

**Lesson Summary**

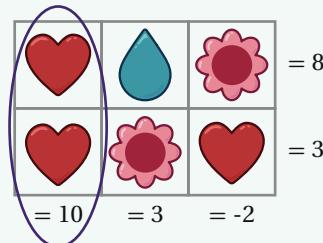
There are many different ways to solve problems and puzzles using math. Let's look at a few strategies for determining the values of shapes in a puzzle.

**Strategy 1: Look for single-shape rows/columns.**

If we know the value of two hearts, we can determine the value of one heart and substitute that value in the other parts of the puzzle.

$$2(\text{Heart}) = 10$$

$$\text{Heart} = 5$$

**Strategy 2: Substitute known shape values to solve for missing values.**

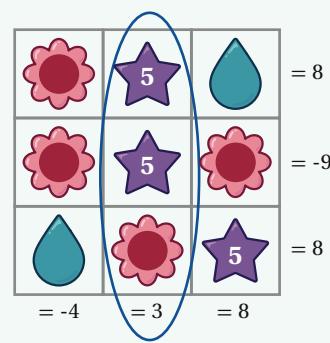
If we know the value of one star, then we can substitute that value in other parts of the puzzle.

$$2(\text{Star}) + \text{Flower} = 3$$

$$2(5) + \text{Flower} = 3$$

$$10 + \text{Flower} = 3$$

$$\text{Flower} = -7$$

**Strategy 3: Look for repeating shape patterns.**

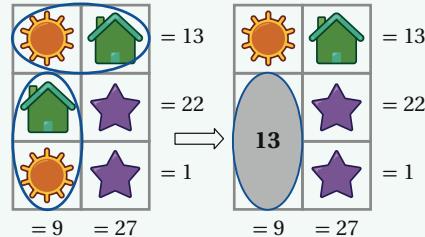
If we know the value of a shape combination and we see it repeat in the puzzle, then we can substitute the value for the shape combination.

$$\text{Sun} + \text{House} = 13$$

$$\text{Sun} + (\text{House} + \text{Sun}) = 9$$

$$\text{Sun} + 13 = 9$$

$$\text{Sun} = -4$$

**Things to Remember:**

# Lesson Practice

A1.5.01

Name: ..... Date: ..... Period: .....

**Problems 1–2:** Here is a shape puzzle. The sum of each row and column is shown.

1. Select all the true statements.

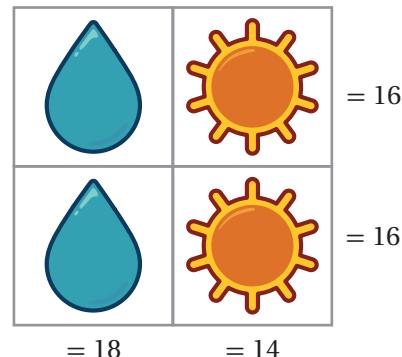
A.  +  = 18

B.  +  = 14

C.  = 14

D.  +  = 18

E.  +  = 16

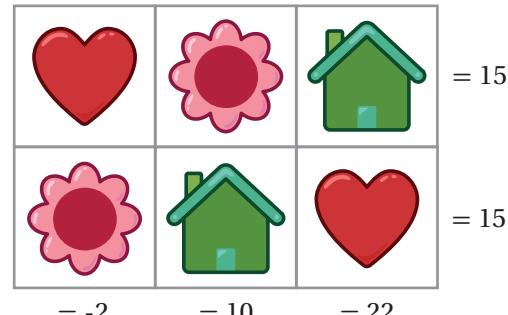


2. Show or explain why this statement is *false*:  = 8.

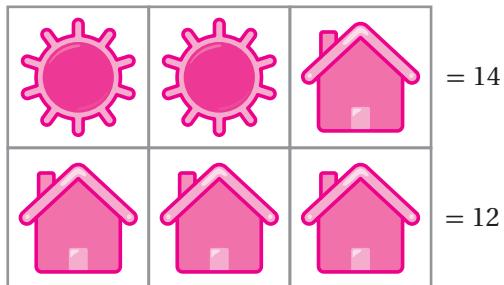
**Responses vary.** If one water drop were equal to 8, then the left column would add up to 16, not 18.

3. Determine the solution for this puzzle.

Shape	Value
Heart	5
Flower	-7
House	17



**Problems 4–5:** Use these two equations:



$$x + x + y = 14$$

$$y + y + y = 12$$

4. Draw a shape puzzle to represent these equations. **Responses vary.**

5. Determine the values of  $x$  and  $y$ .

$x = \underline{\hspace{2cm}} 5 \underline{\hspace{2cm}}$

$y = \underline{\hspace{2cm}} 4 \underline{\hspace{2cm}}$

# Lesson Practice

A1.5.01

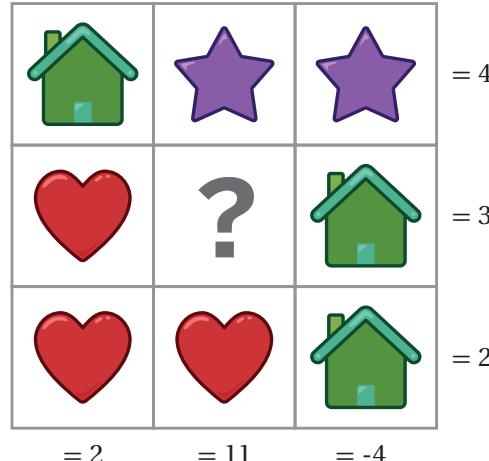
Name: ..... Date: ..... Period: .....

6. Determine the missing shape in the center of this puzzle. Circle your choice.

Heart      House      Star

Show or explain your thinking.

*Explanations vary. Based on the middle and bottom rows, I know the missing shape is one greater than the heart. When I compare the top row to the left column, I can figure out that the star is the shape that is one greater than the heart.*



## Spiral Review

Problems 7–10: Determine the value of the variable that makes each equation true.

7.  $2.5 + (-3) = a$

*a = -0.5*

8.  $12 + b = -9$

*b = -21*

9.  $2c + 3 = 15$

*c = 6*

10.  $3d + 2 = 35$

*d = 11*

## Reflection

1. Put a heart next to the problem you found most interesting.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

A *system of equations* is two or more equations that represent the same constraints using the same variables. The *solution to a system of equations* is the set of all values that makes every equation in the system true. Just like in a shape puzzle where the value of each shape stays the same throughout the puzzle, in a system of equations, the value of the variables stays the same throughout the entire system. There are many solutions to a linear equation in two variables, but there might be only one solution, no solution, or infinitely many solutions to a system of linear equations in two variables.

There are many strategies for determining the solution to a system of equations that makes all equations in a system true. One strategy is called **elimination**, where you add or subtract the equations to produce a new equation with one variable. After finding the value of the first variable, you substitute that value into any original equation to find the value of the second variable. Let's look at some examples.

If the equations in the system share the same *coefficient* with opposite signs on the same variable, you can eliminate a variable by adding. You can solve this system by adding to eliminate the  $y$ -variable.

$$\begin{array}{r} -2x + y = 9 \\ +(8x - y = 3) \\ \hline 6x + 0 = 12 \\ x = 2 \end{array}$$

$$\begin{array}{r} -2(2) + y = 9 \\ y = 13 \end{array}$$

If the equations in the system share the same *coefficient* with the same signs on the same variable, you can eliminate a variable by subtracting. You can solve this system by subtracting to eliminate the  $x$ -variable.

$$\begin{array}{r} x + 2y = 30 \\ -(x + y = 23) \\ \hline y = 7 \end{array}$$

$$\begin{array}{r} x + (7) = 23 \\ x = 16 \end{array}$$

**Things to Remember:**

# Lesson Practice

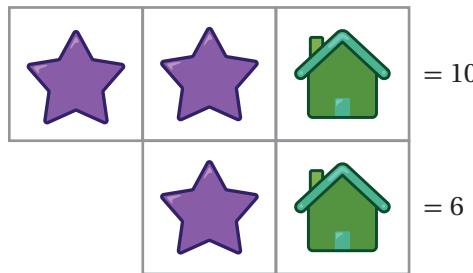
A1.5.02

Name: ..... Date: ..... Period: .....

1. Solve this system of equations. Use the shape puzzle if it helps with your thinking.

$$2x + y = 10$$

$$x + y = 6$$



$$x = \underline{\quad 4 \quad}$$

$$y = \underline{\quad 2 \quad}$$

**Problems 2–3:** Mateo made a mistake while solving this system of equations.

2. Describe one thing Mateo did correctly.

**Responses vary.** Mateo correctly subtracted  $2x - x$  and  $19 - 11$ . Also, trying to eliminate  $y$  is a good way to solve this system of equations.

$$\begin{array}{r} 2x + y = 19 \\ -(x - y = 11) \\ \hline x + 0 = 8 \\ x = 8 \end{array}$$

3. Describe one thing Mateo did incorrectly.

**Responses vary.** Mateo did not subtract correctly. Subtracting one equation from the other won't eliminate  $y$ ; adding the equations will, because the two  $y$ -variables have opposite signs.

**Problems 4–5:** Determine the solution for each system of equations.

4.  $3x + 4y = 6$   
 $3x + 2y = 18$

5.  $5x + 6y = 26$   
 $-5x + 2y = -18$

$$x = \underline{\quad 10 \quad} \quad y = \underline{\quad -6 \quad}$$

$$x = \underline{\quad 4 \quad} \quad y = \underline{\quad 1 \quad}$$

## Lesson Practice

A1.5.02

Name: ..... Date: ..... Period: .....

### Spiral Review

**Problems 6–8:** Write an equivalent expression by combining like terms.

6.  $5a + 3b - 2a$

**$3a + 3b$**

7.  $3(c - 2) + 2c$

**$5c - 6$**

8.  $5d - 2(7d + 3g)$

**$-9d - 6g$**

9. The function  $f(t)$  models a hiker's elevation above or below sea level, in meters,  $t$  hours after noon. Circle the equation that represents this statement:

*At 7 PM, the hiker was 3 meters below sea level.*

- A.  $f(7) = 3$       B.  $f(19) = -3$       C.  **$f(7) = -3$**       D.  $f(-3) = 7$
10. Which line represents  $x - 2y = -6$ ?

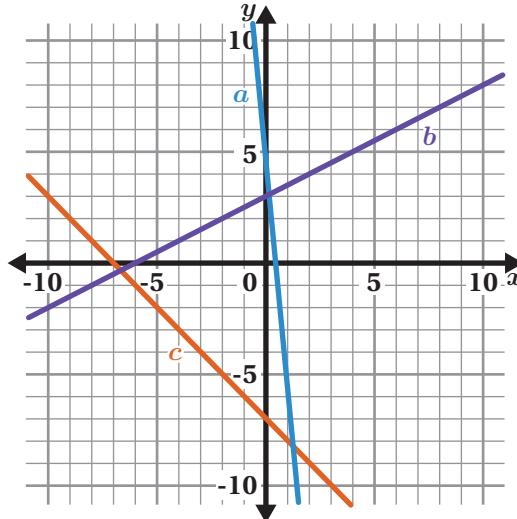
A. Line  $a$

**B.** Line  $b$

C. Line  $c$

Explain your thinking.

*Explanations vary. I found the  $x$ -intercept and  $y$ -intercept of the equation to help me pick the correct line. The  $x$ -intercept is  $-6$  and the  $y$ -intercept is  $3$ , so line  $b$  represents the equation  $x - 2y = -6$ .*



### Reflection

1. Put a star next to the problem you spent the most time on.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

It can be helpful to write *equivalent equations* when using elimination to solve systems of equations. You can create equivalent equations by multiplying each term of the first or second equation by a number. Your goal is to end up with a system of equations where one variable has the same or opposite coefficients so you can add or subtract them to eliminate a variable.

Here is a system of equations:

$$9x - 4y = 2$$

$$3x + y = 10$$

You can multiply the second equation by -3 to eliminate the  $x$ -variables.

$$\begin{array}{r} 9x - 4y = 2 \\ -3(3x + y = 10) \\ \hline 9x - 4y = 2 \\ -9x - 3y = -30 \\ \hline 0 - 7y = -28 \\ y = 4 \end{array}$$

$$\begin{array}{r} 3x + (4) = 10 \\ 3x = 6 \\ x = 2 \end{array}$$

Or you can multiply the second equation by 4 to eliminate the  $y$ -variables.

$$\begin{array}{r} 9x - 4y = 2 \\ 4(3x + y = 10) \\ \hline 9x - 4y = 2 \\ + 12x + 4y = 40 \\ \hline 21x + 0 = 42 \\ x = 2 \end{array}$$

$$\begin{array}{r} 9(2) - 4y = 2 \\ 18 - 4y = 2 \\ -4y = -16 \\ y = 4 \end{array}$$

**Things to Remember:**

# Lesson Practice

A1.5.03

Name: ..... Date: ..... Period: .....

1. Select *all* expressions that are equivalent to  $6x + 8y = 10$ .

A.  $12x + 16y = 20$

B.  $2x + 6y = 10$

C.  $3x + 4y = 5$

D.  $1.5x + 2y = 2.5$

E.  $4x + 5y = 4$

2. Arnav and Omari are solving this system of equations.

They disagree about what the first step should be  
to eliminate a variable.

$$4x + 2y = 62$$

$$-8x - y = 59$$

**Arnav's strategy:** Multiply  $4x + 2y = 62$  by 2

**Omari's strategy:** Multiply  $-8x - y = 59$  by 2

Whose strategy will eliminate a variable once the equations are added? Circle your choice.

Arnav's

Omari's

Both

Neither

Explain your thinking.

*Explanations vary. Both strategies will create a term that can be eliminated by adding the equations together. Arnav's strategy will create  $8x + (-8x)$  to eliminate  $x$ . Omari's strategy will create  $2y - (2y)$  to eliminate  $y$ .*

**Problems 3–4:** Determine the solution to each system of equations.

3.  $2x - 4y = 10$   
 $x + 5y = 40$

4.  $5x + 2y = 20$   
 $2x - 3y = -11$

$x = \underline{\hspace{2cm}} 15 \underline{\hspace{2cm}}$        $y = \underline{\hspace{2cm}} 5 \underline{\hspace{2cm}}$

$x = \underline{\hspace{2cm}} 2 \underline{\hspace{2cm}}$        $y = \underline{\hspace{2cm}} 5 \underline{\hspace{2cm}}$

# Lesson Practice

A1.5.03

Name: ..... Date: ..... Period: .....

5. Using the digits 0–9, without repeating, fill in each blank to create two equivalent equations.

$$\boxed{\phantom{0}}x + \boxed{\phantom{0}}y = \boxed{\phantom{0}}$$

$$\boxed{\phantom{0}}x + \boxed{\phantom{0}}y = \boxed{\phantom{0}}$$

*Responses vary.*

$$4x + 3y = 1, 8x + 6y = 2$$

## Spiral Review

Problems 6–8: Solve each equation.

6.  $x - 2 = 5$

$$x = 7$$

7.  $3(x - 2) = 3 \cdot 5$

$$x = 7$$

8.  $8x - 16 = 40$

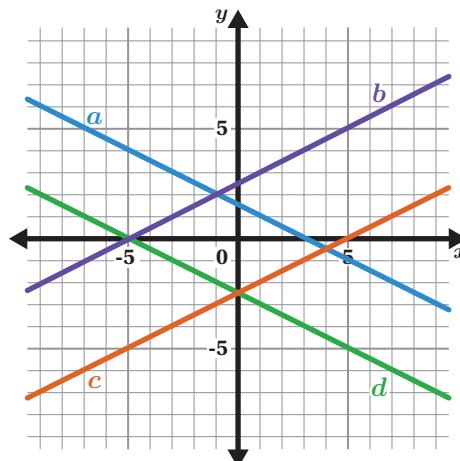
$$x = 7$$

9. Solve the equation  $3x - 9y = 72$  for  $y$ .

$$y = \frac{1}{3}x - 8$$

10. Which line represents  $5x + 10y = 15$ ?

- A. Line  $a$
- B. Line  $b$
- C. Line  $c$
- D. Line  $d$



## Reflection

1. Put a question mark next to a response you'd like to compare with a classmate's.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

One strategy you can use to solve a system of equations is **substitution**, where you replace a variable with an equivalent expression. Substitution is a useful strategy when one variable is already isolated in at least one equation in the system.

Here are two examples of systems of equations where substitution may be a useful strategy. Notice that once you substitute, your new equation only has one variable.

In this system, both  $y$ -variables are already isolated. You can substitute the expression  $-4x + 6$  in for  $y$  in the second equation. The new equation sets both expressions equal to each other.

$$\begin{aligned} y &= -4x + 6 \\ y &= 3x - 15 \\ y = -4x + 6 &\quad y = 3x - 15 \\ -4x + 6 &= 3x - 15 \\ -7x &= -21 \\ x &= 3 \\ y = 3(3) - 15 & \\ y &= -6 \end{aligned}$$

In this system of equations,  $y$  is already isolated, so you can substitute the expression  $2x - 5$  in for  $y$  in the first equation.

$$\begin{aligned} -3x - 2y &= 3 \\ y &= 2x - 5 \\ y = 2x - 5 &\quad -3x - 2y = 3 \\ -3x - 2(2x - 5) &= 3 \\ -3x - 4x + 10 &= 3 \\ -7x + 10 &= 3 \\ -7x &= -7 \\ x &= 1 \\ y = 2(1) - 5 & \\ y &= -3 \end{aligned}$$

## Things to Remember:

## Lesson Practice

A1.5.04

Name: ..... Date: ..... Period: .....

**Problems 1–2:** Show or explain what your first step would be for solving each system of equations.

1.  $4x - y = 20$

$$x + y = 5$$

**Responses vary.** I would add the equations to eliminate  $y$ . I would get a new equation with just one variable:  $5x = 25$ .

2.  $6x - 12y = 24$

$$y = 2x - 1$$

**Responses vary.** I would substitute the second equation into the first equation. I would have a new equation with just one variable:  $6x - 12(2x - 1) = 24$ .

3. Determine the solution to this system of equations:

$$7x - y = -3$$

$$y = x - 3$$

$$x = -1 \quad y = -4$$

**Problems 4–5:** Alma made a mistake as she started to solve this system of equations.

$$y = \frac{1}{2}x - 1$$

$$4x - 2y = 11$$

4. What is the error in Alma's work?

**Responses vary.** The error is in the second line of Alma's work, where she is distributing the  $-2$ . The operation in front of the  $2$  should be  $+$  instead of  $-$ .

Alma

$$\begin{aligned} 4x - 2\left(\frac{1}{2}x - 1\right) &= 11 \\ 4x - x - 2 &= 11 \\ 3x - 2 &= 11 \\ 3x &= 13 \\ x &= \frac{13}{3} \end{aligned}$$

5. Determine the solution to this system of equations.

$$x = 3 \quad y = 0.5$$

# Lesson Practice

A1.5.04

Name: ..... Date: ..... Period: .....

## Spiral Review

**Problems 6–7:** Kadeem made a mistake as he started to solve this system of equations.

6. Show or explain one thing Kadeem did correctly.

*Responses vary.* He multiplied the second equation by 5, which makes it so that both equations have a 5x.

**Kadeem**

$$\begin{array}{r} 5x - 4y = 6 \\ 5(x + y = 25) \\ \hline 5x - 4y = 6 \\ 5x + 5y = 125 \\ \hline -1y = 131 \\ y = -131 \end{array}$$

7. Show or explain Kadeem's mistake.

*Responses vary.* It isn't clear if Kadeem is adding or subtracting the two equations. He subtracted some terms but added others.

**Problems 8–10:** Solve each equation for the given variable.

8. Solve for  $k$ .

$$2t + k = 6$$

$$k = 6 - 2t \quad (\text{or equivalent})$$

9. Solve for  $x$ .

$$4x + 3y = 12$$

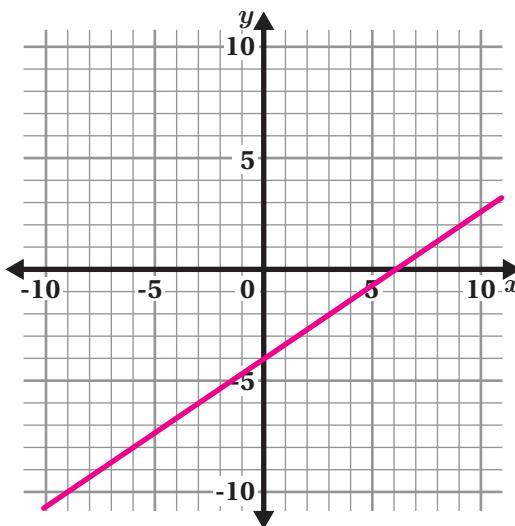
$$x = 3 - \frac{3}{4}y \quad (\text{or equivalent})$$

10. Solve for  $y$ .

$$4x + 3y = 12$$

$$y = \frac{4}{3}x + 4 \quad (\text{or equivalent})$$

11. Graph the equation  $4x - 6y = 24$ .



## Reflection

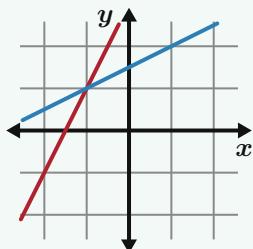
- Put a heart next to the problem you're most proud of.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

On a coordinate plane, you can see the solution of a system of equations at the point(s) where the two lines intersect. A system of linear equations can have:

**One Solution**

The lines intersect at  $(-2, 2)$ .

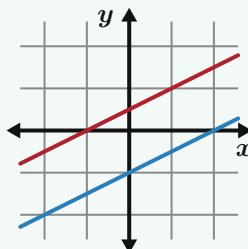


$$\begin{aligned}y &= 2x + 6 \\y &= \frac{1}{2}x + 3\end{aligned}$$

The equations have *different slopes*.

**No Solution**

The lines are *parallel*.

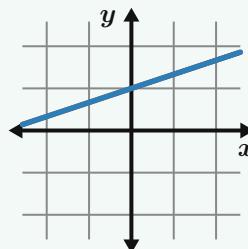


$$\begin{aligned}y &= \frac{1}{2}x + 1 \\y &= \frac{1}{2}x - 2\end{aligned}$$

The equations have the same slope and different *y*-intercepts.

**Infinitely Many Solutions**

The lines are the same.



$$\begin{aligned}y &= \frac{1}{3}x + 2 \\3y - x &= 6\end{aligned}$$

The equations are equivalent.

**Things to Remember:**

# Lesson Practice

A1.5.05

Name: ..... Date: ..... Period: .....

1. The point  $(-2, 2)$  is on the line  $y = x + 4$ . Explain how you can determine if this point is the solution to this system of equations:

$$y = x + 4$$

$$y = 2x - 1$$

**Explanations vary.** You need to test if the point  $(-2, 2)$  is also on the line  $y = 2x - 1$ . You can either substitute the point into the equation or check if it is on the graph of the line.

2. Solve this system of equations. Write the solution as a coordinate pair.

$$y = -\frac{1}{2}x - 8$$

$$y = 3x + 6$$

**(-4, -6)**

3. Match each system of equations to the number of solutions it has.

a.  $y = -2x + 1$  ..... b. No solution

$$2y = -4x + 2$$

b.  $y = -2x + 1$  ..... c. One solution

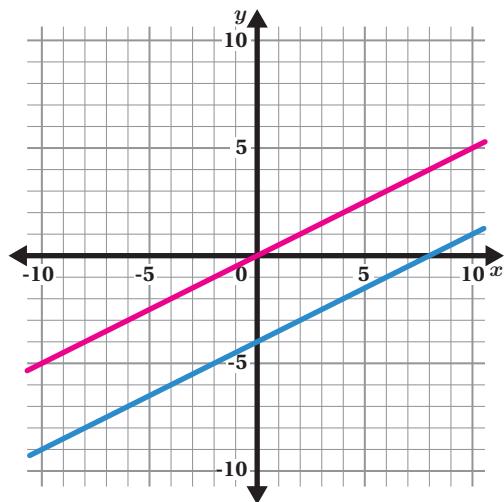
$$y = -2x + 4$$

c.  $y = -2x + 1$  ..... a. Infinitely many solutions

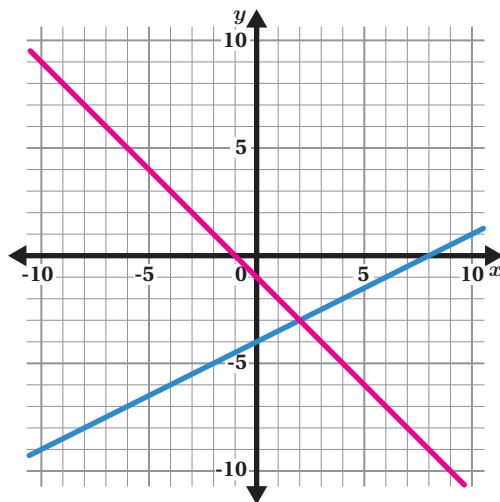
$$y = 2x + 1$$

**Problems 4–5:** Here is a graph of  $y = \frac{1}{2}x - 4$ . Graph a second line to make this system of equations have:

4. No solution. **Lines vary.** Line should have a slope of  $\frac{1}{2}$  and a  $y$ -intercept different than  $-4$ .



5. One solution at  $(2, -3)$ . **Lines vary.** Line should include the point  $(2, -3)$ .



6. Here is one equation in a system of equations:  $y = 3x - 2$ . Write a different second equation so that the system of equations has infinite solutions.

**Equations vary.** Equation should have a slope of 3 and  $y$ -intercept of -2.

# Lesson Practice

A1.5.05

Name: ..... Date: ..... Period: .....

7. A system of linear equations has no solution. Select *all* of the statements that must be true about the equations in this system.

- A. The equations have different slopes.  
 B. The equations have the same slope.  
 C. The equations have different  $y$ -intercepts.  
 D. The equations have the same  $y$ -intercepts.

## Spiral Review

8. Here is a shape puzzle. What is the value of each shape?

Shape	Value
Star	3
House	-5
Flower	10

		= -2
		= 13
		= 20
= 23	= 8	

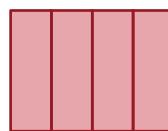
9. Solve this inequality:  $3(x - 3) > 2x - 6$ .

$$x > 3$$

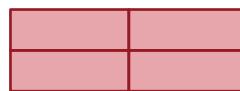
10. Here are two arrangements of identical rectangles. Determine the dimensions  $a$  and  $b$ .

$$a = \underline{\quad 4 \quad} \text{ units}$$

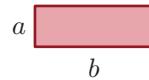
$$b = \underline{\quad 12 \quad} \text{ units}$$



Perimeter: 56 units



Perimeter: 64 units



## Reflection

- Put a star next to a problem that looked more difficult than it really was.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can solve systems of equations symbolically using either substitution or elimination. Looking for specific structures in the equations can help you decide which strategy to use.

- It may be helpful to use substitution when at least one of the equations has an isolated variable or at least one is in *slope-intercept form*.
- It may be helpful to use elimination when both equations are in the same form or if the equations have a pair of same or opposite terms.

When solving a system of equations symbolically, sometimes all of the variables are eliminated.

**No Solution**

When the result of eliminating all of the variables is a false statement, there is *no solution* to the system of equations. This means the lines are parallel and will never intersect.

$$\begin{array}{l} y = 3x + 6 \\ \quad y = 3x - 6 \\ 3x + 6 = 3x - 6 \\ 3x + 12 = 3x \\ 12 = 0 \end{array}$$

**Infinitely Many Solutions**

When the result of eliminating all of the variables is a true statement, there are *infinitely many solutions* to the system of equations. The equations are equivalent and represent the same line.

$$\begin{array}{r} 2 \cdot (2x + 4y = 6) \\ -4x - 8y = -12 \\ 4x + 8y = 12 \\ + -4x - 8y = -12 \\ \hline 0 + 0 = 0 \\ 0 = 0 \end{array}$$

**Things to Remember:**

## Lesson Practice

A1.5.08

Name: ..... Date: ..... Period: .....

**Problems 1–3:** Show or explain what your *first step* would be to solve each system of equations.

**Responses vary.**

1.  $6x + 21y = 103$

$$-6x + 23y = 51$$

Add the equations together to eliminate  $x$ .

2.  $2x + y = 10$

$$y = 6$$

Substitute 6 for  $y$  in the first equation.

3.  $y = \frac{2}{3}x + 7$

$$y = \frac{2}{3}x - 3$$

Notice that the lines have the same slope and then inspect the  $y$ -intercept.

**Problems 4–5:** Determine how many solutions each equation has. If there is one solution, what is the solution?

4.  $5x + 2y = 29$

$$-58 = -10x - 4y$$

No Solution

One Solution

Infinitely many solutions

5.  $2x + 3y = 2$

$$x = 4y + 12$$

No Solution

One Solution

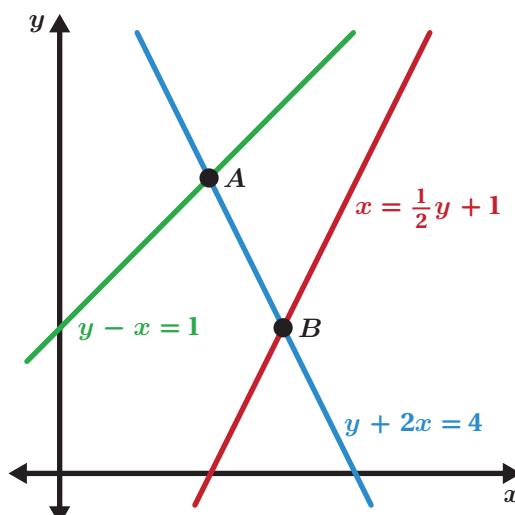
Infinitely many solutions

(4, -2)

6. Determine the coordinates of points  $A$  and  $B$ , the intersections of the lines on the graph.

$A: \underline{\hspace{2cm}} (1, 2) \underline{\hspace{2cm}}$

$B: \underline{\hspace{2cm}} (1.5, 1) \underline{\hspace{2cm}}$



## Lesson Practice

### A1.5.08

Name: ..... Date: ..... Period: .....

7. Here is a system of equations.

$$\begin{aligned}y &= 6x - 7 \\3y &= 6x + 15\end{aligned}$$

Juliana claims that this system has no solutions. Do you agree with Juliana's claim? Explain your thinking.

No. *Explanations vary.* When you rewrite the second equation in slope-intercept form, you get  $y = 2x + 5$ . The slopes of the two equations are different, 6 and 2, so the lines will intersect, giving one solution.

