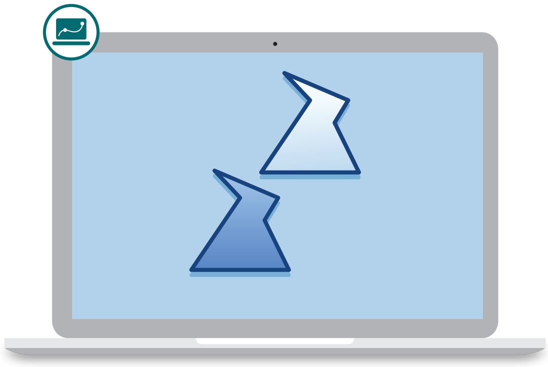
Unit 1
Lesson
2

Name: ______ Date: ______ Period: ______



Spinning, Flipping, Sliding

Let's learn some ways to describe how figures move.

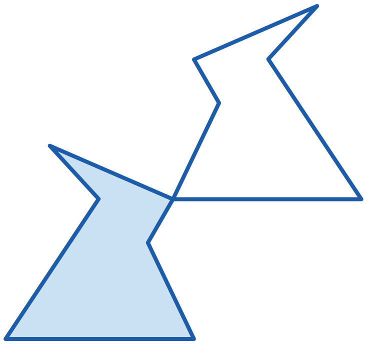
Warm-Up

1 Annika created this shaded figure in the previous lesson.

Let's watch a video of a **transformation**.

What happened to the figure?

***Responses vary*. It moved up, then flipped to the right.**



Activity
1

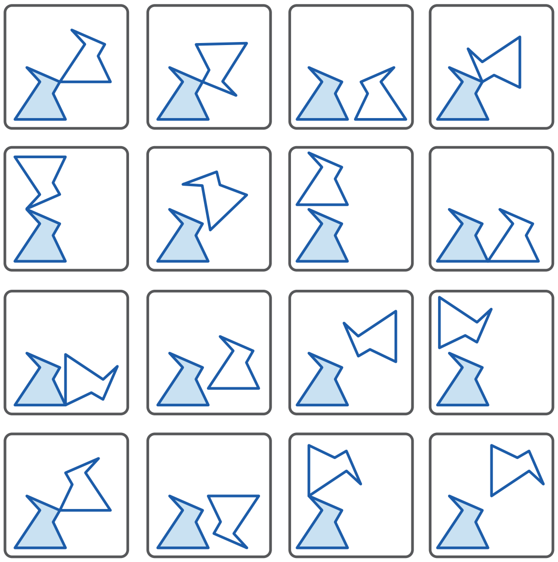
Name: .... Date: .... Period: ....

Describing Transformations

2 Play a few rounds of Polygraph with your classmates!

You will use an Activity 1 Sheet with different *transformation* images for four rounds. For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select an image from the Activity 1 Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating images until you're ready to guess which image the Picker chose.



Record helpful questions from each round in this workspace:

Responses vary.

Activity
1

Name: ______ Date: ______ Period: ______

Describing Transformations (continued)

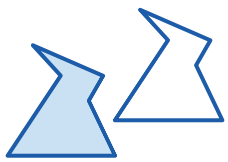
In Polygraph, you saw three types of transformations: **rotations**, **reflections**, and **translations**.

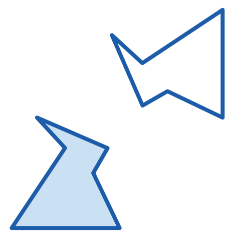
3 Match each word with one of these transformations.

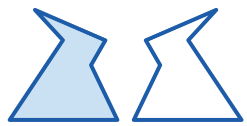
Flip	Mirror	Slide	Spin	Turn

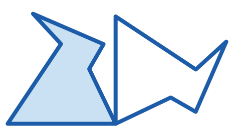
Rotation	Reflection	Translation
Spin Turn	Flip Mirror	Slide

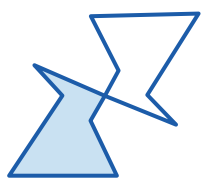
4 For each picture, circle the word that best describes how one figure can move onto the other in a single transformation.

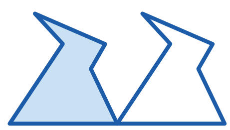
a

Rotation Reflection
Translation

b

Rotation Reflection
Translation

c

Rotation Reflection
Translation

d

Rotation Reflection
Translation

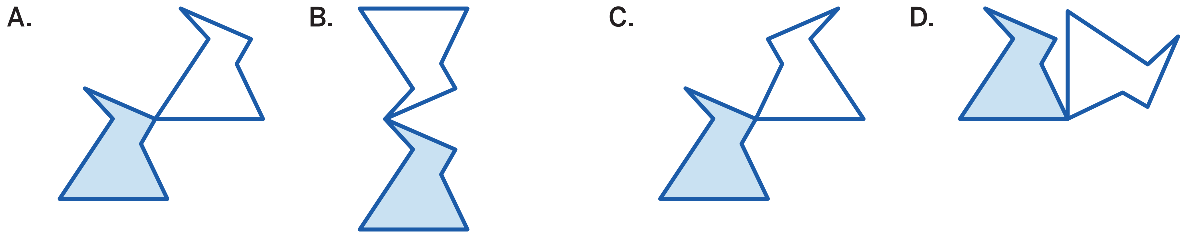
e

Rotation Reflection
Translation

f

Rotation Reflection
Translation

Name: .... Date: .... Period: ....

Rotations, Reflections, Translations

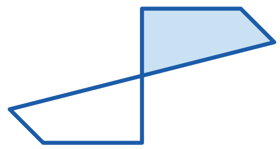
5 Circle an image. Then describe how to move the shaded figure onto the unshaded one. Use at least one of these words in your description: *reflection*, *rotation*, *translation*.



Responses vary.

- **A: Translate to the right and then translate up.**
- **B: Reflect over a horizontal line.**
- **C: Reflect over a vertical line and then translate up.**
- **D: Rotate clockwise 90°.**

6 Mar says you can use one *reflection* to move the shaded figure onto the unshaded one.



Dyani says you can use *one rotation*.

Whose claim is correct? Circle one.

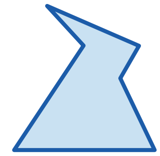
Mar's Dyani's Both Neither

Explain your thinking.

Explanations vary.

- **You could spin the shaded figure to the right 180° around the center point and it would look like the unshaded figure.**
- **You could use reflections like Mar said, except you would need two reflections because you would need to flip the shaded figure down and then to the left for it to look like the unshaded figure.**

7 Synthesis



In your own words, describe what each *transformation* does to a figure.

Responses vary.

Rotation: **A *rotation* spins the figure around. You can spin the figure from the middle or from another point.**

Reflection: **A *reflection* flips the figure over like a mirror, or it's like taking a piece of paper and flipping it over.**

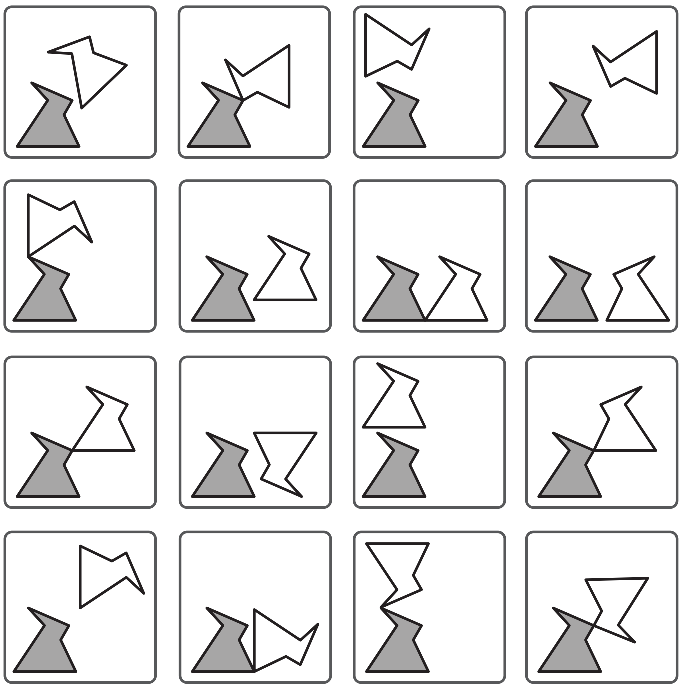
Translation: **A *translation* moves the figure without turning or flipping it.**

Things to Remember:

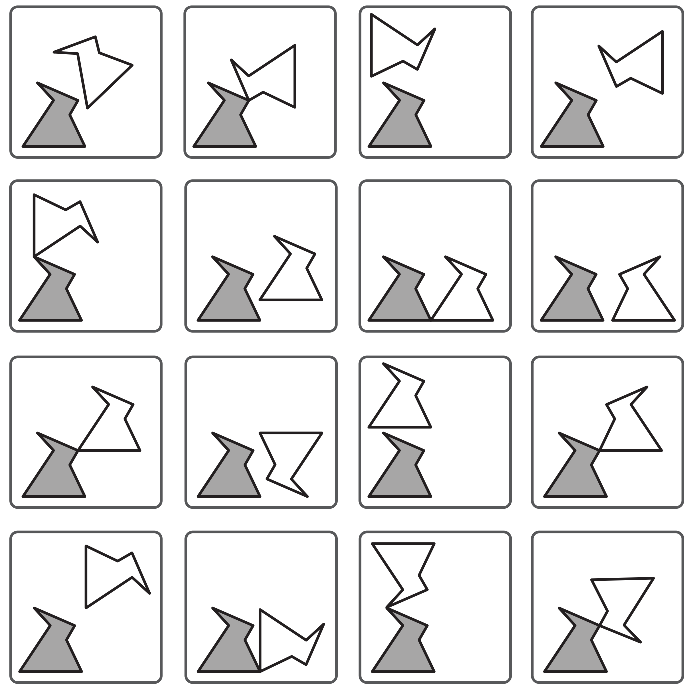
Name: .... Date: .... Period: ....

Polygraph Set A

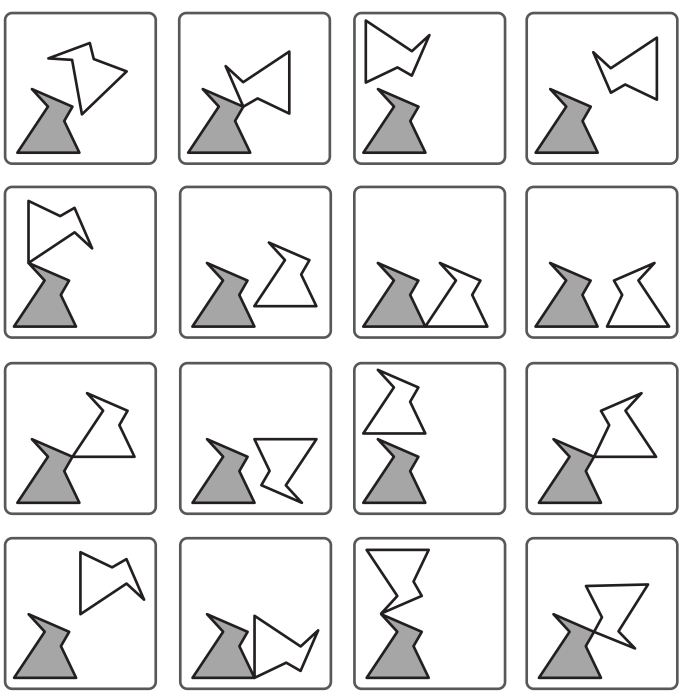
Round 1



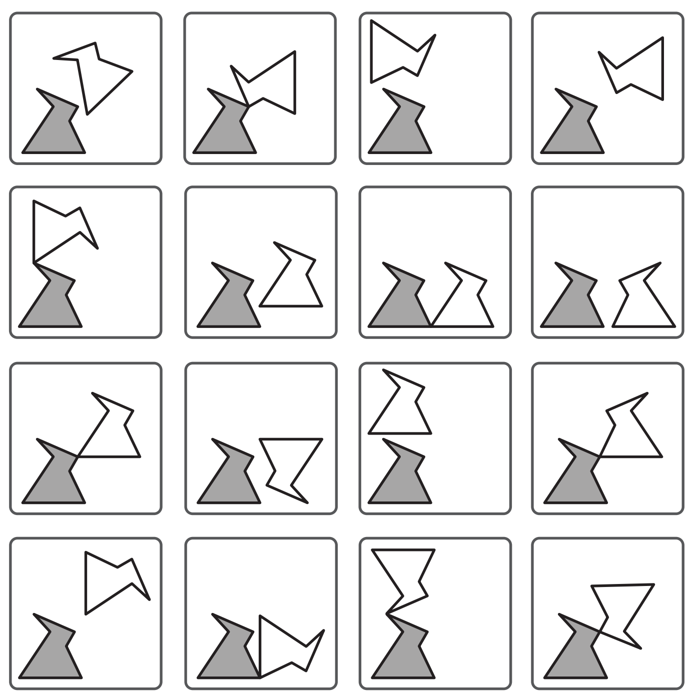
Round 2



Round 3



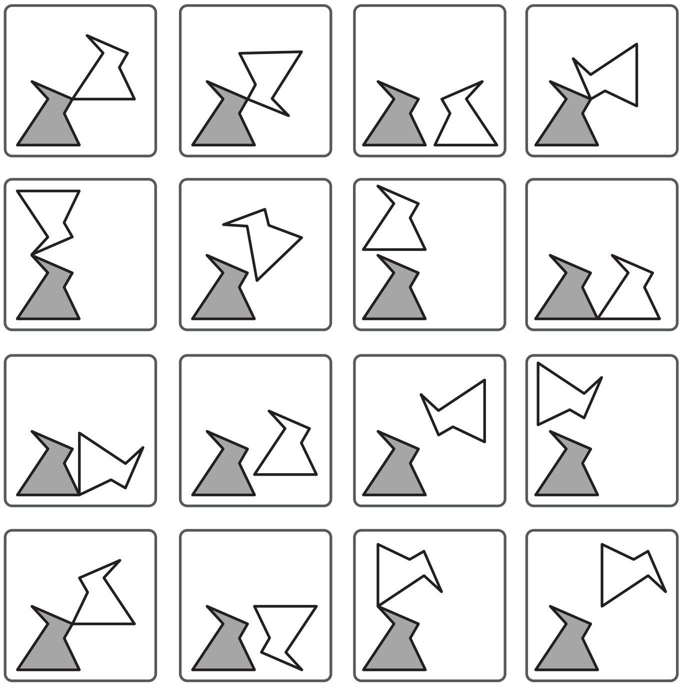
Round 4



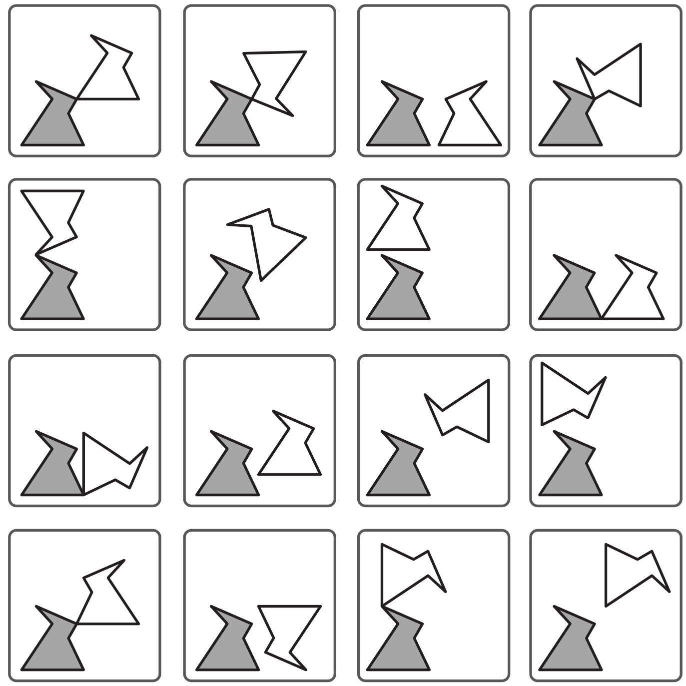
Name: ______________ Date: ______________ Period: ______________

Polygraph Set B

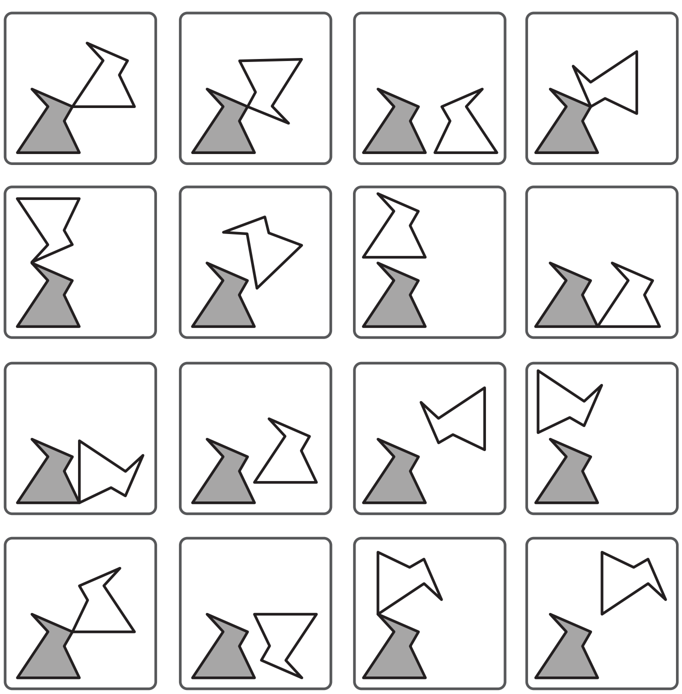
Round 1



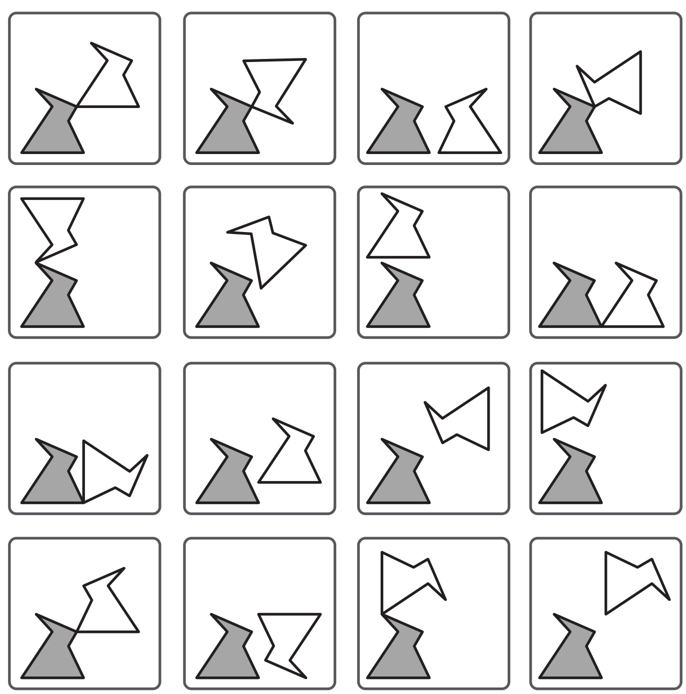
Round 2



Round 3

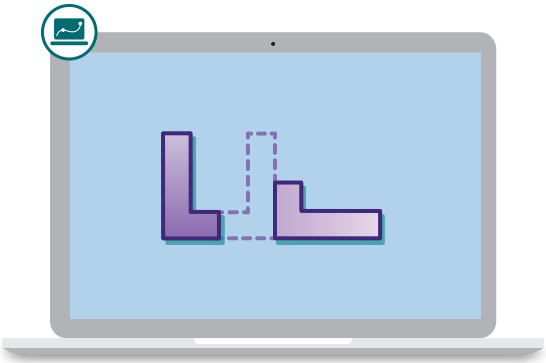


Round 4



Unit 1
Lesson
3

Name: ______ Date: ______ Period: ______

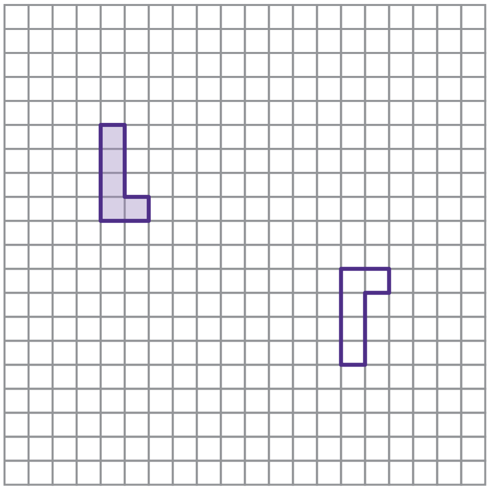


Transformation Targets

Let's explore sequences of transformations.

Warm-Up

1 We have previously seen three types of transformations: translation, rotation, and reflection.



a Let's watch the shaded figure move onto the unshaded figure.

b Describe the **sequence of transformations** you saw.

Responses vary.

- **The shaded figure is translated to the right and then flipped.**
- **The shaded figure is translated 10 units to the right, then reflected.**

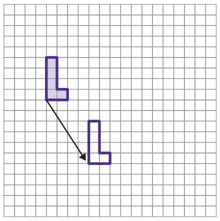
Activity 1

Name: ______ Date: ______ Period: ______

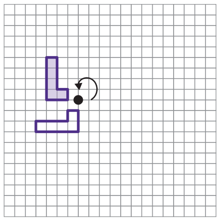
Transformation Targets

Take a look at examples of a translation, rotation, and reflection.

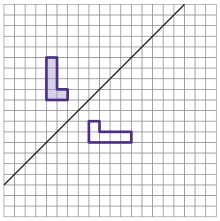
A Translation Down and to the Right



A Rotation 90° Counterclockwise



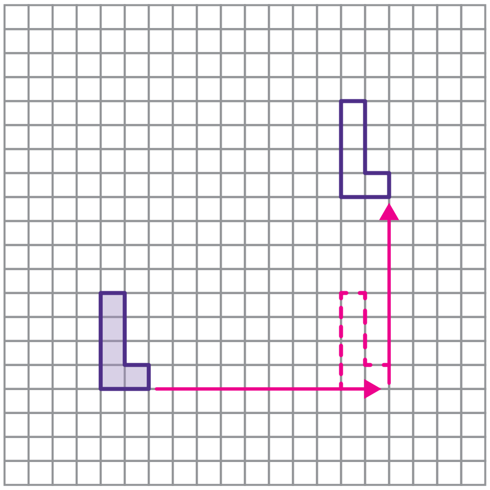
A Reflection Over the Line



You will solve a variety of challenges. For each challenge:

- Show or describe a *sequence of transformations* that moves the shaded figure onto the unshaded figure.
- If it's helpful, draw the new figure after *each* transformation.
- Try to use two or fewer transformations.

2 Challenge #1



Responses vary.
- **A translation right and a translation up. (Sample shown on graph.)**
- **A translation up and to the right.**

3 Let's look at the different translations Ama and Basheera used on the previous challenge. Whose translation is correct? Circle one.

Ama's   Basheera's   Both   Neither

Explain your reasoning.

***Explanations vary.* Ama's translation is correct because the top-left corner of the shaded figure goes to the top-left corner of the unshaded figure. Basheera's translation will *almost* work, but not quite.**

Activity 2

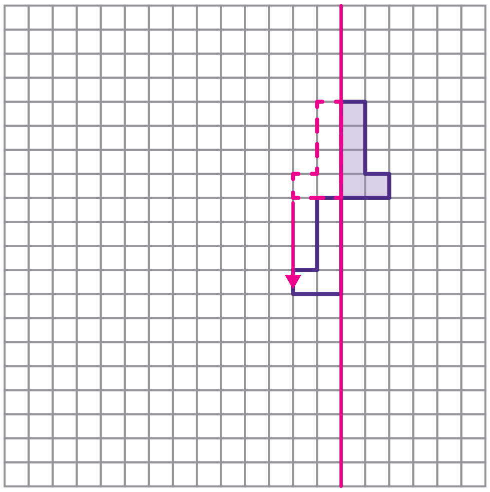
Name: Date: Period:

Sequences of Transformations

4 Challenge #2

Responses vary.

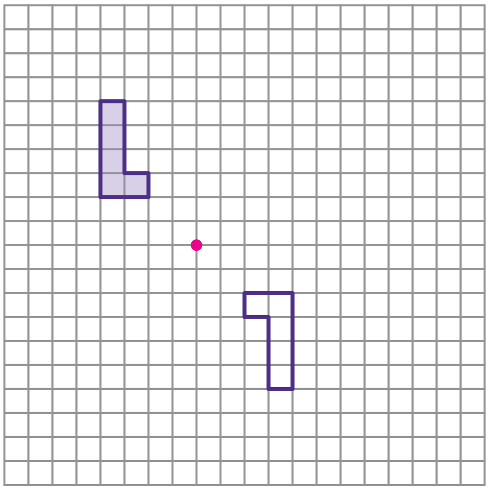
- A reflection over a vertical line and a translation down. (Sample shown on graph.)
- A translation down and a reflection over a vertical line.
- A rotation 180° clockwise around the bottom-left point and a reflection over a horizontal line.



5 Challenge #3

Responses vary.

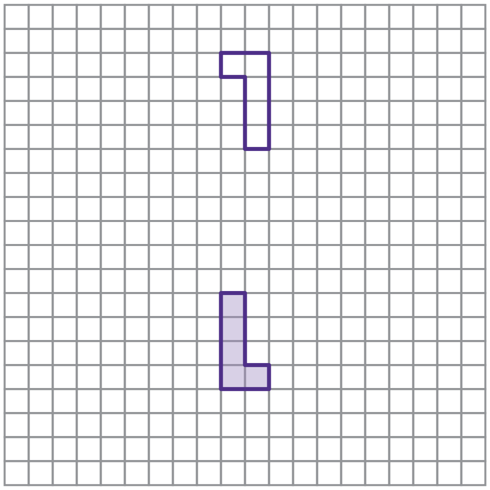
- A reflection over a vertical line and then a reflection over a horizontal line.
- A 180° rotation. (Sample shown on graph.)



6 Describe a sequence of transformations to complete this challenge.

Responses vary.

- Rotate 180° around the center of the grid.
- Imagine a rectangle that encloses the shaded figure. Rotate 180° around the center of that rectangle and then translate up.
- Reflect over a horizontal line that passes through the center of the grid. Then reflect over a vertical line that passes through the center of the grid.



Activity 3

Name: Date: Period:

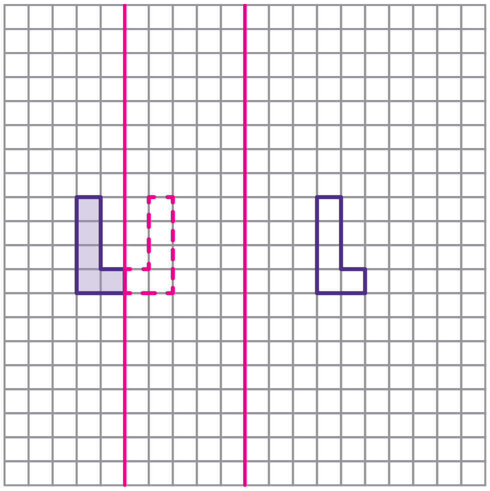
More Transformation Targets

7 Let's try a sequence of transformations for the previous challenge.

8 Challenge #4: Try to solve this challenge using only rotations or only reflections.

Responses vary.

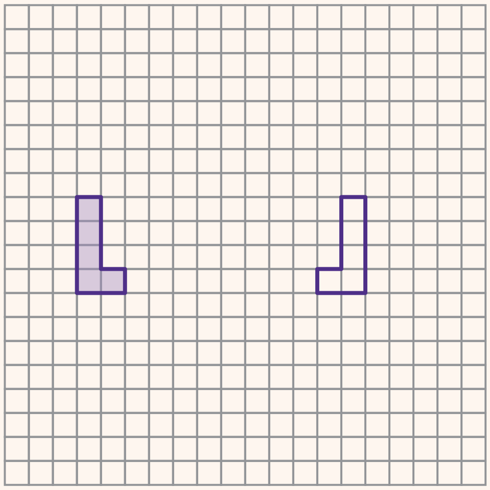
- **Only reflections: Reflect over two vertical lines. (Sample shown on graph.)**
- **Only rotations: Use two 180° rotations.**



Explore More

9 **Discuss:** Is it possible to solve this challenge using only translations and rotations? Explain your thinking.

No. *Explanations vary.* The orientation of the shaded figure and the unshaded figure don't match, so the solution must include a reflection.



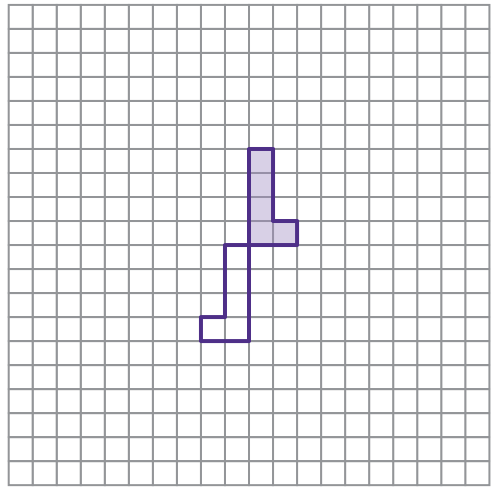
10 Synthesis

Describe some strategies to determine a sequence of transformations to move a shaded figure onto an unshaded figure.

Use the example if it helps with your thinking.

Responses vary.

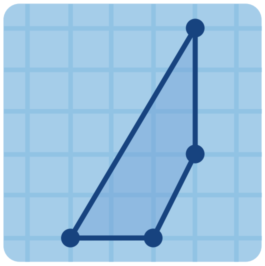
- **Look at which way the figure is facing to determine if a rotation or a reflection could be part of the sequence.**
- **If the location changes but the figure is facing the same direction, use a translation.**



Things to Remember:

Unit 1
Lesson
4

Name: Date: Period:



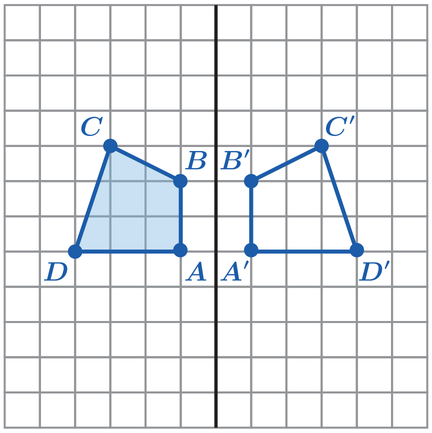
Moving Day

Let's do transformations by hand.

Warm-Up

1. Here is a transformation. The **pre-image** is shaded and the **image** is unshaded.

What do you notice? What do you wonder?



I notice:

Responses vary.

- **I notice the pre-image is reflected over a vertical line on a grid.**
- **I notice that for the image, there is a ′ next to each point label.**

I wonder:

Responses vary.

- **What does the ′ symbol mean in the image point labels?**
- **Does the placement of A', B', C', and D' matter?**
- **Why is it called a pre-image?**

Activity 1

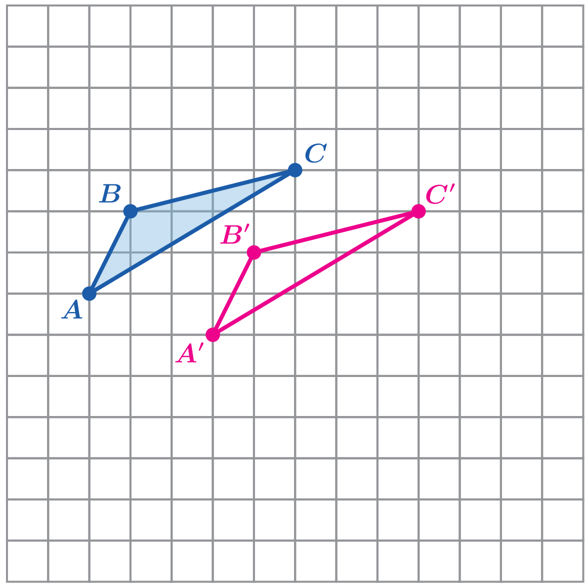
Name: ________ Date: ________ Period: ________

Move It

Perform each transformation. Then label the points in the image to **correspond** with the points in the pre-image.

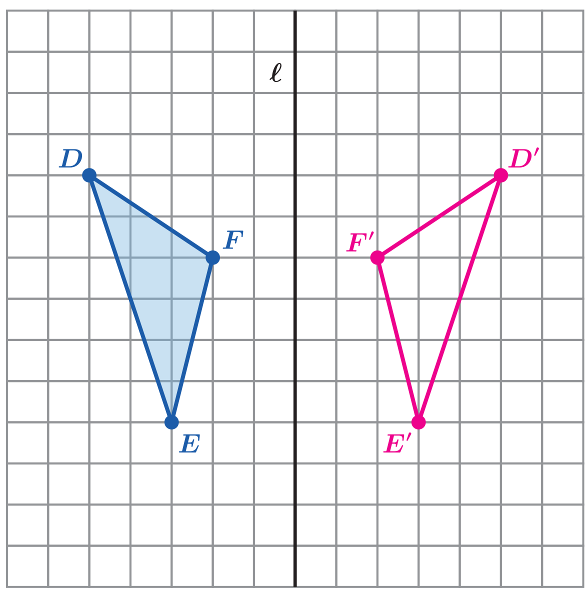
2. Translate triangle ABC 3 units right and 1 unit down.

Response shown on grid.



3. Reflect triangle DEF over line ℓ.

Response shown on grid.

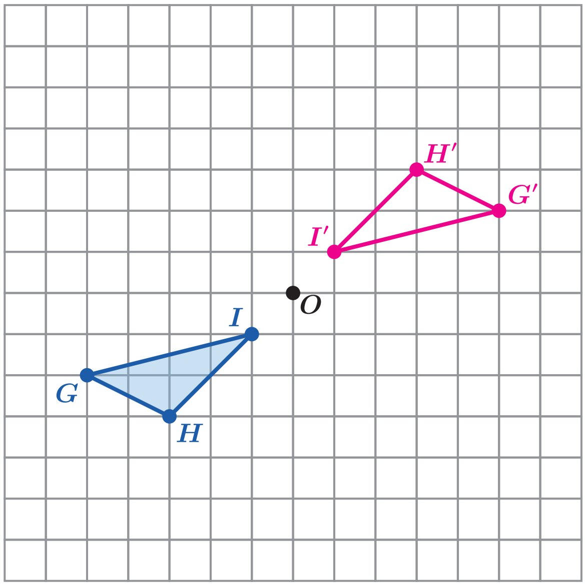


Name: ________ Date: ________ Period: ________

Move It (continued)

4. Rotate triangle GHI 180° counterclockwise around point O.

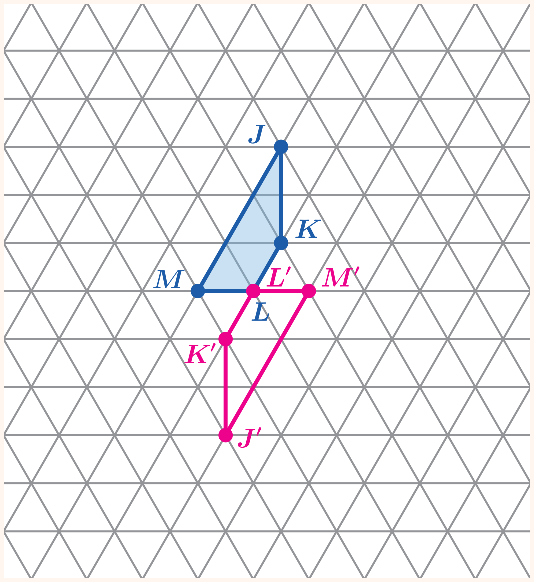
Response shown on grid.



Explore More

5. Rotate figure $JKLM$ 180° clockwise around point L.

Response shown on grid.



Activity 2

Name: .... Date: .... Period: ....

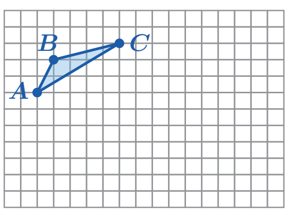
Transformation Information

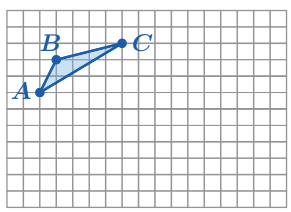
6. You and your partner will get a set of transformation cards. Place them grid-side down without looking at them.

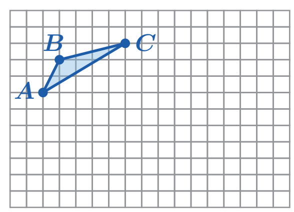
- Decide who will describe the transformation on a card and who will sketch the image. Start with Card 1.
- Describer: Give enough information about the transformation so that the Sketcher can sketch it.
- Sketcher: Pause after sketching and share what you think the transformation is.
- Together: Compare the card with the sketch and make adjustments as needed. Write a precise description of the transformation.
- Switch roles for Card 2 and repeat. Then do the same for Cards 3 and 4.

a **Sketch 1:** Card 1 or Card 2 (Circle one.)

Use as many grids as you need to revise your work.





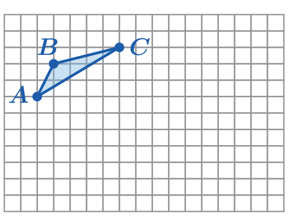


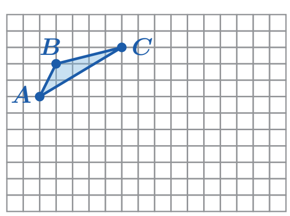
Description of Transformation: ***Responses vary.***

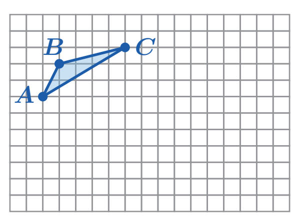
- **Card 1: Reflect triangle ABC over a vertical line that goes through the point 1 unit to the right of point C.**
- **Card 2: Translate triangle ABC 6 units to the right.**

b **Sketch 2:** Card 3 or Card 4 (Circle one.)

Use as many grids as you need to revise your work.







Description of Transformation: ***Responses vary.***

- **Card 3: Translate triangle ABC 2 units down and 7 units to the right.**
- **Card 4: Rotate triangle ABC 90° counterclockwise around the point 2 units to the right of point C.**

Synthesis

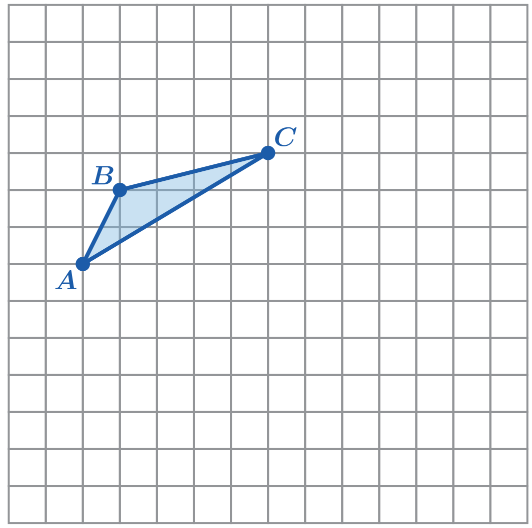
7. What details are helpful to include when precisely describing a transformation?

Responses vary.

Translation: **Describe the direction and distance of the translation.**

Reflection: **Describe the line of reflection, including the orientation of the line and its location in relation to the pre-image.**

Rotation: **Describe the center, angle, and direction of the rotation.**

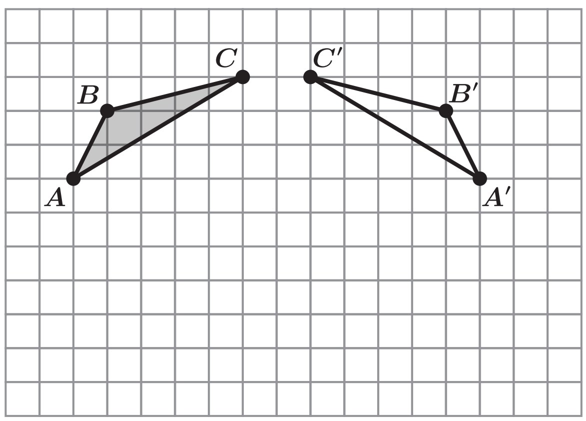


Things to Remember:

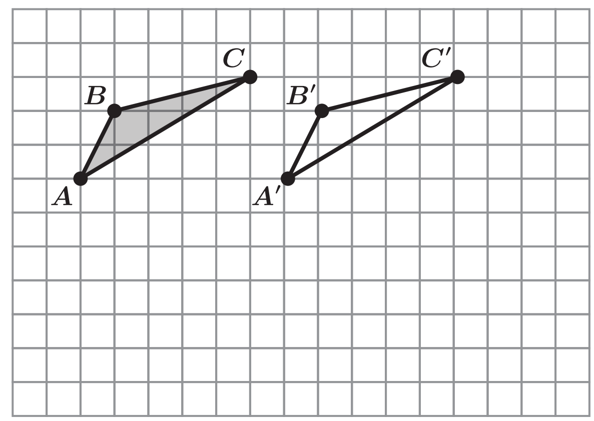
Transformation Information

Directions: Make one copy per pair of students. Then pre-cut the cards and give each pair one set.

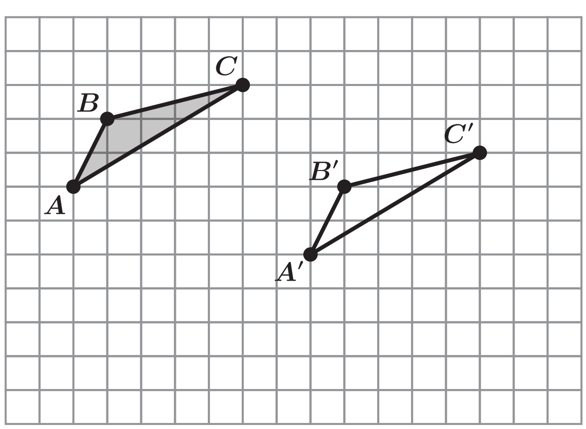
Card 1



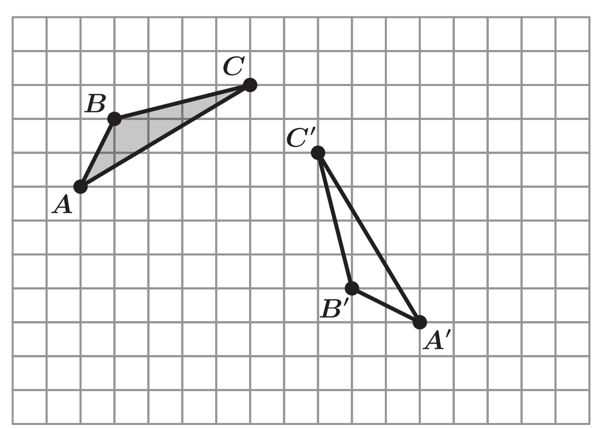
Card 2



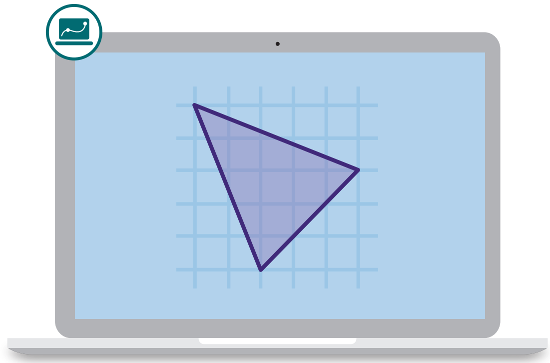
Card 3



Card 4



Name: .... Date: .... Period: ....



Getting Coordinated, Part 1

Let's explore how translations and reflections affect points on the coordinate plane.

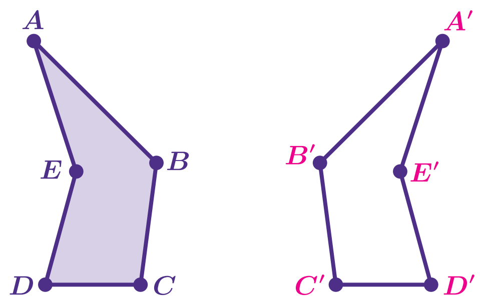
Warm-Up

1 The pre-image (shaded) has been reflected to create the image (unshaded).

Label each corresponding point on the image.

A'	B'	C'	D'	E'

Response shown on image.

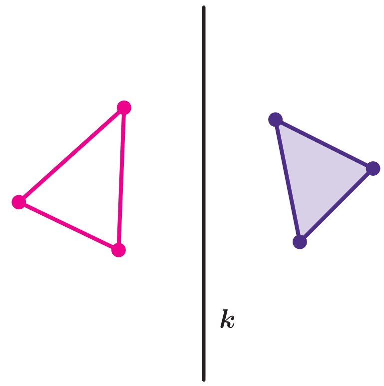


Name: .... Date: .... Period: ....

Do Coordinates Help?

2 Draw the image of the triangle after a reflection over line k.

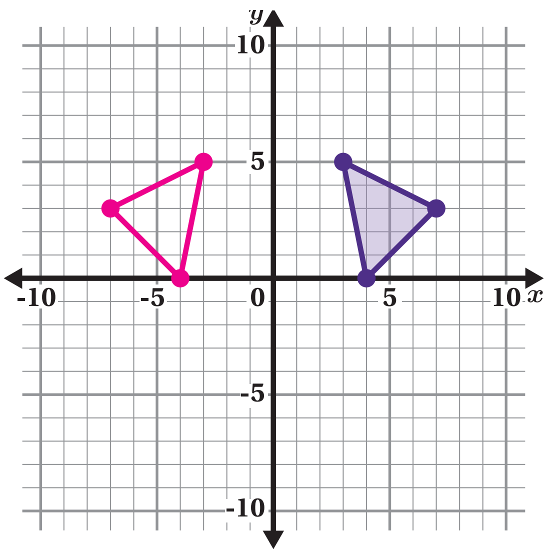
Sample shown on image.



3 Let's try that again, but with a coordinate plane.

Draw the image of the triangle after a reflection over the y-axis.

Response shown on image.



4 Let's look at some responses to the previous two problems.

Discuss: What do you notice about the two sets of reflections?

Responses vary.

- Without a grid, the transformed triangles are much more spread out.
- With a grid, the transformed triangles are much more aligned and accurate.
- The grid makes it easier to place each point at its precise location.

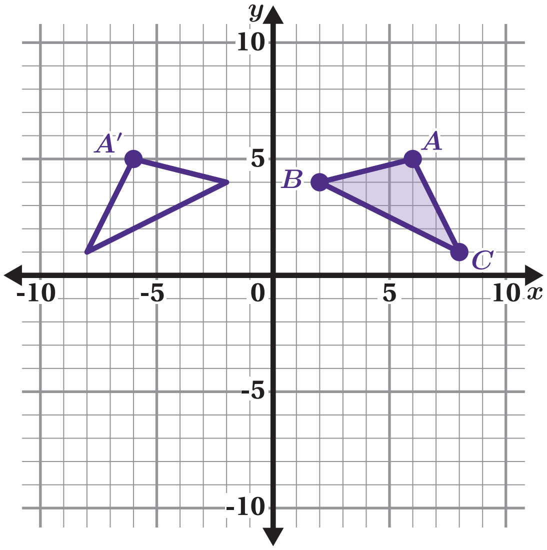
Activity 2

Name: ________ Date: ________ Period: ________

Coordinate Patterns, Part 1

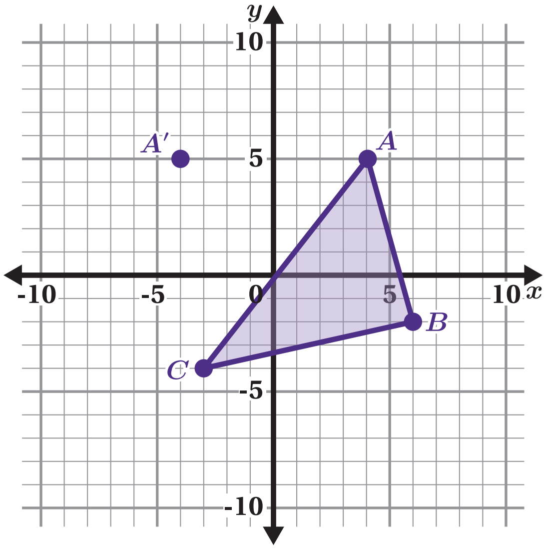
5 Identify the coordinates of each point after a reflection over the y-axis.

Pre-Image Coordinates	Image Coordinates
(6, 5)	(-6, 5)
(2, 4)	**(-2, 4)**
(8, 1)	**(-8, 1)**



6 Determine the coordinates of each point after a reflection over the y-axis.

Pre-Image Coordinates	Image Coordinates
(4, 5)	(-4, 5)
(6, -2)	**(-6, -2)**
(-3, -4)	**(3, -4)**



7 Take a look at your tables from Problems 5 and 6. Those points show a reflection over the y-axis.

a **Discuss:** What patterns do you see between the pre-image coordinates and the image coordinates?

***Responses vary.* I noticed that the sign of the x-coordinate changed, but the sign of the y-coordinate stayed the same.**

b Complete the table for the same transformation.

Pre-Image Coordinates	Image Coordinates
(3, 1)	**(-3, 1)**
(x, y)	**(-x, y)**

Activity 3

Name: .... Date: .... Period: ....

Coordinate Patterns, Part 2

8 Amari says that figure $A'B'C'D'$ is the image of figure $ABCD$ after a reflection over the x-axis.

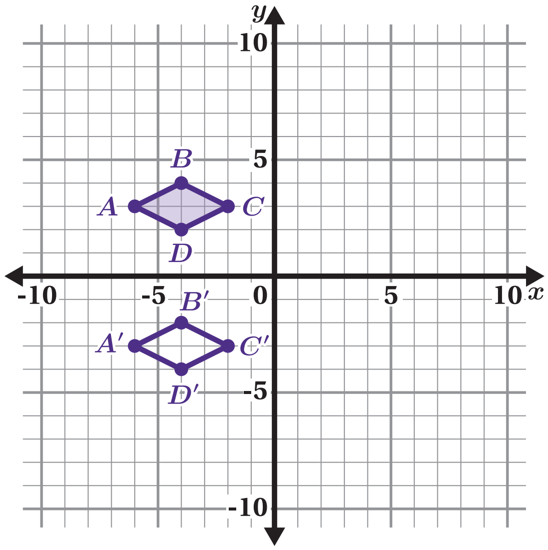
Is Amari's claim correct? Circle one.

Yes No I'm not sure

Explain your thinking.

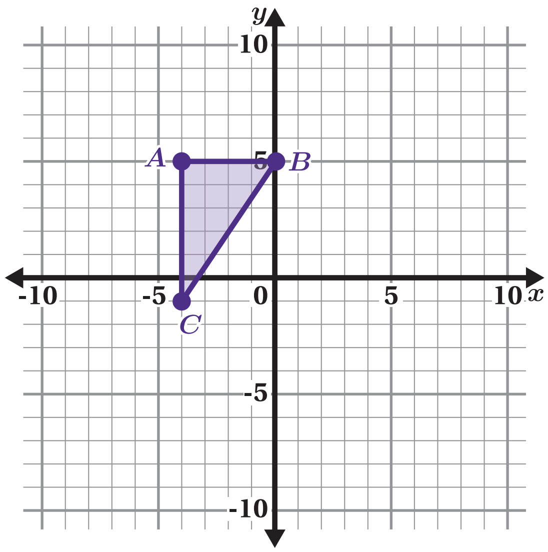
Explanations vary.

- **The corresponding points don't align.**
- **It looks the same, but I think if you try to reflect point D over the x-axis, it would land where point B' is right now.**
- **This is a translation, not a reflection.**



9 Determine the coordinates of each point after a translation 4 units right and 3 units down.

Pre-Image Coordinates	Image Coordinates
(-4, 5)	**(0, 2)**
(0, 5)	**(4, 2)**
(-4, -1)	**(0, -4)**



10 Take a look at your table from Problem 9. Those points show a translation 4 units right and 3 units down.

a **Discuss:** What patterns do you see between the pre-image coordinates and the image coordinates?

Responses vary. **I notice that the y-coordinate went down by 3 and the x-coordinate increased by 4.**

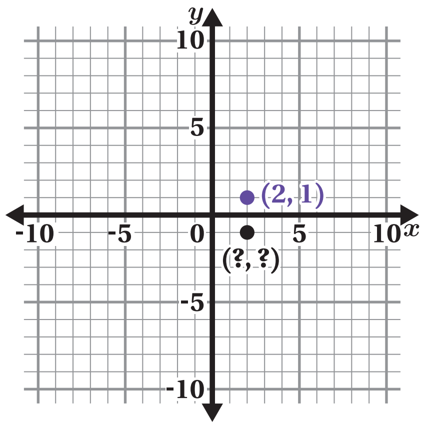
b Complete the table for the same transformation.

Pre-Image Coordinates	Image Coordinates
(3, 1)	**(7, -2)**
(x, y)	**$(x + 4, y - 3)$**

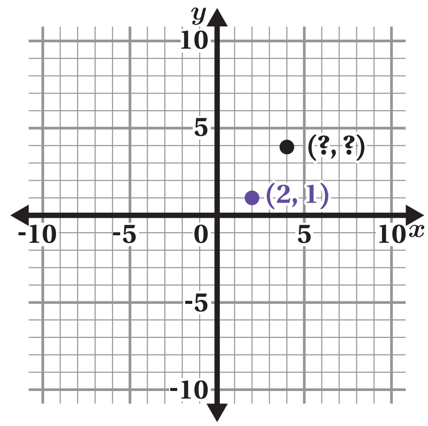
11 Synthesis

If you know the pre-image coordinates, how can you find the image coordinates for any reflection or translation?

Reflection



Translation



Use the examples if they help with your thinking. ***Responses vary.***

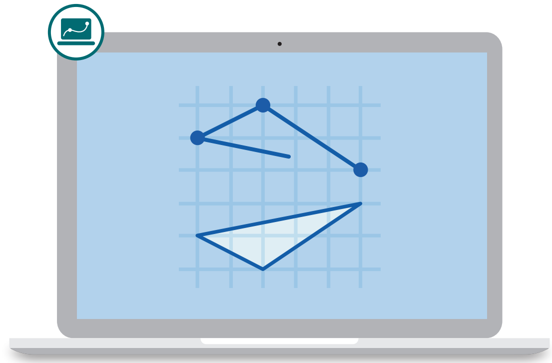
Reflection: **For a reflection over the x-axis, change the sign of the y-value in the ordered pair. For a reflection over the y-axis, change the sign of the x-value in the ordered pair. In other words, (x, y) is reflected over the x-axis to $(x, -y)$ and over the y-axis to $(-x, y)$.**

Translation: **For the translation 2 units right and 3 units up, add 2 to the x-value in the ordered pair and add 3 to the y-value. In other words, (x, y) is translated to $(x + 2, y + 3)$.**

Things to Remember:

Unit 1
Lesson
6

Name: ______ Date: ______ Period: ______



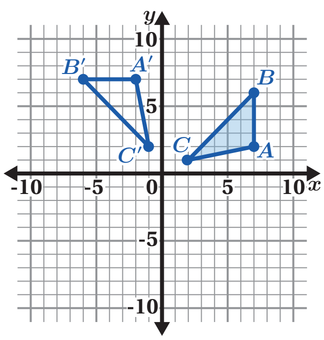
Getting Coordinated, Part 2

Let's explore how rotations affect coordinates.

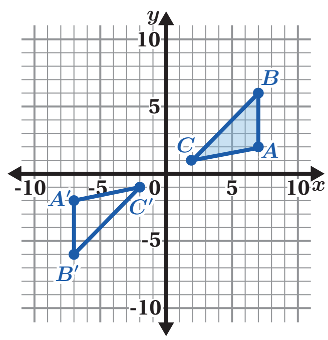
Warm-Up

1 a Take a look at several different rotations.

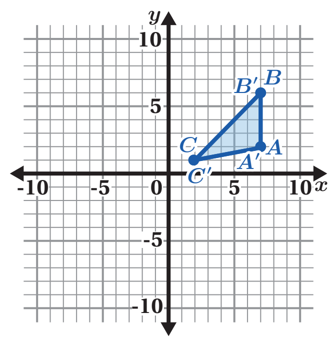
Counterclockwise 90°



Clockwise 180°



Counterclockwise 360°



b **Discuss:** What do you notice? What do you wonder?

Responses vary.

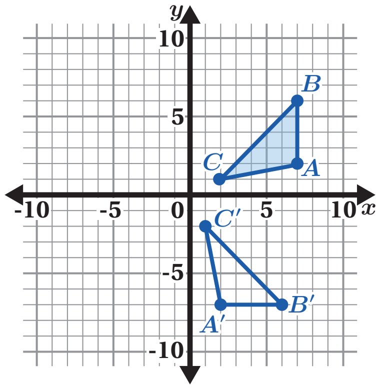
- **I notice that several of the different rotations create the same image.**
- **I wonder what happens to the coordinates for different rotations?**

Activity 1

Name: ______ Date: ______ Period: ______

Coordinate Patterns

2 Tyani says this is a rotation of figure ABC $90°$ clockwise around center $(0, 0)$. Anushka says this is a rotation of figure ABC $270°$ counterclockwise around the *origin*.



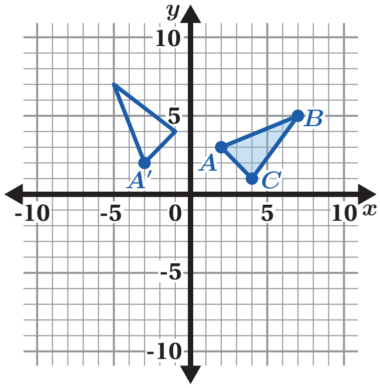
Whose claim is correct? Circle one.

Tyani's   Anushka's   Both   Neither

Explain your thinking.

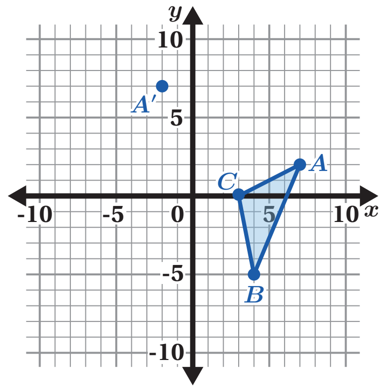
***Explanations vary.* The image of a 90° clockwise rotation is the same as the image of a 270° counterclockwise rotation.**

3 Identify the coordinates for the image of figure ABC after a rotation $90°$ counterclockwise around center $(0, 0)$.



Pre-Image Coordinates	Image Coordinates
$(2, 3)$	$(-3, 2)$
$(7, 5)$	$\mathbf{(-5, 7)}$
$(4, 1)$	$\mathbf{(-1, 4)}$

4 Determine the coordinates for the image of figure ABC after a rotation $90°$ counterclockwise around center $(0, 0)$.

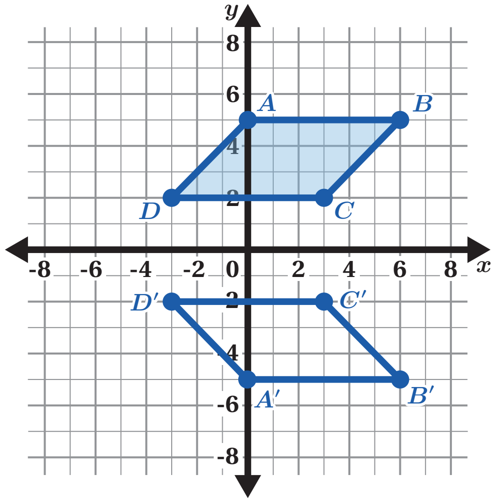


Pre-Image Coordinates	Image Coordinates
$(7, 2)$	$(-2, 7)$
$(4, -5)$	$\mathbf{(5, 4)}$
$(3, 0)$	$\mathbf{(0, 3)}$

Name: .... Date: .... Period: ....

Coordinate Patterns (continued)

5 Ayo says this is a $180°$ clockwise rotation of figure $ABCD$ around center $(0, 0)$. Cameron says this is a reflection over the x-axis.



Whose claim is correct? Circle one.

Ayo's (Rotation) | Cameron's (Reflection) | Both | Neither

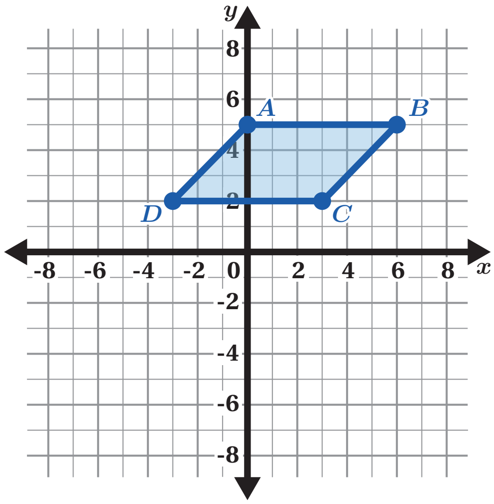
Explain your thinking.

Explanations vary.

- **Each point flips over the x-axis, so Cameron is correct: this is a reflection over the x-axis.**
- **If I rotate figure $ABCD$ $180°$ around $(0, 0)$, point B would end up in the bottom-left corner of the coordinate plane.**

6 Determine the coordinates for the image of figure $ABCD$ after a rotation $180°$ clockwise around center $(0, 0)$.

Pre-Image Coordinates	Image Coordinates
$(0, 5)$	**$(0, -5)$**
$(6, 5)$	**$(-6, -5)$**
$(3, 2)$	**$(-3, -2)$**
$(-3, 2)$	**$(3, -2)$**



Activity 2

Name: ________ Date: ________ Period: ________

Challenge Creator

7 You will use the Activity 2 Sheet to create your own transformation challenge.

a **Make It!** On the Activity 2 Sheet, create a transformation challenge.

b **Solve It!** On this sheet, write the pre-image and image coordinates for your transformation. ***Responses vary.***

Pre-Image Coordinates	Image Coordinates

c **Swap It!** Swap your challenge with one or more partners. Write the pre-image and image coordinates for their transformation. ***Responses vary.***

Partner 1's Challenge

Pre-Image Coordinates	Image Coordinates

Partner 2's Challenge

Pre-Image Coordinates	Image Coordinates

Partner 3's Challenge

Pre-Image Coordinates	Image Coordinates

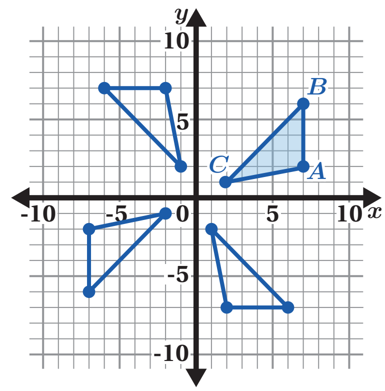
Partner 4's Challenge

Pre-Image Coordinates	Image Coordinates

8 Synthesis

Describe some patterns in the pre-image and image coordinates for rotations around center $(0, 0)$.

Use the examples if they help with your thinking.



Responses vary.

- **For $90°$ and $270°$ rotations, the x- and y-values switch and some of the values are opposites.**
- **For $180°$ rotations, each image coordinate has opposite x- and y-values.**

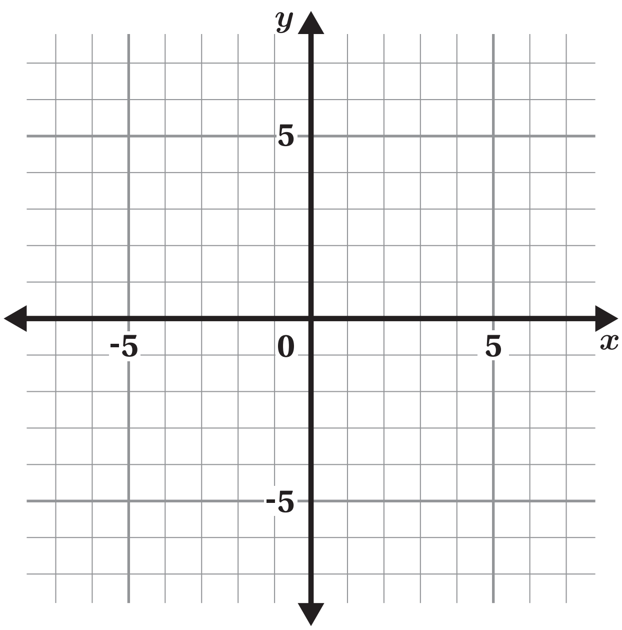
Things to Remember:

Name: .... Date: .... Period: ....

Challenge Creator

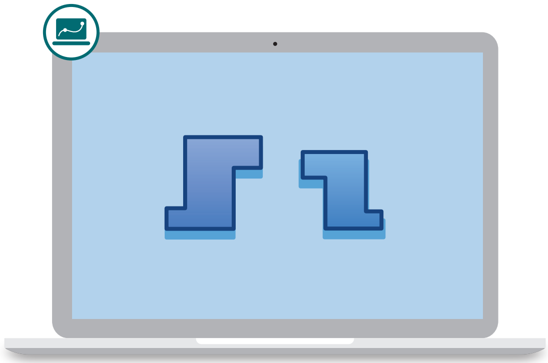
- On the graph, plot four points and connect them to make your own figure. Your figure will be the pre-image.
- Label each point with its coordinate pair (x, y).
- Choose and define a transformation.
  - For a translation, include the number of units and the direction.
  - For a rotation, include the direction and degrees.
  - For a reflection, include whether it is over the x- or y-axis.
- Don't show the image or write its coordinates on this sheet. You and your classmates will determine the image coordinates of each other's transformations on the lesson sheet.

My Pre-Image Polygon:



My Transformation:

Name: ______ Date: ______ Period: ______

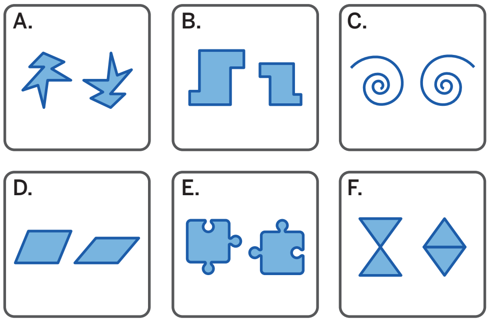


Are They the Same?

Let's explore a type of sameness.

Warm-Up

1 Here are six pairs of figures.



a Circle *all* the pairs with figures that are the same.

b **Discuss:** Which pairs did you choose? Why?

Responses vary. Note: Each pair has a way that it could be "the same."

- Pair B: Both figures have the same shape.
- Pair C: The shapes are both curly arrows and look like they are the same size. They also appear to be reflections of each other.
- Pair D: The side lengths are all the same.

2 Why might someone say Pair B's figures are the same? Are *not* the same?
Responses vary.
They are the same because . . . both figures have the same shape.

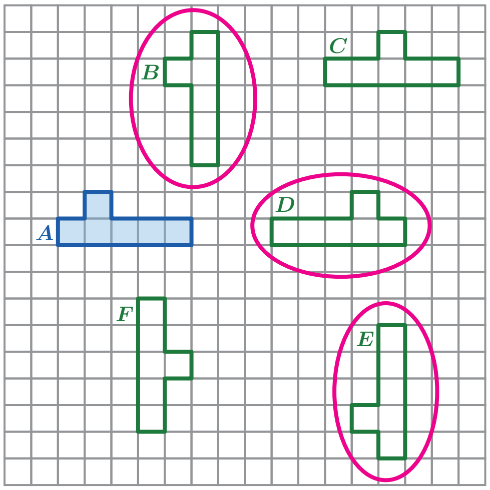
They are not the same because . . . the figures are different sizes.

Name: ______ Date: ______ Period: ______

Defining Congruence

3 One figure is **congruent** to another if it can be moved with translations, rotations, and reflections to fit exactly over the other.

Circle *all* the figures that are congruent to figure A.



4 Describe a sequence of rigid transformations that would move figure A onto one of the congruent figures you circled in the previous problem.

Responses vary.

- **To move figure A onto figure B, translate figure A up 1 unit, then reflect it over a diagonal line between figures A and B.**
- **To move figure A onto figure B, rotate figure A $90°$ clockwise around its bottom-right point, translate it 1 unit to the left and 3 units up, and then reflect it over a vertical line between figures A and B.**

5 Kweku says figures A and G are congruent. Lan says figures A and H are congruent.

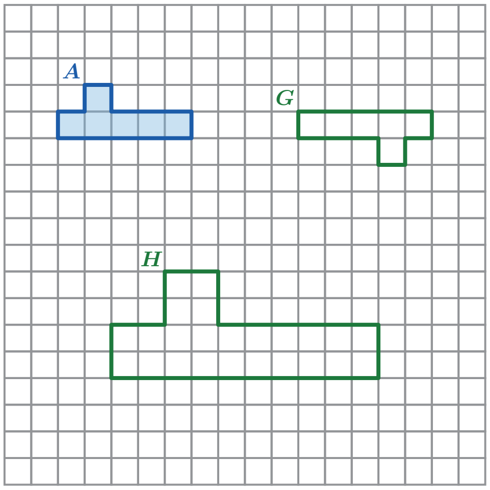
Whose claim is correct? Circle one.

Kweku's   Lan's   Both   Neither

Explain your thinking.

Explanations vary.

- **If you rotate figure A $180°$ and then translate it, you can make figure A land directly on top of figure G so that they match perfectly.**
- **Figure H is the same shape as figure A but a different size. Congruent figures must be the same size *and* shape.**

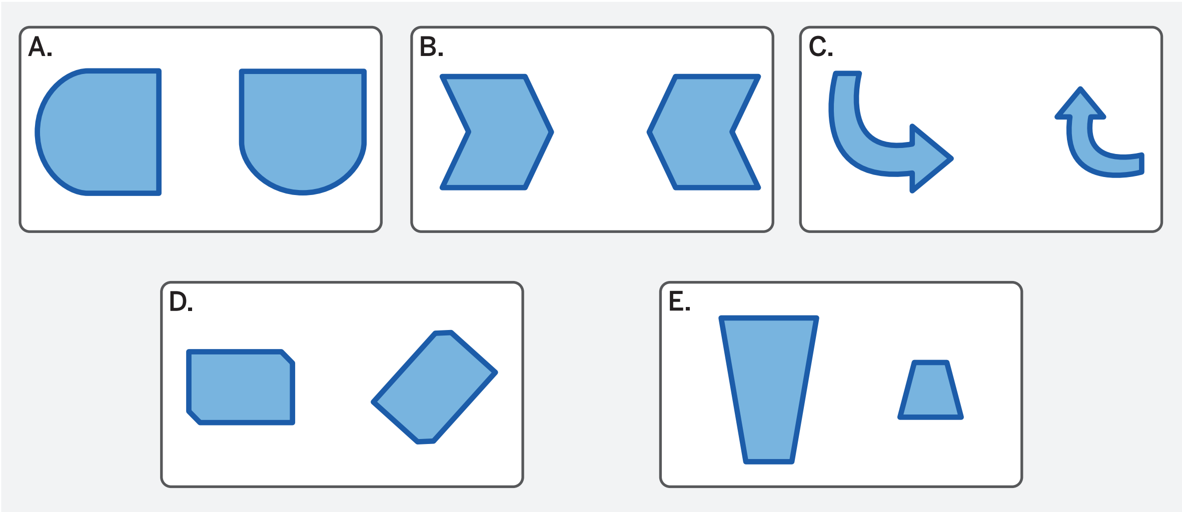


Activity
1

Name: ______________ Date: ______________ Period: ______________

Defining Congruence (continued)

6 Group the pairs of figures based on whether you think they are congruent or not congruent.



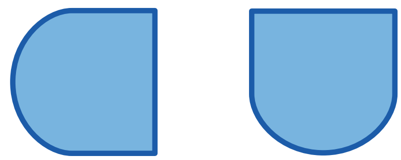
Congruent	Not Congruent
Card A, Card B, Card D	Card C, Card E

Activity 2

Name: ______ Date: ______ Period: ______

Sequence of Rigid Transformations

7 Which sequence of rigid transformations could be used to show that these two figures are congruent?



A. A translation

B. A rotation and a translation

C. A reflection and a translation

D. None. They're not congruent.

Explain your thinking.

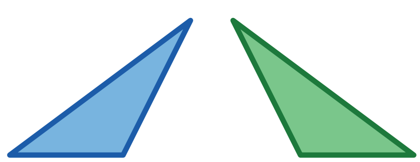
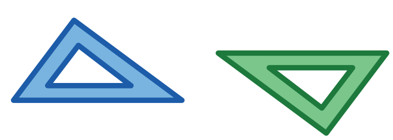
Explanations vary.

- **A rotation and a translation because the figure can be rotated to give the same orientation as the other figure. Then a translation can move one figure onto the other.**
- **A reflection and a translation because the figure can be reflected over a diagonal line and then translated.**

8 For each pair of figures, name a sequence of rigid transformations that could be used to show that they are congruent or write that they aren't congruent. Decide with a partner who will complete Column A and who will complete Column B.

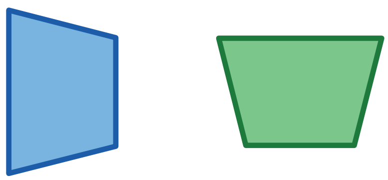
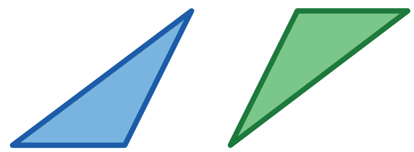
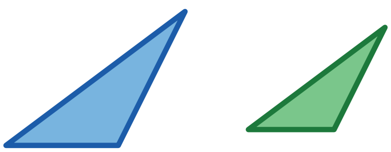
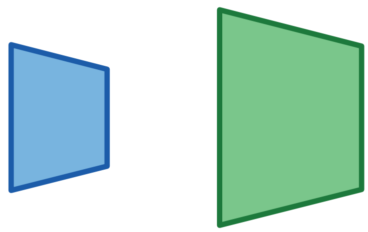
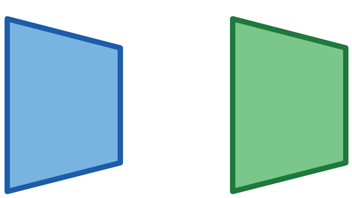
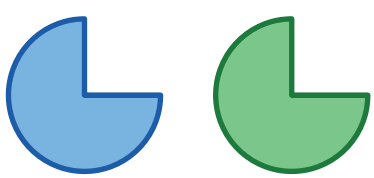
- After each problem, compare your answers. The answers in each row should be the same. Discuss and resolve any differences.
- Complete as many problems as you have time for.

Responses vary.

Column A	Column B
 Sequence: **A reflection and a translation**	 Sequence: **A reflection and a translation**

Name: ______________ Date: ______ Period: ______

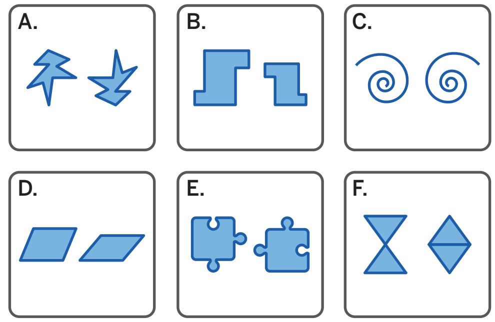
Sequence of Rigid Transformations (continued)

Column A	Column B
 Sequence: A rotation and a translation	 Sequence: A rotation and a translation
 Sequence: None. They’re not congruent.	 Sequence: None. They’re not congruent.
 Sequence: A translation	 Sequence: A translation

9 Synthesis

How can you determine whether two figures are congruent?

Use the examples if they help with your thinking.



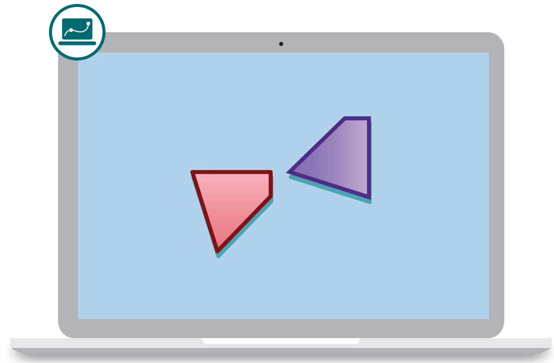
Responses vary. **Two figures are congruent when there is a sequence of translations, rotations, and reflections that takes one figure onto the other.**

Things to Remember:

Name: ______________________ Date: __________ Period: __________

Are They Congruent?

Let's make arguments about whether two figures are congruent.



Warm-Up

1 Here are four pairs of figures.

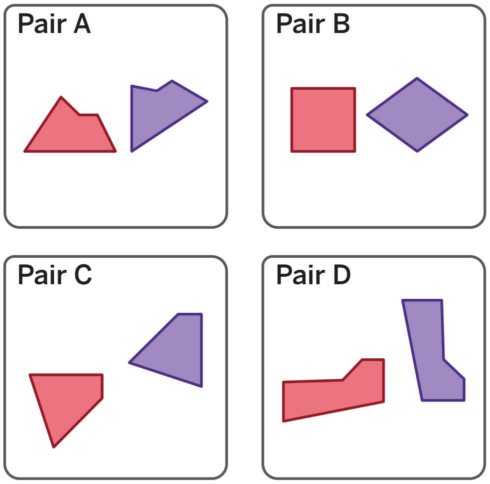
a Make a prediction! Circle *all* pairs of figures that look congruent. ***Responses vary.***

Pair A Pair B Pair C Pair D

b **Discuss:** How can you be more sure which pairs of figures are congruent?

