Problem Set 3: Computer Exercises

Jenny Petrova

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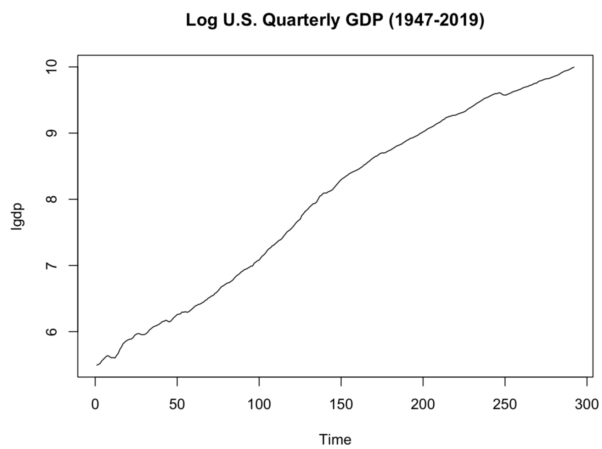
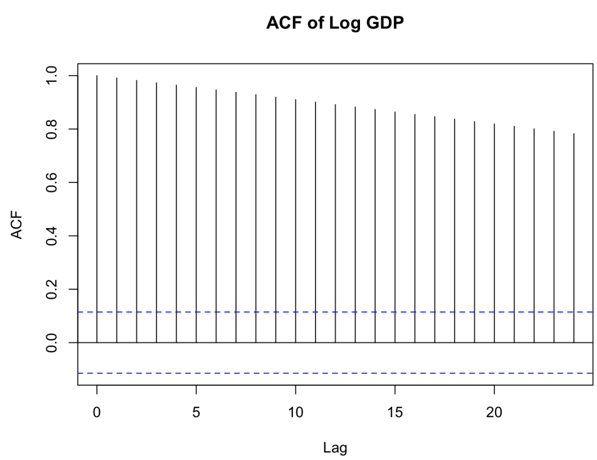
5. The following problems examine the US quarterly real GDP from 1947Q1 to 2019Q4. This data is accessed using the **quantmod** package in **R**. The outputs presented below are generated using the **R** programming language.

   (a) Examine the time series and ACF plots of the logarithm of US quarterly real GDP:

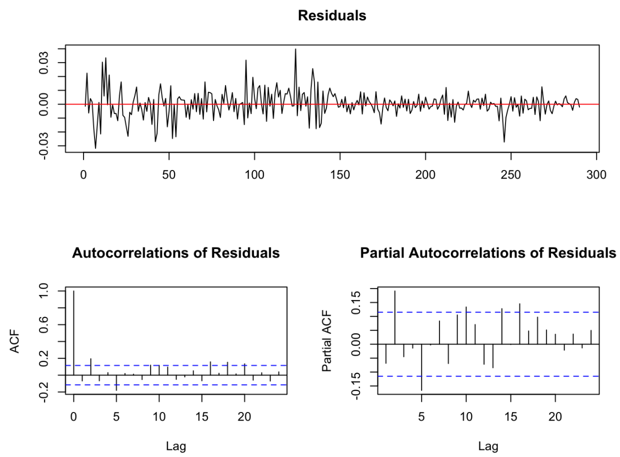




We observe a smooth, upward trend in the time series plot above. The series does not fluctuate around the trend, suggesting a finite and time-invariant variance. The autocorrelations in the ACF plot decay slowly with each lag h, remaining above the confidence interval bounds for all lags observed. The plots indicate the data is potentially trend-stationary.

Perform the augmented Dickey-Fuller test on the log of GDP:

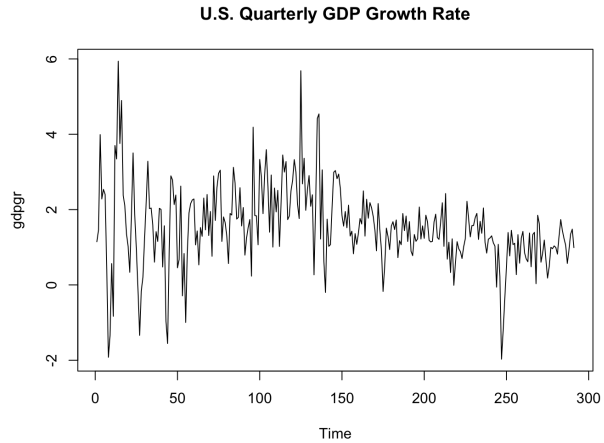
$$H_0 = \text{series is difference stationary} \qquad (y_t \sim I(1) \text{ with drift})$$
$$H_1 = \text{series is trend stationary} \qquad (y_t \sim I(0) \text{ with a deterministic time trend})$$

