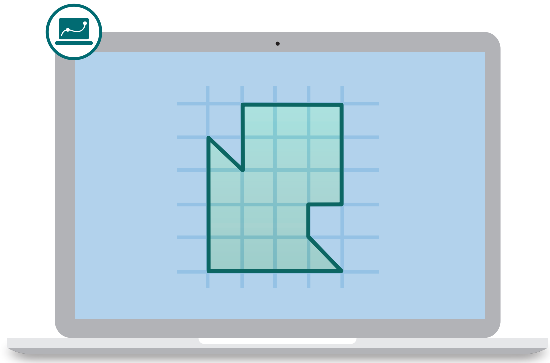
Name: .... Date: .... Period: ....

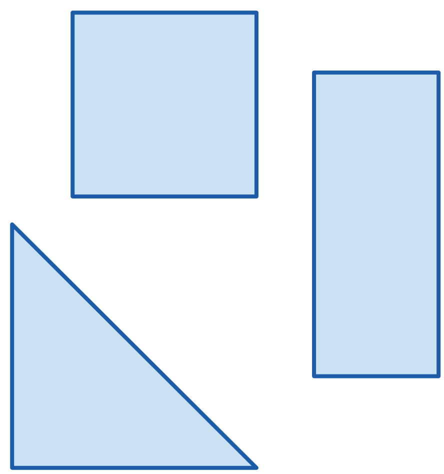


Shapes on a Plane

Let's play with shapes and find their areas.

Warm-Up

1 Trevon made a shape with these pieces.



a Let's watch the shape come together.

b Describe what the shape reminds you of.

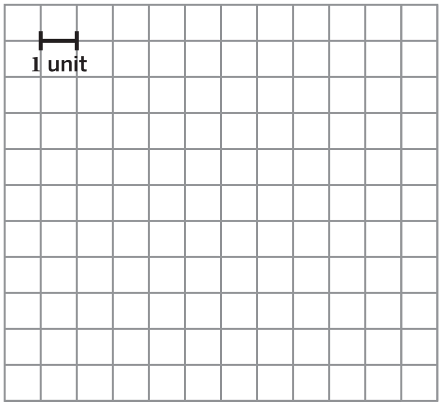
Responses vary.
- A basketball hoop
- A computer monitor
- A cat

Name: ________ Date: ________ Period: ________

Areas of Non-Rectangular Shapes

2 **a** You will use a set of shape cutouts to make your own fun shape.

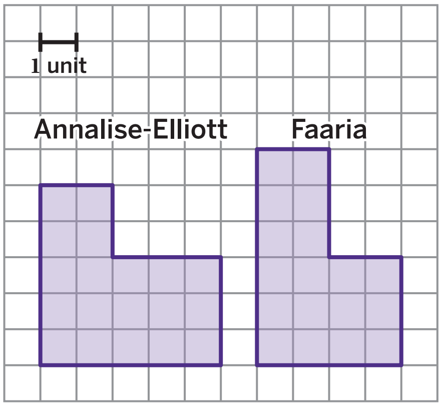
Designs vary.



b Describe your shape and what it reminds you of.

Responses vary depending on the shapes students create.

3 Annalise-Elliott and Faaria both made shapes that look like boots.



Whose boot shape is larger? Circle one.

Annalise-Elliott Faaria I'm not sure

Explain your thinking.

Responses and explanations vary.

- Annalise-Elliott's boot is made of $5 \cdot 3 + 2 \cdot 2 = 19$ squares. Faaria's is made of $4 \cdot 3 + 3 \cdot 2 = 18$ squares.
- Annalise-Elliott's boot has 3 more units on the bottom, and Faaria's boot has only 2 more units on the top.
- Both boots have a perimeter of 20 units, so they are the same size.

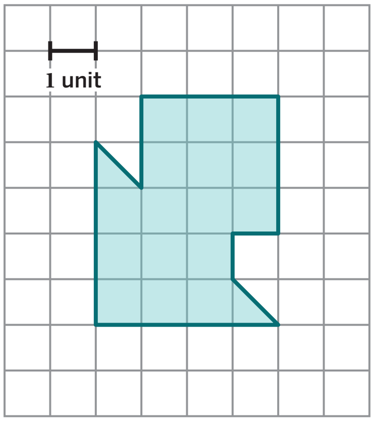
Name: .... Date: .... Period: ....

Areas of Non-Rectangular Shapes (continued)

4 The *area* of a shape is one way to describe its size.

Determine the area of Trevon's shape.

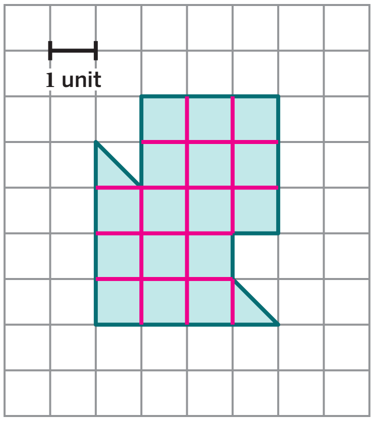
17 square units



5 There is often more than one way to determine area.

a Draw another way to determine the area of Trevon's shape.

Sketches vary.



b Describe your strategy.

Responses vary.

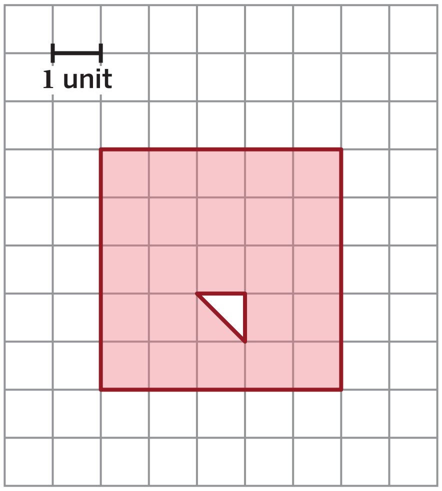
- **I broke the shape into a 3-by-3 square, a 3-by-2 rectangle with one extra square, and two triangles. This gave me $9 + 6 + 1 + 0.5 + 0.5 = 17$ square units.**
- **The cat is boxed into a big 4-by-5 rectangle with a few pieces cut out. The big rectangle has an area of 20 square units, but I have to take away the parts that aren't shaded. $20 - 3 = 17$ square units.**

Name: .... Date: .... Period: ....

Area Challenges

6 **a** **Discuss:** What does Nur's shape remind you of?

Responses vary. **It reminds me of a face with a mouth.**



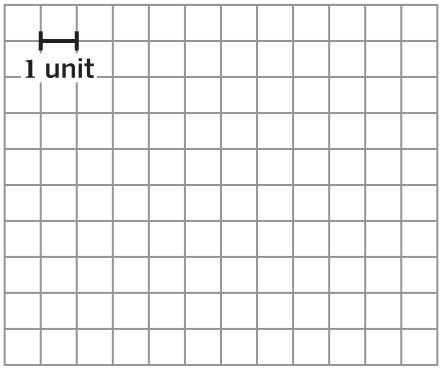
b Determine the area of Nur's shape. Draw on the shape if it helps with your thinking.

24.5 square units

7 How many different areas can you make with your shape cutouts? Use the grid if it helps with your thinking.

Responses vary.

- **The smallest possible area is 13.5 square units.**
- **The largest possible area is 27 square units.**



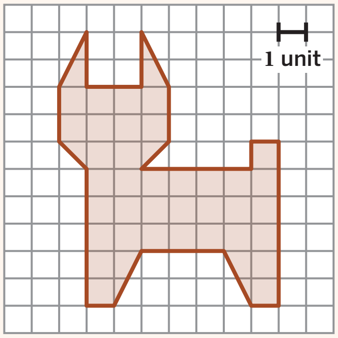
Explore More

8 Determine the area of the dog.

41 square units

Explain your thinking.

Explanations vary. **There are 13 square units in the dog's head, 21 square units in the body because $7 \cdot 3 = 21$, 6 square units in the feet, and 1 square unit in the tail.**



9 Synthesis

Discuss both questions, then select *one* and write your response.

- What's something you learned today?
- What do you want to learn more about?

Responses vary.

- **I learned that you can break a shape into parts to find the area of each part.**
- **I learned that sometimes shapes can look like cats!**
- **I want to learn more about what this class will be like this year.**
- **I want to learn more about finding complicated areas.**

Things to Remember:

Areas of Non-Rectangular Shapes

Directions: Make one copy per four students. Then pre-cut the cards and give each student one set of shapes. Have students cut out the shapes.

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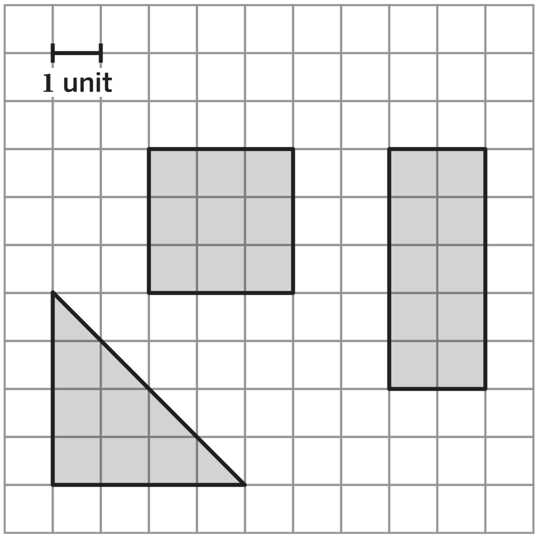
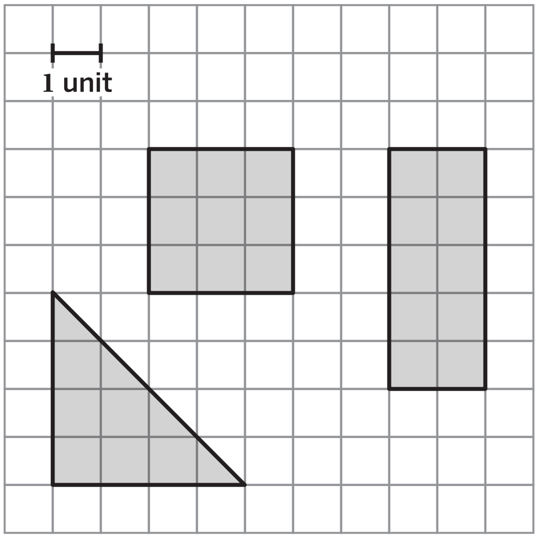
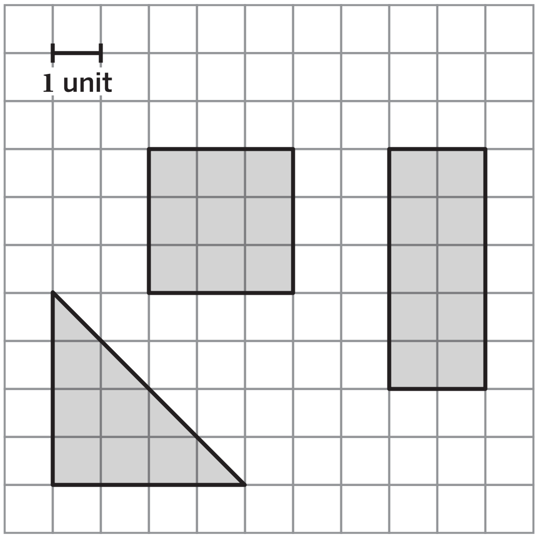
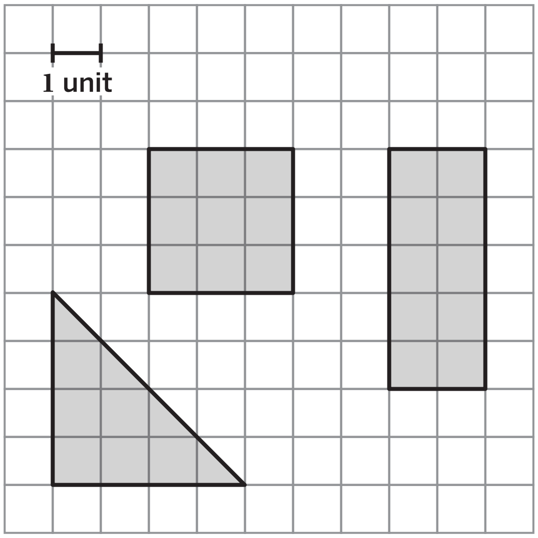
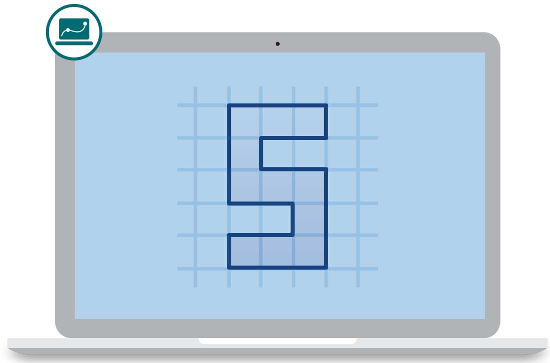








Name: .... Date: .... Period: ....



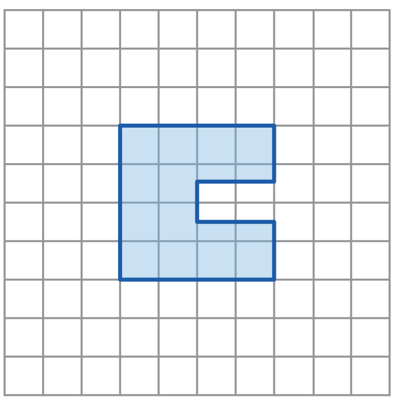
Letters

Let's explore the area of shapes.

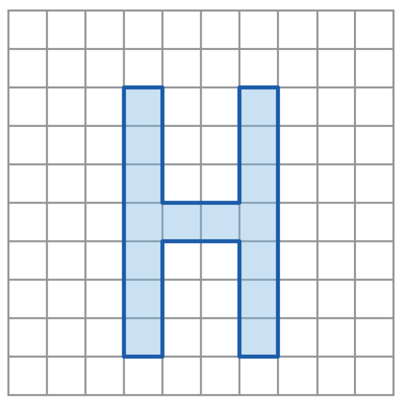
Warm-Up

1 Which figure doesn't belong? Explain your thinking.

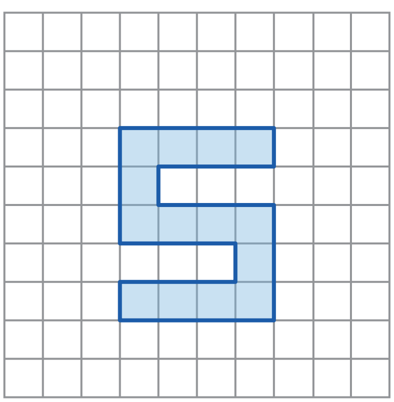
A.



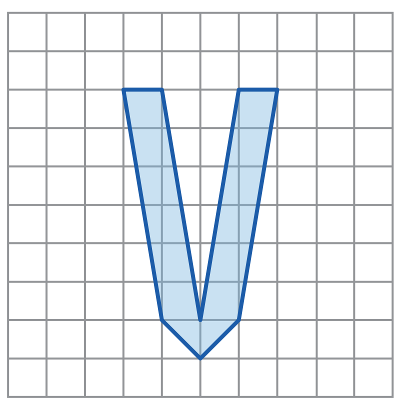
B.



C.



D.



Responses vary.

- **Choice A is the only letter that has the same height and width.**
- **Choice B is the only letter that has multiple lines of symmetry and an area that is not 14 square units.**
- **Choice C is the only letter that doesn't have a line of symmetry.**
- **Choice D is the only letter that's not made out of rectangles.**

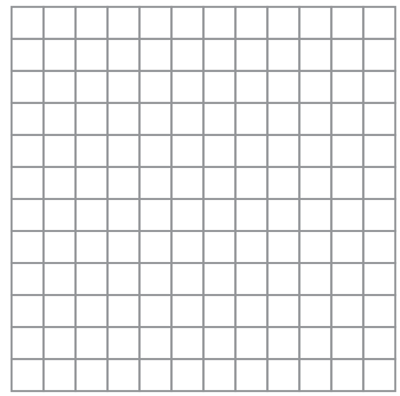
Activity
1

Name: ________ Date: ________ Period: ________

Rearranging Shapes

2 **a** Draw the first letter of your name on the grid.

Sketches vary.



b Tell a story about your name.

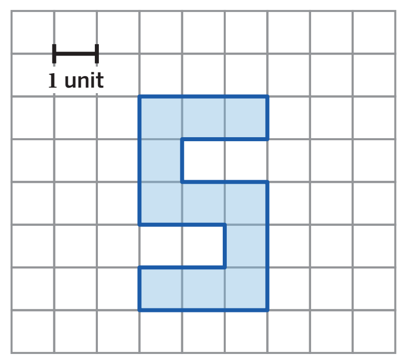
Responses vary.

- **My name means "song" or "poetry" in Hebrew.**
- **I am named for my uncle. He lives in Kansas.**
- **I don't know how I got my name.**

3 Saanvi sketched an "S" and colored it in.

What is the area of the shape Saanvi colored?

11 square units



4 Ichiro and Isabella each drew an "I." Ichiro cut his "I" into pieces to see how much area to color.

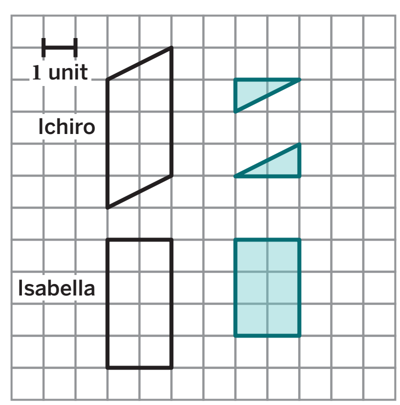
Whose letter has a greater area? Circle one.

Ichiro Isabella They are the same

Explain your thinking.

Explanations vary.

- **The pieces fill up both shapes without any gaps or overlaps so both areas must be the same.**
- **If you move the top triangle in Ichiro's "I" down to the bottom, you create Isabella's "I."**



Name: .... Date: .... Period: ....

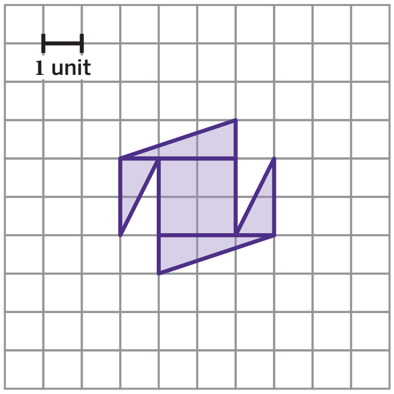
Rearranging Shapes (continued)

5 Zahra also cut up her "Z" to see how much area it covered.

What is the area of the shape she colored?

Use arrows to show how you could rearrange the pieces, if it helps with your thinking.

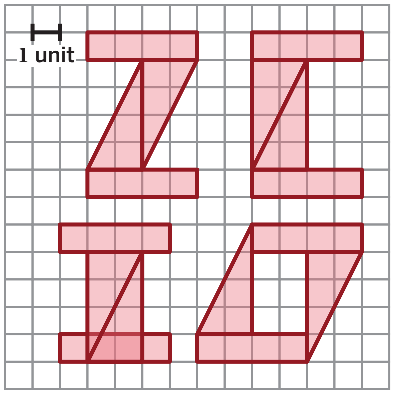
9 square units



6 Zola cut a "Z" into pieces and rearranged it to make new letters.

Select *all* the new letters that have the same area as the "Z."

- [x] C
- [ ] I
- [x] O



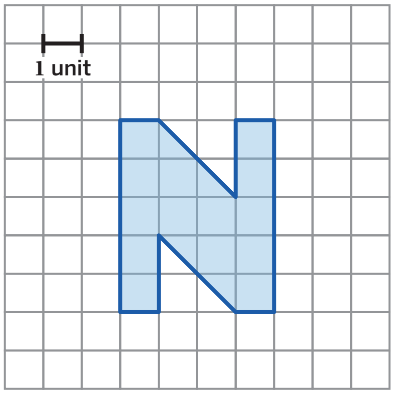
7 Nathan made an "N."

a What is the area of the shape Nathan colored? Sketch on the grid if it helps to show your thinking.

16 square units

b **Discuss:** What strategy did you use?

Responses vary.



Activity 2

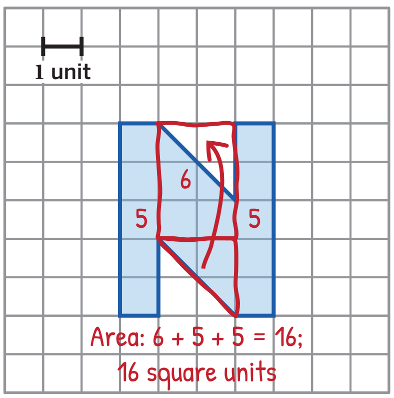
Name: ________ Date: ________ Period: ________

Area Strategies

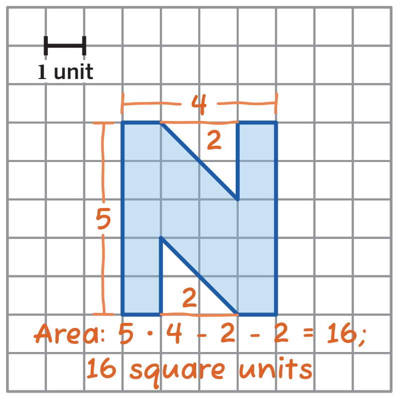
8 Omar and Jayla used different strategies to determine the area of "N."

a Take a look at each student's work.

Omar



Jayla



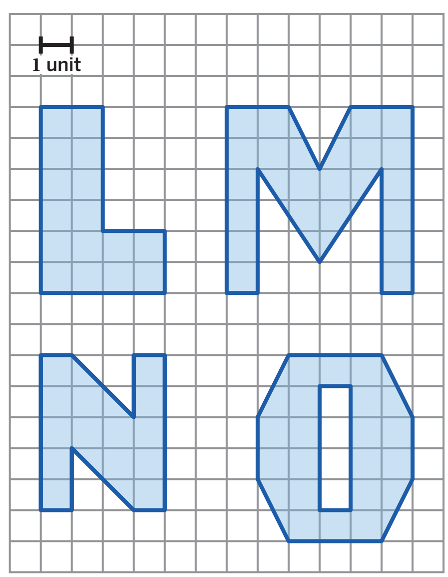
b Pick one student and explain how you think they determined the area.

Responses vary.

- **Omar moved the bottom triangle up to make a rectangle.**
- **Jayla made a big rectangle, then took away the areas of the parts that weren't shaded.**

9 Complete the table.

Letter	Area (sq. units)
L	**16**
M	**24**
N	16
O	**22**



10 Synthesis

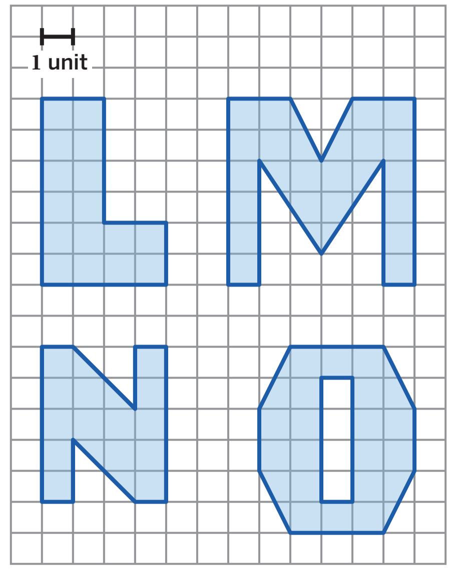
a Which area calculation are you most proud of? Circle one. ***Responses vary.***

L M N O

b Write some advice for someone determining this area.

Responses vary.

- **L: You can break the shape up into two rectangles.**
- **M: You can cut triangle shapes out of the M, make copies of them, and then use them to fill in the gaps and create rectangles.**
- **O: You can find the area of the rectangle that encloses the letter, then subtract the four triangles at the corners and the rectangle at the center.**

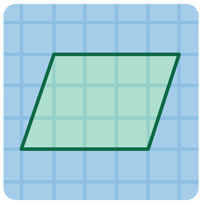


Things to Remember:

Unit 1
Lesson
3

Name: ______ Date: ______ Period: ______

Exploring Parallelograms, Part 1

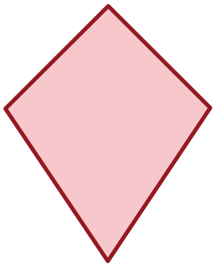


Let's investigate features of parallelograms.

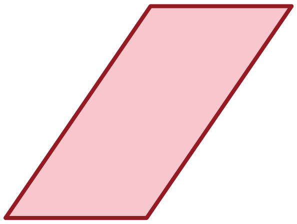
Warm-up

1. Which one doesn't belong? Explain your thinking.

A.

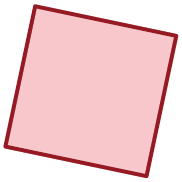


B.



C.

D.



Responses vary.

- **Choice A doesn't belong because it's the only shape without parallel sides.**
- **Choice B doesn't belong because it's the only shape without a line of symmetry.**
- **Choice C doesn't belong because it's the only shape without any slanted sides.**
- **Choice D doesn't belong because it's the only shape that is a square.**

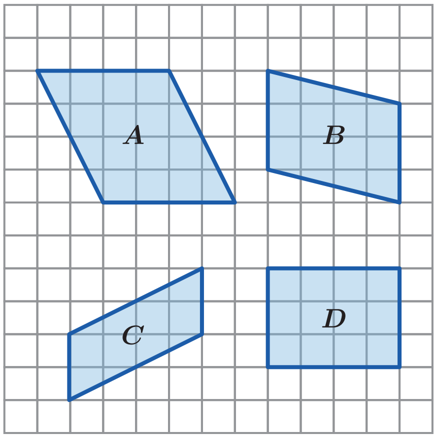
Activity 1

Name: .... Date: .... Period: ....

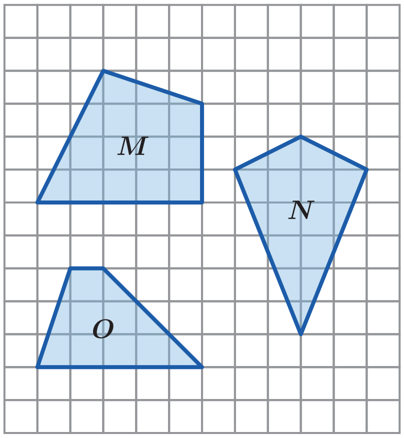
Parallelograms

2. Figures A, B, C, and D are *parallelograms*. Figures M, N, and O are *quadrilaterals* that are *not* parallelograms. What do you notice? What do you wonder?

Parallelograms



Not Parallelograms



I notice:

Responses vary.

- **Parallelograms have four sides.**
- **The opposite sides of parallelograms have the same length and are parallel.**

I wonder:

Responses vary.

- **Do the opposite angles of parallelograms have the same angle measure?**
- **Can a parallelogram have equal side lengths and not be a square?**

3. What do you think makes a shape a parallelogram?

a Write a first draft of your definition.

Responses vary.

b Meet with a partner to discuss your first drafts. Use the questions on the screen to help you provide feedback to each other.

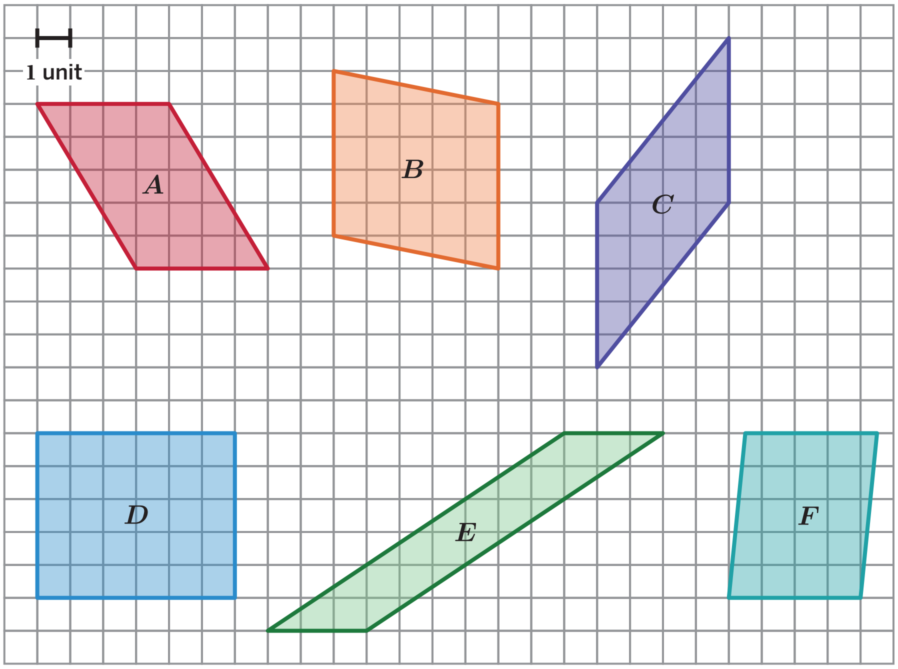
c Write a second draft that is stronger and clearer.

Responses vary. **If the opposite sides of a quadrilateral are parallel and have the same length, then the shape is a parallelogram.**

Name: .... Date: .... Period: ....

Area Strategies

4. Use any strategy to determine the area of these parallelograms.



Parallelogram	A	B	C	D	E	F
Area (sq. units)	**20**	**25**	**20**	**30**	**18**	**20**

5. Describe your strategy for determining the area of parallelogram C.

***Responses vary.* I cut the parallelogram horizontally and rearranged the triangles to make a rectangle with sides measuring 4 units and 5 units.**

6. What other parallelograms would your strategy work for? Explain your thinking.

***Responses vary.* I could also cut parallelogram B horizontally and rearrange the pieces to make a rectangle.**

Name: .... Date: .... Period: ....

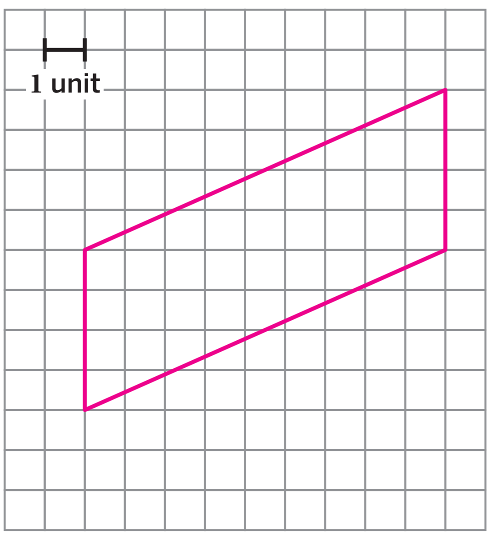
Area Strategies (continued)

7. Show or describe a classmate's strategy that was different from your own.

***Responses vary.* My classmate drew a rectangle around the parallelogram so it includes two right triangles. They rearranged the two right triangles to form a smaller rectangle. Then they subtracted the area of the smaller rectangle from the area of the larger rectangle.**

8. Draw a parallelogram with an area of 36 square units that is *not* a rectangle.

***Responses vary.* Sample shown on grid.**

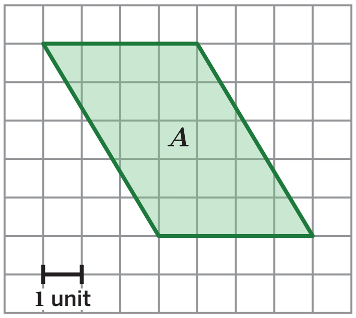


9. Explain how you know your parallelogram has an area of 36 square units.

***Responses vary.* I know my parallelogram has an area of 36 square units because I can break it into two triangles that can be rearranged to make a rectangle with an area of 36 square units.**

Synthesis

10. Show or describe a strategy for calculating the area of a parallelogram. Use parallelogram A if it helps with your thinking.



Responses vary.

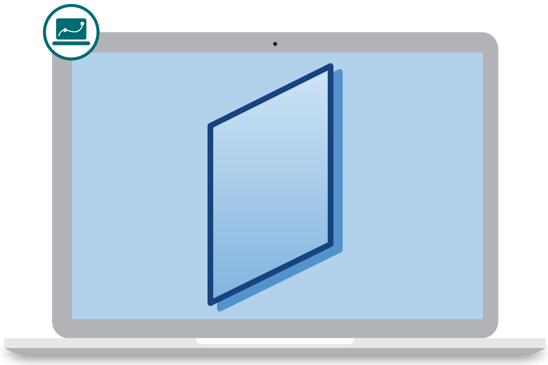
- **I can calculate the area by drawing a rectangle around the parallelogram, then subtracting the area of the non-shaded sections.**
- **I can calculate the area by breaking off a right triangle and rearranging it on the other side to form a rectangle.**

Things to Remember:

Name: .... Date: .... Period: ....

Off the Grid, Part 1

Let's practice determining the area of parallelograms.



Warm-Up

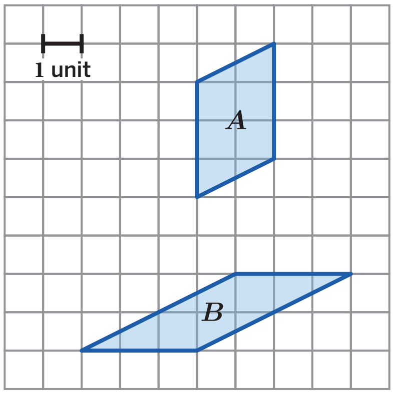
1 Which parallelogram has a greater area? Circle one.

A B They have the same area

Show or explain your thinking.

Explanations vary.

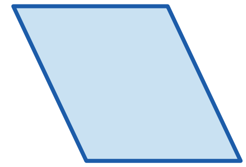
- **They both have an area of 6 square units.**
- **Both parallelograms have a base of 3 units and a height of 2 units, so even though B is more stretched out, the areas are the same.**



Name: .... Date: .... Period: ....

Measuring to Determine Area

2 In this lesson, you'll measure different parts of parallelograms.



a Let's watch how the measuring tool works.

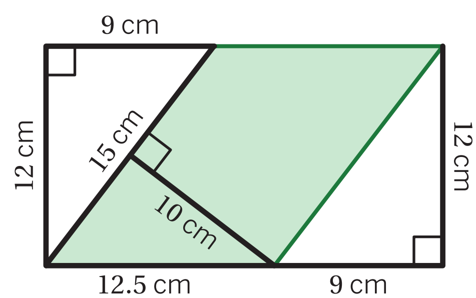
b Label as many measurements on this parallelogram as you want.

Responses vary.

3 What is the area of this parallelogram?

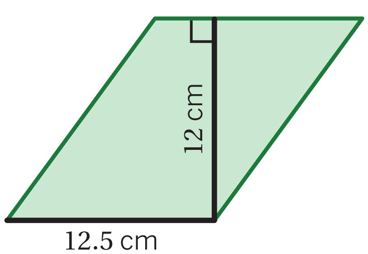
Use as many measurements as you need to calculate the area.

150 square centimeters

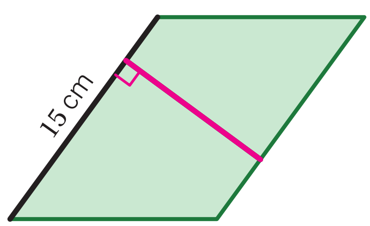


4 Here are some measurements that Angel and Ebony took. Sketch a line that Ebony can measure next to help calculate the area.

Angel



Ebony



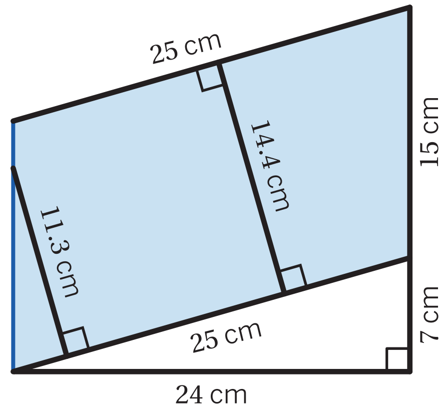
Name: .... Date: .... Period: ....

Measuring to Determine Area (continued)

5 What is the area of this parallelogram?

Use as many measurements as you need to help with your thinking.

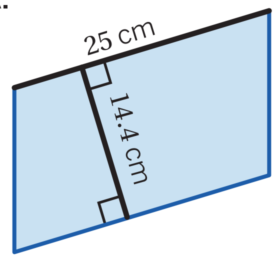
360 square centimeters



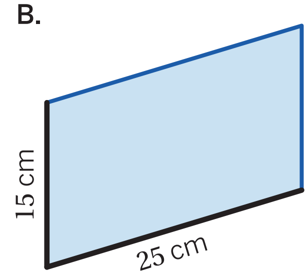
6 Here are some measurements taken by four different students.

Select *all* the parallelograms with measurements that can be used as a base and height pair.

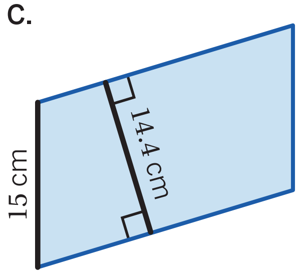
☑ **A.**



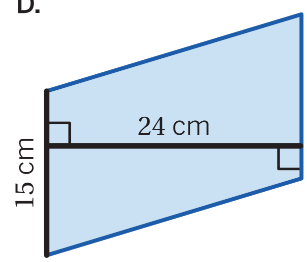
☐ **B.**



☐ **C.**



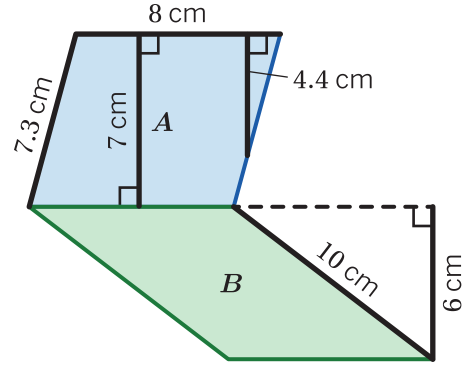
☑ **D.**



Name: .... Date: .... Period: ....

More Parallelograms

7 Which parallelogram has a greater area?



Use as few measurements as you can to help you decide.

A B They have the same area

Show or explain your thinking.

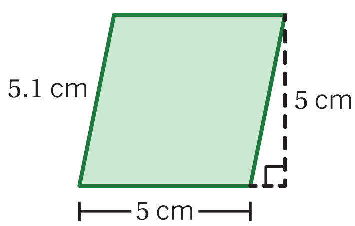
Explanations vary.

- **The area of parallelogram A is $8 \cdot 7 = 56$ square centimeters. The area of parallelogram B is $8 \cdot 6 = 48$ square centimeters.**
- **They have the same base, but A has a longer height, so A has to have a greater area.**

8 Draw a new parallelogram with the same base and a different height, so the area measures 40 square centimeters.

***Responses vary.* Any parallelogram with a height of 8 centimeters.**

Original Parallelogram   **New Parallelogram**



Explore More

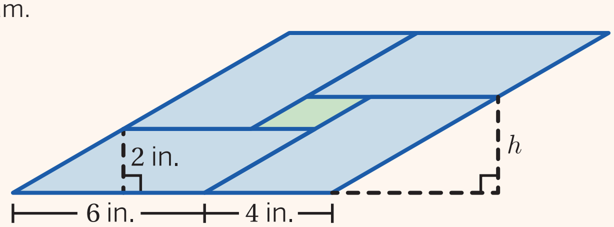
9 The shaded region in this diagram is composed of four identical parallelograms and a smaller parallelogram.

a What is the value of h?

3 inches

b What is the total shaded area?

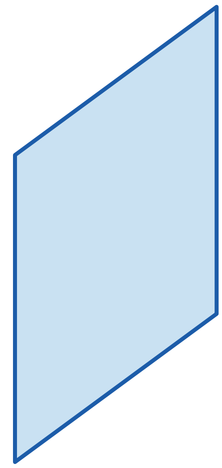
50 square inches



10 Synthesis

Describe how you can determine the area of any parallelogram.

Draw on this image if it helps to show your thinking.



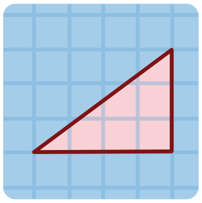
***Responses vary*. Pick any side to be the base, and measure it. Then draw a measurement that's perpendicular to your base. That will be the height. Multiply the base and height to determine the area.**

Things to Remember:

Unit 1
Lesson
6

Name: ________ Date: ________ Period: ________

Exploring Triangles



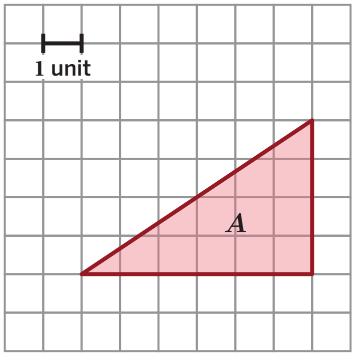
Let's explore the area of triangles.

Warm-Up

1. Determine the area of triangle A. Show or describe your thinking.

12 square units. *Explanations vary.*

- I counted the number of shaded squares.
- I drew a rectangle around triangle A and calculated the area. Then I divided the area by 2 because the triangle is half of the rectangle.



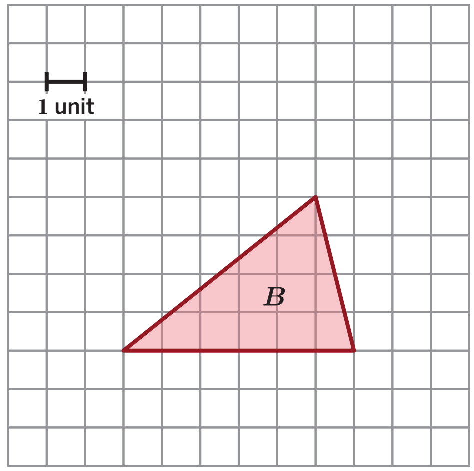
Activity 1

Name: ________ Date: ________ Period: ________

Area Strategies

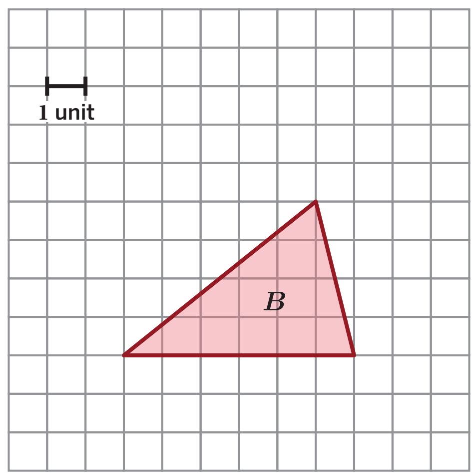
2. Determine the area of triangle B. Show or describe your thinking.

12 square units. *Explanations vary.* I created a parallelogram with the same base and height as triangle B. I determined that the area of this parallelogram was 24 square units. Then I divided by 2 to get the area of triangle B.



3. Find a classmate who calculated the area of triangle B using a different strategy. Show or describe how your partner calculated the area.

Responses vary. My partner drew two rectangles around the triangle and calculated the area of each rectangle. They divided each area by 2 because the triangle is half of the rectangle. Then they added the two smaller triangle areas together to find the area of the larger triangle.



4. Let's look at two strategies for calculating the area of triangle B.

Discuss: How are these two strategies alike? How are they different?

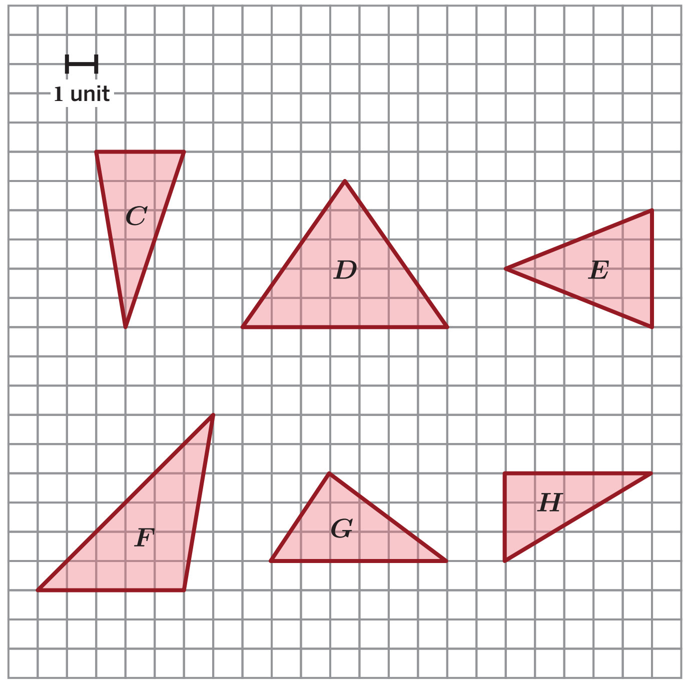
Responses vary. Both strategies use the triangle to build other shapes. The first strategy encloses the triangle using two rectangles. The second strategy uses the triangle to make a parallelogram.

Name: .... Date: .... Period: ....

Lots of Triangles

5. Determine the area of as many of these triangles as you can.

Triangle	Area (sq. units)
C	**9**
D	**17.5**
E	**10**
F	**15**
G	**9**
H	**7.5**

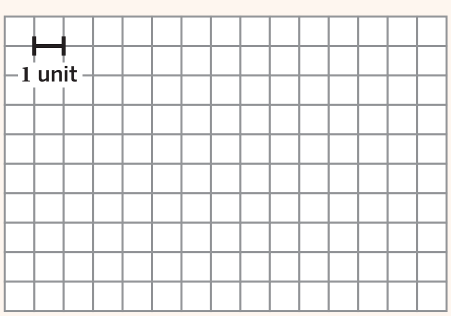


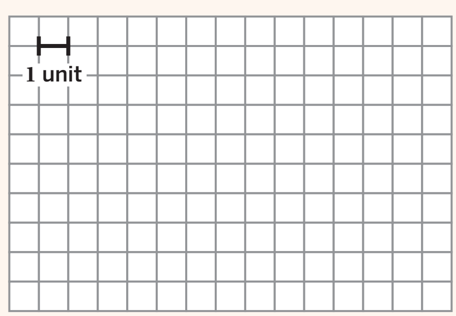
6. Describe the strategy that was most helpful to you. Did this strategy work for *all* the triangles?

Responses vary. **I drew one or more rectangles around the triangles. I found the total area and then divided the total area by 2. My strategy didn't work for triangle F.**

Explore More

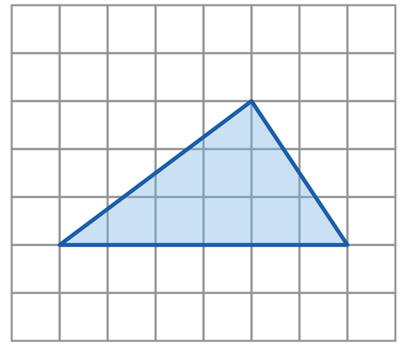
7. Draw two different triangles that both have an area of 18 square units. ***Responses vary.***





Synthesis

8. Describe a strategy to determine the area of a triangle. Use the example if it helps with your thinking.



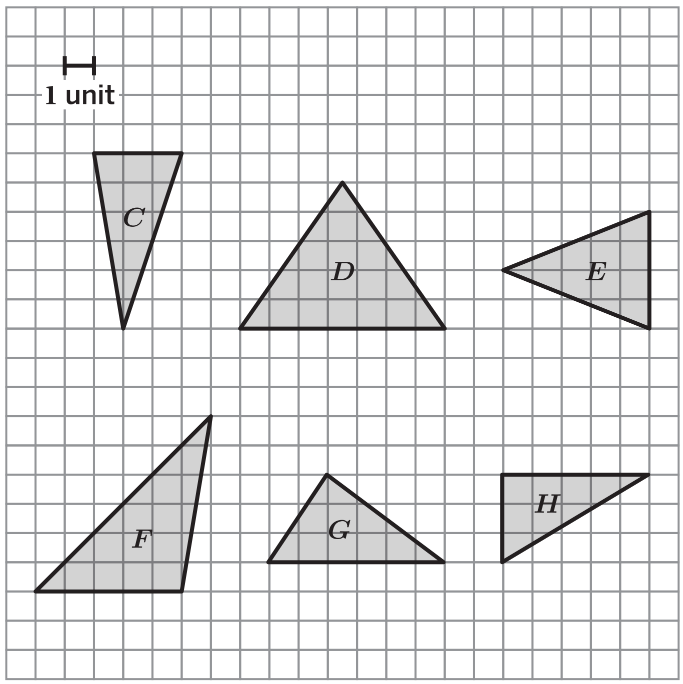
***Responses vary.* I can make a copy of the triangle and rearrange the triangles to make a parallelogram. I can determine the area of the parallelogram and then divide that area by 2 to find the area of one triangle.**

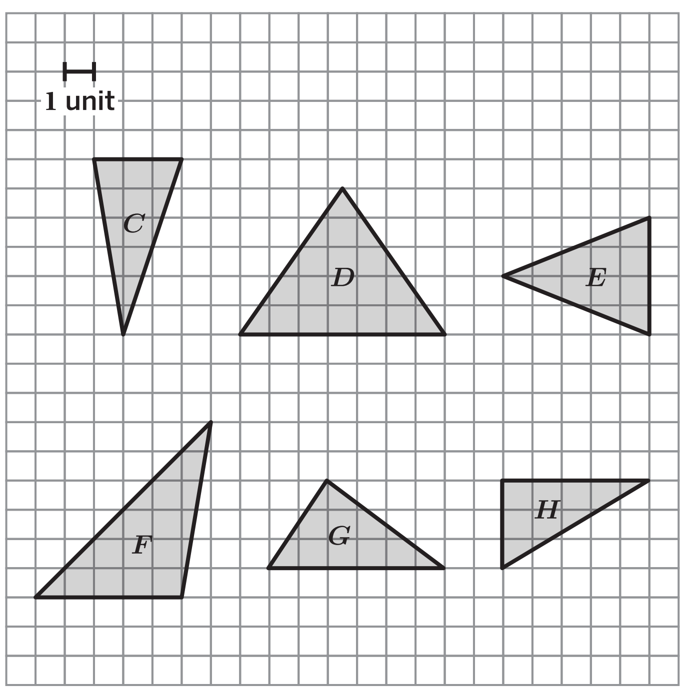
Things to Remember:

Lots of Triangles

Cut out the bottom set of triangles and use them to form parallelograms with the top set of triangles. As you create parallelograms, consider discussing these questions with your classmates:

- Which triangle did you start with?
- Where did you get stuck?
- What did you try?
- Which strategies were most helpful to you?





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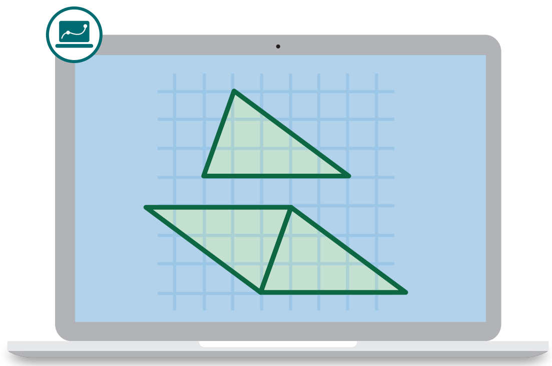
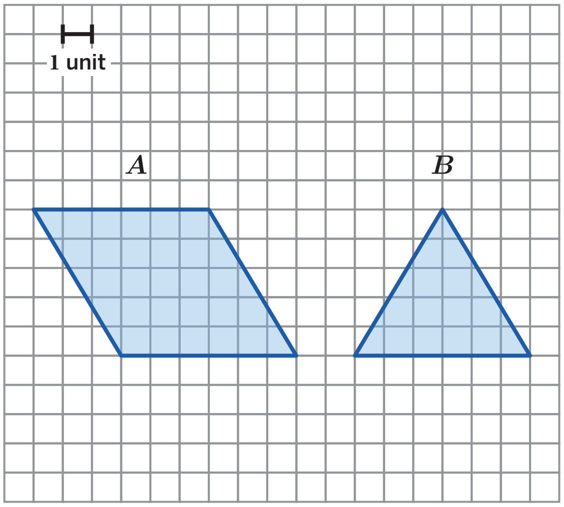


Triangles and Parallelograms

Let's explore the relationship between triangles and parallelograms.

Warm-Up

1 List two things that are the same about these figures.



Responses vary.

- Both shapes are blue.
- Both shapes are 5 units tall.
- The side on the bottom of each shape is 6 units long.

List two things that are different.

Responses vary.

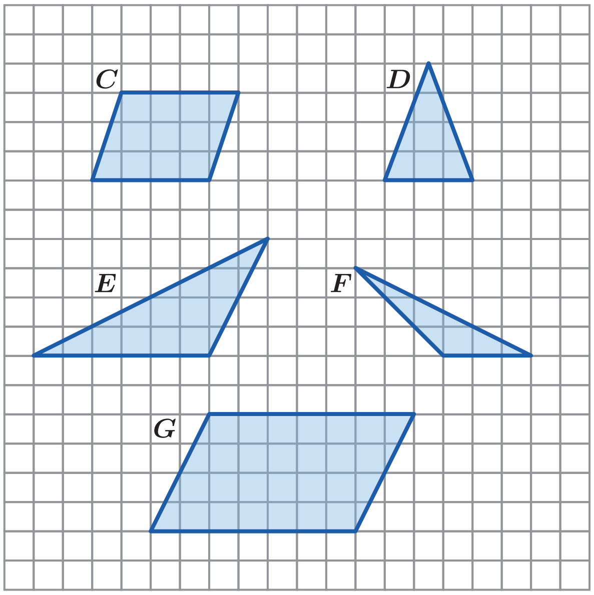
- Figure A is a parallelogram, and figure B is a triangle.
- Figure A is bigger than figure B. It has a larger area.
- The area of figure A is double the area of figure B.

Name: .... Date: .... Period: ....

Triangles and Parallelograms

2 You can determine the heights of triangles, just like you can with parallelograms.

Select a set of shapes that have the same height.

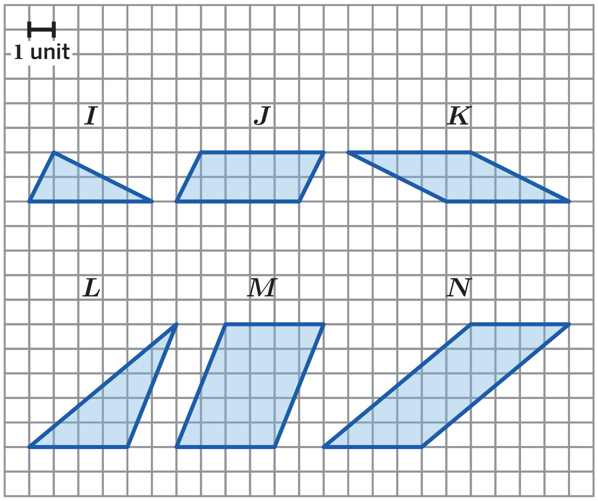


- ☐ C
- ☐ D
- ☐ E
- ☐ F
- ☐ G

Responses vary.
- C and F
- D, E, and G

3 **a** Determine the base, height, and area of each shape.

Shape	Base (units)	Height (units)	Area (sq. units)
I	5	2	5
J	5	2	10
K	5	2	10
L	4	5	10
M	4	5	20
N	4	5	20



b **Discuss:** What patterns do you notice?

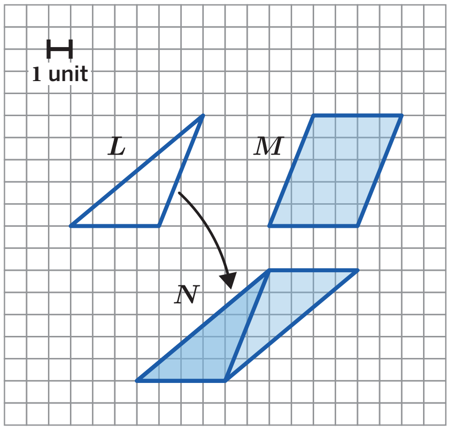
Responses vary. I notice that the area of each triangle is half the area of the parallelograms beside it. I also notice that the shapes can look different but have the same area.

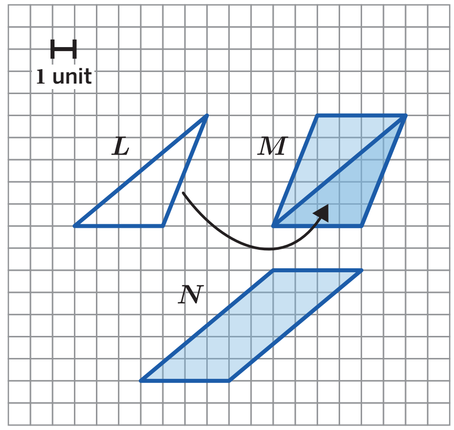
Name: Date: Period:

Triangles and Parallelograms (continued)

4 Here is a triangle and two parallelograms from the previous problem.

a Take a look at how the shapes compare.





b What is the relationship between the areas of these three shapes?

***Responses vary.* The triangle fills up half the area of each parallelogram, so the area of the triangle is half the area of each parallelogram.**

Activity
2

Name: .... Date: .... Period: ....

Generalizing Triangle Area

5 Let's see if we can always combine two copies of a triangle to form a parallelogram.

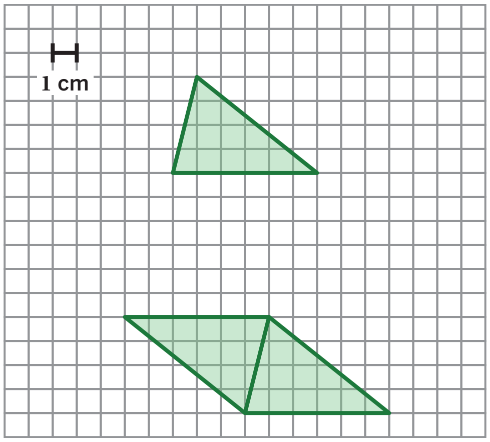
a Let's create a triangle.

Triangles vary.

b **Discuss:** How many different parallelograms can you create using two copies of your triangle?

***Responses vary.* I can create 3 different parallelograms using the two triangles.**

6 Here is a triangle and a parallelogram.



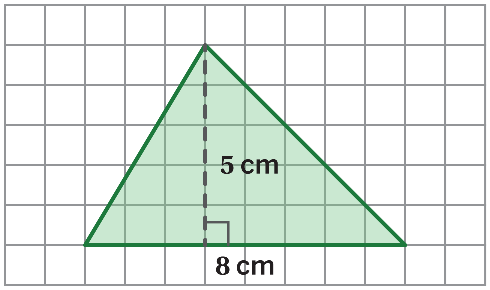
a What is the area of the triangle?

12 square centimeters

b What is the area of the parallelogram?

24 square centimeters

7 Here's the expression that Alisha entered to find the area of this triangle.



$8 \cdot 5 \cdot \frac{1}{2}$

Explain what each number represents in the expression.

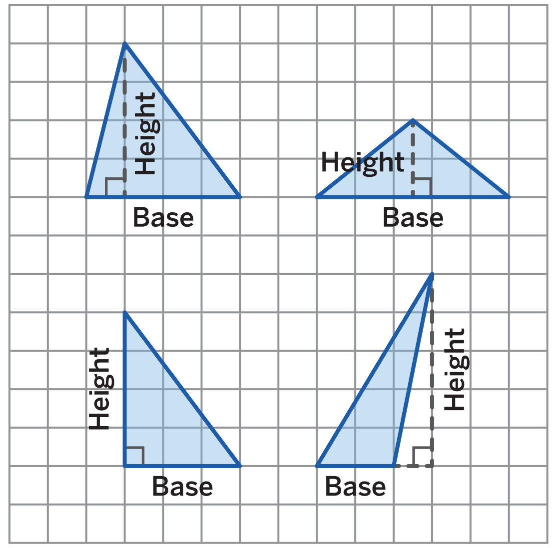
Responses vary.

- **8 represents the base of the triangle.**
- **5 represents the height of the triangle.**
- **$\frac{1}{2}$ represents that the area of this triangle is half the area of the parallelogram that can be created using two identical triangles with a base of 8 centimeters and a height of 5 centimeters.**

8 Synthesis

How can you use the base and height of any triangle to calculate its area?

Responses vary. **I can multiply the base and height of any triangle to determine the area of a parallelogram made with two identical triangles. To determine the area of the triangle, you then have to divide the parallelogram's area by 2 or multiply by $\frac{1}{2}$.**



Things to Remember:

Triangles and Parallelograms

Directions: Make one copy per pair of students. Then pre-cut the cards and give each student one set of shapes.

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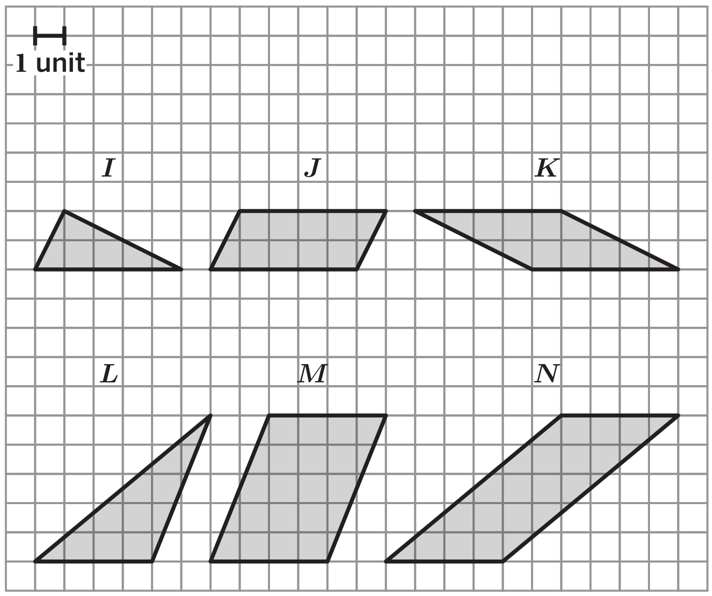

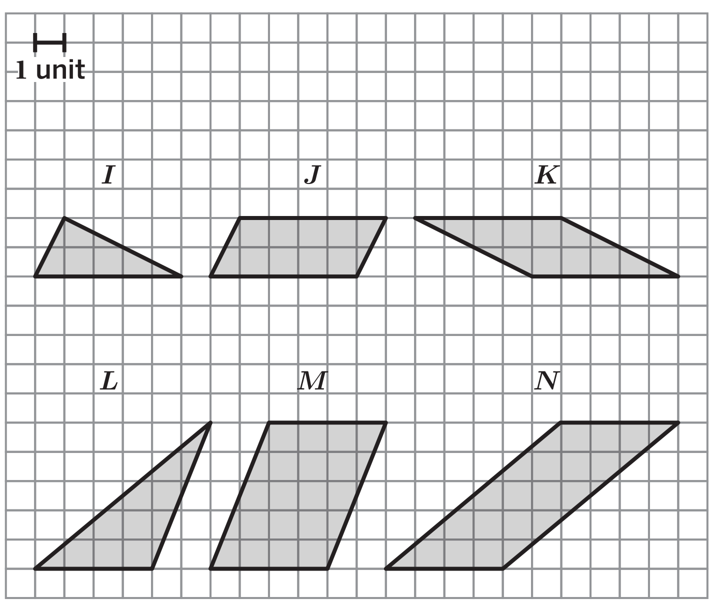

Unit 1
Lesson
8

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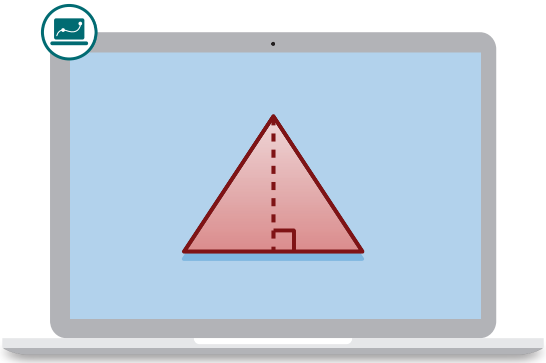


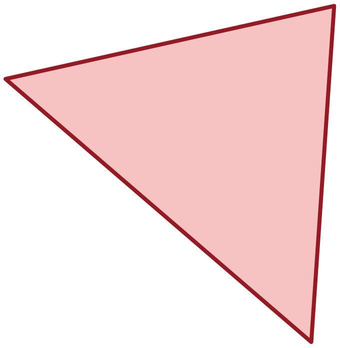
Off the Grid, Part 2

Let's practice calculating the area of triangles.

Warm-Up

1 Let's look at the different sides of a triangle.

What is one thing that changes? What is one thing that stays the same?



Responses vary.

- **One thing that changes is how long the base of the triangle is.**
- **One thing that changes is where you draw the height. Sometimes it's near the middle, sometimes it's not.**
- **One thing that stays the same is the area.**
- **One thing that stays the same is the triangle itself, no matter which way you turn it.**

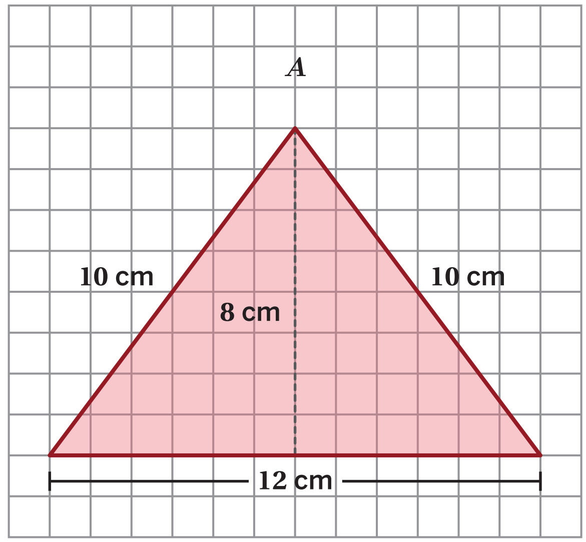
Activity 1

Name: ______________________ Date: __________ Period: __________

Base, Height, and Area

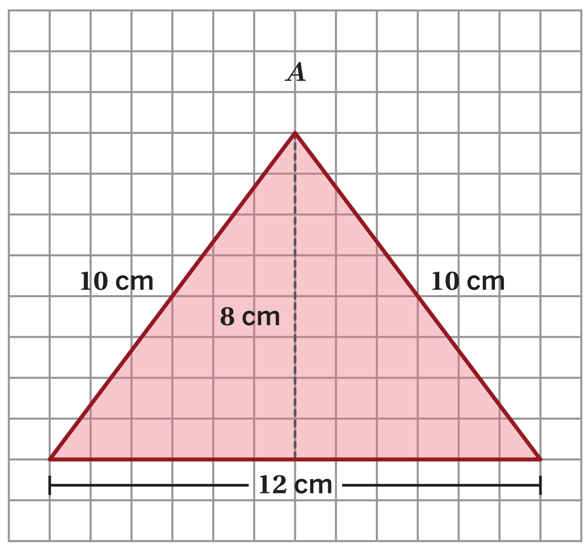
2 Use any strategy to determine the area of triangle A.

48 square centimeters

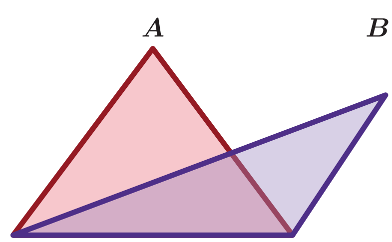


3 Select *all* the expressions that could represent the area of this triangle. Draw on the triangle if it helps with your thinking.

- ☑ A. $\frac{1}{2} \bullet 12 \bullet 8$
- ☐ B. $\frac{12 \bullet 10}{2}$
- ☑ C. $12 \bullet 8 \div 2$
- ☑ D. $6 \bullet 8$
- ☐ E. $6 \bullet 4$



4 Here is triangle A from the previous problem, along with a new triangle. Which triangle has the greater area? Circle one.



Triangle A Triangle B

They are the same Not enough information

Explain your thinking.

Responses and explanations vary.

- **Triangle A. The area of a triangle depends on the base and the height. The bases of both triangles are the same, but triangle A is taller, so its area must be greater.**
- **Not enough information. I can't know the area of these triangles because there are no measurements.**

Activity 2

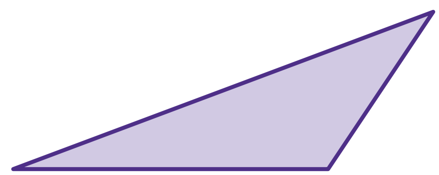
Name: ____________ Date: ________ Period: ________

Choose Your Measurements

5 Ishaan wants to calculate the area of this triangle, but the measurements are not labeled.

Draw on the triangle to show what Ishaan should measure to calculate the area.

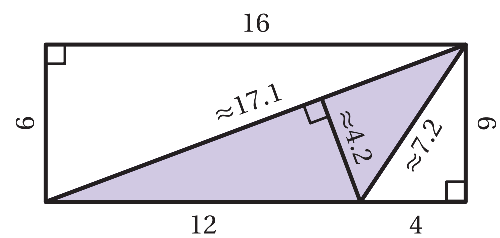
Measurements vary.



6 Use as many measurements as you want to calculate the area of the triangle.

All measurements are in centimeters.

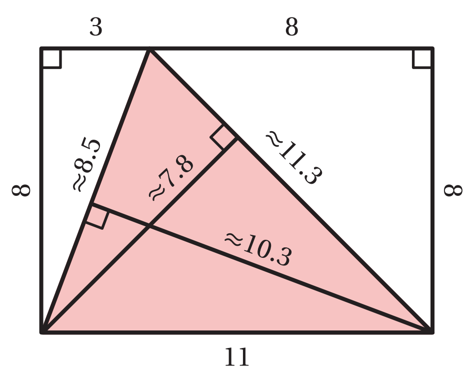
36 square centimeters



7 Use as many measurements as you want to calculate the area of the triangle.

All measurements are in centimeters.

44 square centimeters



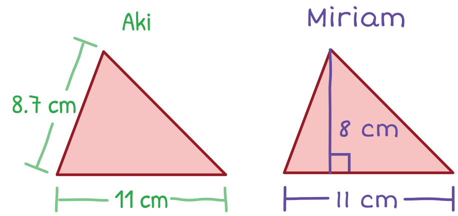
Name: .... Date: .... Period: ....

Choose Your Measurements (continued)

8 Aki and Miriam found different areas for the same triangle.

Here are the measurements they took.

Whose measurements lead to the correct area? Circle one.



Aki's Miriam's Both Neither

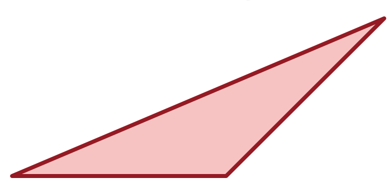
Explain your thinking.