Responses vary.

- **Tracing paper could help us compare corresponding side lengths and angle measurements.**
- **A grid could help us figure out a series of transformations.**



Activity

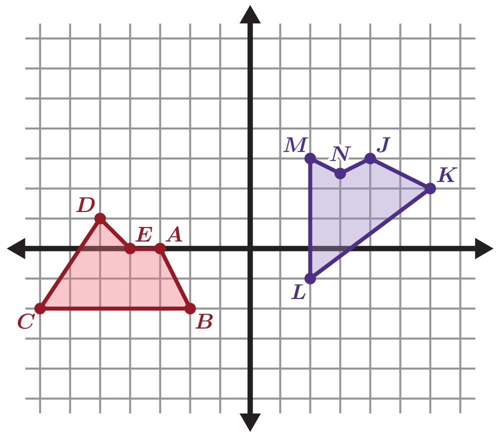
1

Name: .... Date: .... Period: ....

Congruent or Not?

Let's revisit each of the pairs from the Warm-Up.

2 Let's watch a sequence of transformations.



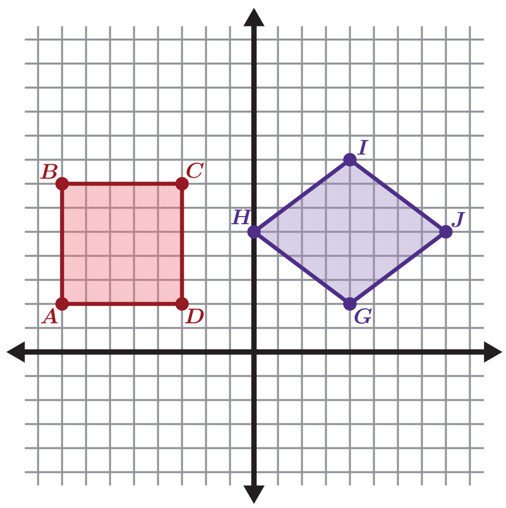
a Precisely describe the sequence of transformations.

***Responses vary.* The figure on the right is translated left 9 units and down 1 unit. Then it's rotated so side LK lines up with side CB.**

b Explain how you know the figures are *not* congruent.

***Responses vary.* It's clear that the figures aren't congruent because point M doesn't line up with point D and point N doesn't line up with point E.**

3 Mauricio noticed that each side length in figure $ABCD$ is equal to each side length in figure $GHIJ$. He says this proves that $ABCD \cong GHIJ$ ($ABCD$ is congruent to $GHIJ$).



Is his claim correct? Circle one.

Yes No

Explain your thinking.

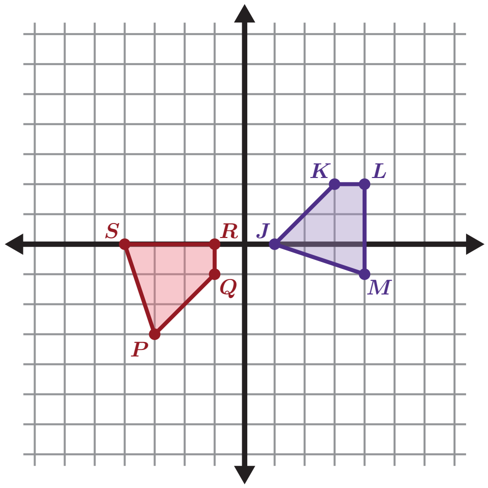
Explanations vary.

- **Knowing that the side lengths in one figure are equal to the side lengths in another figure is not enough to determine congruence. The angles should also be equal, and these angles are not equal.**
- **I couldn't place figure $ABCD$ perfectly on top of figure $GHIJ$ using tracing-paper transformations, which means the figures are not congruent. The fact that side lengths in one figure are equal to side lengths in the other does not change that.**

Name: .... Date: .... Period: ....

Congruent or Not? (continued)

4 Students in another class were asked to convince their peers that the figures in Pair C are congruent. Which of their arguments do you think is most convincing?



A. Both figures have 4 sides and an area of 5.5 square units.

B. I can move the figures right on top of each other by translating figure $JKLM$ down 3 and left 4, and then reflecting over side QP.

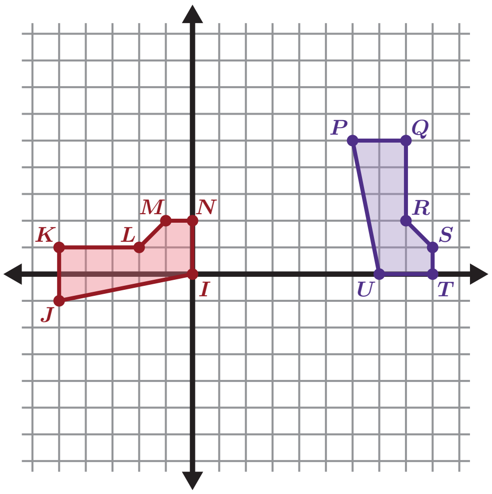
C. When I measure the side lengths of figures $JKLM$ and $PQRS$, I get the same measurements.

Explain your thinking.

***Explanations vary.* This argument is most convincing because knowing the side lengths and the area is not enough information to show that the two figures are congruent.**

5 Describe a sequence of rigid transformations that will convince your classmates that the figures from Pair D are congruent.

Use tracing paper if it helps with your thinking.



Responses vary.

- **Translate figure $PQRSTU$ 7 units left. Then rotate 90° counterclockwise around $(0, 0)$.**
- **Translate figure $PQRSTU$ to move point U onto point I. Then rotate it left, moving point P onto point J.**

Activity 2

Name: ________________ Date: ________ Period: ________

Prove It!

6 Angel says that any two rectangles with the same area are congruent.

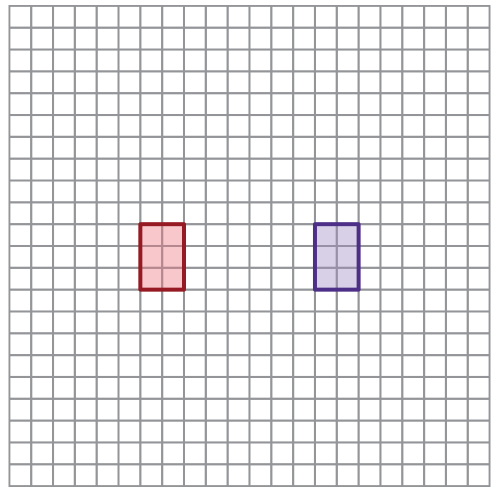
Is Angel's claim correct? Circle one.

Draw some rectangles to help you decide.

Yes   No   I'm not sure

Explain your thinking.

***Explanations vary.* A 4-by-2 rectangle and a 1-by-8 rectangle both have an area of 8 square units, but they are not congruent. That means rectangles that have the same area are not always congruent.**

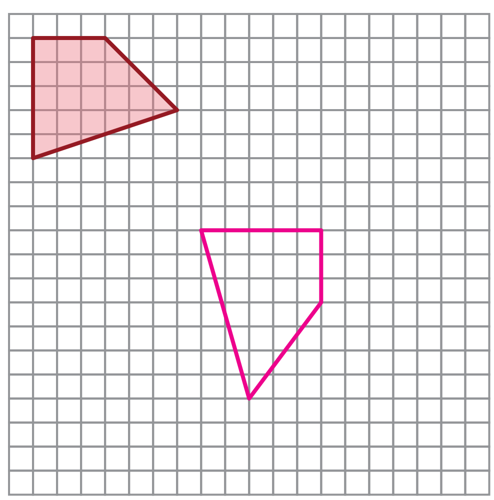


7 Which statement(s) are enough to prove that two figures are congruent? Select *all* that apply.

- ☐ **A.** The figures are both right triangles.
- ☑ **B.** You can trace one figure on tracing paper and move it perfectly on top of the other.
- ☐ **C.** The figures that have areas of 8 square units are both rectangles.
- ☑ **D.** You can move one figure right on top of the other by translating 8 units left.
- ☐ **E.** The figures are both isosceles right triangles.

8 **a** Draw a second figure to create two figures that are *not* congruent but that someone might think are.

***Responses vary.* Sample shown on grid.**



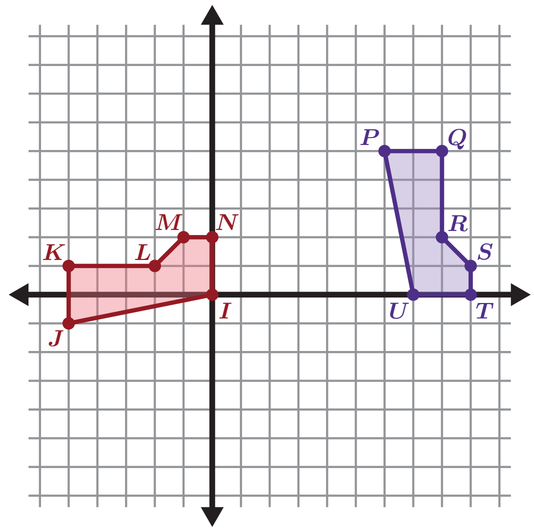
b Explain how you know the two figures are not congruent.

***Responses vary.* These figures aren't congruent because I can't use a sequence of rigid transformations to move one on top of the other.**

9 Synthesis

How can you prove that two figures are congruent?

Use the example if it helps with your thinking.



Responses vary.

- I could use tracing paper to determine that the corresponding side lengths and angle measures are congruent.
- I could describe a sequence of transformations that moves one figure onto the other.

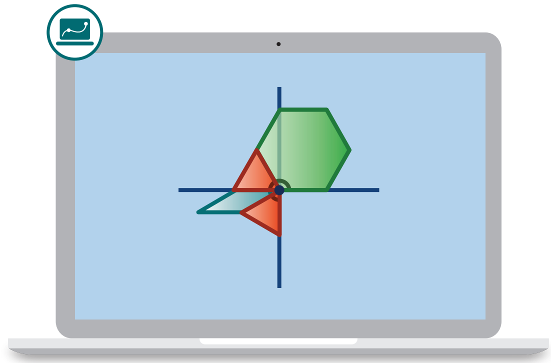
Things to Remember:

Unit 7
Lesson
2

Name: ______ Date: ______ Period: ______

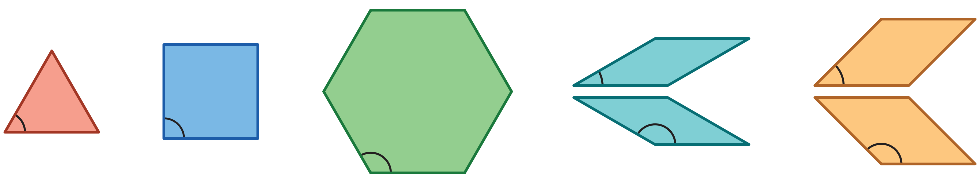
Friendly Angles

Let's explore complementary and supplementary angles.



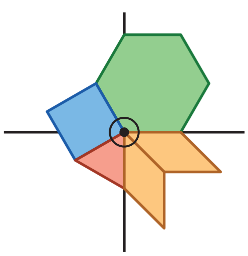
Warm-Up

1 These shapes were used to create four $360°$ designs.

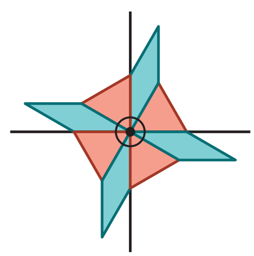


a Pick a design that you like. ***Responses vary.***

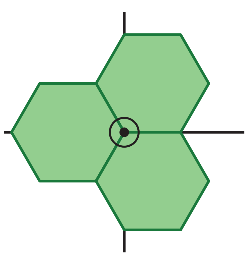
A.



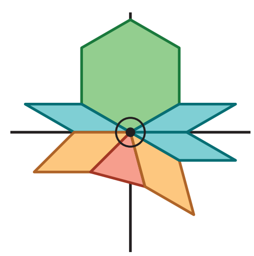
B.



C.



D.



b **Discuss:** What do you like about the design you chose?

Responses vary.

- **I picked Design A because it uses a lot of different shapes.**
- **I picked Design B because I like the repeating patterns.**
- **I picked Design C because it used only hexagons.**
- **I picked Design D because it looks like a person wearing an orange dress.**

Activity 1

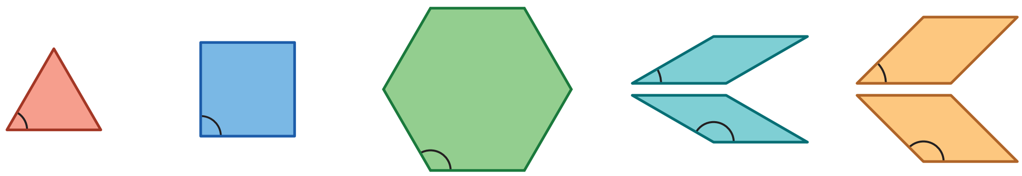
Name: .... Date: .... Period: ....

Mystery Measures

2 You and your partner will use a set of shapes or the digital screen for this activity.

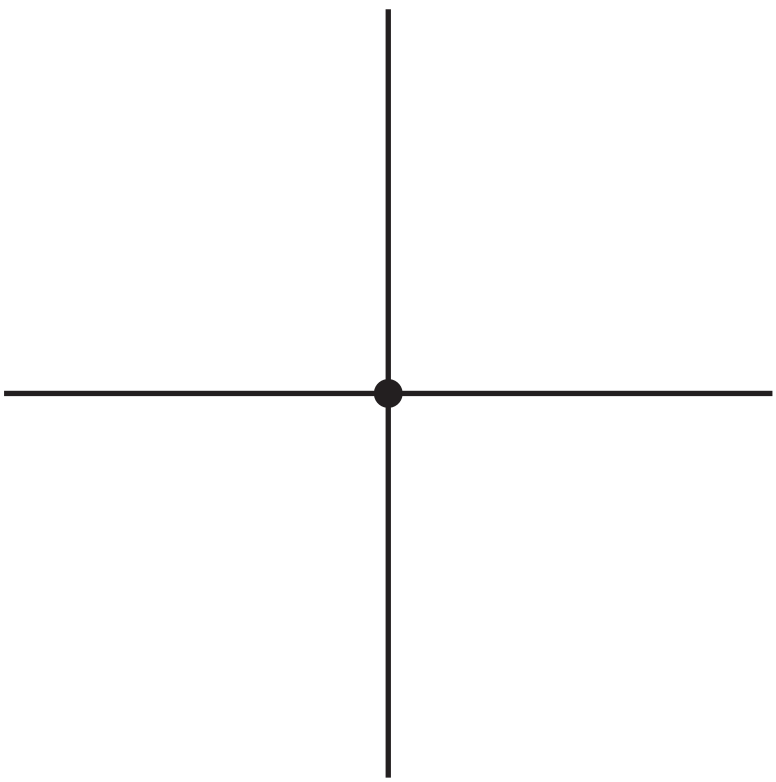
Each shape has at least one unknown angle measure.

a Determine as many angle measures as you can by creating designs with your shapes. Label each shape with its angle measure.



60° | **90°** | **120°** | **30° and 150°** | **45° and 135°**

Workspace:



b Choose two angles. Explain how you determined their measures.

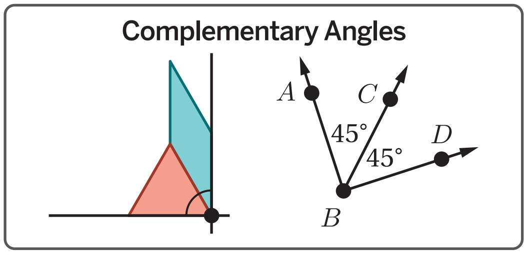
Responses vary. **I created a design with four red triangles and one green hexagon. I saw that three red triangles cover half of the circle, so the measure of the angle in the red triangle is $\frac{180}{3} = 60$ degrees. Then I noticed that the red triangle and the green hexagon add up to 180°, so the measure of the angle in the green hexagon is $180 - 60 = 120$ degrees.**

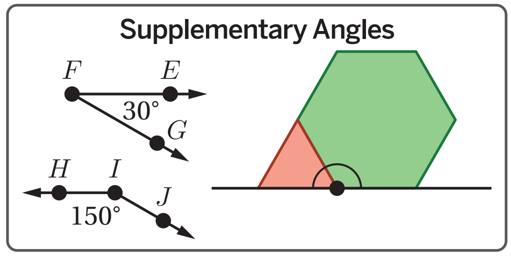
Activity 2

Name: ______ Date: ______ Period: ______

Relationships and Equations

3 The terms **complementary** and **supplementary** describe special pairs of angles.





Describe what you think these terms mean. ***Descriptions vary.***

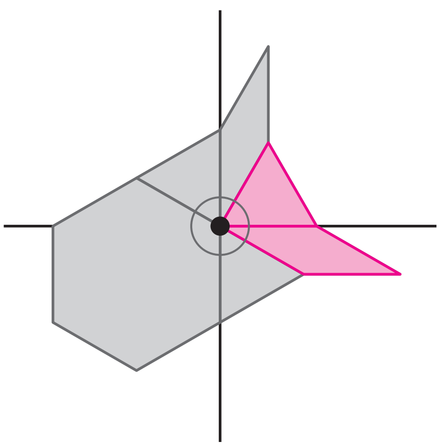
Complementary angles . . . **add up to 90°, making a right angle.**

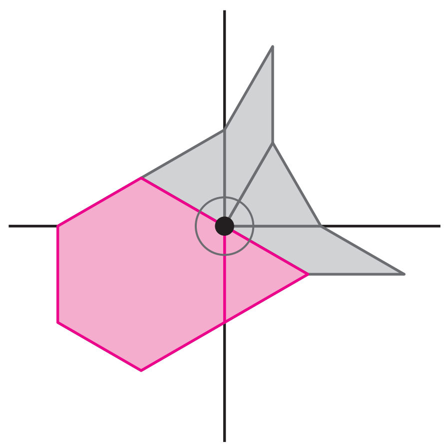
Supplementary angles . . . **add up to 180°, like a line or straight angle.**

4 Here is a new design.

a Shade in a pair of complementary angles.
***Responses vary.* Sample shown on image.**

b Shade in a pair of supplementary angles.
***Responses vary.* Sample shown on image.**

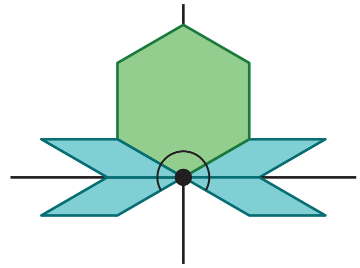




Name: ____________ Date: ________ Period: ________

Relationships and Equations (continued)

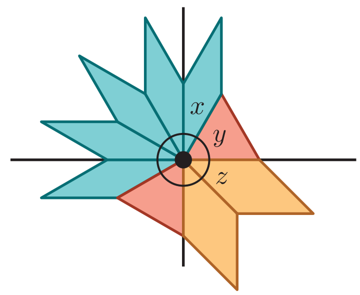
5 Ivory used the equation $2x + 120 = 180$ to determine one angle measure in this diagram.



Explain or show what each part of Ivory's equation represents in the diagram.

Responses vary.
- **$2x$ represents two small teal angles that touch the center.**
- **120 represents the one angle on the green hexagon.**
- **180 represents the total. Two small teal angles that touch the center and the hexagon's angle add up to make a line, which measures 180°.**

6 Here is a new diagram.



a Select *all* the true equations.

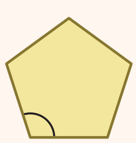
- [x] **A.** $3x = 90$
- [x] **B.** $x + y = 90$
- [ ] **C.** $5x = 180$
- [x] **D.** $x + y + 2z = 180$
- [ ] **E.** $x + y + z = 360$

b **Discuss:** How did you decide which equations are true?

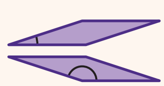
Responses vary.
- **$3x = 90$ is true because there are three angles of $x°$ that make up a right angle.**
- **$x + y = 90$ is true because the teal angle ($x°$) and the red angle ($y°$) are complementary.**
- **$x + y + 2z = 180$ is true because when I combine the teal angle ($x°$), the red angle ($y°$), and the two orange angles ($z°$), they add up to half a circle, which measures 180°.**

Explore More

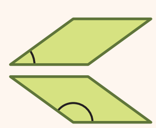
7 Use the Explore More Sheet to determine as many of these unknown angle measures as you can by creating designs in the workspace provided. Record each angle measure below its shape.



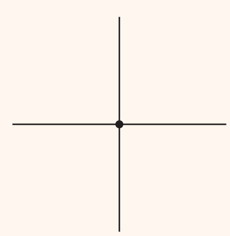
108°



18° and 162°



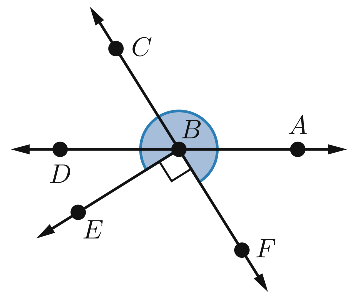
36° and 144°



8 Synthesis

Here is a diagram. Describe or show as many angle relationships as you can.

Use the terms *complementary* and *supplementary* in your description.

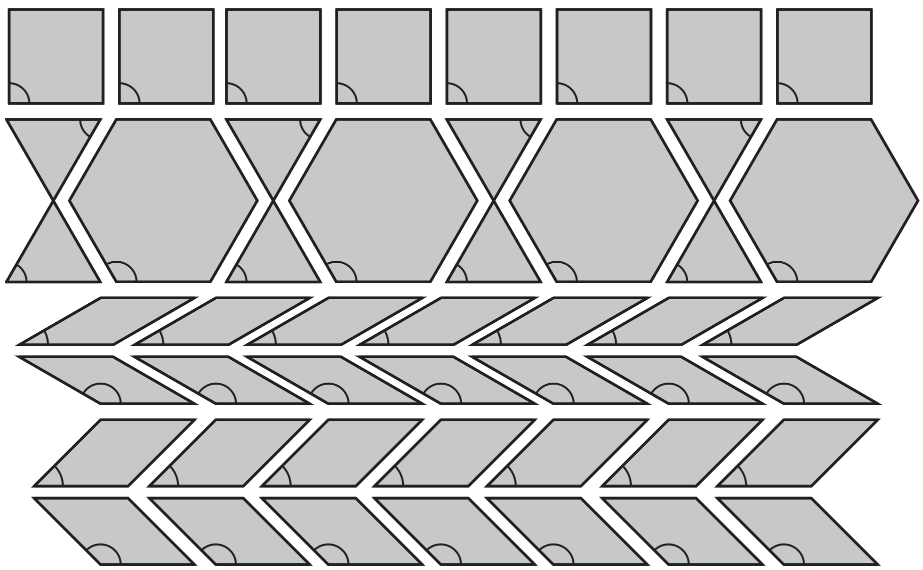


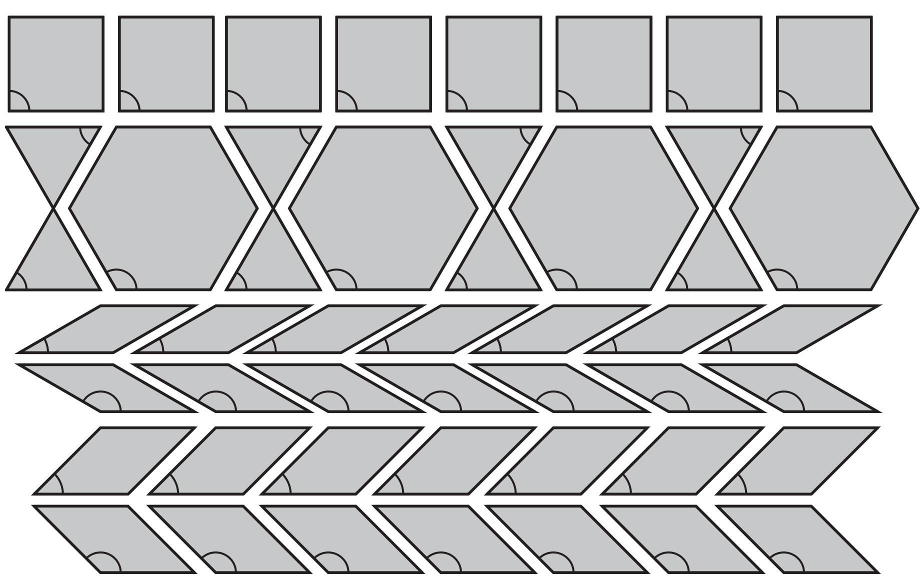
Responses vary. **Angles ABC and DBC are supplementary because they add up to 180° (or make a straight angle). So are angles CBA and ABF. Angles EBD and DBC are complementary because they make a right angle together.**

Things to Remember:

Mystery Measures

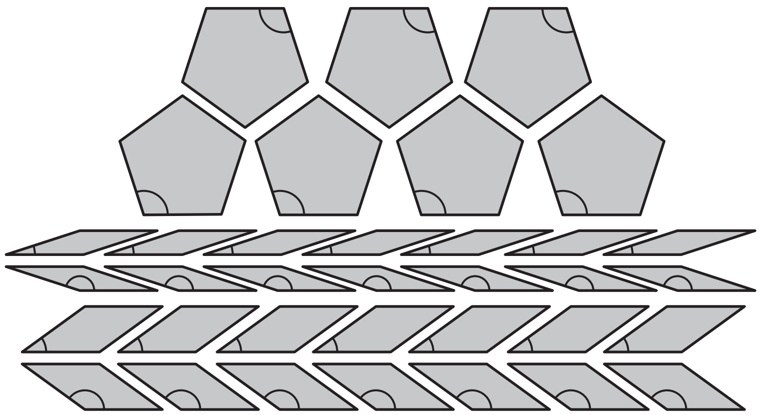
Directions: Make one copy per two pairs of students. Then pre-cut the shapes and give each pair of students one set.

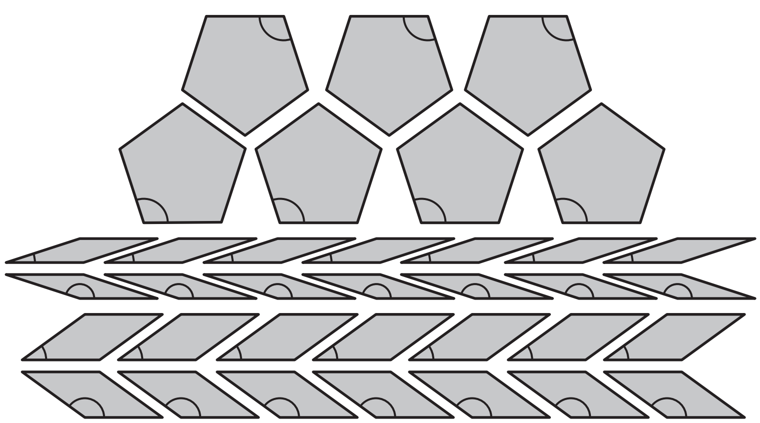




Explore More

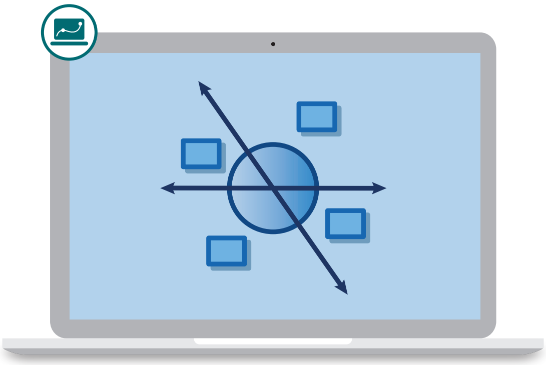
Directions: Make one copy per two pairs of students. Then pre-cut the shapes and give each pair of students one set.





Unit 7
Lesson
3

Name: Date: Period:



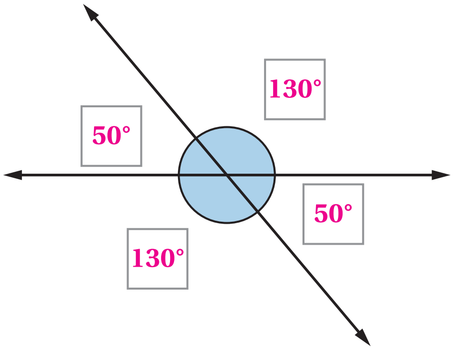
Angle Diagrams

Let's explore vertical angles.

Warm-Up

1 a Estimate each angle measure.

Responses vary. **Sample shown in diagram.**



b **Discuss:** How did you choose your estimates?

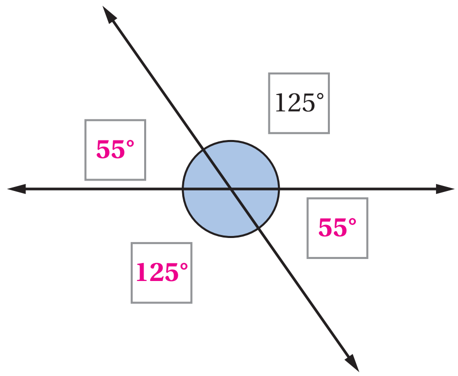
Responses vary. **I knew that the angles needed to add up to 360°. I also knew that two angles needed to be larger than 90° and two angles needed to be smaller than 90°.**

Activity
1

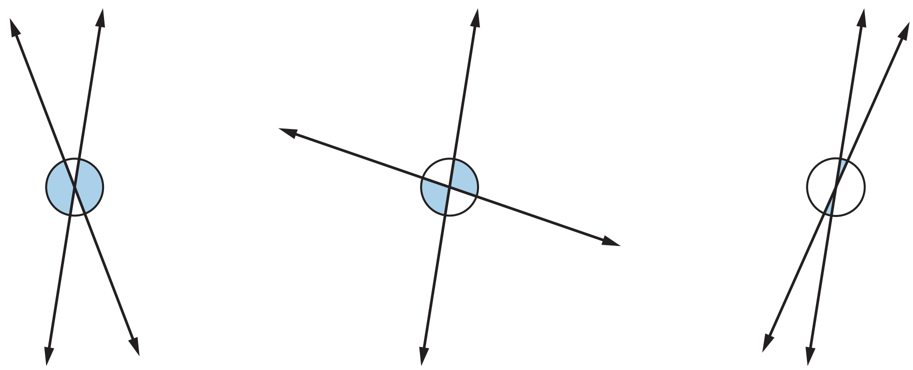
Name: Date: Period:

Vertical Angles

2 Here is an angle puzzle. Use the given angle measure to determine all the angle measures.
Responses shown in diagram.



3 Lola noticed that when two lines cross, the angles that are opposite each other have the same measure. These angles are called **vertical angles**.



Are the measures of vertical angles *always*, *sometimes*, or *never* the same? Circle one.

Always   Sometimes   Never

Explain your thinking.

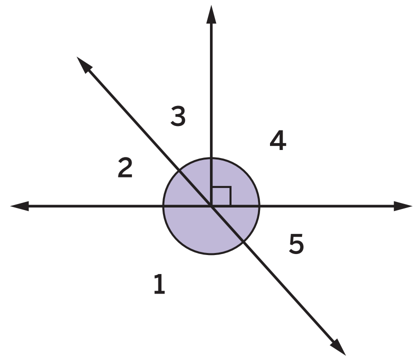
***Explanations vary.* No matter how the lines cross, the shaded angles always look the same size, and the unshaded angles always look the same size.**

Activity 2

Name: ______________________ Date: __________ Period: __________

Vertical Angle Puzzle

4 Here is a new angle puzzle. Which of these is a pair of vertical angles?



A. 1 and 4

B. 2 and 3

C. 2 and 5

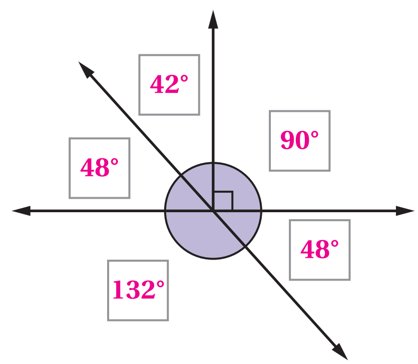
D. 3 and 5

Discuss: Choose one of the other pairs. How do you know they are *not* vertical angles?

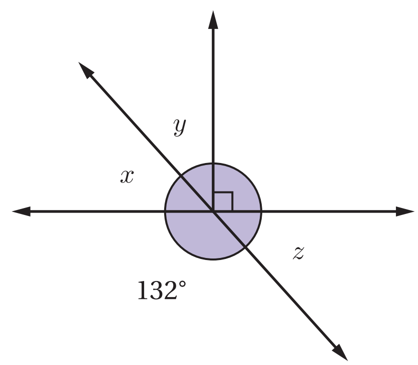
***Responses vary.* 1 and 4 are not vertical angles because even though they share a vertex, they are not across from each other on intersecting lines.**

5 Here is a new angle puzzle. You can ask for the measure of an angle. Determine all the angle measures using as few hints as you can.

Responses shown in diagram.



6 Kwasi and Lola wrote equations to help them solve the previous angle puzzle.



Kwasi's equation: $x + 132 = 180$

Lola's equation: $132 + z = 180$

Whose equation is correct? Circle one.

Kwasi Lola Both Neither

Explain your thinking.

***Explanations vary.* The angles that measure $x°$ and 132° make a line, so they are supplementary. The angles that measure $z°$ and 132° also make a line, so they are supplementary.**

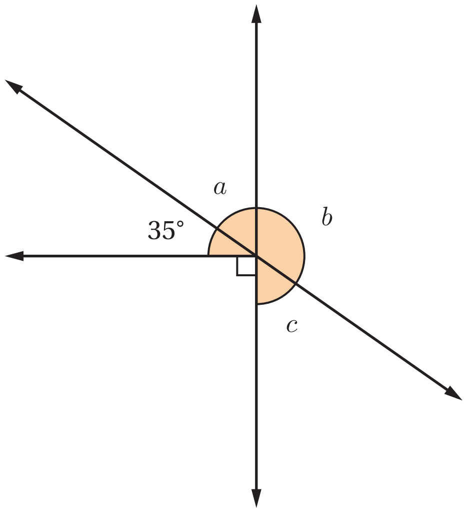
Note: Students who select "Kwasi," "Lola," or "Both" will be marked correct.

Activity 3

Name: ______ Date: ______ Period: ______

Writing and Using Equations

Use the diagram for Screens 7–8.



7 Write a true equation based on this angle puzzle. Try to write an equation none of your classmates will.

Equations vary.

- $a + 35 = 90$
- $a + b = 180$
- $b + c = 180$
- $35 + 90 + c = 180$
- $a + b + c + 35 + 90 = 360$

8 Determine the values of a, b, and c.

$a = 55°$, $b = 125°$, $c = 55°$

Activity 3

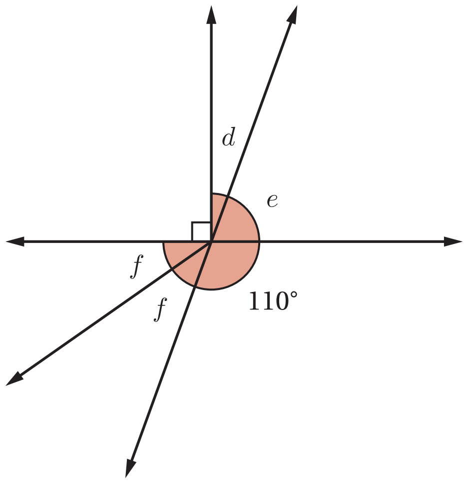
Name: ______ Date: ______ Period: ______

Writing and Using Equations (continued)

9 Here is a new angle puzzle.

Determine the values of d, e, and f.

$d = 20°$, $e = 70°$, $f = 35°$



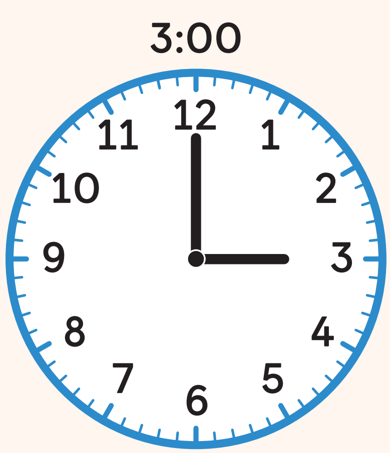
10 Kwasi wrote the equation $f + 110 = 180$ for the previous puzzle. Change Kwasi's equation to make it true.

Responses vary.

- $2f + 110 = 180$
- $e + 110 = 180$
- $f + 110 = 145$
- $e = 2f$

Explore More

11 Here is a clock.



a What is the angle between the clock's hour hand and minute hand at 3:00?

90°

b What is the angle between the clock hands at 2:20? (Hint: It is not 60°.)

50°

c What is one time when the angle between the clock hands is 40°?

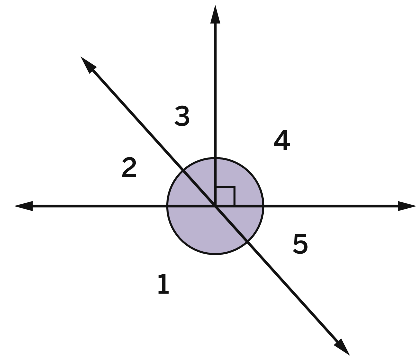
Responses vary. 5:20 and 6:40

12 Synthesis

Describe what you know about vertical angles.

Use the example if it helps with your thinking.

Responses vary. **Vertical angles are on opposite sides of lines that cross. These angles are always the same size.**



Things to Remember:

Unit 1
Lesson
10

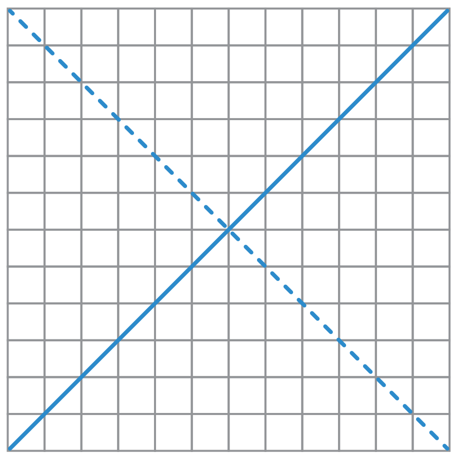
Name: .... Date: .... Period: ....

Transforming Angles

Let's explore congruent angles.

Warm-up

1 Select *all* the transformations that can be used to move the solid line onto the dashed line.



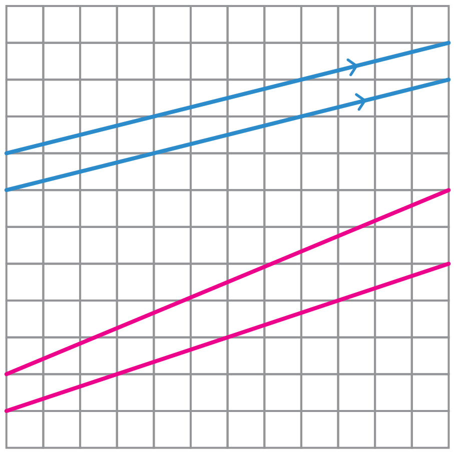
- ☐ **A.** Only translations
- ☑ **B.** Only rotations
- ☑ **C.** Only reflections

2 This pre-image is a pair of lines.

The arrowheads mean that the lines are *parallel*.

Draw two lines that *cannot* be a translation, rotation, or reflection of these lines.

***Responses vary*. Sample shown on graph.**



Activity 1

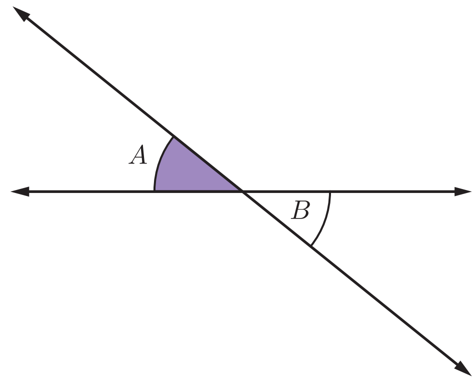
Name: ______ Date: ______ Period: ______

Vertical Angles

Angles that are opposite each other when two lines cross, like A and B, are called *vertical angles*. We can use transformations to prove whether angles are congruent.

3 Describe a single transformation to show that vertical angles A and B are congruent.

***Responses vary.* Rotate angle A 180° around the point of intersection.**



4 Here are another two lines that cross. $\angle C$ has a measure of $70°$.

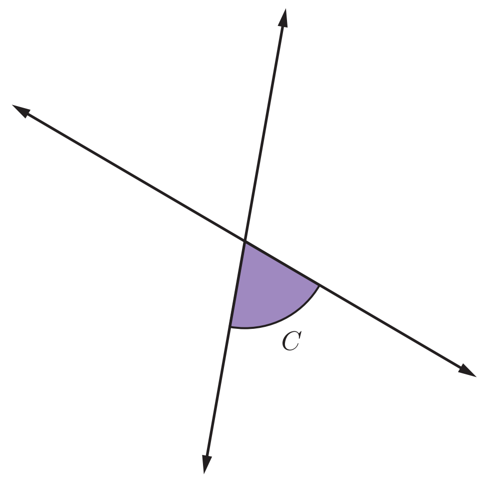
With this information, Ariel says it's possible to determine the measure of the three remaining angles. Vihaan says it's only possible to determine the measure of one more angle.

Whose claim is correct? Circle one.

Ariel's   Vihaan's   Neither

Explain your reasoning.

***Explanations vary.* Ariel's claim is correct because $\angle C$'s vertical angle is congruent. The measure of each of the other two angles is $110°$ because each of those angles creates a straight line with $\angle C$.**

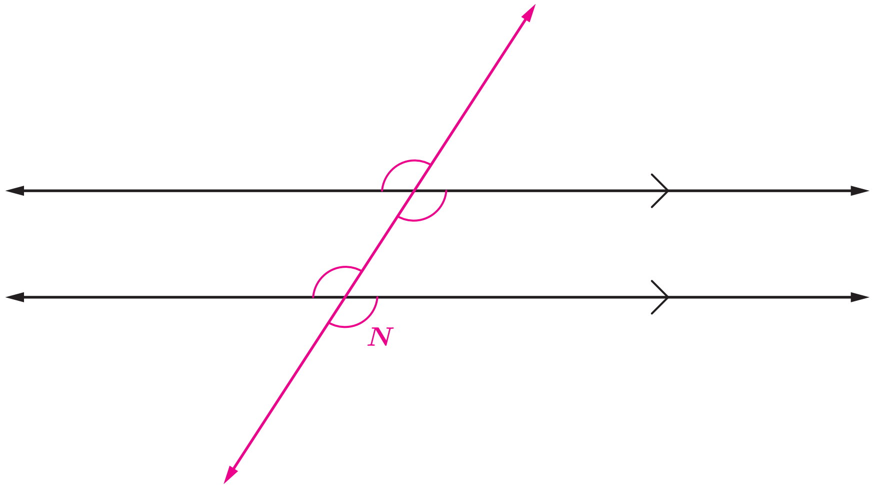


Activity 2

Name: ______ Date: ______ Period: ______

Parallel Lines and a Transversal

5 Here are two parallel lines.



a Draw a third line that intersects both the parallel lines. This line is called a **transversal**.
Responses vary. **Sample shown.**

b Choose one angle. Label it with your favorite letter.
Responses vary. **Sample shown.**

c Mark all of the angles that are congruent to your angle.
Responses vary. **Sample shown.**

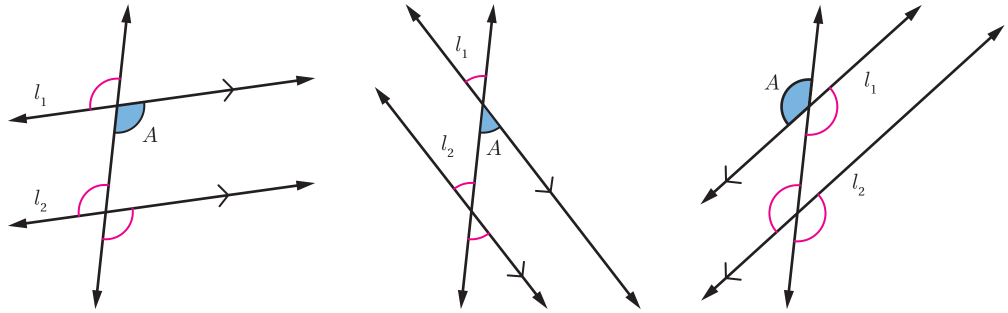
d Compare your drawing with a classmate's.

Activity 3

Name: .... Date: .... Period: ....

Congruent Angles?

6 Lines l_1 and l_2 are parallel. Take a look at three different positions for l_1 and l_2.



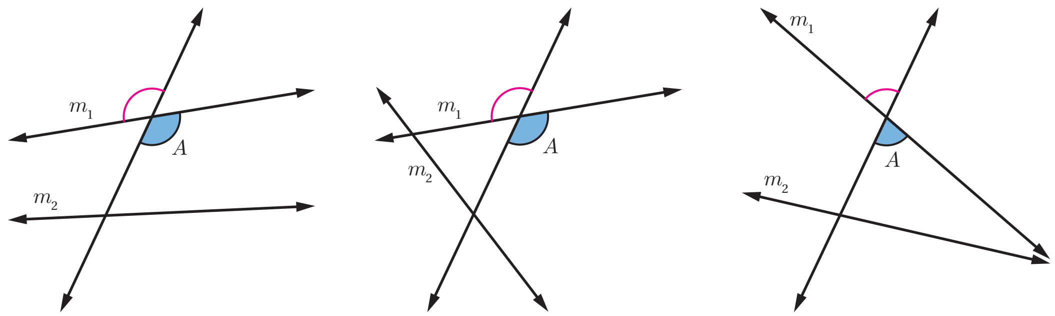
a **Discuss:** How many angles in each diagram are *always* congruent to angle A?

Three angles are always congruent to angle A: the angle vertical to angle A and the two corresponding vertical angles on l_2.

b In each diagram, mark the angles that are congruent to angle A.

Response shown.

7 Lines m_1 and m_2 are *not* parallel. Take a look at three different positions for m_1 and m_2.



a **Discuss:** How many angles in each diagram are *always* congruent to angle A?

One angle is always congruent to angle A (the angle vertical to angle A).

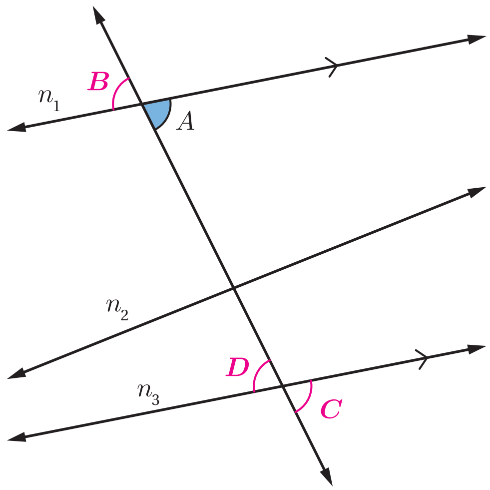
b In each diagram, mark the angles that are congruent to angle A.

Response shown.

Name: .... Date: .... Period: ....

Congruent Angles? (continued)

8 Here are three lines: n_1, n_2, and n_3.



a **Discuss:** How many angles in the diagram *must* be congruent to angle A?

***Responses vary.* Three angles must be congruent to angle A.**

b Mark the angles that are congruent to angle A. Label each one with a different letter.

Response shown.

9 For each angle you marked in the previous problem, describe a single transformation to show that the angle is congruent to angle A.

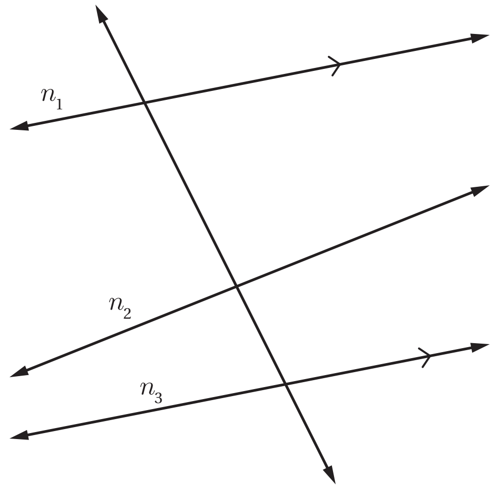
Responses vary.

- **Angle B: Rotate angle A 180° clockwise around the point where the transversal crosses line n_1.**
- **Angle C: Translate angle A along the transversal line until it lies on top of angle C.**
- **Angle D: Rotate angle A 180° clockwise around the halfway point on the transversal between lines n_1 and n_3.**

10 Synthesis

How can you use transformations to show that two angles are congruent?

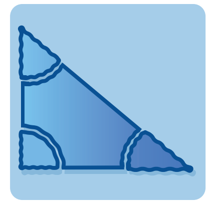
Use the example if it helps with your thinking.



Responses vary. **If I can use a rigid transformation to move angle A directly onto another angle, then the two angles are congruent. I can use rotations to show that vertical angles are congruent and translations to show that corresponding angles on parallel lines are congruent.**

Things to Remember:

Name: Date: Period:



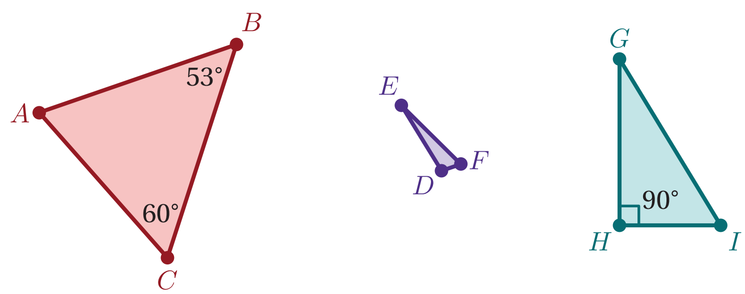
Tearing It Up

Let's explore the interior angles of triangles.

Warm-Up

1. Here are three triangles.

Triangle 1 **Triangle 2** **Triangle 3**



If you add up the three interior angles of each triangle, which triangle do you think has the greatest sum? Explain your thinking.

Responses vary.

- **I think Triangle 1 has the greatest angle sum because it's the largest triangle.**
- **I think Triangle 3 has the greatest interior angle sum because 90° is the greatest angle labeled.**
- **I think all three triangles have the same angle sum because the sum of the interior angles of any triangle is 180 degrees. (Note: Some students may already be familiar with the interior angle sum of triangles.)**

Activity 1

Name: ________ Date: ________ Period: ________

Find All Three

You will get a card with a picture of a triangle.

2. The measurement of one of the angles is labeled. Estimate the measures of the other two angles. ***Responses vary.***

Labeled angle measure: ________

Estimated angle measure: ________ Estimated angle measure: ________

3. Find two other students with triangles that look congruent to yours, but with a different labeled angle.

Name: ________ Card number: ________

Name: ________ Card number: ________

4. Confirm that all three triangles are congruent and that each card has a different labeled angle. How did you know that the triangles were congruent?
***Responses vary.* I knew I found congruent triangles when I traced my triangle on tracing paper and it fit exactly on the other triangles.**

5. Record the three angle measures for your triangle in the table.

Card Numbers	Angle 1	Angle 2	Angle 3	Angle Sum
1, 9, 14	40°	50°	90°	180°

Additional card and angle pairings:

- **Cards: 2, 10, and 27. Angles: 30°, 70°, 80°. Angle Sum: 180°**
- **Cards: 3, 12, and 20. Angles: 65°, 70°, 45°. Angle Sum: 180°**
- **Cards: 4, 18, and 26. Angles: 90°, 30°, 60°. Angle Sum: 180°**
- **Cards: 5, 22, and 24. Angles: 40°, 120°, 20°. Angle Sum: 180°**
- **Cards: 6, 16, and 21. Angles: 45°, 25°, 110°. Angle Sum: 180°**
- **Cards: 7, 15, and 25. Angles: 100°, 30°, 50°. Angle Sum: 180°**
- **Cards: 8, 28, and 30. Angles: 85°, 65°, 30°. Angle Sum: 180°**
- **Cards: 11, 13, and 23. Angles: 20°, 100°, 60°. Angle Sum: 180°**
- **Cards: 17, 19, and 29. Angles: 40°, 80°, 60°. Angle Sum: 180°**
- **Cards: 31, 34, and 35. Angles: 55°, 75°, 50°. Angle Sum: 180°**
- **Cards: 32, 33, and 36. Angles: 95°, 30°, 55°. Angle Sum: 180°**

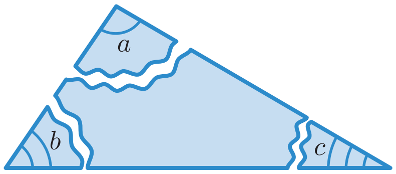
Activity
2

Name: .... Date: .... Period: ....

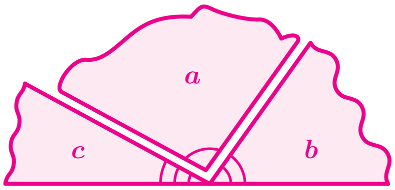
Tear It Up

6. You will use a blank sheet of paper to complete this activity. Use a straightedge to draw a triangle that you think will be different from the triangles your classmates will draw.

7. Label the angles of your triangle with the letters a, b, and c. Cut out the triangle, then tear the three angles off of the triangle like in this picture.



8. Rearrange the angles so that the three vertices meet with no overlap.

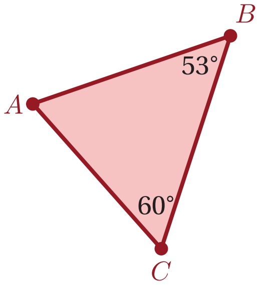


9. Compare your results with your classmates' results. What do you notice about your angles? What does this mean about the sum of the angles in a triangle?

***Responses vary*. The angles in each triangle always make a line when they are arranged with the vertices of the angles all meeting at the same point. Since there are 180 degrees in a line, there must also be 180 degrees in a triangle.**

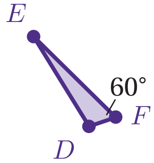
10. Here are the triangles from the Warm-Up, with some additional angle measurements labeled. For each triangle, determine a possible value for the angle listed.

Triangle 1



$m\angle A =$ **67°**

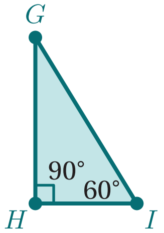
Triangle 2



Responses less than 120° are considered correct.

$m\angle D =$ **100°**

Triangle 3

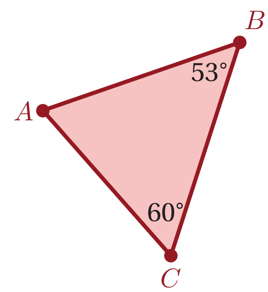


$m\angle G =$ **30°**

Synthesis

11. What is true about the sum of the three angle measures in a triangle?

Use the example if it helps with your thinking.



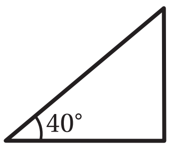
Responses vary. **The measures of the three interior angles in any triangle have a sum of 180 degrees. I know this because the angles form a straight line, which has a measure of 180°. This fact can help us determine the measurements of unknown angles.**

Things to Remember:

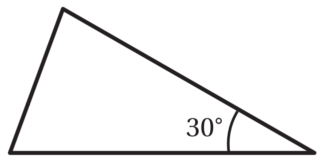
Find All Three

Directions: Make one copy for every 36 students. Then pre-cut the cards and give each student one card.

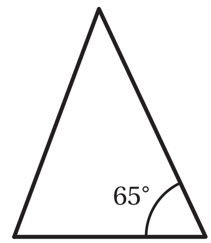
Card 1



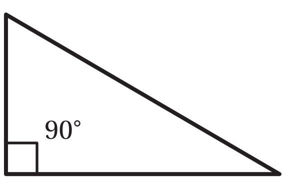
Card 2



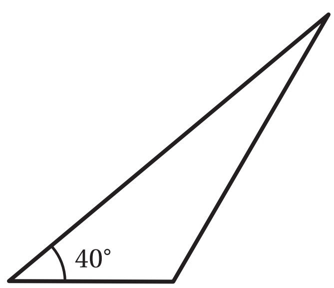
Card 3



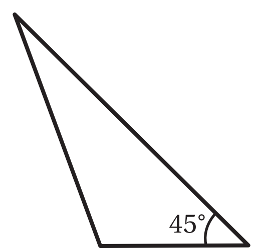
Card 4



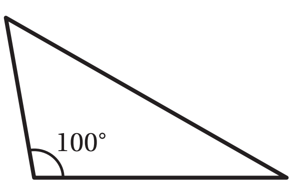
Card 5



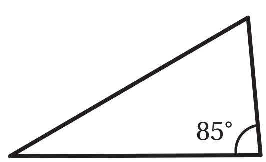
Card 6



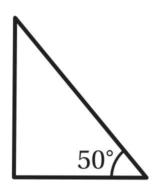
Card 7



Card 8

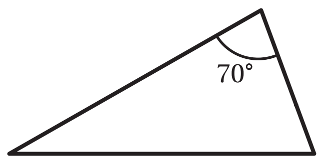


Card 9

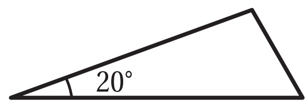


Find All Three

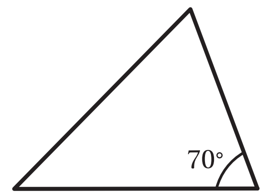
Card 10



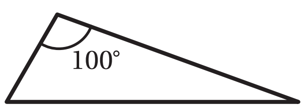
Card 11



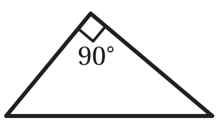
Card 12



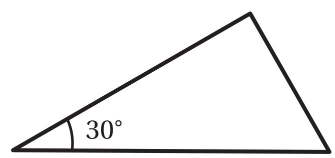
Card 13



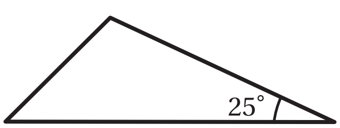
Card 14



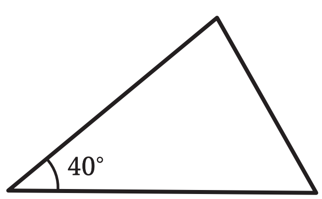
Card 15



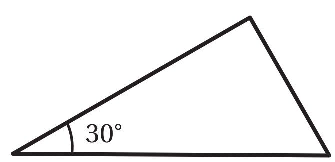
Card 16



Card 17

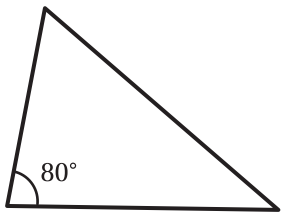


Card 18

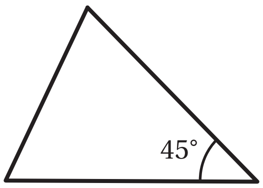


Find All Three

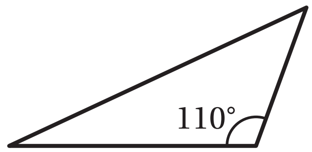
Card 19



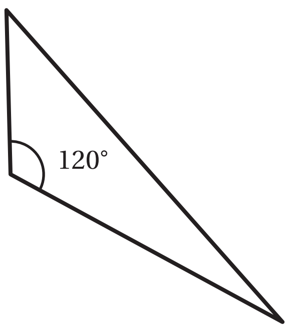
Card 20



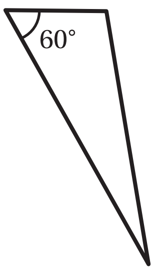
Card 21



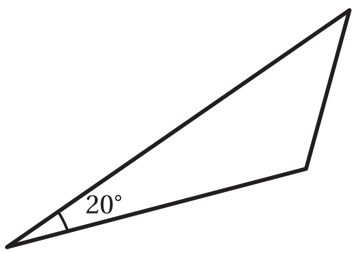
Card 22



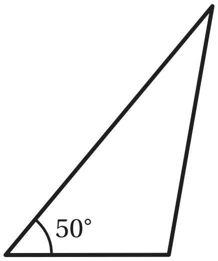
Card 23



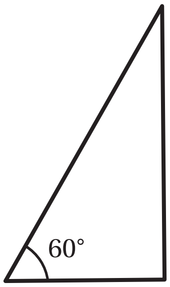
Card 24



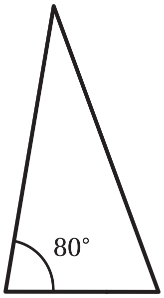
Card 25



Card 26

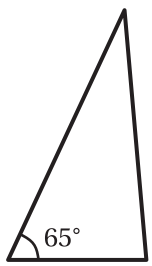


Card 27

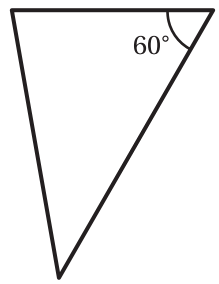


Find All Three

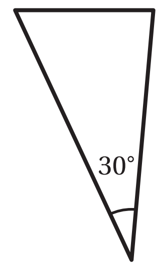
Card 28



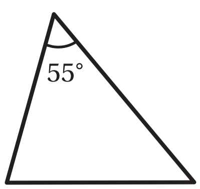
Card 29



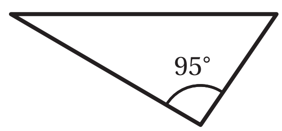
Card 30



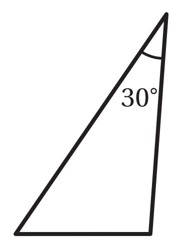
Card 31



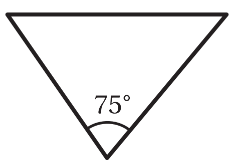
Card 32



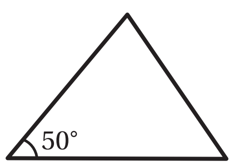
Card 33



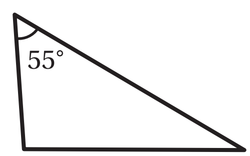
Card 34



Card 35

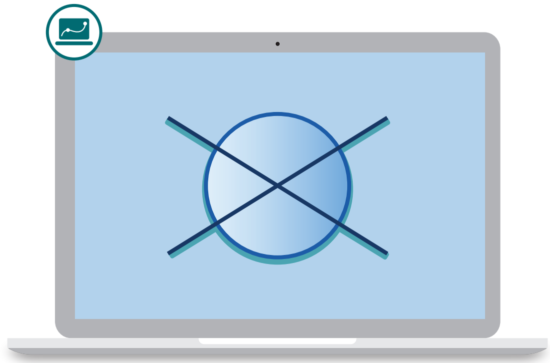


Card 36



Unit 1
Lesson
12

Name: Date: Period:



Puzzling It Out

Let's solve some puzzles using angle relationships.

Warm-Up

1 Point D is on line AC.

What are possible measures for angles ADB and CDB?

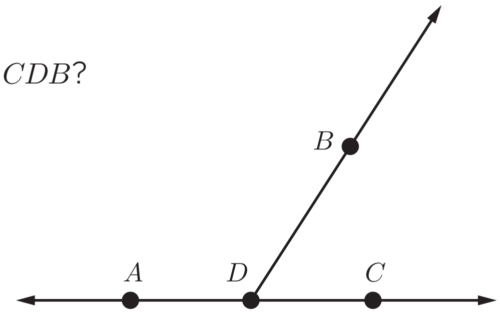


Figure may not be drawn to scale.

Responses vary.
- **100° and 80°**
- **110° and 70°**
- **104° and 76°**

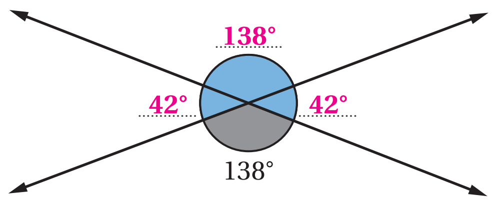
Activity
1

Name: .... Date: .... Period: ....

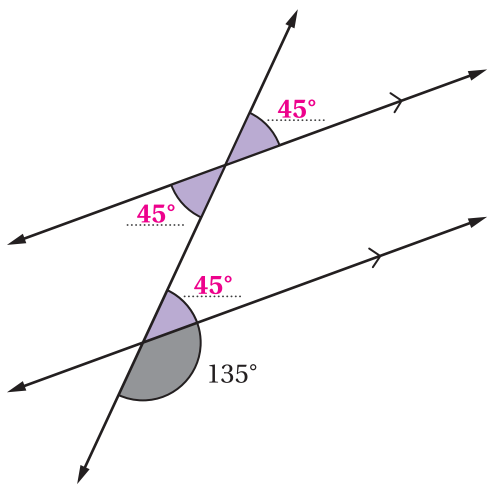
Angle Puzzles

Here are some angle puzzles. For each puzzle, one angle is revealed. You can ask for the measure of one or two more angles, if needed. Determine all the angle measures using as few hints as you can.

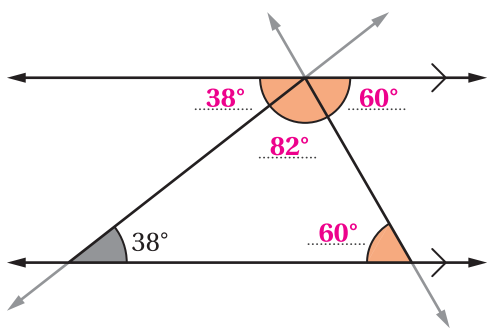
2 Angle Puzzle #1



3 Angle Puzzle #2



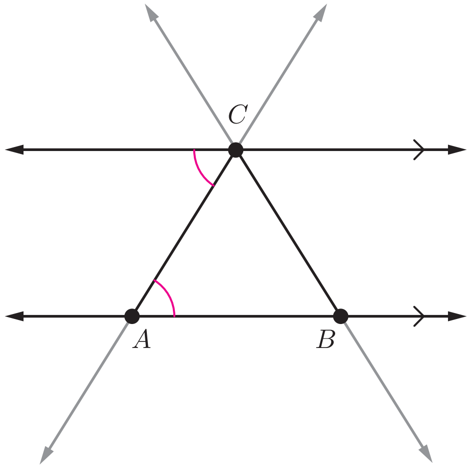
4 Angle Puzzle #3



Name: .... Date: .... Period: ....

Triangles and Parallel Lines

5 Imagine that the two lines stay parallel, but you can drag points A, B, and C to make any triangle.



a Mark one set of two or more angles inside the parallel lines that you think will *always* be congruent no matter where you drag the points.

Responses vary.

b Explain your thinking.

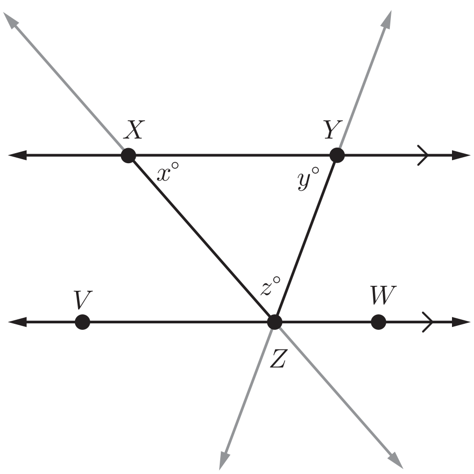
***Explanations vary.* If you rotate the diagram by 180°, it will put one angle on top of the other.**

6 Let's see what happens when you drag points A, B, and C.

Discuss: Are the angles that you thought will always be congruent actually congruent?

***Responses vary.* Yes because even when the points move, one angle can still be moved onto the other with a 180° rotation.**

7 Fabiana claims that if you tell her the value of x and y, she can use *transformations* to determine the value of z.



What might Fabiana be thinking? Do you believe her claim?

***Responses vary.* We can use rotations to see that angles VZX and YXZ are congruent and that angles WZY and XYZ are congruent. So the angles that form line VW have measures $x°$, $y°$, and $z°$. If Fabiana knows two of those angle measures, she can find the third one so they all add up to 180 degrees.**

Activity 3

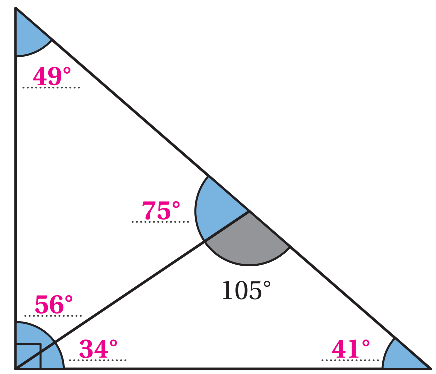
Name: .... Date: .... Period: ....

More Angle Puzzles

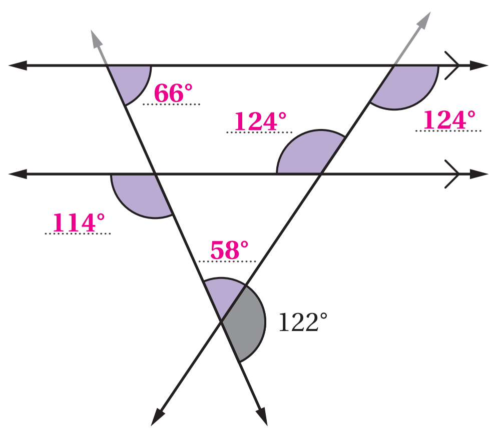
Here are some more angle puzzles. For each puzzle, one angle is revealed. You can ask for the measure of up to two more angles, if needed. Determine all the angle measures using as few hints as you can.

8 Angle Puzzle #4

Note: The square indicates a right angle (90°).



9 Angle Puzzle #5



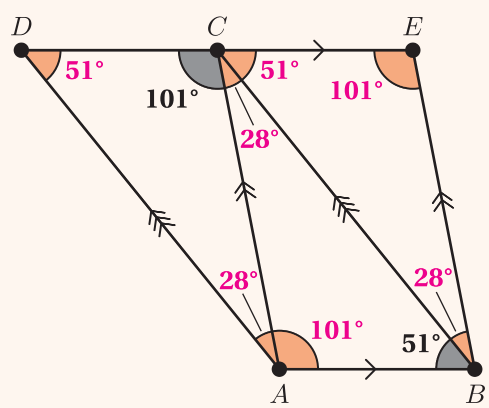
Explore More

10 Here is a diagram with three pairs of parallel lines. Is it possible to determine all of the angle measures with the given information?

Yes

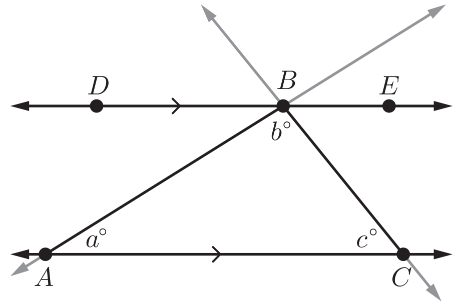
Show or explain your thinking.

Responses shown in image.



11 Synthesis

What are some angle relationships that can help you determine unknown angle measures?

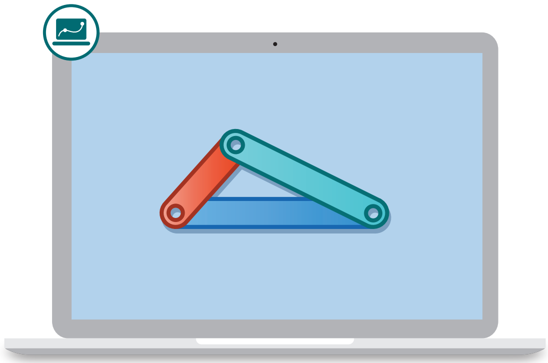


Responses vary.

- **The measures of the interior angles of a triangle add up to 180 degrees.**
- **The angles that make up a line add up to 180 degrees.**
- **Vertical angles, which are created by two intersecting lines, are congruent.**
- **There are several sets of congruent angles that are created by a transversal through a set of parallel lines.**

Things to Remember:

Name: .... Date: .... Period: ....



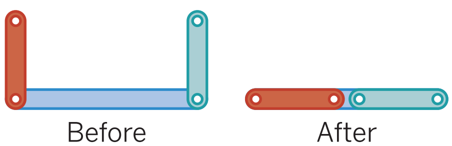
Can You Build It?

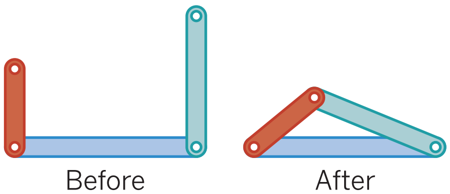
Let's explore what combinations of three line segments form a triangle.

Warm-Up

1 Here are two sets of line segments that were used to try to form a triangle.

Set 1 **Set 2**





What do you notice? What do you wonder?

I notice:

***Responses vary.* I notice that in Set 1, the segments are not long enough to form a triangle.**

I wonder:

***Responses vary.* I wonder what length the segments have to be in order to form a triangle.**

Activity
1

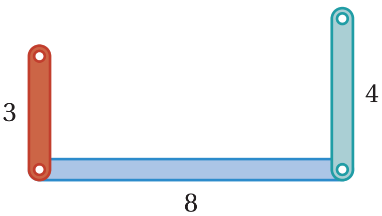
Name: ________ Date: ________ Period: ________

Length of the Third Side

You will use a set of line segments to complete this activity.

2 Will these three line segments form a triangle? Circle one.

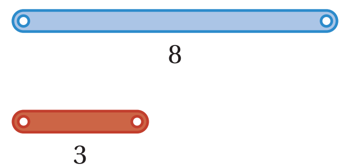
Yes No I'm not sure



Explain your thinking.

***Explanations vary.* The two shorter segments are too short to connect.**

3 Set 1 and Set 2 from the Warm-Up each have one line segment that is 8 units long and one that is 3 units long. Set 1 does not form a triangle. Set 2 does.



Try other lengths for the third segment. Try to find several that do and do not form a triangle.

Forms a Triangle

- 10 units

Responses vary.

- **6 units**
- **8 units**
- **9 units**

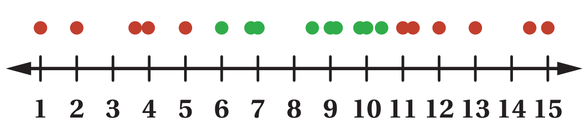
Does Not Form a Triangle

- 2 units

Responses vary.

- **3 units**
- **11 units**
- **14 units**

4 Here is a graph of other lengths that students tried. The lengths that form a triangle are represented by green dots.



Describe what you notice about those lengths.

***Responses vary.* All the segments that form a triangle are longer than 5 units but shorter than 11 units.**

Activity
2

Name: .... Date: .... Period: ....

Not Too Long, Not Too Short

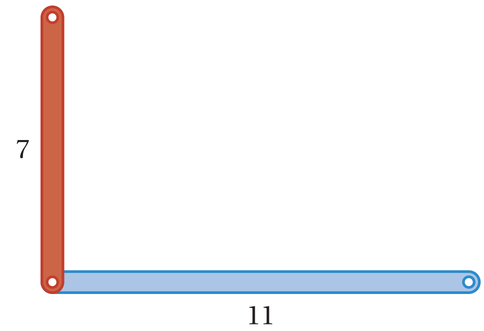
You will use a set of line segments to complete this activity.

5 Jamya is convinced that a third segment that is 19 units long will form a triangle.

Mohamed thinks that 19 units is too long.

Whose claim is correct? Circle one.

Jamya Mohamed

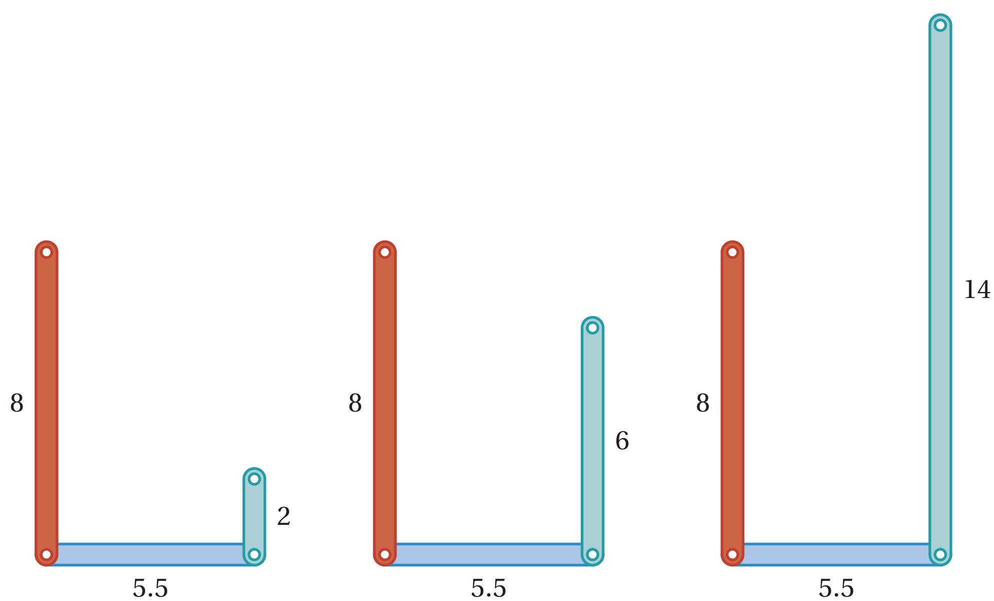


Explain your thinking.