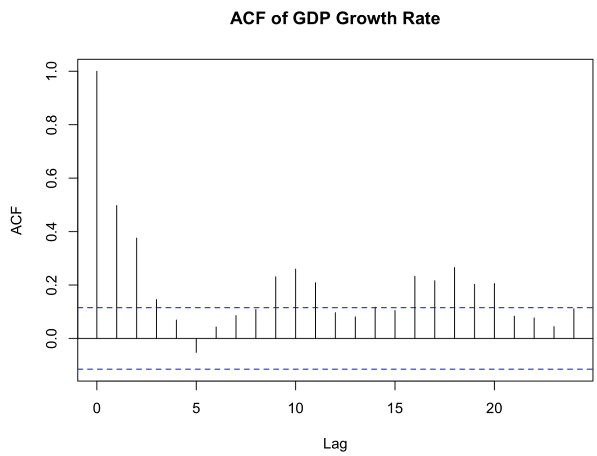



```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression trend

Value of test-statistic is: 0.5576 24.2128 3.4307

Critical values for test statistics:
      1pct  5pct 10pct
tau3 -3.98 -3.42 -3.13
phi2  6.15  4.71  4.05
phi3  8.34  6.30  5.36
```

The values of the test statistics are $\tau_3 = 0.5576$, $\phi_3 = 24.2128$, and $\phi_2 = 3.4307$. We mainly observe the τ_3 test statistic, which is less negative (larger) than the critical values for τ_3. Therefore we fail to reject the null hypothesis. We do not have sufficient evidence to suggest the series is trend stationary (with a deterministic time trend).

(b) Examine the time series and ACF plots of the growth rate of US quarterly real GDP:

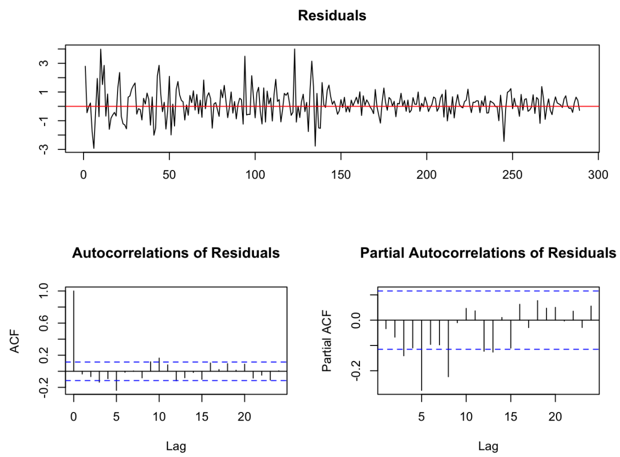




In time series plot above, we observe a decreasing magnitude in the variance over time. The autocorrelations in the ACF plot decay to zero by lag 4, but we observe spikes outside the confidence interval bounds at lags 9-11 and lags 16-20. The plots indicate the data is potentially nonstationary.

Perform the augmented Dickey-Fuller test on GDP growth rate:

$$H_0 = \text{series has a unit root} \qquad (y_t \sim I(1))$$
$$H_1 = \text{series is stationary} \qquad (y_t \sim I(0))$$

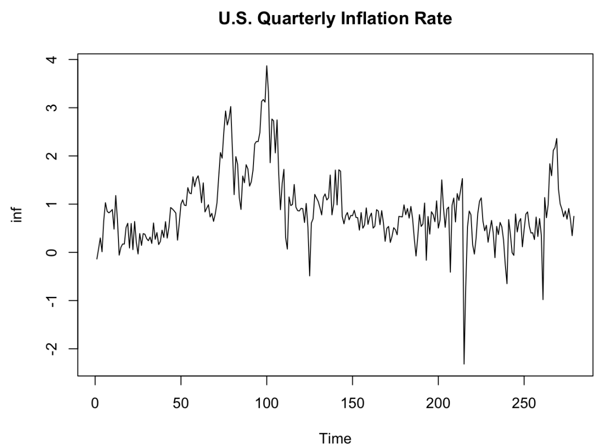


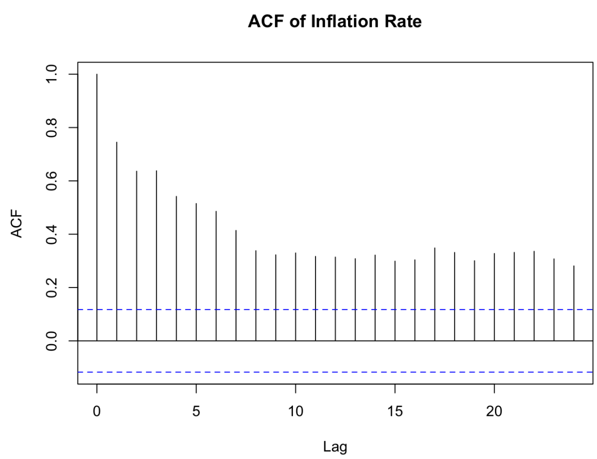

```
################################################
# Augmented Dickey-Fuller Test Unit Root Test #
################################################

Test regression none

Value of test-statistic is: -3.5033

Critical values for test statistics:
      1pct  5pct 10pct
tau1 -2.58 -1.95 -1.62
```

