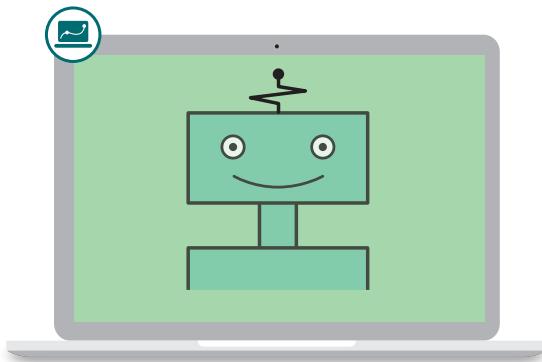Unit 2
Lesson

4

Name: _____ Date: _____ Period: _____

Robot Factory

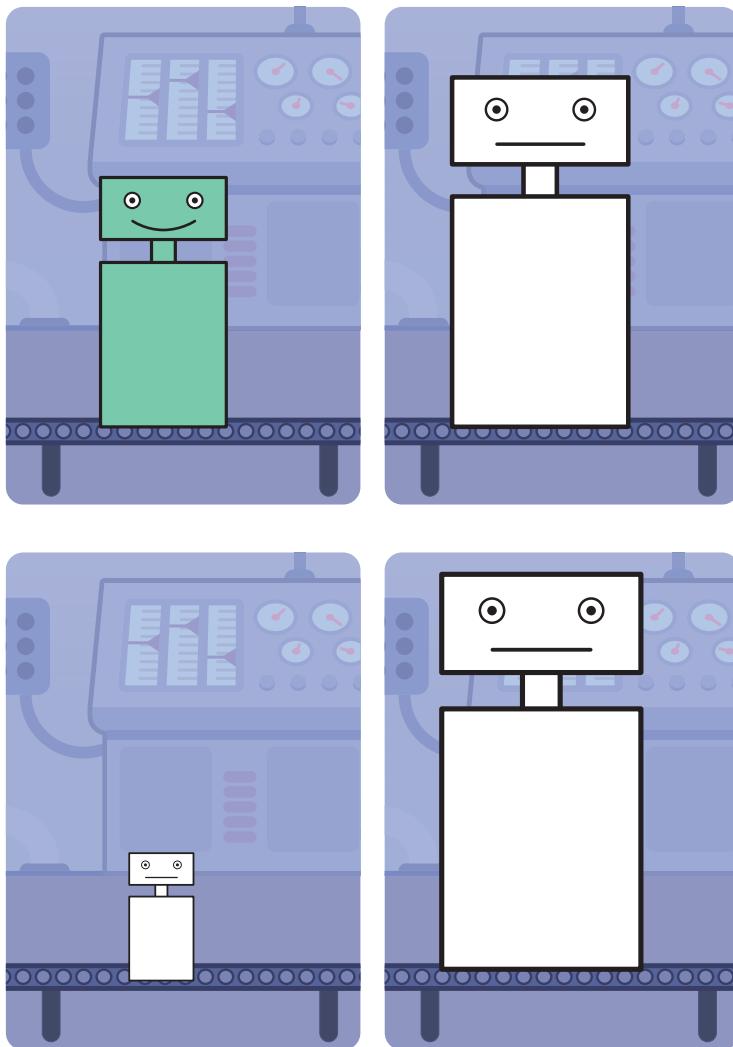Let's write equations for proportional relationships.

Warm-Up

1-2 The robot factory can make robots of all different sizes.

This robot's shade of green is made by mixing green and white paint using the amounts in the table.

Complete the table so that all four robots have the same shade of green.

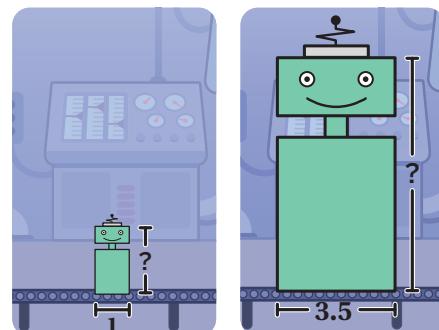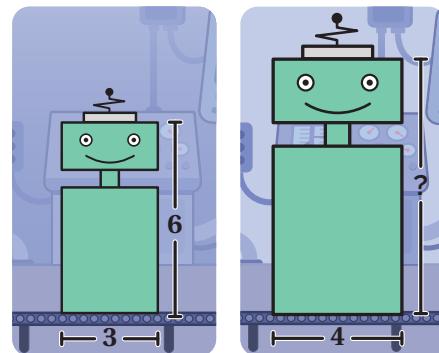| Green Paint (cups)	White Paint (cups)
4	3
8	**6**
1	**$\frac{3}{4}$**
10	**$\frac{30}{4}$**

Name: _____   Date: _____   Period: _____

Robot Hats

3-4 Let's make 30 *scaled copies* of this robot.

The width of this robot is 3 inches. Its hat is 6 inches off the ground.

Complete the table with the height for placing the hat on each robot.

Robot Width (in.)	Height for Placing Hat (in.)
3	6
4	**8**
1	**2**
3.5	**7**
2.18	**4.36**
1.76	**3.52**
3.425	**6.85**
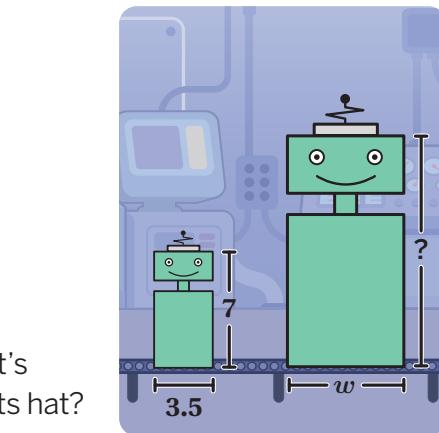
5 Here is a student's work from the previous problem.

Robot Width (in.)	Height for Placing Hat (in.)
3.5	3.5 • 2
2.18	2.18 • 2
1.76	1.76 • 2

Help someone know how to finish the job. For any robot's width, how could you determine the height for placing its hat?

Responses vary. **To know the height for placing the robot's hat, take its width and double it.**

6 Instructions with words are useful for humans, but machines understand mainly numbers and symbols.

Which equation tells the factory the relationship between a robot's width, w, and the height for placing its hat, h?
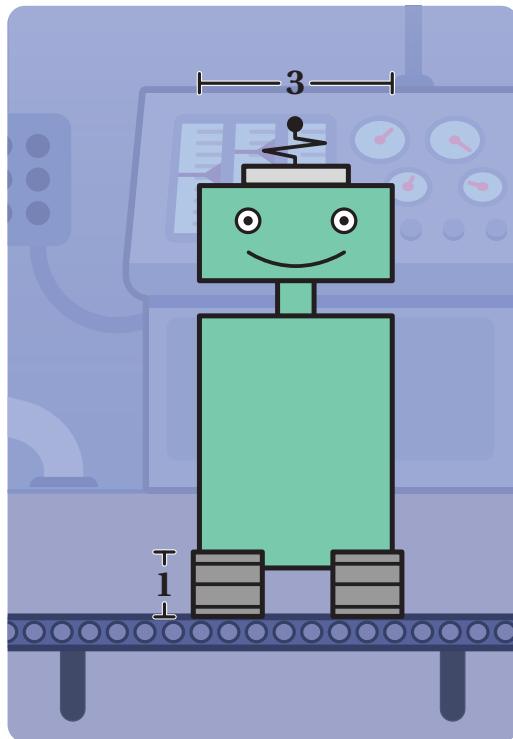
A. $h = \frac{1}{2}w$ **B.** $h = w + 3$ **C.** $w = 2h$ **D.** $h = 2w$

Name: _____ Date: _____ Period: _____

More Robot Parts

7-8 This robot is 3 inches wide. Its shoes are 1 inch tall.

Enter the shoe height for each robot.

Robot Width (in.), w	Shoe Height (in.), s
3	1
6	**2**
5	$\dfrac{5}{3}$
1	$\dfrac{1}{3}$

9 Write an equation the factory could use to put shoes on the rest of the robots.

Use s for the shoe height and w for the robot's width.

$s = $ ____$\dfrac{w}{3}$ **or** $\dfrac{1}{3}w$ **(or equivalent)**____

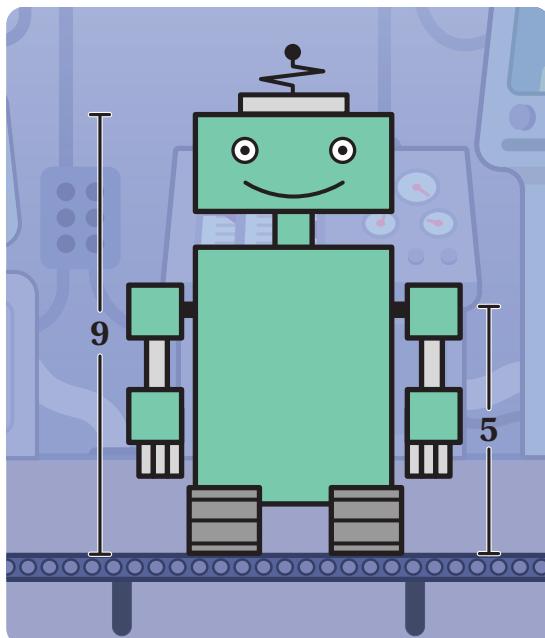**10-11** This robot has a height of 9 inches. Its arms are 5 inches off the ground.

Write an equation that the factory could use to proportionally attach arms to the robots.

Use a for the height to place the arms and r for the robot height.

$a = $ ____$\dfrac{5}{9}r$ **or** $0.56r$ **(or equivalent)**____

Use this table if it helps with your thinking.

Robot Height (in.), r	Height for Placing Arms (in.), a
9	5
5	2.78
1	0.56

12 Synthesis

Describe a strategy for writing an equation of a proportional relationship when given a table.

Use one or both of the examples if they help with your thinking.

Responses vary.

- **First, find the constant of proportionality for the relationship. Then write an equation that looks like: (2nd variable) = (constant of proportionality) • (1st variable).**
- **Use the given values to determine the second value when the first value is 1. For instance, if I know the pair $(9, 5)$, then I can divide both values by 9 to get $(1, \frac{5}{9})$. This number, $\frac{5}{9}$, is the constant of proportionality, which I can use to write the equation.**

Hat

Robot Width (in.), w	Hat Height (in.), h
3	6
1	2

The constant proportionally is 2.

An equation for this relationship is $h = 2w$.

Arms

Robot Width (in.), r	Arm Height (in.), a
9	5
1	$\frac{5}{9}$

The constant proportionally is $\frac{5}{9}$.
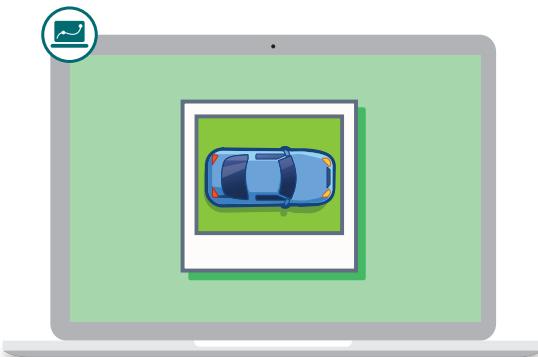
An equation for this relationship is $a = \frac{5}{9}r$.

Things to Remember:

Unit 5
Lesson

5

Name: .... Date: .... Period: ....
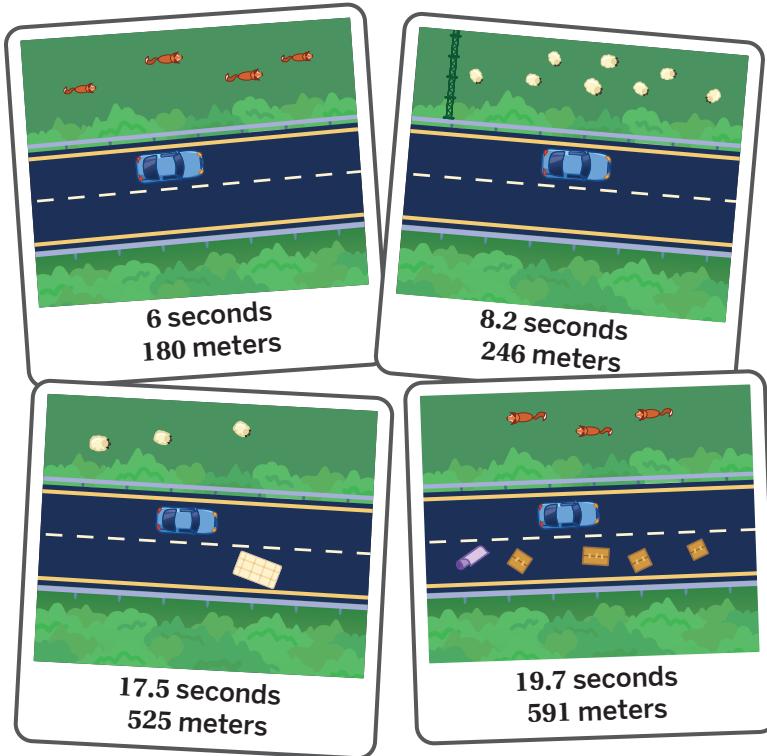
Snapshots

Let's use equations to make sense of proportional relationships in the world.

Warm-Up

1 Write a story about this car's trip.

Responses vary. **The car drove in one direction, and the longer it drove, the farther it went. The farther it went, the more interesting and unusual things it came across.**

6 seconds
180 meters

8.2 seconds
246 meters

17.5 seconds
525 meters

19.7 seconds
591 meters

Name: _____ Date: _____ Period: _____

Travel Times

2 The car travels at a constant speed. After **6** seconds, it travels **180** meters.

6 seconds
180 meters
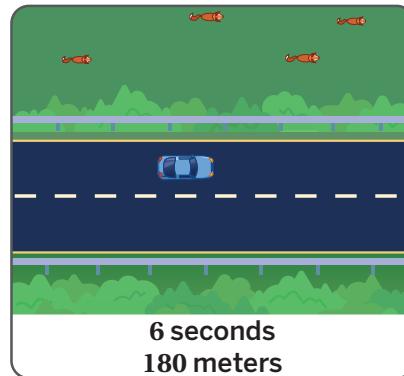
a Write an equation for the car's distance, d, at any time, t.

$d =$ _____**30t**_____

b Enter the time and distance for three moments during the car's trip.
Responses vary.

Time (sec), t	Distance (m), d
6	180
8.2	246
17.5	525

3 Ethan says that a row in a table is like a picture and an equation is like a video. Explain what Ethan might be thinking.

Responses vary.
- **A row in a table is just one pair of values in a relationship and an equation can tell you about all possible pairs of values.**
- **A row in a table is a single moment. An equation can provide all of the moments.**

4 Use the equation you wrote to complete the table.

Time (sec), t	Distance (m), d
6	180
3	**90**
2	60

Name: .... Date: .... Period: ....

Cakes

5 A cake recipe uses the equation $m = 6c$, where c is the number of cakes and m is ounces of milk.

Explain what the constant of proportionality means in this situation.

Responses vary. **The constant of proportionality is 6, which means each cake uses 6 ounces of milk.**
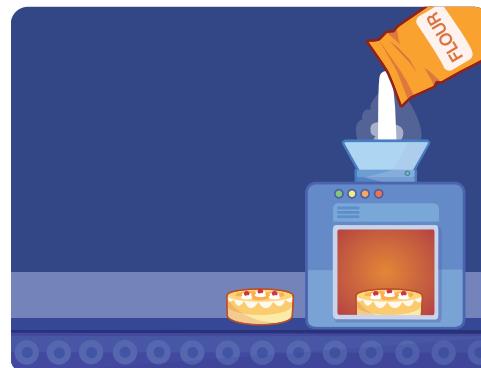
6 How many ounces of milk are needed to bake 12 cakes?

72 ounces

7 A cake recipe says to use 3 cups of flour for every 2 cakes.

a 💬 **Discuss:** How much flour do you need for 1 cake?

Responses vary. **You need 1.5 cups of flour.**

b Write an equation to calculate the amount of flour needed, f, for any number of cakes, c.

$f = $ ....$\dfrac{3}{2}c$ **(or equivalent)**.........

2 cakes
3 cups of flour

Name: .... Date: .... Period: ....
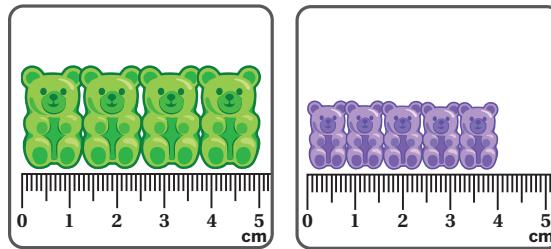
Gummy Bears

8 Here is an equation: $\ell = \frac{5}{4}g$. ℓ is the total length and g is the number of gummy bears.

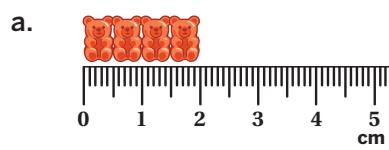Which size gummy bear does the equation represent? Circle one.

(Large)   Small   Both   Neither

Explain your thinking. *Explanations vary.*
The equation represents a situation where every gummy bear is $\frac{5}{4}$ inches long. The large gummy bear looks to be that length.
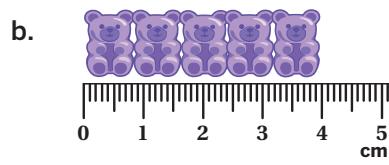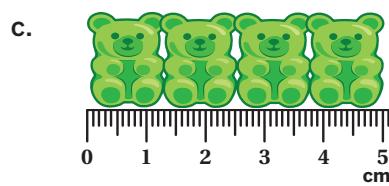
$$\ell = \frac{5}{4}g$$

9 Match each image with one or more equations or descriptions. In each equation, ℓ is the total length and g is the number of gummy bears.

a.

b.

c.

.......**b**....... $\ell = \frac{4}{5}g$

.......**a**....... $\ell = \frac{1}{2}g$

.......**c**....... $\ell = \frac{5}{4}g$

.......**c**....... A line of 80 of these gummy bears is 100 centimeters long.

Explore More

10 Use the Explore More Sheet to answer a question about a truck's trip.

18 hours.
Explanations vary. **Since the truck can travel 6 miles for each gallon and can hold 150 gallons of gas, it can travel 150 • 6 = 900 miles on each tank of gas. Traveling 50 miles per hour, the truck could travel for $\frac{900}{50} = 18$ hours.**

11 Synthesis

Here is a snapshot from a situation represented by the equation $p = 8g$, where p represents a number of pints and g represents a number of gallons.

How can an equation of a proportional relationship represent a situation? Use the example if it helps you explain your thinking.

Responses vary. **I think the equation $p = 8g$ represents the situation because it tells us that there are 8 pints in a gallon.**

Pint   Gallon

Things to Remember:

Name: .... Date: .... Period: ....

Explore More

Here are some facts about this truck:

- It travels at an average rate of 50 miles per hour.

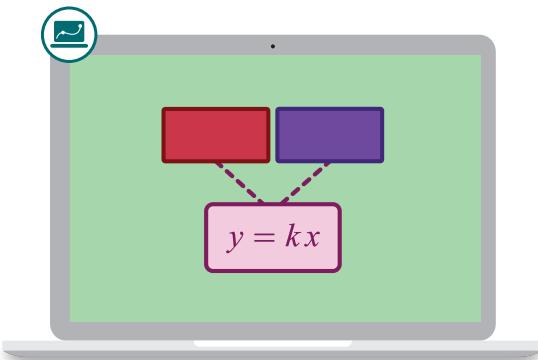- It can travel 6 miles for each gallon of gas.

How many hours can the truck travel without stopping if it has a full tank of 150 gallons?

Explain your thinking.

Name: _____ Date: _____ Period: _____

All Kinds of Equations

Let's use equations to decide whether a relationship is proportional.

$$y = kx$$

Warm-Up

1 This table represents the equation $y = 3x + 1$.

a Use the equation to complete the table.

b Does the equation represent a proportional relationship? Circle one.

Yes   (No)   Not enough information

Explain your thinking.

Explanations vary.
- **There is no constant of proportionality.**
- **In the second row, in order for y to equal 4, I need to multiply 1 by 4. In the third row, in order for y to equal 10, I need to multiply 3 by $\frac{10}{3}$.**

x	y
0	1
1	4
3	10
5	16
2.5	**8.5**

Name: _____   Date: _____   Period: _____

Stories, Equations, Tables

2 Use the story and equation to complete the table. Then decide whether the relationship is proportional. Complete either Story 1 or Story 2.

Story 1

Story: Trinidad earns $12 per hour.

Equation: $y = 12x$

Is the relationship proportional? Circle one.

(Yes)   No

Explain your thinking.
Explanations vary. **The constant of proportionality is 12.**

Time Worked (hr), x	Pay ($), y
0	**0**
1	**12**
2.5	30
3	**36**

Responses in the last row vary. **Sample shown.**

Story 2

Story: A recipe recommends 1 banana for every 2 smoothies.

Equation: $y = \dfrac{1}{2}x$

Is the relationship proportional? Circle one.

(Yes)   No

Explain your thinking.
Explanations vary. **The constant of proportionality is 0.5.**

Number of Smoothies, x	Number of Bananas, y
0	0
1	**0.5**
5	2.5
8	**4**

Responses in the last row vary. **Sample shown.**

Find a partner who completed the other story.

💬 **Discuss:** How are your responses alike? How are they different?

Responses vary. **The equations, tables, and stories were different, but both relationships were proportional even though they had different constants of proportionality.**

Name: _____ Date: _____ Period: _____

Stories, Equations, Tables (continued)

Complete either Story 3 or Story 4.

Story 3

Story: A cell phone costs $500, plus $35 per month for the plan.

Equation: $y = 500 + 35x$

Is the relationship proportional? Circle one.

Yes   (No)

Explain your thinking.
There is no constant of proportionality.

Number of Months, x	Total Cost ($), y
0	**500**
1	**535**
3	605
5	675

Responses in the last row vary. **Sample shown.**

Story 4

Story: The area of a square is the side length multiplied by itself.

Equation: $y = x^2$

Is the relationship proportional? Circle one.

Yes   (No)

Explain your thinking.
There is no constant of proportionality.

Side Length (units), x	Area (sq. units), y
0	**0**
1	**1**
10	100
5	25

Responses in the last row vary. **Sample shown.**

3 Here are the equations that represent the four stories.

a Select *all* the equations that represent a proportional relationship.

☑ **A.** $y = 12x$ ☐ **B.** $y = 500 + 35x$

☑ **C.** $y = \frac{1}{2}x$ ☐ **D.** $y = x^2$

b Explain one way to decide if an equation represents a proportional relationship.
Explanations vary. **One way to decide if a relationship is proportional is to look at the equation and see if it has the form $y = $ (something)x.**

Name: _____ Date: _____ Period: _____

Equations and Proportionality

4 Decide whether each equation, table, or story represents a proportional relationship by placing a checkmark in the appropriate column.

	Proportional	Not Proportional
$4 + x = y$		✔
$y = 4x$	✔	
Jacy walked 4 miles in 100 minutes at a steady pace.	✔	
$0.04x = y$	✔	
$y = \dfrac{x}{4}$	✔	
$\dfrac{4}{x} = y$		✔

x	y
1	4
2	2
3	$\frac{4}{3}$
4	1

(This table row: Not Proportional ✔)

5 Maki thinks $\frac{4}{x} = y$ is proportional. Karima thinks $y = \frac{x}{4}$ is proportional. Whose thinking is correct? Circle one.

Maki's (Karima's) Both Neither

Explain your thinking.

Explanations vary. **The constant of proportionality for $y = \frac{x}{4}$ would be $\frac{1}{4}$, but $\frac{4}{x} = y$ doesn't have a constant of proportionality because it's not the same as multiplying by 4.**

Proportional

$\frac{4}{x} = y$ $y = \frac{x}{4}$

Explore More

6 Use the Explore More Sheet to answer questions about a relationship and proportionality.

7 Synthesis

a Write two equations: one that represents a proportional relationship and one that does not. ***Responses vary.***

Proportional Relationship	Not a Proportional Relationship
• $d = 58t$ • $a = 0.12B$ • $W = \dfrac{n}{2}$	• $d = t + 58$ • $a = 0.12B - 5$ • $W = \dfrac{2}{n}$

b Describe how you know whether an equation represents a proportional relationship.

Explanations vary. **Equations of proportional relationships can always be made to have the form $y = $ something $\bullet\, x$, and the "something" is the constant of proportionality.**

Things to Remember:

Name: _____   Date: _____   Period: _____

Explore More

Observe how the number of circles changes at each stage.
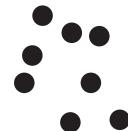
Stage 0 **Stage 1** **Stage 2** **Stage 3** **Stage 4**

a Complete the table.

Stage	Number of Circles
0	1
1	
2	
3	
4	

b Does this represent a proportional relationship? Circle one.

Yes   No

Explain your thinking.

Unit 2
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8
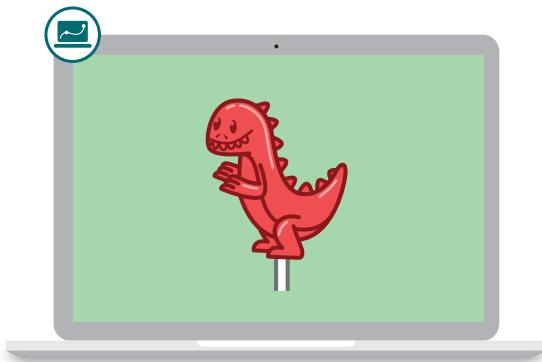
Name: _____   Date: _____   Period: _____
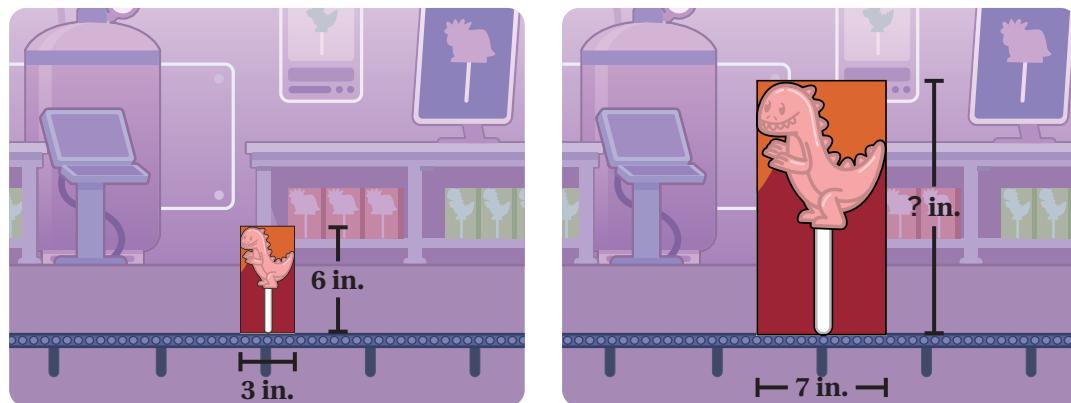
DinoPops

Let's explore what a proportional relationship looks like on a graph.

Warm-Up

1 Here are two DinoPops in their boxes.

DinoPops come in all sizes between 2 and 200 inches tall. They are always scaled copies of one another.
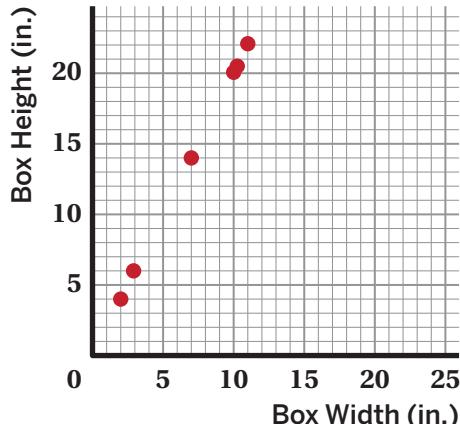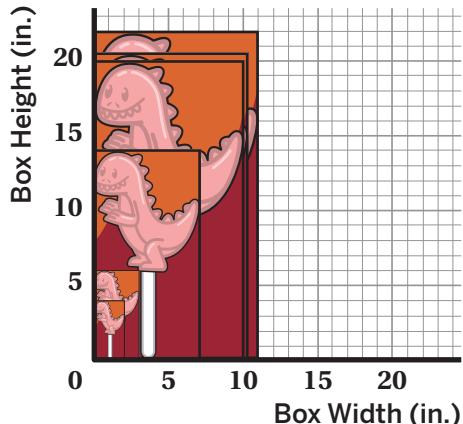

Complete the table to determine the height of the box for the large DinoPop.

Box Width (in.)	Box Height (in.)
3	6
7	**14**

Name: _____   Date: _____   Period: _____

DinoPops

2 Here are some DinoPop boxes and a graph of some points.

What do you notice? What do you wonder?

I notice: *Responses vary.*
- **I notice that each point on the graph corresponds to one DinoPop box.**
- **I notice that you can draw a straight line through all of the points.**
- **I notice that the imaginary line passes through the lower-left corner of the graph, (0, 0).**

I wonder: *Responses vary.*
- **I wonder if all of the DinoPop boxes will be on this imaginary line.**
- **I wonder if other types of boxes make points that line up nicely.**
- **I wonder if other types of boxes form a line through (0, 0).**

3 A box that is 5 inches wide and 10 inches tall is a perfect fit for a DinoPop. A box that is 7 inches wide and 14 inches tall is also a perfect fit.
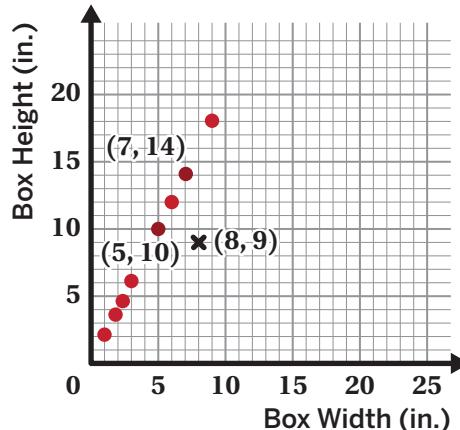

Write ordered pairs for two other boxes that are a perfect fit for a DinoPop. *Responses vary.*

(__4__, __8__)   (__8__, __16__)

A box that is 8 inches wide and 9 inches tall is *not* a perfect fit for a DinoPop.

Write ordered pairs for two other boxes that are *not* a perfect fit for a DinoPop. *Responses vary.*

(__2__, __10__)   (__7__, __5__)
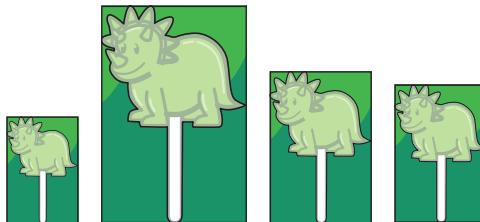
Name: _____ Date: _____ Period: _____

TriceraPops

4-5 Here are several TriceraPops in boxes.
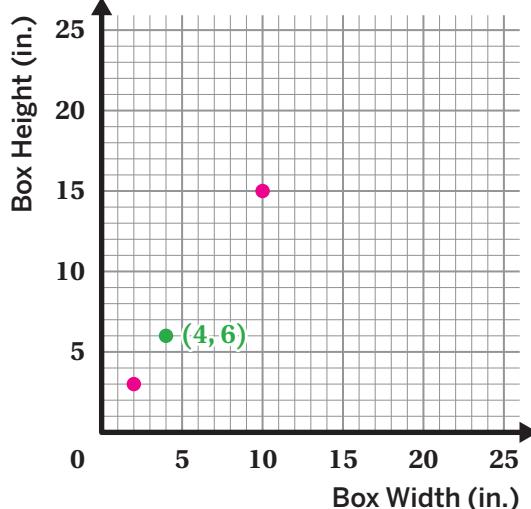
The graphed point represents *one* of these boxes.

a 💬 **Discuss:** What do you know about this box?
Responses vary. **I know that this box has a width of 4 inches and a height of 6 inches.**

b Add at *least two* more points to the graph to represent other boxes that fit a TriceraPop.
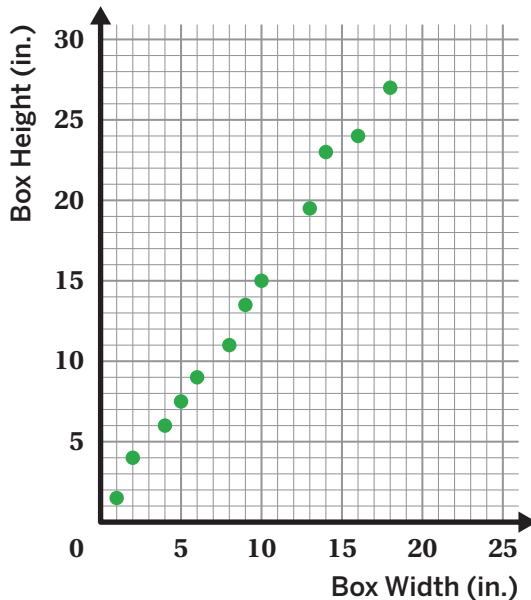*Responses vary.* **Sample shown on graph.**

6 A student made some TriceraPop boxes, as shown on this graph.

Some of the boxes were not a good fit.

Describe how to use the graph to find the bad boxes.

Draw on the graph if it helps to show your strategy.
Responses vary. **Sketch a line that goes through the origin and most of the other points. The few points that do not fall on this line are the bad boxes.**

Name: _____ Date: _____ Period: _____

TriceraPops (continued)

7 Here is a graph of box sizes for a new lollipop.

Is there a proportional relationship between the height and the width of these boxes? Circle one.
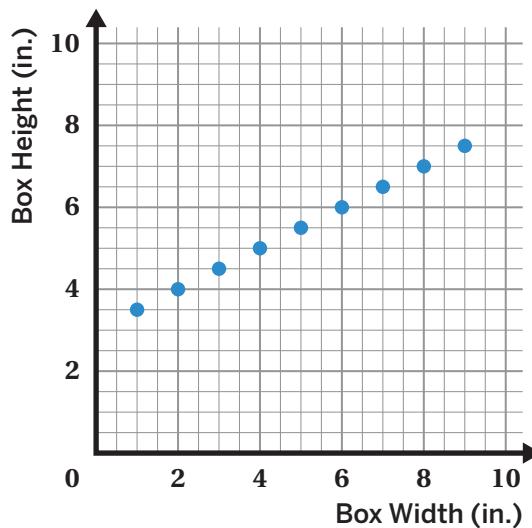
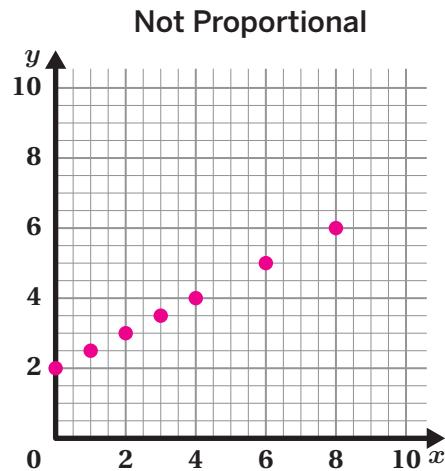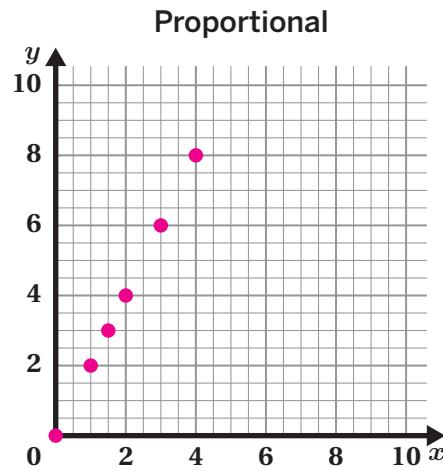Yes   (No)   I'm not sure

Explain your thinking.

Explanations vary.

- **I see that this relationship includes a box that is 6 inches by 6 inches and also a box that is 2 inches by 4 inches. Those are not equivalent ratios, so the relationship can't be proportional.**
- **Some boxes are short and wide, and others are tall and narrow. Therefore, the width and height are not proportional to one another.**
- **The points do lie on a single line, but that line does not include the origin, so the relationship is not proportional.**
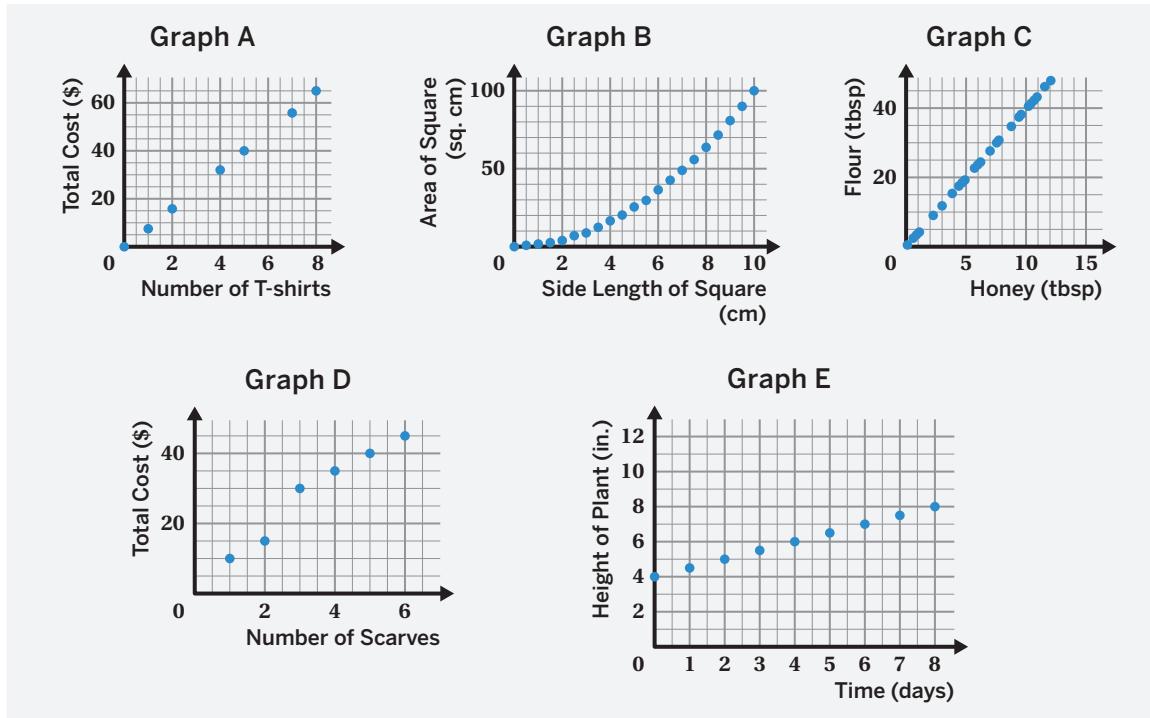
8 Make one graph that represents a proportional relationship and one graph that does not.
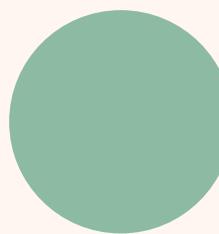
Responses vary.

Proportional

Not Proportional

Name: _____ Date: _____ Period: _____

Graphs

9 Decide whether each graph represents a proportional relationship.

Graph A

Graph B

Graph C

Graph D

Graph E

Proportional	Not Proportional
Graph A, Graph C	Graph B, Graph D, and Graph E

Explore More

10 This color green is made by mixing 3 cups of white paint and 2 cups of green paint.
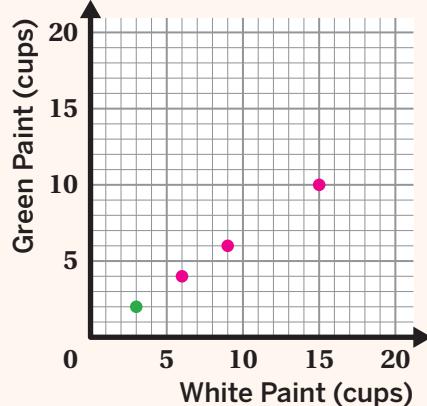
3 white cups

2 green cups

Graph points that represent *at least* two more ways to make this color.
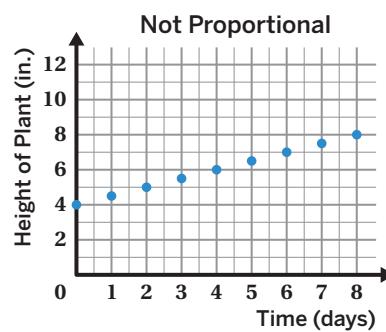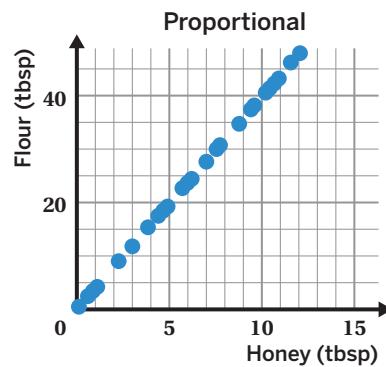
Responses vary. **Sample shown on graph.**

11 Synthesis

How can you use a graph to decide whether a relationship is proportional?

Use the examples if they help with your thinking.

Responses vary.
- **A relationship is proportional if all its points lie on a single line that goes through (0, 0).**
- **A relationship is not proportional if all its points lie on a single line that does not go through the origin. The graph representing the relationship between time and the height of a plant isn't proportional since it contains the point (0, 4).**
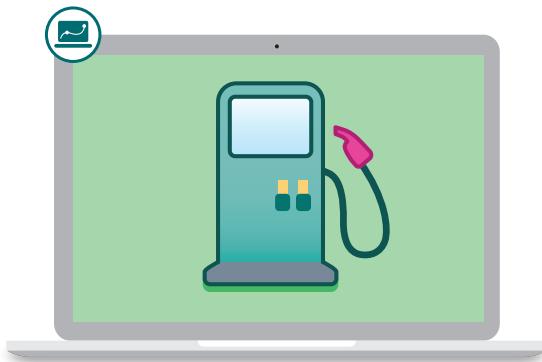- **A relationship is not proportional if it has points that do not lie on a single line.**

Proportional

Not Proportional

Things to Remember:

Unit 2
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9

Name: _____ Date: _____ Period: _____
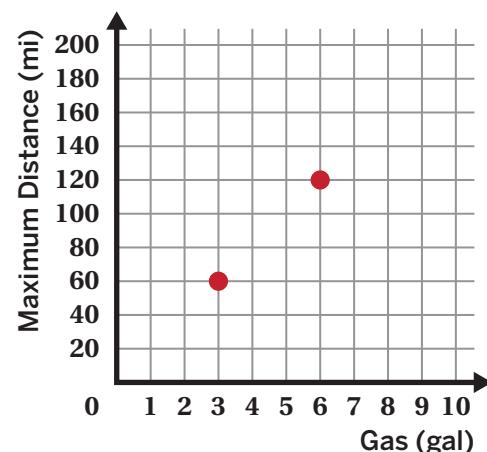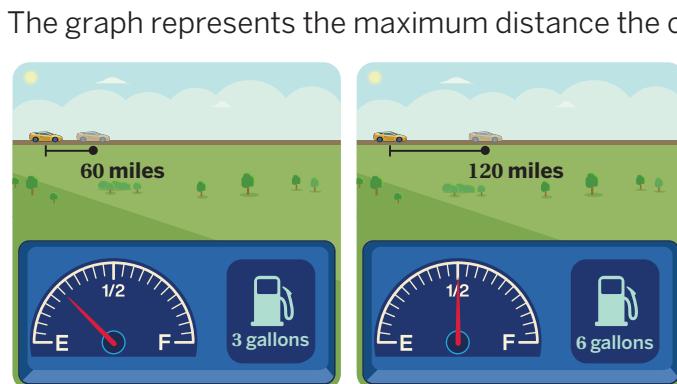
Gallon Challenge

Let's identify constants of proportionality using a graph.

Warm-Up

1   Here are two images showing the amount of gas in a car's tank.

The graph represents the maximum distance the car can go using each amount of gas.

💬 **Discuss:** What do you notice? What do you wonder?

Responses vary.
- **I notice that the number of miles is always 20 times the number of gallons.**
- **I notice that you can drive further when there is more gas in the tank to start.**
- **I wonder how often a car actually travels the maximum distance.**
- **I wonder what is the furthest you could drive on one tank of gas.**

Name: .... Date: .... Period: ....

On the Road

2 Here are several points representing the maximum distance the car can go using different amounts of gas.
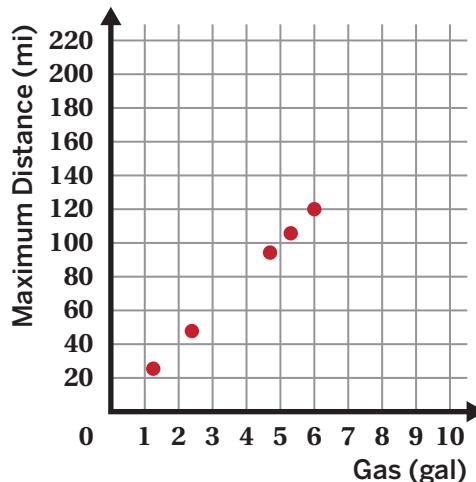
What would the graph look like if it included every possible point for this car?

Responses vary. **It would look like a line through the origin.**

3 The car can travel 240 miles with a full 12-gallon tank of gas. Which line represents this relationship? Circle one.

A   (B)   C   D

4 A car's *gas mileage* is the maximum distance it can go using 1 gallon of gas (measured in miles per gallon).
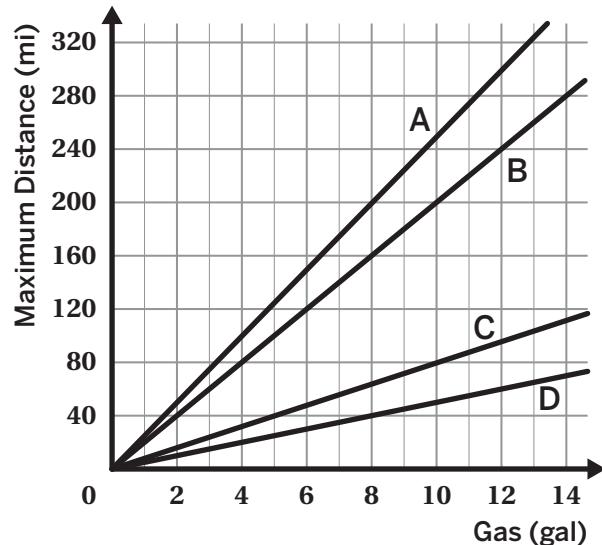
Based on the graph, what is this car's gas mileage?

........**20**........ miles per gallon

Where do you see this number in the graph?

Responses vary.
• **The point (1, 20) on the graph tells me that the car can drive 20 miles using 1 gallon of gas.**
• **I can use the point (12, 240) on the graph and divide 240 by 12 to determine the number of miles for each gallon of gas.**

Name: _____ Date: _____ Period: _____
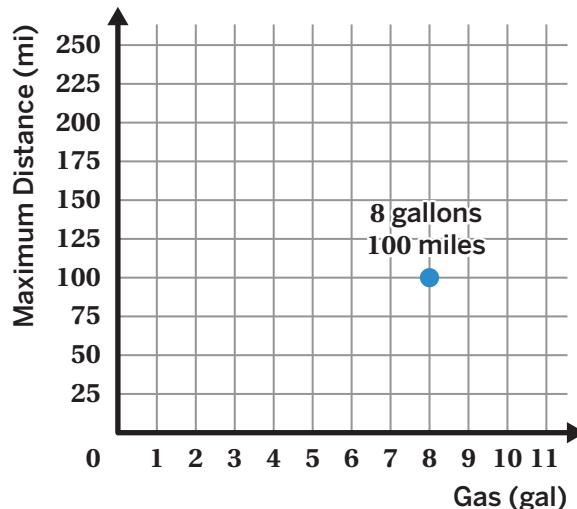
Gas Mileage

5 Yasmine's truck travels 100 miles using 8 gallons of gas.

What is the gas mileage for her truck?
12.5 miles per gallon

6 Yasmine wants to buy a new vehicle that gets better gas mileage than her truck.

Which vehicle should she pick?
Circle one.

(Vehicle A)   Vehicle B   Either Vehicle A or Vehicle B

Explain your thinking.
Explanations vary.
- **Vehicle A's gas mileage is 25 miles per gallon. Vehicle B's gas mileage is less than 25 miles per gallon because the line for Vehicle B goes through a point lower on the graph than (1, 25).**
- **Using Vehicle A, you can go 125 miles using 5 gallons of gas. Using Vehicle B, you can't even go 75 miles.**

Gas Mileage (continued)

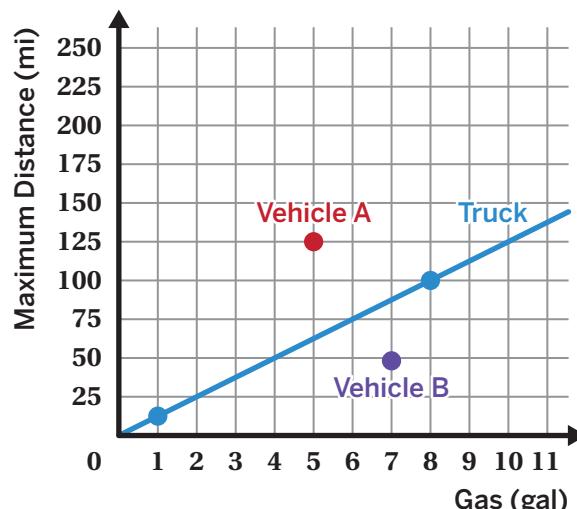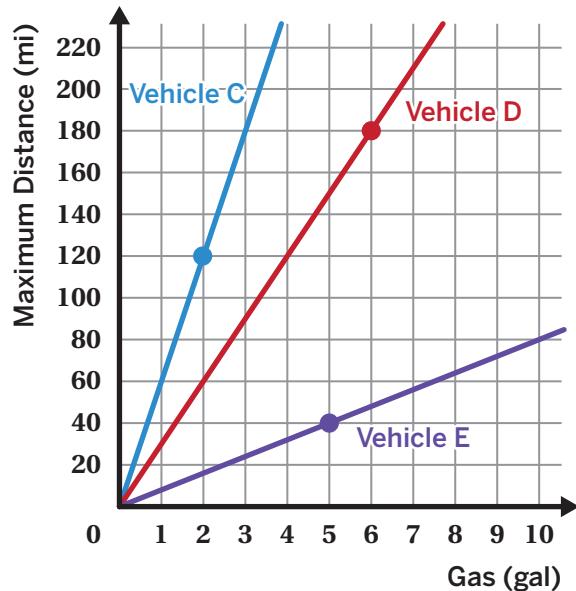**7** In the relationship between amount of gas and maximum distance, the vehicle's gas mileage is a *constant of proportionality*.

What is that constant of proportionality for each vehicle?

Vehicle	Constant of Proportionality (mi per gal)
C	60
D	30
E	8

Explore More

8 Here is the maximum distance traveled by three vehicles using a certain amount of gas.

Plane	Cruise Ship	Train

1,500 miles — 3,000 gallons

500 miles — 25,000 gallons

1,800 miles — 9,000 gallons

Note: The average plane has 100 passengers.

Note: The average cruise ship has 3,000 passengers.

Note: The average train has 300 passengers.

Compare each vehicle's environmental impact to the impact of driving a car.

Responses vary. **The plane gets 0.5 miles per gallon and carries 100 passengers. The cruise ship gets 0.02 miles per gallon and carries 3,000 passengers. The train gets 0.2 miles per gallon and carries 300 passengers. That means a full cruise ship and train can carry passengers the same distance per gallon of gas, while the plane is less fuel efficient. If a car is carrying 4 people and gets 20 miles per gallon, then it's more fuel efficient than any of these options. But a car carrying just one person is less fuel efficient than a plane, a cruise ship, or a train.**
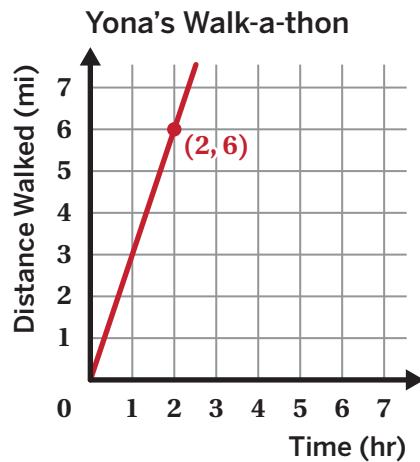
9 Synthesis

What are *two* different ways you can find a constant of proportionality using a graph?

Use the example if it helps with your thinking.

First method: *Responses vary.* **Figure out what the y-value is when the x-value is 1. In this case, the graph passes through the point $(1, 3)$, so the constant of proportionality is 3.**

Second method: *Responses vary.* **Divide a y-value by its corresponding x-value to find out how much y there is per x. For example, $\frac{6}{2} = 3$.**

Yona's Walk-a-thon

Things to Remember:

Unit 2
Lesson
10

Name: _____ Date: _____ Period: _____
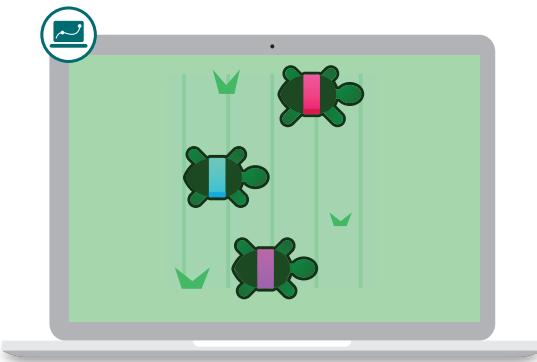
Three Turtles

Let's compare proportional relationships.

Warm-Up

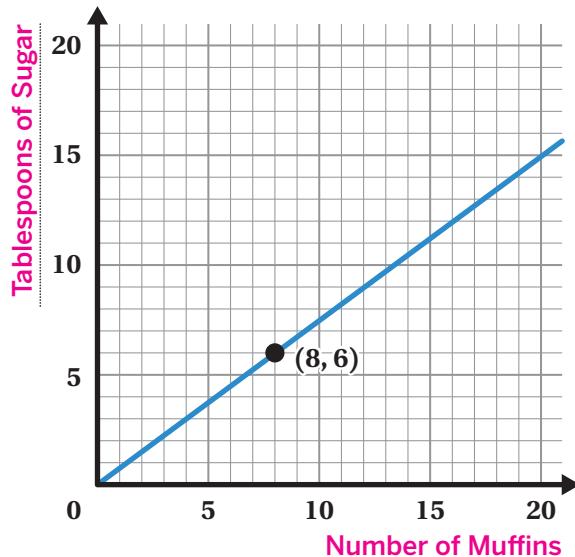**1** Here is a graph that represents a proportional relationship.

a Label the axes with any quantities you'd like.

Responses vary.

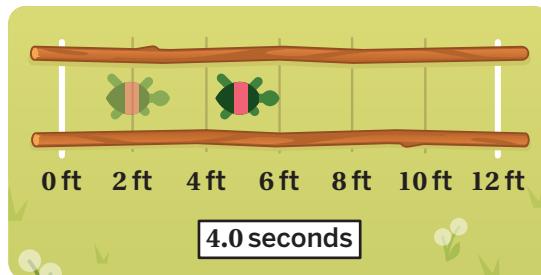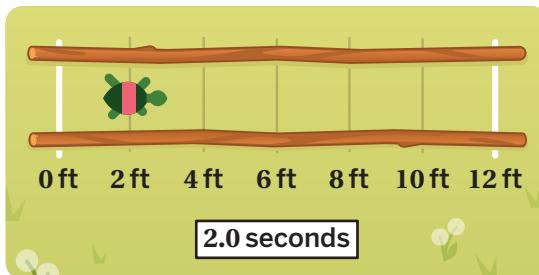**b** Write a true statement about the quantities based on the graph.

Responses vary. **To make 8 muffins, use 6 tablespoons of sugar.**

Graph: Tablespoons of Sugar (y-axis, 0 to 20) vs. Number of Muffins (x-axis, 0 to 20), with point (8, 6) marked.

Name: _____ Date: _____ Period: _____

Traveling Turtles

2 This turtle walks at a constant rate. The turtle's distance is measured at the front of its head.

0 ft   2 ft   4 ft   6 ft   8 ft   10 ft   12 ft

2.0 seconds

0 ft   2 ft   4 ft   6 ft   8 ft   10 ft   12 ft

4.0 seconds
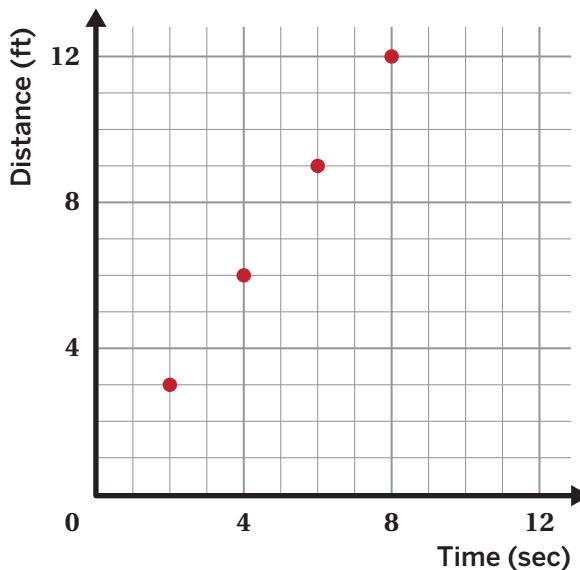
a Complete the table.

b What is a constant of proportionality for this relationship?

$\frac{3}{2}$ **(or equivalent) or** $\frac{2}{3}$ **(or equivalent)**

Time (sec)	Distance (ft)
2	3
4	6
6	**9**
8	**12**

3 This graph shows the points from the table in the previous problem.
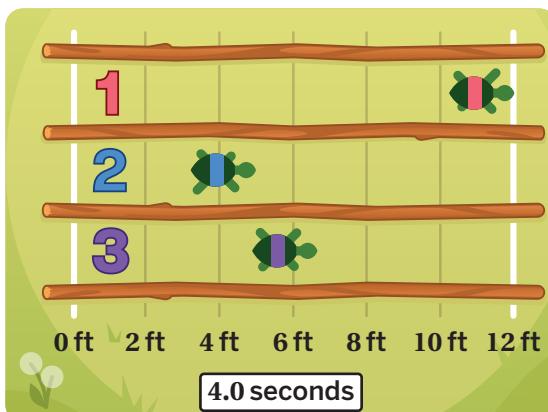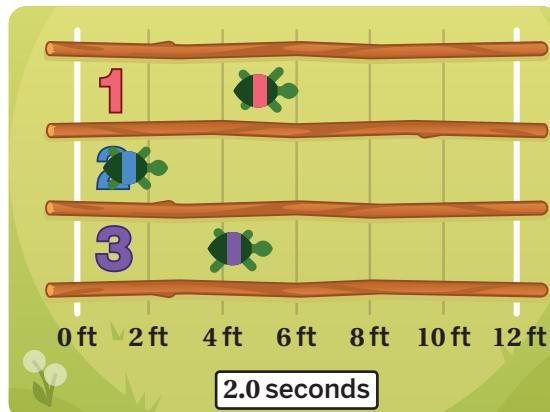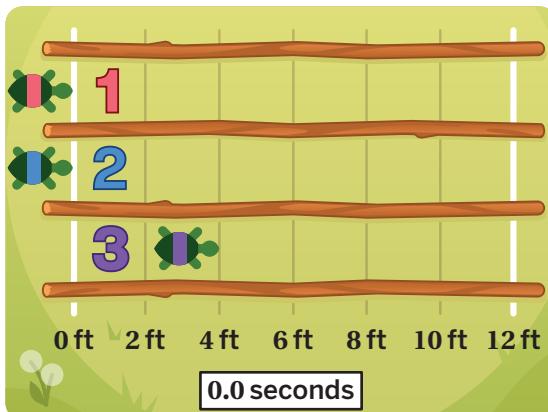
Write an equation for this relationship, using d for distance and t for time.

$d = \frac{3}{2}t$ **or** $t = \frac{2}{3}d$ **(or equivalent)**

Name: _____ Date: _____ Period: _____

Three Turtles

4 These images show three turtles walking. Each turtle walks at a constant rate.

0 ft 2 ft 4 ft 6 ft 8 ft 10 ft 12 ft

0.0 seconds

2.0 seconds

4.0 seconds

8.0 seconds

Label each line with the turtle it represents: Turtle 1, Turtle 2, or Turtle 3.

5 Match each equation to its graph.
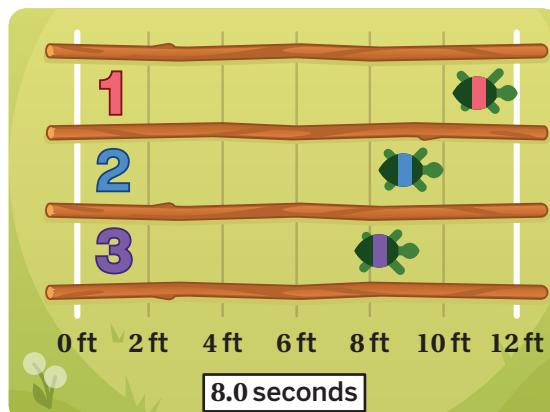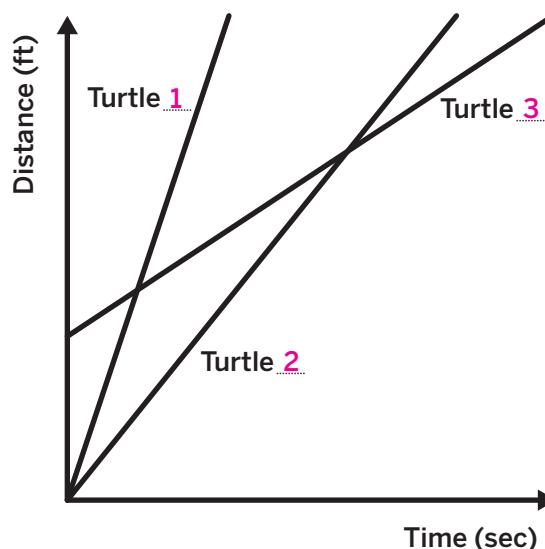
a. $d = 3t$ _____ **a** Turtle 1

b. $d = \frac{2}{3}t + 4$ _____ **c** Turtle 2

c. $d = 1.25t$ _____ **b** Turtle 3

Explain your thinking.

Explanations vary. **The turtle with a head start has an equation representing a relationship that is not proportional. The slower proportional turtle matches the equation that has the smaller constant of proportionality.**

Distance (ft)

Turtle **1**

Turtle **3**

Turtle **2**

Time (sec)

Name: _____ Date: _____ Period: _____

Turtle Challenges
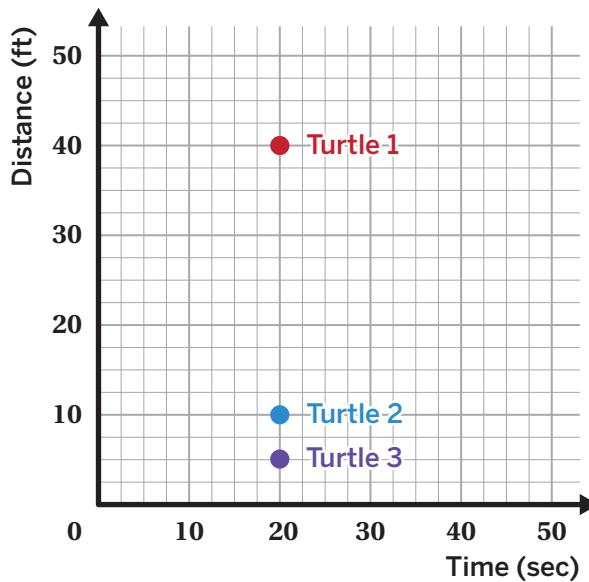
6 Here are three new turtles and their distances at 20 seconds.

- Turtle 1 is 40 feet from the start.
- Turtle 2 is 10 feet from the start.
- Turtle 3 is 5 feet from the start.

Write an equation for each turtle, using d for distance and t for time. (One equation has been written for you.)

Responses vary.

Turtle	Equation
Turtle 1	$d = 2t$ $d = 30 + \frac{t}{2}$
Turtle 2	$d = \frac{1}{2}t$
Turtle 3	$d = \frac{1}{4}t$ $d = 25 - t$

7 Irelle wrote three equations for the new turtles.

a Which of these relationships is *not* proportional?

A. $d = 1t + 20$

B. $d = \frac{1}{2}t$

C. $d = \frac{5}{20}t$

b Describe this turtle's race.

Responses vary. **This turtle began the race with a 20-foot head start, and then traveled 1 foot per second for 30 seconds.**

Name: _____ Date: _____ Period: _____

Turtle Challenges (continued)
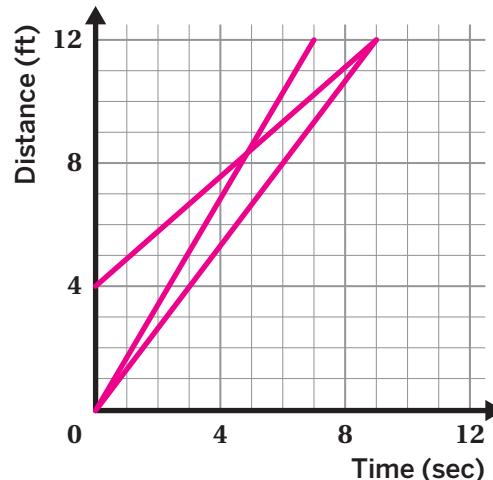
8 Sort these cards into three groups that each represent the same turtle.

Card A

Card B

Card C

Time (sec)	Turtle Distance (ft)
0	0
1	4
2	8
4	16
8	32

Card D

Time (sec)	Turtle Distance (ft)
0	0
1	$\frac{1}{2}$
2	1
4	2
8	4

Card E

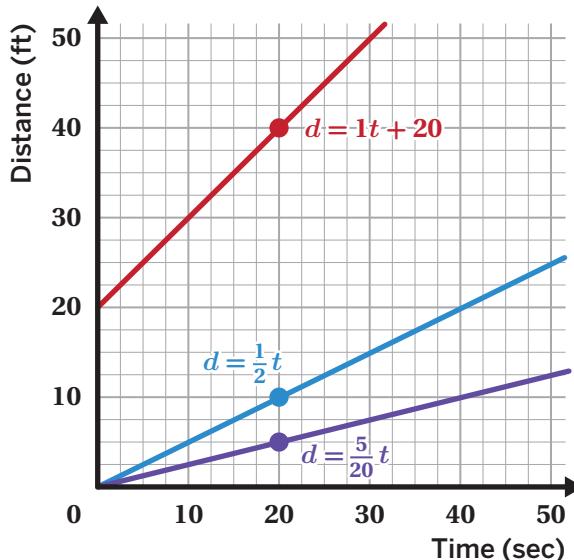At 8 seconds, the turtle's distance is 2 feet.

Card F

At 2 seconds, the turtle's distance is 8 feet.

Card G

$d = 4t$

Card H

$d = \frac{1}{4}t$

Group 1	Group 2	Group 3
B, D	A, E, H	C, F, G

9 Create your own turtle race by sketching 3 lines that represent 3 different turtles. Your turtle race must include *at least* two of the following features:

☐ A turtle that stays still.

☐ A turtle that has a head start.

☐ Two turtles that finish at the exact same time.

☐ A turtle that travels backward.

☐ Two turtles that travel at the same pace.

Responses vary.

Explore More

10 Use the Explore More Sheet to explore another turtle race.

11 Synthesis
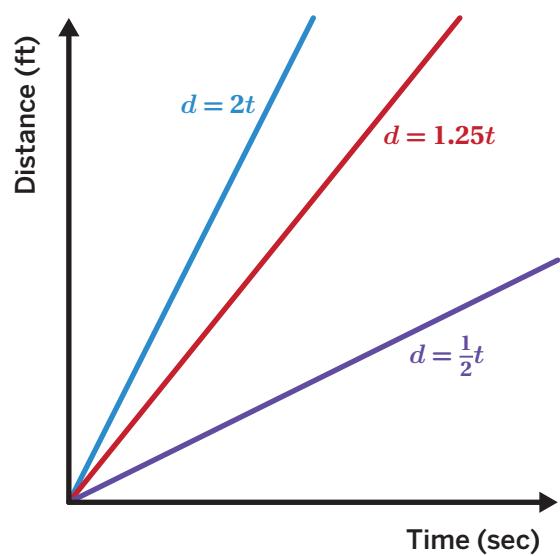
This graph shows the distance traveled over time by three different turtles.

Discuss both questions. Then select *one* and write your response.

☐ How can you tell from the graphs which turtle moved the fastest?
☐ How can you tell from the equations which turtle moved the fastest?

Responses vary.
- **I can tell from the graphs which turtle moved the fastest by comparing how steep the lines are. The steeper the line, the faster the turtle.**
- **I can tell from the equations which turtle moved the fastest by looking at the constant of proportionality. When the equations are all written as $d =$, the greater the constant of proportionality, the faster the turtle.**

Distance (ft)

$d = 2t$

$d = 1.25t$

$d = \frac{1}{2}t$

Time (sec)

Things to Remember:

Name: _____ Date: _____ Period: _____
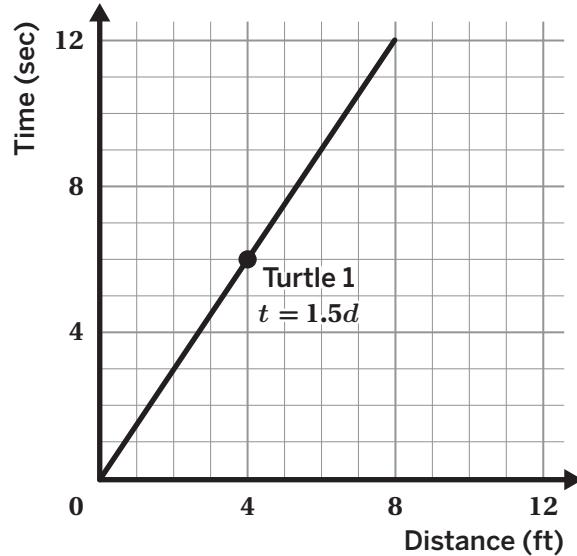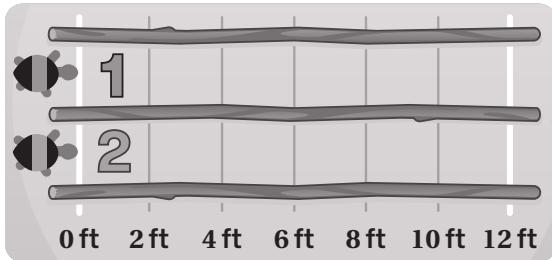
Explore More
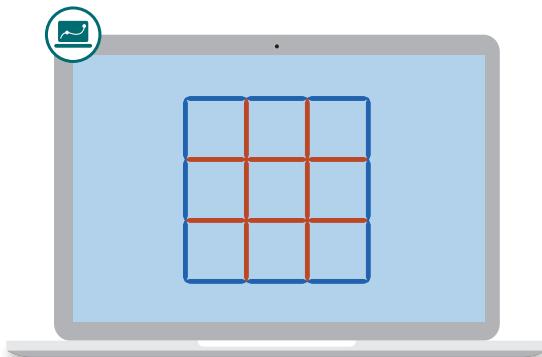
The graph shows the line for Turtle 1.

Draw a line for Turtle 2 so that it's slower than Turtle 1. Notice the axis labels!

0 ft   2 ft   4 ft   6 ft   8 ft   10 ft   12 ft

Turtle 1
$t = 1.5d$

Name: .... Date: .... Period: ....
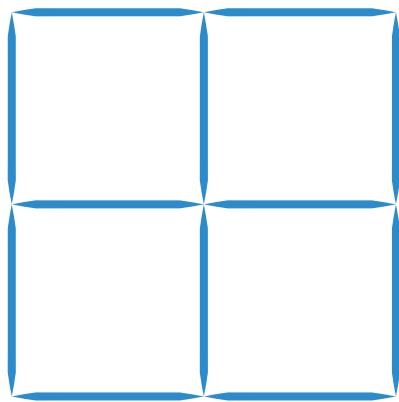
Toothpicks

Let's examine relationships between side length, diagonal length, and the perimeter of a square.
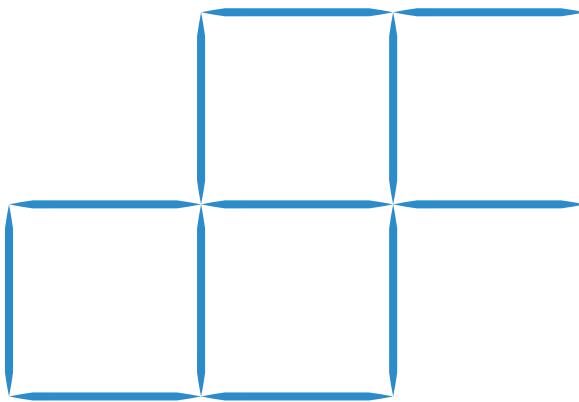
Warm-Up

1 These two figures were built with toothpicks.

Figure A

Figure B

Which figure used more toothpicks? Circle one.

Figure A   (Figure B)   They used the same

Explain your thinking.
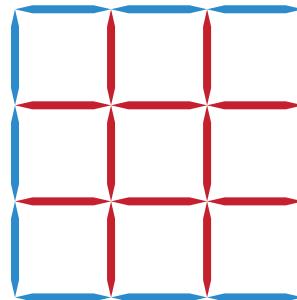
Explanations vary.
- **Figure A needs 12 toothpicks and Figure B needs 13 toothpicks.**
- **I can make Figure A by moving over 2 toothpicks from Figure B to fill in the top-left corner. That leaves 1 toothpick left over.**

Name: .... Date: .... Period: ....

Proportional or Not?

2 To determine the number of toothpicks needed to build this figure, we count the blue perimeter toothpicks and the red interior toothpicks.
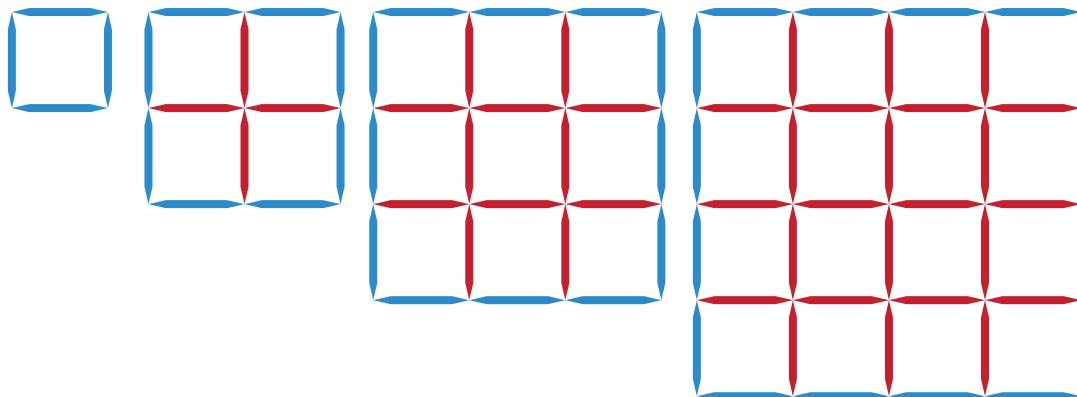
Write the number of each type of toothpick.

Perimeter (blue toothpicks)	Interior (red toothpicks)
12	12

3 Let's explore how changing the size of the square changes the number of toothpicks.

a Take a look at this square and its scaled copies.
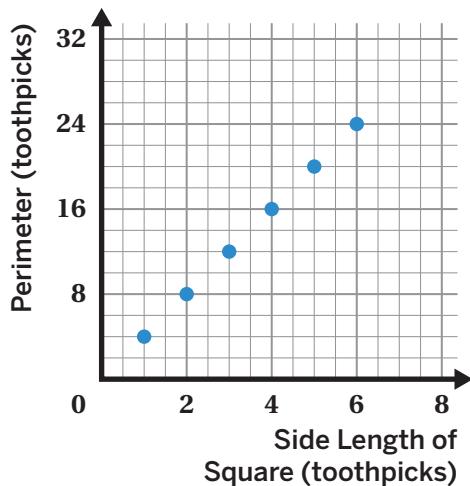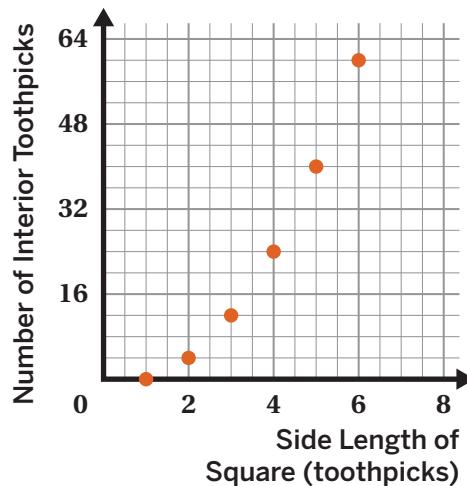
b Write the number of each type of toothpick needed to build each square.