***Explanations vary*. 19 units is too long because if segments measuring 7 units and 11 units were attached to a segment measuring 19 units, they would not be able to connect to form a triangle.**

6 Which of these lengths will form a triangle with side lengths 5.5 and 8 units? Circle one.

2 units 6 units 14 units



Name: ______ Date: ______ Period: ______

Not Too Long, Not Too Short (continued)

7 You will use a challenge card to create your own triangle challenge.

- Choose two line segments of any length.
- Determine three multiple choice options. One option should form a triangle with your other line segments, and two options should not.
- Swap your triangle challenge with one or more partners.
- Determine the lengths that will form a triangle for each of your partners' challenges.
- Complete as many challenges as you have time for.

Responses vary.

	Given Length	Given Length	Correct Third Length
______'s Challenge			
______'s Challenge			
______'s Challenge			
______'s Challenge			
______'s Challenge			

Name: .... Date: .... Period: ....

Does It Add Up

You will use a set of line segments to complete this activity.

8 A triangle has a perimeter of 24 units.

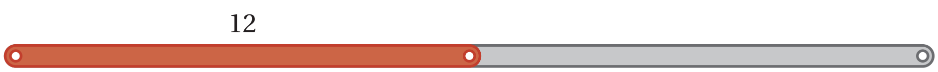
Perimeter = 24 units

What are three possible lengths for the sides of this triangle?

***Responses vary.* Sample shown in table.**

Length 1	Length 2	Length 3
8	**7**	**9**

9 Abena made the first side 12 units long.



Perimeter = 24 units

Will Abena be able to form a triangle with a perimeter of 24 units? Explain your thinking.

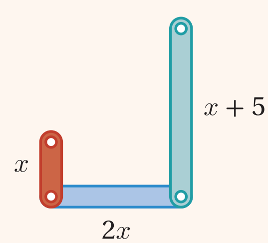
No. *Explanations vary.* If the perimeter is 24 units and one segment is 12 units long, then the lengths of the other two segments need to add up to 12 units. That means there are no combinations of segment lengths that would form a triangle. For example, $4 + 8 = 12$ but 4, 8, and 12 units would form a straight line, not a triangle.

Explore More

10 A triangle has sides that are x, $2x$, and $x + 5$ units.

What are some values of x that would form a triangle?

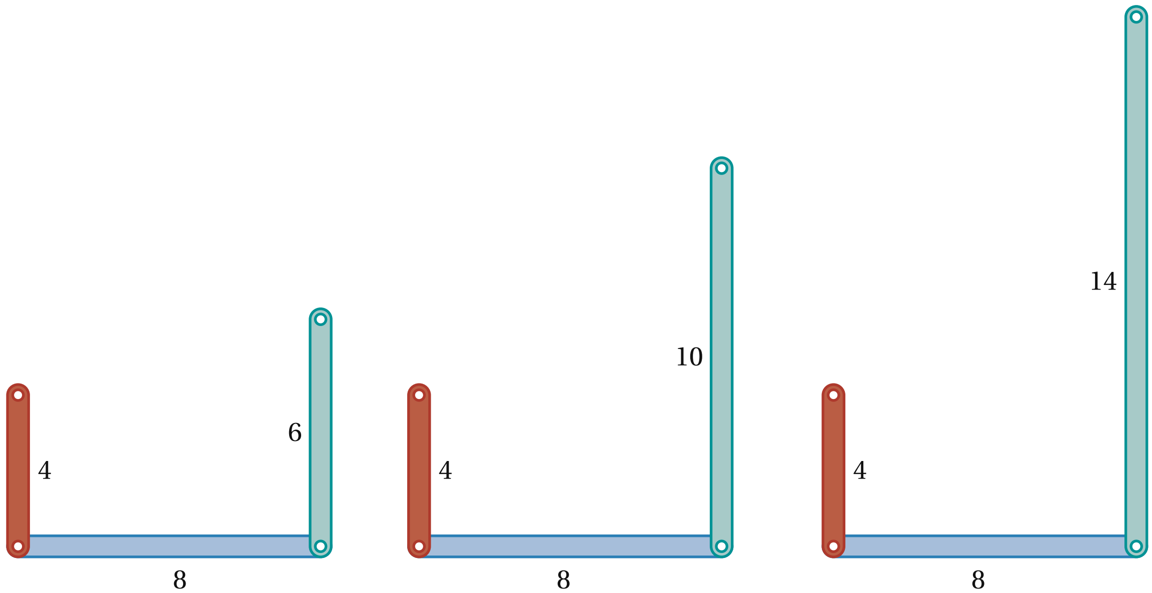
***Responses vary.* If x is 4, then the segment lengths would be 4, 8, and 9 units. Segments of those lengths would form a triangle.**



11 Synthesis

Explain how you can determine whether three line segments will form a triangle.

***Responses vary.* I can form a triangle if the two shorter segments added together are longer than the third segment.**



Things to Remember:

Not Too Long, Not Too Short

Lesson 7.05
Activity 2

Directions: Make one copy per four students. Then pre-cut the cards and give each student one card to record their own challenge.

........................'s Challenge

Given Length	Given Length

Select the correct length of the third segment.

A. .... units

B. .... units

C. .... units

........................'s Challenge

Given Length	Given Length

Select the correct length of the third segment.

A. .... units

B. .... units

C. .... units

........................'s Challenge

Given Length	Given Length

Select the correct length of the third segment.

A. .... units

B. .... units

C. .... units

........................'s Challenge

Given Length	Given Length

Select the correct length of the third segment.

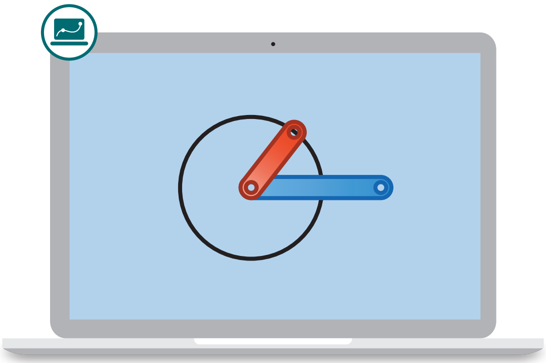
A. .... units

B. .... units

C. .... units

Unit 7
Lesson
6

Name: Date: Period:



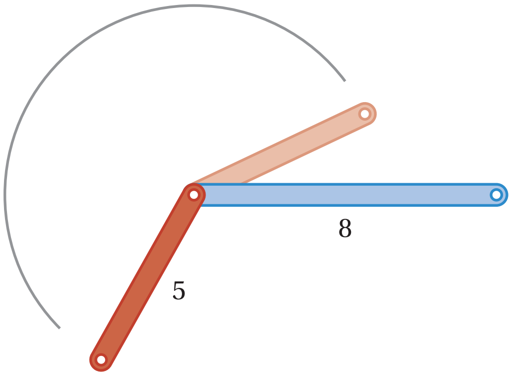
Is It Enough?

Let's explore connections between line segments and circles.

Warm-Up

1 Let's watch a line segment rotate around.

What shape does its path create?
Explain why this makes sense. ***Responses vary.***



The shape the path makes is . . . **a circle.**

This makes sense because . . . **it is going around and around from the center. The 5 is like the radius of the circle.**

Activity 1

Name: .... Date: .... Period: ....

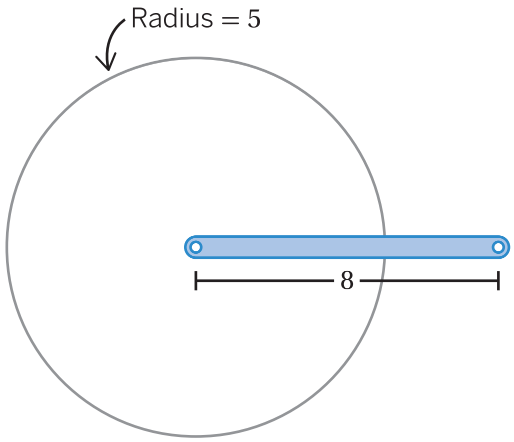
Building Triangles

You will use a set of line segments to help with your thinking.

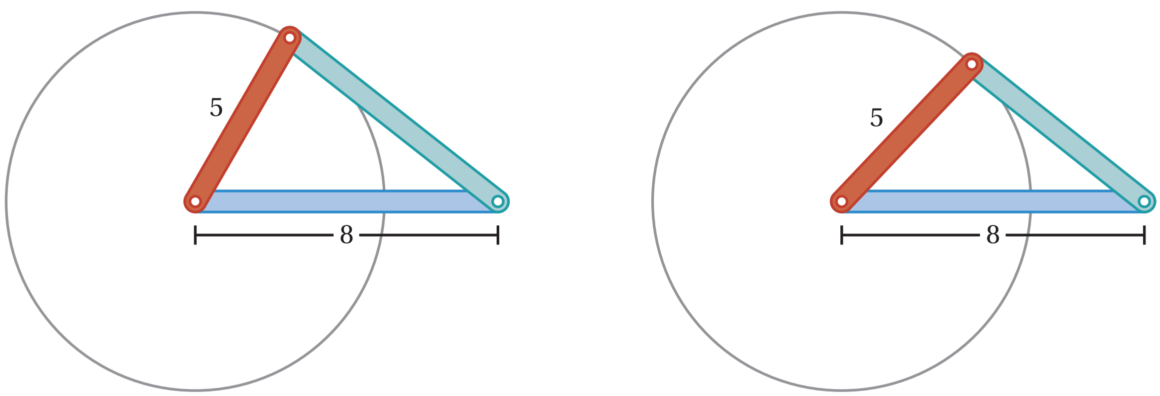
2 Use the circle and the segments to create different triangles.

Select each triangle you're able to make.

- [x] A triangle with a very long third side
- [x] A triangle with a short third side
- [x] A triangle with a $90°$ angle
- [x] A triangle with all acute angles
- [x] An isosceles triangle



3 A student made these two triangles.



Discuss: How can you tell these triangles are not the same?

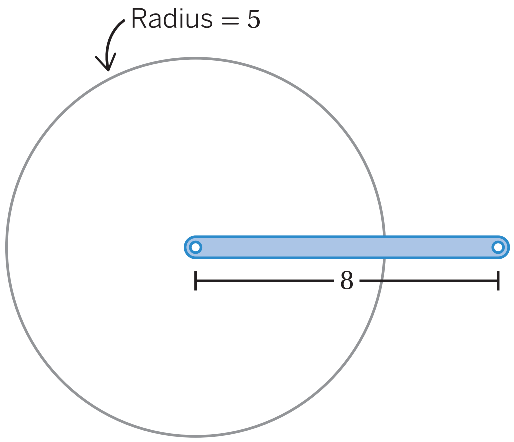
Responses vary. **Each 5-unit side intersects the circle at a different point, which means the third sides will be different lengths so the two triangles are not the same.**

4 Angel wants to draw a triangle with sides that are 5, 8, and 9 units long.

Describe or show how Angel might draw a third side that is 9 units long.

Responses vary.

- **Angel might make a line that is 9 units long and then move it until it intersects with the circle.**
- **Angel might make another circle with a radius that's 9 units long.**



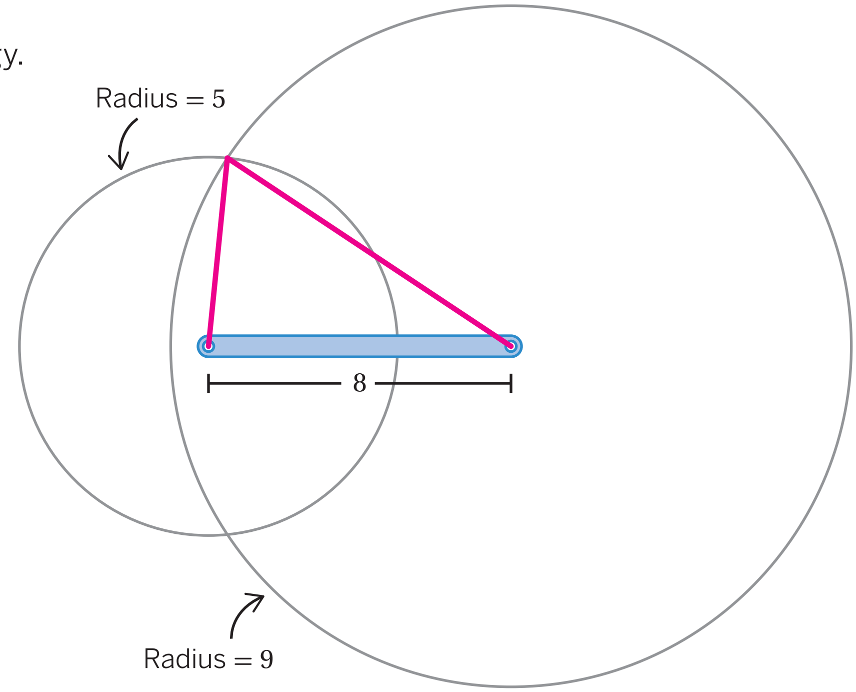
Name: ______ Date: ______ Period: ______

Building Triangles (continued)

5 Here is Angel's strategy.

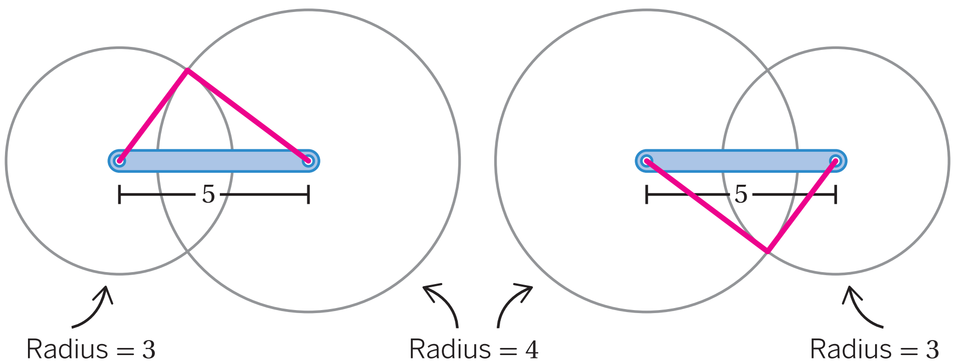
Draw a triangle that Angel could have drawn with side lengths that are 5, 8, and 9 units.

Responses vary.

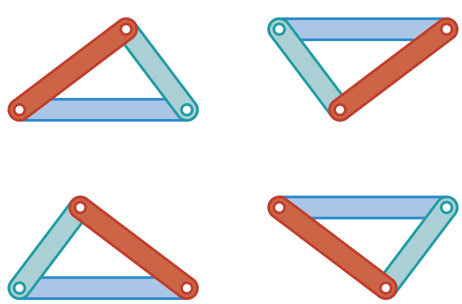


6 Use Angel's strategy to create two triangles with sides that are 3, 4, and 5 units long.

Responses vary.



7 Here are four possible triangles with side lengths of 3, 4, and 5 units. Angel thinks these triangles are **identical copies**.



Do you agree? Circle one.

Yes No I'm not sure

Explain your thinking.

Explanations vary. **You can move each triangle so that it fits right on top of the others, which means all the triangles are the same shape and size.**

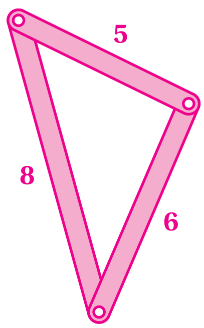
Name: ______________ Date: ________ Period: ________

Uniqueness

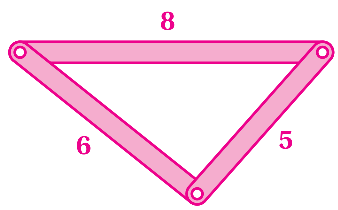
You will use a set of line segments to help with your thinking.

8 **a** Create two different triangles with the same side lengths. Then draw your triangles. ***Responses vary.***

Triangle A



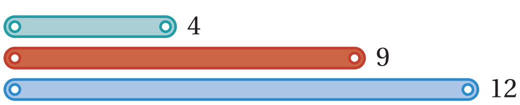
Triangle B



b **Discuss:** Will these three side lengths always create *identical copies*?
***Responses vary*. Yes. I might need to flip or turn triangle A, but I can place it on top of triangle B.**

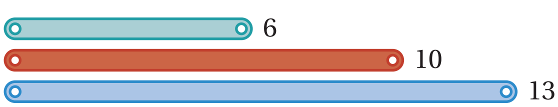
9 Circle the number of non-identical polygons that can be made using each set of line segments.

a



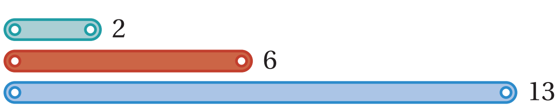
Zero One More than one

b



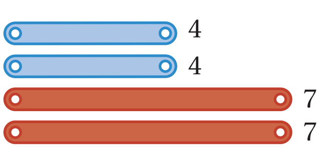
Zero One More than one

c



Zero One More than one

d



Zero One More than one

Activity
3

Name: Date: Period:

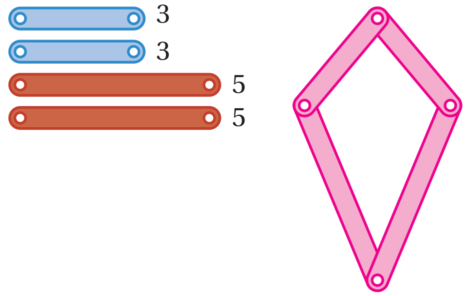
Quadrilaterals

You will use a set of line segments to help with your thinking.

10 Lukas made a quadrilateral with sides that are 3, 3, 5, and 5 units long.

Draw the shape Lukas might have made.

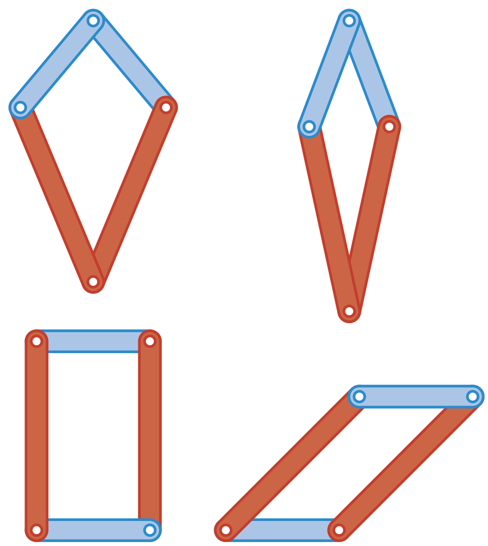
Drawings vary.



11 Here are four quadrilaterals that Lukas made with the sides that are 3, 3, 5, and 5 units long.

Describe why it's possible for Lukas to create quadrilaterals that are not identical copies.

***Responses vary.* Lukas can change which segments are next to each other (he can put the two short segments next to each other or across from each other). Also, Lukas can change the angle between two segments, tilting the shape or squishing it.**

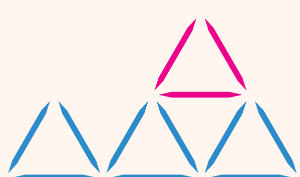


Explore More

12 In the diagram, 9 toothpicks are used to make three equilateral triangles.

Draw or describe how you can move only 3 of the toothpicks to make a diagram that has exactly 5 equilateral triangles.

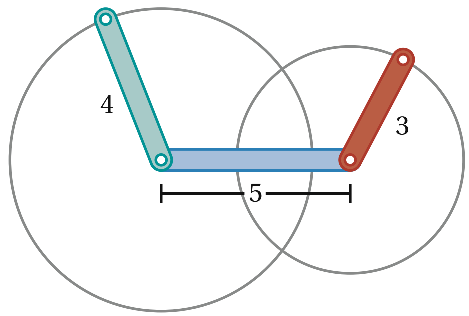
***Responses vary*. I can move one of the triangles on top between the other two triangles to create one large triangle with side lengths of 2 toothpicks. That makes 4 small triangles and 1 large triangle.**



13 Synthesis

a Describe how to create a triangle given three side lengths.

Responses vary. **I can draw two circles with the radius of the two smallest segments and make a triangle where those two circles cross.**



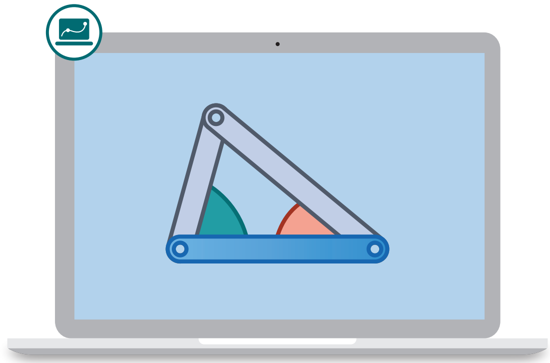
b Explain why there will be only one possible triangle.

Responses vary. **There will only be one possible triangle because all the triangles created this way will match up exactly when placed on top of each other.**

Things to Remember:

Unit 7
Lesson
7

Name: .... Date: .... Period: ....



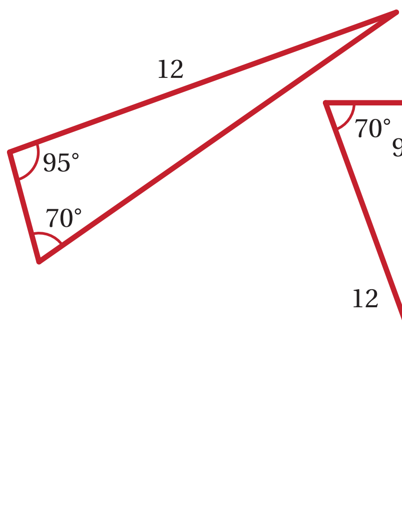
More Than One?

Let's build and compare triangles.

Warm-Up

1 Are these triangles identical? Circle one.

Yes No I'm not sure



Explain your thinking.

Responses and explanations vary.

- **Yes. They're identical because they are both long and skinny, and they have all of the same measurements.**
- **No. They're not identical because the 12 is in different places. On the left triangle, the 12 is next to the 95° and on the right, it is next to the 70°.**
- **I'm not sure. To check if they're identical, I need to be able to drag one on top of the other to see if they are exactly the same size and shape.**

Activity 1

Name: .... Date: .... Period: ....

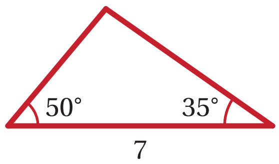
Two Angles, One Side

You will use a set of line segments and angles to help you with your thinking.

2 Sol made a triangle with these measurements:

- A side length of 7 units
- A 35° angle
- A 50° angle

Sol's Triangle



Roberto wants to make a triangle with the same measurements. Will Roberto's triangle be identical to Sol's? Circle one.

Definitely Maybe No way

Explain your thinking.

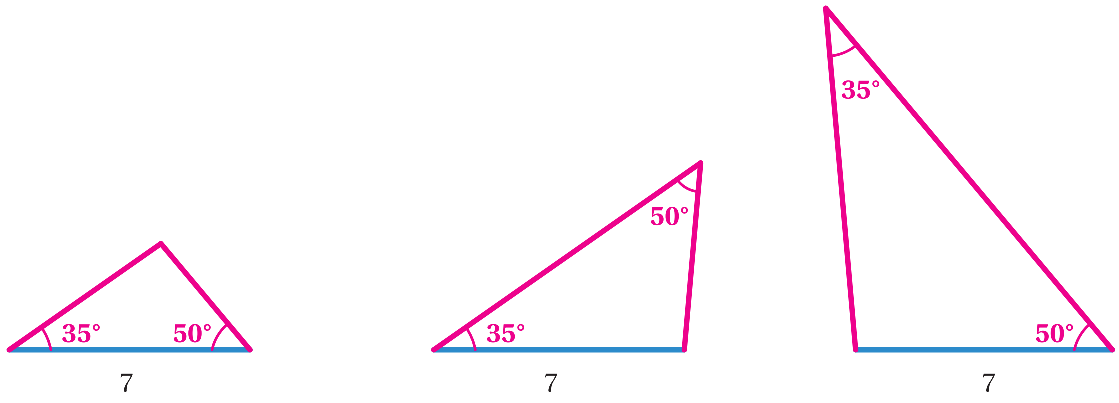
Responses and explanations vary.

- **Definitely. Roberto's triangle will be the same because his triangle will have the same three measurements as Sol's triangle.**
- **Maybe. The two triangles could be the same, but maybe the triangles will be different sizes.**
- **No way. I only know one side length, so there is no way Roberto's triangle will be the same as Sol's.**

3 Create triangles with the same measurements as Sol's triangle.

Try to create more than one non-identical triangle.

Responses vary.



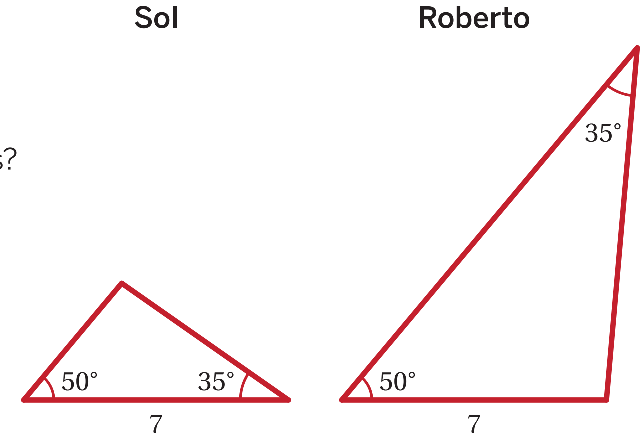
Name: Date: Period:

Two Angles, One Side (continued)

4 Here are the triangles that Sol and Roberto created.

What could you say to help Roberto create a triangle that is identical to Sol's?

***Responses vary.* I would tell Roberto that the 7 has to go in the middle between the 35° angle and the 50° angle.**



Activity 2

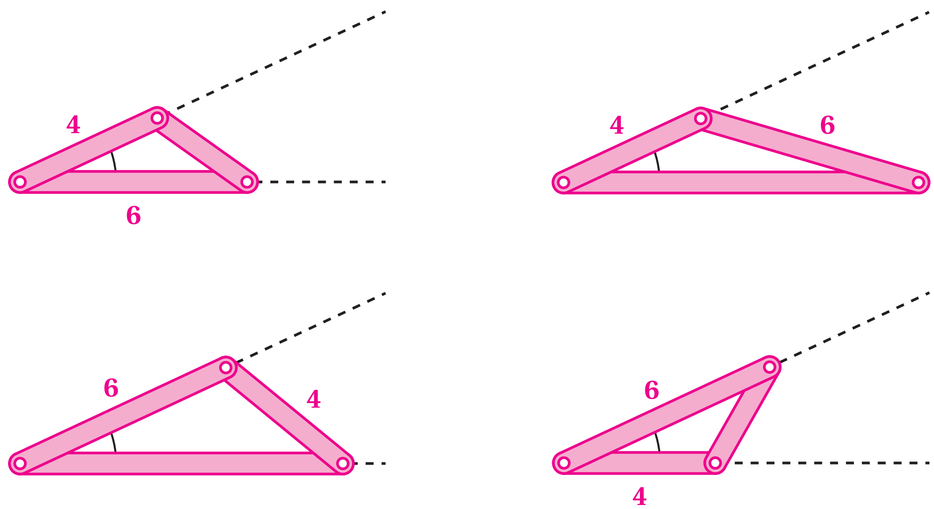
Name: .... Date: .... Period: ....

Side Angle Side

You will use a set of line segments and angles to help you with your thinking.

5 **a** Create as many non-identical triangles as you can with these measurements:

- Side lengths of 4 and 6 units
- One angle of 25°



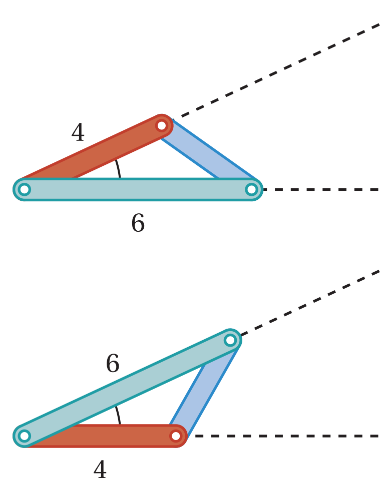
b **Discuss:** What was your strategy for making non-identical triangles?

***Responses vary.* I changed where the side length of 6 was compared to the 25° angle.**

6 Malik claims he will always get *identical copies* if the 25° angle is between the side lengths of 4 and 6 units.

Is Malik's claim correct? Explain your thinking.

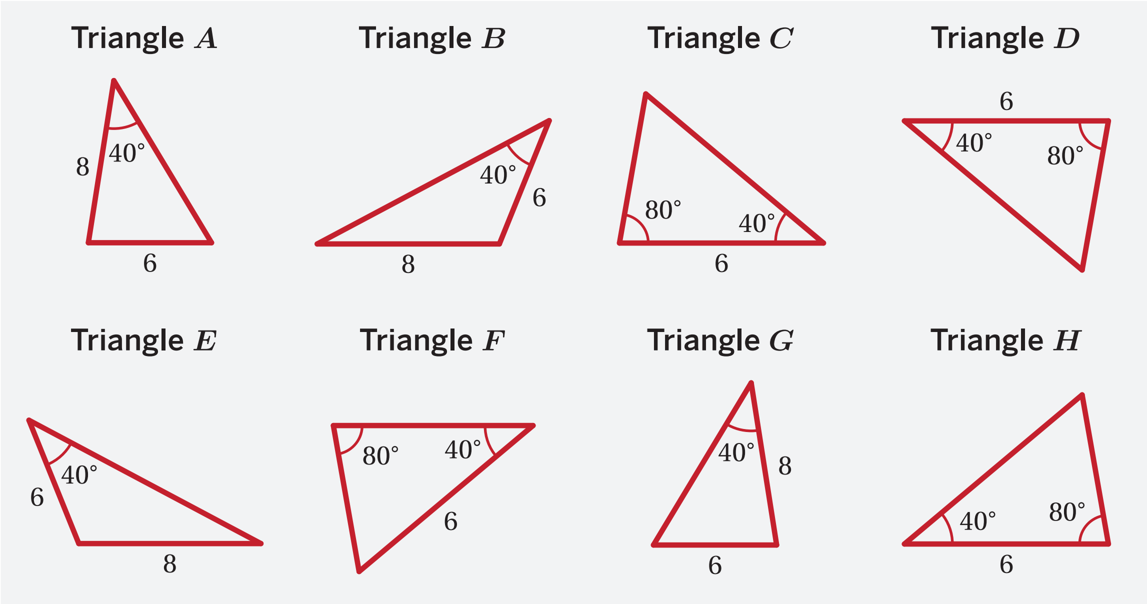
Yes. *Explanations vary.* All of the triangles I created with the angle measure between the two sides were identical, even if some were upside down.



Name: .... Date: .... Period: ....

Side Angle Side (continued)

7 Make groups of identical copies. One triangle will not have a group.



Group 1	Group 2	Group 3
Triangles A and G	Triangles B and E	Triangles C, D, and H

8 Isabelle claims Triangles A and E are identical copies and belong in the same group. Is Isabelle's claim correct? Circle one.

Yes, they're identical

No, they're not identical

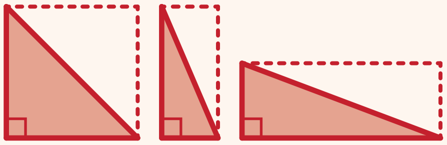
Explain your thinking.

***Explanations vary.* One triangle has a side length of 6 next to the 40° angle, and the other triangle has a side length of 6 on the opposite side of the triangle.**

Explore More

9 Fatima thinks that in a right triangle, the other two angles are *complementary* (add up to 90°).

Is Fatima correct? Explain your thinking.



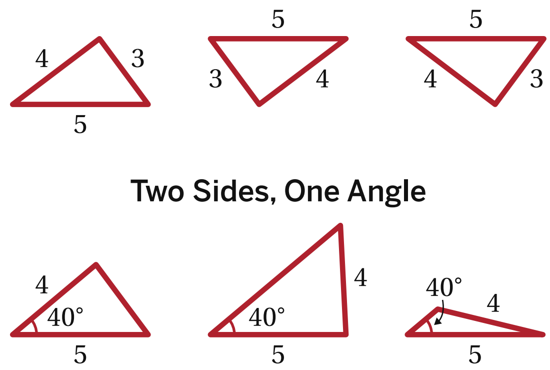
Yes. *Explanations vary.* Fatima can take two identical copies of the same right triangle and form a rectangle. Together, the two non-right angles of the triangle form the right angle of a rectangle.

10 Synthesis

Explain why triangles with three of the same side lengths are *identical copies*, but triangles with the same two side lengths and one angle measure aren't always identical copies.

Three Sides

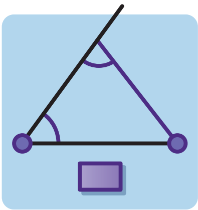
Two Sides, One Angle



Responses vary. **With all three side lengths, everything is locked in because every side length is connected to every other side length. With two side lengths and one angle, I don't know where the angle goes. The angle could go between the two side lengths or it could go next to only one of the side lengths, which changes the shape of the triangle.**

Things to Remember:

Name: .... Date: .... Period: ....



Can You Draw It?

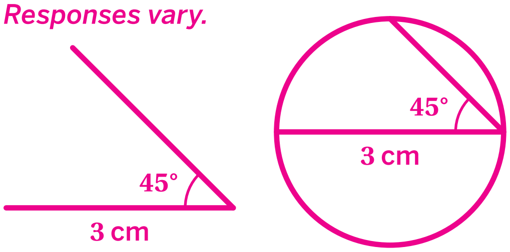
Let's draw triangles.

Warm-Up

1. Sketch a shape or figure that includes:

- A line segment with a length of 3 centimeters.
- A 45° angle.

Responses vary.



Activity 1

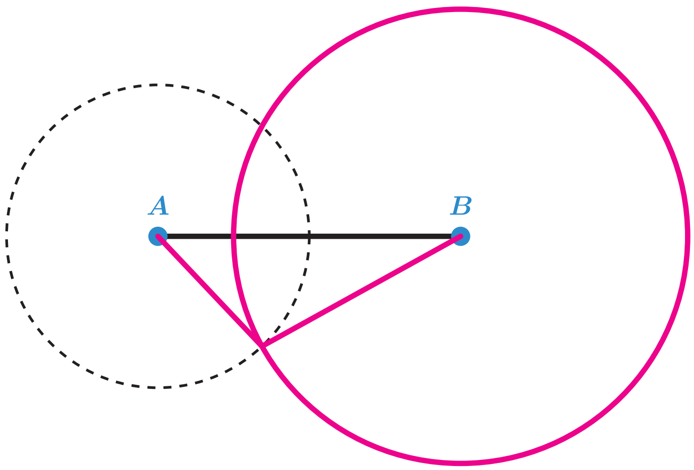
Name: ____ Date: ____ Period: ____

Complete the Triangles

2. Sadia is trying to draw a triangle with sides measuring 3 centimeters, 2 centimeters, and 4 centimeters. Here are the steps Sadia took so far:

- Step 1: Draw a 4-centimeter line segment AB.
- Step 2: Use a compass to draw a circle around point A with a radius of 2 centimeters.

Describe or show how Sadia can finish drawing this triangle.

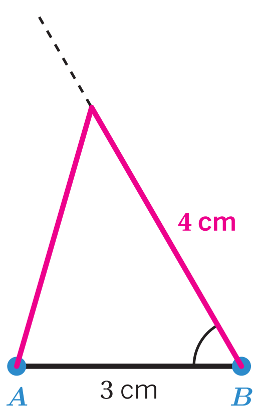


***Responses vary.* Sadia could draw a circle with a radius of 3 centimeters around point B. The circles will intersect twice. Then from one intersection point, Sadia can draw a line segment to point A and a line segment to point B.**

3. Nekeisha is trying to draw a triangle with one 3-centimeter side, one 4-centimeter side, and one $60°$ angle. Here are the steps Nekeisha took so far:

- Step 1: Draw a 3-centimeter line segment.
- Step 2: Use a protractor to draw a $60°$ angle at one end of the line segment.

Describe or show how Nekeisha can finish this drawing.



***Responses vary.* Nekeisha can draw a 4-centimeter line segment starting at point B along the $60°$ angle. Then draw another line segment that connects the end of the 4-centimeter line segment to point A.**

Name: .... Date: .... Period: ....

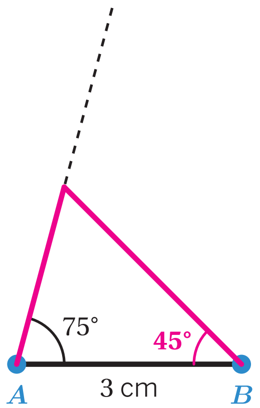
Complete the Triangles (continued)

4. Ahmed is trying to draw a triangle with one $45°$ angle, one $75°$ angle, and one 3-centimeter side.

Here are the steps Ahmed took so far:

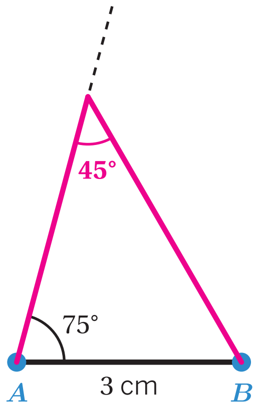
- Step 1: Draw a 3-centimeter line segment.
- Step 2: Use a protractor to draw a $75°$ angle.

Describe or show how Ahmed can finish this drawing.



***Responses vary.* Ahmed can draw a $45°$ angle at point B.**

5. Is it possible for Ahmed to draw another non-identical triangle that matches this description? Use the diagram to show or explain your thinking.



Yes. *Explanations vary.* Changing the position of the $45°$ angle creates a different non-identical triangle.

Activity 2

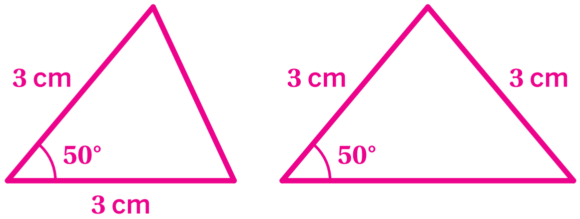
Name: .... Date: .... Period: ....

Drawing Challenges

For each description:

- Draw as many non-identical triangles as you can.
- Determine how many unique triangles can be made with these measurements.

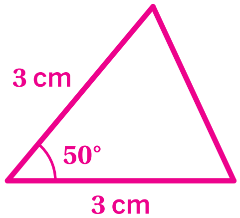
6. Two 3-centimeter sides and one $50°$ angle. ***Drawings vary.***



How many unique triangles can be made with these measurements? Circle one.

Zero One More than one

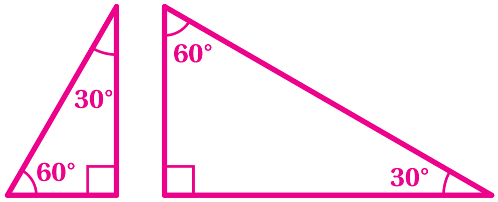
7. Two 3-centimeter sides with a $50°$ angle between them. ***Drawings vary.***



How many unique triangles can be made with these measurements? Circle one.

Zero One More than one

8. One $30°$ angle, one $60°$ angle, and one $90°$ angle. ***Drawings vary.***



How many unique triangles can be made with these measurements? Circle one.

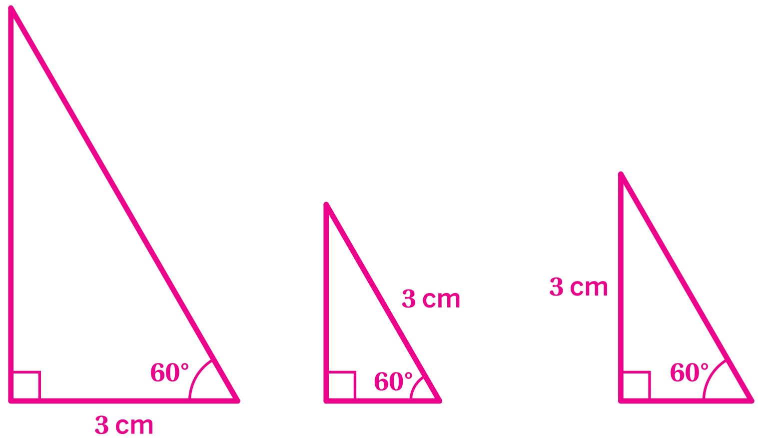
Zero One More than one

Activity 2

Name: .... Date: .... Period: ....

Drawing Challenges (continued)

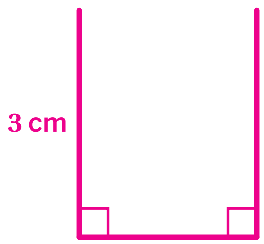
9. One 60° angle, one 90° angle, and one 3-centimeter side. ***Drawings vary.***



How many unique triangles can be made with these measurements? Circle one.

Zero One More than one

10. Two 90° angles and one 3-centimeter side. ***Drawings vary.***



How many unique triangles can be made with these measurements? Circle one.

Zero One More than one

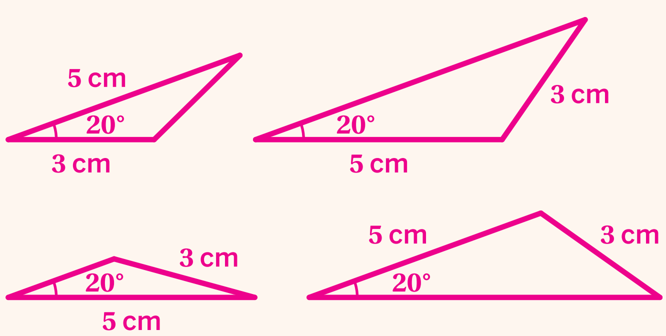
Explore More

11. What is the greatest number of non-identical triangles possible when you know the measurements of two sides and one angle?

Four triangles

Show or explain your thinking.

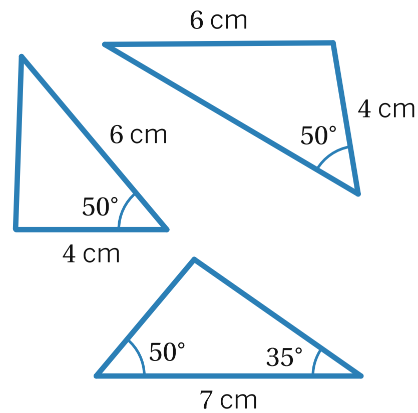
Explanations vary.



Figures may not be drawn to scale.

Synthesis

12. Describe how many non-identical triangles can be made with different combinations of measurements.



Responses vary.

- Three side lengths can only create one unique triangle.
- Three angle measures can create many non-identical triangles.
- One side length and two angle measures can create many non-identical triangles.
- Two side lengths and one angle measure can create many non-identical triangles.
- One angle measure in between two side lengths can only create one unique triangle.

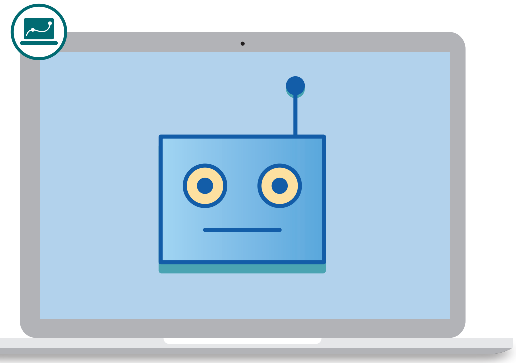
Things to Remember:

Unit 1

Lesson

2

Name: .... Date: .... Period: ....



Scaling Robots

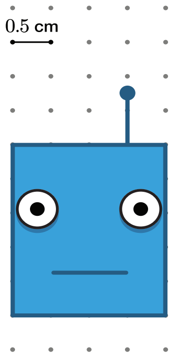
Let's explore scaled copies further.

Warm-Up

1 Create a robot face that you like. Your robot face should be a rectangle and include two eyes and an antenna. (See the example robot.) Then complete the table.

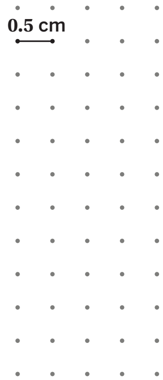
Responses vary.

Example Robot



Height (cm)	2.5
Width (cm)	2
Eye Distance (cm)	1
Antenna (cm)	0.75

My Robot



Height (cm)	
Width (cm)	
Eye Distance (cm)	
Antenna (cm)	

Activity 1

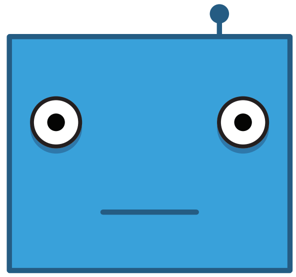
Name: .... Date: .... Period: ....

Scaling Robots

2 Here's Felipe's robot. He wants to make a scaled copy of his robot. Complete the table so that the new measurements represent a scaled copy of his original robot.

	Original Robot	New Robot
Height (cm)	5	10
Width (cm)	4	**8**
Eye Distance (cm)	2.5	**5**
Antenna (cm)	0.5	**1**

Felipe's Robot



3 Scaled copies always have a **scale factor**.

Imani and Anh built a robot and made a table for a scaled copy.

Imani

	Original Robot	New Robot
Height (cm)	5 $\xrightarrow{\times 2}$	10
Width (cm)	3	6
Eye Distance (cm)	2	4
Antenna (cm)	1.5 $\xrightarrow{\times 2}$	3

Anh

	Original Robot	New Robot
Height (cm)	5	10
Width (cm)	3	6
Eye Distance (cm)	2	4
Antenna (cm)	1.5	3

$$\frac{10}{5} = \frac{6}{3} = \frac{4}{2} = \frac{3}{1.5}$$

Show or describe where in their work you can see the scale factor is 2.

Responses vary.

- **Imani's work shows the scale factor as the number you need to multiply the values in the first column by to get the values in the second column. For example, $5 \cdot 2 = 10$ and $3 \cdot 2 = 6$.**
- **Anh wrote all the lengths as ratios, and those ratios are equivalent. They all have the same value, 2, which must be the scale factor.**

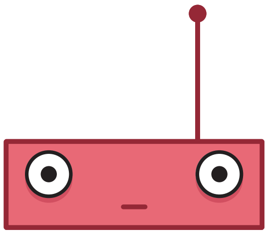
Activity

2

Name: Date: Period:

Analyzing Robots

4-5 Imani made another robot and tried to make a scaled copy.



	Original Robot	New Robot
Height (cm)	2	8
Width (cm)	6	12
Eye Distance (cm)	4	10
Antenna (cm)	1	7

Do you think the new robot will be a scaled copy?

If yes, explain your thinking.

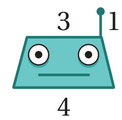
If no, cross out and replace one or more measurements so that the new robot *is* a scaled copy.

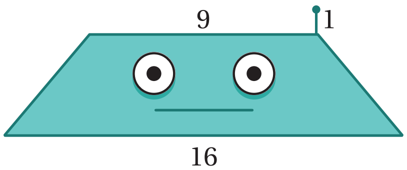
Not a scaled copy. ***Explanations vary.*** **There should be a constant scale factor between each measurement of Imani's robot and each corresponding measurement of the new robot.**

Name: ______ Date: ______ Period: ______

Analyzing Robots (continued)

6 Anushka made the small robot. Then she tried to make a scaled copy of the robot.





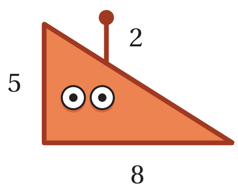
a **Discuss:** What might Anushka's strategy have been?

***Responses vary.* I think Anushka's strategy was multiplying each length by itself. For example, $3 \cdot 3 = 9$.**

b How could you help Anushka revise her work?

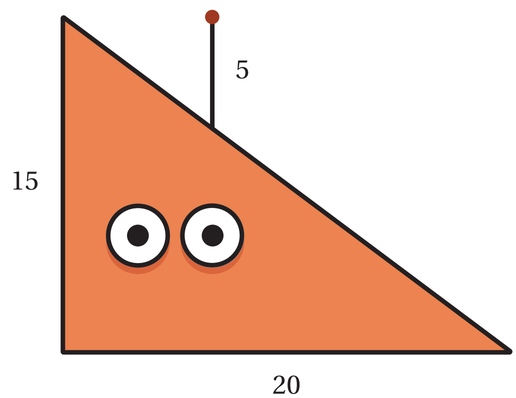
***Responses vary.* I would help Anushka revise her work by making sure that the lengths in the original and in the scaled copy are proportional, which means that the scale factor would be the same from the original to the new robot. It doesn't matter what scale factor you choose as long as you multiply each length by the same scale factor.**

7 Na'ilah drew a small and a large robot. Help her make the large robot a scaled copy.



a Cross out and replace one or more measurements on the large robot that could make it a scaled copy.

Responses vary.



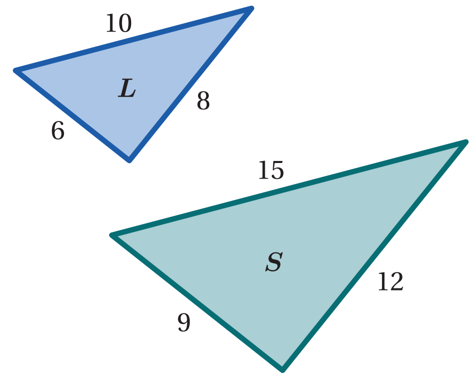
b What is the scale factor from the small to the large robot?

Responses vary.

8 Synthesis

How can you use lengths to determine whether a figure is a scaled copy of another figure?

Use the example if it helps with your thinking.



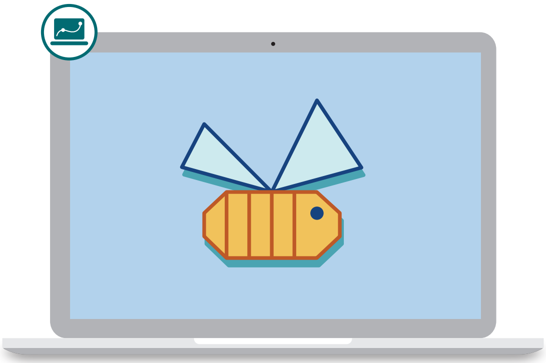
Responses vary.

- **If the ratios of the side lengths between the two figures are equivalent, then one figure is a scaled copy of the other. For example, $\frac{15}{10} = \frac{9}{6} = \frac{12}{8}$, so L and S are scaled copies of each other.**
- **If the scale factor is the same between each length in the original figure and its matching length in the copy, then the lengths are proportional and the new figure is a scaled copy. For example, if I multiply each length in figure L by 1.5, I get all of the lengths in figure S.**

Things to Remember:

Unit 1
Lesson
3

Name: Date: Period:

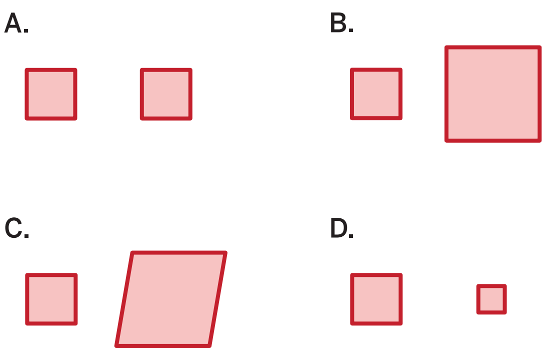


Make It Scale

Let's draw scaled copies.

Warm-Up

1 Which pair doesn't belong? Explain your thinking.



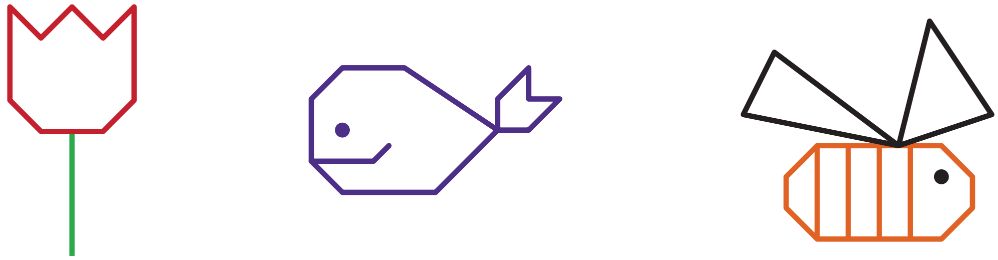
Responses and explanations vary.

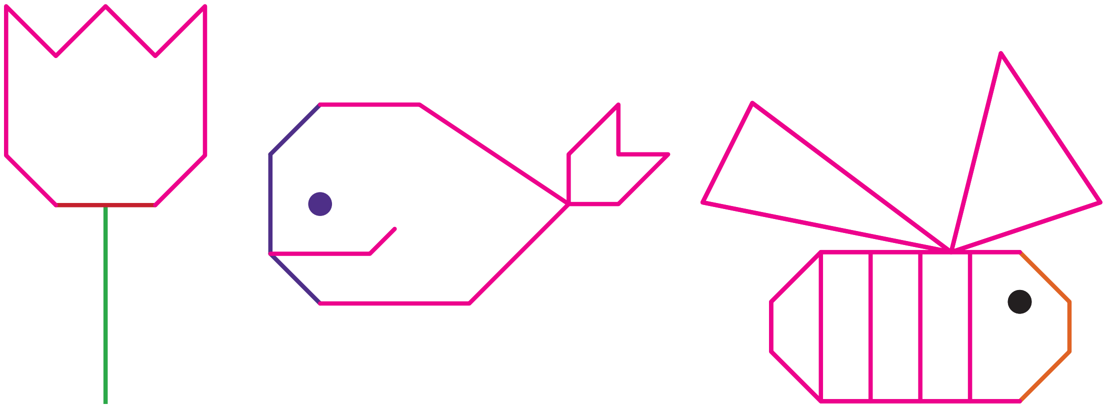
- **Pair A doesn't belong because it's the only pair where the two figures are the same shape and size.**
- **Pair B doesn't belong because it's the only pair where the figure on the right is a larger scaled copy of the figure on the left.**
- **Pair C doesn't belong because it's the only pair where the figure on the right is not a scaled copy because the angles are not all the same.**
- **Pair D doesn't belong because it's the only pair where the figure on the right is a smaller scaled copy of the figure on the left.**

Name: .... Date: .... Period: ....

Drawing Scaled Copies Without a Grid

2-3 Choose *one* figure that you'd like to make a scaled copy of. Complete the scaled copy of the figure you chose. Use a scale factor of 2.





4 What might help you make a more accurate scaled copy?

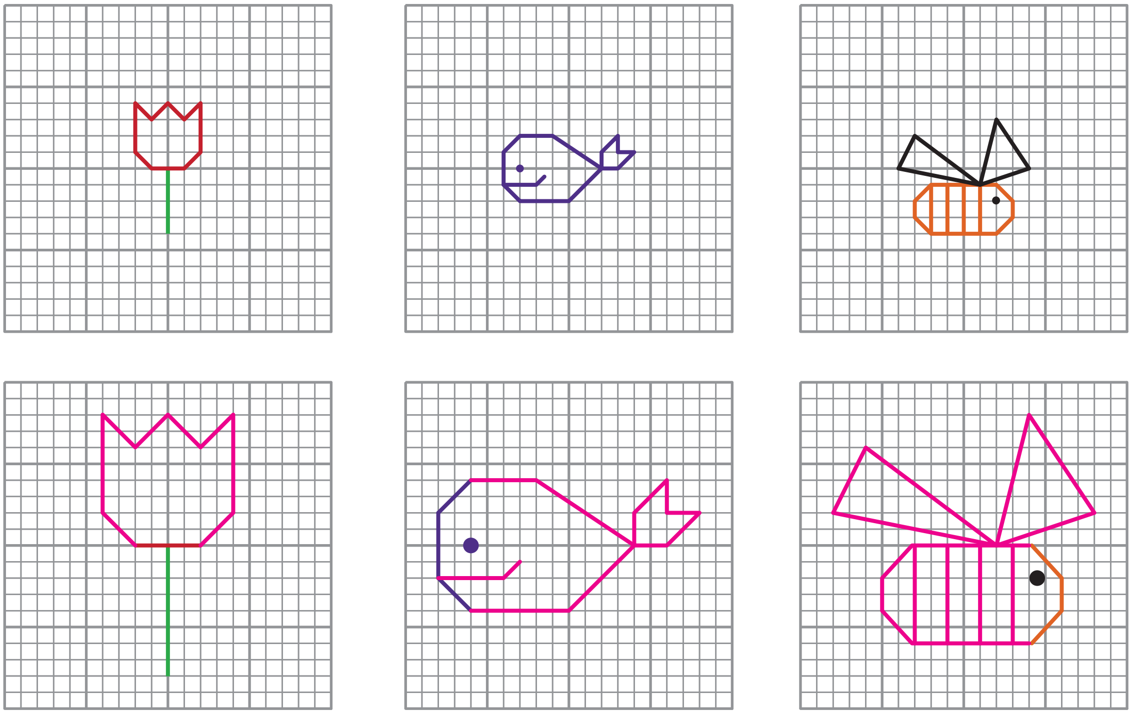
Responses vary.
- **It would be helpful to know the side measurements of the original figure. That way we could calculate how long the lengths of the scaled copy should be.**
- **A grid would help me draw the scaled lengths more accurately.**

Activity 2

Name: ______ Date: ______ Period: ______

Drawing Scaled Copies With a Grid

5 Make a new scaled copy of the figure you chose. Use a scale factor of 2.



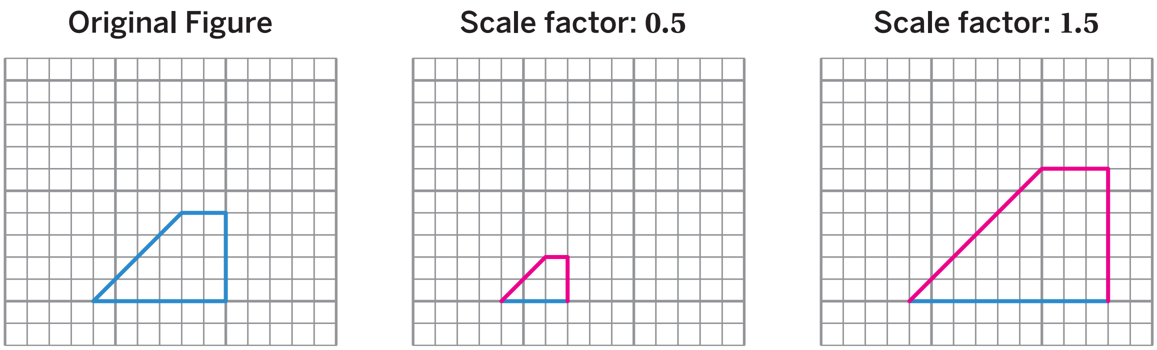
6 Explain the strategy you used to draw the new scaled copy.

***Responses vary*. First, I figured out the length of one side in the original figure. Next, I multiplied that number by 2 to get the length of that side in the scaled copy. Then, I drew that side in my scaled copy. For diagonal lines, I had to pay extra attention to get the angle just right. I repeated these steps for each line in the figure.**

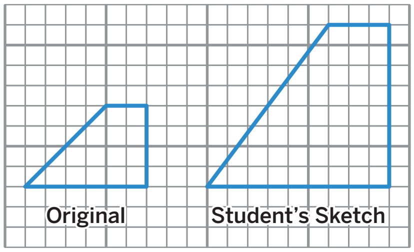
Name: ______ Date: ______ Period: ______

Drawing Scaled Copies With a Grid (continued)

7 Choose a scale factor of 0.5 or 1.5. Then complete the scaled copy.



8 Here is one student's sketch. Sasha thinks the student used a scale factor of 2. Randy thinks the student used a scale factor of 1.5. Who is correct? Circle one.



Sasha Randy Both Neither

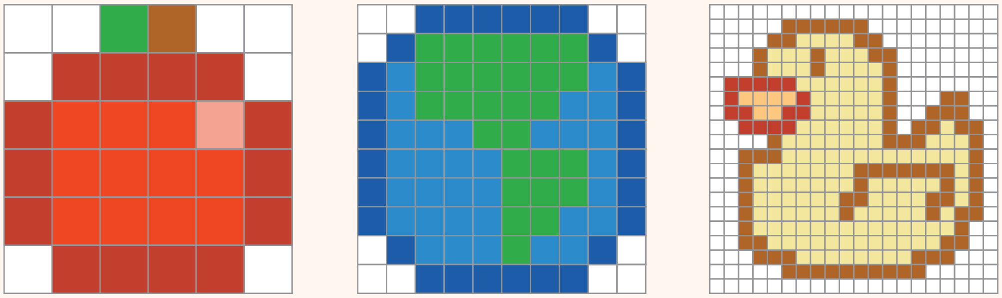
Explain your thinking.

Responses and explanations vary.

- **Sasha is correct because the height of the sketch is multiplied by 2.**
- **Randy is correct because the width is multiplied by 1.5.**
- **Both Sasha and Randy are correct because the scale factors of 2 and 1.5 are both used in this sketch.**
- **Neither student is correct because more than one scale factor was used, and the sketch is not a scaled copy.**

Explore More

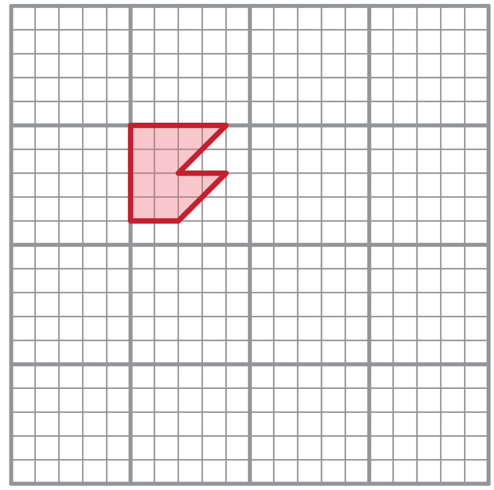
9 On a piece of graph paper, draw one of these images using a scale factor of 1.5. Or draw your own image and a scaled copy. ***Responses vary.***



10 Synthesis

Describe how to draw a scaled copy.

Use the example if it helps with your thinking.



Responses vary.

- **When making a scaled copy, make sure all the side lengths are multiplied by the same scale factor.**
- **When drawing the new figure, keep in mind how you draw the angles so that the angles remain the same in both figures.**

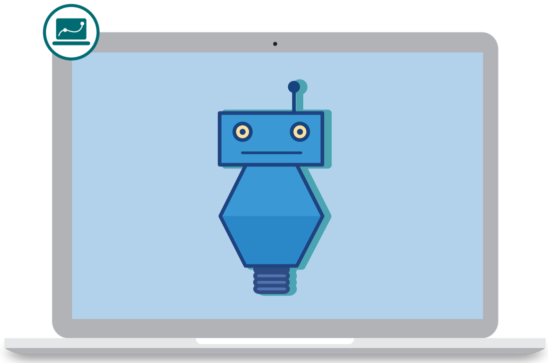
Things to Remember:

Unit 1
Lesson
4

Name: Date: Period:

Scale Factor Challenges

Let's explore how scale factors affect the size of scaled copies.



Warm-Up

1 a Discuss: How are these equations alike? How are they different?

A. $8x = 80$

B. $8x = 8$

C. $8x = 1$

D. $\frac{1}{8}x = 1$

Responses vary.
- **Alike: The equations all involve multiplying one known number and one unknown number.**
- **Different: Of the four solutions, two are greater than 1, one is equal to 1, and one is less than 1.**

b Use what you noticed to solve each equation mentally.

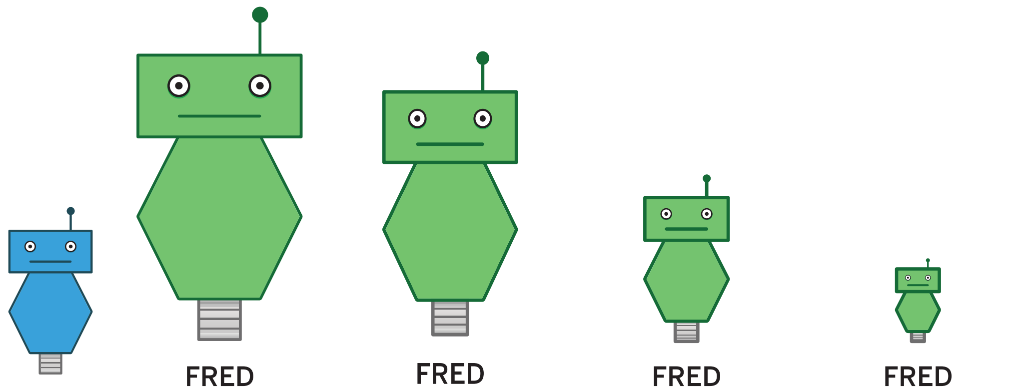
A: $x = 10$, B: $x = 1$, C: $x = \frac{1}{8}$, D: $x = 8$

Activity 1

Name: ______ Date: ______ Period: ______

Exploring Scale Factors

2 Here is a robot: TED. The other robots are scaled copies of TED with different scale factors.



Discuss: What do you notice?

Responses vary.
- I notice that scale factors larger than 1 make a scaled copy that's larger than TED.
- I notice that scale factors less than 1 make a scaled copy that's smaller than TED.

3 Here are two new robots: ED and NED. In this lesson, all measurements are in grid units.

What scale factor will make ED match NED?

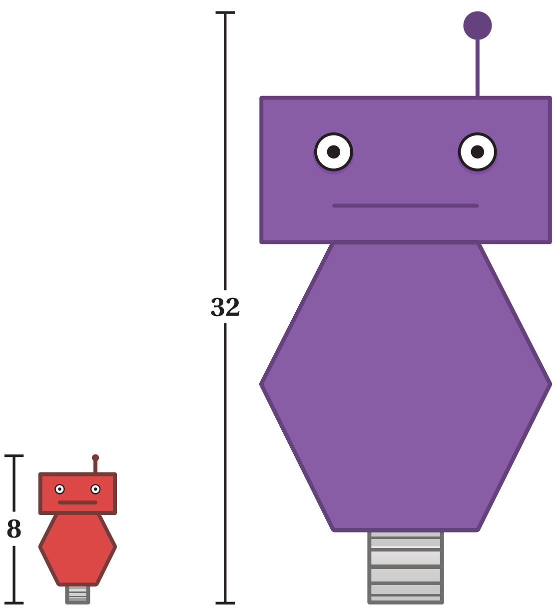
4 (or equivalent, such as $\frac{32}{8}$)

4 What scale factor will make NED match ED?

$\frac{1}{4}$ (or equivalent, such as 0.25 or $\frac{8}{32}$)

Explain your thinking.

Explanations vary. To make ED match NED, we had to multiply by 4. To go from NED back to ED, we have to go in reverse. So instead of multiplying by 4, we divide by 4, which is the same as multiplying by $\frac{1}{4}$.



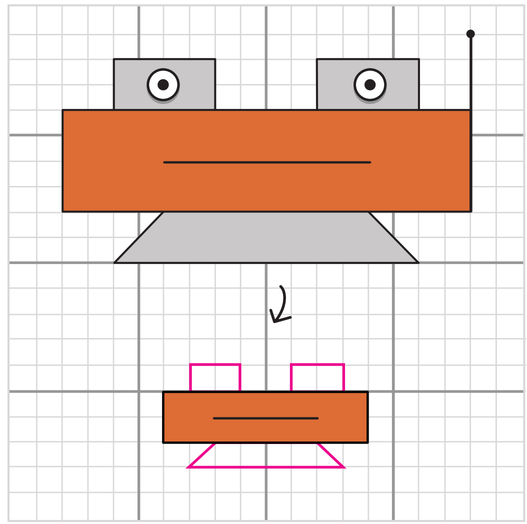
Activity 2

Name: .... Date: .... Period: ....

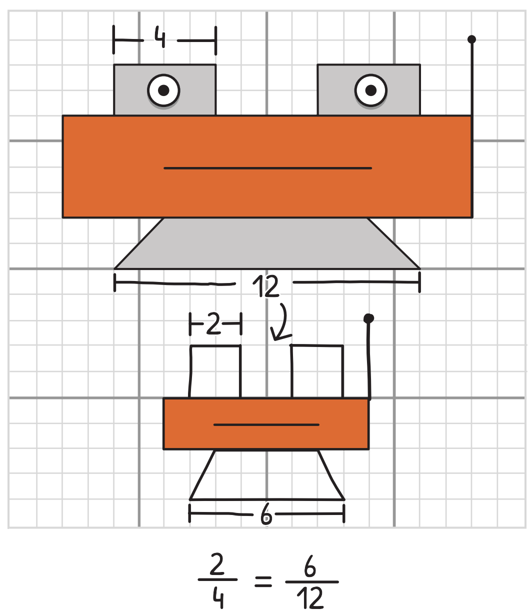
Scaled Down and Back Again

5 Here is a robot called ROVER.

Complete the scale drawing of ROVER using a scale factor of $\frac{1}{2}$.



6 Here is Adhira's drawing from the previous problem.



a What is something Adhira did well?

***Responses vary.* Adhira used equivalent ratios $\left(\frac{2}{4} = \frac{6}{12}\right)$ to figure out how wide to make everything in the new drawing.**

b What is something Adhira can improve?

***Responses vary.* Adhira forgot to apply the scale factor to how tall things are. The eyes and trapezoid body should each be 1 unit tall (instead of 2), and the antenna should be $3 \cdot \frac{1}{2} = 1.5$ units tall (instead of 3).**

7 The scale factor from the original to the copy is $\frac{1}{2}$.

a What scale factor could you use to scale the copy back to the original?

2

b How are these two scale factors related?

Responses vary.
- **The scale factors 2 and $\frac{1}{2}$ balance each other out. One scales up and the other scales down by the same amount.**
- **The scale factors multiply to make 1.**
- **You can think of these two scale factors as "multiply by 2" and "divide by 2," and dividing by 2 is the same as multiplying by $\frac{1}{2}$.**

Activity 3

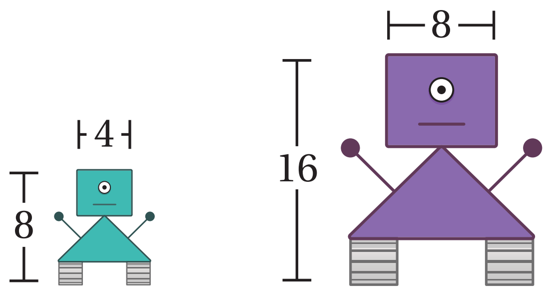
Name: Date: Period:

Practicing With Scale Factors

8 In this activity, all pairs of bots have corresponding measurements that are proportional.

Determine a scale factor to make the bot on the left match the bot on the right.

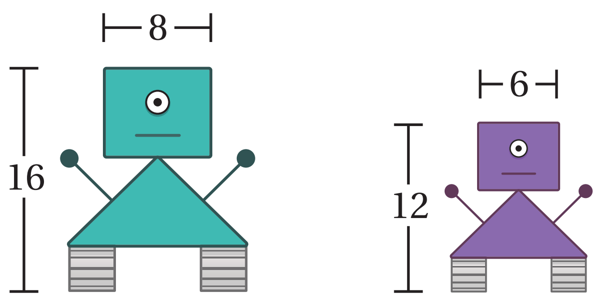
a



Scale factor: $\frac{16}{8}$

(or equivalent)

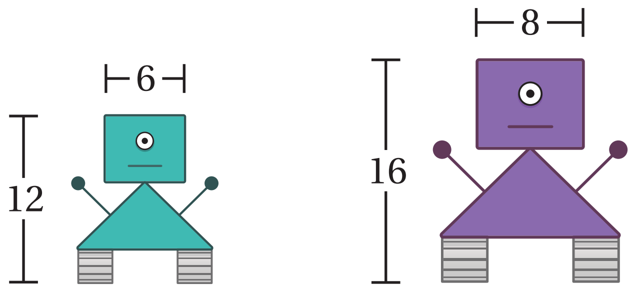
b



Scale factor: $\frac{3}{4}$

(or equivalent)

c



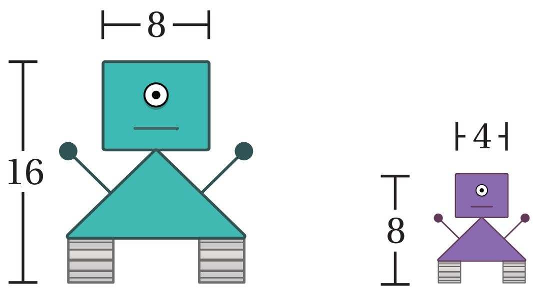
Scale factor: $\frac{4}{3}$

(or equivalent)

Name: Date: Period:

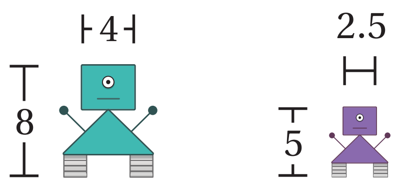
Practicing With Scale Factors (continued)

d



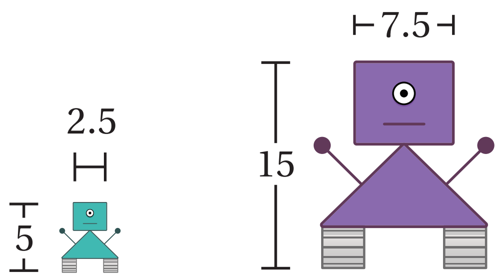
Scale factor: $\frac{1}{2}$
(or equivalent)

e



Scale factor: $\frac{5}{8}$
(or equivalent)

f



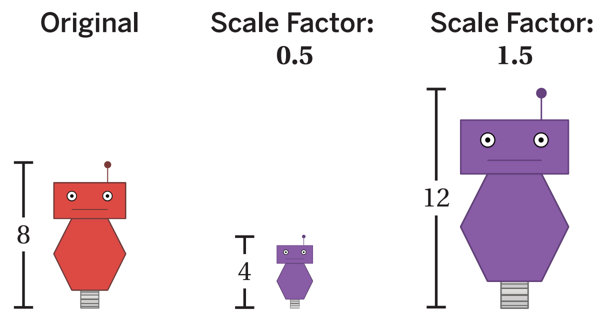
Scale factor: $\frac{15}{5}$
(or equivalent)

Explore More

9 Use the Explore More Sheet to design your own robot. Then complete its scaled copy.
Responses vary.

10 Synthesis

Describe how you can tell from the scale factor whether a scaled copy will be larger than, smaller than, or the same size as the original.



Responses vary.

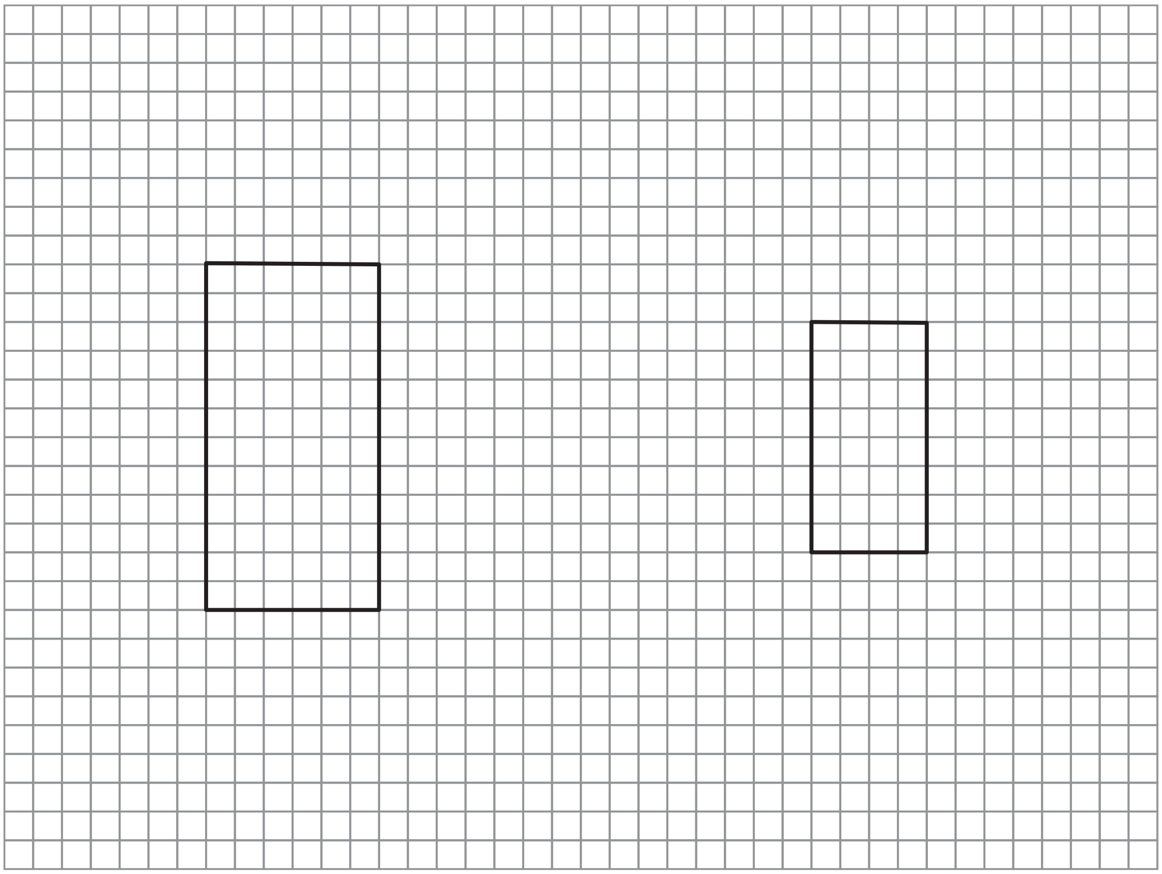
- **If the scale factor is greater than 1, the scaled copy will be larger than the original figure.**
- **If the scale factor is between 0 and 1, the scaled copy will be smaller than the original figure.**
- **If the scale factor is equal to 1, the scaled copy will be the same size as the original figure.**

Things to Remember:

Name: .... Date: .... Period: ....