The value of the test statistic is $\tau_1 = -3.5033$, which is more negative (smaller) than the critical values for τ_1. Therefore we reject the null hypothesis of a unit root. There is evidence to suggest the series is stationary.

We conclude the log of real GDP appears to be difference stationary and the first difference (growth rate) of real GDP appears to be stationary. Based on these results, it would be appropriate to apply stationary ARMA models to the difference of the log of real GDP.

(c) Examine the time series and ACF plots of US quarterly inflation rates from 1960Q2 to 2019Q4:

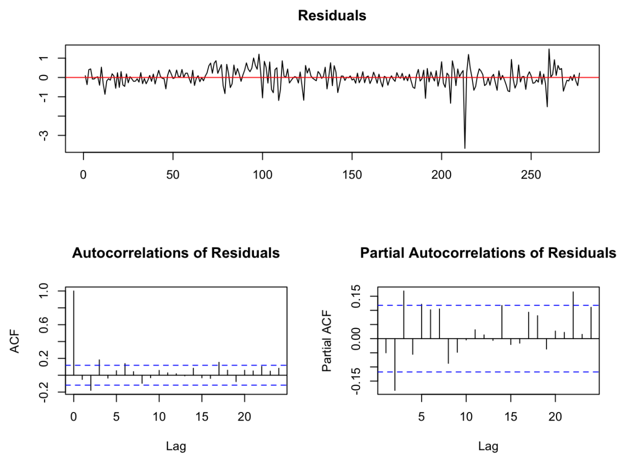




We observe a non-constant variance in the time series plot of inflation rates. The autocorrelations in the ACF plot decay with each lag h, decaying at a much slower rate after lag 8, remaining above the confidence interval bounds for all lags observed. The plots indicate the data is potentially nonstationary.

Perform the augmented Dickey-Fuller test on inflation rate:

$$H_0 = \text{series has a unit root} \qquad (y_t \sim I(1) \text{ with drift})$$
$$H_1 = \text{series is stationary} \qquad (y_t \sim I(0) \text{ with a nonzero mean})$$




```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression drift

Value of test-statistic is: -4.9561 12.2863

Critical values for test statistics:
      1pct  5pct 10pct
tau2 -3.44 -2.87 -2.57
phi1  6.47  4.61  3.79
```

The value for the test statistics are $\tau_2 = -4.9561$ and $\phi_1 = 12.2863$. We mainly observe the τ_2 test statistic, which is more negative (smaller) than the critical values for τ_2. Therefore we reject the null hypothesis of a unit root. There is evidence to suggest the series is stationary (with a nonzero mean).

Perform the KPSS test on inflation rate:

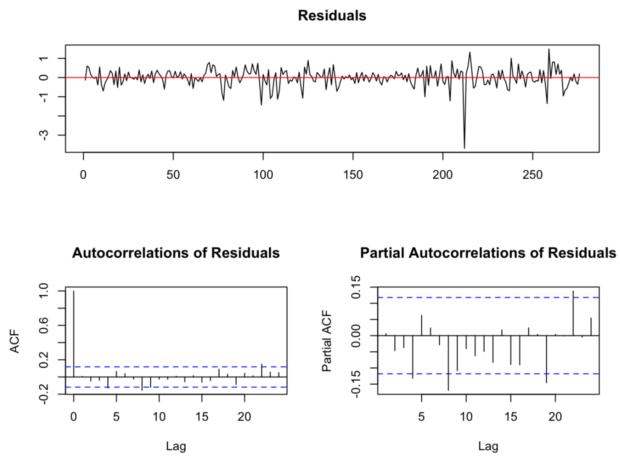
$$H_0 = \text{series is stationary}$$
$$H_1 = \text{series has a unit root}$$

```
 KPSS Test for Level Stationarity

data:  inf
KPSS Level = 0.62381, Truncation lag parameter = 5, p-value = 0.02047


#######################
# KPSS Unit Root Test #
#######################

Test is of type: mu with 5 lags.

Value of test-statistic is: 0.6238

Critical value for a significance level of:
                10pct  5pct 2.5pct  1pct
critical values 0.347 0.463  0.574 0.739
```

At the 5% significance level, the p-value = 0.02047. The value of the test statistic is 0.6238, which is larger than the critical value at the 5% confidence significance level. As such, we reject the null hypothesis. There is evidence to suggest the series has a unit root (i.e. the series is nonstationary). This evidence contradicts with the results found in the ADF test, meaning further investigation is necessary.