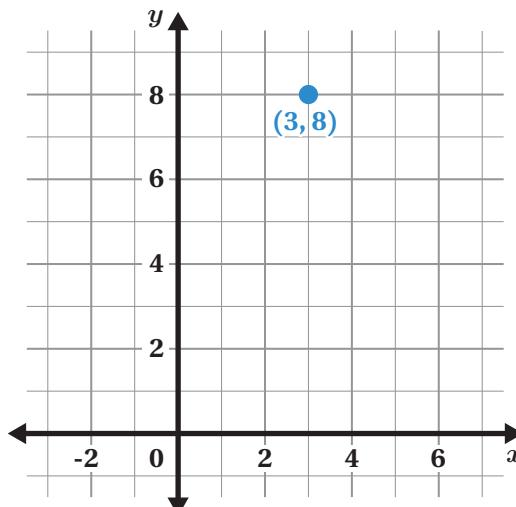
8. Write a system of equations where  $(3, 8)$  is the solution.

Use the graph if it helps your thinking.

*Equations vary.*

Equation 1:  $y = x + 5$

Equation 2:  $y = 2x + 2$



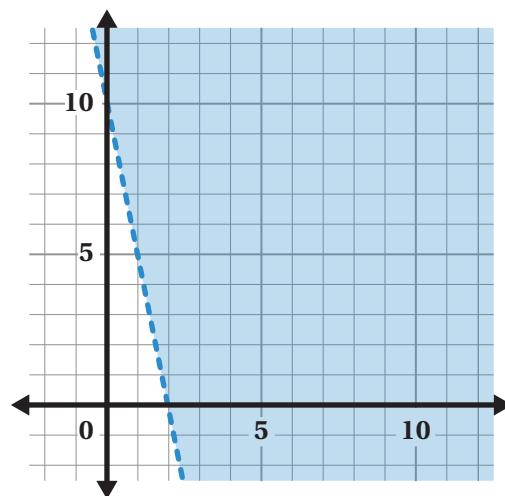
## Spiral Review

9. Select all the ordered pairs that are solutions to the inequality  $6y < 30 - 5x$ .

A.  $(0, 0)$      B.  $(6, 3)$      C.  $(0, 5)$      D.  $(4, 1)$      E.  $(-5, 0)$

10. Which inequality represents this graph?

- A.  $5x + y < 10$   
B.  $5x + y \leq 10$   
 C.  $5x + y > 10$   
D.  $5x + y \geq 10$



## Reflection

- Circle the problem that was the most challenging for you.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

A **system of inequalities** is a system of two or more inequalities that represent the *constraints* on a shared set of variables.

You can use different strategies to determine if a point is a solution to a system of inequalities.

- If the point is in the shaded region for both inequalities, then it is a solution to the system.
- If the  $x$ - and  $y$ -values of the point are substituted into both inequalities and the inequalities are true, then the point is a solution to the system.

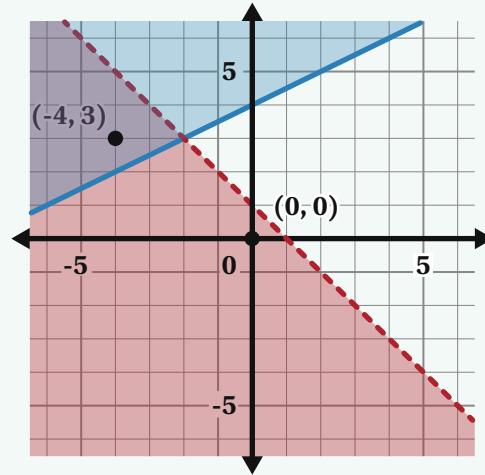
Here is a graph for this system of inequalities.

$$x + y < 1$$

$$y \geq \frac{1}{2}x + 4$$

You can see that the point  $(-4, 3)$  is a solution because it is in the shaded region for both inequalities.

You can also substitute points into both inequalities to determine if they are solutions.  $(0, 0)$  is not a solution and  $(-4, 3)$  is a solution.



## Things to Remember:

# Lesson Practice

A1.5.09

Name: ..... Date: ..... Period: .....

1. The graph shows this system of inequalities.

$$x + y \geq 5$$

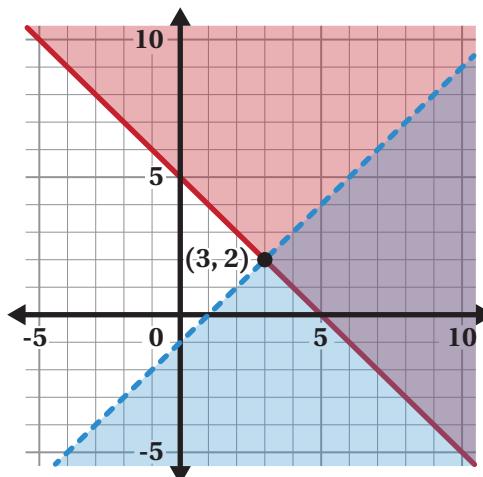
$$x - y > 1$$

Is the point  $(3, 2)$  a solution to the system?

No.

Explain your thinking.

*Explanations vary. The point  $(3, 2)$  satisfies the first inequality, but not the second one.*



**Problems 2–4:** It costs Lukas \$5.00 to mail a package. Lukas has postcard stamps,  $p$ , that are worth \$0.34 each and first-class stamps,  $f$ , that are worth \$0.49 each.

2. Lukas wrote the inequality  $0.34p + 0.49f \geq 5$ . What does this inequality represent?

*Responses vary. The inequality represents the combinations of postcard stamps and first-class stamps that add up to a total value \$5 or more.*

3. Lukas wrote another inequality:  $p + f \leq 12$ . What does this inequality represent?

*Responses vary. The inequality represents the combinations of postcard stamps and first-class stamps that add up to a total of 12 stamps or fewer.*

4. If Lukas uses 1 postcard stamp and 9 first-class stamps, will this satisfy both constraints? Explain your thinking.

*No. Explanations vary. 1 postcard stamp and 9 first-class stamps would satisfy the second constraint about having 12 or fewer stamps. But  $0.34(1) + 0.49(9) = 4.75$ , which is less than \$5.*

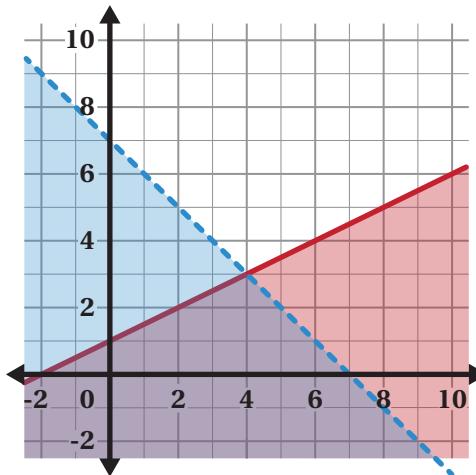
5. Using the digits 0–9 without repeating, fill in each blank such that each statement is true:

- $A$  is a solution to both inequalities.
- $B$  is a solution to only one inequality.
- $C$  is a solution to only the other inequality.
- $D$  is not a solution to either inequality.

*Responses vary.*

$$A = (\boxed{4}, \boxed{2}) \quad B = (\boxed{0}, \boxed{3})$$

$$C = (\boxed{9}, \boxed{5}) \quad D = (\boxed{8}, \boxed{6})$$



# Lesson Practice

A1.5.09

Name: ..... Date: ..... Period: .....

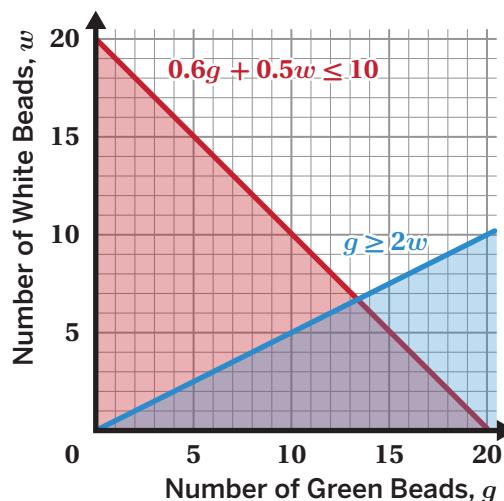
**Problems 6–7:** Arjun is making a bracelet.

He has \$10 to spend on beads.

Green beads cost \$0.60 each and white beads cost \$0.50 each.

His bracelet design needs at least twice as many green beads as white beads.

The graph shows the system of inequalities that represents this situation.



6. What is a combination of green and white beads that meets both constraints?

**Responses vary.** 10 green beads and 4 white beads

7. What is a combination of green and white beads that meets *only* one constraint?

**Responses vary.** 5 green beads and 5 white beads

## Spiral Review

8. Solve this system of equations. Write your solution as a coordinate pair.

$$2x + y = 8$$

$$y = 2x + 4$$

(1, 6)

9. Guiying is at the market with \$14 to buy fruit. Guiying decides to buy apples and grapes. Apples,  $a$ , cost \$1.67 per pound and grapes,  $g$ , cost \$1.87 per pound. Write an inequality to represent this situation.

**1.67a + 1.87g ≤ 14 (or equivalent)**

10. Rio claims that (5, 10) is *not* a solution to the inequality  $y < 3x - 5$ . Do you agree with Rio's claim? Explain your thinking.

**Yes. Explanations vary.** If you substitute the values into the inequality you get  $10 < 3(5) - 5$ . This simplifies to  $10 < 10$  which is not a true statement.

## Reflection

- Put a star next to the problem you understood best.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

The **solutions to a system of inequalities** are all the points that make both inequalities true. The solutions can be seen in the region that is shaded by all inequalities in the system, called the **solution region**.

One strategy for determining the location of the solution region is to test a point. Choose a point that is not on either *boundary line*, substitute the  $x$ - and  $y$ -values into each inequality to see if it makes the statement true, and shade based on the results of the test.

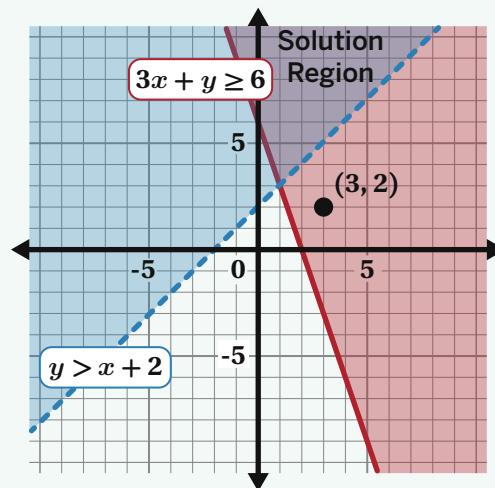
Let's look at this system of inequalities:

$$\begin{aligned}3x + y &\geq 6 \\y &> x + 2\end{aligned}$$

You can test the point  $(3, 2)$  to help determine the solution region.

<b>Solid Line</b> $3x + y \geq 6$ $3(3) + 2 \geq 6$ $11 \geq 6$ <b>True ✓</b>	<b>Dashed Line</b> $y > x + 2$ $2 > 3 + 2$ $2 > 5$ <b>False X</b>
---	---

**Shade the side of the solid line that includes  $(3, 2)$**



**Shade the side of the dashed line that does not include  $(3, 2)$**

## Things to Remember:

# Lesson Practice

A1.5.10

Name: ..... Date: ..... Period: .....

**Problems 1–2:** Here is the graph of a system of inequalities.

$$y > -x + 2$$

$$3x + y \geq 1$$

1. Which letter represents the solution region to the system of inequalities?  
Circle one:

A

B

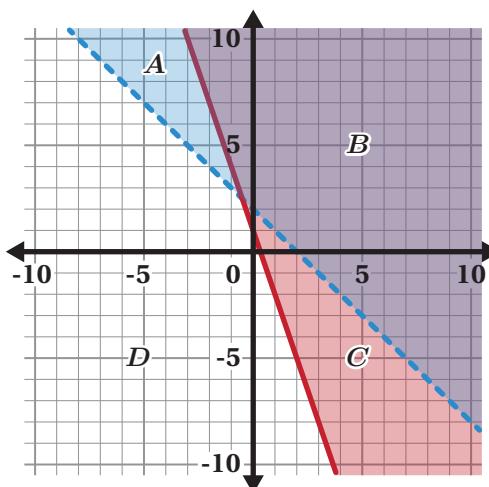
C

D

2. Is the point  $(5, -4)$  a solution to the system?  
Circle one:

Yes

No



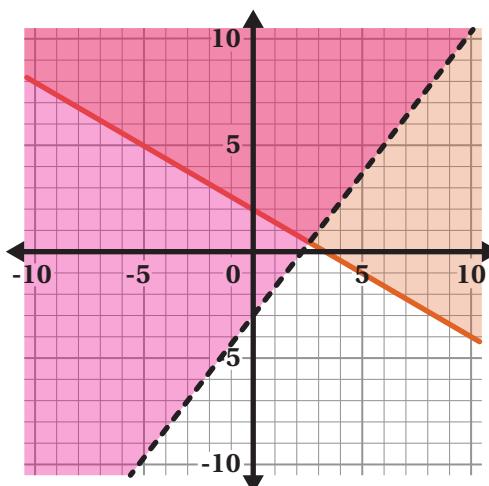
**Problems 3–4:** Javier graphed the first inequality and the boundary line of the second inequality.

$$3x + 5y \geq 10$$

$$4x - 3y < 9$$

3. Complete the graph of the second inequality.  
4. Explain how you knew where to shade the second inequality.

**Responses vary.** I plugged  $(0, 0)$  into  $4x - 3y < 9$ . Since  $(0, 0)$  makes the inequality true, that side of the boundary line should be shaded.



**Problems 5–6:** Here is a system of inequalities that has no solutions.

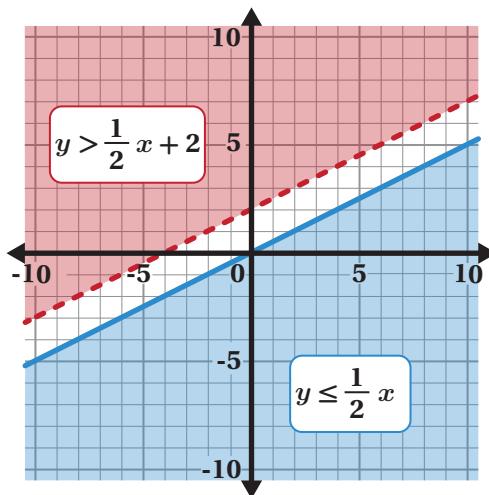
5. Explain how you know the system has no solutions.

**Responses vary.** The system has no solutions because the regions of each inequality do not overlap. Since the boundary lines are parallel, they won't intersect.

6. Describe one change you could make to the system of inequalities that would have solutions.

**Responses vary.**

- Change the direction of the shading of one or both inequalities by changing the inequality symbol.
- Change the slope of one of the inequalities.



## Lesson Practice

A1.5.10

Name: ..... Date: ..... Period: .....

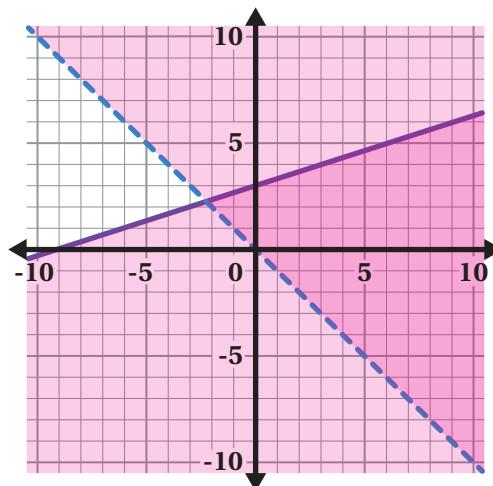
**Problems 7–8:** Nyanna graphed the boundary lines of this system of inequalities:

$$\begin{aligned}x + y &> 0 \\ -x + 3y &\leq 9\end{aligned}$$

7. Complete the graph of the system of inequalities.

8. Identify a coordinate pair that is in the solution region.

**Responses vary.** (2, 1)



**Problems 9–10:** Fill in each blank with an inequality symbol such that:

9. The system has no solutions.

$$x - y \boxed{<} 0$$

$$x - y \boxed{>} 0$$

10. Only points with matching  $x$ - and  $y$ -coordinates are a solution to the system.

$$x - y \boxed{\leq} 0$$

$$x - y \boxed{\geq} 0$$

## Spiral Review

**Problems 11–13:** Determine the value of each expression.

11.  $\frac{4}{6} \cdot 8$

$\frac{16}{3}$  (or equivalent)

12.  $\frac{3}{5} \cdot 2 \cdot \frac{10}{9}$

$\frac{4}{3}$  (or equivalent)

13.  $\frac{3}{6} \cdot 3 \cdot \frac{6}{9} \cdot 2$

2 (or equivalent)

## Reflection

- Circle the problem you feel least confident about.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

Situations that include repeated multiplication can be modeled using an equation of the form  $y = a \cdot b^x$ , where  $a$  represents the initial value and  $b$  represents the base. The base determines whether the quantity in the situation will increase or decrease over time. You can use the equation to solve problems about the situation.

For example, Carlos bought a new mega-growing fish. The fish weighed 4 grams when Carlos bought it, and its mass grows 1.5 times greater every hour.

He wrote the equation  $m = 4 \cdot 1.5^t$  to represent the relationship, where:

- $m$  represents the mass of Carlos's fish in grams.
- $t$  represents the time in hours.

You can use the equation to determine the mass of the fish after 4 hours by substituting  $t = 4$  and solving for  $m$ .

$$m = 4 \cdot 1.5^t$$

$$m = 4 \cdot 1.5^4$$

$$m = 4 \cdot 5.0625$$

$$m = 20.25$$

After 4 hours, the fish will have a mass of 20.25 grams.

**Things to Remember:**

# Lesson Practice

A1.6.01

Name: ..... Date: ..... Period: .....

**Problems 1–3:** Carlos has a pack of toy fish whose mass doubles every hour when you add water. This table shows the mass of a toy fish over time.

1. What was the mass of the toy fish before it was in water?

**15 grams**

2. What will the mass of the toy fish be after 6 hours?

**960 grams**

Time (hr)	Mass (g)
0	?
1	30
2	60
3	120
4	240
...	...
6	?

3. Wohali buys another brand of toy fish that claims to grow faster than Carlos's fish. Wohali wrote this equation:  $m = 10 \cdot 3^t$ . He used  $m$  for the mass, in grams, and  $t$  for time, in hours. Explain what the 10 and 3 mean in this situation.

**Responses vary.** 10 means that the mass of the toy fish is 10 grams before it goes in the water. 3 means that the fish's mass grows 3 times bigger every hour.

**Problems 4–5:** Jamar had 80 followers on social media. His number of followers tripled every month for 4 months.

4. Select all the expressions that represent Jamar's followers after 4 months.

- A.  $80 \cdot 3 \cdot 3 \cdot 3 \cdot 3$        B.  $80 + 4^3$        C.  $80 \cdot 3^4$   
 D.  $80 + 3 + 3 + 3 + 3$        E.  $80 \cdot 4 \cdot 4 \cdot 4$

5. Complete the table.

Time (months)	0	1	2	3	4
Followers	80	240	720	2,160	6,480

## Lesson Practice

A1.6.01

Name: ..... Date: ..... Period: .....

**Problems 6–7:** A group of biologists tracked the number of squirrels in a town.

They wrote the equation  $n = 40 \cdot 1.5^t$ , where  $n$  is the total number of squirrels and  $t$  is the number of years since the biologists started counting.

6. Explain what the 40 and 1.5 mean in this situation.

**Responses vary.** 40 means that there are 40 squirrels in the town when the biologists start counting. 1.5 means that the number of squirrels in town grows by 1.5 times every year.

7. How many squirrels do the biologists predict there will be 2 years after they started counting?

**90 squirrels**

8. A baby chicken weighs 32 grams when it hatches. The chicken's mass increases by 45% each week for the first 12 weeks of its life. Complete the table of the chicken's mass over time.

Time (weeks)	0	1	2	3	4	...	12
Mass (g)	32	46.4	67.28	97.56	141.46	...	2,764.18

## Spiral Review

**Problems 9–12:** Determine the value of each expression.

9.  $3^3 = 27$

10.  $2(3^3) = 54$

11.  $3^3 + 4 = 31$

12.  $2 \cdot 3^3 + 4 = 58$

## Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

An **exponential function** increases or decreases by a **constant ratio**. The constant ratio will multiply consecutive  $y$ -values. Another name for the constant ratio is the **growth factor**.

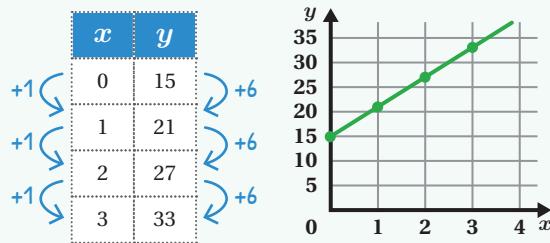
A **linear function** increases or decreases by a **constant difference**. Another name for the constant difference is the **rate of change**.

Here are two examples.

## Linear Function

This pattern has a constant difference of 6, so the rate of change is 6.

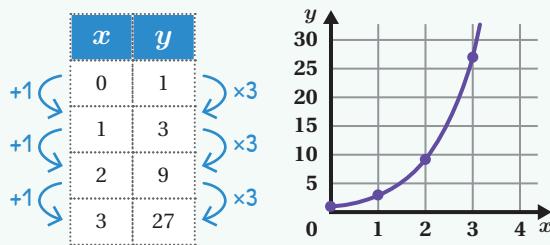
The graph of this linear function is a straight line.



## Exponential Function

This pattern has a constant ratio of 3, so the growth factor is 3.

The graph of this exponential function is a curve that gets steeper and steeper.



## Things to Remember:

# Lesson Practice

A1.6.02

Name: ..... Date: ..... Period: .....

**Problems 1–3:** These tables show the number of red and yellow globs each day.

- How many of each type of glob will there be on day 4?

Day	0	1	2	3	4
Red Globs	50	70	90	110	130

Day	0	1	2	3	4
Yellow Globs	5	10	20	40	80

- Will there be more red or yellow globs on day 10? Show or explain your thinking.

**Yellow globs.** Explanations vary. There will be more yellow globs on day 10. Even though there are fewer yellow globs in the beginning, they are growing faster.

- Which group of globs changes by a constant growth factor? Show or explain how you know.

**Yellow globs.** Explanations vary. The number of yellow globs is multiplied by 2 each day.  $10 \div 5 = 2$  and  $40 \div 20 = 2$ , which shows a constant ratio. The red globs grow by adding 20 every day, which represents a constant difference.

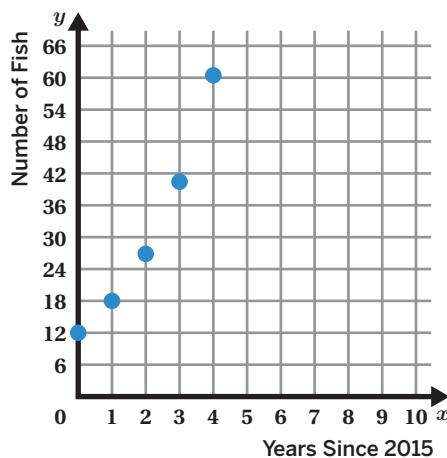
**Problems 4–5:** This graph shows the number of fish in a pond from 2015 to 2019.

- How many fish are in the pond in 2015?

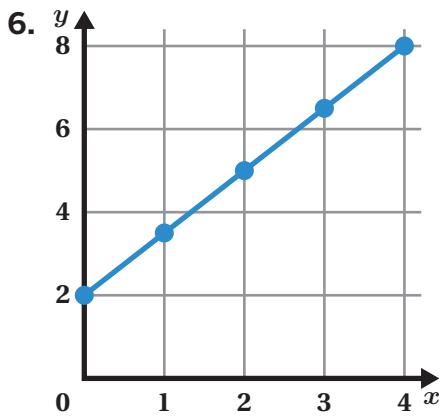
12 fish

- Does the number of fish grow by a constant difference? Show or explain how you know.

No. Explanations vary. A constant difference would mean that the same number of fish were added every year. There were 6 fish added the first year and more than 6 fish added the second year, so there is no constant difference.

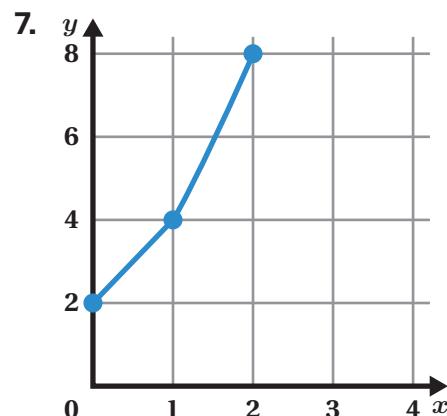


**Problems 6–7:** Determine whether each graph shows a constant rate of change or a constant growth rate. Circle your choice.



Constant rate  
of change

Constant  
growth rate



Constant rate  
of change

Constant  
growth rate

# Lesson Practice

A1.6.02

Name: ..... Date: ..... Period: .....

**Problems 8–9:** Determine whether each table shows a linear or exponential function. Circle your choice.

8.

x	y
0	4
1	8
2	16
3	32

Linear

Exponential

9.

x	y
0	4
1	8
2	12
3	16

Linear

Exponential

## Spiral Review

10. Determine the value of each expression when  $n = 4$ .

a  $n^2 - 5 = 11$

b  $n(n + 6) = 40$

c  $3n^2 = 48$

11. Approximate the value of each radical expression.

a  $\sqrt{10}$

b  $\sqrt{54}$

c  $\sqrt{41}$

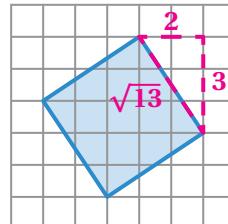
Between 3 & 4

Between 7 & 8

Between 6 & 7

12. Each grid square represents 1 square unit. What is the exact side length of the shaded square?

$\sqrt{13}$  units



## Reflection

- Put a question mark next to a response you'd like to compare with a classmate's.
- Use this space to ask a question or share something you're proud of.

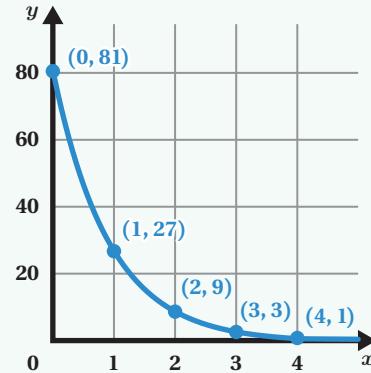
**Lesson Summary**

In the exponential function  $f(x) = a \cdot b^x$ ,  $a$  represents the  $y$ -intercept and  $b$  represents the growth factor. Both parts can be seen on a graph.

Let's look at two examples.

Here is the graph of the exponential function  $g(x) = 81 \cdot \left(\frac{1}{3}\right)^x$ .

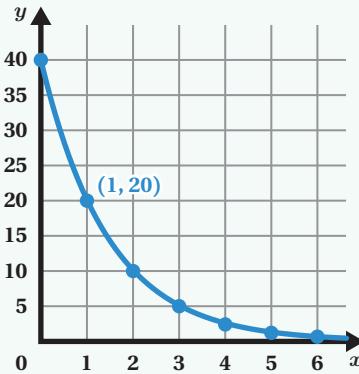
- 81 means that  $(0, 81)$  is the  $y$ -intercept.
- $\frac{1}{3}$  is the growth factor. As  $x$  increases by 1, the  $y$ -values are multiplied by a factor of  $\frac{1}{3}$ .  $81 \cdot \left(\frac{1}{3}\right) = 27$ .



You can also write an exponential equation to represent a graph.

- The  $y$ -intercept is  $(0, 40)$ , so 40 is the  $a$ -value.
- The points  $(1, 20)$ ,  $(2, 10)$ , and  $(3, 5)$  are on the graph of the function. Since 10 is  $\frac{1}{2}$  of 20 and 5 is  $\frac{1}{2}$  of 10, this function has a growth factor of  $\frac{1}{2}$ , which is the  $b$ -value in the equation.

One way to write the equation of this exponential function is  $f(x) = 40 \cdot \left(\frac{1}{2}\right)^x$ .

**Things to Remember:**

# Lesson Practice

A1.6.03

Name: ..... Date: ..... Period: .....

**Problems 1–3:** Determine the value of each expression when  $x = 2$ .

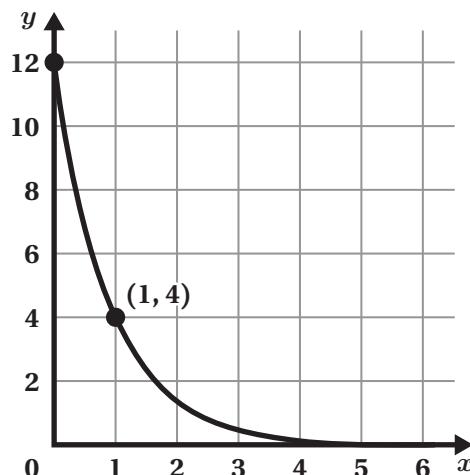
1.  $4^x = 16$

2.  $\left(\frac{1}{3}\right)^x = \frac{1}{9}$  (or equivalent)

3.  $5(6^x) = 180$

4. Here is a graph of  $y = 12 \cdot \left(\frac{1}{3}\right)^x$ . Explain where you can see the 12 and the  $\frac{1}{3}$  in the graph.

*Responses vary. The 12 is the starting value. You can see it in the point  $(0, 12)$  in the graph. The  $\frac{1}{3}$  is the constant ratio. You can see that because 4 is  $\frac{1}{3}$  of 12.*



**Problems 5–7:** Match each equation to the graph that represents it.

Equation A

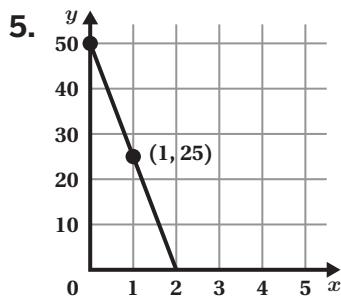
$$y = 50 \cdot \left(\frac{1}{2}\right)^x$$

Equation B

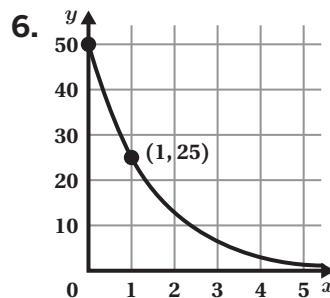
$$y = 50 \cdot 2^x$$

Equation C

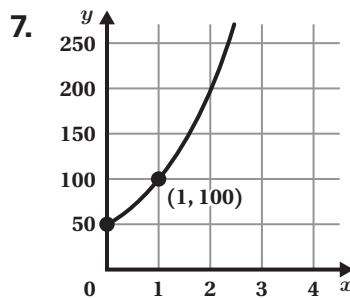
$$y = 50 - 25x$$



Equation C



Equation A



Equation B

8. Explain how you determined which equation to match with the graph in problem 5.

*Responses vary.*

- The graph starts at 50 and has a constant difference of 25.  $y = 50 - 25x$  is the only equation with a constant difference.
- The graph shows a linear relationship, and I know that linear equations do not have exponents.  $y = 50 - 25x$  is the only equation without an exponent.