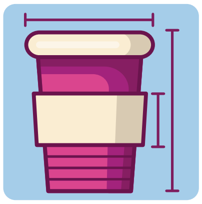
Explore More

Design your own robot on the left. Then complete its scaled copy on the right.



Name: ______ Date: ______ Period: ______

Will It Fit?

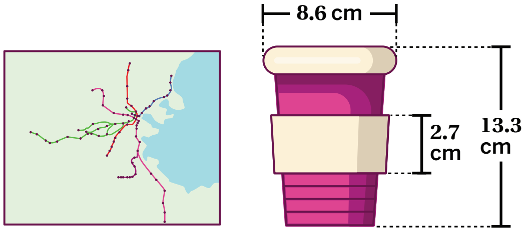


Let's analyze scale drawings.

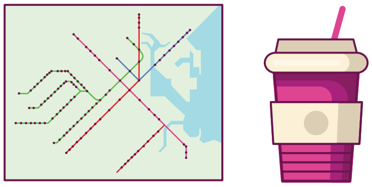
Warm-Up

Here are some drawings of the Boston subway system and a coffee cup.

Scale Drawings



Might Not Be Scale Drawings



1. **Discuss:** What makes a drawing a **scale drawing**?

Responses vary.

- **A scale drawing is a drawing that shows the object accurately, and all of the parts in the drawing correspond to the parts in the actual object.**
- **A scale drawing is like a scaled copy of a real object, but only shows one flat surface of the actual object.**

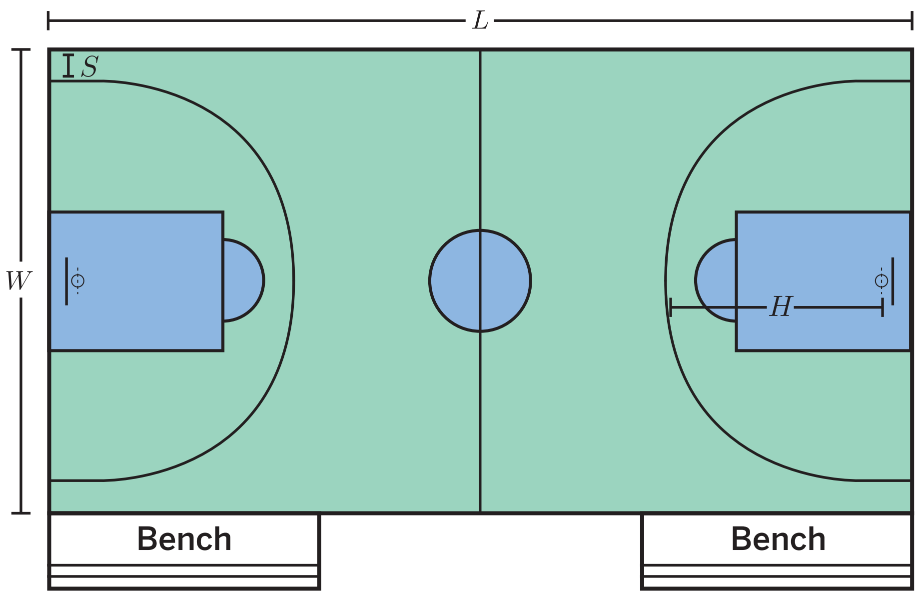
Activity 1

Name: .... Date: .... Period: ....

Will the Court Fit?

Karima heard from students that they would like a basketball court in their community park. When Karima presented the idea to the park's board of directors, they approved building the court in a 20-by-20-meter area of the park.

Here is the scale drawing that Karima presented.



2. The scale for Karima's drawing is 2 centimeters to 5 meters. Explain what this means in your own words.

***Responses vary.* It means that 2 centimeters on the scale drawing represents 5 meters on the actual court.**

Name: Date: Period:

Will the Court Fit? (continued)

Will Karima's court fit in the 20-by-20-meter square area the park directors designated for the court?

3. Use your ruler to measure the scale drawing. Record the measurements in the table. Then determine the dimensions of the actual court.

	Length of Court, L	Width of Court, W	Hoop to 3-Point Line, H	3-Point Line to Side Line, S
Scale Drawing	**11.2 cm**	**6 cm**	**2.8 cm**	**0.4 cm**
Actual Court	**28 m**	**15 m**	**7 m**	**1 m**

4. Explain how you know whether the court will fit.

***Responses vary*. The actual dimensions of Karima's court are 28 meters by 15 meters and will not fit in the designated 20-by-20-meter park area because 28 meters is longer than 20 meters.**

Explore More

5. On an actual basketball court, the bench area is typically 9 meters long. Determine how long the bench area should be on the scale drawing.

Does your answer match Karima's drawing?

The bench should be about 3.6 centimeters long, which matches the length of the bench area in Karima's drawing.

Name: .... Date: .... Period: ....

Superior Court

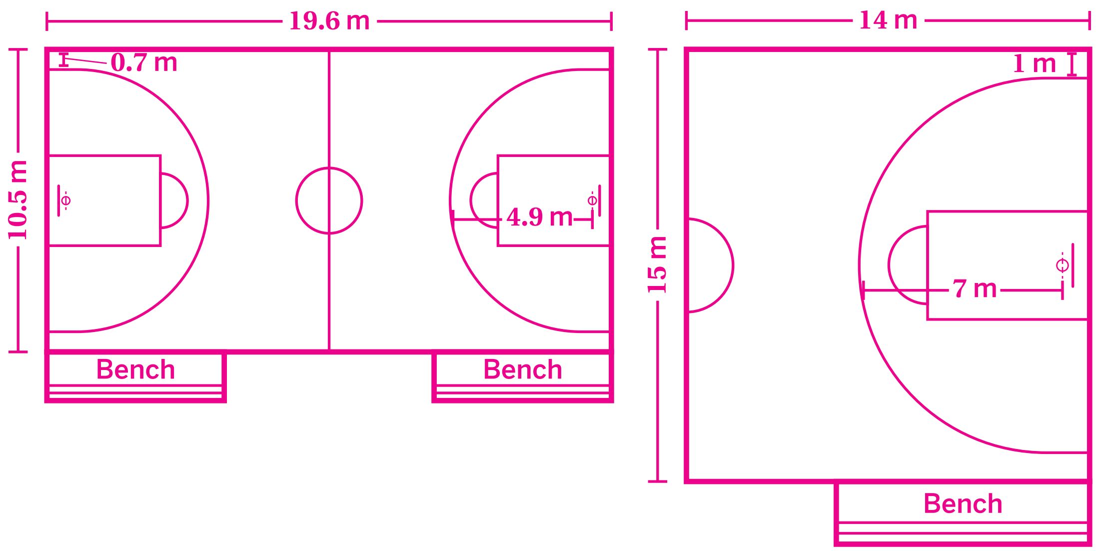
6. How would you recommend Karima adjust her proposal so that it fits in the 20-by-20-meter park area? Explain your thinking.

Responses vary.

- **Karima could scale down all of the lengths in her court by the same scale factor so that the length of the court fits in the designated area.**
- **I recommend that Karima propose building a half court since the dimensions of a half court would fit in the designated area.**

7. Sketch your proposed court that would fit in the 20-by-20-meter park area. Label your sketch with all necessary distances.

Responses vary.



8. The basketball court will share the 20-by-20-meter park area with an outdoor seating area. After your court is built, how much area will remain for outdoor seating? Explain your thinking.

Responses vary.

- **Scaled full court: The scaled full court will have an area of 205.8 square meters because $19.6 \cdot 10.5 = 205.8$. Since the original full area is 400 square meters, there will be 194.2 square meters remaining for both the outdoor seating and the players' benches.**
- **Half court: The half court will have an area of 210 square meters because $14 \cdot 15 = 210$. Since the original full area is 400 square meters, there will be 190 square meters remaining for both the outdoor seating and the players' bench.**

Synthesis

9. Explain how you could use Karima's scale drawing to calculate the actual distance across the center court circle. Remember that the scale for Karima's drawing was 2 centimeters to 5 meters.

	Distance Across the Center Court Circle
Scale Drawing	1.8 cm
Actual Court	**4.5 m**

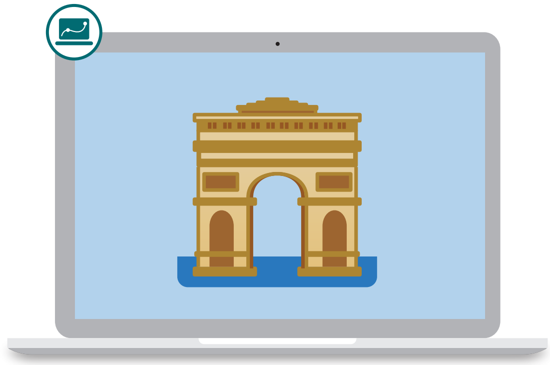
Responses vary.

- **Each centimeter represents 2.5 meters, and $2.5 \cdot 1.8 = 4.5$ meters.**
- **The scale is 2 centimeters to 5 meters. $2 \cdot 0.9 = 1.8$ centimeters, so the court is actually $5 \cdot 0.9 = 4.5$ meters.**

Things to Remember:

Unit 1
Lesson
10

Name: .... Date: .... Period: ....



Scaling Buildings

Let's see how different scales can describe the same thing.

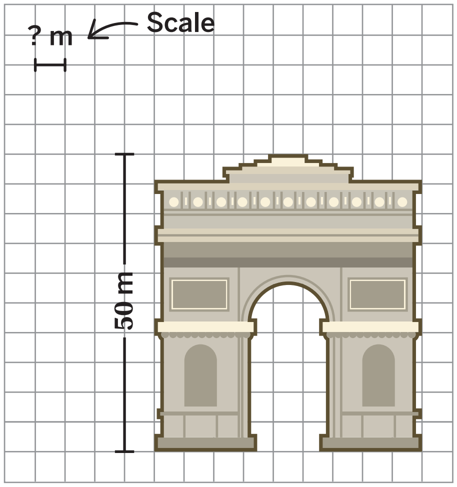
Warm-up

1 Here is a scale drawing of the Arc de Triomphe in Paris, France.

The Arc de Triomphe is 50 meters tall.

What is the unknown value in the scale?

5 meters



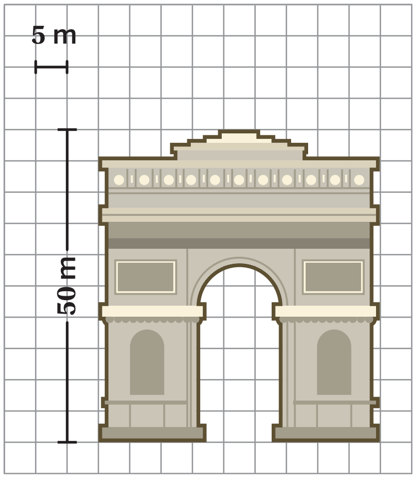
Arc de Triomphe, France (1836)

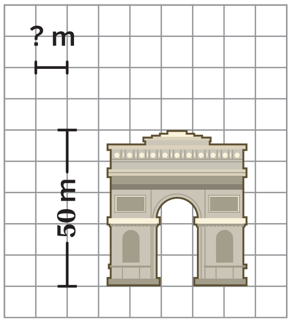
Activity 1

Name: .... Date: .... Period: ....

Same Object, Different Scale

Here are two scale drawings of the Arc de Triomphe: one from the Warm-Up and a new scale drawing.





2 On the new scale drawing, the unknown segment represents:

A. More than 5 meters.

B. Less than 5 meters.

C. Exactly 5 meters.

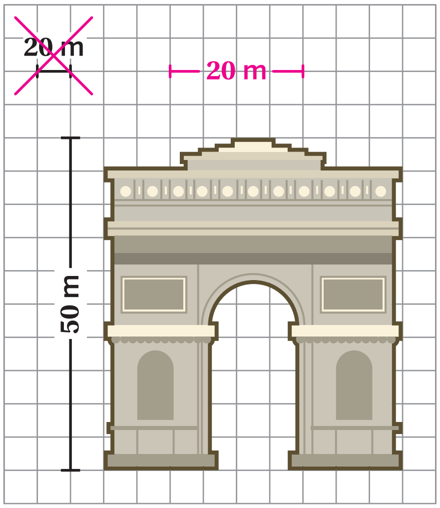
Explain your thinking.

***Explanations vary.* The new drawing is smaller, so each segment needs to represent a greater length to represent the same total height.**

3 What is the unknown value in the scale?

10 meters

4 The scale in this scale drawing is the wrong size. Update the scale so that it shows the correct number of grid units for 20 meters.



Explain your thinking.

Explanations vary.

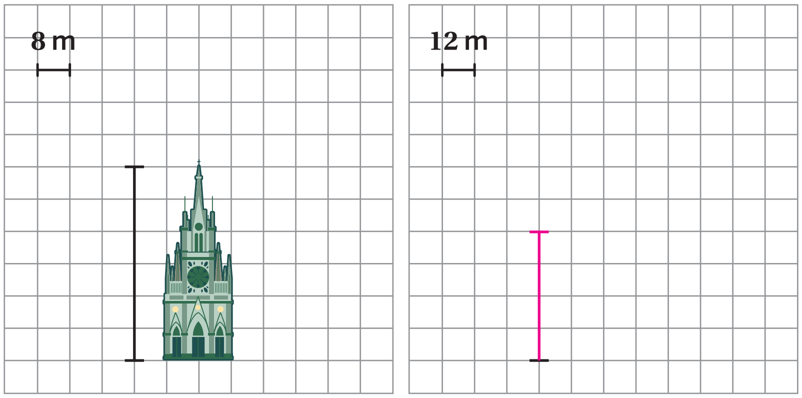
- **The building is 50 meters tall. On the grid, this is represented by 10 units, so each unit must represent 5 meters. That means 20 meters must represent 4 units because $5 \cdot 4 = 20$.**
- **$20 \cdot 2.5 = 50$ and $4 \cdot 2.5 = 10$, so the 20-meter scale must be 4 units wide.**

Name: ________ Date: ________ Period: ________

Same Object, Different Scale (continued)

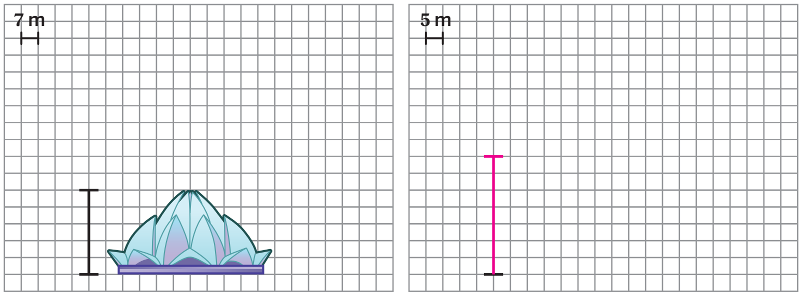
For each scale drawing, look at the building's height. Then show what the height of each building would be with the new scale.

5



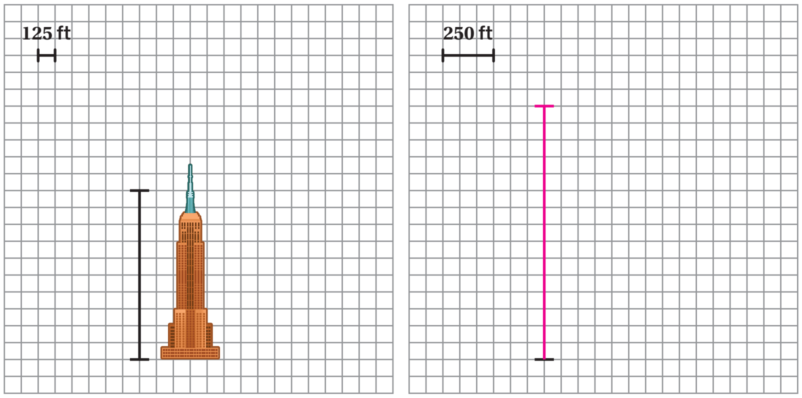
Las Lajas Sanctuary, Colombia (1949)

6



Lotus Temple, India (1986)

7



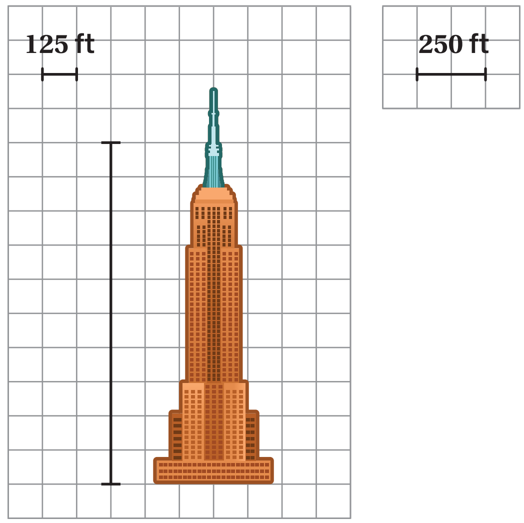
Empire State Building, U.S.A. (1931)

Activity 2

Name: .... Date: .... Period: ....

When are Scales Equivalent?

8 Here is a scale drawing of the Empire State Building, with a scale of 1 unit to 125 feet. Imagine a new scale drawing with a scale of 2 units to 250 feet.



Will the new drawing be smaller, larger, or the same size as this drawing?

A. Smaller

B. Larger

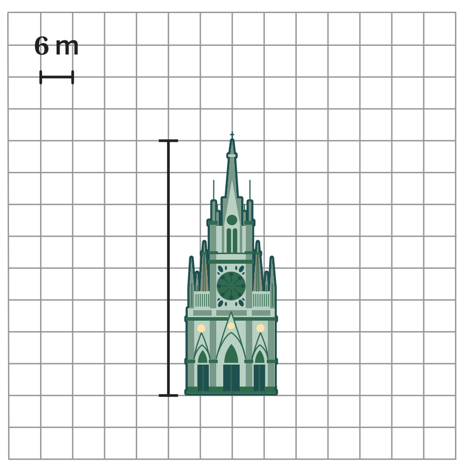
C. The same size

Explain your thinking.

Explanations vary.

- **The building ends up at the same height using either scale.**
- **If 1 unit represents 125 feet, then $1 \cdot 2 = 2$ units represent $125 \cdot 2 = 250$ feet. That means the scales are the same.**

9 Here is a scale drawing of Las Lajas Sanctuary with a scale of 1 unit to 6 meters.



Select *all* the scales that will make a scale drawing of the same size.

- [ ] A. 2 units to 3 meters
- [ ] B. 2 units to 6 meters
- [x] C. 2 units to 12 meters
- [ ] D. 3 units to 9 meters
- [x] E. 3 units to 18 meters

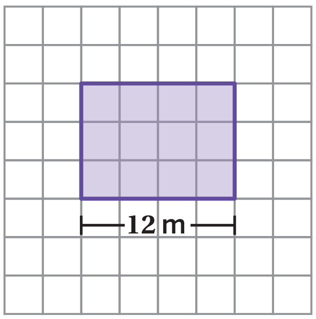
Name: .... Date: .... Period: ....

When are Scales Equivalent? (continued)

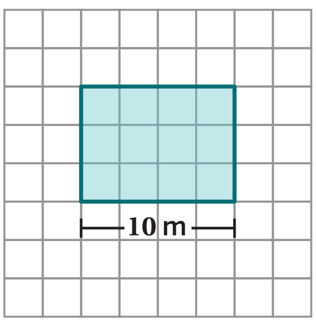
10 Match each scale drawing with one or more possible ways of describing its scale. One scale description will have no match.

Scale Drawing

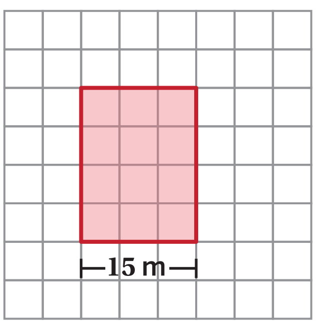
a.



b.



c.



Scale Description

c 2 units to 10 meters

a 2 units to 6 meters

a 3 units to 9 meters

b 1 unit to 2.5 meters

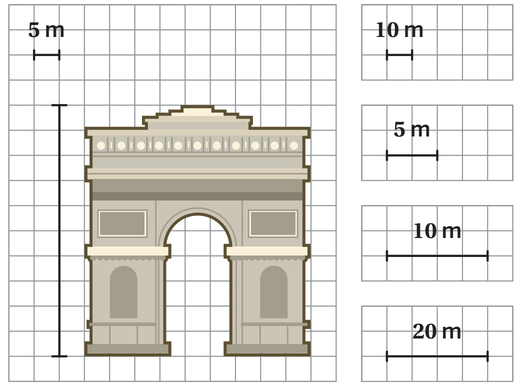
...... 3 units to 5 meters

b 2 units to 5 meters

11 Synthesis

How can you tell whether a new scale drawing will be smaller than, larger than, or the same size as the original just by looking at the scales?

Use these examples if they help with your thinking.



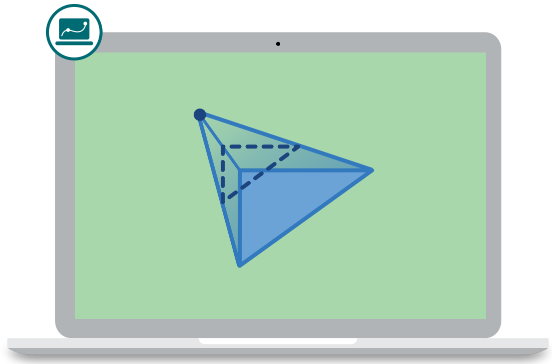
Responses vary.

- **If two segments are the same length but one represents a smaller distance, that scale drawing will be larger. For example, 1 unit to 5 meters will produce a larger scale drawing than 1 unit to 10 meters.**
- **If two segments represent the same distance, but one of the segments is shorter, that scale drawing will be smaller. For example, 1 unit to 10 meters will produce a smaller scale drawing than 4 units to 10 meters.**

Things to Remember:

Unit 2
Lesson
1

Name: .... Date: .... Period: ....

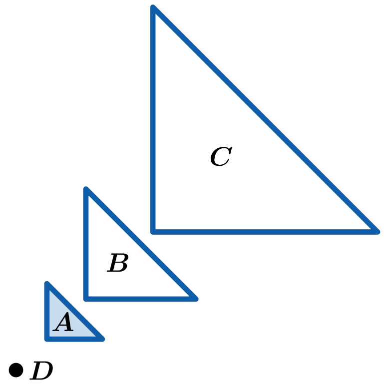


Sketchy Dilations

Let's explore scaled copies and dilations.

Warm-Up

1 Triangles B and C are images of triangle A.



What do you notice? What do you wonder?

Responses vary.

I notice:

- **I notice the triangles appear to be scaled copies.**
- **I notice there is no rotation, reflection, or translation that moves triangle A onto triangle B or triangle C.**
- **I notice triangle B looks about twice as large as triangle A and about twice as far from point D.**

I wonder:

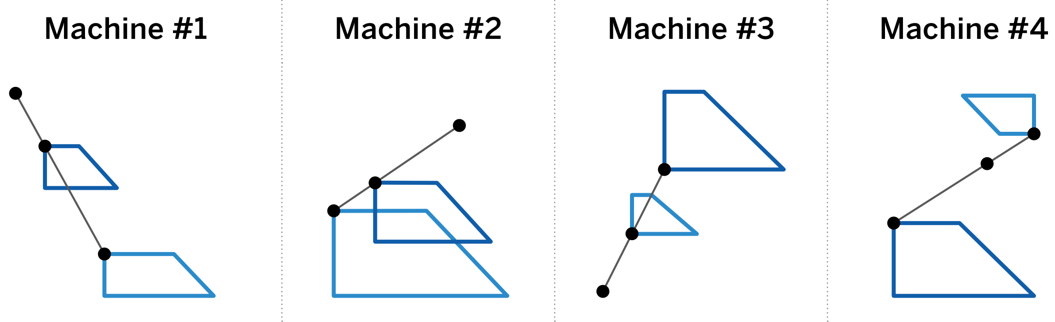
- **I wonder whether the triangles are scaled copies of each other.**
- **I wonder why point D is included.**
- **I wonder if there is a transformation that changes the size of a figure.**

Activity 1

Name: .... Date: .... Period: ....

Dilations and Scaled Copies

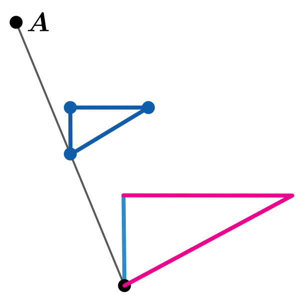
2 **a** Take a look at these sketches, each created by a different stretching machine.



b **Discuss:** How does each stretching machine work? ***Responses vary.***

- **Machine #1: It creates a copy of the original about twice as large and twice as far.**
- **Machine #2: It creates a copy that is about 1.5 times as large and 1.5 times as far away from the closed point.**
- **Machine #3: It creates a copy about half as large and half as far away from the point.**
- **Machine #4: It creates a copy about half the size of the pre-image but rotated $180°$ around the point.**

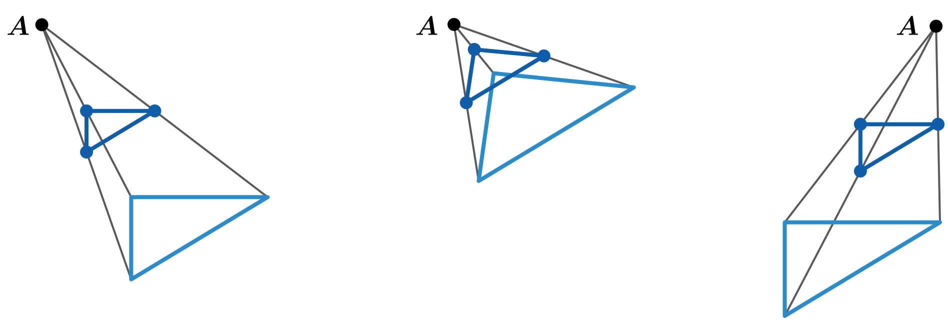
3 Here is a *pre-image* and part of its *image* in a stretching machine.



a Sketch the rest of its image. ***Image shown.***

b How would the image be different if point A were closer to the triangle? ***Responses vary.* If point A were closer to the triangle, the image would also be closer to the triangle, but it would still be about twice as large as the outlined figure.**

4 Stretching machines create **dilations**. A dilation is a new type of transformation that creates a scaled copy from a given point.



Does moving point A change the size of the image, its location, or both? Circle one.

Size Location Both

Explain your thinking. ***Explanations vary*. Moving point A changes the location of the dilation but not the size of the dilation. Each of the points on the dilation are still about twice as far away from point A as the corresponding points on my sketch, but they are in different locations.**

Activity
2

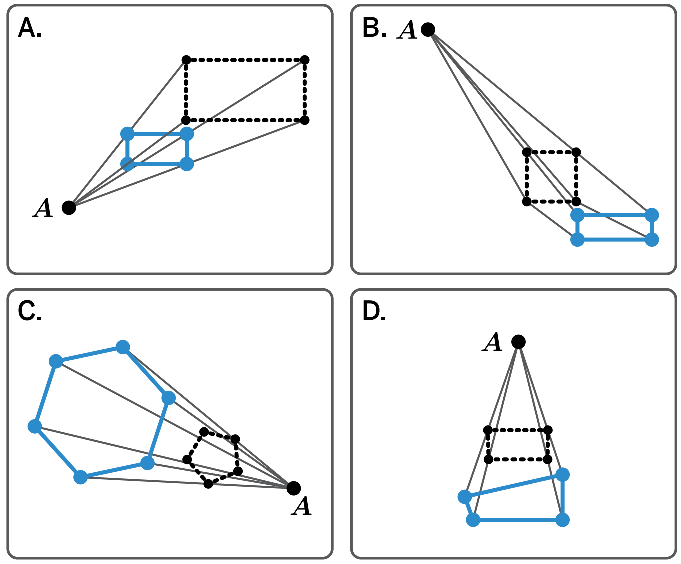
Name: .... Date: .... Period: ....

Dilation Play

5 Circle one image that is *not* a dilation (in other words, it could not have been created using a stretching machine). **B, C, or D**

Explain your thinking.

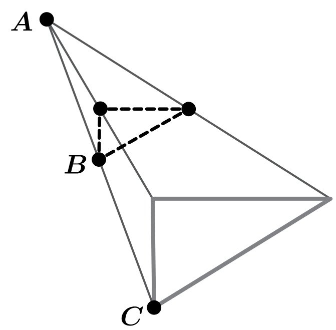
Explanations vary.
- **B because the lines are bent.**
- **C because one figure has five sides and the other has six sides.**
- **D because one figure is a rectangle and the other is not.**



6 Let's watch what happens when points A and C move.

Here are several dilation challenges. Select *all* the ones you think are possible.

- [x] **A.** Make the image smaller than the pre-image.
- [x] **B.** Make the image the same size as the pre-image, but in a different location.
- [x] **C.** Make the distance between A and B twice the distance between B and C.
- [ ] **D.** Make points A, B, and C form a triangle.

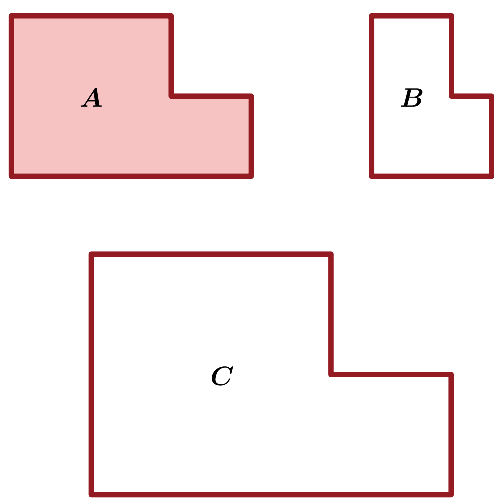


7 Which figure could be a *dilation* of figure A? Circle one.

Figure B Figure C Both   Neither

Explain your thinking.

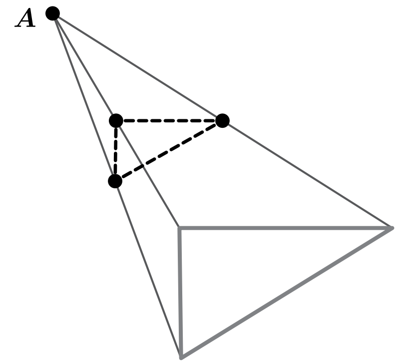
***Explanations vary.* Figures A and C appear to be scaled copies of one another, which means there's probably a point off-screen that figure A could be dilated from to get figure C.**



8 Synthesis

Describe how dilations work.

Use this example if it helps with your thinking.

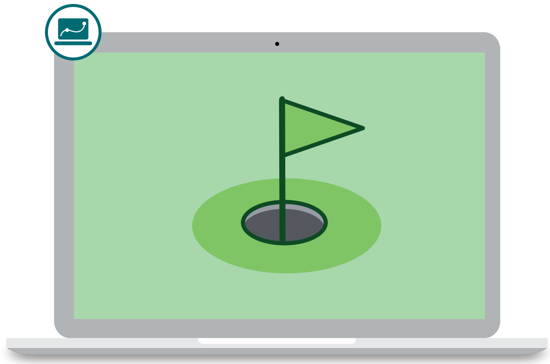


***Responses vary*. Dilations create scaled copies. They require a point, and then lines extend from that point through the vertices of the figure being dilated. The dilated figure could be anywhere along those lines.**

Things to Remember:

Unit 2
Lesson
2

Name: ________ Date: ________ Period: ________



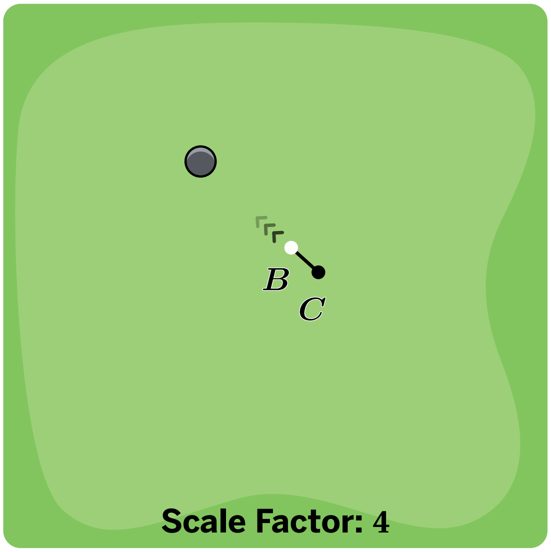
Dilation Mini Golf

Let's dilate points using measurement tools.

Warm-Up

1 Welcome to Dilation Mini Golf.

Your goal is to get the ball into the hole by dilating point B.



a Let's watch an animation to see what we mean.

b **Discuss:**

- In this situation, what is the pre-image? What is the image?
- What do you think *scale factor:* 4 means?

Responses vary.

- **Point B is the pre-image and the ball in the hole is the image.**
- ***Scale factor:* 4 represents the number of segments between the center and the hole.**
- ***Scale factor:* 4 is the number of times the distance between point B and point C is multiplied.**

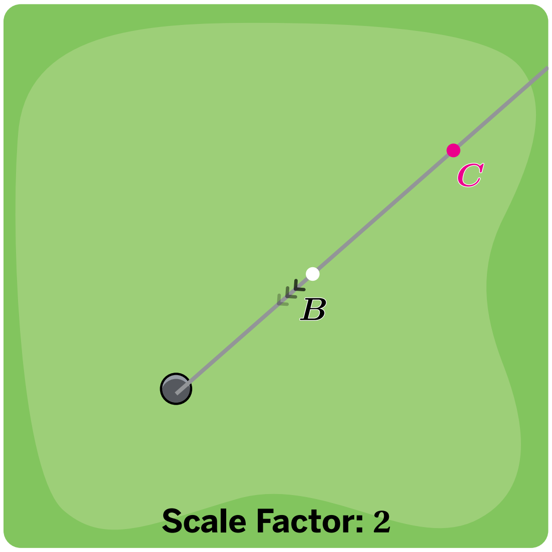
Activity 1

Name: ______ Date: ______ Period: ______

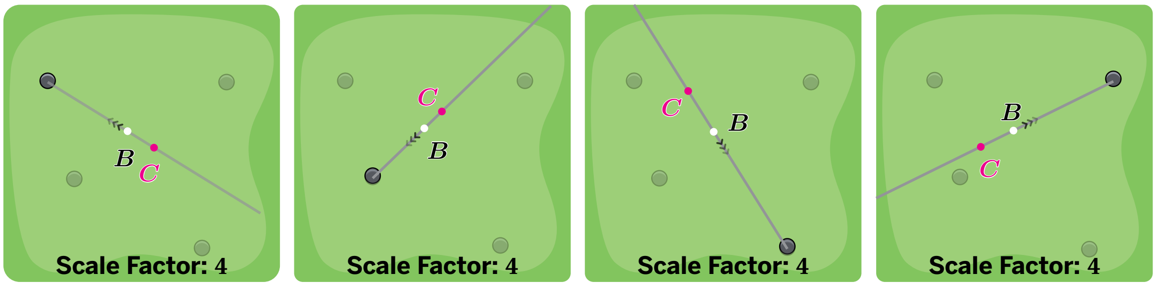
Rounds of Mini Golf

2 Point B will be dilated using point C as the **center of dilation** and a *scale factor* of 2.

Mark where point C should be so that the image of point B lands in the hole.



3 Point B will be dilated using point C as the center of dilation and a scale factor of 4. For each hole, mark where point C should be so that the image of point B lands in the hole.

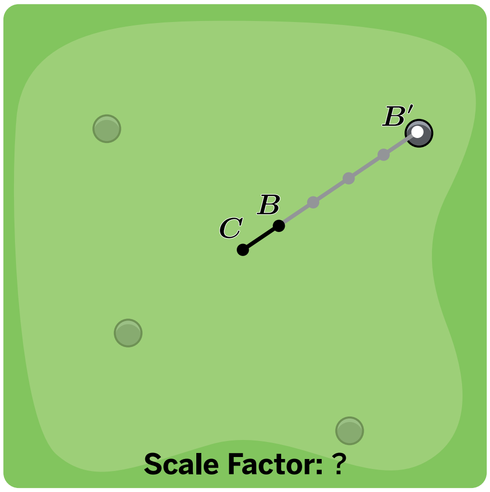


4 Point B was dilated using point C as the center of dilation.

What is the scale factor? 5

Explain your thinking.

***Explanations vary.* The scale factor is 5 because the distance between the center and the point is 5 times the original distance.**



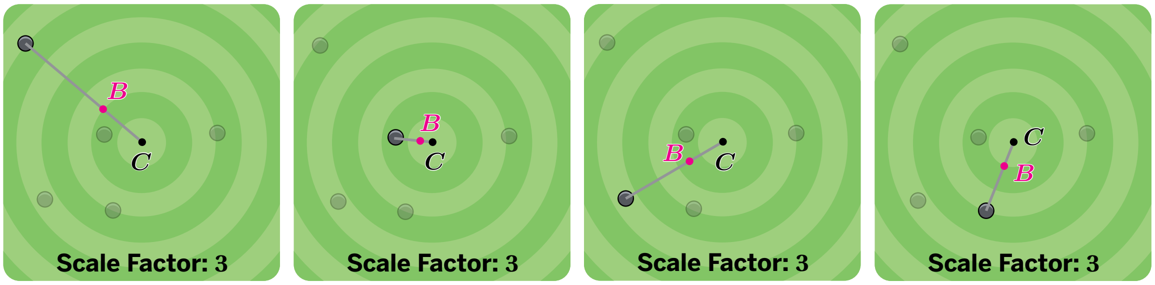
Name: .... Date: .... Period: ....

Dilation Distances

5 Now let’s place the ball (point B). This will be your pre-image.

Point B will be dilated using point C as the center of dilation and a scale factor of 3.

For each hole, mark point B so that its image lands in the hole.



6 Point B will be dilated using point C as the center of dilation and a scale factor of 4.

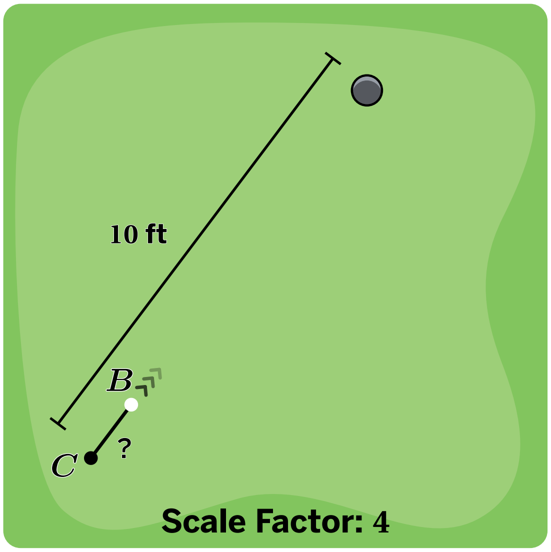
What should the distance between point C and point B be so that the image is in the hole?

2.5 feet

Explain your thinking.

Explanations vary.

- **The correct distance is 2.5 feet because 2.5 multiplied by 4 is 10 feet.**
- **Point B should be 2.5 feet from the center because the scale factor is 4, and $2.5 \cdot 4 = 10$.**



7 We can dilate more than one point at a time!

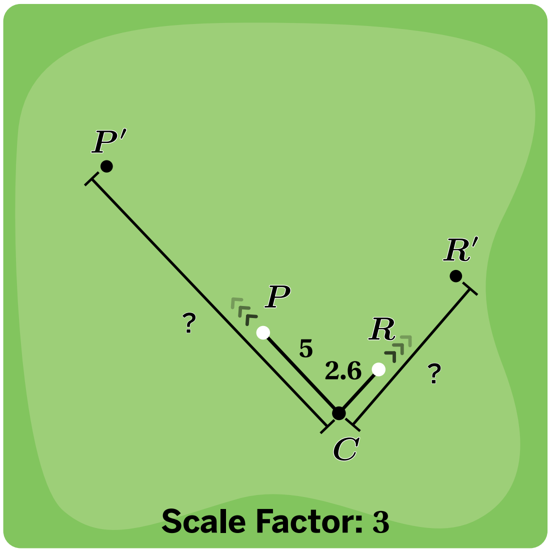
Points P and R will be dilated using point C as the center of dilation and a scale factor of 3.

What is the distance between point C and the image of point P?

15 feet

What is the distance between point C and the image of point R?

7.8 feet

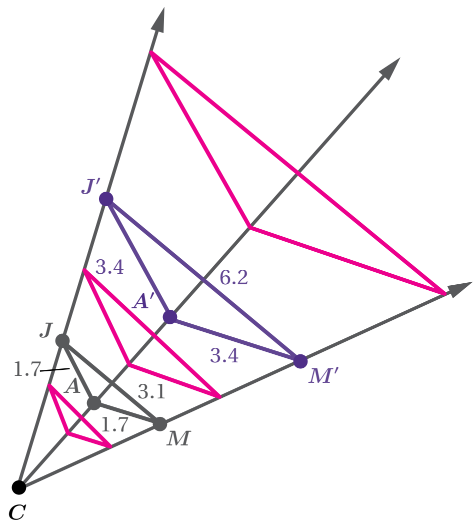


Activity 3

Name: ______ Date: ______ Period: ______

Dilating a Triangle

8 Let's watch an animation to see how triangle JAM is dilated using point C as the center of dilation and a scale factor of 2.



a Describe all of the ways you see a scale factor of 2 represented in this diagram.

***Responses vary.* I see a scale factor of 2 with the distance from point C to point M' being double the distance from point C to point M. The same is true for the distance from point C to point A' and point C to point A, and point C to point J' and point C to point J.**

b Select at least one more scale factor. ***Responses vary.***

3 1.5 $\frac{3}{4}$ Other: ______

c On the same diagram, dilate triangle JAM using center C and each scale factor you chose. **Sample shown on diagram.**

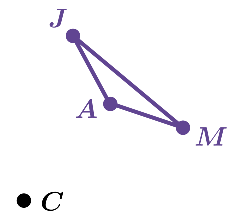
d List everything that's alike about triangle JAM and its dilations.

Responses vary.
- **The line segments in triangle JAM look parallel to the line segments in triangle $J'A'M'$.**
- **The angles in triangle JAM look congruent to the angles in triangle $J'A'M'$.**

9 Synthesis

Describe how to dilate a point or a figure given a center of dilation and a scale factor.

Use the example if it helps you with your thinking.

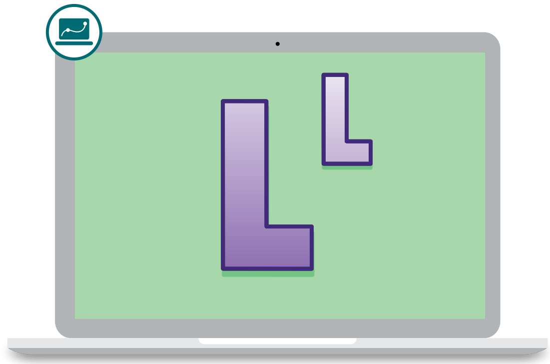


Scale Factor: 2

Responses vary. **To dilate the figure in the example, you can: First, measure the distance between point A and the center of dilation, C, then multiply that distance by the scale factor. Next, measure your new distance from point C along the line between points A and C. Mark the new point A'. You can repeat with all of the points in the figure to get the image.**

Things to Remember:

Name: .... Date: .... Period: ....



Transformation Targets With Dilations

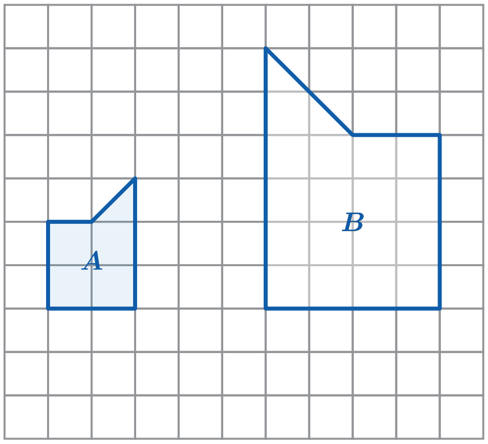
Let's play Transformation Targets with dilations.

Warm-Up

1 Describe a *sequence of transformations* that could move polygon A onto polygon B.

Responses vary.

- Dilate polygon A by a scale factor of 2, do a horizontal reflection, then translate to the right so that the figures align.
- Reflect polygon A over the longest side of the polygon. Then dilate the figure by a scale factor of 2, using a point to the left of the figure as the center of dilation.



Activity 1

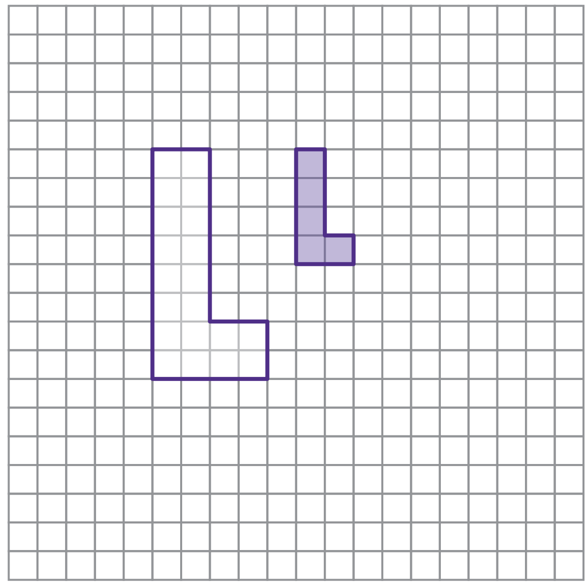
Name: ____________ Date: ____________ Period: ____________

Transformation Targets

2 **Challenge #1:** Show or describe a sequence of transformations to move the pre-image (shaded) onto the image.

Draw any points or lines that you use in your sequence of transformations.

Responses vary.
- **One solution for this challenge involves a single dilation by a scale factor of 2.**
- **Another solution involves a translation (down and to the left), followed by a dilation by a scale factor of 2.**



3 Aditi says you can complete the previous challenge with a dilation and a translation. Emiliano says you can complete this challenge with only a dilation. Whose claim is correct?

Aditi's Emiliano's Both Neither

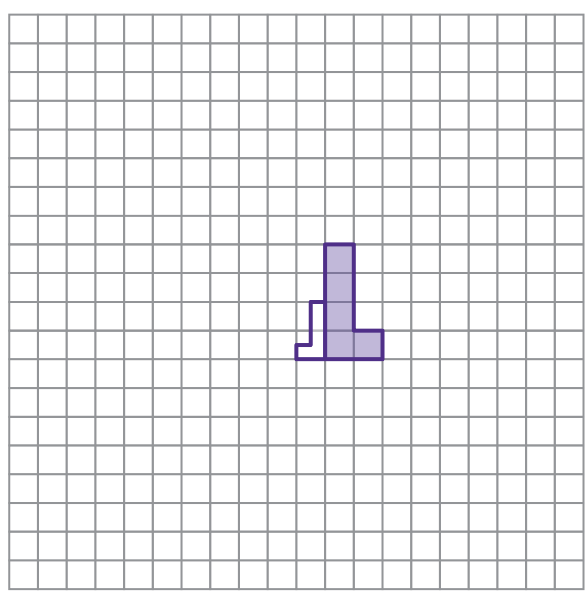
Explain your thinking.

Explanations vary. **You can complete the challenge by dilating using a scale factor of 2, using the top-right corner of the pre-image as the center of dilation, then translating left 4 units. You can also complete the challenge with only a dilation using a scale factor of 2 and a center of dilation that is 4 units to the right of the top-right corner of the pre-image.**

4 **Challenge #2:** Show or describe a sequence of transformations to move the pre-image (shaded) onto the image.

Draw any points or lines that you use in your sequence of transformations.

Responses vary. **One solution for this challenge involves a single dilation by a scale factor of $\frac{1}{2}$, followed by a reflection.**

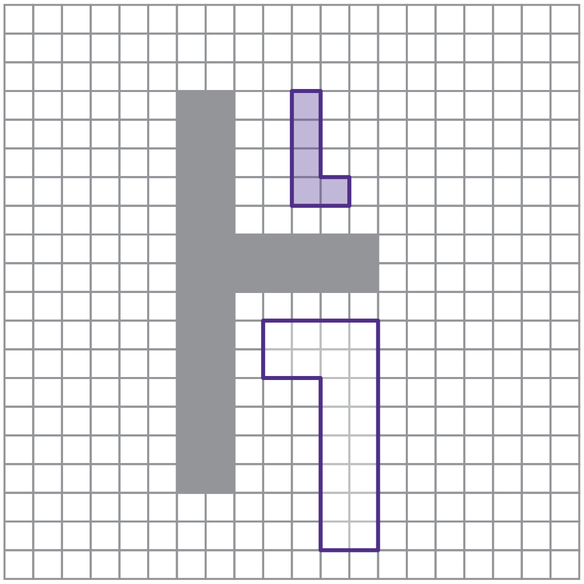


Activity 2

Name: .... Date: .... Period: ....

Now With Obstacles!

5 **Challenge #3:** Show or describe a sequence of transformations to move the pre-image (shaded) onto the image.

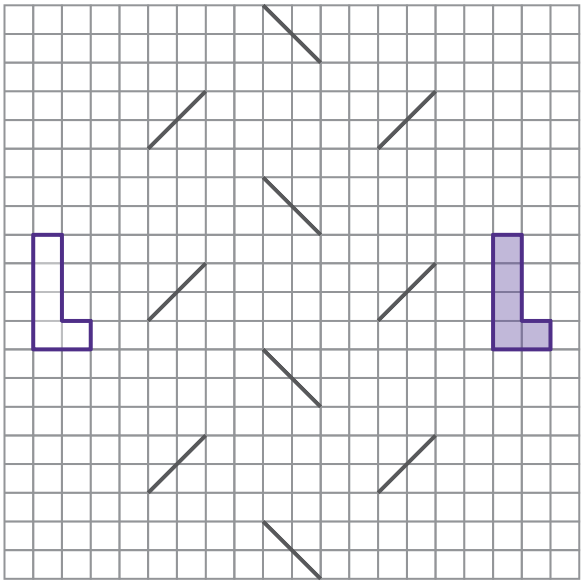


Avoid the obstacles!

Draw any points or lines that you use in your sequence of transformations.

Responses vary. **One solution for this challenge involves a rotation 180° clockwise, followed by a dilation using a scale factor of 2.**

6 Describe a sequence of transformations that you think will move the pre-image (shaded) onto the image.



Avoid the obstacles!

Responses vary. **First, I'll dilate the figure by $\frac{1}{4}$ so that I can translate from right to left without hitting the obstacles. After translating, I'll dilate the figure by 4 so that it can go back to its original size.**

7 Sketch the result of each transformation you described in the previous problem on the grid.

Draw any points or lines that you use in your sequence of transformations.

Responses vary.

Name: .... Date: .... Period: ....

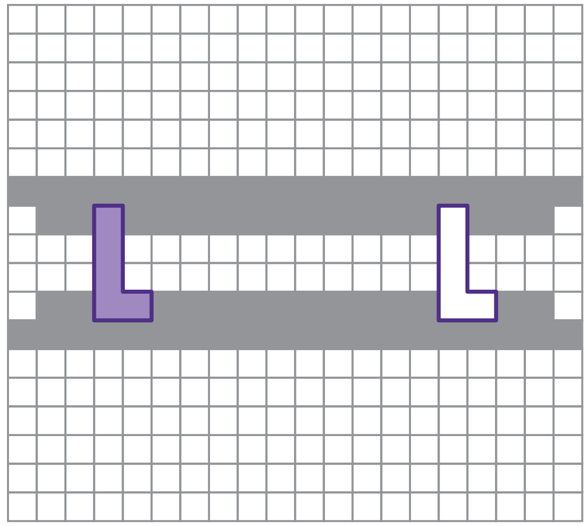
Now With Obstacles! (continued)

8 **Challenge #4:** Describe a sequence of transformations to transform the pre-image (shaded) onto the image.

Avoid the obstacles!

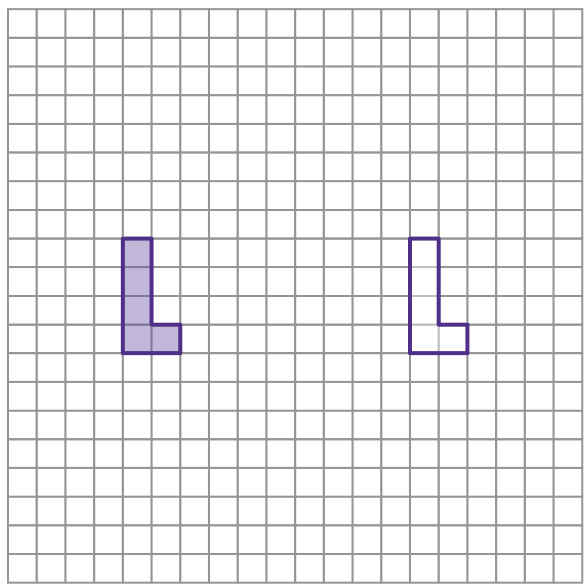
Draw any points or lines that you use in your sequence of transformations.

***Responses vary.* One solution for this challenge involves a dilation by a scale factor of $\frac{1}{2}$, followed by a translation to the right, and then a dilation by a scale factor of 2.**



9 **Challenge #5:** Use only dilations to transform the pre-image (shaded) onto the image!

***Responses vary.* One solution for this challenge involves a dilation of scale factor $\frac{1}{2}$ with the center of dilation to the right of the shaded L-shape, followed by a dilation of scale factor 2 with a center of dilation to the left of the shaded L-shape.**

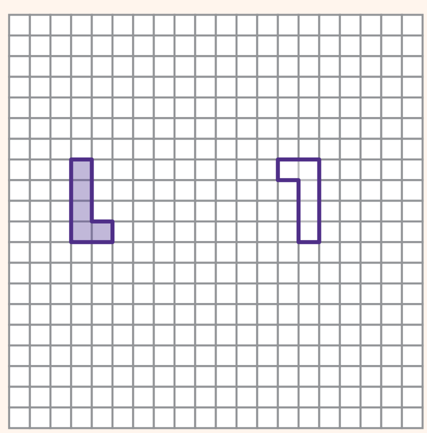


Explore More

10 Describe a sequence of transformations that uses *only* dilations to move the pre-image (shaded) onto the image!

Draw the center of dilation and the result of each dilation.

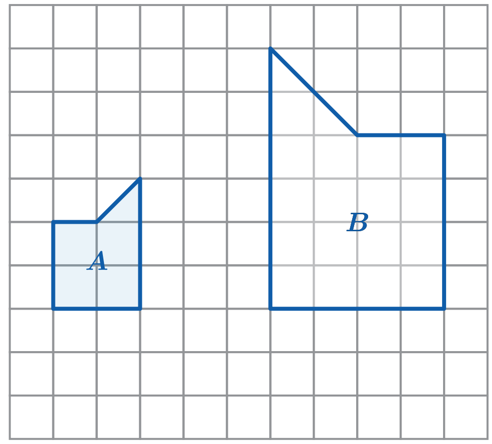
***Responses vary.* One solution for this challenge involves a dilation with a scale factor of $\frac{1}{2}$ and a center of dilation to the right of the shaded L-shape, followed by a dilation with a scale factor of -2 and a center of dilation to the right of the shaded L-shape.**



11 Synthesis

Describe some strategies for determining a sequence of transformations that moves a pre-image onto an image.

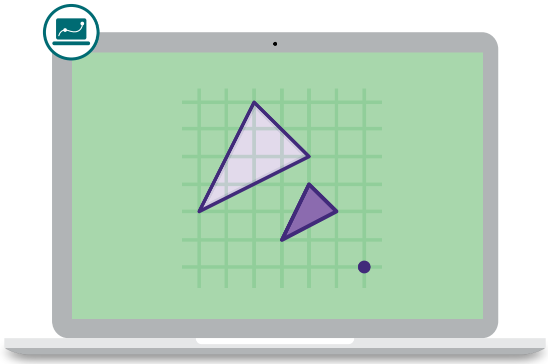
Use the example if it helps you with your thinking.



Responses vary. **There are lots of different ways to move a pre-image onto an image. One strategy is to use translations to line up corresponding corners, then use dilations or reflections. Sometimes you can just use dilations to move the pre-image onto an image. With dilations you can change the size and location of an image by putting the center of dilation in different places.**

Things to Remember:

Name: ________ Date: ________ Period: ________



Match My Dilation

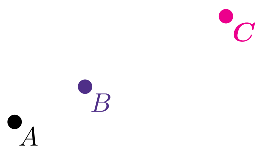
Let's dilate figures on a square grid.

Warm-Up

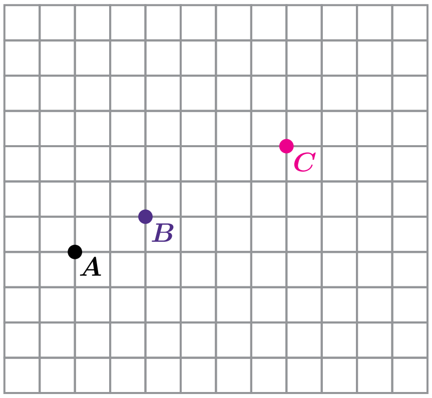
Plot a point C that is the image of point B dilated using point A as the center of dilation and a scale factor of 3.

1 Try it *without* a grid.

Responses vary.



2 Try it *with* a grid.



Describe your strategy for dilating *with* a grid.

Responses vary.

- The distance between points A and C should be 3 times the distance from point A to point B. I used the grid to count squares. To move from point A to point B, I went 1 square up and 2 squares to the right. So point C should be $1 \cdot 3 = 3$ squares up and $2 \cdot 3 = 6$ squares to the right of point A.
- To get from point A to point B on the grid, I went up 1 and right 2. To get from point B to point C, I went up 1 and right 2 two more times.

Activity 1

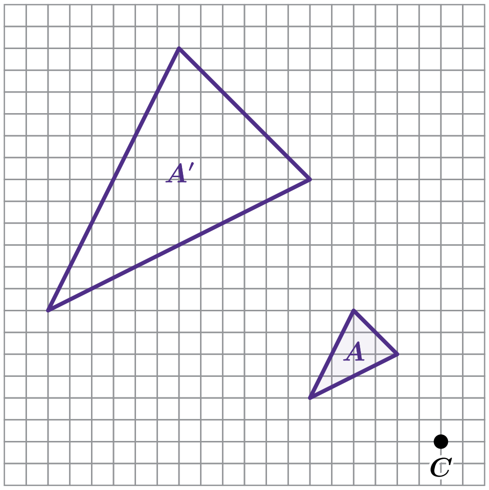
Name: .... Date: .... Period: ....

Dilation Challenges

3 Triangle A' is the image of triangle A dilated using point C as the center of dilation.

What was the scale factor used in the dilation?

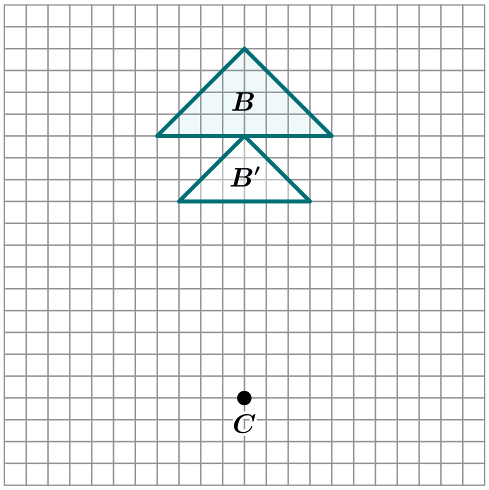
3



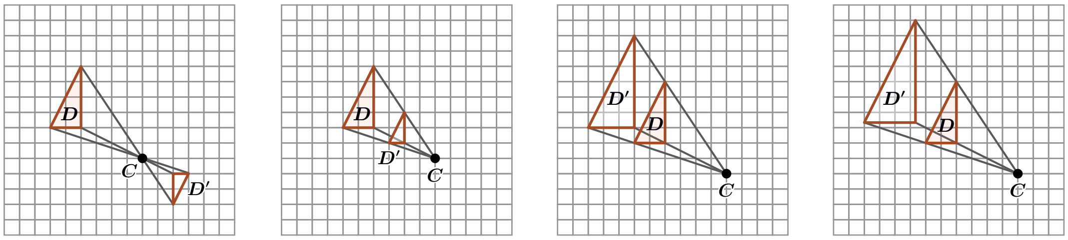
4 Triangle B' is the image of triangle B dilated using point C as the center of dilation.

What was the scale factor used in the dilation?

$\frac{3}{4}$ **(or equivalent)**



5 **a** Take a look at these dilations of triangle D.



Scale factor: -0.5 Scale factor: 0.5 Scale factor: 1.5 Scale factor: 1.8

b How does the scale factor affect the size of the image? How does it affect the location of the image?

Responses vary. **For scale factors greater than 1, the image is bigger than the pre-image and further from the center of dilation. For scale factors between 0 and 1, the image is smaller than the pre-image and in between the pre-image and center of dilation. For scale factors less than 0, the image and pre-image are on opposite sides of the center of dilation, and the image is also flipped.**

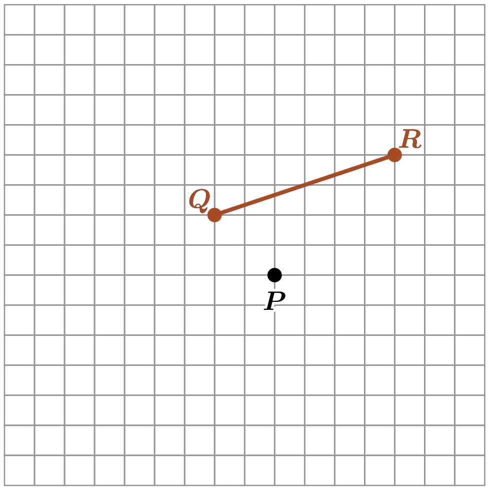
Activity 2

Name: .... Date: .... Period: ....

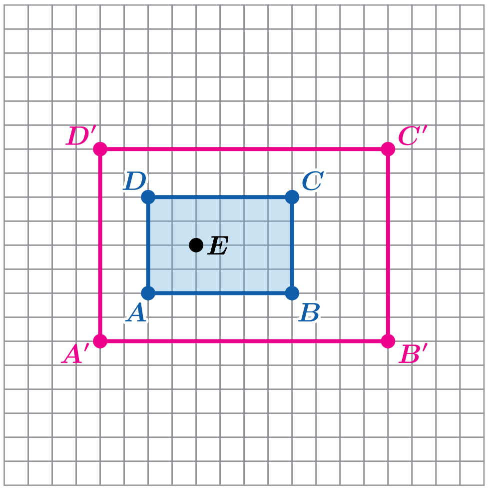
Match My Dilation

6 Show or explain how to dilate segment QR using point P as the center of dilation and a scale factor of $\frac{1}{2}$.

***Responses vary.* Count the horizontal distance (2 units) and vertical distance (2 units) from point P to point Q. Then multiply those distances by $\frac{1}{2}$ and place point Q' that distance away from point P. Use a similar process with point R to get point R'.**



7 **a** Dilate quadrilateral $ABCD$ using point E as the center of dilation and a scale factor of 2.



b **Discuss:** How would the size and location of the image change if point E were in a different location?

***Responses vary.* The size of the image would be the same, but the location of the image would change based on where the center of dilation is.**

Name: .... Date: .... Period: ....

Challenge Creator

8 You will use the Activity 3 Sheet to complete this activity.

a **Make It!** On the Activity 3 Sheet, create a dilation challenge.

b **Solve It!** On this page, redraw your pre-image. Then draw your image using the center of dilation and scale factor you chose. Label the vertices A', B', and C'.

c **Swap It!** Swap your challenge with one or more partners. Draw your partners' pre-images. Then draw each image using the center of dilation and scale factor.

Responses vary.

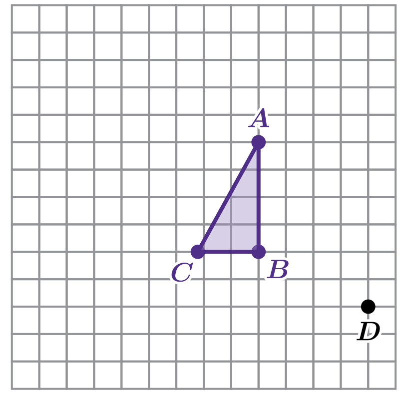
My Dilation Challenge	Partner 1
Scale Factor:	Scale Factor: ....

Partner 2	Partner 3
Scale Factor:	Scale Factor: ....

9 Synthesis

What is important to remember when dilating a figure using a center of dilation and a scale factor?

Use the example if it helps you with your thinking.



Responses vary. **It's important to remember to draw a line from the center of dilation through a point in the pre-image. Then count the horizontal distance and vertical distance from the center of dilation to the vertex. Next, multiply those distances by the scale factor and place a point for the image that distance away from the center of dilation. Use a similar process for all the other vertex points in the pre-image. Then label the vertices of the image A', B', and C'.**

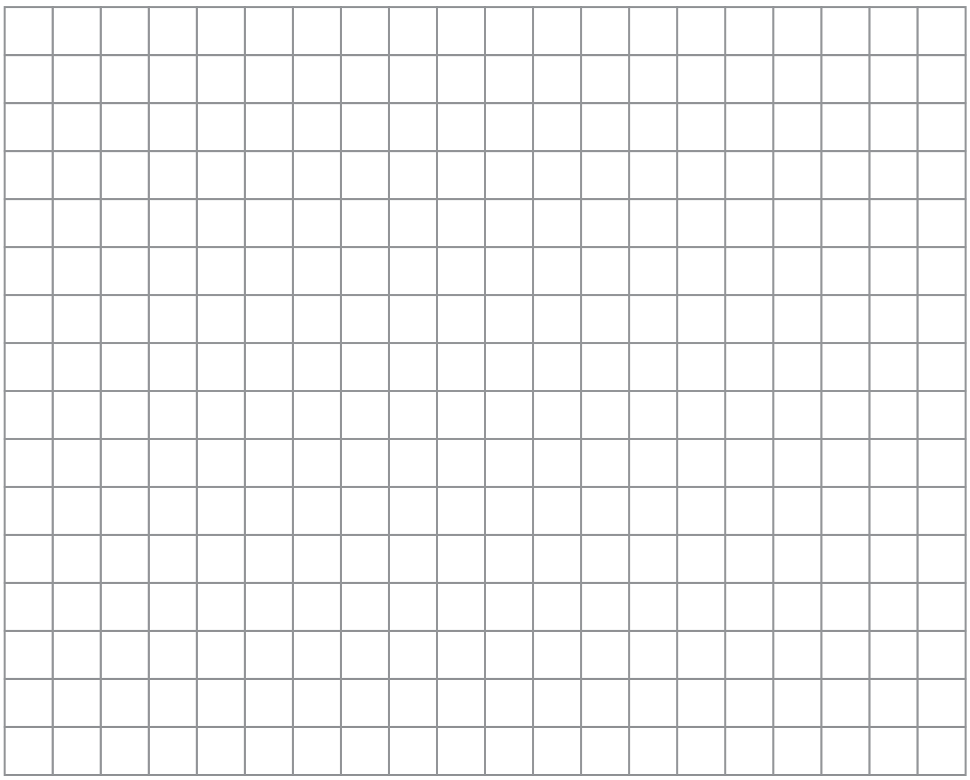
Things to Remember:

Name: .... Date: .... Period: ....

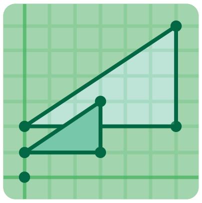
Challenge Creator

Create your own dilation challenge by completing these steps:

- Draw a triangle and label the vertices A, B, and C.
- Choose a center of dilation and label it D.
- Choose a scale factor: ___________



Name: ______ Date: ______ Period: ______



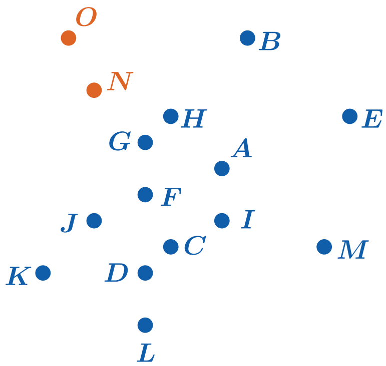
Dilations on a Plane

Let's look at dilations on the coordinate plane.

Warm-Up

1. Determine which point is a dilation of point N using point O as the center of dilation and a scale factor of 3.

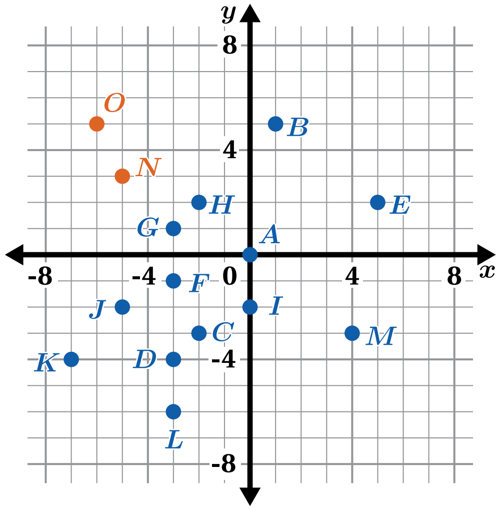
Responses vary.



2. Here is the same task, but with a coordinate plane.

Determine which point is a dilation of point N using point O as the center of dilation and a scale factor of 3.

Point F



Activity 1

Name: ________ Date: ________ Period: ________

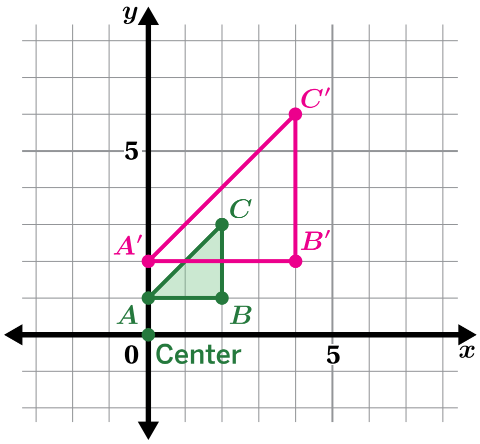
Dilate It!

Use any strategy or tool to perform each dilation. Label the corresponding points in the image using the ′ symbol.

3. **a** Dilate triangle ABC using $(0, 0)$ as the center of dilation and a scale factor of 2.

b Write the image coordinates in the table.

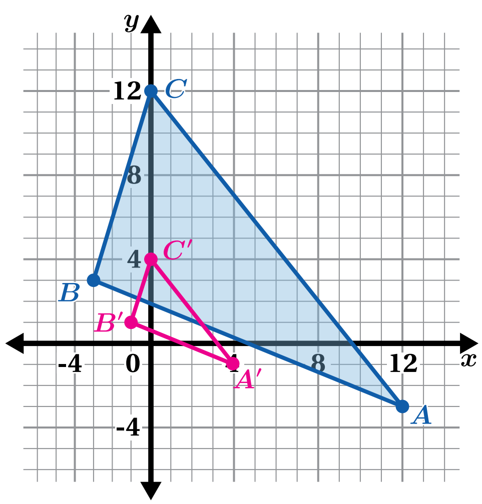
Pre-Image Coordinates	Image Coordinates
(0, 1)	**(0, 2)**
(2, 1)	**(4, 2)**
(2, 3)	**(4, 6)**



4. **a** Dilate triangle ABC using $(0, 0)$ as the center of dilation and a scale factor of $\frac{1}{3}$.

b Write the image coordinates in the table.

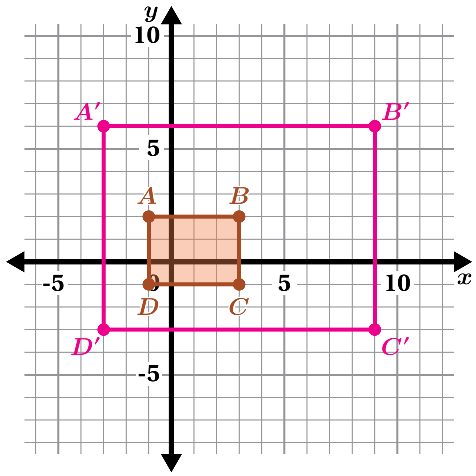
Pre-Image Coordinates	Image Coordinates
(12, -3)	**(4, -1)**
(-3, 3)	**(-1, 1)**
(0, 12)	**(0, 4)**



Name: .... Date: .... Period: ....

Dilate It! (continued)

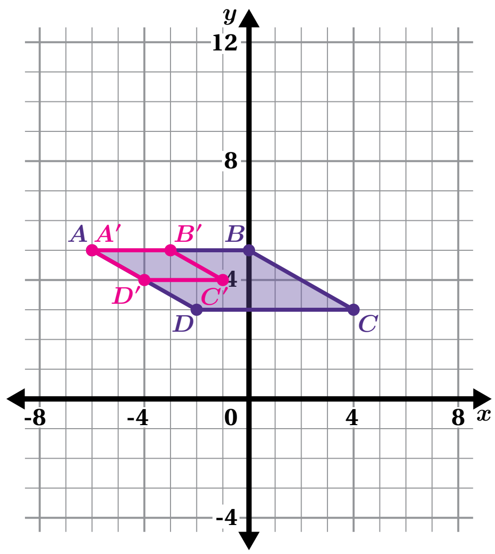
5. **a** Dilate rectangle $ABCD$ using $(0, 0)$ as the center of dilation and a scale factor of 3.



b Write the image coordinates in the table.

Pre-Image Coordinates	Image Coordinates
(-1, 2)	**(-3, 6)**
(3, 2)	**(9, 6)**
(3, -1)	**(9, -3)**
(-1, -1)	**(-3, -3)**

6. **a** Dilate parallelogram $ABCD$ using (-6, 5) as the center of dilation and a scale factor of $\frac{1}{2}$.



b Write the image coordinates in the table.

Pre-Image Coordinates	Image Coordinates
(-6, 5)	**(-6, 5)**
(0, 5)	**(-3, 5)**
(4, 3)	**(-1, 4)**
(-2, 3)	**(-4, 4)**

c How is this problem different from the other problems in this activity?

***Responses vary.* Problems 1–3 all have a center of dilation at (0, 0), and this problem has a center of dilation at (-6, 5).**

Activity 2

Name: ______ Date: ______ Period: ______

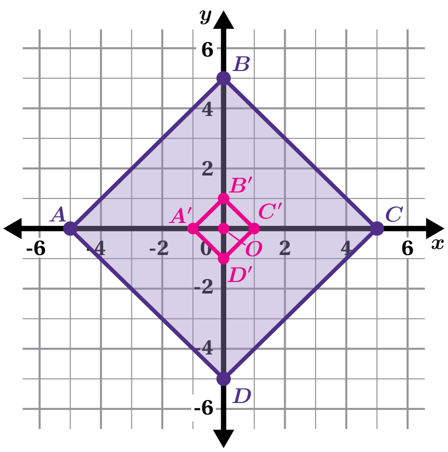
Dilation Information

You and your partner will get a set of dilation cards. Place them grid-side down without looking at them.

- Decide who will describe the dilation on a card and who will sketch the image. Start with Card 1.
- Describer: Give enough information about the dilation so that the Sketcher can sketch it.
- Sketcher: Pause after sketching and share what you think the dilation is.
- Together: Compare the card with the sketch and make adjustments as needed. Write a precise description of the dilation.
- Switch roles for Card 2 and repeat. Then do the same for Cards 3 and 4.

7. Sketch 1: Card 1 or Card 2 (Circle one.)

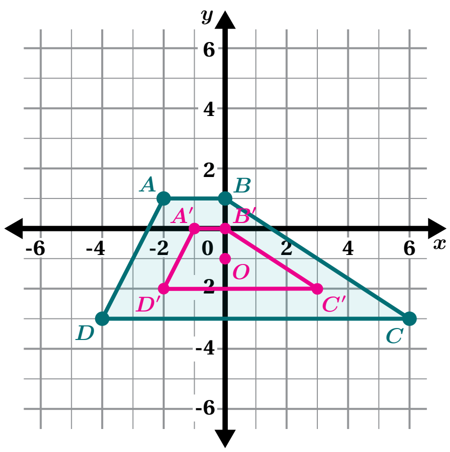
Card 1



Description of Transformation:
Dilate figure $ABCD$ by . . .

