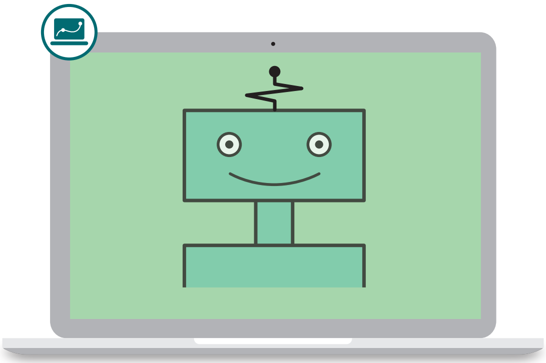
Unit 2

Lesson

4

Name: ______________ Date: ________ Period: ________



Robot Factory

Let's write equations for proportional relationships.

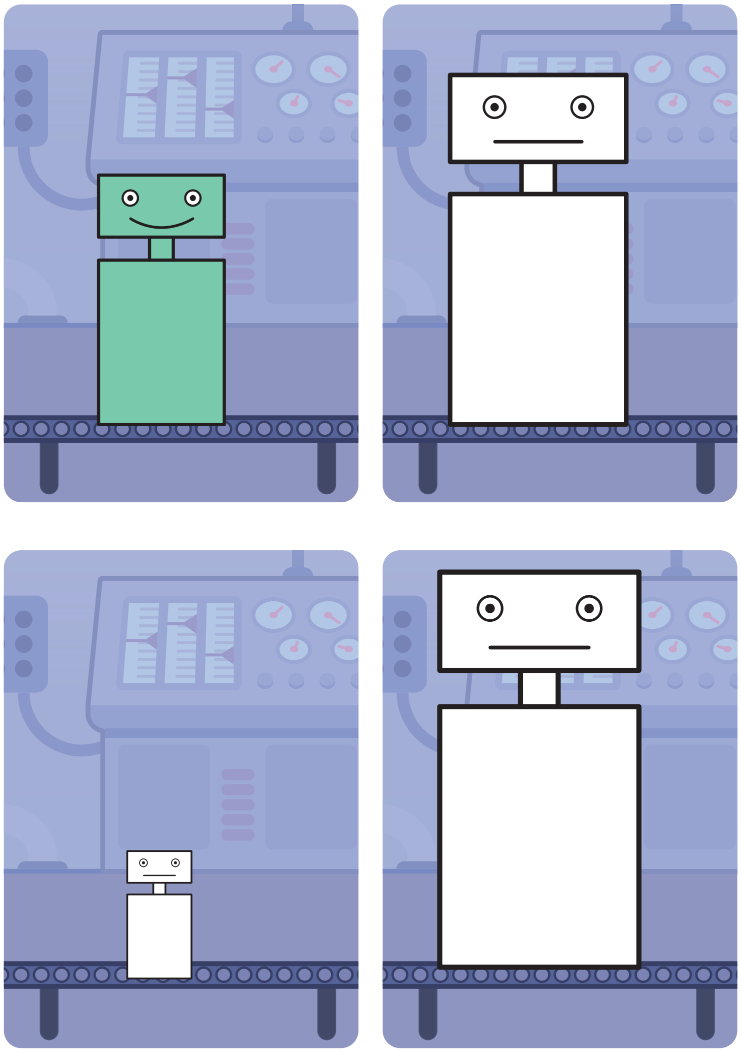
Warm-Up

1-2 The robot factory can make robots of all different sizes.

This robot's shade of green is made by mixing green and white paint using the amounts in the table.

Complete the table so that all four robots have the same shade of green.

Green Paint (cups)	White Paint (cups)
4	3
8	
1	
10	



Activity 1

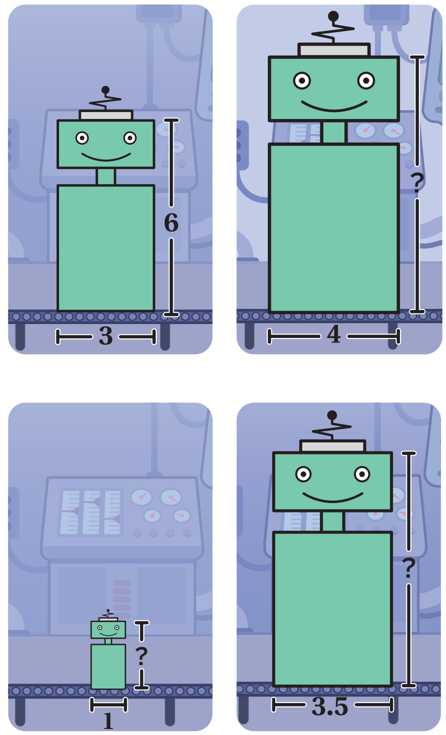
Name: ______________ Date: ________ Period: ________

Robot Hats

3-4 Let's make 30 *scaled copies* of this robot.

The width of this robot is 3 inches. Its hat is 6 inches off the ground.

Complete the table with the height for placing the hat on each robot.

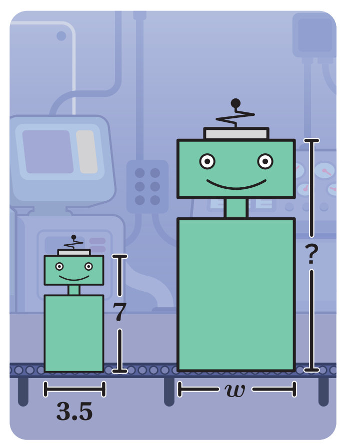


Robot Width (in.)	Height for Placing Hat (in.)
3	6
4	
1	
3.5	
2.18	
1.76	
3.425	

5 Here is a student's work from the previous problem.

Robot Width (in.)	Height for Placing Hat (in.)
3.5	3.5 • 2
2.18	2.18 • 2
1.76	1.76 • 2

Help someone know how to finish the job. For any robot's width, how could you determine the height for placing its hat?



6 Instructions with words are useful for humans, but machines understand mainly numbers and symbols.

Which equation tells the factory the relationship between a robot's width, w, and the height for placing its hat, h?

A. $h = \frac{1}{2}w$ **B.** $h = w + 3$ **C.** $w = 2h$ **D.** $h = 2w$

Activity 2

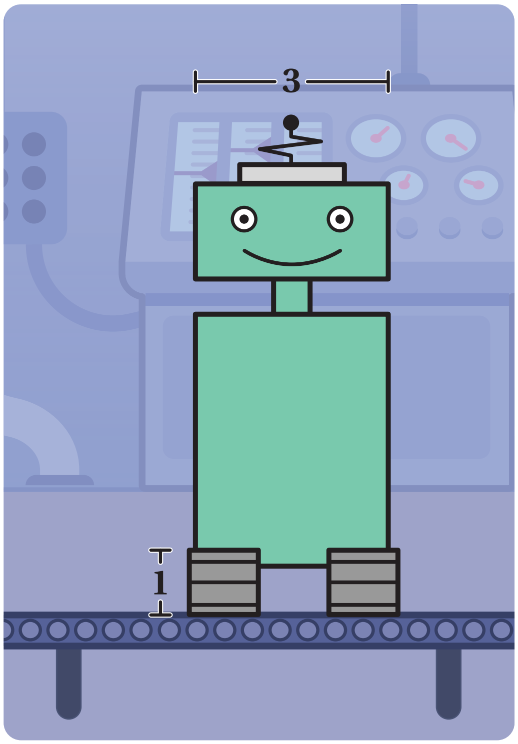
Name: ______ Date: ______ Period: ______

More Robot Parts

7-8 This robot is 3 inches wide. Its shoes are 1 inch tall.

Enter the shoe height for each robot.

Robot Width (in.), w	Shoe Height (in.), s
3	1
6	
5	
1	

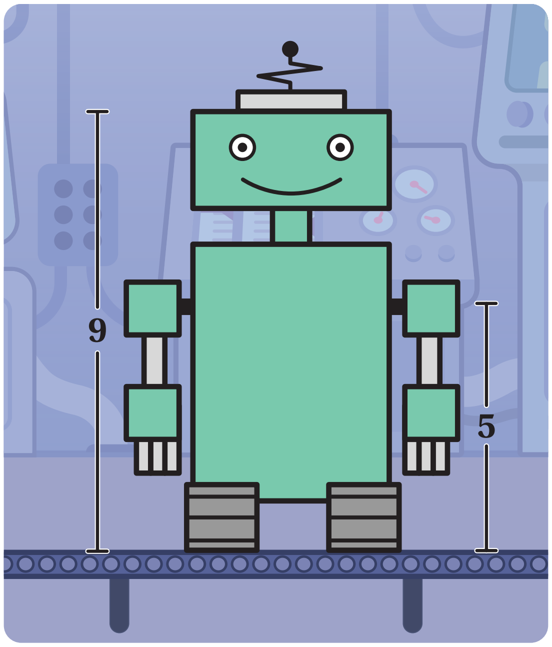


9 Write an equation the factory could use to put shoes on the rest of the robots.

Use s for the shoe height and w for the robot's width.

$s =$ ______

10-11 This robot has a height of 9 inches. Its arms are 5 inches off the ground.



Write an equation that the factory could use to proportionally attach arms to the robots.

Use a for the height to place the arms and r for the robot height.

$a =$ ______

Use this table if it helps with your thinking.

Robot Height (in.), r	Height for Placing Arms (in.), a
9	5
5	2.78
1	0.56

12 Synthesis

Describe a strategy for writing an equation of a proportional relationship when given a table.

Use one or both of the examples if they help with your thinking.

Hat

Robot Width (in.), w	Hat Height (in.), h
3	6
1	2

The constant proportionally is 2.

An equation for this relationship is $h = 2w$.

Arms

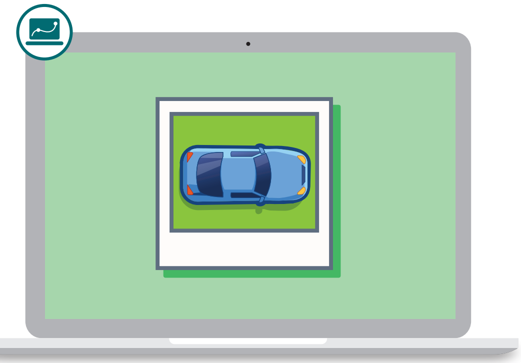
Robot Width (in.), r	Arm Height (in.), a
9	5
1	$\frac{5}{9}$

The constant proportionally is $\frac{5}{9}$.

An equation for this relationship is $a = \frac{5}{9}r$.

Things to Remember:

Name: ________ Date: ________ Period: ________

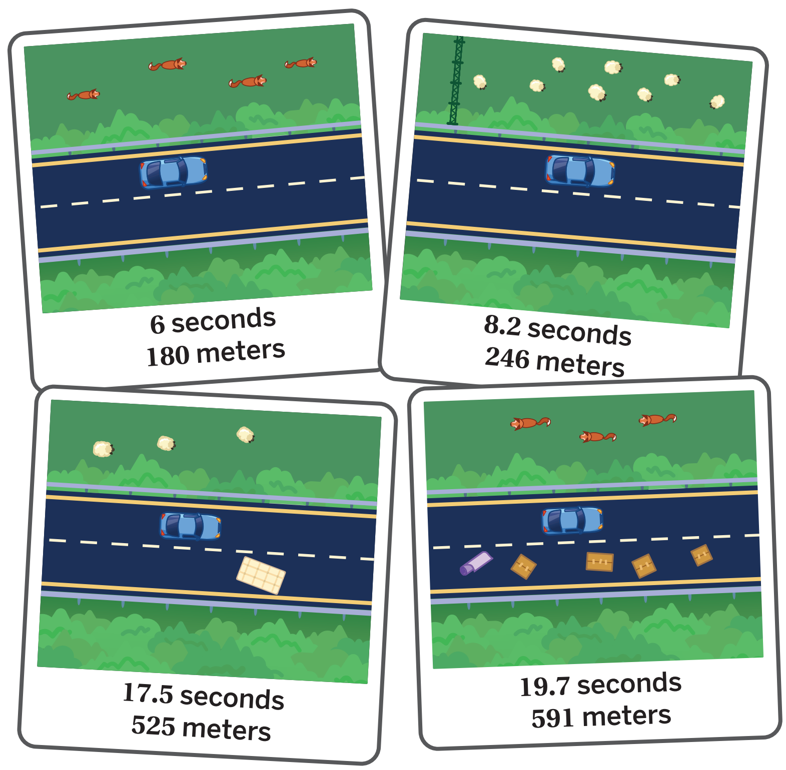


Snapshots

Let's use equations to make sense of proportional relationships in the world.

Warm-Up

1 Write a story about this car's trip.

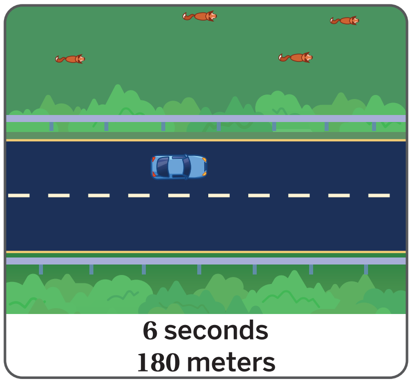


Activity 1

Name: .... Date: .... Period: ....

Travel Times

2 The car travels at a constant speed. After 6 seconds, it travels 180 meters.



a Write an equation for the car's distance, d, at any time, t.

$d =$ ....

b Enter the time and distance for three moments during the car's trip.

Time (sec), t	Distance (m), d

3 Ethan says that a row in a table is like a picture and an equation is like a video. Explain what Ethan might be thinking.

4 Use the equation you wrote to complete the table.

Time (sec), t	Distance (m), d
6	180
3	
	60

Activity 2

Name: ______ Date: ______ Period: ______

Cakes

5 A cake recipe uses the equation $m = 6c$, where c is the number of cakes and m is ounces of milk.

Explain what the constant of proportionality means in this situation.

6 How many ounces of milk are needed to bake 12 cakes?

7 A cake recipe says to use 3 cups of flour for every 2 cakes.

2 cakes
3 cups of flour

a **Discuss:** How much flour do you need for 1 cake?

b Write an equation to calculate the amount of flour needed, f, for any number of cakes, c.

$f =$ ______

Name: .... Date: .... Period: ....

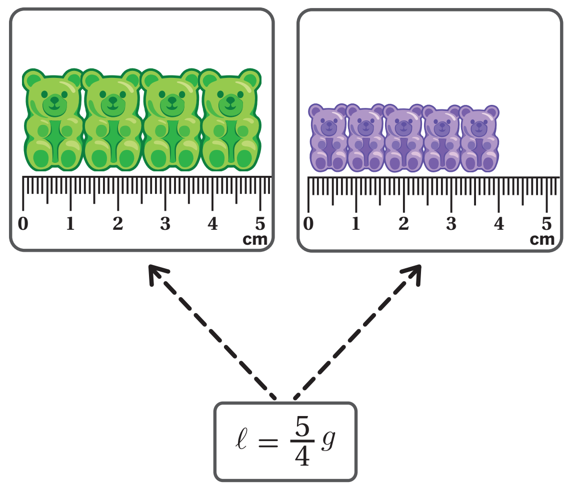
Gummy Bears

8 Here is an equation: $\ell = \frac{5}{4}g$. ℓ is the total length and g is the number of gummy bears.

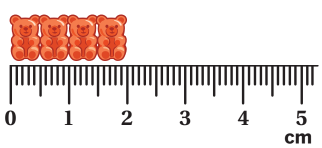
Which size gummy bear does the equation represent? Circle one.

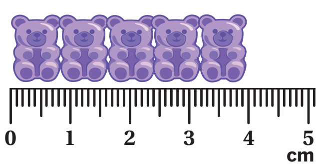
Large Small Both Neither

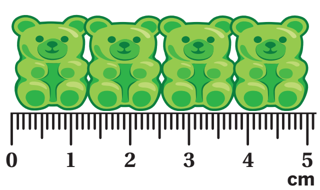
Explain your thinking.



9 Match each image with one or more equations or descriptions. In each equation, ℓ is the total length and g is the number of gummy bears.

a. 

b. 

c. 

.... $\ell = \frac{4}{5}g$

.... $\ell = \frac{1}{2}g$

.... $\ell = \frac{5}{4}g$

.... A line of 80 of these gummy bears is 100 centimeters long.

Explore More

10 Use the Explore More Sheet to answer a question about a truck's trip.

11 Synthesis

Here is a snapshot from a situation represented by the equation $p = 8g$, where p represents a number of pints and g represents a number of gallons.

How can an equation of a proportional relationship represent a situation? Use the example if it helps you explain your thinking.

Things to Remember:

Name: .... Date: .... Period: ....

Explore More

Here are some facts about this truck:

- It travels at an average rate of 50 miles per hour.
- It can travel 6 miles for each gallon of gas.

How many hours can the truck travel without stopping if it has a full tank of 150 gallons?

Explain your thinking.

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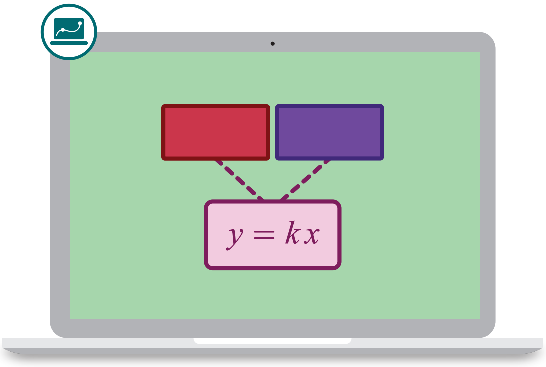
Name: ____________ Date: ________ Period: ________

All Kinds of Equations

Let's use equations to decide whether a relationship is proportional.



Warm-Up

1 This table represents the equation $y = 3x + 1$.

a Use the equation to complete the table.

b Does the equation represent a proportional relationship? Circle one.

Yes No Not enough information

Explain your thinking.

x	y
0	1
1	4
3	10
	16
2.5	

Name: ______ Date: ______ Period: ______

Stories, Equations, Tables

2 Use the story and equation to complete the table. Then decide whether the relationship is proportional. Complete either Story 1 or Story 2.

Story 1

Story: Trinidad earns \$12 per hour.

Equation: $y = 12x$

Is the relationship proportional? Circle one.

Yes No

Explain your thinking.

Time Worked (hr), x	Pay (\$), y
0	
1	
	30

Story 2

Story: A recipe recommends 1 banana for every 2 smoothies.

Equation: $y = \frac{1}{2}x$

Is the relationship proportional? Circle one.

Yes No

Explain your thinking.

Number of Smoothies, x	Number of Bananas, y
0	0
1	
	2.5

Find a partner who completed the other story.

Discuss: How are your responses alike? How are they different?

Name: .... Date: .... Period: ....

Stories, Equations, Tables (continued)

Complete either Story 3 or Story 4.

Story 3

Story: A cell phone costs \$500, plus \$35 per month for the plan.

Equation: $y = 500 + 35x$

Is the relationship proportional? Circle one.

Yes No

Explain your thinking.

Number of Months, x	Total Cost (\$), y
0	
1	
	605

Story 4

Story: The area of a square is the side length multiplied by itself.

Equation: $y = x^2$

Is the relationship proportional? Circle one.

Yes No

Explain your thinking.

Side Length (units), x	Area (sq. units), y
0	
1	
	100

3 Here are the equations that represent the four stories.

a Select *all* the equations that represent a proportional relationship.

- ☐ **A.** $y = 12x$
- ☐ **B.** $y = 500 + 35x$
- ☐ **C.** $y = \frac{1}{2}x$
- ☐ **D.** $y = x^2$

b Explain one way to decide if an equation represents a proportional relationship.

Activity 2

Name: ______ Date: ______ Period: ______

Equations and Proportionality

4 Decide whether each equation, table, or story represents a proportional relationship by placing a checkmark in the appropriate column.

	Proportional	Not Proportional
$4 + x = y$		
$y = 4x$		
Jacy walked 4 miles in 100 minutes at a steady pace.		
$0.04x = y$		
$y = \frac{x}{4}$		
$\frac{4}{x} = y$		
<table><tr><th>x</th><th>y</th></tr><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>2</td></tr><tr><td>3</td><td>$\frac{4}{3}$</td></tr><tr><td>4</td><td>1</td></tr></table>		

5 Maki thinks $\frac{4}{x} = y$ is proportional. Karima thinks $y = \frac{x}{4}$ is proportional. Whose thinking is correct? Circle one.

Proportional

$\frac{4}{x} = y$ $y = \frac{x}{4}$

Maki's Karima's Both Neither

Explain your thinking.

Explore More

6 Use the Explore More Sheet to answer questions about a relationship and proportionality.

7 Synthesis

a. Write two equations: one that represents a proportional relationship and one that does not.

Proportional Relationship	Not a Proportional Relationship

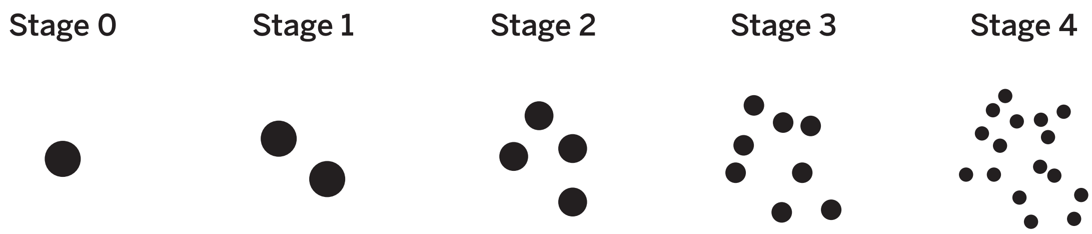
b. Describe how you know whether an equation represents a proportional relationship.

Things to Remember:

Name: ______________ Date: ______________ Period: ______________

Explore More

Observe how the number of circles changes at each stage.



a Complete the table.

Stage	Number of Circles
0	1
1	
2	
3	
4	

b Does this represent a proportional relationship? Circle one.

Yes No

Explain your thinking.

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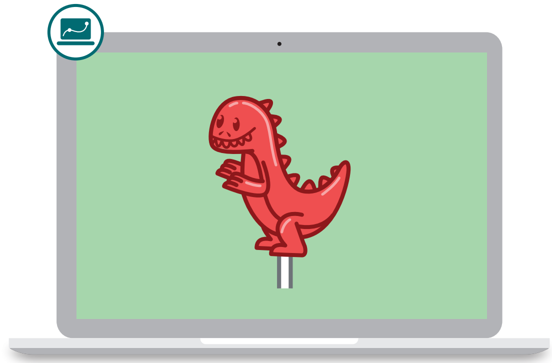


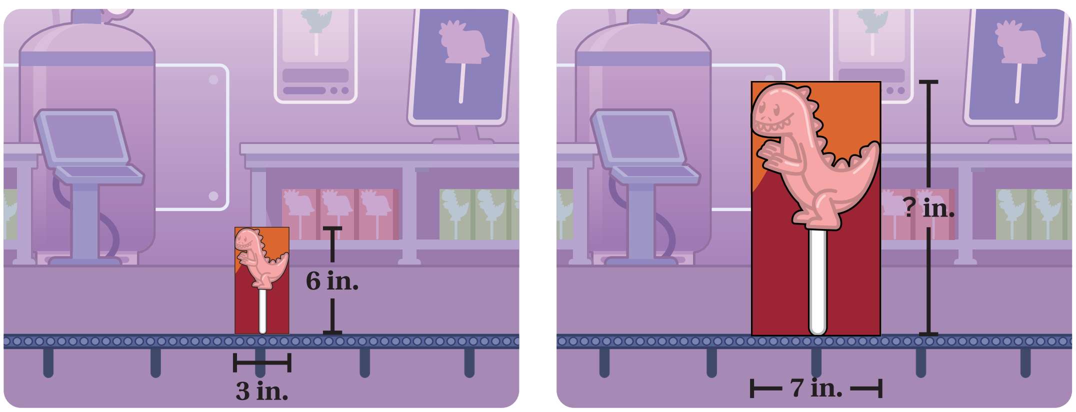
DinoPops

Let's explore what a proportional relationship looks like on a graph.

Warm-Up

1 Here are two DinoPops in their boxes.

DinoPops come in all sizes between 2 and 200 inches tall. They are always scaled copies of one another.



Complete the table to determine the height of the box for the large DinoPop.

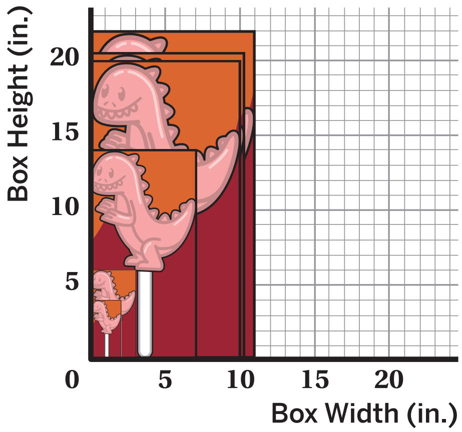
Box Width (in.)	Box Height (in.)
3	6
7	

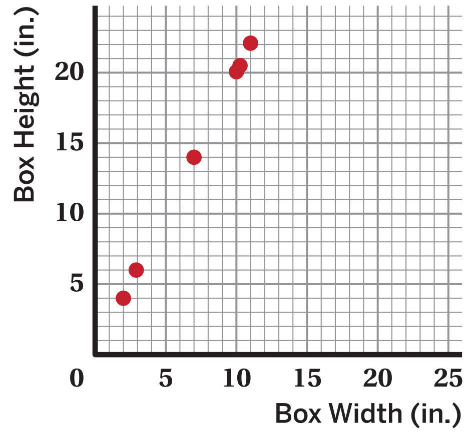
Activity 1

Name: ____________ Date: ____________ Period: ____________

DinoPops

2 Here are some DinoPop boxes and a graph of some points.





What do you notice? What do you wonder?

I notice:

I wonder:

3 A box that is 5 inches wide and 10 inches tall is a perfect fit for a DinoPop. A box that is 7 inches wide and 14 inches tall is also a perfect fit.

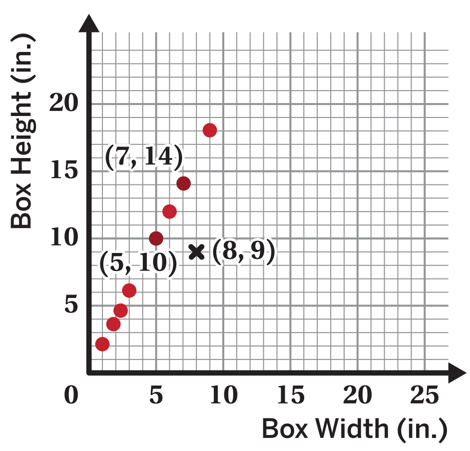
Write ordered pairs for two other boxes that are a perfect fit for a DinoPop.

(____, ____) (____, ____)

A box that is 8 inches wide and 9 inches tall is *not* a perfect fit for a DinoPop.

Write ordered pairs for two other boxes that are *not* a perfect fit for a DinoPop.

(____, ____) (____, ____)



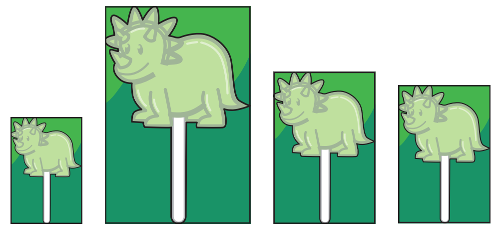
Activity 2

Name: ______________ Date: ______ Period: ______

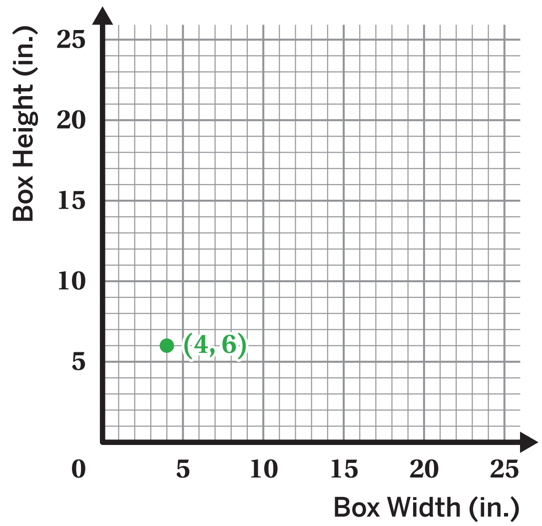
TriceraPops

4-5 Here are several TriceraPops in boxes.

The graphed point represents *one* of these boxes.



a **Discuss:** What do you know about this box?



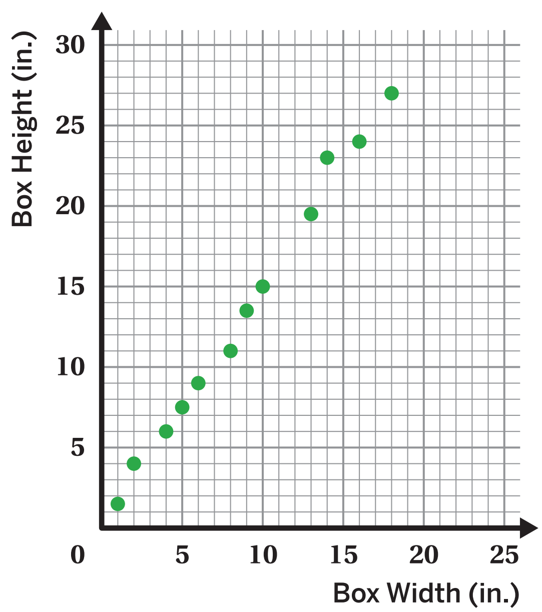
b Add at *least two* more points to the graph to represent other boxes that fit a TriceraPop.

6 A student made some TriceraPop boxes, as shown on this graph.

Some of the boxes were not a good fit.

Describe how to use the graph to find the bad boxes.

Draw on the graph if it helps to show your strategy.



Activity 2

Name: ______ Date: ______ Period: ______

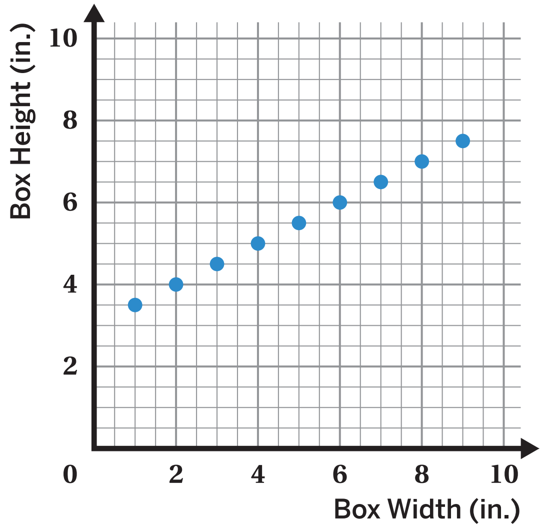
TriceraPops (continued)

7 Here is a graph of box sizes for a new lollipop.

Is there a proportional relationship between the height and the width of these boxes? Circle one.

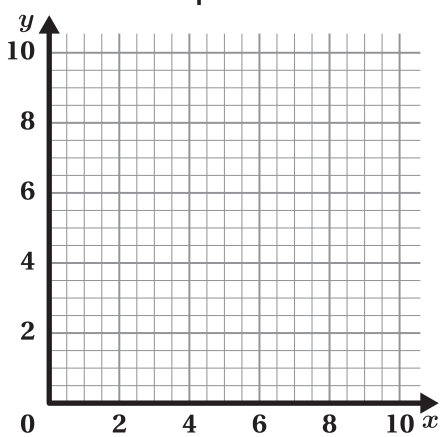
Yes No I'm not sure

Explain your thinking.

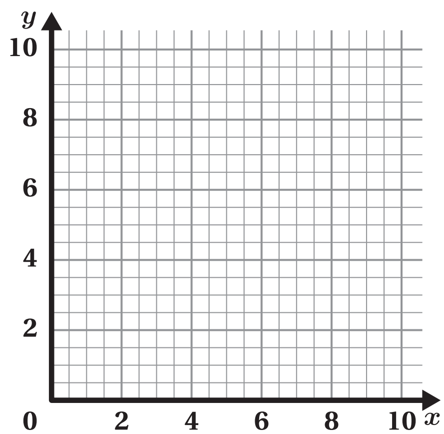


8 Make one graph that represents a proportional relationship and one graph that does not.

Proportional



Not Proportional

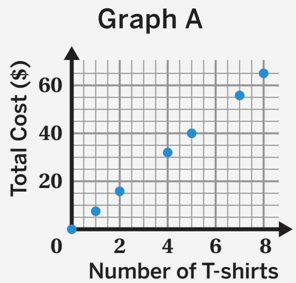


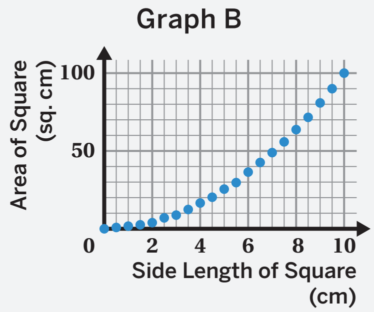
Activity 3

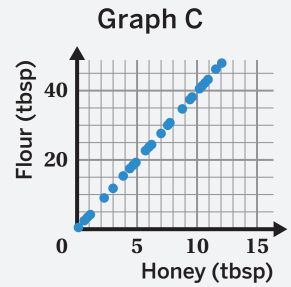
Name: .... Date: .... Period: ....

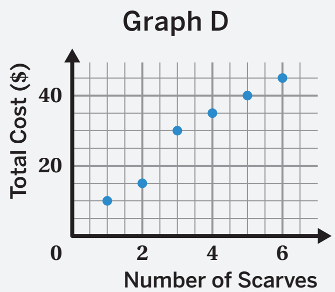
Graphs

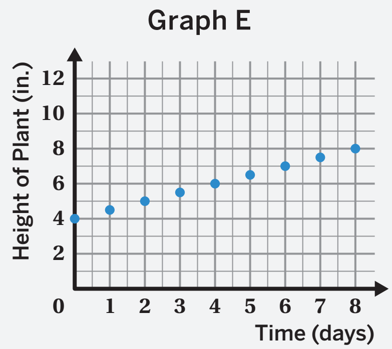
9 Decide whether each graph represents a proportional relationship.









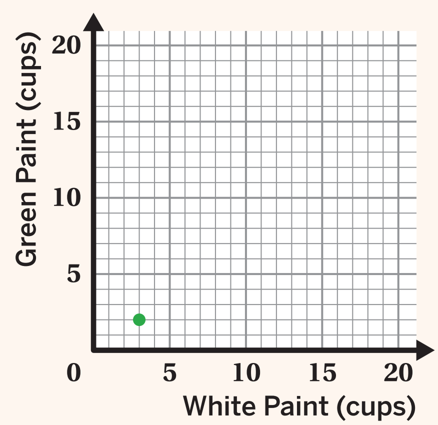


Proportional	Not Proportional

Explore More

10 This color green is made by mixing 3 cups of white paint and 2 cups of green paint.

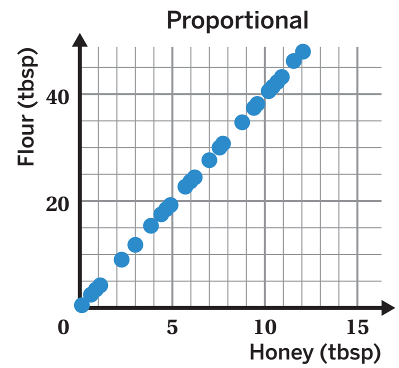
Graph points that represent *at least* two more ways to make this color.

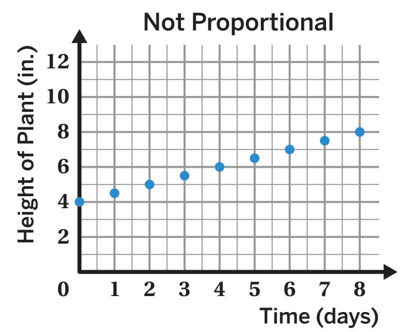


11 Synthesis

How can you use a graph to decide whether a relationship is proportional?

Use the examples if they help with your thinking.





Things to Remember:

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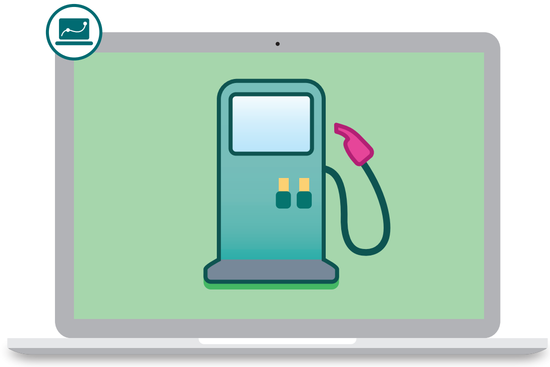


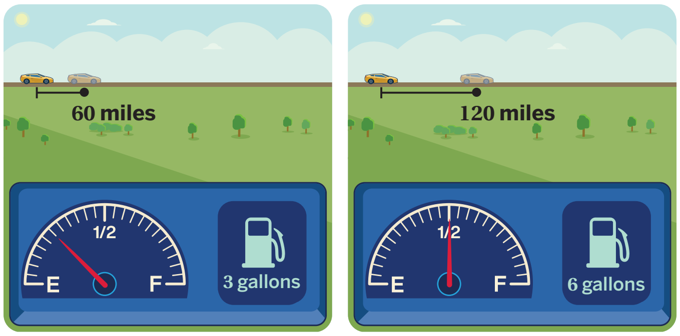
Gallon Challenge

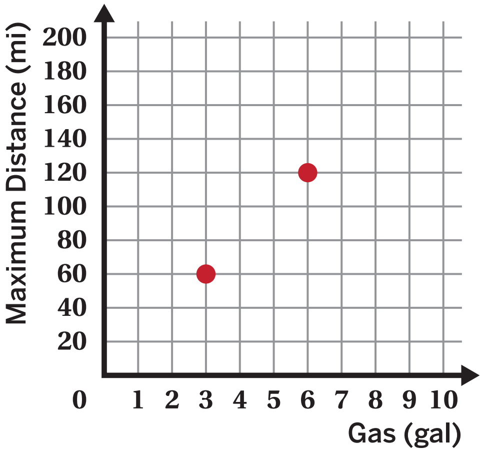
Let's identify constants of proportionality using a graph.

Warm-Up

1 Here are two images showing the amount of gas in a car's tank.

The graph represents the maximum distance the car can go using each amount of gas.





Discuss: What do you notice? What do you wonder?

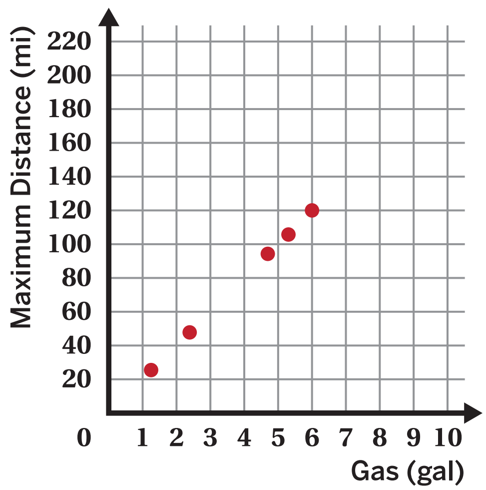
Activity 1

Name: ________ Date: ________ Period: ________

On the Road

2 Here are several points representing the maximum distance the car can go using different amounts of gas.

What would the graph look like if it included every possible point for this car?



3 The car can travel 240 miles with a full 12-gallon tank of gas. Which line represents this relationship? Circle one.

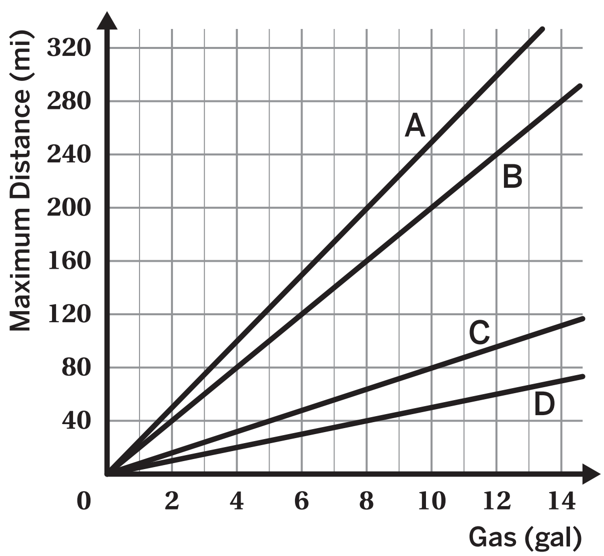
A B C D

4 A car's *gas mileage* is the maximum distance it can go using 1 gallon of gas (measured in miles per gallon).

Based on the graph, what is this car's gas mileage?

________ miles per gallon

Where do you see this number in the graph?



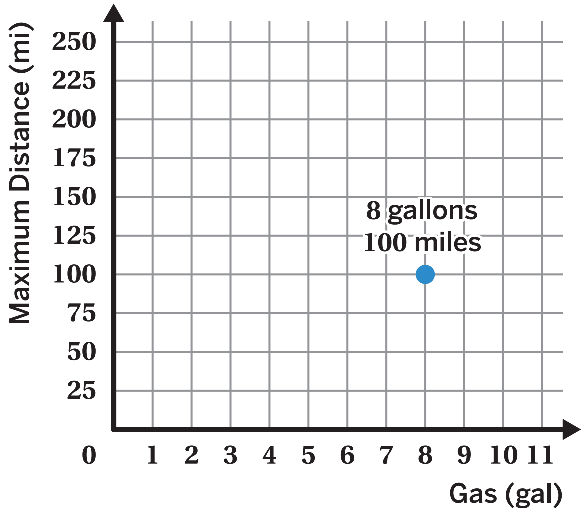
Activity
2

Name: .... Date: .... Period: ....

Gas Mileage

5 Yasmine's truck travels 100 miles using 8 gallons of gas.

What is the gas mileage for her truck?

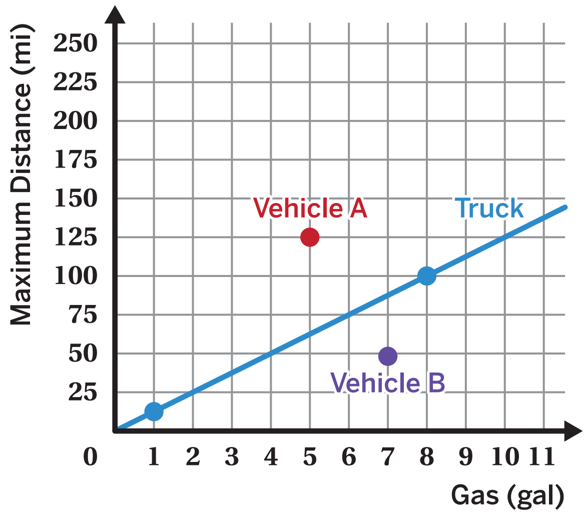


6 Yasmine wants to buy a new vehicle that gets better gas mileage than her truck.

Which vehicle should she pick?
Circle one.

Vehicle A   Vehicle B   Either Vehicle A or Vehicle B

Explain your thinking.



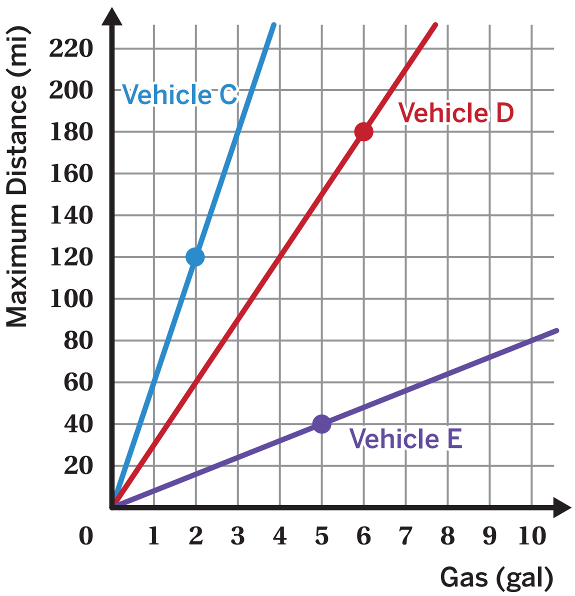
Activity 2

Name: ________ Date: ________ Period: ________

Gas Mileage (continued)

7 In the relationship between amount of gas and maximum distance, the vehicle's gas mileage is a *constant of proportionality*.

What is that constant of proportionality for each vehicle?



Vehicle	Constant of Proportionality (mi per gal)
C	
D	
E	

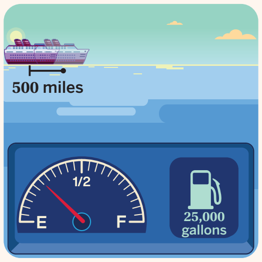
Explore More

8 Here is the maximum distance traveled by three vehicles using a certain amount of gas.

Plane

Note: The average plane has 100 passengers.

Cruise Ship



Note: The average cruise ship has 3,000 passengers.

Train

Note: The average train has 300 passengers.

Compare each vehicle's environmental impact to the impact of driving a car.

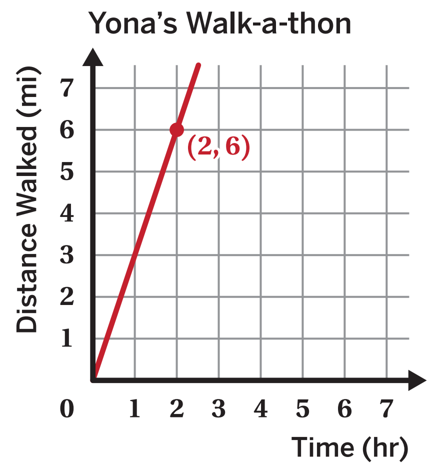
9 Synthesis

What are *two* different ways you can find a constant of proportionality using a graph?

Use the example if it helps with your thinking.

First method:

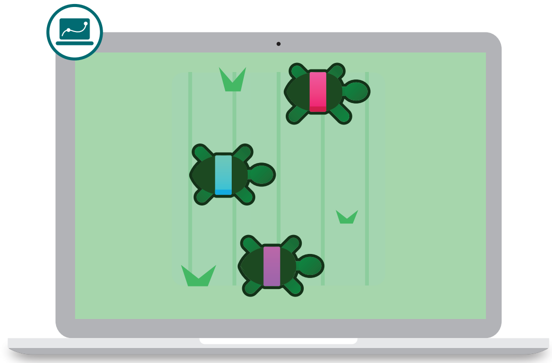
Second method:



Things to Remember:

Unit 2
Lesson
10

Name: .... Date: .... Period: ....



Three Turtles

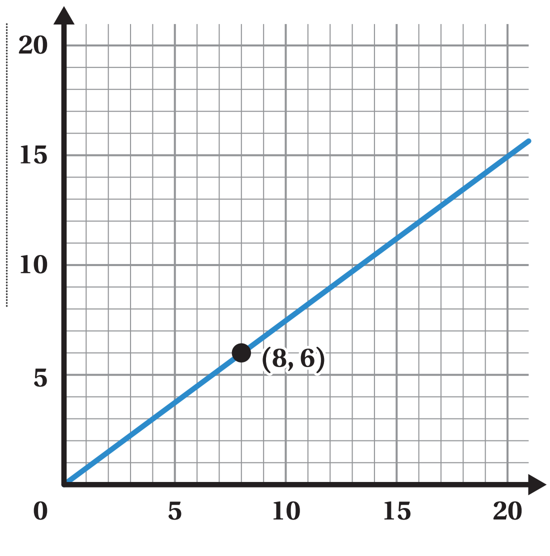
Let's compare proportional relationships.

Warm-Up

1 Here is a graph that represents a proportional relationship.

a Label the axes with any quantities you'd like.

b Write a true statement about the quantities based on the graph.

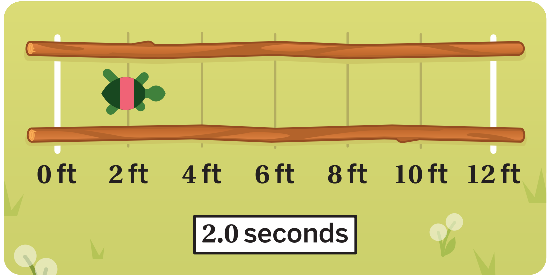


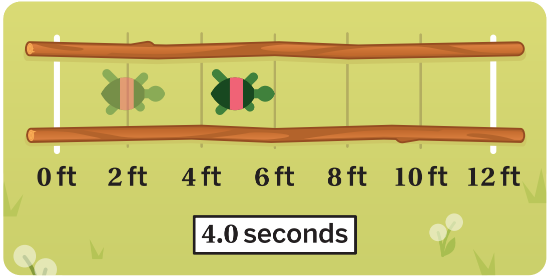
Activity
1

Name: .... Date: .... Period: ....

Traveling Turtles

2 This turtle walks at a constant rate. The turtle's distance is measured at the front of its head.





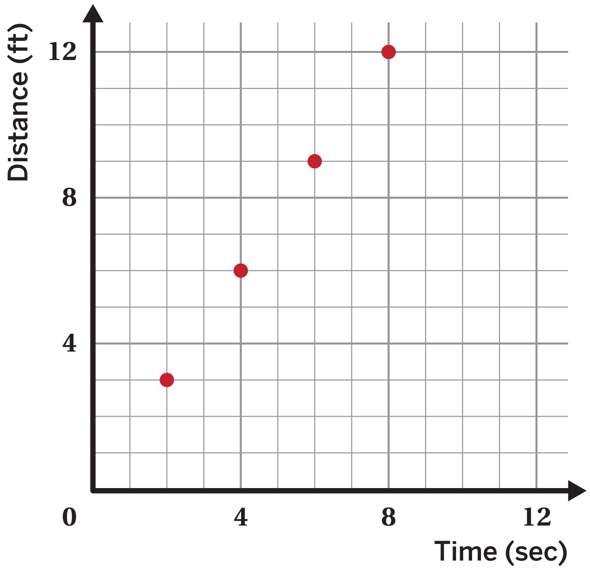
a Complete the table.

b What is a constant of proportionality for this relationship?

Time (sec)	Distance (ft)
2	3
4	6
6	
8	

3 This graph shows the points from the table in the previous problem.

Write an equation for this relationship, using d for distance and t for time.

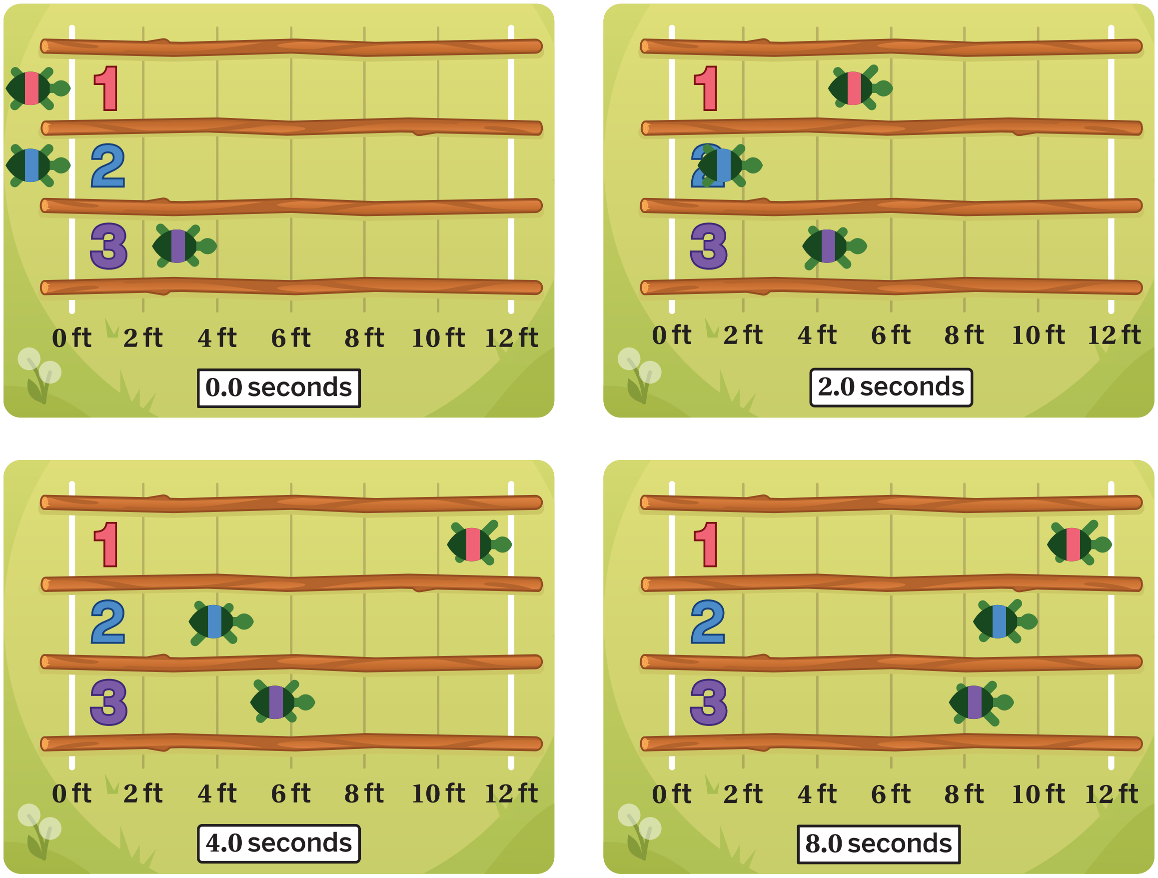


Activity 2

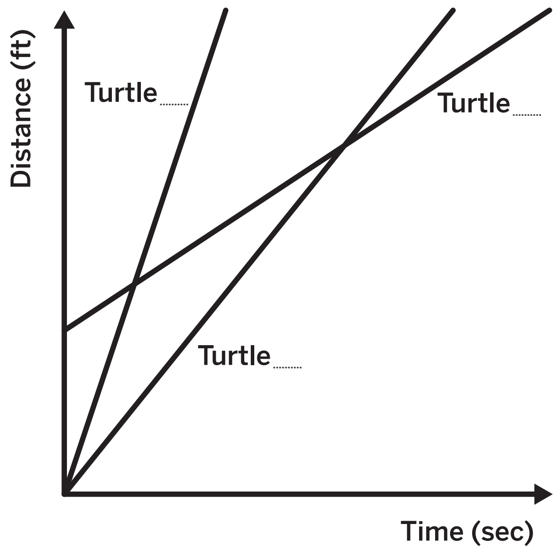
Name: ______ Date: ______ Period: ______

Three Turtles

4 These images show three turtles walking. Each turtle walks at a constant rate.



Label each line with the turtle it represents: Turtle 1, Turtle 2, or Turtle 3.



5 Match each equation to its graph.

a. $d = 3t$ ______ Turtle 1

b. $d = \frac{2}{3}t + 4$ ______ Turtle 2

c. $d = 1.25t$ ______ Turtle 3

Explain your thinking.

Activity 3

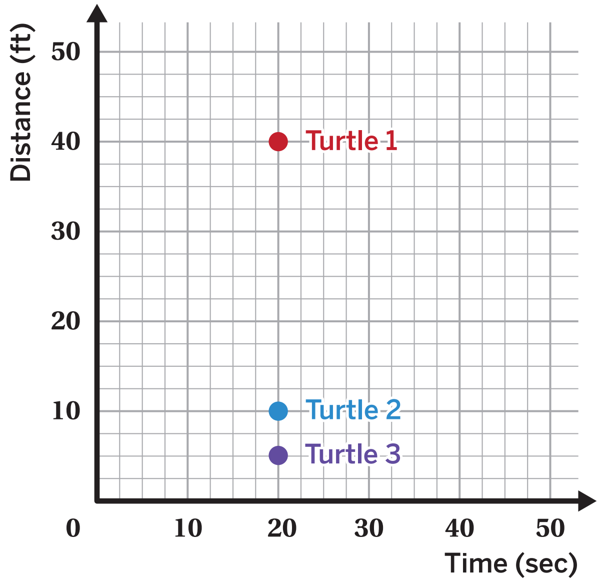
Name: ______ Date: ______ Period: ______

Turtle Challenges

6 Here are three new turtles and their distances at 20 seconds.

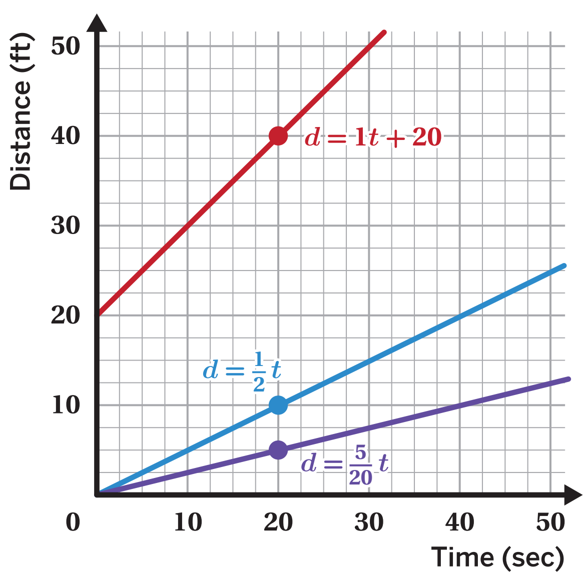
- Turtle 1 is 40 feet from the start.
- Turtle 2 is 10 feet from the start.
- Turtle 3 is 5 feet from the start.

Write an equation for each turtle, using d for distance and t for time. (One equation has been written for you.)



Turtle	Equation
Turtle 1	
Turtle 2	$d = \frac{1}{2}t$
Turtle 3	

7 Irelle wrote three equations for the new turtles.



a Which of these relationships is *not* proportional?

A. $d = 1t + 20$

B. $d = \frac{1}{2}t$

C. $d = \frac{5}{20}t$

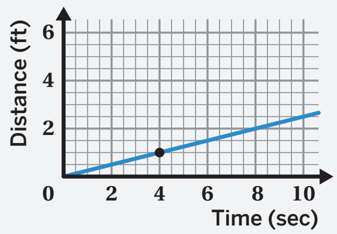
b Describe this turtle's race.

Name: ________ Date: ________ Period: ________

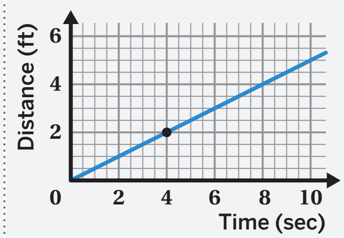
Turtle Challenges (continued)

8 Sort these cards into three groups that each represent the same turtle.

Card A



Card B



Card C

Time (sec)	Turtle Distance (ft)
0	0
1	4
2	8
4	16
8	32

Card D

Time (sec)	Turtle Distance (ft)
0	0
1	$\frac{1}{2}$
2	1
4	2
8	4

Card E

At 8 seconds, the turtle's distance is 2 feet.

Card F

At 2 seconds, the turtle's distance is 8 feet.

Card G

$d = 4t$

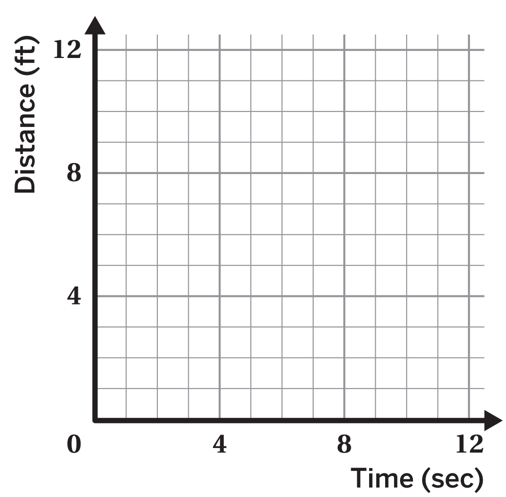
Card H

$d = \frac{1}{4}t$

Group 1	Group 2	Group 3

9 Create your own turtle race by sketching 3 lines that represent 3 different turtles. Your turtle race must include *at least* two of the following features:

- ☐ A turtle that stays still.
- ☐ A turtle that has a head start.
- ☐ Two turtles that finish at the exact same time.
- ☐ A turtle that travels backward.
- ☐ Two turtles that travel at the same pace.



Explore More

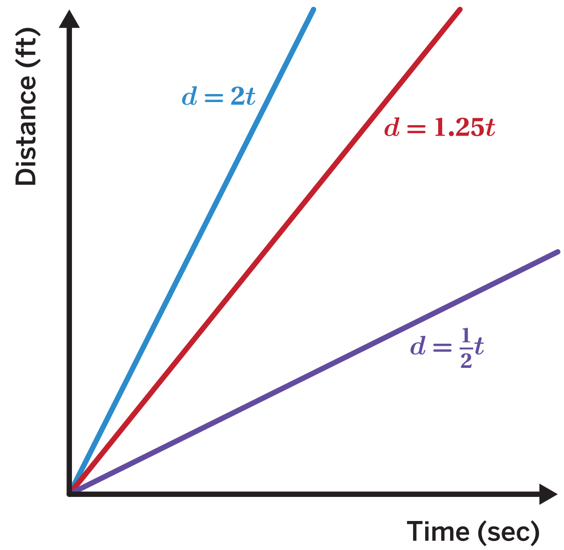
10 Use the Explore More Sheet to explore another turtle race.

11 Synthesis

This graph shows the distance traveled over time by three different turtles.

Discuss both questions. Then select *one* and write your response.

- ☐ How can you tell from the graphs which turtle moved the fastest?
- ☐ How can you tell from the equations which turtle moved the fastest?



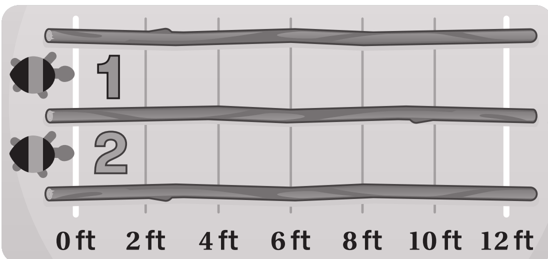
Things to Remember:

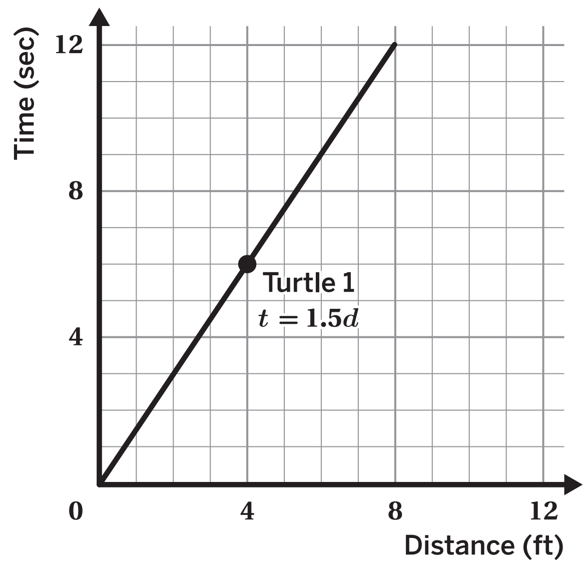
Name: ______________________ Date: __________ Period: __________

Explore More

The graph shows the line for Turtle 1.

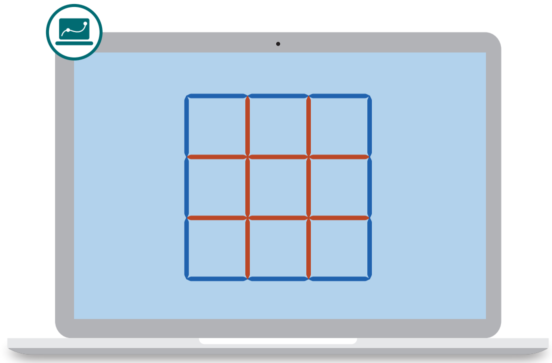
Draw a line for Turtle 2 so that it's slower than Turtle 1. Notice the axis labels!





Unit 3
Lesson
1

Name: ________ Date: ________ Period: ________

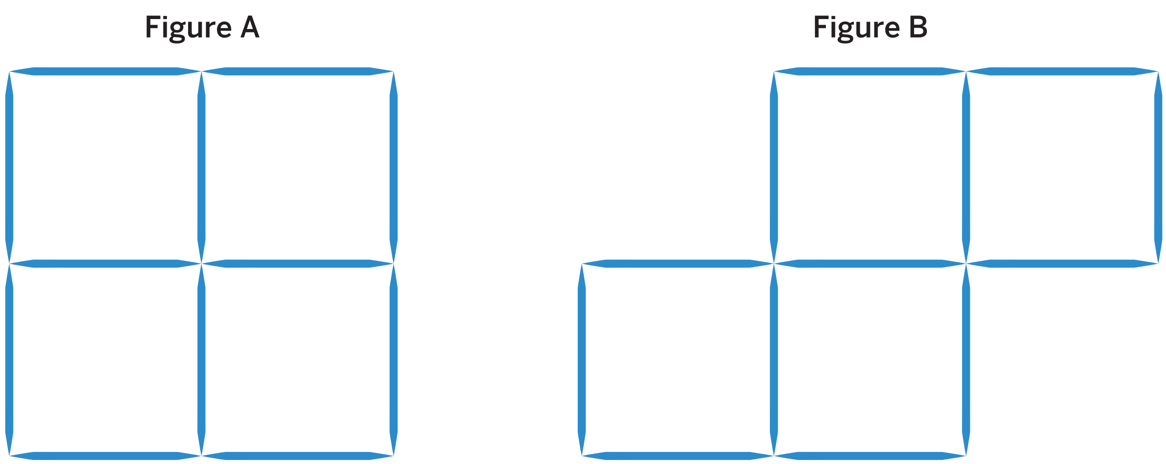


Toothpicks

Let's examine relationships between side length, diagonal length, and the perimeter of a square.

Warm-Up

1 These two figures were built with toothpicks.



Which figure used more toothpicks? Circle one.

Figure A   Figure B   They used the same

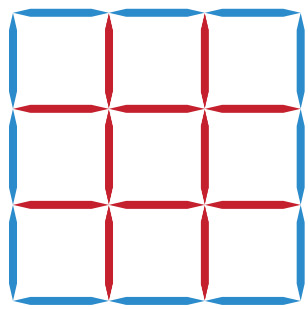
Explain your thinking.

Activity 1

Name: .... Date: .... Period: ....

Proportional or Not?

2 To determine the number of toothpicks needed to build this figure, we count the blue perimeter toothpicks and the red interior toothpicks.

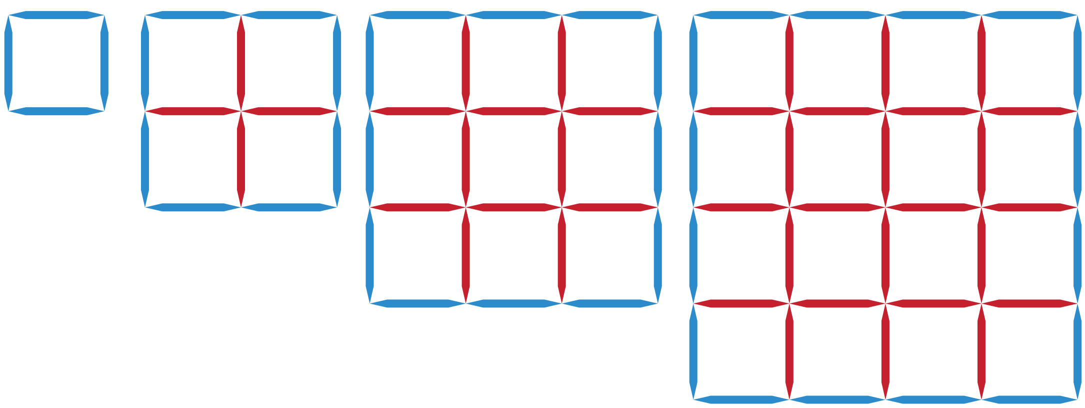


Write the number of each type of toothpick.

