Lesson Summary

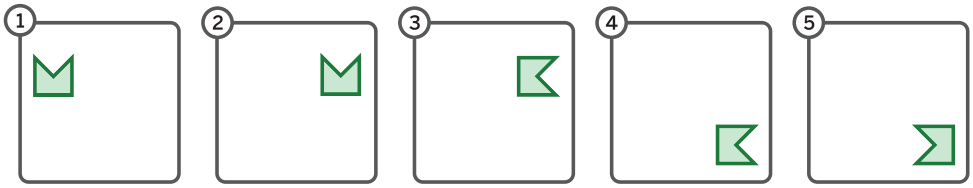
Transformations are actions that you can perform to change a figure. They are applied to every point on the figure. Here are some examples:

- A **rotation** turns or spins a figure.
- A **reflection** flips or mirrors a figure over a line by moving every point to a point directly on the opposite side of the line.
- A **translation** slides a figure without turning it.

Things to Remember:

Name: .... Date: .... Period: ....

1. These five frames show a figure's different positions.

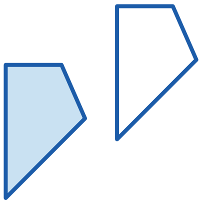


Describe how the figure moves to get from its position in each frame to the next.

From	To	Description of Moves
Frame 1	Frame 2	**The figure is translated to the right.**
Frame 2	Frame 3	**The figure reflects over a horizontal line.**
Frame 3	Frame 4	**The figure is translated to the left.**
Frame 4	Frame 5	**The figure reflects over a horizontal line, returning to its original position in Frame 1.**

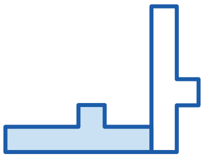
Problems 2–4: Determine whether each transformation shows a translation, reflection, or rotation.

2.



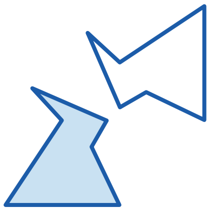
Translation

3.



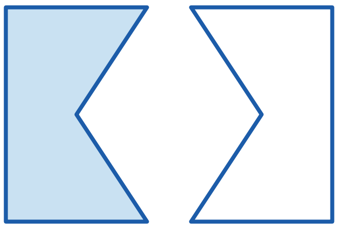
Rotation

4.



Reflection

5. Which transformation could move the shaded figure to the unshaded one? Select *all* that apply.



- ☑ **A.** Reflection
- ☑ **B.** Rotation
- ☐ **C.** Translation
- ☐ **D.** None of these

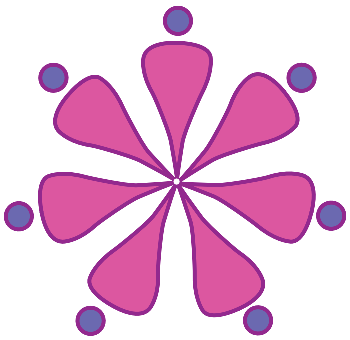
Lesson Practice
8.1.02

Name: .... Date: .... Period: ....

Problems 6–7: A kaleidoscope is a tube with mirrors and colored glass inside. When you point it toward the light and look through it as you turn it, you can see patterns appear.

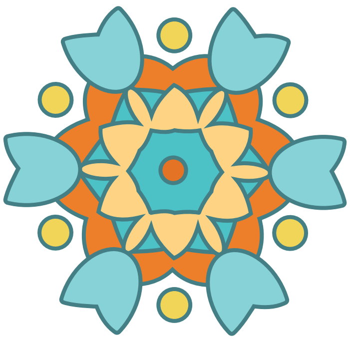
Analyze the designs and describe how the pattern could be created using reflections, rotations, or translations.

6.



Responses vary. **Seven copies of the same design rotate around a circle.**

7.



Responses vary. **The top half reflects over a horizontal line to create the bottom half.**

Spiral Review

Problems 8–11: Evaluate each expression.

8. $-5 \cdot (-2.4) = 12$

9. $-7.4 \div 10 = -0.74$ **(or equivalent)**

10. $-\frac{4}{7} \div (-2) = \frac{2}{7}$ **(or equivalent)**

11. $4 \cdot \left(-\frac{3}{8}\right) = -\frac{3}{2}$ **(or equivalent)**

Reflection

1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

A **sequence of transformations** is a set of translations, rotations, and/or reflections that you can perform on a figure. Each sequence of transformations has a meaningful order.

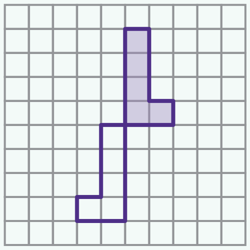
Here are important details to consider when choosing your sequence of transformations:

- For translations, every point on the figure moves the same distance in the same direction.
- For reflections, the new point will be the same distance from the line as it was in the original figure.
- Rotations are performed around a point by a given angle and in a specific direction. The direction of a rotation can be **clockwise**, traveling in the same direction as the hands on a clock, or **counterclockwise**, traveling in the opposite direction as the hands on a clock.

You can use different sequences of transformations on a figure and get the same result.

For example, here are two sequences of transformations that both move the shaded figure onto the unshaded figure:

- Sequence #1: Reflect over the longest edge and then translate the figure 4 units down.
- Sequence #2: Rotate $180°$ clockwise around the lower left corner of the shaded figure, then reflect over the horizontal line that divides the height into two halves.



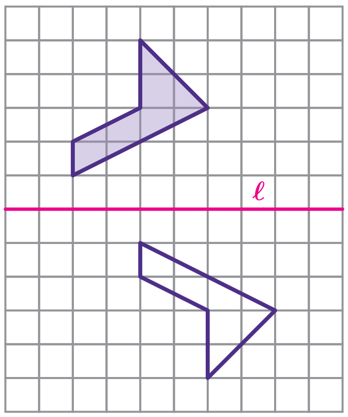
Things to Remember:

Lesson Practice
8.1.03

Name: ______________________ Date: __________ Period: __________

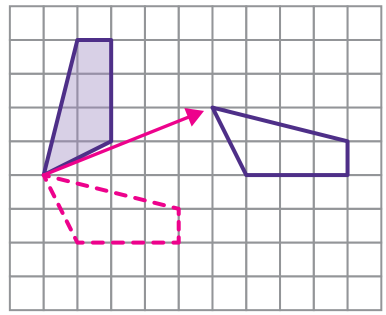
Problems 1–4: Describe a sequence of transformations that moves the shaded figure onto the unshaded figure. ***Responses vary*. Samples shown on graphs.**

1.



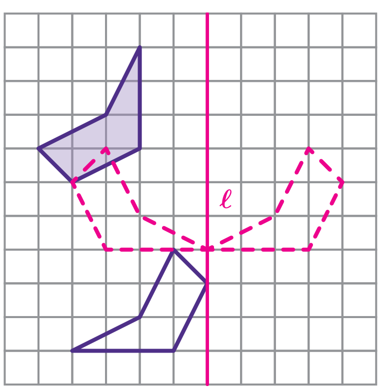
Reflect the shaded figure over line ℓ, and then translate 2 units to the right.

2.



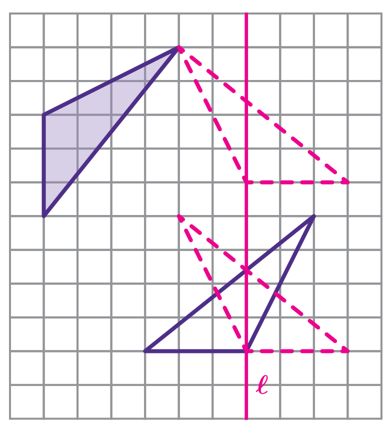
Rotate the shaded figure 90° clockwise around the bottom-left point, and then translate 5 units right and 2 units up.

3.



Rotate the shaded figure 90° clockwise around its bottom point, reflect over line ℓ, and then translate 4 units left and 3 units down.

4.



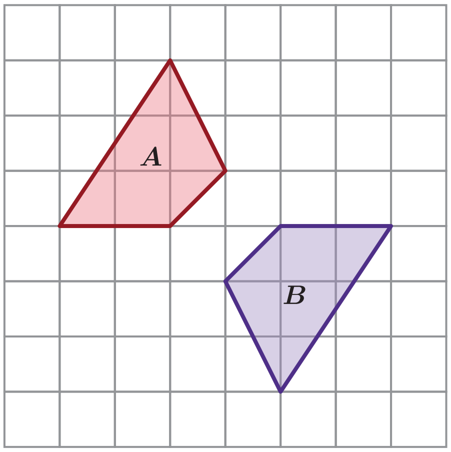
Rotate the shaded figure 90° counterclockwise around its top-right point, translate 5 units down, and then reflect over line ℓ.

Name: ________ Date: ________ Period: ________

5. Jacy says that figure A can move onto figure B using only reflections.

Is Jacy correct? Explain your thinking.

Yes. *Explanations vary*. Reflecting figure A over a vertical line and then over a horizontal line will move figure A onto figure B.



6. Select *all* the sequences of transformations that could return a figure to its original position.

- ☑ **A.** Reflect the figure over a line and then reflect back over the same line.
- ☑ **B.** Translate the figure 1 unit to the right, then 4 units to the left, and then 3 units to the right.
- ☐ **C.** Reflect the figure over one line and then reflect over a different line.
- ☑ **D.** Rotate the figure 90° counterclockwise around a point and then 270° counterclockwise around the same point.
- ☐ **E.** Rotate the figure 90° counterclockwise around a point and then 270° clockwise around the same point.

Spiral Review

Problems 7–9: Write an operation in the box to make each equation true.

7. $12 \boxed{-} (-8) = 20$

8. $-17 \boxed{-} 9 = -26$

9. $24 \boxed{+} (-29) = -5$

Reflection

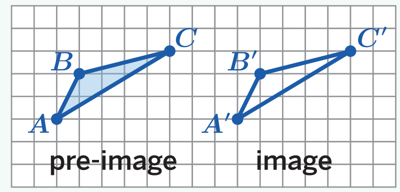
1. Put a question mark next to a response you'd like to compare with a classmate's.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice 8.1.04

Science Mom Lesson 3

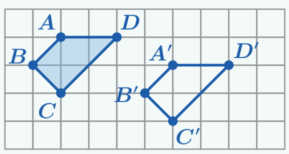
Lesson Summary

When you transform a figure, the original figure is called the **pre-image** and the new figure is called the **image**. All of the points in the image **correspond** to the points in the pre-image. The points in the image are named after the pre-image point they correspond to. For example, point A' corresponds to point A.



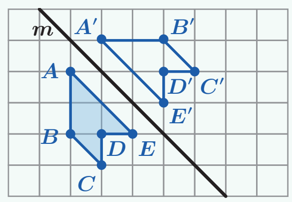
Here are important details to help you describe transformations:

Translations



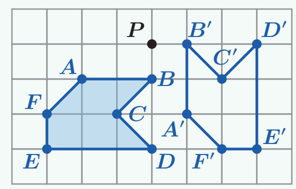
Describe the direction (up or down, left or right) and the number of units.
E.g., Figure $ABCD$ is translated 4 units right and 1 unit down.

Reflections



Describe the line of reflection.
E.g., Figure $ABCDE$ is reflected over line m.

Rotations



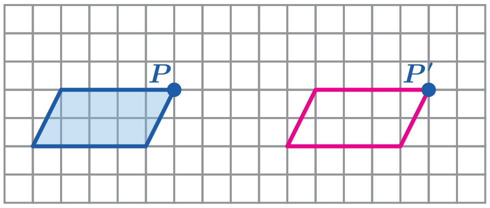
Describe the center of rotation, angle of rotation, and direction (clockwise or counterclockwise).
E.g., Figure $ABCDEF$ is rotated 90° counterclockwise around point P.

Things to Remember:

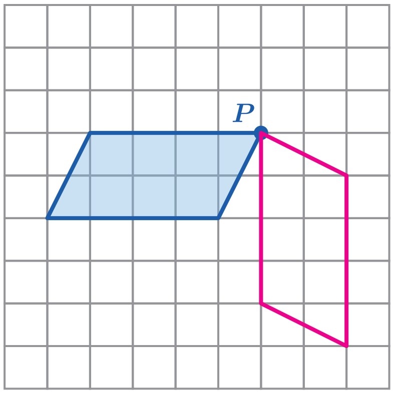
Lesson Practice
8.1.04

Name: ______ Date: ______ Period: ______

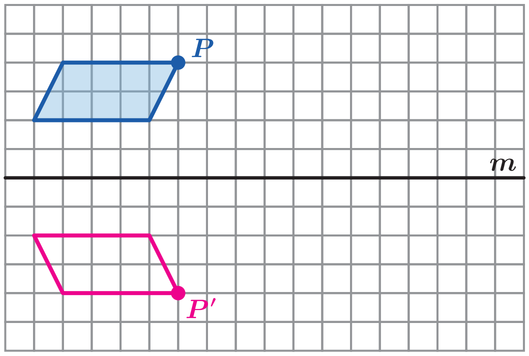
1. Translate this figure to move point P onto P'.



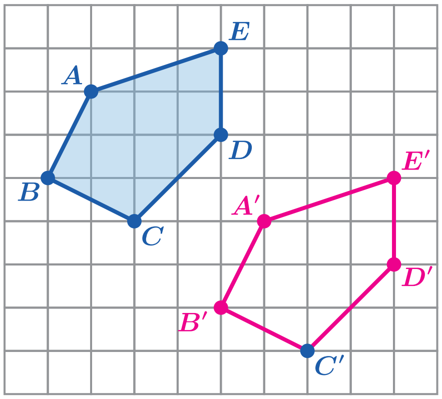
2. Rotate this figure $90°$ counterclockwise around point P.



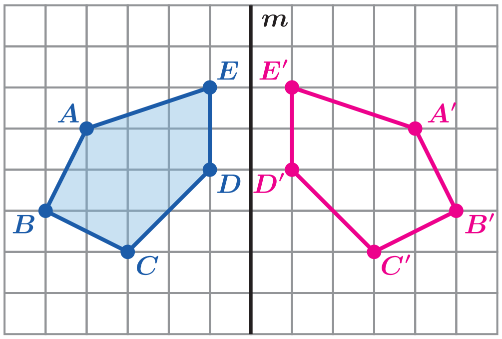
3. Reflect this figure over line m.



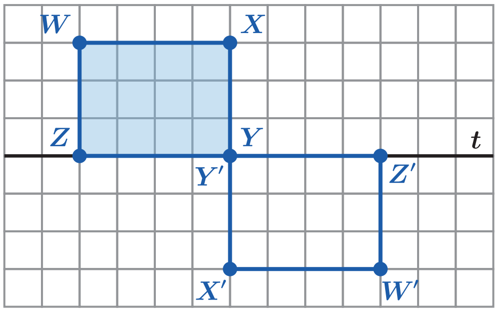
4. Translate figure $ABCDE$ to move point C onto C'.



5. Reflect figure $ABCDE$ over line m.



6. Figure $WXYZ$ was transformed to create figure $W'X'Y'Z'$. Which transformation took place?



A. A reflection over line t.

B. A translation 6 units down and 4 units to the right.

C. A rotation $90°$ clockwise around point Y.

D. A rotation $180°$ clockwise around point Y.

Spiral Review

7. Match each expression with an equivalent expression.

Expressions	Equivalent Expressions
a. $-3x - 7$	b ..... $-0.9 + 5.7x$
b. $-3.4 + 5.7x + 2.5$	d ..... $-\frac{1}{2}(6x - 14)$
c. $1.8x - 5.9 + 3.9x$	c ..... $5.7x - 5.9$
d. $-3x + 7$	a ..... $\left(-\frac{7}{2} - \frac{3}{2}x\right) \bullet 2$

Reflection

1. Put a star next to a problem you could explain to a classmate.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice 8.1.05

Science Mom Lesson 4

Lesson Summary

When you compare the coordinates of corresponding points in the image and pre-image, you might notice patterns in their values.

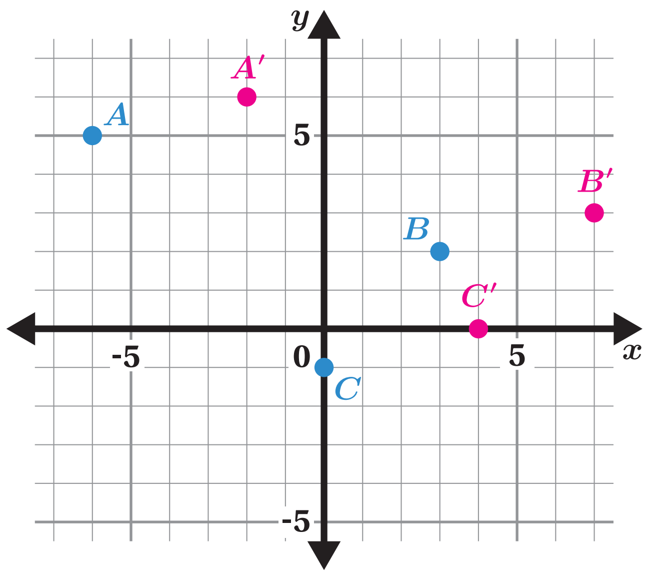
- When you translate a point to the left or right, it changes the value of the x-coordinate.
- When you translate a point up or down, it changes the value of the y-coordinate.
- When you reflect a point over the x-axis, it changes the sign of the y-coordinate. The x-coordinate remains the same.
- When you reflect a point over the y-axis, it changes the sign of the x-coordinate. The y-coordinate remains the same.

Things to Remember:

Name: ______________ Date: ________ Period: ________

1. Plot the location of points A, B, and C after a translation 4 units to the right and 1 unit up. Label the points A', B', and C'. Then write the coordinates in the table.

Point	Coordinates
A'	(-2, 6)
B'	(7, 3)
C'	(4, 0)



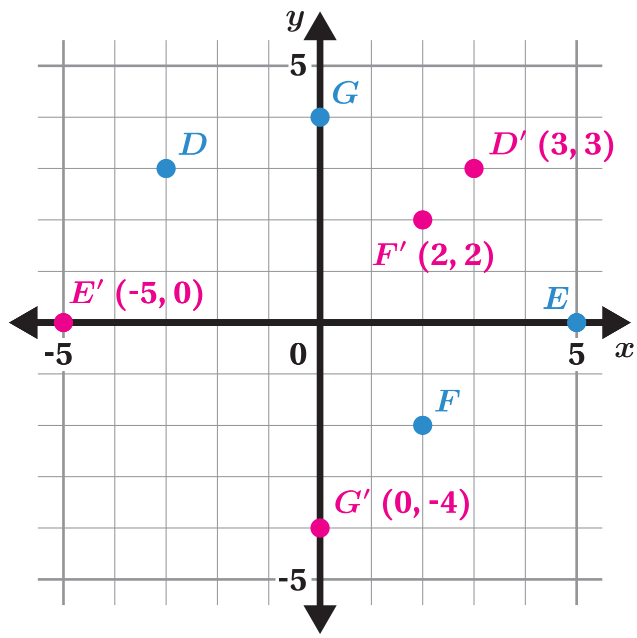
Problems 2–3: Points D, E, F, and G are plotted on the coordinate plane.

2. Plot the coordinates of points D and E after a reflection over the y-axis. Label the images D' and E'. Include the coordinates.

Responses shown on graph.

3. Plot the coordinates of points F and G after a reflection over the x-axis. Label the images F' and G'. Include the coordinates.

Responses shown on graph.

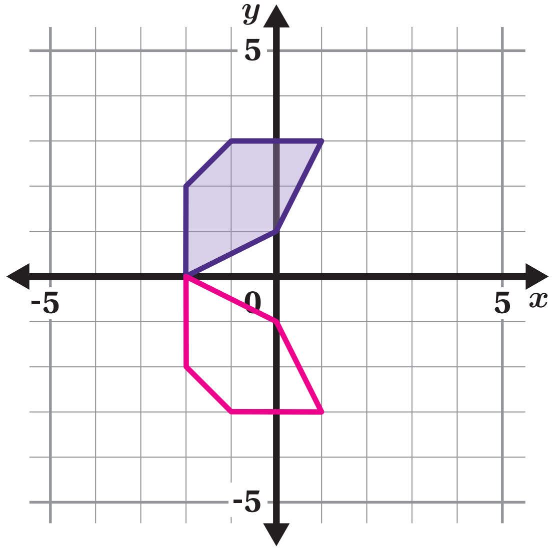


Lesson Practice
8.1.05

Name: .... Date: .... Period: ....

4. Determine the coordinates of each point after the shaded pre-image is reflected over the x-axis. Use the graph if it helps with your thinking.

Pre-Image Coordinates	Image Coordinates
(-2, 0)	**(-2, 0)**
(-2, 2)	**(-2, -2)**
(-1, 3)	**(-1, -3)**
(1, 3)	**(1, -3)**
(0, 1)	**(0, -1)**



5. Point $H(5, 3)$ is translated 4 units down and 3 units to the left to get point H'. Which of the following are the coordinates of point H'?

A. (-2, 1) **B.** (1, 0) **C.** (1, 6) **D.** (2, -1)

6. Point (x, y) is reflected over the y-axis, and then translated 5 units to the right and 2 units down. Write the generalized coordinates of the image.

$(-x + 5, y - 2)$

Spiral Review

Problems 7–12: Compare each of these values using the symbols <, =, or >.

7. -11 ...>... -15

8. 8.01 ...>... 8

9. $\frac{2}{3}$ ...>... $-\frac{3}{2}$

10. -8.01 ...<... -8

11. -(-6) ...=... 6

12. -2.5 ...=... $-\frac{10}{4}$

Reflection

1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

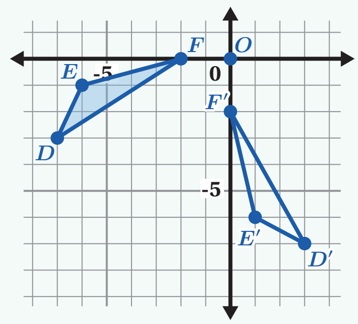
Lesson Summary

When you compare the coordinates of corresponding points in an image and pre-image, you might notice patterns in their values.

For a $90°$ or $270°$ rotation, the x- and y-coordinates switch and one changes sign. For a $180°$ rotation, both coordinates change signs. For a $360°$ rotation, both coordinates stay the same.

For example, triangle DEF was rotated $90°$ counterclockwise.

For triangles DEF and $D'E'F'$, the x- and y-coordinates of each point switch places, and some of the signs change.



Pre-Image Coordinates	Image Coordinates
(-2, 0)	(0, -2)
(-6, -1)	(1, -6)
(-7, -3)	(3, -7)

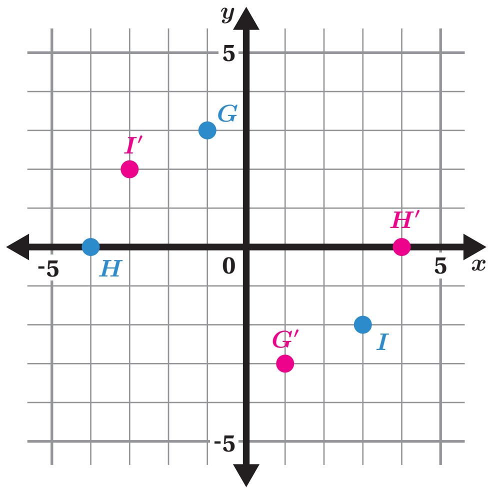
Things to Remember:

Lesson Practice
8.1.06

Name: .... Date: .... Period: ....

1. Plot the location of points G, H, and I after a $180°$ rotation around the origin. Label the points G', H', and I'. Then write the coordinates in the table.

Point	Coordinates
G'	**(1, -3)**
H'	**(4, 0)**
I'	**(-3, 2)**



2. Point $P(5, 3)$ is rotated $90°$ counterclockwise around the origin and the image is labeled P'. Which of the following are the coordinates of point P'?

A. (1, -3) **B.** (-3, -5) **C.** (3, -5) **D.** (-3, 5)

3. Here are the pre-image and image coordinates of points on a graph. Describe the transformation.

Responses vary.

- **90° clockwise rotation around the origin**
- **270° counterclockwise rotation around the origin**

Pre-Image Coordinates	Image Coordinates
(0, 5)	(5, 0)
(-2, 1)	(1, 2)
(4, 3)	(3, -4)
(6, 0)	(0, -6)
(-5, -1)	(-1, 5)

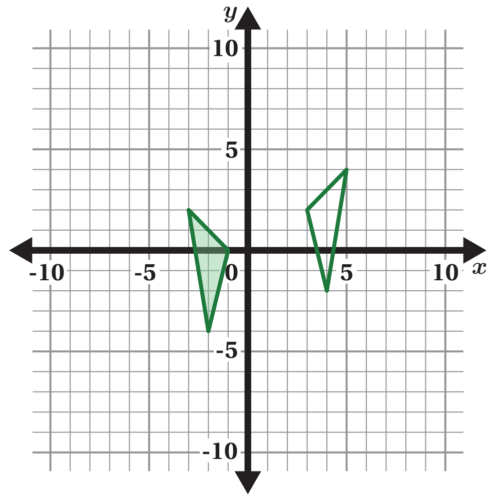
4. Point (x, y) is rotated $90°$ counterclockwise around the origin, and then translated 1 unit to the left and 3 units up. Write the generalized coordinates of the image.

$\mathbf{(-y - 1, x + 3)}$

Name: ______ Date: ______ Period: ______

5. Describe a sequence of transformations that will move the pre-image (shaded) onto the image (unshaded).

Responses vary. **Reflect the pre-image over the y-axis. Then translate 2 units to the right and 2 units up.**



Spiral Review

Problems 6–10: Write an expression that has a value of $\frac{3}{5}$ based on the given rule.

Responses vary.

6. An expression that is a sum.

$\frac{1}{5}+\frac{2}{5}$

7. An expression that is a difference.

$1\frac{2}{5}-\frac{4}{5}$

8. An expression that is a product.

$3 \cdot \frac{1}{5}$

9. An expression that is a quotient.

$6 \div 10$

10. An expression that involves at least two operations.

$2\left(\frac{1}{10}+\frac{2}{10}\right)$

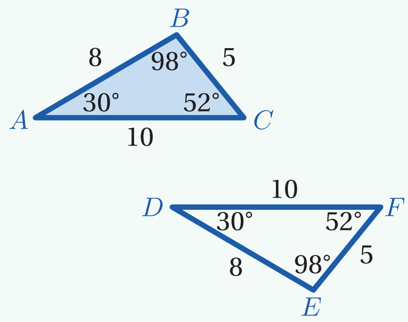
Reflection

1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Two figures are **congruent** if you can use a sequence of rigid transformations to move one exactly on top of the other.



You don't need to check that all corresponding angle measures and side lengths are equal if you can show a sequence of rigid transformations.

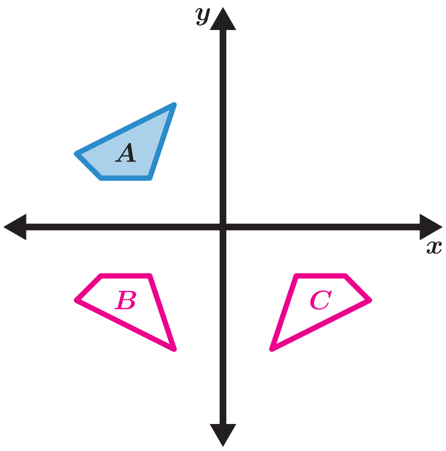
For example, figure DEF is congruent to figure ABC because you can reflect ABC over a horizontal line and translate it to fit exactly on top of figure DEF.

Things to Remember:

Lesson Practice
8.1.08

Name: ________ Date: ________ Period: ________

Problems 1–3: This coordinate plane shows figure A.



1. Reflect figure A over the x-axis. Label the image B.

Sample shown on graph.

2. Reflect figure B over the y-axis. Label the image C.

Sample shown on graph.

3. Are figures A and C congruent? Explain your thinking.

Yes. *Explanations vary.* Because there is a sequence of rigid transformations taking quadrilateral A to quadrilateral C, the two figures are congruent.

4. Pentagon $ABCDE$ was graphed on a coordinate plane and then rotated $90°$ counterclockwise around the origin to form pentagon $A'B'C'D'E'$. Which statement is true?

A. Pentagon $A'B'C'D'E'$ is not congruent to pentagon $ABCDE$.

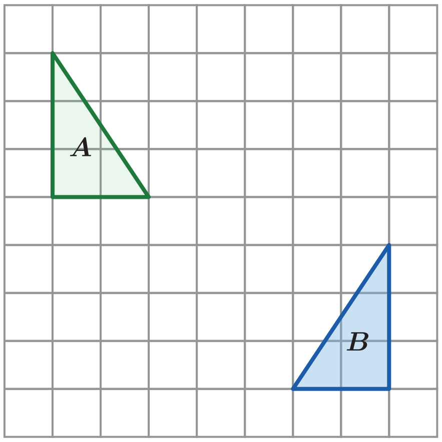
B. The area of pentagon $A'B'C'D'E'$ is not equal to the area of pentagon $ABCDE$.

C. The angle measures of pentagon $A'B'C'D'E'$ are congruent to the corresponding angle measures of pentagon $ABCDE$.

D. The perimeter of pentagon $A'B'C'D'E'$ is greater than the perimeter of pentagon $ABCDE$.

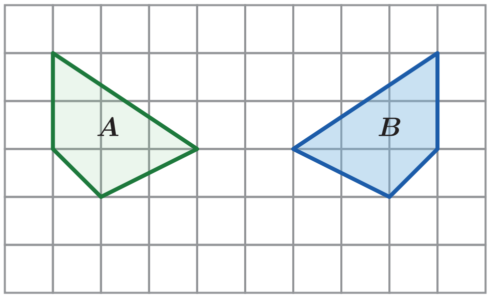
Problems 5–8: Determine whether figure A is congruent to figure B. Explain your thinking.

5.



Congruent. *Explanations vary.* Figure A can be reflected over a vertical line halfway between figures A and B, then translated down 4 units.

6.

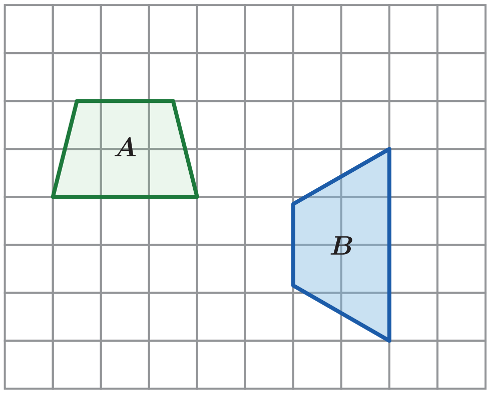


Congruent. *Explanations vary.* Figure A can be reflected over a vertical line halfway between figures A and B.

Lesson Practice
8.1.08

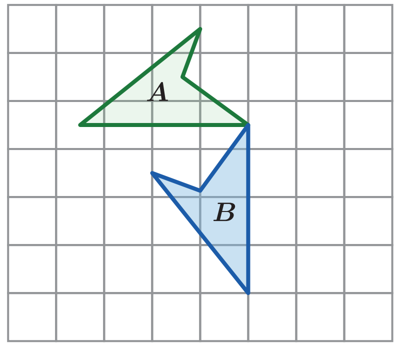
Name: ________ Date: ________ Period: ________

7.



Not congruent. ***Explanations vary.*** **There is no sequence of rigid transformations that will move figure A onto figure B.**

8.



Congruent. ***Explanations vary.*** **Figure A can be rotated $90°$ counterclockwise with the vertex shared by the two figures as the center of rotation.**

Spiral Review

Problems 9–10: Determine the coordinates of the image of point A $(2, -5)$ after each transformation.

9. A reflection over the x-axis.

$(2, 5)$

10. A reflection over the y-axis.

$(-2, -5)$

Reflection

1. Circle one problem, word, or concept that you want to know more about.

2. Use this space to ask a question or share something you're proud of.

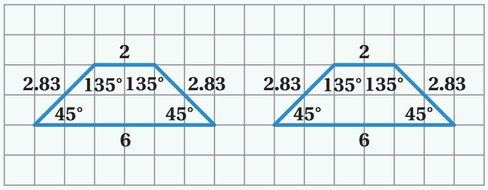
Lesson Summary

Here are two ways to determine whether two figures are congruent:

- You can determine a sequence of transformations to move one exactly onto the other.
- You can determine that *all* the corresponding sides have the same length and *all* the corresponding angles have the same measure.

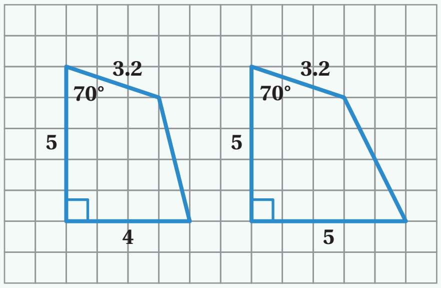
Two figures are not congruent if their corresponding side lengths or angle measures are not the same, or if they have different perimeters or areas.

Congruent



These figures are congruent because all the corresponding side lengths and angle measures are the same.

Not Congruent



These figures are not congruent because *some* of the side lengths and measures are the same, but some are different.

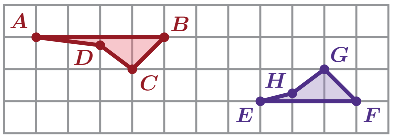
Things to Remember:

Lesson Practice

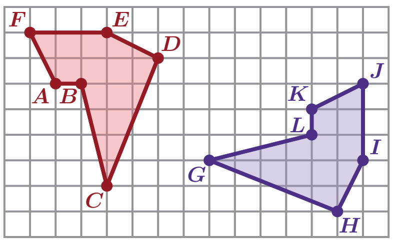
8.1.09

Name: ______________________ Date: __________ Period: __________

Problems 1–2: For each problem, determine whether the figures are congruent and explain how you know. ***Explanations vary.***

1. 

No. Figure $ABCD$ is not congruent to figure $EFGH$ because the side length of AB is 4 units and the side length of EF is 3 units. AB and EF are the longest sides of each figure and need to be the same length for the figures to be congruent.

2. 

Yes. When figure $ABCDEF$ is rotated 90° clockwise around point C and translated 1 unit up and 4 units to the right, it will move onto figure $KLGHIJ$.

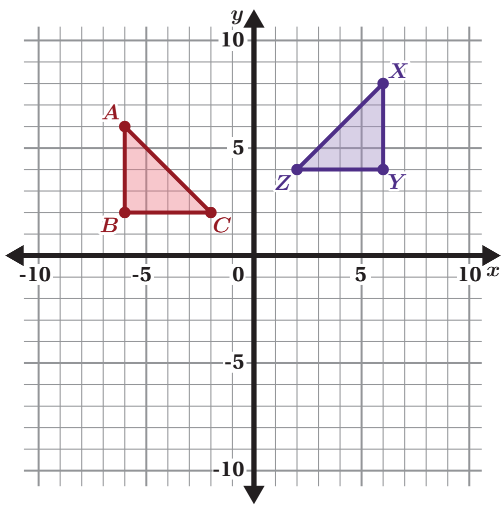
3. Triangle ABC and triangle XYZ are congruent. Which sequence of transformations will move triangle ABC onto triangle XYZ?

A. A reflection over the y-axis, and then a translation 2 units up.

B. A reflection over the x-axis, and then a translation 2 units down.

C. A reflection over the y-axis, and then a translation 2 units right.

D. A reflection over the x-axis, and then a translation 2 units left.



4. If two rectangles have the same perimeter, will they always be congruent? Explain your thinking.

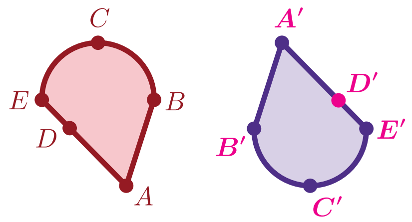
No. *Explanations vary.* A rectangle with a length of 3 feet and width of 2 feet will have the same perimeter as a rectangle with a length of 4 feet and a width of 1 foot. However, their corresponding sides do not have the same length, so the rectangles will not be congruent.

Lesson Practice
8.1.09

Name: ________ Date: ________ Period: ________

Problems 5–7: Here are two congruent figures.

Figure 1 Figure 2



5. Label Figure 2 with points A', B', C', and E' so that they correspond to points A, B, C, and E in Figure 1.

6. If segment AB is 2 centimeters long, how long is segment $A'B'$? Explain your thinking.

2 centimeters. *Explanations vary*. The figures are congruent, so the corresponding segments of congruent figures are also congruent.

7. Plot point D' where you think it is located. Explain your thinking.

***Explanations vary*. Because the figures are congruent, point D' will be on the corresponding side. It will be in the same location on $A'E'$ as D is on AE.**

Spiral Review

Problems 8–10: A rectangular room has a length of 12 feet and a width of 15 feet. A contractor creates a blueprint of the room using a scale factor of $\frac{1}{6}$.

8. Determine the length and width of the room on the blueprint.

Length of 2 feet and width of 2.5 feet

9. How does the perimeter of the actual room compare to the perimeter on the blueprint?

***Responses vary*. The perimeter of the actual room is 6 times the perimeter on the blueprint.**

10. How does the area of the actual room compare to the area on the blueprint?

***Responses vary*. The area of the actual room is 36 times the area on the blueprint.**

Reflection

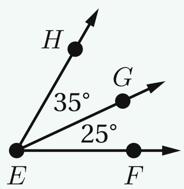
1. Put a question mark next to a problem you're feeling unsure of.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

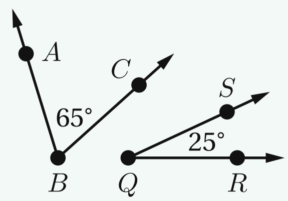
Here are three angle relationships that can help you determine missing angle measures.

Adjacent angles share a side and a vertex.



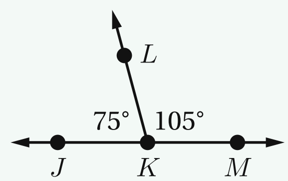
$\angle HEG$ and $\angle FEG$ are adjacent angles.

Complementary angles have measures that add up to $90°$.



$\angle ABC$ and $\angle RQS$ are complementary angles.

Supplementary angles have measures that add up to $180°$.



$\angle JKL$ and $\angle MKL$ are supplementary angles.

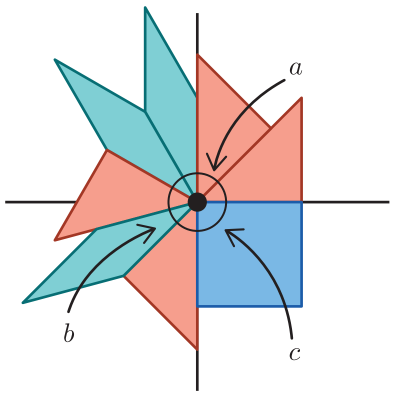
Things to Remember:

Lesson Practice
7.7.02

Name: .... Date: .... Period: ....

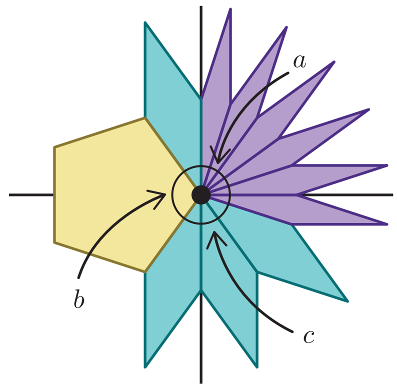
Problems 1–2: Determine the values of a, b, and c in each diagram.

1.



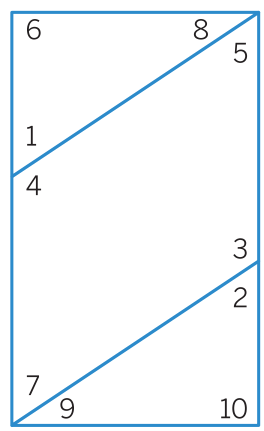
$a = 45°$, $b = 30°$, $c = 90°$

2.



$a = 18°$, $b = 108°$, $c = 36°$

Problems 3–6: Here is a rectangle.



3. List a pair of angles that are complementary.

Responses vary.

- 7 and 9
- 8 and 5

4. List a pair of angles that are supplementary.

Responses vary.

- 1 and 4
- 2 and 3
- 6 and 10

5. If Angle 7 measures 56°, determine the value of one other angle.

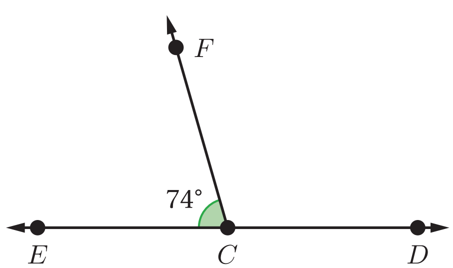
Responses vary. ∠9 measures 34°.

6. If Angle 1 measures 56°, determine the value of one other angle.

Responses vary. ∠4 measures 124°.

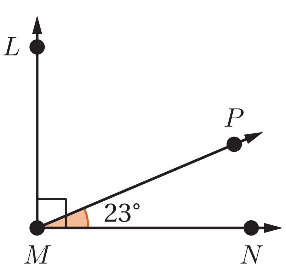
Problems 7–8: Determine the missing angle measures.

