ________ Date: ________ Period: ________

6. Hoang is buying 5 concert tickets that cost $\$95.00$ each. The concert venue gives Hoang a coupon with a 10% discount. A sales tax of 8% is applied after the discount. How much did Hoang pay after the discount and sales tax?

A. $\$92.34$ **B.** $\$427.50$ **C.** $\$461.70$ **D.** $\$513.00$

7. Amal bikes x kilometers. Kimora bikes $\frac{3}{10}$ less than that. Using y for Kimora's distance, write an equation that describes the relationship between the two quantities.

Spiral Review

Problems 8–11: Complete each blank using the symbols >, <, or =.

8. $-\frac{3}{4}$ ________ $\frac{4}{3}$

9. 3.24 ________ $-(-3.24)$

10. $-\frac{2}{3}$ ________ $-\frac{5}{6}$

11. $-|4|$ ________ $-|-4|$

Reflection

1. Put a question mark next to a response you'd like to compare with a classmate's.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Situations involving percent increase and decrease are everywhere in our society.

For example, news articles often contain facts and statistics about pollution, such as:

In 2019, the U.S. generated 72.8 million tons of plastic waste.
This was 55% more waste than in 2000.

Information like this can be used to generate interesting questions. When writing these questions, it's important to be precise with language. We could ask: *How much waste was there in 2000?* But a more precise question might be: *How many tons of plastic waste did the U.S. generate in 2000?*

We can use strategies from this unit, such as equations, double number lines, tables, and tape diagrams, to answer these kinds of questions.

Things to Remember:

Lesson Practice
7.4.10

Name: ______ Date: ______ Period: ______

Problems 1–2: A city has a 5% sales tax.

1. A toothbrush costs $3.40 before tax. How much does it cost including tax?

2. A book costs $32.55 after tax. How much did it cost before tax?

Problems 3–5: Oliver went to the store and purchased these items.

Milk (1 gal)	$3.61
Beef (1 lb)	$7.30
Apples (1 lb)	$2.39
Bananas (1 lb)	$0.99
Oranges (1 lb)	$1.96
Potatoes (1 lb)	$1.75
Total	$18.00

3. Oliver set a monthly grocery budget for himself. This grocery list costs 6% of the budget. What is his monthly budget?

4. Complete each sentence:

Milk made up about ______% of Oliver's spending on this trip.

This was about ______% of his grocery budget for the month.

5. Use the given information to write another sentence about Oliver's spending. Include a percentage.

Name: .... Date: .... Period: ....

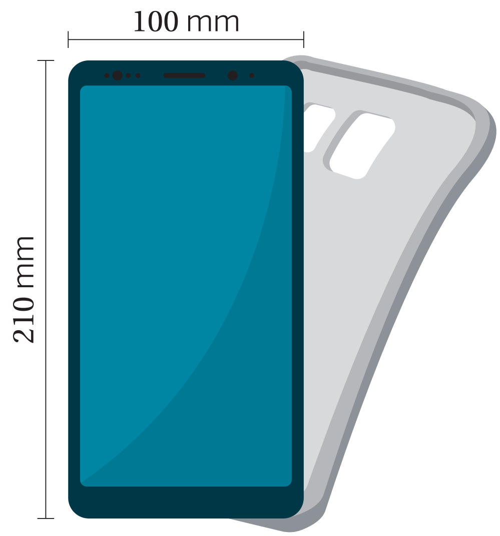
6. A greeting card costs $6 before tax. A customer has a coupon for a 10% discount. Then a 5% sales tax is added. How much will the customer pay for the greeting card?

7. The price of gold is often reported per ounce. At the end of 2005, gold was $513 per ounce. At the end of 2015, it was $1,060 per ounce. By what percent did the price increase?

8. A grocery store allows customers to use multiple coupons when checking out. Suppose you have a $5 off coupon and a 10% off coupon. The register will calculate the new price after each coupon is used. Does the order you use the coupons make a difference? Explain your thinking.

Spiral Review

9. Here are the dimensions of a phone case designed by the Soft Shield company. The size of a case can be up to 1% off and still fit a phone. Determine the largest and smallest width and height of the case that will fit a phone.



10. What is the decimal equivalent of $\frac{13}{100}$?

Reflection

1. Put a star next to a problem that looked more difficult than it really was.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

When a problem involves a proportional relationship, determining the constant of proportionality or scale factor can be helpful.

You can see this relationship between the columns of this table. The heights are multiplied by $2\frac{1}{2}$ to get the widths.

Height (in.)	Width (in.)
2	$\xrightarrow{\cdot 2\frac{1}{2}}$ 5
$3\frac{1}{2}$	$\xrightarrow{\cdot 2\frac{1}{2}}$ $8\frac{3}{4}$

When you multiply one quantity in a proportional relationship by a value, the other quantity will change by the same factor whether or not the values are whole numbers.

You can see this relationship between the rows of the table. When the height is multiplied by $1\frac{3}{4}$, the width is multiplied by the same number.

Height (in.)	Width (in.)
2	5
$\cdot 1\frac{3}{4}$ $3\frac{1}{2}$	$8\frac{3}{4}$ $\cdot 1\frac{3}{4}$

Things to Remember:

Lesson Practice
7.4.11

Name: .... Date: .... Period: ....

Problems 1–3: A snail is moving away from a rock at a constant rate. This table shows the distance the snail is from the rock at certain times.

Distance (in.)	Time (min)
0	0
1	$1\frac{1}{3}$

1. How many minutes does it take for the snail to reach a distance of 9 inches from the rock?

2. How far will the snail be from the rock after 9 minutes?

3. Select *all* the equations that represent the relationship between the distance in inches, d, and time in minutes, t.

- ☐ **A.** $d = \frac{4}{3}t$
- ☐ **B.** $d = \frac{3}{4}t$
- ☐ **C.** $t = \frac{4}{3}d$
- ☐ **D.** $t = \frac{3}{4}d$

Problems 4–5: At a deli counter, a customer buys:

- $1\frac{3}{4}$ pounds of ham for $\$14.50$.
- $2\frac{1}{2}$ pounds of turkey for $\$26.25$.
- $\frac{3}{8}$ pounds of roast beef for $\$5.50$.

4. Which deli meat is the least expensive per pound? Show or explain your thinking.

5. Which deli meat is the most expensive per pound?

Name: ________ Date: ________ Period: ________

6. To make a shade of paint called Jasper Green, mix 4 quarts of green paint with $\frac{2}{3}$ of a cup of black paint.

How much green paint should be mixed with 4 cups of black paint to make Jasper Green?

Spiral Review

Problems 7–8: This table shows a relationship between x and y.

x	y
1	8
2	16
3	24

7. What is a constant of proportionality in the relationship?

8. Write an equation to represent the relationship.

9. Angel checks out 12 library books, and Shikoba checks out $\frac{1}{3}$ less than that. How many books does Shikoba check out?

10. Which is greater? Circle one.

40% of 12 12% of 40 They are the same

Show or explain your thinking.

11. 12 is 40% of what number? Show or explain your thinking.

Reflection

1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Proportional relationships may involve fractional amounts. You can solve problems involving fractions by using the same strategies you use to solve problems with whole numbers.

- To determine the constant of proportionality within a recipe, divide the amount of an ingredient by the total number of servings.
- Constants of proportionality can help to compare proportional relationships involving fractional quantities.

Here is a recipe for banana bread. To find the amount of sugar per serving, divide $\frac{3}{4}$ cups of sugar by 6 servings. This gives you $\frac{3}{4} \div 6$, or $\frac{1}{8}$ cups of sugar per serving.

Banana Bread Recipe

Number of servings: 6

- 2 lb of bananas
- $\frac{1}{2}$ cups of butter
- $\frac{3}{4}$ cups of sugar
- $2\frac{1}{2}$ cups of flour
- 1 tsp of baking soda

Things to Remember:

Lesson Practice
7.4.12

Name: ____________ Date: ________ Period: ________

1. A recipe calls for $\frac{1}{2}$ cups of sugar and 1 cup of flour. Complete the table to show how much sugar and flour is needed for different batches of the recipe.

Sugar (cups)	Flour (cups)
$\frac{1}{2}$	1
$\frac{3}{4}$	
	$1\frac{3}{4}$
1	
	$2\frac{1}{2}$

Problems 2–4: A punch recipe calls for $1\frac{1}{2}$ quarts of sparkling water and $\frac{3}{4}$ quarts of grape juice.

2. How much sparkling water would you need to mix with 9 quarts of grape juice?

3. How much grape juice would you need to mix with $3\frac{3}{4}$ quarts of sparkling water?

4. How much of each ingredient would you need to make 75 quarts of punch?

Sparkling water: ____________

Grape juice: ____________

Problems 5–6: To make a specific color of green paint, a painter mixes $\frac{1}{2}$ of a gallon of blue paint with $\frac{4}{5}$ of a gallon of yellow paint.

5. How many gallons of yellow paint are needed to mix with 3 gallons of blue paint?

6. How many gallons of each color are needed to make 26 total gallons of this green paint?

Blue paint: ____________

Yellow paint: ____________

Lesson Practice
7.4.12

Name: .... Date: .... Period: ....

Spiral Review

7. Complete the table to represent $y = \frac{2}{3}x$.

x	y
12	
	16

8. Select *all* the ratios that are equivalent to $4 : 5$.

- ☐ **A.** $2 : 2.5$
- ☐ **B.** $3 : 4$
- ☐ **C.** $3 : 3.75$
- ☐ **D.** $8 : 10$
- ☐ **E.** $14 : 27.5$

9. On a map, DeAndre's house is 2.5 inches from his grandparent's house. The map has a scale of 1 inch to 20 miles. How far apart, in inches, would DeAndre's house be from his grandparent's house on a map that has a scale of 1 inch to 80 miles?

A. 0.5 **B.** 0.625 **C.** 1.60 **D.** 1.75

10. What is 40% of 160?

11. What is 160% of 40?

Reflection

1. Put a heart next to the problem you found most interesting.

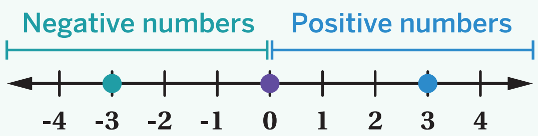
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Positive numbers are numbers that are greater than 0. **Negative numbers** are numbers that are less than 0. Zero is neither positive nor negative.

You can extend a number line to the right of 0 to show positive numbers, and you can extend a number line to the left of 0 to show negative numbers.

For example:



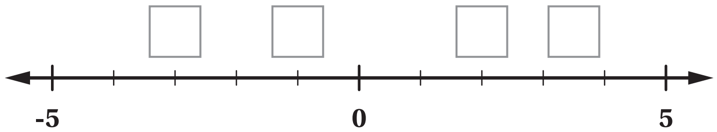
The number 3 is 3 units to the right of 0 on the number line.

The number -3 is 3 units to the left of 0 on the number line.

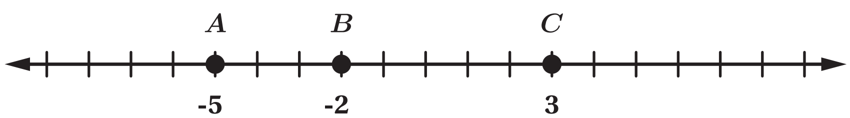
Things to Remember:

Name: ______________ Date: ________ Period: ________

1. Fill in the blanks on the number line.



Problems 2–6: Here is a number line.



2. Describe where you would plot -100 on the number line.

3. Point D is 1 unit to the left of point A. Plot point D.

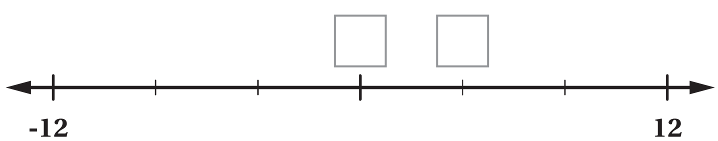
4. Point E is at 0. Plot point E.

5. List both locations that are 4 units away from point B.

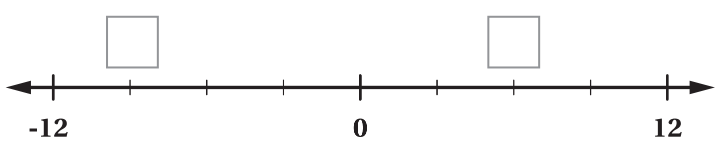
6. Point F is the same distance from point A and point C. Plot point F.

Problems 7–9: Fill in the blanks on the number lines.

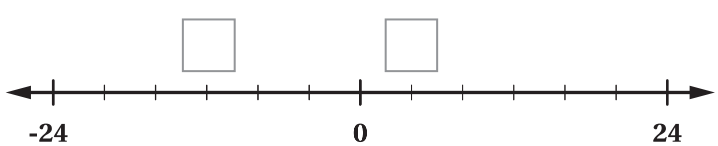
7.



8.



9.



10. Which pair of numbers are on opposite sides of zero on the number line?

A. 0 and 5

B. 2 and -5

C. -2 and -5

D. 2 and 5

Lesson Practice
6.7.01

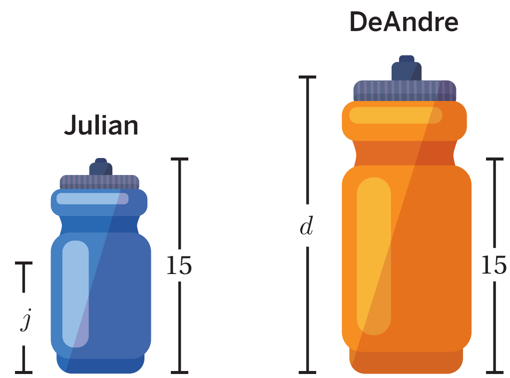
Name: .... Date: .... Period: ....

Spiral Review

11. A rectangle has an area of 24 square centimeters. If one side is $2\frac{2}{5}$ centimeters long, how long is the other side?

Explain your thinking.

Problems 12–14: Two friends at soccer practice are drinking from their water bottles. Julian drinks $\frac{3}{5}$ of his 15-ounce bottle. DeAndre drinks 15 ounces of water, which is $\frac{3}{5}$ of his bottle.



12. How are these two situations alike? How are they different?

13. Write an equation to represent the amount of water Julian drinks, j.

14. Write an equation to represent the total amount of water in DeAndre's bottle, d.

Reflection

1. Put a smiley face next to a problem you were stuck on and then figured out.

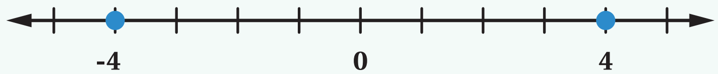
2. Use this space to ask a question or share something you're proud of.

Lesson Practice 6.7.02

Science Mom Lesson 92

Lesson Summary

Two numbers are **opposites** if they are the same distance from 0 on different sides of the number line. For example, -4 and 4 are opposites because they are both 4 units away from 0.



Every number has an opposite, including fractions and decimals. 0 is its own opposite. The opposite of the opposite of a number is the number itself. For example, $-(-2) = 2$.

All positive and negative whole numbers and 0 are a group of numbers called *integers*. All positive and negative numbers that can be written as fractions, including whole numbers, are called *rational numbers*.

2 and -2 are both integers and rational numbers.

8.3, -8.3, $\frac{3}{2}$, and $-\frac{3}{2}$ are rational numbers, but *are not* integers.

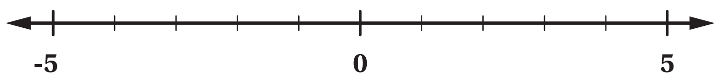
Things to Remember:

Lesson Practice
6.7.02

Name: .... Date: .... Period: ....

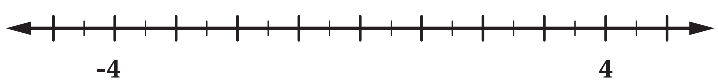
1. Plot each number in its approximate location on the number line.

-3	$\frac{3}{2}$	$-\frac{4}{3}$	$\frac{9}{4}$



2. Plot each number in its approximate location on the number line.

-2	$\frac{5}{4}$	-3.4	0



Problems 3–6: Complete each statement below.

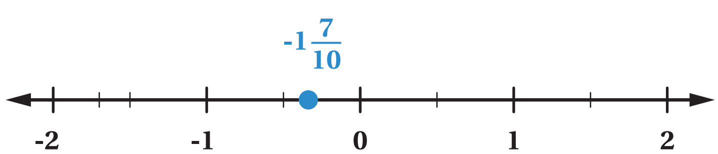
3. The opposite of -2 is ....

4. The opposite of $\frac{5}{4}$ is ....

5. The opposite of -3.4 is ....

6. The opposite of 0 is ....

Problems 7–8: Rebecca incorrectly plotted the point $-1\frac{7}{10}$ on the number line.

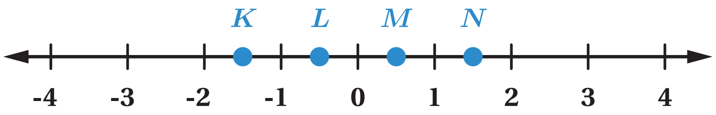


7. What question could you ask to help Rebecca understand her mistake?

8. Plot $-1\frac{7}{10}$ in the correct location on the number line.

Name: ________________ Date: ________ Period: ________

9. Points K, L, M, and N are plotted on the number line.

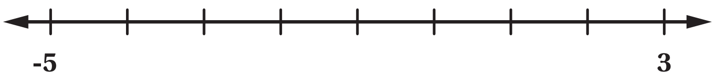


What point represents the location of -0.5?

A. Point K **B.** Point L **C.** Point M **D.** Point N

Problems 10–12: Plot and label where zero is located on each number line.

10.



11.

12.

Spiral Review

Problems 13–16: Solve each equation.

13. $3x = 6$

14. $\frac{1}{3}x = 6$

15. $\frac{1}{3} = 6x$

16. $\frac{1}{3} = \frac{1}{6}x$

Reflection

1. Circle a problem you want to talk to a classmate about.

2. Use this space to ask a question or to share something you're proud of.

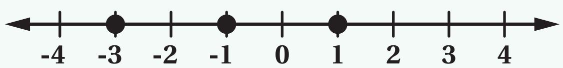
Lesson Summary

You can use a number line to compare numbers with different **signs** (like -4 and 3) or numbers with the same sign (like -4 and -3).

The order of numbers from least to greatest is the same order as they appear on the number line from left to right. This means that negative numbers farther from 0 are less than negative numbers that are closer to 0.