Side Length (toothpicks)	Perimeter (blue toothpicks)	Interior (red toothpicks)
1	4	0
2	8	4
3	12	12
4	16	24

Name: _____ Date: _____ Period: _____

Proportional or Not? (continued)

4 Here are two graphs based on the toothpick squares from the previous problem.

Side Length vs. Perimeter

Side Length vs. Interior

Use the graphs to decide which relationships are proportional. Circle one.

(A.) Side length of a square vs. perimeter

B. Side length of a square vs. number of interior toothpicks

C. Both

D. Neither

Explain your thinking.

Explanations vary.
- **The points on the perimeter graph form a line through the origin, but the points on the other graph form a curve.**
- **Every perimeter is 4 times the side length, so it has a constant of proportionality.**

5 **a** For any relationship you said was proportional, what is its constant of proportionality?

A constant of proportionality for the relationship between side length of a square and perimeter is 4 $\left(\text{or } \frac{1}{4}\right)$.

b What does the constant of proportionality mean in this context?

Responses vary.
- **The constant of proportionality is what you multiply the side length by to figure out the perimeter.**
- **A square has 4 equal sides, so the perimeter is 4 times the side length.**

Explore More

6 Use the Explore More Sheet to further explore these toothpick relationships.

See the Teacher Edition for sample responses.

Name: .... Date: .... Period: ....

Across and Around
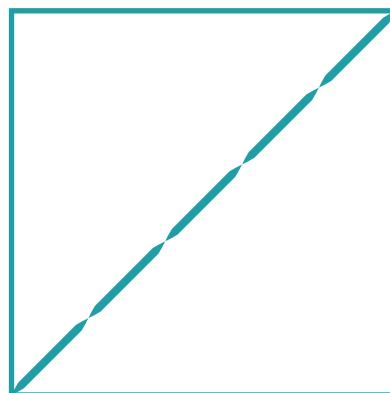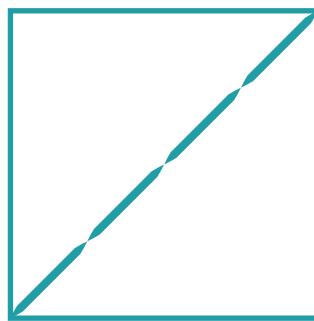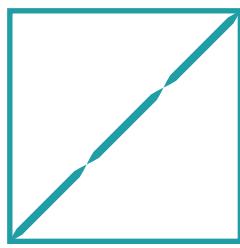
7 This square has a diagonal that is 1 toothpick long. About how many toothpicks long is its perimeter? Parts of toothpicks are allowed. Get as close as you can.

Responses vary. ≈ **2.83**

Responses between 2.82 and 2.84 toothpicks are considered correct. The exact perimeter is $2\sqrt{2}$.

8 Here are squares with different diagonal lengths.

Complete the table to show the number of toothpicks you would need to build the perimeter of each square. *Responses vary.* **Sample shown in table.**

Diagonal Length (toothpicks)	Perimeter (toothpicks)
1	**2.83**
2	5.66
3	**8.49**
4	**11.32**
5	14.15

9 **a** 💬 **Discuss:** Is the relationship between diagonal length and perimeter proportional? How do you know?

Yes. *Responses vary.* **The relationship is proportional because I can multiply each diagonal length by the same value, 2.83, to get the number of toothpicks needed for the perimeter.**

b How can you estimate the perimeter of a square with a diagonal of 100 toothpicks.

Responses vary. **I can multiply 100 by the constant of proportionality, which is 2.83. This means there will be about 2.83 • 100 = 283 toothpicks along the perimeter when the diagonal is 100 toothpicks.**
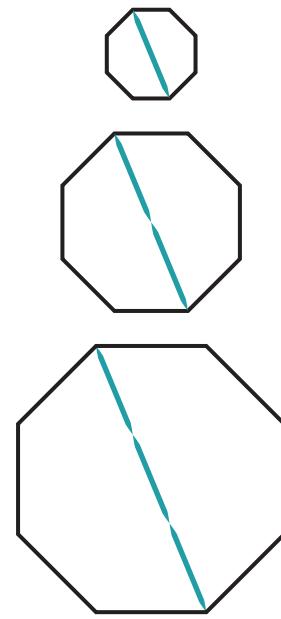
10 Synthesis

This table shows the relationship between the diagonal length and perimeter of an octagon.

How can you tell if two quantities in a geometry situation are proportional?

Responses vary. **You can tell if a relationship is proportional if there is a constant of proportionality. In the octagon example, the table has a constant of proportionality of 3.06. If you multiply the diagonal length by 3.06, you get the perimeter.**

Diagonal Length (toothpicks)	Perimeter (toothpicks)
1	3.06
2	6.12
3	9.18
4	12.24
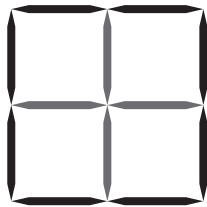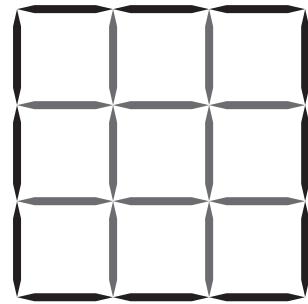
Things to Remember:

Name: _____ Date: _____ Period: _____
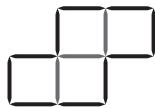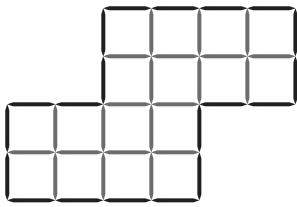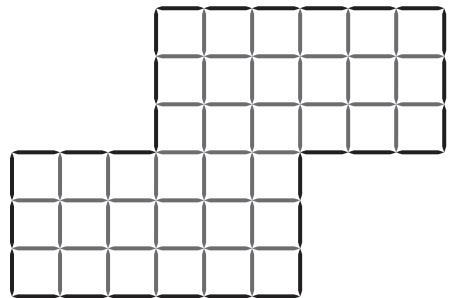
Explore More

Determine the number of toothpicks needed to build the perimeter and interior of the 100th stage of each pattern.

Pattern A

Stage 1   Stage 2   Stage 3

Perimeter of Stage 100: _____   Interior of Stage 100: _____

Pattern B

Stage 1   Stage 2   Stage 3

Perimeter of Stage 100: _____   Interior of Stage 100: _____

Name: .... Date: .... Period: ....

Measuring Around
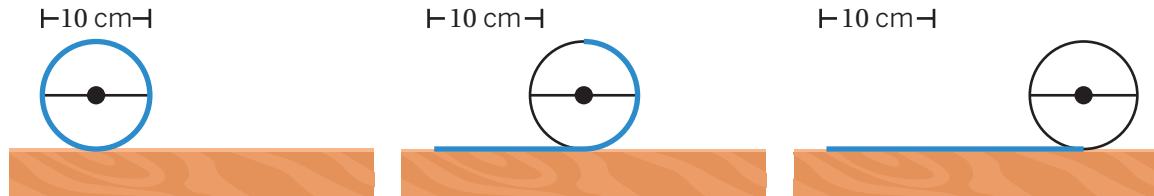
Let's explore the relationship between the diameter of a circle and its circumference.

Warm-Up

1 The **circumference** of a circle is the distance all the way around it.

a Imagine unrolling the circumference to show it as a line segment.

⊢10 cm⊣   ⊢10 cm⊣   ⊢10 cm⊣

b Estimate the length of the circumference.

Responses vary. **Students may reason that there are about 3 diameters and estimate the length as 30 centimeters.**

Name: ....   Date: ....   Period: ....

So Many Circles
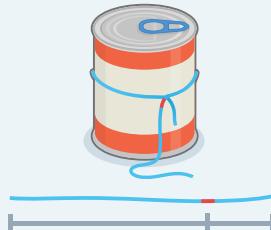
2 Measure the diameter and the circumference of at least three circular objects. Record your results in the table.
Objects and measurements vary.

Object	Diameter (cm)	Circumference (cm)
Toilet paper tube	4.1	12.9

Three Ways to Measure the Circumference
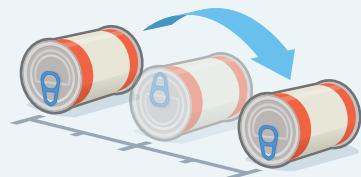
String

Tape Measure

Roll

3 This graph shows the diameter and circumference of several circular objects.
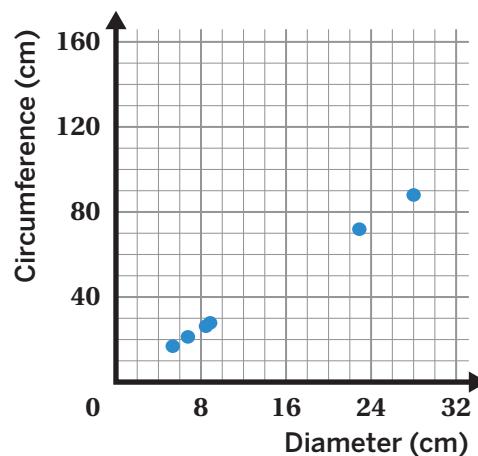
Do you think the relationship between circumference and diameter is proportional? Circle one. *Responses vary.*

Yes   No   I'm not sure

Explain your thinking.
Explanations vary.

- The points are close to a straight line through the origin, so the relationship is proportional. Some of the points are not exactly on the line, but that's probably due to measurement error.

- The points are not in a perfectly straight line which means that there is not a constant of proportionality you can multiply the diameter by to get the circumference.

Name: _____ Date: _____ Period: _____
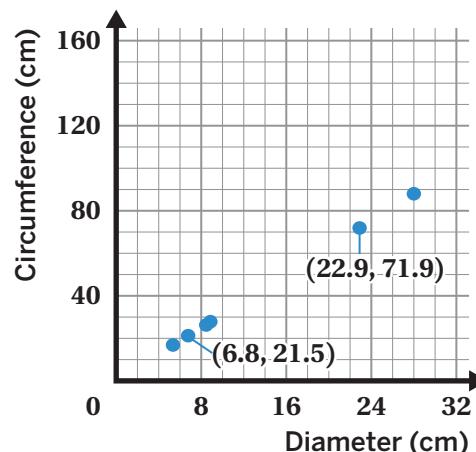
Introducing π

4 The relationship between circumference, C, and diameter, d, is proportional.

Estimate the *constant of proportionality* that completes the equation relating circumference and diameter.

$C = $ _____ d

Responses between 3.1 and 3.2 are considered correct.
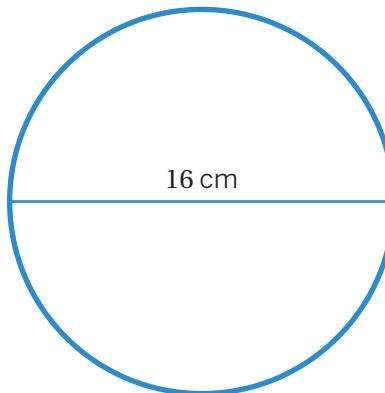
5 The constant of proportionality between a circle's diameter, d, and its circumference, C, is π (written as **pi** and pronounced "pie").

In other words, $C = \pi d$.

Common approximations for π are 3.14, $\frac{22}{7}$, and 3.14159. None of these are exactly π.

Calculate the circumference of a circle with a diameter of 16 centimeters.

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations. **16π centimeters, 50.24 centimeters, or $\frac{352}{7}$ centimeters**

16 cm
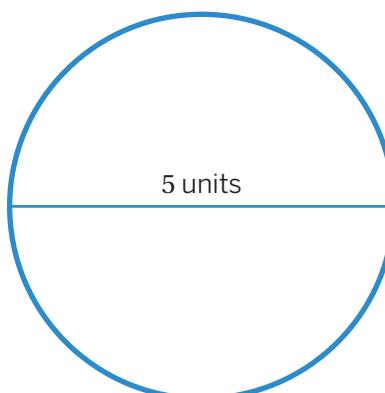
6 This circle has a diameter of 5 units.

Four students tried to calculate its circumference.

Order their answers starting with the *farthest* from the exact circumference to the *closest* to the exact circumference.

| 15.7 units | 15 units | 5π units | 15π units |

5 units

| **15π units** | **15 units** | **15.7 units** | **5π units** |

Farthest From Exact Circumference   **Closest to Exact Circumference**

Name: .... Date: .... Period: ....
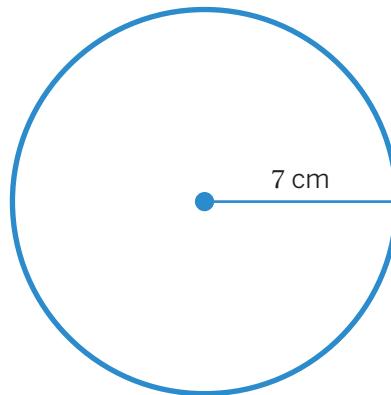
Proportionality and π

---

7 The radius of this circle is 7 centimeters.

What is its circumference?

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

- **14π centimeters**
- **43.96 centimeters**
- **44 centimeters**

7 cm

---

8 The constant of proportionality in the relationship between diameter and circumference is π.

a Complete the table.

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

Object	Radius (cm)	Diameter (cm)	Circumference (cm)
Cookie	3	6	**6π**
Small plate	9	**18**	**18π**
Quarter	**1.2**	2.4	**2.4π**
Frisbee	**10.5**	**21**	21π
Tennis ball can	$\frac{7}{2}$	**7**	**7π**
Vinyl record	$\frac{100}{2\pi} \approx 15.92$	$\frac{100}{\pi} \approx 31.83$	100

b When you finish, compare your answers with a classmate.
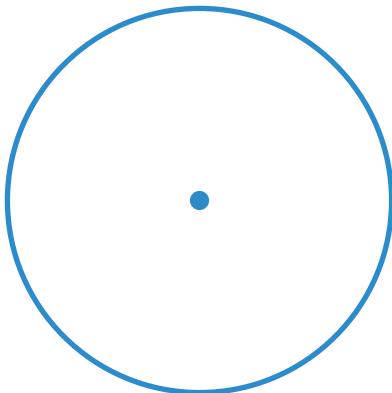
💬 **Discuss:** How are your answers alike? How are they different?

Responses vary.

10 Synthesis

Describe the relationship between radius, diameter, and circumference.
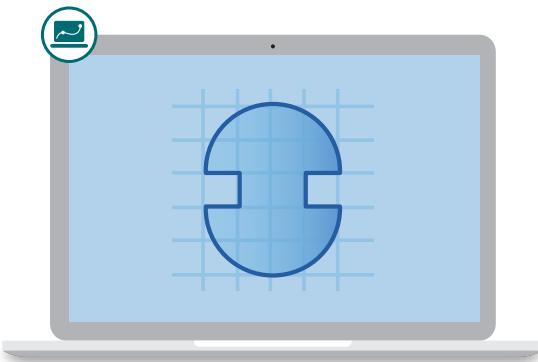
Responses vary. The circumference of a circle is π times the diameter of the circle. If I know the radius, I can first multiply by 2 to figure out the diameter, then I can multiply by π to figure out the circumference.

Things to Remember:

Unit 3
Lesson
4

Name: _____ Date: _____ Period: _____

Perimeter Challenges

Let's calculate the perimeters of complex shapes.
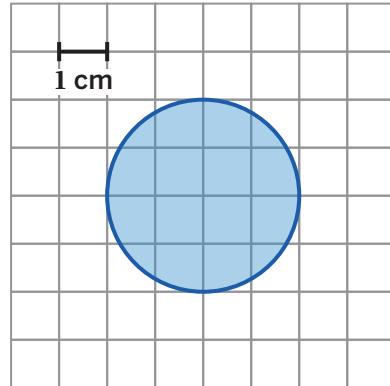
Warm-Up

1 What is the circumference of this circle?
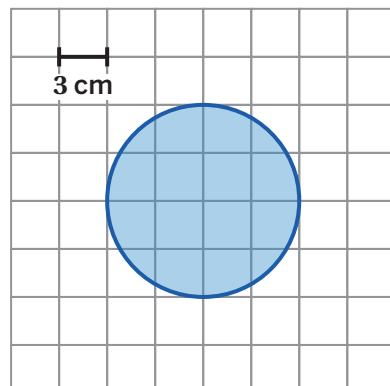
Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.
- **12.56 centimeters**
- **$\frac{88}{7}$ centimeters**
- **4π centimeters**

2 Now, what is the circumference of *this* circle?

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.
- **37.68 centimeters**
- **$\frac{264}{7}$ centimeters**
- **12π centimeters**

Name: _____ Date: _____ Period: _____

Calculating Perimeters

3 This circle and semicircle have the same diameter.

Dyani claims that the semicircle's perimeter is *half* the circle's perimeter.
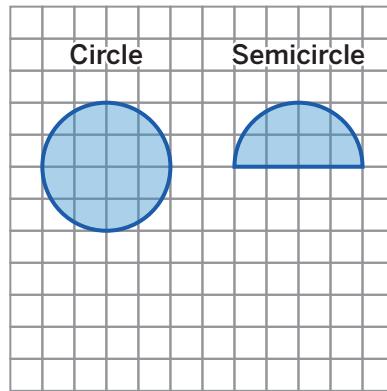
Odalis says the perimeters are *equal*.

Whose claim is correct? Circle one.

Dyani's   Odalis's   Both   (Neither)

Explain your thinking.
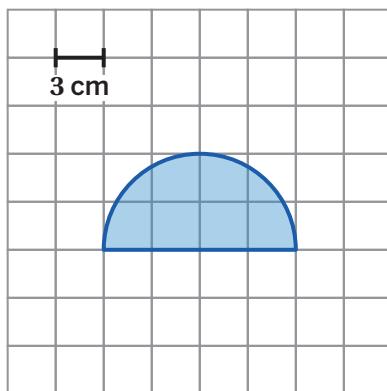
Explanations vary. **The top halves of the two shapes have the same perimeter. On the bottom half, the curve is longer than the straight line, so the perimeter of the semicircle is larger than half but not equal to the perimeter of the circle.**

Circle   Semicircle

4 What is the perimeter of this semicircle?
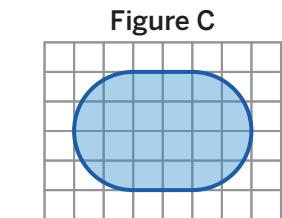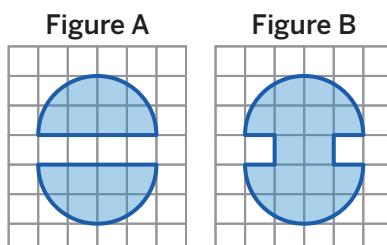
Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

- **30.84 centimeters**
- **$\frac{216}{7}$ centimeters**
- **$6\pi + 12$ centimeters**

3 cm

5 Order the figures by their total perimeter.

Figure C	Least Total Perimeter
Figure B	
Figure A	Greatest Total Perimeter

Figure A   Figure B

Figure C

Name: ....   Date: ....   Period: ....
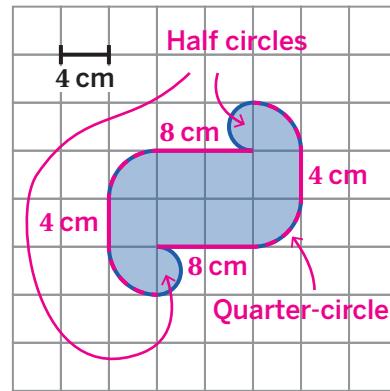
More Perimeters, More Problems

6 This shape is made of squares and parts of circles.

Nakia said the perimeter of this shape is made of these parts:

- Four quarter-circles with a radius of 4 centimeters
- Two half circles with a radius of 2 centimeters
- Two 4-centimeter pieces
- Two 8-centimeter pieces

Show or explain where Nakia might see each of these parts.

Responses vary. **The 4 rounded corners of the shape are created by quarter-circles with a radius of 4 centimeters. The 2 "tails" of the shape are made up of half circles with a radius of 2 centimeters. The short sides on the left and right are both 4 centimeters long. The long sides on the top and bottom are both 8 centimeters long.**

(Diagram: Half circles, 4 cm, 8 cm, 4 cm, 4 cm, 8 cm, Quarter-circle)
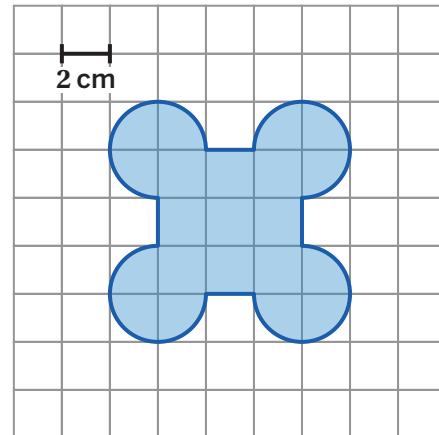
7 a 💬 **Discuss:** What parts of circles and squares do you see in this shape?

Responses vary. **I see four $\frac{3}{4}$-circles with a radius of 2 centimeters and four 2-centimeter pieces.**

b Determine the perimeter of this shape.
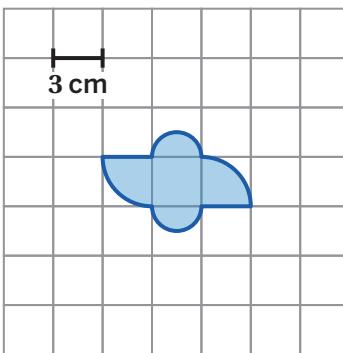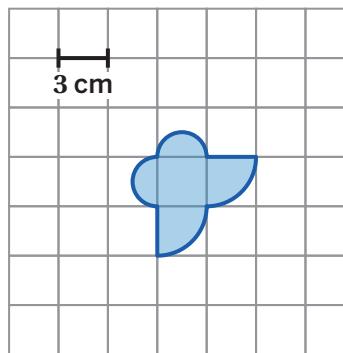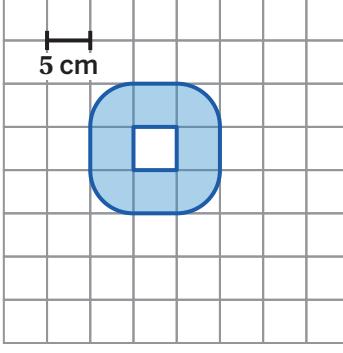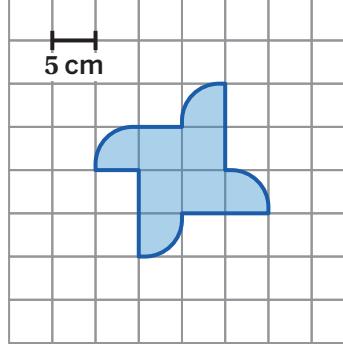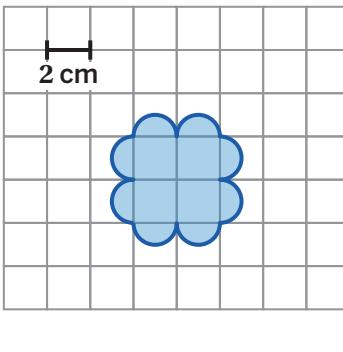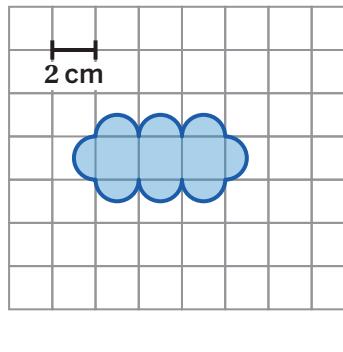
Responses vary as students may use π, *3.14, or* $\frac{22}{7}$ *in their calculations.*
- **45.68 centimeters**
- **$\frac{320}{7}$ centimeters**
- **$12\pi + 8$ centimeters**

(Diagram: 2 cm)

Name: _____ Date: _____ Period: _____

Sum of Its Parts

8 You will work with a partner.

- Decide who will complete Column A and who will complete Column B.
- Determine the perimeter of each shape. The perimeters in each row should be equal.
- Compare your solutions, then discuss and resolve any differences.

Column A	Column B
a Perimeter _____ $6\pi + 6$ centimeters _____	**a** Perimeter _____ $6\pi + 6$ centimeters _____
3 cm	3 cm
b Perimeter _____ $10\pi + 40$ centimeters _____	**b** Perimeter _____ $10\pi + 40$ centimeters _____
5 cm	5 cm
c Perimeter _____ 8π centimeters _____	**c** Perimeter _____ 8π centimeters _____
2 cm	2 cm

9 Synthesis

Describe a strategy for determining the perimeter of a shape that is made of squares and parts of circles.

4 cm
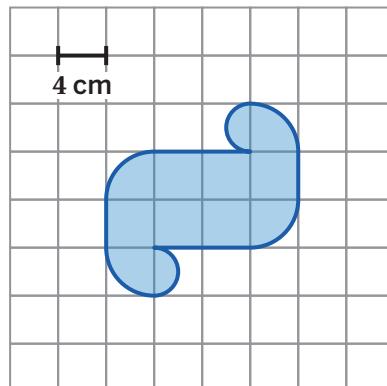
Use the example if that helps you to show your thinking.

Responses vary. First, figure out how many quarter-circles, semicircles, and straight edges there are. In this example, there are 4 quarter-circles, 2 semicircles, and 6 straight edges.

Then, figure out the perimeter of each kind of piece. For example, for a quarter-circle, multiply its radius by 2 to know the diameter, then multiply by π to get the circumference of the entire circle, then multiply by $\frac{1}{4}$ since it is a quarter-circle. In this example, $4 \cdot 2 \cdot \pi = 8\pi$ and $8\pi \cdot \frac{1}{4} = 2\pi$.
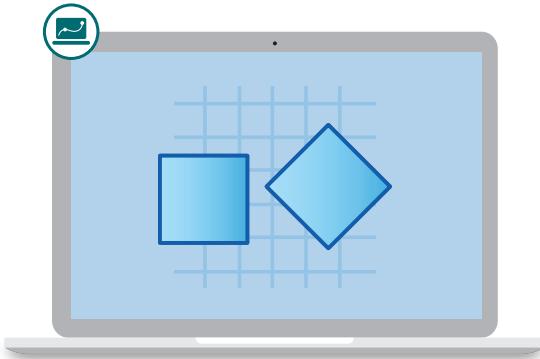
Finally, add up the perimeters of all the pieces to get the total perimeter. In this example, the perimeter is $12\pi + 24$ centimeters.

Things to Remember:

Unit 3

Lesson

5

Name: _____ Date: _____ Period: _____

Area Strategies

Let's estimate and calculate areas.
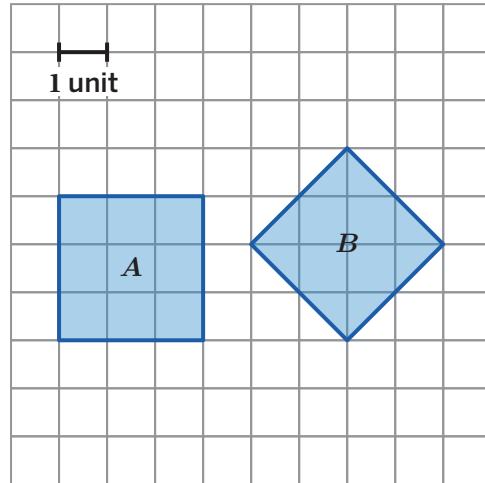
Warm-Up

1 Which square has a larger *area*? Circle one.

$\boxed{\text{Square } A}$ Square B They're the same

Explain your thinking.

Explanations vary. **Square A has a side that is 3 units long, but square B has a side that is less than 3 units long.**

1 unit

2 The area of square A is 9 square units.

Work with a partner to determine square B's area. Sketch if it helps with your thinking.

Be prepared to describe your strategy.

8 square units. *Explanations vary.*

- **I know that each full square is 1 square unit, and there are 4 full squares. Two of the partial squares make a whole, and there are 8 partial squares, so that makes 4 full squares: $4 + 4 = 8$ in total.**
- **I split the shape vertically and horizontally to create 4 smaller triangles. Each triangle's area is $\frac{1}{2} \cdot 2 \cdot 2 = 2$ square units. Since there are 4 triangles, the total area is $2 \cdot 4 = 8$ square units.**
- **I put the shape inside of a 4-by-4 square and then subtracted out the parts that were not shaded.**

Warm-Up

Name: _____   Date: _____   Period: _____

Calculating Area
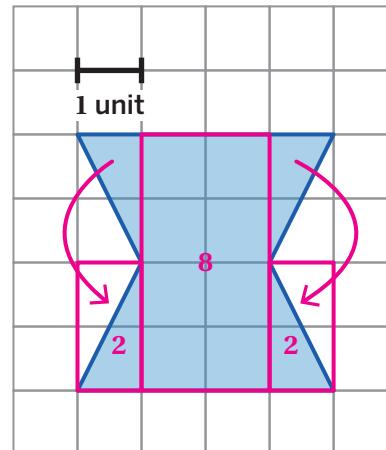
2 What is the area of this shape?

Sketch to help show your thinking.
12 square units

Explain your thinking.
Explanations vary. **I put two triangles together to form a 2-by-1 rectangle, so I ended up with a 2-by-4 rectangle and two 2-by-1 rectangles. $8 + 2 + 2 = 12$ square units**
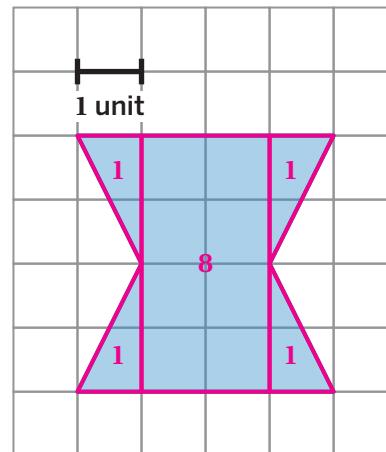
4 There is often more than one way to determine area.

Sketch to show another way of determining the area of this shape.

Describe your strategy.
Responses vary. **I broke the shape up into one long rectangle and four small triangles. The area of the rectangle is $2 \cdot 4 = 8$ square units, and the area of each triangle is 1 square unit. $8 + 4 \cdot 1 = 12$ square units**

5 Zamari says these two shapes have the same area.

Is Zamari correct? Circle one.
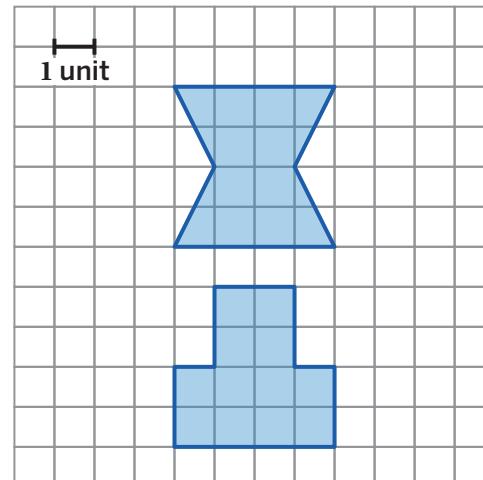
⬭Yes⬭   No   I'm not sure

Explain your thinking.
Explanations vary. **The shapes have the same area because you can rearrange the top shape to look like the bottom shape, which means they cover the same total number of squares.**

Name: _____ Date: _____ Period: _____

Estimating Area
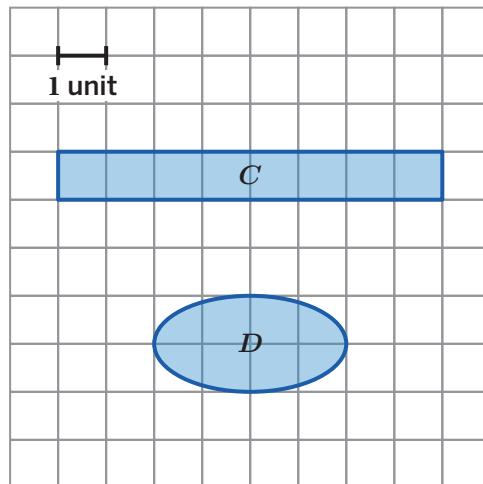
6 Which shape has a larger area? Circle one.

(Shape C)   Shape D They're the same

Explain your thinking.

Explanations vary.
- **If you cut shape D into a top half and bottom half, and move each half on top of shape C, all of shape D fits inside of shape C, so shape C must have a larger area.**
- **Shape D can fit entirely in a 2-by-4 rectangle, which means that its area is less than 8 square units, which is the area of shape C.**
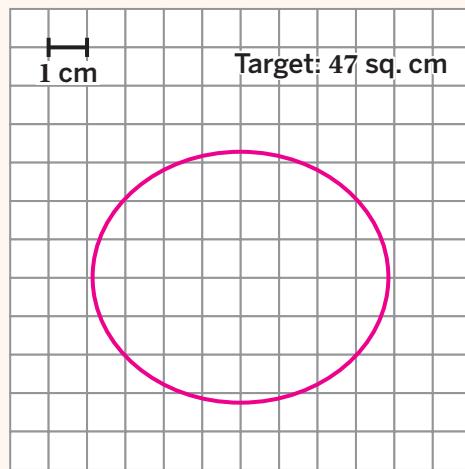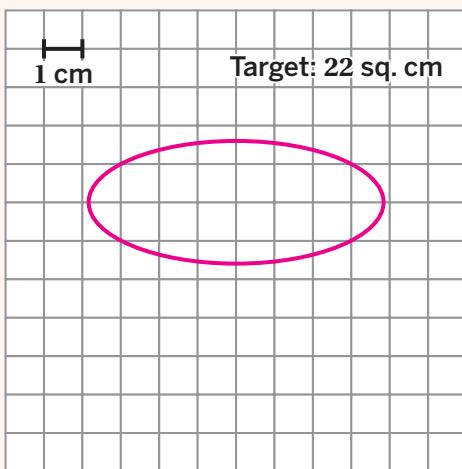
7 What is the approximate area of shape D?

Sketch if it helps with your thinking.

Responses between 6 and 6.6 square units are considered correct.

Explore More
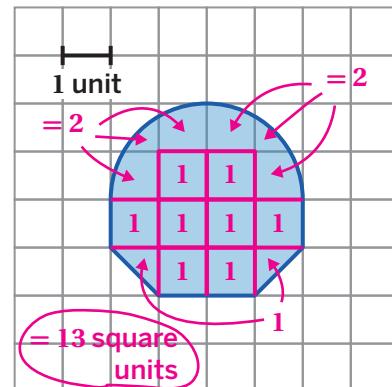
8 Draw an oval with an area that is approximately equal to each target area. *Responses vary.*

Target: 22 sq. cm

Target: 47 sq. cm

9 Synthesis

Show or describe a strategy for estimating the area of this shape.
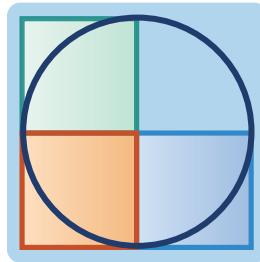
Responses vary. **The area is about 13 square units. I counted the total number of whole squares, then combined each of the half squares to make a whole square. For the curved edge, I assumed that the squares that were less than half-shaded could be cut up and filled into the spaces that were missing in the squares that were mostly shaded, so I counted each of the mostly shaded squares: $8 + 1 + 4 = 13$ square units.**

Things to Remember:

Unit 3
Lesson

7

Name: .... Date: .... Period: ....
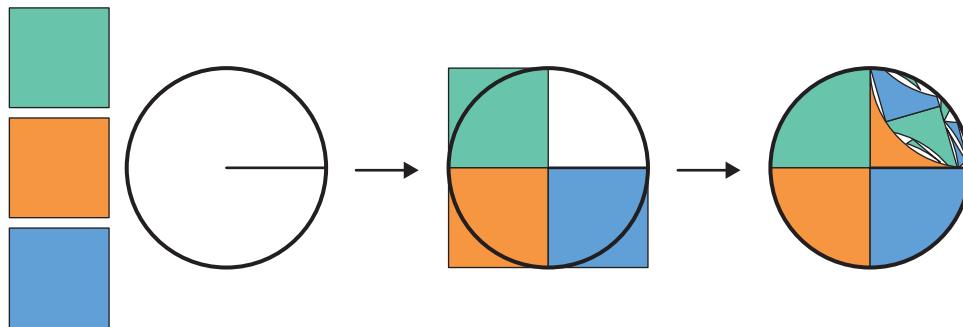
Circle Area

Let's determine a formula for the relationship between the radius of a circle and its area.

Warm-Up

Let's watch an animation.

1. What do you notice? What do you wonder?

I notice:

Responses vary.
- **I notice that 3 radius squares do not exactly equal the area of the circle.**
- **I notice there is white space left over, so the exact number of radius squares is slightly more than 3.**

I wonder:
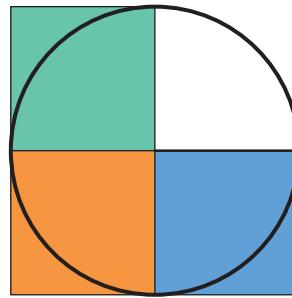
Responses vary.
- **I wonder how much more the area of the circle is compared to the area of the 3 radius squares.**
- **I wonder if there is a formula that I can use to more accurately calculate the area of a circle.**

Name: .... Date: .... Period: ....

Finding a Formula

2. Diya says that you can find the approximate area of a circle by calculating $3 \cdot r^2$.

What do you think each part of this expression means? Draw on the diagram if it helps with your thinking.

Responses vary. **The r^2 represents the area of a radius square. It takes about 3 radius squares to cover the circle, so the area of the radius square is being multiplied by 3.**

3. Jaleel says you can find the approximate area of a circle by calculating $r \cdot r \cdot 3$.

Do you agree with Jaleel? Circle one.

(Yes)   No   I'm not sure
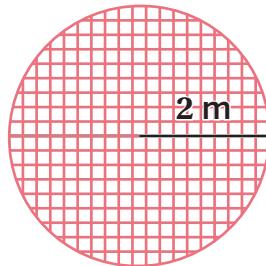
Explain your thinking.

Explanations vary. $r \cdot r$ **gives the area of a radius square, just like the r^2 in Diya's expression. Then the area of the radius square is multiplied by 3 since it takes about 3 radius squares to cover the circle.**

4. Here are four circles, their radius or diameter lengths, and their areas.

4 in.

2 m

35 cm

50 cm
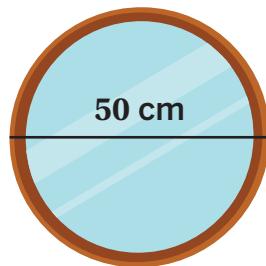
$A \approx 12.57$ square meters

$A \approx 50.27$ square inches

$A \approx 3848.45$ square centimeters

$A \approx 1963.50$ square centimeters

Write a formula to calculate the exact area of a circle, A, with a radius, r. Use these examples and your work from Problems 2–3 if it is helpful.

$A =$ ....$\pi \cdot r^2$ **(or equivalent)**.........

Name: _____   Date: _____   Period: _____
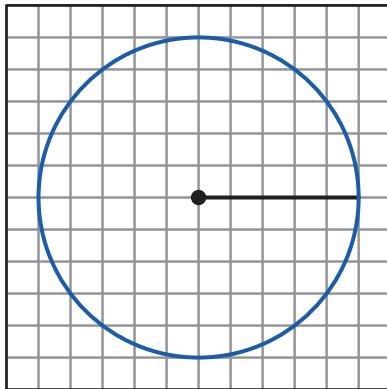
Circle Area

5. Rewrite the formula for the exact area of a circle you developed in Activity 1. Use A for the area of the circle and r for the length of the radius.
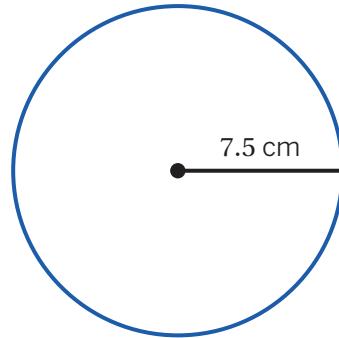
$A = $ ....$\pi \cdot r^2$ **(or equivalent)**........

6. Calculate the exact area of each circle.
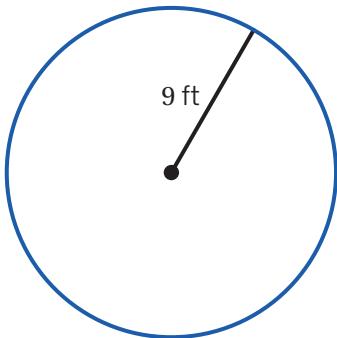
a Area: ....**25π sq. units**........

b Area: ....**56.25π sq. cm**........

7.5 cm

c Area: ....**81π sq. ft**........

9 ft

d Area: ....**64π sq. cm**........
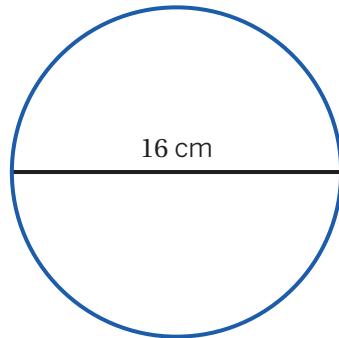
16 cm
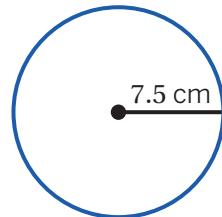
Synthesis

7. **a** Describe a strategy to calculate the area of a circle if you know its *radius*.
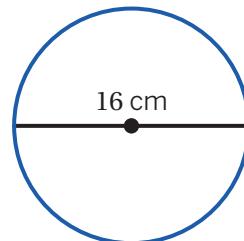
Responses vary.
- **Multiply the radius length by itself and then multiply by π.**
- **Replace r in the formula $A = \pi \cdot r^2$ with the length of the radius and calculate the result.**

7.5 cm

b How does your strategy change if you know a circle's *diameter*?

Responses vary. **Divide the diameter length by 2 to get the radius. Then you can use the same strategy.**

16 cm
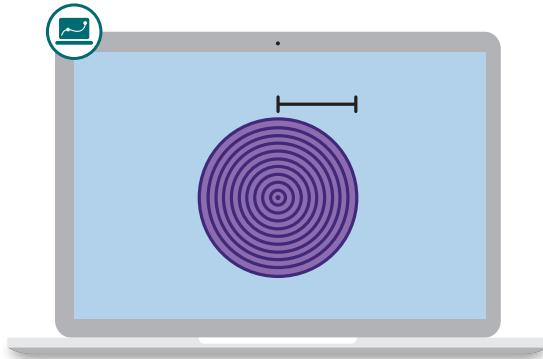
Things to Remember:

Name: .... Date: .... Period: ....

Why Pi?

Let's explore why the formula for the area of a circle makes sense.
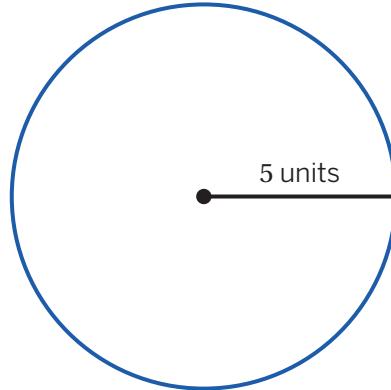
Warm-Up

1 This circle has a radius of 5 units.

Four students tried to calculate the area.

Order their answers starting with the farthest from the exact area to the closest to the exact area.

5 units

$25\pi^2$ square units	25π square units	10π square units	78.5 square units

$25\pi^2$ square units	10π square units	78.5 square units	25π square units

Farthest From Exact Area   Closest to Exact Area
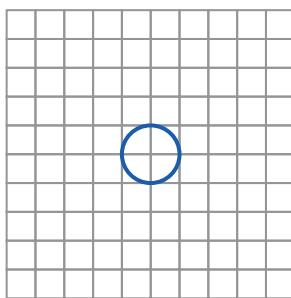
Note: It is also acceptable to swap the positions of $25\pi^2$ square units and 10π square units.

Name: _____ Date: _____ Period: _____
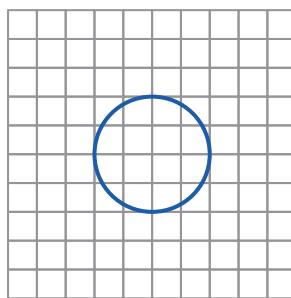
Proportional or Not?

2 Here are some circles.
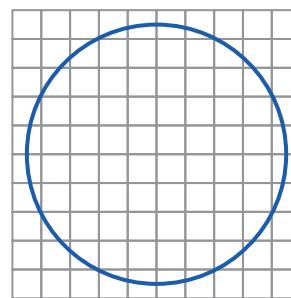
Circle A	Circle B	Circle C

Record the area of each circle in the table.
Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

Circle	Radius (units)	Area (sq. units)
A	1	**π**
B	2	**4π**
C	4.5	**20.25π**

3 This graph shows the radius and area of several circles.

Is there a proportional relationship between the radius and the area of a circle?
Circle one.

Yes   (No)   I'm not sure
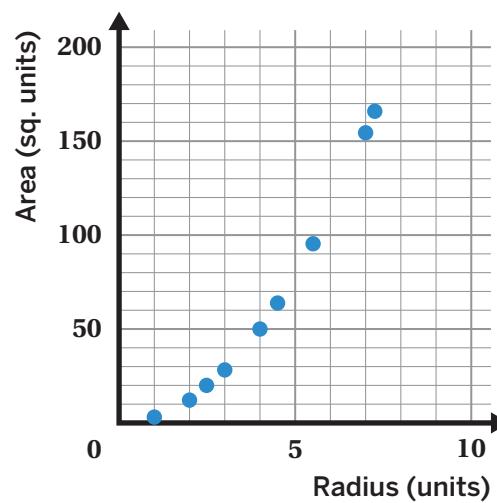
Explain your thinking.
Explanations vary.
- **The points don't make a line through the origin, so it's not proportional.**
- **If I look back at my table, there is no constant of proportionality.**

Unrolling a Circle
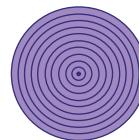
4 Let's watch an animation.
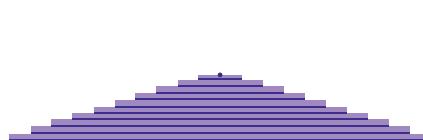
Rolled   **Unrolled**

💬 **Discuss:**
- How are the rolled and unrolled figures alike?
- How are they different?
- What parts of the original circle do you see in the unrolled figure?
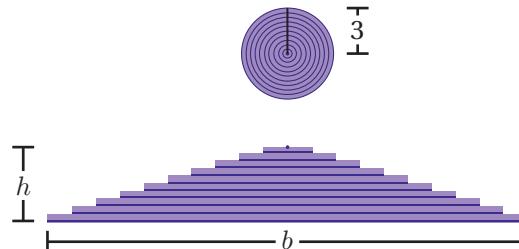
Responses vary.
- **The unrolled figure has the same area as the rolled figure.**
- **The rolled figure is a circle, and the unrolled figure is more like a triangle.**
- **The unrolled figure's base is the same length as the circumference of the circle.**
- **The height of the unrolled figure is the same length as the radius of the circle.**

5 As the circle is split into thinner rings, the stack of unrolled rings looks closer to a triangle.

What is the base and the height of the triangle?

Base b: _____**6π**_____ units
(or equivalent)
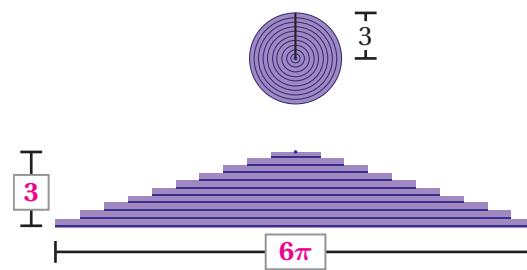
Height h: _____**3**_____ units

6 Label the diagram with the base and height you just determined.

What is the area of the triangle?
9π square units (or equivalent)

Name: _____ Date: _____ Period: _____
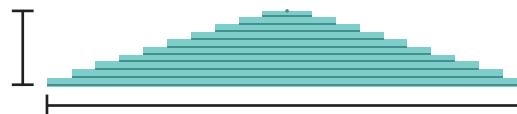
Unrolling a Circle (continued)

2 This diagram shows a new rolled and unrolled circle.

Calculate the area of the circle.

Write measurements on the triangle if it helps with your thinking.

25π square units (or equivalent)
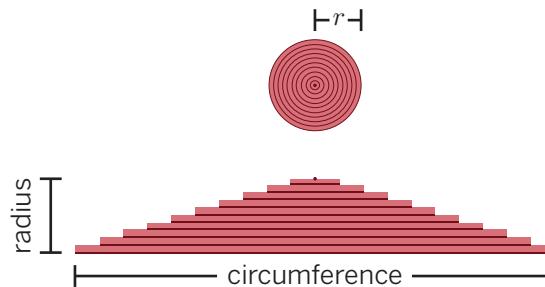
8 Haruto says the area of the triangle can be calculated using this formula:

$A = \frac{1}{2} \cdot$ circumference $\cdot$ radius

Use this fact to convince a friend that the area of a circle is $A = \pi \cdot r^2$.

Responses vary. **The radius of a circle is r and the circumference is $\pi \cdot d$, so you can substitute those values into the equation to get $A = \frac{1}{2} \cdot (\pi \cdot d) \cdot r$. Since the diameter is twice as long as the radius, we can write $\pi \cdot 2 \cdot r$ instead of $\pi \cdot d$. Another way to write $r \cdot r$ is r^2. Another way to write $\frac{1}{2} \cdot 2$ is 1. If you put all of this together, you get $A = 1\pi r^2$ or $A = \pi r^2$.**

radius

circumference

Explore More

9 Use the Explore More Sheet to explore a different way to find the area of a circle.

Responses vary. **See the Teacher Edition for sample responses.**
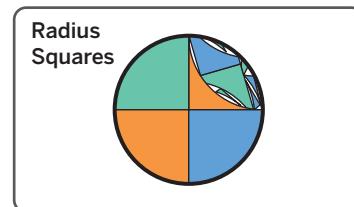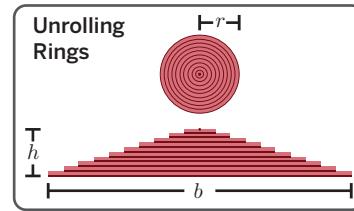
10 Synthesis

Choose one of these representations.

How does the representation you chose show that the area of a circle is πr^2?

Responses vary.
- **Unrolling rings: When you unroll the rings of a circle, it makes a triangle. The height of the triangle is r and the base of the triangle is $2\pi r$. If you use the triangle area formula, you can show that the triangle's area is $A = \pi r^2$. Since the triangle and circle have the same area, this means that the area of the circle is also $A = \pi r^2$.**
- **Radius squares: It takes π copies of a radius square to cover a circle, so the area of the circle is $\pi \cdot$ area of a radius square, or $\pi \cdot r^2$.**

Unrolling Rings
$\vdash r \dashv$
h
b

Radius Squares
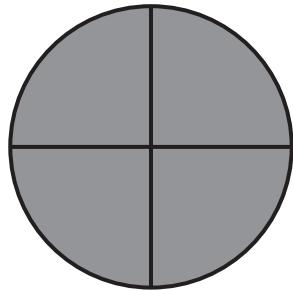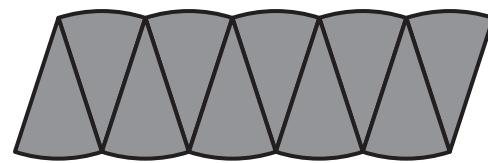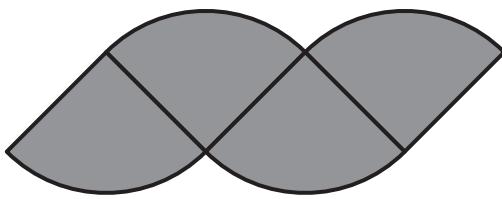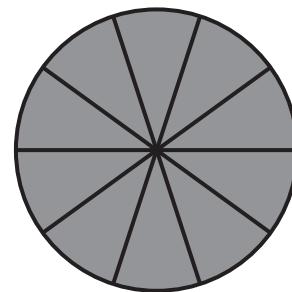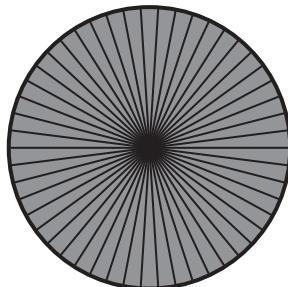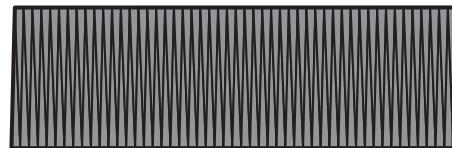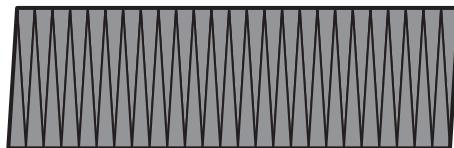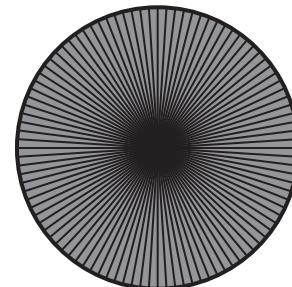
Things to Remember:

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Explore More

Here is a different way to find the area of a circle. As we make more slices for the circle, the rearranged slices get closer to a rectangle.
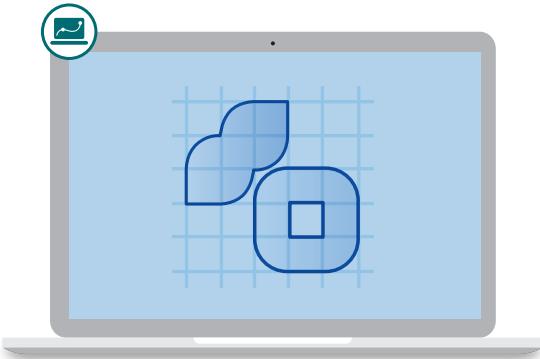
4 slices

10 slices

50 slices

100 slices

Explain how to use the resulting rectangle to find the area of the circle.

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Area Challenges

Let's calculate the areas of complex shapes.
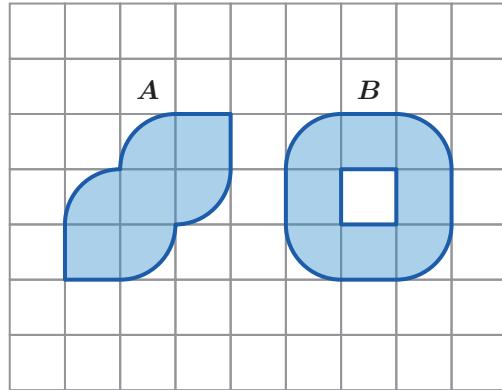
Warm-Up

1 Which shape has the greater area? Circle one.

Shape A (Shape B)   They're the same

Explain your thinking.

Explanations vary.
- **Shape B is made up of 4 squares and 4 quarter-circles. Shape A has 4 quarter circles but only 3 squares.**
- **You can make shape A by moving around the parts of shape B. You will have one square left over.**

Calculating Areas

2 Here is shape A from the Warm-Up.

What is its area?
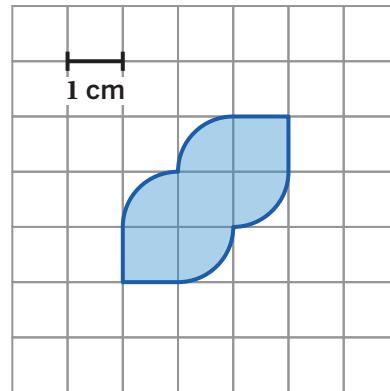
Responses vary as students may use π, *3.14,*
or $\frac{22}{7}$ *in their calculations.*
- $\pi + 3$ **square centimeters**
- $\frac{43}{7}$ **square centimeters**
- **6.14 square centimeters**

1 cm

Explain your thinking.

Explanations vary.
- **There are 3 squares. The area of each square is 1, so their total area is $3 \cdot 1 = 3$. There are 4 quarter-circles, which makes one whole circle. The area of the whole circle is π.**
- **The shape has 3 squares and 4 quarter-circles. The area of each square is 1 square centimeter. The area of each quarter-circle is $\frac{1}{4}\pi$. In all, the area is $3 \cdot 1 + 4 \cdot \frac{1}{4}\pi = 3 + \pi$.**
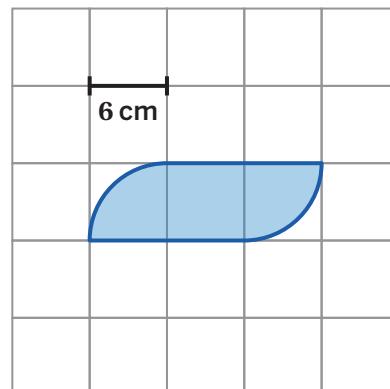
3 Here is a new shape with a new scale.

What is its area?
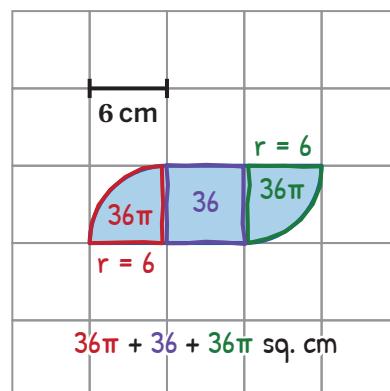
Responses vary as students may use π, *3.14, or* $\frac{22}{7}$ *in their calculations.*
- $18\pi + 36$ **square centimeters**
- $\frac{648}{7}$ **square centimeters**
- **92.52 square centimeters**

6 cm

4 Taylor calculated the area of the shape using the expression $36\pi + 36 + 36\pi$, but made a mistake.

Identify the mistake and explain why it is incorrect.
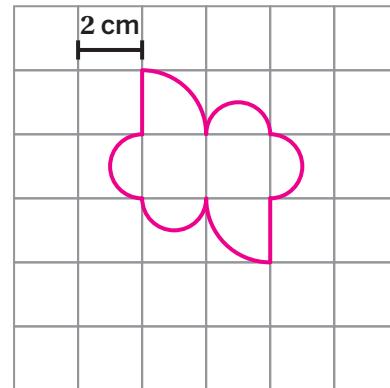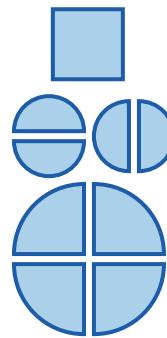
Responses vary. **Taylor labeled the quarter-circles as each having an area of 36π square centimeters. 36π is the area of the entire circle, so 36π needs to be multiplied by $\frac{1}{4}$ to get the area of a single quarter-circle.**

6 cm

r = 6

36π 36 36π

r = 6

36π + 36 + 36π sq. cm

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Create a Shape

5 Create a shape made up of squares and parts of circles that has an area of $8 + 4\pi$ square centimeters.
Shapes vary. **Sample shown on grid.**
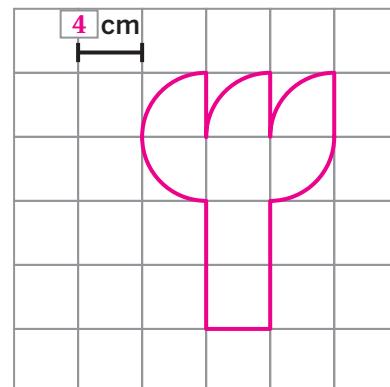
2 cm

6 Create your own challenge!

My Challenge

4 cm

a Circle the scale you'd like to use, then label the diagram with your choice. *Responses vary.*

   2 cm   4 cm   6 cm   8 cm

b Draw a shape made up of squares and parts of circles. *Responses vary.* **Sample shown on grid.**

c Determine the exact area of your shape.
Responses vary. **$20\pi + 48$ square centimeters**
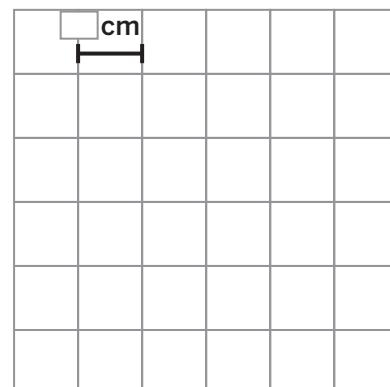
Now it's time to complete your partner's challenge!

My Partner's Challenge

☐ cm

d Ask your partner what scale they used, then label the diagram with what they chose. *Responses vary.*

e Ask your partner the area of their shape and record it.

   ....*Responses vary.*............. square centimeters

f Draw a shape made up of squares and parts of circles that has the same area as your partner's shape.
Responses vary.

g When you're done, compare your shape with your partner's shape. How are they alike? How are they different?
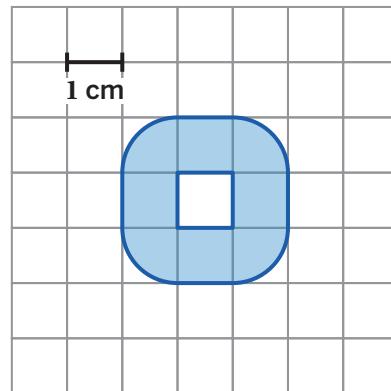
Synthesis

Describe a strategy for determining the area of a shape that's made up of squares and parts of circles.

Use the example if it helps to show your thinking.
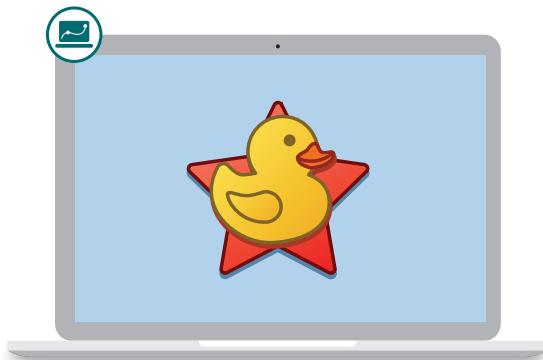
Responses vary. **Count the number of squares, semicircles, and quarter-circles. If possible, combine parts of circles into whole circles. Use the scale to figure out what the radius of the circles are and how long the sides of the square are. Then figure out the area of each piece and add up all the pieces to determine the total area.**

1 cm

Things to Remember:

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Lucky Duckies

Let's learn about benchmark percentages with rubber duckies.
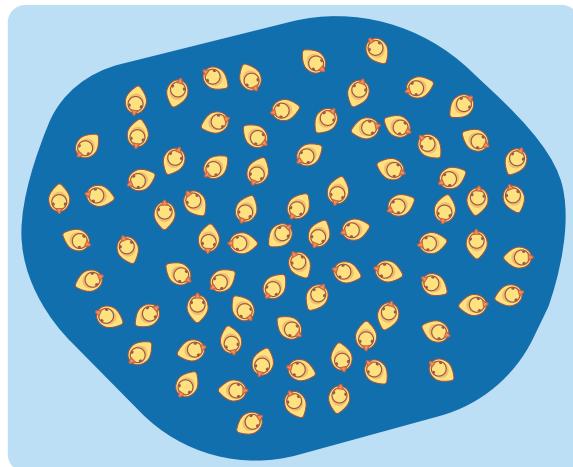
Warm-Up

1 Here is a carnival game called
Duck, *Duck*, *Choose*.

Players win a prize if they catch a
rubber ducky with a star on the bottom.

Let's try catching a ducky with a star.
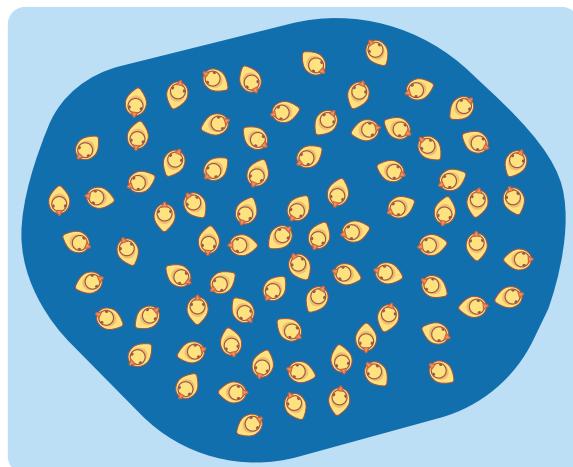
2 There are two games that both have
80 duckies. Which game has more
duckies with stars?

    A. The game where 50% of duckies have stars

    B. The game where 50 duckies have stars

    C. They are the same

Explain your thinking.

Explanations vary. **50% of 80 is half,
so the first game has 40 duckies with
stars. That means the second game
has more duckies with stars.**

Ducky Game Design
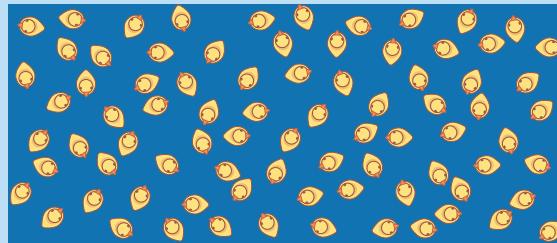
3 **a** Use the digital activity to move the dividers so that each game has about the right number of duckies with stars.
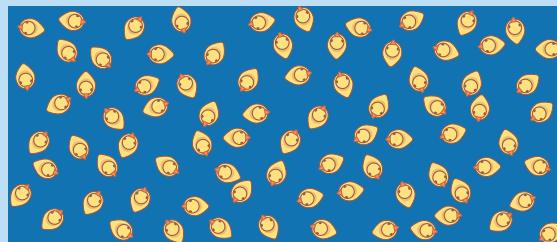
b 💬 **Discuss:** How did you decide where to place each divider?

Responses vary. **I know that 50% is half, so I put the divider in the middle of the duckies. I know that 25% is half of 50%, so I put the divider for 25% about halfway to the divider for 50%.**

Game A: 50% have stars
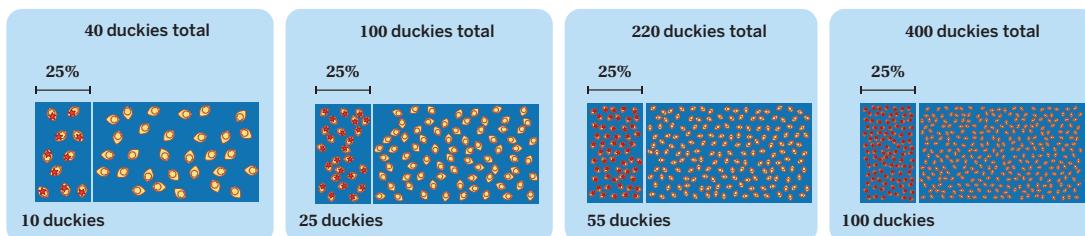
Game B: 25% have stars

4 Here are some games where 25% of the duckies have stars.

a Take a look at the total number of duckies in each game.

40 duckies total	100 duckies total	220 duckies total	400 duckies total
25%	25%	25%	25%
10 duckies	25 duckies	55 duckies	100 duckies

b Describe what 25% of a number means.

Responses vary.

• **25% means 25 out of every 100. If you have 300 duckies, $25 \cdot 3 = 75$ of them would have stars.**

• **25% means $\frac{1}{4}$ of something. If you have 25% of the duckies, then you have $\frac{1}{4}$ of all the duckies.**
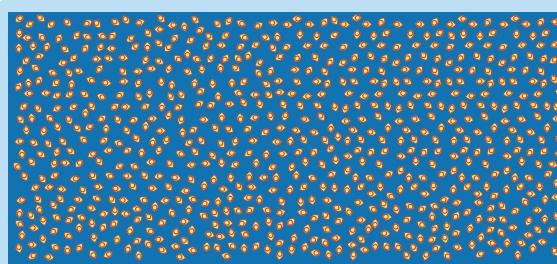
5 10 **percent** (10%) means 10 for every 100.

This game has 800 duckies. 10% of them have stars.

How many of the duckies have stars?

80 duckies

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Ducky Game Design (continued)

6 Here is how Santiago figured out the number of duckies that are winners when 10% out of 800 duckies win.

Show or explain what Santiago may have been thinking.

Responses vary. **Santiago knew that there are ten 10s in 100, so he split the tape diagram into 10 pieces and put an equal number in each piece so the total was 800.**

7 Group these choices based on what percentage they represent. One choice will have no match.

a. 10% is shaded.

b. $\frac{3}{4}$ is shaded.

c.

d. $\frac{1}{2}$ is shaded.

e. 25% is shaded.

f.

g. 75% is shaded.

h. $\frac{1}{4}$ is shaded.
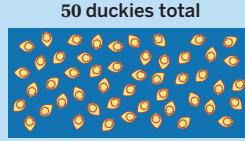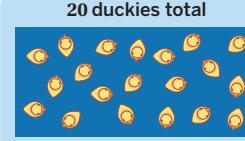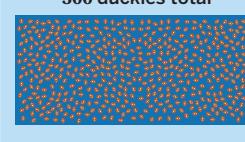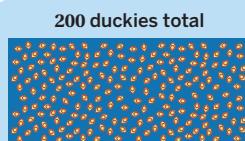
Group 1	Group 2	Group 3
a c	e f h	b g

d has no match.

Name: _____   Date: _____   Period: _____

Repeated Challenges
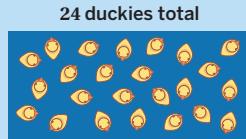
8
- Pair up with a classmate. Decide who will complete Column A and who will complete Column B.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

Column A	Column B
10% of 50 duckies have stars. How many duckies have stars? **5 duckies** 50 duckies total	25% of 20 duckies have stars. How many duckies have stars? **5 duckies** 20 duckies total
25% of 200 duckies have stars. How many duckies have stars? **50 duckies** 200 duckies total	10% of 500 duckies have stars. How many duckies have stars? **50 duckies** 500 duckies total
75% of 200 duckies have stars. How many duckies have stars? **150 duckies** 200 duckies total	50% of 300 duckies have stars. How many duckies have stars? **150 duckies** 300 duckies total

Name: _____ Date: _____ Period: _____

Repeated Challenges (continued)

50% of 24 duckies have stars. How many duckies have stars?
12 duckies

24 duckies total

75% of 16 duckies have stars. How many duckies have stars?
12 duckies

16 duckies total
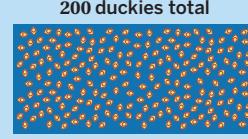
25% of 80 duckies have stars. How many duckies have stars?
20 duckies

80 duckies total

10% of 200 duckies have stars. How many duckies have stars?
20 duckies

200 duckies total

50% of 300 duckies have stars. How many duckies have stars?
150 duckies

300 duckies total
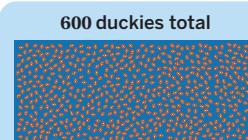
25% of 600 duckies have stars. How many duckies have stars?
150 duckies

600 duckies total

Explore More

9 Use the digital activity to make several different games that all have 30 winning duckies.
Responses vary. **10% of 300 duckies, 25% of 120 duckies, 50% of 60 duckies, 75% of 40 duckies**
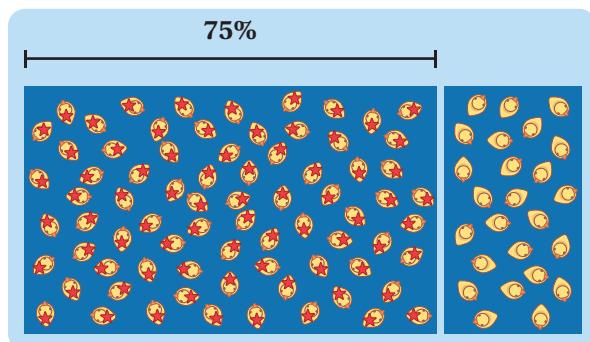
🔟 Synthesis

In your own words, explain what 75% of a number means.

Responses vary.

- **75% means 75 out of every 100.
  If you have 200 duckies, 75 · 2 = 150 of them would have stars.**
- **75% means $\frac{3}{4}$ of something.
  If you've done 75% of your homework, then you've done $\frac{3}{4}$ of your homework.**

Things to Remember:

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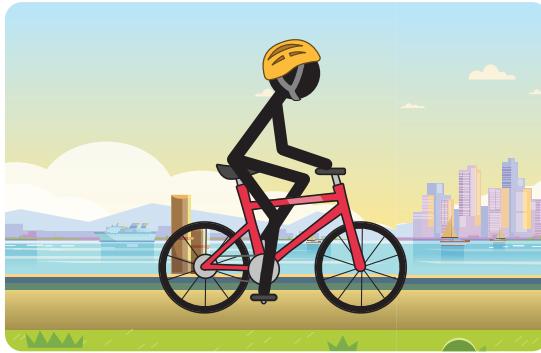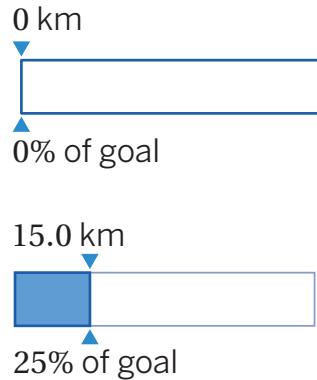
Bicycle Goals

Let's connect percentages and ratios.

Warm-Up

1 Let's watch the percentage change as the biker tries to beat their goal.

0 km

0% of goal

15.0 km

25% of goal

2 What is this biker's goal distance?

60 kilometers

Explain your thinking.

Explanations vary.

- **25% is $\frac{1}{4}$, so the whole goal would be 100%, or $15 \cdot 4 = 60$ kilometers.**
- **There are four 25 percents in 100%, so you need to multiply by 4 to get the goal distance.**

Name: .... Date: .... Period: ....

Bicycle Goals

3 Alejandro's goal was to ride 36 kilometers.
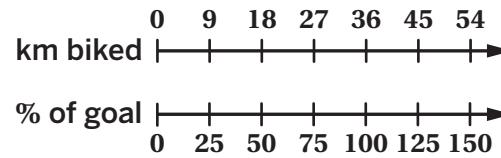
His app says he rode 75% of his goal.

How far did he ride?
27 kilometers

? km

75% of goal

4 **a** Take a look at the double number line that represents Alejandro's goal and progress.

km biked
0   9   18   27   36   45   54

% of goal
0   25   50   75   100   125   150

b Describe how you can tell that the goal distance is 36 kilometers.

Responses vary. **The goal is always 100%, and 36 kilometers on the top number line lines up with 100% on the bottom number line.**

Name: _____ Date: _____ Period: _____
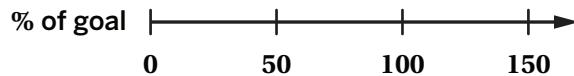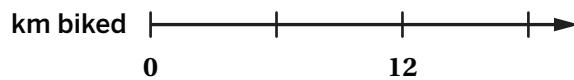
Bicycle Goals (continued)

5 Basheera's goal was to ride 12 kilometers.

Her app says she rode 150% of her goal.

How far did she ride?

18 kilometers

km biked

```
|-------+-------+-------+------->
0               12
```

% of goal

```
|-------+-------+-------+------->
0       50      100     150
```
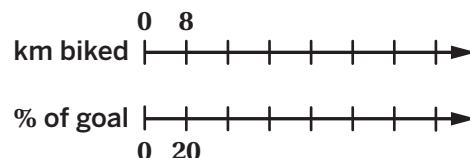
6 According to an app, Callen and Folami biked 8 kilometers, which is 20% of their goal.
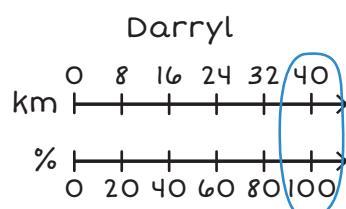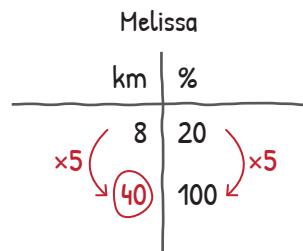
What was their goal distance?

40 kilometers

```
            0   8
km biked  |-+-+-+-+-+-+-+->

% of goal |-+-+-+-+-+-+-+->
            0  20
```

7 Here are two different strategies for calculating the goal when 20% of the goal is 8 kilometers.

💬 **Discuss:** How did each student use ratios to calculate the goal?

Melissa

km	%
8	20
(40)	100

×5 (...) ×5

Darryl

```
      0   8  16  24  32 (40)
km  |--+--+--+--+--+--+-->

%   |--+--+--+--+--+--+-->
    0  20  40  60  80 (100)
```
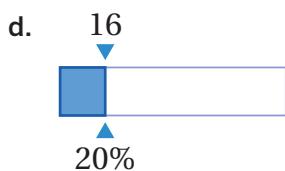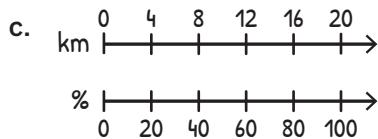
Responses vary. **Melissa made a table to find equivalent ratios. She multiplied by 5 so that the percent in the right column was 100%. Darryl used equivalent ratios to fill in the double number line until he found the number that lined up with 100%.**

Name: .... Date: .... Period: ....

Percentages and Ratios

8 Match the strategies that represent the same percentage problem.

a.

km	%
80	100
8	10
16	20

b.

km	%
20	100
2	10
16	80

c.

d.

20%
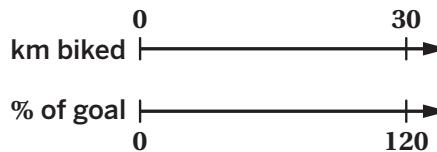
20% of 80 km	80% of 20 km
a	b
d	c

9 Miko's app says he biked 30 kilometers, which is 120% of his goal.

What was his goal distance?
25 kilometers

Explore More

10 How many ways can you make a biker ride 10 kilometers?
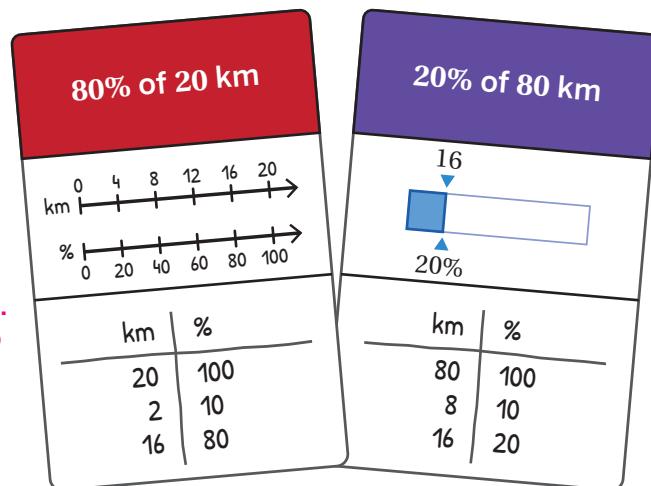
Enter a goal and a percent of that goal for the biker to ride.