7. $\angle FCD$



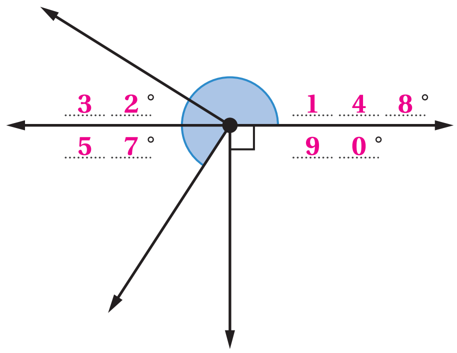
106°

8. $\angle LMP$



67°

9. Fill in each blank using the digits 0 to 9 only once. One digit will not be used.
Responses vary. **Sample shown.**



Spiral Review

Problems 10–13: Solve each equation.

10. $x + 40 = 180$
$x = 140$

11. $x + 40 = 90$
$x = 50$

12. $2x + 40 = 180$
$x = 70$

13. $2(x + 40) = 180$
$x = 50$

Problems 14–15: A small dog gets fed $\frac{3}{4}$ of a cup of dog food twice a day.

14. Write an equation representing the relationship between the number of days, d, and the number of cups of food, f.
$f = 1.5d$ **(or equivalent)**

15. How many days will a large bag of dog food last if a new bag contains 210 cups of food?
140 days

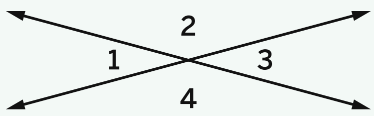
Reflection

1. Put a star next to a problem you're still wondering about.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice 7.7.03

Lesson Summary

When two lines cross, the angles that are opposite each other have the same measure. These angles are called **vertical angles**.



$\angle 1$ and $\angle 3$ are a pair of vertical angles. Another pair is $\angle 2$ and $\angle 4$.

Using vertical angles can help you determine unknown angle measures.

For example, if the measure of Angle 1 is $30°$, then:

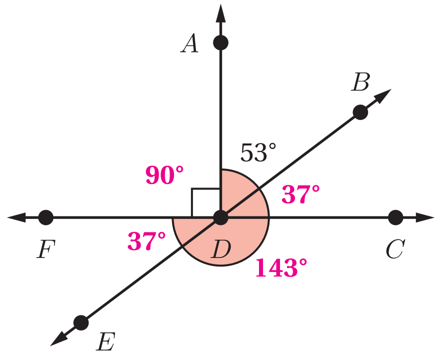
- The measure of Angle 3 is $30°$ because $\angle 1$ and $\angle 3$ are vertical angles.
- The measure of Angle 2 is $150°$ because $\angle 1$ and $\angle 2$ are supplementary angles.
- The measure of Angle 4 is $150°$ because $\angle 2$ and $\angle 4$ are vertical angles.

Things to Remember:

Lesson Practice
7.7.03

Name: ______________ Date: ________ Period: ________

Problems 1–3: Here is a diagram.



1. Determine the measure of each angle.

Angle	Measure (degrees)
ADB	53
BDC	**37**
CDE	**143**
FDE	**37**
FDA	**90**

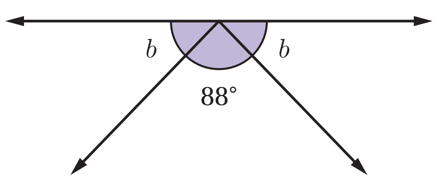
2. Identify one pair of vertical angles in the diagram.

***Responses vary.* Angles BDC and FDE**

3. Explain how you know they are vertical angles.

***Responses vary.* Because they are angles opposite each other where two lines cross.**

Problems 4–5: Here is a new diagram.



4. Which equation represents the relationship between the angles in the figure?

A. $88 + b = 90$

B. $88 + b = 180$

C. $2b + 88 = 90$

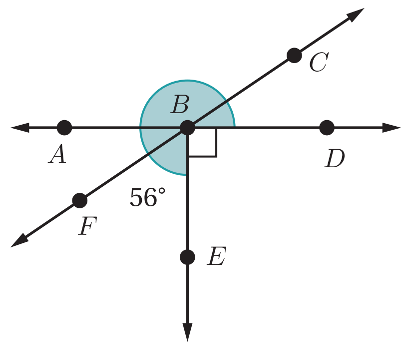
D. $2b + 88 = 180$

5. Dakota says that the angles marked b are vertical angles. Paz disagrees. Who is correct? Explain your thinking.

Paz. *Explanations vary.* The angles are equal but not vertical. The angles are made from one line and two segments meeting at a point. To be vertical angles, they would need to be made from two lines crossing.

Name: ________ Date: ________ Period: ________

6. This diagram includes supplementary, complementary, and vertical angles.
Determine each of the missing angle measures.



Angle	Measure (degrees)
ABC	**146**
CBD	**34**
DBE	**90**
FBA	**34**

Spiral Review

7. Select *all* of the equations that are equivalent to $3x + 45 = 180$.

- ☐ **A.** $3(x + 45) = 180$
- ☑ **B.** $3(x + 15) = 180$
- ☐ **C.** $3(x + 15) = 60$
- ☑ **D.** $x + 15 = 60$
- ☑ **E.** $3x = 135$

Problems 8–10: Paz is solving the equation $4\left(x + \frac{3}{2}\right) = 8$.

Paz says: *I can subtract* $\frac{3}{2}$ *from each side to get* $4x = \frac{13}{2}$ *and then divide by* 4 *to get* $x = \frac{13}{8}$. Dakota says: *I think you made a mistake.*

8. How can Dakota determine that Paz's solution is incorrect?

***Responses vary.* Dakota could substitute $\frac{13}{8}$ for x in the equation and check if the expression on the left-hand side is equal to 8.**

9. Describe the error that Paz might have made.

***Responses vary.* Paz might not have realized that the 4 was multiplied by both x and $\frac{3}{2}$.**

10. Determine the correct solution for x.

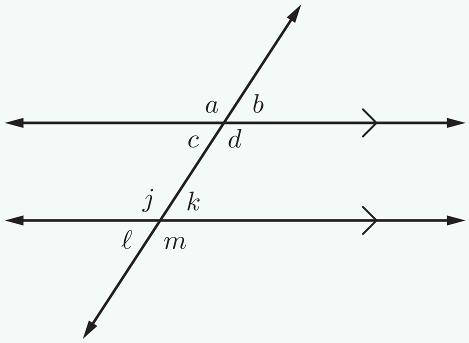
$x = \frac{1}{2}$

Reflection

1. Put a question mark next to a problem you were feeling stuck on.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary



Rigid transformations of a single line result in a line. Rigid transformations of parallel lines result in parallel lines. We can use these properties to help show which angles in a diagram are congruent.

We can use transformations to show that *vertical angles* (angles opposite each other when two lines cross) are congruent. For example, vertical angles a and d are congruent because you can rotate angle a 180 degrees around where the lines intersect and it will move onto angle d.

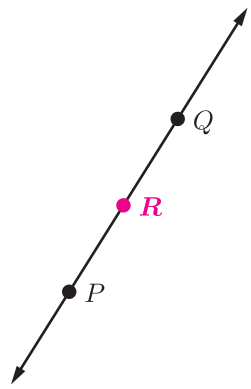
Transformations can also help show which angles are congruent when a **transversal** intersects parallel lines. Angle b is congruent to angle k because you can translate angle b along the transversal until it moves exactly onto angle k.

Things to Remember:

Lesson Practice
8.1.10

Name: ________ Date: ________ Period: ________

Problems 1–2: Points P and Q are plotted on a line.

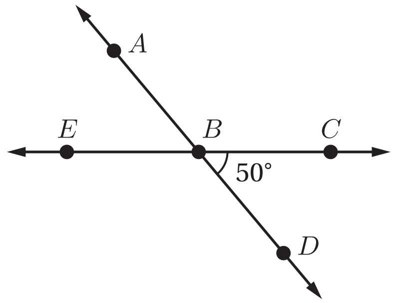


1. Plot point R so that a $180°$ rotation with center R moves point P onto point Q and point Q onto point P.

2. Is there more than one point R that works? Explain your thinking.

No. *Explanations vary.* The midpoint of PQ is the only point that works. $180°$ rotations with any other center will not send point P to point Q or point Q to point P.

Problems 3–5: Determine the measure of each angle in the diagram. Explain your thinking.



3. $\angle ABC$

$130°$. *Explanations vary.* $\angle ABC$ and $\angle CBD$ make a line, so the two angles add up to $180°$.

4. $\angle EBD$

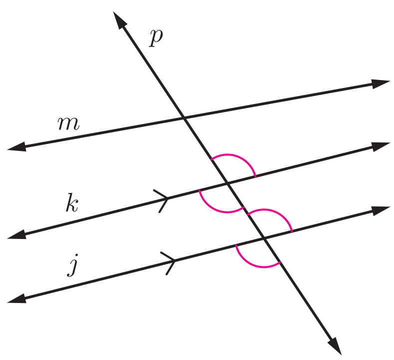
$130°$. *Explanations vary.* $\angle EBD$ and $\angle CBD$ make a line, so the two angles add up to $180°$.

5. $\angle ABE$

$50°$. *Explanations vary.* $\angle ABE$ and $\angle DBC$ are vertical angles.

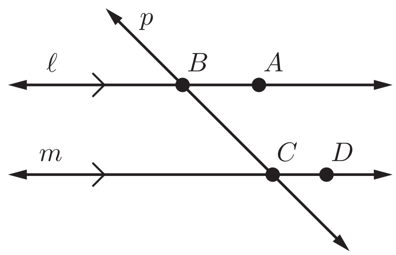
6. Lines j and k are parallel. Mark any four congruent angles.

Responses vary.



Name: .... Date: .... Period: ....

7. Transversal p intersects parallel lines ℓ and m. If the measure of $\angle BCD$ is three times as much as the measure of $\angle ABC$, what is the measure of $\angle ABC$? Show or explain your thinking.



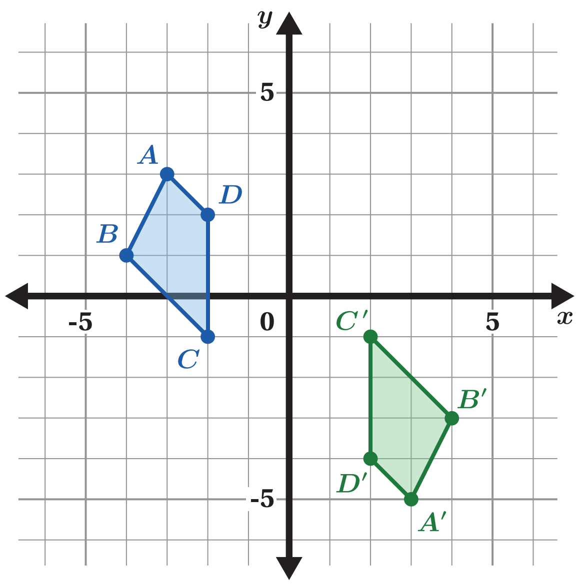
45°. *Explanations vary.* The sum of the measures of $\angle BCD$ and $\angle ABC$ are equal to 180°.

If x represents the measure of $\angle ABC$, in degrees, then:

$$x + 3x = 180$$
$$4x = 180$$
$$x = 45$$

Spiral Review

Problems 8–9: This graph shows two figures.



8. Describe a sequence of transformations that moves figure $ABCD$ onto figure $A'B'C'D'$.

***Responses vary.* Rotate 180° around point C, and then translate the image 4 units to the right.**

9. Is figure $ABCD$ congruent to figure $A'B'C'D'$? Explain your thinking.

Yes. *Explanations vary.* There is a sequence of transformations that moves one onto the other, so the two figures are congruent.

Reflection

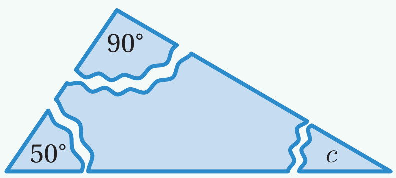
1. Put a star next to your favorite problem.

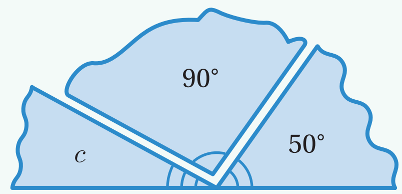
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

The interior angle measures of any triangle always sum to 180 degrees. We can demonstrate this by rearranging the angles of any triangle to form a straight line, which has a measure of $180°$.

If you know the measures of two angles in a triangle, you can determine the third angle by subtracting the sum of the two known angle measures from $180°$. Here is an example.





$$c = 180 - 90 - 50$$
$$c = 40$$

Things to Remember:

Lesson Practice
8.1.11

Name: .... Date: .... Period: ....

1. If triangle ABC is a right triangle and $m\angle A = 40°$, what are possible measures for angles B and C?

Responses vary.
- $m\angle B = 50°$ **and** $m\angle C = 90°$
- $m\angle B = 90°$ **and** $m\angle C = 50°$

2. Triangle ABC is an isosceles triangle and the measure of one of the angles is $40°$. List a set of possible angle measures in this triangle.

Responses vary.
- **70°, 70°, 40°**
- **40°, 40°, 100°**

3. Select *all* of the sets of angles that could make a triangle.

- [x] **A.** 60°, 60°, 60°
- [ ] **B.** 90°, 90°, 45°
- [ ] **C.** 30°, 40°, 50°
- [x] **D.** 90°, 45°, 45°
- [x] **E.** 120°, 30°, 30°

4. Angle A in triangle ABC is obtuse. Can angle B or angle C be obtuse?

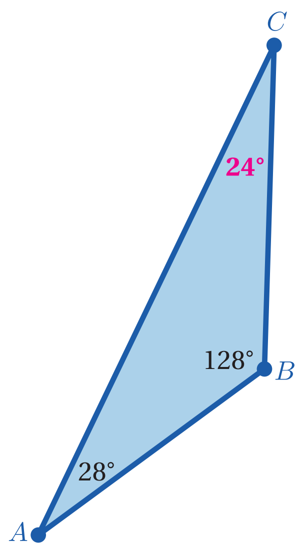
No

Explain your thinking.

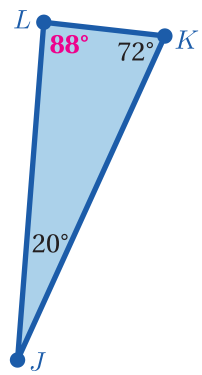
Explanations vary. **The angles in a triangle need to add to 180 degrees. If two of the angles are obtuse, then their sum is already more than 180 degrees.**

Problems 5–6: Determine the measure of each missing angle.

5.



6.



Name: .... Date: .... Period: ....

7. Can there be a triangle with two right angles?

No

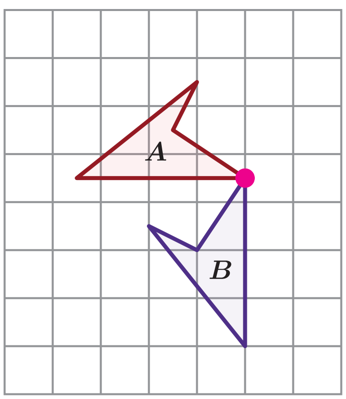
Explain your thinking.

Explanations vary. **The sum of the three interior angles of a triangle is 180 degrees. Because two right angles already have a sum of 180 degrees, the measure of the third angle must be $0°$, which is not possible for a triangle.**

Spiral Review

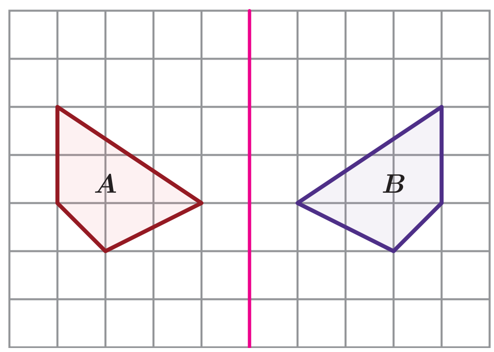
Problems 8–9: For each pair of polygons, describe a transformation that shows polygon A is congruent to polygon B.

8.



Responses vary. **Rotate polygon A $90°$ counterclockwise around the bottom-right vertex of polygon A.**

9.



Responses vary. **Reflect polygon A over a vertical line that lies halfway between the two polygons.**

Reflection

1. Circle a problem you're still curious about.

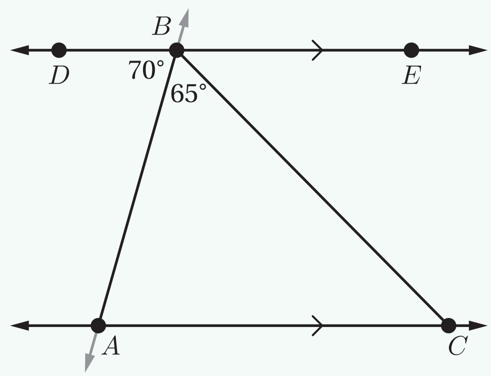
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

There are several strategies you can use to determine unknown angle measures in a diagram.

- Use translations to show that corresponding angles on parallel lines are congruent.
- Use rotations to show that vertical angles are always congruent.
- Use the fact that the sum of three interior angles in any triangle is 180 degrees.
- Use the fact that any straight line has an angle measure of $180°$.

Here is an example. The measure of angle DBA is $70°$ and the measure of angle ABC is $65°$. Let's determine the remaining angle measures.



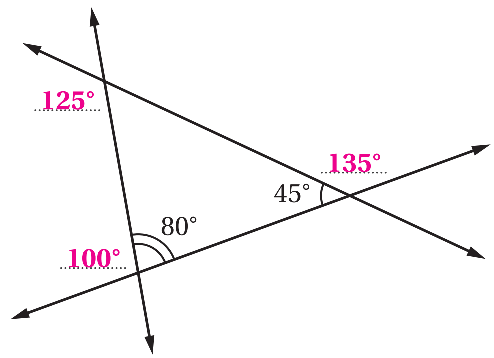
- The measure of $\angle BAC$ must also be $70°$ because you can translate and then rotate $\angle DBA$ $180°$ to fit exactly on top of $\angle BAC$.
- $\angle EBC$ must be $45°$ because it makes a straight line with $\angle DBA$ and $\angle ABC$, and $70 + 65 + 45 = 180$.
- $\angle BCA$ must also be $45°$ because you can translate and rotate $\angle EBC$ exactly on top of it and also because it is part of a triangle with the $70°$ and $65°$ angles.

Things to Remember:

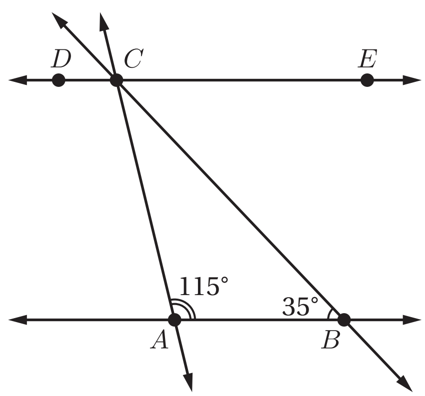
Lesson Practice
8.1.12

Name: .... Date: .... Period: ....

1. Here are three intersecting lines. Determine the missing angle measurements.



Problems 2–4: Line AB is parallel to line DE.

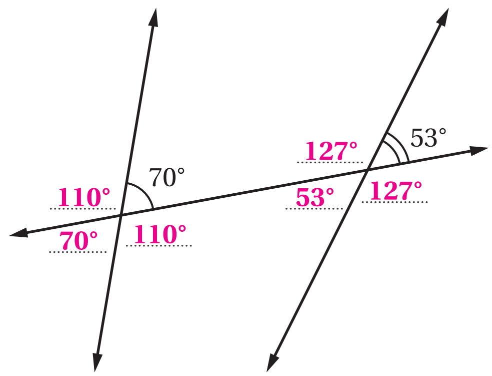


2. What is the measure of $\angle ACD$?
   115°

3. What is the measure of $\angle ECB$?
   35°

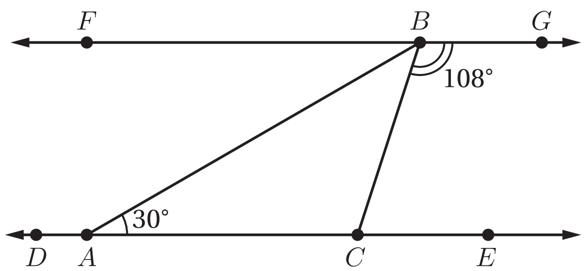
4. What is the measure of $\angle ACB$?
   30°

5. This diagram shows three lines with some missing angle measures. Complete the diagram by writing in all of the missing angle measures.



6. Line DE is parallel to line FG.

Is it possible to determine all five missing angle measures with the given information? Show or explain your thinking.



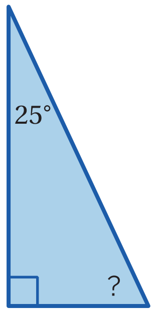
Yes. *Explanations vary.*

- $\angle DAB$ and $\angle BAC$ are supplementary angles, so $m\angle DAB = 150°$.
- $\angle BAC$ and $\angle FBA$ are congruent, so $m\angle FBA = 30°$.
- $\angle FBA$, $\angle ABC$, and $\angle CBG$ form a straight line measuring 180°, so $m\angle ABC = 42°$.
- $\angle BAC$, $\angle ABC$, and $\angle BCA$ are interior angles in a triangle, so $m\angle BCA = 108°$.
- $\angle BCA$ and $\angle BCE$ are supplementary angles, so $m\angle BCE = 72°$.

Spiral Review

Problems 7–9: Determine the missing angle measure(s) in each triangle. Show or explain your thinking.

7. Right triangle:

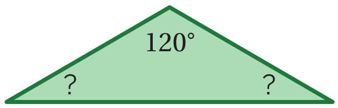


65°. *Explanations vary.*

$90 + 25 + x = 180$

$x = 65$

8. Isosceles triangle:

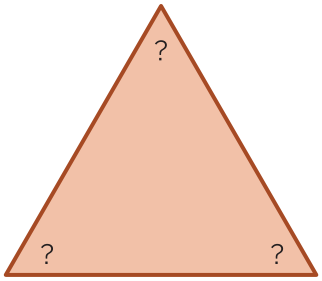


Each missing angle is 30°.
Explanations vary.

$2x + 120 = 180$

$x = 30$

9. Equilateral triangle:



Each missing angle is 60°.
Explanations vary.

$3x = 180$

$x = 60$

Reflection

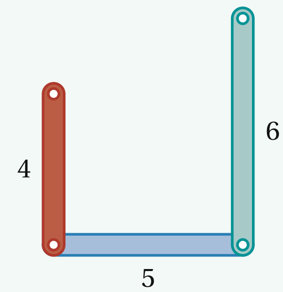
1. Put a smiley face next to a problem you were stuck on and then figured out.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Three line segments do not always make a triangle.

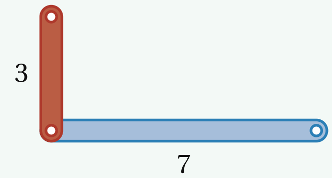
In order for three line segments to form a triangle, the sum of the two shorter segments' lengths must be greater than the third segment's length.

For example, $4 + 5 > 6$, so these three line segments would make a triangle.



You can use this relationship to determine what possible lengths would create a triangle.

For example, if two side lengths of a triangle are 3 units and 7 units, then the third side must be greater than 4 and less than 10 units.



Things to Remember:

Lesson Practice
7.7.05

Name: ________ Date: ________ Period: ________

1. Select *all* the sets of line segments that will form a triangle.

- ☐ **A.** 3, 4, and 8 units
- ☑ **B.** 7, 6, and 12 units
- ☑ **C.** 5, 13, and 11 units
- ☐ **D.** 12, 6, and 4 units
- ☐ **E.** 4, 6, and 10 units

2. Choose one of the other sets of segments from Problem 1 and explain why it will *not* form a triangle.

***Responses vary.* 4 units, 6 units, and 10 units will not form a triangle because $4 + 6$ is not greater than 10.**

3. One side of a triangle is 5.5 inches long and another side is 10.5 inches long. Select *all* possible side lengths for the third side.

- ☐ **A.** 3 inches
- ☐ **B.** 5 inches
- ☑ **C.** 7 inches
- ☑ **D.** 10 inches
- ☑ **E.** 12 inches
- ☐ **F.** 20 inches

Problems 4–5: A triangle has one side that is 4 centimeters long and another side that is 9 centimeters long. The third side is a whole number of centimeters.

4. What is the shortest possible length of the third side? **6** centimeters

5. What is the longest possible length of the third side? **12** centimeters

6. The direct distance between Airport A and Airport B is 120 miles. The direct distance between Airport B and Airport C is 240 miles. Which of these could be the direct distance between Airport A and Airport C?

A. 370 miles **B.** 300 miles **C.** 120 miles **D.** 110 miles

7. Create a set of four segment lengths so that:

- Each length is different.
- Each length is a whole number (in centimeters).
- No matter which of the three lengths you choose, you will *always* be able to form a triangle.

Explain your thinking.

***Explanations vary.* No matter which three lengths you choose, the sum of the lengths of the two shortest segments will always be greater than the length of the longest segment. That means they'll always form a triangle.**

Segment	Length (cm)
A	**3**
B	**4**
C	**5**
D	**6**

Lesson Practice
7.7.05

Name: .... Date: .... Period: ....

Spiral Review

Problems 8–11: Evaluate each expression.

8. $5 - 8$ $\mathbf{= -3}$

9. $5 + (\text{-}8)$ $\mathbf{= -3}$

10. $\text{-}5 + 8$ $\mathbf{= 3}$

11. $\text{-}5 + (\text{-}8)$ $\mathbf{= -13}$

Problems 12–13: Solve each equation.

12. $1.5 = 0.6(w + 0.3)$

$\mathbf{w = 2.2}$

13. $1.5 = 0.6w + 0.3$

$\mathbf{w = 2}$

14. Arjun says that because $\frac{1}{12}$ and $\frac{2}{12}$ produce repeating decimal values, any fraction with a denominator of 12 will also produce a repeating decimal.

Is Arjun correct? Explain your thinking.

No. *Explanations vary.*

- **I know that $\frac{12}{12}$ equals 1, which is not a repeating decimal.**
- **Some fractions with a denominator of 12 produce repeating decimals. But others, like $\frac{3}{12}$ and $\frac{6}{12}$, do not.**

Reflection

1. Put a star next to the problem you understood best.
2. Use this space to ask a question or share something you're proud of.

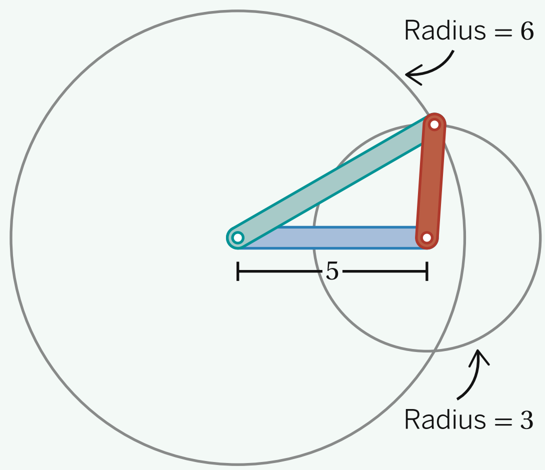
Lesson Practice 7.7.06

Lesson Summary

You can use circles to draw triangles.

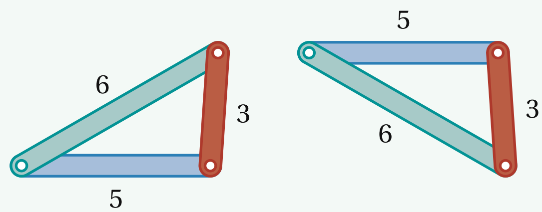
Here are the steps to draw a triangle with sides that are 5, 3, and 6 units long:

- Draw a line segment that is 5 units long.
- Draw a circle with a *radius* of 3 units centered at one end point.
- Draw a circle with a radius of 6 units centered at the other end point.
- Draw line segments connecting the end points of the 5 unit segment to a point where the two circles intersect.



All the triangles whose sides are 5, 3, and 6 units long will be **identical copies** because they have the same shape and size.

In fact, it is only possible to create one unique triangle if you know its three side lengths (unless you can't make a triangle at all).



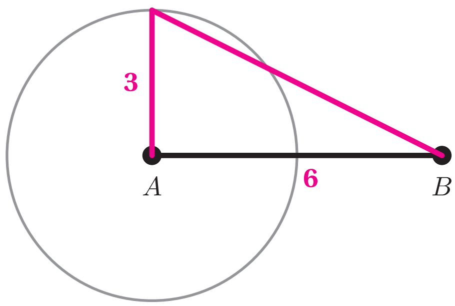
Things to Remember:

Lesson Practice
7.7.06

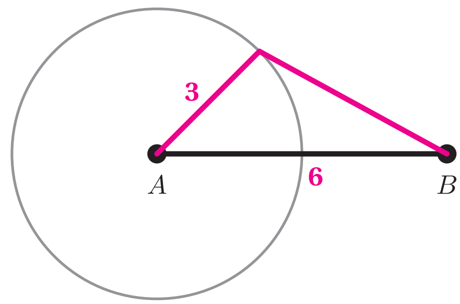
Name: .... Date: .... Period: ....

1. Segment AB is 6 units long and the radius of circle A is 3 units. Draw two different triangles where one side is 6 units long and another side is 3 units long.

Triangle 1



Triangle 2



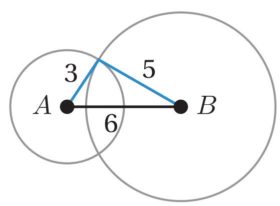
Responses vary.

2. Faith drew two triangles with side lengths of 3, 5, and 6 units.

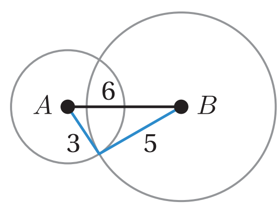
Are the triangles identical? Explain your thinking.

Yes. *Explanations vary.* The triangles are identical copies because if you moved or turned one triangle on top of the other, they would be the same size and shape.

Triangle 1

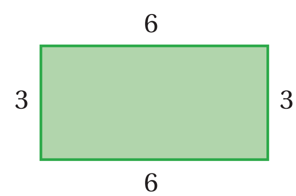


Triangle 2



3. Antwon drew this rectangle. Write an argument to convince him that this is not the only quadrilateral he can draw with these side lengths.

***Responses vary.* It is possible to draw a parallelogram or a kite using these side lengths.**



4. A box contains wood planks of several different lengths. There is one 3-foot plank, one 4-foot plank, one 5-foot plank, and one 7-foot plank. What is the maximum number of different triangles that can be made using these planks as sides?

A. 1

B. 2

C. 3

D. 4

Lesson Practice
7.7.06

Name: ______ Date: ______ Period: ______

5. Create a set of five side lengths so that:

- Each length is different.
- Each length is a whole number (in inches).
- No matter which three you choose, you will never be able to make a triangle.

Explain how you know that your set of lengths meet all the requirements.

***Responses vary.* No matter which three lengths are used, the sum of the two shortest sides will always be shorter than or equal to the length of the longest side. For example, 10 inches is too long for 3 inches and 6 inches, 6 inches is too long for 2 inches and 3 inches, and 3 inches is too long for 1 inch and 2 inches.**

Side	Length (in.)
A	**1**
B	**2**
C	**3**
D	**6**
E	**10**

Spiral Review

6. A triangle has one side that is 6 units long and another side that is 3 units long. Select *all* the possible lengths for the third side.

- [ ] **A.** 2 units
- [ ] **B.** 3 units
- [x] **C.** 4 units
- [x] **D.** 6 units
- [x] **E.** 8 units
- [ ] **F.** 10 units

Problems 7–10: Evaluate each expression.

7. $4 - 9$ **= -5**

8. $-4 - 9$ **= -13**

9. $4 - (-9)$ **= 13**

10. $-4 - (-9)$ **= 5**

Problems 11–12: If you deposit $300 in an account and it grows by 6% each year, how much money will be in your account after . . .

11. 1 year?
$318

12. 2 years?
$337.08

Reflection

1. Circle a problem you want to talk to a classmate about.

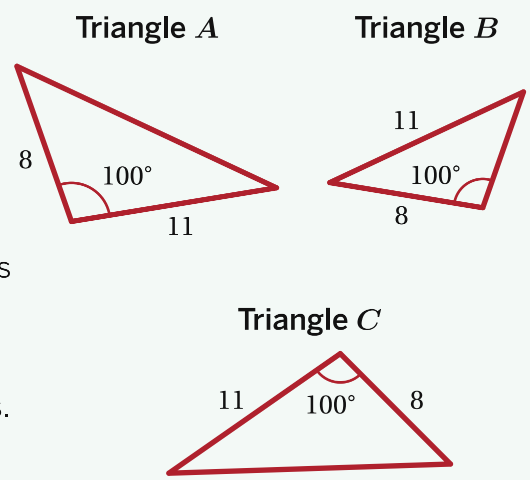
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

If three side lengths make a triangle, all triangles with those side lengths are identical copies. But what about a combination of side lengths and angles? Some combinations of measurements can make more than one unique triangle.

Here are three triangles: A, B, and C.

Each triangle has three of the same measurements: one side that is 8 units long, one side that is 11 units long, and one $100°$ angle.



Only triangles A and C are *identical copies* because the $100°$ angle is in between the side lengths of 8 and 11 units.

This means that there is more than one unique triangle that can be made with these measurements.

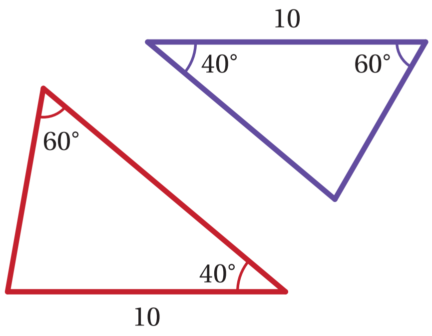
In general, knowing the order or placement of the sides and angles can help you determine whether two triangles are *identical copies* and how many unique triangles there are.

Things to Remember:

Lesson Practice
7.7.07

Name: .... Date: .... Period: ....

1. Are these two triangles identical? Circle one.

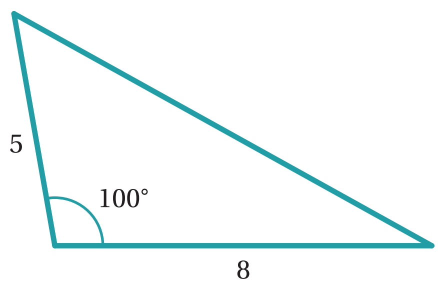


Yes No Not enough information

Explain your thinking.

***Explanations vary.* The side that's 10 units in the first triangle is adjacent to the $40°$, but it's between the two angles in the second triangle. The triangle on the left is also larger.**

2. This triangle has a side length of 5 units, a side length of 8 units, and a $100°$ angle.



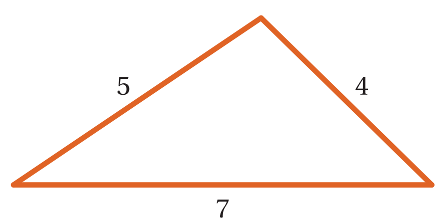
Is this the only triangle that can be created with these three measurements? Circle one.

Yes No Not enough information

Explain your thinking.

***Explanations vary.* It is possible to create other triangles using these measurements if you change where the angle is related to the sides.**

3. This triangle has side lengths of 7, 4, and 5 units.



Is this the only triangle that can be created with these three measurements? Circle one.

Yes No Not enough information

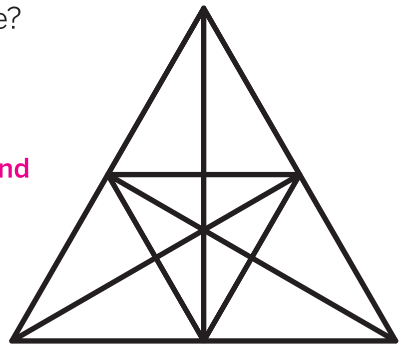
Explain your thinking.

***Explanations vary.* Any other triangle with the same three side lengths will be identical, even if it is turned around.**

Name: ________ Date: ________ Period: ________

4. How many non-identical triangles are there in this image? Show or explain your thinking.

8 non-identical triangles (47 triangles total). ***Explanations vary.*** **I noticed that the triangle is split into many identical triangles. I started with the smallest triangle and then found the next biggest triangle I could form until I counted 8.**



Spiral Review

5. Angle H and angle K are supplementary angles.

- The measure of angle H is $63°$.
- The measure of angle K is $(2x)°$.

Which equation can be used to determine the value of x?

A. $63 = 2x$ **B.** $63 + 2x = 180$ **C.** $63 + 2x = 90$ **D.** $63 + 2x = 360$

Problems 6–9: For each given angle, determine the measure of the supplementary angle.

6. $80°$ angle

$\mathbf{100°}$

7. $25°$ angle

$\mathbf{155°}$

8. $119°$ angle

$\mathbf{61°}$

9. $x°$ angle

$\mathbf{(180 - x)°}$

Problems 10–11: Two months ago, the price of a cell phone was c dollars. Last month, the price of the phone increased by 10%.

10. Write an expression for the price of the phone last month.

$\mathbf{1.1c}$

11. This month, the price of the phone decreased by 10%. Is the price of the phone this month the same as it was two months ago? Explain your thinking.

No. ***Explanations vary.*** **Increasing by 10% is 110% of the original price. Decreasing by 10% will be 99% of the original (because $1.1 \times 0.9 = 0.99$).**

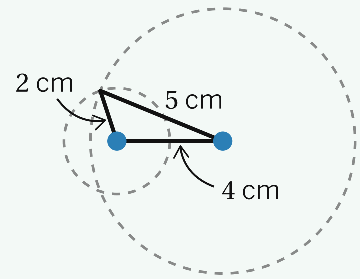
Reflection

1. Circle a problem you are still curious about.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

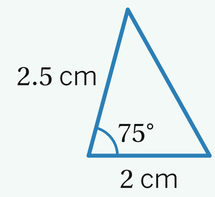
You can use a protractor, ruler, and compass to draw triangles with given measurements.



For example, when given three side lengths, using a ruler and a compass to draw circles with radiuses that match the given lengths can help you draw a triangle.

There is only one unique triangle that can be drawn when given three side lengths. When given two side lengths and an angle measure, multiple non-identical triangles can be drawn.

For example, here is one strategy for drawing a triangle that has a 75° angle, a side length of 2 centimeters, and a side length of 2.5 centimeters.

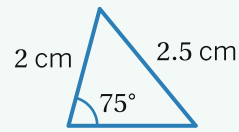


Step 1: Draw a 2-centimeter line segment.

Step 2: Draw a 75° angle using a protractor.

Step 3: Measure 2.5 centimeters along the other ray of the angle and connect the triangle.

Here is another non-identical triangle that has the same three measurements. The 75° angle can be positioned in different places in relation to the 2- and 2.5-centimeter sides.



Things to Remember:

Lesson Practice
7.7.08

Name: .... Date: .... Period: ....

Problems 1–2: A triangle has a 90° angle, a 60° angle, and one 6-centimeter side. Circle true or false for each statement about this triangle.

1. The triangle contains two angles that are complementary.

True False

2. Many non-identical triangles can be made using these measurements.

True False

Problems 3–5: For each set of three measurements, decide whether you can create zero triangles, one triangle, or more than one non-identical triangle. Circle your choice, then explain your thinking.

3. One 4-centimeter side, one 6-centimeter side, and one 50° angle.

Zero One More than one

***Explanations vary.* I can change the position of the angle to create more than one triangle.**

4. One 4-centimeter side, one 5-centimeter side, and one 6-centimeter side.

Zero One More than one

***Explanations vary.* Any two triangles with all the same side lengths will be identical.**

5. One 90° angle, one 100° angle, and one 30° angle.

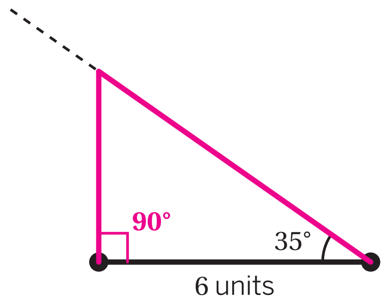
Zero One More than one

***Explanations vary.* The angles of a triangle must have a sum of 180°.**

6. A triangle has a 90° angle, a 35° angle, and a side that is 6 units long.

The 6-unit side is in between the 90° and 35° angles.

Complete the diagram and label your diagram with the given measurements.



Lesson Practice
7.7.08

Name: .... Date: .... Period: ....

7. Write one side length and two angles so that only one unique triangle is possible.

Explain your thinking.

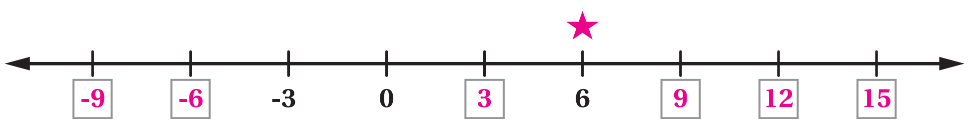
***Explanations vary.* If all of the angle measures are equal then the triangle must have all the same side lengths. If one side length is 5 units, then all side lengths must be 5 units. When all sides are the same length only one unique triangle is possible.**

Side Length	Angle	Angle
5 units	**60°**	**60°**

Side lengths vary.