For example, let's say you want to compare -3 and -1. On a number line, -1 is to the right of -3. This means that -1 is greater than -3, or $-3 < -1$. This also makes sense because -1 is closer to 0 than -3 is.

A number line can also help you order numbers from least to greatest. 1 is greater than -1 and -3 because it is the farthest to the right on the number line.



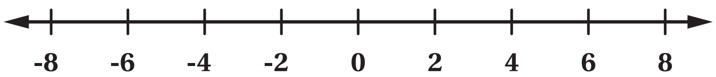
Things to Remember:

Lesson Practice
6.7.04

Name: ______ Date: ______ Period: ______

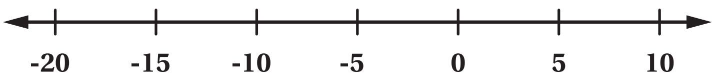
1. Complete each statement with a number that makes it true. Use the number line if it helps with your thinking.

a $\square < 5$ **b** $\square < -5$ **c** $-5 < \square$ **d** $-5 > \square$



2. Complete each statement with the symbol <, >, or =. Use the number line if it helps with your thinking.

a -5 ______ 2 **b** 5 ______ -5 **c** -12 ______ -15 **d** -12.5 ______ -12



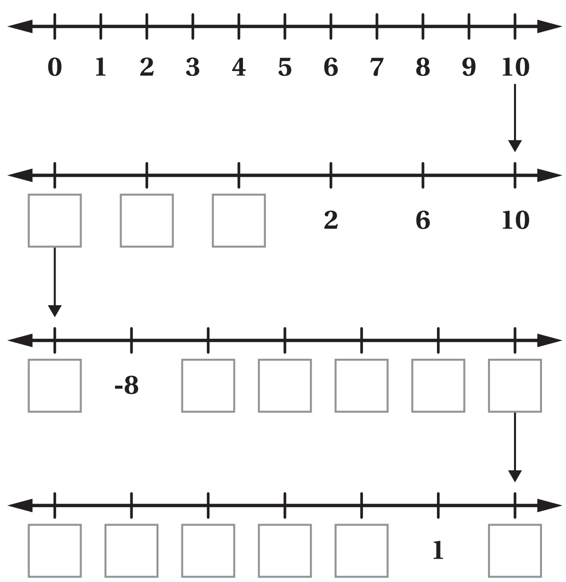
3. Circle whether each statement is *true* or *false*.

Statement A: -8.4 is to the right of -8.7 on the number line.	True	False
Statement B: -2.4 is greater than -2.3.	True	False
Statement C: $-\frac{11}{12} < -\frac{7}{12}$	True	False

Choose one statement and explain your thinking.

4. Here is a number line maze.

Each number line has a different scale but at least one matching number, which is labeled with an arrow. Determine the missing values in this number line maze.



Name: ______ Date: ______ Period: ______

5. Here are five numbers: $-\frac{2}{5}$, -1, $\frac{4}{3}$, 1, $-\frac{3}{2}$.

These numbers are plotted on a horizontal number line. Which statement about the locations of the numbers is true?

A. $-\frac{2}{5}$ is the farthest to the left, and 1 is farthest to the right.

B. -1 is the farthest to the left, and 1 is farthest to the right.

C. $\frac{4}{3}$ is the farthest to the left, and $-\frac{3}{2}$ is farthest to the right.

D. $-\frac{3}{2}$ is the farthest to the left, and $\frac{4}{3}$ is farthest to the right.

Spiral Review

6. Solve each equation. Write each solution as a fraction and as a decimal.

Equation	Solution (Fraction)	Solution (Decimal)
$2x = 3$		
$5y = 3$		
$0.3z = 0.09$		

Problems 7–10: Each lap around a track is 400 meters. How many meters will someone run in:

7. 2 laps?

8. 5 laps?

9. x laps?

10. Sol ran $7{,}600$ meters. How many laps is that? Explain your thinking.

Reflection

1. Put a star next to one problem you are still wondering about.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can use a vertical number line to represent positive and negative numbers. On a vertical number line, points above 0 are positive and points below 0 are negative.

When talking about elevation, 0 feet represents sea level. This means that a positive elevation is above sea level and a negative elevation is below sea level.

When talking about temperature, 0°C means the temperature is freezing. If the temperature in Mt. Olympus is -10°C, that means it has a temperature of 10°C below 0°C, or below freezing.

If the temperature in Cotopaxi is -3°C, you can write $-10 < -3$, which means that it is colder in Mt. Olympus than it is in Cotopaxi.

Things to Remember:

Lesson Practice
6.7.03

Name: .... Date: .... Period: ....

Problems 1–3: Determine if each statement is true or false.

Statement	True	False
1. An elevation of 35 feet is the same as an elevation of -35 feet.		
2. A city that has an elevation of -17 meters is closer to sea level than a city that has an elevation of -40 meters.		
3. A temperature of -4°F is the same as a temperature of 4°F below zero.		

Problems 4–7: Here is a table that shows some elevations in several cities around the world.

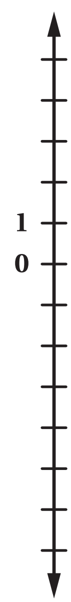
Location	Elevation (ft)
San Juan, Puerto Rico	-4
New Orleans, Louisiana	-7
Amsterdam, Netherlands	-2
Jakarta, Indonesia	3
Taipei, Taiwan	5
Tunis, Tunisia	0

4. A city, not listed in the table, has a higher elevation than San Juan, Puerto Rico.

Select *all* the numbers that could represent the city's elevation in feet.

- ☐ **A.** 0
- ☐ **B.** 4
- ☐ **C.** -2
- ☐ **D.** -10
- ☐ **E.** -7

5. Plot all of the elevations in the table on the number line.



6. Mio says: *I know that 2 is less than 4, so -2 must be less than -4. This means Amsterdam has a lower elevation than San Juan*. Is Mio correct? Explain your thinking.

7. Nasir says: 3 *is less than* -7 *because* 3 *is closer to* 0 *on the number line*. Is Nasir correct? Explain your thinking.

Name: ________ Date: ________ Period: ________

Problems 8–10: Here is a table that shows some of the lowest temperatures recorded in five U.S. locations.

Location	Temperature (°F)
Boca, CA	-45
Coventry, CT	-32
Tallahassee, FL	-2
Mt. Carroll, IL	-38
CCC Fire Camp F-16, GA	-17

8. Which of these locations had the lowest record temperature?

9. Which location had a lower record temperature: Tallahassee, FL, or CCC Fire Camp F-16, GA? Write a statement using < or > to compare the recorded temperatures.

10. Which location had a lower record temperature: Coventry, CT, or Mt. Carroll, IL? Write a statement using < or > to compare the recorded temperatures.

Spiral Review

Problems 11–14: Determine the value of each expression.

11. $2^3 \bullet 4$

12. $\frac{2^3}{4}$

13. $2^4 - 4$

14. $2^3 + 4^3$

Reflection

1. Put a heart next to a problem you feel most confident about.
2. Use this space to ask a question or to share something you're proud of.

Lesson Practice 6.7.05

Science Mom Lesson 95

Lesson Summary

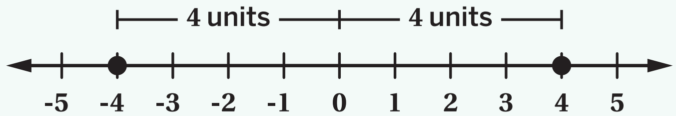
The **absolute value** of a number is a way to describe its distance from 0. For example:

The absolute value of -4 is 4, because it is 4 units away from 0.

$|-4| = 4$

The absolute value of 4 is also 4, because it is 4 units away from 0.

$|4| = 4$



The distance from 0 to itself is 0, so $|0| = 0$.

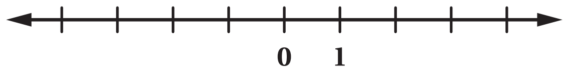
Absolute values are helpful when you are interested in the size of a difference or measurement but its direction is not important.

Things to Remember:

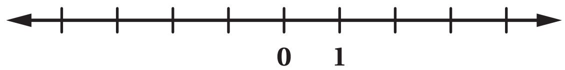
Lesson Practice
6.7.05

Name: .... Date: .... Period: ....

1. Plot and label all numbers that are 4 units away from 0.



2. Plot and label all numbers with an absolute value of $\frac{5}{2}$.



3. Ivory claims that a number and its opposite will always have the same absolute value. Is Ivory correct? Explain your thinking.

Problems 4–11: Complete each number sentence with the symbol $<$, $>$, or $=$.

4. -3.2 .... 1.5

5. $|-3.2|$ .... $|1.5|$

6. 2 .... -1.5

7. $|2|$ .... $|-1.5|$

8. $\frac{3}{2}$ .... -1.5

9. $\left|\frac{3}{2}\right|$ .... $|-1.5|$

10. $|-2.7|$ .... $|-4.5|$

11. $|-2.7|$ .... -4.5

12. Which list of absolute value expressions is ordered from *least* to *greatest* value?

A. $|1|, \left|\frac{9}{4}\right|, |-5|, |-7|$

B. $|-7|, |-5|, |1|, \left|\frac{9}{4}\right|$

C. $|-7|, |-5|, \left|\frac{9}{4}\right|, |1|$

D. $\left|\frac{9}{4}\right|, |1|, |-5|, |-7|$

13. Make each statement true by using each number at most once.

$|\square| = \square$

$|\square| > \square$

$\square < |\square|$

-3 -2 -1

0 1 2 3

Spiral Review

Problems 14–17: Determine the value of each quotient.

14. $24 \div 15$

15. $0.24 \div 0.15$

16. $0.24 \div 0.015$

17. $0.024 \div 0.015$

Reflection

1. Circle the problem you feel most confident about.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

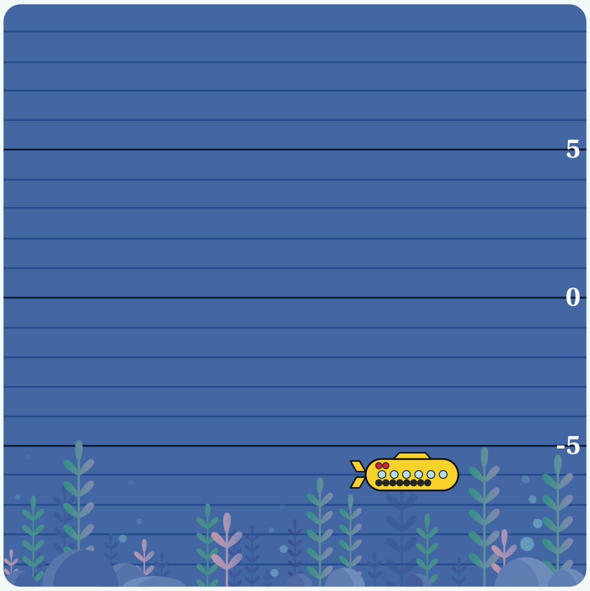
Using models, such as floats and anchors on a vertical number line, can be useful when representing addition and subtraction of positive and negative numbers.

For example, imagine a submarine whose position is at -6 units. The submarine will move from its position as 3 floats are added and 2 anchors are removed.

- 3 floats being added represents moving up 3 units or adding 3.
- 2 anchors being removed represents moving up 2 units because $-(-2) = 2$.

The submarine's new position would be $-6 + 3 + 2 = -1$ units.

To move the submarine to 0 units from -1 units, 1 float can be added. -1 and 1 are an opposite pair, which means they sum to 0.

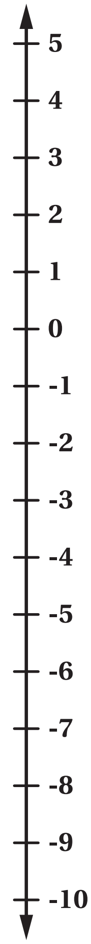


Things to Remember:

Lesson Practice
7.5.01

Name: ______________ Date: ________ Period: ________

Problems 1–5: One moment in December, it was -8°C in Harbin, China, and -2°C in Beijing, China. Use the number line if it helps with your thinking.



1. Which city was colder?

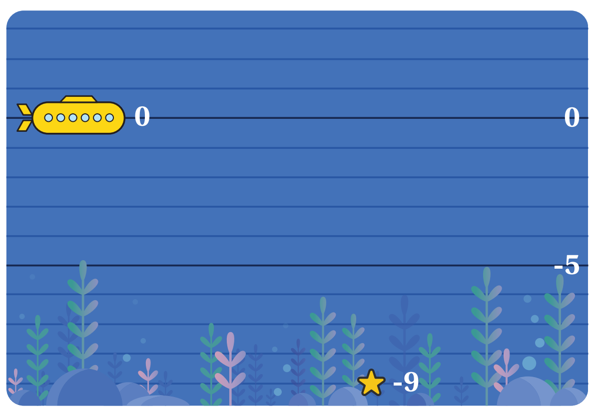
2. At the same moment, it was 7 degrees warmer in Shanghai than it was in Beijing. What was the temperature in Shanghai?

3. How many degrees warmer was it in Shanghai than in Harbin?

4. Later in the day, Beijing got 5 degrees colder. What was Beijing's new temperature?

5. Later in the day, Harbin's temperature was 1°C. By how much did Harbin's temperature change? Explain your thinking.

Problems 6–7: This submarine has 0 floats and 0 anchors. The submarine can hold up to 10 floats and 10 anchors.



6. List *all* the combinations of floats and anchors that could collect the star at -9 units.

7. How do you know there are no other combinations?

Name: .... Date: .... Period: ....

Spiral Review

8. Fill in each blank using the symbols >, <, or =.

3	☐	-3
12	☐	24
-12	☐	-24
7	☐	7.2
-7	☐	-7.2

Problems 9–11: A color of green paint is made by mixing 2 cups of yellow paint with 3.5 cups of blue paint.

9. Complete the table to show amounts of yellow and blue paint that will make the same color of green but in a smaller amount.

Yellow Paint (cups)	Blue Paint (cups)

10. Complete the table to show amounts of yellow and blue paint that will make the same color of green but in a larger amount.

Yellow Paint (cups)	Blue Paint (cups)

11. Will a mixture that is 3 cups of yellow and 4.5 cups of blue be more blue, more yellow, or the same color of green as the original mixture? Explain your thinking.

Reflection

1. Put a question mark next to a response you'd like to compare with a classmate's.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Different combinations of floats and anchors can give you the same result. Here are some examples:

- If a submarine starts at 4 units, adding 2 floats or removing 2 anchors will both result in the submarine moving up to 6 units. So adding a positive number is the same as subtracting a negative number.

Adding Floats	Removing Anchors
$4 + 2 = 6$	$4 - (-2) = 6$

- If a submarine starts at 5 units, removing 1 float or adding 1 anchor will both result in the submarine moving down to 4 units. So subtracting a number is the same as adding its opposite.

Removing Floats	Adding Anchors
$5 - 1 = 4$	$5 + (-1) = 4$

When you add two values that are opposites, the sum is always 0. These numbers are also called *additive inverses* of each other.

Things to Remember:

Lesson Practice
7.5.02

Name: .... Date: .... Period: ....

Problems 1–4: Determine the value of each expression.

1. $5 + (-3)$

2. $-5 + 3$

3. $-5 - 3$

4. $-5 - (-3)$

5. The temperature was 13°F and then dropped 5 degrees. What was the final temperature?

6. The temperature was -13°F and then dropped 5 degrees. What was the final temperature?

7. The temperature was -13°F and then rose to 5°F. What was the *change* in temperature? Explain your thinking.

8. A swimmer was 8 feet underwater. Then he swam 3 feet deeper. Hikari wrote the expression $-8 - 3$. Charlie wrote the expression $-8 + (-3)$. Explain why both Hikari and Charlie are correct.

Lesson Practice
7.5.02

Name: ________ Date: ________ Period: ________

Problems 9–10: The table shows eight expressions.

9. Determine the value of each expression.

Expression	Value
$1 + 2 - 3$	
$1 + 2 - 3 + 4$	
$1 + 2 - 3 + 4 - 5$	
$1 + 2 - 3 + 4 - 5 + 6$	
$1 + 2 - 3 + 4 - 5 + 6 - 7$	
$1 + 2 - 3 + 4 - 5 + 6 - 7 + 8$	
$1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 - 9$	
$1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 - 9 + 10$	

10. What is the value of the next expression? The 10th expression? The 20th expression?

Spiral Review

Problems 11–13: Complete each statement with a value that makes the statement true.

11. ________ < 13

12. ________ < -0.1

13. ________ > -2

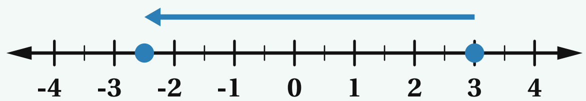
Reflection

1. Put a star next to a problem you're still wondering about.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

When you add positive and negative decimals and fractions, it might help to use a number line and think of each equation as representing start $+$ change $=$ end.

For example, in the equation $3 + (-5.5) = x$, 3 represents the starting location, -5.5 represents the change (moving 5.5 units to the left), and x represents the end location.



Things to Remember:

Lesson Practice
7.5.03

Name: .... Date: .... Period: ....

Problems 1–3: Determine the value of the variable that makes each equation true.

1. $40 + a = 30$

2. $-3.5 + c = 4.5$

3. $d + 2.4 = 0.9$

Problems 4–5: Evaluate each expression.

4. $2 - 3$

5. $-2 - 3$

6. Select the equation that is represented by this number line.

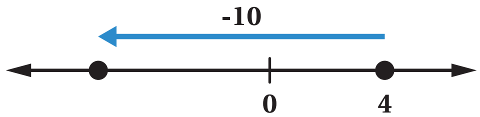
A. $-1 + x = 4$

B. $4 + x = -1$

C. $-1 + 4 = x$

D. $4 + (-1) = x$

7. Select the equation that is represented by this number line.



A. $4 + x = -10$

B. $-10 + x = 4$

C. $4 + (-10) = x$

D. $x + (-10) = 4$

Lesson Practice
7.5.03

Name: .... Date: .... Period: ....

Spiral Review

8. Last week, it rained g inches. This week, the amount of rain decreased by 5%. Which expressions represent the amount of rain that fell this week? Select *all* that apply.

- ☐ **A.** $g - 0.05$
- ☐ **B.** $g - 0.05g$
- ☐ **C.** $0.95g$
- ☐ **D.** $0.05g$
- ☐ **E.** $(1 - 0.05)g$

9. Here is an equation representing the relationship between the volume measured in cups, c, and the same volume measured in ounces, z: $c = \frac{1}{8}z$

Does this equation represent a proportional relationship? Circle one.