Perimeter (blue toothpicks)	Interior (red toothpicks)

3 Let's explore how changing the size of the square changes the number of toothpicks.

a Take a look at this square and its scaled copies.



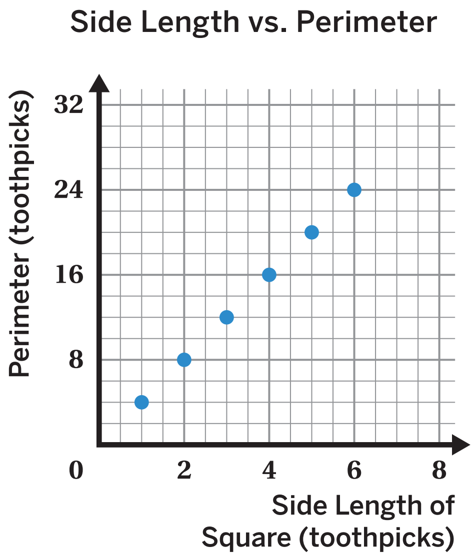
b Write the number of each type of toothpick needed to build each square.

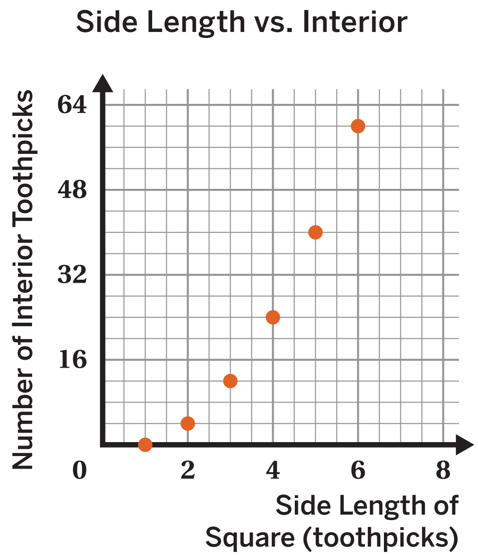
Side Length (toothpicks)	Perimeter (blue toothpicks)	Interior (red toothpicks)
1		
2		
3		
4		

Name: ______ Date: ______ Period: ______

Proportional or Not? (continued)

4 Here are two graphs based on the toothpick squares from the previous problem.





Use the graphs to decide which relationships are proportional. Circle one.

A. Side length of a square vs. perimeter

B. Side length of a square vs. number of interior toothpicks

C. Both

D. Neither

Explain your thinking.

5 **a** For any relationship you said was proportional, what is its constant of proportionality?

b What does the constant of proportionality mean in this context?

Explore More

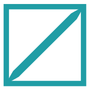
6 Use the Explore More Sheet to further explore these toothpick relationships.

Activity 2

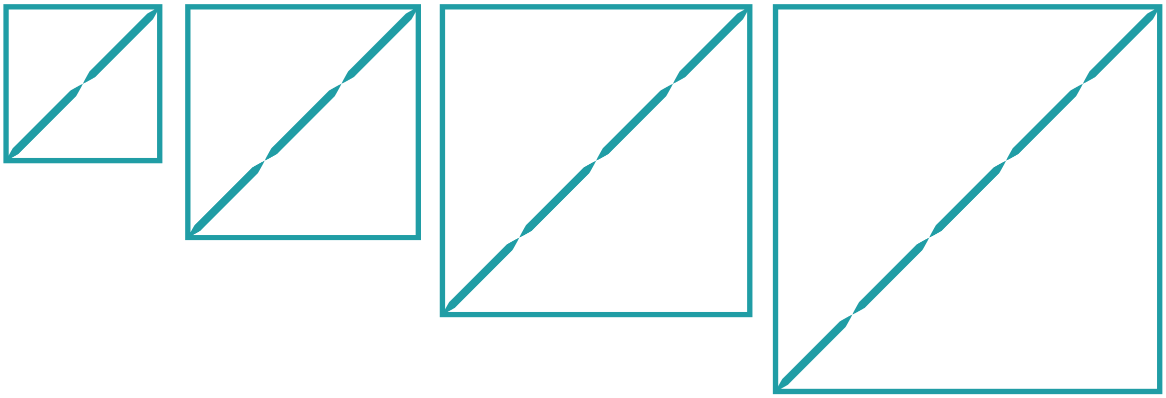
Name: .... Date: .... Period: ....

Across and Around

7 This square has a diagonal that is 1 toothpick long. About how many toothpicks long is its perimeter? Parts of toothpicks are allowed. Get as close as you can.



8 Here are squares with different diagonal lengths.



Complete the table to show the number of toothpicks you would need to build the perimeter of each square.

Diagonal Length (toothpicks)	Perimeter (toothpicks)
1	
2	5.66
3	
4	
5	14.15

9 **a** **Discuss:** Is the relationship between diagonal length and perimeter proportional? How do you know?

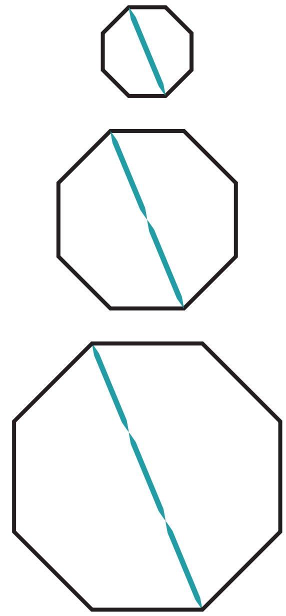
b How can you estimate the perimeter of a square with a diagonal of 100 toothpicks.

10 Synthesis

This table shows the relationship between the diagonal length and perimeter of an octagon.

How can you tell if two quantities in a geometry situation are proportional?

Diagonal Length (toothpicks)	Perimeter (toothpicks)
1	3.06
2	6.12
3	9.18
4	12.24



Things to Remember:

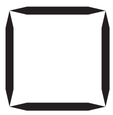
Name: .... Date: .... Period: ....

Explore More

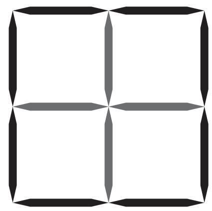
Determine the number of toothpicks needed to build the perimeter and interior of the 100th stage of each pattern.

Pattern A

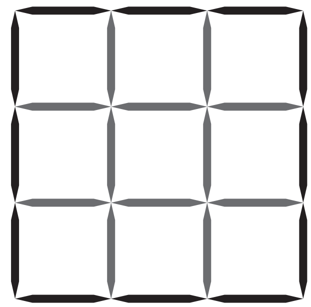
Stage 1



Stage 2



Stage 3

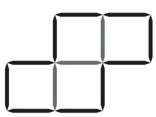


Perimeter of Stage 100: ....

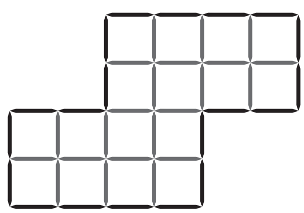
Interior of Stage 100: ....

Pattern B

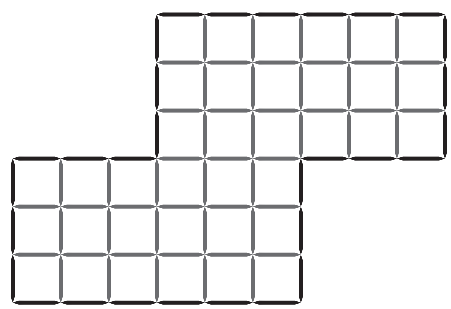
Stage 1



Stage 2



Stage 3

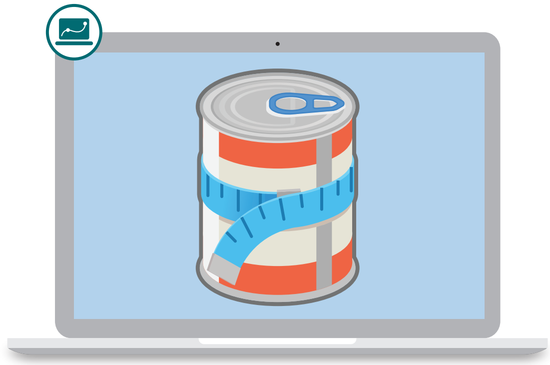


Perimeter of Stage 100: ....

Interior of Stage 100: ....

Unit 3
Lesson
3

Name: .... Date: .... Period: ....



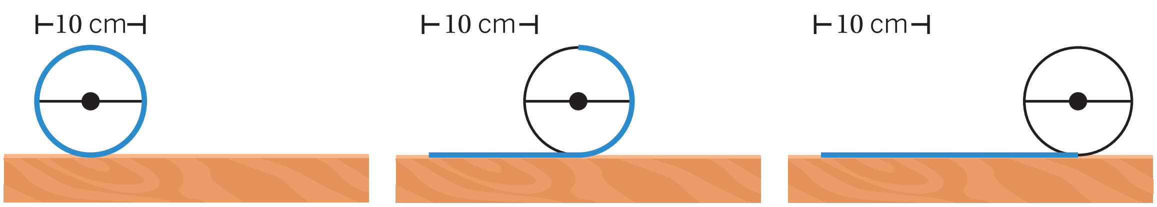
Measuring Around

Let's explore the relationship between the diameter of a circle and its circumference.

Warm-Up

1 The **circumference** of a circle is the distance all the way around it.

a Imagine unrolling the circumference to show it as a line segment.



b Estimate the length of the circumference.

Activity
1

Name: .... Date: .... Period: ....

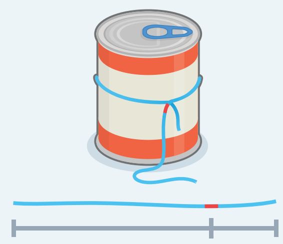
So Many Circles

2 Measure the diameter and the circumference of at least three circular objects. Record your results in the table.

Object	Diameter (cm)	Circumference (cm)
Toilet paper tube	4.1	12.9

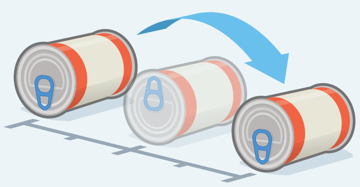
Three Ways to Measure the Circumference

String



Tape Measure

Roll

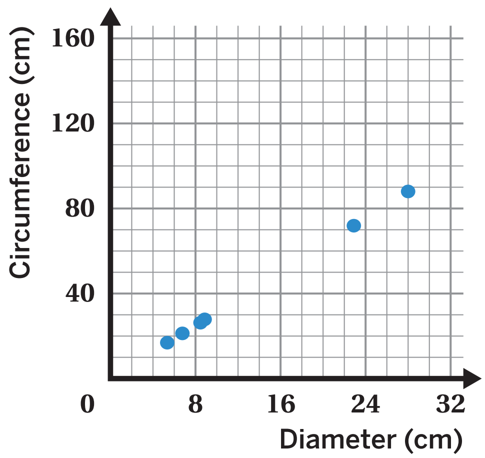


3 This graph shows the diameter and circumference of several circular objects.

Do you think the relationship between circumference and diameter is proportional? Circle one.

Yes No I'm not sure

Explain your thinking.



Activity 2

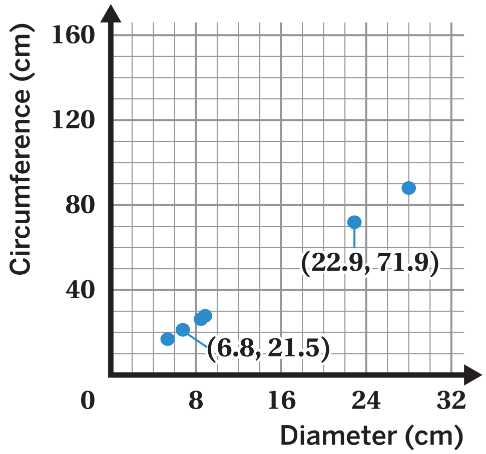
Name: ____ Date: ____ Period: ____

Introducing π

4 The relationship between circumference, C, and diameter, d, is proportional.

Estimate the *constant of proportionality* that completes the equation relating circumference and diameter.

$C =$ ______ d

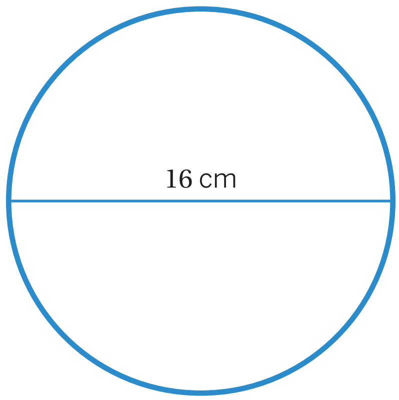


5 The constant of proportionality between a circle's diameter, d, and its circumference, C, is π (written as **pi** and pronounced "pie").

In other words, $C = \pi d$.

Common approximations for π are 3.14, $\frac{22}{7}$, and 3.14159. None of these are exactly π.

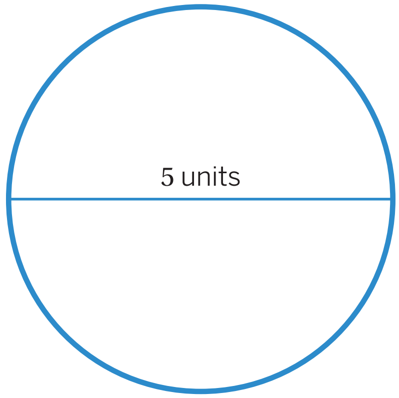
Calculate the circumference of a circle with a diameter of 16 centimeters.



6 This circle has a diameter of 5 units.

Four students tried to calculate its circumference.

Order their answers starting with the *farthest* from the exact circumference to the *closest* to the exact circumference.



15.7 units	15 units	5π units	15π units

Farthest From Exact Circumference **Closest to Exact Circumference**

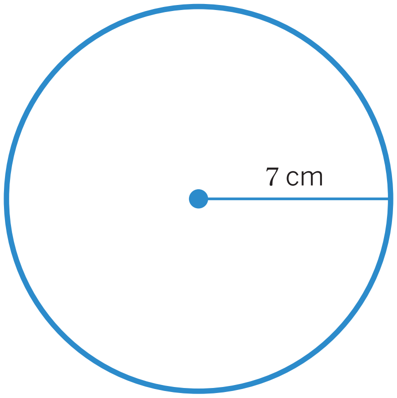
Activity 3

Name: .... Date: .... Period: ....

Proportionality and π

7 The radius of this circle is 7 centimeters.

What is its circumference?



8 The constant of proportionality in the relationship between diameter and circumference is π.

a Complete the table.

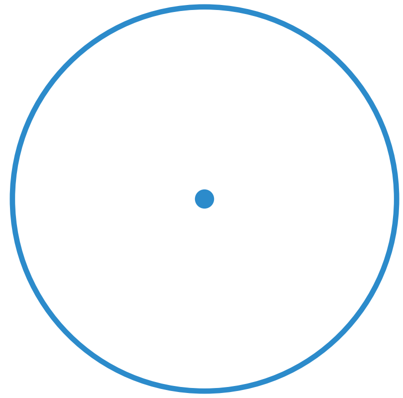
Object	Radius (cm)	Diameter (cm)	Circumference (cm)
Cookie	3	6	
Small plate	9		
Quarter		2.4	
Frisbee			21π
Tennis ball can	$\frac{7}{2}$		
Vinyl record			100

b When you finish, compare your answers with a classmate.

Discuss: How are your answers alike? How are they different?

10 Synthesis

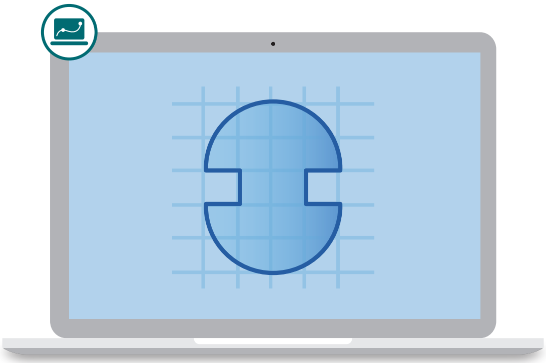
Describe the relationship between radius, diameter, and circumference.



Things to Remember:

Unit 3
Lesson
4

Name: ________ Date: ________ Period: ________

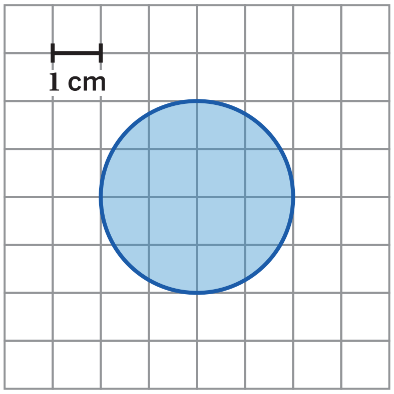


Perimeter Challenges

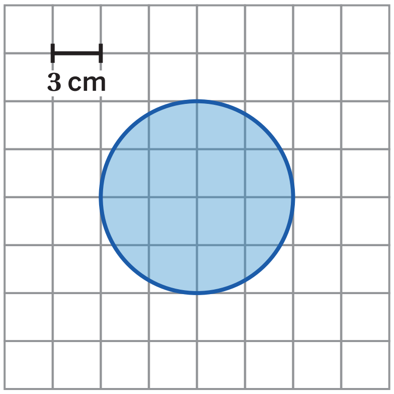
Let's calculate the perimeters of complex shapes.

Warm-Up

1 What is the circumference of this circle?



2 Now, what is the circumference of *this* circle?



Name: .... Date: .... Period: ....

Calculating Perimeters

3 This circle and semicircle have the same diameter.

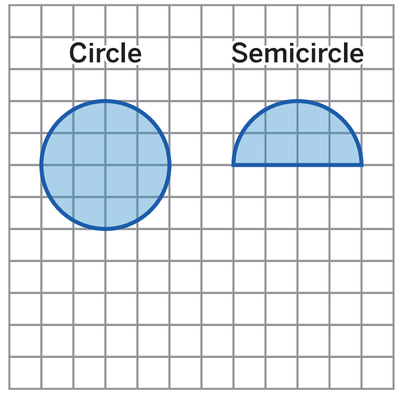
Dyani claims that the semicircle's perimeter is *half* the circle's perimeter.

Odalis says the perimeters are *equal*.

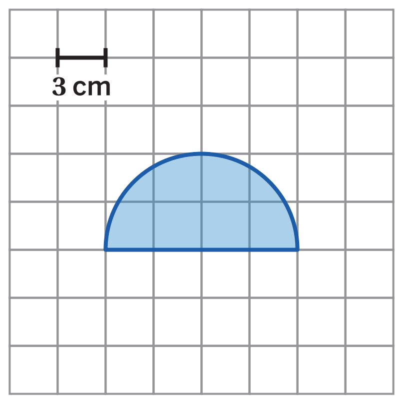
Whose claim is correct? Circle one.

Dyani's   Odalis's   Both   Neither

Explain your thinking.

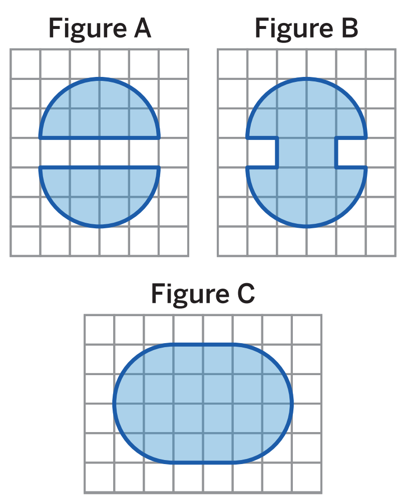


4 What is the perimeter of this semicircle?



5 Order the figures by their total perimeter.

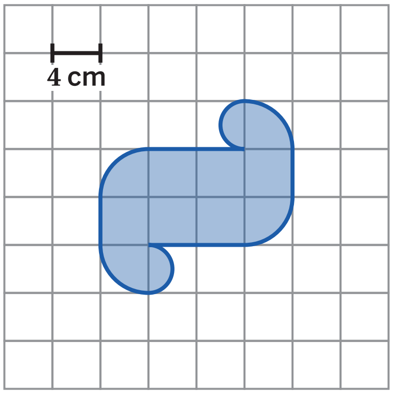
	Least Total Perimeter
	Greatest Total Perimeter



Name: .... Date: .... Period: ....

More Perimeters, More Problems

6 This shape is made of squares and parts of circles.

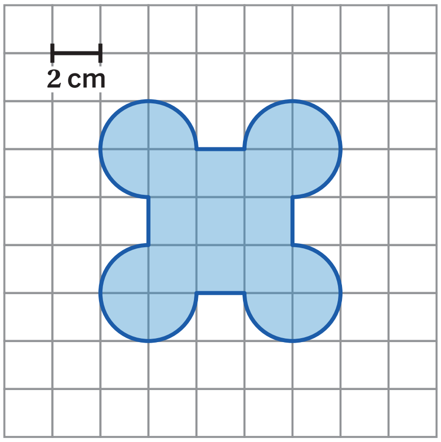


Nakia said the perimeter of this shape is made of these parts:

- Four quarter-circles with a radius of 4 centimeters
- Two half circles with a radius of 2 centimeters
- Two 4-centimeter pieces
- Two 8-centimeter pieces

Show or explain where Nakia might see each of these parts.

7 **a** **Discuss:** What parts of circles and squares do you see in this shape?



b Determine the perimeter of this shape.

Activity 3

Name: ________ Date: ________ Period: ________

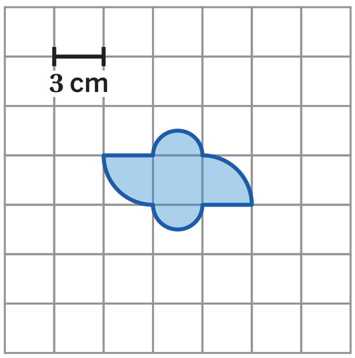
Sum of Its Parts

8 You will work with a partner.

- Decide who will complete Column A and who will complete Column B.
- Determine the perimeter of each shape. The perimeters in each row should be equal.
- Compare your solutions, then discuss and resolve any differences.

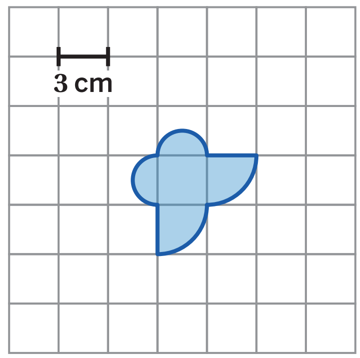
Column A

a Perimeter ________



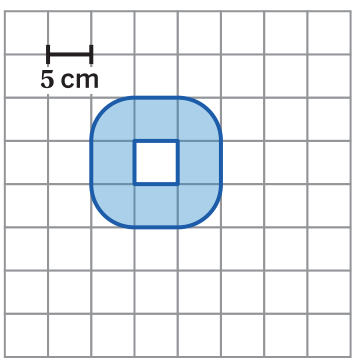
Column B

a Perimeter ________



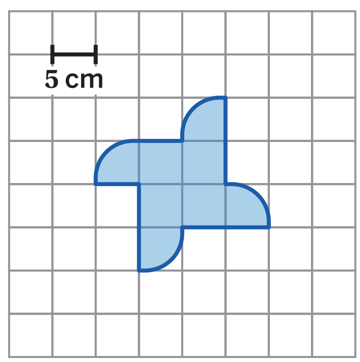
Column A

b Perimeter ________



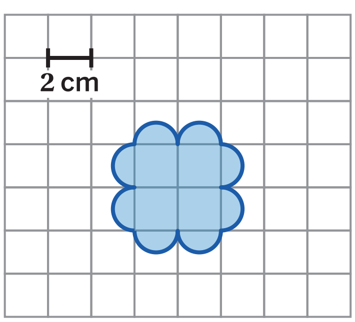
Column B

b Perimeter ________



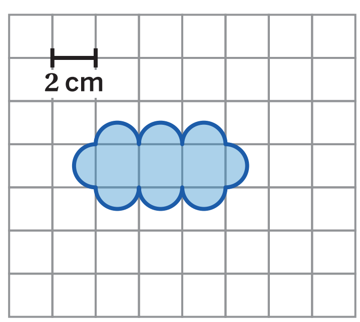
Column A

c Perimeter ________



Column B

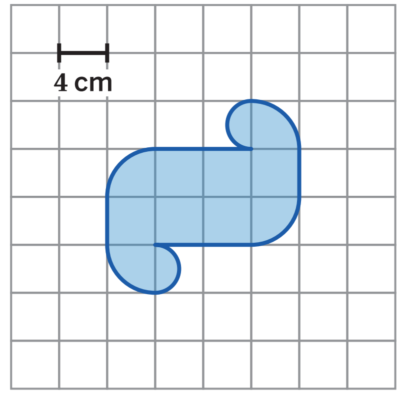
c Perimeter ________



9 Synthesis

Describe a strategy for determining the perimeter of a shape that is made of squares and parts of circles.

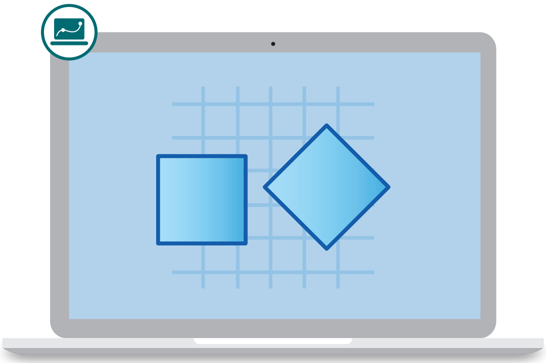
Use the example if that helps you to show your thinking.



Things to Remember:

Unit 3
Lesson
5

Name: ________ Date: ________ Period: ________



Area Strategies

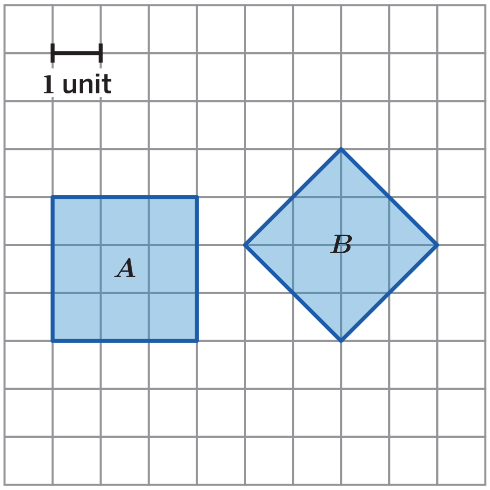
Let's estimate and calculate areas.

Warm-Up

1 Which square has a larger *area*? Circle one.

Square A Square B They're the same

Explain your thinking.



2 The area of square A is 9 square units.

Work with a partner to determine square B's area. Sketch if it helps with your thinking.

Be prepared to describe your strategy.

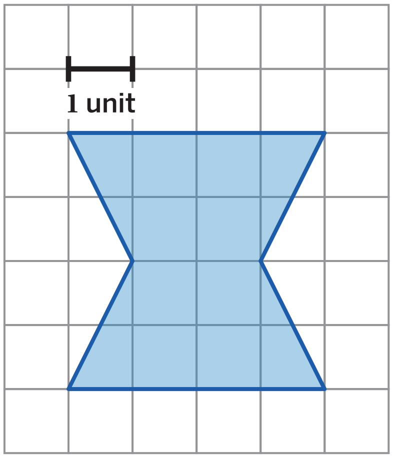
Activity
1

Name: .... Date: .... Period: ....

Calculating Area

2 What is the area of this shape?

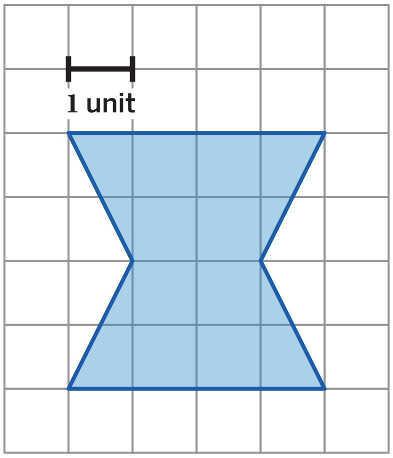
Sketch to help show your thinking.



Explain your thinking.

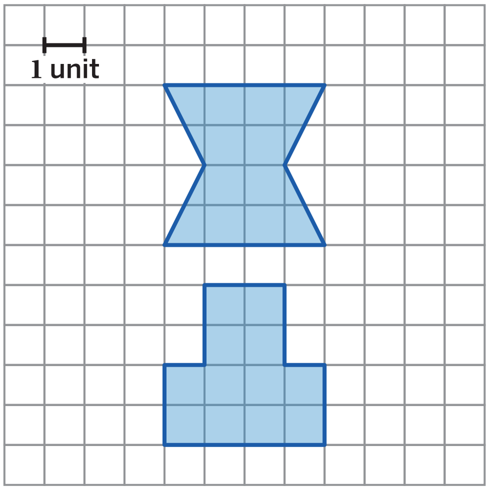
4 There is often more than one way to determine area.

Sketch to show another way of determining the area of this shape.



Describe your strategy.

5 Zamari says these two shapes have the same area.



Is Zamari correct? Circle one.

Yes No I'm not sure

Explain your thinking.

Activity 2

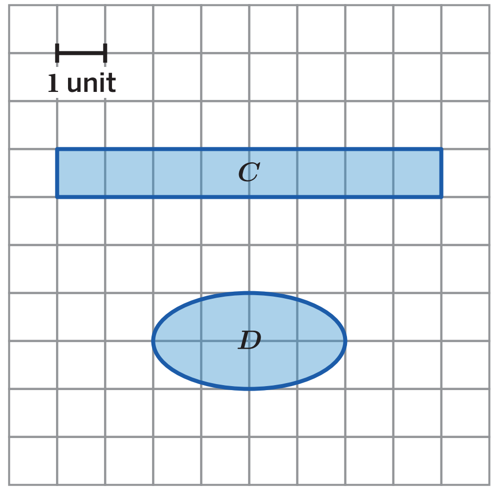
Name: .... Date: .... Period: ....

Estimating Area

6 Which shape has a larger area? Circle one.

Shape C Shape D They're the same

Explain your thinking.

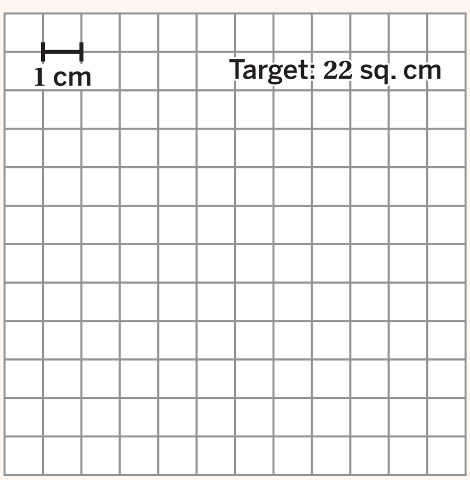


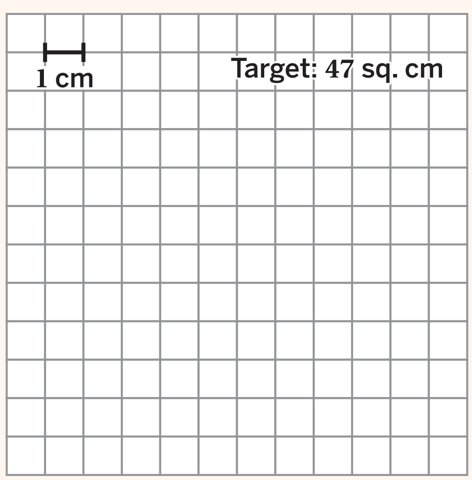
7 What is the approximate area of shape D?

Sketch if it helps with your thinking.

Explore More

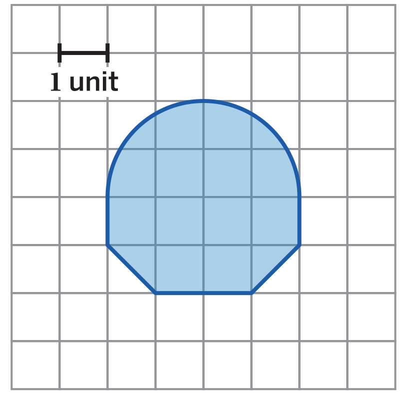
8 Draw an oval with an area that is approximately equal to each target area.





9 Synthesis

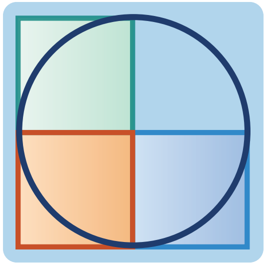
Show or describe a strategy for estimating the area of this shape.



Things to Remember:

Unit 3
Lesson
7

Name: .... Date: .... Period: ....

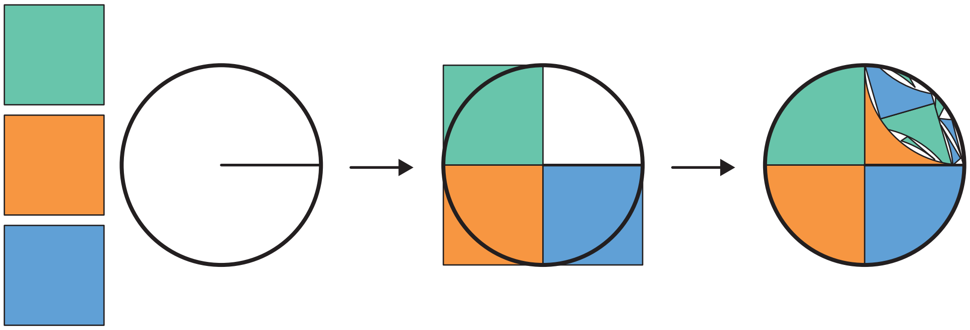


Circle Area

Let's determine a formula for the relationship between the radius of a circle and its area.

Warm-Up

Let's watch an animation.



1. What do you notice? What do you wonder?

I notice:

I wonder:

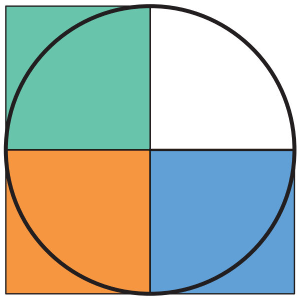
Activity 1

Name: ________ Date: ________ Period: ________

Finding a Formula

2. Diya says that you can find the approximate area of a circle by calculating $3 \cdot r^2$.

What do you think each part of this expression means? Draw on the diagram if it helps with your thinking.



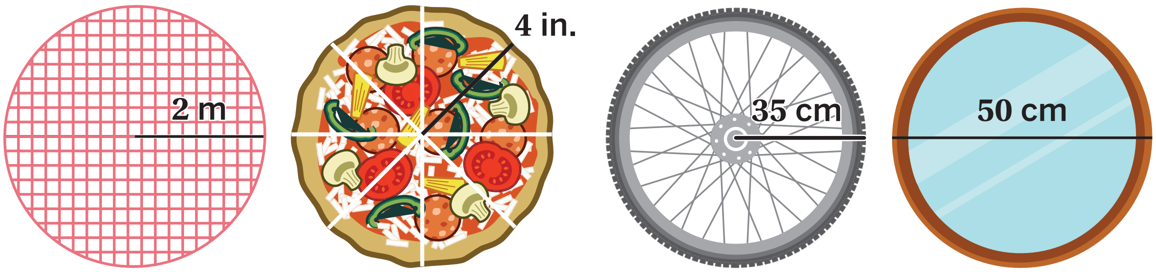
3. Jaleel says you can find the approximate area of a circle by calculating $r \cdot r \cdot 3$.

Do you agree with Jaleel? Circle one.

Yes No I'm not sure

Explain your thinking.

4. Here are four circles, their radius or diameter lengths, and their areas.



$A \approx 12.57$ square meters

$A \approx 50.27$ square inches

$A \approx 3848.45$ square centimeters

$A \approx 1963.50$ square centimeters

Write a formula to calculate the exact area of a circle, A, with a radius, r. Use these examples and your work from Problems 2–3 if it is helpful.

$A =$ ________

Activity 2

Name: ____ Date: ____ Period: ____

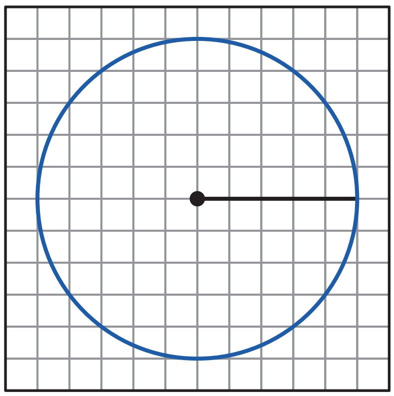
Circle Area

5. Rewrite the formula for the exact area of a circle you developed in Activity 1. Use A for the area of the circle and r for the length of the radius.

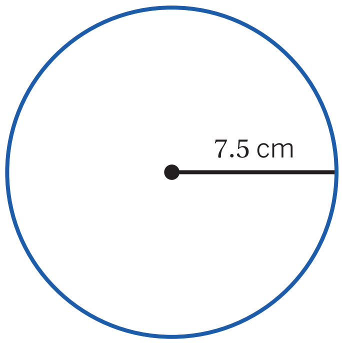
$A =$ ____

6. Calculate the exact area of each circle.

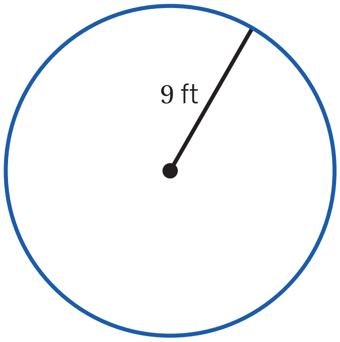
a Area: ____



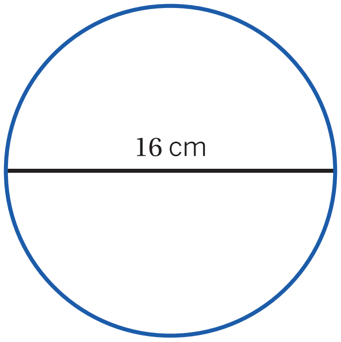
b Area: ____



c Area: ____

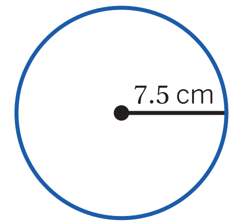


d Area: ____

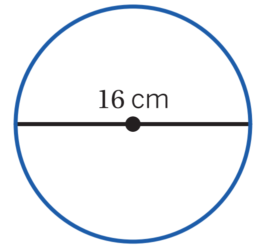


Synthesis

7. **a** Describe a strategy to calculate the area of a circle if you know its *radius*.



b How does your strategy change if you know a circle's *diameter*?



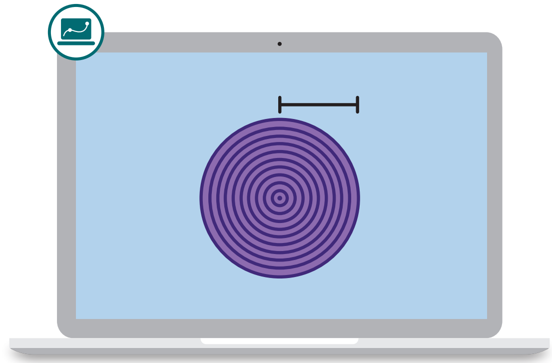
Things to Remember:

Unit 3

Lesson

8

Name: ______________ Date: ________ Period: ________

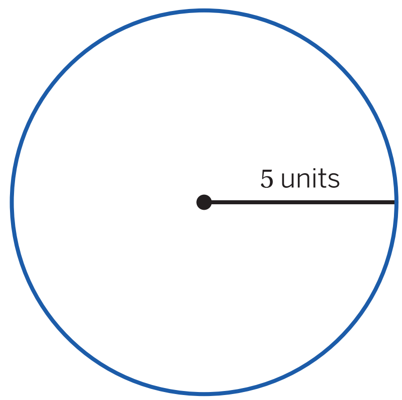


Why Pi?

Let's explore why the formula for the area of a circle makes sense.

Warm-Up

1 This circle has a radius of 5 units.



Four students tried to calculate the area.

Order their answers starting with the farthest from the exact area to the closest to the exact area.

$25\pi^2$ square units	25π square units	10π square units	78.5 square units

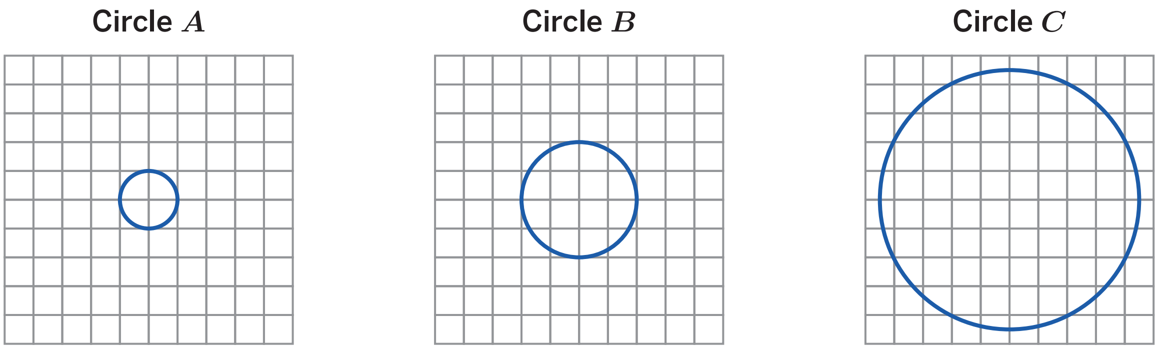
Farthest From Exact Area **Closest to Exact Area**

Activity 1

Name: .... Date: .... Period: ....

Proportional or Not?

2 Here are some circles.



Record the area of each circle in the table.

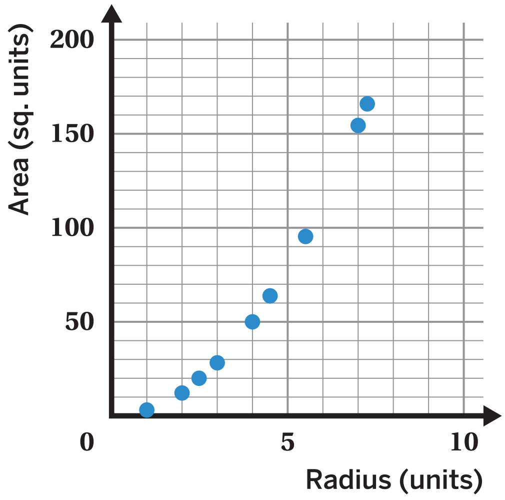
Circle	Radius (units)	Area (sq. units)
A	1	
B	2	
C	4.5	

3 This graph shows the radius and area of several circles.

Is there a proportional relationship between the radius and the area of a circle? Circle one.

Yes No I'm not sure

Explain your thinking.



Activity 2

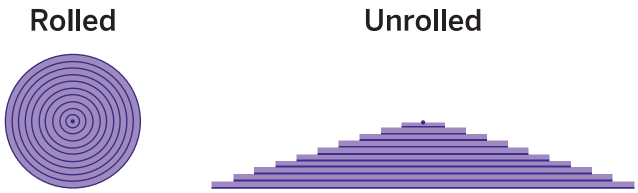
Name: .... Date: .... Period: ....

Unrolling a Circle

4 Let's watch an animation.

Discuss:

- How are the rolled and unrolled figures alike?
- How are they different?
- What parts of the original circle do you see in the unrolled figure?

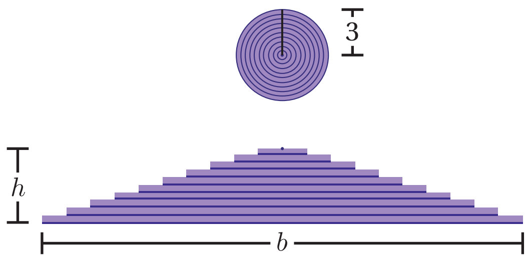


5 As the circle is split into thinner rings, the stack of unrolled rings looks closer to a triangle.

What is the base and the height of the triangle?

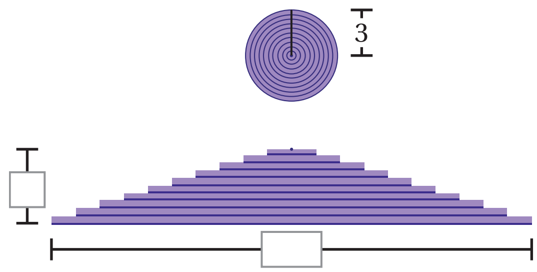
Base b: .... units

Height h: .... units



6 Label the diagram with the base and height you just determined.

What is the area of the triangle?



Activity 2

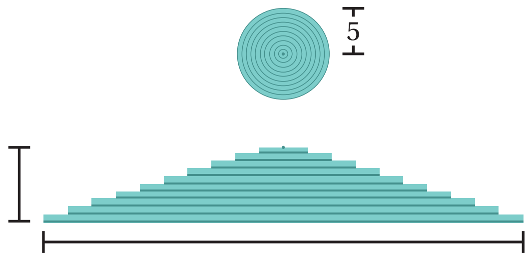
Name: .... Date: .... Period: ....

Unrolling a Circle (continued)

2 This diagram shows a new rolled and unrolled circle.

Calculate the area of the circle.

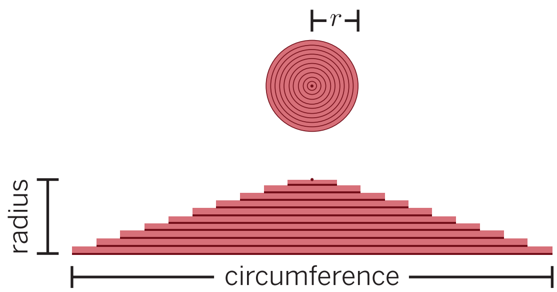
Write measurements on the triangle if it helps with your thinking.



8 Haruto says the area of the triangle can be calculated using this formula:

$A = \frac{1}{2} \cdot$ circumference $\cdot$ radius

Use this fact to convince a friend that the area of a circle is $A = \pi \cdot r^2$.



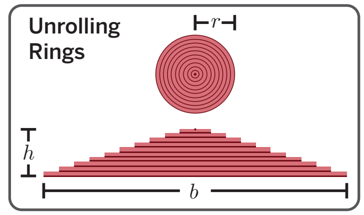
Explore More

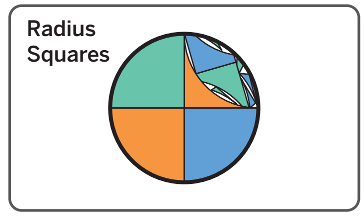
9 Use the Explore More Sheet to explore a different way to find the area of a circle.

10 Synthesis

Choose one of these representations.

How does the representation you chose show that the area of a circle is πr^2?



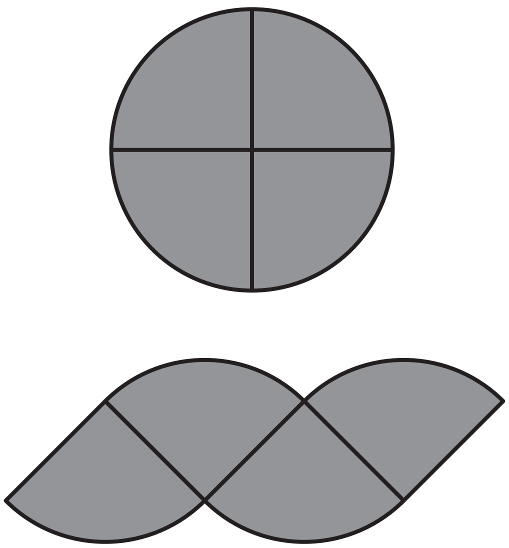


Things to Remember:

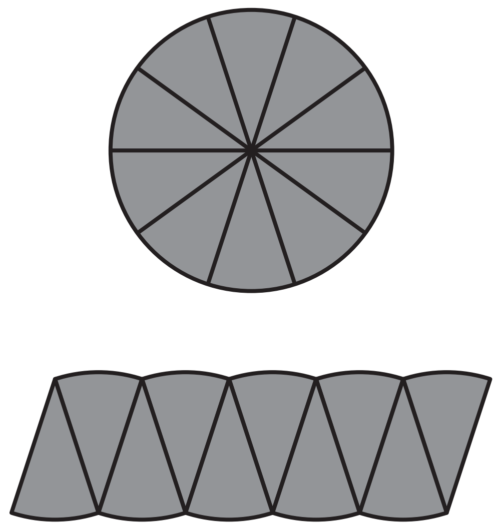
Explore More

Here is a different way to find the area of a circle. As we make more slices for the circle, the rearranged slices get closer to a rectangle.

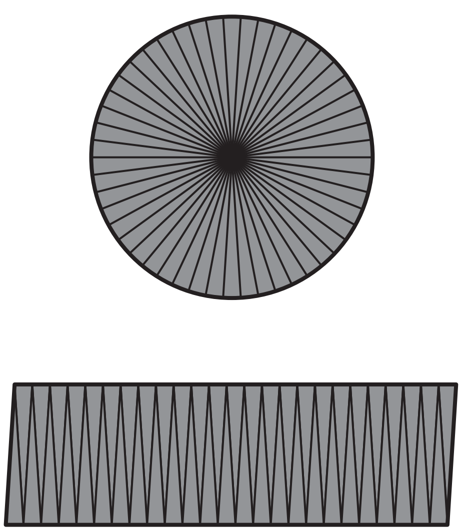
4 slices



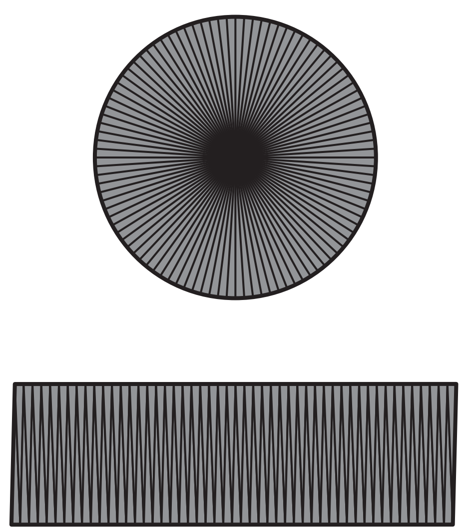
10 slices



50 slices



100 slices



Explain how to use the resulting rectangle to find the area of the circle.

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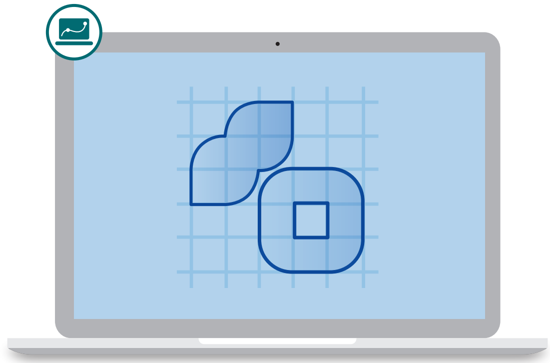


Area Challenges

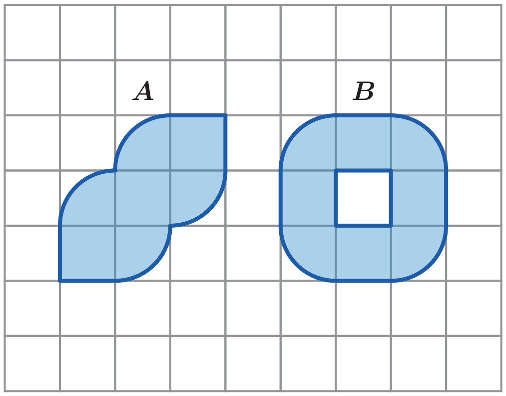
Let's calculate the areas of complex shapes.

Warm-Up

1 Which shape has the greater area? Circle one.

Shape A Shape B They're the same

Explain your thinking.



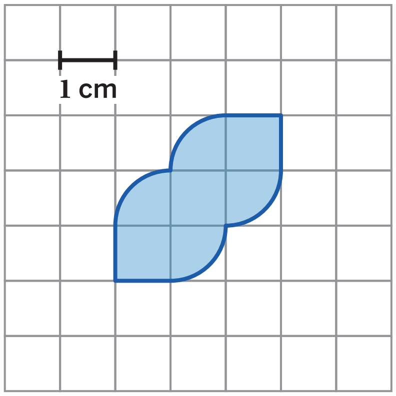
Activity 1

Name: .... Date: .... Period: ....

Calculating Areas

2 Here is shape A from the Warm-Up.

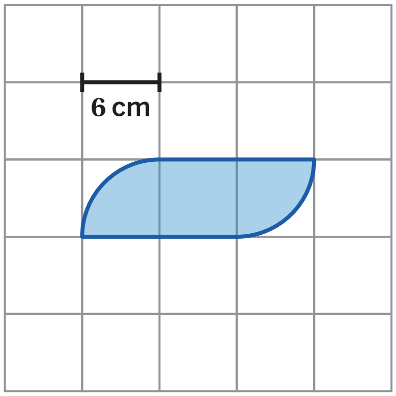
What is its area?



Explain your thinking.

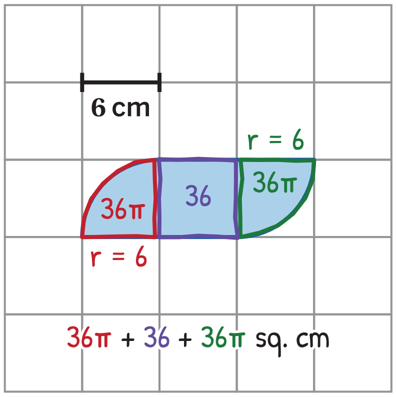
3 Here is a new shape with a new scale.

What is its area?



4 Taylor calculated the area of the shape using the expression $36\pi + 36 + 36\pi$, but made a mistake.

Identify the mistake and explain why it is incorrect.

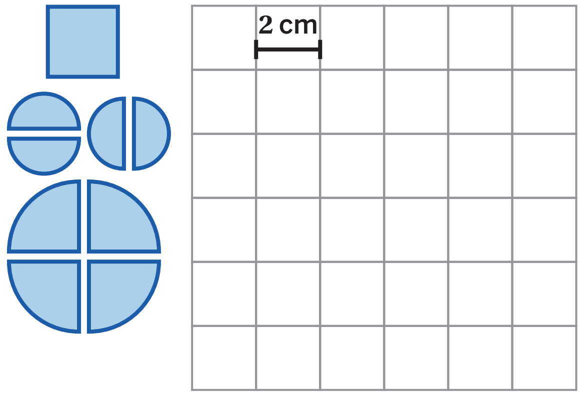


Activity 2

Name: .... Date: .... Period: ....

Create a Shape

5 Create a shape made up of squares and parts of circles that has an area of $8 + 4\pi$ square centimeters.



6 Create your own challenge!

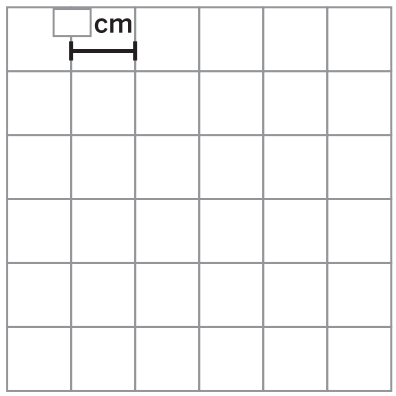
a Circle the scale you'd like to use, then label the diagram with your choice.

2 cm 4 cm 6 cm 8 cm

b Draw a shape made up of squares and parts of circles.

c Determine the exact area of your shape.

My Challenge



Now it's time to complete your partner's challenge!

d Ask your partner what scale they used, then label the diagram with what they chose.

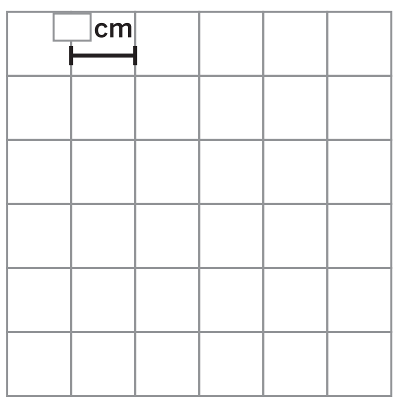
e Ask your partner the area of their shape and record it.

.... square centimeters

f Draw a shape made up of squares and parts of circles that has the same area as your partner's shape.

g When you're done, compare your shape with your partner's shape. How are they alike? How are they different?

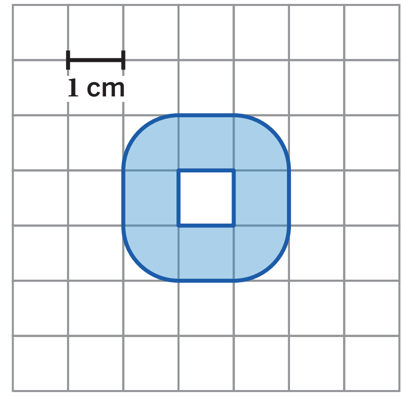
My Partner's Challenge



9 Synthesis

Describe a strategy for determining the area of a shape that's made up of squares and parts of circles.

Use the example if it helps to show your thinking.



Things to Remember:

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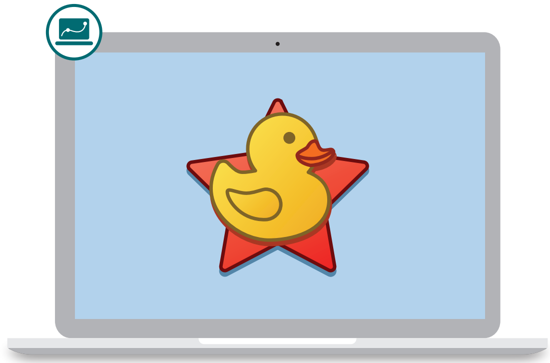
Name: ______ Date: ______ Period: ______



Lucky Duckies

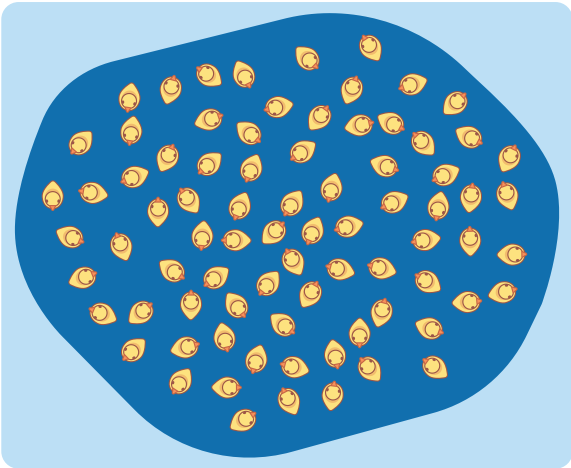
Let's learn about benchmark percentages with rubber duckies.

Warm-Up

1 Here is a carnival game called *Duck, Duck, Choose*.

Players win a prize if they catch a rubber ducky with a star on the bottom.

Let's try catching a ducky with a star.



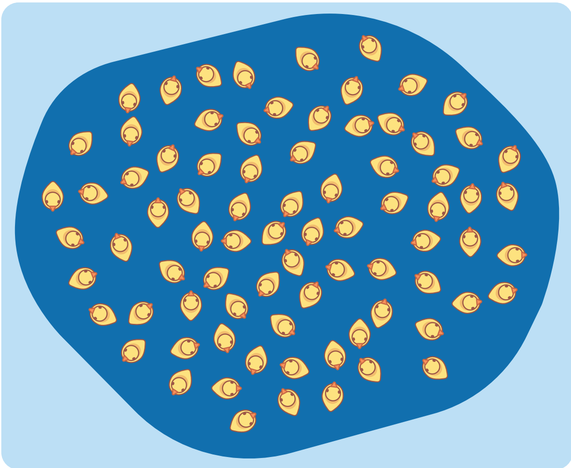
2 There are two games that both have 80 duckies. Which game has more duckies with stars?

A. The game where 50% of duckies have stars

B. The game where 50 duckies have stars

C. They are the same

Explain your thinking.



Activity 1

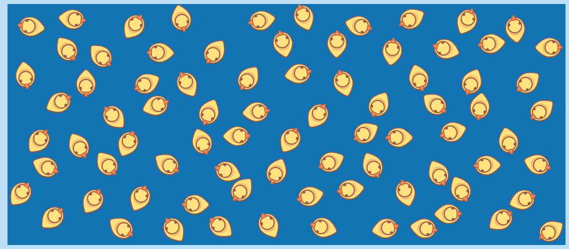
Name: ____________ Date: ________ Period: ________

Ducky Game Design

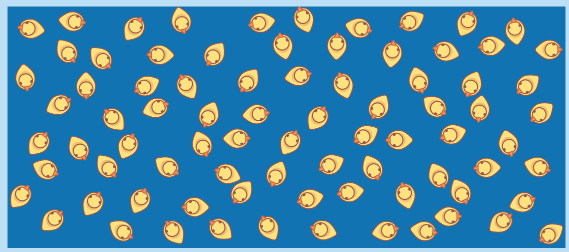
3 **a** Use the digital activity to move the dividers so that each game has about the right number of duckies with stars.

b **Discuss:** How did you decide where to place each divider?

Game A: 50% have stars

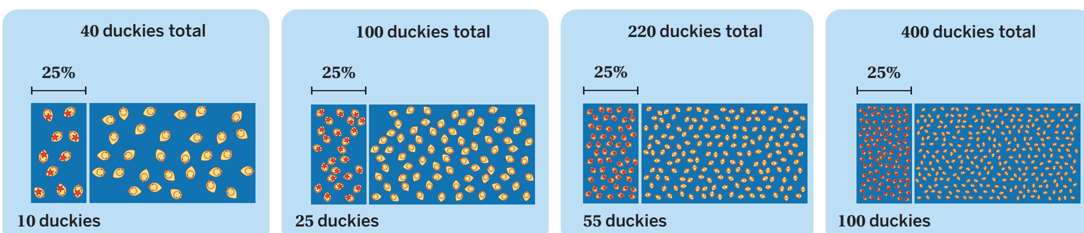


Game B: 25% have stars



4 Here are some games where 25% of the duckies have stars.

a Take a look at the total number of duckies in each game.

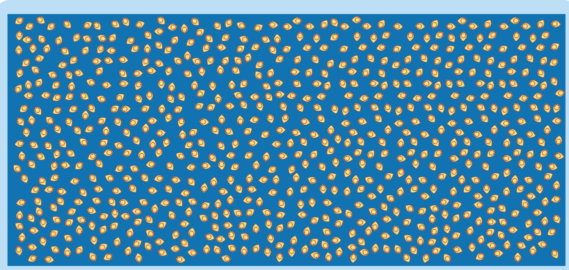


b Describe what 25% of a number means.

5 10 **percent** (10%) means 10 for every 100.

This game has 800 duckies. 10% of them have stars.

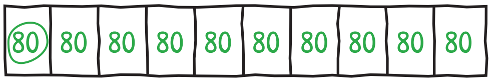
How many of the duckies have stars?



Name: ______ Date: ______ Period: ______

Ducky Game Design (continued)

6 Here is how Santiago figured out the number of duckies that are winners when 10% out of 800 duckies win.

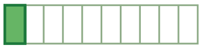


Show or explain what Santiago may have been thinking.

7 Group these choices based on what percentage they represent. One choice will have no match.

a. 10% is shaded.

b. $\frac{3}{4}$ is shaded.

c. 

d. $\frac{1}{2}$ is shaded.

e. 25% is shaded.

f. 

g. 75% is shaded.

h. $\frac{1}{4}$ is shaded.

Group 1	Group 2	Group 3

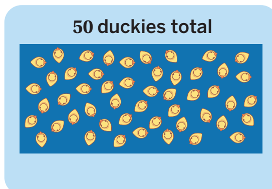
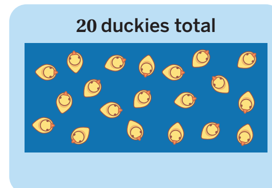
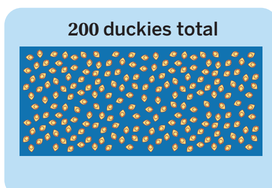
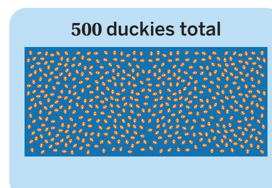
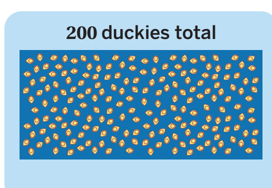
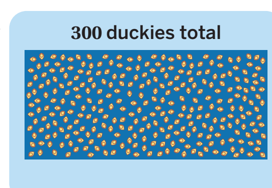
Activity 2

Name: .... Date: .... Period: ....

Repeated Challenges

8

- Pair up with a classmate. Decide who will complete Column A and who will complete Column B.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

Column A	Column B
10% of 50 duckies have stars. How many duckies have stars? 	25% of 20 duckies have stars. How many duckies have stars? 
25% of 200 duckies have stars. How many duckies have stars? 	10% of 500 duckies have stars. How many duckies have stars? 
75% of 200 duckies have stars. How many duckies have stars? 	50% of 300 duckies have stars. How many duckies have stars? 

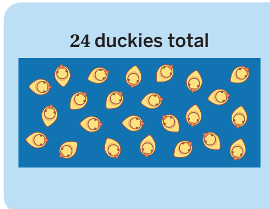
Activity

2

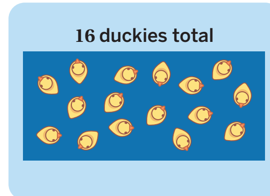
Name: .... Date: .... Period: ....

Repeated Challenges (continued)

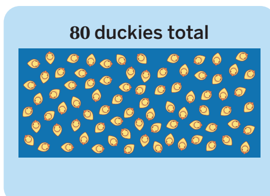
50% of 24 duckies have stars. How many duckies have stars?



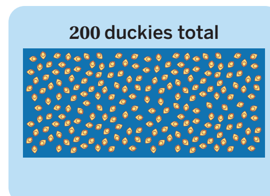
75% of 16 duckies have stars. How many duckies have stars?



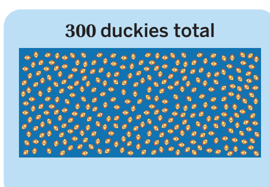
25% of 80 duckies have stars. How many duckies have stars?



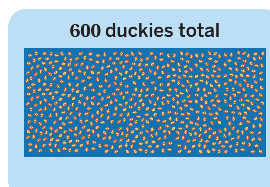
10% of 200 duckies have stars. How many duckies have stars?



50% of 300 duckies have stars. How many duckies have stars?



25% of 600 duckies have stars. How many duckies have stars?