Goal	50 kilometers	100 kilometers	10 kilometers	20 kilometers
Percent of Goal	20%	10%	100%	50%

Synthesis

Describe how solving problems with percentages is like solving problems with ratios.

Responses vary. A percentage is like a ratio but with the number 100 in it somewhere. You can make a double number line for a ratio and also for a percentage. Tables and tape diagrams can also help with ratios and percentages. The whole tape diagram is another way to say 100%.

80% of 20 km

km
0 4 8 12 16 20

%
0 20 40 60 80 100

km	%
20	100
2	10
16	80

20% of 80 km

16

20%

km	%
80	100
8	10
16	20

Things to Remember:

Unit 3
Lesson
12

Name: _____ Date: _____ Period: _____
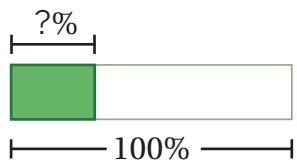
Cost Breakdown
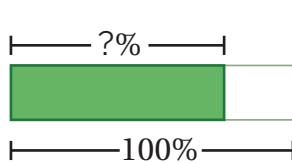
Let's calculate any percentage of a number.

Warm-Up
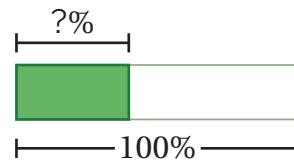
1 For each challenge, write your best estimate of the missing percent.

?%	?%	?%
100%	100%	100%
Responses vary. 30%	*Responses vary. 75%*	*Responses vary. 40%*

Name: _____ Date: _____ Period: _____

Break It Down

2 Ada and Bao run a clothing store.

The price of each item includes the profit that Ada and Bao make, the cost of inventory, and the cost to run the store.
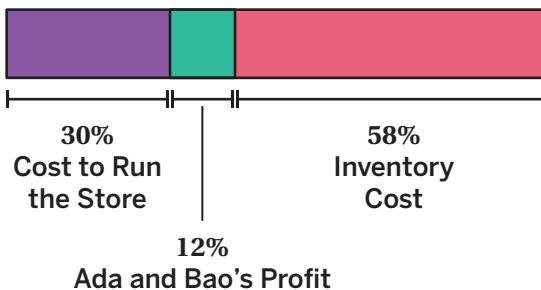
The diagram shows where the money goes when Ada and Bao sell a T-shirt.

What do you notice? What do you wonder?

Responses vary.

I notice:

- **I notice that the profit is the smallest percentage.**
- **I notice that they all add up to 100%.**
- **I notice that the inventory cost is the biggest percentage.**

I wonder:
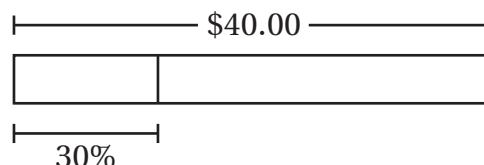
- **I wonder how much of the cost to run the store goes to paying the people who work at the store.**
- **I wonder if Ada and Bao share the profit equally.**

30%
Cost to Run the Store

58%
Inventory Cost

12%
Ada and Bao's Profit

3 30% of the price of each shirt goes to running the store.

How much of a $40 shirt goes to running the store?

Use the tape diagram if it helps with your thinking.

$12

$40.00

30%
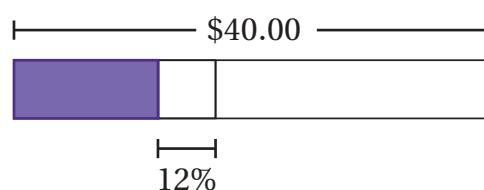
4 Ada and Bao keep 12% of the price of each shirt as profit.

What is their profit on a $40 shirt?

$4.80

$40.00

12%

Name: _____   Date: _____   Period: _____

Break It Down (continued)

5 Here is how Bao calculated 12% of $40.
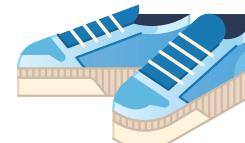
Explain how you could use Bao's strategy to calculate how much of the price of each shirt goes to inventory cost (58% of $40).

Responses vary. **You would still start by dividing by 100 to get $\frac{40}{100}$. But then you would multiply by 58 instead of 12. The expression would be $\frac{40}{100} \cdot 58$.**
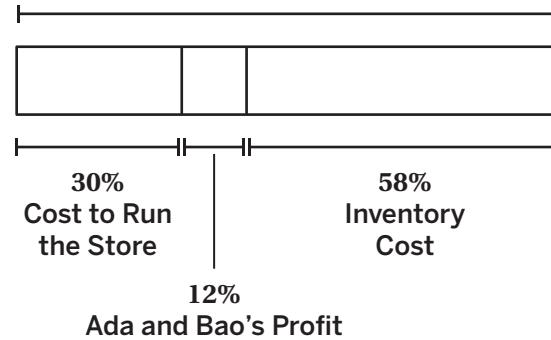
Cost (dollars)	Percentage
40	100%
$\frac{40}{100}$	1%
$\frac{40}{100} \cdot 12$	12%

÷100 ×12   ÷100 ×12

6 Here is a $75 pair of shoes. Calculate each value.

Cost to Run the Store	$22.50
Ada and Bao's Profit	$9
Inventory Cost	$43.50

$75.00

30% Cost to Run the Store

12% Ada and Bao's Profit
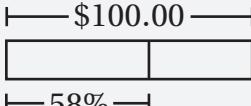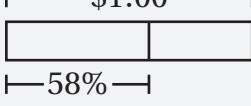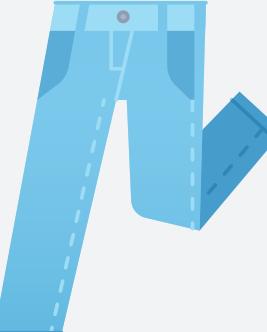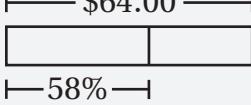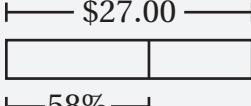
58% Inventory Cost

Another Strategy

7 Ada and Bao's inventory costs are 58% of the total cost. What is their inventory cost for each item in this table?
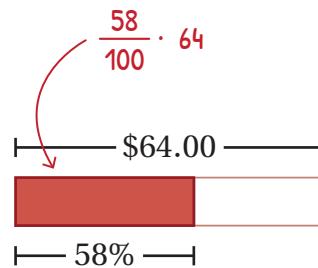
Item	Representation	Total Cost (dollars)	Inventory Cost (dollars)
Dress	⊢—— $100.00 ——⊣ ⬚ ⊢—58%—⊣	$100	**$58**
Sticker	⊢—— $1.00 ——⊣ ⬚ ⊢—58%—⊣	$1	**$0.58**
Jeans	⊢—— $64.00 ——⊣ ⬚ ⊢—58%—⊣	$64	**$37.12**
Hat	⊢—— $27.00 ——⊣ ⬚ ⊢—58%—⊣	$27	**$15.66**

Name: _____ Date: _____ Period: _____

Another Strategy (continued)

8 Ada thinks of 58% as 58 cents for every dollar. So he writes $\frac{58}{100} \cdot 64$ to calculate 58% of $64.

$\frac{58}{100} \cdot 64$

Describe how Ada might calculate 36% of $15.

Responses vary. **He would still start by dividing by 100 to get $\frac{36}{100}$. But then he would multiply by 15 instead of 64. The expression would be $\frac{36}{100} \cdot 15$.**

— $64.00 —

— 58% —

9 Match each expression with a question. One expression will have no match.

a. $\frac{36}{100} \cdot 70$

b. $\frac{15}{100} \cdot 36$

c. $\frac{100}{36} \cdot 48$

d. $\frac{36}{100} \cdot 15$

e. $70 \div 100 \cdot 36$

f. $\frac{36}{100} \cdot 48$

What is 36% of $15?	What is 36% of $48?	What is 36% of $70?
b	f	a
d		e

c has no match.

Name: _____   Date: _____   Period: _____

Repeated Challenges

10 Solve as many challenges as you have time for.
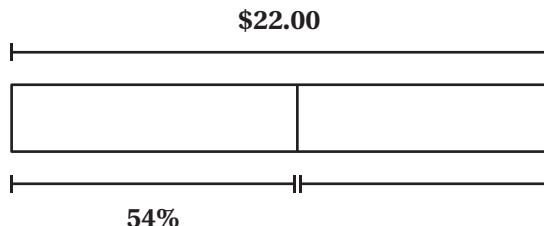
Problem	Representation	Answer
The price of a space T-shirt is $22. 16% of every sale goes to material cost. Calculate the material cost.	⊢— $22.00 —⊣ [bar diagram] ⊢⊣ 16%	**$3.52**
The price of a striped button-up shirt is $30. 9% of every sale goes to clothing company profit. Calculate the clothing company profit.	⊢— $30.00 —⊣ [bar diagram] ⊢⊣ 9%	**$2.70**
The price of a striped long-sleeve T-shirt is $48. 8% of every sale goes to transport cost. Calculate the transport cost.	⊢— $48.00 —⊣ [bar diagram] ⊢⊣ 8%	**$3.84**
The price of a blue pair of shoes is $65. 5% of every sale goes to factory profit. Calculate the factory profit.	⊢— $65.00 —⊣ [bar diagram] ⊢⊣ 5%	**$3.25**

Synthesis

Describe a strategy for calculating a percentage of a number.

Use the example if it helps to explain your thinking.
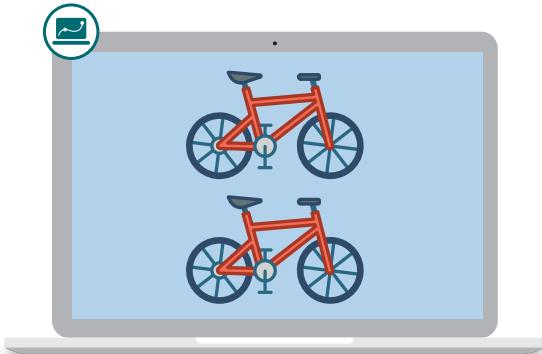
Responses vary. **One strategy is to figure out how much 1% would be, then multiply that number by the percent you actually have. Another strategy is to think about the percent as a fraction out of 100 and then multiply by the total number you have.**

$22.00

54%

Things to Remember:

Unit 3
Lesson

13

Name: _____ Date: _____ Period: _____

More Bicycle Goals

Let's calculate unknown percentages.

Warm-Up

Evaluate each expression mentally. Try to think of more than one strategy.

1 $\frac{1}{3}$ of $\frac{1}{4}$

$\frac{1}{12}$ **(or equivalent)**

2 $\frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$ **(or equivalent)**

3 $\frac{2}{3} \cdot \frac{1}{4} = \frac{2}{12}$ **(or equivalent)**

4 $\frac{2}{3} \cdot \frac{5}{4} = \frac{10}{12}$ **(or equivalent)**

Name: ....   Date: ....   Period: ....
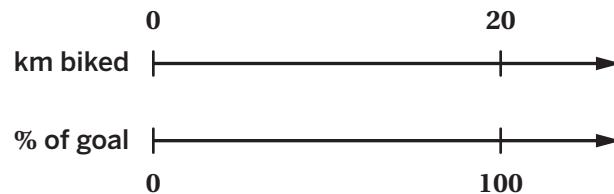
Chasing Goals

5 Alejandro's goal for Monday was to ride 20 kilometers.

His app says he rode 40% of his goal.

How far did he ride?
8 kilometers

km biked |————————————|————————▶
0   20

% of goal |————————————|————————▶
0   100
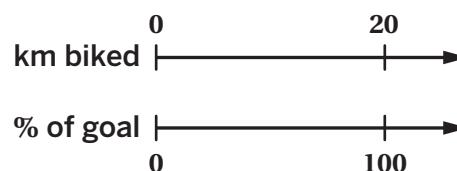
6 Alejandro's goal for Tuesday was to ride 20 kilometers.
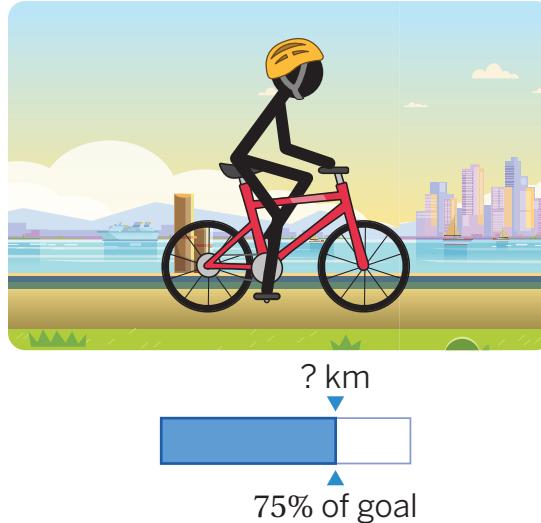
His app says he rode 10 kilometers.

What percent of his goal did he ride?
50%

km biked |————————————|————————▶
0   20

% of goal |————————————|————————▶
0   100

7 On Wednesday, Alejandro and Basheera rode 17 kilometers.

- Alejandro's goal was 20 kilometers.
- Basheera's goal was 50 kilometers.

Who rode a greater percent of their goal? Circle one.
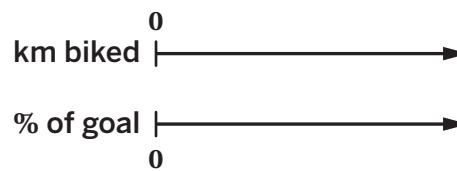
(Alejandro)   Basheera   Same percent
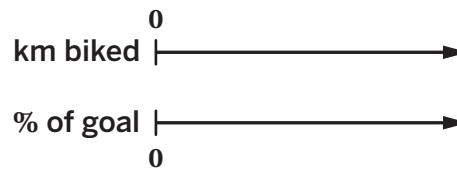
Explain your thinking.
Explanations vary.
- **Alejandro had a smaller goal, so even though he and Basheera biked the same distance, Alejandro biked a bigger portion of his goal.**
- **Alejandro biked more than 50% of his goal and Basheera biked less than 50% of her goal.**

Alejandro

km biked |————————————————————▶
0

% of goal |————————————————————▶
0

Basheera

km biked |————————————————————▶
0

% of goal |————————————————————▶
0

Name: _____ Date: _____ Period: _____
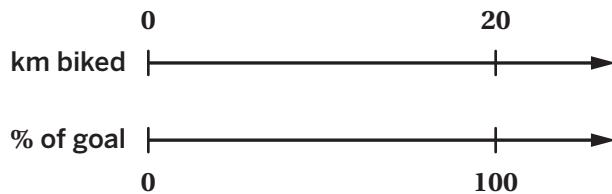
Chasing Goals (continued)

8 On Wednesday, Alejandro and Basheera rode 17 kilometers.

Alejandro's goal was 20 kilometers.

What percent of his goal did he ride?

85%


```
          0                    20
km biked  |--------------------|------------->

% of goal |--------------------|------------->
          0                    100
```

9 Here is how Alejandro calculated 17 out of 20 as a percentage.

Explain how you could use Alejandro's strategy to calculate 13 out of 20 as a percentage.

Responses vary. **You would still start by dividing 100 by 20 to determine what percent of the goal 1 kilometer is. Since you want to know what percent of the goal 13 kilometers is, you would multiply by 13 instead of 17. The expression would be $\frac{100}{20} \cdot 13$.**

Distance (km) | Percent of Goal

$$\times \frac{1}{20} \Big(\begin{array}{c} 20 \\ 1 \end{array}$$

$$\times 17 \Big(\begin{array}{c} 1 \\ 17 \end{array}$$

$$100 \Big) \times \frac{1}{20}$$

$$\frac{100}{20}$$

$$\times 17$$

$$\boxed{\frac{100}{20} \cdot 17}$$

Name: _____ Date: _____ Period: _____
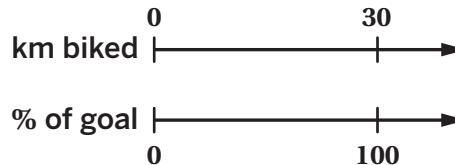
Reaching Goals

10 Alejandro and Basheera set a new goal for Saturday: 30 kilometers.

They rode 36 kilometers.

What percent of their goal did they ride?
120%

11 Here are the expressions Alejandro and Basheera used to calculate 36 out of 30 as a percentage.

Whose expression is correct? Circle one.

Alejandro   Basheera
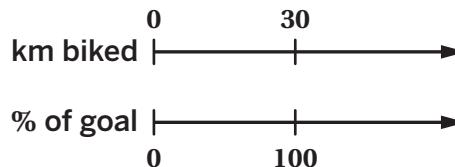
(Both)   Neither

Alejandro   Basheera

$$\frac{100}{30} \cdot 36 \qquad \frac{36}{30} \cdot 100$$

Explain your thinking.
Explanations vary. **Even though their expressions look different, they have the same value (120). Both expressions multiply 36 and 100 and divide by 30.**

12 On Sunday, Alejandro, Basheera, and Callen rode 40 kilometers. They each had different goals, as shown in the table. Calculate what percent of their goal each person rode.

Representation	Distance Traveled (km)	Goal (km)	Percent of Goal
Alejandro	40	50	**80%**
Basheera	40	25	**160%**
Callen	40	64	**62.5%**

Name: _____ Date: _____ Period: _____

Reaching Goals (continued)

13 Match each question with a double number line and an expression. One choice will have no match.

a. $\frac{32}{100} \cdot 45$ b. $\frac{32}{45} \cdot 100$ c. $\frac{45}{32} \cdot 100$

d.

| 0 | 32 | 45 |
Number

%
0 100 (?)

e.

| 0 | 32 | 45 |
Number

%
0 (?) 100

What is 32 out of 45 as a percentage?	What is 45 out of 32 as a percentage?
b **e**	**c** **d**
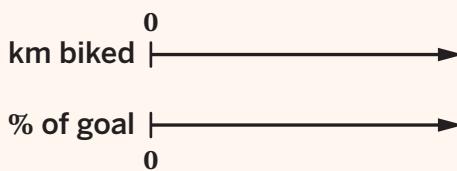
a has no match.

Explore More
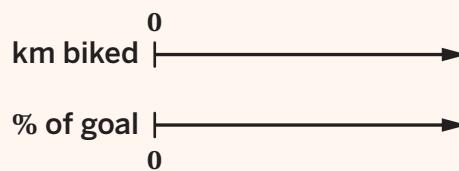
14 Basheera had a goal of riding 30 kilometers and rode 51 kilometers.
Callen had a goal of 20 kilometers and rode 41 kilometers.

Determine who rode a greater percent of their goal, in as many different ways as you can.

Basheera

0
km biked ├─────────────────►

% of goal ├─────────────────►
0

Callen

0
km biked ├─────────────────►

% of goal ├─────────────────►
0

Callen

Explain your thinking.
Explanations vary.
• **Callen rode more than twice 20 kilometers. Basheera rode less than twice 30 kilometers.**
• **Callen rode $\frac{41}{20} \cdot 100 = 205\%$ of 20. Basheera rode $\frac{51}{30} \cdot 100 = 170\%$ of 30.**

15 Synthesis
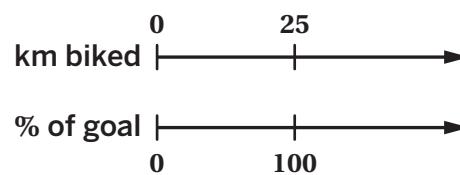
Here is what Basheera wrote to solve a bicycle challenge.

$$\frac{31}{25} \cdot 100 = 124$$

What do 31, 25, and 124 represent in this situation?

Responses vary.
- **31 represents the distance that Basheera biked.**
- **25 represents the distance of Basheera's goal.**
- **124 represents the percent of Basheera's goal that she biked.**

km biked

% of goal

Things to Remember:

Unit 3
Lesson
14

Name: _____   Date: _____   Period: _____
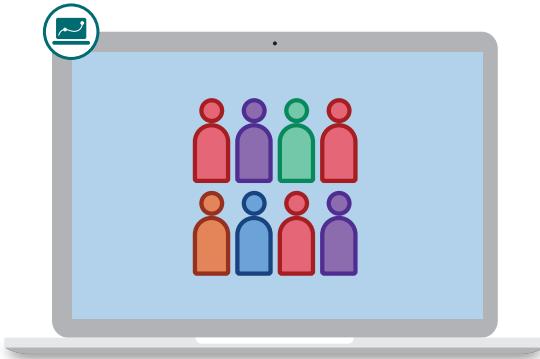
A Country as a Village

Let's explore different countries.
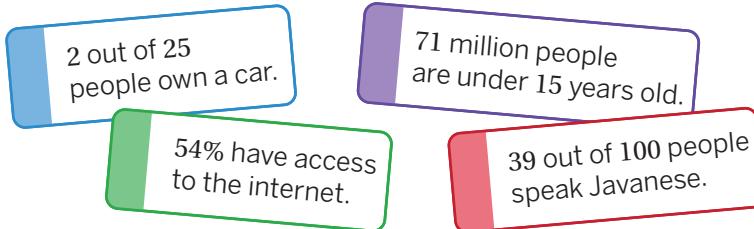
Warm-Up

1 Here are some facts about Indonesia from the year 2020.

What do you notice? What do you wonder?

Responses vary.
- **I notice more than half of the people in Indonesia have access to the internet.**
- **I notice less than 10% of people own a car.**
- **I wonder if the population of Indonesia has changed since 2020.**
- **I wonder what percentage of people are under 15 years old.**

2 out of 25 people own a car.

71 million people are under 15 years old.

54% have access to the internet.

39 out of 100 people speak Javanese.

Sources: World Bank, CEIC, WorldData.info

Name: _____ Date: _____ Period: _____

Exploring Indonesia

2 The population of Indonesia was about 272 million people in 2020.[1]

How many people in Indonesia have each of these characteristics?

Characteristic	Number of People (millions)
Speak Javanese	**Between 106 and 107**
Own a car	**Between 21 and 22**
Have internet access	**Between 146 and 147**
Under 15 years old	**71**

Population of Indonesia: 272 million

2 out of 25 people own a car.

71 million people are under 15 years old.

54% have access to the internet.

39 out of 100 people speak Javanese.

3 It's hard to picture 272 million people.

Imagine Indonesia was a village with just 100 people.

How many people would have each of these characteristics?

Characteristic	Number of People in Village
Speak Javanese	**39**
Own a car	**8**
Have internet access	**54**
Under 15 years old	**26**

[1] Source: *World Bank*

Name: _____ Date: _____ Period: _____

A Country as a Village

4 You and your partner will need the Activity 2 Sheet for this activity.

Pick a country. Use the information from 2020 on the Activity 2 Sheet to make a poster describing what this country would look like as a 100-person village.

Brazil   China   India   Nigeria   United States

Be sure to include these items in your poster:
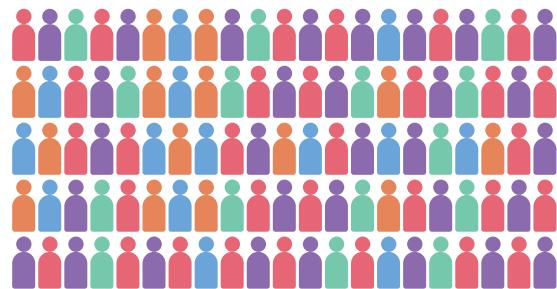
☐ Your names and the name of the country you selected.

☐ An answer to the question: *If this country were a village of* 100 *people, how many of them would have each of these characteristics?*

☐ Your thinking and calculations for each characteristic.

☐ Something that you are curious to learn more about.

☐ *At least* two other characteristics of this country you are interested in knowing about.
Responses vary.

5 Synthesis

How is working with percentages like working with a village of 100 people?

Responses vary. Percentages are a number out of 100, so a percentage is kind of the same as the number of people out of a village of 100. For example, if 62 out of 100 people wear glasses, then 62% of people wear glasses.

Things to Remember:

Name: _____ Date: _____ Period: _____

A Country as a Village

Here is some information about five of the most populated countries in the world in the year 2020. Use this information to make a poster.

Population of Brazil: 213 million

193 out of every 1,000 people own a car.

42.6 million people are under 15 years old.

81% have access to the internet.

93 out of 100 people speak Portuguese.

Sources: World Bank, Helgi Library, WorldData.info

Population of China: 1.41 billion

2 out of every 9 people own a car.

254 million people are under 15 years old.

70% have access to the internet.

91 out of 100 people speak Mandarin.

Sources: World Bank, Ministry of Public Security (China), WorldData.info

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Population of India: 1.4 billion

3 out of 50 people own a car.

364 million people are under 15 years old.

43% have access to the internet.

40 out of 100 people speak Hindi.

Sources: World Bank, CEIC, WorldData.info

Population of Nigeria: 208 million

55 out of 1,000 people own a car.

91.5 million people are under 15 years old.

51% have access to the internet.

24 out of 100 people speak Hausa.

Sources: World Bank, CEIC, WorldData.info

Population of the United States: 332 million

819 per 1,000 people own a car.
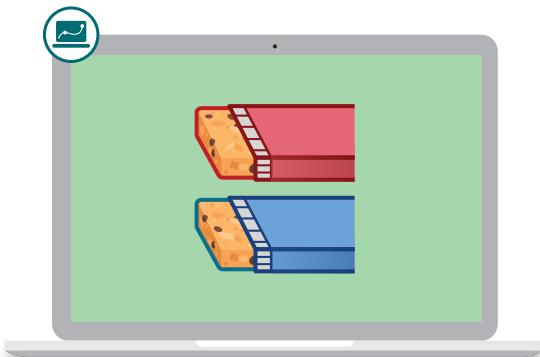
61 million people are under 15 years old.

97% have access to the internet.

92 out of 100 people speak English.

Sources: World Bank, Federal Highway Administration, US Census Bureau

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Unit 4

Lesson

2

Name: .... Date: .... Period: ....
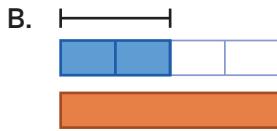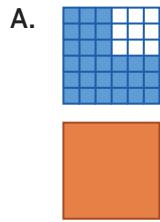
More and Less

Let's visualize what it means to increase or decrease by a percentage.

Warm-Up

1 Which one doesn't belong?

A.   B.   C.   D.

Explain your thinking.

Responses and explanations vary.
- **A is the only pair where there are squares. The top is $\frac{3}{4}$ of the bottom.**
- **B is the only pair where one is half of the other. The top is $\frac{1}{2}$ of the bottom.**
- **C is the only pair where the top is more than the bottom. The top is $\frac{5}{4}$ of the bottom.**
- **D is the only pair that's in a proportion of 1.**

Name: _____ Date: _____ Period: _____

Granola Bars

2 DesBest Granola bars are now 20% longer.
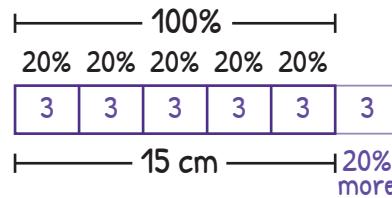
If the original bar was 15 centimeters long, how long is the new granola bar?
18 centimeters

DesBest Granola

3 Let's take a look at how DeAndre found the new length of the 15-centimeter granola bar after a 20% increase.

Explain what DeAndre may have been thinking.
Responses vary. **Since 5 copies of 20% make 100%, DeAndre split 15 centimeters into 5 pieces that were 3 centimeters each. Since this bar has 20% more, it is 3 more than 15, or 18.**

Name: _____ Date: _____ Period: _____

Granola Bars (continued)

4 In order to make more money, DesWorst Granola bars are now 10% shorter.

If the original bar was 15 centimeters long, how long is the new granola bar?
13.5 centimeters
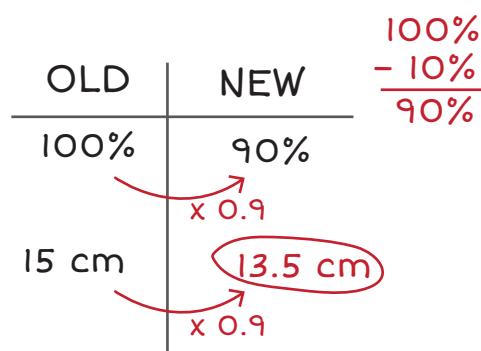
DesWorst
Granola

5 Let's take a look at how Afia found the length of the 15-centimeter granola bar after a 10% decrease.

Explain what Afia may have been thinking.
Responses vary. **She figured out that 10% less is another way of saying 90% there, so Afia multiplied by the constant of proportionality 0.9 to figure out how long the new bar was.**

OLD	NEW
100%	90%
15 cm	13.5 cm

x 0.9

x 0.9

100%
− 10%
90%

6 Two different boxes of granola bars are the same price.

One box has 40% more bars. The other has 4 more bars.

Which is the better deal? Circle one.

40% more bars   4 more bars   Not enough information
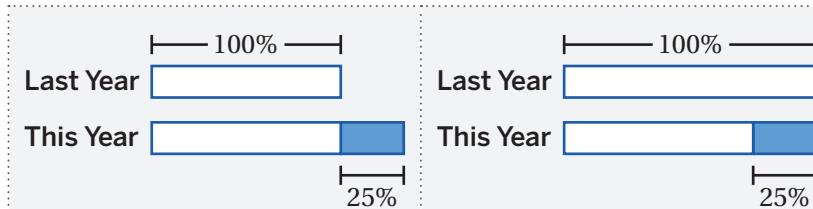
Explain your thinking.
Explanations vary.
- **I don't know how many bars there are, so I don't know how much 40% more is.**
- **If the number of bars is small, then 4 more bars is better. If the number of bars is large, like 20, then 40% would be better.**

Des-Granola
Now with
40%
more bars!

Granola-Mos
Now with
4
more bars!

Name: _____   Date: _____   Period: _____

Fruitful Percentages

7 Decide which tape diagram could represent each situation by placing a check mark in the appropriate column.

	⊢— 100% —⊣ Last Year ☐ This Year ☐ 25%	⊢— 100% —⊣ Last Year ☐ This Year ☐ 25%
This year's apple harvest is $\frac{3}{4}$ of last year's.		✔
There are $\frac{1}{4}$ more raspberries this year compared to last year.	✔	
This year's plum harvest is 125% of last year's.	✔	
This year's blueberry harvest is 75% of last year's.		✔
Compared to last year's strawberry harvest, this year's harvest is a 25% increase.	✔	
Compared to last year, this year's peach harvest decreased 25%.		✔

8 Here's a situation from the previous problem.

If 80 pounds of peaches were harvested last year, how many pounds were harvested this year?

Explain your thinking.

60 pounds. *Explanations vary*. I know this year's harvest is 75% of 80, and 0.75 • 80 = 60.

> Compared to last year, this year's peach harvest decreased by 25%.

Name: _____ Date: _____ Period: _____

Percent Practice

9 Order these values from *least* to *greatest*.

75% less than 60	25 more than 20	100% of 20	25% of 20	25% more than 20
25% of 20	**75% less than 60**	**100% of 20**	**25% more than 20**	**25 more than 20**

Least   **Greatest**

Explore More

10 Evan can use three coupons to buy a $25 shirt. The store will apply these coupons one at a time.

Order the coupons so that Evan gets the lowest price.

$10 off	20% off	5% off

20% off or 5% off	First Coupon
5% off or 20% off	
$10 off	Last Coupon

Explain your thinking.

***Explanations vary*. We want the biggest discount, so the percent-offs should use the largest number, which is at the beginning.**
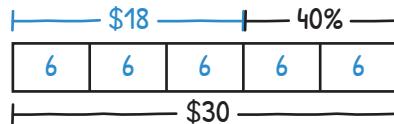
11 Synthesis

Here are two different strategies to determine the new price of a hat after a discount of 40%.

The hat was originally $30.
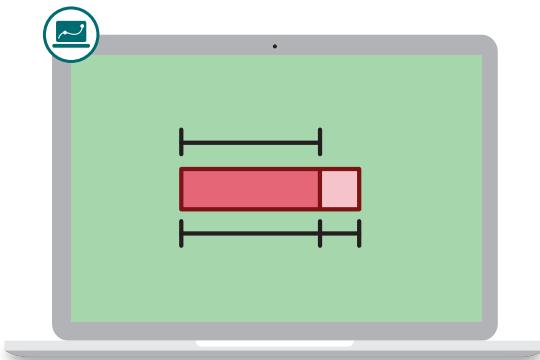
💬 **Discuss:** How are these strategies alike and how they are different?

Responses vary. **These strategies are the same because they both calculate 60% of $30. In the tape diagram, you think of 60% as $\frac{3}{5}$ and determine that fraction of $30. In the table, you multiply $30 by 0.6, which is another way of writing 60%.**

$18	40%

6	6	6	6	6

$30

OLD	NEW
100%	60%
	x 0.6
$30	$18
	x 0.6

$$\begin{array}{r} 100\% \\ -\ 40\% \\ \hline 60\% \end{array}$$

Things to Remember:

Unit 4

Lesson

3

Name: _____ Date: _____ Period: _____

All the Equations

Let's use equations to represent percent increase or decrease.

Warm-Up

1 Here is a rectangle.

Which rectangle is 21% longer than the original rectangle?

A.

B.

C.

D.

|—— 100% ——|

276

Name: ‎ Date: ‎ Period: ‎
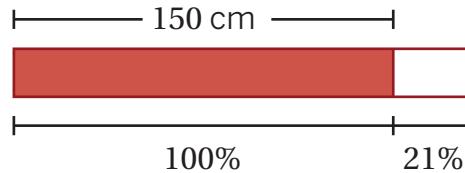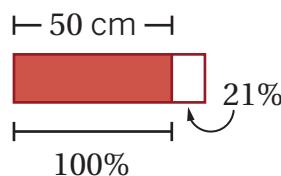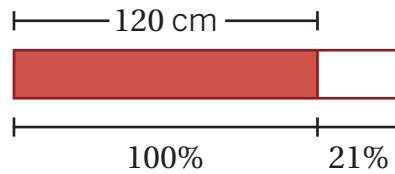
Percent Increase

Each new rectangle is 21% longer than the original.

2 Complete the table with the length of each new rectangle.

Original Rectangle Length (cm)	Length After 21% Increase (cm)
120	**145.2**
50	**60.5**
150	**181.5**

⊢——— 120 cm ———⊣

⊢————— 100% —————⊣   ⊢— 21% —⊣

⊢— 50 cm —⊣

⊢—— 100% ——⊣ 21%

⊢——— 150 cm ———⊣

⊢———— 100% ————⊣   ⊢— 21% —⊣

3 We can use equations to represent relationships involving percent increase and decrease.

Write an equation to represent the relationship between the length of an original rectangle, b, and the length of a new rectangle, c.

$c = $ ‎

Responses vary.
- $c = 1.21b$
- $c = 1b + 0.21b$
- $c = (1 + 0.21)b$

Name: _____   Date: _____   Period: _____

Percent Decrease

4 Here is a rectangle.

Which shaded rectangle is 72% shorter than the original rectangle?

├──────── 100% ────────┤

A.

B.

C.

D.

5 The original rectangle is 120 centimeters.

Calculate the length of a rectangle that is 72% shorter.

33.6 centimeters

├──────── 120 cm ────────┤

├──────── 72% ────────┤

├──────── 100% ────────┤
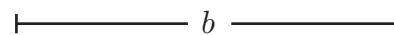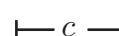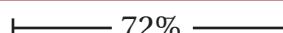
6 Select the *three* equations that represent the relationship between the length of the original rectangle, b, and the length of the new rectangle, c.

├──────── b ────────┤

├── c ──┤

├──────── 72% ────────┤

├──────── 100% ────────┤

☑ **A.** $(100 - 72)b = c$

☐ **B.** $c = 0.28b$

☑ **C.** $0.72b = c$

☐ **D.** $(1 - 0.72)b = c$

☑ **E.** $c = b - 0.72$

☐ **F.** $c = 1b - 0.72b$

Name: _____   Date: _____   Period: _____

Percents and Equations

7 For each equation, put a check under the percent increase or decrease it represents. One equation will be neither.

Let b represent the original value and c represent the final value.

Equation	A Decrease of 88%	An Increase of 12%	Neither
$(1 - 0.88)b = c$	✔		
$(1 + 0.12)b = c$		✔	
$c = 0.12b$	✔		
$1b - 0.88b = c$	✔		
$0.88b = c$			✔
$c = 1.12b$		✔	
$c = 1b + 0.12b$		✔	

8 Manuel paid off 88% of his debt. He originally owed $1,950. How much does Manuel owe now?

Use an equation from the column "A Decrease of 88%" if it helps with your thinking.
234 dollars

Explore More

9 Use the Explore More Sheet to answer a question about some magical goo.
121.5 grams

10 Synthesis

Here are two equations that could be used to represent a problem about *percent increase* or *percent decrease*.

Circle an equation and write a story about a situation it could represent.

$$1.25 \cdot b = c \qquad\qquad\qquad 0.80 \cdot b = c$$

Responses vary.
- **If someone works overtime they could get an extra 25% per hour.**
- **My friend always eats 20% of whatever I bring for lunch and I eat what's left.**

Things to Remember:

Name: _____   Date: _____   Period: _____
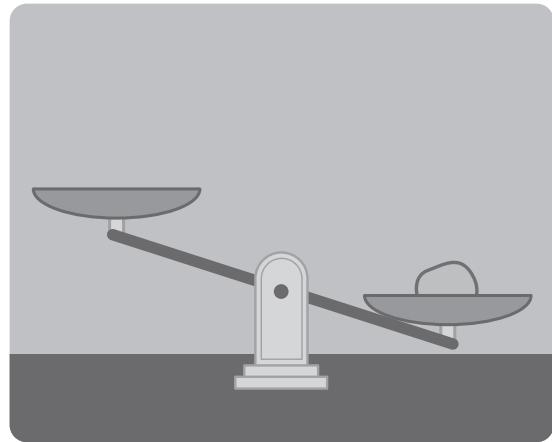
Explore More

An astronaut brought a 16-gram sample of goo from another planet to her laboratory.

When the goo is exposed to light, the amount of goo increases by the same percentage every hour.
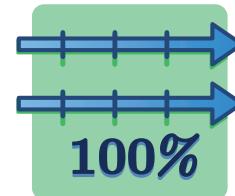
Complete the table.

Time (hr)	Goo (g)
0	16
1	24
2	36
…	…
5	

Unit 4
Lesson
4

Name: _____ Date: _____ Period: _____
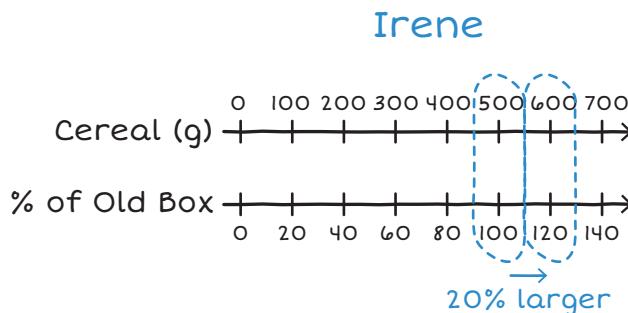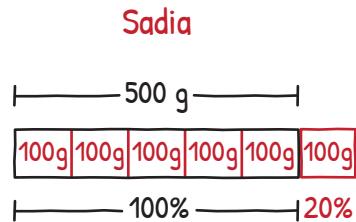
100%

Let's make connections between double number lines and percent problems.

100%

Warm-Up

1. A box of cereal used to weigh 500 grams. Now it's 20% larger. Sadia and Irene each worked to determine the new weight. What do you notice? What do you wonder?

Sadia

├──── 500 g ────┤

| 100g | 100g | 100g | 100g | 100g | 100g |

├──── 100% ────┤ 20%

Irene

Cereal (g) — 0 100 200 300 400 500 600 700

% of Old Box — 0 20 40 60 80 100 120 140

20% larger

Responses vary.

I notice:
- **In Sadia's representation, if you add up all the 100-gram parts, you get 600 grams total.**
- **In Irene's representation, you can see 120% of the old box matches up with 600 grams.**
- **The increase was a move to the right on the double number line.**

I wonder:
- **What would a decrease look like on the double number line?**
- **What are some other ways of showing a percent increase?**
- **Did the price increase when the box of cereal got bigger?**

Name: .... Date: .... Period: ....

Double Number Lines
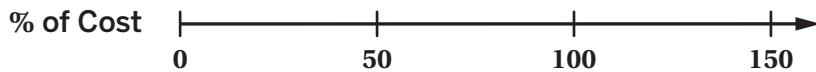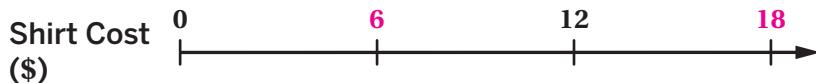
For each problem:

- Complete the double number line to show the percentages that correspond to the original amount and to the new amount.
- Answer the question.

2. A store sells all of its clothes for 50% more than it costs to make (also called a "markup").

If a shirt costs $12 to make, what is the price of the shirt?
$18

Shirt Cost ($)
0   **6**   12   **18**

% of Cost
0   50   100   150
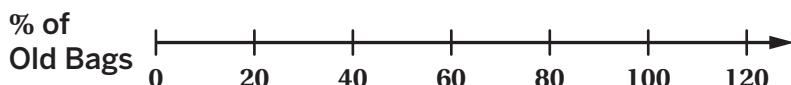
3. At the movie theater, the size of popcorn bags decreased by 20%.

If the original bags held 15 cups of popcorn, how much do the new bags hold?
12 cups

Popcorn (cups)
0   **3**   **6**   **9**   **12**   15   **18**

% of Old Bags
0   20   40   60   80   100   120
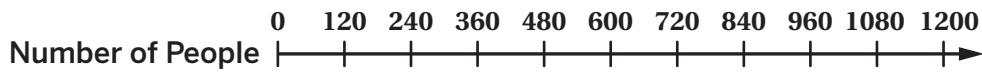
4. A school had 1,200 students last year and only 1,080 students this year.

What was the percent decrease in the number of students?
10% decrease

Number of People
0   120   240   360   480   600   720   840   960   1080   1200

% of Students Last Year
0   **10**   **20**   **30**   **40**   **50**   **60**   **70**   **80**   **90**   **100**

Name: _____ Date: _____ Period: _____
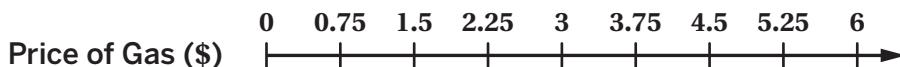
Double Number Lines (continued)
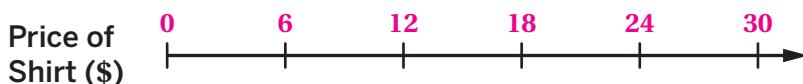
5. Last week, gas was $3.75 per gallon. This week, gas was $4.50 per gallon. What was the percent increase?

20% increase

| Price of Gas ($) | 0 | 0.75 | 1.5 | 2.25 | 3 | 3.75 | 4.5 | 5.25 | 6 |

| % of Last Week's Gas Price | 0 | 20 | 40 | 60 | 80 | 100 | 120 | 140 | 160 |
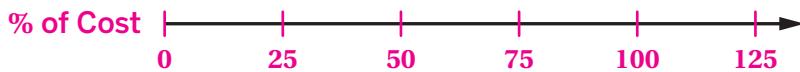
6. After a 20% discount, the price of a T-shirt is $24. What was the price before the discount?

$30

| Price of Shirt ($) | 0 | 6 | 12 | 18 | 24 | 30 |

| % of Price | 0 | 20 | 40 | 60 | 80 | 100 |

7. A used car dealer has a 25% markup on its cars. If the dealer sells the car for $6,600 after the markup, what was the cost before the markup?

$5,280

| Cost of Car ($) | 0 | 1320 | 2640 | 3960 | 5280 | 6600 |

| % of Cost | 0 | 25 | 50 | 75 | 100 | 125 |

Name: _____ Date: _____ Period: _____
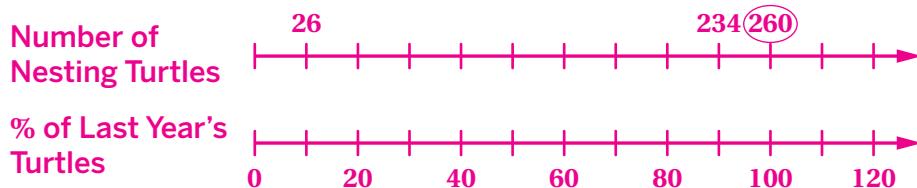
Green Sea Turtles

Some beaches are protected sanctuaries so that green sea turtles can come to shore to lay eggs without being disturbed.

This year, there are 234 nesting turtles at a sanctuary. This is 10% less than the number of nesting turtles at the same sanctuary last year.

8. Create at least two representations that show how many nesting turtles were at the sanctuary last year. *Responses vary.*

Double Number Line

Number of Nesting Turtles

26   234 260

% of Last Year's Turtles

0   20   40   60   80   100   120

Table

Old	New
100%	90%
260	234

×0.9

÷0.9

Equation

Let x represent the number of nesting turtles at the sanctuary.

$$0.9 \cdot x = 234$$
$$x = \frac{234}{0.9}$$
$$x = 260$$

9. How many nesting turtles were at the sanctuary last year?

260 nesting turtles

Explore More
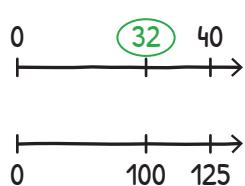
10. If the population decreases another 10%, how many nesting turtles will be at the sanctuary next year? Explain your thinking.

About 210 nesting turtles. *Explanations vary.* I multiplied 234, the amount for this year, by 0.9 to find the amount after another 10% decrease, which gives a decimal that is about 210.

Synthesis

11. A number increases by 25%. The new number is 40. Here are three strategies for finding the original number.

Double Number Line	Table	Equation

Choose one of the strategies and explain it in your own words.
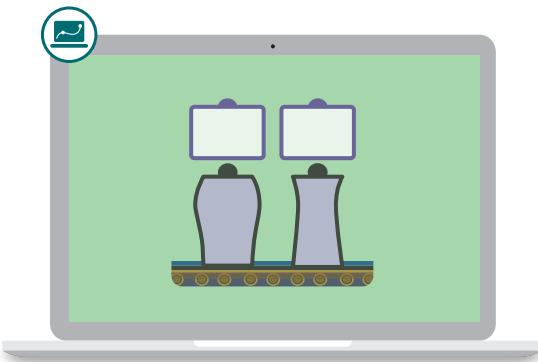
Responses vary.

- **For the double number line, the new value of 40 lines up with 125%, so 32 lines up with 100%.**
- **For the table, 100% gets multiplied by 1.25 to get to 125%, and 32 also gets multiplied by 1.25 to get 40.**
- **For the equation, we divide both sides by 1.25 to isolate b, which is 32.**

Things to Remember:

Unit 4
Lesson
5

Name: _____ Date: _____ Period: _____

Percent Machines

Let's explore problems with multiple percent changes.

Warm-Up

1 This is a percent machine.
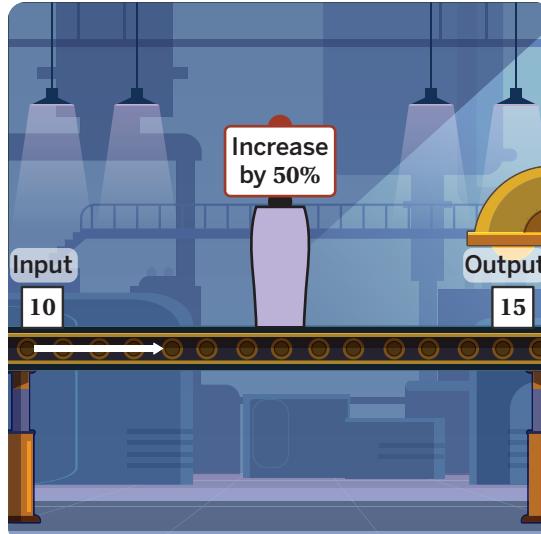
It takes an input and increases it by 50% to make an output.

What number is put into the machine?
10

What number comes out of the machine?
15

Increase
by 50%

Input

10

Output

15

2 A machine was built to follow these rules:
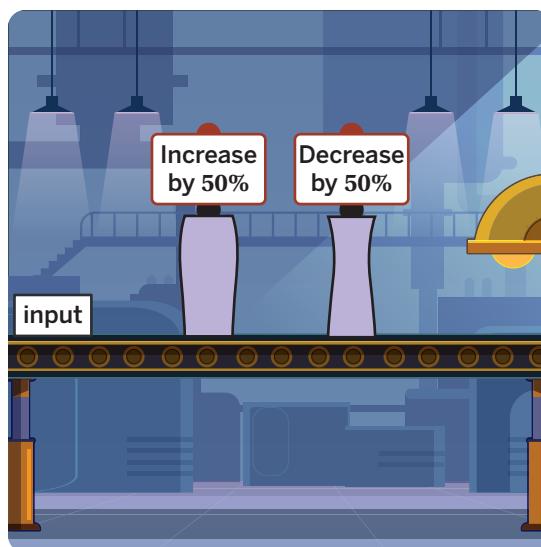
- Increase by 50%
- Decrease by 50%

Do you think the machine's output will be less than, greater than, or equal to a positive input? Circle one.

Less than   Greater than   Equal to

Explain your thinking.
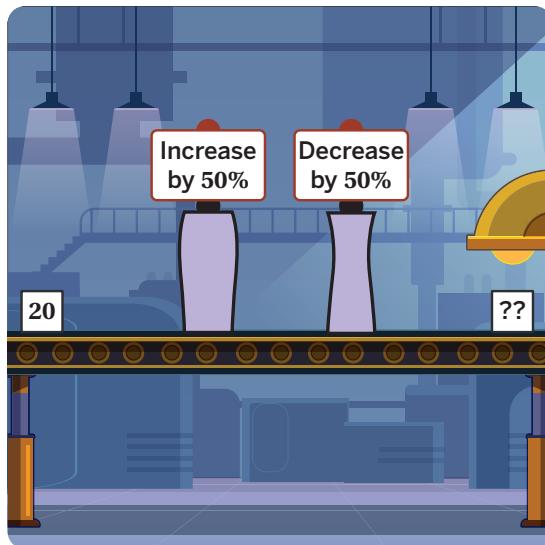
Responses and explanations vary. **The output will be less than a positive input because we are decreasing a larger value, so the final value will be smaller.**

Increase
by 50%

Decrease
by 50%

input

Name: _____ Date: _____ Period: _____

Putting Inputs In

3 If 20 is put into this machine, what will come out?
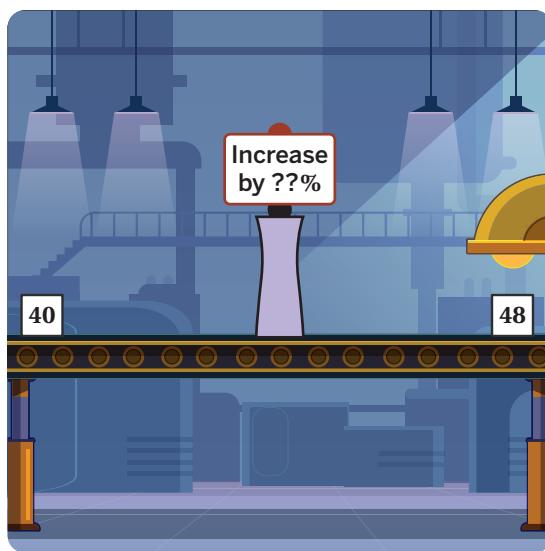
15

4 40 went into this machine and 48 came out.

What percent increase does this machine use?

20%

5 Take a look at Rudra's and Ivory's strategies for determining the percent increase from the previous problem.

💬 **Discuss:** How are Rudra's and Ivory's strategies alike? How are they different?
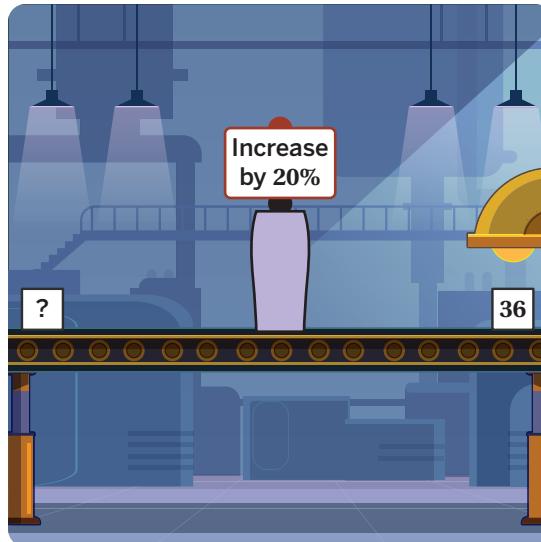
Responses vary. **They both found an increase of 20%, but Rudra used double number lines and Ivory used a table.**

Name: _____ Date: _____ Period: _____

Getting Outputs Out

6 A number went into the machine and 36 came out.

What number went in?

30

7 Kai used a table to solve the previous challenge but then made an error.

a Describe something Kai did well.

Responses vary.
- **He figured out the output was 120% of the original.**
- **He put the 36 in the output column of the table.**
- **He figured out that the constant of proportionality is 1.20.**
- **He got a value less than 36.**

b Describe what you would change.
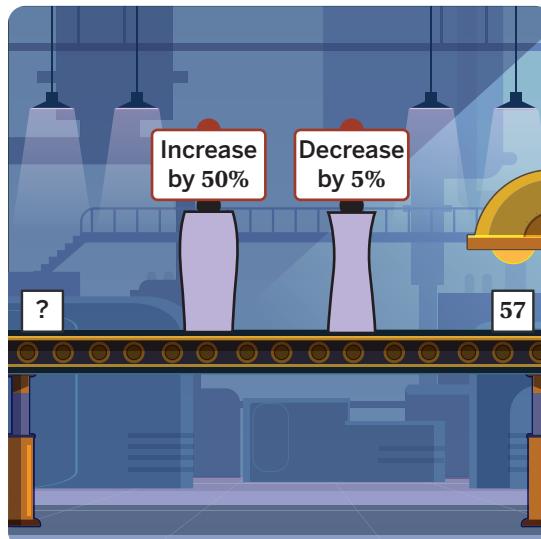
Responses vary.
- **Keep the 1.2 (the constant of proportionality) the same in both rows.**
- **Divide by 1.2 to work backward instead of taking away 20% to get 0.80.**

8 A number went into this machine and 57 came out.

What number went in?

40

Name: _____ Date: _____ Period: _____

My Percent Machine

9 You will use the Activity 3 Sheet to create your own percent machine.

a Make It!

- On the activity sheet, create your own percent machine and choose an output.

- You and your classmates will determine the input that produces the output you chose for your machine.

- In this table, fill in the percent change(s) and the output for your machine. Then determine the input that will give you the output.

Responses vary.

My Machine

Input	Step 1	Step 2 (optional)	Output

b Swap It!

- Share your machine with a partner. Look at each other's machines, but keep your own input a secret.

- What is your partner's input?

- As time allows, find more partners.

Responses vary.

Partner 1's Machine

Input	Step 1	Step 2 (optional)	Output

Partner 2's Machine

Input	Step 1	Step 2 (optional)	Output

Partner 3's Machine
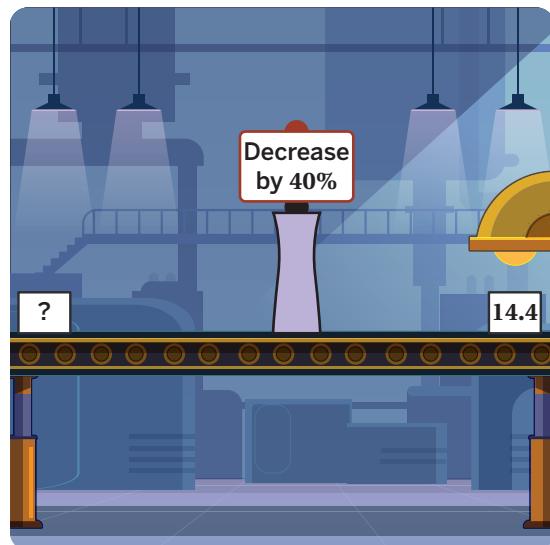
Input	Step 1	Step 2 (optional)	Output

🔟 Synthesis

Describe a strategy for determining the input when you know the percent change and the output.

Use this example if it helps you explain your thinking.

Responses vary. **To figure out the original value, you need to know what the going-forward relationship is. You can write this in a table, as an equation, or in a double number line. In this example, the equation is $x \cdot 0.60 = 14.4$. Then divide by 0.6 to determine the input.**

Decrease by 40%

?

14.4

Things to Remember:

Name: _____ Date: _____ Period: _____
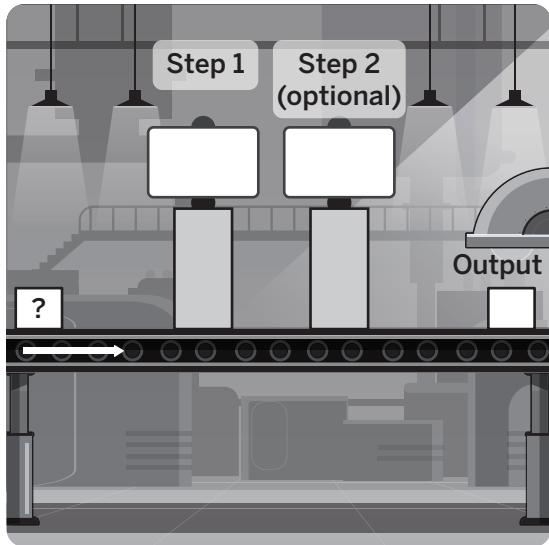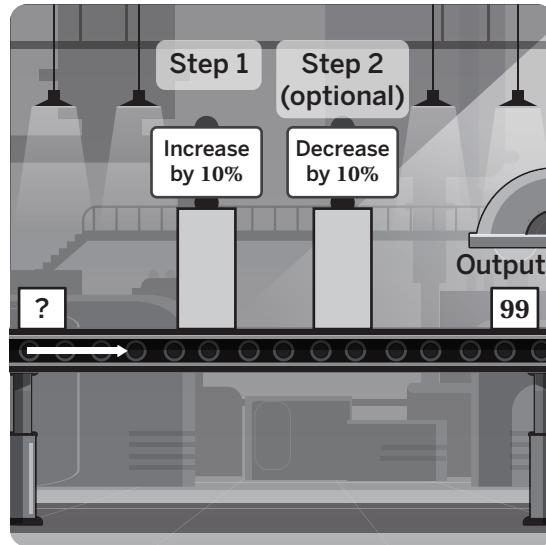
My Percent Machine

It's time to create your own percent machine!

- Choose whether your machine will have one or two steps.
- Write in the percent increase or decrease for each step. See the example.
- Write an output.
- Fill in the table for your machine in your Student Edition.

My Machine

Example

Name: _____ Date: _____ Period: _____

Tax and Tip

Let's solve multistep percent problems involving sales tax and tip.

Warm-Up

1-2 Customers are reporting problems with the receipts at the Des-Cafe.

The Des-Cafe

Soup	Salad
Pizza	Sandwich
Donut	Tea

Soup:	$5.00
Tax:	+ $0.35
Total:	$5.35

Salad:	$8.00
Tax:	+ $0.56
Total:	$8.56

Pizza:	$18.00
Tax:	$ _____
Total:	$ _____

Sandwich:	$13.00
Tax:	$ _____
Total:	$ _____

Here's the information from the two receipts that printed out correctly.

What percent is the sales tax? Explain your thinking.

Item	Price	Tax	Total
Soup	5.00	0.35	5.35
Salad	8.00	0.56	8.56

7%. _Explanations vary._
- **I divided the total cost by the price to figure out the constant of proportionality, which was 1.07. The 1 represents 100%, and the 0.07 represents a 7% increase.**
- **I divided the tax ($0.56) by the original price ($8.00), which was 0.07. This as a percent is 7%.**

Name: .... Date: .... Period: ....

Tax

3 The sales tax rate is 7%.

Determine the remaining values for these receipts.

Item	Price	Tax	Total
Soup	5.00	0.35	5.35
Salad	8.00	0.56	8.56
Pizza	18.00	**1.26**	**19.26**
Sandwich	13.00	**0.91**	**13.91**

Soup: $5.00
Tax: + $0.35
Total: $5.35

Salad: $8.00
Tax: + $0.56
Total: $8.56

Pizza: $18.00
Tax: $
Total: $

Sandwich: $13.00
Tax: $
Total: $

4 The Des-Cafe got a new cash register!

Write expressions that the cash register can use to determine the tax and total for any item.

Price	Tax	Total
$5.00	$0.35	$5.35
$8.00	$0.56	$8.56
c	**$0.07c$**	**$1.07c$**

Soup: $5.00
7% Tax: + $0.35
Total: $5.35

Soup: $8.00
7% Tax: + $0.56
Total: $8.56

Name: _____ Date: _____ Period: _____

Tip

5 Customers at restaurants usually leave a tip for the server. Sometimes restaurants add a tip automatically.

Write expressions that the cash register can use to determine an 18% tip and the total after tip.

Total Before Tip	18% Tip	Total After Tip
$5.35	$0.96	$6.31
$8.56	$1.54	$10.10
t	**0.18t**	**1.18t**

Total Before Tip:	$5.35
18% Tip:	+ $0.96
Total After Tip:	$6.31

Total Before Tip:	$8.56
18% Tip:	+ $1.54
Total After Tip:	$10.10

6 Kiran and Ava had a bill for $100. The tax rate is 5% and they want to tip 20%.
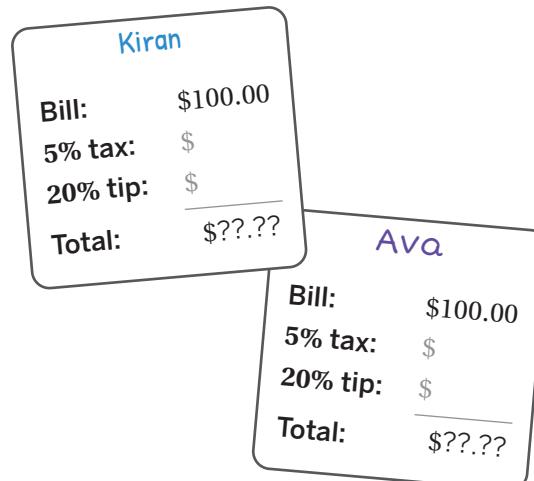
- Kiran calculates the tip *after* tax is added to the bill.
- Ava calculates the tip *before* tax is added to the bill.

Whose strategy would result in a greater tip? Circle one.

(Kiran's)   Ava's   They're the same

Explain your thinking.

Explanations vary. **Ava is calculating 20% of $100. Kiran is calculating 20% of more than $100.**

Kiran

Bill:	$100.00
5% tax:	$
20% tip:	$
Total:	$??.??

Ava

Bill:	$100.00
5% tax:	$
20% tip:	$
Total:	$??.??

Name: .... Date: .... Period: ....

Tip (continued)

7 A store offers a 20% off coupon.

The tax rate is 7.5% of the subtotal.

If an item is listed at $15, what is the total after the coupon and tax have been applied?
$12.90

Explain your thinking.

Explanations vary. **To determine the price after the coupon, I multiplied 15 by 0.8 and got 12. Then, to determine the price after tax, I multiplied $12 by 1.075 to get 12.9.**

Price:	$15.00
20% Off Coupon:	$ _____
Subtotal:	$
7.5% Tax:	$
Total:	$??.??

Name: _____ Date: _____ Period: _____

Repeated Challenges

8 Find the total the customer pays for each receipt. Answer as many as you have time for.

a This meal's bill is $28.00.

A 7% sales tax is applied, followed by an automatic tip of 18% (on the after-tax amount).

$35.35

Bill:	$28.00
7% Tax:	$
Total Before Tip:	$
18% Tip:	$
Total With Tip:	$?.??

b A store sells an item for $18.00.

A 45% off coupon is applied, followed by a 7% sales tax on the subtotal.

$10.59

Price:	$18.00
45% Off Coupon:	$
Subtotal:	$
7% Tax:	$
Total:	$?.??

c A store sells an item for $31.00.

A 15% off coupon is applied, followed by a 7% sales tax on the subtotal.

$28.19

Price:	$31.00
15% Off Coupon:	$
Subtotal:	$
7% Tax:	$
Total:	$?.??

d This meal's bill is $23.00.

A 7% sales tax is applied, followed by an automatic tip of 18% (on the after-tax amount).

$29.04

Bill:	$23.00
7% Tax:	$
Total Before Tip:	$
18% Tip:	$
Total With Tip:	$?.??

Explore More

9 A store sells an item for $75.00.

Fatima uses a 20% off coupon and then tax is applied to the subtotal.

The total after coupon and tax is $75.00.

What is the tax rate? Explain your thinking.

25%. *Explanations vary.* I wrote the equation $75 \cdot 0.8 \cdot x = 75$ and then solved it by dividing. I found that x must be 1.25, which means the tax rate is 25%.

Price:	$75.00
20% Off Coupon:	$
Subtotal:	$
?% Tax:	$
Total:	$75.00

10 Synthesis

Describe a strategy for calculating the total of an item after different discounts and tax rates.

Use the receipts if they help with your thinking.

Responses vary. **First, figure out the subtotal with the discount, which means multiplying by a number less than 1. Then, use the answer that you got to figure out the total with tax. To do so, you will multiply by a number greater than 1. Be careful about tax percentages like 6.25%, which can be confusing.**

Price:	$12.00
20% Discount:	− $2.40
Subtotal:	$9.60
5% Tax:	+ $0.48
Total:	$10.08

Price:	$12.00
15% Discount:	− $1.80
Subtotal:	$10.20
15% Tax:	+ $1.53
Total:	$11.73

Price:	$12.00
10% Discount:	− $1.20
Subtotal:	$10.80
8% Tax:	+ $0.86
Total:	$11.66

Price:	$12.00
5% Discount:	− $0.60
Subtotal:	$11.40
6.25% Tax:	+ $0.71
Total:	$12.11

Things to Remember:

Unit 4
Lesson
8

Name: .... Date: .... Period: ....

Plate Rate

Let's use percent change to analyze an issue in society.

Warm-Up

The minimum wage is the lowest amount of money a worker can earn per hour.

In 2023, the federal Minimum wage was $7.25 per hour.

There is a different federal minimum wage for workers who receive tips, such as restaurant servers. In 2023, this was $2.13 per hour.

1. Which way would you prefer to be paid? Explain your thinking.

Responses and explanations vary.
- **I'd prefer the standard minimum wage because I want my income to be predictable and not have it depend on how many customers I have and how generous they are.**
- **I'd prefer to receive a lower wage with the possibility for tips. The wage is only about $5 lower per hour, and I think I could get more than that in tips.**
- **It depends on what sort of job I have. If I'm working at a restaurant that has a lot of customers and where tipping is expected, that would be my preference. Otherwise, I'd take the normal minimum wage.**

Name: .... Date: .... Period: ....

Waiting Tables

Here is some information about four servers who work at different restaurants: Laila, Tiana, Peter, and Julian. Choose *one* server to complete the activity.

☐ **Laila** is 35 years old. She is married and has two children. She has worked at the same restaurant for 7 years. She works 40 hours per week and makes $2.13 per hour. In a typical week, she serves 75 tables. The average bill at the restaurant is $41 per table, and she typically receives an 18% tip.

☐ **Tiana** is 25 years old. She lives with a roommate and a dog. She has worked at a fancy restaurant for 6 months. She works 40 hours per week and makes $2.13 per hour. She usually serves 45 tables per week. The average bill at the restaurant is $130 per table, and she typically receives a 20% tip.

☐ **Julian** is 29 years old. He is a single father with a 3-year-old son. He just finished his third year as a server. He works 40 hours per week and makes $2.13 per hour. In a typical week, he serves 95 tables. The average bill at the restaurant is $22 per table, and he typically receives a 15% tip.

☐ **Peter** is 19 years old. He lives at home with his parents and goes to college part-time. He recently started as a server, working 40 hours per week. Where Peter lives, the minimum wage for tipped workers is $7.25 per hour. In a typical week, he serves 90 tables. The average bill at the restaurant is $21 per table, and he typically receives a 15% tip.

2. How much money does your server make in a typical week? Show or explain your thinking.

Laila: $638.70 per week. *Work varies.* (40 • 2.13) + (75 • 41 • 0.18) = $638.70

Tiana: $1,255.20 per week. *Work varies.* (40 • 2.13) + (45 • 130 • 0.20) = $1255.20

Julian: $398.70 per week. *Work varies.* (40 • 2.13) + (95 • 22 • 0.15) = $398.70

Peter: $573.50 per week. *Work varies.* (40 • 7.25) + (90 • 21 • 0.15) = $573.50

3. Let's say that customers at your server's restaurant raise their tips by 5 percentage points. (So for example, 18% becomes 23%.)

How much would your server make now in a typical week? And by what *percent* would their pay increase? Show your thinking. ***Work varies.***

Laila: (40 • 2.13) + (75 • 41 • 0.23) = $792.45. This is about a 24% increase from $638.70.

Tiana: (40 • 2.13) + (45 • 130 • 0.25) = $1547.70. This is about a 23% increase from $1,255.20.

Julian: (40 • 2.13) + (95 • 22 • 0.2) = $503.20. This is about a 26% increase from $398.70.

Peter: (40 • 7.25) + (90 • 21 • 0.2) = $668. This is about a 16% increase from $573.50.

Name: _____ Date: _____ Period: _____

Waiting Tables (continued)

4. Now, let's say that customers at the restaurant *lower* their tips by 5 percentage points (for example, 18% becomes 13%).

How much would your server make now in a typical week? And by what percent would their pay decrease (compared to Problem 2)? Show your thinking. ***Work varies.***

Laila: $(40 \cdot 2.13) + (75 \cdot 41 \cdot 0.13) = \$484.95$. This is about a 24% decrease from $638.70.

Tiana: $(40 \cdot 2.13) + (45 \cdot 130 \cdot 0.15) = \$962.70$. This is about a 23% decrease from $1,255.20.

Julian: $(40 \cdot 2.13) + (95 \cdot 22 \cdot 0.1) = \$294.20$. This is about a 26% decrease from $398.70.

Peter: $(40 \cdot 7.25) + (90 \cdot 21 \cdot 0.1) = \$479$. This is about a 16% decrease from $573.50.

Gabriel gets paid $11.75 per hour to work in the kitchen. Unlike servers, restaurant workers in the kitchen do not usually receive tips.

5. How much money does Gabriel earn in a 40-hour week?

$470

6. Compare Gabriel's pay to the server you chose. Consider the server's pay in a typical week as well as when tips are high or low. Use percentages in your comparison.

***Responses vary.* Gabriel's pay is consistent week to week, whereas Laila's varies. In a typical week, Laila makes $638.70, which is about 36% more than Gabriel. If tips are higher, Laila might make $792.45, which is 69% higher than Gabriel. If tips are low, Laila might make $484.95, which is still 3% higher than Gabriel.**

7. Some restaurants have experimented with different ways of paying workers. One restaurant pays all workers $16 per hour, but doesn't allow tipping.

How do you expect people might feel about this policy? Consider how restaurant workers, restaurant owners, and customers might feel.

***Responses vary.* Under this policy, each worker would earn $640 per week. Laila, Julian, Peter, and Gabriel might be happy because it would be a pay increase. Tiana might be unhappy because it would be a pay decrease (when considering the usual tip amounts). But Tiana might be happy knowing that people like Julian will earn enough money to live on. Additionally, many workers might be happier knowing how much money to expect each week. I wonder if restaurant owners would like this policy, because where does all the new money going to workers come from? If it means raising prices, customers might not be happy.**

Name: _____ Date: _____ Period: _____

A New Way to Pay

Think about the three ways to pay servers that we've discussed so far:

- Servers get paid $2.13 per hour, plus tips.
- Servers get paid $7.25 per hour, plus tips.
- Servers get paid $16 per hour, with no tips.

8. Invent and describe a new way to determine a server's pay that you think is better. Explain your thinking.

Responses vary. **Servers should make enough money to live without needing to rely on customers to leave optional tips. In my system, all servers make at least $13 per hour, and then certain servers can make a higher wage in a few ways, such as:**

- **Making an extra $2 per hour for every child or family member they support.**
- **Making an extra $1 per hour for every year they've worked at the restaurant (because they'll be more experienced and valuable to the restaurant).**

There would be no tipping, but menu prices would go up to help pay these higher wages.

9. Calculate what each of the four servers from Activity 1 would earn under your system. Show or explain your thinking. *Responses vary.*

Laila: **She has 2 children and 7 years experience, so she would earn $24 per hour, or $960 a week.**

Tiana: **She would earn the minimum $13 per hour, or $520 a week.**

Julian: **He has 1 child and 3 years experience, so he would earn $18 per hour, or $720 a week.**

Peter: **He would earn the same as Tiana, $13 per hour, or $520 per week.**

Explore More

10. Some restaurants have eliminated tipping and raised the price of menu items by 20%. They say this allows worker pay to be fairer. Do you think restaurants should do this? Explain your thinking.

Responses and explanations vary.
- **No. As a customer, I'd like to be able to choose how much to tip.**
- **Yes. Tipping can lead to differences in pay that are unpredictable and unfair. Everyone is better off without it.**

Synthesis

11. Describe how you can determine how much a restaurant server makes in a week. Use the example if it helps with your thinking.

Responses vary. **To determine how much a server makes, first multiply the number of hours they work by their hourly rate. Then add that to the amount they make from tips. You can calculate that amount by multiplying the average number of tables they serve, the average bill at those tables, and the percentage that people usually tip.**
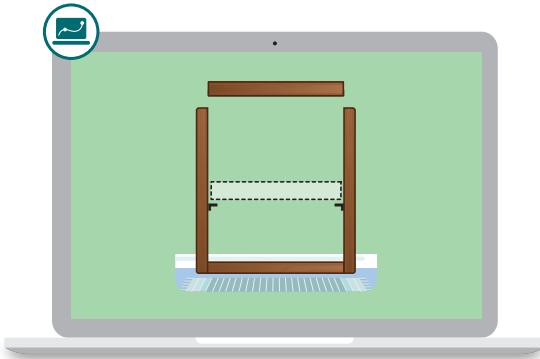
> **Tiana** works 40 hours per week and makes $2.13 per hour. She usually serves 45 tables per week. The average bill at the restaurant is $130 per table, and she typically receives a 20% tip.

Things to Remember:

Name: .... Date: .... Period: ....

Bookcase Builder

Let's explore situations where errors occur and represent those errors using percentages.

Warm-Up

1 Dakota and Ebony are buying deli meat.

Dakota orders 2 ounces of ham, but ends up with 2.3 ounces.
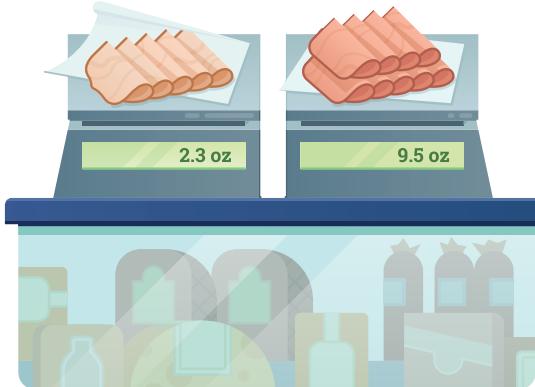
Ebony orders 10 ounces of turkey, but ends up with 9.5 ounces.

Whose order had a bigger error? Circle one. *Responses vary.*

2.3 oz   9.5 oz

Dakota's order   Ebony's order

Explain your thinking.

Explanations vary.
- **Dakota's order has a bigger error. The error on her order is 15% compared to 5% on Ebony's order.**
- **Ebony's order has a bigger error. She got 0.5 ounces less than she expected.**
- **Dakota's order has a bigger error. I multiplied Dakota's order by 5 so that I could better compare them.**
- **Ebony's order has a bigger error. Getting less than you ordered is always worse than getting more than you ordered.**

Name: .... Date: .... Period: ....
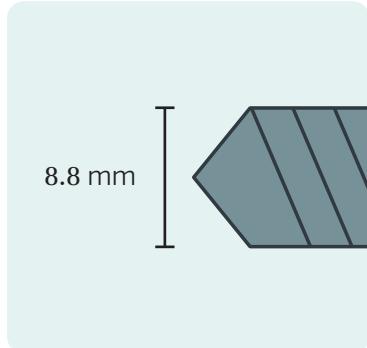
Bookcase Blunders

2 This bookcase was designed for 10-millimeter screws, but screws that are a little bigger or smaller are acceptable.
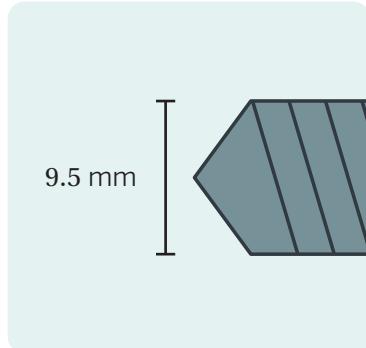
Let's watch the screen to find out which screws work for the bookcase.