***Explanations vary*. Both measured a base correctly, but Aki measured a different side as if it were the height. The height has to be perpendicular to the base. Aki could use the 8.7-centimeter measurement as a base, but then Aki would need a matching height.**

9 Sketch as many different triangles as you can with the same area as triangle C.

***Triangles vary*. The base multiplied by the height should be 48 square centimeters.**

Triangle C
Area = 24 sq. cm

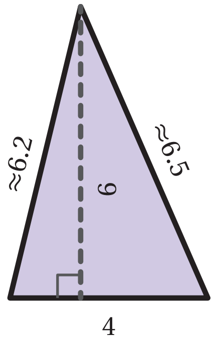


Name: ______ Date: ______ Period: ______

Repeated Challenges

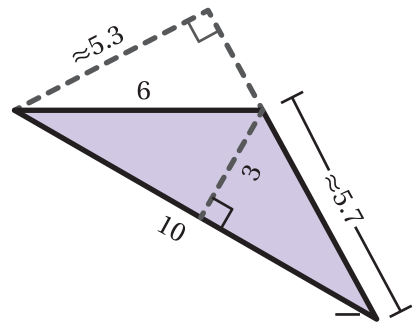
10 Calculate the area of each triangle. Use as many measurements as you need. All measurements are in centimeters.

a



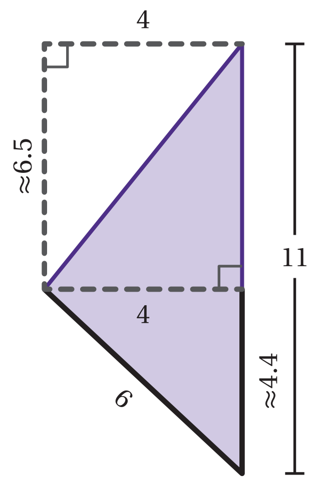
12 square centimeters

b



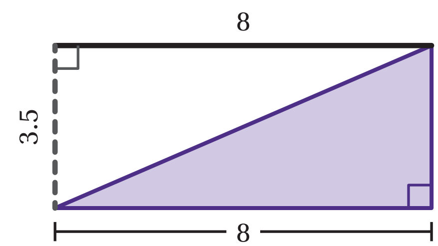
15 square centimeters

c



22 square centimeters

d

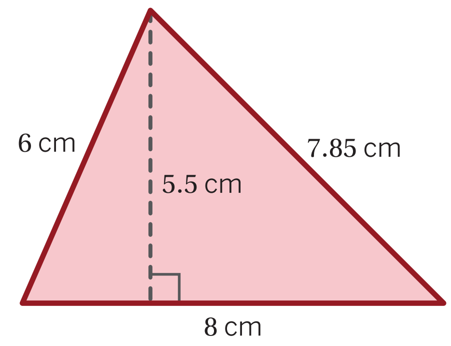


14 square centimeters

11 Synthesis

Describe how to calculate the area of a triangle.
Use this example if it helps with your thinking.

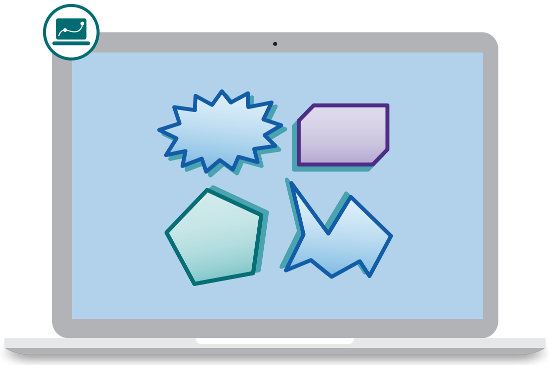
***Responses vary*. To find the area, I always need a base and a height. The base can be any side of the triangle, and the height is always perpendicular to the base. I can calculate the area by multiplying the base by the height and dividing by two.**



Things to Remember:

Unit 1
Lesson
9

Name: ________ Date: ________ Period: ________



Pile of Polygons

Let’s play with polygons.

Warm-Up

1 Play a few rounds of Polygraph with your classmates!

You will use the Warm-Up Sheet with shapes for four rounds. In each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a shape from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating shapes until you’re ready to guess which shape the Picker chose.

Record helpful questions from each round in this workspace:

Responses vary.

Activity 1

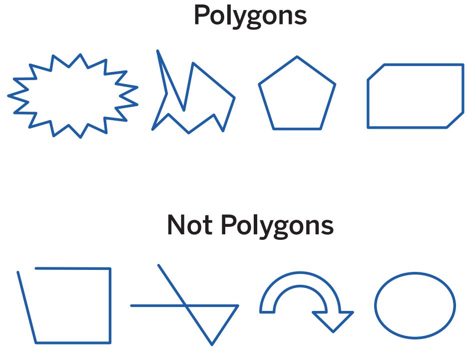
Name: Date: Period:

Polygons and Not Polygons

2 How are **polygons** different from shapes that are not polygons?

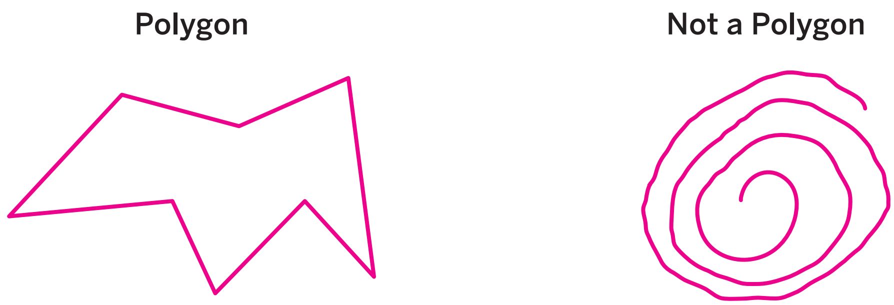
Responses vary.

- Polygons don't have any curvy sides.
- Some of the shapes that aren't polygons have sides that cross or don't close.
- The sides of polygons always close and never cross.



3 Sketch a shape that is a polygon and a shape that is not a polygon.

Sketches vary. Sample shown.



Activity

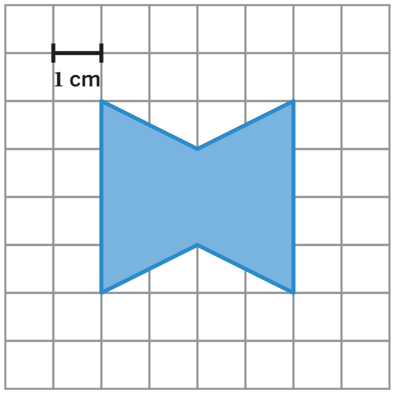
2

Name: .... Date: .... Period: ....

What is the Area?

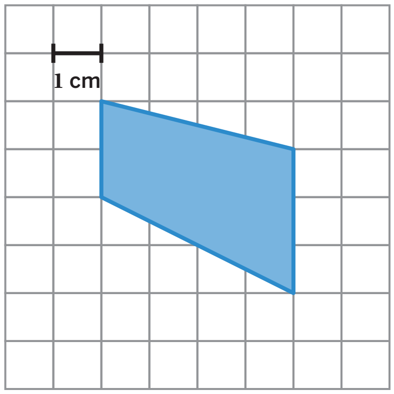
4 What is the area of this polygon? Draw on the shape if it helps with your thinking.

12 square centimeters



5 Use any strategy to determine the area of this polygon.

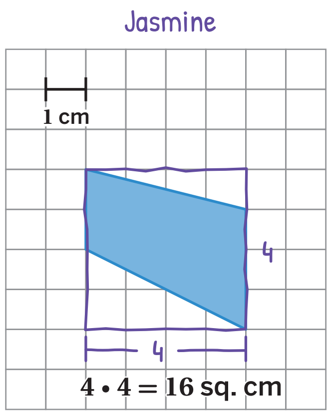
10 square centimeters

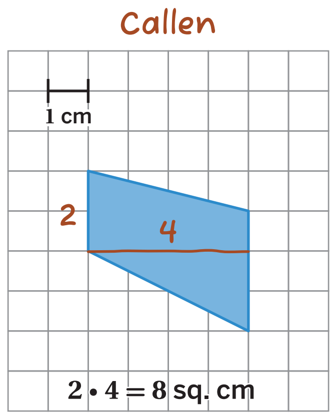


Name: .... Date: .... Period: ....

What is the Area? (continued)

6 Jasmine and Callen both made mistakes when they calculated the area for this polygon.





a Choose your favorite mistake. Circle one.

Jasmine Callen

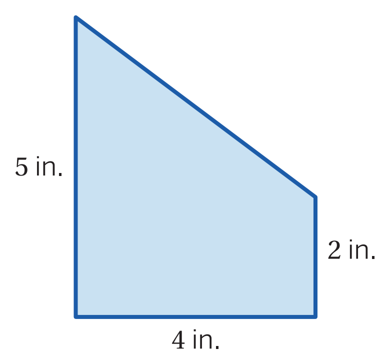
b Explain what you think is incorrect about the work you chose.

Responses vary.

- **Jasmine thought about drawing a rectangle around the shape. She didn't subtract the parts that weren't shaded.**
- **Callen calculated the area of the shape as if it was a parallelogram. But the shape isn't a parallelogram because the opposite sides don't have the same length.**

7 Determine the area of this polygon.

14 square inches



Activity 3

Name: ______ Date: ______ Period: ______

Challenge Creator

8 You will use the Challenge Creator Sheet to create your own area challenge.

a. **Make It!** On the Challenge Creator Sheet, create your own area challenge.

b. **Solve It!** On this page, determine the area of your polygon. ***Responses vary.***

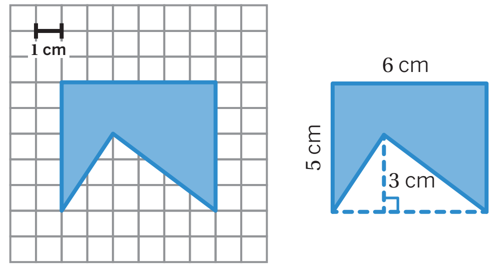
My Area

c. **Swap It!** Swap your challenge with one or more partners. Determine the area of each partner's polygon. ***Responses vary.***

	Partners' Areas
Partner 1	
Partner 2	
Partner 3	

9 Synthesis

Describe a strategy for calculating the area of this polygon.



Responses vary.

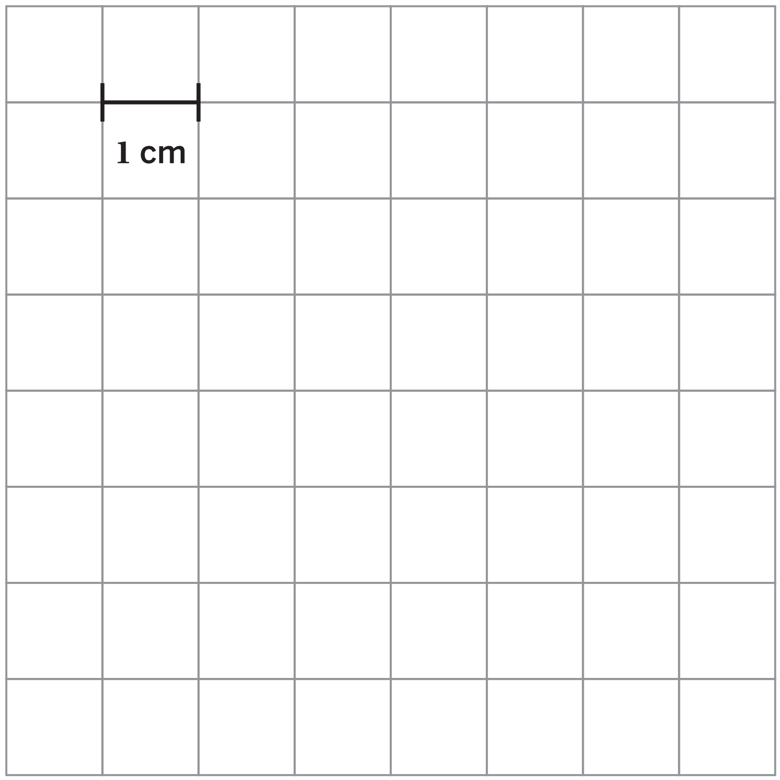
- I can break the polygon into shapes I know how to find the area of, like rectangles and triangles.
- I can make one big rectangle, then subtract all the parts that aren't shaded.
- I can either count the number of squares on the grid or use the measurements to figure out how long each side is.

Things to Remember:

Name: ____________ Date: ____________ Period: ____________

Challenge Creator

- Create a 4-sided, 5-sided, or 6-sided polygon on this grid.
- Calculate the area of your polygon in your Student Edition. (Don't write it on this page!)



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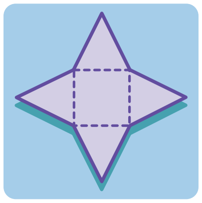
Unit 1
Lesson
12

Name: Date: Period:

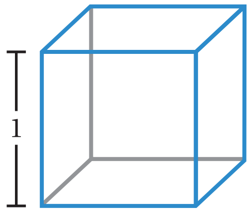
Nothing But Nets



Let's make connections between polyhedra and their nets.

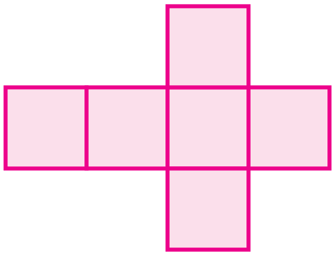
Warm-Up

Here is a polyhedron.



1. What could you call this type of polyhedron?
Cube, square prism, or rectangular prism

2. Draw its net.



Activity 1

Name: ________ Date: ________ Period: ________

Nets and Polyhedra

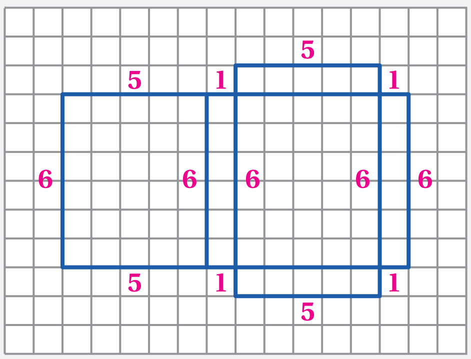
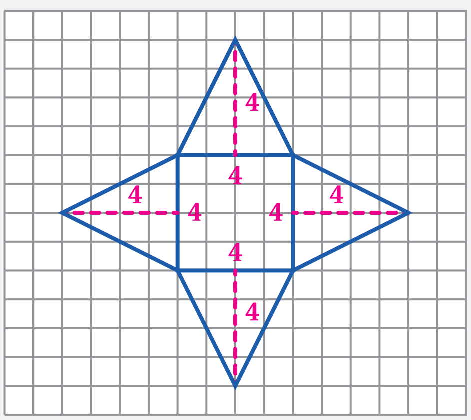
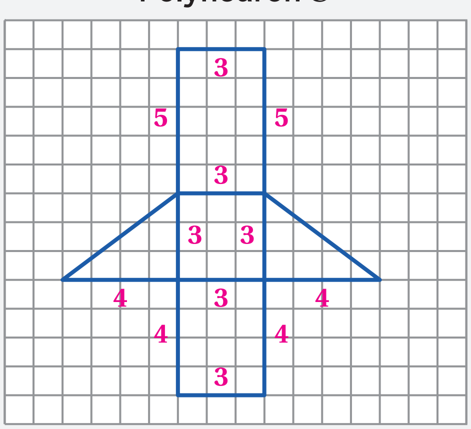
3. You will use a set of cards for this activity. Match each polyhedron to its net. Record your matches in the table below and circle the name of the polyhedron.

Polyhedron	Net	Name		
Card 6	Card 1	Triangular pyramid	Triangular prism	Rectangular prism
Card 7	Card 2	Triangular pyramid	Triangular prism	Rectangular prism
Card 8	Card 3	Triangular pyramid	Triangular prism	Rectangular prism
Card 5	Card 4	Triangular pyramid	Triangular prism	Rectangular prism

Name: ______________ Date: ________ Period: ________

Make Polyhedra

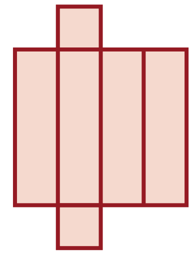
4. You will use the Activity 2 Sheet for this activity. Take a look at the nets for polyhedra A, B, and C. Cut out, assemble, and name each polyhedron. Then calculate its surface area. Record your responses and show your thinking in the table below.

Net	Name	Surface Area
Polyhedron A 	Rectangular prism	82 square units. *Work varies.* $2(6 \cdot 5) + 2(1 \cdot 6) + 2(1 \cdot 5) = 82$
Polyhedron B 	Rectangular pyramid or square pyramid	48 square units. *Work varies.* $4\left(\frac{1}{2} \cdot 4 \cdot 4\right) + (4 \cdot 4) = 48$
Polyhedron C 	Triangular prism	48 square units. *Work varies.* $(3 \cdot 5) + 2\left(\frac{1}{2} \cdot 3 \cdot 4\right) + (3 \cdot 3) + (3 \cdot 4) = 48$

Synthesis

5. How can a net help you calculate surface area?

Responses vary. **A net helps me calculate the surface area of a polyhedron by showing me its "unfolded" two-dimensional representation. This allows me to see all the faces so that I can determine the total surface area.**



Things to Remember:

Nets and Polyhedra

Directions: Make one copy per pair of students. Then pre-cut the cards and give each pair of students one set.

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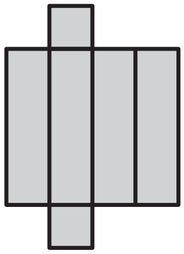
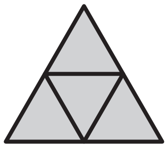
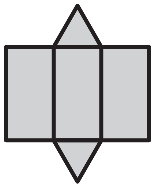
Card 1



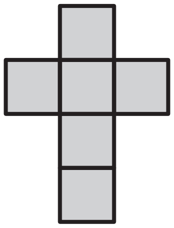
Card 2



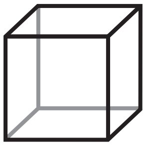
Card 3



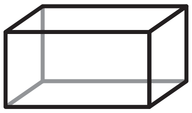
Card 4



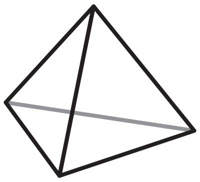
Card 5



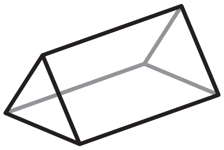
Card 6



Card 7

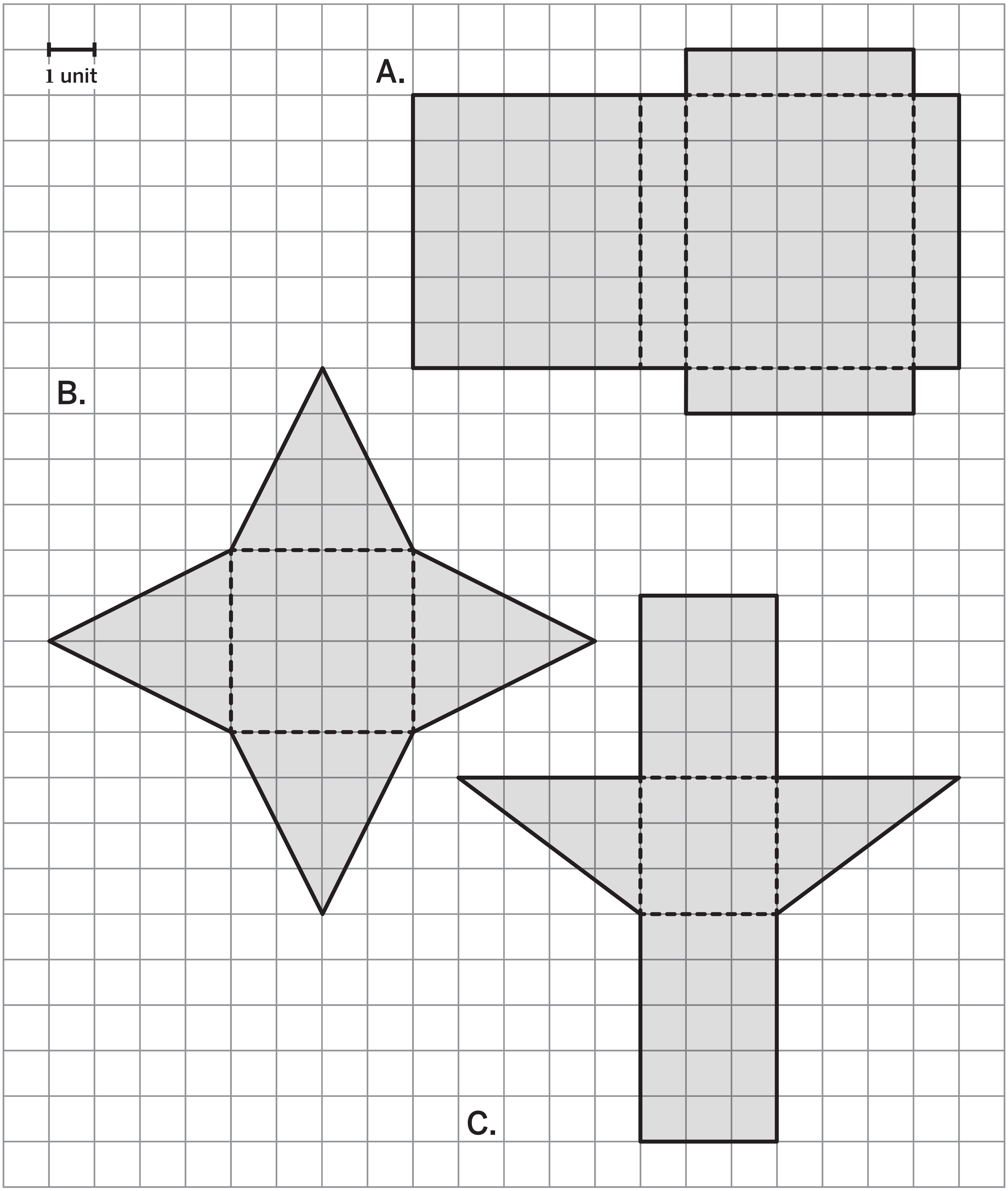


Card 8



Make Polyhedra

Cut out each of these nets along the solid lines and fold them along the dotted lines to assemble the polyhedra.



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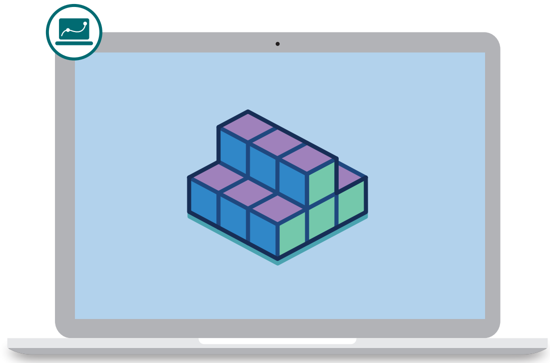
Unit 1
Lesson
13

Name: ________ Date: ________ Period: ________



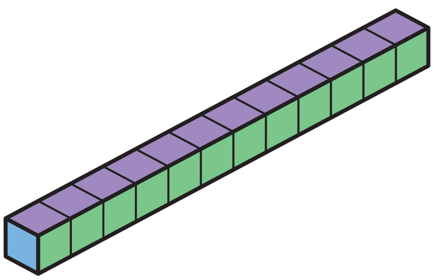
Face Value

Let's determine the surface area of prisms and pyramids.

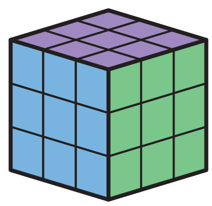
Warm-Up

1 Which one doesn't belong? Explain your thinking.

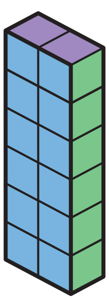
A.



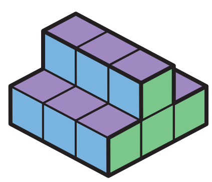
B.



C.



D.



Responses vary.

- Choice A: It's the only one with both a height and width of 1 unit.
- Choice B: It's the only one with the same length, width, and height. It's the only cube. It's the only one that doesn't have a volume of 12 cubic units.
- Choice C: It's the only one with a height greater than its width and length.
- Choice D: It's the only one that's not a rectangular prism. It's the only prism that has a surface area of less than 40 square units.

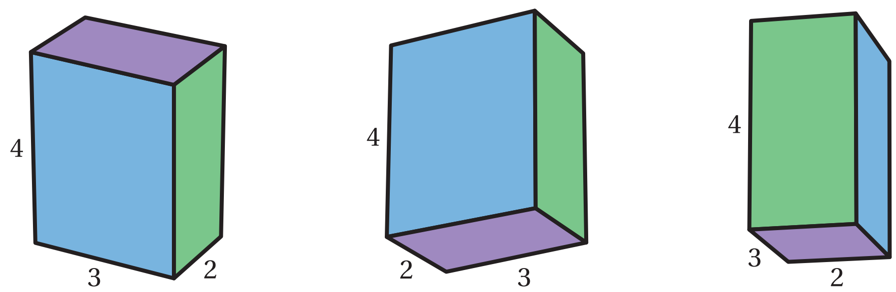
Activity 1

Name: ________ Date: ________ Period: ________

Surface Area Without a Grid

2 Note: All measurements in this lesson are in units.

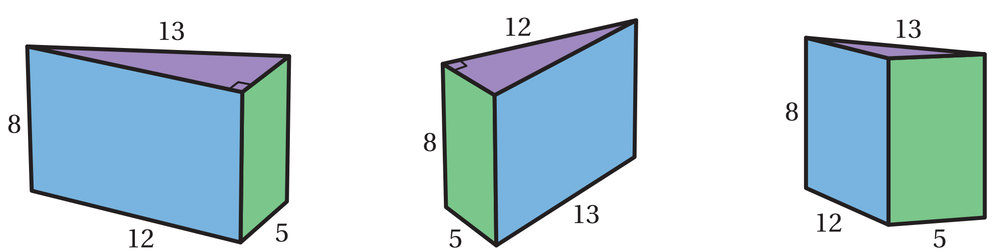
a Take a look at all the faces of this rectangular prism in square units.



b Calculate its surface area.

52 square units

3 Calculate the surface area of this triangular prism.



300 square units

Activity 1

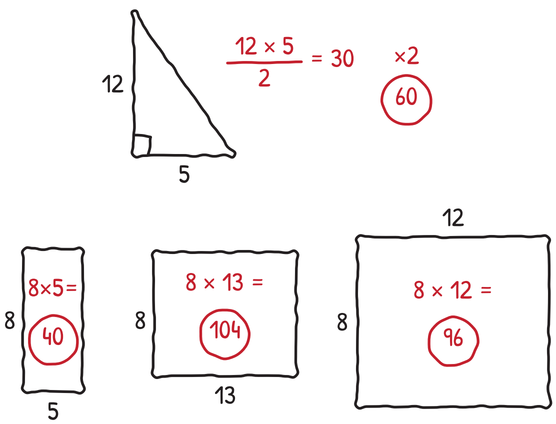
Name: ________ Date: ________ Period: ________

Surface Area Without a Grid (continued)

4 Here are two different strategies for finding the surface area of the prism from the previous problem.

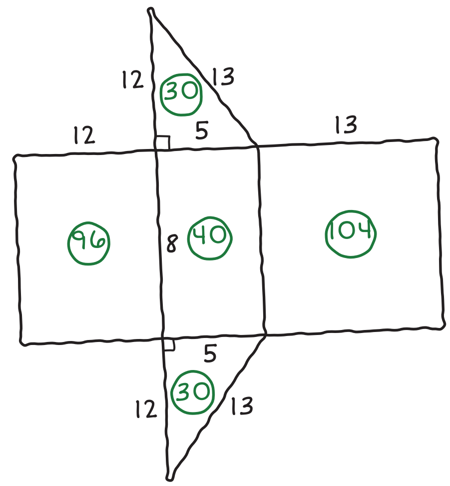
Discuss: What did each student do? How are their strategies alike? How are they different?

Ebele



60 + 40 + 104 + 96 = 300 square units

Kiri



96 + 40 + 104 + 30 + 30 = 300 square units

Responses vary.

- **Ebele drew each type of face separately. Ebele drew one triangle, plus three rectangles because each one is a different size. Then, Ebele calculated the area of each face and added them together. Ebele multiplied the area of the triangle by 2 because there are two identical triangular faces. Meanwhile, Kiri drew a net and then calculated the area of each face.**
- **Their strategies are alike because they both found the area of the faces and added them together to find the total surface area.**
- **Their strategies are different because Kiri drew a net and Ebele did not.**

Activity 2

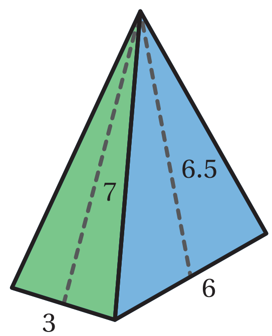
Name: .... Date: .... Period: ....

From Polyhedra to Nets

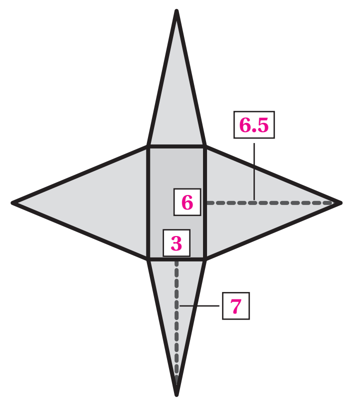
5 Here is a rectangular pyramid. The dotted lines represent the heights of the triangles.

Fill in the lengths to make a net that can be folded to create this pyramid.

Rectangular Pyramid



Net



6 Calculate the surface area of the rectangular pyramid from the previous problem. Use the pyramid's net if it helps with your thinking.

78 square units

7 Which of these polyhedra has a greater surface area? Circle one.

Prism A

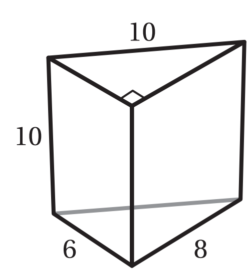
Prism B

They are the same

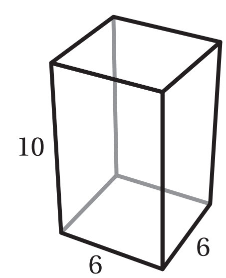
Explain your thinking.

***Explanations vary*. The surface area of Prism A is $60 + 80 + 100 + 24 + 24 = 288$ square units. The surface area of Prism B is $60 + 60 + 60 + 60 + 36 + 36 = 312$ square units.**

Prism A



Prism B

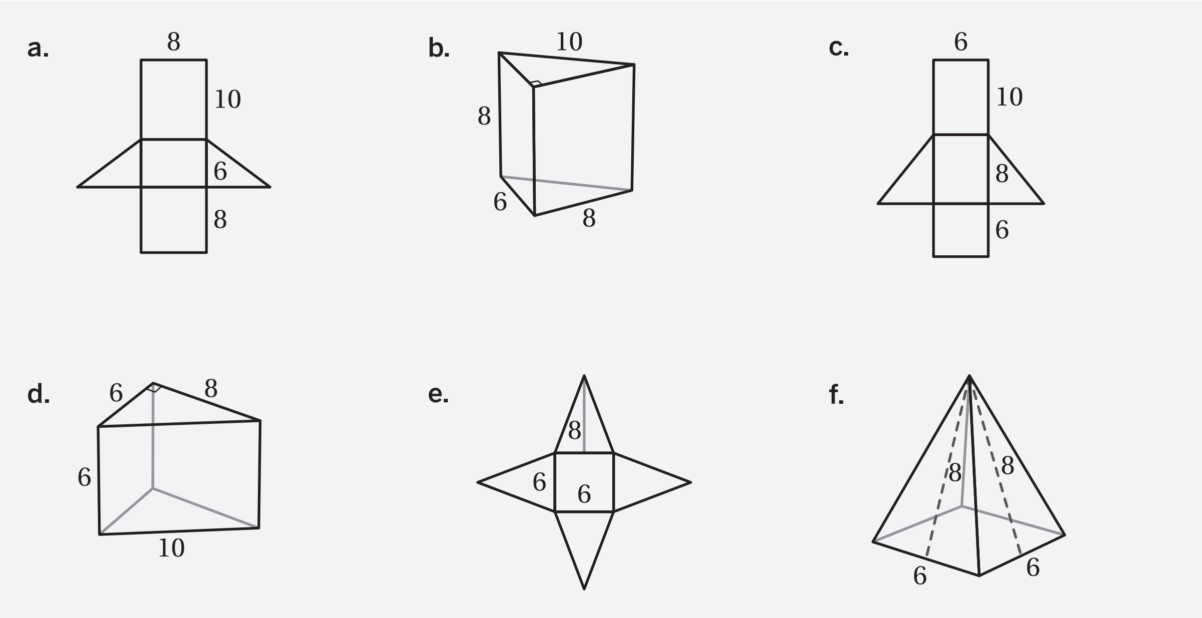


Activity 2

Name: ______ Date: ______ Period: ______

Nets of Prisms and Pyramids (continued)

8 Match each polyhedron and net to their surface area.



The surface area is 132 square units.	The surface area is 240 square units.	The surface area is 192 square units.
e	a	c
f	b	d

Explore More

9 Select one question and write your response.

a What are the dimensions of two prisms that have the same surface area but different volumes?

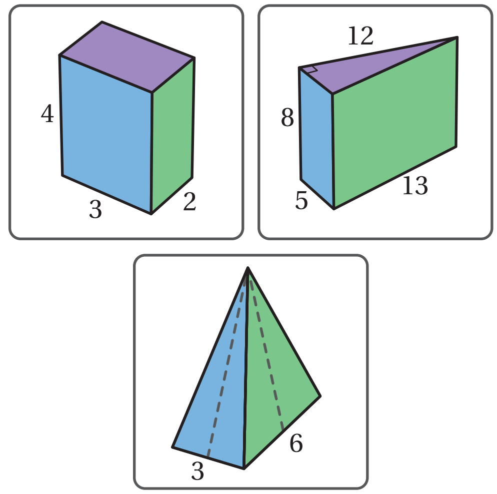
***Responses vary.* A rectangular prism that measures 1-by-1-by-5 units has a surface area of 22 square units and a volume of 5 cubic units. A rectangular prism that measures 1-by-2-by-3 units has the same surface area but a volume of 6 cubic units.**

b What are the dimensions of two prisms that have the same volume but different surface areas?

***Responses vary.* A rectangular prism that measures 1-by-2-by-2 units has a volume of 4 cubic units and a surface area of 16 square units. A rectangular prism that measures 1-by-1-by-4 units has the same volume but a surface area of 18 square units.**

10 Synthesis

How can you calculate the surface area of a prism or a pyramid from a picture?

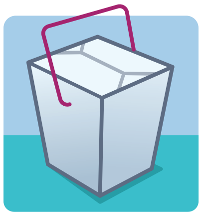


Responses vary.

- **I can make a net for the prism or pyramid, calculate the area of each face, then add the areas together.**
- **Calculate the area of each face of the prism or pyramid and add them together. Sometimes there are faces that have the same area, so the area of those faces need to be multiplied by how many of them there are.**

Things to Remember:

Name: .... Date: .... Period: ....



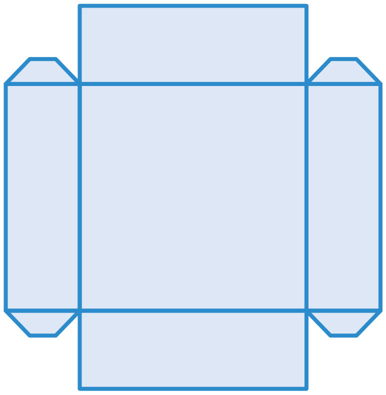
Take It To Go

Let's design a to-go container.

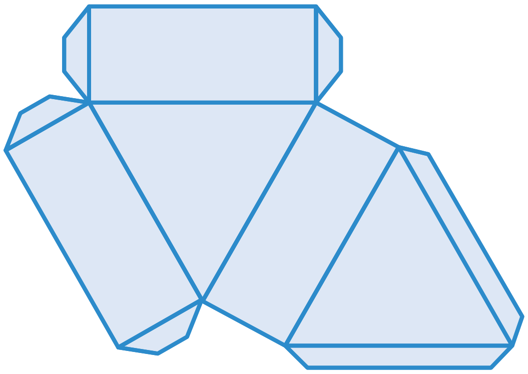
Warm-Up

1. DeAndre is opening a new restaurant. He is making patterns that can be folded into to-go containers for the different foods he will sell.

Pattern A



Pattern B



Discuss:

a How are the patterns alike? How are they different?

***Responses vary.* The patterns are alike because they need to be folded to be useful. They also both look like nets with extra little shapes along the edges. The patterns are different because one has a square base and the other has a triangular base.**

b Which container would you prefer to hold a bagel?

***Responses vary.* I prefer the container made from Pattern A because I think a bagel would fit better in a container with a square base that doesn't have a lid.**

c Which pattern do you think requires more material?

***Responses vary.* I think Pattern B requires more material because it includes a lid.**

Activity 1

Name: Date: Period:

Lots of Triangles

DeAndre's restaurant will serve sandwiches, salads, and single slices of pizza. He needs to design to-go containers for each item.

Your task is to design a to-go container for one of the food items and calculate the amount of material you need to make it.

Use this information to help you create your design.

- A sandwich is roughly 4 inches by 4 inches by 2 inches.
- A salad is roughly 120 cubic inches.
- A slice is roughly the shape of a triangle with a height of 8 inches and a base of 5 inches.

2. Which food item are you designing a container for? Circle one. ***Responses vary.***

Sandwich Salad Slice of pizza

3. What is the shape of the base of your container?

***Responses vary.* The sandwich and salad containers will likely have square or rectangular bases, and the pizza container will likely have a triangular base.**

4. How many faces does your container have?

***Responses vary.* The sandwich and salad containers will likely have 6 faces and the pizza container will likely have 5 faces.**

5. Draw or describe how you want your container to look. Be sure to include all the necessary measurements.

Responses vary.

6. Calculate how much material you need to make your container.

Responses vary.

Name: .... Date: .... Period: ....

Make It!

7. Share your design with a partner. Discuss how you might improve your design and write down what adjustments you want to make.

Responses vary.

8. Draw your revised pattern on blank paper using the measurements you designed.

Responses vary.

9. Cut out and fold your pattern to create your container.

Responses vary.

Explore More

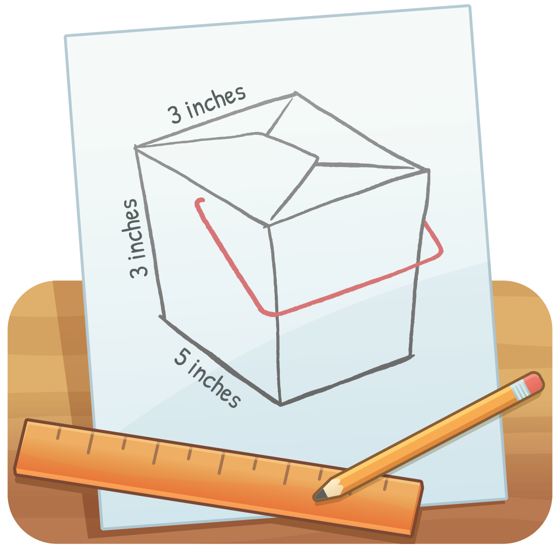
10. To-go containers can be made out of different materials, like cardboard, styrofoam, or aluminum. Research two different types of materials. Then write a pitch to DeAndre about which material(s) he should use for his containers, and why.

Responses vary.

Synthesis

11. **a** How was surface area related to the work you did today?

***Responses vary.* I needed to calculate the surface area to determine how much material I'd use to create the containers.**



b Now that you have seen your classmates' designs, what would you have done differently if you had more time?

Responses vary.

Things to Remember:

Unit 2
Lesson
2

Name: ______ Date: ______ Period: ______

Ratio Relationships

Let's describe how to compare pizza toppings.

Warm-Up

Evaluate each expression mentally.

1. $2 \cdot 15 = 30$

2. $4 \cdot 15 = 60$

3. $6 \cdot 15 = 90$

4. $12 \cdot 15 = 180$

Activity 1

Name: ______ Date: ______ Period: ______

Ratio Rounds

5. You will use pizza cards to complete Rounds 1–3.

Round 1: Write down as many **ratios** as you can about your pizza card.

My Ratios

Complete as many statements as you can about your pizza card. ***Responses vary.***

For every 3 pizzas, there are 6 mushrooms.

For every 4 mushrooms, there are 2 pineapples.

For every ______ ______, there are ______ ______.

For every ______ ______, there are ______ ______.

For every ______ ______, there are ______ ______.

For every ______ ______, there are ______ ______.

Round 2: Find a classmate whose card has a pizza that is *exactly the same* as one of your pizzas. Then write down the ratio relationship between the two toppings on each of your cards.

My Ratio

The ratio of tomatoes to mushrooms is 3 to 2. ***Responses vary.***

______'s Ratio (Classmate)

The ratio of tomatoes to mushrooms is 18 to 12. ***Responses vary.***

What is the same about your ratios?
Responses vary.

What is different about your ratios?
Responses vary.

Name: ________ Date: ________ Period: ________

Ratio Rounds (continued)

Round 3: Form a group with 2–3 classmates whose cards each have the same *total number of mushrooms* as your card. Then write down the ratio relationship between the two toppings on each of your cards.

My Ratios

What is the ratio of mushrooms : pepperonis? 4 : 10

What is the ratio of pepperonis : mushrooms? 10 : 4

Responses vary.

________'s Ratio (Classmate)

What is the ratio of mushrooms : pineapples? 4 : 2

What is the ratio of pineapples : mushrooms? 2 : 4

Responses vary.

________'s Ratio (Classmate)

What is the ratio of mushrooms : tomatoes? 4 : 6

What is the ratio of tomatoes : mushrooms? 6 : 4

Responses vary.

Activity
2

Name: ________ Date: ________ Period: ________

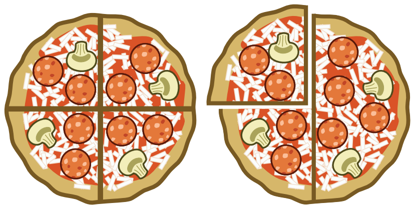
Two Truths and a Lie

6. Which statement is false?

   A. The ratio of mushrooms to pepperonis is $2 : 1$.

   B. For every 4 mushrooms, there are 8 pepperonis.

   C. The ratio of pepperonis to mushrooms is 12 to 6.

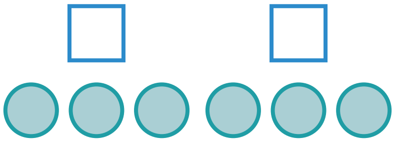


7. Which statement is false?

   A. The ratio of circles to squares is $1 : 3$.

   B. There are 2 squares for every 6 circles.

   C. For every square, there are 3 circles.

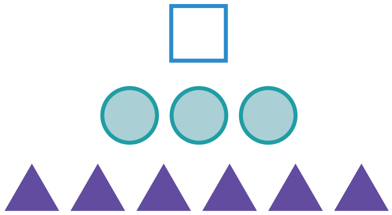


8. Which statement is false?

   A. For every circle, there are 2 triangles.

   B. The ratio of circles to squares is $3 : 1$.

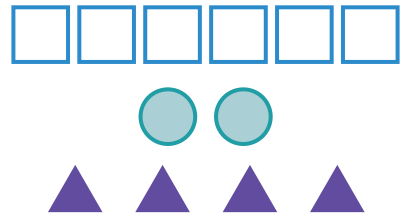
   C. The ratio of squares to triangles is 1 to 2.



Name: ______ Date: ______ Period: ______

Two Truths and a Lie (continued)

9. Here is another set of shapes.



a Write three statements about these shapes: two that are true and one that is false.

Responses vary.

- **True: The ratio of squares to triangles is 3 to 2.**
- **False: The ratio of triangles to circles is 1 : 2.**
- **True: For every circle, there are 2 triangles.**

b Trade your statements with a classmate. Which of their statements is false?

Responses vary.

10. Now create your own challenge!

a Draw your own set of shapes.

Drawings vary.

b Write three statements about your drawing: two that are true and one that is false.

Responses vary.

c Trade your challenge with a classmate. Which of their statements is false?

Responses vary.

Synthesis

11. **a** Describe the ratio between these moons and stars in as many different ways as you can.

Responses vary.

- **For every 3 moons, there are 6 stars.**
- **The ratio of stars to moons is 6 to 3.**
- **The ratio of moons : stars is 1 : 2.**

b Which way of describing a ratio is your favorite? Explain your reasoning.

Responses vary.

Things to Remember:

Ratio Rounds

Directions: Make one copy for the whole class. Then pre-cut the cards and give each student one card.

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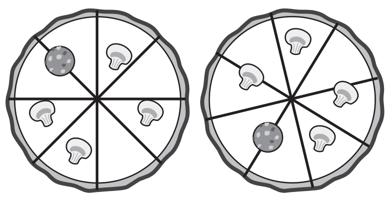

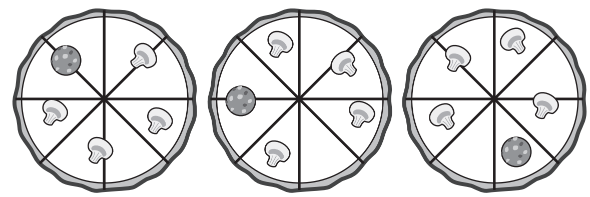

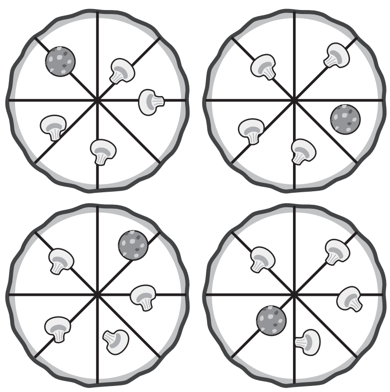

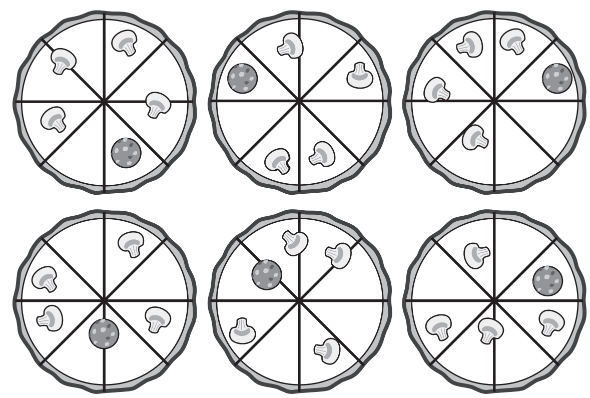

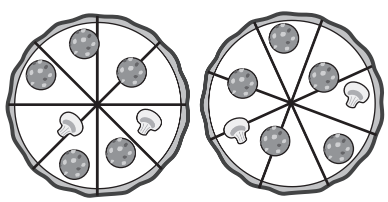

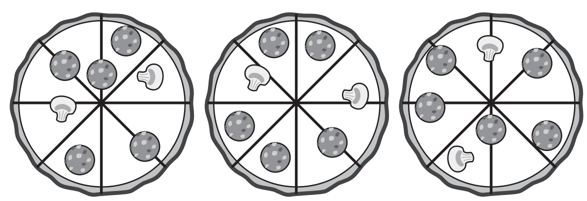

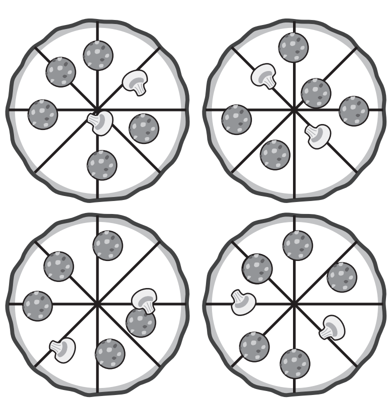

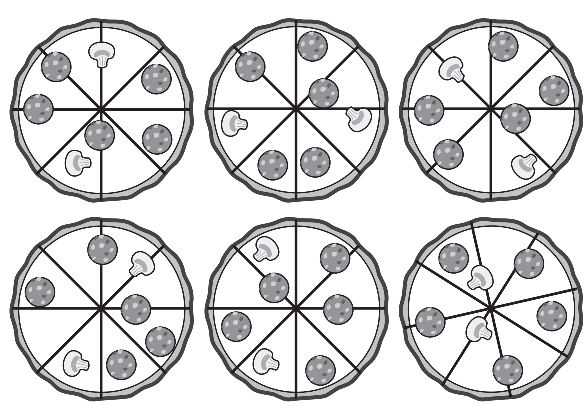

Ratio Rounds

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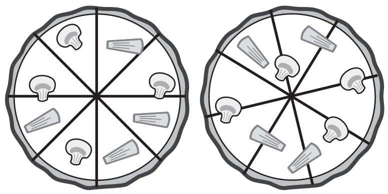

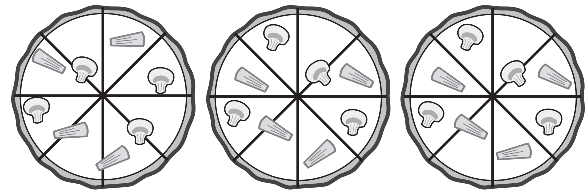

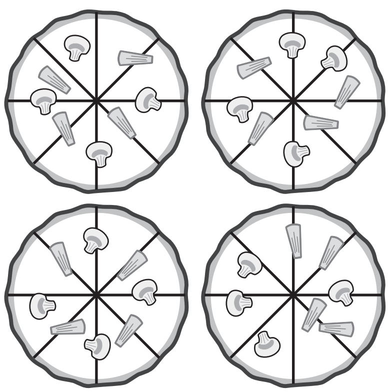

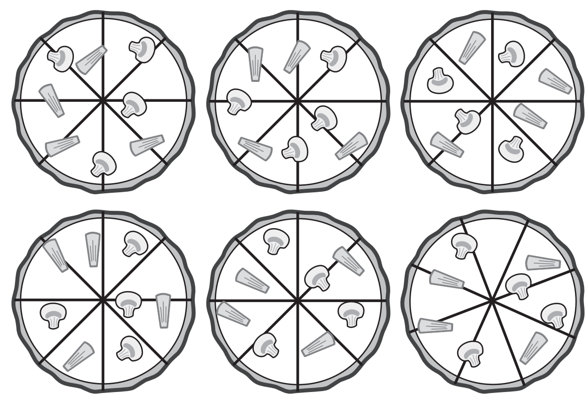

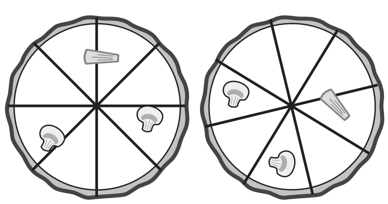

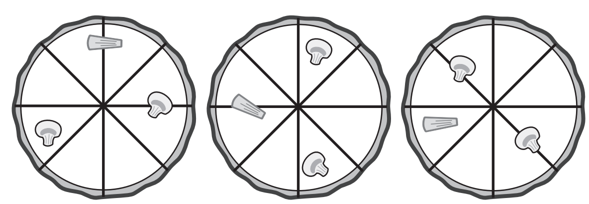

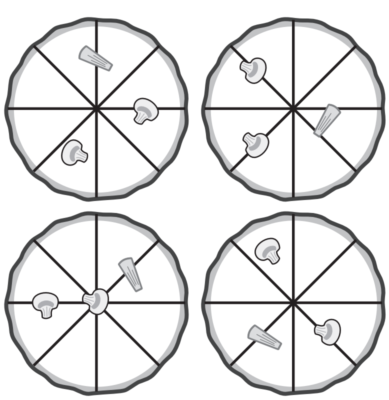

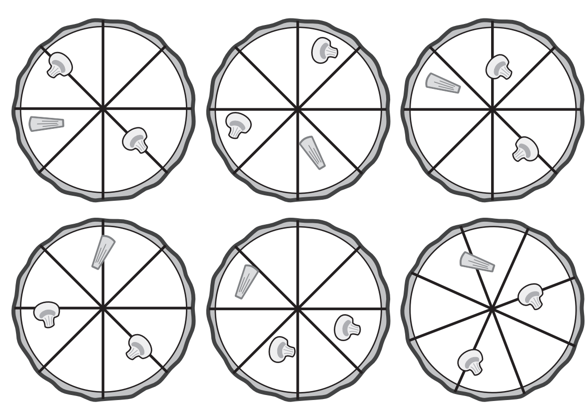

Ratio Rounds

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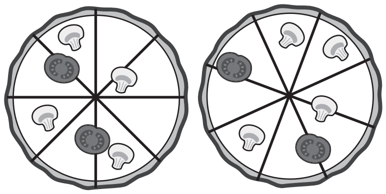

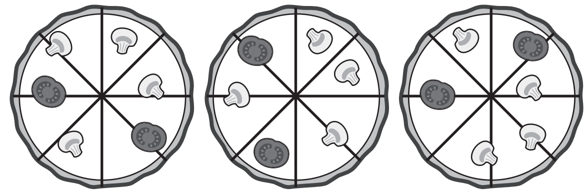

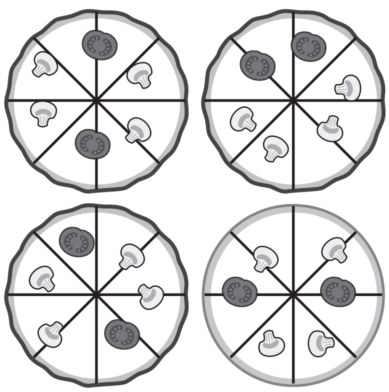

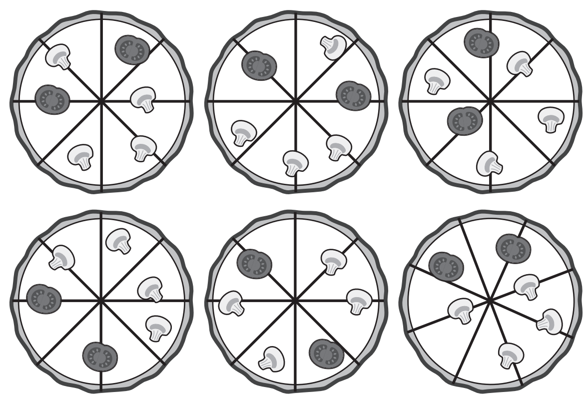

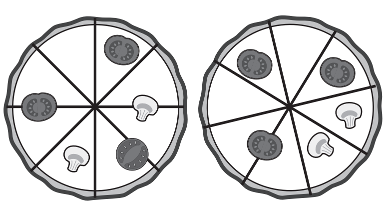

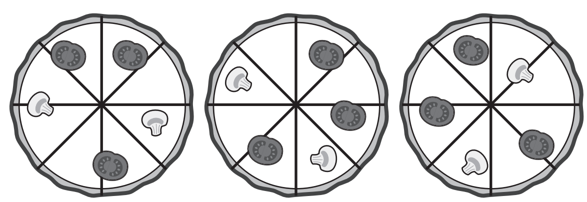

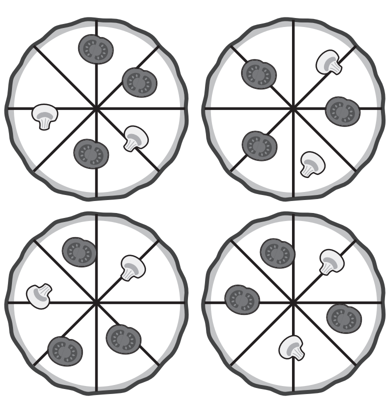

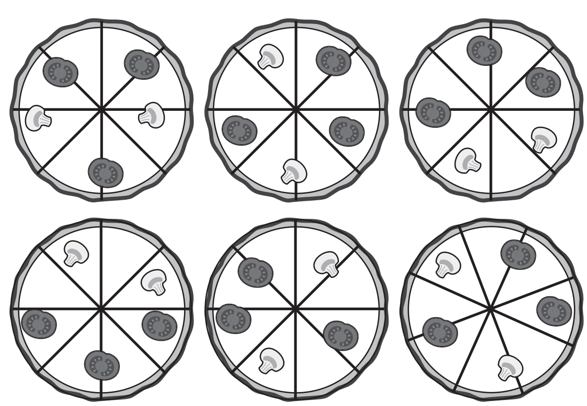

Ratio Rounds

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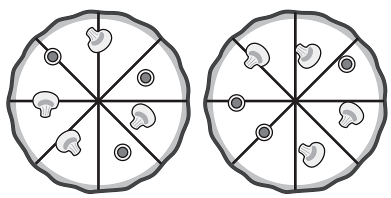

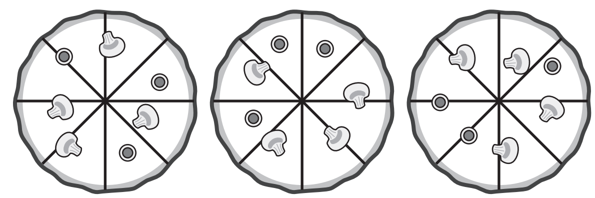

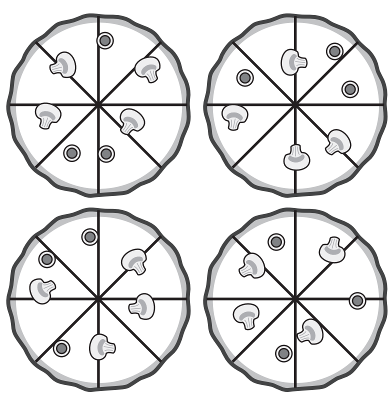

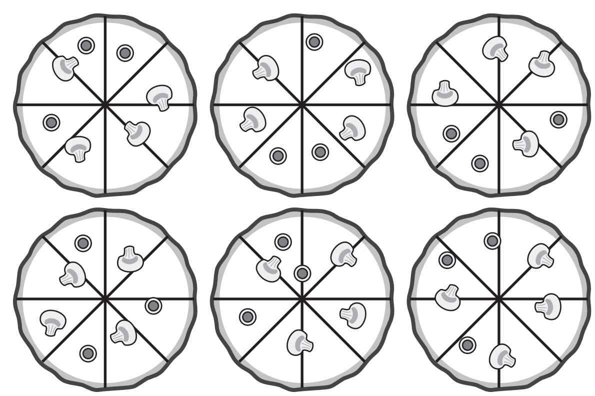

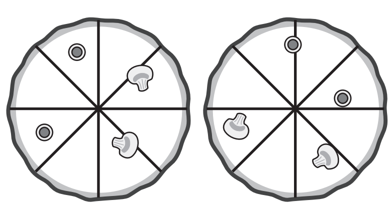

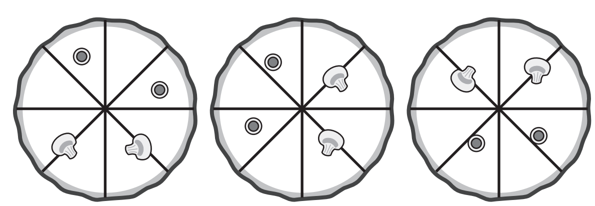

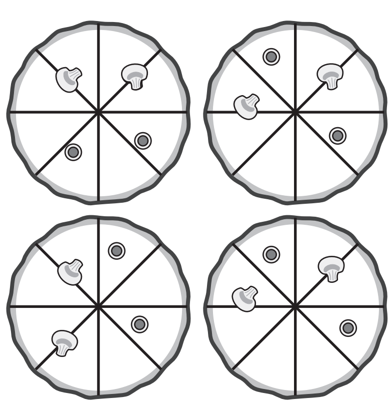

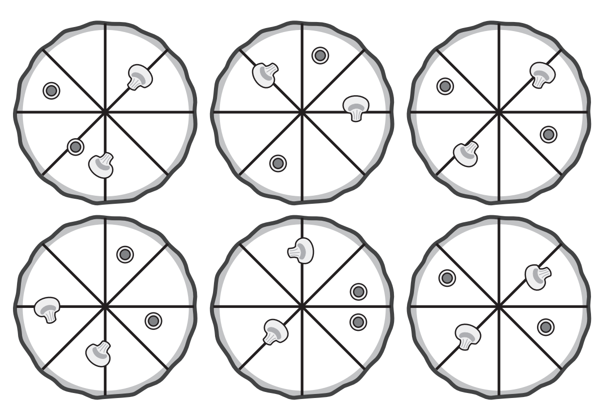

Ratio Rounds

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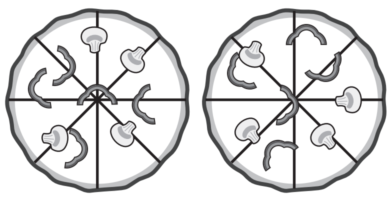
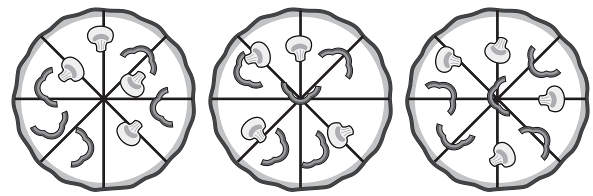
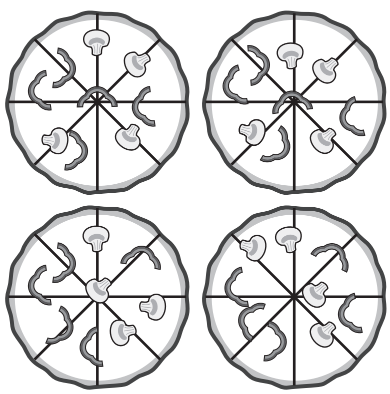
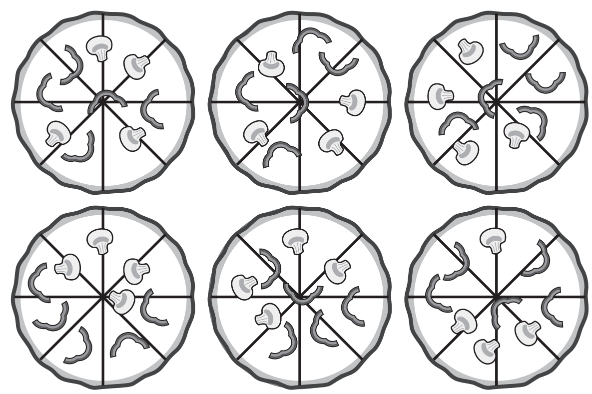
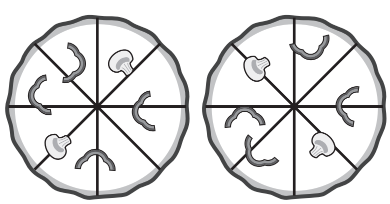
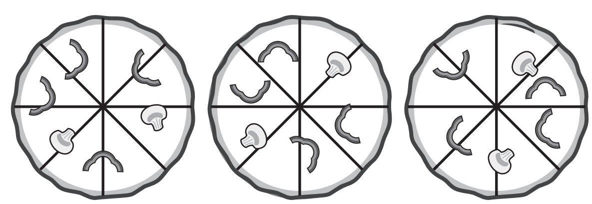


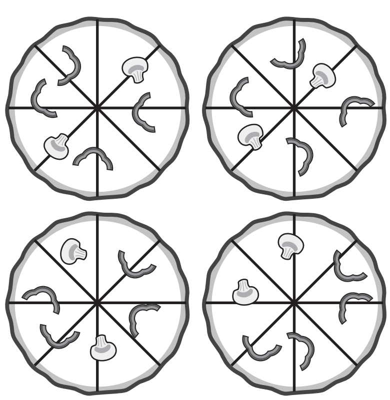


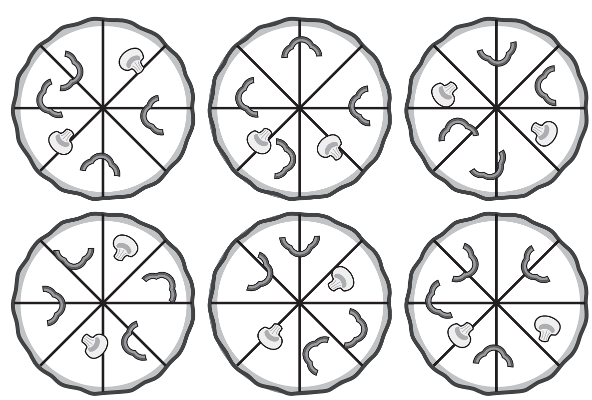












Unit 2
Lesson
3

Name: Date: Period:

Rice Ratios

Let's explore ratios in recipes..

Warm-Up

Evaluate each expression mentally.

1. $4 \cdot 8 = 32$

2. $4 \cdot 10 = 40$

3. $4 \cdot 18 = 72$

4. $4 \cdot 30 = 120$

5. $4 \cdot 38 = 152$

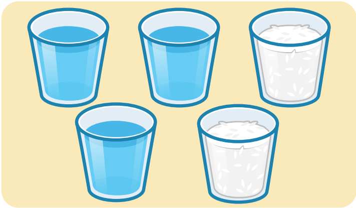
Activity 1

Name: ____________ Date: ________ Period: ________

Rice Advice

6. Here are the cooking instructions for three different bags of basmati rice.

Bag A	Bag B	Bag C
Boil 3 cups of water for every 2 cups of rice.	Boil $1\frac{1}{2}$ cups of water for every 1 cup of rice.	Boil 4 cups of water for every 2 cups of rice.



a The ratios for Bag A and Bag B are called **equivalent ratios**. Why do you think they're called that?

***Responses vary.* They're called equivalent ratios because the relationship between rice and water is the same in each set of instructions. The instructions for Bag A uses double the amounts of water and rice as the instructions for Bag B.**

b Marco wants to follow the directions for Bag A but use more rice. What is another ratio of water to rice that Marco could use? Explain your thinking.

***Responses and explanations vary.* Marco could use 6 cups of water and 4 cups of rice. That's the same as cooking twice as much as the original instructions suggest.**

c The recipe for Bag A says it makes rice for 6 people. What ratio of water to rice would you use to feed 18 people?

9 cups of water to 6 cups of rice

Activity 2

Name: .... Date: .... Period: ....

Rice Around the World

Here are the recipes for four rice dishes from around the world.

7. Jamar invited a friend over for dinner. How much of each ingredient does Jamar need to make 2 large bowls of jollof rice?

8 cups of rice

6 tablespoons of tomato paste

2 bell peppers

10 tomatoes

4 onions

$\frac{2}{3}$ cups of oil

Jollof Rice

Jollof rice is a tomato-based rice dish from Senegal, Ghana, and Nigeria.

Ingredients

Makes one large bowl

- 4 cups of rice
- 3 tablespoons of tomato paste
- 1 bell pepper
- 5 tomatoes
- 2 onions
- $\frac{1}{3}$ cups of oil

8. Nia wants to cook arroz con leche for 12 people.

a How much of each ingredient does Nia need?

6 cups of rice

12 cups of milk

1 cups of sugar

3 handfuls of raisins

3 cinnamon sticks

b Valeria wrote that Nia needs 9 cinnamon sticks. Why might Valeria think this?

***Responses vary.* Valeria might have thought that Nia needs 8 more of everything, including 8 more cinnamon sticks, because the recipe was changed to feed 8 more people.**

c What advice would you give Valeria?

***Responses vary.* I would tell Valeria that the recipe suggests 1 cinnamon stick for every 4 people, not every 1 person.**

Arroz Con Leche

Arroz con leche is a creamy dessert from Mexico and Spain.

Ingredients

Serves 4 people

- 2 cups of rice
- 4 cups of milk
- $\frac{1}{3}$ cups of sugar
- 1 handful of raisins
- 1 cinnamon stick

Name: ______ Date: ______ Period: ______

Rice Around the World (continued)

9. Julian has 1 cup of sugar and wants to use all of it to make champorado.

a How much of the other ingredients does he need?

$\frac{1}{2}$ cups of rice

2 cups of water

1 cans of coconut milk

$\frac{1}{4}$ cups of cocoa powder

b How many people will Julian's champorado serve?

2 people

Champorado

Champorado is a chocolate rice porridge eaten in the Philippines.

Ingredients

Serves 4 people

- 1 cup of rice
- 4 cups of water
- 2 cans of coconut milk
- $\frac{1}{2}$ cups of cocoa powder
- 2 cups of sugar

10. Ariana says this recipe makes too much risotto.

a How much of each ingredient would it take to make a smaller amount of risotto?

Responses vary.

$1\frac{1}{2}$ cups of rice

5 cups of chicken broth

2 tablespoons of olive oil

1 tablespoons of butter

4 ounces of Parmesan cheese

b How many people will this serve?

***Responses vary.* 4 people**

Risotto

Risotto is an Italian rice dish that uses broth to create a creamy texture.

Ingredients

Serves 8 people

- 3 cups of rice
- 10 cups of chicken broth
- 4 tablespoons of olive oil
- 2 tablespoons of butter
- 8 ounces of Parmesan cheese

Synthesis

11. The cooking instructions on Bag A and Bag B call for equivalent ratios of water to rice.

Bag A

Boil 4 cups of water for every 2 cups of rice.

Bag B

Boil 2 cups of water for every 1 cup of rice.

a Explain what equivalent ratios are in your own words.

Responses vary. **Equivalent ratios are when two or more ratios show the same relationship between two quantities.**

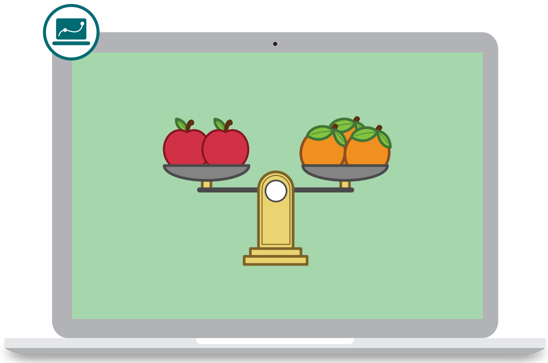
b Create a new ratio of water to rice that is equivalent to the ratios for Bag A and Bag B.

Responses vary. **6 cups of water to 3 cups of rice**

Things to Remember:

Unit 2
Lesson
4

Name: Date: Period:

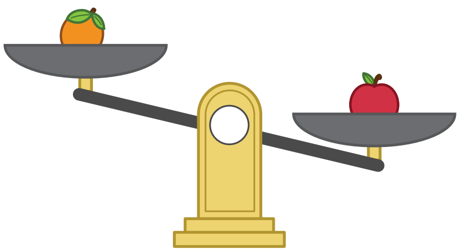


Fruit Lab

Let's investigate equivalent ratios by balancing fruit on scales.

Warm-Up

1 Let's watch apples and oranges balance on a scale.



a When the scale balances, record the values in the **table**.

b Find as many ways as you can to balance the scale. ***Responses vary.***

Number of Oranges	Number of Apples
3	2
6	4
9	6
12	8
15	10
18	12

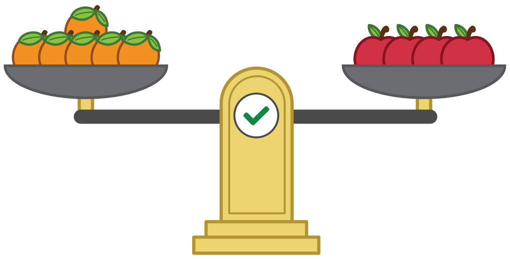
Activity 1

Name: ______ Date: ______ Period: ______

Apples to Oranges

2 Here is Victor's table from the Warm-Up. What do you notice about the table? What do you wonder?

Number of Oranges	Number of Apples
15	10
3	2
6	4
9	**6**



I notice:

Responses vary.