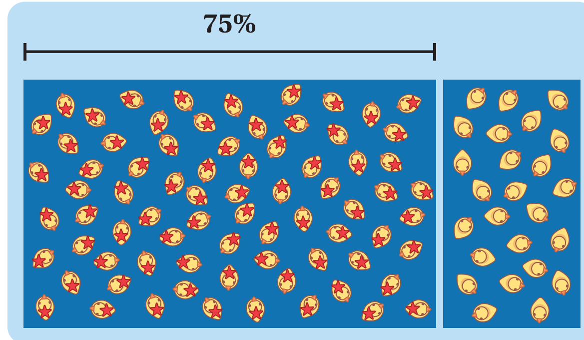


Explore More

9 Use the digital activity to make several different games that all have 30 winning duckies.

10 Synthesis

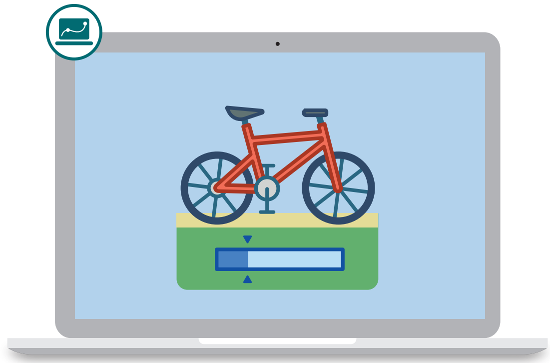
In your own words, explain what 75% of a number means.



Things to Remember:

Unit 3
Lesson
10

Name: .... Date: .... Period: ....

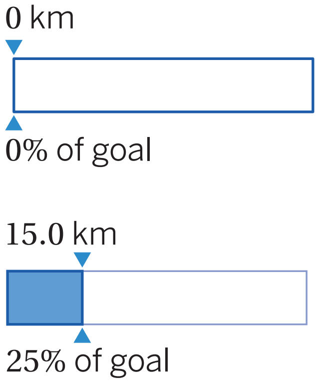


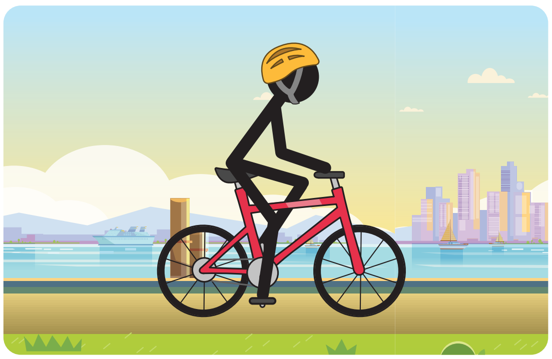
Bicycle Goals

Let's connect percentages and ratios.

Warm-Up

1 Let's watch the percentage change as the biker tries to beat their goal.





2 What is this biker's goal distance?

Explain your thinking.

Activity
1

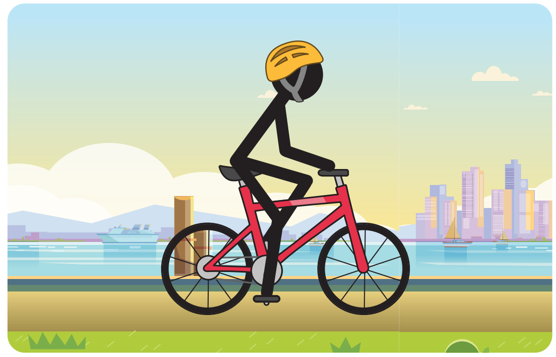
Name: .... Date: .... Period: ....

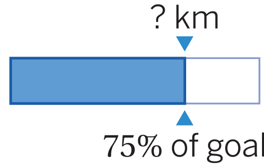
Bicycle Goals

3 Alejandro's goal was to ride 36 kilometers.

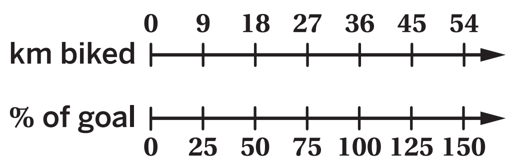
His app says he rode 75% of his goal.

How far did he ride?





4 **a** Take a look at the double number line that represents Alejandro's goal and progress.



b Describe how you can tell that the goal distance is 36 kilometers.

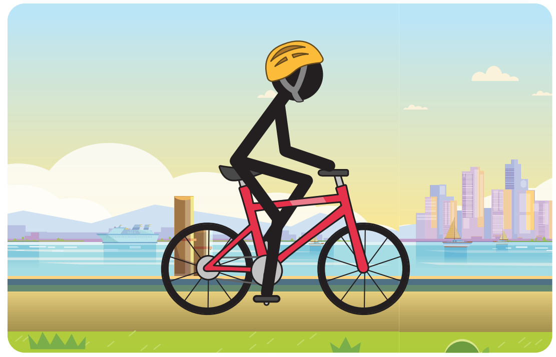
Name: ________ Date: ________ Period: ________

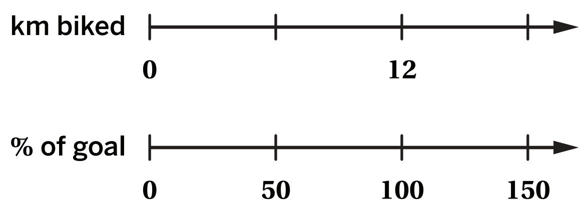
Bicycle Goals (continued)

5 Basheera's goal was to ride 12 kilometers.

Her app says she rode 150% of her goal.

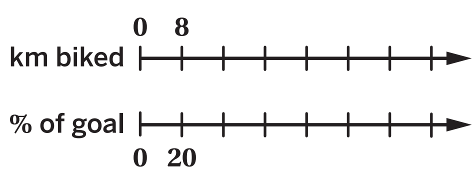
How far did she ride?





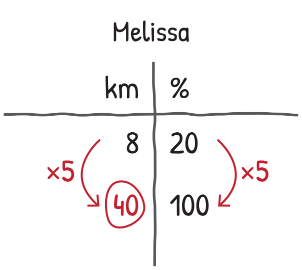
6 According to an app, Callen and Folami biked 8 kilometers, which is 20% of their goal.

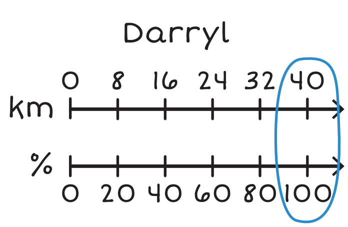
What was their goal distance?



7 Here are two different strategies for calculating the goal when 20% of the goal is 8 kilometers.

Discuss: How did each student use ratios to calculate the goal?





Activity
2

Name: ________ Date: ________ Period: ________

Percentages and Ratios

8 Match the strategies that represent the same percentage problem.

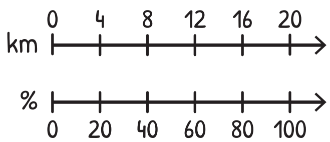
a.

km	%
80	100
8	10
16	20

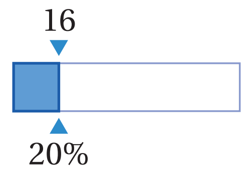
b.

km	%
20	100
2	10
16	80

c.



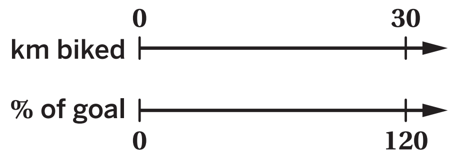
d.



20% of 80 km	80% of 20 km

9 Miko's app says he biked 30 kilometers, which is 120% of his goal.

What was his goal distance?



Explore More

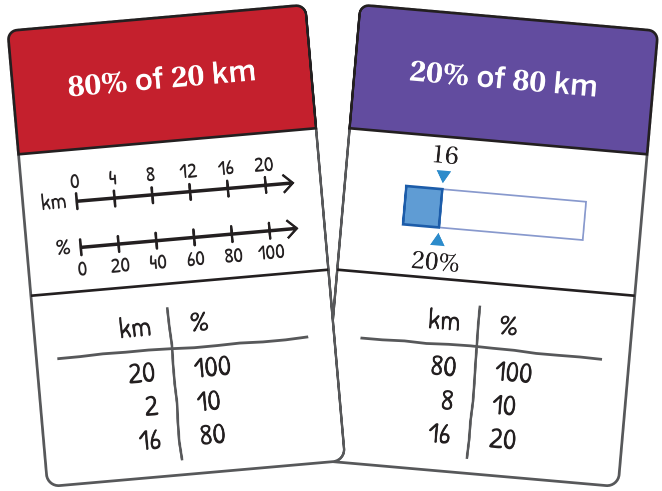
10 How many ways can you make a biker ride 10 kilometers?

Enter a goal and a percent of that goal for the biker to ride.

Goal				
Percent of Goal				

11 Synthesis

Describe how solving problems with percentages is like solving problems with ratios.



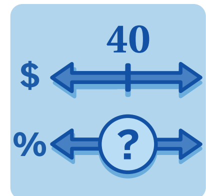
Things to Remember:

Unit 3
Lesson
11

Name: .... Date: .... Period: ....

What's Missing?

Let's use ratio reasoning to find unknown quantities.



Warm-Up

Evaluate each expression mentally.

1. $\frac{3}{10} \cdot 20$

2. $\frac{3}{10} \cdot 25$

3. $\frac{3}{10} \cdot 5$

4. $\frac{3}{10} \cdot \frac{5}{2}$

Activity 1

Name: .... Date: .... Period: ....

Card Sort: What's Missing?

5. You will use a set of cards for this activity. Match each card to its place in the table. Then fill in all the empty spaces that remain.

	Question	Representation	Answer
a	I have a 40% off coupon. If I use it to buy a shirt that costs \$20, how much money would I save?		
b	I have a 20% off coupon. If I use it to buy a shirt and save \$40, what was the original price of the shirt?		
c	I paid \$40 for a jacket with an original price of \$50. What percent of the original price did I pay?		
d			

Name: .... Date: .... Period: ....

Sale Price and Original Price

6. Complete the table.

	Question	Representation	Answer
a	Eliza bought a hat for $21. The original price is $30. What percent of the original price did Eliza pay?		
b	A discount store sells items at 80% of the original price. If the original price of pants is $55, what is the sale price?		
c	A discount store sells items at 80% of the original price. If the sale price of sneakers is $96, what is the original price?		

Explore More

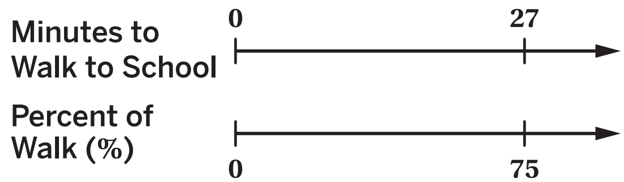
7. Precious biked 125% of her daily goal on Monday. What percent of her total weekly goal did she bike on Monday?

Precious's Biking Goals

Day	Su	M	T	W	Th	F	S
Goal (km)	0	8	4	10	0	8	20

Synthesis

8. Explain how this double number line can help you calculate the total time Zee takes to walk to school.



Things to Remember:

Card Sort: What's Missing?

Directions: Make one copy per four students. Then pre-cut the cards and give each pair one set of Cards 1–5.

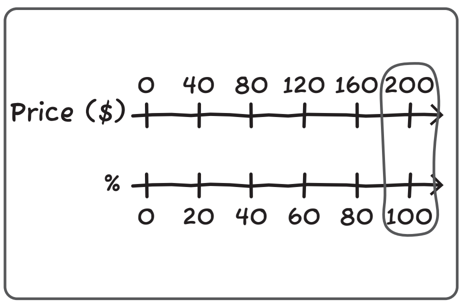
Card 1

	Price ($)	Percentage (%)	
	40	100	
$\times \frac{1}{10}$	4	10	$\times \frac{1}{10}$
$\times 5$	20	50	$\times 5$

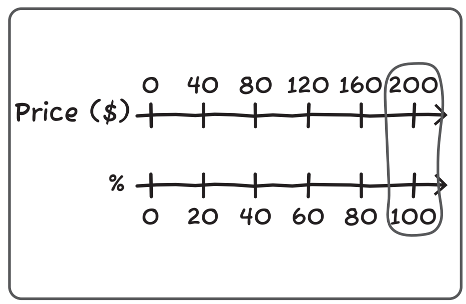
Card 1

	Price ($)	Percentage (%)	
	40	100	
$\times \frac{1}{10}$	4	10	$\times \frac{1}{10}$
$\times 5$	20	50	$\times 5$

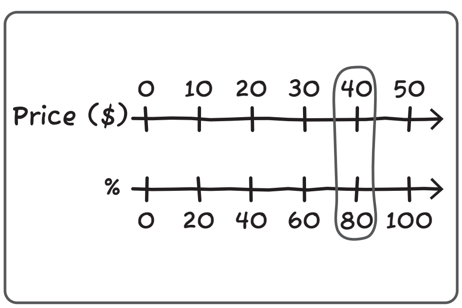
Card 2



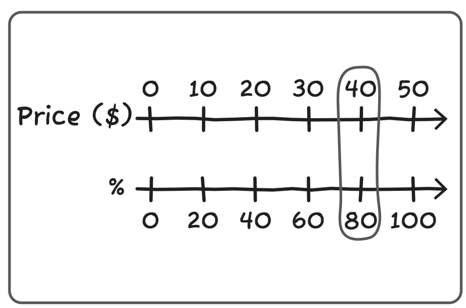
Card 2



Card 3



Card 3



Card 4

50%

Card 4

50%

Card 5

$8.00

Card 5

$8.00

Unit 3
Lesson
12

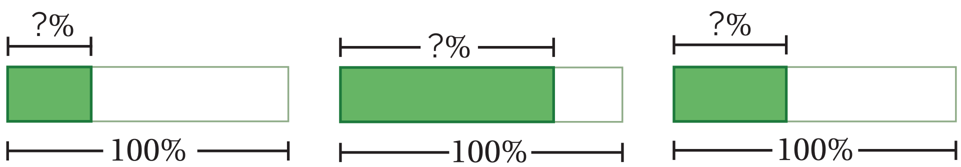
Name: .... Date: .... Period: ....

Cost Breakdown

Let's calculate any percentage of a number.

Warm-Up

1 For each challenge, write your best estimate of the missing percent.



Activity 1

Name: .... Date: .... Period: ....

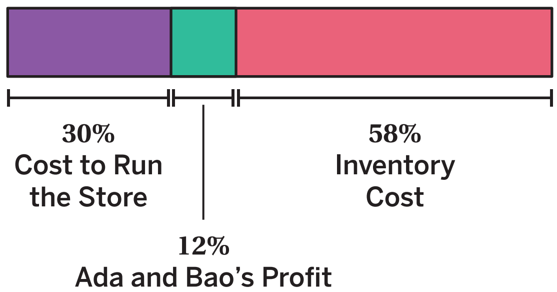
Break It Down

2 Ada and Bao run a clothing store.

The price of each item includes the profit that Ada and Bao make, the cost of inventory, and the cost to run the store.

The diagram shows where the money goes when Ada and Bao sell a T-shirt.

What do you notice? What do you wonder?

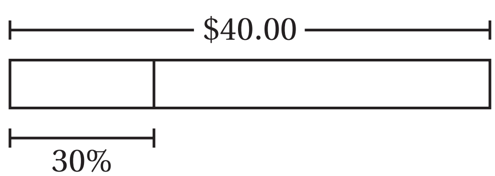


I notice:

I wonder:

3 30% of the price of each shirt goes to running the store.

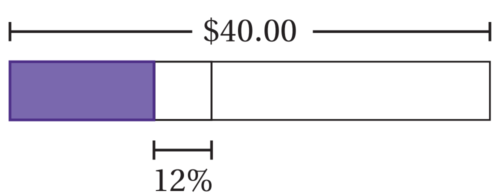
How much of a $40 shirt goes to running the store?



Use the tape diagram if it helps with your thinking.

4 Ada and Bao keep 12% of the price of each shirt as profit.

What is their profit on a $40 shirt?



Activity 1

Name: ________ Date: ________ Period: ________

Break It Down (continued)

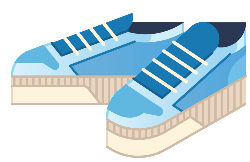
5 Here is how Bao calculated 12% of $\$40$.

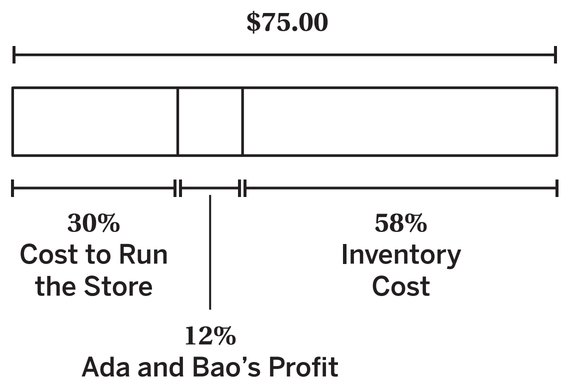
Explain how you could use Bao's strategy to calculate how much of the price of each shirt goes to inventory cost (58% of $\$40$).

	Cost (dollars)	Percentage	
	40	100%	
$\div 100$	$\frac{40}{100}$	1%	$\div 100$
$\times 12$	$\frac{40}{100} \cdot 12$	12%	$\times 12$

6 Here is a $\$75$ pair of shoes. Calculate each value.

Cost to Run the Store	
Ada and Bao's Profit	
Inventory Cost	



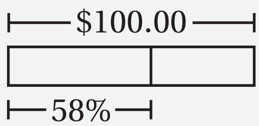
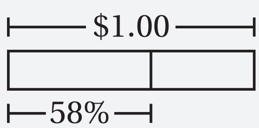
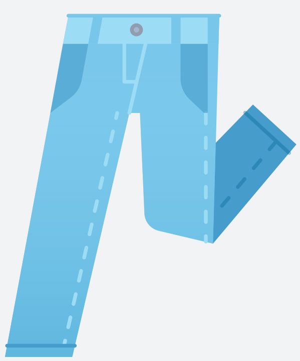
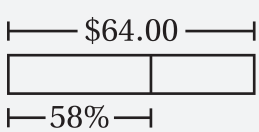
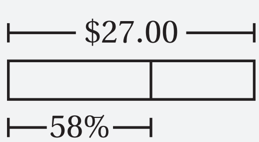


Activity 2

Name: ____________ Date: ________ Period: ________

Another Strategy

7 Ada and Bao's inventory costs are 58% of the total cost. What is their inventory cost for each item in this table?

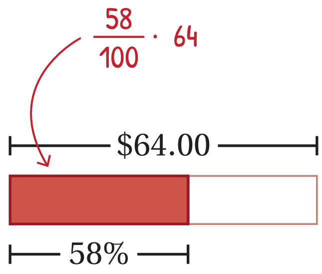
Item	Representation	Total Cost (dollars)	Inventory Cost (dollars)
Dress 		$100	
Sticker 		$1	
Jeans 		$64	
Hat 		$27	

Name: ______ Date: ______ Period: ______

Another Strategy (continued)

8 Ada thinks of 58% as 58 cents for every dollar. So he writes $\frac{58}{100} \cdot 64$ to calculate 58% of \$64.

Describe how Ada might calculate 36% of \$15.



9 Match each expression with a question. One expression will have no match.

a. $\frac{36}{100} \cdot 70$

b. $\frac{15}{100} \cdot 36$

c. $\frac{100}{36} \cdot 48$

d. $\frac{36}{100} \cdot 15$

e. $70 \div 100 \cdot 36$

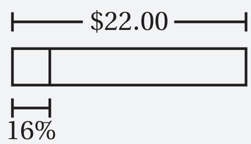
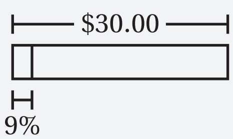
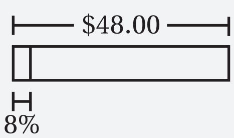
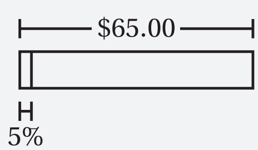
f. $\frac{36}{100} \cdot 48$

What is 36% of \$15?	What is 36% of \$48?	What is 36% of \$70?

Name: ______ Date: ______ Period: ______

Repeated Challenges

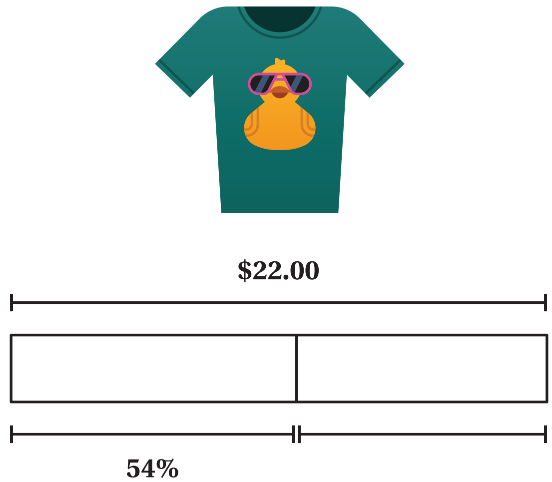
10 Solve as many challenges as you have time for.

Problem	Representation	Answer
The price of a space T-shirt is $22. 16% of every sale goes to material cost. Calculate the material cost.		
The price of a striped button-up shirt is $30. 9% of every sale goes to clothing company profit. Calculate the clothing company profit.		
The price of a striped long-sleeve T-shirt is $48. 8% of every sale goes to transport cost. Calculate the transport cost.		
The price of a blue pair of shoes is $65. 5% of every sale goes to factory profit. Calculate the factory profit.		

11 Synthesis

Describe a strategy for calculating a percentage of a number.

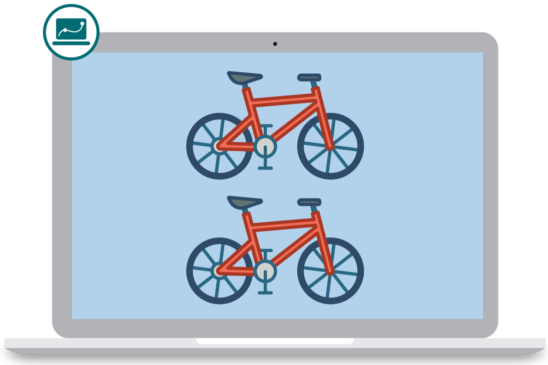
Use the example if it helps to explain your thinking.



Things to Remember:

Unit 3
Lesson
13

Name: ________ Date: ________ Period: ________



More Bicycle Goals

Let's calculate unknown percentages.

Warm-Up

Evaluate each expression mentally. Try to think of more than one strategy.

1 $\frac{1}{3}$ of $\frac{1}{4}$

2 $\frac{1}{3} \cdot \frac{1}{4}$

3 $\frac{2}{3} \cdot \frac{1}{4}$

4 $\frac{2}{3} \cdot \frac{5}{4}$

Activity 1

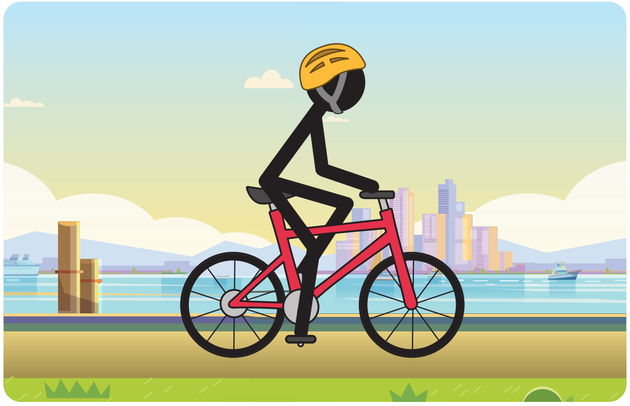
Name: .... Date: .... Period: ....

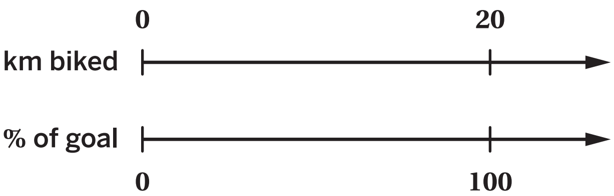
Chasing Goals

5 Alejandro's goal for Monday was to ride 20 kilometers.

His app says he rode 40% of his goal.

How far did he ride?

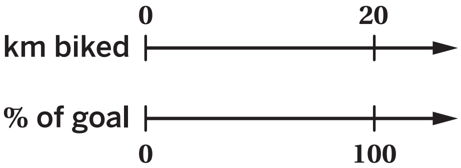




6 Alejandro's goal for Tuesday was to ride 20 kilometers.

His app says he rode 10 kilometers.

What percent of his goal did he ride?



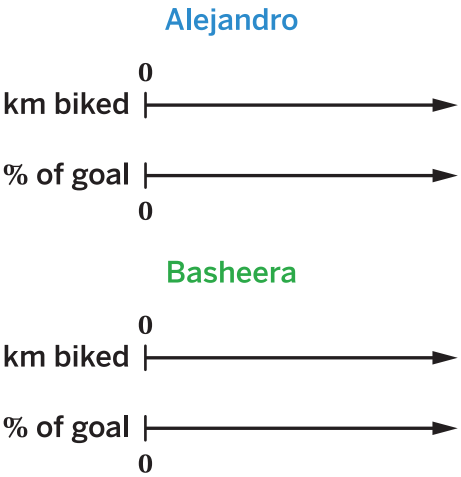
7 On Wednesday, Alejandro and Basheera rode 17 kilometers.

- Alejandro's goal was 20 kilometers.
- Basheera's goal was 50 kilometers.

Who rode a greater percent of their goal? Circle one.

Alejandro Basheera Same percent

Explain your thinking.



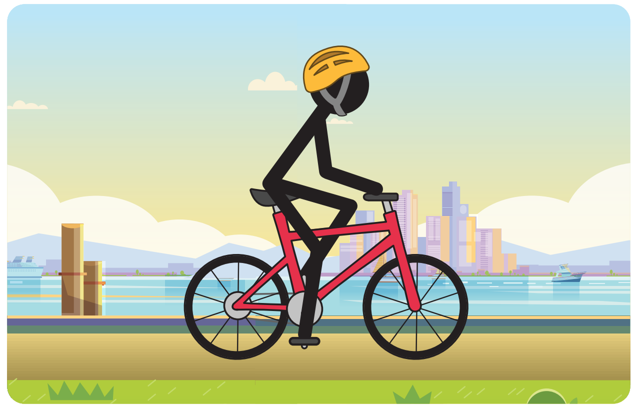
Name: ________ Date: ________ Period: ________

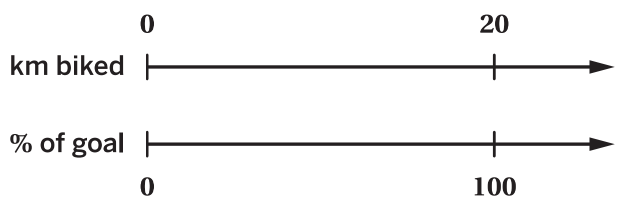
Chasing Goals (continued)

8 On Wednesday, Alejandro and Basheera rode 17 kilometers.

Alejandro's goal was 20 kilometers.

What percent of his goal did he ride?





9 Here is how Alejandro calculated 17 out of 20 as a percentage.

Explain how you could use Alejandro's strategy to calculate 13 out of 20 as a percentage.

	Distance (km)	Percent of Goal	
	20	100	
$\times \frac{1}{20}$			$\times \frac{1}{20}$
	1	$\frac{100}{20}$	
$\times 17$			$\times 17$
	17	$\frac{100}{20} \cdot 17$	

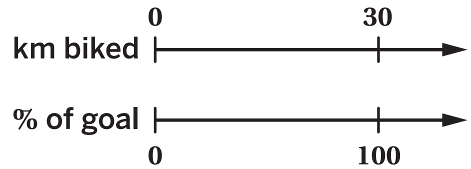
Name: ______ Date: ______ Period: ______

Reaching Goals

10 Alejandro and Basheera set a new goal for Saturday: 30 kilometers.

They rode 36 kilometers.

What percent of their goal did they ride?



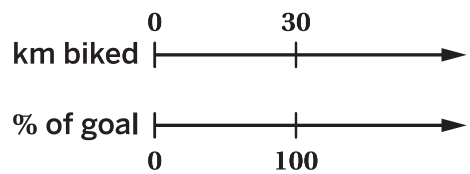
11 Here are the expressions Alejandro and Basheera used to calculate 36 out of 30 as a percentage.

Alejandro

$$\frac{100}{30} \cdot 36$$

Basheera

$$\frac{36}{30} \cdot 100$$

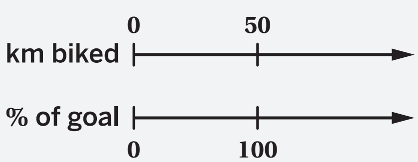
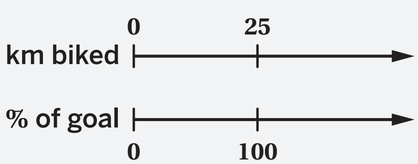
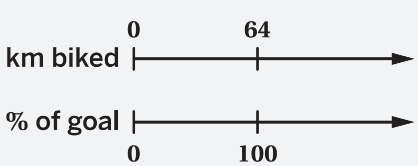


Whose expression is correct? Circle one.

Alejandro	Basheera
Both	Neither

Explain your thinking.

12 On Sunday, Alejandro, Basheera, and Callen rode 40 kilometers. They each had different goals, as shown in the table. Calculate what percent of their goal each person rode.

Representation	Distance Traveled (km)	Goal (km)	Percent of Goal
Alejandro 	40	50	
Basheera 	40	25	
Callen 	40	64	

Name: .... Date: .... Period: ....

Reaching Goals (continued)

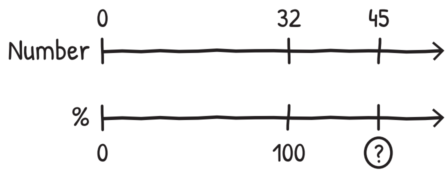
13 Match each question with a double number line and an expression. One choice will have no match.

a. $\frac{32}{100} \cdot 45$

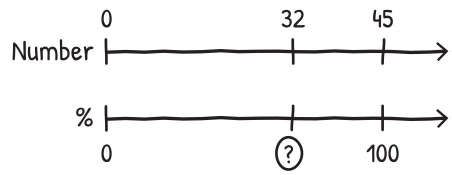
b. $\frac{32}{45} \cdot 100$

c. $\frac{45}{32} \cdot 100$

d.



e.



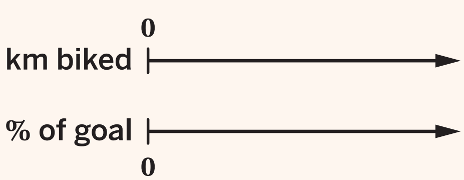
What is 32 out of 45 as a percentage?	What is 45 out of 32 as a percentage?

Explore More

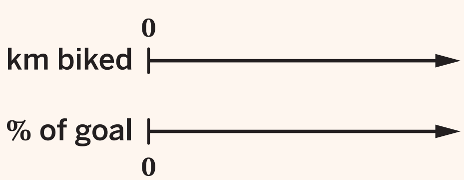
14 Basheera had a goal of riding 30 kilometers and rode 51 kilometers.
Callen had a goal of 20 kilometers and rode 41 kilometers.

Determine who rode a greater percent of their goal, in as many different ways as you can.

Basheera



Callen



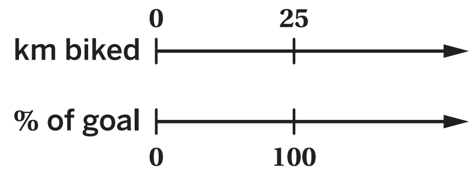
Explain your thinking.

15 Synthesis

Here is what Basheera wrote to solve a bicycle challenge.

$$\frac{31}{25} \cdot 100 = 124$$

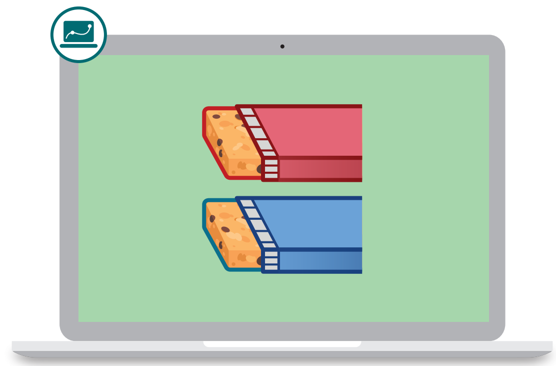
What do 31, 25, and 124 represent in this situation?



Things to Remember:

Unit 4
Lesson
2

Name: ______ Date: ______ Period: ______

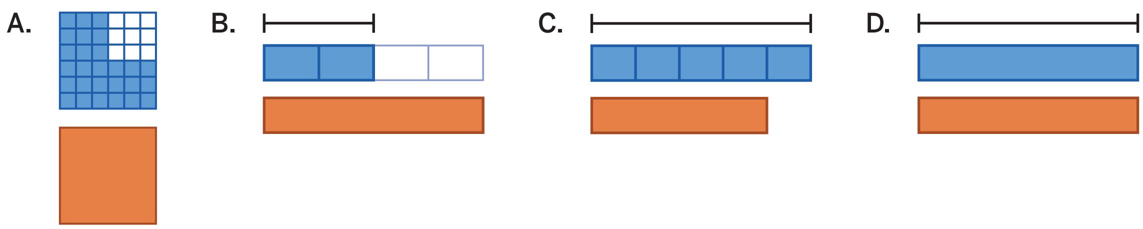


More and Less

Let's visualize what it means to increase or decrease by a percentage.

Warm-Up

1 Which one doesn't belong?



Explain your thinking.

Activity 1

Name: .... Date: .... Period: ....

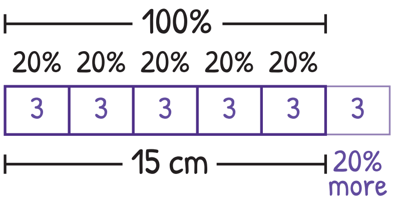
Granola Bars

2 DesBest Granola bars are now 20% longer.

If the original bar was 15 centimeters long, how long is the new granola bar?

3 Let's take a look at how DeAndre found the new length of the 15-centimeter granola bar after a 20% increase.

Explain what DeAndre may have been thinking.



Name: .... Date: .... Period: ....

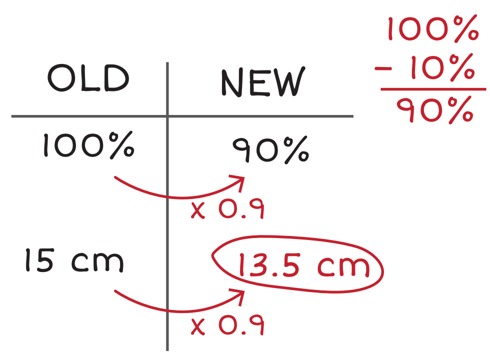
Granola Bars (continued)

4 In order to make more money, DesWorst Granola bars are now 10% shorter.

If the original bar was 15 centimeters long, how long is the new granola bar?

5 Let's take a look at how Afia found the length of the 15-centimeter granola bar after a 10% decrease.

Explain what Afia may have been thinking.



6 Two different boxes of granola bars are the same price.

One box has 40% more bars. The other has 4 more bars.

Which is the better deal? Circle one.

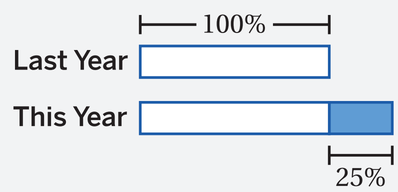
40% more bars   4 more bars   Not enough information

Explain your thinking.

Name: ______ Date: ______ Period: ______

Fruitful Percentages

7 Decide which tape diagram could represent each situation by placing a check mark in the appropriate column.



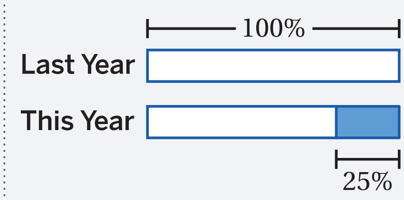


	Diagram 1	Diagram 2
This year's apple harvest is $\frac{3}{4}$ of last year's.		
There are $\frac{1}{4}$ more raspberries this year compared to last year.		
This year's plum harvest is 125% of last year's.		
This year's blueberry harvest is 75% of last year's.		
Compared to last year's strawberry harvest, this year's harvest is a 25% increase.		
Compared to last year, this year's peach harvest decreased 25%.		

8 Here's a situation from the previous problem.

If 80 pounds of peaches were harvested last year, how many pounds were harvested this year?

Explain your thinking.

Compared to last year, this year's peach harvest decreased by 25%.

Activity 3

Name: .... Date: .... Period: ....

Percent Practice

9 Order these values from *least* to *greatest*.

75% less than 60	25 more than 20	100% of 20	25% of 20	25% more than 20

Least **Greatest**

Explore More

10 Evan can use three coupons to buy a $25 shirt.
The store will apply these coupons one at a time.

Order the coupons so that Evan gets the lowest price.

$10 off	20% off	5% off

	First Coupon
	Last Coupon

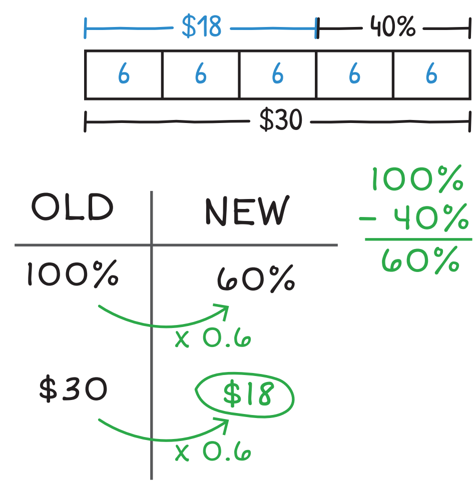
Explain your thinking.

11 Synthesis

Here are two different strategies to determine the new price of a hat after a discount of 40%.

The hat was originally $30.

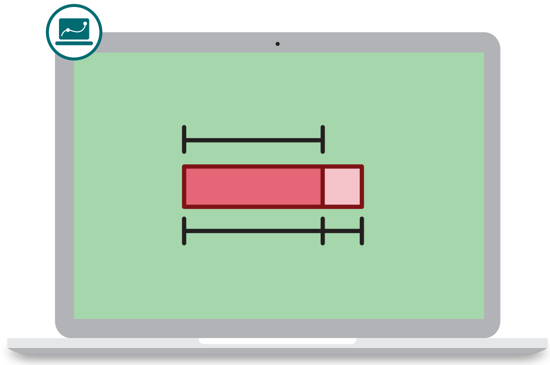
Discuss: How are these strategies alike and how they are different?



Things to Remember:

Unit 4
Lesson
3

Name: .... Date: .... Period: ....

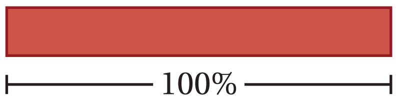


All the Equations

Let's use equations to represent percent increase or decrease.

Warm-Up

1 Here is a rectangle.



Which rectangle is 21% longer than the original rectangle?

A. 

B. 

C. 

D. 

Activity 1

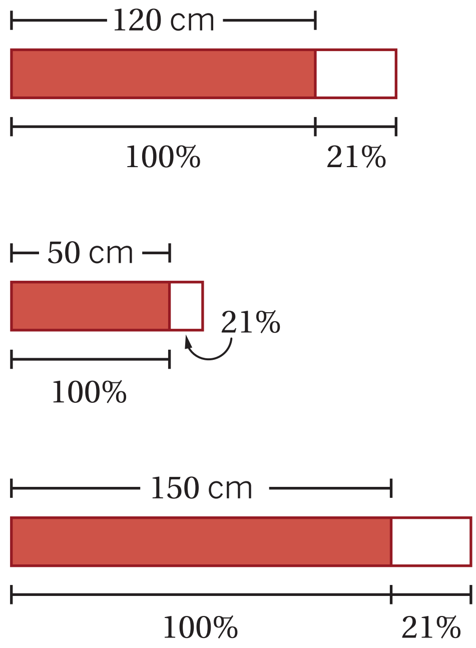
Name: ______ Date: ______ Period: ______

Percent Increase

Each new rectangle is 21% longer than the original.

2 Complete the table with the length of each new rectangle.

Original Rectangle Length (cm)	Length After 21% Increase (cm)
120	
50	
150	



3 We can use equations to represent relationships involving percent increase and decrease.

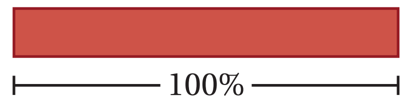
Write an equation to represent the relationship between the length of an original rectangle, b, and the length of a new rectangle, c.

$c =$ ______

Name: ______ Date: ______ Period: ______

Percent Decrease

4 Here is a rectangle.



Which shaded rectangle is 72% shorter than the original rectangle?

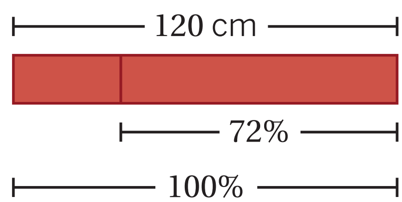
A. 

B. 

C. 

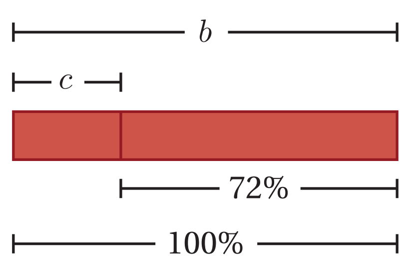
D. 

5 The original rectangle is 120 centimeters.



Calculate the length of a rectangle that is 72% shorter.

6 Select the *three* equations that represent the relationship between the length of the original rectangle, b, and the length of the new rectangle, c.



- ☐ **A.** $(100 - 72)b = c$
- ☐ **B.** $c = 0.28b$
- ☐ **C.** $0.72b = c$
- ☐ **D.** $(1 - 0.72)b = c$
- ☐ **E.** $c = b - 0.72$
- ☐ **F.** $c = 1b - 0.72b$

Activity 3

Name: ________ Date: ________ Period: ________

Percents and Equations

7 For each equation, put a check under the percent increase or decrease it represents. One equation will be neither.

Let b represent the original value and c represent the final value.

Equation	A Decrease of 88%	An Increase of 12%	Neither
$(1-0.88)b=c$			
$(1+0.12)b=c$			
$c=0.12b$			
$1b-0.88b=c$			
$0.88b=c$			
$c=1.12b$			
$c=1b+0.12b$			

8 Manuel paid off 88% of his debt. He originally owed $1,950. How much does Manuel owe now?

Use an equation from the column "A Decrease of 88%" if it helps with your thinking.

Explore More

9 Use the Explore More Sheet to answer a question about some magical goo.

10 Synthesis

Here are two equations that could be used to represent a problem about *percent increase* or *percent decrease*.

Circle an equation and write a story about a situation it could represent.

$1.25 \cdot b = c$

$0.80 \cdot b = c$

Things to Remember:

Name: .... Date: .... Period: ....

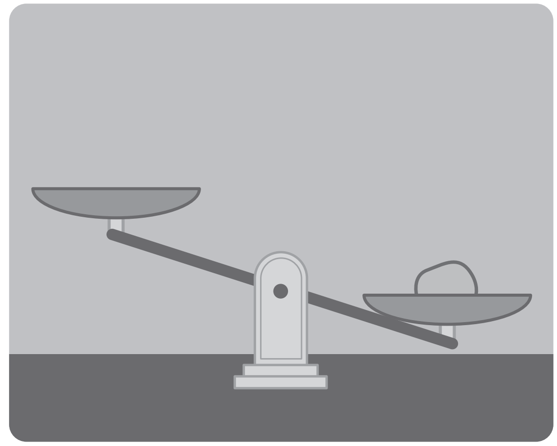
Explore More

An astronaut brought a 16-gram sample of goo from another planet to her laboratory.

When the goo is exposed to light, the amount of goo increases by the same percentage every hour.

Complete the table.

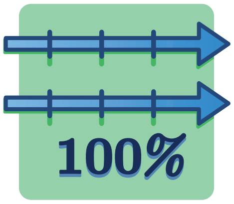
Time (hr)	Goo (g)
0	16
1	24
2	36
...	...
5	



Name: .... Date: .... Period: ....

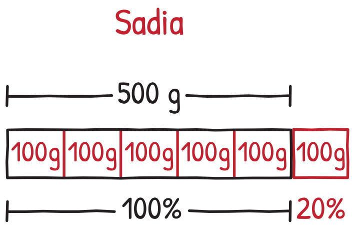
100%

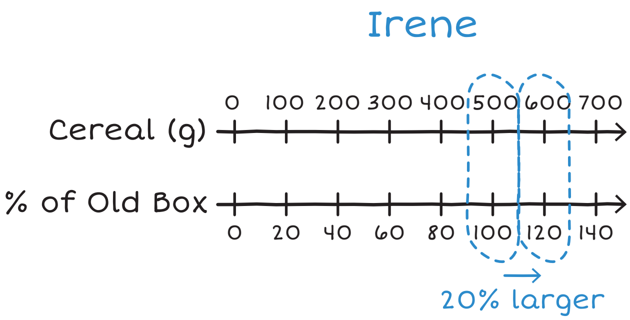
Let's make connections between double number lines and percent problems.



Warm-Up

1. A box of cereal used to weigh 500 grams. Now it's 20% larger. Sadia and Irene each worked to determine the new weight. What do you notice? What do you wonder?





I notice:

I wonder:

Activity 1

Name: Date: Period:

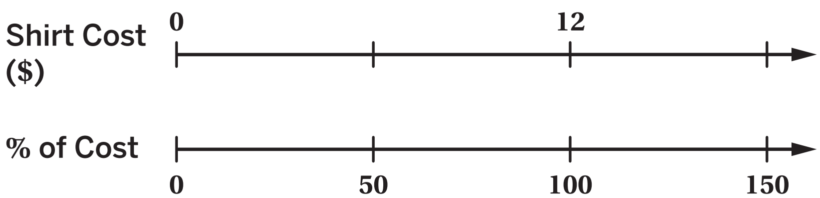
Double Number Lines

For each problem:

- Complete the double number line to show the percentages that correspond to the original amount and to the new amount.
- Answer the question.

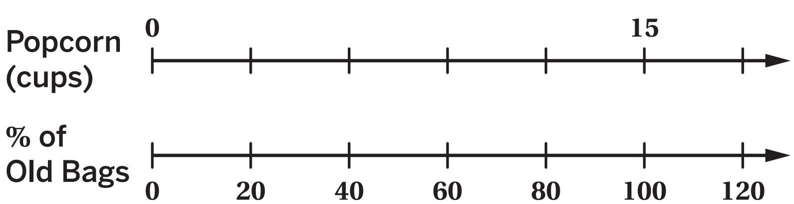
2. A store sells all of its clothes for 50% more than it costs to make (also called a “markup”).

If a shirt costs \$12 to make, what is the price of the shirt?



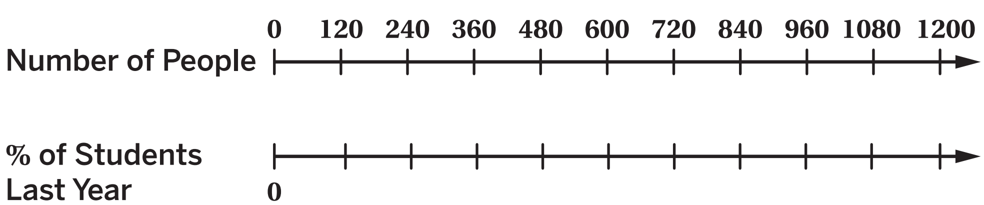
3. At the movie theater, the size of popcorn bags decreased by 20%.

If the original bags held 15 cups of popcorn, how much do the new bags hold?



4. A school had 1,200 students last year and only 1,080 students this year.

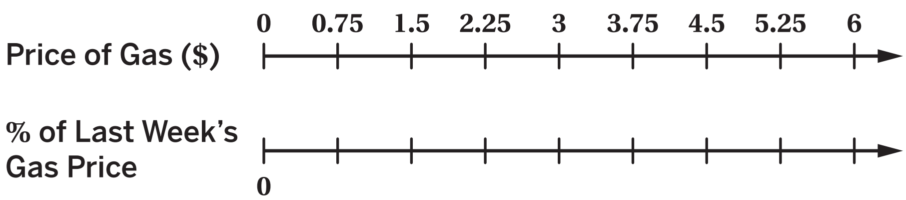
What was the percent decrease in the number of students?



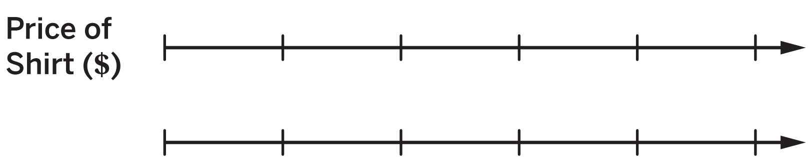
Name: .... Date: .... Period: ....

Double Number Lines (continued)

5. Last week, gas was $3.75 per gallon. This week, gas was $4.50 per gallon. What was the percent increase?



6. After a 20% discount, the price of a T-shirt is $24. What was the price before the discount?



7. A used car dealer has a 25% markup on its cars. If the dealer sells the car for $6,600 after the markup, what was the cost before the markup?

Activity 2

Name: ________ Date: ________ Period: ________

Green Sea Turtles

Some beaches are protected sanctuaries so that green sea turtles can come to shore to lay eggs without being disturbed.

This year, there are 234 nesting turtles at a sanctuary. This is 10% less than the number of nesting turtles at the same sanctuary last year.

8. Create at least two representations that show how many nesting turtles were at the sanctuary last year.

Double Number Line

Table

Equation

9. How many nesting turtles were at the sanctuary last year?

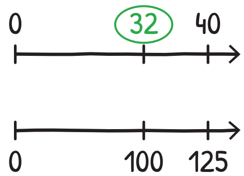
Explore More

10. If the population decreases another 10%, how many nesting turtles will be at the sanctuary next year? Explain your thinking.

Synthesis

11. A number increases by 25%. The new number is 40. Here are three strategies for finding the original number.

Double Number Line



Table

OLD	NEW
100%	× 1.25 → 125%
32	← ÷ 1.25 40

Equation

$$\frac{40}{1.25} = \frac{\cancel{1.25}^{1} \cdot b}{\cancel{1.25}}$$

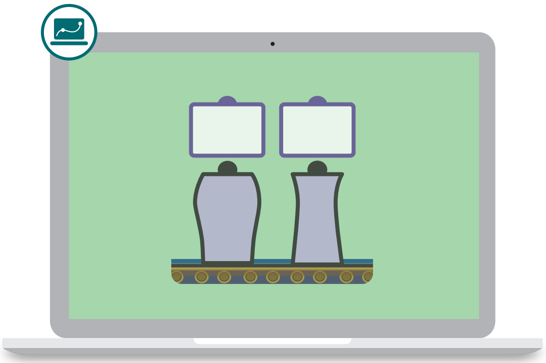
$$32 = b$$

Choose one of the strategies and explain it in your own words.

Things to Remember:

Unit 4
Lesson
5

Name: ________ Date: ________ Period: ________



Percent Machines

Let's explore problems with multiple percent changes.

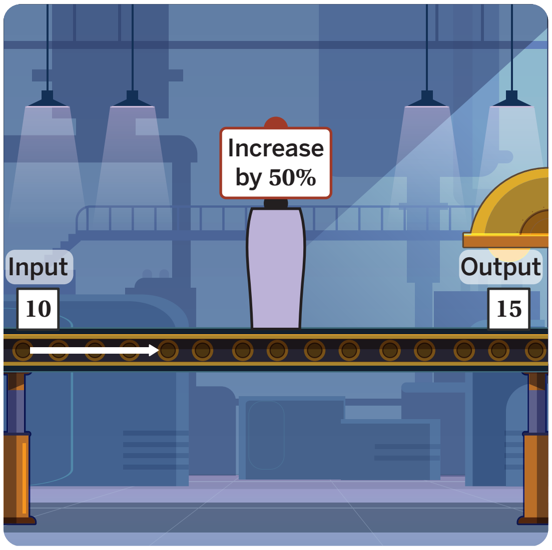
Warm-Up

1 This is a percent machine.

It takes an input and increases it by 50% to make an output.

What number is put into the machine?

What number comes out of the machine?



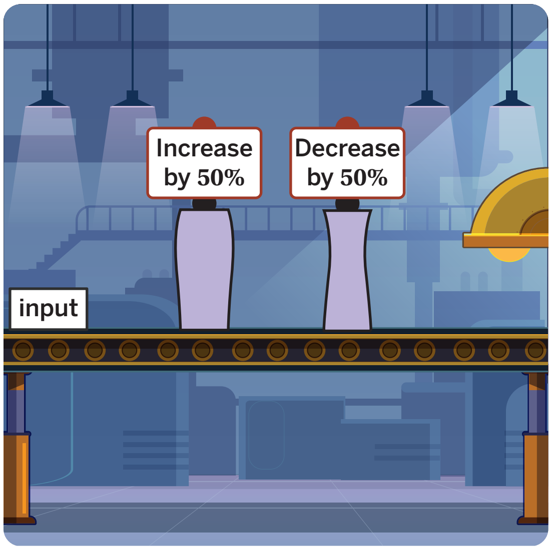
2 A machine was built to follow these rules:

- Increase by 50%
- Decrease by 50%

Do you think the machine's output will be less than, greater than, or equal to a positive input? Circle one.

Less than Greater than Equal to

Explain your thinking.

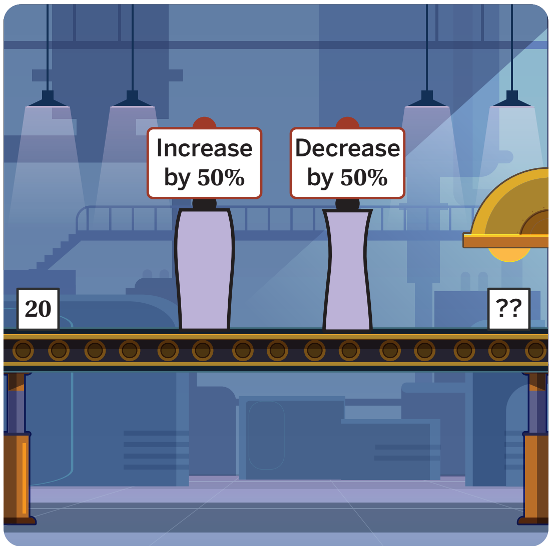


Activity 1

Name: ________ Date: ________ Period: ________

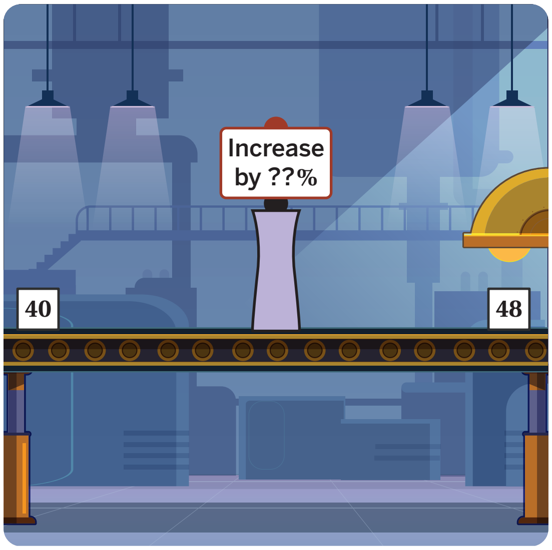
Putting Inputs In

3 If 20 is put into this machine, what will come out?



4 40 went into this machine and 48 came out.

What percent increase does this machine use?



5 Take a look at Rudra's and Ivory's strategies for determining the percent increase from the previous problem.

Discuss: How are Rudra's and Ivory's strategies alike? How are they different?

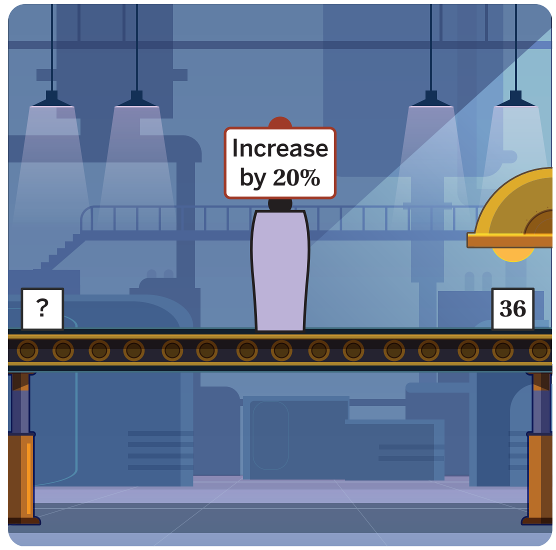
Activity 2

Name: ______ Date: ______ Period: ______

Getting Outputs Out

6 A number went into the machine and 36 came out.

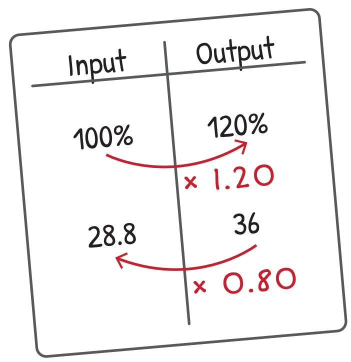
What number went in?



7 Kai used a table to solve the previous challenge but then made an error.

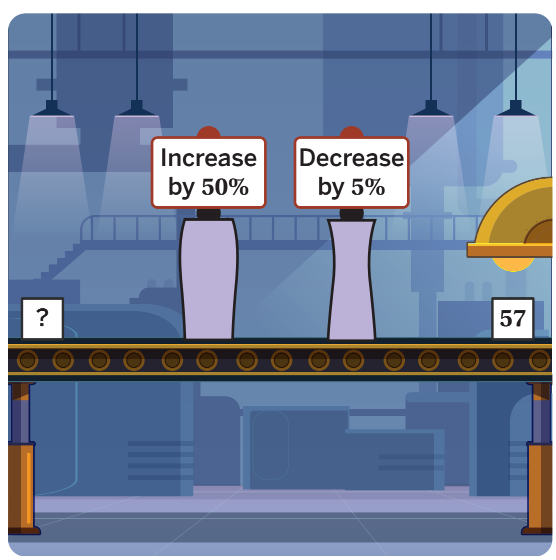
a Describe something Kai did well.

b Describe what you would change.



8 A number went into this machine and 57 came out.

What number went in?



Activity
3

Name: ____________ Date: ________ Period: ________

My Percent Machine

9 You will use the Activity 3 Sheet to create your own percent machine.

a **Make It!**

- On the activity sheet, create your own percent machine and choose an output.
- You and your classmates will determine the input that produces the output you chose for your machine.
- In this table, fill in the percent change(s) and the output for your machine. Then determine the input that will give you the output.

My Machine

Input	Step 1	Step 2 (optional)	Output

b **Swap It!**

- Share your machine with a partner. Look at each other's machines, but keep your own input a secret.
- What is your partner's input?
- As time allows, find more partners.

Partner 1's Machine

Input	Step 1	Step 2 (optional)	Output

Partner 2's Machine

Input	Step 1	Step 2 (optional)	Output

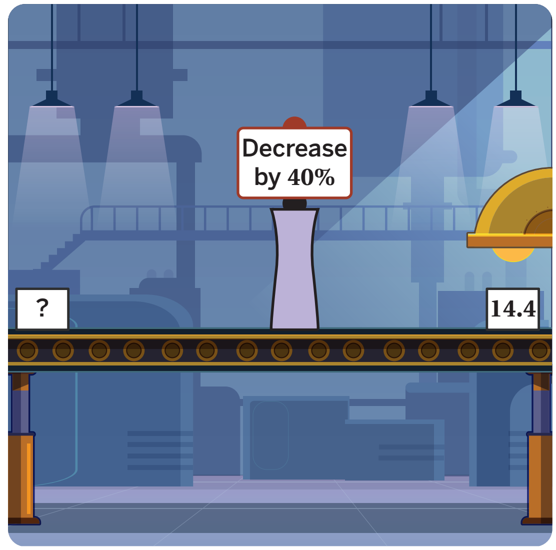
Partner 3's Machine

Input	Step 1	Step 2 (optional)	Output

10 Synthesis

Describe a strategy for determining the input when you know the percent change and the output.

Use this example if it helps you explain your thinking.



Things to Remember:

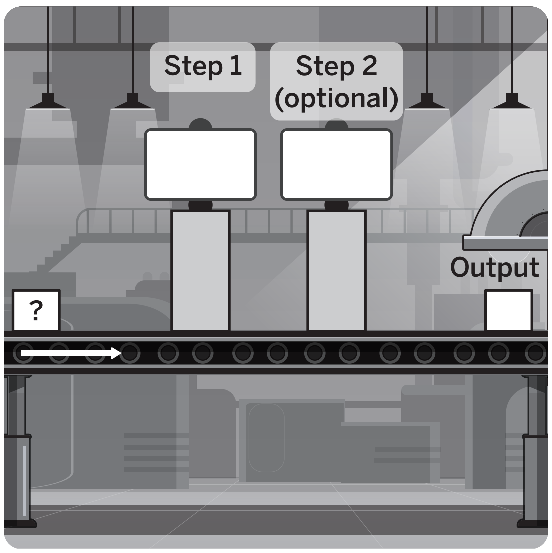
Name: ________ Date: ________ Period: ________

My Percent Machine

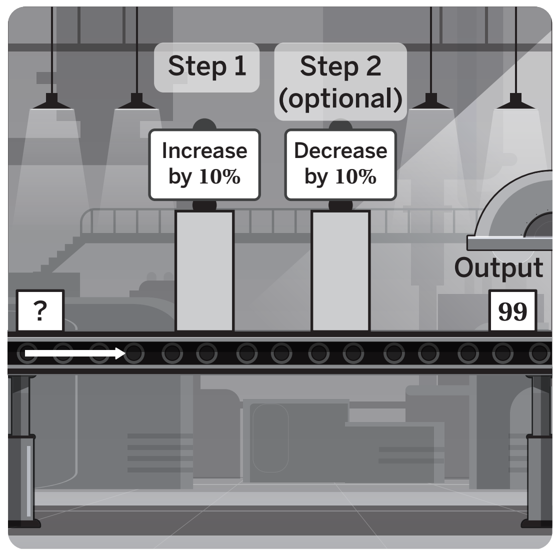
It's time to create your own percent machine!

- Choose whether your machine will have one or two steps.
- Write in the percent increase or decrease for each step. See the example.
- Write an output.
- Fill in the table for your machine in your Student Edition.

My Machine



Example



Unit 4
Lesson
7

Name: .... Date: .... Period: ....

Tax and Tip

Let's solve multistep percent problems involving sales tax and tip.

Warm-Up

1-2 Customers are reporting problems with the receipts at the Des-Cafe.

Soup:	$5.00
Tax:	+ $0.35
Total:	$5.35

Salad:	$8.00
Tax:	+ $0.56
Total:	$8.56

Pizza:	$18.00
Tax:	$
Total:	$

Sandwich:	$13.00
Tax:	$
Total:	$

Here's the information from the two receipts that printed out correctly.

What percent is the sales tax? Explain your thinking.

Item	Price	Tax	Total
Soup	5.00	0.35	5.35
Salad	8.00	0.56	8.56

Activity 1

Name: ________ Date: ________ Period: ________

Tax

3 The sales tax rate is 7%.

Determine the remaining values for these receipts.

Item	Price	Tax	Total
Soup	5.00	0.35	5.35
Salad	8.00	0.56	8.56
Pizza	18.00		
Sandwich	13.00		

Soup:	$\$5.00$
Tax:	$+\ \$0.35$
Total:	$\$5.35$

Salad:	$\$8.00$
Tax:	$+\ \$0.56$
Total:	$\$8.56$

Pizza:	$\$18.00$
Tax:	\$
Total:	\$

Sandwich:	$\$13.00$
Tax:	\$
Total:	\$

4 The Des-Cafe got a new cash register!

Write expressions that the cash register can use to determine the tax and total for any item.

Price	Tax	Total
$5.00	$0.35	$5.35
$8.00	$0.56	$8.56
c		

Soup:	$\$5.00$
7% Tax:	$+\ \$0.35$
Total:	$\$5.35$

Soup:	$\$8.00$
7% Tax:	$+\ \$0.56$
Total:	$\$8.56$

Activity 2

Name: ____________ Date: ________ Period: ________

Tip

5 Customers at restaurants usually leave a tip for the server. Sometimes restaurants add a tip automatically.

Write expressions that the cash register can use to determine an 18% tip and the total after tip.

Total Before Tip	18% Tip	Total After Tip
$5.35	$0.96	$6.31
$8.56	$1.54	$10.10
t		

Total Before Tip:	$5.35
18% Tip:	+ $0.96
Total After Tip:	$6.31

Total Before Tip:	$8.56
18% Tip:	+ $1.54
Total After Tip:	$10.10

6 Kiran and Ava had a bill for $100. The tax rate is 5% and they want to tip 20%.

- Kiran calculates the tip *after* tax is added to the bill.
- Ava calculates the tip *before* tax is added to the bill.

Whose strategy would result in a greater tip? Circle one.

Kiran's Ava's They're the same

Explain your thinking.

Kiran

Bill:	$100.00
5% tax:	$
20% tip:	$
Total:	$??.??

Ava

Bill:	$100.00
5% tax:	$
20% tip:	$
Total:	$??.??

Name: Date: Period:

Tip (continued)

7 A store offers a 20% off coupon.

The tax rate is 7.5% of the subtotal.

If an item is listed at $15, what is the total after the coupon and tax have been applied?

Price:	$15.00
20% Off Coupon:	$
Subtotal:	$
7.5% Tax:	$
Total:	$??.??

Explain your thinking.

Activity 3

Name: .... Date: .... Period: ....

Repeated Challenges

8 Find the total the customer pays for each receipt. Answer as many as you have time for.

a This meal's bill is $28.00.

A 7% sales tax is applied, followed by an automatic tip of 18% (on the after-tax amount).

Bill:	$28.00
7% Tax:	$
Total Before Tip:	$
18% Tip:	$
Total With Tip:	$?.??

b A store sells an item for $18.00.

A 45% off coupon is applied, followed by a 7% sales tax on the subtotal.

Price:	$18.00
45% Off Coupon:	$
Subtotal:	$
7% Tax:	$
Total:	$?.??

c A store sells an item for $31.00.

A 15% off coupon is applied, followed by a 7% sales tax on the subtotal.

Price:	$31.00
15% Off Coupon:	$
Subtotal:	$
7% Tax:	$
Total:	$?.??

d This meal's bill is $23.00.

A 7% sales tax is applied, followed by an automatic tip of 18% (on the after-tax amount).

Bill:	$23.00
7% Tax:	$
Total Before Tip:	$
18% Tip:	$
Total With Tip:	$?.??

Explore More

9 A store sells an item for $75.00.

Fatima uses a 20% off coupon and then tax is applied to the subtotal.

The total after coupon and tax is $75.00.

What is the tax rate? Explain your thinking.

Price:	$75.00
20% Off Coupon:	$
Subtotal:	$
?% Tax:	$
Total:	$75.00

10 Synthesis

Describe a strategy for calculating the total of an item after different discounts and tax rates.

Use the receipts if they help with your thinking.

Price:	$12.00
20% Discount:	− $2.40
Subtotal:	$9.60
5% Tax:	+ $0.48
Total:	$10.08

Price:	$12.00
15% Discount:	− $1.80
Subtotal:	$10.20
15% Tax:	+ $1.53
Total:	$11.73

Price:	$12.00
10% Discount:	− $1.20
Subtotal:	$10.80
8% Tax:	+ $0.86
Total:	$11.66

Price:	$12.00
5% Discount:	− $0.60
Subtotal:	$11.40
6.25% Tax:	+ $0.71
Total:	$12.11

Things to Remember:

Unit 4
Lesson
8

Name: .... Date: .... Period: ....

