Using Cost Pass-Through to Calibrate Demand

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Abstract

We demonstrate that cost pass-through can be used to inform demand calibration, potentially eliminating the need for data on margins, diversion, or both. We derive the relationship between cost pass-through and consumer demand using a general oligopoly model of Nash-Bertrand competition and develop specific results for four demand systems: linear demand, logit demand, the Almost Ideal Demand System (AIDS), and log-linear demand. The methods we propose may be useful to researchers and antitrust authorities when reliable measures of margins or diversion are unavailable.

Keywords: cost pass-through; demand calibration; merger simulation
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1 Introduction

Researchers in industrial economics frequently conduct counter-factual experiments based on parameterized systems of consumer demand. The functional form of demand is assumed, and the structural parameters are either estimated from data or calibrated. Our focus in this paper is on demand calibration. Heretofore, calibration for differentiated product industries has been thought to require information price-cost margins and consumer diversion, which together are sufficient to recover the structural parameters of many demand systems.[1]

We develop that cost pass-through can be used to inform demand calibration, potentially obviating the need for margins, diversion, or both. As a motivating example, suppose an economist seeks to calibrate a linear demand system to facilitate merger simulation. Price-cost margins are available so the own-price elasticities of demand are obtainable through first order conditions. Unfortunately, the available data are insufficient to estimate diversion and the documentary evidence is unhelpful. We demonstrate that cross-price elasticities nonetheless can be selected to rationalize cost pass-through, perhaps obtained from documents or estimated with reduced-form regressions of prices on cost factors. In this example, cost pass-through replaces information on diversion in the calibration process.

The connection between cost pass-through and the properties of demand has been emphasized in the recent theoretical literature. Jaffe and Weyl (2011) propose using cost pass-through to inform the second order properties of demand (i.e., demand curvature) given knowledge of the first order properties (i.e., demand elasticities).[2] Our findings flip the intuition: cost pass-through can inform the first order properties of demand provided the economist is willing to assume the functional form of demand and thereby fix the second order properties. In the motivating example of linear demand, cost pass-through is informative because the cross-elasticities of demand for any two products relate to the degree to which the products' prices are strategic compliments, in the sense of Bulow, Geanakoplos, and Klemperer (1985), which in turn relates to cost pass-through.

The paper proceeds in three parts. We first derive the relationship between cost pass-through and the properties of demand in a general oligopoly model of Nash-Bertrand competition, following Jaffe and Weyl (2011). We then develop how cost pass-through can inform

[1] Consumer diversion from one product to another can be defined as the proportion of consumers leaving the first product, in response to a small price increase, that switch to the second product. Knowledge of quantities and prices is also necessary for calibration. Relative to demand estimation, calibration is more common among antitrust practitioners because it can utilize confidential information that becomes available to the U.S. Department of Justice and the Federal Trade Commission under the Hart-Scott-Rodino Act.

[2] See also the discussion in Miller, Remer, Ryan, and Sheu (2012).

the calibration of four specific demand systems: linear demand, logit demand, the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980), and log-linear demand. These systems are commonly employed in antitrust analysis of mergers involving differentiated products (e.g., Werden, Froeb, and Scheffman (2004); Werden and Froeb (2008)). The results provide methods of calibration that may be useful when reliable measures of margins or diversion are unavailable but when cost pass-through can be estimated from data or discerned from other sources. Finally, we provide a numerical example.

2 Cost Pass-Through and Demand

Consider a model of Bertrand-Nash competition in which firms face well-behaved and twice-differentiable demand functions. Each firm i produces some subset of the products available to consumers and sets prices to maximize short-run profits, taking as given the prices of its competitors. The first order conditions that characterize firm i's profit-maximizing prices can be expressed

$$f_i(P) \equiv - \left[\frac{\partial Q_i(P)}{\partial P_i}^T \right]^{-1} Q_i(P) - (P_i - MC_i) = 0, \tag{1}$$

where Q_i is a vector of firm i's sales, P_i is a vector of firm i's prices, P is a vector of all prices, and MC_i is a vector of firm i's marginal cost.