- **I notice that there are always more oranges than apples.**
- **I notice that the number of apples is always even.**
- **I notice that the top row is 5 times the middle row, and the last row is 2 times the middle row.**
- **I notice that the rows are all equivalent ratios.**

I wonder:

Responses vary.

- **I wonder how many apples you could fit on the scale.**
- **I wonder if you could balance 3 apples or some other odd number of apples.**
- **I wonder if there are an unlimited number of combinations that could balance.**

3 Write another equivalent ratio in the last row. Try to find one that you think no one else will think of.

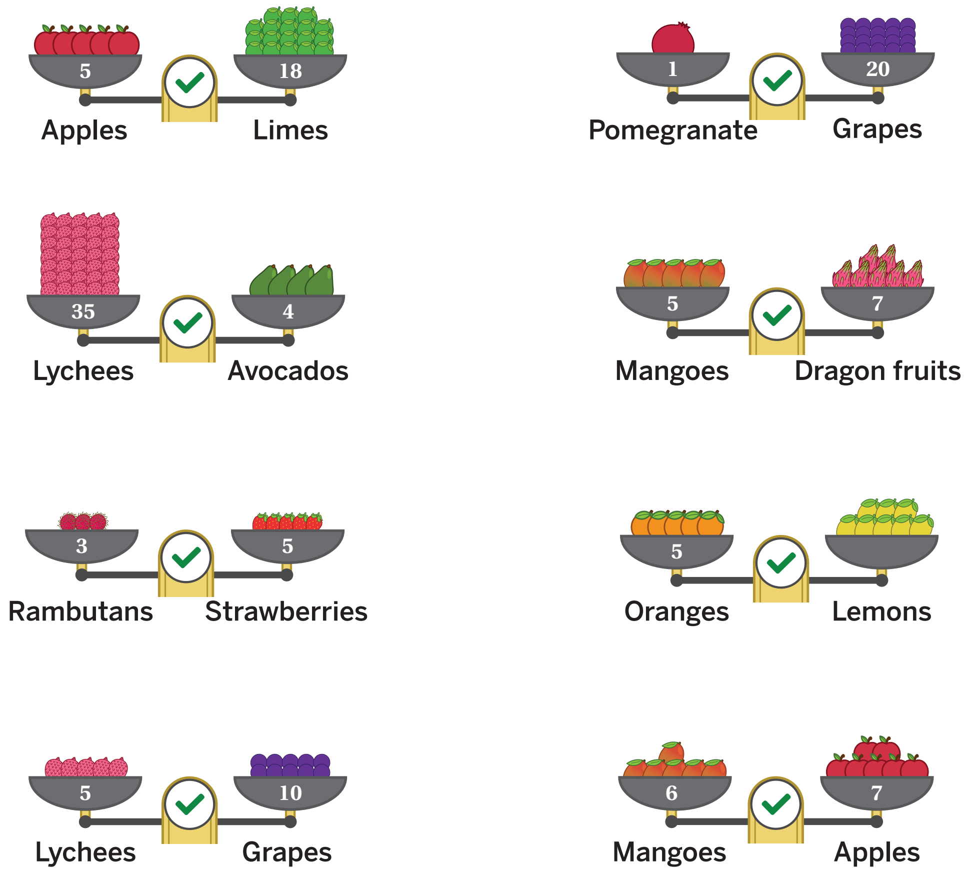
Responses vary. **Any ratio of oranges to apples that's equivalent to 3 : 2 is considered correct.**

Activity 2

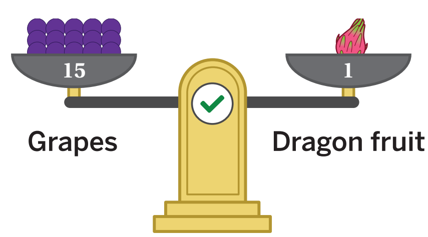
Name: .... Date: .... Period: ....

Fruit Lab

4 You will use the Activity 2 Sheet to complete this activity. Choose a pair of fruits to see how they balance. Then record several equivalent ratios for that pair of fruits. Repeat with different combinations of fruits.



5 A student knows that 15 grapes balance with 1 dragon fruit. They say that 16 grapes will balance with 2 dragon fruits. Will this 16 : 2 ratio balance the scale? Circle one.



Yes No I'm not sure

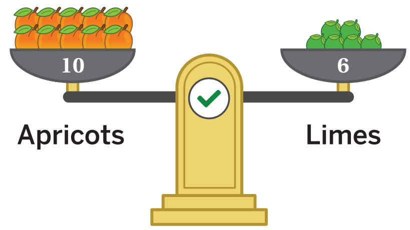
Explain your thinking.

***Explanations vary*. The student doubled the number of dragon fruits but didn't double the number of grapes, so the dragon fruit side will be heavier than the grape side.**

Name: ______ Date: ______ Period: ______

Fruit Lab (continued)

6 The scale balances with a ratio of 10 apricots to 6 limes. Select *all* of the equivalent ratios.



- ☐ **A.** 20 apricots to 16 limes
- ☑ **B.** 50 apricots to 30 limes
- ☐ **C.** 7 apricots to 3 limes
- ☑ **D.** 5 apricots to 3 limes
- ☐ **E.** 11 apricots to 7 limes

7 The table shows some ratios of limes to lychees that balance the scale. Dyani says that 22 limes will balance with 55 lychees. Will the 22 : 55 ratio balance? Circle one.

Number of Limes	Number of Lychees
2	5
20	50

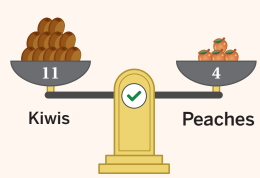
Yes   No   I'm not sure

Explain your thinking.

***Explanations vary*. If 2 limes balance with 5 lychees, that means 22 limes will also balance with 55 lychees because 2 • 11 = 22 and 5 • 11 = 55.**

Explore More

8 A ratio of 11 kiwis : 4 peaches balances. So does a ratio of 15 pears : 6 peaches.



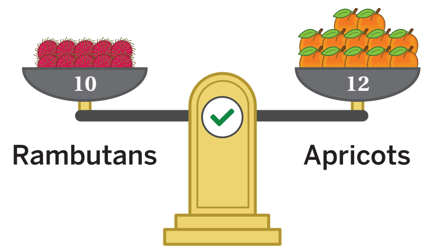
Write a ratio of kiwis to pears that would balance. Explain your thinking.

***Responses and explanations vary*. See Teacher Edition for sample responses.**

9 Synthesis

When you know a ratio balances a scale, how can you create equivalent ratios that also balance the scale?

Use the example if it helps with your thinking.



Responses vary. **I can create equivalent ratios by multiplying or dividing both numbers in the ratio by the same number. For example, the ratio of rambutans to apricots is $10:12$. This means that $20:24$ and $5:6$ will also balance because those ratios are the same as $10:12$, just multiplied or divided by 2. $100:120$ will also balance because that's just $10:12$ multiplied by 10.**

Things to Remember:

Name: ________________ Date: ________ Period: ________

Fruit Lab

- Choose a pair of fruits. Record their names in the first row of a table.
- Record several equivalent ratios for those fruits in the table.
- Repeat for as many pairs of fruits as you like!

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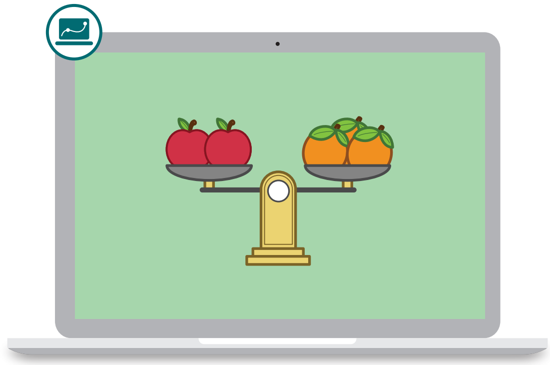
Unit 2
Lesson
4

Name: ______ Date: ______ Period: ______

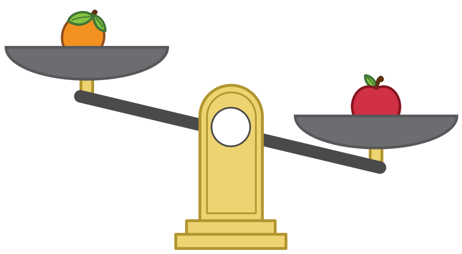


Fruit Lab

Let's investigate equivalent ratios by balancing fruit on scales.

Warm-Up

1 Let's watch apples and oranges balance on a scale.



a When the scale balances, record the values in the **table**.

b Find as many ways as you can to balance the scale. ***Responses vary.***

Number of Oranges	Number of Apples
3	2
6	4
9	6
12	8
15	10
18	12

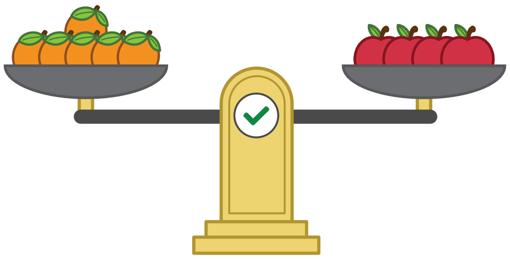
Activity 1

Name: .... Date: .... Period: ....

Apples to Oranges

2 Here is Victor's table from the Warm-Up. What do you notice about the table? What do you wonder?

Number of Oranges	Number of Apples
15	10
3	2
6	4
9	**6**



I notice:

Responses vary.
- **I notice that there are always more oranges than apples.**
- **I notice that the number of apples is always even.**
- **I notice that the top row is 5 times the middle row, and the last row is 2 times the middle row.**
- **I notice that the rows are all equivalent ratios.**

I wonder:

Responses vary.
- **I wonder how many apples you could fit on the scale.**
- **I wonder if you could balance 3 apples or some other odd number of apples.**
- **I wonder if there are an unlimited number of combinations that could balance.**

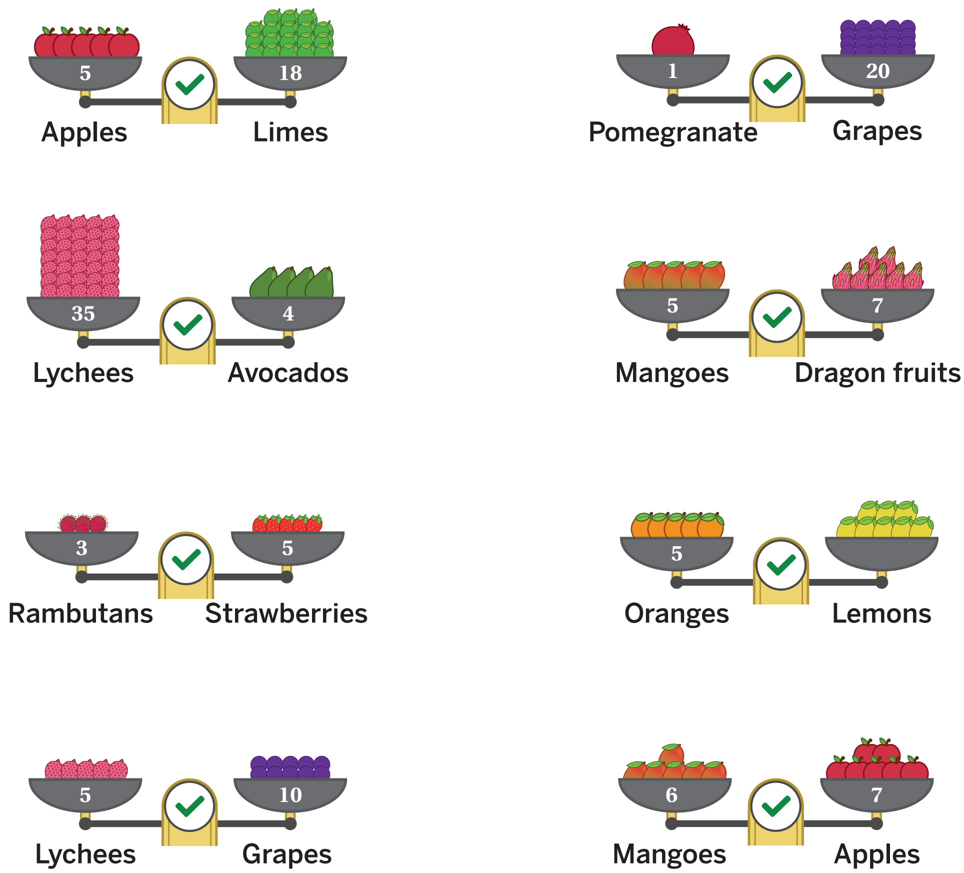
3 Write another equivalent ratio in the last row. Try to find one that you think no one else will think of.

***Responses vary.* Any ratio of oranges to apples that's equivalent to 3 : 2 is considered correct.**

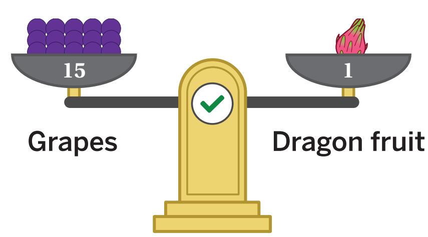
Name: .... Date: .... Period: ....

Fruit Lab

4 You will use the Activity 2 Sheet to complete this activity. Choose a pair of fruits to see how they balance. Then record several equivalent ratios for that pair of fruits. Repeat with different combinations of fruits.



5 A student knows that 15 grapes balance with 1 dragon fruit. They say that 16 grapes will balance with 2 dragon fruits. Will this 16 : 2 ratio balance the scale? Circle one.



Yes No I'm not sure

Explain your thinking.

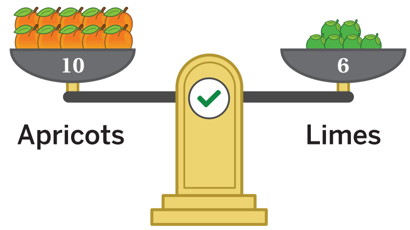
***Explanations vary*. The student doubled the number of dragon fruits but didn't double the number of grapes, so the dragon fruit side will be heavier than the grape side.**

Activity 2

Name: ________ Date: ________ Period: ________

Fruit Lab (continued)

6 The scale balances with a ratio of 10 apricots to 6 limes. Select *all* of the equivalent ratios.



- [ ] **A.** 20 apricots to 16 limes
- [x] **B.** 50 apricots to 30 limes
- [ ] **C.** 7 apricots to 3 limes
- [x] **D.** 5 apricots to 3 limes
- [ ] **E.** 11 apricots to 7 limes

7 The table shows some ratios of limes to lychees that balance the scale. Dyani says that 22 limes will balance with 55 lychees. Will the 22 : 55 ratio balance? Circle one.

Number of Limes	Number of Lychees
2	5
20	50

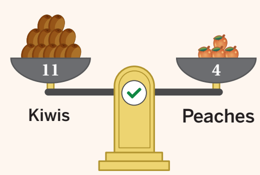
Yes   No   I'm not sure

Explain your thinking.

***Explanations vary.* If 2 limes balance with 5 lychees, that means 22 limes will also balance with 55 lychees because $2 \cdot 11 = 22$ and $5 \cdot 11 = 55$.**

Explore More

8 A ratio of 11 kiwis : 4 peaches balances. So does a ratio of 15 pears : 6 peaches.



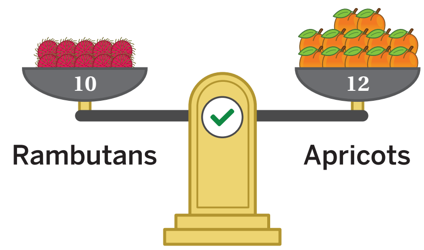
Write a ratio of kiwis to pears that would balance. Explain your thinking.

***Responses and explanations vary.* See Teacher Edition for sample responses.**

9 Synthesis

When you know a ratio balances a scale, how can you create equivalent ratios that also balance the scale?

Use the example if it helps with your thinking.



Responses vary. **I can create equivalent ratios by multiplying or dividing both numbers in the ratio by the same number. For example, the ratio of rambutans to apricots is $10 : 12$. This means that $20 : 24$ and $5 : 6$ will also balance because those ratios are the same as $10 : 12$, just multiplied or divided by 2. $100 : 120$ will also balance because that's just $10 : 12$ multiplied by 10.**

Things to Remember:

Name: .... Date: .... Period: ....

Fruit Lab

- Choose a pair of fruits. Record their names in the first row of a table.
- Record several equivalent ratios for those fruits in the table.
- Repeat for as many pairs of fruits as you like!

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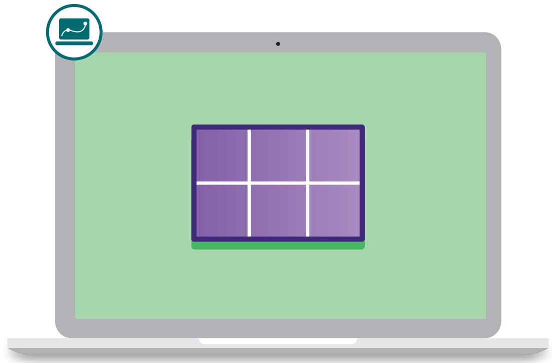
Unit 2
Lesson
8

Name: Date: Period:

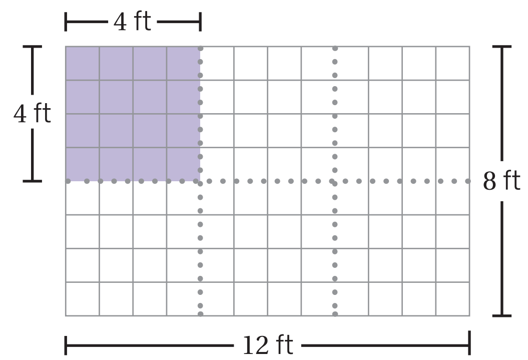


Common Factors

Let's explore factors.

Warm-Up

1 A 4-by-4-foot square will tile the floor of this 8-by-12-foot room.



a Take a look at the diagram to see what we mean.

b Find other square sizes that tile the room.

1-by-1-foot or 2-by-2-foot squares

c **Discuss:** What does it mean to *tile*?

Responses vary. **Tiling means that the squares can fit evenly in both directions. For example, a 4-by-4-foot square tiles a 12-by-8-foot rectangle because 4 fits evenly into 12 three times and into 8 two times.**

Activity 1

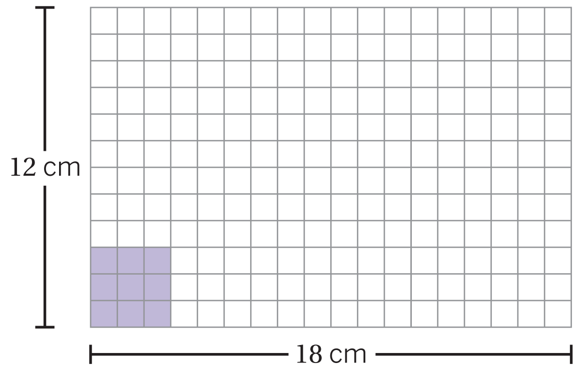
Name: ______ Date: ______ Period: ______

Greatest Common Factors

2 A 3-by-3-centimeter square will tile this 12-by-18-centimeter rectangle. This means that 3 is a **common factor** of 12 and 18.

Select *all* the other common factors of 12 and 18.

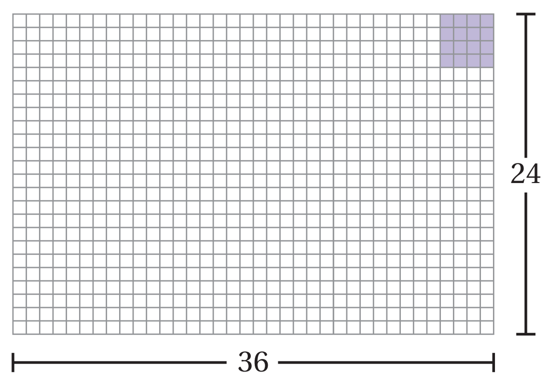
- [x] **A.** 1
- [x] **B.** 2
- [ ] **C.** 4
- [x] **D.** 6
- [ ] **E.** 12



3 4 is a common factor of 24 and 36.

Determine as many common factors of 24 and 36 as you can.

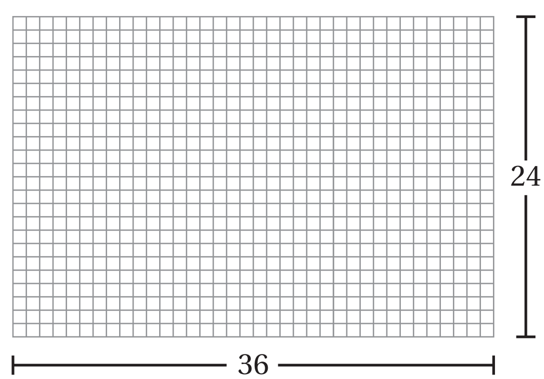
***Responses vary.* Common factors of 24 and 36: 1, 2, 3, 4, 6, 12**



4 The **greatest common factor (GCF)** is the greatest number that is a common factor of two numbers.

What is the greatest common factor of 24 and 36?

12

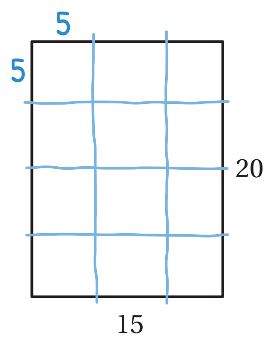


Name: ________ Date: ________ Period: ________

Greatest Common Factors (continued)

5 Here are Latifa's and Tameeka's strategies for determining the greatest common factor of 20 and 15.

Latifa



Tameeka

Factors of 15: 1, 3, 5, 15

Factors of 20: 1, 2, 4, 5, 10, 20

Discuss: What are the advantages and disadvantages of each strategy?

Responses vary.

- **Latifa's strategy is helpful because you can see how many times 5 goes into each number, but this strategy might be hard to use if the numbers were really big.**
- **Tameeka's strategy is helpful because you could use it for any number. However, you can't double check to make sure the numbers you listed are actually factors, like you can with Latifa's strategy.**

6 What is the greatest common factor of 27 and 36? Explain your thinking.

9. *Explanations vary.*

- **I wrote a list of factors for 27 (1, 3, 9, and 27) and a list of factors for 36 (1, 2, 3, 4, 6, 9, 12, 18, and 36). I used the two lists to determine that the greatest common factor is 9.**
- **I drew a diagram of a rectangle with a width of 27 and a length of 36. I was able to tile the entire rectangle with 12 complete 9-by-9 squares.**

Activity 2

Name: ______ Date: ______ Period: ______

Common Factors and Multiples

7 The greatest common factor of 20 and 15 is 5.

What is the *least common multiple (LCM)* of 20 and 15?

60

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

8 How are *GCF* and *LCM* alike? How are they different?

Responses vary.

Alike:

- **They both involve two numbers.**
- **They both involve thinking about multiplication.**
- **Lists can be really helpful for figuring both out.**

Different:

- **The greatest common factor can't be bigger than both numbers. The least common multiple can't be smaller than both numbers.**
- **When determining the greatest common factor, you think about what numbers multiply to become the numbers you have. When determining the least common multiple, you think about multiplying the numbers you have.**

Activity 3

Name: .... Date: .... Period: ....

Repeated Challenges

9 Determine the least common multiple or greatest common factor. Use a 100-grid or draw a diagram if it helps with your thinking.

a What is the least common multiple of 6 and 4?

12

b What is the greatest common factor of 9 and 12?

3

c What is the least common multiple of 10 and 6?

30

d What is the least common multiple of 2 and 16?

16

e What is the greatest common factor of 16 and 2?

2

f What is the greatest common factor of 18 and 27?

9

Explore More

10 Jamir and Kimaya each wrote a question about greatest common factor (GCF) and least common multiple (LCM).

Jamir

Does every pair of numbers have a GCF and a LCM?

Kimaya

Is the GCF of two numbers always smaller than the LCM?

Discuss your answer to at least one question with a classmate.

Responses vary.

- **Jamir's question: Yes. Since all numbers are divisible by 1, 1 is a common factor of every pair of numbers. You can also always multiply two numbers to get a common multiple, which means every pair of numbers has a LCM, too.**
- **Kimaya's question: Yes. The largest a GCF can be is the smallest of two numbers, and the smallest a LCM can be is the largest of two numbers. For example, the GCF of 4 and 12 is 4, and the LCM of 4 and 12 is 12. So, unless the two numbers are the same, the GCF is always smaller than the LCM.**

11 Synthesis

Discuss these questions with a classmate. Sketch a diagram if it helps with your thinking.

a What does *greatest common factor* mean?

Responses vary. **The greatest common factor of two numbers is the largest number that divides into both numbers evenly.**

b Why do you think we don't study the *least* common factor?

Responses vary. **The least common factor is not that interesting because 1 divides evenly into all numbers, so the least common factor is always 1.**

Things to Remember:

Name: .... Date: .... Period: ....

Mixing Paint, Part 1

Let's see how mixing colors relates to ratios.

Warm-Up

Mentally determine the missing value that makes each pair of fractions equivalent.

1. $\frac{1}{5} = \frac{\boxed{2}}{10}$

2. $\frac{2}{5} = \frac{6}{\boxed{15}}$

3. $\frac{4}{\boxed{5}} = \frac{12}{15}$

4. $\frac{\boxed{10}}{8} = \frac{15}{12}$

Name: .... Date: .... Period: ....

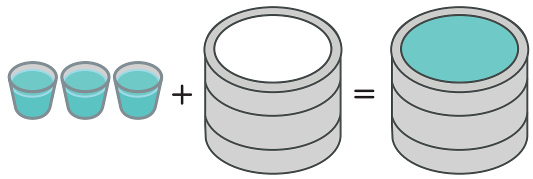
Comparing Ratios

5 Paint stores create different colors by using different ratios of white paint to tint.

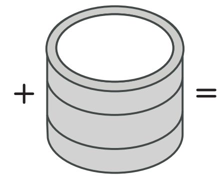
a Choose *one* tint color.

b Circle the amount of tint you want to add to 2 gallons of white paint.

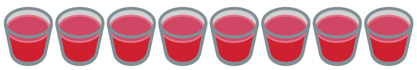
Example Paint Mix



My Paint Mix

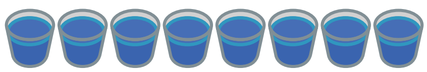


Use the digital activity to see your paint mix.

Red 

Teal 

Purple 

Blue 

6 Write the ratio you created. ***Responses vary.* Samples shown in fill-in-the-blanks.**

5 ounces tint : 2 gallons white paint

a Can you find two different ways to make a *darker* color? Use the digital activity to check your work.

Responses depend on the ratio students create.

5 ounces tint : **1** gallons white

6 ounces tint : **2** gallons white

b Can you find two different ways to make a *lighter* color? Use the digital activity to check your work.

Responses depend on the ratio students create.

5 ounces tint : **2.5** gallons white

4 ounces tint : **2** gallons white

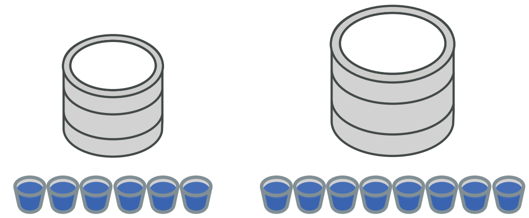
Activity
1

Name: .... Date: .... Period: ....

Comparing Ratios (continued)

7 Here are Luca's and Marc's ratios. Which will make a darker blue? Circle one.

Luca's ratio   Marc's ratio   They'll make the same blue



Luca's Ratio	Marc's Ratio
6 ounces blue	8 ounces blue
2 gallons white	4 gallons white

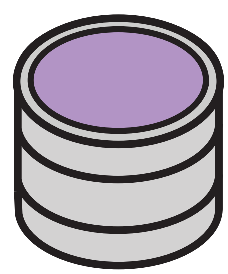
Explain your thinking.

Explanations vary.
- **Luca uses 3 ounces of blue for every gallon of white, while Marc only uses 2 ounces of blue for every gallon of white.**
- **If I double Luca's ratio, both ratios would have 4 gallons of white paint, but Luca's would have 12 ounces of blue, while Marc's would only have 8 ounces of blue.**
- **If I multiply Luca's ratio by 4 and Marc's ratio by 3, they would have the same amount of blue tint. Luca's ratio would have 24 ounces of blue and 8 gallons of white. Marc's ratio would have 24 ounces of blue and 12 gallons of white. Luca uses less white paint, so his ratio would make a darker blue.**

8 **a** Let's watch to see which ratio makes a darker blue.

b **Discuss:** What is a different strategy you could use to compare the ratios?
Responses vary.

9 Here is Amoli's ratio:



8 ounces purple : 4 gallons white

Select *all* of the choices that will result in a darker purple.

☐ **A.** Adding white paint

☑ **B.** Using less white paint

☑ **C.** Adding purple tint

☐ **D.** Using less purple tint

☐ **E.** Adding 2 ounces of purple tint and 2 gallons of white paint

Activity 2

Name: ________ Date: ________ Period: ________

Lighter or Darker Paint

10 Order the ratios from *darkest* blue to *lightest* blue.

A. 5 ounces blue : 4 gallons white

B. 4 ounces blue : 3 gallons white

C. 10 ounces blue : 6 gallons white

D. 9 ounces blue : 6 gallons white

C	D	B	A

Darkest Blue **Lightest Blue**

11 Luca says that these two ratios make the same shade of blue.

≡ 4 ounces blue : 3 gallons white

≡ 5 ounces blue : 4 gallons white

What would you recommend Luca change?

Responses vary.

- It's easier to compare the colors when they have the same amount of white paint or blue tint. You could make both colors have 12 gallons of white by multiplying the first ratio by 4 and the second ratio by 3.
- You can figure out if the colors are the same shade by calculating how much blue there is for every gallon of white. The first color has $\frac{4}{3}$ or about 1.333 ounces of blue for every gallon of white. The second color has $\frac{5}{4}$ or 1.25 ounces of blue for every gallon of white.

12 Solve all six challenges. For each pair of ratios, choose which ratio makes a darker blue.

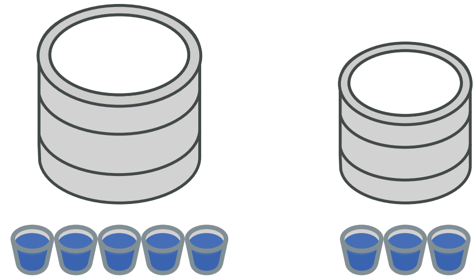
Ratio A	Ratio B	Ratio A	Ratio B	They'll make the same blue.
2 oz blue : 4 gal white	3 oz blue : 4 gal white		✓	
4 oz blue : 3 gal white	4 oz blue : 5 gal white	✓		
3 oz blue : 2 gal white	5 oz blue : 4 gal white	✓		
5 oz blue : 2 gal white	15 oz blue : 6 gal white			✓
7 oz blue : 3 gal white	5 oz blue : 2 gal white		✓	
5 oz blue : 4 gal white	9 oz blue : 7 gal white		✓	

13 Synthesis

Describe a strategy for comparing two ratios. Use the example if it helps with your thinking.

Responses vary.

- **I can compare two ratios by making both ratios have the same quantity for one of its numbers. In this example, I could multiply Ratio B by 2 so that both Ratio A and Ratio B have the same amount of white paint. Then I can compare the number of ounces of blue tint.**
- **I can compare ratios by calculating how much blue tint each has for every gallon of white. Here, Ratio A has $\frac{5}{4} = 1.25$ ounces of blue for each gallon of white, and Ratio B has $\frac{3}{2} = 1.5$ ounces of blue for each gallon of white.**

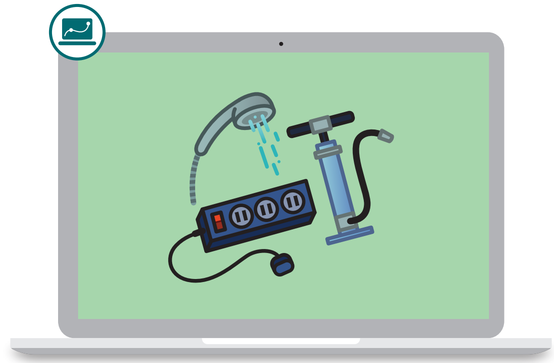


Ratio A
5 ounces blue
4 gallons white

Ratio B
3 ounces blue
2 gallons white

Things to Remember:

Name: .... Date: .... Period: ....



Disaster Preparation

Let's use ratio tables to help prepare for disasters.

Warm-Up

1 Cities need to prepare for possible disasters.

What are *three* things a city should have for its people in case of a disaster?

1.

2.

3.

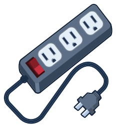
Responses vary.

- Toilets
- Beds
- Food
- Blankets
- Water
- Extra clothes
- Shelter
- Medical supplies

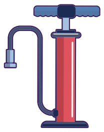
2 There are national recommendations for the items that cities should stock up on in case of a disaster. Here are three of those items.[1]

Handheld Shower

Power Strip



Air Pump



How many of each item do you think a city with a population of 100 should have?

Responses vary.

Population	Handheld Showers	Power Strips	Air Pumps
100			

[1] Source: *Commonly Used Sheltering Items Catalog*

Activity 1

Name: ____________ Date: ________ Period: ________

Shower, Power, and Air

3 Here are the recommendations for a city of 100 people.

How many of each item would you recommend for Lucas, Wisconsin?

	Population	Handheld Showers	Power Strips	Air Pumps
Recommendations	100	4	5	1
Lucas, Wisconsin	700	**28**	**35**	**7**

Responses vary. **If students create equivalent ratios, their answer will be 28 handheld showers, 35 power strips, and 7 air pumps.**
Note: The recommendations for handheld showers, power strips, and air pumps follow a ratio relationship, but students don't need to assume that at this stage in the lesson, as long as they can defend their answer.

4 How many of each item would you recommend for Blue Ridge, Georgia?

	Population	Handheld Showers	Power Strips	Air Pumps
Recommendations	100	4	5	1
Blue Ridge, Georgia	1,200	**48**	**60**	**12**

Responses vary. **If students create equivalent ratios, their answer will be 48 handheld showers, 60 power strips, and 12 air pumps.**

5 How many of each item would you recommend for Hamlin City, Kansas?

	Population	Handheld Showers	Power Strips	Air Pumps
Recommendations	100	4	5	1
Hamlin City, Kansas	25	**1**	**1.25 or 2**	**0.25 or 1**

Responses vary. **If students create equivalent ratios, their answer will be 1 handheld shower, 1.25 power strips, and 0.25 air pumps. Students might choose to round to whole numbers based on reasoning, and their answer will be 1 handheld shower, 2 power strips, and 1 air pump.**

Name: ________ Date: ________ Period: ________

Shower, Power, and Air (continued)

6 Taylor recommended that Hamlin City, Kansas should buy 1 handheld shower, 1 power strip, and 1 air pump.

	Population	Handheld Showers	Power Strips	Air Pumps
Recommendations	100	4	5	1
Lucas, Wisconsin	700	28	35	7
Blue Ridge, Georgia	1,200	48	60	12
Hamlin City, Kansas	25	1	1	1

a **Discuss:** Why do you think Taylor made this recommendation?

Responses vary. **I think Taylor wants to ensure that Hamlin City has at least one of each item, even though it's not equivalent to the national recommendations for a city of 25 people.**

b What do you agree with about Taylor's recommendations? What do you disagree with?

Responses vary. **I agree with Taylor that Hamlin City should have 1 handheld shower. I disagree with Taylor about the air pumps, though. I don't think Hamlin City needs any air pumps since only 25 people live there.**

7 Here are the actual recommendations for Lucas, Wisconsin; Blue Ridge, Georgia; and Hamlin City, Kansas.

Discuss: What was their strategy for calculating the number of each item? Are there any recommendations you disagree with?

	Population	Handheld Showers	Power Strips	Air Pumps
Lucas, Wisconsin	700	28	35	7
Blue Ridge, Georgia	1,200	48	60	12
Hamlin City, Kansas	25	1	2	1

Responses vary.
- **Their strategy is to use equivalent ratios to their recommendations for 100 people.**
- **I think 60 power strips feel like a lot. You need outlets for all of them to plug into and finding 60 outlets seems tough.**

Activity
2

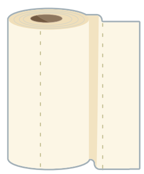
Name: .... Date: .... Period: ....

Make a Poster

8 Here are the national recommendations about other items to stock up on in case of disaster.

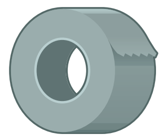
Paper Towels

For every 5 people, have 1 roll of paper towels.



Duct Tape

Have 3 rolls of duct tape for every 25 people.



Magnifying Glass

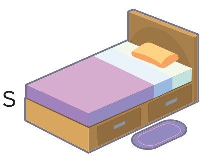
Have 1 magnifying glass for every 50 people.

Cotton Balls

For every 100 people, have 4 bags of 50 cotton balls each.

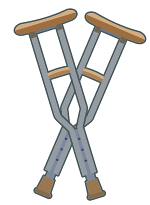
Beds

Have 1 bed for each person, plus 10 extra beds for volunteers.



Crutches

Have 6 pairs of crutches.



a Use these recommendations to make a disaster preparedness proposal for these 3 cities.

	Population	Rolls of Paper Towels	Magnifying Glasses	Cotton Balls	Pairs of Crutches
Branch City, Arkansas	300	**60**	**6**	**600 cotton balls (or 12 bags)**	**6**
Bennington City, Nebraska	2,000	**400**	**40**	**4,000 cotton balls (or 80 bags)**	**6**
Harrisburg, Pennsylvania	50,200	**10,040**	**1,004**	**100,400 cotton balls (or 2,008 bags)**	**6**

b Is there anything you disagree with about these recommendations? If so, explain which numbers you think should change and why. If not, explain why not.

Responses vary.

- **I don't think Harrisburg really needs 1,004 magnifying glasses. I can understand keeping a few magnifying glasses in every city, but I'm not sure a city needs 1 for every 50 people.**
- **The national recommendations for crutches seems like a mistake. There should be a ratio between the number of crutches and the number of people, like 6 pairs of crutches for every 100 people.**

Name: ________ Date: ________ Period: ________

Make a Poster (continued)

c Complete these steps and make a poster of your work: ***Posters vary.***

- ☐ Choose a city or town that is meaningful to you and look up its population.

City, State	Population (to the nearest 10 people)
________	________

- ☐ Make a proposal for items that this city should stock up on. Choose *at least* four different supplies from the national recommendations list. Then determine how many of each item the city should have on hand in case of a disaster.

Item 1:	**Item 2:**
Item 3:	**Item 4:**

- ☐ Show or explain how you determined the amount of each item your city will need.

- ☐ Explain *at least* two changes or additions you would make to the national recommendations.

9 Synthesis

Explain how to use a table of equivalent ratios to determine unknown values. Use the example if it helps with your thinking.

	Population	Handheld Showers	Power Strips
Recommendations	100	4	5
Lucas, Wisconsin	700	28	35
Blue Ridge, Georgia	1,200	48	60

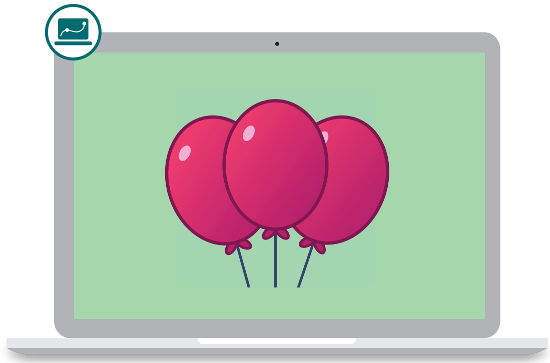
Responses vary.

- **I can make an equivalent ratio by multiplying or dividing the numbers as needed. For example, if there are 4 handheld showers for every 100 people, I can multiply both quantities by 7 to determine that we need $7 \cdot 4 = 28$ handheld showers for 700 people.**
- **I can find the ratio between the population and 1 item, then use that ratio to figure out how many items would be needed for other populations. In the example, there are 25 people for every handheld shower. So a town with 700 people would need $700 \div 25 = 28$ handheld showers.**

Things to Remember:

Unit 2
Lesson
11

Name: Date: Period:



Balloons

Let's develop and use tools to solve problems involving equivalent ratios.

Warm-Up

Evaluate each expression mentally.

1. $2 \cdot 31 = 62$

2. $8 \cdot 31 = 248$

3. $9 \cdot 31 = 279$

4. $11 \cdot 31 = 341$

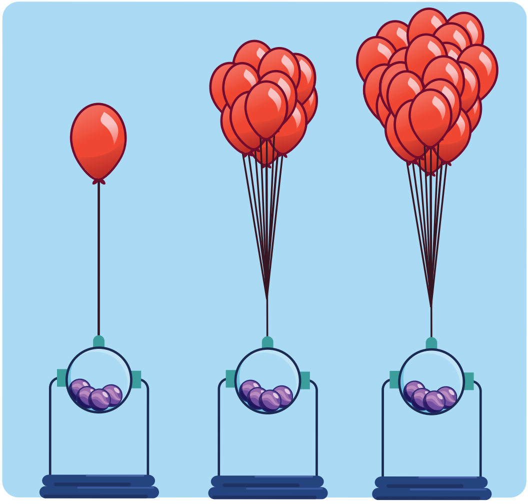
Activity 1

Name: ______ Date: ______ Period: ______

Balloon Float

5 Helium balloons can make objects float, but too many balloons will make objects fly away!

Let's watch an animation to see the *middle* container float.



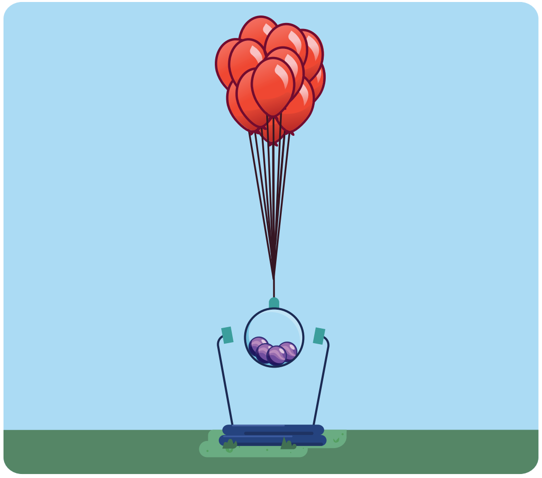
6 Red balloons float purple marbles at a ratio of $12 : 4$.

What will happen to the marbles if we add 1 balloon and 1 marble? Circle one.

Sink down   Float in place   Fly up

Explain your thinking.

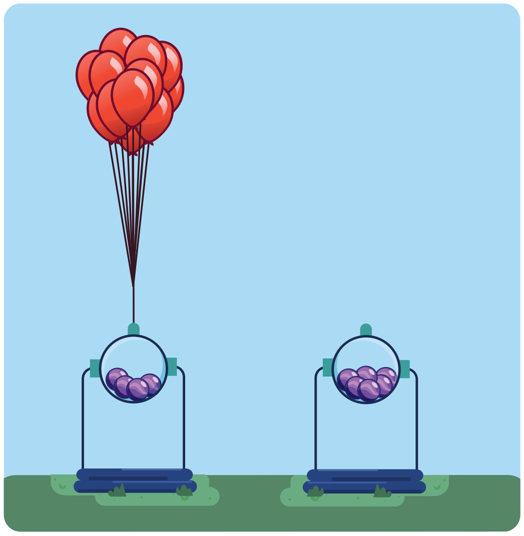
***Explanations vary.* 12 balloons float 4 marbles, so I need 3 balloons for every marble. If I add 1 marble, I need to add 3 balloons to keep the marbles afloat. 1 balloon isn't enough to float 1 marble, so it would sink.**



7 Red balloons float purple marbles at a ratio of $12 : 4$.

How many red balloons will float 6 purple marbles?

18 red balloons



Name: ____________ Date: ________ Period: ________

Balloon Float (continued)

8 Here are Charlie's and Daeja's strategies for determining how many red balloons will float 6 purple marbles.

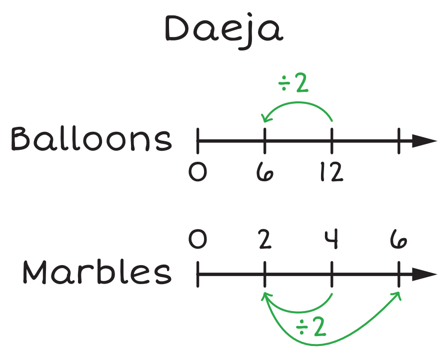
a Look at each student's strategy.

Charlie

Balloons	Marbles
12	4
3	1
	6

÷4 ÷4

Daeja



b Select one student by circling their name, then explain how they could finish their strategy to solve the problem.

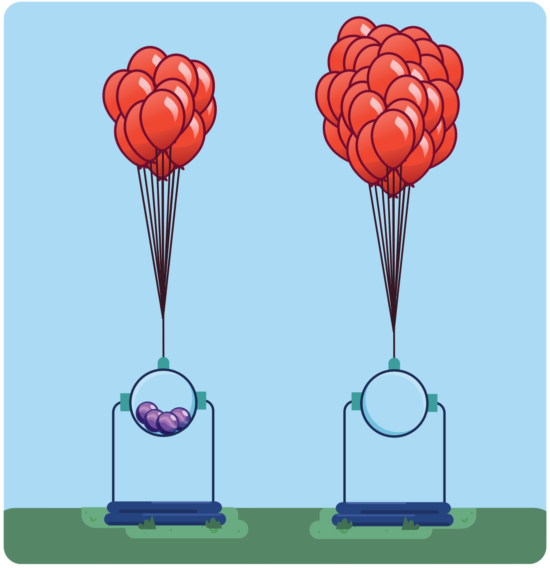
Explanations vary.

- **Charlie could finish by multiplying both the 3 and the 1 by 6 to get 18 balloons.**
- **Daeja could finish by multiplying both the 6 balloons and the 2 marbles by 3 to get 18 balloons. Daeja could also just continue counting by 6 along the top number line and get to 18 balloons for 6 marbles.**

9 Red balloons float purple marbles at a ratio of 12 : 4.

How many purple marbles will 30 red balloons float?

10 purple marbles



Name: ________ Date: ________ Period: ________

Marble Float

10 Red balloons float purple marbles at a ratio of $12 : 4$.

Red balloons float green marbles at a ratio of $15 : 6$.

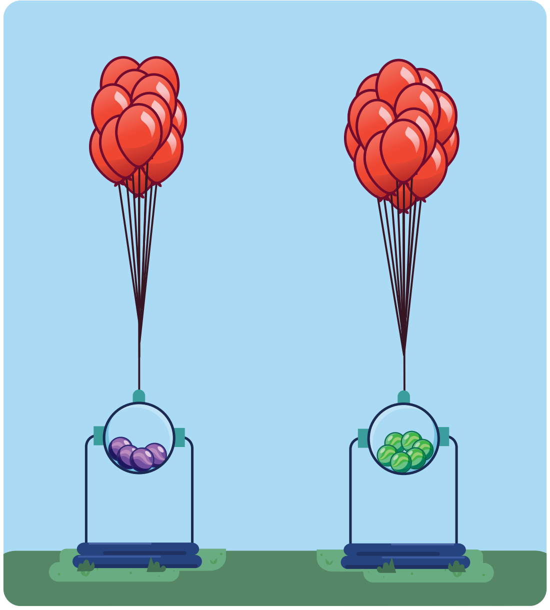
Which is heavier: a purple marble or a green marble?

Purple Green They're the same

Explain your thinking.

Explanations vary.

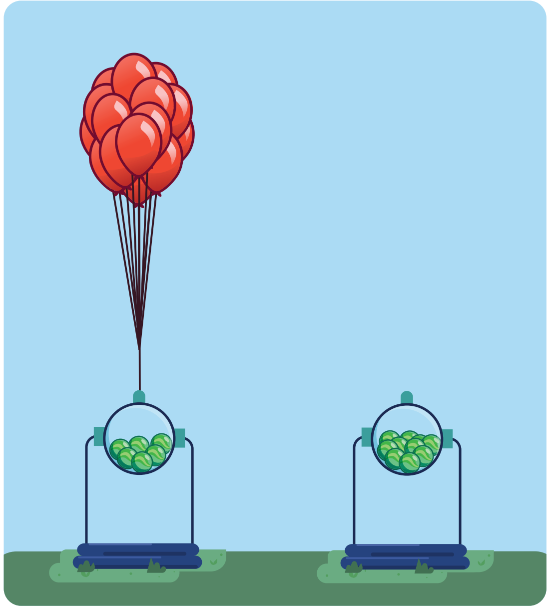
- **I need $\frac{12}{4} = 3$ balloons to float every purple marble but only $\frac{15}{6} = 2.5$ balloons to float every green marble.**
- **I would need $12 \cdot 3 = 36$ balloons to float 12 purple marbles and only $15 \cdot 2 = 30$ balloons to float 12 green marbles.**



11 Red balloons float green marbles at a ratio of $15 : 6$.

How many red balloons will float 10 green marbles?

25 red balloons



Activity 2

Name: ______ Date: ______ Period: ______

Marble Float (continued)

12 Here are Charlie's and Daeja's strategies for determining how many red balloons will float 10 green marbles.

Discuss: How are their strategies alike? How are they different?

Charlie

	Balloons	Marbles	
	15	6	
$\times\frac{1}{6}$	2.5	1	$\times\frac{1}{6}$
$\times 10$	25	10	$\times 10$

Daeja

	Balloons	Marbles	
	15	6	
$\times 5$	75	30	$\times 5$
$\div 3$	25	10	$\div 3$

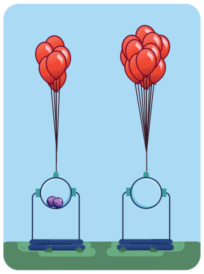
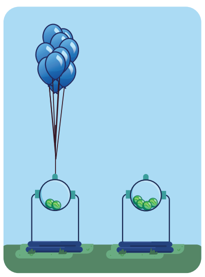
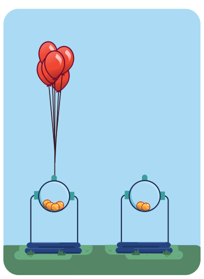
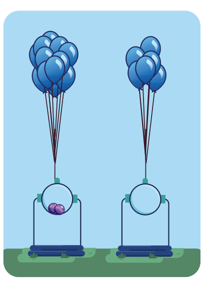
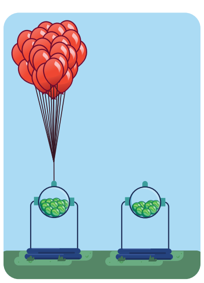
Responses vary.
Alike:
- **Both students used a table. They also both found a different ratio before finding the ratio they needed to solve the problem.**
- **Both students used multiplication.**

Different:
- **Charlie only used multiplication, while Daeja used both multiplication and division.**
- **Charlie calculated how many balloons for 1 marble and then multiplied to get the number of balloons for 10 marbles.**
- **Daeja found a number that works well with both 6 marbles and 10 marbles. Daeja multiplied to figure out how many balloons float 30 marbles, then divided to determine the number of balloons for 10 marbles.**

Name: ____________ Date: ________ Period: ________

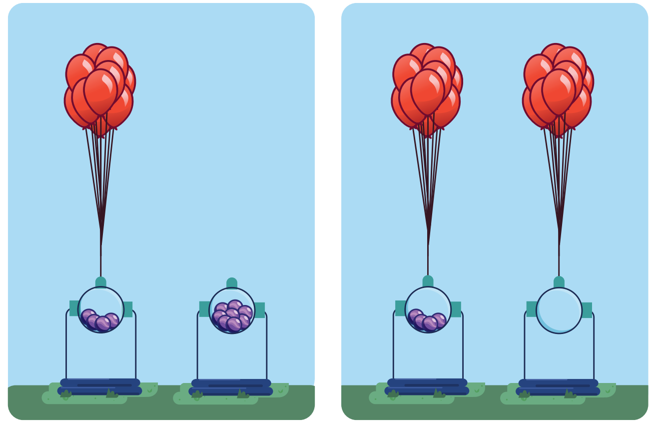
Repeated Challenges

13 For each ratio, create an equivalent ratio to make the balloons float.

	Ratio	Number of Balloons	Number of Marbles
	Red balloons float purple marbles at a ratio of 6 : 2.	12 red balloons	**4 purple marbles**
	Blue balloons float green marbles at a ratio of 10 : 2.	**20 blue balloons**	4 green marbles
	Red balloons float orange marbles at a ratio of 6 : 4.	**3 red balloons**	2 orange marbles
	Blue balloons float purple marbles at a ratio of 12 : 2.	6 blue balloons	**1 purple marble**
	Red balloons float green marbles at a ratio of 25 : 10.	**20 red balloons**	8 green marbles

14 Synthesis

Describe a strategy for determining missing values in equivalent ratios, like an unknown number of balloons or marbles..



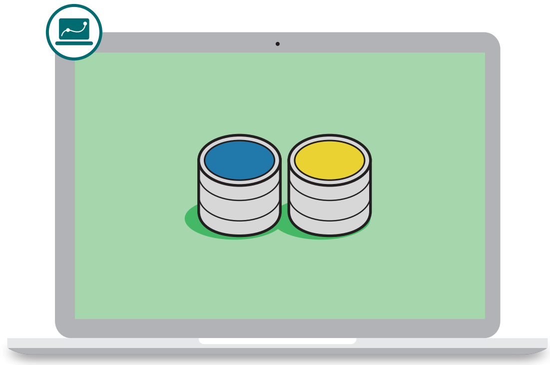
Responses vary.

- **One strategy is to figure out the number of balloons for every marble or the number of marbles for every balloon. Then I can multiply this ratio by the number of marbles or balloons I know to figure out the missing number.**
- **Another strategy is to use a double number line or a table to determine a different equivalent ratio that would be helpful as a middle step.**

Things to Remember:

Unit 2
Lesson
13

Name: Date: Period:



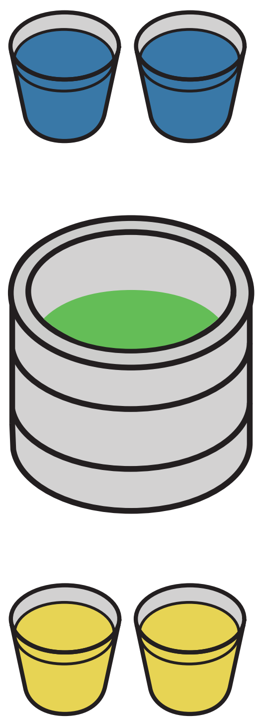
Mixing Paint, Part 2

Let's use tape diagrams to represent ratios.

Warm-Up

1 Let's watch how to make a new color.

What do you notice? What do you wonder?



I notice:

Responses vary.

- I notice that the more paint you add, the more the bucket fills up.
- I notice it takes 8 cups of paint to fill the bucket.
- I notice the more yellow you add, the lighter the green gets.

I wonder:

Responses vary.

- I wonder what happens if you make equivalent ratios of yellow and blue paint.
- I wonder if the bucket of paint is always the same size.
- I wonder what would happen if you mixed half a cup of yellow with half a cup of blue.

Activity 1

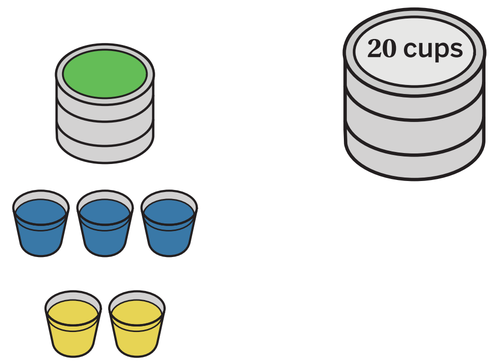
Name: ____________ Date: ________ Period: ________

How Much of Each?

2 Tyrone makes a green paint by mixing 3 cups of blue with 2 cups of yellow.

He needs 20 more cups of green paint to finish painting a mural.

How much of each color should he mix?



Blue (cups)	Yellow (cups)	Total (cups)
12	**8**	20

3 Tyrone drew a *tape diagram* to help determine that he needs 12 cups of blue and 8 cups of yellow to make 20 cups of green paint.

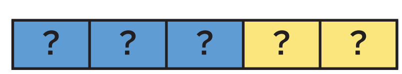
Where do you see the 3 : 2 ratio, 20, and 12 represented in Tyrone's diagram?

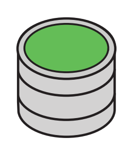
The 3 : 2 ratio is shown by . . . ***Responses vary.*** **The 3 : 2 ratio is shown by the three blue boxes and the two yellow boxes.**

The 20 total cups are shown by . . . ***Responses vary.*** **The 20 total cups are shown by the total number in the diagram: $4 + 4 + 4 + 4 + 4 = 20$ total cups.**

The 12 cups of blue are shown by . . . ***Responses vary.*** **The 12 blue cups are shown by the total number in the blue boxes.**

4 Kayla needs 35 gallons of the same green paint.





She used this tape diagram to determine how much of each paint color she needs.

How many gallons should go into each box in the tape diagram?

7 gallons

Activity 2

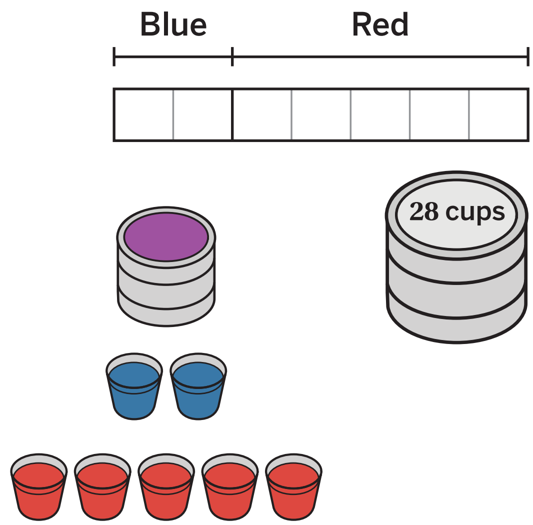
Name: ________ Date: ________ Period: ________

Same Color?

5 Sai makes purple paint by mixing 2 cups of blue and 5 cups of red.

How much of each color should Sai mix to get 28 cups of purple paint?

Use the tape diagram if it helps with your thinking.



Blue (cups)	Red (cups)	Total (cups)
8	20	28

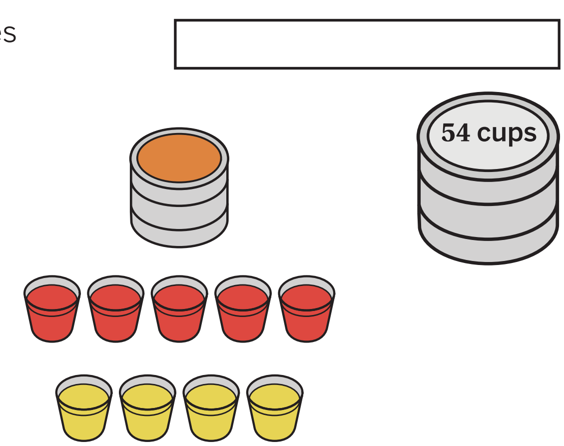
6 Select *all* of the combinations that would make the same color.

- [x] **A.** 1 cup blue and 2.5 cups red
- [ ] **B.** 2.5 cups blue and 1 cup red
- [x] **C.** 2 quarts blue and 5 quarts red
- [ ] **D.** 2 cups blue and 5 gallons red
- [ ] **E.** 1 gallon blue and 1 cup red

7 Dylan has a recipe for orange paint that mixes 5 parts red paint and 4 parts yellow paint.

How much of each color should Dylan mix to get 54 cups of orange paint?

Draw your own tape diagram if it helps with your thinking.

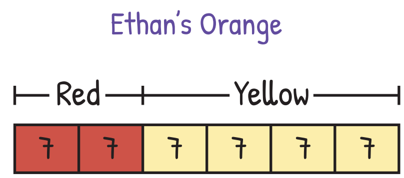


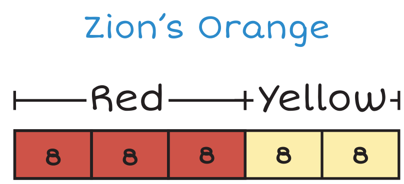
Red (cups)	Yellow (cups)	Total (cups)
30	24	54

Name: .... Date: .... Period: ....

Same Color? (continued)

8 Here are Ethan's and Zion's recipes for orange paint.





Which student made more paint? Circle one.

Ethan   Zion   Same amount

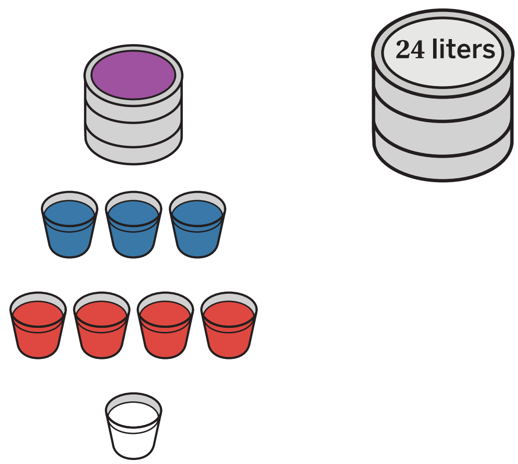
Explain your thinking.

Explanations vary. **Ethan made $7 \cdot 6 = 42$ cups of paint. Zion only made $5 \cdot 8 = 40$ cups of paint.**

9 A recipe for purple paint calls for 3 parts blue, 4 parts red, and 1 part white paint.

Kimora needs 24 liters of purple paint to paint a mural. How much of each color will Kimora need?

Draw your own tape diagram if it helps with your thinking.



Blue (L)	Red (L)	White (L)	Total (L)
9	**12**	**3**	24

Explore More

10 Create *three* equivalent ratios by filling in each blank using the numbers 0 to 9 only once.

☐ : ☐

☐ : ☐

☐ : 1☐

Responses vary.
- **1 : 2, 3 : 6, 9 : 18**
- **2 : 4, 3 : 6, 5 : 10**

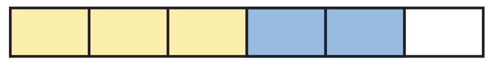
11 Synthesis

Here are the ingredients for a mango lassi drink.

Explain how the tape diagram represents this situation.

Ingredients for Mango Lassi

- 6 cups of mango
- 4 cups of yogurt
- 2 cups of milk



Responses vary. **The mango lassi has 12 cups of liquid, and there are 6 boxes in the tape diagram, so every box represents 2 cups. The 6 cups of mango are shown by the 3 yellow boxes, because $3 \cdot 2 = 6$. The 4 cups of yogurt are the 2 blue boxes, and the 2 cups of milk are represented by the 1 white box.**

Things to Remember:

Unit 2
Lesson
14

Name: .... Date: .... Period: ....

City Planning

Let's explore city planning using ratios.

Warm-Up

1 Imagine that you're moving to a new city.

What would be important to you when looking for a place to live?

Responses vary.
- Close to work and school
- Affordable
- Near my friends and family
- Has a grocery store and a park nearby

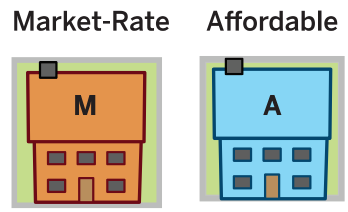
Name: .... Date: .... Period: ....

Affordable and Market-Rate Housing

2 Many cities have a shortage of housing affordable enough for residents to have money left over for other necessities, like food and healthcare.

One approach cities use is to create *affordable housing units* that have cost limits.

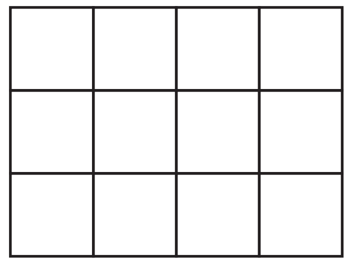
Housing with no cost limit is called *market-rate*.



Design a neighborhood.

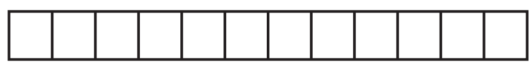
Write **M** in each square that you want to represent a market-rate house and **A** in each square that you want to represent an affordable house.

Responses vary.



Create a tape diagram to represent your neighborhood.

Responses vary.

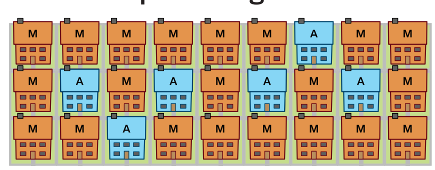


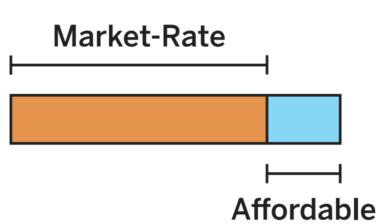
3 Metropolis requires a 7 : 2 ratio of market-rate housing units to affordable housing units.

How does your neighborhood compare to Metropolis's requirement?

***Responses vary.* The two neighborhoods have the same number of affordable housing units, but the Metropolis neighborhood has more units overall. My neighborhood has a larger portion of affordable housing than the Metropolis neighborhood does.**

Metropolis Neighborhood

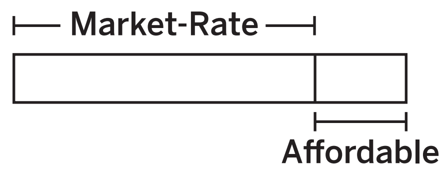




Name: ________ Date: ________ Period: ________

Affordable and Market-Rate Housing (continued)

4 Imagine you are a member of a city council, part of whose job is to help develop new neighborhoods. The city has a law that says each neighborhood must have a 7 : 2 ratio of market-rate units to affordable units.



The city wants to develop a new neighborhood with 36 units of land. How many units of each type are needed to meet the requirement?

Use the tape diagram or the grid if it helps with your thinking.

Market-Rate	Affordable	Total
28	**8**	36

5 Metropolis requires a 7 : 2 ratio of market-rate units to affordable units.

Market-Rate	Affordable	Total
62	10	72

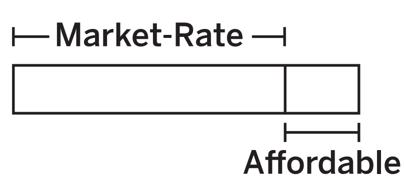
Does this neighborhood meet the requirement? Circle one.

Yes No I'm not sure

Explain your thinking. If you're not sure, what would help you be more sure?

Explanations vary. **If there are 10 affordable units, then there should be 35 market-rate units because $2 \cdot 5 = 10$ and $7 \cdot 5 = 35$. This neighborhood has more than 35 market-rate units, so it does not meet the requirement.**

6 How many units of each type of housing does this neighborhood need to meet Metropolis's requirement?



Use the tape diagram or the grid if it helps with your thinking.

Market-Rate	Affordable	Total
56	**16**	72

Activity 2

Name: ________ Date: ________ Period: ________

Green Space

7 Urban green spaces, such as parks and gardens, give people space for physical activity, relaxation, peace, and an escape from the heat.

Here are two neighborhoods in Evergreen City.

Where would you prefer to live? Circle one.

Neighborhood A Neighborhood B

Explain your thinking.

Neighborhood A

Neighborhood B

Responses and explanations vary.

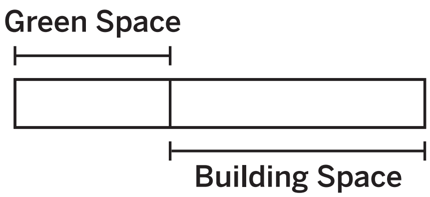
- **I would rather live in Neighborhood A because you can do more things with a big green space. I would love to have a big park where I can run around.**
- **I would prefer to live in Neighborhood B because then every house is close to at least one bit of green space.**

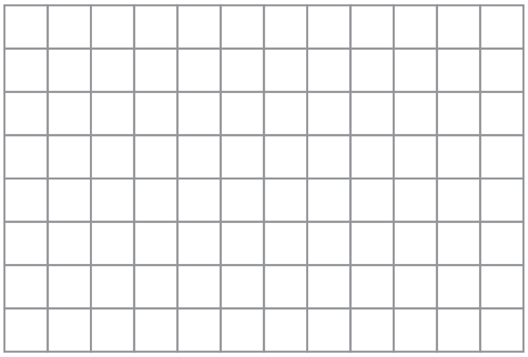
8 Evergreen City requires a $3 : 5$ ratio of units of green space to units of building space.

The city is developing 96 units of land for a new neighborhood.

How many of each type of space should the city plan for?

Use the tape diagram or the grid if it helps with your thinking.





Green Space	Building Space	Total
36	**60**	96

Activity 2

Name: ________________ Date: ________ Period: ________

Green Space (continued)

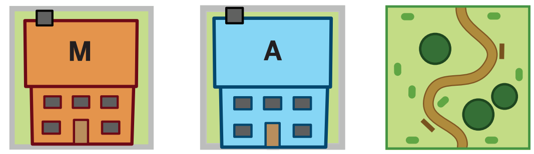
9 Overall, Evergreen City requires a $4 : 1 : 3$ ratio of market-rate housing to affordable housing to green space.

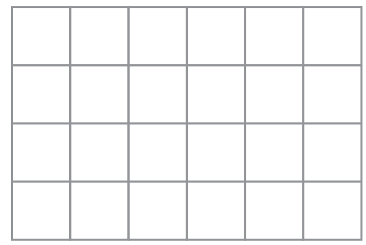
Here are 24 units of land.

Design a neighborhood that meets Evergreen City's requirements.
Check your work using the digital activity.

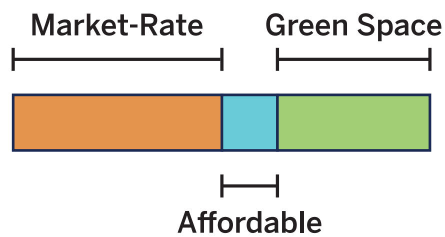
***Responses vary.* All neighborhoods should have 12 units of market-rate housing, 3 units of affordable housing, and 9 units of green space.**

Market-Rate **Affordable** **Green**





10 This neighborhood in Evergreen City meets the requirement of a $4 : 1 : 3$ ratio of market-rate housing to affordable housing to green space.



However, residents claim the neighborhood is not fair.

a Why might the residents feel it's not fair?

***Responses vary.* The residents might think this neighborhood is not fair because almost all of the green space is by the market-rate housing. Also, there is a lot more market-rate housing than affordable housing.**

b What changes do you think should be made?

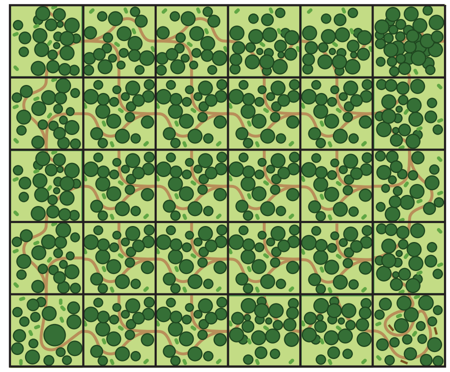
***Responses vary.* I would change how affordable housing is distributed so that everyone benefits from the green space. I would also increase the number of units of affordable housing.**

11 Synthesis

Des-Town requires a $3:2$ ratio of building space to green space.