Plate Rate

Let's use percent change to analyze an issue in society.

Warm-Up

The minimum wage is the lowest amount of money a worker can earn per hour.

In 2023, the federal minimum wage was $7.25 per hour.

There is a different federal minimum wage for workers who receive tips, such as restaurant servers. In 2023, this was $2.13 per hour.

1. Which way would you prefer to be paid? Explain your thinking.

Activity
1

Name: .... Date: .... Period: ....

Waiting Tables

Here is some information about four servers who work at different restaurants: Laila, Tiana, Peter, and Julian. Choose *one* server to complete the activity.

- ☐ **Laila** is 35 years old. She is married and has two children. She has worked at the same restaurant for 7 years. She works 40 hours per week and makes \$2.13 per hour. In a typical week, she serves 75 tables. The average bill at the restaurant is \$41 per table, and she typically receives an 18% tip.
- ☐ **Tiana** is 25 years old. She lives with a roommate and a dog. She has worked at a fancy restaurant for 6 months. She works 40 hours per week and makes \$2.13 per hour. She usually serves 45 tables per week. The average bill at the restaurant is \$130 per table, and she typically receives a 20% tip.
- ☐ **Julian** is 29 years old. He is a single father with a 3-year-old son. He just finished his third year as a server. He works 40 hours per week and makes \$2.13 per hour. In a typical week, he serves 95 tables. The average bill at the restaurant is \$22 per table, and he typically receives a 15% tip.
- ☐ **Peter** is 19 years old. He lives at home with his parents and goes to college part-time. He recently started as a server, working 40 hours per week. Where Peter lives, the minimum wage for tipped workers is \$7.25 per hour. In a typical week, he serves 90 tables. The average bill at the restaurant is \$21 per table, and he typically receives a 15% tip.

2. How much money does your server make in a typical week? Show or explain your thinking.

3. Let's say that customers at your server's restaurant raise their tips by 5 percentage points. (So for example, 18% becomes 23%.)

How much would your server make now in a typical week? And by what *percent* would their pay increase? Show your thinking.

Name: Date: Period:

Waiting Tables (continued)

4. Now, let's say that customers at the restaurant *lower* their tips by 5 percentage points (for example, 18% becomes 13%).

How much would your server make now in a typical week? And by what percent would their pay decrease (compared to Problem 2)? Show your thinking.

Gabriel gets paid $\$11.75$ per hour to work in the kitchen. Unlike servers, restaurant workers in the kitchen do not usually receive tips.

5. How much money does Gabriel earn in a 40-hour week?

6. Compare Gabriel's pay to the server you chose. Consider the server's pay in a typical week as well as when tips are high or low. Use percentages in your comparison.

7. Some restaurants have experimented with different ways of paying workers. One restaurant pays all workers $\$16$ per hour, but doesn't allow tipping.

How do you expect people might feel about this policy? Consider how restaurant workers, restaurant owners, and customers might feel.

Activity
2

Name: Date: Period:

A New Way to Pay

Think about the three ways to pay servers that we've discussed so far:

- Servers get paid $2.13 per hour, plus tips.
- Servers get paid $7.25 per hour, plus tips.
- Servers get paid $16 per hour, with no tips.

8. Invent and describe a new way to determine a server's pay that you think is better. Explain your thinking.

9. Calculate what each of the four servers from Activity 1 would earn under your system. Show or explain your thinking.

Laila:

Tiana:

Julian:

Peter:

Explore More

10. Some restaurants have eliminated tipping and raised the price of menu items by 20%. They say this allows worker pay to be fairer. Do you think restaurants should do this? Explain your thinking.

Synthesis

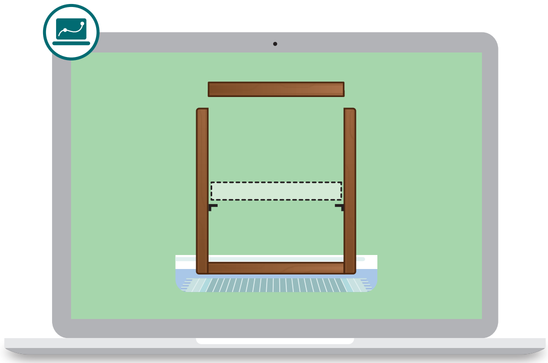
11. Describe how you can determine how much a restaurant server makes in a week. Use the example if it helps with your thinking.

> **Tiana** works 40 hours per week and makes $\$2.13$ per hour. She usually serves 45 tables per week. The average bill at the restaurant is $\$130$ per table, and she typically receives a 20% tip.

Things to Remember:

Unit 4
Lesson
9

Name: Date: Period:



Bookcase Builder

Let's explore situations where errors occur and represent those errors using percentages.

Warm-Up

1 Dakota and Ebony are buying deli meat.

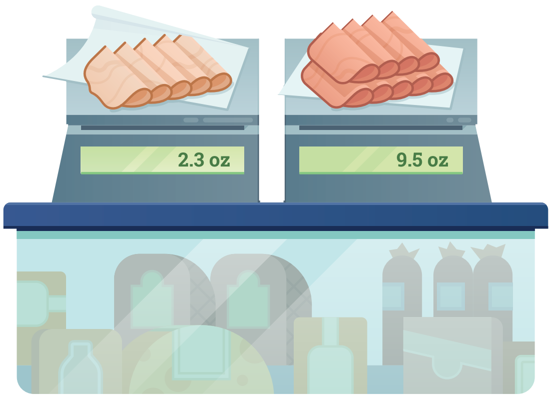
Dakota orders 2 ounces of ham, but ends up with 2.3 ounces.

Ebony orders 10 ounces of turkey, but ends up with 9.5 ounces.

Whose order had a bigger error? Circle one.

Dakota's order Ebony's order

Explain your thinking.



Activity 1

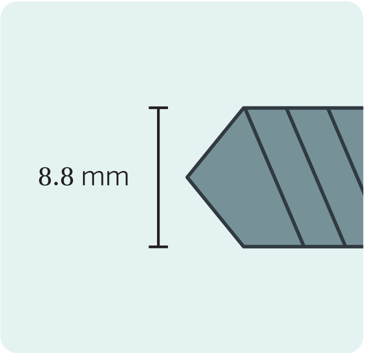
Name: ______ Date: ______ Period: ______

Bookcase Blunders

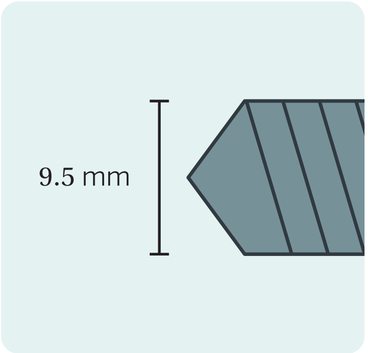
2 This bookcase was designed for 10-millimeter screws, but screws that are a little bigger or smaller are acceptable.

Let's watch the screen to find out which screws work for the bookcase.

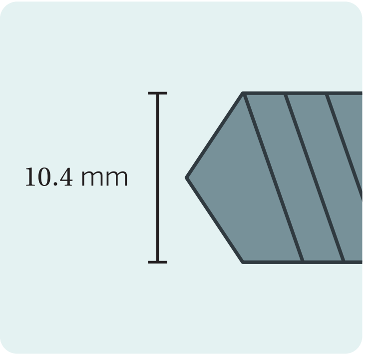
☐ **A.**



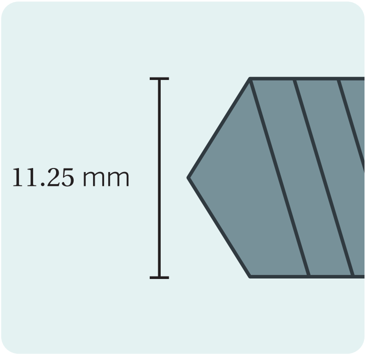
☐ **B.**



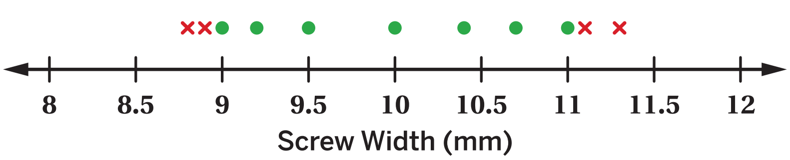
☐ **C.**



☐ **D.**



3 The screw widths that worked are represented with green dots on this number line. The widths that didn't work are represented with red X's.



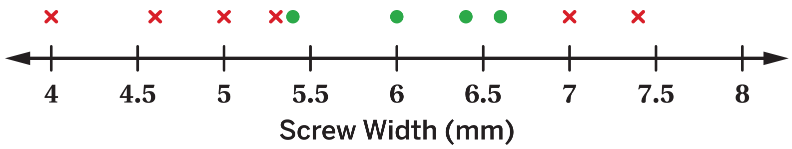
Discuss: What are the smallest and largest screws that will work?

Name: Date: Period:

Bookcase Blunders (continued)

4 Imagine a new bookcase designed for 6-millimeter screws.

Here is a number line that represents the widths of screws that work and don't work for the new bookcase.



What are the smallest and largest screw widths that work?

5 Here are the smallest and largest acceptable screw widths for the two previous bookcases.

How are the acceptable screw widths different in each row? How are they alike?

Desired Width (mm)	Smallest Acceptable Width (mm)	Largest Acceptable Width (mm)
10	9	11
6	5.4	6.6

Different:

Alike:

6 A factory tries to make a 6 mm screw. It ends up being 5.7 mm instead.

One way to describe the error is 0.3 mm.

Another way is to describe it as a **percent error**.

This screw has a percent error of $\frac{0.3}{6} = 5\%$.

Complete the table to decide if each screw will work (its percent error must be 10% or less).

Desired Width (mm)	Screw Width (mm)	Percent Error	Will It Work?
6	5.7	5%	Yes
10	10.9		
6	7.2		
15	13.8		

Activity 2

Name: Date: Period:

Challenge Your Shelf

7 Nikolai is making a shelf for a new bookcase.

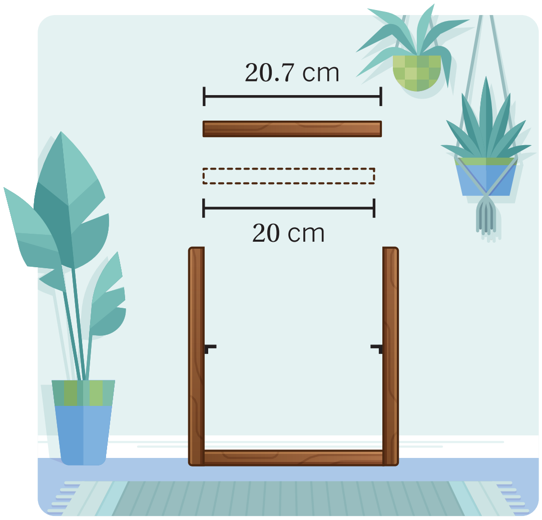
The shelf needs to be 20 centimeters long. It ends up being 20.7 centimeters long.

The acceptable percent error is 5% for a shelf to fit.

Will the shelf fit? Circle one.

Yes No

Explain your thinking.



8 Determine whether the shelf will fit given the acceptable error.

Answer as many as you have time for.

a
- Shelf length: 19.3 cm
- Desired shelf length: 20 cm
- Acceptable error: 5%

Will the shelf fit? Circle one.

Yes No

b
- Shelf length: 12 cm
- Desired shelf length: 15 cm
- Acceptable error: 5%

Will the shelf fit? Circle one.

Yes No

c
- Shelf length: 19.45 cm
- Desired shelf length: 18 cm
- Acceptable error: 5%

Will the shelf fit? Circle one.

Yes No

d
- Shelf length: 21 cm
- Desired shelf length: 20.5 cm
- Acceptable error: 10%

Will the shelf fit? Circle one.

Yes No

Activity 2

Name: .... Date: .... Period: ....

Challenge Your Shelf (continued)

e
- Shelf length: 20.6 cm
- Desired shelf length: 17.6 cm
- Acceptable error: 3%

Will the shelf fit? Circle one.

Yes No

f
- Shelf length: 17.2 cm
- Desired shelf length: 20.7 cm
- Acceptable error: 10%

Will the shelf fit? Circle one.

Yes No

g
- Shelf length: 25.9 cm
- Desired shelf length: 23.7 cm
- Acceptable error: 7%

Will the shelf fit? Circle one.

Yes No

h
- Shelf length: 21.8 cm
- Desired shelf length: 21.7 cm
- Acceptable error: 2%

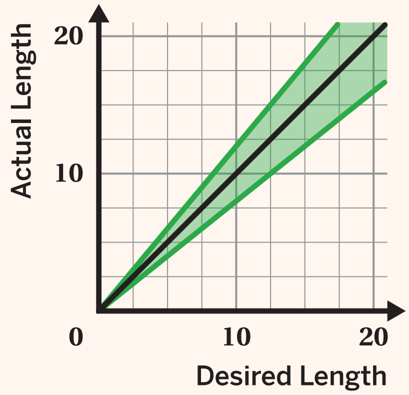
Will the shelf fit? Circle one.

Yes No

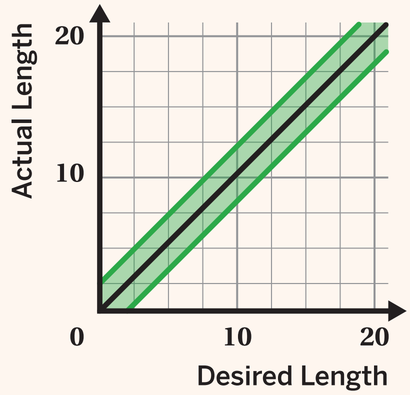
Explore More

9 Nikolai is building shelves for bookcases of different sizes and is allowing for a 25% error. Circle the graph where the green area represents shelves that will fit.

A.



B.



Explain your thinking.

10 Synthesis

Explain how to calculate percent error.

Use the examples if they help with your thinking.

Dakota

- Orders 2 ounces of ham
- Ends up with 2.3 ounces

Ebony

- Orders 10 ounces of turkey
- Ends up with 9.5 ounces

Things to Remember:

Unit 4

Lesson

10

Name: ______ Date: ______ Period: ______

Population & Pollution

Let's use what we know about percentages to generate and answer questions about our world.

Warm-Up

1 Here is some information.

Write a question that you could figure out using this information and whose answer is not already given.

There are about 336 million people living in the United States.

The U.S. makes up about 4.2% of the global population.

In 1900, the population of the U.S. was about 76 million people, and the population of the world was 1.6 billion people.

Activity 1

Name: ________ Date: ________ Period: ________

Pollution Percentages

2 Population increase is not the only change the planet has seen. Here is some information about two other topics.

Plastics

Plastic has countless uses, from household items to food packaging to medical instruments. But plastic has always been tricky to dispose of.

After use, plastic waste might be recycled or sent to a landfill; or it might be mismanaged, like being dumped into a body of water or burned.

Fast Fashion

Fast fashion is a business model where companies quickly produce new and low-priced clothes in large quantities.

To cut costs, the companies often use cheaper materials, such as polyester, and methods that have negative consequences for the environment and for workers.

Discuss:

- What do you know about each topic?
- What might you want to know?

3 **a** Select one topic. Read through some more information about that topic.

Plastics

- The amount of plastic waste generated in the U.S. grew by 400% from 1980 to 2019.
- In 1980, the U.S. recycled $20{,}000$ tons of plastic waste. In 2019, it was 3.1 million tons.
- In 2019, 73% of U.S. plastic waste went to landfills, 0.5% ended up in the ocean, and 4% was recycled.
- In 2019, the U.S. generated 72.8 million tons of plastic waste. This was 55% more waste than in 2000.
- In 2000, the U.S. mismanaged 3.6 million tons of plastic waste. In 2019, the number was 33% lower.

Fast Fashion

- Globally, 80 billion new items of clothing were bought in 2015. This was 400% more than was bought in 1995.
- In 2015, the fashion industry emitted 1.2 billion tons of carbon dioxide.
- Doubling the life of clothing from one year to two years reduces carbon dioxide emissions by 24%.
- In 2015, 60% of new clothes contained polyester. Clothes makers used 21.3 million tons of polyester, which was a 157% increase from 2000.
- The average person bought 60% more clothes in 2015 than in 2000.

Name: .... Date: .... Period: ....

Pollution Percentages (continued)

b Write two questions about the topic you selected that you could figure out using the given information and whose answer is not already given.

Question 1:

Question 2:

4 **Discuss** the questions you wrote with two partners:

- Would the answer to this question be interesting or useful?
- Can you answer this question using only the information given?
- Is the answer to the question not obvious from the information given?

Revise your questions after each conversation.

Partner 1 Conversation Notes	Revision of Both Questions
	Question 1: Question 2:
Partner 2 Conversation Notes	**Final Version of Both Questions**
	Question 1: Question 2:

Activity 2

Name: .... Date: .... Period: ....

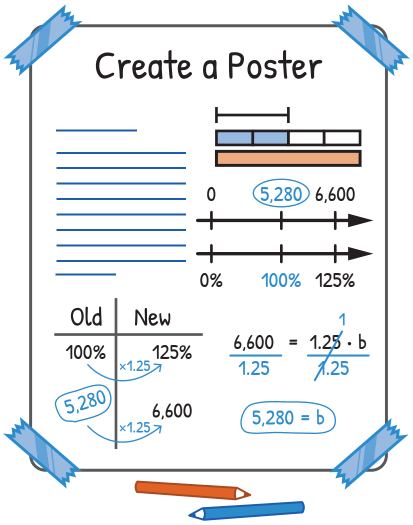
Make a Poster

5 Find a small group who chose the same topic. Discuss the questions you wrote and choose two to answer on a poster.

Create the poster. Here is what your poster should include:

(Check each item as you add it to your work.)

- ☐ A descriptive title
- ☐ The two questions you selected
- ☐ At least one equation or other representation (tape diagram, double number line, or table) that helps you answer your questions
- ☐ Your answers to each question (with units)
- ☐ An explanation of how you calculated each answer
- ☐ Two new questions that you have about this topic



You can use the space below for work or other calculations.

6 Look at your classmates' posters.

a What features of your classmates' posters helped you understand their thinking?

b Describe something you would change about your poster now that you have seen other groups' work.

7 Synthesis

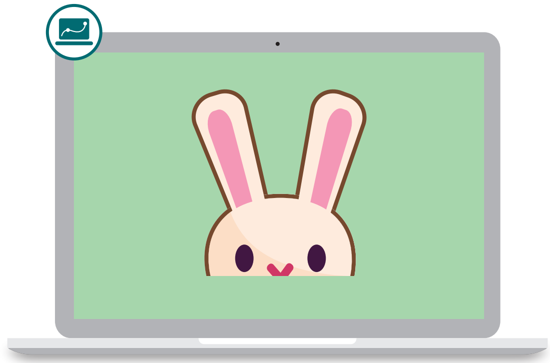
Discuss both questions. Then select one question and write your response.

- ☐ What did you learn about the topic you chose?
- ☐ What questions do you still have about your topic, pollution, or the population?

Things to Remember:

Unit 4
Lesson
11

Name: ________ Date: ________ Period: ________



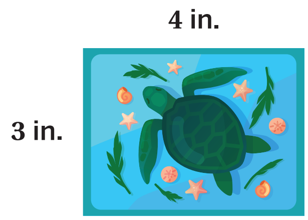
Sticker Sizes

Let's use constants of proportionality to determine unknown values in proportional relationships.

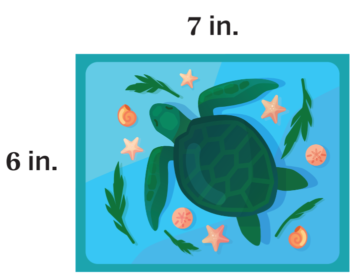
Warm-Up

1 StuckStickers is a company that makes stickers of all different sizes.

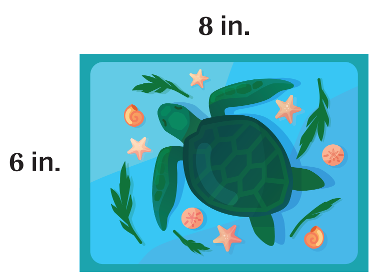
Which size will make a scaled copy of this logo?



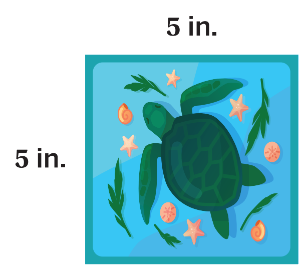
A. 6 in. by 7 in.



B. 6 in. by 8 in.



C. 5 in. by 5 in.



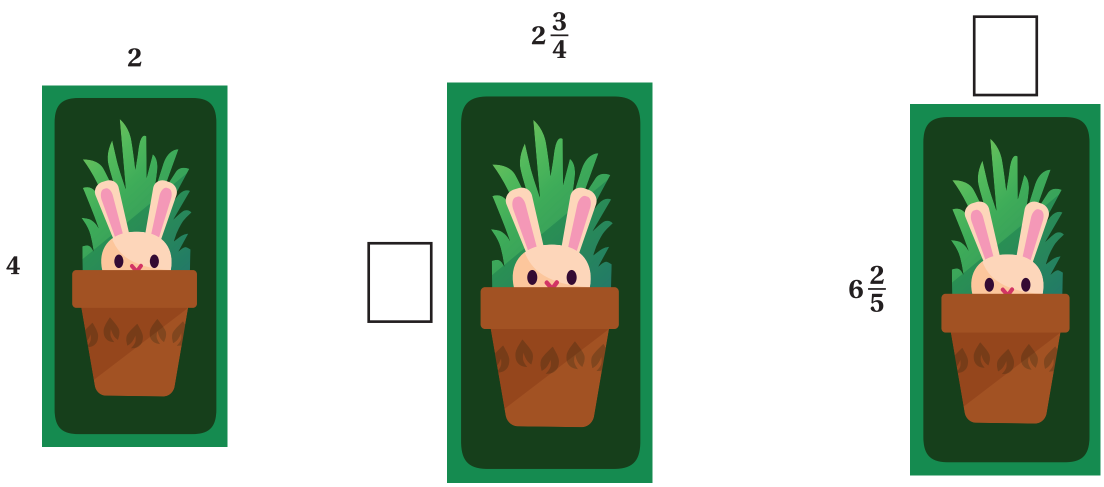
Explain your thinking.

Name: .... Date: .... Period: ....

Scaling Stickers

2 Here's a logo that Aditi is making into stickers.

a Enter the missing values so that each sticker is a scaled copy of the logo. (Measurements are in centimeters.)



b Describe your strategy.

3 Hamza wants to use this design to create a bumper sticker.

If the bumper sticker has a height of $3\frac{1}{2}$ inches, what will its width be?

Name: ______ Date: ______ Period: ______

Scaling Stickers (continued)

4 Polina and Jamir both used tables to determine the width of Hamza's bumper sticker.

Polina

Height (in.)	Width (in.)
2	5
$\cdot 1\frac{3}{4}$ $3\frac{1}{2}$	$\cdot 1\frac{3}{4}$

Jamir

Height (in.)	Width (in.)
2 $\xrightarrow{\cdot 2\frac{1}{2}}$	5
$3\frac{1}{2}$ $\xrightarrow{\cdot 2\frac{1}{2}}$	

Discuss each student's strategy.

5 Alex wants to use this logo to create a sticker.

If the sticker is $\frac{3}{5}$ inches wide, what does the height need to be?

Height (in.)	Width (in.)
3	$4\frac{1}{2}$
	$\frac{3}{5}$

3 in.

$4\frac{1}{2}$ in.

Activity 2

Name: .... Date: .... Period: ....

Sticker Sheets

6 Alex is thinking about buying stickers by the sheet. Four sheets cost $14.

How much would $11\frac{1}{2}$ sheets cost?

Number of Sheets	Total Cost ($)
4	14
$11\frac{1}{2}$	

7 Alex has $70 to spend on stickers.

How many sheets of stickers can Alex buy?

Number of Sheets	Total Cost ($)
4	14
	70

Name: .... Date: .... Period: ....

Sticker Sheets (continued)

8 Select *all* the equations that represent the relationship between the total cost, t, and number of sheets of stickers, s.

- ☐ **A.** $t = \frac{7}{2}s$
- ☐ **B.** $s = \frac{7}{2}t$
- ☐ **C.** $t = \frac{2}{7}s$
- ☐ **D.** $s = \frac{2}{7}t$

Explore More

9 It takes about $1\frac{1}{4}$ seconds for light to travel from Earth to the Moon.

The Moon is about $238{,}000$ miles from Earth.

The Sun is about $94{,}500{,}000$ miles from Earth.

Determine approximately how long it would take for light to get from the Sun to Earth.

10 Synthesis

Describe how to use a table of a proportional relationship to determine missing values.

Use the example if it helps with your thinking.

Height (in.)	Width (in.)
$2\frac{2}{3}$	$1\frac{1}{3}$
	$5\frac{1}{2}$

Things to Remember:

Name: ________ Date: ________ Period: ________

Peach Cobbler

Let's use a constant of proportionality to compare relationships that involve fractional quantities.

Warm-Up

Determine each quotient mentally. Be prepared to share your strategy.

1. $1 \div \frac{1}{3}$

2. $5 \div \frac{1}{3}$

3. $\frac{1}{2} \div \frac{1}{3}$

4. $2\frac{1}{2} \div \frac{1}{3}$

Activity 1

Name: ______ Date: ______ Period: ______

Which Recipe?

Amara and Callen want to make a peach cobbler that isn't too sweet. They have three recipes and are deciding which one to make.

Recipe A

Number of servings: 9

- $2\frac{7}{10}$ lb of peaches
- $\frac{1}{2}$ cups of butter
- 1 cup of flour
- $1\frac{1}{8}$ cups of sugar
- $\frac{1}{2}$ tsp of lemon juice

Recipe B

Number of servings: 12

- 4 lb of peaches
- $\frac{3}{4}$ cups of butter
- $\frac{3}{4}$ cups of flour
- $1\frac{1}{3}$ cups of sugar
- $\frac{1}{2}$ tsp of lemon juice

Recipe C

Number of servings: $4\frac{1}{2}$

- $1\frac{4}{5}$ lb of peaches
- $\frac{1}{4}$ cups of butter
- $\frac{2}{3}$ cups of flour
- $\frac{3}{4}$ cups of sugar
- 1 tsp of lemon juice

Amara and Callen want to make a recipe that isn't too sweet.

5. They think Recipe C will be the least sweet because it has the least amount of sugar. Do you agree? Explain your thinking.

6. Which recipe should they make? Explain your thinking.

7. Is the relationship between number of servings and total amount of sugar a proportional relationship? Explain your thinking.

Activity 2

Name: ________ Date: ________ Period: ________

Adjusting a Recipe

Jamar wants to make peach cobbler using Recipe B.

8. Determine how much of each ingredient Jamar needs for one serving. Show your thinking.

Recipe B

Number of servings: 12

- 4 lb of peaches
- $\frac{3}{4}$ cups of butter
- $\frac{3}{4}$ cups of flour
- $1\frac{1}{3}$ cups of sugar
- $\frac{1}{2}$ tsp of lemon juice

Single Serving of Recipe B

Number of servings: 1

- ________ lb of peaches
- ________ cups of butter
- ________ cups of flour
- ________ cups of sugar
- ________ tsp of lemon juice

9. Jamar plans to make just enough for 3 adults and 3 children. The children will eat less than the adults.

a. How many servings should he make? Explain your thinking.

b. Use the number of servings you chose to adjust Recipe B.

Recipe B for Jamar's Family

Number of servings: ________

- ________ lb of peaches
- ________ cups of butter
- ________ cups of flour
- ________ cups of sugar
- ________ tsp of lemon juice

10. Jamar has a measuring spoon that is $\frac{1}{8}$ teaspoons. How many spoonfuls of lemon juice does he need to make this recipe? Show or explain your thinking.

Synthesis

11. What are some strategies you can use to show that Haru's soup will have more carrots per pint than Mohamed's soup?

Mohamed's Vegetable Soup Recipe

$\frac{1}{3}$ of a cup of carrots for every

$\frac{1}{5}$ of a pint of broth

Haru's Vegetable Soup Recipe

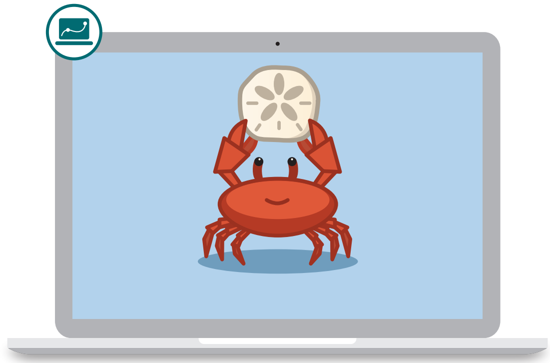
$\frac{7}{8}$ of a cup of carrots for every

$\frac{1}{3}$ of a pint of broth

Things to Remember:

Unit 7
Lesson
1

Name: ________ Date: ________ Period: ________



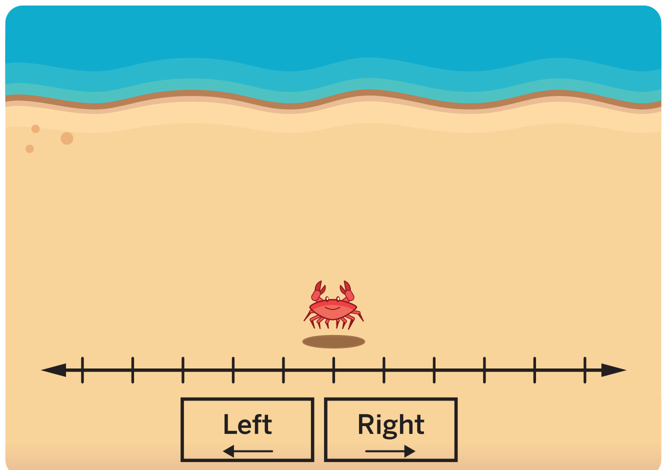
Can You Dig It?

Let's find the hidden sand dollars.

Warm-Up

1 a Let's help the crab look for two sand dollars.

b Write a clue to help someone else find them.



Activity 1

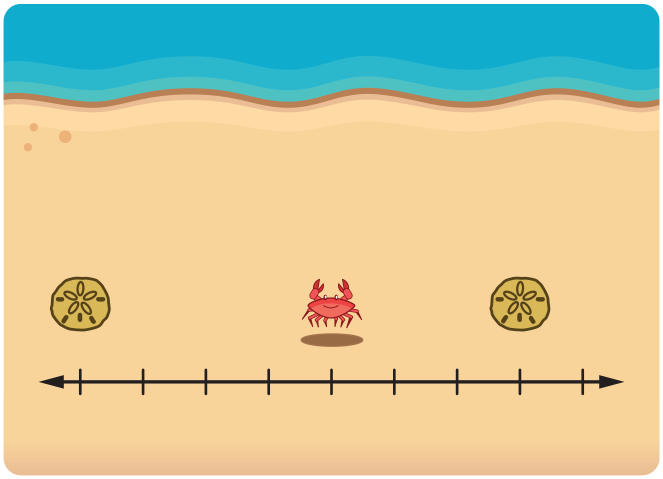
Name: .... Date: .... Period: ....

Finding Sand Dollars

2 Odalis and Kai each wrote correct clues.

Odalis: *One sand dollar is at positive* 3 *from the crab, and the other is at negative* 4*.*

Kai: *Go* 3 *steps to the right of the crab to find the first sand dollar, then* 7 *steps to the left to find the other one.*



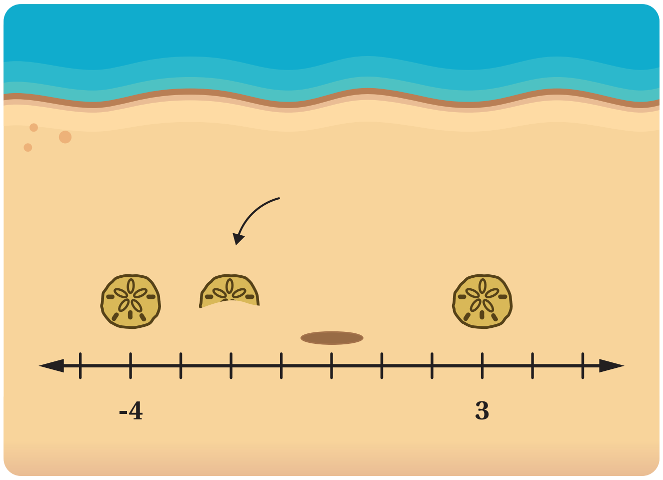
Discuss:

- How are their clues alike? How are they different?

- What do you think the "negative 4" in Odalis's clue means?

3 Here is a new sand dollar.

Write a clue that describes its location.



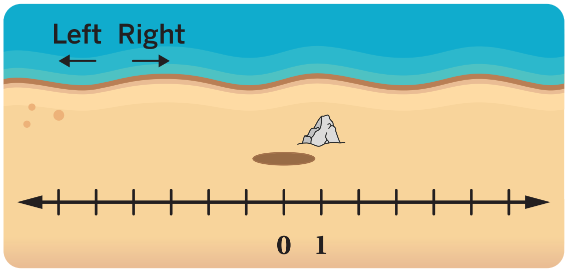
Activity 2

Name: ________ Date: ________ Period: ________

Sand Dollar Challenges

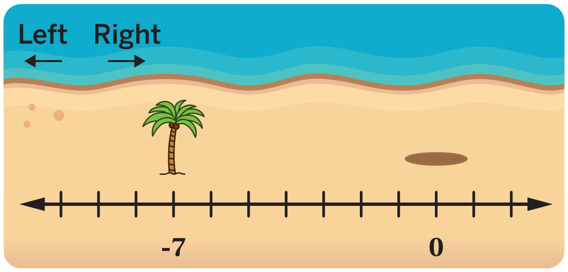
4 The rock is at 1 on the number line. A sand dollar is 4 units to the left of the rock.

Where is the sand dollar?



5 The palm tree is at -7 on the number line. A sand dollar is 3 units to the right of the palm tree.

Where is the sand dollar?



6 Odalis and Kai made mistakes on the previous problem.

Odalis said the sand dollar was at -5.
Kai said the sand dollar was at -10.

Circle your favorite mistake.

Odalis Kai

What could you tell this student to help them revise their answer?

Activity 3

Name: ______ Date: ______ Period: ______

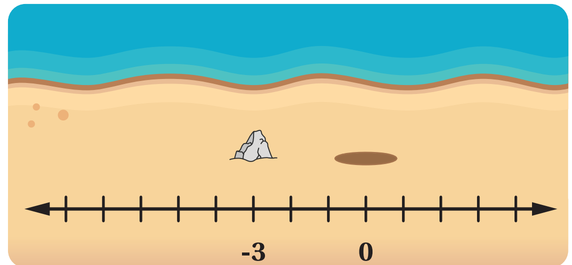
More Sand Dollar Challenges

7 The rock is at -3 on the number line. A sand dollar is 2 units away from the rock.

Lan says the sand dollar has to be at a **negative number**. Is she correct? Circle one.

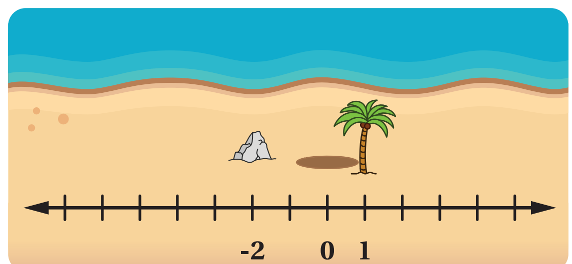
Yes No I'm not sure

Explain your thinking.



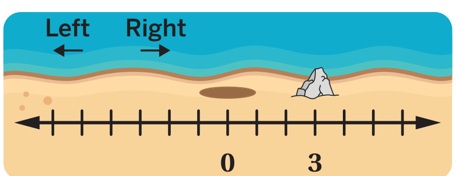
8 A sand dollar is 2 units away from the rock and 5 units away from the palm tree.

Where is the sand dollar?

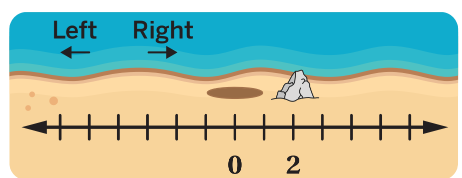


9 For each challenge, write the location of the sand dollar on the number line using the clue. Complete as many challenges as you can.

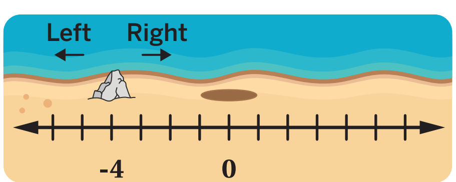
a 2 units to the right of the rock



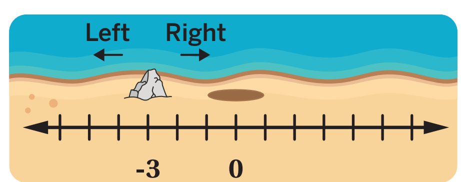
b 5 units to the left of the rock



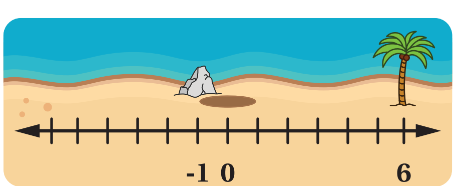
c 2 units to the left of the rock



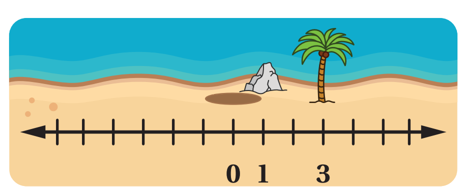
d 6 units to the right of the rock



e 3 units away from the rock and 4 units away from the palm tree



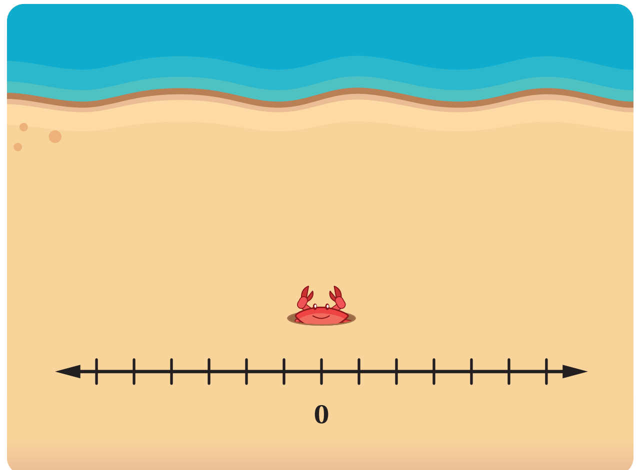
f 3 units away from the rock and 5 units away from the palm tree



10 Synthesis

List at least two things you know about positive and negative numbers on a number line.

Draw on the image if it helps to show your thinking.



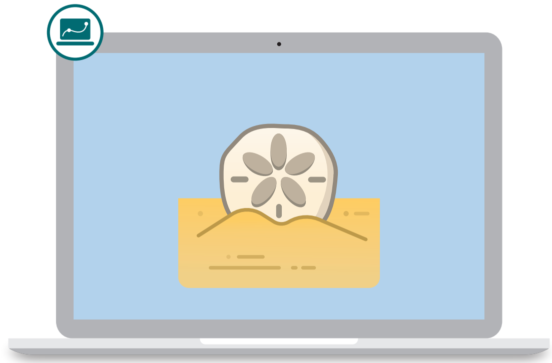
Things to Remember:

Unit 7
Lesson
2

Name: .... Date: .... Period: ....

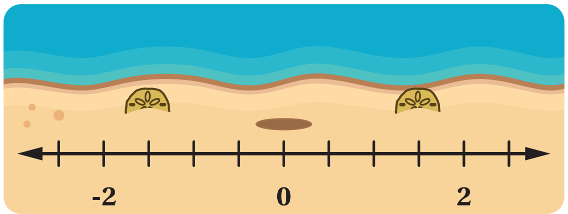
Digging Deeper

Let's plot positive and negative numbers on the number line.

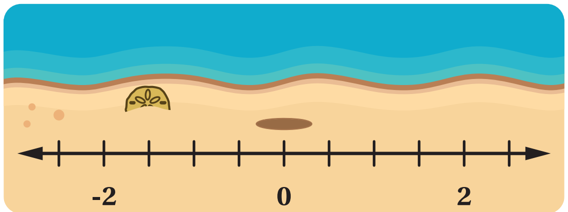


Warm-Up

1 Here are two buried sand dollars. Write clues to describe their locations.



2 Select *all* the numbers that describe the location of the sand dollar.



- ☐ **A.** $-\frac{3}{2}$
- ☐ **B.** -1.5
- ☐ **C.** $-1\frac{1}{2}$
- ☐ **D.** -2.5
- ☐ **E.** -3

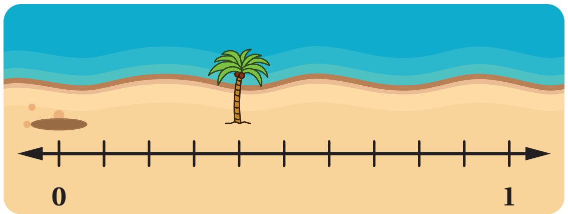
Activity 1

Name: ________ Date: ________ Period: ________

Rational Numbers and Their Opposites

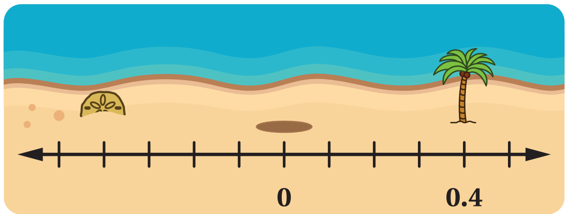
3 A sand dollar is buried under the tree.

Where is the sand dollar on the number line?



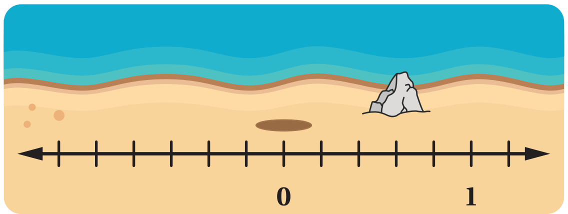
4 A new sand dollar is buried at the **opposite** of the palm tree.

What do you think opposite means here?



5 A new sand dollar is buried at the opposite of the rock.

Where is the new sand dollar?



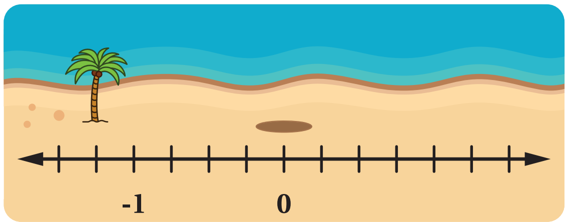
Activity 1

Name: .... Date: .... Period: ....

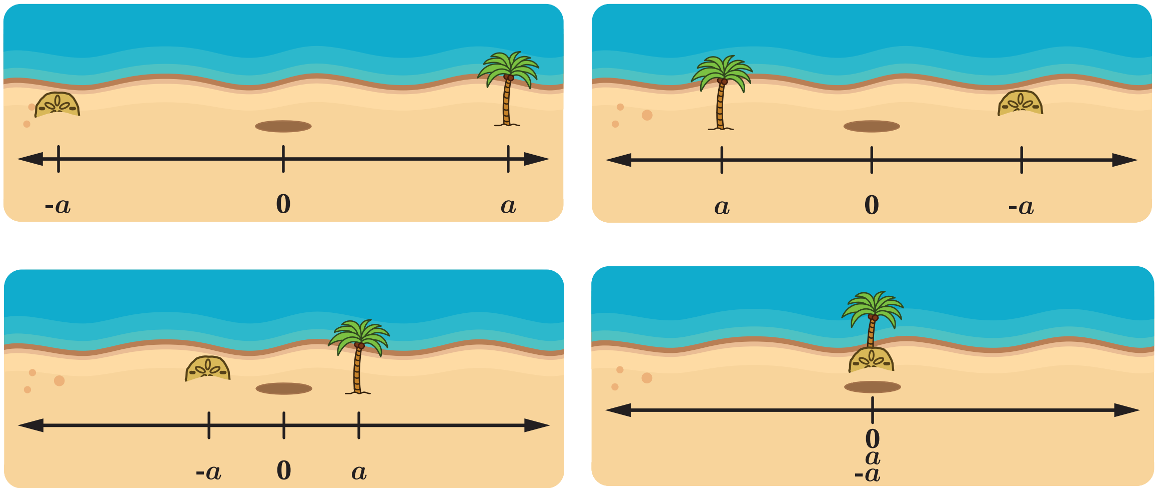
Rational Numbers and Their Opposites (continued)

6 A new sand dollar is buried at the opposite of the palm tree.

Where is the new sand dollar?



7 The palm tree is at a. The sand dollar is buried at the opposite of a, which you can write as $-a$. When a is in different locations, what do you notice about $-a$?



8 Pair opposite numbers together.

$-(-5)$	$\frac{3}{4}$	$\frac{4}{3}$	-0.75
$-1\frac{1}{3}$	-5	$\frac{1}{5}$	$-\frac{1}{5}$

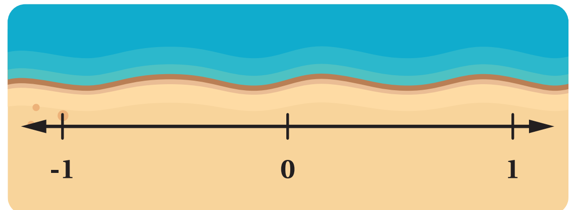
Pair 1	Pair 2	Pair 3	Pair 4

Activity 2

Name: ____________ Date: ________ Period: ________

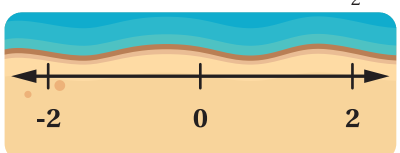
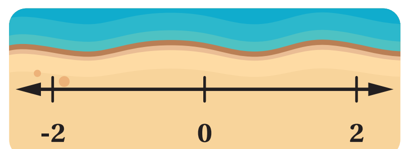
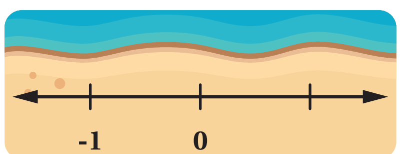
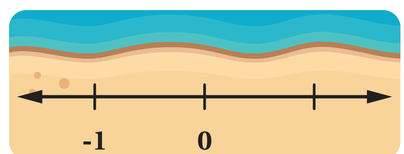
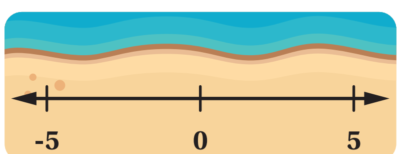
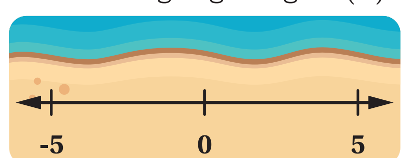
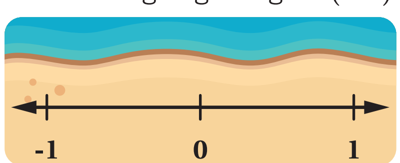
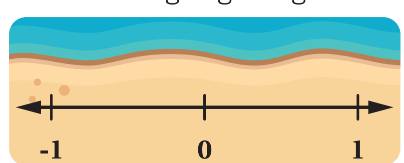
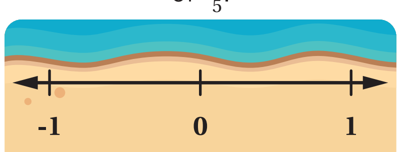
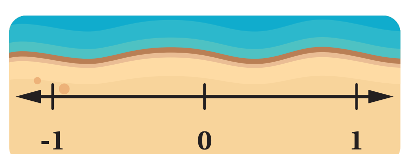
Rational Number Challenges

9 The crab is going to dig at $-\left(-\frac{3}{4}\right)$. Plot a sand dollar where the crab will find it.



10 Plot a point where the crab will dig to find each sand dollar.

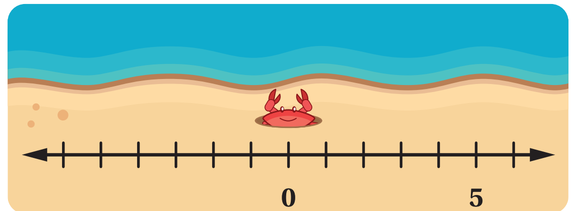
- Decide with your partner who will complete Column A and who will complete Column B.
- After plotting the location of each sand dollar, compare your solutions. The solutions in each row should be the same. Discuss and resolve any differences.

	Column A	Column B
a	The crab is going to dig at $\frac{3}{2}$. 	The crab is going to dig at 1.5. 
b	The crab is going to dig at -1.1. 	The crab is going to dig at the opposite of 1.1. 
c	The crab is going to dig at the opposite of -2. 	The crab is going to dig at $-(-2)$. 
d	The crab is going to dig at $-(-0.1)$. 	The crab is going to dig at 0.1. 
e	The crab is going to dig at the opposite of $-\frac{4}{5}$. 	The crab is going to dig at the opposite of -0.8. 

11 Synthesis

Describe where -3.2 is on a number line.

Draw on the image if it helps to show your thinking.



Things to Remember:

Name: .... Date: .... Period: ....

Order in the Class

Let's compare positive and negative numbers.

Warm-Up

1. Which one doesn't belong?

A. $+3 > \frac{5}{4}$

B. $-\frac{5}{4} < 3$

C. $\frac{5}{4} > -\frac{5}{4}$

D. $-\frac{5}{4} > -3$

Explain your thinking.

Name: ____________ Date: ________ Period: ________

Greater Than?

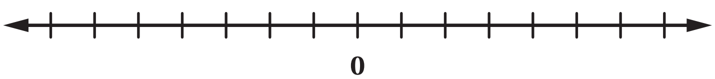
You will use your number card to compare numbers with your classmates.

My number: ____________

2. For each round, compare your number with different classmates' numbers. Use the number lines if they help with your thinking.

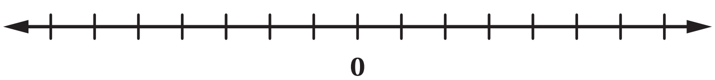
Round 1: Find a person whose number has a *different* **sign** than yours.

Partner's Name	Partner's Number	Comparison in Words	Comparison in Symbols
		____ is greater than ____.	____ > ____



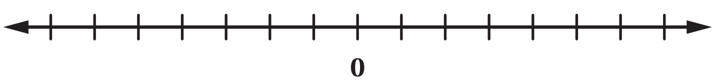
Round 2: Find a person whose number has the *same* sign as yours.

Partner's Name	Partner's Number	Comparison in Words	Comparison in Symbols
		____ is greater than ____.	____ > ____



Round 3: Find a person whose number is the opposite of yours.

Partner's Name	Partner's Number	Comparison in Words	Comparison in Symbols
		____ is greater than ____.	____ > ____



Activity
2

Name: ____ Date: ____ Period: ____

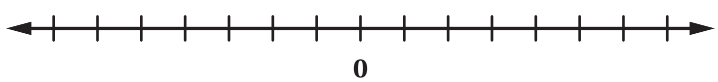
Least to Greatest

3. For each round, form a group of three or four people.

Round 4: Record the values from each of your cards: ____, ____, ____, and ____. Write three different inequalities comparing these numbers.

Inequality 1	Inequality 2	Inequality 3
____ $<$ ____	____ $<$ ____	____ $<$ ____

Round 5: Record the values from each of your cards: ____, ____, ____, and ____. Plot and label each number at its approximate location on the number line.



Round 6: Record the values from each of your cards: ____, ____, ____, and ____. Order the values from *least* to *greatest*.

Least **Greatest**

Round 7: Form a line with everyone in the class so that the numbers are in order from *least* to *greatest*.

Activity 3

Name: ______ Date: ______ Period: ______

Ordering Numbers

4. Here are four more numbers.

-2	$1\frac{1}{2}$	-1.5	-1

a Complete each statement using $>$ or $<$.

-2 ______ $1\frac{1}{2}$ -1.5 ______ -1

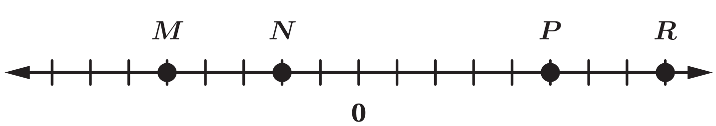
b Use two of the numbers to complete this sentence.

______ is the opposite of ______ because . . .

c Order the numbers from *least* to *greatest*.

Least **Greatest**

5. Use the number line to complete each sentence.



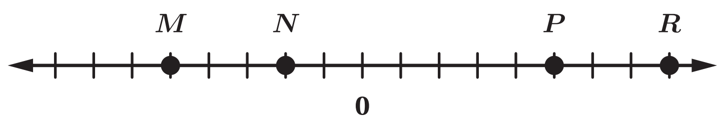
a ______ is the opposite of ______ because . . .

b ______ is greater than ______ because . . .

c ______ is the least of the numbers because . . .

Synthesis

6. Describe how to compare numbers on a number line. Use the example if it helps with your thinking.



Things to Remember:

Unit 7
Lesson
3

Name: ______ Date: ______ Period: ______

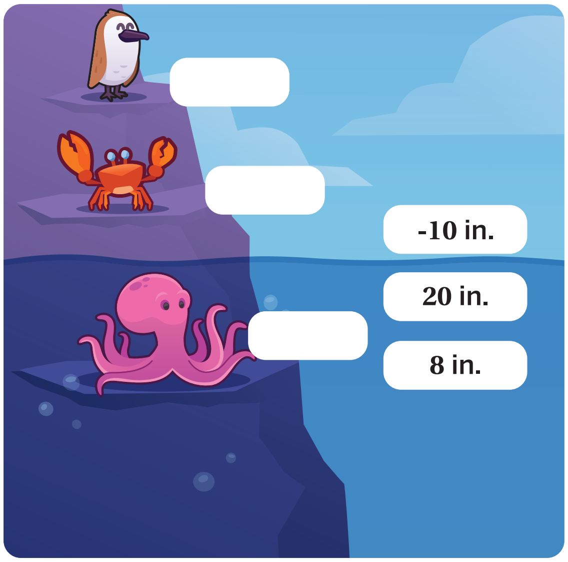
Sub-Zero

Let's use positive and negative numbers in context.

Warm-Up

1 Here are three animals sitting on rocks at different elevations.

Match each elevation with the correct rock.



2 What do you think 0 inches represents in this situation?

Activity

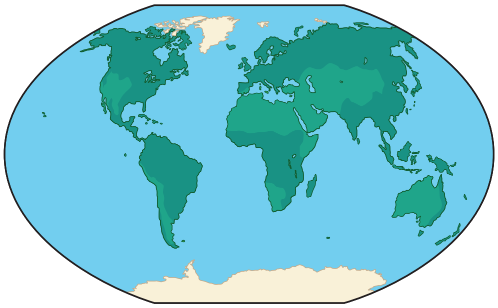
1

Name: Date: Period:

World Temperatures

3 Let's look at different temperatures around the world on one particular day.

Choose a location you'd like to visit and explain your choice to a classmate.

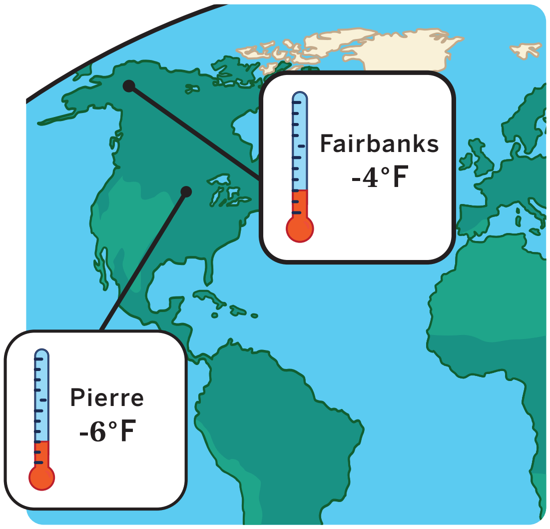


4 Neel says Fairbanks is warmer because $-4 > -6$. Yunuen says Pierre is warmer because 6 is greater than 4.

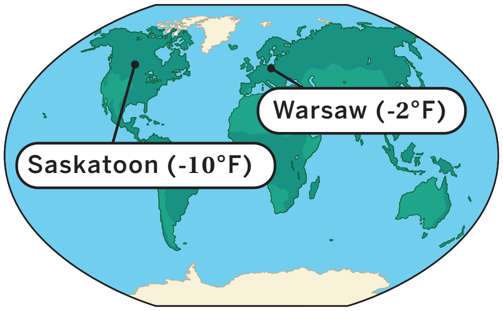
Whose thinking is correct? Circle one.

Neel's Yunuen's Both Neither

Explain your thinking.



5 Write a temperature that is colder than Warsaw's and warmer than Saskatoon's.

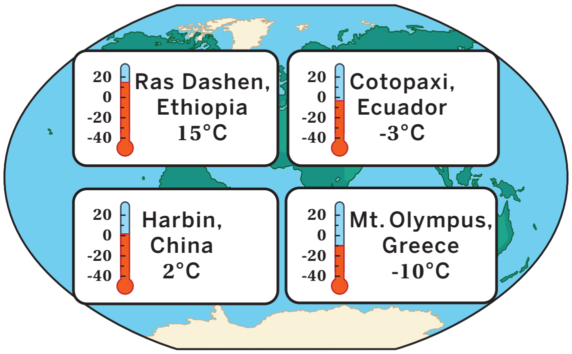


Activity 2

Name: Date: Period:

Colder Temperatures

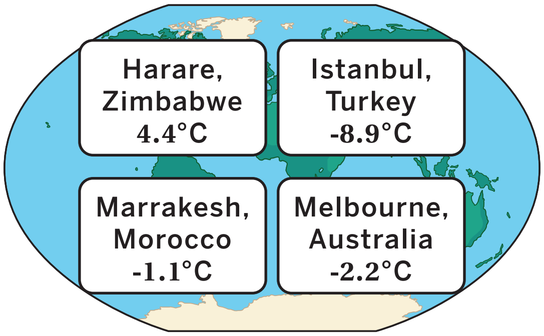
6 Here are the temperatures of different places around the world on another day.



Select *all* of the true statements.

- ☐ **A.** Ras Dashen is warmer than Cotopaxi.
- ☐ **B.** Cotopaxi is -3 degrees below 0.
- ☐ **C.** Mt. Olympus is the coldest.
- ☐ **D.** Mt. Olympus is 12 degrees colder than Harbin.
- ☐ **E.** Cotopaxi is colder than Mt. Olympus.

7 Here are some of the coldest recorded temperatures for four cities.



Order these temperatures from *warmest* to *coldest*.

Warmest 🔥

Coldest ❄️

Activity 2

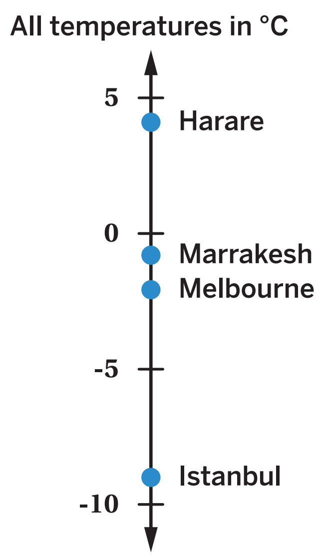
Name: .... Date: .... Period: ....

Colder Temperatures (continued)

8 This number line shows some of the lowest recorded temperatures for the same four cities.

One of the lowest recorded temperatures in Rome, Italy, is -7.2°C.

Plot the point for Rome at its approximate location on the number line.



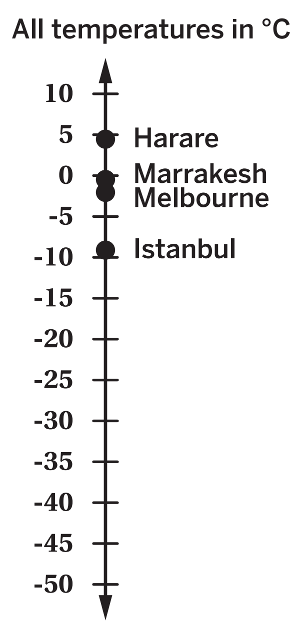
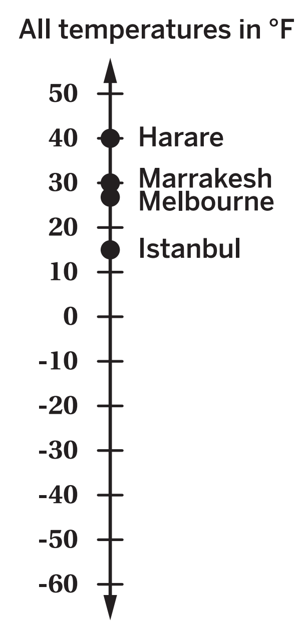
9 One way to write that Harare's temperature is warmer than Marrakesh's is $4.4 > -1.1$.

Write your own *inequality* comparing Rome's temperature to another city's temperature.

Explore More

10 Look up the lowest recorded temperature in a city near you.

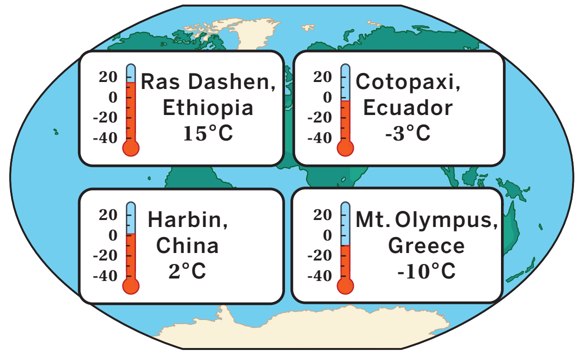
Plot and label a point to represent this temperature on one of the number lines.

Celsius	Fahrenheit
	

11 Synthesis

When comparing two temperatures, how can you tell which is warmer and which is colder?

Use the examples if they help with your thinking.



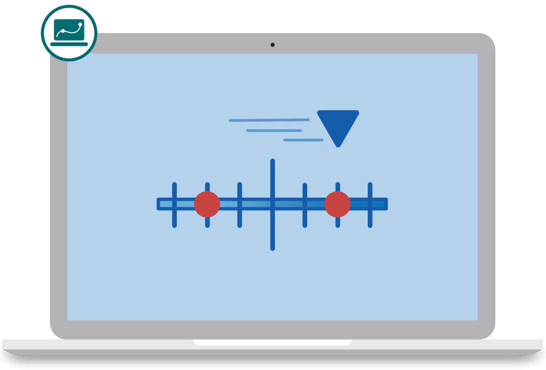
Things to Remember:

Unit 7
Lesson
5

Name: ______ Date: ______ Period: ______

Distance on the Number Line

Let's explore the absolute value of rational numbers.



Warm-Up

1 Let's play a game to get a score.

Discuss: How do you think scores are determined?

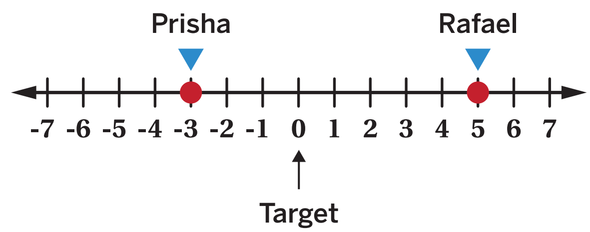
Activity 1

Name: ____________ Date: ________ Period: ________

What Is Absolute Value?

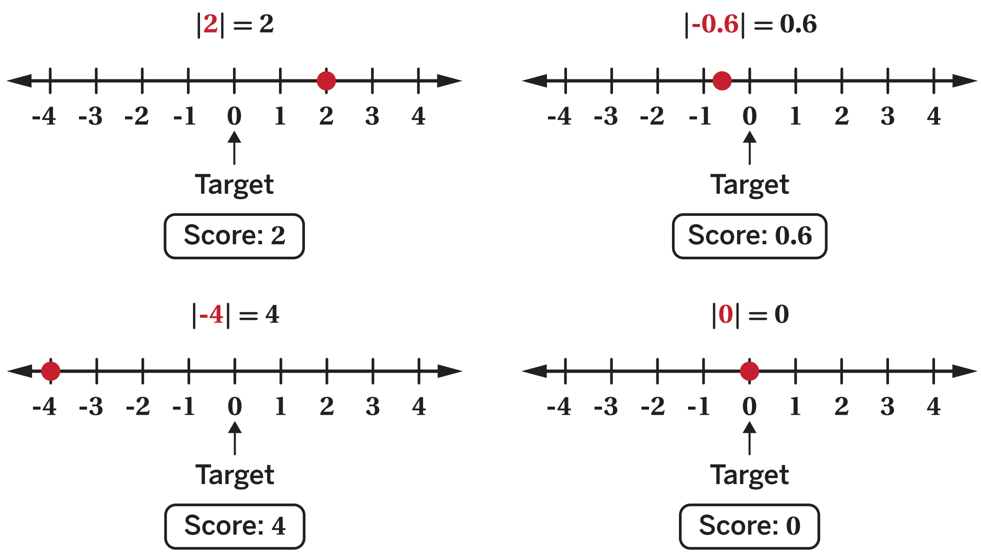
2 Prisha landed at -3. Rafael landed at 5. What was each student's score?

Student	Value
Prisha	
Rafael	



3 The game uses **absolute value** to calculate the score. We read |-2| as "the absolute value of negative 2."

a Take a look at some scores for different stopping points.



b Describe what you think absolute value means.

4 Prisha says $|x|$ means "the opposite of x." Rafael says $|x|$ means "how far x is from zero." Whose thinking is correct? Circle one.

Prisha's　　Rafael's　　Both　　Neither

Explain your thinking.

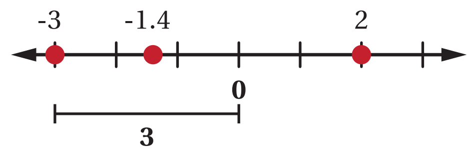
Activity
2

Name: ______ Date: ______ Period: ______

Comparing Absolute Values

5 Determine the value of each expression. Use the number line if it helps with your thinking.

Expression	Value
$\lvert -3 \rvert$	3
$\lvert -1.4 \rvert$	
$\lvert 2 \rvert$	

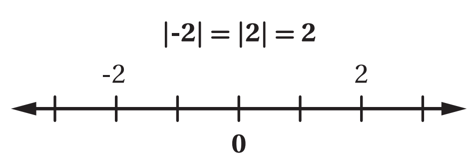


6 Determine whether each statement is true or false.

Statement	True	False
$\lvert 0 \rvert = 0$		
$\lvert 6 \rvert = -6$		
$\lvert 6 \rvert > \lvert -6 \rvert$		
$-5 < -4$		
$\lvert -5 \rvert < \lvert -4 \rvert$		
$\frac{3}{2} > -\frac{1}{2}$		
$\left\lvert \frac{3}{2} \right\rvert > \left\lvert -\frac{1}{2} \right\rvert$		

7 Isaiah says -2 and 2 have the same absolute value.

Explain why Isaiah's claim is correct.



Activity 3

Name: ________ Date: ________ Period: ________

Absolute Value Puzzles

8 a) Make a true inequality by using each number at most once.

$$|\square| > \square$$

-2 -1 1 2

b) Explain how you know your inequality is true.