Perform the augmented Dickey-Fuller test on the first difference of inflation rate:

$$H_0 = \text{series has a unit root} \qquad (y_t \sim I(1) \text{ with drift})$$
$$H_1 = \text{series is stationary} \qquad (y_t \sim I(0) \text{ with a nonzero mean})$$




```
################################################
# Augmented Dickey-Fuller Test Unit Root Test #
################################################

Test regression drift

Value of test-statistic is: -18.635 173.6317

Critical values for test statistics:
      1pct  5pct 10pct
tau2 -3.44 -2.87 -2.57
phi1  6.47  4.61  3.79
```

The value for the test statistics are $\tau_2 = -4.9561$ and $\phi_1 = 12.2863$. We again observe the τ_2 test statistic to be more negative (smaller) than the critical values for τ_2 at each significance level. Therefore we reject the null hypothesis of a unit root. There is evidence to suggest the first difference of the series is stationary.

Perform the KPSS test on the first difference of inflation rate:

$$H_0 = \text{series is stationary}$$
$$H_1 = \text{series has a unit root}$$

```
 KPSS Test for Level Stationarity

data:  dinf
KPSS Level = 0.038787, Truncation lag parameter = 5, p-value = 0.1


#######################
# KPSS Unit Root Test #
#######################

Test is of type: mu with 5 lags.

Value of test-statistic is: 0.0388

Critical value for a significance level of:
                10pct  5pct 2.5pct  1pct
critical values 0.347 0.463  0.574 0.739
```

At the 5% significance level, the p-value is > 0.1. The value of the test statistic is 0.0388, which is smaller than the critical value at the 5% confidence significance level. As such, we fail to reject the null hypothesis of level stationary. We have evidence to support the first difference of the series is stationary. This evidence agrees with the results found in the ADF test, suggesting that the series potentially has a unit root, since both tests indicate the first difference of the series is stationary.

Conclusion

In practice, many forecasters treat US inflation as $I(1)$. Our findings are consistent with this view, as we have evidence to believe inflation rates contain a unit root (require a 1^{st} difference to be stationary), meaning inflation rates are an integrated process of order 1, denoted by $I(1)$.

Corresponding Code

```
install.packages('quantmod')
library(quantmod)
library(tseries)
library(urca)

getSymbols('GDP', src = 'FRED')

# 5(a)
# (i)
lgdp <- log(GDP)['/2019']
plot.ts(lgdp, main='Log U.S. Quarterly GDP (1947-2019)')
acf(lgdp, main='ACF of Log GDP')

# (ii)
# ADF test
result_lgdp <- ur.df(lgdp, type = "trend", selectlags = "AIC")
summary(result_lgdp)
plot(result_lgdp)


# 5(b)
# (i)
gdpgr <- 100 * diff(lgdp)[-1]
plot.ts(gdpgr, main='U.S. Quarterly GDP Growth Rate')
acf(gdpgr, main='ACF of GDP Growth Rate')

# (ii)
# ADF test
result_gdpgr <- ur.df(gdpgr, type = "none", selectlags = "AIC")
summary(result_gdpgr)
plot(result_gdpgr)


# 5(c)
# (i)
cpi <- getSymbols("CPALTT01USQ661S", src = "FRED", auto.assign = FALSE)
inf <- 100 * diff(log(cpi))
inf <- na.omit(inf)
plot.ts(inf, main = "U.S. Quarterly Inflation Rate")
acf(inf, main = "ACF of Inflation Rate")

# (ii)
# ADF test
adf_inf <- ur.df(inf, type = "drift", selectlags = "AIC")
summary(adf_inf)
plot(adf_inf)

# KPSS test
?kpss.test
kpss.test(inf, "Level")
?ur.kpss
kpss_inf <- ur.kpss(inf, type="mu", lags="short")
summary(kpss_inf)

# (iii)
# ADF test - 1st difference
dinf <- diff(inf)
dinf <- na.omit(dinf)
```

```
adf_dinf <- ur.df(dinf, type = "drift", selectlags = "AIC")
summary(adf_dinf)
plot(adf_dinf)

# KPSS test - 1st difference
?kpss.test
kpss.test(dinf, "Level")
?ur.kpss
kpss_dinf <- ur.kpss(dinf, type="mu", lags="short")
summary(kpss_dinf)
```