☐ **A.**

8.8 mm

☑ **B.**

9.5 mm

☑ **C.**

10.4 mm

☐ **D.**

11.25 mm
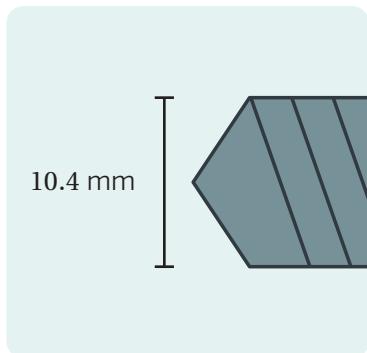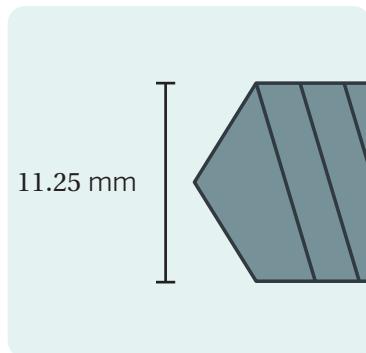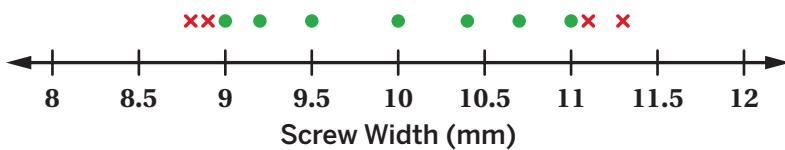
3 The screw widths that worked are represented with green dots on this number line. The widths that didn't work are represented with red X's.

Screw Width (mm)
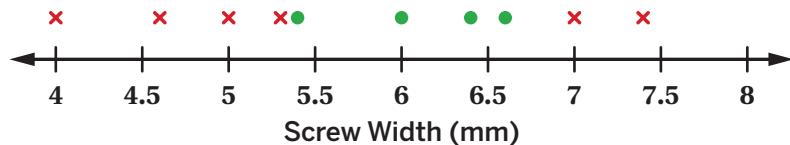
💬 **Discuss:** What are the smallest and largest screws that will work?

The smallest screw that works is 9 mm and the largest screw that works is 11 mm.

Name: .... Date: .... Period: ....

Bookcase Blunders (continued)

4 Imagine a new bookcase designed for 6-millimeter screws.

Here is a number line that represents the widths of screws that work and don't work for the new bookcase.

Screw Width (mm)

What are the smallest and largest screw widths that work?
All screws between 5.4 and 6.6 millimeters work.

5 Here are the smallest and largest acceptable screw widths for the two previous bookcases.

How are the acceptable screw widths different in each row? How are they alike? *Responses vary.*

Desired Width (mm)	Smallest Acceptable Width (mm)	Largest Acceptable Width (mm)
10	9	11
6	5.4	6.6

Different:

- **The range of acceptable screws is different for each situation. When 10 mm is desired, it can be 1 mm bigger or smaller. When 6 mm is desired, it can only be 0.6 mm bigger or smaller.**

Alike:

- **In both situations, the screws are allowed to be bigger or smaller by a specific amount.**
- **In both situations, the acceptable widths are 10% above and below the desired width.**

6 A factory tries to make a 6 mm screw. It ends up being 5.7 mm instead.

One way to describe the error is 0.3 mm.

Another way is to describe it as a **percent error**.

This screw has a percent error of $\frac{0.3}{6} = 5\%$.

Complete the table to decide if each screw will work (its percent error must be 10% or less).

Desired Width (mm)	Screw Width (mm)	Percent Error	Will It Work?
6	5.7	5%	Yes
10	10.9	**9%**	**Yes**
6	7.2	**20%**	**No**
15	13.8	**8%**	**Yes**

Name: .... Date: .... Period: ....

Challenge Your Shelf

7 Nikolai is making a shelf for a new bookcase.
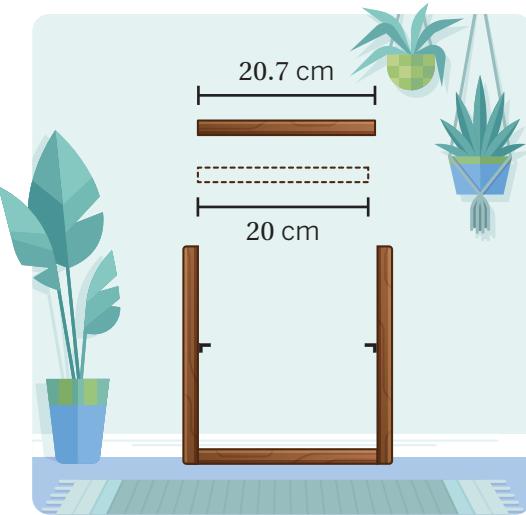
The shelf needs to be 20 centimeters long. It ends up being 20.7 centimeters long.

The acceptable percent error is 5% for a shelf to fit.

Will the shelf fit? Circle one.

(Yes)   No

Explain your thinking.

Explanations vary. **The error is 0.7 centimeters and the original is 20 centimeters. 0.7 is 3.5% of 20, which is less than 5%.**

8 Determine whether the shelf will fit given the acceptable error.

Answer as many as you have time for.

a
- Shelf length: 19.3 cm
- Desired shelf length: 20 cm
- Acceptable error: 5%

Will the shelf fit? Circle one.

(Yes)   No

b
- Shelf length: 12 cm
- Desired shelf length: 15 cm
- Acceptable error: 5%

Will the shelf fit? Circle one.

Yes   (No)

c
- Shelf length: 19.45 cm
- Desired shelf length: 18 cm
- Acceptable error: 5%

Will the shelf fit? Circle one.

Yes   (No)

d
- Shelf length: 21 cm
- Desired shelf length: 20.5 cm
- Acceptable error: 10%

Will the shelf fit? Circle one.

(Yes)   No

e
- Shelf length: 20.6 cm
- Desired shelf length: 17.6 cm
- Acceptable error: 3%

Will the shelf fit? Circle one.

Yes   (No)

f
- Shelf length: 17.2 cm
- Desired shelf length: 20.7 cm
- Acceptable error: 10%

Will the shelf fit? Circle one.

Yes   (No)

g
- Shelf length: 25.9 cm
- Desired shelf length: 23.7 cm
- Acceptable error: 7%

Will the shelf fit? Circle one.

Yes   (No)

h
- Shelf length: 21.8 cm
- Desired shelf length: 21.7 cm
- Acceptable error: 2%
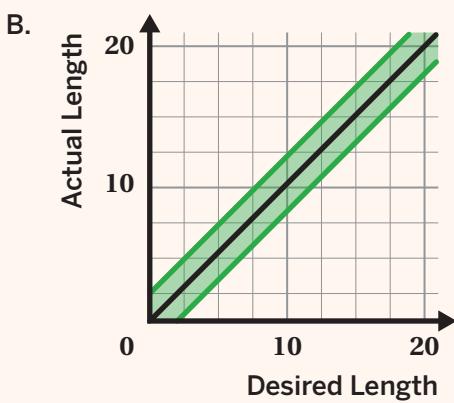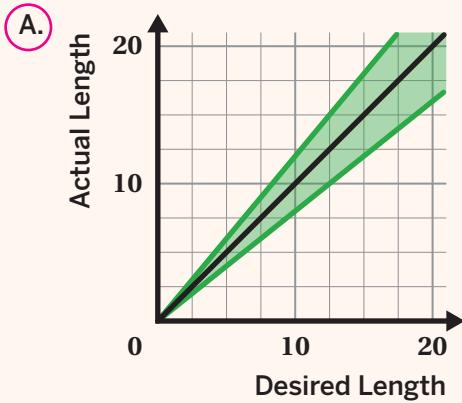
Will the shelf fit? Circle one.

(Yes)   No

Explore More

9 Nikolai is building shelves for bookcases of different sizes and is allowing for a 25% error. Circle the graph where the green area represents shelves that will fit.

A. (circled)
Actual Length vs Desired Length graph

B.
Actual Length vs Desired Length graph

Explain your thinking.

Explanations vary. **The acceptable range will get bigger as the desired length gets bigger. For example, when the desired length is 10 centimeters, the acceptable error will be 2.5 centimeters above or below 10 centimeters. But when the desired length is 20 centimeters, the acceptable error will be 5 centimeters.**

10 Synthesis

Explain how to calculate percent error.

Use the examples if they help with your thinking.

Responses vary. **You can calculate percent error by subtracting the amount you want and the amount you get and then dividing by the amount you want. Then you take that fraction and convert it to a percent. For Dakota, it would be $\frac{0.3}{2} = 15\%$, and for Ebony it would be $\frac{0.5}{10} = 5\%$.**

Dakota
- Orders 2 ounces of ham
- Ends up with 2.3 ounces
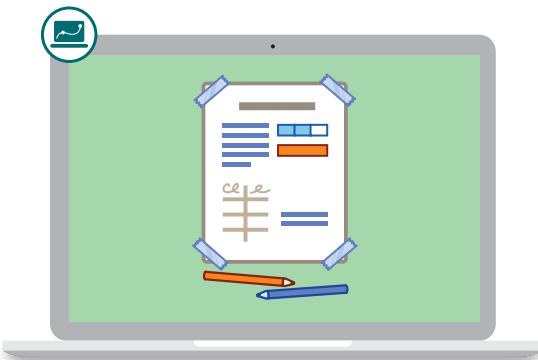
Ebony
- Orders 10 ounces of turkey
- Ends up with 9.5 ounces

Things to Remember:

Unit 4
Lesson

10

Population & Pollution

Let's use what we know about percentages to generate and answer questions about our world.

Warm-Up

1 Here is some information.

Write a question that you could figure out using this information and whose answer is not already given.

Responses vary.
- **What is the global population today?**
- **By what percent has the U.S. population grown from 1900 to today?**
- **By what percent has the world population grown from 1900 to today?**
- **The U.S. made up what percent of the world population in 1900?**

> There are about 336 million people living in the United States.

> The U.S. makes up about 4.2% of the global population.

> In 1900, the population of the U.S. was about 76 million people, and the population of the world was 1.6 billion people.

Name: _____ Date: _____ Period: _____

Pollution Percentages

2 Population increase is not the only change the planet has seen. Here is some information about two other topics.

Plastics

Plastic has countless uses, from household items to food packaging to medical instruments. But plastic has always been tricky to dispose of.

After use, plastic waste might be recycled or sent to a landfill; or it might be mismanaged, like being dumped into a body of water or burned.

Fast Fashion

Fast fashion is a business model where companies quickly produce new and low-priced clothes in large quantities.

To cut costs, the companies often use cheaper materials, such as polyester, and methods that have negative consequences for the environment and for workers.

💬 **Discuss:**

- What do you know about each topic?
- What might you want to know?

Responses vary. **I know that a lot of plastic doesn't get recycled and that making clothes leads to pollution, but I don't know how big the pollution problems are.**

3 **a** Select one topic. Read through some more information about that topic.

Plastics

- The amount of plastic waste generated in the U.S. grew by **400%** from 1980 to 2019.
- In 1980, the U.S. recycled **20,000** tons of plastic waste. In 2019, it was **3.1** million tons.
- In 2019, 73% of U.S. plastic waste went to landfills, **0.5%** ended up in the ocean, and 4% was recycled.
- In 2019, the U.S. generated 72.8 million tons of plastic waste. This was 55% more waste than in 2000.
- In 2000, the U.S. mismanaged 3.6 million tons of plastic waste. In 2019, the number was 33% lower.

Fast Fashion

- Globally, **80** billion new items of clothing were bought in 2015. This was **400%** more than was bought in 1995.
- In 2015, the fashion industry emitted 1.2 billion tons of carbon dioxide.
- Doubling the life of clothing from one year to two years reduces carbon dioxide emissions by 24%.
- In 2015, **60%** of new clothes contained polyester. Clothes makers used 21.3 million tons of polyester, which was a 157% increase from 2000.
- The average person bought **60%** more clothes in 2015 than in 2000.

Name: _____ Date: _____ Period: _____

Pollution Percentages (continued)

b Write two questions about the topic you selected that you could figure out using the given information and whose answer is not already given. ***Responses vary.***

Question 1:

- **How much plastic did we mismanage before?**
- **How much clothing did we buy before?**

Question 2:

- **How much plastic was in the ocean before?**
- **How much carbon dioxide would we emit if we buy less clothing?**

4 💬 **Discuss** the questions you wrote with two partners:

- Would the answer to this question be interesting or useful?
- Can you answer this question using only the information given?
- Is the answer to the question not obvious from the information given?

Revise your questions after each conversation.

Partner 1 Conversation Notes	Revision of Both Questions
Responses vary.	***Responses vary.*** Question 1: • **How much plastic did the U.S. mismanage before?** • **How much clothing did the average person buy 20 years ago?** Question 2: • **How much plastic was in the ocean in 2019?** • **How much carbon dioxide would we emit if we doubled the life of our clothing?**
Partner 2 Conversation Notes ***Responses vary.***	**Final Version of Both Questions** ***Responses vary.*** Question 1: • **How much plastic waste did the U.S. mismanage in 2019?** • **How many items of clothing were bought globally in 1995?** Question 2: • **How much plastic waste from the U.S. ended up in the ocean in 2019?** • **How much carbon dioxide would the fashion industry emit if people doubled the life of their clothing?**

Name: _____ Date: _____ Period: _____

Make a Poster

5 Find a small group who chose the same topic. Discuss the questions you wrote and choose two to answer on a poster.
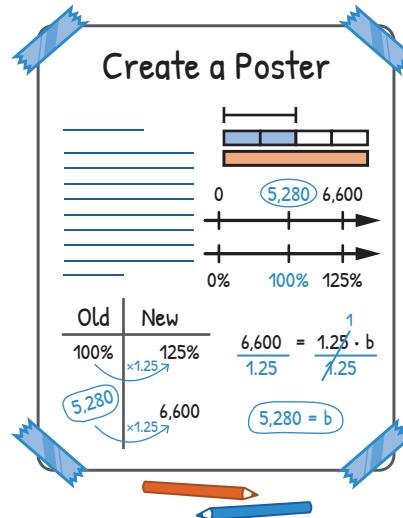
Create the poster. Here is what your poster should include:

(Check each item as you add it to your work.)

☐ A descriptive title

☐ The two questions you selected

☐ At least one equation or other representation (tape diagram, double number line, or table) that helps you answer your questions

☐ Your answers to each question (with units)

☐ An explanation of how you calculated each answer

☐ Two new questions that you have about this topic

You can use the space below for work or other calculations.
Posters vary.

Create a Poster

6 Look at your classmates' posters.

a What features of your classmates' posters helped you understand their thinking?

Responses vary. **When the posters contained visuals and diagrams, I was able to better make sense of the stats.**

b Describe something you would change about your poster now that you have seen other groups' work.

Responses vary. **I would get more precise with our language. For example, I used "we" in some places where maybe I should have specified "the fashion industry" or "the U.S."**

7 Synthesis

Discuss both questions. Then select one question and write your response.

☐ What did you learn about the topic you chose?

☐ What questions do you still have about your topic, pollution, or the population?
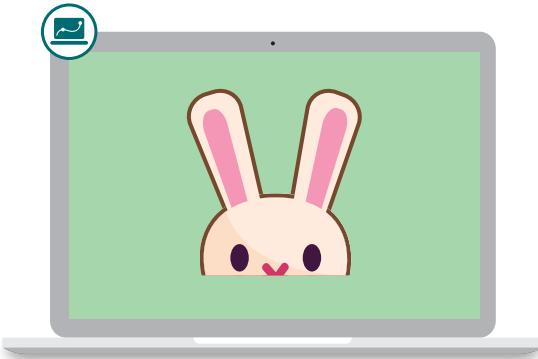
Responses vary.
- **I learned that plastic has had a rapid rise. We're producing way more plastic than in past decades, and even though management of the plastic waste seems to have improved, not very much plastic actually gets recycled.**
- **We've looked at plastic and fashion. I want to know what other categories of pollution there are, and which have the biggest impact on the planet. I want to know if global pollution is generally getting better or worse.**

Things to Remember:

Unit 4
Lesson
11

Sticker Sizes

Let's use constants of proportionality to determine unknown values in proportional relationships.

Warm-Up

1 StuckStickers is a company that makes stickers of all different sizes.

Which size will make a scaled copy of this logo?

4 in.

3 in.

A. 6 in. by 7 in.   **B.** 6 in. by 8 in.   **C.** 5 in. by 5 in.

7 in.

6 in.

8 in.

6 in.

5 in.

5 in.

Explain your thinking.
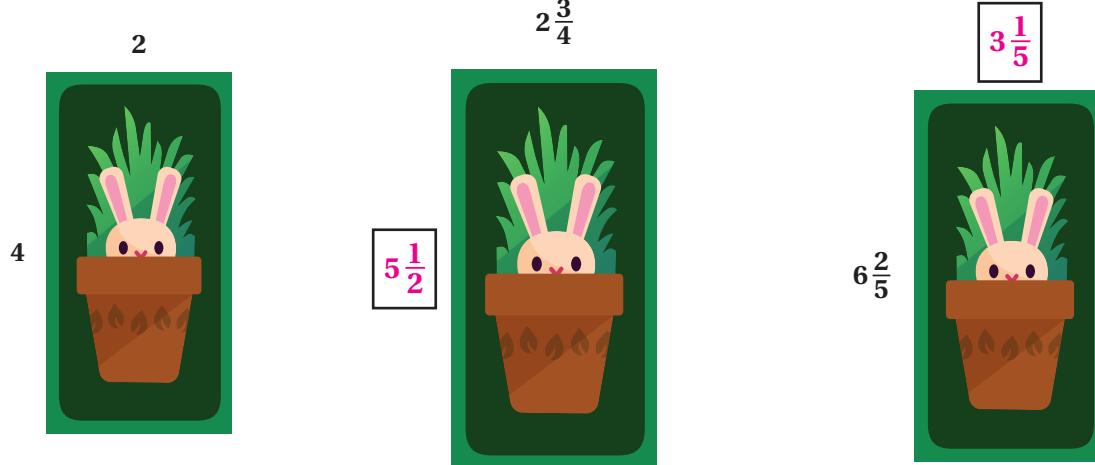
Explanations vary.
- **This is the only sticker size where the image stays the same. In the others, the image is squished a little.**
- **This is the only sticker size where you can multiply the length and the width of the original logo by 2 and get the larger version.**

Name: .... Date: .... Period: ....

Scaling Stickers

2 Here's a logo that Aditi is making into stickers.

$1\frac{1}{3}$

a Enter the missing values so that each sticker is a scaled copy of the logo. (Measurements are in centimeters.)

$2\frac{2}{3}$

$2\frac{3}{4}$

2

$\boxed{3\frac{1}{5}}$

4

$\boxed{5\frac{1}{2}}$

$6\frac{2}{5}$

b Describe your strategy.

Explanations vary. **I noticed that the height is 2 times as long as the width. So to find the height, I multiplied the width by 2. To find the width, I multiplied the height by $\frac{1}{2}$.**

3 Hamza wants to use this design to create a bumper sticker.

If the bumper sticker has a height of $3\frac{1}{2}$ inches, what will its width be?

$8\frac{3}{4}$ inches

2 in.

5 in.

Name: _____   Date: _____   Period: _____

Scaling Stickers (continued)

4 Polina and Jamir both used tables to determine the width of Hamza's bumper sticker.

Polina

Height (in.)	Width (in.)
2	5
$3\frac{1}{2}$	

$\cdot 1\frac{3}{4}$ (height) $\cdot 1\frac{3}{4}$ (width)

Jamir

Height (in.)	Width (in.)
2	$\xrightarrow{\cdot 2\frac{1}{2}}$ 5
$3\frac{1}{2}$	$\xrightarrow{\cdot 2\frac{1}{2}}$

Discuss each student's strategy.

Responses vary.

- Polina knows that to enlarge the design, she will need to apply a scale factor. Changing the height from 2 to $3\frac{1}{2}$ requires a scale factor of $1\frac{3}{4}$, which has to also apply to the width.
- Jamir is looking at the constant of proportionality in the relationship between the height and width of the design. The width is $2\frac{1}{2}$ times the height. So to find the width of the bumper sticker, he has to multiply $3\frac{1}{2} \cdot 2\frac{1}{2}$.

5 Alex wants to use this logo to create a sticker.

If the sticker is $\frac{3}{5}$ inches wide, what does the height need to be?

Height (in.)	Width (in.)
3	$4\frac{1}{2}$
$\frac{2}{5}$	$\frac{3}{5}$

3 in.

$4\frac{1}{2}$ in.

Name: ....   Date: ....   Period: ....

Sticker Sheets

6 Alex is thinking about buying stickers by the sheet. Four sheets cost $14.

How much would $11\frac{1}{2}$ sheets cost?

Number of Sheets	Total Cost ($)
4	14
$11\frac{1}{2}$	**40.25**

Cost:
$14.00

Sheets:
4

7 Alex has $70 to spend on stickers.

How many sheets of stickers can Alex buy?

Number of Sheets	Total Cost ($)
4	14
20	70

Activity

2

Name: .... Date: .... Period: ....

Sticker Sheets (continued)

8 Select *all* the equations that represent the relationship between the total cost, t, and number of sheets of stickers, s.

☑ **A.** $t = \frac{7}{2}s$

☐ **B.** $s = \frac{7}{2}t$

☐ **C.** $t = \frac{2}{7}s$

☑ **D.** $s = \frac{2}{7}t$

Explore More

9 It takes about $1\frac{1}{4}$ seconds for light to travel from Earth to the Moon.

The Moon is about **238,000** miles from Earth.

The Sun is about **94,500,000** miles from Earth.

Determine approximately how long it would take for light to get from the Sun to Earth.
Responses between 496 and 500 seconds are considered correct.

10 Synthesis

Describe how to use a table of a proportional relationship to determine missing values.

Use the example if it helps with your thinking.

Responses vary. **I can figure out a constant of proportionality using the top row. In this table, $\frac{1}{2}$ is a constant of proportionality. Then I can use that to either multiply or divide to figure out the missing number in the second row.**

Height (in.)	Width (in.)
$2\frac{2}{3}$	$1\frac{1}{3}$
	$5\frac{1}{2}$

Things to Remember:

Unit 4

Lesson

12

Name: _____ Date: _____ Period: _____

Peach Cobbler

Let's use a constant of proportionality to compare relationships that involve fractional quantities.

Warm-Up

Determine each quotient mentally. Be prepared to share your strategy.

1. $1 \div \frac{1}{3} = \mathbf{3}$

2. $5 \div \frac{1}{3} = \mathbf{15}$

3. $\frac{1}{2} \div \frac{1}{3} = \mathbf{1\frac{1}{2}}$ **(or equivalent)**

4. $2\frac{1}{2} \div \frac{1}{3} = \mathbf{7\frac{1}{2}}$ **(or equivalent)**

Name: .... Date: .... Period: ....

Which Recipe?

Amara and Callen want to make a peach cobbler that isn't too sweet. They have three recipes and are deciding which one to make.

Recipe A	**Recipe B**	**Recipe C**
Number of servings: 9	**Number of servings: 12**	**Number of servings: $4\frac{1}{2}$**
• $2\frac{7}{10}$ lb of peaches	• 4 lb of peaches	• $1\frac{4}{5}$ lb of peaches
• $\frac{1}{2}$ cups of butter	• $\frac{3}{4}$ cups of butter	• $\frac{1}{4}$ cups of butter
• 1 cup of flour	• $\frac{3}{4}$ cups of flour	• $\frac{2}{3}$ cups of flour
• $1\frac{1}{8}$ cups of sugar	• $1\frac{1}{3}$ cups of sugar	• $\frac{3}{4}$ cups of sugar
• $\frac{1}{2}$ tsp of lemon juice	• $\frac{1}{2}$ tsp of lemon juice	• 1 tsp of lemon juice

Amara and Callen want to make a recipe that isn't too sweet.

5. They think Recipe C will be the least sweet because it has the least amount of sugar. Do you agree? Explain your thinking.

No. Explanations vary. Recipe A has $\frac{1}{8}$ cups of sugar per serving. Recipe B has $\frac{1}{9}$ cups of sugar per serving, and Recipe C has $\frac{1}{6}$ cups of sugar per serving. Recipe C has the least total amount of sugar, but it has the most sugar per serving. This means it will be the sweetest tasting recipe.

6. Which recipe should they make? Explain your thinking.

Recipe B. Explanations vary. Recipe B has the least amount of sugar per serving.

7. Is the relationship between number of servings and total amount of sugar a proportional relationship? Explain your thinking.

Yes. Explanations vary. The constant of proportionality between the amount of sugar and number of servings stays the same. For example, if the number of servings is doubled, the amount of sugar will need to be doubled.

Name: _____ Date: _____ Period: _____

Adjusting a Recipe

Jamar wants to make peach cobbler using Recipe B.

8. Determine how much of each ingredient Jamar needs for one serving. Show your thinking.

Recipe B

Number of servings: 12

- 4 lb of peaches
- $\frac{3}{4}$ cups of butter
- $\frac{3}{4}$ cups of flour
- $1\frac{1}{3}$ cups of sugar
- $\frac{1}{2}$ tsp of lemon juice

Single Serving of Recipe B

Number of servings: 1

- _____ $\frac{1}{3}$ _____ lb of peaches
- _____ $\frac{1}{16}$ _____ cups of butter
- _____ $\frac{1}{16}$ _____ cups of flour
- _____ $\frac{1}{9}$ _____ cups of sugar
- _____ $\frac{1}{24}$ _____ tsp of lemon juice

9. Jamar plans to make just enough for 3 adults and 3 children. The children will eat less than the adults.

a How many servings should he make? Explain your thinking.

Responses vary. **4 because each adult may want 1 serving and then the three children split one additional adult serving.**

b Use the number of servings you chose to adjust Recipe B. *Responses vary.*

Recipe B for Jamar's Family

Number of servings: _____ **4** _____

- _____ **$1\frac{1}{3}$** _____ lb of peaches
- _____ $\frac{1}{4}$ _____ cups of butter
- _____ $\frac{1}{4}$ _____ cups of flour
- _____ $\frac{4}{9}$ _____ cups of sugar
- _____ $\frac{1}{6}$ _____ tsp of lemon juice

10. Jamar has a measuring spoon that is $\frac{1}{8}$ teaspoons. How many spoonfuls of lemon juice does he need to make this recipe? Show or explain your thinking.

$1\frac{1}{3}$ spoonfuls. *Explanations vary.* **To make 4 servings, Jamar will need $\frac{1}{6}$ teaspoons of lemon juice, which is $1\frac{1}{3}$ of his measuring spoon because $\frac{1}{6} \div \frac{1}{8} = 1\frac{1}{3}$.**

Synthesis

11. What are some strategies you can use to show that Haru's soup will have more carrots per pint than Mohamed's soup?

Mohamed's Vegetable Soup Recipe

$\frac{1}{3}$ of a cup of carrots for every

$\frac{1}{5}$ of a pint of broth

Haru's Vegetable Soup Recipe

$\frac{7}{8}$ of a cup of carrots for every

$\frac{1}{3}$ of a pint of broth
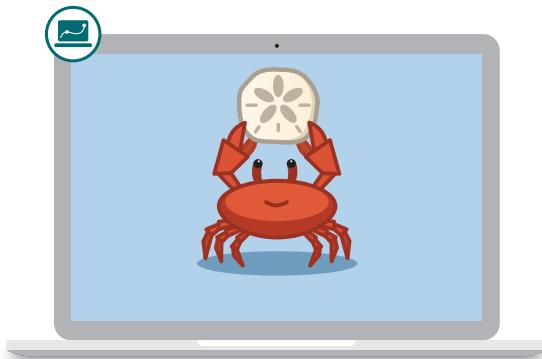
Responses vary. I could use a double number line or a table or calculate the unit rate to compare the amount of carrots each person's recipe has in one pint of broth.

Things to Remember:

Unit 7
Lesson
1

Name: _____ Date: _____ Period: _____

Can You Dig It?
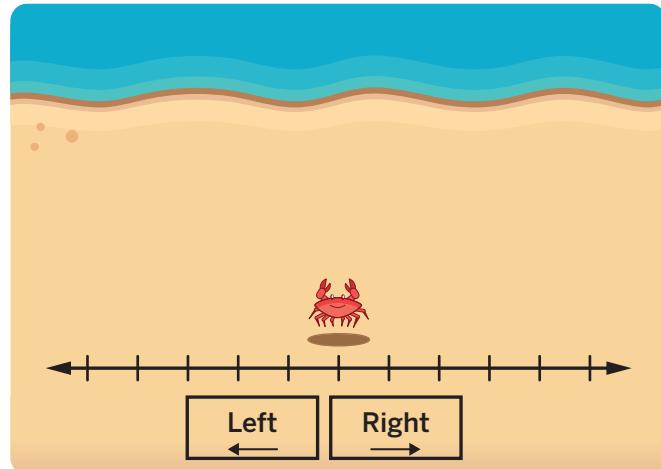
Let's find the hidden sand dollars.

Warm-Up

1 **a** Let's help the crab look for two sand dollars.

b Write a clue to help someone else find them.

Responses vary. **One sand dollar is 4 steps to the left of the hole and one is 3 steps to the right of the hole.**

Name: .... Date: .... Period: ....
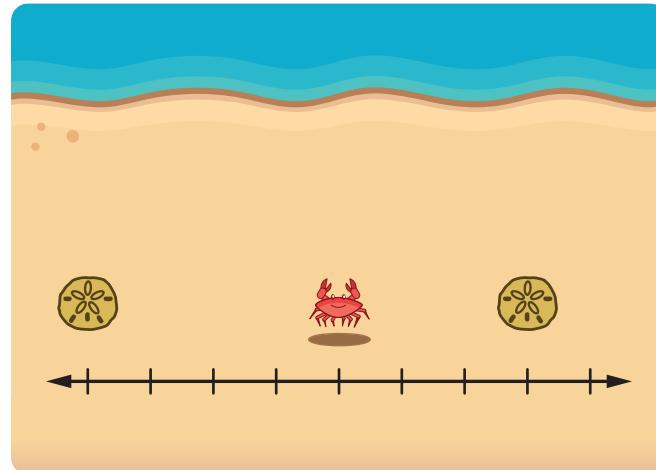
Finding Sand Dollars

2 Odalis and Kai each wrote correct clues.

Odalis: *One sand dollar is at positive* 3 *from the crab, and the other is at negative* 4*.*

Kai: *Go* 3 *steps to the right of the crab to find the first sand dollar, then* 7 *steps to the left to find the other one.*

💬 **Discuss:**

• How are their clues alike? How are they different?

Responses vary. **Both of their clues mention the number 3, which represents 3 steps to the right of the crab. But when describing the location of the second sand dollar, Odalis bases it on where the crab originally started, while Kai bases it on the location of the first sand dollar.**

• What do you think the "negative 4" in Odalis's clue means?
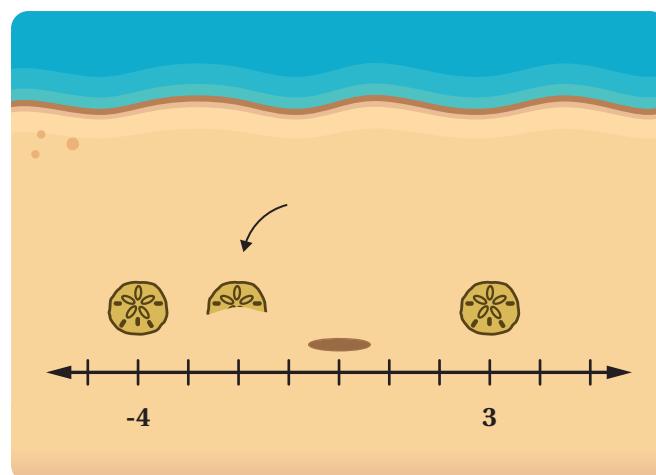
Responses vary. **The -4 represents 4 steps to the left of the crab.**

3 Here is a new sand dollar.

Write a clue that describes its location.

Responses vary.
• **The new sand dollar is at 2 on the negative side.**
• **The sand dollar is 2 units to the left of the hole.**
• **The sand dollar is at -2.**

Name: _____   Date: _____   Period: _____
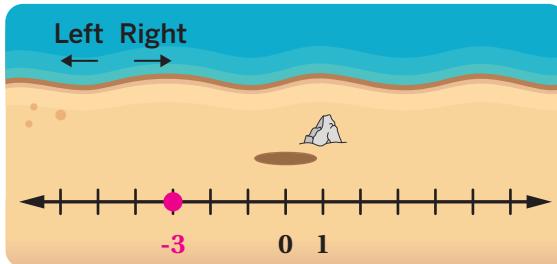
Sand Dollar Challenges

4 The rock is at 1 on the number line. A sand dollar is 4 units to the left of the rock.

Where is the sand dollar?

-3

Left Right

-3   0 1
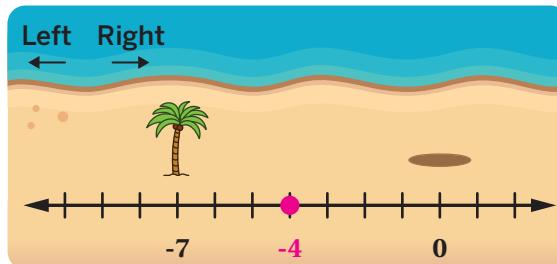
5 The palm tree is at -7 on the number line. A sand dollar is 3 units to the right of the palm tree.

Where is the sand dollar?

-4

Left Right

-7   -4   0

6 Odalis and Kai made mistakes on the previous problem.

Odalis said the sand dollar was at -5.
Kai said the sand dollar was at -10.

Circle your favorite mistake.

Odalis   Kai

What could you tell this student to help them revise their answer?
Responses vary.
• **For Odalis: Count the number of spaces to the right of -7.**
• **For Odalis: From -7, count 3 numbers toward 0.**
• **For Kai: The sand dollar is to the right of the palm tree.**
• **For Kai: Negative numbers get closer to 0 as you go to the right.**

Name: _____   Date: _____   Period: _____

More Sand Dollar Challenges
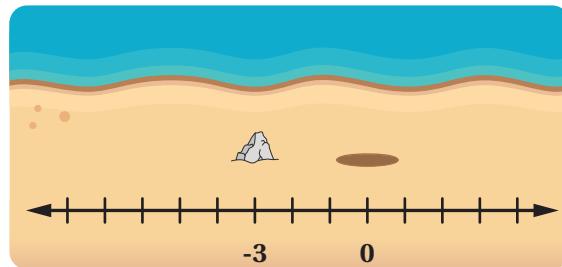
7 The rock is at -3 on the number line. A sand dollar is 2 units away from the rock.

Lan says the sand dollar has to be at a **negative number**. Is she correct? Circle one.

(Yes)   No   I'm not sure
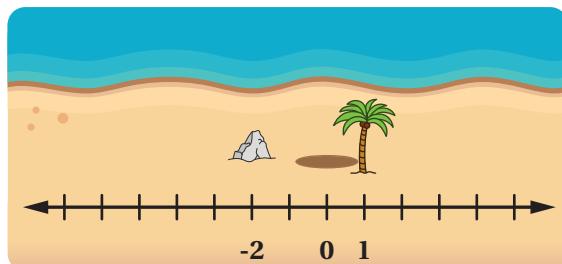
Explain your thinking.
Explanations vary. **The sand dollar could be at -5 or -1. Both of those numbers are to the left of 0.**

8 A sand dollar is 2 units away from the rock and 5 units away from the palm tree.
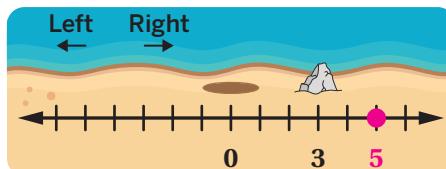
Where is the sand dollar?
-4
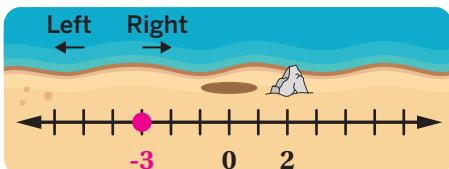
9 For each challenge, write the location of the sand dollar on the number line using the clue. Complete as many challenges as you can.
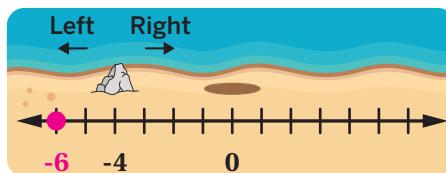
a 2 units to the right of the rock
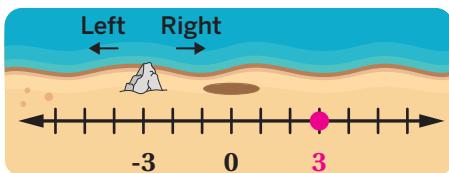
b 5 units to the left of the rock
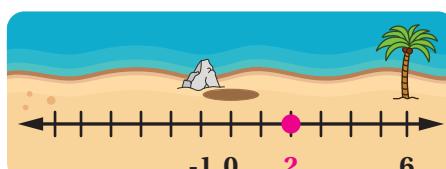
c 2 units to the left of the rock

d 6 units to the right of the rock

e 3 units away from the rock and 4 units away from the palm tree

f 3 units away from the rock and 5 units away from the palm tree
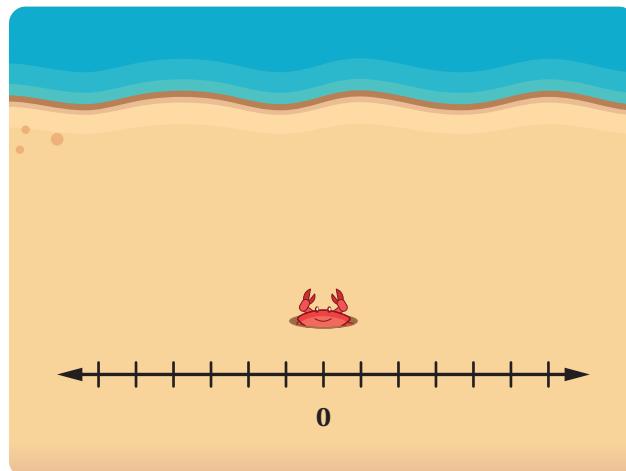
10 Synthesis

List at least two things you know about positive and negative numbers on a number line.

Draw on the image if it helps to show your thinking.

Responses vary.

- **Any number to the left of the hole is a negative number.**
- **The number tells you how many units you are away from 0 and the direction to go.**
- **Negative numbers have a - sign in front of them.**

Things to Remember:

Unit 7
Lesson
2

Name: _____ Date: _____ Period: _____

Digging Deeper

Let's plot positive and negative numbers on the number line.
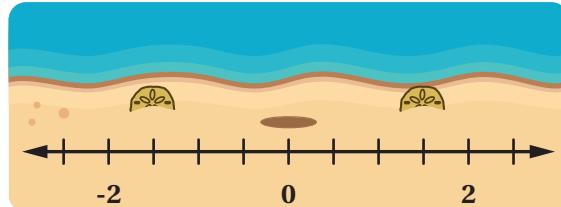
Warm-Up

1 Here are two buried sand dollars. Write clues to describe their locations.
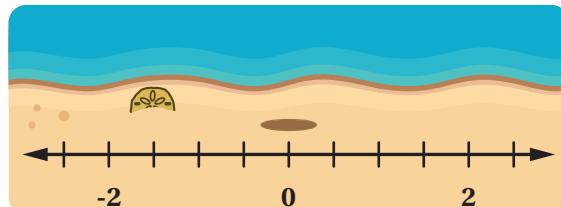
Responses vary.

- **The sand dollars are one tick mark to the left of 2 and one tick mark to the right of -2.**
- **The sand dollars are both three tick marks from 0, one to the left and one to the right.**
- **The sand dollars are at 1.5 and -1.5.**

2 Select *all* the numbers that describe the location of the sand dollar.

☑ **A.** $-\frac{3}{2}$

☑ **B.** -1.5

☑ **C.** $-1\frac{1}{2}$

☐ **D.** -2.5

☐ **E.** -3

Name: _____ Date: _____ Period: _____
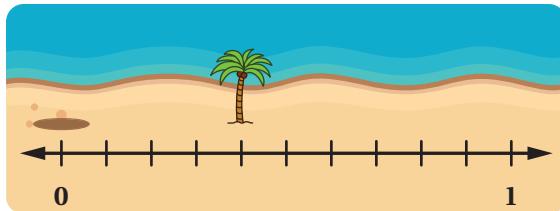
Rational Numbers and Their Opposites

3 A sand dollar is buried under the tree.

Where is the sand dollar on the number line?
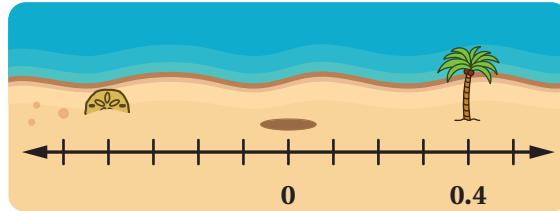
0.4 (or equivalent)

4 A new sand dollar is buried at the **opposite** of the palm tree.

What do you think opposite means here?
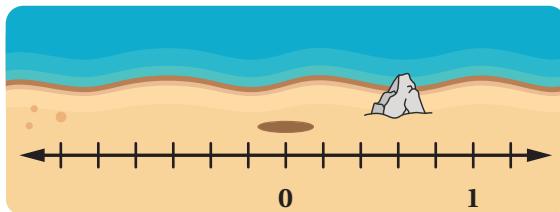
Responses vary. **Opposite means the same distance from 0, but on the other side.**

5 A new sand dollar is buried at the opposite of the rock.

Where is the new sand dollar?

-$\frac{3}{5}$ (or equivalent)

Name: _____ Date: _____ Period: _____
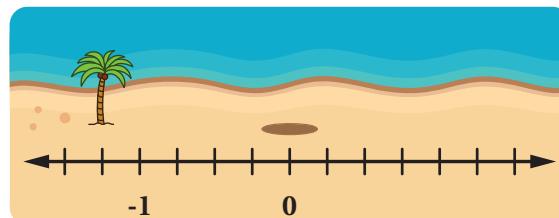
Rational Numbers and Their Opposites (continued)
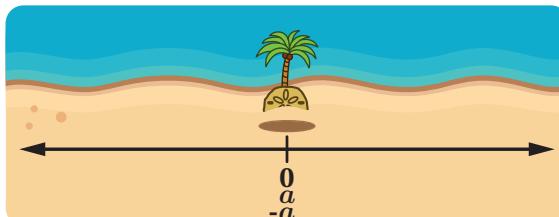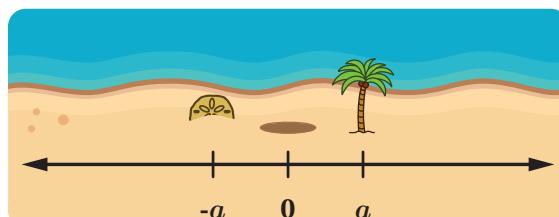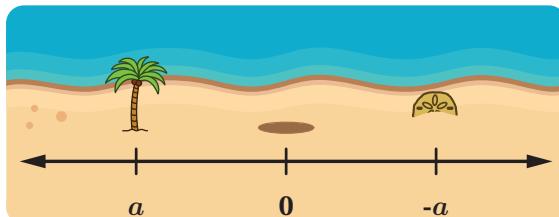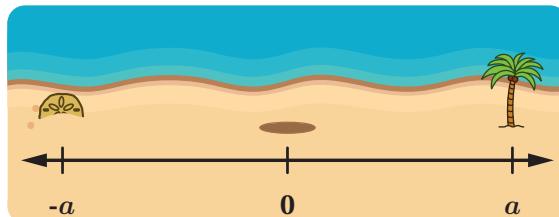
6 A new sand dollar is buried at the opposite of the palm tree.

Where is the new sand dollar?

1.25 (or equivalent)

7 The palm tree is at a. The sand dollar is buried at the opposite of a, which you can write as $-a$. When a is in different locations, what do you notice about $-a$?

Responses vary.
- When a is positive, $-a$ is negative. When a is negative, $-a$ is positive.
- If a is 0, then $-a$ is also 0.
- $-a$ is always on the opposite side of 0 from a.

8 Pair opposite numbers together.
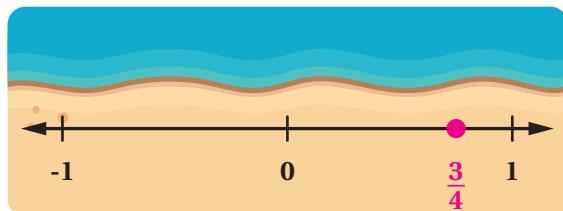
$-(-5)$	$\frac{3}{4}$	$\frac{4}{3}$	-0.75
$-1\frac{1}{3}$	-5	$\frac{1}{5}$	$-\frac{1}{5}$

Pair 1	Pair 2	Pair 3	Pair 4
$-(-5)$ and -5	$\frac{3}{4}$ and -0.75	$-1\frac{1}{3}$ and $\frac{4}{3}$	$\frac{1}{5}$ and $-\frac{1}{5}$

Name: _____   Date: _____   Period: _____

Rational Number Challenges

9 The crab is going to dig at $-\left(-\frac{3}{4}\right)$. Plot a sand dollar where the crab will find it.

10 Plot a point where the crab will dig to find each sand dollar.

• Decide with your partner who will complete Column A and who will complete Column B.

• After plotting the location of each sand dollar, compare your solutions. The solutions in each row should be the same. Discuss and resolve any differences.

Column A	Column B
a The crab is going to dig at $\frac{3}{2}$.	The crab is going to dig at 1.5.
b The crab is going to dig at -1.1.	The crab is going to dig at the opposite of 1.1.
c The crab is going to dig at the opposite of -2.	The crab is going to dig at -(-2).
d The crab is going to dig at -(-0.1).	The crab is going to dig at 0.1.
e The crab is going to dig at the opposite of $-\frac{4}{5}$.	The crab is going to dig at the opposite of -0.8.
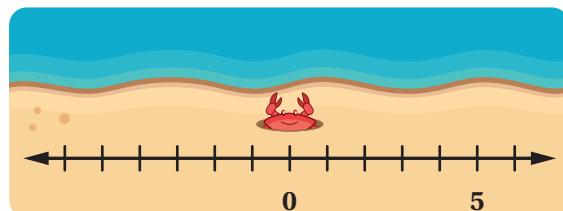
🖥 11 Synthesis

Describe where -3.2 is on a number line.

Draw on the image if it helps to show your thinking.

Responses vary.
- -3.2 is between -4 and -3, but closer to -3.
- -3.2 is slightly to the left of -3.
- Start at 0. Count a little more than 3 to the left.
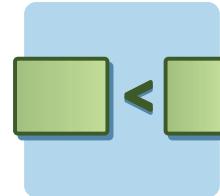
Things to Remember:

Name: _____ Date: _____ Period: _____

Order in the Class

Let's compare positive and negative numbers.

Warm-Up

1. Which one doesn't belong?

A. $+3 > \dfrac{5}{4}$

B. $-\dfrac{5}{4} < 3$

C. $\dfrac{5}{4} > -\dfrac{5}{4}$

D. $-\dfrac{5}{4} > -3$

Explain your thinking.

Responses and explanations vary.
- **Choice A is the only inequality with two positive numbers.**
- **Choice B is the only inequality with a < symbol.**
- **Choice C is the only inequality that does not include a 3.**
- **Choice D is the only inequality with two negative numbers.**

Name: .... Date: .... Period: ....

Greater Than?

You will use your number card to compare numbers with your classmates.
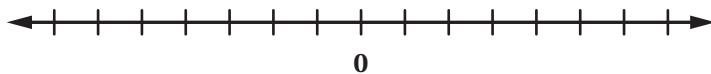
My number: ....

2. For each round, compare your number with different classmates' numbers. Use the number lines if they help with your thinking.
Responses vary.
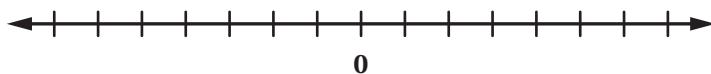
Round 1: Find a person whose number has a *different* **sign** than yours.

Partner's Name	Partner's Number	Comparison in Words	Comparison in Symbols
		.... is greater than	.... > ....
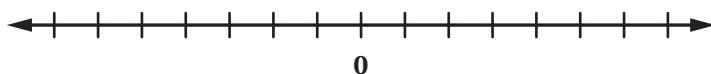
Round 2: Find a person whose number has the *same* sign as yours.

Partner's Name	Partner's Number	Comparison in Words	Comparison in Symbols
		.... is greater than	.... > ....

Round 3: Find a person whose number is the opposite of yours.

Partner's Name	Partner's Number	Comparison in Words	Comparison in Symbols
		.... is greater than	.... > ....

Name: _____ Date: _____ Period: _____

Least to Greatest
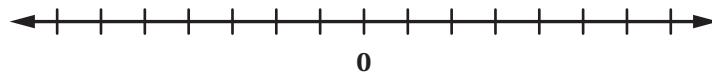
3. For each round, form a group of three or four people.
Responses vary.

Round 4: Record the values from each of your cards: _____, _____, _____, and _____. Write three different inequalities comparing these numbers.

Inequality 1	Inequality 2	Inequality 3
_____ < _____	_____ < _____	_____ < _____

Round 5: Record the values from each of your cards: _____, _____, _____, and _____. Plot and label each number at its approximate location on the number line.

0

Round 6: Record the values from each of your cards: _____, _____, _____, and _____. Order the values from *least* to *greatest*.

Least   **Greatest**

Round 7: Form a line with everyone in the class so that the numbers are in order from *least* to *greatest*.

$-9.2, -9.02, -5, -3, -\dfrac{8}{3}, -2\dfrac{1}{2}, -2.10, -2.01, -2, -1\dfrac{1}{2}, -\dfrac{5}{4}, -1, -\dfrac{6}{7}, -0.4, -\dfrac{1}{3},$

$-\dfrac{1}{7}, \dfrac{1}{7}, \dfrac{1}{3}, \dfrac{2}{5}, -\left(-\dfrac{6}{7}\right), -(-1), +1.25, +1.5, +2, +2.01, 2.10, 2.5, 2\dfrac{2}{3}, 3, +5, 9.02, +9.2$

Name: _____ Date: _____ Period: _____

Ordering Numbers

4. Here are four more numbers.

-2	$1\frac{1}{2}$	-1.5	-1

a Complete each statement using > or <.

-2 _____ **<** _____ $1\frac{1}{2}$ -1.5 _____ **<** _____ -1

b Use two of the numbers to complete this sentence.
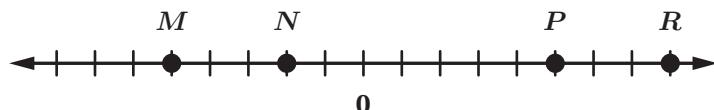
_____**-1.5**_____ is the opposite of _____**$1\frac{1}{2}$**_____ because . . . **it is the same distance from 0 and on the opposite side of the number line.**

c Order the numbers from *least* to *greatest*.

-2	**-1.5**	**-1**	**$1\frac{1}{2}$**

Least   **Greatest**

5. Use the number line to complete each sentence.
Responses and explanations vary.
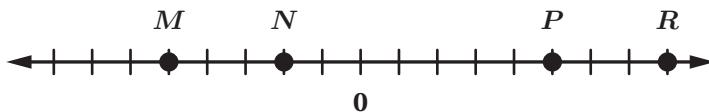
a _**M**_ is the opposite of _**P**_ because . . . **it is the same distance from 0 and on the opposite side of the number line.**

b _**R**_ is greater than _**N**_ because . . . **it is farther to the right on the number line.**

c _**M**_ is the least of the numbers because . . . **it is the farthest to the left on the number line.**

Synthesis

6. Describe how to compare numbers on a number line. Use the example if it helps with your thinking.

$$M \quad N \qquad\qquad P \quad R$$

Responses vary. **The numbers on a number line become greater as you move from left to right. For example, R is greater than P because it is farther to the right on the number line. M is the least of the numbers because it is the farthest to the left on the number line.**

Things to Remember:

Unit 7
Lesson
3

Name: .... Date: .... Period: ....

Sub-Zero

Let's use positive and negative numbers in context.

Warm-Up

1 Here are three animals sitting on rocks at different elevations.

Match each elevation with the correct rock.
Responses shown.

2 What do you think 0 inches represents in this situation?
Responses vary. **0 inches represents the surface of the water.**

20 in.

8 in.

-10 in.

20 in.

-10 in.

8 in.

Name: .... Date: .... Period: ....
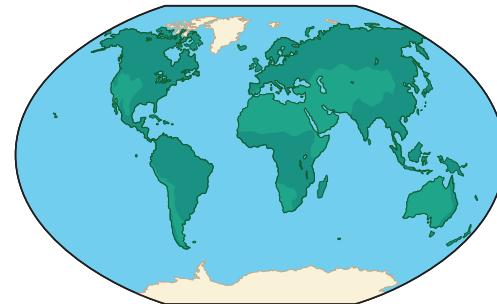
World Temperatures

3 Let's look at different temperatures around the world on one particular day.

Choose a location you'd like to visit and explain your choice to a classmate.
Responses vary.

4 Neel says Fairbanks is warmer because -4 > -6. Yunuen says Pierre is warmer because 6 is greater than 4.
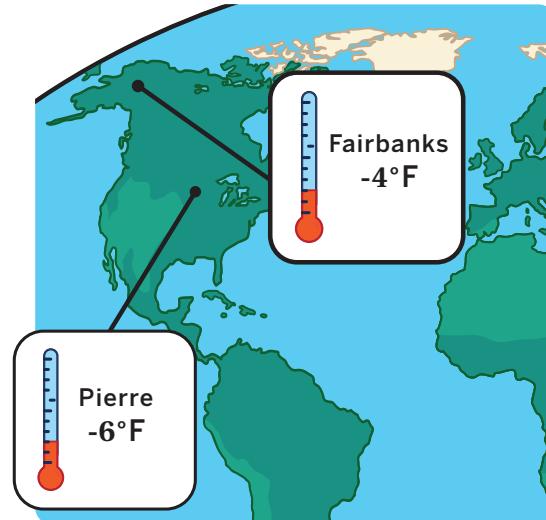
Whose thinking is correct? Circle one.

(Neel's)   Yunuen's   Both   Neither
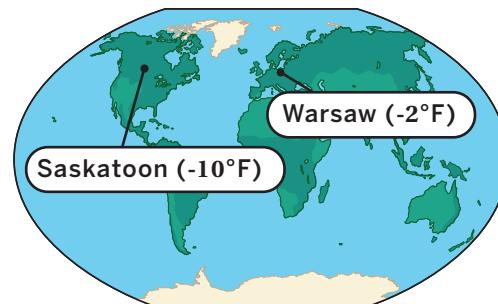
Explain your thinking.
Explanations vary.
- **Neel is correct that -4 is greater than -6 because -4 is above -6 on a number line, which means that the temperature in Fairbanks is warmer.**
- **Yunuen might be thinking that -6 is farther away from 0 than -4 is, which is true. However, this does not mean that the temperature is warmer.**

Fairbanks -4°F

Pierre -6°F

5 Write a temperature that is colder than Warsaw's and warmer than Saskatoon's.
Responses vary. **Any temperature less than -2°F and greater than -10°F, such as -3°F or -8°F**

Warsaw (-2°F)

Saskatoon (-10°F)

Name: _____   Date: _____   Period: _____
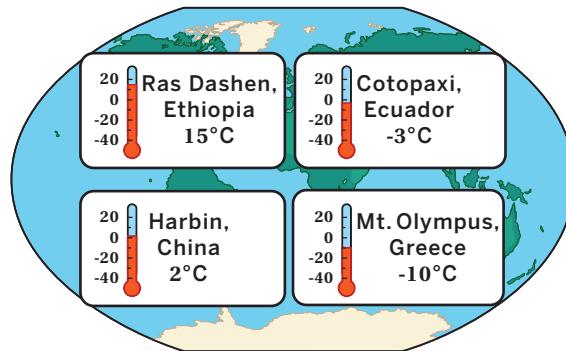
Colder Temperatures

6 Here are the temperatures of different places around the world on another day.

Select *all* of the true statements.

☑ **A.** Ras Dashen is warmer than Cotopaxi.

☐ **B.** Cotopaxi is -3 degrees below 0.

☑ **C.** Mt. Olympus is the coldest.

☑ **D.** Mt. Olympus is 12 degrees colder than Harbin.

☐ **E.** Cotopaxi is colder than Mt. Olympus.

| 20 0 -20 -40 | Ras Dashen, Ethiopia 15°C | 20 0 -20 -40 | Cotopaxi, Ecuador -3°C |
| 20 0 -20 -40 | Harbin, China 2°C | 20 0 -20 -40 | Mt. Olympus, Greece -10°C |
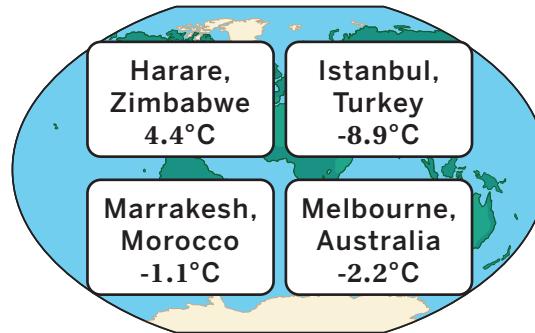
7 Here are some of the coldest recorded temperatures for four cities.

Order these temperatures from *warmest* to *coldest*.

| Harare, Zimbabwe 4.4°C | Istanbul, Turkey -8.9°C |
| Marrakesh, Morocco -1.1°C | Melbourne, Australia -2.2°C |

Warmest 🔥

| 4.4°C (Harare) |
| -1.1°C (Marrakesh) |
| -2.2°C (Melbourne) |
| -8.9°C (Istanbul) |

Coldest ❄️

Name: _____ Date: _____ Period: _____

Colder Temperatures (continued)

8 This number line shows some of the lowest recorded temperatures for the same four cities.

One of the lowest recorded temperatures in Rome, Italy, is -7.2°C.

Plot the point for Rome at its approximate location on the number line.
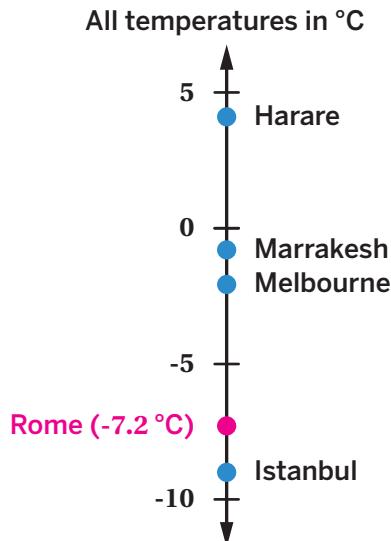
Response shown on the number line. Responses between -6 and -8 are considered correct.

9 One way to write that Harare's temperature is warmer than Marrakesh's is 4.4 > -1.1.

Write your own *inequality* comparing Rome's temperature to another city's temperature.

Inequalities vary.
- **-7.2 > -8.9**
- **-7.2 < -2.2**
- **-1.1 > -7.2**

All temperatures in °C

```
        5 ──┼── ● Harare

        0 ──┼── ● Marrakesh
               ● Melbourne

       -5 ──┼──
Rome (-7.2 °C) ●
               ● Istanbul
      -10 ──┼──
```
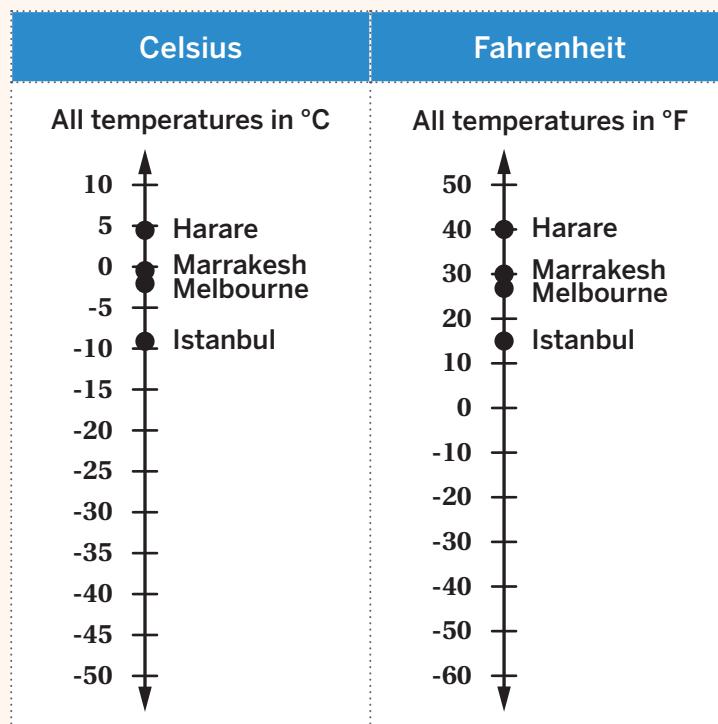
Explore More

10 Look up the lowest recorded temperature in a city near you.

Plot and label a point to represent this temperature on one of the number lines.

Responses vary.

Celsius	Fahrenheit
All temperatures in °C	All temperatures in °F

Celsius — All temperatures in °C
```
  10 ──┼──
   5 ──┼── ● Harare
   0 ──┼── ● Marrakesh
  -5 ──┼── ● Melbourne
 -10 ──┼── ● Istanbul
 -15 ──┼──
 -20 ──┼──
 -25 ──┼──
 -30 ──┼──
 -35 ──┼──
 -40 ──┼──
 -45 ──┼──
 -50 ──┼──
```

Fahrenheit — All temperatures in °F
```
  50 ──┼──
  40 ──┼── ● Harare
  30 ──┼── ● Marrakesh
  20 ──┼── ● Melbourne
  10 ──┼── ● Istanbul
   0 ──┼──
 -10 ──┼──
 -20 ──┼──
 -30 ──┼──
 -40 ──┼──
 -50 ──┼──
 -60 ──┼──
```
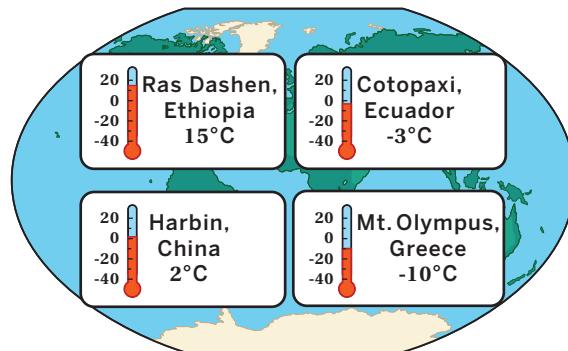
Synthesis

When comparing two temperatures, how can you tell which is warmer and which is colder?

Use the examples if they help with your thinking.

Responses vary.

- **A warmer temperature is another way to say a greater temperature, so whatever number is greater will be warmer. For example, -3 > -10, so -3°C is warmer than -10°C.**

- **Any positive number is greater than any negative number, so any positive temperature is warmer than any negative temperature.**

Things to Remember:

Unit 7
Lesson

5
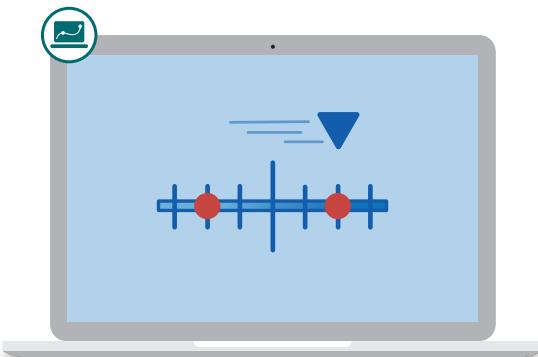
Name: _____ Date: _____ Period: _____

Distance on the Number Line

Let's explore the absolute value of rational numbers.

Warm-Up

1 Let's play a game to get a score.
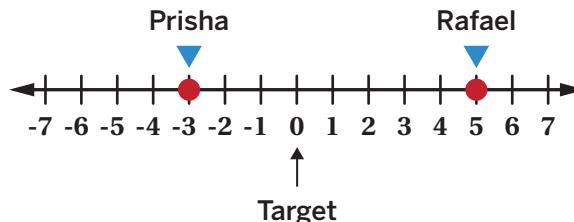
💬 **Discuss:** How do you think scores are determined?

Responses vary. **The score is the number I land on except without the sign.**

Name: .... Date: .... Period: ....

What Is Absolute Value?
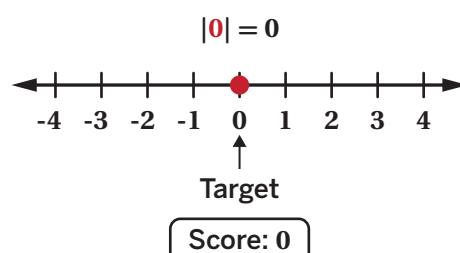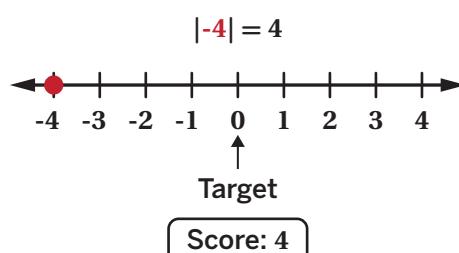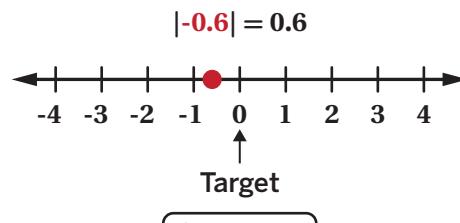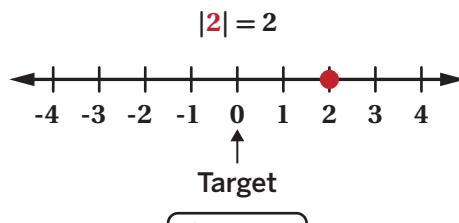
2 Prisha landed at -3. Rafael landed at 5. What was each student's score?

Student	Value
Prisha	**3**
Rafael	**5**

3 The game uses **absolute value** to calculate the score. We read |-2| as "the absolute value of negative 2."

a Take a look at some scores for different stopping points.

$|2| = 2$

Score: 2

$|-0.6| = 0.6$

Score: 0.6

$|-4| = 4$

Score: 4

$|0| = 0$

Score: 0

b Describe what you think absolute value means.

Responses vary.
- **Absolute value means the number without the sign.**
- **Absolute value means how far away the number is from 0.**
- **Absolute value means take all the negative numbers and make them positive.**

4 Prisha says $|x|$ means "the opposite of x." Rafael says $|x|$ means "how far x is from zero." Whose thinking is correct? Circle one.

Prisha's   (Rafael's)   Both   Neither

Explain your thinking.
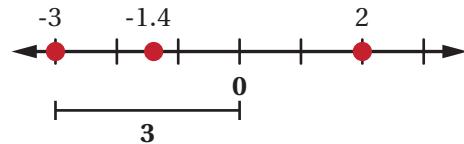
Explanations vary.
- **Sometimes $|x|$ has the same value as the opposite of x, and sometimes it does not. For example, the opposite of 3 is -3, but $|3|$ is 3.**
- **The absolute value is always a positive number or 0. "How far x is from zero" is also always positive or 0 because you can't be a negative distance away from something. The opposite of a number can be positive or negative.**

Name: ....   Date: ....   Period: ....

Comparing Absolute Values

5 Determine the value of each expression. Use the number line if it helps with your thinking.

Expression	Value
|-3|	3
|-1.4|	**1.4**
|2|	**2**

6 Determine whether each statement is true or false.

Statement	True	False
$\|0\| = 0$	✔	
$\|6\| = -6$		✔
$\|6\| > \|-6\|$		✔
$-5 < -4$	✔	
$\|-5\| < \|-4\|$		✔
$\frac{3}{2} > -\frac{1}{2}$	✔	
$\left\|\frac{3}{2}\right\| > \left\|-\frac{1}{2}\right\|$	✔	
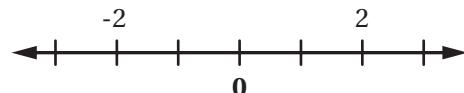
7 Isaiah says -2 and 2 have the same absolute value.

$|-2| = |2| = 2$

Explain why Isaiah's claim is correct.

Explanations vary.
- **Both -2 and 2 are the same distance from 0.**
- **-2 and 2 are opposites so they have the same absolute value.**

Name: .... Date: .... Period: ....

Absolute Value Puzzles

8 **a** Make a true inequality by using each number at most once.

Responses vary.
- $|-2| > 1$
- $|1| > -1$

$$\left|\,\boxed{}\,\right| > \boxed{}$$

| -2 | -1 | 1 | 2 |

b Explain how you know your inequality is true.

Explanations vary.
- $|-2| > 1$ is true because $|-2| = 2$ and $2 > 1$.
- $|1| > -1$ is true because $|1|$ is positive and all positive numbers are greater than all negative numbers.

9 Make true statements by using each number exactly once.

Responses vary.
- $|-5| > 3$ and $4 = |-4|$
- $|3| > -5$ and $4 = |-4|$

$$\left|\,\boxed{}\,\right| > \boxed{}$$

$$\boxed{} = \left|\,\boxed{}\,\right|$$

| 3 | 4 | -4 | -5 |

10 Make true statements by using each number exactly once.

$3 > |-2|$, $|-1| < 2$, and $|-4| = 4$

$$\boxed{} > \left|\,\boxed{}\,\right|$$

$$\left|\,\boxed{}\,\right| < \boxed{}$$

$$\left|\,\boxed{}\,\right| = \boxed{}$$

| -1 | -2 | -4 |

| 2 | 3 | 4 |

Name: .... Date: .... Period: ....

Absolute Value Puzzles (continued)

11 Is it possible to make true statements using each number exactly once? Circle one.

Yes　　　　　　(No)　　　　　　I'm not sure

Explain your thinking.

Explanations vary. **The only way to make the first sentence true is to use -6 and 6. This leaves two negative numbers. It can't be true that a negative number is greater than the absolute value of a number because distance (and absolute value) are always positive or zero.**

$$\left| \boxed{} \right| = \boxed{}$$

$$\boxed{} > \left| \boxed{} \right|$$
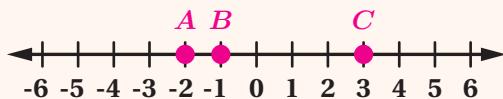
-5　　6　　-6　　-7

Explore More

12 Use these clues to determine the values of A, B, and C. Use the number line to record your answers and to help you with your thinking.
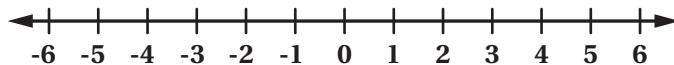
Responses shown on the number line.

A B　　　C

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

Clues

- The absolute value of A is 2.
- B is greater than A.
- Point B is closer to zero than point A is.
- C is positive.
- The distance between A and B is 1.
- The distance between B and C is 4.

Explain 2–3 things you know about absolute value. Use the number line if it helps to show your thinking.

Responses vary.
- Absolute value is how far away a number is from zero.
- Positive numbers and zero stay the same, but negative numbers become positive.
- Numbers that are opposites, like -6 and 6, have the same absolute value.
- You use the symbols | | to show absolute value.

Things to Remember:

Unit 5
Lesson
1

Name: _____ Date: _____ Period: _____
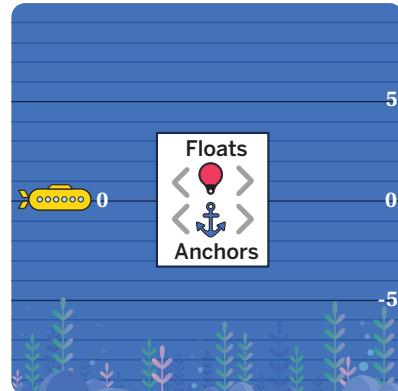
Floats and Anchors

Let's use floats and anchors to represent values on a number line.
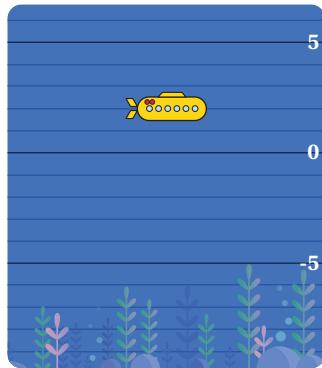
Warm-Up

1 This submarine is controlled by floats and anchors.
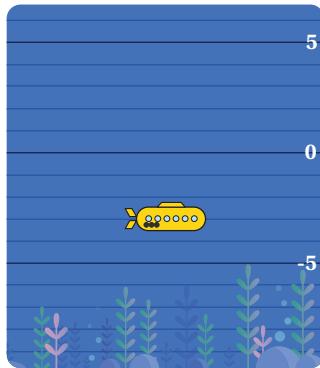
a Take a look at these different combinations.
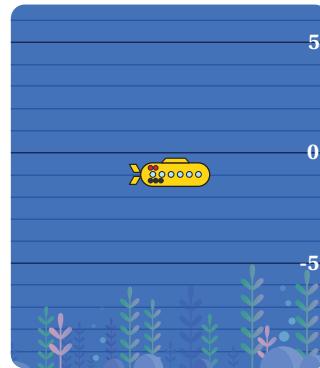
Add 2 floats | Add 3 anchors | Add 2 floats, Add 3 anchors

b 💬 **Discuss:** What do you notice? What do you wonder?

Responses vary. **I notice floats make the submarine go up, anchors make the submarine go down, and when both are added, whichever there's more of is the direction it moves.**

Name: ....   Date: ....   Period: ....

Collect the Star
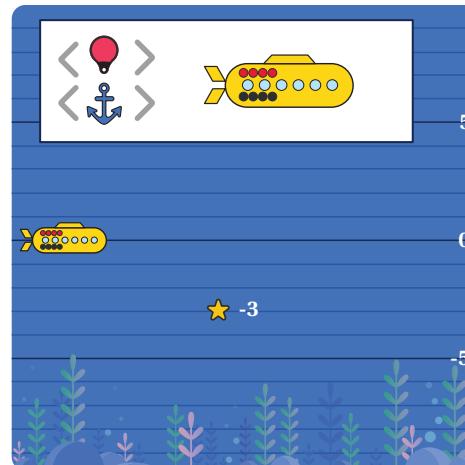
2-3 This submarine starts with 4 floats and 4 anchors.

a 💬 **Discuss:** Why do you think this submarine's current position is at 0 units?

Responses vary. **The anchors and floats are balanced out so it doesn't go up or down.**

b The table shows one way to move the submarine to -3 to get the star. Write three more actions to get the submarine to -3.

Responses vary.
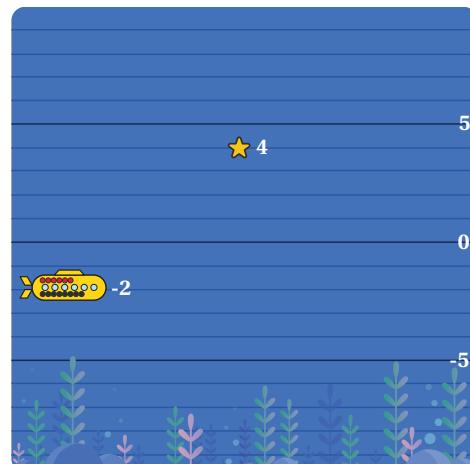
Start	Action	Final
0	Remove 1 float Add 2 anchors	-3
0	**Remove 3 floats**	-3
0	**Remove 2 floats Add 1 anchor**	-3
0	**Remove 4 floats Remove 1 anchor**	-3

4 This submarine starts with 6 floats and 8 anchors.

The submarine has space for up to 10 floats and up to 10 anchors.

Write an action that could move the submarine to 4 units to collect the star.
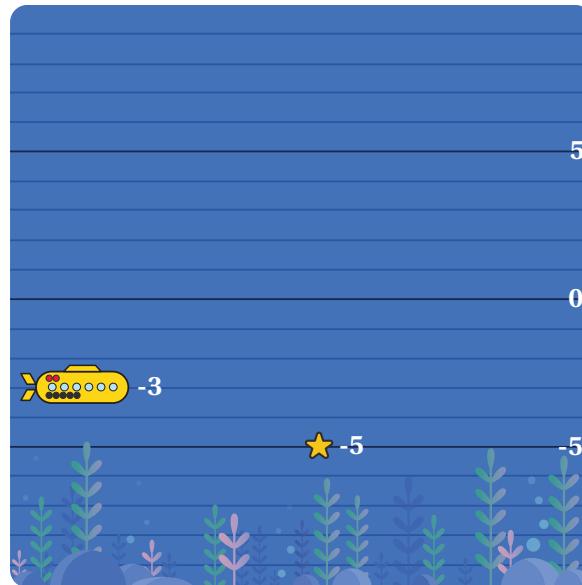
Start	Action	Final
-2	*Responses vary.* • **Remove 6 anchors** • **Add 4 floats and remove 2 anchors**	4

Name: _____   Date: _____   Period: _____

Collect the Star (continued)

5 This submarine starts at -3 units.

Select *all* the actions that would move it to -5 units.

☐ **A.** Add 2 floats

☑ **B.** Add 2 anchors

☑ **C.** Remove 1 float and add 1 anchor

☑ **D.** Add 3 floats and add 5 anchors

☐ **E.** Remove 2 floats and add 4 anchors

6 Imagine a new submarine. For each action, put a check for whether the submarine would go up, go down, or stay in the same position.