Yes   No

Explain your thinking.

10. Here is an equation representing the length, l, and width, w, for a rectangle whose area is 60 square units: $l = \frac{60}{w}$

Does this equation represent a proportional relationship? Circle one.

Yes   No

Explain your thinking.

11. Tiam bought a train ticket online. The original price of the train ticket was $\$83.00$. Tiam used a coupon to receive a 20% discount. A sales tax of 9% was applied after the discount. How much did Tiam end up paying?

A. $\$18.09$ **B.** $\$56.44$ **C.** $\$72.38$ **D.** $\$75.00$

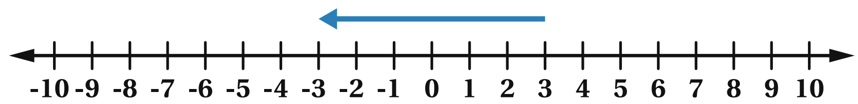
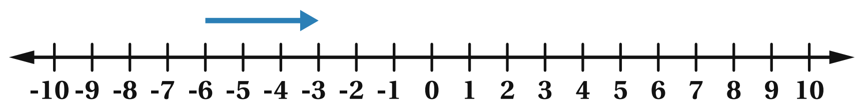
Reflection

1. Put a heart next to the problem you feel most confident about.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

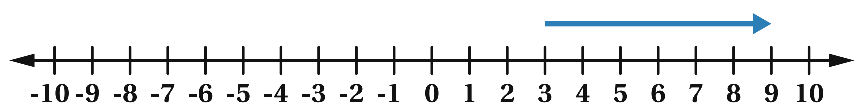
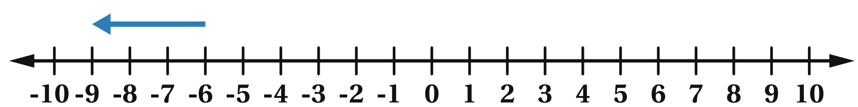
When determining the sum of terms in an expression, the order of the values does not change the final result. This is an example of the *commutative property*.

To represent addition on a number line, start at one of the values and use the other value for the direction and distance of the change.

3 + (-6)	
-6 + 3	

When determining the difference, the order of the values in an expression *does* affect the final result.

To represent subtraction on a number line, start at the first value and then use the second value to determine the direction and distance of the change. If subtracting a positive number, move to the left; if subtracting a negative number, move to the right.

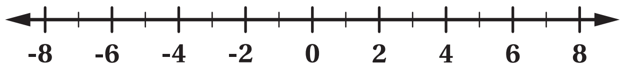
3 – (-6)	
-6 – 3	

Things to Remember:

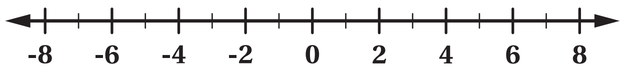
Name: ______ Date: ______ Period: ______

Problems 1–3: Use the number lines to determine the value of each expression.

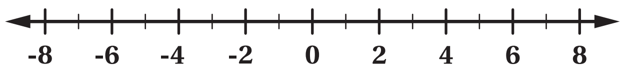
1. 4 – 6



2. -3 + (-2)



3. -8 – (-3)



Problems 4–9: Complete the tables and answer the follow-up questions.

4.

Expression	Value
2 + (-3)	
-8 + 4	
-2 + (-7)	-9
-3.6 + (-2.6)	

5.

Expression	Value
-3 + 2	-1
4 + (-8)	-4
-7 + (-2)	
-2.6 + (-3.6)	

6. Use your work from the previous tables to describe any patterns you notice.

7.

Expression	Value
3 – 2	1
5 – (-9)	14
-6 – (-3)	
-1.5 – (-4.7)	

8.

Expression	Value
2 – 3	
-9 – 5	
-3 – (-6)	3
-4.7 – (-1.5)	

9. Use your work from the previous tables to describe any patterns you notice.

Lesson Practice
7.5.04

Name: ____________ Date: ________ Period: ________

Spiral Review

10. According to the U.S.D.A., hamburger patties should be cooked until the internal temperature is 160°F, for safety. Valeria takes out a frozen patty to cook. Its current temperature is -7°F. How much does its temperature need to increase to reach 160°F?

Problems 11–13: Bettie's Boutique is having a 20% off sale.

11. Complete the table to determine the sale price for each item.

12. Explain why the relationship between original price and sale price is proportional.

Item	Original Price	Sale Price
Hat	$15	
Tie	$25	
Scarf	$35	

13. Select *all* the equations that represent the relationship between the original price, x, and the sale price, y.

- ☐ **A.** $y = 0.8x$
- ☐ **B.** $y = x - 0.2$
- ☐ **C.** $y = x - 0.2x$
- ☐ **D.** $y = 0.2x$
- ☐ **E.** $y = (1 - 0.2)x$

Reflection

1. Circle a problem you want to talk to a classmate about.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

When adding and subtracting **integers**, fractions, and decimals, there are multiple paths to the same value. Here are some strategies for adding and subtracting positive and negative numbers:

- Imagine the problem as floats and anchors or think about it on a number line. For example, $(-3) + (-4)$ is like starting with (-3) and adding 4 anchors or moving 4 to the left.
- Rewrite subtraction as addition. For example, $-3 - 4$ can be rewritten as $-3 + (-4)$, which is -7.
- Combine numbers that add or subtract to make 0. For example, when adding -5 and 6, you can break 6 into $5 + 1$. Using properties of operations, we can add $-5 + 5 + 1$ in pieces. The $-5 + 5$ portion of the expression adds to 0 and $0 + 1 = 1$, so the final value is 1.

Things to Remember:

Name: ________ Date: ________ Period: ________

Problems 1–2: Order the expressions from *least* to *greatest*.

1.

$-5 + (-4)$	$4 + 5$	$-4 + 5$	$4 + (-5)$

Least **Greatest**

2.

$-5 - (-4)$	$-4 - (-5)$	$-4 - 5$	$4 - (-5)$

Least **Greatest**

3. Is the solution to $-2.7 + x = -3.5$ positive or negative? Explain your thinking.

Problems 4–7: Determine the value of the variable that makes each equation true.

4. $33 + a = -33.8$

5. $-9 - b = 3.5$

6. $c = \left(-\frac{3}{4}\right) + \frac{3}{2}$

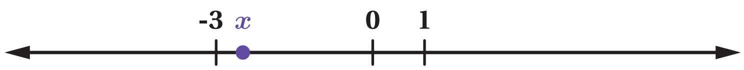
7. $d + 0.7 = -4$

Lesson Practice
7.5.05

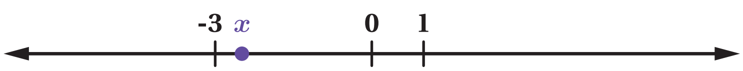
Name: .... Date: .... Period: ....

Spiral Review

8. Plot the approximate location of $x + 4$ on the number line.



9. Plot the approximate location of $4 - x$ on the number line.



10. Aditi's Attic is having a \$5 off sale. Is the relationship between original price and sale price proportional? Explain your thinking.

Original Price	Sale Price
\$15	\$10
\$25	\$20
\$35	\$30

11. Select the expression that is equivalent to $2\frac{3}{4} - \left(-\frac{7}{8}\right)$.

A. $2\frac{3}{4} - \frac{7}{8}$

B. $2\frac{3}{4} + \frac{7}{8}$

C. $2\frac{3}{4} + \left(-\frac{7}{8}\right)$

D. $-2\frac{3}{4} + \left(-\frac{7}{8}\right)$

Reflection

1. Put a smiley face next to the problem you learned from most.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

We can use models, such as floats and anchors, to make sense of multiplying integers. Here are some examples for a submarine that starts at 0 units.

Action	Representation	Submarine's Direction	Final Value
Adding 2 groups of 3 floats	$2 \cdot 3$	Up	6
Removing 2 groups of 3 floats	$-2 \cdot 3$	Down	-6
Adding 2 groups of 4 anchors	$2 \cdot (-4)$	Down	-8
Removing 2 groups of 4 anchors	$-2 \cdot (-4)$	Up	8

Things to Remember:

Name: .... Date: .... Period: ....

Problems 1–3: Determine the value of the variable that makes each equation true.

1. $3 \cdot a = 12$

2. $-3 \cdot 4 = b$

3. $-3 \cdot c = 12$

Problems 4–6: A weather station on top of a mountain reports that the temperature is currently 0°C and has been decreasing at a constant rate of 3 degrees per hour.

4. What will the temperature be in 5 hours?

5. What was the temperature 1 hour ago?

6. What was the temperature 3 hours ago?

Problems 7–9: For each equation, check the box to show whether it is true or false. If the equation is false, change one value of the equation to make it true, and write the revised equation on the line.

	Equation	True	False	Revised Equation
7.	$3 \cdot (-6) = -18$			
8.	$5 \cdot (-2) = 10$			
9.	$(-4) \cdot (-1) = -4$			

Name: .... Date: .... Period: ....

Spiral Review

Problems 10–13: The sales tax rate in Tyler's state is 6.6%.

10. Write an equation to represent how the cost of an item changes after sales tax is added. Let x represent an original cost and y represent the cost after sales tax.

11. Tyler orders a meal that costs $\$15$. What is the after-tax amount?

12. If Tyler leaves an 18% tip on the after-tax amount, how much will he end up paying?

13. If Tyler pays with a $\$20$ bill and gets no change, what percent tip did he leave?

Reflection

1. Put a question mark next to a problem you're feeling unsure of.

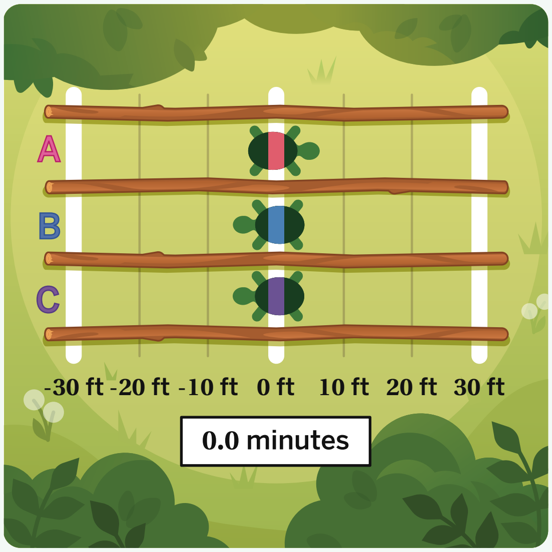
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Multiplying positive and negative numbers can help represent position, rate, and time. The position of an object is equal to the walking rate multiplied by the time or rate • time = change in position.

Here are three turtles. They are all together at 0 feet.

- Turtle A walks to the right 6 feet per minute.
  3 minutes ago, Turtle A was at -18 feet because
  $6 \cdot (-3) = -18$.
- Turtle B walks to the left 5 feet per minute.
  6 minutes ago, Turtle B was at 30 feet because
  $(-5) \cdot (-6) = 30$.
- Turtle C walks to the left 3 feet per minute.
  In 4 minutes, Turtle C will be at -12 feet because
  $(-3) \cdot 4 = -12$.



Things to Remember:

Name: ______ Date: ______ Period: ______

Problems 1–4: Determine the value of each expression.

1. $5(-3)$

2. $-5 \cdot 3$

3. $(-5)(-3)$

4. $-5 \cdot (-0.3)$

Problems 5–8: Anton the Ant can travel up and down an ant nest. Currently, his crawling rate is -7 centimeters per second, which means he's crawling down the ant nest. Anton passes ground level at position 0.

5. Which equation represents Anton's position 8 seconds *after* he passes ground level?

A. $-7 \cdot 8 = 56$ **B.** $-7 \cdot 8 = -56$ **C.** $-7 \cdot -8 = 56$ **D.** $-7 \cdot -8 = -56$

6. Which equation represents Anton's position 8 seconds *before* he passes ground level?

A. $-7 \cdot 8 = 56$ **B.** $-7 \cdot 8 = -56$ **C.** $-7 \cdot -8 = 56$ **D.** $-7 \cdot -8 = -56$

7. Write an ant story that could be represented by the equation $5 \cdot (-6) = -30$.

8. Anton has friends in nearby nests named Anto and Anty. For each description, write an equation in the table that represents it. Assume each ant starts at position 0.

Description	Equation
Anto crawls for 28 seconds at -2 centimeters per second.	
Anty crawls for 3 seconds at -8 centimeters per second.	

Name: .... Date: .... Period: ....

Spiral Review

Problems 9–10: A tank of water breaks and begins draining at a constant rate.

9. Complete the table.

Time (min)	Water in the Tank (L)
0	770
1	756
2	
. . .	. . .
5	

10. Is there a proportional relationship between time and the amount of water left in the tank? Explain your thinking.

11. Nuka buys a pair of shoes. The shoes cost $\$40$. The store is having a 20% off sale on everything in the store. How much money does Nuka save from the sale?

Reflection

1. Circle the problem you feel most confident about.

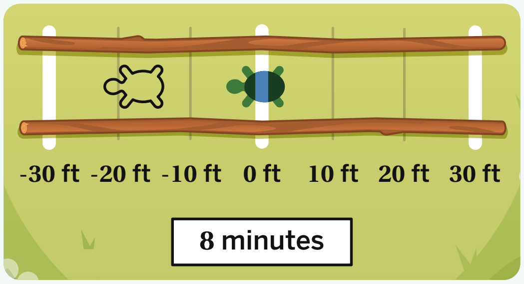
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

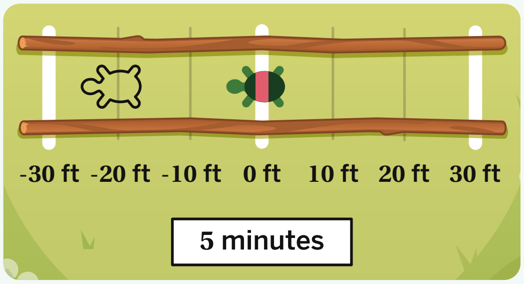
You can use your knowledge of position, rate, and time to multiply and divide positive and negative numbers.

Here are two examples.

Turtle A walks -2 feet per minute. If it starts at 0 feet, how long will it take to walk to -16 feet? Dividing position by walking rate tells us the time, so $\frac{-16}{-2} = 8$ minutes.



Turtle B takes 5 minutes to walk 20 feet to the left, or -20 feet. This means that each minute, Turtle B walks -4 feet. Dividing distance by time tells us the walking rate, so $\frac{-20}{5} = -4$.



Thinking about an expression in terms of position, rate, and time can help you determine whether the value is negative or positive.

Things to Remember:

Lesson Practice
7.5.08

Name: ______ Date: ______ Period: ______

1. Select *all* the expressions that have a negative value.

☐ **A.** $\frac{-15}{12}$ ☐ **B.** $\frac{15}{-12}$ ☐ **C.** $\frac{-15}{-12}$

☐ **D.** $-\frac{15}{12}$ ☐ **E.** $\frac{15}{12}$

Problems 2–3: Determine the value of x that makes each equation true.

2. $-3x = 6.3$

3. $\frac{x}{-1.2} = -0.3$

Problems 4–5: A machine that drills holes for wells drilled at a constant rate to a depth of -72 feet in one day (24 hours).

4. How much did the depth change each hour? Make sure to show whether the change is positive or negative.

5. What was the depth after 15 hours?

6. Select *all* the values that are equivalent to $-\frac{12}{7}$.

☐ **A.** $-\frac{-12}{-7}$ ☐ **B.** $-5\frac{1}{7}$ ☐ **C.** $-1\frac{5}{7}$

☐ **D.** $1\frac{5}{7}$ ☐ **E.** $\frac{-12}{-7}$

Lesson Practice
7.5.08

Name: .... Date: .... Period: ....

Spiral Review

7. The equation $30 + (-30) = 0$ is an example of two numbers whose sum is 0. Write a different equation with two numbers whose sum is 0.

8. Write an equation with three numbers whose sum is 0.

9. Write an equation with *four* numbers, none of which are opposites, whose sum is 0. One example of opposites is -30 and 30.

10. Write an equation using *four* numbers that multiply to 0.

Reflection

1. Put a star next to a problem you could explain to a classmate.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can use properties of operations, order of operations, and multiplication and division of integers as strategies to solve integer puzzles.

For example, if we want to make this inequality true, it may be helpful to think about the signs of the sum inside the parentheses and the number outside of the parentheses.

Make the inequality true.

$\underline{\quad\quad}(\underline{\quad\quad} + \underline{\quad\quad}) > 0$

$-8(-4 + 1) > 0$ is true because $-8(-3)$ is positive.

$-8(4 + 1) > 0$ is false because $-8(5)$ is negative.

Things to Remember:

Lesson Practice
7.5.10

Name: ________________ Date: ________ Period: ________

Problems 1–4: Determine the value of the variable that makes each equation true.

1. $-22 + a = -5$

2. $-22 - 5 = b$

3. $-5c = -22$

4. $\frac{d}{-5} = 22$

5. Which expression has the greater value?

A. $(-22) - (-5)$

B. $(-5) - (-22)$

C. They have the same value

Explain your thinking.

Problems 6–7: Let $x = -2$, $y = 4$, and $z = 2$.

6. Order these expressions from *least* to *greatest*.

$x - z$	$x - 2y$	$x \cdot y$	xyz

Least **Greatest**

7. Would your order be different if the value of x was 2 instead? Explain your thinking.

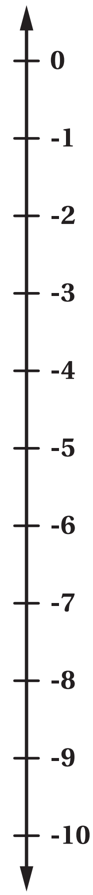
8. For the expressions $\frac{a}{b}$ and $a + b$, choose values for a and b so that $\frac{a}{b}$ is positive and $a + b$ is negative.

Lesson Practice
7.5.10

Name: ________ Date: ________ Period: ________

Spiral Review

Problems 9–11: Lucia held a penny underwater in a pool and let it go. The penny traveled downward at a rate of 0.8 feet per second. Use the number line if it helps with your thinking.



9. How many seconds did it take for the penny to move -4.4 feet (4.4 feet downward)?

10. 3 seconds after letting go, the penny was at position -3.4 feet. What position was the penny let go from?