Now suppose that a per-unit tax is levied on each product in the model – the tax perturbs marginal costs and allows for the derivation of cost pass-through. The post-tax first order conditions are

$$f(P) + t = 0,$$

where t is the vector of taxes and $f(P) = [f_1(P)' \ f_2(P)' \ \ldots]'$. Differentiating with respect to t obtains

$$\frac{\partial P}{\partial t} \frac{\partial f(P)}{\partial P} + I = 0,$$

and algebraic manipulations then yield

$$\frac{\partial f(P)}{\partial P} = - \left(\frac{\partial P}{\partial t} \right)^{-1}. \tag{2}$$

Thus, the Jacobian of $f(P)$ equals the opposite inverse of the cost pass-through matrix. This Jacobian depends on both the first and second derivatives of demand, as can be ascertained from equation 1, and it follows that cost pass-through similarly relates to both the first-order and second-order properties of demand.[3]

3 General Procedure and Specific Demand Systems

The premise of this paper is that, provided one is willing to specify the functional form of demand, equation 2 can be used to calibrate the structural demand parameters, i.e., to select demand parameters that rationalize observed cost pass-through.[4] The procedure identifies at most N^2 parameters, where N is the number of products, due to the dimensionality of the matrices in equation 2. Thus, it is conceptually possible for cost pass-through to identify all of the demand elasticities even for demand systems that are fully first-order flexible.

Below we develop how cost pass-through can inform the calibration of four specific demand systems: linear demand, logit demand, the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980), and log-linear demand. A key result is that, in practice, properties of demand that can be identified from cost pass-through depend on the assumed functional form of demand. In what follows, we restrict attention to single-product firms to simplify the exposition; the results can be extended to the multiproduct case though in some cases Slutsky symmetry or other additional assumptions are required.

3.1 Linear Demand

The linear demand system has the form

$$Q_i = \alpha_i + \sum_j \beta_{ij} P_j. \tag{3}$$

[3] See Miller, Remer, Ryan, and Sheu (2012) for an explicit derivation of $\partial f(P)/\partial P$. We find it useful to keep in mind that the matrix has dimensionality $N \times N$, where N is the number of products, and takes the form

$$\frac{\partial f(P)}{\partial P} = \begin{bmatrix} \frac{\partial f_1(P)}{\partial P_1} & \cdots & \frac{\partial f_1(P)}{\partial P_J} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_J(P)}{\partial P_1} & \cdots & \frac{\partial f_J(P)}{\partial P_J} \end{bmatrix}.$$

where J is the number of firms.

[4] Precise calibration requires knowledge of all cost pass-through rates because each element of $\partial f(P)/\partial P$ depends on all the elements of $\partial P/\partial t$.

The parameters to be calibrated include J product-specific intercepts and J^2 price coefficients. It can be derived that the elements of the Jacobian of $f(P)$ are

$$\frac{\partial f_i(P)}{\partial P_j} = \begin{cases} -2 & \text{if } i = j \\ -\beta_{ij}/\beta_{ii} & \text{otherwise} \end{cases} \tag{4}$$

Cost pass-through identifies at most $J \times (J-1)$ price coefficients in this case because the Jacobian of $f(P)$ has constants along the diagonal. To calibrate the demand system, the own-price coefficients can be inferred from margins and the firms' first order conditions.[5] The cross-price coefficients then can be identified using cost pass-through and equation 2, and the intercepts can be recovered from price and quantity data. The linear demand system can be calibrated given (1) cost pass-through and (2) margins, prices, and quantity; cost pass-through relieves the need for diversion. Section 4 provides a numerical example.

3.2 Logit Demand

The logit demand system has the form

$$Q_i = \frac{e^{(\eta_i - P_i)/\tau}}{\sum_k e^{(\eta_k - P_k)/\tau}} M, \tag{5}$$

where M is the size of the market and the parameters include J product-specific terms (η_i) and a single scaling/price coefficient (τ). It is standard to normalize the market size to one, so that quantities have the interpretation of being market shares. It can be derived that the elements of the Jacobian of $f(P)$ are

$$\frac{\partial f_i(P)}{\partial P_j} = \begin{cases} -\frac{M}{M-Q_i} & \text{if } i = j \\ \frac{Q_i Q_j}{(M-Q_i)^2} & \text{otherwise} \end{cases} \tag{6}$$

The relationship between the quantities and cost pass-through is over-identified; a minimum distance estimator could be invoked to recover the quantities given cost pass-through. The quantities paired with the margin of a single product are sufficient to obtain the demand parameters. Thus, the logit demand system can be calibrated given (1) cost pass-through and (2) prices and a single margin; cost pass-through relieves the need for shares (since logit assumes diversion proportional to share, this is equivalent to relieving the need for diversion).

[5] The first oder conditions provide that $\beta_{ii} = -\frac{Q_i}{P_i}\frac{1}{m_i}$ where m_i is the margin.