Spiral Review

Problems 8–9: Here is a number line.



8. Complete the number line. The markings on the number line are equally spaced.
Response shown on number line.

9. Put a star over the point on the number line that represents the value of (-3)(-2).
Response shown on number line.

Problems 10–13: Calculate each value.

10. $\frac{24}{-4} = -6$

11. $\frac{-24}{-4} = 6$

12. $\frac{-24}{4} = -6$

13. $\frac{-24 - 4}{-4} = 7$

14. A factory produces 3 bottles of sparkling water for every 7 bottles of plain water. If those are the only two products they produce, what percent of their production is sparkling water?
30%

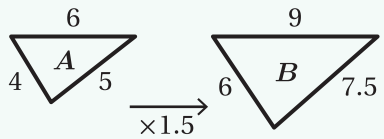
Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

When we create scaled copies, the **scale factor** is the number that every length of the original shape is multiplied by to produce the scaled copy.

For example, the scale factor from triangle A to triangle B is 1.5.



Another way to see the scale factor is to look for the ratio between corresponding sides in the two shapes. The ratio of the new length to the original length is the scale factor.

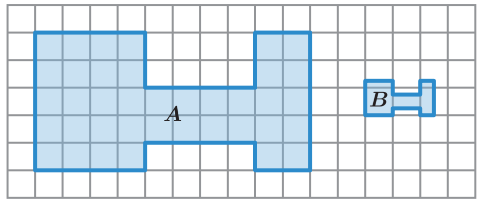
For triangle A and triangle B, the ratios are $\frac{9}{6}$, $\frac{7.5}{5}$, and $\frac{6}{4}$. Since those ratios are all equivalent, any one can be used as the scale factor. You can also use another equivalent ratio as the scale factor, like $\frac{3}{2}$ or 1.5.

Things to Remember:

Lesson Practice
7.1.02

Name: ______________ Date: ________ Period: ________

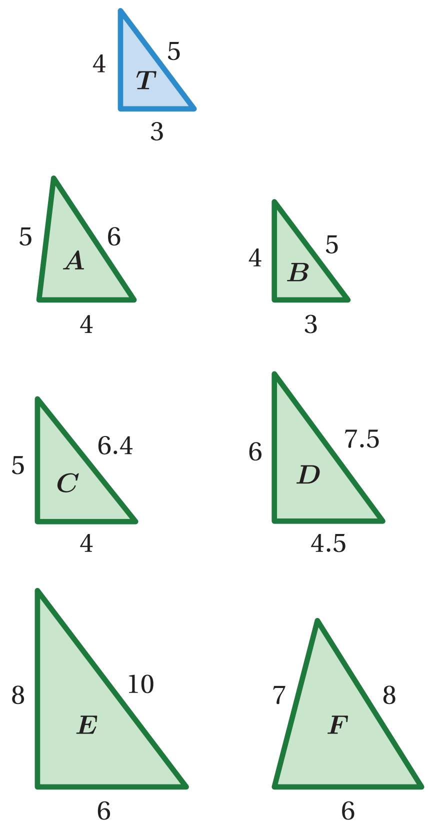
1. Figures A and B are scaled copies.



What scale factor takes A to B? Explain your thinking.

$\frac{1}{4}$. *Explanations vary.* All of the corresponding side lengths in the scaled copy are $\frac{1}{4}$ the length of the original.

Problems 2–4: Here are seven triangles.



2. Which triangles are scaled copies of triangle T?

Triangles B, D, and E

3. For each scaled copy, write the scale factor that takes triangle T to that triangle. Leave blank if it is not a scaled copy.

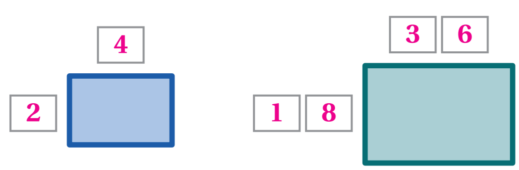
Triangle	Scale Factor
A	
B	1
C	
D	1.5
E	2
F	

4. List two triangles whose lengths can be represented by the equivalent ratios $\frac{4}{8} = \frac{5}{10} = \frac{3}{6}$.

Triangles T and E or triangles B and E

5. Using the digits 0 to 9 without repetition, fill in the blanks to make rectangles that are scaled copies.

Responses vary.



Lesson Practice
7.1.02

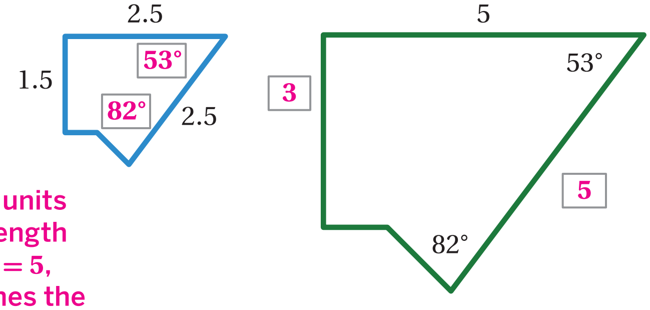
Name: ______ Date: ______ Period: ______

Problems 6–8: Polygon B is a scaled copy of polygon A.

6. What is the scale factor that takes polygon A to polygon B? Explain your thinking.

2. *Explanations vary.* The side length of 5 units in polygon B corresponds with the side length of 2.5 units in polygon A. Because $2.5 \cdot 2 = 5$, all the side lengths in polygon B are 2 times the corresponding side lengths in polygon A.

Polygon A **Polygon B**



7. Enter the missing lengths in polygon B.

Responses shown on diagram.

8. Enter the missing angle measurements in polygon A.

Responses shown on diagram.

9. Tyler says that figure K is a scaled copy of figure G. Is Tyler correct? Explain your thinking.

Figure G

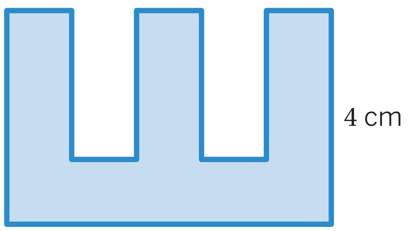
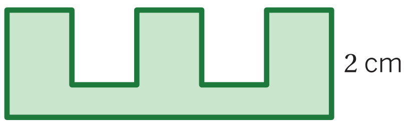


Figure K



No. *Explanations vary.* The heights suggest a scale factor of 0.5 from figures G to K, but that scale factor doesn't seem like it was applied to *all* dimensions of figure G.

Spiral Review

Problems 10–13: Evaluate each expression.

10. $\frac{1}{4} \cdot 32 = 8$ **11.** $\frac{1}{4} \cdot 5.6 = 1.4$ **12.** $7.2 \cdot \frac{1}{9} = 0.8$ **13.** $2 \div \frac{1}{4} = 8$

Reflection

1. Put a heart next to the problem you feel most confident about.

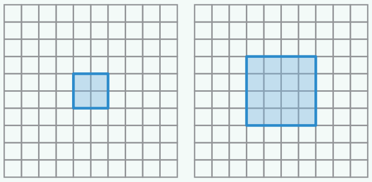
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

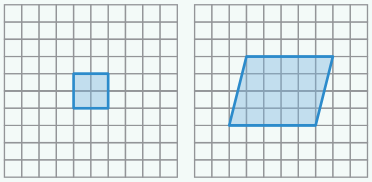
To create a scaled copy, we multiply all the side lengths in a shape by the same *scale factor*. This will create new side lengths, while keeping the angle measures and ratio between sides the same as the original.

To draw an accurate scaled copy, it's helpful to use measuring tools or a grid to make sure your drawing has the correct side lengths and angles.

Scaled Copies



Not Scaled Copies

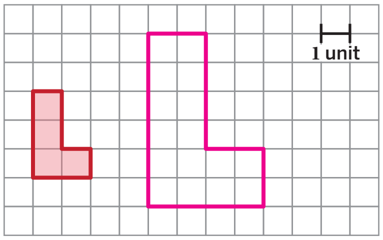


Things to Remember:

Lesson Practice
7.1.03

Name: .... Date: .... Period: ....

Problems 1–2: Here is a polygon.



1. Draw a scaled copy of the polygon using a scale factor of 2.

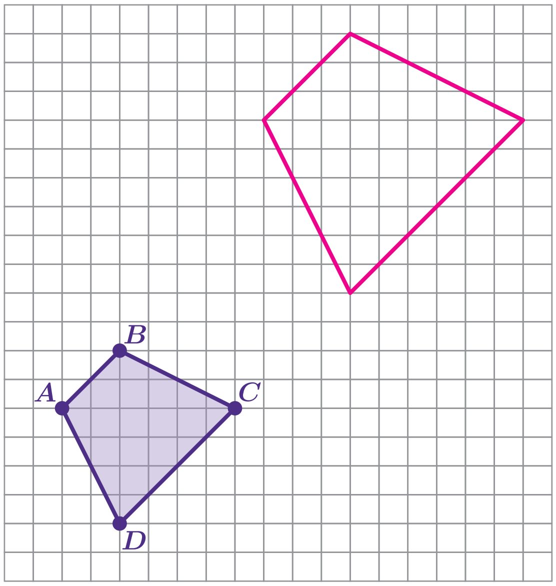
Sample shown on grid.

2. What is the area and perimeter of your scaled copy?

The area is 16 square units. The perimeter is 20 units.

3. Draw a scaled copy of figure $ABCD$ using a scale factor of 1.5.

Sample shown on grid.



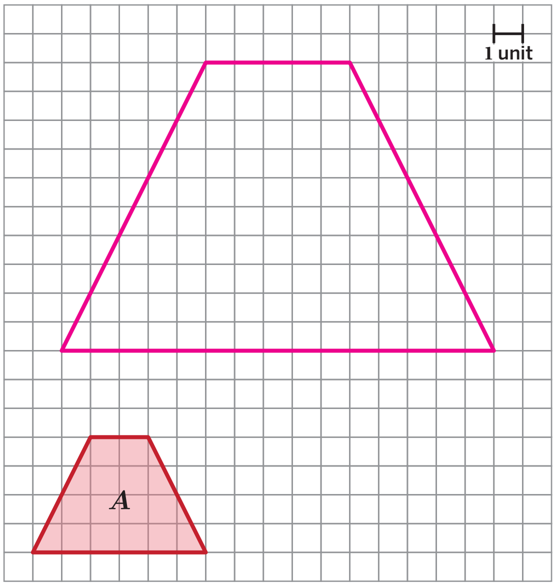
Problems 4–5: Imagine there is a quadrilateral B that is a scaled copy of quadrilateral A. Its shortest side is 5 units long.

4. What is the scale factor from quadrilateral A to B?

2.5

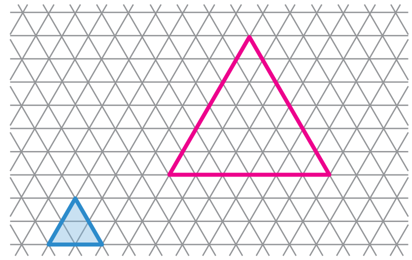
5. Draw quadrilateral B.

Sample shown on grid.



Name: .... Date: .... Period: ....

Problems 6–7: Here is an equilateral triangle.



6. Draw a scaled copy of this equilateral triangle using a scale factor of 3.

Sample shown on grid.

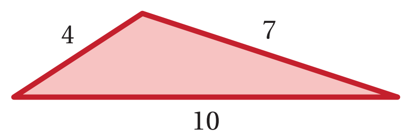
7. Equilateral triangles are always scaled copies. What other shapes are always scaled copies?

Squares, circles, or any regular shape (shapes where all sides and all angles are equal)

Spiral Review

8. Triangle Z is a scaled copy of triangle M. Select *all* sets of values that could be the side lengths of triangle Z.

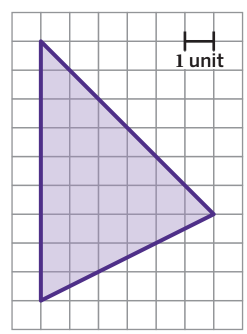
Triangle M



- ☐ **A.** 8, 11, and 14
- ☑ **B.** 10, 17.5, and 25
- ☐ **C.** 6, 9, and 11
- ☑ **D.** 6, 10.5, and 15
- ☑ **E.** 8, 14, and 20

9. Determine the area of this triangle.

27 square units



Problems 10–12: Solve each equation. Show or explain your thinking.

10. $6x = 156$

$x = 26$

Work varies.

$6x \div 6 = 156 \div 6$

$x = 26$

11. $16x = 8$

$x = \frac{1}{2}$

Work varies.

$16x \div 16 = 8 \div 16$

$x = \frac{1}{2}$

12. $\frac{1}{5}x = 1$

$x = 5$

Work varies.

$\frac{1}{5}x \div \frac{1}{5} = 1 \div \frac{1}{5}$

$x = 5$

Reflection

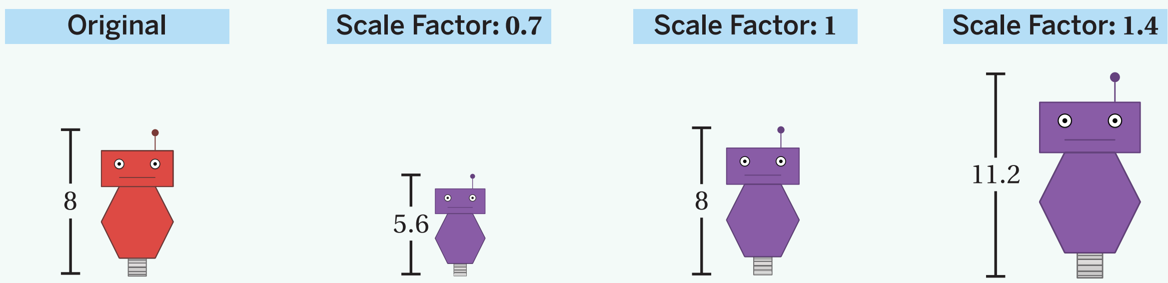
1. Put a question mark next to a response you'd like to compare with a classmate's.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can use different scale factors to create copies that are smaller, larger, or the same size as the original.

If the scale factor is:

- *Less* than 1, the copy will be *smaller* than the original.
- *Equal to* 1, the copy will be *the same size* as the original.
- *Greater* than 1, the copy will be *larger* than the original.

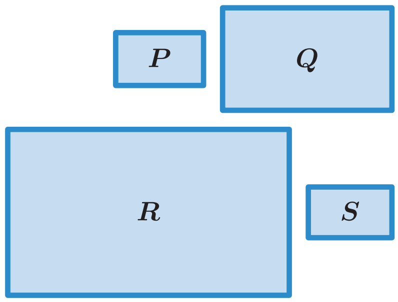


Things to Remember:

Lesson Practice
7.1.04

Name: .... Date: .... Period: ....

1. Rectangles P, Q, R, and S are scaled copies of one another. For each pair, state whether the scale factor that takes one figure to another is greater than 1, equal to 1, or less than 1.



Rectangle P to rectangle R **Greater than 1**

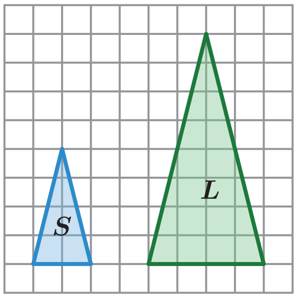
Rectangle Q to rectangle S **Less than 1**

Rectangle Q to rectangle R **Greater than 1**

Rectangle S to rectangle P **Equal to 1**

Rectangle R to rectangle P **Less than 1**

Problems 2–4: Triangle S and triangle L are scaled copies of one another.



2. What is the scale factor that takes triangle S to triangle L?

2

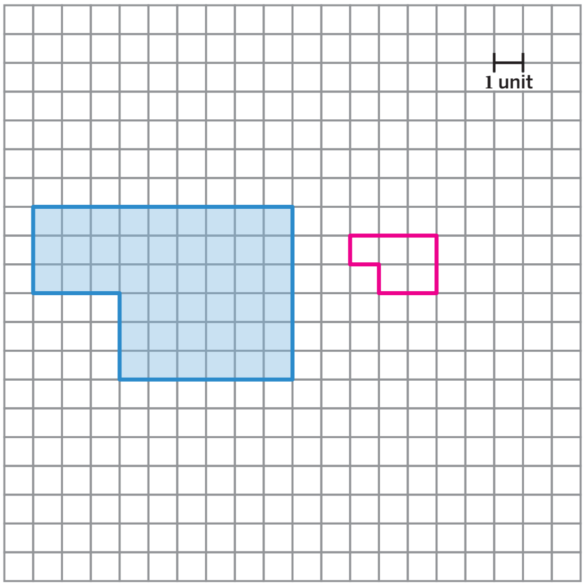
3. What is the scale factor that takes triangle L to triangle S?

$\frac{1}{2}$

4. Triangle M (not shown) is also a scaled copy of triangle S. The scale factor that takes triangle S to triangle M is $\frac{3}{2}$. What is the scale factor that takes triangle M to triangle S?

$\frac{2}{3}$

Problems 5–7: Here is a polygon.



5. Draw a scaled copy of the polygon that has a perimeter of 10 units.

Sample shown on grid.

6. What is the scale factor from the original polygon to your scaled copy?

$\frac{1}{3}$

7. What is the scale factor from the scaled copy back to the original polygon?

3

Name: ________ Date: ________ Period: ________

8. Will any two squares always be scaled copies of one another?

Yes

Explain your thinking.

Explanations vary. **To be scaled copies, two figures need to have the same angles as one another and proportional sides. In any square, all the angles are 90° and the ratios between the sides are always equal to 1. Therefore, any two squares are scaled copies of one another.**

9. Figure B is a scaled copy of figure A with a scale factor of $5\frac{1}{2}$.

What is the value of x?

22

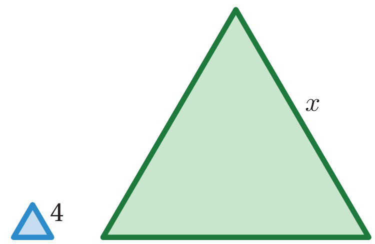


Figure A **Figure** B

Spiral Review

10. Select *all* the ratios that are equivalent to $12 : 3$.

- ☐ **A.** $6 : 1$
- ☐ **B.** $1 : 4$
- ☑ **C.** $4 : 1$
- ☑ **D.** $24 : 6$
- ☐ **E.** $15 : 6$
- ☑ **F.** $1{,}200 : 300$
- ☐ **G.** $112 : 13$

Reflection

1. Put a star next to a problem that looked more difficult than it really was.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Scale drawings are two-dimensional representations of actual objects or places. Floor plans and maps are examples of scale drawings you might have seen before.

On a scale drawing:

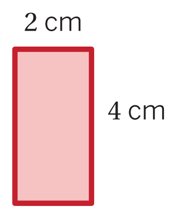
- Every part of the drawing matches up with a part of the actual object.
- Distances on the drawing are proportional to their matching distances in real life.
- A scale tells you how actual measurements are represented on the drawing. For example, if a drawing has a scale of "1 inch to 8 feet," then a 0.5-inch line segment on that drawing would represent an actual distance of 4 feet.

Things to Remember:

Lesson Practice
7.1.07

Name: ________________ Date: ________ Period: ________

Problems 1–4: Here is part of a scale drawing of Zahra's office. The scale is 6 centimeters to 15 feet.



1. What are the dimensions of the actual office?

10 feet by 5 feet

2. What is the actual area of her office?

50 square feet

3. Zahra wants to put a couch in her office that is 3 feet wide. How wide would the couch be if it were drawn on the scale drawing?

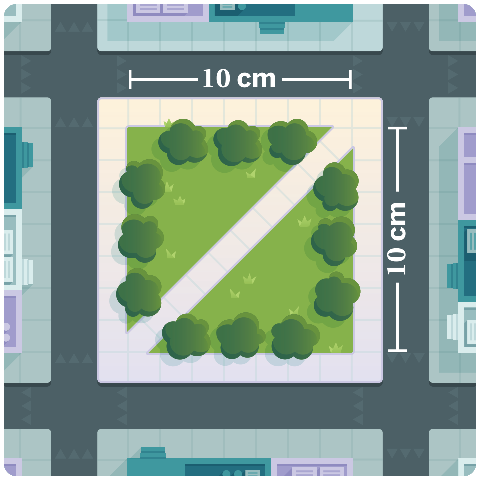
1.2 centimeters

4. Fabiana's office is near Zahra's office and measures 4 centimeters by 8 centimeters in the scale drawing. Is Fabiana's office twice the area of Zahra's office? Explain your thinking.

No. *Explanations vary.*

- **The actual dimensions of Fabiana's office are 10 feet by 20 feet, so the area is 200 square feet, or 4 times the area of Zahra's office.**
- **Since each dimension of Zahra's office is doubled, the area of Fabiana's office would be more than doubled.**

Problems 5–7: Jada is looking at a map of a square park that has a scale of 5 centimeters to 200 feet. On the map, each side of the park is 10 centimeters long.



5. Jada lives 500 feet from the park. How long would this distance be on the map?

12.5 centimeters

6. If Jada ran around the perimeter of the park once, what distance would she run?

1,600 feet

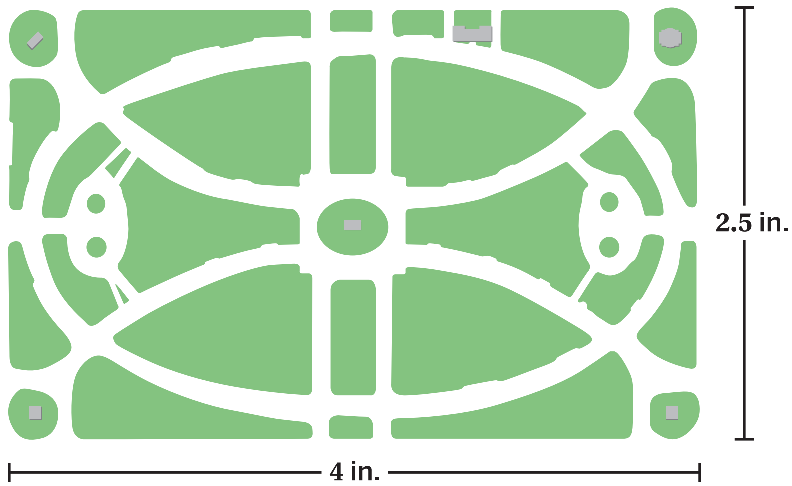
7. Jada wants to run a mile (5,280 feet). About how many times would she need to run around the park in order to reach her goal?

About 3.3 times

Name: ________ Date: ________ Period: ________

8. Here is a scale drawing of Lafayette Square in Washington, D.C. The scale is 1 inch to 200 feet. Determine the actual dimensions of Lafayette Square.

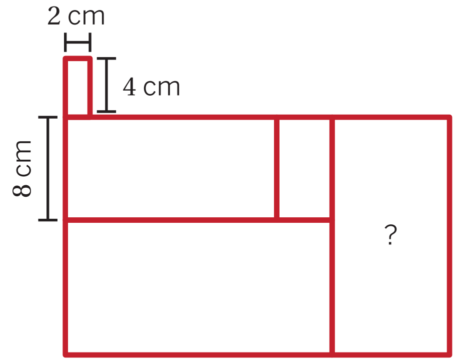
About 800 feet long and 500 feet wide



9. Here is the whole floor that Zahra's office is on. Each room is a scaled copy of every other room. The scale for this blueprint is 6 centimeters to 15 feet.

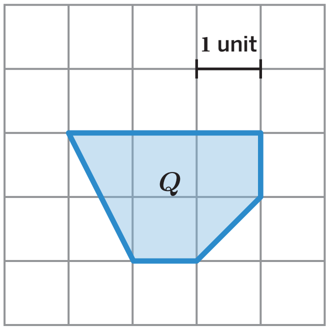
Use any strategy to determine the *actual area* of the room with the question mark.

The dimensions of the room with the question mark are 18 centimeters by 9 centimeters, so its actual area is $45 \cdot 22.5 = 1012.5$ square feet.



Spiral Review

Problems 10–11: Here is polygon Q.



10. What is the area of polygon Q?

4.5 square units

11. Lola drew a scaled copy of polygon Q using a scale factor of 2. What is the area of Lola's polygon? Explain your thinking.

18 square units. *Explanations vary.* When comparing areas, you use the square of the scale factor. So the area of Lola's polygon is 4 times greater than the area of polygon Q. $4.5 \cdot 4 = 18$.

Reflection

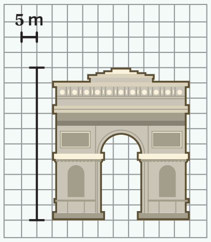
1. Put a smiley face next to a problem you were stuck on and then figured out.

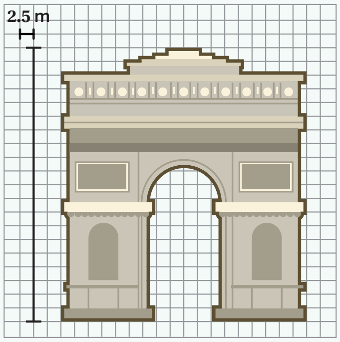
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Here is a scale drawing with a scale of 1 unit to 5 meters.

If you wanted to change this to a scale of 1 unit to 2.5 meters, you could do so in a couple of ways:





Strategy 1

- Use the original scale to find the actual dimensions.

  The height in the drawing is 10 units. $10 \cdot 5 = 50$

  The actual height of the building is 50 meters.
- Use the original dimensions and the new scale to find the dimensions of the new scale drawing. $\frac{50}{2.5} = 20$

The height in the new drawing should be 20 units.

Strategy 2

- Determine how the two scales are related. $2.5 \cdot 2 = 5$

  Because $2.5 \cdot 2 = 5$, each length in the new drawing should be 2 times as long as they are in the original drawing.
- Use this relationship to calculate the dimensions for the new drawing.

  The height in the original drawing is 10 units. $10 \cdot 2 = 20$

The height in the new drawing should be 20 units.

Things to Remember:

Lesson Practice
7.1.10

Name: .... Date: .... Period: ....

Problems 1–4: Ali and Kiana buried treasure together on their school's field. The field is 400 feet wide. Ali made a map that is 8 inches wide to record its location.

1. Write two possible scales Ali could have used to make this map.

***Responses vary.* 1 inch to 50 feet or 2 inches to 100 feet.**

2. Kiana made her own map using a scale of 1 inch to 20 feet. Whose map is larger: Kiana's or Ali's? Explain your thinking.

Kiana's. *Explanations vary.* On Ali's map, the field is 8 inches wide. On Kiana's, the field is 20 inches wide because $400 \div 20 = 20$.

3. On Kiana's map, the treasure is 2 inches from the south edge of the field. How far is the treasure from the south edge on Ali's map?

0.8 inches (or equivalent)

4. On Kiana's map, the area of the field is 16 square inches. Kiana says that the actual area of the field is 320 square feet. Is Kiana correct? Explain your thinking.

No. *Explanations vary.* When you create a scaled copy, the area is not scaled by the same scale factor, so the area would not be $16 \cdot 20 = 320$.

5. Select *all* the scales that are equivalent to 3 centimeters to 15 meters.

- ☐ **A.** 3 inches to 15 inches
- ☑ **B.** 1 centimeter to 5 meters
- ☐ **C.** 3 meters to 15 centimeters
- ☑ **D.** 4 millimeters to 2 meters
- ☐ **E.** 1 inch to 5 feet

Explain your thinking for the scale(s) you selected.

Explanations vary.
- **Scale B is equivalent because 3 centimeters with this scale would be $3 \cdot 5 = 15$ meters.**
- **Scale D is equivalent because 4 millimeters can be rewritten as 0.4 centimeters. If each value in the scale 0.4 centimeters to 2 meters is multiplied by 7.5, the result is 3 centimeters to 15 meters.**

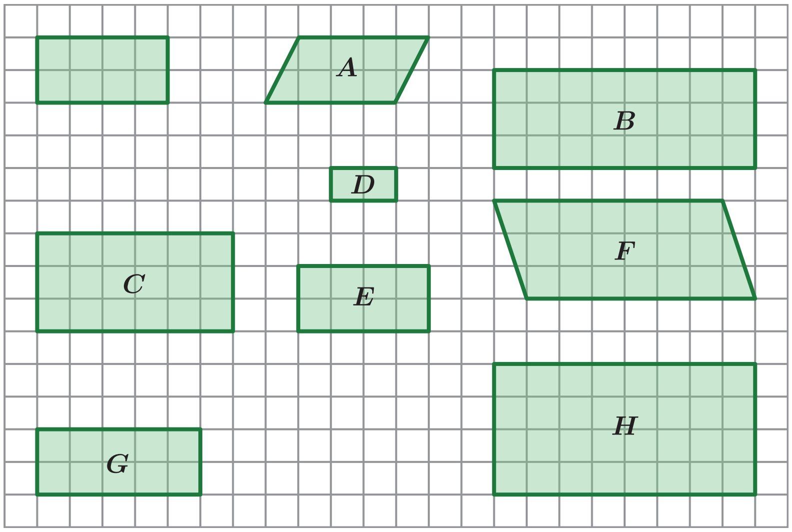
6. On a blueprint, the living room is 2.1 inches wide. The blueprint has a scale of 1 inch to 10 feet. How wide would the living room be on a blueprint that has a scale of 1 inch to 15 feet?

A. 1.4 inches **B.** 2.1 inches **C.** 3.15 inches **D.** 21 inches

Name: .... Date: .... Period: ....

Spiral Review

7. Here is an unlabeled rectangle and several quadrilaterals that are labeled.



In the table, mark the quadrilaterals that are scaled copies of the unlabeled rectangle. Then, for all scaled copies, write the scale factor used to create it.

	A	B	C	D	E	F	G	H
Is it a scaled copy?			✓	✓	✓			✓
Scale Factor			1.5	0.5	1			2

Reflection

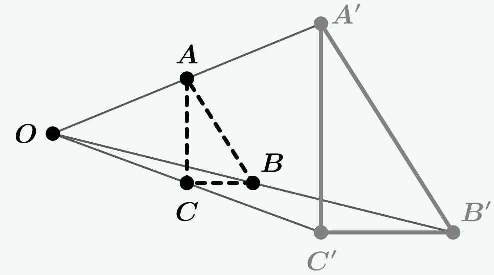
1. Put a star next to a problem where you revised your thinking.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

A **dilation** is a type of transformation that creates *scaled copies*. Dilating a figure means moving each of its vertices along a line that's extended from a given point. The original distance from the given point to each vertex on the *pre-image* is multiplied by the same number to create the dilated image.

For example, Triangle ABC was dilated from the point O to create triangle $A'B'C'$.

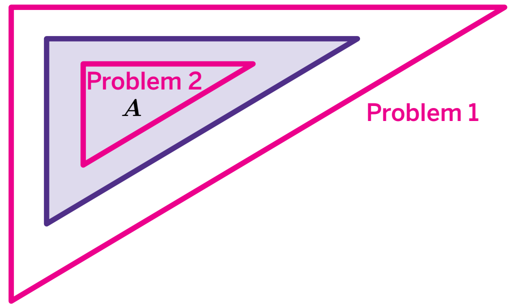


Things to Remember:

Lesson Practice
8.2.01

Name: ______________________ Date: __________ Period: __________

Problems 1–2: Here is triangle A.



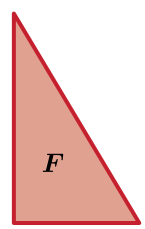
1. Draw a dilation of triangle A where the image has a larger area. ***Responses vary.***

2. Draw a dilation of triangle A where the image has a smaller area. ***Responses vary.***

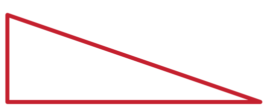
3. Rectangle A is 12 centimeters by 3 centimeters. Rectangle B is a scaled copy of rectangle A. Select *all* the pairs that could be the dimensions of rectangle B.

- ☑ **A.** 6 centimeters by 1.5 centimeters
- ☐ **B.** 10 centimeters by 1 centimeter
- ☑ **C.** 18 centimeters by 4.5 centimeters
- ☐ **D.** 6 centimeters by 1 centimeter
- ☑ **E.** 80 centimeters by 20 centimeters

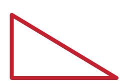
4. Select *all* the figures that could be a scaled copy of triangle F.



☐ **A.**

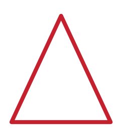


☑ **B.**

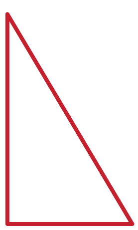


☐ **C.**

☐ **D.**



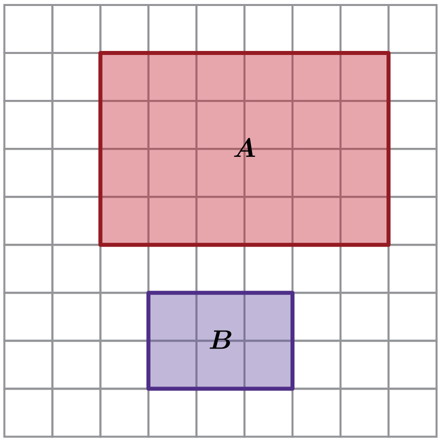
☑ **E.**



Name: ______________ Date: ________ Period: ________

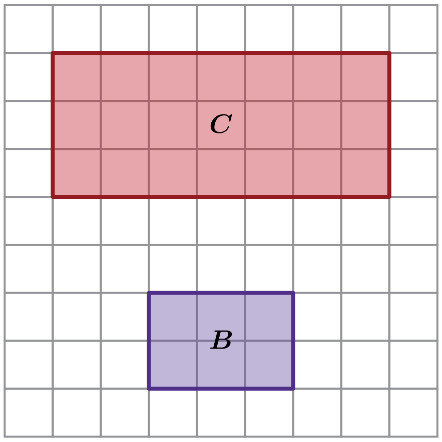
Problems 5–6: Determine whether each pair of figures represents an original figure and its scaled copy. Explain your thinking.

5.



Yes. *Explanations vary*. Rectangle A is double the height and width of rectangle B.

6.



No. *Explanations vary*. The corresponding side lengths of the rectangles are not proportional.

Spiral Review

7. Which of these sets of angle measures could be the three interior angle measures of a triangle?

A. $40°, 50°, 60°$

B. $50°, 60°, 70°$

C. $60°, 70°, 80°$

D. $70°, 80°, 90°$

8. What makes two figures congruent? Explain your thinking.

Two figures are congruent if one can move onto the other using translations, rotations, or reflections. Congruent figures are the same size and shape.

Reflection

1. Star the problem you spent the most time on.

2. Use this space to ask a question or share something you're proud of.

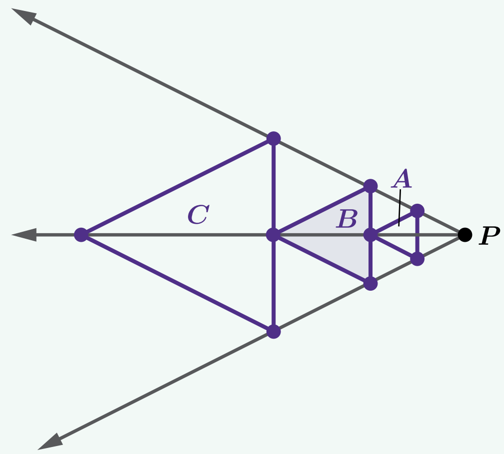
Lesson Summary

A *dilation* is a transformation that involves a **center of dilation** and a **scale factor**.

One strategy for dilating a figure is to measure the distance between the center of dilation and one of the pre-image points, multiply that distance by the scale factor, then place the image point that distance away from the center of dilation along the same line. Repeat this strategy with all the other points in the pre-image.

In this example, triangle B is the pre-image.

- Triangle A is a dilation of triangle B using point P as the center of dilation and a scale factor of $\frac{1}{2}$.
- Triangle C is a dilation of triangle B using point P as the center of dilation and a scale factor of 2.



Things to Remember:

Lesson Practice
8.2.02

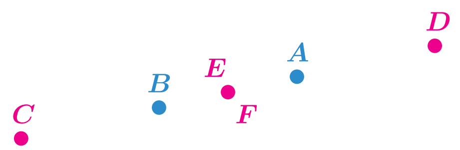
Name: ______________ Date: ________ Period: ________

Problems 1–2: Segment AB is 3 centimeters long. Point O is the center of dilation.

1. How long is the image of segment AB after a dilation with a scale factor of 5?
15 centimeters

2. How long is the image of segment AB after a dilation with a scale factor of 3.7?
11.1 centimeters

Problems 3–6: Here are points A and B. Plot the points after each dilation.
Responses shown on diagram.



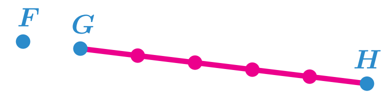
3. Point C is the image of point B using point A as the center of dilation and a scale factor of 2.

4. Point D is the image of point A using point B as the center of dilation and a scale factor of 2.

5. Point E is the image of point B using point A as the center of dilation and a scale factor of $\frac{1}{2}$.

6. Point F is the image of point A using point B as the center of dilation and a scale factor of $\frac{1}{2}$.

Problems 7–8: Here is a diagram.



7. Point H is the image of point G using point F as the center of dilation. What is the scale factor?
6

8. If the distance from point F to point H is 9 units, what is the distance from point F to point G?
1.5 units

Show or explain your thinking.
***Explanations vary.* The scale factor is 6. $1.5 \cdot 6 = 9$.**

Lesson Practice
8.2.02

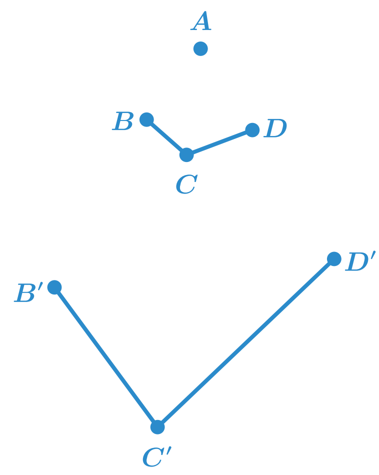
Name: ______ Date: ______ Period: ______

9. Isaiah claims that figure $B'C'D'$ is a dilation of figure BCD using point A as the center of dilation. Is Isaiah's claim correct?

No

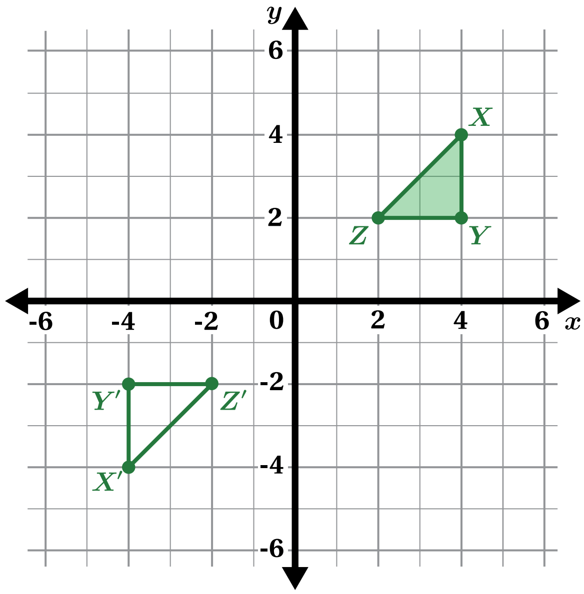
Explain your thinking.

Explanations vary. **I drew a ray from point A through points B and B' and then drew another ray from point A through point C to point C'. The ratio of the distance between points A and B' to the distance between points A and B is not the same as the ratio of the distance between points A and C' to the distance between points A and C.**



Spiral Review

Problems 10–12: Determine whether each transformation moves triangle XYZ onto triangle $X'Y'Z'$. Write *true* or *false*.



10. Triangle XYZ is reflected over the y-axis, followed by a reflection over the x-axis.

True

11. Triangle XYZ is reflected over the x-axis, followed by a reflection over the y-axis.

True

12. Triangle XYZ is reflected over the x-axis, followed by a translation 3 units left.

False

Reflection

1. Circle a problem you want to talk to a classmate about.
2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 24

Lesson Summary

Dilations can be combined with other *sequences of transformations*.

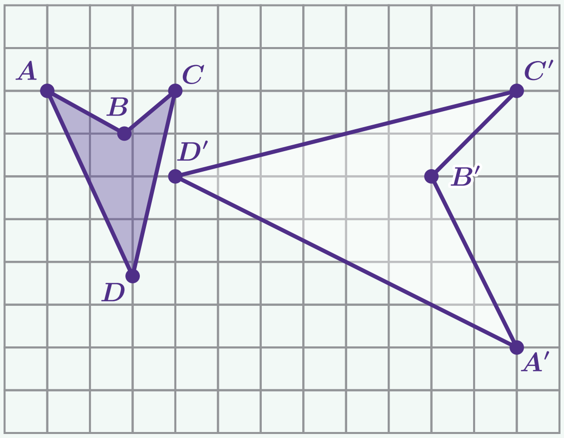
Here is a sequence of transformations that moves figure $ABCD$ onto figure $A'B'C'D'$

Step 1: Dilate figure $ABCD$ using point D as the center of dilation and a scale factor of 2.