Action	Up	Same Position	Down
Add 3 floats Add 4 anchors			✔
Remove 10 anchors	✔		
Remove 5 floats Remove 5 anchors		✔	
Add 8 floats Remove 8 anchors	✔		
Add 6 floats Add 2 anchors	✔		
Remove 7 floats Add 3 anchors			✔

Name: .... Date: .... Period: ....

Sea-king Stars

7 The table shows the submarine's starting position and the action that will change its position.

What will be the submarine's final position?

Start	Action	Final
-2	Add 3 floats Remove 5 anchors	**6**

8 Crow and Mai wrote expressions to answer the previous question.

Crow's expression: -2 + 3 − 5

Mai's expression: -2 + 3 − (-5)

Who wrote a correct expression? Circle one.

Crow   (Mai)   Both   Neither

Explain your thinking.
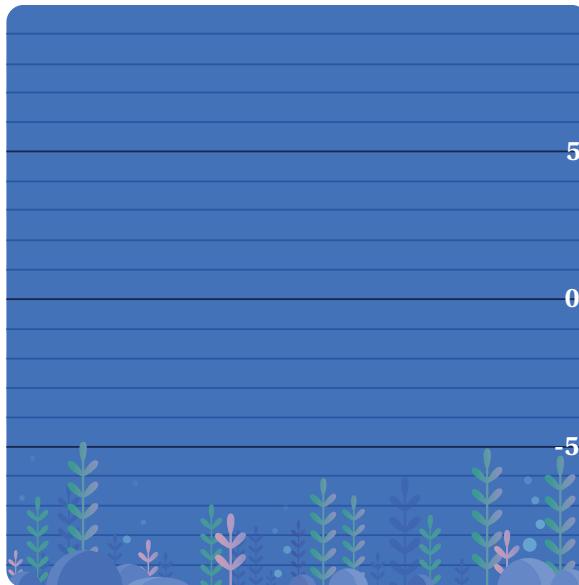
Explanations vary.
- **Crow's expression is subtracting 5, which is like adding 5 anchors.**
- **Mai's expression has two negatives: one for removing and one for the anchors.**

Name: _____   Date: _____   Period: _____

Captain's Challenge

9 What is the final position of each submarine? Complete as many challenges as you have time for. Use the image if it helps with your thinking.

Start	Action	Final
0	Add 3 floats Add 7 anchors	**-4**
-9	Add 8 floats Remove 6 anchors	**5**
1	Remove 3 floats Add 4 anchors	**-6**
-3	Remove 1 float Add 3 anchors	**-7**
-4	Remove 1 float	**-5**
5	Remove 8 floats Remove 3 anchors	**0**
-2	Add 5 floats Remove 2 anchors	**5**
-5	Add 1 float Remove 2 anchors	**-2**

5

0

-5
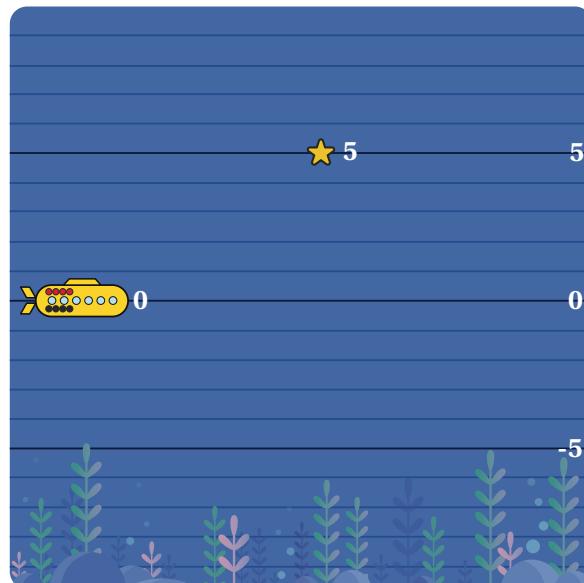
10 Synthesis

Describe a set of actions that would allow this submarine to collect the star at 5 units.

Try to come up with something none of your classmates will.
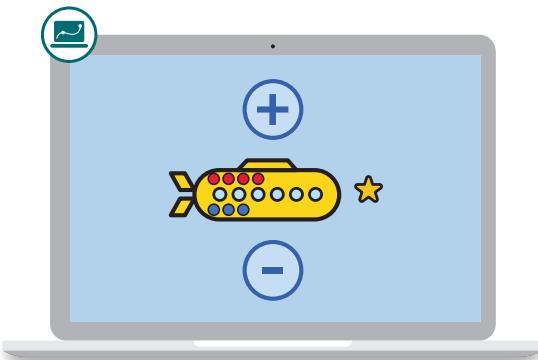
Responses vary.
- **Add 5 floats**
- **Add 1 float and remove 4 anchors**
- **Add 3 floats and remove 2 anchors**
- **Add 6 floats and add 1 anchor**

Things to Remember:

Unit 5
Lesson
2

Name: _____ Date: _____ Period: _____

More Floats and Anchors

Let's use floats and anchors to reason about adding and subtracting positive and negative numbers.

Warm-Up

1 *If you add some floats and remove some anchors, the submarine will go up.*

Is this statement *always*, *sometimes*, or *never* true? Circle one.
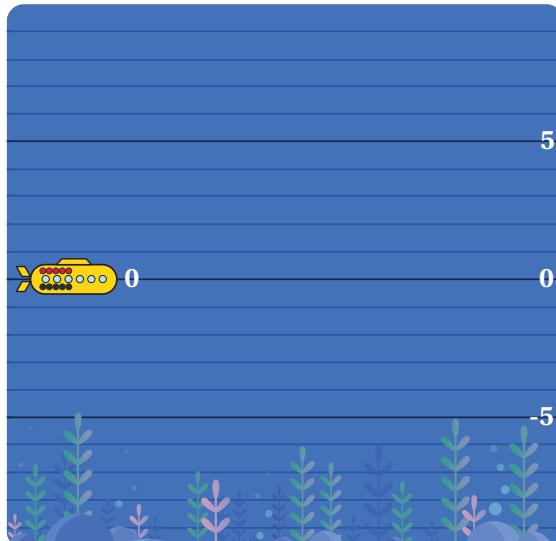
(Always)   Sometimes   Never

Explain your thinking.
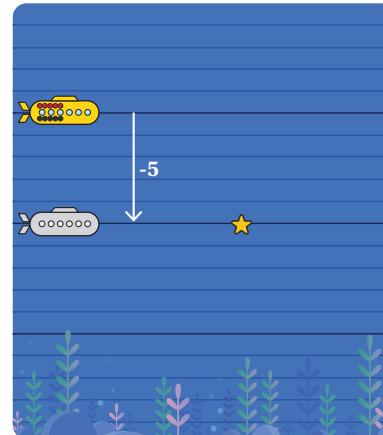
Explanations vary. **Adding floats makes the submarine go up and removing anchors makes the submarine go up, so it will always go up.**

Name: .... Date: .... Period: ....

Ups and Downs
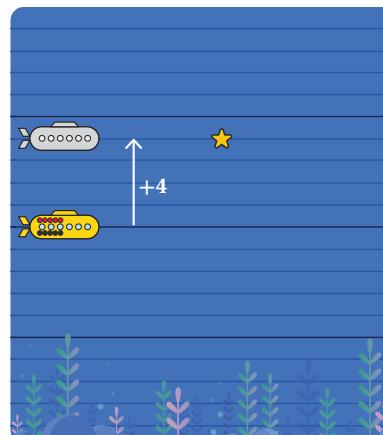
2 Select *all* the actions that would make this submarine go down 5 units.

- ☐ **A.** Add 5 floats
- ☑ **B.** Remove 5 floats
- ☑ **C.** Add 5 anchors
- ☐ **D.** Remove 5 anchors
- ☐ **E.** Add 3 anchors and add 2 floats

3 Select *all* the actions that would make this submarine go up 4 units.

- ☑ **A.** Add 4 floats
- ☐ **B.** Remove 4 floats
- ☐ **C.** Add 4 anchors
- ☑ **D.** Remove 4 anchors
- ☑ **E.** Add 3 floats and remove 1 anchor

4-5 Here are the details for four new submarine scenarios.

Complete the table.

Start	Action	Final Expression	Final Value
3	Add 7 anchors	$3 + (-7)$	**-4**
3	Remove 7 floats	$3 - 7$	**-4**
-2	**Add 8 floats**	$-2 + 8$	**6**
-2	**Remove 8 anchors**	$-2 - (-8)$	**6**

💬 **Discuss:** What do you notice? What do you wonder?

Responses vary.
- I notice the final value for $3 + (-7)$ is the same as the final value for $3 - 7$.
- I notice the final value for $-2 + 8$ is the same as the final value for $-2 - (-8)$.
- I wonder if plus a negative is always the same as minus a positive.
- I wonder if minus a negative is always the same as plus a positive.

Name: _____ Date: _____ Period: _____

Depths of Understanding

6 Marc and Naoki are trying to evaluate $3 - (-2)$.

Marc says: *This is like adding* 2 *anchors, so the submarine goes down to* 1.
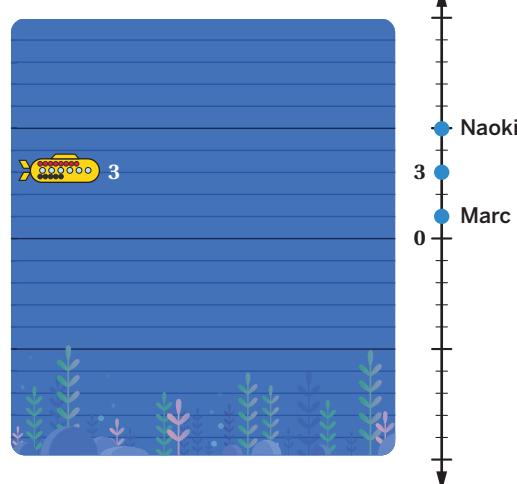
Naoki claims: *This is like removing* 2 *anchors, so the submarine goes up to* 5.

Whose thinking is correct? Circle one.

Marc (1)   (Naoki (5))   Both   Neither

Explain your thinking.

Explanations vary. **Since there are two negative signs in -(-2), one is for removing and the other one is for anchors. Removing anchors makes the submarine go up, so Naoki is correct.**

7 **a** What is the value of $-1 + (-4)$?

-5
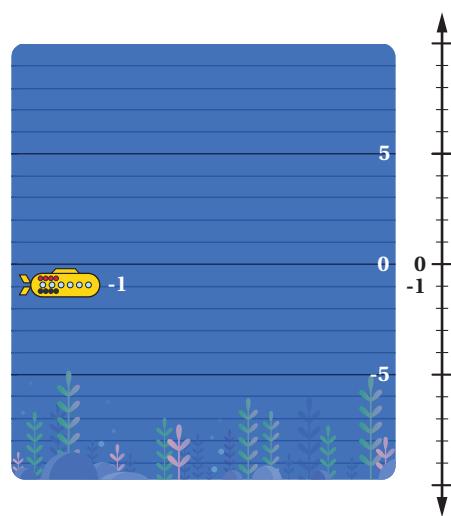
b What is the value of $-1 - (-4)$?

3

c Show or explain your thinking.

Explanations vary. **The first problem is like adding 4 anchors, which makes the submarine go down 4. The second problem is like removing 4 anchors, which makes the submarine go up 4.**

Activity
2

Name: ....   Date: ....   Period: ....

Depths of Understanding (continued)

8 Group the expressions into pairs that have the same value.

$-4 - (-10)$	$4 + (-10)$	$-4 + 10$	$-4 - 10$
$-4 + (-10)$	$4 + 10$	$4 - (-10)$	$4 - 10$

Pair 1	Pair 2
$-4 - (-10)$	$-4 + (-10)$
$-4 + 10$	$-4 - 10$
Pair 3	**Pair 4**
$4 + (-10)$	$4 - (-10)$
$4 - 10$	$4 + 10$

Explore More

9 **a** Determine the value of each expression.

Expression	Value
1	**1**
$1 - 2$	**-1**
$1 - 2 + 3$	**2**
$1 - 2 + 3 - 4$	**-2**
$1 - 2 + 3 - 4 + 5$	**3**

b Describe any patterns you notice.

Responses vary. **I noticed that the numbers in the first and second rows were the same, except that one was negative. This also happened in the third and fourth rows.**

c What is the value of the next expression? The 10th expression? The 100th expression?

6th expression: -3
10th expression: -5
100th expression: -50
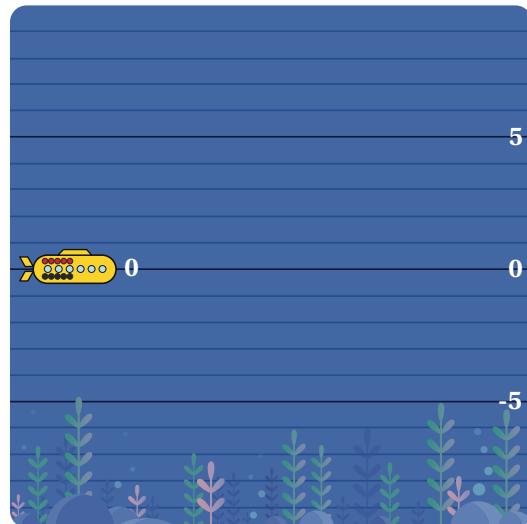
10 Synthesis

Explain why it makes sense that $0 - (-5)$ is equivalent to $0 + 5$.

Use a floats and anchors situation if it helps with your thinking.

Responses vary. **It makes sense that these expressions are the same because they have the same start and they are two different ways of going up. $0 - (-5)$ is like starting from 0 and removing 5 anchors. $0 + 5$ is like starting from 0 and adding 5 floats.**

Things to Remember:

Name: _____   Date: _____   Period: _____

Bumpers

Let's add numbers including decimals and fractions on a number line.
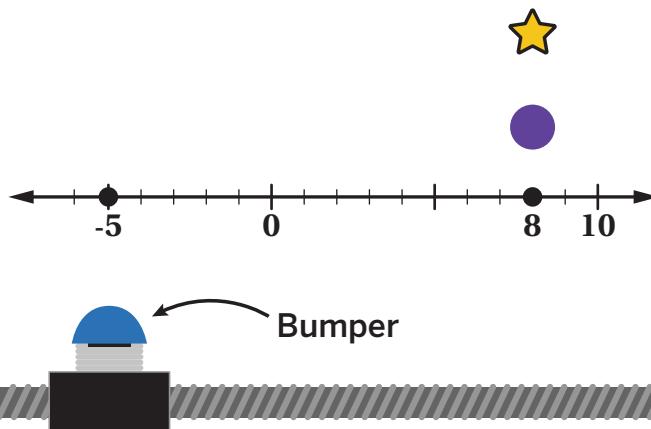
Warm-Up

1 Hitting the ball with the bumper collects the star.

The bumper is at -5 units and the ball is at 8 units.

Write the number that would move the bumper and hit the ball.

13 units

Bumper

Name: _____ Date: _____ Period: _____
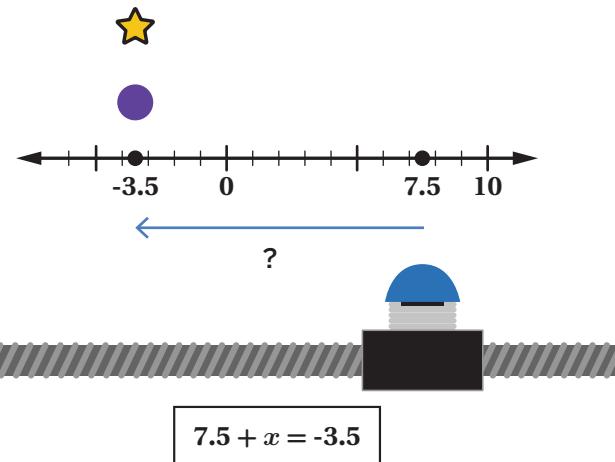
Things Could Get Bumpy

2 The bumper is at 7.5 units and the ball is at -3.5 units.

$$7.5 + x = \text{-}3.5$$

a Write the number that would move the bumper and hit the ball.

-11 units

b 💬 **Discuss** your strategy with a classmate.

Strategies vary. **I know that it takes -7.5 units to get back to 0 and then another -3.5 units to get to -3.5, so in total, it is -11 units.**

3 Dalia says that the expression $7.5 + 3.5$ can be used to know how to move the bumper.

Do you agree? Circle one.

Agree   Disagree   I'm not sure

Explain your thinking.
Responses and explanations vary.
- **I agree because 11 units is the distance the bumper must move to the left.**
- **I disagree because the bumper must move to the left, which is -11.**

Name: _____ Date: _____ Period: _____
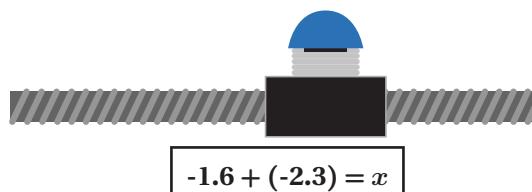
Carefully Placed

4 This bumper is at -1.6 units.

It's programmed to move -2.3 units.

Where should the ball be placed so that the bumper hits it?

-3.9 units

$-1.6 + (-2.3) = x$
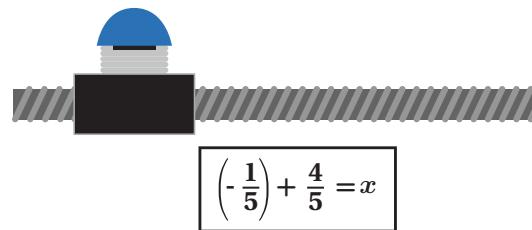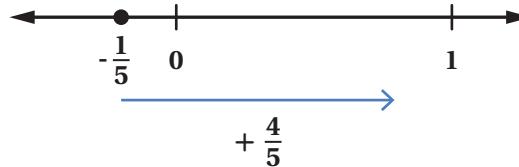
5 This bumper is at $-\frac{1}{5}$ units.

It's programmed to move $\frac{4}{5}$ units.

Where should the ball be placed?

$\frac{3}{5}$ **units (or equivalent)**

$\left(-\frac{1}{5}\right) + \frac{4}{5} = x$
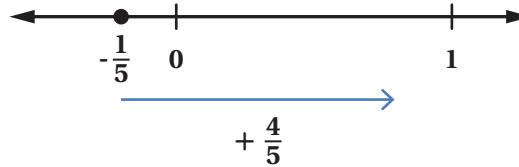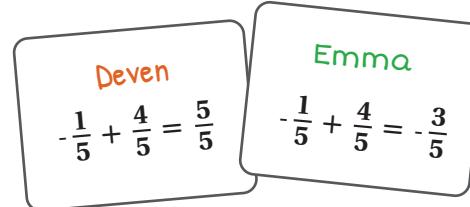
6 Two students made mistakes on the previous challenge.

Circle the card with your favorite mistake.

What could you say to help the student understand their mistake?

Responses vary.
- **Deven: If you start at 0 and add $\frac{4}{5}$, you still wouldn't get all the way to $\frac{5}{5}$, so if you start to the left of 0, you won't make it that far to the right.**
- **Emma: If you start at $-\frac{1}{5}$ and move to the right, there's no way you can get to $-\frac{3}{5}$.**

Deven

$-\frac{1}{5} + \frac{4}{5} = \frac{5}{5}$

Emma

$-\frac{1}{5} + \frac{4}{5} = -\frac{3}{5}$

Name: _____   Date: _____   Period: _____
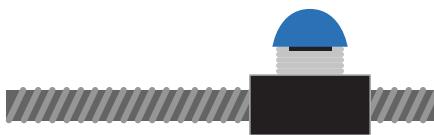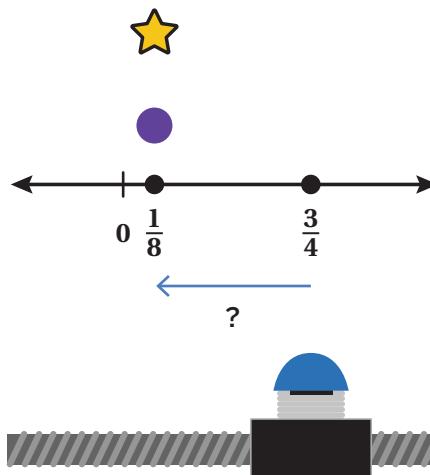
Bumper Challenge

7 What is the value of x that makes this equation true?

$\frac{3}{4} + x = \frac{1}{8}$

$-\frac{5}{8}$ **(or equivalent)**

8 What is the value of x that makes each equation true? Solve as many challenges as you have time for.

a $4 + x = 10$

$x = 6$

b $7 + x = 2$

$x = -5$

c $-1.3 + x = 7.2$

$x = 8.5$

d $\frac{9}{5} + x = \frac{3}{5}$

$x = -\frac{6}{5}$ **(or equivalent)**

e $10 + (-2.5) = x$

$7.5 = x$

f $8.1 + x = -1$

$x = -9.1$

g $x + 8.4 = -4.2$

$x = -12.6$

h $x + \frac{1}{6} = -\frac{2}{3}$

$x = -\frac{5}{6}$ **(or equivalent)**

i $x + \left(-\frac{9}{2}\right) = -1$

$x = \frac{7}{2}$ **(or equivalent)**

j $x + (-13.2) = -7.6$

$x = 5.6$

k $\frac{1}{2} + 2 = x$

$\frac{5}{2} = x$ **(or equivalent)**

l $5.7 + 0.4 = x$
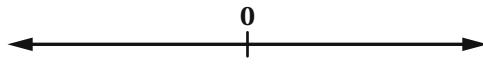
$6.1 = x$

9 Synthesis

Describe how you can use a number line to determine the value of x in an equation like $3.1 + x = $ -2.

Use the number line if it helps you show your thinking.

Responses vary. **3.1 is the start and -2 is the end, and I don't know the change. Because I am going from positive to negative, there is an arrow going to the left from the start to the end. Then I can count or figure out how long the arrow should be.**

Things to Remember:

Unit 5

Lesson

4
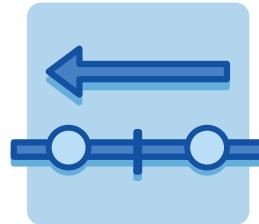
Name: _____ Date: _____ Period: _____

Draw Your Own
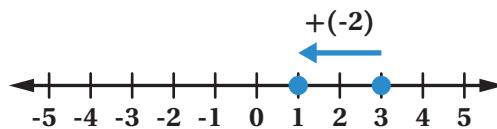
Let's use number lines to reason about addition and subtraction.
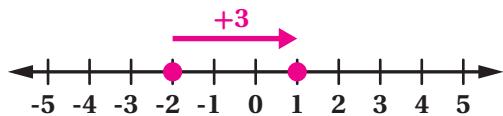
Warm-Up

1. Renata drew a number line for 3 + (-2). Use her thinking to complete the number line to determine the value of -2 + 3.

$$3 + (-2) = 1$$

+(-2)

-5 -4 -3 -2 -1   0   1   2   3   4   5

-2 + 3 = __1__

+3

-5 -4 -3 -2 -1   0   1   2   3   4   5

Name: _____ Date: _____ Period: _____

Pluses and Minuses
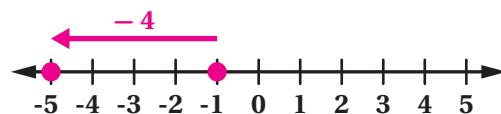
2. Renata says her number line for 3 + (-2) could also represent 3 − 2 because adding two anchors has the same effect as removing two floats. Do you agree? Circle one.

Yes　　　　　　No　　　　　　　　I'm not sure
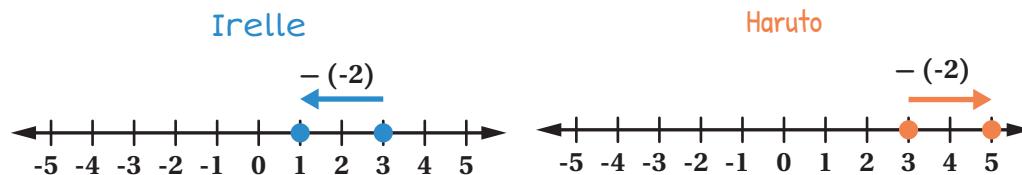
Explain your thinking.
Responses and explanations vary. **In the Warm-Up, Renata's number line starts at 3 and ends at 1. 3 − 2 = 1 starts at 3 and ends at 1 too.**

3. Use your thinking from the previous problem to represent -1 − 4 on this number line.

4. Irelle and Haruto each drew a number line for 3 − (-2).

Whose number line is correct? Circle one.

Irelle　　　　　(Haruto)　　　　　Both　　　　　Neither

Explain your thinking.
Explanations vary. **Irelle's diagram is the same as Renata's from earlier, which shows 3 + (-2). The problem 3 − (-2) is a subtraction problem, and subtraction is the opposite of addition, so we have to go the other direction on the number line, like in Haruto's diagram.**

5. Match each addition expression to a subtraction expression with the same value.

a. 9 − 7　　　　　　　　　　　__**b**__ -9 + (-7)

b. -9 − 7　　　　　　　　　　__**d**__ -9 + 7

c. 9 − (-7)　　　　　　　　　__**c**__ 9 + 7

d. -9 − (-7)　　　　　　　　　__**a**__ 9 + (-7)

6. 💬 **Discuss:** How can addition be helpful when representing subtraction on a number line? Use the expressions from the previous problem if they help with your thinking.

Responses vary. **Subtraction on the number line can be rewritten as addition. I can start at a number and then go either left or right, so rewriting subtraction as addition helps me decide which way to go on the number line. -9 − 7 is the same as -9 + (-7), which means start at -9 and go 7 units left.**

Name: .... Date: .... Period: ....

Draw Your Own Diagram
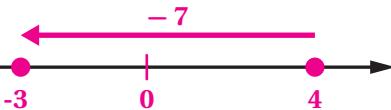
7. For each pair of expressions, complete the number lines and determine the value of each expression. *Work varies.*

a $7 - 4 =$ ....**3**.... $\qquad$ $4 - 7 =$ ....**-3**....
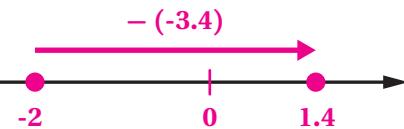
b $3 - (\text{-}9) =$ ....**12**.... $\qquad$ $\text{-}9 - 3 =$ ....**-12**....

c $\text{-}3.4 - (\text{-}2) =$ ....**-1.4**.... $\qquad$ $\text{-}2 - (\text{-}3.4) =$ ....**1.4**....

d $\text{-}3.4 + (\text{-}2) =$ ....**-5.4**.... $\qquad$ $\text{-}2 + (\text{-}3.4) =$ ....**-5.4**....

8. Describe 2–3 patterns you notice in the previous problem's number lines or their values.

Responses vary.

- I notice that when the order of the numbers is switched when subtracting, the value is the opposite of what it was before.
- I notice that the order of the numbers in the addition expression doesn't change the value.
- I notice that if the larger number is first in a subtraction problem, then the value is positive, but if the larger number is second, the value is negative.

Name: ....   Date: ....   Period: ....

Draw Your Own Conclusion

9. Luke-Josephine says: *The distance between x and y on a number line is $x - y$.*

💬 **Discuss:** Is this statement *always*, *sometimes*, or *never* true? How would you convince someone else?

Sometimes. Responses vary. If $x \geq y$, it's true. Otherwise, the distance will be the opposite of $x - y$.

For each problem, select one statement. Explain whether it is *always, sometimes,* or *never* true. Use examples, words, or number lines to support your claim.

10.   **Statement A**   **Statement B**   **Statement C**

$x + 1$ is positive. $x + y$ has the same value $x + (\text{-}x)$ is positive.
                              as $y + x$.

a Statement .... is (always/sometimes/never) true.

Statement A is sometimes true. Statement B is always true. Statement C is never true.

b My reasoning: *Responses vary.*
  - **Statement A: $x + 1$ is negative when $x = \text{-}3$, and $x + 1$ is positive when $x = 3$.**
  - **Statement B: $x + y = 0 + x + y$. So if you start at 0 and move x then move y on a number line, it's the same as starting at 0 then moving y then moving x since the arrows have the same distances and directions.**
  - **Statement C: When you add an opposite pair you always get 0, and 0 isn't positive.**

11.   **Statement D**   **Statement E**   **Statement F**

$x - y$ is the opposite x is less than $x + y$. $x - y$ is greater than
of $y - x$. $x + y$.

a Statement .... is (always/sometimes/never) true.

Statement D is always true. Statement E is sometimes true. Statement F is sometimes true.

b My reasoning: *Responses vary.*
  - **Statement D: If $x - y$ is added to $y - x$, the value is 0. This means $x - y$ and $y - x$ are an opposite pair.**
  - **Statement E: If y is positive, the statement is true. If y isn't positive, it's false.**
  - **Statement F: If y is negative, the statement is true. If y isn't negative, it's false.**

Explore More

12. Write your own statement about adding or subtracting positive and negative numbers that is either *always* or *never* true. Then trade statements with a classmate and decide if their statement is *always* or *never* true.

Responses vary.

Synthesis

13. What happens to the value of a subtraction expression when you rearrange the order of the numbers? Use the examples if they help with your thinking.

Responses vary. **When the order is switched, you get opposite values. For example, $7 - 4 = 3$ and $4 - 7 = $ -3, and 3 and -3 are opposites.**

$7 - 4$ and $4 - 7$

$-2 - (-3.5)$ and $-3.5 - (-2)$

Things to Remember:

Unit 5
Lesson

5

Name: _____ Date: _____ Period: _____

Number Puzzles

Let's solve some puzzles involving positive and negative numbers.

Warm-Up

1 The equation $1 + 2 = 3 + 4$ is false.

a Circle one or more numbers and change them to the opposite number to make the equation true.

Responses vary. **Sample shown on diagram. Students may also circle and add negative signs to 1 and 3.**

$$1 \ + \ \boxed{-2} \ = \ 3 \ + \ \boxed{-4}$$
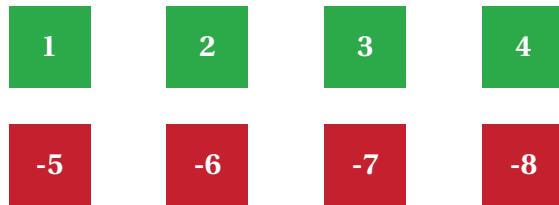
b Explain how you know your equation is true.

Responses vary. **Both sides of the equation equal -1.**

Name: _____ Date: _____ Period: _____

Number Puzzles

For each number puzzle:

- Make true equation(s) by using the numbers to fill in the blanks.
- Use each number only once per puzzle.
- You can circle a number and change it to its opposite. Try to circle as few numbers as possible.

1	**2**	**3**	**4**
-5	**-6**	**-7**	**-8**

2 ☐ + ☐ = ☐ − ☐

Responses vary.
Zero circles:
- $4 + (\text{-}5) = 2 - 3$
- $3 + (\text{-}6) = 1 - 4$

One circle:
- $1 + 2 = (\text{-}3) - (\text{-}6)$
- $2 + (\text{-}1) = 4 - 3$

Two circles:
- $1 + 5 = 8 - 2$

3 ☐ + **5** = ☐
     ☐ − ☐ = **9**

Responses vary.
Zero circles: Impossible
One circle:
- $\text{-}3 + 5 = 2$
  $4 - (\text{-}5) = 9$
- $(\text{-}7) + 5 = (\text{-}2)$
  $3 - (\text{-}6) = 9$
- $1 + 5 = 6$
  $2 - (\text{-}7) = 9$

4 Is it possible to solve the previous puzzle using 0 circles?

Yes   (No)   I'm not sure.
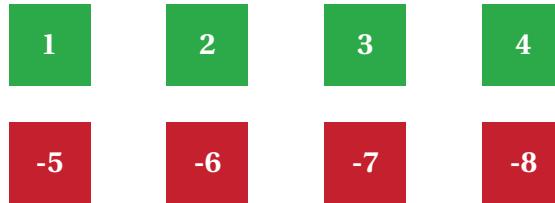
Explain your thinking.

***Explanations vary.* You can't even do the first line without circling because none of the cards are exactly five more than any of the others.**

Name: .... Date: .... Period: ....

Number Puzzles (continued)

For each number puzzle:

- Make true equation(s) by using the numbers to fill in the blanks.
- Use each number only once per puzzle.
- You can circle a number and change it to its opposite. Try to circle as few numbers as possible.

1	**2**	**3**	**4**
-5	**-6**	**-7**	**-8**

5 $\square + \square + \square = 0$

Responses vary.
Zero circles:
- $3 + 4 + (\text{-}7) = 0$

One circle:
- $1 + 3 + (\text{-}4) = 0$

6 $\square + \square + \square = 0$

$\square - \square - \square = 9$

Responses vary.
Zero circles:
- $3 + 4 + (\text{-}7) = 0$
  $(\text{-}5) - (\text{-}8) - (\text{-}6) = 9$

One circle:
- $2 + 4 + (\text{-}6) = 0$
  $(\text{-}3) - (\text{-}5) - (\text{-}7) = 9$

Two circles:
- $2 + 3 + (\text{-}5) = 0$
  $(\text{-}1) - (\text{-}6) - (\text{-}4) = 9$

Name: _____ Date: _____ Period: _____

More Number Puzzles

The previous puzzles used only **integers**. These puzzles do not. For each number puzzle:

- Make true equation(s) by using the numbers to fill in the blanks.
- Use each number only once per puzzle.
- You can circle a number and change it to its opposite. Try to circle as few numbers as possible.

7

| -0.5 | 1 | -1.5 | 2 |
| -2.5 | 3 | -3.5 | 4 |

$$\square + 5 + \square = \square$$

$$-4.5 - \square - \square = \square$$

Responses vary.

One circle:

- $(-0.5) + 5 + (-1.5) = 3$
  $(-4.5) - (-3.5) - (-2) = 1$

Two circles:

- $(-4) + 5 + 2 = 3$
  $(-4.5) - (-2.5) - (-3.5) = 1.5$
- $1 + 5 + (-4) = 2$
  $(-4.5) - (-3.5) - (-2.5) = 1.5$

8

| $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{1}{8}$ |
| $-\frac{3}{8}$ | $-\frac{5}{8}$ | $-\frac{7}{8}$ | -1 |

$$\square + \square - \square = 0$$

$$\square - \square + \square = 1$$

Responses vary.

Zero circles:

- $\left(-\frac{3}{8}\right) + \left(-\frac{5}{8}\right) - (-1) = 0$
  $\frac{3}{4} - \frac{1}{4} + \frac{1}{2} = 1$

One circle:

- $\frac{1}{8} + \left(-\frac{3}{4}\right) - \left(-\frac{5}{8}\right) = 0$
  $\left(-\frac{3}{8}\right) - \left(-\frac{7}{8}\right) + \frac{1}{2} = 1$

Two circles:

- $\left(-\frac{1}{8}\right) + \left(-\frac{1}{4}\right) - \left(-\frac{3}{8}\right) = 0$
  $\left(-\frac{5}{8}\right) - \left(-\frac{7}{8}\right) + \frac{3}{4} = 1$

Explore More

9 Use the Explore More Sheet to explore another number puzzle.

Responses vary. $\left(-\frac{1}{24}\right) + \left(-\frac{1}{12}\right) + \frac{1}{8} = 0$

10 Synthesis

Imagine subtracting a pair of numbers.

Describe how you can tell whether the result will be positive or negative.

$$\boxed{} - \boxed{} = \boxed{?}$$

Responses vary. **A bigger number minus a smaller number (like 3 — 2) will give a positive result. A smaller number minus a bigger number (like 2 — 3) will give a negative result. Things get tricky when both numbers are negative, though. For example, you have to remember that -4 is bigger than -10, so -4 — (-10) is positive while -10 — (-4) is negative.**

Things to Remember:

Name: _____ Date: _____ Period: _____

Explore More

Solve this puzzle in as many different ways as you can, using as many circles as you want.
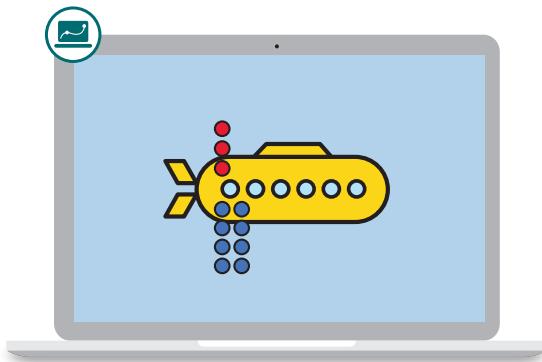
You can use each number only once per solution.

$-\dfrac{1}{48}$	$-\dfrac{1}{24}$	$-\dfrac{1}{16}$	$-\dfrac{1}{12}$
$\dfrac{1}{8}$	$\dfrac{1}{6}$	$\dfrac{1}{4}$	$\dfrac{1}{3}$

$$\boxed{} + \boxed{} + \boxed{} = \boxed{0}$$

My solutions:

Unit 5
Lesson
6

Name: _____ Date: _____ Period: _____
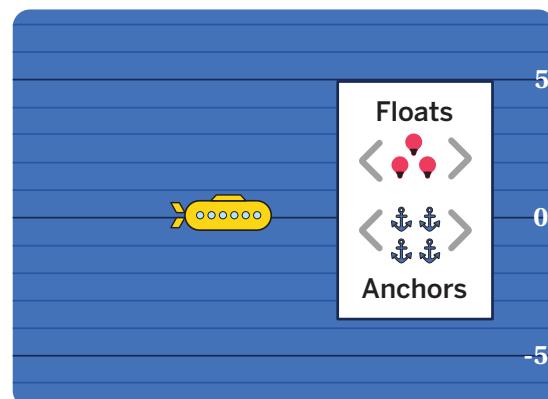
Floating in Groups

Let's use floats and anchors to make sense of multiplying integers.

Warm-Up

1 This submarine is controlled by *groups* of floats and anchors.
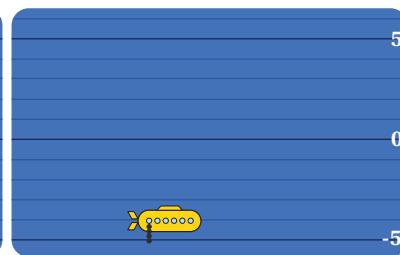
a Take a look at these different combinations.

Add 1 group of 3 floats

Add 1 group of 4 anchors

Add 1 group of 3 floats
Add 1 group of 4 anchors

b Choose a number. How can you add groups of 3 floats and 4 anchors to get the submarine to that number?

Responses vary.
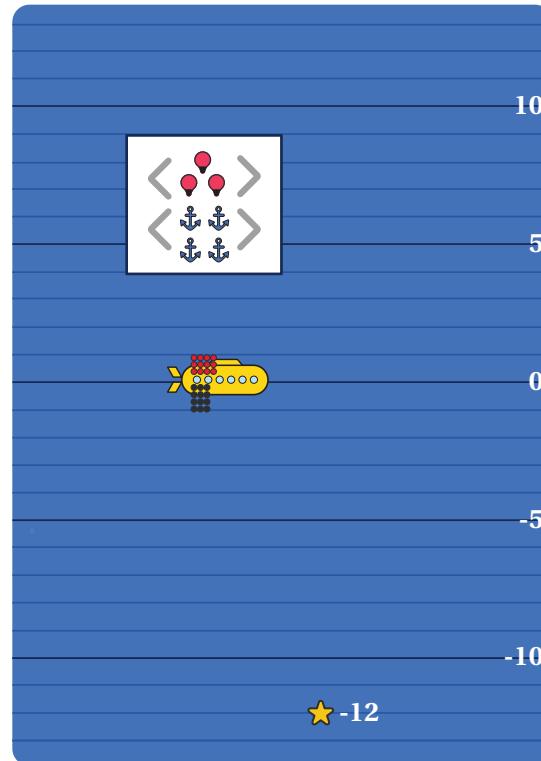
Name: _____ Date: _____ Period: _____

Star Power

2–3 This submarine starts at an elevation of 0 units. Floats can be added or removed in groups of 3. Anchors can be added or removed in groups of 4.
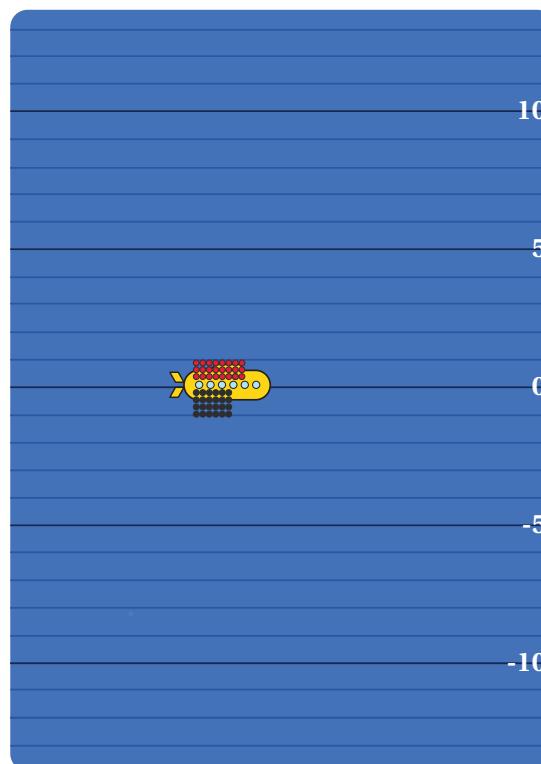
The table shows one way to get the submarine to -12 to collect the star. Write two more actions to get the submarine to -12. *Responses vary.*

Start	Action	Final
0	Add 3 groups of 4 anchors	-12
0	**Add 4 groups of 3 floats and 6 groups of 4 anchors**	-12
0	**Remove 4 groups of 3 floats**	-12

4 This submarine starts at 0 units.

Where will it end up after removing 6 groups of 3 floats?

-18 units

Star Power (continued)

5 Demetrius wrote this equation to help him solve the situation from the previous problem:
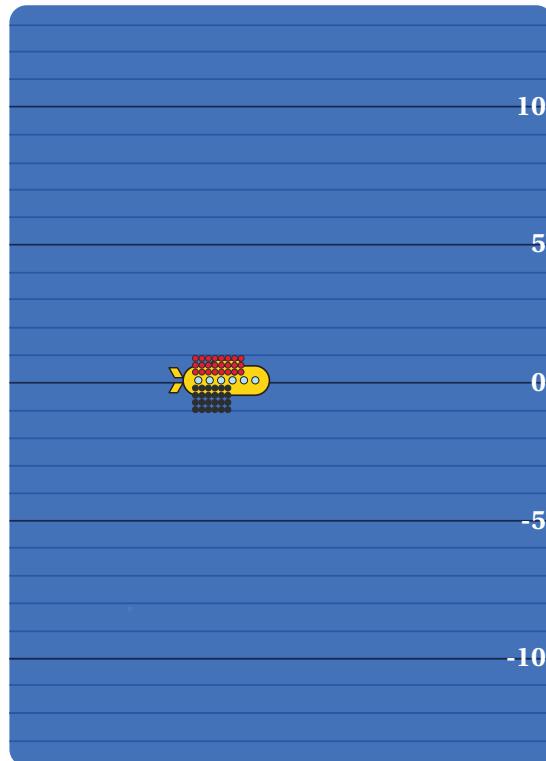
$(-6) \cdot 3 = -18$

Explain what each number represents in the situation. *Explanations vary.*

-6 represents . . . **removing 6 groups. The 6 is negative because they are being removed.**

3 represents . . . **how many floats are in each group.**

-18 represents . . . **the end position of the submarine.**

6 This submarine starts at 0 units.

Where will it end up after removing 5 groups of 4 anchors? **20 units**

7 Callen and Demetrius each wrote an expression to help solve this submarine situation.
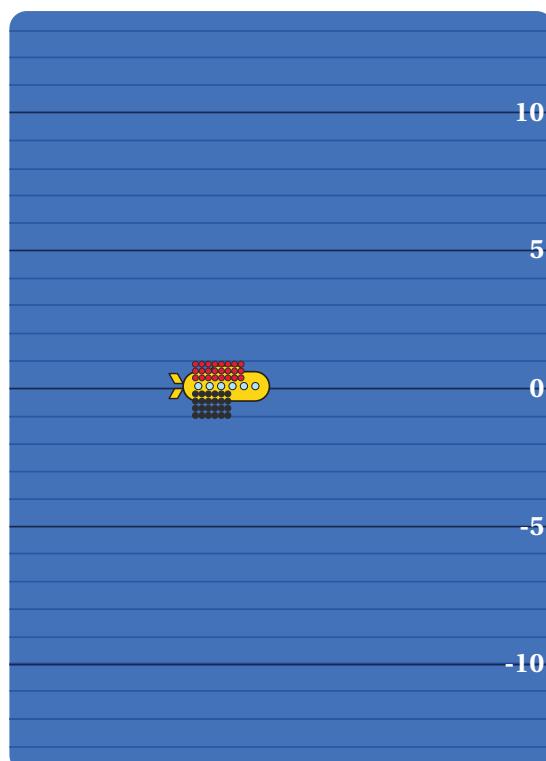
Callen's expression: $0 - 5 \cdot (-4)$

Demetrius's expression: $(-5)(-4)$

Explain why each person's expression makes sense. *Responses vary.*

Callen's expression makes sense because . . .
the submarine starts at 0 and then 5 groups of 4 anchors are removed. Anchors are negative, so the 4 is negative.

Demetrius's expression makes sense because . . . **-5 is like removing 5 groups and -4 is like 4 anchors.**

Name: _____ Date: _____ Period: _____
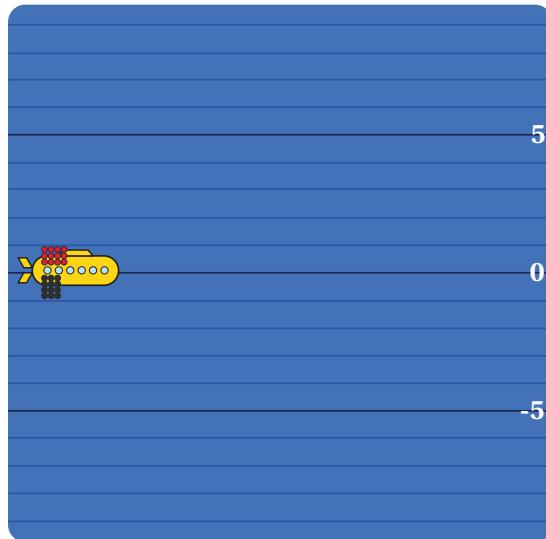
Underwater Expressions

8 What is the value of the expression 2 • (-4)?

-8

Explain your thinking. Use this submarine if it helps with your thinking.

Explanations vary. **This is like adding 2 groups of 4 anchors, so the submarine will go down to -8 units.**

9 Group the expressions into pairs that have the same value. Some expressions will have no match.

6 • (-2)	-6 • (-4)	4 • (-4)	(-2) • (-12)
(-8) • (-2)	-3 • 4	4 • 4	8 • (-3)

Pair 1	Pair 2
-6 • (-4)	**(-8) • (-2)**
(-2) • (-12)	**4 • 4**

Pair 3	Expressions With No Match
6 • (-2)	**4 • (-4)**
-3 • 4	**8 • (-3)**

Explore More

10 Use the digital activity to collect as many stars as you can.
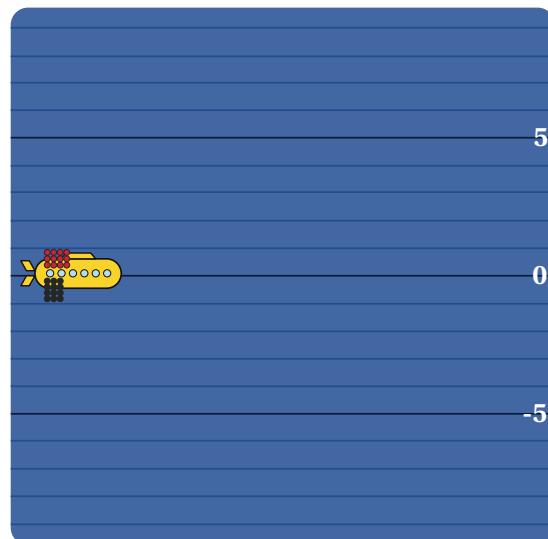
Responses vary. **It is possible to collect all the stars. Here are three examples:**
- **To collect the star at 3 units, add 1 group of floats.**
- **To collect the star at -2, add 2 groups of floats and 2 groups of anchors.**
- **To collect the star at 5, add 3 groups of floats and 1 group of anchors.**

11 Synthesis

Use the floats and anchors scenario to explain why it makes sense that (-2)(-4) is positive.
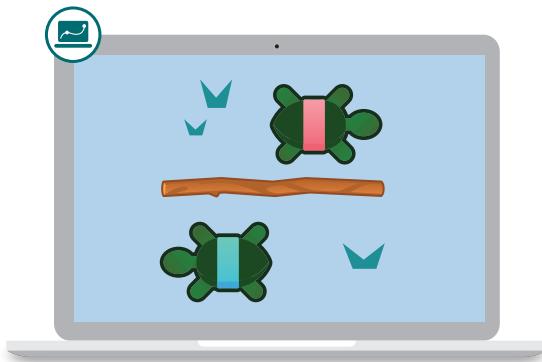
Responses vary. **The -2 is like removing 2 groups and the -4 represents 4 anchors. If you remove anchors, the submarine goes up, so if you remove 2 groups of 4 anchors, the submarine goes up by 8.**

Things to Remember:

Unit 5
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Name: _____ Date: _____ Period: _____
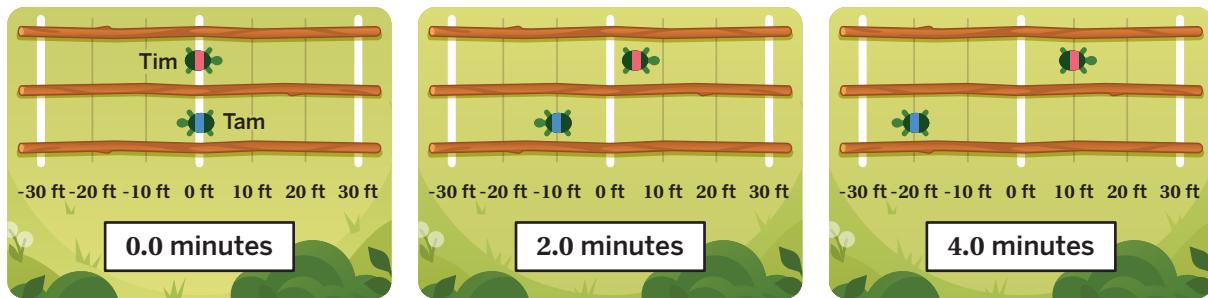
Back in Time

Let's use position, rate, and time to represent multiplying positive and negative numbers.

Warm-Up

1 Here are a few moments from an animation.

Tim 🐢 🐢 Tam -30 ft -20 ft -10 ft 0 ft 10 ft 20 ft 30 ft **0.0 minutes**	🐢 🐢 -30 ft -20 ft -10 ft 0 ft 10 ft 20 ft 30 ft **2.0 minutes**	🐢 🐢 -30 ft -20 ft -10 ft 0 ft 10 ft 20 ft 30 ft **4.0 minutes**
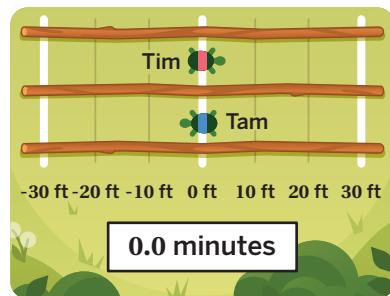
Write a story about Tim and Tam.

Responses vary. **Tim and Tam are taking some time to process an argument they just had, so they are walking away from each other toward their houses. Tam is walking faster than Tim is.**

Name: .... Date: .... Period: ....

Time for Turtles

2 Tim and Tam each walk at a constant rate.
Complete the last row of the table.

Time (min)	Tim's Position (ft)	Tam's Position (ft)
0	0	0
1	2.5	-5
2	5	-10
3	7.5	-15
. . .	. . .	. . .
6	**15**	**-30**

3 Tim's walking rate is 2.5 feet per minute.

What is Tam's walking rate?

-5 feet per minute

Explain your thinking.

Explanations vary.
- **Each minute, Tam moves 5 feet to the left.**
- **After moving for 1 minute, Tam was at position -5 feet.**
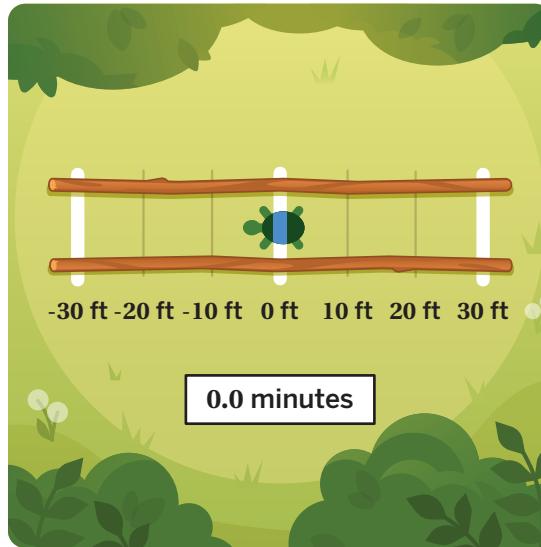
Name: _____ Date: _____ Period: _____

Time for Turtles (continued)

Use the value you calculated for Tam's walking rate for the following problems.

4 What will Tam's position be in 3.2 minutes?

-16 feet

5 What was Tam's position 3.2 minutes ago?

16 feet

6 One student wrote this equation to determine Tam's position on the previous problem:
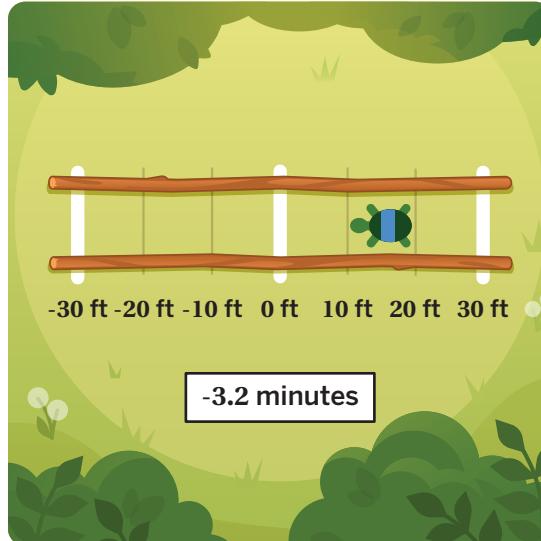
$(-5)(-3.2) = 16$

Explain what each number represents in the situation. ***Responses vary.***

-5 represents . . . **that Tam is walking left 5 feet every minute.**

-3.2 represents . . . **3.2 minutes back in time.**

16 represents . . . **where Tam was at that time.**

Name: _____ Date: _____ Period: _____

Positives and Negatives

7 For each expression, put a check for whether it has a positive or negative value.

Expression	Positive	Negative
$(-7) \cdot (-8)$	✔	
$(-7) \cdot 8$		✔
$3.5 \cdot (-12)$		✔
$(-3.5) \cdot (-12)$	✔	
$\left(\frac{1}{2}\right)(-20)$		✔
$\left(-\frac{1}{2}\right)(-20)$	✔	

Name: .... Date: .... Period: ....

Turtles Through Time

8 Tommy walks at a rate of -2 feet per minute.
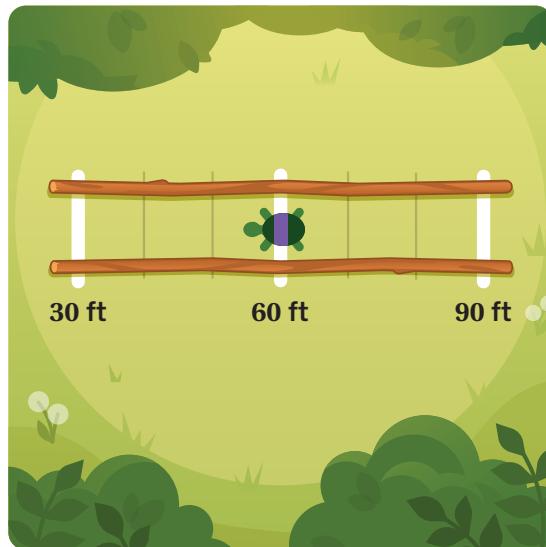
Right now he is at 60 feet.

Where was Tommy 10 minutes ago?
80 feet

30 ft   60 ft   90 ft

Explain how you know.
Explanations vary. **Tommy was at 80 feet because he started 20 feet to the right of where he is now.**

9 Select *all* the expressions that describe where Tommy was 10 minutes ago.

☐ **A.** $60 + 2 \cdot (-10)$

☑ **B.** $60 + (-2)(-10)$

☐ **C.** $60 - 20 \cdot 10$

☐ **D.** $60 - 20$

☑ **E.** $60 + 20$

Explore More
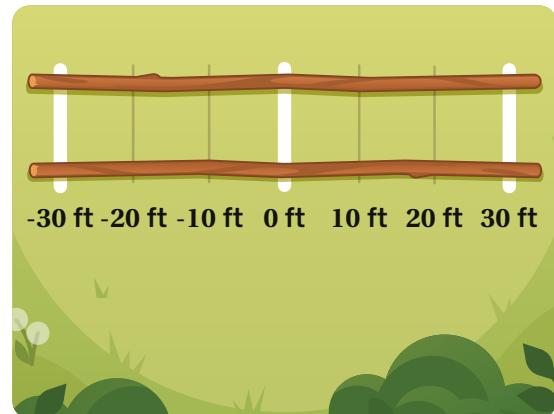
10 Use the Explore More Sheet to explore another turtle puzzle.
Responses vary. **See the Teacher Edition for sample responses.**

11 Synthesis

Use a situation with turtles starting at 0 to explain why 5 • (-2) is negative and (-5) • (-2) is positive.

Responses vary. **5 represents a walking rate to the right, while -5 represents a walking rate to the left. -2 represents where a turtle was 2 minutes ago. Starting from 0, 2 minutes ago, the turtle with a walking rate of 5 was at -10, and the turtle with a walking rate of -5 was at 10.**

-30 ft -20 ft -10 ft   0 ft   10 ft   20 ft   30 ft

Things to Remember:

Name: _____ Date: _____ Period: _____

Explore More
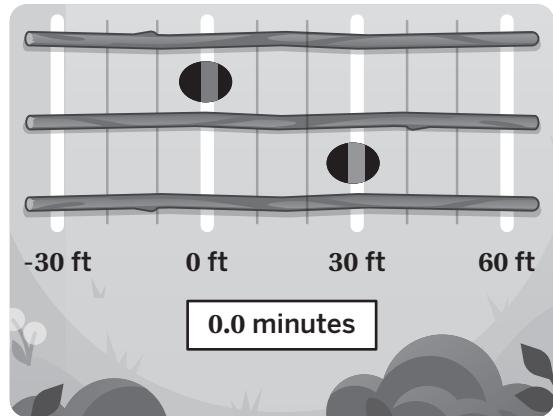
5 minutes ago, these turtles were at the same position.

What could their walking rates be?

Determine several possible pairs of rates.

-30 ft   0 ft   30 ft   60 ft

0.0 minutes

Pair 1:

Turtle	Position (ft)	Rate (ft/min)
Top	0	
Bottom	30	

Pair 2:

Turtle	Position (ft)	Rate (ft/min)
Top	0	
Bottom	30	

Pair 3:

Turtle	Position (ft)	Rate (ft/min)
Top	0	
Bottom	30	
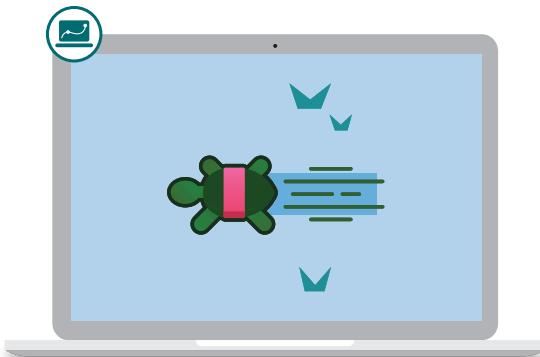
Pair 4:

Turtle	Position (ft)	Rate (ft/min)
Top	0	
Bottom	30	

Unit 5
Lesson

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Name: _____   Date: _____   Period: _____

Speeding Turtles

Let's use position, rate, and time to represent dividing integers.

Warm-Up

1 Which one doesn't belong? Circle one.

$3 \cdot 2$ $-\dfrac{12}{2}$

$(-3)(-2)$ $12 \div 2$

Explain your thinking.

Responses and explanations vary.
- **Top left: It's the only expression with two positive numbers multiplied together.**
- **Top right: It's the only expression that is equal to -6.**
- **Bottom left: It's the only expression that has two negative signs.**
- **Bottom right: It's the only expression that includes a division symbol (but it's not the only expression involving division).**

Name: _____ Date: _____ Period: _____

Return of the Turtles

2 Tim walks -8 feet per minute for 3.25 minutes.

Where will Tim end up?

Rate (ft/min)	Time (min)	Position (ft)
-8	3.25	**-26**
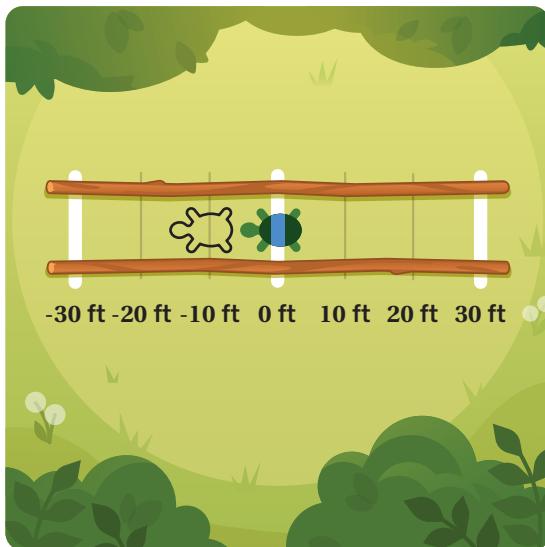
3 Tam walked -4 feet per minute.

She began at 0 feet and ended up at -10 feet.

For how much time was Tam walking?

Rate (ft/min)	Time (min)	Position (ft)
-4	**2.5**	-10

Activity

1

Name: .... Date: .... Period: ....
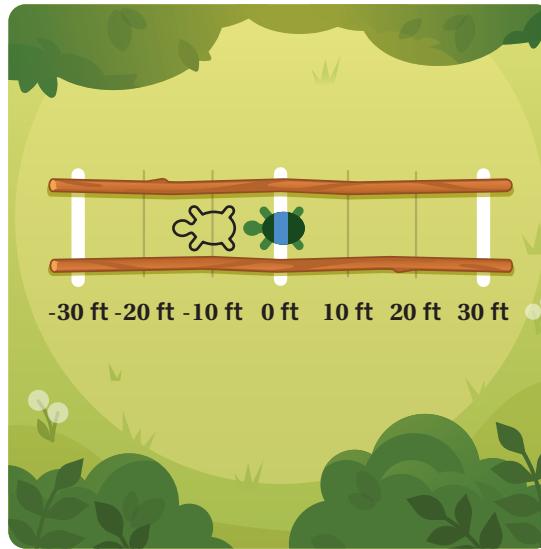
Return of the Turtles (continued)

4 Fabiana wrote this equation to determine the time from the previous problem:

$$\frac{-10}{-4} = t$$

Explain why her equation makes sense.
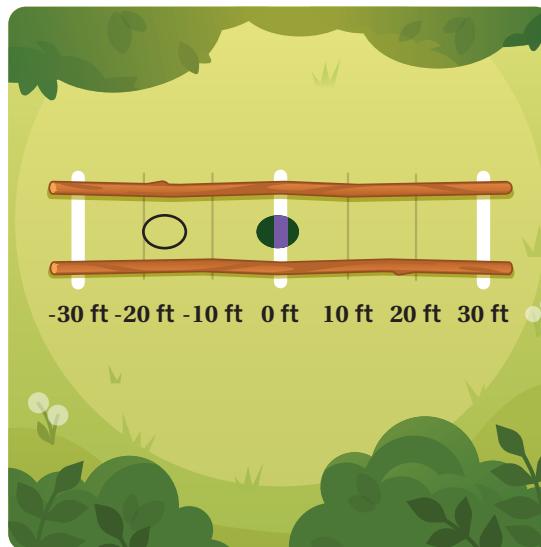
Responses vary. **Her equation makes sense because division is the reverse of multiplication, so to figure out what times -4 makes -10, you would need to use division.**

-30 ft -20 ft -10 ft 0 ft   10 ft 20 ft 30 ft

5 Tommy walks -17 feet in 5 minutes.

What is Tommy's walking rate?

Rate (ft/min)	Time (min)	Position (ft)
$\frac{-17}{5}$ (or equivalent)	5	-17

-30 ft -20 ft -10 ft 0 ft   10 ft 20 ft 30 ft

Name: _____ Date: _____ Period: _____
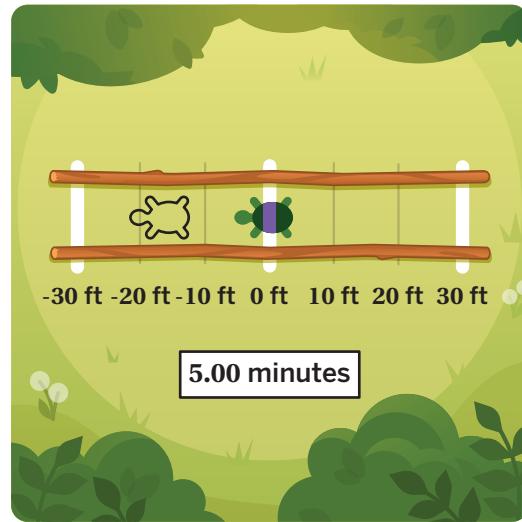
Turtle Express

6 Tommy walks -17 feet in 5 minutes.

Select *all* the equations that represent this situation, where r represents Tommy's walking rate.

☑ **A.** $r \cdot 5 = -17$

☐ **B.** $(-17) \cdot 5 = r$

☑ **C.** $r = -\dfrac{17}{5}$

☑ **D.** $r = \dfrac{-17}{5}$

☐ **E.** $r = \dfrac{5}{-17}$

-30 ft -20 ft -10 ft 0 ft 10 ft 20 ft 30 ft

5.00 minutes

7 Group the expressions that have the same value. One expression will have no match.

$6 \div 3$	$-\dfrac{3}{6}$	$-\dfrac{6}{3}$	$\dfrac{-3}{6}$
$\dfrac{-3}{-6}$	$\dfrac{-6}{-3}$	$3 \div (-6)$	$\dfrac{6}{-3}$

Group 1	Group 2	Group 3
$\dfrac{-6}{-3}$ $6 \div 3$	$-\dfrac{3}{6}$ $\dfrac{-3}{6}$ $3 \div (-6)$	$-\dfrac{6}{3}$ $\dfrac{6}{-3}$

Explore More

8 Determine a current position and rate for three new turtles so that in 5 minutes all three turtles will be in the same position.

No two turtles should have the same starting position and rate as each other.
Responses vary.

Turtle	Position (ft)	Rate (ft/min)
A	3	0
B	-7	2
C	18	-3

A
B
C

-30 ft -20 ft -10 ft 0 ft 10 ft 20 ft 30 ft

0.0 minutes

9 Synthesis

Explain why these expressions have the same value.

Responses vary. **They are both equal to -2. The top one is the opposite of $\frac{6}{3}$, which is the opposite of 2. The bottom one is 6 divided by -3, which is like walking at a rate of -3 and ending up at 6. You have to go back in time.**

$$-\frac{6}{3}$$

$$\frac{6}{-3}$$

Things to Remember:

Unit 5

Lesson

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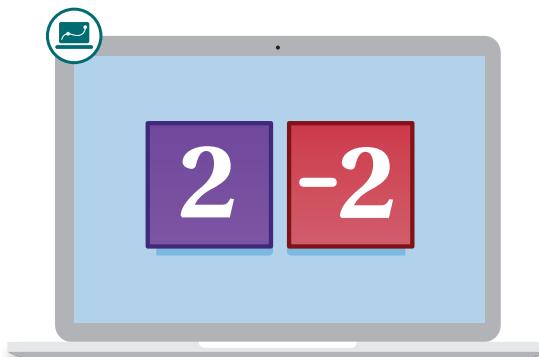
Name: _____ Date: _____ Period: _____

Integer Puzzles

Let's reason about the signs and values of integer expressions using the four operations.

2 **-2**

Warm-Up

1 **a** Make a true inequality by using these numbers to fill in the blanks.

1 **-2** **3** **-4**

☐ × ☐ < ☐ × ☐

Responses vary.
- $1 \times 3 < (-2) \times (-4)$
- $3 \times (-4) < 1 \times (-2)$

b Explain how you know this inequality is true.

Explanations vary. **A negative times a negative is positive, so $1 \times 3 = 3$ and $(-2) \times (-4) = 8$. 3 is less than 8.**

Name: _____ Date: _____ Period: _____

Puzzling Values

For each number puzzle:

- Fill in the blanks with these numbers.
- You may use each number once per puzzle.

| -1 | 2 | 3 | -4 | -5 | 6 | 7 | -8 |

2 Make this inequality true.

$$\square \left(\square + \square \right) > \boxed{0}$$

Responses vary.
- $2(3 + 6) > 0$
- $-1(-4 + 3) > 0$
- $3(-5 + 6) > 0$

3 **a** Make an expression.

$$\square \left(\square + \square \right)$$

Responses vary. -8(-4 + (-5))

b What is the value of your expression? Can you create an expression with this puzzle that has a greater value?

Responses vary. -8(-4 + -5) = 72

4 Josiah and Wey Wey made this expression. They used different strategies to determine the value of the expression.

| -5 | (| 2 | + | -4 |) |

Josiah

$(-5)(2) + (-5)(-4)$
$-10 + 20$
10

| -5 | (| 2 | + | -4 |) |

Wey Wey

$(-5)(2 - 4)$
$(-5)(-2)$
10

💬 **Discuss:** What do you notice? What do you wonder?

Responses vary.
- **I notice that Josiah uses the distributive property and multiplies -5 by both numbers, 2 and -4, inside the parentheses and then he adds the two together.**
- **I notice that Wey Wey calculates the sum of the numbers, 2 and -4, inside the parentheses and then multiplies this sum by -5.**
- **I wonder if the distributive property always works with both positive and negative numbers.**
- **I wonder if there are always two strategies for solving these types of problems.**

Name: _____ Date: _____ Period: _____

Greatest Puzzling Values

For each number puzzle:

- Fill in the blanks with these numbers.
- You may use each number once per puzzle.

| 1 | -2 | 3 | -4 | 5 | -6 | 7 | -8 |

5 **a** Make an expression with the greatest possible value.

☐ × ☐ + ☐ × ☐

Responses vary.

b What is the value of your expression?

Responses vary. **Greatest possible value:** $5 \times 7 + (-6) \times (-8) = 83$

6 Kiri created this expression which has a value of 26.

| 7 | × | 3 | + | 1 | × | 5 |

Write a hint or question to help her create an expression with an even greater value.

Responses vary.
- **Try using the biggest numbers together. 7 groups of 5 is bigger than 7 groups of 3.**
- **Remember that a negative number times a negative number is a positive number.**

7 Fill in the blanks with these numbers.

| -1 | 2 | 3 | -4 | -5 | 6 | 7 | -8 |

a Make an expression with the greatest possible value.

Responses vary.

$$\frac{\boxed{} - \boxed{}}{\boxed{} + \boxed{}}$$

b What is the value of your expression?

Responses vary. **Greatest possible value:** $\frac{7 - (-8)}{2 + (-1)} = 15$

Name: .... Date: .... Period: ....

Greatest Puzzling Values (continued)

8 Fill in the blanks with these numbers.

| 2 | 3 | 4 | 5 | -2 | -3 | -4 | -5 |

a Make an expression with the greatest possible value.

$$\frac{\boxed{} + \boxed{} \times \boxed{}}{\boxed{} - \boxed{}}$$

Responses vary.

b What is the value of your expression?

Responses vary.

Greatest possible value: $\frac{-4 + 5 \times (-5)}{2 - 3} = 29$

Second greatest possible value: $\frac{5 + (-5) \times (-4)}{3 - 2} = 25$

Explore More

9 Here are six copies of a new puzzle. Solve it in as many ways as you can.

| -1 | -2 | -3 | -4 | 5 | 6 | 7 | 8 |

Responses vary. **Three responses are shown.**

$-2 \left(\boxed{5} + \boxed{6} \right) = \boxed{-3} \times \boxed{7} + \boxed{-1}$

$5 \left(\boxed{-1} + \boxed{-3} \right) = \boxed{8} \times \boxed{-2} + \boxed{-4}$

$7 \left(\boxed{-4} + \boxed{6} \right) = \boxed{-2} \times \boxed{-3} + \boxed{8}$

$\boxed{} \left(\boxed{} + \boxed{} \right) = \boxed{} \times \boxed{} + \boxed{}$

$\boxed{} \left(\boxed{} + \boxed{} \right) = \boxed{} \times \boxed{} + \boxed{}$

$\boxed{} \left(\boxed{} + \boxed{} \right) = \boxed{} \times \boxed{} + \boxed{}$

🔟 Synthesis

Describe something you learned today that might help someone else as they complete puzzles like these.

Responses vary.
- I learned that I can make a fraction's value bigger by making the denominator smaller.
- I remembered that I can make a large positive number by multiplying two large negative numbers.

$$\square \, (\square + \square)$$

-1	2	3	-4
-5	6	7	-8

$$\square \times \square + \square \times \square$$

1	-2	3	-4
5	-6	7	-8

$$\frac{\square + \square \times \square}{\square - \square}$$

2	3	4	5
-2	-3	-4	-5

$$\frac{\square - \square}{\square + \square}$$

-1	2	3	-4
-5	6	7	-8

Things to Remember:

Name: _____ Date: _____ Period: _____

Puzzling Values

Use this space to record all of your attempts and thinking as you work on each puzzle. After each attempt, think about what you learned and how your strategy might change on the next attempt.

Puzzle #1	Puzzle #2

Use this space to record things you learned while solving these puzzles, including advice to yourself or others.

Name: _____   Date: _____   Period: _____

Lesson 5.10
Activities 1–2
(p. 2 of 2)

Puzzling Values

Use this space to record all of your attempts and thinking as you work on each puzzle. After each attempt, think about what you learned and how your strategy might change on the next attempt.

Puzzle #3   **Puzzle #4**

Explore More

Use this space to record things you learned while solving these puzzles, including advice to yourself or others.

Unit 5
Lesson

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Name: _____ Date: _____ Period: _____

Temperatures Around the World
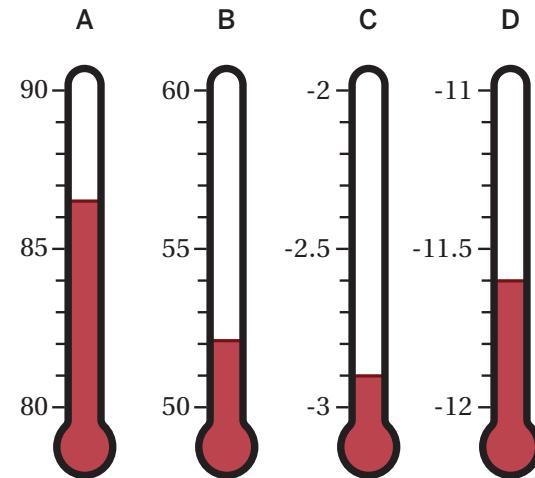
Let's add and subtract positive and negative numbers to compare temperatures and ice mass.

Warm-Up

1 What is the approximate temperature on each thermometer? *Responses vary.*

Thermometer	Temperature (°F)
A	**86.5**
B	**52.1**
C	**-2.9**
D	**-11.6**

A	B	C	D
90	60	-2	-11
85	55	-2.5	-11.5
80	50	-3	-12

All temperatures are in °F.

2 These temperatures are the average annual temperatures of different locations as of 2014. Match each temperature to a location and record it on the map.

💬 **Discuss** your choices with a partner.

Responses vary. **I remembered learning that Antarctica is the coldest place on Earth, so I put -11.6°F there.**

Greenland
-2.9 °F

New York City
52.1 °F

Khartoum
86.5 °F

-11.6 °F
Antarctica

Name: _____ Date: _____ Period: _____

Changing Temperatures

3 How do you think the average annual temperatures of the places shown in the Warm-Up have changed over the last century? Circle one.

Gotten warmer   Gotten colder   Stayed the same

Explain your thinking.

Responses and explanations vary.
- **It doesn't feel like the temperatures where I live have changed much in my lifetime, so I think they stayed about the same.**
- **I think it has gotten warmer over time because I've noticed more warm days than what I remember from years ago.**

4 This table shows how the average temperature in these places have changed from 1960 to 2014. Complete the table.

Location	Average Temperature in 1960 (°F)	Average Temperature in 2014 (°F)	Change From 1960 to 2014 (°F)
NYC, U.S.	49.8	52.1	2.3
Khartoum, Sudan	84.6	86.5	**1.9**
Greenland	-6	-2.9	**3.1**
Antarctica	**-12.9**	-11.6	1.3

Source: *Berkeley Earth*

5 Look at the column "Change From 1960 to 2014."

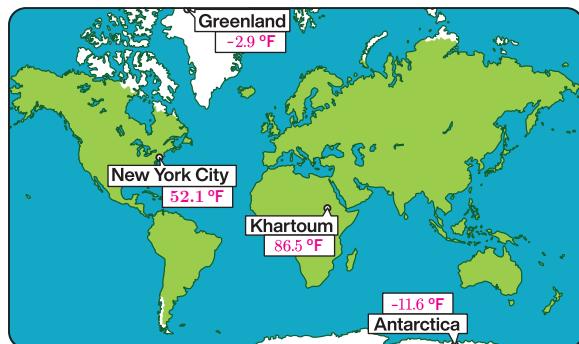**a** What do you notice?

