



Science Mom Lesson 45

Unit A1.5, Systems of Linear Equations and Inequalities: Notes Name _____

Lessons 1–2: Introduction to Elimination

Summary

A *system of equations* is two or more equations representing constraints on shared variables.

Elimination is a method of solving systems of equations where you add or subtract the equations to produce a new equation with fewer variables.

Here are two systems of equations that have been solved by elimination.

Use the elimination method to solve the final system.

$\begin{array}{r} 2x + y = 30 \\ - (x + y = 23) \\ \hline x + 0 = 7 \\ x = 7 \end{array}$ $(7) + y = 23$ $y = 16$	$\begin{array}{r} -2x + y = 9 \\ + (8x - y = 3) \\ \hline 6x + 0 = 12 \\ x = 2 \end{array}$ $-2(2) + y = 9$ $y = 13$	$\begin{array}{l} x + 2y = 10 \\ x - y = 7 \end{array}$
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Describe how to use elimination to solve a system of equations.

Things I Want to Remember

Lessons 1–2: Introduction to Elimination**Try This!**

Circle whether adding, subtracting, or either can be used to eliminate a variable.

1.1 $3x + 5y = 20$ Adding Subtracting Either
 $x + 5y = 4$

1.2 $-2x + 3y = 12$ Adding Subtracting Either
 $2x - 6y = 3$

1.3 $-9x + y = 5$ Adding Subtracting Either
 $9x + y = 3$

2. Solve this system of equations:

$$\begin{aligned} 6x + 2y &= 4 \\ 2x + 2y &= 12 \end{aligned}$$

$$x = \underline{\hspace{2cm}} \quad y = \underline{\hspace{2cm}}$$

- I can solve shape puzzles.
- I can solve systems of equations by adding or subtracting the equations to eliminate a variable.
- I recognize that adding or subtracting the equations in a system creates a new equation that shares a solution with that system.



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Lesson 3: Elimination Using Equivalent Equations

Summary

Writing equivalent equations can be useful when using elimination to solve systems of equations.

Here is one student's first step. Why do you think they got stuck?

$$\begin{array}{r} 9x - 4y = 2 \\ -(3x + y = 10) \\ \hline 6x - 5y = -8 \end{array}$$

Here is how a different student eliminated x .

$$\begin{array}{r} 9x - 4y = 2 \\ -3(3x + y = 10) \\ \hline \end{array}$$

$$\begin{array}{r} 9x - 4y = 2 \\ + -9x - 3y = -30 \\ \hline 0 - 7y = -28 \\ y = 4 \end{array}$$

$$\begin{array}{r} 3x + (4) = 10 \\ 3x = 6 \\ x = 2 \end{array}$$

Try eliminating y instead.

$$\begin{array}{r} 9x - 4y = 2 \\ -(3x + y = 10) \\ \hline \end{array}$$

Why do you think they multiplied the second equation by -3 ?

How did you decide what to multiply by?

Things I Want to Remember



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Lesson 3: Elimination Using Equivalent Equations

Try This!

Show the first step that you would use to solve each system using elimination.

$$\begin{aligned} 1.1 \quad 2x - y &= 5 \\ -x + 3y &= 5 \end{aligned}$$

$$\begin{aligned} 1.2 \quad -3x + 12y &= 15 \\ 4x + 3y &= -1 \end{aligned}$$

Solve this system of equations using elimination:

$$\begin{aligned} 2.1 \quad 4x + 3y &= 3 \\ 8x + y &= 1 \end{aligned}$$

$$x = \underline{\hspace{2cm}} \quad y = \underline{\hspace{2cm}}$$

- I can solve a system of equations using elimination.
- I recognize that multiplying an equation by a number creates a new system with the same solution.

Lesson 4: Solving Systems by Substitution**Summary**

Substitution is a method of solving systems of equations where you replace a variable with an expression it is equal to.

Here is a complete example of the substitution method and an example with the first step shown.

Complete the second example using substitution.

$$\begin{aligned}y &= -4x + 6 & y &= 3x - 15 \\-4x + 6 &= 3x - 15 \\-7x &= -21 \\x &= 3 \\y &= 3(3) - 15 \\y &= -6\end{aligned}$$

$$\begin{aligned}y &= 2x - 5 & -3x - 2y &= 3\end{aligned}$$

What is important to remember when using substitution to solve a system of equations?

When might using substitution be a useful strategy for solving a system?

Things I Want to Remember



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Lesson 4: Solving Systems by Substitution

Try This!

Show the first step you would complete to solve each system using substitution.

$$1.1 \quad y = -3x - 10$$

$$x + 2y = 10$$

$$1.2 \quad y = -x + 3$$

$$y = 2x - 12$$

Solve the system of equations using substitution.

$$2.1 \quad y = 2x - 9$$

$$y = -x + 9$$

$$2.2 \quad b = 3a - 8$$

$$2a + b = 2$$

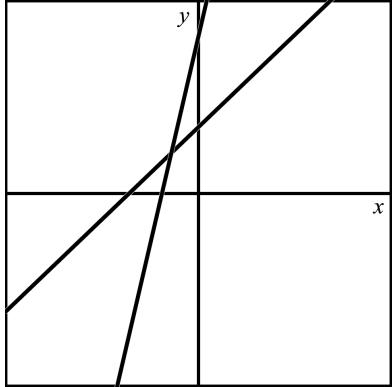
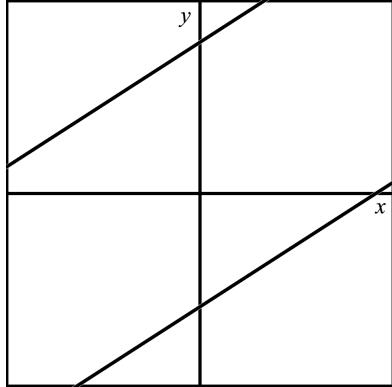
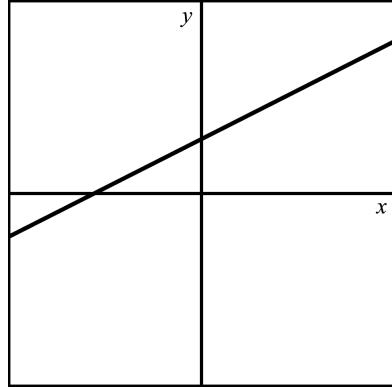
$$x = \underline{\hspace{2cm}} \quad y = \underline{\hspace{2cm}}$$

$$a = \underline{\hspace{2cm}} \quad b = \underline{\hspace{2cm}}$$

I can solve a system of equations using substitution.

Lesson 5: Graphing Systems of Linear Equations**Summary**

A system of equations can have one solution, no solutions, or infinite solutions.

Number of Solutions	One Solution	No Solutions	Infinite Solutions
Graph			
How can you tell from a graph?			
Equation	$y = 2x + \frac{1}{4}$ $y = 4x + \frac{1}{4}$	$y = \frac{2}{3}x + 10$ $y = \frac{2}{3}x - 7$	$y = \frac{1}{2}x + 3$ $2y = x + 6$
How can you tell from the equations?			



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Lesson 5: Graphing Systems of Linear Equations

Try This!

One equation in a system is $y = 7x - 12$. Write another equation to create a system with:

1.1 No solutions. _____

1.2 One solution. _____

1.3 Infinite solutions. _____

Determine if each system has no solutions, one solution, or infinite solutions. Circle your choice.

2.1 $y = 2x + 3$ No solutions One solution Infinite solutions
 $3y = 6x + 9$

2.2 $y = 2x + \frac{1}{2}$ No solutions One solution Infinite solutions
 $y = \frac{1}{2}x + 2$

2.3 $y = 3x + 6$ No solutions One solution Infinite solutions
 $y = 3x - 6$

- I can solve a system of linear equations using a graph.
 - I can determine if a system of linear equations has no solutions, one solution, or infinite solutions by looking at a graph or at the equations.

Lesson 8: Strategically Solving Systems of Equations**Summary**

Elimination or substitution can each be used to solve systems of linear equations.

$$\begin{array}{r} 3x + 4y = 3 \\ + -3x + 3y = 18 \\ \hline 7y = 21 \\ y = 3 \end{array}$$

$$\begin{array}{l} 3x + 4(3) = 3 \\ x = -3 \end{array}$$

$$(3, -3)$$

$$\begin{aligned} y &= 3x + 6 & 2x + 2y &= 20 \\ 2x + 2y &= 20 & 2x + 2(3x+6) &= 20 \\ && 2x + 6x + 12 &= 20 \\ y &= 3(1) + 6 & 8x + 12 &= 20 \\ y &= 9 & 8x &= 8 \\ && (1, 9) & x = 1 \end{aligned}$$

Why do you think elimination was used here?

Why do you think substitution was used here?

Choose either elimination or substitution to solve this system of equations.

Solve the system and explain your choice.

$$\begin{aligned} y &= 2x - 4 \\ y &= 0.5x + 5 \end{aligned}$$

Things I Want to Remember

desmos

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Lesson 8: Strategically Solving Systems of Linear Equations

Try This!

Circle which method (elimination, substitution, or either) you would use to solve each system.

1.1	$y = 3x - 10$ $2x - 3y = 16$	Elimination	Substitution	Either
1.2	$y = 4x - 10$ $y = x + 2$	Elimination	Substitution	Either
1.3	$9x + y = 25$ $3x + 2y = 5$	Elimination	Substitution	Either
1.4	$y = -2x + 3$ $x + 2y = 9$	Elimination	Substitution	Either

Pick one system and solve using the method of your choice.

Solution: (____ , ____)

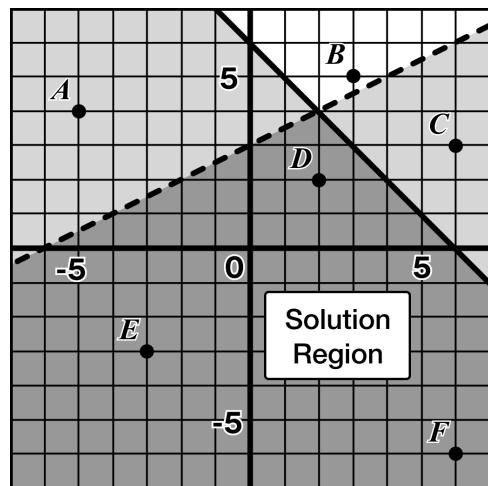
I can solve systems of equations using substitution and elimination.

Lessons 9–10: Solutions to Systems of Inequalities**Summary**

The *solutions to a system of inequalities* are all the points that make both inequalities true. The solutions can be seen in the region where the graphs overlap, called the *solution region*.

$$\begin{aligned}y &\leq -x + 6 \\-2x + 4y &< 12\end{aligned}$$

Which point(s) are in the solution region?