***Responses vary.* Dilate figure $ABCD$ by using $(0, 0)$ as the center of dilation and a scale factor of $\frac{1}{5}$.**

Card 2



Description of Transformation:
Dilate figure $ABCD$ by . . .

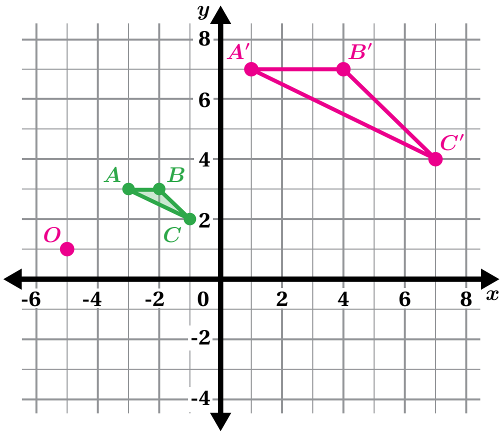
***Responses vary.* Dilate figure $ABCD$ by using $(0, -1)$ as the center of dilation and a scale factor of $\frac{1}{2}$.**

Name: .... Date: .... Period: ....

Dilation Information (continued)

8. Sketch 2: Card 3 or Card 4 (Circle one.)

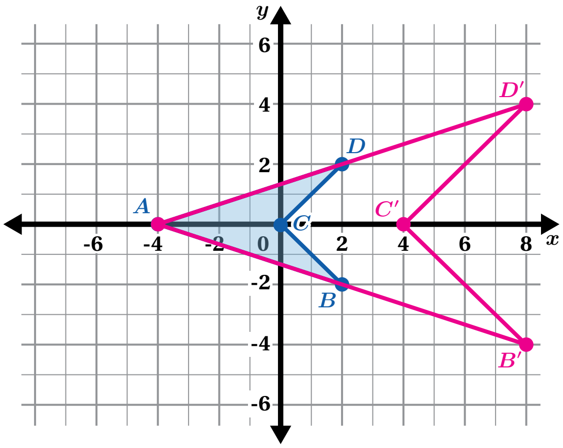
Card 3



Description of Transformation:
Dilate figure ABC by . . .

***Responses vary.* Dilate figure ABC by using (-5, 1) as the center of dilation and a scale factor of 3.**

Card 4



Description of Transformation:
Dilate figure $ABCD$ by . . .

***Responses vary.* Dilate figure $ABCD$ by using (-4, 0) as the center of dilation and a scale factor of 2.**

Explore More

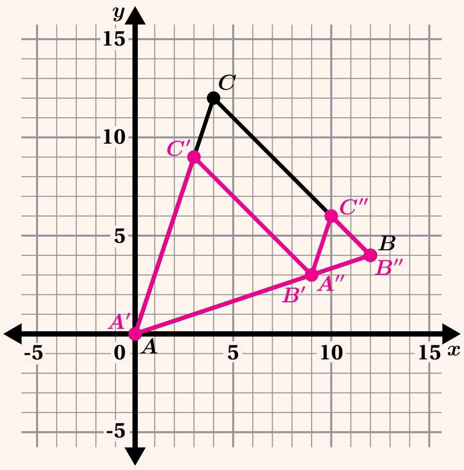
9. Here is triangle ABC.

a Dilate triangle ABC using center $(0, 0)$ and a scale factor of $\frac{3}{4}$. Label the vertices $A'B'C'$.

b Dilate triangle ABC using center $(12, 4)$ and a scale factor of $\frac{1}{4}$. Label the vertices $A''B''C''$.

c Explain why A'' and B' must be at the same coordinates.

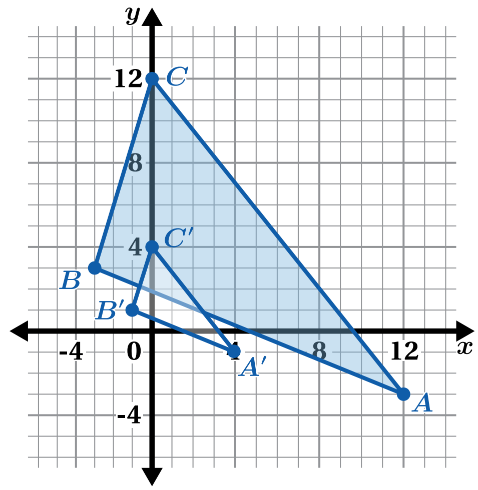
A'' and B' must be the same coordinates because the sum of the scale factors $\left(\frac{3}{4} \text{ and } \frac{1}{4}\right)$ is 1.



Synthesis

10. What is important to remember when dilating figures on a coordinate plane?

Use the example if it helps with your thinking.



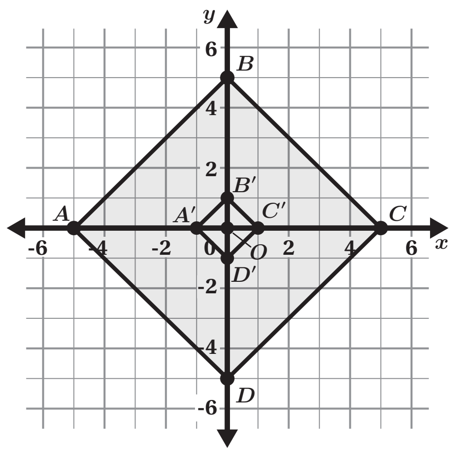
Responses vary. **It's important to include the location of the center of dilation and a scale factor. The scale factor 10 helps communicate whether the image will be smaller or larger than the pre-image. The center of dilation helps to communicate where the image will be located. The coordinate plane can be used to measure distance between pre-image points, the center of dilation, and image points.**

Things to Remember:

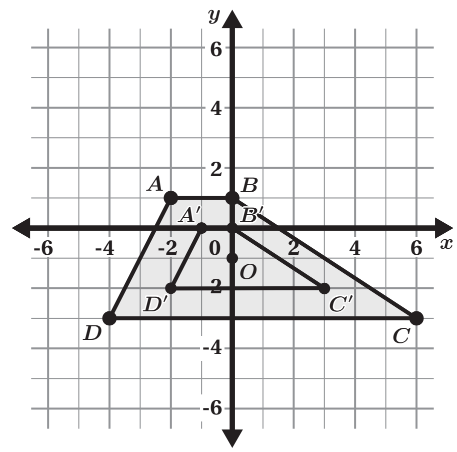
Dilation Information

Directions: Make one copy per pair of students. Then pre-cut the cards and give each pair one set.

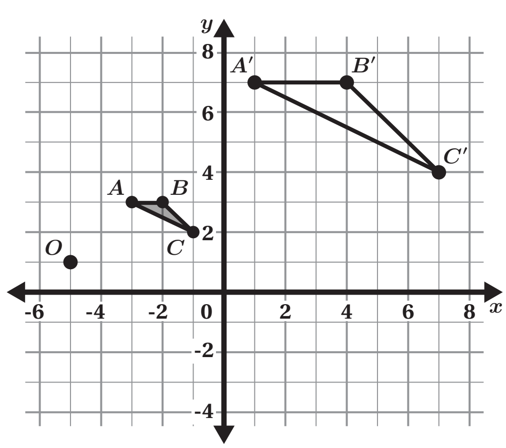
Card 1



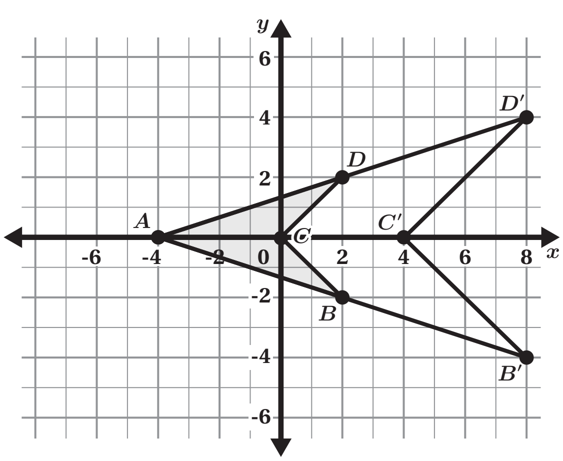
Card 2



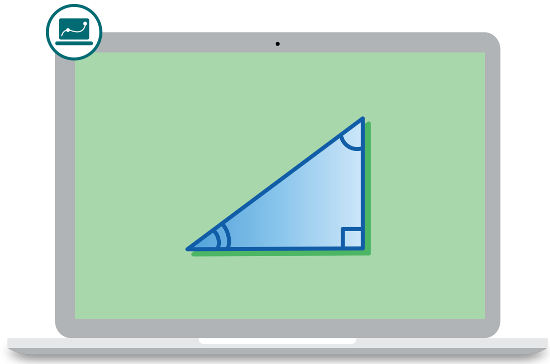
Card 3



Card 4



Name: ______________ Date: ________ Period: ________

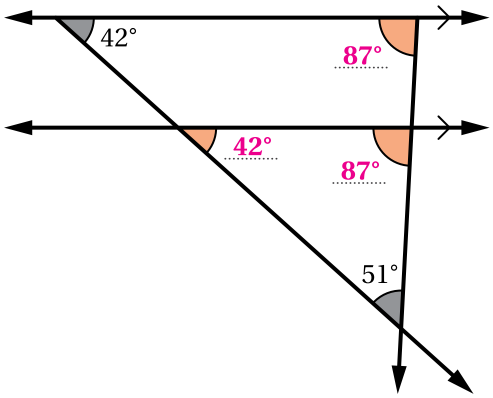


Are Angles Enough?

Let’s determine whether congruent corresponding angles are enough to know whether triangles are similar.

Warm-Up

1 Determine the measure for each missing angle.
Responses shown on diagram.

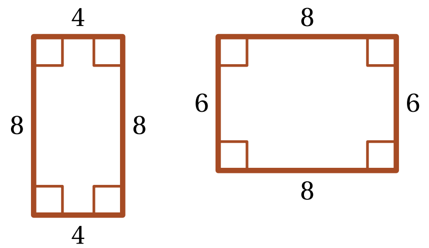


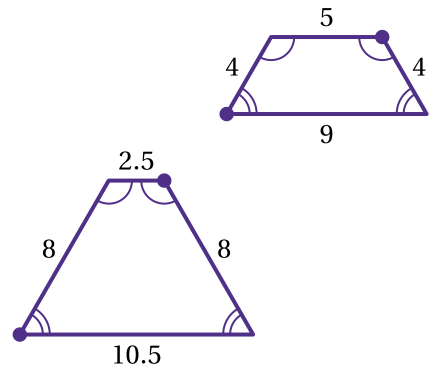
Activity
1

Name: .... Date: .... Period: ....

Are Angles Enough?

2 Take a look at these pairs of figures that have congruent corresponding angles.

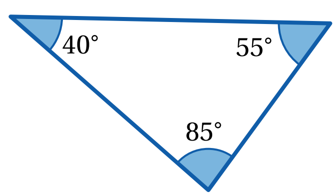




Discuss: How do you know that the figures in each pair are *not* similar?

***Responses vary.* In each figure, the corresponding sides have different scale factors.**

3 **a** Take a look at this triangle with angle measures $40°$, $55°$, and $85°$.



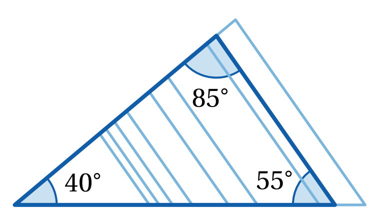
b If all of your classmates made triangles with the same angle measures as this one, would all the triangles be similar? Circle one. ***Responses vary.***

Yes No I'm not sure

Explain your thinking.

Explanations vary.
- **Yes, all of the triangles would be similar because the angles are congruent.**
- **No. It's not enough for only the corresponding angles to be congruent. We need both congruent angles and proportional side lengths.**

4 Here are some triangles that all have $40°$, $85°$, and $55°$ angles.



a Let's watch an animation of these triangles.

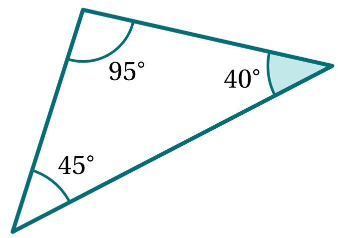
b **Discuss:** Are these triangles all similar?

Yes. *Responses vary.* All of the triangles are similar because if you dilate one of the triangles it fits onto the other triangles.

Name: .... Date: .... Period: ....

Are Angles Enough? (continued)

5 **a** Take a look at this triangle with one angle measuring $40°$.



b If all of your classmates made triangles with one angle measuring $40°$, would all the triangles be similar? Circle one.

Responses vary.

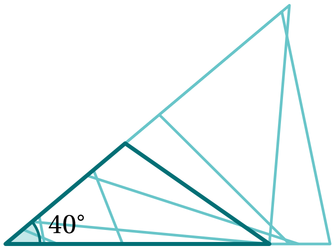
Yes   No   I'm not sure

Explain your thinking.

Explanations vary.

- **Yes, all of the triangles would be similar because one angle in everyone's triangle would be congruent.**
- **No, one angle being the same in each triangle is not enough to make all the triangles similar.**

6 Here are some triangles that all have a $40°$ angle.



a Let's watch an animation of these triangles.

b **Discuss:** Are these triangles all similar?

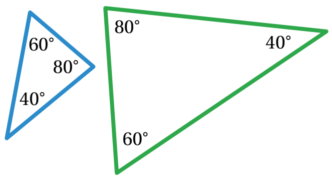
No. *Responses vary.* They are not similar because there isn't a sequence of transformations that moves one triangle onto the other.

Name: .... Date: .... Period: ....

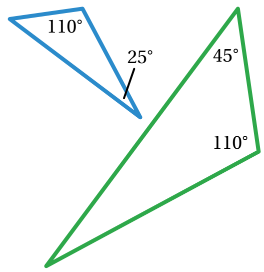
Similar Sort

7 Sort the pairs of figures into three groups. (Images are not to scale.)

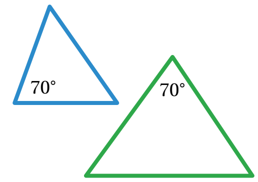
A.



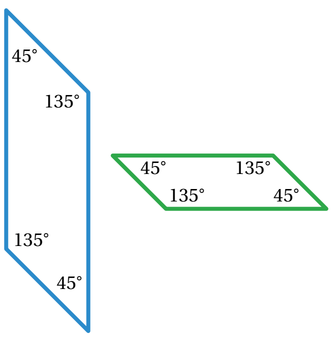
B.



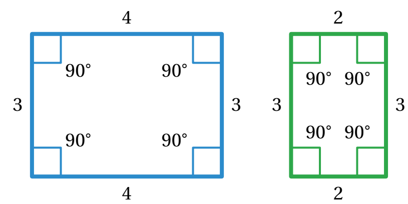
C.



D.



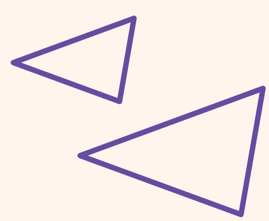
E.



Similar	Not Similar	Not Enough Information
A and B	E	C and D

Explore More

8 Here are two similar triangles. Rio says that in similar triangles, if you match up two pairs of sides at a vertex, then the third sides are always parallel. Is Rio's claim correct? **Yes**



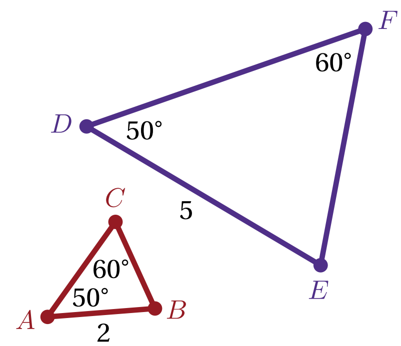
Explain your thinking.

***Explanations vary*. Rio can show that these triangles are similar through dilation. Using a common vertex as a center of dilation, a dilation of the third side will always take lines to parallel lines.**

9 Synthesis

Describe how you can use angles to determine whether triangles are similar.

Use the example if it helps you with your thinking.



***Responses vary.* Triangles are similar if two pairs of corresponding angles are congruent.**

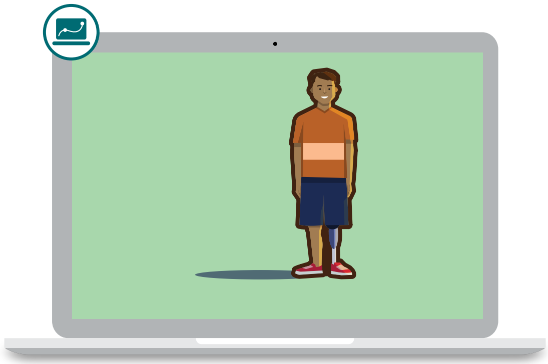
Things to Remember:

Unit 2

Lesson

8

Name: .... Date: .... Period: ....

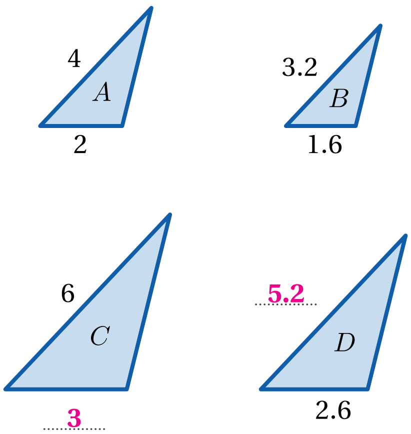


Shadows

Let's use what we know about similar triangles to determine missing side lengths.

Warm-Up

1 Here are four similar triangles.



a Examine the given side lengths. Then write in the missing values.

Responses shown on diagram.

b **Discuss:** How did you determine these values?

***Responses vary.* I noticed that in each triangle, the shortest side is always half the length of the longest side.**

Activity 1

Name: ______ Date: ______ Period: ______

Similar Triangles in Shadows

2 Let’s watch a slider control the time of day.

What do you notice? What do you wonder? ***Responses vary.***

I notice:

- **I notice that all shadows increase in the same way as the time of day changes.**
- **I notice there are different shadow lengths for the person, mailbox, and lamppost.**

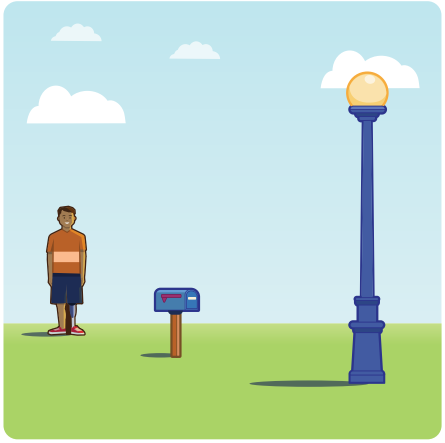
I wonder:

- **I wonder how the shadows relate to the objects.**

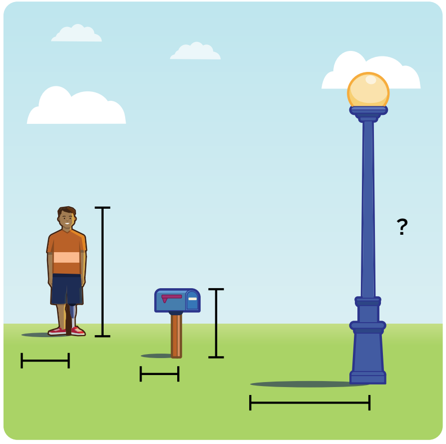
3 Kayla noticed that similar triangles could be formed using the shadows of these figures.

Where do you think Kayla saw triangles? Why might Kayla think these triangles are similar? Draw on the picture if it helps to show your thinking.

***Responses vary.* Each object and its shadow form two sides of a right triangle. The triangles are all similar because the shadows have the same scale factor based on the time of day.**



4 Your task is to determine the height of this lamppost.



a. Decide which of these measurements you need and request them from your teacher:

- ☐ Height of the person
- ☐ Length of the person’s shadow
- ☐ Height of the mailbox
- ☐ Length of the mailbox’s shadow
- ☐ Length of the lamppost’s shadow

***Selections vary.* To answer part b, students need to select the length of the lamppost’s shadow, and both the height and length of at least one of the other objects.**

b. Once you have enough information, determine the height of the lamppost.

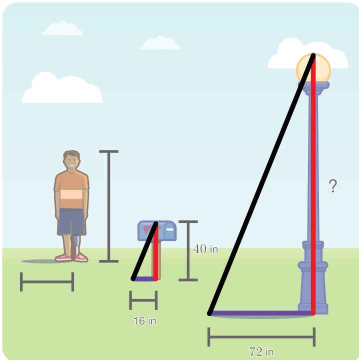
180 inches (or equivalent)

Name: .... Date: .... Period: ....

Similar Triangles in Shadows (continued)

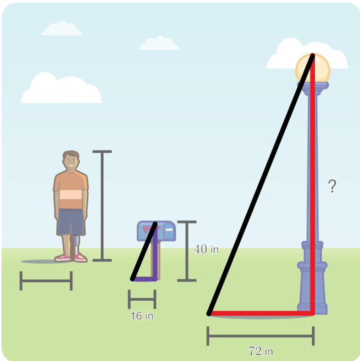
5 Here are Ama's and Neena's strategies for determining the height of the lamppost.

Ama



$\frac{40}{?} = \frac{16}{72}$

Neena



$\frac{40}{16} = \frac{?}{72}$

Discuss: How are their strategies alike? How are they different?

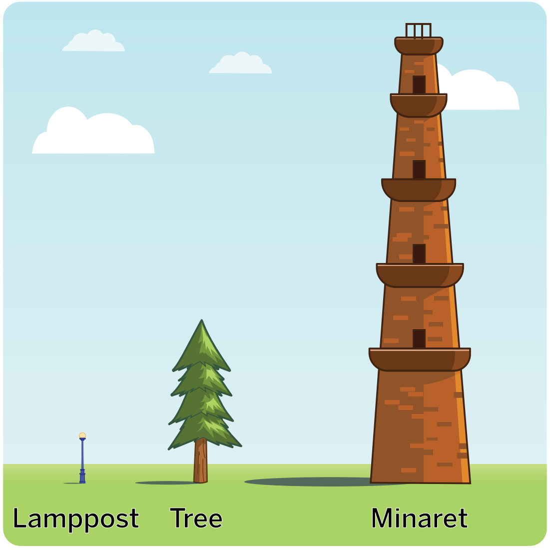
Responses vary.

- **Their strategies are alike because they're using the same triangles and values.**
- **Their strategies are different because Ama's ratio is based on the proportions of the corresponding legs of two triangles, while Neena's ratio uses the proportions of the legs within a triangle.**

6 Here is the lamppost from the previous problem, as well as two new objects.

Determine the missing heights.

	Lamppost	Tree	Minaret
Height (ft)	15	**50**	**140**
Shadow Length (ft)	6	20	56

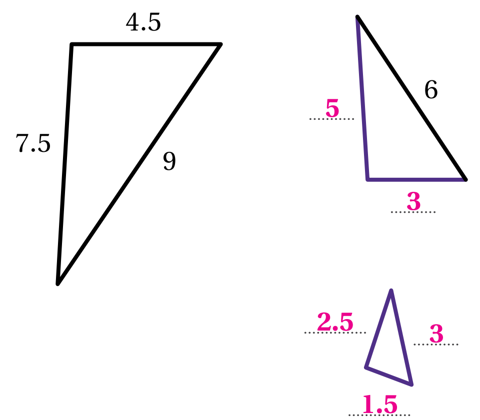


Activity
2

Name: ______ Date: ______ Period: ______

Similar Triangles Puzzle

7 Here are three similar triangles.



a Determine all the side lengths using as few hints as you can. You can ask for the measure of up to two side lengths, if needed.

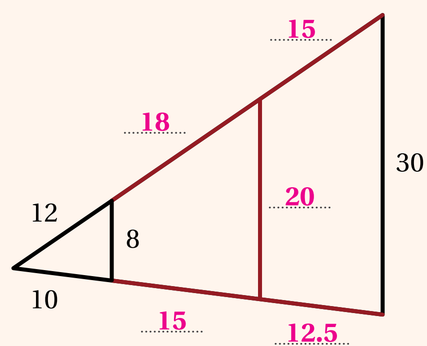
Responses shown on diagram.

b What was your strategy?

***Responses vary.* I noticed that the longest side of the biggest triangle was 1.5 times the length of the corresponding side in the medium triangle. I used this ratio to determine the missing side lengths of the medium triangle. Then, I asked for the longest side of the smallest triangle to determine the ratio of 3 between the largest and smallest triangle. I used this ratio to calculate the other missing side lengths.**

Explore More

8 Here are three similar triangles.



a Determine all the side lengths using as few hints as you can. You can ask for the measure of two or three additional side lengths, if needed.

Responses shown on diagram.

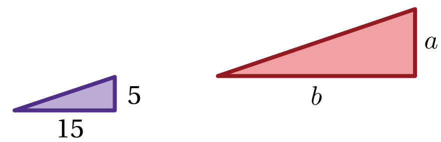
b What was your strategy?

***Responses vary.* I was able to find the scale factor of 3.75 from the smallest triangle to the largest triangle using the vertical side length measurements. Then, I asked for the vertical side of the medium triangle to find the scale factor for the medium triangle. From there I have enough information to find the remaining sides.**

9 Synthesis

What are some strategies for determining unknown side lengths in similar triangles?

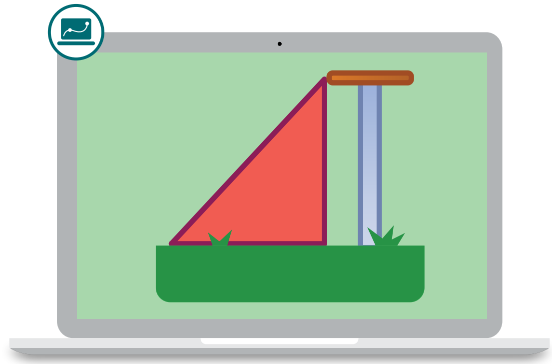
Use the example if it helps with your thinking.



Responses vary. **We can set up ratios between triangles because they are similar. Ratios can be set up so that corresponding sides of the triangles are matching. Alternatively, you can apply ratios between the side lengths of a triangle because they are equivalent for similar triangles.**

Things to Remember:

Name: ________ Date: ________ Period: ________



Water Slide

Let's look at similar triangles and lines.

Warm-Up

1 Let's watch someone try two different slides.

Discuss: What makes a smooth slide? What makes a bumpy slide?

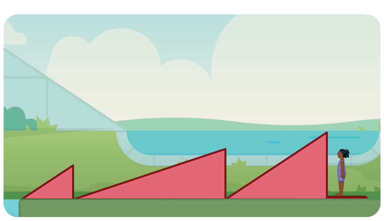
Responses vary.

- **There is a smooth slide when the surface of the slide makes a straight line.**
- **There is a bumpy slide when the surfaces of the slide don't form a straight line.**

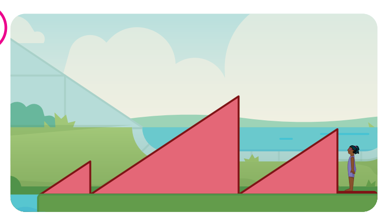
2 Your goal is to create a smooth slide.

Which set of ramps do you think would make a smooth slide?

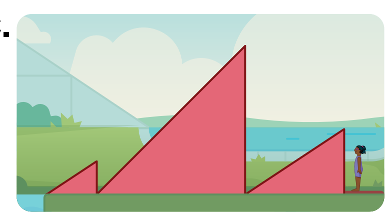
A.



B.



C.



Activity 1

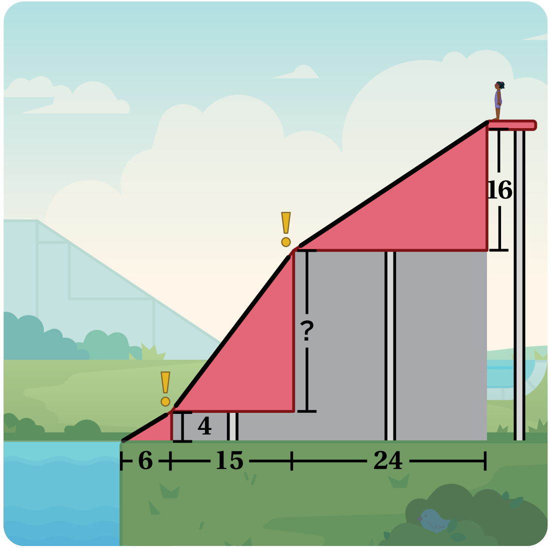
Name: .... Date: .... Period: ....

Smooth Slides

3 These ramps will make a bumpy slide!

Fill in the height for Ramp 2 to make a smooth slide.

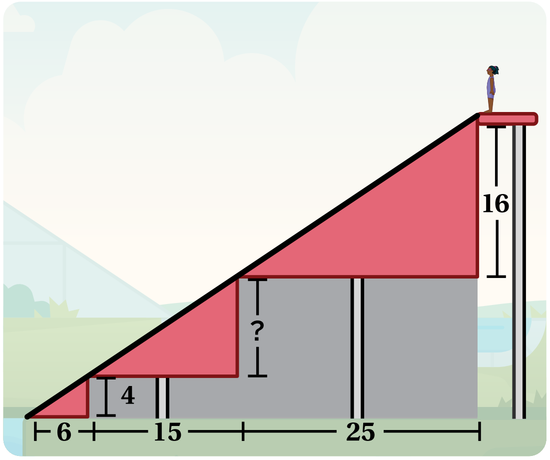
Ramp	Base (ft)	Height (ft)
Ramp 1	6	4
Ramp 2	15	**10**
Ramp 3	24	16



4 Jada says: *The ramps are all similar triangles*.

How can Jada use the properties of similar triangles to find the height of the middle ramp?

Responses vary. **Jada can determine what scale factor, when multiplied by the base of Ramp 1, results in the base of Ramp 2. Then they can determine the height of Ramp 2 by multiplying that scale factor by the height of Ramp 1.**



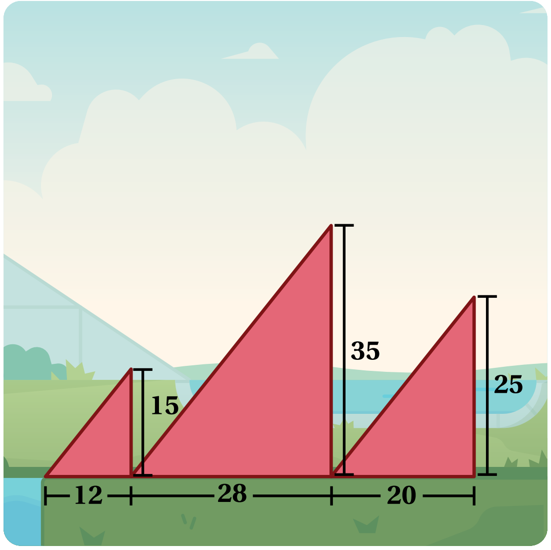
5 Here is a new set of ramps.

Will these ramps make a smooth slide? Circle one.

Yes No I'm not sure

Explain your thinking.

Explanations vary. **These ramps will make a smooth slide because the triangles are similar. I can tell they're similar because the corresponding sides are proportional.**



Activity 2

Name: ________ Date: ________ Period: ________

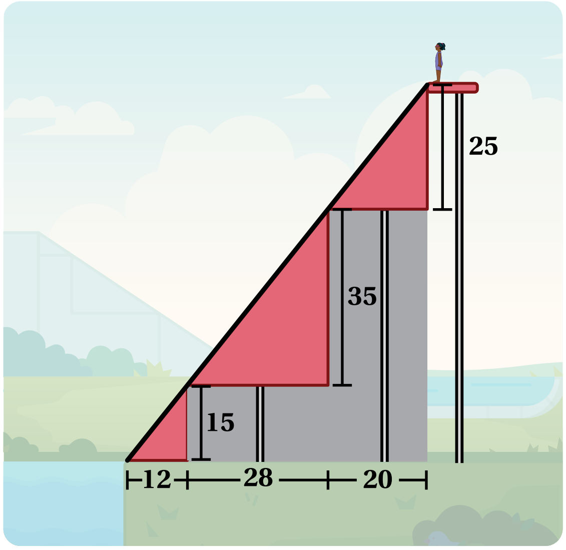
Introducing Slope

6 **Slope** measures the steepness of a line.

This slide forms a line with a slope of $\frac{5}{4}$.

How do you think slope is calculated?

***Responses vary.* You can calculate slope by finding the height-to-base ratio of one of the similar triangles.**

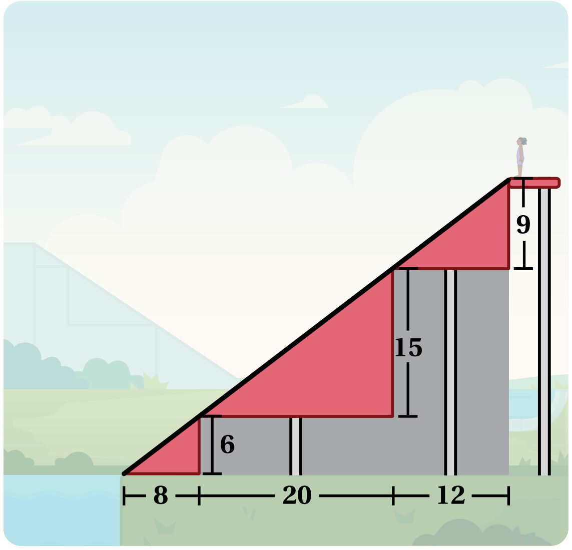


7 What is the slope of this slide?

$\frac{3}{4}$ (or equivalent)

Explain your thinking.

***Explanations vary.* I found the slope by calculating the height-to-base ratio of the smallest triangle, which was $\frac{6}{8}$ or $\frac{3}{4}$.**



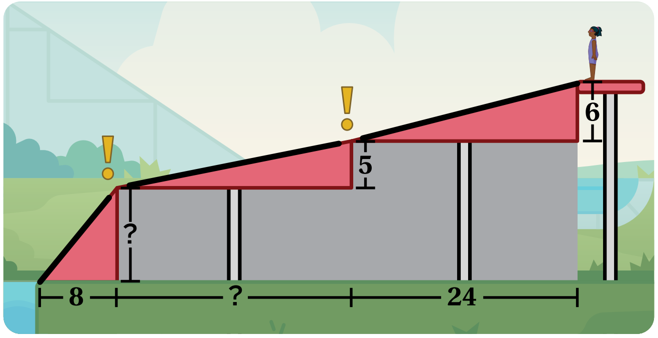
Name: ______ Date: ______ Period: ______

Introducing Slope (continued)

8 These ramps will make a bumpy slide!

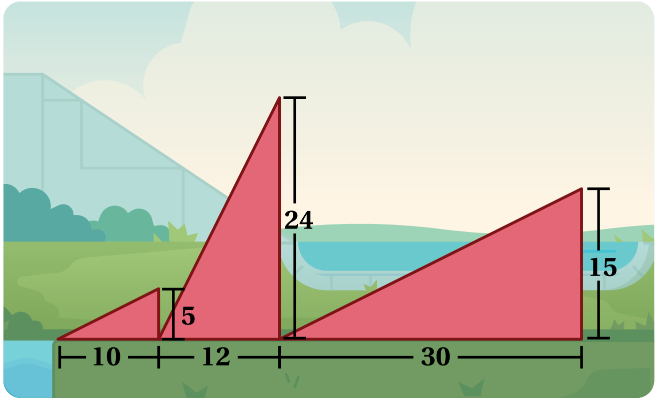
Fill in the missing values so that the slide has a slope of $\frac{1}{4}$.

Ramp	Base (ft)	Height (ft)
Ramp 1	8	**2**
Ramp 2	**20**	5
Ramp 3	24	6



9 Issa tried to create a slide with a slope of $\frac{1}{2}$.

Ramp	Base (ft)	Height (ft)
Ramp 1	10	5
Ramp 2	12	24
Ramp 3	30	15



These ramps didn't create a smooth slide!

a **Discuss:** What do you think Issa did well?

***Responses vary.* All the ramps are similar, but they are not in the same orientation.**

b Describe what the mistake might be in Issa's work.

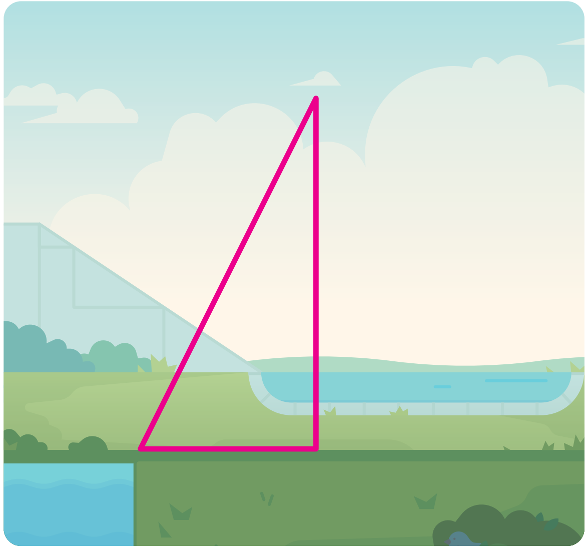
***Responses vary.* The height-to-base ratio for Ramps 1 and 3 is $\frac{1}{2}$, but for Ramp 2, the height-to-base ratio is 2 or $\frac{2}{1}$. I would tell Issa to be sure to put the height over the base when making his ratio.**

Name: .... Date: .... Period: ....

Your Water Slide

10 Draw a triangle to create a slide that will be fun, but not too scary.

Sample shown in image.



Decide and write the slope of your slide.

Responses vary. $\frac{5}{2}$

11 Now create three possible ramps for a smooth ride using the slope from the previous question. ***Responses vary.***

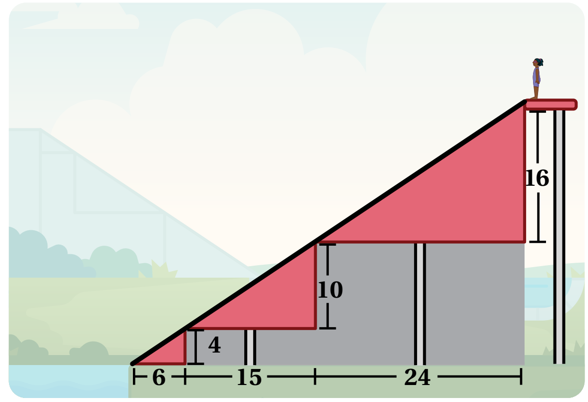
Ramp	Base (ft)	Height (ft)
Ramp 1	2	5
Ramp 2	4	10
Ramp 3	5	12.5

Students are successful here when the height-to-base ratios of all three ramps are equivalent to the slope chosen in the previous question.

12 Synthesis

Define *slope* in your own words and describe how to calculate it.

Responses vary. **Slope measures how steep a line is. A large slope means a steep line. The closer the slope is to 0, the less steep the line is. You calculate slope by finding the height-to-base ratio of the triangle.**



Things to Remember:

Unit 2
Lesson
10

Name: ____ Date: ____ Period: ____

Slope Challenges

Let's figure out the slopes of lines.

Warm-Up

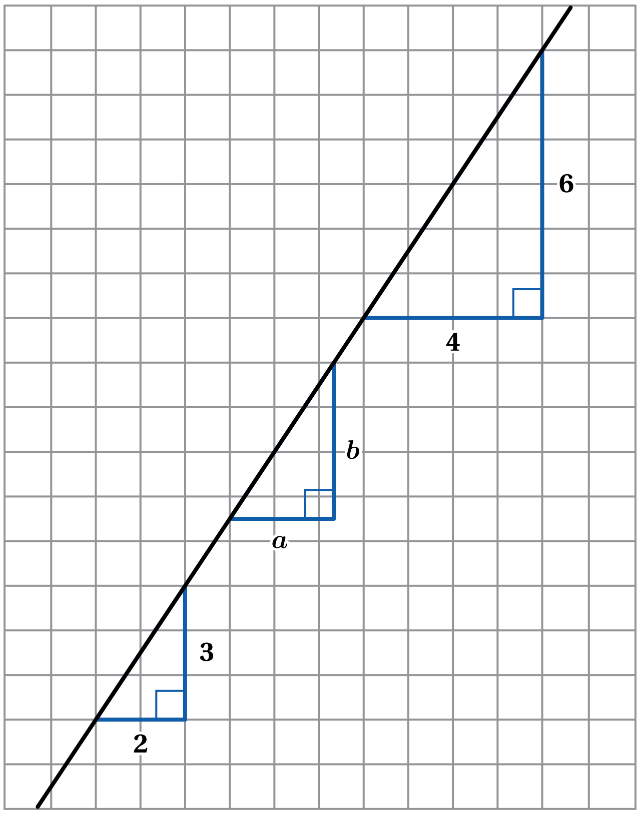
1 Here are three **slope triangles**.

What do you notice? What do you wonder?

Responses vary.

I notice:

- I notice there are three triangles.
- I notice that the largest triangle has side lengths that are twice as long as the side lengths of the smallest triangle.
- I notice all of the triangles appear to be scaled copies of each other.
- I notice that the longest side of each triangle all lie on the same line.



I wonder:

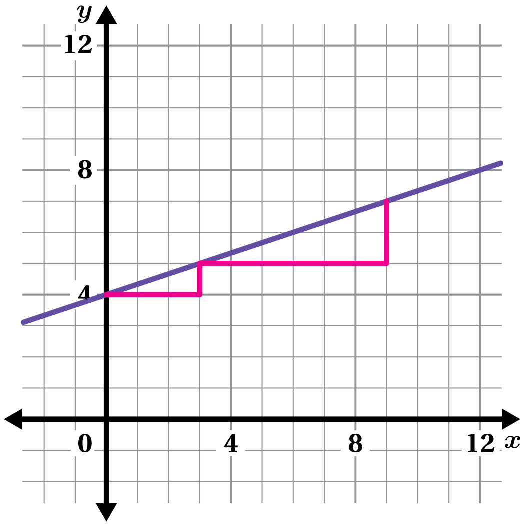
- I wonder what the values of a and b are.
- I wonder if all triangles drawn along this line are scaled copies of each other.

Activity 1

Name: .... Date: .... Period: ....

Determining Slope

2 Here is a line.



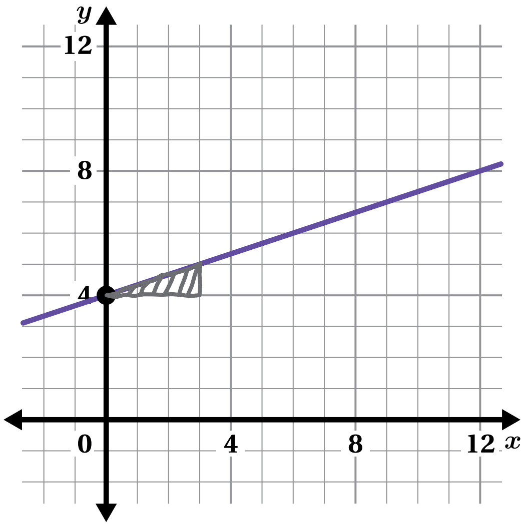
a Draw at least two *slope triangles* along the line.

***Responses vary.* Sample shown on graph.**

b What is the slope of the line?

$\frac{1}{3}$ **(or equivalent)**

3 Luke-Josephine says the slope of this line is 3.



What would you say to help Luke-Josephine understand that the slope is $\frac{1}{3}$?

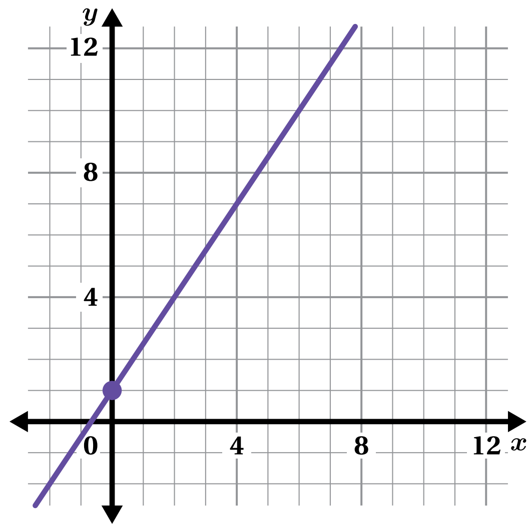
***Responses vary.* I would tell Luke-Josephine that the slope of the line is the height-to-base ratio of the slope triangle. The height is 1 and the base is 3, so the slope is $\frac{1}{3}$.**

Activity
1

Name: .... Date: .... Period: ....

Determining Slope (continued)

4 What is the slope of this line?



$\frac{3}{2}$ **(or equivalent)**

5 Kweku says the slope of the line in the previous problem is $\frac{6}{4}$.

Liam says the slope is $\frac{2}{3}$.

Whose thinking is correct? Circle one.

Kweku's   Liam's   Both   Neither

Explain your thinking.

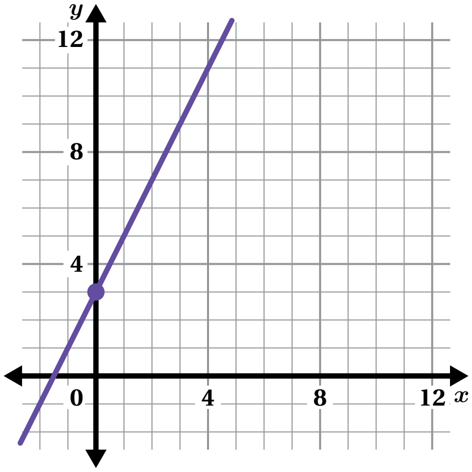
***Explanations vary.* Kweku's thinking is correct because the slope triangle will have a height of 6 and a base of 4, which matches the steepness of the line. Liam's slope triangle will have a height of 2 and a base of 3, which is less steep than the line.**

Name: ________ Date: ________ Period: ________

Repeated Challenges

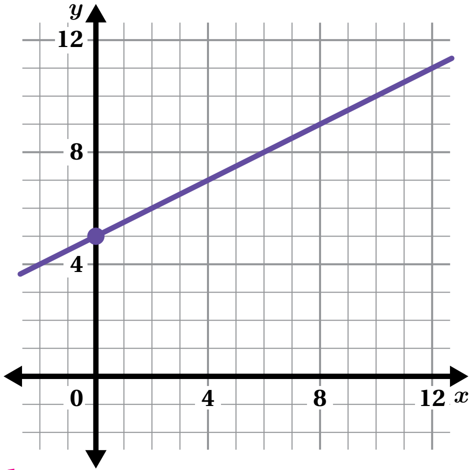
6 What is the slope of each line? Solve as many challenges as you have time for. It's more important to make sense of the challenges than it is to work quickly!

a



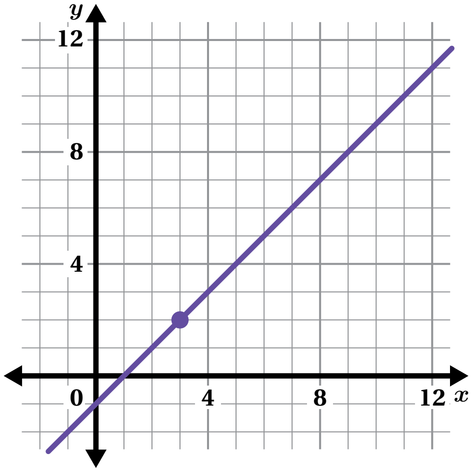
2 (or equivalent)

b



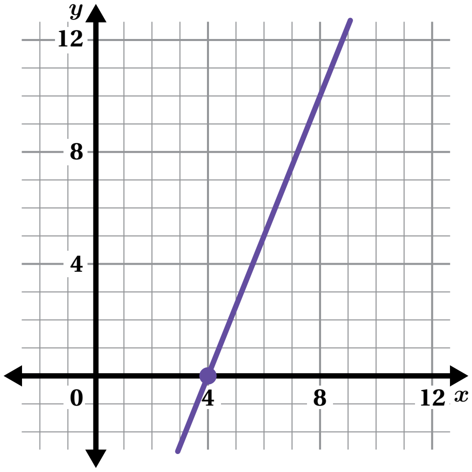
$\frac{1}{2}$ (or equivalent)

c



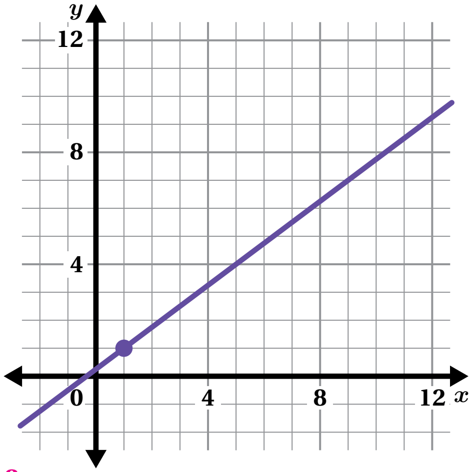
1 (or equivalent)

d



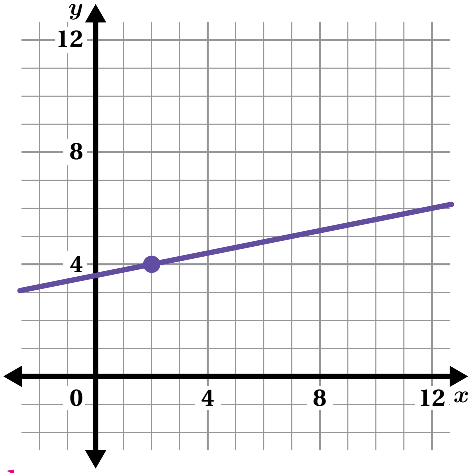
2.5 (or equivalent)

e



$\frac{3}{4}$ (or equivalent)

f

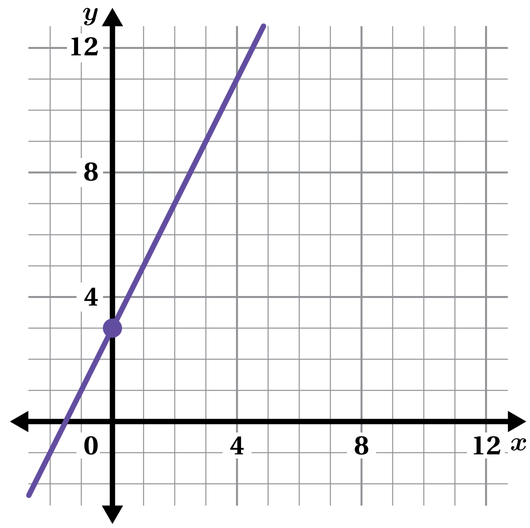


$\frac{1}{5}$ (or equivalent)

7 Synthesis

Describe a strategy for determining the slope of any line.

Use the example if it helps you with your thinking.



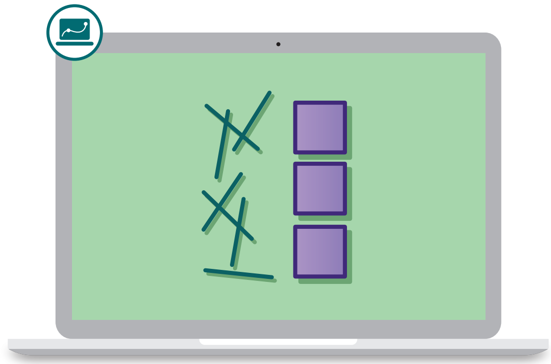
Responses vary.

- **Draw a slope triangle between two points on the line. The slope is the height of the triangle over the base of the triangle.**
- **To determine the slope of a line, find the height-to-base ratio of the slope triangle.**

Things to Remember:

Unit 6
Lesson
1

Name: .... Date: .... Period: ....



Toothpicks and Tiles

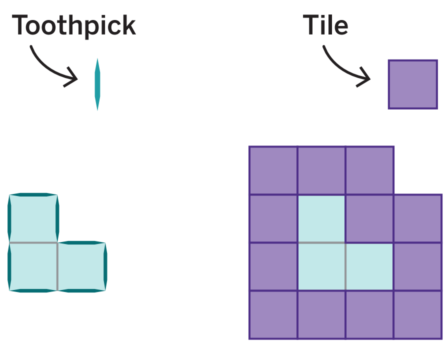
Let's make predictions about relationships.

Warm-Up

1 Here are two identical copies of the same shape. One has a border of toothpicks around it. The other has a border of tiles.

Border toothpicks: 8

Border tiles: 12

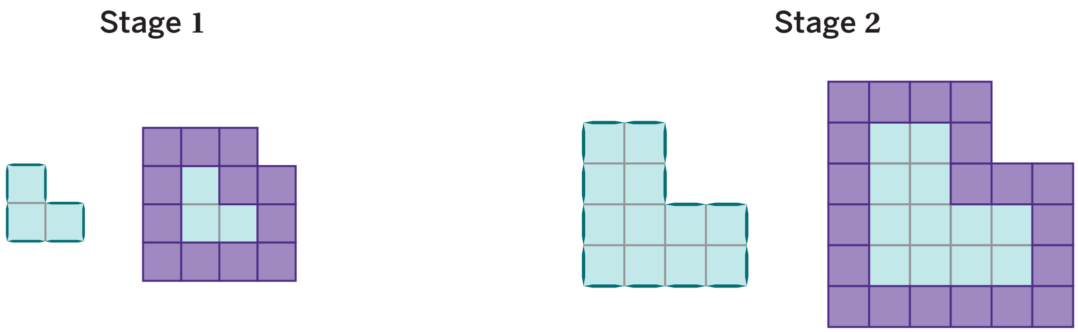


Activity 1

Name: ______ Date: ______ Period: ______

Patterns and Predictions

2 Stage 1 shows two figures. Stage 2 is a *scaled copy* of Stage 1. Stage x is a scaled copy of Stage 1 with a *scale factor* of x.



How many border toothpicks and tiles are in Stage 2? How many will there be in Stage 3?

Stage	Border Toothpicks	Border Tiles
1	8	12
2	**16**	**20**
3	**24**	**28**

3 Will there ever be a stage with exactly 100 toothpicks? Explain your thinking.

No. *Explanations vary.* The number of toothpicks is always a multiple of 8, and 100 is not a multiple of 8.

4 There is a stage that uses 100 border tiles. Which stage? Explain your thinking.

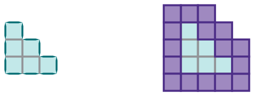
Stage 12. *Explanations vary.* I know that the number of border tiles goes up by 8 with each stage. So, if I keep adding 8, I get to 100 at Stage 12.

Name: Date: Period:

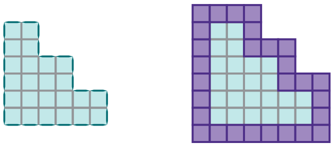
Patterns and Predictions (continued)

5 Here is a new design.

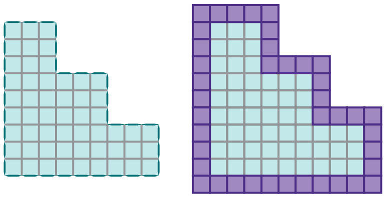
Stage 1



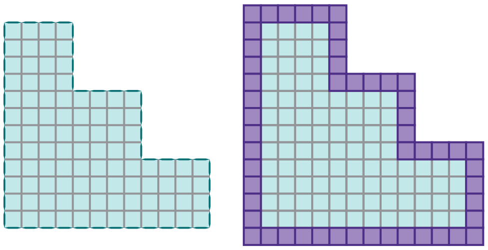
Stage 2



Stage 3



Stage 4



How many border tiles will there be in Stage 5? Explain your thinking.

64. *Explanations vary.* There are 12 more toothpicks at each stage, so there would be 60 toothpicks at Stage 5. Then I added 4 to get 64 because there are always 4 more tiles than toothpicks.

Stage	Border Toothpicks	Border Tiles
1	12	16
2	24	28
3	36	40
4	48	52

6 Which stage uses 100 border tiles?

Stage 8

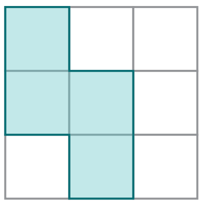
Activity 2

Name: Date: Period:

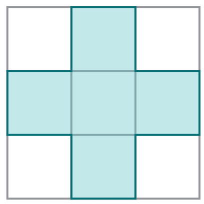
Unique Designs

7 Choose one of these Stage 1 designs or create your own.

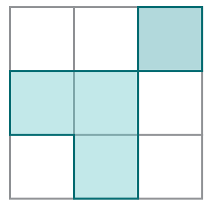
Design A



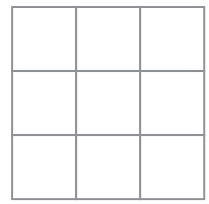
Design B



Design C



Create Your Own



a Use the Activity 2 Sheet to determine the number of border toothpicks and tiles for Stages 1–3 of the design you chose.

***Designs vary.* Sample shown in table for Design A.**

Stage	Border Toothpicks	Border Tiles
1	**10**	**14**
2	**20**	**24**
3	**30**	**34**

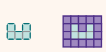
b Predict how many border tiles are used in Stage 4.

***Responses vary.* 44 tiles for Design A.**

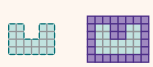
Explore More

8 There's something unusual about the number of border tiles in Stage 1 of this design.

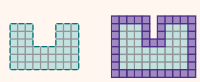
Stage 1



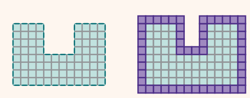
Stage 2



Stage 3



Stage 4



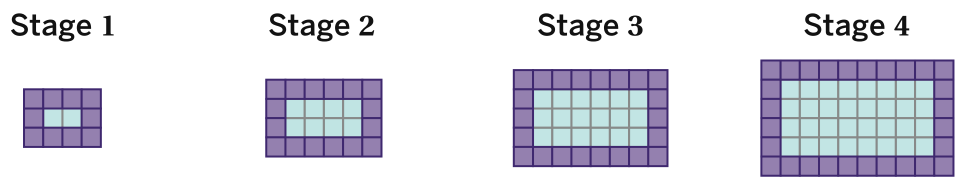
Why is Stage 1 different from Stages 2 and beyond?

***Responses vary.* The difference between Stage 1 and all the other stages happens in the top row of tiles. In Stages 2 and beyond, there is an even number of tiles in the top row (8, 10, 12, etc.). In Stage 1, the pattern breaks. Instead of 6 tiles, there are 5 because some tiles "collide."**

Stage	Border Toothpicks	Border Tiles
1	12	15
2	24	28
3	36	40
4	48	52

9 Synthesis

Here is a new pattern.



Border tiles: 10 | Border tiles: 16 | Border tiles: 22 | Border tiles: 28

Describe how you can determine the number of border tiles at any stage.

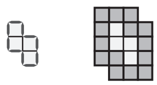
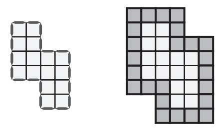
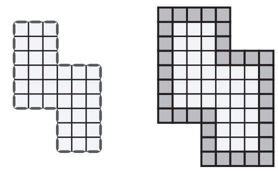
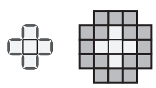
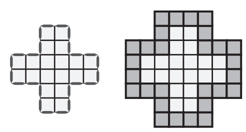
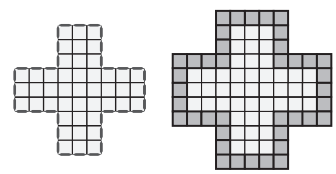
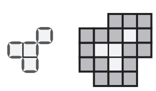
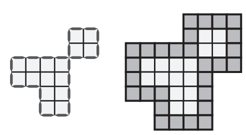
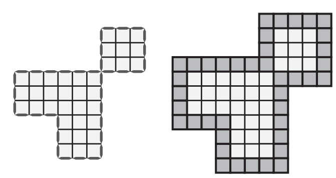
Responses vary. **I can determine the number of toothpicks because it's like the perimeter of the shape, or 6 times the stage number. The number of tiles is 4 more than the number of toothpicks.**

Things to Remember:

Name: .... Date: .... Period: ....

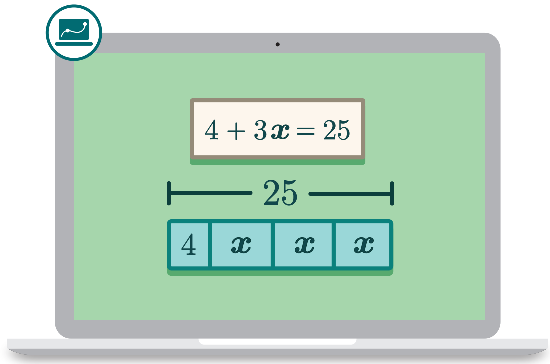
Unique Designs

Use the table to help you determine how many border toothpicks and tiles are used for Stages 1–3 for the design you chose.

	Stage 1	Stage 2	Stage 3
Design A			
Design B			
Design C			
Create Your Own			

Unit 6
Lesson
3

Name: ________ Date: ________ Period: ________



Equations

Let's connect representations of relationships.

Warm-Up

1 Here is a tape diagram.

30

$x + 6$	$x + 6$	$x + 6$

a Which equation matches the tape diagram?

A. $3x + 6 = 30$

B. $3 + 6x = 30$

C. $3(x + 6) = 30$

b Draw a tape diagram for one of the equations that you *did not* select.

Responses vary.

$3x + 6 = 30$

30

x	x	x	6

$3 + 6x = 30$

30

3	x	x	x	x	x	x

Activity

1

Name: ____________ Date: ____________ Period: ____________

Connecting Representations

Liam plans to bake 25 cookies. He wants to keep 4 cookies for himself, and then split the rest evenly between his 3 friends.

2 Which equation and tape diagram match Liam's situation?

A. $4 + 3x = 25$

25

4	x	x	x

B. $4x + 3 = 25$

25

x	x	x	x	3

C. $3(x + 4) = 25$

25

$x + 4$	$x + 4$	$x + 4$

D. $4(x + 3) = 25$

25

$x + 3$	$x + 3$	$x + 3$	$x + 3$

3 How many cookies should each of Liam's friends receive?

7 cookies

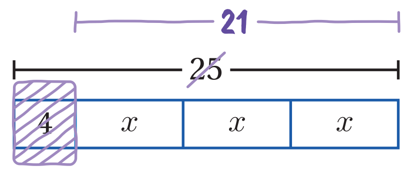
Activity 1

Name: ____________ Date: ____________ Period: ____________

Connecting Representations (continued)

4 Liam used this tape diagram and equation to represent his situation. Here is his first step. What equation would represent his new tape diagram? Explain why this equation is helpful.

$$4 + 3x = 25$$



Responses vary.

- $3x = 21$. This equation is helpful because it is getting closer to an equation like "$x = \ldots$" It also helps to see that x must be 7 because $3 \cdot 7 = 21$.
- $x + x + x = 21$. This equation is helpful because it shows that an unknown number added 3 times is 21, so the number must be 7.
- $4 + 3x - 4 = 25 - 4$. This equation helps keep track of the steps that Liam used to solve.

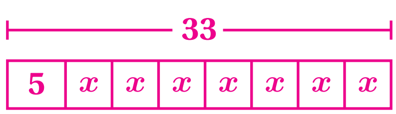
5 Liam ended up baking 33 cookies. He kept 5 for himself and split the rest evenly between 7 friends.

a Write an equation to represent Liam's new situation.

$5 + 7x = 33$ (or equivalent)

b Draw a tape diagram for Liam's new situation.

Diagrams vary.

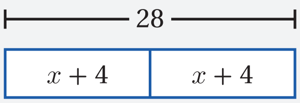


Name: .... Date: .... Period: ....

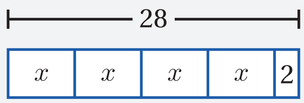
Missing Representations

6 Make groups that represent the same situation. Some groups will be missing a representation.

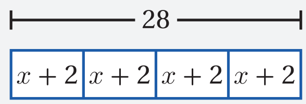
a.



b.



c.



d. A cake-baking kit says: 28 tablespoons of sugar is provided for 2 cakes. For each cake, save 4 tablespoons of sugar for frosting and put the rest in the batter.

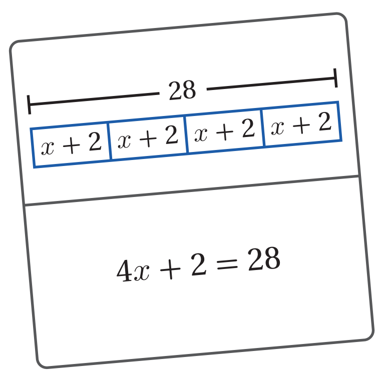
e. Riku's mom buys 4 cinnamon buns, one per family member. Each person also gets \$2 to spend on a beverage. The bill is \$28.

f. $2(x + 4) = 28$

g. $4x + 2 = 28$

Group 1	Group 2	Group 3
a	b	c
d	g	e
f		

7 Irene incorrectly matched these two representations. What could you say to convince Irene that these don't match?



Responses vary. **In $4x + 2 = 28$, the 2 is by itself, so that means that the 4 groups are only x. The 2 is its own section.**

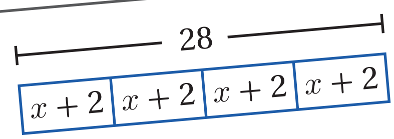
Name: ______ Date: ______ Period: ______

Missing Representations (continued)

8 One group did not have a matching equation. Write an equation that matches.

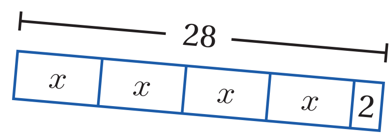
$4(x+2)=28$ (or equivalent)

Riku's mom buys 4 cinnamon buns, one per family member. Each person also gets \$2 to spend on a beverage. The bill is \$28 total.



9 One group did not have a matching situation. Write a situation that matches.

Responses vary. **I was playing around at home with a scale. I put 4 bricks on the scale, and then my 2-pound cat jumped on the scale. The scale read: 28 pounds.**



$$4x+2=28$$

Explore More

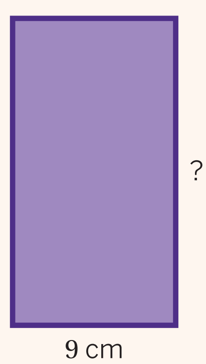
10 This rectangle has an unknown length, a width of 9 centimeters, and a perimeter of 52 centimeters.

a Write an equation or draw a tape diagram to represent this situation.

$2(x+9)=52$ (or equivalent)

b Determine the length of the rectangle.

17 centimeters



11 Synthesis

Here is a new situation. Explain how the number 9 is important in each representation. ***Responses vary.***

Story

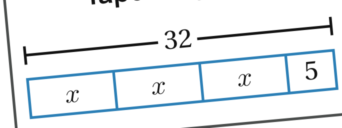
Jaylin buys 3 bags of bagels.

The store gives her 5 bagels for free, making it 32 bagels total.

Equation

$3x + 5 = 32$

Tape Diagram



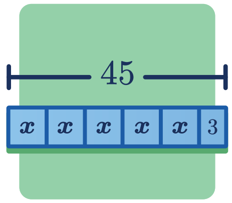
In the story . . . **there must be 9 bagels in each bag. If I have 3 bags of 9 and add 5 bagels, I get 32 bagels.**

In the equation . . . **9 makes sense as the value for x because $3(9) + 5 = 32$.**

In the tape diagram . . . **9 is the length of the x segments because $9 + 9 + 9 + 5 = 32$.**

Things to Remember:

Name: ________ Date: ________ Period: ________



Seeing Structure

Let's analyze and solve questions in context.

Warm-Up

Here are six equations.

Equation A $100 = 8(x + 9)$	**Equation B** $9(x + 7) = 100$	**Equation C** $100 = 8x + 72$
Equation D $9x + 63 = 100$	**Equation E** $100 = 72 + 8x$	**Equation F** $(x + 7) \cdot 9 = 100$

1. Select two equations that have something in common. How are the two equations alike?

Responses vary.

- **Equations A and B are alike because they are both equations that have an expression equal to 100, and the expression is some number times a quantity in parentheses.**
- **Equations C and E are alike because the equations use all of the same numbers, and they have the same solution: $x = 3.5$.**

2. Create two groups so that the equations in each group have something in common. Then explain what your groups have in common.

Group 1	Group 2
Equations: **A, B, F**	Equations: **C, D, E**
All the equations in this group . . . **have an expression that looks like $\_(x + \_)$.**	All the equations in this group . . . **have an expression that looks like $\_x + \_$.**

Activity 1

Name: .... Date: .... Period: ....

Which Diagram?

For each situation, first choose the diagram that best represents it. Then write an equation, determine the solution, and explain what the solution means in the situation.

Diagram A

45

x	x	x	x	x	3

Diagram B

45

$x+3$	$x+3$	$x+3$	$x+3$	$x+3$

3. A postal worker weighs 5 identical cardboard packages and a 3-pound plastic box. Everything weighs a total of 45 pounds.

Diagram	Equation	Solution	Meaning of Solution
A	$5x+3=45$	$x=8.4$	Each cardboard package weighs 8.4 pounds.

4. Tyani is making 5 gift bags. Each bag contains x pencils. Tyani adds 3 more pencils to each bag. Altogether, the gift bags contain 45 pencils.

Diagram	Equation	Solution	Meaning of Solution
B	$5(x+3)=45$	$x=6$	Each bag starts with 6 pencils.

5. A national park charges \$3 for each car that enters and also a fee for each person that enters. A family of 5 enters the park in 1 car and pays a total of \$45.

Diagram	Equation	Solution	Meaning of Solution
A	$5x+3=45$	$x=8.4$	The fee per person is \$8.40.

Activity 2

Name: .... Date: .... Period: ....

Questions and Answers

6. Natalia's family wants to inflate a total of 60 balloons for a party. Yesterday, they inflated 24 balloons. Today, they want to split the remaining balloons equally between 4 family members.

a Use this information to write a question that you don't already have the answer to.

***Questions vary.* How many balloons will each family member inflate?**

b Write an equation or draw a tape diagram to represent the situation and your question.

Responses vary. $60 = 24 + 4x$

c Solve the equation to answer your question.

$x = 9$. ***Responses vary.* Each family member will inflate 9 balloons.**

d Use the equation to check your solution.

$24 + 4(9) = 24 + 36 = 60$

7. An art class charges each student $\$15$ to attend, plus a fee for supplies. The instructor hopes to collect $\$240$ total from the 12 students who attend the class.

a Use this information to write a question that you don't already have the answer to.

***Questions vary.* How much should the instructor charge per person for supplies in order to make $\$240$?**

b Write an equation or draw a tape diagram to represent the situation and your question.

Responses vary. $240 = 12(x + 15)$

c Solve the equation to answer your question.

$x = 5$. ***Responses vary.* The instructor should charge $\$5$ per person for supplies.**

d Use the equation to check your solution.

$12(5 + 15) = 12(20) = 240$

Explore More

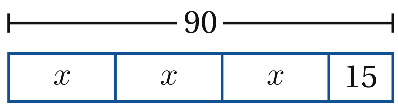
8. Write your own problem that can be solved with an equation or tape diagram. Then swap problems with a classmate and solve your classmate's problem.

Responses vary.

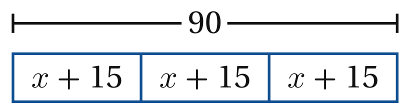
Synthesis

9. Here are two equations and their tape diagrams.

$3x + 15 = 90$



$3(x + 15) = 90$

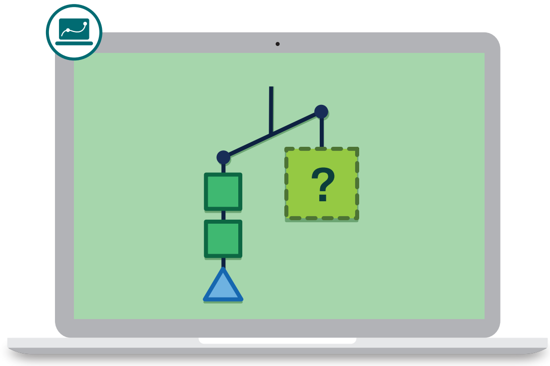


Describe how the tape diagrams are alike and different.

Responses vary. **Both diagrams have the same length: 90. The first tape diagram has one unknown value three times, plus 15 more. The second diagram has three identical pieces of tape, with each piece representing an unknown plus 15.**

Things to Remember:

Name: .... Date: .... Period: ....



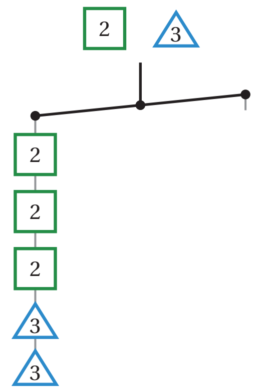
Balancing Moves

Let's see how hangers can represent balanced relationships.

Warm-Up

1 Balance the hanger by adding shapes to either side. Be sure to make the sides different.

***Responses vary.* The hanger is balanced when the total weight on each side of the hanger is the same.**

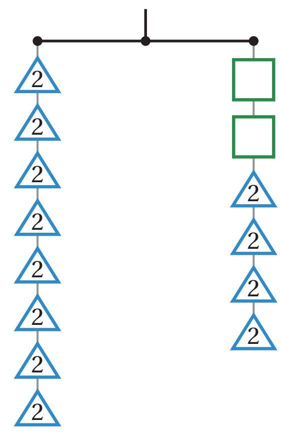


Activity 1

Name: .... Date: .... Period: ....

Unknown Weights

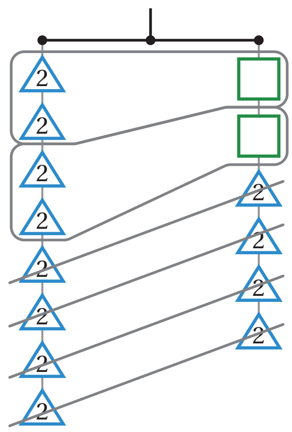
2 Determine the weight of the square so that the hanger stays balanced. Describe your strategy.



Weight of Triangle (lb)	Weight of Square (lb)
2	**4**

Responses vary. **I crossed off 4 triangles on each side. I noticed that 4 triangles with a total weight of 8 were the same weight as 2 squares. I divided 8 by 2 to determine the weight of one square.**

3 This diagram shows Adnan's strategy for determining the weight of a square in the previous problem. Describe this strategy.



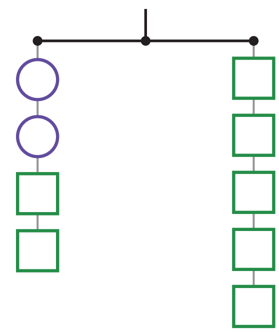
Responses vary. **Adnan first crossed out 4 triangles from each side, so only squares were left on the right. Then he divided each side into 2 equal groups because there were 2 squares.**

Name: ______ Date: ______ Period: ______

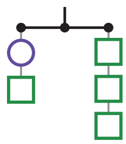
Unknown Weights (continued)

4 Hanger A is balanced. Select *all* the other hangers that must also be balanced.

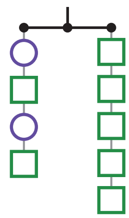
Hanger A



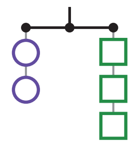
☐ **Hanger B**



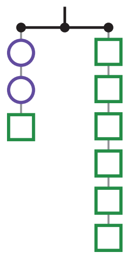
☑ **Hanger C**



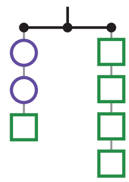
☑ **Hanger D**



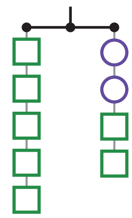
☐ **Hanger E**



☑ **Hanger F**

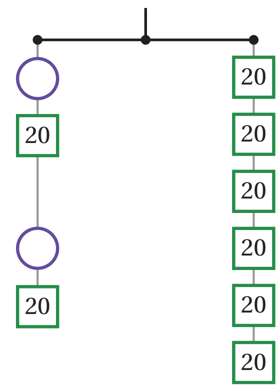


☑ **Hanger G**



5 **a** Determine the weight of a circle so the hanger stays balanced.

Weight of Square (lb)	Weight of Circle (lb)
20	**40**

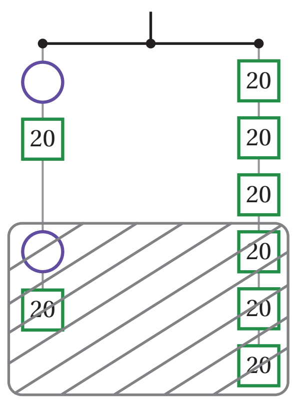


b Describe your strategy.

***Responses vary.* First, I crossed off 40 pounds from each side, leaving 2 circles balanced with 80 pounds. Then, I divided both sides by 2 and determined that one circle is balanced with 40 pounds.**

6 This is how Theo made a hanger with fewer objects. Will the new hanger be balanced? Explain your thinking.

Yes. *Explanations vary*. Theo split each side of the hanger into halves, so it will still be balanced. What remains on each side is equal to what was taken away.



Activity 2

Name: ________ Date: ________ Period: ________

Challenge Creator

7 Now it's your turn to create your own hanger diagram challenge. ***Responses vary.***

a **Make It!** On the Activity 2 Sheet, design your challenge.

b **Solve It!** On this page, copy the weight of the first shape from the challenge you designed. Then determine the weight of the second shape.

