Portfolio Optimization Using Time Series

Time Series Analysis

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Introduction

1. 'An approach to portfolio optimization with time series forecasting algorithms and machine learning techniques' by Jyotirmayee Behera and Pankaj Kumar.
2. Motivation: optimal asset selection and portfolio construction
3. Challenge: selecting an efficient model (balancing model limitations) to model stock price fluctuations

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Model Selection

ARIMA(p,d,q) Model

$$\phi(L)(1-L)^d y_t = \theta(L)\varepsilon_t \tag{1}$$

$$\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p \tag{2}$$

$$\theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \cdots - \theta_p L^p \tag{3}$$

$$\varepsilon_t \sim WN(0, \sigma^2)$$

- Linear combination of past values and white noise, differenced to stationarity
- Limitation: cannot capture non-linear patterns in time series data

Model Selection

GARCH(p,q) Model

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{4}$$

$$\varepsilon_t = \sqrt{h_t} v_t, \quad v_t \sim IID(0,1) \tag{5}$$

- Captures volatility clustering, forecasts time-varying volatility
- Limitation: limited forecast horizon (assume deterministic and symmetric volatility, and normally distributed errors)

ARIMA+GARCH Model

- Effective in modeling volatility, but lacks in capturing complex price dynamics

Model Selection

Neural Network Models

- types: ANN, RNN, MLP, LSTM
- Limitations: data hungry, high tuning and computational complexity

ARIMA+Neural Network Models

- Efficient in time series modeling / forecasting, but not suitable for financial applications

Model Selection

SVM Model

- Supervised ML algorithm
- High accuracy, effective high-dimensional performance
- Limitations: scalability, requires careful cross-validation, sensitive to extreme outliers

ARIMA+SVM Model

- Handles non-linearity and more robust to outliers, but not as flexible as neural networks

Model Selection

LS-SVM Model

- Enhances speed of solving optimization problems

ARIMA+LS-SVM Model

- Balances performance, efficiency, and flexibility
- High scalability and accuracy

Chosen model for portfolio optimization!

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Mean-Variance Portfolio Optimization (MVPO)

- Developed by Harry Markowitz, 'Modern Portfolio Theory'
- Maximize expected return (or minimize risk) of a portfolio

Let P be a portfolio with $j = 1, \ldots, n$ stocks.

$$\min \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} w_i w_j$$

$$\text{subject to } \sum_{j=1}^{n} f_j w_j = f_P,$$

$$\sum_{j=1}^{n} w_j = 1,$$

$$\text{where } \sigma_{ij} = \sigma_i \sigma_j \rho_{ij}, \; 0 \leq w_j \leq 1$$

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ARIMA+LS-SVM Model

- utilized for stock price prediction
- performance of the model determined by obtaining the efficient solution for the mean–variance portfolio optimization model

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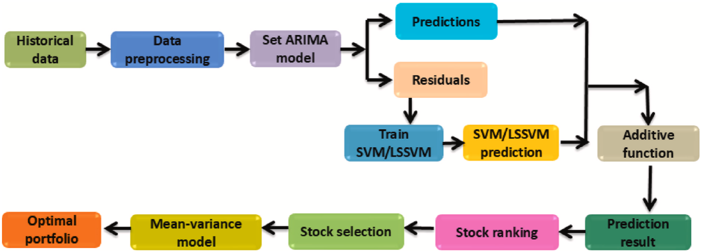
Experimental Design

- Data: closing prices of BSE 100 component stocks, from January 1, 2010, to June 30, 2023

Methodological Framework to Portfolio Optimization:



Experimental Design

Hybrid Model Approach:

1. Employ ARIMA method to fit closing prices (y_t) and generate forecasts ($\hat{f}_t$)
2. Calculate residuals: $\varepsilon_t = y_t - \hat{f}_t$
3. Employ LS-SVM method to fit residual terms (ε_t) and generate error forecasts ($\hat{\varepsilon}_t$), using 8:2 train/test ratio.
4. Compute ultimate predictions: $\hat{y}_t = \hat{f}_t + \hat{\varepsilon}_t$
5. Choose stocks with lowest prediction error ($\min |y_t - \hat{y}_t|$)
6. Integrate MVPO model with chosen high-performing stocks to achieve most efficient portfolio

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Experimental Results

Performance assessment of various models in prediction.