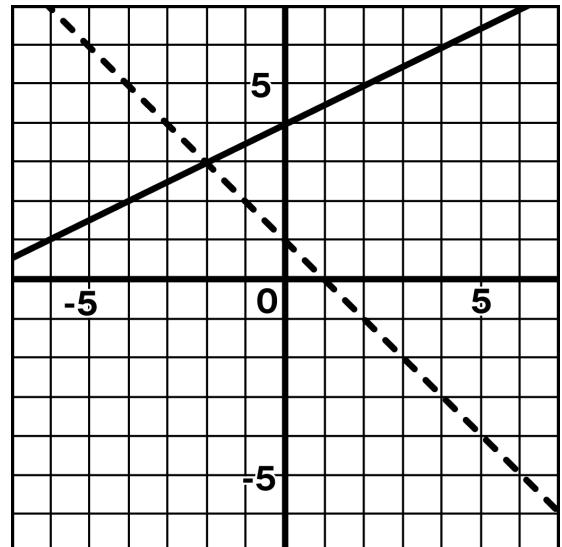
Testing points can be helpful to determine the solution region. The point $(0, 0)$ was tested in the system of inequalities shown.

Test the point $(-4, 3)$ and mark the solution region on the graph.

$(0, 0)$	$(-4, 3)$
$x + y < 1$ $0 + 0 < 1$ $0 < 1$ True ✓	
$y \geq \frac{1}{2}x + 4$ $0 \geq \frac{1}{2}(0) + 4$ $0 \geq 4$ False ✗	

Not in the solution region.

$$\begin{aligned}x + y &< 1 \\y &\geq \frac{1}{2}x + 4\end{aligned}$$

**Things I Want to Remember**

desmos

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Lessons 9–10: Solutions to Systems of Inequalities

Try This!

The graph for this system is shown.

$$-2x - y \geq 3$$

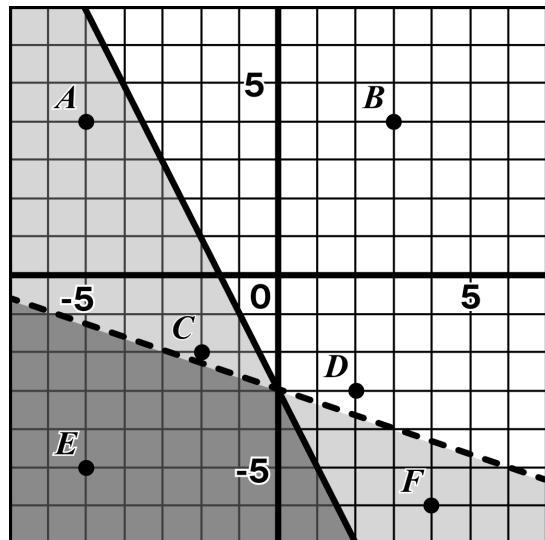
$$y < -\frac{1}{3}x - 3$$

Identify a point that is:

1.1 In the solution region: _____

1.2 A solution to **no** inequalities: _____

1.3 A solution to only **one** inequality: _____



Determine if each point is in the solution region.

2.1 Is (3, 2) in the solution region?

Yes No

2.2 Is (-1, -3) in the solution region?

Yes No

2.3 Is (1, 6) in the solution region?

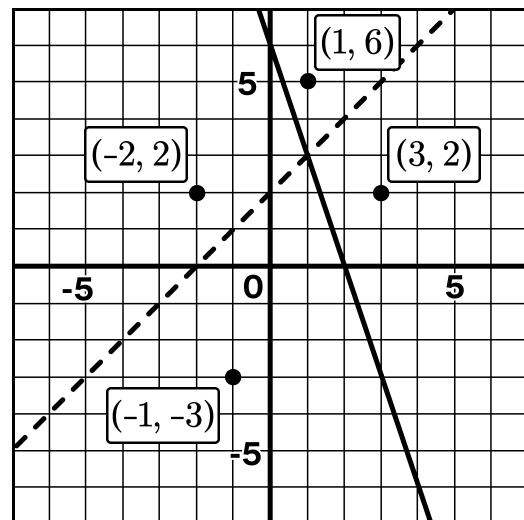
Yes No

2.4 Is (-2, 2) in the solution region?

Yes No

$$3x + y \geq 6$$

$$y > x + 2$$



- I can use a graph to determine if a point is a solution to a system of inequalities.
- I can connect coordinate pairs to constraints in a situation.
- I can describe that solutions to a system of inequalities are all points that make both inequalities true and the region where the graphs overlap.
- I can determine the solution region of a system of inequalities given the boundary lines.

Lessons 1–3: Connecting Representations of Linear and Exponential Functions

Summary

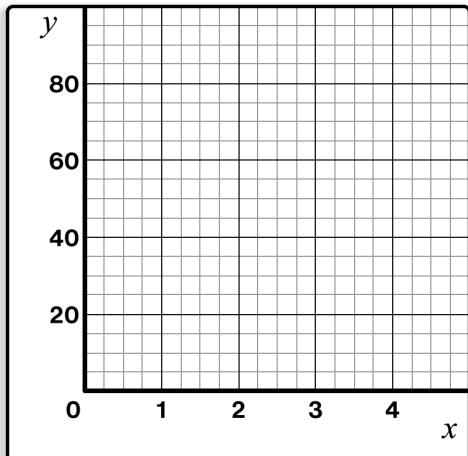
Here are two examples of *linear* and *exponential* relationships. Let's compare the two types.
Fill in the missing information in each example.

Linear

x	y
0	
1	
2	
3	

$$y = 30 + 20x$$

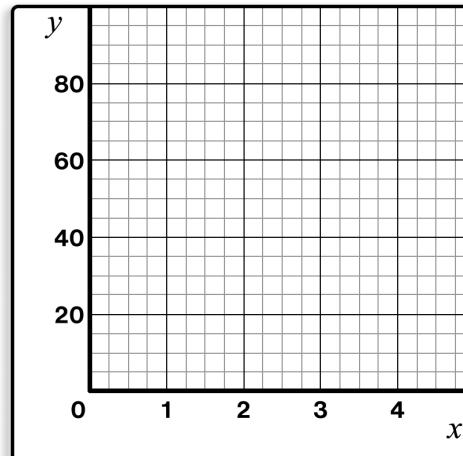
At first there are 30 globs. 20 more globs are added each month.

**Exponential**

x	y
0	
1	
2	
3	40

$$y = 5 \cdot (2)^x$$

At first there are _____ globs. Then they _____ each month.



Circle where you see the initial value in each representation.

Describe or show where you see either the rate of change or growth factor in the table and equation.

Will the linear growth or exponential growth produce more globs after 10 months?
Show or explain your thinking.

Things I Want to Remember

Lessons 1–3: Connecting Representations of Linear and Exponential Functions

Try This!

Determine if each table shows a linear relationship, exponential relationship, or something else. Circle your choice.

1.1

x	y
3	5
4	15
5	45

1.2

x	y
1	1
2	11
3	111

1.3

x	y
2	25
3	50
4	75

Linear / Exponential /
Something else

Linear / Exponential /
Something else

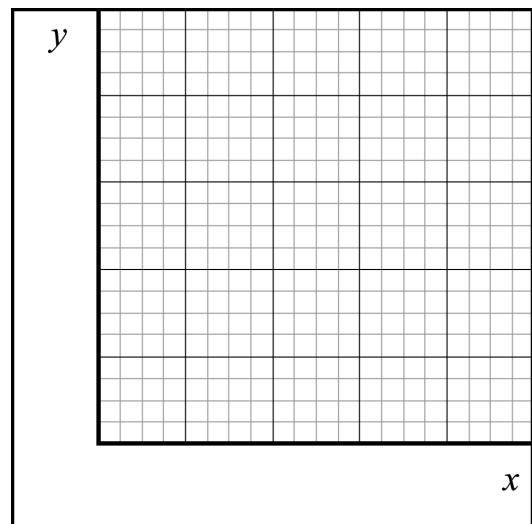
Linear / Exponential /
Something else

2.1 Select one equation and sketch its graph. (Circle it.)

$$f(x) = 2 \cdot (3)^x$$

$$g(x) = 3 \cdot (2)^x$$

x	y



2.2 Is $f(6) < g(6)$ true? Explain how you know.

- I understand that linear patterns have a constant rate of change and exponential patterns have a constant growth factor.
- I can determine whether a pattern represents a linear or exponential relationship.
- I can use graphs and equations of linear and exponential functions to make predictions.
- I can describe exponential functions using their key features and statements written in function notation.
- I can create equations, tables, and graphs for linear and exponential functions.

Lesson 4: Introducing Simple and Compound Interest**Summary**

Here are examples of *simple interest* and *compound interest*. Let's compare the two.

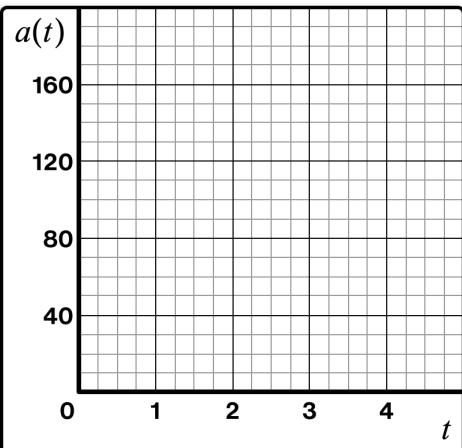
Fill in the missing information (table, graph, description) for each type of interest account.

Account A: Simple Interest

t	$a(t)$
0	
1	
2	
3	

$$a(t) = 100 + 10t$$

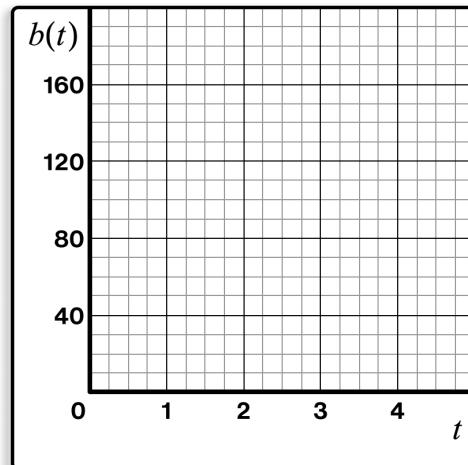
Account A starts with \$100 and \$10 is added each year.

**Account B: Compound Interest**

t	$b(t)$
0	
1	
2	
3	

$$b(t) = 50 \cdot (1.1)^t$$

Account B starts with _____ and increases by _____ each year.



Use a calculator to determine how long it would take each account to reach \$200.

How do simple interest and compound interest relate to linear and exponential relationships?

Things I Want to Remember

Lesson 4: Introducing Simple and Compound Interest**Try This!**

Adah invests \$75 in an account that earns 5% simple interest yearly. The function $a(t) = 75 + 3.75t$ models the account balance after t years.	Jamir invests \$75 in an account that earns 3% compound interest yearly. The function $j(t) = 75(1.03)^t$ models the account balance after t years.
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Determine the balance of each account at:

1.1 10 years Adah: _____ Jamir: _____

1.2 12 years Adah: _____ Jamir: _____

Determine how many years it will take for each account to reach a balance of:

1.3 \$100 Adah: _____ Jamir: _____

1.4 \$200 Adah: _____ Jamir: _____

1.5 Which person's account would you prefer to have? Justify your choice.

- I can make connections between simple and compound interests, and linear and exponential functions.
- I can use a graphing calculator to solve problems about linear and exponential functions.