Explain how a city planner might determine how many units of building space can be developed in this neighborhood.

Draw on the diagram if it helps with your thinking.



Responses vary. **The total number of units in the ratio is $3 + 2 = 5$, so the city planner can divide the total number of units in the neighborhood by 5 and then multiply by either 3 or 2 to figure out how many of each type of housing unit they need. According to the diagram, there are 30 total units in the neighborhood, so there would need to be $\frac{30}{5} \cdot 3 = 18$ units of building space and $\frac{30}{5} \cdot 2 = 12$ units of green space.**

Things to Remember:

Unit 3
Lesson
1

Name: .... Date: .... Period: ....

Many Measurements

Let's connect units of measure with everyday objects.

Warm-Up

1. Which do you think is taller? Circle one.

Responses vary.

A coconut | A pineapple

2. Which do you think is larger? Circle one.

A grapefruit | A plum

3. Which do you think is heavier? Circle one.

Responses vary.

A cherry | A grape

Activity 1

Name: .... Date: .... Period: ....

Describe It

4. **Discuss:** Use words, drawings, hand gestures, familiar objects, or other strategies to answer the question: *How much is* ....?

- ☐ 1 foot
- ☐ 1 meter
- ☐ 1 gallon
- ☐ 1 millimeter
- ☐ 1 cup
- ☐ 1 square foot
- ☐ 1 yard
- ☐ 1 pound

5. Which measurements were *less* complicated to describe? Which measurements were *more* complicated to describe? ***Responses vary.***

Less Complicated	More Complicated

6. Sort the Activity 1 Cards based on whether they measure length, volume, or weight. There will be four cards in each group.

Length	Volume	Weight
Card 3 Card 4 Card 7 Card 12	Card 5 Card 9 Card 10 Card 11	Card 1 Card 2 Card 6 Card 8

7. Sort the measurements in each group from the *smallest* unit to the *largest* unit.

	Smallest Unit			Largest Unit
Length	1 millimeter	1 centimeter	1 kilometer	1 mile
Volume	1 milliliter	1 cup	1 liter	1 gallon
Weight	1 gram	1 ounce	1 pound	1 kilogram

Activity 2

Name: ______ Date: ______ Period: ______

Match It

8. Match each Activity 2 Card with the unit of measurement that best represents it.

1 Kilogram Card I	**1 Ounce** Card H	**1 Millimeter** Card C
1 Mile Card G	**1 Liter** Card F	**1 Gram** Card B
1 Kilometer Card A	**1 Pound** Card E	**1 Cup** Card L
1 Milliliter Card J	**1 Gallon** Card D	**1 Centimeter** Card K

9. **Discuss:** Choose *one* of the measurements from Problem 8. What else could you measure with this unit of measurement?

Responses vary.

Explore More

10. Here are four unit conversions arranged by what they're measuring. Add any other unit conversions you can think of for each category. ***Responses vary.***

Length	Weight
1 foot = 12 inches **100 centimeters = 1 meter**	1 kilogram = 1000 grams **16 ounces = 1 pound**

Volume	Time
1 gallon = 4 quarts **1000 milliliters = 1 liter**	1 hour = 60 minutes **60 seconds = 1 minute**

Synthesis

11. **a** List several things you could measure about this can.

Responses vary. **I could measure its volume, height, width, and weight.**

b What units would you use to measure each of those things?

Responses vary. **For the height, width, and circumference, I would use a unit that measures length, like centimeters or inches. I'm guessing this is a pretty small can, so I would use milliliters or fluid ounces to measure the volume. For the weight, I'd use grams.**

Things to Remember:

Describe It

Directions: Make one copy per pair of students. Then pre-cut the 12 cards and give each pair of students one set.

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Card 1

1 kilogram (kg)

Card 2

1 ounce (oz)

Card 3

1 millimeter (mm)

Card 4

1 mile (mi)

Card 5

1 liter (L)

Card 6

1 gram (g)

Card 7

1 kilometer (km)

Card 8

1 pound (lb)

Card 9

1 cup

Card 10

1 milliliter (mL)

Card 11

1 gallon (gal)

Card 12

1 centimeter (cm)

Match It

Directions: Make one copy per pair of students. Then pre-cut the 12 cards and give each pair of students one set.

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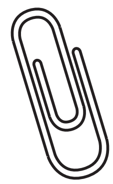
Card A

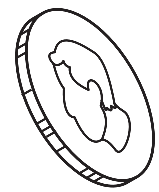
Distance walked in
10 minutes

Card B



Weight of a paper clip

Card C



Thickness of a dime

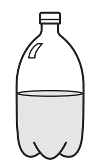
Card D

Volume of milk in
a large milk jug

Card E

Weight of a hooded
sweatshirt

Card F

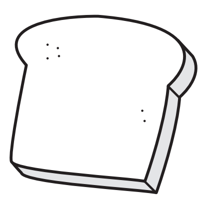


Volume of soda in a large
soda bottle that is half full

Card G

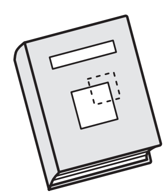
Distance ran in 10 minutes

Card H



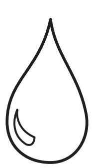
Weight of a slice of bread

Card I



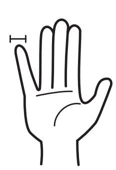
Weight of a textbook

Card J



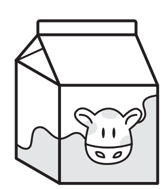
Volume of water in
a raindrop

Card K



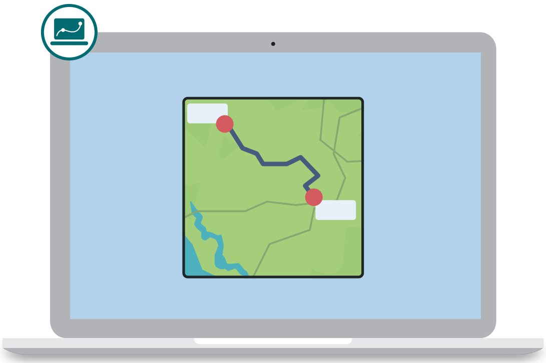
Width of a pinky finger

Card L



Volume of milk in a school
milk carton

Name: .... Date: .... Period: ....



Pen Pals

Let's compare measurements in different units.

Warm-Up

1 Four pen pals share letters with each other.

Discuss: What do you notice? What do you wonder?

Name	Eva	Ayaan	Thiago	Binta
Country	United States	India	Brazil	Liberia
Favorite Food	Bubble tea	Mango lassi	Quindim	Spaghetti
Favorite Animal	Horse	Dog	Horse	Bird
Favorite Sport	Football	Cricket	Futebol	Football

Responses vary. I notice that most of the pen pals like football. I wonder why football is different in different countries.

Name: ________________ Date: ________ Period: ________

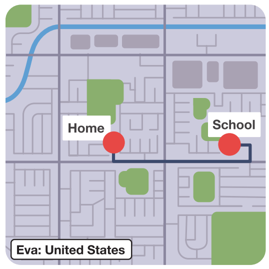
Traveling to School

2 The pen pals discuss how far they each live from school. Use your best estimates to order the pen pals from *closest* to *farthest* from school.

Thiago: Brazil

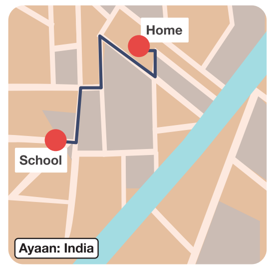
20 kilometers

Eva: United States



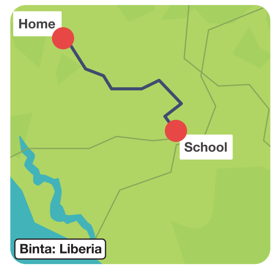
2,000 feet

Ayaan: India



900 meters

Binta: Liberia



15 miles

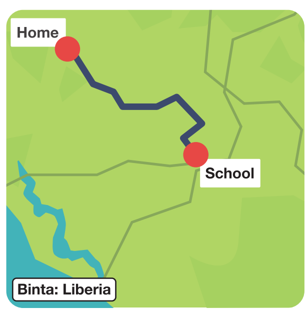
Closest **Farthest**

***Responses vary.* There are no correct answers for this screen. Students should use their intuition about the sizes of the given units to help them estimate the closest and farthest distances from school.**

3 Binta lives 15 miles from her school in Liberia. Thiago lives 20 kilometers from his school in Brazil.

Who lives closer to their school? Circle one and explain your thinking.

Binta Thiago About the same distance



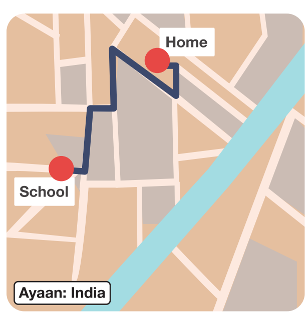
8 kilometers ≈ 5 miles

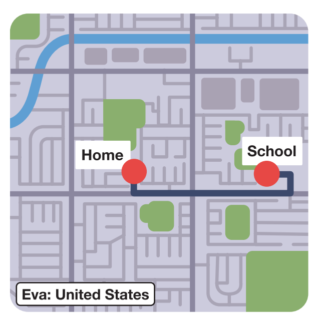
***Explanations vary.* Binta lives 15 miles, or 24 kilometers, from school. Thiago lives 20 kilometers from his school, so Thiago lives closer.**

4 Ayaan lives 900 meters from his school in India. Eva lives 2,000 feet from her school in the United States.

Who lives closer to their school? Circle one and explain your thinking.

Ayaan Eva About the same distance





3 meters ≈ 10 feet

***Explanations vary.* Ayaan lives 900 meters from school, which is about 3,000 feet. Eva lives 2,000 feet from her school, so she lives closer.**

Activity 2

Name: .... Date: .... Period: ....

Weighing Strategies

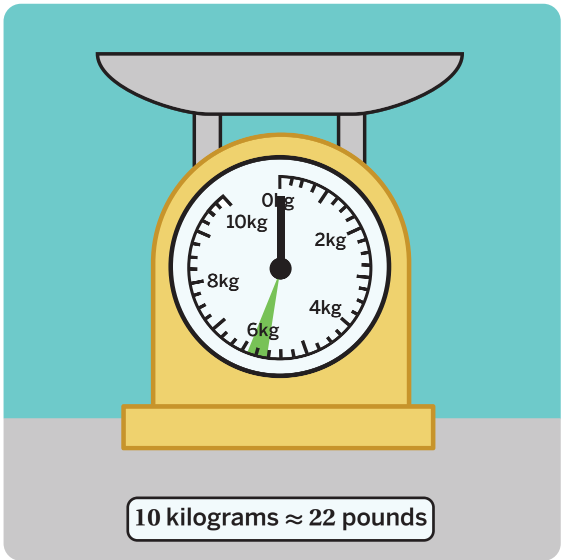
5 Thiago's horse eats about 6 kilograms of hay per day.

Eva wants to know how many pounds that is.

About how many pounds is 6 kilograms?

13.2 pounds

Note: Responses between 12.8 pounds and 13.6 pounds are considered correct.



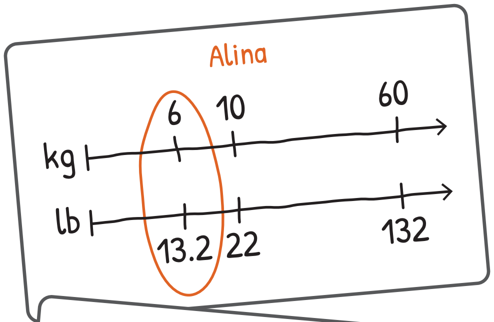
6 Alina and Maia both determined how many pounds of hay Thiago's horse eats per day.

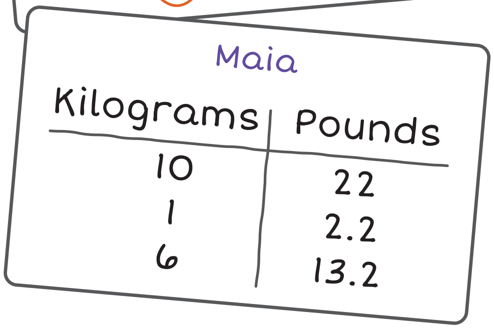
Alina used a double number line and Maia used a table.

Choose a student and explain their thinking.

Responses vary.

- **Alina multiplied both 10 and 22 by 6 to get 60 kilograms ≈ 132 pounds. Then she divided both of those numbers by 10 to get 6 kilograms ≈ 13.2 pounds.**
- **Maia divided the numbers in the conversion rate by 10 to determine the number of pounds per kilogram. Then she multiplied by 6 to calculate the number of pounds for 6 kilograms.**





Name: .... Date: .... Period: ....

Favorite Things

7 Binta decides to make Ayaan's recipe for mango lassi.

The recipe calls for 135 milliliters of milk.

About how many tablespoons of milk should Binta use?

9 tablespoons

Note: Responses between 8.8 tablespoons and 9.2 tablespoons are considered correct.

MANGO LASSI
Ingredients
Serves 2 people

- 250 milliliters mango pulp
- 240 milliliters yogurt
- 135 milliliters milk
- 20 grams sugar
- 1 gram cardamom powder

30 milliliters ≈ 2 tablespoons

8 Thiago decides to make Binta's recipe for spaghetti.

The recipe calls for 450 grams of bell pepper.

About how many ounces of bell pepper should Thiago use?

15.75 ounces

Note: Responses between 15.25 ounces and 16.25 ounces are considered correct.

SPAGHETTI
Ingredients

- 1 box of spaghetti
- 450 grams of bell pepper
- 200 grams of tomato
- 150 grams of onion
- 3 bouillon cubes
- 400 grams of ground beef
- 800 grams of Italian sausage
- Habanero peppers, curry powder, ginger, salt, oil

200 grams ≈ 7 ounces

Explore More

9 People have known for over 2,000 years that Earth is round, but it took a long time to discover how big it is.

A Greek mathematician named Eratosthenes was the first known person to calculate the distance around Earth's equator. In about 240 BCE, he calculated the distance around Earth's equator to be about 250,000 stadia using an estimated distance from Alexandria to Syene, along with the lengths of shadows.

The actual distance is about 24,901 miles.

What is the difference, in miles, between Eratosthenes's calculation and the actual distance? Explain your thinking.

1,299 miles. *Explanations vary.* 250,000 stadia is about 524 • 50 = 26200 miles. This is 26200 − 24901 = 1299 miles more than the actual distance around Earth.

10 Synthesis

Describe a strategy for converting a measurement from one unit to another.

Use the examples if they help you with your thinking.

8 kilometers ≈ 5 miles

200 grams ≈ 7 ounces

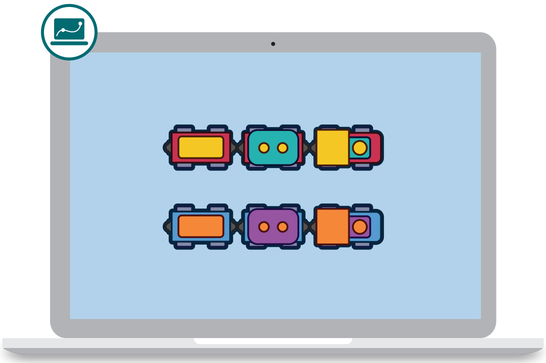
30 milliliters ≈ 2 tablespoons

Responses vary.

- **To convert measurements from one unit to another, you can use a table of values or a double number line. You need a ratio to start with, and then you can multiply and divide to get to the number you're trying to convert to.**
- **You can find how many per one and then multiply to get your answer. For example, if I want to convert kilometers to miles, I know there is $\frac{5}{8}$ of a mile for every 1 kilometer. Then I multiply $\frac{5}{8}$ by the number of kilometers I have to find the number of miles.**

Things to Remember:

Name: .... Date: .... Period: ....



Model Trains

Let's use ratios to compare speeds.

Warm-Up

1 Which one doesn't belong? Circle one. ***Responses vary.***

5 miles in 15 minutes	20 miles per 1 hour	3 minutes per mile	32 kilometers per 1 hour

Explain your thinking.

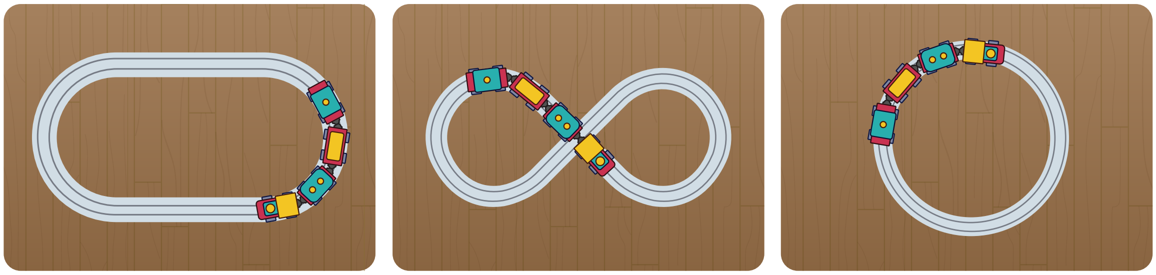
Explanations vary.

- **5 miles in 15 minutes is the only rate that's not expressed as a unit rate.**
- **20 miles per 1 hour is the only rate that sounds like how I talk about speeds.**
- **3 minutes per mile is the only rate expressed as a pace instead of a speed.**
- **32 kilometers per 1 hour is the only rate that uses metric units.**

Name: .... Date: .... Period: ....

How Fast?

2 A children's museum has three types of model train sets for students to build and play with. Let's watch how the train moves on each track.



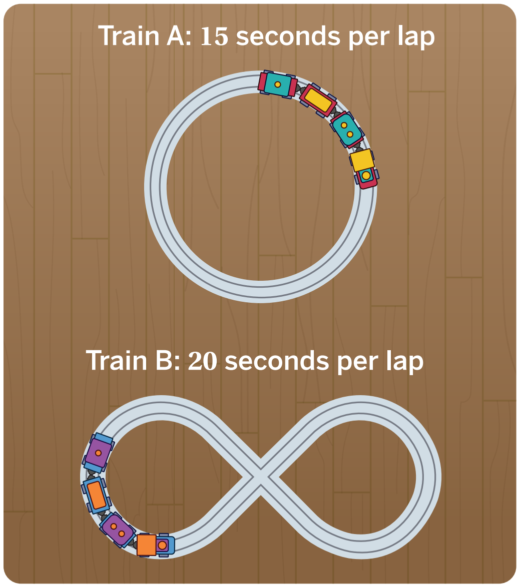
3 Here are trains from two students.

Which train is faster? Circle one.

Train A   Train B   Not enough information

Explain your thinking. If you don't have enough information, what information would help you determine which train travels faster?

***Explanations vary.* We can determine which train travels faster if we know how long each of the tracks is. If we know the length of each track, we can find and compare the speed of each train.**

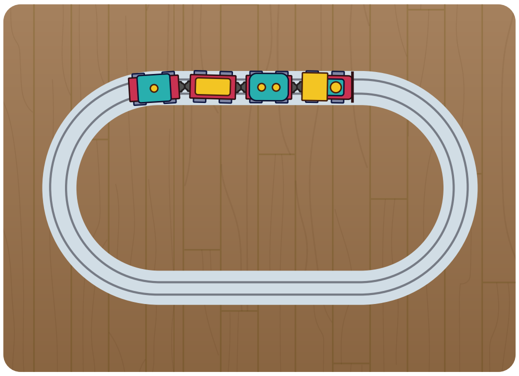


4 Here is a track. It is 325 centimeters long.

This train takes 10 seconds per lap.

What is its speed in centimeters per second?

32.5 centimeters per second



Activity
1

Name: ______________ Date: ______________ Period: ______________

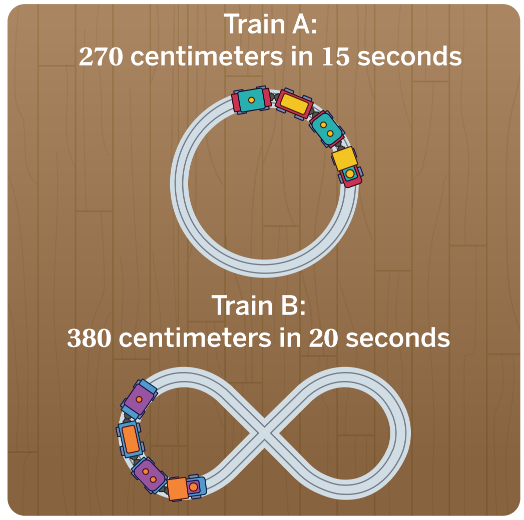
How Fast? (continued)

5 Which train is faster? Circle one.

Train A   Train B   They go the same speed

Explain your thinking.

***Explanations vary.* Train A is traveling $\frac{270}{15} = 18$ centimeters per second, and Train B is traveling $\frac{380}{20} = 19$ centimeters per second, so Train B is faster.**



6 Amoli and Tiam used different strategies to determine which train was faster.

Discuss: How are their strategies alike? How are they different?

Amoli

Train A

$270 \div 15 = 18$ cm per sec

Train B

$380 \div 20 = 19$ cm per sec

Train B is faster.

Tiam

Train A

cm	sec
270	15
1080	60

Train B

cm	sec
380	20
1140	60

Train B is faster.

Responses vary.
- **Amoli divided the number of centimeters by the number of seconds to calculate how far each train travels in 1 second. This is called the unit rate.**
- **Tiam used equivalent ratios to determine how far each train travels in 60 seconds. The train that travels farther in 60 seconds is faster.**

Activity 2

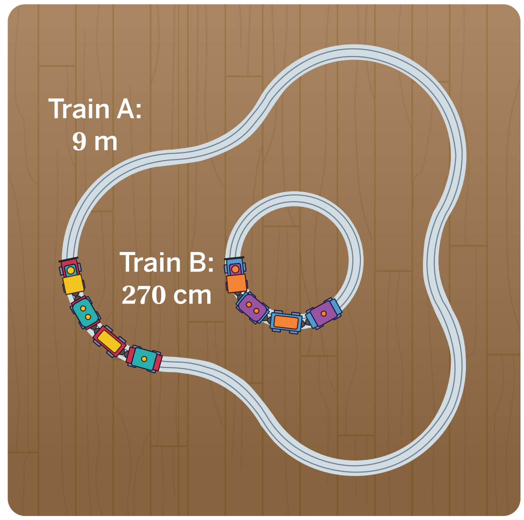
Name: ________ Date: ________ Period: ________

Which is Faster?

7 Here are two trains. They each complete a lap in 20 seconds.

What is each train's speed in centimeters per second?

	Speed (centimeters per second)
Train A	45
Train B	13.5



8 Here are distances and times for four model trains.

1 meter = 100 centimeters

1 minute = 60 seconds

Order the trains by speed.

A. 3.25 meters in 1 minute

B. 3.25 meters in 20 seconds

C. 270 centimeters in 20 seconds

D. 325 centimeters in 30 seconds

A	D	C	B

Slowest **Fastest**

Explore More

9 A train's speed is 60 centimeters per second.

Write a track length. Then determine the number of laps the train can complete in 10 seconds.

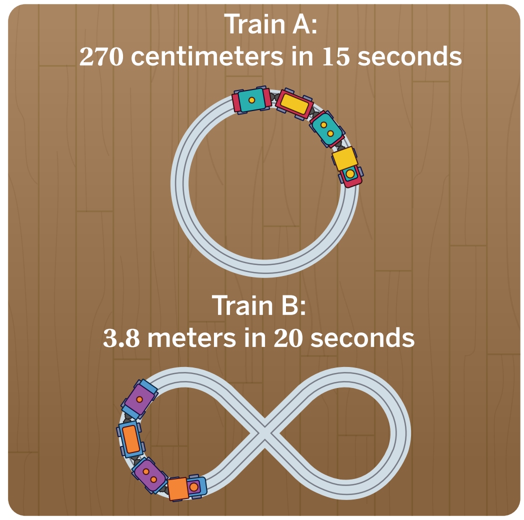
Responses vary.

Track Length (cm)	Laps in 10 Seconds
600	1

10 Synthesis

Describe *two* strategies for determining which of two trains is faster.

Use the examples if they help with your thinking.



Responses vary.

- **To determine which of two trains is faster, you can find a unit rate for each train, which measures how far the train travels in 1 second.**
- **Use equivalent ratios to see how far each train goes in the same amount of time. The train that goes farther is the faster train.**

Things to Remember:

Name: ________ Date: ________ Period: ________

Soft Serve

Let's compare soft serve prices using unit rates.

Warm-Up

1 a Take a look at the prices for different sizes of soft serve sold at a store.

S	M	L
\$2	\$2.80	\$3.60
5 oz	7 oz	9 oz

b **Discuss:** Which size offers the best deal?

***Responses vary*. Each size costs 40 cents per ounce, so the rate is the same for all sizes and there is no best deal.**

Activity 1

Name: ______ Date: ______ Period: ______

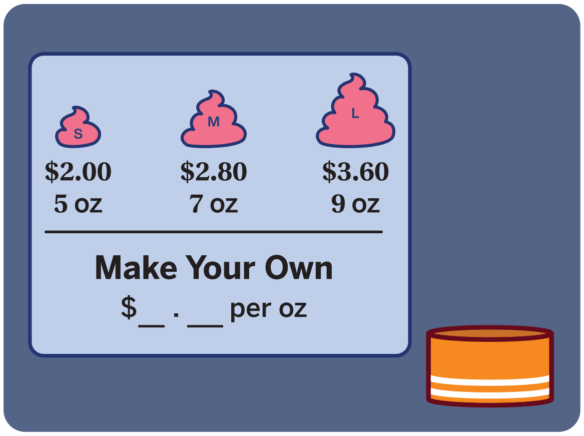
Two Unit Rates

2 Kala notices that soft serve costs the same per ounce no matter what size you get.

She suggests that the store put the rate on the menu.

How much does soft serve cost per ounce?

0.40 dollars per ounce



3 The store added the price per ounce, or **unit price**, to the menu.

A customer asks for 8 ounces of soft serve.

How much will this cost?

3.20 dollars

4 A new customer comes in with $3 and wants to spend it all on soft serve.

How many ounces can they get for $3?

7.5 ounces

Activity
1

Name: ______ Date: ______ Period: ______

Two Unit Rates (continued)

5 Here is how Neena figured out how much soft serve you can get for $3.

	Cost (dollars)	Weight (ounces)	
	2	5	
÷2	1	2.5	÷2
×3	3	7.5	×3

a **Discuss:** What was Neena's strategy?

***Responses vary.* Neena used a table to determine how many ounces per dollar, and then multiplied by 3.**

b Explain or show where you can see *ounces per dollar* in Neena's work.

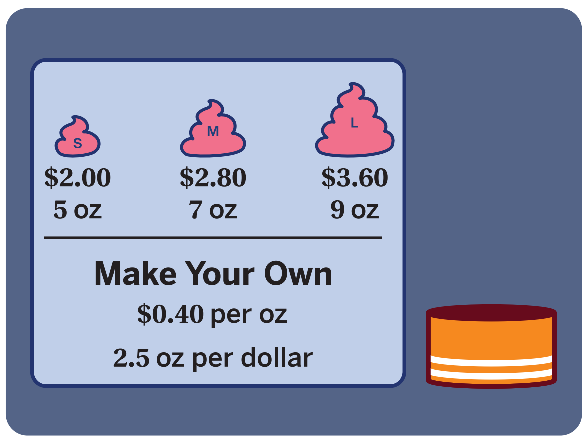
***Responses vary.* I can see in the second row that the number of ounces for 1 dollar is 2.5.**

6 The store's menu now includes *both* unit rates.

A new customer comes in with $7 and wants to spend it all on soft serve.

How much soft serve can they get for $7?

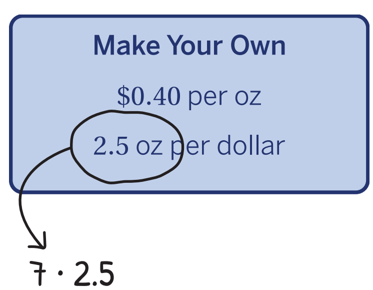
17.5 ounces



7 Here is how Tamiya figured out how much soft serve you can get for $7.

How do you think Tamiya knew which unit rate to use?

***Responses vary.* The soft serve is 2.5 ounces per dollar, which means for every dollar, you can buy 2.5 ounces. There is $7 to spend, so I think that's why Tamiya multiplied 7 by 2.5.**

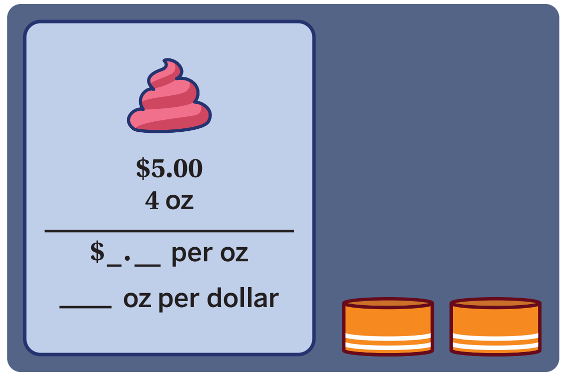


Activity
2

Name: .... Date: .... Period: ....

New Flavors

8 The store offers a new flavor, Swirl, with this pricing: $5 for every 4 ounces.



a How much does Swirl cost per ounce?

1.25 dollars per ounce

b How many ounces can you get per dollar?

0.8 or $\frac{4}{5}$ ounces per dollar

9 How much does 7 ounces of Swirl cost?

8.75 dollars

Explain your thinking.

***Explanations vary.* Since I know the number of ounces, I can multiply by the dollars per ounce. The cost is $1.25 per ounce, so $7 \cdot 1.25 = 8.75$.**

10 Match each rate with either chocolate or vanilla.

$2 per ounce	$0.50 per ounce	2 ounces per dollar	$\frac{1}{2}$ ounces per dollar	$9 for 4.5 ounces

Chocolate

2 oz for $4

$2 per ounce

$\frac{1}{2}$ ounces per dollar

$9 for 4.5 ounces

Vanilla

8 oz for $4

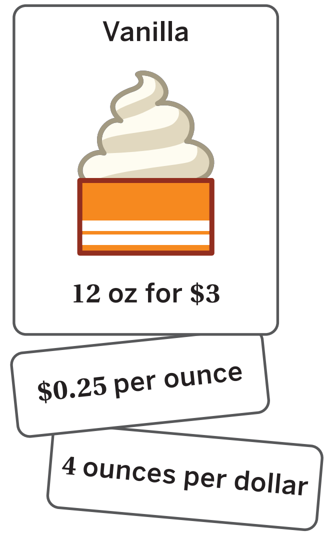
$0.50 per ounce

2 ounces per dollar

11 Synthesis

Explain how to calculate the two unit rates for vanilla soft serve.

Responses vary. **I can calculate the amount of soft serve for 1 dollar by dividing 12 ounces by $3.00 to get 4 ounces per dollar. Then I can calculate the unit price of the soft serve by dividing $3.00 by 12 ounces to get $0.25 per ounce.**



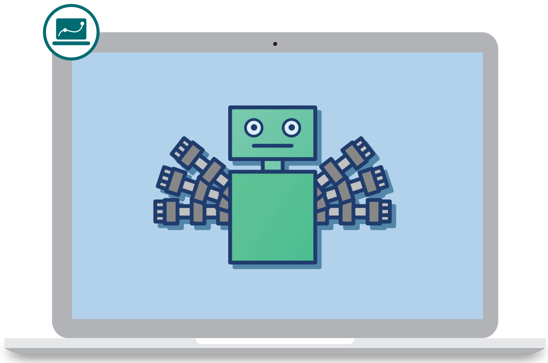
Things to Remember:

Unit 3
Lesson
7

Name: .... Date: .... Period: ....

Welcome to the Robot Factory

Let's determine unknown values using unit rates.



Warm-Up

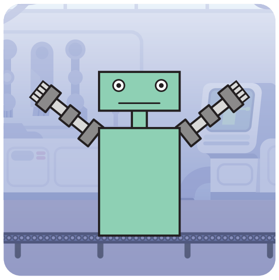
1 This table shows some lengths in both inches and feet.

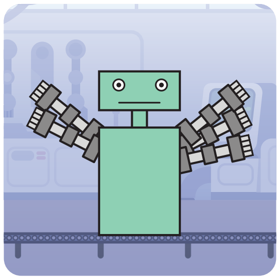
What are *three* things you notice about the table?

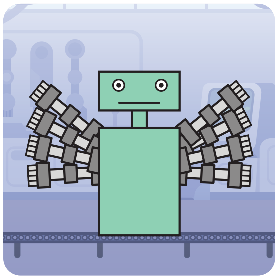
Responses vary.
- **I notice that every length is different.**
- **I notice all of the lengths in inches are even numbers.**
- **I notice that the number on the right is the number on the left times 12.**

Length (ft)	Length (in.)
1	12
3	36
5	60
10	120

2 Welcome to the Robot Factory! Take a look at how many arms the robot has in each image.







Activity 1

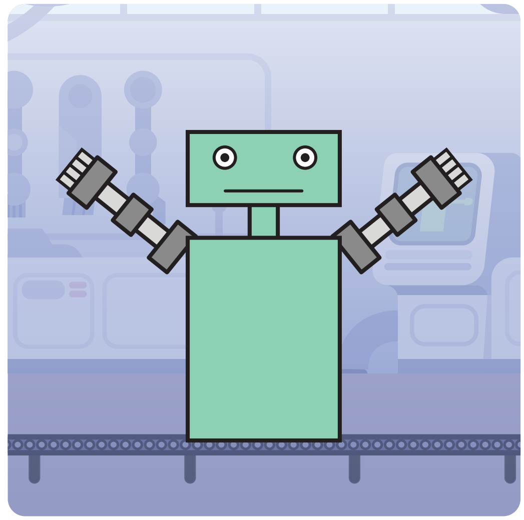
Name: ________ Date: ________ Period: ________

Arms and Fingers

3 This robot has 2 arms and 8 fingers.

Here are some other robots with different numbers of arms.

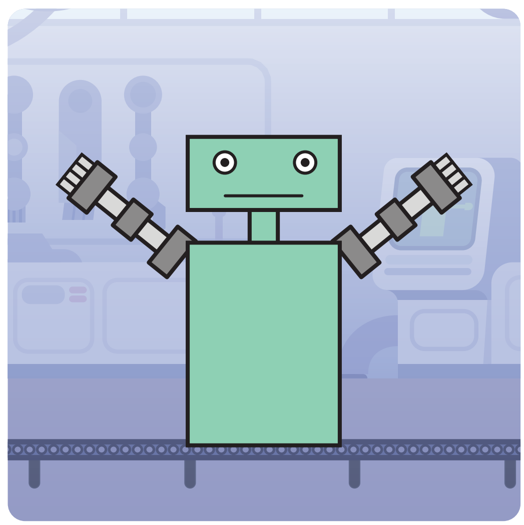
Complete the table to show the number of fingers on each robot.



Arms	Fingers
2	8
7	28
3	12
9	36

4 A new row has been added to the table.

How many arms go with this many fingers?



Arms	Fingers
2	8
11	44

5 Choose *one* question and write your response.

☐ If you know the number of *fingers*, how can you determine the number of arms?

☐ If you know the number of *arms*, how can you determine the number of fingers?

Responses vary.

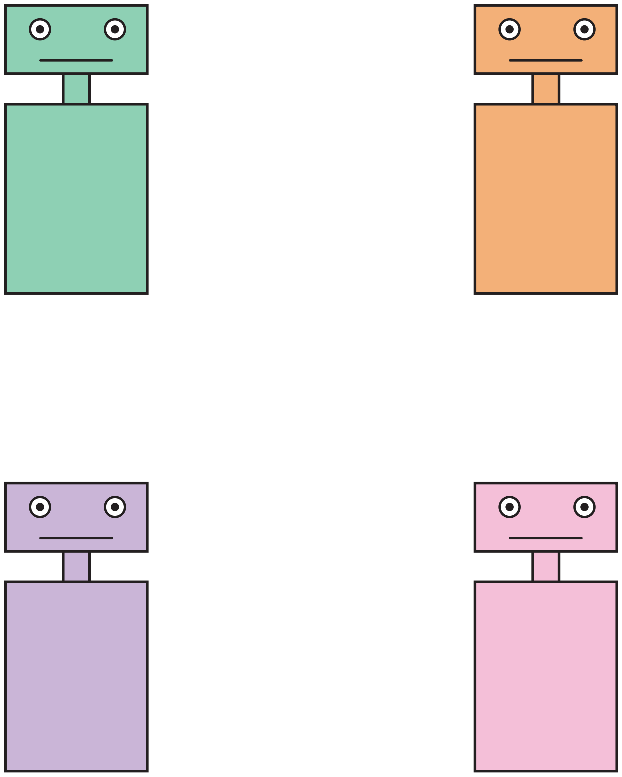
- I can determine the number of arms by dividing by 4.
- I can determine the number of fingers by multiplying by 4.

Activity 2

Name: ________ Date: ________ Period: ________

Painting Robots

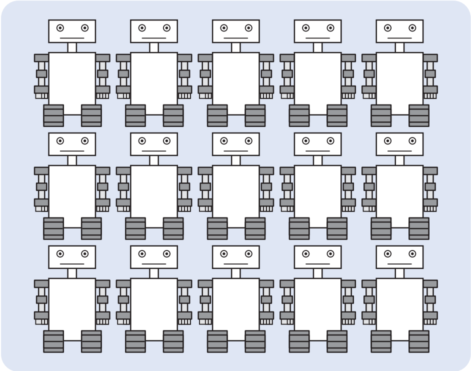
6 Choose a color to paint your robot.



7 6 robots need 2 gallons of paint.

Complete the table.

Number of Robots	Amount of Paint (gal)
6	2
15	5
21	7
11	$\frac{11}{3}$



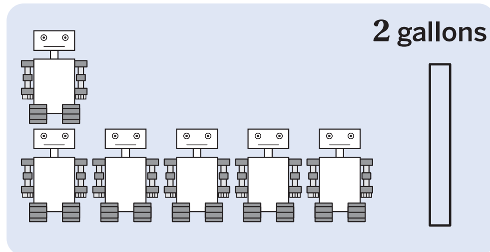
Name: .... Date: .... Period: ....

Painting Robots (continued)

8 Write instructions for how you could determine the amount of paint needed for *any* number of robots.

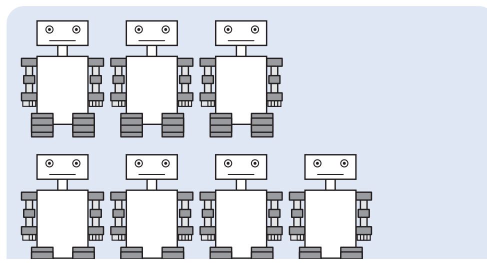
Use your table from the previous problem if it helps with your thinking.

***Responses vary.* Take the number of robots and multiply by the number of gallons per robot, which is $\frac{1}{3}$.**



9 Here are some extra-large robots. 4 robots need 10 gallons of paint.

Complete the table.



Number of Robots	Amount of Paint (gal)
4	10
7	**17.5**
9	**22.5**
13	**32.5**

Explore More

10 Joud wrote down the amount of paint and the painting time needed for different numbers of robots. Some of the values are missing. Complete the table.

Number of Robots	Amount of Paint (gal)	Painting Time (min)
5	2	**4**
12.5	5	10
15	**6**	12
2.5	1	**2**

11 Synthesis

Explain how you can use a table of equivalent ratios to determine unknown values, like the amount of paint needed for different numbers of robots.

Use this table if it helps with your thinking.

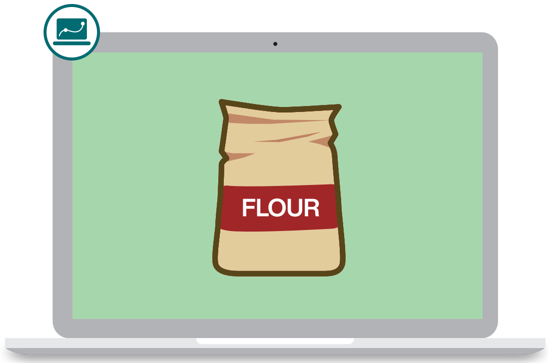
***Responses vary.* In a table of equivalent ratios, you can multiply by a unit rate to go from one column to another.**

Number of Robots	Amount of Paint (gal)
1	$\frac{1}{3}$
6	2
33	11
18	6
9	3

Things to Remember:

Unit 4
Lesson
3

Name: ______ Date: ______ Period: ______



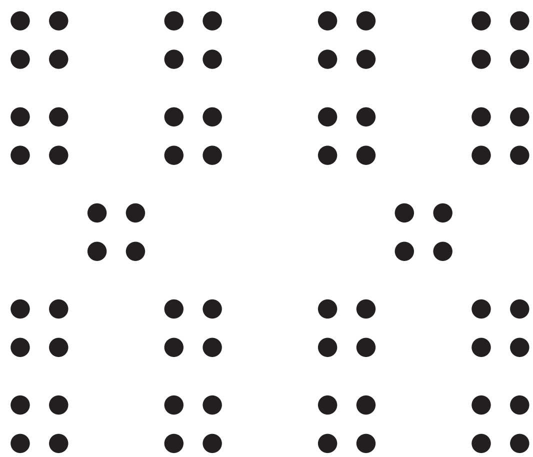
Flour Planner

Let's think about fractions by using drawings and diagrams to ask, "How many groups?"

Warm-Up

1 a How many dots are in this image?

72 dots



b Explain or show how you saw them.

Explanations vary.

- There are 8 groups of 8, plus the remaining dots in the middle make 9 groups of 8.
- There are 4 groups of 18.
- There are 2 groups of 36, so 72 dots in total.

Activity 1

Name: .... Date: .... Period: ....

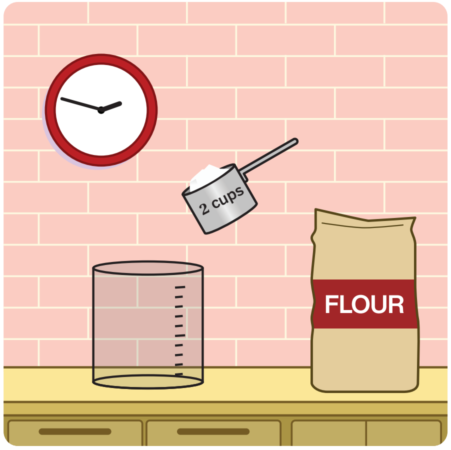
Fractional Scoops

2 Tres leches cake is a popular dessert in Mexico and Central America that's made with three kinds of milk.

Alexis needs 6 cups of flour to make tres leches cake but only has a 2-cup measuring scoop.

How many scoops does Alexis need?

3 scoops



3 Circle an equation that you could use to determine how many 2-cup scoops make 6 cups of flour.

$6 \cdot ? = 2$ $\quad$ $6 \div 2 = ?$ $\quad$ $2 \div 6 = ?$ $\quad$ $2 \cdot ? = 6$

Explain your thinking.

Responses and explanations vary.

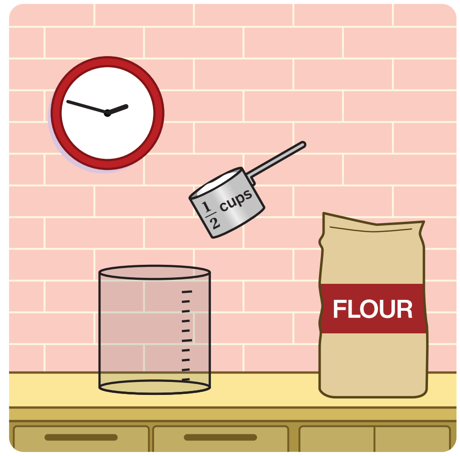
- **$6 \div 2 = ?$ because I want to know how many 2s go into 6.**
- **$2 \cdot ? = 6$ because I want to know how many groups of 2 make 6.**

Activity 1

Name: .... Date: .... Period: ....

Fractional Scoops (continued)

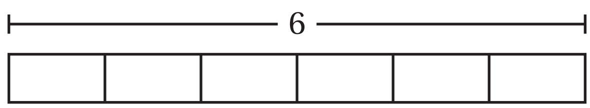
4 LaShawn also needs 6 cups of flour to make tres leches cake but only has a $\frac{1}{2}$-cup measuring scoop.



How many scoops does LaShawn need?

12 scoops

5 **a** Let's watch how LaShawn determined the number of $\frac{1}{2}$-cup scoops needed for 6 cups of flour.



b Explain how this tape diagram helped LaShawn decide to use 12 scoops.

***Responses vary*. LaShawn made a tape diagram representing 6 and then figured out how many $\frac{1}{2}$s fit in 6.**

Activity 2

Name: Date: Period:

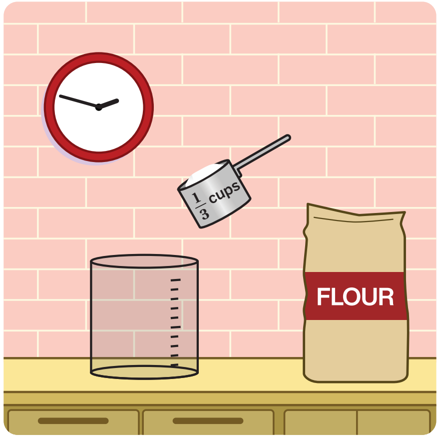
A Bigger Scoop

6 Sirnee is a sweet dish that is often made for Islamic celebratory feasts.

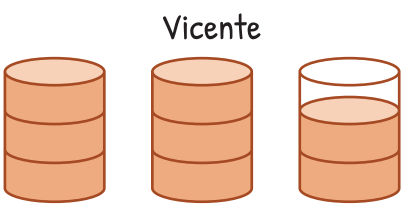
Hamza needs $2\frac{2}{3}$ cups of flour to make sirnee but only has a $\frac{1}{3}$-cup measuring scoop.

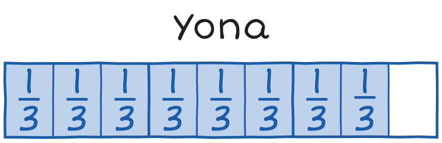
How many scoops does Hamza need?

8 scoops



7 Vicente and Yona each sketched a diagram to determine how many $\frac{1}{3}$-cup scoops they need to measure $2\frac{2}{3}$ cups of flour.





Discuss: How could each diagram help us calculate the number of scoops needed?

Responses vary.

- **Three $\frac{1}{3}$-cup scoops make up one full cup, and there are 2 cups so that makes 6 scoops. Then two more $\frac{1}{3}$-cup scoops make $\frac{2}{3}$ of a cup. So they need 8 scoops in total.**
- **There are 8 sections in the tape diagram, so they need 8 scoops.**

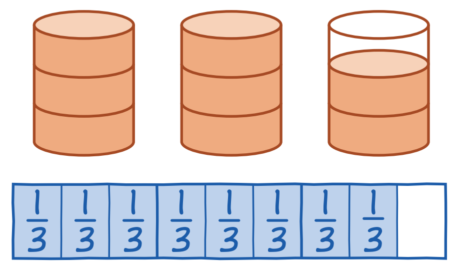
Name: ________ Date: ________ Period: ________

A Bigger Scoop (continued)

8 Hamza found a $\frac{2}{3}$-cup measuring scoop to use to make sirnee.

How many of these scoops would Hamza need to measure $2\frac{2}{3}$ cups of flour?

Use the cups diagram and the tape diagram if they help with your thinking.



4 scoops

Explain your thinking.

Explanations vary.

- **I circled groups of $\frac{2}{3}$ in the tape diagram, and there were 4 groups, so he'd need 4 scoops.**
- **The $\frac{2}{3}$-cup is twice as big as the $\frac{1}{3}$-cup, so he'd need half as many scoops.**

9 Group together the choices that represent the same situation. Two choices will have no match.

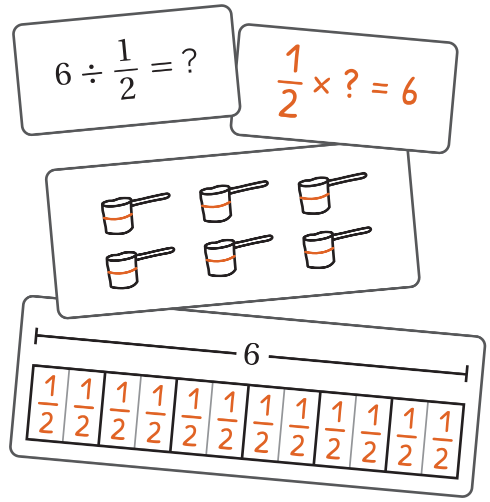
$3 \div \frac{3}{4} = ?$	$\frac{3}{4} \div 3 = ?$	$\frac{3}{4} \cdot ? = 3$	$3 \cdot ? = \frac{3}{4}$
$? \div \frac{3}{4} = 3$	$\frac{3}{4} \cdot 3 = ?$	4 scoops	$\frac{1}{4}$ scoops

Alexis needs 3 cups of flour and has a $\frac{3}{4}$-cup measuring scoop.	LaShawn needs $\frac{3}{4}$ cups of flour and has a 3-cup measuring scoop.
$3 \div \frac{3}{4} = ?$	$\frac{3}{4} \div 3 = ?$
$\frac{3}{4} \cdot ? = 3$	$3 \cdot ? = \frac{3}{4}$
4 scoops	**$\frac{1}{4}$ scoops**

$? \div \frac{3}{4} = 3$ and $\frac{3}{4} \cdot 3 = ?$ do not have a match.

10 Synthesis

How can you use an equation or a diagram to determine how many $\frac{1}{2}$-cup scoops you need to make 6 cups?

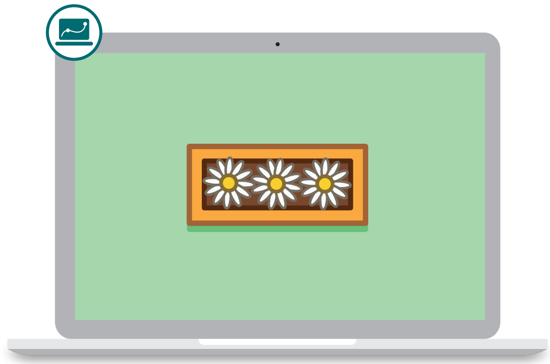


Responses vary.

- **I can make a tape diagram that's 6 units long and break it into $\frac{1}{2}$-unit pieces, then count the number of pieces I need.**
- **I can write a multiplication equation like $\frac{1}{2} \cdot ? = 6$ and figure out what the ? needs to be to make the equation true.**

Things to Remember:

Name: ______ Date: ______ Period: ______



Flower Planters

Let's use flower planters to answer the question "How many in *one* group?"

Warm-Up

1 Order these expressions from *least* to *greatest* by the value of the quotient.

$12 \div 12$	$12 \div \frac{2}{3}$	$12 \div 1$
$12 \div 3$	$12 \div \frac{1}{4}$	

Least

$12 \div 12$

$12 \div 3$

$12 \div 1$

$12 \div \frac{2}{3}$

$12 \div \frac{1}{4}$

Greatest

Activity 1

Name: .... Date: .... Period: ....

Plenty of Planters

2 Write a story that could be represented by the expression $12 \div \frac{1}{3}$.

Draw a sketch if it helps you illustrate your story.

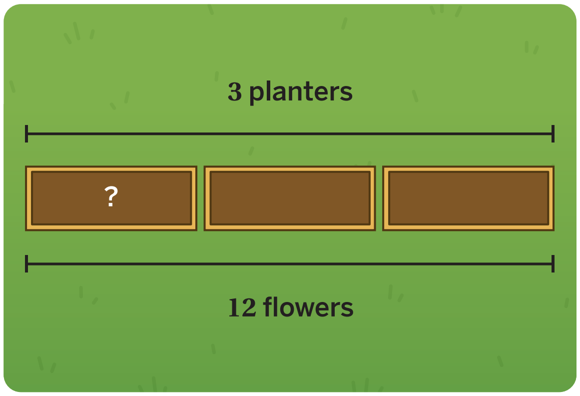
***Responses vary.* Every day, I eat $\frac{1}{3}$ of a chocolate bar. How long will it take me to eat 12 chocolate bars?**

3 Brianna is planting flowers in the school garden.

12 flowers fill 3 small planters.

How many flowers fill 1 small planter?

4 flowers

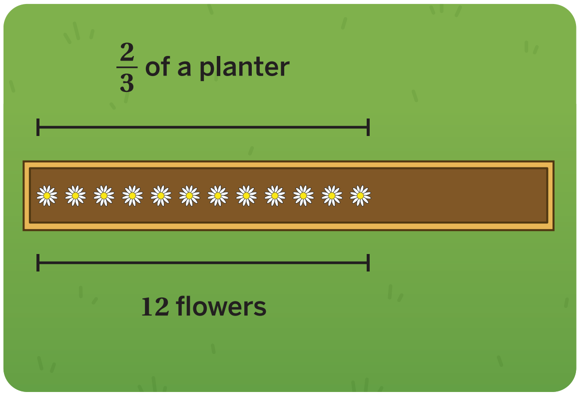


4 Brianna also put flowers in a big planter.

12 flowers fill $\frac{2}{3}$ of a big planter.

How many flowers fill 1 big planter?

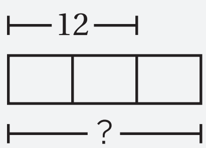
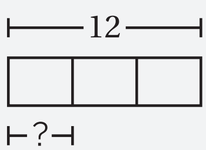
18 flowers



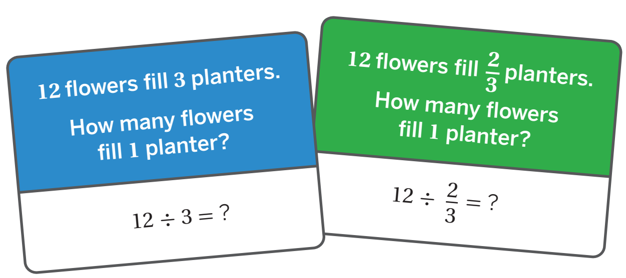
Name: .... Date: .... Period: ....

Plenty of Planters (continued)

5 Match each representation with a question.

	12 flowers fill 3 planters. How many flowers fill 1 planter?	12 flowers fill $\frac{2}{3}$ of a planter. How many flowers fill 1 planter?
$12 \div 3 = ?$	✓	
$12 \div \frac{2}{3} = ?$		✓
$\frac{2}{3} \cdot ? = 12$		✓
$3 \cdot ? = 12$	✓	
		✓
	✓	

6 How are these expressions alike? How are they different?



Responses vary.

Alike:

- **Both start with 12 flowers.**
- **They're both 12 divided by something.**
- **They have the same question: How many flowers fill 1 planter?**

Different:

- **In the one on the right, the number of planters is a fraction.**
- **In the one on the left, the answer is less than 12. In the one on the right, the answer is more than 12.**

Name: ________ Date: ________ Period: ________

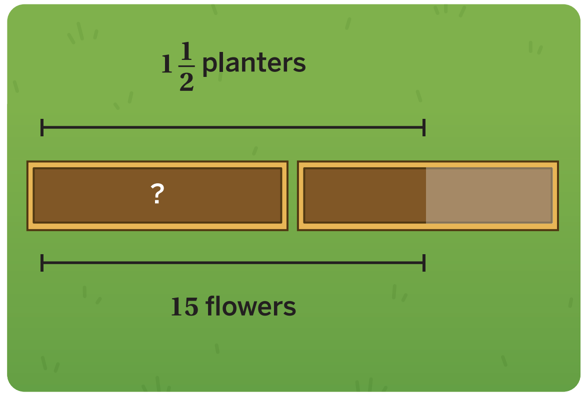
Practicing With Planters

7 Brianna has 15 flowers to put in these planters.

The flowers fill $1\frac{1}{2}$ planters.

How many flowers fill 1 planter?

10 flowers

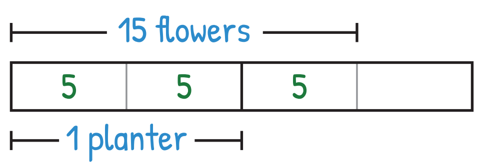


8 Here is a diagram Brianna made to calculate how many flowers fill 1 planter when 15 flowers fill $1\frac{1}{2}$ planters.

Explain how Brianna can use this diagram to help her answer the question.

***Responses vary.* Brianna figured out that there were 5 flowers in each $\frac{1}{2}$ of a planter. Since there are two $\frac{1}{2}$s in 1 planter, that means that there are 10 flowers in 1 planter.**

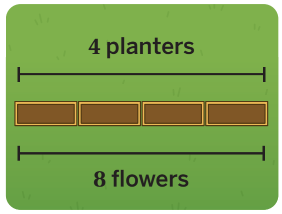
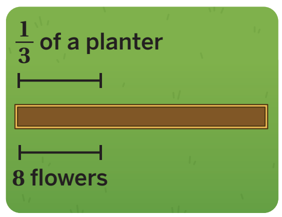
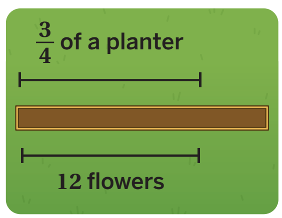
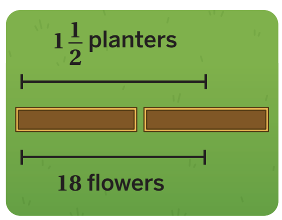
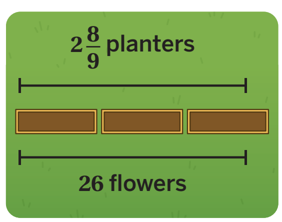
$$15 \div 1\frac{1}{2} = ?$$



Name: ________________ Date: ________ Period: ________

Practicing With Planters (continued)

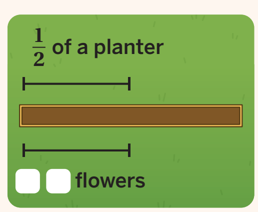
9 Solve as many challenges as you have time for.

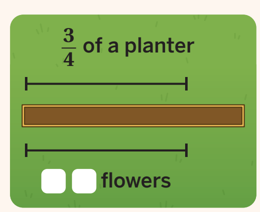
	Situation	Diagram	How many flowers fill 1 planter?
a	8 flowers fill 4 planters.		2
b	8 flowers fill $\frac{1}{3}$ of a planter.		24
c	12 flowers fill $\frac{3}{4}$ of a planter.		16
d	18 flowers fill $1\frac{1}{2}$ planters.		12
e	26 flowers fill $2\frac{8}{9}$ of a planter.		9

Explore More

10 Fill in each blank using the digits 0 to 9 only once, so that the same number of flowers fill each planter.

Responses vary.
- Left planter: 18 flowers
- Right planter: 27 flowers

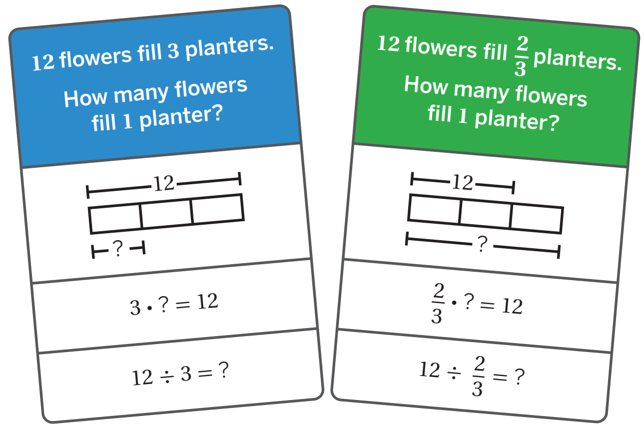




11 Synthesis

Describe how a tape diagram can represent a division problem.

Responses vary. **You can use a tape diagram to show the total divided into equal parts, to determine how many are in 1 group. You can also use the tape diagram to show how many are in a part of the whole, to determine the total.**



Things to Remember:

Connecting Tape Diagrams

Directions: Make one copy for the whole class. For classes with more than 36 students, create multiple copies. Then pre-cut the cards and give each student one card. For Round 3, both cards in each row must be distributed.

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Card 1

6

3	3

Card 2

6

2	2	2

Card 3

8

2	2	2	2

Card 4

8

4	4

Card 5

15

5	5	5

Card 6

15

3	3	3	3	3

Card 7

20

4	4	4	4	4

Card 8

20

5	5	5	5

Card 9

12

6	6

Card 10

12

2	2	2	2	2	2

Connecting Tape Diagrams

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Card 11

14

7	7

Card 12

14

2	2	2	2	2	2	2

Card 13

21

7	7	7

Card 14

21

3	3	3	3	3	3	3

Card 15

18

6	6	6

Card 16

18

3	3	3	3	3	3

Card 17

10

5	5

Card 18

10

2	2	2	2	2

Card 19

30

5	5	5	5	5	5

Card 20

30

6	6	6	6	6

Connecting Tape Diagrams

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Card 21

24

6	6	6	6

Card 22

24

4	4	4	4	4	4

Card 23

35

5	5	5	5	5	5	5

Card 24

35

7	7	7	7	7

Card 25

28

7	7	7	7

Card 26

28

4	4	4	4	4	4	4

Card 27

16

8	8

Card 28

16

2	2	2	2	2	2	2	2

Card 29

40

8	8	8	8	8

Card 30

40

5	5	5	5	5	5	5	5

Connecting Tape Diagrams

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Card 31

32

4	4	4	4	4	4	4	4

Card 32

32

8	8	8	8

Card 33

72

9	9	9	9	9	9	9	9

Card 34

72

8	8	8	8	8	8	8	8	8

Card 35

36

4	4	4	4	4	4	4	4	4

Card 36

36

9	9	9	9

Unit 4
Lesson
5

Name: ________ Date: ________ Period: ________

Garden Bricks

Let's use tape diagrams to think about, "How many groups?"

Warm-Up

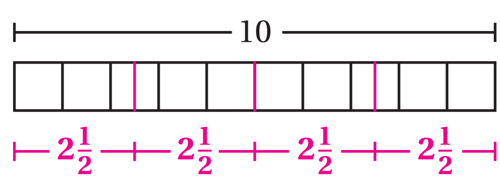
Question	Expression
How many groups of $2\frac{1}{2}$ are in 10?	$10 \div 2\frac{1}{2}$

1. **Discuss:** How do you know that the expression represents the question?

Responses vary. **I know $10 \div 2\frac{1}{2}$ represents the question because division lets us figure out how many groups of $2\frac{1}{2}$ are in 10.**

2. Use the tape diagram to answer the question.

4. *Drawings vary.* **Sample shown on tape diagram.**

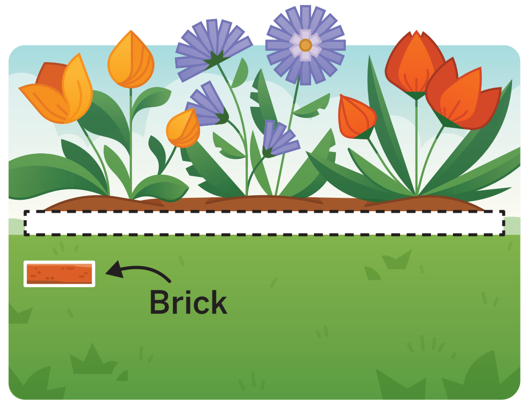


Activity 1

Name: ______ Date: ______ Period: ______

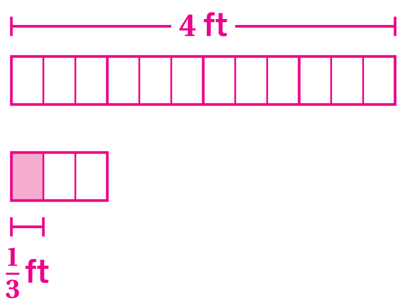
How Many Bricks?

Deja and CK are upgrading their class gardens by placing bricks along the front of each garden.



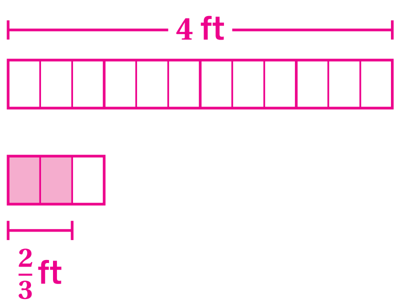
3. The first garden is 4 feet long. Deja is using small bricks, which are $\frac{1}{3}$ of a foot long. How many small bricks does Deja need? Draw a tape diagram to show your thinking.

12 small bricks



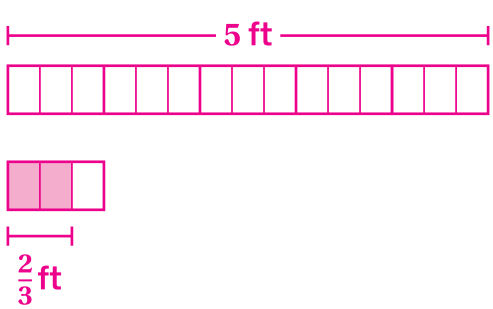
4. The second garden is also 4 feet long. CK is using large bricks, which are $\frac{2}{3}$ of a foot long. How many large bricks does CK need? Draw a tape diagram to show your thinking.

6 large bricks



5. The third garden is 5 feet long. How many large bricks do Deja and CK need? Draw a tape diagram to show your thinking.

$7\frac{1}{2}$ large bricks



Name: .... Date: .... Period: ....

How Many Bricks? (continued)

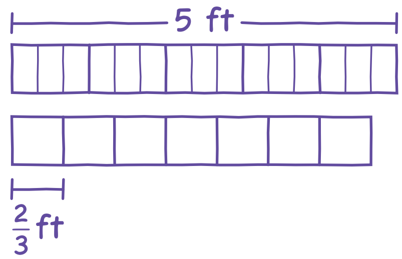
6. Deja and CK are working on Problem 5. Each student's work contains accurate and inaccurate parts.

Deja

$5 \div \frac{2}{3}$

"$5 \div \frac{2}{3}$ is less than 5 because I'm dividing."

CK



"I need $7\frac{1}{3}$ bricks because there are 7 whole bricks and $\frac{1}{3}$ left over."

a **Discuss:** How are their methods alike? How are they different?

Responses vary.

- Alike: Both have 5 as the dividend and $\frac{2}{3}$ as the divisor.
- Different: Deja used a division expression, and CK used a tape diagram.

b Pick *one* student's work. What do you think they did well? What question could you ask to help them understand their mistake?

Responses vary.

- Deja wrote the correct expression. To help her understand her mistake, I'd ask, "What does it mean to divide a whole number by a number less than one?"
- CK correctly drew 5 feet, split each foot into thirds, then counted by $\frac{2}{3}$. To help explain CK's mistake, I'd ask, "Does $\frac{1}{3}$ of a large brick close the gap?

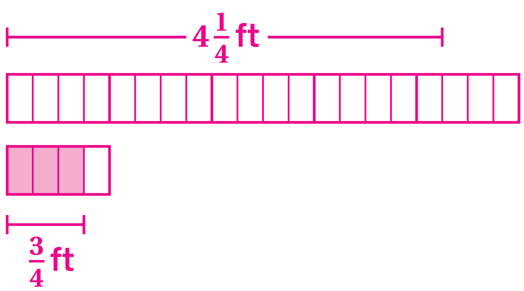
7. CK wrote $4\frac{1}{4} \div \frac{3}{4}$ to help answer a different question about bricks and gardens.

a Explain what $4\frac{1}{4}$ and $\frac{3}{4}$ mean in this situation.

Responses vary. $4\frac{1}{4}$ is the length of the garden in feet. $\frac{3}{4}$ is the length of each brick in feet.

b Draw a tape diagram and use it to determine the value of $4\frac{1}{4} \div \frac{3}{4}$.

$5\frac{2}{3}$

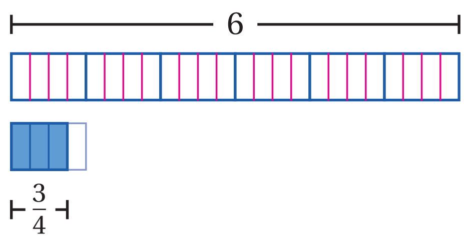
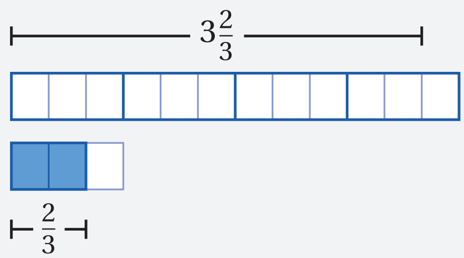
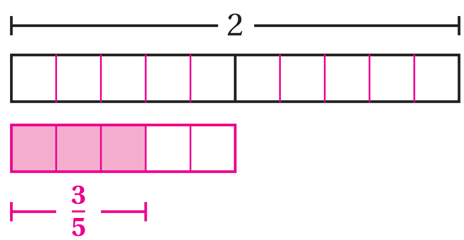
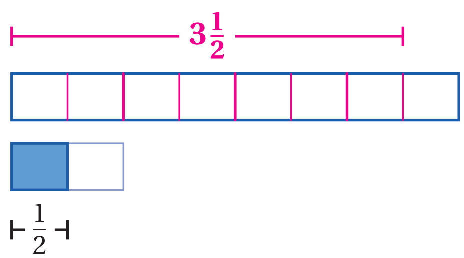


Activity 2

Name: ______ Date: ______ Period: ______

What's Missing?

8. Complete each row in the table.

	Expression	Tape Diagram	Quotient
a	$6 \div \frac{3}{4}$		**8**
b	$3\frac{2}{3} \div \frac{2}{3}$		$5\frac{1}{2}$
c	$2 \div \frac{3}{5}$		$3\frac{1}{3}$
d	$3\frac{1}{2} \div \frac{1}{2}$		7

Explore More

9. a. Write a division expression.

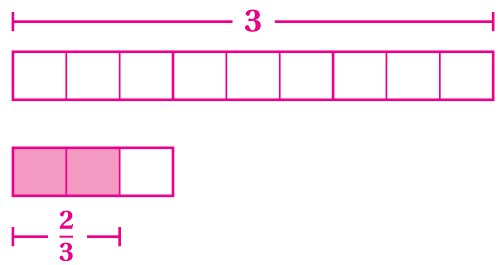
b. On a separate piece of paper, draw a tape diagram that represents your expression.

c. Trade tape diagrams with a partner. Determine their division expression and calculate its quotient.

Responses and tape diagrams vary.

Synthesis

10. **a** Draw a tape diagram to represent $3 \div \frac{2}{3}$.



b Describe how you can use the tape diagram to help determine the value of $3 \div \frac{2}{3}$.

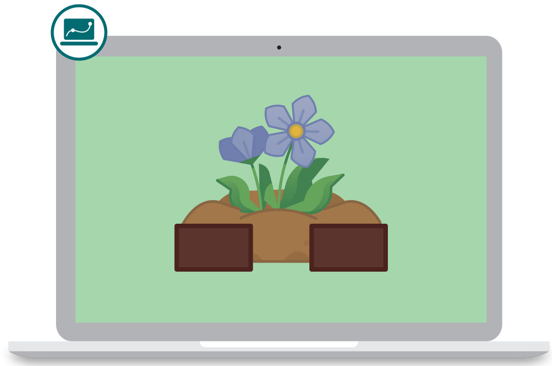
Responses vary.