11. What was the position of the penny after it sank for 10 seconds? Explain your thinking.

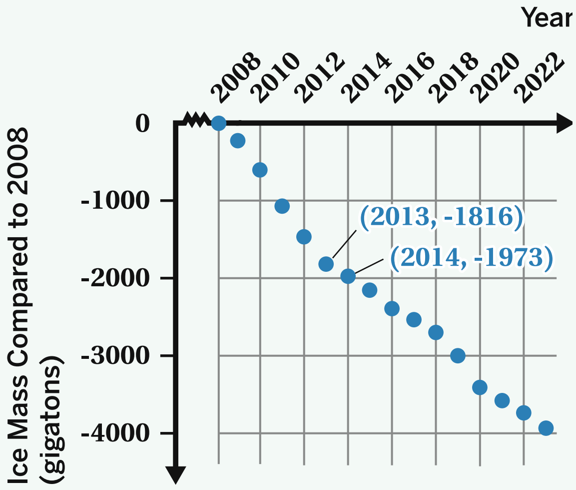
Reflection

1. Circle the problem you enjoyed doing the most.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Adding and subtracting positive and negative numbers can help you solve problems involving real-world situations.

You can tell from the graph that the ice mass in Greenland is decreasing each year. Subtraction lets you determine how much it decreased between any two years.



For example, to determine the change in the ice mass from 2013 to 2014, you need to find the difference between -1,973 and -1,816.

Since -1,973 is less than -1,816, the difference will be negative.

$-1973 - (-1816) = -1973 + 1816 = -157$

The change in ice mass from 2013 to 2014 is -157 gigatons. You can use this information to determine if the ice mass is continuing to change at the same rate in future years, or if the ice mass is changing at a faster or slower rate.

Things to Remember:

Name: ______ Date: ______ Period: ______

Problems 1–3: Determine the value of the variable that makes each equation true.

1. $3 \cdot (-2.5) = x$

2. $-3y = 33$

3. $-3 - \frac{1}{2} = z$

4. This table shows the transactions, in dollars, in a checking account for the month of January.

Determine the total of the transactions for the month. Show or explain your thinking.

January
-\$38.50
\$126.30
\$429.40
-\$265.00

Problems 5–7: On January 22, 1943, the town of Spearfish, South Dakota, set the record for the world's fastest temperature change.

- At 7:30 AM, the temperature was -4°F.
- By 7:32 AM, the temperature was 45°F.
- By 9:00 AM, the temperature was 54°F.
- By 9:27 AM, the temperature was -4°F.

5. What was the temperature change from 7:30 AM to 7:32 AM? Make sure to show whether the change was positive or negative.

6. What was the temperature change from 9:00 AM to 9:27 AM? Make sure to show whether the change was positive or negative.

7. Years later, the town of Bristol had a big temperature drop. Between 6:02 PM and 6:10 PM, the temperature went from 9°F to -11°F. How does this temperature change compare to Spearfish's big drop that started at 9 AM?

Lesson Practice
7.5.11

Name: .... Date: .... Period: ....

Spiral Review

Problems 8–10: For each fraction, determine whether it terminates or repeats when written as a decimal.

8. $\frac{4}{9}$ Terminating Repeating

9. $\frac{5}{8}$ Terminating Repeating

10. $\frac{50}{75}$ Terminating Repeating

Problems 11–12: These are equation puzzles. Fill in the blanks so that each row and column makes a true equation.

11. Use numbers to complete the puzzle.

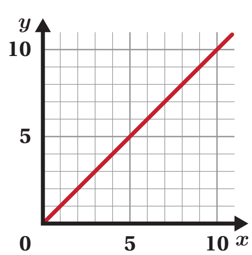
4	÷		×		=	0
×		+		+		
	×	2	−		=	0
+		−		+		
4	÷		+	2	=	0
=		=		=		
0		0		0		

12. Use the symbols +, −, ×, and ÷.

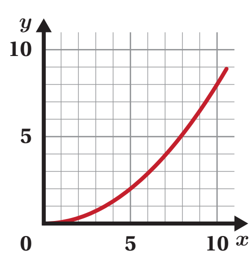
-4		-1		3	=	1
1		-3		-3	=	1
4		1		-2	=	1
=		=		=		
1		1		1		

13. Select *all* the graphs that don't represent a proportional relationship.

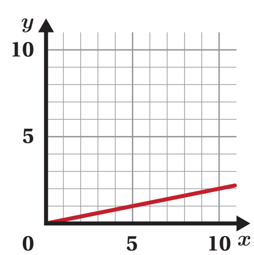
☐ **A.**



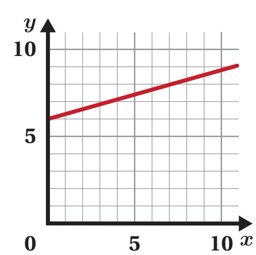
☐ **B.**



☐ **C.**



☐ **D.**



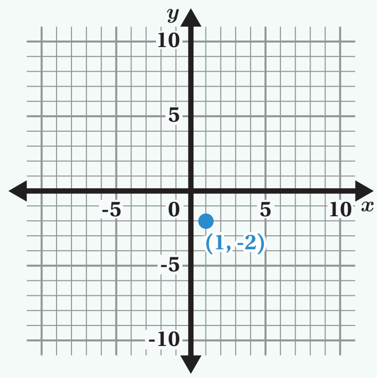
Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

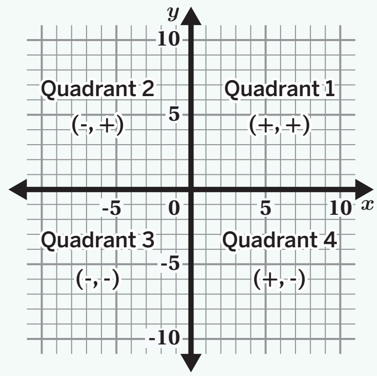
You can include positive and negative numbers along the x- and y-axes, just like on a number line. The x- and y-axes cross at the *origin*, or the point $(0, 0)$.

Ordered pairs are written (x, y), where the x-value is the horizontal location (left and right) and the y-value is the vertical location (up and down). For example, the point $(1, \text{-}2)$ is 1 unit to the right and
2 units down from the origin.



The four regions of the coordinate plane are called **quadrants**. They are numbered 1–4 starting with the top right quadrant and going in a circle counter-clockwise.

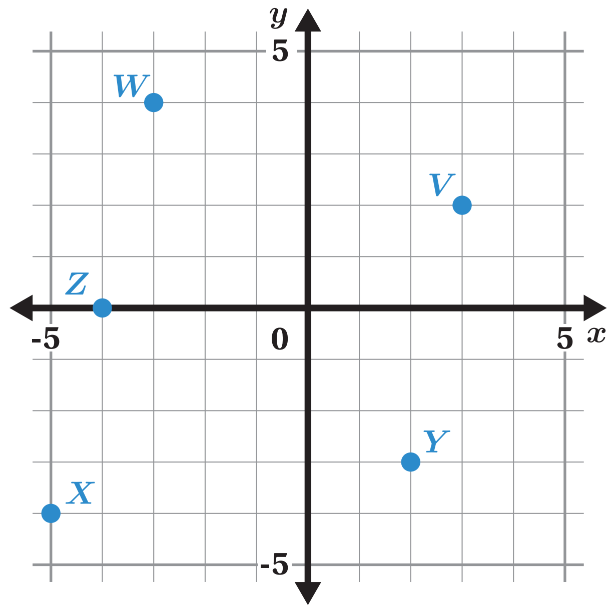
The image shows each quadrant, along with the sign of the x- and y-values in that quadrant.



Things to Remember:

Name: ______________ Date: ________ Period: ________

Problems 1–6: Here is a coordinate plane.



1. Write the coordinates of each point in the table.

Point	Coordinates
V	
W	
X	
Y	
Z	

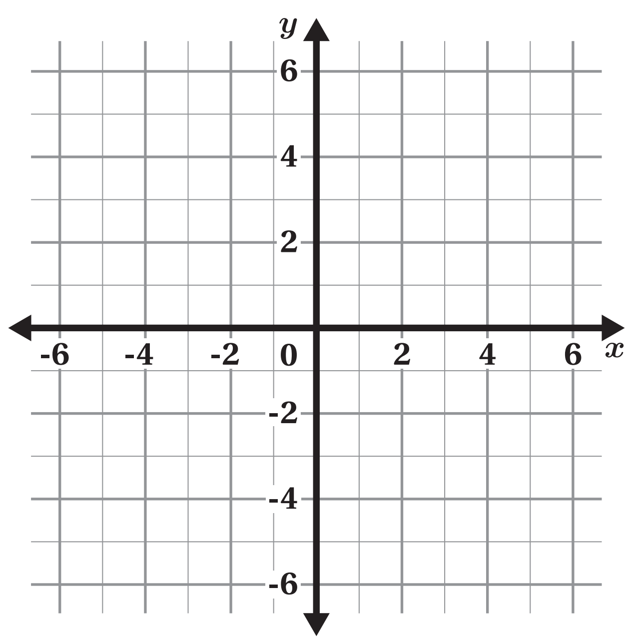
2. Plot and label point A at $(-2, 1)$.

3. Plot and label point P at $(1, -2)$.

4. Plot two points that are each 2 units away from point V. Label each point with its coordinates.

5. Point Q is more than 3 units directly to the left of point Y. Write at least one thing you know about the coordinates of point Q.

6. Draw the first letter of your name on the coordinate plane so that at least part of your drawing is in each quadrant.



Lesson Practice
6.7.10

Name: ______________ Date: ________ Period: ________

Spiral Review

7. The height requirement for an amusement park ride is written as $h > 42$, where h represents a rider's height in inches. Write a sentence or sketch a sign that describes these rules as clearly as possible.

8. Select *all* the values of x that are solutions to the inequality $x >$ -2.

- ☐ **A.** -1
- ☐ **B.** -2
- ☐ **C.** -3
- ☐ **D.** -2.1
- ☐ **E.** -1.8

Problems 9–12: Solve each equation.

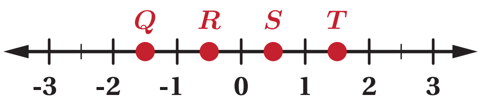
9. $3a = 12$

10. $b + 3.3 = 8.9$

11. $1 = \frac{1}{4}c$

12. $2d = 6.4$

13. Here is a number line with points Q, R, S, and T. Which point represents $-1\frac{1}{2}$?



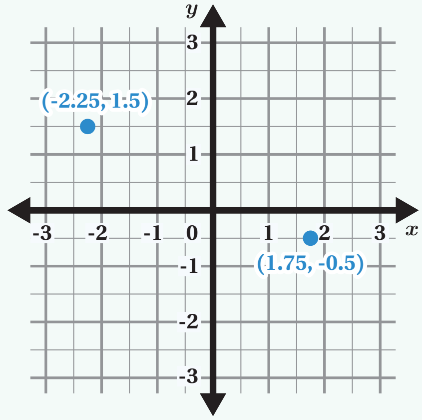
Reflection

1. Put a smiley face next to a problem you were stuck on and then figured out.

2. Use this space to ask a question or share something you're proud of.

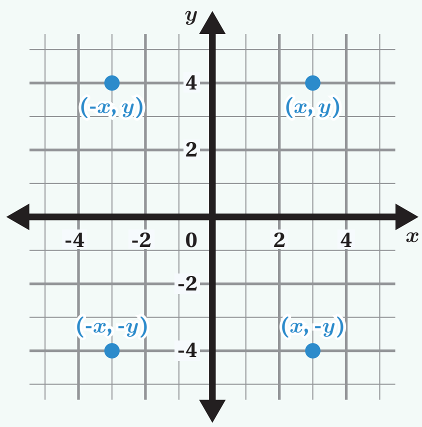
Lesson Summary

You can use different scales to show very big or very small numbers on a coordinate plane. In these cases, the interval is still consistent (e.g., goes by 2s or 0.5s). Sometimes points are plotted in between tick marks. Consider the points (1.75, -0.5) and (-2.25, 1.5) and where they appear on the graph shown.



The points (3, 4) and (3, -4) have the same x-coordinate and the y-coordinates only differ by their sign. We can see on the graph that those points are a reflection, or a mirror, of each other across the x-axis.

The points (-3, -4) and (3, -4) have the same y-coordinate and the x-coordinates only differ by their sign. We can see on the graph that those points are a reflection, or a mirror, of each other across the y-axis.

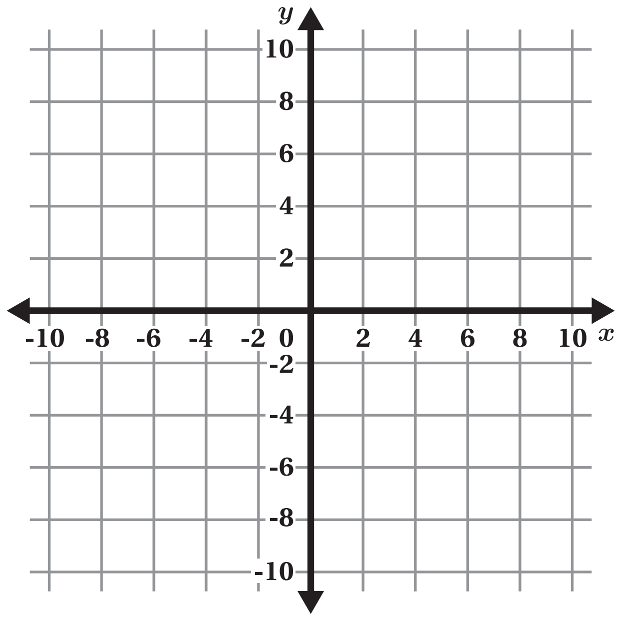


Things to Remember:

Lesson Practice
6.7.11

Name: ____________ Date: ________ Period: ________

Problems 1–2: Here is a graph.

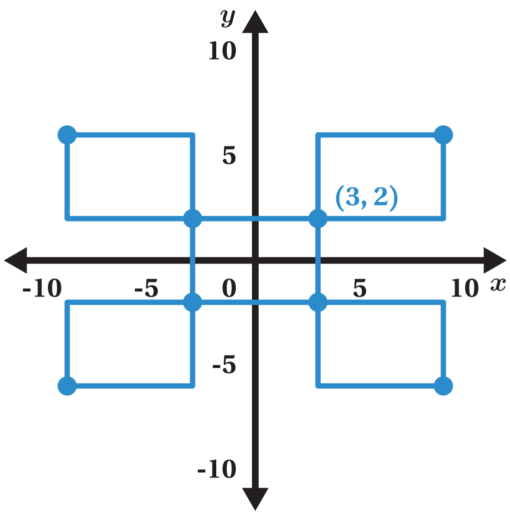


1. Plot and connect each point in order. Plot all of Column 1 first, then Column 2.

Column 1	Column 2
(-2, -9)	(-2, 4)
(-2, -3)	(0, 2)
(-4, -3)	(2, 4)
(-6, -1)	(2, -1)
(-6, 4)	(0, -3)
(-4, 2)	(-2, -3)

2. Describe your strategy for plotting the point (-2, -9) in Problem 1.

Problems 3–4: Here are identical rectangles on a coordinate plane. The origin is in the center of the middle rectangle.

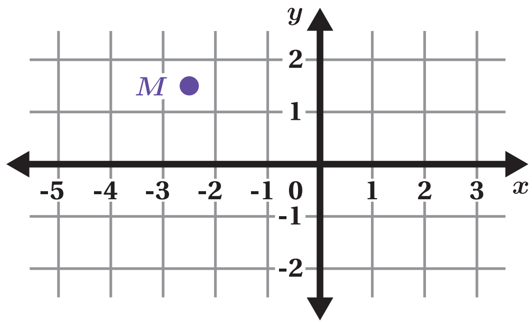


3. Label the coordinates of the remaining points.

4. What patterns do you notice?

Name: ________________ Date: ________ Period: ________

5. What is the value of the x-coordinate of point M? Your answer should be a decimal rounded to the nearest 0.5.



6. Point A is located at $(3, n)$ on the coordinate plane. Point B is located at $(\text{-}3, n)$. What do you know about points A and B?

Spiral Review

Problems 7–10: Complete each number sentence with the symbol $<$, $>$, or $=$. Use a number line if it helps with your thinking.

7. $\left|\text{-}\frac{3}{2}\right| \square \text{-}\frac{2}{3}$

8. $\text{-}\frac{3}{2} \square \text{-}\frac{2}{3}$

9. $\text{-}\frac{3}{2} \square \frac{2}{3}$

10. $\frac{3}{2} \square \left|\text{-}\frac{2}{3}\right|$

Problems 11–12: DeShawn's dog weighs 34 pounds. Jacy's dog weighs 12 pounds more than DeShawn's dog.

11. Select *all* the equations that show the weight of Jacy's dog, j.

- ☐ **A.** $j = 34 + 12$
- ☐ **B.** $j = 34 - 12$
- ☐ **C.** $j + 12 = 34$
- ☐ **D.** $j - 12 = 34$
- ☐ **E.** $j = 34 \bullet 12$

12. Determine how much Jacy's dog weighs.

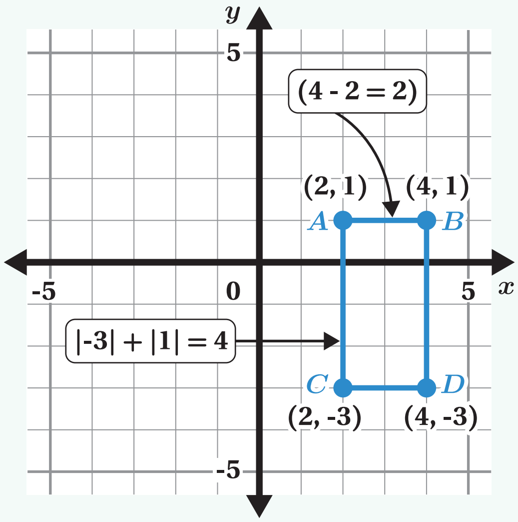
Reflection

1. Put a question mark next to a response you'd like to compare with a classmate's.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can plot points on the coordinate plane to create *polygons*. When the vertices of a polygon are horizontally or vertically aligned on the graph, you can count the number of units between them to determine the length of that side.



You can also calculate the side lengths using the coordinates of each vertex. Here are two calculation strategies:

- If the coordinates are in the same quadrant, like points A and B, find the length by subtracting the coordinates that are different.
- If the coordinates are in different quadrants, like points A and C, use the absolute value to determine the distance each point is from the axis between them.