Lesson 5: Evaluating Exponential Functions**Summary**

Exponential functions can be used to model exponential behavior.

Let's compare the two examples. Fill in the missing parts of the representations.

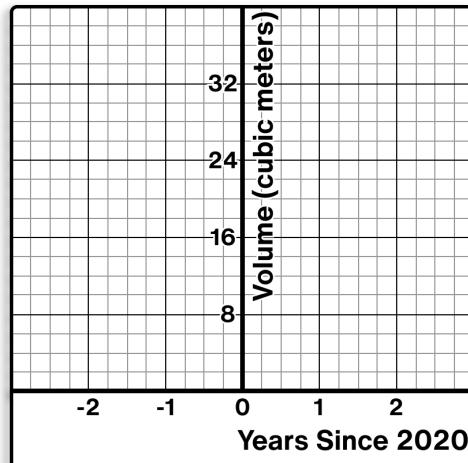
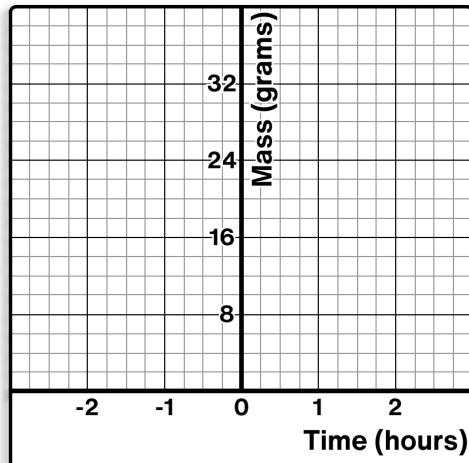
t	$m(t)$
-2	
-1	
0	
1	
2	

Carlos bought a new toy fish that triples every hour when placed in water.

$$m(t) = 4 \cdot (3)^t$$

t	$v(t)$
-2	
-1	
0	
1	
2	

In 2020, a coral reef had a volume of 10 cubic meters and _____ by a factor of _____.
 $v(t) = 10 \cdot \left(\frac{1}{2}\right)^t$



Describe how you evaluate exponential functions for negative inputs and zero.

Compare and contrast the domains of $m(t)$ and $v(t)$.

Things I Want to Remember

Lesson 5: Evaluating Exponential Functions

Try This!

Marc's video channel had 18 subscribers in 2020. The function $m(x)$ represents his total subscriber count x years after 2020.

$$m(x) = 18\left(\frac{3}{2}\right)^x$$

- 1.1 Complete the table.

x	$m(x)$
-2	
-1	
0	
1	

- 1.3 Sketch the graph of either $m(x)$ or $k(x)$.

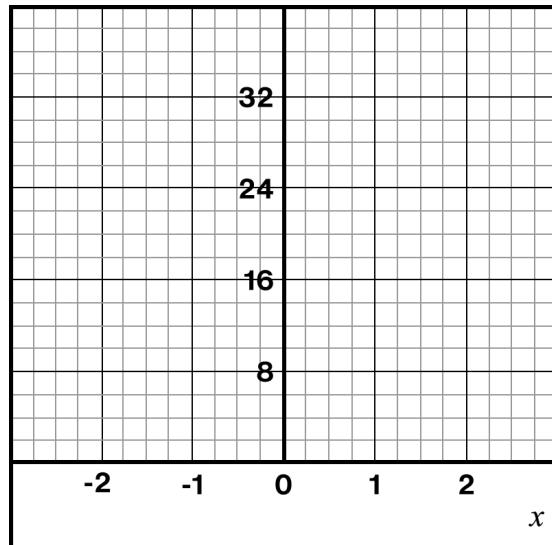
- 1.4 Explain whether $m(-1)$ and $k(-1)$ make sense in their situation.

Kiana drank a beverage that had 20 milligrams of caffeine. The function $k(x)$ represents the total caffeine in Kaina's system x hours after she consumed the drink.

$$k(x) = 20\left(\frac{4}{5}\right)^x$$

- 1.2 Complete the table.

x	$k(x)$
-2	
-1	
0	
1	



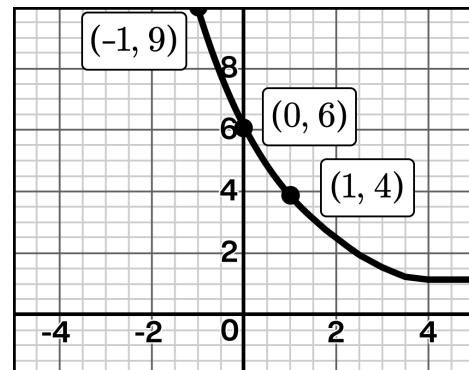
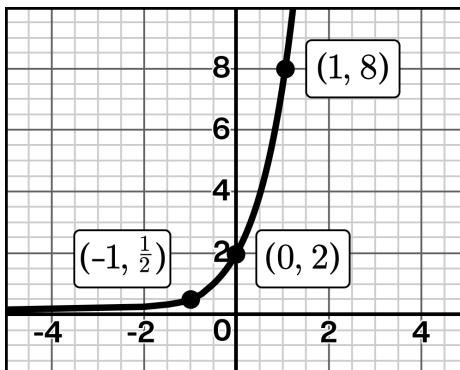
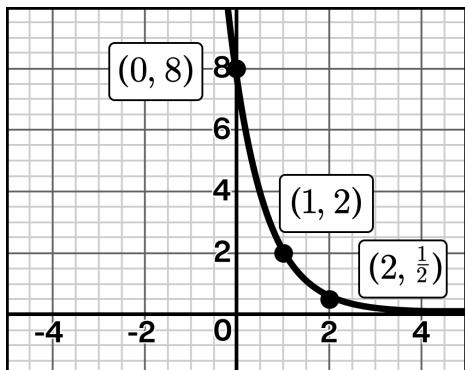
- I can evaluate exponential functions with positive, zero, and negative inputs.
- I can describe the domain of an exponential function in context.

Lesson 6: Writing Equations of Exponential Functions**Summary**

Exponential functions can be written in the form $f(x) = a \cdot b^x$, where:

- The a -value is the y -intercept or initial value.
- The b -value is the growth factor.

Write the exponential equation for each graph. Show your thinking.



x	$f(x)$
0	8
1	2
2	$\frac{1}{2}$

x	$g(x)$
-1	$\frac{1}{2}$
0	2
1	8

x	$h(x)$
-1	9
0	6
1	4

$$f(x) = \underline{\hspace{2cm}}$$

$$g(x) = \underline{\hspace{2cm}}$$

$$h(x) = \underline{\hspace{2cm}}$$

Describe how to determine the a - and b -value of an exponential function given a graph.

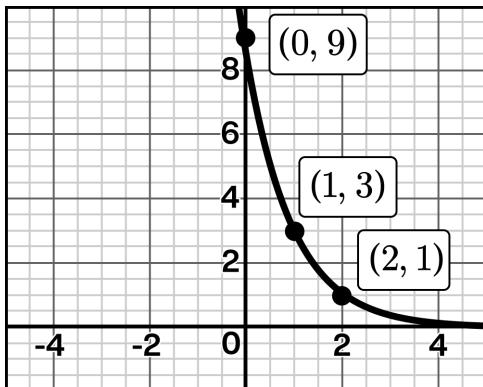
Things I Want to Remember

Lesson 6: Writing Equations of Exponential Functions

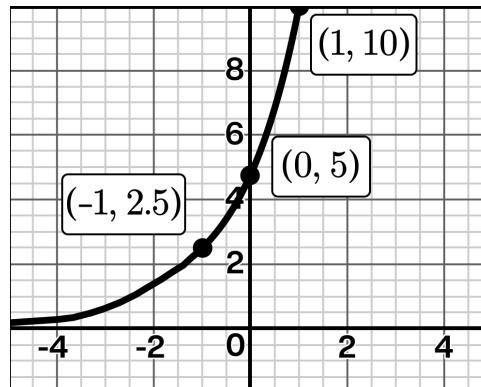
Try This!

Select the equation that matches each exponential graph. Use a table if it helps with your thinking.

1.1



1.2



A. $y = 9\left(\frac{2}{3}\right)^x$

B. $y = 9\left(\frac{1}{3}\right)^x$

C. $y = 9(3)^x$

D. $y = 9(-3)^x$

x	y

A. $y = 5\left(\frac{1}{2}\right)^x$

B. $y = -2.5(2)^x$

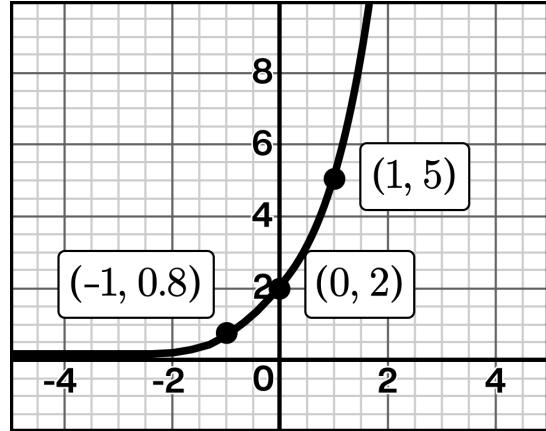
C. $y = 2.5(4)^x$

D. $y = 5(2)^x$

x	y

2. Write the exponential equation that represents this graph. Use the table if it helps with your thinking.

x	y



$y = \underline{\hspace{2cm}}$

- I can write equations of exponential functions from a graph.
 I can describe how changing a and b in $f(x) = a \cdot b^x$ affects its graph.

Lessons 7–9: Percent and Fractional Increase and Decrease**Summary**

We can use the growth factor, b , in an exponential equation like $y = a \cdot b^x$ to determine the type of exponential relationship: growth or decay.

- When $b > 1$, this is an example of exponential _____.
- When $0 < b < 1$, this is an example of exponential _____.

Let's look at a bank account that starts with \$100. Complete the matching information for each row of the table. The first row is done for you.

Description	Growth or Decay?	Equation
Doubles	Growth	$y = 100(2)^x$
Increases by 5% every year		
		$y = 100(0.97)^x$
Triples every year		
Retains 63% each year		
		$y = 100(1 - 0.03)^x$
Halves every year		

How can you use the description of an exponential situation to write an equation?

Which two situations are equivalent?

Lessons 7–9: Percent and Fractional Increase and Decrease

Try This!

Determine which equation matches each situation. Circle your choice.

- 1.1 The value of a Desmon card collection increases 4% every year. In 2020, the price of the collection was valued at \$500.

Use y to represent the cost of the collection and x to represent time in years since 2020.

- A. $y = 500(0.4)^x$
- B. $y = 500(0.04)^x$
- C. $y = 500(1.4)^x$
- D. $y = 500(1.04)^x$

- 1.2 A potted plant receives 24 mL of fertilizer. It will lose 1% of the fertilizer every hour.

Use y to represent the amount of fertilizer left in the potted plant and x to represent time in hours since receiving the fertilizer.

- A. $y = 24(0.01)^x$
- B. $y = 24(1.01)^x$
- C. $y = 24(0.99)^x$
- D. $y = 24(0.1)^x$

A laptop battery can currently stay on for 480 minutes. The battery's capacity is decreasing by 8% each year.