9 Make true statements by using each number exactly once.

$$|\square| > \square$$

$$\square = |\square|$$

3 4 -4 -5

10 Make true statements by using each number exactly once.

$$\square > |\square|$$

$$|\square| < \square$$

$$|\square| = \square$$

-1 -2 -4

2 3 4

Activity 3

Name: ______ Date: ______ Period: ______

Absolute Value Puzzles (continued)

11 Is it possible to make true statements using each number exactly once? Circle one.

Yes   No   I'm not sure

Explain your thinking.

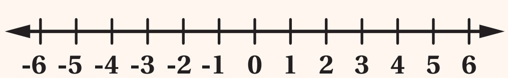
$$|\square| = \square$$

$$\square > |\square|$$

-5   6   -6   -7

Explore More

12 Use these clues to determine the values of A, B, and C. Use the number line to record your answers and to help you with your thinking.

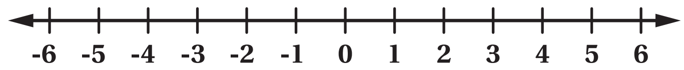


Clues

- The absolute value of A is 2.
- B is greater than A.
- Point B is closer to zero than point A is.
- C is positive.
- The distance between A and B is 1.
- The distance between B and C is 4.

13 Synthesis

Explain 2–3 things you know about absolute value. Use the number line if it helps to show your thinking.



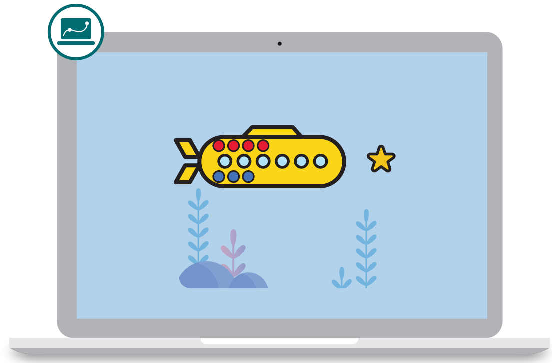
Things to Remember:

Unit 5
Lesson
1

Name: ______________________ Date: __________ Period: __________

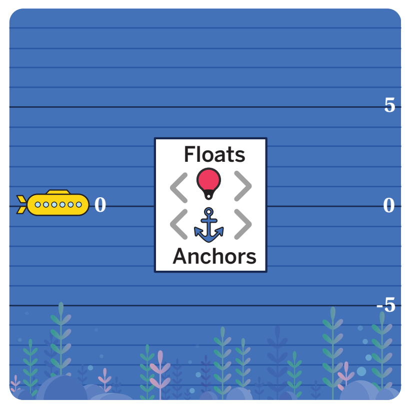
Floats and Anchors

Let's use floats and anchors to represent values on a number line.



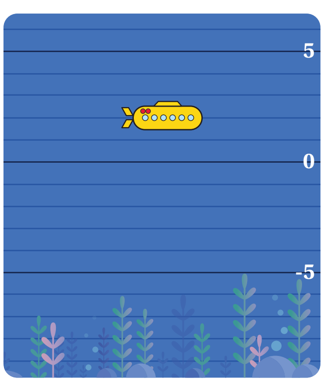
Warm-Up

1 This submarine is controlled by floats and anchors.

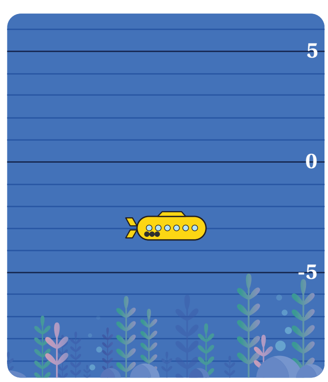


a Take a look at these different combinations.

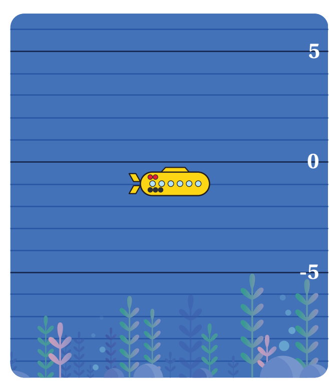
Add 2 floats



Add 3 anchors



Add 2 floats, Add 3 anchors



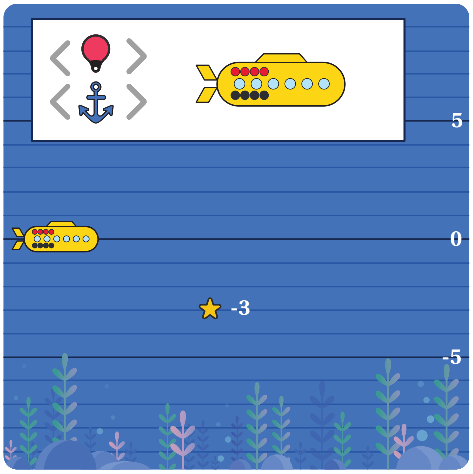
b **Discuss:** What do you notice? What do you wonder?

Activity 1

Name: .... Date: .... Period: ....

Collect the Star

2-3 This submarine starts with 4 floats and 4 anchors.

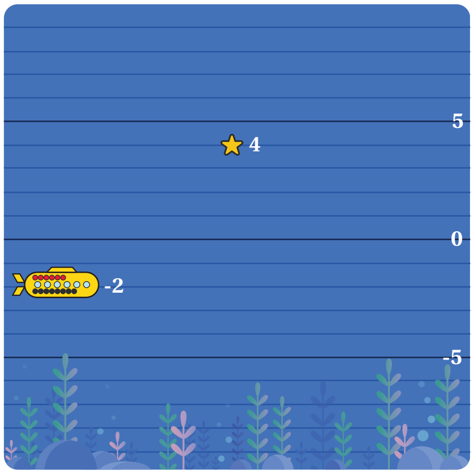


a. **Discuss:** Why do you think this submarine's current position is at 0 units?

b. The table shows one way to move the submarine to -3 to get the star. Write three more actions to get the submarine to -3.

Start	Action	Final
0	Remove 1 float Add 2 anchors	-3
0		-3
0		-3
0		-3

4 This submarine starts with 6 floats and 8 anchors.



The submarine has space for up to 10 floats and up to 10 anchors.

Write an action that could move the submarine to 4 units to collect the star.

Start	Action	Final
-2		4

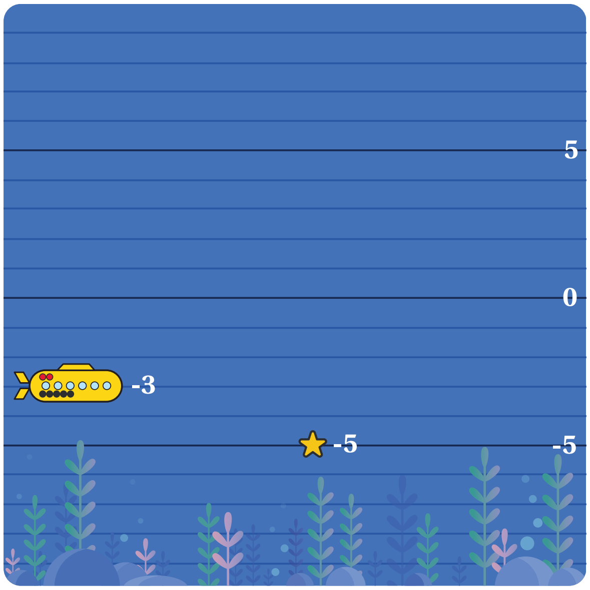
Name: ____________ Date: ________ Period: ________

Collect the Star (continued)

5 This submarine starts at -3 units.

Select *all* the actions that would move it to -5 units.

- ☐ **A.** Add 2 floats
- ☐ **B.** Add 2 anchors
- ☐ **C.** Remove 1 float and add 1 anchor
- ☐ **D.** Add 3 floats and add 5 anchors
- ☐ **E.** Remove 2 floats and add 4 anchors



6 Imagine a new submarine. For each action, put a check for whether the submarine would go up, go down, or stay in the same position.

Action	Up	Same Position	Down
Add 3 floats Add 4 anchors			
Remove 10 anchors			
Remove 5 floats Remove 5 anchors			
Add 8 floats Remove 8 anchors			
Add 6 floats Add 2 anchors			
Remove 7 floats Add 3 anchors			

Activity
2

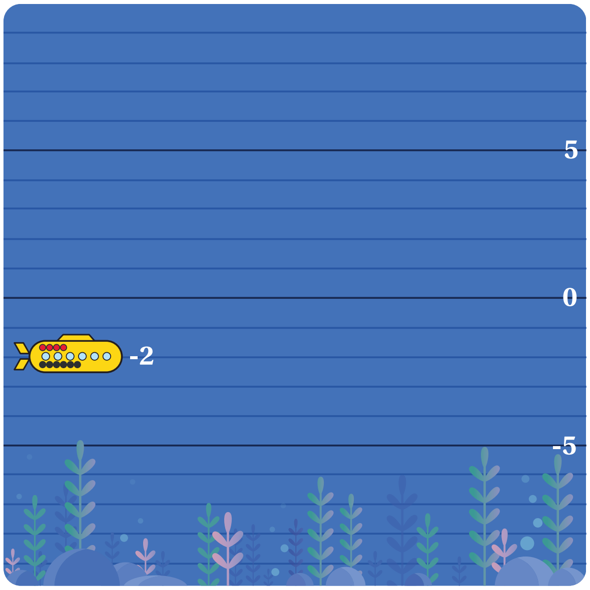
Name: ________ Date: ________ Period: ________

Sea-king Stars

7 The table shows the submarine's starting position and the action that will change its position.

What will be the submarine's final position?

Start	Action	Final
-2	Add 3 floats Remove 5 anchors	



8 Crow and Mai wrote expressions to answer the previous question.

Crow's expression: $-2 + 3 - 5$

Mai's expression: $-2 + 3 - (-5)$

Who wrote a correct expression? Circle one.

Crow Mai Both Neither

Explain your thinking.

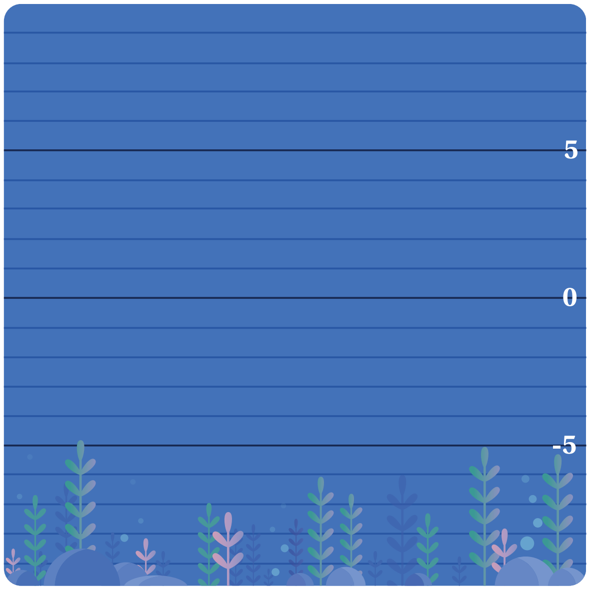
Activity 3

Name: ________ Date: ________ Period: ________

Captain's Challenge

9 What is the final position of each submarine? Complete as many challenges as you have time for. Use the image if it helps with your thinking.

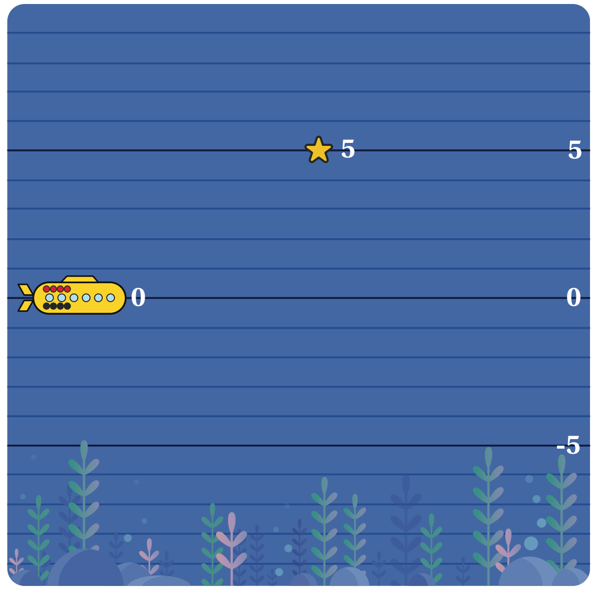
Start	Action	Final
0	Add 3 floats Add 7 anchors	
-9	Add 8 floats Remove 6 anchors	
1	Remove 3 floats Add 4 anchors	
-3	Remove 1 float Add 3 anchors	
-4	Remove 1 float	
5	Remove 8 floats Remove 3 anchors	
-2	Add 5 floats Remove 2 anchors	
-5	Add 1 float Remove 2 anchors	



10 Synthesis

Describe a set of actions that would allow this submarine to collect the star at 5 units.

Try to come up with something none of your classmates will.

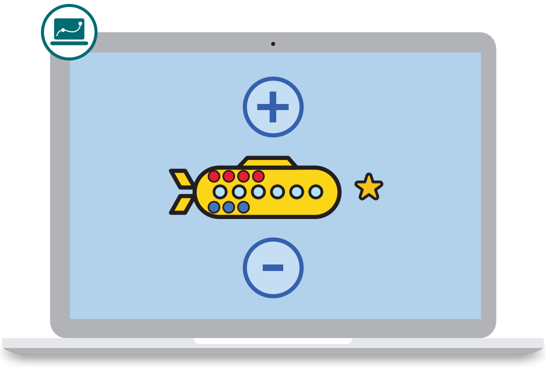


Things to Remember:

Name: ______________ Date: ________ Period: ________

More Floats and Anchors

Let's use floats and anchors to reason about adding and subtracting positive and negative numbers.



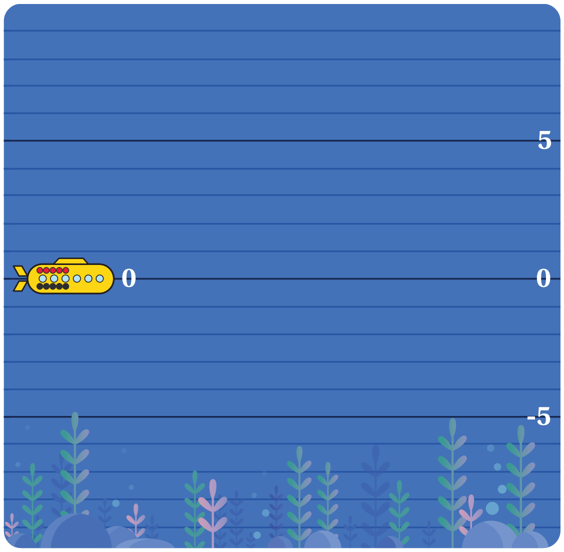
Warm-Up

1 *If you add some floats and remove some anchors, the submarine will go up.*

Is this statement *always*, *sometimes*, or *never* true? Circle one.

Always Sometimes Never

Explain your thinking.

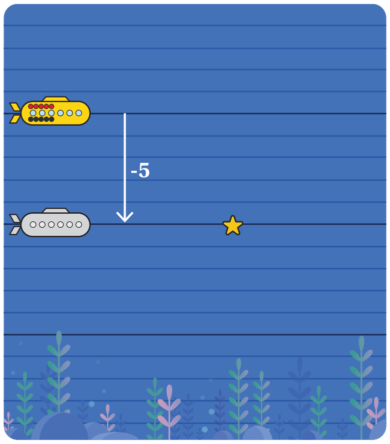


Activity 1

Name: ________ Date: ________ Period: ________

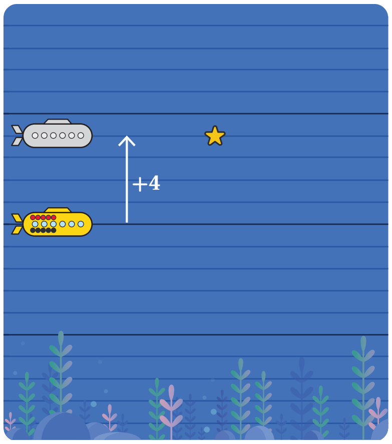
Ups and Downs

2 Select *all* the actions that would make this submarine go down 5 units.



- ☐ **A.** Add 5 floats
- ☐ **B.** Remove 5 floats
- ☐ **C.** Add 5 anchors
- ☐ **D.** Remove 5 anchors
- ☐ **E.** Add 3 anchors and add 2 floats

3 Select *all* the actions that would make this submarine go up 4 units.



- ☐ **A.** Add 4 floats
- ☐ **B.** Remove 4 floats
- ☐ **C.** Add 4 anchors
- ☐ **D.** Remove 4 anchors
- ☐ **E.** Add 3 floats and remove 1 anchor

4-5 Here are the details for four new submarine scenarios.

Complete the table.

Start	Action	Final Expression	Final Value
3	Add 7 anchors	$3 + (-7)$	
	Remove 7 floats	$3 - 7$	
-2		$-2 + 8$	
		$-2 - (-8)$	

Discuss: What do you notice? What do you wonder?

Activity 2

Name: ______ Date: ______ Period: ______

Depths of Understanding

6 Marc and Naoki are trying to evaluate $3 - (-2)$.

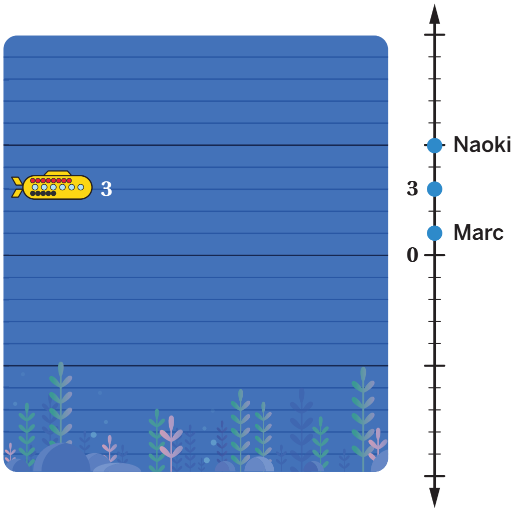
Marc says: *This is like adding* 2 *anchors, so the submarine goes down to* 1.

Naoki claims: *This is like removing* 2 *anchors, so the submarine goes up to* 5.

Whose thinking is correct? Circle one.

Marc (1) Naoki (5) Both Neither

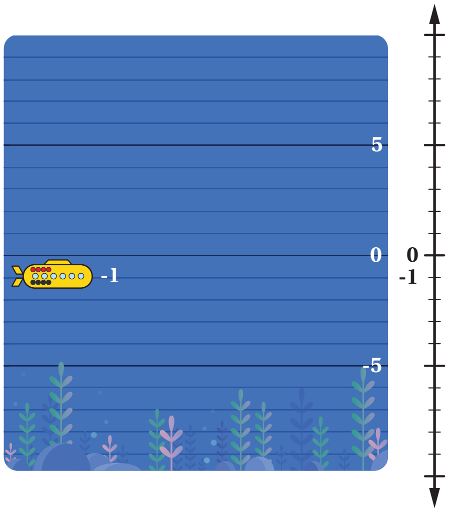
Explain your thinking.



7 **a** What is the value of $-1 + (-4)$?

b What is the value of $-1 - (-4)$?

c Show or explain your thinking.



Name: ________ Date: ________ Period: ________

Depths of Understanding (continued)

8 Group the expressions into pairs that have the same value.

$-4 - (-10)$	$4 + (-10)$	$-4 + 10$	$-4 - 10$
$-4 + (-10)$	$4 + 10$	$4 - (-10)$	$4 - 10$

Pair 1	Pair 2
Pair 3	**Pair 4**

Explore More

9 a Determine the value of each expression.

Expression	Value
1	
$1 - 2$	
$1 - 2 + 3$	
$1 - 2 + 3 - 4$	
$1 - 2 + 3 - 4 + 5$	

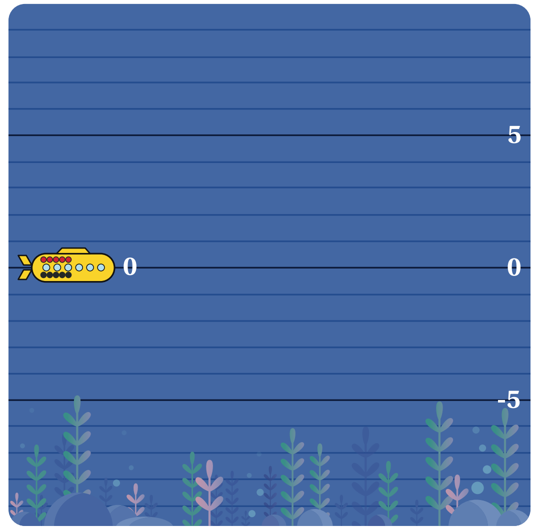
b Describe any patterns you notice.

c What is the value of the next expression? The 10th expression? The 100th expression?

10 Synthesis

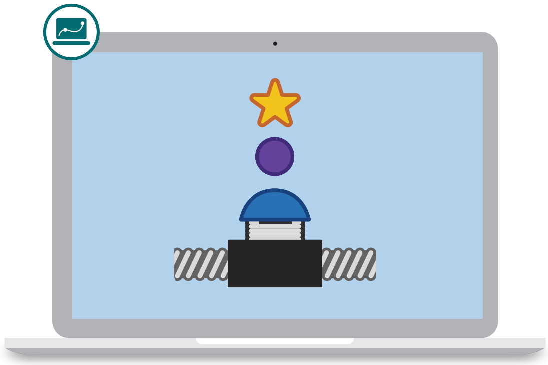
Explain why it makes sense that $0 - (-5)$ is equivalent to $0 + 5$.

Use a floats and anchors situation if it helps with your thinking.



Things to Remember:

Name: .... Date: .... Period: ....



Bumpers

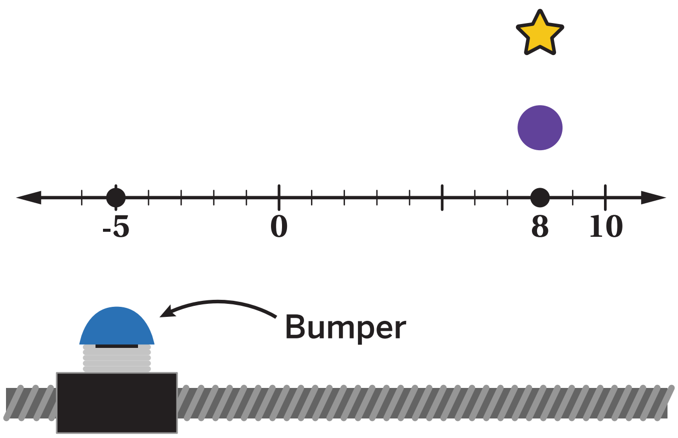
Let's add numbers including decimals and fractions on a number line.

Warm-Up

1 Hitting the ball with the bumper collects the star.

The bumper is at -5 units and the ball is at 8 units.

Write the number that would move the bumper and hit the ball.



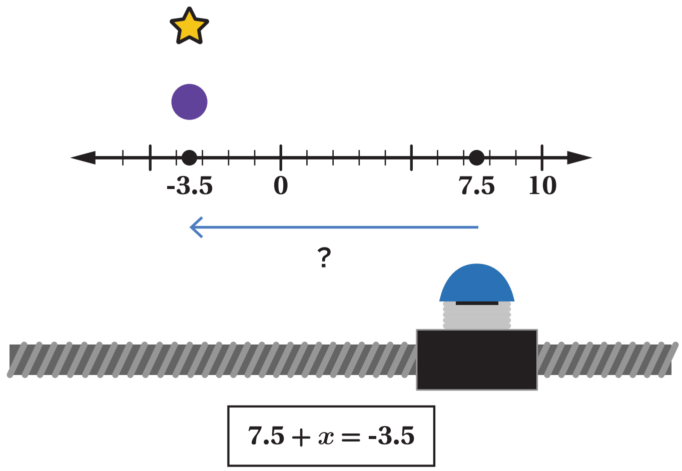
Activity 1

Name: .... Date: .... Period: ....

Things Could Get Bumpy

2 The bumper is at 7.5 units and the ball is at -3.5 units.

a Write the number that would move the bumper and hit the ball.



b **Discuss** your strategy with a classmate.

3 Dalia says that the expression $7.5 + 3.5$ can be used to know how to move the bumper.

Do you agree? Circle one.

Agree   Disagree   I'm not sure

Explain your thinking.

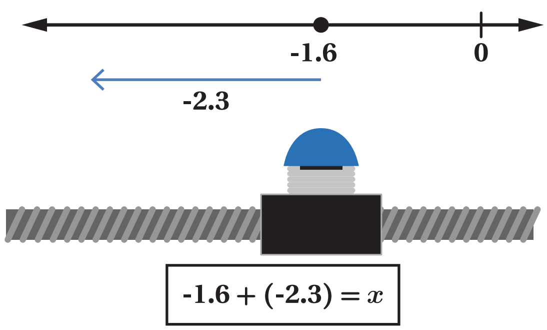
Name: Date: Period:

Carefully Placed

4 This bumper is at -1.6 units.

It's programmed to move -2.3 units.

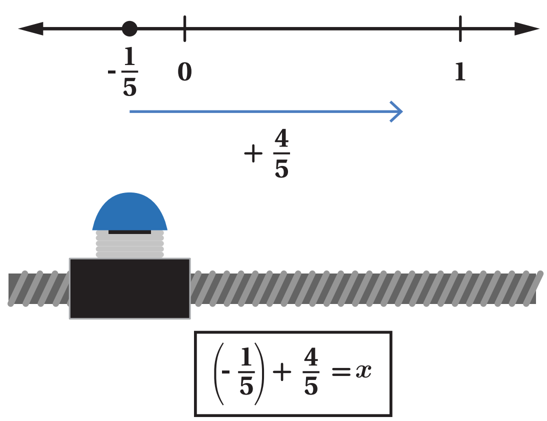
Where should the ball be placed so that the bumper hits it?



5 This bumper is at $-\frac{1}{5}$ units.

It's programmed to move $\frac{4}{5}$ units.

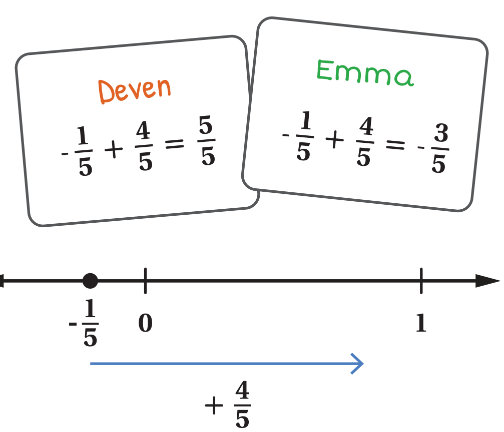
Where should the ball be placed?



6 Two students made mistakes on the previous challenge.

Circle the card with your favorite mistake.

What could you say to help the student understand their mistake?



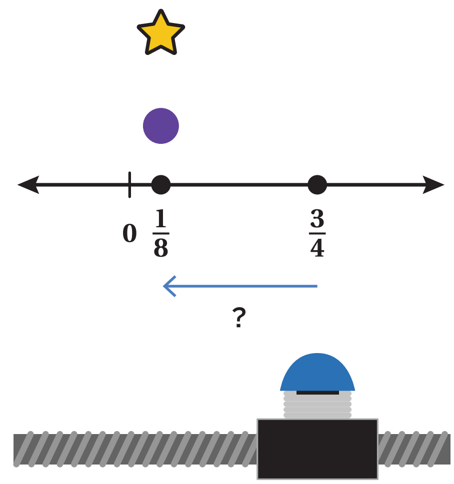
Activity 3

Name: ________ Date: ________ Period: ________

Bumper Challenge

7 What is the value of x that makes this equation true?

$$\frac{3}{4} + x = \frac{1}{8}$$



8 What is the value of x that makes each equation true? Solve as many challenges as you have time for.

a $4 + x = 10$

b $7 + x = 2$

c $-1.3 + x = 7.2$

d $\frac{9}{5} + x = \frac{3}{5}$

e $10 + (-2.5) = x$

f $8.1 + x = -1$

g $x + 8.4 = -4.2$

h $x + \frac{1}{6} = -\frac{2}{3}$

i $x + \left(-\frac{9}{2}\right) = -1$

j $x + (-13.2) = -7.6$

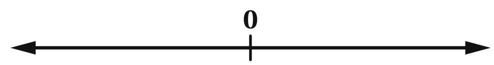
k $\frac{1}{2} + 2 = x$

l $5.7 + 0.4 = x$

9 Synthesis

Describe how you can use a number line to determine the value of x in an equation like $3.1 + x = -2$.

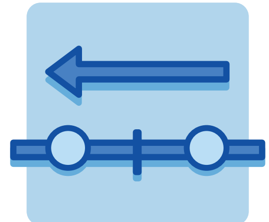
Use the number line if it helps you show your thinking.



Things to Remember:

Name: ______ Date: ______ Period: ______

Draw Your Own

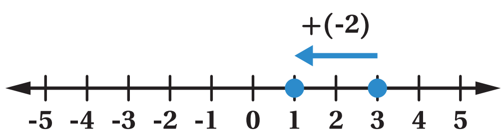


Let's use number lines to reason about addition and subtraction.

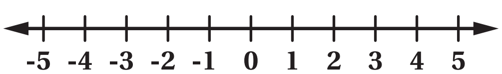
Warm-Up

1. Renata drew a number line for $3 + (-2)$. Use her thinking to complete the number line to determine the value of $-2 + 3$.

$3 + (-2) = 1$



$-2 + 3 =$ ______



Name: ________ Date: ________ Period: ________

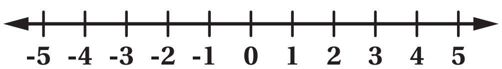
Pluses and Minuses

2. Renata says her number line for $3 + (\text{-}2)$ could also represent $3 - 2$ because adding two anchors has the same effect as removing two floats. Do you agree? Circle one.

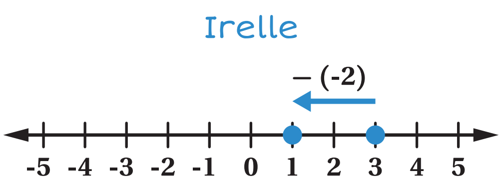
Yes No I'm not sure

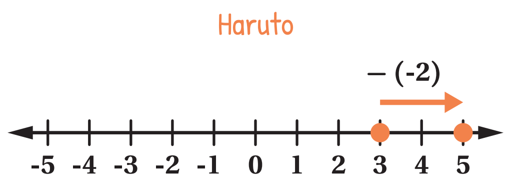
Explain your thinking.

3. Use your thinking from the previous problem to represent $\text{-}1 - 4$ on this number line.



4. Irelle and Haruto each drew a number line for $3 - (\text{-}2)$.





Whose number line is correct? Circle one.

Irelle Haruto Both Neither

Explain your thinking.

5. Match each addition expression to a subtraction expression with the same value.

a. $9 - 7$ .... $\text{-}9 + (\text{-}7)$

b. $\text{-}9 - 7$ .... $\text{-}9 + 7$

c. $9 - (\text{-}7)$ .... $9 + 7$

d. $\text{-}9 - (\text{-}7)$ .... $9 + (\text{-}7)$

6. **Discuss:** How can addition be helpful when representing subtraction on a number line? Use the expressions from the previous problem if they help with your thinking.

Name: ______ Date: ______ Period: ______

Draw Your Own Diagram

7. For each pair of expressions, complete the number lines and determine the value of each expression.

a $7 - 4 =$ ______ $4 - 7 =$ ______

b $3 - (-9) =$ ______ $-9 - 3 =$ ______

c $-3.4 - (-2) =$ ______ $-2 - (-3.4) =$ ______

d $-3.4 + (-2) =$ ______ $-2 + (-3.4) =$ ______

8. Describe 2–3 patterns you notice in the previous problem's number lines or their values.

Activity 3

Name: .... Date: .... Period: ....

Draw Your Own Conclusion

9. Luke-Josephine says: *The distance between x and y on a number line is $x - y$.*

Discuss: Is this statement *always*, *sometimes*, or *never* true? How would you convince someone else?

For each problem, select one statement. Explain whether it is *always, sometimes,* or *never* true. Use examples, words, or number lines to support your claim.

10.

Statement A	Statement B	Statement C
$x + 1$ is positive.	$x + y$ has the same value as $y + x$.	$x + (\text{-}x)$ is positive.

a. Statement .... is (always/sometimes/never) true.

b. My reasoning:

11.

Statement D	Statement E	Statement F
$x - y$ is the opposite of $y - x$.	x is less than $x + y$.	$x - y$ is greater than $x + y$.

a. Statement .... is (always/sometimes/never) true.

b. My reasoning:

Explore More

12. Write your own statement about adding or subtracting positive and negative numbers that is either *always* or *never* true. Then trade statements with a classmate and decide if their statement is *always* or *never* true.

Synthesis

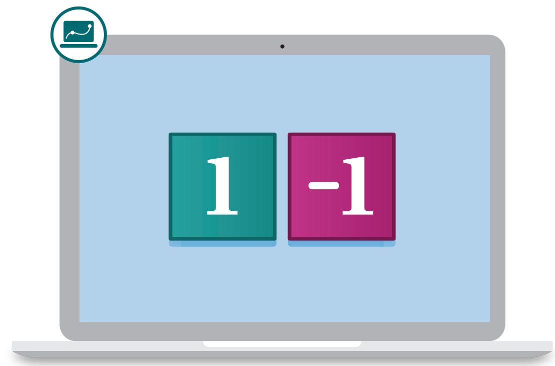
13. What happens to the value of a subtraction expression when you rearrange the order of the numbers? Use the examples if they help with your thinking.

$7 - 4$ and $4 - 7$

$-2 - (-3.5)$ and $-3.5 - (-2)$

Things to Remember:

Name: .... Date: .... Period: ....



Number Puzzles

Let's solve some puzzles involving positive and negative numbers.

Warm-Up

1 The equation $1 + 2 = 3 + 4$ is false.

a Circle one or more numbers and change them to the opposite number to make the equation true.

$$\boxed{1} + \boxed{2} = \boxed{3} + \boxed{4}$$

b Explain how you know your equation is true.

Activity 1

Name: .... Date: .... Period: ....

Number Puzzles

For each number puzzle:

- Make true equation(s) by using the numbers to fill in the blanks.
- Use each number only once per puzzle.
- You can circle a number and change it to its opposite. Try to circle as few numbers as possible.

1	2	3	4
-5	-6	-7	-8

2 $\square + \square = \square - \square$

3 $\square + 5 = \square$

$\square - \square = 9$

4 Is it possible to solve the previous puzzle using 0 circles?

Yes No I'm not sure.

Explain your thinking.

Name: .... Date: .... Period: ....

Number Puzzles (continued)

For each number puzzle:

- Make true equation(s) by using the numbers to fill in the blanks.
- Use each number only once per puzzle.
- You can circle a number and change it to its opposite. Try to circle as few numbers as possible.

1	2	3	4
-5	-6	-7	-8

5. ☐ + ☐ + ☐ = 0

6. ☐ + ☐ + ☐ = 0

☐ − ☐ − ☐ = 9

Activity 2

Name: .... Date: .... Period: ....

More Number Puzzles

The previous puzzles used only **integers**. These puzzles do not. For each number puzzle:

- Make true equation(s) by using the numbers to fill in the blanks.
- Use each number only once per puzzle.
- You can circle a number and change it to its opposite. Try to circle as few numbers as possible.

7

-0.5	1	-1.5	2
-2.5	3	-3.5	4

$\square + 5 + \square = \square$

$-4.5 - \square - \square = \square$

8

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{8}$
$-\frac{3}{8}$	$-\frac{5}{8}$	$-\frac{7}{8}$	-1

$\square + \square - \square = 0$

$\square - \square + \square = 1$

Explore More

9 Use the Explore More Sheet to explore another number puzzle.

10 Synthesis

Imagine subtracting a pair of numbers.

Describe how you can tell whether the result will be positive or negative.

$\square - \square = ?$

Things to Remember:

Name: .... Date: .... Period: ....

Explore More

Solve this puzzle in as many different ways as you can, using as many circles as you want.

You can use each number only once per solution.

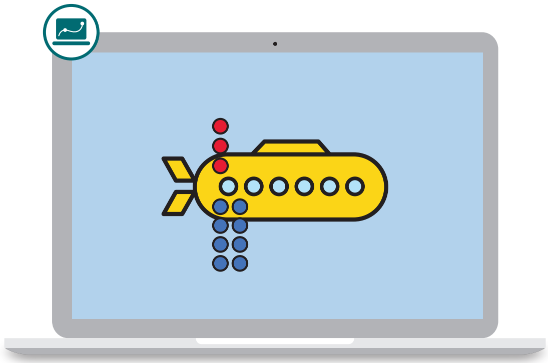
$-\frac{1}{48}$	$-\frac{1}{24}$	$-\frac{1}{16}$	$-\frac{1}{12}$
$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$

$$\square + \square + \square = 0$$

My solutions:

Unit 5
Lesson
6

Name: .... Date: .... Period: ....

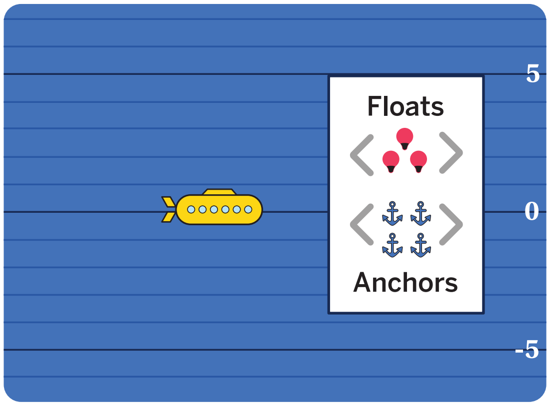


Floating in Groups

Let's use floats and anchors to make sense of multiplying integers.

Warm-Up

1 This submarine is controlled by *groups* of floats and anchors.



a Take a look at these different combinations.

Add 1 group of 3 floats	Add 1 group of 4 anchors	Add 1 group of 3 floats Add 1 group of 4 anchors

b Choose a number. How can you add groups of 3 floats and 4 anchors to get the submarine to that number?

Activity 1

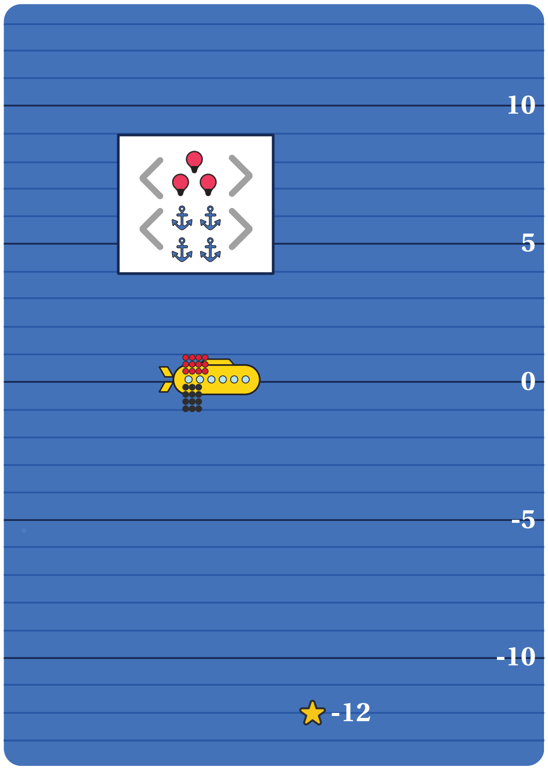
Name: .... Date: .... Period: ....

Star Power

2–3 This submarine starts at an elevation of 0 units. Floats can be added or removed in groups of 3. Anchors can be added or removed in groups of 4.

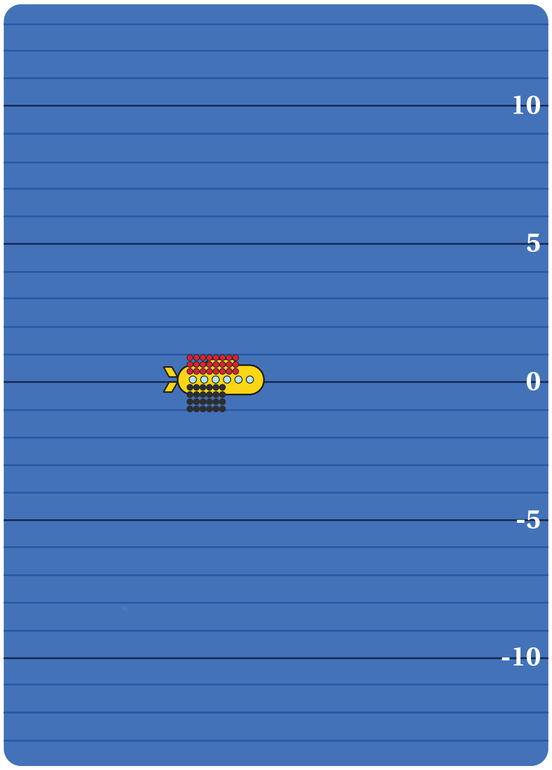
The table shows one way to get the submarine to -12 to collect the star. Write two more actions to get the submarine to -12.

Start	Action	Final
0	Add 3 groups of 4 anchors	-12
0		-12
0		-12



4 This submarine starts at 0 units.

Where will it end up after removing 6 groups of 3 floats?



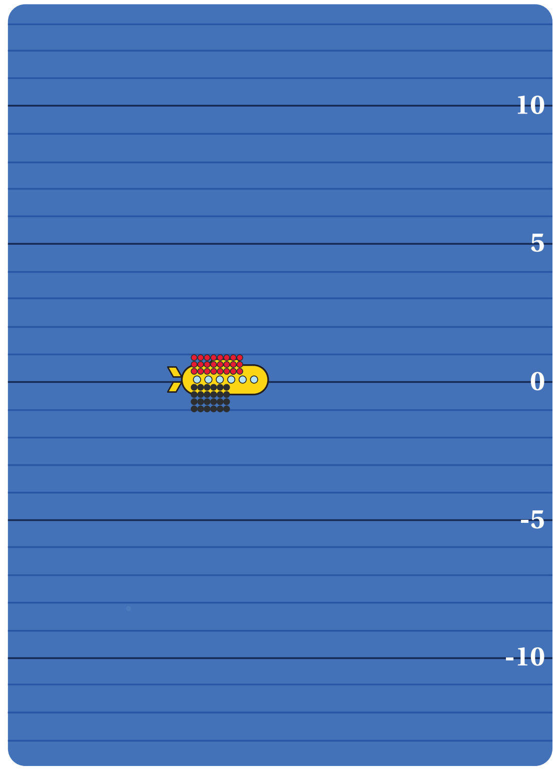
Name: .... Date: .... Period: ....

Star Power (continued)

5 Demetrius wrote this equation to help him solve the situation from the previous problem:

$(-6) \cdot 3 = -18$

Explain what each number represents in the situation.



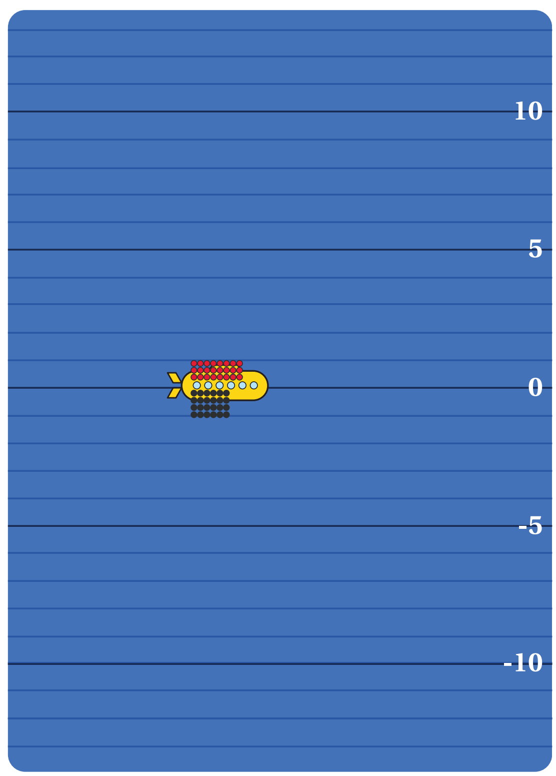
-6 represents . . .

3 represents . . .

-18 represents . . .

6 This submarine starts at 0 units.

Where will it end up after removing 5 groups of 4 anchors?



7 Callen and Demetrius each wrote an expression to help solve this submarine situation.

Callen's expression: $0 - 5 \cdot (-4)$

Demetrius's expression: $(-5)(-4)$

Explain why each person's expression makes sense.

Callen's expression makes sense because . . .

Demetrius's expression makes sense because . . .

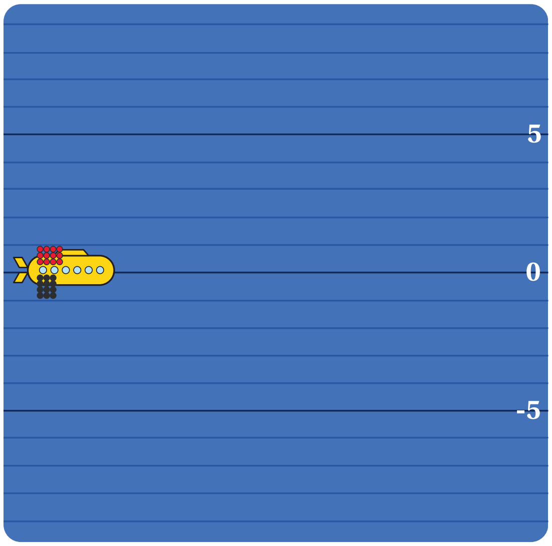
Activity
2

Name: .... Date: .... Period: ....

Underwater Expressions

8 What is the value of the expression $2 \cdot (-4)$?

Explain your thinking. Use this submarine if it helps with your thinking.



9 Group the expressions into pairs that have the same value. Some expressions will have no match.

$6 \cdot (-2)$	$-6 \cdot (-4)$	$4 \cdot (-4)$	$(-2) \cdot (-12)$
$(-8) \cdot (-2)$	$-3 \cdot 4$	$4 \cdot 4$	$8 \cdot (-3)$

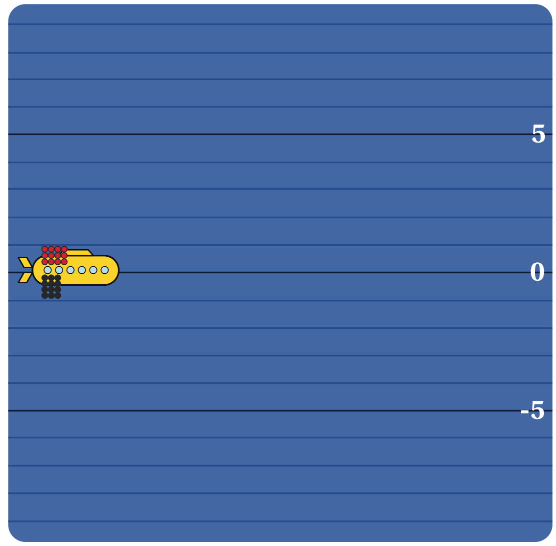
Pair 1	Pair 2
Pair 3	**Expressions With No Match**

Explore More

10 Use the digital activity to collect as many stars as you can.

11 Synthesis

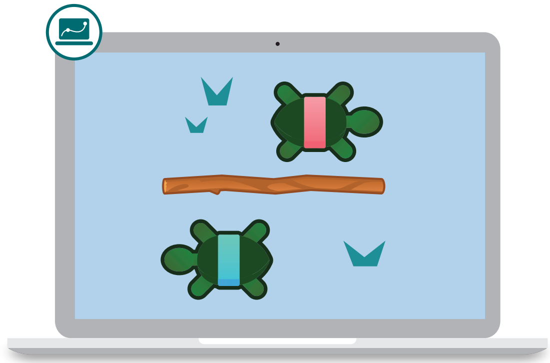
Use the floats and anchors scenario to explain why it makes sense that $(-2)(-4)$ is positive.



Things to Remember:

Unit 5
Lesson
7

Name: ________ Date: ________ Period: ________

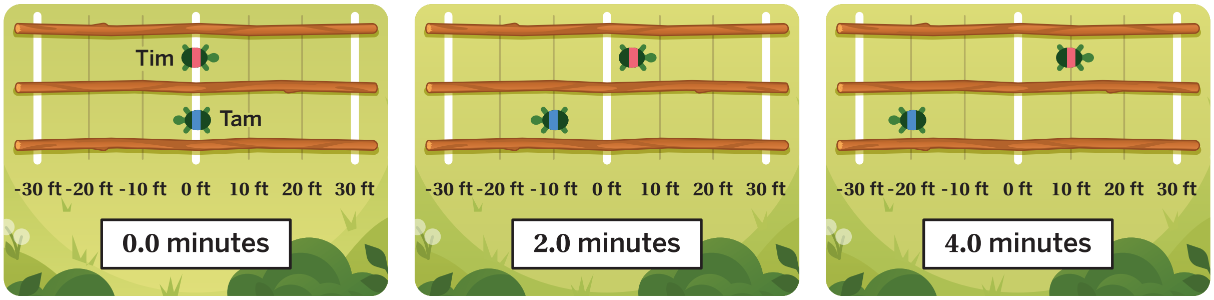


Back in Time

Let's use position, rate, and time to represent multiplying positive and negative numbers.

Warm-Up

1 Here are a few moments from an animation.



Write a story about Tim and Tam.

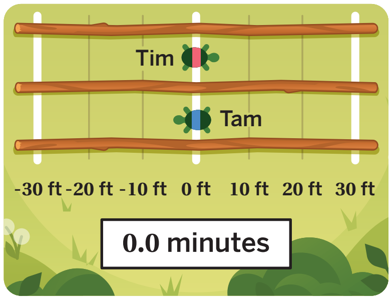
Activity 1

Name: .... Date: .... Period: ....

Time for Turtles

2 Tim and Tam each walk at a constant rate. Complete the last row of the table.

Time (min)	Tim's Position (ft)	Tam's Position (ft)
0	0	0
1	2.5	-5
2	5	-10
3	7.5	-15
. . .	. . .	. . .
6		



3 Tim's walking rate is 2.5 feet per minute.

What is Tam's walking rate?

Explain your thinking.

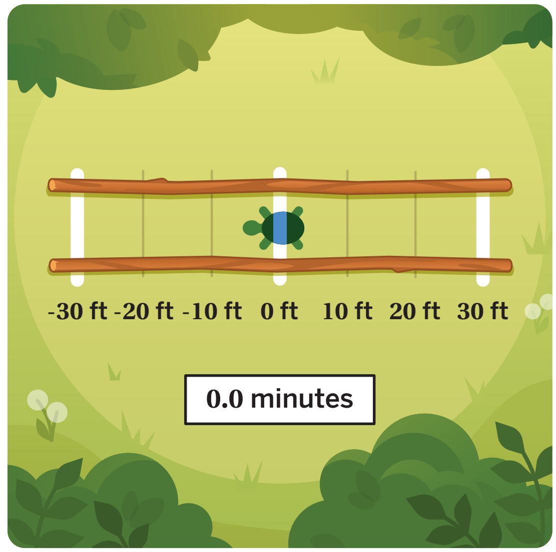
Activity
1

Name: Date: Period:

Time for Turtles (continued)

Use the value you calculated for Tam's walking rate for the following problems.

4 What will Tam's position be in 3.2 minutes?



5 What was Tam's position 3.2 minutes ago?

6 One student wrote this equation to determine Tam's position on the previous problem:

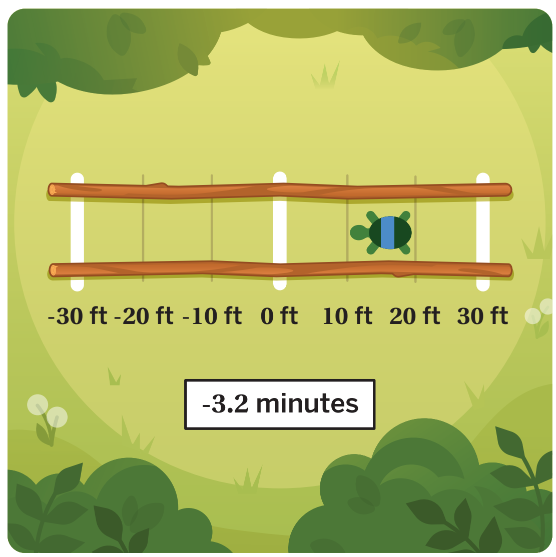
$(-5)(-3.2) = 16$

Explain what each number represents in the situation.

-5 represents . . .

-3.2 represents . . .

16 represents . . .



Name: .... Date: .... Period: ....

Positives and Negatives

7 For each expression, put a check for whether it has a positive or negative value.

Expression	Positive	Negative
$(-7) \cdot (-8)$		
$(-7) \cdot 8$		
$3.5 \cdot (-12)$		
$(-3.5) \cdot (-12)$		
$\left(\frac{1}{2}\right)(-20)$		
$\left(-\frac{1}{2}\right)(-20)$		

Activity 3

Name: .... Date: .... Period: ....

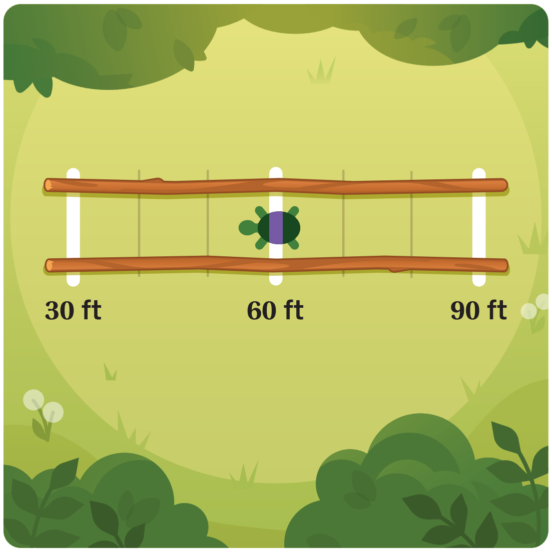
Turtles Through Time

8 Tommy walks at a rate of -2 feet per minute.

Right now he is at 60 feet.

Where was Tommy 10 minutes ago?

Explain how you know.



9 Select *all* the expressions that describe where Tommy was 10 minutes ago.

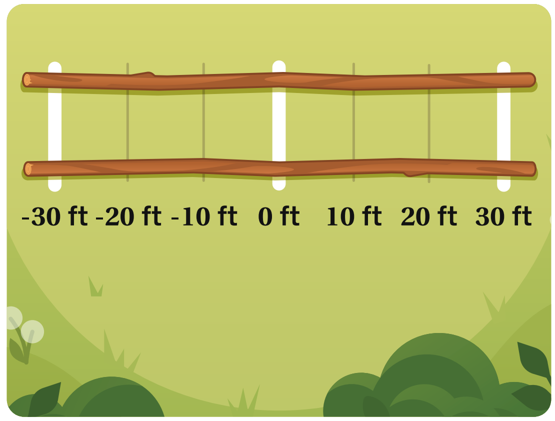
- ☐ **A.** $60 + 2 \cdot (-10)$
- ☐ **B.** $60 + (-2)(-10)$
- ☐ **C.** $60 - 20 \cdot 10$
- ☐ **D.** $60 - 20$
- ☐ **E.** $60 + 20$

Explore More

10 Use the Explore More Sheet to explore another turtle puzzle.

11 Synthesis

Use a situation with turtles starting at 0 to explain why $5 \cdot (-2)$ is negative and $(-5) \cdot (-2)$ is positive.



Things to Remember:

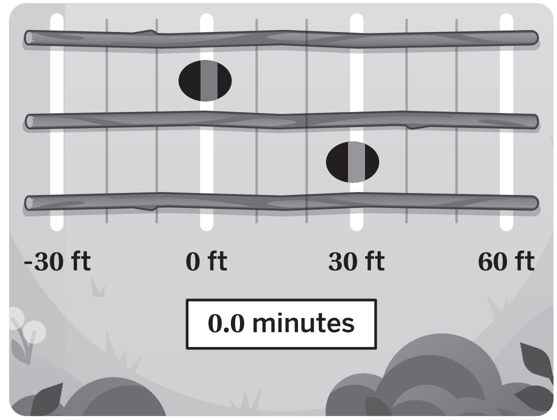
Name: .... Date: .... Period: ....

Explore More

5 minutes ago, these turtles were at the same position.

What could their walking rates be?

Determine several possible pairs of rates.



Pair 1:

Turtle	Position (ft)	Rate (ft/min)
Top	0	
Bottom	30	

Pair 2:

Turtle	Position (ft)	Rate (ft/min)
Top	0	
Bottom	30	

Pair 3:

Turtle	Position (ft)	Rate (ft/min)
Top	0	
Bottom	30	

Pair 4:

Turtle	Position (ft)	Rate (ft/min)
Top	0	
Bottom	30	

Unit 5

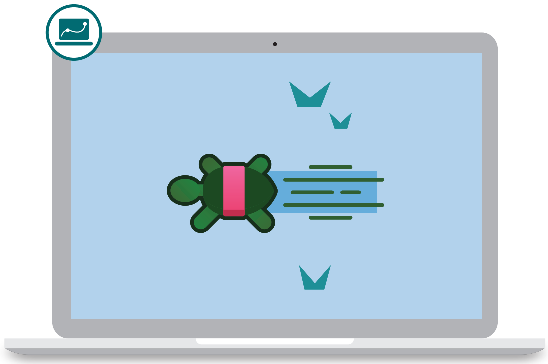
Lesson

8

Name: .... Date: .... Period: ....

Speeding Turtles

Let's use position, rate, and time to represent dividing integers.



Warm-Up

1 Which one doesn't belong? Circle one.

$3 \cdot 2$

$-\frac{12}{2}$

$(-3)(-2)$

$12 \div 2$

Explain your thinking.

Activity 1

Name: .... Date: .... Period: ....

Return of the Turtles

2 Tim walks -8 feet per minute for 3.25 minutes.

Where will Tim end up?

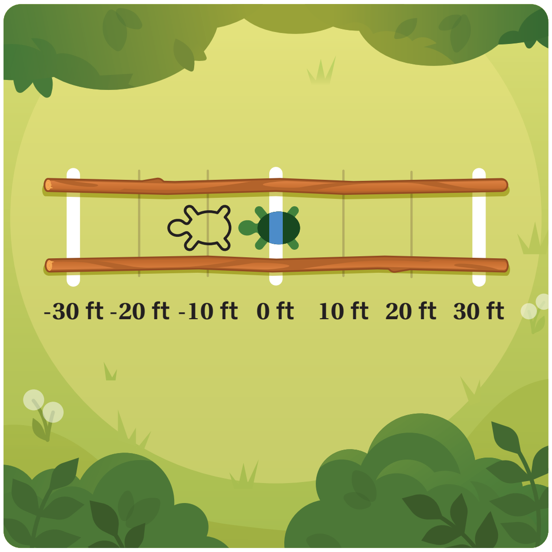
Rate (ft/min)	Time (min)	Position (ft)
-8	3.25	

3 Tam walked -4 feet per minute.

She began at 0 feet and ended up at -10 feet.

For how much time was Tam walking?

Rate (ft/min)	Time (min)	Position (ft)
-4		-10



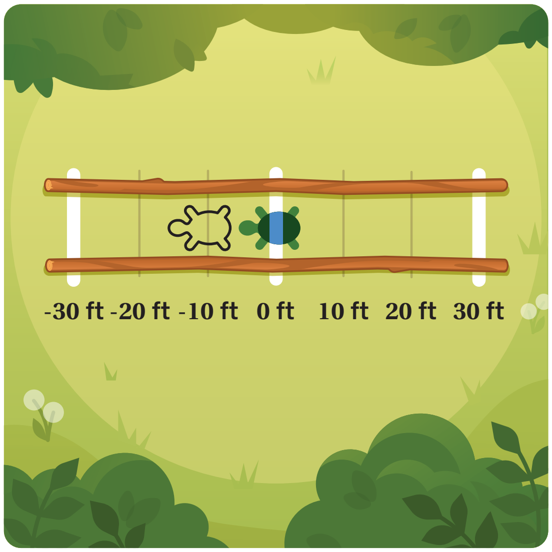
Name: .... Date: .... Period: ....

Return of the Turtles (continued)

4 Fabiana wrote this equation to determine the time from the previous problem:

$$\frac{-10}{-4} = t$$

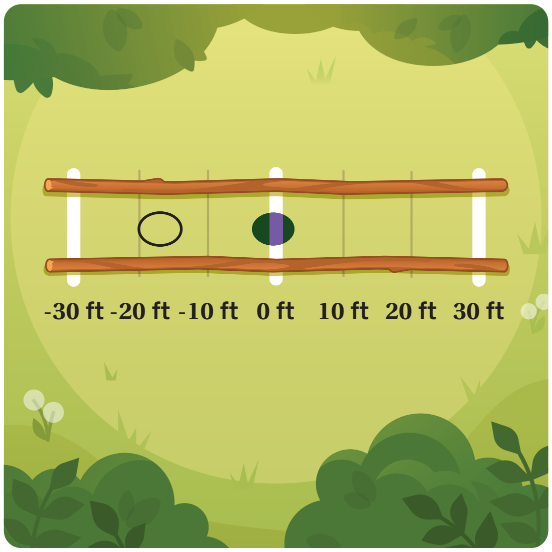
Explain why her equation makes sense.



5 Tommy walks -17 feet in 5 minutes.

What is Tommy's walking rate?

Rate (ft/min)	Time (min)	Position (ft)
	5	-17



Activity 2

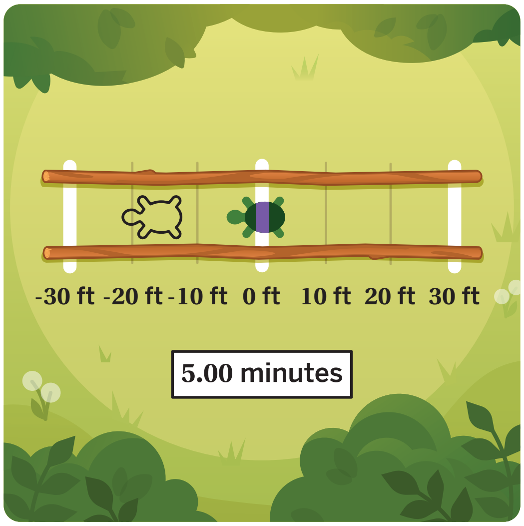
Name: ______ Date: ______ Period: ______

Turtle Express

6 Tommy walks -17 feet in 5 minutes.

Select *all* the equations that represent this situation, where r represents Tommy's walking rate.

- ☐ **A.** $r \cdot 5 = \text{-}17$
- ☐ **B.** $(\text{-}17) \cdot 5 = r$
- ☐ **C.** $r = \text{-}\frac{17}{5}$
- ☐ **D.** $r = \frac{\text{-}17}{5}$
- ☐ **E.** $r = \frac{5}{\text{-}17}$



7 Group the expressions that have the same value. One expression will have no match.

$6 \div 3$	$\text{-}\frac{3}{6}$	$\text{-}\frac{6}{3}$	$\frac{\text{-}3}{6}$
$\frac{\text{-}3}{\text{-}6}$	$\frac{\text{-}6}{\text{-}3}$	$3 \div (\text{-}6)$	$\frac{6}{\text{-}3}$

Group 1	Group 2	Group 3

Explore More

8 Determine a current position and rate for three new turtles so that in 5 minutes all three turtles will be in the same position.

No two turtles should have the same starting position and rate as each other.

Turtle	Position (ft)	Rate (ft/min)
A		
B		
C		

9 Synthesis

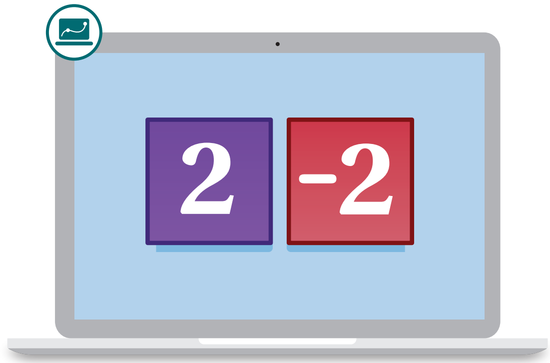
Explain why these expressions have the same value.

$-\frac{6}{3}$
$\frac{6}{-3}$

Things to Remember:

Unit 5
Lesson
10

Name: ______________________ Date: __________ Period: __________



Integer Puzzles

Let's reason about the signs and values of integer expressions using the four operations.

Warm-Up

1 **a** Make a true inequality by using these numbers to fill in the blanks.

1 | -2 | 3 | -4

☐ × ☐ < ☐ × ☐

b Explain how you know this inequality is true.

Activity 1

Name: ____________ Date: ________ Period: ________

Puzzling Values

For each number puzzle:

- Fill in the blanks with these numbers.
- You may use each number once per puzzle.

-1 | 2 | 3 | -4 | -5 | 6 | 7 | -8

2 Make this inequality true.

$\square \left(\square + \square\right) > 0$

3 **a** Make an expression.

$\square \left(\square + \square\right)$

b What is the value of your expression? Can you create an expression with this puzzle that has a greater value?

4 Josiah and Wey Wey made this expression. They used different strategies to determine the value of the expression.

$-5\,(2 + -4)$

Josiah

$(-5)(2) + (-5)(-4)$

$-10 + 20$

10

$-5\,(2 + -4)$

Wey Wey

$(-5)(2 - 4)$

$(-5)(-2)$

10

Discuss: What do you notice? What do you wonder?

Activity 2

Name: ______ Date: ______ Period: ______

Greatest Puzzling Values

For each number puzzle:

- Fill in the blanks with these numbers.
- You may use each number once per puzzle.

1	-2	3	-4	5	-6	7	-8

5 a. Make an expression with the greatest possible value.

$\square \times \square + \square \times \square$

b. What is the value of your expression?

6 Kiri created this expression which has a value of 26.

$7 \times 3 + 1 \times 5$

Write a hint or question to help her create an expression with an even greater value.

7 Fill in the blanks with these numbers.

-1	2	3	-4	-5	6	7	-8

a. Make an expression with the greatest possible value.

$$\frac{\square - \square}{\square + \square}$$

b. What is the value of your expression?

Name: .... Date: .... Period: ....

Greatest Puzzling Values (continued)

8 Fill in the blanks with these numbers.

2 3 4 5 -2 -3 -4 -5

a Make an expression with the greatest possible value.

$$\frac{\square + \square \times \square}{\square - \square}$$

b What is the value of your expression?