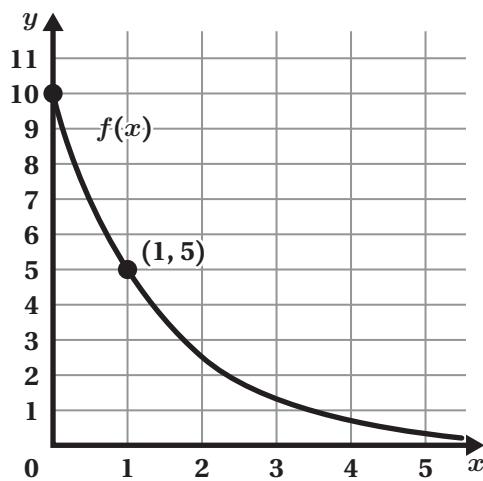
# Lesson Practice

A1.6.03

Name: ..... Date: ..... Period: .....

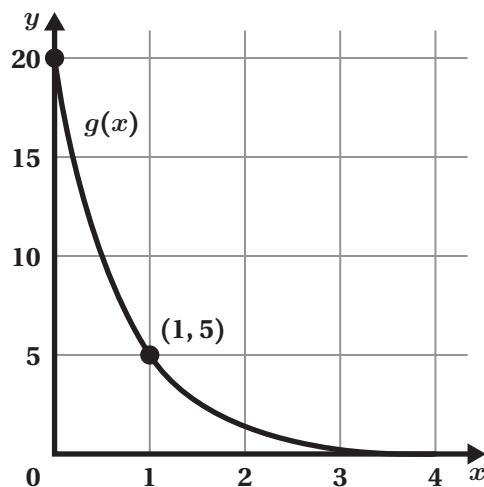
**Problems 9–11:** Here are graphs of two different exponential relationships. Write an equation to represent each graph.

9.



Equation .....  $f(x) = 10 \cdot \left(\frac{1}{2}\right)^x$

10.



Equation .....  $g(x) = 20 \cdot \left(\frac{1}{4}\right)^x$

11. Pick Problem 9 or 10 and explain how you determined the initial value and the growth factor. *Responses vary.*

**Problem 9:** The point  $(0, 10)$  shows an initial value of 10. I compared  $(0, 10)$  to the point  $(1, 5)$ . Since 5 is half of 10, I know that the growth factor is  $\frac{1}{2}$ .

**Problem 10:** The point  $(0, 20)$  shows an initial value of 20. I compared  $(0, 20)$  to the point  $(1, 5)$ . Since 5 is  $\frac{1}{4}$  of 20, I know that the growth factor is  $\frac{1}{4}$ .

## Spiral Review

**Problems 12–15:** Rewrite each expression as a single power.

12.  $4^4 \cdot 4^8$

$4^{12}$

13.  $\frac{3^5}{3^8}$

$3^{-3}$  or  $\frac{1}{3^3}$

14.  $(12^3)^5$

$12^{15}$

15.  $\frac{7^3 \cdot 7^4}{7^5}$

$7^2$

## Reflection

- Put a smiley face next to the problem you learned from most.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

You can evaluate exponential functions for inputs that are positive, negative, or zero.

Let's evaluate the function  $f(x) = 9 \cdot 4^x$  for  $f(-3)$  using the equation or a table.

You can substitute a value into the equation.

Steps	Explanation
$f(-3) = 9 \cdot 4^{(-3)}$	Substitute $x = (-3)$ into the function.
$f(-3) = 9 \cdot \left(\frac{1}{4}\right)^3$	Apply the property of negative exponents to rewrite the expression with a positive exponent.
$f(-3) = 9 \cdot \left(\frac{1}{64}\right)$	Rewrite the expression without any exponents.
$f(-3) = \frac{9}{64}$	Multiply to determine the value of $f(-3)$ .

The domain of an exponential function can include values like zero, all positive, or all negative values. It is important to use the information in the situation to decide which values make sense and which do not. For example, a function that models the amount of caffeine remaining in the body after drinking coffee does not include negatives in the domain. However, an exponential model of the population of ants can include negative numbers in the domain.

You can move backward in a table.

$x$	$f(x)$
-3	$\frac{9}{64}$
-2	$\frac{9}{16}$
-1	$\frac{9}{4}$
0	9

Multiply 9 by  $\frac{1}{4}$  three times to determine the value of  $f(-3)$ .

## Things to Remember:

# Lesson Practice

A1.6.05

Name: ..... Date: ..... Period: .....

1. Which equation best models the data in the table?

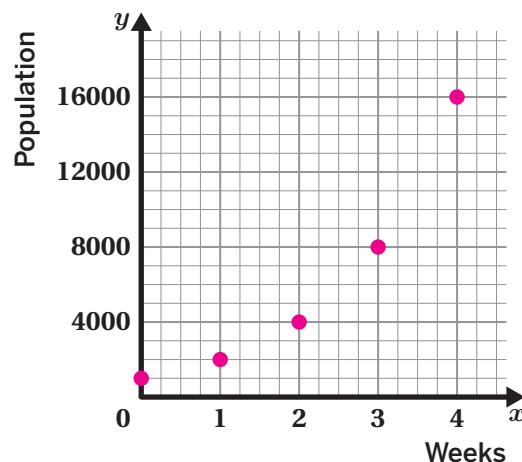
- A.  $f(x) = 80(1.25)^x$
- B.  $f(x) = 64(1.25)^x$
- C.  $f(x) = 64 + 1.25x$
- D.  $f(x) = 60 + 20x$

$x$	$f(x)$
1	80
2	100
3	125
4	156.25

**Problems 2–4:** The equation  $p(w) = 1000 \cdot 2^w$  models a population of mosquitos,  $p(w)$ , where  $w$  is the number of weeks after the population was first measured.

2. Complete the table and plot the values on the graph.

Weeks, $w$	Population, $p(w)$
0	1,000
1	2,000
2	4,000
3	8,000
4	16,000



3. Where on the graph do you see the 1,000 from the equation?

**Responses vary.** I see that 1,000 is the  $y$ -intercept of the graph.

4. Determine the value of  $p(-2)$  and explain what it means in this situation.

$p(-2) = 250$ . **Explanations vary.** This means that there were 250 mosquitos 2 weeks before the population was first measured.

**Problems 5–7:** The equation  $f(t) = 800 \cdot \left(\frac{1}{2}\right)^t$  models a fish population,  $f(t)$ , where  $t$  is time in years since the beginning of 2015.

5. What is the population of fish at the beginning of 2015?

**800**

6. What is the population of fish at the beginning of 2018?

**100**

7. What is the population of fish at the beginning of 2012? Do you think this makes sense in this situation?

**6,400.** **Responses vary.** Maybe, it depends on how big the lake is. For one of the Great Lakes, this number makes sense. If it is a small lake then it is unlikely that there were 6,400 fish in it.

## Lesson Practice

A1.6.05

Name: ..... Date: ..... Period: .....

### Spiral Review

**Problems 8–9:** Charlie's gaming club wants to make at least \$300 selling boxes of cookies and pies. They make \$9 for each box of cookies and \$15 for each pie.

8. Write an inequality to represent the number of boxes of cookies,  $c$ , and pies,  $p$ , the team can sell to make at least \$300.

$$9c + 15p \geq 300$$

9. If the team sells 5 boxes of cookies, what is the minimum number of pies they need to sell in order to meet their goal?

$$17 \text{ pies}$$

**Problems 10–13:** Determine the value of each expression.

10.  $2^4 = 16$

11.  $-2^4 = -16$

12.  $(-2)^4 = 16$

13.  $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

### Reflection

- Star a problem you're still feeling confused about.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

Exponential functions can be written in the form  $f(x) = a \cdot b^x$ , where  $a$  is the *initial value* and  $b$  is the growth factor. Exponential functions represent repeated *percent increase* when the growth factor is larger than 1.

In situations that model repeated percent increase, the growth factor  $b$  can be written as 1 (representing 100%) plus the percent increase in decimal form.

For example, the value of a baseball card collection increases by 4% every year. In 2020, the collection was valued at \$500. Let  $f(x)$  represent the value of the collection and  $x$  represent the years since 2020.

- At first, the collection was valued at \$500. The initial value, or  $a$ , is 500.
- The value increases by 4% every year. Because the value of the cards increases by a repeated percent each year, we can represent the growth by changing the percent to a decimal (4% to 0.04) and adding 1. The growth factor, or  $b$ , will be 1.04.

We can write the exponential function that represents this situation as  $f(x) = 500 \cdot (1.04)^x$ .

**Things to Remember:**

# Lesson Practice

## A1.6.07

Name: ..... Date: ..... Period: .....

**Problems 1–3:** A group of biologists tracked the number of deer in a forest over several years. There were 600 deer when they first counted. The population has increased by 15% each year.

1. How many deer are in the forest 1 year after the biologists first counted?

**690 deer**

2. Write an expression that represents the deer population after 3 years.

**$600 \cdot 1.15^3$  (or equivalent)**

3. Write an expression that represents the deer population after  $t$  years.

**$600 \cdot 1.15^t$  (or equivalent)**

4. Sothy's family paid \$1,300 in property tax last year. This year, the county will increase the property tax by 2.1%.

Select *all* the expressions that represent Sothy's family's property taxes this year.

- A.  $1300 + (1.021)$
- B.  $1300(1.21)$
- C.  $1300(1.021)$
- D.  $1300(1.0021)$
- E.  $1300 + 1300(0.021)$

**Problems 5–6:** Sai gets a \$500 loan from the bank with an annual interest rate of 6%.

5. Write a function,  $f(t)$ , to represent the amount Sai will owe, in dollars, after  $t$  years.

**$f(t) = 500 \cdot 1.06^t$**

6. Complete the table to determine how much money Sai will owe over time without making any payments.

Time (yr)	Amount Owed (\$)
0	500
1	<b>530</b>
2	<b>561.80</b>
3	<b>595.51</b>
4	<b>631.24</b>

## Lesson Practice

### A1.6.07

Name: ..... Date: ..... Period: .....

**Problems 7–9:** Three cities have the same initial population and different percent increases each year. Match each function  $p(t)$ , representing the population after  $t$  years, with its correct description.

$$p(t) = 5000 \cdot (1.20)^t$$

$$p(t) = 5000 \bullet (1.02)^t$$

$$p(t) = 5000 \bullet (1.002)^t$$

- 7.** City A has a 0.2% annual increase in population.

**8.** City B has a 20% annual increase in population.

**9.** City C has a 2% annual increase in population.

$$p(t) = 5000 \cdot (1.002)^t$$

$$p(t) = 5000 \cdot (1.20)^t$$

$$p(t) = 5000 \cdot (1.02)^t$$

## Spiral Review

- 10.** Order these values from least to greatest.

75% of 12

25% of 32

50% of 20

10% of 95

25% of 32

75% of 12

10% of 95

50% of 20

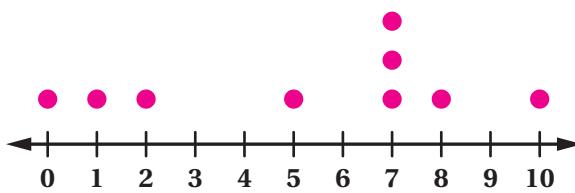
Least

## Greatest

- 11.** Create a dot plot that has:

- At least 5 data points.
  - A median of 7.
  - A mean that is less than the median.

## *Responses vary.*



## Reflection

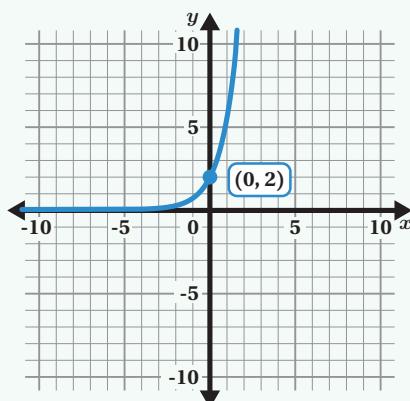
1. Circle the problem you enjoyed doing the most.
  2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

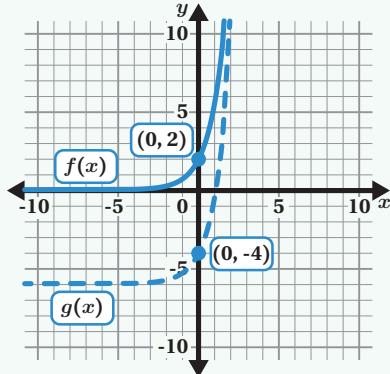
Here is the graph of  $f(x) = 2 \cdot 3^x$ .

Functions can be translated horizontally and vertically.

Here are two examples of *translations* of  $f(x)$ .

**Vertical Translations**

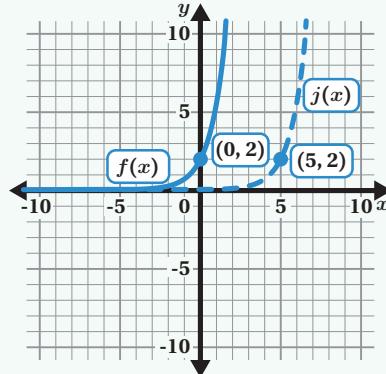
The equation  $f(x) = 2 \cdot 3^x + k$  represents a vertical translation by  $k$  units.



Here  $f(x)$  was translated 6 units down and can be represented with the new equation:  $g(x) = 2 \cdot 3^x - 6$ .

**Horizontal Translations**

The equation  $f(x) = 2 \cdot 3^{(x - h)}$  represents a horizontal translation by  $h$  units.



Here  $f(x)$  was translated 5 units to the right and can be represented with the new equation:  $j(x) = 2 \cdot 3^{(x - 5)}$ .

**Things to Remember:**

# Lesson Practice

A1.6.10

Name: ..... Date: ..... Period: .....

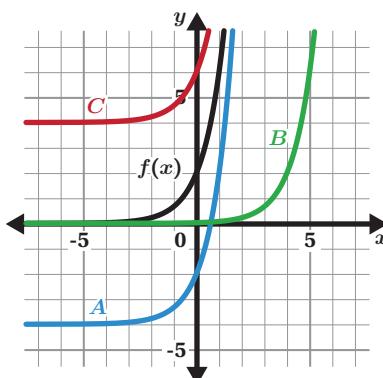
1. Here is the graph of  $f(x) = 2 \cdot 3^x$ .

Match each function with its graph.

$$f(x) = 2 \cdot 3^x + 4 \quad \text{_____} \quad \textcolor{red}{C}$$

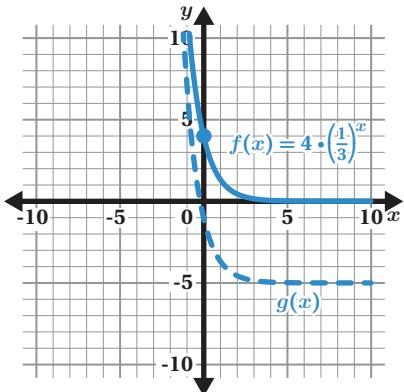
$$f(x) = 2 \cdot 3^x - 4 \quad \text{_____} \quad \textcolor{blue}{A}$$

$$f(x) = 2 \cdot 3^{(x-4)} \quad \text{_____} \quad \textcolor{blue}{B}$$



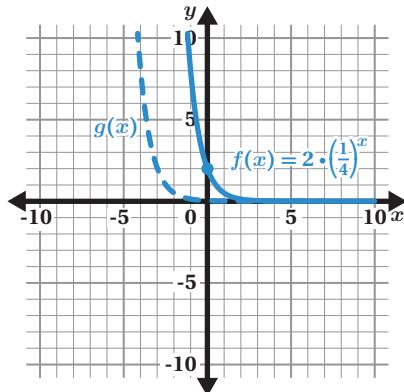
**Problems 2–3:** For each set of graphs shown, write an equation for the dotted curve,  $g(x)$ .

2.  $f(x) = 4 \cdot \left(\frac{1}{3}\right)^x$



$$g(x) = \textcolor{red}{4} \cdot \left(\frac{1}{3}\right)^x - 5$$

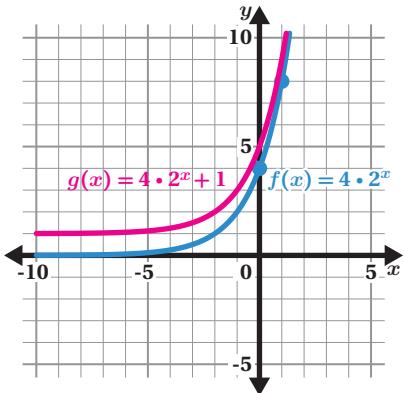
3.  $f(x) = 2 \cdot \left(\frac{1}{4}\right)^x$



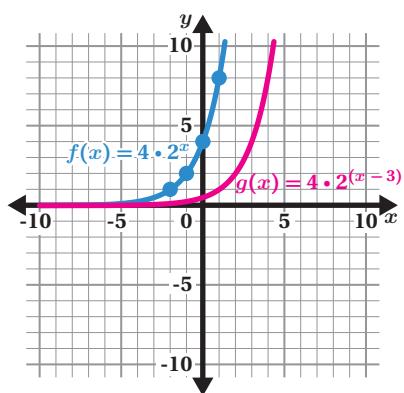
$$g(x) = 2 \cdot \left(\frac{1}{4}\right)^{(x+3)}$$

**Problems 4–5:** The function  $g(x)$  is a transformation of  $f(x) = 4 \cdot 2^x$ .

4. Graph  $g(x) = 4 \cdot 2^x + 1$ .



5. Graph  $g(x) = 4 \cdot 2^{(x-3)}$ .



## Lesson Practice

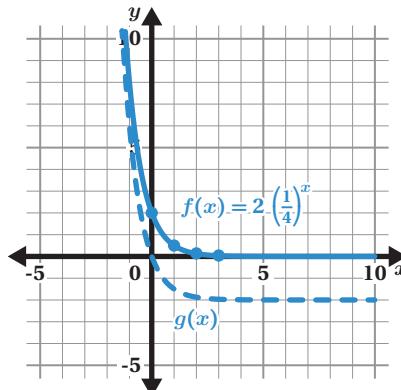
### A1.6.10

Name: ..... Date: ..... Period: .....

**Problems 6–8:** The function  $g(x)$  is a transformation of  $f(x) = 2 \cdot \left(\frac{1}{4}\right)^x$ .

6. Ariana says  $g(x) = 2 \cdot \left(\frac{1}{4}\right)^x - 4$  because the  $y$ -intercept of  $f(x)$  shifted down 4 units. Explain why Ariana's thinking is incorrect.

**Explanations vary.** Ariana is correct that the  $y$ -intercept shifted down, but no point on  $f(x)$  shifted down 4 units, so Ariana's equation does not represent  $g(x)$ .



7. Kyrie says  $f(x)$  transformed using both vertical and horizontal translations. Explain why Kyrie's thinking is incorrect.

**Explanations vary.** Kyrie is incorrect because each point should transform from  $f(x)$  to  $g(x)$  using the same translations. If there had been a horizontal translation then the  $y$ -intercept of  $f(x)$  wouldn't translate straight down to the  $y$ -intercept of  $g(x)$ , there would need to be a horizontal movement as well.

8. Write the correct equation for  $g(x)$ .

$$g(x) = 2 \cdot \left(\frac{1}{4}\right)^x - 2$$

## Spiral Review

9. What is the solution to this system of equations?

$$5x + y = 18$$

$$x - 3y = 10$$

- A.  $(-2, -4)$   
**B.**  $(4, -2)$   
C. There is no solution.  
D. There are an infinite number of solutions.

## Reflection

- Put a star next to a problem you could explain to a classmate.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can invest money in accounts that earn simple or compound interest. Accounts that earn **simple interest** can be modeled by linear functions, while accounts that earn **compound interest** can be modeled by exponential functions. Which account earns the most interest depends on how much time there is to invest and other variables.

Let's look at an example. Adah has \$100 to invest in an account. Which account should she choose if she has 12 years to invest?

Simple Interest	Compound Interest
Adah could invest \$100 in an account that earns 10% simple interest annually. The function $a(t) = 100 + 10t$ models the account balance after $t$ years.	Adah could invest \$100 in an account that earns 10% compound interest annually. The function $b(t) = 100 \cdot (1.10)^t$ models the account balance after $t$ years.
To determine the balance of the account after 12 years, substitute $t = 12$ into each function and solve for $a(t)$ .	
$a(12) = 100 + 10(12)$ $a(12) = 220$ After 12 years, the account balance will be \$220.	$b(12) = 100 \cdot (1.10)^{12}$ $b(12) = 313.84$ After 12 years, the account balance will be about \$313.84.

Adah may choose to invest in the account that earns compound interest because it earns more money over 12 years.

**Things to Remember:**

# Lesson Practice

## A1.6.13

Name: ..... Date: ..... Period: .....

**Problems 1–4:** Determine if each equation or table represents simple or compound interest. Circle your choice.

1.  $b(t) = 1000(1.03)^t$

Simple

Compound

2.  $b(t) = 1000 + 30t$

Simple

Compound

3.

Time (yr)	Account Balance (\$)
0	300
1	330
2	360

Simple

Compound

4.

Time (yr)	Account Balance (\$)
0	200
1	230
2	264.50

Simple

Compound

**Problems 5–7:** Jin invests \$4,000 in an account that earns 5% compound interest per year.

5. Complete the table.

6. Which function represents the amount of money in Jin's account after  $x$  years?

A.  $f(x) = 4000 + 1.05x$

B.  $f(x) = 4000(1.05)^x$

C.  $f(x) = 4000(0.05)^x$

D.  $f(x) = 4000 + (1.05)^x$

7. What will the balance of the account be after 10 years?

**\$6,515.58**

Time (yr)	Account Balance (\$)
0	<b>4,000</b>
1	4,200
2	4,410
3	<b>4,630.50</b>
4	<b>4,862.03</b>

## Lesson Practice

A1.6.13

Name: ..... Date: ..... Period: .....

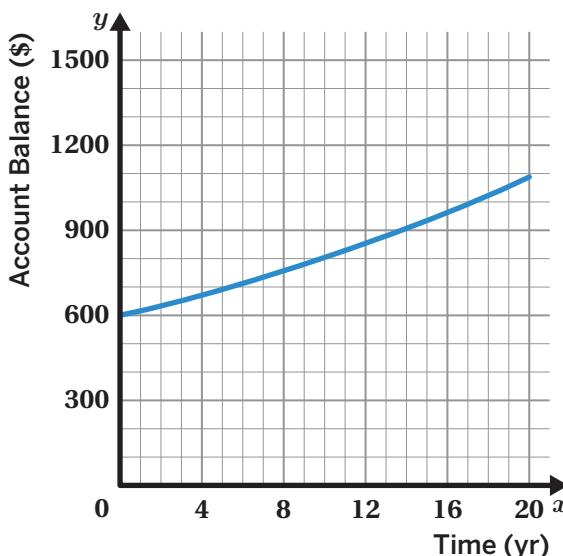
**Problems 8–9:** Keya invests \$600 in an account that earns 3% compound interest per year. The graph shows the function  $f(t) = 600(1.03)^t$ , which gives Keya's account balance after  $t$  years.

8. About how many years will it take for her account balance to reach \$1,000?

**About 17 years.**

9. Use the graph to determine the value of  $f(14)$ .

**$f(14)$  is equal to about 900.**



What does that tell you about the situation?

**Responses vary. This tells me that after 14 years, Keya's account balance will be about \$900.**

10. You just won a contest and have two prize options.

- **Option A:** One payment of \$20 million
- **Option B:** 2 cents on day one, 4 cents on day two, 8 cents on day three, and so on, for 30 days

Which option would you choose? Explain your choice.

**Responses and explanations vary.**

- I would choose Option B. I calculated the value after 30 days and it was \$21,474,836.46.
- I would choose Option A because I would want the money all at once.

## Spiral Review

**Problems 11–13:** Determine whether each function is linear, exponential, or something else. Circle your choice.

11.  $f(x) = x^2 + 5$

Linear

Exponential

Something else

12.  $g(x) = 2x + 5$

Linear

Exponential

Something else

13.  $h(x) = 2^x + 5$

Linear

Exponential

Something else

## Reflection

1. Put a star next to a problem you want to understand better.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can write exponential expressions representing compound interest in multiple equivalent ways to reveal different information about the account and situation.

Here is an example.

The amount owed on a \$400 loan has a monthly interest rate of 3%. Let  $t$  represent the number of years since taking out the loan if no payments are made.

- The expression  $400 \cdot 1.03^{12t}$  represents a 3% monthly interest rate 12 times each year,  $t$ .
- The expression  $400 \cdot (1.03^{12})^t$  uses the powers of powers property to help us think about the interest rate for every  $t$  year.
- Since  $1.03^{12} = 1.4258$ , the expression  $400 \cdot (1.4258)^t$  shows the annual interest rate of 42.58%.

Each expression reveals different information about the monthly and annual interest rates applied to the account. While a monthly interest rate of 3% may not seem like it impacts the account balance much, the annual interest rate reveals that the loan amount is increasing by 42.58% each year, and that really adds up!

**Things to Remember:**

## Lesson Practice

### A1.6.14

Name: ..... Date: ..... Period: .....

**Problems 1–2:** Alina takes out a \$1,000 loan with a monthly interest rate of 5%. She makes no additional payments, deposits, or withdrawals.

1. Select *all* the expressions that can be used to calculate her balance after  $t$  years.

- A.  $1000 \cdot 1.05^t$
- B.  $1000 \cdot 1.05^{12t}$
- C.  $1000(1.05^{12})^t$
- D.  $1000 \cdot 1.7959^t$
- E.  $1000(1.7959)$

2. What is the interest rate per year for this loan?

79.59%

**Problems 3–7:** Alejandro invests money into a college savings account. He writes the expression  $750(1.006^{12})^3$  to help him calculate what the account balance will be in 3 years.

3. Explain what 750 represents in the expression.

*Responses vary. 750 represents the initial amount Alejandro invested.*

4. Explain what 1.006 represents in the expression.

*Responses vary. 1.006 represents the growth factor of the investment, or 0.6% interest earned on the investment.*

5. Explain what 12 represents in the expression.

*Responses vary. 12 represents that this is a monthly interest rate.*

6. Explain what 3 represents in the expression.

*Responses vary. 3 represents the number of years.*

7. Write an equivalent expression that could represent Alejandro's account balance in 3 years.

*750 • 1.006<sup>36</sup> or 750(1.0744)<sup>3</sup> (or equivalent)*

## Lesson Practice

A1.6.14

Name: ..... Date: ..... Period: .....

**Problems 8–9:** Rebecca is considering taking out a payday loan that has a 17% monthly interest rate.

8. Complete the table.

9. If Rebecca takes out a \$300 payday loan, how much would she owe after 2 years if she made no additional payments?

**\$12,989.19**

Monthly Interest Rate	17%
Monthly Growth Factor	<b>1.17</b>
Growth Factor per Year	<b>6.58</b>
Interest Rate per Year	<b>558%</b>

10. Wohali takes out an \$8,000 federal student loan with a monthly interest rate of 0.38% for professional trade school. Write a function,  $g(t)$ , to calculate the amount Wohali owes after making no payments for  $t$  years.

**$g(t) = 8000 \cdot (1.0038)^{12t}$  (or equivalent)**

### Spiral Review

**Problems 11–13:** Determine the value of each function when  $n = 2$ .

11.  $f(n) = 4 \cdot 2^n$   **$f(2) = 16$**

12.  $g(n) = 2 \cdot 4^n$   **$g(2) = 32$**

13.  $h(n) = 8 + 2^n$   **$h(2) = 12$**

14. Using the digits 0 to 9, without repeating, fill in each blank to create four equivalent expressions.

$$7^{\square} = 7^{\square} \times 7^{\square} = 7^{\square} \times 7^{\square} \times 7^{\square} = (7^{\square})^{\square}$$

**Responses vary.**  $7^8 = 7^7 \times 7^1 = 7^5 \times 7^3 \times 7^0 = (7^4)^2$

### Reflection

- Put a star next to a problem that looked more difficult than it really was.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

When you take out a loan on a credit card, the annual interest rate may be compounded at different *intervals*, or different lengths of time.

You can use the formula  $P\left(1 + \frac{r}{n}\right)^{nt}$  to calculate the total amount in an account that accrues compound interest.

- $P$  represents the initial amount of the loan. In finance this is often called the *principal*.
- $r$  represents the interest rate in decimal form.
- $n$  represents the number of compounding intervals in a year.
- $t$  represents the time in years.

Common compounding periods:

Annually	Semi-annually	Quarterly	Monthly	Daily
$n = 1$	$n = 2$	$n = 4$	$n = 12$	$n = 365$

Let's look at the impact of compound interest applied at different intervals on a loan for \$1,000.

Interest	Owed in	Compounded Monthly	Compounded Quarterly	Compounded Annually
15% annually	5 years	$1000\left(1 + \frac{0.15}{12}\right)^{12 \cdot 5}$ ≈ \$2,107.18	$1000\left(1 + \frac{0.15}{4}\right)^{4 \cdot 5}$ ≈ \$2,088.15	$1000(1 + 0.15)^5$ ≈ \$2,011.36

**Things to Remember:**

## Lesson Practice

A1.6.15

Name: ..... Date: ..... Period: .....

1. Tyrone puts \$2,500 into a savings account with a 1.2% annual interest rate, compounded semi-annually. He makes no additional payments, deposits, or withdrawals.

Select *all* the expressions that can be used to calculate his balance after 3 years.