- 2.1 Write a function that represents the battery's capacity, $b(x)$, after x years.

- 2.2 Use the function you wrote to determine how long the laptop battery will stay on after 5 years.

- I can write equations of exponential functions from a graph.
- I can describe how changing a and b in $f(x) = a \cdot b^x$ affects its graph.
- I can compare and write equivalent equations of functions that represent exponential decay.
- I can calculate the percent change of an exponential function given a graph.
- I can use equations of exponential functions to solve problems in context.
- I can interpret what the values of a , b , and k mean for exponentials in the form $y = a \cdot b^x$ and $y = b^x + k$.

Lessons 10–11: Different Compounding Intervals**Summary**

Compound interest can be earned in different time intervals, such as daily, monthly, and annually.

Write three equivalent expressions that will calculate the total value of a \$1 000 loan with a 7% monthly interest rate after 2 years, with no additional payments.

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When you take out a loan from the bank, the annual interest rate may be compounded at different intervals.

Complete the table for the total value of the \$1 000 loan for different compounding intervals.

Interest	Owed In	Compounded Monthly	Compounded Quarterly	Compounded Annually
15% annually	5 years	$1000\left(1 + \frac{0.15}{12}\right)^{12 \cdot 5}$ $\approx \$2\,107.18$		
9% annually	6 months		$1000\left(1 + \frac{0.09}{4}\right)^{4 \cdot 0.5}$ $\approx \$1\,045.51$	

How does the compounding interval affect the total value?

The formula $P\left(1 + \frac{r}{n}\right)^{nt}$ calculates the total amount in an account with compound interest.

Select **two** variables from the formula and describe what they represent.

Lessons 10–11: Different Compounding Intervals**Try This!**

For each situation, select **all** the expressions that can be used to calculate the balance after 5 years. Assume that no additional payments, deposits, or withdrawals are made.

- 1.1 A savings account has a starting balance of \$100 and earns 4% annual interest compounded annually.

$100(1.04)^5$

$100(1 + 0.04)^5$

$100(1.1699)^5$

$100(1 + 0.04^5)$

$100(1.04)^{20}$

- 1.2 A \$100 loan with a monthly interest rate of 8%.

$100(1.08)^5$

$100(1 + 0.08)^5$

$100(1.08^{12})^5$

$100(1.08)^{60}$

$100(2.51817)^5$

A bank offers different interest rates for their checking accounts.

- Option A: 2% annual interest rate compounded daily.
 - Option B: 3% annual interest rate compounded semi-annually.
2. If you deposit \$900, which option will give you a greater balance in 4 years?
Show your reasoning.

- | |
|---|
| <input type="checkbox"/> I can write and interpret exponential functions that represent compound interest. |
| <input type="checkbox"/> I can use the properties of exponents to interpret equivalent exponential expressions. |
| <input type="checkbox"/> I can interpret exponential expressions for accounts that compound at different intervals. |
| <input type="checkbox"/> I can calculate balances given an annual interest rate and a compounding interval. |

Lessons 13–14: Modeling Exponential Data**Summary**

Similar to a line of best fit, a calculator can compute an equation for an exponential curve of best fit.

Kyrie used the population of Detroit from 1950–2020 data to generate a line and curve of best fit.

Describe how to determine whether to use a line or exponential curve of best fit.

Describe what the slope and growth factor means in each.

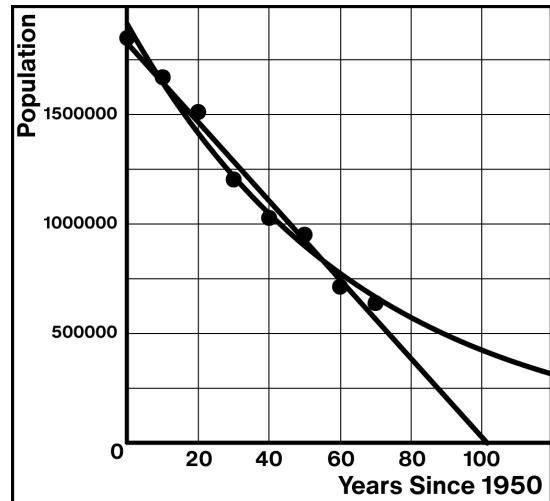
Slope of Linear Model:

This means...

Growth Factor of Exponential Model:

This means...

Use Kyrie's graphs to estimate the population of Detroit in 2040 (90 years after 1950).

Linear Model Prediction:**Exponential Model Prediction:**

$$y_1 \sim mx_1 + b$$

PARAMETERS

$$m = -17\,989.3$$

$$b = 1\,825\,460$$

$$y_1 \sim a \cdot b^{x_1}$$

PARAMETERS

$$a = 1\,912\,612$$

$$b = 0.985051$$

Things I Want to Remember

Lessons 13–14: Modeling Exponential Data

Try This!

Aki was curious about the population changes in Colorado. They generated this line and exponential curve of best fit.

Describe what the slope and growth factor means in each.

1.1 Slope of linear model:

This means...

1.2 Growth factor of exponential model:

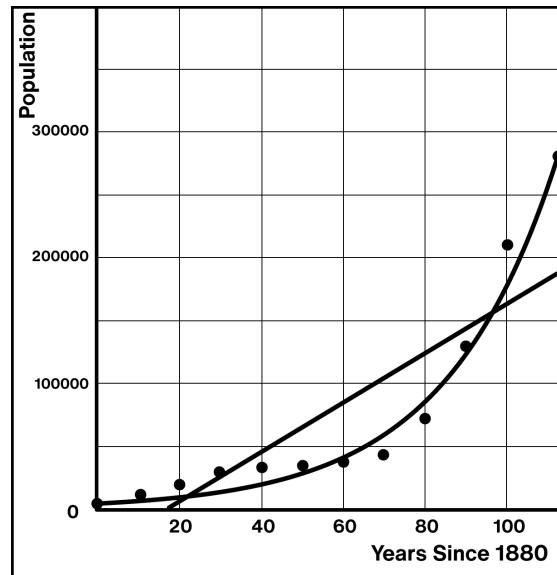
This means...

1.3 Which model do you think better fits the data for the population of Colorado from 1880–1990?

Use Aki's graphs to estimate the population of Colorado in 1945 (65 years after 1980).

1.4 Linear model prediction:

1.5 Exponential model prediction:



$$y_1 \sim mx_1 + b$$

PARAMETERS

$$m = 2075.25$$

$$b = -38082.20$$

$$y_1 \sim a \cdot b^{x_1}$$

PARAMETERS

$$a = 3991.89$$

$$b = 1.03967$$

- I can fit an exponential or linear function to data.
- I can interpret and use models to make predictions.
- I can use a graphing calculator to create a linear or exponential model to fit a function to data.
- I can informally assess the fit of a function to data.
- I can use a linear or exponential model to make predictions and solve problems in context.

Lessons 1–2: Quadratic Visual Patterns**Summary**

Let's explore a new type of relationship. Here is a visual pattern.

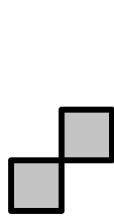
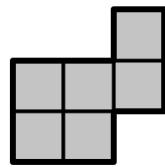
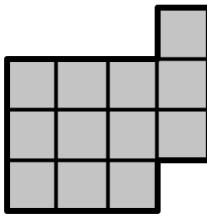
**Figure 1****Figure 2****Figure 3**

Figure	Number of Tiles
1	
2	$2^2 + 2$
3	
4	
10	
n	

Sketch the pattern for figure 4.

The relationship between figure number and number of tiles is *quadratic* because . . .

Things I Want to Remember

Lessons 1–2: Quadratic Visual Patterns

Try This!

- 1.1 Sketch or describe what figure 10 would look like.

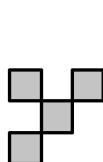


Figure 1

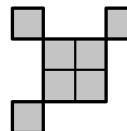


Figure 2

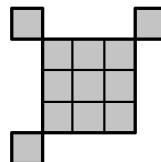


Figure 3

- 1.2 How many tiles would figure n have?

- 1.3 Is this relationship quadratic? Explain your thinking.

- 2.1 Sketch or describe what figure 10 would look like.

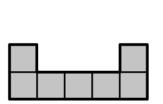


Figure 1

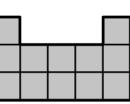


Figure 2

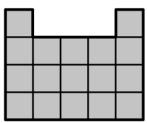


Figure 3

- 2.2 How many tiles would figure n have?

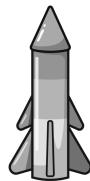
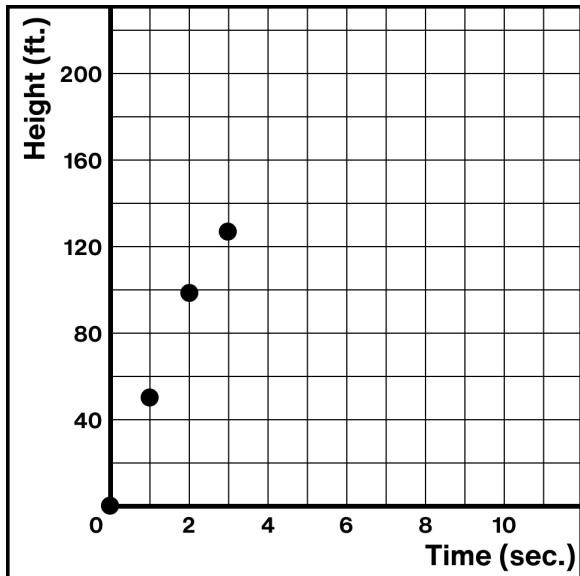
- 2.3 Is this relationship quadratic? Explain your thinking.

- I can describe a nonlinear pattern using words or an expression.
- I can compare linear and quadratic relationships.
- I can write an expression for a quadratic relationship from a pattern.
- I understand that quadratic relationships contain a square or squared term.

Lessons 4–5: Quadratic Relationships in Tables and Graphs

Summary

Use symmetry and *constant second differences* to complete the table and graph for this rocket.



Time (sec.)	Height (ft.)
0	0
1	52
2	94
3	126
4	
5	

Graphs of quadratic relationships have a *line of symmetry* that is . . .

Things I Want to Remember

Lessons 4–5: Quadratic Relationships in Tables and Graphs

Try This!