- The tape diagram shows that $4\frac{1}{2}$ groups of $\frac{2}{3}$ makes 3.
- I can show that 3 is equivalent to $\frac{9}{3}$ by cutting every whole into 3 equal-sized groups. I know $\frac{9}{3} \div \frac{2}{3} = 4\frac{1}{2}$ because $\frac{2}{3}$ fits 4 whole times with $\frac{1}{3}$ left over, and $\frac{1}{3}$ is half of $\frac{2}{3}$.

Things to Remember:

Unit 4
Lesson
6

Name: ________ Date: ________ Period: ________

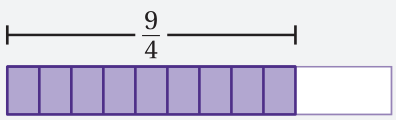
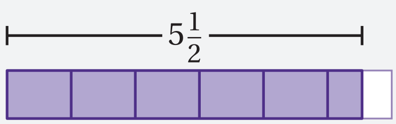
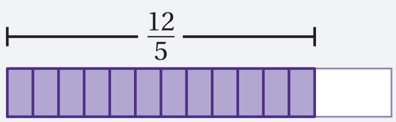
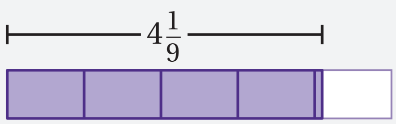


Fill the Gap

Let's use garden bricks to determine whether the number of groups is greater or less than 1.

Warm-Up

1-2 Complete the table.

Tape Diagram	Fraction	Mixed Number
	$\frac{9}{4}$	$2\frac{1}{4}$
	$\frac{11}{2}$	$5\frac{1}{2}$
	$\frac{12}{5}$	$2\frac{2}{5}$
	$\frac{37}{9}$	$4\frac{1}{9}$

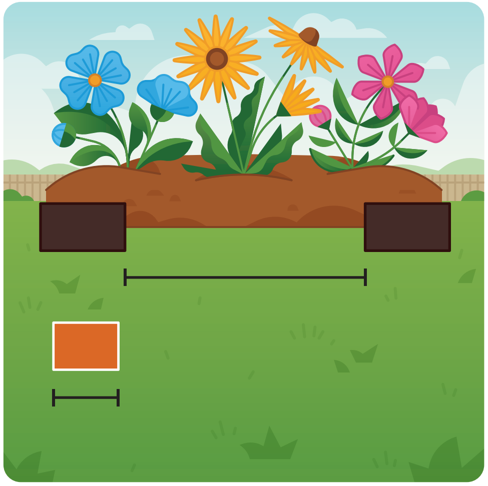
Name: ______ Date: ______ Period: ______

More or Less Than One Group

3 Deja is filling a gap along the front of this garden.

About how many bricks does Deja need?

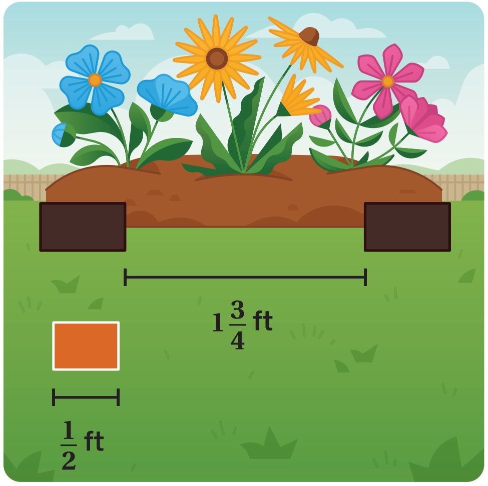
***Responses vary.* 3 bricks**



4 The gap in Deja's garden is $1\frac{3}{4}$ feet long. Each brick is $\frac{1}{2}$ of a foot long.

How many bricks does Deja need to fill the gap?

$3\frac{1}{2}$ bricks



5 Deja and CK each wrote an expression to represent the number of bricks needed to fill the gap.

Deja wrote $1\frac{3}{4} \div \frac{1}{2}$. CK wrote $\frac{1}{2} \div 1\frac{3}{4}$. Whose expression is correct? Circle one.

Deja's CK's Both Neither

Explain your thinking.

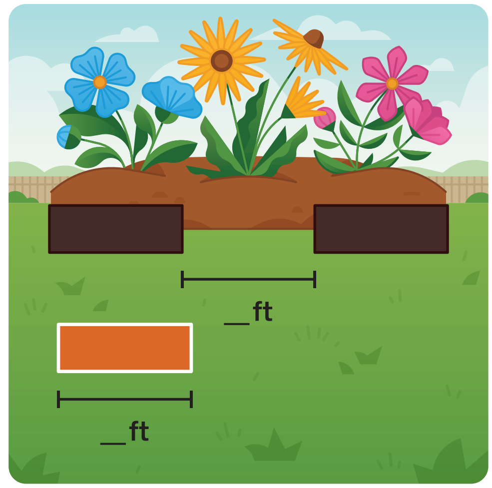
Explanations vary.

- **The first number in the expression is how long the gap is and the second is how big the brick is. For example, $6 \div 2$ would be like putting 2-foot bricks into a 6-foot gap.**
- **The gap is longer than the brick, so more than one brick fits into the gap. Deja's expression also shows that the answer is more than 1.**

Name: ______ Date: ______ Period: ______

More or Less Than One Group (continued)

6 Here is CK's expression: $\frac{1}{2} \div 1\frac{3}{4}$.



a Draw a sketch to represent this expression in the garden situation.

b The value of $\frac{1}{2} \div 1\frac{3}{4}$ is:

Less than 1   Greater than 1   Equal to 1

7 Sort these expressions by the value of their quotient.

$2\frac{1}{4} \div \frac{3}{4}$	$\frac{1}{4} \div \frac{3}{8}$	$\frac{3}{8} \div \frac{1}{4}$	$1 \div \frac{1}{4}$
$\frac{5}{4} \div 1\frac{1}{4}$	$\frac{3}{8} \div \frac{3}{8}$	$1 \div 4$	

Less than 1	Greater than 1	Equal to 1
$1 \div 4$	$1 \div \frac{1}{4}$	$\frac{5}{4} \div 1\frac{1}{4}$
$\frac{1}{4} \div \frac{3}{8}$	$\frac{3}{8} \div \frac{1}{4}$	$\frac{3}{8} \div \frac{3}{8}$
	$2\frac{1}{4} \div \frac{3}{4}$	

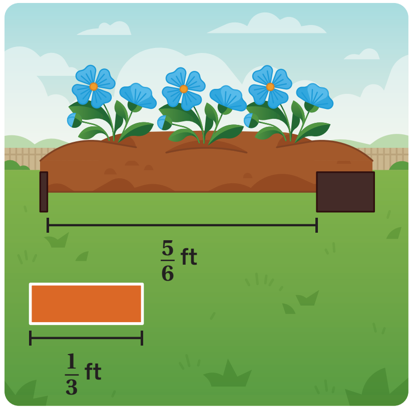
Name: .... Date: .... Period: ....

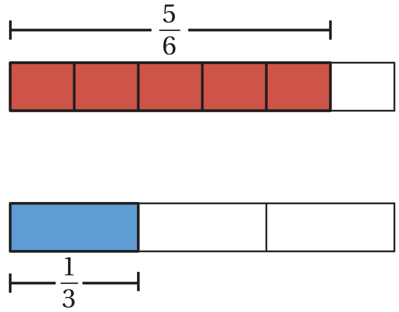
Equal-Sized Pieces

8 Here is a new expression: $\frac{5}{6} \div \frac{1}{3}$.

Use the garden or tape diagram to determine its value.

$\frac{5}{2}$ (or equivalent)

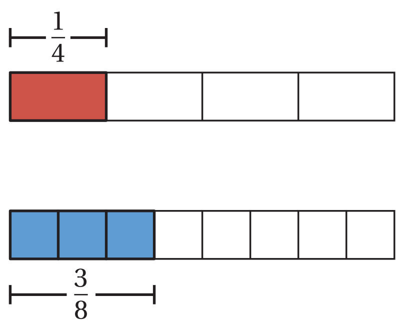




9 What is $\frac{1}{4} \div \frac{3}{8}$?

$\frac{2}{3}$ (or equivalent)

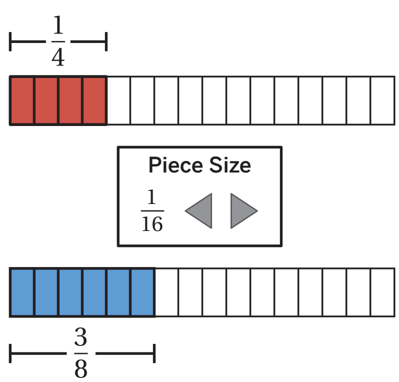
Note: Responses between 0.66 and 0.67 are considered correct.



10 Deja broke $\frac{1}{4}$ and $\frac{3}{8}$ into $\frac{1}{16}$-sized pieces.

a **Discuss:** How does Deja's strategy show that $\frac{1}{4} \div \frac{3}{8} = \frac{4}{6}$?

Responses vary. When the pieces are the same size, you can count how many of one fit in the other.



b Let's determine other helpful ways to break up $\frac{1}{4}$ and $\frac{3}{8}$.

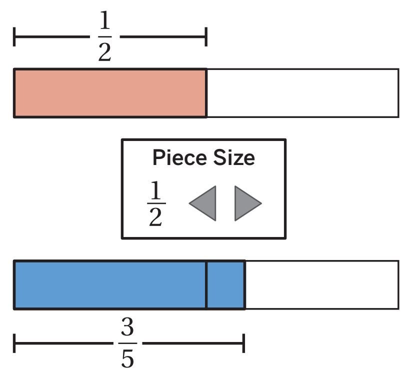
Responses vary. Other helpful ways to break up each fraction include $\frac{1}{8}$, $\frac{1}{24}$, $\frac{1}{32}$, etc.

Activity
3

Name: ________ Date: ________ Period: ________

The Return of Common Denominators

11 **a** Let's look at how we can break these fractions into equal pieces and set up a *common denominator*.



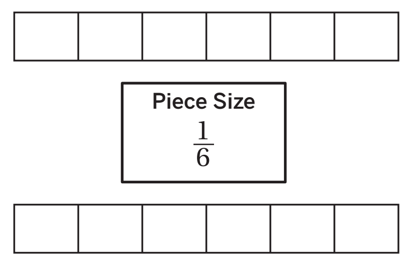
b Draw on the diagram to break the fractions into equal pieces. Then calculate $\frac{1}{2} \div \frac{3}{5}$.

$\frac{5}{6}$ (or equivalent)

Note: Responses between 0.83 and 0.84 are considered correct.

12 Calculate $\frac{2}{3} \div \frac{1}{2}$.

Use the diagram if it helps you with your thinking.

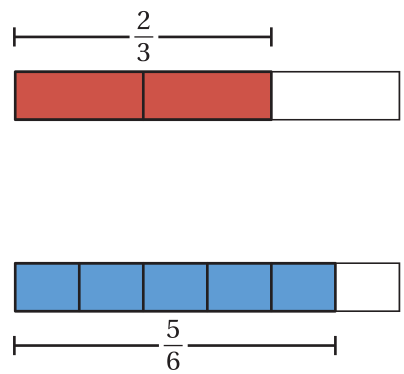


$\frac{4}{3}$ (or equivalent)

Note: Responses between 1.33 and 1.34 are considered correct.

13 Synthesis

Explain how you can show that $\frac{2}{3} \div \frac{5}{6} = \frac{4}{5}$. Use the tape diagrams if they help with your thinking.



Responses vary. **If you break both fractions into $\frac{1}{6}$-sized pieces, then you can see how many groups of $\frac{5}{6}$ you need. The top tape diagram is $\frac{4}{6}$ and the bottom is $\frac{5}{6}$, so $\frac{4}{5}$ of the bottom will fit in the top.**

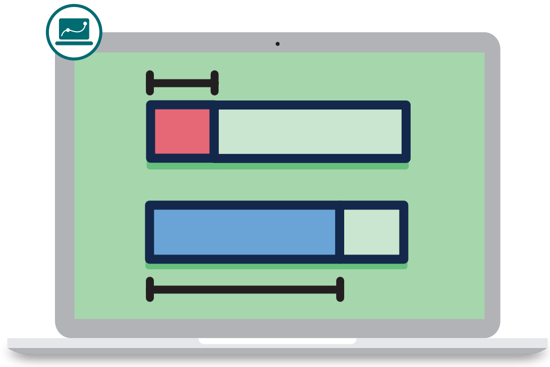
Things to Remember:

Unit 4

Lesson

7

Name: ______ Date: ______ Period: ______



Break It Down

Let's divide fractions by rewriting with common denominators.

Warm-Up

1 Calculate the following:

a $12 \div 3 = 4$

b $\frac{12}{5} \div \frac{3}{5} = 4$

How are these problems alike? How are they different?

Responses vary.

Alike:

- **They are both division problems.**
- **They both have a 12 and a 3.**
- **Both of the answers are 4.**

Different:

- **The first problem divides whole numbers and the second problem divides fractions.**

Activity 1

Name: .... Date: .... Period: ....

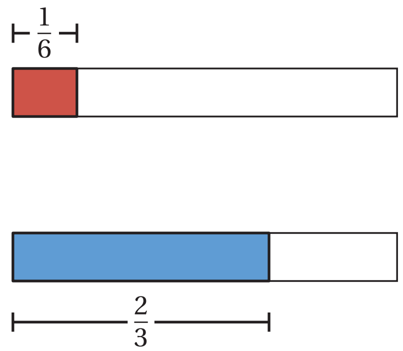
Common Denominators

2 The value of $\frac{1}{6} \div \frac{2}{3}$ is:

Less than 1   Greater than 1   Equal to 1

Explain your thinking.

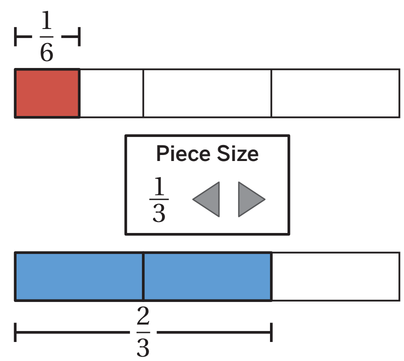
***Explanations vary.* You're dividing a smaller number by a larger number, so there will be less than 1 group of $\frac{2}{3}$s that can fit into $\frac{1}{6}$.**



3 **a** Let's look at how we can break both of these fractions into equal pieces and make common denominators.

b Draw on the diagram to break the fractions into equal pieces. Then calculate $\frac{1}{6} \div \frac{2}{3}$.

$\frac{1}{4}$ (or equivalent)

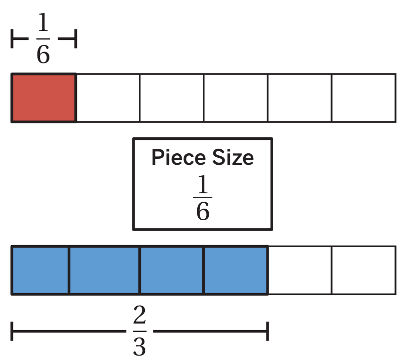


4 Here's how Ahmed calculated $\frac{1}{6} \div \frac{2}{3}$.

Discuss: Why do you think Ahmed used $\frac{1}{6}$-sized pieces?

Responses vary.

- **Using $\frac{1}{6}$-sized pieces is helpful because both fractions can be broken into $\frac{1}{6}$s.**
- **If you write both fractions with the same denominator, then you turn the problem into a whole number division problem like in the Warm-Up.**



$\frac{1}{6} \div \frac{2}{3}$

$\frac{1}{6} \div \frac{4}{6}$

$1 \div 4$

$\frac{1}{4}$

Activity 1

Name: ________ Date: ________ Period: ________

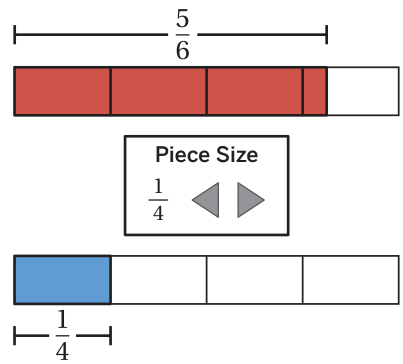
Common Denominators (continued)

5 **a** Let's look at how to make common denominators.

b Draw on the diagram to break the fractions into equal pieces. Then calculate $\frac{5}{6} \div \frac{1}{4}$.

$\frac{10}{3}$ (or equivalent)

Note: Responses between 3.33 and 3.34 are considered correct.



6 Ahmed and Zoe calculated the previous problem without a diagram. Their calculations are both correct.

Ahmed	Zoe
$\frac{5}{6} \div \frac{1}{4}$	$\frac{5}{6} \div \frac{1}{4}$
$\frac{10}{12} \div \frac{3}{12}$	$\frac{20}{24} \div \frac{6}{24}$
$10 \div 3$	$20 \div 6$
$\frac{10}{3}$	$\frac{20}{6}$

How are their strategies alike? How are they different?

Responses vary.

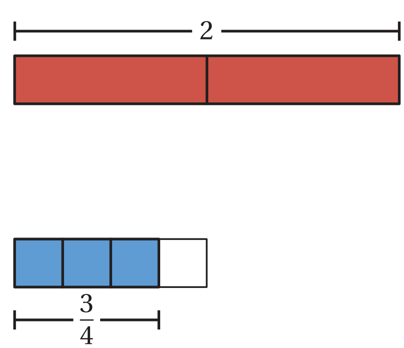
Alike:

- **Both strategies start by creating common denominators.**
- **Both strategies use only whole numbers in the third row.**
- **Both strategies involve dividing the numerators.**

Different:

- **Ahmed's strategy uses 12ths and Zoe's uses 24ths.**
- **Ahmed's numerators in the second row are 10 and 3. Zoe's are 20 and 6.**

7 Zoe says you can't use common denominators to calculate $2 \div \frac{3}{4}$ because 2 is a whole number.



What advice would you give Zoe?

***Responses vary.* If you write 2 as $\frac{2}{1}$ then you can use the common denominator strategy!**

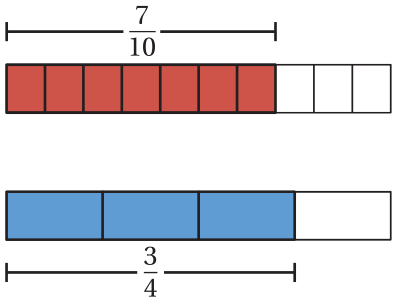
Name: ______ Date: ______ Period: ______

Dividing With Common Denominators

8 **a** Calculate $\frac{7}{10} \div \frac{3}{4}$.

$\frac{14}{15}$ (or equivalent)

Note: Responses between 0.93 and 0.94 are considered correct.



b **Discuss:** What was your strategy?

Strategies vary. I found the common denominator of 4 and 10, which is 20. Then I cut both tape diagrams into 20 total parts. The dividend became $\frac{14}{20}$ and the divisor became $\frac{15}{20}$. That means the dividend is $\frac{14}{15}$ parts of the divisor.

9 Solve as many challenges as you have time for.

a $\frac{4}{3} \div \frac{2}{3} = 2$ (or equivalent)

b $\frac{1}{6} \div \frac{5}{6} = \frac{1}{5}$ (or equivalent)

c $\frac{3}{8} \div \frac{1}{4} = \frac{3}{2}$ (or equivalent)

d $2 \div \frac{1}{3} = 6$

e $\frac{3}{10} \div \frac{2}{5} = \frac{3}{4}$ (or equivalent)

f $\frac{5}{6} \div \frac{3}{4} = \frac{10}{9}$ (or equivalent)

g $4 \div \frac{3}{4} = \frac{16}{3}$ (or equivalent)

h $\frac{11}{4} \div \frac{2}{3} = \frac{33}{8}$ (or equivalent)

i $2\frac{1}{2} \div \frac{2}{3} = \frac{15}{4}$ (or equivalent)

j $1\frac{4}{5} \div \frac{1}{2} = \frac{18}{5}$ (or equivalent)

10 Synthesis

Describe how finding a common denominator can help you divide a fraction with another fraction.

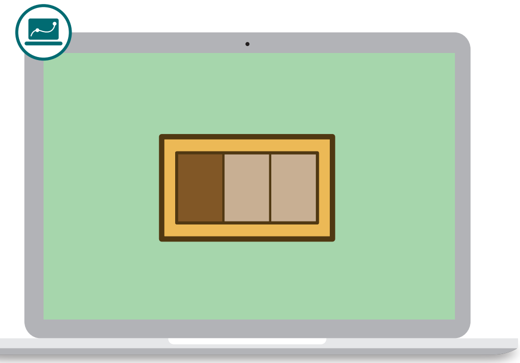
$\frac{4}{3} \div \frac{2}{3}$ $\frac{5}{2} \div \frac{4}{3}$

Use these examples if they help you explain your thinking.

***Responses vary.* If the denominators of the fractions are the same, then you can just divide the numerators to figure out the quotient. If the denominators are not the same, then you can rewrite the fractions with common denominators to make the problem easier.**

Things to Remember:

Name: ________________ Date: ________ Period: ________

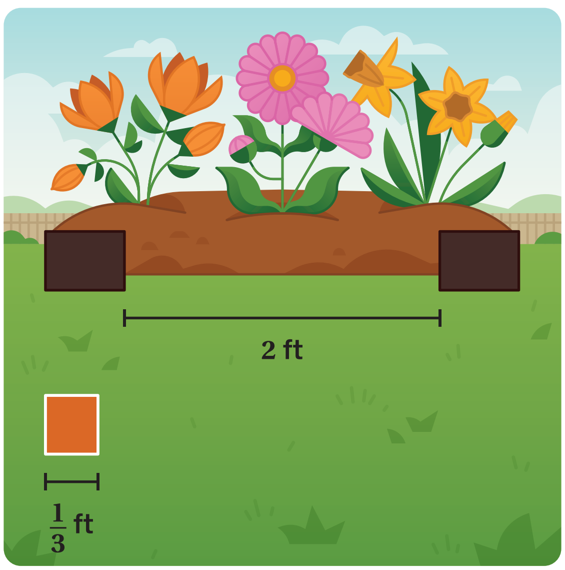


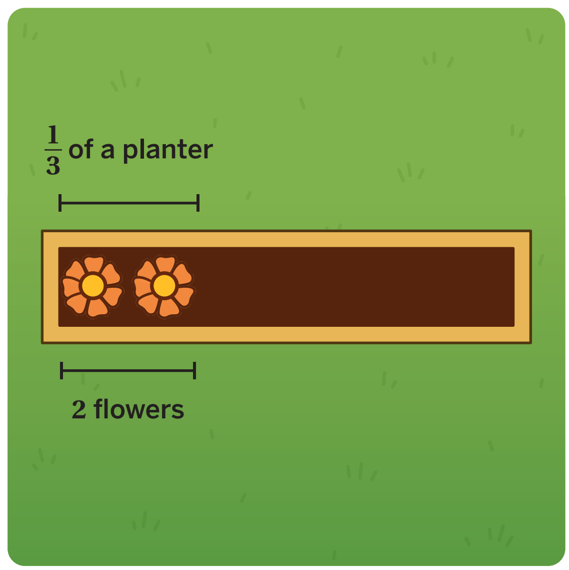
Potting Soil

Let's explore another strategy for dividing fractions.

Warm-Up

1 Habib says $2 \div \frac{1}{3}$ represents the brick situation. Inola says $2 \div \frac{1}{3}$ represents the flower situation.





Discuss: Why are they both correct?

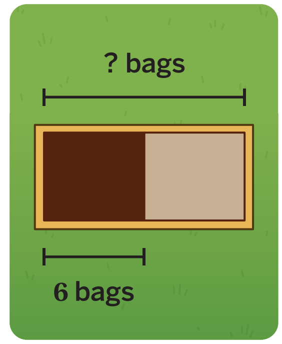
Responses vary. **They're both correct because the situations are just different ways of seeing division. The brick situation is asking how many groups of $\frac{1}{3}$ make 2. The flower situation is asking how many flowers are in 1 whole group, if 2 flowers are in $\frac{1}{3}$ of a group.**

Activity 1

Name: ______________ Date: ______ Period: ______

Digging Into Fraction Division

2 Habib and Inola are filling planters with potting soil so that their class can grow vegetables.

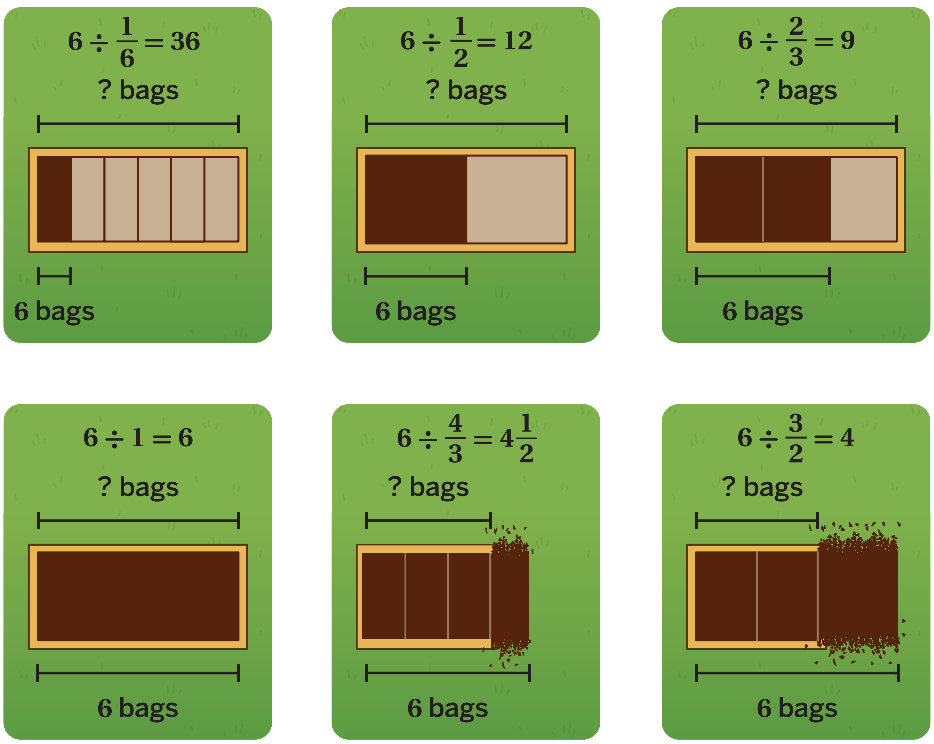


a Let's take a look at how many bags of soil fill $\frac{1}{2}$ of a planter.

b How many bags does it take to fill 1 planter?

12 bags

3 **a** Take a look at six different soil situations.



b What do you notice? What do you wonder? ***Responses vary.***

I notice:

- **There are always 6 bags of soil.**
- **As you fill the planter with more soil, the number after the division sign gets larger and the quotient gets smaller.**
- **If you add soil past the end of the box, the number after the division sign will be greater than 1.**

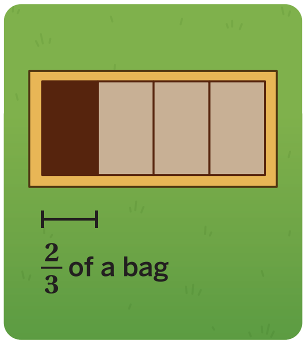
I wonder:

- **Where does the number after the division sign come from?**
- **How is the quotient calculated?**
- **Why is the planter box sometimes split into different numbers of sections?**
- **What does it mean when each bag of soil represents a smaller section of the planter?**

Name: ____ Date: ____ Period: ____

Digging Into Fraction Division (continued)

4 It takes $\frac{2}{3}$ of a bag of soil to fill $\frac{1}{4}$ of this planter.



How many bags does it take to fill 1 planter?

$\frac{8}{3}$ bags (or equivalent)

Note: Responses between 2.66 and 2.67 are considered correct.

5 Habib wrote $\frac{2}{3} \div \frac{1}{4} = \frac{8}{3}$ to solve the previous problem.

What does each fraction mean in this situation?

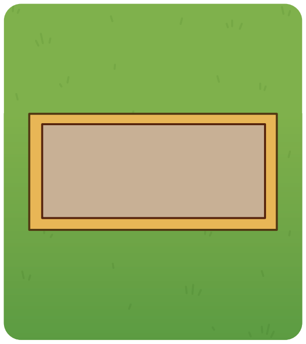
Responses vary.

$\frac{2}{3}$ means . . . **the amount of bags of soil that's in the planter right now.**

$\frac{1}{4}$ means . . . **the portion of the planter that the soil fills up.**

$\frac{8}{3}$ means . . . **the amount of soil you need to fill 1 whole planter.**

6 Inola wrote $5\frac{1}{3} \div \frac{1}{2}$ to solve a new problem.



Draw or describe a situation about planters and potting soil that represents Inola's expression.

Responses vary. **$5\frac{1}{3}$ bags of soil fill up $\frac{1}{2}$ of a giant planter. How many bags does Inola need to fill 1 giant planter?**

Activity 2

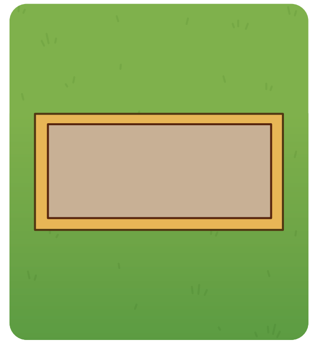
Name: ________ Date: ________ Period: ________

Different Operation, Same Value

7 **Discuss:** How could you think about the expression $\frac{9}{2} \div \frac{1}{3}$ in terms of a planter?

Draw a diagram if it helps you with your thinking.

***Responses vary.* It takes $\frac{9}{2}$ bags of soil to fill $\frac{1}{3}$ of a planter.**



8 What is $\frac{9}{2} \div \frac{1}{3}$?

$\frac{27}{2}$ (or equivalent)

9 Habib says that $\frac{9}{2} \div \frac{1}{3}$ has the same value as $\frac{9}{2} \cdot 3$.

Habib

$\frac{9}{2} \div \frac{1}{3}$

$\frac{9}{2} \cdot 3$

$\frac{27}{2}$

a **Discuss:** How would you show Habib's strategy using a tape diagram?

***Responses vary.* I would draw a tape diagram showing $\frac{9}{2}$ is $\frac{1}{3}$ of the whole. Since I would need three groups of $\frac{9}{2}$ to complete 1 whole, I could multiply $\frac{9}{2}$ by 3 to determine the total.**

b Use Habib's strategy to calculate $\frac{2}{3} \div \frac{1}{7}$.

$\frac{14}{3}$ (or equivalent)

Note: Responses between 4.66 and 4.67 are considered correct.

Activity 2

Name: ________ Date: ________ Period: ________

Different Operation, Same Value (continued)

10 **a** Calculate the value of each expression.

Expression	Value
$\frac{4}{3} \div \frac{1}{3}$	**4**
$\frac{4}{3} \div \frac{1}{6}$	**8**
$\frac{4}{3} \div \frac{1}{5}$	$\frac{20}{3}$ **(or equivalent)**
$1\frac{2}{3} \div \frac{1}{4}$	$\frac{20}{3}$ **(or equivalent)**

Note: In Rows 3–4, responses between 6.66 and 6.67 are considered correct.

b Discuss your answers and strategies with a classmate.

11 Synthesis

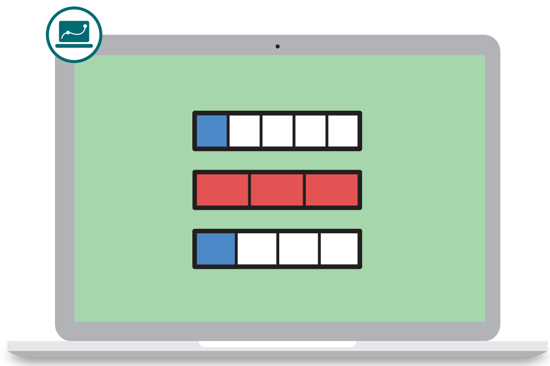
Describe a strategy for dividing a number by a unit fraction, such as $2\frac{1}{3} \div \frac{1}{5}$.

***Responses vary*. To divide by a unit fraction, you can multiply by the unit fraction's denominator. For example, $2\frac{1}{3} \div \frac{1}{5}$ has the same value as $2\frac{1}{3} \cdot 5$ because there are 5 groups of $2\frac{1}{3}$ that make up 1 whole.**

Things to Remember:

Unit 4
Lesson
9

Name: ____________ Date: ________ Period: ________

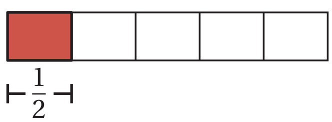
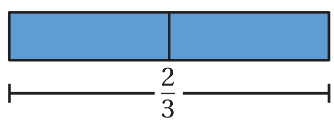
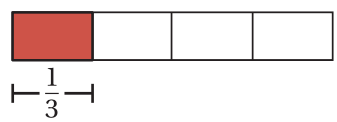
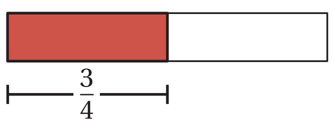
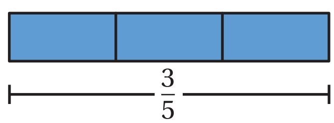
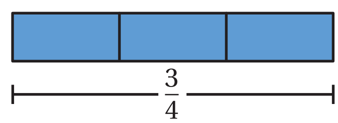
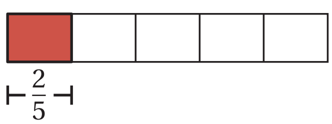
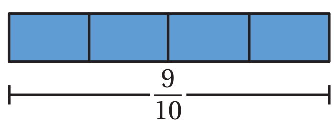


Division Challenges

Let's compare strategies for dividing fractions with and without tape diagrams.

Warm-Up

1 Solve as many challenges as you have time for. Try some problems of each type.

Multiplying	Dividing	Surprise Me!
 What is the value of $5 \cdot \frac{1}{2}$? **$\frac{5}{2}$ (or equivalent)**	 What is the value of $\frac{2}{3} \div 2$? **$\frac{1}{3}$ (or equivalent)**	 What is the value of $4 \cdot \frac{1}{3}$? **$\frac{4}{3}$ (or equivalent)**
 What is the value of $2 \cdot \frac{3}{4}$? **$\frac{3}{2}$ (or equivalent)**	 What is the value of $\frac{3}{5} \div 3$? **$\frac{1}{5}$ (or equivalent)**	 What is the value of $\frac{3}{4} \div 3$? **$\frac{1}{4}$ (or equivalent)**
 What is the value of $5 \cdot \frac{2}{5}$? **2 (or equivalent)**	 What is the value of $\frac{9}{10} \div 4$? **$\frac{9}{40}$ (or equivalent)**	 What is the value of $3 \cdot \frac{2}{5}$? **$\frac{6}{5}$ (or equivalent)**

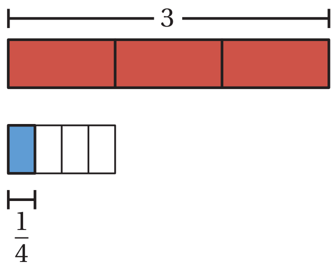
Name: .... Date: .... Period: ....

Two Strategies With Tape Diagrams

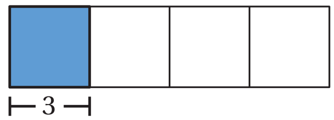
2 Jalen and Katie drew diagrams to calculate $3 \div \frac{1}{4}$.

a Take a look at each student's diagram.

Jalen



Katie



b Calculate $3 \div \frac{1}{4}$.

12

3 Here is a new expression: $\frac{4}{3} \div \frac{1}{5}$.

Jalen says the quotient is $\frac{20}{3}$. Katie says the quotient is $\frac{4}{15}$.

Diagram 1

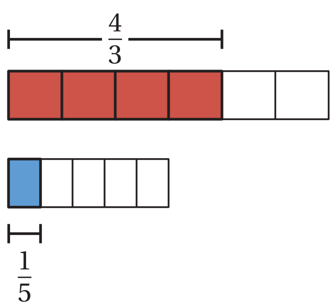
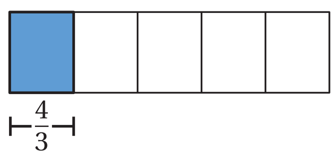


Diagram 2



a Whose quotient is correct? Circle one.

Jalen's   Katie's   Both   Neither

b Use one of the diagrams to help explain your thinking.

Explanations vary.

- **Looking at Diagram 1, I can tell the quotient has to be more than 1 because there is more than one $\frac{1}{5}$ in $\frac{4}{3}$. So Katie can't be right.**
- **If you look at Diagram 2, you can see that dividing by $\frac{1}{5}$ is like multiplying by 5, and $\frac{4}{3} \cdot 5 = \frac{20}{3}$.**

Activity 2

Name: ______ Date: ______ Period: ______

Two Strategies Revisited

4 Here are four expressions.

a Order these expressions by value from *least* to *greatest*.

$\frac{4}{3} \div \frac{2}{5}$	$\frac{4}{3} \div 1$	$\frac{4}{3} \div \frac{1}{5}$	$\frac{4}{3} \div 2$

$\frac{4}{3} \div 2$	$\frac{4}{3} \div 1$	$\frac{4}{3} \div \frac{2}{5}$	$\frac{4}{3} \div \frac{1}{5}$

Least **Greatest**

b **Discuss:** How are these expressions alike? How are they different?

Responses vary.

- **Alike: Each of the expressions has a dividend of $\frac{4}{3}$.**
- **Different: Some of the fractions are divided by whole numbers, while others are divided by fractions.**

5 Here is an expression from the previous problem:

$\frac{4}{3} \div \frac{2}{5}$

Calculate its value.

$\frac{10}{3}$ (or equivalent)

Note: Responses between 3.33 and 3.34 are considered correct.

Name: ________ Date: ________ Period: ________

Two Strategies Revisited (continued)

6 Here is how Jalen and Katie calculated $\frac{4}{3} \div \frac{2}{5}$.

a Take a look at each of their strategies.

Jalen

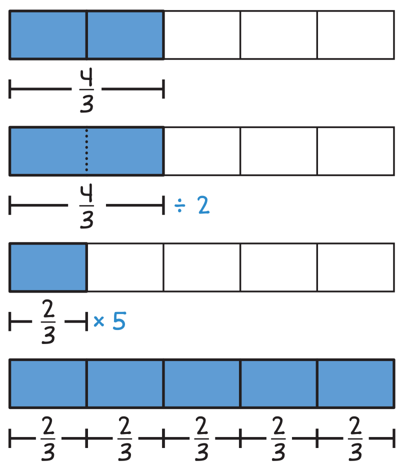
$\frac{4}{3} \div \frac{2}{5}$

$\frac{20}{15} \div \frac{6}{15}$

$20 \div 6$

$\frac{10}{3}$

Katie



$\frac{4}{3} \div \frac{2}{5}$

$\frac{4}{3} \div \frac{2}{5}$

$\frac{2}{3} \div \frac{1}{5}$

$\frac{10}{3}$

b Which strategy would you use to calculate $\frac{9}{10} \div \frac{3}{4}$? Circle one.

Jalen's   Katie's   My own

c If you chose Jalen's or Katie's strategy, what would your first step be for the strategy you chose? Otherwise, describe your own strategy.

Responses and explanations vary.

- **Jalen's. My first step would be to rewrite $\frac{9}{10}$ and $\frac{3}{4}$ with common denominators: $\frac{18}{20}$ and $\frac{15}{20}$.**
- **Katie's. My first step would be to divide by 3 to get $\frac{3}{10} \div \frac{1}{4}$.**

Name: .... Date: .... Period: ....

Fraction Fluency

7 Here is an expression from the previous problem:

$$\frac{9}{10} \div \frac{3}{4}$$

Calculate its value.

$\frac{6}{5}$ (or equivalent)

8 Here is a new expression: $\frac{6}{5} \div \frac{2}{3}$.

The three answers below are *not* correct.

$\frac{12}{15}$ $\frac{6}{5}$ 3

Circle your favorite (wrong) answer and explain why it cannot be correct.

Responses and explanations vary.

- **$\frac{12}{15}$ can't be correct because it's less than 1, and there's more than 1 group of $\frac{2}{3}$ in $\frac{6}{5}$.**
- **$\frac{6}{5}$ can't be correct because $\frac{6}{5} \div 1 = \frac{6}{5}$, so $\frac{6}{5} \div \frac{2}{3}$ must be greater than $\frac{6}{5}$.**
- **3 can't be correct because $\frac{2}{3} \cdot 3 = \frac{6}{3}$, not $\frac{6}{5}$.**

Name: ________ Date: ________ Period: ________

Fraction Fluency (continued)

9 Solve as many challenges as you have time for. Calculate each expression.

a. $5 \div \frac{2}{3} = \frac{15}{2}$ (or equivalent)

b. $\frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$ (or equivalent)

c. $\frac{6}{5} \div \frac{2}{3} = \frac{9}{5}$ (or equivalent)

d. $\frac{1}{2} \div \frac{5}{3} = \frac{3}{10}$ (or equivalent)

e. $\frac{3}{4} \div \frac{3}{5} = \frac{5}{4}$ (or equivalent)

f. $\frac{9}{4} \div \frac{7}{10} = \frac{45}{14}$ (or equivalent)

g. $\frac{5}{9} \div \frac{5}{3} = \frac{1}{3}$ (or equivalent)

h. $\frac{1}{9} \div \frac{3}{5} = \frac{5}{27}$ (or equivalent)

i. $\frac{9}{7} \div \frac{1}{7} = 9$ (or equivalent)

j. $\frac{6}{7} \div \frac{4}{7} = \frac{3}{2}$ (or equivalent)

k. $2 \div \frac{8}{9} = \frac{9}{4}$ (or equivalent)

l. $\frac{6}{5} \div \frac{2}{5} = 3$ (or equivalent)

10 Synthesis

Describe a strategy for calculating the quotient of two fractions, such as $\frac{2}{5} \div \frac{3}{4}$.

Draw a diagram if it helps you with your thinking.

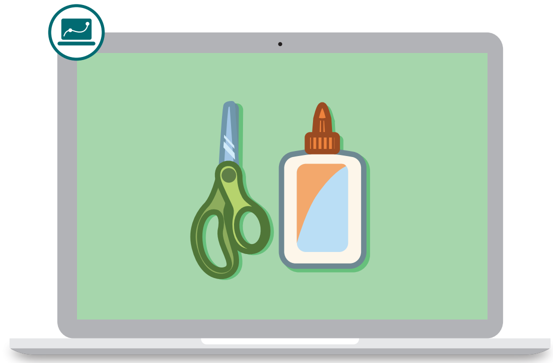
Responses vary.

- **First, create a common denominator. Then determine the quotient by dividing the numerator of the first fraction by the numerator of the second fraction. In this example, you can write $\frac{8}{20} \div \frac{15}{20} = \frac{8}{15}$.**
- **First, divide the first fraction by the numerator of the second fraction so you can get a unit fraction: $\frac{2}{15} \div \frac{1}{4}$. Then multiply by the denominator: $\frac{2}{15} \cdot 4 = \frac{8}{15}$.**

Things to Remember:

Unit 4
Lesson
12

Name: ________ Date: ________ Period: ________

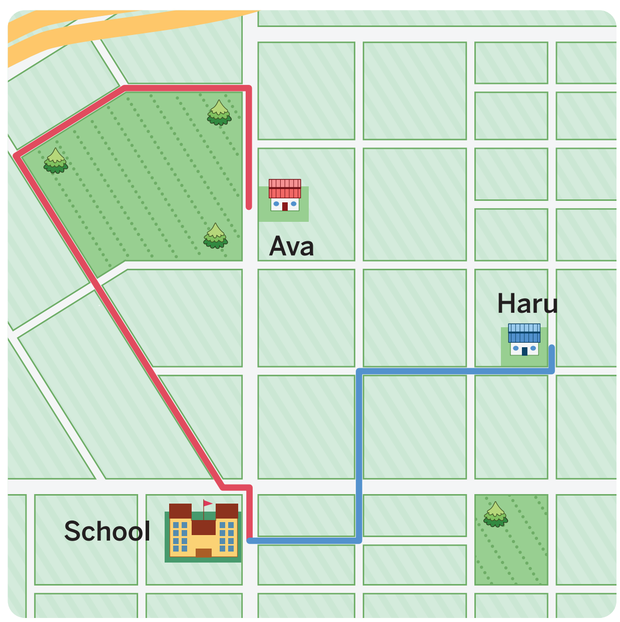


Classroom Comparisons

Let's compare the size of familiar objects by asking, "How many times as long?"

Warm-Up

1 Here is how Ava and Haru walked to school on Monday.



What do you notice? What do you wonder?

Responses vary.

I notice:

- **Ava's walk looks longer than Haru's.**
- **Both Ava and Haru avoided the big roads.**
- **Ava went around the park even though she probably didn't have to.**

I wonder:

- **How long does it take each of them to walk to school?**
- **Why didn't Ava take the most direct route?**
- **Why don't Ava and Haru meet up to walk to school together?**

Activity
1

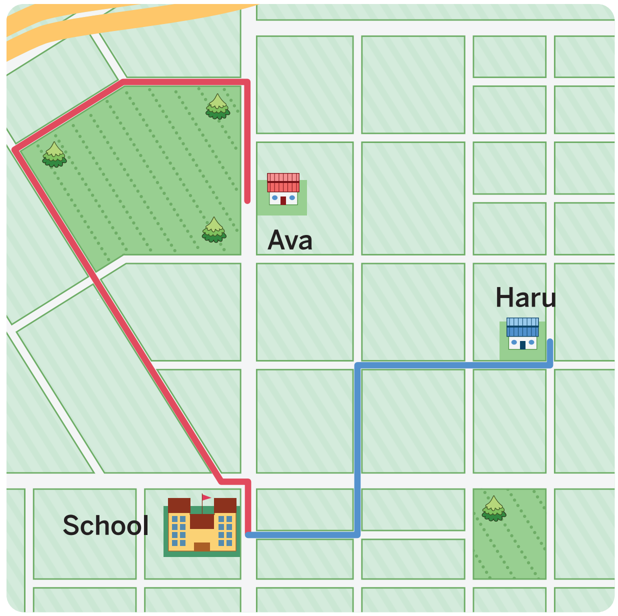
Name: ________ Date: ________ Period: ________

Comparing Distances

2 Ava walked farther than Haru.

About how many times as far do you think Ava walked?

Responses vary.



3 Ava walked $1\frac{1}{4}$ miles. Haru walked $\frac{3}{4}$ of a mile.

How many times as far did Ava walk?

$\frac{5}{3}$ times as far as Haru.

Note: Responses between 1.66 and 1.67 are considered correct.

4 Select *all* the expressions that represent how many times as far Ava walked compared to Haru.

- ☐ **A.** $\frac{5}{4} - \frac{3}{4}$
- ☐ **B.** $1\frac{1}{4} \cdot \frac{3}{4}$
- ☑ **C.** $1\frac{1}{4} \div \frac{3}{4}$
- ☐ **D.** $\frac{3}{4} \div 1\frac{1}{4}$
- ☑ **E.** $\frac{5}{4} \div \frac{3}{4}$

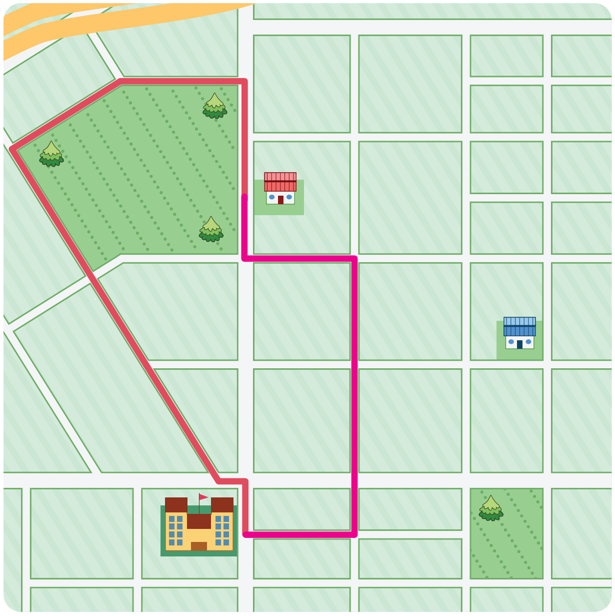
Name: .... Date: .... Period: ....

Comparing Distances (continued)

5 Ava decides to walk a different path to school on Tuesday.

Draw a path Ava could walk.

Paths vary.



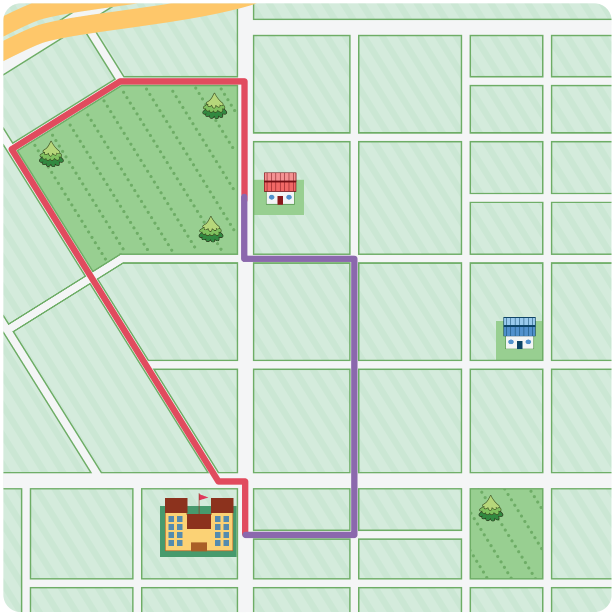
6 The map shows Ava's paths on Monday and Tuesday.

Monday: Ava walked $1\frac{1}{4}$ miles.

Tuesday: Ava walked $\frac{7}{8}$ of a mile.

How many times as far did Ava walk on Tuesday than on Monday?

$\frac{7}{10}$

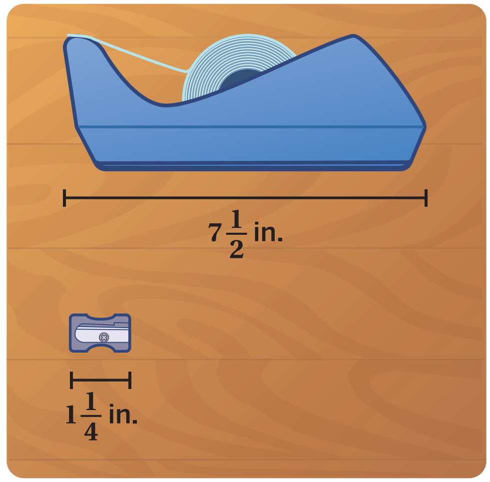


Activity 2

Name: ____________ Date: ________ Period: ________

Comparing Classroom Objects

7 Ava and Haru are comparing two objects in their classroom.



Ava says: *The tape dispenser is* 6 *times as long*.

Haru says: *The pencil sharpener is* $\frac{1}{6}$ *times as long*.

Whose thinking is correct? Circle one.

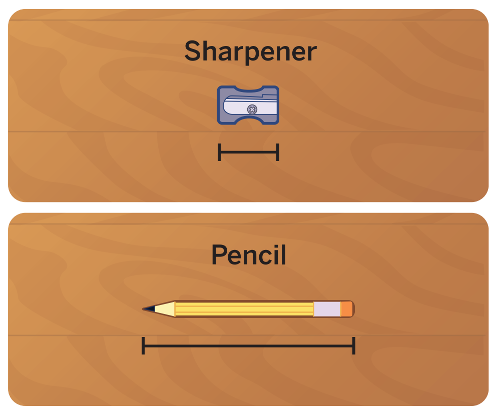
Ava's   Haru's   Both   Neither

Explain your thinking.

Explanations vary.

- **Ava is comparing the tape dispenser to the pencil sharpener, so it makes sense that the answer is more than 1, since the tape dispenser is longer than the pencil sharpener. Haru is comparing the pencil sharpener to the tape dispenser, so it makes sense that the answer is less than 1.**
- **$7\frac{1}{2} \div 1\frac{1}{4} = 6$, so Ava is right. $1\frac{1}{4} \div 7\frac{1}{2} = \frac{1}{6}$, so Haru is also right.**

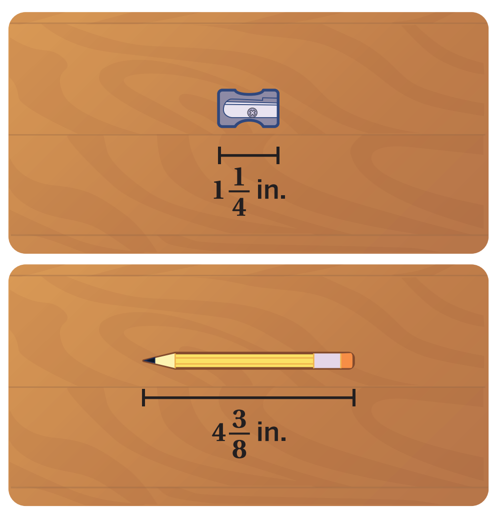
8 The sharpener is ________ times as long as the pencil.



Estimate a value.

Responses less than 1 are considered correct.

9 The sharpener is how many times as long as the pencil?



$\frac{10}{35}$ (or equivalent)

Note: Responses between 0.28 and 0.29 are considered correct.

Name: ____________ Date: ________ Period: ________

Comparing Classroom Objects (continued)

10 Here is a collection of classroom objects, along with their lengths (in inches).

Scissors	$6\frac{1}{4}$	**Eraser**	$1\frac{7}{8}$
Marker	$5\frac{5}{8}$	**Stapler**	5
Red pen	$6\frac{7}{8}$	**Pencil sharpener**	$1\frac{1}{4}$
Tape dispenser	$7\frac{1}{2}$	**Glue bottle**	$3\frac{1}{8}$
Highlighter	$3\frac{3}{4}$	**Calculator**	$2\frac{1}{2}$
Large pencil	$8\frac{1}{8}$	**Small pencil**	$4\frac{3}{8}$

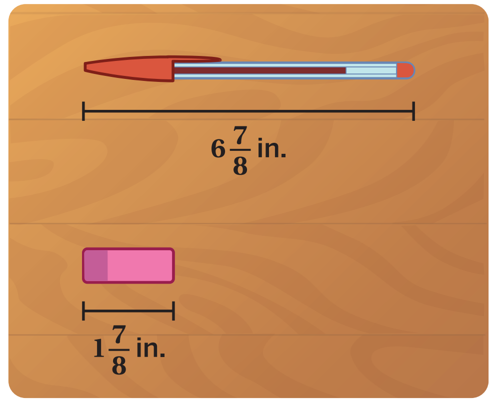
Choose pairs of objects to compare. Write an expression and a statement comparing their lengths. Solve as many challenges as you have time for!

Responses vary.

	Objects	Expression	Statement
a	Large pencil and Stapler	$8\frac{1}{8} \div 5$	The large pencil is $1\frac{5}{8}$ times as long as the stapler.
b	____ and ____		The ____ is ____ times as long as the ____.
c	____ and ____		The ____ is ____ times as long as the ____.
d	____ and ____		The ____ is ____ times as long as the ____.

11 Synthesis

Describe a strategy for solving problems like this:
The pen is how many times as long as the eraser?

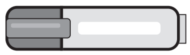
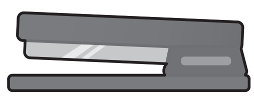
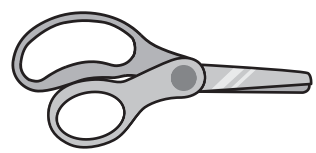
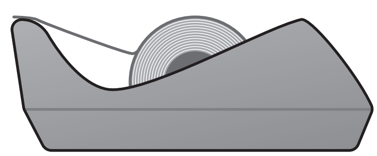


Responses vary. **Divide the length of the first object by the length of the second object. If the measurements are given as mixed numbers, it can help to convert them to fractions before dividing.**

Things to Remember:

Comparing Classroom Objects

Select two classroom items to compare using the table in Problem 10.

Highlighter  $3\frac{3}{4}$ in.	**Eraser**  $1\frac{7}{8}$ in.	**Small pencil**  $4\frac{3}{8}$ in.
Marker  $5\frac{5}{8}$ in.	**Pencil sharpener**  $1\frac{1}{4}$ in.	**Red pen**  $6\frac{7}{8}$ in.
Large pencil  $8\frac{1}{8}$ in.	**Stapler**  5 inches	**Scissors**  $6\frac{1}{4}$ in.
Glue bottle  $3\frac{1}{8}$ in.	**Calculator**  $2\frac{1}{2}$ in.	**Tape dispenser**  $7\frac{1}{2}$ in.

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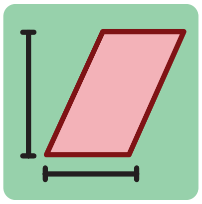
Unit 4
Lesson
13

Name: .... Date: .... Period: ....

Puzzling Areas

Let's explore the areas of rectangles and triangles with fractional side lengths.



Warm-Up

Evaluate each expression mentally.

1. $3 \cdot 4 = 12$

2. $\frac{1}{3} \cdot 4 = \frac{4}{3}$

3. $\frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$

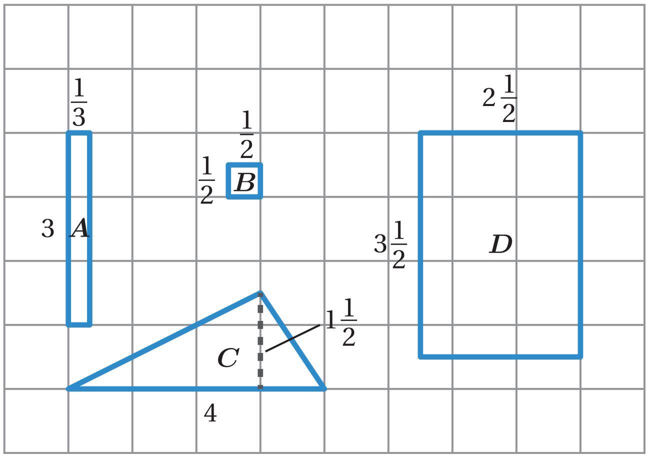
4. $2 \cdot \frac{1}{3} \cdot \frac{1}{4} = \frac{2}{12}$ or $\frac{1}{6}$

5. $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{24}$

Name: .... Date: .... Period: ....

Areas

6. Use any strategy to determine the area of as many figures as you can. Use the workspace below if it helps with your thinking.



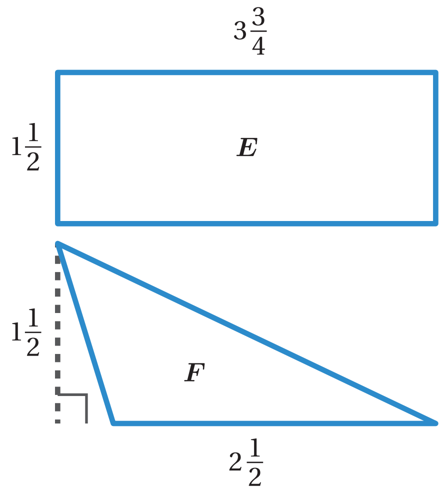


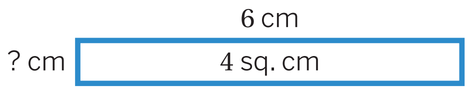
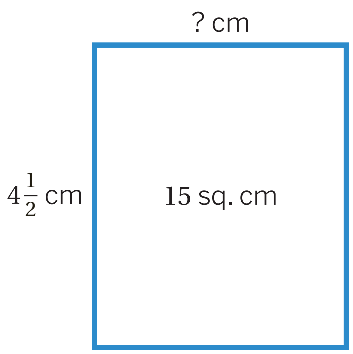
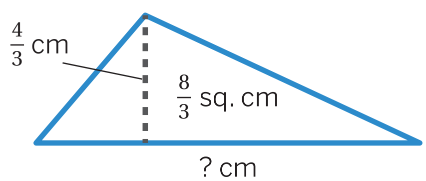
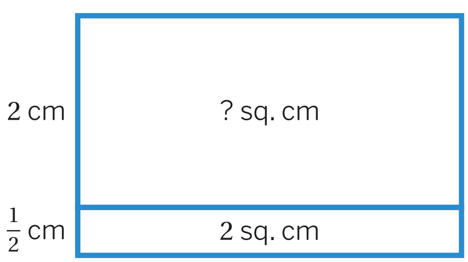
Figure	A	B	C	D	E	F
Area (sq. units)	1	$\frac{1}{4}$	3	$8\frac{3}{4}$	$5\frac{5}{8}$	$\frac{15}{8}$

Workspace:

Name: ______ Date: ______ Period: ______

Level Up Area Puzzles

7. Use any strategy to determine the unknown side length or area.

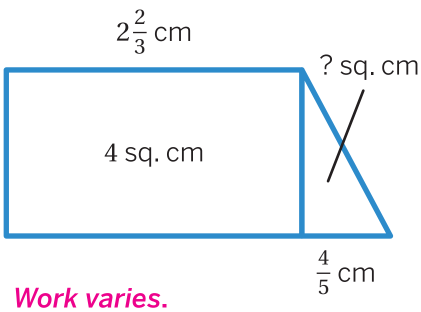
<table>
<tr><th></th><th>Puzzle</th><th>Workspace</th></tr>
<tr><td>a</td><td>
</td><td>Work varies.
$4 \div 6 = \frac{4}{6}$
$= \frac{2}{3}$
? = $\frac{2}{3}$ centimeters</td></tr>
<tr><td>b</td><td>
</td><td>Work varies.
$15 \div 4\frac{1}{2} = 15 \div \frac{9}{2}$
$= \frac{30}{2} \div \frac{9}{2}$
$= 30 \div 9$
$= \frac{30}{9}$ or $3\frac{1}{3}$
? = $3\frac{1}{3}$ centimeters</td></tr>
<tr><td>c</td><td>
</td><td>Work varies.
$\frac{8}{3} \div \frac{4}{3} = 8 \div 4$
$= 2$
$2 \div \frac{1}{2} = \frac{4}{2} \div \frac{1}{2}$
$= 4 \div 1$
$= 4$
? = 4 centimeters</td></tr>
<tr><td>d</td><td>
</td><td>Work varies.
Length of rectangle:
$2 \div \frac{1}{2} = 2 \cdot 2 = 4$
Area:
$4 \cdot 2 = 8$
? = 8 square centimeters</td></tr>
</table>

Name: .... Date: .... Period: ....

Level Up Area Puzzles (continued)

8. Solve as many puzzles as you have time for. You can work on them in any order.

Puzzle A



Work varies.

Height of rectangle and triangle:

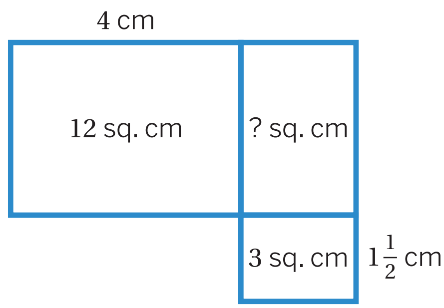
$$4 \div 2\frac{2}{3} = 4 \div \frac{8}{3}$$
$$= \frac{12}{3} \div \frac{8}{3}$$
$$= 12 \div 8 = \frac{12}{8} = \frac{3}{2}$$

Area of triangle:

$$\frac{3}{2} \cdot \frac{4}{5} = \frac{12}{10} = \frac{6}{5}$$
$$\frac{6}{5} \cdot \frac{1}{2} = \frac{6}{10} = \frac{3}{5}$$

? = $\frac{3}{5}$ square centimeters

Puzzle B



Work varies.

Height of unknown rectangle:

$12 \div 4 = 3$

Length of unknown rectangle:

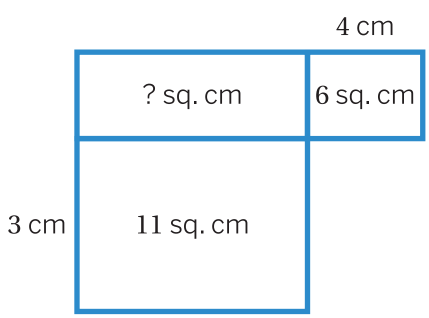
$$3 \div 1\frac{1}{2} = 3 \div \frac{3}{2}$$
$$= \frac{6}{2} \div \frac{3}{2} = 2$$

Unknown area:

$3 \cdot 2 = 6$

? = 6 square centimeters

Puzzle C



Work varies.

Length of unknown rectangle:

$11 \div 3 = \frac{11}{3}$

Height of unknown rectangle:

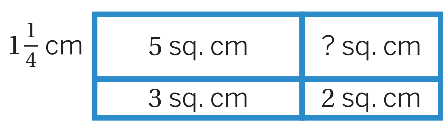
$6 \div 4 = \frac{6}{4} = \frac{3}{2}$

Unknown area:

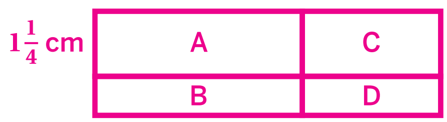
$\frac{11}{3} \cdot \frac{3}{2} = \frac{33}{6} = \frac{11}{2}$ or $5\frac{1}{2}$

? = $5\frac{1}{2}$ square centimeters

Puzzle D



Work varies.



Length of A:

$$5 \div 1\frac{1}{4} = 5 \div \frac{5}{4}$$
$$= 5 \cdot \frac{4}{5} = 4$$

Length of D:

$$2 \div \frac{3}{4} = 2 \cdot \frac{4}{3} = \frac{8}{3}$$

Height of B:

$$3 \div 4 = \frac{3}{4}$$

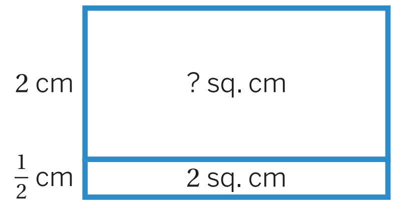
Area of C:

$$1\frac{1}{4} \cdot \frac{8}{3} = \frac{5}{4} \cdot \frac{8}{3}$$
$$= \frac{40}{12} = \frac{10}{3} \text{ or } 3\frac{1}{3}$$

? = $3\frac{1}{3}$ square centimeters

Synthesis

9. **a** When is multiplication helpful for solving problems about areas?



***Responses vary*. Multiplication is helpful when I know the base and height, and I want to calculate the area.**

b When is division helpful for solving problems about areas?

***Responses vary*. Division is helpful when I know the area, and I want to determine a side length.**

Things to Remember:

Name: .... Date: .... Period: ....

Decimal Diagrams and Algorithms

Let's add and subtract decimals.

Warm-Up

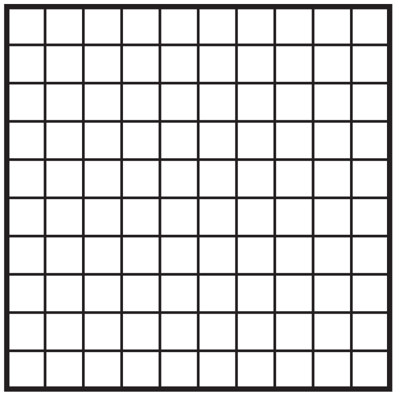
1 This large square represents 1.

Write 0.425 using different quantities of tenths, hundredths, and/or thousandths.

Write as many combinations as you can think of.

Responses vary.

Tenths	Hundredths	Thousandths
4	2	5
0	0	425
4	0	25
4.2	0	5
3	12	5



Activity

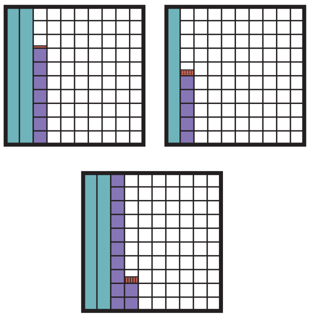
1

Name: .... Date: .... Period: ....

More Than One Way to Add

2 Prisha and Omar used different strategies to add $0.271 + 0.154$.

Prisha



$= 0.271 + 0.154$

Omar

$$\begin{array}{r} 0.271 \\ +\,0.154 \\ \hline 0.3 \\ 0.12 \\ 0.005 \\ \hline 0.425 \end{array}$$

a Whose calculation is correct? Circle one.

Prisha's   Omar's   Both   Neither

b Explain your thinking.

Explanations vary. **Both students added by place value. First, they added 2 tenths and 1 tenth to get 3 tenths. Then they calculated the sum of 7 hundredths and 5 hundredths, which is 12 hundredths. Prisha correctly showed this on her model using 12 purple squares, while Omar showed it as 0.12. They added 1 thousandth with 4 thousandths to get 5 thousandths. Omar then added 12 hundredths with 3 tenths to get 4 tenths, 2 hundredths, and 5 thousandths. Prisha also represented this in her third diagram.**

Note: Students who select "Prisha's," "Omar's," or "Both" are considered correct.

Activity 1

Name: ______ Date: ______ Period: ______

More Than One Way to Add (continued)

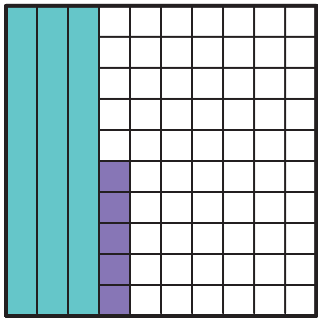
3 Group the representations that have the same value. You will have three groups.

a. $0.048 + 0.302$

b. $0.04 + 0.010$

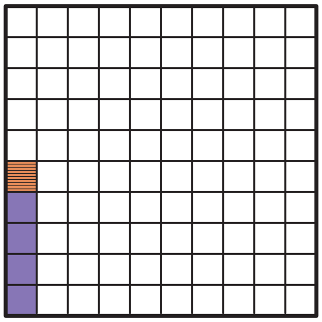
c. $0.499 + 0.001$

d.



e. 50 hundredths

f.



g. 0.5

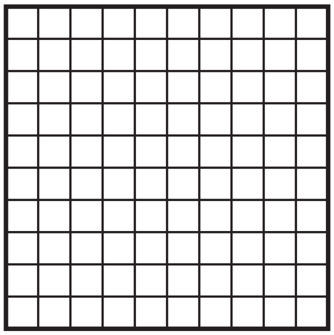
h. 4 hundredths and 10 thousandths

i. 0.35

Group 1	Group 2	Group 3
a	b	c
d	f	e
i	h	g

4 Determine the value of $0.275 + 0.135$.

0.41



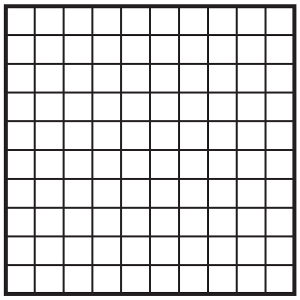
Activity 2

Name: .... Date: .... Period: ....