Explore More

9 Here are six copies of a new puzzle. Solve it in as many ways as you can.

-1 -2 -3 -4 5 6 7 8

$\square(\square + \square) = \square \times \square + \square$

$\square(\square + \square) = \square \times \square + \square$

$\square(\square + \square) = \square \times \square + \square$

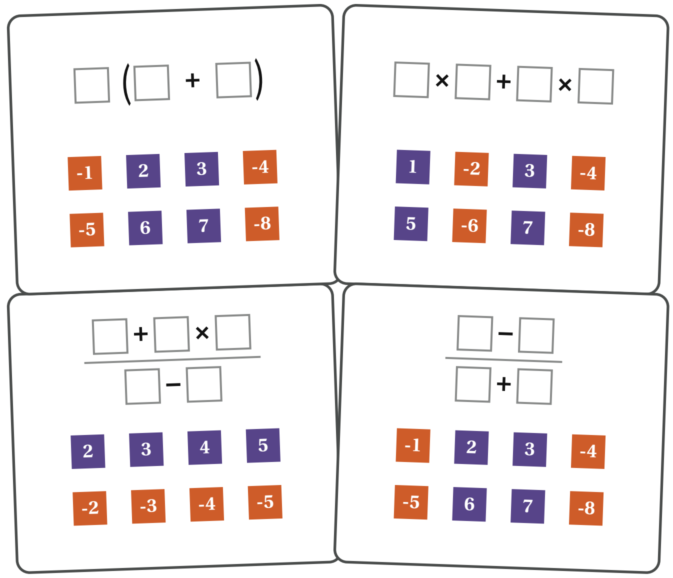
$\square(\square + \square) = \square \times \square + \square$

$\square(\square + \square) = \square \times \square + \square$

$\square(\square + \square) = \square \times \square + \square$

10 Synthesis

Describe something you learned today that might help someone else as they complete puzzles like these.



Things to Remember:

Name: .... Date: .... Period: ....

Puzzling Values

Use this space to record all of your attempts and thinking as you work on each puzzle. After each attempt, think about what you learned and how your strategy might change on the next attempt.

Puzzle #1	Puzzle #2

Use this space to record things you learned while solving these puzzles, including advice to yourself or others.

Name: .... Date: .... Period: ....

Puzzling Values

Use this space to record all of your attempts and thinking as you work on each puzzle. After each attempt, think about what you learned and how your strategy might change on the next attempt.

Puzzle #3	Puzzle #4

Explore More

Use this space to record things you learned while solving these puzzles, including advice to yourself or others.

Name: ______ Date: ______ Period: ______

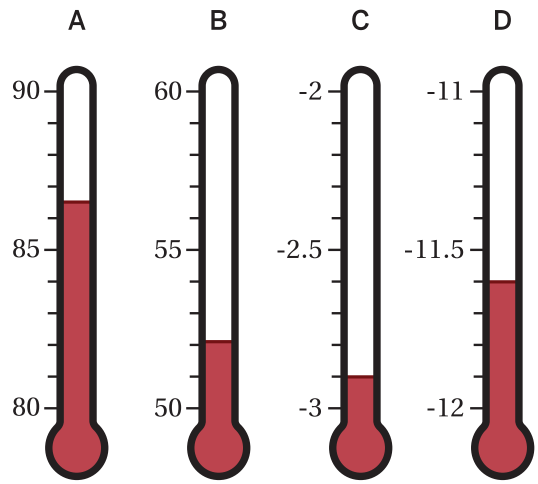
Temperatures Around the World

Let's add and subtract positive and negative numbers to compare temperatures and ice mass.

Warm-Up

1 What is the approximate temperature on each thermometer?

Thermometer	Temperature (°F)
A	
B	
C	
D	



All temperatures are in °F.

2 These temperatures are the average annual temperatures of different locations as of 2014. Match each temperature to a location and record it on the map.

Discuss your choices with a partner.

Activity 1

Name: ________ Date: ________ Period: ________

Changing Temperatures

3 How do you think the average annual temperatures of the places shown in the Warm-Up have changed over the last century? Circle one.

Gotten warmer   Gotten colder   Stayed the same

Explain your thinking.

4 This table shows how the average temperature in these places have changed from 1960 to 2014. Complete the table.

Location	Average Temperature in 1960 (˚F)	Average Temperature in 2014 (˚F)	Change From 1960 to 2014 (˚F)
NYC, U.S.	49.8	52.1	2.3
Khartoum, Sudan	84.6	86.5	
Greenland	-6	-2.9	
Antarctica		-11.6	1.3

Source: *Berkeley Earth*

5 Look at the column "Change From 1960 to 2014."

a What do you notice?

b What might be some impacts of these temperature changes?

Activity

2

Name: .... Date: .... Period: ....

Slippery Slope

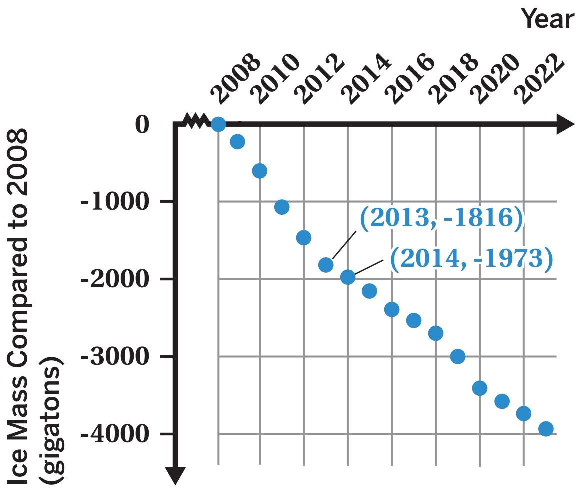
6 One impact of rising temperatures is that the amount of ice in the world is decreasing each year.

Greenland is mostly covered by ice.

This graph shows the mass of ice in Greenland in different years compared to the mass in 2008.

Tell a story about this graph.

Include the point $(2013, -1816)$ in your story.



Source: *Climate.NASA.gov*

7 What is the approximate change in Greenland's ice mass from 2013 to 2014?

Show whether the change is positive or negative.

Name: .... Date: .... Period: ....

Slippery Slope (continued)

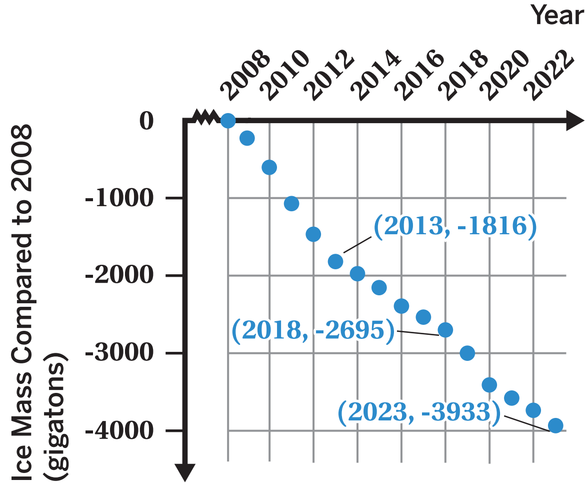
8 During which time period was ice melting fastest? Circle one.

2008 to 2013

2013 to 2018

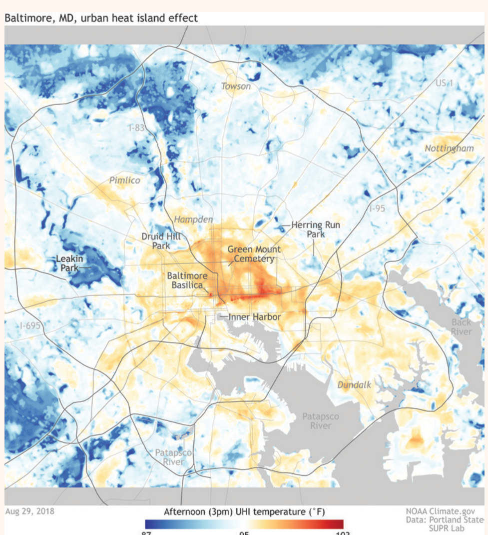
2018 to 2023

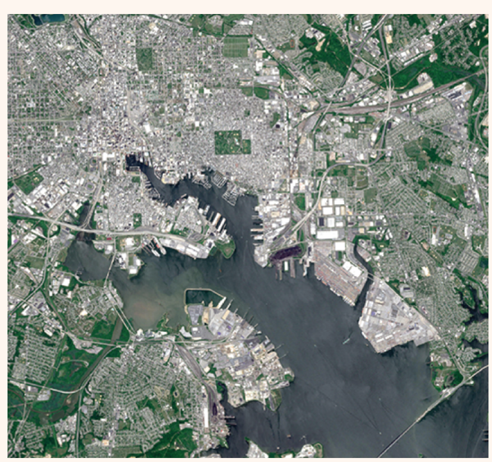
Explain your thinking.



Explore More

9 In this activity, we've looked at average temperatures across entire regions. Temperatures can also vary between different parts of a small local area, even neighborhood to neighborhood. Here are two maps of Baltimore, Maryland. One shows the temperatures at 3 PM on August 29, 2018 and the other shows the satellite view.





Sources: *NOAA*, *Portland State SUPR Lab*

What story might this data tell?

10 Synthesis

Select one question. Then record your response.

- ☐ Why is it important to study the temperature in places both near and far from where you live?
- ☐ How can positive and negative numbers help us make sense of topics like changing temperatures and ice mass?
- ☐ What other questions might you ask to investigate these topics further?

Things to Remember:

Unit 7

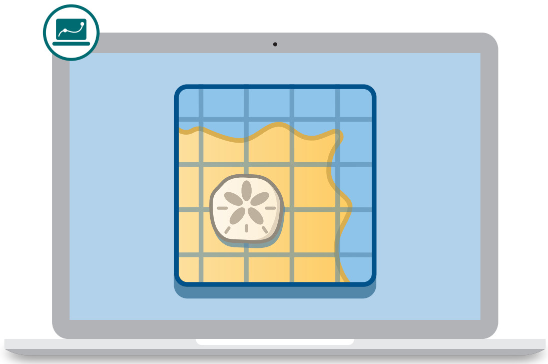
Lesson

10

Name: .... Date: .... Period: ....

Sand Dollar Search

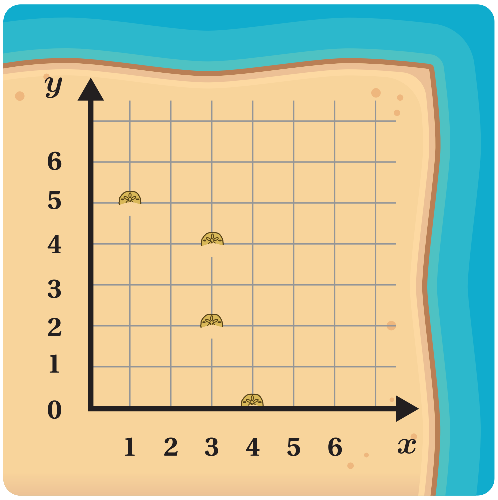
Let's explore negative numbers on the coordinate plane.



Warm-Up

1 Here is a map of part of an island.

Collect all four sand dollars by labeling each one with its coordinates.



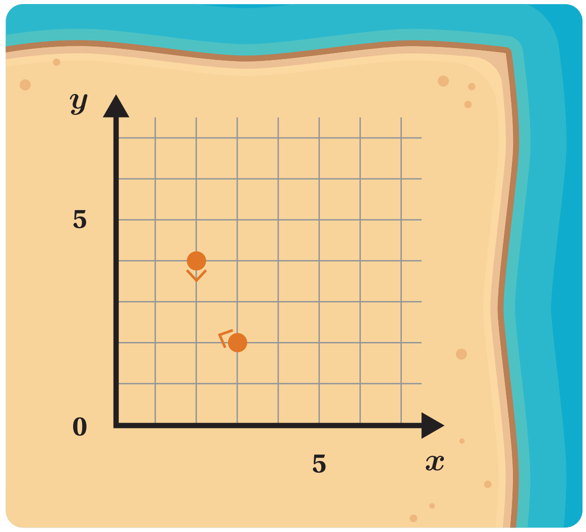
Name: .... Date: .... Period: ....

Find the Sand Dollar

2 Another sand dollar is buried on the island.

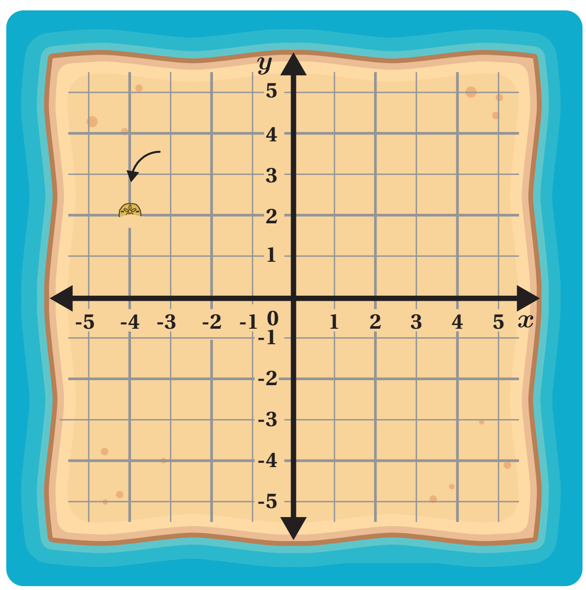
Here are two guesses for where the sand dollar is. Write coordinates for where you would search for it.

Coordinates (x, y)
(3, 2)
(2, 4)



3 You find a new map of the entire island.

A new sand dollar is visible. Write a clue that describes its location.



Activity 2

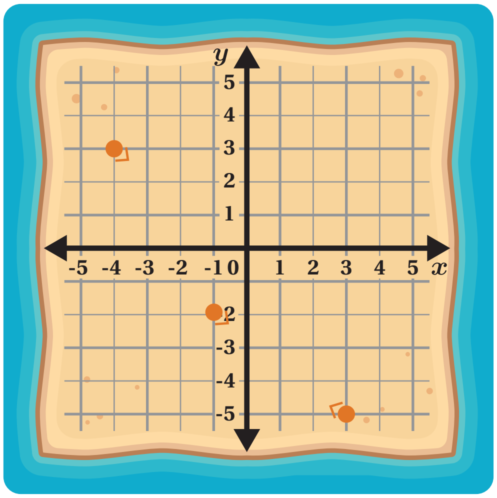
Name: .... Date: .... Period: ....

The Search Continues

4 There is another sand dollar buried somewhere on the island.

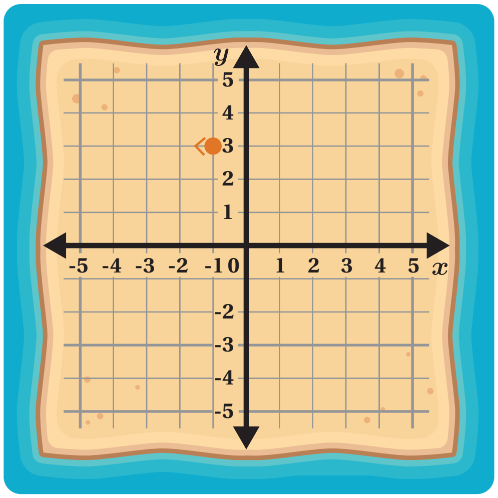
Here are three guesses for where the sand dollar is. Write coordinates for where you would search for it.

Coordinates (x, y)
(-4, 3)
(-1, -2)
(3, -5)



5 Esi was searching for another sand dollar. She entered (-1, 3) and the arrow pointed to the left.

What point would you recommend Esi try next? Explain your thinking.

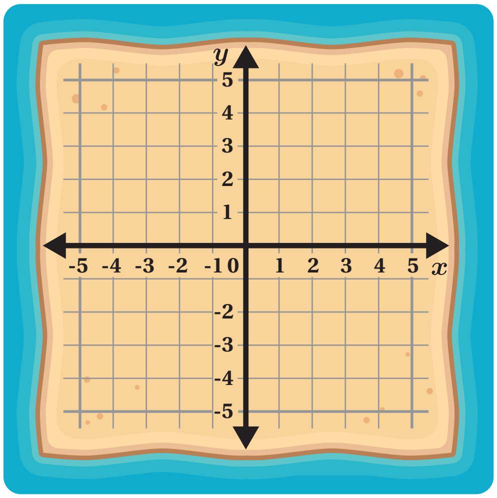


6 A sand dollar is hidden at a location that has two negative coordinates.

a Draw the sand dollar at a possible location.

Compare your point with a partner.

b **Discuss:** What do your points have in common?



Activity 3

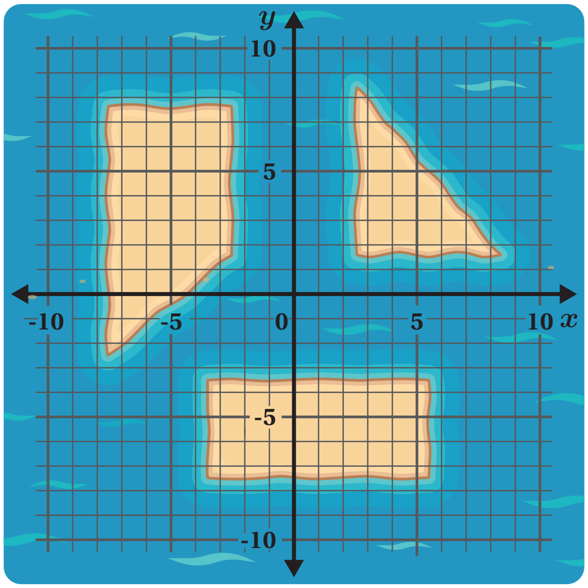
Name: Date: Period:

Find That Sand Dollar!

You will use support cards for this activity.

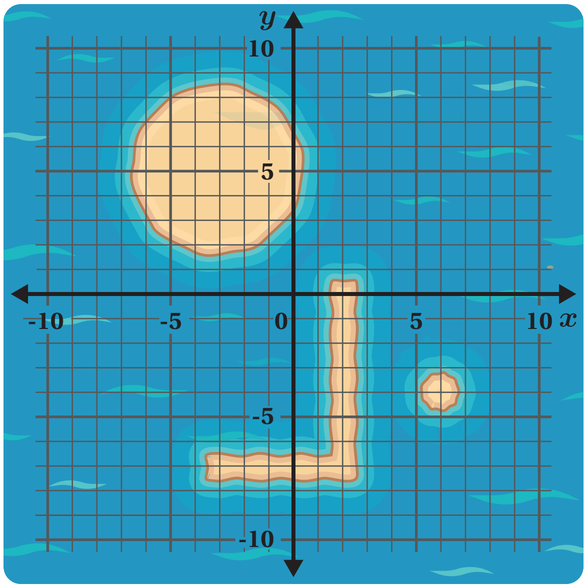
7 **Round 1:** There is a sand dollar on each of these islands.

When it is your turn to find the sand dollar, label each guess and the feedback you get until you find all three sand dollars.



8 **Round 2:** Here are three new islands.

Label each guess and the feedback you get until you find all three sand dollars.



Name: .... Date: .... Period: ....

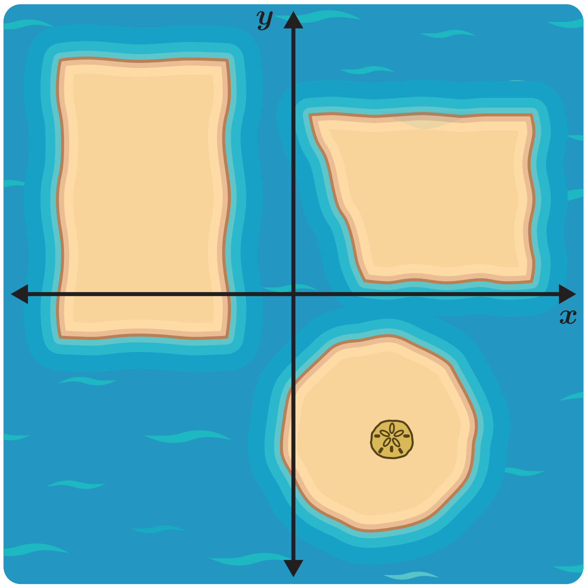
Find That Sand Dollar! (continued)

9 Which of these could be the location of the sand dollar?

A. $(4, 6)$ **B.** $(4, -6)$

C. $(-6, -4)$ **D.** $(-6, 4)$

Explain your thinking.



Explore More

10 Ariel was searching for a sand dollar and the arrow pointed up.

a What must be true about the sand dollar's x-coordinate?

A. $x = -5$

B. $x < -5$

C. $x > -5$

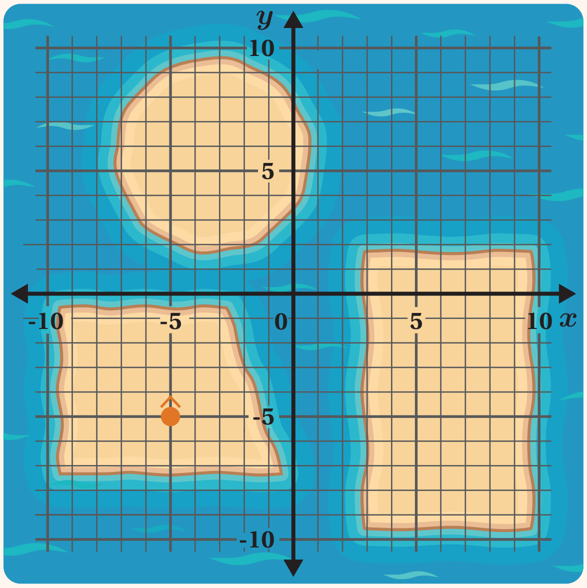
b What must be true about the sand dollar's y-coordinate?

A. $y = -5$

B. $y < -5$

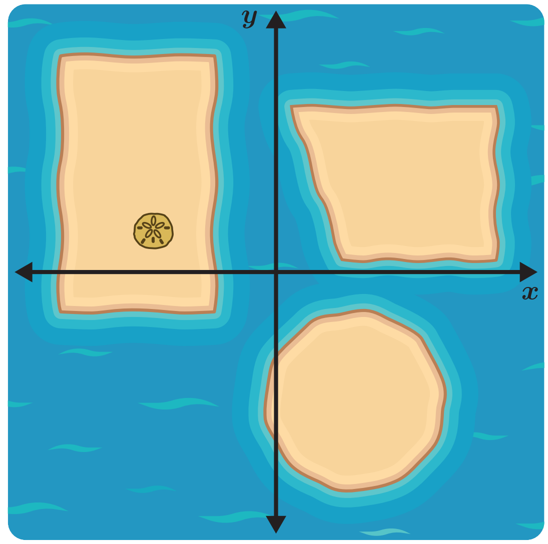
C. $y > -5$

Explain your thinking.



11 Synthesis

Explain what you know about the coordinates of this sand dollar.



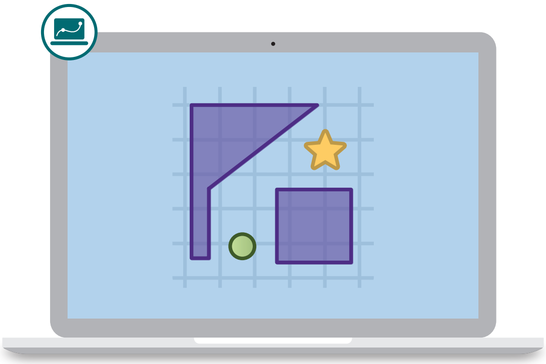
Things to Remember:

Unit 7

Lesson

11

Name: Date: Period:



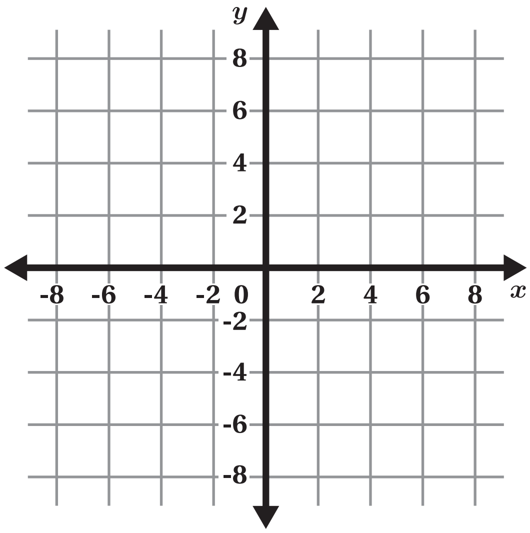
The A-maze-ing Coordinate Plane

Let's practice with the coordinate plane using mazes.

Warm-Up

1 a Plot a point at the coordinates $(3, -1)$.

b Explain how you chose where to plot the point.



Activity 1

Name: ____________ Date: ________ Period: ________

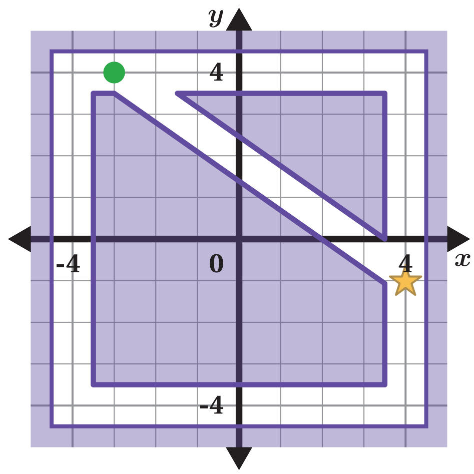
Maze Challenges

The goal of the maze is to move the ball to the star without hitting any walls.
For each challenge:

- Write the coordinates of a path to move the ball to the star. Your path can have several coordinate points.
- Pass your paper to a partner to draw the path using your coordinates.

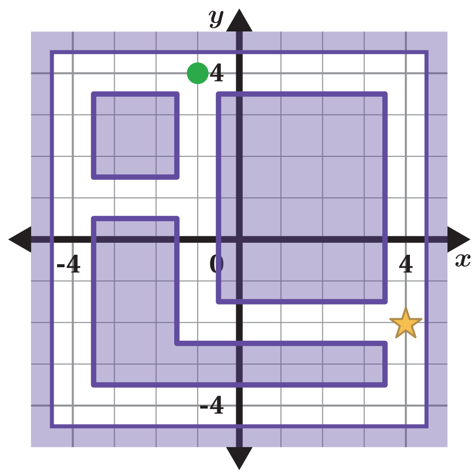
2 Let's watch the ball collect the star in this example. Finish writing the coordinates of the path.

Your Path
(-3, 4)



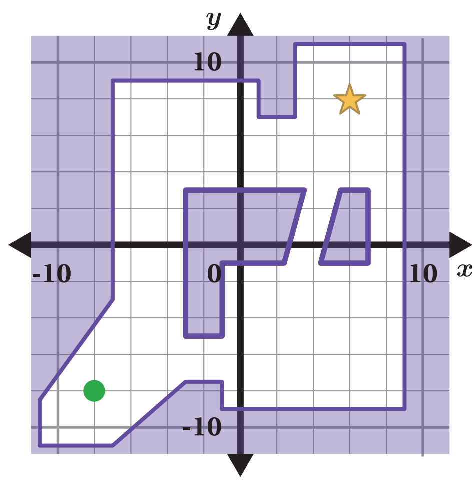
3 Write the coordinates of a path to move the ball to the star. Note: You do not need to use all the rows.

Your Path
(-1, 4)



4 Write the coordinates of a path to move the ball to the star.

Your Path
(-8, -8)



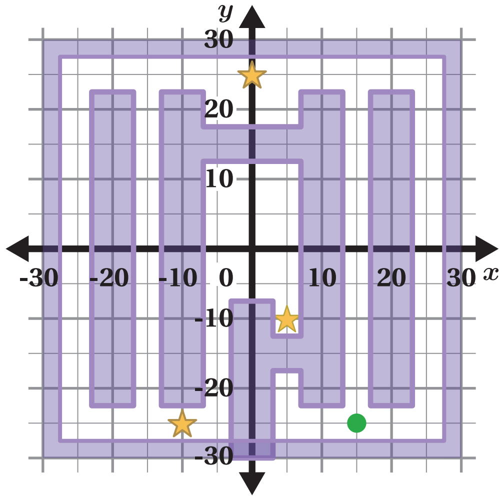
Activity 2

Name: ______ Date: ______ Period: ______

More Mazes

5 In this maze, there are multiple stars.

a Sketch a path to collect them all.



b Write the coordinates that create your path.

Your Path
(15, -25)

Name: ________ Date: ________ Period: ________

More Mazes (continued)

6 Trinidad and Annika were working on the previous maze.

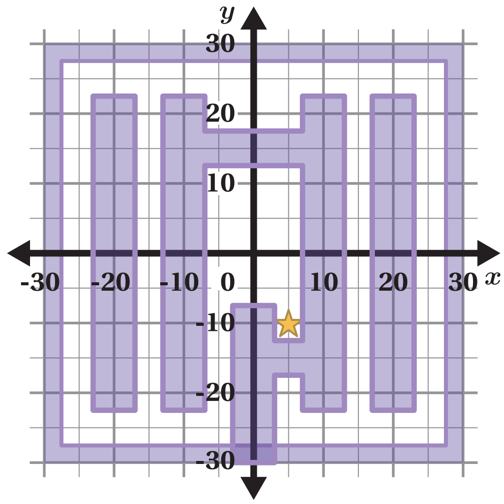
Trinidad says: *The star's coordinates are* $(1, -2)$.

Annika says: *The coordinates are* $(1, -10)$.

Whose claim is correct? Circle one.

Trinidad's   Annika's   Neither

Explain your thinking.

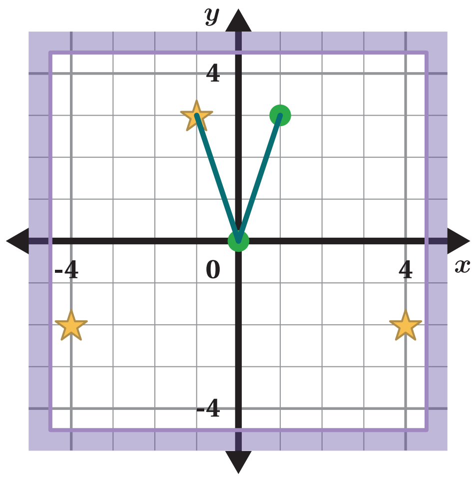


Activity 3

Name: .... Date: .... Period: ....

Reflections on the Plane

7 Let's look at a new way of collecting stars.

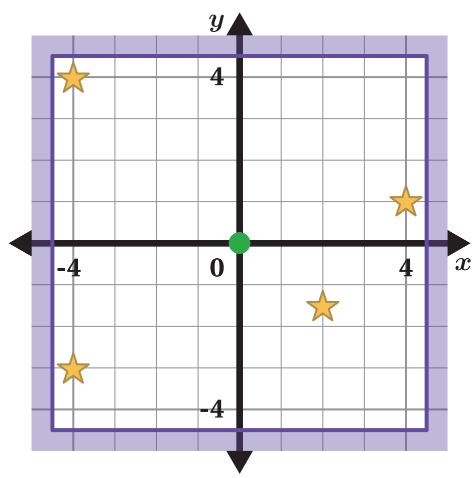


a **Discuss:** What happens when we try the point $(1, 3)$?

b Complete the table to collect all the stars. Sketch the new paths on the graph.

Your Path	Mirrored Path
(0, 0)	(0, 0)
(1, 3)	(-1, 3)

8 The ball is mirrored across the y-axis. Write coordinates to collect all the stars. Choose your two points wisely!



Your Path	Mirrored Path
(0, 0)	(0, 0)

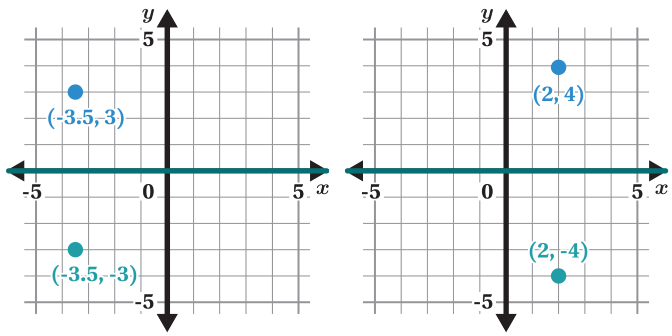
Name: .... Date: .... Period: ....

Reflections on the Plane (continued)

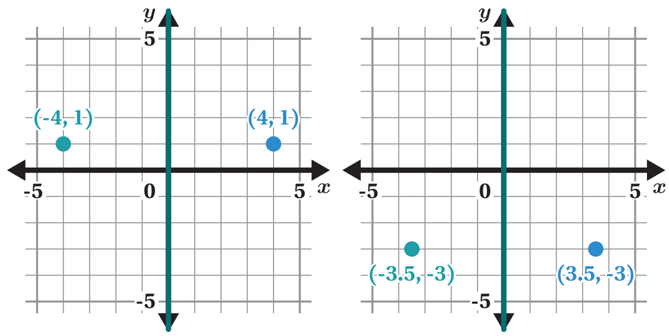
9 Here are several coordinates of a ball and its reflection.

Write at least two patterns about the coordinates that you found interesting.

Reflections Across the x-axis

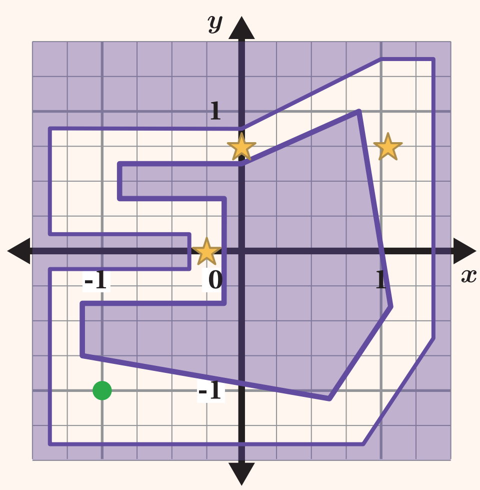


Reflections Across the y-axis



Explore More

10 Sketch a path to collect all the stars. Then write the coordinates of your path.



11 Synthesis

Look back over the challenges and select the one you are most proud of solving.

Write some advice for someone solving that challenge.

Things to Remember:

Unit 7

Lesson

12

Name: ________ Date: ________ Period: ________

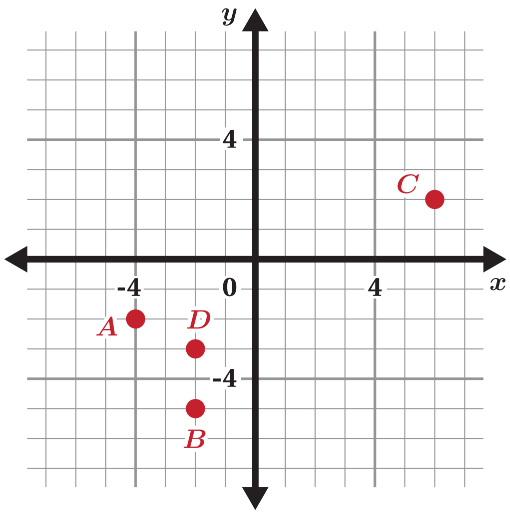
Polygon Maker

Let's explore polygons on the coordinate plane.

Warm-Up

1 Let's watch how a Polygon Maker works.

Point	Coordinates
A	(-4, -2)
B	(-2, -5)
C	(6, 2)
D	(-2, -3)



a Connect the coordinates to create a *polygon*. Tell a classmate what the polygon reminds you of.

b Change one coordinate and connect the points to make a new polygon.

Activity 1

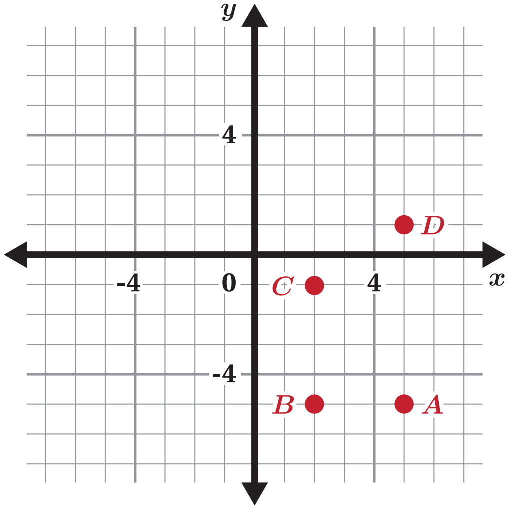
Name: ____ Date: ____ Period: ____

A Polygon

2 A student wants to make a rectangle.

Change some of their coordinates so that the Polygon Maker makes a rectangle.

Point	Coordinates
A	(5, -5)
B	(2, -5)
C	(2, -1)
D	(5, 1)



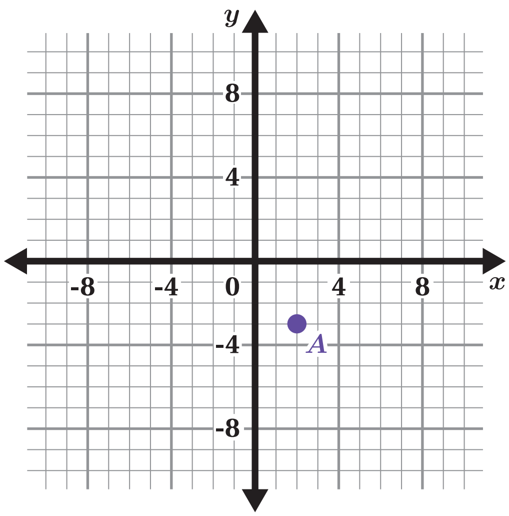
3 Calculate the perimeter of the rectangle you made.

Explain your thinking.

4 Vicente wants to make a square.

Complete the table with coordinates that will make a square. Sketch the square you made on the coordinate plane.

Point	Coordinates
A	(2, -3)
B	
C	
D	

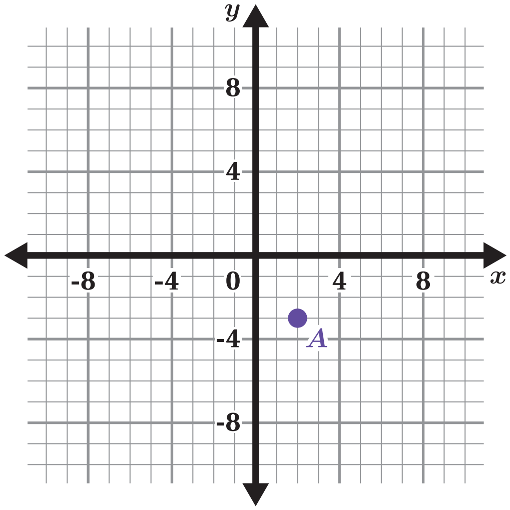


Name: ________ Date: ________ Period: ________

A Polygon (continued)

5 Here are Vicente's coordinates.

Point	Coordinates
A	(2, -3)
B	(2, 4)
C	(9, 4)
D	(9, -3)



Do they make a square? Circle one.

Yes No I'm not sure

Show or explain your thinking.

6 Vicente says: *My square has side lengths of* 7.

Sketch or describe where you see 7 in the table and graph.

Point	Coordinates
A	(2, -3)
B	(2, 4)
C	(9, 4)
D	(9, -3)

Name: .... Date: .... Period: ....

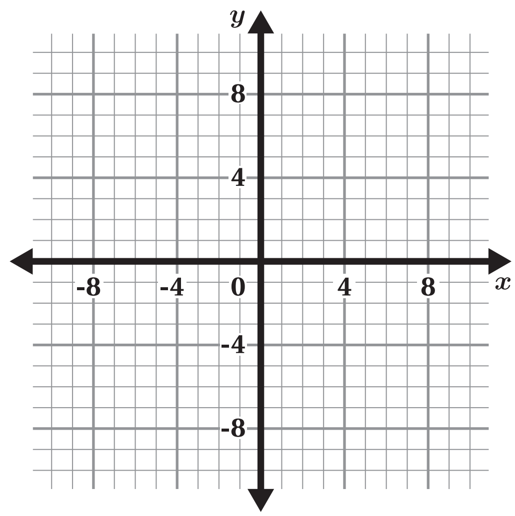
Make Your Own Polygon

7 Here are three descriptions:

- Looks like a house that's 15 units tall
- Looks like a capital L that's 6 units wide
- A rectangle with an area of 24 square units

Choose one description. Write and sketch coordinates to make a polygon that matches that description.

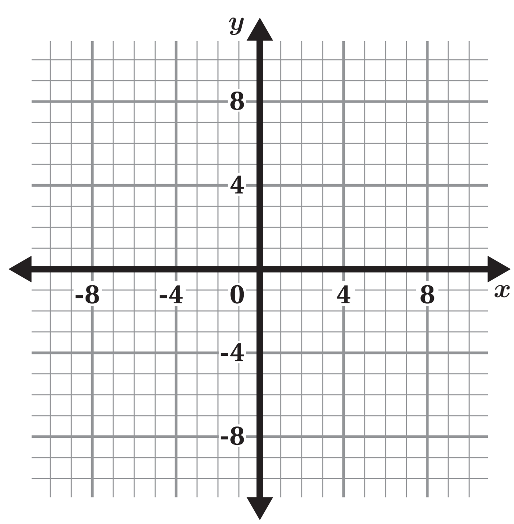
Point	Coordinates
A	
B	
C	
D	
E	



8 You will create your own polygon challenge.

- Write a description of a polygon on a slip of paper. Make it fun (like "looks like a pinecone") and include the size (like "must have a perimeter of 20 units").
- On this page, write the coordinates and sketch a polygon that matches your description.

Point	Coordinates
A	
B	
C	
D	



Name: ______ Date: ______ Period: ______

Make Your Own Polygon (continued)

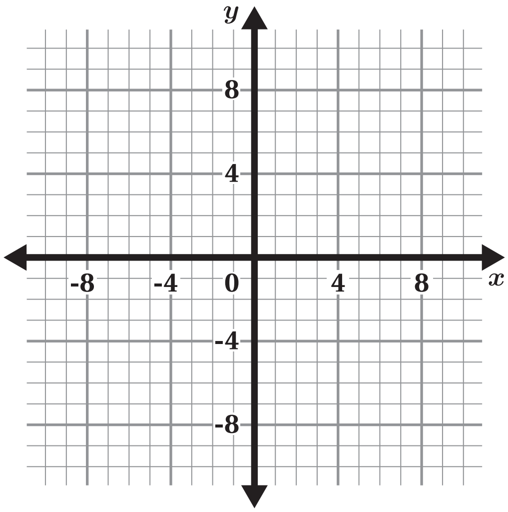
8 You will swap your polygon challenge with two classmates.

- Hand your slip of paper to a partner and invite them to use coordinates to create a polygon on their paper.
- When they finish, share your original polygon and compare.
- Write the coordinates and graph your partners' polygon challenges on this page.

Partner 1

Description:

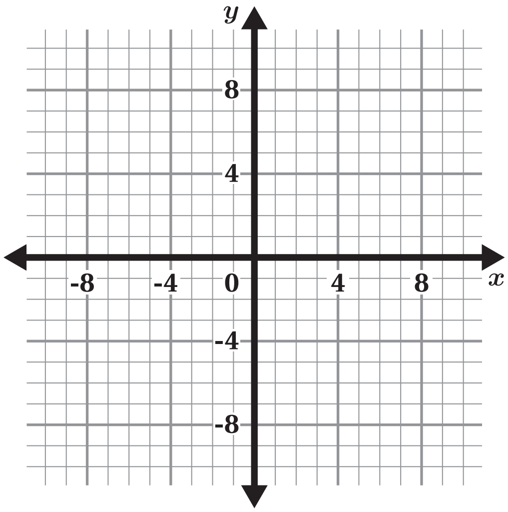
Point	Coordinates
A	
B	
C	
D	



Partner 2

Description:

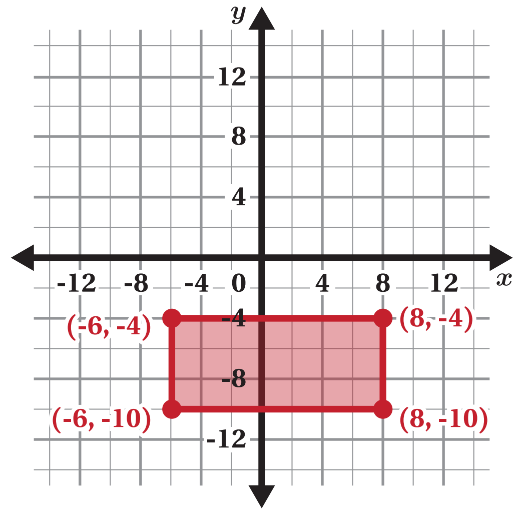
Point	Coordinates
A	
B	
C	
D	



9 Synthesis

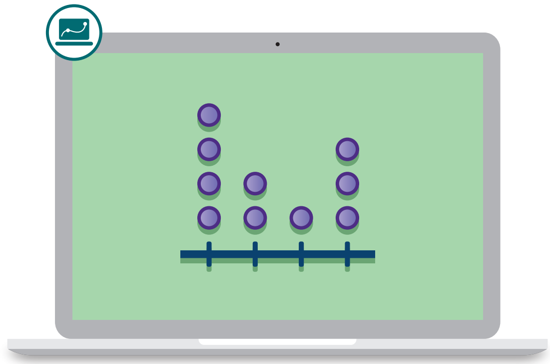
Describe how you can use the coordinates to calculate the side lengths of a rectangle. Use the table and graph if they help with your explanation.

Point	Coordinates
A	(-6, -4)
B	(8, -4)
C	(8, -10)
D	(-6, -10)



Things to Remember:

Name: ____________ Date: ________ Period: ________



Dot Plots

Let's turn data into dot plots.

Warm-Up

1 Antwon used a survey to collect data about some of his friends.

What question might he have asked?

Friend	?
Maria	20
Citlalli	22
Carlos	17
Laila	10
Troy	45
Omar	15
Issa	20

Name: .... Date: .... Period: ....

Dot Plots

2 Meet Antwon's classmates!

Antwon claims that most of these students spend more than 4 hours on their phones each day.

Do you agree? Circle one.

Agree   Disagree   I'm not sure

Explain your thinking.

How many hours do you spend on your phone each day?

Terrance 0 hr	Shanice 3 hr	Dylan 4 hr	Felipe 0 hr	Farah 0 hr
Omar 0 hr	Maria 4 hr	Esteban 5 hr	Emma 6 hr	Alma 2 hr
Diamond 5 hr	Mauricio 6 hr	Eva 2 hr	Kai 9 hr	Troy 3 hr
Rio 3 hr	Citlalli 7 hr	Carlos 4 hr	Laila 0 hr	Issa 8 hr

3 Let's look at another way to visualize numerical data: as a **dot plot**.

What do you notice? What do you wonder?

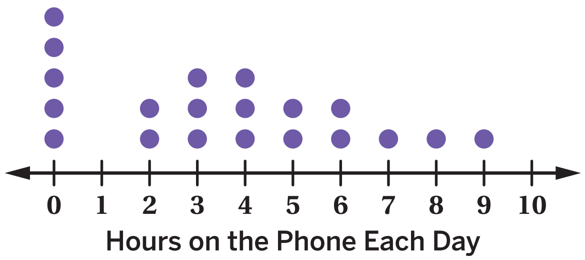
I notice:

I wonder:

Name: .... Date: .... Period: ....

Dot Plots (continued)

Here is the dot plot of the data from Antwon's classmates.



4 Raven is a new student in Antwon's class. She spends 3 hours on her phone each day. Draw a dot to add Raven's data to the dot plot.

5 Dylan says: *I spend 4 hours on my phone each day.*

a Circle a dot on the dot plot that could represent Dylan.

b Is this the only dot that could represent Dylan?

Yes No I'm not sure

Explain your thinking.

6 Here is Antwon's claim from earlier.

He said: *Most students spend more than 4 hours on their phones each day.*

How can a dot plot help you decide if his claim is true?

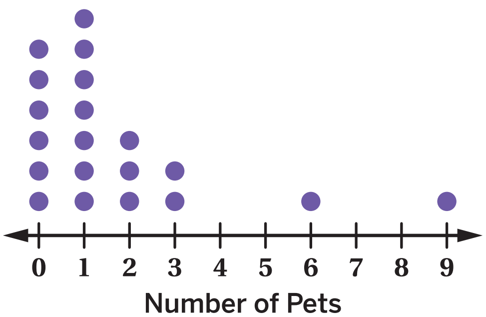
Activity
2

Name: ________ Date: ________ Period: ________

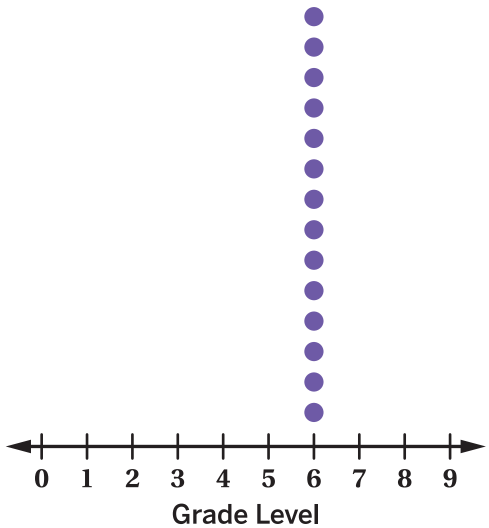
Statistical Questions

7 Antwon asked his classmates a few more questions and made dot plots of their answers.

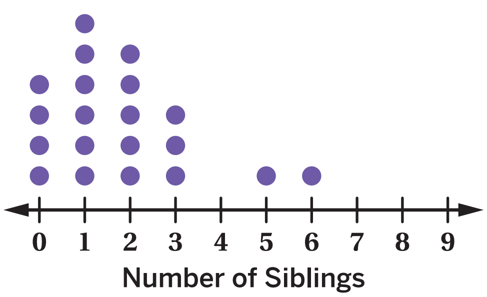
How many pets do you have?



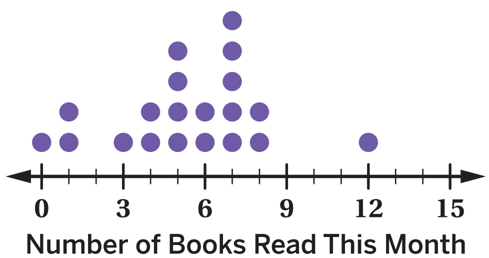
What grade are you in?



How many siblings do you have?



How many books have you read this month?



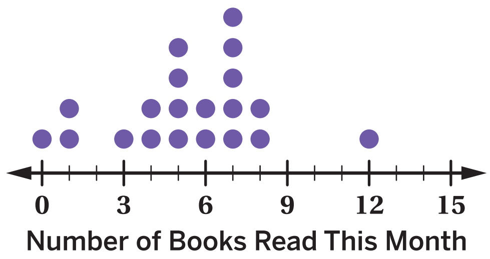
Discuss: What are two things you learned about Antwon's classmates from this data?

Name: ______ Date: ______ Period: ______

Statistical Questions (continued)

8 This dot plot shows the number of books Antwon's classmates read this month.

Write a question that this dot plot could help you answer.



9 Which question would a dot plot be *less* helpful in answering? Circle one.

How many books has Farah read this month?

Have most students read more than 3 books this month?

Explain your thinking.

10 A **statistical question** requires more than one piece of data to answer it.

Select *all* the statistical questions.

- ☐ **A.** Have most students read more than 3 books this month?
- ☐ **B.** How many books has Farah read this month?
- ☐ **C.** Do students in this class prefer fiction or nonfiction?
- ☐ **D.** Is the book that Citlalli read fiction or nonfiction?
- ☐ **E.** How long are the books that Kai read?

11 Synthesis

What are the advantages and disadvantages of visualizing a data set as a dot plot?

Use the examples if they help with your thinking.

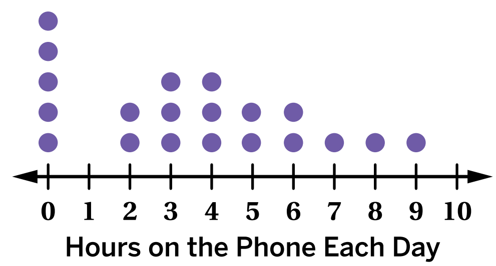
Advantages:

Disadvantages:

Data Set

0, 3, 4, 0, 0, 0, 4, 5, 6, 2, 5, 6, 2, 9, 3, 3, 7, 4, 0, 8

Dot Plot



Things to Remember:

Unit 8
Lesson
5

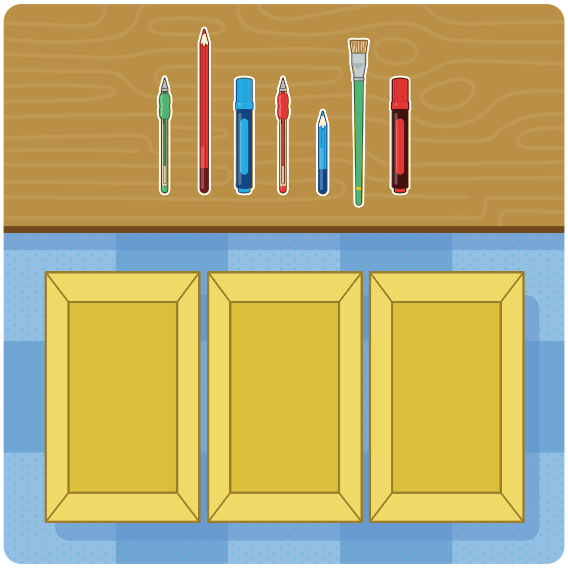
Name: .... Date: .... Period: ....

The Plot Thickens

Let's use histograms to represent data sets.

Warm-Up

1 a. Organize the art supplies into the bins.



b. Explain how you organized the supplies.

Activity 1

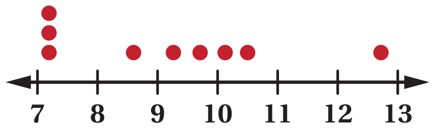
Name: ________ Date: ________ Period: ________

Introduction to Histograms

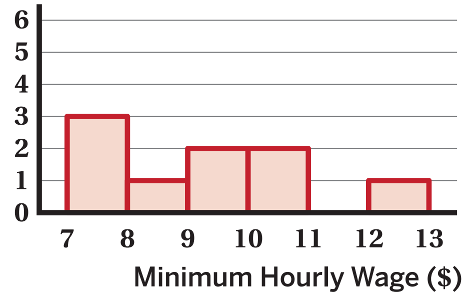
2 Here are two different ways to show the minimum wages of 9 U.S. states in 2020.

Discuss: How do you think the **histogram** was made?

Dot Plot



Histogram



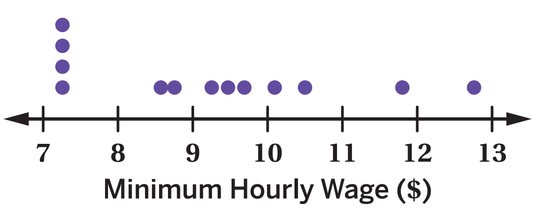
Source: *United States Department of Labor*

3 How are the histogram and the dot plot alike?

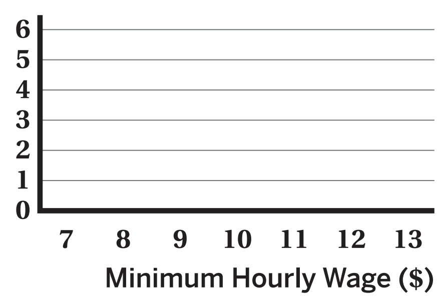
How are they different?

4 More states were added to the dot plot. Each bar in a histogram is called a **bin**.

Draw bins to make a histogram with this new data.



Source: *United States Department of Labor*

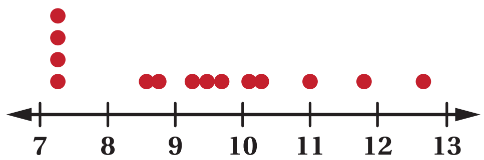


Name: ______ Date: ______ Period: ______

Creating Histograms

5 In statistics, each bin includes the smaller of the two numbers and goes up to but does not include the larger number. When a data point is on the edge between two bins, it goes into the bin with the larger values.

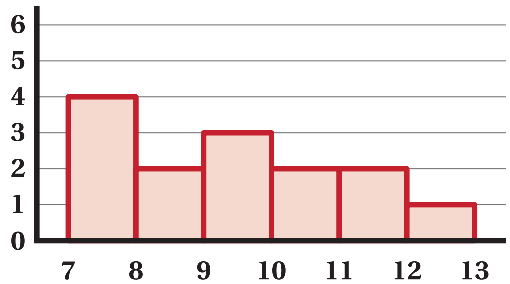
a $11 was added to this dot plot. Let's watch which bin the $11 goes into.



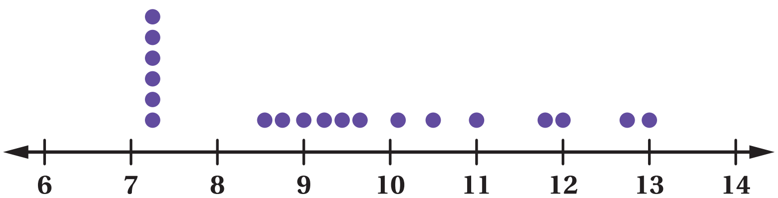
b Which bin would $8 go into? Circle one.

7 to 8   8 to 9   Somewhere else

Explain your thinking.

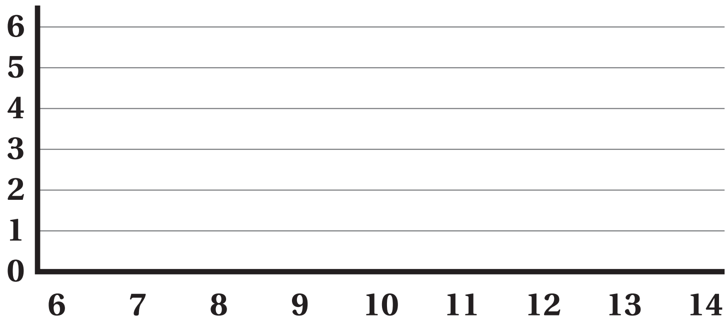


6 So far, we've seen histograms with a bin size of 1. Here is a dot plot with some more minimum wages.



Source: *United States Department of Labor*

Make a histogram with a bin size of 2.



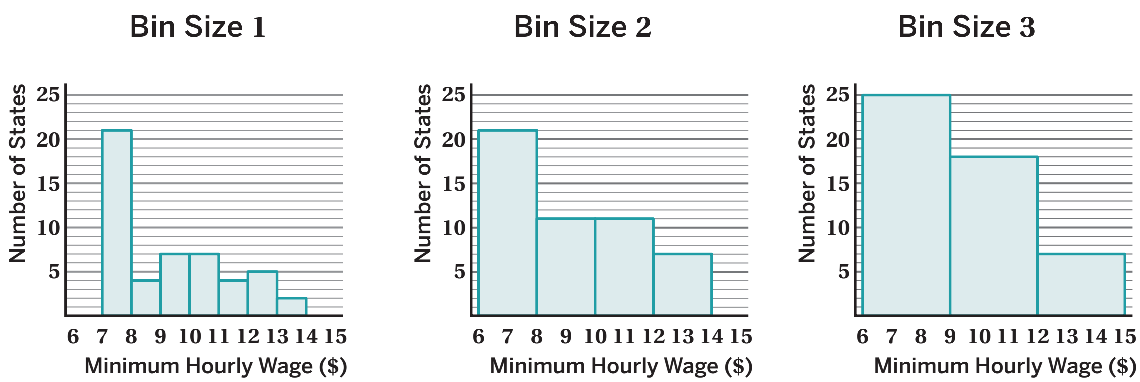
Activity 3

Name: .... Date: .... Period: ....

Using Histograms

7 Let's look at the minimum wages for all 50 U.S. states. Why might these data points be difficult to visualize as a dot plot?

8 Each of these histograms show the minimum wages of all 50 states in 2020.



Source: *United States Department of Labor*

Discuss: What do you notice? What do you wonder?

9 Different visualizations like dot plots and histograms help answer different questions.

a Which of these questions can a histogram help answer?

A. Which state has the highest minimum wage?

B. What is the highest minimum wage?

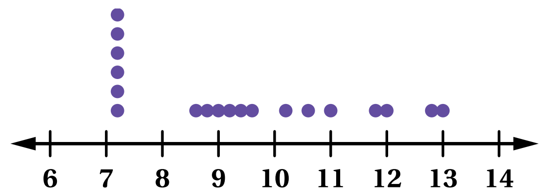
C. How many states have a minimum wage of $7.25?

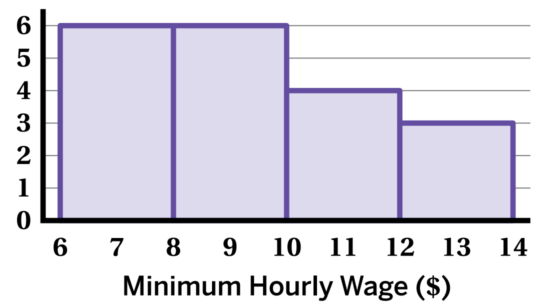
D. How many states have a minimum wage of $10 or more?

b Answer the question you chose.

10 Synthesis

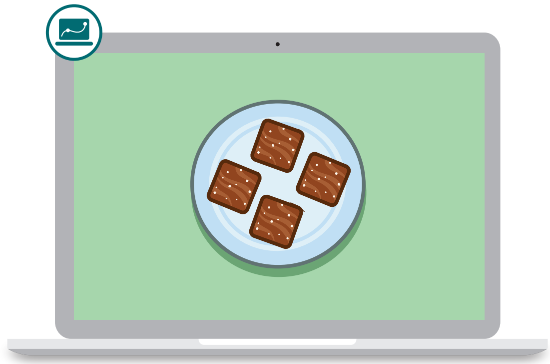
What is important to remember about a histogram?





Things to Remember:

Name: .... Date: .... Period: ....

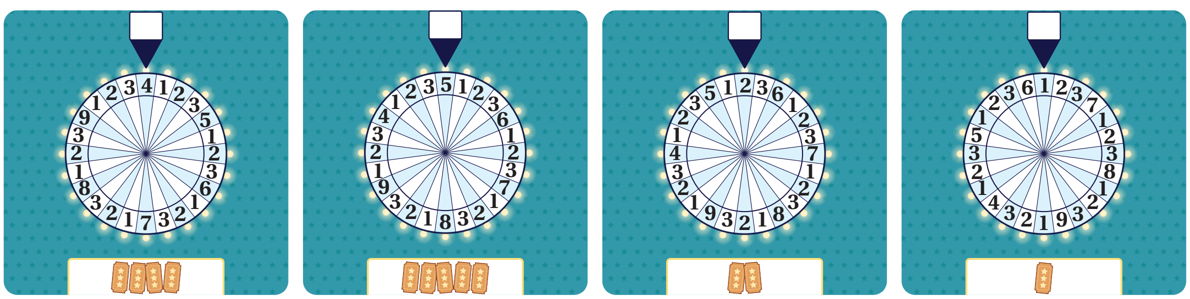


Snack Time

Let's explore the mean of a data set and what it tells us.

Warm-Up

1 Here are four images of an arcade game.



Discuss: What do you notice? What do you wonder?

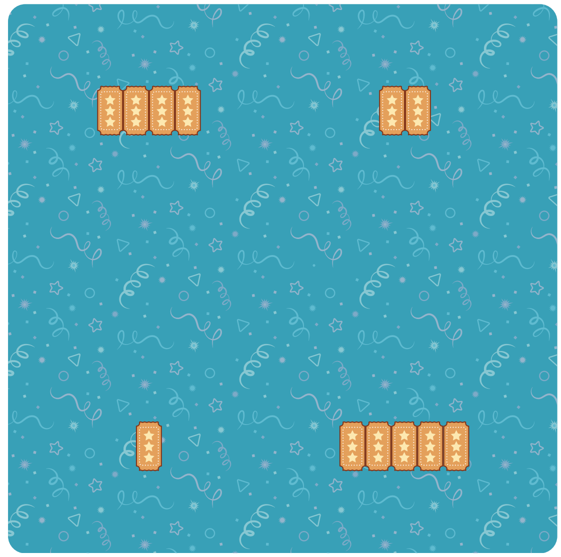
Activity
1

Name: ______ Date: ______ Period: ______

Mean as an Equal Share

2 4 friends played this game at the arcade. Here are the tickets each friend won.

They decided to share the tickets equally. How many tickets should each friend get? Explain your thinking.



3 Here is Ava's work for determining how many tickets each friend gets.

Discuss: What was Ava's strategy?

Ava

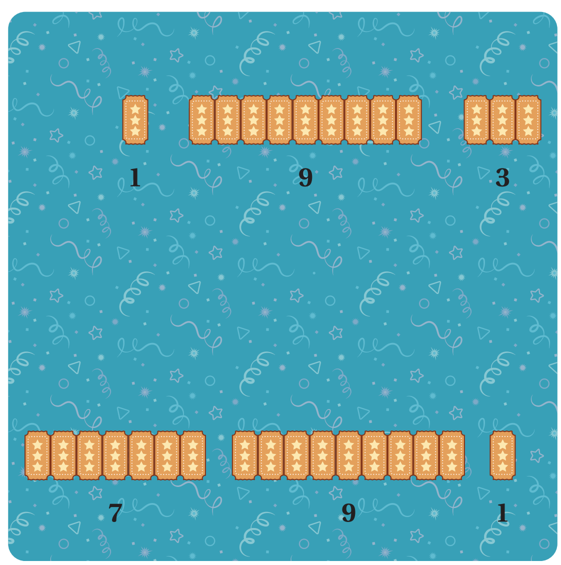
$4 + 2 + 1 + 5 = 12$

$12 \div 4$

4 The **mean**, or average, is the number of tickets each friend gets if the tickets are distributed equally.

Here are the tickets that 6 other friends won.

Calculate the mean number of tickets.



Activity
2

Name: ____________ Date: ____________ Period: ____________

Mean as a Statistic

5 A **statistic** is a single number that measures something about a data set.

The mean is an example of a statistic.

What does the mean tell us about Ishaan's snacks?

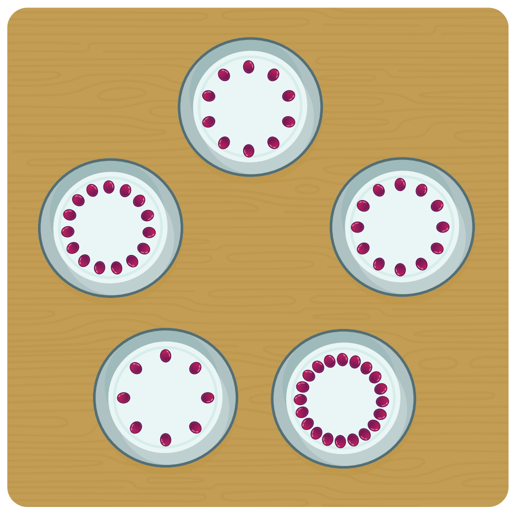
Ishaan's Snacks

Day	Number of Carrots
Monday	10
Tuesday	8
Wednesday	5
Thursday	12
Friday	10

Mean: 9 carrots

6 Here is how many grapes Crow ate each day last week. Calculate the mean of this data.

Day	Number of Grapes
Monday	10
Tuesday	12
Wednesday	20
Thursday	8
Friday	15



7 Ali and Makayla ate blueberries each day last week. Ali ate 12 blueberries per day. The table shows how many blueberries Makayla ate.

Who ate more blueberries per day on average? Circle one.

Ali　　Makayla　　They ate the same amount on average

Explain your thinking.