- A.  $2500\left(1 + \frac{0.012}{2}\right)^{3 \cdot 2}$
- B.  $2500\left(1 + \frac{0.012}{6}\right)^6$
- C.  $2500(1 + 0.012)^3$
- D.  $2500(1 + 0.006)^6$
- E.  $2500\left(1 + \frac{0.012}{3}\right)^{3 \cdot 2}$

**Problems 2–4:** Maneli wants to take out a \$5,000 loan to help pay for a new washing machine and dryer. The bank offers her the loan with an 18% annual interest rate, compounded quarterly.

Maneli wrote this expression to calculate the balance of the loan in 2 years, but she made an error.

$$5000\left(1 + \frac{0.18}{2}\right)^{(4 \cdot 2)}$$

2. Find the error and explain why it is incorrect.

*Explanations vary. Since the interest is compounded quarterly, the 0.18 should be divided by 4.*

3. Write a correct expression to represent Maneli's balance after 2 years.

$$5000\left(1 + \frac{0.18}{4}\right)^{4 \cdot 2} \quad (\text{or equivalent})$$

4. What will her balance be in 2 years?

\$7,110.50

**Problems 5–6:** A payday loan company offers a \$1,000 loan with a 25% annual interest rate.

5. If no other charges or payments are made, what will the balance of the loan be after 1 year at each compounding period?

Compounding Period	Balance (\$)
Annually	1,250
Monthly	1,280.73
Daily	1,283.92

6. Describe how changing the compounding period affects the balance of the loan.

*Responses vary. The more often a loan is compounded, the higher the balance will be.*

## Lesson Practice

A1.6.15

Name: ..... Date: ..... Period: .....

**Problems 7–8:** Xavier has \$5,000 to invest and has to choose between three investment options.

- Option A: 2.25% interest applied each quarter
- Option B: 3% interest applied every 4 months
- Option C: 4.5% interest applied twice each year

7. Write an expression for each account to represent Xavier's balance after 5 years.

$$a(x) = 5000 \cdot \left(1 + \frac{0.0225}{4}\right)^{4 \cdot 5}$$

$$b(x) = 5000 \cdot \left(1 + \frac{0.03}{3}\right)^{3 \cdot 5}$$

$$c(x) = 5000 \cdot \left(1 + \frac{0.045}{2}\right)^{2 \cdot 5}$$

8. Which option will have the largest balance after 5 years?

**Option C will have the largest balance after 5 years with \$6,246.02**

## Spiral Review

**Problems 9–10:** Irene needs to make at least 25 dinners for a party, including chicken dinners and vegetarian dinners. She has \$250 to spend. Chicken dinners cost \$8.75 each and vegetarian dinners cost \$5.50 each.

- $c$  represents the number of chicken dinners.
- $v$  represents the number of vegetarian dinners.

9. Write a system of inequalities that represents Irene's constraints.

$$c + v \geq 25$$

$$8.75c + 5.50v \leq 250$$

10. Can Irene make 5 chicken dinners and 20 vegetarian dinners? Explain your thinking.

**Yes. Explanations vary.**

- 5 chicken dinners and 20 vegetarian dinners cost \$153.75, which is less than \$250.
- When  $c = 5$  and  $v = 20$ , the inequality  $8.75(5) + 5.50(20) \leq 250$  is true.

## Reflection

1. Put a heart next to a problem you understand well.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

Exponential and linear functions are often used to model the population growth of a city or country. *Models* can help us predict unknown data values, including future values. While some models can be useful, they also have limitations.

- Exponential functions increase toward infinity, but populations are limited by space, time, and available land.
- Linear models may generate a negative  $y$ -intercept, but populations do not have less than 0 people.
- Models may only be useful for predicting unknown values within a specific *domain*.

Let's look at an example. Here is data about the population of a city since 1880.

An exponential model is a better fit than a linear model. The data fits the shape of a curve more closely than it follows a line.

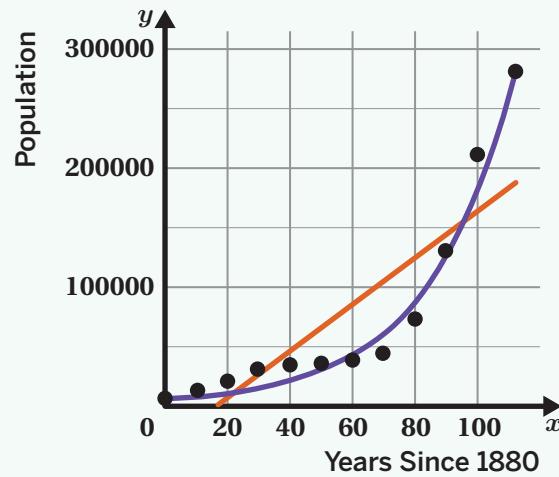
You can use the exponential function  $p(t) = 3992(1.0397)^x$  to model the number of people in 1945.

Since 1945 is 65 years after 1880, substitute 65 into  $p(t)$  to estimate the unknown value.

$$p(65) = 3992(1.0397)^{65}$$

$$p(65) = 50143$$

The model estimates that there were 50,143 people in 1945.

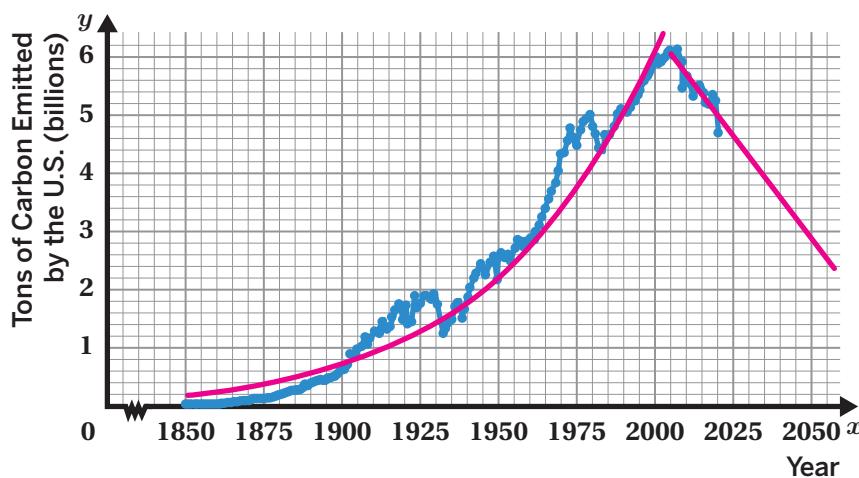
**Things to Remember:**

# Lesson Practice

## A1.6.16

Name: ..... Date: ..... Period: .....

**Problems 1–5:** The graph shows the number of tons of carbon, in billions, that the United States emitted from 1850 to 2020.



Source: Our World in Data

- How would you describe the data in this graph?

*Responses vary. The amount of carbon emitted seems to increase exponentially from 1850 until about 2005. After 2005, the amount of carbon emitted seems to decrease significantly.*

- What change occurs in the data around 2005? What do you think may have caused that change?

*Responses vary.*

- The data has a negative trend; there is less carbon emitted. Maybe there was a new law passed that made companies and factories change their practices.
- The amount of carbon emitted starts to decrease after 2005. Maybe new kinds of cars came out that emit less carbon, like electric cars.

- Sketch a line or exponential curve of best fit to model the data from 1850 to 2005.

*Responses vary. Sample shown on graph.*

- Sketch a line or exponential curve of best fit to model the data after 2005.

*Responses vary. Sample shown on graph.*

- Use your model to predict how many tons, in billions, the United States will emit in 2050.

*Responses vary.*

- Linear model: About 2.5 billion tons of carbon
- Exponential model: About 4 billion tons of carbon

# Lesson Practice

A1.6.16

Name: ..... Date: ..... Period: .....

**Problems 6–8:** The graph shows the number of weekly confirmed cases of COVID-19 per million people in Italy. Emma modeled the given data with a linear function.

6. Describe an advantage of using Emma's model.

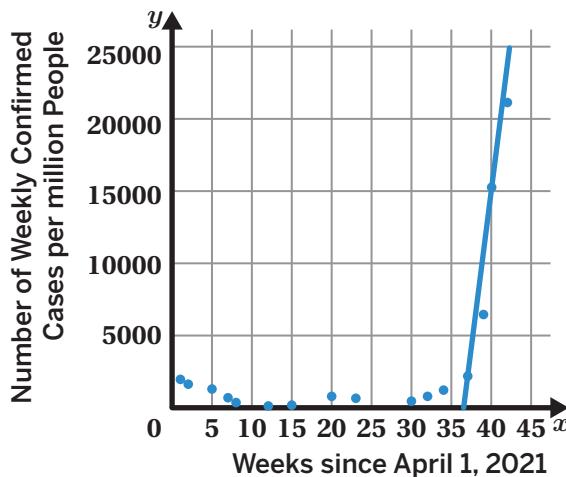
**Responses vary.** This model does a good job of representing the drastic increase in COVID-19 cases between 37 and 42 weeks after April 1, 2021.

7. Describe a disadvantage of using Emma's model.

**Responses vary.** This model does not represent the majority of the given data.

8. Provide a suggestion for Emma to improve her model.

**Responses vary.** I would suggest for Emma to fit an exponential function to the data.



Source: Our World in Data

## Spiral Review

9. The line of best fit  $y = 8.23x - 1.84$  was calculated for a data set. Which value could be the  $r$ -value of the data?

- A.  $r = 0.72$
- B.  $r = -0.72$
- C. Both are possible.
- D. Neither is possible.

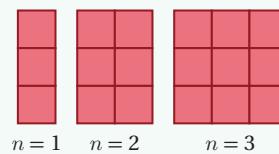
## Reflection

1. Put a star next to the problem you understood best.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

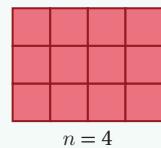
You can investigate visual patterns to determine how to build a particular figure or write an expression to represent the pattern.

Here is an example of the first three steps of a visual pattern.



You can describe what changes and what stays the same to help you draw the next figure, where  $n = 4$ .

Describing how you see a visual pattern changing can help you write a rule or expression to determine the other values in the pattern.



Here are two ways you may determine value of the tenth step of this pattern, or how many tiles are in  $n = 10$ .

- If you see the pattern increasing by a column of 3 each step, you could add 3 more until you reach  $n = 10$ .
- If you see each step in the pattern as a rectangle that has the dimensions 3-by- $n$ , you could multiply 3 by 10 to find the total number of tiles.
- In both cases, you would determine that there will be 30 tiles when  $n = 10$ .

$n$	Number of Tiles
1	3
2	6
3	9
...	...
10	30

$\times 3$

$+3$

There are many ways to see and describe visual patterns accurately.

**Things to Remember:**

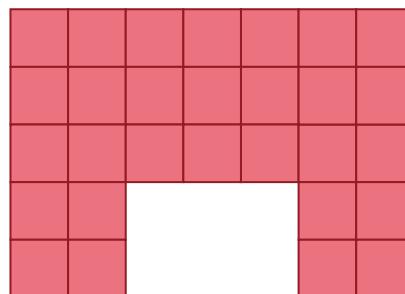
# Lesson Practice

A1.7.01

Name: ..... Date: ..... Period: .....

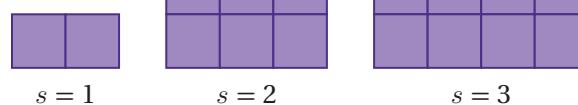
1. Select all of the expressions that could represent the number of tiles in this diagram.

- A.  $5 \cdot 7$
- B.  $5 \cdot 7 - 6$
- C.  $3 \cdot 7 + 2 \cdot 2$
- D.  $5 \cdot 7 - 2 \cdot 3$
- E.  $2 \cdot 5 + 3 \cdot 3 + 2 \cdot 5$



2. Here are the first three steps in a pattern. How many tiles will there be when  $s = 10$ ?

110 tiles



$s = 1$        $s = 2$

$s = 3$

Explain your thinking.

*Explanations vary. I saw that each step is a square with the side length of  $s$  and an extra column with  $s$  squares. So  $s = 10$  would be a 10-by-10 square with an extra column of 10.  $10 \cdot 10 + 10 = 110$  tiles.*

3. What type of relationship does the pattern in the table represent? Circle one.

Linear

Exponential

Neither

$s$	Number of Tiles
1	3
2	9
3	27
4	81

Explain your thinking.

*Explanations vary. As  $s$  increases by 1, the number of tiles changes by a common ratio of 3.*

# Lesson Practice

A1.7.01

Name: ..... Date: ..... Period: .....

**Problems 4–6:** A teacher gives her class a table with only the first two rows in the pattern.

4. Rishi says the pattern is an exponential relationship.

Ichiro says there is not enough information to be sure.

Whose thinking is correct? Explain your thinking.

**Ichiro's. Explanations vary.** If the number of tiles is increasing by 20, then the pattern is linear. If the number of tiles is being multiplied by 5, then the pattern is exponential.

<i>s</i>	Number of Tiles
1	5
2	25

5. How many tiles would be in the next step if the relationship were *linear*?

45 tiles

6. How many tiles would be in the next step if the relationship were *exponential*?

125 tiles

## Spiral Review

7. Juana began hiking at 6:00 AM. At noon, she had hiked 12 miles. At 4:00 PM, Juana finished her hike with a total distance of 26 miles.

On average, during which time interval was Juana hiking faster? Circle one.

6:00 AM to noon

Noon to 4:00 PM

Explain your thinking.

**Explanations vary.** From 6:00 AM to noon, Juana hiked 12 miles in 6 hours, so her average speed was 2 miles per hour. From noon to 4:00 PM, Juana hiked another 14 miles in 4 hours. I knew that because  $26 - 12 = 14$ . That means her average speed was 3.25 miles per hour.

**Problems 8–10:** Use the distributive property to write an equivalent expression for each.

8.  $3(n + 5)$

3*n* + 15 (or equivalent)

9.  $16x + 20$

4(4*x* + 5) (or equivalent)

10.  $2(4m - 6)$

8*m* – 12

4(2*m* – 3) (or equivalent)

## Reflection

- Circle the problem you feel least confident about.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can determine if a visual pattern represents a linear, exponential, or quadratic relationship, or something else.

If you can observe a square that is changing throughout a pattern, the relationship might be quadratic and you can use a squared term to write a quadratic expression.

Here are some strategies you might use to write an expression to represent a visual pattern:

You can look for what is changing and staying the same.

- The 3 outside tiles stay the same.
- The interior square is growing from 1 to 4 to 9.

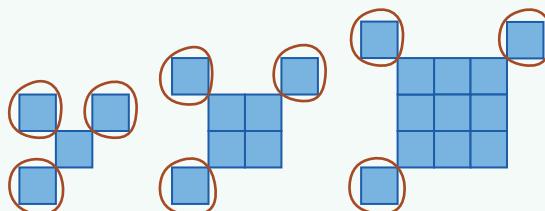


Figure 1

Figure 2

Figure 3

You can look for where you see the figure number,  $n$ , in each diagram.

- You can see the figure number as the side length of the growing square.
- You can write each number of tiles as an expression.

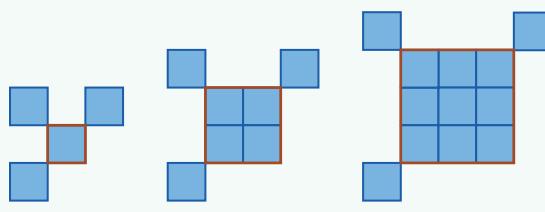


Figure 1

Figure 2

Figure 3

$$1^2 + 3$$

$$2^2 + 3$$

$$3^2 + 3$$

You can write a quadratic expression to represent the number of tiles in Figure  $n$  as  $n^2 + 3$ .

**Things to Remember:**

# Lesson Practice

A1.7.02

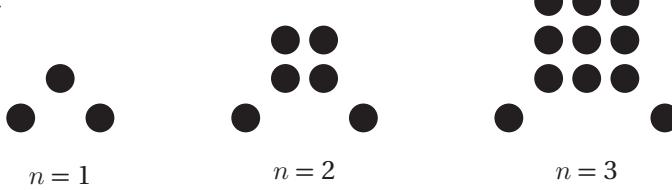
Name: ..... Date: ..... Period: .....

**Problems 1–4:** Circle whether each expression represents a quadratic relationship, linear relationship, or neither.

- |                |           |        |         |
|----------------|-----------|--------|---------|
| 1. $2^n + n$   | Quadratic | Linear | Neither |
| 2. $2n^2 + 2$  | Quadratic | Linear | Neither |
| 3. $2(n - 5)$  | Quadratic | Linear | Neither |
| 4. $2n(n + 3)$ | Quadratic | Linear | Neither |

5. Does this pattern show a quadratic relationship?

Yes



Explain your thinking.

*Explanations vary. This pattern is quadratic because each figure contains a square that is related to the figure number ( $1^2, 2^2, 3^2$ ).*

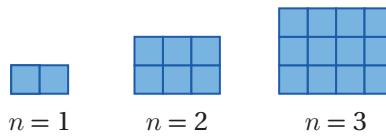
6. Karima says that she sees a square plus one more row in each pattern.

Circle all the expressions Karima could use to represent the number of tiles in this pattern.

$$n^2 + 1$$

$$n^2 + n$$

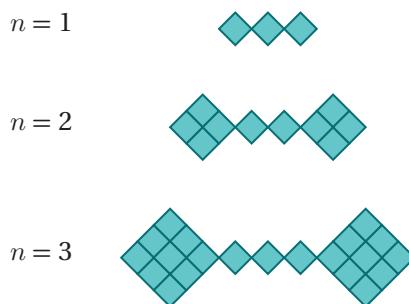
$$n(n + 1)$$



7. Write an expression to represent the relationship between the figure number,  $n$ , and the total number of tiles.

*Responses vary.*

- $2n^2 + n$
- $n^2 + n + n^2$
- $n(2n + 1)$



# Lesson Practice

A1.7.02

Name: ..... Date: ..... Period: .....

8. Three students each wrote different, but correct, expressions to represent this pattern.

Ethan

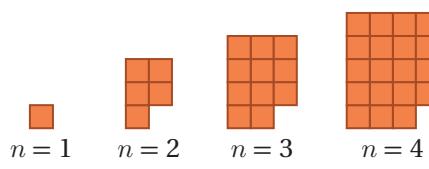
$$n^2 + (n - 1)$$

Ama

$$n(n + 1) - 1$$

Nasir

$$n^2 + n - 1$$



Choose a student and describe how they see the expression in the pattern.

*Responses vary.*

- Ethan sees a square,  $n^2$ , and an extra row that is 1 tile shorter than the square,  $n - 1$ .
- Ama sees a rectangle that is  $n$  units on one side and 1 tile longer on the other side,  $n + 1$ , but the corner piece is removed,  $-1$ .
- Nasir sees a square,  $n^2$ , and an extra row,  $n$ , but a piece is removed from the bottom row,  $-1$ .

## Spiral Review

9. Select all the expressions that are equivalent to  $6m + 3q$ .

- A.  $4m + 2m + 5q - 2q$        B.  $3(2m + q)$        C.  $(6 + 3)(m + q)$   
 D.  $3q + 2m + 3q + m$        E.  $q + 15m + 2q - 9m$

10. Complete the table for the function  $h(x) = 5(2)^x$ .

$x$	-2	-1	0	1	2
$h(x)$	1.25	2.5	5	10	20

## Reflection

1. Put a smiley face next to a problem you were stuck on and then figured out.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

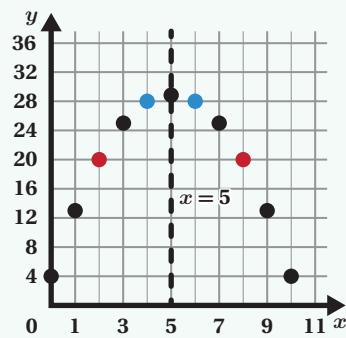
The graph of a quadratic function is called a **parabola**. Parabolas have a line of symmetry that goes through the maximum or minimum point. If you fold a parabola along this line, you get two identical halves. Here is a table and graph that represent a quadratic relationship.

Table

$x$	$y$
2	20
4	28
5	29
6	28
8	20

You can see the points are symmetrical across the line of symmetry at  $x = 5$ .

Graph



You can see the points create the shape of a parabola and are symmetrical across the line of symmetry at  $x = 5$ .

## Things to Remember:

# Lesson Practice

## A1.7.04

Name: ..... Date: ..... Period: .....

**Problems 1–3:** For each pair of symmetrical points on a parabola, determine the equation for the line of symmetry.

1. (0, 0) and (13, 0)

$x = \dots$  **6.5**

2. (4, 2) and (20, 2)

$x = \dots$  **12**

3. (7, 10) and (28, 10)

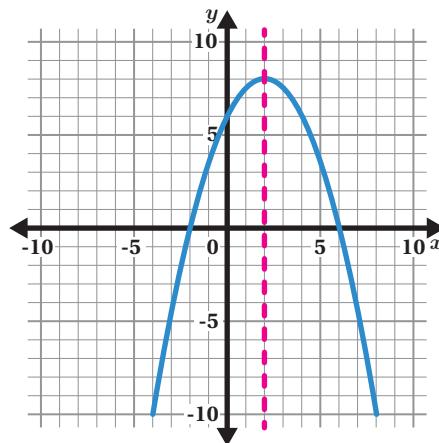
$x = \dots$  **17.5**

**Problems 4–5:** Here is a graph of a parabola.

4. Draw the *line of symmetry* where you think it is located on this parabola.

5. Write the equation for the line of symmetry.

$x = \dots$  **2**



**Problems 6–8:** Here are a few points that belong to a function  $g(x)$ .

6. Does  $g(x)$  represent a quadratic relationship? Circle your response and explain your thinking.

Yes

No

Not enough information

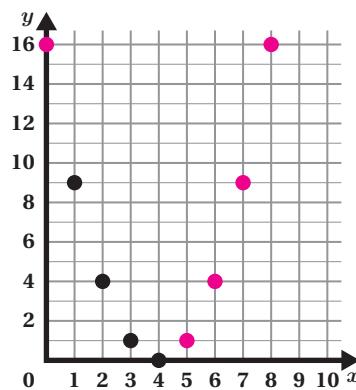
*Explanations vary. I know that  $g(x)$  is quadratic because the second differences of the values in the table are all 2. Quadratic relationships have constant second differences.*

7. Complete the table for  $g(x)$  and plot the new points on the graph.

8. Write the equation for the line of symmetry.

$x = \dots$  **4**

$x$	$g(x)$
0	<b>16</b>
1	9
2	4
3	1
4	0
5	<b>1</b>
6	<b>4</b>
7	<b>9</b>
8	<b>16</b>



# Lesson Practice

## A1.7.04

Name: ..... Date: ..... Period: .....

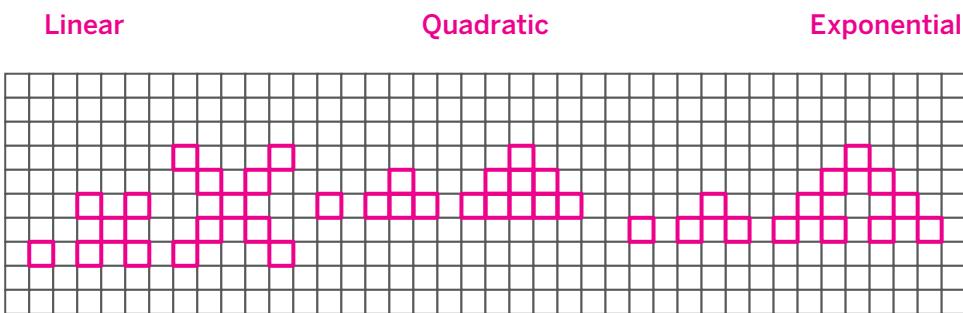
**Problems 9–10:** Here is an incomplete table that could represent several types of functions.

9. Select a function type and determine the number of tiles that would be in Figure 2. Circle one. *Responses vary.*

Linear      Quadratic      Exponential

Figure	Number of Tiles
1	1
2	<i>Responses vary.</i>
3	9

10. Draw three figures to match the pattern in the table. *Responses vary.*



## Spiral Review

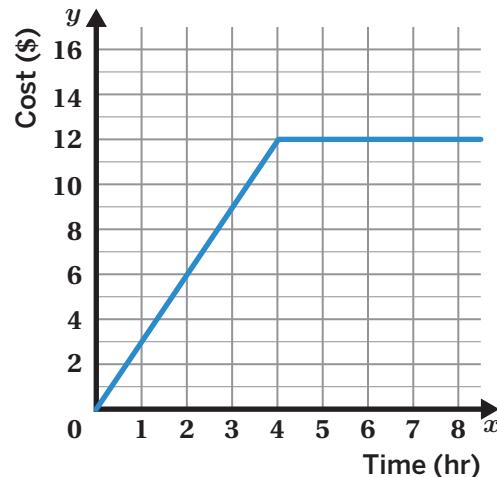
**Problems 11–12:** The graph represents the relationship between the amount of time a car is parked, in hours, and the cost of parking, in dollars.

11. Is the relationship a function?

Yes

12. Describe the relationship between the amount of time a car is parked and the cost of parking.

*Explanations vary. The cost of parking is \$3 per hour, until 4 hours. For any additional time after 4 hours, you will pay \$12 total.*



## Reflection

- Put a star next to a problem you're still wondering about.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

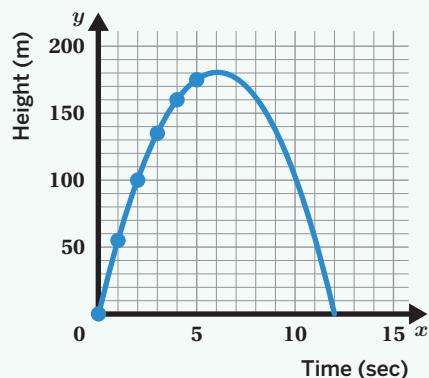
You can use tables and graphs to make predictions about quadratic relationships in context.

This table and graph show the height of a stomp rocket at various times.

**Table**

Time (sec)	Height (m)
0	0
1	55
2	100
3	135
4	160
5	175

+55  
+45  
+35  
+25  
+15  
-10  
-10  
-10  
-10  
-10

**Graph**

You can extend the pattern of the table using the second difference.

From the table, you can see how the rocket is 160 meters high after 4 seconds and 175 meters high after 5 seconds.

You can use the graph to determine the maximum height of the rocket by looking for the highest point on the parabola. The maximum height of the rocket is 180 meters at 6 seconds.

You can also see from the graph that it takes 12 seconds for the rocket to land.

**Things to Remember:**

# Lesson Practice

A1.7.05

Name: ..... Date: ..... Period: .....

**Problems 1–3:** This table and graph show the height of a stomp rocket at various times.

Time (sec)	0	1	2	3
Height (ft)	0	144	256	336

1. What is the maximum height that the stomp rocket reached?

**400 feet**

2. How long did it take for the stomp rocket to land?

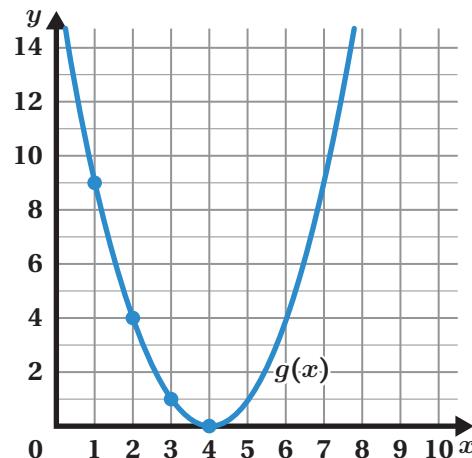
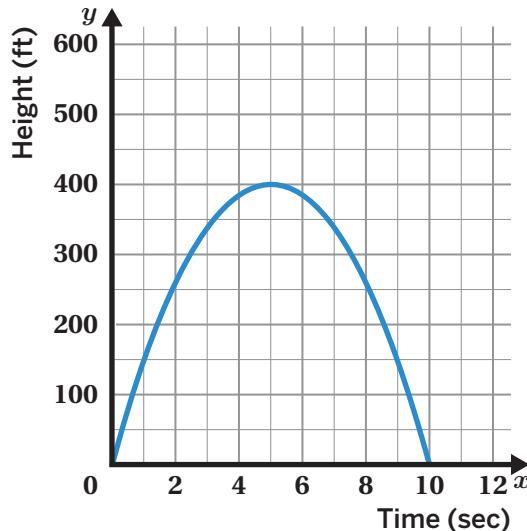
**10 seconds**

3. How high was the stomp rocket after 4 seconds?

**384 feet**

4. Here is the graph of the function  $g(x)$ . Write the equation for the line of symmetry of  $g(x)$ .

$x = \dots$  **4**.....



**Problems 5–6:** Oliver jumps off a diving board into a swimming pool. The relationship between his height and time can be modeled by a quadratic relationship.

Time (sec)	0	0.2	0.4	0.6	0.8	1	1.2	1.4
Height (m)	3	4.8	6.2	7.2	<b>7.8</b>	<b>8</b>	<b>7.8</b>	<b>7.2</b>

5. Complete the table.

6. After how many seconds will Oliver reach his maximum height?

**1 second**

# Lesson Practice

A1.7.05

Name: ..... Date: ..... Period: .....

7. The table shows the height of a stomp rocket from the time it is launched. How many seconds will it take for the rocket to land? Circle one.

8 seconds

9 seconds

Between 8  
and 9 seconds

Between 9 and  
10 seconds

Explain your thinking.

Time (sec)	Height (m)
0	0
1	42
2	74
3	96

*Explanations vary. There is a constant second difference of -10. If I continue the pattern using the constant second differences, then I get heights of 108, 110, 100, 80, 50, 10, and -40 meters. The rocket will land when the height is 0 meters, which is between 10 and -40. That is between 9 and 10 seconds.*

## Spiral Review

**Problems 8–10:** Mohamed can run a mile in 9 minutes. The number of miles Mohamed runs is a function of the amount of time he has been running.

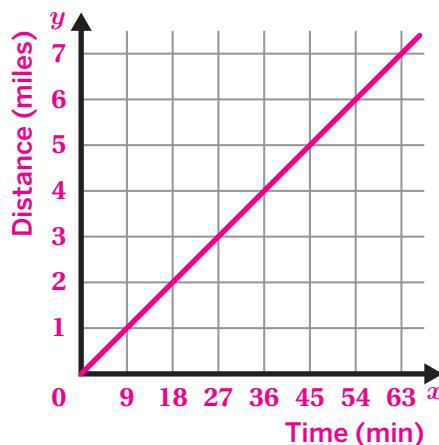
8. Graph this function. Be sure to label the axes.