Responses vary.
- **I notice that all of the changes are positive, so that means it's getting warmer.**
- **I notice that the change in temperature was the biggest in Greenland.**

b What might be some impacts of these temperature changes?

Responses vary.
- **If it's getting warmer, then the ice will melt in lots of places.**
- **One impact could be that plants and animals are not comfortable in the same places anymore.**

Slippery Slope

6 One impact of rising temperatures is that the amount of ice in the world is decreasing each year.
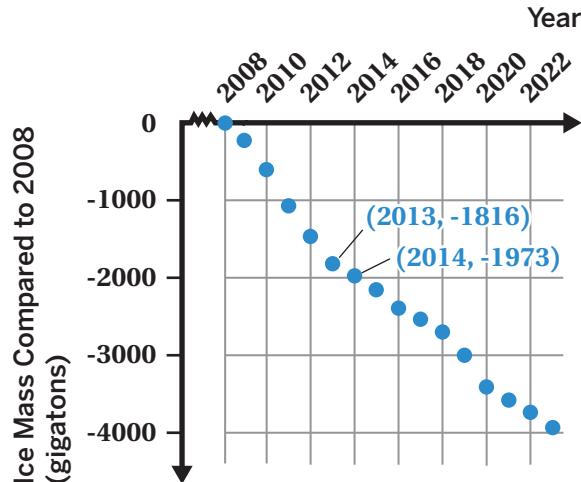
Greenland is mostly covered by ice.

This graph shows the mass of ice in Greenland in different years compared to the mass in 2008.

Tell a story about this graph.

Include the point (2013, -1816) in your story.

Responses vary. **The ice in Greenland is melting! In only 5 years, it decreased by 1,816 gigatons. By 2019, it lost about 3,000 gigatons! Who knows what will happen in the future.**

Year

(2013, -1816)
(2014, -1973)

Source: *Climate.NASA.gov*

7 What is the approximate change in Greenland's ice mass from 2013 to 2014?

Show whether the change is positive or negative.

-157 gigatons

Name: ....   Date: ....   Period: ....

Slippery Slope (continued)

8 During which time period was ice melting fastest? Circle one.

(2008 to 2013)
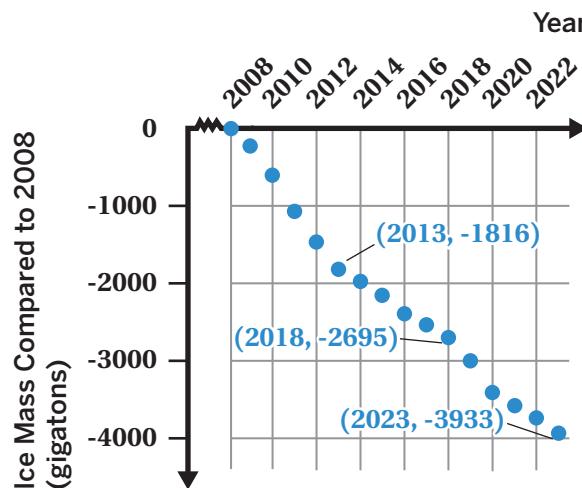
2013 to 2018

2018 to 2023

Explain your thinking.
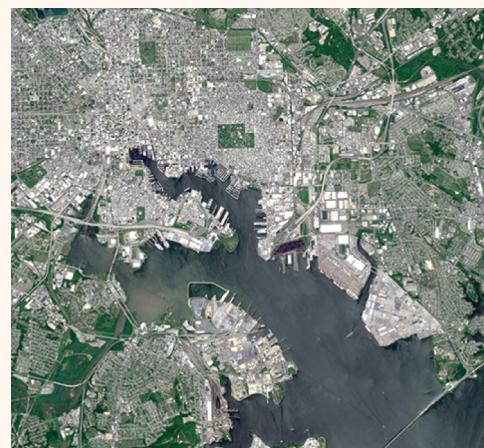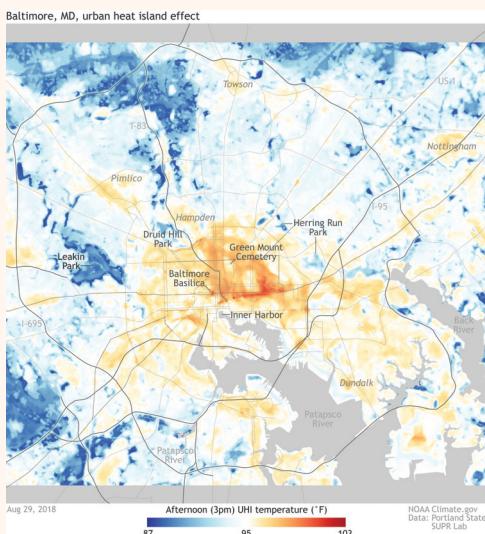
Explanations vary.

- **That part of the graph is really steep, which means the amount of ice was changing the fastest.**
- **The amount of ice changed by -1,816 between 2008 and 2013, by -879 between 2013 and 2018, and by -1,238 between 2018 and 2023. The period from 2008 to 2013 had the most change.**

Graph: Year (x-axis: 2008, 2010, 2012, 2014, 2016, 2018, 2020, 2022) vs Ice Mass Compared to 2008 (gigatons) (y-axis: 0, -1000, -2000, -3000, -4000). Labeled points: (2013, -1816), (2018, -2695), (2023, -3933).

Explore More

9 In this activity, we've looked at average temperatures across entire regions. Temperatures can also vary between different parts of a small local area, even neighborhood to neighborhood. Here are two maps of Baltimore, Maryland. One shows the temperatures at 3 PM on August 29, 2018 and the other shows the satellite view.

Baltimore, MD, urban heat island effect

Aug 29, 2018   Afternoon (3pm) UHI temperature (°F)   87   95   103   NOAA Climate.gov Data: Portland State SUPR Lab

Sources: *NOAA, Portland State SUPR Lab*

What story might this data tell?

Responses vary. **The parts on the map where there is less green are also the hottest. Maybe green things like trees and plants keep places cooler.**

🔟 Synthesis

Select one question. Then record your response.

☐ Why is it important to study the temperature in places both near and far from where you live?

☐ How can positive and negative numbers help us make sense of topics like changing temperatures and ice mass?

☐ What other questions might you ask to investigate these topics further?

Responses vary.
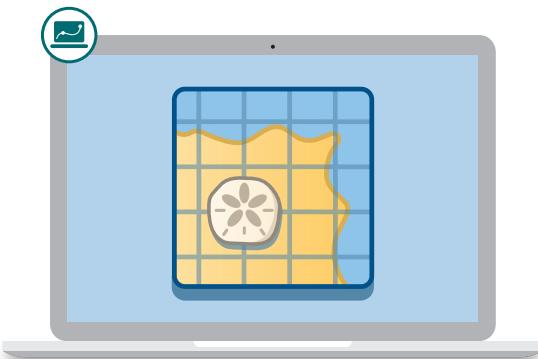
- It's important to study the temperature where I live to help me understand the impact my choices (and the choices of those around me) have on the plants, animals, and people in my region. It's also important to study the temperature in places far away because residents of those places might feel the impact of the choices we make before I do.

- Knowing how to compare, add, and subtract positive and negative numbers will better equip me to interpret data about changing temperatures and ice mass. When I understand this data more deeply, I can be intentional in my choices that impact these issues.

- What impact does the changing temperature and ice mass have on plants and animals in Greenland? How does the changing temperature and ice mass impact places much closer to the equator? Is there a relationship between changing temperatures and severe weather?

Things to Remember:

Unit 7
Lesson

10

Name: .... Date: .... Period: ....

Sand Dollar Search

Let's explore negative numbers on the coordinate plane.
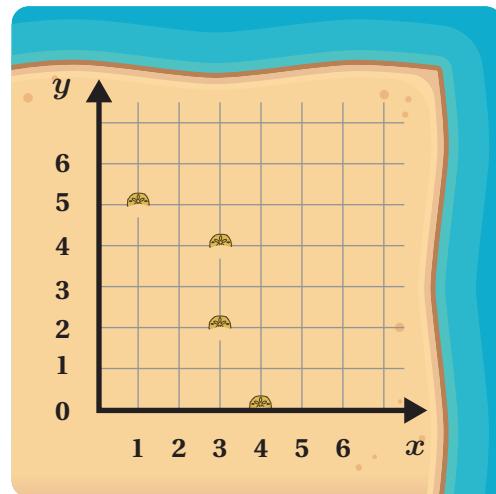
Warm-Up

1 Here is a map of part of an island.

Collect all four sand dollars by labeling each one with its coordinates.

(4, 0), (3, 2), (3, 4), (1, 5)

Name: .... Date: .... Period: ....
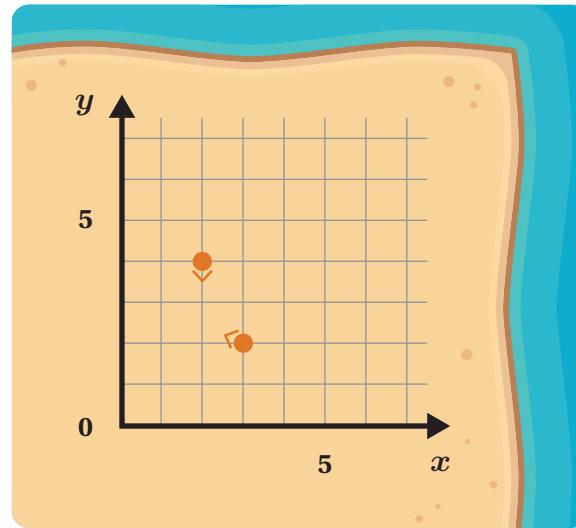
Find the Sand Dollar

2 Another sand dollar is buried on the island.

Here are two guesses for where the sand dollar is. Write coordinates for where you would search for it.

Responses vary.

Coordinates (x, y)
$(3, 2)$
$(2, 4)$
(2, 3)

3 You find a new map of the entire island.

A new sand dollar is visible. Write a clue that describes its location.

Responses vary.

- **The sand dollar is 4 units to the left and 2 units up from (0, 0).**
- **The sand dollar is 6 units to the left and 1 unit down from the first sand dollar.**
- **The new sand dollar has an x-coordinate of -4 and a y-coordinate of 2.**
- **The sand dollar is at (-4, 2).**

Name: _____   Date: _____   Period: _____

The Search Continues

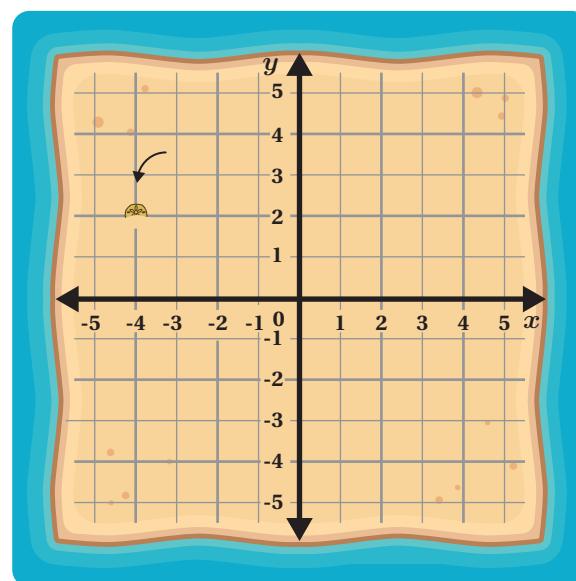**4** There is another sand dollar buried somewhere on the island.

Here are three guesses for where the sand dollar is. Write coordinates for where you would search for it. *Responses vary.*
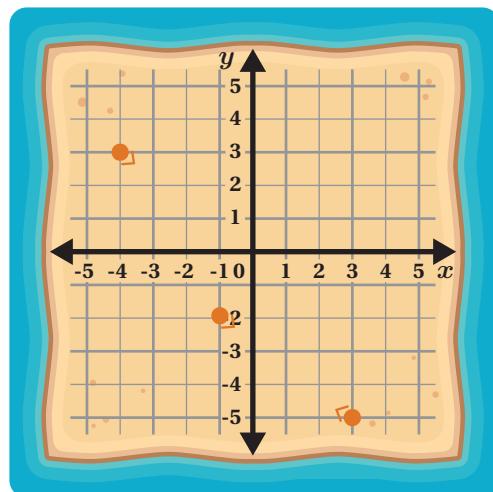
Coordinates (x, y)
(-4, 3)
(-1, -2)
(3, -5)
(2, -4), (1, -3), (0, -3)
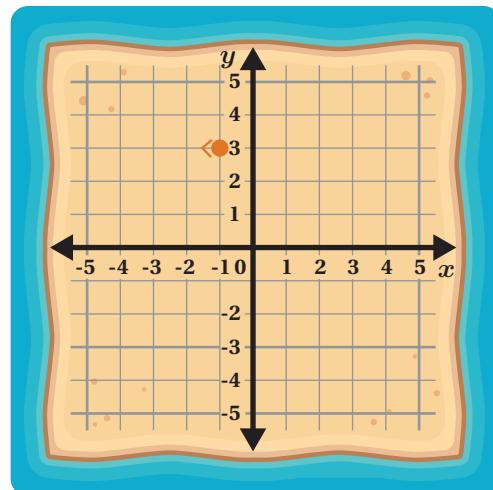
5 Esi was searching for another sand dollar. She entered (-1, 3) and the arrow pointed to the left.

What point would you recommend Esi try next? Explain your thinking.

Responses and explanations vary. **(-3, 3). This point is kind of in the middle between the left edge of the island and Esi's point.**

6 A sand dollar is hidden at a location that has two negative coordinates.
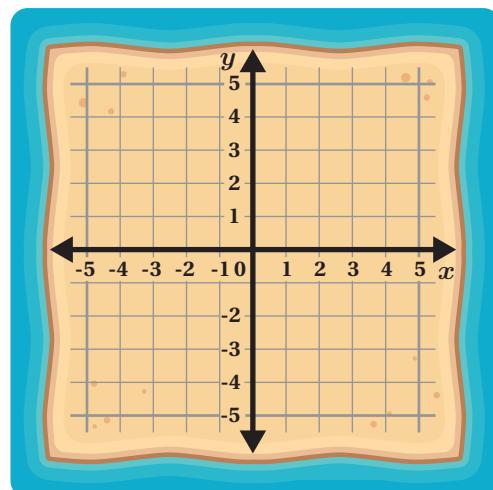
a Draw the sand dollar at a possible location.

Responses vary. **All correct sand dollars will be in Quadrant 3.**

Compare your point with a partner.

b **Discuss:** What do your points have in common?

Responses vary. **They go left on the x-axis and down on the y-axis.**

Name: _____ Date: _____ Period: _____

Find That Sand Dollar!
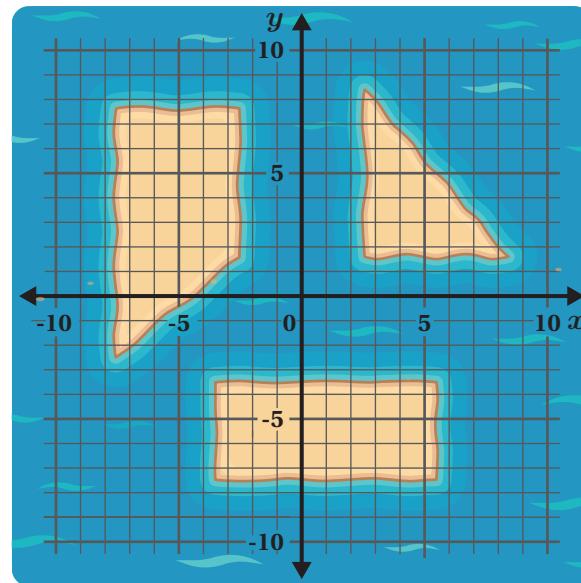
You will use support cards for this activity.

7 **Round 1:** There is a sand dollar on each of these islands.

When it is your turn to find the sand dollar, label each guess and the feedback you get until you find all three sand dollars.

Partner A: Sand dollars are at (-7, -1), (5, 4), and (0, -7).
Partner B: Sand dollars are at (-7, -2), (5, 2), and (0, -5).
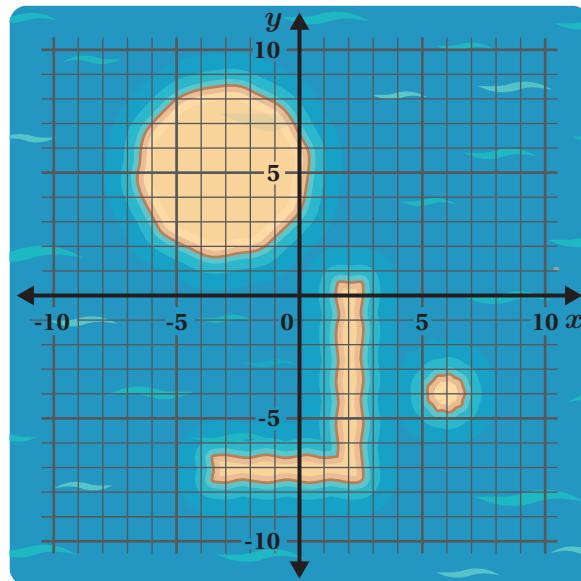
8 **Round 2:** Here are three new islands.

Label each guess and the feedback you get until you find all three sand dollars.

Partner A: Sand dollars are at (6, -4), (-2, -7), (-2, 6).

Partner B: Sand dollars are at (6, -4), (-3, -7), (-5, 5).

Name: _____ Date: _____ Period: _____
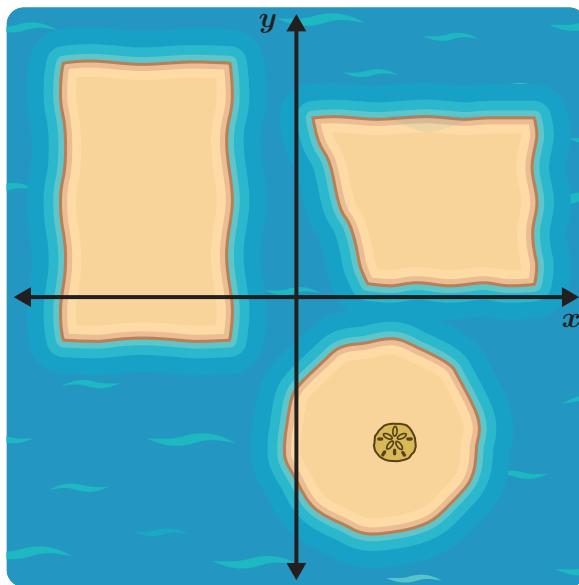
Find That Sand Dollar! (continued)

9 Which of these could be the location of the sand dollar?

A. (4, 6) **B.** (4, -6)

C. (-6, -4) D. (-6, 4)

Explain your thinking.

Explanations vary. **The sand dollar is to the right of the origin, so the x-coordinate is positive. It is below the origin, so the y-coordinate is negative.**

Explore More

10 Ariel was searching for a sand dollar and the arrow pointed up.

a What must be true about the sand dollar's x-coordinate?
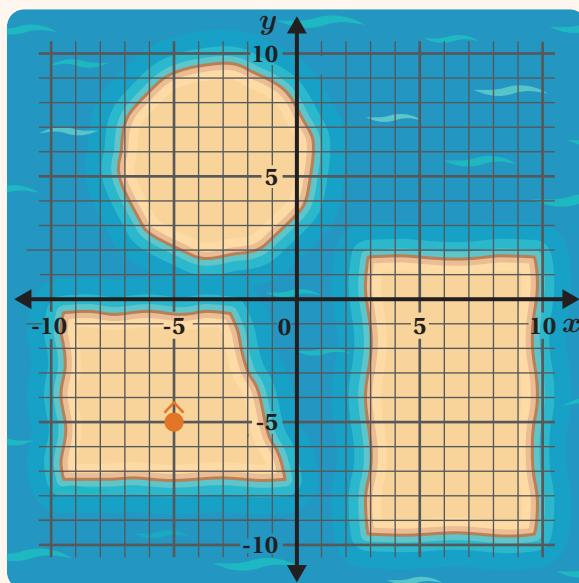
A. $x = -5$

B. $x < -5$

C. $x > -5$

b What must be true about the sand dollar's y-coordinate?

A. $y = -5$

B. $y < -5$

C. $y > -5$

Explain your thinking.

Explanations vary. **The current guess is at (-5, -5) and the arrow is pointing up. That means that the x-coordinate will stay the same, but the y-coordinate will be greater than what it is now.**
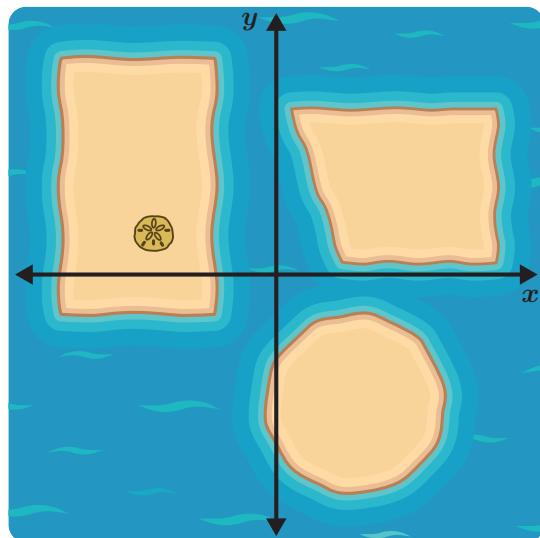
Synthesis

Explain what you know about the coordinates of this sand dollar.

Responses vary.
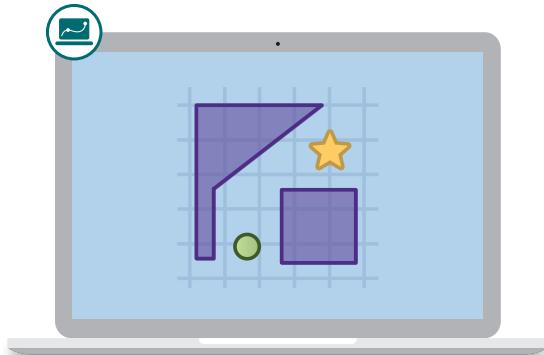
- **The x-coordinate is negative.**
- **The y-coordinate is positive.**
- **If the scale is the same as on the other island maps, then the coordinates might be close to (-6, 2).**
- **It's located in Quadrant 2.**

Things to Remember:

Unit 7
Lesson
11

Name: _____ Date: _____ Period: _____

The A-maze-ing Coordinate Plane

Let's practice with the coordinate plane using mazes.

Warm-Up
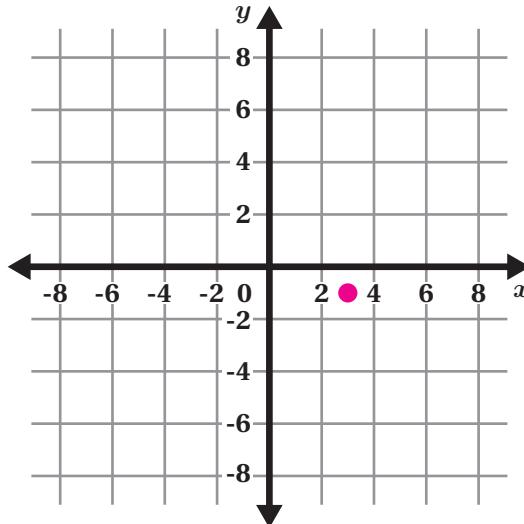
1 **a** Plot a point at the coordinates (3, -1).

Response shown on graph.

b Explain how you chose where to plot the point.

Explanations vary.
- **I know the point should be 3 to the right and 1 down. Because the graph is going by 2s that means that it is really 1.5 squares to the right and 0.5 squares down.**
- **I know that 3 is between 2 and 4, so I put it in the middle of those two numbers horizontally. I know that -1 is between 0 and -2, so I put it in the middle of those two numbers vertically.**

Name: _____   Date: _____   Period: _____

Maze Challenges

The goal of the maze is to move the ball to the star without hitting any walls.
For each challenge:

- Write the coordinates of a path to move the ball to the star. Your path can have several coordinate points.
- Pass your paper to a partner to draw the path using your coordinates.
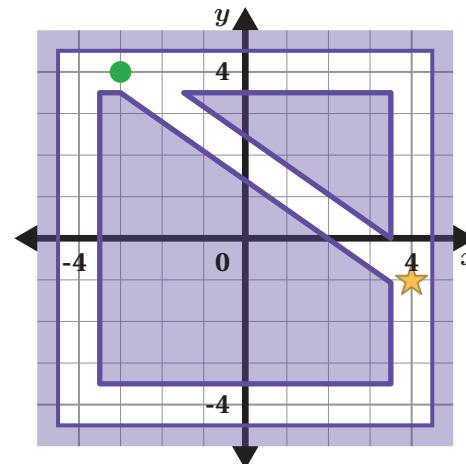
2 Let's watch the ball collect the star in this example. Finish writing the coordinates of the path.

Your Path
(-3, 4)
(4, -1)
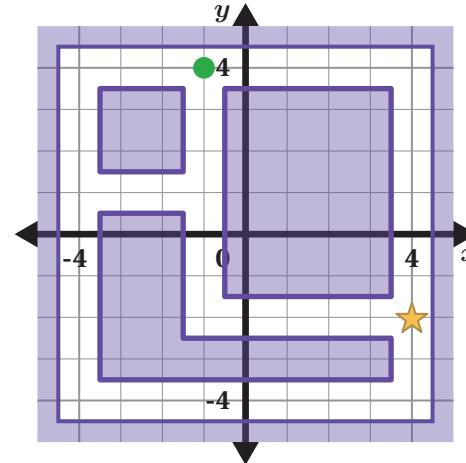
3 Write the coordinates of a path to move the ball to the star. Note: You do not need to use all the rows. *Responses vary.*

Your Path
(-1, 4)
(-1, -2)
(4, -2)
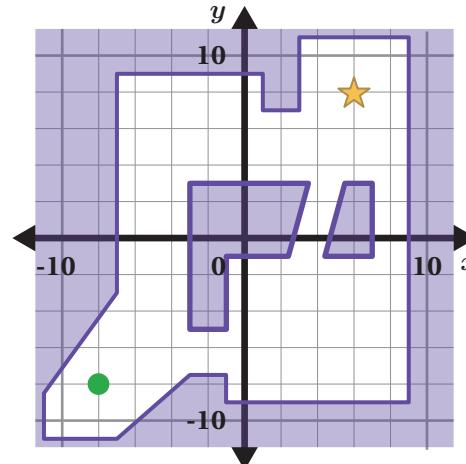
4 Write the coordinates of a path to move the ball to the star. *Responses vary.*

Your Path
(-8, -8)
(2, -5)
(6, 8)

Name: _____   Date: _____   Period: _____

More Mazes

5 In this maze, there are multiple stars.

a Sketch a path to collect them all.
Sketches vary.
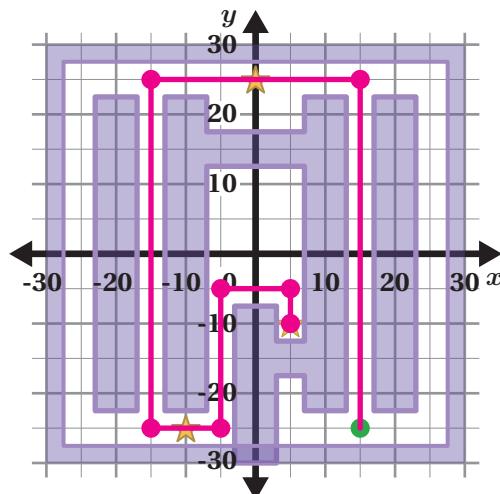
b Write the coordinates that create your path.
Responses vary.

Your Path
(15, -25)
(15, 25)
(-15, 25)
(-15, -25)
(-5, -25)
(-5, -5)
(5, -5)
(5, -10)

Name: .... Date: .... Period: ....

More Mazes (continued)

6 Trinidad and Annika were working on the previous maze.

Trinidad says: *The star's coordinates are* (1, -2).

Annika says: *The coordinates are* (1, -10).

Whose claim is correct? Circle one.
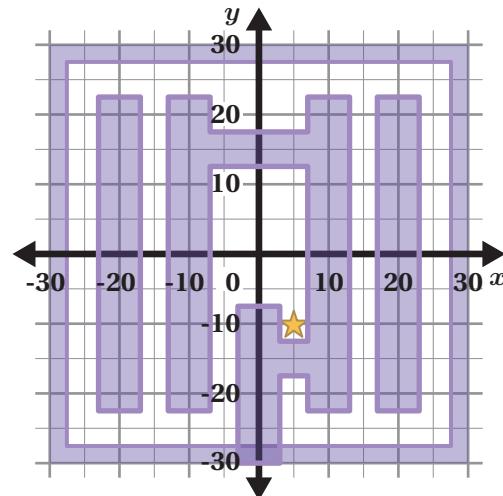
Trinidad's   Annika's   (Neither)

Explain your thinking.

Explanations vary. **They are both wrong because the graph goes by 5s. This means that 1 over and 2 down from (0, 0) is (5, -10).**

Name: _____ Date: _____ Period: _____

Reflections on the Plane

7 Let's look at a new way of collecting stars.
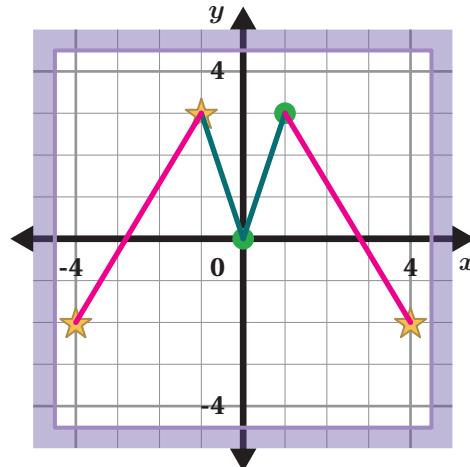
a 💬 **Discuss:** What happens when we try the point $(1, 3)$?

Responses vary. **Two lines extend from the origin out to $(1, 3)$ and $(-1, 3)$.**

b Complete the table to collect all the stars. Sketch the new paths on the graph.

Responses vary.
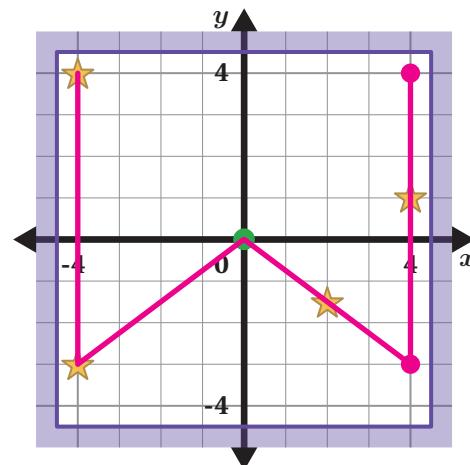
Your Path	Mirrored Path
$(0, 0)$	$(0, 0)$
$(1, 3)$	$(-1, 3)$
$(4, -2)$	**$(-4, -2)$**

8 The ball is mirrored across the y-axis. Write coordinates to collect all the stars. Choose your two points wisely!

Responses vary.

Your Path	Mirrored Path
$(0, 0)$	$(0, 0)$
$(4, -3)$	**$(-4, -3)$**
$(4, 4)$	**$(-4, 4)$**

Name: _____ Date: _____ Period: _____

Reflections on the Plane (continued)
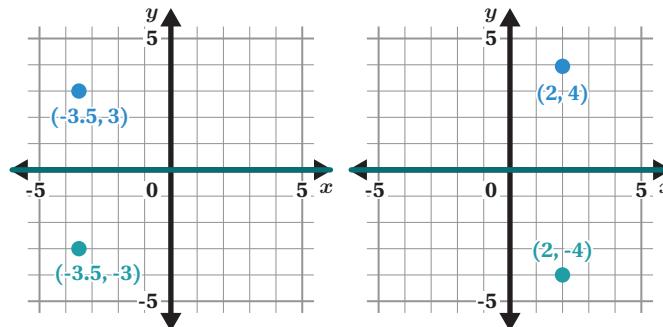
9 Here are several coordinates of a ball and its reflection.

Write at least two patterns about the coordinates that you found interesting.
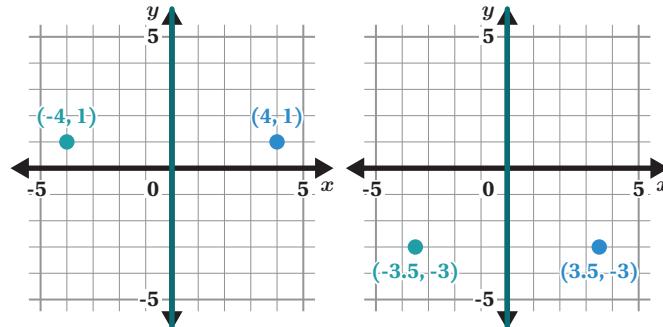
Responses vary.

- **When I reflect a point over the y-axis, the y-coordinate stays the same but the x-coordinate changes signs.**
- **When I reflect over the x-axis, the x-coordinate stays the same and the y-coordinate changes signs.**
- **When I reflect, one coordinate stays the same and the other coordinate is the same but has the opposite sign.**

Reflections Across the x-axis
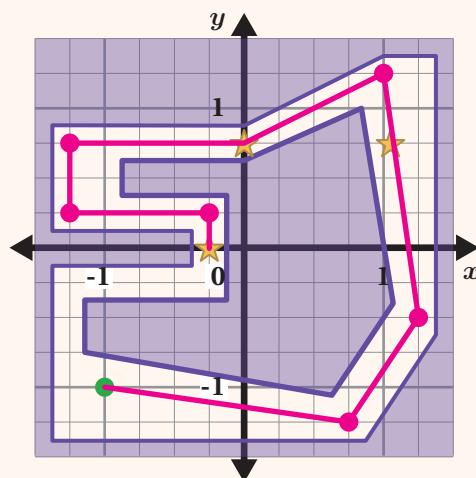
Reflections Across the y-axis

Explore More

10 Sketch a path to collect all the stars. Then write the coordinates of your path.

Responses vary. **(-1, -1), (0.75, -1.25), (1.25, -0.5), (1, 1.25), (0, 0.75), (-1.25, 0.75), (-1.25, 0.25), (-0.25, 0.25), (-0.25, 0)**

11 Synthesis

Look back over the challenges and select the one you are most proud of solving.

Write some advice for someone solving that challenge.

Responses and explanations vary.

- **I made sure to pay attention to the scale of the graph. If the scale went by 2s, then my points would be different.**
- **Try to take shortcuts by using diagonal lines.**
- **If I want to go to the same point on the other side of an axis, I can just change the sign of one of the coordinates.**

Things to Remember:

Unit 7
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Name: _____ Date: _____ Period: _____
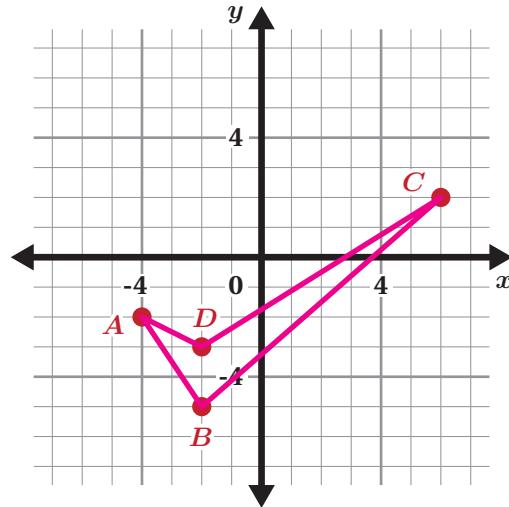
Polygon Maker

Let's explore polygons on the coordinate plane.

Warm-Up

1 Let's watch how a Polygon Maker works.

Point	Coordinates
A	(-4, -2)
B	(-2, -5)
C	(6, 2)
D	(-2, -3)

a Connect the coordinates to create a *polygon*. Tell a classmate what the polygon reminds you of.

Responses vary.
- **Check mark**
- **Boomerang**

b Change one coordinate and connect the points to make a new polygon.

Polygons vary.

Name: ....   Date: ....   Period: ....
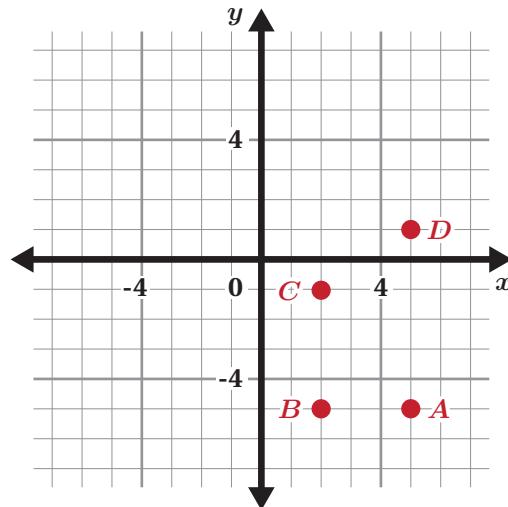
A Polygon

2 A student wants to make a rectangle.

Change some of their coordinates so that the Polygon Maker makes a rectangle.

Point	Coordinates
A	(5, -5)
B	(2, -5)
C	(2, -1)
D	(5, 1)

Responses vary.
- **Change C from (2, -1) to (2, 1).**
- **Change D from (5, 1) to (5, -1).**

3 Calculate the perimeter of the rectangle you made.
Responses vary. **18 units or 14 units**
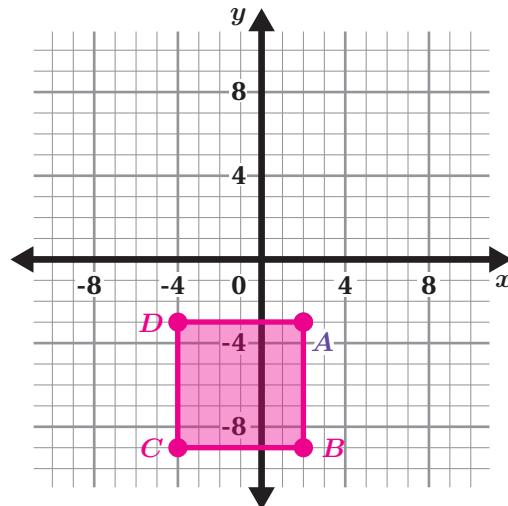
Explain your thinking.
Explanations vary. **For the horizontal sides, going from 2 to 5 is 3 units. For the vertical sides, it is 4 units from -5 to -1. This makes the perimeter 14 units.**

4 Vicente wants to make a square.

Complete the table with coordinates that will make a square. Sketch the square you made on the coordinate plane.
Points and squares vary.

Point	Coordinates
A	(2, -3)
B	**(2, -9)**
C	**(-4, -9)**
D	**(-4, -3)**

Name: ....   Date: ....   Period: ....

A Polygon (continued)

5 Here are Vicente's coordinates.
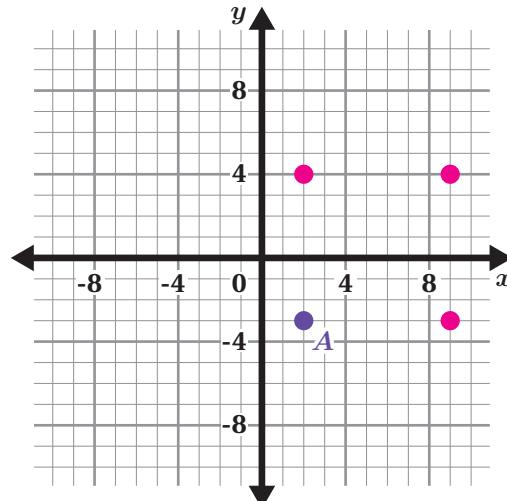
Point	Coordinates
A	(2, -3)
B	(2, 4)
C	(9, 4)
D	(9, -3)

Do they make a square? Circle one.

(Yes)   No   I'm not sure

Show or explain your thinking.

Explanations vary. **Vicente's polygon is made up of straight horizontal and vertical lines and every side is 7 units long. So Vicente did build a square.**

6 Vicente says: *My square has side lengths of* 7.

Sketch or describe where you see 7 in the table and graph.

Responses vary.
- **7 is how far 9 is from 2 and also how far 4 is from -3.**
- **For the horizontal sides, going from 9 to 2 is 7 units. For the vertical sides, it's 3 units from -3 to 0 and then another 4 units from 0 to 4.**

Point	Coordinates
A	(2, -3)
B	(2, 4)
C	(9, 4)
D	(9, -3)

Name: _____ Date: _____ Period: _____

Make Your Own Polygon
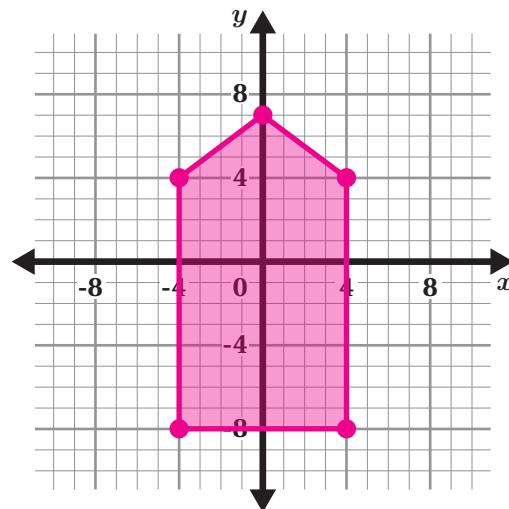
7 Here are three descriptions:

- Looks like a house that's 15 units tall
- Looks like a capital L that's 6 units wide
- A rectangle with an area of 24 square units

Choose one description. Write and sketch coordinates to make a polygon that matches that description.

Points and polygons vary.
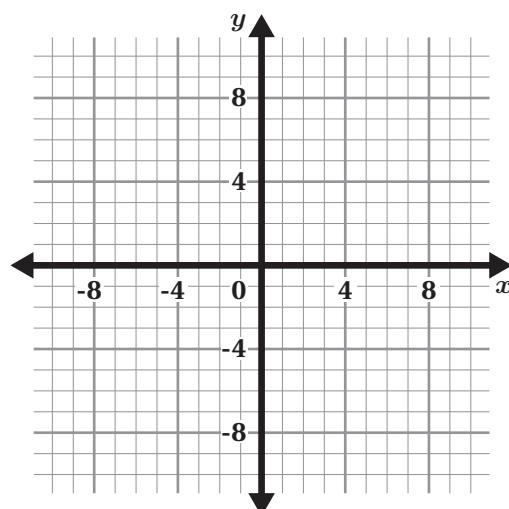
Point	Coordinates
A	**(0, 7)**
B	**(4, 4)**
C	**(4, -8)**
D	**(-4, -8)**
E	**(-4, 4)**

8 You will create your own polygon challenge.

- Write a description of a polygon on a slip of paper. Make it fun (like "looks like a pinecone") and include the size (like "must have a perimeter of 20 units").
- On this page, write the coordinates and sketch a polygon that matches your description.

Points and polygons vary.

Point	Coordinates
A	
B	
C	
D	

Name: _____ Date: _____ Period: _____

Make Your Own Polygon (continued)

8 You will swap your polygon challenge with two classmates.

- Hand your slip of paper to a partner and invite them to use coordinates to create a polygon on their paper.
- When they finish, share your original polygon and compare.
- Write the coordinates and graph your partners' polygon challenges on this page.

Descriptions, points, and polygons vary.

Partner 1
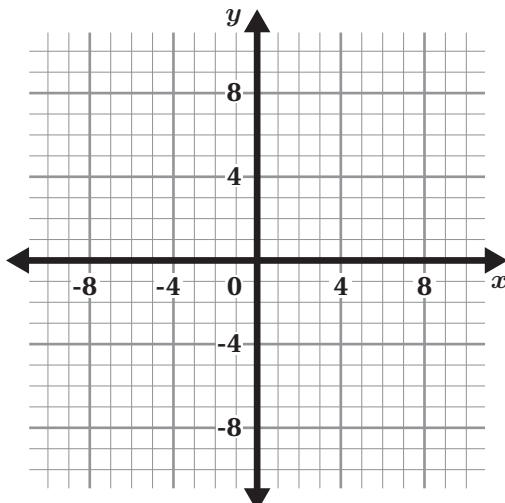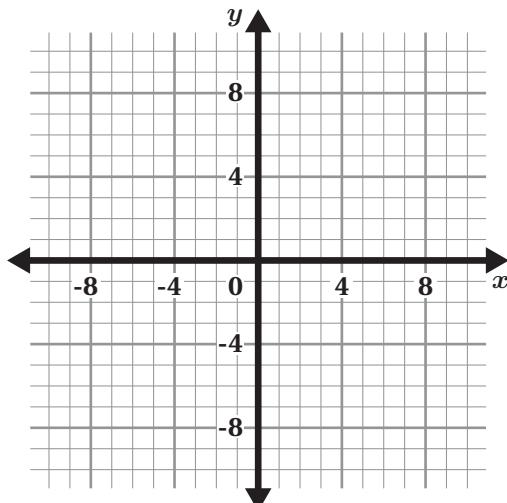
Description:

Point	Coordinates
A	
B	
C	
D	

Partner 2

Description:
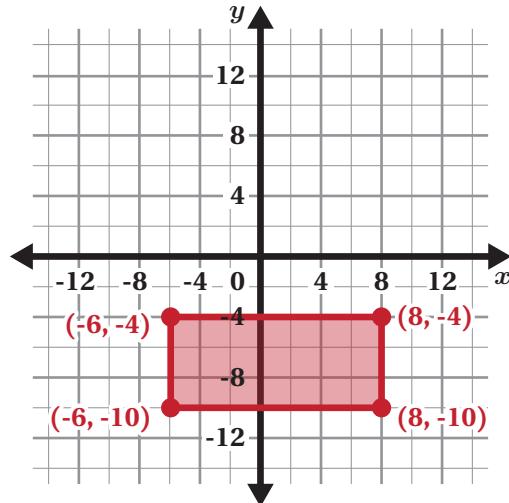
Point	Coordinates
A	
B	
C	
D	

9 Synthesis

Describe how you can use the coordinates to calculate the side lengths of a rectangle. Use the table and graph if they help with your explanation.

Point	Coordinates
A	(-6, -4)
B	(8, -4)
C	(8, -10)
D	(-6, -10)
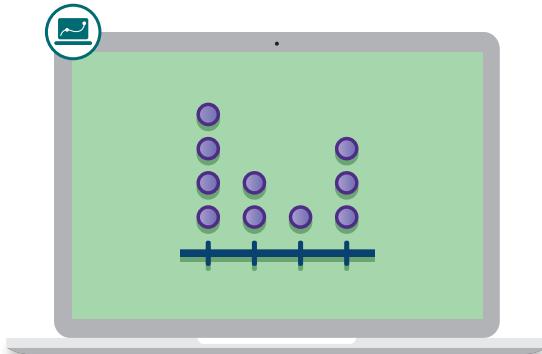
Responses vary. You can find one side length by counting the distance between the two y-coordinates and find the other by counting the distance between the two x-coordinates. The short side is 6 units because from -10 to -4 is 6. The long side is 14 units because it's 6 units from -6 to 0 and then 8 more units from 0 to 8.

Things to Remember:

Unit 8
Lesson
2

Name: .... Date: .... Period: ....

Dot Plots

Let's turn data into dot plots.

Warm-Up

1 Antwon used a survey to collect data about some of his friends.

What question might he have asked?

Responses vary.
- **How many games did you play this weekend?**
- **How many minutes does it take you to get to school?**
- **How old do you think you'll be when you live the best year of your life?**

Friend	?
Maria	20
Citlalli	22
Carlos	17
Laila	10
Troy	45
Omar	15
Issa	20

Name: _____   Date: _____   Period: _____

Dot Plots

2 Meet Antwon's classmates!

Antwon claims that most of these students spend more than 4 hours on their phones each day.

Do you agree? Circle one.
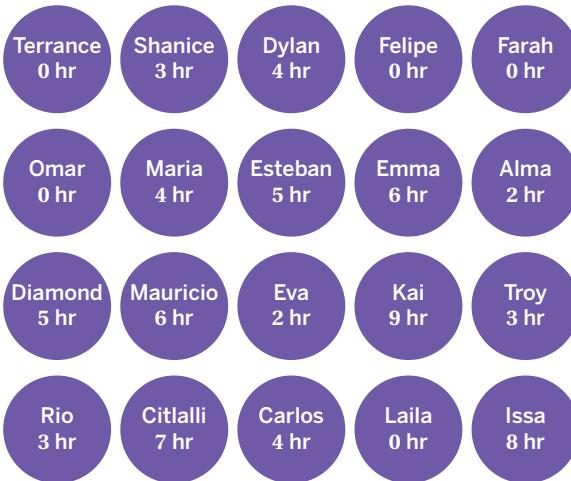
Agree   Disagree   I'm not sure

Explain your thinking.

Responses and explanations vary.

- **Agree. Many of these students spend more than 4 hours a day on their phones.**
- **Disagree. Some of these students don't spend any time on their phone.**
- **I'm not sure. There are a lot of students here, so it's not really clear what "most" would mean.**

How many hours do you spend on your phone each day?

Terrance 0 hr	Shanice 3 hr	Dylan 4 hr	Felipe 0 hr	Farah 0 hr
Omar 0 hr	Maria 4 hr	Esteban 5 hr	Emma 6 hr	Alma 2 hr
Diamond 5 hr	Mauricio 6 hr	Eva 2 hr	Kai 9 hr	Troy 3 hr
Rio 3 hr	Citlalli 7 hr	Carlos 4 hr	Laila 0 hr	Issa 8 hr

3 Let's look at another way to visualize numerical data: as a **dot plot**.

What do you notice? What do you wonder?

I notice:

Responses vary.

- **I notice that the dot plot goes from 0 to 10.**
- **I notice that there are 5 students who didn't spend any time on their phones.**
- **I notice that there are the same number of dots as students.**
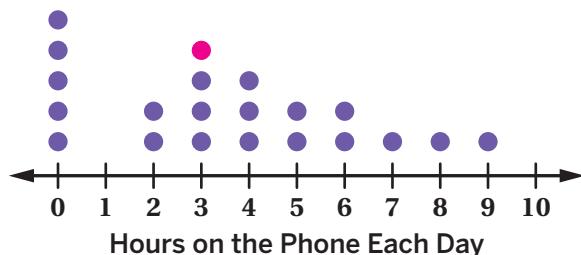
I wonder:

Responses vary.

- **I wonder why someone would want to make a dot plot.**
- **I wonder why the dot plot goes from 0 to 10.**
- **I wonder why there are so many 0s on the dot plot.**

Name: _____ Date: _____ Period: _____

Dot Plots (continued)

Here is the dot plot of the data from Antwon's classmates.

Hours on the Phone Each Day

4 Raven is a new student in Antwon's class. She spends 3 hours on her phone each day. Draw a dot to add Raven's data to the dot plot.
Response shown on the dot plot.

5 Dylan says: *I spend* 4 *hours on my phone each day.*

a Circle a dot on the dot plot that could represent Dylan.
Any dot above 4 is considered correct.

b Is this the only dot that could represent Dylan?

Yes   (No)   I'm not sure

Explain your thinking.
Explanations vary. **Dylan could be any of the three dots that are above the 4.**

6 Here is Antwon's claim from earlier.

He said: *Most students spend more than* 4 *hours on their phones each day.*

How can a dot plot help you decide if his claim is true?
Responses vary.
- **I can count how many students spend more than 4 hours on their phone and how many don't and then compare them.**
- **I can tell that there are more dots on the left side of the dot plot, so Antwon's claim probably isn't true.**
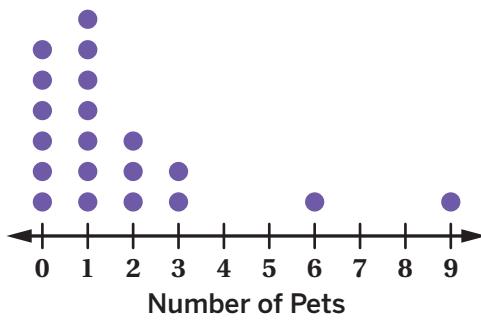
Name: .... Date: .... Period: ....
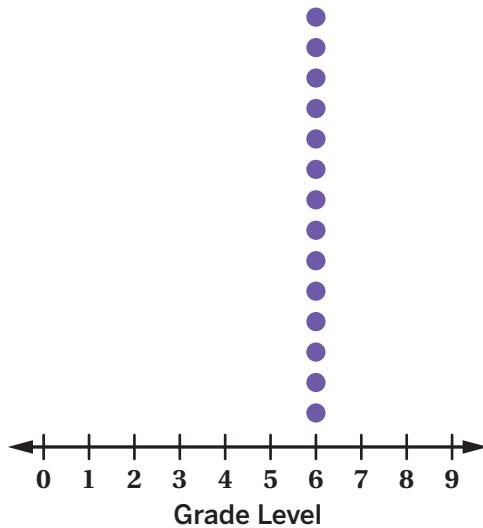
Statistical Questions

7 Antwon asked his classmates a few more questions and made dot plots of their answers.
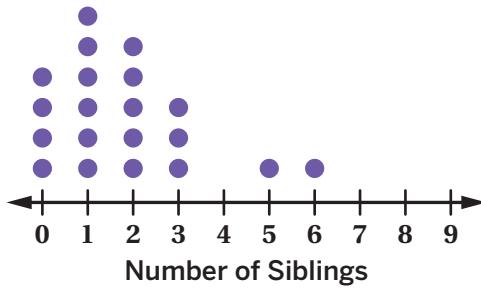
How many pets do you have?
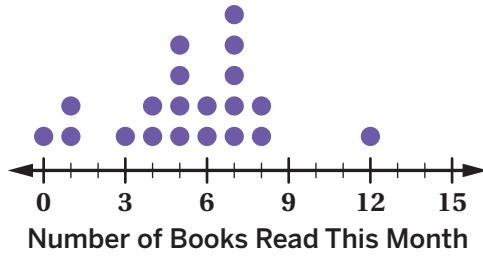
Number of Pets

What grade are you in?

Grade Level

How many siblings do you have?

Number of Siblings

How many books have you read this month?

Number of Books Read This Month

💬 **Discuss:** What are two things you learned about Antwon's classmates from this data?

Responses vary.
- Most students have 1 pet or 0 pets.
- The greatest number of pets a student has is 9.
- The greatest number of books a student read this month is 12.
- All of Antwon's classmates are in 6th grade.
- There are 4 only-children in Antwon's class.
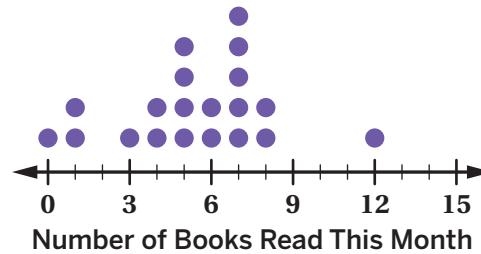
8 This dot plot shows the number of books Antwon's classmates read this month.

Write a question that this dot plot could help you answer.

Number of Books Read This Month

Responses vary.
- **How many books did Antwon's classmates read this month?**
- **What is the most common number of books Antwon's classmates read?**
- **Do students in Antwon's class read a lot?**
- **How much do Antwon's classmates read?**

9 Which question would a dot plot be *less* helpful in answering? Circle one.

How many books has Farah read this month?

Have most students read more than 3 books this month?

Explain your thinking.

Explanations vary.
- **We can't use a dot plot to answer the question about Farah because we don't know which dot Farah is.**
- **A dot plot tells us about a lot of numbers, so it would help us know about all the students, not just Farah.**

10 A **statistical question** requires more than one piece of data to answer it.

Select *all* the statistical questions.

☑ **A.** Have most students read more than 3 books this month?

☐ **B.** How many books has Farah read this month?

☑ **C.** Do students in this class prefer fiction or nonfiction?

☐ **D.** Is the book that Citlalli read fiction or nonfiction?

☑ **E.** How long are the books that Kai read?

11 Synthesis

What are the advantages and disadvantages of visualizing a data set as a dot plot?

Use the examples if they help with your thinking. *Responses vary.*

Advantages:

- **A dot plot helps me organize data so that I can see whether a claim is true.**
- **A dot plot helps me see how many people had the same response.**
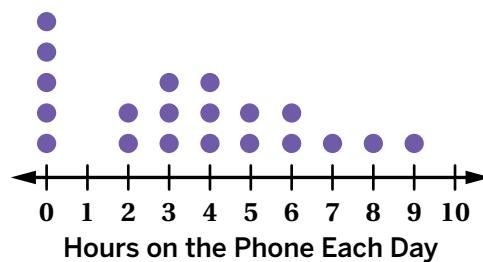
Disadvantages:

- **When I use a dot plot, I can't see which dot represents which person.**

Data Set

0, 3, 4, 0, 0, 0, 4, 5, 6, 2, 5, 6, 2, 9, 3, 3, 7, 4, 0, 8

Dot Plot

Hours on the Phone Each Day

Things to Remember:

**Unit 8
Lesson
5**

Name: _____ Date: _____ Period: _____

The Plot Thickens
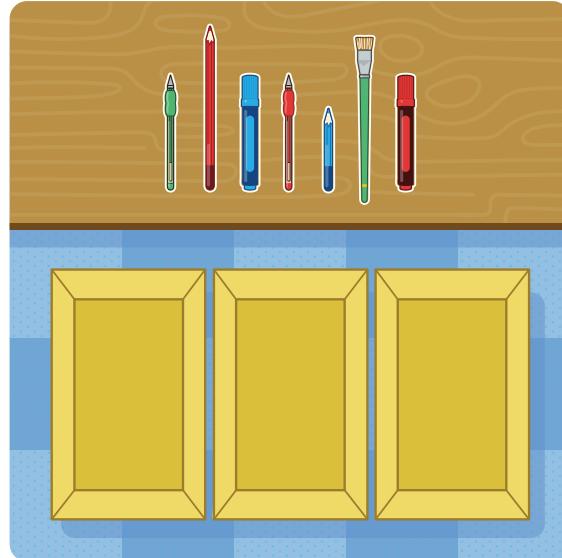
Let's use histograms to represent data sets.

Warm-Up

1 **a** Organize the art supplies into the bins.

Organizations vary.

b Explain how you organized the supplies.
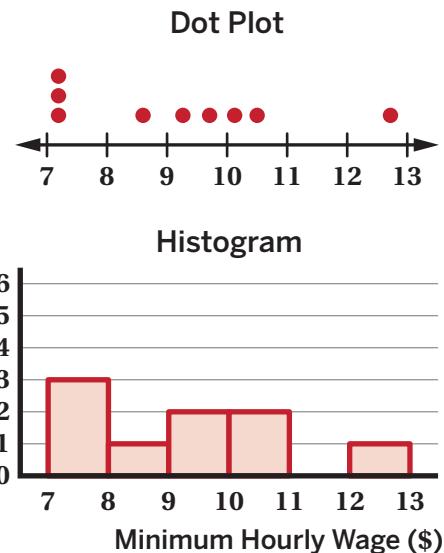
Responses vary.
- **I put things that are alike in the same bin.**
- **I put all the pencils in the same bin.**

Name: _____   Date: _____   Period: _____

Introduction to Histograms

2 Here are two different ways to show the minimum wages of 9 U.S. states in 2020.

💬 **Discuss:** How do you think the **histogram** was made?

Responses vary. **The histogram was made by drawing bars as tall as the number of dots between two values. For example, there are 2 dots between $9 and $10, so the bar is 2 units tall.**

Dot Plot

Histogram

Source: *United States Department of Labor*

3 How are the histogram and the dot plot alike?

Responses vary.
- **They show the same data.**
- **They both have the same scale: from 7 to 13.**
- **The number of dots between each value is equal to the height of each bar.**
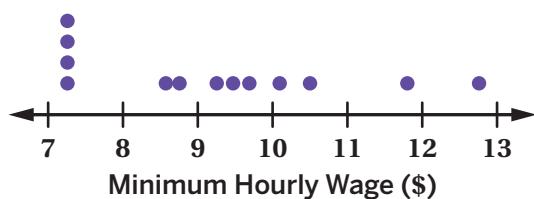
How are they different?
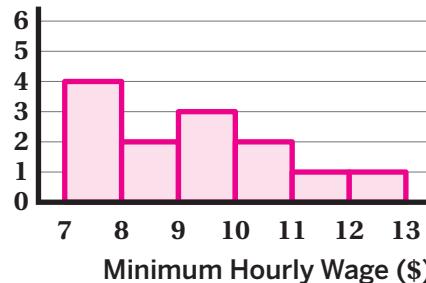
Responses vary.
- **The histogram uses bars to show data, while the dot plot uses individual dots.**
- **The histogram has vertical and horizontal number lines, while the dot plot only has a horizontal one.**

4 More states were added to the dot plot. Each bar in a histogram is called a **bin**.

Draw bins to make a histogram with this new data.

Minimum Hourly Wage ($)

Source: *United States Department of Labor*

Minimum Hourly Wage ($)

Name: .... Date: .... Period: ....

Creating Histograms

5 In statistics, each bin includes the smaller of the two numbers and goes up to but does not include the larger number. When a data point is on the edge between two bins, it goes into the bin with the larger values.
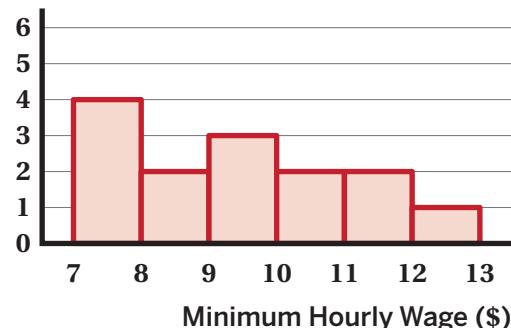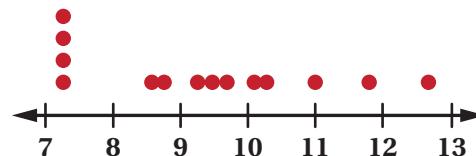
a $11 was added to this dot plot. Let's watch which bin the $11 goes into.

b Which bin would $8 go into? Circle one.

7 to 8   (8 to 9)   Somewhere else
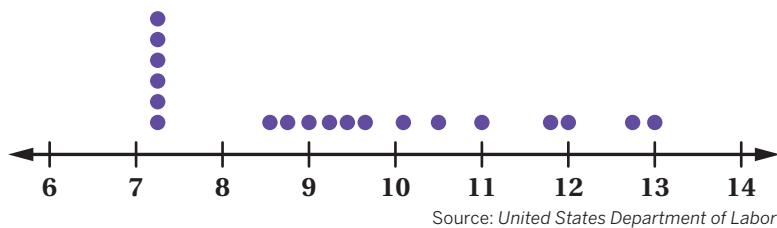
Explain your thinking.

Explanations vary.
- **$11 went into the 11 to 12 bin, so $8 should go into the 8 to 9 bin.**
- **Each bin includes the lesser value but not the greater value, so $8 would not go in the 7 to 8 bin.**

Minimum Hourly Wage ($)
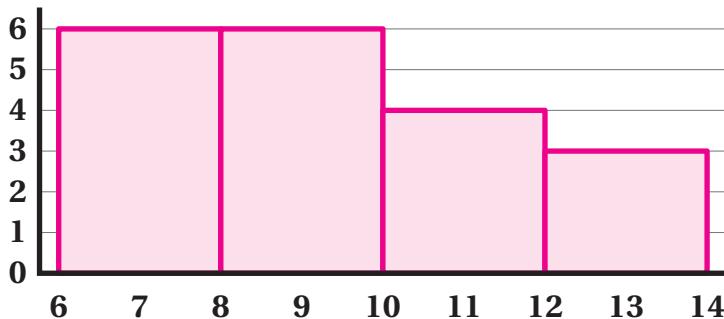
6 So far, we've seen histograms with a bin size of 1. Here is a dot plot with some more minimum wages.

Source: *United States Department of Labor*

Make a histogram with a bin size of 2.

Name: _____ Date: _____ Period: _____
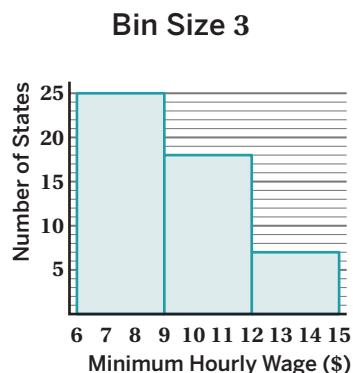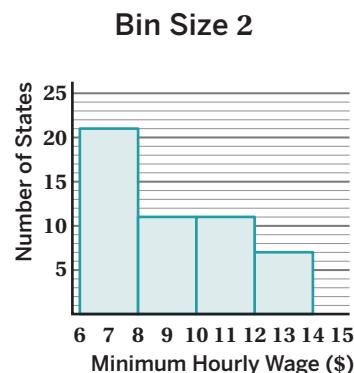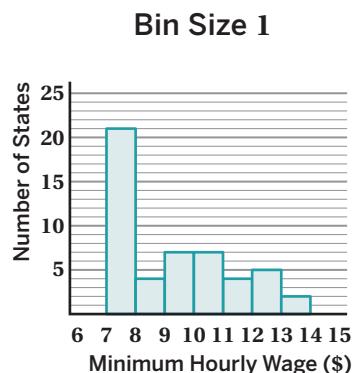
Using Histograms

7 Let's look at the minimum wages for all 50 U.S. states. Why might these data points be difficult to visualize as a dot plot?

Responses vary.
- **There are a lot of data points. The number of dots on a dot plot could be overwhelming.**
- **Some of the values might be hard to put on a dot plot, like $8.56 or $10.10.**
- **There might be dots that are too close together, like $8.56 and $8.70.**

8 Each of these histograms show the minimum wages of all 50 states in 2020.

Bin Size 1

Bin Size 2

Bin Size 3

Source: *United States Department of Labor*

💬 **Discuss:** What do you notice? What do you wonder?

Responses vary.
- **I notice that the scale of both axes stays the same even when the bin size changes. The horizontal axis goes from 6 to 15, and the vertical axis goes from 0 to 25.**
- **I notice that when the bin size increases, the bin gets wider.**
- **I notice that if you add the heights of the smaller bins, you get the height of the larger bin.**
- **I wonder how you pick the best bin size.**
- **I wonder why someone would want a bin size of 3.**
- **I wonder if the histograms show 50 states no matter the bin size.**

9 Different visualizations like dot plots and histograms help answer different questions.

a Which of these questions can a histogram help answer?

A. Which state has the highest minimum wage?

B. What is the highest minimum wage?

C. How many states have a minimum wage of $7.25?

(D.) How many states have a minimum wage of $10 or more?

b Answer the question you chose.
18 states
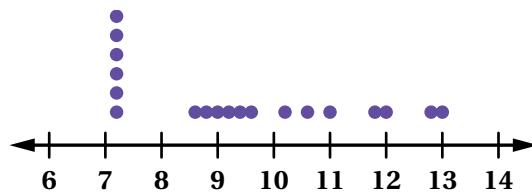
What is important to remember about a histogram?

Responses vary.
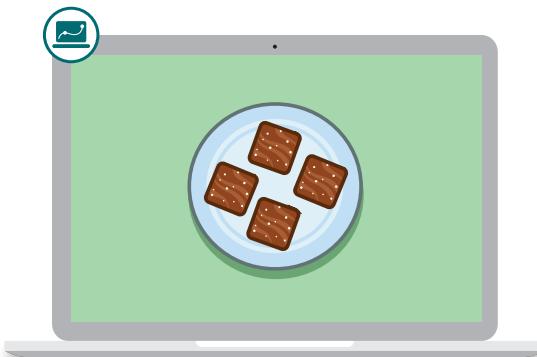
- I can use the same data to make histograms with different bin sizes.
- When I have a data point on the edge of two bins, it goes into the bin with the greater values. So the data point 12 goes into the 12–14 bin.
- Histograms are like dot plots but are helpful when there are lots of data points.

Minimum Hourly Wage ($)

Things to Remember:

Unit 8
Lesson

7

Name: .... Date: .... Period: ....
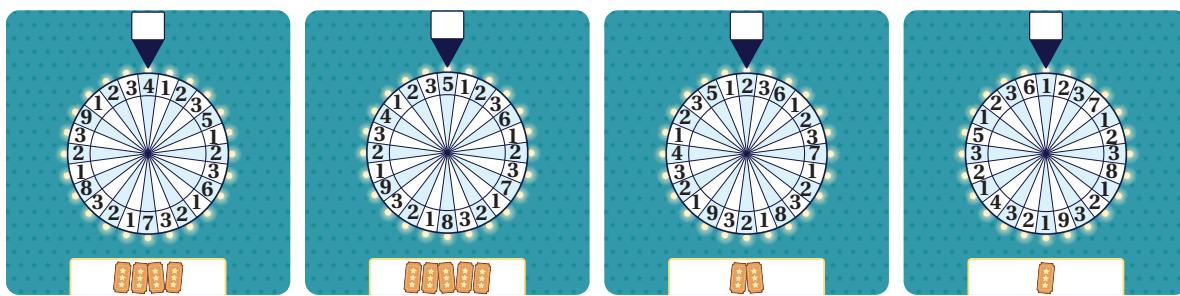
Snack Time

Let's explore the mean of a data set and what it tells us.

Warm-Up

1 Here are four images of an arcade game.

💬 **Discuss:** What do you notice? What do you wonder?
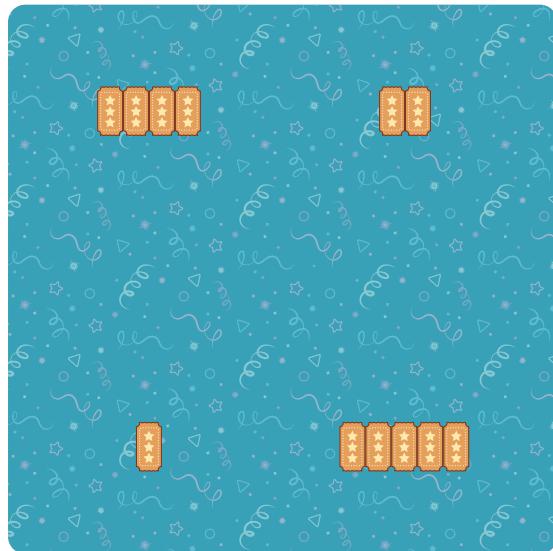
Responses vary.
- **I notice that the number of tickets you win is the same as the number the wheel lands on.**
- **I notice that the most number of tickets you can win is 9.**
- **I notice it's harder to win a larger number of tickets.**
- **I wonder if there's a game like this at an arcade near me.**
- **I wonder what kinds of prizes you can buy with the tickets.**
- **I wonder what happens when the spinner lands between numbers.**

Name: .... Date: .... Period: ....

Mean as an Equal Share

2 4 friends played this game at the arcade. Here are the tickets each friend won.

They decided to share the tickets equally. How many tickets should each friend get? Explain your thinking.

3 tickets. *Explanations vary*. I rearranged the tickets until every person had the same number of tickets. When I did that, there were 3 tickets per person.

3 Here is Ava's work for determining how many tickets each friend gets.

💬 **Discuss:** What was Ava's strategy?

Responses vary. **To determine the number of tickets per friend, Ava added the tickets and divided by 4, because there are 4 friends.**

Ava

$4 + 2 + 1 + 5 = 12$

$12 \div 4$
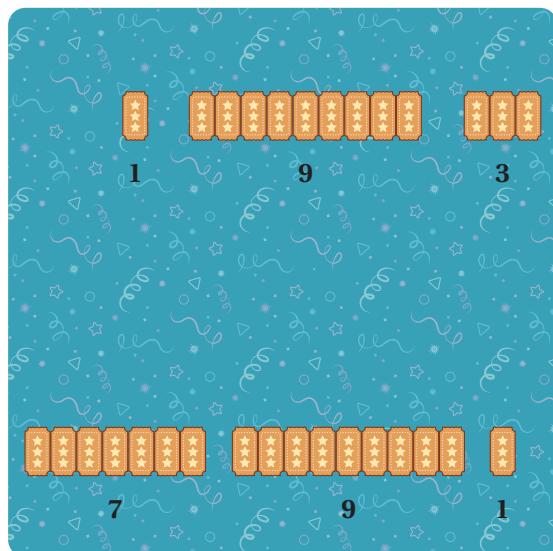
4 The **mean**, or average, is the number of tickets each friend gets if the tickets are distributed equally.

Here are the tickets that 6 other friends won.

Calculate the mean number of tickets.

5 tickets

1   9   3

7   9   1

Name: _____   Date: _____   Period: _____

Mean as a Statistic

5 A **statistic** is a single number that measures something about a data set.

The mean is an example of a statistic.

What does the mean tell us about Ishaan's snacks?
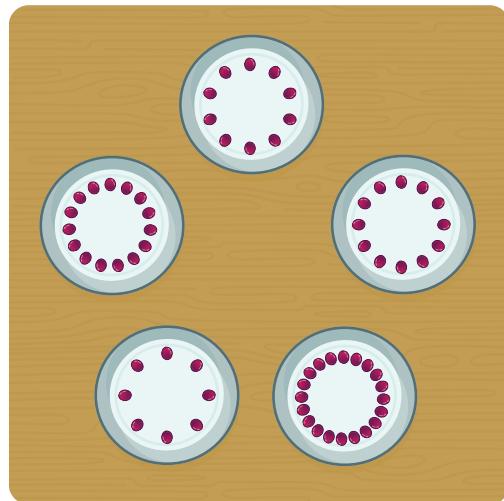
Responses vary.
- **The mean tells us that if Ishaan evened out the number of carrots he ate per day, he would have 9 carrots per day.**
- **The mean tells us that Ishaan ate an average of 9 carrots per day during the week he recorded his data.**

Ishaan's Snacks

Day	Number of Carrots
Monday	10
Tuesday	8
Wednesday	5
Thursday	12
Friday	10

Mean: 9 carrots

6 Here is how many grapes Crow ate each day last week. Calculate the mean of this data.

Day	Number of Grapes
Monday	10
Tuesday	12
Wednesday	20
Thursday	8
Friday	15

13 grapes

7 Ali and Makayla ate blueberries each day last week. Ali ate 12 blueberries per day. The table shows how many blueberries Makayla ate.

Who ate more blueberries per day on average? Circle one.

(Ali)   Makayla   They ate the same amount on average

Explain your thinking.
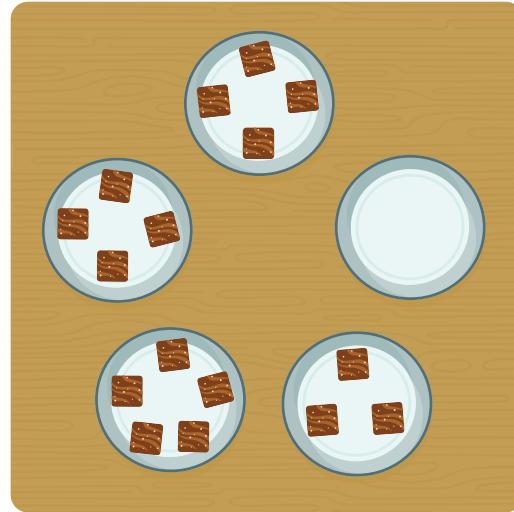
Explanations vary. **The mean of Makayla's data is** $\frac{1 + 15 + 13 + 11 + 15}{5} = 11$. **That means Makayla ate 11 blueberries per day on average. Ali ate 12 blueberries per day, which is more than Makayla.**

Ali's Snacks
12 blueberries per day

Makayla's Snacks

Day	Number of Blueberries
Monday	1
Tuesday	15
Wednesday	13
Thursday	11
Friday	15

Name: _____ Date: _____ Period: _____

Mean as a Statistic (continued)

8 Here is how many brownies Omari ate.

Day	Number of Brownies
Monday	4
Tuesday	0
Wednesday	3
Thursday	5
Friday	4

Calculate the mean of this data.

3.2 brownies

9 Omari calculated the mean incorrectly.

💬 **Discuss:**

• What is correct about Omari's work?

• What is incorrect about Omari's work?

Responses vary.
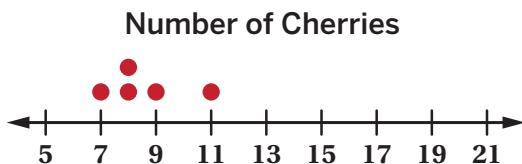
• **Omari correctly calculated the mean for the four plates with brownies on them.**

• **Omari did not include the empty brownie plate in his calculations. The plate with 0 brownies doesn't change the total number of brownies, but it does change the number Omari should divide by from 4 to 5.**

Omari

4 + 3 + 5 + 4 = 16

16 ÷ 4 = 4

The mean is 4 brownies.

Name: _____   Date: _____   Period: _____
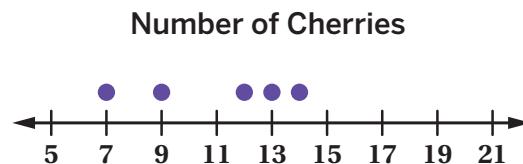
Mean as a Measure of Center

10 For 5 days, Oliver recorded the number of cherries he had as a snack. The mean for his data is 11 cherries. Circle a data set that could *not* be Oliver's.