Things to Remember:

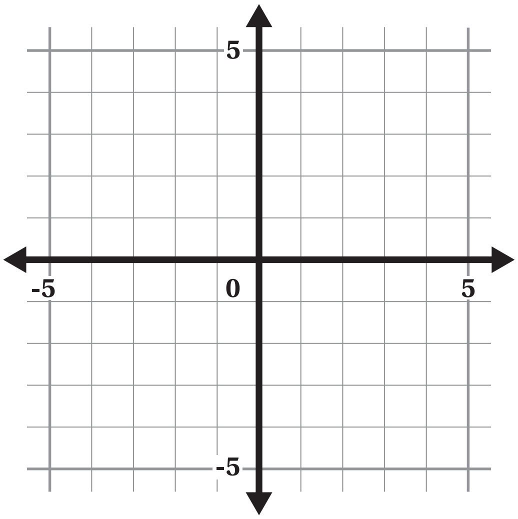
Lesson Practice
6.7.12

Name: ______________ Date: ________ Period: ________

Problems 1–4: Here is a set of coordinates.

Point	Coordinates
A	(-3, 1)
B	(3, 1)
C	(3, -4)
D	(-1, -4)
E	(-1, -2)
F	(-3, -2)

1. Plot each point.

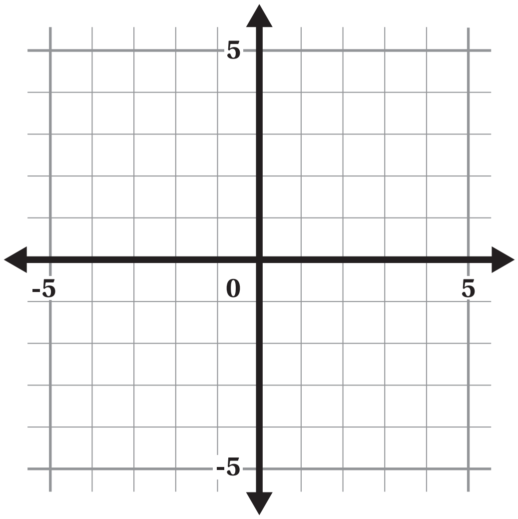


2. Connect the points to create polygon $ABCDEF$.

3. Determine the length of the segment between point A and point B.

4. Determine the perimeter of polygon $ABCDEF$.

Problems 5–7: Three points of a rectangle are (3, 0), (3, -5), and (-4, -5).



5. What are the coordinates of the missing point?

6. Sketch the rectangle and calculate its perimeter.

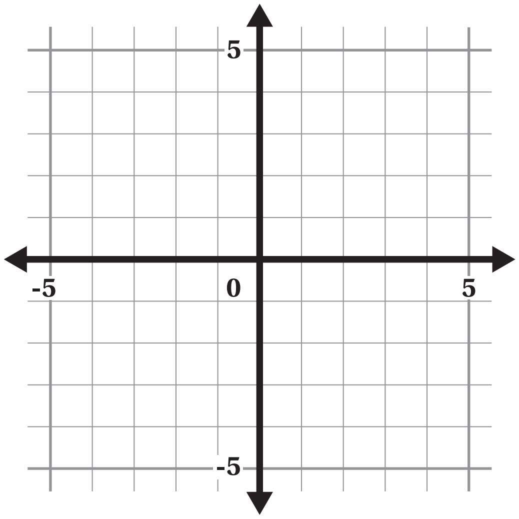
7. Calculate the area of the rectangle.

Name: .... Date: .... Period: ....

8. Gabriela drew a quadrilateral with these descriptions:

- It has exactly one pair of parallel sides.
- Two of the vertices are $(-4, 5)$ and $(2, -3)$.
- At least one side has a length of 5 units.

Create a quadrilateral that Gabriela could have made.



Spiral Review

9. Select *all* the values of x that are solutions to the inequality $-0.5 > x$.

- ☐ **A.** 0
- ☐ **B.** -1
- ☐ **C.** -0.40
- ☐ **D.** -0.6
- ☐ **E.** -0.55

10. Is $\frac{12}{5} \div \frac{3}{5}$ greater than, less than, or equal to 1?

11. What is the value of $\frac{9}{5} \div \frac{3}{5}$?

12. At top speed, an elephant can run 25 miles per hour and a giraffe can run 16 miles in $\frac{1}{2}$ hour. Which animal runs faster?

Reflection

1. Put a heart next to a problem you understand well.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice 6.8.02

Science Mom Lesson 109

Lesson Summary

A **statistical question** is a question that needs more than one piece of data to answer it.

Here is an example:

- "Which classroom in your school has the most books?" is a statistical question because you need to know the number of books in *each* classroom to answer it.
- "How many books are in your classroom?" is not a statistical question because you only need to know the number of books in *one* classroom to answer it.

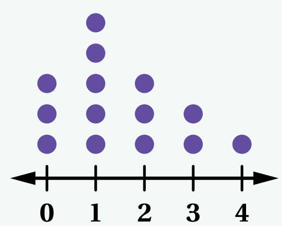
You can organize data that answers a statistical question into a list or a **dot plot**.

List

$0, 0, 0, 1, 1, 1, 1, 1, 2, 2, 2, 3, 3, 4$

Lists allow you to see all of the data. Lists can be used for both numerical and categorical data.

Dot Plot



Dot plots are a visual representation of numerical data and allow you to compare multiple data sets.

Things to Remember:

Lesson Practice
6.8.02

Name: ______ Date: ______ Period: ______

Problems 1–5: Five sixth-grade students at a school were each asked the following survey questions:

Question A: What grade are you in?

Question B: How many books did you read in the last year?

Question C: How many inches are in 1 foot?

Question D: How many dogs and cats do you have?

Their answers are shown in the table. Write the letter of the question that could have produced each line of data.

	Question	Diya	Chey	Tiana	Shep	Callen
1.		0	1	1	3	0
2.		12	12	12	12	12
3.		6	6	6	6	6
4.		11	5	18	20	9

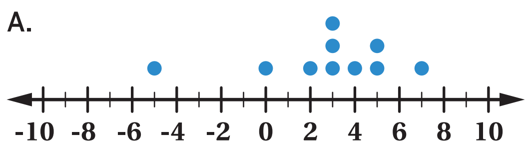
5. How are Questions A and C different from the other questions?

Problems 6–7: Zee asked 10 students how many minutes it takes them to get to school each morning.

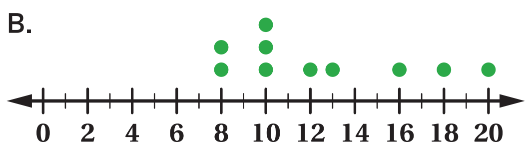
6. Which dot plot could represent the data that Zee collected?

7. Which dot plot could not represent Zee's data? Explain your thinking.

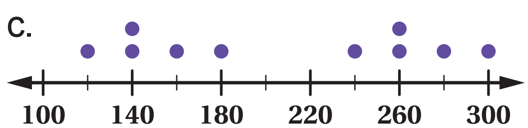
A.



B.



C.



Name: .... Date: .... Period: ....

8. Which question is a statistical question?

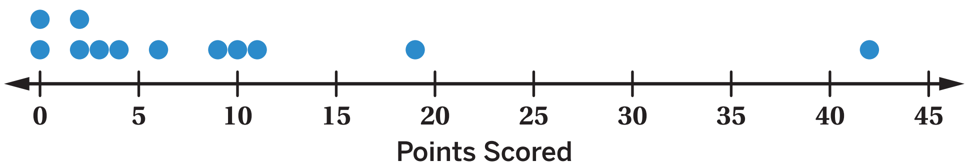
A. How old are you?

B. How old are the students in our school?

C. How old is your teacher?

D. What is the difference between your age and your teacher's age?

9. This dot plot represents the points scored by 12 players on a basketball team during an important game.



Write a story about the basketball game.

Spiral Review

10. Order these values from *least* to *greatest*.

$|-17|$	$|-18|$	-18	$|19|$	20

Least **Greatest**

Problems 11–13: Determine each quotient.

11. $45052 \div 28$ **12.** $6052 \div 17$ **13.** $60.52 \div 1.7$

Reflection

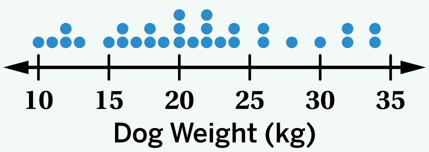
1. Put a star next to your favorite problem.

2. Use this space to ask a question or share something you're proud of.

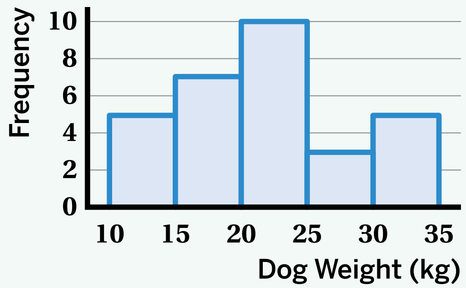
Lesson Summary

You can use dot plots and **histograms** to visualize numerical data. Here is an example of a data set of the weights of 30 dogs presented in a dot plot and in a histogram.

Dot Plot



Histogram



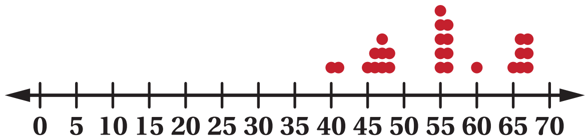
In a histogram, data values are grouped into bins that cover a range of values, and each **bin** has the same width. The height of each bar represents the total number of values in that range, including the left boundary (least value) but excluding the right boundary (greatest value). For example, the height of the tallest bar, from 20 to 25, represents weights of 20 kilograms up to (but not including) 25 kilograms.

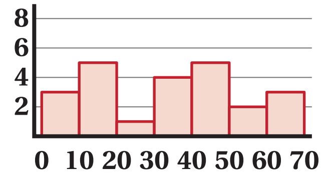
Things to Remember:

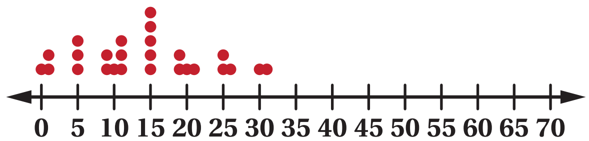
Lesson Practice
6.8.05

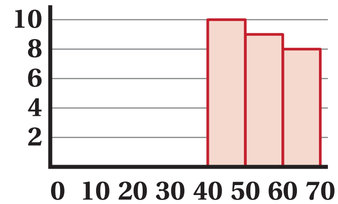
Name: ______________ Date: ________ Period: ________

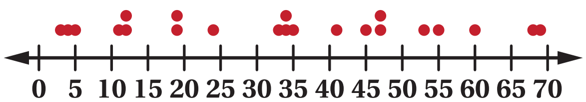
1. Match each histogram with the dot plot that represents the same data set.

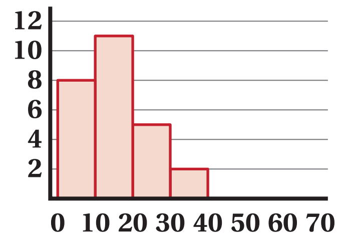
a. 

______ 

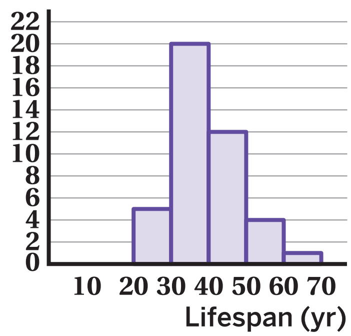
b. 

______ 

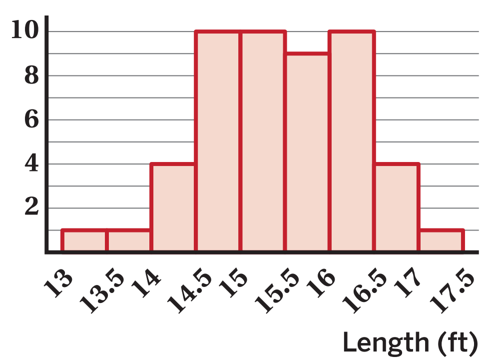
c. 

______ 

2. Here is a histogram showing the lifespans of 42 chimpanzees that lived in the wild. How many chimpanzees lived at least 50 years and less than 70 years?



Problems 3–4: A marine biologist is studying a group of sharks. She made a histogram of the lengths of a group of adult sharks.



3. Select *all* of the false statements

- ☐ **A.** A total of 9 sharks were measured.
- ☐ **B.** Two of the sharks are less than 14 feet long.
- ☐ **C.** A typical shark is about 14.5 to 16.5 feet long.
- ☐ **D.** The longest shark measured was 10 feet long.
- ☐ **E.** The smallest shark measured was 11 feet long.

4. Explain your thinking for one of the false statements.

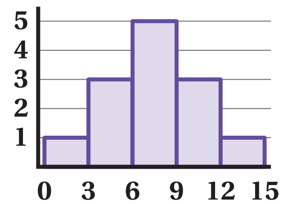
Name: .... Date: .... Period: ....

Problems 5–6: Here are three representations of the same data set.

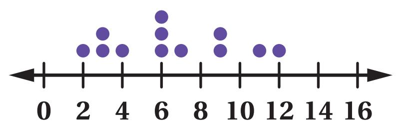
List

2, 3, 3, 4, 6, 6, 6,
7, 9, 9, 11, 12

Histogram



Dot Plot



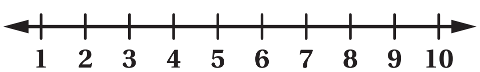
5. Circle the representation that doesn't match the other two.

6. Revise your circled representation to make it match the other two representations.

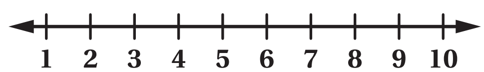
Spiral Review

Problems 7–8: Graph each inequality on a number line.

7. $m > 6$



8. $3 > n$



Problems 9–12: Determine the value of each expression.

9. $3.727 + 1.384$

10. $3.727 - 1.384$

11. $5.01 \cdot 4.8$

12. $5.01 \div 4.8$

Reflection

1. Put a question mark next to a response you'd like to compare with a classmate's.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

A **statistic** is a single number that measures something about a data set. One way to measure the center of a data set is by determining the **mean**, or average, of all the data values. You can think of the mean as “an equal share.”

For example, suppose this data set represents how many liters of water are in 5 bottles: $1, 4, 2, 3, 0$. To calculate the mean, you first add up all of the values to determine the total (10 liters), then divide that sum by the number of values (5 bottles). This example can be represented by the expression $(1 + 4 + 2 + 3 + 0) \div 5$, or $10 \div 5$. So, the mean amount of water in the 5 bottles is 2 liters (per bottle). The mean is a whole number in this example, but it is possible for the mean to be a decimal number.

Things to Remember:

Lesson Practice
6.8.07

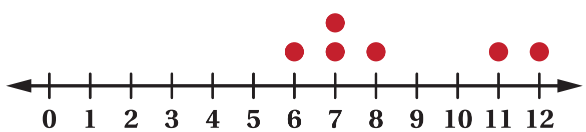
Name: ______________ Date: ______ Period: ______

1. A preschool teacher plans to reorganize these 4 boxes of playing blocks so that each box contains an equal number of blocks. How could the teacher determine the number of blocks to put in each box?

Box 1	Box 2	Box 3	Box 4
32 blocks	18 blocks	41 blocks	9 blocks

2. 3 classes worked together to raise money for their classroom libraries. They agreed to share the money equally. The first class raised $25.50, the second class raised $49.75, and the third class raised $37.25. What is each class's equal share? Show or explain your thinking.

3. Kimaya guesses that 11 is the mean of this data set. Without calculating, determine if Kimaya's guess is correct. Explain your thinking.



Problems 4–5: For 12 days, Mai recorded how many minutes long her bus rides to school were. Here are the times she recorded.

Time on the Bus (min)											
9	12	6	9	10	7	6	12	9	8	10	10

4. Determine the mean for Mai's data. Show or explain your thinking.

5. What does the mean tell us about Mai's trip to school?

6. In English class, Hanjun's teacher gives 4 quizzes, each worth 5 points. After 3 quizzes, Hanjun has scores of 4, 3, and 4. How many points does Hanjun need to get on the last quiz to have an average score of 4? Show or explain your thinking.

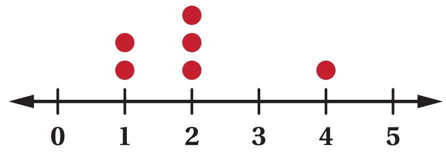
Name: .... Date: .... Period: ....

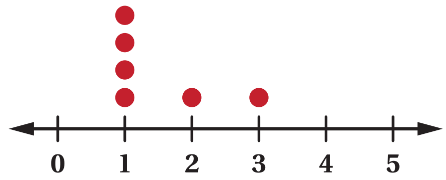
7. Select a dot plot that matches each statement. You can only select each dot plot once.

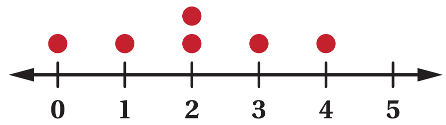
a. The mean is 2.

b. Only one value is greater than the mean.

c. Two values are greater than the mean.

.... 

.... 

.... 

8. In a round of mini golf, Angel records the number of putts on each hole.

Number of Putts								
2	3	1	4	5	2	3	4	3

What is the mean number of putts per hole? Show or explain your thinking.

Spiral Review

Problems 9–11: Evaluate the expression $4x^3$ for each value of x.

9. 1

10. 2

11. $\frac{1}{2}$

12. Select *all* of the values of x that are solutions to the inequality $x > -3$.

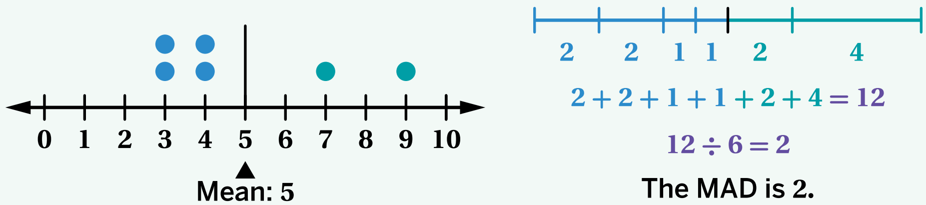
☐ **A.** -2
☐ **B.** $|-4|$
☐ **C.** -4
☐ **D.** 3.5
☐ **E.** -3.5

Reflection

1. Put a smiley face next to a problem you were stuck on and then figured out.
2. Use the space to ask a question or share something you're proud of.

Lesson Summary

You can describe how spread out the values in a data set are with a single number, the **mean absolute deviation (MAD)**. The MAD is calculated by determining the mean of the absolute deviations (i.e., the average of the distances between each data value and the mean).