Ali's Snacks

12 blueberries per day

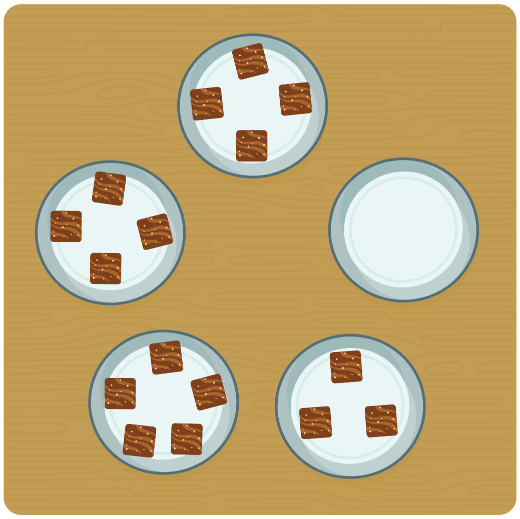
Makayla's Snacks

Day	Number of Blueberries
Monday	1
Tuesday	15
Wednesday	13
Thursday	11
Friday	15

Name: ______ Date: ______ Period: ______

Mean as a Statistic (continued)

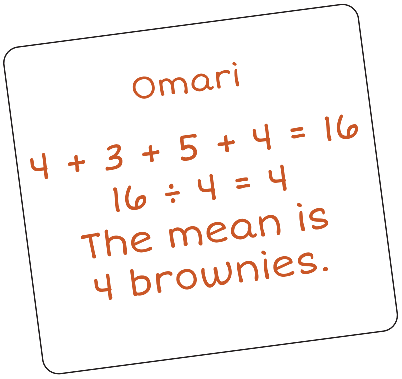
8 Here is how many brownies Omari ate.



Day	Number of Brownies
Monday	4
Tuesday	0
Wednesday	3
Thursday	5
Friday	4

Calculate the mean of this data.

9 Omari calculated the mean incorrectly.



Discuss:

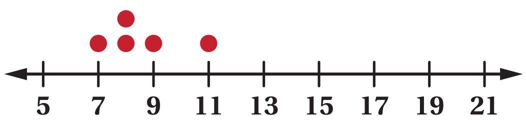
- What is correct about Omari's work?
- What is incorrect about Omari's work?

Name: .... Date: .... Period: ....

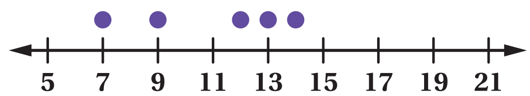
Mean as a Measure of Center

10 For 5 days, Oliver recorded the number of cherries he had as a snack. The mean for his data is 11 cherries. Circle a data set that could *not* be Oliver's.

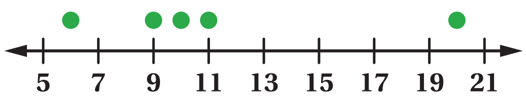
A. Number of Cherries



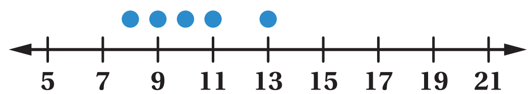
B. Number of Cherries



C. Number of Cherries



D. Number of Cherries

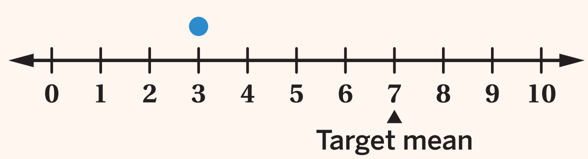


Explain your thinking.

Explore More

11 Add at least *four* more points to create a dot plot that has a mean of 7. Then check your work.

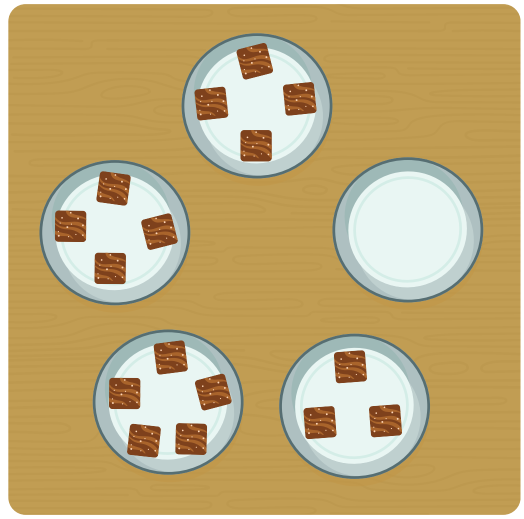
Create as many different dot plots as you have time for.



12 Synthesis

How can you determine the mean of a data set?

Use the example if it helps with your thinking.



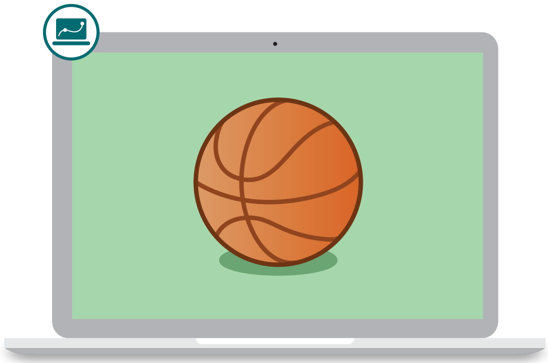
Things to Remember:

Unit 8
Lesson
9

Name: ________ Date: ________ Period: ________

Hoops

Let's explore how to measure the spread of a data set.



Warm-Up

1–2 You will use the Warm-Up Sheet for this activity.

How many baskets can you make?

- Flick the counter from each of the starting points. If the counter lands on top of the hoop, you made a basket.
- You get 8 shots per round.
- Try to make as many as you can!

Round 1: ________ baskets Round 2: ________ baskets

3 Let's look at the Round 1 and Round 2 results from the class.

Discuss: What do you notice?

Activity 1

Name: .... Date: .... Period: ....

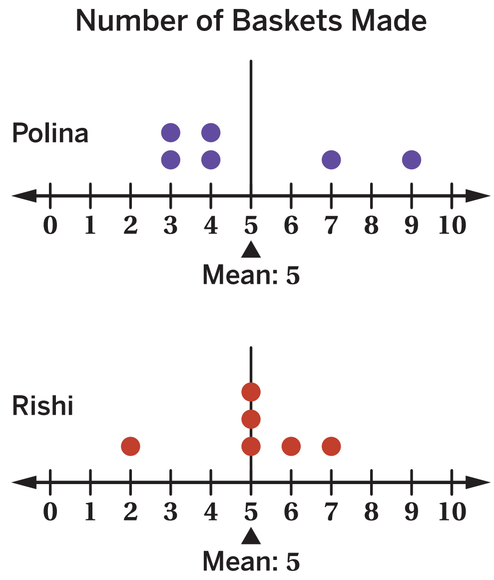
Mean Absolute Deviation

4 Polina and Rishi practice 10 free throws everyday. Each dot plot shows the number of baskets they made on 6 different days.

Which player is more consistent? Circle one.

Polina Rishi

Explain your thinking.



5 One way to measure consistency is to calculate the average of the absolute deviations. This is known as the **mean absolute deviation (MAD)**.

a Let's watch how to calculate the mean absolute deviation.

b Explain how to calculate the mean absolute deviation in your own words.

6 The mean absolute deviation (MAD) is a statistic that measures the spread of a data set. Polina's data has a MAD of 2 baskets.

Will the MAD of Rishi's data be larger than, smaller than, or equal to the MAD of Polina's data? Circle one.

Larger Smaller Equal

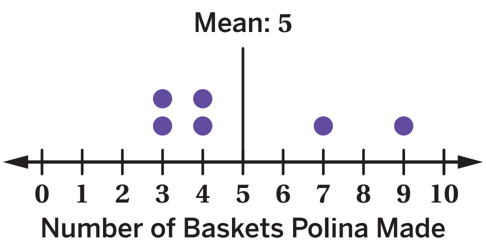
Activity 2

Name: ____________ Date: ________ Period: ________

Calculating MAD

7 Here is Polina's data.

Number of Baskets	3	3	4	4	7	9
Absolute Deviation (distance from the mean, 5)	2	2	1	1	2	4



Mean Absolute Deviation (MAD)

Sum of absolute deviations:

$2 + 2 + 1 + 1 + 2 + 4 = 12$

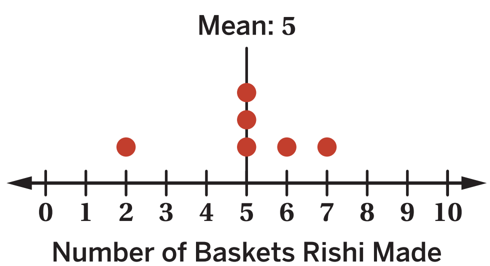
Average or mean of the absolute deviations:

$12 \div 6 = 2$

Discuss: What do you notice? What do you wonder?

8 Here is Rishi's data.

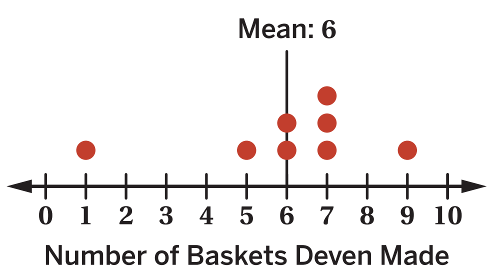
Calculate the MAD. Use the table to help you organize your thinking.



Number of Baskets	2	5	5	5	6	7
Absolute Deviation (distance from the mean, 5)						

9 Deven also practices free throws everyday.

Calculate the MAD of Deven's data. Use the table to help you organize your thinking.



Number of Baskets	1	5	6	6	7	7	7	9
Absolute Deviation (distance from the mean, 6)								

Name: Date: Period:

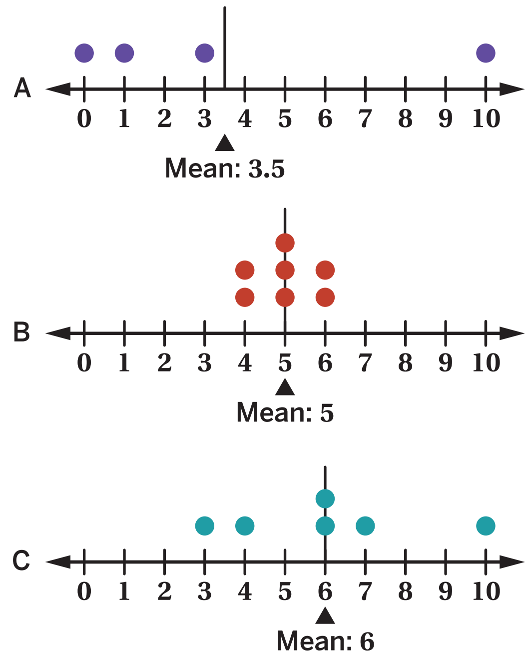
Calculating MAD (continued)

10 Here are the number of baskets that three new players made.

Order the data sets from *smallest* to *largest* MAD.

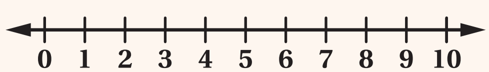
	Smallest MAD
	Largest MAD

Number of Baskets



Explore More

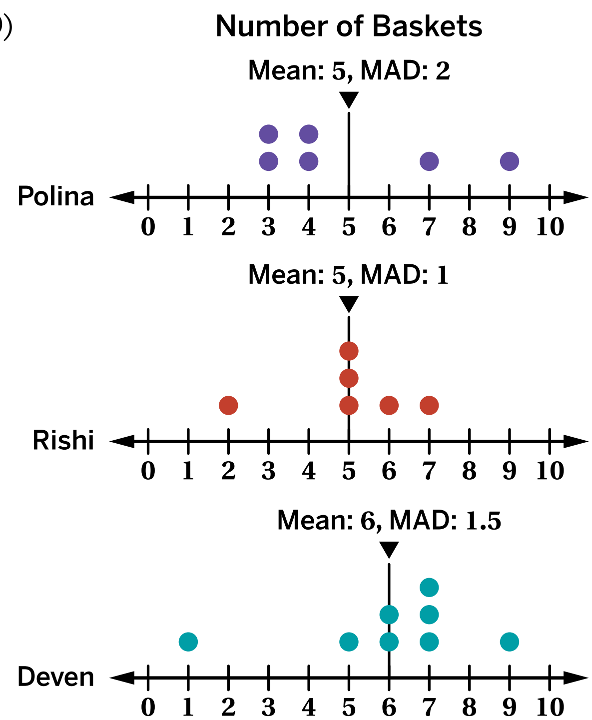
11 Create a dot plot with at least *four* points, a mean of 6, and a MAD of 1.



12 Synthesis

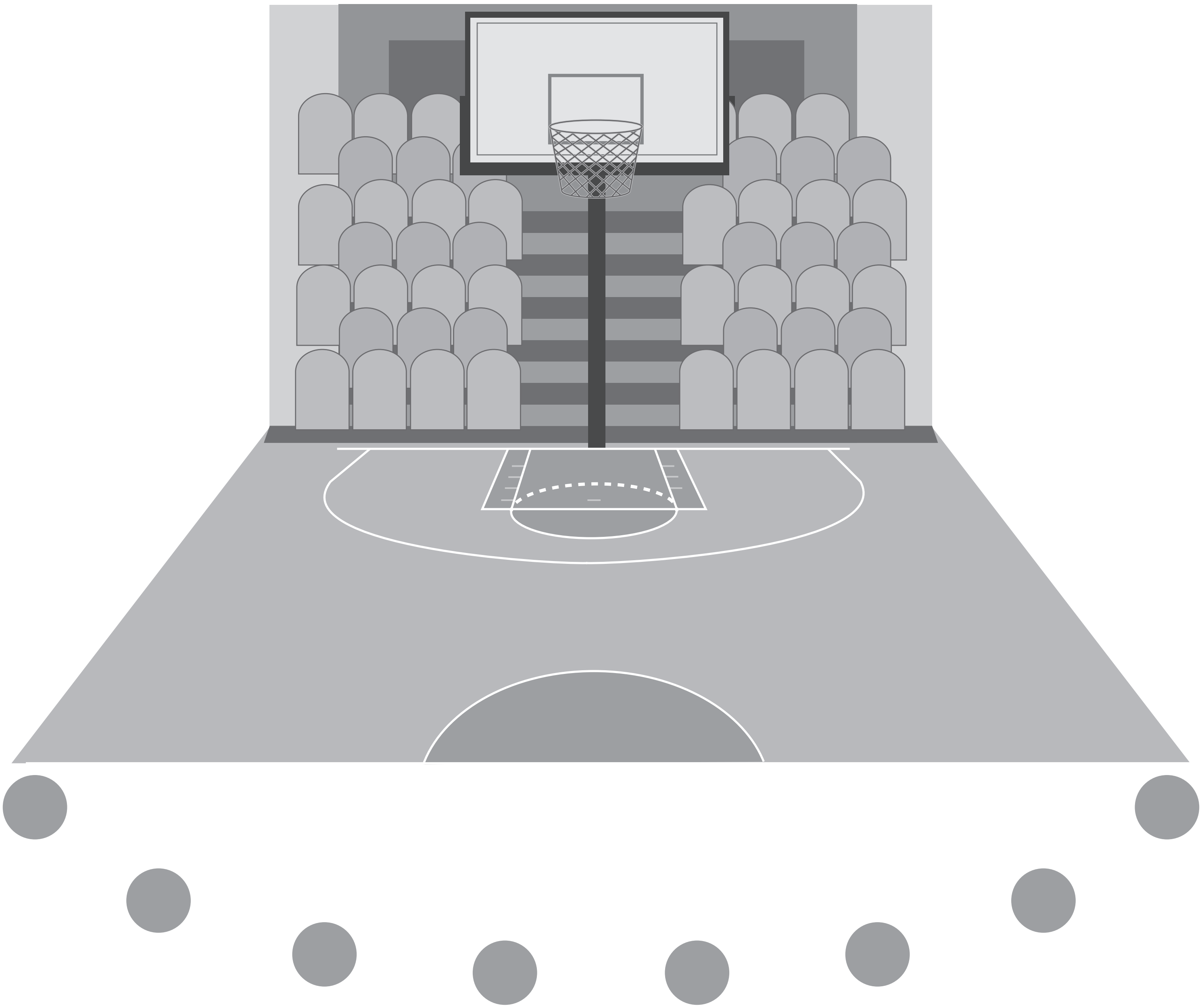
How does the mean absolute deviation (MAD) help you compare data sets?

Use the examples if they help with your thinking.



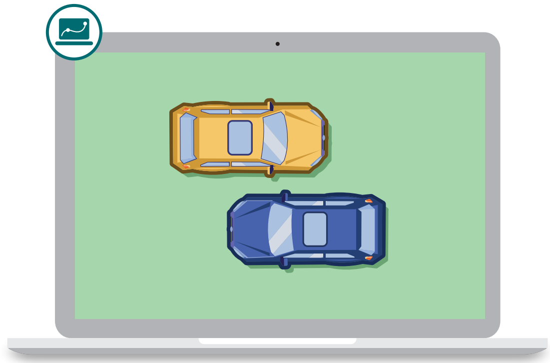
Things to Remember:

Warm-Up



Unit 8
Lesson
11

Name: ______ Date: ______ Period: ______



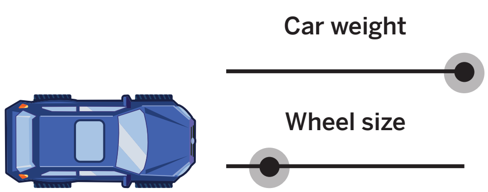
Toy Cars

Let's explore the median of a data set and what it tells us.

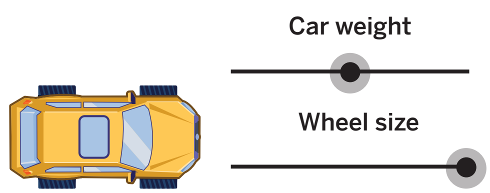
Warm-Up

1–2 Here are two toy cars with different colors, weights, and wheel sizes.

Car A

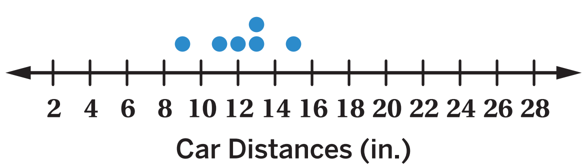


Car B

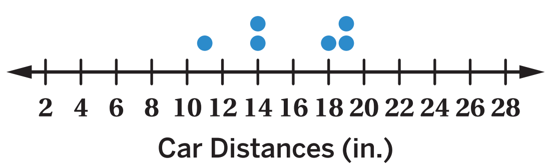


a Each car was launched 6 times. Compare their results.

Car A



Car B

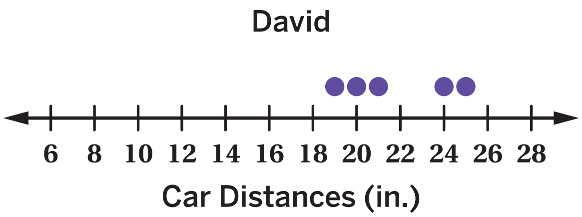


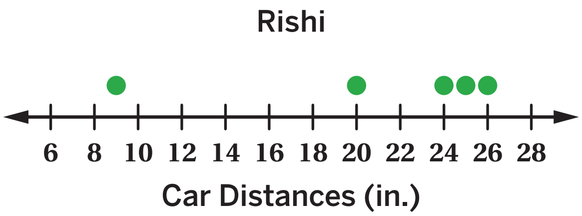
b **Discuss:** Which car generally travels farther? How do you know?

Name: .... Date: .... Period: ....

A New Measure of Center

3 David and Rishi launched their cars 5 times each.





Which car do you think generally travels farther? Circle one.

David's car   Rishi's car   I'm not sure

Explain your thinking.

4 David and Rishi each think their car travels farther.

David says: *When we compare the mean distance for each car, my car travels farther.*

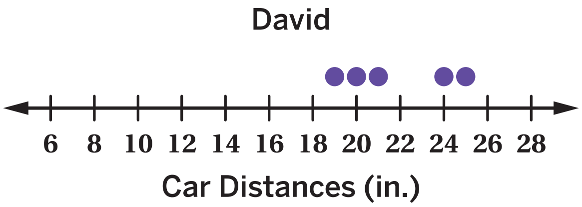
Rishi says: *When we compare the middle distance for each car, my car travels farther.*

Whose argument do you agree with? Circle one.

David's   Rishi's   Both   Neither

Explain your thinking.

5 The middle value of a data set is called the **median**. The median distance for Rishi's car is 24 inches. What is the median distance for David's car?



Activity
2

Name: .... Date: .... Period: ....

Many Medians

6 Yona launched her car 7 times and recorded the distances, in inches, on a notepad. She calculated the median but made a mistake.

Yona
~~19~~, ~~14~~, ~~18~~, 28, ~~21~~, ~~12~~, ~~14~~

a **Discuss:** What did Yona do well?

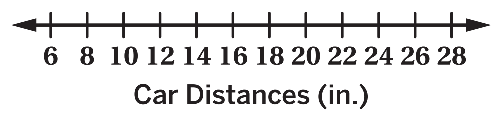
b What could you change to make all her work correct?

7 Here is Yona's data.

19, 14, 18, 28, 21, 12, 14

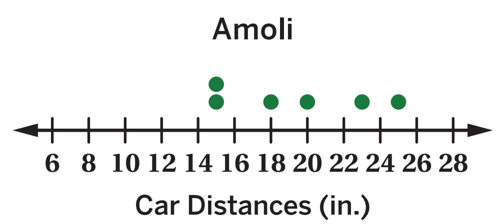
What is the median distance of this data set?

Create a dot plot if it helps with your thinking.



8 Amoli launched her car 6 times.

What do you think is the median of her data set? Explain your thinking.



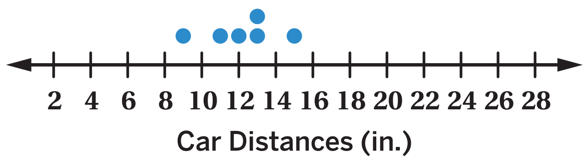
Name: ________ Date: ________ Period: ________

Many Medians (continued)

9 Statisticians agree that when there's an even number of data points, the median is the average of the middle two numbers.

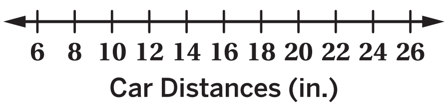
The median distance for Amoli's car is 19 inches because the average of 18 and 20 is 19.

The dot plot shows the data on Car A. What is the median distance that Car A traveled?



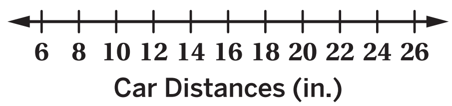
10 Calculate the median distance for each car. Complete as many problems as you have time for. Create a dot plot if it helps with your thinking.

a 14, 19, 15, 20, 17



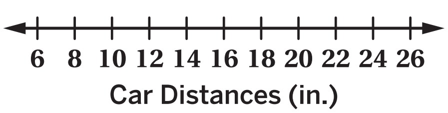
________ inches

b 25, 22, 19, 21, 14, 14



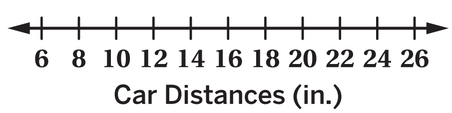
________ inches

c 18, 12, 24, 12, 18, 22, 22



________ inches

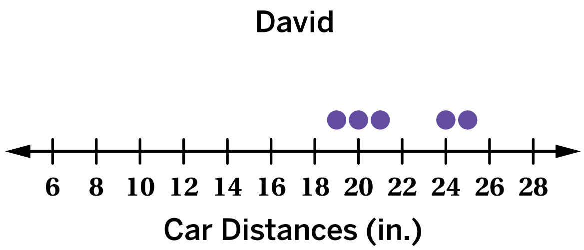
d 16, 6, 26, 14, 23, 27, 28, 19, 23, 17

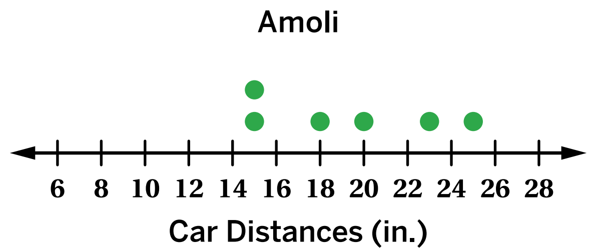


________ inches

11 Synthesis

Describe how to determine the median of a data set. Use the examples if they help with your thinking.

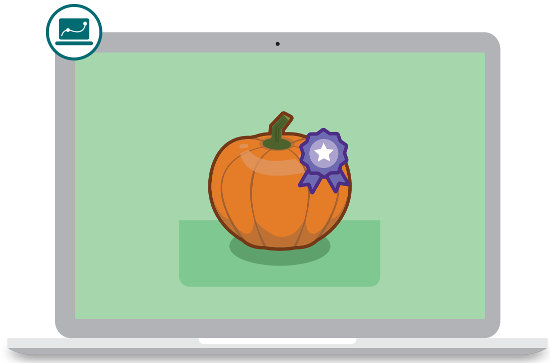




Things to Remember:

Unit 8
Lesson
13

Name: ______ Date: ______ Period: ______



Pumpkin Patch

Let's determine and interpret the quartiles of a data set.

Warm-Up

1 A farmer sells different sizes of pumpkins.

A customer wants a medium-sized pumpkin.

Circle the pumpkins the farmer could sell to this customer.

Activity 1

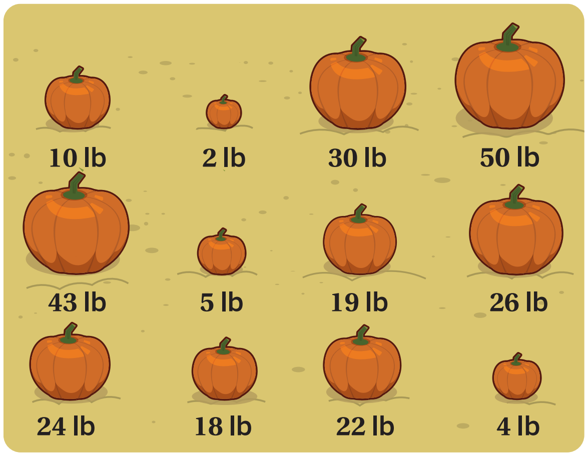
Name: .... Date: .... Period: ....

Introduction to Quartiles

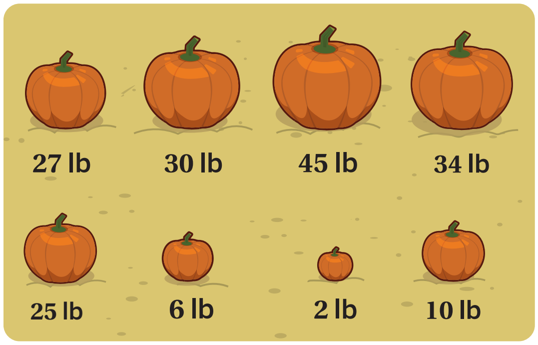
2 Farmer Na'ilah always marks the middle half of her pumpkins as medium-sized.

a Let's watch an animation to see how Na'ilah decided which pumpkins were in the middle half.

b Explain how she decided.



3 Here are some pumpkins on a different farm. Circle the pumpkins with weights that are in the middle half.



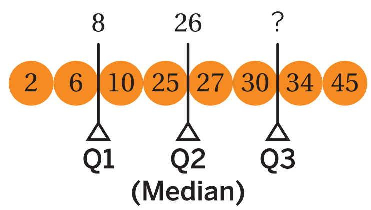
Name: .... Date: .... Period: ....

Introduction to Quartiles (continued)

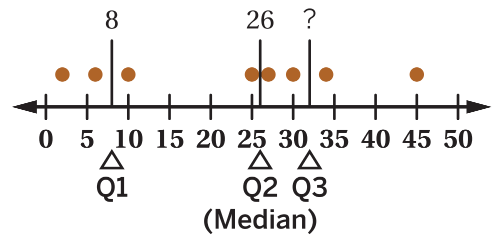
4 **Quartiles** divide a data set into four sections. They can help us identify and describe the middle half of a data set.

Here are Quartile 1 (Q1) and Quartile 2 (Q2) for Na'ilah's pumpkins.

Pumpkin Weights (lb)



Pumpkin Weights (lb)



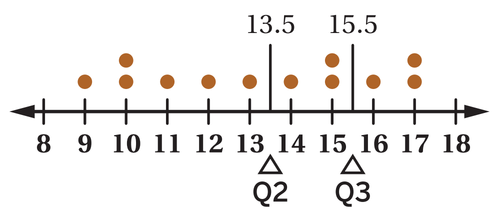
What do you think the value of Quartile 3 is? Explain your thinking.

5 Quartile 1 is the median of the lower half of a data set.

Here are the weights of 12 pumpkins on Tasia's farm.

What is the value of Quartile 1?

Pumpkin Weights (lb)

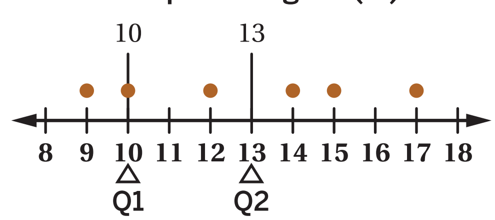


6 Quartile 3 is the median of the upper half of a data set.

Here are the weights of 6 pumpkins on Tyler's farm.

What is the value of Quartile 3?

Pumpkin Weights (lb)

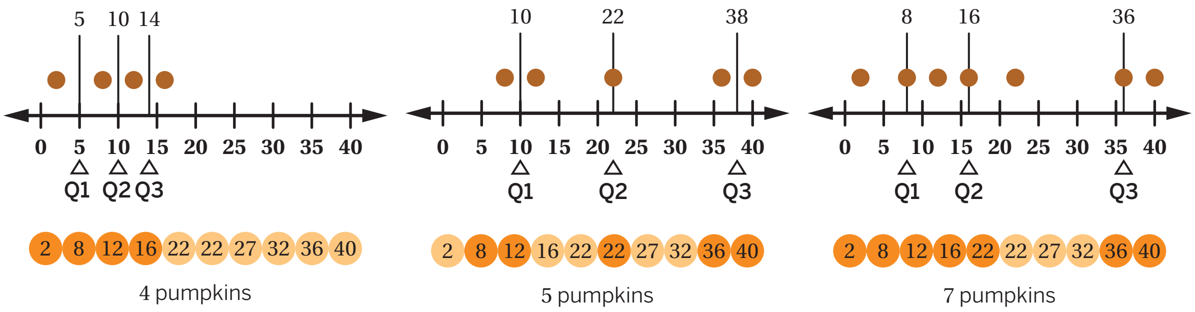


Activity 2

Name: .... Date: .... Period: ....

Reasoning About Quartiles

7 Here are some dot plots with different pumpkin weights.



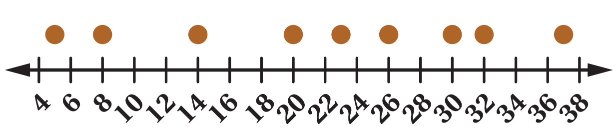
a **Discuss:** What do you notice?

b Here are several challenges. Select *all* the ones you think are possible. Use a device to check your answers.

- ☐ Q2 is not equal to a pumpkin weight.
- ☐ Q1, Q2, and Q3 are equal to pumpkin weights.
- ☐ Q1, Q2, and Q3 are not equal to pumpkin weights.
- ☐ Q1 is equal to a pumpkin weight, but Q3 is not.
- ☐ Q1 and Q2 have the same value.

8 Here are 9 pumpkins on Adrian's farm. What do you think the values of Q1 and Q3 are? Explain your thinking. Use the dot plot if it helps to show your thinking.

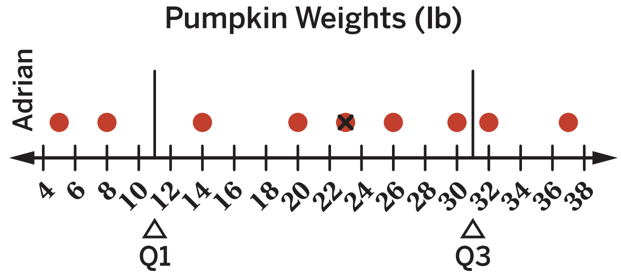
Pumpkin Weights (lb)



Name: Date: Period:

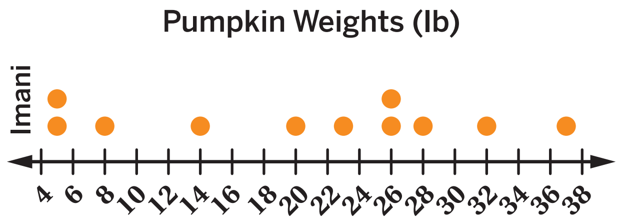
Reasoning About Quartiles (continued)

9 Statisticians agree that when you determine Q1 and Q3 for a data set that has an odd number of points, you do *not* include the median in the lower or upper half of the data.



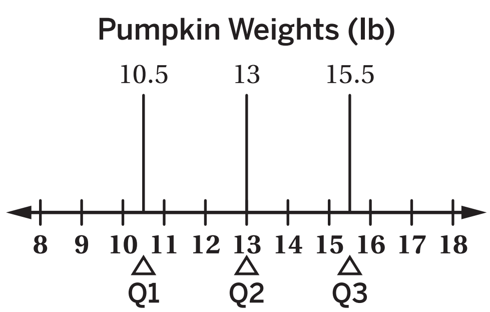
This dot plot shows the weight of 11 pumpkins on Imani's farm.

What is the value of Q3 for these pumpkin weights?



10 A store has 80 pumpkins for sale. Here are the values of the quartiles.

About how many of the 80 pumpkins would you expect to weigh *less* than 15.5 pounds? Explain your thinking.

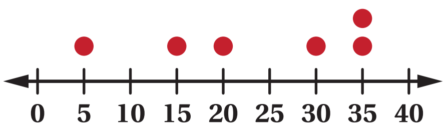


Name: ______ Date: ______ Period: ______

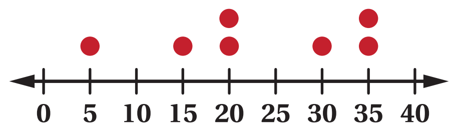
Repeated Challenges

11 Solve as many problems as you have time for. Sensemaking is more important than speed.

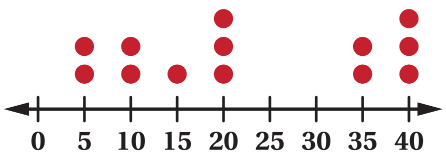
a The dot plot shows 6 data points.
What is the value of Q1?



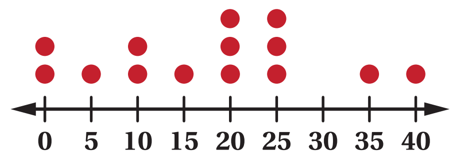
b The dot plot shows 7 data points.
What is the value of Q2?



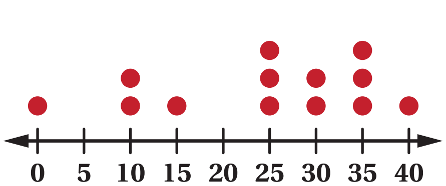
c The dot plot shows 13 data points.
What is the value of Q1?



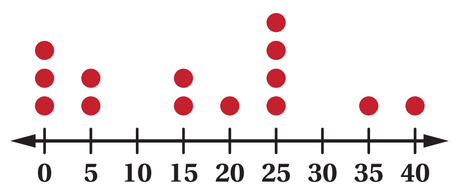
d The dot plot shows 14 data points.
What is the value of Q3?



e The dot plot shows 13 data points.
What is the value of Q2?



f The dot plot shows 14 data points.
What is the value of Q3?



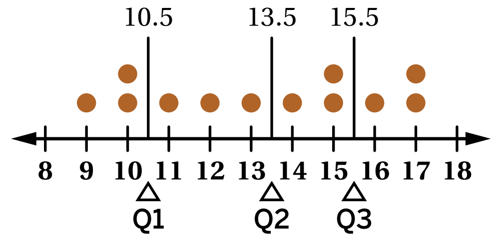
12 Synthesis

Discuss both questions with a partner. Then select *one* and write your response.

Use the example if it helps to show your thinking.

- How do quartiles relate to the middle half of a data set?
- How can you determine the value of the quartiles for a data set?

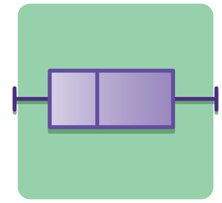
Pumpkin Weights (lb)



Things to Remember:

Name: Date: Period:

Car, Plane, Bus, or Train?



Let's explore box plots to visualize data.

Warm-Up

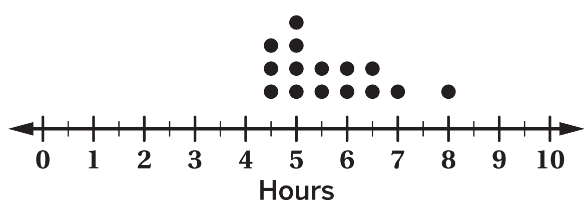
Jalen's family lives in St. Louis. They often visit relatives in Chicago.

1. **Discuss:**

- What modes of transportation do you think Jalen's family could take?
- Which would *you* take? Why?

2. Jalen recorded how long it took to drive from St. Louis to Chicago the last 15 times his family drove there.

Determine the values of Q1, Q2, and Q3. Label them on the dot plot.



Activity 1

Name: ________ Date: ________ Period: ________

Car or Plane?

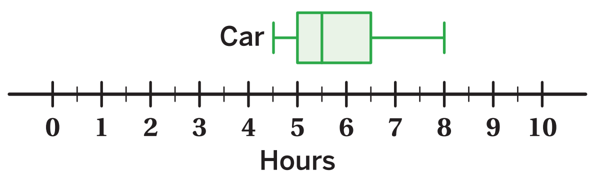
Jalen's family wonders: *What mode of transportation should we use to travel between Chicago and St. Louis?*

3. One way to compare data sets is by using **box plots**. Let's watch how a box plot is drawn.

a **Discuss:** What do you notice? What do you wonder?

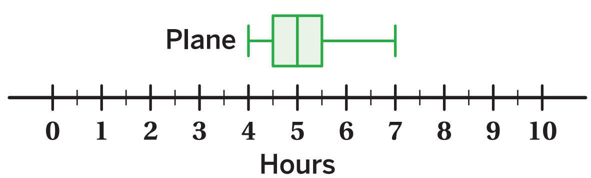
b Label the box plot with each statistic.

- Minimum (Min.)
- Quartile 1 (Q1)
- Quartile 2 (Q2)
- Quartile 3 (Q3)
- Maximum (Max.)



c What percent of the drives are between 5 and 5.5 hours? Explain your thinking.

4. Jalen's family sometimes takes a plane to get to St. Louis. Here is a box plot that represents some of their travel times when they flew.



a Determine each statistic for the plane data.

- Minimum: ________
- Quartile 1: ________
- Median: ________
- Quartile 3: ________
- Maximum: ________

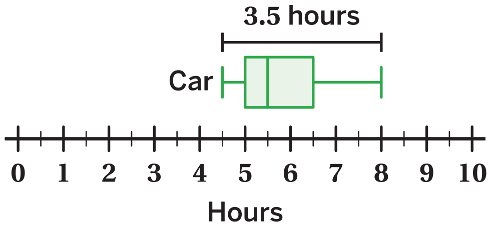
b What percent of the plane data was less than 5 hours long? Explain your thinking.

Name: .... Date: .... Period: ....

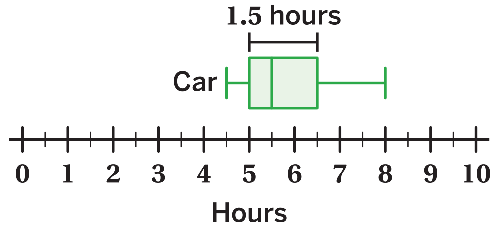
Measures of Spread

5. Jalen's family is also wondering: *How spread out are our travel times when we travel by car?*

Let's look at two ways to describe the spread of a box plot.



The **range** is the distance from the *minimum* to the *maximum*.



The **interquartile range (IQR)** is the distance from Q1 to Q3.

a Read the definitions of range and IQR.

How are they alike?

How are they different?

b Jalen says that one of these measures of spread describes the middle 50% of the data points.

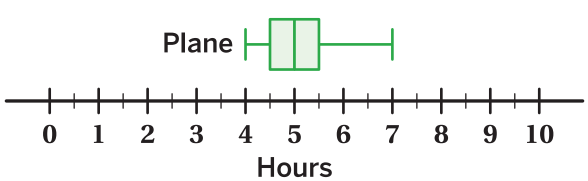
Which measure of spread do you think he is talking about? Circle one.

IQR Range Neither

Explain your thinking.

6. What is the range and IQR for the plane data?

Range: .... IQR: ....



7. Use the car data and the plane data to help you answer: *Which is more consistent, driving or flying?* Explain your thinking.

Activity 3

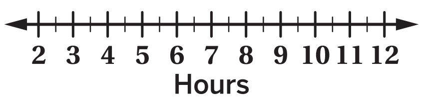
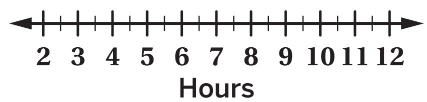
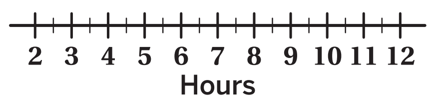
Name: .... Date: .... Period: ....

Bus or Train?

8. Jalen's family has also traveled to Chicago by bus and by train.

You will use a set of cards for this activity.

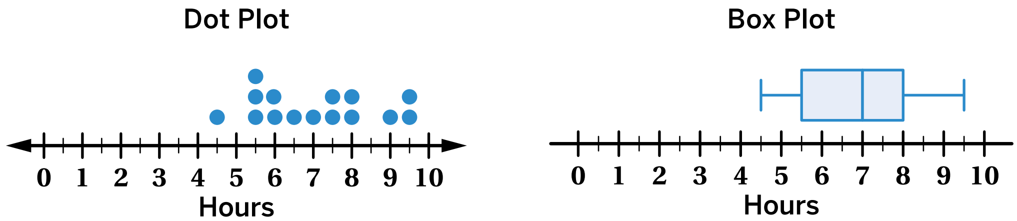
- Match each dot plot, box plot, and statistic to a mode of transportation. There will be two empty spaces in the table for you to complete.
- Draw a box plot on Card A and determine the values of the missing statistics to complete the table.

	Bus	Train
Travel Times (hours)	7.5	10.5	9.5	9.5	6 7	6.5	7.5	11	8.5	6.5	8.5	6	10.5	6.5 8	7	7	9	8
Dot Plot		
Box Plot		
Median		
IQR		
Range		

9. Which mode of transportation (car, train, bus, or plane) would you recommend Jalen's family use to travel between Chicago and St. Louis? Use evidence to support your argument.

Synthesis

10. Here is a dot plot, a box plot, and several statistics for the car data.



Statistics			
Median: 7 hours	Number of Data Points: 15	Range: 5 hours	IQR: 2.5 hours

Which statistics are more visible in the dot plot? In the box plot? Why do you think that is?

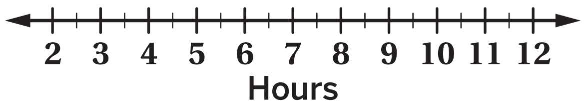
Things to Remember:

Bus or Train?

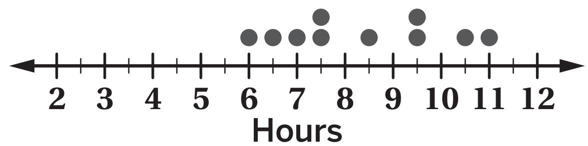
Lesson 8.14
Activity 3

Directions: Make one copy per pair of students. Then pre-cut the cards and give each student one set.

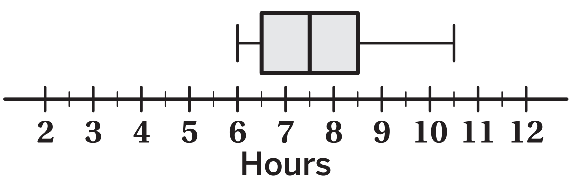
Card A



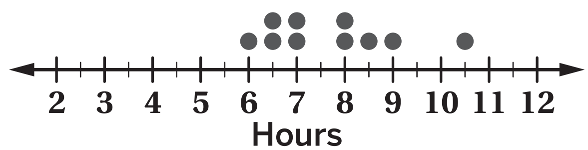
Card B



Card C



Card D



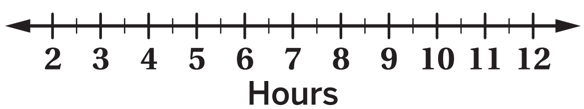
Card E
4.5 hours

Card F
5 hours

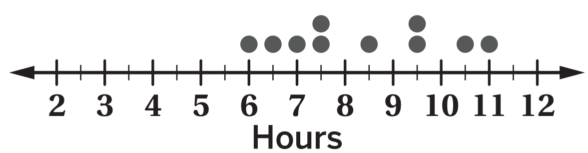
Card G
7.5 hours

Card H
2.5 hours

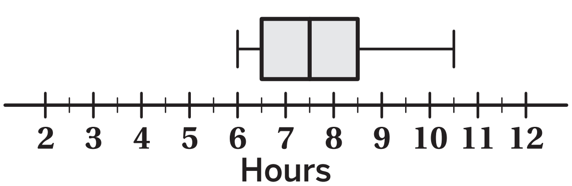
Card A



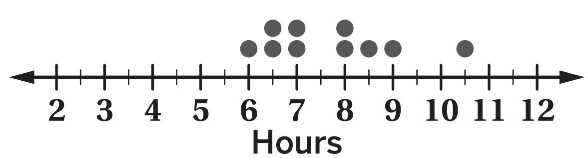
Card B



Card C



Card D



Card E
4.5 hours

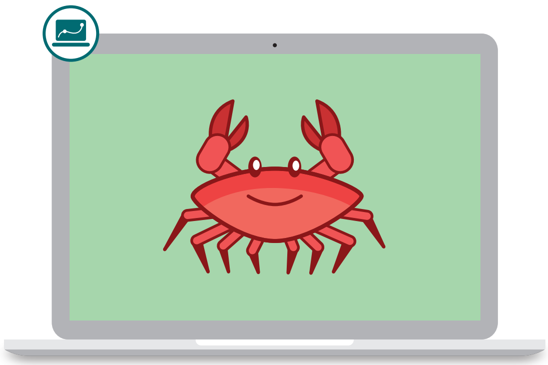
Card F
5 hours

Card G
7.5 hours

Card H
2.5 hours

Unit 8
Lesson
10

Name: Date: Period:



Crab Island

Let's compare large populations of data.

Warm-Up

1 Which one doesn't belong? Explain your thinking.

A. $\frac{1+5+9}{3}$

B. $\frac{1+2+3+4+5}{3}$

C. $\frac{2+4+5+6+8}{5}$

D. $\frac{3+5+13}{3}$

Activity 1

Name: Date: Period:

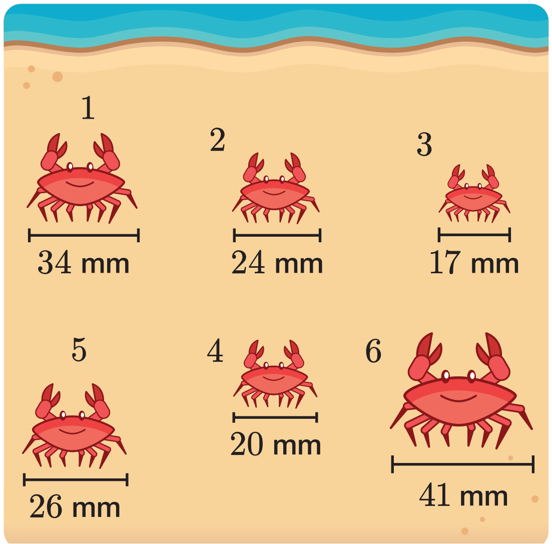
Sampling

2 Crab Island has a particular species of crabs.

What is a question about these crabs that we could collect data for?

3 **a** Take a look at this group of crabs Varuna is studying.

Crab	Width (mm)
1	34
2	24
3	17
4	20
5	26
6	41



b **Discuss:** What question could this group of crabs help answer?

4 Calculate the mean of Varuna's group of crabs.

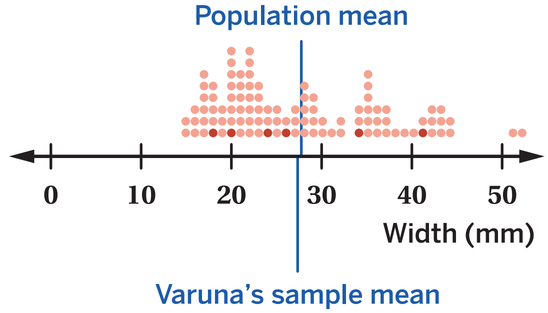
Name: .... Date: .... Period: ....

Sampling (continued)

5 Here is a dot plot of the widths of the crabs in the **population**.

The crabs Varuna chose are highlighted. This is a **sample**.

What do you notice? What do you wonder?

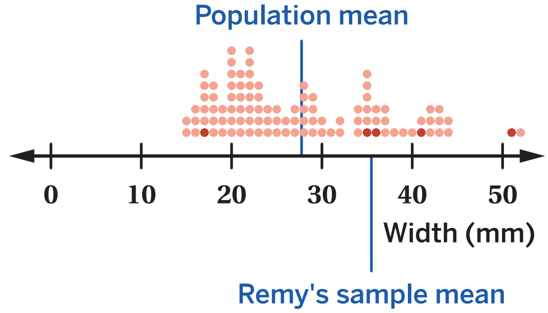


I notice:

I wonder:

6 Remy chose a different sample. The dot plot shows Remy's sample and sample mean.

What might happen if someone used Remy's sample to study the crabs on Crab Island?

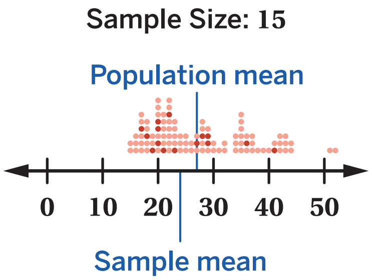


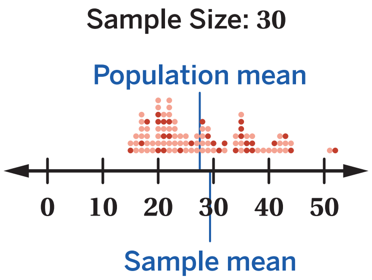
Activity
2

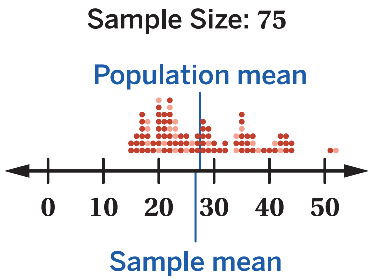
Name: Date: Period:

Explore More Samples

7 Here are three different samples of crab widths.

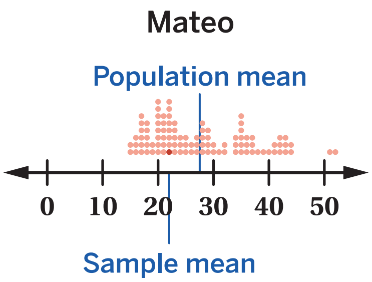


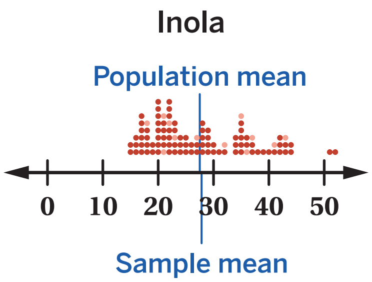


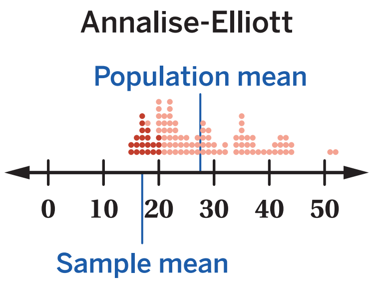


Discuss: What are advantages and disadvantages of using a large sample?

8 Mateo, Inola, and Annalise-Elliott each collected different samples of crabs. Select a sample and explain what advice you would give to that person.







Activity 3

Name: ______ Date: ______ Period: ______

Populations and Samples

9 Kris wonders how students at this school feel about the cafeteria food. Kris asked 50 students. In this situation, what is the population and what is the sample?

Place a P next to the population and an S next to the sample.

______ The students in the school

______ The cafeteria food

______ The head cook

______ The 50 students

10 Complete each row of possible questions, populations, and samples.

Question	Population	Sample
What is the average circumference of a tree in my state?	All of the trees in the state	
	The eggs in San Diego	One egg from each store in the city
What is the average weight of an apple?		The apples in the school cafeteria

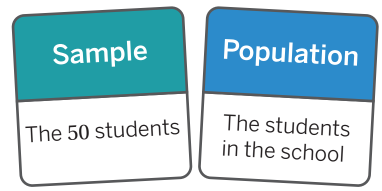
Explore More

11 **a** What is a question you could ask where the population is all of the books in the school?

b What is a question you could ask where a sample is all of the books in the school?

12 Synthesis

What are some advantages and disadvantages of using samples to answer a question about a population?



Things to Remember:

Unit 8
Lesson
11

Name: .... Date: .... Period: ....

Headlines

Let's see what makes a good sample.

Warm-Up

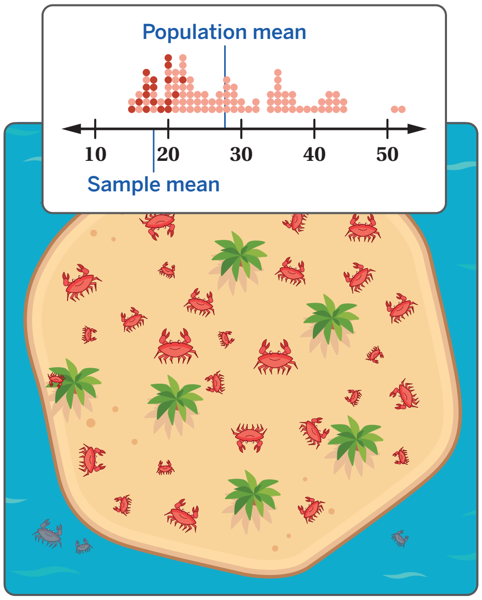
1 Adrian collected this sample of crabs and then wrote this headline:

"Crabs Are Smaller Than Ever!"

Do you agree with Adrian's headline? Circle one.

Yes No I'm not sure

Explain your thinking.



Activity 1

Name: .... Date: .... Period: ....

Graduation Rates

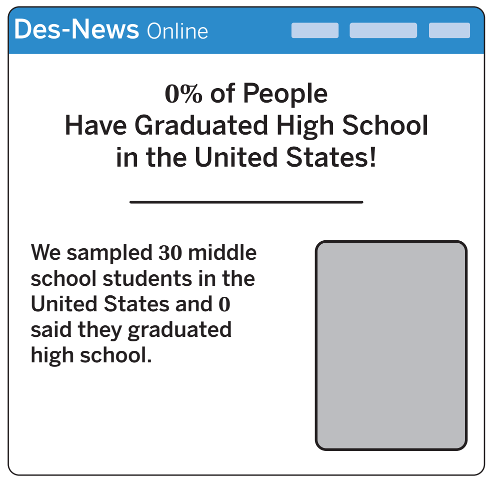
2 We are going to use your class as a sample to learn more about people in the United States.

Answer this question: *Have you graduated high school yet?* Circle one.

Yes No

3 Read the news headline and the information about the sample that the newspaper used.

What do you notice about this sample?


Des-News Online

0% of People Have Graduated High School in the United States!

We sampled 30 middle school students in the United States and 0 said they graduated high school.

4 Read the news headline and the information about the sample that the newspaper used.

Why is this sample more likely to be **representative** of the population?


Des-News Online

90% of People Have Graduated High School in the United States!

We sampled 57,000 random census surveys from Americans 25 and older. 51,300 reported graduating high school.

Name: .... Date: .... Period: ....

Representative Sampling

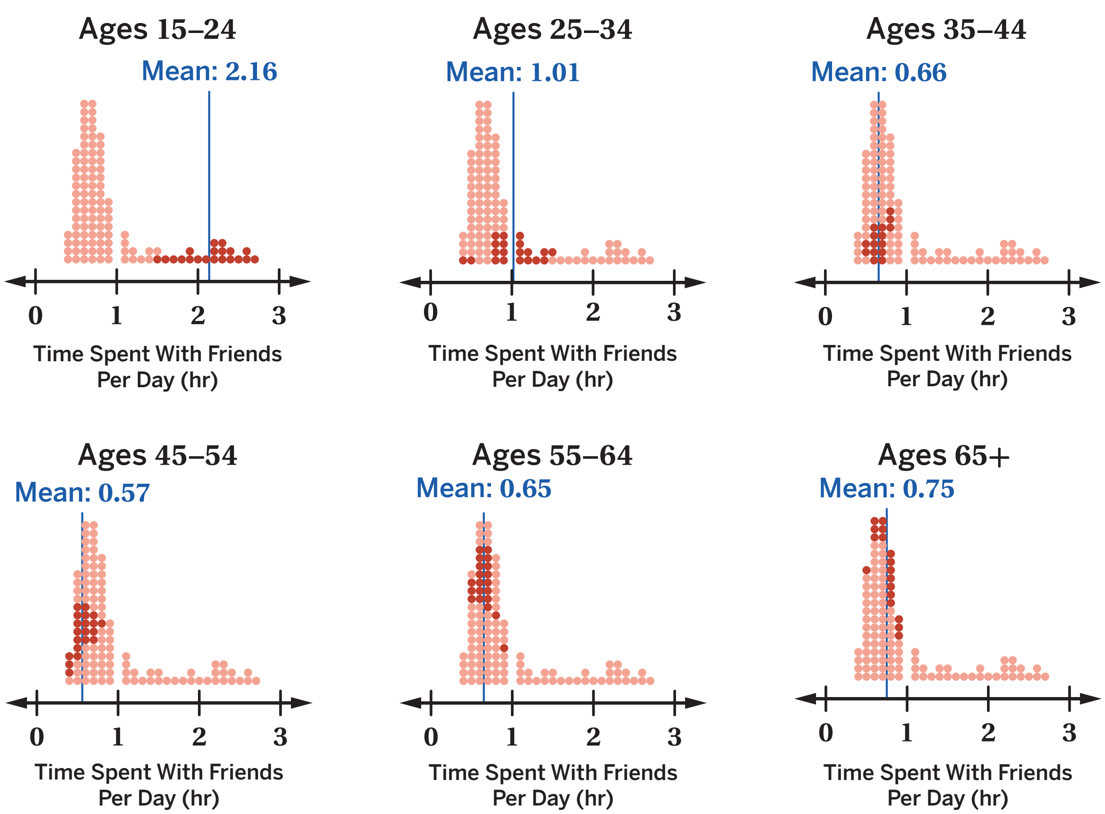
5 The American Time Use Survey asks about 9,400 randomly selected people how they use their time each year.

One of the questions they ask is: *How many hours do you spend with friends on average per day?*

Discuss: How do you think people of different ages might answer this question?

6 Here is a representative sample of 112 responses from the American Time Use Survey.

a Take a look at the results for different ages.



Source: *U.S. Bureau of Labor Statistics*

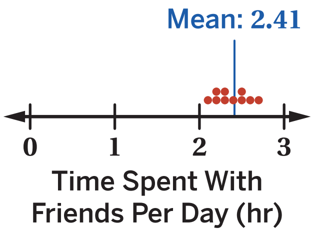
b Describe what you notice about the sample.

Name: ______ Date: ______ Period: ______

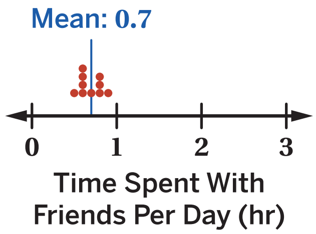
Representative Sampling (continued)

7 Here are three sampling methods and the samples that were collected.

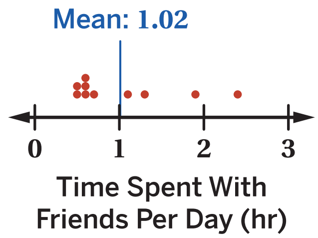
10 students in one high school class



10 random people at a nursing home



10 random phone numbers and ask whoever answers



Choose a sampling method and use the sample to write a headline about how much time people in the United States spend with their friends per day.

Your headline: ______

8 Ada wants to collect new data to see how people in the United States spend their time. Order the strategies from *least likely* to *most likely* to produce a representative sample of the population.

Least Likely

Most Likely

a. Post a poll on social media and look at the first 20 responses.

b. Ask 20 random people at the grocery store.

c. Ask every single person in the country.

d. Call 20 phone numbers at random.

e. Ask 20 friends.

Explore More

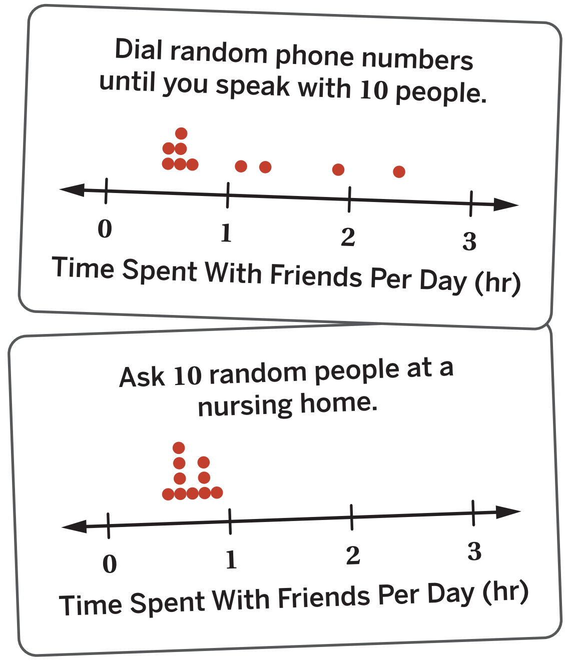
9 On the Explore More Sheet there are dot plots showing how much time people in the United States spend alone each day. Explore the mean response for different ages.

Describe an *incorrect* conclusion someone might make from an unrepresentative sample.

10 Synthesis

Explain how collecting a representative sample or an unrepresentative sample can affect someone's understanding of a population.

Use the examples if they help with your thinking.

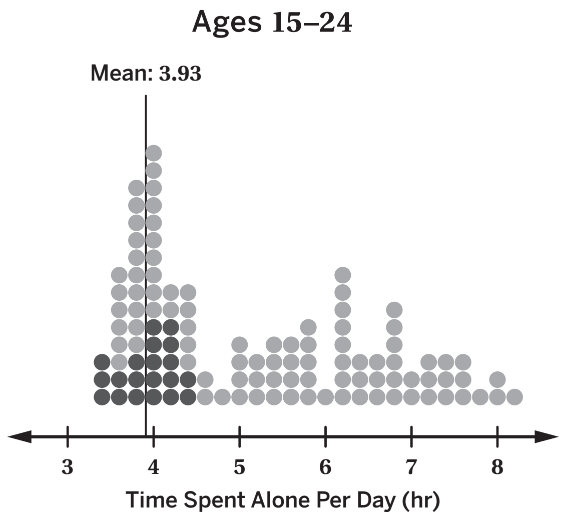


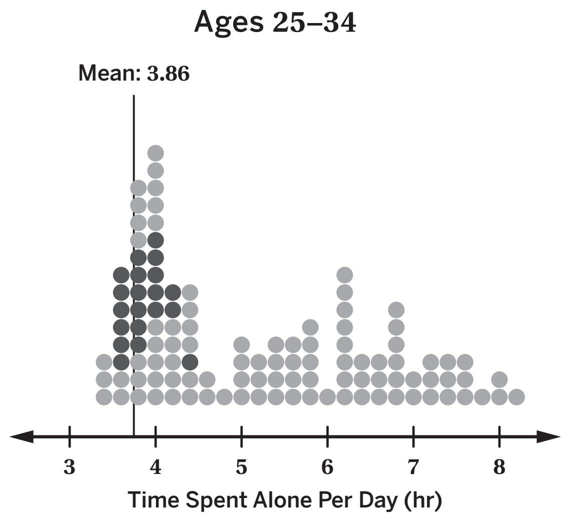
Things to Remember:

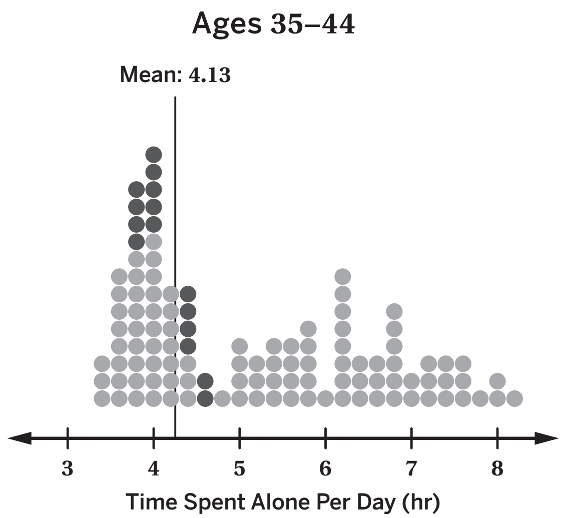
Name: Date: Period:

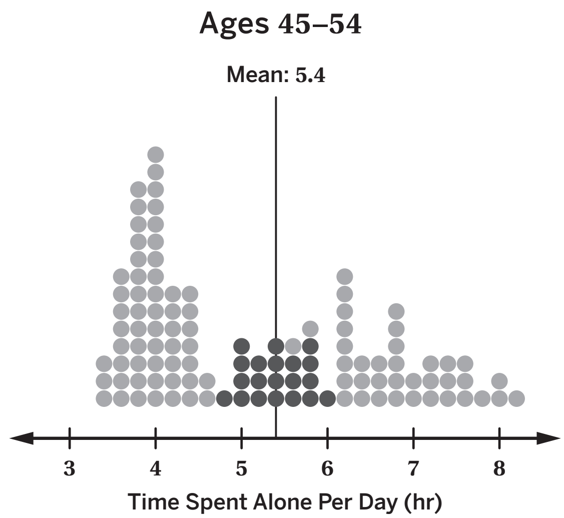
Explore More

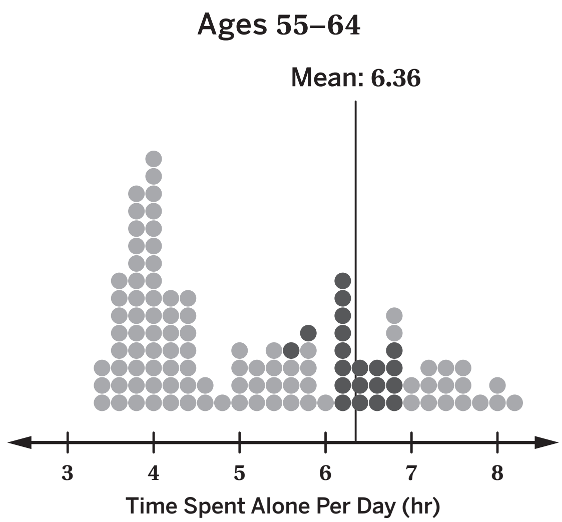
Use these dot plots based on the American Time Use Survey to complete Problem 9 in your Student Edition.

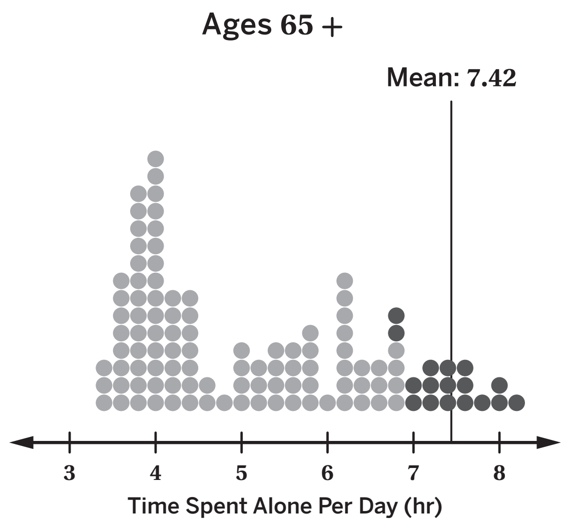












Name: .... Date: .... Period: ....

