1. There are two sets A and B.

Assume that graph G is a bipartite graph with odd cycles, and there is a cycle with point $X_1, X_2, X_3 \cdots X_{(2k-1)}$. $k \in \mathbb{Z}$. $k \geq 1$.

Each points are connected with adjacent one, and X_1 is connected with $X_{(2k-1)}$.

According to the definition of bipartite graph, X_1 is in the set A. X_2 is in the set B. and X_3 is in the set A because of its relation with X_2.

In a similar way, odd points are in the set A. and even points are in the set B. so both X_1 and $X_{(2k-1)}$ are in the set A. but we have assumed that X_1 is connected with $X_{(2k-1)}$.

As stated above, it reach a contradiction.

$\Rightarrow$ A graph G is bipartite iff it contains no cycles of odd length.