The mean absolute deviation is an example of a measure of spread. A measure of spread is a way to measure the consistency of the values in a data set. The smaller the value of the MAD, the less spread out the data points are around the mean, and the more consistent the data is. The larger the MAD, the more spread out the data points are around the mean, and the less consistent the data is.

Things to Remember:

Lesson Practice
6.8.09

Name: ______________________ Date: __________ Period: __________

Problems 1–2: This table shows the amount of time it takes 6 students to get to school. Their mean travel time is 22 minutes.

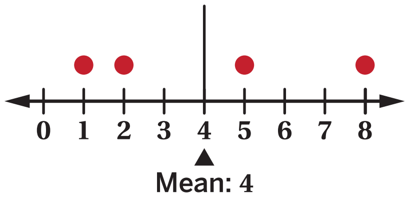
Time (minutes)	10	10	18	20	30	44
Absolute Deviation						

1. Complete the table by calculating the absolute deviation of each value from the mean.

2. Calculate the mean absolute deviation (MAD) of this data set.

3. Calculate the MAD of this data set.

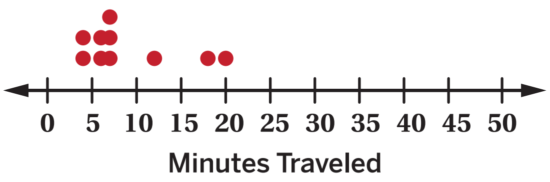
Use the table if it helps with your thinking.



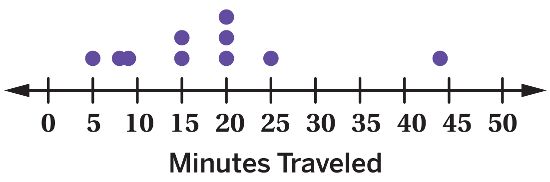
Data Point				
Absolute Deviation				

Problems 4–5: These dot plots show the travel times for 10 students from two cities.

City A



City B



4. The MADs have been calculated for you. Match each MAD to the correct city.

MAD	City
4.4	
7.8	

5. Without calculating, explain how you matched the MADs to the data sets.

Name: ________ Date: ________ Period: ________

6. Here are two dot plots showing the recorded speeds of two manatees.

Which of these statements is true?

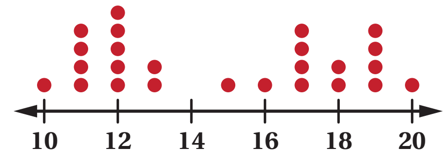
A. The data for Dot Plot B is more spread out than the data for Dot Plot A.

B. The manatee recorded in Dot Plot A swims at a more consistent speed.

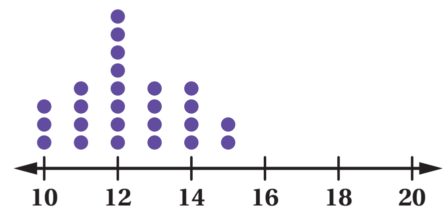
C. The MAD of Dot Plot A is greater than the MAD of Dot Plot B.

D. Both dot plots have approximately the same mean.

Dot Plot A

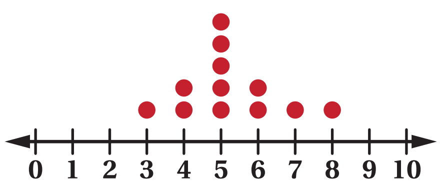


Dot Plot B



Problems 7–8: Nathan and Esi recorded the number of baskets they each made out of 10 attempts. They each collected 12 data points. Nathan's mean number of baskets was 5.25, and his MAD was 1. Esi's mean number of baskets was 5.

Nathan



Esi	7	8	9	0	1	2	1	3	2	10	9	8
Distance From 5												

7. Calculate Esi's MAD. Use the table if it helps with your thinking.

8. Which player is more consistent?

Spiral Review

9. Select *all* the expressions that are equivalent to $2(6 + 3x)$.

☐ **A.** $8 + 5x$

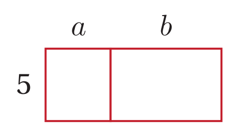
☐ **B.** $12x + 6$

☐ **C.** $12 + 6x$

☐ **D.** $6(2 + x)$

☐ **E.** $2(9x)$

10. Select *all* the expressions that represent the total area of the rectangle.



☐ **A.** $5(a + b)$

☐ **B.** $5 + ab$

☐ **C.** $5a + 5b$

☐ **D.** $2(5 + a + b)$

☐ **E.** $5ab$

Reflection

1. Put a heart next to a problem that you understand well.

2. Use the space to ask a question or share something you're proud of.

Science Mom Lesson 113

Lesson Summary

You can describe a data set using another measure of center called the **median**. The median is the “middle” value in a data set when the values are listed in order from least to greatest (or greatest to least). Half of the data values are less than or equal to the median, and half of the data values are greater than or equal to the median.

To determine the median from an ordered representation of the data, you can repeat a process of eliminating the pairs of least and greatest values.

Here are some examples.

Odd Number of Values	$\cancel{0}$ $\cancel{1}$ $\cancel{1}$ 2 $\cancel{2}$ $\cancel{4}$ $\cancel{5}$ Once all pairs have been eliminated, only one value remains in the middle, making it the median. Median: 2
Even Number of Values	$\cancel{0}$ $\cancel{1}$ $\cancel{1}$ 1 2 $\cancel{2}$ $\cancel{4}$ $\cancel{5}$ Once all pairs have been eliminated, two values remain. Their average is the median. $(1 + 2) \div 2 = 1.5$ Median: 1.5

Things to Remember:

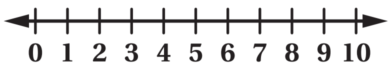
Lesson Practice
6.8.11

Name: .... Date: .... Period: ....

Problems 1–2: Here is a data set.

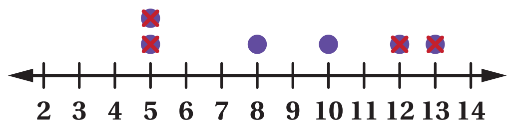
1	6	7	6	2	9	3

1. Create a dot plot for this data.



2. Determine the median of this data.

Problems 3–4: Kayla wants to determine the median of the data in this dot plot. She starts solving the problem but isn't sure what to do next.



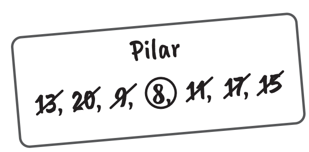
3. What could you say to help Kayla determine the median?

4. What is the median of this data?

5. The table shows Prisha's scores after attempting the first level of a video game 10 times. What is her median score?

130	150	120	170	130
120	160	160	190	140

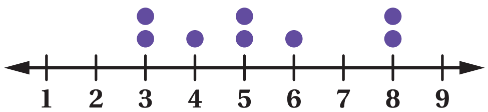
6. Pilar recorded the number of points she scored in her last 7 basketball games. She says that her median score was 8 points. Is she correct? Explain your thinking.



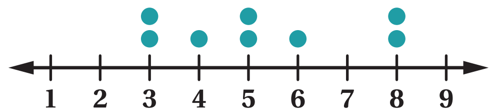
Lesson Practice
6.8.11

Name: .... Date: .... Period: ....

7. *Add* points to this dot plot to make the median 4.



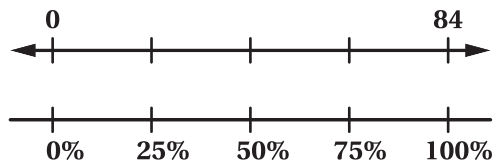
8. *Remove* points from this dot plot to make the median 4.



Spiral Review

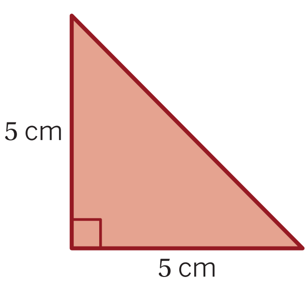
9. Determine 25%, 50%, and 75% of 84.

Use the double number line if it helps with your thinking.

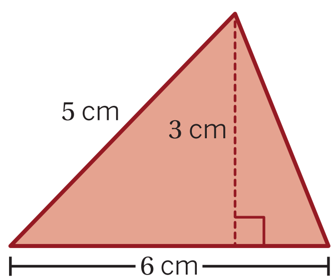


Problems 10–12: Calculate the area of each triangle.

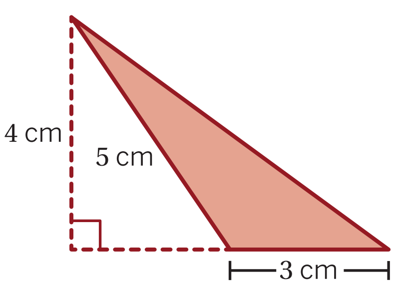
10.



11.



12.



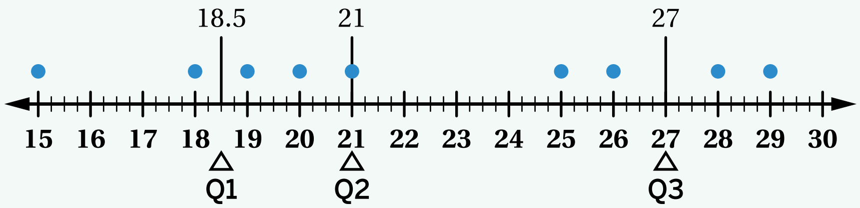
Reflection

1. Put a star next to a problem you could explain to a classmate.
2. Use the space to ask a question or share something you're proud of.

Lesson Summary

You can describe the middle half of a data set by dividing it into four equal sections called **quartiles**.

You can determine the value of the quartiles by splitting the entire data set in half and then splitting the halves again. The middle half is all the data points that are between Q1 and Q3. Representations such as dot plots are helpful for identifying quartiles to describe data sets.



The *first quartile (Q1)* is the median of the lower half of the data set.

The *second quartile (Q2)* is the median of the entire data set.

The *third quartile (Q3)* is the median of the upper half of the data set.

Things to Remember:

Name: ________ Date: ________ Period: ________

Problems 1–2: Here are the ages of 20 people at a family reunion, ordered from youngest to oldest.

3, 8, 9, 10, 11, 11, 12, 18, 18, 28, 30, 35, 37, 40, 53, 54, 58, 65, 70, 72

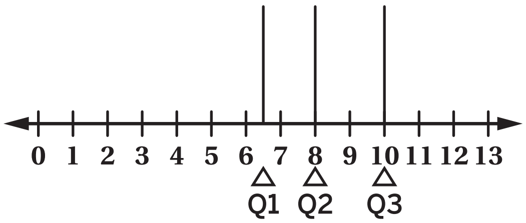
1. The value of Quartile 2 (Q2) is 29. What does that tell us about the people at the family reunion?

2. Determine the value of Q1 and Q3.

Quartile 1 (Q1): ________ Quartile 3 (Q3): ________

Problems 3–5: Haru recorded how long his bus ride to school took for 16 days. Here are the values of the quartiles of his data.

Bus Ride Length (min)



3. About how many rides would you expect to be less than 6.5 minutes long?

4. About how many rides would you expect to be less than 10 minutes long?

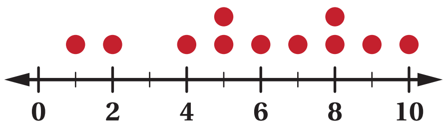
5. About what percent of the rides would you expect to be between 6.5 minutes and 10 minutes long?

6. The heights, in inches, of Javier's classmates are 60, 68, 56, 60, 62, 58, 55, 67, 59, 61, 62, 64, 63, 63, 59, 62, 66, and 61. Determine the values of Q1, Q2 (median), and Q3.

	Height (in.)
Q1	
Q2 (Median)	
Q3	

Name: ______ Date: ______ Period: ______

7. Makayla and Axel both try to determine the quartiles for this dot plot.



Makayla says:

Q1: 4.5 Q2: 6 Q3: 8

Axel says:

Q1: 4 Q2: 6 Q3: 8

Who is correct? Explain your thinking.

Spiral Review

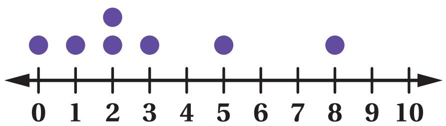
Problems 8–10: Calculate each percentage.

8. 25% of 40

9. 25% of 120

10. 25% of 90

11. Add 5 points to this dot plot without changing the mean absolute deviation (MAD).



Reflection

1. Circle the problem that was the most challenging for you.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can create a **box plot** to visualize a data set. While a box plot shows the same data as a dot plot, it gives us new information about the data. Rather than showing every data point, a box plot separates the data into quartiles.

We can use box plots to describe the spread of the data in two ways.

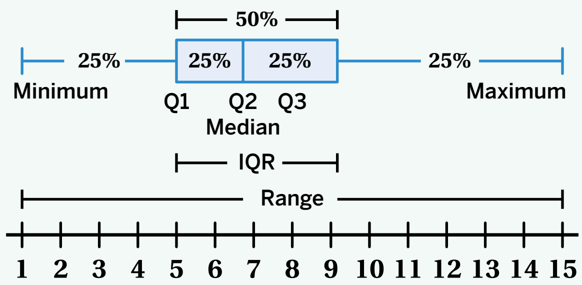
Min.	Q1	Median	Q3	Max.
1	5	6.75	9.25	15

- The **range** represents the difference between the *maximum* and *minimum* values of a data set. It describes the overall spread of the data.

  Range: $15 - 1 = 14$

- The **interquartile range (IQR)** represents the range of the middle 50% of the data (between Q3 and Q1). It describes how spread out the middle of the data is.

  IQR: $9.25 - 6.75 = 2.5$



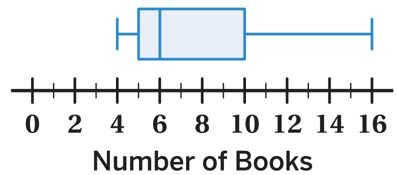
Box plots do not show how many data points are in each set, or the values of any individual data points, except the minimum and maximum.

Things to Remember:

Lesson Practice
6.8.14

Name: ________________ Date: ________ Period: ________

Problems 1–3: Each student in a class recorded how many books they read in a school year.



1. What is the greatest number of books that a student read this year?

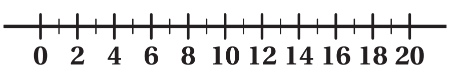
2. What is the median of this data?

3. What is the range of this data?

Problems 4–5: Here are five statistics about a data set.

Minimum	Q1	Median	Q3	Maximum
4	6	9	13	19

4. Create a box plot that represents this data set.



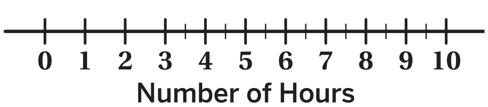
5. What is the IQR of this data set?

Problems 6–9: A group of seventh-grade students recorded the number of hours they spent doing homework in one week. Here is their data: 3, 4, 5, 5, 6, 7, 7, 9, 9, 10.

6. Determine each of the statistics.

Minimum	Q1	Median	Q3	Maximum

7. Create a box plot of this data.



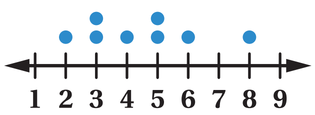
8. Based on the data above, is the following statement true or false? Most seventh-graders do less than 5 hours of homework in a week.

9. Based on the data above, is the following statement true or false? 25% of seventh-grade students spent between 9 and 10 hours doing homework in a week.

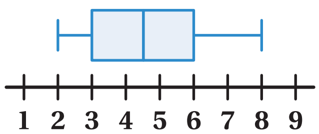
Name: ________ Date: ________ Period: ________

Problems 10–11: Here are three representations of the same data set.

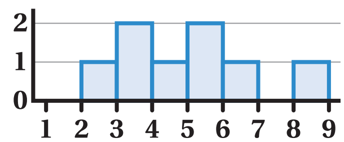
Dot Plot



Box Plot



Histogram



10. One representation has a mistake. Circle the incorrect representation.

11. Describe the mistake made in creating the representation you chose in the previous problem.

Spiral Review

12. Which of these has the largest value?

A. 25% of 80

B. 100% of 60

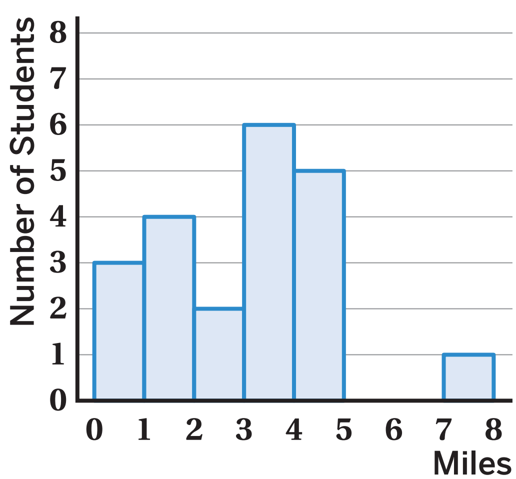
C. 75% of 100

D. 50% of 120

13. Nasir surveyed his classmates to determine how many miles they travel to school each day. This histogram shows the results. Select *all* the true statements.

- ☐ **A.** Most of the data is between 4 and 7 miles.
- ☐ **B.** The median of the data values is between 3 and 4 miles.
- ☐ **C.** The interquartile range of the data values is greater than 5.
- ☐ **D.** No students travel between 2 and 3 miles.
- ☐ **E.** Most of the data is between 3 and 5 miles.

Distance Traveled to School



Reflection

1. Circle the problem you think will help you most on the End-of-Unit Assessment.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

To answer a question about a population, it is sometimes not realistic to collect data from the entire **population**. Instead, you can collect data from a **sample** of the population.

- A *population* is a set of people or things that we want to study.
- A *sample* is part of a population.

The sample you choose should be large enough to be able to draw conclusions about the population.

Here are some examples of populations and samples.

Population	Sample
All of the people who watch basketball	The people at a basketball game
All 7th grade students in your school	The 7th graders in your school who are in a band
All oranges grown in the U.S.	The oranges in your local grocery store

Things to Remember:

Lesson Practice
7.8.10

Name: .... Date: .... Period: ....

Problems 1–3: Aniyah wonders: How much time do 7th graders at my school spend outdoors on a typical day?

1. What is the population for Aniyah's question?

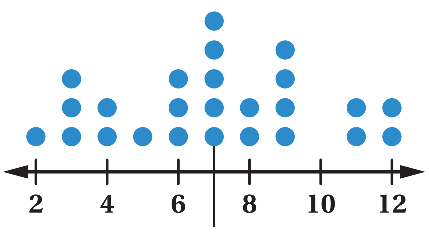
2. Select *all* possible samples for Aniyah's question.