Flower Power

Let's use percentages to make predictions about populations.

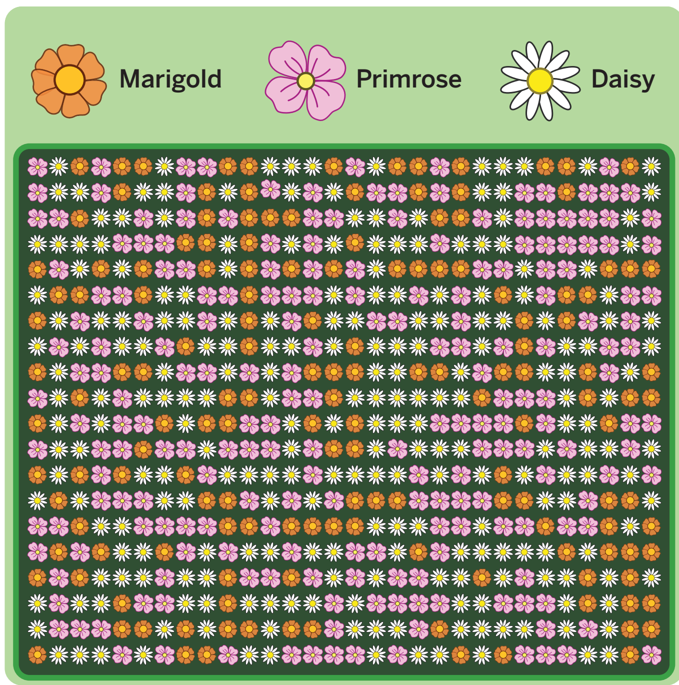
Warm-Up

1 A gardener planted some seeds from a bag of Wildflower Seed Mix.

What do you notice? What do you wonder?

I notice:

I wonder:



2 The gardener wonders how many of each flower type are in the garden. He says there are too many to count one by one.

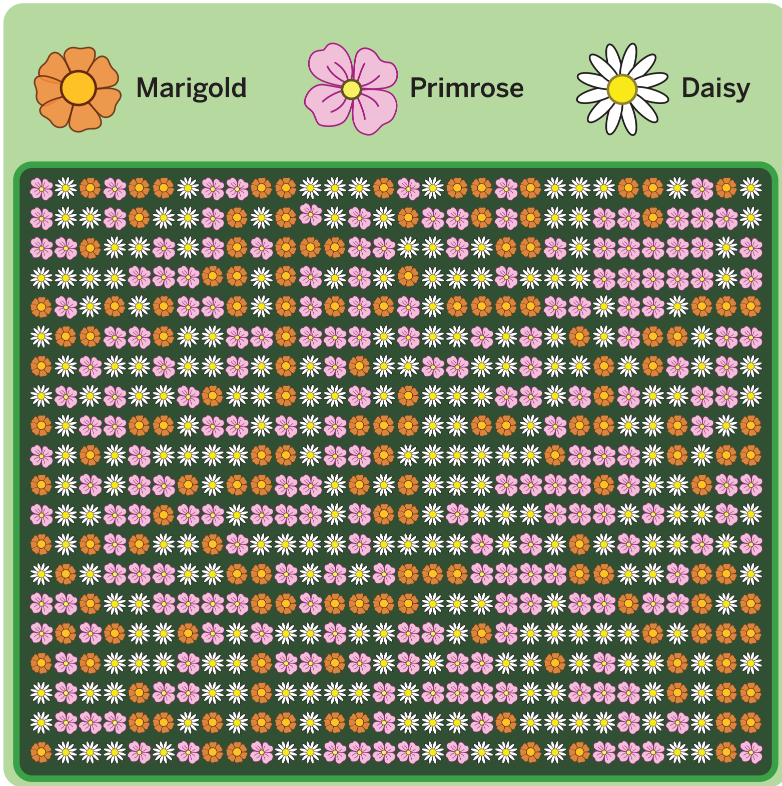
Describe a method he could use to estimate how many of each flower type there are.

Activity
1

Name: .... Date: .... Period: ....

Estimating the Marigolds

3 Here are all 600 flowers in the garden.



a Choose a sample of at least 25 flowers from the garden.

b Complete the table for your sample.

Flower Type	Count	Percentage
Marigold		
Primrose		
Daisy		
Total		100%

4 Use your table to help you estimate how many of the 600 flowers in the garden are marigolds.

Activity
1

Name: .... Date: .... Period: ....

Estimating the Marigolds (continued)

5 Eliza and Javier chose a 5-by-5 sample by having Eliza close her eyes and place her finger on the image.

Here are their strategies for estimating the total number of marigolds.

Explain each student's strategy.

Eliza

Marigolds are 28% of the sample.

Marigolds are probably 28% of the garden.

28% of 600 is 168 marigolds.

Javier

In the sample, 7 of 25 are marigolds.

Marigolds	Total
7	25
168	600

$\cdot 24$ $\cdot 24$

6 Let's watch an animation to reveal how many marigolds are in the entire garden.

Discuss:

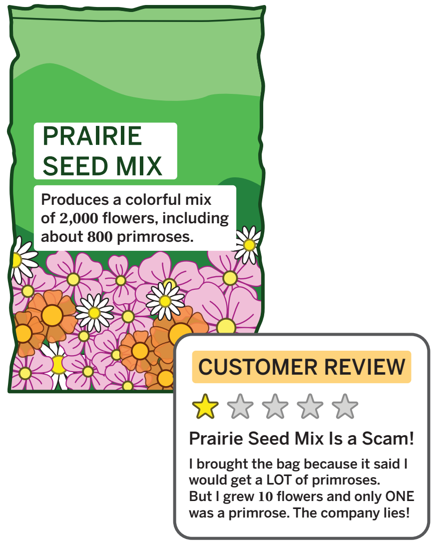
- How did the actual number of marigolds in the garden compare to the earlier estimates?
- What do you think could have improved the estimates?

Activity 2

Name: ________ Date: ________ Period: ________

The Bad Review

7 Here is a new bag of seeds and a customer review.



a. **Discuss:**
- What does the company claim about the bag?
- Why did the customer leave a one-star review?

b. How could you investigate the company's claim about primroses?

8 Let's grow a sample of up to 100 flowers from the Prairie Seed Mix.

a. Write down the number of each type of flower in your sample in the table.

Flower Type	Marigold	Primrose	Daisy	Total
Count				

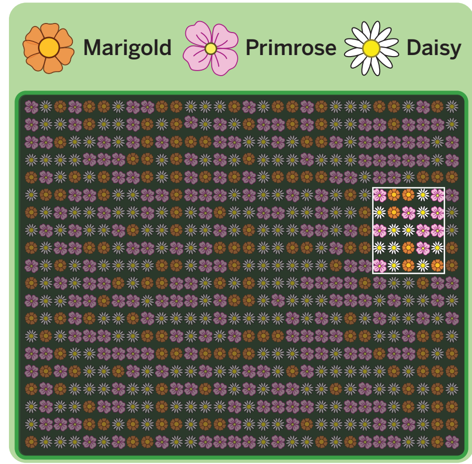
b. How many of the 2,000 flowers do you expect to be primroses based on your sample? Explain your thinking.

9 Based on your sample, what do you think about the company's claim now?

10 Synthesis

Describe how you can use a sample and *proportional* reasoning to estimate information about a population.

Use this sample and the population if that helps with your thinking.



Things to Remember:

Unit 8

Lesson

14

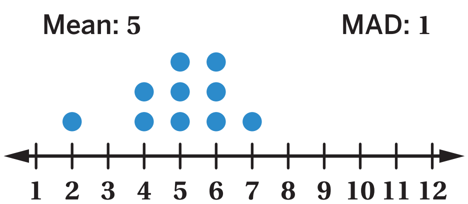
Name: ______ Date: ______ Period: ______

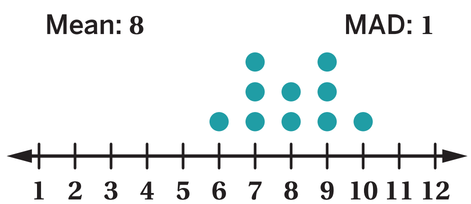
School Newspaper

Let's compare populations by analyzing the difference between the measures of center and variability.

Warm-Up

Here are two dot plots.





1. What do you notice? What do you wonder?

I notice:

I wonder:

Activity 1

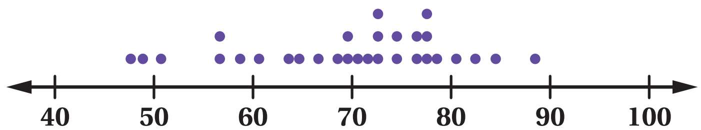
Name: .... Date: .... Period: ....

Homework Headline

2. Students at Median Middle School are investigating issues for the school newspaper.

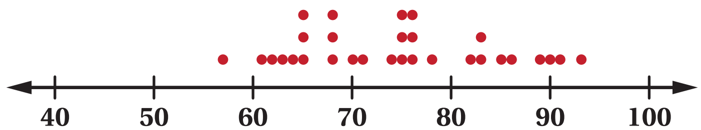
They surveyed an equal number of randomly selected 6th and 7th graders out of the 500 students at their school. Here are the results of the survey and a headline based on the results.

Mean: 70.1 **MAD: 8.3**



Minutes of Homework Per Night: 6th Grade

Mean: 74.5 **MAD: 8.1**



Minutes of Homework Per Night: 7th Grade

a The mean of the 6th grader data set is 70.1 minutes. What does this statistic represent?

b The MAD of each data set is about 8 minutes. What does this statistic represent?

c Based on this data, why might someone believe this headline?

d Based on this data, why might someone *not* believe this headline?

Name: Date: Period:

Homework Headline (continued)

3. Wohali and Ama use different strategies to decide if they believe the headline.

Wohali

$74.5 - 70.1 = 4.4$

I believe the headline because the mean of the 6th graders' data is 4.4 minutes less than the mean of the 7th graders' data.

Ama

$74.5 - 70.1 = 4.4$

$\frac{4.4}{8.3} \approx 0.53$

I don't believe the headline. Even though the mean is 4.4 minutes less, 4.4 minutes isn't a big difference when looking at the spread of the data. The difference is not even 1 MAD, it's about half of that! There will be a lot of overlap between the data sets.

Whose claim do you agree with? Explain your thinking.

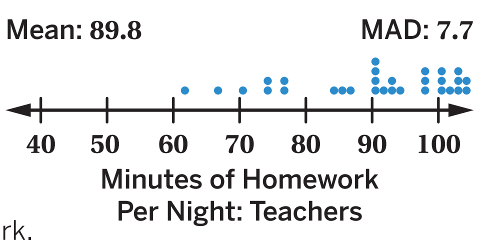
4. The survey results for 8th graders show a mean of 90 minutes and a MAD of 8 minutes.

Let's use Ama's strategy to investigate whether there is a big difference in homework time for 7th and 8th graders.

a Calculate how many MADs apart the means are. Use the larger MAD in your calculation.

b What might Ama say about whether 8th graders have more homework than 7th graders?

5. Teachers use the word "prep" to describe the work they do at home. Here are some data and statistics about their homework time.



a Calculate how many MADs apart the means are for the time teachers work at home with either the time 6th, 7th, or 8th graders spend on homework.

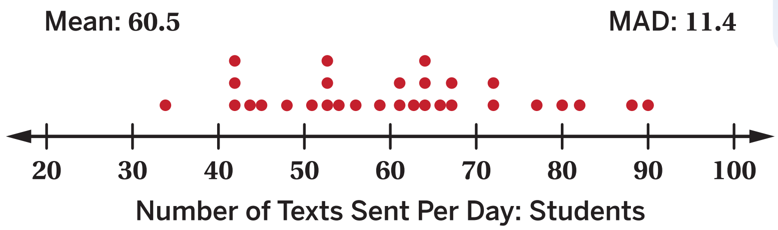
b Write your own headline about the two data sets you compared.

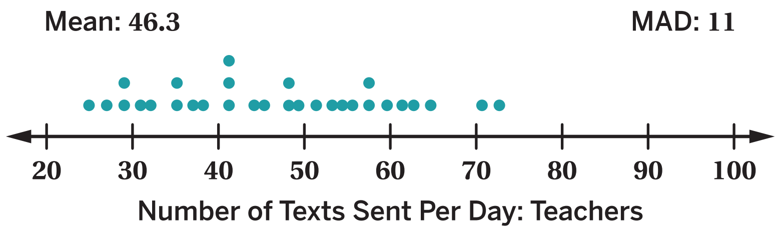
Name: .... Date: .... Period: ....

Texting Title

6. Students at Median Middle School surveyed a random sample of 30 students and 30 teachers.

Here are the survey results and the headline they wrote.





a Based on this data, why might someone believe this headline?

b Based on this data, why might someone *not* believe this headline?

7. Let's investigate further.

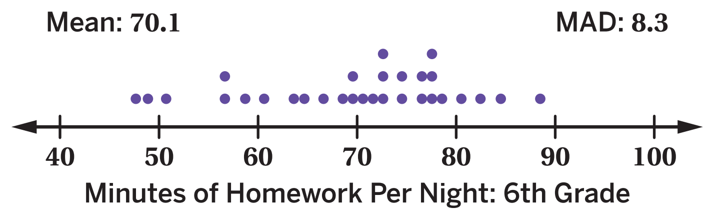
a How many MADs apart are the means? Show your thinking.

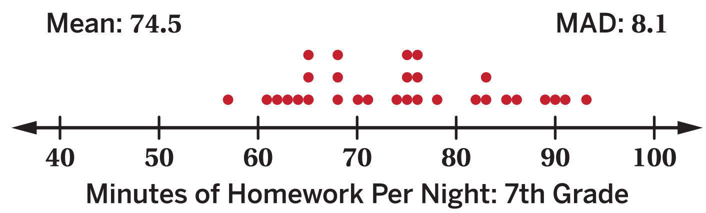
b Does that calculation make the headline more or less believable? Explain your thinking.

Synthesis

8. How can you use the MAD to determine how different two populations are?

Use the data from Median Middle School if it helps with your thinking.





Things to Remember:

Unit 8

Lesson

2

Name: .... Date: .... Period: ....

Prob-bear-bilities

Let's express probabilities as numbers.

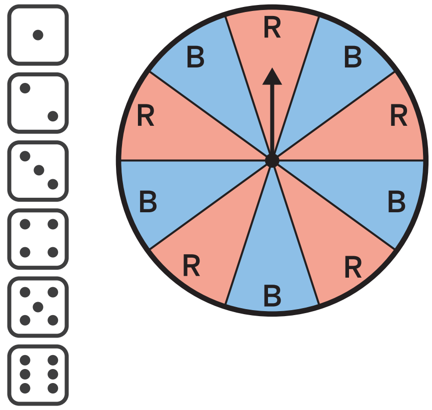
Warm-Up

1 Which game would you rather play?

A. Roll a number cube. Win if it lands on a number greater than 1.

B. Spin this spinner. Win if it lands on red.

Explain your thinking.



Activity 1

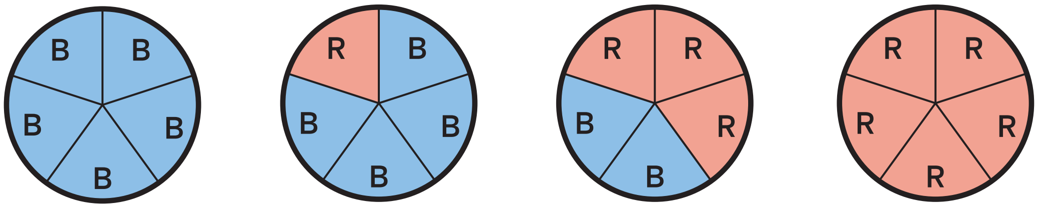
Name: ______ Date: ______ Period: ______

Intro to Probability

2 Here is another spinner with different sections of red and blue.

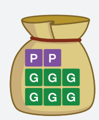
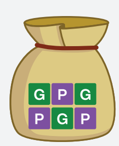
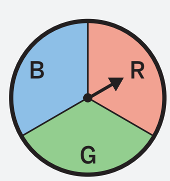
Probability of Spinning Red

$\frac{0}{5}$ or 0% $\frac{1}{5}$ or 20% $\frac{3}{5}$ or 60% $\frac{5}{5}$ or 100%



Describe how you think **probability** is determined.

3 For each event, put a check mark under the probability that matches it.

	Probability		
Event	**0.25**	$\frac{1}{3}$	**50%**
Picking a purple block from this bag 			
Picking a purple block from this bag 			
Randomly selecting the letter E from this word D I C E			
A number cube lands on an even number 			
This spinner lands on red 			

Name: ______ Date: ______ Period: ______

Intro to Probability (continued)

4 For each challenge, order the events by likelihood. Complete as many challenges as you have time for.

a
- The probability of an event is 50%.
- A spinner with 5 equal sections, 2 of which are red, lands on red.
- The probability of an event is $\frac{3}{4}$.

Least Likely **Most Likely**

b
- Equally likely as not
- The probability of an event is $\frac{1}{4}$.
- A spinner with 5 equal sections, 4 of which are red, lands on red.

Least Likely **Most Likely**

c
- A spinner with 5 equal sections, 4 of which are red, lands on red.
- The probability of an event is 75%.
- A fair coin lands tails up.

Least Likely **Most Likely**

d
- You draw a red block from a bag with 6 green blocks and 2 purple blocks.
- The probability of an event is 20%.
- The outcome of an event is certain.

Least Likely **Most Likely**

Activity 2

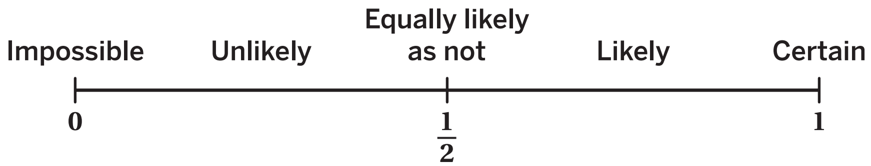
Name: Date: Period:

Probabilities and Sample Spaces

5 Here is a randomizer that chooses one character on each spin.

a Use the Activity 2 Sheet to try out the randomizer.

b Plot a point on the line to show how likely you think it is to get a bear on one spin.



6 A **sample space** is the collection of all possible outcomes of an experiment.

The sample space in the previous problem has 4 characters.

Explain why the probability of spinning a bear is $\frac{1}{4}$.

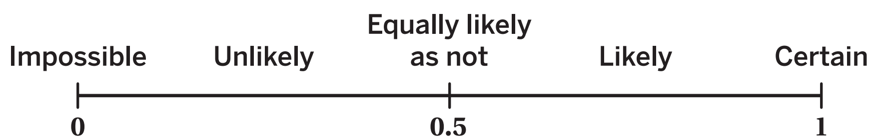
Name: ____________ Date: ____________ Period: ____________

Prob-bear-ly Mismatched

7 Here is a different randomizer that chooses one head and one body on each spin.

a Use the Activity 3 Sheet to try out the randomizer.

b Plot a point on the line to show how likely you think it is to get a whole bear on one spin.



8 How many different characters are in this randomizer's sample space?

Explain your thinking.

Activity
3

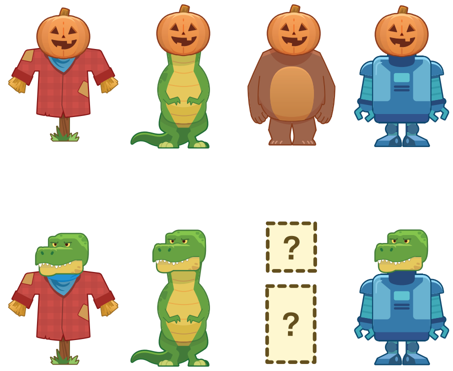
Name: .... Date: .... Period: ....

Prob-bear-ly Mismatched (continued)

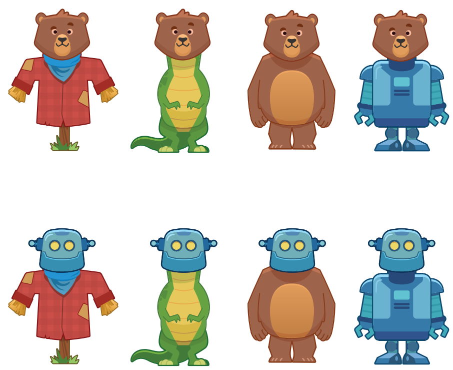
9 This drawing of a sample space is missing one character.

Describe the head and body of the character that will complete the sample space.

Draw a sketch if you have time.



10 Use the completed sample space to answer:



a What is the probability of spinning a character that is at least part bear?

b What is the probability of spinning a character with mismatched parts?

Explore More

11 Here is a different randomizer that chooses one head, one body, and one set of legs on each spin. Use the Explore More Sheet to try out the randomizer.
What is the probability of the randomizer choosing all three parts from the same character?

Explain your thinking.

12 Synthesis

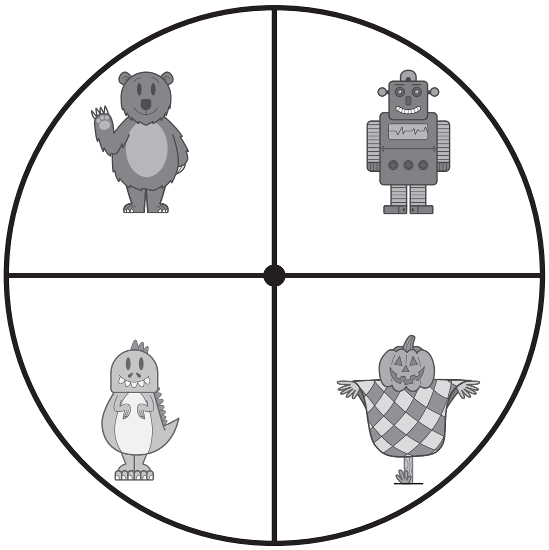
Explain how you can use a sample space to help you determine the *probability* of an event.

Things to Remember:

Name: .... Date: .... Period: ....

Probabilities and Sample Spaces

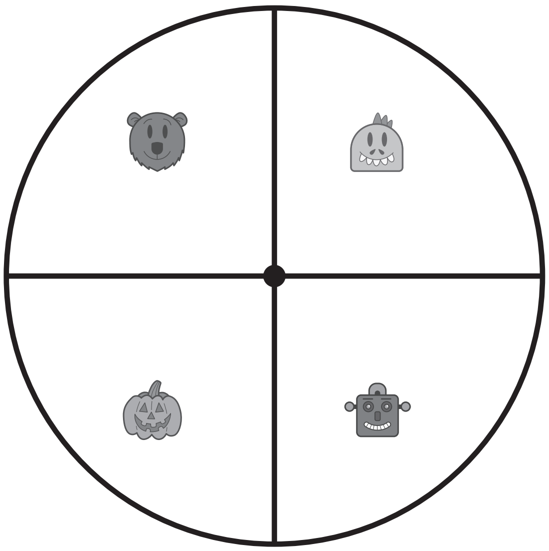
Unfold one end of a paper clip to use as the pointer. Use a pencil to hold the rounded (closed) end of the paper clip in the center of the spinner. Spin the paper clip around the pencil.

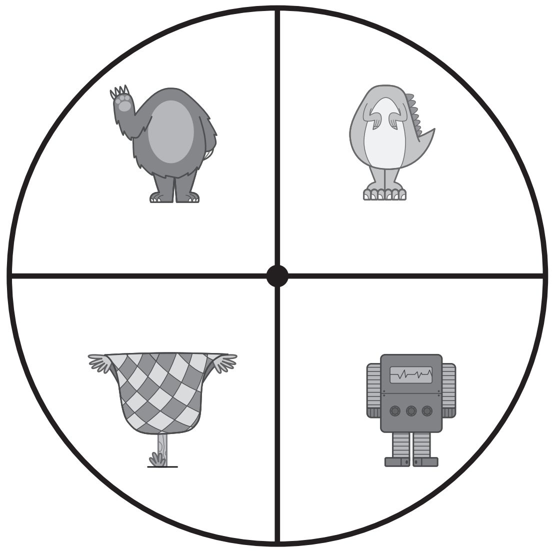


Name: Date: Period:

Prob-bear-ly Mismatched

Unfold one end of a paper clip to use as the pointer. Use a pencil to hold the rounded (closed) end of the paper clip in the center of the spinner. Spin the paper clip around the pencil. Spin both spinners to complete one spin.

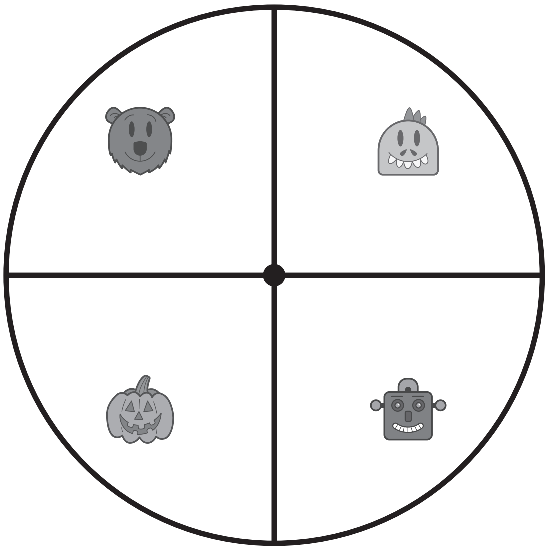


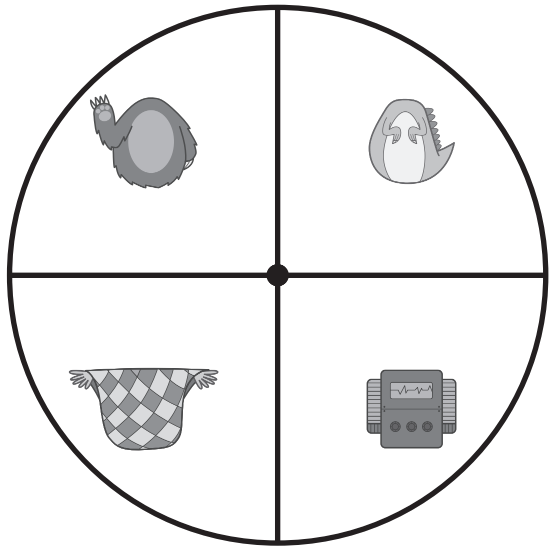


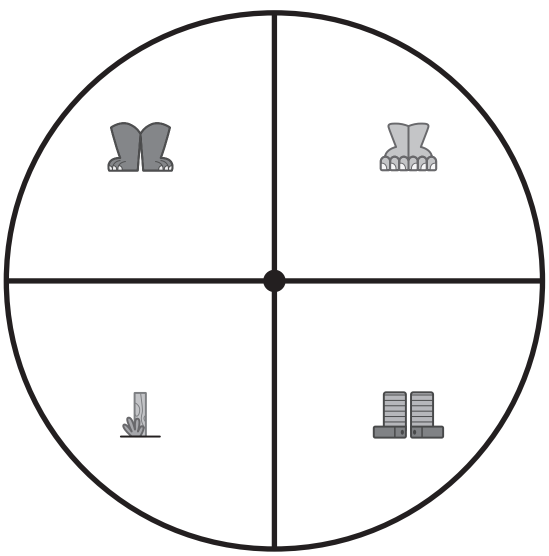
Name: .... Date: .... Period: ....

Explore More

Unfold one end of a paper clip to use as the pointer. Use a pencil to hold the rounded (closed) end of the paper clip in the center of the spinner. Spin the paper clip around the pencil. Spin all three spinners to complete one spin.







Unit 8
Lesson
3

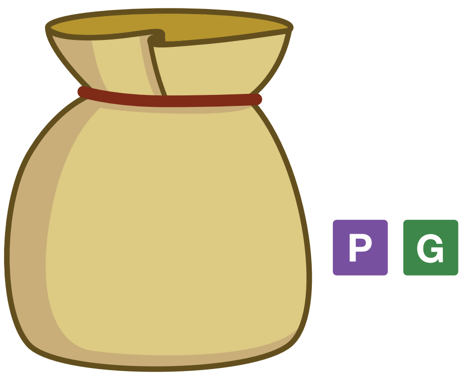
Name: Date: Period:

Mystery Bag

Let's use repeated experiments and proportionality to predict what is inside a mystery bag.

Warm-Up

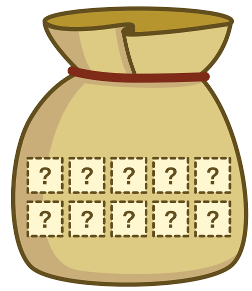
1 Sketch purple (P) and green (G) blocks on the bag so the probability of picking a green block (G) is 40%.



Name: .... Date: .... Period: ....

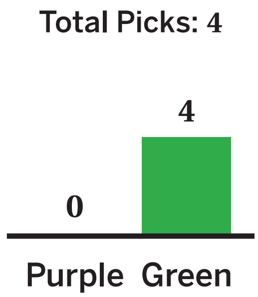
What's in the Bag?

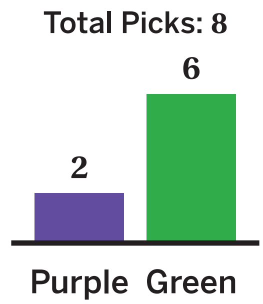
2 Here is Rishi's bag. It has 10 blocks.



These are the results after 4 picks and after 8 picks.

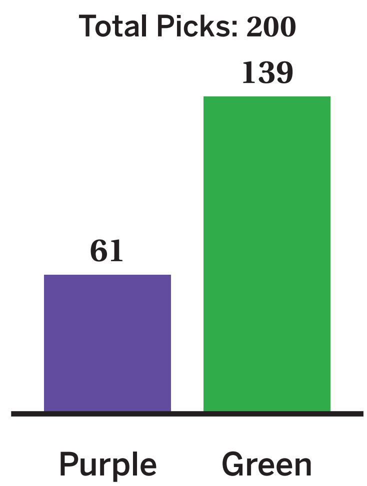
How many of the 10 blocks do you think are green?





3 This graph shows the results from 200 picks.

Use these results to decide how many of the 10 blocks you think are green. Explain your thinking.



4 Here is how Nasir decided the number of green blocks in the bag.

Explain Nasir's strategy.

Nasir

$$\frac{139}{200} = \frac{x}{10}$$

$$10 \cdot 0.7 = \frac{x}{10} \cdot 10$$

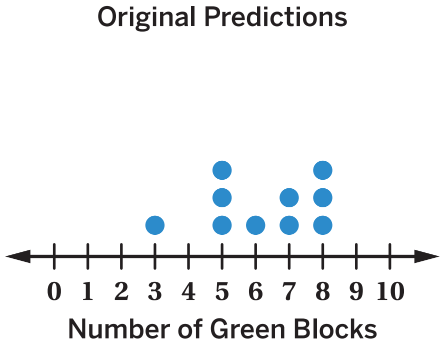
$$7 = x$$

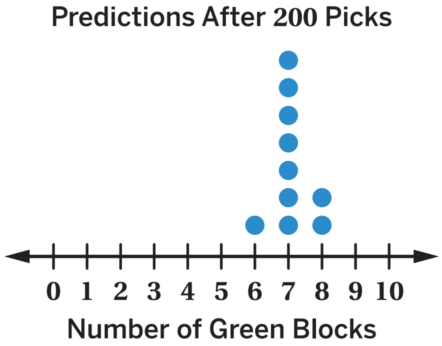
Activity 1

Name: Date: Period:

What's in the Bag? (continued)

5 These graphs show the original predictions and updated predictions from another class.





Discuss: What do you notice and wonder?

6 Let's reveal what's in the bag.

Discuss: How did this compare to what you predicted?

Activity 2

Name: ______ Date: ______ Period: ______

Blocks in the Bag

7 Demari's bag has 8 blocks.

The graph shows the results from 20 picks.

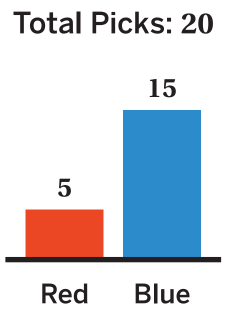
Order the contents of Demari's bag from *least likely* to *most likely*.

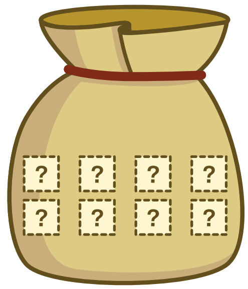
a. 1 red, 7 blue

b. 2 red, 6 blue

c. 6 red, 2 blue

d. 0 red, 8 blue





Least Likely **Most Likely**

8 Keya and Ash were trying to decide which color blocks were in Demari's bag.

- Keya says there are 2 red and 6 blue blocks.
- Ash says there is 1 red and 7 blue blocks.

Who do you agree with? Circle one.

Keya Ash Neither

Explain your thinking.

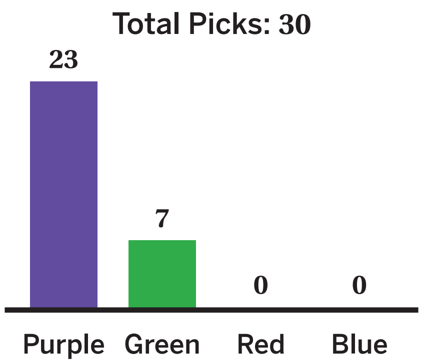
Activity 2

Name: ________ Date: ________ Period: ________

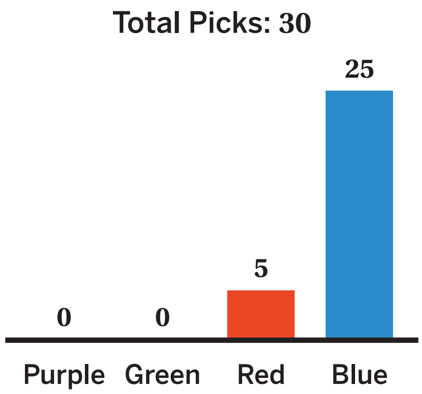
Blocks in the Bag (continued)

9 Use the results to determine how many blocks of one color are likely to be in each bag. Complete as many problems as you have time for.

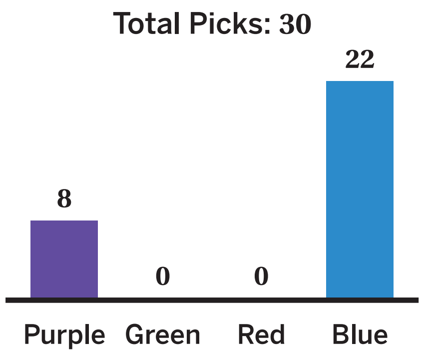
a There are 4 blocks in a bag. How many of the blocks are likely to be purple?



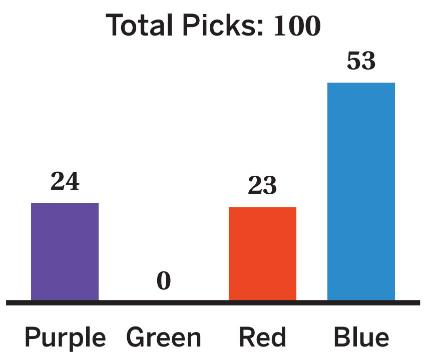
b There are 4 blocks in a bag. How many of the blocks are likely to be red?



c There are 6 blocks in a bag. How many of the blocks are likely to be purple?



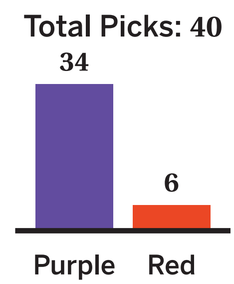
d There are 8 blocks in a bag. How many of the blocks are likely to be blue?

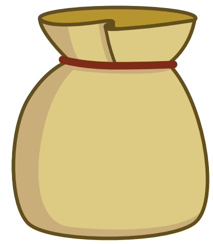


10 Synthesis

Describe how you can use results from a repeated experiment to make predictions.

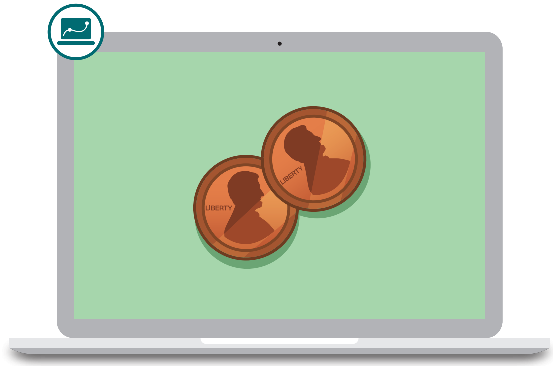
Use the results shown if that helps with your thinking.





Things to Remember:

Name: .... Date: .... Period: ....



Is It Fair?

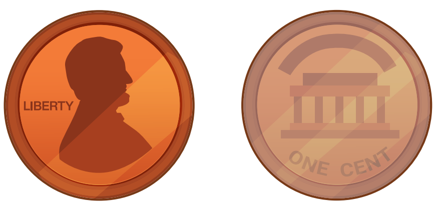
Let's decide what it means to be fair.

Warm-Up

1 Some of these coins are *unfair*.

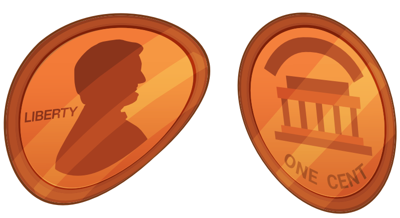
a Take a look at each coin.

Coin A



Coin B

Coin C



Coin D

b Select one coin and describe why you think it's unfair.

Activity 1

Name: .... Date: .... Period: ....

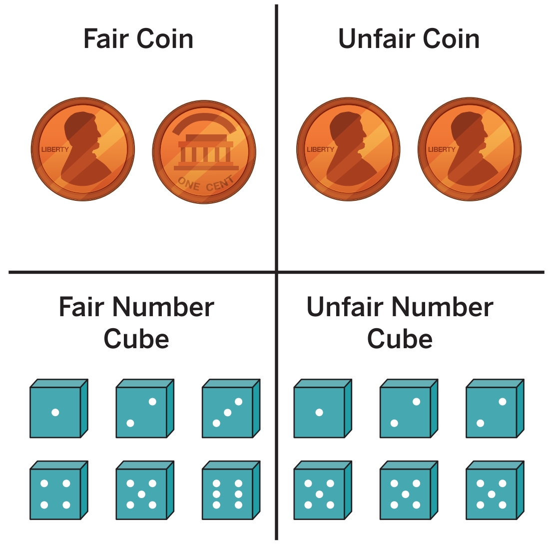
Fair or Not?

2 Let's investigate *fair* and *unfair* objects.

Fair objects have:

- All the expected outcomes.
- An equal probability for each outcome to occur.

Flip a fair coin 10 times and record your results.



a Did you get the results you expected?

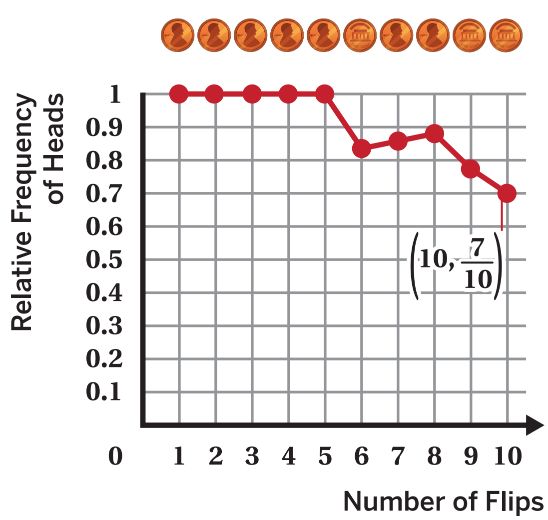
b Select *all* the results where you would suspect a coin is unfair.

- ☐ **A.** 5 heads, 5 tails
- ☐ **B.** 6 heads, 4 tails
- ☐ **C.** 8 heads, 2 tails
- ☐ **D.** 9 heads, 1 tails
- ☐ **E.** 10 heads, 0 tails

3 One coin was flipped 10 times. The image shows that the coin landed 7 times heads up and 3 times tails up.

Do you think this is a fair coin?

4 Here is a graph of the flips from the previous problem.



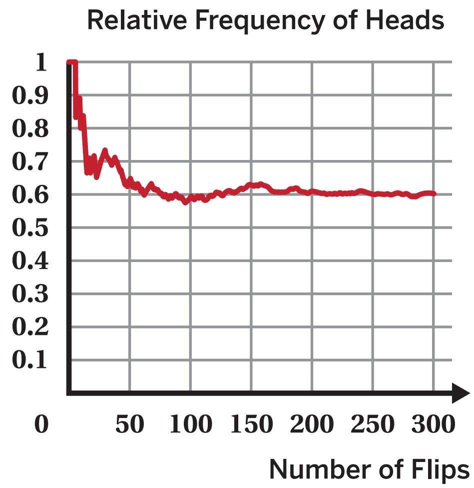
a What does the point $\left(10, \frac{7}{10}\right)$ mean?

b How might a graph help you determine if a coin is fair?

Name: .... Date: .... Period: ....

Fair or Not? (continued)

5 A coin was flipped 300 times. Based on the results, is the coin fair? Explain your thinking.



6 Based on these results, what is the probability of the next flip landing heads up?

Activity 2

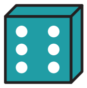
Name: .... Date: .... Period: ....

How Likely?

7 Here is a game involving a number cube.

What do you think is the probability that Player 2 wins on any single roll?

Roll an odd number: Player 1 wins	Roll an even number: Player 2 wins

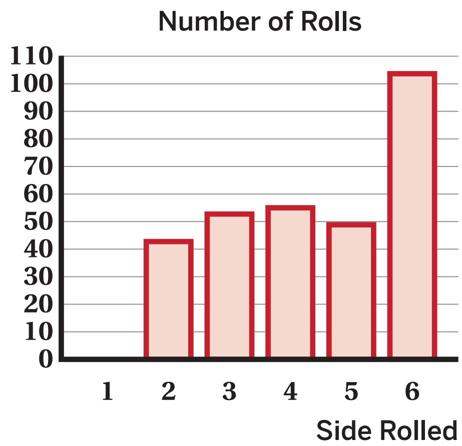


8 **a** Play a few rounds of the game using a number cube. Record your results in the table.

Player 1	Player 2

b Do you think this is a fair number cube? Explain your thinking.

9 Here are the results from two students who played the game and rolled a number cube 300 times. What do you think is the probability that Player 2 wins? Explain your thinking.



Name: .... Date: .... Period: ....

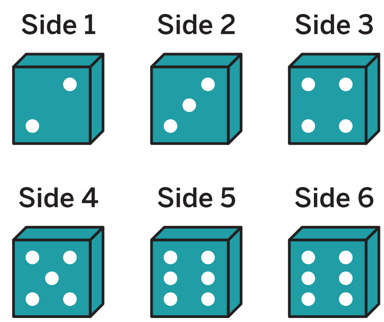
How Likely? (continued)

10 Here is the number cube used by the two students in the previous problem.

Create a game with this number cube that would be fair.

Player 1 wins if . . .

Player 2 wins if . . .



Explore More

11 In this game, players will flip the same coin twice.

The rules are:

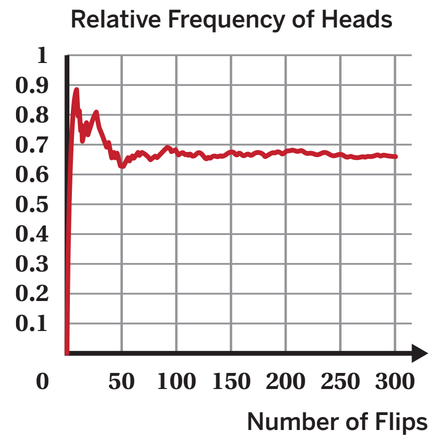
- Flip heads, then tails: Player 1 wins.
- Flip tails, then heads: Player 2 wins.
- If the results match, no one wins.

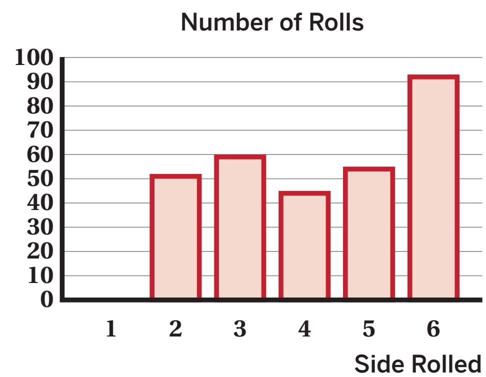
Explain why this game is fair even if the coin is unfair.

12 Synthesis

Describe how you can use a repeated experiment to decide whether an object is fair.

Use the results of these experiments if they help with your thinking.

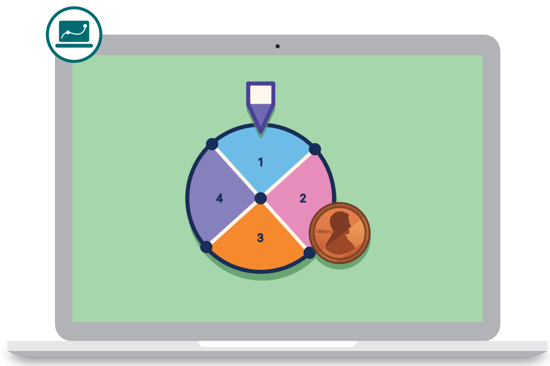




Things to Remember:

Unit 8
Lesson
6

Name: ______ Date: ______ Period: ______



Fair Games

Let's determine whether or not games are fair.

Warm-Up

1 Here is a game involving two fair coins. Player 1 wins if both coins land heads up or both coins land tails up. Otherwise, Player 2 wins.

Play a few rounds, then answer: *Which player would you rather be in the game?*

2 Charlie made a list and claims that the game is not fair because Player 1 is twice as likely to win as Player 2.

H H → Player 1 wins!

T T → Player 1 wins!

H T → Player 2 wins!

Is Charlie's claim correct? Circle one.

Yes No I'm not sure

Explain your thinking.

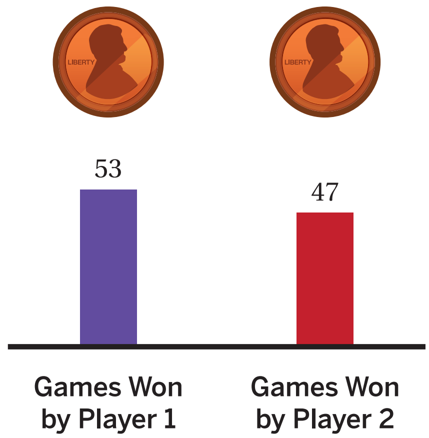
Activity
1

Name: .... Date: .... Period: ....

Coins and Tree Diagrams

3 The game from the Warm-Up was played 100 times. Here are the results.

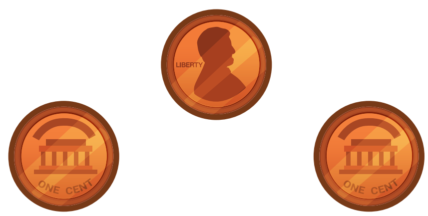
Discuss: Why is this a fair game?



4 Here's a twist: there are 3 coins to flip.

Like before, Player 1 wins if the coins land all heads up or all tails up.

Otherwise, Player 2 wins.



a Play the game up to 10 times. Record your results in the scoreboard.

Player 1	Player 2

b Explain whether you think this game is fair.

5 Andrea wants to determine the probability of winning.

In any round, what is the probability that Player 1 wins (all heads or all tails)?

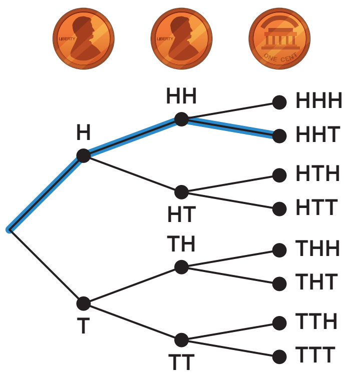
Activity 1

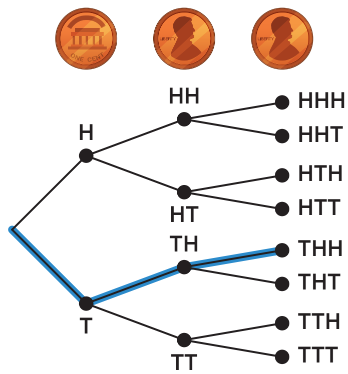
Name: ______ Date: ______ Period: ______

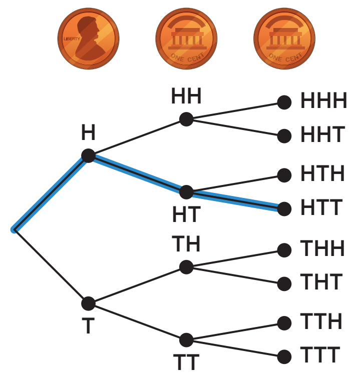
Coins and Tree Diagrams (continued)

6 A **tree diagram** is one way to represent the sample space of **compound events**. This tree diagram represents flipping 3 coins.

a Take a look at the tree diagrams.





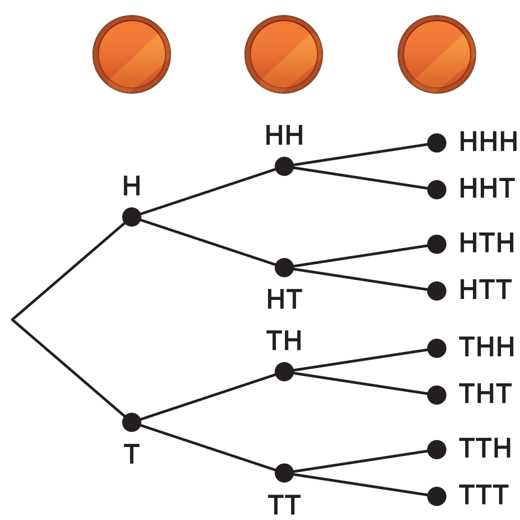


b Explain why the probability of Player 1 winning is $\frac{2}{8}$.

7 Use the tree diagram to help answer:

a What is the probability of *exactly* 1 coin landing tails up?

b What is the probability of *at least* 1 coin landing tails up?

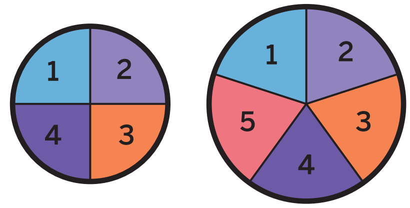


Activity 2

Name: .... Date: .... Period: ....

Spinners and Tables

8 Here is a new game involving these two spinners. Player 1 wins if the numbers are *the same or one apart*. Otherwise, Player 2 wins.



a Use the Activity 2 Card to play a few rounds of the game with a partner. Record the results in the table.

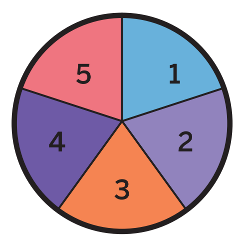
Player 1	Player 2

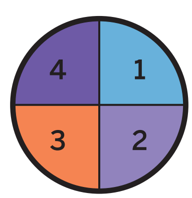
b Make a tree diagram to represent the sample space.

9 Here is a table representing the game. Each space in the table represents a possible outcome. Compare this table to the tree diagram you made.

How are they alike?

How are they different?





	1	2	3	4	5
1	1, 1	1, 2	1, 3	1, 4	1, 5
2	2, 1	2, 2	2, 3	2, 4	2, 5
3	3, 1	3, 2	3, 3	3, 4	3, 5
4	4, 1	4, 2	4, 3	4, 4	4, 5

Name: Date: Period:

Spinners and Tables (continued)

10 Player 1 wins if the numbers are *the same or one apart*. Otherwise, Player 2 wins. Use your tree diagram, the table, or another tool to help you answer:

a. What is the probability of Player 1 winning?

b. What is the probability of Player 2 winning?

	1	2	3	4	5
1	1, 1	1, 2	1, 3	1, 4	1, 5
2	2, 1	2, 2	2, 3	2, 4	2, 5
3	3, 1	3, 2	3, 3	3, 4	3, 5
4	4, 1	4, 2	4, 3	4, 4	4, 5

Explore More

11 Here are some other games that you could play using the same two spinners. Get a feel for the games by playing each one with your partner using the Activity 2 Card.

Four Score	You Spin Sum, You Lose Sum	Lucky 7	Big Wheel, Big Deal
• *Multiply* the numbers. • Player 1 wins if the result is a *multiple of* 4. • Player 2 wins if it's anything else.	• *Add* the two numbers. • Player 1 wins if the sum is 6 *or greater.* • Player 2 wins if the sum is 5 *or less.*	• *Multiply* the numbers. • Player 1 wins if the result is *greater than* 7. • Player 2 wins if the result is *less than* 7.	• Player 1 wins if the *big wheel's number is greater* than the small wheel's number. • Otherwise, Player 2 wins.

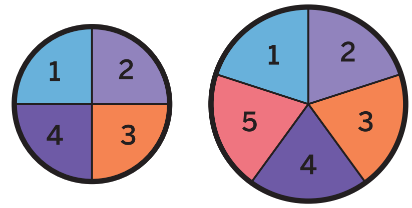
Select *all* the games that are fair (Player 1 and 2 have an equal chance of winning).

- ☐ **A.** Four Score
- ☐ **B.** You Spin Sum, You Lose Sum
- ☐ **C.** Lucky 7
- ☐ **D.** Big Wheel, Big Deal

12 Synthesis

Here are three representations of the sample space for this pair of spinners.

Choose a representation and describe an advantage and a disadvantage.



List

- Left: 1, Right: 1
- Left: 1, Right: 2
- Left: 1, Right: 3
- Left: 1, Right: 4
- Left: 1, Right: 5

- Left: 2, Right: 1
- Left: 2, Right: 2
- Left: 2, Right: 3
- Left: 2, Right: 4
- Left: 2, Right: 5

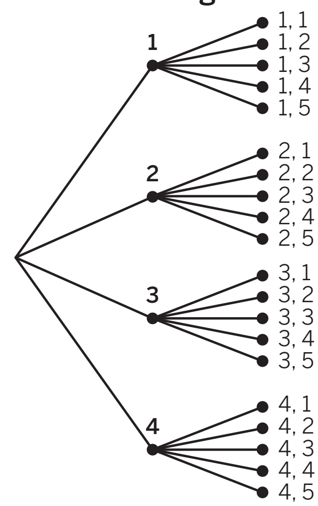
- Left: 3, Right: 1
- Left: 3, Right: 2
- Left: 3, Right: 3
- Left: 3, Right: 4
- Left: 3, Right: 5

- Left: 4, Right: 1
- Left: 4, Right: 2
- Left: 4, Right: 3
- Left: 4, Right: 4
- Left: 4, Right: 5

Table

	1	2	3	4	5
1	1, 1	1, 2	1, 3	1, 4	1, 5
2	2, 1	2, 2	2, 3	2, 4	2, 5
3	3, 1	3, 2	3, 3	3, 4	3, 5
4	4, 1	4, 2	4, 3	4, 4	4, 5

Tree Diagram



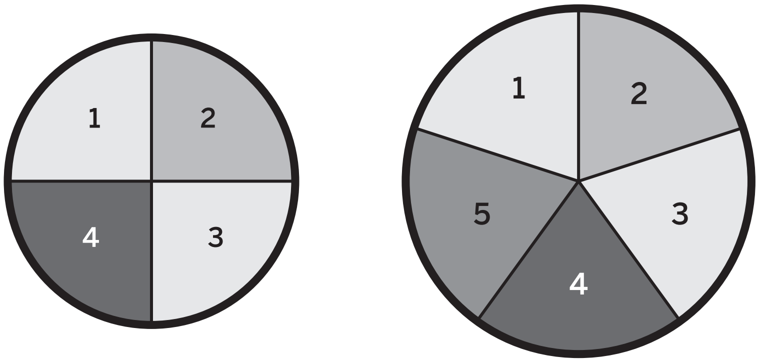
Things to Remember:

Spinners and Tables

Directions: Make one copy per two pairs of students. Then pre-cut the cards and give each pair of students one set of spinners.

Unfold one end of a paper clip to use as the pointer. Use a pencil to hold the rounded (closed) end of the paper clip in the center of the spinner. Spin the paper clip around the pencil. Spin both spinners to complete one spin.

Player 1 wins if the numbers are the same or one apart. Otherwise, Player 2 wins. Play a few rounds of the game with a partner.



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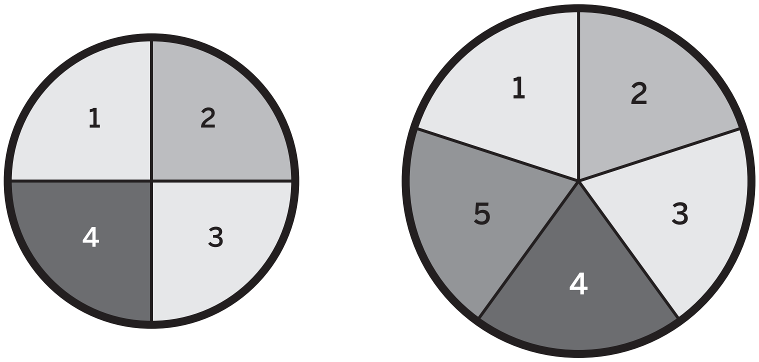
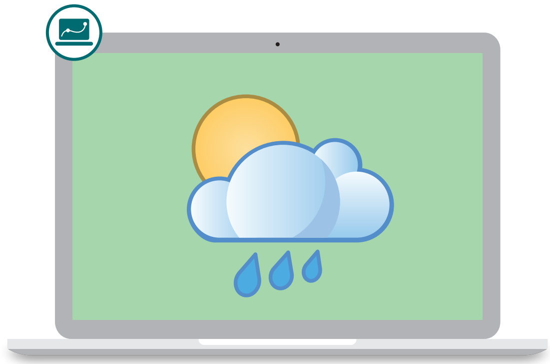


Unit 8
Lesson
7

Name: .... Date: .... Period: ....



Weather or Not

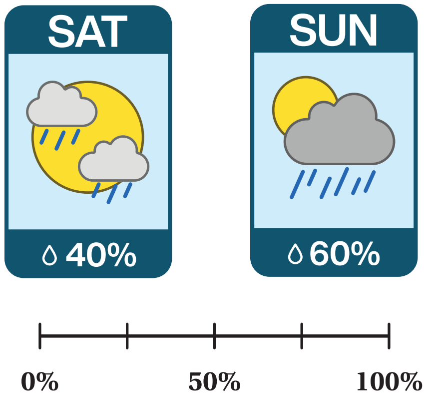
Let's use probability tools to simulate real-world events.

Warm-Up

1 Aniyah saw this forecast for the weekend's weather.

What do you think is the probability that it will rain *at least once* this weekend?

Plot a point on the line to show your guess. Explain your thinking.

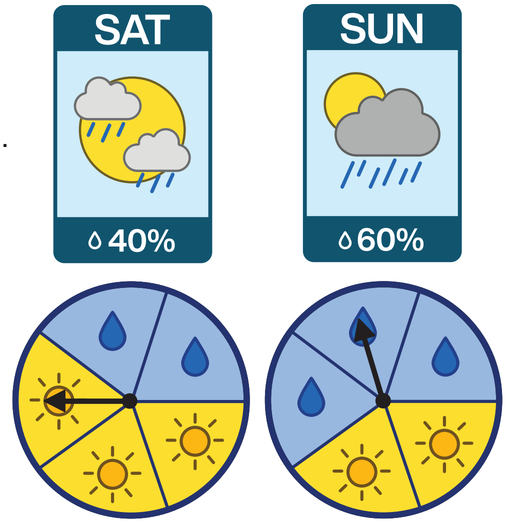


Activity 1

Name: ______ Date: ______ Period: ______

Introducing Simulation

2 Aniyah wanted to run a **simulation**: an experiment designed to estimate the probability of a real-world event. She created these spinners to represent the weather forecast.



a Use the spinners on the Activity 1 Card or digital screen to run one or more experiments.

b Describe how Aniyah could use these spinners to estimate the probability that it will rain this weekend.

3 Aniyah ran 500 experiments. Here are the results.

Discuss: Choose a row and explain what it tells you about this situation.

Experiments with:	Count	Relative Frequency
No rain	109	21.8%
1 day of rain	260	52%
2 days of rain	131	26.2%

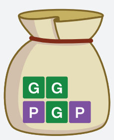
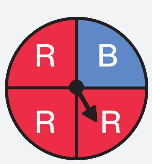
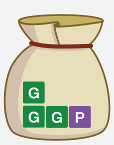
4 What would you tell Aniyah about the probability of rain this weekend? Use evidence from the simulation to support your claim.

Activity 2

Name: ________ Date: ________ Period: ________

Make a Simulation

5 Match each simulation to the weather forecast it could represent.

Simulation	 TODAY 40%	 TODAY 50%	 TODAY 75%
A fair coin lands heads up. 			
A purple block is picked from this bag. 			
A number cube lands on an odd number. 			
The spinner lands on red. 			
A green block is picked from this bag. 			

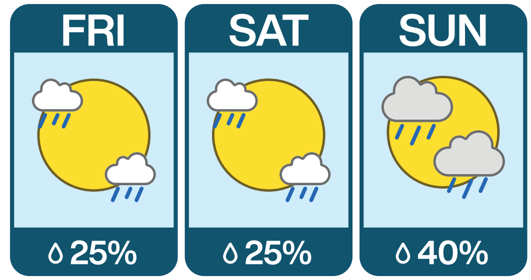
Activity 2

Name: ______ Date: ______ Period: ______

Make a Simulation (continued)

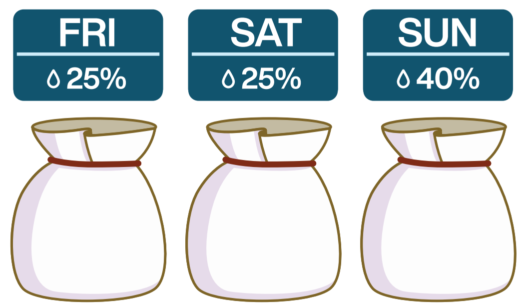
6 A tourist is planning a 3-day vacation. Here is the forecast at their destination.

What do you think is the probability that it will rain at least once during these 3 days? Plot a point on the line to show your guess.



7 The tourist wants to design a simulation to estimate the probability of rain during their vacation.

Draw blocks on each bag to match the probability of rain on each day.



8 Let's watch some simulated experiments on screen.

a Record the results of the simulation you watched in the table.

Experiments with:	Count	Relative Frequency
No rain		
1 day of rain		
2 days of rain		
3 days of rain		

b Estimate the probability that it rains at least once on the tourist's vacation.

Activity
3

Name: ______ Date: ______ Period: ______

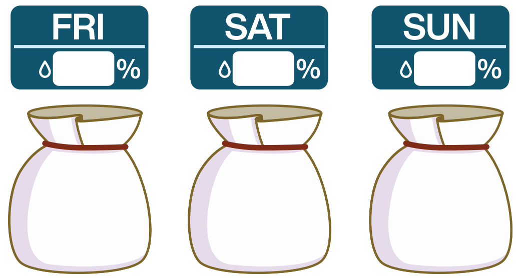
Meet the Criteria

For each challenge:

- Consider the given criteria for a 3-day period.
- Make a weather forecast for each day by writing a percentage in the white box that you think will meet the criteria.
- Draw blocks on the bag that match your forecast.
- Use the digital screen to run the simulation and test your forecast.

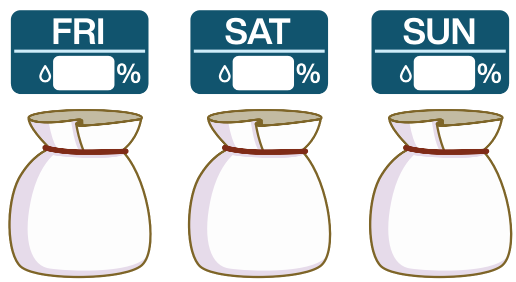
9 Criteria:

- The chance that it rains *all three days* is between 50% and 75%.



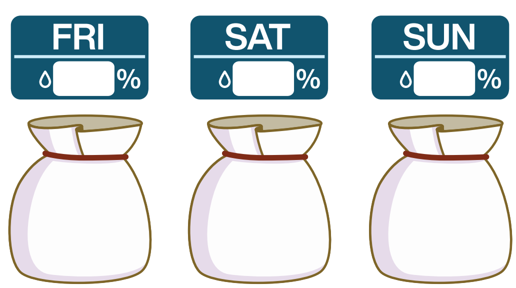
10 Criteria:

- The most likely weather is 2 *days of rain*.
- The next most likely weather is 1 *day of rain*.



11 Criteria:

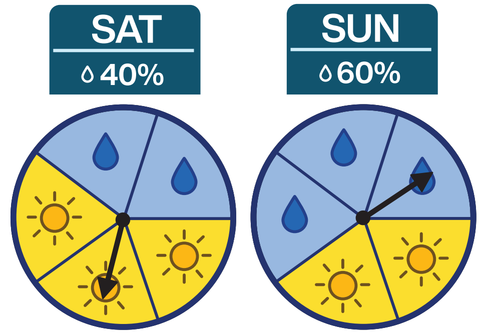
- The chance of *no rain at all* is about 25%.



12 Synthesis

Describe how simulations can be designed and used to estimate the probability of a real-world event.

Use these spinners if they help with your thinking.



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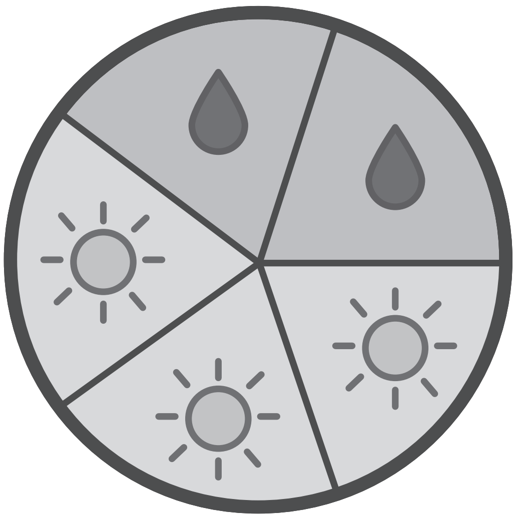

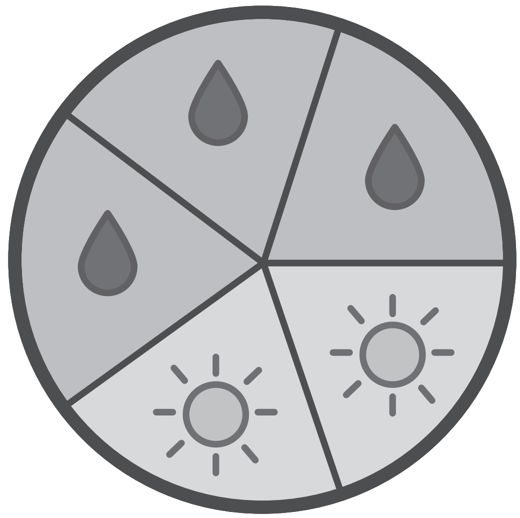

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