Weight of ________	**Weight of** ________

c **Swap It!** Swap your challenge with one or more partners. Write the weight of your partner's first shape. Then determine the weight of your partner's second shape.

Partner 1: ________

Weight of ________	**Weight of** ________

Partner 2: ________

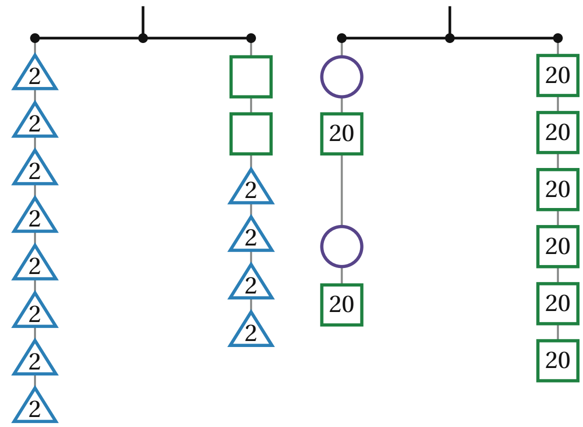
Weight of ________	**Weight of** ________

Partner 3: ________

Weight of ________	**Weight of** ________

8 Synthesis

Describe strategies for making a balanced hanger with fewer objects. Use the diagrams if they help with your thinking.



Responses vary.

- **Take away the same objects from both sides.**
- **Divide both sides into equal parts.**

Things to Remember:

Name: .... Date: .... Period: ....

Challenge Creator

Follow the steps below to create your own hanger diagram challenge.

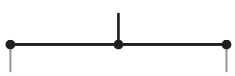
a Choose two shapes for your hanger.

Square Circle Triangle Pentagon

b Write the names of the shapes you chose below. Then write a weight for the *first shape only*.

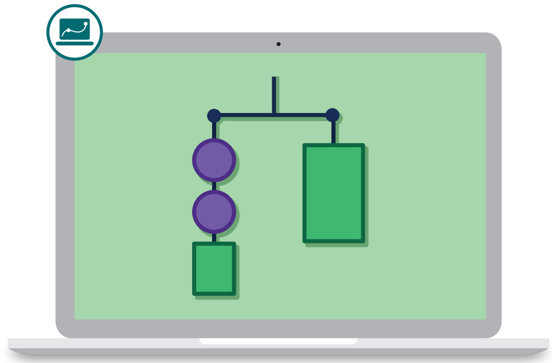
Weight of	**Weight of** ....

c Create a balanced hanger using your two chosen shapes.



d Do not determine the weight of the second shape on this page. You and your classmates will determine the weight in the Student Edition.

Name: .... Date: .... Period: ....



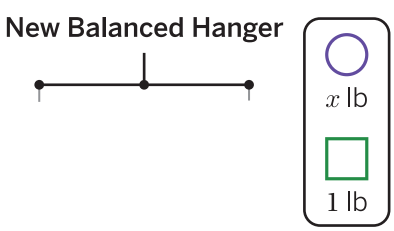
Balancing Equations

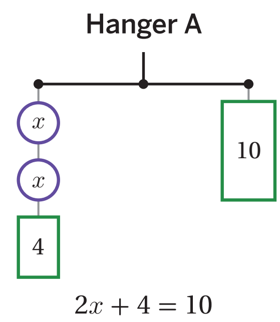
Let's use hanger diagrams to help us solve equations.

Warm-Up

1 Hanger A is balanced.

Create a new balanced hanger that has the same weight for x.





Responses vary.

- x and 2 on the left and 5 on the right
- $2x$ on the left and 6 on the right

Activity 1

Name: .... Date: .... Period: ....

Connecting Hangers to Equations

2 The equation $25 = 4x + 15$ represents Hanger A.

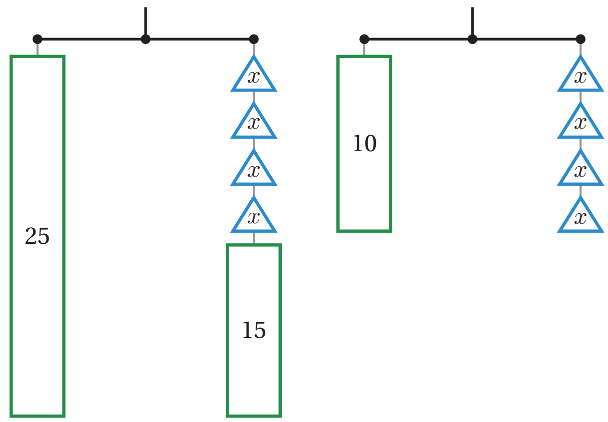
a Write an equation that represents Hanger B.

$10 = 4x$ or $4x = 10$

b What balanced move gets you from Hanger A to Hanger B?

***Responses vary.* To get from Hanger A to Hanger B, I could take away 15 from each side.**

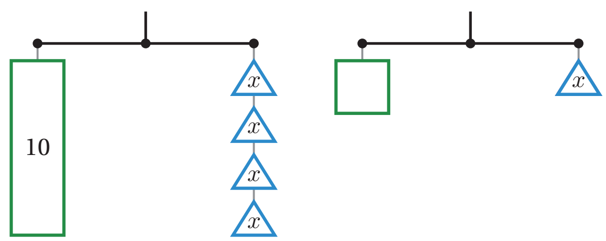
Hanger A **Hanger B**



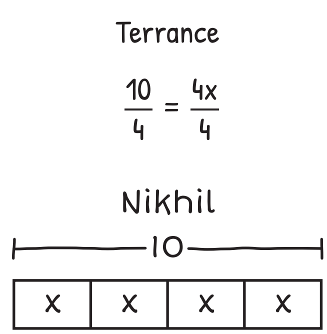
3 What is the weight of one triangle?

2.5 pounds

Hanger B **Hanger C**



4 Here are Terrance's and Nikhil's strategies for determining the weight of one triangle on Hanger B.



Discuss: How are the two strategies alike? How are they different?

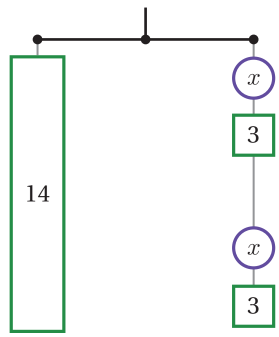
***Responses vary.* Terrance and Nikhil both have a total of 10 and 4 equal, unknown parts. Nikhil used a tape diagram to solve, and Terrance used an equation.**

Name: .... Date: .... Period: ....

Connecting Hangers to Equations (continued)

5 Here is a new hanger. What is the value of x?

$x = 4$



6 Anand and Darius used equations to determine the value of x in the previous problem.

Darius wrote the equation $14 = 2x + 6$.
Anand wrote the equation $14 = 2(x + 3)$.

Who is correct? Circle one.

Darius Anand Both Neither

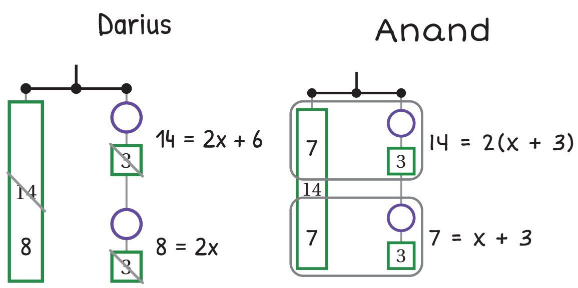
Explain your thinking.

***Explanations vary.* Anand noticed that there are two groups of $x + 3$, and Darius noticed that there are two x's and a total of 6 on the right side of the hanger. These both represent the same total objects in different ways.**

Note: Students who select "Darius," "Anand," or "Both" will be marked correct.

7 Here are Darius's and Anand's work. Select a question to answer.

- ☐ Why did Darius write $8 = 2x$?
- ☐ Why did Anand write $7 = x + 3$?



Responses vary.

- **Darius wrote $8 = 2x$ because he subtracted 6 from each side of the hanger.**
- **Anand wrote $7 = x + 3$ because he divided both sides of the hanger by 2.**

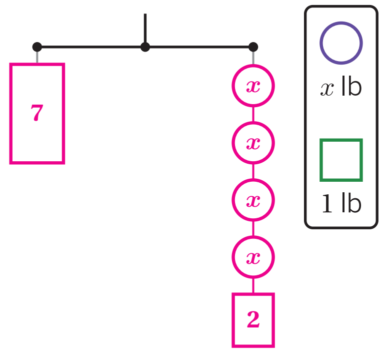
Activity 2

Name: .... Date: .... Period: ....

Solving Equations

8 **a** Create a hanger to represent $7 = 4x + 2$.

***Responses vary.* Sample diagram shown.**



b What value of x makes the equation true?

$x = \frac{5}{4}$ **(or equivalent)**

9 **a** Determine the value of x that makes the equation $4(x + 2) = 40.4$ true.

$x = 8.1$

b Describe the steps you used to determine the value of x.

***Responses vary.* Divide both sides by 4 to get $x + 2 = 10.1$. Subtract 2 from both sides to get $x = 8.1$.**

10 What value of x makes each equation true? Solve as many challenges as you have time for.

a $3x + 1 = 7$

$x = 2$

b $2(x + 5) = 16$

$x = 3$

c $2x + 2.2 = 6.8$

$x = 2.3$

d $4(x + 1.1) = 20.8$

$x = 4.1$

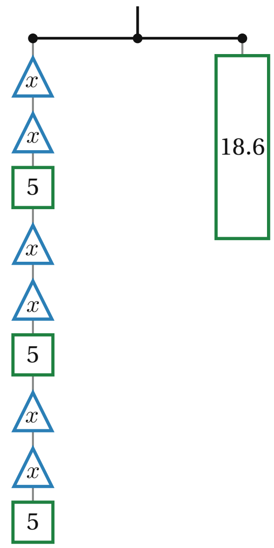
e $4x + \frac{3}{2} = \frac{17}{2}$

$x = \frac{7}{4}$

11 Synthesis

Describe how solving an equation is like solving for the weight of an object on a balanced hanger. Use the diagram if it helps with your thinking.

Responses vary. For an equation and a hanger, both sides need to stay balanced. I can subtract weight from each side or divide each side by the same number to create a balanced hanger or simpler equation.



$3(2x + 5) = 18.6$

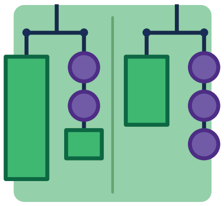
Things to Remember:

Unit 6
Lesson
7

Name: .... Date: .... Period: ....

Keeping It True

Let's solve equations with positive and negative numbers.



Warm-Up

Solve each equation mentally. Try to think of more than one strategy.

1. $x + 4 = 6$

$x = 2$

2. $x + 6 = 4$

$x = -2$

3. $-2x = 4$

$x = -2$

4. $-2x + 6 = 4$

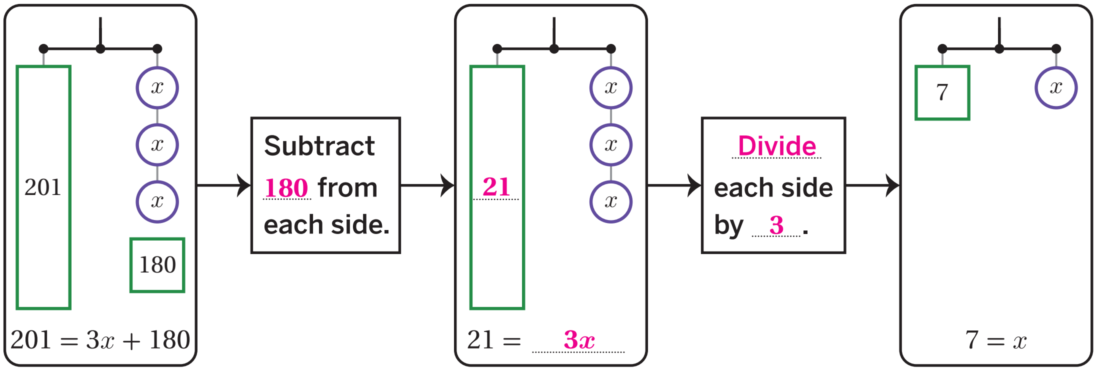
$x = 1$

Name: .... Date: .... Period: ....

Keep It True

Solve each equation by completing the blanks in the hangers, equations, and descriptions.

5.



6. **Equation** **Steps**

$5 = 2x + 8$

-3 $= 2x$

-1.5 $= x$

Step 1: Subtract 8 from each side.

Step 2: Divide each side by 2.

7. **Equation** **Steps**

$2(x - 5) = \text{-}6$

$x - 5 = \text{-}3$

x = 2

Step 1: Divide each side by 2.

Step 2: Add 5 to each side.

8. How can you check if the *solutions to the equations* in Problems 5–7 are correct?

***Responses vary.* I can check if the solutions are correct by replacing x with my solution and evaluating the expressions on both sides to make sure they are equivalent.**

Name: ________ Date: ________ Period: ________

Solve 'em

Here are two groups of equations.

Group A	Group B
$x - (-4) = -6$	$2(x - 1) = -200$
$50x + 200 = 1700$	$900 = -100(x - 3)$
$8.6 = 3x - 3.4$	$3(x + 4.5) = 36$

9. **Discuss:** How are the equations in each group alike or different?
Responses vary.
- Group A has equations that can be solved by adding or subtracting as the first step.
- Group B has equations that can be solved by dividing as the first step.
- Both groups have equations that contain positive numbers, negative numbers, and decimals.

10. Which group do the equations $-3x + \left(-\frac{1}{6}\right) = \frac{5}{6}$ and $-\frac{1}{2}(2x - 6) = -2$ belong to? Explain your thinking.
Responses vary. The equation $-3x + \left(-\frac{1}{6}\right) = \frac{5}{6}$ belongs in Group A because it could be solved by adding first. The equation $-\frac{1}{2}(2x - 6) = -2$ belongs in Group B because it could be solved by dividing first.

11. Choose two equations from *each* group to solve. *Responses vary.*

	Group A	Group B
Equation 1	$x - (-4) = -6$, $x = -10$ $50x + 200 = 1700$, $x = 30$	$2(x - 1) = -200$, $x = -99$ $900 = -100(x - 3)$, $x = -6$
Equation 2	$8.6 = 3x - 3.4$, $x = 4$ $-3x + \left(-\frac{1}{6}\right) = \frac{5}{6}$, $x = -\frac{1}{3}$	$3(x + 4.5) = 36$, $x = 7.5$ $-\frac{1}{2}(2x - 6) = -2$, $x = 5$

Synthesis

12. a Write an equation that would belong in Group B.

Responses vary. $-9(0.2x - 3) = \frac{7}{9}$

b What advice would you give to help someone solve an equation like yours?

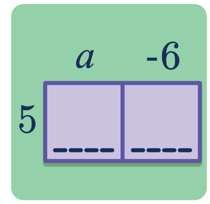
Responses vary. To solve an equation like this, you can divide both sides of the equation first.

Group B
$2(x - 1) = -200$
$900 = -100(x - 3)$
$3(x + 4.5) = 36$

Things to Remember:

Name: ______________ Date: ________ Period: ________

Factoring and Expanding

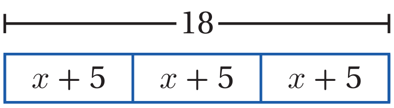


Let's think about efficient ways to solve equations with parentheses.

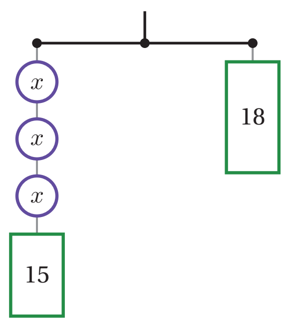
Warm-Up

1. Which one doesn't belong? Explain your thinking.

A.



B.



C. $3(x+5)=18$

D. $3x+5=6$

Responses and explanations vary.

- The tape diagram does not belong because it's the only one with three 5s.
- The hanger diagram doesn't belong because it's the only one with circles.
- The equation $3(x+5)=18$ doesn't belong because it's the only one with parentheses.
- The equation $3x+5=6$ doesn't belong because it represents a different relationship than the other three.

Name: .... Date: .... Period: ....

Factoring Puzzles

Complete the missing information in each puzzle.

2.

	a	-6
5	$\boxed{5a}$	$\boxed{-30}$

Factored	Expanded
$5(a - 6)$	$5a - 30$

3.

	$\boxed{3a}$	$\boxed{-1b}$
2	$6a$	$-2b$

Factored	Expanded
$2(3a - 1b)$	$6a - 2b$

4.

	$5x$	-3
-5	$-25x$	15

Factored	Expanded
$-5(5x - 3)$	$-25x + 15$

5.

	$2c$	3
-1	$-2c$	-3

Factored	Expanded
$-(2c + 3)$	$-2c - 3$

Activity 2

Name: ________ Date: ________ Period: ________

Step by Step by Step by Step

6. Here are Amir's and Sadia's first steps for solving $2(x - 9) = 10$.

Amir	Sadia
$2(x - 9) = 10$	$2(x - 9) = 10$
$2x - 18 = 10$	$x - 9 = 5$

a Are each of their first steps correct? Explain your thinking.

Yes, each of their first steps are correct. *Explanations vary.* Amir expanded the left side of the equation first and Sadia divided each side first, but they both kept the equation balanced.

b Finish solving each equation. Show your thinking.

Amir

$x = 14$

Work varies.

$$2x - 18 = 10$$
$$2x - 18 + 18 = 10 + 18$$
$$2x = 28$$
$$2x \div 2 = 28 \div 2$$
$$x = 14$$

Sadia

$x = 14$

Work varies.

$$x - 9 = 5$$
$$x - 9 + 9 = 5 + 9$$
$$x = 14$$

Activity 3

Name: Date: Period:

Different First Steps

Solve these equations for x using both Amir's and Sadia's methods. Check the box when your solutions match.

7. $3(x+2)=21$

Expand first:

$3x+6=21$

$3x=15$

$x=5$

Divide first:

$x+2=7$

$x=5$

☑

8. $200(x-0.3)=600$

Expand first:

$200x-60=600$

$200x=660$

$x=3.3$

Divide first:

$x-0.3=3$

$x=3.3$

☑

9. $-10\left(x-\frac{7}{10}\right)=-3$

Expand first:

$-10x+7=-3$

$-10x=-10$

$x=1$

Divide first:

$x-\frac{7}{10}=0.3$

$x=1$

☑

Synthesis

10. **a** What are two possible first steps you could use when solving an equation like $6(x+4)=30$?

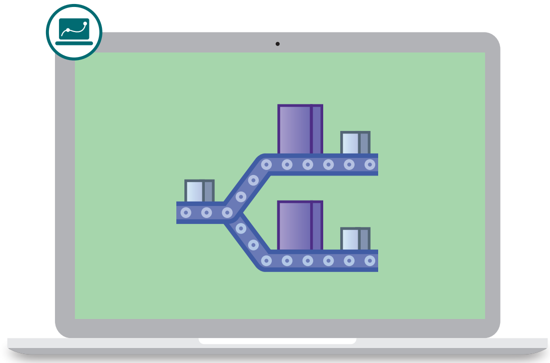
***Responses vary.* I could divide both sides by 6 first, or I could expand $6(x+4)$ and replace the left side with $6x+24$.**

b What are some advantages to having different ways to solve an equation?

***Responses vary.* Having different ways to solve an equation can be helpful because I can decide which way requires fewer steps.**

Things to Remember:

Name: ________ Date: ________ Period: ________



Always-Equal Machines

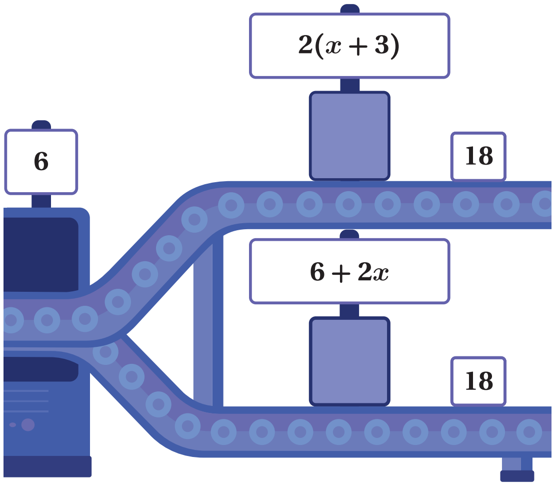
Let's explore equivalent expressions using always-equal machines.

Warm-Up

1 Here are two number machines. Let's watch what happens when an input goes into the machines. Record the results in the table.

Responses depend on the values on the digital screen.

x	$2(x+3)$	$6+2x$
6	18	18
14	34	34
2	10	10
8	22	22

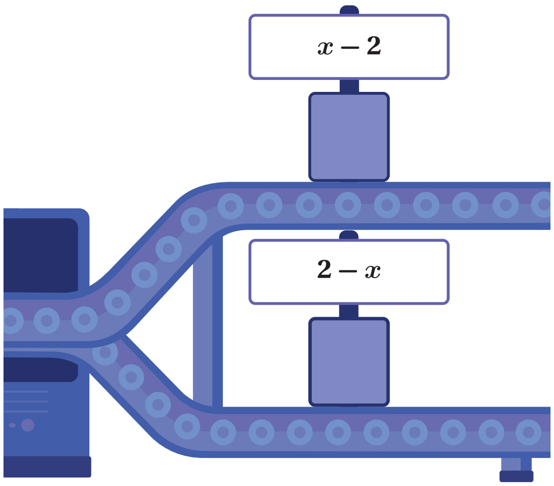


2 Here are two more number machines. When will these machines have equal outputs?

Always Sometimes Never

Explain your thinking.

Explanations vary. These machines will only have equal outputs when the input is 2.



Activity 1

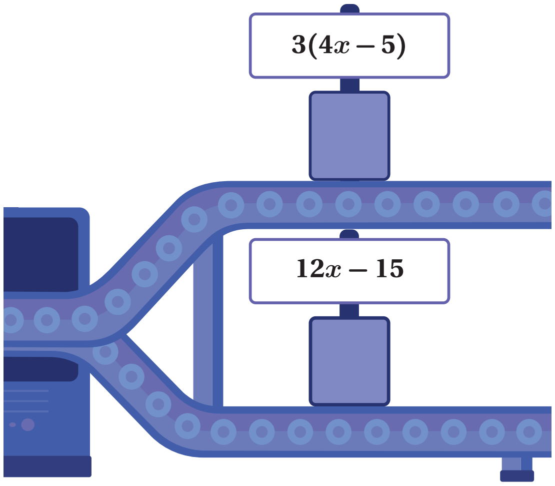
Name: ________ Date: ________ Period: ________

Equivalent Expressions

3 Expressions that give the same output for every input are called **equivalent expressions**.

Are these equivalent expressions? Explain how you know.

Yes. *Explanations vary.* If I expand $3(4x - 5)$, I get $12x - 15$. These expressions are equivalent, so they will give equal outputs for any input.



4 **a** Which expression is equivalent to $2(10 - x)$?

A. $20 - x$

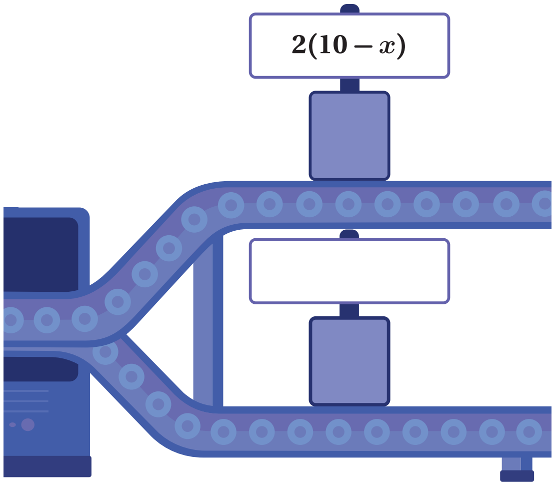
B. $20 - 2x$

C. $2(x - 10)$

D. $2x - 20$

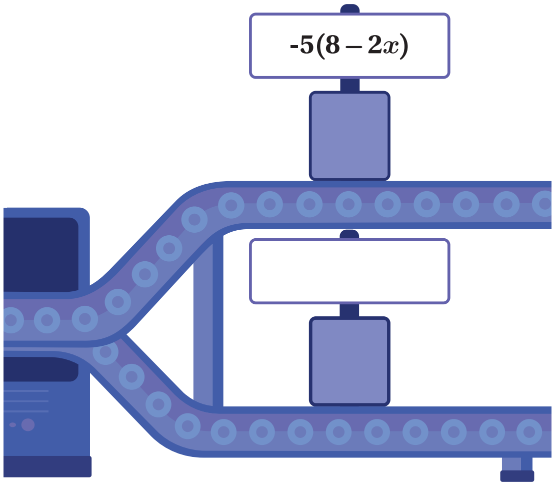
b Choose one input to test your expression.

Responses vary.



5 Write an equivalent expression to $-5(8 - 2x)$.

***Responses vary.* $-40 + 10x$**



Name: ______ Date: ______ Period: ______

Equivalent Expressions (continued)

6 Three students made mistakes writing an equivalent expression to $-5(8 - 2x)$.

Circle your favorite mistake.

Responses vary.

$\boxed{-5(8 - 2x)}$

Zion	Juliana	Nickolas
$-5(2x - 8)$	$-40 - 10x$	$-40 + 2x$

Explain what you think is incorrect about the student's expression.

Explanations vary.

- **Zion's expression is incorrect because you can't just switch the order of the terms with subtraction. $2x - 8$ is not equivalent to $8 - 2x$.**
- **Juliana's expression is incorrect because she multiplied $-2x$ by 5, not -5.**
- **Nickolas's expression isn't equivalent because he didn't multiply -5 by $-2x$. He only multiplied -5 by 8.**

7 **a** Select *all* of the expressions equivalent to $-15 + 6x$.

- [x] **A.** $-3(5 - 2x)$
- [x] **B.** $3(2x - 5)$
- [ ] **C.** $3(5 - 2x)$
- [x] **D.** $6x + (-15)$
- [ ] **E.** $15 + (-6x)$

b Choose one of the equivalent expressions. Explain how you know it is equivalent to $-15 + 6x$.

I know ______ is equivalent because . . .

***Explanations vary.* I know $3(2x - 5)$ is equivalent because $3 \cdot 2x = 6x$ and $3 \cdot (-5) = -15$, so the $6x$ is positive and the 15 is negative.**

Name: ______ Date: ______ Period: ______

More Than One Way

8 Group the equivalent expressions. One expression will have no group.

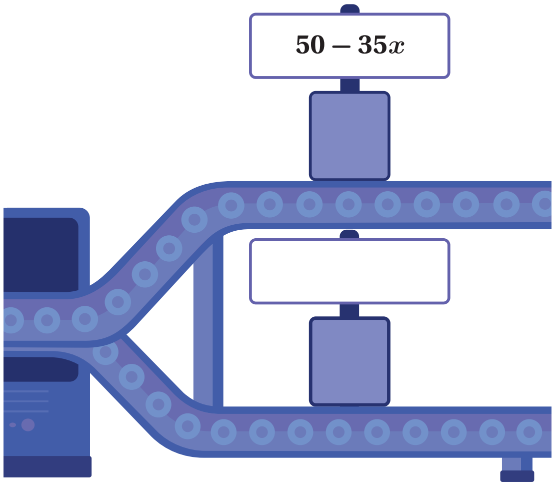
$8(x - 3)$	$-4(-6 + 2x)$	$-8(x - 24)$	$8x - 24$
$-24 + 8x$	$24 - 8x$	$\frac{1}{2}(16x - 48)$	

- **Group 1: $-4(-6 + 2x)$ and $24 - 8x$**
- **Group 2: $8(x - 3)$, $-24 + 8x$, $8x - 24$, and $\frac{1}{2}(16x - 48)$**
- **Expression with no group: $-8(x - 24)$**

9 Write an equivalent expression to $50 - 35x$.

Try to write more than one.

Responses vary.
- $-35x + 50$
- $5(10 - 7x)$
- $10(5 - 3.5x)$



Name: ________ Date: ________ Period: ________

More Than One Way (continued)

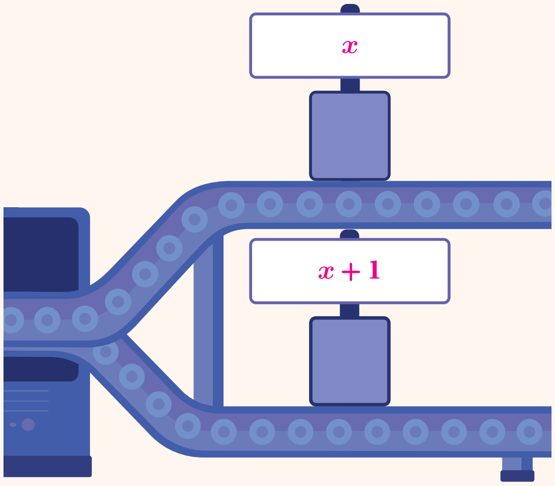
10 Write at least three different equivalent expressions to $64x - 16$.

Responses vary.
- $-16 + 64x$
- $-1(-64x + 16)$
- $16(4x - 1)$

Explore More

11 Make a never-equal machine that will never give the same output for any input. Write your expressions on the two machines.

Responses vary. Sample shown in the number machine.



Explain how you know these are never equal.

Explanations vary. These are never equal because one machine will always output a value that is one greater than the other machine.

12 Synthesis

How can you determine whether two expressions are equivalent to each other? Use these examples if they help with your thinking.

Responses vary. Two expressions are equivalent to each other if one can be rearranged, factored, or expanded to be the same as the other.

$24 - 8x$

$-4(-6 + 2x)$

$-8(x - 24)$

$\frac{1}{2}(16x - 48)$

$8(x - 3)$

$-24 + 8x$

$8x - 24$

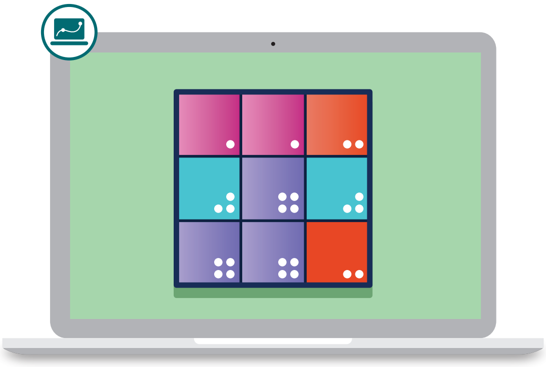
Things to Remember:

Unit 6
Lesson
10

Name: ______ Date: ______ Period: ______

Collect the Squares

Let's write equivalent expressions with fewer terms.



Warm-Up

Solve each equation mentally. Try to think of more than one strategy.

1. $6x = 12$
   $x = 2$

2. $3x + 3x = 12$
   $x = 2$

3. $4x + 2x = 12$
   $x = 2$

4. $6x + 3 = 27$
   $x = 4$

5. $2x + 4x + 3 = 27$
   $x = 4$

Name: .... Date: .... Period: ....

Collect the Squares

6 Collect all the squares by choosing two or more expressions to combine. Then write an equivalent expression using the fewest number of *terms*. There is an example in the first row.

Responses vary depending on the expressions students choose.

$4x + 1$	$9x$
$-2x$	$-10x$

Original Expression	Equivalent Expression
$4x + 1 + (-10x)$ $= 4x - 10x + 1$ $= -6x + 1$	$-6x + 1$
$\mathbf{-2x + (-10x)}$	$\mathbf{-12x}$
$\mathbf{4x + 1 + 9x}$	$\mathbf{13x + 1}$

Name: ________ Date: ________ Period: ________

Collect the Squares (continued)

7 Collect all the squares by choosing two or more expressions to combine. Then write an equivalent expression using the fewest number of terms.

$-2x - 2$	$8x + 8$
-3	$3(x + 1)$

Responses vary depending on the expressions students choose.

Original Expression	Equivalent Expression
$-2x - 2 + (-3)$	$-2x - 5$
$8x + 8 + 3(x + 1)$	$11x + 11$

8 Leslie combined these expressions. Some of his work is correct and some is incorrect.

$-2x - 2$	$8x + 8$
-3	$3(x + 1)$

Original Expression	Equivalent Expression
$-2x - 2 + 8x + 8$ $= -2x + 8x - 2 + 8$ $= 6x + 6$ $= 12x$	$12x$

Explain what you think is incorrect about Leslie's work.

Responses vary. **In the last step, Leslie added $6x$ and 6 together, but he can't because he doesn't know what x is.**

Activity 2

Name: ________ Date: ________ Period: ________

Collect More Squares

9 Collect all the squares by choosing two or more expressions to combine. Then write an equivalent expression using the fewest number of terms.

Responses vary depending on the expressions students choose.

Original Expression	Equivalent Expression
$3x + 6 + x - 2$	$4x + 4$
$-4x - 4 + 6 + x + 1$	$-3x + 3$
$8(x - 3) + (-8x)$	-24
$-2(x + 2) + 7x$	$5x - 4$

$3x + 6$	$x - 2$	$8(x - 3)$
$-4x - 4$	$x + 1$	$-2(x + 2)$
6	$7x$	$-8x$

Activity 2

Name: ______ Date: ______ Period: ______

Collect More Squares (continued)

10 Collect all the squares by choosing two or more expressions to combine. Then write an equivalent expression using the fewest number of terms.

Responses vary depending on the expressions students choose.

Original Expression	Equivalent Expression
$3x + 12 + (-1(x - 9))$	$2x + 21$
$6(x + 1) + 8 - x + (-x - 12)$	$4x + 2$
$7(2x + 1) + (-4(x - 5))$	$10x + 27$
$\frac{1}{4}x - 20 + (-2x)$	$-\frac{7}{4}x - 20$

$-4(x - 5)$	$6(x + 1)$	$\frac{1}{4}x - 20$
$3x + 12$	$-1(x - 9)$	$-x - 12$
$8 - x$	$-2x$	$7(2x + 1)$

Explore More

11 Use these squares to create expressions that are equivalent to $5x - 8$. Create as many different expressions as you can.

Responses vary.

- $-3(x + 2) + 2(4x - 1)$
- $-(3x + 2) + 8x - 6$

$-(3x + 2)$	$-3(x + 2)$	$8x - 6$
$-2x - 6$	$2(4x - 1)$	$-3x + 2$
$8x - 10$	$3x - 6$	$2x + 2 + x$

12 Synthesis

Describe how to write an equivalent expression using the fewest number of terms. Use this expression if it helps with your thinking.

$5x - 2(6x - 4)$

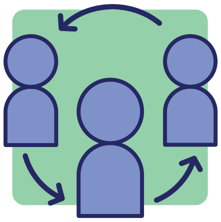
Responses vary. **First, I look for places where I can expand the expression. In the example, -2 times $6x - 4$ is -$12x + 8$. Then, I add together any terms that are alike. In this case, $5x$ and -$12x$ are alike, so $5x +$ (-$12x$) = -$7x$. The final expression is -$7x + 8$.**

Things to Remember:

Name: .... Date: .... Period: ....

Pass the Equation

Let's practice solving equations multiple ways.



Warm-Up

1. Explain some possible first steps you could take to solve the equation.

$2(4x - 3) = 30$

Responses vary.

- **Divide both sides by 2.**
- **Distribute the 2 by multiplying it by $4x$ and -3.**

Name: ______ Date: ______ Period: ______

Equation Roundtable, Round 1

2. Follow the instructions on the Presentation Screen to solve these equations with your group.

$2x - 18 = 10$	$2(3 - x) = -8$
$5(x - 1) = 45$	$-6 = \frac{1}{2}(x - 8)$

Responses vary.

Equation 1	Equation 2
Equation: $2x - 18 = 10$ $2x = 28$ $x = 14$	Equation: $2(3 - x) = -8$ $6 - 2x = -8$ $-2x = -14$ $x = 7$
Check: $2(14) - 18 = 10$ $28 - 18 = 10$ $10 = 10$	Check: $2(3 - 7) = -8$ $2(-4) = -8$ $-8 = -8$

Equation 3	Equation 4
Equation: $5(x - 1) = 45$ $5x - 5 = 45$ $5x = 50$ $x = 10$	Equation: $-6 = \frac{1}{2}(x - 8)$ $-6 = \frac{1}{2}x - 4$ $-2 = \frac{1}{2}x$ $-4 = x$
Check: $5(10 - 1) = 45$ $5(9) = 45$ $45 = 45$	Check: $-6 = \frac{1}{2}(-4 - 8)$ $-6 = \frac{1}{2}(-12)$ $-6 = -6$

3. What do you think is important to remember when solving these types of equations?

Responses vary.

- **When solving equations, you have to keep each side of the equation balanced.**
- **When solving equations, you can expand first if there are parentheses.**

Activity 2

Name: ______ Date: ______ Period: ______

Equation Roundtable, Round 2

4. Here are four new equations. Solve them using the same instructions as Activity 1.

$8x - 6x - 18 = 10$

$55 = 5(x - 1) + 10$

$-10.5 = 6\left(x + \frac{1}{4}\right)$

$-2.8(x - 3) = 9\frac{4}{5}$

Responses vary.

Equation 1

Equation: $8x - 6x - 18 = 10$

$2x - 18 = 10$

$2x = 28$

$x = 14$

Check: $8(14) - 6(14) - 18 = 10$

$112 - 84 - 18 = 10$

$10 = 10$

Equation 2

Equation: $55 = 5(x - 1) + 10$

$45 = 5(x - 1)$

$45 = 5x - 5$

$50 = 5x$

$10 = x$

Check: $55 = 5(10 - 1) + 10$

$55 = 5(9) + 10$

$55 = 45 + 10$

$55 = 55$

Equation 3

Equation: $-10.5 = 6\left(x + \frac{1}{4}\right)$

$-\frac{21}{2} = 6x + \frac{6}{4}$

$-\frac{42}{4} = \frac{24}{4}x + \frac{6}{4}$

$-42 = 24x + 6$

$-48 = 24x$

$-2 = x$

Check: $-10.5 = 6\left(-2 + \frac{1}{4}\right)$

$-\frac{21}{2} = 6\left(-\frac{8}{4} + \frac{1}{4}\right)$

$-\frac{21}{2} = 6\left(-\frac{7}{4}\right)$

$-\frac{42}{4} = -\frac{42}{4}$

Equation 4

Equation: $-2.8(x - 3) = 9\frac{4}{5}$

$-2.8x + 8.4 = 9\frac{4}{5}$

$-2.8x + 8.4 = 9.8$

$-2.8x = 1.4$

$x = -0.5$

(or equivalent)

Check: $-2.8(-0.5 - 3) = 9\frac{4}{5}$

$-2.8(-3.5) = 9.8$

$9.8 = 9.8$

Synthesis

5. There are different ways to solve the equation $2(-3 + 8x) = -10$.

a List two different first steps you could take to solve this equation.

Responses vary.

- Expand $2(-3 + 8x)$ to get $-6 + 16x$.
- Divide both sides by 2.

b Which first step do you prefer? Explain your thinking.

Responses vary.

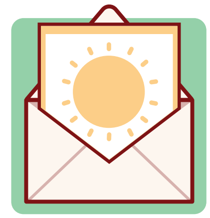
- I prefer to expand first so I don't have parentheses anymore.
- I prefer to divide both sides by 2 to simplify the equation.

Things to Remember:

Name: ______ Date: ______ Period: ______

Community Day

Let’s represent and answer questions about situations in context.

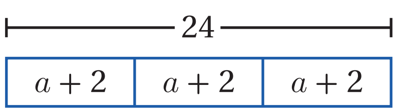


Warm-Up

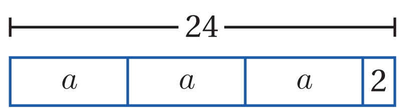
A baker put an equal number of cookies into 3 boxes. Then they put 2 more cookies in each box. They used 24 cookies total.

1. Which tape diagram best represents this situation?

A.



B.



2. Write an equation that represents the tape diagram you chose.

$3(a+2)=24$ (or equivalent)

Activity
1

Name: ______ Date: ______ Period: ______

Three Reads

3. Here is a situation. Let's make sense of it together as a class.

Kyrie's class is making _____ invitations to their school's Community Day.

They have already made _____ invitations and want to finish the rest of them within 7 days.

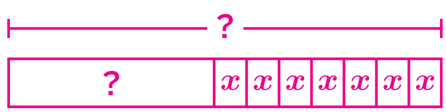
The class plans to spread out the remaining work so that they make the same number of invitations each day.

a **Discussion:** What is this situation about?

***Responses vary.* Kyrie's class is making invitations for their school's Community Day, and they are trying to plan how many they will need to make each day.**

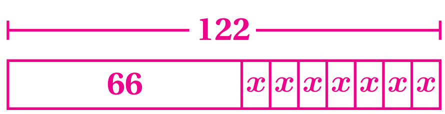
b Create a tape diagram or sketch that represents this situation.

Responses vary.



c Let's look at the missing information. Adjust your diagram to represent that Kyrie's class is making 122 invitations and has already made 66 invitations.

Responses vary.



d Determine how many invitations Kyrie's class should make each day.

The class should make 8 invitations each day.

Activity 2

Name: ________ Date: ________ Period: ________

Similar Problems

4. You will use the Activity 2 Sheet to explore a set of situations. Choose Set 1, Set 2, or Set 3.

Responses vary.

Set ________

5. Create a poster. Here is what your poster should include *for each situation* in your set:

- ☐ The situation in words
- ☐ A visual representation of the situation (tape diagram, hanger, etc.)
- ☐ An equation that represents the situation, along with your work for solving it
- ☐ A check for the solution to the equation
- ☐ The answer (with units) to the question in the situation
- ☐ Connections between the visual representation, equation, and situation

Representations vary for each situation. Samples shown in the table.

	Situation A	Situation B
Set 1	$6(x - 4.5) = 40.5$ $x = 11.25$ Each ticket costs \$11.25 without the coupon.	$40.50 = 6x - 4.50$ or $6x = 40.50 + 4.50$ $x = 7.5$ Each ticket costs \$7.50 without the coupon.
Set 2	$13x + 8x = 15$ $x = \frac{5}{7}$ Each bean bag weighs $\frac{5}{7}$ of a pound.	$8 + 13x = 15$ $x = \frac{7}{13}$ Each small bean bag weighs $\frac{7}{13}$ of a pound.
Set 3	$25(x - 15) = 320$ $x = 27.8$ They have to sell each T-shirt for \$27.80 to make a profit of \$320 on 25 shirts.	$15x - 25 = 320$ $x = 23$ They have to sell each T-shirt for \$23 to make a profit of \$320 on 15 shirts.

Synthesis

6. What do you think is important to remember when solving problems using visual representations and equations?

   Responses vary.

   - **Check to make sure that the tape diagram and equation give the same answer after solving.**
   - **It's helpful to remember that sometimes there are single costs (like supplies) and sometimes the cost applies every time (like the cost to make a shirt). These look different when writing the equation.**

Things to Remember:

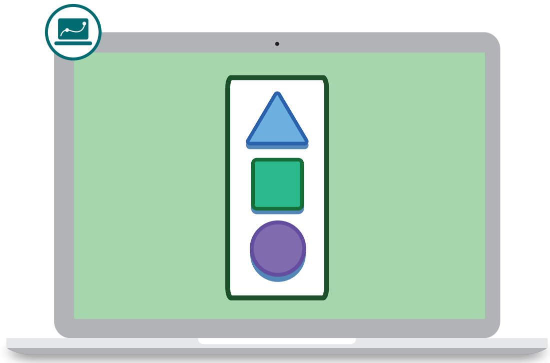
Name: ______ Date: ______ Period: ______

Similar Problems

Here are three sets of related situations.

	Situation A	Situation B
Set 1	6 members of the Martinez family are going to their school's Community Day. They have a coupon for $4.50 off each ticket. If they pay $40.50 for all their tickets, how much does one ticket cost without the coupon?	6 members of the Benton family are going to their school's Community Day. They have a coupon for $4.50 off their total. If they pay $40.50 for all their tickets, how much does one ticket cost without the coupon?
Set 2	Kwabena and Trevon are working together tossing bean bags to one side of a scale in order to balance a giant 15-pound stuffed animal. They're successful after Kwabena tosses 13 bean bags and Trevon tosses 8 bean bags onto the scale. How much does each bean bag weigh?	Adah and Samnang are working together tossing bean bags to one side of a scale in order to balance a giant 15-pound stuffed animal. They're successful after Adah tosses 13 small bean bags and Samnang tosses one giant 8-pound bean bag onto the scale. How much does each small bean bag weigh?
Set 3	Marquis and Yolanda plan to sell T-shirts at their school's Community Day. They make 25 shirts and each costs $15 to make. If they would like to make $320 in profit, how much should they sell each T-shirt for?	Moon and Cameron plan to sell T-shirts at their school's Community Day. They spend $25 on supplies and make 15 shirts. If they would like to make $320 in profit, how much should they sell each T-shirt for?

Name: .... Date: .... Period: ....



Balanced Moves

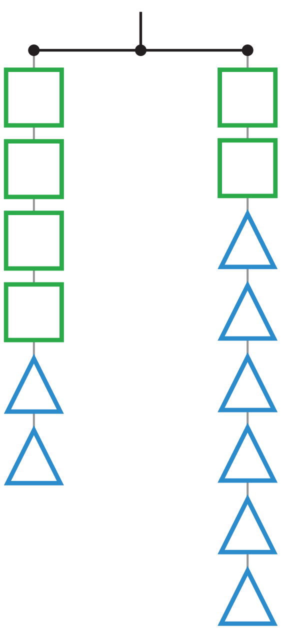
Let's rewrite equations while keeping the same solutions.

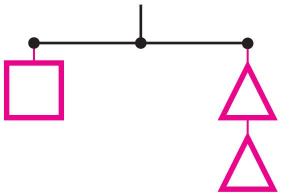
Warm-Up

1 Hanger A is balanced. Draw a different balanced hanger using the same shapes.

Responses vary.

Hanger A



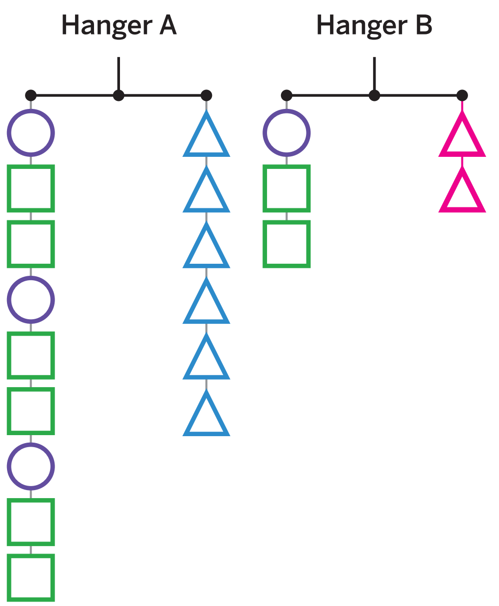


Name: .... Date: .... Period: ....

Multiplication and Division

2 We can balance hangers by adding or subtracting shapes from each side, or by multiplying or dividing.

If Hanger A is balanced, build the right side of Hanger B so that it also balances.



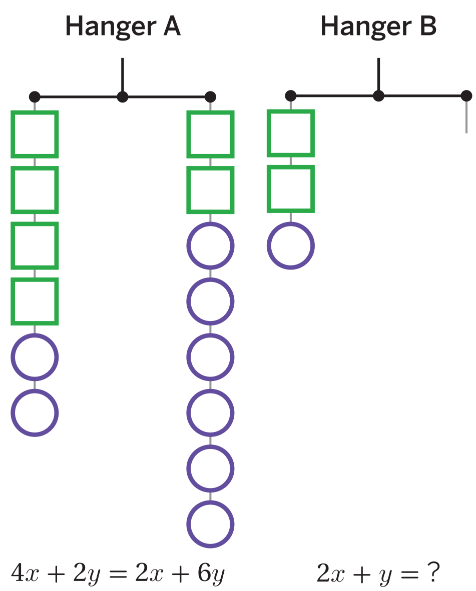
3 In this diagram:

- x represents the weight of each square.
- y represents the weight of each circle.

Ethan divided each side of Hanger A by 2 to make Hanger B.

Write an *expression* to represent the right side of Hanger B.

$x + 3y$



Name: ________ Date: ________ Period: ________

Multiplication and Division (continued)

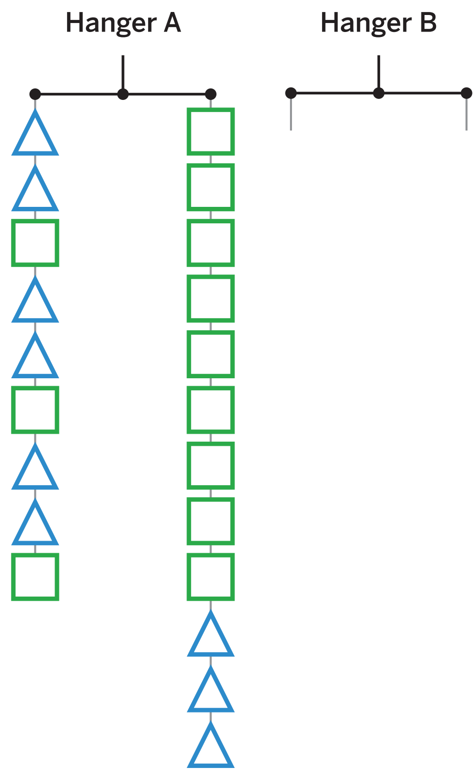
4 In this diagram:

- x represents the weight of each triangle.
- y represents the weight of each square.

Jamir changed the number of shapes on each side of Hanger A to make Hanger B.

Write an *equivalent equation* that could represent a balanced Hanger B.

***Responses vary.* $x = 2y$ or other equation equivalent to $3(2x + y) = 9y + 3x$**



$3(2x + y) = 9y + 3x$? = ?

5 In this diagram:

- x represents the weight of each triangle.
- 2 pounds is the weight of each circle.

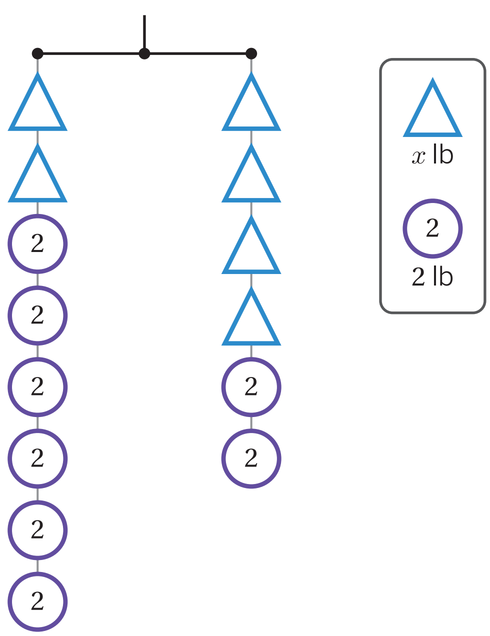
Select an equation that could represent a balanced hanger.

A. $x + 6 = 2x + 2$

B. $12 = 2x + 4$

C. $8 = 2x$

D. $2x + 8 = 4x$



Show or explain your thinking.

Responses and explanations vary.
- **$x + 6 = 2x + 2$: Divide each side of the hanger by 2.**
- **$12 = 2x + 4$: Subtract two triangles ($2x$) from each side of the hanger.**
- **$8 = 2x$: Subtract two triangles ($2x$) and two circles (4) from each side of the hanger.**
- **$2x + 8 = 4x$: Subtract two circles (4) from each side of the hanger.**

Activity 2

Name: .... Date: .... Period: ....

Solving Equations

6 Here's the first step Dalia took to solve the equation $6x + 12 = 10x - 4$.

What did she do to both sides to create a simpler equivalent equation?

***Responses vary.* Dalia subtracted $6x$ from both sides of the original equation.**

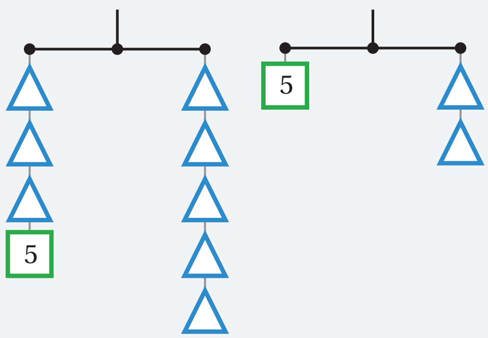
Dalia

$6x + 12 = 10x - 4$

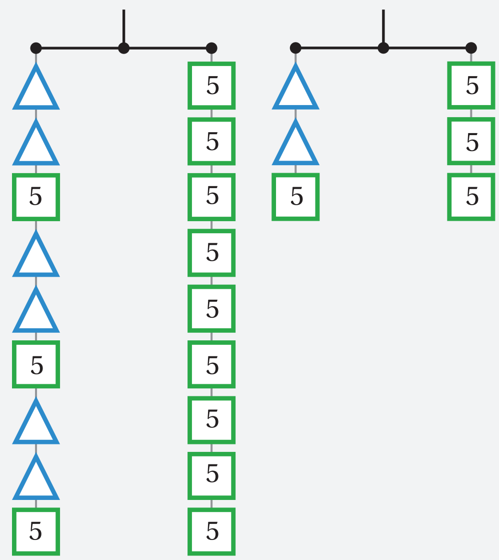
$12 = 4x - 4$

7 Match each hanger set or equation set with the balanced equation move that describes it.

Hanger Set A



Hanger Set B



Equation Set C

$$\frac{5x}{-3} = \frac{12}{1}$$

$$5x = -36$$

Equation Set D

$$15 - 7x = 3 + 5x$$
$$12 - 7x = 5x$$

Equation Set E

$$3x + 5 = 5x$$
$$5 = 2x$$

Equation Set F

$$6x + 15 = 45$$
$$2x + 5 = 15$$

Multiply each side by -3	Divide each side by 3	Subtract 3 from each side	Subtract $3x$ from each side
Equation Set C	**Equation Set F** **Hanger Set B**	**Equation Set D**	**Hanger Set A** **Equation Set E**

Activity 2

Name: ______ Date: ______ Period: ______

Solving Equations (continued)

8 Jaylin solved this equation from the card sort: $15 - 7x = 3 + 5x$.

Jaylin

Equation Set D

$15 - 7x = 3 + 5x$

$12 - 7x = 5x$

$12 = 12x$

$1 = x$

Is Jaylin's solution correct? Circle one.

Yes   No   I'm not sure

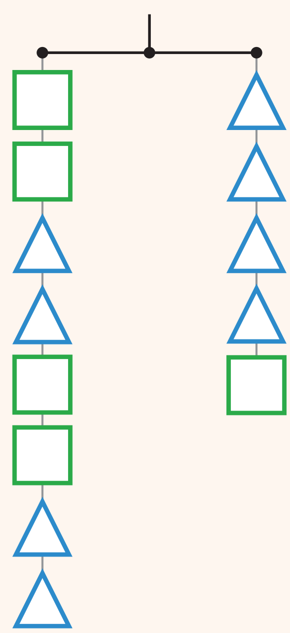
Explain your thinking.

Explanations vary.

- **If I substitute $x = 1$ into the original equation, I get $15 - 7(1) = 3 + 5(1)$ or $8 = 8$, which is a true statement.**
- **Jaylin used balanced moves for each of his steps, which means that the value of the variable didn't change.**

Explore More

9 What is the weight (in pounds) of a square in this balanced hanger?



0 pounds

Explain your thinking.

Explanations vary. **If I remove all of the extra squares and triangles from each side, I get three squares on the left and no shapes on the right. The only way this could be true is if the square's weight is 0 pounds.**

10 Synthesis

How can balanced equation moves be helpful when solving equations?

Use the example if it helps with your thinking.

Responses vary. **They're helpful because they let me make a simpler version of the equation. Because the equation is still equivalent to the original, the variable will have the same value.**

Jaylin

Equation Set D

$15 - 7x = 3 + 5x$

$12 - 7x = 5x$

$12 = 12x$

$1 = x$

Things to Remember:

Name: .... Date: .... Period: ....

More Balanced Moves

Let's solve some equations.

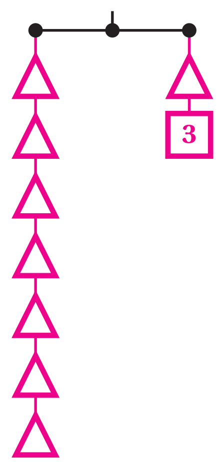
Warm-Up

1. Here are several equations.

- Equation A: $12x + 3 = 3(5x + 9)$
- Equation B: $2x + 5x = x + 3$
- Equation C: $-3x + 12 = 9x - 4$
- Equation D: $x - 4 = \frac{1}{3}(6x - 54)$

a Draw a hanger diagram to represent one of the equations.

***Responses vary.* Equation B sample shown in diagram.**



b **Discuss:** Why might it be useful to represent an equation with a hanger diagram? What limitations might this representation have?

Responses vary.

- **It might be useful to have a visual to think about applying balanced moves to.**
- **Equations that have big numbers, negative numbers, fractions, and/or subtraction are difficult to draw hangers for.**

Activity 1

Name: .... Date: .... Period: ....

Step by Step by Step by Step

Sadia and Amir started solving the same equation. Here is their work.

Sadia	Amir
$12x + 3 = 3(5x + 9)$	$12x + 3 = 3(5x + 9)$
$4x + 1 = 5x + 9$	$12x + 3 = 15x + 27$

2. In what ways are the steps they took alike and different?

***Responses vary.* Both students tried to eliminate the parentheses. Sadia eliminated the parentheses by dividing both sides of the equation by 3. Amir eliminated the parentheses by using the distributive property to create an equivalent expression.**

Caleb and Roberto also solved the equation $12x + 3 = 3(5x + 9)$. Some of their work is correct and some of their work is incorrect.

Caleb	Roberto
$12x + 3 = 3(5x + 9)$	$12x + 3 = 3(5x + 9)$
$7x + 3 = 3(9)$	$12x + 3 = 15x + 27$
$7x + 3 = 27$	$27x + 3 = 27$
$7x = 24$	$27x = 24$
$x = \frac{24}{7}$	$x = \frac{24}{27}$

3. What are some moves they made that kept the equation balanced?

Responses vary.

- **From the third equation to the fourth, Caleb subtracted 3 from each side.**
- **From the first equation to the second, Roberto applied the distributive property.**

4. In each student's work, circle and explain the mistake you think they made.

Caleb's mistake: ***Responses vary.* Caleb made a mistake moving from the first equation to the second by subtracting $5x$ from each side. Since the $5x$ was inside the parentheses, it really represented 3 groups of $5x$, so Caleb's move didn't keep the equation balanced.**

Roberto's mistake: ***Responses vary.* Roberto made a mistake moving from the second equation to the third because he combined the $12x$ and the $15x$ even though they weren't on the same side of the equation.**

Activity 2

Name: ____ Date: ____ Period: ____

Make Your Own Steps

5. Solve each equation for x. Show your thinking.

a $2x + 5x = x + 3$

$x = \frac{1}{2}$ (or equivalent). *Work varies.*

$2x + 5x = x + 3$

$7x = x + 3$

$6x = 3$

$x = \frac{3}{6}$

b $-3x + 12 = 9x - 4$

$x = \frac{4}{3}$ (or equivalent). *Work varies.*

$-3x + 12 = 9x - 4$

$12 = 12x - 4$

$16 = 12x$

$\frac{16}{12} = x$

c $-4(x - 3) = 12x - 4$

$x = 1$. *Work varies.*

$-4(x - 3) = 12x - 4$

$x - 3 = -3x + 1$

$4x - 3 = 1$

$4x = 4$

$x = 1$

d $8x + 7 = 6x - 13$

$x = -10$. *Work varies.*

$8x + 7 = 6x - 13$

$2x + 7 = -13$

$2x = -20$

$x = -10$

e $x - 4 = \frac{1}{3}(6x - 54)$

$x = 14$. *Work varies.*

$x - 4 = \frac{1}{3}(6x - 54)$

$x - 4 = 2x - 18$

$x + 14 = 2x$

$x = 14$

f $3(x - 2) + 2x = 25$

$x = \frac{31}{5}$ (or equivalent). *Work varies.*

$3(x - 2) + 2x = 25$

$3x - 6 + 2x = 25$

$5x - 6 = 25$

$5x = 31$

$x = \frac{31}{5}$

Explore More

6. There are 24 pencils and 3 cups. The second cup holds one more pencil than the first cup. The third cup holds one more pencil than the second cup. How many pencils does each cup contain? Show or explain your thinking.

The cups have 7, 8, and 9 pencils in them. *Explanations vary.* I wrote the equation $x + (x + 1) + (x + 2) = 24$ and solved the equation to determine how many pencils were in the first cup.

Synthesis

7. What are some helpful moves when solving equations?

$12x + 3 = 3(5x + 9)$

$2x + 5x = x + 3$

$-3x + 12 = 9x - 4$

$x - 4 = \frac{1}{3}(6x - 54)$

Responses vary.

- I can create a simpler equivalent equation by applying a balanced equation move to each side.
- I can use the distributive property to make a simpler equivalent expression.
- I can combine like terms to make a simpler equivalent expression.
- I can subtract a variable term from both sides of the equation so that there is only a variable term on one side.

Things to Remember:

Equation Roundtable

Directions: Make one copy per group. Then pre-cut the cards and give each group one set.

Card 1

$$-4x - 7 - 2x = 4x - 2$$

Card 2

$$\frac{1}{2}(7x - 6) = 6x - 8$$

Card 3

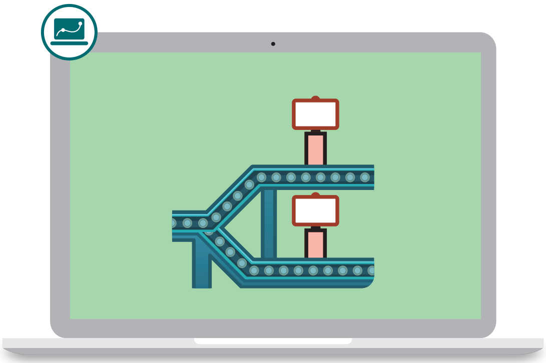
$$\frac{3}{4}x + 7 = x + 13$$

Card 4

$$-4x + 14 = 2(x + 7)$$

Unit 4
Lesson
6

Name: ________ Date: ________ Period: ________



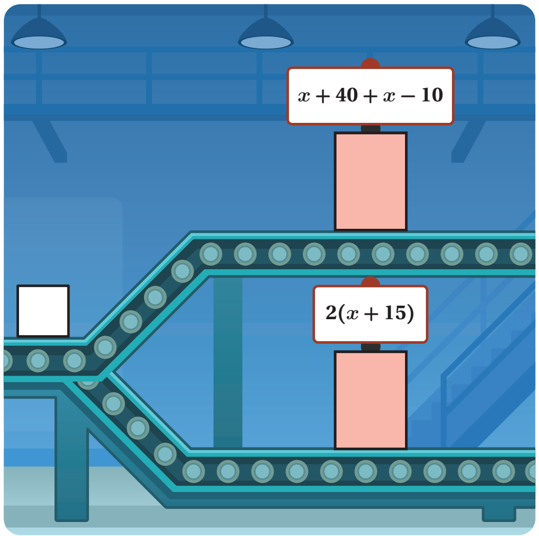
All, Some, or None? Part 1

Let's think about how many solutions an equation can have.

Warm-Up

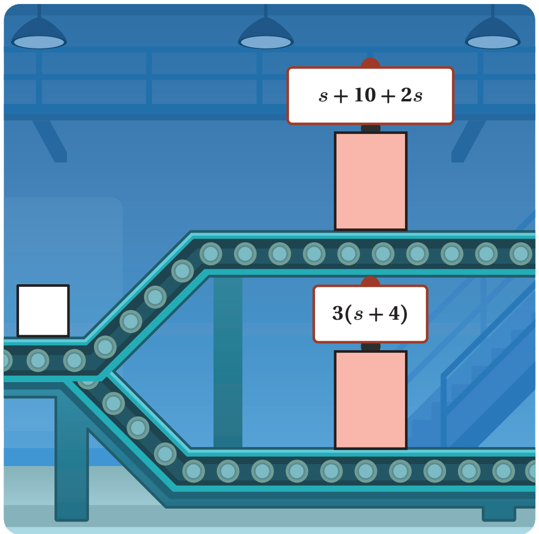
1 Here are two number machines. Tasia put a number into both machines, and the outputs were the same. What was Tasia's input?

Responses vary. Note: Since the expression $x + 40 + x - 10$ is equivalent to $2(x + 15)$, any number put into both machines will give the same output number.



2 Here are two new number machines. Try to find a number to put into both machines to get the same outputs.

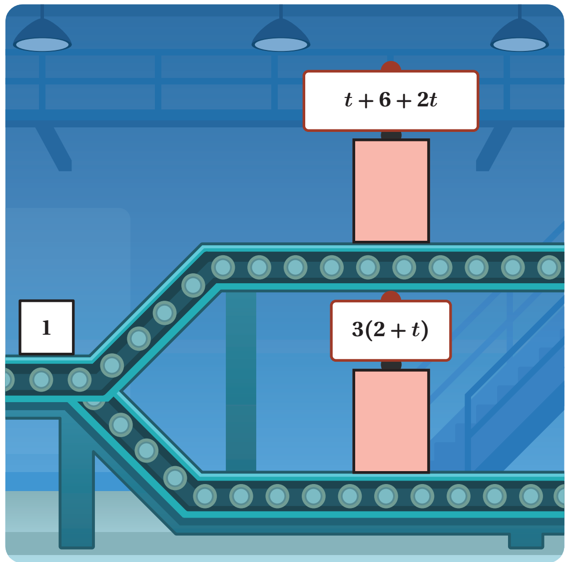
Note: Since there is no value for s that will make the expression $s + 10 + 2s$ equal to $3(s + 4)$, there is no number to put into both machines to get the same output number.



Name: ________ Date: ________ Period: ________

Always-Equal and Never-Equal

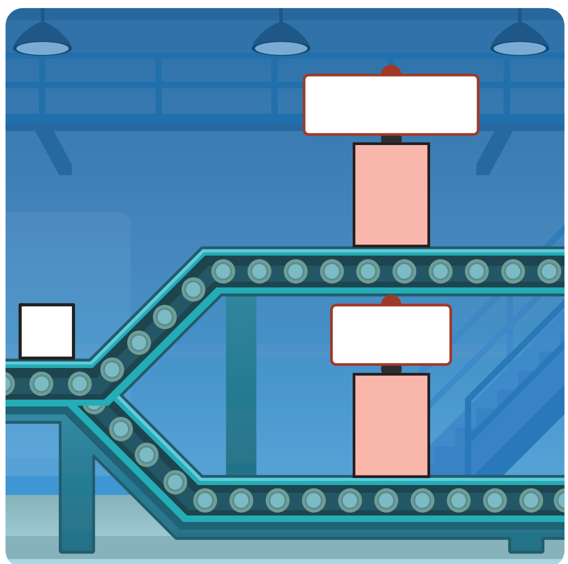
3 Let's look at two number machines that *always* give the same outputs for any input.



Use the two expressions to explain why both machines will give the same outputs for any input.

***Responses vary.* Note: Since both expressions can be simplified to $3t + 6$, they are equivalent. So any input will produce the same value for either machine.**

4 Write two expressions to create two new number machines that will give the same outputs for any input.



Responses vary.

- **$2x + 2$ and $x + x + 10 - 8$**
- **$5(x + 10)$ and $x + x + x + x + x + 50$**

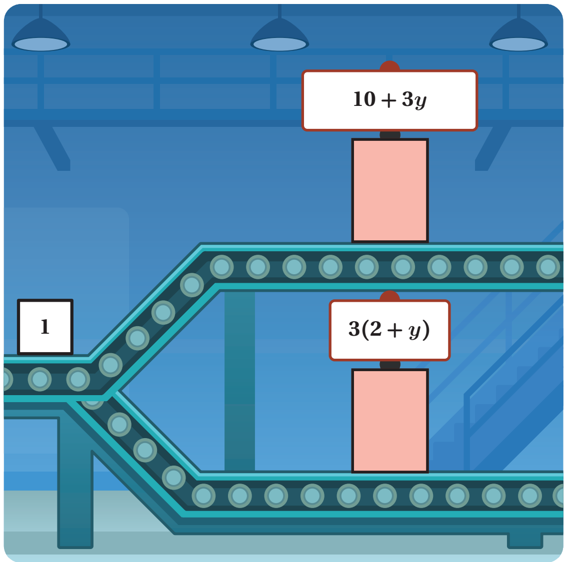
Name: ______ Date: ______ Period: ______

Always-Equal and Never-Equal (continued)

5 Let's look at two number machines that *never* give the same outputs for any input.

Use the two expressions to explain why both machines will *never* give the same outputs for any input.

Responses vary. **I distributed the 3 in the bottom expression and got $6 + 3y$. There is no value of y where $10 + 3y$ will equal $6 + 3y$, so they will never produce the same value for both machines.**



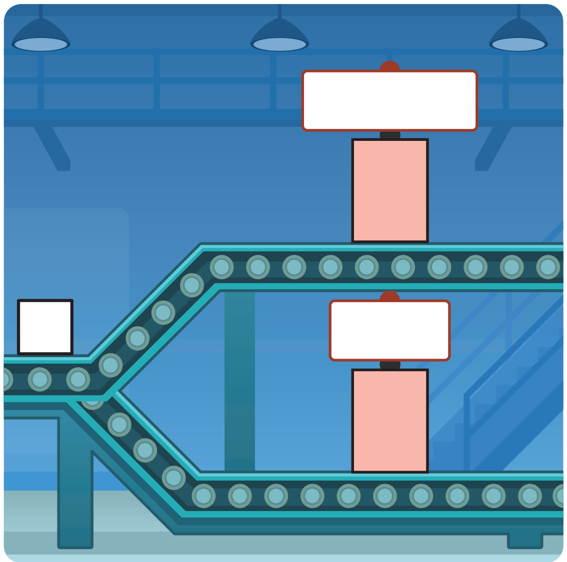
6 Select an equation made of two expressions that create number machines that will *never* give the same outputs for any input.

A. $2x + 3 = 3 + 2x$

B. $2x + 3 = 5 + 2x$

C. $2x + 3 = 2 + 3x$

D. $2x + 2 = 3 + 3x$

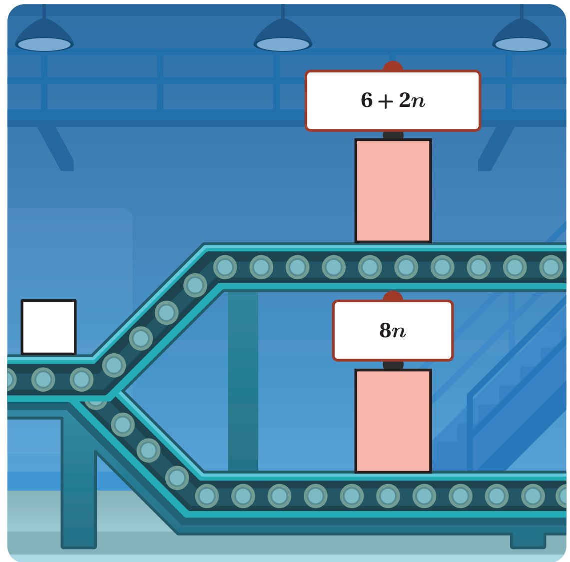


Activity 2

Name: .... Date: .... Period: ....

Number of Solutions

7 Here are two new number machines.



a Write an equation to find an input number that will produce the same outputs for each machine.

$6 + 2n = 8n$ (or equivalent)

b For how many values of n will these machines produce the same output?

Circle one.

All values of n | One value of n | No values of n

8 Group the equations based on their number of solutions.

$v + 2 = v - 2$	$2n = n$	$7 - r = r - 7$	$\frac{1}{2} + x = \frac{1}{3} + x$
$y(-6) \cdot (-3) = 2 \cdot y \cdot 9$	$2t + 6 = 2(t + 3)$	$2n = 2n$	$3(n + 1) = 3n + 1$

No Solution (True for No Values)	One Solution (True for One Value)	Infinitely Many Solutions (True for All Values)
$v + 2 = v - 2$ $\frac{1}{2} + x = \frac{1}{3} + x$ $3(n + 1) = 3n + 1$	$2n = n$ $7 - r = r - 7$	$y \cdot (-6) \cdot (-3) = 2 \cdot y \cdot 9$ $2t + 6 = 2(t + 3)$ $2n = 2n$

Name: .... Date: .... Period: ....

Number of Solutions (continued)

9 Kiandra looked at this equation and, without writing anything, said it must have no solution. What might she have noticed to lead her to this conclusion?

No Solution (True for No Values)
$\frac{1}{2} + x = \frac{1}{3} + x$

Responses vary. **Kiandra probably noticed that there are different constants being added to x on each side of the equation, so there is no single number that x can be that would make both sides the same.**

10 Write an equation for each number of solutions. ***Responses vary.***

No solution: $\mathbf{2 + x = 10 + x}$

One solution: $\mathbf{2 + x = 2}$

Infinitely many solutions: $\mathbf{2 + x = x + 2}$

11 Synthesis

How can you determine whether an equation has no solution, one solution, or infinitely many solutions?

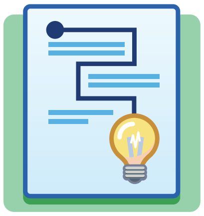
Responses vary.

- **Equations with no solution have expressions that can never be equal on either side of the equal sign.**
- **Equations with one solution have expressions on either side of the equal sign that are only equal for one value.**
- **Equations with infinitely many solutions have equivalent expressions on either side of the equal sign that will always be equal for any value.**

Things to Remember:

Name: ____________ Date: ________ Period: ________

Strategic Solving, Part 1



Let's solve linear equations with no solution, one solution, and infinitely many solutions.

Warm-Up

1. Here are three equations:

- $13x = 3.25$
- $13x = 385x$
- $13x = 10x + 3x$

Choose one and write a situation that it could represent.

Responses vary.

- **A water cooler holds 3.25 gallons of water. The water from the cooler fills 13 cups with the same volume and then runs out. How much water is in each cup?**
- **Gabriel saves 13 dollars a month, and Imani saves 385 dollars a month. When will they have the same amount of savings?**
- **I have two number machines, one with the expression $13x$ and one with the expression $10x + 3x$. What input gives the same output?**

Activity 1

Name: Date: Period:

Predicting Solutions

2. Predict whether each equation has no solution, one solution, or infinitely many solutions. For equations with one solution, predict whether the solution will be *positive*, *negative*, or *zero*.

Equation	No Solution	One Solution			Infinite Solutions
$13x = 3.25$		+	–	0	
$13x = 385x$		+	–	0	
$13x = 10x + 3x$		+	–	0	✔
$13x + 42 = -584$		+	–	0	
$13x + 42 = 13x + -42$	✔	+	–	0	

3. Choose one equation and explain how you made a prediction about its solution.

***Responses vary.* For $13x = 3.25$, I knew the equation had one positive solution because multiplying 13 by a positive number will give me another positive number.**

4. Why do you think it might be helpful to pause and try to predict the number of solutions or the sign of the solution before you start solving an equation?

***Responses vary.* If I've made a prediction about the solution before I start solving, it can help me decide if my solution is reasonable once I'm done solving.**

Name: ______ Date: ______ Period: ______

What Happened?

5. Deven tried to solve the equation $13x + 42 = 13x + -42$. But Sam thinks Deven made a mistake.

Deven

$13x + 42 = 13x + -42$
$13x = 13x + -84$
$0 = -84$

Do you agree? Circle one.

Yes   No   I'm not sure

Explain your thinking.

***Explanations vary.* Deven didn't make a mistake. Deven used balanced equation moves: subtracting 42 from each side and then subtracting $13x$ each side. This created a false equation because this equation has no solution.**

6. Write an equation with infinitely many solutions.

***Responses vary.* $13x = 13x$**

7. What happens when you try to solve the equation you wrote?

***Responses vary.* When I tried to solve my equation using balanced equation moves, I ended up with a true equation with the variable gone, $0 = 0$.**

Activity 3

Name: .... Date: .... Period: ....

The Choice Is Yours

Equation A	Equation B	Equation C
$2r + 49 = \text{-}8(\text{-}r - 5)$	$\frac{n}{7} - 12 = 5n + 5$	$\frac{4m - 16}{4} = \frac{\text{-}16 + 8m}{8}$

Equation D	Equation E
$p - 5(p + 4) = p - (8 - p)$	$3(c - 1) + 2(c - 1) = 5(c - 1)$

Equation F	Equation G	Equation H
$\text{-}\frac{1}{2}(t + 3) - 10 = \text{-}6.5$	$\frac{10 - v}{4} = 2(v + 17)$	$2(2q + 1.5) = 18 - q$

8. Examine these equations. Organize the equations into two or three groups based on the patterns you notice. ***Responses vary.***

Group 1	Group 2	Group 3
Equations B, C, F, and G	Equations A and H	Equations E and D

9. **Discuss:** How did you group the equations?

Responses vary. **Group 1 includes equations that have fractions. Group 2 includes equations where an integer could be distributed once. Group 3 includes equations where the distributive property could be applied more than once.**

10. Choose *three* equations to solve. (Choose at least one from each group.) Show your thinking.

Work varies.