A.   **Number of Cherries**

B.   **Number of Cherries**

C.   **Number of Cherries**

D.   **Number of Cherries**
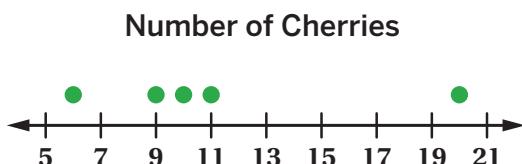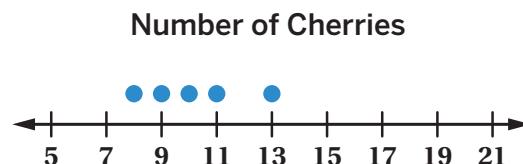
Explain your thinking.
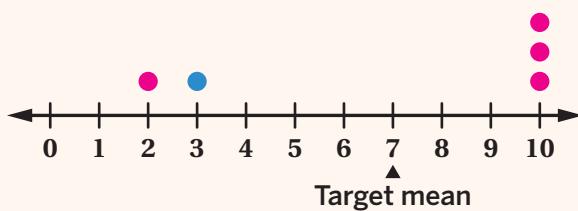
Responses and explanations vary.
- **Choice A: The mean of this data set can't be 11 because the largest data point is at 11 and all the others are smaller.**
- **Choice C: The mean of this data set is 11.2, not 11.**
- **Choice D: The mean of this data set can't be 11 because that's not the center of the data. It looks like the center of the data will be smaller than 11 because that's where most of the points are.**

Explore More

11 Add at least *four* more points to create a dot plot that has a mean of 7. Then check your work.

Create as many different dot plots as you have time for.

***Responses vary.* Sample shown on dot plot.**

Target mean
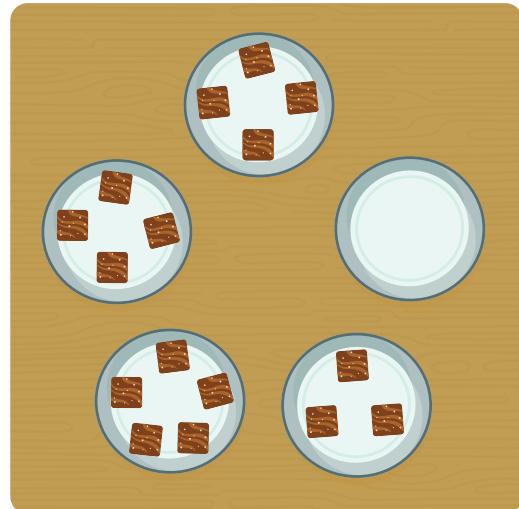
▣ Synthesis

How can you determine the mean of a data set?

Use the example if it helps with your thinking.

Responses vary.

- **Add all the values to get the total and then divide by how many data points there are. In this example, there are 16 total brownies that get shared by 5 plates. So the mean is $\frac{16}{5} = 3.2$.**
- **Rearrange the brownies until there's an equal number of brownies on each plate. That number is the mean.**

Things to Remember:

Unit 8
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Name: .... Date: .... Period: ....

Hoops

Let's explore how to measure the spread of a data set.

Warm-Up

1–2 You will use the Warm-Up Sheet for this activity.

How many baskets can you make?

- Flick the counter from each of the starting points. If the counter lands on top of the hoop, you made a basket.

- You get 8 shots per round.

- Try to make as many as you can!

Round 1: .... baskets   Round 2: .... baskets

Scores vary.

3 Let's look at the Round 1 and Round 2 results from the class.

💬 **Discuss:** What do you notice?

Responses vary.
- **I notice that the mean of our scores got higher in Round 2. That means we got better at making baskets.**
- **I notice that our data was more spread out in Round 1. This means our scores were less consistent, probably because some of us were already good at making baskets and some of us were still learning.**

Name: _____   Date: _____   Period: _____

Mean Absolute Deviation

4 Polina and Rishi practice 10 free throws everyday. Each dot plot shows the number of baskets they made on 6 different days.
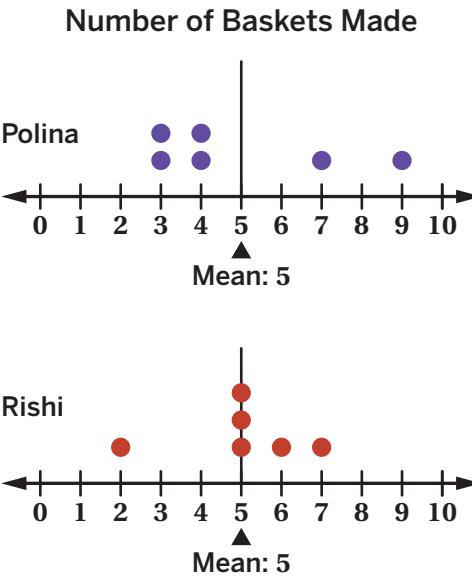
Which player is more consistent? Circle one.

Polina   (Rishi)

Explain your thinking.
Explanations vary. **Rishi is more consistent. I can tell because his dots are less spread out and most of his scores are close to the mean.**

Number of Baskets Made

Polina

0 1 2 3 4 5 6 7 8 9 10
▲
Mean: 5

Rishi

0 1 2 3 4 5 6 7 8 9 10
▲
Mean: 5

5 One way to measure consistency is to calculate the average of the absolute deviations. This is known as the **mean absolute deviation (MAD)**.

a Let's watch how to calculate the mean absolute deviation.

b Explain how to calculate the mean absolute deviation in your own words.
Responses vary.
- **Add all of the distances from the mean. Then divide by 6 because we're trying to calculate the mean of 6 distances.**
- **Even out the absolute deviations by adding them all and dividing by the total number of data points.**

6 The mean absolute deviation (MAD) is a statistic that measures the spread of a data set. Polina's data has a MAD of 2 baskets.

Will the MAD of Rishi's data be larger than, smaller than, or equal to the MAD of Polina's data? Circle one.
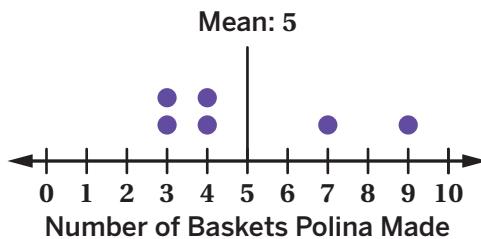
Larger   (Smaller)   Equal

Calculating MAD

Name: _____ Date: _____ Period: _____

7 Here is Polina's data.

Number of Baskets	3	3	4	4	7	9
Absolute Deviation (distance from the mean, 5)	2	2	1	1	2	4

Mean: 5

Number of Baskets Polina Made

Mean Absolute Deviation (MAD)

Sum of absolute deviations:
$2 + 2 + 1 + 1 + 2 + 4 = 12$

Average or mean of the absolute deviations:
$12 \div 6 = 2$
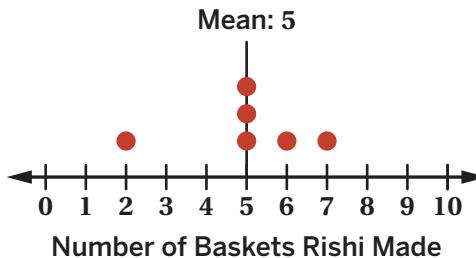
💬 **Discuss:** What do you notice? What do you wonder?
Responses vary.

8 Here is Rishi's data.

Calculate the MAD. Use the table to help you organize your thinking.
1 basket

Mean: 5

Number of Baskets Rishi Made
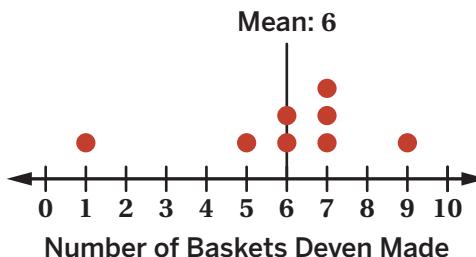
Number of Baskets	2	5	5	5	6	7
Absolute Deviation (distance from the mean, 5)	3	0	0	0	1	2

9 Deven also practices free throws everyday.

Calculate the MAD of Deven's data. Use the table to help you organize your thinking.
1.5 baskets

Mean: 6

Number of Baskets Deven Made
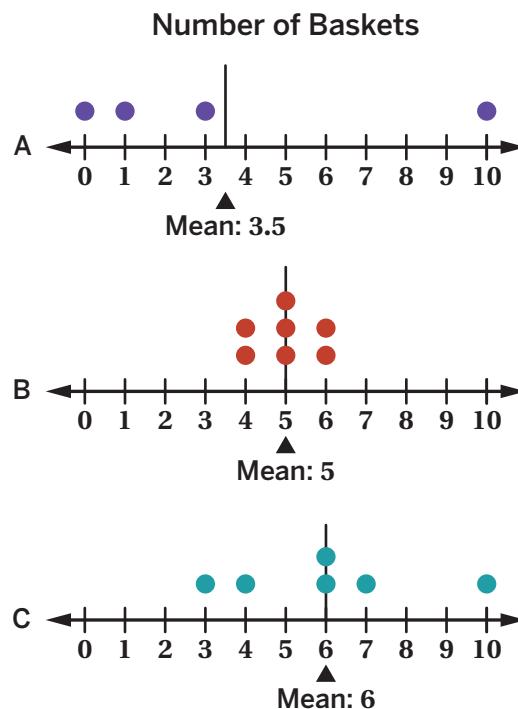
Number of Baskets	1	5	6	6	7	7	7	9
Absolute Deviation (distance from the mean, 6)	5	1	0	0	1	1	1	3

Name: _____ Date: _____ Period: _____

Calculating MAD (continued)

10 Here are the number of baskets that three new players made.

Order the data sets from *smallest* to *largest* MAD.
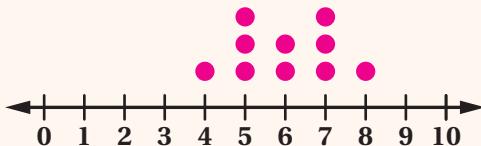
B	Smallest MAD
C	
A	Largest MAD

Number of Baskets

A   0 1 2 3 4 5 6 7 8 9 10
▲ Mean: 3.5

B   0 1 2 3 4 5 6 7 8 9 10
▲ Mean: 5

C   0 1 2 3 4 5 6 7 8 9 10
▲ Mean: 6

Explore More

11 Create a dot plot with at least *four* points, a mean of 6, and a MAD of 1.

Dot plots vary. **Sample dot plot shown.**

0 1 2 3 4 5 6 7 8 9 10
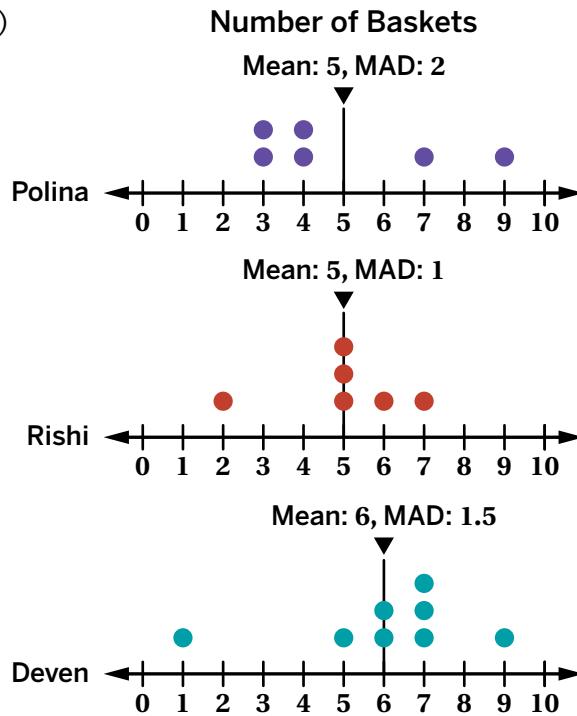
12 Synthesis

How does the mean absolute deviation (MAD) help you compare data sets?

Use the examples if they help with your thinking.
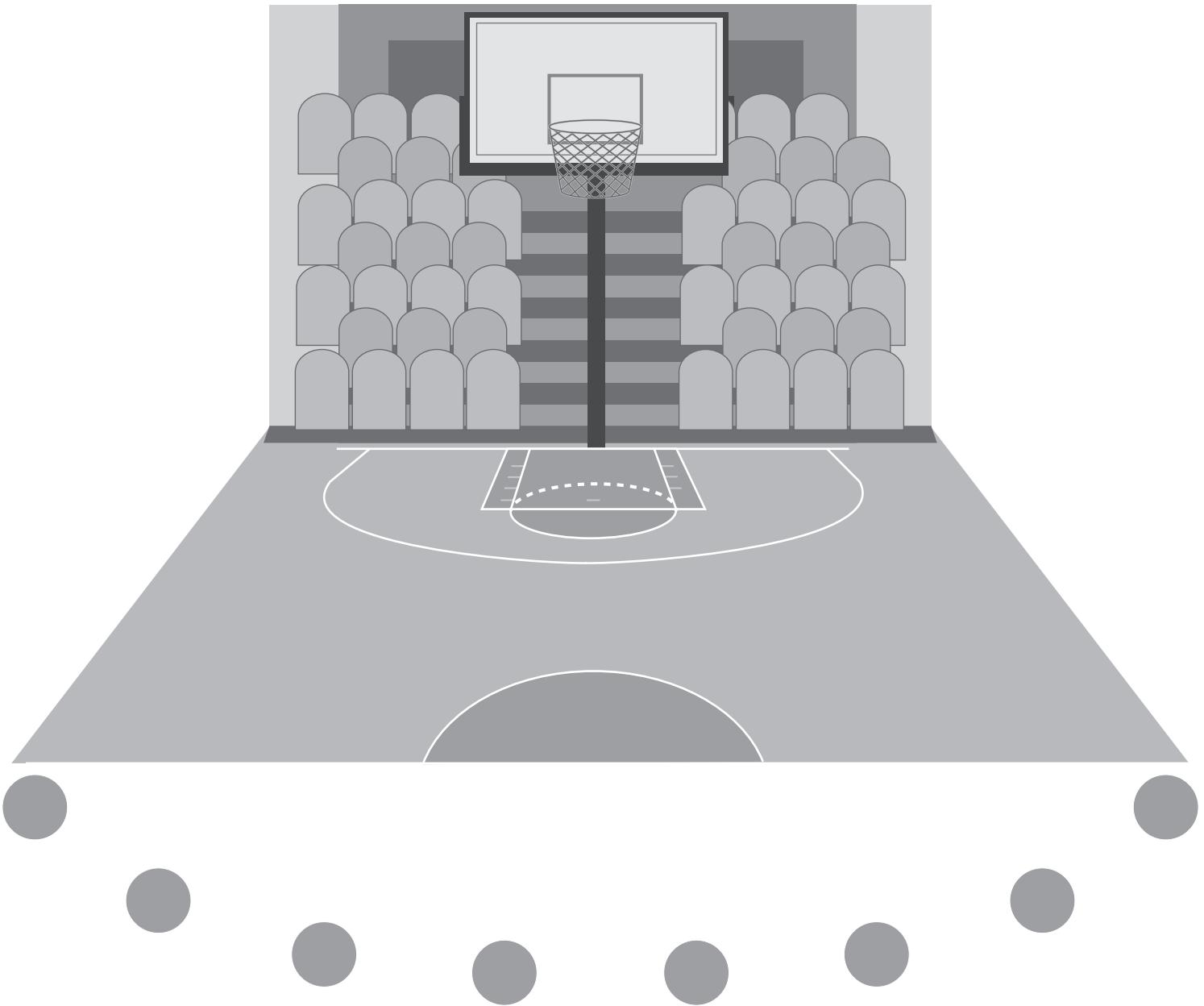
Responses vary.
- **The MAD helps me know how spread out the data is in a data set. The data set with the larger MAD is more spread out.**
- **If I have two data sets and one of them has a smaller MAD, that means the data is less spread out.**

Number of Baskets

Mean: 5, MAD: 2

Polina

0 1 2 3 4 5 6 7 8 9 10

Mean: 5, MAD: 1

Rishi

0 1 2 3 4 5 6 7 8 9 10

Mean: 6, MAD: 1.5

Deven

0 1 2 3 4 5 6 7 8 9 10

Things to Remember:

Name: _____ Date: _____ Period: _____

Warm-Up

Unit 8
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Name: .... Date: .... Period: ....
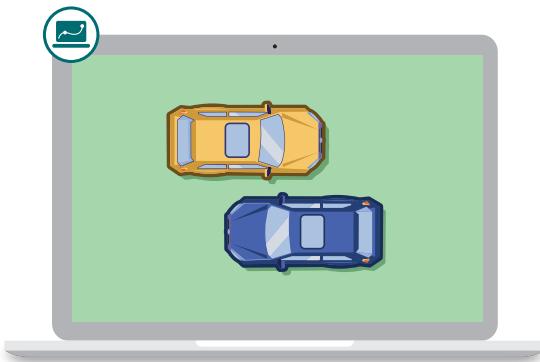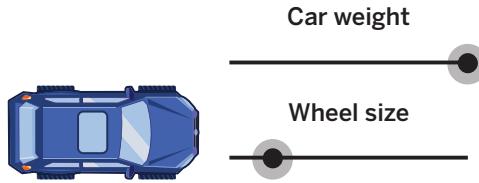
Toy Cars

Let's explore the median of a data set and what it tells us.
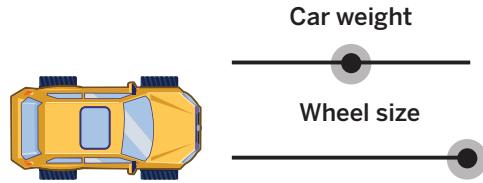
Warm-Up

1–2 Here are two toy cars with different colors, weights, and wheel sizes.

Car A

Car weight

Wheel size

Car B

Car weight

Wheel size
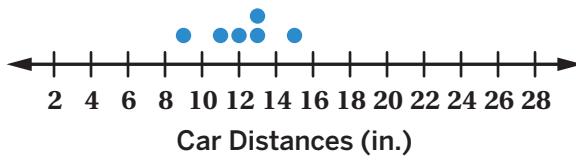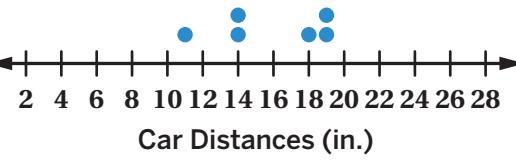
a Each car was launched 6 times. Compare their results.

Car A

2 4 6 8 10 12 14 16 18 20 22 24 26 28
Car Distances (in.)

Car B

2 4 6 8 10 12 14 16 18 20 22 24 26 28
Car Distances (in.)
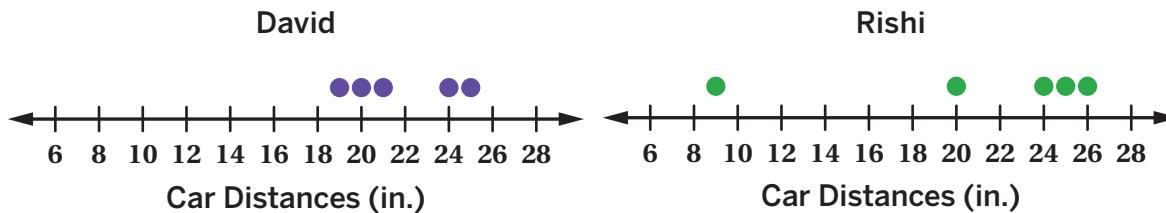
b 💬 **Discuss:** Which car generally travels farther? How do you know?

Responses and explanations vary.
- **Car B generally goes farther because more of its distances were farther than Car A's.**
- **Car B generally goes farther because its mean distance traveled is greater than the mean distance for Car A.**

Name: _____   Date: _____   Period: _____

A New Measure of Center

3 David and Rishi launched their cars 5 times each.

David

Car Distances (in.)

Rishi

Car Distances (in.)

Which car do you think generally travels farther? Circle one.

David's car   Rishi's car   I'm not sure

Explain your thinking.

Responses and explanations vary.
- David's car because it has a greater mean distance than Rishi's car.
- Rishi's car. It had one bad trial (9 inches), but other than that, most of its distances are 24 or greater but most of the distances for David's car are 21 or less.
- I'm not sure. Three of their distances are exactly the same (20, 24, 25). David's other distances are 19 and 21, and Rishi's are 9 and 26. I don't know what to make of that. They should run their cars a few more times to gather more data.

4 David and Rishi each think their car travels farther.

David says: *When we compare the mean distance for each car, my car travels farther.*

Rishi says: *When we compare the middle distance for each car, my car travels farther.*

Whose argument do you agree with? Circle one.

David's   Rishi's   Both   Neither
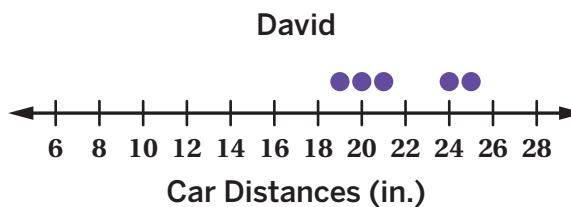
Explain your thinking.

Responses and explanations vary.
- David's because the mean is a number that shows the center of each data set.
- Rishi's because most of his launches went farther than David's. He had one bad launch, which is why his mean distance is lower.
- Both because both statements are true.
- Neither. I think the car that travels farther is the one with the single farthest launch, which is Rishi's car.

5 The middle value of a data set is called the **median**. The median distance for Rishi's car is 24 inches. What is the median distance for David's car?

21 inches

David

Car Distances (in.)

Name: _____ Date: _____ Period: _____

Many Medians

6 Yona launched her car 7 times and recorded the distances, in inches, on a notepad. She calculated the median but made a mistake.

Yona

19, 14, 18, (28), 21, 12, 14

a 💬 **Discuss:** What did Yona do well?

Yona found the middle value of the list.

b What could you change to make all her work correct?

***Responses vary*. I would put the numbers in order first, then determine the middle number.**
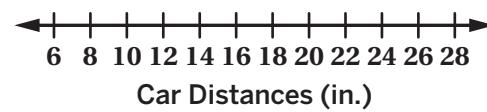
7 Here is Yona's data.

What is the median distance of this data set?

19, 14, 18, 28, 21, 12, 14

Create a dot plot if it helps with your thinking.

18 inches

6 8 10 12 14 16 18 20 22 24 26 28
Car Distances (in.)
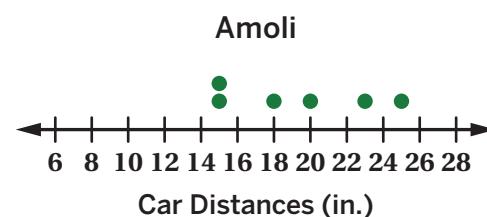
8 Amoli launched her car 6 times.

What do you think is the median of her data set? Explain your thinking.

Responses and explanations vary.

• **19 inches because it's halfway in between the two numbers in the middle of the data set.**

• **18 inches and 20 inches because those are the two numbers in the middle.**

• **There is no median because there is no one value in the middle.**

Amoli

6 8 10 12 14 16 18 20 22 24 26 28
Car Distances (in.)

Name: _____ Date: _____ Period: _____

Many Medians (continued)

9 Statisticians agree that when there's an even number of data points, the median is the average of the middle two numbers.
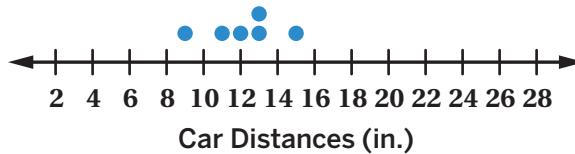
The median distance for Amoli's car is 19 inches because the average of 18 and 20 is 19.

The dot plot shows the data on Car A. What is the median distance that Car A traveled?

12.5 inches

Car Distances (in.)
2 4 6 8 10 12 14 16 18 20 22 24 26 28
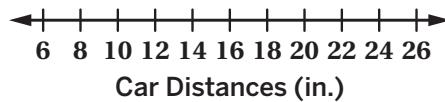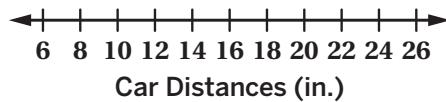
10 Calculate the median distance for each car. Complete as many problems as you have time for. Create a dot plot if it helps with your thinking.

a   14, 19, 15, 20, 17

6 8 10 12 14 16 18 20 22 24 26
Car Distances (in.)

...........**17**........... inches

b   25, 22, 19, 21, 14, 14

6 8 10 12 14 16 18 20 22 24 26
Car Distances (in.)
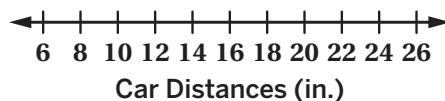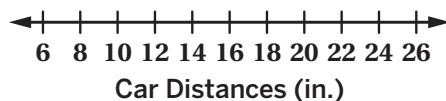
...........**20**........... inches

c   18, 12, 24, 12, 18, 22, 22

6 8 10 12 14 16 18 20 22 24 26
Car Distances (in.)

...........**18**........... inches

d   16, 6, 26, 14, 23, 27, 28, 19, 23, 17

6 8 10 12 14 16 18 20 22 24 26
Car Distances (in.)
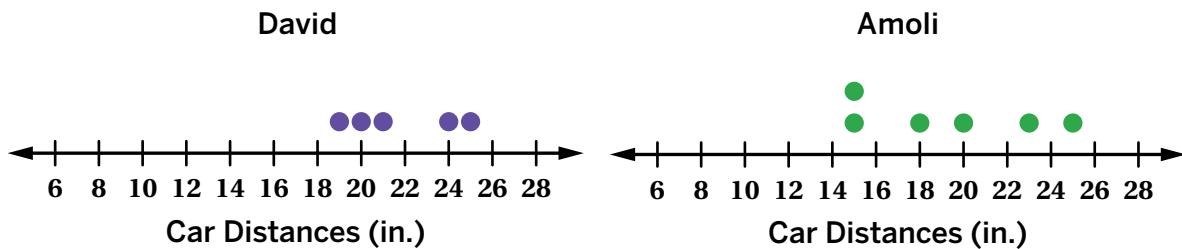
...........**21**........... inches

11 Synthesis

Describe how to determine the median of a data set. Use the examples if they help with your thinking.

David

Car Distances (in.)

Amoli

Car Distances (in.)

Responses vary. **First, put your data points in order from least to greatest, or make a dot plot. Then cross off the largest and the smallest data points. Continue crossing off the largest and smallest points until you get to the points in the middle. If there is only one number in the middle, that's the median. If there are two numbers, calculate their average to determine the median.**

Things to Remember:

Unit 8
Lesson
13

Name: _____ Date: _____ Period: _____

Pumpkin Patch

Let's determine and interpret the quartiles of a data set.

Warm-Up

1 A farmer sells different sizes of pumpkins.

A customer wants a medium-sized pumpkin.

Circle the pumpkins the farmer could sell to this customer. *Choices vary.*

Name: _____ Date: _____ Period: _____
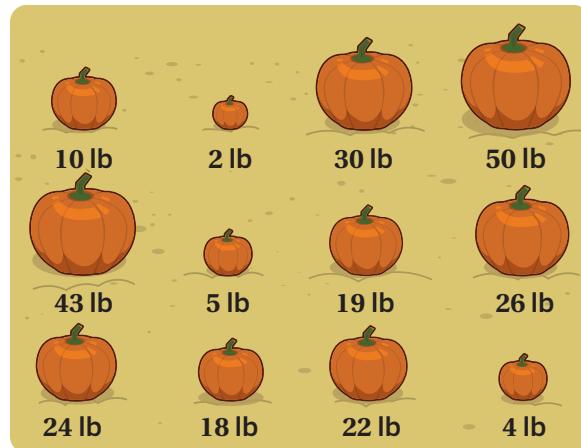
Introduction to Quartiles

2 Farmer Na'ilah always marks the middle half of her pumpkins as medium-sized.

a Let's watch an animation to see how Na'ilah decided which pumpkins were in the middle half.

b Explain how she decided.

Explanations vary. **She lined up the pumpkins from the lightest to the heaviest. Since there are 12 pumpkins, the middle half are the 6 pumpkins in the middle.**

10 lb 2 lb 30 lb 50 lb

43 lb 5 lb 19 lb 26 lb

24 lb 18 lb 22 lb 4 lb

3 Here are some pumpkins on a different farm. Circle the pumpkins with weights that are in the middle half.
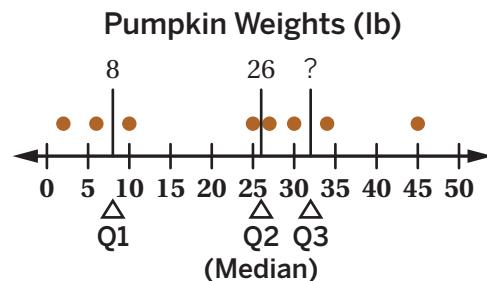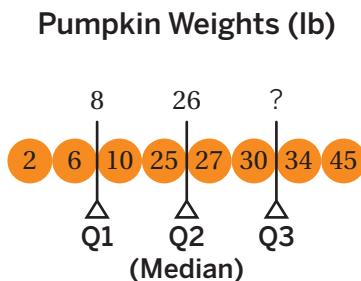
10 pounds, 25 pounds, 27 pounds, and 30 pounds

27 lb 30 lb 45 lb 34 lb

25 lb 6 lb 2 lb 10 lb

Name: _____ Date: _____ Period: _____

Introduction to Quartiles (continued)

4 **Quartiles** divide a data set into four sections. They can help us identify and describe the middle half of a data set.

Here are Quartile 1 (Q1) and Quartile 2 (Q2) for Na'ilah's pumpkins.

Pumpkin Weights (lb)

8   26   ?

2 6 10 25 27 30 34 45

Q1   Q2   Q3
(Median)

Pumpkin Weights (lb)

8   26   ?

0 5 10 15 20 25 30 35 40 45 50

Q1   Q2 Q3
(Median)

What do you think the value of Quartile 3 is? Explain your thinking.
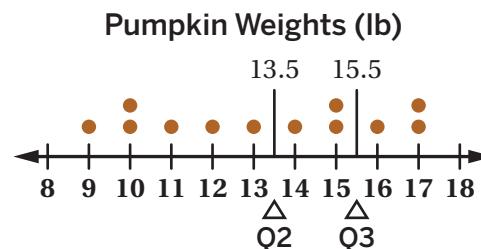
32 pounds. *Explanations vary*. Q3 is in between the 30-pound pumpkin and the 34-pound pumpkin, and 32 pounds is right in the middle of those numbers.

5 Quartile 1 is the median of the lower half of a data set.

Here are the weights of 12 pumpkins on Tasia's farm.

What is the value of Quartile 1?

10.5 pounds

Pumpkin Weights (lb)

13.5   15.5

8 9 10 11 12 13 14 15 16 17 18

Q2   Q3
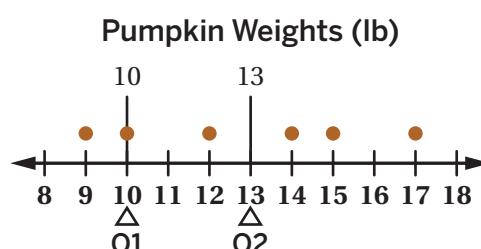
6 Quartile 3 is the median of the upper half of a data set.

Here are the weights of 6 pumpkins on Tyler's farm.
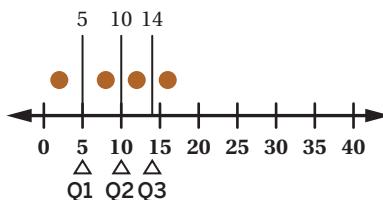
What is the value of Quartile 3?

15 pounds

Pumpkin Weights (lb)

10   13

8 9 10 11 12 13 14 15 16 17 18

Q1   Q2

Name: .... Date: .... Period: ....
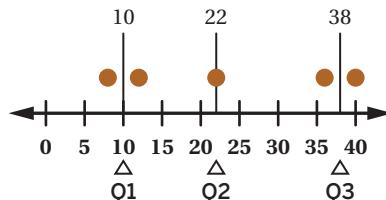
Reasoning About Quartiles
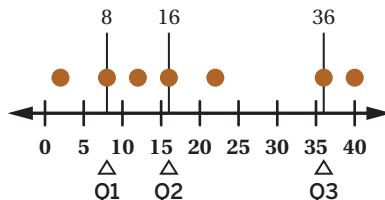
7 Here are some dot plots with different pumpkin weights.

4 pumpkins   5 pumpkins   7 pumpkins

a 💬 **Discuss:** What do you notice?

Responses vary.
- **I notice that when I add a pumpkin, all of the quartiles tend to change.**
- **I notice that a quartile can be on a pumpkin or between two pumpkins.**
- **I notice that if you only have one pumpkin, then all the quartiles are on top of each other.**
- **I notice that if you select both pumpkins that weigh 22 pounds plus one heavier other pumpkin, then Q1 and Q2 have the same value.**

b Here are several challenges. Select *all* the ones you think are possible. Use a device to check your answers.

☐ Q2 is not equal to a pumpkin weight.

☐ Q1, Q2, and Q3 are equal to pumpkin weights.

☐ Q1, Q2, and Q3 are not equal to pumpkin weights.

☐ Q1 is equal to a pumpkin weight, but Q3 is not.

☐ Q1 and Q2 have the same value.
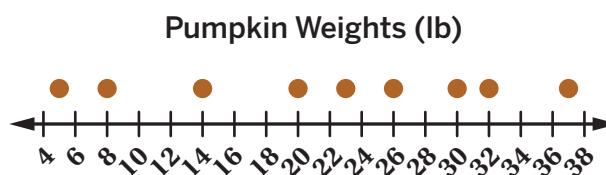
All challenges are possible.

8 Here are 9 pumpkins on Adrian's farm. What do you think the values of Q1 and Q3 are? Explain your thinking. Use the dot plot if it helps to show your thinking.

Pumpkin Weights (lb)

Responses vary.
- **I think Q1 is 11 and Q3 is 31. There are 9 data points, so I can't really split them in half. If I ignore the point in the middle, then there are 4 on each side. 11 is halfway between 8 and 14. 31 is halfway between 30 and 32.**
- **23 is the median, so 14 is the middle number in the lower half and 30 is the middle number in the upper half.**

Note: Students are introduced to the convention of excluding the median when determining the quartiles of a data set with an odd number of points in the next problem.

Name: _____ Date: _____ Period: _____
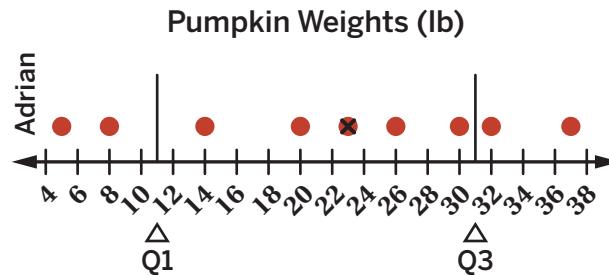
Reasoning About Quartiles (continued)

9 Statisticians agree that when you determine Q1 and Q3 for a data set that has an odd number of points, you do *not* include the median in the lower or upper half of the data.

Pumpkin Weights (lb)

This dot plot shows the weight of 11 pumpkins on Imani's farm.

What is the value of Q3 for these pumpkin weights?

28 pounds

Pumpkin Weights (lb)
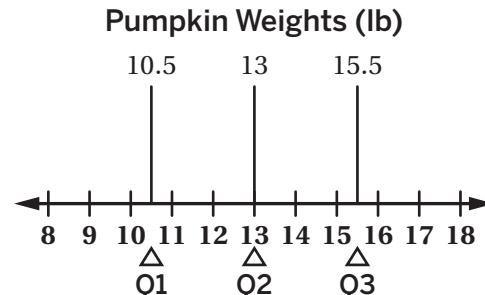
10 A store has 80 pumpkins for sale. Here are the values of the quartiles.

About how many of the 80 pumpkins would you expect to weigh *less* than 15.5 pounds? Explain your thinking.
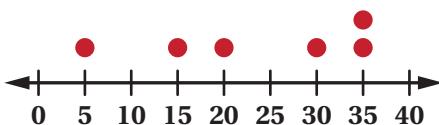
60 pumpkins. *Explanations vary.* There are 80 pumpkins in total, which means there are 20 pumpkins between each quartile on the number line. 15.5 pounds is Q3, so there are 3 sections that weigh less than 15.5 pounds. $20 \cdot 3 = 60$ pumpkins that weigh less than 15.5 pounds.

Pumpkin Weights (lb)
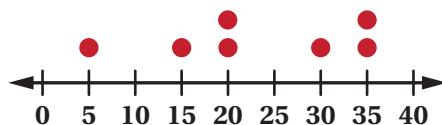
Repeated Challenges

11 Solve as many problems as you have time for. Sensemaking is more important than speed.
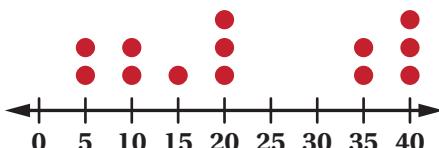
a The dot plot shows 6 data points. What is the value of Q1? **15**
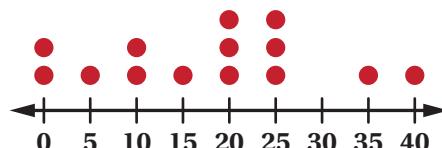
b The dot plot shows 7 data points. What is the value of Q2? **20**
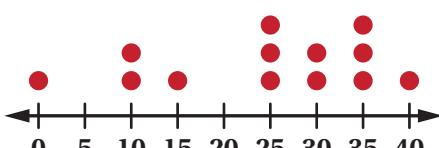
c The dot plot shows 13 data points. What is the value of Q1? **10**
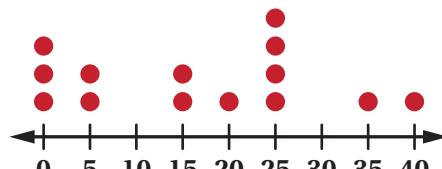
d The dot plot shows 14 data points. What is the value of Q3? **25**

e The dot plot shows 13 data points. What is the value of Q2? **25**

f The dot plot shows 14 data points. What is the value of Q3? **25**

12 Synthesis
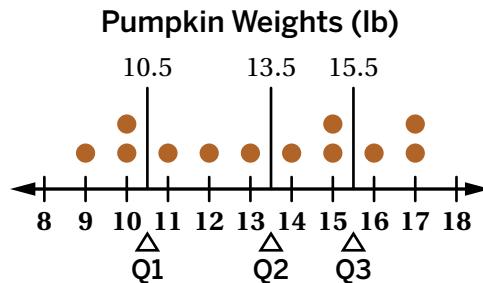
Discuss both questions with a partner. Then select *one* and write your response.

Use the example if it helps to show your thinking.

- How do quartiles relate to the middle half of a data set?
- How can you determine the value of the quartiles for a data set?

Pumpkin Weights (lb)
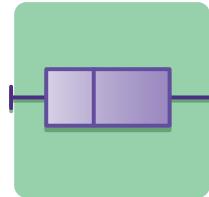
Responses vary.
- Quartiles 1 and 3 are on either side of the middle half of a data set. The middle half includes all the data points that are between Q1 and Q3.
- You determine the value of the quartiles by splitting the data in half and then in half again on each side. If there is a data point in the middle, then don't include that in either half.

Things to Remember:

**Unit 8
Lesson**

14

Name: _____ Date: _____ Period: _____

Car, Plane, Bus, or Train?

Let's explore box plots to visualize data.

Warm-Up

Jalen's family lives in St. Louis. They often visit relatives in Chicago.

1. 💬 **Discuss:**
   - What modes of transportation do you think Jalen's family could take?
   - Which would *you* take? Why?

   Responses vary. **Trains, buses, cars, or planes. I would take a car because then I can be the most flexible.**
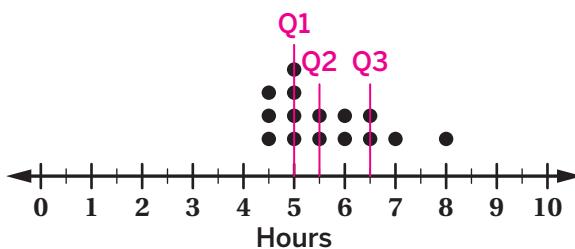
2. Jalen recorded how long it took to drive from St. Louis to Chicago the last 15 times his family drove there.

   Determine the values of Q1, Q2, and Q3. Label them on the dot plot.

   Q1: 5 hours

   Q2: 5.5 hours

   Q3: 6.5 hours

Name: _____ Date: _____ Period: _____

Car or Plane?

Jalen's family wonders: *What mode of transportation should we use to travel between Chicago and St. Louis?*

3. One way to compare data sets is by using **box plots**. Let's watch how a box plot is drawn.
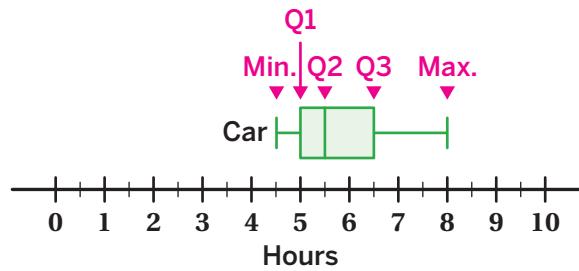
a 💬 **Discuss:** What do you notice? What do you wonder?

Responses vary.
• **I notice that there's a rectangle with a line through it, or maybe two rectangles next to each other, with lines coming out the sides.**
• **I wonder why people use box plots.**

b Label the box plot with each statistic.

• Minimum (Min.)
• Quartile 1 (Q1)
• Quartile 2 (Q2)
• Quartile 3 (Q3)
• Maximum (Max.)
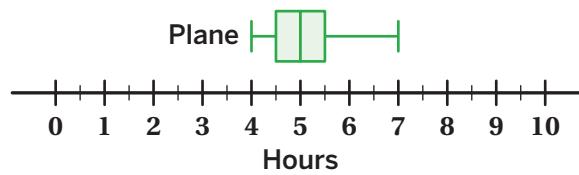
c What percent of the drives are between 5 and 5.5 hours? Explain your thinking.

25%. *Explanations vary.* **5 hours is the value of Q1, and 5.5 hours is the value of Q2 (the median). The percentage of data points between Q1 and Q2 in a data set is about 25%.**

4. Jalen's family sometimes takes a plane to get to St. Louis. Here is a box plot that represents some of their travel times when they flew.

a Determine each statistic for the plane data.

• Minimum: **4 hours**
• Quartile 1: **4.5 hours**
• Median: **5 hours**
• Quartile 3: **5.5 hours**
• Maximum: **7 hours**
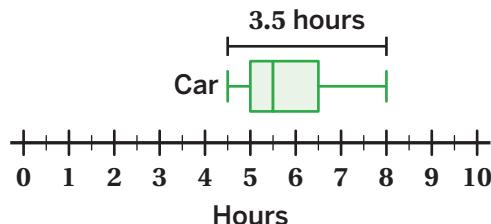
b What percent of the plane data was less than 5 hours long? Explain your thinking.

50% of the plane data. *Explanations vary.* **5 hours is the median, meaning that 50% of the data is less than it.**

Name: _____ Date: _____ Period: _____
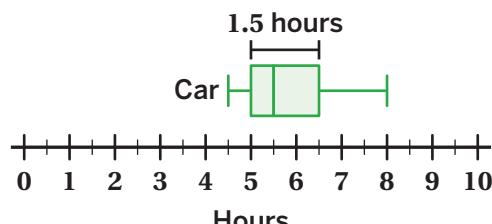
Measures of Spread

5. Jalen's family is also wondering: *How spread out are our travel times when we travel by car?*

Let's look at two ways to describe the spread of a box plot.

3.5 hours

Car [box plot on number line 0–10 Hours]

1.5 hours

Car [box plot on number line 0–10 Hours]

The **range** is the distance from the *minimum* to the *maximum*.

The **interquartile range (IQR)** is the distance from Q1 to Q3.

a Read the definitions of range and IQR.

How are they alike? *Responses vary.* **Range and IQR are alike because they're both measures of spread that tell us the distance from a "low" part of the data to a "high" part of the data.**

How are they different? *Responses vary.* **Range and IQR are different because range is calculated using the least and greatest values, while IQR is calculated using the least and greatest quartiles.**

b Jalen says that one of these measures of spread describes the middle 50% of the data points.

Which measure of spread do you think he is talking about? Circle one.

(IQR)   Range   Neither
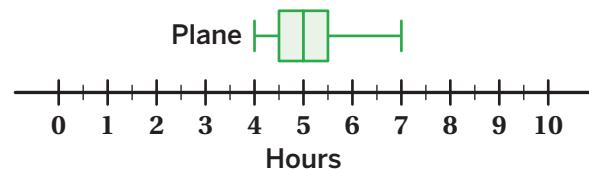
Explain your thinking.

Explanations vary. **Half of the data points are between Quartile 1 and Quartile 3, and the distance between those quartiles is what IQR measures.**

6. What is the range and IQR for the plane data?

Range: **3 hours**   IQR: **1 hour**

Plane [box plot on number line 0–10 Hours]

7. Use the car data and the plane data to help you answer: *Which is more consistent, driving or flying?* Explain your thinking.
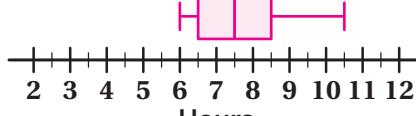
Flying. *Explanations vary.* **Both the range and IQR are less for flying, which means it's a more consistent way to travel.**

Name: .... Date: .... Period: ....

Bus or Train?

8. Jalen's family has also traveled to Chicago by bus and by train.

You will use a set of cards for this activity.

- Match each dot plot, box plot, and statistic to a mode of transportation. There will be two empty spaces in the table for you to complete.
- Draw a box plot on Card A and determine the values of the missing statistics to complete the table.

	Bus					Train				
Travel Times (hours)	7.5	10.5	9.5	9.5	6	6.5	8.5	6	10.5	6.5
	7	6.5	7.5	11	8.5	8	7	7	9	8
Dot Plot	Card B					Card D				
Box Plot	Card A					Card C				
Median	8 hours					Card G 7.5 hours				
IQR	Card H 2.5 hours					2 hours				
Range	Card F 5 hours					Card E 4.5 hours				

9. Which mode of transportation (car, train, bus, or plane) would you recommend Jalen's family use to travel between Chicago and St. Louis? Use evidence to support your argument.
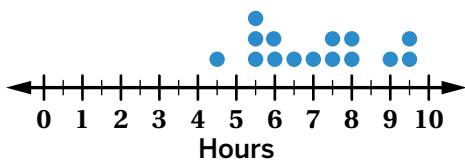
Responses vary.
- I would recommend a plane because its median is the lowest and its spread is low, too.
- I would recommend that Jalen's family go by plane if they weren't worried about cost. The median travel time is 5 hours compared to 7 hours, but flying by plane might be more expensive.
- I would recommend a train because the median is not that different from the others. Plus, trains avoid traffic (unlike cars and buses) and train tickets are probably cheaper than plane tickets.
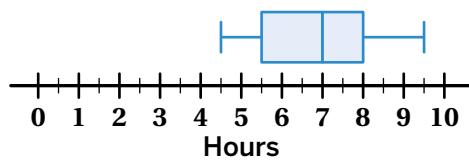
Synthesis

10. Here is a dot plot, a box plot, and several statistics for the car data.

Dot Plot

0 1 2 3 4 5 6 7 8 9 10
Hours

Box Plot

0 1 2 3 4 5 6 7 8 9 10
Hours

Statistics			
Median: 7 hours	Number of Data Points: 15	Range: 5 hours	IQR: 2.5 hours

Which statistics are more visible in the dot plot? In the box plot? Why do you think that is?
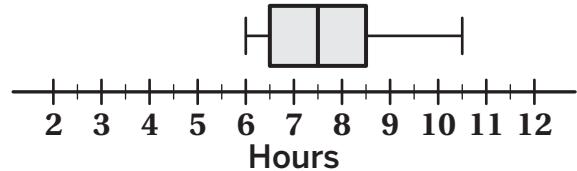
The dot plot is best for determining the number of data points because each data point is represented with one dot. The median, range, and IQR are most visible in the box plot because they are all easy to see directly on the graph or by doing a small calculation with the numbers on the graph.

Things to Remember:

Bus or Train?

Directions: Make one copy per pair of students. Then pre-cut the cards and give each student one set.
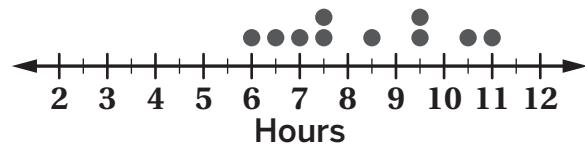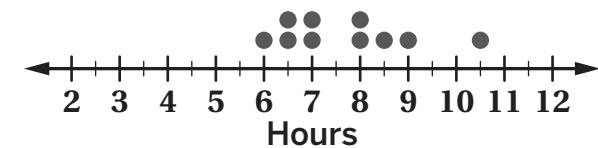
Card A

Hours

Card B

Hours

Card C

Hours

Card D

Hours

Card E
4.5 hours

Card F
5 hours

Card G
7.5 hours

Card H
2.5 hours

Card A

Hours

Card B

Hours

Card C

Hours

Card D

Hours

Card E
4.5 hours

Card F
5 hours

Card G
7.5 hours

Card H
2.5 hours

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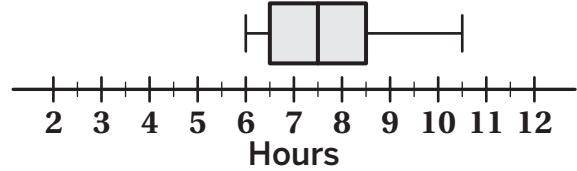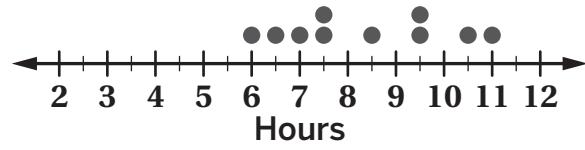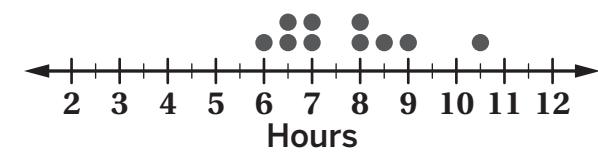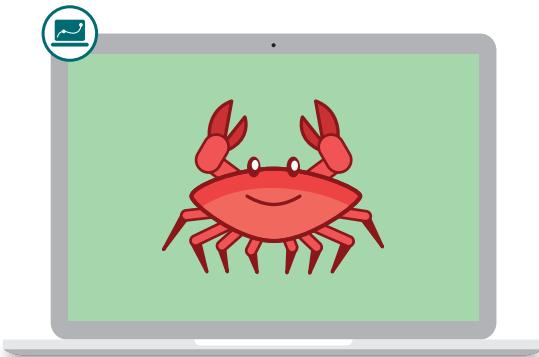
Name: _____ Date: _____ Period: _____

Crab Island

Let's compare large populations of data.

Warm-Up

1 Which one doesn't belong? Explain your thinking.

A. $\dfrac{1 + 5 + 9}{3}$

B. $\dfrac{1 + 2 + 3 + 4 + 5}{3}$

C. $\dfrac{2 + 4 + 5 + 6 + 8}{5}$

D. $\dfrac{3 + 5 + 13}{3}$

Responses and explanations vary.
- **Expression A is the only one that has a 9.**
- **Expression B is the only one that is not a mean because there are more numbers being added than divided by.**
- **Expression C is the only one that is the mean of 5 numbers.**
- **Expression D is the only one that does not have a value of 5.**

Name: .... Date: .... Period: ....

Sampling

2 Crab Island has a particular species of crabs.

What is a question about these crabs that we could collect data for?
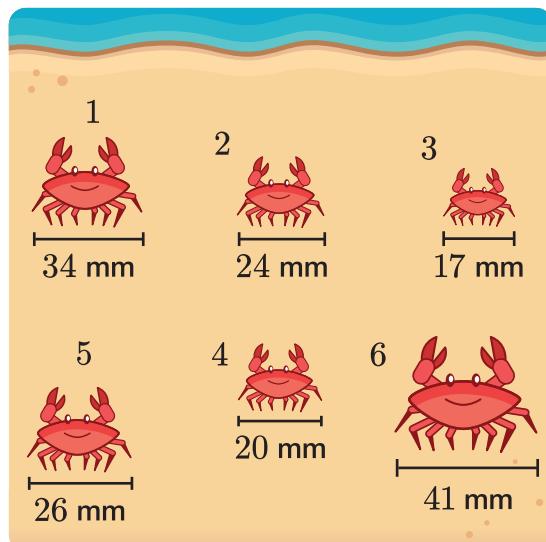
Responses vary.
- **How big are the crabs' claws?**
- **What proportion of the crabs are babies?**
- **Are there more babies than adults?**

3 **a** Take a look at this group of crabs Varuna is studying.

Crab	Width (mm)
1	34
2	24
3	17
4	20
5	26
6	41

b 💬 **Discuss:** What question could this group of crabs help answer?
Responses vary. **How wide are the crabs on this island?**

4 Calculate the mean of Varuna's group of crabs.
27 millimeters

Name: .... Date: .... Period: ....

Sampling (continued)

5 Here is a dot plot of the widths of the crabs in the **population**.

The crabs Varuna chose are highlighted. This is a **sample**.

What do you notice? What do you wonder?
Responses vary.
I notice:
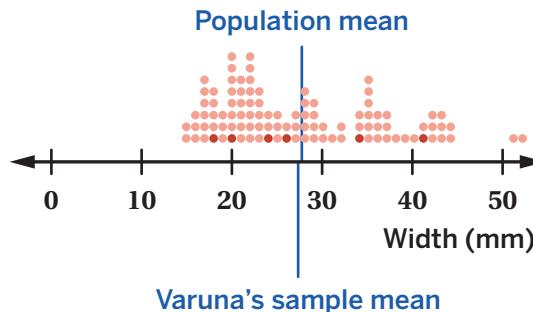
- I notice that the population has A LOT of crabs.
- I notice that the population has two crabs that are really big, and those two weren't in my sample.
- I notice that the sample mean and the population mean are close but not exactly the same.

I wonder:
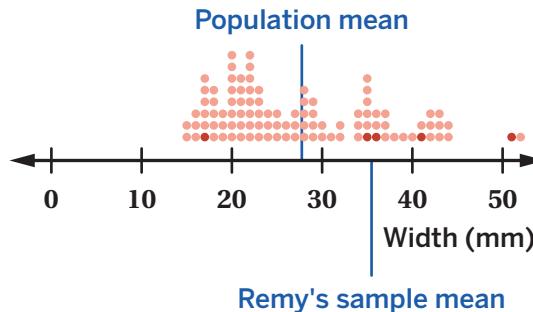
- I wonder what this dot plot looks like for other students who collected different samples.
- I wonder how many total crabs are in the population.
- I wonder how my sample mean would have changed if it included one or both of the big crabs.

Population mean

Width (mm)

Varuna's sample mean

6 Remy chose a different sample. The dot plot shows Remy's sample and sample mean.
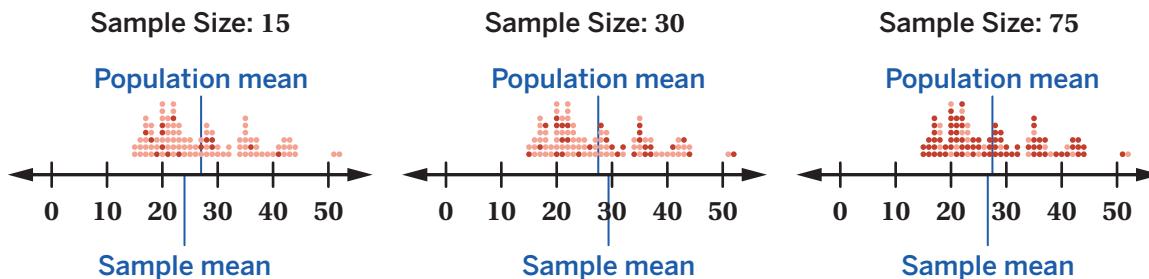
What might happen if someone used Remy's sample to study the crabs on Crab Island?

Responses vary. **Because Remy's sample includes one of the island's two big crabs, the sample mean is much greater than the population mean (about 36 millimeters compared to 28). If someone looked only at this sample, they might think that Crab Island crabs are much bigger on average than they really are.**

Population mean

Width (mm)

Remy's sample mean

Name: _____   Date: _____   Period: _____

Explore More Samples

7 Here are three different samples of crab widths.

Sample Size: 15   Sample Size: 30   Sample Size: 75

Population mean   Population mean   Population mean

Sample mean   Sample mean   Sample mean
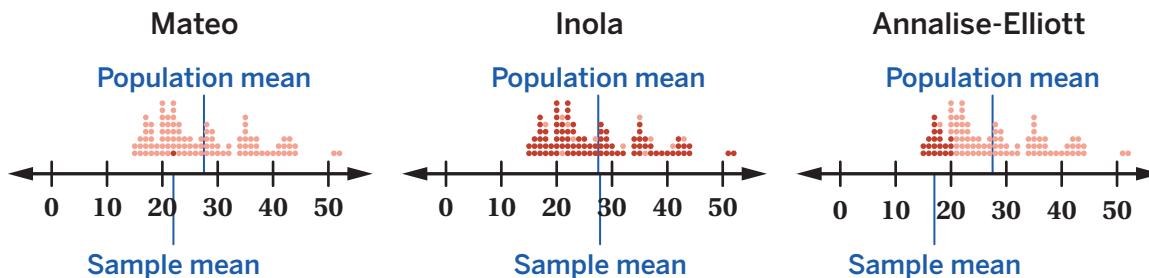
💬 **Discuss:** What are advantages and disadvantages of using a large sample?

Responses vary.
- **An advantage of using a large sample is that you are more likely to get a sample mean that is closer to the population mean because you are including more of the data.**
- **A disadvantage of using a large sample is that it might be hard to do. Tagging and measuring a lot of crabs is a lot of work!**

8 Mateo, Inola, and Annalise-Elliott each collected different samples of crabs. Select a sample and explain what advice you would give to that person.

Mateo   **Inola**   **Annalise-Elliott**

Population mean   Population mean   Population mean

Sample mean   Sample mean   Sample mean

Responses vary.

Advice for Mateo: Choose more crabs in your sample.

Advice for Inola: Take a smaller sample. Even with a smaller sample, your approximate sample mean can be close to the population mean.

Advice for Annalise-Elliott: It looks like your sample includes a lot of small crabs. If you choose randomly, then your sample might include crabs that are more likely to represent the population.

Name: _____ Date: _____ Period: _____

Populations and Samples

9 Kris wonders how students at this school feel about the cafeteria food. Kris asked 50 students. In this situation, what is the population and what is the sample?

Place a P next to the population and an S next to the sample.

_____**P**_____ The students in the school   _____ The cafeteria food

_____ The head cook   _____**S**_____ The 50 students

10 Complete each row of possible questions, populations, and samples.
Responses vary.

Question	Population	Sample
What is the average circumference of a tree in my state?	All of the trees in the state	**A few trees from each forest in my state**
How large are the egg yolks in San Diego?	The eggs in San Diego	One egg from each store in the city
What is the average weight of an apple?	**All of the apples in the world**	The apples in the school cafeteria

Explore More

11 **a** What is a question you could ask where the population is all of the books in the school?
Responses vary. **How long is the average book in my school library?**

b What is a question you could ask where a sample is all of the books in the school?
Responses vary. **How long is the average teen book?**

12 Synthesis

What are some advantages and disadvantages of using samples to answer a question about a population?

Responses vary.

Sample — The 50 students

Population — The students in the school

Advantages:
- **You don't need to find as many people to ask.**
- **You can answer a question more quickly because you don't need as much data.**
- **You are less likely to make errors because of large calculations.**

Disadvantages:
- **It's hard to know exactly how close the information about the sample is to the information about the whole population.**
- **If you choose a bad sample, like all students who don't eat in the cafeteria or students who only like pizza, then you might get information that doesn't match how people actually feel.**

Things to Remember:

Unit 8
Lesson

11

Name: .... Date: .... Period: ....

Headlines

Let's see what makes a good sample.

Warm-Up

1 Adrian collected this sample of crabs and then wrote this headline:

"Crabs Are Smaller Than Ever!"
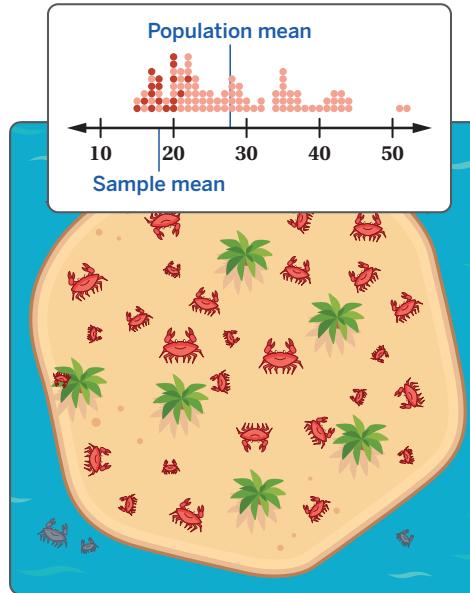
Do you agree with Adrian's headline? Circle one.

Yes   No   I'm not sure

Explain your thinking.
Responses and explanations vary.
- **Yes, according to Adrian's sample, they are smaller than other crabs in the population.**
- **No, Adrian's sample doesn't represent the entire population of crabs because he only sampled small crabs.**
- **I'm not sure. I need more information.**

Name: _____ Date: _____ Period: _____

Graduation Rates

2 We are going to use your class as a sample to learn more about people in the United States.

Answer this question: *Have you graduated high school yet?* Circle one.

Yes   ⬭No⬭

3 Read the news headline and the information about the sample that the newspaper used.

What do you notice about this sample?
Responses vary.
- **I notice that this sample is only of middle school students, but it's asking about graduating high school! That's not fair!**
- **I notice that they only used 30 people in their sample. There are over 300 million people in the U.S. 30 is so small!**

> **Des-News** Online
>
> **0% of People**
> **Have Graduated High School**
> **in the United States!**
> _____
>
> We sampled 30 middle school students in the United States and 0 said they graduated high school.

4 Read the news headline and the information about the sample that the newspaper used.

Why is this sample more likely to be **representative** of the population?
Responses vary. **This sample is more likely to be representative because it asked a lot of people and those people were randomly chosen from all the adults, not middle school students.**

> **Des-News** Online
>
> **90% of People**
> **Have Graduated High School**
> **in the United States!**
> _____
>
> We sampled 57,000 random census surveys from Americans 25 and older. 51,300 reported graduating high school.

Name: _____ Date: _____ Period: _____

Representative Sampling

5 The American Time Use Survey asks about 9,400 randomly selected people how they use their time each year.

One of the questions they ask is: *How many hours do you spend with friends on average per day?*
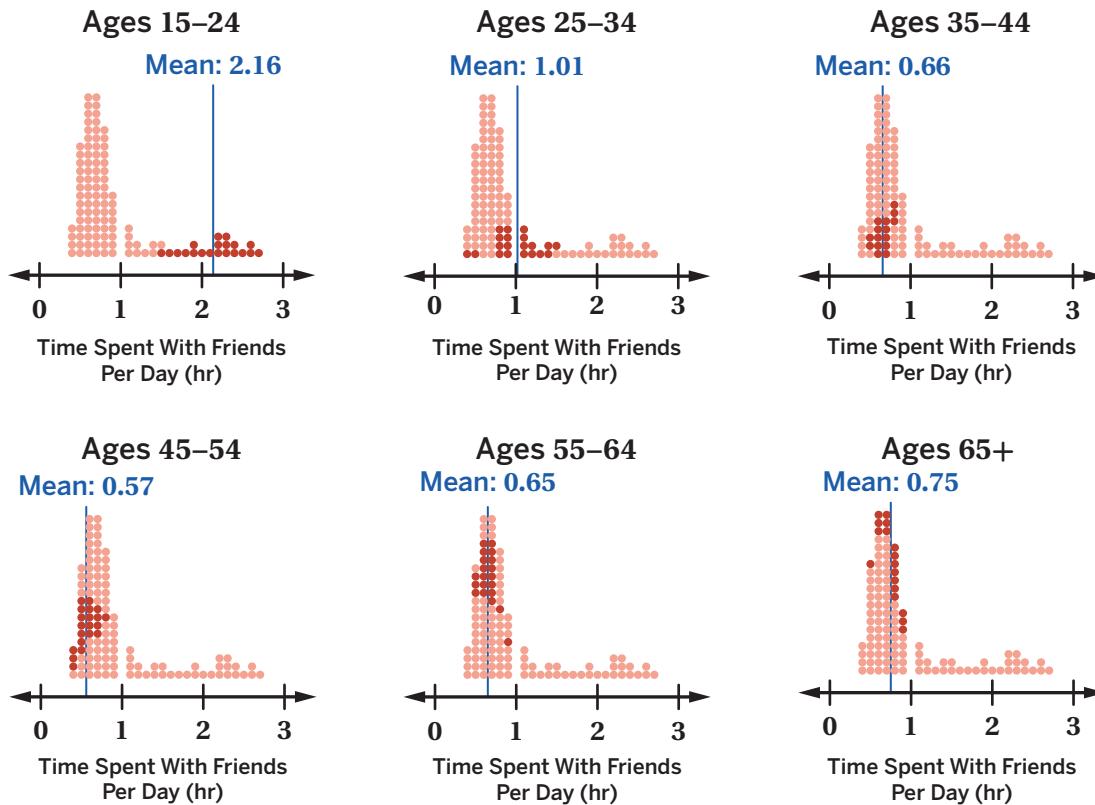
💬 **Discuss:** How do you think people of different ages might answer this question?

Responses vary.
- **I think younger people will spend more time with friends than adults because most adults have to go to work.**
- **I think younger people and older people who are retired will see their friends more.**
- **I think babies won't spend any time with friends because most babies don't have friends.**
- **I think that as people get older they have less time to spend with friends.**

6 Here is a representative sample of 112 responses from the American Time Use Survey.

a Take a look at the results for different ages.

Ages 15–24
Mean: 2.16
Time Spent With Friends Per Day (hr)

Ages 25–34
Mean: 1.01
Time Spent With Friends Per Day (hr)

Ages 35–44
Mean: 0.66
Time Spent With Friends Per Day (hr)

Ages 45–54
Mean: 0.57
Time Spent With Friends Per Day (hr)

Ages 55–64
Mean: 0.65
Time Spent With Friends Per Day (hr)

Ages 65+
Mean: 0.75
Time Spent With Friends Per Day (hr)

Source: U.S. Bureau of Labor Statistics

b Describe what you notice about the sample.
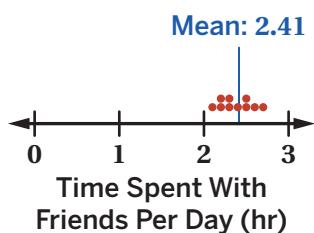
Responses vary.
- **I notice that young people spend a lot more time with friends than adults do.**
- **I notice that people above 65 actually spend more time with friends than people between 35 and 64.**
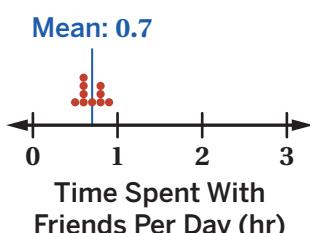
Representative Sampling (continued)

7 Here are three sampling methods and the samples that were collected.
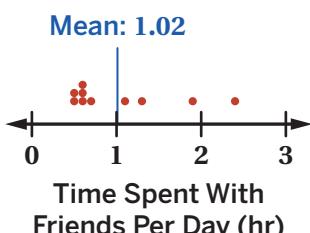
10 students in one high school class	10 random people at a nursing home	10 random phone numbers and ask whoever answers
Mean: 2.41	**Mean: 0.7**	**Mean: 1.02**

Time Spent With Friends Per Day (hr)

Choose a sampling method and use the sample to write a headline about how much time people in the United States spend with their friends per day.

Your headline: _____

Responses depend on students' choice of sampling method. People in the U.S. Spend Over 2 Hours Each Day With Friends! Chores Never Finished!; People in the U.S. Are Lonely: They Spend Less Than 1 Hour a Day With Friends!; An Hour a Day! People in the U.S. Spend About an Hour Each Day With Friends.

8 Ada wants to collect new data to see how people in the United States spend their time. Order the strategies from *least likely* to *most likely* to produce a representative sample of the population. *Responses vary.*

Least Likely

a. Post a poll on social media and look at the first 20 responses.

b. Ask 20 random people at the grocery store.

c. Ask every single person in the country.

d. Call 20 phone numbers at random.

e. Ask 20 friends.

Most Likely

Explore More

9 On the Explore More Sheet there are dot plots showing how much time people in the United States spend alone each day. Explore the mean response for different ages.

Describe an *incorrect* conclusion someone might make from an unrepresentative sample.

Responses vary. Someone might incorrectly assume that many people in the United States spend more than 7 hours a day alone.
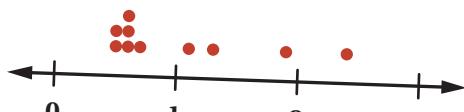
Explain how collecting a representative sample or an unrepresentative sample can affect someone's understanding of a population.

Use the examples if they help with your thinking.
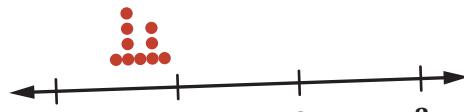
Responses vary. **If a person wants to understand a population based on a sample, the sample needs to be representative. If the sample isn't representative, the person is more likely to make inaccurate conclusions.**

Dial random phone numbers until you speak with 10 people.

Time Spent With Friends Per Day (hr)

Ask 10 random people at a nursing home.
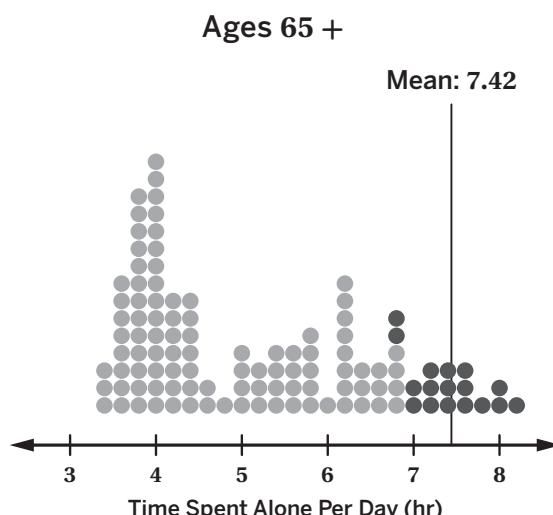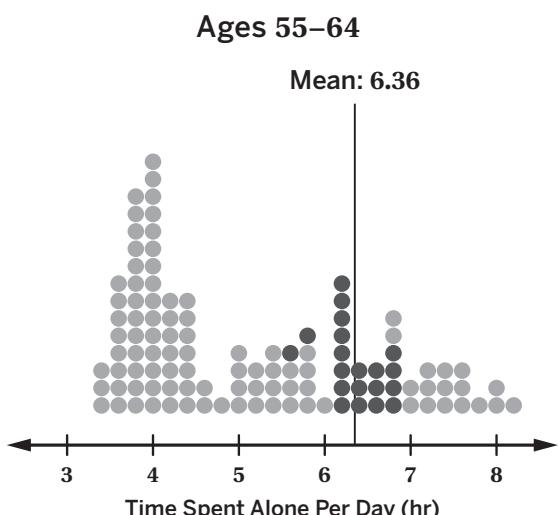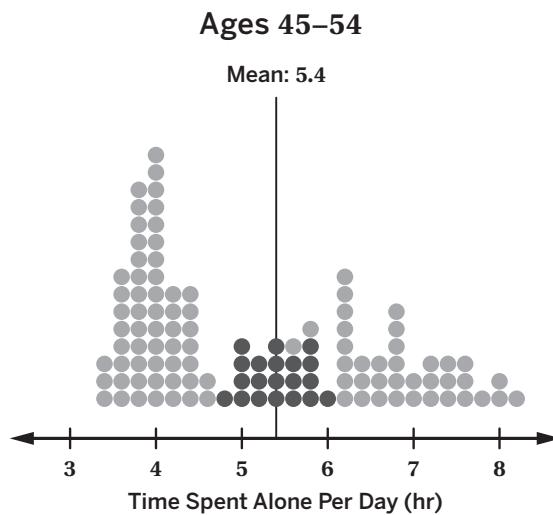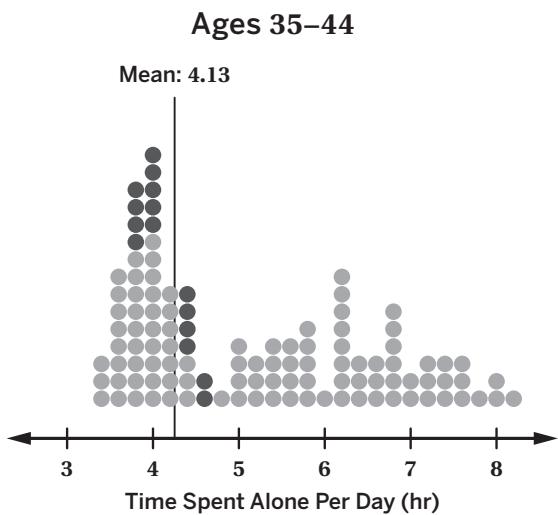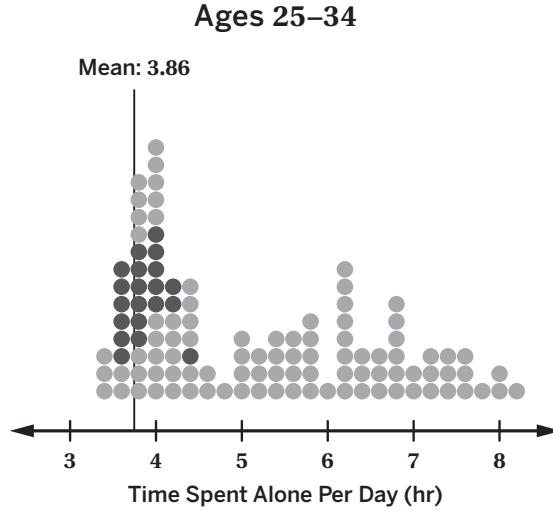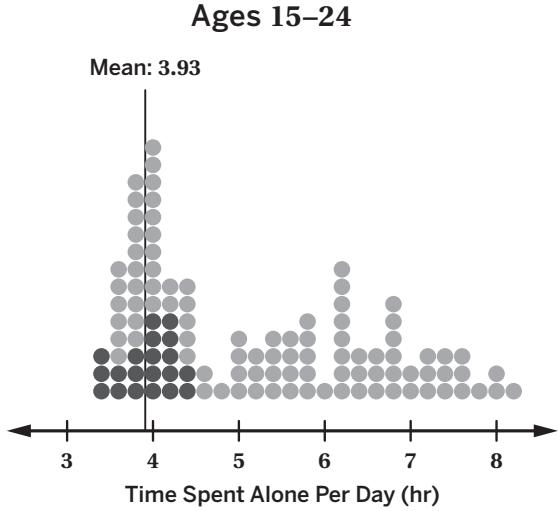
Time Spent With Friends Per Day (hr)

Things to Remember:

Name: _____ Date: _____ Period: _____

Explore More

Use these dot plots based on the American Time Use Survey to complete Problem 9 in your Student Edition.

Ages 15–24

Mean: 3.93

Time Spent Alone Per Day (hr)

Ages 25–34

Mean: 3.86

Time Spent Alone Per Day (hr)

Ages 35–44

Mean: 4.13

Time Spent Alone Per Day (hr)

Ages 45–54

Mean: 5.4

Time Spent Alone Per Day (hr)

Ages 55–64

Mean: 6.36

Time Spent Alone Per Day (hr)

Ages 65 +

Mean: 7.42

Time Spent Alone Per Day (hr)

Unit 8
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12

Name: .... Date: .... Period: ....

Flower Power

Let's use percentages to make predictions about populations.

Warm-Up

1 A gardener planted some seeds from a bag of Wildflower Seed Mix.

What do you notice? What do you wonder?

Responses vary.

I notice:

- **I notice there are three different types of flowers in the garden.**
- **I notice the location of the flowers appears to be random.**

I wonder:

- **I wonder how many of each type of flower are in the garden.**
- **I wonder which flower type appears the most often in the garden.**

🌼 Marigold   🌸 Primrose   🌼 Daisy
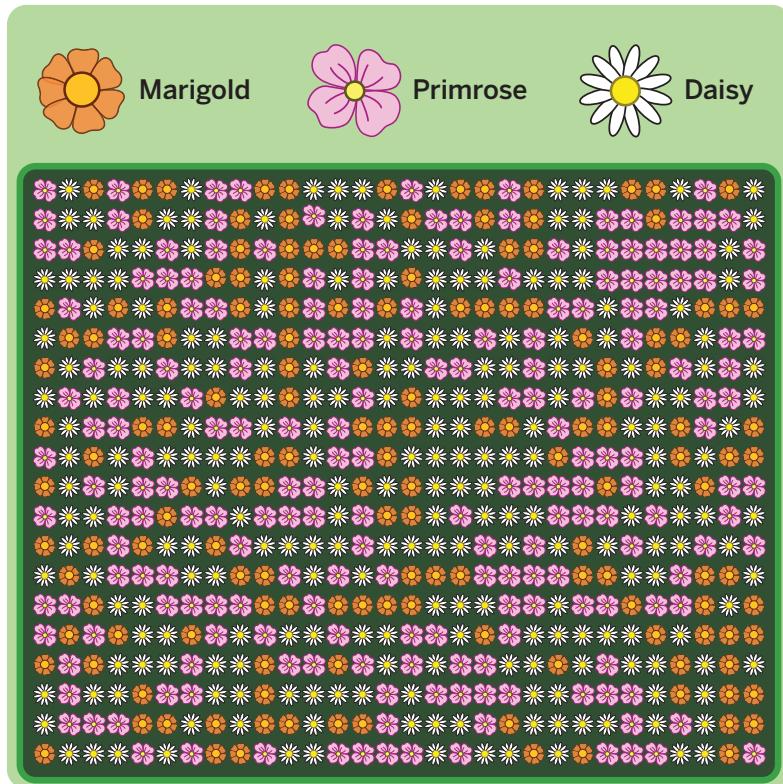
2 The gardener wonders how many of each flower type are in the garden. He says there are too many to count one by one.