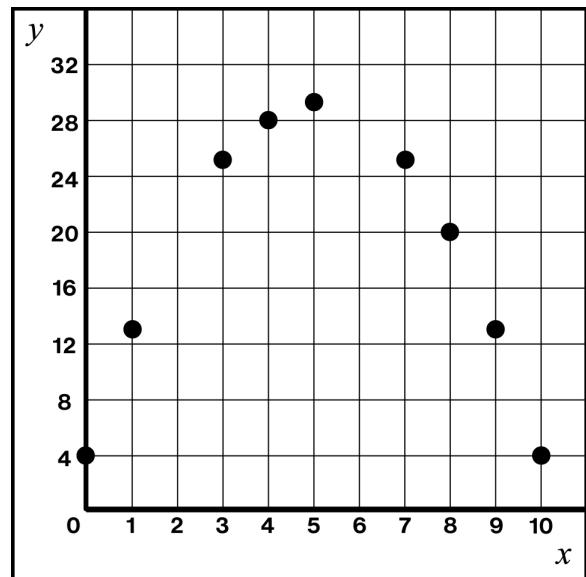
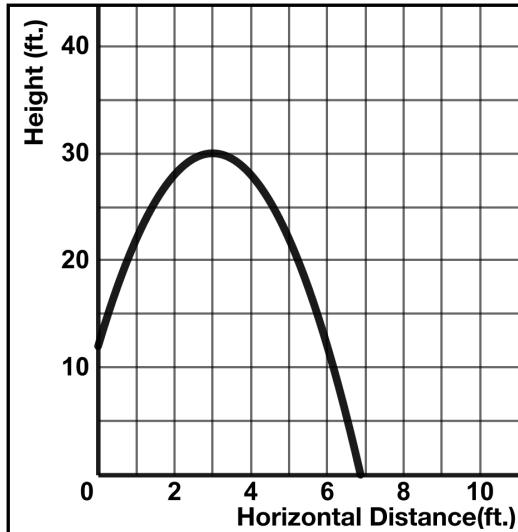
Here are some of the points on a parabola.

- 1.1 Draw the line of symmetry on the graph.

- 1.2 Use the line of symmetry to determine 2 more points that fall on this parabola.

Plot the points on the graph **and** list them below.

x	y

**Balloon A****Balloon B**

Horizontal Distance (ft.)	Height (ft.)
1	28
2	29
3	28
4	25
5	20

2. Which water balloon went higher?

Explain or show how you know.

- I can create tables and graphs to represent a quadratic relationship in context.
- I can describe that parabolas are symmetrical around a line of symmetry.
- I can use tables and graphs to make predictions about quadratic relationships in context.

**Lessons 6–7: Key Features of Parabolas****Summary**

Quadratic functions have many key features we have used for other functions and some new ones.

Label each of these terms on the parabola: x -intercept(s), y -intercept, vertex, concave up/down.

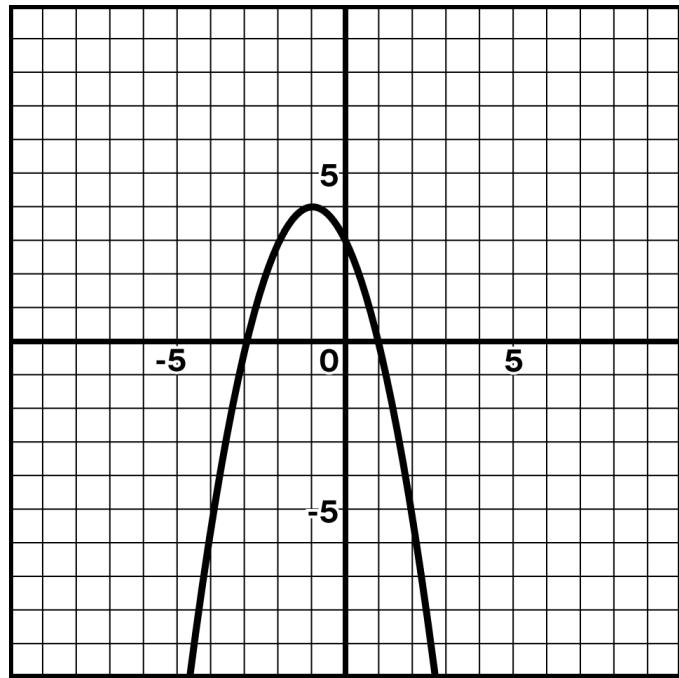
Write a definition of each new term.

Vertex:

Line of symmetry:

Concave up:

Concave down:



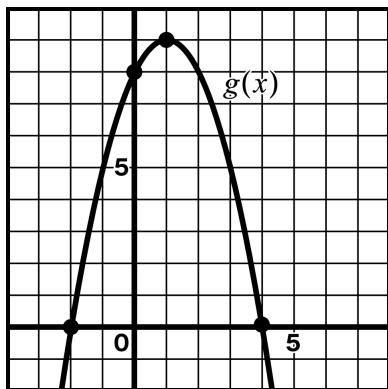
Things I Want to Remember

Lessons 6–7: Key Features of Parabolas

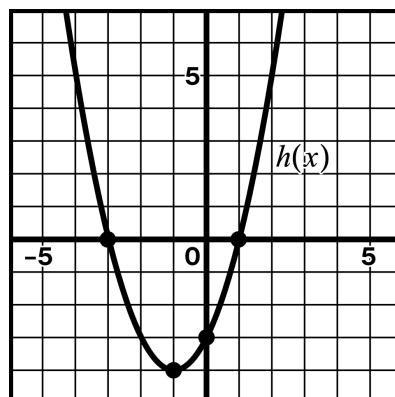
Try This!

Describe each parabola using the terms x -intercept(s), y -intercept, vertex, concave up/down.

1.1



1.2



x -intercept(s)	Concave Up/Down
y -intercept	Vertex

x -intercept(s)	Concave Up/Down
y -intercept	Vertex

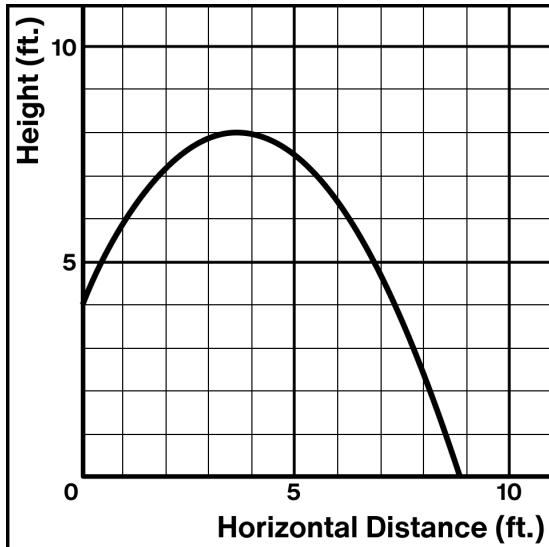
The path of a stomp rocket is modeled by this graph.

Which key feature describes:

2.1 The starting height of the stomp rocket?

2.2 The maximum height of the stomp rocket?

2.3 Where the stomp rocket lands?



- I can describe the key features of a parabola.
- I can create graphs when given a description of a parabola.
- I can compare properties of quadratic functions represented in different ways.

Lesson 10: Intercepts in Factored and Standard Form**Summary**

Different forms of quadratic equations help us see different key features of a parabola.

Factored form: $p(x) = (2x - 10)(x + 3)$

x	$(2x - 10)$	$(x + 3)$	$(2x - 10)(x + 3)$
-3			
5			
0			

How are the x -intercepts related to the equation in factored form?

Standard form: $p(x) = 2x^2 - 4x - 30$

x	$2x^2$	$4x$	-30	$2x^2 - 4x - 30$
0				

How is the y -intercept related to the equation in standard form?

Things I Want to Remember

Lesson 10: Intercepts in Factored and Standard Form

Try This!

1. The same function is written in factored and standard form.

Factored form: $f(x) = (2x - 2)(x + 5)$

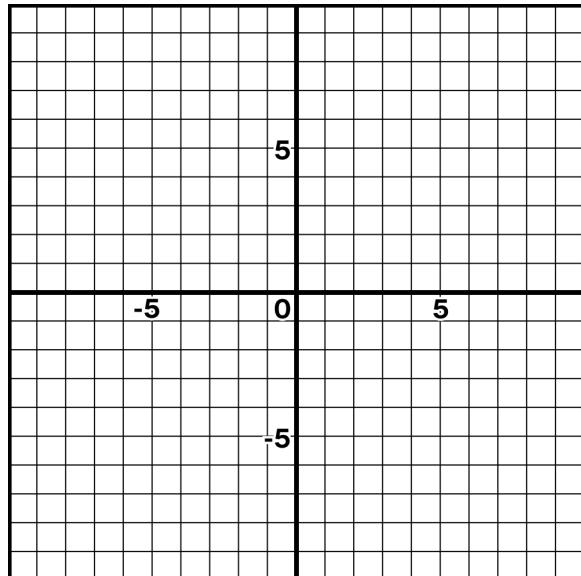
Standard form: $2x^2 + 8x - 10$

Determine the x - and y -intercepts of $f(x)$.

Here is a function: $g(x) = (x - 3)(2x + 2)$.

2. Sketch a graph of $g(x)$ on the coordinate plane.

Include the x - and y -intercepts.



- | |
|--|
| <input type="checkbox"/> I can determine the x -intercepts of a parabola from its equation in factored form. |
| <input type="checkbox"/> I can determine the y -intercept of a parabola from its equation in standard form. |

Lessons 11–12: Graphing Parabolas in Factored Form**Summary**

We can use what we know about key features in factored form to create graphs.

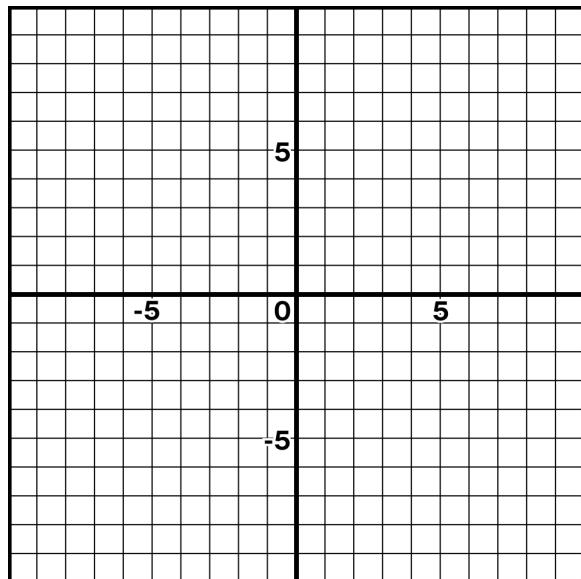
Here is a function: $f(x) = (x + 5)(x - 1)$. Its x -intercepts are (,) and (,).

The vertex is in the middle of the two x -intercepts.

This means the x -value of the vertex is ___ because . . .

A table can help determine the y -value of the vertex.

x	$(x + 5)$	$(x - 1)$	$(x + 5)(x - 1)$
			0
			0



How is the graph of $g(x) = -(x + 5)(x - 1)$ different from the graph of $f(x) = (x + 5)(x - 1)$?

Sketch both graphs on the axes above. Label each function.

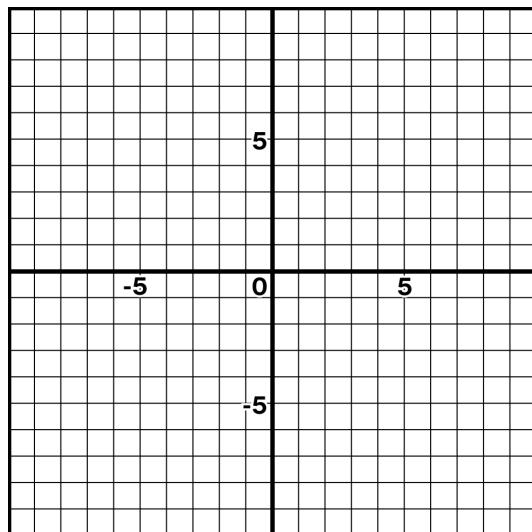
Things I Want to Remember

Lessons 11–12: Graphing Parabolas in Factored Form

Try This!