- ☐ **A.** The 20 students in a 7th grade math class
- ☐ **B.** The first 20 people to arrive at Aniyah's middle school on a particular day
- ☐ **C.** The 7th graders participating in a science fair with students from four middle schools
- ☐ **D.** The 10 7th graders on the school soccer team
- ☐ **E.** The students on the high school debate team

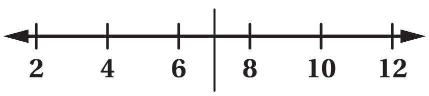
3. Select two samples and list a different possible population each sample could belong to.

Sample	Possible Population
The 20 students in a 7th grade math class	
The first 20 people to arrive at Aniyah's middle school on a particular day	
The 7th graders participating in a science fair with students from four middle schools	
The 10 7th graders on the school soccer team	
The students on the high school debate team	

4. Here is a dot plot of a population. Create a dot plot of a sample with a mean that's the same as the population mean. Your sample should have more than 6, but fewer than 20 data points.



Population mean: 7



Sample mean

Lesson Practice
7.8.10

Name: .... Date: .... Period: ....

Spiral Review

5. Select *all* the measures of center.

- ☐ **A.** Mean
- ☐ **B.** IQR (interquartile range)
- ☐ **C.** Range
- ☐ **D.** Median
- ☐ **E.** MAD (mean absolute deviation)

6. There are 50 marbles in a bag. Students picked a marble, recorded its color, and put it back in. Each student carried out a different number of experiments. The table shows their results. Estimate the probability of getting a green marble from this bag. Explain your thinking.

	Number of Experiments	Green Marbles
Student 1	4	1
Student 2	12	5
Student 3	9	3

Problems 7–9: Calculate the mean of each data set.

7. 8, 9, 9, 9, 10

8. 2, 6, 12, 16

9. 5, 6, 12, 13

Problems 10–11: A bookstore has a 15% discount on all books.

10. How much money is the 15% discount worth on a book that normally costs $18?

11. After the discount, how much would the book cost?

Reflection

1. Put a star next to a problem where you revised your thinking.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Samples are useful when a population is too large to survey or measure. Depending on the strategy you use to sample, your sample might or might not be **representative** of the population. Some samples are not good representations of the population.

A representative sample has a distribution that closely resembles the distribution of the population. Representative samples are useful for making predictions about the whole population.

For example, if you were curious about all middle school students' favorite sport to play, the population would be all middle schoolers. A representative sample of this population might be randomly selecting 5 students from each class or 15 students from each grade to ask. A sample that is *not* representative of this population would be asking students in the tennis club because their responses might lead someone to believe that tennis is the favorite sport among all middle schoolers.

Things to Remember:

Name: .... Date: .... Period: ....

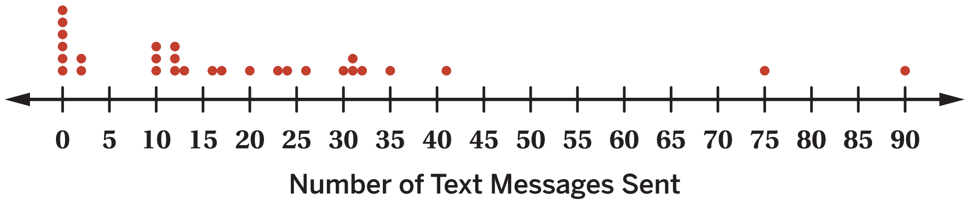
1. Select *all* the reasons why random samples tend to produce a more representative sample.

- ☐ **A.** You can determine how many people you want in the sample.
- ☐ **B.** It's the easiest way to select a sample from a population.
- ☐ **C.** It avoids the bias that can occur with other sampling methods.
- ☐ **D.** Each person in the population has an equal chance of being selected.
- ☐ **E.** The sample mean will always be the same as the population mean.

2. Jada wants to learn about the percentage of students who like the food in the cafeteria. Jada asks the first 25 students who purchase lunch at the cafeteria if they like the food.

Is Jada's method likely to produce a representative sample? Explain your thinking.

Problems 3–4: This dot plot shows the number of text messages sent on one day for a sample of students at a high school. 29 random students were sampled.



3. What do the six dots at 0 on the dot plot represent?

4. Because this sample is representative of the population, describe what a dot plot for the entire high school might look like.

Lesson Practice
7.8.11

Name: .... Date: .... Period: ....

Problems 5–8: Think of a new situation.

5. Write a question you're interested in finding the answer to.

6. What is the population for your question?

7. Describe a strategy that is unlikely to produce a representative sample to answer your question.

8. Describe a strategy to get a representative sample to answer your question.

Spiral Review

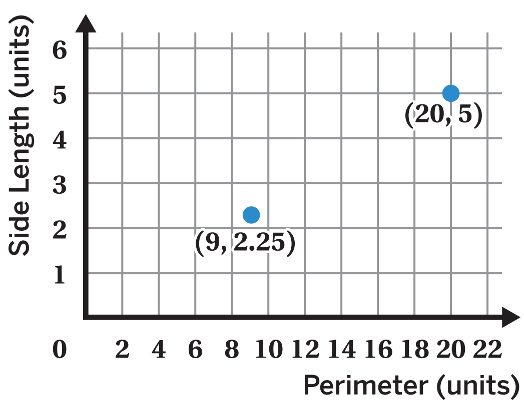
9. Based on a weather report, the probability that it will snow tomorrow is 0.9. Which word describes the likelihood that it will snow tomorrow?

A. Certain **B.** Impossible **C.** Likely **D.** Unlikely

Problems 10–11: This graph shows the side length of a square and its perimeter.

10. Plot and label two more points on the graph.

11. Is there a proportional relationship between the perimeter and side length? Explain your thinking.



Reflection

1. Put a question mark next to a problem you're feeling unsure of.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice 7.8.12

Lesson Summary

How can we use data from a sample to make claims about a population? One way is to use proportional reasoning.

Let's say someone is wondering how many students at their school might vote for a candidate for student council. It would be challenging and time consuming to ask all 500 students at the school, so they collect a sample of 25 students. It is important to gather the sample in a way that makes sure the sample is likely to be representative, like asking one student from each homeroom or asking 25 students at random.

If 10 out of the 25 students in the sample said they would vote for this candidate, there are several strategies for making a prediction about the population.

Strategy A

10 out of 25 is equal to $\frac{10}{25}$ or 40% of the sample.

40% of the population (500 students) would be $0.4 \cdot 500 = 200$ students.

Strategy B

The population is $\frac{500}{25} = 20$ times as large as the sample, so multiply the number of votes by 20 to determine the number of students in the population who would vote for them.

Votes	Total Students
10	25
200	500

Things to Remember:

Name: ________ Date: ________ Period: ________

Problems 1–3: Faaria and Ariel wondered what percentage of students at school would dye their hair blue. They each surveyed a different random sample of the students at school.

1. Faaria asked 10 students, and 2 of them said they would. Based on Faaria's sample, what percent of the students would dye their hair blue?

2. Ariel asked 100 students, and 17 of them said they would. Based on Ariel's sample, what percent of the students would dye their hair blue?

3. Whose percentage is likely to be closest to the percentage of all the students? Explain how you know.

Problems 4–6: In a school of 580 students, one class was asked which hand they write with. "L" means they use their left hand, and "R" means they use their right hand.

Here are the results: L, R, R, R, R, R, R, R, R, L, R, R, R, R, R

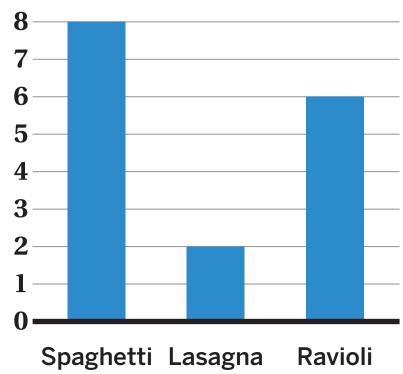
4. Based on this sample, estimate the percentage of students at the school who write with their left hand.

5. Estimate the number of students at the school who write with their left hand.

6. A different class of 18 students is surveyed. Estimate how many write with their left hand. Explain your thinking.

7. 16 students at a school were asked about their favorite pasta dish. The results are shown in the bar graph. Create a new bar graph showing the possible results for all 400 students in the school. Be sure to scale and label the vertical axis.

Class Data



School Data

Lesson Practice
7.8.12

Name: .... Date: .... Period: ....

Spiral Review

8. Match each expression in the first list with an equivalent expression from the second list.

a. $(8x + 6y) - (2x + 4y)$ .... $10x - 10y$

b. $(8x + 6y) - (2x - 4y)$ .... $10x - 2y$

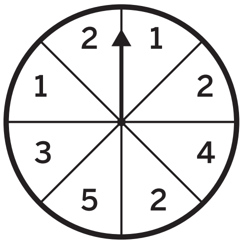
c. $8x - 6y - (\text{-}2x + 4y)$ .... $6x + 2y$

d. $8x - 6y - (\text{-}2x - 4y)$ .... $6x + 10y$

9. Complete the table so that each column has an equivalent fraction, decimal, and percent.

Fraction	$\frac{2}{5}$		$\frac{3}{10}$	
Decimal		0.75		0.125
Percent	40%	75%		

10. A spinner is divided into 8 equal sections.



If the arrow is spun only once, what is the probability that it will land on a number greater than 3?

A. $\frac{1}{4}$

B. $\frac{2}{3}$

C. $\frac{5}{8}$

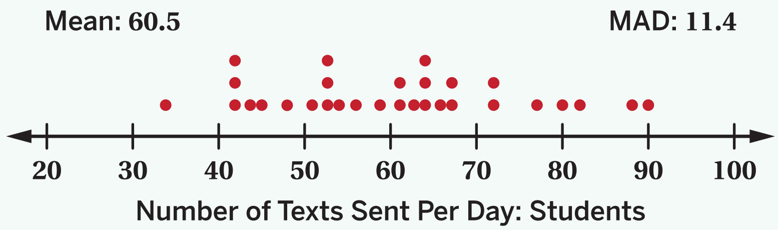
D. $\frac{1}{2}$

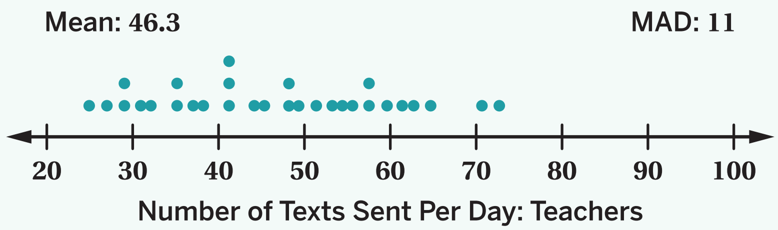
Reflection

1. Put a star next to a problem that looked more difficult than it really was.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

In general, it is easier to compare two individuals or objects than it is to compare two populations. For example, you can answer the question, "Which 7th grader is taller?" by measuring the heights of two 7th grade students and comparing them directly. However, to answer the question, "Do middle school students send more texts per day than their teachers?" you need to collect samples and analyze the measures of center and variability.





Here are the results of a random sample of 30 students and 30 teachers surveyed about the number of texts they send per day. To decide if the data sets are very different from each other, we can calculate the difference in their means and compare it to the larger MAD. The difference is $60.5 - 46.3 = 14.2$, which is about 1.25 times the MAD of 11.4. When the difference is more than 1 times the larger MAD, the data sets are very different. This suggests that students do send more texts than teachers.

Things to Remember:

Name: ______________ Date: ________ Period: ________

Problems 1–3: A school's art club held a fundraiser to raise money for art supplies. This data shows the number of T-shirts sold each week during each season.

Fall

20	26	25	24	29
20	19	19	24	24

MAD: 2.8

Spring

19	27	29	21	25
22	26	21	25	25

MAD: 2.6

1. Determine the mean number of T-shirts sold in the fall and in the spring.

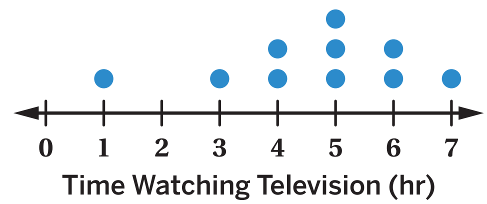
2. Calculate how many MADs apart the means are. Use the larger MAD in your calculation.

3. Based on this data, are sales generally higher in the spring than in the fall? Explain your thinking.

Problems 4–5: Abdullah compared the time that students and teachers spent watching television over the weekend. Abdullah took a random sample of 10 students and 10 teachers and made a dot plot of their responses.

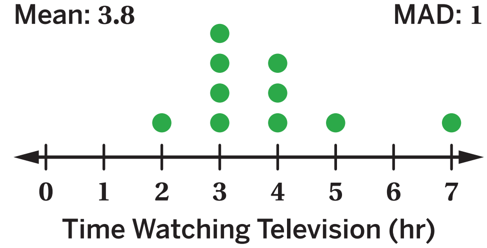
Students

Mean: 4.6 **MAD: 1.3**



Teachers

Mean: 3.8 **MAD: 1**



4. Is there a big difference between the students' data and the teachers' data? Explain your thinking.

5. Abdullah then took a random sample of 10 parents and found that they watched a mean of 2.5 hours of television, with a MAD of 1 hour.

Is there a big difference between how much television parents and students watch? Explain your thinking.

Name: ______________ Date: ________ Period: ________

6. These dot plots represent the time that it took new runners to run a mile.

Which statement is *best* supported by the information in the dot plots?

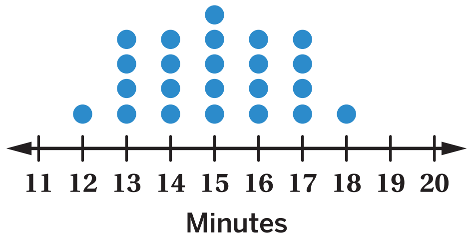
A. The two populations have an equal number of runners.

B. The two populations have the same range.

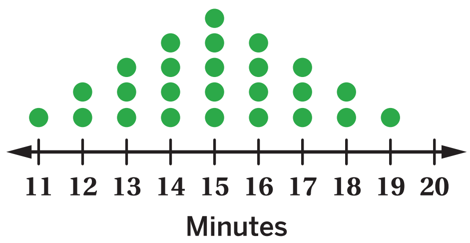
C. The two populations have different medians.

D. The two populations have the same mean.

Population 1



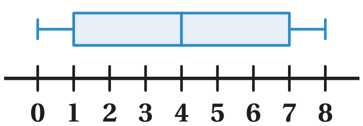
Population 2



Spiral Review

Problems 7–9: Determine the interquartile range (IQR) for each box plot.

7.



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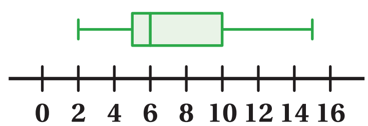

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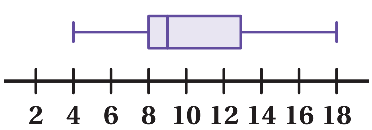


10. Sora says that 0.77 is a *repeating decimal* because both digits are the same. Is Sora's statement correct? Explain your thinking.

Reflection

1. Star the problem you spent the most time on.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

The **probability** of an event is a number that represents how likely the event is to occur. One way to calculate the probability is to look at all of the possible outcomes for an experiment, which is known as the **sample space**.

When all of the outcomes are equally likely, the probability of an event is a ratio.

$$\frac{\textit{number of favorable outcomes}}{\textit{total possible number of outcomes}}$$

Probabilities are numbers between 0 and 1 written as fractions, decimals, or *percentages*. A probability of 1 means the event will always happen. A probability of 0 means the event will never happen.

Here are some examples of events and their probabilities.

Example	Probability
Picking a green marble out of a bag that contains only red and yellow marbles	0
Rolling a 1 on a number cube	$\frac{1}{6}$ (or equivalent)
Flipping a coin and it landing heads up	50% (or equivalent)
Picking a yellow marble from a bag of 10 marbles, where 8 of the marbles are yellow	0.8 (or equivalent)
Picking a green marble in a bag that only contains green marbles	1

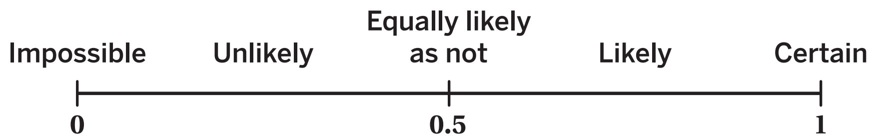
Things to Remember:

Lesson Practice
7.8.02

Name: ________ Date: ________ Period: ________

1. Place a point on the line to show each probability's likelihood from impossible to certain.

60%	8 out of 10	0.37	20%	$\frac{5}{6}$



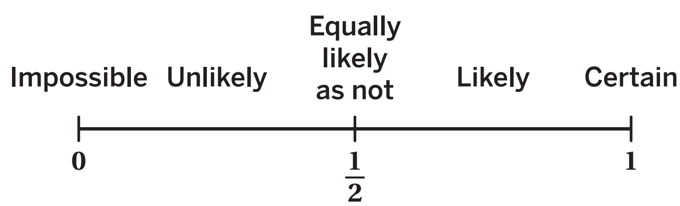
Problems 2–4: List the sample space for each experiment. For example, the sample space of flipping one fair coin is heads or tails.

2. Selecting a random season of the year

3. Selecting a random day of the week

4. Rolling a number cube

Problems 5–10: Determine the probability of each event. Use the line if it helps with your thinking.



5. You read this sentence.

6. A fair coin lands heads up when flipped.

7. A weekend day is selected randomly from the days of the week.

8. You see a live unicorn outside today.

9. A spinner with equal parts red, yellow, and green lands on green on the next spin.

10. You pick a green block when you randomly pick one block from a bag with 7 green blocks and 3 blue blocks.

Name: .... Date: .... Period: ....

Problems 11–13: A computer randomly selects a letter from the English alphabet.

11. How many different outcomes are possible?

12. What is the probability the computer selects a vowel (A, E, I, O, or U)?

13. What is the probability the computer selects the first letter of your first name?

14. This table shows the number of bags of different flavors of crackers in a large box. A bag will be selected at random from the box.

Flavor of Cracker	Number of Bags
Plain	12
Barbeque	16
Ranch	8
Cheddar Cheese	20

Which statement about the flavor of the crackers chosen is best supported by the information in the table?