Step 2: Translate the image after Step 1 so that point D moves onto point D'.

Step 3: Rotate the new image $90°$ clockwise around point D'.

Step 4: Reflect the new image across a horizontal line that contains points D' and B'.



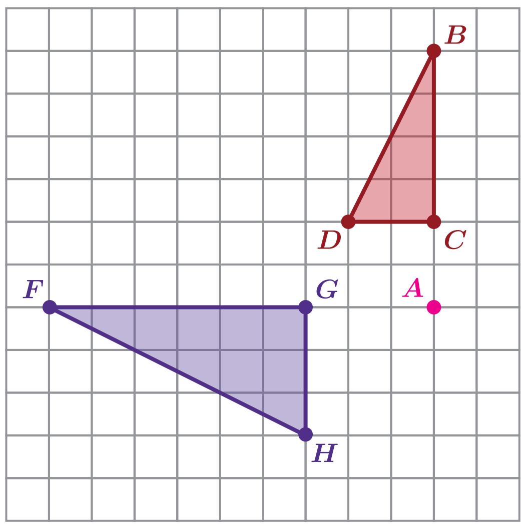
Things to Remember:

Lesson Practice
8.2.03

Name: ______ Date: ______ Period: ______

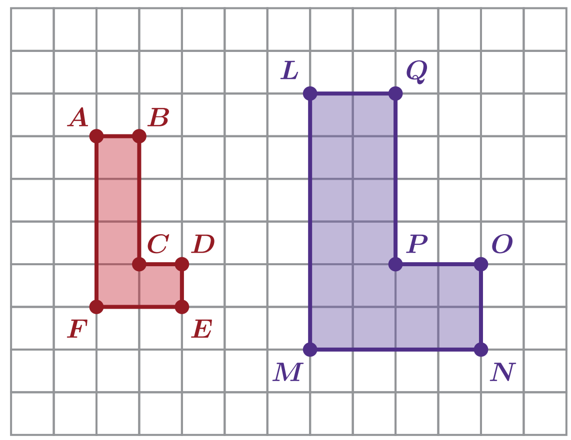
Problems 1–4: Is there a transformation or sequence of translations, rotations, reflections, or dilations that moves one figure onto the other? (Write *yes* or *no*.) If you wrote *yes*, describe the sequence of transformations. If you wrote *no*, describe how you know it's not possible.

1.



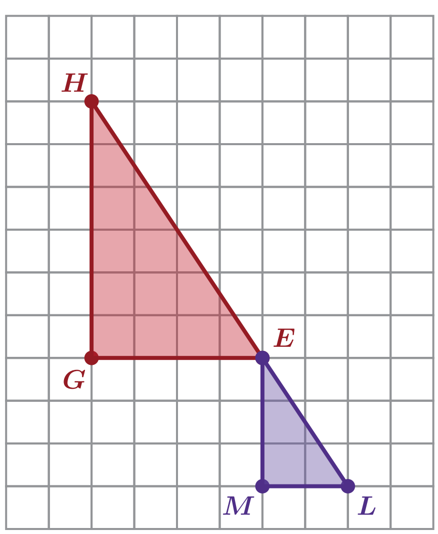
Yes. *Explanations vary.* Rotate triangle BCD $90°$ counterclockwise around point A and then dilate using A as the center of dilation and a scale factor of 1.5.

2.



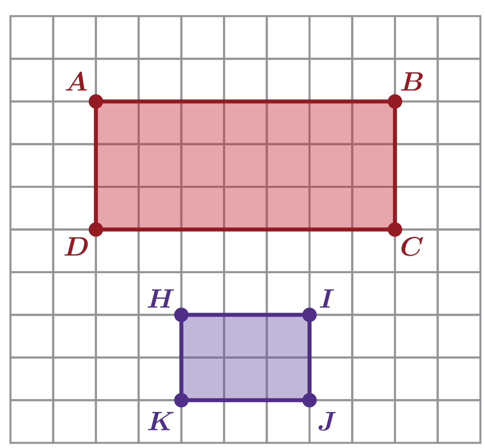
No. *Explanations vary.* The corresponding sides have different scale factors, so there is no sequence of transformations that moves one figure onto the other.

3.



Yes. *Explanations vary.* Dilate triangle ELM using point L as the center of dilation with a scale factor of 2 and then translate the image 3 units up and 2 units to the left.

4.



No. *Explanations vary.* The corresponding sides have different scale factors, so there is no sequence of transformations that moves one figure onto the other.

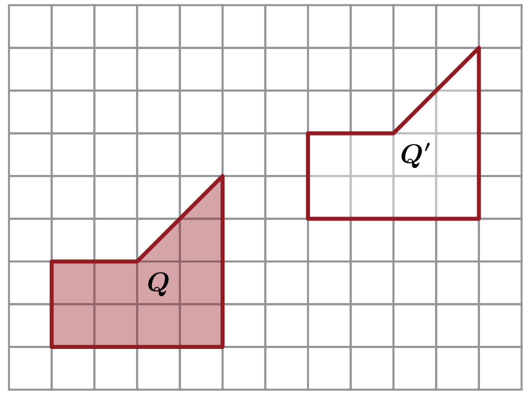
5. Which transformation will move the pre-image onto the image?

A. Translating right and then up.

B. Rotating $180°$.

C. Dilating by a scale factor of 4.

D. Reflecting across a horizontal line.



6. Hikmat says that dilating a figure by a scale factor of $\frac{3}{2}$ will produce an image that is smaller than the pre-image. Is Hikmat's claim correct? Explain your thinking.

No. ***Explanations vary.*** **$\frac{3}{2}$ is a scale factor that is greater than 1, so the image will be larger, not smaller.**

Spiral Review

Problems 7–10: Point G is located at $(5, \text{-}2)$. Determine the coordinates of the image after each transformation.

7. A reflection over the y-axis.

$(\text{-}5, \text{-}2)$

8. A reflection over the x-axis.

$(5, 2)$

9. A $180°$ rotation around the origin.

$(\text{-}5, 2)$

10. A translation 2 units to the left and 1 unit up.

$(3, \text{-}1)$

Reflection

1. Circle the problem that was the most challenging for you.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

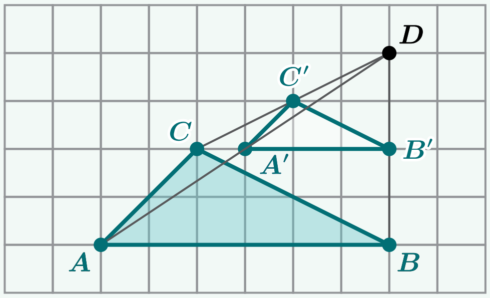
Another strategy for dilating a figure is to use a grid. Count the vertical and horizontal grid squares between the center of dilation and the pre-image, multiply each value by the scale factor, then count that number of grid squares away from the center to get the image.

If the scale factor of the dilation is:

- Greater than 1, the image will be *larger* than the pre-image and further from the center of dilation.
- Equal to 1, the image will be *the same size* as the pre-image and just as far from the center.
- Between 0 and 1, the image will be *smaller* than the pre-image and closer to the center.

For example, triangle ABC is dilated using point D as the center of dilation and a scale factor of $\frac{1}{2}$.

Since the scale factor is less than 1, the image is smaller than the pre-image and closer to the center of dilation.



Things to Remember:

Lesson Practice

8.2.04

Name: .... Date: .... Period: ....

Problems 1–2: Segment AB measures 3 centimeters. Point O is the center of dilation.

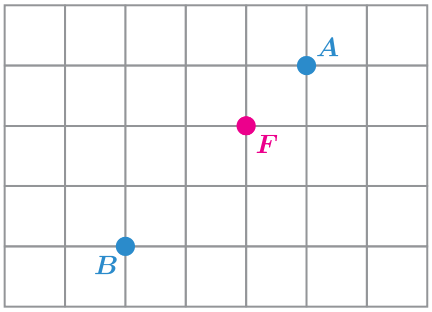
1. How long is the image of AB after a dilation with a scale factor of $\frac{1}{5}$?

$\frac{3}{5}$ **centimeters**

2. How long is the image of AB after a dilation with a scale factor of s?

$3s$ centimeters

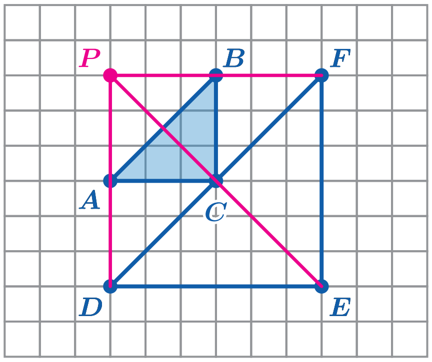
3. Here are points A and B. Point F is the image of point B using point A as the center of dilation and a scale factor of $\frac{1}{3}$. Plot point F.



4. Triangle ABC was transformed into triangle DFE using a dilation.

Label the center of dilation P. Then determine the scale factor.

Center of dilation P shown on diagram.
The scale factor is 2.

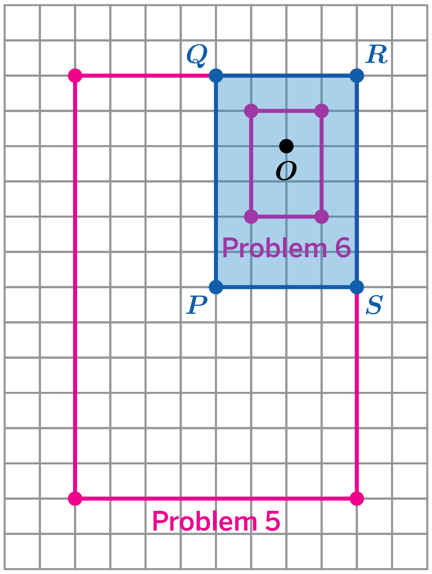


Problems 5–6: Here is rectangle $PQRS$.

Draw the image of rectangle $PQRS$ after each dilation.

5. A dilation using point R as the center of dilation and a scale factor of 2.

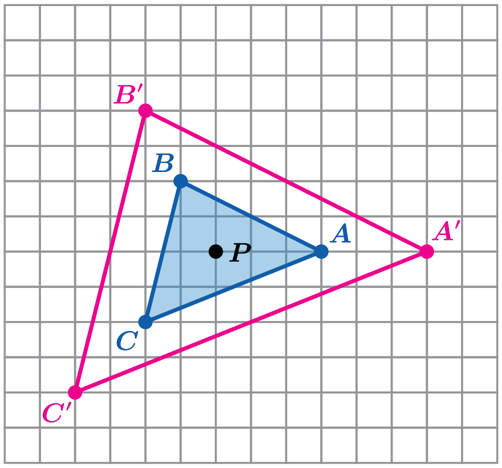
6. A dilation using point O as the center of dilation and a scale factor of $\frac{1}{2}$.



Lesson Practice
8.2.04

Name: ________ Date: ________ Period: ________

Problems 7–9: Here is triangle ABC.



7. Dilate each vertex of triangle ABC using point P as the center of dilation and a scale factor of 2. Draw the image and label the vertices $A'B'C'$.

Response shown on grid.

8. If the length of side AB is 4.5 units and the length of side BC is 4.1 units, what are the lengths of side $A'B'$ and side $B'C'$?

- **Side $A'B'$ is 9 units long.**
- **Side $B'C'$ is 8.2 units long.**

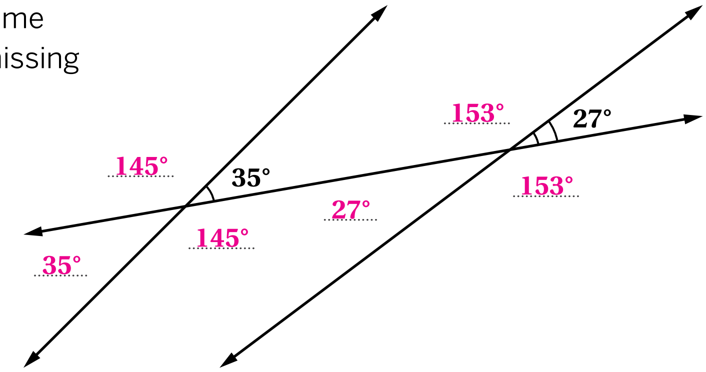
9. How would the size and location of the image change if point P were in a different location?

***Responses vary.* The location of the image would change depending on where the center of dilation was located. The distance from point A' to point P will always be twice the distance from point A to point P.**

Spiral Review

10. Here are three lines, along with some angle measures. Determine the missing angle measures.

Responses shown on diagram.



Reflection

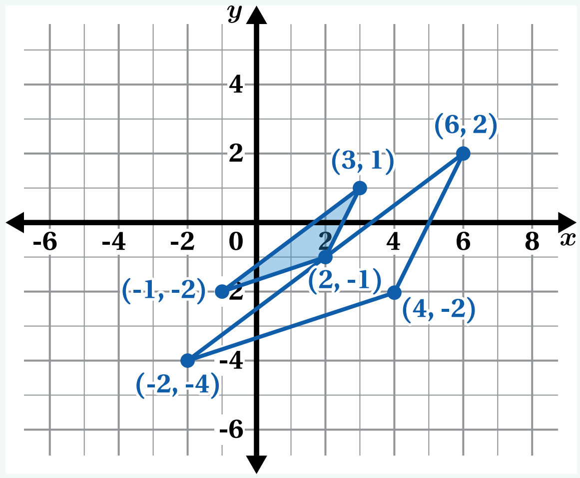
1. Star a problem you're still feeling confused about.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can use coordinates to perform dilations precisely.

Let's say you're dilating this shaded pre-image using the center of dilation $(0, 0)$ and a scale factor of 2.



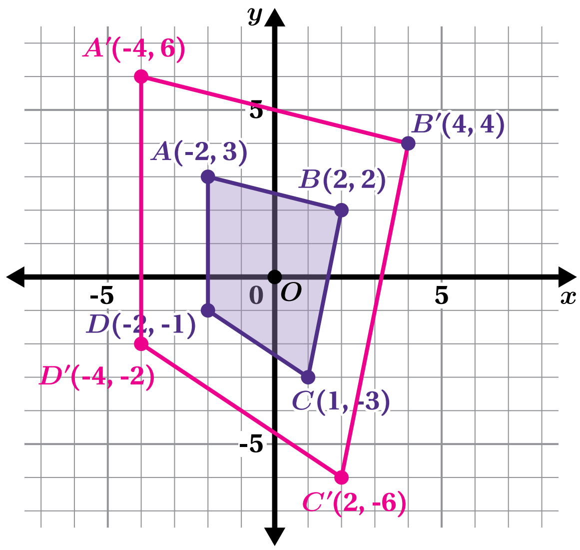
You can use the coordinate plane to measure the horizontal and vertical distances of each point from the center of dilation. Then you can multiply those distances by the scale factor to get the distances between the center of dilation and the points on the image.

In this example, the center of dilation is $(0, 0)$, so you can multiply the pre-image coordinates by the scale factor to get the image coordinates.

Things to Remember:

Name: .... Date: .... Period: ....

Problems 1–3: Quadrilateral $ABCD$ is dilated with the origin as the center of dilation, moving point B onto point B'.

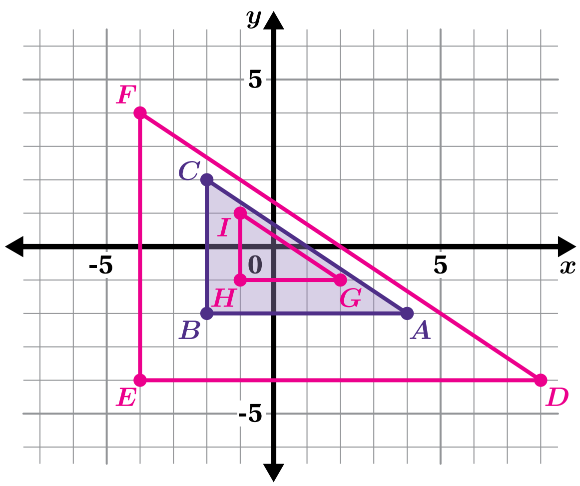


1. What is the scale factor of the dilation?
The scale factor is 2.

2. Draw quadrilateral $A'B'C'D'$.
Response shown on graph.

3. Label the coordinate points for A', C', and D'.
Response shown on graph.

Problems 4–6: Here is triangle ABC on the coordinate plane.



4. Using the origin as the center and a scale factor of 2, draw the dilation of triangle ABC. Label the image triangle DEF.
Response shown on graph.

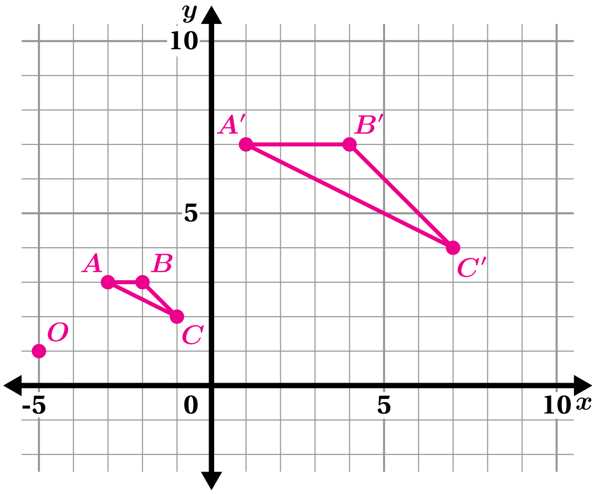
5. Using the origin as the center and a scale factor of $\frac{1}{2}$, draw the dilation of triangle ABC. Label the image triangle GHI.
Response shown on graph.

6. Triangle GHI is a dilation of triangle DEF. Identify the center of dilation and the scale factor.
The center of dilation is the origin and the scale factor is $\frac{1}{4}$.

Lesson Practice
8.2.05

Name: ________ Date: ________ Period: ________

Problems 7–8: Triangle ABC has vertices located at A(-3, 3), B(-2, 3), and C(-1, 2). Triangle $A'B'C'$ is the result of dilating triangle ABC using point O(-5, 1) as the center of dilation and a scale factor of 3.



7. Draw triangle ABC and its image on the coordinate plane.

Response shown on graph.

8. What are the coordinates of triangle $A'B'C'$?

A'(1, 7), B'(4, 7), and C'(7, 4)

9. Here are the pre-image and image coordinates of points on a graph. Describe the transformation.

This is a dilation with (0, 0) as the center of dilation and a scale factor of 3.

Pre-Image Coordinates	Image Coordinates
(0, 4)	(0, 12)
(-1, 3)	(-3, 9)
(5, -6)	(15, -18)

Spiral Review

Problems 10–11: Use what you know about the interior angle measures of triangles to complete these problems.

10. Triangle JKL is a right triangle, and the measure of angle J is 28°. What are the measures of the other two angles?

The measures of the other two angles are 90° and 62°.

11. Triangle PQR is an obtuse triangle, and the measure of angle Q is 72°. What are possible measures of the other two angles?

Responses vary. The sum of the other two angle measures should equal 108°, and one angle measure should be greater than 90°.

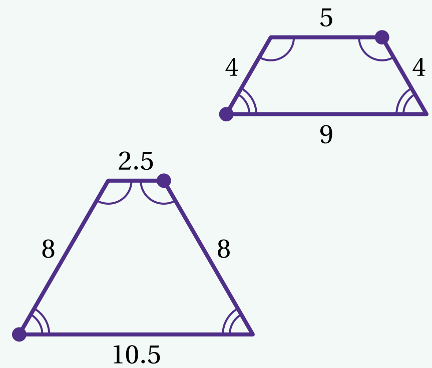
Reflection

1. Circle the problem you enjoyed doing the most.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Similar figures have congruent corresponding angles. Is knowing that the corresponding angles in two figures are congruent enough to show that the figures are also similar? It depends!

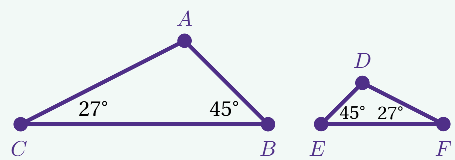
Here are two figures that have congruent corresponding angles but are not similar figures.



For triangles, knowing that the corresponding angles are congruent is enough to know that the triangles are similar.

This is even true if you only know two angle measures, because we can use the fact that the sum of the interior angles of any triangle is $180°$ to figure out the third angle measure.

For example, the unknown angles in triangles CAB and FDE are each $108°$. All the corresponding angles are congruent, which means triangle CAB is similar to triangle FDE.



Things to Remember:

Lesson Practice
8.2.07

Name: ______________ Date: ______ Period: ______

Problems 1–3: Here are some angle measures for different pairs of triangles. Determine whether each pair is *similar*, *not similar*, or if there is *not enough information* to know.

1. Triangle A: $53°$, $71°$
Triangle B: $53°$, $71°$
Similar

2. Triangle E: $63°$, $45°$
Triangle F: $14°$, $71°$
Not Similar

3. Triangle G: $100°$
Triangle H: $70°$
Not enough information

4. Draw two equilateral triangles to fit each category. If it's not possible, write "Not possible."

Not Congruent

Triangles vary.

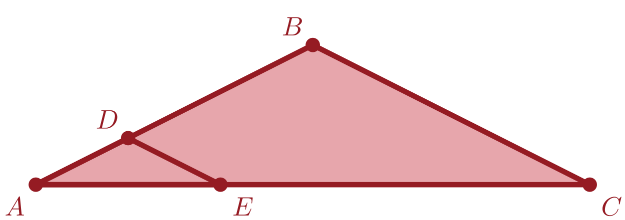
Not Similar

Not possible

5. Do you think two equilateral triangles will be *always*, *sometimes*, or *never* similar? Explain your thinking.

Always. *Explanations vary.* An equilateral triangle will always have three angles that measure 60°, so two equilateral triangles will always have at least two angles that have the same measurements (in fact, they'll always have three). That means they will always be similar.

6. In this figure, segment BC is parallel to segment DE. How do you know that triangle ABC is similar to triangle ADE?



***Responses vary.* Segment BC and segment DE are parallel and segment AB is a transversal, so $\angle ABC$ is congruent to $\angle ADE$. $\angle DAE$ is also congruent to $\angle BAC$. Both triangles have two congruent angles, which means they're similar.**

Lesson Practice
8.2.07

Name: .... Date: .... Period: ....

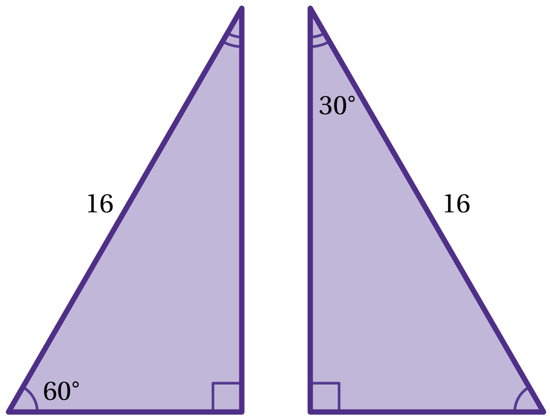
7. Which of the following *best* describes the triangles shown?

   A. Congruent but not similar

   B. Similar but not congruent

   C. Both similar and congruent

   D. Neither similar nor congruent



8. Triangle A has two angles that measure $90°$ and $33°$. Triangle B has two angles that measure $90°$ and $57°$. Determine whether the triangles are *similar* or *not similar*. Explain your thinking.

   The triangles are similar. *Explanations vary*. The sum of the interior angle measures of any triangle is $180°$, so the third angle measure must be $57°$ for triangle A and $33°$ for triangle B. There are three pairs of corresponding angles that are congruent, which means the triangles are similar.

Spiral Review

9. A figure is translated 3 units to the right and then dilated by a scale factor of 3. Determine whether the pre-image and image are *congruent*, *similar*, or *neither*.

   Similar

Reflection

1. Put a heart next to the problem you're most proud of.
2. Use this space to ask a question or share something you're proud of.

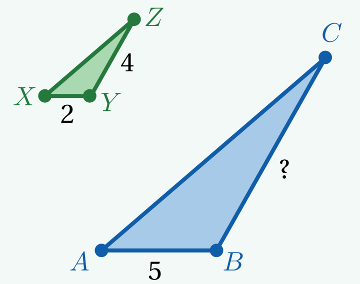
Lesson Summary

There are a variety of strategies for determining unknown side lengths of similar triangles.

You can use ratios that compare two triangles using the scale factor between them.

You can also determine the ratio of two side lengths in the same triangle, then apply that ratio to the corresponding sides in a similar triangle.

Here are two similar triangles, triangle ABC and triangle XYZ. You can use each of these strategies to determine the length of side BC.



Use the scale factor between the triangles.
Side XY and side AB are corresponding sides. The ratio of their side lengths is $\frac{5}{2}$, which means the scale factor is 2.5.

To calculate the length of side BC, you can multiply the length of side YZ by the scale factor.
$4 \bullet 2.5 = 10$, so side BC is 10 units long.

Use ratios of side lengths within one triangle.
The ratio of side YZ to side XY is $4 : 2$, or 2. That means the length of side BC is twice the length of side AB. $5 \bullet 2 = 10$, so side BC is 10 units long.

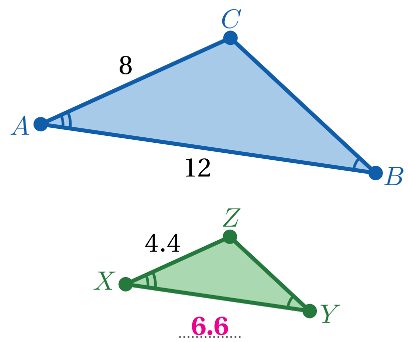
Things to Remember:

Lesson Practice
8.2.08

Name: .... Date: .... Period: ....

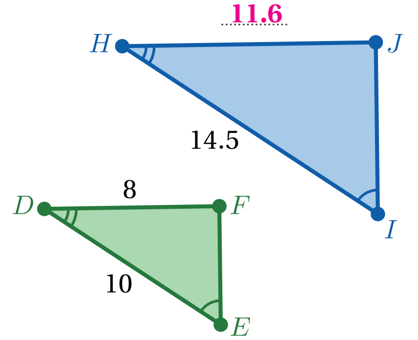
Problems 1–2: Triangle ABC is similar to triangle XYZ. Triangle DEF is similar to triangle HIJ. Determine the missing values. Explain your thinking. The figures may not be drawn to scale.

1.



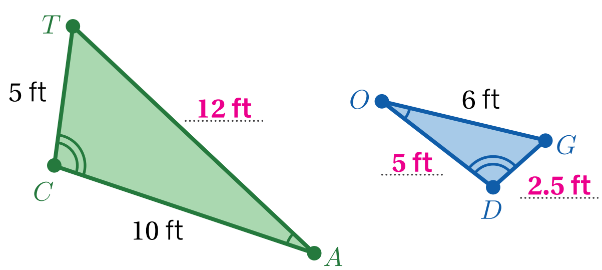
Explanations vary. **The ratio of side AB to side AC is $12:8$ or 1.5. That means the length of side XY is 1.5 times the length of side XZ. $4.4 \cdot 1.5 = 6.6$.**

2.



Explanations vary. **The ratio of the corresponding side lengths HI to DE is $14.5:10$, so the scale factor is 1.45. I can multiply the length of side DF by the scale factor to determine the length of side HJ: $8 \cdot 1.45 = 11.6$.**

3. Triangle CAT is similar to triangle DOG. Triangle CAT is dilated by a scale factor of $\frac{1}{2}$, creating triangle DOG. Determine the missing side lengths. Show or explain your thinking.



Explanations vary.

$5 \cdot \frac{1}{2} = 2.5$

$10 \cdot \frac{1}{2} = 5$

$6 \div \frac{1}{2} = 12$

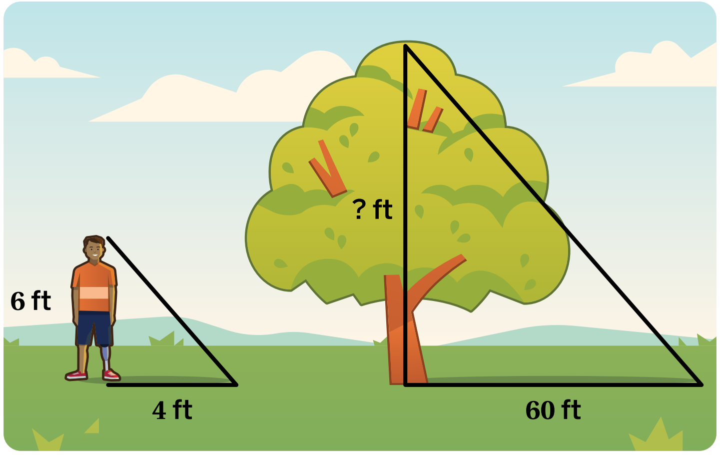
4. On a sunny day, a 6-foot-tall person has a shadow with a length of 4 feet. At the same time, a nearby tree has a shadow 60 feet long. What is the height of the tree?

A. 80 feet

B. 90 feet

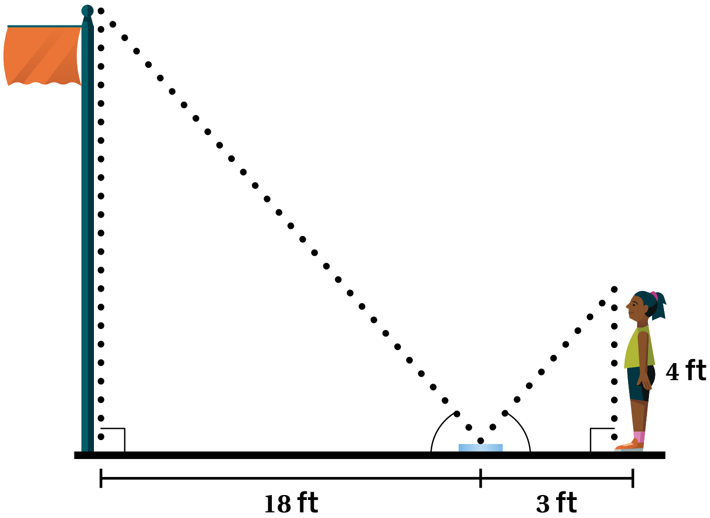
C. 120 feet

D. 180 feet



Name: ________ Date: ________ Period: ________

5. Zoe uses a mirror to measure the height of a flagpole. She places the mirror flat on the ground, then walks backward until she can see the flagpole in the mirror. As the diagram shows, the way that light reflects in the mirror creates a set of congruent angles. Now she can use her own height, the mirror's distance from the flagpole, and her distance from the mirror to calculate the height of the flagpole.

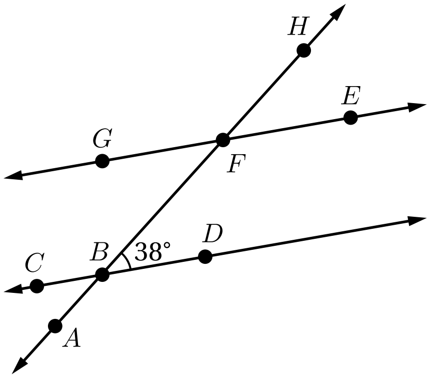


Use the measurements in the diagram to calculate the flagpole's height. Show or explain your thinking.

24 feet. *Explanations vary*. The ratio of Zoe's height to the distance from Zoe's feet to the mirror is the same as the ratio of the flagpole's height to the distance from the base of the flagpole to the mirror. $\frac{4}{3} = \frac{24}{18}$.

Spiral Review

Problems 6–11: Lines GE and CD are parallel. Determine the measures of each angle.



6. $\angle ABC$ **38°**

7. $\angle ABD$ **142°**

8. $\angle GFB$ **38°**

9. $\angle BFE$ **142°**

10. $\angle EFH$ **38°**

11. $\angle GFH$ **142°**

Reflection

1. Put a question mark next to a problem you're feeling unsure of.

2. Use this space to ask a question or share something you're proud of.

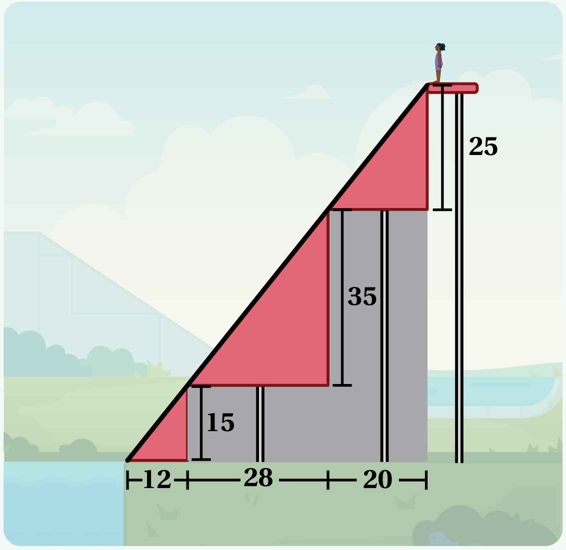
Lesson Summary

Here are three ramps made up of similar triangles with proportional corresponding side lengths. Because of this, they line up perfectly to create a smooth slide.

To line up the longest side of each triangle, the **slope** of each triangle must be the same. Slope is the height-to-base *ratio* of a triangle, which describes the steepness of its longest side.

For example, the slope of this slide is $\frac{5}{4}$ because $\frac{15}{12} = \frac{35}{28} = \frac{25}{20} = \frac{5}{4}$.

You could sketch infinite triangles on the same line that all have the same height-to-base ratio. Any of those triangles can be used to determine the slope.



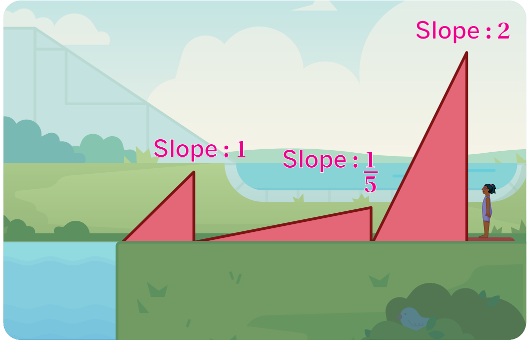
Things to Remember:

Lesson Practice
8.2.09

Name: .... Date: .... Period: ....

1. Here are three ramps. One ramp has a slope of 1, one ramp has a slope of 2, and one ramp has a slope of $\frac{1}{5}$. Label each ramp with its slope.

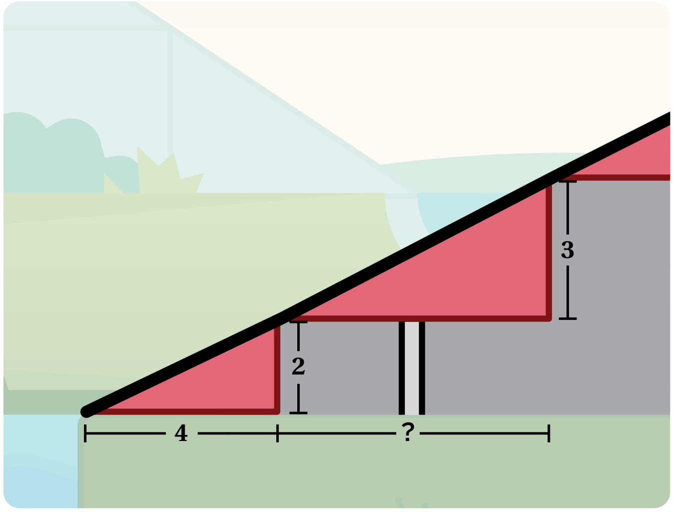
Responses shown in image.



2. Jaylin created a slide with a slope of $\frac{1}{3}$. LaShawn created a slide with a slope of $\frac{1}{2}$. Whose slide was steeper? Explain your thinking.

LaShawn's slide. *Explanations vary*. Both slides have a slope with the same vertical distance, but Jaylin's slope has a greater horizontal distance, which means it's less steep than LaShawn's slide.

Problems 3–5: Here are three ramps that make a smooth slide.



3. How long is the base of the second ramp?

6 units

4. Explain how you know the triangles that form the ramps are similar.

***Explanations vary*. The second triangle ramp is a dilation of the first triangle ramp by a scale factor of 1.5 and then translated.**

5. Which statement about the slopes of the ramps is true?

A. The slope of the first ramp is $\frac{2}{3}$ the slope of the second ramp because the height of the first ramp is $\frac{2}{3}$ the length of the second ramp.

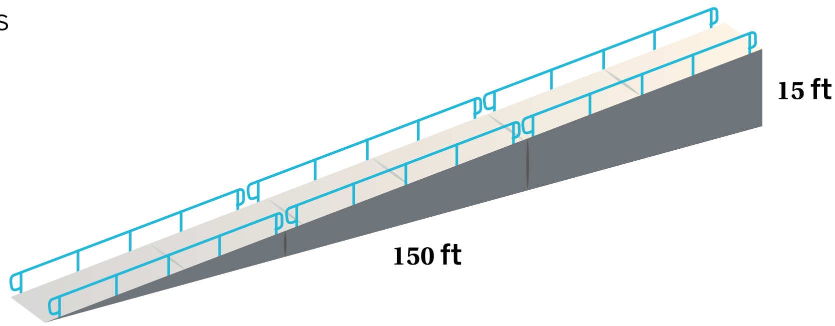
B. The slope of the first ramp is the same as the slope of the second ramp because their triangles are similar.

C. The slope of the first ramp is $\frac{4}{9}$ the slope of the second ramp because the area of the first ramp triangle is $\frac{4}{9}$ the area of the second ramp triangle.

D. The slope of the second ramp is 2 more than the slope of the first ramp because the difference between the bases of the ramps is 2.

Name: .... Date: .... Period: ....

6. The Americans With Disabilities Act of 1990, or A.D.A., is a civil rights act that protects Americans with disabilities from discrimination. It requires that public entities make their programs, services, and activities accessible. One requirement is that public wheelchair ramps must have a maximum slope of $\frac{1}{12}$.



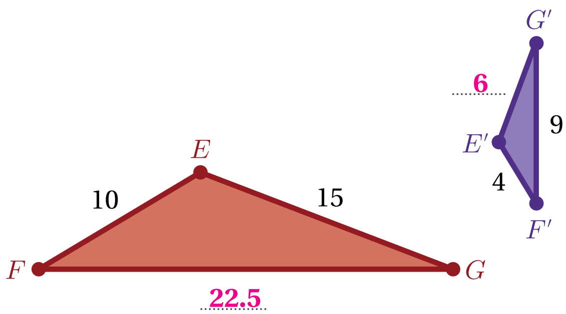
Does this ramp fit the requirement? Explain your thinking.

No. *Explanations vary.* The slope of the ramp is $\frac{15}{150}$, or $\frac{1}{10}$, which is greater than $\frac{1}{12}$.

Spiral Review

7. Triangle EFG is similar to triangle $E'F'G'$. Determine the missing values and explain your thinking.

Explanations vary.



I used the corresponding sides to find the scale factor that takes triangle $E'F'G'$ to triangle EFG, which is $\frac{10}{4} = 2.5$. Then I used the scale factor to find the missing side lengths, $9 \cdot 2.5 = 22.5$ and $15 \div 2.5 = 6$, so the side lengths are 22.5 and 6.

Reflection

1. Circle the problem you're most interested in knowing more about.

2. Use this space to ask a question or share something you're proud of.

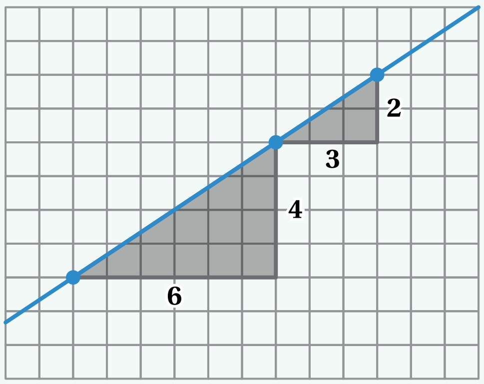
Lesson Summary

You can determine the slope of a line by drawing similar right triangles, called **slope triangles**, between any two points on the line. The height of the slope triangle represents the vertical distance between the points, and the base of the triangle represents the horizontal distance between the points.

Slope is the ratio of the height of a slope triangle to its base.

Here's an example of two possible slope triangles that you could use to calculate the slope of this line.

The slope of this line is $\frac{4}{6}$, or $\frac{2}{3}$, or any equivalent value.

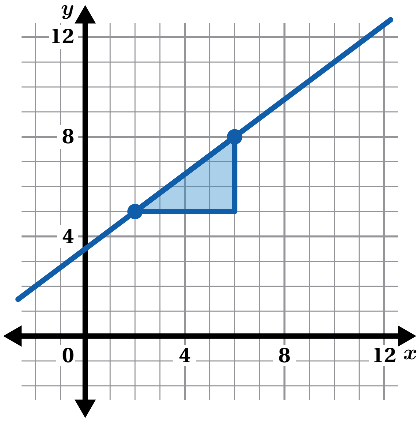


Things to Remember:

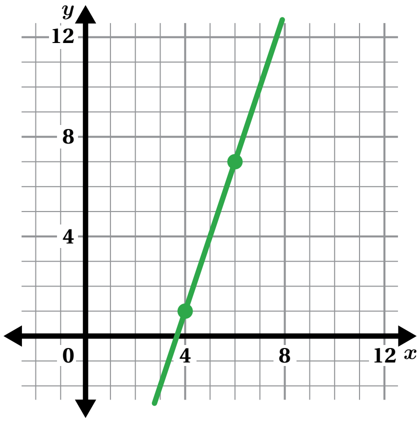
Name: ______ Date: ______ Period: ______

Problems 1–4: Determine the slope of each line.