1. Sketch a graph of $f(x) = (4x - 4)(x - 3)$.

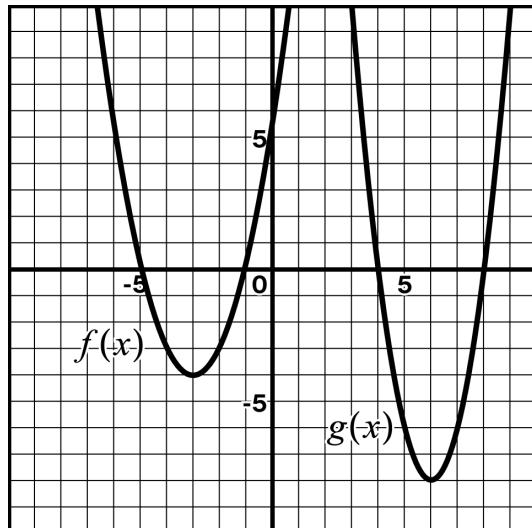
Include the x -intercepts and vertex.



Write a quadratic equation to match each graph.

2.1 $f(x) =$

2.2 $g(x) =$



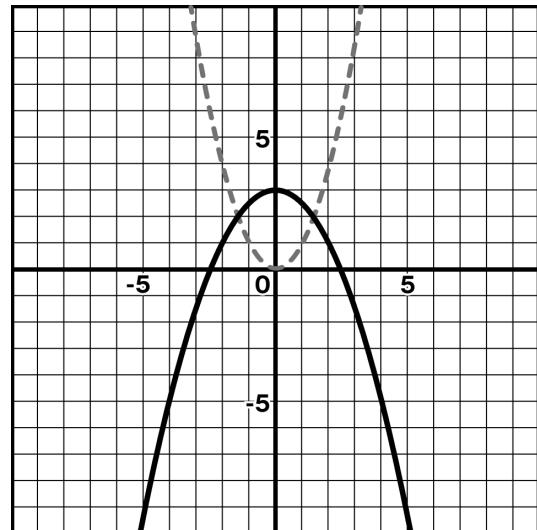
- I can explain how to determine the vertex of a parabola given its equation in factored form.
 - I can use key features to graph quadratic functions in factored form.
 - I can write a quadratic equation in factored form when given its graph.
 - I can describe what the value of a in $a(x - m)(x - n)$ tells us about a parabola.

Lesson 14: Vertical Translations and Stretches of Quadratic Functions**Summary**

Two ways to change parabolas are using *translations* and *vertical stretches*.

The dashed line is a graph of $y = x^2$. The solid line is a graph of $y = -\frac{1}{2}x^2 + 3$.

x	x^2	$-\frac{1}{2}x^2$	$-\frac{1}{2}x^2 + 3$
-2	4	-2	1
-1	1	$-\frac{1}{2}$	$2\frac{1}{2}$
0	0	0	3
1	1	$-\frac{1}{2}$	$2\frac{1}{2}$
2	4	-2	1



Describe how the graph of $y = -\frac{1}{2}x^2 + 3$ compares to the graph of $y = x^2$. Where do you see each transformation in the table?

Things I Want to Remember

Lesson 14: Vertical Translations and Stretches of Quadratic Functions

Try This!

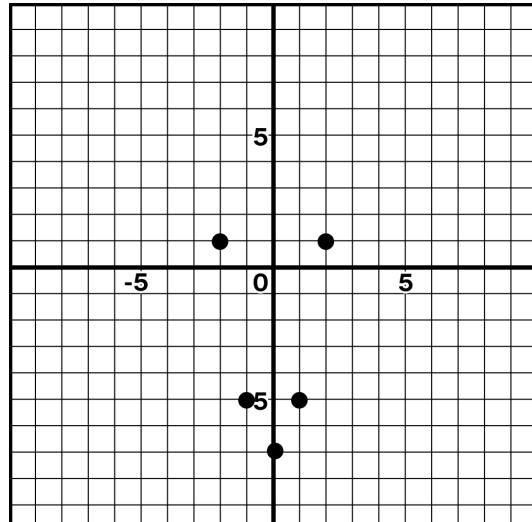
1. Describe how the graph of $y = -x^2 + 8$ compares to the graph of $y = x^2$.

x	x^2	$-x^2$	$-x^2 + 8$
-2	4	-4	4
-1	1	-1	7
0	0	0	8
1	1	-1	7
2	4	-4	4

2. Write an equation for this transformation of $y = x^2$.

Use a table if it helps you with your thinking.

x	x^2		
-2	4	8	1
-1	1	2	-5
0	0	0	-7
1	1	2	-5
2	4	8	1



Equation:

- I can describe what vertical translations and stretches do to the graph of a quadratic function.
- I can write equations of the form $y = ax^2 + k$ to model a quadratic function.

Lesson 15: Vertex Form**Summary**

There are three common forms of quadratic equations. Here is the same function in all three forms:

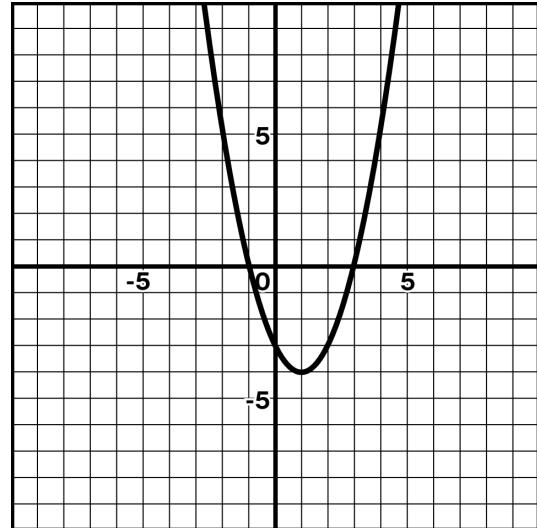
Standard form: $f(x) = x^2 - 2x - 3$

Factored form: $f(x) = (x + 1)(x - 3)$

Vertex form: $f(x) = (x - 1)^2 - 4$

A table can help show why the vertex of $f(x)$ is $(1, -4)$.

x	$x - 1$	$(x - 1)^2$	$(x - 1)^2 - 4$



The minimum has to be -4 because . . .

Write a different equation in vertex form with a vertex at $(1, -4)$. Show or explain your thinking.

Things I Want to Remember

Lesson 15: Vertex Form

Try This!

- 1.1 Determine the vertex of $f(x) = (x + 3)^2 + 1$. Use a table if it helps you with your thinking.

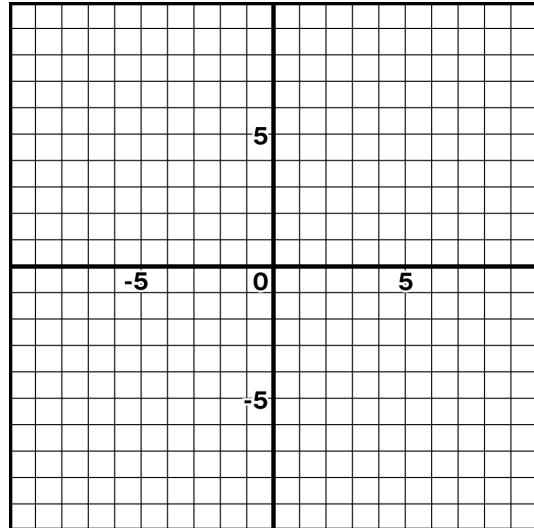
x			

- 1.2 Determine the vertex of $y = -2(x - 4)^2 - 8$. Use a table if it helps you with your thinking.

x				

2. Write an equation of a parabola that has a vertex at (5, 2).

Use the graph if it helps you with your thinking.



I can identify the maximum, minimum, or vertex from a quadratic function in vertex form.

Lessons 16–17: Writing Equations of Quadratic Functions**Summary**

We can use what we know about the key features of a parabola to write equations of its graph.

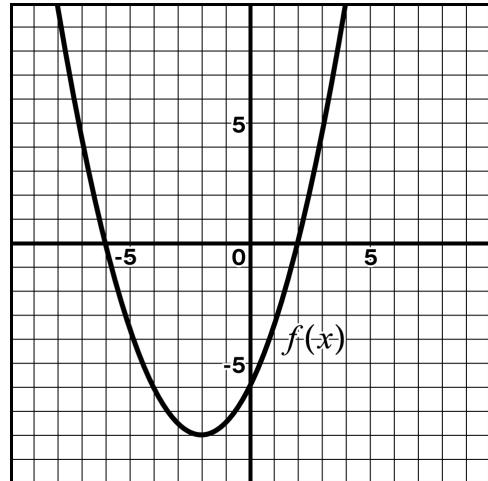
Here are two different equations of the parabola $f(x)$.

Factored form: $f(x) = \frac{1}{2}(x + 6)(x - 2)$

Factored form helps us see the x -intercepts of a parabola.

Vertex form: $f(x) = \frac{1}{2}(x + 2)^2 - 8$

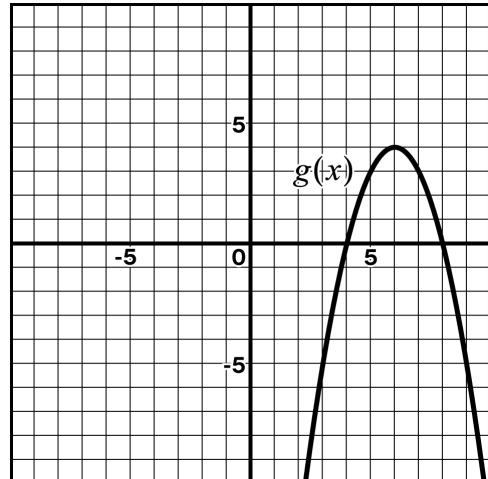
Vertex form helps us see the vertex of a parabola.



Use the key features to write two equations of $g(x)$.

Factored form:

Vertex form:

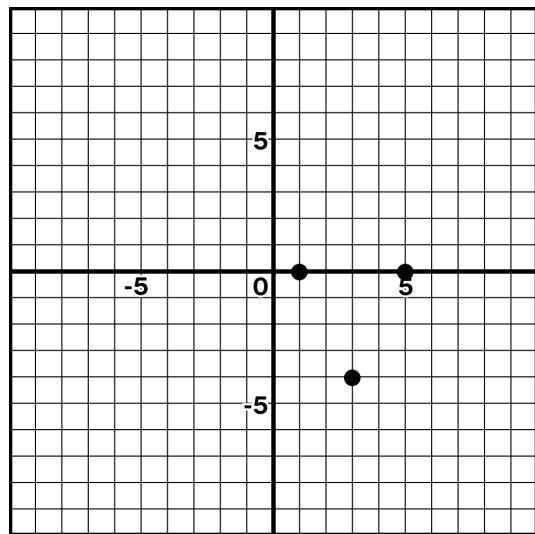


Things I Want to Remember

Lessons 16–17: Writing Equations of Quadratic Functions

Try This!