3.3 AIDS Demand

The AIDS of Deaton and Muellbauer (1980) takes the form

$$W_i = \psi_i + \sum_j \phi_{ij} \log P_j + \beta_i \log(x/P^*), \tag{7}$$

where W_i is an expenditure share (i.e., $W_i = P_i Q_i / \sum_k P_k Q_k$), x is the total expenditure and P^* is a price index given by

$$\log(P^*) = \psi_0 + \sum_k \psi_k \log(P_k) + \frac{1}{2} \sum_k \sum_l \phi_{kl} \log(P_k) \log(P_l).$$

We focus on the special case of $\beta_i = 0$, consistent with common practice in antitrust applications (e.g, Epstein and Rubinfeld (1999)). The restriction is equivalent to imposing an income elasticity of one. It can be derived that the elements of the Jacobian of $f(P)$ are

$$\frac{\partial f_i(P)}{\partial P_j} = \begin{cases} \frac{\phi_{ii}(W_i - 2\phi_{ii})}{(\phi_{ii} - W_i)^2} & \text{if } i = j \\ -\frac{P_i \phi_{ii} \phi_{ij}}{P_j(\phi_{ii} - W_i)^2} & \text{otherwise} \end{cases}. \tag{8}$$

Cost pass-through is sufficient to identify all $J \times J$ price coefficients provided that expenditure shares and prices are available. To illustrate, the own-price coefficients are obtainable from the diagonal elements of $\partial f(P)/\partial P$ and expenditure shares. The cross-price coefficients then are obtainable from the off-diagonal elements and prices, and the demand intercepts are obtainable from equation 7.[6] Thus, the AIDS can be calibrated fully given (1) cost pass-through and (2) expenditure shares and prices; cost pass-through relieves the need for both margins and diversion.

[6]This result requires that total expenditure be constant with respect to price changes, an assumption that could be reasonable for applications dealing with small price movements. An alternative approach would be to add an elasticity of total expenditure parameter to the model as an extra unknown to calibrate. This elasticity would appear in both the own-price and cross-price terms in equation 8. Then cost pass-through would identify the price coefficients as a function of the added elasticity. Information on one firm's margin could then be used to recover the added elasticity, making use of the firm's first order condition. This derivation is available on request.

3.4 Log-linear Demand

The log-linear demand system takes the form

$$\ln(Q_i) = \gamma_i + \sum_j \epsilon_{ij} \ln P_j. \tag{9}$$

The parameters to be calibrated include J product-specific intercepts and J^2 price coefficients; the price coefficients are the own-price and cross-price elasticities of demand. It can be derived that the elements of the Jacobian of $f(P)$ are

$$\frac{\partial f_i(P)}{\partial P_j} = \begin{cases} -\frac{1+\epsilon_{ii}}{\epsilon_{ii}} & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

That the Jacobian of $f(P)$ is diagonal reflects the unique property of log-linear demand that prices are neither strategic substitutes nor strategic complements.[7] This prevents identification of the cross-price elasticities without information on diversion. Still, the own-price elasticities can be identified with cost pass-through through equation 2. The log-linear demand system can be calibrated fully given (1) cost pass-through and (2) diversion, prices and quantities; cost pass-through relieves the need for margins.

4 An Example

Suppose there are three single-product firms, demand is linear, and the following cost pass-through matrix is estimated from data:

$$\frac{\partial P}{\partial t} = \begin{bmatrix} .58 & .15 & .17 \\ .23 & .61 & .20 \\ .21 & .25 & .61 \end{bmatrix}.$$

Further suppose that the unit sales of the three firms are 200, 175, and 150, respectively, that prices are \$10, \$9, and \$8, respectively, and that each firm has a 50% margin. The first order

[7] The implied own-cost pass-through rate of firm i is given by $\epsilon_{ii}/(1+\epsilon_{ii})$ and exceeds one for any elasticity consistent with profit maximization. This can be obtained by re-arranging the first order conditions as follows:

$$P_i = \left(\frac{\epsilon_{ii}}{1+\epsilon_{ii}}\right) MC_i,$$

and noting that demand elasticities are constant with log-linear demand.

conditions imply that the firms' own-price coefficients are -40, -39, and -38. Placing these own-price coefficients into the Jacobian of $f(P)$, cross-price terms can be selected to equate the Jacobian with the opposite inverse of the pass-through rate matrix. This produces the following price coefficient matrix:

$$\beta = \begin{bmatrix} -40 & 12 & 18 \\ 24 & -39 & 19 \\ 17 & 27 & -38 \end{bmatrix}.$$

Combining this matrix with prices and unit sales, following equation 3, yields demand intercepts of 342, 137, and 45.

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