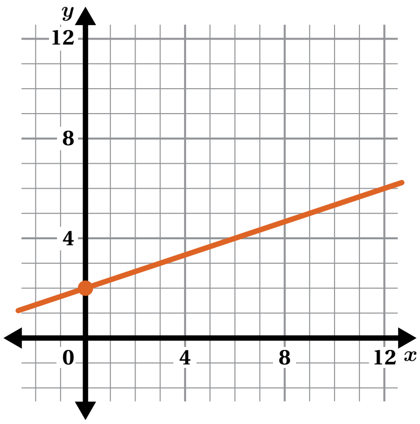
1. Slope: $\frac{3}{4}$ **(or equivalent)**



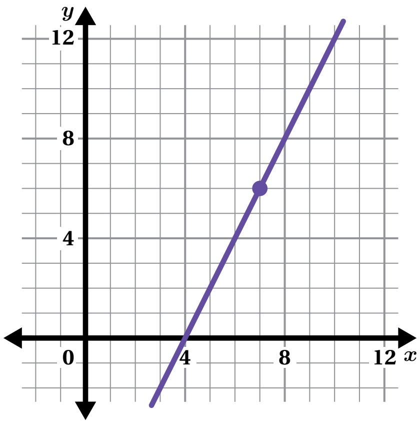
2. Slope: 3 **(or equivalent)**



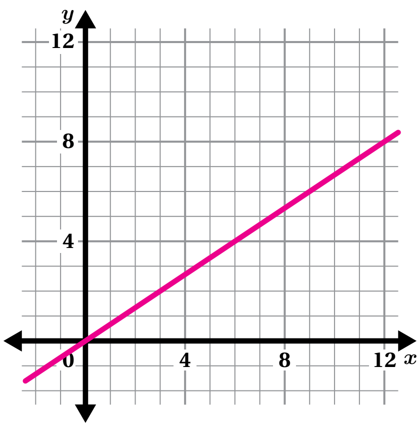
3. Slope: $\frac{1}{3}$ **(or equivalent)**



4. Slope: 2 **(or equivalent)**



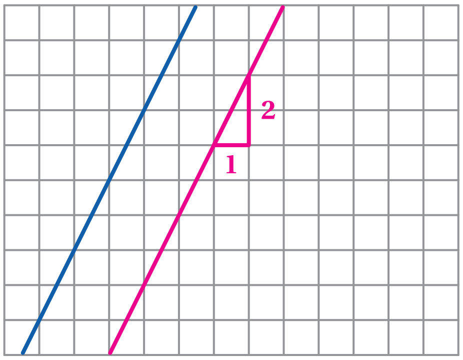
5. Draw a line with a slope of $\frac{2}{3}$.



Responses vary.

6. Here is a line. Draw a line that is parallel. What is the slope of each line?

Explain your thinking.



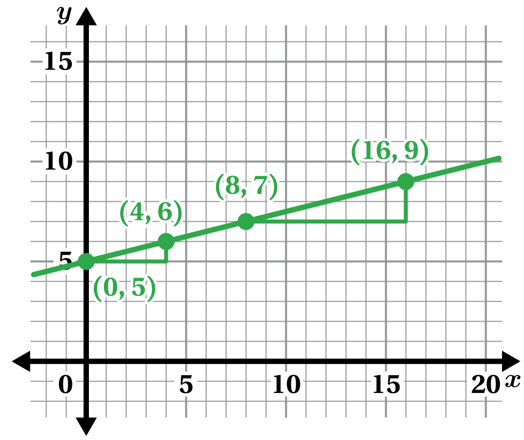
***Explanations vary.* Parallel lines have the same slope. The slope of each line is 2.**

Lesson Practice
8.2.10

Name: ______________________ Date: __________ Period: __________

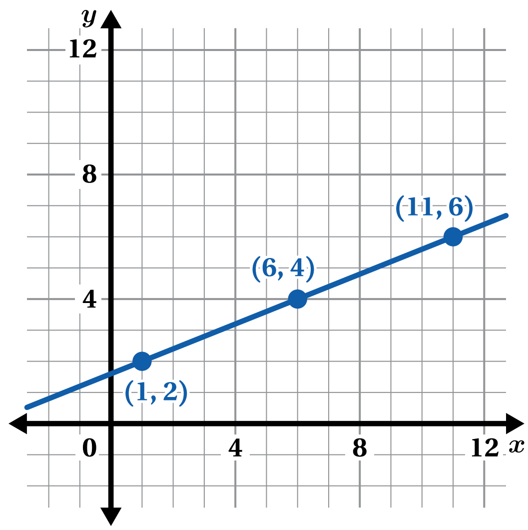
7. Select *all* the expressions that could represent the slope of this line.

- [x] A. $\frac{1}{4}$
- [ ] B. $\frac{4}{6}$
- [ ] C. 4
- [x] D. $\frac{2}{8}$
- [ ] E. $\frac{8}{2}$



8. Joud says the slope of this line is $\frac{2}{5}$. Daniela says the slope of this line is 2.5. Whose claim is correct? Explain your thinking.

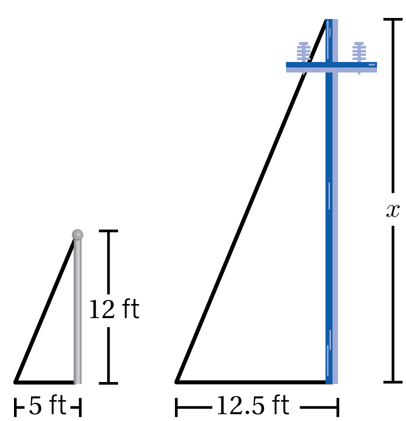
Joud's. *Explanations vary.* If I draw a slope triangle, the vertical distance, 2, divided by the horizontal distance, 5, would be $\frac{2}{5}$.



Spiral Review

9. A 12-foot flagpole casts a shadow that is 5 feet long. At the same time, a nearby telephone pole casts a shadow that is 12.5 feet long. What is the height of the telephone pole, x?

A. 5.2 feet
B. 15 feet
C. 18.5 feet
D. 30 feet



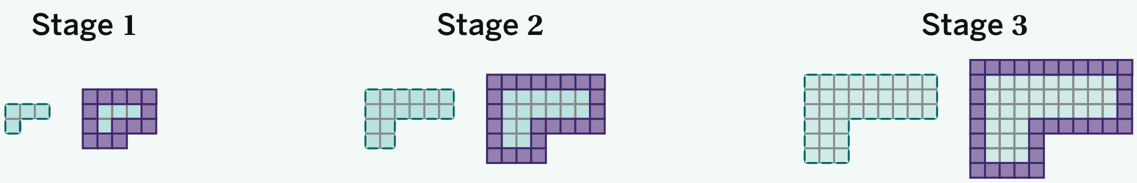
Reflection

1. Put a star next to a problem where you revised your thinking.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Analyzing shape patterns can help you understand number patterns. Here is a design made with border toothpicks and border tiles.



In Stage 1, there are 10 toothpicks. In Stage 2, there are 20 while in Stage 3, there are 30. The number of toothpicks increases by 10 each time. The table shows that the number of tiles is always 4 more than the number of toothpicks.

Stage	Border Toothpicks	Border Tiles
1	10	14
2	20	24
3	30	34

We can extend these rules to make predictions about any stage of the pattern. For example, in Stage 5 there will be 50 toothpicks and the number of border tiles will be 4 greater, 54.

Things to Remember:

Lesson Practice
7.6.01

Name: .... Date: .... Period: ....

Problems 1–4: A sandwich store charges $20 to have 3 subs delivered and $26 to have 4 subs delivered. This includes the delivery fee.

1. How much does the store charge for each additional sub?

$6

2. How much is the delivery fee?

$2

3. Is the relationship between the number of subs delivered and the total amount charged proportional? Explain your thinking.

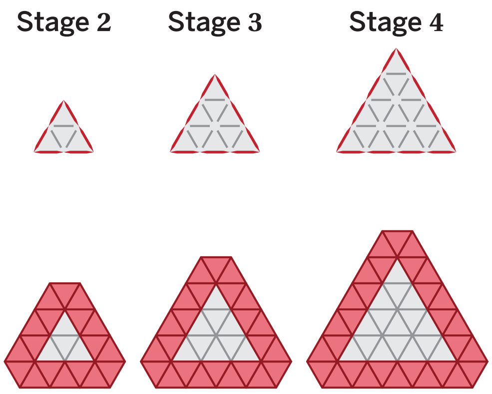
No. *Explanations vary.* If they deliver 3 subs, they charge $6.67 per sub, but for 4 subs, they charge $6.50 per sub.

4. If the total charge is $56, how many subs are in the order? Explain your thinking.

9 subs. *Explanations vary.* The store charges $2 for delivery, leaving $54 for the subs. Since the store charges $6 per sub, 9 subs cost $54.

5. Here are scaled copies of a figure. The top three have a toothpick border and the bottom three have a tile border. Complete the table to show the number of toothpicks and tiles for different stages.

Stage	Border Toothpicks	Border Tiles
2	**6**	**18**
3	**9**	**24**
4	**12**	**30**
5	**15**	**36**
6	**18**	**42**



Lesson Practice
7.6.01

Name: ______ Date: ______ Period: ______

Spiral Review

6. Make each equation true by filling in the correct operation.

$48 \boxed{\div} (-8) = -6$ $(-40) \boxed{\div} 8 = -5$

$12 \boxed{-} (-2) = 14$ $18 \boxed{+} (-12) = 6$

Problems 7–8: Maneli and Trinidad are trying to solve the equation $\frac{2}{3} + x = \frac{1}{3}$.

- Maneli says: *I think we should multiply each side by* $\frac{3}{2}$ *because that is the reciprocal of* $\frac{2}{3}$.
- Trinidad says: *I think we should add* $-\frac{2}{3}$ *to each side because that is the opposite of* $\frac{2}{3}$.

7. Which person's strategy should they use?

Trinidad's

8. What is an equation that can be solved using the other person's strategy?

Responses vary. $\frac{2}{3}x = 4$

Problems 9–11: Rewrite each fraction in decimal form.

9. $\frac{13}{4} = 3.25$ **10.** $\frac{15}{8} = 1.875$ **11.** $\frac{22}{6} = 3.\overline{6}$

Reflection

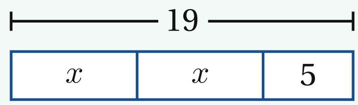
1. Put a heart next to the problem you feel most confident about.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

We can use tape diagrams and equations to make sense of stories and determine unknown amounts.

Here is a story with the equation and tape diagram that represent it.

Story	Equation	Tape Diagram
Two students go to the movie theater. They purchase two tickets and a $5 popcorn to share. In total, they spend $19.	$2x + 5 = 19$	

In the equation and tape diagram, x represents the unknown price of a movie ticket, 2 represents the number of tickets that were purchased, 5 represents the $5 spent on popcorn, and 19 represents the total amount spent.

Things to Remember:

Lesson Practice
7.6.03

Name: .... Date: .... Period: ....

Problems 1–3: Here are two different situations.

1. Match each equation and situation with its tape diagram.

Equation/Situation

a. $3x + 6 = 27$

b. $6x + 3 = 27$

c. A family buys 6 tickets to a show. They also pay a $3 parking fee. They spend $27 total.

d. Diego has 27 ounces of juice. He pours equal amounts for each of his 3 friends and has 6 ounces left for himself.

Diagram A

a, d

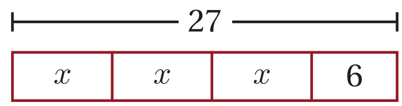
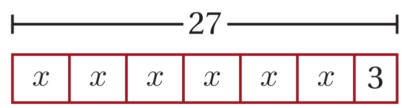


Diagram B

b, c



2. Using your responses from Problem 1, determine the value and meaning of x in Diagram A. Explain what x represents in context.

$x = 7$. x represents the number of ounces of juice he gives each friend, so Diego gives each friend 7 ounces of juice.

3. Using your responses from Problem 1, determine the value and meaning of x in Diagram B. Explain what x represents in context.

$x = 4$. x represents the cost of the ticket, so each ticket costs $4.

4. Match each equation with a tape diagram.

Equation

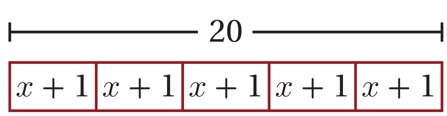
a. $5x + 1 = 20$

b. $5(x + 1) = 20$

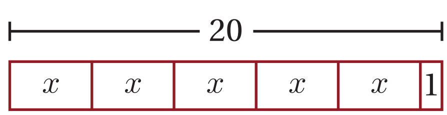
c. $5 + x + 1 = 20$

Tape Diagram

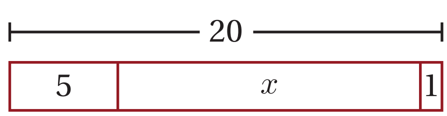
b



a



c

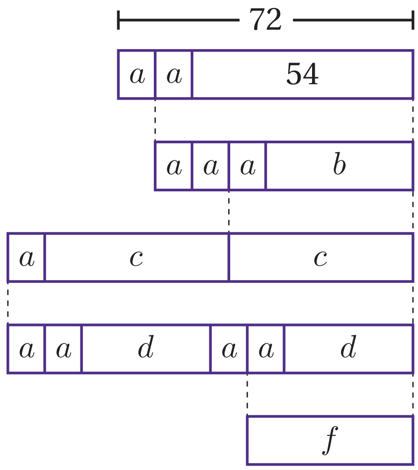


Lesson Practice
7.6.03

Name: ____________ Date: ________ Period: ________

5. Determine the value of each variable.

Variable	Value
a	**9**
b	**36**
c	**45**
d	**31.5**
f	**40.5**



Spiral Review

6. Determine the number of miles each car can travel in 1 hour assuming the operator drives at a constant speed.

Rate	Number of Miles in 1 Hour
135 miles in 3 hours	**45**
22 miles in $\frac{1}{2}$ hour	**44**
7.5 miles in $\frac{1}{4}$ hour	**30**
$97\frac{1}{2}$ miles in $\frac{3}{2}$ hours	**65**

Problems 7–10: Determine the value of each expression.

7. $100 \cdot (-0.09)$ **= -9**

8. $-7 \cdot (-1.1)$ **= 7.7**

9. $-7.3 \cdot 5$ **= -36.5**

10. $-0.2 \cdot (-0.3)$ **= 0.06**

11. Select *all the* expressions that have products that are positive.

- [x] **A.** $-\frac{5}{2} \cdot (-4) \cdot 1\frac{1}{2} \cdot 2$
- [x] **B.** $-2\frac{1}{5} \cdot \left(-\frac{1}{6}\right) \cdot \left(-\frac{3}{4}\right) \cdot \left(-9\frac{7}{8}\right)$
- [x] **C.** $-4 \cdot 0.3 \cdot (-8)$
- [ ] **D.** $-2.1 \cdot (-5.3) \cdot 7.2 \cdot (-0.4)$
- [ ] **E.** $\frac{4}{3} \cdot \frac{3}{4} \cdot \left(-\frac{1}{2}\right)$

Reflection

1. Circle a problem you are still curious about.

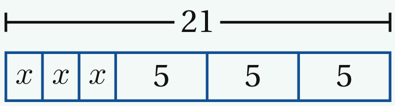
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

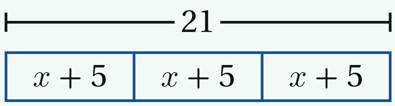
We can use a tape diagram to help us make sense of a situation. Two different tape diagrams and equations can often represent the same situation.

For example, Elena is training for a race. She trained 3 days this week for a total of 21 miles. On each training day, she ran several miles and biked 5 miles. If x represents the number of miles Elena ran, then we can model this situation using two equations and tape diagrams.

$$3x + 15 = 21$$



$$3(x + 5) = 21$$



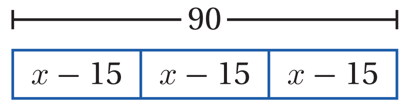
We can use either of the tape diagrams and equations to determine that Elena ran 2 miles on each training day.

Things to Remember:

Lesson Practice
7.6.04

Name: ____________ Date: ________ Period: ________

1. A school ordered 3 large boxes of markers. 3 teachers got 15 markers each, so there were 90 markers left. This tape diagram represents the situation. How many markers were originally in each box?



45 markers

Problems 2–4: A family buys 4 tickets to a show. Each family member also spends $\$2$ on a snack. They spend $\$24$ total.

2. Which equation represents this situation?

A. $2(x+4)=24$

B. $4(x+2)=24$

3. What does x represent in the equation you chose?

x represents the cost of a ticket.

4. Solve the equation you chose. What does the solution tell you about this situation?

$x=4$. Tickets to the show cost $\$4$ each.

Problems 5–7: Amir has 24 ounces of juice. He pours equal amounts for each of his 2 friends and then adds 4 more ounces for each person, using all of the juice.

5. Which equation represents this situation?

A. $2(x+4)=24$

B. $4(x+2)=24$

6. What does x represent in the equation you chose?

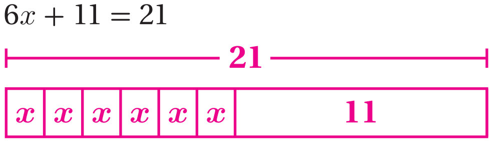
x represents the number of ounces of juice Amir originally poured for each friend.

7. Solve the equation you chose. What does the solution tell you about this situation?

$x=8$. Amir originally poured 8 ounces of juice for each friend.

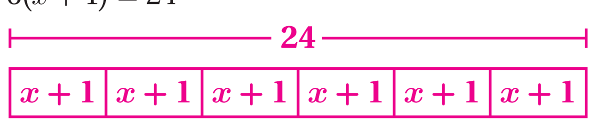
Problems 8–9: Draw a tape diagram and find the solution for each equation.

8. $6x+11=21$



$x=\frac{5}{3}$ (or equivalent)

9. $6(x+1)=24$



$x=3$

Lesson Practice
7.6.04

Name: .... Date: .... Period: ....

10. Each tape diagram is the same length. Write an equation for each tape diagram and find the solution to the equation.

Diagram	Equation	Solution
56 $a+3$ $a+3$ $a+3$ $a+3$	$4(a+3)=56$	$a=11$
56 b b b b 3	$4b+3=56$	$b=\frac{53}{4}$
56 $c+4$ $c+4$ $c+4$	$3(c+4)=56$	$c=\frac{44}{3}$
56 d d d 4	$3d+4=56$	$d=\frac{52}{3}$

Spiral Review

Problems 11–14: Determine the value of each expression.

11. $\frac{2}{3} \cdot \left(\frac{-4}{5}\right) = -\frac{8}{15}$

12. $\left(\frac{-5}{7}\right) \cdot \left(\frac{7}{5}\right) = -1$

13. $\left(\frac{-2}{39}\right) \cdot 39 = -2$

14. $\left(\frac{2}{5}\right) \cdot \left(\frac{-3}{4}\right) = -\frac{3}{10}$

Reflection

1. Circle a problem you want to talk to a classmate about.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

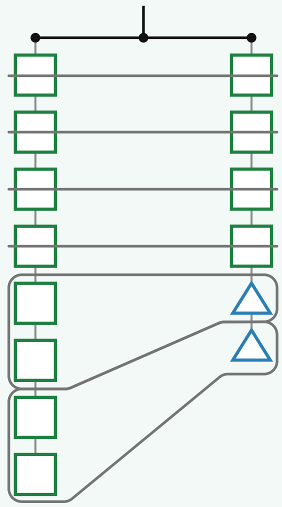
Hangers are balanced when the weight on both sides is the same. When you remove the same weight from both sides, the hanger will remain balanced.

Here are two examples of strategies you can use to determine an unknown value.

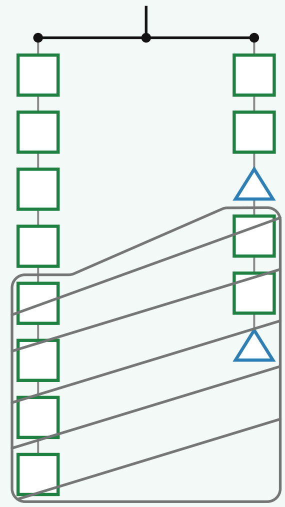
In Hanger A, you can cross off 4 squares on each side while still keeping the sides balanced. Then, 4 squares to the left are balanced with 2 triangles to the right. This means they have the same weight, so 1 triangle has the same weight as 2 squares.

In Hanger B, we can divide each side into two equal groups and remove one group on each side. This leaves 4 squares on the left and a cluster of 1 triangle and 2 squares on the right. Removing 2 more squares from each side leaves the weight of 1 triangle equal to the weight of 2 squares.

Hanger A



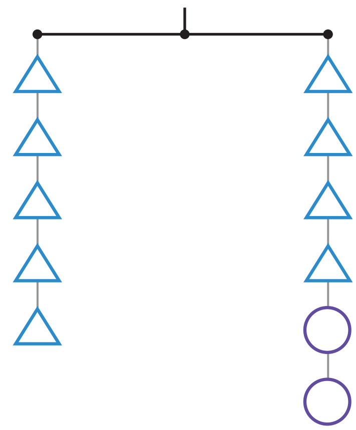
Hanger B



Things to Remember:

Name: ______________________ Date: __________ Period: __________

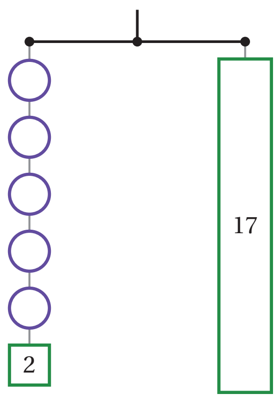
Problems 1–5: Determine the weight of a circle based on these different weights for a triangle.



1. 2 pounds
   1 pound
2. 1 pound
   0.5 pounds
3. 0.5 pounds
   0.25 pounds
4. 3 pounds
   1.5 pounds
5. 100 pounds
   50 pounds

Problems 6–7: Determine the weight of a circle so the hanger stays balanced.

6.



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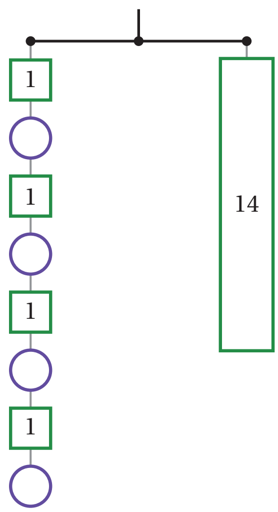
7.

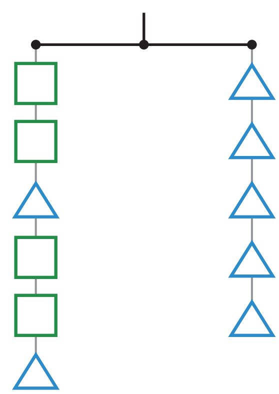


2.5 (or equivalent)

8. Here is a balanced hanger. Darryl says the weight of four squares is equal to the weight of three triangles.

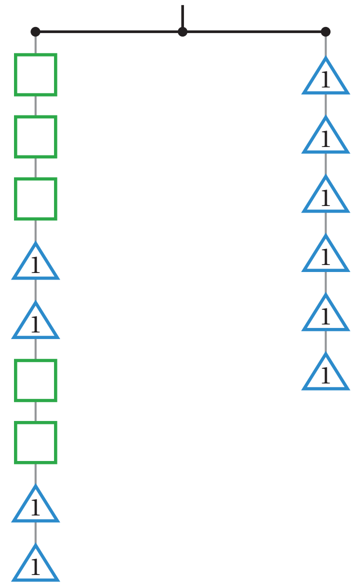
Is Darryl correct? Explain your thinking.

Yes. *Explanations vary.* If he removes two triangles from each side, the hanger will stay balanced. Then, he will have four squares equal to three triangles.



Name: .... Date: .... Period: ....

9. Use this model to determine the value of a square.



A. 0.4

B. 0.5

C. 2.5

D. 4

Spiral Review

Problems 10–11: Solve each equation.

10. $8.5 \cdot 3 = a$

$25.5 = a$

11. $c - 3 = 15$

$c - 3 + 3 = 15 + 3$

$c = 18$

12. Select *all* the expressions that are equivalent to $2(x + 3)$.

- [x] A. $2 \cdot (x + 3)$
- [x] B. $(x + 3) \cdot 2$
- [x] C. $2 \cdot x + 2 \cdot 3$
- [ ] D. $2x + 3$
- [ ] E. $(2 \cdot x) + 3$

Reflection

1. Star the problem you spent the most time on.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

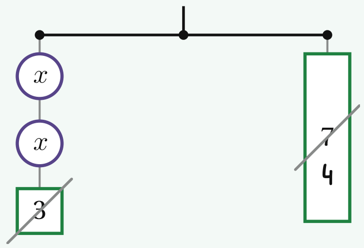
We can use a hanger diagram to represent an equation and help us understand how to find an unknown value in that equation. You can write the steps for finding an unknown value without using a hanger.

For example, the equation $2x + 3 = 7$ can be solved using these steps:

Subtract 3 from both sides.

$$2x + 3 = 7$$
$$2x + 3 - 3 = 7 - 3$$
$$2x = 4$$

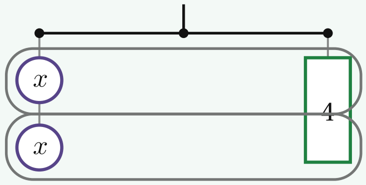
Remove 3 from both sides.



Divide both sides by 2.

$$2x = 4$$
$$2x \div 2 = 4 \div 2$$
$$x = 2$$

Divide into two equal groups.



Things to Remember:

Lesson Practice
7.6.06

Name: ______ Date: ______ Period: ______

Problems 1–3: Solve each equation.

1. $x - 1 = 5$

$x = 6$

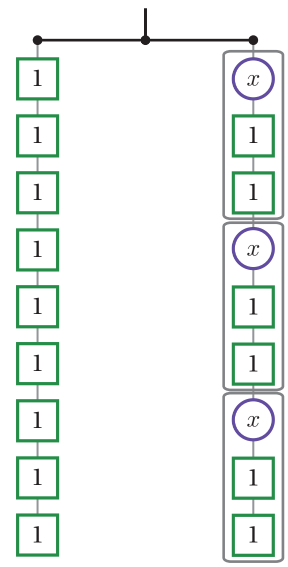
2. $2(x - 1) = 10$

$x = 6$

3. $500 = 100(x - 1)$

$x = 6$

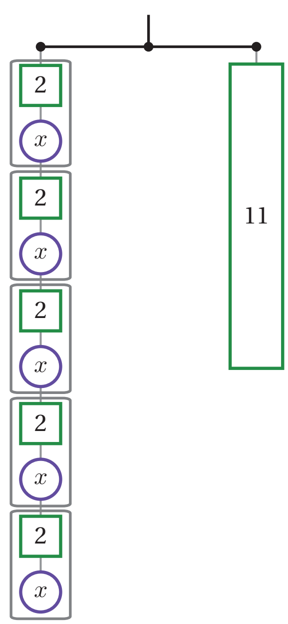
4. Explain how each part of the equation $9 = 3(x + 2)$ is represented in the hanger.



Responses vary.

- **The circle has an unknown weight, so we use x to represent it.**
- **The 9 in the equation is represented by the left side of the hanger with 9 squares, each weighing 1 unit.**
- **There are 3 groups of $x + 2$ on the right side. This represents the $3(x + 2)$ in the equation.**
- **The equal sign represents that the hanger is balanced.**

Problems 5–6: Here is a balanced hanger.



5. Write an equation that represents this hanger.

$5(x + 2) = 11$ (or equivalent)

6. What is the value of x that makes the equation true?

$x = \frac{1}{5}$ (or equivalent)

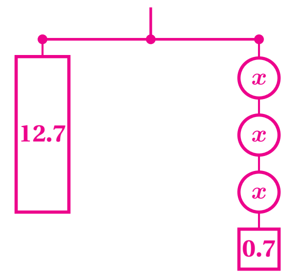
Lesson Practice
7.6.06

Name: .... Date: .... Period: ....

Problems 7–8: Consider the equation $12.7 = 3x + 0.7$.

7. Draw a hanger to represent the equation in the space provided.
Diagrams vary. **Sample diagram shown.**

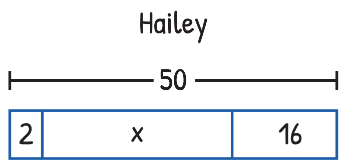
Your Diagram



8. What is the value of x that makes the equation true?
$x = 4$

Spiral Review

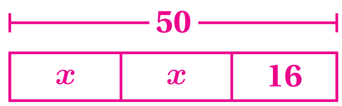
Problems 9–11: Hailey drew a diagram to represent the equation $2x + 16 = 50$, but she made a mistake.



9. Explain the mistake Hailey made.
Responses vary. **Hailey shows $2 + x$ instead of $2 \cdot x$.**

10. Draw a new tape diagram in the space provided to represent the equation $2x + 16 = 50$.
Diagrams vary. **Sample diagram shown.**

New Tape Diagram



11. Determine the value of x using your tape diagram from Problem 10.
$x = 17$

12. Which value is equivalent to $-9 + 5 + 3 - (-8)$?

A. -19 **B.** -9 **C.** -1 **D.** 7

Reflection

1. Put a question mark next to a problem you were feeling stuck on.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

When you solve an equation, apply the same operations to both sides of the equation at each step so that the equation remains true.

Here are two examples.

Equation 1

$$\begin{aligned} 3x - 6 &= 9 \\ 3x - 6 + 6 &= 9 + 6 \\ 3x &= 15 \\ 3x \div 3 &= 15 \div 3 \\ x &= 5 \end{aligned}$$

Equation 2

$$\begin{aligned} 3(x - 6) &= 9 \\ 3(x - 6) \div 3 &= 9 \div 3 \\ x - 6 &= 3 \\ x - 6 + 6 &= 3 + 6 \\ x &= 9 \end{aligned}$$

A *solution to an equation* is a value of a variable that makes the equation true. You can check your solution by substituting the value in for the variable and evaluating.

Equation 1

$$\begin{aligned} 3(5) - 6 &= 9 \\ 15 - 6 &= 9 \\ 9 &= 9 \text{ ✅} \end{aligned}$$

Equation 2

$$\begin{aligned} 3(9 - 6) &= 9 \\ 3(3) &= 9 \\ 9 &= 9 \text{ ✅} \end{aligned}$$

Things to Remember:

Lesson Practice
7.6.07

Name: .... Date: .... Period: ....

Problems 1–3: Solve each equation by filling in the blanks.

1. $15x - 10 = 65$

$15x =$ **75**

$x =$ **5**

2. $3(x + 7) = -12$

$x + 7 =$ **-4**

$x =$ **-11**

3. $-100x - 100 = 0$

$-100x =$ **100**

$x =$ **-1**

Problems 4–7: Solve each equation.

4. $-4x = -28$

$x = 7$

5. $-4(x + 1) = -28$

$x = 6$

6. $x - (-7) = -1$

$x = -8$

7. $-3x + 7 = -1$

$x = \frac{8}{3}$

(or equivalent)

8. Here is an equation: $\frac{1}{4}(x + 8) = -6$. What value of x makes the equation true?

$x = -32$

9. Fill in each blank using the numbers 0 to 9 only once so that the solution for x in both equations is the same.

Responses vary.

3 $x -$ **4** $=$ **8**

2 $(x -$ **1** $) =$ **6**

Lesson Practice
7.6.07

Name: .... Date: .... Period: ....

Spiral Review

10. Match each situation to an equation.

a. Mariana has an 8-foot piece of ribbon. She cuts off a piece that is $\frac{1}{4}$ of a foot long and cuts the remainder into four equal pieces.

b. A baker uses 4 cups of flour. She uses $\frac{1}{4}$ cups to flour the counters and the rest to make 8 muffins.

c. A stack of paper cups is 8 inches tall. The first cup is 4 inches tall and each of the rest of the cups adds $\frac{1}{4}$ inches to the height of the stack.

......b...... $8x + \frac{1}{4} = 4$

......c...... $4 + \frac{1}{4}x = 8$

......a...... $\frac{1}{4} + 4x = 8$

Problems 11–12: There are 88 seats in a theater, and they are split into 4 identical sections. Each section has 14 red seats and an equal number of blue seats.

11. Draw a tape diagram or hanger to represent the situation. ***Diagrams vary.***

88

$14 + x$	$14 + x$	$14 + x$	$14 + x$

12. Write an equation to represent the situation.

$88 = 4(14 + x)$ (or equivalent)

Reflection

1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Some equations have the form $p(x + q) = r$. We call this the **factored** form of the equation because it shows the product of two factors, p and $(x + q)$.

You can solve these equations by **expanding** or dividing as a first step. When expanding first, you can use the distributive property to multiply p by each term inside the parentheses. When dividing first, you can divide both sides of the equation by the factor p.

For example, here are two ways to solve the equation $3(x + 1) = 9$. The first steps are different, but the value of x, the solution to the equation, is the same.

Expanding First (Using the Distributive Property)

$$3(x+1) = 9$$
$$3x + 3 = 9$$
$$3x + 3 - 3 = 9 - 3$$
$$3x = 6$$
$$3x \div 3 = 6 \div 3$$
$$x = 2$$

Dividing First

$$3(x+1) = 9$$
$$3(x+1) \div 3 = 9 \div 3$$
$$x + 1 = 3$$
$$x + 1 - 1 = 3 - 1$$
$$x = 2$$

Things to Remember:

Lesson Practice
7.6.08

Name: ________ Date: ________ Period: ________

Problems 1–3: Write each expression in expanded form.

1. $-2(-6)$ $= 12$

2. $-2(-y)$ $= 2y$

3. $-2(-6 - y)$ $= 12 + 2y$

Problems 4–6: Complete the missing information in the puzzle and complete the table.

4.

	a	-5
4	$\boxed{4a}$	$\boxed{-20}$

Factored	Expanded
$4(a - 5)$	$4a - 20$

5.

	$\boxed{-4a}$	$\boxed{\frac{1}{3}b}$
-3	$12a$	$-b$

Factored	Expanded
$-3\left(-4a + \frac{1}{3}b\right)$	$12a - b$

6.

	$\boxed{3x}$	$\boxed{-7}$
$\boxed{3}$	$\boxed{9x}$	$\boxed{-21}$

Factored	Expanded
$3(3x - 7)$	$9x - 21$

Responses vary.

Lesson Practice
7.6.08

Name: ______ Date: ______ Period: ______

Problems 7–9: Solve each equation. Show your thinking.

7. $2(x-3)=14$

$2(x-3)\div 2=14\div 2$

$x-3=7$

$x-3+3=7+3$

$x=10$

8. $-5(x-1)=40$

$-5(x-1)\div(-5)=40\div(-5)$

$x-1=-8$

$x-1+1=-8+1$

$x=-7$

9. $\frac{5}{7}(x-9)=25$

$\frac{5}{7}(x-9)\div\frac{5}{7}=25\div\frac{5}{7}$

$x-9=35$

$x-9+9=35+9$

$x=44$

10. Emmanuel and Mar started solving the equation $7(x-2)=91$. Finish solving each equation.

Emmanuel

$7(x-2)=91$

$7x-14=91$

$7x=105$

$x=15$

Mar

$7(x-2)=91$

$x-2=13$

$x=15$

Spiral Review

11. Use long division to write $\frac{2}{5}$ as a decimal.

$$\begin{array}{r} 0.4 \\ 5\overline{)2.0} \\ -2\ 0 \\ \hline 0 \end{array}$$

12. Which equation is true when $x=2$?

A. $3x+4=8$

B. $5x-3=12$

C. $6x+4=16$

D. $4x+5=17$

Reflection

1. Circle the problem you enjoyed doing the most.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Expressions that give the same output for every input are called **equivalent expressions**. To determine whether expressions are equivalent, you can test several inputs to see if they produce the same output.

In this example, $3x + 6$ and $3(x + 2)$ are equivalent because they give the same output for every input. You can test other values and they will always give matching outputs. $6(x + 3)$ is not equivalent because it doesn't give the same output for every input (it is only the same when $x =$ -4).

x	$3x + 6$	$3(x + 2)$	$6(x + 3)$
10	36	36	78
7	27	27	60
-4	-6	-6	-6

Things to Remember:

Lesson Practice
7.6.09

Name: .... Date: .... Period: ....

1. Alejandro says that $10x + 6$ and $5x + 11$ are equivalent because they equal 16 when x is 1. Do you agree with Alejandro? Explain your thinking.

No. *Explanations vary.* Equivalent expressions are equal for any value of the variable. When x is 0, these are not equal.

2. Write at least three different expressions that are equivalent to $16a - 24$.

Responses vary.

- $\mathbf{-24 + 16a}$
- $\mathbf{8(2a - 3)}$
- $\mathbf{-2(-8a + 12)}$

3. Write at least three different expressions that are equivalent to $\frac{-1}{2}(-12x + 30)$.

Responses vary.

- $\mathbf{6x - 15}$
- $\mathbf{3(2x - 5)}$
- $\mathbf{6x + (-15)}$

Problems 4–7: Write an equivalent expression in expanded form.

4. $8\left(-x + \frac{1}{4}\right) = \mathbf{-8x + 2}$ **(or equivalent)**

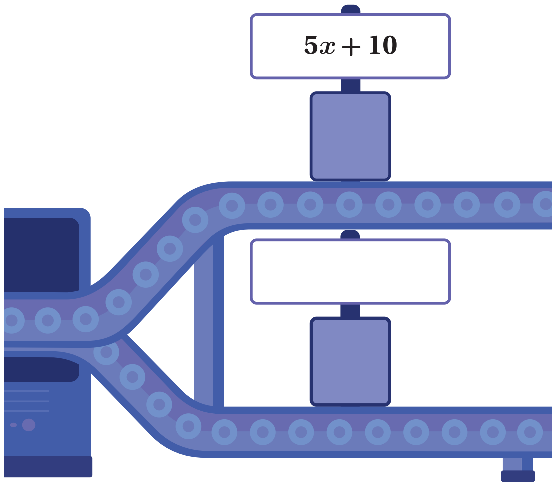
5. $-2(-6x - 1) = \mathbf{12x + 2}$ **(or equivalent)**

6. $\frac{1}{5}(20y - 13) = \mathbf{4y - \frac{13}{5}}$ **(or equivalent)**

7. $9\left(4x + 3y + \frac{2}{3}\right) = \mathbf{36x + 27y + 6}$ **(or equivalent)**

8. Write an expression that will always be equal to $5x + 10$ and an expression that will never be equal. Use x as the variable in each expression.

***Responses vary.* $\mathbf{5(x + 2)}$ will always be equal and $\mathbf{5(x + 10)}$ will never be equal.**



Name: ________ Date: ________ Period: ________

Spiral Review

Problems 9–11: The tables show the energy output, in megawatt-hours (MWh), from a wind farm and a solar power plant.

Wind Farm

Energy (MWh)	Number of Days
1,200	2.4
1,800	3.6
4,000	8
10,000	20

9. Is the energy output of the wind farm proportional to the number of days?

Yes

10. Write an equation showing the relationship. Use E to represent energy and d to represent the number of days.

$E = 500d$ **(or equivalent)**

11. Is the energy output of the solar power plant proportional to the number of days? Explain your thinking.

No. *Explanations vary*. The relationship is not proportional because there is no constant of proportionality between the energy output and the number of days.

Solar Power Plant

Energy (MWh)	Number of Days
100	2.4
650	3.6
1,200	8
1,750	20

12. Match each expression to its value.

a. -30 • (-10)

b. -10 + (-30)

c. -30 –10

d. 10 – (-30)

b, c -40

a 300

d 40

Reflection

1. Put a heart next to the problem you found most interesting.

2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 39

Lesson Summary

You can expand, add, and subtract terms to write equivalent expressions. This helps us write expressions with the same value that have fewer terms. Terms with variables and exponents that are the same are called *like terms* and can be combined. For example, let's find the sum of $-\frac{1}{3}(3x - 6)$ and $2x - 4x + 5$.

We can write the sum as $-\frac{1}{3}(3x - 6) + 2x - 4x + 5$.

Let's write an equivalent expression with fewer terms.