1. Write two **different** quadratic equations whose graph passes through these points.



2.1 Write an equation of a parabola that is concave down with a vertex at $(-3, 5)$.

2.2 Write an equation of a parabola that is concave up with x -intercepts at -3 and 5 .

- | |
|--|
| <input type="checkbox"/> I can write quadratic functions in vertex and factored forms.
<input type="checkbox"/> I can use quadratic equations, tables, and graphs to analyze an issue in society. |
|--|

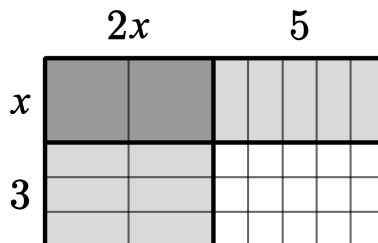
Lessons 1–2: Patterns in Factored-Form and Standard-Form Expressions**Summary**

Quadratic expressions written in *factored form*, which look like $a(x - m)(x - n)$, can be rewritten in *standard form*, which looks like $ax^2 + bx + c$.

Where on the area model do you see the expression's factored form? What about its standard form?

Factored Form Standard Form

$$(x + 3)(2x + 5) \quad 2x^2 + 11x + 15$$



An area model can be useful in rewriting a quadratic expression from factored form to standard form.

Draw an area model of each quadratic expression. Then rewrite the expressions in standard form.

$$(3x - 4)(x + 2)$$

$$x(3x + 1)$$

Standard form: _____**Standard form:** _____

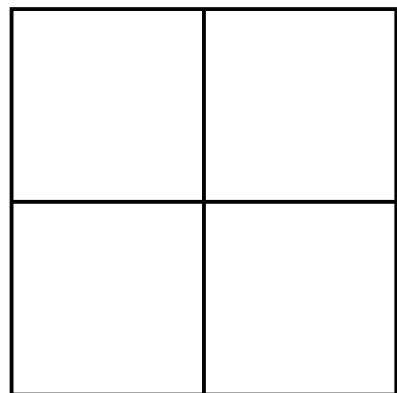
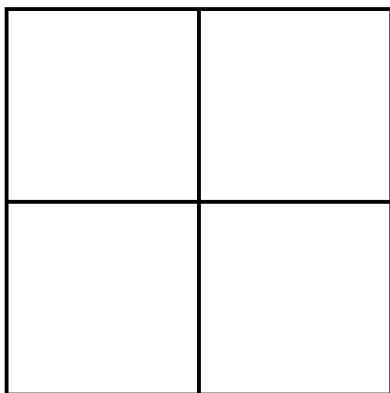
Describe how to write a factored-form expression in standard form.

Lessons 1–2: Patterns in Factored-Form and Standard-Form Expressions**Try This!**

Rewrite the expressions in standard form. Use the diagrams if they help with your thinking.

1.1 $(2x - 5)(x + 3)$

1.2 $(3x - 4)(2x + 1)$

**Standard form:** _____**Standard form:** _____

Identify the a -, b -, and c - values of these standard-form quadratic expressions:

2.1 $5x^2 + 9x - 15$

$a = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$ $c = \underline{\hspace{2cm}}$

2.2 $x^2 - 1$

$a = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$ $c = \underline{\hspace{2cm}}$

2.3 $3x^2 + x$

$a = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$ $c = \underline{\hspace{2cm}}$

- I can use area models and diagrams to rewrite a factored-form quadratic expression in standard form.
- I can identify the a -, b -, and c -values in a standard-form quadratic expression.
- I can use a factored-form expression to predict what an equivalent expression in standard form will look like.

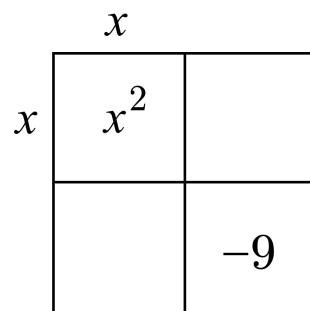
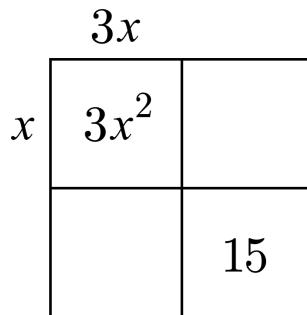
Lessons 3–4: Factoring Quadratic Expressions**Summary**

When factoring quadratic expressions, you can look to the values of the terms to determine potential strategies.

Let's look at two different quadratic expressions. Complete the diagrams, then rewrite the expressions in factored form.

$$3x^2 + 14x + 15$$

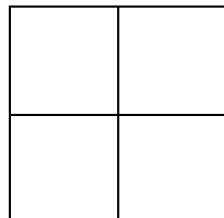
$$x^2 - 9$$

**Factored form:** _____**Factored form:** _____

If there is a common factor in a quadratic expression, it may be helpful to write an equivalent expression first. For example, rewriting $2x^2 + 4x - 16$ as $2(x^2 + 2x - 8)$.

Rewrite $3x^2 - 9x - 30$ as an equivalent expression, then factor it.

Use the diagram if it helps your thinking.

**Factored form:** _____

Describe how to write a standard-form expression in factored form.

Things I Want to Remember

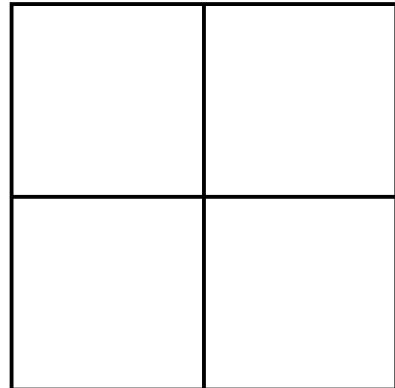
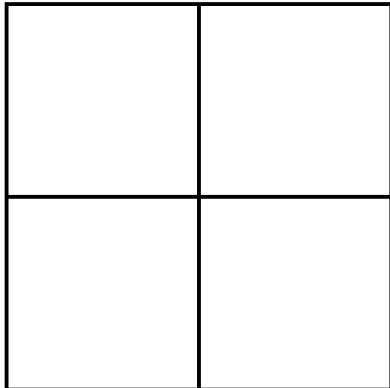
Lessons 3–4: Factoring Quadratic Expressions

Try This!

Factor the expressions. Use the diagrams if they help with your thinking.

1.1 $x^2 + 10x + 21$

1.2 $2x^2 + 3x - 27$

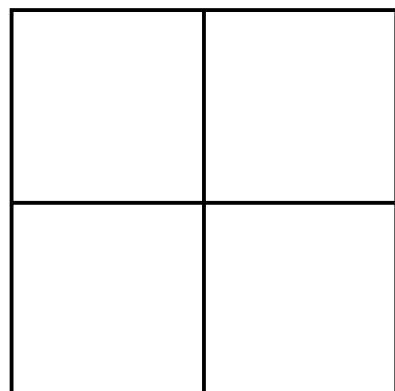
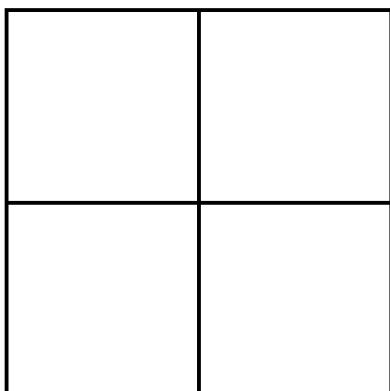


Factored form: _____

Factored form: _____

1.3 $x^2 - 81$

1.4 $2x^2 + 22x + 60$



Factored form: _____

Factored form: _____

- | |
|---|
| <input type="checkbox"/> I can factor to rewrite standard-form quadratic expressions in factored form. |
| <input type="checkbox"/> I can use a standard-form expression to predict what an equivalent expression in factored form will look like. |

**Lessons 5–6: Solving Quadratic Equations Using the Zero-Product Property****Summary**

The *zero-product property* says that if the product of two or more factors is 0, then at least one of the factors is 0. This property can be used to determine the x -intercepts of a function or the solutions to quadratic equations.

Describe how to determine the x -intercepts of quadratic functions using the zero-product property. Use the examples if they help with your thinking.

$$f(x) = (x - 3)(x + 9)$$

$$g(x) = x^2 + 9x + 18$$

Describe how to solve quadratic equations using the zero-product property. Use the examples if they help with your thinking.

$$x^2 + 12x + 20 = 0$$

$$x^2 + 8x = 33$$

What are the similarities and differences between solving for the x -intercepts of a quadratic function and solving a quadratic equation?

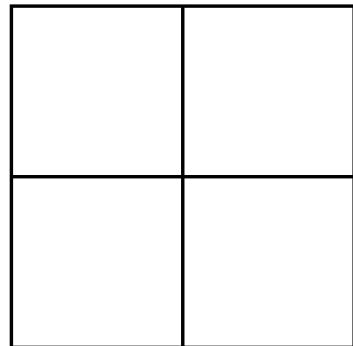
Things I Want to Remember

Lessons 5–6: Solving Quadratic Equations Using the Zero-Product Property**Try This!**

Determine the x -intercepts of the quadratic functions. Use the diagram if it helps with your thinking.

1.1 $h(x) = (x - 1)(x + 6)$

1.2 $k(x) = x^2 - 6x - 40$



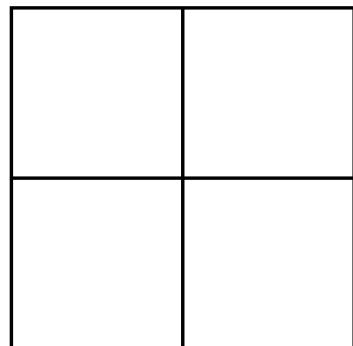
x -intercepts: _____

x -intercepts: _____

Determine the solutions to the quadratic equations. Use the diagram if it helps with your thinking.

2.1 $(5x - 3)(2x + 3) = 0$

2.2 $2x^2 - x = 21$



Solution: _____

Solution: _____

- I can determine the x -intercepts of quadratic functions written in factored form and standard form.
- I understand the zero-product property, which states that if the product of two or more factors is 0, then at least one of its factors is 0.
- I can use factoring and the zero-product property to solve quadratic equations.

**Lesson 7: Solving Equations by Reasoning****Summary**

The structure of a quadratic equation can help determine the number of solutions.

Here are examples of quadratic equations with no solutions, one solution, and two solutions. Write an additional example of each type of equation.

No Solution	One Solution	Two Solutions
$x^2 + 4 = 0$ $x^2 = -25$ $(x - 3)^2 + 1 = 0$	$(x + 4)^2 = 0$ $x^2 + 9 = 9$ $(x - 3)(x - 3) = 0$	$(x + 4)^2 = 1$ $x^2 - 12 = -3$ $(x - 3)(x - 3) = 1$

How can you determine if a quadratic equation has no solutions? One solution? Two solutions?

How can you check if your solution solves the equation? How can you avoid missing a solution?

Things I Want to Remember

Lesson 7: Solving Equations by Reasoning**Try This!**

Circle whether each equation has no solutions, one solution, or two solutions. Solve the equation if there are solutions.