Decimal Differences

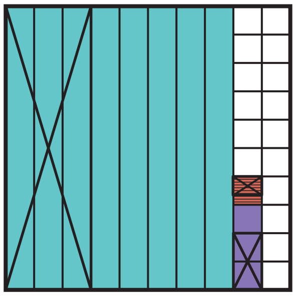
5 Determine the value of $0.2 - 0.11$.

0.09



6 Adhara and Malik used different strategies to calculate $0.84 - 0.327$.

Adhara



Malik

$$\begin{array}{r} {\scriptstyle 3\,10} \\ 0.8\cancel{4}\cancel{0} \\ -\ 0.327 \\ \hline 0.513 \end{array}$$

Discuss:

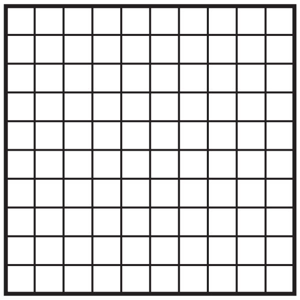
- How are these strategies alike? How are they different?
- Where do you see place value and regrouping in each strategy?

Responses vary.

- **Both strategies rewrite 1 hundredth as 10 thousandths and then subtract 7 thousandths from 10 thousandths.**
- **The strategies are different because Adhara used a grid and Malik used a vertical calculation.**
- **On the grid, I see tenth-, hundredth-, and thousandth-sized pieces to represent place value. Adhara regroups 1 hundredths piece as 10 thousandths pieces. In the vertical calculation, Malik regroups 4 hundredths as 3 hundredths and 10 thousandths.**

7 Determine the value of $0.562 - 0.17$.

0.392



Activity 3

Name: ______ Date: ______ Period: ______

Repeated Challenges

8 Determine the value for each sum or difference. Solve as many challenges as you have time for.

a $0.203 + 0.105 = \mathbf{0.308}$

b $0.15 - 0.1 = \mathbf{0.05}$

c $0.155 + 0.015 = \mathbf{0.17}$

d $0.5 - 0.151 = \mathbf{0.349}$

e $0.25 + 0.15 = \mathbf{0.4}$

f $0.01 + 0.002 = \mathbf{0.012}$

g $0.946 - 0.041 = \mathbf{0.905}$

h $0.589 - 0.187 = \mathbf{0.402}$

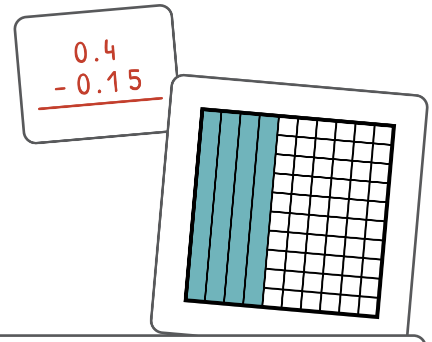
9 Synthesis

Describe a strategy for subtracting decimals.

Use the example $0.4 - 0.15$ if it helps with your thinking.

Responses vary.

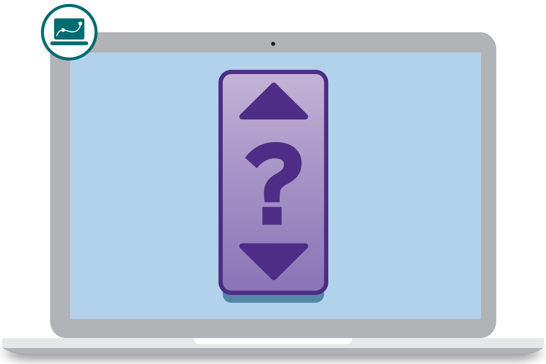
- **You can use a hundredths chart. Start by drawing 4 tenths, then regroup them as 40 hundredths. This will allow you to cross out 15 hundredths and find the difference.**
- **You can use a vertical calculation to subtract. You can regroup 4 tenths as 3 tenths and 10 hundredths. Then you can subtract 5 hundredths from the 10 hundredths, and 1 tenth from the 3 tenths to find the difference.**



40 hundredths minus 15 hundredths

Things to Remember:

Name: ____________ Date: ________ Period: ________



Missing Digits

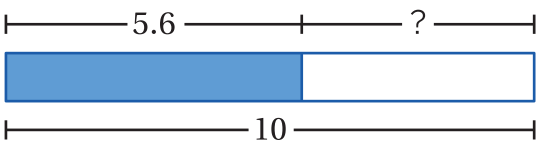
Let's solve number puzzles.

Warm-Up

1 Determine the missing number for each challenge.

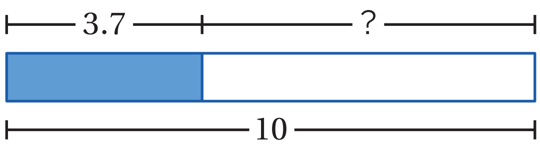
a 5.6 plus what number equals 10?

4.4



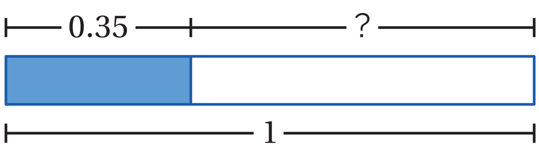
b 3.7 plus what number equals 10?

6.3



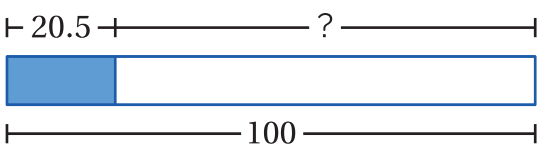
c 0.35 plus what number equals 1?

0.65



d 20.5 plus what number equals 100?

79.5

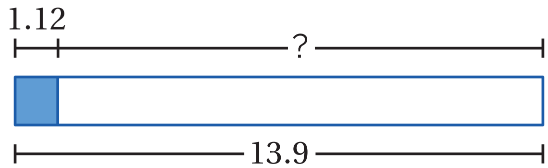


Name: ________ Date: ________ Period: ________

Vertical Calculations

2 1.12 plus what number equals 13.9?

12.78



3 Here is how Kayleen calculated $13.9 - 1.12$.

$$\begin{array}{r} 13.9 \\ -1.12 \\ \hline 2.7 \end{array}$$

What could you say to her to help her understand her mistake?

Responses vary. **I would tell Kayleen to line up the same place values. This will help her subtract the ones from the ones, the tenths from the tenths, and the hundredths from the hundredths. It's really like we're subtracting $13.90 - 1.12$, which is like subtracting $13 - 1$ and 90 hundredths minus 12 hundredths.**

Name: .... Date: .... Period: ....

Number Puzzles

4 Kayleen made a number puzzle.

$$\begin{array}{r} 2.5 \\ +\,7.\square \\ \hline 9.8 \end{array}$$

Fill in the missing digit that makes the statement true.

The missing digit is 3.

5 Here is Yosef's number puzzle.

$$\begin{array}{r} 0.0\square 85 \\ -\,0.00\square 9 \\ \hline 0.005\square \end{array}$$

Fill in the missing digits that make the statement true.

Hundredths place: 0

Thousandths place: 2

Ten-thousandths place: 6

6 Match each number puzzle to its missing digit. Two digits will have no match.

1	2	3	5	6

a.
$$\begin{array}{r} 0.58 \\ -\,0.322 \\ \hline 0.2?8 \end{array}$$

5

b.
$$\begin{array}{r} 59.4? \\ -\,40.6 \\ \hline 18.83 \end{array}$$

3

c.
$$\begin{array}{r} 9.874 \\ +\,0.?26 \\ \hline 10 \end{array}$$

1

Activity 2

Name: .... Date: .... Period: ....

Number Puzzles (continued)

7 Here is Aba's puzzle.

$$5.8 - \square.3\square = 3.\square 3$$

Fill in the missing digits to make the statement true.

Ones place: 2

Tenths place: 4

Hundredths place: 7

8 Aba used addition to check her solution to her number puzzle.

$$\begin{array}{r} {\scriptstyle 1} \\ 3.43 \\ +\ 2.37 \\ \hline 5.80 \end{array}$$

Does this sum prove that her solution is correct? Circle one.

Yes   No   I'm not sure

Explain your thinking.

***Explanations vary*. Aba's solution is correct because she added the difference, 3.43, and the number that was getting subtracted, 2.37, and got 5.80. This is the same as 5.8 because 80 hundredths is equivalent to 8 tenths.**

Activity 3

Name: .... Date: .... Period: ....

Repeated Challenges

9 Fill in the missing digits to make each statement true. Solve as many challenges as you have time for.

a $2.5 + \square.6 = 9.\square$

Ones place: 6, tenths place: 1

b $9.5 - 5.\square5 = \square.15$

Ones place: 4, tenths place: 3

c $0.404 + \square.\square\square6 = 1.000$

Ones place: 0, tenths place: 5, hundredths place: 9

d $\square4.\square3 + 224.17 = 318.80$

Tens place: 9, tenths place: 6

e $7 - \square.\square\square33 = 3.4567$

Ones place: 3, tenths place: 5, hundredths place: 4

f $0.7 - 0.68\square = 0.\square12$

Tenths place: 0, thousandths place: 8

Explore More

10 How many different solutions are there to this problem? Explain your thinking.

$$\begin{array}{r} 5.? \\ -\ ?.?3 \\ \hline 2.2? \end{array}$$

10 solutions. *Explanations vary*. To determine the missing digit on the far right, I will need to regroup from the tenths place to make 10 hundredths. 10 hundredths minus 3 hundredths is 7 hundredths, so the missing digit on the far right is 7. The missing digits in the middle column will always have a difference of 3, like 8 and 5. There are ten possible ways to make a difference of 3, including ways involving regrouping, like 12 – 9. Once I pick the pair of numbers in the tenths place, there is only one choice for the missing number in the ones place. That means there are 10 different solutions to this problem in total.

11 Synthesis

Describe a strategy for determining the missing digits in this subtraction problem.

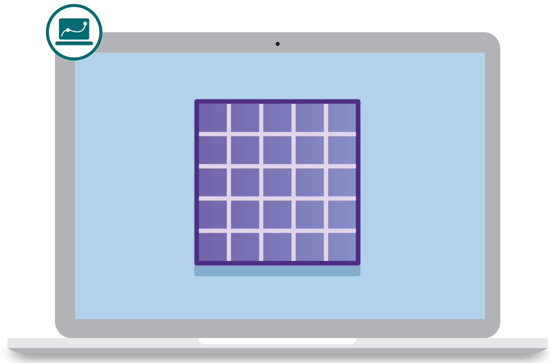
$$\begin{array}{r} 3 \\ -\,1.0\boxed{?} \\ \hline \boxed{?}.\boxed{?}8 \end{array}$$

Responses vary.

- **I know the bottom two numbers need to add up to 3.00. There must be a 2 missing from the hundredths place because 8 hundredths and 2 hundredths will give me a 0 in the hundredths place and an additional tenth. Because I have 1 tenth (from my hundredths), I know the missing tenths digit must be 9. This gives me an additional one, so the missing ones value is 1.**
- **Add a decimal point and two 0s after the 3. You can borrow from the 3 to get 9 tenths and 10 hundredths. Then you can subtract the hundredths from the hundredths, the tenths from the tenths, and the ones from the ones to determine the missing digits.**

Things to Remember:

Name: ______________ Date: ________ Period: ________



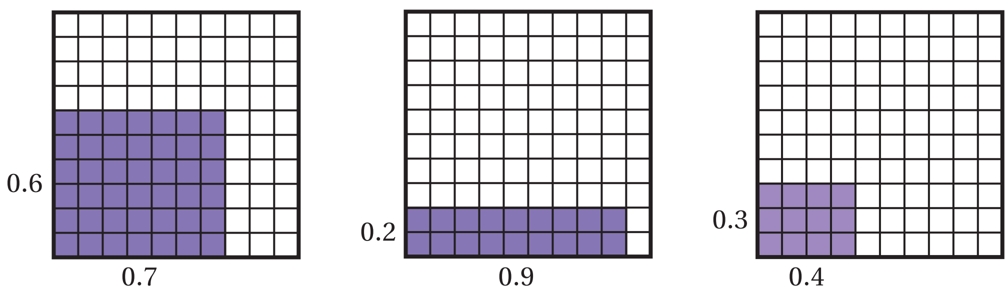
Decimal Multiplication

Let's explore decimal multiplication using place value.

Warm-Up

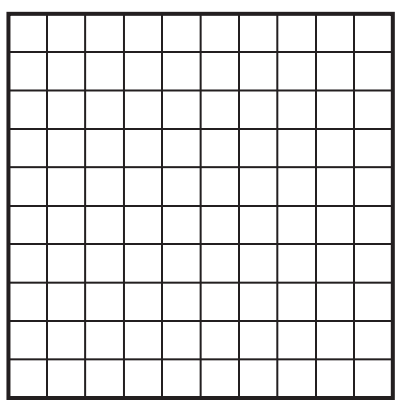
1 The large square has an area of 1.

a Take a look at these shaded rectangles with different lengths and widths.



b Try to create shaded rectangles with the following areas: ***Responses vary.***

Length	Width	Area
		0.16
		0.24
		0.30



Activity
1

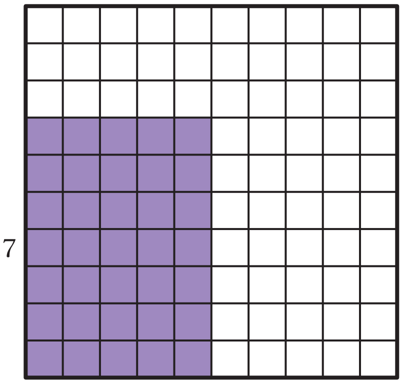
Name: ______ Date: ______ Period: ______

Keepin' It One Hundredth

2 Jamya says that $0.5 \cdot 0.7 = 0.35$.

Use the diagram to show or explain why this makes sense.

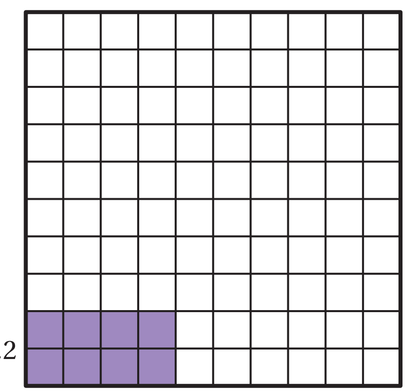
***Responses vary.* This makes sense because there are 35 small squares shaded in the chart, and each small square is 0.01. So the total area is 0.35.**



3 Multiply $0.4 \cdot 0.2$.

Write your answer as a decimal.

0.08

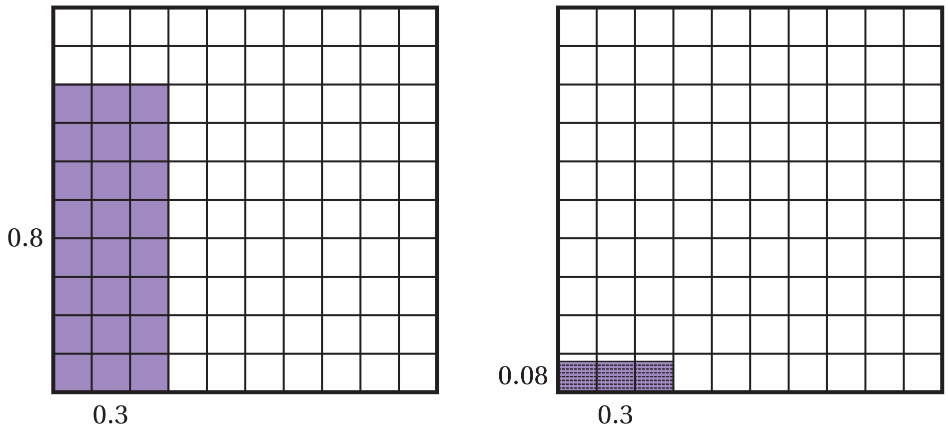


Activity 1

Name: ______ Date: ______ Period: ______

Keepin' It One Hundredth (continued)

4 Here are two diagrams that represent different multiplication problems.



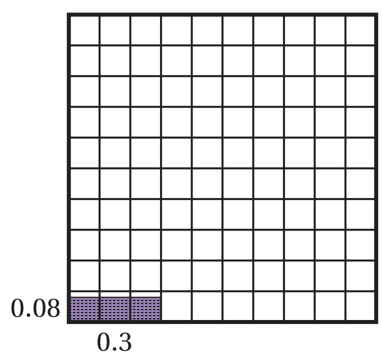
Discuss: How are the diagrams alike? How are they different?

Alike:
- **Both have one side that is 0.3 units long.**
- **Both are rectangles.**
- **Both have one side that has a length with an 8 in it.**
- **Both areas are split into 24 pieces.**

Different:
- **The area on the right is much smaller than the area on the left.**
- **The diagram on the left represents multiplication by 8 tenths and the one on the right represents multiplication by 8 hundredths.**
- **In both diagrams, each square is 1 hundredth. Each tiny rectangle on the right diagram is 1 thousandth.**

5 What is $0.3 \cdot 0.08$?

0.024



6 This is Jayden's work for multiplying $0.3 \cdot 0.08$.

$$0.3 \cdot 0.08$$
$$\frac{3}{10} \cdot \frac{8}{100}$$
$$\frac{24}{1000} = 0.024$$

a **Discuss:** What is Jayden's strategy?

***Responses vary.* Jayden's strategy is to write the decimals as equivalent fractions.**

b Show or explain how you would use this strategy to multiply $0.02 \cdot 0.9$.

***Responses vary.* I would write $\frac{2}{100} \cdot \frac{9}{10}$, which equals $\frac{18}{1000}$ or 0.018.**

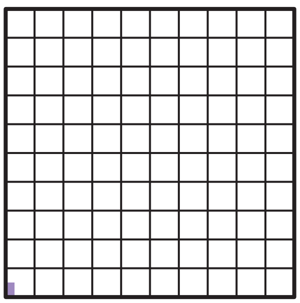
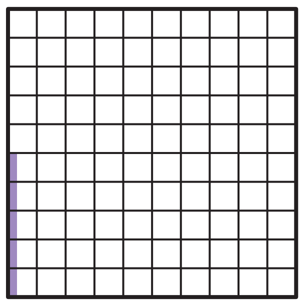
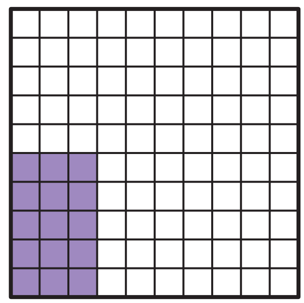
Activity
2

Name: ______ Date: ______ Period: ______

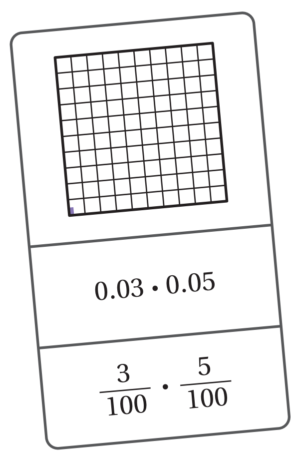
Clicking Into Place Value

7 Match each area with its equivalent expressions.

$\frac{15}{100}$	$0.3 \cdot 0.5$	0.015
$0.03 \cdot 0.05$	$\frac{3}{100} \cdot \frac{5}{100}$	$0.03 \cdot 0.5$

Area 1	Area 2	Area 3
		
$0.03 \cdot 0.05$ $\frac{3}{100} \cdot \frac{5}{100}$	$0.03 \cdot 0.5$ 0.015	$0.3 \cdot 0.5$ $\frac{15}{100}$

8 Jayden matched these choices in the previous problem and noticed that the product was missing.



Calculate $0.03 \cdot 0.05$.

0.0015

Activity
3

Name: ______ Date: ______ Period: ______

Repeated Challenges

9 Solve as many challenges as you have time for.

a. $0.04 \cdot 0.6 = \mathbf{0.024}$

b. $0.03 \cdot 0.02 = \mathbf{0.0006}$

c. $0.2 \cdot 0.007 = \mathbf{0.0014}$

d. $0.003 \cdot 0.3 = \mathbf{0.0009}$

e. $0.5 \cdot 0.4 = \mathbf{0.2}$

f. $0.001 \cdot 0.08 = \mathbf{0.00008}$

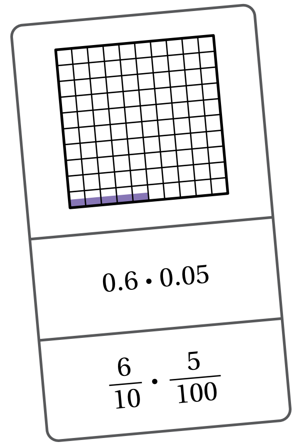
g. $1 \cdot 0.05 = \mathbf{0.05}$

h. $0.4 \cdot 0.005 = \mathbf{0.002}$

10 Synthesis

Show or describe how decimals, fractions, and the hundredths chart are related.

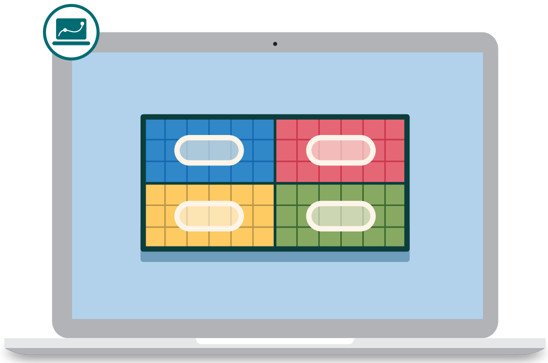
Use the example if it helps with your thinking.



***Responses vary.* The hundredths chart, decimals, and fractions are three ways to show multiplication of numbers less than 1. $0.6 \cdot 0.05$ can also be written as $\frac{6}{10} \cdot \frac{5}{100}$. It can also be shown on a hundredths chart, where each small square is 1 hundredth. If you draw a rectangle with one side that's 0.6 units long and one side that's 0.05 units long, you can calculate the area of the rectangle to determine the answer to $0.6 \cdot 0.05$, which is 0.030, or $\frac{30}{1000}$.**

Things to Remember:

Name: .... Date: .... Period: ....

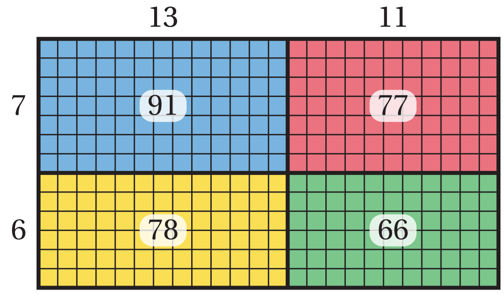


Multiplying With Areas

Let's use area models to multiply decimals.

Warm-Up

1 Diego likes using area models to multiply whole numbers.



a Let's look at different ways to split this 24-by-13 rectangle.

b What do you notice? What do you wonder?

Responses vary.

I notice:

- **I notice that there are sometimes four sections, sometimes two sections, and sometimes one section.**
- **I notice that the two numbers on the top always add up to 24.**
- **I notice that the number inside each rectangle is the area of the rectangle.**

I wonder:

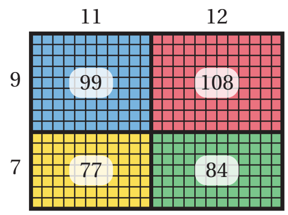
- **I wonder what problem Diego was trying to solve.**
- **I wonder if there's something special about the numbers 24 and 13.**
- **I wonder what the circled numbers mean.**

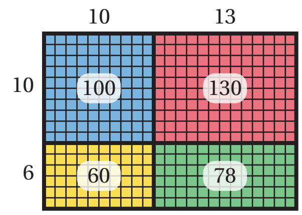
Activity 1

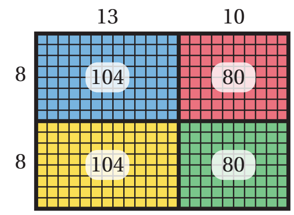
Name: .... Date: .... Period: ....

Creating an Area Model

2 Here are some new area models. Use them to multiply 23 • 16.





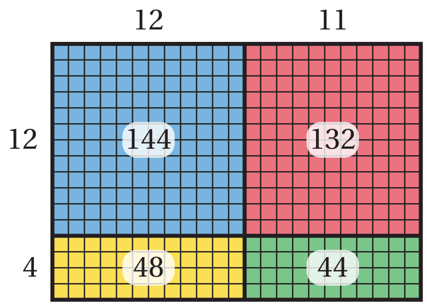


368

3 Mayra and Taylor made different area models to multiply 23 • 16.

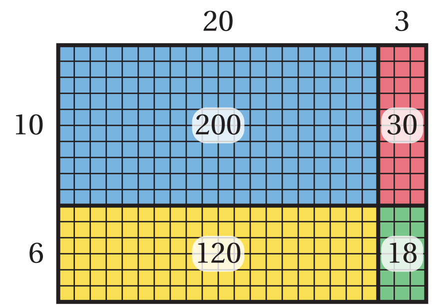
Discuss: Which area model is more helpful? Explain your thinking.

Mayra



144 + 132 + 48 + 44 = 368

Taylor



200 + 30 + 120 + 18 = 368

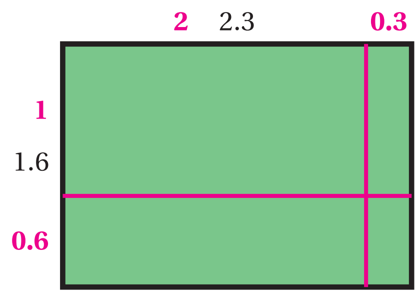
Responses and explanations vary.

- **Taylor's area model would be more helpful because it's split by tens and ones, which makes it easier to multiply each pair of numbers.**
- **Mayra's work would be more helpful for me because I know all my multiplication facts up to 12 • 12.**

Name: .... Date: .... Period: ....

Creating an Area Model (continued)

4 Diego wonders if area models could also help him multiply decimals like $2.3 \cdot 1.6$.



a Show how you would split this rectangle into smaller parts to multiply $2.3 \cdot 1.6$.

***Responses vary.* Sample shown on diagram.**

b Explain why you split it that way.

Explanations vary.

- **I split the rectangle so that the ones were separate from the tenths, kind of like Taylor's diagram which is split by tens and ones.**
- **I split it in half (0.8 and 0.8) so that I could calculate the same area on each side.**

5 Use your area model to calculate $2.3 \cdot 1.6$.

3.68

Activity
2

Name: .... Date: .... Period: ....

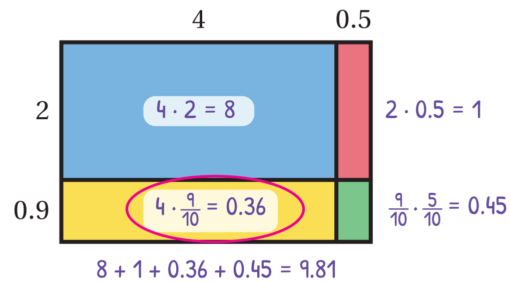
Calculating With an Area Model

6 **a** Use the digital activity to create an area model that helps you multiply $4.5 \cdot 2.9$.

b Calculate the area of each part of your area model. The total area will be calculated for you.

Responses vary depending on how students split the rectangle. **The total area of the rectangle is 13.05.**

7 Diego made an error while multiplying $4.5 \cdot 2.9$.



a Circle the error in Diego's work.

Response shown on diagram.

b What would you say to help him understand his mistake?

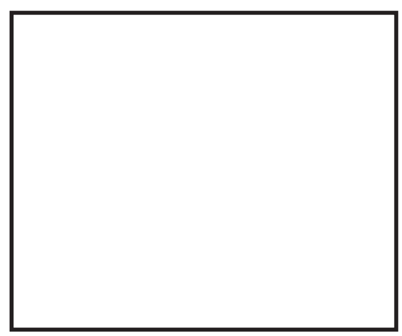
Responses vary.

- **The bottom left part is larger than the bottom right part so you know that the area of the bottom left part has to be more than 0.45.**
- **You wrote 36 hundredths, but you actually have 36 tenths.**

8 Multiply $3.4 \cdot 2.8$.

Use the diagram if it helps with your thinking.

9.52



Name: .... Date: .... Period: ....

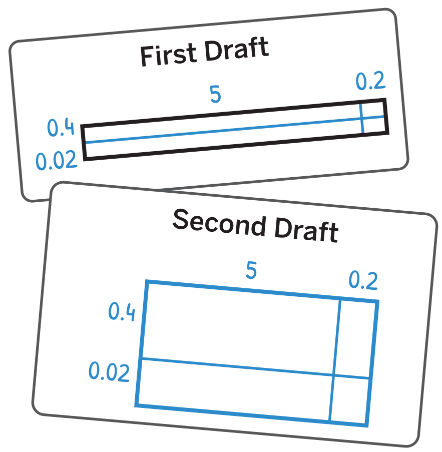
Calculating With an Area Model (continued)

9 Multiply $5.2 \cdot 0.42$.

Use the diagram if it helps with your thinking.

2.184

10 Paz made two drafts to calculate $5.2 \cdot 0.42$.



What are the advantages and disadvantages of Paz's second draft?

Responses vary.

Advantages:

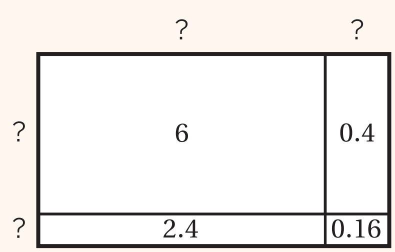
- **It was easier for Paz to write the area of each part inside because the areas are larger in the second draft.**
- **Paz could use the same rectangle for other multiplication problems.**

Disadvantages:

- **It could be harder to estimate the total area because the rectangle doesn't really fit its measurements.**
- **This isn't what a rectangle with those measurements actually looks like.**

Explore More

11 Here is an area model that is missing some labels.



What multiplication problem could this help you solve? Explain your thinking.

Responses and explanations vary. **6.4 • 1.4, 3.2 • 2.8. I filled in the outside numbers with 6, 0.4, 1, and 0.4 so the area in each part was correct.**

12 Synthesis

Describe how you can use an area model to multiply decimals like $2.7 \cdot 1.4$.

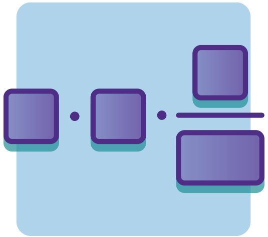
Draw on the diagram if it helps with your thinking.

Responses vary. **First, split the area model into parts. In this example, I would split it into $2 + 0.7$ and $1 + 0.4$. Then calculate the area of each part. Finally, add all the areas together.**

Things to Remember:

Name: ________ Date: ________ Period: ________

Multiplication Methods



Let's use place value to multiply decimals vertically.

Warm-Up

1. Calculate $84 \cdot 13$. Show your thinking.

1,092. *Work varies.* $80 \cdot 10 + 80 \cdot 3 + 4 \cdot 10 + 4 \cdot 3$

2. Find someone who used a different strategy than you did. Discuss how you each calculated $84 \cdot 13$.

Responses vary.

- I determined $84 \cdot 10$ and $84 \cdot 3$, then added the products.
- I used an area model and split the rectangle into 80 and 4 by 10 and 3.
- I used a vertical calculation.

Activity 1

Name: ______ Date: ______ Period: ______

Multiple Methods

Kwame and Tiara used different strategies to multiply $8.4 \cdot 1.3$

Kwame

	8	0.4
1	8	0.4
0.3	2.4	0.12

$8 + 0.4 + 2.4 + 0.12 = 10.92$

Tiara

$8.4 \cdot 1.3$

$84 \cdot 13 \cdot \frac{1}{10} \cdot \frac{1}{10}$

$$\begin{array}{r} 84 \\ \times 13 \\ \hline 12 \\ 240 \\ 40 \\ +800 \\ \hline 1092 \end{array} \cdot \frac{1}{100}$$

10.92

3. **Discuss:** How are their strategies alike? How are they different?

Responses vary. **Both strategies break the problem into four parts. They're different because Kwame is multiplying with decimals the entire time while Tiara is not. Tiara multiplies whole numbers and fractions, then changes her answer into a decimal at the end.**

4. Show how each student might set up $8.4 \cdot 0.13$. *Work varies.*

Kwame

	8	0.4
0.1		
0.03		

Tiara

$8.4 \cdot 0.13$

$84 \cdot 13 \cdot \frac{1}{10} \cdot \frac{1}{100}$

5. Use either strategy to finish calculating $8.4 \cdot 0.13$.

1.092

6. Calculate $0.352 \cdot 0.25$.

0.088

Activity 2

Name: ________ Date: ________ Period: ________

Multiple Multiplication Methods

7. Tiara wrote this expression to try and calculate $2.9 \cdot 0.015$.

$29 \cdot 15 \cdot \frac{1}{10} \cdot \frac{1}{1000}$

If $29 \cdot 15 = 435$, then what is $2.9 \cdot 0.015$?

A. 4.35 B. 0.435 C. 0.0435 D. 0.00435

8. If $165 \cdot 12 = 1980$, then what is $16.5 \cdot 1.2$? Explain your thinking.

19.8. ***Explanations vary.*** **I'm multiplying tenths by tenths, so my answer is in hundredths because $\frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$. This means that the product is 1,980 hundredths or 19.8.**

9. Select all the expressions that have a product of 0.024.

- ☑ A. $0.06 \cdot 0.4$
- ☑ B. $0.6 \cdot 0.04$
- ☐ C. $0.04 \cdot 0.06$
- ☑ D. $2 \cdot 0.012$
- ☑ E. $1.2 \cdot 0.02$

10. Write another expression that has a product of 0.024.

Responses vary. **$0.08 \cdot 0.3$**

Activity 3

Name: Date: Period:

Scavenger Hunt

11. Start with any of the scavenger hunt sheets.

- Record the sheet shape, solve the problem, and write your answer.
- Look for your answer at the top of another scavenger hunt sheet. Solve that problem.
- Repeat until you make it back to your starting sheet.

The sheet students start with varies.

Sheet: Triangle *Work varies.* Answer: 1.148 →	Sheet: Star *Work varies.* ↙ Answer: 13.68
Sheet: Oval *Work varies.* Answer: 1.228 →	Sheet: Trapezoid *Work varies.* ↙ Answer: 0.01368
Sheet: Crescent *Work varies.* Answer: 11.48 →	Sheet: Pentagon *Work varies.* Answer: 12.28

Synthesis

12. Describe a strategy that helps you multiply decimals like $0.039 \cdot 3.2$.

***Responses vary*. One strategy is to rewrite each number as a whole number times a fraction. That way, you can multiply the whole numbers and then determine the place value.**

Things to Remember:

Scavenger Hunt

Triangle Sheet

Answer

12.28

Problem:

Calculate $2.8 \cdot 0.41$.

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Scavenger Hunt

Star Sheet

Answer

1.148

Problem:

Multiply $19 \bullet 0.72$.

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Scavenger Hunt

Oval Sheet

Answer

13.68

Problem:

Calculate $4 \bullet 0.307$.

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Scavenger Hunt

Trapezoid Sheet

Answer

1.228

Problem:

Determine the value of $0.0019 \cdot 7.2$.

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Scavenger Hunt

Crescent Sheet

Answer

0.01368

Problem:

Calculate $4.1 \bullet 2.8$.

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Scavenger Hunt

Pentagon Sheet

Answer

11.48

Problem:

Determine the value of $30.7 \cdot 0.4$.

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Name: .... Date: .... Period: ....

Just Keep Dividing

Let's use long division to divide whole numbers when there is a remainder.

Warm-Up

1. Here is the work that Peter and Alina did to calculate $584 \div 4$.

Peter

```
      6
     4 0
    1 0 0
 4) 5 8 4
   -4 0 0  ←— 4 groups of 100
    1 8 4
   -1 6 0  ←— 4 groups of 40
      2 4
     -2 4  ←— 4 groups of 6
        0
```

Alina

```
   1 4 6
4) 5 8 4
  -4
   1 8 4
  -1 6
     2 4
    -2 4
       0
```

Discuss:

- Where do you see Peter's work in Alina's strategy?
- How is Alina's strategy different from Peter's partial quotients strategy?

Responses vary.

- **In Alina's work, I see her subtracting the 4 in the hundreds place from the 5, which is the same as Peter subtracting 400 from 584. I also see Alina subtracting 16 from 18, which is the same as Peter subtracting 160 from 184.**
- **Alina doesn't use the zeros to show 400 or 160. Instead, she keeps 4 in the hundreds place value and 16 in the tens place value. It looks like she subtracts one digit at a time and then brings down the number in the next place value. She also doesn't split up the quotient into parts. Instead, she writes the number in the next place value in the quotient.**

Activity 1

Name: ________ Date: ________ Period: ________

Digit-by-Digit

2. Alina calculates 685 ÷ 5 using a strategy called **long division**.

$$\begin{array}{r} 1 \\ 5\,\overline{)\,685} \\ -5 \\ \hline 1 \end{array} \longleftarrow \text{5 groups of 1 (hundred)}$$

$$\begin{array}{r} 13 \\ 5\,\overline{)\,685} \\ -5\downarrow \\ \hline 18 \\ -15 \\ \hline 3 \end{array} \longleftarrow \text{5 groups of 3 (tens)}$$

$$\begin{array}{r} 137 \\ 5\,\overline{)\,685} \\ -5\downarrow \\ \hline 18 \\ -15 \\ \hline 35 \\ -35 \\ \hline 0 \end{array} \longleftarrow \text{5 groups of 7 (ones)}$$

a **Discuss:** Alina explains this long division strategy as "dividing one digit at a time, from left to right." What do you think Alina means by this?

Responses vary. **I think this means that Alina focuses first on dividing the digit 6 into 5 groups of 1. This leaves a remainder of 1, so she brings down the next digit to the right, 8. Alina then focuses on dividing 18 into 5 groups of 3, which leaves a remainder of 3. She will likely repeat the process of bringing down the next digit, 5, to divide 35 into 5 equal groups.**

b Complete the work using this strategy.

Sample shown on diagram.

3. Use long division to complete each of these division problems.

a 792 ÷ 6

$$\begin{array}{r} 132 \\ 6\,\overline{)\,792} \\ -6\downarrow \\ \hline 19 \\ -18\downarrow \\ \hline 12 \\ -12 \\ \hline 0 \end{array}$$

b 1736 ÷ 8

$$\begin{array}{r} 217 \\ 8\,\overline{)\,1736} \\ -16\downarrow \\ \hline 13 \\ -\ 8\downarrow \\ \hline 56 \\ -56 \\ \hline 0 \end{array}$$

c 2933 ÷ 7

$$\begin{array}{r} 419 \\ 7\,\overline{)\,2933} \\ -28\downarrow \\ \hline 13 \\ -\ 7\downarrow \\ \hline 63 \\ -63 \\ \hline 0 \end{array}$$

Activity 2

Name: .... Date: .... Period: ....

Dicey Decimals

4. Here is Peter's work for calculating $318 \div 4$.

$$\begin{array}{r} 79.5 \\ 4\overline{)318.0} \\ -28\downarrow \\ \hline 38 \\ -36\downarrow \\ \hline 20 \\ -20 \\ \hline 0 \end{array} \longleftarrow \text{4 groups of 5 (tenths)}$$

Peter says that 2 is the same as 20 tenths.

Discuss:

- Where do you see the 20 tenths represented in this work?
- Why do you think Peter might find it more helpful to divide 20 tenths by 4 instead of dividing 2 by 4?

***Responses vary.* I can see the remainder 2 becomes 20 tenths when he writes the decimal in the quotient and brings down the 0. Peter might feel more comfortable dividing numbers that result in whole number quotients.**

5. Use Peter's strategy to find the quotient of each expression.

a $162 \div 5$

$$\begin{array}{r} 32.4 \\ 5\overline{)162.0} \\ -15\downarrow \\ \hline 12 \\ -10\downarrow \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

b $70 \div 8$

$$\begin{array}{r} 8.75 \\ 8\overline{)70.00} \\ -64\downarrow \\ \hline 60 \\ -56\downarrow \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

6. Nur is working on $2369 \div 8$ using Peter's strategy. Nur wrote the remainder of 1 as 10 tenths, but that still leaves a remainder of 2 (tenths).

What could Nur do to complete the division? Show or explain your thinking.

***Responses vary.* Nur could add another zero to the dividend and bring it down. This would convert 2 tenths to 20 hundredths. 20 hundredths can be divided into 8 groups of 2 hundredths with a remainder of 4 hundredths. Then Nur could repeat the process to convert 4 hundredths to 40 thousandths, which can be divided into 8 groups of 5 thousandths. The quotient would be 296.125.**

$$\begin{array}{r} 296.125 \\ 8\overline{)2369.000} \\ -16\downarrow \\ \hline 76 \\ -72\downarrow \\ \hline 49 \\ -48\downarrow \\ \hline 10 \\ -\ \ 8\downarrow \\ \hline 20 \\ -16\downarrow \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

Name: .... Date: .... Period: ....

Division Detectives

7. Decide with your partner who will complete Column A and who will complete Column B. The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

Column A	Column B
$13 \div 4$	$26 \div 8$
3.25	**3.25**
$2166 \div 12$	$722 \div 4$
180.5	**180.5**
$11 \div 8$	$33 \div 24$
1.375	**1.375**
$4 \div 16$	$2 \div 8$
0.25	**0.25**

Name: .... Date: .... Period: ....

Division Detectives (continued)

Column A	Column B
$102 \div 25$	$204 \div 50$
4.08	**4.08**
$36 \div 32$	$18 \div 16$
1.125	**1.125**

Explore More

8. a. Calculate the quotient for $46 \div 3$.
15.3...

b. Do you think this pattern will continue? Explain your thinking.
Yes. *Explanations vary.* I noticed that the remainder continues to be the same and the decimal in the quotient keeps repeating after each step in the long division.

Synthesis

9. What are some things you think are important to remember when dividing whole numbers that have remainders?

Use the examples if they help with your thinking.

$62 \div 5$

$183 \div 15$

$1 \div 8$

***Responses vary.* If there is a remainder in a division expression, you can keep dividing. Just add a zero to the dividend by inserting a decimal point in the quotient, then continue to divide. When you divide the remainder, you will get a decimal.**

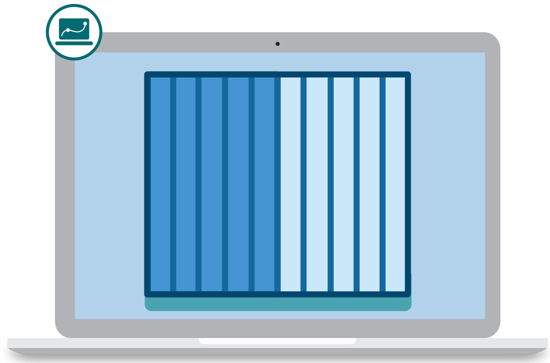
Things to Remember:

Unit 5

Lesson

11

Name: .... Date: .... Period: ....



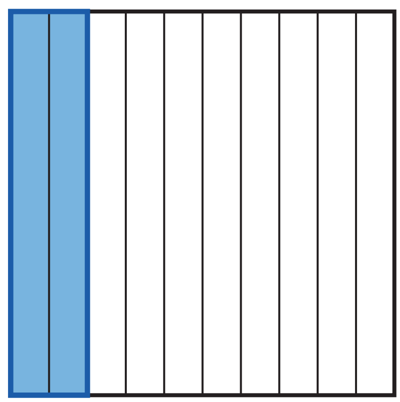
Division Diagrams

Let's divide decimals using hundredths charts and new expressions.

Warm-Up

1 This large square represents 1.

a Select *all* the equations you could use to determine how many blue pieces you need to fill the large square.



- ☑ **A.** $1 \div 0.2 = ?$
- ☐ **B.** $0.2 \div 1 = ?$
- ☑ **C.** $10 \div 2 = ?$
- ☑ **D.** $0.2 \cdot ? = 1$
- ☐ **E.** $2 \div 10 = ?$

b Pick *one* equation and explain how it represents the diagram.

Responses vary.

- **$1 \div 0.2 = ?$: The whole diagram is 1, while the blue piece is 0.2 or $\frac{2}{10}$. $1 \div 0.2$ is like asking how many 0.2s go into 1.**
- **$10 \div 2 = ?$: If I want to know how many blue pieces fill the large square, I can calculate $10 \div 2$ because there are 10 tenths and the blue piece fills 2 of those tenths.**
- **$0.2 \cdot ? = 1$: Each blue piece is 0.2, so if I multiply 0.2 by the number of blue pieces that fill the large square, I will get 1.**

Activity 1

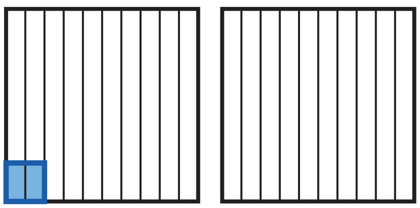
Name: ________ Date: ________ Period: ________

Decimal Division Strategies

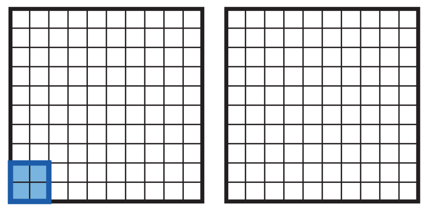
2 Each large square represents 1.

Determine the value of $2 \div 0.04$. Use the tenths or hundredths chart if it helps with your thinking. **50**

Tenths



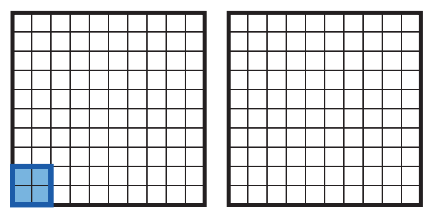
Hundredths



3 Diamond claims that $2 \div 0.04$ has the same value as $200 \div 4$.

Explain why this makes sense.

***Responses vary*. The division problem $2 \div 0.04$ is like asking how many 0.04s go into 2. If we think about 2 as 200 hundredths, that would make each blue piece 4 hundredths. That means $200 \div 4$ should give me the same value.**



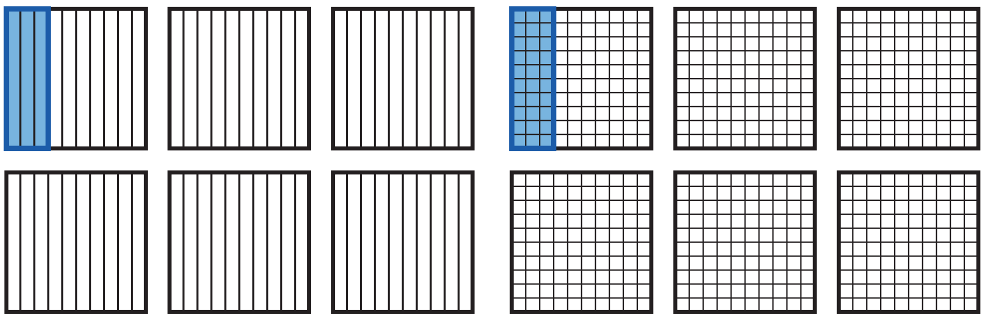
Name: ________ Date: ________ Period: ________

Decimal Division Strategies (continued)

4 Each large square represents 1.

Determine the value of $6 \div 0.3$. Use the tenths or hundredths chart if it helps with your thinking. **20**

Tenths **Hundredths**



5 Here is how Arjun determined the value of $6 \div 0.3$.

$$6 \div 0.3 = \frac{6}{1} \div \frac{3}{10} = \frac{60}{10} \div \frac{3}{10} = 60 \div 3 = 20$$

a **Discuss:** Describe Arjun's strategy.

***Responses vary*. Arjun wrote 6 and 0.3 as fractions and then found a common denominator. Then he just divided the numerators.**

b Use his strategy to determine the value of $5 \div 0.02$.

250

Activity 2

Name: ______ Date: ______ Period: ______

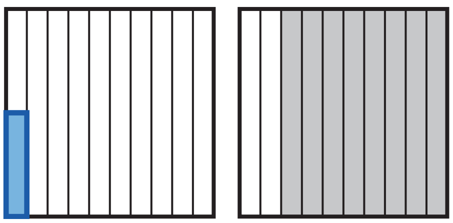
Different Expression, Same Value

6 Select *all* the expressions that have the same value as $1.2 \div 0.05$.

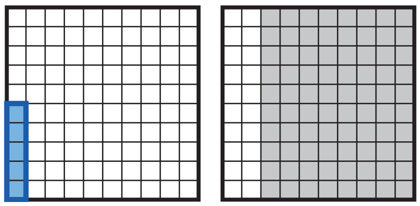
Use the tenths or hundredths chart if it helps with your thinking.

- ☐ **A.** $12 \div 5$
- ☑ **B.** $12 \div 0.5$
- ☑ **C.** $120 \div 5$
- ☑ **D.** $\frac{120}{100} \div \frac{5}{100}$
- ☐ **E.** $\frac{12}{100} \div \frac{5}{100}$

Tenths



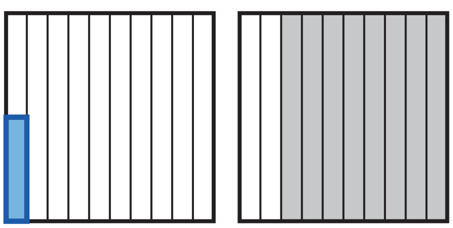
Hundredths



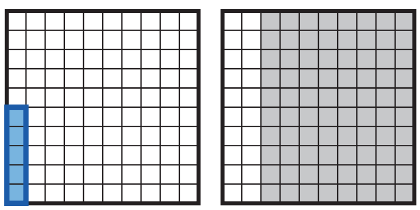
7 Calculate $1.2 \div 0.05$. **24**

Use the tenths or hundredths chart if it helps with your thinking.

Tenths



Hundredths



Name: .... Date: .... Period: ....

Different Expression, Same Value (continued)

8 Write an expression with the same value as $3.6 \div 0.012$ using fractions.

Responses vary. $\frac{3600}{1000} \div \frac{12}{1000}$ **(or equivalent)**

9 Here is Raven's work for $3.6 \div 0.012$.

$$3.6 \div 0.012 = \frac{36}{10} \div \frac{12}{1000}$$
$$= 36 \div 12$$
$$= 3$$

What would you say to help her understand her mistake?

Responses vary.

- **I would tell Raven to look at the denominators of each fraction. Right now, Raven is eliminating denominators that are not the same.**
- **You can rewrite the problem using fractions with common denominators, then you can eliminate the denominators to rewrite the problem using whole numbers.**

Activity 3

Name: ____________ Date: ________ Period: ________

Card Sort

10 Match each division problem with its equivalent representations.

a. $\frac{240}{100} \div \frac{8}{100}$ **b.** $\frac{240}{100} \div \frac{80}{100}$ **c.** $\frac{2400}{1000} \div \frac{8}{1000}$

d. 30 **e.** 300 **f.** $\frac{2400}{1000} \div \frac{800}{1000}$

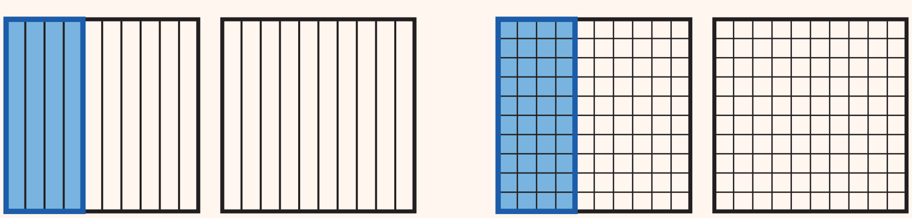
$2.4 \div 0.8$	$2.4 \div 0.08$	$2.4 \div 0.008$
b	a	c
f	d	e

Explore More

11 The value of $2 \div 0.4$ is 5.

How many other decimal division expressions can you write that have a value of 5?

Tenths **Hundredths**



Record a dividend and divisor for each expression. ***Responses vary.***

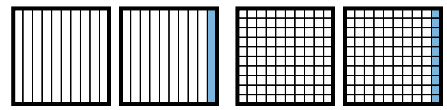
Dividend	2	0.2	4	3	6
Divisor	0.4	0.04	0.8	0.6	1.2

12 Synthesis

Circle an expression that has the same value as $1.9 \div 0.1$.

$\frac{19}{10} \div \frac{1}{10}$ $19 \div 1$ $190 \div 10$

Explain how you know it has the same value.
Use the diagrams if they help with your thinking.



Responses and explanations vary.

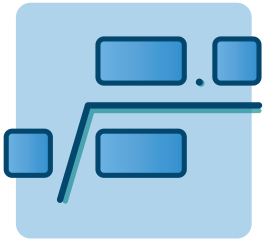
- **$\frac{19}{10} \div \frac{1}{10}$: Another way to think about 1.9 is as 19 tenths. $1.9 \div 0.1$ is like asking how many 1 tenths go into 19 tenths.**
- **$19 \div 1$: If I think about this expression in tenths, there are 19 tenths. Since I'm dividing by 0.1 or 1 tenth, I can calculate $19 \div 1$ to get the quotient.**
- **$190 \div 10$: If I use the hundredths chart, the whole is 190 and the part is 10. Then the division problem is like calculating $190 \div 10$.**

Things to Remember:

Name: .... Date: .... Period: ....

Return of the Long Division

Let's use long division to divide decimals.



Warm-Up

1. **Discuss:** What is the value of each expression?

a. $0.8 \div 4$ **= 0.2**

b. $0.24 \div 3$ **= 0.08**

c. $0.035 \div 5$ **= 0.007**

Activity 1

Name: ____________ Date: ____________ Period: ____________

Nur's and Shep's Strategies

Round 1

2. Use any strategy to calculate $26.5 \div 5$.

5.3

3. Let's look at Nur's and Shep's strategies for calculating $26.5 \div 5$.

Discuss: How are their strategies alike? How are they different?

Responses vary.

- **Alike: Nur and Shep both started by thinking about how 5 groups of 5 go into 26.**
- **Different: Nur kept the decimal point all the way down, thinking about how many groups of 5 go into 15 tenths. Shep, meanwhile, thought about place value at the end.**

Round 2

4. Use any strategy to calculate $106 \div 0.8$.

132.5

5. Let's look at Nur's and Shep's strategies for calculating $106 \div 0.8$.

Discuss: Why was it helpful to rewrite this expression as $1060 \div 8$?

Responses vary. **When Nur and Shep rewrote the expression this way, instead of dividing a whole number by a decimal, they divided a whole number by a whole number.**

Activity 1

Name: ______ Date: ______ Period: ______

Nur's and Shep's Strategies (continued)

Round 3

6. Calculate $19.8 \div 1.5$.

13.2

7. Let's look at Nur's and Shep's strategies for calculating $19.8 \div 1.5$.

Discuss: Why did they each write $198 \div 15$ instead of $1980 \div 15$?

***Responses vary.* To keep the place values the same, Shep and Nur multiplied both 19.8 and 1.5 by 10. If they wrote $1980 \div 15$, that would be like multiplying 19.8 by 100 and 1.5 by 10. Same with writing $198 \div 1.5$. That would be like multiplying 19.8 by 10 and 1.5 by 1.**

8. Use any strategy to show that these equations are true.

a. $0.7 \div 0.4 = 1.75$ ***Work varies.***

$$\frac{7}{10} \div \frac{4}{10} = 7 \div 4$$

$$\begin{array}{r} 1.75 \\ 4\overline{)7.00} \\ -4 \\ \hline 3\,0 \\ -2\,8 \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

b. $22.5 \div 0.04 = 562.5$ ***Work varies.***

$$\frac{2250}{100} \div \frac{4}{100} = 2250 \div 4$$

$$\begin{array}{r} 562.5 \\ 4\overline{)2250.0} \\ -20 \\ \hline 25 \\ -24 \\ \hline 10 \\ -8 \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

Activity 2

Name: ______ Date: ______ Period: ______

Finding Expressions

You will use the Activity 2 Cards to complete this table. You can use each card more than once.

9. Write down at least *one* expression that . . . ***Responses vary.***

. . . includes division by a number:	. . . has a quotient:
Greater than 1. **Cards A, B, C, G, H, I, J, and K**	Less than 1. **Cards C and I**
Less than 1. **Cards D, E, F, and L**	Greater than 15. **Cards D, E, K, and L**
In the hundredths place. **Cards D and E**	Close to 10. **Cards A, B, F, H, and J**

10. Work with a partner to calculate the value of at least three expressions each. Show all of your thinking. Make sure you and your partner select different expressions. When you're finished, compare your thinking with your partner. ***Responses and work vary.***

Expression: ______ Expression: ______ Expression: ______

My work: My work: My work:

Card A: 12.4 **Card B: 10.25** **Card C: 0.04**

Card D: 157.5 **Card E: 102.5** **Card F: 9.25**

Card G: 7.5 **Card H: 12.4** **Card I: 0.875**

Card J: 10.765 **Card K: 15.48** **Card L: 24.51**

Synthesis

11. What are some things you think are important to remember when dividing with decimals?

$26.5 \div 5$

$57 \div 1.5$

$5.11 \div 0.05$

Use the examples if they help with your thinking.

Responses vary.

- **It's important to remember that you can rewrite the division problem so you aren't dividing by a decimal.**
- **You can use a hundredths chart or write both numbers as fractions to help you think about how to rewrite the problem.**
- **If the quotient isn't a whole number, you can just keep dividing. When you divide the remainder, it'll be a decimal.**

Things to Remember:

Finding Expressions

Directions: Make one copy per pair of students. Then pre-cut the cards and give each pair of students one set.

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Card A

$62 \div 5$

Card B

$41 \div 4$

Card C

$1 \div 25$

Card D

$12.6 \div 0.08$

Card E

$5.125 \div 0.05$

Card F

$3.7 \div 0.4$

Card G

$9 \div 1.2$

Card H

$18.6 \div 1.5$

Card I

$7 \div 8$

Card J

$53.825 \div 5$

Card K

$77.4 \div 5$

Card L

$7.353 \div 0.3$

Name: ________ Date: ________ Period: ________

Grocery Prices

Let's explore the cost of groceries in different cities.

Warm-Up

In 2021, the average household in Austin, Texas, made about $1,500.00 per week.[1]

1. **Discuss:** What percentage of their income do you think a family might spend to buy the groceries on this list?

***Responses vary.* About 5% of their income.**

Groceries	
Milk (1 gal)	Apples (1 lb)
Bread (1 loaf)	Bananas (1 lb)
Rice (1 lb)	Oranges (1 lb)
Eggs (1 dozen)	Tomatoes (1 lb)
Cheese (1 lb)	Potatoes (1 lb)
Chicken (1 lb)	Onion (1 lb)
Beef (1 lb)	Lettuce (1 head)

2. Complete this statement using the screen:

These groceries would cost an average household in Austin **2.2** % of their income.

3. Would this list of groceries cost more than $100.00 or less than $100.00? Explain your thinking.

Less than $100.00. *Explanations vary.* I know 1% of $1,500.00 is $15.00, so 2% is $30.00, and 3% is $45.00. So 2.2% would be between $30.00 and $45.00.

[1] Source: *U.S. Bureau of Labor Statistics*

Activity 1

Name: ______ Date: ______ Period: ______

Grocery Prices

Tyani and Anika were asked to calculate approximately how much these groceries would cost for the average household in Austin, Texas. They each wrote expressions to help them. Here is their work.

Tyani	Anika
2.2% of 1500	2.2% of 1500
$= 0.22 \cdot 1500$	$= 0.022 \cdot 1500$

4. Whose work is correct? Explain your thinking.

Anika's. *Explanations vary.* 2.2% is the same as 2.2 per 100, or 22 thousandths. 22 thousandths is equivalent to 0.022, which Anika used in her work.

5. Calculate 2.2% of \$1,500.00. Does this number make sense for this situation?

\$33.00. *Responses vary.*
- **This number makes sense because it's close to how much money my family spends on those items.**
- **This number doesn't make sense because groceries for a family would cost more than \$33.00.**

Here is the approximate average weekly household income in 2021 for three other cities.[1]

Honolulu, Hawaii	Seattle, Washington	Jackson, Mississippi
\$1,700.00	\$2,100.00	\$700.00

6. If the average household in each city spent 2.2% of their weekly income on these groceries, how much would they spend? Complete the table and show your thinking.

	Honolulu	Seattle	Jackson
Expression for 2.2% of Income	$0.022 \cdot 1700$	$0.022 \cdot 2100$	$0.022 \cdot 700$
Money Spent	\$37.40. *Work varies.* 1% is \$17. So, 2% is \$34, and 0.2% is \$3.40. $34 + 3.40 = \$37.40$.	\$46.20. *Work varies.* $2100 \cdot \frac{22}{1000} = \frac{46200}{1000}$ $\frac{46200}{1000} = \$46.20$	\$15.40. *Work varies.* $700 \cdot 0.02 = 14$ $700 \cdot 0.002 = 1.40$ $14 + 1.40 = \$15.40$

[1] Source: *U.S. Bureau of Labor Statistics*

Name: Date: Period:

Grocery Prices (continued)

You'll use the Activities 1 & 2 Sheet to complete this activity.

7. In which cities can someone buy all the groceries on the list using 2.2% or less of their weekly income? In which cities can they not?

People in Seattle, Washington and Austin, Texas can buy all of the groceries using 2.2% or less of their income. People in Honolulu, Hawaii and Jackson, Mississippi cannot.

8. **Discuss:**

a Why do you think the cost of these groceries is different in different places?

***Responses vary.* The cost of living in certain cities is more expensive than in other cities. Local stores might raise their prices in cities where there's a higher average income. It's also possible that shipping groceries to remote locations gets expensive, so grocery stores increase their prices to match.**

b What do you think the impact might be on families who spend more than 2.2% of their weekly income on these groceries?

***Responses vary.* If families spend more than 2.2% of their income on food, then they'll have less money for other things, like housing, education, or even fun stuff. It might also mean they can't buy as many healthy food options for their families.**

Activity
2

Name: ______ Date: ______ Period: ______

Bought Milk?

You'll use the Activities 1 & 2 Sheet to complete this activity.

9. How much does a gallon of milk cost in Austin, Texas?
$\$3.19$

10. What percentage of the total cost of the grocery list is that? Show your thinking.

A. 0.0967% **B.** 0.967% **C.** 9.67% **D.** 96.7%

***Work varies*. $3.19 \div 33 \approx 0.0967$ and $0.0967 = 9.67\%$**

Yunuen says: *Milk is too expensive in Jackson. It's* 11% *of the total cost of that grocery list!*

11. Show or explain where the 11% comes from.
***Work varies*. $4.68 \div 42.29 \approx 0.11$ and $0.11 = 11\%$**

12. Do you agree with Yunuen? If you do, explain what you think would be a fair price for milk in Jackson. If you do not, explain why you think milk in Jackson is priced fairly.
Responses vary.
- **I agree with Yunuen. I think milk in Jackson should be the same percentage of the total grocery cost as milk in Austin: 9.67%. That means milk should cost $42.29 \cdot 0.0967 \approx \$4.09$ in Jackson.**
- **I disagree with Yunuen. Maybe there aren't many dairy farms in Mississippi, so grocery stores have to transport the milk from other places. That would be an extra cost for the stores, which might be why the milk costs more for customers, too.**

13. Choose a different city. What percentage of the total grocery bill is milk? Do you think milk is too expensive in this city? Show or explain your thinking.
Responses and explanations vary.
- **In Honolulu, milk is about 11.3% of the grocery bill. I'd say milk is too expensive here because it costs more than 9.67% of the grocery list. $6.77 \div 59.95 \approx 0.113$ and $0.113 = 11.3\%$.**
- **In Seattle, milk is about 8.3% of the grocery bill. I think milk is affordable here because it makes up a smaller percentage of the grocery cost compared to the other cities. $3.60 \div 43.39 \approx 0.083$ and $0.083 = 8.3\%$.**

Explore More

14. Choose a different food that has a price that you think is too high. Use percentages, decimals, rates, or ratios to propose a fairer price for that food.
Responses vary.

Synthesis

15. Choose *one* question and write your response.

a What do you think is a fair way to determine the price for groceries in different places?

***Responses vary.* A fair way to determine prices is to try to make them about the same percentage of a person's food budget, no matter where they live.**

b What can people do if they live in a place where the price of groceries is high?

Responses vary.

- **They can try to buy groceries somewhere where they're less expensive.**
- **They can buy groceries in bulk and share them with other families. Usually, bulk groceries cost less per item.**
- **They can help others by donating to a local food bank.**

c What new questions do you have about food costs around the world?

Responses vary.

- **Why is food so expensive in Jackson? How can we make it so food is less expensive?**
- **What happens when there isn't enough food in a place? Does it get even more expensive?**
- **What are the most affordable foods people can eat that are also very nutritious?**

Things to Remember:

Name: ____________ Date: ____________ Period: ____________

Grocery Prices

All prices are based on data from the USDA in 2021, as collected by Balancing Everything.[1]

Seattle, Washington

Milk (1 gal)	\$3.60
Bread (1 loaf)	\$3.06
Rice (1 lb)	\$2.03
Eggs (1 dozen)	\$3.01
Cheese (1 lb)	\$7.29
Chicken (1 lb)	\$5.58
Beef (1 lb)	\$6.67
Apples (1 lb)	\$2.30
Bananas (1 lb)	\$0.81
Oranges (1 lb)	\$2.11
Tomatoes (1 lb)	\$2.59
Potatoes (1 lb)	\$1.06
Onion (1 lb)	\$1.19
Lettuce (1 head)	\$2.09
Total	**\$43.39**

Honolulu, Hawaii

Milk (1 gal)	\$6.77
Bread (1 loaf)	\$4.82
Rice (1 lb)	\$2.65
Eggs (1 dozen)	\$4.52
Cheese (1 lb)	\$7.21
Chicken (1 lb)	\$6.37
Beef (1 lb)	\$7.55
Apples (1 lb)	\$3.30
Bananas (1 lb)	\$1.70
Oranges (1 lb)	\$3.13
Tomatoes (1 lb)	\$3.16
Potatoes (1 lb)	\$2.52
Onion (1 lb)	\$2.62
Lettuce (1 head)	\$3.63
Total	**\$59.95**

Austin, Texas

Milk (1 gal)	\$3.19
Bread (1 loaf)	\$2.21
Rice (1 lb)	\$1.34
Eggs (1 dozen)	\$2.77
Cheese (1 lb)	\$5.00
Chicken (1 lb)	\$3.88
Beef (1 lb)	\$5.86
Apples (1 lb)	\$1.57
Bananas (1 lb)	\$0.61
Oranges (1 lb)	\$1.49
Tomatoes (1 lb)	\$1.38
Potatoes (1 lb)	\$1.12
Onion (1 lb)	\$1.02
Lettuce (1 head)	\$1.56
Total	**\$33.00**

Jackson, Mississippi

Milk (1 gal)	\$4.68
Bread (1 loaf)	\$3.66
Rice (1 lb)	\$2.00
Eggs (1 dozen)	\$1.63
Cheese (1 lb)	\$5.99
Chicken (1 lb)	\$4.75
Beef (1 lb)	\$3.98
Apples (1 lb)	\$3.66
Bananas (1 lb)	\$0.69
Oranges (1 lb)	\$2.41
Tomatoes (1 lb)	\$2.82
Potatoes (1 lb)	\$2.03
Onion (1 lb)	\$1.82
Lettuce (1 head)	\$2.17
Total	**\$42.29**

[1] Source: *Balancing Everything*

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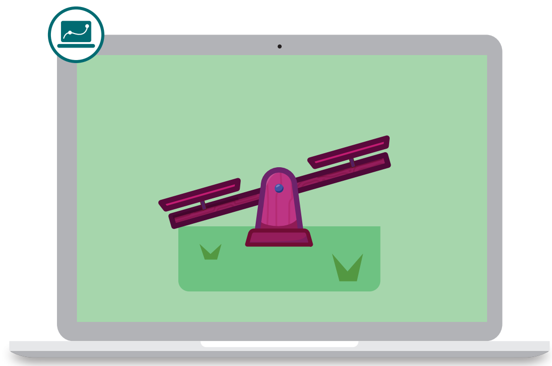
Unit 6
Lesson
1

Name: ________ Date: ________ Period: ________

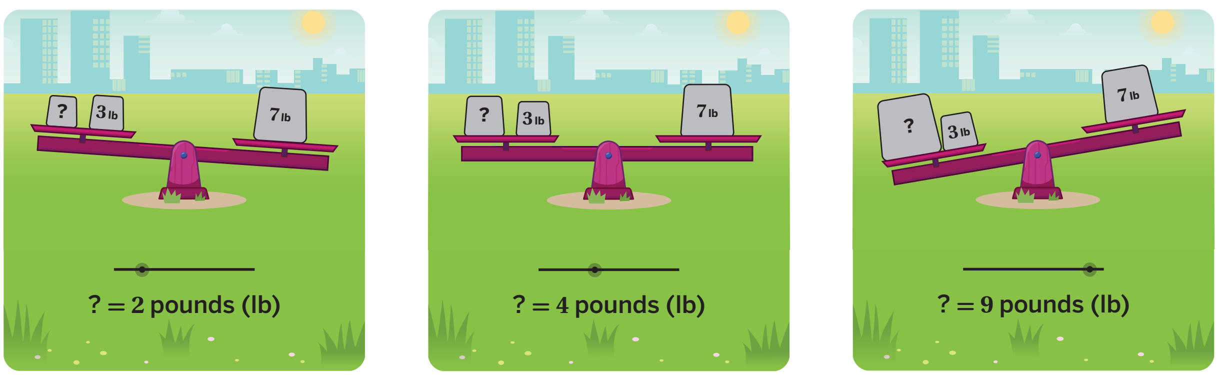
Weight for It

Let's use a seesaw to determine the weights of different animals.



Warm-Up

1 **a** Take a look at some weights on a seesaw.



b What do you notice? What do you wonder? ***Responses vary.***

I notice:

- **When the weight is 4 pounds, the seesaw balances.**
- **When the weight is 2 pounds or 9 pounds, the seesaw is not balanced.**

I wonder:

- **What is special about the number 4.**
- **How many pounds the seesaw can hold.**
- **Whether I would fall off if a giant weight fell on the other end of the seesaw.**

Activity 1

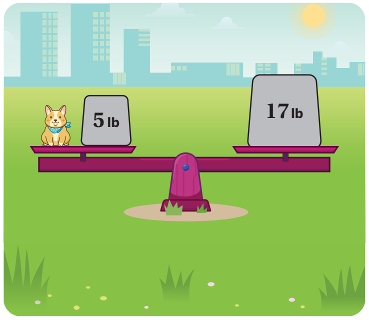
Name: ________ Date: ________ Period: ________

Equations and Tape Diagrams

2 This dog and a 5-pound weight balance a 17-pound weight.

How much does the dog weigh?

12 pounds



3 Tariq wrote an *equation* to represent the situation. He used the **variable** d to represent the dog's weight.

Tariq

$d + 5 = 17$

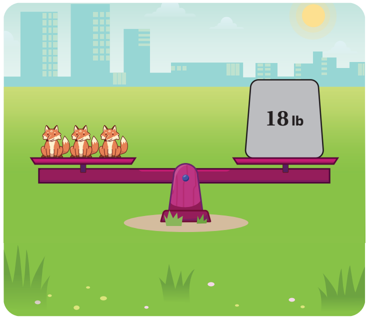
Explain how Tariq's equation is like the seesaw situation.