9. Describe the domain of this function.

Domain:  $x \geq 0$

10. Describe the range of this function.

Range:  $y \geq 0$

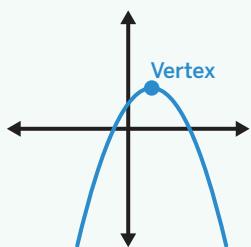


## Reflection

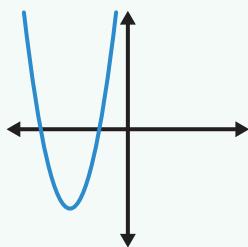
- Put a star next to a problem you could explain to a classmate.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

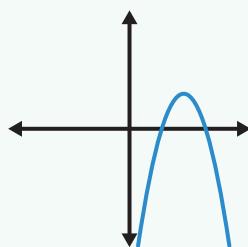
You can describe key features of *parabolas* with terms like: vertex, concave up, or concave down.

**Vertex**

The vertex is the *maximum* or *minimum* point on a parabola (where a parabola changes from increasing to decreasing, or vice versa).

**Concave Up**

A parabola that opens upward is concave up.

**Concave Down**

A parabola that opens downward is concave down.

**Things to Remember:**

# Lesson Practice

A1.7.06

Name: ..... Date: ..... Period: .....

1. The key features of this parabola are labeled *A*, *B*, *C*, and *D*.

Match each key feature with a term from the word bank.

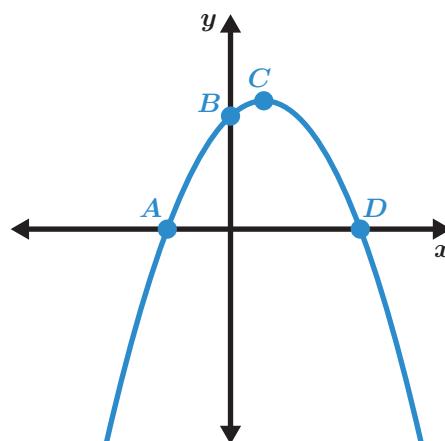
vertex     $x$ -intercept     $y$ -intercept

*A*: .....  **$x$ -intercept**.....

*B*: .....  **$y$ -intercept**.....

*C*: ..... **vertex**.....

*D*: .....  **$x$ -intercept**.....

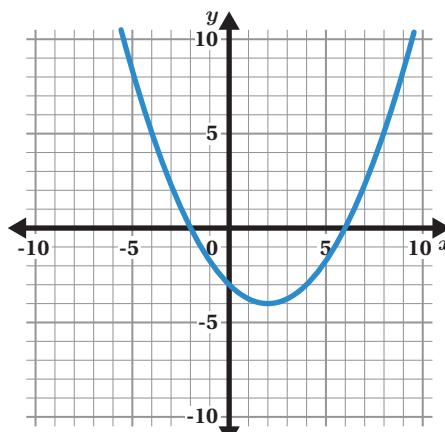


**Problems 2–4:** Use the graph to determine the coordinates of each key feature.

2. vertex: **(2, -4)**

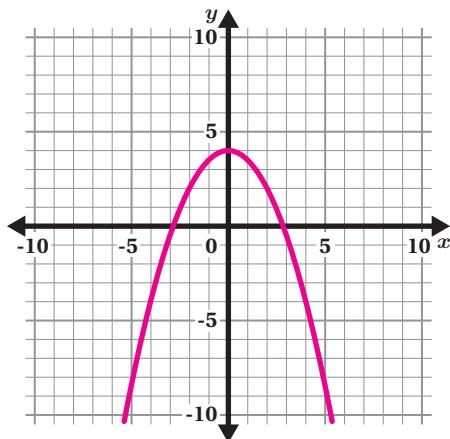
3.  $x$ -intercept(s): **(-2, 0) and (6, 0)**

4.  $y$ -intercept: **(0, -3)**

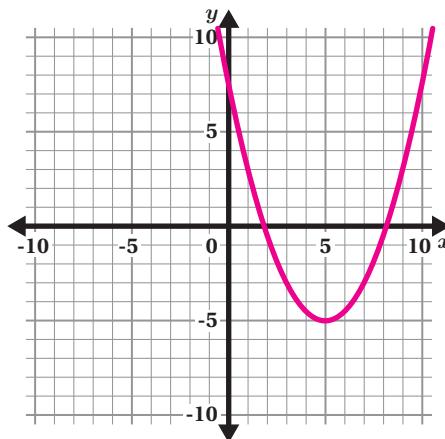


**Problems 5–6:** Graph a parabola that fits each description. **Responses vary.**

5. Concave down with a positive  $y$ -intercept



6. Concave up and vertex at (5, -5)



## Lesson Practice

A1.7.06

Name: ..... Date: ..... Period: .....

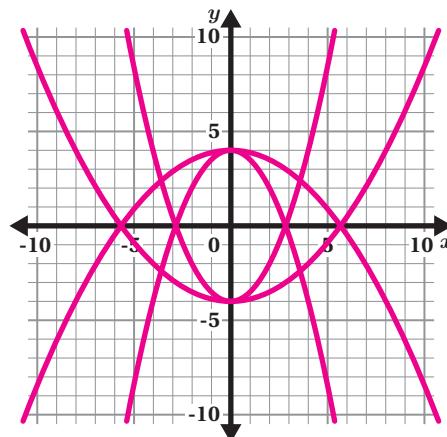
7. A parabola has a vertex at  $(4, 2)$ . Give two possible coordinates for its  $x$ -intercepts.

**Responses vary.**  $(2, 0)$  and  $(6, 0)$ . Correct responses will be an equal distance from  $x = 4$ .

8. Using only parabolas, make a design that:

- Looks the same when reflected over the  $x$ -axis.
- Looks the same when reflected over the  $y$ -axis.

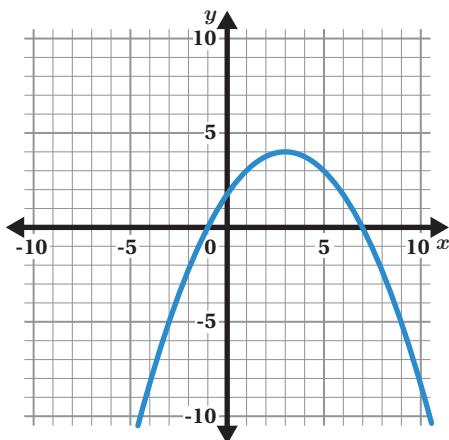
**Designs vary.**



### Spiral Review

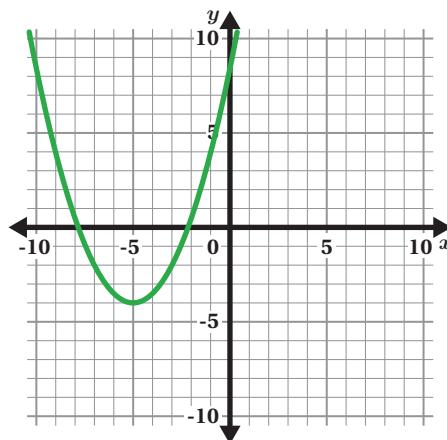
Problems 9–10: Write an equation for the line of symmetry of each parabola.

9.



Equation: .....  $x = 3$  .....

10.



Equation: .....  $x = -5$  .....

### Reflection

1. Put a question mark next to a response you'd like to compare with a classmate's.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

Different forms of quadratic equations help us see different key features of a parabola.

In a standard form quadratic equation, the *y*-*intercept* is the constant term in the equation.

In a factored form quadratic equation, the *x*-*intercepts* are the values that make either factor equal to 0.

Here is an example of the same function written in standard and factored form. We can use the different forms to determine key features and graph the parabola.

**Standard Form**

$$f(x) = 2x^2 - 4x - 6$$

$$f(x) = 2x^2 - 4x \boxed{- 6}$$

$$f(0) = 2(0)^2 - 4(0) - 6$$

The *y*-intercept is (0, -6).

**Factored Form**

$$f(x) = (2x + 2)(x - 3)$$

$x$	$(2x + 2)$	$(x - 3)$	$(2x + 2)(x - 3)$
-1	$2(-1) + 2 = 0$	$(-1) - 3 = -4$	$(0)(-4) = 0$
3	$2(3) + 2 = 8$	$(3) - 3 = 0$	$(8)(0) = 0$

The *x*-intercepts are (-1, 0) and (3, 0).

**Things to Remember:**

# Lesson Practice

A1.7.10

Name: ..... Date: ..... Period: .....

**Problems 1–2:** Determine an  $x$ -intercept(s) from each equation.

1.  $a(x) = (x - 2)(x + 1)$

(2, 0) or (-1, 0)

2.  $b(x) = (x + 3)^2$

(-3, 0)

3. Here is a graph of a quadratic function.

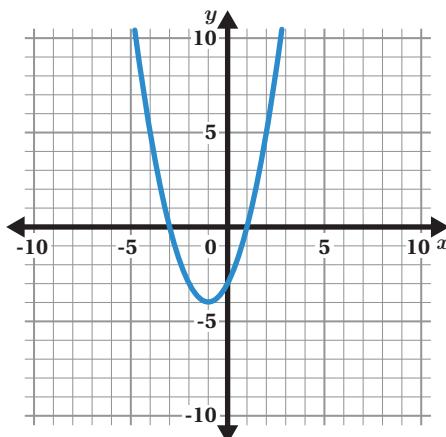
Which function could this be?

A.  $y = (x - 3)(x + 1)$

B.  $y = (x + 3)(x - 1)$

C.  $y = (x - 3)(x - 1)$

D.  $y = (x + 3)(x + 1)$



4. Here is the same function written in two forms.

**Factored form:**  $g(x) = (x + 5)(x - 2)$

**Standard form:**  $g(x) = x^2 + 3x - 10$

What are the  $x$ - and  $y$ -intercepts of the function?

$x$ -intercept: (-5, 0).....

$x$ -intercept: (2, 0).....

$y$ -intercept: (0, -10)....

5. Crow and Ariel were working on a problem together.

Crow said: *The  $y$ -intercept of  $y = (x - 3)^2$  is (0, 3).*

Ariel said: *The  $x$ -intercept of  $y = (x - 3)^2$  is (3, 0).*

Whose thinking is correct? Circle one.

Crow's

Ariel's

Explain your thinking

*Explanations vary. If I make  $x = 0$ , then  $(0 - 3)^2 = 9$ , not 3. If I make  $x = 3$ , then  $(3 - 3)^2 = 0$ , so Ariel is correct.*

## Lesson Practice

### A1.7.10

Name: ..... Date: ..... Period: .....

**Problems 6–7:** Here is a function:  $h(x) = x(x + 6)$ .

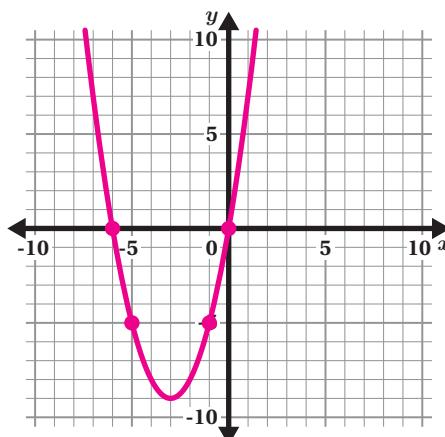
6. Determine the intercepts of  $h(x)$ .

$x$ -intercept: ..... **(-6, 0)**.....

$x$ -intercept: ..... **(0, 0)**.....

$y$ -intercept: ..... **(0, 0)**.....

7. Draw the graph of  $h(x)$ .



**Problems 8–10:** Use a graphing calculator to graph  $f(x) = (x + 3)(x + 1)(x - 2)$ . Change the numbers in the equation to determine an equation whose graph ... **Responses vary.**

8. Has two  $x$ -intercepts.

$$f(x) = (x - 1)(x - 2)(x - 2)$$

Any response with two identical factors is considered correct.

9. Has one  $x$ -intercept.

$$f(x) = (x - 2)(x - 2)(x - 2)$$

Any response with three identical factors is considered correct.

10. Has an  $x$ -intercept at  $(7, 0)$ .

$$f(x) = (x - 1)(x - 3)(x - 7)$$

Any response that includes a factor of  $(x - 7)$  is considered correct.

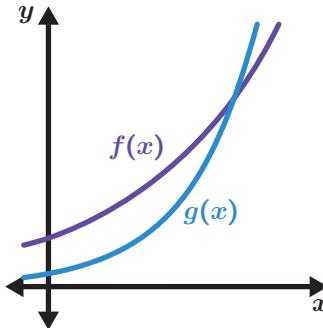
## Spiral Review

11. Here are the graphs of two functions,  $f(x)$  and  $g(x)$ .

$$f(x) = 100 \cdot 2^x$$

Which equation could represent  $g(x)$ ?

- A.  $g(x) = 25 \cdot 4^x$   
B.  $g(x) = 50 \cdot 1.5^x$   
C.  $g(x) = 100 \cdot 4^x$   
D.  $g(x) = 200 \cdot 1.5^x$



**Problems 12–13:** Complete each equation with a number that makes the equation true.

12.  $2 \cdot \underline{-2} + 4 = 0$

13.  $3(3 \cdot \underline{-\frac{4}{3}} + 4) = 0$

## Reflection

- Star a problem you're still feeling confused about.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

We can use key features of a quadratic function to create graphs.

Here is an example. Determine the  $x$ -intercepts and the vertex of the function  $f(x) = (x + 5)(x - 1)$ .

The  $x$ -intercepts of the graph are the  $x$ -values that make each factor equal to 0.

$x$	$(x + 5)$	$(x - 1)$	$(x + 5)(x - 1)$
-5	$(-5) + 5 = 0$	$(-5) - 1 = -6$	$(0)(-6) = 0$
1	$(1) + 5 = 6$	$(1) - 1 = 0$	$(6)(0) = 0$

The  $x$ -intercepts are  $(-5, 0)$  and  $(1, 0)$ .

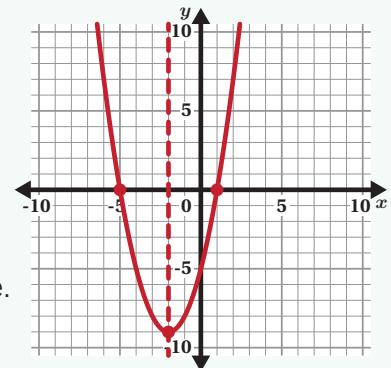
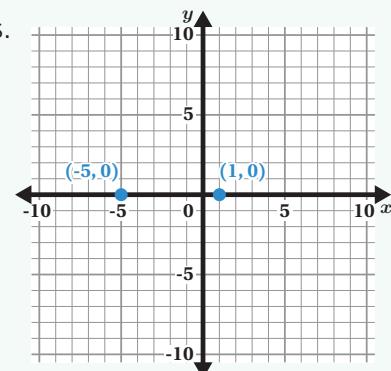
The vertex is always directly between the two  $x$ -intercepts.

The  $x$ -value of the vertex is -2 because -2 is exactly in the middle of the two  $x$ -intercepts.

We substitute  $x = -2$  into the equation to determine the  $y$ -value.

$x$	$(x + 5)$	$(x - 1)$	$(x + 5)(x - 1)$
-2	$(-2) + 5 = 3$	$(-2) - 1 = -3$	$(3)(-3) = -9$

The vertex is at the point  $(-2, 9)$ .

**Things to Remember:**

# Lesson Practice

A1.7.11

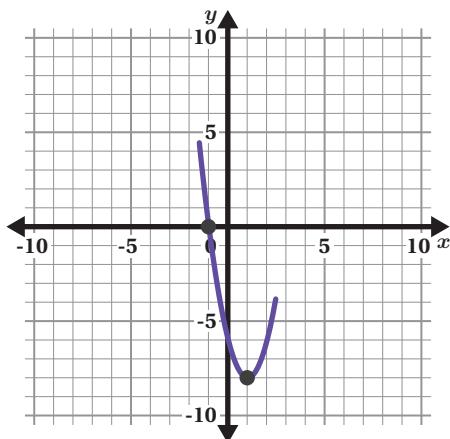
Name: ..... Date: ..... Period: .....

**Problems 1–2:** Determine one point to zap that will light up the entire parabola.

**Responses vary.**

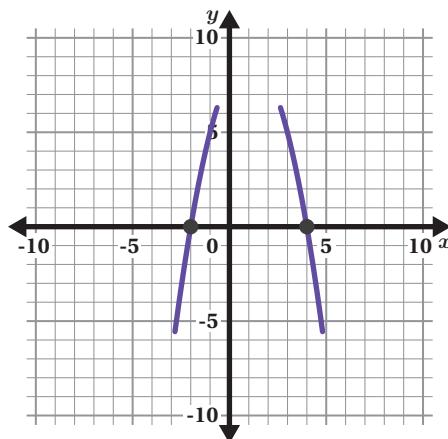
1. Equation:  $y = 2(x - 3)(x + 1)$

Point: **(3, 0)**



2. Equation:  $y = -1(x - 4)(x + 2)$

Point: **(1, 9)**



**Problems 3–5:** A parabola has  $x$ -intercepts at  $(3, 0)$  and  $(7, 0)$ . Determine whether each statement is true or false, or if there is not enough information.

3. The vertex of the parabola is at  $(5, -4)$ .      True      False      **Not enough information**
4. The line of symmetry is at  $x = 5$ .      **True**      False      Not enough information
5. The parabola is concave up.      True      False      **Not enough information**

**Problems 6–7:** Here is a function:  $g(x) = (-2x + 4)(x - 6)$ .

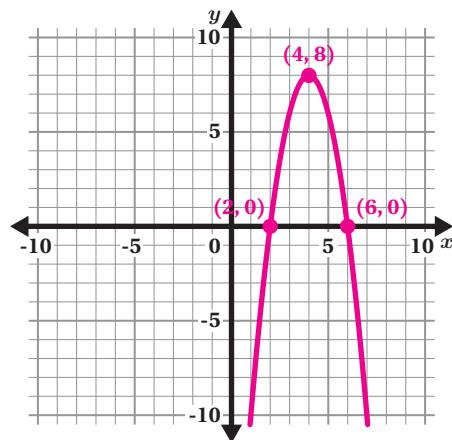
6. Determine the  $x$ -intercepts and vertex of  $g(x)$ .

$x$ -intercept: **(2, 0)**

$x$ -intercept: **(6, 0)**

Vertex: **(4, 8)**

7. Draw the graph of the function  $g(x)$ .



## Lesson Practice

A1.7.11

Name: ..... Date: ..... Period: .....

8. Zahra and Santino were graphing  $p(x) = (x + 3)^2$ .

Zahra says: *This graph doesn't have a vertex because there's only one  $x$ -intercept.*

Santino says: *The vertex is the same as the  $x$ -intercept.*

Whose thinking is correct? Circle one.

Zahra's

Santino's

Both

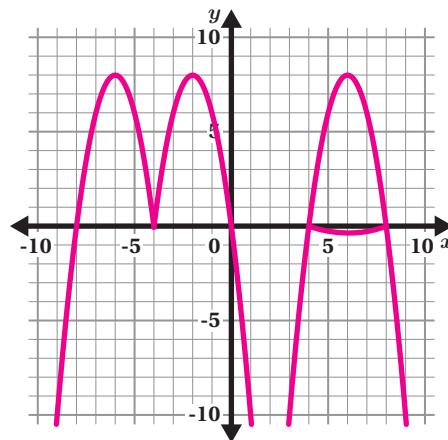
Neither

Explain your thinking. **Explanations vary.** There is only one  $x$ -intercept at  $(-3, 0)$ .

The points on either side of that are  $(-2, 1)$  and  $(-4, 1)$ . This means that  $(-3, 0)$  is also the vertex because the points on either side of it are symmetrical.

9. Using only parabolas, write the equations of your initials. Use the graph and/or graphing technology to help with your thinking. **Responses vary.**

- $y = -2(x + 8)(x + 4) \{x < -4\}$
- $y = -2(x + 4)(x + 0) \{x > -4\}$
- $y = -2(x - 4)(x - 8)$
- $y = \frac{1}{10}(x - 4)(x - 8) \{4 < x < 8\}$



## Spiral Review

Problems 10–11: Determine the  $x$ - and  $y$ -intercepts for each equation.

10.  $y = 4x + 8$

$x$ -intercept:  $(-2, 0)$

$y$ -intercept:  $(0, 8)$

11.  $2x - 3y = 9$

$x$ -intercept:  $(4.5, 0)$

$y$ -intercept:  $(0, -3)$

## Reflection

1. Put a smiley face next to the problem you learned from most.
2. Use this space to ask a question or share something you're proud of.

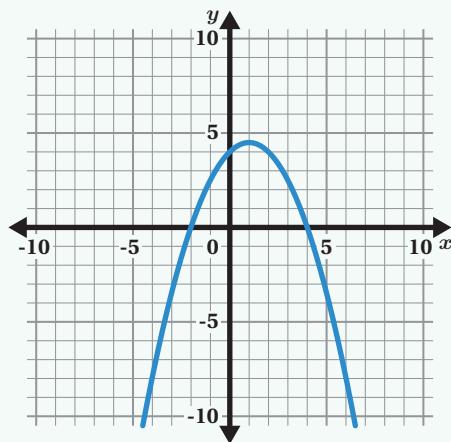
**Lesson Summary**

We can write quadratic equations to match graphs and key features. One way to do this is in the form  $y = a(x - m)(x - n)$ .

- Use the  $x$ -intercepts to write the factors of the equation.
- If the parabola is concave down, make the  $a$ -value negative.
- Adjust the  $a$ -value to match the vertical position of the vertex and the  $y$ -intercept.

Here is an example strategy for writing an equation to match this graph.

- The  $x$ -intercepts are  $(-2, 0)$  and  $(4, 0)$ . You can write the factors as  $y = (x + 2)(x - 4)$ .
- Since the parabola is concave down, multiply by a negative number, like  $y = -(x + 2)(x - 4)$ .
- Adjust the  $a$ -value to make the vertical position of the vertex match the graph, like  $y = -\frac{1}{2}(x + 2)(x - 4)$ .

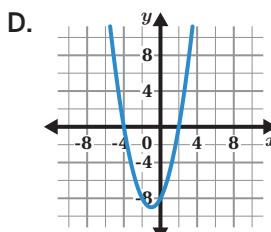
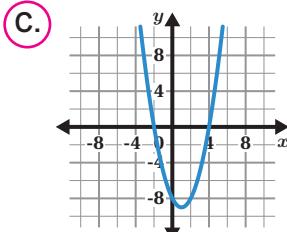
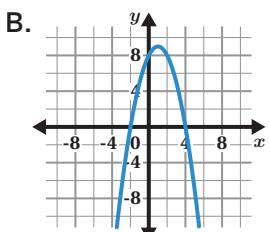
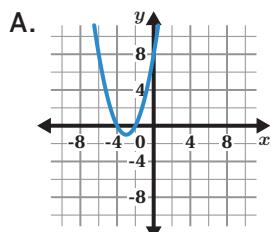
**Things to Remember:**

# Lesson Practice

A1.7.12

Name: ..... Date: ..... Period: .....

1. Which graph shows the function  $y = (x - 4)(x + 2)$ ?



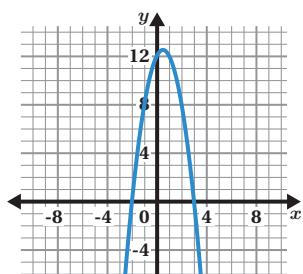
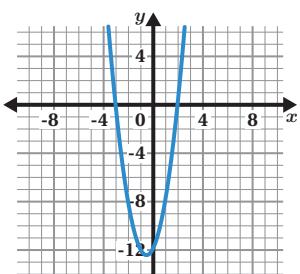
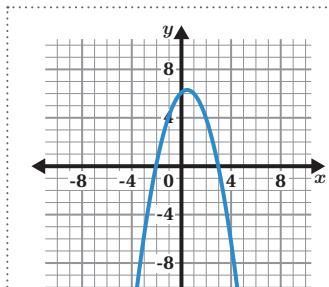
2. Match each equation to the graph it represents. One equation will have no match.

$$y = -(x + 2)(x - 3)$$

$$y = 2(x + 3)(x - 2)$$

$$y = -2(x + 2)(x - 3)$$

$$y = (x + 2)(x - 3)$$



$y = -(x + 2)(x - 3)$

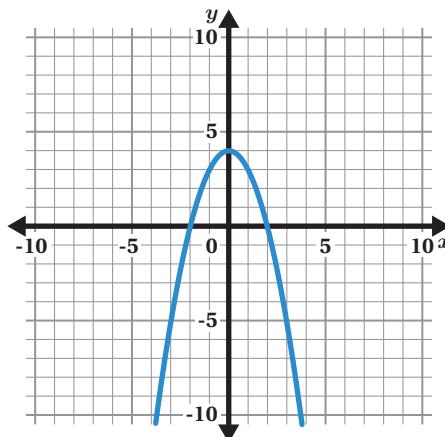
$y = 2(x + 3)(x - 2)$

$y = -2(x + 2)(x - 3)$

Problems 3–4: Here is the graph of  $y = -1(x + 2)(x - 2)$ . Change the equation so the vertex goes through:

3.  $(0, 8)$

$y = -2(x + 2)(x - 2)$



4.  $(0, -2)$

$y = \frac{1}{2}(x + 2)(x - 2)$

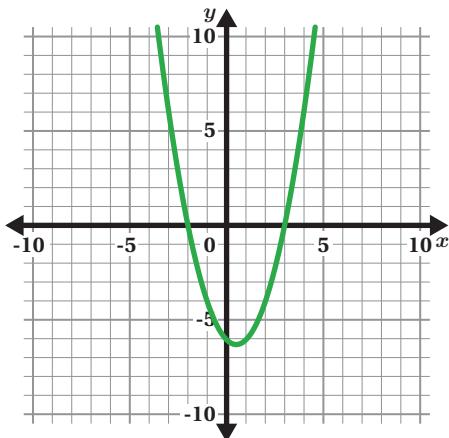
# Lesson Practice

A1.7.12

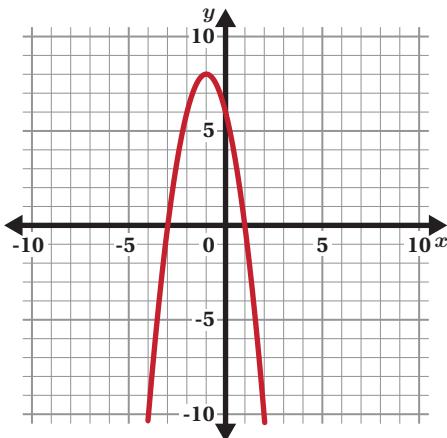
Name: ..... Date: ..... Period: .....

**Problems 5–6:** Write an equation to match each graph.

5. Equation:  $y = (x + 2)(x - 3)$



6. Equation:  $y = -2(x - 1)(x + 3)$



7. Write the equations of three different parabolas that have the same vertex but different  $x$ -intercepts. *Responses vary.*

Equation 1	Equation 2	Equation 3
$y = (x + 2)(x - 2)$	$y = \frac{1}{4}(x + 4)(x - 4)$	$y = 4(x + 1)(x - 1)$

## Spiral Review

**Problems 8–9:** Here are two linear functions:  $f(x)$  and  $h(x)$ .

$$f(x) = 3x + 2$$

8. Which function has a greater  $y$ -intercept? Circle one.

$f(x)$

$h(x)$

9. Which function has a greater slope? Circle one.

$f(x)$

$h(x)$

$x$	$h(x)$
1	5
2	9
3	13

## Reflection

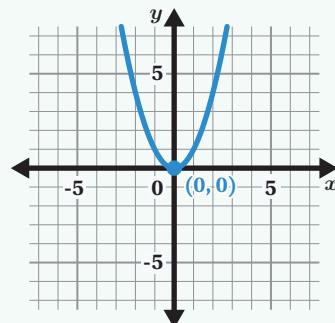
- Put a heart next to a problem you understand well.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

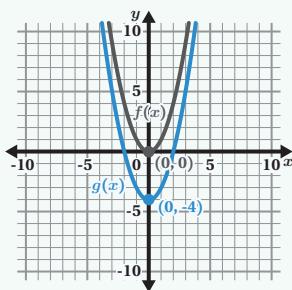
Here is the graph of  $f(x) = x^2$ .

Functions can be translated horizontally and vertically.

Here are three examples of *translations* of  $f(x)$ . The equations for these translations are written in **vertex form**, which highlights the coordinates of the vertex in the equation.



## Vertical Translations

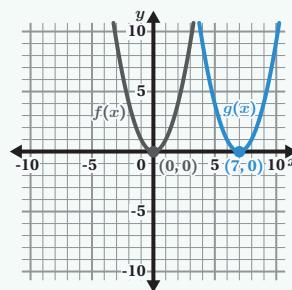


$f(x)$  is translated 4 units down.

Its equation is  

$$g(x) = x^2 - 4.$$

## Horizontal Translations

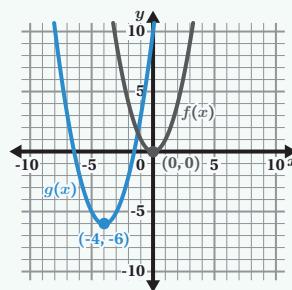


$f(x)$  is translated 7 units right.

Its equation is  

$$h(x) = (x - 7)^2.$$

## Vertical and Horizontal Translations



$f(x)$  is translated 4 units left and 6 units down.

Its equation is  

$$j(x) = (x + 4)^2 - 6.$$

## Things to Remember:

# Lesson Practice

A1.7.14

Name: ..... Date: ..... Period: .....

**Problems 1–2:** Circle if the transformation of  $y = x^2$  is a *horizontal translation*, *vertical translation*, or *neither*.

1.  $y = (x - 3)^2$

Horizontal  
Translation

Vertical  
Translation

Neither

2.  $y = x^2 - 5$

Horizontal  
Translation

Vertical  
Translation

Neither

3. These parabolas are translations of  $y = x^2$ .

Select *all* of the equations shown in the graph.