- Equation A: $r = \frac{3}{2}$
- Equation B: $n = \text{-}3.5$
- Equation C: No solution
- Equation D: $p = \text{-}2$
- Equation E: Infinitely many solutions
- Equation F: $t = \text{-}10$
- Equation G: $v = \text{-}14$
- Equation H: $q = 3$

Synthesis

11. What are some strategies for solving equations like these?

Equation B

$\frac{n}{7} - 12 = 5n + 5$

Equation C

$\frac{4m - 16}{4} = \frac{-16 + 8m}{8}$

Equation D

$p - 5(p + 4) = p - (8 - p)$

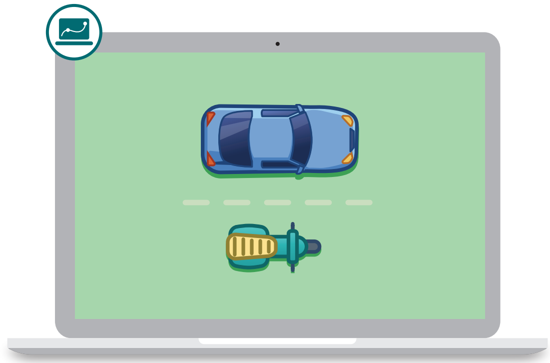
Responses vary.

- **It can be helpful to predict the number of solutions or the sign of the solution before I get started.**
- **I can look at the features of the equation to decide what moves would be most helpful. For example, for an equation with a fraction, I might start by multiplying both sides of the equation by the denominator.**
- **I can create simpler equivalent expressions on one side of the equation by applying the distributive property or combining like terms.**

Things to Remember:

Unit 4
Lesson
8

Name: .... Date: .... Period: ....

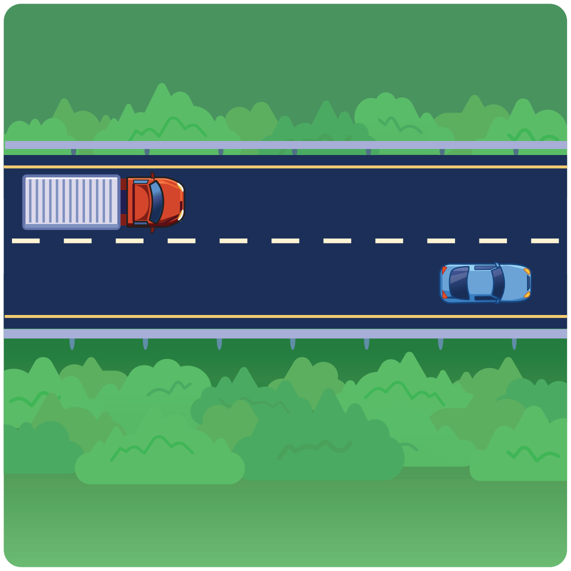


When Will They Meet?

Let's use equations to think about situations.

Warm-Up

1 Let's watch an animation of a truck and a car.



a Do you think the truck will meet the car? Circle one.

Yes No I'm not sure

b What information could help you prove your answer?

***Responses vary.* I would need to know how fast each vehicle is moving.**

Activity 1

Name: .... Date: .... Period: ....

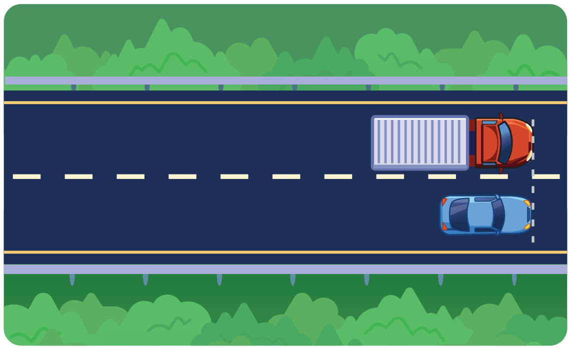
Distance and Time

2 The table shows each vehicle's position at certain times. The vehicles are moving at a constant *rate*. Fill in the missing information in the table.

Time (sec)	Truck Position (m)	Car Position (m)
0	0	18
1	15	29
2	30	40
3	**45**	**51**
4	**60**	**62**
...	...	...
t	**$15t$**	**$11t + 18$**

3 When will the truck meet the car?

4.5 seconds

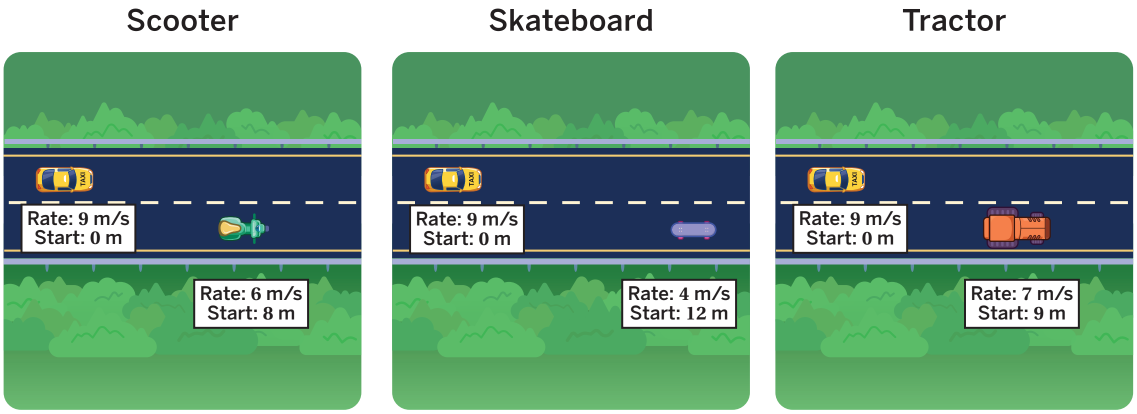


Activity 2

Name: Date: Period:

Choose Your Vehicle

4 Choose and circle a vehicle to compare to the taxi. The rate (in meters per second) and the starting point are displayed for each vehicle.



5 Let t represent time in seconds. For the vehicle you chose, which equation could you solve to determine when the two vehicles meet?

Scooter

A. $6 + 8t = 9t$

B. $8t + 6 = 9$

C. $6t + 8 = 9t$

Skateboard

A. $4t + 12 = 9t$

B. $12t + 4 = 9$

C. $4 + 12t = 9t$

Tractor

A. $7 + 9t = 9t$

B. $7t + 9 = 9t$

C. $9t + 7 = 9$

Explain your thinking.

***Explanations vary.* The rate of the taxi is 9 m/s, so the taxi's position can be represented with the expression $9t$. The skateboard's position can be represented by the expression $4t + 12$. Setting the expressions equal to one another will help me find exactly when the vehicles will meet.**

6 When will the vehicle you chose meet the taxi?

Responses vary.

- **Scooter: $\frac{8}{3}$ seconds (or equivalent)**
- **Skateboard: 2.4 seconds (or equivalent)**
- **Tractor: 4.5 seconds (or equivalent)**

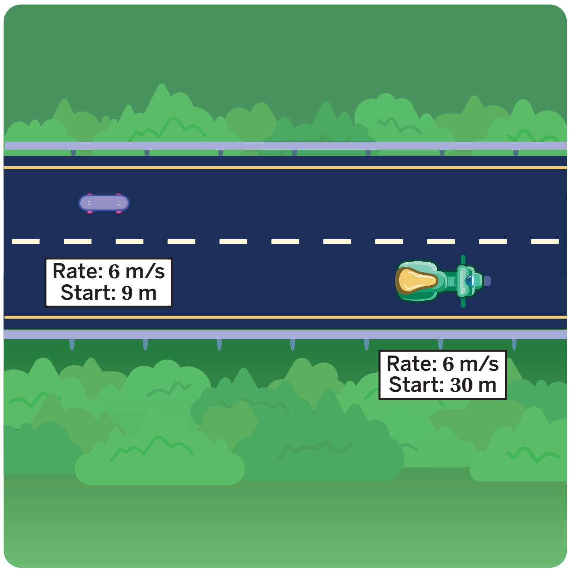
Activity
3

Name: ________ Date: ________ Period: ________

Meet Up

7 Demetrius wants to figure out when these vehicles will meet, so he wrote these expressions.

Skateboard Position (m)	Scooter Position (m)
$6t + 9$	$6t + 30$



Without solving an equation, Demetrius knew the vehicles would never meet. How might he have figured this out?

***Responses vary.* He could tell from looking at the expressions that the vehicles are both moving at 6 m/s. Since they are both moving at the same rate, they will never meet.**

Activity 3

Name: ________ Date: ________ Period: ________

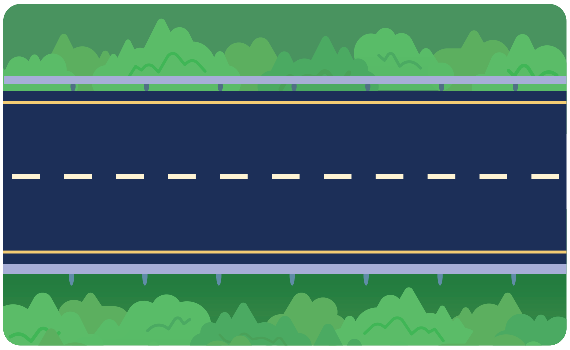
Meet Up (continued)

8 Write expressions, in terms of t, that could represent two vehicles traveling at different rates with different starting positions that will eventually meet.

Responses vary.

Truck position expression: $4t + 3$

Car position expression: $6t + 2$



9 When will the two vehicles meet?

Responses vary. 0.5 seconds

Explore More

10 A tractor and a scooter are in a race. Write expressions, in terms of t, for each vehicle so that the vehicles start separated and meet at 10 seconds. *Responses vary.*

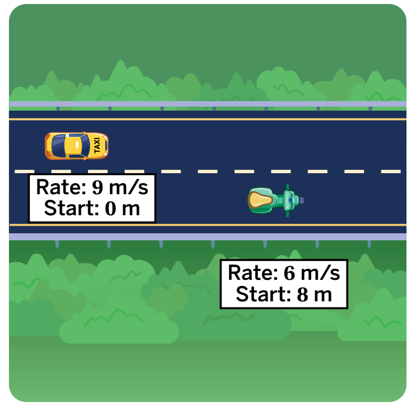
Tractor position expression: $5t + 30$

Scooter position expression: $20t - 120$

11 Synthesis

How can writing expressions to represent the position of vehicles at time t help you determine when they will meet?

Use the example if it helps with your thinking.



Responses vary. **If I write one expression for each vehicle and set them equal to each other, the solution to the equation is the time when the vehicles are in the same position.**

Things to Remember:

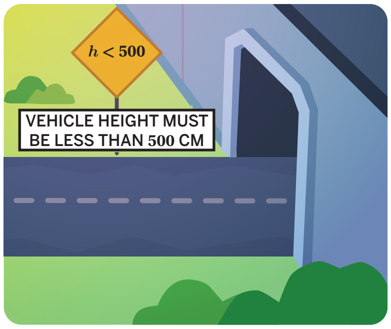
Name: .... Date: .... Period: ....

Tunnel Travels

Let's explore inequalities using words, symbols, and a number line.

Warm-Up

1 Select *all* of the vehicles that can fit in this tunnel.

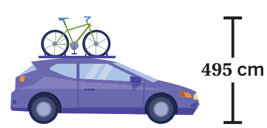


- [x] A.

- [ ] B.

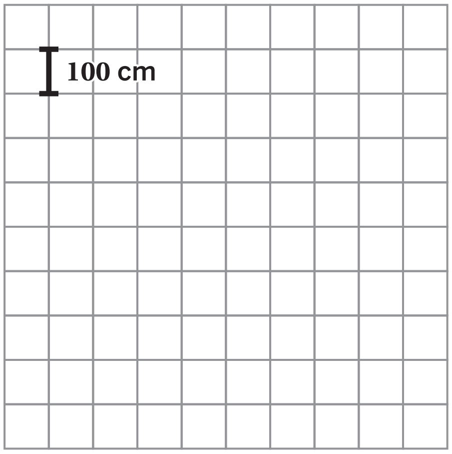
- [ ] C.

- [x] D.



2 Sketch a vehicle that will fit in the tunnel.

Drawings vary. **Vehicles under 500 centimeters will fit.**

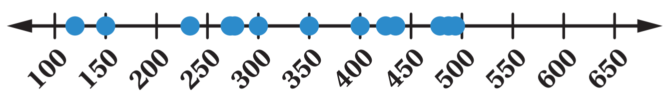


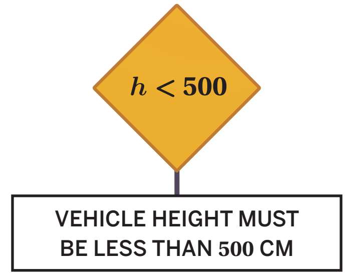
Activity 1

Name: Date: Period:

Inequalities in Context

3 Here are the heights of several vehicles that fit in the tunnel.



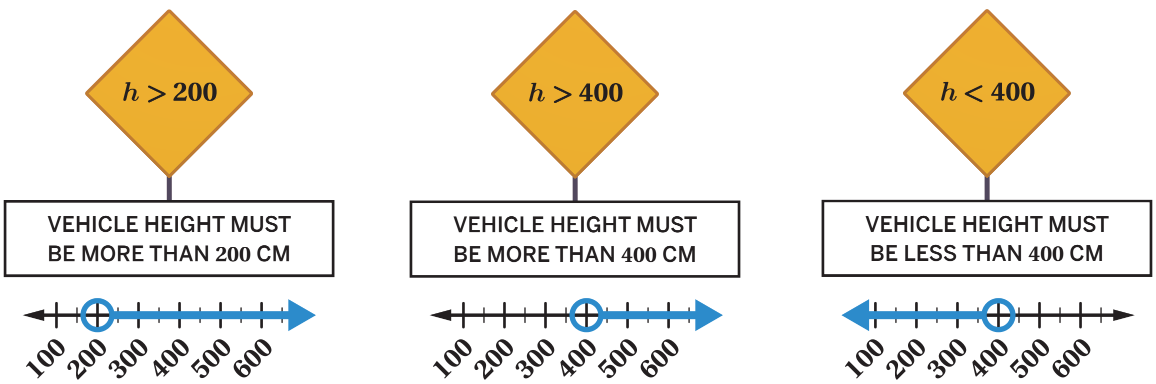


What do you think a graph of *all* the vehicle heights that fit would look like?

Responses vary.

- **It would look like a lot of dots squished together, all to the left of 500.**
- **All the vehicles' heights would make a thick line from the right of 0 (because you can't have a vehicle with a negative height or no height) to the left of 500.**

4 Here are three signs with inequalities and their number line graphs.

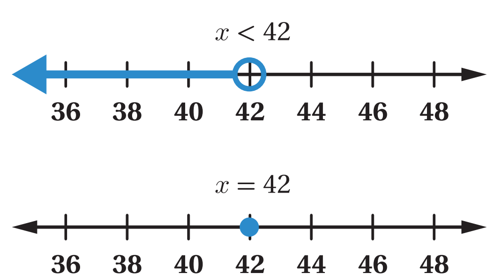


Discuss: What do you notice? What do you wonder?

Responses vary.

- **I notice that when the shaded part of the number line is on the left side of the circle, the vehicle height is less than the number in the inequality.**
- **I notice the circle is at 200 for the inequality $h > 200$. I wonder if 200 is a solution.**
- **I wonder how I can change the inequality $h > 200$ to include a vehicle height of 200 centimeters.**

5 Here are the graphs for $x < 42$ and $x = 42$.



Discuss: How are the graphs alike? How are they different?

Responses vary.

- **Both graphs have a circle at 42.**
- **One graph has a circle that is filled in and the other has a circle that is not filled in.**
- **$x < 42$ has a line with an arrow going to the left and a circle that is not filled in at 42. $x = 42$ does not.**

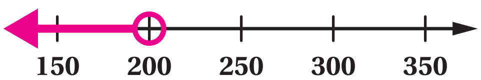
Activity 1

Name: .... Date: .... Period: ....

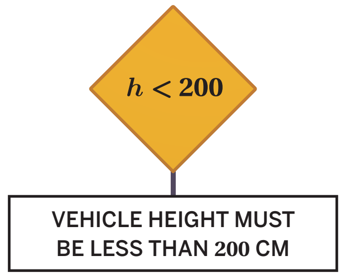
Inequalities in Context (continued)

6 Norma Merrick Sklarek was the first Black woman to become a licensed architect in California. In 1975, she helped design the Pacific Design Center in Los Angeles, California. The parking garage at the Center can fit vehicles that are less than 200 centimeters tall.

Graph all the possible vehicle heights that fit in this parking garage.

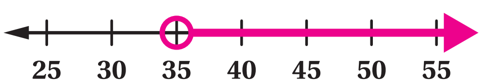


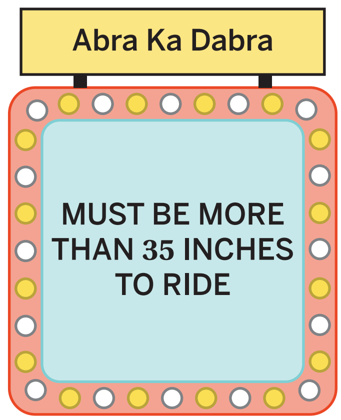
Pacific Design Center



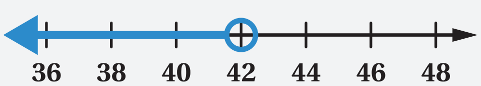
7 Fri Forjindam is co-owner and chief development officer of a company that develops theme parks. In 2016, she designed Bollywood Parks in Dubai. One ride at that park, Abra Ka Dabra, only allows passengers who are taller than 35 inches.

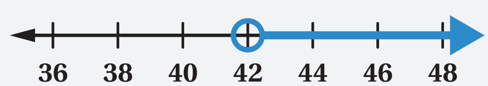
Graph all the possible heights for this ride.





8 Group the choices that represent the same situation. One choice will have no match.

a. 

b. 

c. You must be over 42 inches tall to ride The Whipper.

d. You must be under 42 inches tall to ride the kiddie swings.

e. You must be at least 42 inches to ride the roller coaster.

f. $x < 42$

g. $x > 42$

Group 1	Group 2
a, d, f	b, c, g

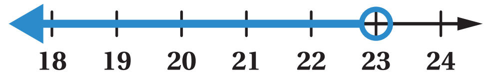
Choice with no match: e

Activity 2

Name: .... Date: .... Period: ....

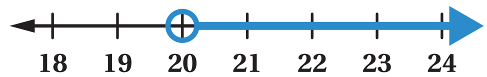
Inequalities Out of Context

9 Rewrite the inequality $x > 23$ so that it matches the graph.



$x < 23$ or $23 > x$

10 To represent this graph:



- Martina wrote the inequality $20 < x$.
- Nasir wrote the inequality $x < 20$.

Whose inequality is correct? Circle one.

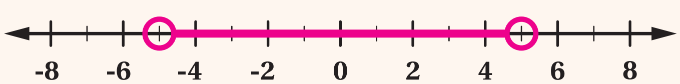
Martina's   Nasir's   Both   Neither

Explain your thinking.

***Explanations vary*. Because the graph is to the right, x has to include all the numbers that are greater than 20, which means that 20 must be less than x.**

Explore More

11 Here is a number line.



a Determine three possible values for x if $|x| < 5$.

***Responses vary*. Values should be between -5 and 5.**

b Plot these values on the number line.

***Responses vary*. Values should satisfy $-5 < x < 5$.**

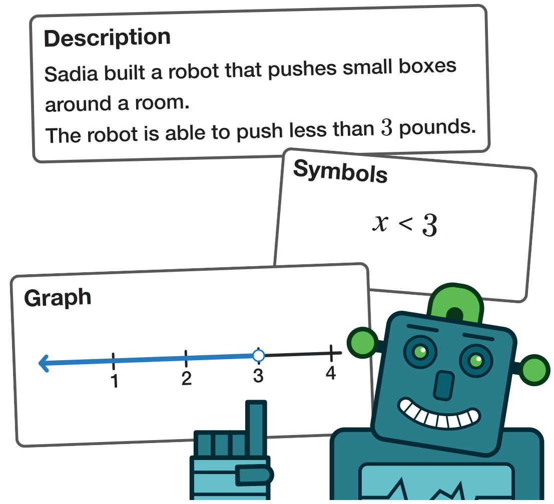
c Plot as many other possible values for x as you can.

Response shown on graph. The number line should be shaded between -5 and 5.

12 Synthesis

Circle one representation and explain how it shows that Sadia's robot can push a 2-pound box.

Description Symbols Graph



Explain your thinking.

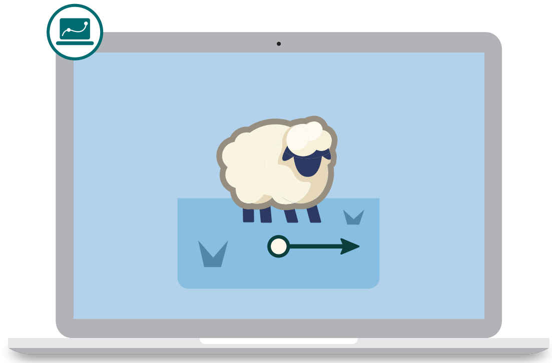
Responses and explanations vary.

- **From the description, I know that Sadia's robot can push a 2-pound box because it can push less than 3 pounds, and 2 is less than 3.**
- **From the symbols, I know that Sadia's robot can push a 2-pound box because $2 < 3$.**
- **From the graph, I know that Sadia's robot can push a 2-pound box because 2 is in the shaded part of the number line.**

Things to Remember:

Unit 7
Lesson
8

Name: .... Date: .... Period: ....



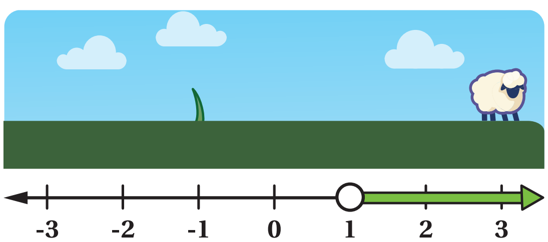
Shira's Solutions

Let's find solutions to an inequality using a number line.

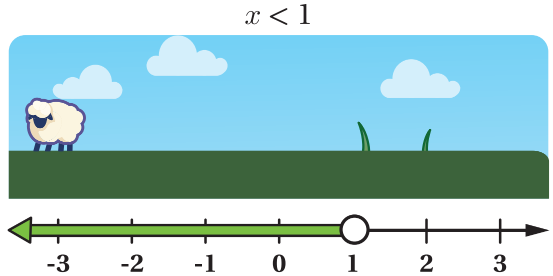
Warm-Up

1 Shira the Sheep loves eating all the blades of grass. These graphs show what happens when Shira eats grass based on different inequalities.

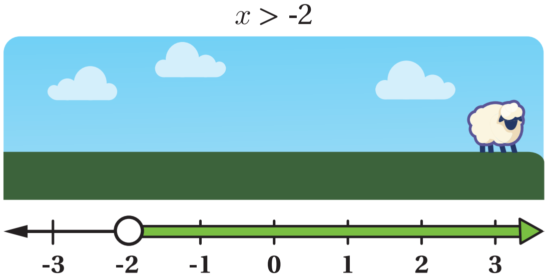
$x > 1$



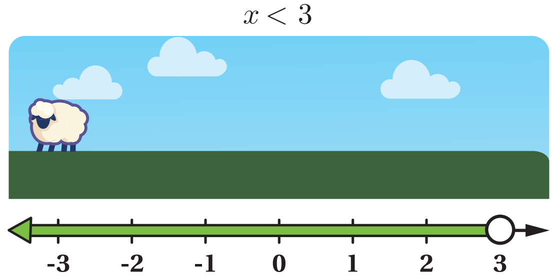
$x < 1$



$x > -2$



$x < 3$



Discuss: What do you notice?

Responses vary.

- I notice there are multiple inequalities where Shira eats all the grass.
- I notice for the inequality $x > 1$ Shira only eats some of the grass.
- I notice that sometimes Shira eats grass one way and sometimes she eats grass the other way.

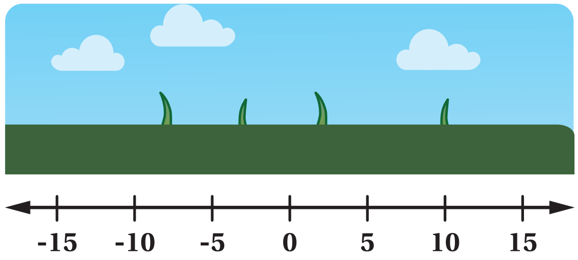
Name: Date: Period:

Connecting Graphs and Inequalities

2 Shira wants to eat these four blades of grass.

The inequality $11 < x$ did not work.

Fix this inequality to help Shira eat all the grass.

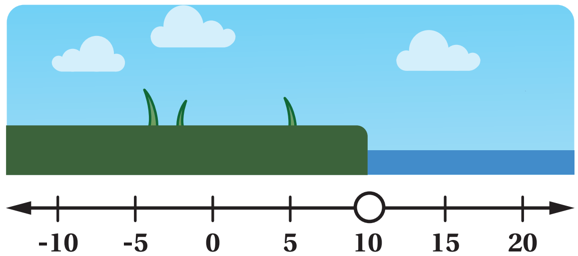


Responses vary.

- $11 > x$
- $-11 < x$
- $x < 11$

3 Shira the Sheep loves eating grass. She does not like water.

Write an inequality to help Shira the Sheep eat all the grass without falling in the water.



Responses vary.

- $x < 10$
- $6 > x$

Explain your thinking.

Explanations vary. **I wrote the inequality $x < 10$ because 10 is where the water starts, and I want Shira to eat away from the water.**

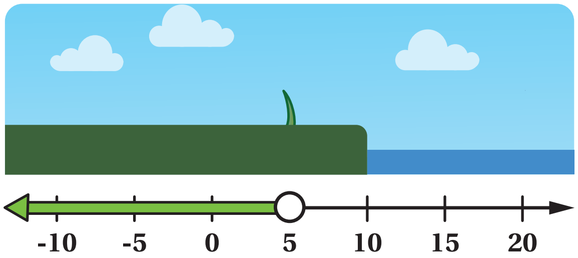
Name: .... Date: .... Period: ....

Connecting Graphs and Inequalities (continued)

4 Kiana wrote $x < 5$ and was surprised that there was one blade of grass remaining.

Explain why 5 is not a **solution to the inequality** $x < 5$.

***Responses vary.* The inequality $x < 5$ is true for all numbers that are less than 5. Because 5 is not less than 5, it is not a solution to this inequality.**

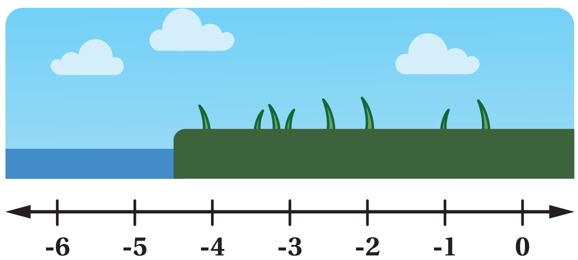


5 Write an inequality so that all the blades of grass are solutions and none of the water is.

Use the number line if it helps with your thinking.

Responses vary.

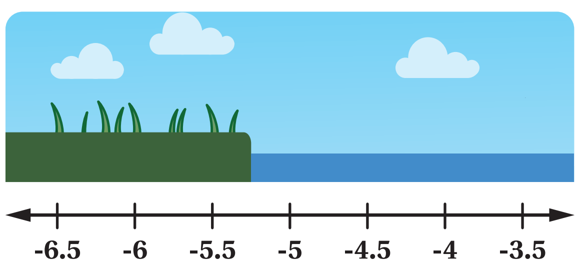
- $x > -4.5$
- $-4.3 < x$



6 Write an inequality so that all the blades of grass are solutions and none of the water is.

Responses vary.

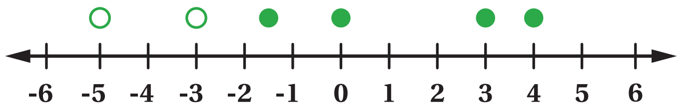
- $x < -5.3$
- $-5.35 > x$



Name: .... Date: .... Period: ....

Solutions to an Inequality

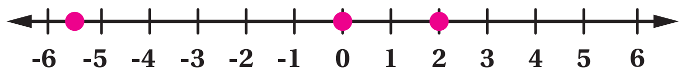
7 Write an inequality so that all of the solid points are solutions and none of the open points are.



Responses vary.

- $x > -2$
- $-2.5 < x$

8 Write at least three solutions to the inequality $2.7 > x$.



Plot the solutions on the number line.

Responses vary. **2, 0, -5.5**

9 Let's look at other solutions to $2.7 > x$.

How many solutions does this inequality have? Explain your thinking.

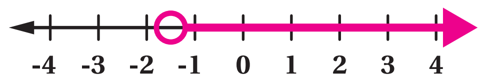
Responses vary. **There are infinite solutions because a solution can be any number that is less than 2.7 and there are infinitely many of those.**

Activity 2

Name: ________ Date: ________ Period: ________

Solutions to an Inequality (continued)

10 Make a graph of *all* the solutions to the inequality $x > -1.5$.



11 Match each inequality or solution with the graph that represents it.

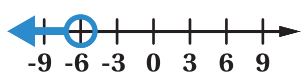
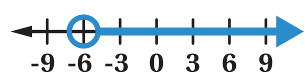
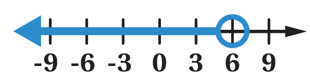
a. Some solutions: 5500, 6.5, -3

b. Some solutions: -100, 0.5, -6

c. $x < -6$

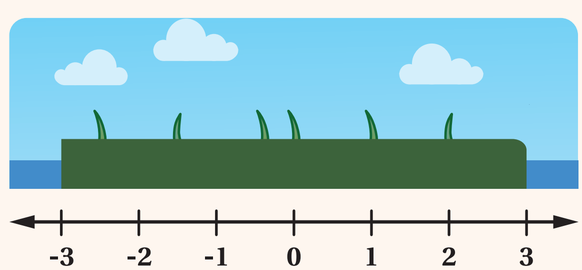
d. $-6 > x$

e. $x < 6$

		
c d	a	b e

Explore More

12 Write inequalities or describe how you can use inequalities to help Shira the Sheep eat all the blades of grass without falling in the water.



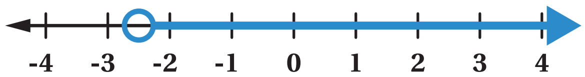
Responses vary.

- $x > -3$ and $x < 3$
- The absolute value of x is less than 3.
- $|x| < 3$

13 Synthesis

What does it mean for a number to be a solution to an inequality?

$x > -2.5$



Responses vary. A number is a solution to an inequality if it makes the inequality true. It's also in the shaded part of the graph. For example, -1 is a solution to $x >$ -2.5 because -1 $>$ -2.5 and because -1 is in the shaded part of the graph.

Things to Remember:

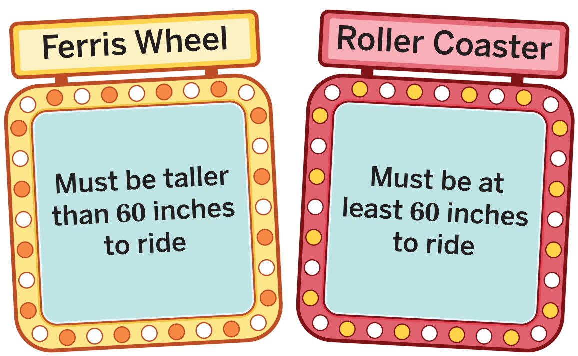
Name: .... Date: .... Period: ....

I Saw the Signs

Let’s work with inequalities.

Warm-Up

1 Here are two signs for two different rides at an amusement park.



Habib is exactly 60 inches tall. Which ride can he go on?

A. Ferris wheel **B.** Roller coaster **C.** Both **D.** Neither

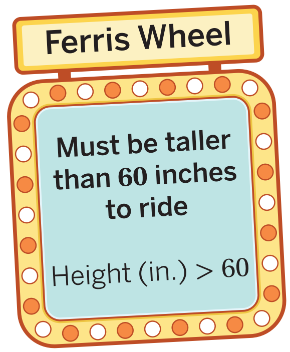
Explain your thinking.

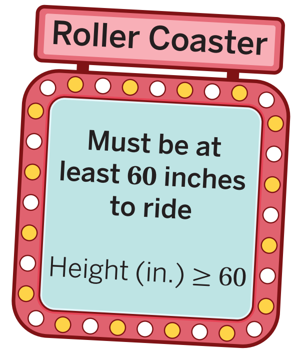
***Explanations vary.* Habib is 60 inches tall, so he can ride the roller coaster. He is not taller than 60 inches, so he can’t ride the Ferris wheel.**

Name: .... Date: .... Period: ....

Riding the Rides

The park added symbols to make the signs clearer. The symbol $>$ means greater than. The symbol $\geq$ means **greater than or equal to**.

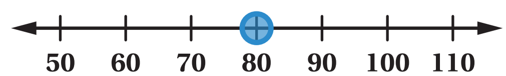
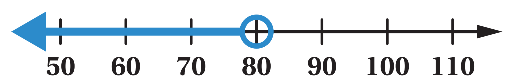
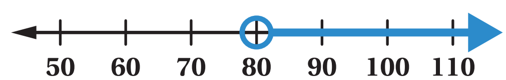
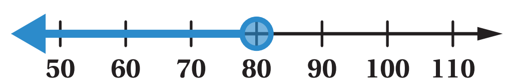
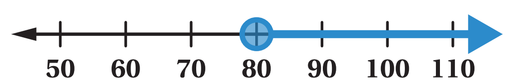




2 What is the shortest height Makayla can be and still ride both rides?

Responses vary.
- **61 inches**
- **60.00001 inches**

3 Here are some equations and inequalities, along with their graphs on a number line.

Equation/Inequality	Graph
$x = 80$	
$x < 80$	
$x > 80$	
$x \leq 80$	
$x \geq 80$	

Discuss: What do you notice?

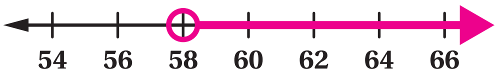
Responses vary.
- **I notice that when the circle is filled in, the symbol is $\leq$ or $\geq$, and when the circle is empty, the symbol is $<$ or $>$.**
- **I notice that when the symbol is $=$, there is no arrow and the circle is filled in.**

Name: ____________ Date: ________ Period: ________

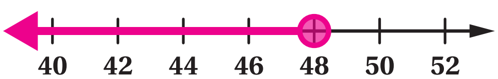
Riding More Rides

4 You are in charge of determining the height restriction for your ride. Create a graph and complete the sign for your ride.

Responses vary.



5 You must be 48 inches or shorter to ride the kiddie swings. Make a graph on the number line to represent the possible heights of the riders.



6 Luis is allowed to ride the kiddie swings. Omar is 6 inches shorter than Luis. Can Omar ride this ride? Explain your thinking.

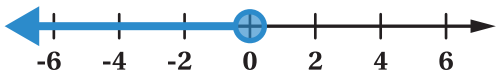
Yes. *Explanations vary.* People who are 48 inches tall or less are allowed to ride the Kiddie Swings. Luis is allowed to ride, so he must be 48 inches tall or less. Anyone shorter than Luis would also be allowed to ride.

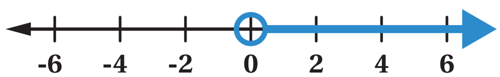
Activity 3

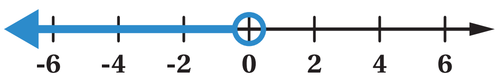
Name: .... Date: .... Period: ....

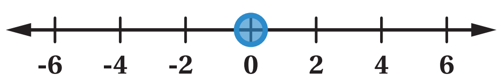
Inequalities Out of Context

7 Match each inequality or description with the graph that represents it. A graph may have more than one match.

a. 

b. 

c. 

d. 

c .... $x < 0$

b .... $0 < x$

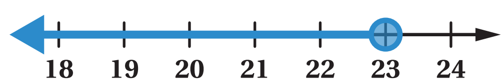
a .... $x \leq 0$

c .... The temperature of a freezer must be less than 0°F.

d .... The temperature of a freezer must be 0°F.

8 Fix the inequality $x > 23$ so that it represents the graph.

$x \leq 23$ or $23 \geq x$

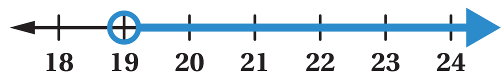


Activity
3

Name: Date: Period:

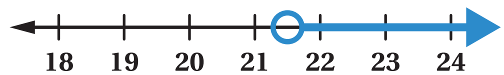
Inequalities Out of Context (continued)

9 Write an inequality that represents this graph.



$x > 19$ or $19 < x$

10 To represent this graph, Tiara wrote the inequality $21.5 < x$. Devon wrote the inequality $x < 21.5$.



Whose inequality is correct? Circle one.

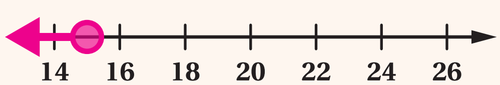
Tiara Devon Both Neither

Explain your thinking.

***Explanations vary.* Because the arrow is to the right, x has to be all the numbers that are greater than 21.5, which means that 21.5 is less than x.**

Explore More

11 Create a graph that represents all the values that make the inequality $x + 10 \leq 25$ true.



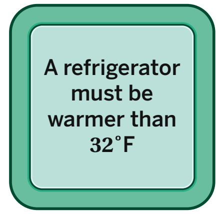
Explain your thinking.

***Explanations vary.* I put the point on 15 because that is the largest number that works. I know that $15 + 10 = 25$ so that means that 15 is the largest number that makes the inequality true, which means that it's the boundary point.**

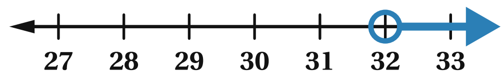
12 Synthesis

Describe how you can tell from each representation that 32 is *not* included in the inequality.

Responses vary. **In the inequality, the symbol is $>$, not $\geq$. On the graph, there is an open circle, so 32 is not included in the inequality. On the sign, it says "warmer than," which is like "more than." 32 is not more than 32.**



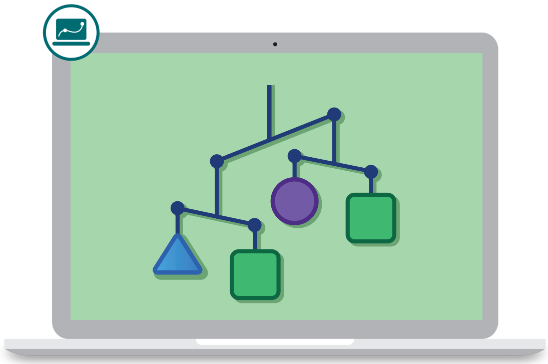
$x > 32$



Things to Remember:

Unit 6
Lesson
14

Name: ______ Date: ______ Period: ______



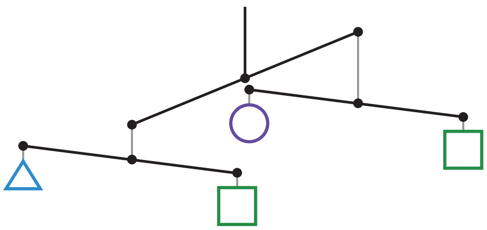
Unbalanced Hangers

Let's solve inequalities using hangers.

Warm-Up

1 Order the shapes in the hanger from *lightest* to *heaviest*.

Circle	**Lightest**
Triangle	
Square	**Heaviest**



Explain how you decided which shape was lightest.

***Explanations vary*. I know the right side is lighter than the left side because it is higher. This means that the circle is lighter than the triangle. I know the circle is lighter than the square because it's higher. This means it has to be the lightest shape.**

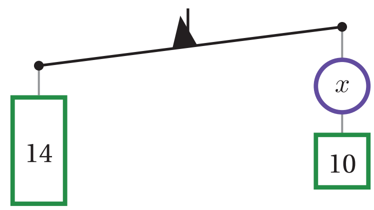
Activity 1

Name: .... Date: .... Period: ....

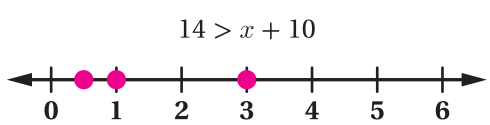
Unbalanced Hangers

2 Here is a hanger that is not balanced. What is one possible value of x?

Responses less than 4 are considered correct.



3 Plot your response from the previous problem on the number line. Determine at least two more possible weights and plot those on the number line.



Sample shown on number line.

4 Describe *all* of the possible values of x that keep the right side lighter.

***Responses vary*. The weight could be any number less than 4. 4 is not included because, if $x = 4$, the hanger would be balanced.**

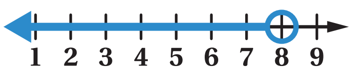
Name: ________ Date: ________ Period: ________

Unbalanced Hangers (continued)

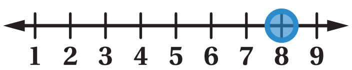
The **solutions to an inequality** include all of the possible values that make an inequality true.

5 This hanger represents the inequality $3x < 24$. What are the solutions to this inequality?

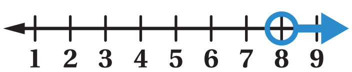
A. $x < 8$

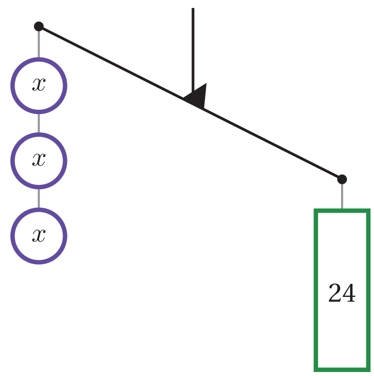


B. $x = 8$



C. $x > 8$

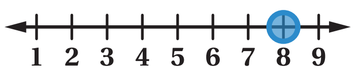




$3x < 24$

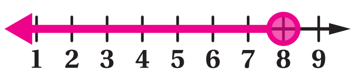
6 Here is the graph of the solution to the equation $3x = 24$.

$3x = 24$



a. Graph what you think the solutions to $3x \leq 24$ look like.

$3x \leq 24$



b. **Discuss:** How are the graphs of the solutions to $3x < 24$, $3x = 24$, and $3x \leq 24$ alike? How are they different?

Responses vary.

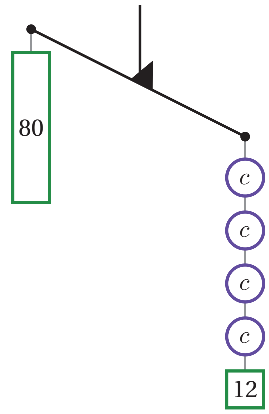
- They all use 8 either as the boundary point or as the solution.
- $3x < 24$ and $3x \leq 24$ have infinitely many solutions. $3x = 24$ only has one solution.
- $3x = 24$ and $3x \leq 24$ both have an "equal to" and $3x < 24$ doesn't.

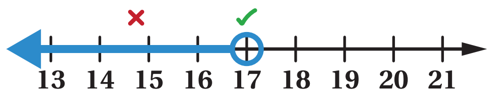
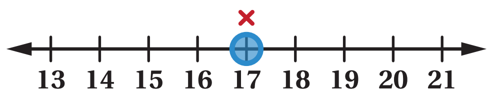
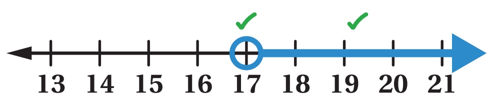
Activity 2

Name: ________ Date: ________ Period: ________

Solving Inequalities

7 Here are three possible solutions to the inequality $80 < 4c + 12$ and their graphs.

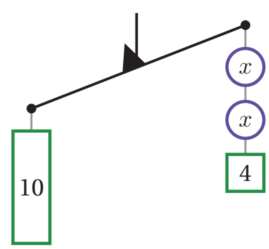


Possible Solution	Graph
$17 > c$	
$17 = c$	
$17 < c$	

Discuss: What do the checks and x's in the graphs mean?

***Responses vary.* In the last row of the table, the check on the open circle on 17 means 17 is the boundary. The x in the middle row means that 17 is not included. The check on the arrow facing right means this is correct because the solution for c includes all of the values that are greater than 17.**

8 What are the solutions to the inequality $10 \geq 2x + 4$? Explain your thinking. Use the hanger if it helps with your thinking.



$x \leq 3$ or $3 \geq x$. *Explanations vary.* First, I determined the boundary point of 3, which keeps the hanger balanced because of the "equal to" in the inequality. A value greater than 3 will make the right side of the hanger heavier. A value less than 3 will keep the hanger lighter on the right as shown in the hanger.

9 Jasmine and Terrance solved the inequality $10 \geq 2x + 4$. Jasmine says the solutions are $x \leq 3$. Terrance says the solutions are $3 \geq x$. Who is correct? Circle one.

Jasmine Terrance Both Neither

Explain your thinking.

***Explanations vary.* These both mean the same thing because $x \leq 3$ and $3 \geq x$ are both true when x is any number less than or equal to 3.**

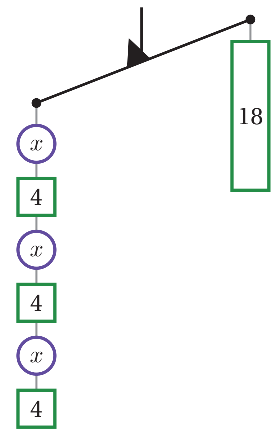
Note: Students who select "Jasmine," "Terrance," or "Both" will be marked correct.

Name: .... Date: .... Period: ....

Repeated Challenges

10 What are the solutions to the inequality $3(x + 4) \geq 18$?

Use the hanger if it helps with your thinking.



$x \geq 2$ or $2 \leq x$

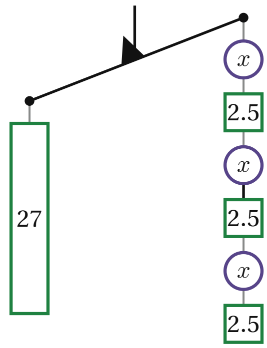
11
- Decide with a partner who will complete Column A and who will complete Column B.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.
- Solve as many inequalities as you have time for. Sense-making is more important than speed.

Column A	Column B
$4x + 2 \leq 10$ $x \leq 2$	$6x + 4 \leq 16$ $x \leq 2$
$12 > 2(x + 1)$ $5 > x$	$24 > 3(x + 3)$ $5 > x$
$10.4 \leq 2(x + 2.2)$ $3 \leq x$	$8(x + 1.1) \geq 32.8$ $x \geq 3$
$2x + \frac{3}{2} > \frac{17}{2}$ $x > \frac{7}{2}$	$4x + \frac{2}{3} > \frac{44}{3}$ $x > \frac{7}{2}$

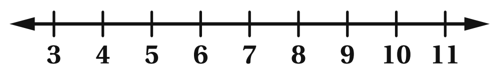
12 Synthesis

Describe a process you can use to determine the solutions to an inequality.
Use the hanger if it helps show your thinking.

***Responses vary*. First, I can determine the boundary point, which is the value of x that will balance the hanger, and whether the boundary point is a solution. Then, I can decide if the solutions are greater than or less than the boundary point by thinking about which side of the inequality is supposed to be greater.**



$27 > 3(x + 2.5)$



Things to Remember:

Unit 6
Lesson
15

Name: ______ Date: ______ Period: ______

Budgeting

Let's solve problems about budgeting and spending money.

Warm-Up

1. Here is a situation with hidden information. Let's make sense of it together as a class.

Mariana is selling magazine subscriptions. She earns ______ per week, plus ______ for every subscription she sells. She plans to buy soccer equipment with the money she earns.

This week, Mariana wants to buy a new ball. The cheapest ball she wants costs ______.

a **Discuss:** What is this situation about?

***Responses vary*. The situation is about a person named Mariana. She has a job where she sells magazine subscriptions. She likes to play soccer and wants to buy equipment. Currently, she wants to buy a ball.**

b Choose a value for each blank that could make sense.

***Responses vary*. She earns \$30 per week, plus \$5 for every subscription she sells. She plans to buy soccer equipment with the money she earns. This week, Mariana wants to buy a new ball. The cheapest ball she wants costs \$35.**

c For the values you chose, how many magazine subscriptions could Mariana sell in order to buy the ball?

***Responses vary*. With the values I chose, Mariana only needs to sell 1 magazine subscription to afford the ball, but she could also sell more, like 10 magazine subscriptions.**

Activity 1

Name: .... Date: .... Period: ....

Mariana's Magazines

2. Mariana is selling magazine subscriptions. She earns \$19 per week, plus \$3 for every subscription she sells. She plans to buy soccer equipment with the money she earns.

This week, Mariana wants to buy a new ball. The cheapest ball she wants costs \$43.

a Write and solve an equation to determine how many magazine subscriptions Mariana needs to sell to make \$43.

$19 + 3x = 43$

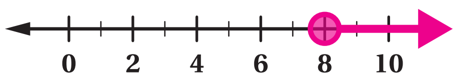
$x = 8$

Mariana needs to sell 8 magazine subscriptions.

b List other numbers of magazine subscriptions Mariana could sell and still buy the ball.

***Responses vary*. Mariana could also sell 10, 15, or 40 magazine subscriptions.**

c Write and graph an inequality to represent *all* the number of subscriptions Mariana could sell and still buy the ball.



$x \geq 8$

3. The next week, Mariana earns \$37. She wants to use it to buy soccer shorts and 5 pairs of socks. The shorts she wants each cost \$22.05.

a What do each pair of socks cost if Mariana spends exactly \$37 on the socks and shorts? (In Mariana's city, there is no sales tax.) Write and solve an equation if it helps you with your thinking.

$37 = 22.05 + 5x$

$x = 2.99$

Each pair of socks would cost \$2.99.

b Write an inequality to represent *all* the sock prices that Mariana could afford with \$37.

$x \leq 2.99$

Activity 2

Name: .... Date: .... Period: ....

Bao's Budgeting

4. Bao has $175 saved in his bank account. He wants to know how much money he can take out each month and still have at least $25 in the account a year from now.

a Which inequality represents Bao's situation?

A. $175 - 12x \leq 25$

B. $175 + 12x \leq 25$

C. $175 - 12x \geq 25$

D. $175 + 12x \geq 25$

b What does 12 represent?

12 represents how many months Bao will be withdrawing money.

c What does x represent?

x represents the amount of money Bao will withdraw each month.

d Bao and his friend try to solve the inequality. Bao's solutions start with $x \leq$. His friend's solutions start with $x \geq$. Which symbol makes sense for this situation? Explain your thinking.

$x \leq$. *Explanations vary.* There will be an amount Bao can withdraw that will get him to exactly $25 a year from now. Withdrawing less than that amount would also let him reach his goal.

e Solve the inequality you chose and explain what the solutions mean in Bao's situation.

$x \leq 12.50$. Bao can withdraw no more than $12.50 each month and reach his goal.

5. Bao is considering getting a part-time job. Instead of taking money out of his account each month, he would put money in. His account still has $175, and his goal is to have at least $1,000 in the account a year from now.

a Write an inequality where x represents the amount of money Bao should put in each month to reach his goal.

$175 + 12x \geq 1000$

b Solve the inequality you wrote and explain what the solutions mean in Bao's situation.

$x \geq 68.75$. Bao should put at least $68.75 in his account each month to reach his goal.

Synthesis

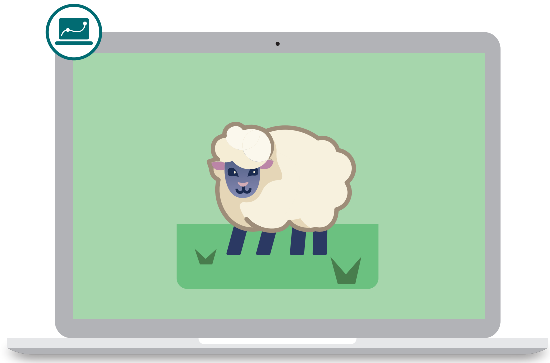
6. A student spends $2.50 on a tasty beverage every school day. They have a $30 gift card to Tea Time Cafe, and want to know how many beverages it can buy. Explain how the inequality $30 - 2.50x \geq 0$ represents this student's situation.

Responses vary. **The amount the student spends on beverages is 2.5 times the number of beverages, x, or $2.5x$. The student can subtract this quantity from 30, the total on the gift card, because the gift card decreases with each purchase. The student can use the gift card as long as the balance doesn't go below $0, which explains the inequality $30 - 2.50x \geq 0$.**

Things to Remember:

Unit 6
Lesson
16

Name: .... Date: .... Period: ....



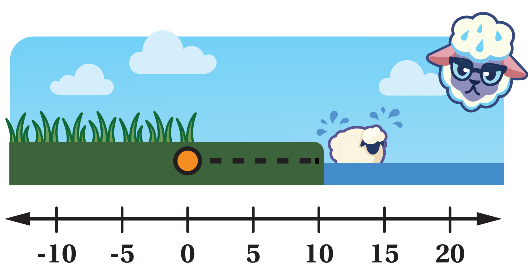
Shira the Sheep

Let's practice solving inequalities with positive and negative coefficients.

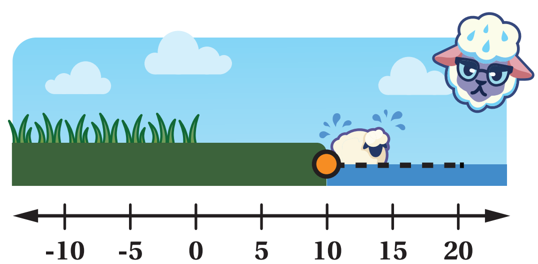
Warm-Up

1 **a** Shira the Sheep loves eating grass. She does not like water. Here are the graphs and results of different inequalities.

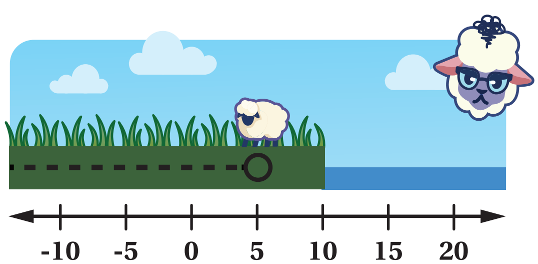
$x \geq 0$



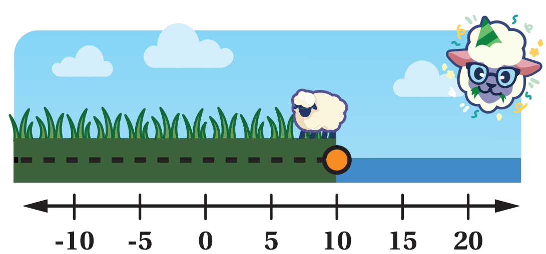
$x \geq 10$



$x < 5$



$x \leq 10$



b **Discuss:** What do you notice?

Responses vary.

- Shira is happy when she eats all of the grass.
- When Shira eats some of the grass, she does not get wet but is still unhappy.

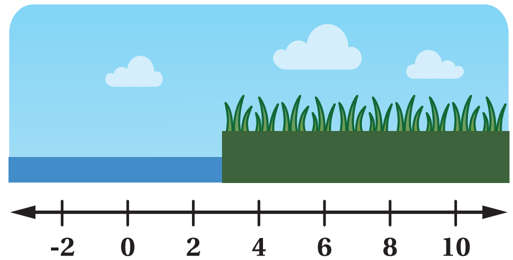
Activity 1

Name: .... Date: .... Period: ....

Shira the Sheep

2 The grass is represented by the inequality $5x > 15$. Solve the inequality to help Shira eat all the grass without falling in the water.

$x > 3$ (or equivalent)

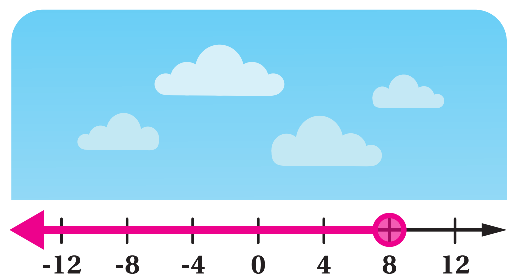


3 **a** Write the solutions to this inequality to help Shira eat all the grass.

$11 \geq 2x - 5$

$x \leq 8$ (or equivalent)

b Sketch the solutions to this inequality on the number line.

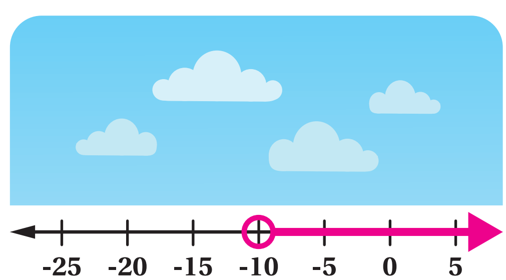


4 **a** Write the solutions to this inequality to help Shira eat all the grass.

$10 - 6x < 70$

$x > -10$ (or equivalent)

b Sketch the solutions to this inequality on the number line.



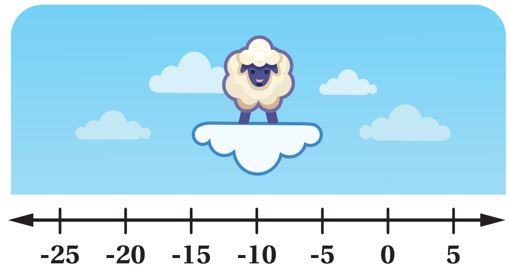
Activity 1

Name: .... Date: .... Period: ....

Shira the Sheep (continued)

5 Alma was solving the previous inequality, $10 - 6x < 70$. She knew the sheep needed to land at -10, but didn't know if the grass was to the right or left.

She wrote $10 - 6(0) < 70$. How might Alma's inequality help her decide where the grass is?



***Responses vary.* 0 is a nice number to test to see if all the values to the right of -10 are solutions. The inequality that Alma wrote, $10 - 6(0) < 70$, is true, which means that 0 is a solution. This tells her that all the numbers to the right of -10 are solutions.**

6 Solve this inequality to help Shira eat all the grass.

$-\frac{1}{2}x + 2 \leq 3$

$x \geq -2$ (or equivalent)

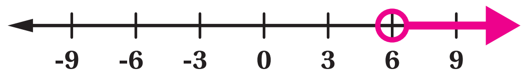
Activity 2

Name: ____________ Date: ________ Period: ________

Help Shira and Chloe

7 **a** Solve $25 - 4x < 1$.

$x > 6$



b Graph its solutions.

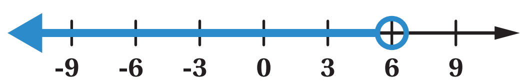
8 Chloe made a mistake solving the inequality $25 - 4x < 1$ and wrote $x < 6$. Explain what you think is incorrect about Chloe's work.

***Responses vary.* If $x < 6$ represents the solutions, then $x = 0$ should make the inequality true. The inequality $25 - 4(0) < 1$ is not true, so the inequality is incorrect.**

Chloe

25 – 4x < 1

x < 6



9 Solve as many inequalities as you have time for to help Shira eat all the grass.

a $8 \geq 3x - 13$

$7 \geq x$

b $-6x - 3 > 15$

$x < -3$

c $\frac{2}{3}x + 9 \leq 15$

$9 \geq x$

d $-89 \geq -12x - 5$

$7 \leq x$

e $3x - 5 > 4$

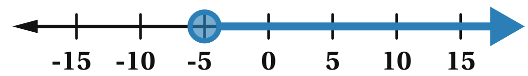
$x > 3$

f $0.2x + 0.6 \geq 0.8$

$x \geq 1$

10 Synthesis

Explain how you solve and graph the solutions to any inequality. Use the inequality $4 - 3x \leq 19$ if it helps you with your thinking.



Responses vary. First, determine the boundary point, which is the value of x when both sides are equal. The circle is filled in if there is a $\leq$ or $\geq$ symbol. Then, pick a different value for x and decide whether it's a solution. If it is, graph from the boundary point toward that number. If it isn't, graph away from that number.

Things to Remember:

Unit 6
Lesson
17

Name: Date: Period:

Write Them and Solve Them

Let's write and solve inequalities and examine what the solutions to those inequalities mean in context.

Warm-Up

1. Jamal volunteers to pass out sandwiches to people who are hungry in his community. He raised $85 and is trying to determine how many sandwiches he can buy for $6.25 each.

He writes the inequality $6.25x \leq 85$.

Then he solves the inequality and gets $x \leq 13.6$.

Select *all* the statements that are true about this situation.

- ☐ **A.** He can buy 13.6 sandwiches.
- ☐ **B.** He can buy 14 sandwiches.
- ☑ **C.** He can buy 12 sandwiches.
- ☑ **D.** He can buy 10 sandwiches.
- ☐ **E.** He can buy -4 sandwiches.

Activity 1

Name: .... Date: .... Period: ....

Orange Juice and Donuts

Kiandra wanted to surprise some friends before school with orange juice and donuts. At the store, an orange juice costs $\$2.15$ and a donut costs $\$0.75$. There is no sales tax. The store has a $\$10$ purchase minimum for credit cards. Kiandra paid with her credit card.

2. Write an inequality that describes Kiandra's situation.

$2.15x + 0.75x \geq 10$ (or equivalent)

3. Solve the inequality you wrote. Show your thinking.

$x \geq 3.45$. *Work varies.*

Solve the related equation:
$2.15x + 0.75x = 10$
$2.9x = 10$
$x = 3.45$

Test a value:
$x = 0$
$2.15(0) + 0.75(0) \geq 10$
$0 \geq 10$
This is not true, so 0 is not a solution and values less than 3.45 are not solutions.

This means that $x \geq 3.45$ are the solutions.

4. What is the meaning of the solutions to your inequality in this situation?

***Responses vary.* The solution means that Kiandra bought a donut and an orange juice for at least 4 people.**

Name: .... Date: .... Period: ....

Solve It!

5. You will use four pairs of cards for this activity. Each pair has a situation card and a corresponding support card.

- Decide with a partner who will have the situation card and who will have the support card.
- Switch roles after each round.

Situation Card Instructions

- Read the situation aloud.
- Write an inequality that represents the situation.
- Solve the inequality you wrote.
- Answer the question on the card using your solutions.

Support Card Instructions

- Help your partner by asking the questions on the card or other questions you think will support them.

You may use this page for workspace.

Situation A	Situation B
Inequality: $15x + 300 \le 750$ **Solution: $x \le 30$** **The marching band can spend up to \$30 for each new uniform.**	**Inequality: $5 - 0.6x \le 0$** **Solution: $x \ge 8\frac{1}{3}$** **The plants will need to be covered after $8\frac{1}{3}$ hours, or after 8:20 PM.**
Situation C	**Situation D**
Inequality: $4(x - 5) \le 65$ **Solution: $x \le 21.25$** **Each person can order a meal that is \$21.25 or less.**	**Inequality: $50 - 1.65x \ge 15$** **Solution: $x \le 21.21$** **Adriana's family can do up to 21 loads of laundry before the card automatically reloads.**

Synthesis

6. Sahana works at the pet store and gets paid $\$9.50$ per hour. She needs to make at least $\$235$ each week in order to pay her bills. Describe how to write an inequality that represents Sahana's situation.

***Responses vary.* Because Sahana gets paid $\$9.50$ every hour, I would write $9.50x$, where x represents the hours she works. Sahana needs to make $\$235$ or more, so $9.50x$ needs to be greater than or equal to $\$235$.**

Things to Remember:

Solve It!

Directions: Make one copy per pair of students. Then pre-cut the cards and give each pair of students one set of four situation cards and four support cards.

Situation Card A

The school marching band has a $750 budget. They paid $300 in competition fees, and still need to buy 15 new uniforms. How much could the marching band spend on each uniform?

Support Card A

After reading the problem:

1. What information is important?

After your partner writes an inequality:

2. How could you start solving?

After your partner solves the inequality:

3. What does your solution mean?
4. Do you need to round your solution?

Situation Card B

LaShawn is a farmer in a city and needs to cover the plants when the temperature gets below 0°C. At noon, the temperature was 5°C and dropped at a steady rate of 0.6 degrees per hour. When do LaShawn's plants need to be covered?

Support Card B

After reading the problem:

1. What is the problem about?
2. What information is important?

After your partner writes an inequality:

3. What does each number represent?

After your partner solves the inequality:

4. Is the boundary included in the solution?
5. What does your solution mean?

Solve It!

Situation Card C

Rudra is taking 3 friends to dinner. If Rudra has $\$65$ and a coupon for $\$5$ off each meal, how much can each person spend?

Support Card C

After reading the problem:

1. What information is important?

After your partner writes an inequality:

2. How is Rudra's total amount represented?
3. How could you start solving?

After your partner solves the inequality:

4. Is the boundary included in the solution?
5. What does your solution mean?

Situation Card D

Adriana's apartment building has a washing machine that uses a card for payment. The card automatically reloads when the balance falls below $\$15$. If the card balance is currently $\$50$ and a load of laundry costs $\$1.65$, how many loads can Adriana's family do before the card reloads?

Support Card D

After reading the problem:

1. What is the problem about?
2. What information is important?

After your partner writes an inequality:

3. How is each load's cost represented?
4. How could you start solving?

After your partner solves the inequality:

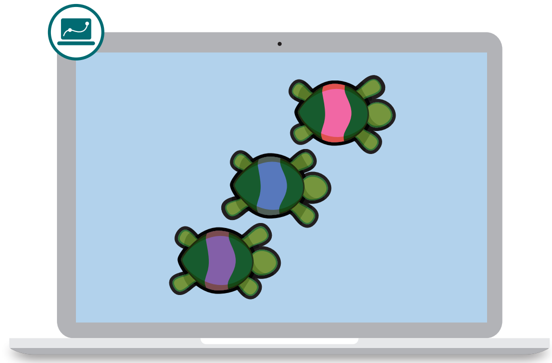
5. What does your solution mean?

Unit 3
Lesson
1

Name: ________ Date: ________ Period: ________

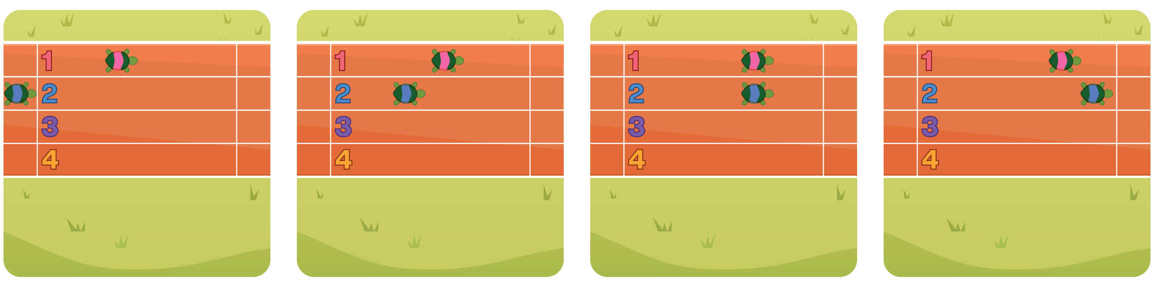
Turtle Time Trials

Let's explore a turtle race with multiple representations.



Warm-Up

1 Let's watch a short animation. Write a story about what you see.



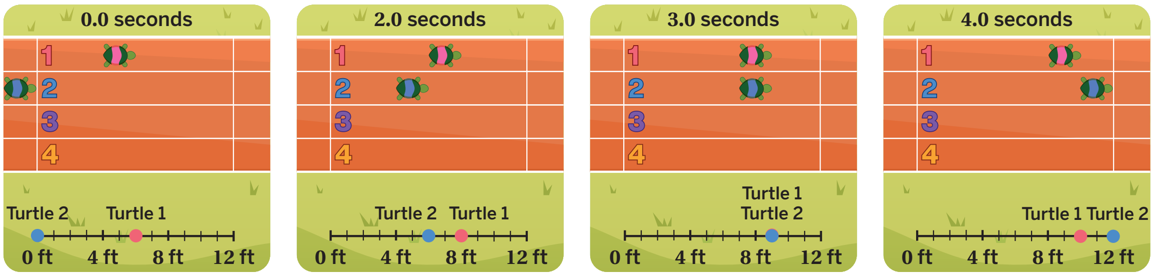
Responses vary. **Two turtles are racing. The turtle in Lane 1 gets a head start, but the turtle in Lane 2 is faster and wins the race.**

Activity 1

Name: ______ Date: ______ Period: ______

Turtle Race

2 Let's watch the same animation from the Warm-Up, but with additional information.



Complete the table.

Time (sec)	Distance of Turtle 1 (ft)	Distance of Turtle 2 (ft)
0	6	0
1	7	3
2	8	**6**
3	9	**9**

3 What is Turtle 2's speed as a *unit rate*? Explain your thinking.

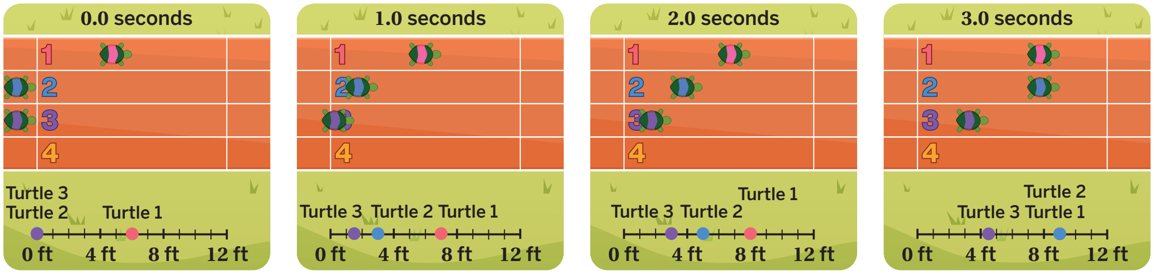
3 feet per second. *Explanations vary*. By looking at Turtle 2's distance at 0 seconds and 1 second, I can see that the distance increases by 3 feet. The same is true between 1 second and 2 seconds, as well as between 2 seconds and 3 seconds.

4 Let's watch an animation showing Turtle 3 running in the same race.

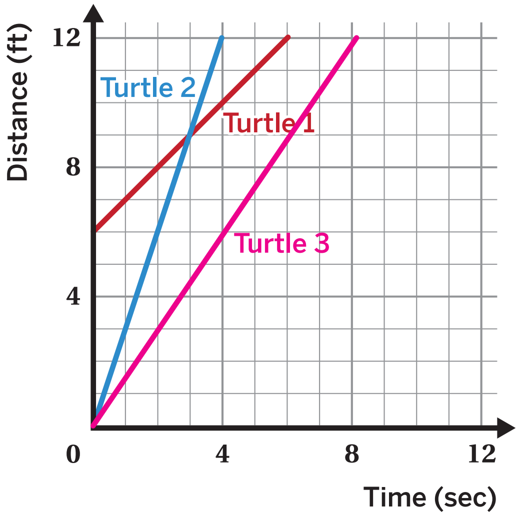
Name: ________ Date: ________ Period: ________

Turtle Race (continued)

5 Here is some information from the race with Turtle 3.



Graph the relationship between distance and time for Turtle 3.



6 Evan says that the relationship between distance and time is *proportional* for all three turtles.

Is Evan's claim correct?

Yes No I'm not sure

Explain your thinking.

***Explanations vary.* The relationship between distance and time is proportional for Turtles 2 and 3 because the graphs are lines that pass through the origin. The relationship between distance and time is not proportional for Turtle 1 because the graph is a line that doesn't pass through the origin.**

7 **a** What is Turtle 3's speed as a *unit rate*? Explain your thinking.

1.5 feet per second

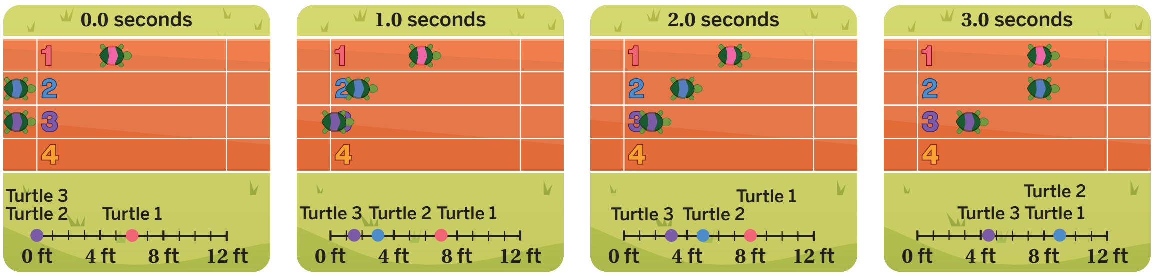
b How does this unit rate relate to the *slope* of Turtle 3's line?

***Responses vary.* The slope has the same value as the unit rate.**

Name: .... Date: .... Period: ....

Ready Turtle Four

8 Here is some information from the race with three turtles. In Lane 4, another turtle is running in the same race.



a Write an equation for Turtle 4 to make it finish in whatever place you want.

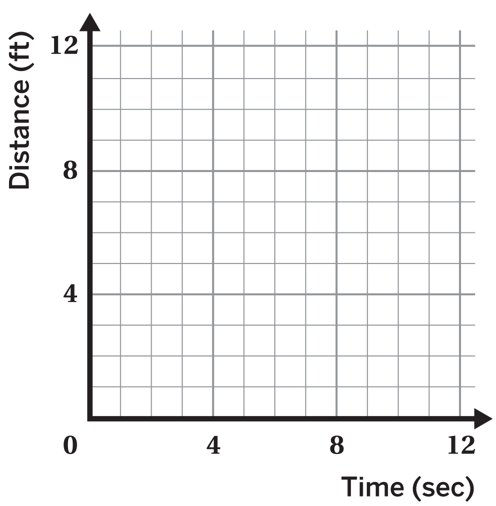
Turtle	Equation
1	$d = 6 + 1t$
2	$d = 3t$
3	$d = 1.5t$
4	*Responses vary.*

b **Discuss:** Based on your equation, how does Turtle 4's race compare to the other turtles?

***Responses vary.* My equation was $d = 0.5t$, which means that the turtle starts the race at the beginning and moves much slower than all of the other turtles.**

9 **a** Create your own turtle race by sketching lines for up to four turtles. At least one line should be proportional. Use a different color for each turtle.

Responses vary.



b **Discuss:** Using the information in your graph, tell a story about your turtle race.

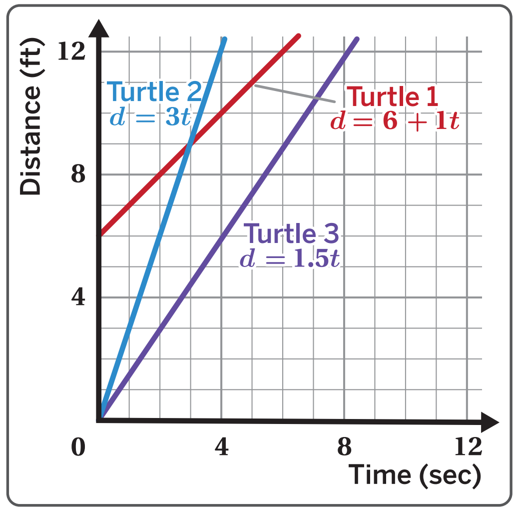
Responses vary based on the graphs students create.

10 Synthesis

How do you use tables, graphs, and equations to compare the turtles in a race?

Use the example if it helps with your thinking.

Responses vary. **In a table, I can calculate the speed of each turtle by seeing how much distance it gains in one second. In a graph, I can determine the speed of each turtle by examining the steepness of each turtle's graph. The steepest one, which is the one with the greatest slope, is fastest. In an equation, the coefficient (the number that is multiplied by t) determines the speed of the turtle. The greatest coefficient is the fastest turtle.**



Time (sec)	Distance of Turtle 1 (ft)	Distance of Turtle 2 (ft)	Distance of Turtle 3 (ft)
0	6	0	0
1	7	3	1.5
2	8	6	3

Things to Remember:

Name: .... Date: .... Period: ....

Water Tank

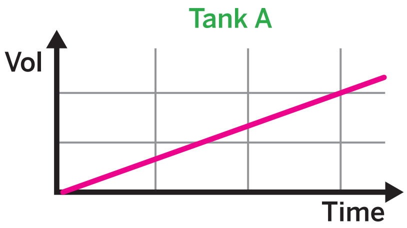
Let's analyze graphs of proportional equations.

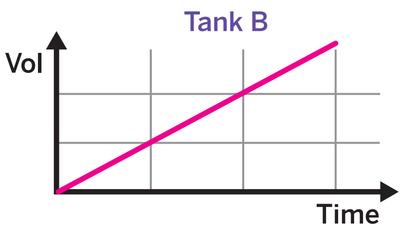
Warm-Up

1 Let's watch two tanks fill up with water.

Draw a graph that represents the relationship between water volume and time for each tank.

Responses vary.





2 Let's watch the tanks fill up with water again, but this time with more information.

Discuss: How are the graphs like your drawing from the previous question? How are they different?

***Responses vary.* The graphs are like my drawing because they both have two lines that pass through (0, 0). My drawing used a different scale for the axes, so my lines looked more/less steep than the lines in these graphs.**

Activity 1

Name: ________ Date: ________ Period: ________

Two Tanks

3 Erendirani and Jamal both calculated slopes.

Erendirani says the slope of Tank A's line is $\frac{10}{2}$.