Responses vary. **The left side of the equation is like the left side of the seesaw. It is the weight of the dog plus a 5-pound weight, so the left side of the equation is $d + 5$. The right side of the equation, 17, is like the 17-pound weight on the right side of the seesaw.**

Name: Date: Period:

Equations and Tape Diagrams (continued)

4 These 3 foxes balance with an 18-pound weight. Each fox weighs the same amount.



a Choose an equation that represents this situation.

A. $3 + x = 18$

B. $3 \cdot x = 18$

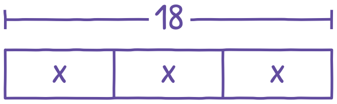
C. $x + x + x = 18$

D. $3 + 18 = x$

b How much does each fox weigh?

6 pounds

5 Tariq drew a tape diagram to determine the weight of each fox.



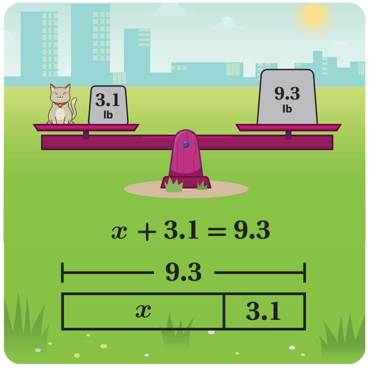
How are the tape diagram and the equation you chose in the previous problem alike?

***Responses vary.* The tape diagram is like the left side of the equation. There are 3 x's, so the tape diagram has 3 parts labeled x. The total in the tape diagram, 18, is like the right side of the equation, which is equal to 18.**

6 This cat and a 3.1-pound weight balance a 9.3-pound weight.

How much does the cat weigh? Use the tape diagram if it helps with your thinking.

6.2 pounds

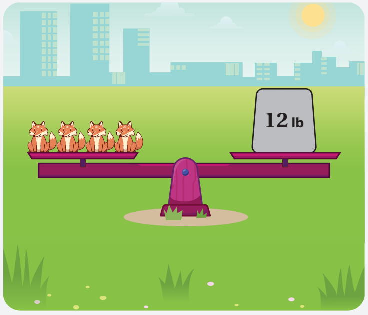
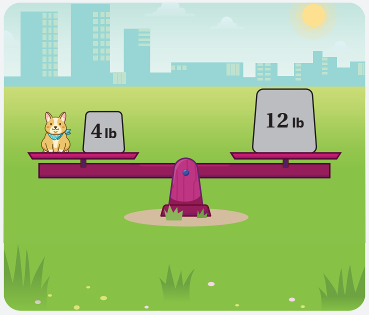


Activity 2

Name: ______ Date: ______ Period: ______

Determining Unknown Weights

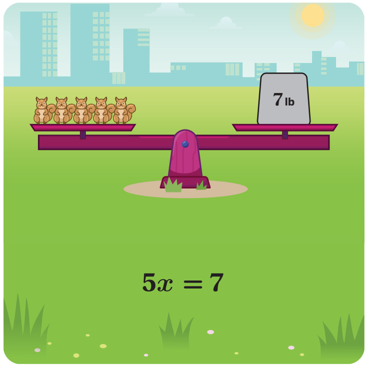
7 For each equation or tape diagram, put a check mark under the balanced seesaw it represents.

		
$4 \cdot x = 12$	✓	
$4 + x = 12$		✓
$x + x + x + x = 12$	✓	
$x = 8$		✓
12 x | x | x | x	✓	
$x = 3$	✓	
12 4 | x		✓

8 These 5 squirrels balance with a 7-pound weight. Each squirrel weighs the same amount.

How much does each squirrel weigh? Draw a tape diagram if it helps with your thinking.

1.4 pounds (or equivalent)



Activity 3

Name: ________ Date: ________ Period: ________

Challenge Creator

9 You will use the Activity 3 Sheet to create your own seesaw challenge.

a **Make It!** On the Activity 3 Sheet, create a balanced seesaw challenge.

b **Solve It!** On this page, write an equation that represents your balanced seesaw problem. Then determine the weight of your animal. Draw a tape diagram if it helps with your thinking.

Responses vary.

My Equation	Weight of My Animal

c **Swap It!** Swap your challenge with one or more partners. Write your partner's equation, then determine the weight of their animal. Draw a tape diagram if it helps with your thinking.

Responses vary.

	Equation	Weight of One Animal
Partner 1		
Partner 2		
Partner 3		
Partner 4		

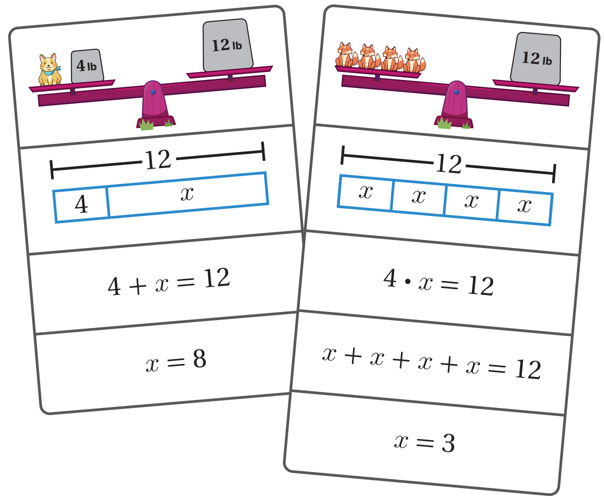
10 Synthesis

How can you tell if an equation and a tape diagram match?

I can tell if an equation and a tape diagram match . . .

Responses vary.

- **If all of the pieces in the tape diagram are the same as one side of an equation and the total length of the tape diagram is the same as the other side of the equation.**
- **By seeing if I can draw the same picture for both of them, like a seesaw or another drawing.**
- **By replacing the variable in the equation and tape diagram with the answer and seeing if they are both correct.**



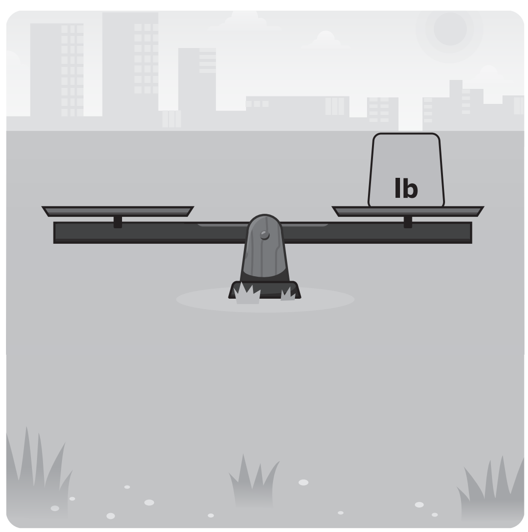
Things to Remember:

Challenge Creator

- Choose one animal from the pictures or make up your own.
- Create a balanced seesaw. Draw copies of your animal on the left side of the seesaw. If you want to, add extra weight on the left side with your animal. Then fill in the weight on the right side.
- Write an equation that represents your balanced seesaw.
- Do not determine the weight of the animal on this page. You and your classmates will determine the weight of each other's animals in your Student Edition.

Weight Cat Dog Squirrel Fox Raccoon Alligator Frog

My seesaw:



My equation: ....

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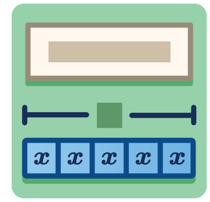
Name: ______________ Date: ________ Period: ________

Five Equations



Let's represent situations with equations and tape diagrams.

Warm-Up

1. Here is a situation. Let's make sense of it together as a class.

_____ cats weigh _____ pounds total.

Each cat weighs the same, so they each weigh _____ pounds.

a **Discuss:** What is this situation about?

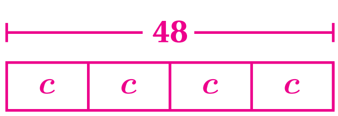
Responses vary. **This situation is about the weight of some cats.**

b Let's look at the missing information. What does the variable c represent?

Responses vary. **The variable c represents the weight of each cat.**

c Create a tape diagram or sketch that represents this situation.

Responses vary.



d Use your tape diagram or sketch to determine the value of c.

$c = 12$ **pounds**

Activity 1

Name: ______ Date: ______ Period: ______

Equations and Tape Diagrams

Here are five equations.

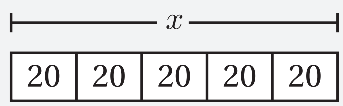
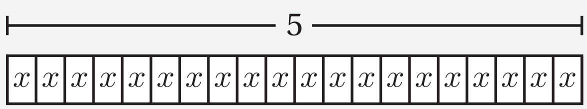
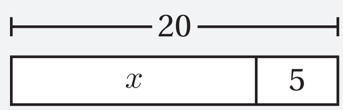
$x + 5 = 20$	$20 = x - 5$	$5 \cdot 20 = x$	$5x = 20$	$20x = 5$

2. Circle two equations that have something in common. ***Responses vary.***

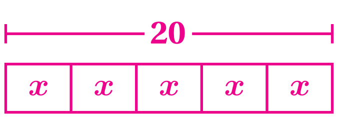
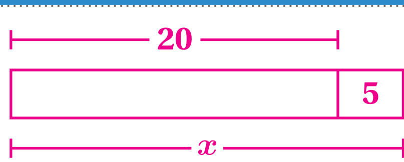
Discuss: How are these equations alike? How are they different?

Responses vary. **$x + 5 = 20$ and $20x = 5$ both have 5, 20, and x. $x + 5 = 20$ involves addition and $20x = 5$ involves multiplication.**

3. Match each tape diagram with one of the equations. Two equations will not have matches.

Tape Diagram	Equation
	**$5 \cdot 20 = x$**
	**$20x = 5$**
	**$x + 5 = 20$**

4. Draw a tape diagram for an equation that did not have a match. ***Responses vary.***

Tape Diagram	Equation
 or 	**$5x = 20$ or $20 = x - 5$**

Activity 2

Name: ______ Date: ______ Period: ______

Which Equation?

4 cats weigh 48 pounds total. Each cat weighs the same, so they each weigh c pounds.

Equation	Solution to the Equation	Solution's Meaning
$4c = 48$	$c = 12$	Each cat weighs 12 pounds.

5. What do you think a **solution to an equation** is?

Responses vary. **The solution to an equation is the number that can replace the variable to make the equation true. 4 times 12 equals 48, so the solution to $4c = 48$ is 12.**

You will use a set of description and situation cards.

6. Match each card with the equation that represents it.

7. Determine the solution to each equation and write the solution's meaning for each situation.

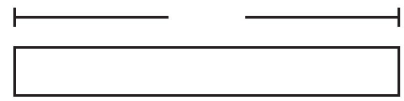
	$x - 9 = 22$	$9x = 22$	$9 + x = 22$
Matching Cards	Card 3 Card 5	Card 4 Card 6	Card 1 Card 2
Solution to the Equation	$x = 31$	$x = \frac{22}{9} = 2\frac{4}{9}$	$x = 13$
Solution's Meaning	Card 3: Mohamed made \$31. Card 5: 9 less than 31 is 22.	Card 4: The product of 9 and $\frac{22}{9}$ is 22. Card 6: Ren can afford to buy 2 day passes because you cannot buy part of a pass.	Card 1: Kwasi has 13 stops left on the bus. Card 2: 22 is 9 more than 13.

Synthesis

8. How can you tell which equation represents a situation? Use the example if it helps with your thinking.

Responses vary. **You can tell which equation represents a situation by deciding which equation shows the same relationship between the values. Also, you can check to see if the solution to the equation makes sense in the situation. For example, the equation $x + 5 = 20$ represents the situation with Kwasi because the solution is $x = 15$ and he would have 5 stops left after 15 stops.**

Kwasi rides the subway 20 stops to get to work. After x stops, he has 5 stops left.



$x + 5 = 20$

$5x = 20$

Things to Remember:

Which Equation?

Lesson 6.02
Activity 2

Directions: Make one copy per two pairs of students. Then pre-cut the cards and give each pair of students one set of Cards 1–6.

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Card 1

Kwasi rides the bus for a total of 22 stops. After 9 stops, there are x stops left.

Card 2

22 is 9 more than x.

Card 3

Mohamed made \$$x$ from mowing lawns. He spent \$9 on a new video game and has \$22 left.

Card 4

The product of 9 and x is 22.

Card 5

9 less than x is 22.

Card 6

Ren has \$22 to spend on day passes to ride the subway. Each day pass costs \$9, and Ren can buy x of them.

Card 1

Kwasi rides the bus for a total of 22 stops. After 9 stops, there are x stops left.

Card 2

22 is 9 more than x.

Card 3

Mohamed made \$$x$ from mowing lawns. He spent \$9 on a new video game and has \$22 left.

Card 4

The product of 9 and x is 22.

Card 5

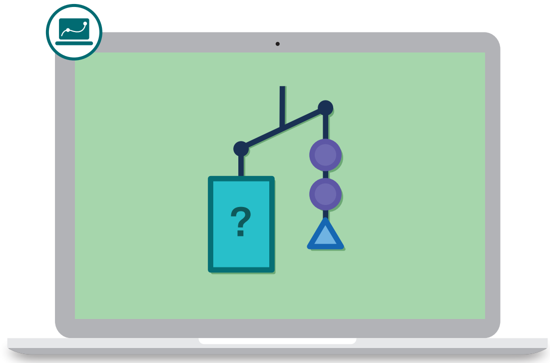
9 less than x is 22.

Card 6

Ren has \$22 to spend on day passes to ride the subway. Each day pass costs \$9, and Ren can buy x of them.

Unit 6
Lesson
3

Name: ____________ Date: ________ Period: ________

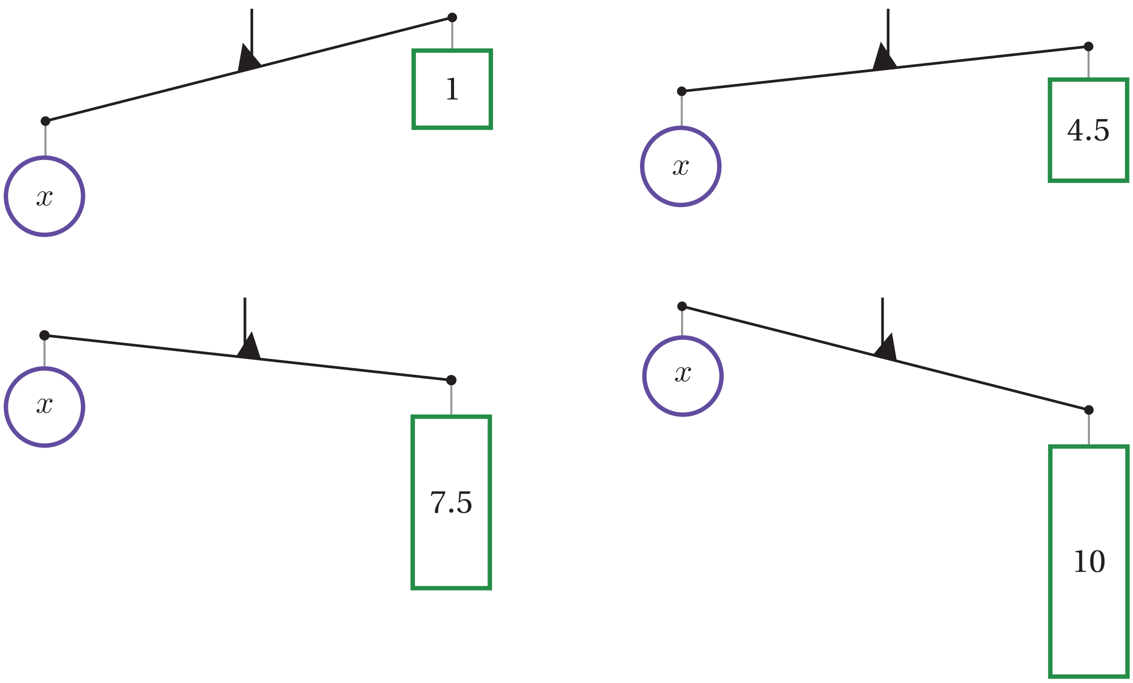


Hanging Around

Let's use balanced hangers to solve equations.

Warm-Up

1 **a** Take a look at the hangers with a circle of weight x on one side and a rectangle of different weights on the other side.



b **Discuss:** What is the weight of the circle? Explain your thinking.

***Responses vary.* It is more than 4.5 and less than 7.5. It is about 6 because that is when the hanger is closest to being balanced.**

Activity

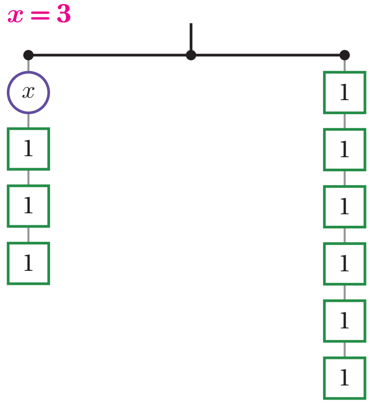
1

Name: Date: Period:

Connect It

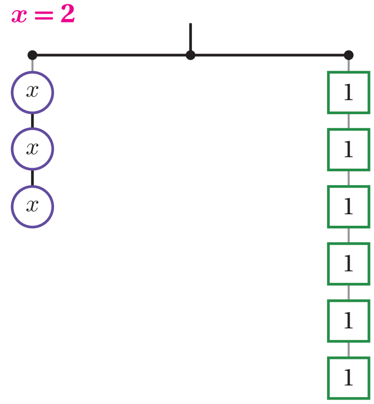
2 What value of x balances the hanger?

$x = 3$

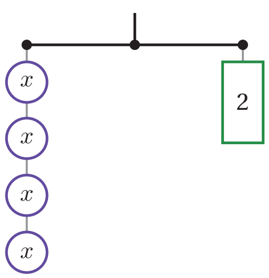


3 What value of x balances the hanger?

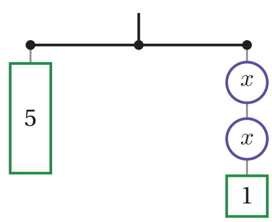
$x = 2$



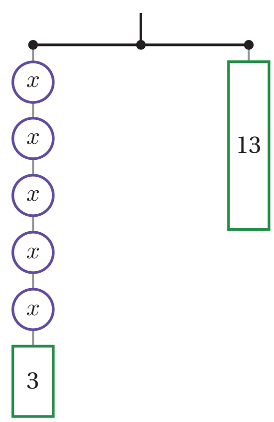
4 **a** Take a look at these hangers and the equations that represent them.



Equation: $4x = 2$



Equation: $5 = 2x + 1$



Equation: $5x + 3 = 13$

b Explain how an equation is like a hanger.

Responses vary. **One side of the equation is like one side of the hanger. If you have 5 x's and a 3 on one side of the hanger, then one side of the equation would be $5x + 3$. The equal sign is like the hanger being balanced.**

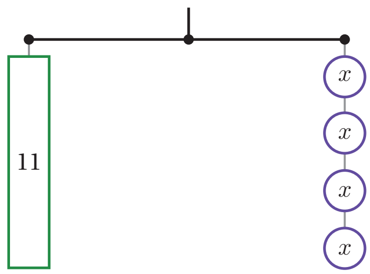
Activity
2

Name: .... Date: .... Period: ....

Make It, Solve It

5 Select an equation that represents this hanger.

Responses vary.



A. $11 + x = 4$

B. $11 = 4x$

C. $11 = x + 4$

D. $11 = x + x + x + x$

Explain your thinking.

***Explanations vary*. There is an 11 on the left side of the hanger and 4 x's on the right side of the hanger.**

6 Use the hanger or the equation from the previous problem to determine the value of x that balances the hanger.

$x = 2\frac{3}{4}$ (or equivalent)

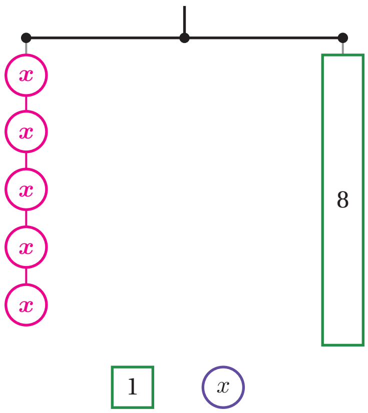
Activity 2

Name: ______ Date: ______ Period: ______

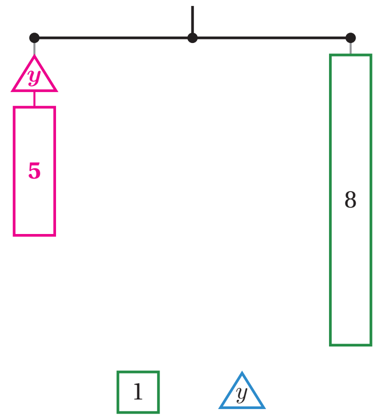
Make It, Solve It (continued)

7 Make a balanced hanger to represent each equation. ***Responses vary.***

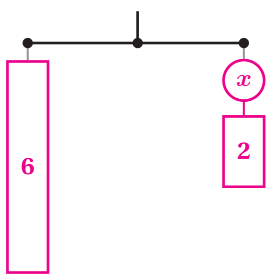
$5x = 8$



$5 + y = 8$



8 **a** Make a balanced hanger that represents $6 = x + 2$. ***Responses vary.***



b What is the value of x that balances the hanger?

$\boldsymbol{x = 4}$

Activity 3

Name: ______ Date: ______ Period: ______

Challenge Creator

9 You will use the Activity 3 Sheet to create your own hanger challenge.

a **Make It!** On the Activity 3 Sheet, create your own balanced hanger challenge.

b **Solve It!** On this page, write the equation that represents your hanger and then determine the value of x that balances your hanger.

Responses vary.

My Equation	Solution to My Equation

c **Swap It!** Swap your challenge with one or more partners. Write their equation, then determine the value of x that balances their hanger.

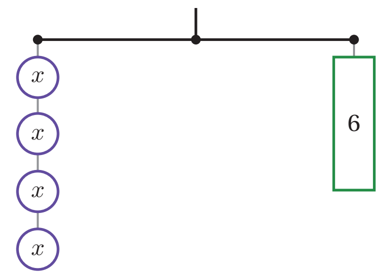
Responses vary.

	Equation	Solution to Their Equation
Partner 1		
Partner 2		
Partner 3		
Partner 4		

10 Synthesis

How can a balanced hanger help determine the solution to an equation?

Use the hanger and equation if that helps you with your thinking.



$4x = 6$

***Responses vary.* A balanced hanger can help determine the solution to an equation because it shows what the value of x must be to make both sides have the same total, which is the same as what value of x would make both sides of the equation equal.**

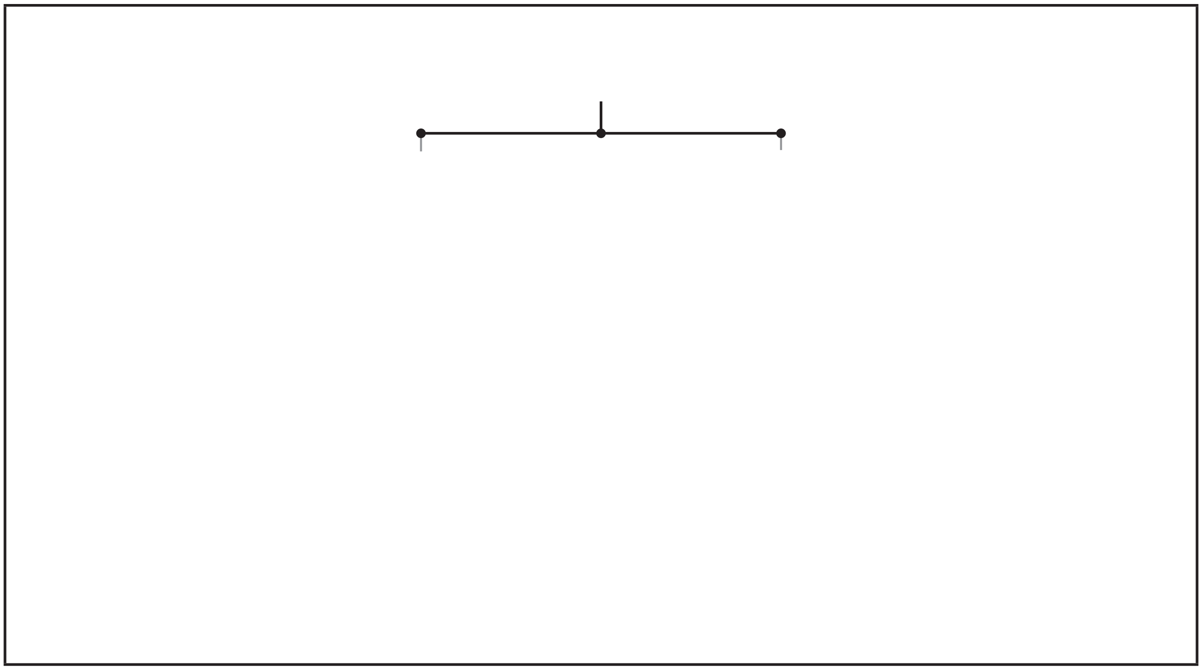
Things to Remember:

Name: .... Date: .... Period: ....

Challenge Creator

Create a balanced hanger using circles and squares. Write an equation that represents your hanger. Do not determine the solution to the equation on this page.

x $\boxed{1}$



Equation: ....

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Name: .... Date: .... Period: ....

Challenge Creator

- Create your own rectangle. Try to create a rectangle none of your classmates will.
- Write one expression to represent its area.
- Your classmates will write an expression that is equivalent to the one you wrote.

Rectangle:

Expression: ....

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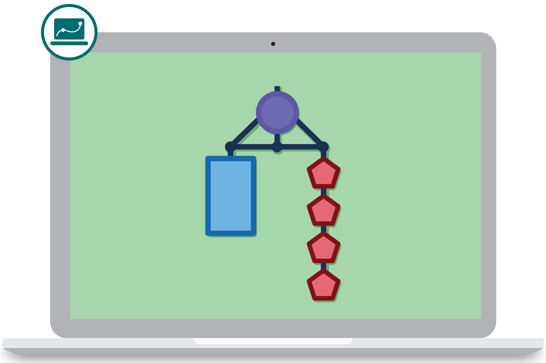
Unit 6
Lesson
4

Name: Date: Period:

Hanging It Up

Let's use a variety of strategies to solve equations.



Warm-Up

Determine the value of each expression mentally. Try to think of more than one strategy.

1. $5 - 2 = \mathbf{3}$

2. $5 - 2.1 = \mathbf{2.9}$

3. $5 - 2.17 = \mathbf{2.83}$

4. $5 - 2.017 = \mathbf{2.983}$

Activity 1

Name: ______ Date: ______ Period: ______

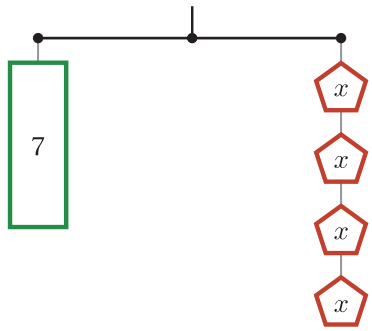
Methods for Solving

5 **a** Write an equation that matches the hanger.

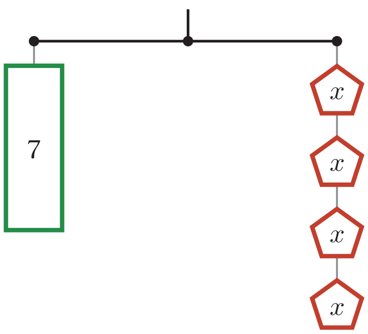
$7 = 4x$

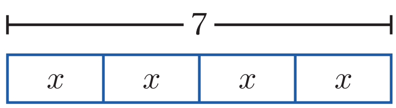
b What is the *solution* to this equation?

$x = 1.75$ (or equivalent)



6 Here are two different representations of the equation $7 = 4x$.





Discuss: How can you determine the solution using each representation?

Responses vary. For the hanger, if you replace each x with 1.75, the right side of the hanger is equal to 7, which balances with the 7 on the left side of the hanger. In the tape diagram, there are 4 sections that total to 7, so each section is $\frac{7}{4}$.

7 Here is how Fabiana solved the equation $7 = 4x$. Describe Fabiana's strategy.

Responses vary. Fabiana divided both sides of the equation by 4.

Fabiana

$$\frac{7}{4} = \frac{4x}{4}$$

Name: .... Date: .... Period: ....

Solving and Solutions

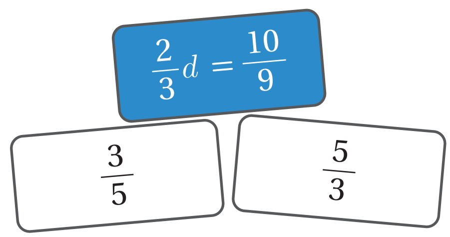
8 Match each solution to its equation. One solution will not have a match.

8.5	0.1	6.1
$\frac{3}{5}$	$\frac{1}{10}$	$\frac{5}{3}$

$\frac{2}{3}d = \frac{10}{9}$	$12.6 = b + 4.1$	$10c = 1$	$10 + a = 16.1$
$\frac{5}{3}$	8.5	$\frac{1}{10}$ 0.1	6.1

Solution with no match: $\frac{3}{5}$

9 Imani and Demari solved this equation.



Imani said the solution is $d = \frac{3}{5}$.

Demari said the solution is $d = \frac{5}{3}$.

Whose solution is correct? Circle one.

Imani's | Demari's | Both | Neither

Explain your thinking.

Explanations vary. **I am trying to determine what number times $\frac{2}{3}$ equals $\frac{10}{9}$, and $\frac{2}{3} \cdot \frac{5}{3} = \frac{10}{9}$. I know it can't be $d = \frac{3}{5}$ because the denominator would be 15 instead of 9.**

Name: .... Date: .... Period: ....

Solving and Solutions (continued)

10 Here are Fabiana's and Alejandro's strategies for solving $10 + a = 16.1$.

Fabiana

$10 + a = 16.1$

$-10 \qquad -10$

$a = 6.1$

Alejandro

$10 + a = 16.1$

$10 + 6 = 16$

so . . .

$10 + 6.1 = 16.1$

Discuss: How are their strategies alike? How are they different?

Responses vary. **The strategies are alike because both students found the value of a that makes the equation true. Fabiana found the value of a by using inverse operations and subtracting 10 from both sides of the equation. Alejandro found the value of a by using number sense.**

11 Use Fabiana's strategy to solve the equation $3.5 = x + 2.01$.

$\mathbf{1.49 = x}$

Activity 3

Name: ____ Date: ____ Period: ____

Repeated Challenges

12

- Decide with your partner who will complete Column A and who will complete Column B.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.
- Solve as many equations as you have time for. Sense-making is more important than speed.

	Column A	Column B
a	$36 = 4x$ $x =$ **9**	$7x = 63$ $x =$ **9**
b	$13 = x + 5$ $x =$ **8**	$21 = x + 13$ $x =$ **8**
c	$\frac{1}{3} = 2x$ $x =$ $\frac{1}{6}$	$3x = \frac{1}{2}$ $x =$ $\frac{1}{6}$
d	$x + 6.17 = 9$ $x =$ **2.83**	$12.22 = x + 9.39$ $x =$ **2.83**

Name: .... Date: .... Period: ....

Repeated Challenges (continued)

	Column A	Column B
e	$x + 1.8 = 14.7$ $x =$ **12.9**	$x + 5.3 = 18.2$ $x =$ **12.9**
f	$\frac{1}{2}x = 16$ $x =$ **32**	$4 = \frac{1}{8}x$ $x =$ **32**
g	$\frac{7}{8} = x + \frac{1}{4}$ $x =$ $\frac{5}{8}$	$x + \frac{1}{16} = \frac{11}{16}$ $x =$ $\frac{5}{8}$

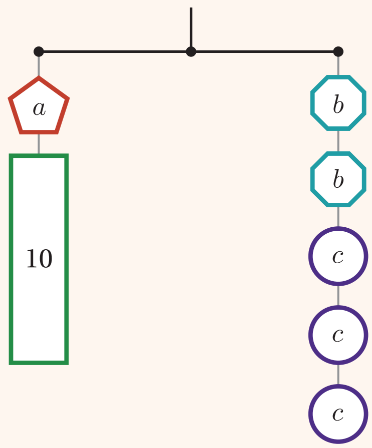
Explore More

13 Complete the table with values for a, b, and c that will make the hanger balance.

Try to determine as many different sets of values as you can.

Responses vary.

a	b	c
2	1.5	3
7	1	5
90	20	20



14 Synthesis

Describe a strategy for solving an equation.

Use the examples if they help with your thinking.

$$3x = 18$$

$$3 + x = 15.6$$

Responses vary.

- I can solve an equation by getting the variable alone on one side of the equation. For the equation $3 + x = 15.6$, I can subtract 3 from both sides. In the equation $3x = 18$, I can divide both sides by 3.
- I can solve an equation by thinking about a tape diagram. For $3x = 18$, I can draw a tape diagram with a total length of 18 and divide my diagram into three equal sections.
- I can solve an equation by thinking about which number makes an equation true. For $3 + x = 15.6$, I can think $3 + 12 = 15$, so $3 + 12.6 = 15.6$.

Things to Remember:

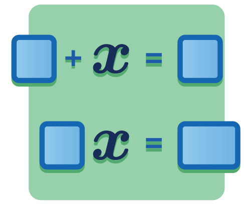
Unit 6

Lesson

5

Name: ______ Date: ______ Period: ______

Swap and Solve



Let's write and solve equations.

Warm-Up

1. Here is a situation with hidden information. Let's make sense of it as a class.

Takeshi has ______ to spend on laundry. It costs ______ to wash and dry each load. Takeshi can wash ______ loads of laundry.

2. Dhruv and Nyanna each wrote an equation to represent this situation.

Dhruv

$p = 21 \cdot 3.50$

Nyanna

$3.50p = 21$

Whose equation is correct? How do you know?

Nyanna's. *Explanations vary.* Each load of laundry costs $3.50, so you can multiply 3.50 by the number of loads to get the total cost.

Name: .... Date: .... Period: ....

Stronger and Clearer Each Time

Here is a set of equations we will use throughout this lesson.

$x+\frac{3}{4}=6$	$\frac{3}{4}x=6$	$6x=\frac{3}{4}$	$x-\frac{3}{4}=6$
$0.25+x=20$	$0.25x=20$	$20 \cdot 0.25=x$	$x-20=0.25$

3. Select an equation and solve it for x. ***Responses vary.***

Equation	Solution
$x+\frac{3}{4}=6$	$x+\frac{3}{4}-\frac{3}{4}=6-\frac{3}{4}$ $x=5\frac{1}{4}$

4. Write a first draft of a situation to match this equation. Make sure to include what the variable represents in your situation.

***Responses vary.* Before my break, I worked for x hours. Then I worked for $\frac{3}{4}$ of an hour. I worked 6 hours total.**

5. Meet with a partner to discuss your first drafts. Use the questions on the screen to help you provide feedback to each other.

6. Write a second draft that is stronger and clearer.

Responses vary.

Name: .... Date: .... Period: ....

Trade and Solve

Takeshi has $21 to spend on laundry. It costs $3.50 to wash and dry each load. Takeshi can wash p loads of laundry.

Equation	Solution	Solution Check	Solution's Meaning
$3.50p = 21$	$p = 6$	$3.50 \cdot 6 = 21$	Takeshi can wash and dry 6 loads of laundry for $21.

7. What do you think a solution's meaning is?

***Responses vary.* A solution's meaning is what that number represents in the situation. Here, the solution is 6 and its meaning is how many loads of laundry Takeshi can do.**

8. You will need several different partners for this activity. With each partner, trade the slips of paper with your situations and complete the table for their situation.

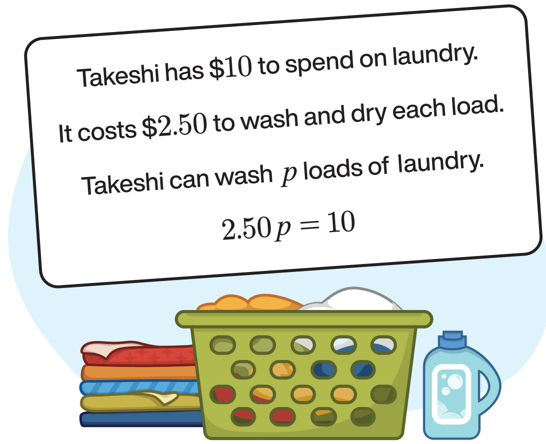
Responses vary.

	Partner A	Partner B	Partner C
Partner's Name			
Equation			
Solution			
Solution Check			
Solution's Meaning			

Synthesis

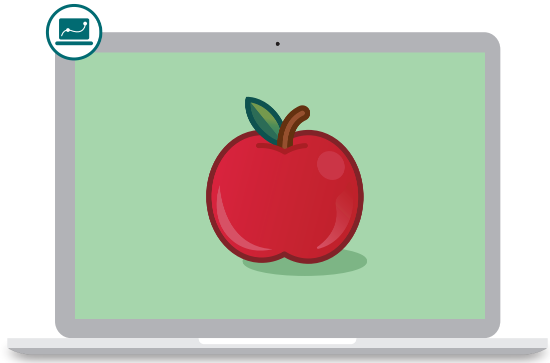
9. What do you think is important to remember when writing equations to represent situations?

Responses vary. **When writing equations to represent situations, it's important to remember the relationship between the numbers and what the variable means.**



Things to Remember:

Name: ________ Date: ________ Period: ________



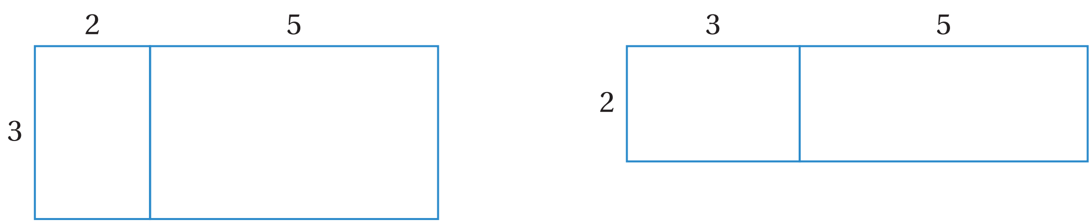
Vari-apples

Let's use variable expressions to represent situations.

Warm-Up

1 Here are two rectangles.

Rectangle A **Rectangle B**



Which rectangle has a greater area? Circle one.

Rectangle A Rectangle B They have the same area

Explain your thinking.

Explanations vary.

- **I added up the areas of the small rectangles. Rectangle A's area is $6 + 15 = 21$ square units. Rectangle B's area is $6 + 10 = 16$ square units.**
- **Rectangle A is a 3-by-7 rectangle and rectangle B is a 2-by-8 rectangle. $3 \cdot 7 = 21$ and $2 \cdot 8 = 16$.**
- **The small rectangle on the left is the same size in rectangles A and B, just turned. We only have to compare the rectangles on the right. A 3-by-5 rectangle is bigger than a 2-by-5 rectangle.**

Activity 1

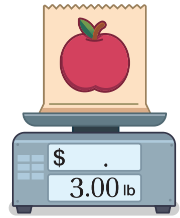
Name: .... Date: .... Period: ....

Intro to Variable Expressions

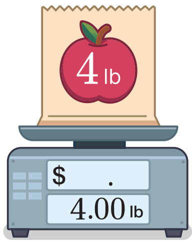
Apples at your store cost $1.50 per pound.

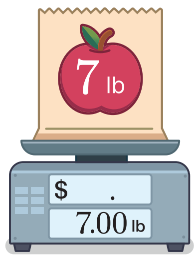
2 A customer orders 3 pounds of apples. How much should you charge them?

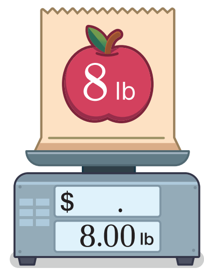
$4.50



3 Here are three new orders. How much should you charge for each order?







Apples (lb)	Cost ($)
4	**6.00**
7	**10.50**
8	**12.00**

4 Describe how to determine the cost of *any* number of pounds of apples.

Responses vary.

- **Multiply the number of pounds you want by 1.50 to get the cost.**
- **Keep adding 1.50 until you get to the number of pounds you want.**

Activity 1

Name: ______ Date: ______ Period: ______

Intro to Variable Expressions (continued)

5 Rudra and Sai each wrote an expression to describe the cost of p pounds of apples.

Rudra: $p + 1.50$

Sai: $1.50p$

Whose expression is correct? Circle one.

Rudra's   Sai's   Both   Neither

Apples (lb)	Cost ($)
3	4.50
4	6.00
7	10.50
8	12.00
p	$1.50p$

Explain your thinking.

Explanations vary.

- **Sai's expression works for every pair of numbers in the table. For example, $1.50(8) = 12$, but $8 + 1.50$ does not equal 12.**
- **Apples are $1.50 per pound, which means you have to multiply to get the total cost.**

6 Limes at your store cost $2.40 per pound.

How much should you charge for p pounds of limes?

$2.40p$

Activity 2

Name: ________ Date: ________ Period: ________

Comparing Variable Expressions

7 For \$5, you can get your groceries delivered. What is the total cost for each of these grocery deliveries?

Cost of Groceries (\$)	Total Cost (\$)
37.95	42.95
50.86	**55.86**
72.11	**77.11**
87.94	**92.94**

Grocery cost:	\$37.95
Delivery fee:	\$5.00
Total:	\$42.95

8 Write an expression for how much you should charge for g dollars worth of groceries, including delivery.

$g + 5$ (or equivalent)

9 Match each situation with an expression that represents its cost. Two expressions will have no match.

$10x$	$15x$	$x + 10$	$x + 15$

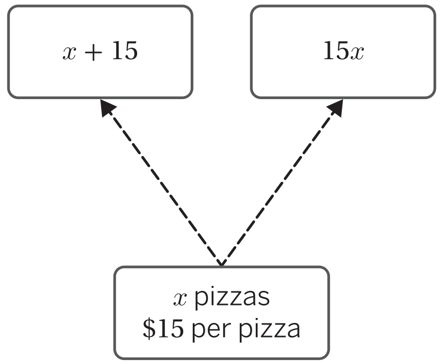
x pizzas \$15 per pizza	x dollars of groceries \$10 for delivery
$15x$	**$x + 10$**

Expressions with no match: $10x$ and $x + 15$

Name: .... Date: .... Period: ....

Comparing Variable Expressions (continued)

10 Which expression represents this situation?



Responses vary.

- **This situation matches with $15x$ because, if you order 2 pizzas, that would be 30 dollars. $2 + 15$ is not 30.**
- **If the cost is $15 per pizza, then for any number of pizzas, multiply 15 by how many pizzas you ordered. That is $15x$ because when a number and a variable are next to each other, that means multiply.**

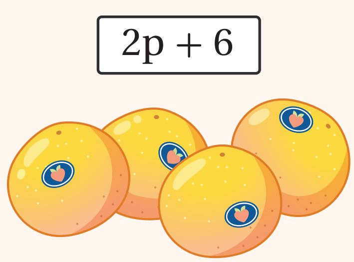
11 The expression $15x$ has one **term**. The expression $15 + x$ has two terms.

Select *all* the expressions that also have two terms.

- [x] **A.** $y - 6$
- [ ] **B.** $\frac{1}{2}x$
- [x] **C.** $3x + 2$
- [x] **D.** $x + y$
- [ ] **E.** $3 \cdot 2$

Explore More

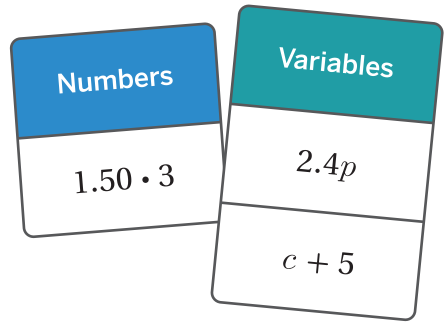
12 Describe a situation that could be represented by the expression $2p + 6$. Create a table if it helps with your thinking.



***Responses vary.* I went to the store for oranges and chocolate. It costs $2 per pound of oranges and $6 for a giant chocolate bar.**

13 Synthesis

Here are two types of expressions: expressions with numbers and expressions with variables.



When might each kind of expression be useful?

Expressions with numbers are useful when . . .

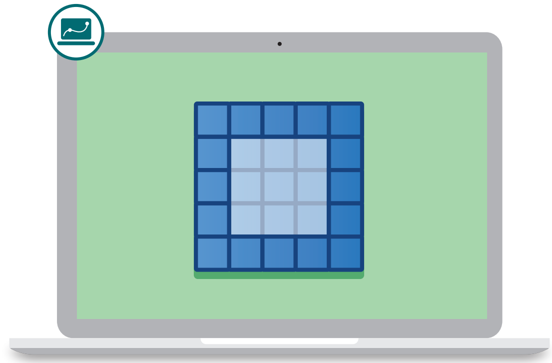
***Responses vary*. Expressions with numbers are useful when you want to figure out a specific number of things, or you already know the information you need to solve a problem.**

Expressions with variables are useful when . . .

***Responses vary*. Expressions with variables are useful when you want to figure out what something would look like for any number, like the cost of any number of pounds of limes.**

Things to Remember:

Name: Date: Period:

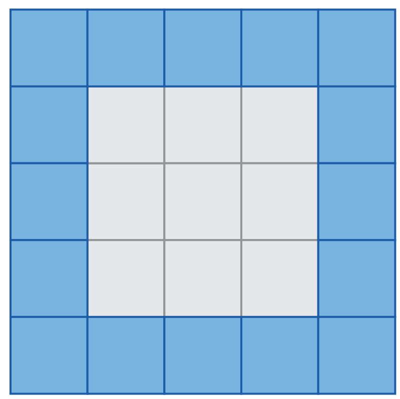


Border Tiles

Let's use diagrams to determine which expressions are equivalent.

Warm-Up

1 Here is a 3-by-3 square surrounded by border tiles.



a Without counting one by one, how many border tiles are there?

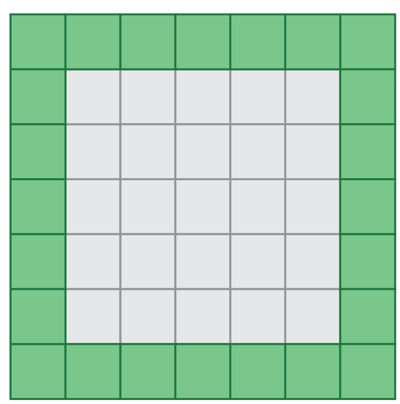
16 border tiles

b Explain how you see it.

Explanations vary.

- **There are 3 tiles on each side plus 4 at the corners.**
- **There are 5 tiles on the top and the bottom plus 3 on each side.**

2 Here is a 5-by-5 square surrounded by border tiles.



Without counting one by one, how many border tiles are there?

24 border tiles

Activity 1

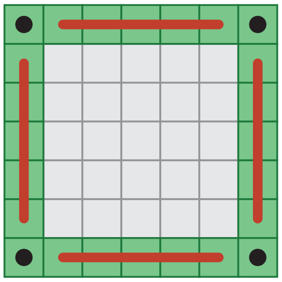
Name: ______ Date: ______ Period: ______

Border Tiles

3 **a** Take a look at Lucia's, Kyrie's, and Manuel's expressions for the 5-by-5 square.

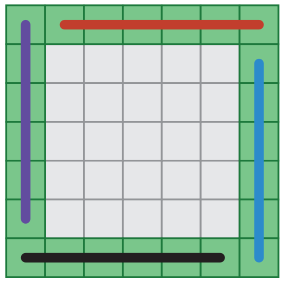
Lucia

$5 + 5 + 5 + 5 + 4$



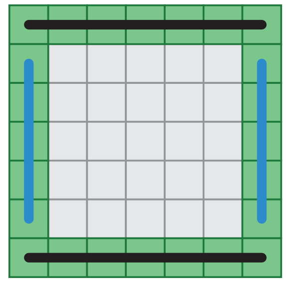
Kyrie

$4(5 + 1)$



Manuel

$7 + 7 + 5 + 5$

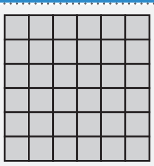
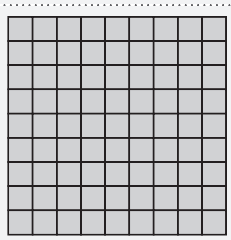
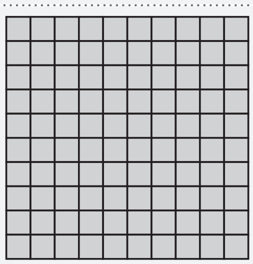


b **Discuss:** How are all of their expressions alike?

Responses vary.

- **They all show ways of counting the number of border tiles.**
- **They all include at least one 5, which is the side length of the square.**
- **They all give the same answer: 24 border tiles.**

4 Here are three new squares. Determine the number of border tiles for each square.

Model	Square	Border Tiles
	6-by-6	**28**
	9-by-9	**40**
	10-by-10	**44**

5 How can you determine the number of border tiles for an n-by-n square? Use your table if it helps with your thinking.

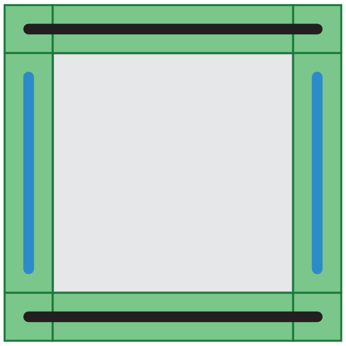
Responses vary.

- **Multiply the side length of the square by 4 for the edges, and then add 4 more for the corners.**
- **Add the side length of the square two times for the sides, and then add two more than the side length of the square for the top and the bottom.**

Name: .... Date: .... Period: ....

Equivalent Expressions

6 Manuel wrote this expression for the number of border tiles in an n-by-n square:

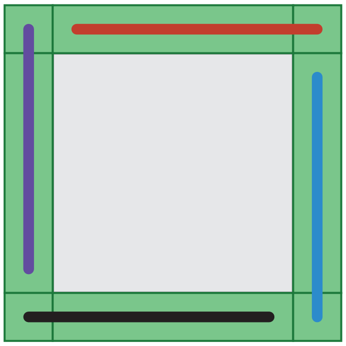


$(n + 2) + (n + 2) + n + n$

Show or explain how Manuel's expression is connected to his sketch.

Responses vary. **The $(n + 2)$'s are the lines on the top and the bottom of the square. The n's are the lines on the left and right side of the square.**

7 Here is Kyrie's sketch for the same square.



What might Kyrie's expression be for the number of border tiles in an n-by-n square?

$(n + 1) + (n + 1) + (n + 1) + (n + 1)$ (or equivalent)

Name: ______ Date: ______ Period: ______

Equivalent Expressions (continued)

8 **Equivalent expressions** are expressions that are equal for every value of a variable.

Here are two expressions.

Kyrie: $4(n+1)$

Manuel: $(n+2)+(n+2)+n+n$

a Use each expression to calculate the number of border tiles when $n = 8$.

Kyrie	**Manuel**
$4(n+1)$	$(n+2)+(n+2)+n+n$
$4(8+1)=36$	$(8+2)+(8+2)+8+8=36$
36 tiles	36 tiles

b **Discuss:** How can you tell that Kyrie's and Manuel's expressions are equivalent?

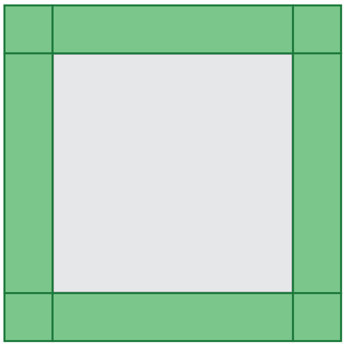
Responses vary. Kyrie's and Manuel's expressions are equivalent because when I replace n with a side length of the square I get the same number of border tiles in each expression.

9 Which expression is also equivalent to Kyrie's and Manuel's expressions?

A. $(n+1)(n+1)$

B. $4n+1$

C. $4n+4$



?

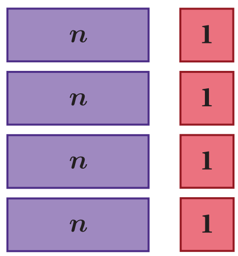
Activity 3

Name: .... Date: .... Period: ....

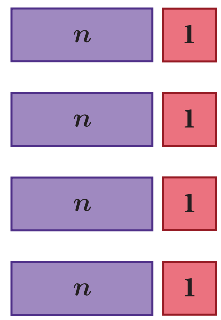
A New Diagram

10 Here is a new way of visualizing expressions.

$4n + 4$



$4(n + 1)$

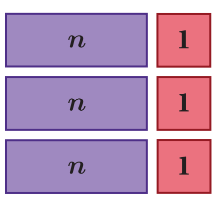
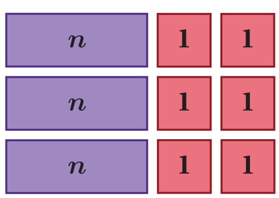


Discuss: How do these diagrams show that $4n + 4$ and $4(n + 1)$ are equivalent?

***Responses vary.* They are the same diagram, just split up in two different ways.**

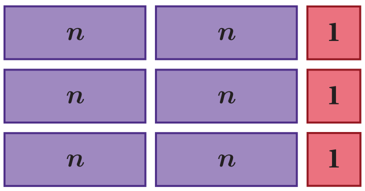
11 Match each expression with the diagram it represents. One expression will have no match.

$3(n + 1)$	$3n + 6$	$3n + 3$
$3(n + 3)$	$(n + 2) + (n + 2) + (n + 2)$	

	
$\mathbf{3(n + 1)}$ $\mathbf{3n + 3}$	$\mathbf{(n + 2) + (n + 2) + (n + 2)}$ $\mathbf{3n + 6}$

Expression with no match: $3(n + 3)$

12 Write an expression to represent this new diagram. Try to write an expression you think none of your classmates will.



Responses vary.

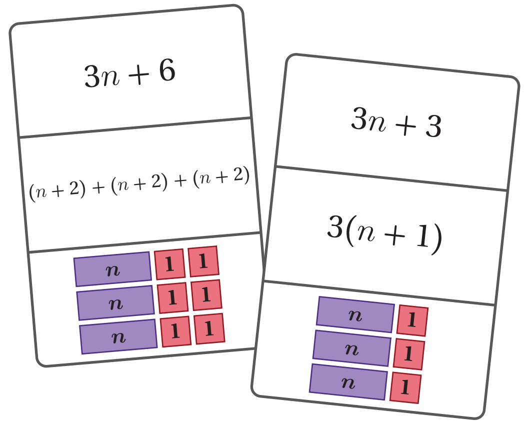
- $\mathbf{6n + 3}$
- $\mathbf{3(2n + 1)}$
- $\mathbf{(2n + 1) + (2n + 1) + (2n + 1)}$
- $\mathbf{3n + 3n + 3}$

13 Synthesis

How can you decide if two expressions are equivalent?

Use the examples if they help you explain your thinking.

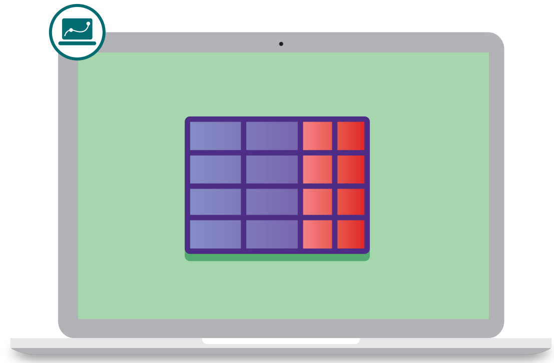
Responses vary. **Two expressions are equivalent if they are different ways of describing the same diagram. For example, $3n + 3$ and $3(n + 1)$ are equivalent because they both represent 3 n-tiles and 3 one-tiles.**



Things to Remember:

Unit 6
Lesson
8

Name: .... Date: .... Period: ....



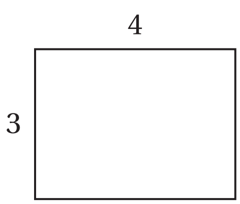
Products and Sums

Let's explore equivalent expressions using rectangle areas.

Warm-Up

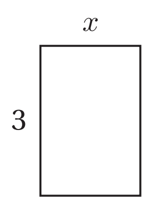
1 Write an expression for the area of each rectangle.

a



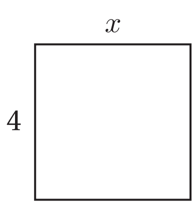
Expression: 12

b



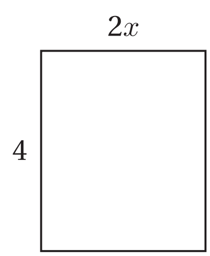
Expression: $3x$

c



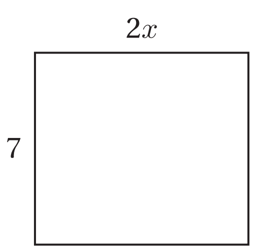
Expression: $4x$

d



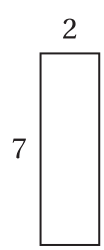
Expression: $8x$

e



Expression: $14x$

f

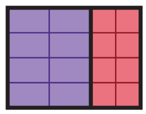
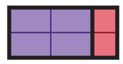
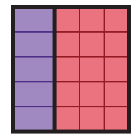


Expression: 14

Name: ______ Date: ______ Period: ______

Rectangles and Equivalent Expressions

2 Here are four rectangles and the *product* and *sum* expressions that represent their areas.

Rectangle	$2x$ 2; 4 	$2x$ 1; 2 	$3x$ 5; 1 	x 3; 5 
Product	$4(2x + 2)$	$2(2x + 1)$	$1(3x + 5)$	$5(x + 3)$
Sum	$8x + 8$	$4x + 2$	$3x + 5$	$5x + 15$

What do you notice about the product expressions?

Responses vary.

- **I notice that the length is always the number in front.**
- **I notice that there is always a + sign between the two parts inside the parentheses.**
- **I notice that the part inside the parentheses matches the width of the rectangle.**

What do you notice about the sum expressions?

Responses vary.

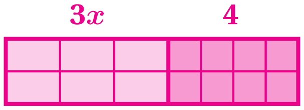
- **I notice that the number of tiles on the left is the left part of the sum and the number of tiles on the right is the right part of the sum.**
- **I notice that it is like putting together two rectangles from the Warm-Up.**
- **I notice there is always a + sign between the terms.**

3 **a** Create a rectangle with an area of $2(3x + 4)$. Label the sides of your rectangle.

Rectangles vary.

$3x$ 4

2



b Write an equivalent expression for the area.

Expressions vary.

- $6x + 8$
- $2(4 + 3x)$
- $8 + 6x$
- $3x + 4 + 3x + 4$

4 How would you convince someone that $2(3x + 4)$ is *not* equivalent to $6x + 4$?

Responses vary.

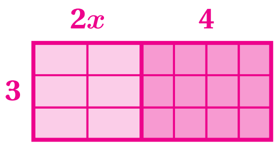
- **Count all the tiles. There are $6x$ and 8, not just 4.**
- **Multiply each of the parts inside the parentheses by the number outside to check your expression.**

Name: .... Date: .... Period: ....

More Rectangles and Equivalent Expressions

5 Create a rectangle with an area of $6x + 12$.

Rectangles vary.



6 Select *all* the expressions that are equivalent to $6x + 12$. Use the rectangle you created or draw a new rectangle if it helps with your thinking.

- [x] **A.** $6(x + 2)$
- [ ] **B.** $6(x + 12)$
- [x] **C.** $3(2x + 4)$
- [ ] **D.** $3(x + 4)$
- [x] **E.** $2(3x + 6)$

7 **a** Here are three new expressions. Select the *two* expressions that are equivalent.

- [x] **A.** $4(x + 2)$
- [ ] **B.** $4(x + 8)$
- [x] **C.** $4x + 8$

b Create a drawing to convince someone that the two expressions you selected are equivalent.

Drawings vary. **Both expressions create the same rectangle.**

Activity 3

Name: ______ Date: ______ Period: ______

Challenge Creator

8 You will use the Activity 3 Sheet to create your own rectangle challenge.

- **a** **Make It!** On the Activity 3 Sheet, create a rectangle challenge.
- **b** **Solve It!** On this page, write two expressions that represent your rectangle. Record one expression on your Activity 3 Sheet.
- **c** **Swap It!** Swap your challenge with one or more partners. Sketch your partner's rectangle and record their expression. Then create an equivalent expression.

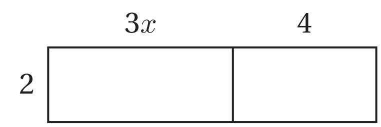
Rectangles and expressions vary.

My First Expression	My Second Expression

	Sketch of Rectangle	Partner's Expression	Equivalent Expression
Partner 1			
Partner 2			
Partner 3			
Partner 4			

9 Synthesis

Describe how you can use the area of a rectangle to write two or more equivalent expressions.



Use the example if it helps to show your thinking.

Responses vary. **I can write one expression by adding the areas of each smaller rectangle, and I can write another expression by multiplying the length by the width of the rectangle. In this rectangle, the sum is $6x + 8$ and the product is $2(3x + 4)$.**

Things to Remember:

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Lesson
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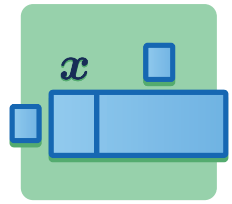
Name: ______ Date: ______ Period: ______

Equivalent Expressions

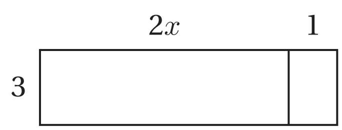


Let's explore equivalent expressions using area models and properties of operations.

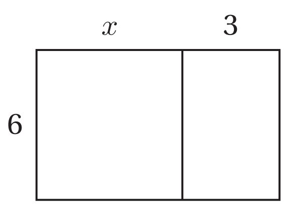
Warm-Up

1. Here are two rectangles.

Rectangle A



Rectangle B



a Which rectangle has an area of $6x + 3$?

Rectangle A

Explain your thinking.

Explanations vary.

$3 \bullet 2x = 6x$

$3 \bullet 1 = 3$

The total area is $6x + 3$.

b What is the area of the other rectangle?

- **$6(x + 3)$**
- **$6x + 18$ (or equivalent)**

2. The expression $6x + 3$ has two terms. The **coefficient** of the term $6x$ is 6.

Select *all* the expressions that also have a coefficient of 6.

- [ ] **A.** $2(2x)$
- [x] **B.** $1(6x)$
- [x] **C.** $6x - 4$
- [ ] **D.** $x - 6$
- [ ] **E.** $6 + x$

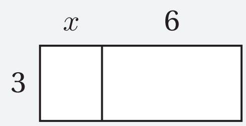
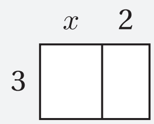
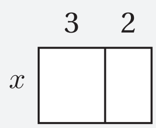
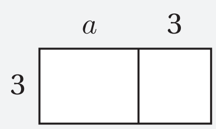
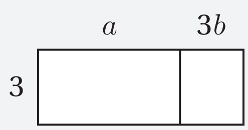
Activity
1

Name: .... Date: .... Period: ....

Card Sort

3. You will use a set of cards for this activity.

- Match each product or sum to its representation. Three expressions will be missing.
- Write in each missing expression.

	Representation	Product	Sum
a		Card A	$3x + 18$
b		Card C	Card B
c		$x(3 + 2)$ or $5x$	Card D
d		Card H	Card F
e		$3(a + 3b)$	Card G
f	The product of 3 and the sum of a and b.	Card I	Card E

4. Explain how you determined one of the missing expressions.

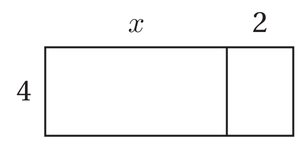
Responses vary. **I knew that $3x + 18$ and $3(x + 6)$ match with a because the areas of the two small rectangles are $3x$ and 18, and the area of the large rectangle is $3(x + 6)$.**

Activity 2

Name: Date: Period:

Two Truths and a Lie

5. Two of these expressions represent the area of this rectangle.



a Which are they?

A. $3(x + 2) + 1(x + 2)$

B. $1 + 3(x + 2)$

C. $4(x + 2)$

b **Discuss:** Which expression is not equivalent to the others? How do you know?

$1 + 3(x + 2)$. *Explanations vary.* $3(x + 2) + 1(x + 2)$ and $4(x + 2)$ are both equivalent to $4x + 8$, but $1 + 3(x + 2)$ is equivalent to $3x + 7$.

6. In each row, two choices are equivalent and one is not. Circle the one that is *not* equivalent.

	Expression A	Expression B	Expression C
a	$6(2 + x)$	$2(6 + x)$	$6(x + 2)$
b	$16x$	$1(8 + x)$	$x + 8$
c	$3x + 4(x + 2)$	$7(x + 2)$	$7x + 8$
d	$4(6x + 3x)$	$36x$	$24x + 3x$

7. Pick one problem and explain how you decided which choice was not equivalent.

Responses vary.

8. Jazz says: $24x$ *and* $3x$ *are* ***like terms****, but* 9 *and* $2x$ *are not.*

What do you think Jazz means?

***Responses vary.* $24x$ and $3x$ are like each other because they have the same variable, x. The terms 9 and $2x$ don't have the same variable, so they are not like terms.**

Synthesis

9. Explain how you can show that two expressions are equivalent.

Use these expressions if they help with your thinking.

***Explanations vary.* I know two expressions are equivalent if I can show that the expression was changed in a way that keeps the value of the expression the same.**

Equivalent Expressions

$3x + 4(x + 2)$
$7x + 8$

Not Equivalent

$7(x + 2)$

Things to Remember:

Card Sort

Directions: Make one copy per two pairs of students. Then pre-cut the cards and give each pair of students one set of Cards A–I.

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Card A
$3(x + 6)$

Card B
$3x + 6$

Card C
$3(x + 2)$

Card D
$3x + 2x$

Card E
3a + 3b

Card F
3a + 9

Card G
3a + 9b

Card H
3(a + 3)

Card I
3(a + b)

Card A
$3(x + 6)$

Card B
$3x + 6$

Card C
$3(x + 2)$

Card D
$3x + 2x$

Card E
3a + 3b

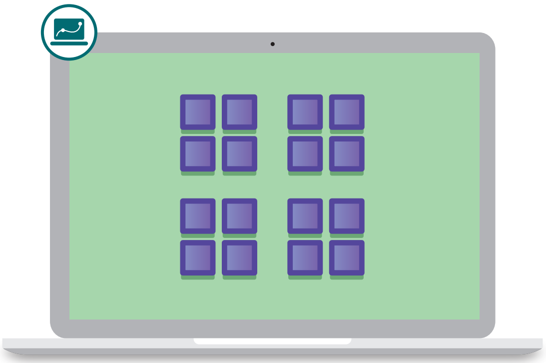
Card F
3a + 9

Card G
3a + 9b

Card H
3(a + 3)

Card I
3(a + b)

Name: ________ Date: ________ Period: ________

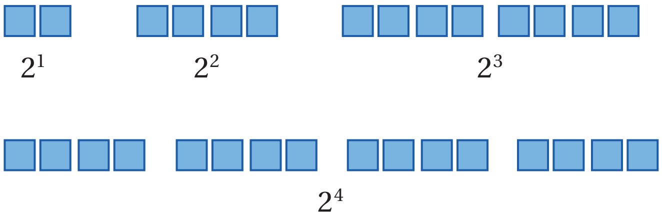


Powers

Let's see how exponents show repeated multiplication.

Warm-Up

1 Here are some images and their matching expressions.



Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that there are always groups of 2 squares.
- I notice that the number goes in a pattern: 2, 4, 8, 16.
- I notice that the small number is not the same as the number of groups of 2.
- I wonder what the small number means.
- I wonder what 2^{10} would look like.
- I wonder if the number of tiles would ever fill up the page.

Activity 1

Name: ______ Date: ______ Period: ______

Powers of 2

2 The expression 2^4 ("2 to the power of 4") is equivalent to $2 \cdot 2 \cdot 2 \cdot 2$.

2^4

How could you determine the value of 2^5?

Responses vary.

- **Double the number of tiles from 2^4.**
- **Multiply 2 five times, so $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$.**

3 Write a number or expression that is equivalent to 2^5.

Responses equivalent to 32 are considered correct.

- **32**
- **$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$**
- **$16 \cdot 2$**
- **$2^4 \cdot 2$**

4 Group the equivalent expressions. One expression will not have a match.

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	$2 + 2 + 2 + 2 + 2$		$2^4 \cdot 2$
2^5	$5 + 5$	$2 \cdot 5$	5^2

Group 1	Group 2
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	$2 + 2 + 2 + 2 + 2$
2^5	$2 \cdot 5$
$2^4 \cdot 2$	$5 + 5$

Expression without a group: 5^2

Name: .... Date: .... Period: ....

Powers of 2 (continued)

5 One expression in this group is not equivalent to the others.

2^5
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$2 + 2 + 2 + 2 + 2$
$2^4 \cdot 2$

Which expression is it?

A. 2^5

B. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

C. $2 + 2 + 2 + 2 + 2$

D. $2^4 \cdot 2$

Explain your thinking.

Explanations vary.

- **The value of $2 + 2 + 2 + 2 + 2$ is 10, which is not equal to $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$.**
- **2^5 means to multiply by 2 five times, so $2 + 2 + 2 + 2 + 2$ isn't equivalent because it has addition.**

Activity 2

Name: .... Date: .... Period: ....

Exponents With Whole Number Bases

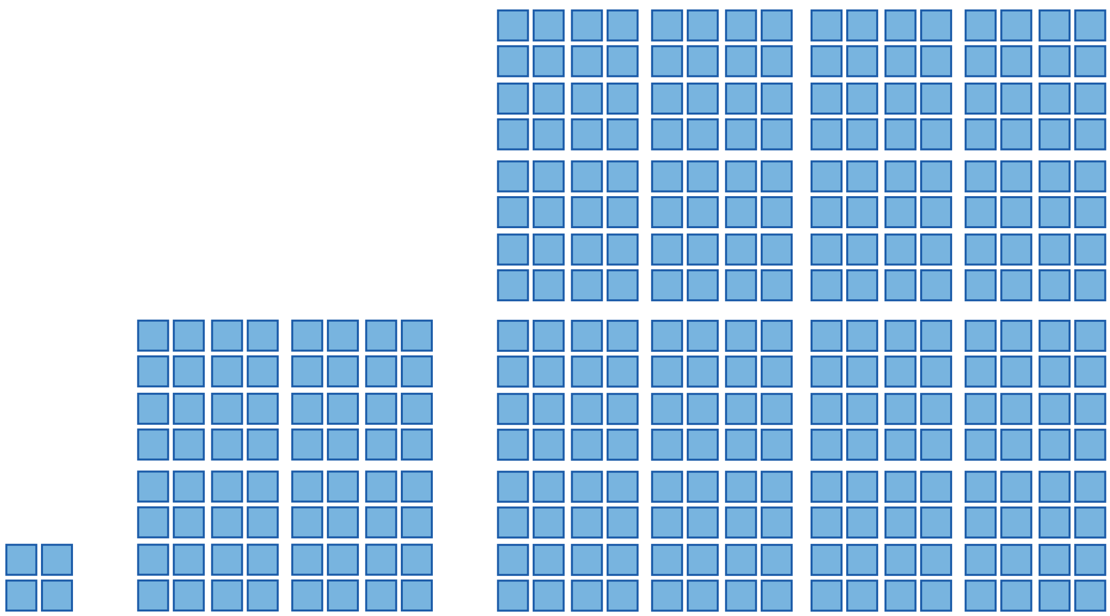
6 Select *one* expression that is equivalent to 3^4.