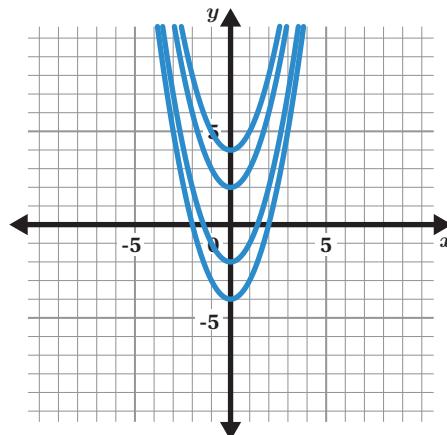
A.  $y = x^2 + 2$

B.  $y = (x - 2)^2$

C.  $y = x^2 - 2$

D.  $y = x^2 - 4$

E.  $y = (x + 4)^2$



4. Match each vertex of a parabola to its equation.

a.  $y = (x - 3)^2 + 5$

e. .... (0, -4)

b.  $y = (x + 7)^2 + 3$

b. .... (-7, 3)

c.  $y = (x - 4)^2$

d. .... (-3, 5)

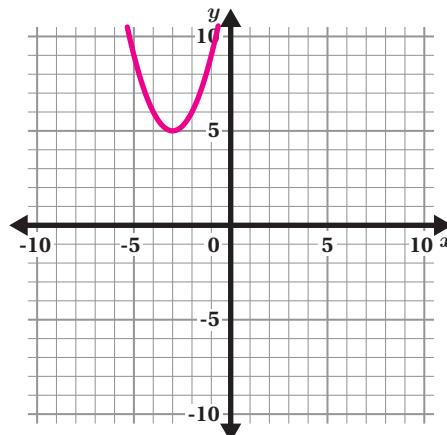
d.  $y = (x + 3)^2 + 5$

a. .... (3, 5)

e.  $y = x^2 - 4$

c. .... (4, 0)

5. Draw the graph of  $y = (x + 3)^2 + 5$ .



# Lesson Practice

A1.7.14

Name: ..... Date: ..... Period: .....

6. Here are four equations in vertex form. Which function has a graph with a vertex at  $(1, 3)$ ?

A.  $y = (x + 1)^2 + 3$

B.  $y = (x - 1)^2 + 3$

C.  $y = (x - 3)^2 + 1$

D.  $y = (x + 3)^2 + 1$

7. Write and graph the equations of five parabolas so that when you connect their vertices, they form a sixth parabola. **Responses vary.**

## Equations

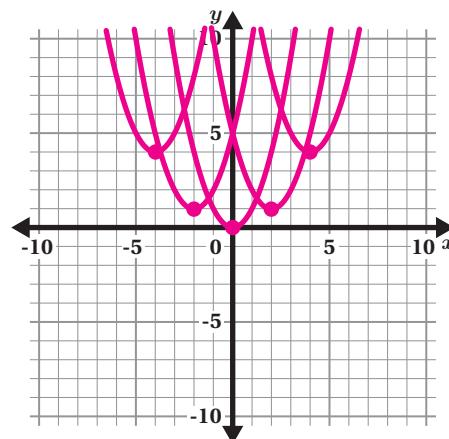
$$y = x^2$$

$$y = (x + 2)^2 + 1$$

$$y = (x - 2)^2 + 1$$

$$y = (x + 4)^2 + 4$$

$$y = (x - 4)^2 + 4$$

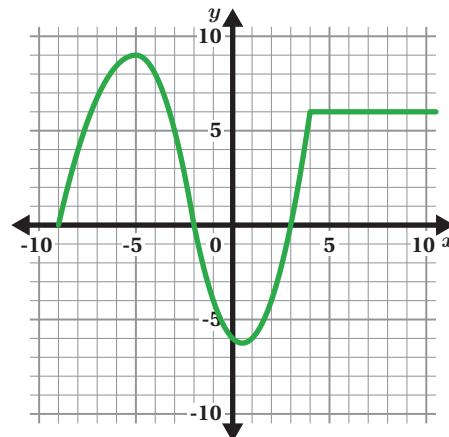


## Spiral Review

Problems 8–10: Here is a graph of  $g(x)$ .

8. At what point does the maximum occur?

$(-5, 9)$



9. At what points does  $g(x) = 0$ ?

$(-9, 0)$ ,  $(-2, 0)$ , and  $(3, 0)$

10. What is the  $y$ -intercept?

$(0, -6)$

## Reflection

- Put a star next to a problem you want to understand better.
- Use this space to ask a question or share something you're proud of.

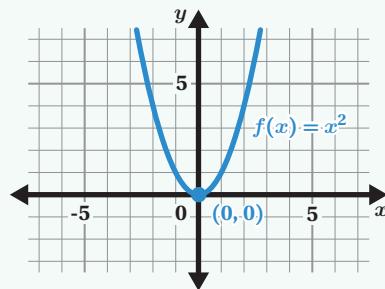
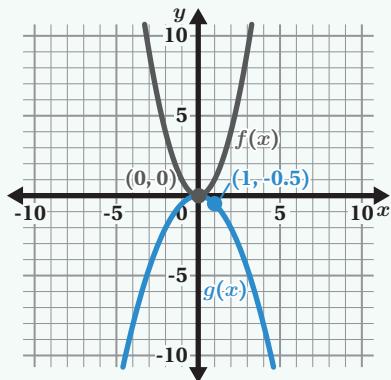
**Lesson Summary**

Quadratic functions like  $f(x) = a(x - h)^2 + k$  are written in vertex form, where  $(h, k)$  is the vertex of the parabola, and  $a$  shows the vertical scale.

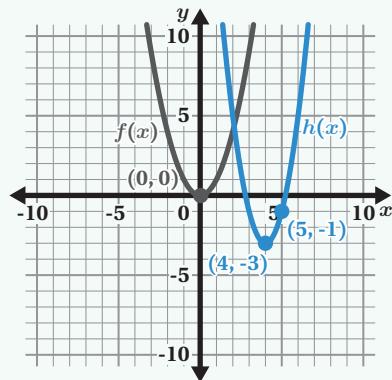
The  $a$ -value:

- Multiplies each output of the function by a constant value.
- Shows by what factor the graph is scaled in the  $y$ -direction.
- Tells whether a parabola is concave up or concave down.

Let's look at some examples of quadratic functions that have been scaled vertically:

**Vertical Scale**

Here  $f(x)$  was scaled vertically by a factor of  $-\frac{1}{2}$ . Its equation is  $g(x) = -\frac{1}{2}x^2$ . This made the parabola wider and concave down.

**Vertical Scale and Translations**

Here  $f(x)$  was scaled vertically by a factor of 2 and translated right 4 units and down 3 units. Its equation is  $h(x) = 2(x - 4)^2 - 3$ .

**Things to Remember:**

# Lesson Practice

A1.7.15

Name: ..... Date: ..... Period: .....

**Problems 1–3:** Here's the graph of  $y = x^2$ . Change one number to make the graph:

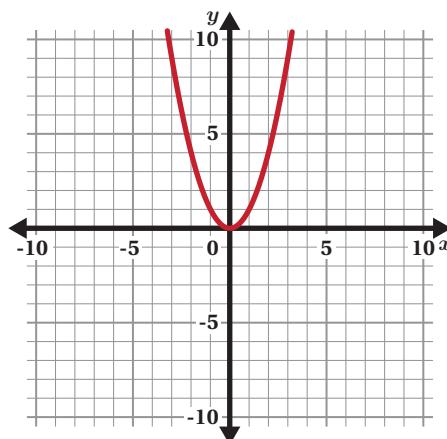
**Responses vary.**

1. Wider:  $y = \frac{1}{2}x^2$  (or equations with  $0 < a < 1$ )

2. Narrower:  $y = 2x^2$  (or equations with  $a > 1$ )

3. Open down:

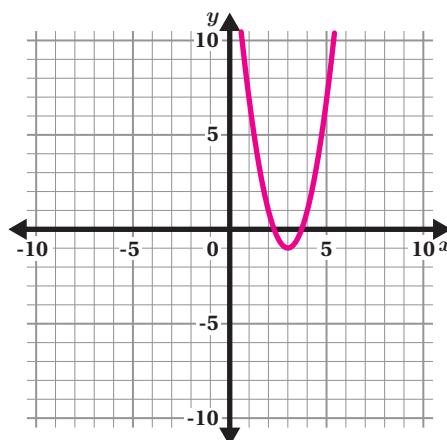
$$y = -1x^2 \text{ (or equations with } a < 0\text{)}$$



4. Describe how the graph of  $y = x^2$  compares to  $y = -4(x - 2)^2 + 7$ .

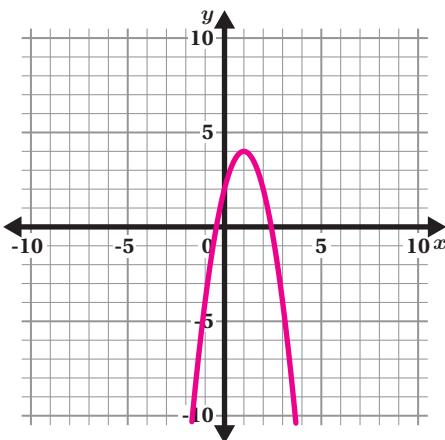
**Responses vary.** The vertex would be translated to (2, 7) and the parabola would open down. There is also a vertical scale so that the vertical distance between the vertex and the next point would be 4 units.

5. Draw a graph of a parabola that has a vertex at (3, -1) and is scaled vertically by a factor of 2.

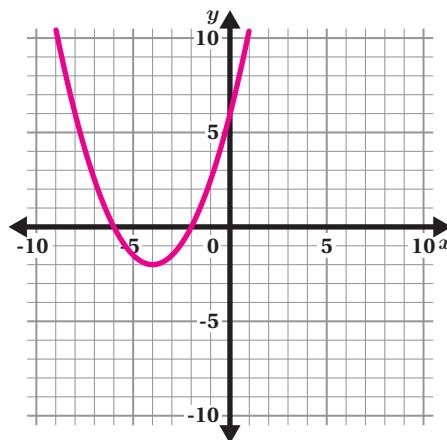


**Problems 6–7:** Draw a graph for each equation.

6.  $a(x) = -2(x - 1)^2 + 4$



7.  $c(x) = 0.5(x + 4)^2 - 2$



# Lesson Practice

A1.7.15

Name: ..... Date: ..... Period: .....

8. Using at least 4 parabolas, draw a graph that reminds you of a rainbow. The space between the  $y$ -intercept of each parabola should remain the same.

*Graphs and equations vary.*

Record the functions you use.

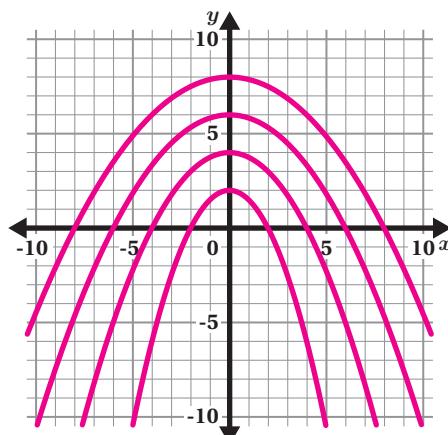
## Functions

$$y = -\frac{1}{8}x^2 + 8$$

$$y = -\frac{1}{6}x^2 + 6$$

$$y = -\frac{1}{4}x^2 + 4$$

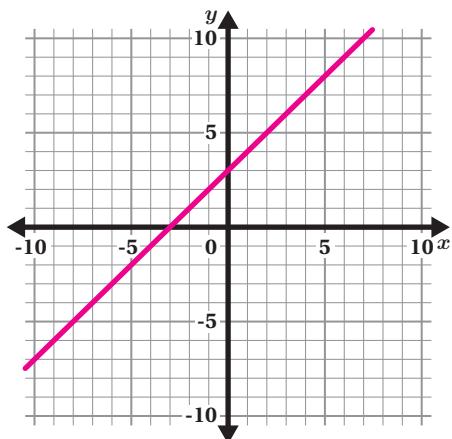
$$y = -\frac{1}{2}x^2 + 2$$



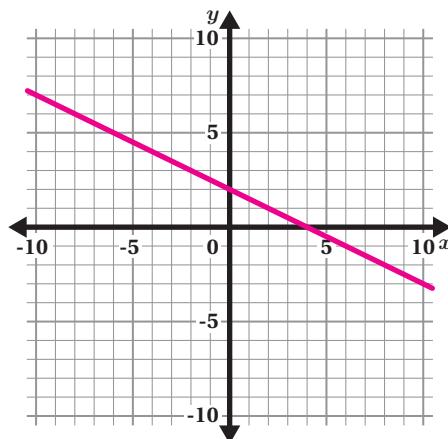
## Spiral Review

Problems 9–10: Draw a graph of each equation.

9.  $-3x + 3y = 9$



10.  $2x + 4y = 8$



## Reflection

1. Circle the problem you enjoyed doing the most.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

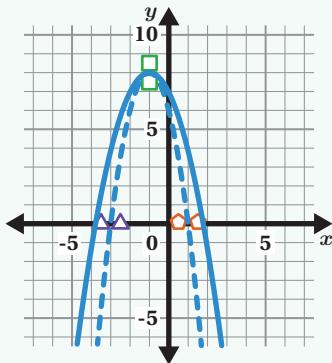
You can use key features to write quadratic functions in standard, factored, or vertex form.

Here are two strategies for using key features to write an equation of a parabola that will pass through these gates:

**Vertex Form:**  $f(x) = a(x - h)^2 + k$

Identify a possible vertex:  $(-1, 8)$

$$f(x) = a(x + 1)^2 + 8$$



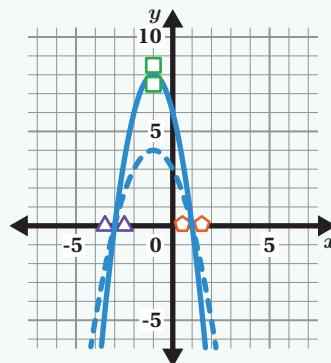
Since the parabola is concave down, the  $a$ -value needs to be negative. Then I can adjust the value to scale the parabola vertically to match the  $x$ -intercepts.

$$f(x) = -2(x + 1) + 8$$

**Factored Form:**  $f(x) = a(x - b)(x - c)$

Identify possible  $x$ -intercepts:  $(-3, 0)$  and  $(1, 0)$  Substitute the  $x$ -values of the  $x$ -intercepts into  $b$  and  $c$ :

$$f(x) = a(x + 3)(x - 1)$$



Since the parabola is concave down, the  $a$ -value needs to be negative. Then I can adjust the value to scale the parabola vertically to adjust the vertical position of the vertex.

$$f(x) = -2(x + 3)(x - 1)$$

## Things to Remember:

# Lesson Practice

A1.7.16

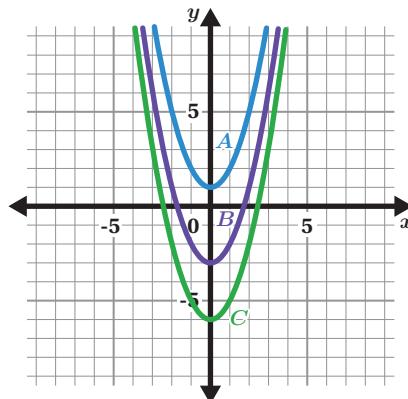
Name: ..... Date: ..... Period: .....

1. Parabolas A, B, and C are translations of  $y = x^2$ . Determine the equation of each parabola.

A:  $y = x^2 + 1$

B:  $y = x^2 - 3$

C:  $y = x^2 - 6$



2. Which equation has a graph with a vertex at (1, 3)?

A.  $f(x) = (x - 1)^2 + 3$

B.  $f(x) = (x - 3)^2 + 1$

C.  $f(x) = (x + 1)^2 + 3$

D.  $f(x) = (x + 3)^2 + 1$

Explain your thinking.

*Explanations vary. When you substitute 1 for  $x$ , then  $(1 - 1)^2 = 0$ . Any other number squared would be larger, so this is the minimum.  $(1 - 1)^2 + 3 = 3$ , so the vertex is (1, 3).*

3. Select all the equations that match this graph.

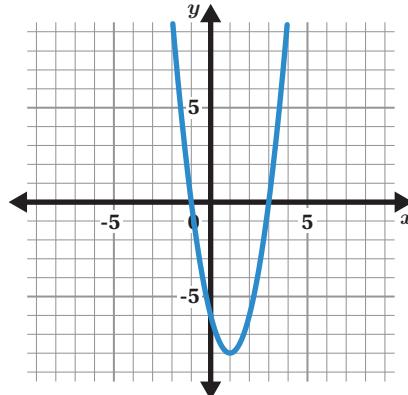
A.  $y = 2(x - 1)^2 - 8$

B.  $y = -(x - 1)^2 - 8$

C.  $y = (2x - 2)(x + 3)$

D.  $y = (2x + 2)(x - 3)$

E.  $y = (x - 1)^2 - 3$



4. Write an equation for a quadratic function with  $x$ -intercepts at (-2, 0) and (6, 0).

*Responses vary.  $y = (x + 2)(x - 6)$*

**Problems 5–6:** Write an equation for a quadratic function that matches each description. Use graphing technology to check your work.

5. Concave down with a vertex at (-2, 6).

*Responses vary.  $y = -(x + 2)^2 + 6$*

6. Concave up with a vertex at (0, -8).

*Responses vary.  $y = x^2 - 8$*

# Lesson Practice

A1.7.16

Name: ..... Date: ..... Period: .....

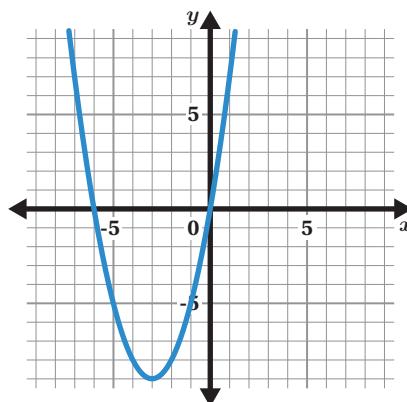
7. Here are two equations:

$$m(x) = x(x + 6)$$

$$p(x) = (x + 3)^2 - 9$$

Show or explain how you know that both equations describe this graph.

**Responses vary.**  $m(x)$  is written in factored form, so I know the  $x$ -intercepts are at 0 and -6. The vertex is exactly between the  $x$ -intercepts, so I plugged in  $x = -3$ . I found the vertex was at  $(-3, -9)$ .  $p(x)$  is written in vertex form and shows the vertex is at  $(-3, -9)$ .



8. Which equation describes this graph?

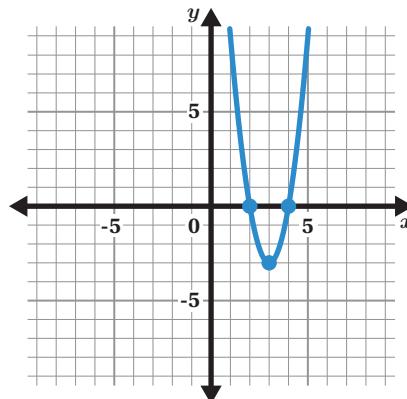
Circle your choice.

$$g(x) = 3(x + 3)^2 + 3$$

$$h(x) = 3(x - 3)^2 - 3$$

Explain your thinking.

**Explanations vary.** I know that  $h(x)$  describes this graph because the vertex of  $h(x)$  is  $(3, -3)$ , which matches the vertex on the graph. The vertex of  $g(x)$  is  $(-3, 3)$ , which is incorrect.



## Spiral Review

**Problems 9–10:** Write an equation that matches each description.

9. Description of  $f(x)$ :

- linear function
- $y$ -intercept of 12
- negative slope

**Responses vary.**  $f(x) = -3x + 12$

10. Description of  $g(x)$ :

- quadratic function
- $x$ -intercepts of  $(4, 0)$  and  $(7, 0)$
- concave up

**Responses vary.**  $g(x) = (x - 4)(x - 7)$

## Reflection

1. Put a star next to a problem that looked more difficult than it really was.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

We can use quadratic equations, tables, and graphs to make sense of situations in society. These mathematical models can help inform the decisions we make about real-world issues. In real-world situations, it is important to be able to create and compare models of different relationships.

For example, in this lesson we explored two models that represent the cost of housing. We used a linear model to explore the relationship between the price of rent and the number of units rented. We used a quadratic model to explore the relationship between rent and revenue generated. A community organization could use these models to determine a fairer price of rent.

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### Things to Remember:

# Lesson Practice

A1.7.17

Name: ..... Date: ..... Period: .....

1. Which function has a graph with a vertex at  $(-1, 4)$ ?

- A.  $y = (x - 1)^2 + 4$       B.  $y = (x - 4)^2 - 1$   
 C.  $y = (x + 1)^2 + 4$       D.  $y = (x + 4)^2 - 1$

2. Select *all* the functions whose graphs have an  $x$ -intercept at  $(3, 0)$ .

- A.  $a(x) = (x + 2)(x - 3)$   
 B.  $b(x) = (x + 3)(x - 2)$   
 C.  $c(x) = 3x(x - 2)$   
 D.  $d(x) = (2x - 6)(x + 2)$   
 E.  $e(x) = (2x - 3)(x + 3)$

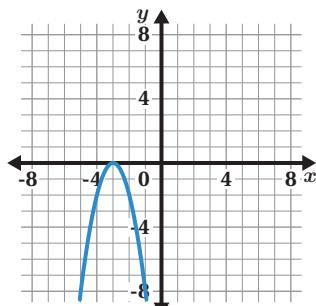
3. Match each graph to the quadratic equation it represents. One equation has no match.

a.  $y = -(x - 3)^2 + 2$

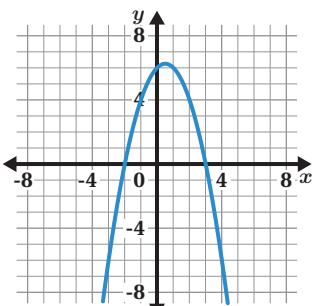
b.  $y = -3x^2 + 2$

c.  $y = -2(x + 3)^2$

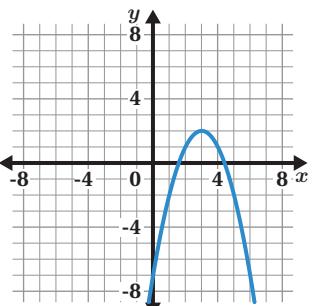
d.  $y = -(x + 2)(x - 3)$



Equation: c



Equation: d



Equation: a

4. Write an equation of a parabola that has a vertex at  $(-2, 1)$ .

**Responses vary.**

- $y = (x + 2)^2 + 1$
- $y = -2(x + 2)^2 + 1$
- $y = -(x + 1)(x + 3)$

# Lesson Practice

A1.7.17

Name: ..... Date: ..... Period: .....

5. Azul and Latifa try to write an equation to describe this graph. Each equation is incorrect in some way.

Azul

$$y = (x - 1)^2 + 4$$

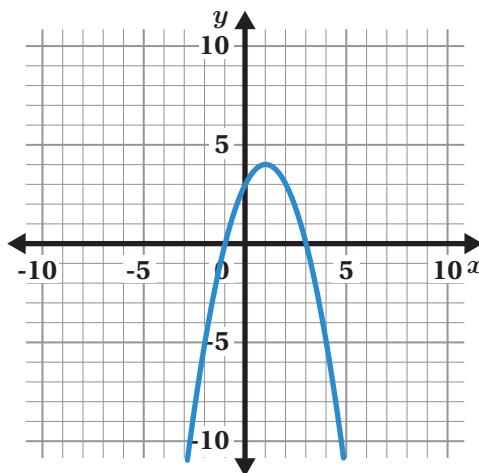
Latifa

$$y = -(x + 3)(x - 1)$$

Choose one equation. Explain how you would change the equation so that it creates this graph.

*Responses vary.*

- Azul: The graph of Azul's equation will be concave up. To make the graph concave down, I would multiply  $(x - 1)^2$  by -1. Her new equation would be  $y = -(x - 1)^2 + 4$ .
- Latifa: The  $x$ -intercepts of Latifa's graph are  $(-3, 0)$  and  $(1, 0)$ . Changing Latifa's equation to  $y = -(x - 3)(x + 1)$  would make the  $x$ -intercepts  $(3, 0)$  and  $(-1, 0)$ .



**Problems 6–7:** Here is a function:  $m(x) = 2x(x - 3)$ .

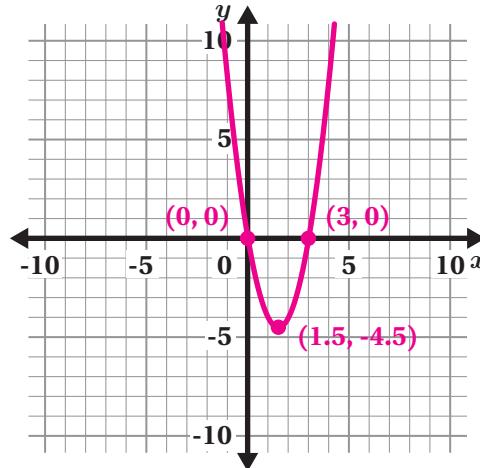
6. Determine the  $x$ -intercepts and vertex of  $m(x)$ .

$x$ -intercept: ..... **(0, 0)**.....

$x$ -intercept: ..... **(3, 0)**.....

vertex: ..... **(1.5, -4.5)**.....

7. Draw the graph of the function  $m(x)$ .



## Spiral Review

**Problems 8–10:** Determine the value of each function when  $x = -1$ .

8.  $f(x) = 2(x - 1)^2 + 3$   
 **$f(-1) = 11$**

9.  $g(x) = 3x^2 - 2x - 1$   
 **$g(-1) = 4$**

10.  $h(x) = (2x + 3)(x - 1)$   
 **$h(-1) = -2$**

## Reflection

1. Put a heart next to the problem you're most proud of.
2. Use this space to ask a question or share something you're proud of.

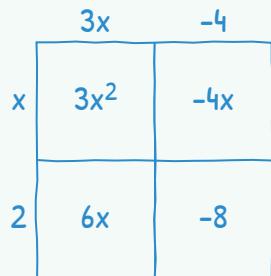
**Lesson Summary**

Quadratic expressions can be written in *factored form* or *standard form*.

You can use an area model to help you rewrite a factored-form quadratic expression into an equivalent expression in standard form.

Here are two examples.

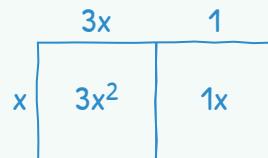
Factored form:  $(3x - 4)(x + 2)$



$$3x^2 + 6x - 4x - 8$$

Standard form:  $3x^2 + 2x - 8$

Factored form:  $x(3x + 1)$



Standard form:  $3x^2 + x$

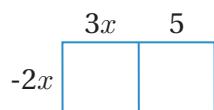
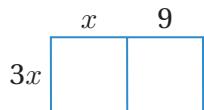
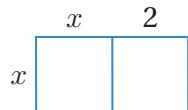
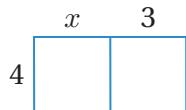
**Things to Remember:**

# Lesson Practice

A1.8.02

Name: ..... Date: ..... Period: .....

1. Write an expression that represents each area model.



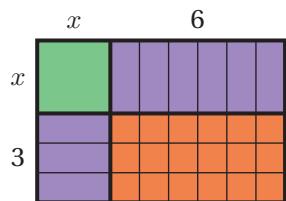
$$4x + 12 \quad (\text{or equivalent})$$

$$x^2 + 2x \quad (\text{or equivalent})$$

$$3x^2 + 27x \quad (\text{or equivalent})$$

$$-6x^2 - 10x \quad (\text{or equivalent})$$

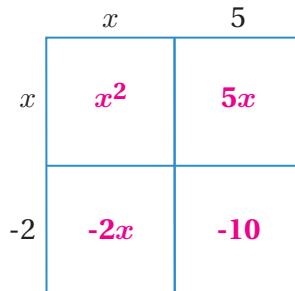
2. Write two expressions that match the area model.



Factored form:  $(x + 3)(x + 6)$

Standard form:  $x^2 + 9x + 18$

3. Multiply to rewrite  $(x + 5)(x - 2)$  in standard form. Use the diagram if it helps with your thinking.



$$x^2 + 3x - 10$$

4. Match each expression to its equivalent expression in standard form.

a.  $(x + 2)(x + 6)$

c.  $x^2 + 12x + 32$

b.  $(2x + 8)(x + 2)$

d.  $2x^2 + 18x + 16$

c.  $(x + 8)(x + 4)$

b.  $2x^2 + 12x + 16$

d.  $(x + 8)(2x + 2)$

a.  $x^2 + 8x + 12$

# Lesson Practice

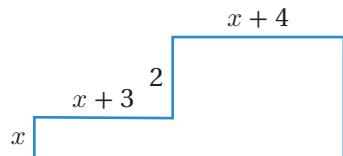
A1.8.02

Name: ..... Date: ..... Period: .....

5. Complete the table by writing each expression in standard form.

Factored Form	Standard Form
$(x + 7)(x + 3)$	$x^2 + 10x + 21$
$(x - 2)(x - 12)$	$x^2 - 14x + 24$

6. Write an expression that represents the area and an expression for the perimeter of this figure:



Area:  $2x^2 + 9x + 8$  (or equivalent)

Perimeter:  $6x + 18$  (or equivalent)

## Spiral Review

Problems 7–9: Complete each equivalent expression:

7.  $14a + 21 = \underline{7} (\underline{2} a + \underline{3})$       8.  $3b + 2 + 3b + 10 = \underline{6} (\underline{1} b + \underline{2})$

9.  $-3(3c + 3) = \underline{-9} c + \underline{-9}$

## Reflection

- Put a star next to the problem you spent the most time on.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can analyze the structure of a quadratic expression written in *factored form* to make predictions about what the equivalent expression in *standard form*,  $ax^2 + bx + c$ , will look like.

Here are two strategies for multiplying a factored-form expression to rewrite quadratic expressions in standard form.

**Strategy 1**

$$(2x - 5)(x + 3)$$

2x	-5
x	$2x^2$
3	$-5x$

Standard form:  $2x^2 + x - 15$

$$a = 2 \quad b = 1 \quad c = -15$$

**Strategy 2**

$$(3x - 4)(3x + 4)$$

$$(3x - 4)(3x + 4)$$

9x<sup>2</sup> + 12x - 12x - 16

Standard form: 9x<sup>2</sup> - 16

$$a = 9 \quad b = 0 \quad c = -16$$

Here are some patterns demonstrated in these two examples:

- If the constants in factored form have opposite signs, the  $c$ -value in standard form will be negative.
- If the factors have the same coefficients but opposite constants, then  $b = 0$  in standard form.

**Things to Remember:**

# Lesson Practice

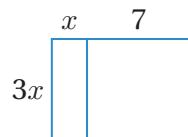
A1.8.03

Name: ..... Date: ..... Period: .....