A. Plain is the least likely flavor.

B. Choosing barbecue is twice as likely as choosing ranch.

C. There is an equal chance of choosing plain, barbeque, ranch, or cheddar cheese.

D. Choosing cheddar cheese is twice as likely as choosing ranch.

Spiral Review

Problems 15–16: e represents an object's weight on Earth and m represents that same object's weight on the Moon. The equation $m = \frac{1}{6}e$ represents the relationship between these quantities.

15. What does $\frac{1}{6}$ represent in this situation?

16. If a person weighs 24 pounds on the Moon, how many pounds would they weigh on Earth?

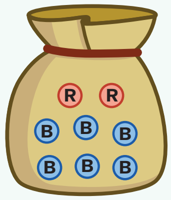
Reflection

1. Star a problem you're still feeling confused about.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice 7.8.03

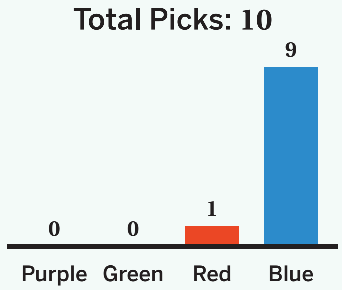
Lesson Summary

In situations where you don't know the sample space, you can use data from experiments and proportional reasoning to predict what the sample space is. Repeating an experiment more times can help make your prediction more accurate.



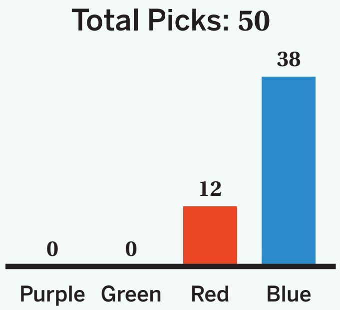
For example, here are the results from picking a marble out of a mystery bag 10 times. The bag has 8 marbles in it.

Only 1 out of the 10 marbles was red, so the *constant of proportionality* is 0.1. Multiplying 0.1 times the number of marbles in the bag (8), may lead someone to predict that there is only $0.1 \cdot 8 = 0.8$ or 1 red marble in the bag.



After 50 picks, the constant of proportionality (0.24) times the number of marbles in the bag is $0.24 \cdot 8 = 1.92$, which is close to 2.

This is a more accurate prediction of the number of red marbles in the bag.



Things to Remember:

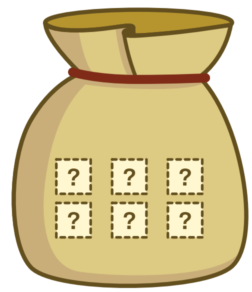
Name: ______ Date: ______ Period: ______

Problems 1–2: A month is chosen at random from the months of the year.

1. What is the probability of getting a month that starts with the letter "A?" Consider listing the sample space to show your thinking.

2. If you repeat this experiment 600 times, how many times do you expect to get a month that starts with "A?"

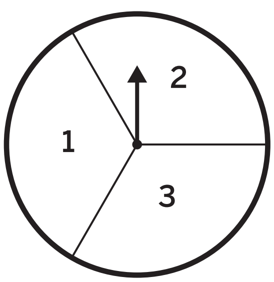
3. A bag has 6 blocks in it. Joel picks a block out of the bag 60 times and gets a green block 43 times. Based on these results, how many blocks do you expect to be green? Explain your thinking.



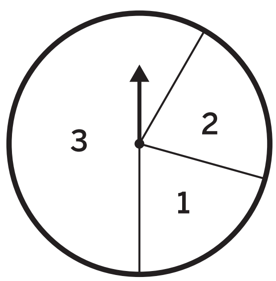
4. Miko spun a spinner with numbered sections 15 times. Here are the results. Which spinner is most likely the one Miko used?

Miko's results:
3, 3, 3, 1, 3, 2, 2, 3,
3, 1, 3, 1, 3, 3, 2

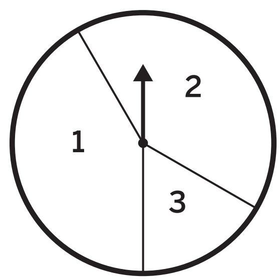
A.



B.



C.



Explain your thinking.

Problems 5–7: A textbook has 428 numbered pages, starting with 1. You are equally likely to stop on any of the pages if you flip through the book randomly.

5. What is the probability that you turn to page 45?

6. What is the probability that you turn to an even-numbered page?

7. If you repeat this experiment 50 times, about how many times do you expect to turn to an even-numbered page?

Name: .... Date: .... Period: ....

8. The table shows the number of different colors of crayons in a case. A 7th grader will randomly choose one crayon from the case.

Based on the information in the table, which statement is true?

Color	Number of Crayons
Blue	2
Red	8
Orange	4
Yellow	5

A. The crayon is least likely to be orange.

B. The crayon is 4 times as likely to be red as it is to be blue.

C. The crayon is equally likely to be orange or yellow.

D. The crayon is more likely to be red than all other colors combined.

Spiral Review

Problems 9–11: For each word, determine the probability of selecting the letter "A" at random.

9. LAMB

10. SAFETY

11. ALABAMA

12. Select *all* the situations in which the surface area would be useful to know.

☐ **A.** Ordering tiles to replace the roof of a house

☐ **B.** Estimating how long it will take to clean the windows of a greenhouse

☐ **C.** Deciding whether leftover soup will fit in a container

☐ **D.** Estimating how long it will take to fill a swimming pool with a garden hose

☐ **E.** Buying fabric to sew a couch cover

Reflection

1. Circle a problem you are still curious about.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Repeated experiments can help you decide if an object is fair.

If an experiment is repeated only a few times, the results may not be what you expect, even if the object is fair. The more times you repeat the experiment (i.e., hundreds or thousands of times), the closer the relative frequency should get to the probability. This allows you to make a better decision about whether the object is fair.

For example, here is a fair coin. The probability of this coin landing heads up is $\frac{1}{2}$.

If you flip the coin only 3 times, it may land heads up all 3 times. You may think the coin is unfair, but continuing to repeat the experiment can change your perspective.

If you flipped the coin $1{,}000$ times, it would land heads up about half of the time because the sample space of this event is "heads" and "tails."

Things to Remember:

Lesson Practice
7.8.05

Name: ____________ Date: ________ Period: ________

Problems 1–3: Deja has a six-sided number cube.

1. If this were a standard number cube, what would be the probability that the cube lands on a five?

2. Deja suspects the six-sided number cube is not standard.

- Deja rolled a five 40 times out of 100.
- Amor rolled a five 21 times out 50.
- Santino rolled a five 11 times out of 30.

Based on these results, estimate the probability of rolling a five.

3. Is it likely this is a standard number cube? Explain your thinking.

Problems 4–5: Santino flips a coin 10 times to see if it is fair. It lands heads up 3 times and tails up 7 times.

4. Are these results enough to determine if the coin is fair? Explain your thinking.

5. What could Santino do to be more sure about these results?

Problems 6–9: A game is played with two tetrahedral (4-sided) dice. The dice are in the shape of a pyramid with the numbers 1, 2, 3, and 4 written on each triangular side. After the dice are rolled, the numbers on the sides facing down are added together.

6. What are all of the possible sums for rolling two tetrahedral dice?

7. What sum do you think you will be most likely to get when rolling these dice? Least likely? Explain your thinking.

8. Create rules for a fair two-player game that uses these dice.

Player A wins if...

Player B wins if...

9. Explain why you think your game in Problem 8 is fair.

10. A random number generator selects a digit from 1 to 5. Troy uses the generator $1{,}500$ times. Which statement best predicts how many times the digit 3 will appear among the $1{,}500$ results?

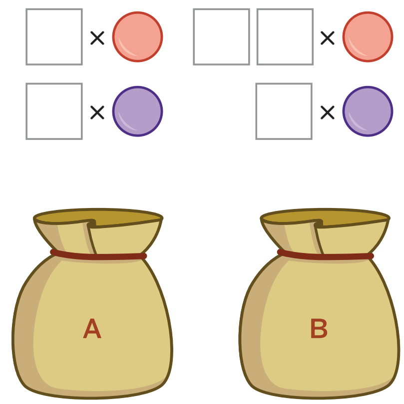
A. It will appear exactly 300 times.

B. It will appear close to 300 times but probably not exactly 300 times.

C. It will appear exactly 340 times.

D. It will appear close to 340 times but probably not exactly 340 times.

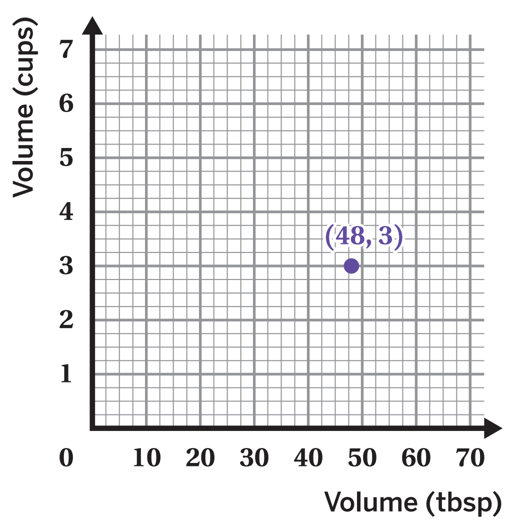
11. Fill in each blank using the digits 0 to 9 only once so that the probability of drawing a purple marble is the same for each bag.



Spiral Review

Problems 12–14: There is a proportional relationship between sugar measured in cups and sugar measured in tablespoons. For example, 48 tablespoons of sugar is equivalent to 3 cups of sugar.

12. Plot and list the coordinates of two more points that represent the relationship.



13. What is the constant of proportionality for this relationship?

14. Write an equation representing this relationship. Use c for cups and t for tablespoons.

Reflection

1. Put a question mark next to a problem you were feeling stuck on.
2. Use this space to ask a question or share something you're proud of.

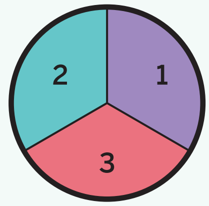
Lesson Practice 7.8.06

Science Mom Lesson 123

Lesson Summary

There are several different ways to make sense of **compound events**, or events that involve multiple steps.

Here is one example: Let's spin a spinner and flip a fair coin. There are 6 outcomes in the *sample space* of this multistep event, which you can see in a list, a table, and a **tree diagram**.



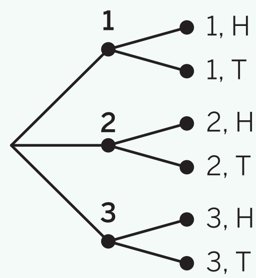
List

1, Heads	1, Tails
2, Heads	2, Tails
3, Heads	3, Tails

Table

	H	T
1	1, H	1, T
2	2, H	2, T
3	3, H	3, T

Tree Diagram

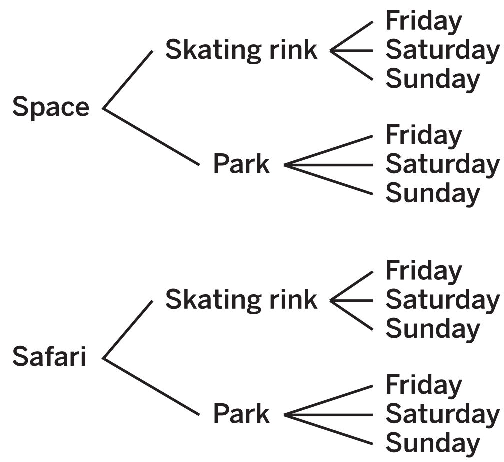


Things to Remember:

Lesson Practice
7.8.06

Name: .... Date: .... Period: ....

Problems 1–5: Farah made a tree diagram to help her choose a theme, location, and day of the week for her birthday party.



1. How many themes is Farah considering?

2. How many locations is Farah considering?

3. How many days is Farah considering?

4. One party Farah is considering is a space theme at the skating rink on Sunday. Write two other possible parties Farah could have.

5. If Farah lets her sibling choose a theme, location, and day at random, what is the probability that Farah's birthday will be a safari at the park on Saturday? Explain your thinking.

Problems 6–7: Isabella selects one type of lettuce and one dressing to make a salad.

- Lettuce types: iceberg, romaine
- Dressings: ranch, Italian, Caesar

6. Create a list, table, or tree diagram to represent all the possible combinations of choices. Then determine the number of possible outcomes.

7. If Isabella picks a type of lettuce and dressing at random, is getting iceberg lettuce or Caesar dressing more likely? Explain your thinking.

Name: .... Date: .... Period: ....

8. Twelve students play a game. Each student is assigned a number 1 to 12. Someone rolls two six-sided number cubes. The student whose number matches the sum gets a point. After 100 rounds, which student is most likely to have the most points? Explain your thinking.

Problems 9–10: Juan and Neo play a game. Each player holds out a hand at the same moment to represent a rock, paper, or scissors. Then they each choose one of the three items.

9. Create a list, table, or tree diagram to represent all the possible combinations of choices.

10. What is the probability that they both choose the same object?

Spiral Review

Problems 11–12: $\frac{1}{3}$ produces a decimal that repeats every one digit. What fraction produces each of the following:

11. A decimal that repeats every two digits.

12. A decimal that terminates after two digits.

13. Select *all* the true equations.

- ☐ **A.** $8 = (8 + 8 + 8 + 8) \div 3$
- ☐ **B.** $(10 + 10 + 10 + 10 + 10) \div 5 = 10$
- ☐ **C.** $(6 + 4 + 6 + 4 + 6 + 4) \div 6 = 5$
- ☐ **D.** $4 = (4 + 2 + 4 + 2) \div 4$
- ☐ **E.** $(2 + 2 + 2 + 2 + 2) \div 2 = 2$

Reflection

1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Simulations are experiments that are used to estimate the probability of a real-world event. They are especially useful for estimating the probabilities of compound events, such as determining the probability that it will rain at least once over a three day period. In order to design a good simulation, first determine the probability of the individual events occuring.

For example, you could use a coin, number cube, or spinner to simulate a 50% probability of rain.

Flipping a Coin

Landing heads up

$\left(\frac{1}{2} = 50\%\right)$

Rolling a Number Cube

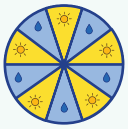
Rolling an even number

$\left(\frac{3}{6} = 50\%\right)$

Using a Spinner

Spinning a raindrop

$\left(\frac{5}{10} = 50\%\right)$



To simulate the probability of rain over three days where each day has a 50% chance of rain, you can use three coins, number cubes, or spinners and repeat the experiment many times.

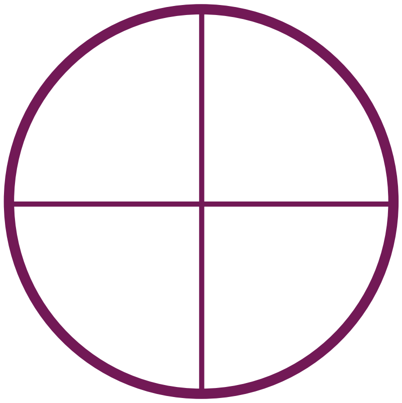
Things to Remember:

Lesson Practice
7.8.07

Name: .... Date: .... Period: ....

Problems 1–3: The weather forecast says there is a 75% chance it will rain today.

1. Design a spinner you could use to simulate a 75% chance of rain.



2. Explain why using a number cube to simulate this probability may be less useful than using a spinner.

3. Describe or draw a different way you could simulate this probability.

4. Paz has 3 kittens. According to the vet, each kitten is born with blue eyes and there is a 50% chance they will change color once the kittens reach three months. Paz designs a simulation using 3 coins, where heads represent the eyes changing color.

The table shows the results of 100 experiments.

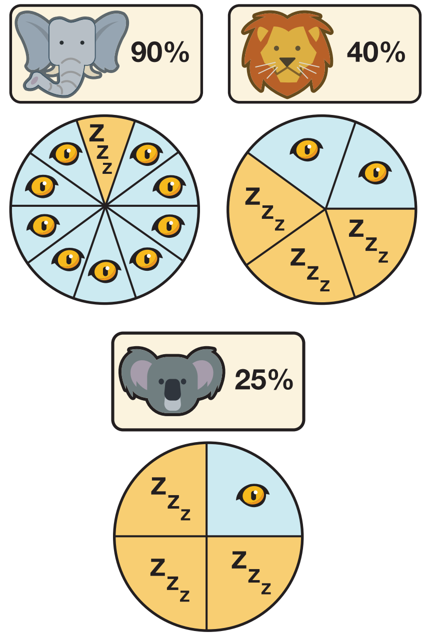
Estimate the probability that at least one of Paz's kittens will still have blue eyes at three months old. Explain your thinking.

Experiments with:	Count
No blue-eyed kittens	11
1 blue-eyed kitten	32
2 blue-eyed kittens	43
3 blue-eyed kittens	14

Lesson Practice
7.8.07

Name: ______________________ Date: __________ Period: __________

Problems 5–6: Brianna designed and used a simulation to help her estimate the probability of seeing her three favorite animals awake when she visits the zoo. She records the results of 300 experiments.



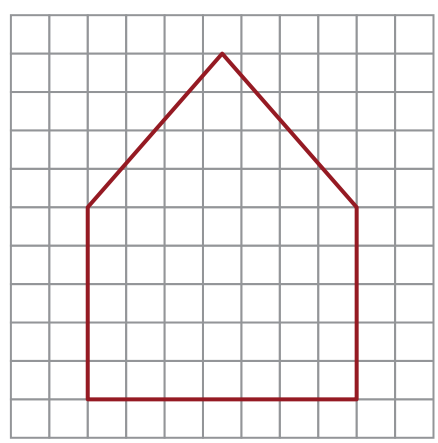
Experiments with:	Count	Relative Frequency
No animals awake	12	4%
1 animal awake	171	57%
2 animals awake	105	35%
3 animals awake	12	4%

5. Estimate the probability that all 3 of her favorite animals will be awake when she visits the zoo.

6. Estimate the probability that at least 2 of her favorite animals will be awake when she visits the zoo.

Spiral Review

Problems 7–8: Here is a diagram of the base of a bird feeder. Each square on the grid represents 1 square inch.



7. What is the area of the base of the bird feeder?

8. The distance between the two bases is 8 inches. What is the volume of the bird feeder?

Problems 9–13: Write each fraction as a percent.

9. $\frac{3}{5}$ **10.** $\frac{1}{50}$ **11.** $\frac{9}{10}$ **12.** $\frac{9}{5}$ **13.** $\frac{18}{60}$

Reflection

1. Put a smiley face next to the problem you learned from most.

2. Use this space to ask a question or share something you're proud of.