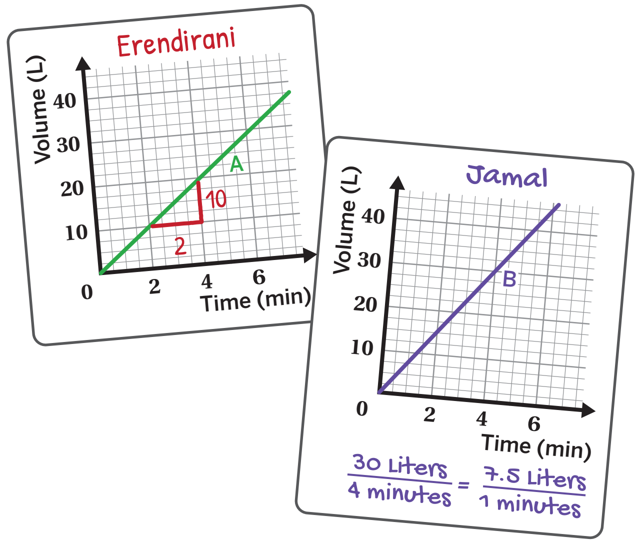
Jamal says the slope of Tank B's line is 7.5.

Whose claim is correct?

Erendirani's Jamal's Both Neither

Explain your thinking.

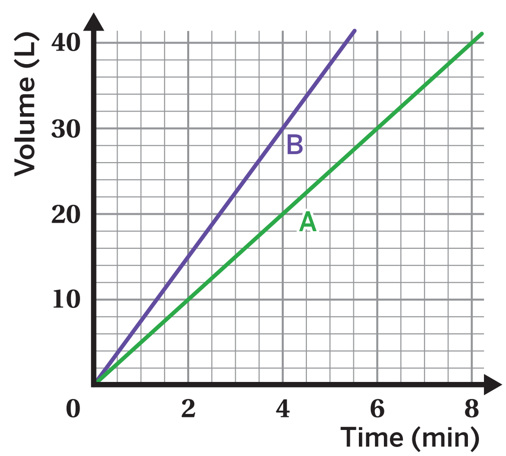
***Explanations vary*. I can use the vertical and horizontal sides of the slope triangle to find the slope. I can also use the slope triangle sides to create a proportion to find the slope. Note: Students who select either option or select both will be marked correct.**



4 Here are the graphs of Tank A and Tank B, shown on the same coordinate plane.

Explain what the slopes of each line represent for Tank A and Tank B.

***Responses vary*. The slopes represent unit rates for filling up each tank. The slope for Tank A, 5, means that Tank A fills up at a rate of 5 liters every minute. The slope for Tank B, 7.5, means that Tank B fills up at a rate of 7.5 liters every minute.**



5 For Tank A, the equation $V = 5t$ represents the volume, V, after t minutes.

a For Tank B, write an equation to represent the volume, V, after t minutes.

$V = 7.5t$ (or equivalent)

b How can you tell from the equations that these relationships are proportional?

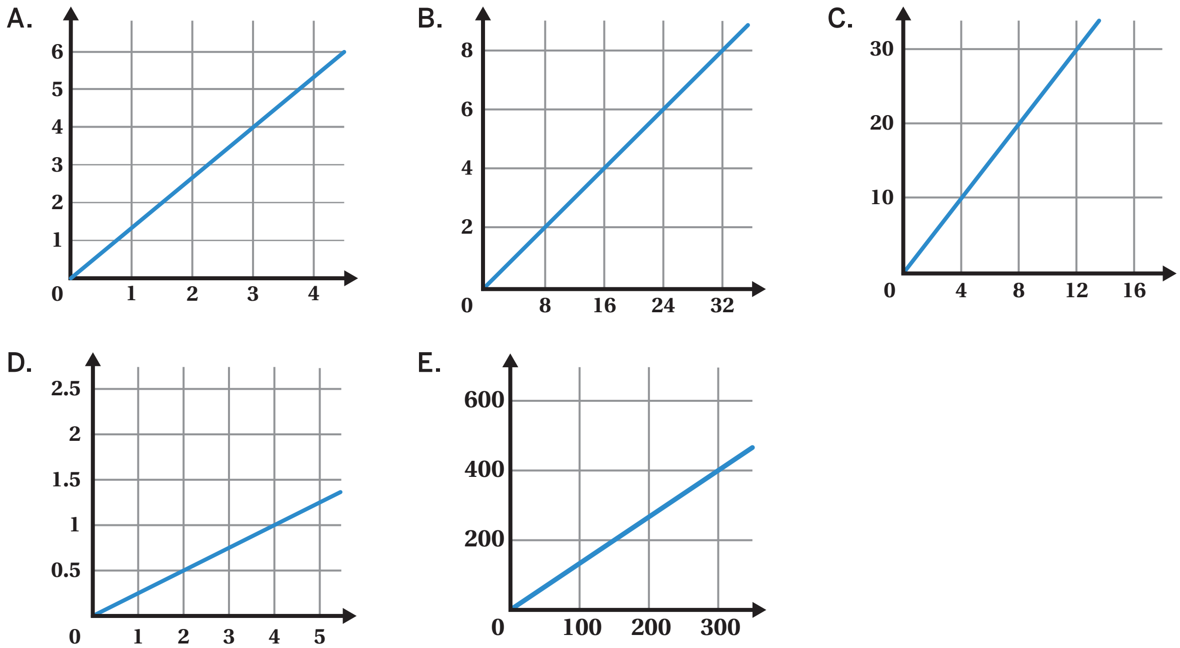
***Responses vary*. For each tank, the volume of water (in liters) can be found by multiplying the time (in minutes) by the slope.**

Activity 2

Name: ______ Date: ______ Period: ______

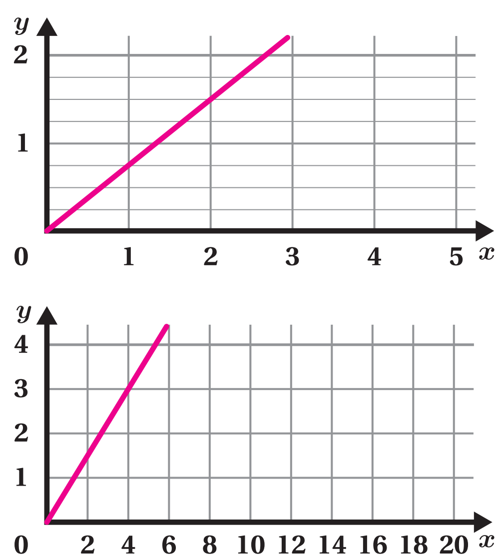
Pairing Graphs and Equations

6 Match each graph to the equation that represents the same proportional relationship.



Equation	$y = \frac{4}{3}x$	$y = \frac{1}{4}x$	$y = 2.5x$
Graph	Graph A Graph E	Graph B Graph D	Graph C

7 **a** Draw the graph of $y = 0.75x$ on each set of axes.



b Explain how you decided where to draw the lines.

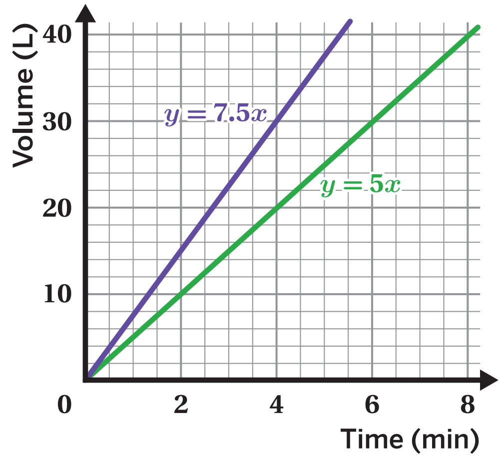
Explanations vary. On the first graph, I found the point $(1, 0.75)$ and connected a line from $(0, 0)$ to $(1, 0.75)$. For the second graph, I converted 0.75 to $\frac{3}{4}$. I started at $(0, 0)$ and made a slope triangle with sides that are 4 units horizontally and 3 units vertically. Then I connected the points to draw the line.

8 Synthesis

How are the equations of proportional relationships related to their graphs?

Use the examples if it helps with your thinking.

Responses vary. **The equations of proportional relationships always have the form $y = mx$, where m is the slope of the line.**



Things to Remember:

Unit 3

Lesson

3

Name: .... Date: .... Period: ....

Proportional Posters

Let's compare proportional relationships.

Warm-Up

1. The table, graph, and equation all represent the same situation.

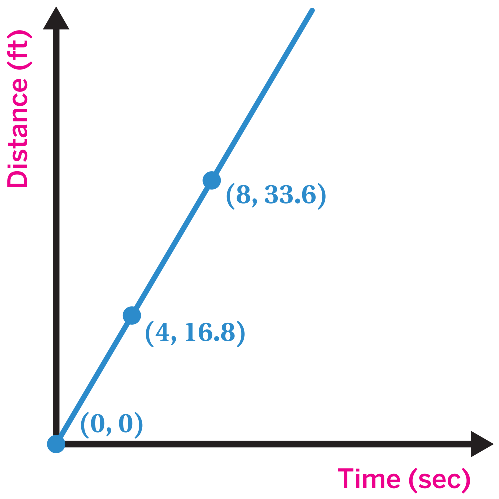
Table

x	y
0	0
4	16.8
8	33.6

Equation

$y = 4.2x$

Graph



a Describe a situation that could be represented by the table, graph, and equation.

***Responses vary.* A frog jumps 16.8 feet every 4 seconds.**

b What does x and y represent in your situation?

***Responses vary.* In my situation, x represents time (in seconds) and y represents distance jumped (in feet).**

c Label the axes of the graph to match your situation.

Activity 1

Name: .... Date: .... Period: ....

Comparing Two Different Representations

2. You will use the Activity 1 Sheet to select a situation to explore.

Situation: ....

3. Create a poster. Here is what your poster should include:

- ☐ Your names.
- ☐ The name of the situation you chose.
- ☐ Your answers to the questions about the situation.
- ☐ Explanations or calculations that show your reasoning for each of the answers.
- ☐ At least two new mathematical representations that allow you to compare the proportional relationships in your situation. Representations can include graphs, equations, and/or tables.

Explore More

Situation A: Maki and Ren

4. Maki and Ren earned a total of \$210. They each worked the same number of hours. Who earned more money? How much more?

If Maki and Ren each worked the same number of hours to earn \$210, then their combined income is \$15.40 per hour. This means they worked about 13.64 hours each since $\frac{210}{15.4} \approx 13.64$. Since Maki makes \$8.40 per hour, Maki earned about \$115. Since Ren makes \$7 per hour, Ren made about \$95. Maki earned \$20 more than Ren.

5. Maki and Ren earned a total of \$315. They each earned the same amount of money. Who worked more hours? How many more?

If Maki and Ren each earned the same amount of money and combined it to get \$315, then they each made $\frac{\$315}{2} = \$157.50$. Maki must have worked $\frac{\$157.80}{8.40} = 18.75$ hours, while Ren worked $\frac{\$157.50}{7} = 22.5$ hours. Ren worked 3.75 more hours.

Situation B: Ahmed's Lemonade

4. Ahmed used 30 cups of water to make some lemonade. According to Recipe 1, how much lemonade mix should Ahmed use?

***Responses vary.* Ahmed should use 7.5 cups of lemonade mix. Using Recipe 1, with $y = 30$, then $30 = 4x$, and $x = \frac{30}{4} = 7.5$ cups of lemonade mix.**

5. Ahmed tried to follow Recipe 2 by using 4 cups of water and 20 cups of lemonade mix. What was the mistake? How can this be fixed?

***Responses vary.* Recipe 2 requires the amount of water to be 5 times the amount of lemonade mix. The mistake was that Ahmed made the amount of mix to be 5 times the amount of water. For 20 cups of mix, Ahmed should use $20 \cdot 5 = 100$ cups of water. Ahmed already added 4 cups of water, so 96 additional cups are needed to fix this mistake.**

Activity 2

Name: ______ Date: ______ Period: ______

Gallery Tour

As a class, you will take a gallery tour.

6. What features of your classmates' posters helped you understand their thinking?

Responses vary.

- **Seeing how my classmates made notes about the different representations**
- **Seeing the specific points labeled on their graphs**
- **Seeing how my classmates labeled the unit rate in tables and graphs**

7. Describe something you would change about your poster now that you have seen other groups' work.

Responses vary.

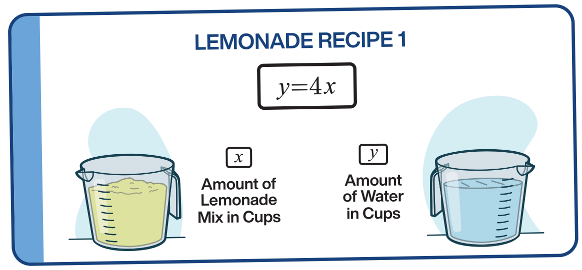
- **I would add more information to my graph to clearly show the unit rate.**
- **I would add arrows to show how different unit rates are connected.**

Synthesis

8. How can you compare proportional relationships when they are represented differently?

Use the example if it helps with your thinking.

Responses vary. You can compare proportional relationships that are represented differently by looking at the unit rate. For example, Recipe 1 gives an equation where the unit rate is 4 lemonade scoops for every 1 cup of water. Recipe 2 gives a table where the unit rate is 5 scoops of lemonade powder for every 1 cup of water.



LEMONADE RECIPE 2

Lemonade Mix (cups)	Water (cups)
10	50
13	65
21	105

Things to Remember:

Name: ______________________ Date: ____________ Period: ____________

Comparing Two Different Representations

Choose one of the situations. Then complete the problems by creating a poster.

Situation A: Maki and Ren

Maki babysits a neighbor's children. Ren mows another neighbor's lawn.

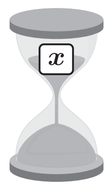
a. Who makes more money after working 12 hours? Show or explain your thinking.

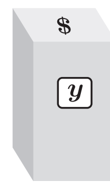
b. What is the rate of change for each situation and what does it mean?

c. How long would it take each person to earn \$150? Show or explain your thinking.

MAKI'S EARNINGS

$\boxed{y = 8.40x}$





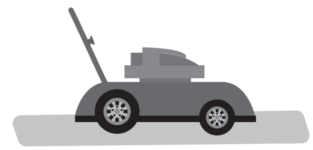
\# of hours worked

Amount of money earned

REN EARNS:

$\boxed{\$7}$

PER HOUR



Situation B: Ahmed's Lemonade

Ahmed plans to start a lemonade stand and is trying to perfect the recipe.

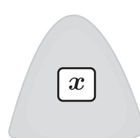
a. If Ahmed has 16 cups of lemonade mix, how many cups of water are needed for each recipe?

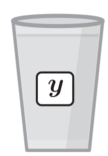
b. What is the rate of change for each situation and what does it mean?

c. Ahmed has 5 gallons (80 cups) of water and 20 cups of lemonade mix. Which lemonade recipe should Ahmed use? Show your thinking.

LEMONADE RECIPE 1

$\boxed{y = 4x}$





Amount of lemonade mix in cups

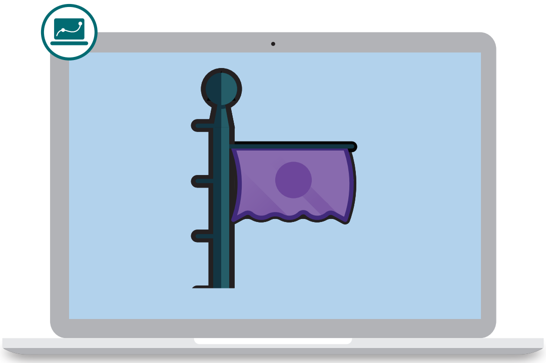
Amount of water in cups

LEMONADE RECIPE 2

Lemonade mix (cups)	Water (cups)
10	50
13	65
21	105

Unit 3
Lesson
4

Name: Date: Period:



Flags

Let's explore non-proportional linear relationships.

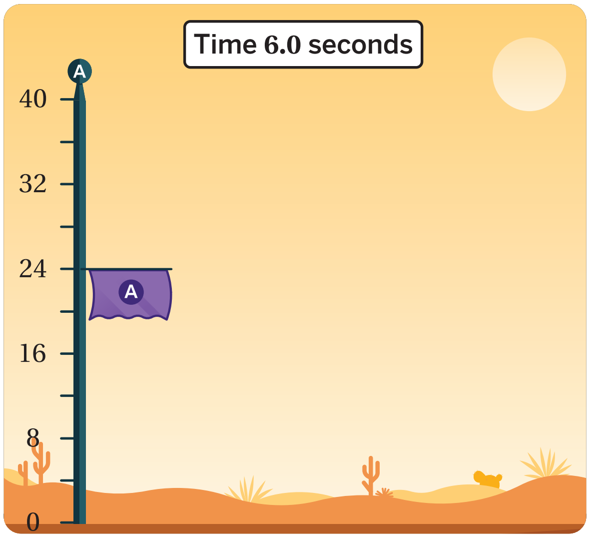
Warm-Up

1 Let's watch an animation of a flag and its graph.

Discuss: What do you notice? What do you wonder?

Responses vary.

- The flag starts on the ground.
- The flag moves up and the line on the graph moves up (as you look from left to right).
- The flag stops at 24 feet, which is 16 feet below the highest mark on the flagpole.
- Does the flag keep going up after 6 seconds?
- How long would it take for the flag to reach the top of the flagpole?
- What would the slope of the graph look like if the rate at which the flag was raised changed as it was rising?



Activity 1

Name: .... Date: .... Period: ....

Forming an Equation

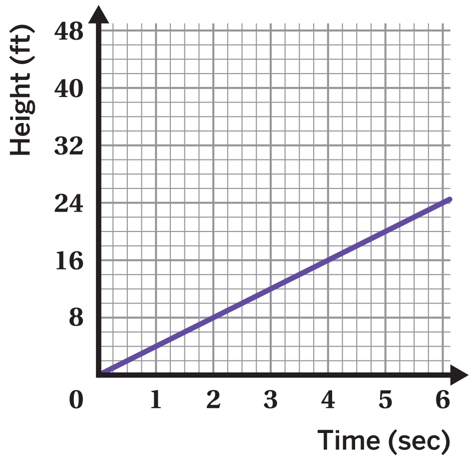
2 This line represents the relationship between height and time for the flag.

a What is the slope of the line?

4 (or equivalent)

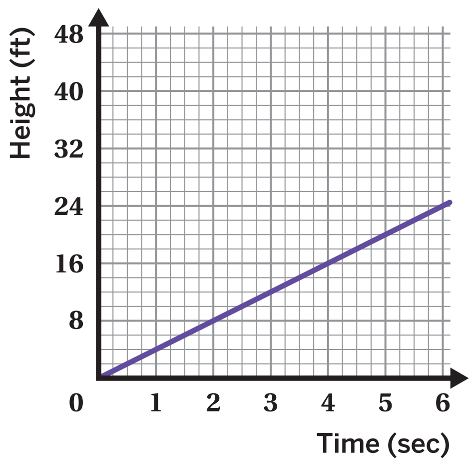
b What does this number represent in the situation?

***Responses vary*. The flag rises at a rate of 4 feet per second.**



3 Write an equation for the flag's height, h, after t seconds.

$h = 4t$ (or equivalent)



Activity
2

Name: .... Date: .... Period: ....

Comparing Two Flags

Let's explore the relationship between height and time for a new flag, Flag B.

4 Let's watch an animation of Flag A and Flag B.

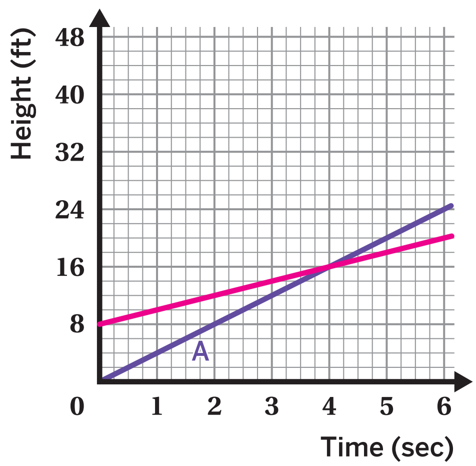
Describe the behavior of Flag B.

Responses vary.

- **Flag B starts higher than Flag A, but ends lower.**
- **Flag B moves up.**
- **Flag B begins at 8 feet, rises for 6 seconds, and ends at 20 feet.**

5 Here is a graph representing the height of Flag A over time.

Draw a graph representing the height of Flag B over time.



6 Here is a graph and a table with information about Flag B.

Cho claims: *Flag B rises at 2 feet per second, so its equation is* $h = 2t$.

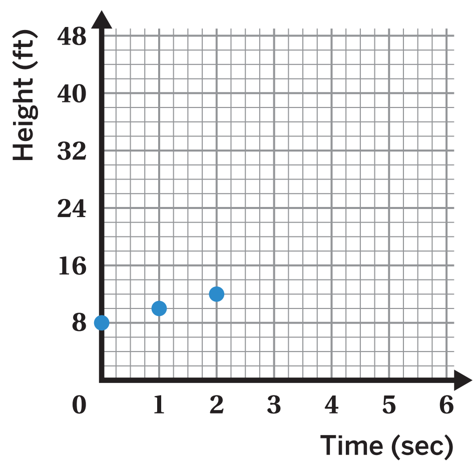
Part of Cho's claim is correct and part of it is incorrect.

- What about Cho's claim is correct?

  ***Responses vary.* Cho's claim is correct in that the flag rises 2 feet per second, so $2t$ will be part of the equation.**

- What is incorrect?

  ***Responses vary.* Cho's claim is incorrect because the equation $h = 2t$ means Flag B would start at 0 feet, instead of at 8 feet. The equation should be $h = 2t + 8$.**



Time (sec)	0	1	2
Height (ft)	8	10	12

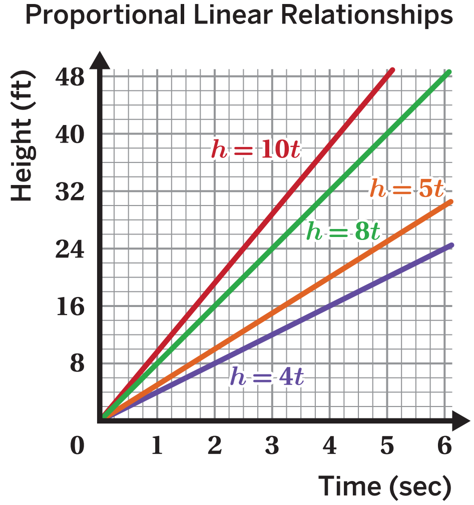
Activity
3

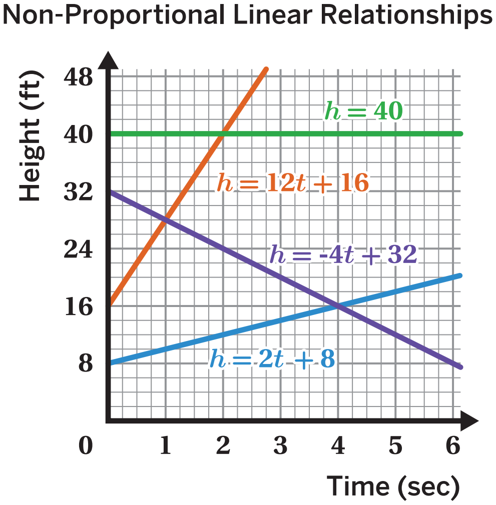
Name: .... Date: .... Period: ....

Exploring Linear Relationships

7 A relationship between two quantities is linear if there is a constant rate of change. It's called a **linear relationship** because its graph is a line.

Some linear relationships are proportional and some are non-proportional. Here are some examples.





What are some differences between proportional and non-proportional linear relationships?

Responses vary. **A line that represents a proportional relationship will pass through the origin. A line that represents a non-proportional linear relationship will not pass through the origin.**

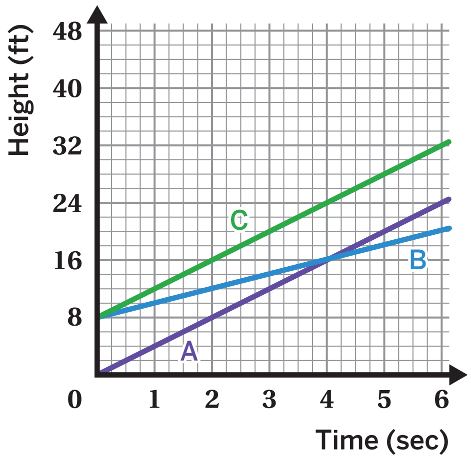
Name: Date: Period:

Exploring Linear Relationships (continued)

Now let's explore the relationship between height and time for a new flag, Flag C.

8 Based on the graph, select *all* the true statements.

- ☐ **A.** Flag C's graph is proportional.
- ☑ **B.** Flag C's graph is linear.
- ☐ **C.** The slope of Flag C's line is 8.
- ☐ **D.** Flag C rises at 8 feet per second.
- ☑ **E.** Flag C rises at 4 feet per second.



9 Write an equation for the height of Flag C.

Flag	Equation
A	$h = 4t$
B	$h = 2t + 8$
C	$h = 4t + 8$ (or equivalent)

10 Can you write a new equation for Flag C so that the flag:

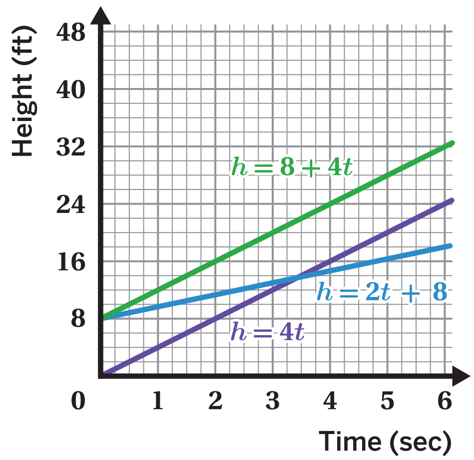
Responses vary.

- Starts high?
  $h = 4t + 20$
- Starts low?
  $h = 4t$
- Moves fast?
  $h = 12t + 8$
- Moves slow?
  $h = t + 8$

11 Synthesis

How can you identify a non-proportional linear relationship in a graph and in an equation?

Use the example if it helps with your thinking.



Responses vary. **I can identify a non-proportional linear relationship in a graph because the line does not pass through the point $(0, 0)$. In an equation, I can look for a starting value, like the value 8 in the equations $h = 8 + 4t$ and $h = 2t + 8$.**

Things to Remember:

Unit 3
Lesson
5

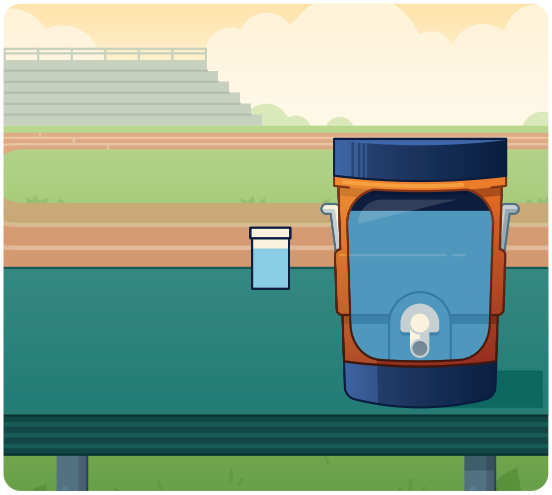
Name: .... Date: .... Period: ....

Water Cooler

Let's explore lines with different slopes.

Warm-Up

1 Let's watch a short animation about a water cooler.



a **Discuss:** What do you notice? What do you wonder?

Responses vary.

- **I notice that water is moving from the cooler to the cups.**
- **I notice that the water level in the cooler is decreasing.**
- **I wonder how many cups it would take to empty the water cooler.**
- **I wonder how much water is poured into each cup and whether it's the same amount of water for each cup.**

b Make an initial prediction. After how many cups do you think the cooler will run out of water?

Responses vary.

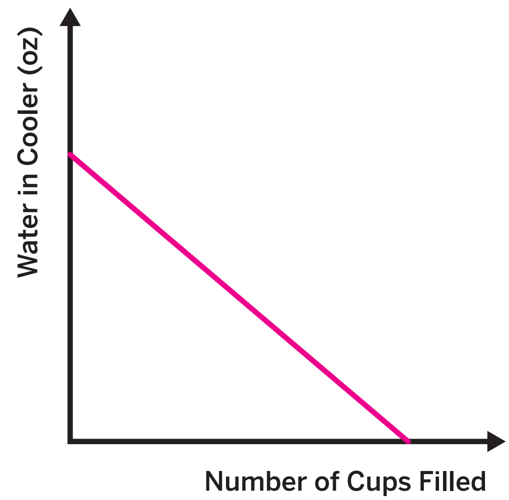
- **100 cups**
- **50 cups**
- **80 cups**

Activity 1

Name: ________ Date: ________ Period: ________

Cooler Cups

2 **a** Sketch the relationship between the ounces of water remaining in the cooler and number of cups filled.



b **Discuss:** How can you see this relationship in your sketch?

***Responses vary.* As the number of cups filled increases, the amount of water left in the cooler decreases. That's why my drawing shows a graph decreasing from left to right.**

3 The table shows the amount of water remaining in the cooler after 0, 1, and 2 cups have been filled.

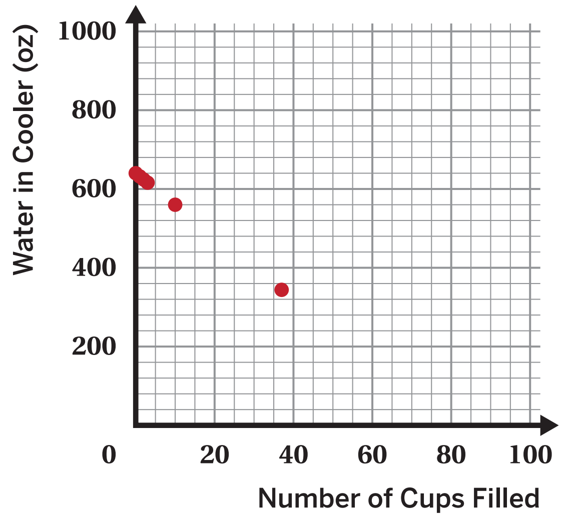
Determine the missing values.

Number of Cups Filled	0	1	2	3	...	10	37
Water in Cooler (oz)	640	632	624	**616**	...	**560**	**344**

4 A classmate plotted their points from the table onto a graph.

Write an equation representing the amount of water in the cooler, y, after filling x cups.

$y = 640 - 8x$ (or equivalent)



Name: .... Date: .... Period: ....

Cooler Cups (continued)

5 Make a final prediction:

After how many cups will the cooler run out of water?

80 cups

Explain your thinking.

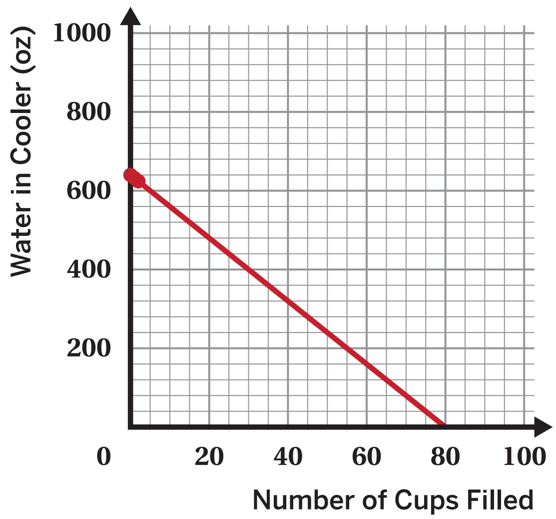
***Explanations vary.* I used the graph and found the value of x when y (the amount of water in the cooler) was 0.**

6 Let's watch an animation to see how many cups the cooler actually fills!

7 Here is the equation that a classmate wrote for the relationship between the water in the cooler and the number of cups:

$y = 640 - 8x$

The **vertical intercept** is the point where the graph of a line crosses the vertical axis.



a What is the vertical intercept of this graph, and what does it represent in this situation?

***Responses vary.* The vertical intercept is the point $(0, 640)$, which represents how many ounces of water were in the cooler before any cups were filled.**

b What is the slope of the line, and what does it represent in this situation?

***Responses vary.* The slope is -8, which means the amount of water in the cooler decreases at a constant rate of 8 ounces each time a cup is filled.**

Activity 2

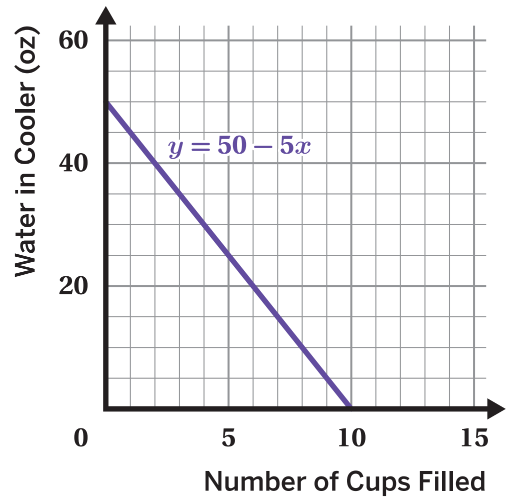
Name: ______ Date: ______ Period: ______

A New Water Cooler

The line $y = 50 - 5x$ represents a different water cooler situation.

8 Write a story that this graph could represent.

***Responses vary.* A water cooler at a soccer practice started with 50 ounces of water in it. Each time someone filled a cup, the amount of water in the cooler decreased.**



9 Caasi says that the water in the cooler decreases by 20 ounces for every 4 cups filled.

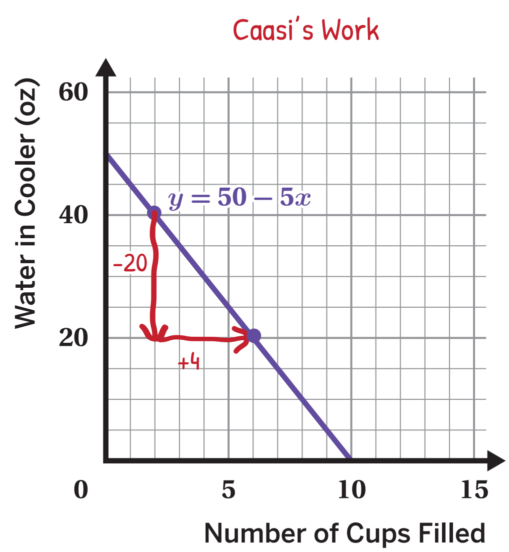
Jamar says that the water in the cooler decreases by 5 ounces for every 1 cup filled.

Whose claim is correct?

Caasi's Jamar's Both Neither

Explain your thinking.

***Explanations vary.* Caasi's and Jamar's claims are both correct because they're giving equivalent rates. The rates can both be represented by similar slope triangles on the graph. Note: Students who select either option or select both will be marked correct.**



10 Look at the graph of $y = 50 - 5x$ from the previous problems. The line crosses the horizontal axis at the point $(10, 0)$. This point is called the **horizontal intercept**.

What does this intercept represent in this situation?

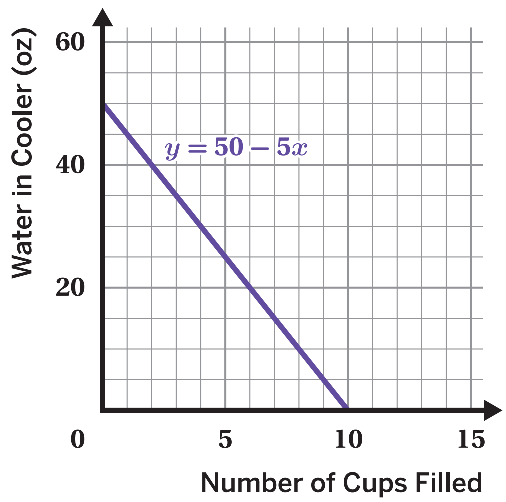
***Responses vary.* $(10, 0)$ represents how many cups it takes to empty the cooler.**

Name: .... Date: .... Period: ....

A New Water Cooler (continued)

11 Revise your original story so that it is stronger and clearer.

Responses vary. A water cooler at a soccer practice started with 50 ounces of water in it, before any cups were filled. Each cup holds 5 ounces of water. After 10 cups were filled, the water cooler ran out of water.

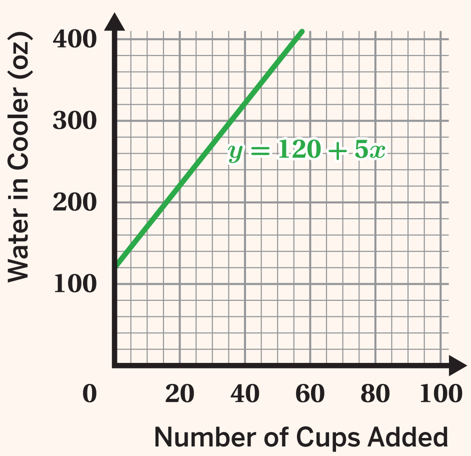


Explore More

12 The line $y = 120 + 5x$ represents a third water cooler situation.

a Write a story that this graph could represent.

Responses vary. A water cooler had 120 ounces of water in it and needed to be filled with more water. 5 ounces of water was added to the cooler for every cup added.



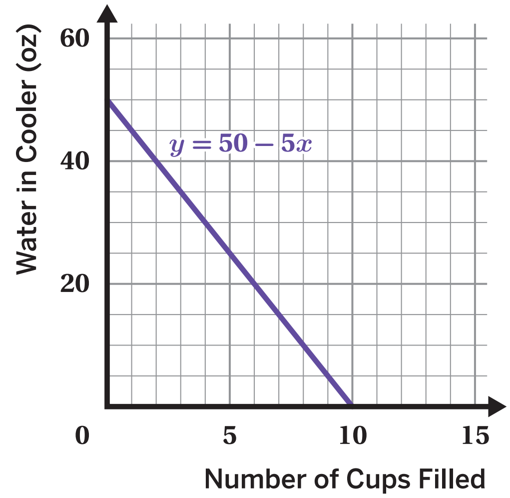
b How is this situation like the other two water cooler situations? How is it different?

Responses vary. This situation is like the second water cooler situation because each cup here also holds 5 ounces of water. This situation is different because the amount of water is increasing instead of decreasing, since the cups of water are added to the cooler instead of taken from it.

13 Synthesis

What are 2–3 things you can determine about a situation from its graph?

Use the graph if it helps with your thinking.



Responses vary.

- The starting point. For example, the water cooler here started with 50 ounces of water in it before any cups were filled.
- Whether there is an increase or decrease in the y-variable. In this case, after each cup was filled, the water cooler lost 5 ounces of water.
- The x-value when y is 0. In this graph, for example, the water cooler ran out of water after 10 cups.

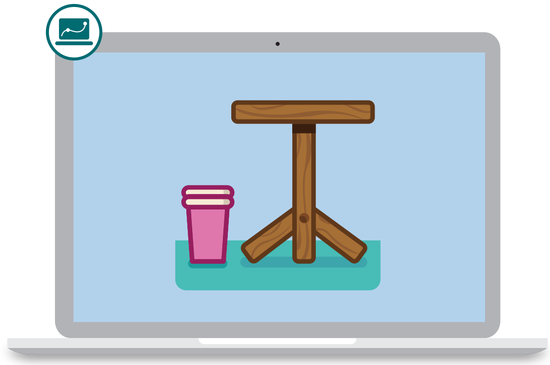
Things to Remember:

Unit 3
Lesson
7

Name: .... Date: .... Period: ....

Stacking Cups

Let's use a linear relationship to make a prediction

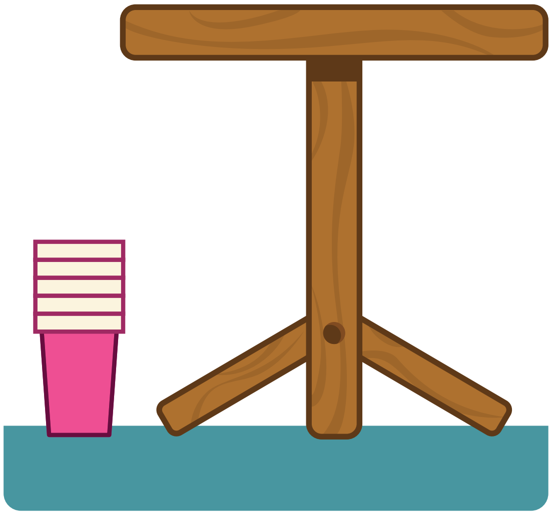


Warm-Up

1 a How many stacked cups do you think you need to reach the top of this table?

Responses vary.

- 10 cups
- 60 cups
- 30 cups (actual answer)



b What information would help you make a more precise prediction?

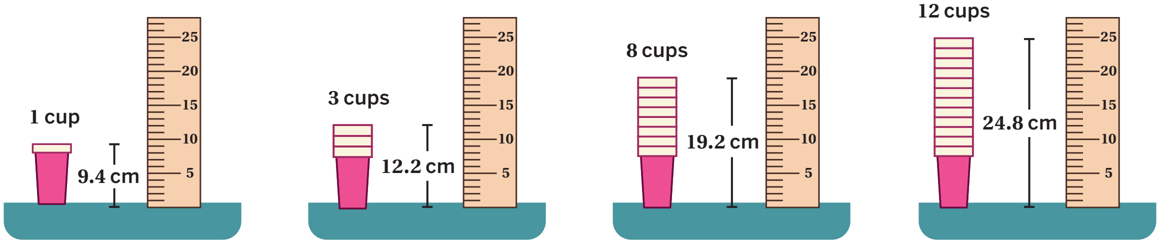
Responses vary.

- It would help to know the total height of a cup, as well as the height of the "lip" of a cup.
- If I knew the height of several stacks of cups (each with a different number of cups), I could determine the answer more precisely.

Name: .... Date: .... Period: ....

Stacking Cups (continued)

2 Let's look at different stacks of cups.



Record *at least two* data points. ***Responses vary.***

Number of Cups	Height (cm)
1	9.4
3	12.2
8	19.2
12	24.8

3 Sylvia found that a stack of 5 cups has a height of 15 centimeters.

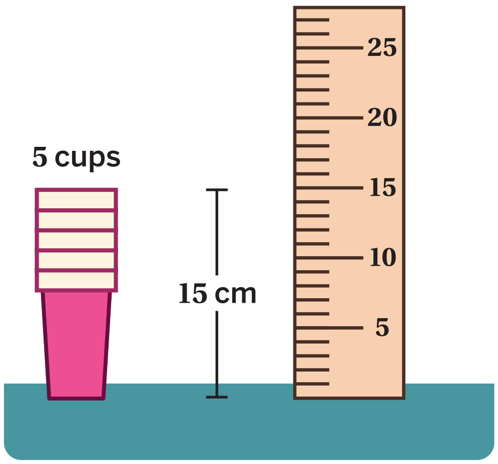
She thinks that a stack of 10 cups will have a height of 30 centimeters.

Is she correct? Circle one.

Yes No

Explain your thinking.

***Explanations vary.* The height cannot double when the number of cups are doubled because the relationship between height and number of cups is not proportional.**



Name: .... Date: .... Period: ....

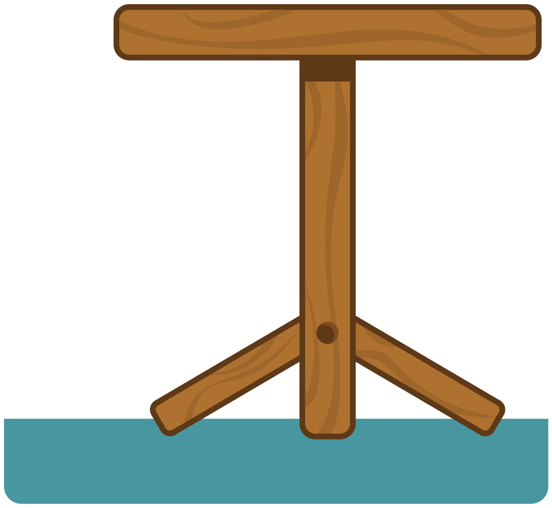
Stacking Cups (continued)

4 The height from the floor to the top of the table is 50 centimeters.

Previously, you predicted a number of cups to reach the top of the table.

Now that you have more information, calculate the exact number of stacked cups you need.

30 cups



5 Let's see how many cups you actually need to reach the top of the table!

Activity
2

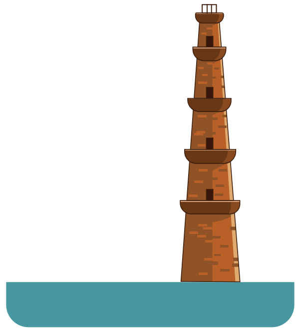
Name: ________ Date: ________ Period: ________

More and More Cups

6 This minaret is 42.5 meters (or 4,250 centimeters) tall.

How many stacked cups do you need to reach the top of the minaret?

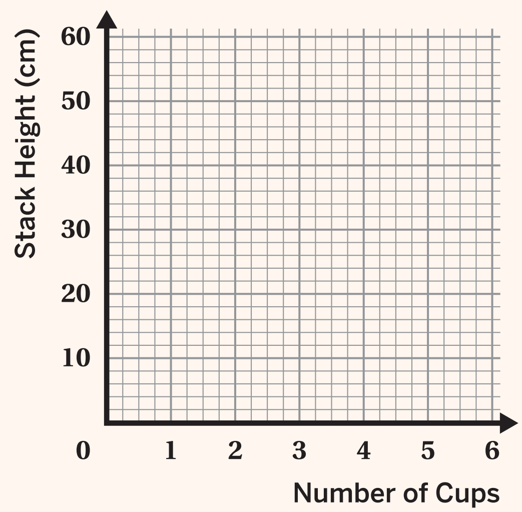
3,030 cups



7 Let's see how many cups you actually need to reach the top of the minaret!

Explore More

8 What cup and lip heights would you use so that 5 of your cups will reach the exact height of the table (50 centimeters)? Use the graph if it helps with your thinking.



***Responses vary.* Cup height: 20 cm, lip height: 6 cm.**

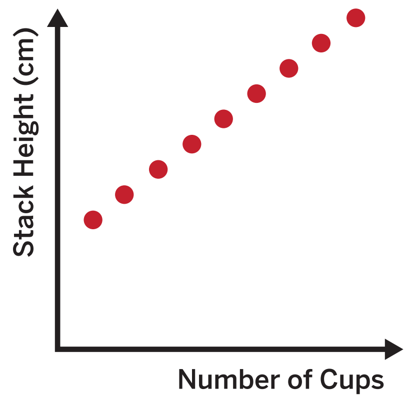
Activity 3

Name: .... Date: .... Period: ....

Modeling the Relationship

9 Graphs can help us understand a relationship better.

Kele and Vicente each made a graph.

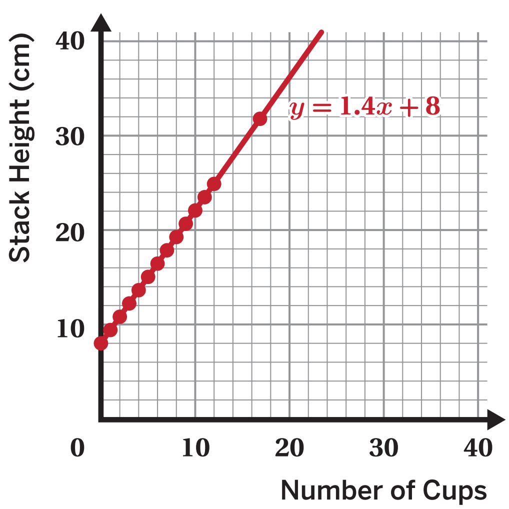


When might Kele's graph be useful? When might Vicente's graph be useful?

Responses vary.

- **Kele's graph might be useful when you're thinking about an equation of a line that models this situation.**
- **Vicente's graph might be useful when you're thinking about each of the individual cups in this situation.**

10 Here is a graph with a line and an equation that represent the relationship between number of cups and stack height.



Discuss:

- What is the slope of the line? What does it represent in this situation?
- What is the y-intercept (vertical intercept) of the line? What does it represent in this situation?

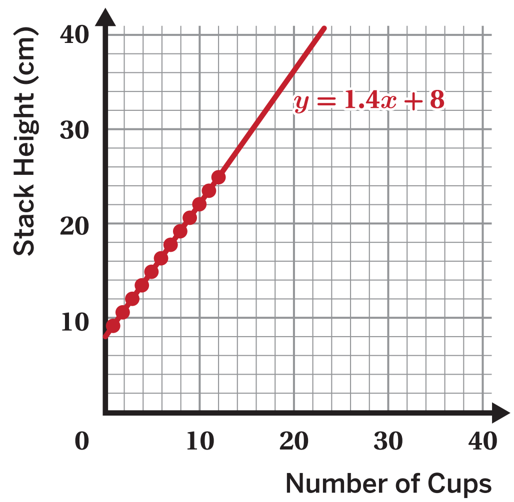
Responses vary.

- **The slope is 1.4. It represents how much the height of the stack increases every time a cup gets added.**
- **The y-intercept is $(0, 8)$. This represents the distance from the bottom of the stack to the lip of the first cup, which is 8 centimeters.**

11 Synthesis

How can you use a linear relationship to model a situation and make predictions?

Responses vary. First I can graph the relationship between the number of cups and the height of the stacked cups. Then I can look at the graph to determine the number of cups, x, that match the height of the table. Let's say the table is 50 centimeters. The graph shows that $x = 30$ when $y = 50$, so that means I need 30 stacked cups to reach the top of the table.



Things to Remember:

Name: .... Date: .... Period: ....

Translations

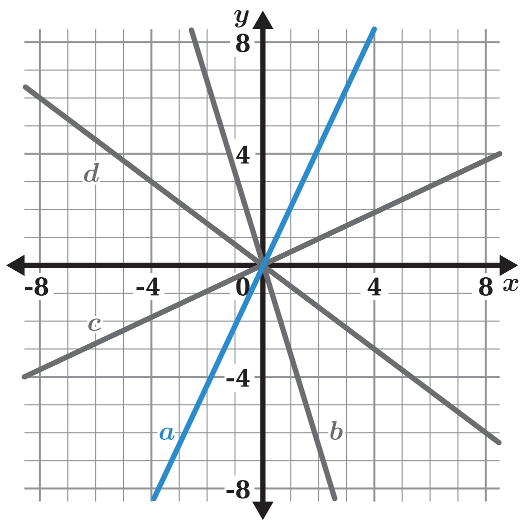
Let's see what happens to the equations of translated lines.

Warm-Up

1 Let's adjust the equation $y = 3x$ to match line a.

$y = 2x$

2 Write equations to match the lines on the graph.



Line	Equation
a	$y = 2x$
b	$y = -3x$
c	$y = \frac{1}{2}x$
d	$y = -\frac{3}{4}x$

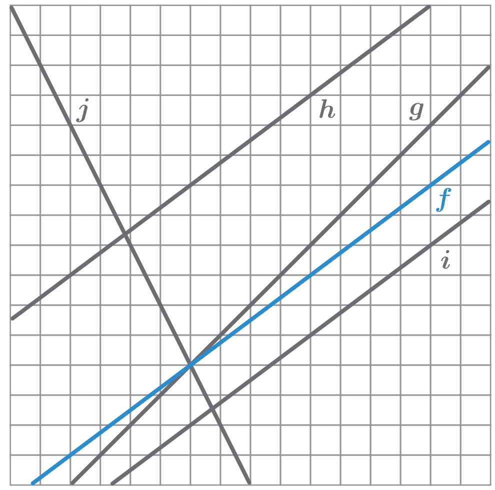
Name: ______________ Date: ______ Period: ______

Translating Lines

3 Here are several lines.

You can only see part of them, but they actually continue forever in both directions.

Which lines are images of line f after a *translation*? Select *all* that apply.



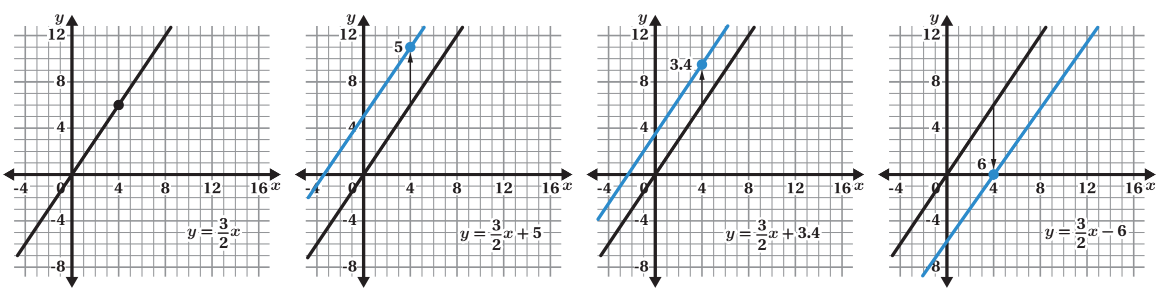
- ☐ **A.** Line g
- ☑ **B.** Line h
- ☑ **C.** Line i
- ☐ **D.** Line j

4 Describe a translation that moves line f onto each of the lines that are images of line f.

Responses vary.

- **Line f has been translated 6 units up to form line h.**
- **Line f has been translated 2 units down to form line i.**

5 Take a look at equations for three different translations of the line $y = \frac{3}{2}x$.



What do you notice about the relationship between the y-intercept (vertical intercept) of the translated line and its equation?

***Responses vary.* The arrow shows the vertical distance between the blue line and the black line. In the equation, the number being added to the x-term represents the number of units the blue line has been translated vertically, or the y-intercept of the blue line.**

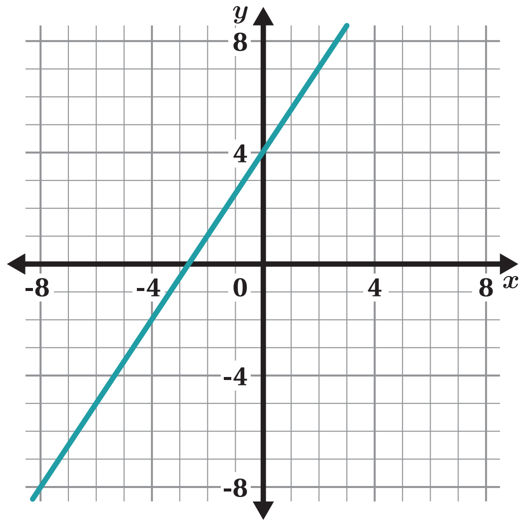
Activity 2

Name: ______ Date: ______ Period: ______

Equations of Lines

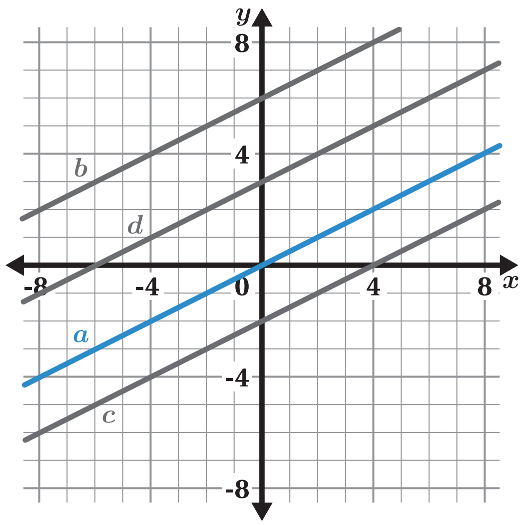
6 Adjust one number in the equation $y = \frac{3}{2}x + 1$ to match the line.

$y = \frac{3}{2}x + 4$ **(or equivalent)**



7 Write equations to match the lines on the graph. The first one has been done for you.

Line	Equation
a	$y = \frac{1}{2}x$
b	$y = \frac{1}{2}x + 6$
c	$y = \frac{1}{2}x - 2$
d	$y = \frac{1}{2}x + 3$



Name: .... Date: .... Period: ....

Equations of Lines (continued)

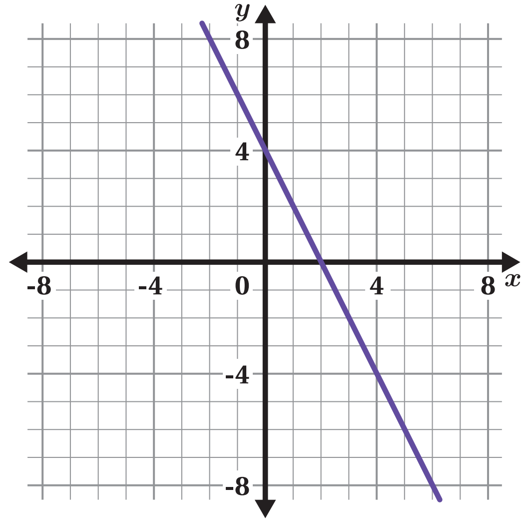
8 What is the equation of this line?

A. $y = -\frac{1}{2}x + 4$

B. $y = 4x - 2$

C. $y = -x + 4$

D. $y = -2x + 4$



Explain your thinking.

Explanations vary. **The graph has a slope of -2 and the y-intercept is 4.**

Explore More

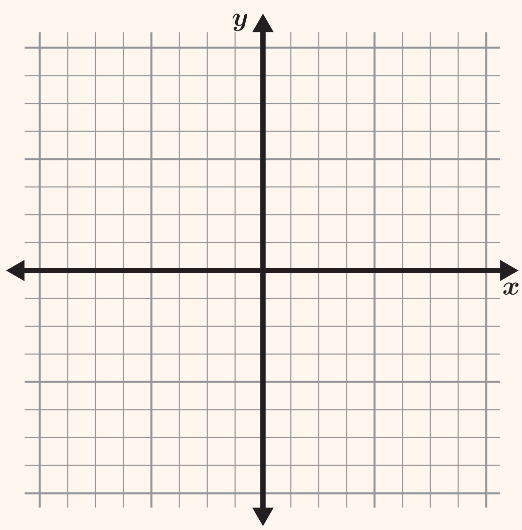
9 Zoe says that the graph of $y = 3(x + 4)$ is the same as the graph of $y = 3x$, but translated up 4 units.

Is Zoe's claim correct?

Yes No

Show or explain your thinking.

Explanations vary. **If the graph was $y = 3x + 4$, then it would be the same as the graph of $y = 3x$ shifted 4 units up. But $y = 3(x + 4)$ is actually the same as $y = 3x + 12$, so its graph would be the graph of $y = 3x$ shifted 12 units up.**

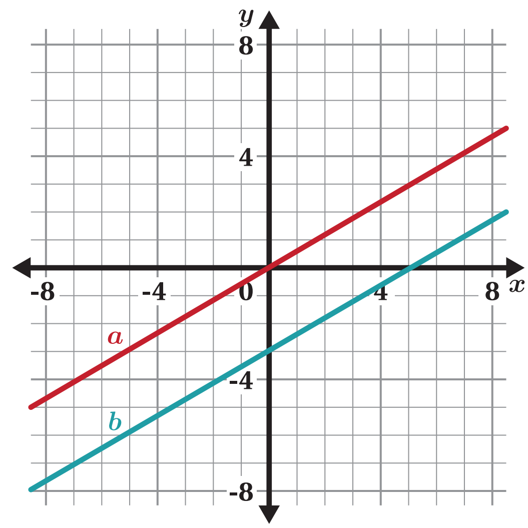


10 Synthesis

Here is a graph of two equations.

How are the equations of line a and line b alike and different?

Responses vary. **The equations are alike because they both have the same slope, $\frac{3}{5}$. The equations are different because you have to translate vertically to get from the graph of line a, $y = \frac{3}{5}x$, to the graph of line b, $y = \frac{3}{5}x - 3$.**



Things to Remember:

Name: ______________ Date: ________ Period: ________

Landing Planes

Let's think about strategies for calculating slope.

Warm-Up

1 Determine possible values for each variable to make the equations true.

$$\frac{a}{b} = -2$$

$$\frac{m}{n} = 2$$

$$q - r = -2$$

Responses vary.

- a and b: $\frac{-6}{3}$; $\frac{6}{-3}$
- m and n: $\frac{-6}{-3}$; $\frac{6}{3}$
- q and r: $6 - 8$; $-6 - (-4)$; $0 - 2$; $-2 - 0$; $-1 - 1$

Activity 1

Name: .... Date: .... Period: ....

Planes, Lines, and Slopes

2 Predict whether the slope of the line through each set of points is positive, negative, or zero.

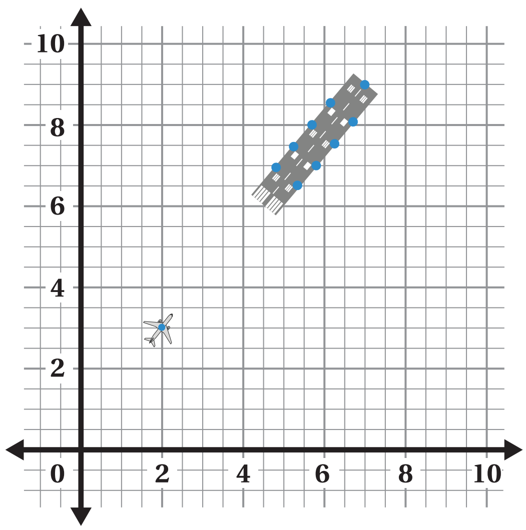
- (600, 500) and (400, 500)
- (7, 1) and (12,7)
- (10, 40) and (30, 20)

Positive Slope	Negative Slope	Zero Slope
(7, 1) and (12, 7)	(10, 40) and (30, 20)	(600, 500) and (400, 500)

3 Your task is to land the plane.

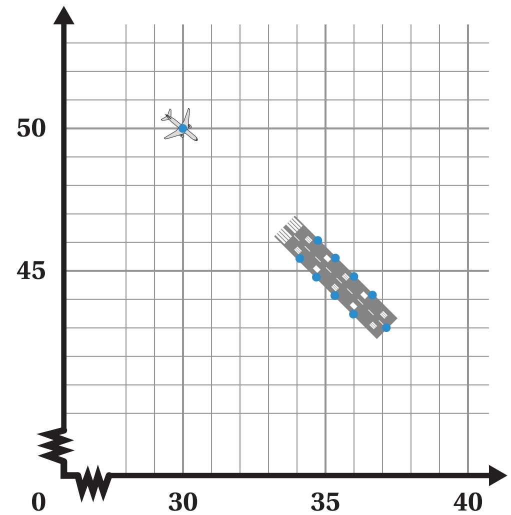
To do that, calculate the slope of the line that goes through (2, 3) and (7, 9).

$\frac{6}{5}$ (or equivalent)



4 To land the plane, calculate the slope of the line that goes through (30, 50) and (37, 43).

-1 (or equivalent)

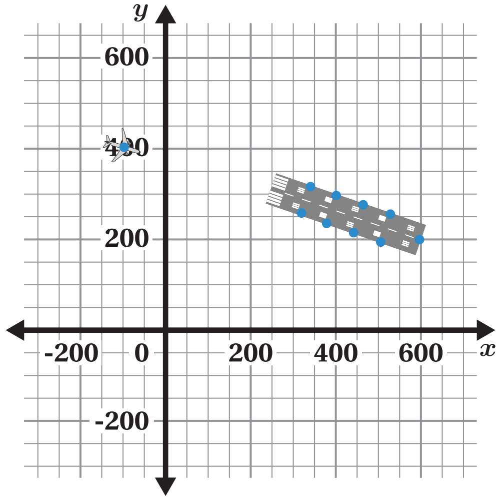


Name: .... Date: .... Period: ....

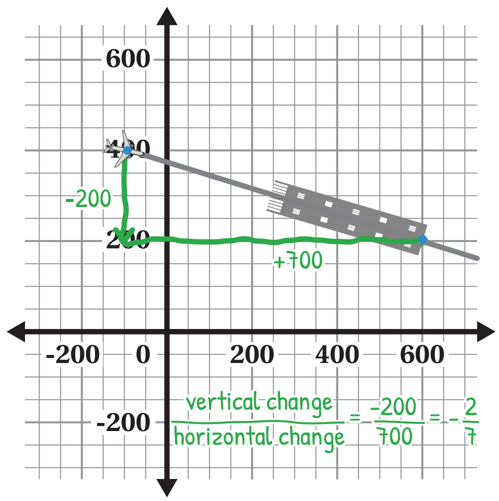
Strategies for Calculating Slope

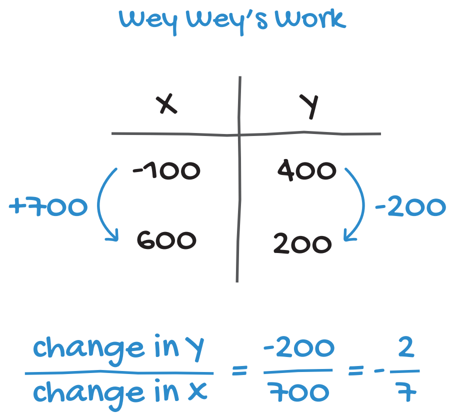
5 To land the plane, calculate the slope of the line that goes through $(-100, 400)$ and $(600, 200)$.

$-\frac{2}{7}$ **(or equivalent)**



6 Here are two students' strategies for calculating the slope of the line that goes through $(-100, 400)$ and $(600, 200)$.





Discuss: How are their strategies alike? How are they different?

Responses vary.

- **Kwame and Wey Wey both determined that the slope was $-\frac{200}{700} = -\frac{2}{7}$.**
- **Kwame found the vertical change and horizontal change using a slope triangle. Wey Wey found the equivalent change in y and change in x using a table.**

Name: ________ Date: ________ Period: ________

Challenge Creator

7 **a** **Make It!** Create your own plane-landing challenge.

- Choose a pair of coordinates to represent the starting position (Point 1) and ending position (Point 2) of your plane. Record the points on this page.
- On graph paper, choose a scale and label your axes, then plot your two points. Label the points with their coordinates.

b **Solve It!** On this page, calculate the slope that passes through your two points.

c **Swap It!** Trade graphs with one or more partners. On this page, calculate the slope of the line that passes through their two points.

d With each partner, compare your strategies for calculating the slope of a line that passes through two given points.

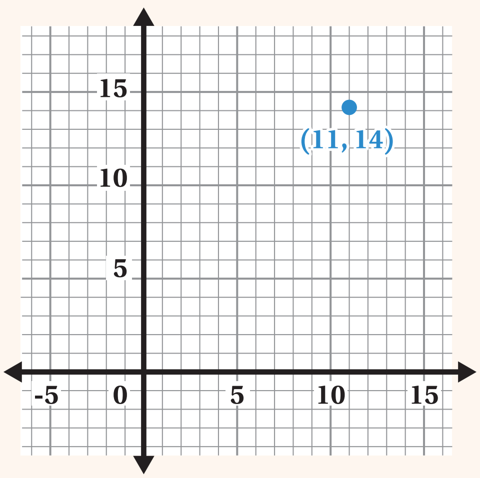
Responses vary.

My Challenge	Partner 1's Challenge	Partner 2's Challenge
Point 1: ________	Point 1: ________	Point 1: ________
Point 2: ________	Point 2: ________	Point 2: ________
Slope: ________	Slope: ________	Slope: ________

Explore More

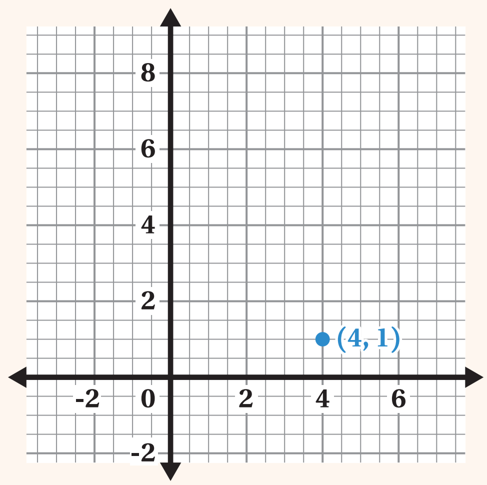
8 We know the slope of the path of each of these planes, but not their starting positions. Determine the value of p so that the line passing through the points has the indicated slope.

a Plane A starts at $(p, 2)$ and stops at $(11, 14)$. Its path has a slope of 2.



$p = 5$

b Plane B starts at $(1, p)$ and stops at $(4, 1)$. Its path has a slope of -2.

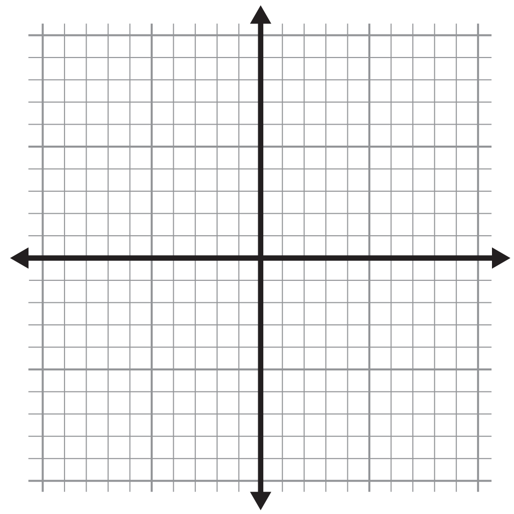


$p = 7$

9 Synthesis

What are some strategies for finding the slope of a line that goes through two given points?

Use the graph if it helps to show your thinking.



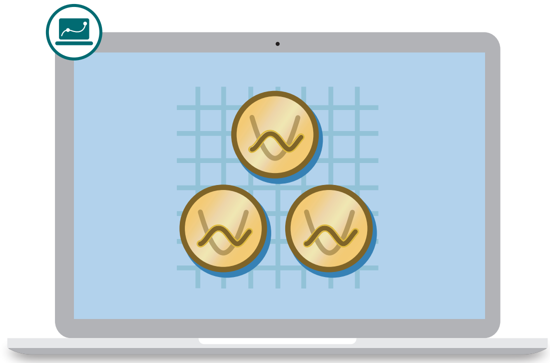
Responses vary.

- One strategy is to draw a slope triangle using the two points on a coordinate plane, then determine the ratio of vertical change to horizontal change.
- Another strategy is to use a table to determine the horizontal change between the x-coordinates and the vertical change between the y-coordinates, then write the ratio of the change in y to the change in x.

Things to Remember:

Unit 3
Lesson
10

Name: ________ Date: ________ Period: ________

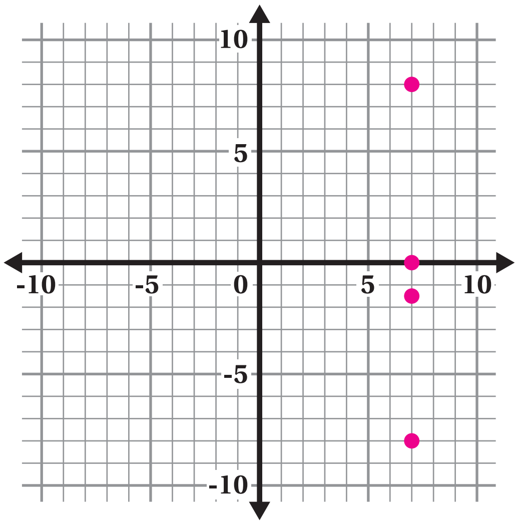


Coin Capture

Let's write equations for vertical and horizontal lines.

Warm-Up

1 **a** Plot four points in different locations. The x-coordinate of each point should be 7.



b **Discuss:** What would your and your classmates' points look like if they were all on one graph?

***Responses vary*. They would look like a vertical line.**

2 Let's look at the points that some other students graphed. Write an equation to represent all the points with an x-coordinate of 7.

$x = 7$

Activity 1

Name: .... Date: .... Period: ....

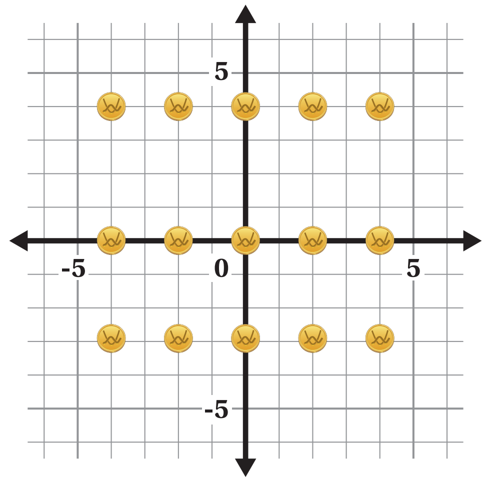
Capture the Coins

3 **a** Draw lines through the coins to "capture" them. Try to draw as few lines as possible.

b Write an equation for each line you drew.

Equations:

Responses vary.
- $y = 4$
- $y = 0$
- $y = -3$

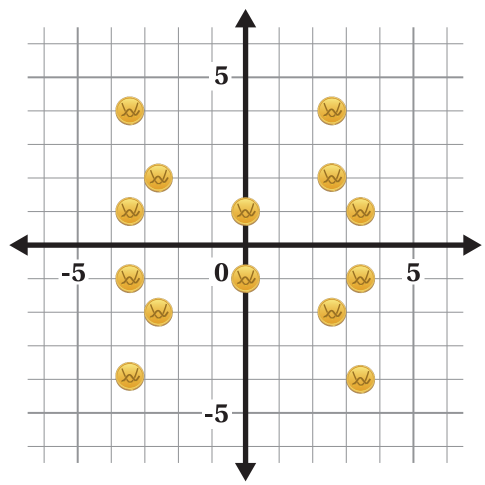


4 **a** Draw lines through the coins to "capture" them. Try to draw as few lines as possible.

b Write an equation for each line you drew.

Equations:

Responses vary.
- $x = 3$
- $x = 0$
- $x = -3$



5 Lupe says that vertical lines have a slope of zero.

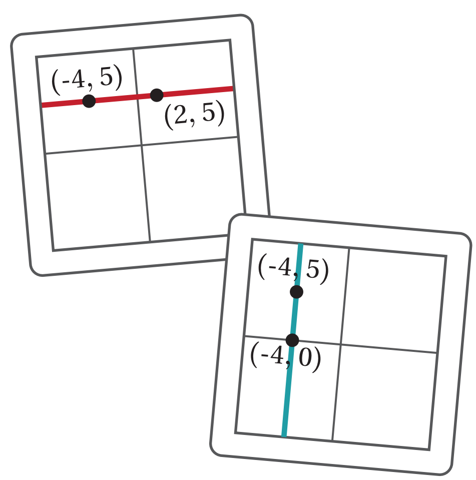
Nekeisha says that horizontal lines have a slope of zero.

Whose claim is correct?

Lupe's Nekeisha's Both Neither

Explain your thinking.

***Explanations vary.* The points on a horizontal line have the same y-coordinate, so the ratio of the vertical change to the horizontal change is always 0 to some number, which equals 0.**

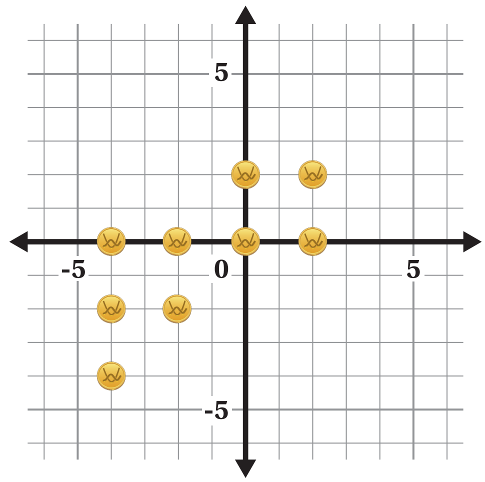


Activity 2

Name: Date: Period:

Challenge Creator

6 **a** Draw lines through the coins to "capture" them. Try to draw as few lines as possible.



b Write an equation for each line you drew.

Equations:

Responses vary.

- $y = x$
- $y = 0$
- $y = x + 2$

7 Create your own Coin Capture challenge!

a **Make It!** Use the Activity 2 Sheet to create your challenge.

b **Solve It!** On this page, write equations for the lines you would use to capture all the coins in your challenge. Try to use as few lines as you can.

c **Swap It!** Trade graphs with a partner and solve each other's challenges.

Responses vary.

My Challenge

Equations:

Partner 1's Challenge

Equations:

Partner 2's Challenge

Equations:

Partner 3's Challenge

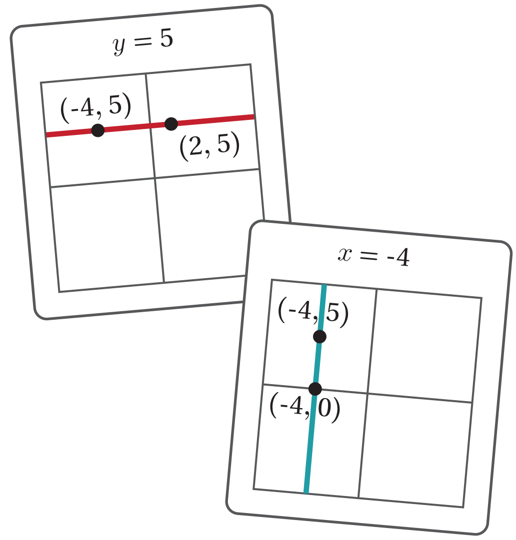
Equations:

8 Synthesis

How can you tell from looking at a linear equation if its graph is a horizontal or vertical line?

Responses vary.

- The graph of a linear equation is a horizontal line when it's in the form $y = a$.
- The graph of a linear equation is a vertical line when it's in the form $x = b$.



Things to Remember:

Name: .... Date: .... Period: ....

Challenge Creator

Draw 8 coins on the graph.