$-\frac{1}{3}(3x - 6) + 2x - 4x + 5$	Expand using the distributive property.
$-x + 2 + 2x - 4x + 5$	Add terms that are alike ($-x$, $2x$, and $-4x$).
$-3x + 2 + 5$	Add terms that are alike (2 and 5).
$-3x + 7$	

Things to Remember:

Lesson Practice
7.6.10

Name: ______ Date: ______ Period: ______

1. Select *all* of the expressions that are equivalent to $4x - 5 + 6$.

- ☑ **A.** $4x + (\text{-}5) + 6$
- ☐ **B.** $4x - 6 + 5$
- ☑ **C.** $4x + 1$
- ☐ **D.** $5x$
- ☐ **E.** $5 + 6 - 4x$

2. Fill in the blanks to make each equation true. ***Responses vary.***

$6x +$ **$4x$** $= 10x$

$6x +$ **$\text{-}4x$** $= 2x$

$6x +$ **$\text{-}16x$** $= \text{-}10x$

$6x +$ **$4x + 5$** $= 10x + 5$

$6x -$ **$4x$** $= 2x$

$6x -$ **$5x$** $= x$

$6x +$ **$\text{-}6x + 10$** $= 10$

$6x - ($ **$2x + 10$** $) = 4x - 10$

3. Fill in each blank with a number or expression so that each row and column has the same sum.

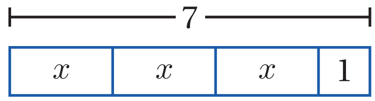
$2x$	$x + 2$	$2 - x$
$5 - x$	x	**$2x - 1$**
$x - 1$	2	**$x + 3$**

Lesson Practice
7.6.10

Name: .... Date: .... Period: ....

Spiral Review

Problems 4–6: This diagram can be represented by the equation $7 = 3x + 1$.



4. Explain where you can see the 3 in the diagram.

There are 3 parts labeled x.

5. Determine the value of x.

$\boldsymbol{x = 2}$

6. Select *all* the situations that could be represented by this equation.

- ☑ **A.** Aaliyah is studying 7 hours this week for end-of-year exams. She spends 1 hour on English and an equal number of hours each on math, science, and history.
- ☐ **B.** Lan spends \$3 on 7 markers and \$1 on a pen.
- ☑ **C.** Sneha shares 7 grapes with 3 friends. Sneha eats 1 grape and gives each friend the same number of grapes.
- ☑ **D.** Kiri read 1 book this week and has a goal to read 7 total books within the next 3 weeks. Kiri will read the same number of books each week.
- ☐ **E.** Adriana runs 7 miles every day for 3 days and then runs 1 mile on the fourth day.

Problems 7–10: Solve each equation.

7. $5(n - 4) = -60$

$\boldsymbol{n = -8}$

8. $-3t + (-8) = 25$

$\boldsymbol{t = -11}$

9. $7p - 8 = -22$

$\boldsymbol{p = -2}$

10. $\frac{2}{5}(j + 40) = -4$

$\boldsymbol{j = -50}$

11. A small town had a population of 960 people last year. The population grew to 1,200 people this year. By what percent did the population grow?

25%

Reflection

1. Put a question mark next to a response you would like to compare with a classmate's.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can solve an equation in many different ways.

$-6(3x - 5) = 75$	Rewrite subtracting 5 as adding (-5).
$-6(3x + (-5)) = 75$	Distribute the -6 to the $3x$ and -5.
$-18x + 30 = 75$	Subtract 30 from both sides.
$-18x = 45$	Divide each side by -18.
$x = -2.5$	

Here is a different strategy for solving the same equation.

$-6(3x - 5) = 75$	Divide both sides of the equation by -6.
$3x - 5 = -12.5$	Add 5 to both sides.
$3x = -7.5$	Divide each side by 3.
$x = -2.5$	

Things to Remember:

Lesson Practice
7.6.11

Name: .... Date: .... Period: ....

Problems 1–3: Kwame and Maia each solved the same equation but used different strategies to do so.

Kwame	Maia
$3.3(x - 10) = 66$	$3.3(x - 10) = 66$
$3.3x - 3.3 \cdot 10 = 66$	$3.3(x - 10) \div 3.3 = 66 \div 3.3$
$3.3x - 33 = 66$	$x - 10 = 20$
$3.3x - 33 + 33 = 66 + 33$	$x - 10 + 10 = 20 + 10$
$3.3x = 99$	$x = 30$
$3.3x \div 3.3 = 99 \div 3.3$	
$x = 30$	

1. How did Kwame begin? Maia?
Kwame began by distributing first. Maia began by dividing each side by 3.3.

2. Show what they might have done to get from one step to another.
Sample shown in work.

3. Which method do you prefer to use for this equation? Why?
Responses vary.

Problems 4–7: Solve each equation. Show your thinking.

4. $4(x - 3) = 16$

$$4(x-3) \div 4 = 16 \div 4$$
$$x - 3 = 4$$
$$x - 3 + 3 = 4 + 3$$
$$x = 7$$

5. $-5(x - 4) = 40$

$$-5(x-4) \div (-5) = 40 \div (-5)$$
$$x - 4 = -8$$
$$x - 4 + 4 = -8 + 4$$
$$x = -4$$

6. $\frac{3}{7}(x - 8) = 15$

$$\frac{3}{7}(x-8) \div \frac{3}{7} = 15 \div \frac{3}{7}$$
$$x - 8 = 35$$
$$x - 8 + 8 = 35 + 8$$
$$x = 43$$

7. $\frac{1}{6}(x + 6) = 11$

$$\frac{1}{6}(x+6) \div \frac{1}{6} = 11 \div \frac{1}{6}$$
$$x + 6 = 66$$
$$x + 6 - 6 = 66 - 6$$
$$x = 60$$

Lesson Practice
7.6.11

Name: .... Date: .... Period: ....

Problems 8–9: Alina and Naoki are each solving the equation $7(x + 2) = 91$. Alina starts by using the distributive property. Naoki starts by dividing each side by 7.

8. Show what Alina's and Naoki's full solution methods might look like.

Alina	**Naoki**
$7(x+2) = 91$	$7(x+2) = 91$
$7x + 14 = 91$	$7(x+2) \div 7 = 91 \div 7$
$7x + 14 - 14 = 91 - 14$	$x + 2 = 13$
$7x = 77$	$x + 2 - 2 = 13 - 2$
$7x \div 7 = 77 \div 7$	$x = 11$
$x = 11$	

9. What is the same and what is different about their methods?

***Responses vary.* Both divided each side by 7 (at different steps) and determined the solution $x = 11$. Alina subtracted 14 from each side while Naoki subtracted 2 from each side.**

Spiral Review

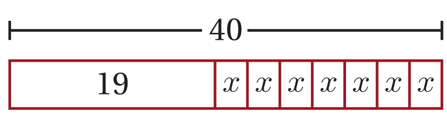
10. Vicente and Zwena are trying to write $9x - 2x + 4x$ using fewer terms.

- Vicente says that $9x - 2x + 4x = 3x$ because the subtraction sign tells us to subtract everything that comes after $9x$.
- Zwena says that $9x - 2x + 4x = 11x$ because the subtraction only applies to $2x$.

Who is correct? Explain your thinking.

Zwena. *Explanations vary.* Rewriting subtraction as addition gives us $9x + (-2x) + 4x$, which shows that the subtraction symbol in front of the $2x$ applies only to the $2x$ and not to the terms that come after it.

11. Write three different equations that represent the tape diagram. Then determine the value of x.



$x = 3$. *Equations vary.*

- $7x + 19 = 40$
- $19 + x + x + x + x + x + x + x = 40$
- $7x = 40 - 19$

12. Choose the expression that is equivalent to $(9x + 5) + (-7x + 2)$.

A. $-2x + 10$ **B.** $-63x + 10$ **C.** $16x + 7$ **D.** $2x + 7$

Reflection

1. Put a star next to a problem where you revised your thinking.

2. Use this space to ask a question or share something you're proud of.

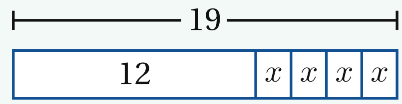
Lesson Practice 7.6.12

Science Mom Lesson 41

Lesson Summary

You can use a visual representation or an equation to answer questions about a situation.

For example, Zahra buys 4 pens and a binder for \$12, paying a total of \$19. This tape diagram represents Zahra's situation.



Zahra can determine the price of each pen by writing and solving the equation $12 + 4x = 19$:

$$12 + 4x = 19$$
$$4x = 7$$
$$x = 1.75$$

Zahra pays \$1.75 for each pen.

Things to Remember:

Lesson Practice

7.6.12

Name: .... Date: .... Period: ....

Problems 1–4: Select an equation to represent the situation. Then solve the equation and explain the solution's meaning in context.

$5x - 7 = 3$	$7 = 3(5 - x)$	$3x + 5 = -7$	$x + 7 = 3 \cdot 5$

	Situation	Equation	Solution	Solution's Meaning
1.	The temperature outside is currently -7°C. Since midnight, the temperature tripled and then rose 5 degrees. What was the temperature at midnight?	$3x + 5 = -7$	$x = -4$	The temperature at midnight was -4°C.
2.	Ama has 7 pink roses plus some white roses. She gives all of her roses away by giving 5 roses to each of her 3 favorite teachers. How many white roses does Ama give away?	$x + 7 = 3 \cdot 5$	$x = 8$	Ama gives away 8 white roses.
3.	A family of 3 goes to a fair. Tickets cost $5 each, but each person has a coupon. They pay $7 altogether. How much money does each person save when buying their ticket?	$7 = 3(5 - x)$	$x = \frac{8}{3}$ (or equivalent)	Each person saved $\frac{8}{3}$ on their ticket cost, or $2.67.
4.	A club puts its members into 5 groups for an activity. 7 students leave early, so there are only 3 students left to finish the activity. How many students were in each group?	$5x - 7 = 3$	$x = 2$	2 students were in each group.

5. 6 soccer teams are practicing on a field. Each team has the same number of players. A coach asks 2 players from each team to leave the field to help move some equipment. Now there are 78 players on the field. Write and solve an equation whose solution is the number of players on each team.

$6(x - 2) = 78$ or $6x - 12 = 78$; $x = 15$ players

Name: .... Date: .... Period: ....

Spiral Review

6. Change the position of the parentheses to create an expression that is equivalent to $(8x - 9 - 12 + 5)$. Explain how you know the two expressions are equivalent.

   Responses vary. **$(8x - 9 - 12) + 5$. I know this expression is equivalent because either way we add 5 as the last step.**

7. Change the position of the parentheses to create an expression that is *not* equivalent to $(8x - 9 - 12 + 5)$.

   Responses vary.
   - $8x - (9 - 12 + 5)$
   - $8x - (9 - 12) + 5$
   - $8x - 9 - (12 + 5)$

Problems 8–11: Determine the value of the variable that makes each equation true.

8. $a \cdot 3 = -30$
$a = -10$

9. $-9 \cdot b = -45$
$b = 5$

10. $-89 \cdot 12 = c$
$c = -1068$

11. $d \cdot 88 = -88000$
$d = -1000$

12. Select *all* the expressions that show x increased by 35%.

   - [x] A. $1.35x$
   - [ ] B. $\frac{35}{100}x$
   - [x] C. $x + \frac{35}{100}x$
   - [x] D. $(1 + 0.35)x$
   - [ ] E. $(100 + 35)x$
   - [x] F. $\frac{100 + 35}{100}x$

13. Alma has a loan of $12,589. This loan has a simple interest rate of 2.1% per year. No payments will be made on the loan until the end of one year.

    About how much interest will Alma pay on this loan at the end of the year?

    A. $2,644
    B. $264
    C. $15,233
    D. $12,853

Reflection

1. Put a heart next to the problem you found most interesting.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can use hanger diagrams to visualize and represent equations. Hanger diagrams can help you see how moves that keep the hanger balanced create *equivalent equations*, such as:

- Adding or subtracting the same term on each side of an equation.
- Multiplying or dividing the expressions on each side of an equation by the same number.

You can use these moves to solve equations for an unknown variable. Here is an example of using balanced moves to solve an equation:

Solving Step	Balanced Equation Move
$4x + 9 = -2x - 3$ $\quad - 9 \qquad - 9$	Subtract 9 from both sides.
$4x = -2x - 12$ $+ 2x + 2x$	Add $2x$ to both sides.
$\frac{6x}{6} = \frac{-12}{6}$ $x = -2$	Divide by 6 on both sides.

You can check if a value is a *solution* to an equation by substituting it into the original equation. If it makes the equation true then the solution is correct.

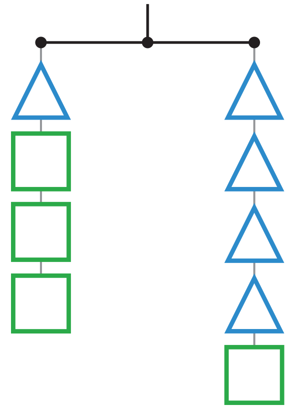
$$4(-2) + 9 = -2(-2) - 3$$
$$-8 + 9 = 4 - 3$$
$$1 = 1$$

Things to Remember:

Lesson Practice
8.4.03

Name: ____________ Date: ________ Period: ________

Problems 1–2: In this balanced hanger diagram, x represents the weight of the triangle and y represents the weight of the square.



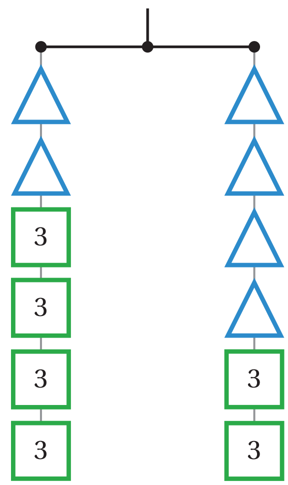
1. Write an equation using x and y to represent the hanger diagram.

Responses vary. $x + 3y = 4x + y$

2. If $x = 6$, what is the value of y? Show or explain your thinking.

$y = 9$. ***Explanations vary.*** **I can remove one triangle and one square from each side, which leaves 2 squares on the left and 3 triangles on the right. Since each triangle weighs 6, that means the remaining weight on the right is 18. I can divide that 18 equally among the two squares, which means each square weighs 9.**

3. In the diagram shown, x represents the weight of each triangle, and 3 represents the weight of each square.



Select *all* the equations that could represent a balanced hanger.

- ☐ **A.** $6x = 6$
- ☑ **B.** $x + 9 = 3x + 3$
- ☑ **C.** $6 = 2x$
- ☐ **D.** $6 + 12x = 12 + 6x$
- ☑ **E.** $2x + 12 = 4x + 6$

4. Match each set of equations with a possible step that turns the first equation into the second equation.

Equations		Possible Steps
a.	$6x + 9 = 4x - 3$ $2x + 9 = -3$	b Divide each side by -4.
b.	$-4(5x - 7) = -18$ $5x - 7 = 4.5$	d Multiply each side by -4.
c.	$8 - 10x = 7 + 5x$ $4 - 10x = 3 + 5x$	e Divide each side by 4.
d.	$-\frac{5}{4}x = 4$ $5x = -16$	a Subtract $4x$ from each side.
e.	$12x + 4 = 20x + 24$ $3x + 1 = 5x + 6$	c Subtract 4 from each side.

Problems 5–6: Bernell and Diego are each trying to solve $2x + 6 = 3x - 8$.

5. The result of Bernell's first step is $-x + 6 = -8$. Describe Bernell's first step.

***Responses vary.* Bernell subtracted $3x$ from each side.**

6. The result of Diego's first step is $6 = x - 8$. Describe Diego's first step.

***Responses vary.* Diego subtracted $2x$ from each side.**

Spiral Review

Problems 7–9: Determine whether each point lies on the graph of the linear equation $4x - y = 3$. Write *yes* or *no*.

7. $(0, 3)$
No

8. $\left(\frac{3}{4}, 0\right)$
Yes

9. $(-1, -7)$
Yes

10. Triangle G is transformed, resulting in triangle H. Which sequence of transformations could be used to show that triangle H is similar but not congruent to triangle G?

A. A translation followed by a dilation

B. A rotation followed by a reflection

C. A reflection followed by a translation

D. A translation followed by a rotation

Reflection

1. Put a smiley face next to the problem you learned from most.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

There are many ways to solve an equation using balanced equation moves. Generally, you want to perform steps that will move you closer to an equivalent equation where the variable is isolated, such as $x = 5$, or $10 = y$. Here are some examples of moves that can be helpful in solving an equation.

Applying the *distributive property*	Combining *like terms* on one side of an equation	Adding or subtracting the same term from both sides	Multiplying or dividing both sides of the equation by the same value
$2x + 4 = 3(2x + 4)$ $2x + 4 = 6x + 12$	$-10x + 3x = 15$ $-7x = 15$	$-x - 8 = 4x + 7$ $-8 = 5x + 7$	$-15x = 5(2x + 4)$ $-3x = 2x + 4$

Things to Remember:

Lesson Practice
8.4.04

Name: ______ Date: ______ Period: ______

1. Here is Anushka's and Lukas's work solving the equation $\frac{2}{5}b + 1 = -11$.

Anushka

$$\frac{2}{5}b + 1 = -11$$
$$\frac{2}{5}b = -10$$
$$b = -10 \cdot \frac{5}{2}$$
$$b = -25$$

Lukas

$$\frac{2}{5}b + 1 = -11$$
$$2b + 1 = -55$$
$$2b = -56$$
$$b = -28$$

Whose solution is correct? Circle one.

Anushka's Lukas's Both Neither

Explain your thinking.

Explanations vary. **Both Anuska and Lukas made errors. Anushka added -1 on the left side and 1 on the right side of the equation. Lukas multiplied both sides of the equation by 5, but forgot to multiply the 1 by 5.**

2. Solve the equation $3(x - 4) = 12x$. Remember to check your solution.

$x = -\frac{4}{3}$. *Work varies.*

$$3(x - 4) = 12x$$
$$x - 4 = 4x$$
$$-4 = 3x$$
$$-\frac{4}{3} = x$$

Solution check:

$$3\left(-\frac{4}{3} - 4\right) = 12\left(-\frac{4}{3}\right)$$
$$-4 - 12 = -16$$
$$-16 = -16$$

This is a true statement, so $x = -\frac{4}{3}$ is a solution.

3. Here is how Zee solved an equation. Circle the mistake and explain why Zee's work is wrong.

Zee

$$-2(3x - 5) = 4(x + 3) + 8$$
$$-6x + 10 = 4x + 12 + 8$$
$$-6x + 10 = 4x + 20$$
$$10 = -2x + 20$$
$$-10 = -2x$$
$$5 = x$$

Explanations vary. **Zee made a mistake in the fourth line. Zee subtracted $6x$ from $4x$ instead of adding.**

What is the correct solution?

$x = -1$. *Work varies.*

$$-2(3x - 5) = 4(x + 3) + 8$$
$$-6x + 10 = 4x + 12 + 8$$
$$-6x + 10 = 4x + 20$$
$$-10x + 10 = 20$$
$$-10x = 10$$
$$x = -1$$

Lesson Practice
8.4.04

Name: ______ Date: ______ Period: ______

4. Elena solved the equation $2(-3x + 4) = 5x + 2$. Describe what Elena did in each step.

Elena

$2(-3x + 4) = 5x + 2$

$-6x + 8 = 5x + 2$

$8 = 11x + 2$

$6 = 11x$

$\frac{6}{11} = x$

Responses vary. **In the first step, Elena used the distributive property and multiplied 2 times $-3x$ and 4. In the second step, Elena added $6x$ to both sides of the equation. In the third step, Elena subtracted 2 from both sides of the equation. In the final step, Elena divided both sides of the equation by 11.**

Problems 5–6: Determine whether $x = -3$ is a solution for each equation. Show or explain your thinking.

5. $-2(x + 2) = -10$

Not a solution. ***Explanations vary.***

$-2(-3 + 2) = -10$

$-2(-1) = -10$

$2 = -10$

False, so $x = -3$ is not a solution.

6. $8(x - 1) = 18x + 22$

Solution. ***Explanations vary.***

$8(-3 - 1) = 18(-3) + 22$

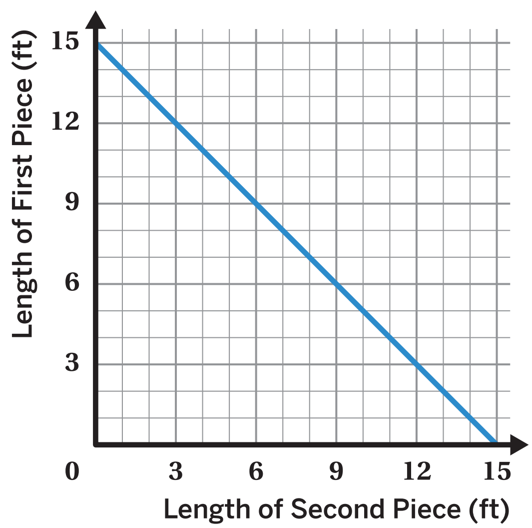
$8(-4) = -54 + 22$

$-32 = -32$

True, so $x = -3$ is a solution.

Spiral Review

Problems 7–9: A length of ribbon is cut into two pieces to use in a craft project. The graph shows the length x of the second piece for each length y of the first piece.



7. How long is the ribbon? Explain your thinking.

15 feet. ***Explanations vary.*** **This is represented by the vertical intercept of the graph.**

8. What is the slope of the line?

-1

9. Explain what the slope of the line represents in this situation.

Responses vary. **For every 1-foot increase in the length of the second piece, the length of the first piece will decrease by 1 foot.**

Reflection

1. Put a heart next to the problem you found most interesting.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Equations can have one solution, no solution, or **infinitely many solutions**.

Here are some examples.

One Solution

$$3x + 8 = 6 + 2 - 3x$$

This equation is only true when $x = 0$.

A linear equation has *one solution* when the expressions on either side of the equation have one value for the variable that makes them equal.

No Solution

$$3(x + 4) = 3x + 7$$

This equation is never true for any value of x.

A linear equation has *no solution* when the expressions on either side of the equation have no value for the variable that make them equal.

Infinitely Many Solutions

$$10 - 3x = 8 - 3x + 2$$

This equation is always true for any value of x.

A linear equation has *infinitely many solutions* when the expressions on either side of the equation are *equivalent*: always equal no matter the value of the variable.

Things to Remember:

Lesson Practice
8.4.06

Name: ______ Date: ______ Period: ______

Problems 1–4: Decide whether the given equation has *one solution*, *no solution*, or *infinitely many solutions*. Explain your thinking.

1. $x - 13 = x + 1$

No solution. *Explanations vary.* There is no value for x that you can subtract 13 from and add 1 to and get the same thing.

2. $x + \frac{1}{2} = x - \frac{1}{2}$

No solution. *Explanations vary.* The expressions $x + \frac{1}{2}$ and $x - \frac{1}{2}$ will never be equal for any value of x.

3. $2(x + 3) = 5x + 6 - 3x$

Infinitely many solutions. *Explanations vary.* This equation is always true for any value of x because the expressions on either side of the equal sign are both equivalent to $2x + 6$.

4. $-(7 - 5x) = 6x - 3$

One solution. *Explanations vary.*

$-7 + 5x = 6x - 3$

$-7 + 3 = x$

$x = -4$

5. Tamiya says that the equation $2x + 2 = x + 1$ has no solution because the left-hand side is double the right-hand side. Is Tamiya's claim correct? Explain your thinking.

No. *Explanations vary.* You can subtract $1x$ and 1 from both sides of the equation to get $x + 1 = 0$, so $x = -1$ is a solution. (This works because 0 is its own double, and it's the only number that is its own double.)

6. Write the other side of this equation so that it is true for all values of x.

$\frac{1}{2}(6x - 10) - x =$ $2x - 5$ (or equivalent)

7. Write the other side of this equation so that it is true for no values of x. *Responses vary.*

$\frac{1}{2}(6x - 10) - x =$ $2x + 5$

8. The equation $5\left(0.2x + \frac{1}{25}\right) = x + \frac{1}{5}$ has infinitely many solutions. Anya creates another equation, $10\left(0.2x + \frac{1}{25}\right) = 2x + \frac{2}{5}$, and says it also has infinitely many solutions. Why might she think that?

Responses vary. Each side of the original equation has been multiplied by 2. This move keeps both sides of the equation in balance, so whatever values of x make the first equation true also make the second equation true.

Lesson Practice
8.4.06

Name: .... Date: .... Period: ....

Spiral Review

Problems 9–10: Solve each equation. Show your thinking.

9. $-4(r+2) = 4(2-2r)$

$r = 4$. *Work varies.*

$-(r+2) = 2 - 2r$

$-r - 2 = 2 - 2r$

$r - 2 = 2$

$r = 4$

10. $1.3 + 6d = 2.7 - 8d$

$d = 0.1$. *Work varies.*

$1.3 + 14d = 2.7$

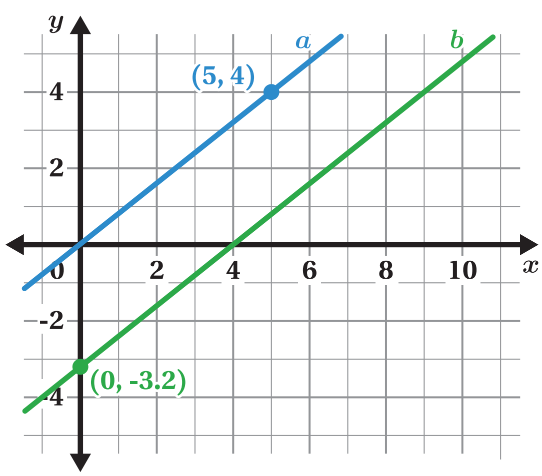
$14d = 1.4$

$d = 0.1$

11. These two lines are parallel. Write an equation for each line.

Line a: $y = 0.8x$ (or equivalent)

Line b: $y = 0.8x - 3.2$ (or equivalent)



Reflection

1. Put a star next to your favorite problem.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Equations can have many different features, including fractions, decimals, negative values, grouping symbols, and multiple terms. Based on the features, it can be helpful to think about what steps might be most useful in solving the equation.

When solving an equation with one solution, the goal is to end up with the variable isolated on one side of the equation and its value on the other. But this doesn't happen when there is no solution or infinitely many solutions.

One Solution	No Solution	Infinitely Many Solutions
$3x + 8 = 6 + 2 - 3x$	$3(x + 4) = 3x + 7$	$10 - 3x = 8 - 3x + 2$
$3x + 8 = 8 - 3x$	$3x + 12 = 3x + 7$	$10 - 3x = 10 - 3x$
$6x + 8 = 8$	$12 = 7$	$10 = 10$
$6x = 0$		
$x = 0$		
This equation is only true when $x = 0$.	This equation is never true for any value of x.	This equation is always true for any value of x.

Things to Remember:

Lesson Practice
8.4.07

Name: .... Date: .... Period: ....

Problems 1–3: Identify whether each equation has a solution that is *positive*, *negative*, or *zero* without solving.

1. $7x = 3.25$
Positive

2. $-7x = 32.5$
Negative

3. $3x + 11 = 11$
Zero

Problems 4–7: Solve each equation. Show your thinking.

4. $2b + 8 - 5b + 3 = -13 + 8b - 5$

$b = \frac{29}{11}$ **(or equivalent).** ***Work varies.***

$-3b + 11 = -18 + 8b$

$-11b + 11 = -18$

$-11b = -29$

$b = \frac{29}{11}$ **(or equivalent)**

5. $2x + 7 - 5x + 8 = 3(5 + 6x) - 12x$

$x = 0$. ***Work varies.***

$-3x + 15 = 15 + 18x - 12x$

$-3x + 15 = 15 + 6x$

$-9x + 15 = 15$

$-9x = 0$

$x = 0$

6. $3(3 - 3x) = 2(x + 3) - 30$

$3 = x$. ***Work varies.***

$9 - 9x = 2x + 6 - 30$

$9 - 9x = 2x - 24$

$9 = 11x - 24$

$33 = 11x$

$3 = x$

7. $\frac{1}{3}(z + 4) - 6 = \frac{2}{3}(5 - z)$

$z = 8$. ***Work varies.***

$z + 4 - 18 = 2(5 - z)$

$z - 14 = 10 - 2z$

$3z - 14 = 10$

$3z = 24$

$z = 8$

8. Kadeem is solving the linear equation $x - 3(2 - 3x) = 2(5x + 3)$. Here are his final two steps. Select the statement that correctly describes Kadeem's solution.

Kadeem

$-6 + 10x = 10x + 6$

$-6 = 6$

A. The solution is the ordered pair $(-6, 6)$.

B. The solution is $x = 0$.

C. There are infinitely many solutions since $-6 = 6$ is a false statement.

D. There is no solution because $-6 = 6$ is a false statement.

Name: ________ Date: ________ Period: ________

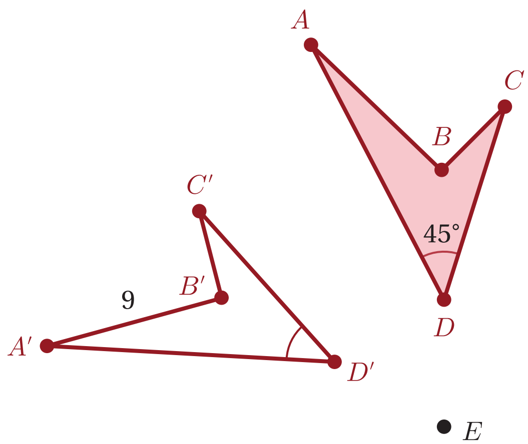
9. What is the solution to this equation?

$-\frac{1}{5}(5x - 15) + 18 = 6x$

A. $x = \frac{7}{20}$ **B.** $x = 2$ **C.** $x = \frac{20}{7}$ **D.** $x = 3$

Spiral Review

Problems 10–11: Figure $A'B'C'D'$ is the image of figure $ABCD$ after a rotation around point E.



10. What is the length of side AB? Explain your thinking.

9 units. ***Explanations vary.*** **The corresponding side lengths of a rotated figure are congruent.**

11. What is the measure of $\angle A'D'C'$? Explain your thinking.

45°. ***Explanations vary.*** **The corresponding angles of a rotated figure are congruent.**

Reflection

1. Circle the problem that was the most challenging for you.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

We can write two expressions in one variable and set them equal to each other to represent a scenario in which two conditions are equal. We can solve this equation to determine the unknown quantity.

For example, imagine two hikers walking in the same direction on a flat trail. The hikers will meet each other when they are at the same location on the trail at the same time.

To determine when this occurs, an expression can be used to represent the location and walking speed of each hiker.

	Location (ft)	Walking Speed (ft/s)	Expression
Hiker 1	30	4	$30 + 4t$
Hiker 2	10	7	$10 + 7t$

You can set these two expressions equal to each other to form one equation that can be solved.

$$30 + 4t = 10 + 7t$$

$$20 = 3t$$

$t = \frac{20}{3}$ or about 6.7 seconds

Things to Remember:

Lesson Practice
8.4.08

Name: ______________ Date: ________ Period: ________

1. For what value of x do the expressions $\frac{2}{3}x + 2$ and $\frac{4}{3}x - 6$ have the same value?
   12

2. Which story could the equation $3x + 6 = 2 + 4x$ represent?

   A. A truck starts 6 meters down a road and is moving at a speed of 3 meters per second. A car starts 2 meters down a road and is moving at a speed of 4 meters per second. After x seconds, the car and the truck meet.

   B. A motorcycle starts 3 meters down a road and is moving at a speed of 6 meters per second. A bicycle starts 4 meters down a road and is moving at a speed of 2 meters per second. After x seconds, the motorcycle and the bike meet.

Problems 3–6: Tiana and Rei are biking in the same direction on the same path.

3. Rei rides at a constant speed of 16 miles per hour. Write an expression that represents the number of miles Rei travels after t hours.
   $16t$ miles

4. Tiana starts riding a half hour before Rei. If Rei has been riding for t hours, how long has Tiana been riding?
   $t + \frac{1}{2}$ hours (or equivalent)

5. Tiana rides at a constant speed of 12 miles per hour. Write an expression that represents the number of miles Tiana travels after Rei has been riding for t hours.
   $12(t + \frac{1}{2})$ miles (or equivalent)

6. Use your expressions to determine when Rei and Tiana meet. Show or explain your thinking.
   1.5 hours. *Explanations vary.*

   $16t = 12(t + \frac{1}{2})$

   $16t = 12t + 6$

   $4t = 6$

   $t = \frac{3}{2}$

Lesson Practice
8.4.08

Name: ______ Date: ______ Period: ______

7. Here are two cell phone plans.

- Plan A costs $\$70$ per month and comes with a free phone worth $\$500$.
- Plan B costs $\$50$ per month, but doesn't come with a free phone.

If you choose Plan B and buy a $\$500$ phone, after how many months will your total cost be the same as the cost of choosing Plan A? Show or explain your thinking.

25 months. *Explanations vary.*

Let x be the number of months. Using Plan A, I spend $70x$ dollars after x months. Using Plan B, I spend $50x + 500$ dollars after x months.

$70x = 50x + 500$

$20x = 500$

$x = 25$

The total cost will be the same after 25 months.

Spiral Review

Problems 8–9: Solve each equation. Show your thinking.

8. $5(x + 2) = 30$

$x = 4$. *Work varies.*

$5(x + 2) \div 5 = 30 \div 5$

$x + 2 = 6$

$x + 2 - 2 = 6 - 2$

$x = 4$

9. $5x + 2 = 30$

$x = \frac{28}{5}$ (or equivalent). *Work varies.*

$5x + 2 - 2 = 30 - 2$

$5x = 28$

$5x \div 5 = 28 \div 5$

$x = \frac{28}{5}$

10. A triangle is graphed on a coordinate grid. Which transformation will result in a triangle that is *not* congruent to the original triangle?

A. A reflection over the y-axis.

B. A translation 5 units to the left.

C. A dilation by a scale factor of 4.

D. A $180°$ counterclockwise rotation around the origin.

Reflection

1. Put a question mark next to a problem you're feeling unsure of.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can use variables, verbal descriptions, symbols, and number lines to represent inequalities related to real-world situations.

To represent an inequality on a number line, you can shade part of the number line to indicate that every point covered by the shaded region is a solution. Then draw an arrow on one end of the number line to show the possible solutions continue on in that direction.

Here is an example:

Situation	Verbal Description	Inequality	Number Line
A two-year-old sleeps more than 9 hours a day.	Any value greater than 9.	$x > 9$	

Things to Remember:

Name: ______________ Date: ________ Period: ________

Problems 1–3: At a book sale, all books cost less than \$5.

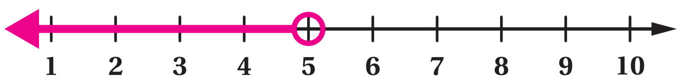
1. List *three* possible prices for a book at this book sale.

Responses vary. **\$1.49, \$3.00, \$4.99**

2. Write an inequality to represent the cost of a book at the book sale, b.

$b < 5$ or $5 > b$

3. Graph all the possible prices of books at the sale.

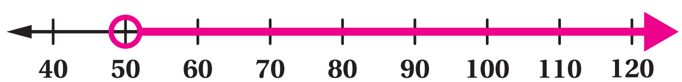


Problems 4–7: n represents the number of eggs a sea turtle lays.

4. What does $50 < n$ mean in this situation?

A sea turtle laid more than 50 eggs.

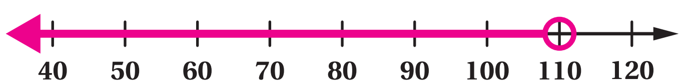
5. Graph this inequality.



6. What does $n < 110$ mean in this situation?

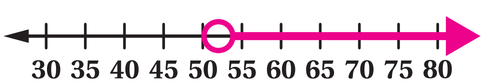
A sea turtle laid fewer than 110 eggs.

7. Graph this inequality.

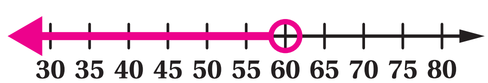


Problems 8–9: One day in Boston, the temperature was above 52°F and below 60°F.

8. Make two graphs, one to represent temperatures above 52°F and another to represent temperatures below 60°F.



Temperatures above 52°F



Temperatures below 60°F

9. Write two inequalities to represent the possible temperatures, T, on that day.

$T > 52$ (or equivalent) and $T < 60$ (or equivalent)

Lesson Practice
6.7.07

Name: ______ Date: ______ Period: ______

Spiral Review

10. Select *all* of the temperatures that are warmer than -10°F.

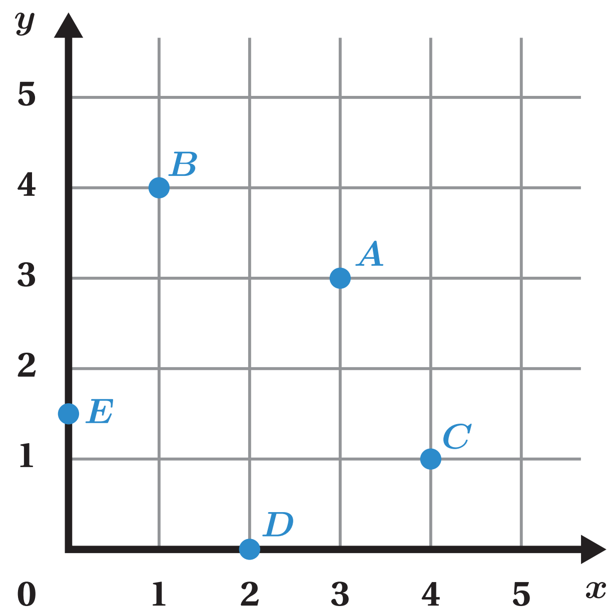
- ☑ **A.** 10°F
- ☑ **B.** 0°F
- ☑ **C.** -5°F
- ☐ **D.** -11°F
- ☐ E. -20°F

11. Anika wrote the number sentence $|-5| = |5|$. Explain what it means in your own words.

***Responses vary.* -5 and 5 are both 5 units away from 0 on the number line.**

12. Write the coordinates of each point shown on the graph.

Point	Coordinates
A	**(3, 3)**
B	**(1, 4)**
C	**(4, 1)**
D	**(2, 0)**
E	**(0, 1.5)**



Reflection

1. Put a star next to a problem you want to understand better.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice 6.7.08

Science Mom Lesson 48

Lesson Summary

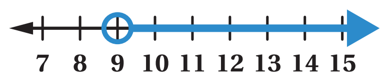
A **solution to an inequality** is any value that makes the inequality true. You can use a number line to represent the solutions to an inequality.

For example, for the inequality $c < 10$, you could say:

- 5 is a solution because $5 < 10$ is a true statement.
- 12 is not a solution because $12 < 10$ is not a true statement.

Some inequalities like $c < 10$ have an infinite number of solutions. We use inequality statements with variables and the symbols $<$ or $>$ to represent all the solutions.

Here is an example:

Inequality	Description	Possible Solutions	Number Line
$x > 9$	Any value greater than 9.	$9.75, 10, 11.3, 82$	

Things to Remember:

Lesson Practice
6.7.08

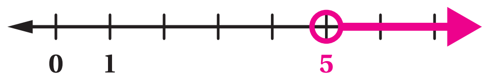
Name: .... Date: .... Period: ....

Problems 1–2: Here is the inequality $k > 5$.

1. Select *all* the values of k that are solutions to the inequality.

- ☐ **A.** 4.9
- ☐ **B.** 5
- ☑ **C.** 6
- ☑ **D.** 5.2
- ☐ **E.** -5.01

2. Make a graph of all the solutions to the inequality.



3. List three numbers that are solutions and three numbers that are not solutions to the inequality $x < -2.25$. Use the number line if it helps with your thinking.

Responses vary.

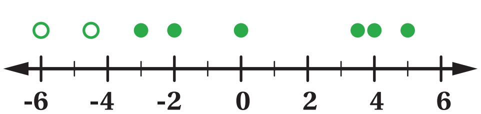
Solutions	Not Solutions
-3	-2
-5.225	0
-2.3	2.3

4. Write an inequality so that $-\frac{2}{3}$, -1.5, and -5 *are* solutions and 2, 0, and 100 *are not* solutions. Use the number line if it helps with your thinking.

Responses vary. $x < -0.5$

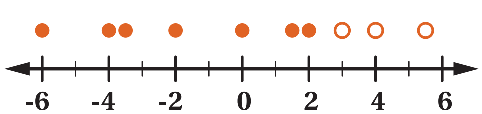
Problems 5–6: Write an inequality so that all of the solid points and none of the open points are solutions.

5.



Responses vary. $x > -4$

6.



Responses vary. $x < 2.5$

Name: .... Date: .... Period: ....

7. Use each number exactly once so that all of the statements are true.

-3 -2 -1 0 1 2 3

$x <$ ☐ and $x >$ ☐

Solutions to both inequalities:

☐ ☐ ☐

Not solutions to both inequalities:

☐ ☐

Responses vary.

- $x < 3$, $x > -1$. Solutions: 0, 1, 2
  Not solutions: -2, -3
- $x < 2$, $x > -2$. Solutions: -1, 0, 1
  Not solutions: -3, 3
- $x < 1$, $x > -3$. Solutions: 0, -1, -2
  Not solutions: 2, 3

Spiral Review

Problems 8–11: Complete each number sentence with the symbol <, >, or =.

