

HW5 Writing problem

1. $M = \text{"Hello!"}$

$$H(M) = 55 = m$$

(a) RSA

$$n = 493 = 17 \times 29 \Rightarrow p = 17, q = 29$$

$$\phi(n) = 16 \times 28 = 448$$

$$PR = (d, n) = (369, 493)$$

$$369 = e^{-1} \bmod 448 \Rightarrow e = 17$$

$$PU = (e, n) = (17, 493)$$

Sign =

$$S = 55^{369} \bmod 493 = 395$$

Verify =

$$m' = 395^{17} \bmod 493 = 55$$

$$\therefore m' = H(M) = m$$

\therefore Pass.

(b) ElGamal

$$PR = (q, \alpha, X_A) = (113, 17, 37)$$

$$Y_A = 17^{37} \bmod 113 = 79$$

$$PU = (q, \alpha, Y_A) = (113, 17, 79)$$

$$k = 13$$

Sign =

$$S_1 = \alpha^k \bmod q = 17^{13} \bmod 113 = 92$$

$$S_2 = k^{-1}(m - X_A S_1) \bmod q-1$$

$$= 13^{-1}(55 - 37 \times 92) \bmod 112 = 69 \times 11 \bmod 112 = 89$$

Verify =

$$\alpha^m \bmod q = 17^{55} \bmod 113 = 93$$

$$Y_A^{S_1} S_1^{S_2} \bmod q = 79^{92} 92^{89} \bmod 113$$

$$= 60 \times 75 \bmod 113 = 93$$

$$\therefore \alpha^m \equiv Y_A^{S_1} S_1^{S_2} \bmod q$$

\therefore Pass

(c) Schnorr

$$PR = (p, q, a, s) = (293, 73, 53, 29)$$

$$\begin{aligned} V &= a^{-s} \bmod p = 53^{-29} \bmod 293 \\ &= (53^{-1})^{29} \bmod 293 = 94^{29} \bmod 293 = 140 \end{aligned}$$

$$PU = (p, q, a, v) = (293, 73, 53, 140)$$

Sign:

$$\text{choose } r=2 \Rightarrow X = a^r \bmod p = 53^2 \bmod 293 = 172$$

$$e = H(M \parallel X) = 17$$

$$y = (2 + 29 \times 17) \bmod 73 = 57$$

Verify:

$$\begin{aligned} X' &= a^y v^e \bmod p = 53^{57} 140^{17} \bmod 293 \\ &= 186 \times 149 \bmod 293 = 172 \end{aligned}$$

$$H(M \parallel X') = 17$$

$$\therefore H(M \parallel X') = e = 17$$

\therefore pass

(d) DSA

$$PR = (p, q, g, x) = (293, 73, 53, 61)$$

$$y = g^x \bmod p = 53^{61} \bmod 293 = 84$$

$$PU = (p, q, g, y) = (293, 73, 53, 84)$$

$$k = 13$$

Sign:

$$r = (g^k \bmod p) \bmod q = (53^{13} \bmod 293) \bmod 73 = 39 \bmod 73 = 39$$

$$\begin{aligned} s &= k^{-1} (H(M) + rx) \bmod q = 13^{-1} (55 + 39 \times 61) \bmod 73 \\ &= 45 \times 75 \bmod 73 = 30 \end{aligned}$$

Verify:

$$(r', s') = (39, 30) \quad H(M') = 55$$

$$w = (s')^{-1} \bmod q = 30^{-1} \bmod 73 = 56$$

$$u_1 = [H(M') w] \bmod q = 55 \times 56 \bmod 73 = 14$$

$$u_2 = (r') w \bmod q = 39 \times 56 \bmod 73 = 67$$

$$\begin{aligned} V &= [(g^{u_1} y^{u_2}) \bmod p] \bmod q = [(53^{14} 84^{67}) \bmod 293] \bmod 73 \\ &= 16 \times 94 \bmod 293 \bmod 73 = 39 \end{aligned}$$

$$\therefore V = r'$$

\therefore pass.