Model		MAE	MSE	RMSE	Time (s)
SVM	Mean	0.001256	0.800960	0.001338	88.221
	Std	0.002416	3.087503	0.002507	
LS-SVM	Mean	0.000122	0.686276	0.000129	22.417
	Std	0.000222	2.616905	0.000229	
ARIMA	Mean	0.000750	1.028893	0.001497	16.740
	Std	0.001322	4.140065	0.002346	
ARIMA+SVM	Mean	0.001329	0.884505	0.001405	94.546
	Std	0.002435	1.433664	0.002523	
ARIMA+LS-SVM	Mean	0.000106	0.051044	0.000113	30.890
	Std	0.000294	0.305830	0.000298	

Experimental Results

Performance assessment of ARIMA+LS-SVM model at different (p,d,q) values.

(p,d,q) values		MAE	MSE	RMSE	Time (s)
(0,1,0)	Mean	0.000106	0.051044	0.000113	30.890
	Std	0.000294	0.305830	0.000298	
(1,1,0)	Mean	0.000110	0.088759	0.000116	33.474
	Std	0.000320	0.187223	0.000324	
(1,1,1)	Mean	0.000112	0.090481	0.000118	64.034
	Std	0.000330	0.187089	0.000333	
(1,1,2)	Mean	0.000108	0.087211	0.000115	84.276
	Std	0.000324	0.188150	0.000328	
(2,1,1)	Mean	0.000108	0.086934	0.000115	84.772
	Std	0.000324	0.187254	0.000328	
(2,1,2)	Mean	0.000109	0.088693	0.000115	162.089
	Std	0.000319	0.184776	0.000322	
(4,1,2)	Mean	0.000107	0.090520	0.000114	221.361
	Std	0.000299	0.191544	0.000303	
(5,2,0)	Mean	0.000139	0.131989	0.000148	66.304
	Std	0.000323	0.220939	0.000329	

Statistical Testing

Friedman Test - used to assess statistically significant differences in prediction accuracy.
H_0: mean performance metrics of all models are identical
H_1: significant disparities in the models' average performances
Results: reject H_0. At least one model's performance differs significantly from the others.

Statistical Testing

Nemenyi Test - post-hoc test for pairwise comparison of models after a global statistical test (such as the Friedman test) has rejected the null hypothesis of similar performance.

Results:

Nemenyi post-hoc test *p*-values for MAE.

	ARIMA	SVM	LS-SVM	ARIMA+SVM	ARIMA+LS-SVM
ARIMA	1	0.650824	0	0.83145	0
SVM	0.650824	1	1.11E−16	0.111246	0
LS-SVM	0	1.11E−16	1	0	0.636464
ARIMA+SVM	0.83145	0.111246	0	1	0
ARIMA+LS-SVM	0	0	0.636464	0	1

Nemenyi post-hoc test *p*-values for MSE.

	ARIMA	SVM	LS-SVM	ARIMA+SVM	ARIMA+LS-SVM
ARIMA	1	2.06E−08	0.914924	5.88E−15	0.000721
SVM	2.06E−08	1	7.79E−11	0.219129	0
LS-SVM	0.914924	7.79E−11	1	1.11E−16	0.016156
ARIMA+SVM	5.88E−15	0.219129	1.11E−16	1	0
ARIMA+LS-SVM	0.000721	0	0.016156	0	1

Nemenyi post-hoc test *p*-values for RMSE.