8. $\left|-\frac{9}{20}\right|$ > -0.5

9. $|-0.5|$ > $\frac{9}{20}$

10. $\left|-\frac{9}{20}\right|$ < 0.5

11. $\left|-\frac{9}{20}\right|$ < $|-0.5|$

12. Different numbers of hot sauce bottles and their costs are given in the table.

What is the unit rate in dollars per bottle?
$3.54 per bottle

Number of Bottles	Cost
3	$10.62
5	$17.70
7	$24.78

Reflection

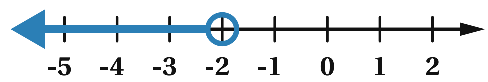
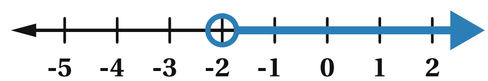
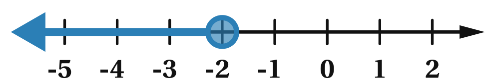
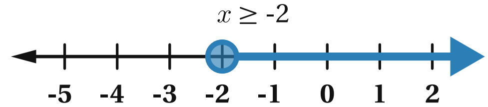
1. Put a question mark next to a problem you were feeling stuck on.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can use an inequality to describe a range of values. For example, the inequality $x > 2$ describes all the values that are greater than 2. The value 2 is not included. The inequality $x \geq 2$ includes the value 2. When graphing, we fill in the circle to represent including the boundary point. An open circle means the boundary point is not included.

Here are the symbols used to write inequalities.

Symbol	Name	Meaning	Example
$<$	Less than	Lower, smaller	$x < -2$ 
$>$	Greater than	More than, higher, larger	$x > -2$ 
$\leq$	**Less-than-or-equal-to**	No more than, the maximum amount	$x \leq -2$ 
$\geq$	**Greater-than-or-equal-to**	At least, the minimum amount	$x \geq -2$ 

Things to Remember:

Lesson Practice
7.6.13

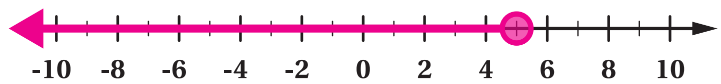
Name: ______________________ Date: __________ Period: __________

1. Select *all* the inequalities that are true when $x = 4$.

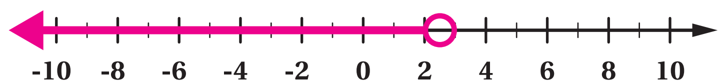
- ☐ **A.** $x < 2$
- ☑ **B.** $x < 10$
- ☐ **C.** $x < 4$
- ☑ **D.** $x \geq 4$
- ☐ **E.** $x \geq 8$

Problems 2–4: Make a graph on the number line that represents each inequality.

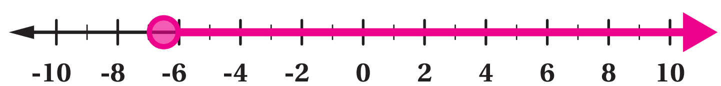
2. $x \leq 5$



3. $x < \frac{5}{2}$

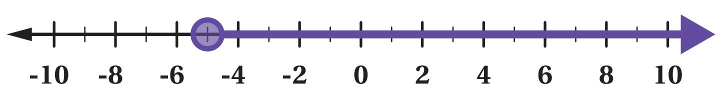


4. $x \geq -6.5$



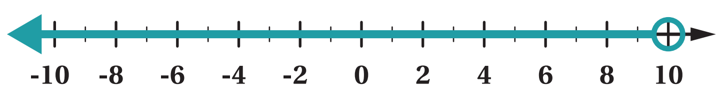
Problems 5–6: Write an inequality that represents each graph.

5.



$x \geq -5$ (or equivalent)

6.



$x < 10$ (or equivalent)

Problems 7–8: Write an inequality that represents each verbal description.

7. Roller coasters have height requirements for safety reasons. The minimum height requirement on a roller coaster at a local theme park, h, is 54 inches.

$h \geq 54$ (or equivalent)

8. Water freezes at any temperature at or below 32 degrees Fahrenheit.

$x \leq 32$ (or equivalent)

9. Write a story that can be represented by the inequality $x < 10$.

***Responses vary.* I was told not to spend all my money. I had $10 to start.**

Name: .... Date: .... Period: ....

Spiral Review

Problems 10–13: Here are two stories.

Story A

This year's freshman class is 10% smaller than last year's class. But during the first week of classes, 20 more students join. There are now 830 students in the freshman class.

Story B

A store reduces the price of a computer by $\$20$. Then during a 10% off sale, a customer pays $\$830$.

10. Determine which story the equations $0.9x + 20 = 830$ and $0.9(x - 20) = 830$ represent.

$0.9x + 20 = 830$ represents Story A and $0.9(x - 20) = 830$ represents Story B.

11. Explain why one equation has parentheses and the other doesn't.

***Explanations vary.* $0.9(x - 20) = 830$ has parentheses because the $\$20$ discount is applied first, then the 10% discount is applied afterward. Parentheses help with performing operations in the right order.**

Solve each equation and interpret its meaning.

12. $0.9x + 20 = 830$

$x = 900$
There were 900 students in last year's freshman class.

13. $0.9(x - 20) = 830$

$x = 942.22$
The original price of the computer was $\$942.22$.

Reflection

1. Put a star next to the problem you understood best.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice 7.6.14

Science Mom Lesson 50

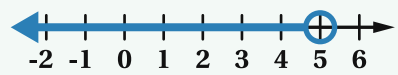
Lesson Summary

You can use many of the same strategies you use for solving equations to solve inequalities. The values of x that make the inequality true are known as the **solutions to an inequality**. You can test values by substituting them into the inequality.

For example, consider the inequality $4x + 2 < 22$.

- To determine the value of x that balances the hanger, solve the equation $4x + 2 = 22$.
- When $x = 5$, the hanger is balanced. All values less than 5 will make the inequality true because $4x + 2$ needs to be less than 22.

The solution shown on the graph means that all values of x *less than* 5 will make the inequality true.



To check the solution, substitute any value less than 5 into the original inequality.

$$4(4) + 2 < 22$$

$$16 + 2 < 22$$

$$18 < 22 \checkmark$$

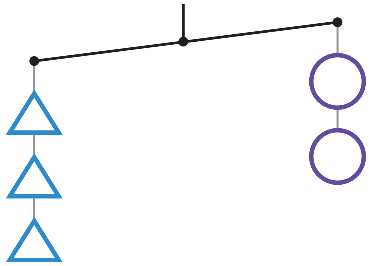
Things to Remember:

Lesson Practice
7.6.14

Name: .... Date: .... Period: ....

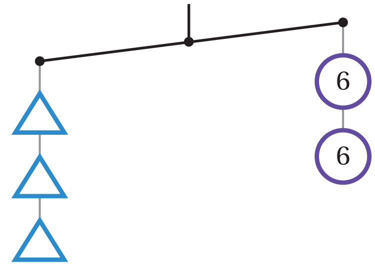
Problems 1–2: Here are two unbalanced hangers. Write an inequality to represent the relationship between the weights on each hanger. Use t to represent the weight of the triangle in grams. Use c to represent the weight of the circle in grams.

1.



$3t > 2c$ **(or equivalent)**

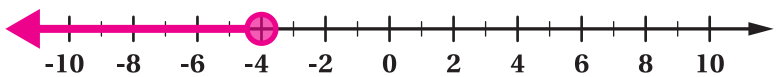
2.



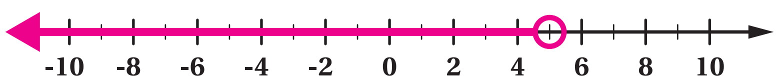
$t > 4$ **(or equivalent)**

Problems 3–5: Use the number line diagrams to graph possible solutions for each inequality.

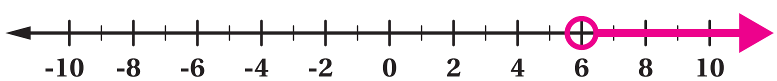
3. $5x \leq -20$



4. $11 > 2x + 1$



5. $2(x + 3) > 18$

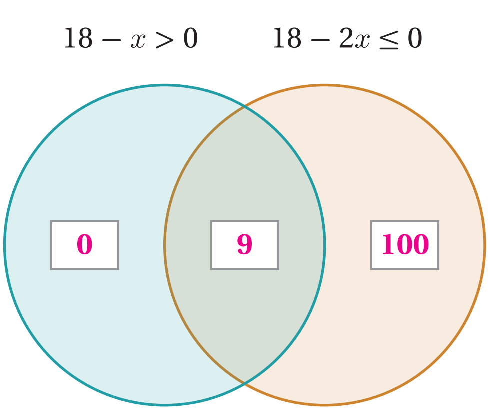


6. List three values for x that would make $5x \leq -20$ true.

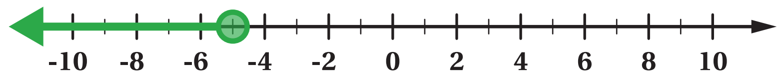
Responses less than or equal to -4 are considered correct.

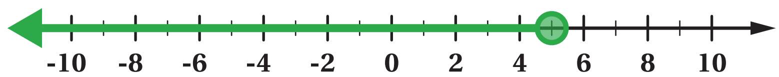
7. Write a value in each region that makes the inequality or inequalities true.

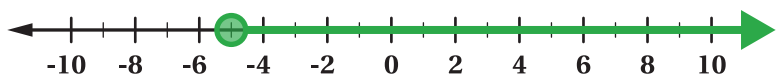
***Responses vary*. Sample shown on diagram.**

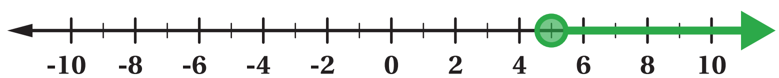


8. Which number line represents the solutions to the inequality $3x - 8 \le 7$?

A.


B.


C.


D.


Spiral Review

9. Select *all* the values that are solutions to $x \le$ -4.

- ☐ A. 4
- ☑ B. -4
- ☐ C. -3.99
- ☑ D. -4.01
- ☐ E. 0

10. Finish writing $\frac{5}{8}$ as a decimal.

0.625
8)5.000
-48
20
-16
40
-40
0

Reflection

1. Put a star next to your favorite problem.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Inequalities can be used in real-world situations, like budgeting for a project.

For example, Aditi has \$5 and sells homemade greeting cards for \$1.50 each. Her goal is to have \$20 total.

The solution to the equation $1.50x + 5 = 20$ represents the number of greeting cards, x, that she needs to sell in order to have exactly \$20. If she sells 10 greeting cards, she will have \$20 because $1.50(10) + 5 = 20$.

What if Aditi wants to have more than \$20? The inequality $1.50x + 5 > 20$ represents this situation.

While the solution to the equation was $x = 10$, the solution to the inequality is $x > 10$. Aditi will need to sell *more than* 10 cards to have *more than* \$20 total.

Aditi

$$1.50x + 5 = 20$$
$$1.50x + 5 - 5 = 20 - 5$$
$$1.50x = 15$$
$$x = 10$$

10 cards makes \$20

$x > 10$ represents making more than \$20

Things to Remember:

Lesson Practice
7.6.15

Name: ______________________ Date: __________ Period: __________

Problems 1–3: Alina donates x dollars out of every $\$9$ that she earns. This happens 7 times. If Alina has exactly $\$42$ remaining for herself, her situation can be represented with the equation $7(9 - x) = 42$.

1. Solve the equation.

$x = 3$

2. Alina wants to have *at least* $\$42$ remaining for herself. Write an inequality to represent this situation.

$7(9 - x) \geq 42$ (or equivalent)

3. Solve the inequality and explain what the solutions mean in the situation.

$x \leq 3$. *Explanations vary.* Alina can donate up to \$3 for every \$9 she earns.

Problems 4–6: Jamir buys a candle that is 9 inches tall and burns down 0.5 inches per minute. He wonders how many minutes it will take until the candle is exactly 6 inches tall, so he writes the equation $9 - 0.5x = 6$.

4. Solve the equation.

$x = 6$

5. Jamir wants to know how many minutes until the candle's height is 6 inches *or less*. Write an inequality to represent this situation.

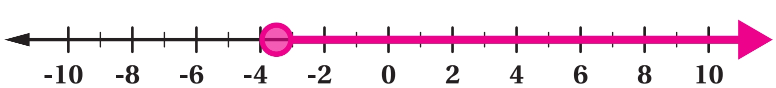
$9 - 0.5x \leq 6$ (or equivalent)

6. Solve the inequality and explain what the solutions mean in the situation.

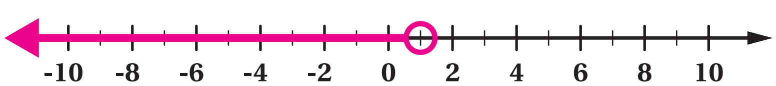
$x \geq 6$. *Explanations vary.* The candle's height will be 6 inches or less at 6 minutes and beyond.

Problems 7–8: Use the number line to graph the solutions to each inequality.

7. $2x \geq -7$



8. $3x + 1 < 4$



Name: ______________________ Date: __________ Period: __________

Spiral Review

Problems 9–12: Evaluate each expression.

9. $\frac{2}{5} \cdot (-10)$ = -4

10. $-8\left(-\frac{3}{2}\right)$ = 12

11. $\frac{10}{6} \cdot 0.6$ = 1

12. $-\frac{100}{37} \cdot (-0.37)$ = 1

Problems 13–16: Here are some prices customers paid for different items at a farmer's market. Determine the cost for 1 pound of each item.

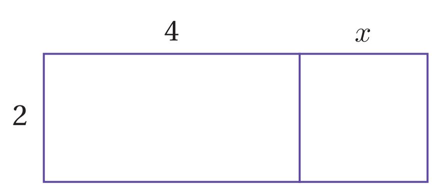
13. \$5 for 4 pounds of apples
\$1.25

14. \$3.50 for $\frac{1}{2}$ pounds of cheese
\$7

15. \$8.25 for $1\frac{1}{2}$ pounds of coffee beans
\$5.50

16. \$6.75 for $\frac{3}{4}$ pounds of fudge
\$9

17. Select *all* the expressions that represent the area of the largest rectangle.



☑ **A.** $2 \cdot 4 + 2 \cdot x$

☐ **B.** $8 + x$

☑ **C.** $8 + 2x$

☑ **D.** $2(4 + x)$

☐ **E.** $2 + 4 + x$

Reflection

1. Put a heart next to the problem you are most proud of.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

When solving an inequality, it can help to start by solving a related equation. The solution to the equation tells you the *boundary point* of the graph of the solutions to the inequality. Once you determine the boundary point, you still need to decide whether the solutions include values greater or less than the boundary point. This can be done by testing a value to the right or left of the boundary point on the number line.

Let's solve the inequality $-3x + 6 < 18$ by starting with the related equation.

Equation	Explanation
$-3x + 6 = 18$	Write the inequality as an equation.
$-3x + 6 - 6 = 18 - 6$	Subtract 6 on both sides.
$-3x \div (-3) = 12 \div (-3)$	Divide by -3 on both sides.
$x = -4$	This is the boundary point.

You can show the solution on a number line by drawing an open circle at -4, the boundary point. To determine if the solutions to the inequality are to the right or left of -4, choose a value, such as 0, to test in the original inequality.

In this case, 0 is to the right of -4 on the number line. $-3(0) + 6 < 18$ is true, which means all values to the right of -4 are solutions to this inequality. The solutions to the inequality are $x > -4$.

Things to Remember:

Lesson Practice
7.6.16

Name: .... Date: .... Period: ....

1. Select *all* the values of x that make the inequality $-x + 6 \geq 10$ true.

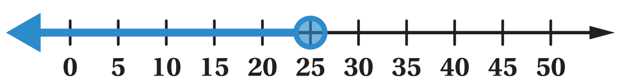
- ☐ **A.** -3.9
- ☐ **B.** 4
- ☑ **C.** -4.01
- ☑ **D.** -4
- ☐ **E.** 4.01
- ☐ **F.** 3.9
- ☐ **G.** 0
- ☑ **H.** -7

2. A library is having a party for any student who read at least 25 books over the summer. Match each situation or graph with an inequality.

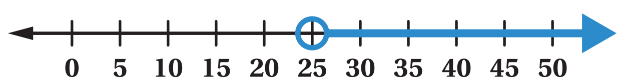
a. Ricardo read x books and was invited to the party.

b. Prisha read x books but was not invited to the party.

c.



d.



b ...... $x < 25$

d ...... $x > 25$

c ...... $x \leq 25$

a ...... $x \geq 25$

Problems 3–4: Here is a new inequality: $100 - 3x \geq -50$.

3. Select *all* the values of x that make the inequality $100 - 3x \geq -50$ true.

- ☑ **A.** 0
- ☑ **B.** 50
- ☑ **C.** -50
- ☑ **D.** 49.9
- ☐ **E.** 50.1

4. In order to solve the inequality $100 - 3x \geq -50$, Makalya solved the equation $100 - 3x = -50$ and got $x = 50$. What is the solution to the inequality?

$x \leq 50$

5. Diego is solving the inequality $-3x \geq 45$. He solves the equation $-3x = 45$ to determine $x = -15$. What is the solution to the inequality?

A. $x < -15$

B. $x > -15$

C. $x \leq -15$

D. $x \geq -15$

Lesson Practice
7.6.16

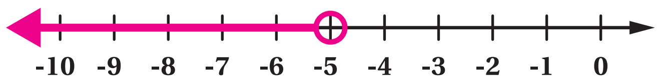
Name: ______________ Date: ________ Period: ________

Problems 6–7: Complete the table to determine the solutions to each inequality. Write the solutions as an inequality and graph them on the number line.

6. $-6x > 30$

$x < -5$

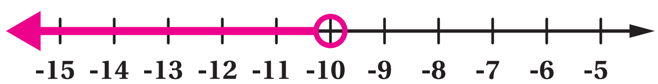
x	-7	-6	-5	-4	-3	-2	-1	0
$-6x$	**42**	**36**	**30**	**24**	**18**	**12**	**6**	**0**



7. $\frac{3}{4}x < -\frac{15}{2}$

$x < -10$

x	-16	-14	-12	-10	-8	-6	-4	-2
$\frac{3}{4}x$	**-12**	$-\frac{21}{2}$	**-9**	$-\frac{15}{2}$	**-6**	$-\frac{9}{2}$	**-3**	$-\frac{3}{2}$



8. Solve the inequality $4x + 8 < 20$.

$x < 3$

Spiral Review

9. Alma makes 5 cups of her favorite color of purple paint by mixing 3 cups of blue paint, $1\frac{1}{2}$ cups of red paint, and $\frac{1}{2}$ of a cup of white paint. Alma has 2 cups of white paint. How much blue paint and red paint will Alma need to use with the 2 cups of white paint?

12 cups of blue paint and 6 cups of red paint

10. A backpack normally costs \$25, but is on sale for \$21. What percent is the discount? Show your thinking.

16%. *Work varies.* $25 - 21 = 4$, $\frac{4}{25} \cdot 100 = 16$

Reflection

1. Put a star next to a problem you want to understand better.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice 7.6.17

Science Mom Lesson 53

Lesson Summary

You can use inequalities to represent and solve real-world problems. When you write an inequality, it can be helpful to first decide what quantity the variable represents. Then, write an inequality based on the relationships between the quantities in the situation.

Once you have solved your inequality, you will need to interpret the solution to ensure it makes sense for the situation. Some quantities only make sense with whole number values (e.g., number of people, number of buses, items that can be purchased, etc.), while other solutions can include decimal or fractional values (e.g., height of a roller-coaster rider, weight of a package, etc.).

Things to Remember:

Lesson Practice
7.6.17

Name: ______________ Date: ________ Period: ________

Problems 1–3: A store sold $\frac{2}{5}$ of the shirts on display and brought out another 30. The store likes to keep at least 150 shirts on display. The manager wrote the inequality $\frac{3}{5}x + 30 \geq 150$ to describe the situation.

1. Explain what $\frac{3}{5}$ means in the manager's inequality.

***Explanations vary.* It represents the amount of shirts that remained in the display. After $\frac{2}{5}$ of the shirts were sold, $\frac{3}{5}$ remained.**

2. Solve the inequality.

$\mathbf{x \geq 200}$

3. What do the solutions to the inequality mean in this situation?

The solution means that the store initially had at least 200 shirts on display.

Problems 4–6: Camila has up to $\$100$ to spend on a swimming pool party. Camila is inviting 15 friends and plans to spend $\$38.50$ on pizza.

4. Write an inequality to represent the situation.

$\mathbf{15x + 38.50 \leq 100}$ **(or equivalent)**

5. Solve the inequality.

$\mathbf{x \leq 4.1}$ **(or equivalent)**

6. What do the solutions to your inequality mean in this situation?

The solution to the inequality is the most Camila can spend per person to access the pool.

Problems 7–8: Write an inequality to represent each situation.

7. In the cafeteria, there is one large 10-seat table and many smaller 4-seat tables. There are enough tables to fit at most 210 students. Write an inequality whose solution is the possible number of 4-seat tables in the cafeteria.

$\mathbf{4x + 10 \leq 210}$ **(or equivalent)**

8. 5 barrels catch rainwater in the schoolyard. 4 barrels are the same size, and the fifth barrel holds 10 liters of water. The Environmental Club is hoping that the 5 barrels can catch at least 210 liters of water to use to water the school's garden. Write an inequality whose solution is the possible size of each of the 4 barrels.

$\mathbf{4x + 10 \geq 210}$ **(or equivalent)**

Lesson Practice
7.6.17

Name: .... Date: .... Period: ....

Spiral Review

9. Select *all* the inequalities that have the same solutions as $-4x < 20$.

☑ **A.** $-x < 5$

☑ **B.** $4x > -20$

☐ **C.** $4x < -20$

☐ **D.** $x < -5$

☑ **E.** $x > -5$

Problems 10–13: Solve each equation.

10. $-1d - 4 = -3$

$d = -1$

11. $-\frac{1}{4}m + 5 = 16$

$m = -44$

12. $10b + (-45) = -43$

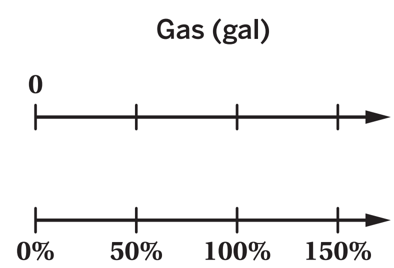
$b = \frac{1}{5}$

13. $-8(y - 1.25) = 4$

$y = 0.75$

14. The gas tank of a truck holds 30 gallons. The gas tank of a passenger car holds 50% less. How many gallons does the car's tank hold? Use the double number line if it helps with your thinking.

15 gallons



Reflection

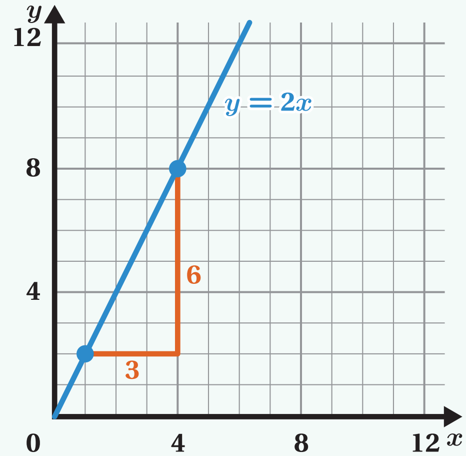
1. What is one math concept from this unit that you have improved on since the unit started? Explain what you did to help yourself improve.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

A line that passes through the origin, $(0, 0)$, represents a *proportional relationship*. The *slope* of the line represents a *unit rate* for this relationship.

Here is a graph of the equation $y = 2x$. The slope of the line is $\frac{6}{3}$, or 2.

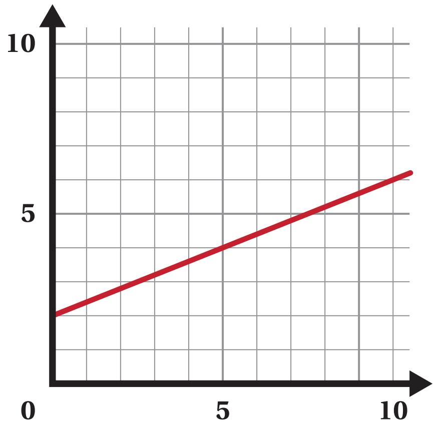


Things to Remember:

Name: .... Date: .... Period: ....

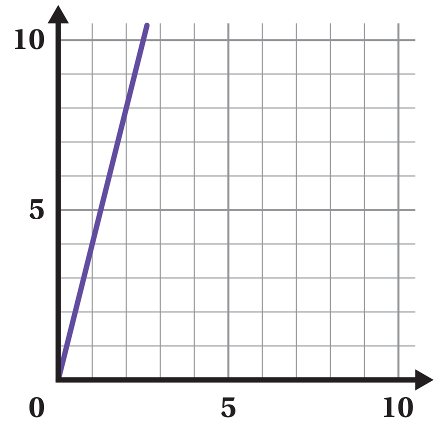
Problems 1–4: Determine whether each graph represents a proportional or non-proportional relationship. Then determine the slope of each line.

1.



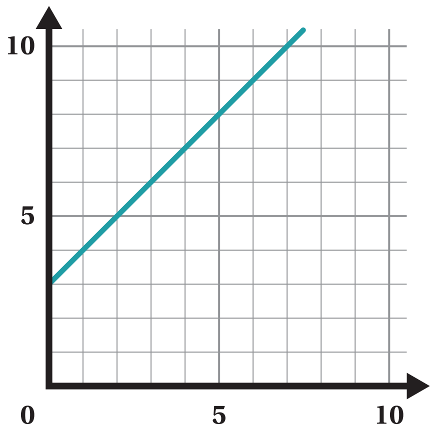
Non-proportional. The slope is $\frac{2}{5}$ (or equivalent).

2.



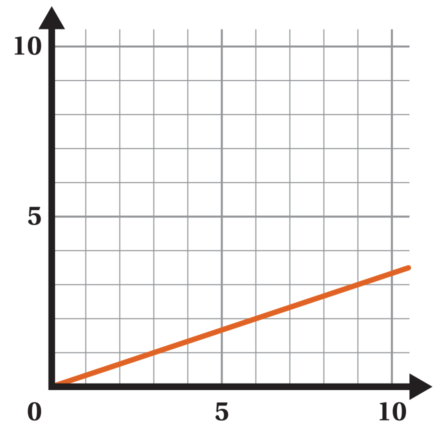
Proportional. The slope is 4.

3.



Non-proportional. The slope is 1.

4.



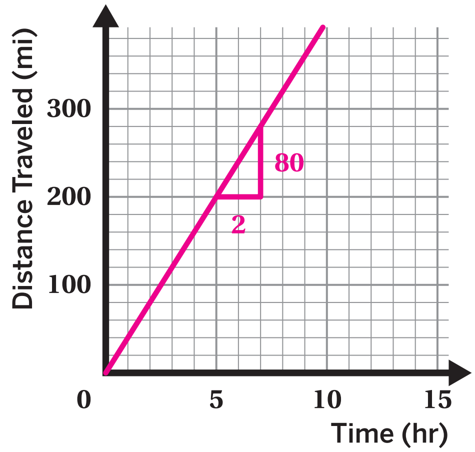
Proportional. The slope is $\frac{1}{3}$ (or equivalent).

Problems 5–6: A car begins traveling at a constant rate. After 2 hours, the car has traveled 80 miles.

5. Graph the line showing the relationship between the car's distance traveled and time.

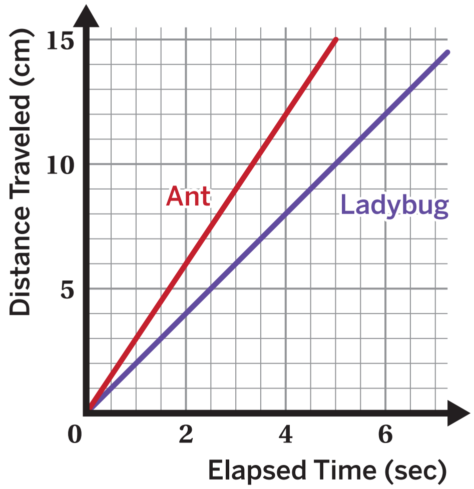
6. What is the slope of the line and what does it represent in context?

The slope is $\frac{80}{2}$, or 40. This means the car travels 40 miles for every 1 hour (or 40 miles per hour).



7. An ant and a ladybug compete in a 15-centimeter race. Write a story that represents the two lines on the graph.

Responses vary. **The ant and the ladybug both start at the starting line. The ant moves at a unit rate of 3 centimeters per second. The ladybug moves at a unit rate of 2 centimeters per second. The ant wins the race.**

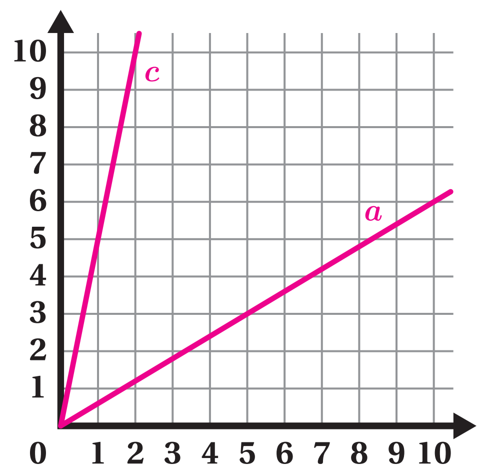


Spiral Review

Problems 8–9: Draw and label a line for each slope.

8. Line a has a slope of $\frac{3}{5}$.

9. Line c has a slope of 5.



10. The table shows a proportional relationship between x and y. What is a constant of proportionality for the relationship between x and y?

x	y
0.25	0.375
1.5	2.250
2.75	4.125
6.50	9.75

A. 0.25

B. 0.50

C. 1.50

D. 1.75

Reflection

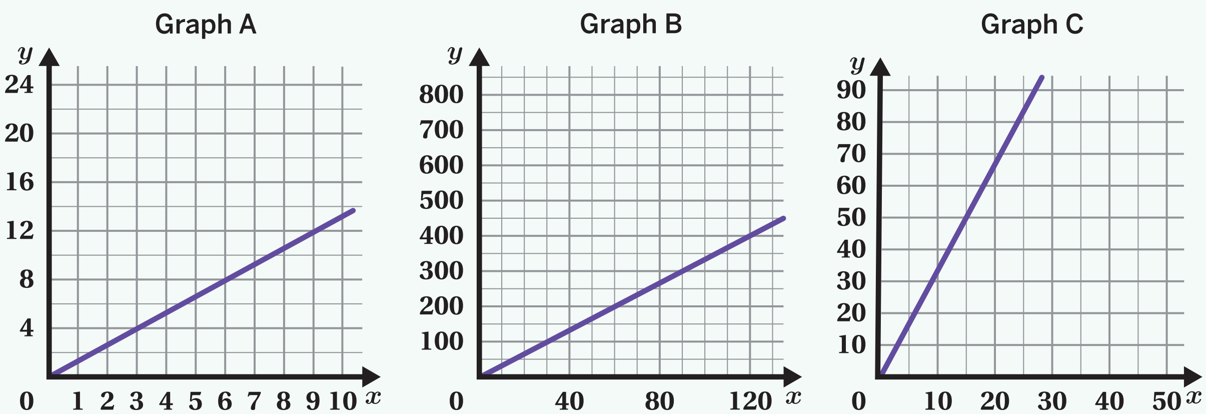
1. Put a heart next to a problem you understand well.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

We can represent a proportional relationship using the equation $y = mx$, where m represents both a unit rate and the slope of the line.

When we represent these relationships on axes with different scales, we can use slope to compare these graphs. For example, you can compare Graphs A, B, and C using their slopes.



- Graph A slope: $\frac{12}{9} = \frac{4}{3}$
- Graph B slope: $\frac{100}{30} = \frac{10}{3}$
- Graph C slope: $\frac{50}{15} = \frac{10}{3}$

You can see that the slopes for Graph B and Graph C are equivalent. This means they have the same proportional relationship, even though the lines may look like they don't have the same steepness.

Things to Remember:

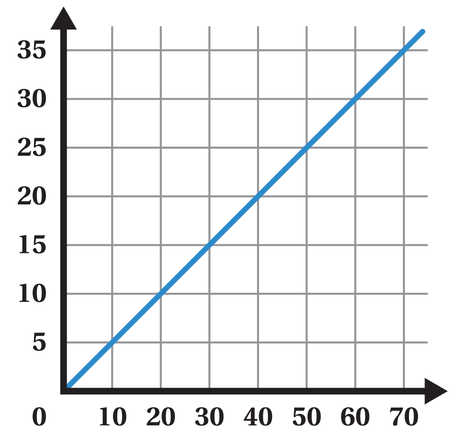
Name: ________ Date: ________ Period: ________

1. Write an equation for the graph of a proportional relationship that passes through the point $(25, 15)$. Explain your thinking.

   $y = \frac{3}{5}x$ (or equivalent). *Explanations vary.* I used a slope triangle to find the slope: $\frac{15}{25} = \frac{3}{5}$. Then I formed the equation using $y = mx$ to represent a proportional relationship.

2. Write an equation of the line graphed. Use $y = mx$ form where m represents the slope of the line.

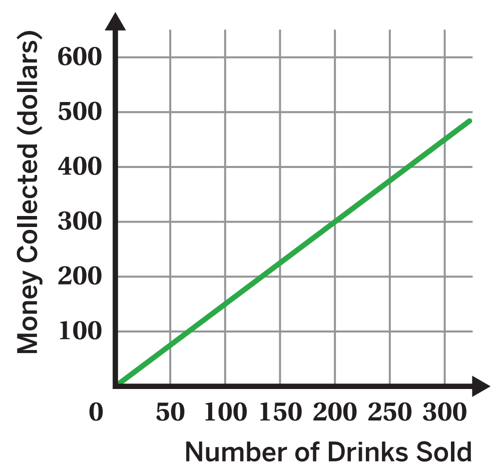
   $y = \frac{1}{2}x$ (or equivalent)



Problems 3–4: Here is the graph of a proportional relationship.

3. Write an equation to represent this relationship, where x is the number of drinks sold at a concession stand and y is the amount of money collected in dollars.

   $y = \frac{3}{2}x$ (or equivalent)

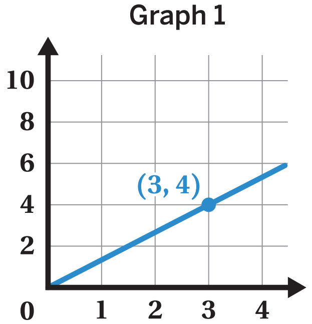


4. Use your equation to complete this table.

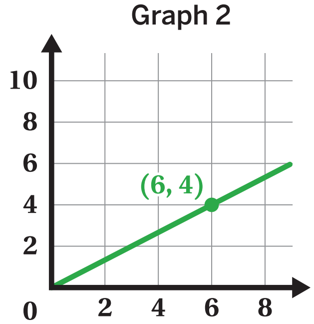
Drinks Sold	220	1000	1
Money (dollars)	330	1500	$\frac{3}{2}$

Problems 5–6: Here are three graphs.

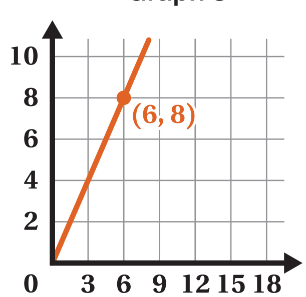
Graph 1



Graph 2



Graph 3



5. Does Graph 2 represent the same relationship as Graph 1? Explain your thinking.

   No. *Explanations vary.* The slope for Graph 1 is $\frac{4}{3}$, while the slope for Graph 2 is $\frac{2}{3}$.

6. Does Graph 3 represent the same relationship as Graph 1? Explain your thinking.

   Yes. *Explanations vary.* The slopes for Graphs 1 and 3 are both equal to $\frac{4}{3}$.

Lesson Practice
8.3.02

Name: .... Date: .... Period: ....

Spiral Review

7. Write an equation for the proportional relationship represented in this table.

$y = 80x$ (or equivalent)

x	y
25	2000
40	3200

Problems 8–9: This table shows a proportional relationship between time, in minutes, and distance walked, in miles.

Time (min)	Distance (mi)
90	5
216	12
162	**9**
18	1

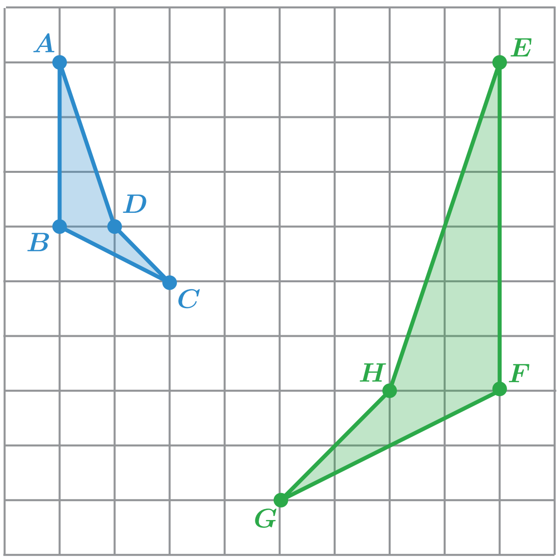
8. Complete the table.

9. Describe the scales you could use on the x- and y-axes of a coordinate grid to show all the times and distances in the table.

***Responses vary.* From 0 to 216 on the horizontal axis (time) and from 0 to 12 on the vertical axis (distance).**

10. Describe a sequence of rotations, reflections, translations, and dilations to show that one figure is similar to the other.

***Responses vary.* Translate figure $EFGH$ up 3 units and left 5 units. Reflect figure $EFGH$ over a vertical line through point D, then dilate it using point D as the center of dilation and a scale factor of $\frac{1}{2}$.**



Reflection

1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Using different representations is helpful when comparing proportional relationships, such as equations, tables, and graphs.

You can represent proportional relationships with the equation $y = mx$, where m is the slope of a line and also represents a unit rate for the situation. You can identify the slope or unit rate using all of the different representations, or by using slope triangles within a graph.

Things to Remember:

Lesson Practice
8.3.03

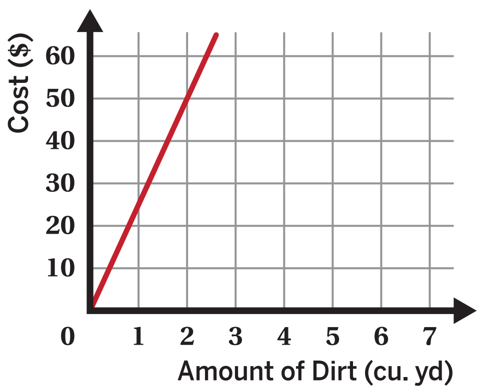
Name: ________ Date: ________ Period: ________

1. A recipe calls for 3 cups of flour for every 2 cups of water. Write an equation for the recipe, where x represents the number of cups of flour and y represents the number of cups of water.

$y = \frac{2}{3}x$ **(or equivalent)**

Problems 2–4: A contractor must haul a large amount of dirt to a work site. She collected cost information from two companies.

Happy Hauling Service



EZ Excavation

Dirt (cu. yd)	Cost ($)
8	196
20	490
26	637

2. Calculate the rate of change for Happy Hauling Service.

The rate of change is $25 per cubic yard.

3. Calculate the rate of change for EZ Excavation.

The rate of change is $24.50 per cubic yard.

4. If the contractor has 40 cubic yards of dirt to haul and only cares about price, which company should she hire? Explain your thinking.

EZ Excavation. *Explanations vary*. It would cost $980, which is less than the cost of Happy Hauling Service.

Problems 5–6: Students are selling raffle tickets for a school fundraiser. They collect $24 for every 10 raffle tickets they sell. Suppose M is the amount of money the students collect for selling r raffle tickets.

5. Complete the table.

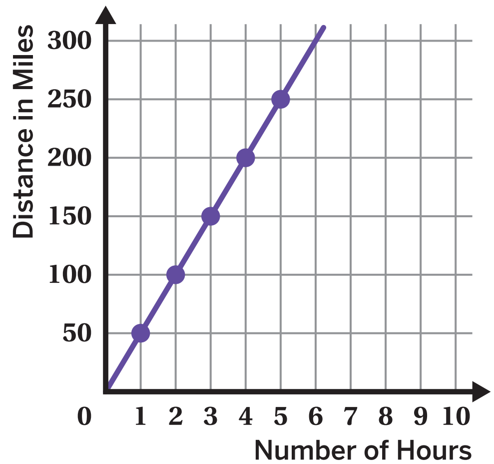
Number of Tickets, r	Amount of Money, M
10	**$24**
20	**$48**
300	**$720**
1000	**$2400**

6. Write an equation that reflects the relationship between M and r.

$M = \frac{12}{5}r$ **(or equivalent)**

Name: ______ Date: ______ Period: ______

7. The graph shows a proportional relationship between the distance traveled on a road trip and the number of hours of travel. Which statement correctly identifies both the slope and the representation of slope for the situation?