Describe a method he could use to estimate how many of each flower type there are.

Responses vary. **Count how many of each type there are in the top row and then multiply by the number of rows to get an estimate of the whole garden.**

Name: _____ Date: _____ Period: _____

Estimating the Marigolds

3 Here are all 600 flowers in the garden.

Marigold   Primrose   Daisy

a Choose a sample of at least 25 flowers from the garden.
Samples vary.

b Complete the table for your sample.
Responses vary based on students' sample.

Flower Type	Count	Percentage
Marigold	9	14%
Primrose	25	39%
Daisy	30	47%
Total	64	100%

4 Use your table to help you estimate how many of the 600 flowers in the garden are marigolds.
Responses vary based on students' sample.

Estimating the Marigolds (continued)

5 Eliza and Javier chose a 5-by-5 sample by having Eliza close her eyes and place her finger on the image.

Here are their strategies for estimating the total number of marigolds.

Explain each student's strategy.

Explanations vary.
- **Eliza's strategy: Figure out the percent of marigolds in the sample. Then calculate that same percent of flowers in the whole garden.**
- **Javier's strategy: Figure out how many times larger the population is than the sample. Multiply the number of marigolds in the sample by the same scale factor.**

Eliza

Marigolds are 28% of the sample.

Marigolds are probably 28% of the garden.

28% of 600 is 168 marigolds.

Javier

In the sample, 7 of 25 are marigolds.

Marigolds	Total
7	25
168	600

·24 (left)   ·24 (right)

6 Let's watch an animation to reveal how many marigolds are in the entire garden.

💬 **Discuss:**
- How did the actual number of marigolds in the garden compare to the earlier estimates?
- What do you think could have improved the estimates?

Responses vary.
- **There are 151 marigolds in the garden, which is less than my estimate of 172 marigolds. This is also 17 marigolds less than Eliza and Javier's estimate.**
- **Using a larger sample size or looking at multiple samples might have provided a more accurate representation of the population.**

The Bad Review

7 Here is a new bag of seeds and a customer review.

a 💬 **Discuss:**

- What does the company claim about the bag?
- Why did the customer leave a one-star review?

Responses vary.
- **The company claims that the prairie seed mix would produce a mix of 2,000 flowers, which would include about 800 primroses.**
- **The customer grew 10 flowers and only 1 was a primrose, which is 10%. That's a lower percentage than is implied from the description on the bag.**

PRAIRIE SEED MIX

Produces a colorful mix of 2,000 flowers, including about 800 primroses.

CUSTOMER REVIEW

⭐☆☆☆☆

Prairie Seed Mix Is a Scam!
I brought the bag because it said I would get a LOT of primroses. But I grew 10 flowers and only ONE was a primrose. The company lies!

b How could you investigate the company's claim about primroses?

Responses vary. **I could plant more flowers and see if about 40% are primroses, as the bag implies.**

8 Let's grow a sample of up to 100 flowers from the Prairie Seed Mix.

a Write down the number of each type of flower in your sample in the table.

Flower Type	Marigold	Primrose	Daisy	Total
Count	27	41	32	100

Responses vary based on sample. **Each flower in the digital garden has the following probabilities: Marigold: 30%, Primrose: 44%, Daisy: 26%.**

b How many of the 2,000 flowers do you expect to be primroses based on your sample? Explain your thinking.

Responses and explanations vary depending on the results of each sample.
- **I predict 820 of the flowers will be primroses since 41% of the flowers in my sample were primroses, and 41% of 2,000 is 820.**
- **I think there will be close to 820 primroses because I grew 100 flowers and 41 were primroses. That's a ratio of $\frac{41}{100}$, which is equivalent to $\frac{820}{2000}$.**

9 Based on your sample, what do you think about the company's claim now?

Responses vary.
- **I think the company's claim is true. In my sample, nearly half of the flowers were primroses, which would be more than 800 for a bag of 2,000.**
- **I estimated that there would be 480 primroses in the whole bag. That is not close to 800. The company's claim is probably untrue.**
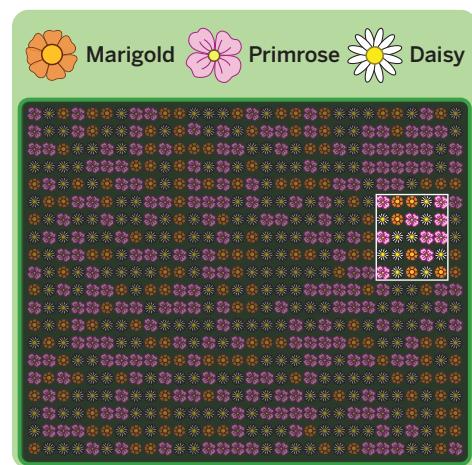
🔟 Synthesis

Describe how you can use a sample and *proportional* reasoning to estimate information about a population.

Use this sample and the population if that helps with your thinking.

Responses vary.

- I can take a sample and calculate the percentage of the thing I'm investigating, like the percent of marigolds in the sample. Then I calculate that same percentage for the whole population.

- Let's say I want to estimate the number of marigolds out of 600 flowers. I know the number of marigolds in the sample should be almost proportional to the number of marigolds in the population. The ratio of marigolds in the sample is $\frac{6}{25}$. I can use this ratio to estimate about 144 marigolds in this population because $\frac{6}{25}$ is equivalent to $\frac{6 \cdot 24}{25 \cdot 24} = \frac{144}{600}$.

Things to Remember:

Unit 8
Lesson
14

Name: _____   Date: _____   Period: _____
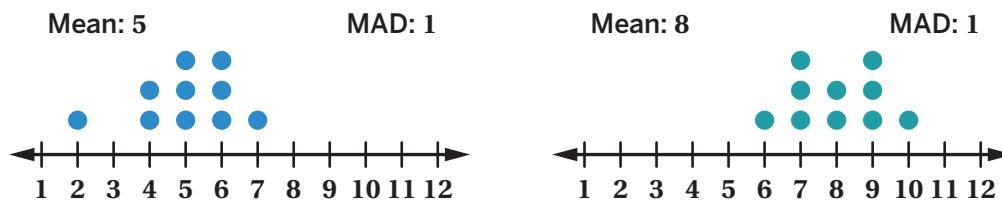
School Newspaper

Let's compare populations by analyzing the difference between the measures of center and variability.

Warm-Up

Here are two dot plots.

Mean: 5   **MAD: 1**   **Mean: 8**   **MAD: 1**

1. What do you notice? What do you wonder?

   I notice:

   Responses vary.
   - **I notice that the data sets have a difference of 3 in their means but have the same MAD.**
   - **I notice that both plots have data at 6 and 7.**
   - **I notice that the data points are close to the mean in each dot plot.**

   I wonder:
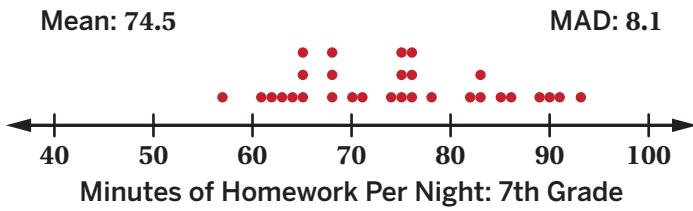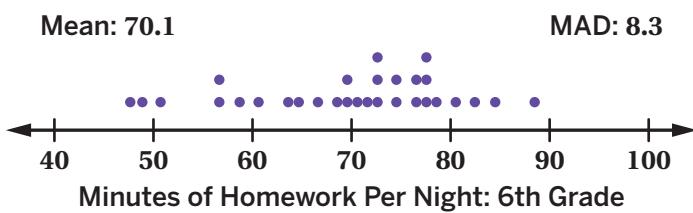
   Responses vary.
   - **I wonder what the data represents.**
   - **I wonder what the dot plots would look like if the MAD was less than 1 or greater than 1.**

Name: .... Date: .... Period: ....

Homework Headline

2. Students at Median Middle School are investigating issues for the school newspaper.

They surveyed an equal number of randomly selected 6th and 7th graders out of the 500 students at their school. Here are the results of the survey and a headline based on the results.

Des-News

Survey Finds That 6th Graders Have Less Homework Than 7th Graders

Mean: 70.1   **MAD: 8.3**

Minutes of Homework Per Night: 6th Grade

Mean: 74.5   **MAD: 8.1**

Minutes of Homework Per Night: 7th Grade

a The mean of the 6th grader data set is 70.1 minutes. What does this statistic represent?
Responses vary. **A 6th grader has 70.1 minutes of homework per night on average.**

b The MAD of each data set is about 8 minutes. What does this statistic represent?
Responses vary. **How spread out the data is**

c Based on this data, why might someone believe this headline?
Responses vary. **Someone might believe this headline because 6th graders have more dots on the left side of the dot plot, which shows less homework.**

d Based on this data, why might someone *not* believe this headline?
Responses vary. **Someone might not believe this headline because there is a lot of overlap between the dots in each data set.**

Name: _____   Date: _____   Period: _____

Homework Headline (continued)

3. Wohali and Ama use different strategies to decide if they believe the headline.

<div style="display:flex">

Wohali

$74.5 - 70.1 = 4.4$

I believe the headline because the mean of the 6th graders' data is 4.4 minutes less than the mean of the 7th graders' data.

Ama

$74.5 - 70.1 = 4.4$

$\dfrac{4.4}{8.3} \approx 0.53$

I don't believe the headline. Even though the mean is 4.4 minutes less, 4.4 minutes isn't a big difference when looking at the spread of the data. The difference is not even 1 MAD, it's about half of that! There will be a lot of overlap between the data sets.

</div>

Whose claim do you agree with? Explain your thinking.

Responses and explanations vary. **I agree with Ama. There isn't a very big difference between 70.1 and 74.5 minutes of homework if the MAD is about 8 minutes.**

4. The survey results for 8th graders show a mean of 90 minutes and a MAD of 8 minutes.

Let's use Ama's strategy to investigate whether there is a big difference in homework time for 7th and 8th graders.

a Calculate how many MADs apart the means are. Use the larger MAD in your calculation.

$\dfrac{90 - 74.5}{8.1} \approx 1.91$
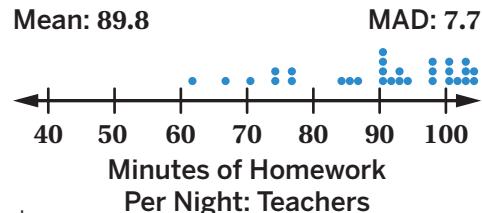
b What might Ama say about whether 8th graders have more homework than 7th graders?

Responses vary. **Ama would say that 8th graders definitely have more homework than 7th graders because the difference between the means is 15.5 minutes, which is close to 2 MADs.**

5. Teachers use the word "prep" to describe the work they do at home. Here are some data and statistics about their homework time.

Mean: 89.8   **MAD: 7.7**

(dot plot, horizontal axis from 40 to 100)

40 50 60 70 80 90 100

Minutes of Homework Per Night: Teachers

a Calculate how many MADs apart the means are for the time teachers work at home with either the time 6th, 7th, or 8th graders spend on homework.

Responses vary based on comparison. **7th Graders to Teachers:** $\dfrac{89.8 - 74.5}{8.1} \approx 1.89$
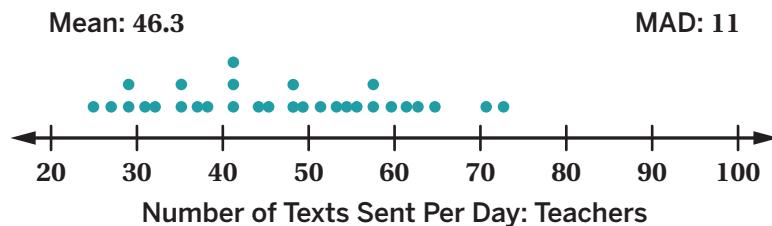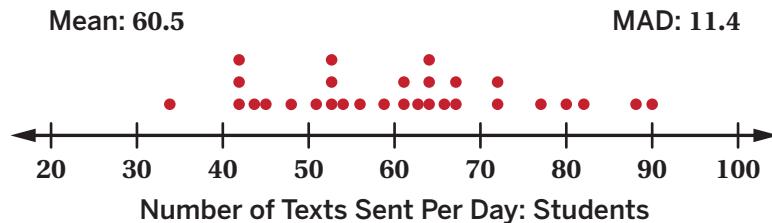
b Write your own headline about the two data sets you compared.

Responses vary. **"Survey Finds That Teachers Do a Lot More School Work Than Students After School!"**

Name: .... Date: .... Period: ....

Texting Title

6. Students at Median Middle School surveyed a random sample of 30 students and 30 teachers.

Here are the survey results and the headline they wrote.

Des-News

Survey Finds That Students Send More Text Messages Than Teachers

Mean: 60.5   **MAD: 11.4**

Number of Texts Sent Per Day: Students

Mean: 46.3   **MAD: 11**

Number of Texts Sent Per Day: Teachers

a Based on this data, why might someone believe this headline?

Responses vary. **Someone might believe this headline because the mean for the student data is higher than the mean for the teacher data.**

b Based on this data, why might someone *not* believe this headline?

Responses vary. **Someone might not believe this headline because the survey doesn't include very many people. Also, there's a lot of overlap between the dot plots.**

7. Let's investigate further.

a How many MADs apart are the means? Show your thinking.

$$\frac{60.5 - 46.3}{11.4} \approx 1.25$$

b Does that calculation make the headline more or less believable? Explain your thinking.
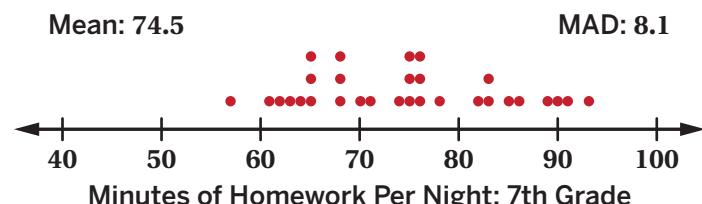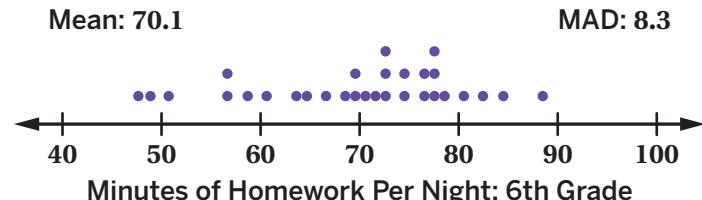
Responses and explanations vary. **The calculations make the headline more believable. Ama's strategy shows that the means are more than 1 MAD apart.**

Synthesis

8. How can you use the MAD to determine how different two populations are?

Use the data from Median Middle School if it helps with your thinking.

Responses vary. **I can use the MAD to decide how different two populations are by calculating if the means are at least one MAD apart. If they are, then the populations are probably quite different. If they aren't, the populations might not be very different.**

Mean: 70.1   **MAD: 8.3**

Minutes of Homework Per Night: 6th Grade

Mean: 74.5   **MAD: 8.1**

Minutes of Homework Per Night: 7th Grade

Things to Remember:

Unit 8
Lesson
2

Name: .... Date: .... Period: ....

Prob-bear-bilities

Let's express probabilities as numbers.
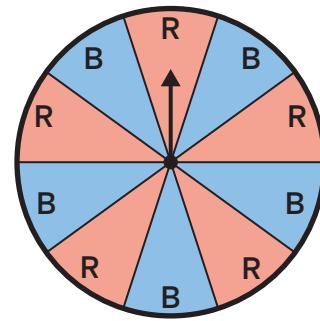
Warm-Up

1 Which game would you rather play?

A. Roll a number cube. Win if it lands on a number greater than 1.

B. Spin this spinner. Win if it lands on red.

Explain your thinking.
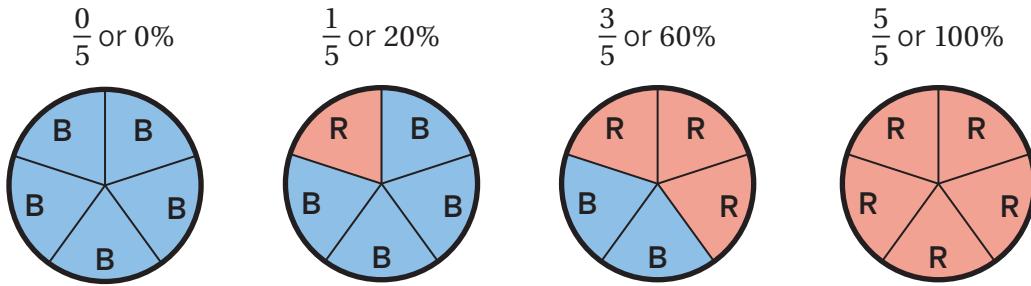
Responses and explanations vary. **I would rather roll a number cube because there are more chances of winning than losing. I am equally likely as not to win with the spinner game.**

Name: .... Date: .... Period: ....

Intro to Probability

2 Here is another spinner with different sections of red and blue.

Probability of Spinning Red

$\frac{0}{5}$ or 0% $\frac{1}{5}$ or 20% $\frac{3}{5}$ or 60% $\frac{5}{5}$ or 100%
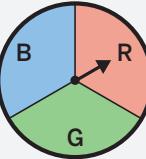
Describe how you think **probability** is determined.

Responses vary. **Probability is the number of red sections divided by the total number of sections, which is always 5 for this spinner.**

3 For each event, put a check mark under the probability that matches it.

Event		Probability		
		0.25	$\frac{1}{3}$	50%
Picking a purple block from this bag		✔		
Picking a purple block from this bag				✔
Randomly selecting the letter E from this word	D I C E	✔		
A number cube lands on an even number				✔
This spinner lands on red			✔	

Name: _____ Date: _____ Period: _____

Intro to Probability (continued)

4 For each challenge, order the events by likelihood. Complete as many challenges as you have time for.

a
- The probability of an event is 50%.
- A spinner with 5 equal sections, 2 of which are red, lands on red.
- The probability of an event is $\frac{3}{4}$.

A spinner with 5 equal sections, 2 of which are red, lands on red.	The probability of an event is 50%.	The probability of an event is $\frac{3}{4}$.

Least Likely   **Most Likely**

b
- Equally likely as not
- The probability of an event is $\frac{1}{4}$.
- A spinner with 5 equal sections, 4 of which are red, lands on red.

The probability of an event is $\frac{1}{4}$.	Equally likely as not	A spinner with 5 equal sections, 4 of which are red, lands on red.

Least Likely   **Most Likely**

c
- A spinner with 5 equal sections, 4 of which are red, lands on red.
- The probability of an event is 75%.
- A fair coin lands tails up.

A fair coin lands tails up.	The probability of an event is 75%.	A spinner with 5 equal sections, 4 of which are red, lands on red.

Least Likely   **Most Likely**

d
- You draw a red block from a bag with 6 green blocks and 2 purple blocks.
- The probability of an event is 20%.
- The outcome of an event is certain.

You draw a red block from a bag with 6 green blocks and 2 purple blocks.	The probability of an event is 20%.	The outcome of an event is certain.

Least Likely   **Most Likely**

Name: _____ Date: _____ Period: _____

Probabilities and Sample Spaces
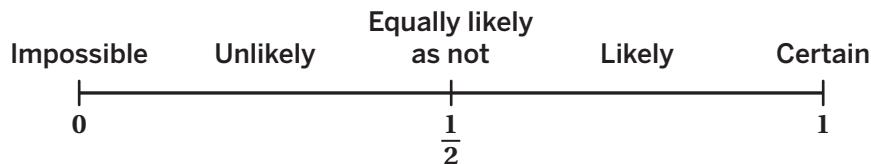
5 Here is a randomizer that chooses one character on each spin.

a Use the Activity 2 Sheet to try out the randomizer.

b Plot a point on the line to show how likely you think it is to get a bear on one spin.

Responses vary.

Impossible	Unlikely	Equally likely as not	Likely	Certain

$$0 \qquad\qquad \frac{1}{2} \qquad\qquad 1$$

6 A **sample space** is the collection of all possible outcomes of an experiment.

The sample space in the previous problem has 4 characters.

Explain why the probability of spinning a bear is $\frac{1}{4}$.

Responses vary. **There is only 1 bear out of 4 possible different characters, so there is a $\frac{1}{4}$ or 25% chance of spinning a bear.**

Name: .... Date: .... Period: ....
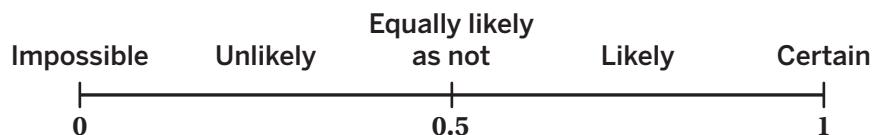
Prob-bear-ly Mismatched

7 Here is a different randomizer that chooses one head and one body on each spin.

a Use the Activity 3 Sheet to try out the randomizer.

b Plot a point on the line to show how likely you think it is to get a whole bear on one spin.

Responses vary.

Impossible	Unlikely	Equally likely as not	Likely	Certain

0   0.5   1

8 How many different characters are in this randomizer's sample space?

Explain your thinking.

16 characters. *Explanations vary*. The bear head can have 4 possible bodies. The dinosaur head can also have 4 possible bodies. Because there are 4 different heads and 4 different bodies, that means 4 • 4 = 16 different characters.
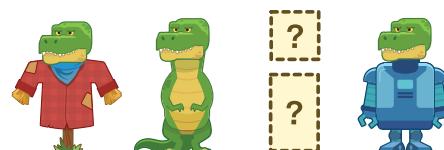
Name: _____ Date: _____ Period: _____

Prob-bear-ly Mismatched (continued)

9 This drawing of a sample space is missing one character.

Describe the head and body of the character that will complete the sample space.

Draw a sketch if you have time.
A dinosaur head with a bear body

10 Use the completed sample space to answer:

a What is the probability of spinning a character that is at least part bear?
$\frac{7}{16}$ **(or equivalent, such as 0.4375 or 43.75%)**

b What is the probability of spinning a character with mismatched parts?
$\frac{12}{16}$ **(or equivalent, such as 0.75 or 75%)**

Explore More

11 Here is a different randomizer that chooses one head, one body, and one set of legs on each spin. Use the Explore More Sheet to try out the randomizer.

What is the probability of the randomizer choosing all three parts from the same character?
$\frac{4}{64}$ **(or equivalent)**

Explain your thinking.

Explanations vary. **There are 4 possible heads, 4 possible bodies, and 4 possible legs, which means 4 • 4 • 4 = 64 possible character combinations in the sample space. Of those 64, only 4 will be all from the same character.**

Synthesis

Explain how you can use a sample space to help you determine the *probability* of an event.

Responses vary. The sample space helps to count the possible outcomes. The probability is the number of outcomes that I want (like part bear) divided by the total. For example, the probability of having a robot head is $\frac{4}{16}$ because there are 16 characters and 4 of them have robot heads.
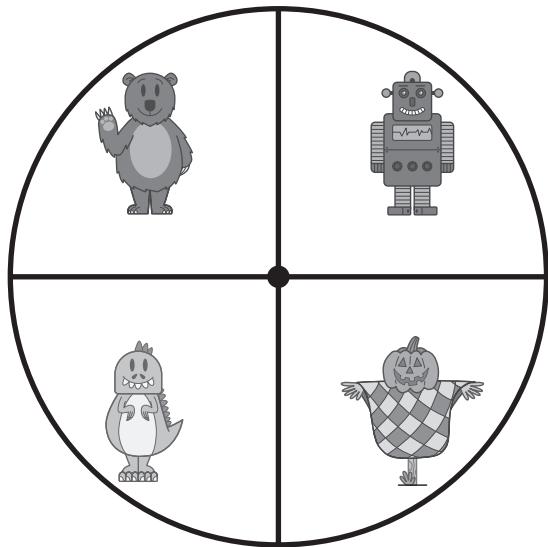
Things to Remember:

Name: .... Date: .... Period: ....

Probabilities and Sample Spaces

Unfold one end of a paper clip to use as the pointer. Use a pencil to hold the rounded (closed) end of the paper clip in the center of the spinner. Spin the paper clip around the pencil.

Name: _____   Date: _____   Period: _____

Prob-bear-ly Mismatched

Unfold one end of a paper clip to use as the pointer. Use a pencil to hold the rounded (closed) end of the paper clip in the center of the spinner. Spin the paper clip around the pencil. Spin both spinners to complete one spin.

Name: .... Date: .... Period: ....
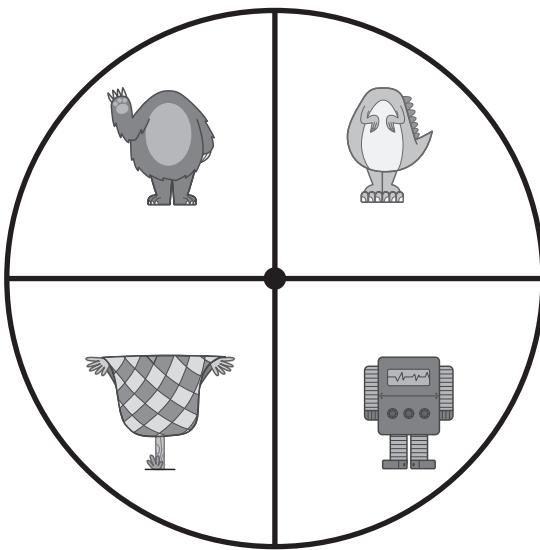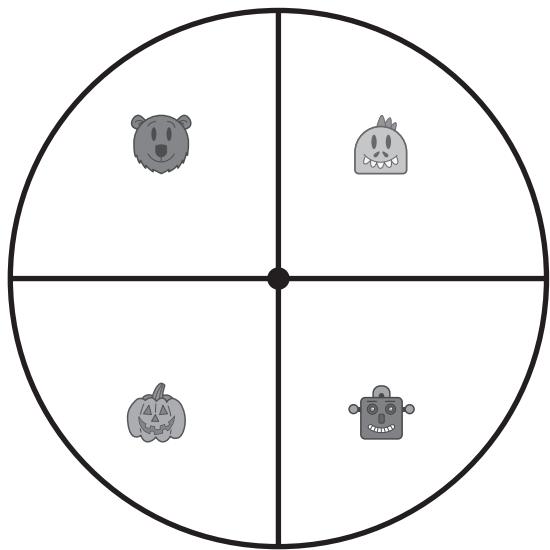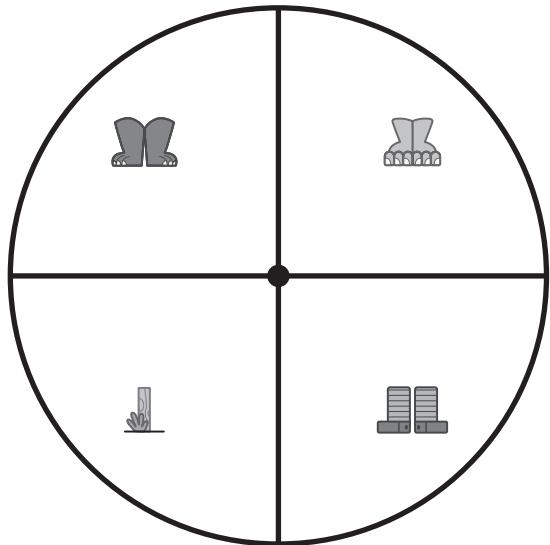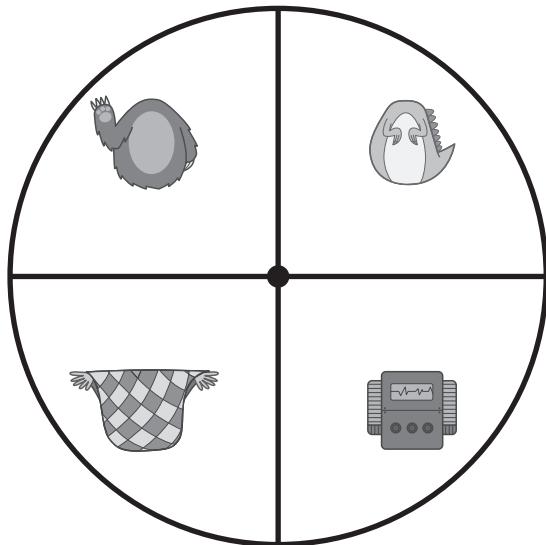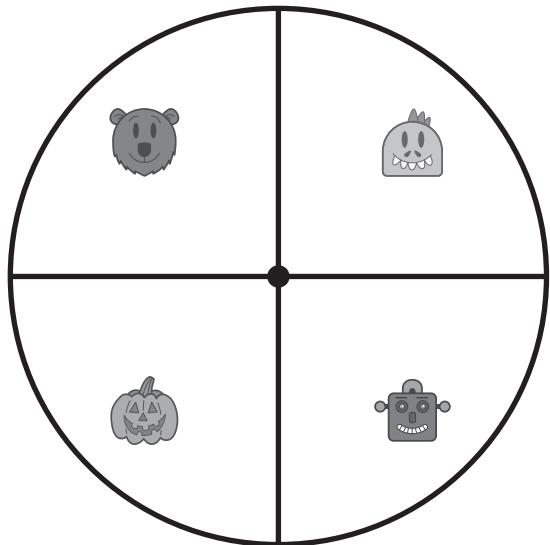
Explore More

Unfold one end of a paper clip to use as the pointer. Use a pencil to hold the rounded (closed) end of the paper clip in the center of the spinner. Spin the paper clip around the pencil. Spin all three spinners to complete one spin.

Unit 8
Lesson
3

Name: .... Date: .... Period: ....

Mystery Bag

Let's use repeated experiments and proportionality to predict what is inside a mystery bag.
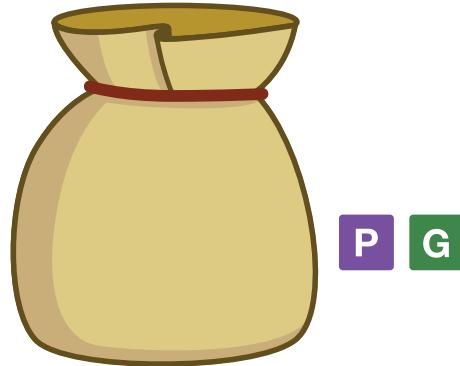
Warm-Up

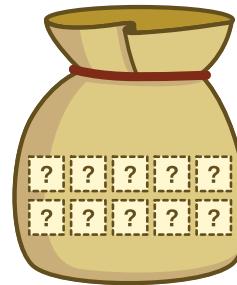**1** Sketch purple (P) and green (G) blocks on the bag so the probability of picking a green block (G) is 40%.

Responses vary.
- **2 green blocks, 3 purple blocks**
- **4 green blocks, 6 purple blocks**
- **6 green blocks, 9 purple blocks**

P G

Name: _____ Date: _____ Period: _____

What's in the Bag?

2 Here is Rishi's bag. It has 10 blocks.
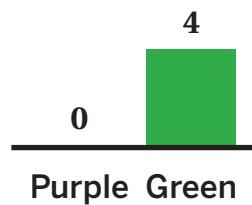
These are the results after 4 picks and after 8 picks.
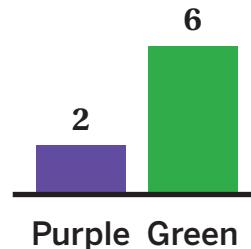
How many of the 10 blocks do you think are green?
Responses vary.

Total Picks: 4

4

0

Purple Green

Total Picks: 8

6

2

Purple Green
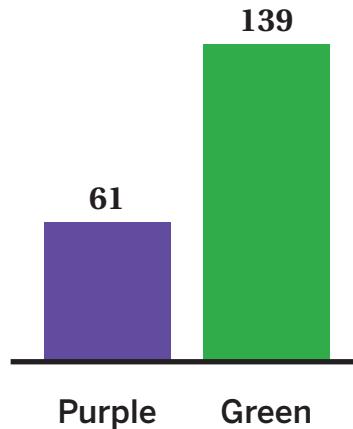
3 This graph shows the results from 200 picks.

Use these results to decide how many of the 10 blocks you think are green. Explain your thinking.

Responses and explanations vary. 7 blocks. There are 139 green blocks out of 200 total, which is about 70% and 70% of 10 is 7.

Total Picks: 200

139

61

Purple   Green

4 Here is how Nasir decided the number of green blocks in the bag.

Explain Nasir's strategy.
Responses vary.
- **Nasir set up a proportion. He divided 139 by 200 and got 0.7 after rounding. Then he multiplied the number of blocks in the bag (10) by 0.7 to determine the number of green blocks.**
- **Nasir calculated a constant of proportionality of about 0.7. To determine the number of green blocks, he multiplied the number of blocks in the bag (10) by the constant of proportionality.**

Nasir
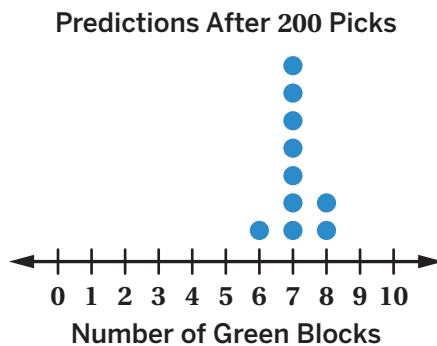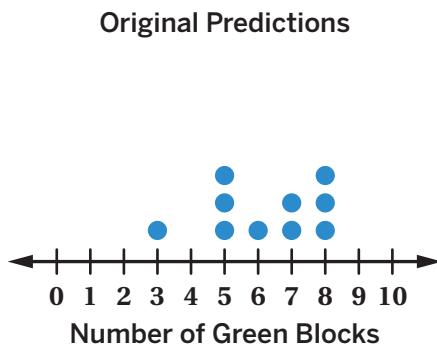
$$\frac{139}{200} = \frac{x}{10}$$

$$10 \cdot 0.7 = \frac{x}{10} \cdot 10$$

$$7 = x$$

Name: .... Date: .... Period: ....

What's in the Bag? (continued)

5 These graphs show the original predictions and updated predictions from another class.

Original Predictions

Predictions After 200 Picks

0 1 2 3 4 5 6 7 8 9 10
Number of Green Blocks

0 1 2 3 4 5 6 7 8 9 10
Number of Green Blocks

💬 **Discuss:** What do you notice and wonder?

Responses vary.

- **I notice that the highest prediction for both graphs was 8 green blocks.**
- **I notice that after 200 picks, more people predicted 7 green blocks.**
- **I notice the dots on the left are more spread out than the dots on the right.**
- **I wonder if this means there are 7 green blocks in the bag.**
- **I wonder if the numbers are always closer together if you pick more times.**
- **I wonder why the numbers are more spread out in the original predictions.**

6 Let's reveal what's in the bag.
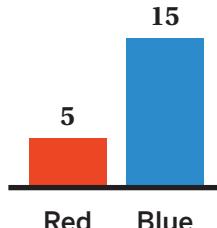
💬 **Discuss:** How did this compare to what you predicted?

Responses vary.

Name: _____   Date: _____   Period: _____

Blocks in the Bag

7 Demari's bag has 8 blocks.

The graph shows the results from 20 picks.

Order the contents of Demari's bag from *least likely* to *most likely*.

a. 1 red, 7 blue

b. 2 red, 6 blue

c. 6 red, 2 blue

d. 0 red, 8 blue

Total Picks: 20

d	c	a	b

Least Likely   **Most Likely**

8 Keya and Ash were trying to decide which color blocks were in Demari's bag.

- Keya says there are 2 red and 6 blue blocks.
- Ash says there is 1 red and 7 blue blocks.

Who do you agree with? Circle one.

Keya   Ash   Neither

Explain your thinking.

Responses and explanations vary.
- **I agree with Keya. $\frac{2}{6} = \frac{1}{3}$ and $\frac{5}{15} = \frac{1}{3}$, so these are equivalent ratios. There is the same constant of proportionality between red and blue in the results and in Keya's prediction.**
- **I don't agree with either of them because 20 picks isn't enough picks to know how many of each color are in the bag.**

Name: _____ Date: _____ Period: _____
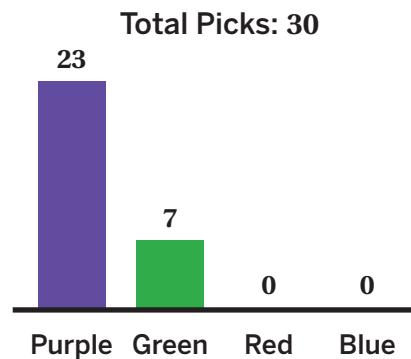
Blocks in the Bag (continued)

9 Use the results to determine how many blocks of one color are likely to be in each bag. Complete as many problems as you have time for.
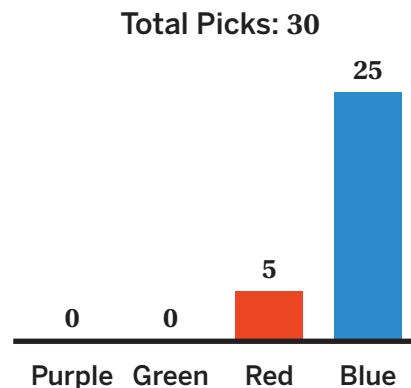
a There are 4 blocks in a bag. How many of the blocks are likely to be purple?

3

Total Picks: 30

23   7   0   0

Purple   Green   Red   Blue

b There are 4 blocks in a bag. How many of the blocks are likely to be red?

1

Total Picks: 30

25

0   0   5

Purple   Green   Red   Blue
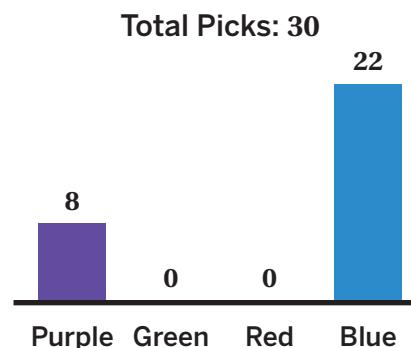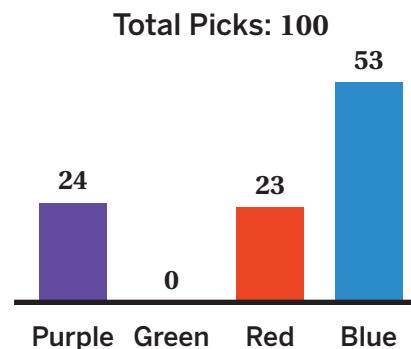
c There are 6 blocks in a bag. How many of the blocks are likely to be purple?

2

Total Picks: 30

22

8   0   0

Purple   Green   Red   Blue

d There are 8 blocks in a bag. How many of the blocks are likely to be blue?

4

Total Picks: 100

53

24   0   23

Purple   Green   Red   Blue

10 Synthesis

Describe how you can use results from a repeated experiment to make predictions.

Use the results shown if that helps with your thinking.

Responses vary. **First, I can use the results of an experiment to set up a ratio. For instance, $\frac{34}{40}$ of the blocks picked were purple. Then, I can multiply that ratio by the number of blocks in the bag to help me make a prediction. If there are 6 blocks, $\frac{34}{40} \cdot 6 = 5.1$, which suggests 5 of the 6 blocks are purple.**

Total Picks: 40

34

6

Purple   Red

Things to Remember:

Unit 8
Lesson
5

Name: .... Date: .... Period: ....

Is It Fair?

Let's decide what it means to be fair.

Warm-Up

1 Some of these coins are *unfair*.

a Take a look at each coin.

Coin A   **Coin B**

Coin C   **Coin D**

b Select one coin and describe why you think it's unfair.

Responses and explanations vary.
- **Coin A looks like it is made of different materials on each side. It might bounce off the gray side.**
- **Coin C is curved, so it probably won't land equally on both sides.**
- **Coin D has heads on both sides, so there is no way of landing on tails.**

Name: _____   Date: _____   Period: _____

Fair or Not?

2 Let's investigate *fair* and *unfair* objects.

Fair objects have:

- All the expected outcomes.
- An equal probability for each outcome to occur.

Flip a fair coin 10 times and record your results.
Responses vary.

a Did you get the results you expected?
Responses vary.

b Select *all* the results where you would suspect a coin is unfair.

☐ **A.** 5 heads, 5 tails

☐ **B.** 6 heads, 4 tails

☐ **C.** 8 heads, 2 tails

☐ **D.** 9 heads, 1 tails

☐ **E.** 10 heads, 0 tails
Responses vary.

Fair Coin	Unfair Coin
Fair Number Cube	**Unfair Number Cube**

3 One coin was flipped 10 times. The image shows that the coin landed 7 times heads up and 3 times tails up.

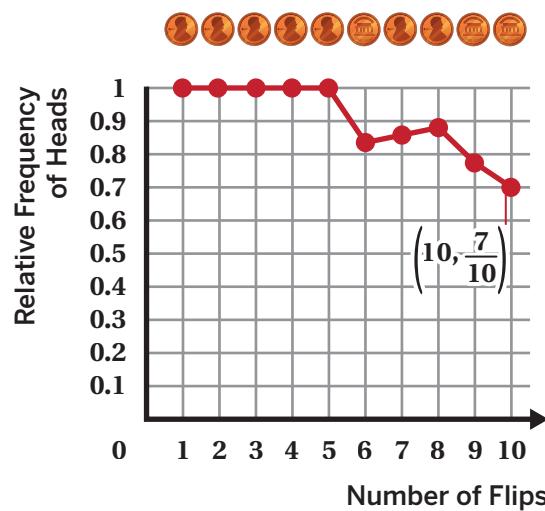Do you think this is a fair coin?
Responses vary.
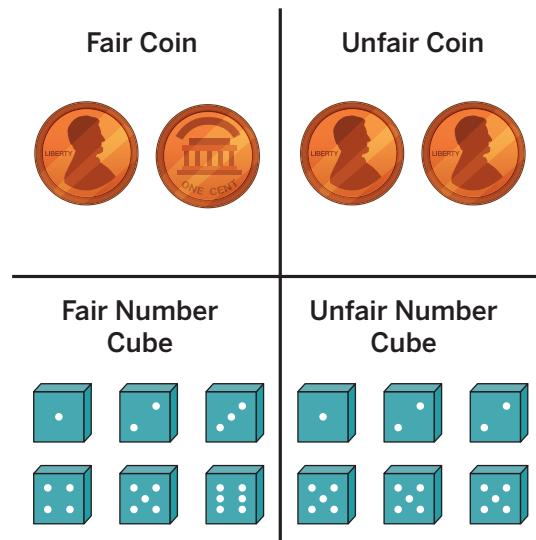
4 Here is a graph of the flips from the previous problem.

a What does the point $\left(10, \frac{7}{10}\right)$ mean?
Responses vary. **The point $\left(10, \frac{7}{10}\right)$ means that after 10 flips, the relative frequency of heads was $\frac{7}{10}$.**

b How might a graph help you determine if a coin is fair?
Responses vary. **If the graph gets close to 0.5, then the coin is fair. If not, then it isn't fair.**

Graph: Relative Frequency of Heads vs. Number of Flips. Point labeled $\left(10, \frac{7}{10}\right)$.

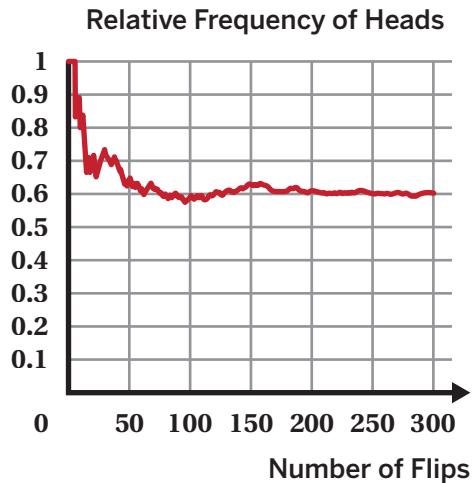Name: .... Date: .... Period: ....

Fair or Not? (continued)

5 A coin was flipped 300 times. Based on the results, is the coin fair? Explain your thinking.

No. Explanations vary. If the coin is fair, then the graph would get closer to 0.5. But as the number of flips increased, the graph got closer to 0.6, which means that landing heads up is more likely than landing tails up.

Relative Frequency of Heads

Number of Flips

6 Based on these results, what is the probability of the next flip landing heads up?

Responses vary. About 60%

Name: ....   Date: ....   Period: ....

How Likely?

7 Here is a game involving a number cube.

What do you think is the probability that Player 2 wins on any single roll?

Responses vary.

| Roll an odd number: Player 1 wins | Roll an even number: Player 2 wins |

8 **a** Play a few rounds of the game using a number cube. Record your results in the table.

Responses vary.

Player 1	Player 2

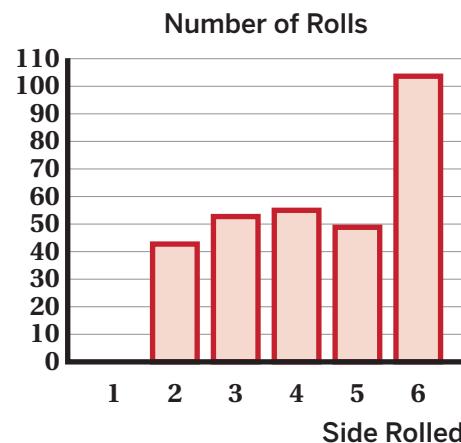**b** Do you think this is a fair number cube? Explain your thinking.

Responses and explanations vary.
- **Yes. I think this is a fair number cube because my partner and I won about an equal number of times.**
- **No. I don't think it's a fair number cube because my partner won more games than I did.**

9 Here are the results from two students who played the game and rolled a number cube 300 times. What do you think is the probability that Player 2 wins? Explain your thinking.

Responses and explanations vary.
- **It looks like the one is never rolled and the six is rolled twice as often as the others, so my guess is that there is 1 two, 1 four, and 2 sixes on this number cube. This would mean the probability of Player 2 winning is $\frac{4}{6}$.**
- **About 40 of the rolls were two, about 50 of them were four, and about 100 were six. So that would be $\frac{(40+50+100)}{300}$, or about a 63% chance of Player 2 winning.**

Number of Rolls

Side Rolled

Name: _____ Date: _____ Period: _____

How Likely? (continued)

10 Here is the number cube used by the two students in the previous problem.
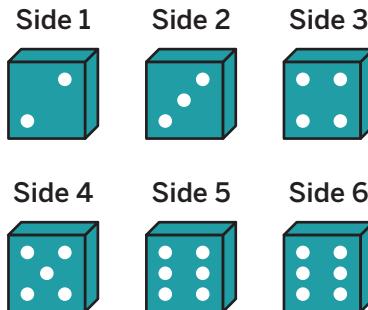
Create a game with this number cube that would be fair.

Player 1 wins if . . .

Player 2 wins if . . .

Responses vary.
- **Player 1 wins if the number is a five or a six.
  Player 2 wins if the number is not a five or a six.**
- **Player 1 wins if the number is divisible by 3.
  Player 2 wins if it is not divisible by 3.**

Side 1   Side 2   Side 3

Side 4   Side 5   Side 6

Explore More

11 In this game, players will flip the same coin twice.

The rules are:
- Flip heads, then tails: Player 1 wins.
- Flip tails, then heads: Player 2 wins.
- If the results match, no one wins.

Explain why this game is fair even if the coin is unfair.

Responses vary. **This game is fair because the probability of the coin landing heads up then tails up is the same as the probability of tails up then heads up. This is true whether the coin is fair or unfair and means that each player has the same chance of winning the game.**

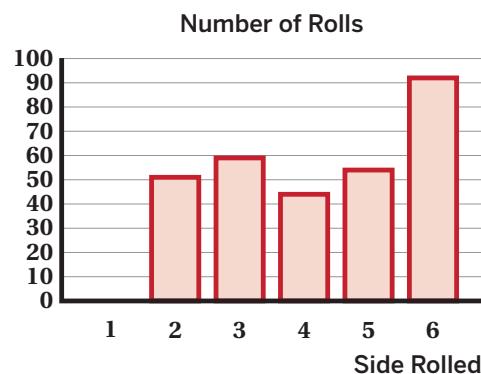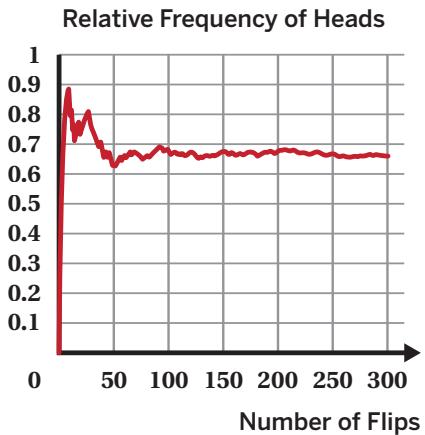| Player 1 Wins	Player 2 Wins
No One Wins	No One Wins

12 Synthesis

Describe how you can use a repeated experiment to decide whether an object is fair.

Use the results of these experiments if they help with your thinking.

Responses vary. **If the results of the experiment are close to the probability for a fair object, then the object is probably fair. If not, then the object is probably not fair. For example, the relative frequency of heads for the coin is close to 0.6, but it should be 0.5. In the cube rolling experiment, all the outcomes should appear about an equal number of times, but they don't, so the object probably isn't fair.**
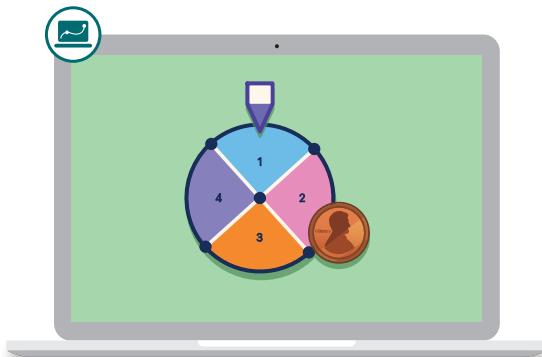
Relative Frequency of Heads

Number of Flips

Number of Rolls

Side Rolled

Things to Remember:

Unit 8
Lesson
6

Name: _____ Date: _____ Period: _____

Fair Games

Let's determine whether or not games are fair.

Warm-Up

1 Here is a game involving two fair coins. Player 1 wins if both coins land heads up or both coins land tails up. Otherwise, Player 2 wins.

Play a few rounds, then answer: *Which player would you rather be in the game?*
Responses vary.

2 Charlie made a list and claims that the game is not fair because Player 1 is twice as likely to win as Player 2.

(H) (H) → Player 1 wins!   (T) (T) → Player 1 wins!   (H) (T) → Player 2 wins!

Is Charlie's claim correct? Circle one.

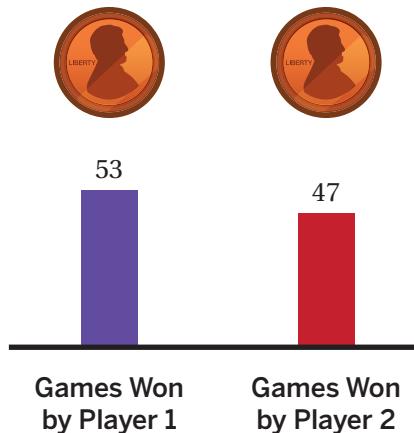Yes   (No)   I'm not sure

Explain your thinking.
***Explanations vary.* Charlie only wrote down 3 of the 4 different possible outcomes. "TH" or "tails, then heads" is missing from the list. If Charlie includes that one, then each player has 2 ways to win.**

Activity 1

Name: .... Date: .... Period: ....

Coins and Tree Diagrams

3 The game from the Warm-Up was played 100 times. Here are the results.

💬 **Discuss:** Why is this a fair game?

Responses vary. **The game is fair because the players have an equal probability of winning the game (0.5). The results of 100 games is close to 50 wins for each player.**
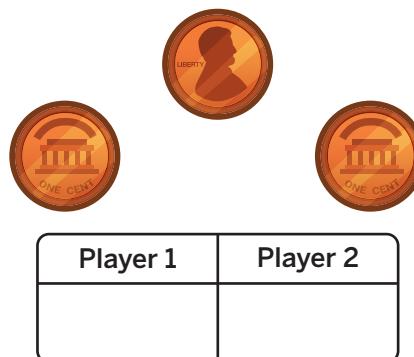
53 | 47

Games Won by Player 1 | **Games Won by Player 2**

4 Here's a twist: there are 3 coins to flip.

Like before, Player 1 wins if the coins land all heads up or all tails up.

Otherwise, Player 2 wins.

a Play the game up to 10 times. Record your results in the scoreboard.

Results vary.

Player 1	Player 2

b Explain whether you think this game is fair.

Responses vary.
- **I don't think the game is fair because when I played, Player 2 won more times than Player 1.**
- **I think it's less likely to get all 3 coins to be the same compared to 2 coins. This is kind of like the bear in Lesson 2. It was more likely to get a whole bear when there were only 4 possible characters but less likely when there were 16 possible characters.**

5 Andrea wants to determine the probability of winning.

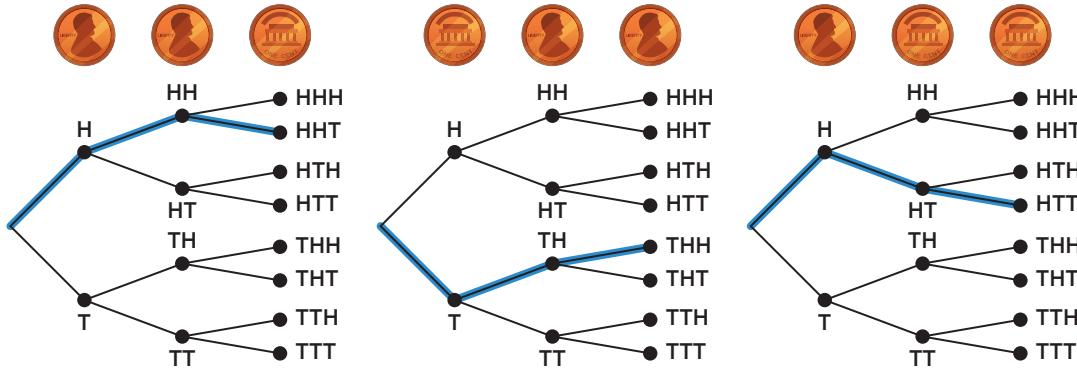In any round, what is the probability that Player 1 wins (all heads or all tails)?

$\frac{2}{8}$ **(or equivalent)**

Name: ....   Date: ....   Period: ....

Coins and Tree Diagrams (continued)

6 A **tree diagram** is one way to represent the sample space of **compound events**. This tree diagram represents flipping 3 coins.

a Take a look at the tree diagrams.

b Explain why the probability of Player 1 winning is $\frac{2}{8}$.

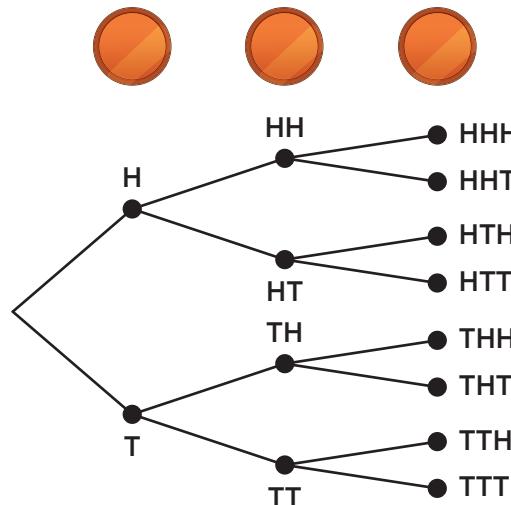*Responses vary.* **There are 8 possible outcomes, and Player 1 wins in 2 of them.**

7 Use the tree diagram to help answer:

a What is the probability of *exactly* 1 coin landing tails up?

$\frac{3}{8}$ **(or equivalent)**

b What is the probability of *at least* 1 coin landing tails up?

$\frac{7}{8}$ **(or equivalent)**

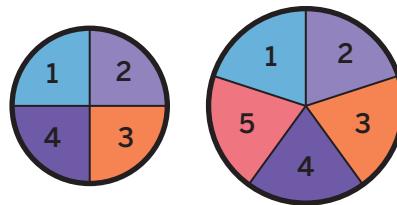Name: _____ Date: _____ Period: _____

Spinners and Tables

8 Here is a new game involving these two spinners.
Player 1 wins if the numbers are *the same or one apart*. Otherwise, Player 2 wins.

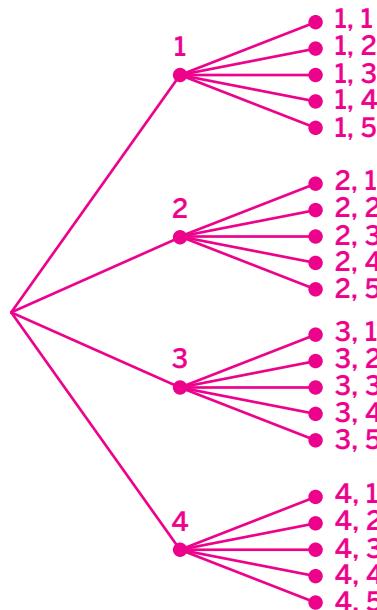**a** Use the Activity 2 Card to play a few rounds of the game with a partner. Record the results in the table.

Results vary.

Player 1	Player 2

b Make a tree diagram to represent the sample space.

Responses vary.

9 Here is a table representing the game. Each space in the table represents a possible outcome. Compare this table to the tree diagram you made.
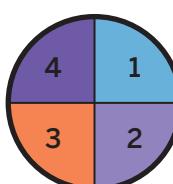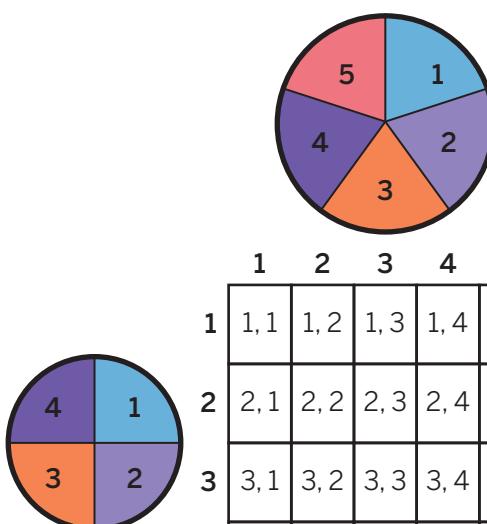

How are they alike?

Responses vary.
- **Both representations have a spot for each outcome.**
- **I can see 20 possible outcomes in each representation.**
- **I can see 4 groups of 5 in each representation.**

How are they different?

Responses vary.
- **The tree diagram goes from left to right, but there isn't really a direction in the table.**
- **A tree diagram would work for more than 2 coins or spinners, but I don't think a table would work for more than 2 coins or spinners.**

	1	2	3	4	5
1	1, 1	1, 2	1, 3	1, 4	1, 5
2	2, 1	2, 2	2, 3	2, 4	2, 5
3	3, 1	3, 2	3, 3	3, 4	3, 5
4	4, 1	4, 2	4, 3	4, 4	4, 5

Spinners and Tables (continued)

10 Player 1 wins if the numbers are *the same or one apart*. Otherwise, Player 2 wins. Use your tree diagram, the table, or another tool to help you answer:

	1	2	3	4	5
1	1, 1	1, 2	1, 3	1, 4	1, 5
2	2, 1	2, 2	2, 3	2, 4	2, 5
3	3, 1	3, 2	3, 3	3, 4	3, 5
4	4, 1	4, 2	4, 3	4, 4	4, 5

a What is the probability of Player 1 winning?

$\frac{11}{20}$ **(or equivalent)**

b What is the probability of Player 2 winning?

$\frac{9}{20}$ **(or equivalent)**

Explore More

11 Here are some other games that you could play using the same two spinners. Get a feel for the games by playing each one with your partner using the Activity 2 Card.

Four Score	**You Spin Sum, You Lose Sum**	**Lucky 7**	**Big Wheel, Big Deal**
• *Multiply* the numbers.	• *Add* the two numbers.	• *Multiply* the numbers.	• Player 1 wins if the *big wheel's number is greater* than the small wheel's number.
• Player 1 wins if the result is a *multiple of* 4.	• Player 1 wins if the sum is 6 *or greater*.	• Player 1 wins if the result is *greater than* 7.	• Otherwise, Player 2 wins.
• Player 2 wins if it's anything else.	• Player 2 wins if the sum is 5 *or less*.	• Player 2 wins if the result is *less than* 7.	

Select *all* the games that are fair (Player 1 and 2 have an equal chance of winning).

☐ **A.** Four Score

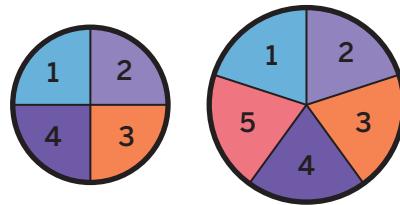☑ **B.** You Spin Sum, You Lose Sum

☐ **C.** Lucky 7

☑ **D.** Big Wheel, Big Deal

Synthesis

Here are three representations of the sample space for this pair of spinners.

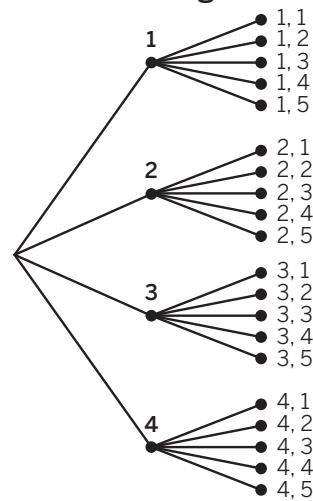Choose a representation and describe an advantage and a disadvantage.

List

- Left: 1, Right: 1
- Left: 1, Right: 2
- Left: 1, Right: 3
- Left: 1, Right: 4
- Left: 1, Right: 5

- Left: 2, Right: 1
- Left: 2, Right: 2
- Left: 2, Right: 3
- Left: 2, Right: 4
- Left: 2, Right: 5

- Left: 3, Right: 1
- Left: 3, Right: 2
- Left: 3, Right: 3
- Left: 3, Right: 4
- Left: 3, Right: 5

- Left: 4, Right: 1
- Left: 4, Right: 2
- Left: 4, Right: 3
- Left: 4, Right: 4
- Left: 4, Right: 5

Responses vary. **See the Teacher Edition for sample responses.**

Table

	1	2	3	4	5
1	1, 1	1, 2	1, 3	1, 4	1, 5
2	2, 1	2, 2	2, 3	2, 4	2, 5
3	3, 1	3, 2	3, 3	3, 4	3, 5
4	4, 1	4, 2	4, 3	4, 4	4, 5

Tree Diagram

1
- 1, 1
- 1, 2
- 1, 3
- 1, 4
- 1, 5

2
- 2, 1
- 2, 2
- 2, 3
- 2, 4
- 2, 5

3
- 3, 1
- 3, 2
- 3, 3
- 3, 4
- 3, 5

4
- 4, 1
- 4, 2
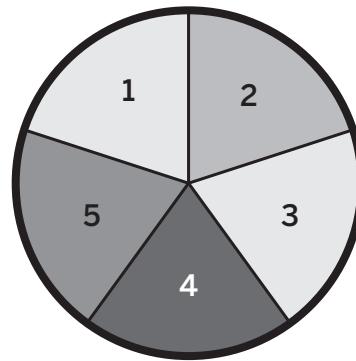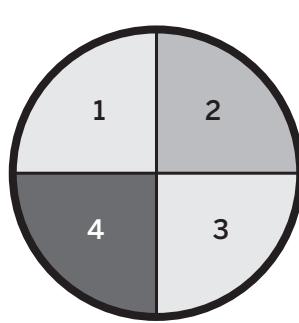- 4, 3
- 4, 4
- 4, 5

Things to Remember:

Spinners and Tables

✂--- **Directions:** Make one copy per two pairs of students. Then pre-cut the cards and give each pair of students one set of spinners.
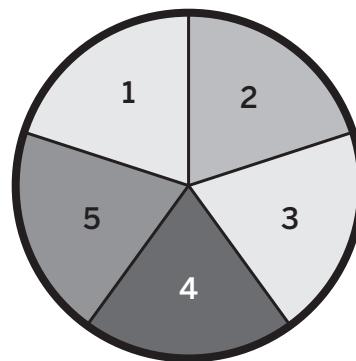
Unfold one end of a paper clip to use as the pointer. Use a pencil to hold the rounded (closed) end of the paper clip in the center of the spinner. Spin the paper clip around the pencil. Spin both spinners to complete one spin.

Player 1 wins if the numbers are the same or one apart. Otherwise, Player 2 wins. Play a few rounds of the game with a partner.
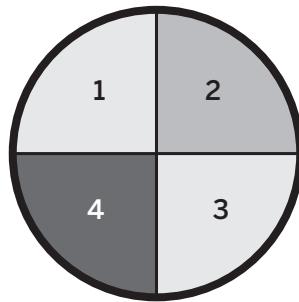

Unfold one end of a paper clip to use as the pointer. Use a pencil to hold the rounded (closed) end of the paper clip in the center of the spinner. Spin the paper clip around the pencil. Spin both spinners to complete one spin.

Player 1 wins if the numbers are the same or one apart. Otherwise, Player 2 wins. Play a few rounds of the game with a partner.

Unit 8
Lesson
7

Name: _____ Date: _____ Period: _____

Weather or Not

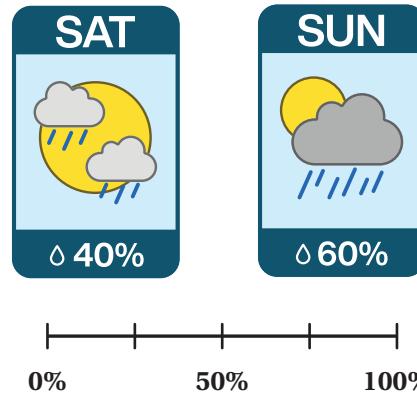Let's use probability tools to simulate real-world events.

Warm-Up

1 Aniyah saw this forecast for the weekend's weather.

What do you think is the probability that it will rain *at least once* this weekend?

Plot a point on the line to show your guess. Explain your thinking.

Responses and explanations vary.
- **Because the percentages are 40% and 60%, the chance it rains is the average, or 50%.**
- **There is a 40% chance of rain on one day and a 60% chance of rain on the next day, so the chance that it rains at all is 40% + 60% = 100%.**
- **It seems more likely than not to rain at least once, so I guessed about 70%.**
- **There are three options: no rain, one day of rain, or two days of rain. This means there is a 66% chance it rains at least once.**

SAT ◊ 40%   SUN ◊ 60%

0%   50%   100%
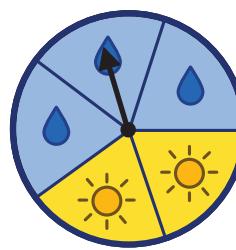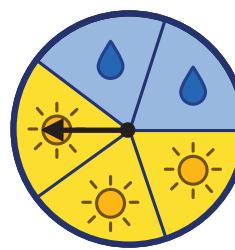
Name: _____ Date: _____ Period: _____

Introducing Simulation

2 Aniyah wanted to run a **simulation**: an experiment designed to estimate the probability of a real-world event. She created these spinners to represent the weather forecast.

SAT ◊40%   **SUN** ◊60%

a Use the spinners on the Activity 1 Card or digital screen to run one or more experiments.

b Describe how Aniyah could use these spinners to estimate the probability that it will rain this weekend.

Responses vary. **Aniyah could run a lot of experiments and record how many of the experiments had at least one spinner land on a blue raindrop.**

3 Aniyah ran 500 experiments. Here are the results.

💬 **Discuss:** Choose a row and explain what it tells you about this situation.

Responses vary.

• **The "no rain" row means that the spinner landed on a sun portion for 109 out of 500 experiments. This is 21.8% of the time.**

• **The "2 days of rain" row represents both spinners landing on rain, which happened 26.2% of the time. This tells me that about a quarter of the time, this weather forecast will result in rain on both Saturday and Sunday.**

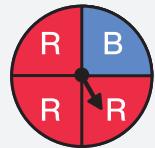| Experiments with:	Count	Relative Frequency
No rain	109	21.8%
1 day of rain	260	52%
2 days of rain	131	26.2%

4 What would you tell Aniyah about the probability of rain this weekend? Use evidence from the simulation to support your claim.

Responses vary.

• **The probability it rains this weekend is about 78%. If Aniyah adds the relative frequency for "1 day of rain" with the relative frequency for "2 days of rain," she would get about 78%.**

• **The probability it rains this weekend is close to 80%. The probability of no rain is close to 20%, so the probability that it does rain must be $100\% - 20\% = 80\%$.**

Make a Simulation

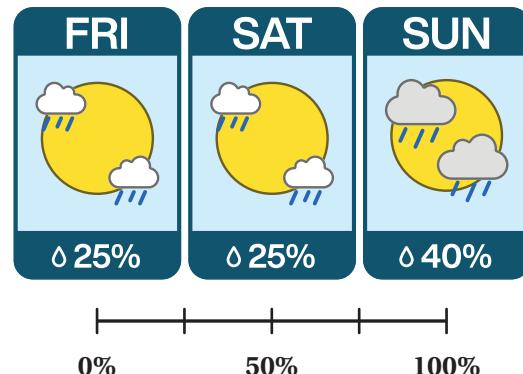**5** Match each simulation to the weather forecast it could represent.

Simulation	☁️ TODAY ☂ 40%	☁️ TODAY ☂ 50%	☁️ TODAY ☂ 75%
A fair coin lands heads up.		✓	
A purple block is picked from this bag.	✓		
A number cube lands on an odd number.		✓	
The spinner lands on red.			✓
A green block is picked from this bag.			✓

Name: _____   Date: _____   Period: _____

Make a Simulation (continued)

6 A tourist is planning a 3-day vacation. Here is the forecast at their destination.

What do you think is the probability that it will rain at least once during these 3 days? Plot a point on the line to show your guess.
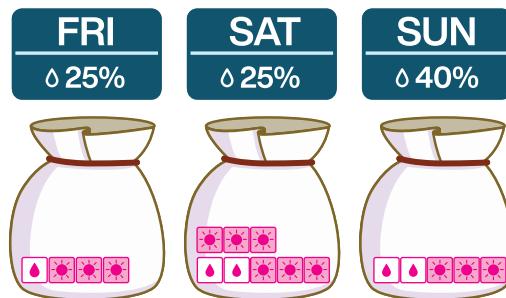*Responses vary.*

FRI	SAT	SUN
◊ 25%	◊ 25%	◊ 40%

0%   50%   100%

7 The tourist wants to design a simulation to estimate the probability of rain during their vacation.

Draw blocks on each bag to match the probability of rain on each day.
Responses vary. **Sample shown in diagram.**

FRI	SAT	SUN
◊ 25%	◊ 25%	◊ 40%

8 Let's watch some simulated experiments on screen.

a Record the results of the simulation you watched in the table.
Responses vary. **Sample results shown in table.**

b Estimate the probability that it rains at least once on the tourist's vacation.
Responses between 0.6 and 0.7 (or equivalent), or responses that match the table, are considered correct.

Experiments with:	Count	Relative Frequency
No rain	33	33%
1 day of rain	48	48%
2 days of rain	18	18%
3 days of rain	1	1%

Name: .... Date: .... Period: ....
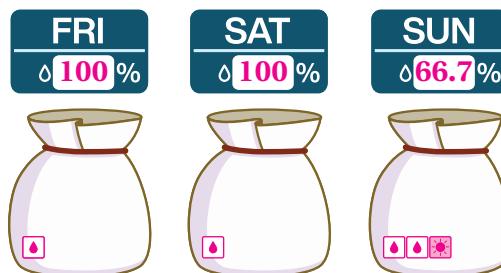
Meet the Criteria

For each challenge:

- Consider the given criteria for a 3-day period.
- Make a weather forecast for each day by writing a percentage in the white box that you think will meet the criteria.
- Draw blocks on the bag that match your forecast.
- Use the digital screen to run the simulation and test your forecast.

9 Criteria:

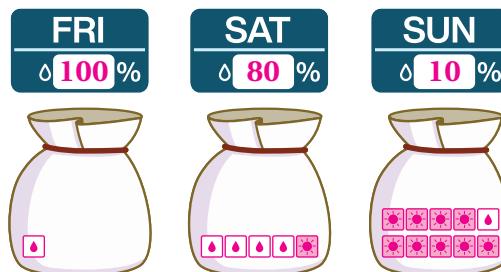- The chance that it rains *all three days* is between 50% and 75%.

  Sample forecast and blocks shown in diagram.

FRI	SAT	SUN
◊ **100**%	◊ **100**%	◊ **66.7**%

10 Criteria:

- The most likely weather is 2 *days of rain*.
- The next most likely weather is 1 *day of rain*.

  Sample forecast and blocks shown in diagram.

FRI	SAT	SUN
◊ **100**%	◊ **80**%	◊ **10**%
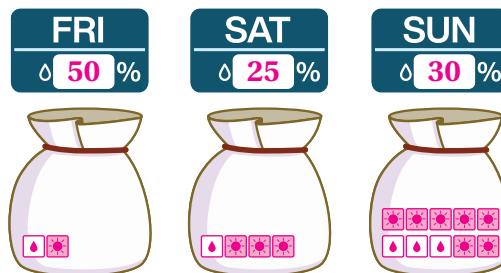
11 Criteria:

- The chance of *no rain at all* is about 25%.

  Sample forecast and blocks shown in diagram.

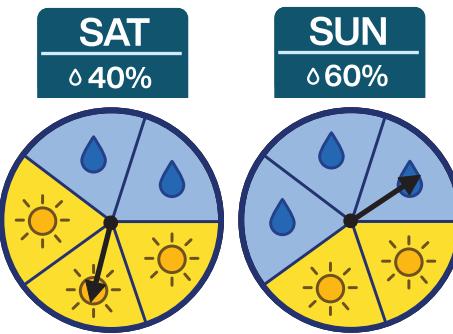| FRI	SAT	SUN
◊ **50**%	◊ **25**%	◊ **30**%

12 Synthesis

Describe how simulations can be designed and used to estimate the probability of a real-world event.

Use these spinners if they help with your thinking.

Responses vary. **When I look at the example weather forecast, I wonder how likely it is to have no rain all weekend. A simulation can help answer that. The spinners represent the forecast for each day. I can spin both spinners and record whether the result represents 0, 1, or 2 days of rain. If I do this hundreds of times, I can see the relative frequency of 0, 1, or 2 days of rain, which is a good approximation for the probability.**

SAT ◊ **40%**

SUN ◊ **60%**

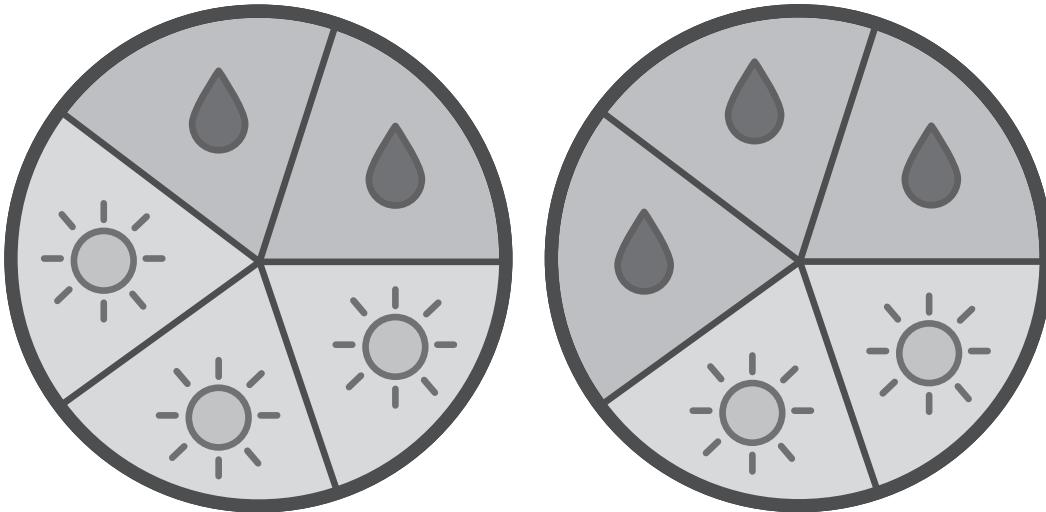| Experiments with:	Count	Relative Frequency
No rain	109	21.8%
1 day of rain	260	52%
2 days of rain	131	26.2%

Things to Remember:

Introducing Simulation

✂--- **Directions:** Make one copy per two pairs of students. Then pre-cut the cards and give each pair of students one set of spinners.

Unfold one end of a paper clip to use as the pointer. Use a pencil to hold the rounded (closed) end of the paper clip in the center of the spinner. Spin the paper clip around the pencil. Spin both spinners to complete one spin.

Unfold one end of a paper clip to use as the pointer. Use a pencil to hold the rounded (closed) end of the paper clip in the center of the spinner. Spin the paper clip around the pencil. Spin both spinners to complete one spin.