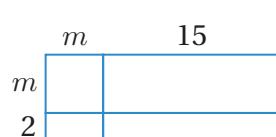
1. Write an expression that represents each area model.



$$5(6 + 12 + 14) \quad (\text{or equivalent})$$

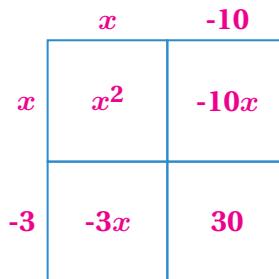


$$3x^2 + 21x \quad (\text{or equivalent})$$



$$m^2 + 17m + 30 \quad (\text{or equivalent})$$

2. Complete the diagram to show that  $(x - 10)(x - 3)$  is equivalent to  $x^2 - 13x + 30$ .



**Problems 3–5:** For each expression in factored form, write an equivalent expression in standard form.

3.  $(x - 2)^2$   
 $x^2 - 4x + 4$

4.  $(x + 1)(x - 1)$   
 $x^2 - 1$

5.  $(2x + 4)(x - 3)$   
 $2x^2 - 2x - 12$

**Problems 6–7:** Write an expression in factored form that has:

6. A  $b$ -value less than 4 and a  $c$ -value greater than 1 when written in standard form.

*Responses vary.*  $(x + 1)(x + 2)$

7. A negative  $a$ -value, negative  $b$ -value, and a positive  $c$ -value when written in standard form.

*Responses vary.*  $(-x + 1)(x + 3)$

# Lesson Practice

A1.8.03

Name: ..... Date: ..... Period: .....

8. Select any number from the inner square.

- Multiply the number to the left of your selection by the number to the right.
- Multiply the number above your selection by the number below.
- Here's an example with the number 12 selected:

$$\begin{array}{ccccc} & & 2 & & \\ 11 & \times & 13 & = 143 \\ & & 22 & & \\ & & = 44 & & \end{array}$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The difference between the numbers will be 99 no matter your selection. Explain why.

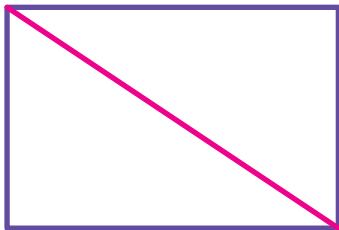
**Responses vary.** If my selection is  $x$ , then the numbers on both sides are  $x - 1$  and  $x + 1$ . Similarly, the numbers above and below are  $x - 10$  and  $x + 10$ . Using algebra to multiply each pair and subtract:

$$\begin{aligned}(x - 1)(x + 1) - (x - 10)(x + 10) &= (x^2 - 1) - (x^2 - 100) \\ &= x^2 - x^2 - 1 + 100 \\ &= 99\end{aligned}$$

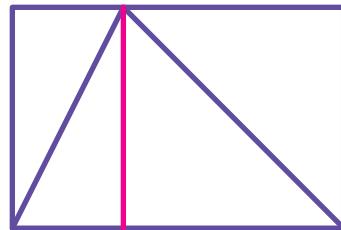
## Spiral Review

9. Draw a line to show that this rectangle is made only from 2 congruent triangles.

**Responses vary.**



10. Draw a line to show that this rectangle is made only from 2 pairs of congruent triangles.



## Reflection

- Put a star next to the problem you spent the most time on.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

A diagram can be a helpful tool for rewriting *standard-form* quadratic expressions in *factored form*. Here is an example: Rewrite the expression  $3x^2 + 14x + 15$  in factored form.

## Work

$3x^2$	
	15

	x	15
3x	$3x^2$	$45x$
1	x	15

## Explanation

Place the  $ax^2$  term in top-left corner of the diagram and the  $c$ -term in the bottom right.

Try different *factors* on the outside of the diagram that multiply to get  $ax^2$  and  $c$  until the inside of the diagram matches standard form.

This attempt didn't work because  $x + 45x \neq 14x$ .

The first attempt might not work and that's okay! Try different factors or switching the positions of the current factors.

	3x	5
x	$3x^2$	$5x$
3	$9x$	15

This attempt works because the linear terms in the diagram combine to match  $bx$  in standard form:  $5x + 9x = 14x$ .

The factored form is  $(3x + 5)(x + 3)$ .

## Things to Remember:

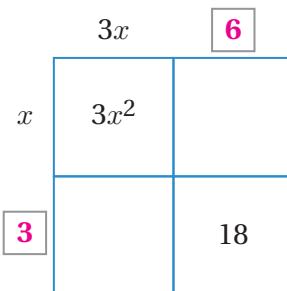
# Lesson Practice

A1.8.04

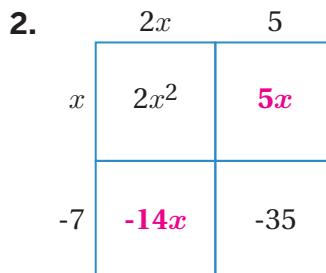
Name: ..... Date: ..... Period: .....

1. Write two possible constants that could complete the outside of the diagram.

**Responses vary.**

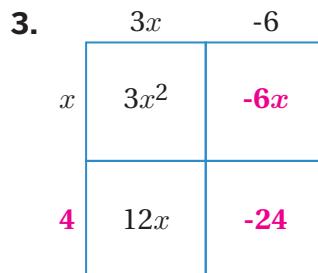


**Problems 2–3:** Complete the diagram puzzles and expressions.



Factored form:  $(2x + 5)(x - 7)$

Standard form:  $2x^2 - 9x - 35$



Factored form:  $(3x - 6)(x + 4)$

Standard form:  $3x^2 + 6x - 24$

4. Rewrite each expression in factored or standard form.

Factored Form	Standard Form
$(x + 3)(x + 6)$	$x^2 + 9x + 18$
$(2x - 3)(2x - 7)$	$4x^2 - 20x + 21$
$(3x - 2)(x + 4)$	$3x^2 + 10x - 8$

5. Determine values that make the equation true.

$$7x^2 - 10x - 8 = (\underline{1}x - 2)(7x + \underline{4})$$

## Lesson Practice

A1.8.04

Name: ..... Date: ..... Period: .....

6. This quadratic expression in standard form has an unknown  $b$ -value. If we know the expression can be factored, list two possibilities for the unknown value.

$$3x^2 + \boxed{?}x - 4$$

*Responses vary. -11, 11, -1, 1, -4, 4*

7. Create and complete a diagram puzzle that only uses factors of 12 as coefficients and constants.

*Responses vary.*

	$3x$	$3$
$2x$	$6x^2$	$6x$
2	$6x$	6

## Spiral Review

Problems 8–10: Fill in the missing values to make each equation true. *Responses vary.*

8.  $3(\underline{\quad} + \underline{\quad}) = 18$

9.  $\underline{\quad}(\underline{\quad} + \underline{\quad}) = 20$

10.  $4(\underline{\quad} + \underline{\quad}) = 3(\underline{\quad}) + 2(\underline{\quad})$

## Reflection

- Put a question mark next to a response you'd like to compare with a classmate's.
- Use the space below to ask one question you have or to share something you're proud of.

**Lesson Summary**

You can use the structure of a quadratic expression written in *standard form* to predict what the *factored form* will look like. Here are some strategies:

- Try to factor out a common factor first.
- If the standard form expression only has two terms, write the missing term with a coefficient of 0.
- If the  $c$ -value is negative, the signs of the constants in factored form will be different.

Here are some examples.

$$3x^2 - 9x - 30$$

3 is a common factor.

$$3(x^2 - 3x - 10)$$

Then factor  $x^2 - 3x - 10$ .

$x$	-5
$x$	$x^2$
2	-10

When  $a = 1$ , the constants in factored form multiply to  $c$  and add to  $b$ .

**Factored form:**

$$3(x - 5)(x + 2)$$

$$x^2 - 81$$

Expressions with this structure are called a *difference of squares*. Rewriting the expression with three terms might be helpful.

Factor  $x^2 + 0x - 81$ .

$x$	9
$x$	$x^2$
-9	-9x

**Factored form:**

$$(x - 9)(x + 9)$$

$$2x^2 + 3x - 27$$

There is not a common factor in this quadratic expression.

Test pairs of expressions that multiply to  $2x^2$  and -27.

The  $c$ -value is negative so the signs of the constants in factored form will be different.

$2x$	9
$x$	$2x^2$
-3	-6x

**Factored form:**

$$(2x + 9)(x - 3)$$

**Things to Remember:**

# Lesson Practice

A1.8.05

Name: ..... Date: ..... Period: .....

1. Write a + or – sign in each box to make true equations.

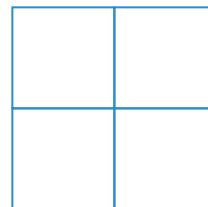
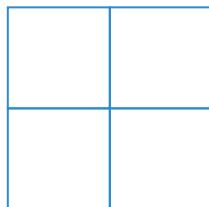
$$(x \boxed{+} 18)(x \boxed{-} 3) = x^2 + 15x - 54$$

$$(x \boxed{-} 18)(x \boxed{-} 3) = x^2 - 21x + 54$$

**Problems 2–5:** Fill in the blanks to make each equation true. Use the diagrams if they help your thinking.

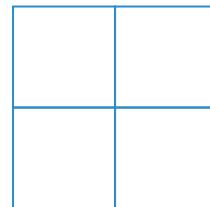
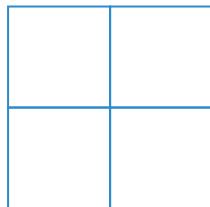
2.  $x^2 \underline{- 12} x \underline{+ 27} = (x - 9)(x - 3)$

3.  $x^2 + 12x \underline{+ 32} = (x + 4)(x \underline{+ 8})$



4.  $2x^2 + 11x + 15 = (2x \underline{+ 5})(x \underline{+ 3})$

5.  $3x^2 - 11x \underline{- 42} = (x - 6)(3x \underline{+ 7})$



**Problems 6–8:** Factor each expression.

6.  $x^2 - x - 30$

$(x - 6)(x + 5)$

7.  $4x^2 + 20x + 25$

$(2x + 5)^2$  (or equivalent)

8.  $4x^2 - 81$

$(2x - 9)(2x + 9)$

## Lesson Practice

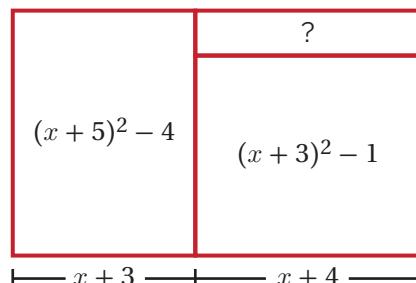
A1.8.05

Name: ..... Date: ..... Period: .....

9. The diagram shows the expressions for areas and lengths. Determine an expression for the unknown area.

*Responses vary.*

$5(x + 4)$  or equivalent



## Spiral Review

Problems 10–12: Solve each equation.

10.  $6 + 2x = 0$

$x = -3$

11.  $2x - 5 = 0$

$x = \frac{5}{2}$

12.  $\frac{1}{2}(x - 87) = 0$

$x = 87$

## Reflection

1. Circle the problem that was the most challenging for you.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

An  $x$ -intercept is the coordinate point where the graph crosses the  $x$ -axis, and a **zero** is the  $x$ -value of the  $x$ -intercept.

To determine the zeros or  $x$ -intercepts, you determine the  $x$ -values that make the function equal to 0. Rewriting the function in factored form is a helpful step for determining those values.

Here are two examples of determining the  $x$ -intercepts of functions in different forms.

Factored Form:  $h(x) = (x - 1)(x + 6)$

When  $x = 1$ , the factor  $x - 1 = 0$ ,  
so  $h(1) = 0$ .

When  $x = -6$ , the factor  $x + 6 = 0$ ,  
so  $h(-6) = 0$ .

The zeros are  $x = 1$  and  $x = -6$ .

The  $x$ -intercepts are  $(1, 0)$  and  $(-6, 0)$ .

Standard Form:  $g(x) = x^2 - 6x - 40$

First, I can factor the function.

$x$	$x^2$	$-10x$
4	$4x$	-40

$$g(x) = (x - 10)(x + 4)$$

When  $x = 10$ , the factor  $x - 10 = 0$ ,  
so  $g(10) = 0$ .

When  $x = -4$ , the factor  $x + 4 = 0$ ,  
so  $g(-4) = 0$ .

The zeros are  $x = 10$  and  $x = -4$ .

The  $x$ -intercepts are  $(10, 0)$  and  $(-4, 0)$ .

## Things to Remember:

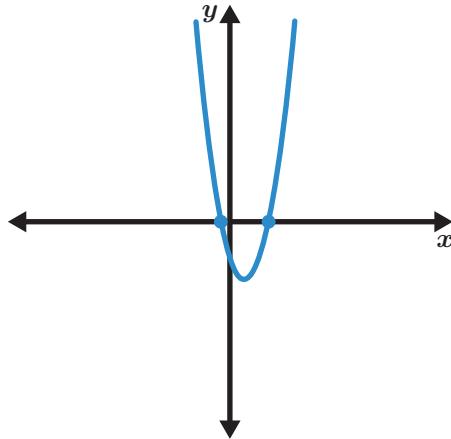
# Lesson Practice

A1.8.06

Name: ..... Date: ..... Period: .....

**Problems 1–3:** What are the  $x$ -intercepts of the function?

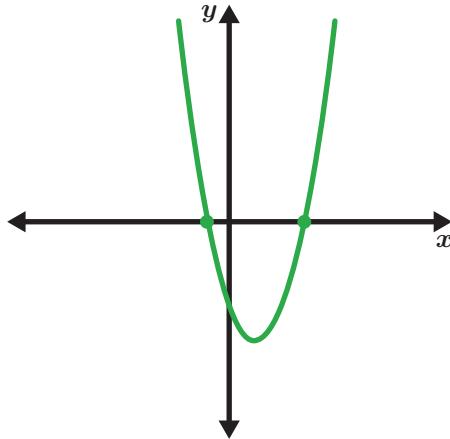
1.  $a(x) = (x - 2)(2x + 1)$



$x$ -intercept #1:  $(2, 0)$

$x$ -intercept #2:  $(-\frac{1}{2}, 0)$

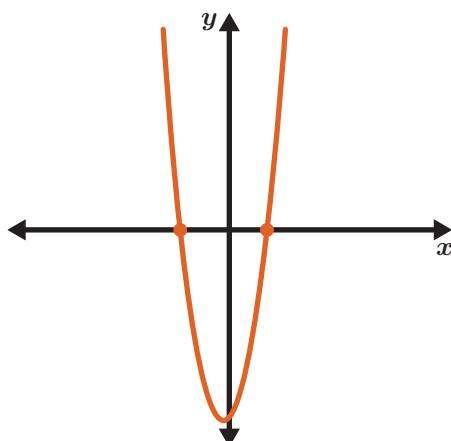
2.  $b(x) = x^2 - 3x - 4$



$x$ -intercept #1:  $(-1, 0)$

$x$ -intercept #2:  $(4, 0)$

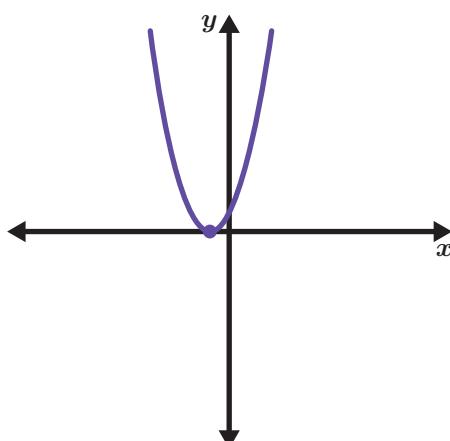
3.  $c(x) = 2x^2 + x - 10$



$x$ -intercept #1:  $(2, 0)$

$x$ -intercept #2:  $(-2\frac{1}{2}, 0)$

4. Write an equation that could represent this graph. **Responses vary.**



$f(m) = \dots$   $(x + 1)^2$

5. Select *all* the functions that have 5 and -1 as their  $x$ -intercepts.

A.  $f(x) = (x + 5)(x - 1)$

B.  $g(x) = (x + 1)(x - 5)$

C.  $h(x) = x^2 + 4x - 5$

D.  $j(x) = 2x^2 - 8x - 10$

E.  $k(x) = (15 - 3x)(4x + 4)$

# Lesson Practice

A1.8.06

Name: ..... Date: ..... Period: .....

6. Change one number so that the function has  $x$ -intercepts at -1 and 3.

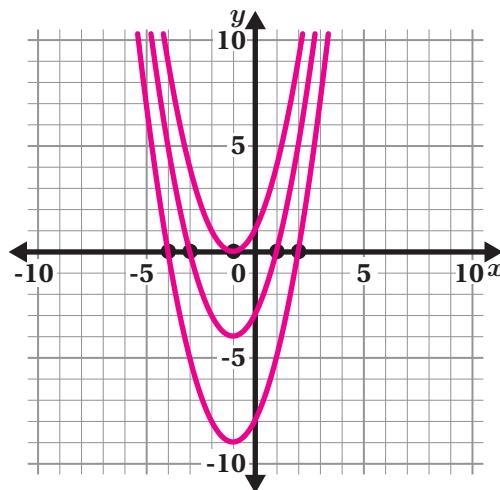
$$g(x) = x^2 - 1x - 3$$

$$g(x) = x^2 - 2x - 3$$

7. Write the equations of 3 parabolas that go through all of the points.

**Responses vary.**

- $y = (x + 4)(x - 2)$
- $y = (x - 1)(x + 3)$
- $y = (x + 1)^2$



## Spiral Review

Problems 8–9: Solve for  $x$ .

8.  $2x - 3 \geq 4x + 7$

$$x \leq -5$$

9.  $3(x - 2) < 2x - 12$

$$x < -6$$

## Reflection

1. Circle the problem that was the most challenging for you.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

The **zero-product property** states that if the product of two or more factors is 0, then at least one of the factors is 0. You can use this property to determine the  $x$ -intercepts of a function or the *solutions* to quadratic equations using the following steps.

- Set the quadratic equation equal to 0.
- Factor the equation.
- Set each factor equal to 0.
- Solve for  $x$ .

Here are two examples of solving quadratic equations.

$$(5x - 3)(2x + 3) = 0$$

$$2x^2 - x = 21$$

Set each factor equal to 0 and solve for  $x$ .

$$\begin{aligned} (5x - 3) &= 0 & (2x + 3) &= 0 \\ 5x &= 3 & 2x &= -3 \\ x &= \frac{3}{5} \text{ and } x &= -\frac{3}{2} \end{aligned}$$

First, rewrite the equation so that it is equal to 0.

$$2x^2 - x - 21 = 0$$

Then factor the equation.  
 $(2x - 7)(x + 3) = 0$

Set each factor equal to 0 and solve for  $x$ .

$$\begin{aligned} (2x - 7) &= 0 & (x + 3) &= 0 \\ 2x &= 7 & x &= -3 \\ x &= \frac{7}{2} \text{ and } x &= -3 \end{aligned}$$

**Things to Remember:**

## Lesson Practice

A1.8.07

Name: ..... Date: ..... Period: .....

1. Rewrite each standard-form quadratic equation in factored form.

a  $x^2 + 7x + 6 = 0$   
 $(x + 6)(x + 1) = 0$

b  $x^2 - 5x + 6 = 0$   
 $(x - 3)(x - 2) = 0$

c  $x^2 + 5x - 6 = 0$   
 $(x + 6)(x - 1) = 0$

2. Rewrite the equation  $x^2 - x = 6$  so that one side is equal to 0. Then solve the equation.

$x^2 - x - 6 = 0$  (or equivalent)

$x = 3$  and  $x = -2$

Problems 3–5: Solve each equation.

3.  $(4 - 5x)(x + 4) = 0$   
 $x = \frac{4}{5}$  and  $x = -4$

4.  $x^2 + 14x + 33 = 0$   
 $x = -3$  and  $x = -11$

5.  $5x + 12 = x^2 - 5x - 12$   
 $x = -2$  and  $x = 12$

6. Solve the equation  $3x^2 - 18x + 24 = 0$ .

Use the diagram if it helps with your thinking.

$x = 4$  and  $x = 2$

	$x$	$-4$
$3x$	$3x^2$	$-12x$
$-6$	$-6x$	$24$

(or equivalent)

Problems 7–8: Solve each equation.

7.  $x^2 - 7x = 0$   
 $x = 0$  and  $x = 7$

8.  $(x + 2)(x + 4) = 3$   
 $x = -5$  and  $x = -1$

## Lesson Practice

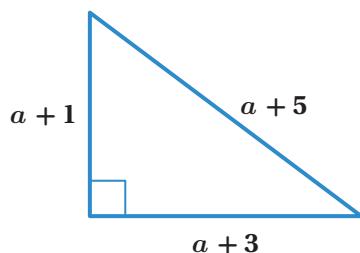
A1.8.07

Name: ..... Date: ..... Period: .....

9. Determine the value of  $a$  in the right triangle.

Explain your thinking.

**$a = 5$ .** Explanations vary. I used the Pythagorean theorem to set up the equation  $(a + 1)^2 + (a + 3)^2 = (a + 5)^2$ . The solutions to this equation are 5 and -3, but -3 isn't a valid solution since it would cause some of the triangle's side lengths to be negative.



10. Write two equations with  $x = \frac{2}{3}$  and  $x = -9$  as solutions.

**Responses vary.**  $(3x - 2)(x + 9) = 0$  and  $\left(x - \frac{2}{3}\right)(x + 9) = 0$

## Spiral Review

11. Decide whether this equation has one solution, no solution, or infinitely many solutions:  $-3(x + 2) = -x + 6 - 2x$ .

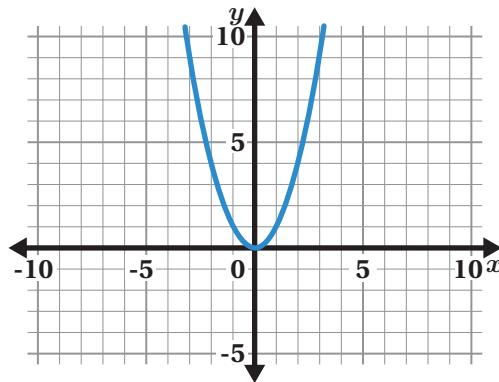
One solution

No solution

Infinitely many solutions

12. Select all the true statements about the function  $f(x) = x^2$ .

- A. The domain has no negative values.
- B. The range has no negative values.
- C. The function has no minimum.
- D. The function has no maximum.



## Reflection

1. Put a star next to a problem you could explain to a classmate.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

To determine the number of solutions to a quadratic equation, you can use reasoning or use the structure of the equation. Here are some examples:

No Solutions	One Solution	Two Solutions
$(x + \dots)^2 = \text{a negative number}$ No value squared will result in a negative number.	$(x + \dots)^2 = 0$ Only one value squared will equal 0.	$(x + \dots)^2 = \text{a positive number}$ There are two values that when squared will equal a positive number.
$(x + 10)^2 = -25$ $(x - 3)^2 + 1 = 0$ $x^2 + 4 = 0$	$(x + 4)^2 = 0$ $x^2 + 9 = 9$ $(x - 3)(x - 3) = 0$	$(x + 4)^2 = 1$ $x^2 - 12 = -3$ $(x - 3)(x - 3) = 1$

**Things to Remember:**

# Lesson Practice

A1.8.08

Name: ..... Date: ..... Period: .....

1. Write a quadratic equation that has . . . **Responses vary.**

a Two solutions .....  $x^2 = 16$

b One solution .....  $x^2 + 5 = 5$

c No solutions .....  $x^2 + 10 = 9$

2. For each equation, determine the number of solutions.

Equation	Number of Solutions
$x(x + 3) = 0$	<b>Two solutions</b>
$(x + 3)(x + 1) = 0$	<b>Two solutions</b>
$(x + 1)(x + 1) = 0$	<b>One solution</b>
$x^2 - 10x = -9$	<b>Two solutions</b>
$(x - 5)(x - 5) = -14$	<b>No solutions</b>
$x^2 - 3 = -3$	<b>One solution</b>
$x^2 + 6 = 2$	<b>No solutions</b>

**Problems 3–4:** Determine the solution for each equation.

3.  $149 + (x - 2)^2 = 149$

$x = 2$

4.  $x^2 + 3x = x - 1$

$x = -1$

**Problems 5–6:** Determine the two solutions for each equation.

5.  $100 + (x - 2)^2 = 149$

$x = 9, x = -5$

6.  $x^2 + 4x = x + 18$

$x = -6, x = 3$

7. Which value for  $x$  is a solution to the equation  $x^2 + 10 = 9$ ?

A.  $x = 1$

B.  $x = -1$

C.  $x = \sqrt{19}$

D. There is no solution to the equation.

# Lesson Practice

A1.8.08

Name: ..... Date: ..... Period: .....

## Spiral Review

8. Order the expressions by value from *least* to *greatest*:  $5^2$ ,  $\sqrt{90}$ , 8,  $3^3$ ,  $\sqrt{27}$ .

$\sqrt{27}$	8	$\sqrt{90}$	$5^2$	$3^3$
-------------	---	-------------	-------	-------

Least

Greatest

**Problems 9–10:** Complete each diagram. Then write the corresponding quadratic expression in factored form and standard form.

9.

	-3x	3
6x	<b>-18x<sup>2</sup></b>	18x
3	<b>-9x</b>	<b>9</b>

Factored form:

$$(6x + 3)(-3x + 3)$$

Standard form:

$$-18x^2 + 9x + 9$$

10.

	6x	-8
5x	<b>30x<sup>2</sup></b>	<b>-40x</b>
2	<b>12x</b>	-16

Factored form:

$$(5x + 2)(6x - 8)$$

Standard form:

$$30x^2 - 28x - 16$$

## Reflection

1. Put a question mark next to a problem you're feeling unsure of.
2. Use this space to ask a question or share something you're proud of.

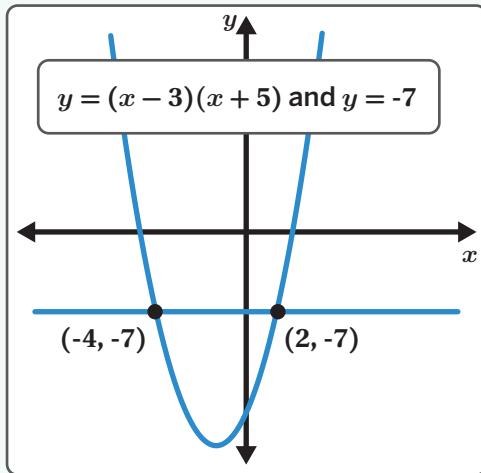
**Lesson Summary**

Graphs can be used to determine the solutions to a quadratic equation.

Here are two strategies using graphs to solve  $(x - 3)(x + 5) = -7$ .

**Strategy 1**

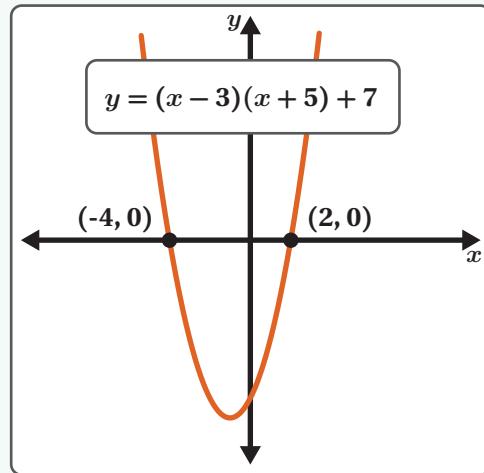
- Graph both sides of the equation as two separate graphs.
- Determine the  $x$ -coordinates where the graphs intersect.



Solutions:  $x = -4$  and  $x = 2$

**Strategy 2**

- Rewrite the equation so that it equals 0.
- Graph the equation. The solutions will be at the  $x$ -intercepts.



Solutions:  $x = -4$  and  $x = 2$

**Things to Remember:**

# Lesson Practice

A1.8.09

Name: ..... Date: ..... Period: .....

1. Here is a graph of  $y = x^2 - 4x - 5$  and  $y = 4$ .

Select all the points that are solutions to

$$x^2 - 4x - 5 = 4.$$

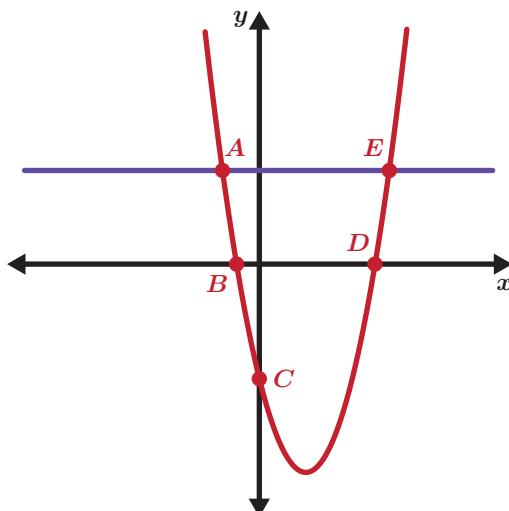
A. Point A

B. Point B

C. Point C

D. Point D

E. Point E



Problems 2–5: Circle how many solutions each equation has. Record any solutions.

2.  $-2x^2 + 2 = -6$

No  
solutions

One  
solution

Two  
solutions

$$x = \underline{\hspace{2cm}} -2 \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}} 2 \underline{\hspace{2cm}}$$

3.  $x(x + 3) = 4$

No  
solutions

One  
solution

Two  
solutions

$$x = \underline{\hspace{2cm}} -4 \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}} 1 \underline{\hspace{2cm}}$$

4.  $0 = (x - 2)(x + 4) + 9$

No  
solutions

One  
solution

Two  
solutions

$$x = \underline{\hspace{2cm}} -1 \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}} \underline{\hspace{2cm}}$$

5.  $2x(x + 1) = -1$

No  
solutions

One  
solution

Two  
solutions

$$x = \underline{\hspace{2cm}} \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}} \underline{\hspace{2cm}}$$

Problems 6–7: Fill in the blank so the equation has:

6. One solution

$$3x(x + 2) = \underline{\hspace{2cm}} -3 \underline{\hspace{2cm}}$$

7. No solutions **Responses vary.**

$$(x - 2)(x + \underline{\hspace{2cm}} 1 \underline{\hspace{2cm}}) = -4$$

# Lesson Practice

A1.8.09

Name: ..... Date: ..... Period: .....