1.1 $x^2 + 10 = 110$

No solutions One solution Two solutions

$x = \underline{\hspace{2cm}}$

1.2 $(x - 8)^2 = 0$

No solutions One solution Two solutions

$x = \underline{\hspace{2cm}}$

1.3 $x(x + 1) = 6$

No solutions One solution Two solutions

$x = \underline{\hspace{2cm}}$

1.4 $(x + 3)(x + 3) = -9$

No solution One solution Two solutions

$x = \underline{\hspace{2cm}}$

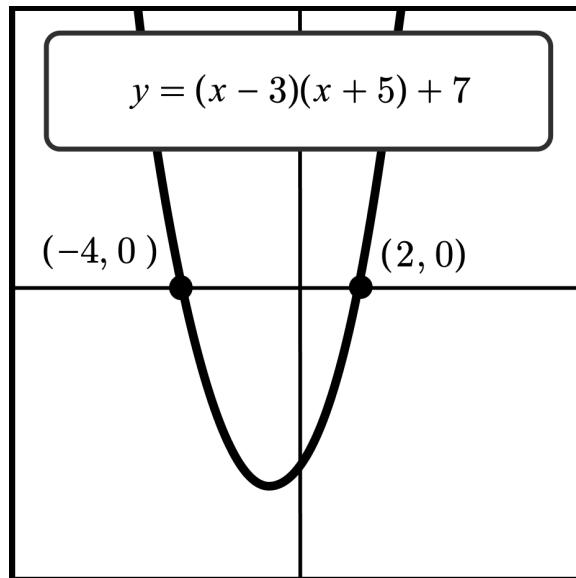
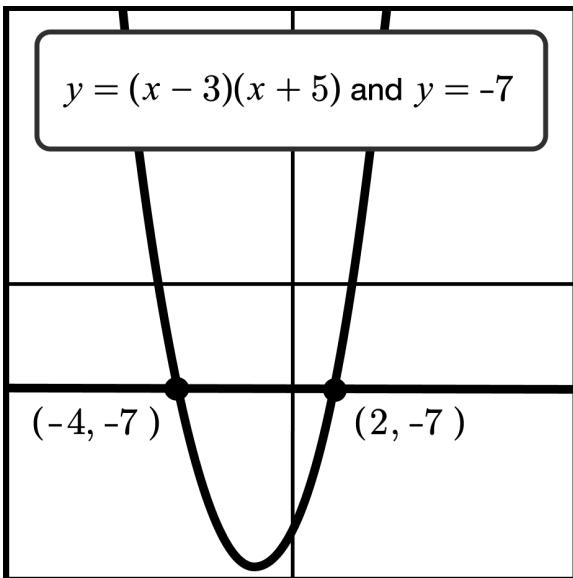
- | |
|--|
| <input type="checkbox"/> I can solve quadratic equations by reasoning. |
| <input type="checkbox"/> I can determine whether a quadratic equation has zero, one, or two solutions. |

Lesson 8: Solving Quadratic Equations by Graphing**Summary**

The graph of a quadratic equation can help determine the solutions.

Let's look at how a graph can help solve the equation $(x - 3)(x + 5) = -7$.

Identify the solutions and describe the strategy used for each approach.



Solutions:

Strategy:

Solutions:

Strategy:

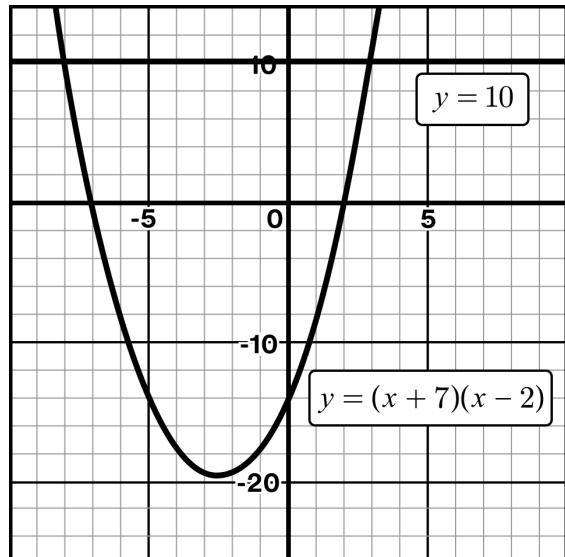
Things I Want to Remember

Lesson 8: Solving Quadratic Equations by Graphing

Try This!

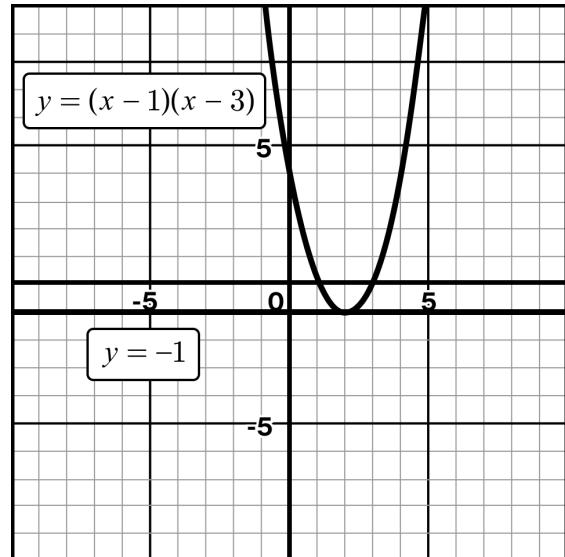
Here is the graph that Peter made to solve $(x + 7)(x - 2) = 10$.

- Determine the solution(s) to this equation.



Here is the graph that Peter made to solve $(x - 1)(x - 3) = -1$.

- Determine the solution(s) to this equation.



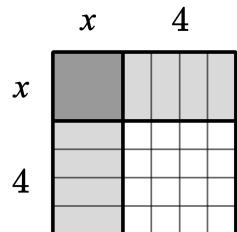
- I can solve quadratic equations by graphing.
- I can use graphs to determine whether a quadratic equation has zero, one, or two solutions.

Lessons 10–11: Solving by Completing the Square**Summary**

A quadratic expression is a *perfect square* if it can be represented as something multiplied by itself.

For example, $(x + 4)^2$ and $x^2 + 8x + 16$ are both perfect squares.

What are other examples of perfect square quadratic expressions?



Equations in the form $(x + \underline{\hspace{1cm}})^2 = \underline{\hspace{1cm}}$ can be solved by taking the square root. *Completing the square* is the process of rewriting a quadratic expression or equation to include a perfect square.

How do you know what constant value to add to make a perfect square?

Use the example if it helps with your thinking.

$$x^2 + 12x = 24$$

$$x^2 + 12x + 36 = 24 + 36$$

$$(x+6)^2 = 60$$

Solve the equation $x^2 - 18x = 10$ by completing the square.

Things I Want to Remember

**Unit A1.8, Quadratic Equations: Notes**

Name _____

Lessons 10–11: Solving by Completing the Square**Try This!**

Fill in the blanks to make each expression a perfect square.

1.1 $x^2 - 14x + \underline{\hspace{2cm}}$

1.2 $x^2 + \underline{\hspace{2cm}} + 81$

1.3 $x^2 + 6x + \underline{\hspace{2cm}}$

Solve the equations by completing the square.

2.1 $x^2 + 10x = 2$

2.2 $x^2 + 20x - 25 = 0$

$x = \underline{\hspace{2cm}}$

$x = \underline{\hspace{2cm}}$

- | |
|---|
| <input type="checkbox"/> I can justify whether a quadratic expression is a perfect square. |
| <input type="checkbox"/> I can determine missing values to make perfect square quadratic expressions. |
| <input type="checkbox"/> I can solve quadratic equations by completing the square. |

Lessons 13–15: Solving Quadratic Equations Using the Quadratic Formula**Summary**

The quadratic formula comes from completing the square.

The solutions to any quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

This formula is known as the quadratic formula.

The solutions to the equation $x^2 - 4x - 12 = 0$ are $x = -2$ and $x = 6$.

Use the quadratic formula to show that the solutions are correct.

You can write quadratic equations to represent a situation and solve problems about it.

For example, the function $h(t) = -1.5t^2 + 12t + 8$ represents the height, in meters, of a stomp rocket t seconds after it has been launched.

Explain why the equation $0 = -1.5t^2 + 12t + 8$ will help us determine when the rocket will touch the ground.

Xavier solved the equation and got the solutions $x \approx -0.619$ and $x \approx 8.619$. Do both of these solutions answer the question of when the rocket will hit the ground? Explain your thinking.

Things I Want to Remember

Lessons 13–15: Solving Quadratic Equations Using the Quadratic Formula

Try This!

Use the quadratic formula to solve the equations.

$$1.1 \quad 0 = x^2 + 6x + 3$$

$$a = \underline{\hspace{2cm}} b = \underline{\hspace{2cm}} c = \underline{\hspace{2cm}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$1.2 \quad 0 = 3x^2 - 5x + 2$$

$$a = \underline{\hspace{2cm}} b = \underline{\hspace{2cm}} c = \underline{\hspace{2cm}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The function $h(t) = -2.5t^2 + 6t - 8$ represents the height, in meters, of a stomp rocket t seconds after it has been launched.

- 2.1 Write an equation that can be solved to determine when the rocket will return to its original height of -8 meters.

- 2.2 How long will it take for the rocket to return to its original height of -8 meters?

- | |
|--|
| <input type="checkbox"/> I understand that the quadratic formula is based on the process of completing the square. |
| <input type="checkbox"/> I can use the quadratic formula to solve quadratic equations. |
| <input type="checkbox"/> I can write quadratic equations to solve problems about a situation. |
| <input type="checkbox"/> I can solve quadratic equations and explain what the solutions mean in a situation. |

Lesson 17: Solving Systems of Linear and Quadratic Equations**Summary**

A system of equations contains two or more equations, each of which represents different constraints on shared variables. A system can contain a variety of equations, including linear and quadratic equations.

Describe a strategy for solving a system.

Use the example if it helps with your thinking.

$$y = 2x - 7 \quad y = x^2 - 5x + 3$$

$$2x - 7 = x^2 - 5x + 3$$

$$0 = x^2 - 7x + 10$$

$$0 = (x - 5)(x - 2)$$

$$\boxed{x = 5}$$

$$\boxed{x = 2}$$

$$y = 2(5) - 7$$

$$\boxed{y = 3}$$

$$y = 2(2) - 7$$

$$\boxed{y = -3}$$

Points of intersection:

$$\boxed{(5, 3)}$$

$$\boxed{(2, -3)}$$

What are the similarities and differences between solving a system of linear equations and solving a system that includes a quadratic equation?

Things I Want to Remember

Lesson 17: Solving Systems of Linear and Quadratic Equations**Try This!**

Solve these systems of equations:

$$\begin{aligned} 1.1 \quad y &= 9x^2 - 7 \\ &y = -3 \end{aligned}$$

$$\begin{aligned} 1.2 \quad y &= x^2 - 3x \\ &y = 2x - 6 \end{aligned}$$

- I can solve systems of linear and quadratic equations.