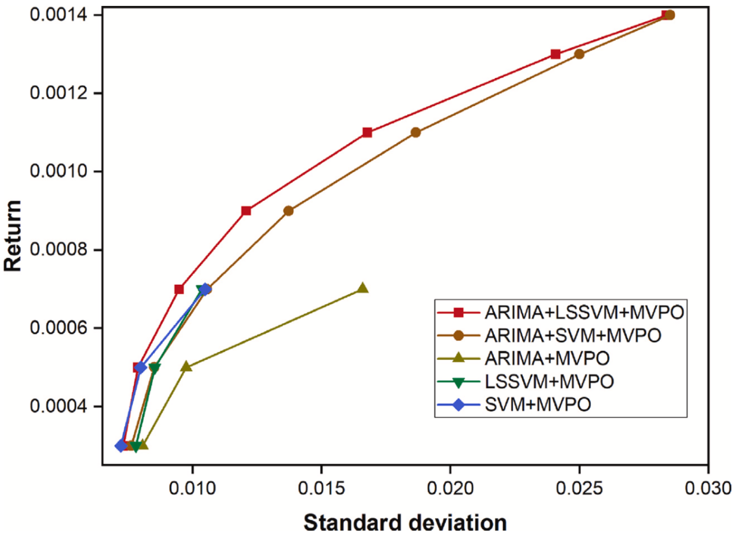
	ARIMA	SVM	LS-SVM	ARIMA+SVM	ARIMA+LS-SVM
ARIMA	1	0.929185	0	0.991739	0
SVM	0.929185	1	0	0.720529	0
LS-SVM	0	0	1	0	0.578283
ARIMA+SVM	0.991739	0.720529	0	1	0
ARIMA+LS-SVM	0	0	0.578283	0	1

MVPO Model Performance

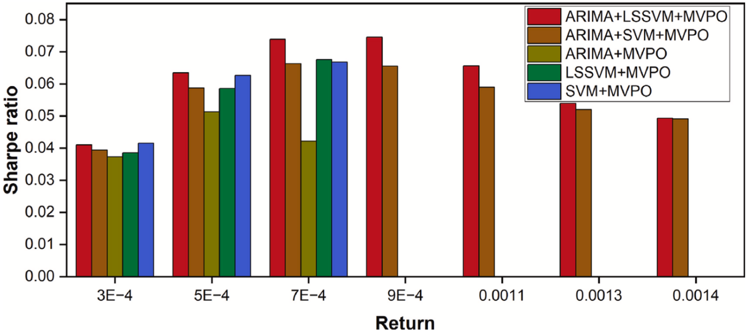
1. Employ [SVM, LS-SVM, ARIMA, ARIMA+SVM, ARIMA+LS-SVM]+MVPO to represent MVPO models incorporating predicted stocks from each SVM, LS-SVM, ARIMA, ARIMA+SVM, ARIMA+LS-SVM model, respectively.
2. Determine optimal portfolio for $f_P = 0.0003, 0.0005, 0.0007, 0.0009, 0.0011, 0.0013, 0.0014$.
3. Compute variance, standard deviation, and Sharpe Ratio for each portfolio.

$$SR = \max\{SR(P)\}, \text{ where } SR(P) = \frac{f_P}{\sigma_p}$$

Efficient Frontier



Sharpe Ratio



Cumulative Return

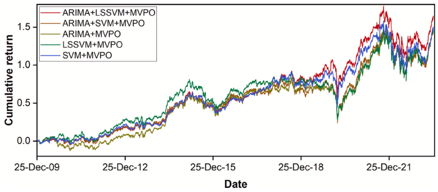


Figure: $f_P = 0.0003$

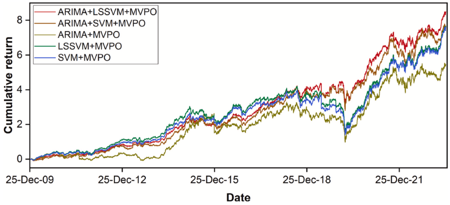


Figure: $f_P = 0.0007$

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Existing Literature

ERC Model:

- 'Equally-weighted Risk Contribution' portfolio by Maillard et al. (2009-2010)
- ensures each asset contributes same marginal risk
- maximizes diversification without relying on expected returns
- limitations: may overweight low-volatility assets

Minimax Portfolio Optimization Model

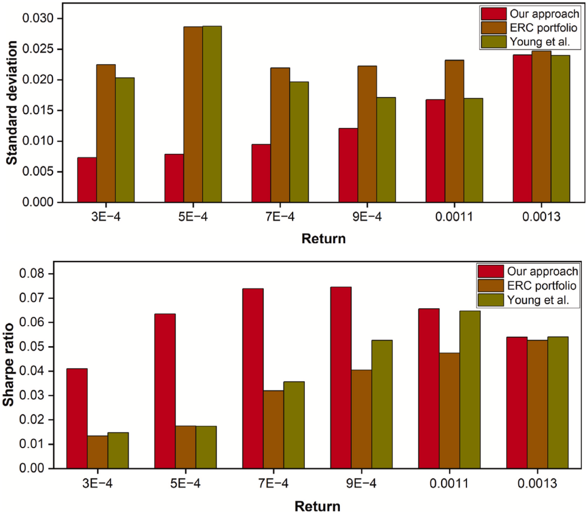
- by Young et al. (1998)
- maximizes worst-case return (equivalently, minimizes maximum loss) across past observation periods
- limitations: overly conservative if past worst cases are outliers, ignores variance and potential upside returns

Comparison Results

Comparison with existing literature.

μ_{fix}	Model	Standard deviation	Variance	Sharpe ratio	Time (s)
0.0003	Our approach	0.007313	0.000053	0.041023	0.00
	ERC portfolio	0.022468	0.000505	0.013352	0.34
	Young et al.	0.020304	0.000412	0.014775	0.00
0.0005	Our approach	0.007875	0.000062	0.063489	0.00
	ERC portfolio	0.028649	0.000821	0.017452	0.34
	Young et al.	0.028746	0.000826	0.017393	0.00
0.0007	Our approach	0.009472	0.000090	0.073904	0.00
	ERC portfolio	0.021923	0.000481	0.031930	0.33
	Young et al.	0.019650	0.000386	0.035624	0.00
0.0009	Our approach	0.012069	0.000146	0.074570	0.00
	ERC portfolio	0.022243	0.000495	0.040462	0.33
	Young et al.	0.017093	0.000292	0.052654	0.00
0.0011	Our approach	0.016770	0.000281	0.065595	0.00
	ERC portfolio	0.023184	0.000537	0.047447	0.33
	Young et al.	0.016979	0.000288	0.064785	0.01
0.0013	Our approach	0.024070	0.000579	0.054009	0.00
	ERC portfolio	0.024673	0.000609	0.052688	0.33
	Young et al.	0.023996	0.000576	0.054076	0.00

Comparison Results



Cumulative Return

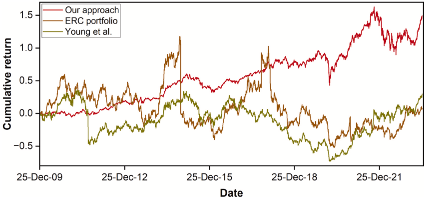


Figure: $f_P = 0.0003$

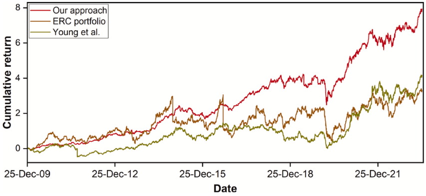


Figure: $f_P = 0.0007$

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Conclusion

- ARIMA+LS-SVM demonstrates superior stock price prediction
- ARIMA+LS-SVM+MVPO achieves superior risk-return efficiency, higher returns, lower variance and standard deviation, higher Sharpe ratio, and improved portfolio cumulative returns $\longrightarrow$ outperforms alternatives
- potential applications in algorithmic trading and risk management
- future research could incorporate additional factors and more diverse stock market datasets

Conclusion

Thank you!