8. The graphs of  $y = -x^2 + 25$ ,  $y = 24$ ,  $y = 21$ , and  $y = 16$  all intersect at integer values.

Write the equations of three more lines that follow this property. **Responses vary.**

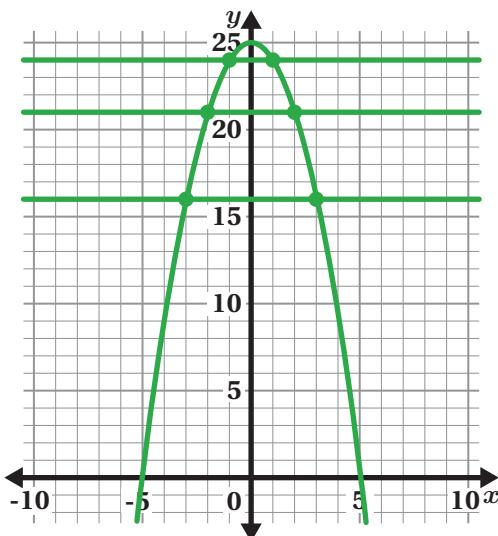
$y = \underline{\hspace{2cm}} 9$

$y = \underline{\hspace{2cm}} 0$

$y = \underline{\hspace{2cm}} -11$

What pattern do you notice?

**Responses vary.** If you subtract each  $y$ -value from 25, it equals a perfect square.



## Spiral Review

9. Fill in the blanks to make each equation true. **Responses vary.**

$6^2 = \underline{\hspace{2cm}} 36$

$(\underline{\hspace{2cm}} 7 \text{ or } -7)^2 = 49$

$2^2 = \underline{\hspace{2cm}} \sqrt{16}$

$(\underline{\hspace{2cm}} \sqrt{11})^2 = 11$

10. Match each expression to an equivalent expression. One expression will have no match.

- |                      |                            |
|----------------------|----------------------------|
| a. $1 - x^2$         | <u>c.</u> $(y + x)(y - x)$ |
| b. $x^2 - y^2$       | <u>a.</u> $(1 + x)(1 - x)$ |
| c. $y^2 - x^2$       | <u>e.</u> $(1 + x)(x - 1)$ |
| d. $x^2 - 2xy + y^2$ | <u>d.</u> $(x - y)(x - y)$ |
| e. $x^2 - 1$         |                            |

## Reflection

- Put a star next to a problem that looked more difficult than it really was.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

A quadratic expression is a **perfect square** if it can be represented as something multiplied by itself, like  $(x + \dots)^2$ .

You can determine the missing constant value to add to make a perfect square by dividing the linear coefficient in half and then squaring that number.

Here are two examples of filling in the blanks to make each expression a perfect square.

$$x^2 + 14x + \dots$$

$$x^2 - \dots + 81$$

In perfect square expressions, the  $b$ -value, 14, is always double the constant in the factored form  $(x + 7)^2$ .

So the missing number must be the constant 7 squared, which is 49.

The  $c$ -value, 81, is a perfect square so the factored form expression must be  $(x - 9)^2$ .

Rewrite the factored form to standard by using an area model or distributive property. The missing number must be  $-18x$ .

$$x^2 + 14x + 49$$

	$x$	7
$x$	$x^2$	$7x$
7	$7x$	49

$$x^2 - 18x + 81$$

	$x$	-9
$x$	$x^2$	$-9x$
-9	$-9x$	81

## Things to Remember:

# Lesson Practice

A1.8.11

Name: ..... Date: ..... Period: .....

1. Which expression is equivalent to  $(x - 6)^2$ ?

A.  $(x + 6)^2$

B.  $x^2 - 12x - 36$

C.  $x^2 - 36$

D.  $x^2 - 12x + 36$

2. Write each perfect square expression in factored form.

a.  $x^2 + 6x + 9$

$(x + 3)^2$

b.  $x^2 - 16x + 64$

$(x - 8)^2$

c.  $x^2 - 12x + 36$

$(x - 6)^2$

d.  $x^2 + 5x + \frac{25}{4}$

$\left(x + \frac{5}{2}\right)^2$   
(or equivalent)

3. Select all the expressions that are perfect squares.

A.  $x^2 + 10x + 25$

B.  $x^2 - 10x + 25$

C.  $x^2 + 4$

D.  $(x + 3.5)(3.5 + x)$

E.  $(x - 10)(10 - x)$

F.  $x^2 + \frac{1}{2}x + \frac{1}{16}$

4. Erendirani says that if a perfect square expression is written in the form  $x^2 + bx + c$ , the value of  $c$  cannot be negative. Why is this true?

**Responses vary.** If the quadratic expression is a perfect square, it can be written like  $(x + h)^2$  or  $(x - h)^2$ . The  $c$ -value in  $x^2 + bx + c$  is equal to  $h^2$ . Any number squared is either positive or 0.

**Problems 5–8:** Fill in the blanks to complete each perfect square.

5.  $x^2 + 24x + \underline{\hspace{1cm}}$

6.  $x^2 - 2x + \underline{\hspace{1cm}}$

7.  $x^2 - \underline{\hspace{1cm}} + 64$

8.  $x^2 + \frac{2}{3}x + \underline{\hspace{1cm}}$

## Lesson Practice

A1.8.11

Name: ..... Date: ..... Period: .....

9. The expressions  $(x - 4)^2$  and  $(4 - x)^2$  are both perfect squares. Are they equivalent to one another? Explain your thinking.

**Yes, they are equivalent. Explanations vary.**

- I converted both expressions to standard form, and they both resulted in  $x^2 - 8x + 16$ .
- I graphed both expressions and noticed they were the same parabola.

10. LaShawn claims that  $x^2 + bx + \frac{b^2}{4}$  is a perfect square. Do you agree? Explain your thinking.

**Yes. Explanations vary. To be a perfect square, the  $c$ -value needs to be equal the  $b$ -value divided by 2 and squared.  $\left(\frac{b}{2}\right)^2 = \frac{b^2}{4}$ .**

## Spiral Review

**Problems 11–12:** The equations  $y = x^2 + 5x + 6$  and  $y = (x + 2)(x + 3)$  are equivalent.

11. Which equation would you use to determine the  $x$ -intercepts? Explain your thinking.

**Responses and explanations vary. I would use  $y = (x + 2)(x + 3)$ . To find the  $x$ -intercepts, I would set  $y = 0$ . Factored-form equations equal to 0 are easy for me to solve.**

12. Which equation would you use to determine the  $y$ -intercept? Explain your thinking.

**Responses and explanations vary. I would use  $y = x^2 + 5x + 6$ . To find the  $y$ -intercept, I would set  $x = 0$ . When I do that in a standard form equation like this one, the  $c$ -term is the  $y$ -intercept.**

13. Without using a graphing calculator, select *all* the equations with a positive  $y$ -intercept.

- A.  $y = x^2 + 3x - 2$
- B.  $y = (x + 1)(x + 5)$
- C.  $y = x^2 - 10x$
- D.  $y = (x - 3)^2$
- E.  $y = -5x^2 + 3x - 12$

## Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can solve quadratic equations by graphing, factoring, or **completing the square**, which is the process of rewriting a quadratic expression or equation to include a perfect square. You can analyze the structure of the equation to help you decide which strategy to use.

Here is an example:  $x^2 + 10x = 2$ . Solving by graphing will not produce exact solutions and factoring is not possible for this equation, so we can solve by completing the square:

**Work**

$$x^2 + 10x + 25 = 2 + 25$$

$$(x + 5)^2 = 27$$

$$x + 5 = \pm\sqrt{27}$$

$$x = -5 \pm \sqrt{27}$$

**Explanation**

$x^2 + 10x + 25$  is a perfect square, so add the constant value 25 to both sides of the equation.

Rewrite the perfect square  $x^2 + 10x + 25$  in factored form.

Take the square root and include both possibilities by writing  $\pm$ .

Solve for  $x$ .

**Things to Remember:**

# Lesson Practice

A1.8.12

Name: ..... Date: ..... Period: .....

1. Select *all* the expressions that are perfect squares.

A.  $(x + 5)(5 + x)$

B.  $(x - 3)^2$

C.  $x^2 - 3^2$

D.  $x^2 + 8x + 64$

E.  $x^2 + 10x + 25$

2. Add the number that would make the expression a perfect square. Then write an equivalent expression in factored form.

$$x^2 - 6x + \boxed{9}$$

$$(x - 3)^2$$

$$x^2 + 2x + \boxed{1}$$

$$(x + 1)^2$$

$$x^2 - 14x + \boxed{49}$$

$$(x - 7)^2$$

3. Match each equation to an equivalent equation.

a.  $x^2 - 12x = 6$

b.  $(x - 6)^2 = 30$

b.  $x^2 - 12x + 6 = 0$

d.  $(x - 3)^2 = 42$

c.  $x^2 - 6x = 6$

a.  $(x - 6)^2 = 42$

d.  $x^2 - 6x = 33$

c.  $(x - 3)^2 = 15$

4. Alexis solved the equation  $x^2 + 12x = 13$  by completing the square, but some parts are blank. Fill in the blanks.

$$x^2 + 12x = 13$$

$$\boxed{x^2 + 12x + 36 = 13 + 36} \text{ (or equivalent)}$$

$$(x + 6)^2 = 49$$

$$x + 6 = \pm 7$$

$$x = \boxed{-13} \text{ and } x = \boxed{1}$$

# Lesson Practice

A1.8.12

Name: ..... Date: ..... Period: .....

**Problems 5–7:** Solve each equation by completing the square.

5.  $x^2 - 2x = 8$

$x = -2$  and  $x = 4$

6.  $7 = x^2 + 4x - 1$

$x = -2 \pm \sqrt{12}$   
(or equivalent)

7.  $x^2 - 18x + 60 = -11$

$x = 9 \pm \sqrt{10}$   
(or equivalent)

8. Write a quadratic equation of the form  $x^2 + bx + c = 0$  with solutions that are

$x = 5 - \sqrt{2}$  and  $x = 5 + \sqrt{2}$ .

$x^2 - 10x + 23 = 0$

## Spiral Review

9. For each equation, determine the number of solutions.

Equation	Number of Solutions
$x^2 + 144 = 0$	No solutions
$x^2 - 144 = 0$	Two solutions
$(x - 7)^2 = 0$	One solution

10. The graph of  $y = (x - 1)^2 + 4$  is the same as the graph of  $y = x^2$ , but:

- A. It is shifted 1 unit to the right and 4 units up.
- B. It is shifted 1 unit to the left and 4 units up.
- C. It is shifted 1 unit to the right and 4 units down.
- D. It is shifted 1 unit to the left and 4 units down.

## Reflection

1. Put a heart next to the problem you found most interesting.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can use the **quadratic formula** to solve quadratic equations written in standard form.

The quadratic formula states that the solutions to any quadratic equation  $ax^2 + bx + c = 0$  are  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

The quadratic formula comes from completing the square.

Here is an example of completing the square and using the quadratic formula to solve  $x^2 + 8x + 13 = 0$ . Even though the answers look different, they are equivalent.

**Completing the Square**

$$\text{Solve } x^2 + 8x + 13 = 0$$

$$x^2 + 8x = -13$$

$$x^2 + 8x + 16 = -13 + 16$$

$$x^2 + 8x + 16 = 3$$

$$(x + 4)^2 = 3$$

$$x + 4 = \pm\sqrt{3}$$

$$x = -4 \pm \sqrt{3}$$

**Quadratic Formula**

$$\text{Solve } x^2 + 8x + 13 = 0$$

$$a = 1, b = 8, c = 13$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{-8 \pm \sqrt{12}}{2}$$

**Things to Remember:**

# Lesson Practice

A1.8.14

Name: ..... Date: ..... Period: .....

1. The quadratic formula is derived by solving  $ax^2 + bx + c = 0$  by . . .

A. Factoring

B. Completing  
the square

C. Graphing

D. Elimination

2. The quadratic formula can be used to find the solutions to any quadratic equation in the form  $ax^2 + bx + c = 0$ .

Which equation represents the solutions to

$$2x^2 - x + 13 = 0?$$

A.  $x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(13)}}{2(2)}$    B.  $x = \frac{-2 \pm \sqrt{(2)^2 - 4(13)(-1)}}{2(13)}$    C.  $x = \frac{-1 \pm \sqrt{(-1)^2 - 4(2)(13)}}{2(-1)}$

**Quadratic Formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Problems 3–5:** The quadratic equation  $x^2 + 7x + 10 = 0$  is in the form  $ax^2 + bx + c = 0$ .

3. What are the values of  $a$ ,  $b$ , and  $c$ ?

$$a = 1$$

$$b = 7$$

$$c = 10$$

4. Substitute the values of  $a$ ,  $b$ , and  $c$  into the quadratic formula.  
(You do not need to perform any operations.)

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(10)}}{2(1)}$$

5. Explain how the expression you wrote is related to solving  $x^2 + 7x + 10 = 0$  by completing the square.

*Responses vary. The expression takes all the steps for solving by completing the square and bundles them up into one expression. Instead of having to evaluate every step, you can do the calculation all at once.*

6. Part of the quadratic formula is  $\pm \sqrt{b^2 - 4ac}$ . What value must this equal to have exactly one solution? Explain your thinking.

*0. Explanations vary. Plus 0 and minus 0 have the same value so there would be one solution for the quadratic if 0 is under the square root sign.*

# Lesson Practice

A1.8.14

Name: ..... Date: ..... Period: .....

7. Kiri is using a quadratic equation to figure out the quadratic formula. Here are her first few steps.

Why did Kiri add  $\left(\frac{b}{2a}\right)^2$  in the bottom row?

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

**Responses vary.** The quadratic formula involves completing the square. To find the value that completes the square, you divide the coefficient of  $x$  by 2 and square it.

The coefficient of  $x$  here is  $\frac{b}{a}$ , so Kiri divides and squares it to get  $\left(\frac{b}{2a}\right)^2$ , then adds that to both sides of the equation.

## Spiral Review

**Problems 8–10:** Rewrite each equation in the form  $(x + \underline{\hspace{1cm}})^2 = \underline{\hspace{1cm}}$ .

An example has been done for you.

Example:

$$x^2 + 10x = 4$$

$$(x + 5)^2 = 29$$

8.  $x^2 + 6x = -2$

$$(x + 3)^2 = 7$$

9.  $x^2 + 12x + 3 = -7$

$$(x + 6)^2 = 26$$

10.  $x^2 - 32 = -20x$

$$(x + 10)^2 = 132$$

**Problems 11–13:** Solve each equation for  $x$ .

11.  $3x^2 - 5 = 0$

$$x = \pm \sqrt{\frac{5}{3}}$$

12.  $-7x^2 + 2 = 0$

$$x = \pm \sqrt{\frac{2}{7}}$$

13.  $ax^2 + c = 0$

$$x = \pm \sqrt{-\frac{c}{a}}$$

## Reflection

- Star a problem you're still feeling confused about.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can solve any quadratic equation written in standard form,

$ax^2 + bx + c = 0$ , using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here is an example of using the quadratic formula to solve the equation  $2x^2 - 9x + 6 = 0$ .

**Work**

$$a = 2, b = -9, c = 6$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(6)}}{2(2)}$$

$$x = \frac{9 \pm \sqrt{33}}{4}$$

$$x = \frac{9 + \sqrt{33}}{4} \text{ and } x = \frac{9 - \sqrt{33}}{4}$$

**Explanation**

Identify the  $a$ -,  $b$ -, and  $c$ -values from the standard form quadratic equation.

Substitute the  $a$ -,  $b$ -, and  $c$ -values into the quadratic formula.

Use order of operations to simplify the expression.

Write the solutions.

**Things to Remember:**

## Lesson Practice

A1.8.15

Name: ..... Date: ..... Period: .....

**Problems 1–2:** Each equation is in the form  $ax^2 + bx + c = 0$ . Determine the values of  $a$ ,  $b$ , and  $c$ .

1.  $x^2 - 5x + 9 = 0$

$a = 1$     $b = -5$     $c = 9$

2.  $-x^2 + 8 = 0$

$a = -1$     $b = 0$     $c = 8$

**Problems 3–5:** Determine the exact solution(s) of each equation.

3.  $2x^2 - 7x = 15$

$x = 5$  and  $x = -1.5$   
(or equivalent)

4.  $2x^2 + 5x - 1 = 0$

$x = \frac{-5 \pm \sqrt{33}}{4}$   
(or equivalent)

5.  $4x^2 = 5$

$x = \pm \frac{\sqrt{5}}{2}$   
(or equivalent)

6. Santiago determined that the solutions to  $3x^2 - 6x - 9 = 0$  are  $x = 3$  and  $x = -1$ . Is Santiago correct? Explain your thinking.

*Yes. Explanations vary.*

- I graphed  $y = 3x^2 - 6x - 9$  and saw that the parabola's  $x$ -intercepts were  $(3, 0)$  and  $(-1, 0)$ .
- I substituted each solution into the original equation to see if it made the equation true.

7. Write a quadratic equation that would be simpler to solve *without* using the quadratic equation.

*Responses vary.*

$(x + 2)(x - 1) = 0$

## Lesson Practice

A1.8.15

Name: ..... Date: ..... Period: .....

8. Write a quadratic function with the following  $x$ -intercepts.

$$\left(\frac{2 \pm \sqrt{40}}{6}, 0\right)$$

**Responses vary.**  $f(x) = 3x^2 - 2x - 3$

## Spiral Review

**Problems 9–10:** Here is the function  $f(x) = (x + 1)(x + 5)$ .

9. What are the coordinates of the  $x$ -intercepts?

**(-5, 0) and (-1, 0)**

10. What are the coordinates of the vertex? Explain your thinking.

**(-3, -4). Explanations vary. The  $x$ -intercepts tell me that the line of symmetry is at  $x = -3$ . When I input -3 into the function, the output is -4.**

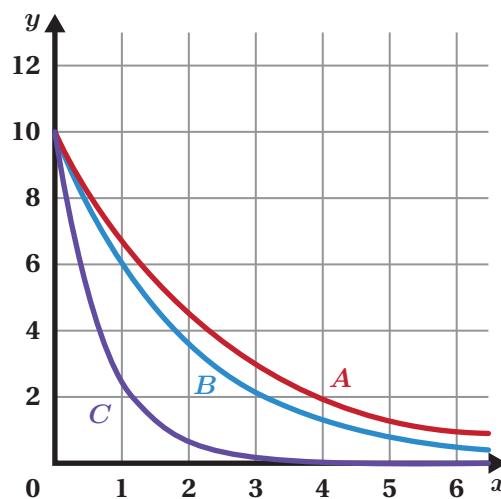
11. Here are the graphs of three equations.

Match each graph with its equation.

**A**  $y = 10\left(\frac{2}{3}\right)^x$

**C**  $y = 10\left(\frac{1}{4}\right)^x$

**B**  $y = 10\left(\frac{3}{5}\right)^x$



## Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can use quadratic functions to represent the paths of objects that are launched in the air, such as a stomp rocket. You can use functions to answer questions about the launch by:

1. Substituting the given information into the equation.
2. Solving for the missing variable using any strategy (e.g., the quadratic formula or any other available strategy).
3. Interpreting if the solution(s) make sense in the situation.

The function  $h(t) = -2.5t^2 + 6t + 8$  represents the height, in meters, of a stomp rocket  $t$  seconds after it has been launched.

When will the rocket touch the ground?	When will the rocket be at a height of 10 meters?	Will the rocket reach a height of 15 meters?
<p>Given information: <math>h(t) = 0</math></p> $0 = -2.5t^2 + 6t + 8$ <p>Using the quadratic formula, I get the following solutions:</p> $t = -0.954 \text{ and } t = 3.354$ <p>Going back in time does not make sense in this situation, so the only answer is 3.354 seconds.</p>	<p>Given information: <math>h(t) = 10</math></p> $10 = -2.5t^2 + 6t + 8$ $0 = -2.5t^2 + 6t - 2$ <p>Using the quadratic formula, I get the following solutions:</p> $t = 0.4 \text{ and } t = 2$ <p>The rocket will be at a height of 10 meters at both .4 seconds and 2 seconds.</p>	<p>Given information: <math>h(t) = 15</math></p> $15 = -2.5t^2 + 6t + 8$ $0 = -2.5t^2 + 6t - 7$ <p>Using the quadratic formula, I get no solution:</p> $t = \frac{-6 \pm \sqrt{-34}}{-5}$ <p>This means the rocket will never reach a height of 15 feet.</p>

**Things to Remember:**

## Lesson Practice

### A1.8.16

Name: ..... Date: ..... Period: .....

**Problems 1–3:** The function  $h(t) = -75t^2 + 60t$  models the height, in inches, of a jumping frog, where  $t$  is the number of seconds after it jumped.

1. Solve the equation  $-75t^2 + 60t = 0$ .

**$t = 0$  and  $t = 0.8$**

2. What do the solutions tell us about the jumping frog?

**Responses vary.** Each solution corresponds to a moment when the frog was on the ground. It was on the ground when it started jumping, and it returned to the ground after 0.8 seconds.

3. When does the frog reach its maximum height of 12 inches? **0.4 seconds**

**Problems 4–5:** The function  $f(t) = 4 + 12t - 16t^2$  models the height of a tennis ball, in feet,  $t$  seconds after it was hit.

4. Select all of the solutions to the equation  $0 = 4 + 12t - 16t^2$ .

A.  $-\frac{1}{4}$

B.  $\frac{1}{4}$

C. 4

D. 1

E. -1

5. How many seconds until the tennis ball hits the ground? Explain how you know.

**1 second.** Explanations vary.  $f(1) = 0$ , which means the ball is on the ground at 1 second. The other solution to the equation is a negative value of  $t$ , which doesn't make sense for this situation.

**Problems 6–8:** Katie is planning to go skydiving. She writes the function  $h(t) = -16t^2 + 13500$  to represent her height, in feet,  $t$  seconds after jumping out of the airplane.

6. According to Katie's function, how high is the airplane when she jumps?

**13,500 feet**

7. It's recommended that skydivers open their parachutes at 5000 feet. Use  $h(t)$  to approximate how many seconds after jumping Katie should open her parachute.

**About 23 seconds**

8. When Katie actually jumps, do you think she will reach 5000 feet in less time, more time, or exactly the amount of time you approximated? Explain your thinking.

**Responses and explanations vary.** I think it will take Katie more than 23 seconds because her body is working against a lot of wind, which might cause her to slow down.

# Lesson Practice

A1.8.16

Name: ..... Date: ..... Period: .....

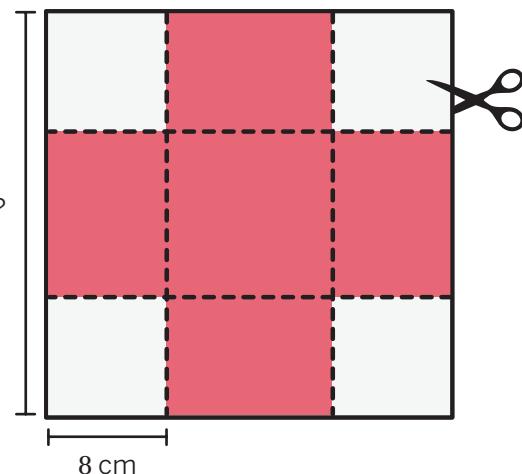
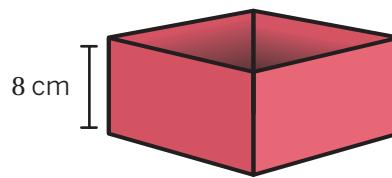
9. A company wants to make a square box with no top. The requirements are:

- It must be 8 cm tall.
- Its volume must be 1,000 cubic cm.

The boxes are made by cutting four corners from a square piece of cardboard and folding the flaps up.

What should the length of the starting square be? Show or explain your thinking.

**About 27.2 cm.** *Explanations vary.* If  $x$  is the length and the width of the square box, I can write an equation for the volume:  $8x^2 = 1000$ . The positive solution to this equation is  $\sqrt{125}$  cm, or about 11.2 cm. So the length of the square is  $8 + 8 + 11.2 = 27.2$  cm.



## Spiral Review

10. Select all the irrational numbers.

A.  $\sqrt{13}$

B.  $-\frac{53}{11}$

C.  $-\sqrt{17}$

D.  $-\sqrt{\frac{28}{7}}$

E.  $\sqrt{81}$

F.  $(-\sqrt{8})^2$

11. Write an equation of each type that starts with  $x^2 + 8x = \dots$

No Solutions	$x^2 + 8x = \dots$
One Solution	$x^2 + 8x = \dots$
Two Solutions	$x^2 + 8x = \dots$

*Equations vary.*

**No solutions**

- $x^2 + 8x = -50$
- $x^2 + 8x = -17$

**One solution**

- $x^2 + 8x = -16$
- $x^2 + 8x = 8x$

**Two solutions**

- $x^2 + 8x = 50$
- $x^2 + 8x = 0$

## Reflection

1. Put a star next to a problem you want to understand better.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

*Rational numbers* are numbers that you can write as a fraction with an integer numerator and denominator. *Irrational numbers* are numbers that are not rational, which means you cannot write them as a fraction with an integer numerator and denominator.

Here are some properties of sums and products of rational and irrational numbers.

	Sums	Products		
Properties	The sum of two rational numbers is always rational. The sum of a rational number and an irrational number is always irrational.	The product of two rational numbers is always rational. The product of a nonzero rational number and an irrational number is always irrational.		
Examples	$\frac{1}{2} + 3$ $8 + \sqrt{25}$	$\sqrt{7} + 2$ $\frac{1}{2} + \sqrt{44}$	$\frac{2}{3} \cdot 0.\overline{1}$ $\frac{4}{5} \cdot \left(-\frac{7}{11}\right)$	$\pi \cdot \frac{1}{3}$ $2 \cdot \sqrt{5}$

**Things to Remember:**

# Lesson Practice

A1.8.17

Name: ..... Date: ..... Period: .....

1. Select *all* the numbers that are rational.

A.  $-2$

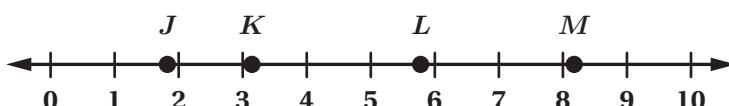
B.  $\frac{1}{3}$

C.  $\sqrt{24}$

D.  $\sqrt{25}$

E.  $\sqrt{-9}$

2. Here is a number line with four points on it. Match each point to the appropriate expression.



$\sqrt{10}$  ..... *K* .....

$\sqrt{34}$  ..... *L* .....

$5 + \sqrt{10}$  ..... *M* .....

$5 - \sqrt{10}$  ..... *J* .....

3. Select *all* the expressions that are rational.

A.  $3 + \frac{8}{7}$

B.  $\sqrt{2} \cdot 9$

C.  $\frac{8}{3} \cdot \sqrt{7}$

D.  $\sqrt{16} - \sqrt{25}$

E.  $\sqrt{-9} + 1\bar{6}$

4. Here are the solutions to some quadratic equations. Decide if the solutions are rational or irrational.

a)  $-2 \pm \sqrt{9}$

Rational

Irrational

b)  $3 \pm \sqrt{2}$

Rational

Irrational

c)  $\frac{1}{2} \pm \frac{3}{2}$

Rational

Irrational

d)  $10 \pm 0.3$

Rational

Irrational

e)  $\frac{1 \pm \sqrt{8}}{2}$

Rational

Irrational

f)  $-7 \pm \sqrt{\frac{4}{9}}$

Rational

Irrational

5. Consider the statement: *An irrational number multiplied by an irrational number always makes an irrational product.* Select *all* the examples that show that this statement is false.

A.  $\sqrt{4} \cdot \sqrt{4}$

B.  $\sqrt{5} \cdot \sqrt{5}$

C.  $\sqrt{3} \cdot \sqrt{5}$

D.  $\sqrt{0} \cdot \sqrt{5}$

E.  $\frac{1}{\sqrt{5}} \cdot \sqrt{5}$

# Lesson Practice

A1.8.17

Name: ..... Date: ..... Period: .....

6. Mark the statements based on whether they are *always*, *sometimes*, or *never* true.

Statement	Always true	Sometimes true	Never true
a The sum of two rational numbers is rational.	✓		
b The sum of a rational number and an irrational number is rational.			✓
c The product of two rational numbers is rational.	✓		
d The product of a rational number and an irrational number is irrational.		✓	

7. Using the digits 0 to 9 without repeating, fill in the boxes to create a true equation.

$$\boxed{\phantom{0}}\sqrt{\boxed{\phantom{0}}}\cdot \boxed{\phantom{0}}\sqrt{\boxed{\phantom{0}}} = \boxed{\phantom{0}}\boxed{\phantom{0}}$$

*Responses vary.*

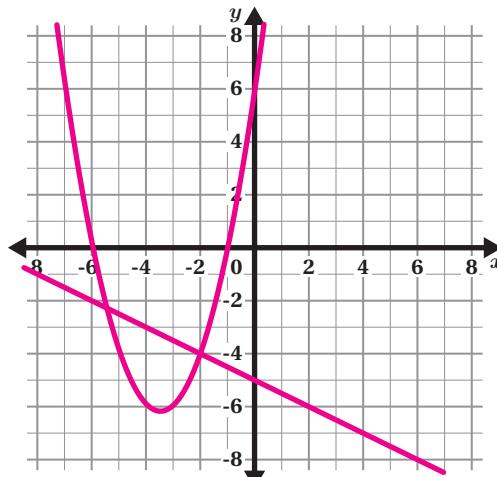
$$3\sqrt{1} \cdot 6\sqrt{9} = 54, 6\sqrt{1} \cdot 3\sqrt{9} = 54, 3\sqrt{1} \cdot 8\sqrt{9} = 72, 8\sqrt{1} \cdot 3\sqrt{9} = 72, 1\sqrt{2} \cdot 9\sqrt{8} = 36, \\ 9\sqrt{2} \cdot 1\sqrt{8} = 36, 3\sqrt{2} \cdot 5\sqrt{8} = 60, 5\sqrt{2} \cdot 3\sqrt{8} = 60, 2\sqrt{4} \cdot 9\sqrt{1} = 36, \text{ or } 9\sqrt{4} \cdot 2\sqrt{1} = 36$$

## Spiral Review

Problems 8–9: Use the coordinate plane.

8. Graph  $y = -\frac{1}{2}x - 5$ .

9. Graph  $y = x^2 + 7x + 6$ .



## Reflection

- Put a star next to a problem you're still wondering about.
- Use this space to ask a question or share something you're proud of.