A. The slope is 50, so the distance traveled is 1 mile every 50 hours.

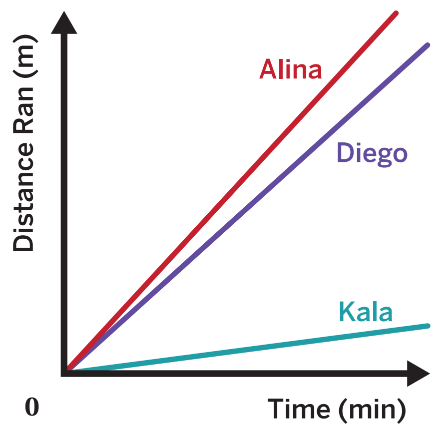
B. The slope is $\frac{1}{50}$, so the distance traveled is 1 mile every 50 hours.

C. The slope is 50, so the distance traveled is 50 miles every hour.

D. The slope is $\frac{1}{50}$, so the distance traveled is 50 miles every hour.

8. Alina runs twice as fast as Diego. Diego runs twice as fast as Kala. Could this graph represent the speeds of Alina, Diego, and Kala? Explain your thinking.

No. *Explanations vary.* In the graph, Alina appears to be running only slightly faster than Diego (not twice as fast), while Diego appears to be running much faster than Kala (more than twice as fast).



Spiral Review

9. The formula for converting temperature in degrees Fahrenheit to degrees Celsius is $C = \frac{5}{9}(F - 32)$. Use this formula to complete the table.

Temperature (°F)	Temperature (°C)
77	25
32	**0**
-0.4	-18
50	**10**

Reflection

1. Put a question mark next to a response you'd like to compare with a classmate's.

2. Use this space to ask a question or share something you're proud of.

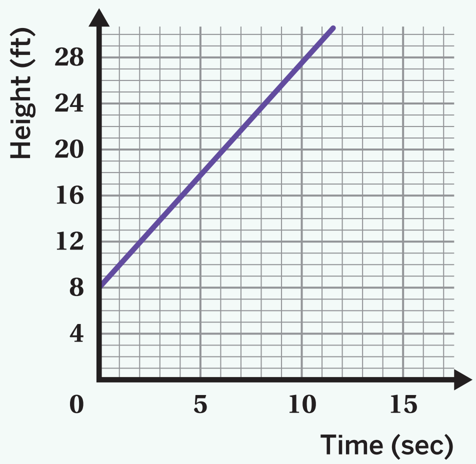
Lesson Summary

Linear relationships are graphs that are lines. Some linear relationships are proportional relationships and some are not.

For example, this graph represents a flag's height, in feet, over time.

- The line starts at $(0, 8)$, which means the flag is at first 8 feet off the ground.
- The slope is 2, which represents the number of feet the flag rises each second.
- The equation $y = 8 + 2x$ represents the height of the flag y after x seconds.

This relationship is linear, but it's non-proportional because the line does not pass through the origin.



Things to Remember:

Lesson Practice
8.3.04

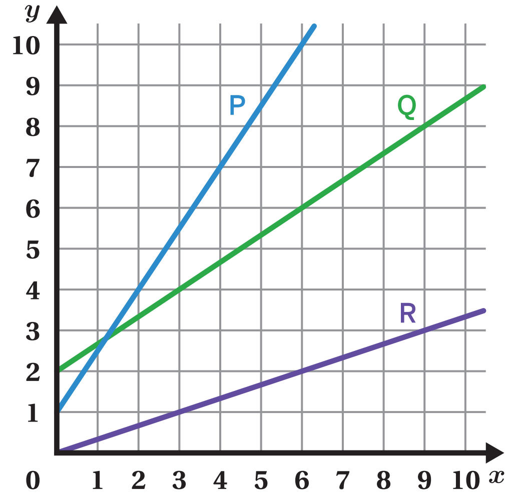
Name: .... Date: .... Period: ....

1. Match each equation with the graph of its line.

a $y = \frac{1}{3}x$ ......R......

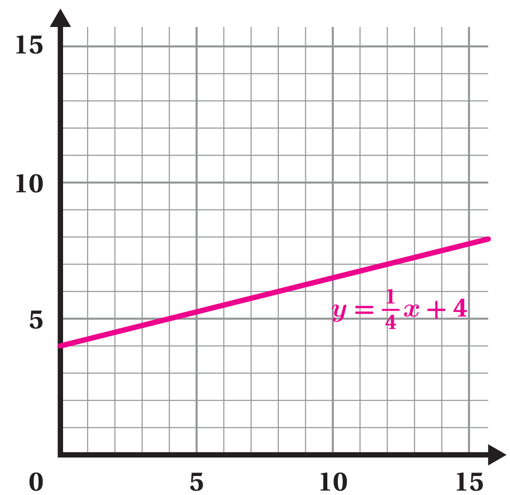
b $y = \frac{3}{2}x + 1$ ......P......

c $y = \frac{2}{3}x + 2$ ......Q......



2. Create a graph that shows a non-proportional linear relationship with a slope of $\frac{1}{4}$. Then write an equation that represents the graph.

Responses vary.



Problems 3–8: Determine whether each of these linear relationships is *proportional* or *non-proportional*.

3. From their initial step, a student walks at a constant speed of 4 miles per hour.

Proportional

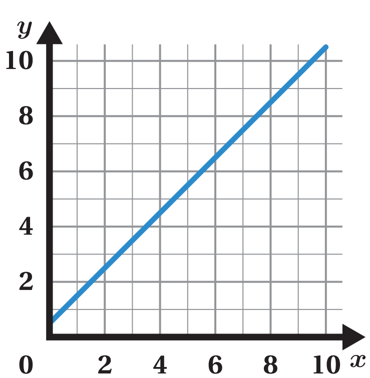
4. $y = 2x$

Proportional

5. A giraffe at birth is 3 feet tall and grows 161 inches every month for a year.

Non-proportional

6.



Non-proportional

7.

x	y
2	4
3	5
5	7

Non-proportional

8. $y = 20 + 4x$

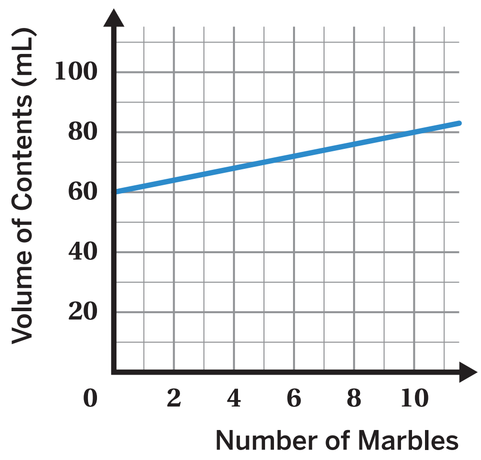
Non-proportional

Lesson Practice
8.3.04

Name: ____________ Date: ________ Period: ________

Problems 9–11: In Aesop's fable "The Crow and the Pitcher," a thirsty crow discovers a pitcher of water but is unable to reach the water with its beak. So the crow adds pebbles to the pitcher to make the water rise higher and higher until it reaches the top. Then it can drink the water!

Thiago recreates this situation using a cylinder containing water and a bunch of marbles. The line represents the volume of the cylinder's contents, y, after x marbles are added.



9. What is the slope of the line?

2 (or equivalent)

10. What does the slope represent in this situation?

***Responses vary.* The volume of the contents increases by 2 milliliters for every 1 marble added.**

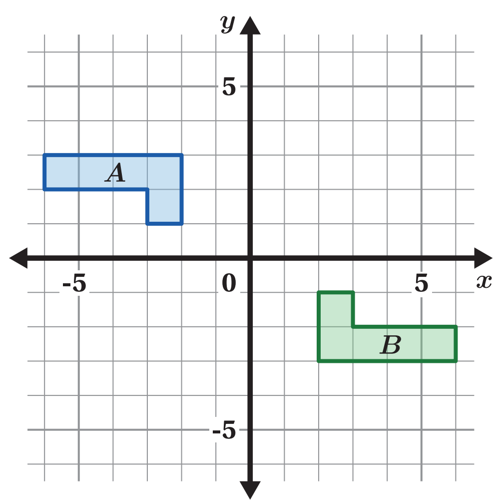
11. Write an equation representing the relationship between x and y.

$y = 2x + 60$ (or equivalent)

Spiral Review

12. Are figures A and B congruent? Explain your thinking.

Yes. *Explanations vary.* There's a sequence of transformations that moves one figure onto the other, which means the figures are congruent.



Reflection

1. Put a smiley face next to a problem you were stuck on and then figured out.

2. Use this space to ask a question or share something you're proud of.

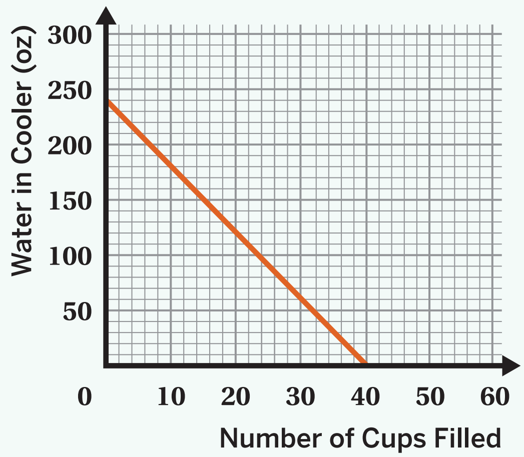
Science Mom Lesson 58

Lesson Summary

When a *linear relationship* has a negative slope, this means that as the x-values increase, the y-values decrease at a constant rate.

Let's say the equation $y = 240 - 6x$ represents the amount of water in a cooler, y, after x cups have been filled.

- The **vertical intercept**, also called the y-*intercept*, is $(0, 240)$. In this situation, the vertical intercept represents the starting amount of water in the cooler.
- The slope is -6. This means that the amount of water decreases by 6 ounces for each cup filled. Because the amount of water decreases each time, the slope is negative.
- The **horizontal intercept**, also called the x-*intercept*, is $(40, 0)$. In this situation, the horizontal intercept represents how many cups can be filled before the cooler runs out of water.

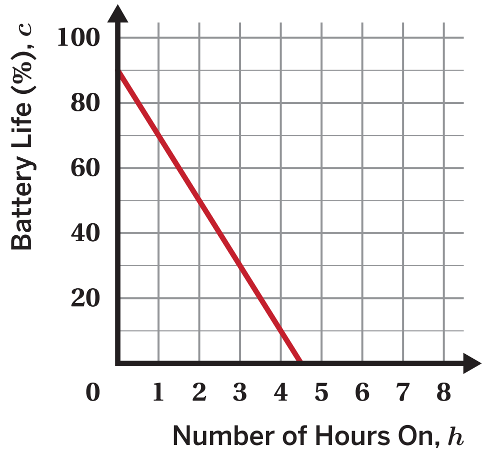


Things to Remember:

Lesson Practice
8.3.05

Name: .... Date: .... Period: ....

Problems 1–4: Tameeka is monitoring her computer's battery life. When the computer is on, the battery loses energy at a constant rate. This graph shows the percentage of battery life remaining, c, after the computer has been on for h hours.



1. What percent of battery life was left when Tameeka turned on the computer?

90%

2. Complete the table.

Number of Hours On	0	1	2	. . .	3.5
Battery Life, %	90	70	**50**	. . .	**20**

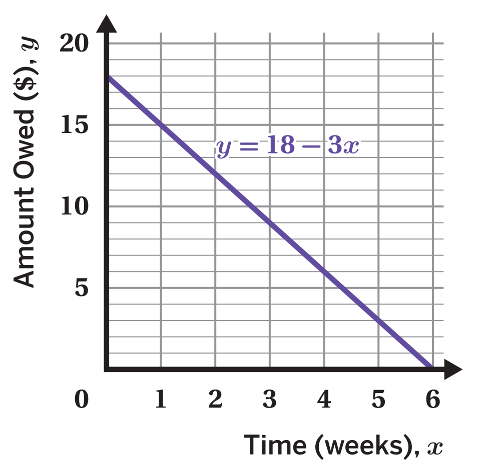
3. Write an equation that represents the percentage of battery life remaining, c, after the computer has been on for h hours.

$c = 90 - 20h$ (or equivalent)

4. After how many hours of being on will Tameeka's computer's battery life run out?

4.5 hours

Problems 5–6: Juliana borrowed some money from her brother. She pays him back by giving him the same amount every week. The graph shows how much she owes him, y, after each week, x.



5. What is the vertical intercept and what does it represent in this situation?

(0, 18). *Responses vary.* It represents the amount of money Juliana borrowed from her brother.

6. What is the horizontal intercept and what does it represent in this situation?

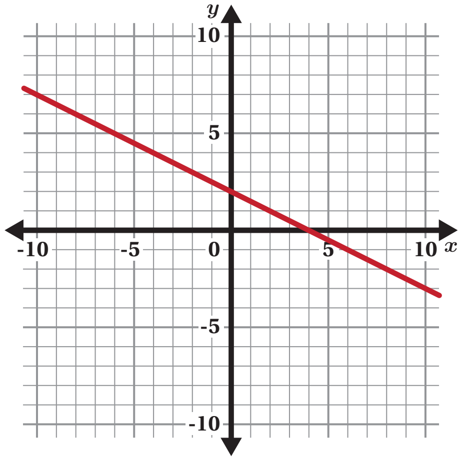
(6, 0). *Responses vary.* It represents the time, in weeks, that it takes for Juliana to pay her brother back.

Lesson Practice
8.3.05

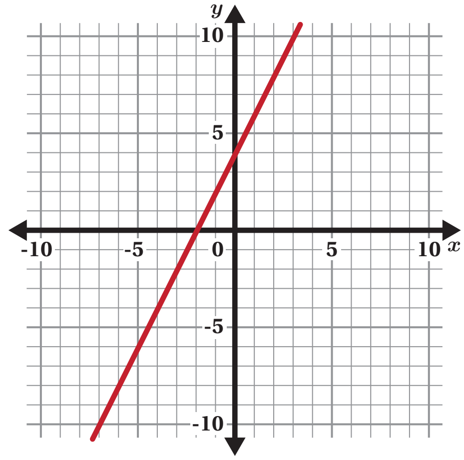
Name: ____________ Date: ________ Period: ________

7. Which graph has a horizontal intercept of $(2, 0)$?

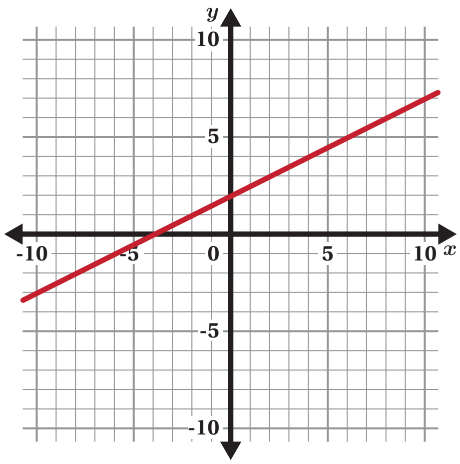
A.



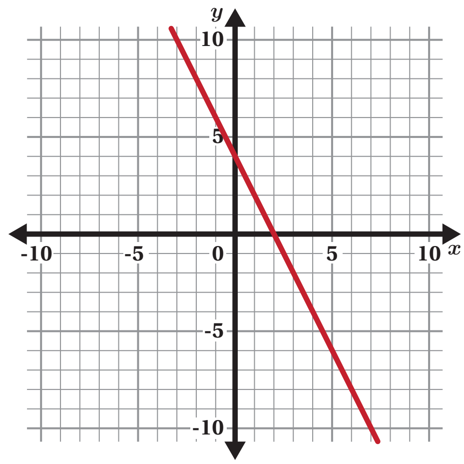
B.



C.

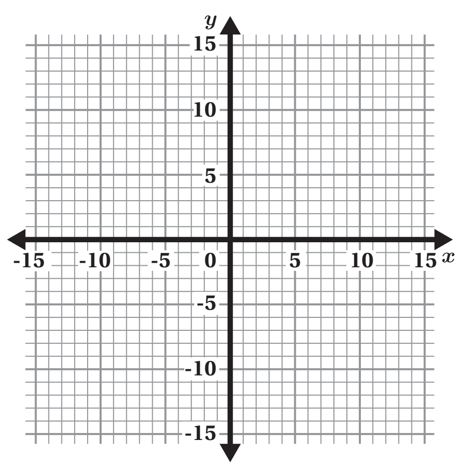


D.



Spiral Review

Problems 8–9: Determine the coordinates of the image after the point is dilated using the origin as the center of dilation and a scale factor of 2.5.



8. $(1, 0)$

$(2.5, 0)$

9. $(-3, 6)$

$(-7.5, 15)$

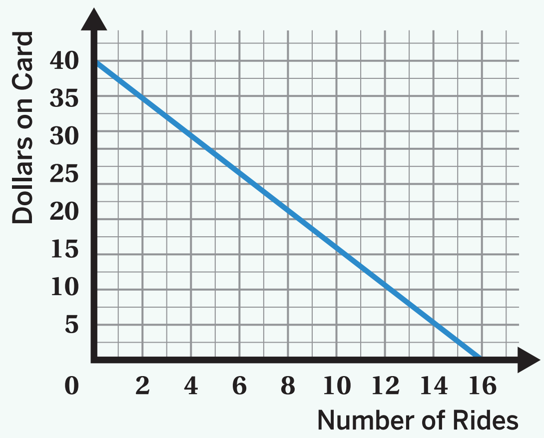
Reflection

1. Circle the problem that was the most challenging for you.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Linear relationships can help us make predictions. We can use the graph or the equation of a linear relationship to determine the value of one variable when we're given the other variable.



Let's say we want to know how much money will be left on a transit card after 10 rides. We can look at a graph like this one and determine the amount of money on the card, y, that corresponds to $x = 10$. In this situation, the slope of the line tells us how much each ride costs.

The y-intercept tells us the amount of money on the card before taking any rides. Unlike a proportional relationship, the graph of this linear relationship doesn't pass through the origin.

Things to Remember:

Name: .... Date: .... Period: ....

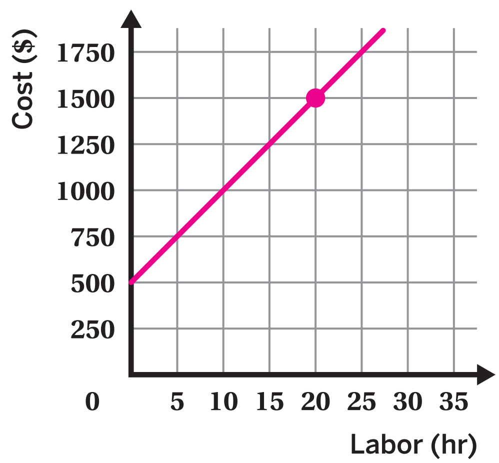
Problems 1–4: To paint a house, a painting company charges a flat rate of $500 for supplies, plus $50 for each hour of labor.

1. How much would the painting company charge to paint a house that needs 20 hours of labor? 50 hours of labor? Write your answers in the table.

Labor (hr)	Cost ($)
20	**1500**
50	**3000**

2. Draw a line representing the relationship between the number of hours of labor needed to paint the house and the total cost.

3. Plot a point to show the cost of 20 hours of labor.



4. What is the slope of the line? What does it represent?

50. *Responses vary.* The slope of the line represents 50 dollars per hour, which is the price per hour that the painting company charges for labor.

5. At a carnival, y represents the amount of money in a cash box after x tickets are purchased for carnival games. The line representing the relationship between x and y has a slope of $\frac{1}{4}$ and a y-intercept of 8. Explain what the slope and y-intercept represent in this situation.

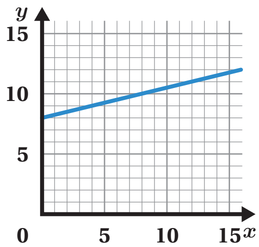
***Responses vary.* The slope represents that each ticket costs $0.25. The y-intercept represents the amount, $8, that's already in the cash box before any tickets are sold.**

Lesson Practice
8.3.07

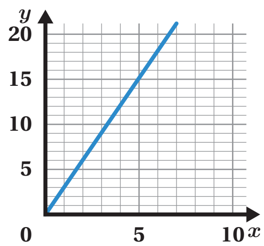
Name: ______ Date: ______ Period: ______

Problems 6–9: For each real-world situation, choose the graph that best represents it.

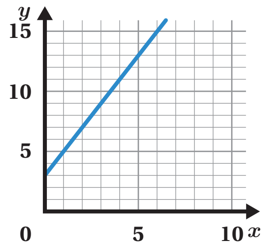
Graph A



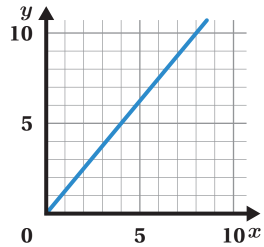
Graph B



Graph C



Graph D



6. y represents the perimeter of an equilateral triangle and x represents its side length. The slope of the line representing the relationship between x and y is 3.
Graph B

7. y represents the amount of money collected after x raffle tickets are purchased. The slope of the line representing the relationship between x and y is 0.25.
Graph A

8. y represents the number of chapters a student has read after x days. The slope of the line representing the relationship between x and y is $\frac{5}{4}$.
Graph D

9. y represents the total cost, in dollars, of x blueberry muffins, including a delivery fee. The slope of the line representing the relationship between x and y is 2.
Graph C

Spiral Review

10. Here is a table showing the values of a linear relationship. Write an equation to represent this relationship.
$y = -5x + 17$ (or equivalent)

x	0	1	2
y	17	12	7

11. Write the equation of a line that has a slope of 2 and a vertical intercept of 8.
$y = 2x + 8$ (or equivalent)

Reflection

1. Put a heart next to the problem you feel most confident about.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

A *translation* of a line that represents a proportional relationship creates a line that is parallel to the pre-image, but changes the location of the vertical intercept, also known as the y-intercept.

The equation $y = mx$ represents a line that passes through the origin. The equation $y = mx + b$ represents a vertical translation of line $y = mx$ by b units.

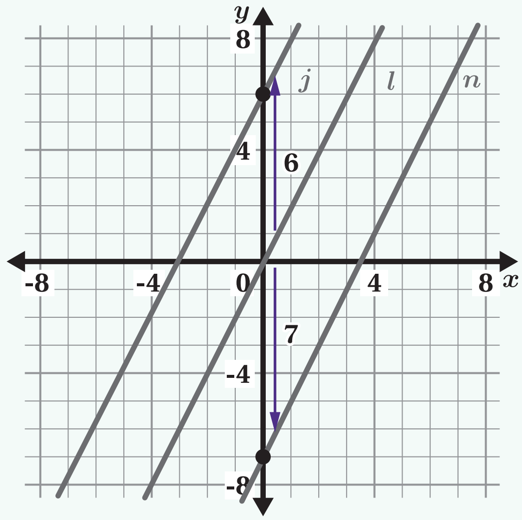
If $b > 0$, the line is translated up.

If $b < 0$, the line is translated down.

For example, the equation of line l is $y = 2x$.

If line l is translated 6 units up to produce line j, the equation of line j is $y = 2x + 6$.

If line l is translated 7 units down to produce line n, the equation of line n is $y = 2x - 7$.



Things to Remember:

Lesson Practice
8.3.08

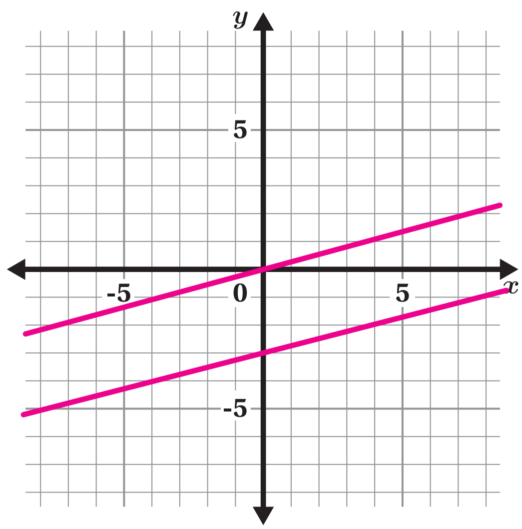
Name: .... Date: .... Period: ....

1. Select *all* the equations whose graphs have the same y-intercept.

- ☑ **A.** $y = 3x - 8$
- ☐ **B.** $y = \text{-}8x + 3$
- ☐ **C.** $y = 3x + 8$
- ☑ **D.** $y = 5x - 8$
- ☑ **E.** $y = 2x - 8$
- ☑ **F.** $y = 13x - 8$

Problems 2–3: Here is a coordinate plane.

2. Graph the equations $y = \frac{1}{4}x$ and $y = \frac{1}{4}x - 3$.



3. How are the graphs alike? How are they different?

***Responses vary.* Both graphs have a slope of $\frac{1}{4}$. The graph of $y = \frac{1}{4}x - 3$ is translated down 3 units from the graph of $y = \frac{1}{4}x$, so the y-intercepts are different.**

Problems 4–5: A cable company charges existing customers $\$70$ per month for cable service. For new customers, there is an additional one-time fee of $\$100$.

4. Write a linear equation representing the relationship between x, the number of months of service, and y, the total amount paid in dollars by a customer.

Existing customer: $y = 70x$

New customer: $y = 70x + 100$

5. When the two equations are graphed on the coordinate plane, how are the graphs similar?

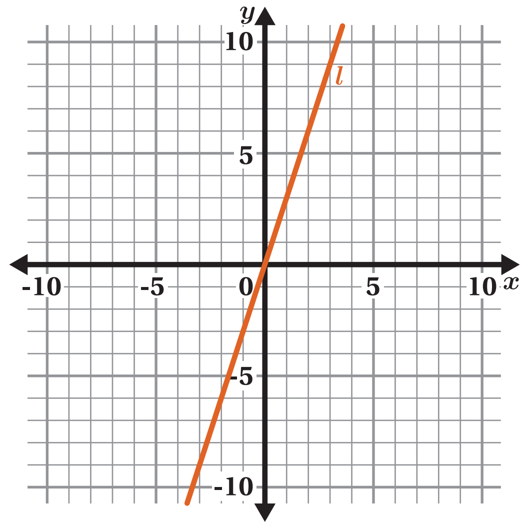
***Responses vary.* They are parallel lines. The graph of $y = 70x + 100$ is translated 100 units up from the graph of $y = 70x$.**

Name: ________ Date: ________ Period: ________

6. Here is the graph of line l. Which equation represents a line that is a translation of l?

A. $y = \frac{1}{3}x - 7$ B. $y = 2x + 5$

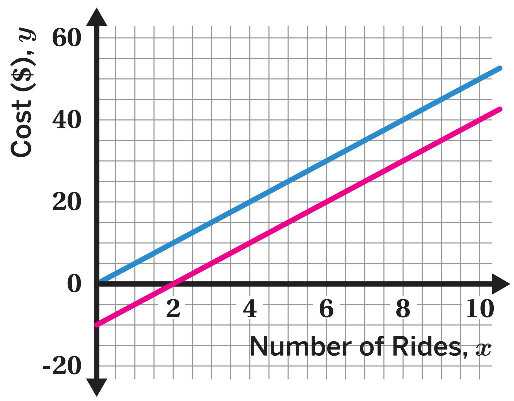
C. $y = 3x - 8$ D. $y = -3x + 1$



Problems 7–8: Jasmine goes to an amusement park. The amusement park charges \$5 for each ride. Here is a graph that represents the cost, y, after x rides.

7. Write an equation that represents the cost, y, after Jasmine goes on x rides.
   $y = 5x$ (or equivalent)

8. Mio goes to the same amusement park and has a coupon that can be used for 2 free rides. On the same coordinate plane, graph the relationship that represents the amount of money, y, that Mio would spend after x rides, if she uses the coupon.
   Response shown on graph.



9. Write an equation for the relationship you graphed in the previous problem.
   $y = 5x - 10$ (or equivalent)

Spiral Review

Problems 9–10: Determine the coordinates of the image after the point is translated 4 units left and 3 units down.

10. $(3, 6)$
    (-1, 3)

11. $(0, 5)$
    (-4, 2)

Reflection

1. Put a smiley face next to the problem you learned from the most.

2. Use this space to ask a question or share something you're proud of.

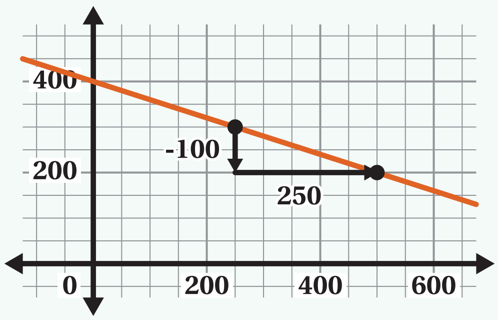
Lesson Summary

You can determine the slope of a line using two points on that line. Lines with positive slopes increase in height from left to right, while lines with negative slopes decrease in height from left to right.

You can use slope triangles to calculate the vertical change and horizontal change between two points on a coordinate plane. You can also calculate the slope by listing the coordinates in a table and then determining the difference between the y-coordinates (the vertical change) and the difference between the x-coordinates (the horizontal change).

The slope is the ratio of the vertical change to the horizontal change.

Using Slope Triangles



Using Coordinates in a Table

	x	y	
+250	250	300	-100
	500	200	

$$\frac{\text{change in } y \text{ (vertical change)}}{\text{change in } x \text{ (horizontal change)}} = \frac{-100}{250} = -\frac{2}{5}$$

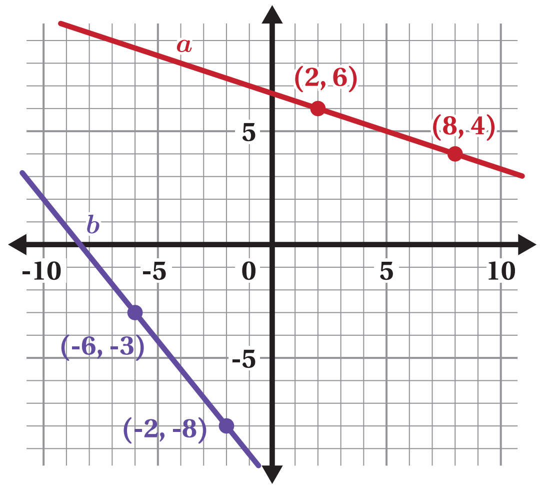
Things to Remember:

Name: .... Date: .... Period: ....

1. Calculate the slope of each line.

line a: $-\frac{1}{3}$ (or equivalent)

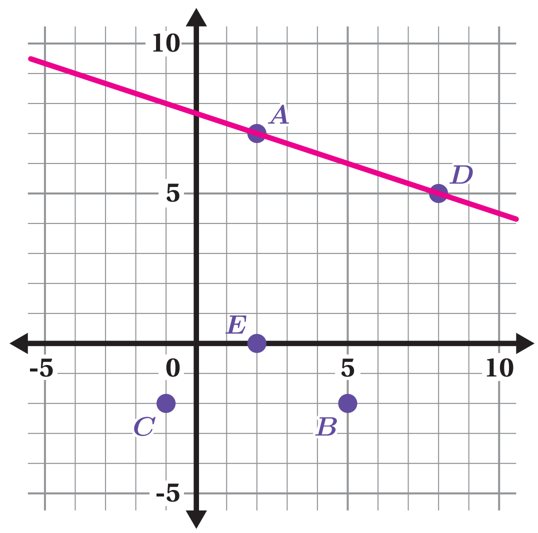
line b: $-\frac{5}{4}$ (or equivalent)



2. Draw a line with a slope of $-\frac{1}{3}$ that passes through point A.

What other point lies on that line?

Point D



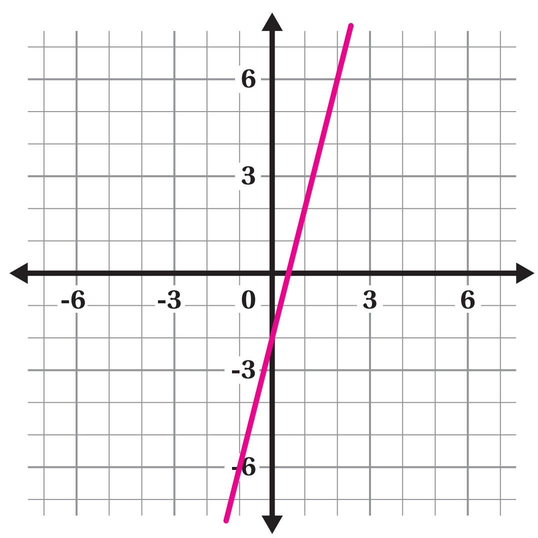
Problems 3–4: Here is a blank graph.

3. Draw a line with a slope of 4 and a negative y-intercept.

***Responses vary.* Sample shown on graph.**

4. Explain how you know the slope of your line is 4.

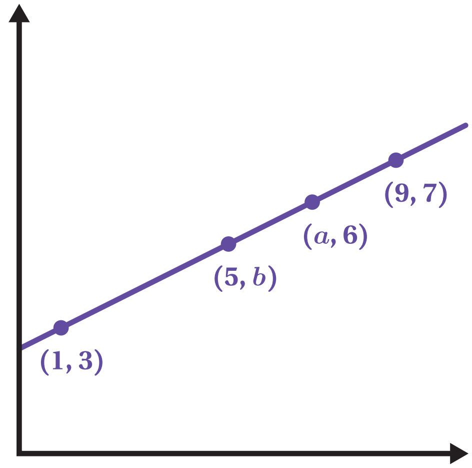
***Responses vary.* I can tell the slope is 4 because from $(0, -2)$ to $(1, 2)$ the change in y is $+4$ and the change in x is $+1$, and $\frac{4}{1} = 4$.**



Lesson Practice
8.3.09

Name: .... Date: .... Period: ....

Problems 5–7: All the points in this graph are on the same line.



5. What is the slope of the line? Explain your thinking.

$\frac{1}{2}$. *Explanations vary.* From $(1, 3)$ to $(9, 7)$, the vertical change is $+4$ and the horizontal change is $+8$, so the slope is $\frac{4}{8} = \frac{1}{2}$.

6. What are the values for a and b?

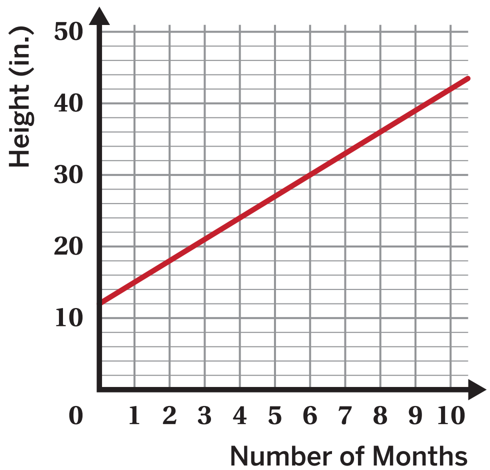
$a = 7, b = 5$

7. What is the x-value when $y = 0$?

$x =$ -5

Spiral Review

Problems 8–9: The graph shows the height of a bamboo plant, h, n months after it has been planted.



8. Write an equation that gives the bamboo's height, h, after n months.

$h = 3n + 12$ (or equivalent)

9. How many inches tall will the bamboo plant be after 18 months? Show or explain your thinking.

66 inches. *Explanations vary.*

$h = 3(18) + 12$

$h = 54 + 12$

$h = 66$

Reflection

1. Circle the problem you enjoyed doing the most.

2. Use this space to ask a question or share something you're proud of.

Science Mom Lesson 62

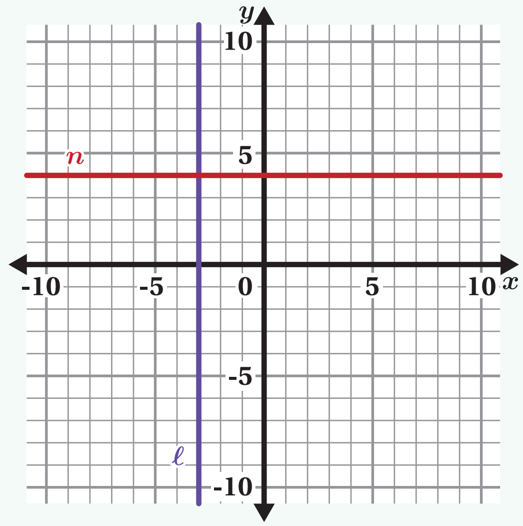
Lesson Summary

On the coordinate plane:

- Horizontal lines represent situations where the y-value is constant and the x-values change. Horizontal lines have a slope of 0.
- Vertical lines represent situations where the x-value is constant and the y-values change. Vertical lines have an *undefined* slope.

For example, the equation $y = 4$ represents the horizontal line n because every point on the line has the same y-coordinate, 4.

The equation $x =$ -3 represents the vertical line ℓ because every point on the line has the same x-coordinate, -3.

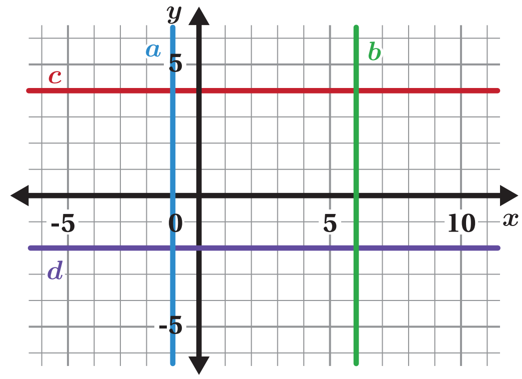


Things to Remember:

Lesson Practice
8.3.10

Name: ____________ Date: ________ Period: ________

Problems 1–4: Here are four lines on a coordinate plane. Write an equation for each line.



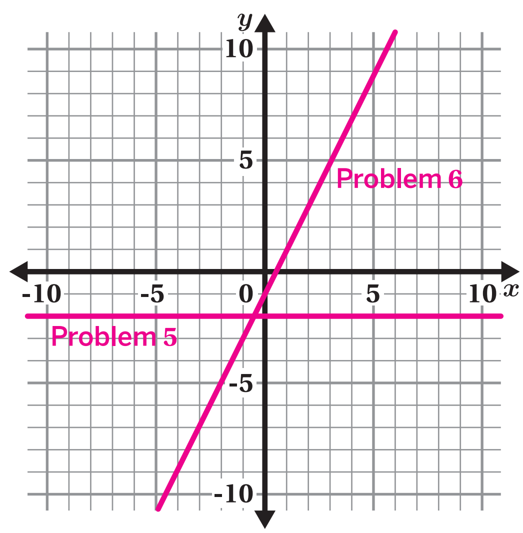
1. line a: $x = -1$

2. line b: $x = 6$

3. line c: $y = 4$

4. line d: $y = -2$

Problems 5–6: Use the coordinate plane to draw lines that have the given slopes and y-intercepts. Then write an equation for each line.



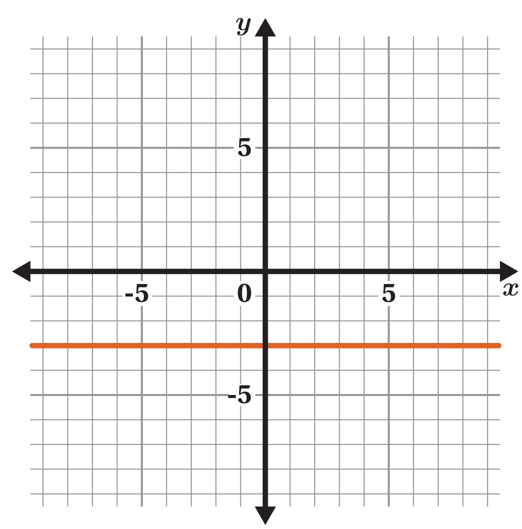
5. Slope: 0
y-intercept: -2

Equation: $y = -2$

6. Slope: 2
y-intercept: -1

Equation: $y = 2x - 1$

7. This graph represents a linear relationship. Choose the equation that matches the line shown on the graph.



A. $x = 3$

B. $x = -3$

C. $y = 3$

D. $y = -3$

Name: ________ Date: ________ Period: ________

8. Write an equation for the line that passes through the points $(4, 3)$ and $(4, 15)$.

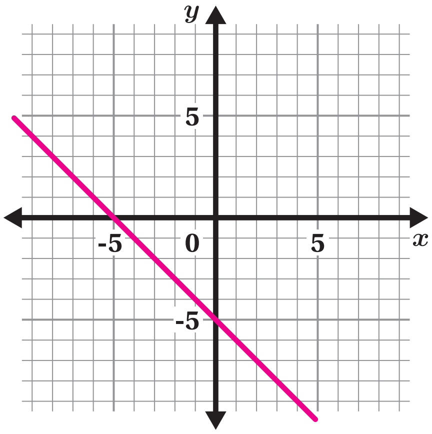
$x = 4$

9. Write an equation for the line that passes through the points $(1, -6)$ and $(-6, -6)$.

$y = -6$

Spiral Review

10. Graph the equation $y = -x - 5$.



11. Select *all* the pairs of points that have lines passing through them with a slope of $\frac{2}{3}$.

- ☐ **A.** $(0, 0)$ and $(2, 3)$
- ☑ **B.** $(0, 0)$ and $(3, 2)$
- ☐ **C.** $(1, 5)$ and $(4, 2)$
- ☑ **D.** $(-2, -2)$ and $(4, 2)$
- ☐ **E.** $(20, 30)$ and $(-20, -30)$

Reflection

1. Put a star next to a problem that looked more difficult than it really was.

2. Use this space to ask a question or share something you're proud of.