A. 12 B. $4 \cdot 4 \cdot 4$ C. $3 \cdot 3 \cdot 3 \cdot 3$ D. 81

Show or explain your thinking.

Explanations vary.

7 $4 \cdot 4$ is equivalent to 4^2, where 2 is the **exponent** and 4 is the **base**.



4^1 4^3 4^4

Write $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ using at least one exponent.

Responses vary.

- 4^5
- $4^4 \cdot 4$

8 Write a number or an expression that is equivalent to 4^3.

Responses equivalent to 64 are considered correct.

- 64
- $4 \cdot 4 \cdot 4$
- $16 \cdot 4$
- $4^2 \cdot 4$

Name: .... Date: .... Period: ....

Exponents With Fractional Bases

9 Here are some images and their matching expressions.

$\left(\frac{1}{2}\right)^1$ $\left(\frac{1}{2}\right)^2$ $\left(\frac{1}{2}\right)^3$

Write two things you know about $\left(\frac{1}{2}\right)^3$.

1.

2.

Responses vary.

- **I know it's less than 1 because 1 would be the whole rectangle.**
- **I know that the numbers keep splitting in half every time.**
- **I know that it has 1 shaded block and 7 unshaded blocks.**
- **I know that it is 1 out of 8.**

10 Quinn wrote two expressions equivalent to $\left(\frac{1}{3}\right)^4$.

Quinn

$\left(\frac{1}{3}\right)^4$

$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$

$\frac{1}{9} \cdot \frac{1}{9}$

Write a different expression equivalent to $\left(\frac{1}{3}\right)^4$.

Responses equivalent to $\frac{1}{81}$ are considered correct.

- $\left(\frac{1}{3}\right)^3 \cdot \frac{1}{3}$
- $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^2$

Explain your thinking.

Explanations vary.

- **I multiplied $\frac{1}{3}$ by itself four times.**
- **I know $3^4 = 81$, so $\left(\frac{1}{3}\right)^4 = \frac{1}{81}$.**

11 Write an expression that is equivalent to $\left(\frac{1}{2}\right)^5$.

Responses equivalent to $\frac{1}{32}$ are considered correct.

- $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
- $\left(\frac{1}{2}\right)^4 \cdot \frac{1}{2}$
- $\left(\frac{1}{2}\right)^3 \cdot \frac{1}{2} \cdot \frac{1}{2}$

Name: .... Date: .... Period: ....

Exponents With Fractional Bases (continued)

12 Determine the value of each expression. Complete as many as you have time for.

a. $2^3 = 8$

b. $4^2 = 16$

c. $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$

d. $3^4 = 81$

e. $5^1 = 5$

f. $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

g. $1^8 = 1$

h. $\left(\frac{1}{4}\right)^3 = \frac{1}{64}$

13 Synthesis

Without calculating, how can you tell whether expressions with exponents are equivalent?

Responses vary. **I can tell if expressions with exponents are equivalent by checking the number of times a value is multiplied. 11^5 is equivalent to $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$ and $11^4 \cdot 11$ because 11 is multiplied five times.**

$11 + 11 + 11 + 11 + 11$

$11^4 \cdot 11$

$11 \cdot 5$

$11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$

5^{11}

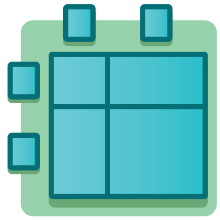
11^5

Things to Remember:

Unit 6
Lesson
11

Name: ________________ Date: ________ Period: ________

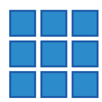
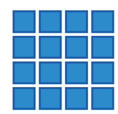
Exponent Expressions



Let's evaluate expressions with exponents.

Warm-Up

1. 4, 9, and 16 are examples of **perfect squares**. Here are some representations of these perfect squares

Perfect Square	4	9	16
Diagram			
Exponent Expression	2^2	3^2	4^2

a **Discuss:** What do you notice? What do you wonder?

Responses vary.
- **I notice that all of the exponents are 2.**
- **I notice that we can make an exact square out of smaller squares.**
- **I wonder what other numbers are perfect squares.**

b Do you think 49 is a perfect square? Explain your thinking.

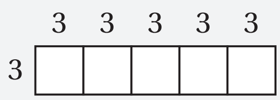
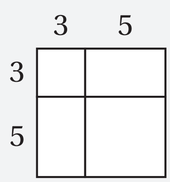
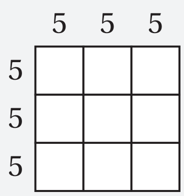
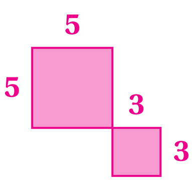
Yes. ***Explanations vary.*** **I could draw an exact square out of 49 smaller squares.**

Name: .... Date: .... Period: ....

What's Missing?

2. You will use a set of cards for this activity.

- Work with a partner to group the matching diagrams, expressions, and values. There will be three blank spaces.
- Complete the table with the missing representations.

Diagram	Expression	Value
	$5 \cdot 3^2$	Card E
	Card B	64
	Card D	225
Responses vary. 	Card A	Card C

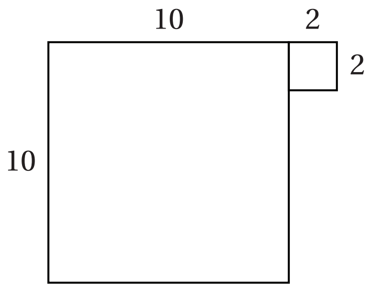
Activity
2

Name: .... Date: .... Period: ....

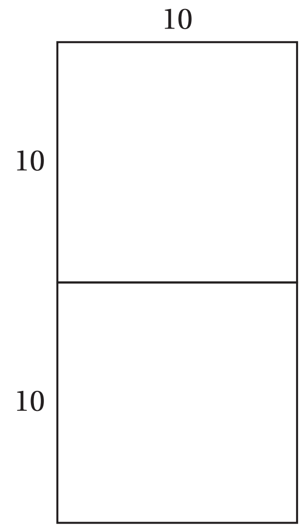
Evaluating Expressions

3. **Discuss:** Which diagram represents $2 + 10^2$?

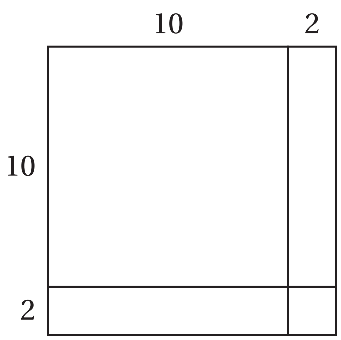
A.



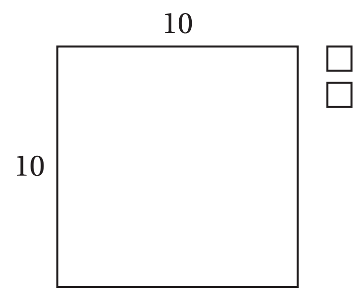
B.



C.



D.



Explanations vary. **The large square represents 10^2 and the two small squares represent 2.**

4. Latifa and Nicolas each got a different value for $2 + 10^2$.

Latifa	Nicolas
$2 + 10^2$	$2 + 10^2$
12^2	$2 + 100$
144	102

Whose work is correct? Circle one.

Nicolas's Latifa's Both Neither

Explain your thinking.

Explanations vary. **Nicolas is correct because his answer matches the total area of Choice D. There is a square with an area of 100 square units and 2 additional squares, so the total area is 102 square units.**

Activity 3

Name: ________ Date: ________ Period: ________

Partner Problems

5. Decide with your partner who will complete Column A and who will complete Column B.

- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

	Column A	Column B
a	$5^2 + 4$ **= 29**	$2^2 + 25$ **= 29**
b	$5^2 - 4$ **= 21**	$25 - 2^2$ **= 21**
c	$\frac{4^2}{2}$ **= 8**	$\frac{2^5}{4}$ **= 8**
d	$(7 - 2)^2 - 3^2$ **= 16**	$\frac{1}{4}(1 + 3)^3$ **= 16**
e	$\frac{8 - 2^2}{4}$ **= 1**	$\frac{8 - 2(3)}{2}$ **= 1**
f	$\frac{2^4 - (3^2 - 5)}{2^2}$ **= 3**	$\frac{1^5 + (6^2 - 10)}{3^2}$ **= 3**

Explore More

6. Write an expression for your partner to evaluate. Swap problems, then write an expression with an exponent that has the same value as your partner's but uses different numbers.

Responses vary.

Synthesis

7. What are some things to remember when determining the value of expressions with exponents?

Use these examples if they help with your thinking.

Responses vary. **If there are parentheses, then those operations should be evaluated first. Then evaluate the parts of the expression with exponents before you complete any other operations.**

$5 \cdot 3^2$

$(3 + 5)^2$

$(3 \cdot 5)^2$

$5^2 + 3^2$

Things to Remember:

What's Missing?

Directions: Make one copy per two pairs of students. Then pre-cut the cards and give each pair of students one set of Cards A–E.

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Card A

$5^2 + 3^2$

Card B

$(3 + 5)^2$

Card C

34

Card D

$(3 \cdot 5)^2$

Card E

45

Card A

$5^2 + 3^2$

Card B

$(3 + 5)^2$

Card C

34

Card D

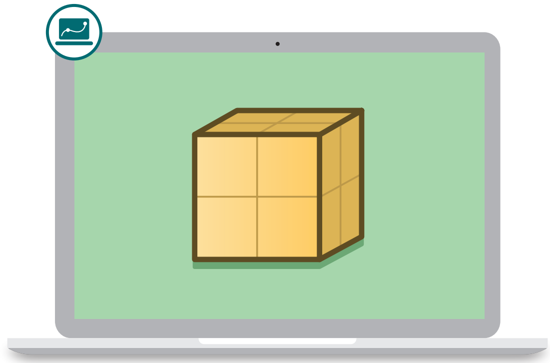
$(3 \cdot 5)^2$

Card E

45

Unit 6
Lesson
12

Name: .... Date: .... Period: ....



Squares and Cubes

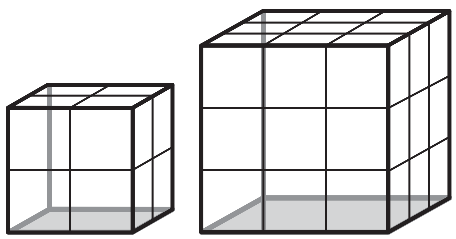
Let's evaluate variable expressions.

Warm-Up

1 8 and 27 are examples of **perfect cubes**.

What other numbers do you think might be perfect cubes?

Responses vary. **1, 64, 125**



8 27

Activity 1

Name: ______ Date: ______ Period: ______

Variable Expressions With Area

2 Match each expression with its area model.

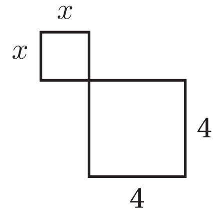
a. $(x + 4)^2$

b. $x^2 + 4$

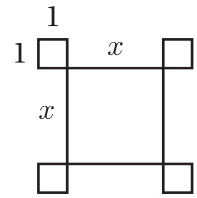
c. $4x^2$

d. $x^2 + 4^2$

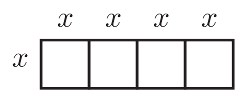
d



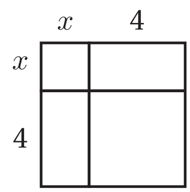
b



c



a



Name: ____________ Date: ________ Period: ________

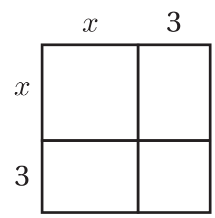
Variable Expressions With Area (continued)

3 The area of figure A is $(x + 3)^2$ square units.

What is its area when $x = 4$?

49 square units

Figure A

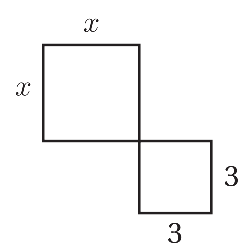


4 The area of figure B is $x^2 + 3^2$ square units.

What is its area when $x = 4$?

25 square units

Figure B



5 Amir says that $(x + 3)^2$ and $x^2 + 3^2$ are equivalent expressions. Help him understand why they are *not* equivalent.

Use figures A and B if they help you show your thinking.

Responses vary.

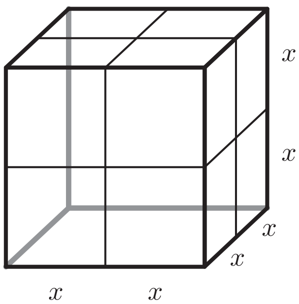
- **When $x = 4$, these expressions have different values so they can't be equivalent. $(4 + 3)^2 = 49$ and $4^2 + 3^2 = 25$**
- **If you draw them, they make different pictures. $(x + 3)^2$ matches figure A and $x^2 + 3^2$ matches figure B.**

Name: Date: Period:

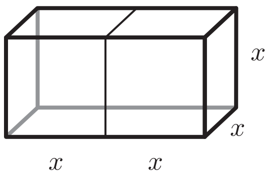
Cubes and Squares

6 Which prism has a volume of $(2x)^3$ cubic units?

Prism C



Prism D



Prism C Prism D Both Neither

Explain your thinking.

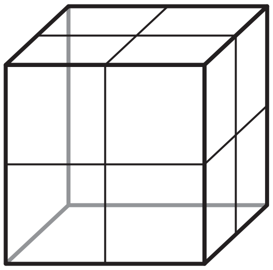
Explanations vary.

- **Prism C is like a cube that is $2x$ on each side, and that's what $(2x)^3$ means.**
- **If you replace x with 1 then $(2 \cdot 1)^3 = 2 \cdot 2 \cdot 2 = 8$. Prism D would only have a volume of 2 cubic units, not 8.**

7 What is the volume of prism C when $x = 3$?

216 cubic units

Prism C



8 Here is a new expression: $2x^3$.

Evaluate $2x^3$ when $x = 5$. Show or explain your thinking.

250. *Explanations vary.*

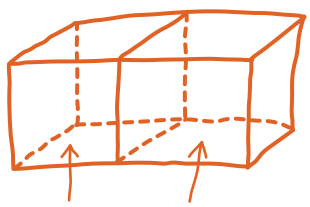
- **I wrote $2 \cdot 5 \cdot 5 \cdot 5$, and then I multiplied 10 by 25.**
- **I calculated the value of 5^3 as 125, then I multiplied it by 2.**

Name: ________ Date: ________ Period: ________

Cubes and Squares (continued)

9 Here are Amir's and Chloe's strategies for evaluating $2x^3$ when $x = 5$.

Amir



$5 \cdot 5 \cdot 5 \quad 5 \cdot 5 \cdot 5$

$125 + 125$

250

Chloe

$2x^3$ when $x = 5$

$2(5)^3 = 2(5 \cdot 5 \cdot 5)$

$= 2 \cdot 125$

$= 250$

Discuss: How are their strategies alike? How are they different?

Responses vary. **Both people calculated $5 \cdot 5 \cdot 5$ at some point, which shows the volume of one cube. Amir drew a picture and Chloe wrote out a sequence of numerical expressions. Chloe substituted 5 where x was and Amir made two 5-by-5-by-5 cubes.**

10 Use Chloe's strategy to evaluate $2x^3$ when $x = \frac{1}{2}$.

$\frac{1}{4}$ (or equivalent)

11 Use Chloe's strategy to determine the value of $5x^2 + 4x + 3$ when $x = 3$.

60

Activity 3

Name: ______ Date: ______ Period: ______

Repeated Challenges

12 Evaluate each expression for the given value of x.

	Expression	Value of x	Value of the Expression
a	$(x + 2)^2$	5	**49**
b	$5x^2$	3	**45**
c	$4x^3$	2	**32**
d	$4 + x^2$	6	**40**
e	$(2x)^3$	2	**64**

Explore More

13 Here are two expressions: $(x + 5)^2$ and $x^2 + 5^2$.

Amir says they *always* have the same value.
Chloe says they *never* have the same value.

Who is correct? Circle one.

Amir   Chloe   Both   Neither

Explain your thinking.

***Explanations vary.* They have the same value if $x = 0$, so they sometimes have the same value.**

14 Synthesis

Describe how to evaluate $(x + 1)^3$ when $x = 3$.

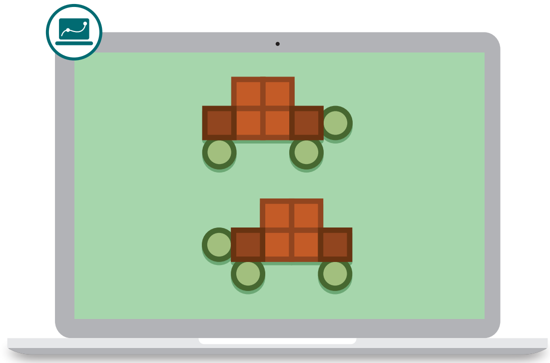
Responses vary.

- Substitute the number 3 where the x is in the expression. First, we need to add $3 + 1$, which is 4. Then calculate 4^3. 4 to the power of 3 means $4 \cdot 4 \cdot 4$, which is $16 \cdot 4 = 64$.
- Draw a picture of a cube that is $3 + 1$ on each side. That means the cube is 4 by 4 by 4. Volume is the base area times the height, so $4 \cdot 4 = 16$ and $16 \cdot 4 = 64$.

Things to Remember:

Unit 6
Lesson
13

Name: .... Date: .... Period: ....

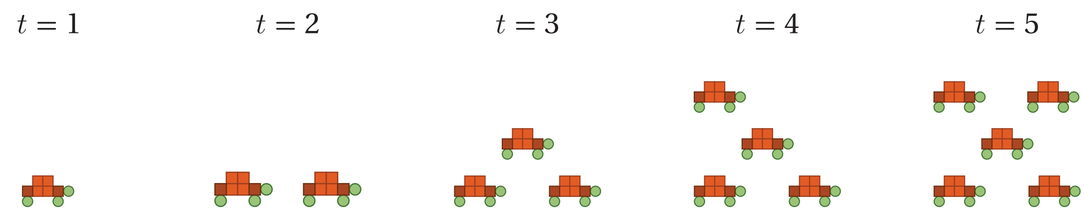


Turtles All the Way

Let's explore relationships between two variables.

Warm-Up

1 Here is a pattern of turtles. What different things can you count in this pattern?



Responses vary.

- The number of turtles
- The number of green circles
- The number of tiles in total
- The number of tiles in one turtle
- The number of rows of turtles

Activity 1

Name: ______ Date: ______ Period: ______

Turtles, Turtles, Turtles

2 The variable t represents the number of turtles.

a Here are three other variables. Circle one.

$s =$ number of square tiles $h =$ height of a turtle $g =$ number of green circles

b Describe how changing t affects the value of the variable you chose.

Responses depend on the variable chosen.

- **s: The number of square tiles always goes up by 6 every time you add a turtle.**
- **h: The height of a turtle is always the same no matter how many turtles there are.**
- **g: There are 3 green circles on every turtle so it's always 3 times the number of turtles.**

3 Saanvi made a table to help make sense of the relationship between t and g.

a Complete the table.

Number of Turtles, t	Number of Green Circles, g
1	3
2	**6**
3	**9**
4	**12**
5	**15**

b **Discuss:** What patterns do you see?

Responses vary.

- **Every time there is a new turtle, there are 3 more green circles.**
- **The number of circles is always 3 times the number of turtles.**
- **The number of green circles are all multiples of 3.**

4 Saanvi and Kadeem wrote equations to represent the relationship between t and g.

Saanvi

$g = 3t$

Kadeem

$t = 3g$

Whose equation is correct? Circle one.

Saanvi's Kadeem's Both Neither

Explain your thinking.

Explanations vary.

- **I used the numbers from the table. Saanvi's equation kept being true and Kadeem's equation didn't. For example, 6 equals 3(2), but 3 doesn't equal 6(2).**
- **If you think about it, Saanvi's equation says that the number of green circles is 3 times the number of turtles. Every turtle has 3 green circles, so the equation makes sense.**

Activity 2

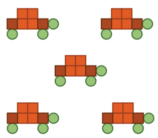
Name: .... Date: .... Period: ....

Dependent or Independent

5 The **independent variable** is the variable in a relationship that is the cause. The **dependent variable** is the effect or result.

In Saanvi's example, the independent variable is the number of turtles, t, and the dependent variable is the number of green circles, g.

Image



Table

Number of Turtles, t	Number of Green Circles, g
1	3
2	6
3	9
4	12

Equation

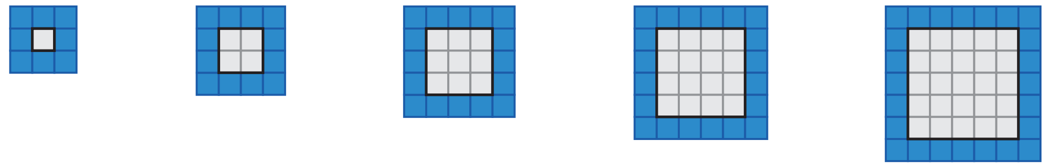
$g = 3t$

Discuss: If the independent variable is t, what other dependent variables could you explore?

***Responses vary.* The total number of brown tiles, the number of tiles in total, or the number of rows of turtles.**

6 Here is a pattern you may have seen before. The independent variable is n, the side length of the lighter square.

$n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$



a Circle one dependent variable to explore. Consider choosing one that's different than your partner's.

$t =$ total area of the tiles $g =$ area of the lighter tiles $p =$ perimeter of the lighter square

b Describe how changing n affects the value of the dependent variable you chose.

Responses depend on the variable chosen.

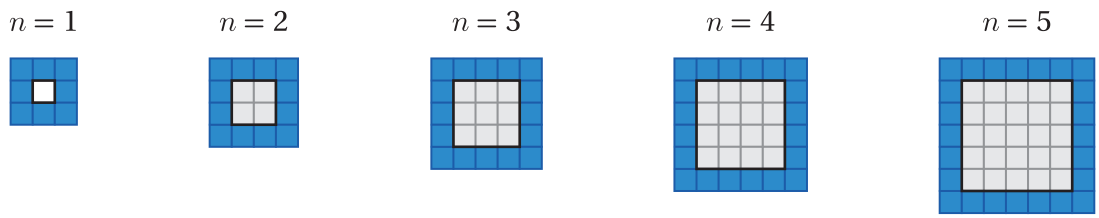
- **t: The total area gets bigger and bigger every time. It's like a big square.**
- **g: The area of the lighter tiles kind of grows in a square. It doesn't go up by the same number every time.**
- **p: The perimeter of the lighter square goes up by 4 every time n increases by 1.**

Activity 3

Name: ______ Date: ______ Period: ______

Border Tiles Revisited

Here is the pattern from the previous screen.



7 **a** Complete the table for the variable you chose.

t = total area of the tiles

n	t
1	**9**
2	**16**
3	**25**
4	**36**
5	**49**

g = area of the gray tiles

n	g
1	**1**
2	**4**
3	**9**
4	**16**
5	**25**

p = perimeter of the gray square

n	p
1	**4**
2	**8**
3	**12**
4	**16**
5	**20**

b **Discuss:** What patterns do you see? ***Responses vary.***

8 Which equation represents your pattern? Explain your thinking.

t = total area of the tiles

A. $t = (n+2)^2$

B. $n = (t+2)^2$

C. $t = n^2 + 2$

Explanations vary.

g = area of the gray tiles

A. $g = n^2$

B. $n = g^2$

C. $g = 2n$

Explanations vary.

p = perimeter of the gray square

A. $p = 4n$

B. $p = 4(n+2)$

C. $n = 4p$

Explanations vary.

9 What is the total area of the tiles when $n = 10$?

$t = 144$

What is the area of the gray tiles when $n = 10$?

$g = 100$

What is the perimeter of the gray square when $n = 10$?

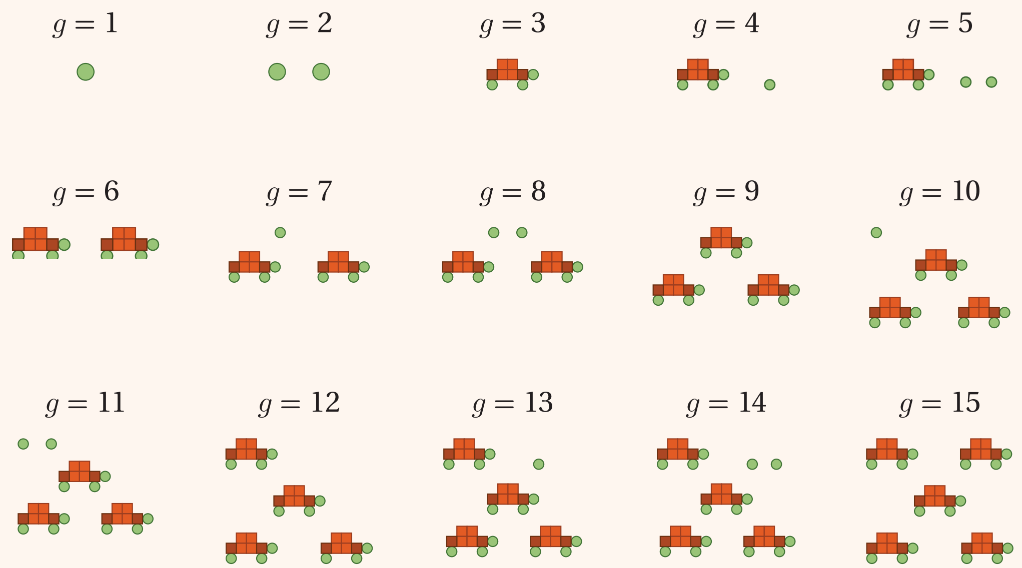
$p = 40$

Name: .... Date: .... Period: ....

Border Tiles Revisited (continued)

Explore More

10 Here is a new pattern where g represents the number of green circles.



a What is the independent variable?

The number of green circles, g

b Choose a dependent variable to explore.

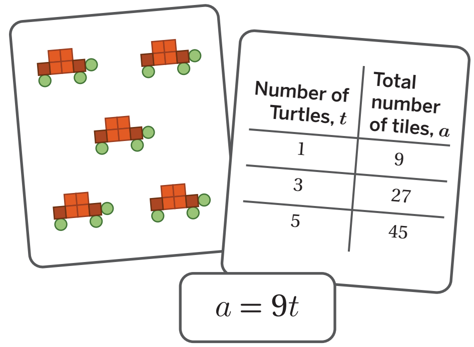
***Responses vary.* The number of turtles, t**

c Describe the relationship between the independent variable and dependent variable you chose.

***Responses vary depending on responses to Problem 10b.* Every time the number of green circles gets to 3 more, the number of turtles jumps up by 1. For example, if $g = 4$, the number of turtles is still 1, but once it gets to $g = 6$, the number of turtles jumps to 2.**

11 Synthesis

How can we see relationships between independent and dependent variables in tables and equations?



Responses vary. Tables show a relationship between the variables in each row. The left column is the independent variable and the right column is the dependent variable. For example, there are 5 turtles in the image and they have 45 total tiles, so the table has the row $t = 5$ and $a = 45$. An equation shows the relationship in general. In this case, the independent variable is 9 times the dependent variable, so we can write the equation $a = 9t$.

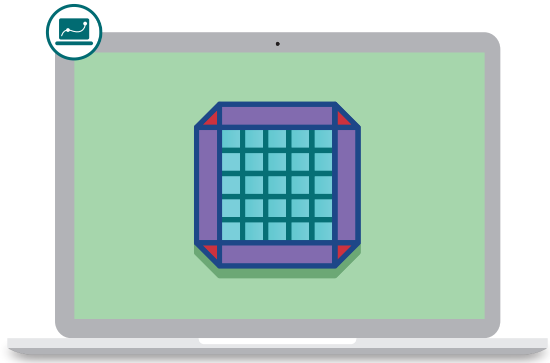
Things to Remember:

Unit 6
Lesson
14

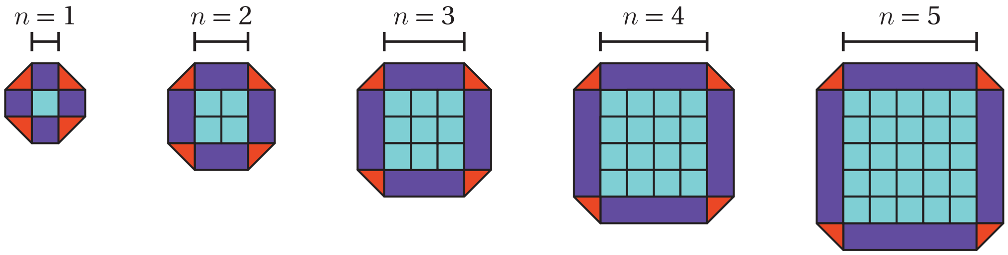
Name: ________ Date: ________ Period: ________

Representing Relationships

Let's use graphs to represent relationships between two variables.



Warm-Up



1 **Discuss:** What do you notice? What do you wonder?

Responses vary.

- I notice that there are always three colors.
- I notice that the middle is a square that keeps growing.
- I notice that there are always 4 red triangles.
- I wonder when the shape would get so big it takes over the page.
- I wonder how many squares I would need to build each shape.

2 In this pattern, the independent variable is the side length of the purple rectangles, n. What dependent variables could you explore?

Responses vary.

- Total area of the shape
- Number of center teal squares
- Height of the shape

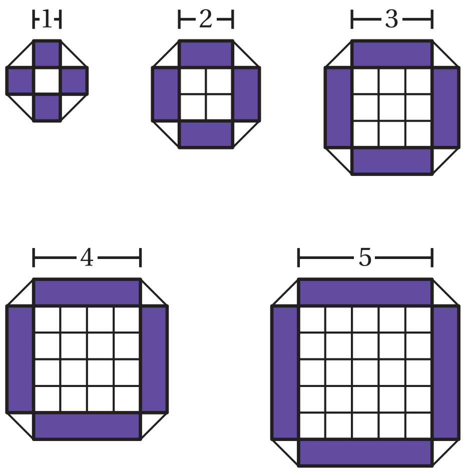
Activity 1

Name: Date: Period:

Introducing Graphs

3 Jayden chose to explore the relationship between the side length of the purple rectangles, n, and the total area of the purple rectangles, p.

Complete the table to represent this relationship.

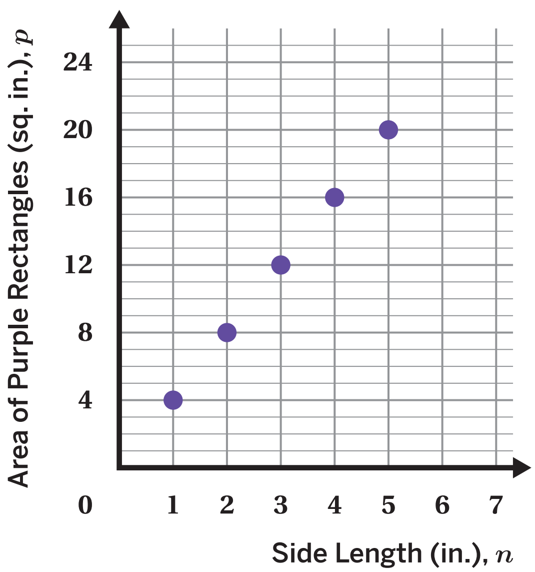


Side Length (in.), n	Area of Purple Rectangles (sq. in.), p
1	4
2	8
3	12
4	16
5	20

4 Rebecca represented the relationship between n and p with a graph.

Discuss: How does the graph show the same information as the images?

Responses vary. **Each point on the graph matches one of the images. The number going horizontally is the side length and the number going vertically is the total area of the purple rectangles.**



5 Explain how you know Jayden's table and Rebecca's graph represent the same relationship.

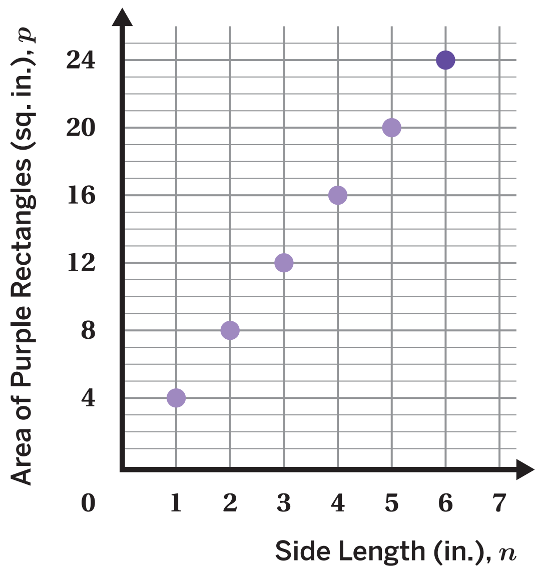
Responses vary. **I know these represent the same relationship because they show the same information in different ways. For example, I can see the row with $n = 2$ and $p = 8$ in the graph 2 units over and 8 units up. The 2 represents the side length and the 8 represents the total area of the purple rectangles.**

Name: ______ Date: ______ Period: ______

Introducing Graphs (continued)

6 Rebecca added the *ordered pair* $(6, 24)$ to the graph. Explain what each value represents in this context.

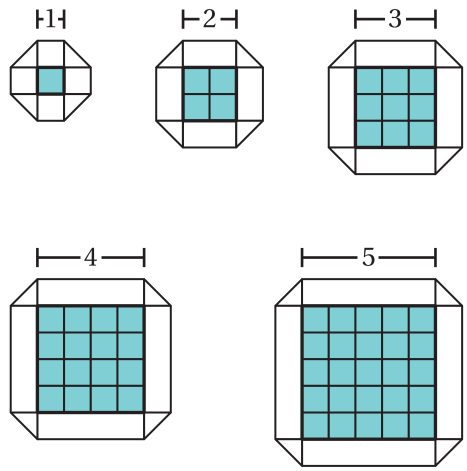
***Responses vary.* It means that when the side length is 6 inches, the total area of the purple rectangles is 24 square inches.**



Name: ______ Date: ______ Period: ______

Plotting Points

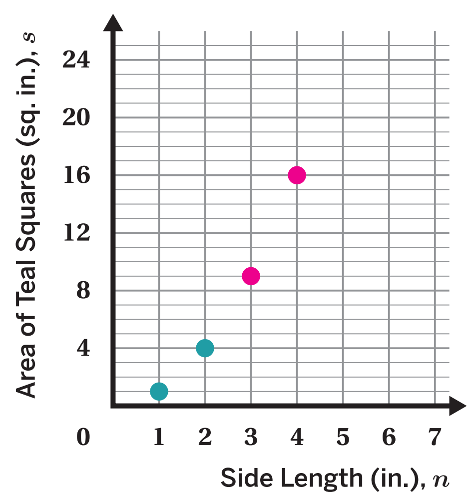
7 Precious explored the relationship between the side length, n, and the total area of the teal squares, s. Complete the table to represent this relationship.



Side Length (in.), n	Area of Teal Squares (sq. in.), s
1	1
2	**4**
3	**9**
4	**16**
5	**25**

8 Plot points that represent the third and fourth rows in the table.

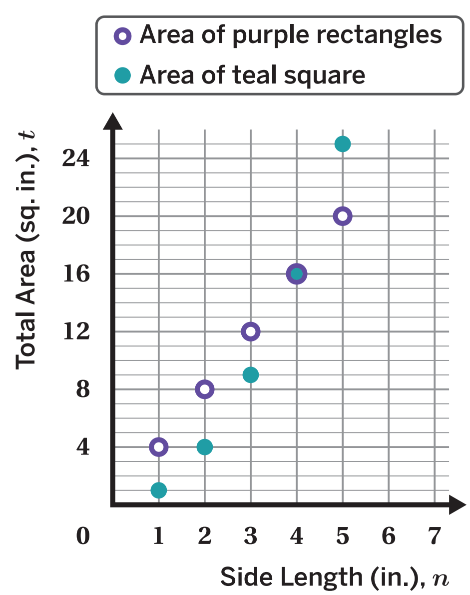
Response shown on the graph.



9 Precious noticed the point $(4, 16)$ was on both graphs.

Discuss: What does this say about the area of purple rectangles and the area of the teal squares?

Responses vary. **It means that when the side length is 4 inches, both the purple rectangles and teal squares have the same area. The area is 16 square inches.**

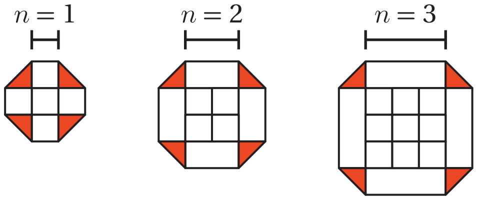


Activity 3

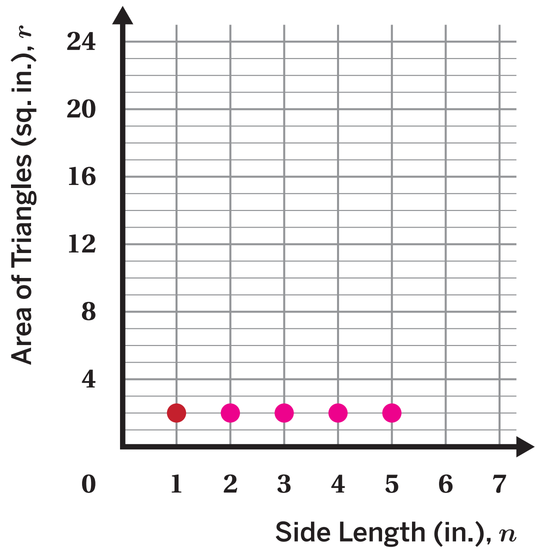
Name: ______ Date: ______ Period: ______

Comparing Relationships

10 Graph four points to represent the relationship between the side length, n, and the total area of the red triangles, r.



Response shown on the graph.



11 Let's look at a graph that shows all three relationships.

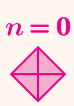
Discuss: How is the triangle relationship different from the other two relationships? How is it alike?

Responses vary. As the length of the purple rectangles gets longer, the total area of the triangles remains the same, while the areas of the purple rectangles and teal squares increase. The graph shows that the red triangle points lie on a horizontal line, whereas the points representing the teal and purple areas follow inclined lines.

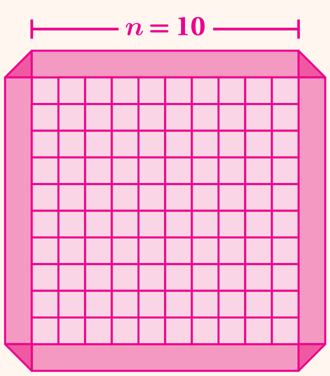
Explore More

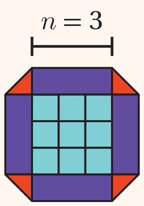
12 Draw an image to represent when:

a $n = 0$



b $n = 10$

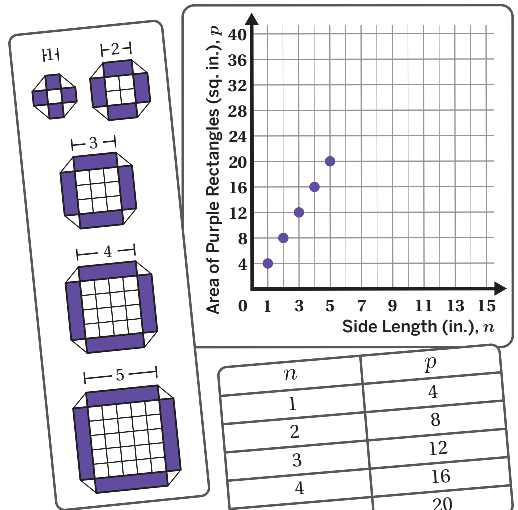




13 Synthesis

How can you tell that a table, a graph, and an image show the same relationship?

Responses vary. **They each should have the same information. Each row of the table represents part of the image and a point on the graph. For example, the image shows that when the side length is 4 inches, the purple area is 16 square inches. This shows up in the table in the row $n = 4$ and $p = 16$ and on the graph in the point $(4, 16)$.**



n	p
1	4
2	8
3	12
4	16
5	20

Things to Remember:

Unit 6
Lesson
15

Name: ______ Date: ______ Period: ______

Connecting Representations

Let's make connections between different representations of the same relationship.

Warm-Up

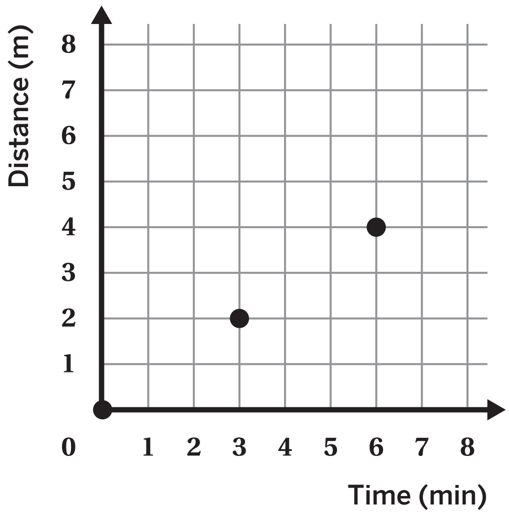
1. Which table represents this graph?

Table A

Time (min)	Distance (m)
0	0
2	3
4	6

Table B

Time (min)	Distance (m)
0	0
3	2
6	4



Explain your thinking.

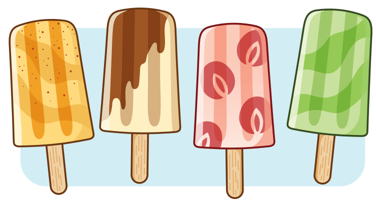
Table B. *Explanations vary.* There is a point on the graph where the time is 3 minutes. Table A does not have a point at 3 minutes.

Name: ______ Date: ______ Period: ______

What's Missing?

You will use a set of cards for this activity.

Paletas

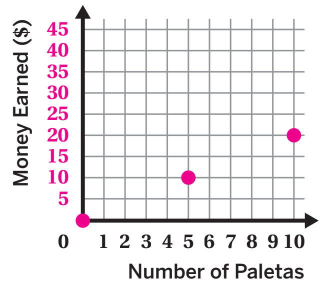


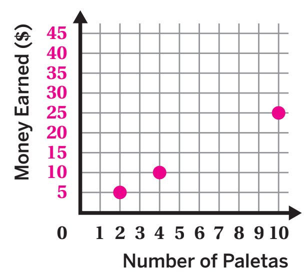
Piraguas

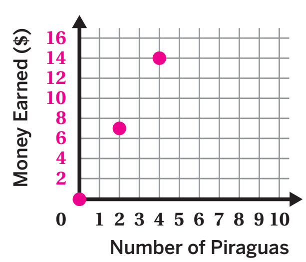
2. With your partner, match each situation with the tables, graphs, and equations that show the same relationship.

3. Complete the missing representations.

<table>
<tr><th>Situation</th><th>Table</th><th>Graph</th><th>Equation</th></tr>
<tr>
<td>Amanda sells paletas, p, for $2 each.

What is the total amount of money, m, Amanda can earn?</td>
<td>Card D<table><tr><th>p</th><th>m</th></tr><tr><td>0</td><td>0</td></tr><tr><td>5</td><td>10</td></tr><tr><td>10</td><td>20</td></tr></table></td>
<td>Card B
</td>
<td>$m = 2p$</td>
</tr>
<tr>
<td>Tameeka sells paletas, p, for $2.50 each.

What is the total amount of money, m, Tameeka can earn?</td>
<td>Card A<table><tr><th>p</th><th>m</th></tr><tr><td>2</td><td>5</td></tr><tr><td>4</td><td>10</td></tr><tr><td>10</td><td>25</td></tr></table></td>
<td></td>
<td>Card F
$m = 2.50p$</td>
</tr>
<tr>
<td>Esteban sells piraguas, p, for $3.50 each.

What is the total amount of money, m, Esteban can earn?</td>
<td><table><tr><th>p</th><th>m</th></tr><tr><td>0</td><td>0</td></tr><tr><td>2</td><td>7</td></tr><tr><td>4</td><td>14</td></tr></table></td>
<td>Card E
</td>
<td>Card C
$m = 3.50p$</td>
</tr>
</table>

Name: Date: Period:

What's Missing? (continued)

4. Choose one of the previous situations. Show or explain where you see the price per item in the matching table, graph, and equation.

Responses vary. **For each equation, the unit price is the coefficient of the independent variable. On the graphs, the point with $x = 1$ would show the price per each item. On the tables, the price per item is the number that is multiplied by the value of p to get the value of m.**

- **In the first situation, the unit price of the paletas is \$2.**
- **In the second situation, the unit price of the paletas is \$2.50.**
- **In the third situation, the unit price of the piraguas is \$3.50.**

5. Ángel sells piraguas for \$4.50 each. How will Ángel's graph be different from Esteban's?

Responses vary. **Each of the points in Ángel's graph will be a little above Esteban's. If you connected the points with a line, Ángel's line would be a little steeper.**

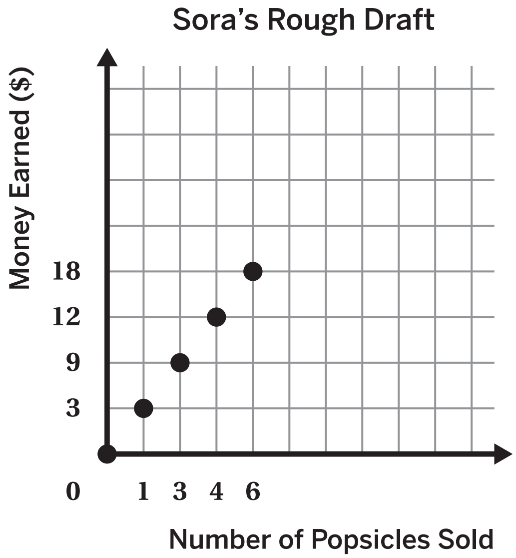
Activity 2

Name: .... Date: .... Period: ....

Critique, Correct, Clarify

Sora sells popsicles for \$3 each. He made a table and a graph to help him understand the relationship between the number of popsicles he sells and the money he earns. His table is correct, but his graph is not quite correct.

Number of Popsicles Sold	Money Earned (\$)
0	0
1	3
3	9
4	12
6	18



6. What do you think Sora did well in his graph?

***Responses vary.* Sora wrote the independent and dependent variables on the correct axes.**

7. What would you recommend Sora change about his graph?

***Responses vary.* I would change how the numbers are scaled on each axis.**

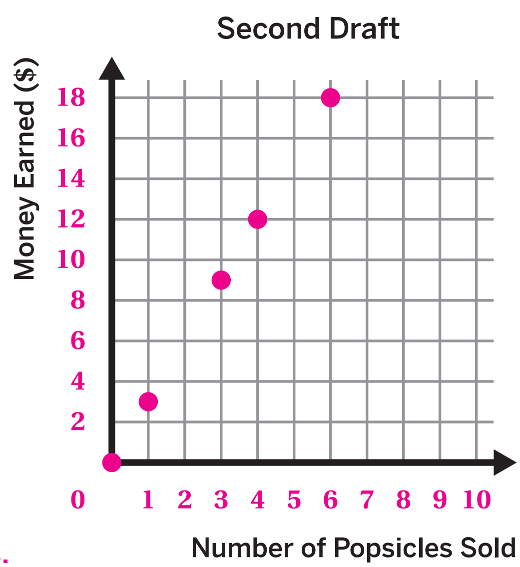
8. Use Sora's table to create a second draft of the graph of the relationship between popsicles sold and money earned.

9. Circle one point on your graph. Explain what that point means in Sora's situation.

***Responses and explanations vary.* I circled the point (3, 9). This point means that Sora sold three popsicles and earned \$9.**

10. What are some other mistakes a person might make when they are creating a graph?

***Responses vary.* One mistake you might make is to swap the x- and y-values when plotting points.**



Synthesis

11. Explain how tables, equations, and graphs represent the same relationship.

Use the example if it helps with your explanation.

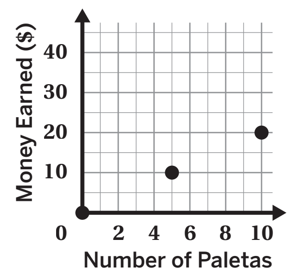
Situation

Amanda sells paletas for $2 each.

Table

p	m
0	0
5	10
10	20

Graph



Equation

$m = 2p$

Responses vary. **First, you want to check that each representation has the same independent and dependent variables. Then you can use the relationship to fill in values in a table. The values in a table become the coordinates in a graph.**

Things to Remember:

What's Missing?

Directions: Make one copy per pair of students. Then pre-cut the cards and give each pair of students one set.

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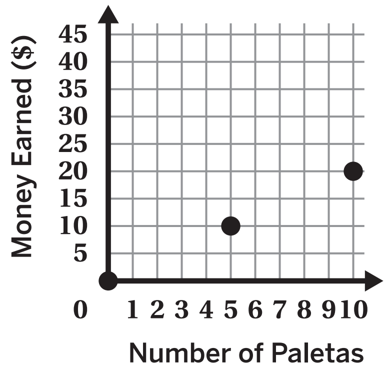
Card A

p	m
2	5
4	10
10	25

Card B



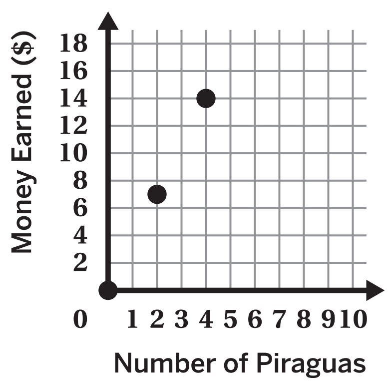
Card C

$m = 3.50p$

Card D

p	m
0	0
5	10
10	20

Card E



Card F

$m = 2.50p$

Unit 6
Lesson
16

Name: ______ Date: ______ Period: ______

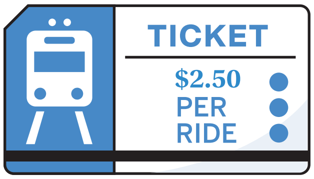
Subway Fares

Let's use tables, graphs, and equations to help customers compare subway fares.

Warm-Up

In Metropolis, there are three ticket options to ride the bus or subway.

Option 1
Regular Fare



Option 2
Unlimited 7-Day Pass

Option 3
Reduced Fare

For people who have low income, are 65 or older, or who have a qualifying disability.

1. For each option, how much will it cost to ride the subway 3 times in the same week?

Option 1: $7.50

Option 2: $30

Option 3: $3.75

Name: ______ Date: ______ Period: ______

Consider the Costs

2. The Metropolis Transit Association (MTA) is in charge of the public buses and subways in Metropolis. Your task is to help an MTA employee show customers how much each ticket option from the Warm-Up costs based on the number of rides.

a As a group, work together to create a table, graph, and equation for each option.

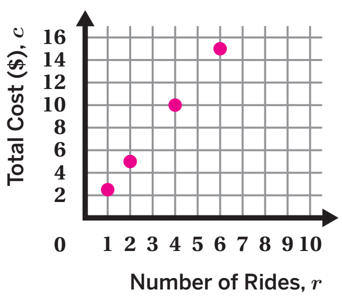
Responses vary.

Option 1 Regular Fare

Table

Number of Rides, r	Total Cost, c (\$)
1	2.5
2	5
4	10

Graph



Equation

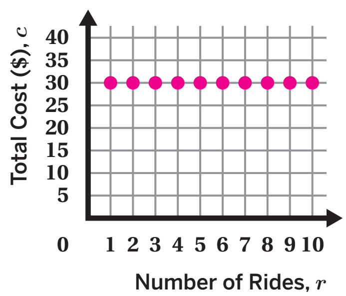
$c = 2.50r$

Option 2 Unlimited 7-Day Pass

Table

Number of Rides, r	Total Cost, c (\$)
1	30
2	30
3	30

Graph



Equation

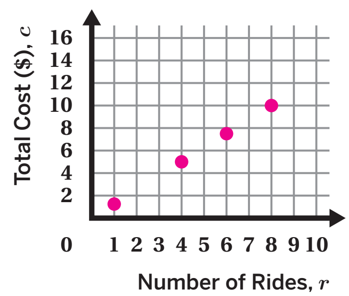
$c = 30$

Option 3 Reduced Fare

Table

Number of Rides, r	Total Cost, c (\$)
1	1.25
4	5
6	7.5

Graph



Equation

$c = 1.25r$

b Which variable is the independent variable? Which is the dependent variable?

r, number of rides, is the independent variable and c, total cost, is the dependent variable.

c Write two to three sentences comparing and contrasting the graphs for the three options.

***Responses vary.* I noticed the graphs for each option seemed to follow a line pattern. The points for Option 2 seem to fall on a horizontal line, while the points for Options 1 and 3 both increase from left to right. All of the graphs have the same independent and dependent variables.**

Activity 2

Name: Date: Period:

Helping Customers

3. Read about four subway customers and choose one to help. Make sure each person in your group chooses a different customer.

Eliza	Nikhil	Sydney	Bao
Eliza is 70 years old. She works at a daycare about 1.5 miles away from her house. Sometimes she walks to work and sometimes she takes the subway. She rides the subway between 2–8 times per week.	Nikhil is 23 years old. He uses a wheelchair and it takes him 20 minutes to get to the closest wheelchair -accessible subway station from his house. Nikhil works as a chef and uses the subway to get to and from work 5 days a week.	Sydney is a 20 year old college student who works part time. Sydney uses the subway to get to school and work and usually rides between 15–20 times per week.	Bao is 16 years old. He walks to school during the week and only uses the subway on the weekends to visit friends.

a. Which fare option should your customer choose? Circle one.

Regular fare Unlimited 7-day pass Reduced fare

Responses vary.

b. Use the tables, graphs, and equations you made in Activity 1 to support your argument.

Responses vary.

- **Eliza qualifies for the reduced fare option. At \$1.25 per ride, riding the subway may cost her between \$2.50 and \$10 a week.**
- **Nikhil qualifies for the reduced fare option. At \$1.25 per ride, riding the subway may cost him about \$12.50 a week.**
- **Sydney should consider purchasing the unlimited 7-day pass. The regular fare would cost Sydney between \$37.50 and \$50 per week. The unlimited pass could save Sydney between \$7.50 and \$20 a week.**
- **Bao should purchase individual regular fare tickets. He doesn't ride the subway enough for the unlimited pass to save him money.**

Activity 3

Name: ________ Date: ________ Period: ________

Increased Fares

The MTA needs more money to help maintain the subway service. The MTA leadership is thinking about raising the regular fare by $0.50.

4. Describe one advantage and one disadvantage of raising the regular fare. Explain your thinking for each.

Responses vary.

One advantage of raising the regular fare is . . . **that the MTA will have more money to fund improvements to the subways and salaries for its employees.**

One disadvantage of raising the regular fare is . . . **that many people may not be able to afford to spend more money to ride the subway.**

5. Look back at your work in Activity 1 for the regular fare. How would raising the fare by $0.50 change the table, graph, and equation? ***Responses vary.***

Table	Graph	Equation
The total costs would all increase.	**All of the points representing the price increase would be higher on the y-axis than the original points.**	**The equation would change to $c = 3r$.**

6. Which of the four customers would be most impacted by the fare increase? Explain your thinking.

Responses and explanations vary. **I think the fare increase would impact Sydney the most. Sydney is a student and works part time, which might mean Sydney doesn't have a lot of extra money to spend on subway fares.**

7. If you were part of the MTA leadership, how would you adjust the fares to get the money you need to maintain service while also charging customers fairly?

Responses vary. **I would suggest first increasing the unlimited 7-day pass because the people who use it every day are already saving money. If I had to, I would increase the regular fare price, but keep the reduced fare option $1.25. This way, the MTA would still be able to earn more money without having to affect everyone, especially people who do not have a lot of money to spend on transportation.**

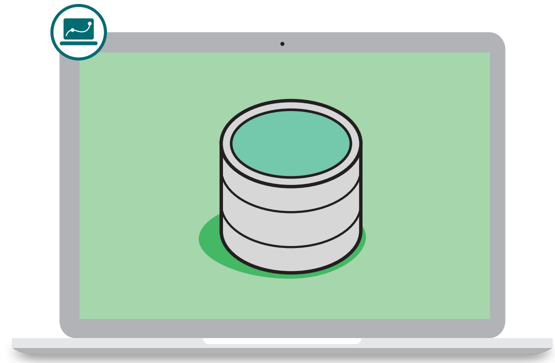
Synthesis

8. How can making a graph and a table help us understand relationships in the world, such as subway fares?

***Responses vary*. Making a table helps us organize different pieces of information. Making a graph helps us see how things change over time and trends, and helps us compare things.**

Things to Remember:

Name: ________________ Date: ________ Period: ________

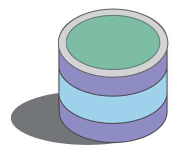


Paint

Let's explore equivalent ratios.

Warm-Up

1 Here is a color made from 5 cups of white paint and 7 cups of green paint.

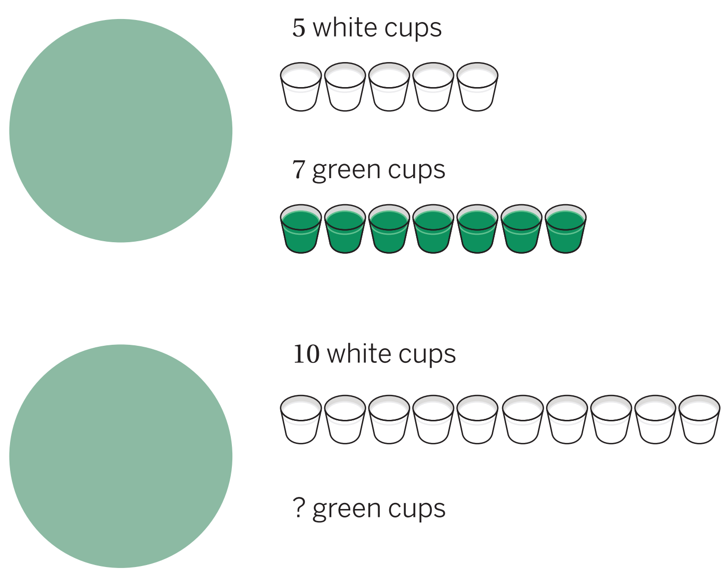


Discuss: What would you name this color? Why?

Responses vary.

2 Brielle wants to match this color.

How many cups of green paint should she mix with 10 cups of white paint to make the same color?



White Paint (cups)	Green Paint (cups)
5	7
10	14

Activity 1

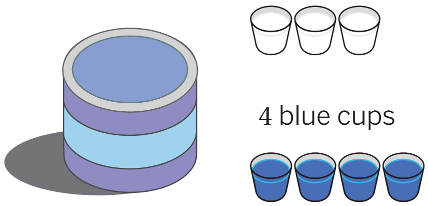
Name: .... Date: .... Period: ....

Color Match

3 Here are two paint mixtures:

- 3 white cups and 4 blue cups
- 6 white cups and 8 blue cups

Both mixtures make the same color because they are *equivalent ratios*.



Which mixture would also make the same color?

A. 5 white cups and 6 blue cups

B. 4 white cups and 3 blue cups

C. 1 white cup and $1\frac{1}{2}$ blue cups

D. 1.5 white cups and 2 blue cups

Explain your thinking.

Explanations vary.

- **Using a unit rate: I figured out that for both mixtures, there are 0.75 cups of white paint for each cup of blue paint. That means for 2 cups of blue paint, there must be $0.75 \cdot 2 = 1.5$ cups of white paint.**
- **Using a scale factor: I started with the ratio in the picture and took half of each color, which is 1.5 cups of white and 2 cups of blue.**

Activity
2

Name: ____________ Date: ____________ Period: ____________

Paint Palooza

4 Darryl mixed 4 cups of white paint with 6 cups of red paint, but he didn't have enough to finish painting his wall.

How much red paint would he need to add to 1 cup of white paint to match the color?

White Paint (cups)	Red Paint (cups)
4	6
1	$\frac{6}{4}$ or 1.5 (or equivalent)

5 Brielle ran out of paint for her room.

Complete the table so that the new mixture matches the original paint color.

Blue Paint (cups)	Red Paint (cups)	White Paint (cups)
12	9	14
4	3	$\frac{14}{3}$ (or equivalent)

Name: .... Date: .... Period: ....

Colorful Challenge

6 You will use the Activity 3 Sheet to create your own paint color challenge!

a **Make It!** Create your challenge on the Activity 3 Sheet.

b **Solve It!** On this page, record the number of cups of paint used in both your original mixture and the new mixture. ***Responses vary.***

	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				

c **Swap It!**

- Swap your challenge with one or more partners.
- Record the information about your partner's original mixture and their new mixture.
- Fill in the missing amounts to complete the new mixture. ***Responses vary.***

Partner 1	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				

Partner 2	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				

Partner 3	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				

7 Synthesis

Explain how equivalent ratios can help make matching paint colors.

Use the example if it helps with your thinking.

Responses vary. **In this example, there is 1 cup of white for 4 cups of green. I can make a new mixture with a matching color by multiplying the cups of white and cups of green by the same value, so that the ratios are equivalent. In the example, the number of white and green cups are multiplied by 2. 1 to 4 is an equivalent ratio to 2 to 8.**

Things to Remember:

Name: .... Date: .... Period: ....

Colorful Challenge

- Create your own paint color by filling in the amounts for *at least* two colors.

	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				

- Name your paint color and describe what you think it looks like.

- Add a new amount of *one* color that you used in your original mixture to the table. Then challenge your classmates to fill in the missing amounts and match your original paint color.

	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
New Mixture That Matches				

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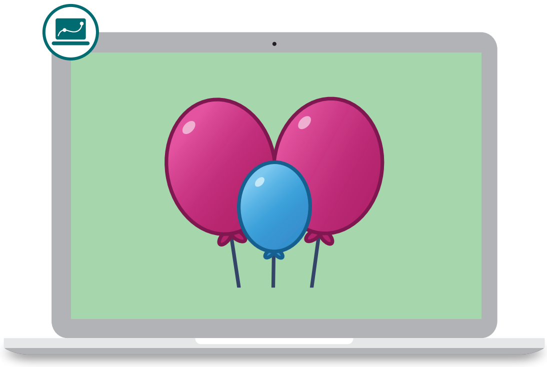
Unit 2
Lesson
2

Name: ______ Date: ______ Period: ______



Balloon Float

Let's explore proportional relationships in tables.

Warm-Up

1 This table shows how many rolls of paper towels a store receives when they order different numbers of cases.

Number of Cases Ordered	Number of Rolls of Paper Towels
1	12
3	36
5	60
10	120

What do you notice? What do you wonder?

I notice:

Responses vary.

- I notice that each row is an equivalent ratio to the other rows.
- I notice there are 12 rolls of paper towels in every case.
- I notice that if I multiply every number in the first column by 12, I get the number in the second column.

I wonder:

Responses vary.

- I wonder what's the largest number of cases that can be ordered.
- I wonder if the relationship between number of cases and number of rolls of paper towels would continue forever.

Activity 1

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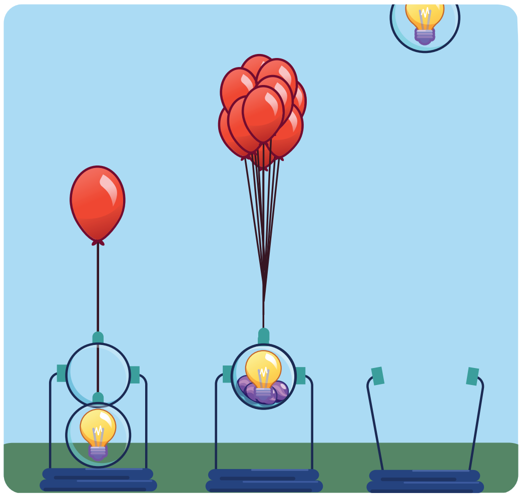
Balloon Float

Helium balloons can make objects float, but too many balloons will make objects fly away!

2 **a** Let's watch the *middle* light bulb float.



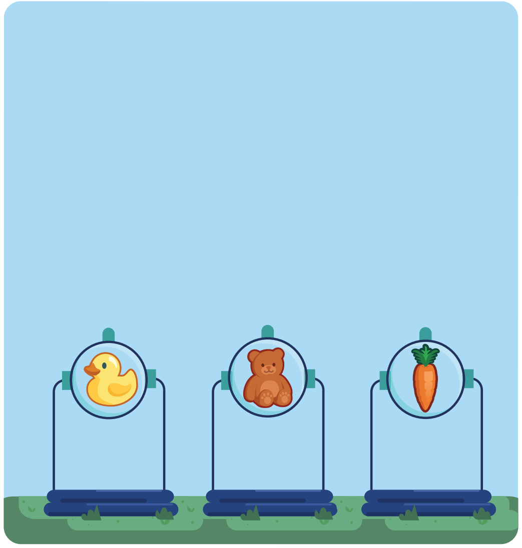
b **Discuss:** Why do you think the number of balloons matters?

***Responses vary.* If there aren't enough balloons, the light bulb will fall to the ground. And if there are too many balloons, the light bulb will fly away. The number of balloons has to be just right to make the light bulb float.**

3 In the previous problem, the light bulb weighed 2 ounces and needed 6 balloons to float.

If each balloon carries the same weight, how many balloons would you need to float each object?

Object	Weight (oz)	Number of Balloons
Light bulb	2	6
Rubber duck	10	**30**
Toy bear	6	**18**
Carrot	3	**9**



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Balloon Float (continued)

4 Here are two strategies for determining the number of balloons needed to make the rubber duck float.

Ariel

Object	Weight (oz)	Number of Balloons
Light bulb	2	6
Rubber duck	×5 10	30 ×5
Toy bear	6	
Carrot	3	

Emma

Object	Weight (oz)	Number of Balloons
Light bulb	2 $\xrightarrow{\times 3}$	6
Rubber duck	10 $\xrightarrow{\times 3}$	30
Toy bear	6	
Carrot	3	

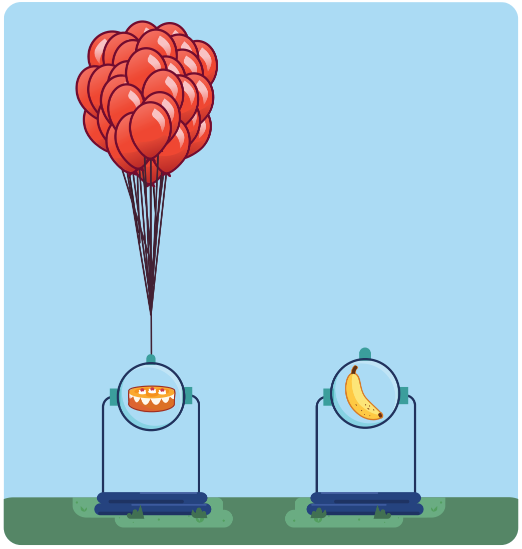
Discuss: How might Ariel and Emma use their strategies to finish their tables?

Responses vary.

- **Ariel might finish the table by scaling the numbers in the first row (2 ounces and 6 balloons). To determine how many balloons would make the toy bear float, Ariel would need to multiply by a scale factor of 3, which would give Ariel 18 balloons. To determine the number of balloons for the carrot, Ariel need to multiply by a scale factor of 1.5, which would give Ariel 9 balloons.**
- **Emma might finish the table by multiplying each weight by 3 to get the number of balloons. The unit rate (balloons per ounce) is 3, which tells us how many balloons are required to carry 1 ounce of weight.**

5 Here are some new objects. Complete the table so that each object floats.

Object	Weight (oz)	Number of Balloons
Light bulb	2	6
Cake	**20**	60
Banana	$3\frac{1}{3}$	**10**



Activity 2

Name: Date: Period:

Proportional Relationships

When two quantities are always in an equivalent ratio, they have what's called a **proportional relationship**.

6 Here are two more tables.

Which of these two tables represents a proportional relationship? Circle one.

Table 1 Table 2 Both Neither

Explain your thinking.

Explanations vary. **The number of balloons is always 3 times the weight in ounces. That means they're always in a 3 to 1 ratio, which means their relationship is proportional.**

Table 1

Weight (oz)	Number of Balloons
3	6
7	10
9	12
30	33

Table 2

Weight (oz)	Number of Balloons
4	12
6	18
42	126
8	24

7 Sort the tables into two groups based on whether they represent proportional relationships.

Table A

x	y
0	0
4	5
8	10
12	15

Table B

x	y
0	0
2	4
4	16
6	36

Table C

x	y
0	2
3	5
6	8
9	11

Table D

x	1	2	3	4
y	10	8	6	4

Table E

x	2	8	1	20
y	5	20	2.5	50

Proportional Relationship	Not a Proportional Relationship
Table A, Table E	**Table B, Table C, Table D**

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Proportional Relationships (continued)

8 How did you decide whether this table represents a proportional relationship?

***Responses vary.* This is not a proportional relationship. If it were a proportional relationship, I could multiply all the x-values by the same number and get all the y-values. That's not possible here.**

Proportional Relationship | Not a Proportional Relationship

x	y
0	0
2	4
4	16
6	36

9 Select *all* the relationships you think are proportional.

- [x] **A.** A person's height in feet and their height in inches
- [ ] **B.** The number of cookies baked and the number of minutes they were in the oven
- [x] **C.** The amount of bread baked and the number of grams of flour needed to bake it
- [ ] **D.** A person's time as they run a marathon and their total distance covered
- [x] **E.** The gallons of gasoline purchased and their total cost

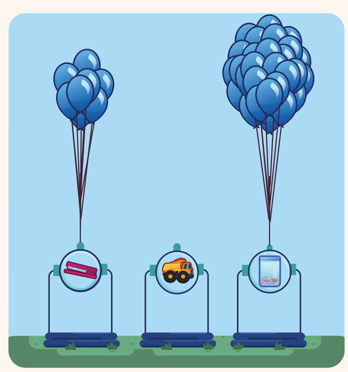
***Responses vary.* Different choices could be correct depending on the assumptions students make.**

Explore More

10 Blue balloons are different from red balloons.
8 blue balloons can float a 10-ounce stapler.

Complete the table so that each object floats.

Object	Weight (oz)	Blue Balloons
Stapler	10	8
Toy truck	15	**12**
Jelly beans	**35**	28



11 Synthesis

Here are some relationships, some of which are proportional and some of which are not.

What determines whether a relationship is proportional?

Use the examples if they help with your thinking.

- A person's height in feet and their height in inches
- The number of cookies baked and the number of minutes they were in the oven
- The amount of bread baked and the number of grams of flour needed to bake it
- A person's time as they run a marathon and their total distance covered
- The gallons of gasoline purchased and their total cost

***Responses vary*. A relationship is proportional if the two quantities are always in the same ratio. For example, if the total cost of gasoline is based on a consistent price (like $3 per gallon), then the gallons of gasoline purchased and the total cost will always be in the same ratio. So that relationship would be proportional.**

Things to Remember: